Mixing statistically independent sources

Variance of mixture is given as

$$Var(x) = \langle (x - \langle x \rangle)^2 \rangle$$

$$= \langle x^2 \rangle - \langle x \rangle^2$$

$$= \left\langle \left(\sum_i w_i s_i \right)^2 \right\rangle - \left\langle \sum_i w_i s_i \right\rangle^2$$

$$= \sum_{i,j} w_i w_j \langle s_i s_j \rangle - \sum_{i,j} w_i w_j \langle s_i \rangle \langle s_j \rangle$$

$$= \sum_i w_i w_j \left(\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle \right) + \sum_{i,j : i \neq j} w_i w_j$$

$$\left(\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle \right)$$

$$= \sum_i w_i^2 \left(\langle s_i s_j \rangle - \langle s_i \rangle^2 \right) + \sum_{i,j : j \neq i} w_i w_j \left(\langle s_i \rangle \langle s_j \rangle - \langle s_i \rangle \langle s_j \rangle \right)$$

S_i and S_j are statistically independent for $i \neq j$

$$\Rightarrow \langle s_i \rangle \langle s_j \rangle - \langle s_i \rangle \langle s_j \rangle = 0$$

Also, $Var(S_i) = 1$

To guarantee that the mixture has unit variance

$$Var(x) = 1$$

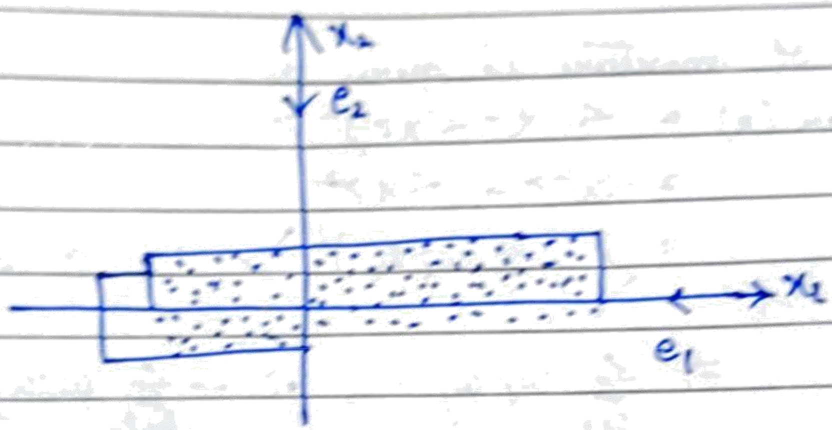
$$\therefore \sum_i w_i^2 = 1$$

$\therefore$ The following constraint has to be impressed on the weights w_i for the mixture to have unit variance.

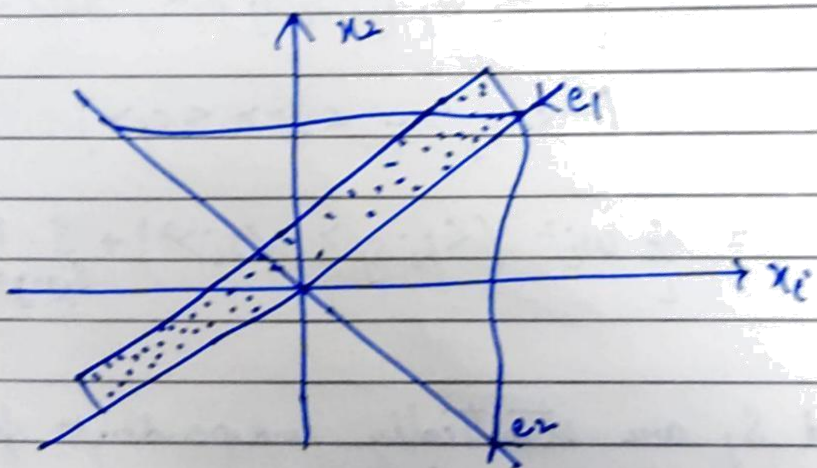
$$\sum_i w_i^2 = 1$$

2. Guess independent components and distribution from data

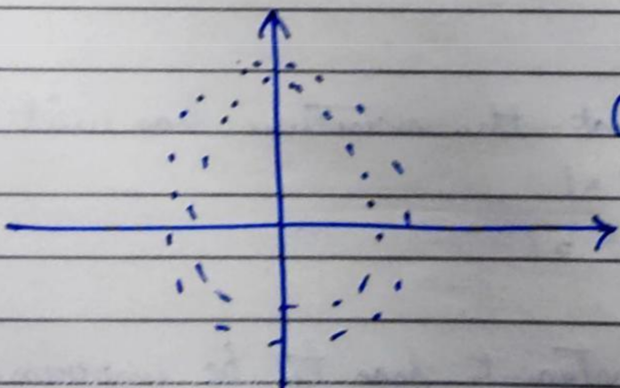
a



b



c



Can not separated into independent component.