1a. $P(H) = \lambda$

$\therefore P(T) = 1-\lambda$

$P(H \text{ at } K+1^{th} \text{ toss}) = P(T \text{ at } K \text{ toss and } H \text{ at } K+1 \text{ th})$

$= (1-\lambda)^K \lambda$

b let M be number of tosses required to get the first head and let $S = E(M)$

As tosses are independent and expectation is additive

$$S = \lambda \times 1 + (1-\lambda) \times (S+1)$$

$$S = \lambda + S + 1 - \lambda S - \lambda$$

$$\therefore S\lambda = 1$$

$$S = \frac{1}{\lambda}$$

2. $X \rightarrow$ Random variable

a. Variance of X : $Var(X) = E[(X - E[X])^2]$

To prove $Var(X) = E[X^2] - E[X]^2$

Given that

$$Var(X) = E[(X - E[X])^2]$$

$$Var(X) = E[X^2 - 2X E[X] + E[X]^2]$$

$$Var(X) = E[X^2] - E[X]^2$$

b. $E[x] = 0$ and $E[x^2] = 1$

To find : 1 Variance of x

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$$Var(X) = E[X^2] - E[X]^2$$
$$= 1 - 0$$
$$Var(X) = 1$$

2 $Y = a + bX$

$$E[Y^2] = E[(a+bX)^2]$$

$$E[Y^2] = E[a^2 + 2abX + b^2X^2]$$
$$E[Y^2] = a^2 + 2abE[X] + b^2 E[X^2]$$
$$E[Y^2] = a^2 + 2ab(0) + b^2(1)$$
$$E[Y^2] = a^2 + b^2$$

$$E[Y] = E[a+bX] = a + bE[X]$$
$$E[Y] = a + b(0)$$
$$\therefore E[Y] = a$$

$$Var(Y) = E[Y^2] - E[Y]^2$$
$$= a^2 + b^2 - a^2$$
$$Var(Y) = b^2$$

3. a Given a horse, the probability that it wins

$$P(B) = P(B, A) + P(B \cap A)$$
$$P(B) = P(B/A)\,P(A) + P(B/A)\,P(\neg A)$$
$$P(B) = 0.99 \times 10^{-5} + (1 - 0.999) \times (1 - 10^{-5})$$
$$P(B) = 1.99 \times 10^{-5} \quad -① $$

Probability that AKU predicts a black beauty is wrong

$$P(A/B) = \frac{0.99 \times 10^{-5}}{1.99 \times 10^{-5}} = 0.497$$