# CL 305
# TRANSPORT PHENOMENA
## VECTORS AND TENSORS


INSTRUCTOR

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# VECTOR MULTIPLICATION

**Vector**: Quantity with a definite magnitude and direction

Magnitude of a vector, $\mathbf{v}$: Represented by either $|\mathbf{v}|$ or $v$

**Scalar Product or Dot Product:**

Consider two vectors $\mathbf{v}$ and $\mathbf{w}$. Their scalar product (or dot product) is defined as

$$(\mathbf{v} \cdot \mathbf{w}) = |\mathbf{v}||\mathbf{w}| \cos \phi_{\mathbf{vw}}$$

$\phi_{\mathbf{vw}}$ is the angle between the vectors $\mathbf{v}$ and $\mathbf{w}$.

Scalar product of a vector with itself is the square of its magnitude:

$$(\mathbf{v} \cdot \mathbf{v}) = |\mathbf{v}||\mathbf{v}| = |\mathbf{v}|^2 = v^2$$

# VECTOR MULTIPLICATION

**Scalar Product or Dot Product:** Governing rules are

**Commutative:** $(\mathbf{v} \cdot \mathbf{w}) = (\mathbf{w} \cdot \mathbf{v})$

**Distributive:** $(\mathbf{u} \cdot \{\mathbf{v} + \mathbf{w}\}) = (\mathbf{u} \cdot \mathbf{v}) + (\mathbf{u} \cdot \mathbf{w})$

**NOT Associative:** $(\mathbf{u} \cdot \mathbf{v})\mathbf{w} \neq \mathbf{u}(\mathbf{v} \cdot \mathbf{w})$

# VECTOR MULTIPLICATION

**Vector Product or Cross Product:**

Consider two vectors $\mathbf{v}$ and $\mathbf{w}$. Their vector product (or cross product) is defined as

$$[\mathbf{v} \times \mathbf{w}] = (|\mathbf{v}||\mathbf{w}| \sin \phi_{\mathbf{vw}})\mathbf{n_{vw}}$$

$\phi_{\mathbf{vw}}$ is the angle between the vectors $\mathbf{v}$ and $\mathbf{w}$, and $\mathbf{n_{vw}}$ is a unit vector perpendicular to both $\mathbf{v}$ and $\mathbf{w}$ and pointing in a direction given by the right hand rule.

Vector product of a vector with itself is zero: $[\mathbf{v} \times \mathbf{v}] = 0$

**NOT Commutative:** $[\mathbf{v} \times \mathbf{w}] = -[\mathbf{w} \times \mathbf{v}]$ (**anti-commutative**)

**Distributive:** $[\mathbf{u} \times \{\mathbf{v} + \mathbf{w}\}] = [\mathbf{u} \times \mathbf{v}] + [\mathbf{u} \times \mathbf{w}]$

**NOT Associative:** $\left[\mathbf{u} \times [\mathbf{v} \times \mathbf{w}]\right] \neq \left[[\mathbf{u} \times \mathbf{v}] \times \mathbf{w}\right]$

# VECTOR OPERATIONS IN TERMS OF COMPONENTS

**Kronecker Delta, $\delta_{ij}$:**

$$\delta_{ij} = +1 \quad \text{if} \quad i = j$$

$$\delta_{ij} = 0 \quad \text{if} \quad i \neq j$$

**Permutation Symbol, $\varepsilon_{ijk}$:**

$$\varepsilon_{ijk} = \frac{1}{2}(i-j)(j-k)(k-i)$$

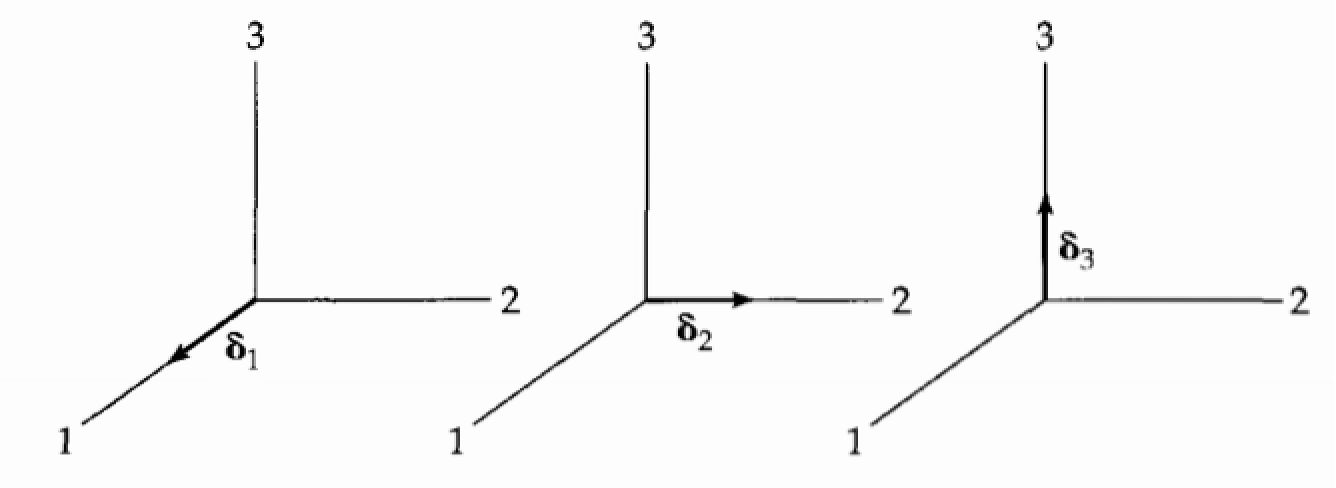
$$\varepsilon_{ijk} = +1 \quad \text{if} \quad ijk = 123, 231 \text{ or } 312$$

$$\varepsilon_{ijk} = -1 \quad \text{if} \quad ijk = 321, 132 \text{ or } 213$$

$$\varepsilon_{ijk} = 0 \quad \text{if any two indices are alike}$$

# UNIT VECTORS

Let $\boldsymbol{\delta}_1, \boldsymbol{\delta}_2$, and $\boldsymbol{\delta}_3$ be the unit vectors along the 1, 2 and 3 axis respectively.




Taken from Transport Phenomena by Bird, Stewart, Lightfoot.

$$(\boldsymbol{\delta}_i \cdot \boldsymbol{\delta}_j) = \delta_{ij}$$

$$[\boldsymbol{\delta}_i \times \boldsymbol{\delta}_j] = \sum_{k=1}^{3} \varepsilon_{ijk} \boldsymbol{\delta}_k$$

# VECTOR IN TERMS OF COMPONENTS

$$\mathbf{v} = \boldsymbol{\delta}_1 v_1 + \boldsymbol{\delta}_2 v_2 + \boldsymbol{\delta}_3 v_3 = \sum_{i=1}^{3} \boldsymbol{\delta}_i v_i$$

$v_1, v_2, v_3$ are the projections of the vector $\mathbf{v}$ on the 1, 2 and 3 axis respectively. These are called the components of the vector $\mathbf{v}$.

Magnitude of a vector is given by

$$|\mathbf{v}| = v = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{\sum_i v_i^2}$$

**Scalar Product or Dot Product:**

$$(\mathbf{v} \cdot \mathbf{w}) = \left(\left\{\sum_i \boldsymbol{\delta}_i v_i\right\} \cdot \left\{\sum_j \boldsymbol{\delta}_j w_j\right\}\right) = \sum_i \sum_j (\boldsymbol{\delta}_i \cdot \boldsymbol{\delta}_j) v_i w_j = \sum_i v_i w_i$$

# VECTOR IN TERMS OF COMPONENTS

**Vector Product or Cross Product:**

$$[\mathbf{v} \times \mathbf{w}] = \left[\left\{\sum_j \boldsymbol{\delta}_j v_j\right\} \times \left\{\sum_k \boldsymbol{\delta}_k w_k\right\}\right] = \sum_i \sum_j \sum_k \varepsilon_{ijk} \boldsymbol{\delta}_i v_j w_k = \begin{vmatrix} \boldsymbol{\delta}_1 & \boldsymbol{\delta}_2 & \boldsymbol{\delta}_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

**Dyadic Product of Vectors:**

$$\mathbf{vw} = \sum_i \sum_j \boldsymbol{\delta}_i \boldsymbol{\delta}_j v_i w_j$$

These are tensors of the second order

$\boldsymbol{\delta}_i \boldsymbol{\delta}_j$ are called **unit dyads**

# UNIT DYADS

**Dyadic Product :** $$\mathbf{vw} = \sum_i \sum_j \boldsymbol{\delta}_i \boldsymbol{\delta}_j v_i w_j$$

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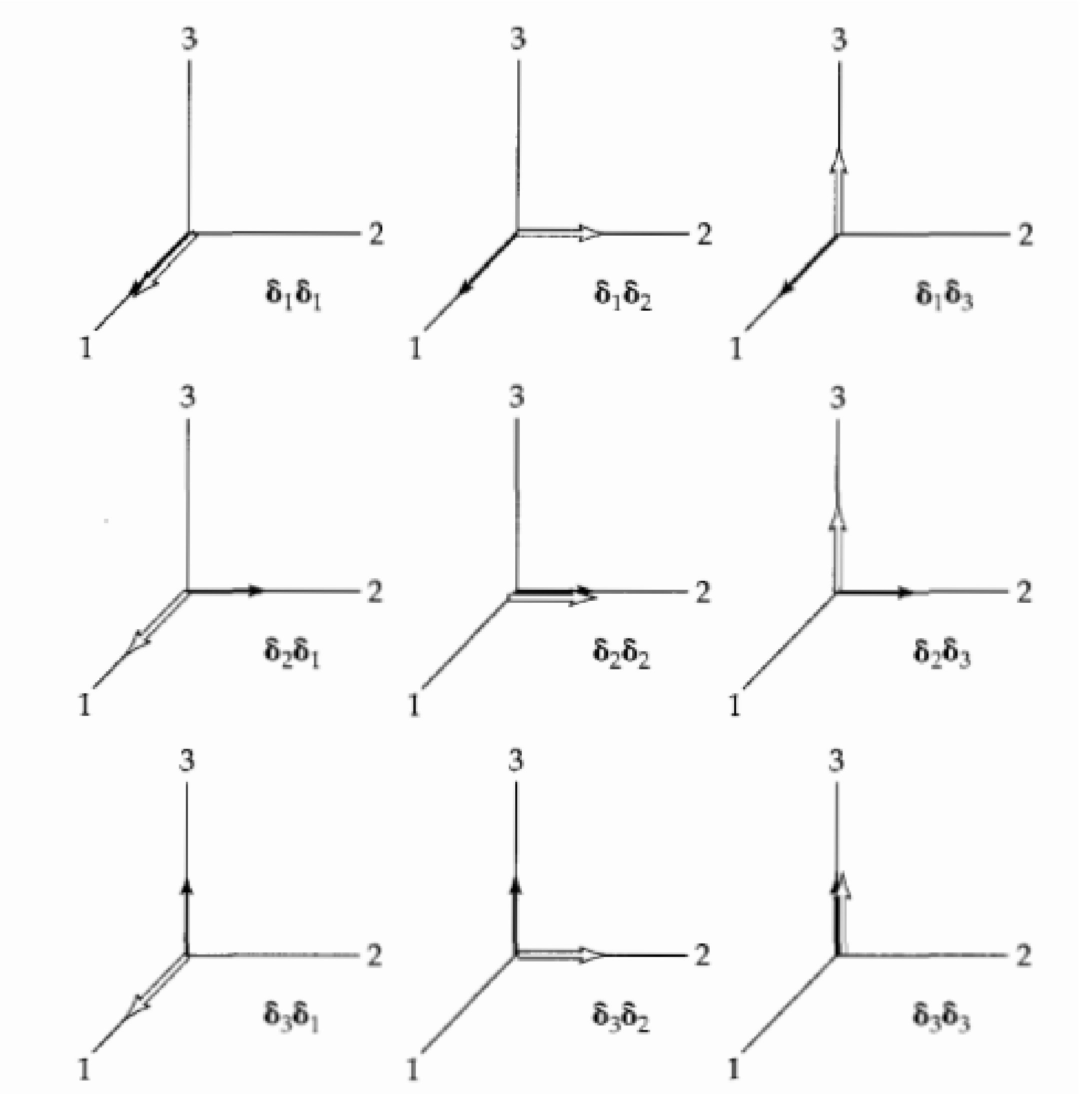


Taken from Transport Phenomena by Bird, Stewart, Lightfoot.

# UNIT DYADS: OPERATIONS

$$(\boldsymbol{\delta}_i\boldsymbol{\delta}_j : \boldsymbol{\delta}_k\boldsymbol{\delta}_l) = (\boldsymbol{\delta}_j \cdot \boldsymbol{\delta}_k)(\boldsymbol{\delta}_i \cdot \boldsymbol{\delta}_l) = \delta_{jk}\delta_{il}$$

$$[\boldsymbol{\delta}_i\boldsymbol{\delta}_j \cdot \boldsymbol{\delta}_k] = \boldsymbol{\delta}_i(\boldsymbol{\delta}_j \cdot \boldsymbol{\delta}_k) = \boldsymbol{\delta}_i\delta_{jk}$$

$$[\boldsymbol{\delta}_i \cdot \boldsymbol{\delta}_j\boldsymbol{\delta}_k] = (\boldsymbol{\delta}_i \cdot \boldsymbol{\delta}_j)\boldsymbol{\delta}_k = \delta_{ij}\boldsymbol{\delta}_k$$

$$\{\boldsymbol{\delta}_i\boldsymbol{\delta}_j \cdot \boldsymbol{\delta}_k\boldsymbol{\delta}_l\} = \boldsymbol{\delta}_i\,(\boldsymbol{\delta}_j \cdot \boldsymbol{\delta}_k)\,\boldsymbol{\delta}_l = \delta_{jk}\boldsymbol{\delta}_i\boldsymbol{\delta}_l$$

$$\{\boldsymbol{\delta}_i\boldsymbol{\delta}_j \times \boldsymbol{\delta}_k\} = \boldsymbol{\delta}_i[\boldsymbol{\delta}_j \times \boldsymbol{\delta}_k] = \sum_{l=1}^{3} \varepsilon_{jkl}\boldsymbol{\delta}_i\boldsymbol{\delta}_l$$

$$\{\boldsymbol{\delta}_i \times \boldsymbol{\delta}_j\boldsymbol{\delta}_k\} = [\boldsymbol{\delta}_i \times \boldsymbol{\delta}_j]\boldsymbol{\delta}_k = \sum_{l=1}^{3} \varepsilon_{ijl}\boldsymbol{\delta}_l\boldsymbol{\delta}_k$$