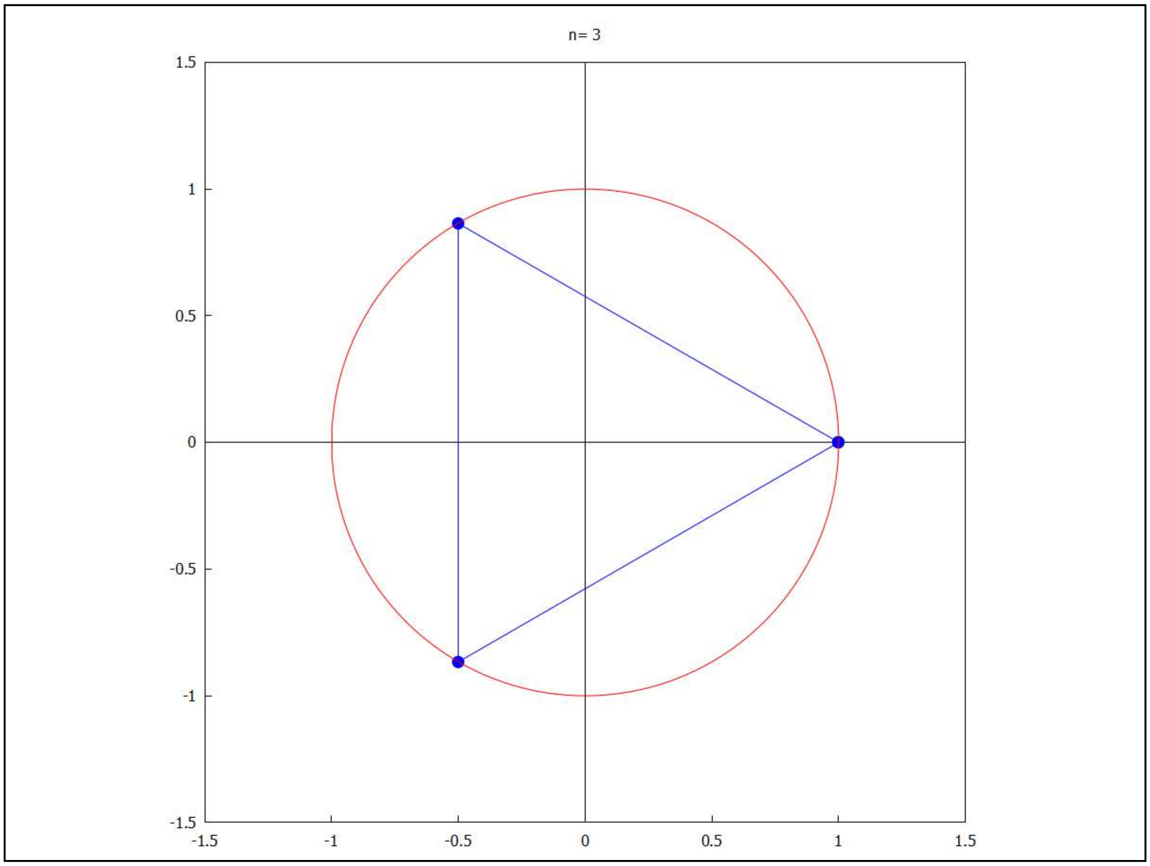
# 1 Practical 1
# Points of unity on a unit circle

```
kill(all)$
plotRoots(n,w):=block(
root:solve(z^n=w,z),
sol:map(rhs,root),
rsol:map(realpart,sol),
isol:map(imagpart,sol),
    m:(cabs(w))^(1/n),
    rsol1:cons(rsol[n],rsol),
    isol1:cons(isol[n],isol),
    wxdraw2d(
    title=concat("n= ",n),
    xaxis=true,xaxis_type=solid,xrange=[−m−0.5,m+0.5],
    yaxis=true,yaxis_type=solid,yrange=[−m−0.5,m+0.5],
    proportional_axes=xy,
    point_size=2,
    point_type=7,
    points_joined=true,
    points(rsol1,isol1),
    color=red,
    nticks=200,
    parametric(m·cos(t),m·sin(t),t,0,2·%pi))
    );
```

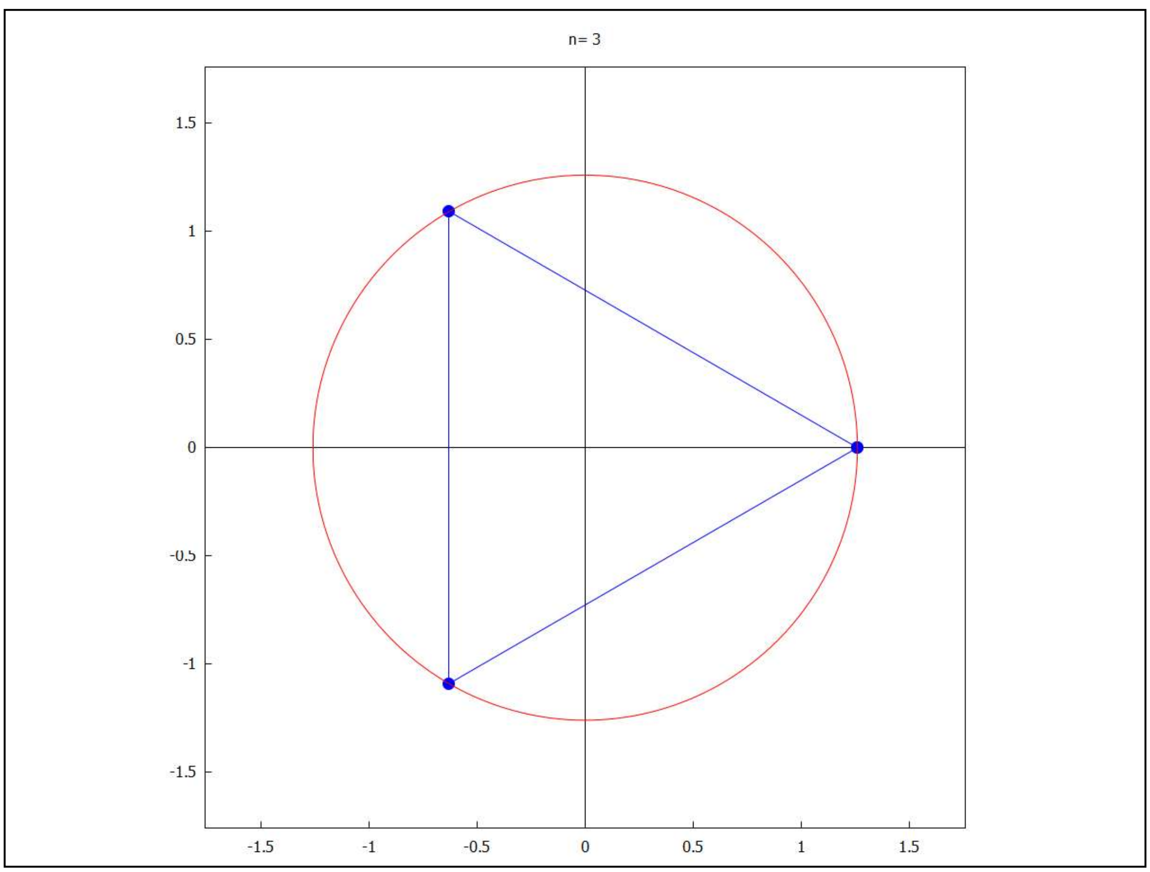
$\mathrm{plotRoots}(n,w):=\mathrm{block}(\mathrm{root}:\mathrm{solve}(z^{n}=w,z),\mathrm{sol}:\mathrm{map}(\mathrm{rhs},\mathrm{root}),\mathrm{rsol}:\mathrm{map}(\mathrm{realpart},\mathrm{sol}),\mathrm{isol}:\mathrm{map}(\mathrm{imagpart},\mathrm{sol}),m:\mathrm{cabs}(w)^{1/n},\mathrm{rsol1}:\mathrm{cons}(\mathrm{rsol}_{n},\mathrm{rsol}),\mathrm{isol1}:\mathrm{cons}(\mathrm{isol}_{n},\mathrm{isol}),\mathrm{wxdraw2d}(\mathrm{title}=\mathrm{concat}(n= ,n),\mathrm{xaxis}=\mathrm{true},\mathrm{xaxis\_type}=\mathrm{solid},\mathrm{xrange}=[-m-0.5,m+0.5],\mathrm{yaxis}=\mathrm{true},\mathrm{yaxis\_type}=\mathrm{solid},\mathrm{yrange}=[-m-0.5,m+0.5],\mathrm{proportional\_axes}=\mathrm{xy},\mathrm{point\_size}=2,\mathrm{point\_type}=7,\mathrm{points\_joined}=\mathrm{true},\mathrm{points}(\mathrm{rsol1},\mathrm{isol1}),\mathrm{color}=\mathrm{red},\mathrm{nticks}=200,\mathrm{parametric}(m\cos(t),m\sin(t),t,0,2\pi)))$

## 1.1 Plotting

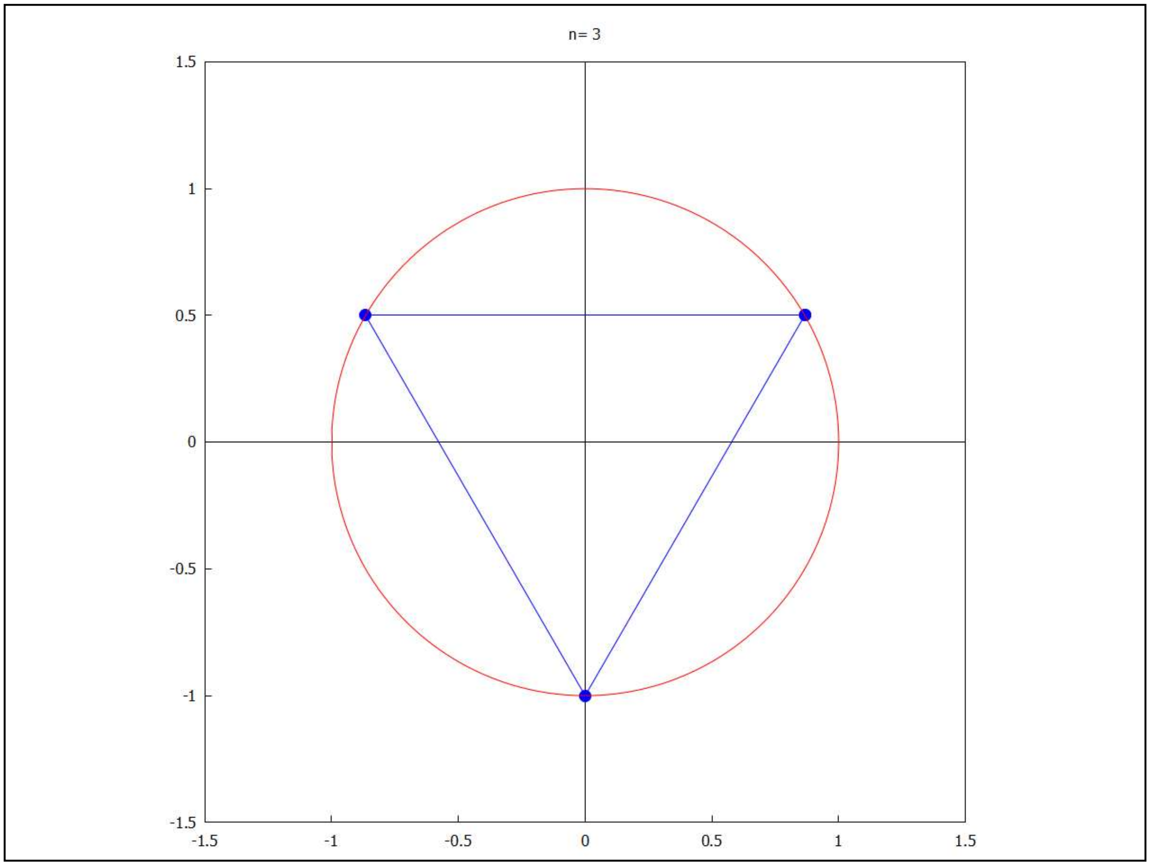
```
plotRoots(3,1);
```



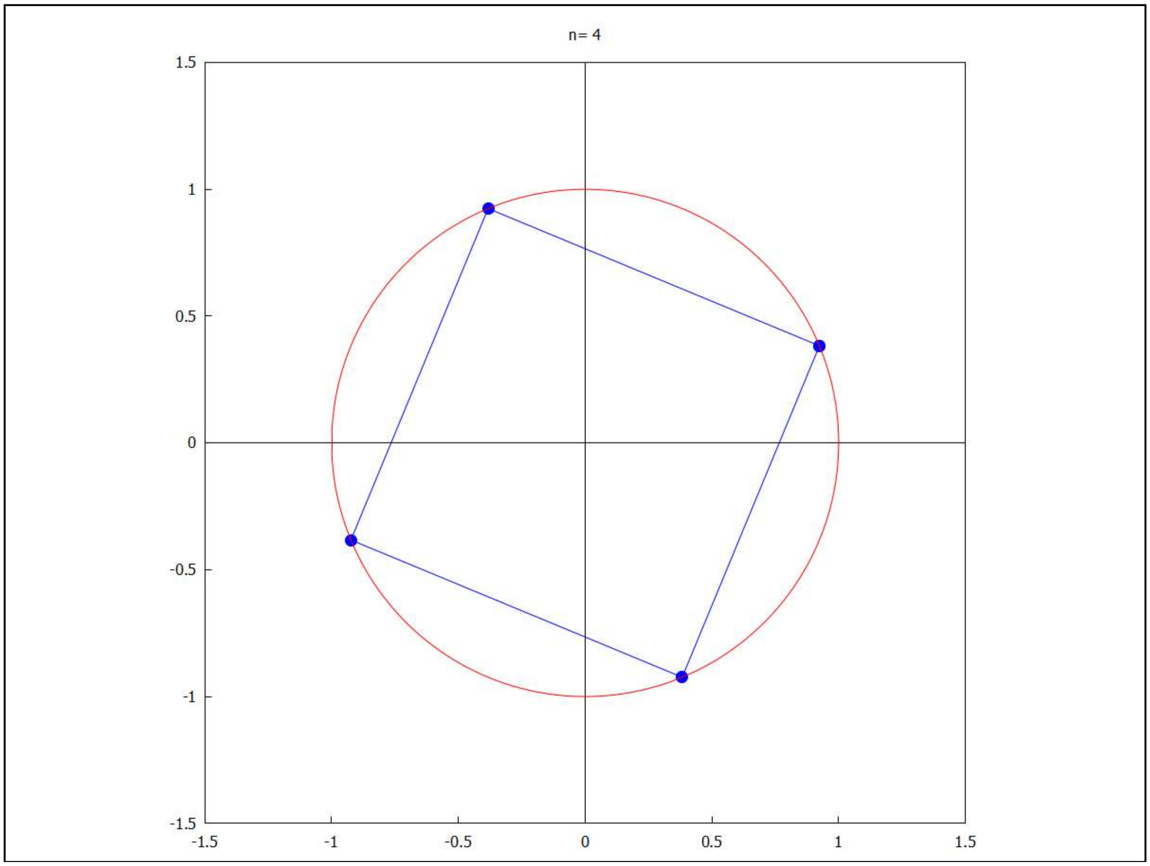

**plotRoots(3,2);**



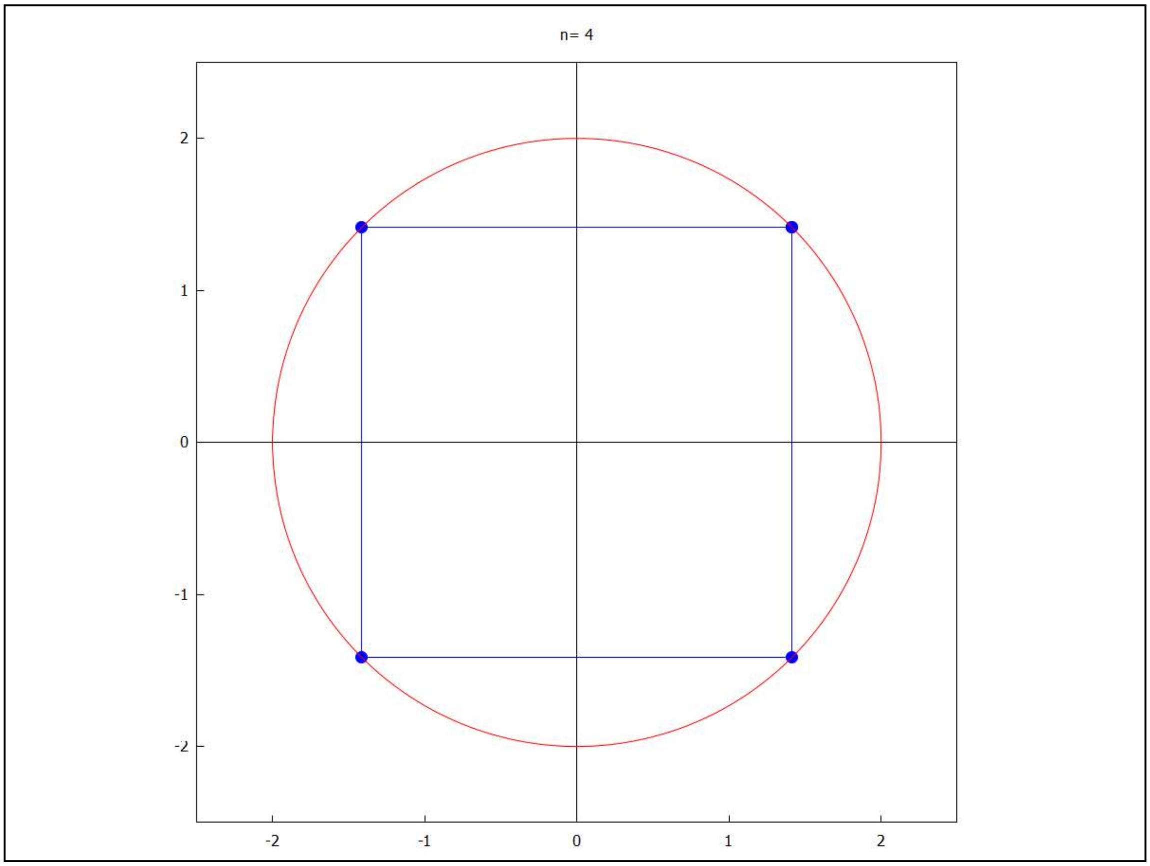
```
plotRoots(3,%i);
```



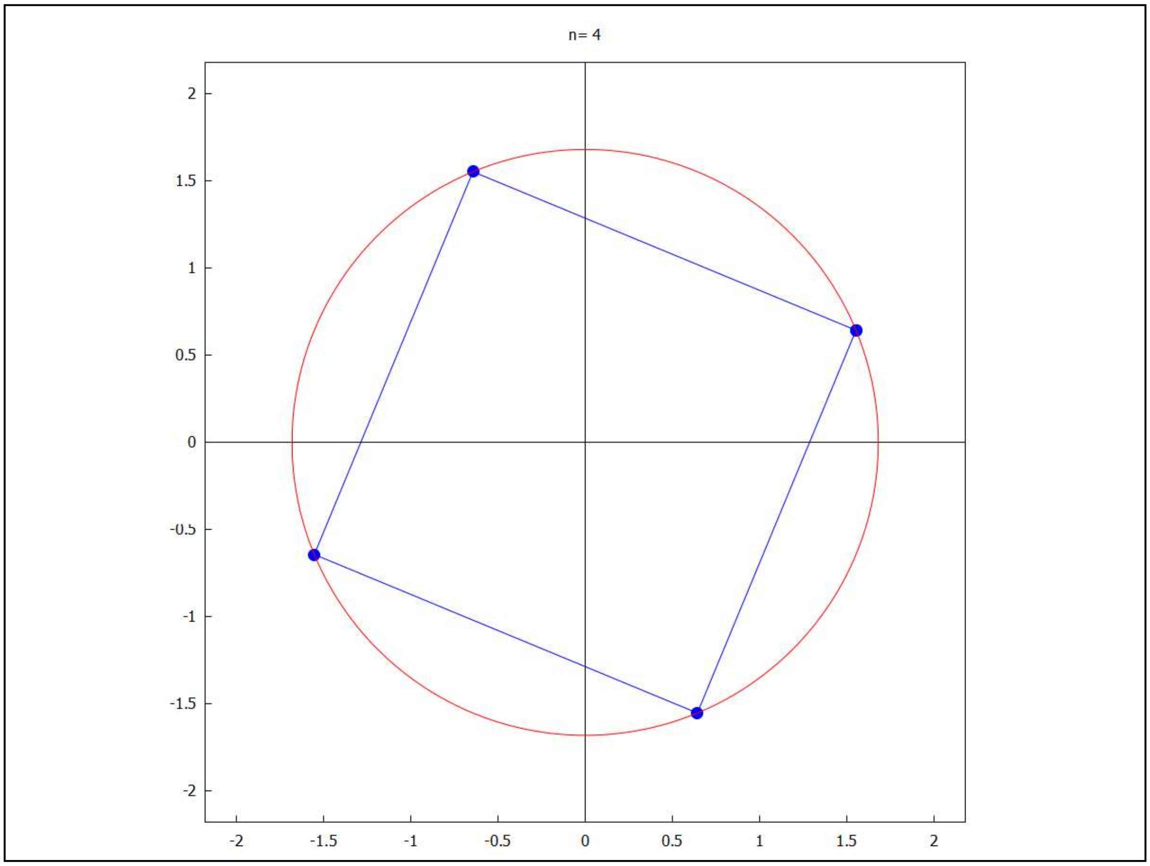

```
plotRoots(4,%i);
```



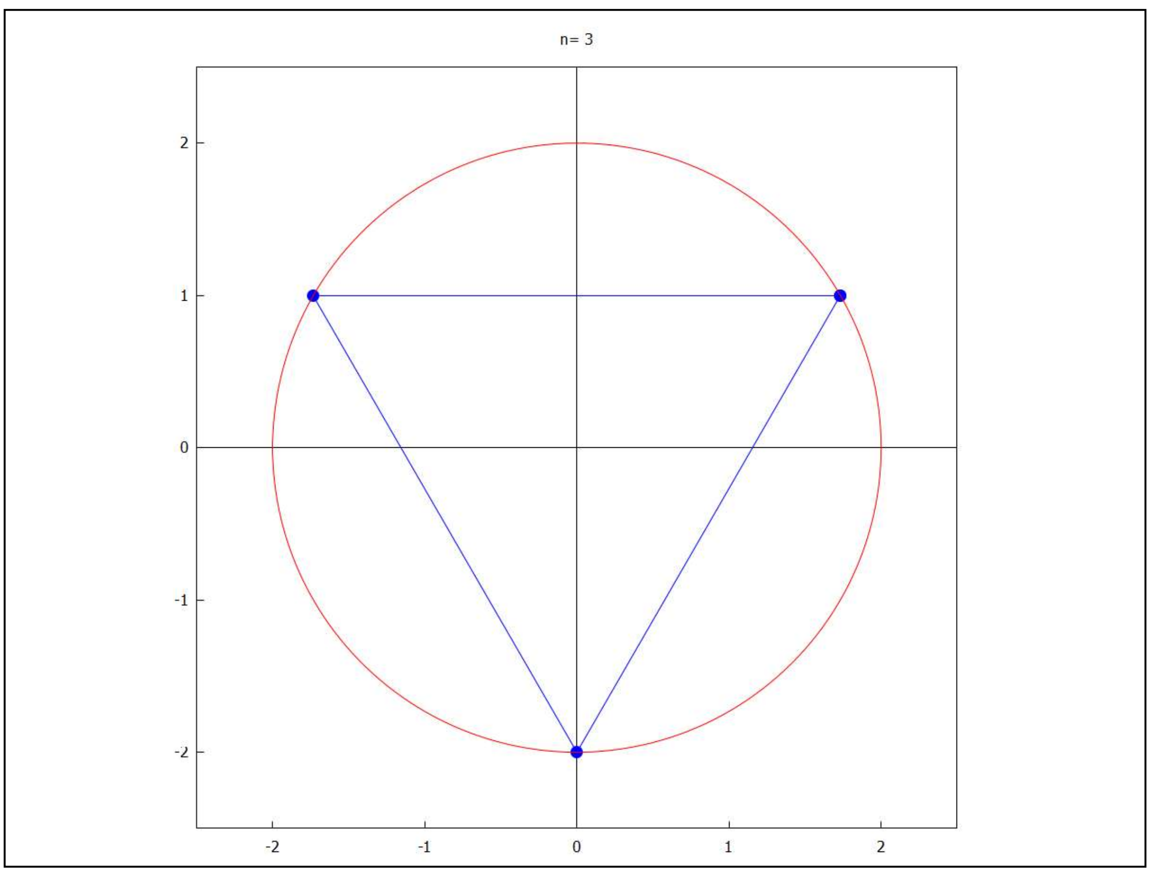
```
plotRoots(4,−16);
```




```
plotRoots(4,8·%i);
```




**plotRoots(3,8·%i);**



```
roots:solve(z^2+2·z+(1−%i)=0,z);
```

$$\left[z=-(-1)^{1/4}-1,z=(-1)^{1/4}-1\right]$$

```
rectform(roots);
```

$$\left[z=-\left(\frac{\%i}{\sqrt{2}}\right)-\frac{1}{\sqrt{2}}-1,z=\frac{\%i}{\sqrt{2}}+\frac{1}{\sqrt{2}}-1\right]$$

# 2 Practical 2

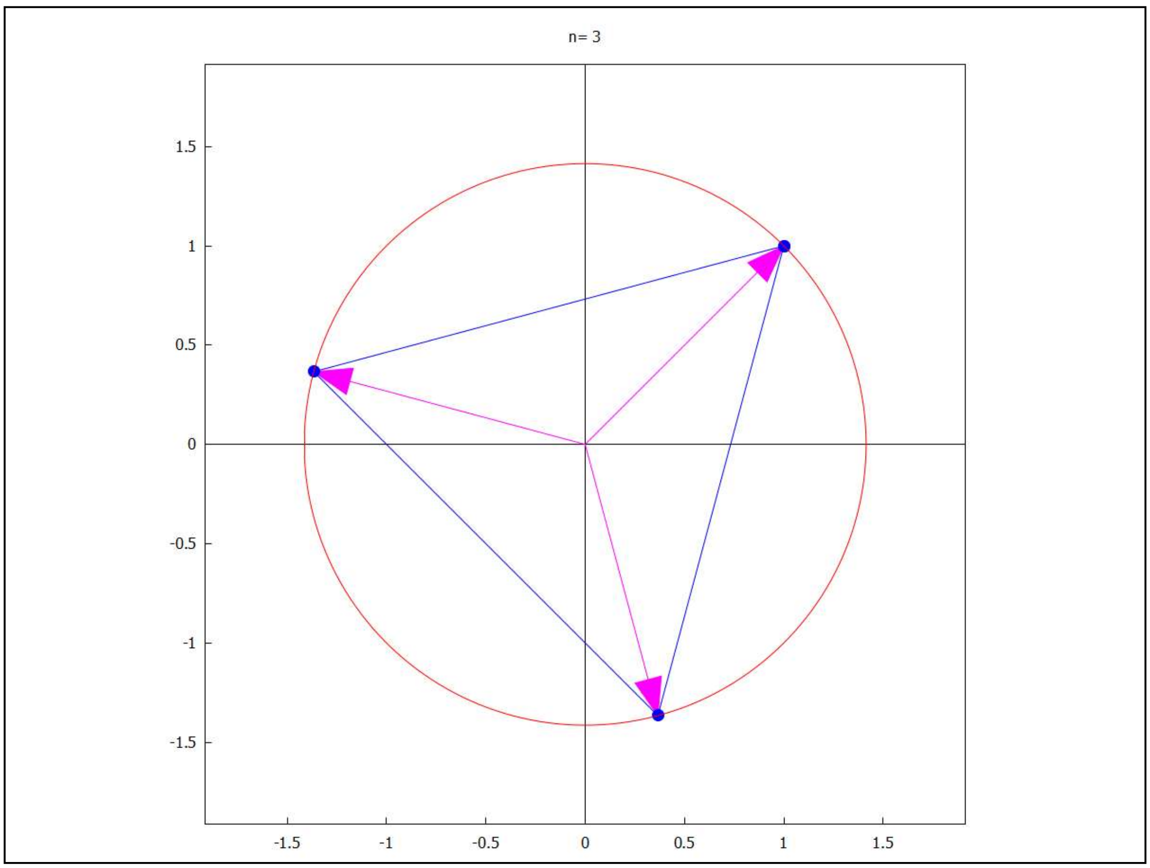
## Solutions of $z^3=8i$

```
kill(all)$
plotRoots(n,w):=block(
root:solve(z^n=w,z),
sol:map(rhs,root),
rsol:map(realpart,sol),
isol:map(imagpart,sol),
v:makelist(vector([0,0],[rsol[k],isol[k]]),k,1,n),
m:(cabs(w))^(1/n),
rsol1:cons(rsol[n],rsol),
isol1:cons(isol[n],isol),
wxdraw2d(
   title=concat("n= ",n),
   xaxis=true,xaxis_type=solid,xrange=[−m−0.5,m+0.5],
   yaxis=true,yaxis_type=solid,yrange=[−m−0.5,m+0.5],
   proportional_axes=xy,
   color=magenta,
   head_length=0.2,
   head_angle=20,
   v,
   color=blue,
   point_size=2,
   point_type=7,
   points_joined=true,
   points(rsol1,isol1),
   color=red,
   nticks=200,
   parametric(m·cos(t),m·sin(t),t,0,2·%pi))
   );
```

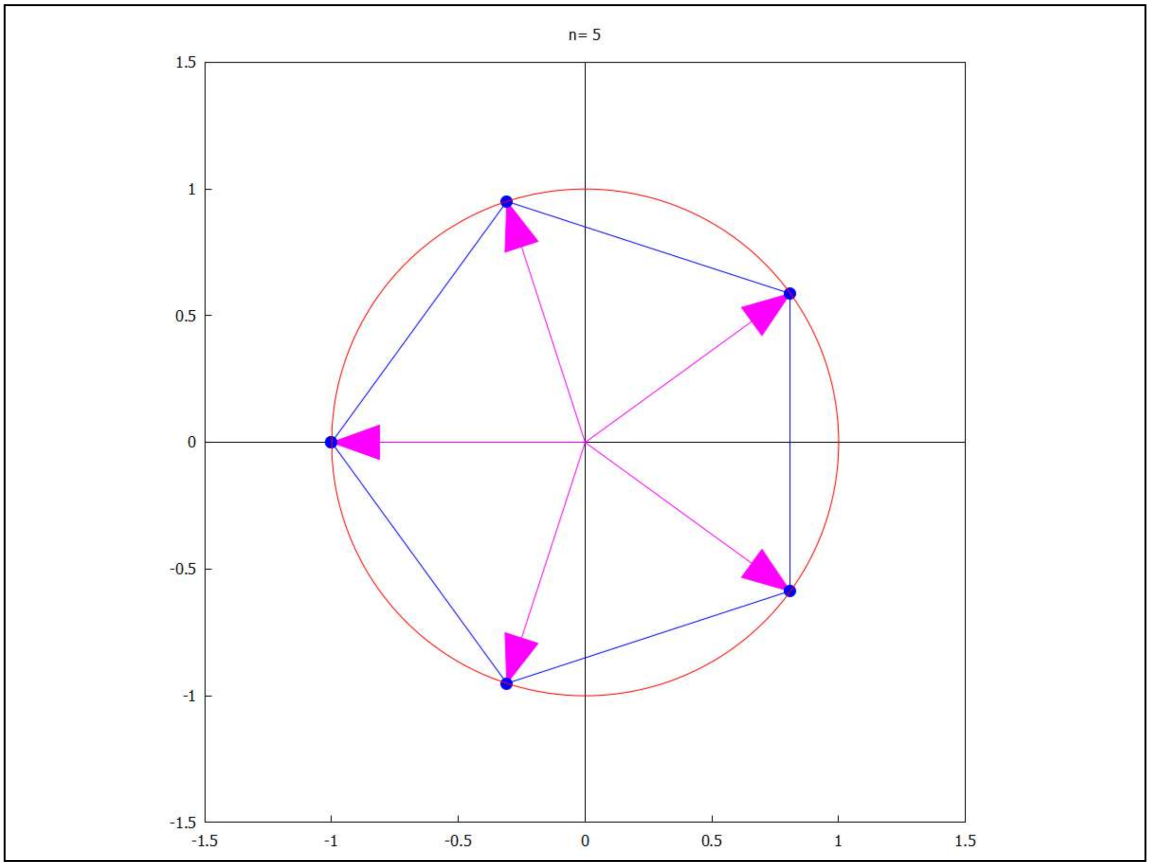
plotRoots(n ,w ):=block(root:solve($z^n = w, z$), sol: map(rhs, root), rsol: map(realpart, sol), isol: map(imagpart, sol), v: makelist$\left(\text{vector}([0,0], \left[rsol_k, isol_k\right]), k, 1, n\right)$, m: cabs$(w)^{1/n}$, rsol1: cons($rsol_n$, rsol), isol1: cons($isol_n$, isol), wxdraw2d(title = concat(n= , n), xaxis = true, xaxis_type = solid, xrange = [ − m − 0.5, m + 0.5], yaxis = true, yaxis_type = solid, yrange = [ − m − 0.5, m + 0.5], proportional_axes = xy, color = magenta, head_length = 0.2, head_angle = 20, v, color = blue, point_size = 2, point_type = 7, points_joined = true, points(rsol1, isol1), color = red, nticks = 200, parametric(m cos(t), m sin(t), t, 0, 2 π)))

## 2.1 Plotting

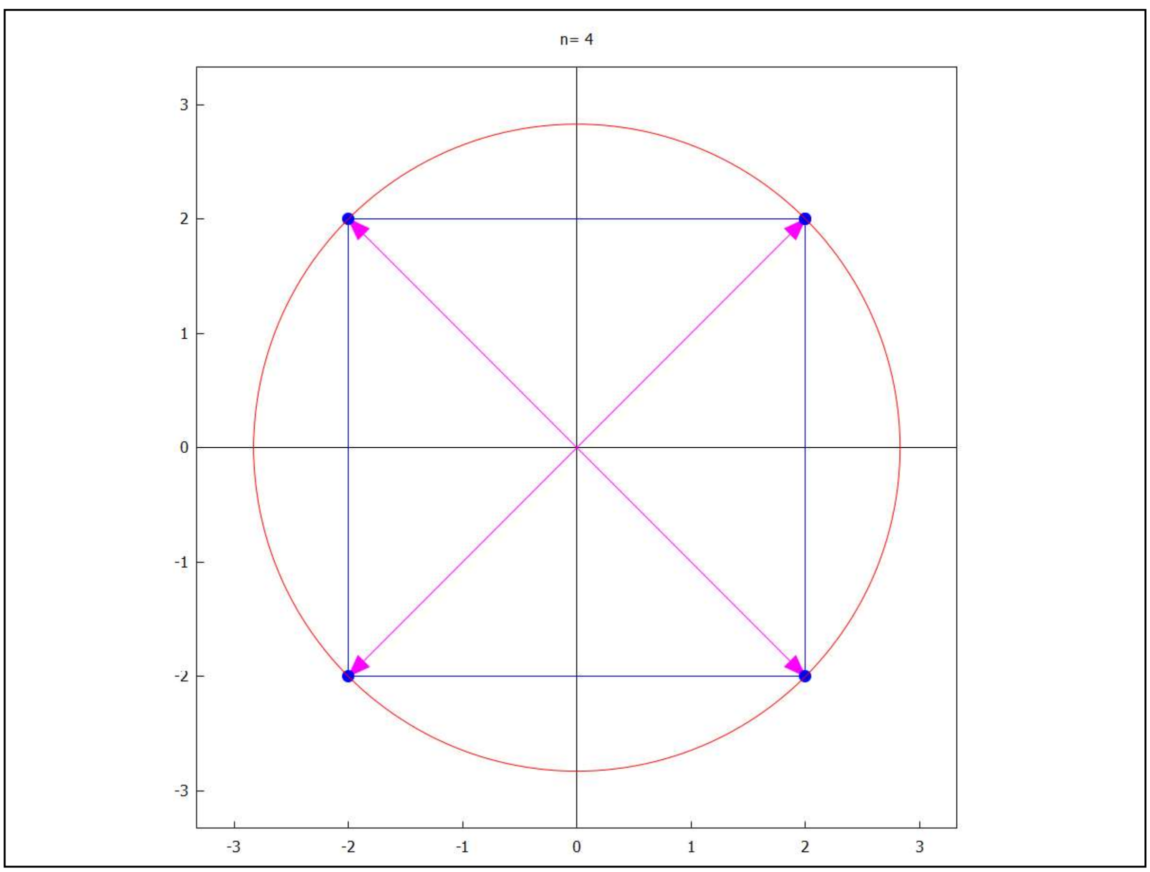
**plotRoots(3,−2+2·%i);**



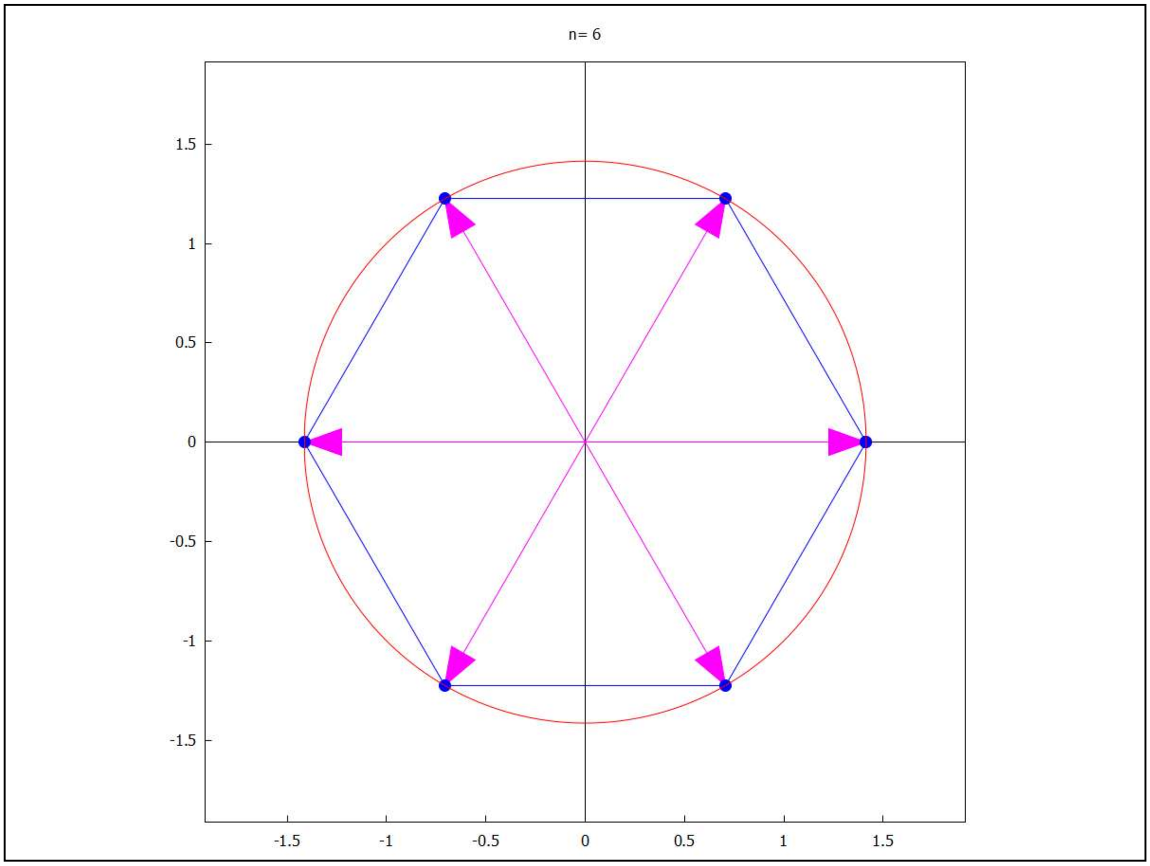

**plotRoots(5,−1);**



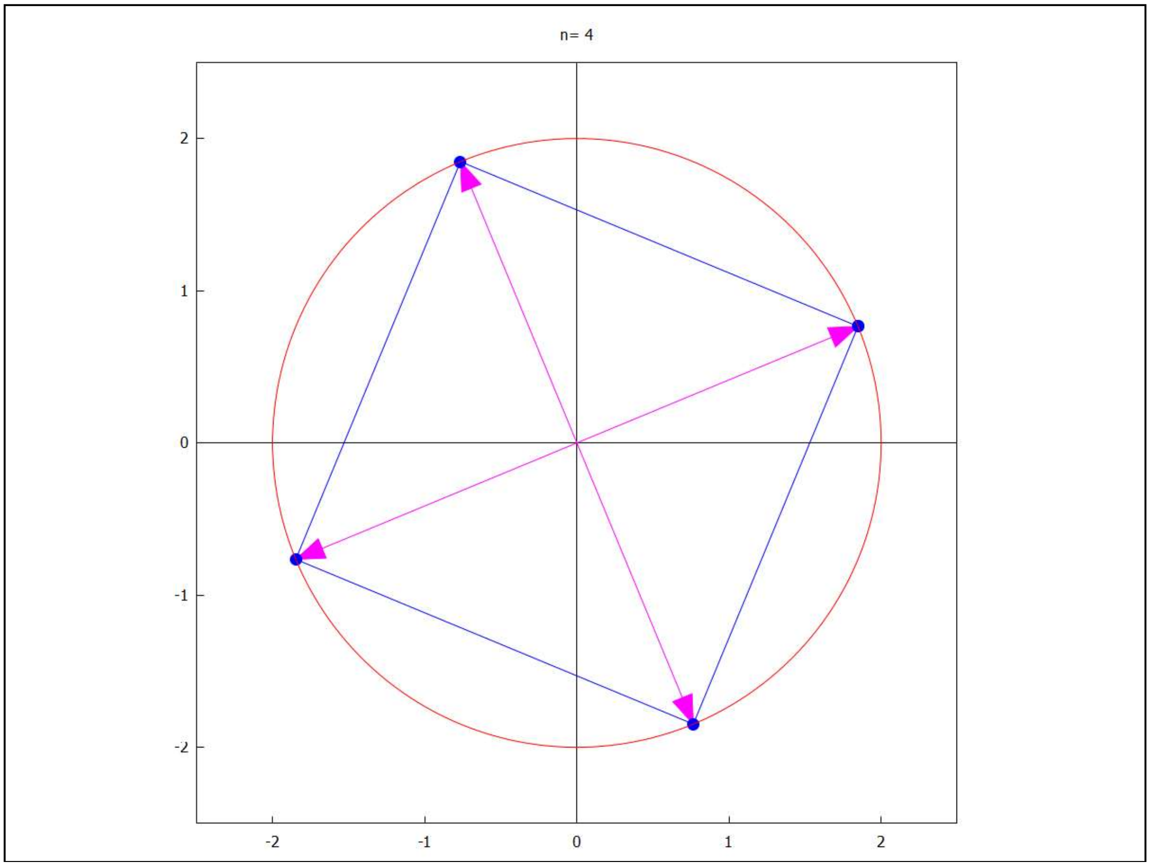

**plotRoots(4,−64);**



```
plotRoots(6,8);
```




```
plotRoots(4,16·%i);
```



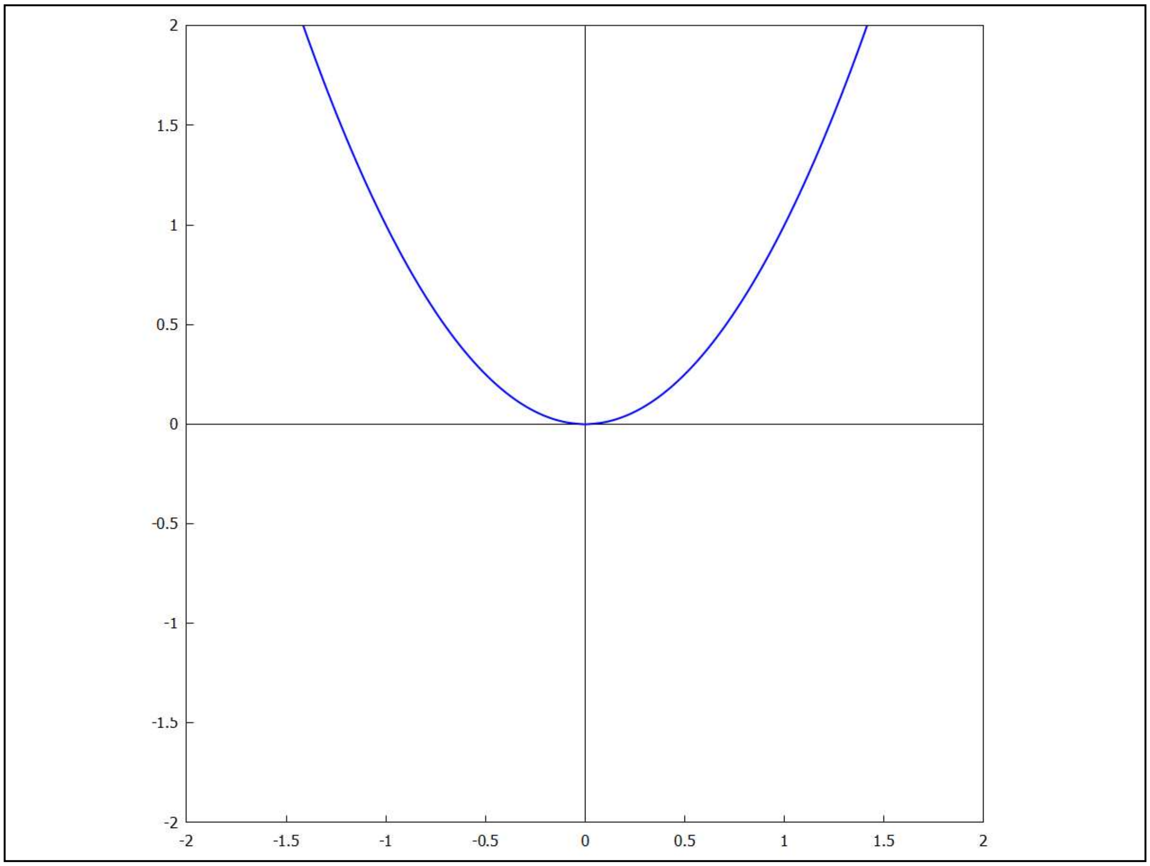
# 3 Practical 3 Curves and shifting them

## 3.1 Parabola

```
kill(all)$
s(t):=t+%i·t^2;
```

$$s(t) := t + \%i\, t^2$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−2,2],
yaxis=true,yaxis_type=solid,yrange=[−2,2],
proportional_axes=xy,
nticks=200,
line_width=2,
parametric(realpart(s(t)),imagpart(s(t)),t,−2,2));
```

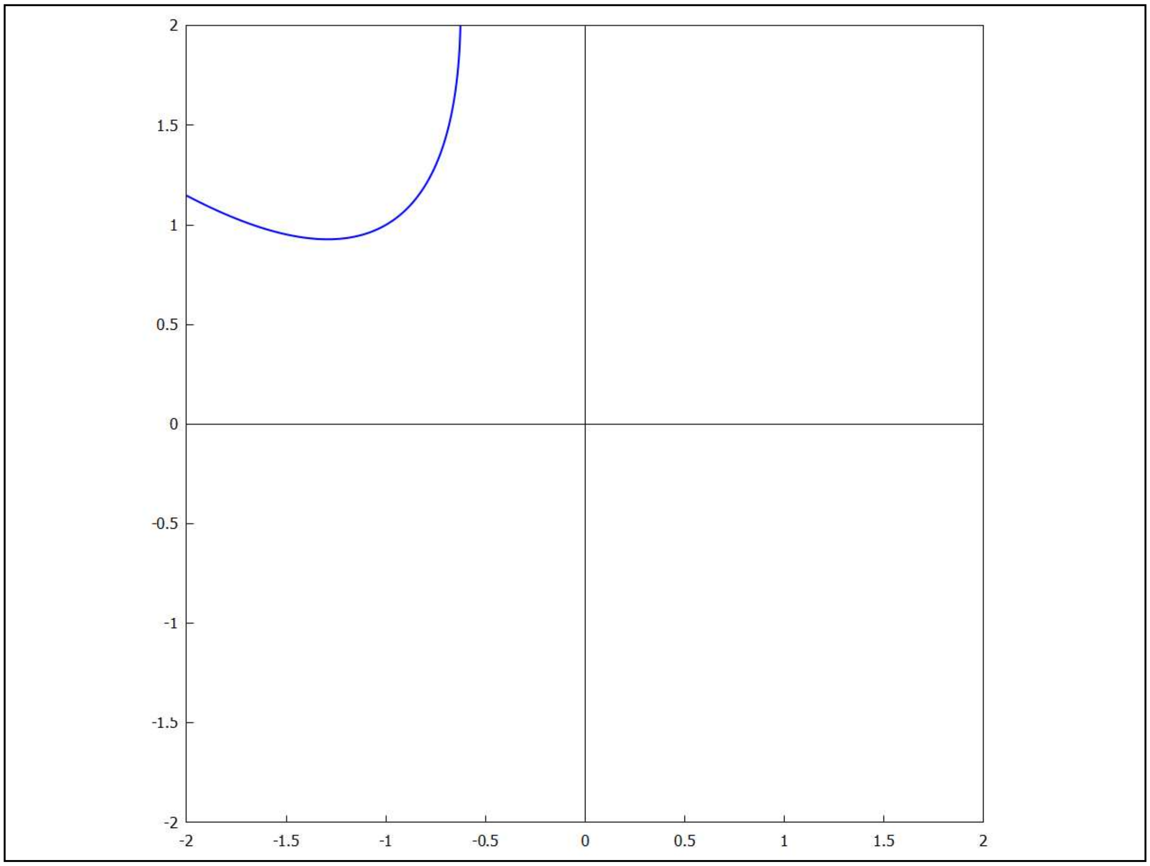



rotating about the origin by π/6

```
theta:%pi/6;
r(t):=s(t)·exp(%i·theta)+(−1+%i);
```

$$\frac{\pi}{6} \qquad r(t):=s(t)\,\exp(\%i\,\mathrm{theta})+(-1+\%i)$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−2,2],
yaxis=true,yaxis_type=solid,yrange=[−2,2],
proportional_axes=xy,
nticks=200,
line_width=2,
parametric(realpart(r(t)),imagpart(r(t)),t,−2,2));
```

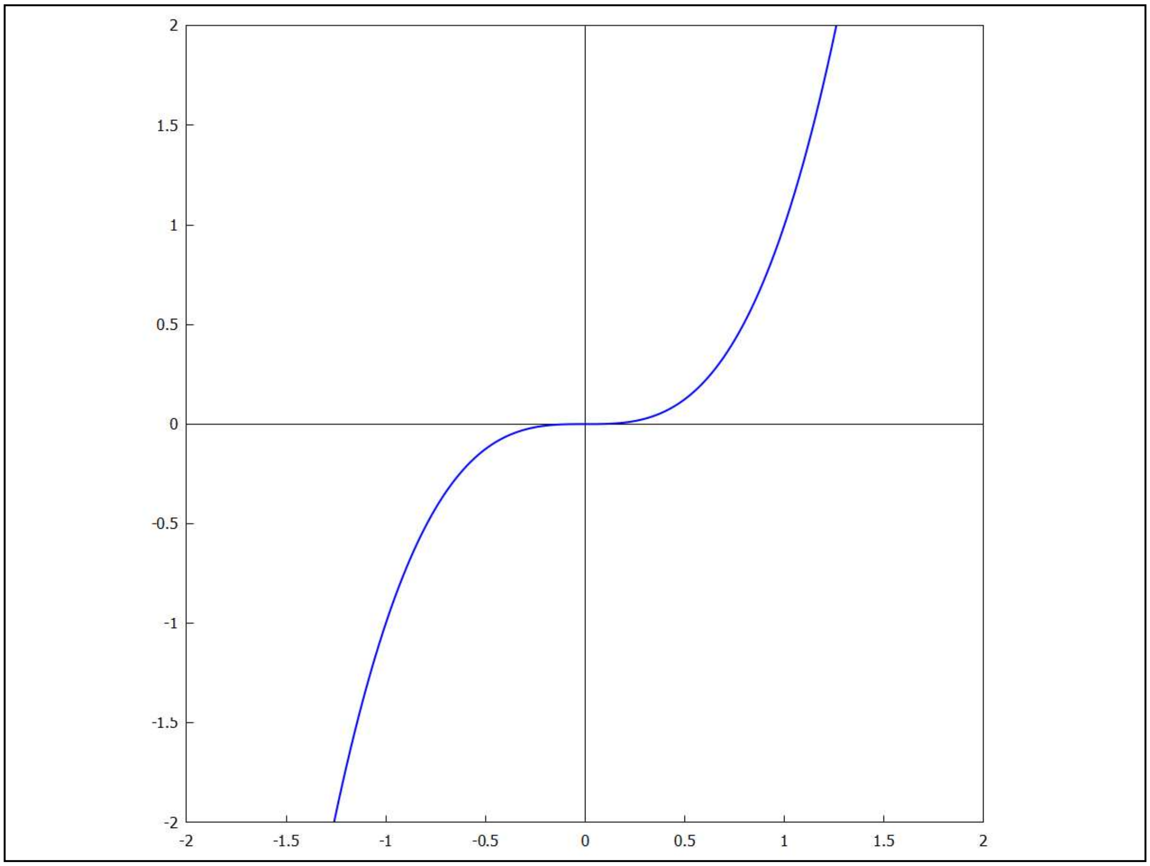



## 3.2 Curve

```
kill(all)$
s(t):=t+%i·t^3;
```

$$s(t):=t+\%i\,t^3$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−2,2],
yaxis=true,yaxis_type=solid,yrange=[−2,2],
proportional_axes=xy,
nticks=200,
line_width=2,
parametric(realpart(s(t)),imagpart(s(t)),t,−2,2));
```

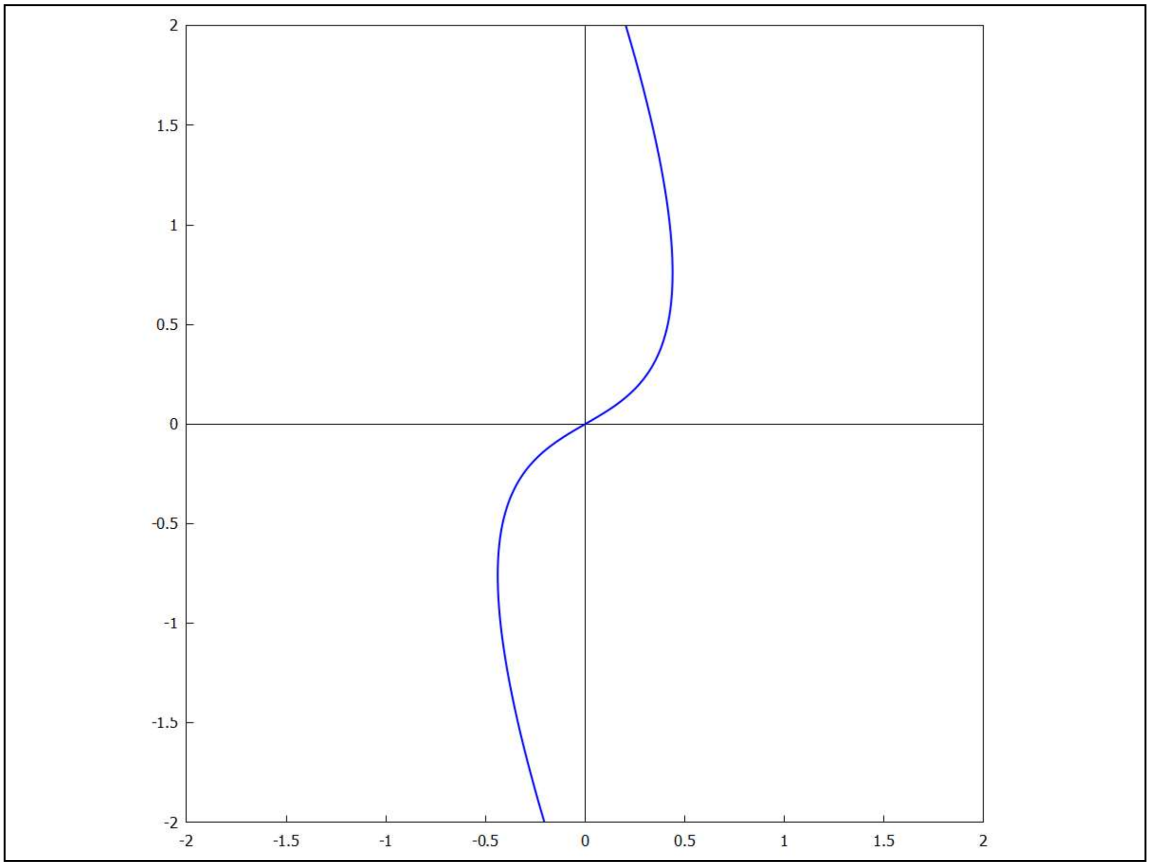



rotating about the origin by π/6

```
theta:%pi/6;
r(t):=s(t)·exp(%i·theta);
```

$$\frac{\pi}{6} \qquad r(t) := s(t)\,\exp(\%i\ \mathrm{theta})$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−2,2],
yaxis=true,yaxis_type=solid,yrange=[−2,2],
proportional_axes=xy,
nticks=200,
line_width=2,
parametric(realpart(r(t)),imagpart(r(t)),t,−2,2));
```

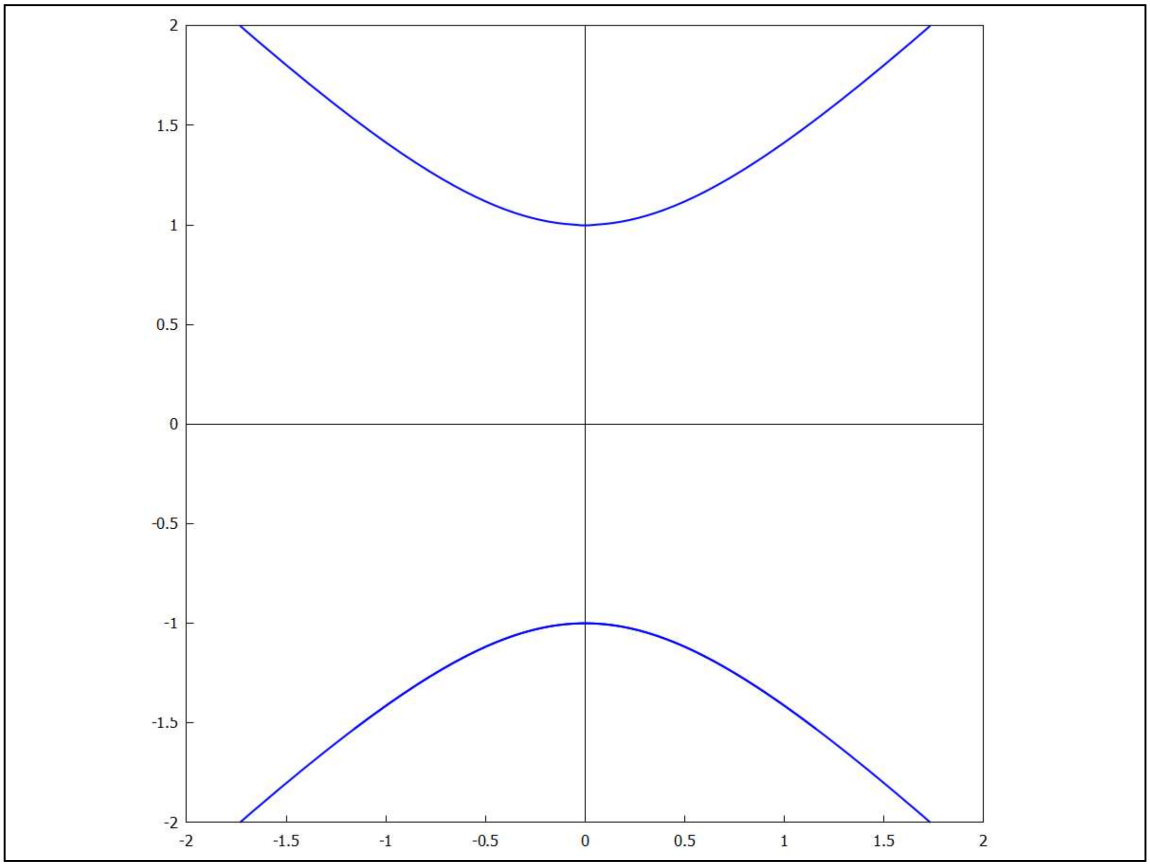



## 3.3 Hyperbola

```
kill(all)$
s(t):=tan(t)+%i·sec(t);
```

$$s(t):=\tan(t)+\%i\ \sec(t)$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−2,2],
yaxis=true,yaxis_type=solid,yrange=[−2,2],
proportional_axes=xy,
nticks=200,
line_width=2,
parametric(realpart(s(t)),imagpart(s(t)),t,−4,4));
```



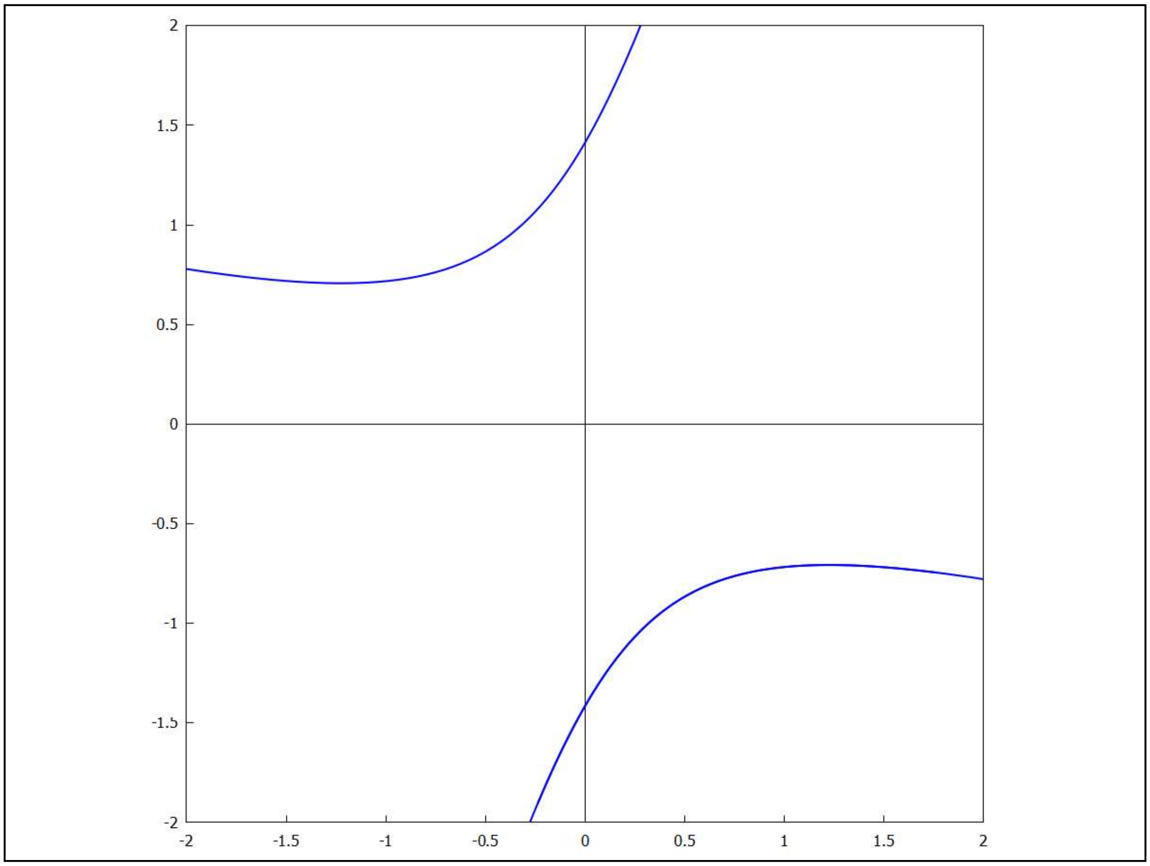

rotating about the origin by π/6

```
theta:%pi/6;
r(t):=s(t)·exp(%i·theta);
```

$$\frac{\pi}{6}$$

r(t):=s(t) exp(%i theta)

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−2,2],
yaxis=true,yaxis_type=solid,yrange=[−2,2],
proportional_axes=xy,
nticks=200,
line_width=2,
parametric(realpart(r(t)),imagpart(r(t)),t,−4,4));
```

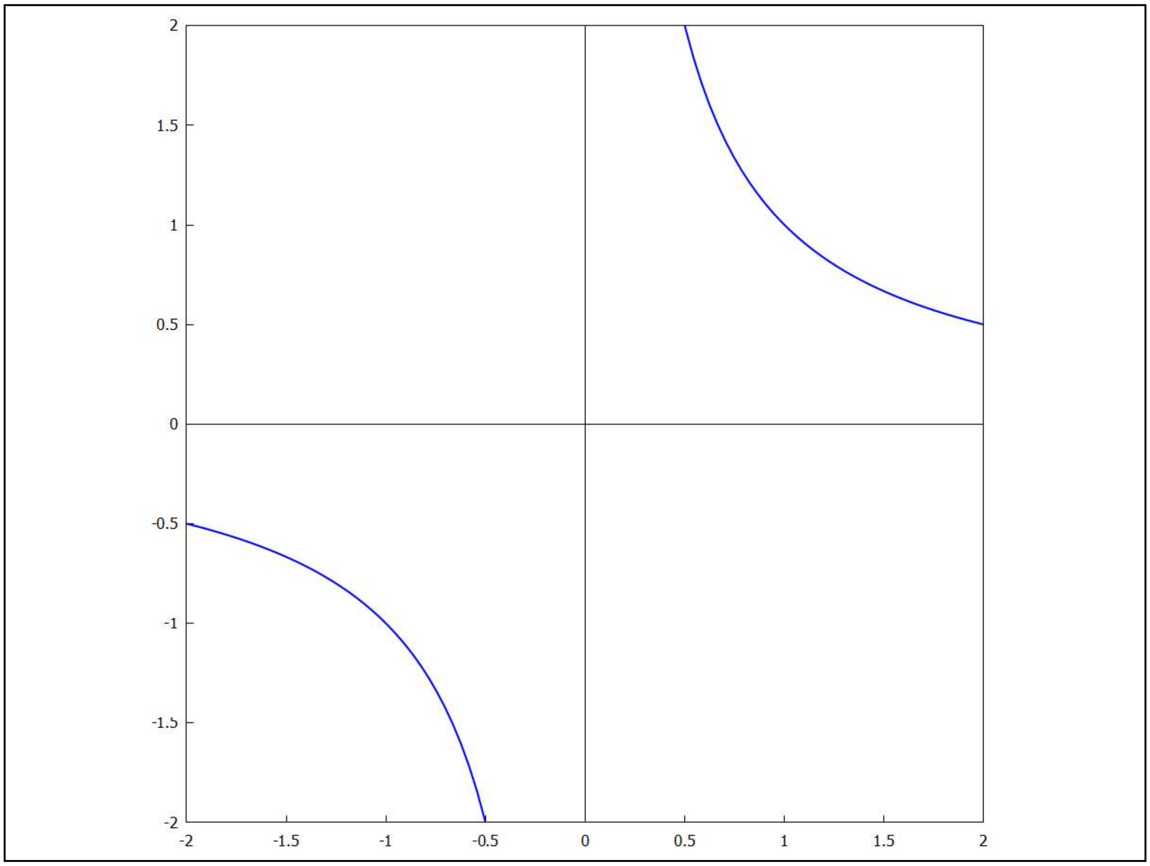



## 3.4 Hyperbola

```
kill(all)$
s(t):=t+%i·(1/t);
```

$$s(t):=t+\%i\,\frac{1}{t}$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−2,2],
yaxis=true,yaxis_type=solid,yrange=[−2,2],
proportional_axes=xy,
nticks=200,
line_width=2,
parametric(realpart(s(t)),imagpart(s(t)),t,−4,4));
```

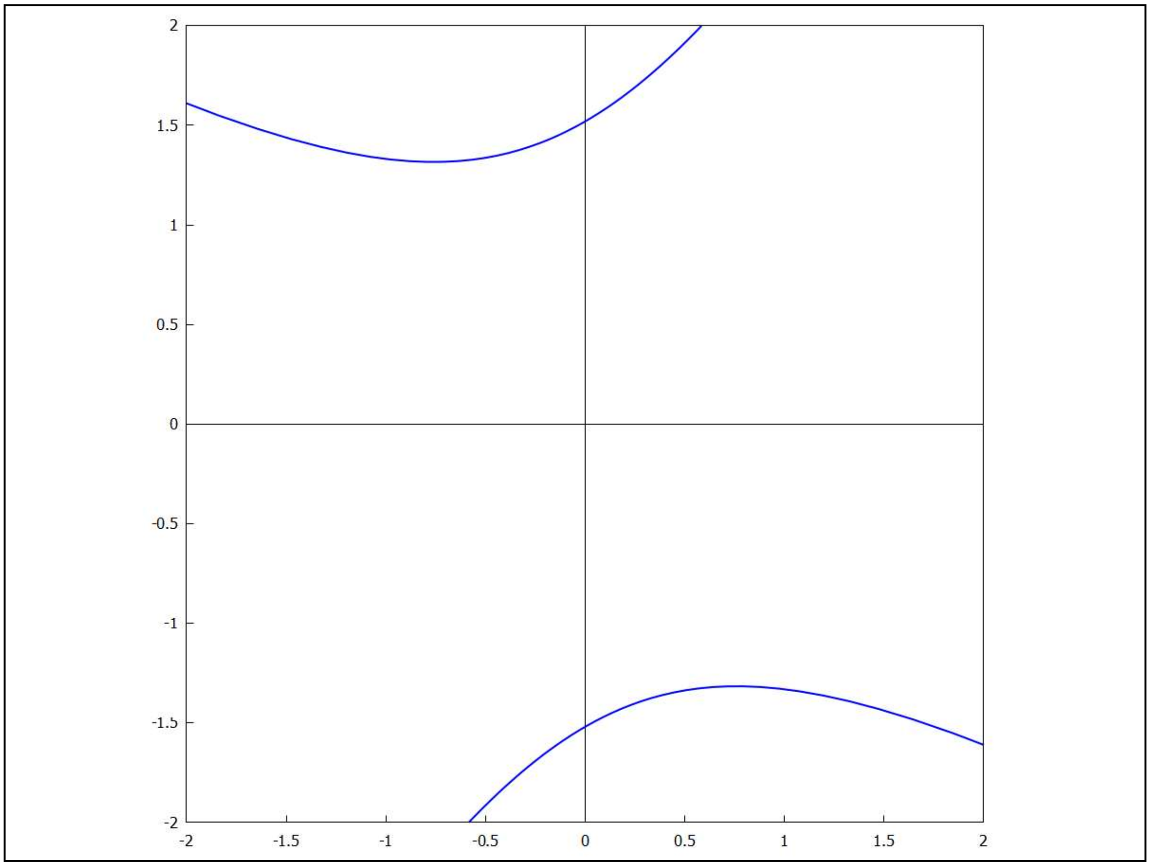



rotating about the origin by π/3

```
theta:%pi/3;
r(t):=s(t)·exp(%i·theta);
```

$$\frac{\pi}{3} \qquad r(t):=s(t)\exp(\%i\ \mathrm{theta})$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−2,2],
yaxis=true,yaxis_type=solid,yrange=[−2,2],
proportional_axes=xy,
nticks=200,
line_width=2,
parametric(realpart(r(t)),imagpart(r(t)),t,−4,4));
```

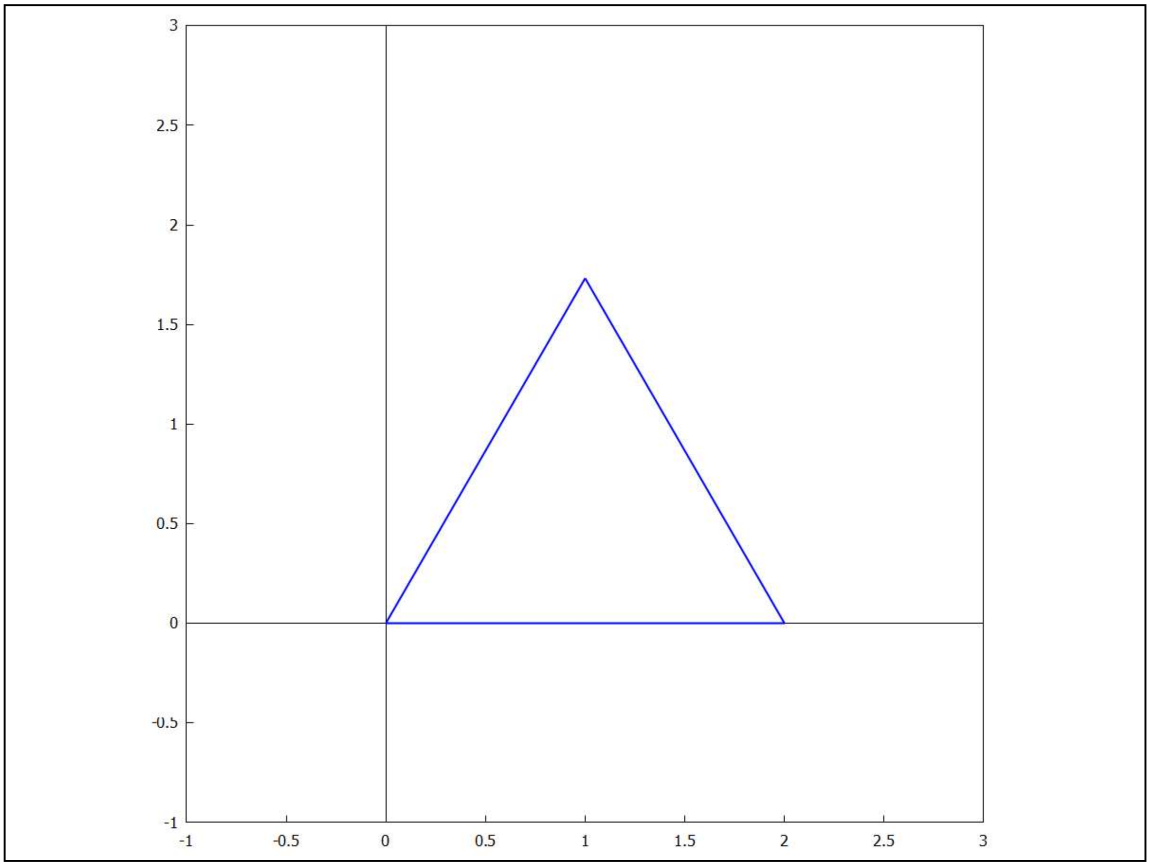



## 3.5 Triangle

```
kill(all)$
s1(t):=t+%i·0;
s2(t):=t+%i·(−t·sqrt(3)+2·sqrt(3));
s3(t):=t+%i·(t·sqrt(3));
```

s1(t):= t + %i 0 s2(t):= t + %i ( − t $\sqrt{3}$ + 2 $\sqrt{3}$ ) s3(t) := t + %i (t $\sqrt{3}$ )

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−1,3],
yaxis=true,yaxis_type=solid,yrange=[−1,3],
proportional_axes=xy,
nticks=200,
line_width=2,
parametric(realpart(s1(t)),imagpart(s1(t)),t,0,2),
parametric(realpart(s2(t)),imagpart(s2(t)),t,1,2),
parametric(realpart(s3(t)),imagpart(s3(t)),t,0,1));
```

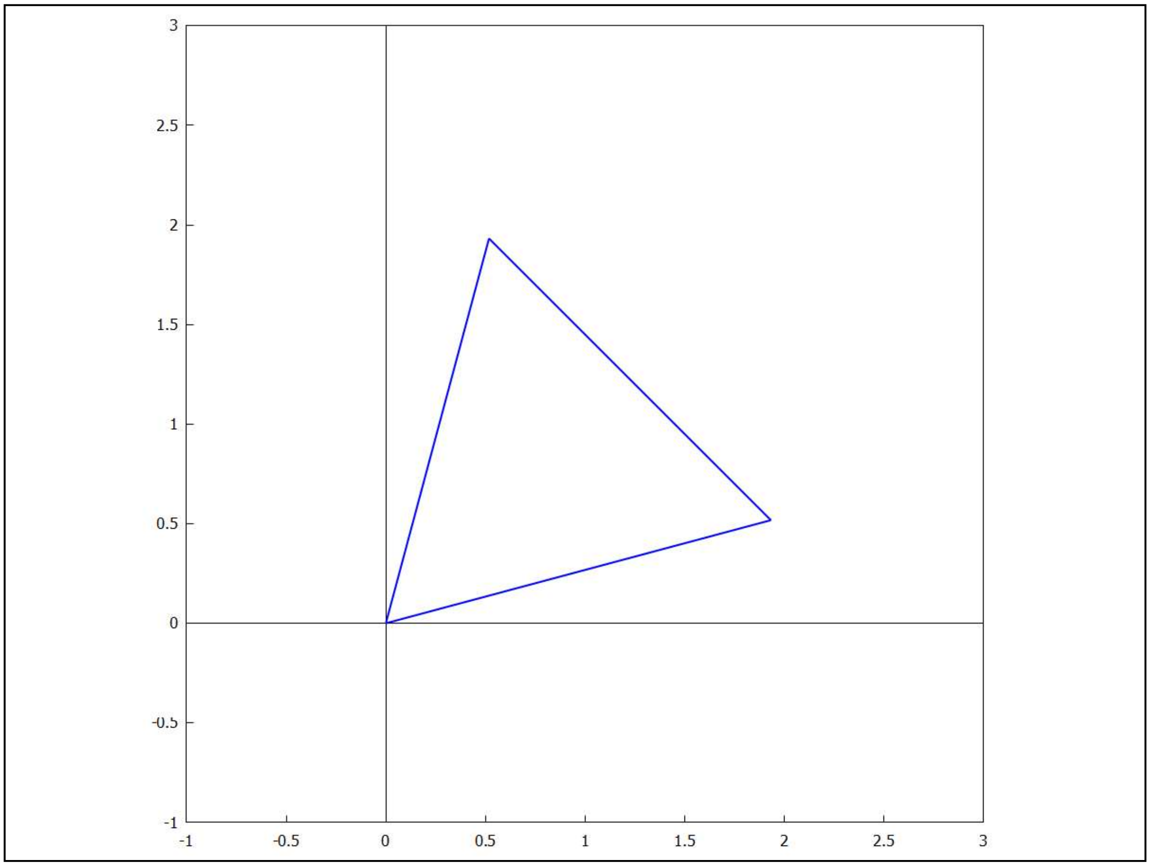



rotating about the origin by π/12

```
theta:%pi/12;
r1(t):=s1(t)·exp(%i·theta);
r2(t):=s2(t)·exp(%i·theta);
r3(t):=s3(t)·exp(%i·theta);
```

$\frac{\pi}{12}$  r1 (t ):= s1 (t ) exp (%i theta)  r2 (t ):= s2 (t ) exp (%i theta)  r3 (t ):= s3 (t ) exp (%i theta)

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−1,3],
yaxis=true,yaxis_type=solid,yrange=[−1,3],
proportional_axes=xy,
nticks=200,
line_width=2,
parametric(realpart(r1(t)),imagpart(r1(t)),t,0,2),
parametric(realpart(r2(t)),imagpart(r2(t)),t,1,2),
parametric(realpart(r3(t)),imagpart(r3(t)),t,0,1));
```



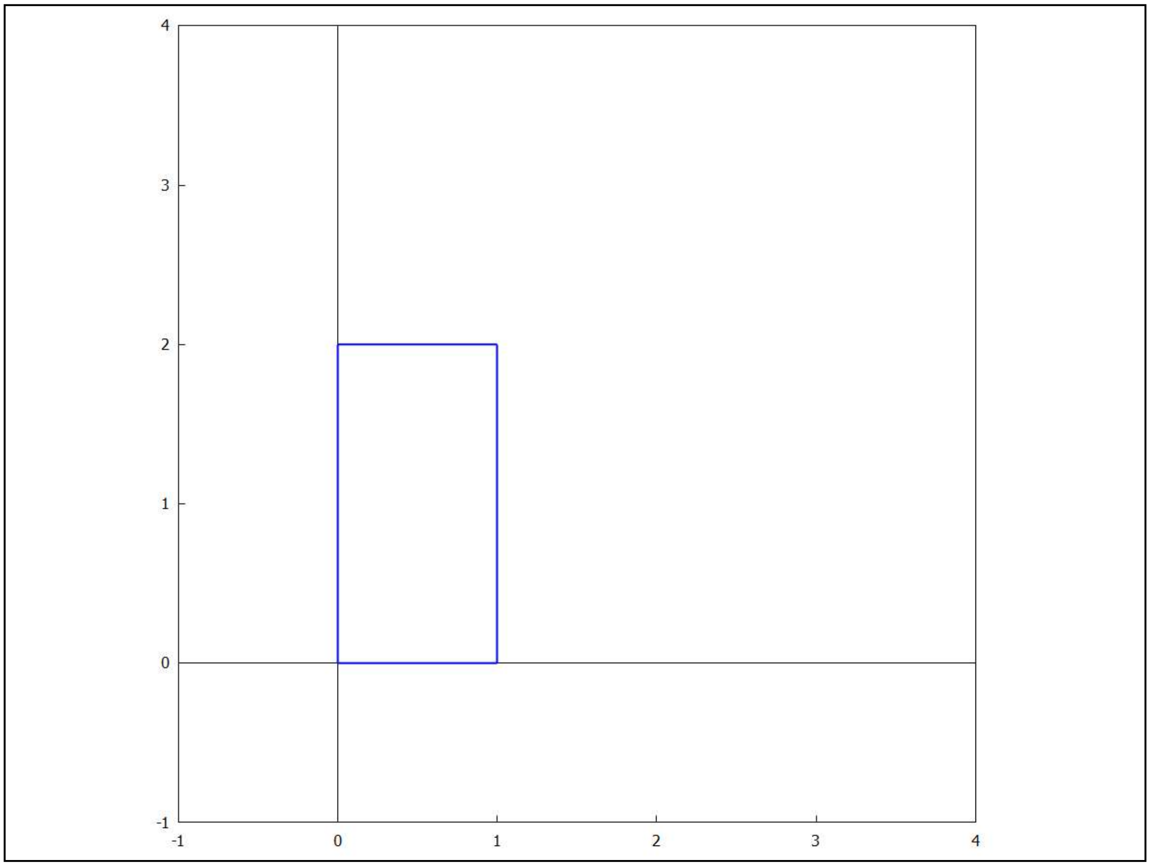

## 3.6 Rectangle

```
kill(all)$
s1(t):=t+%i·0;
s2(t):=1+%i·t;
s3(t):=t+%i·2;
s4(t):=0+%i·t;
```

s1 (t):= t + %i 0    s2 (t):= 1 + %i t    s3 (t):= t + %i 2

s4 (t):= 0 + %i t

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−1,4],
yaxis=true,yaxis_type=solid,yrange=[−1,4],
proportional_axes=xy,
nticks=200,
line_width=2,
parametric(realpart(s1(t)),imagpart(s1(t)),t,0,1),
parametric(realpart(s2(t)),imagpart(s2(t)),t,0,2),
parametric(realpart(s3(t)),imagpart(s3(t)),t,0,1),
parametric(realpart(s4(t)),imagpart(s4(t)),t,0,2));
```

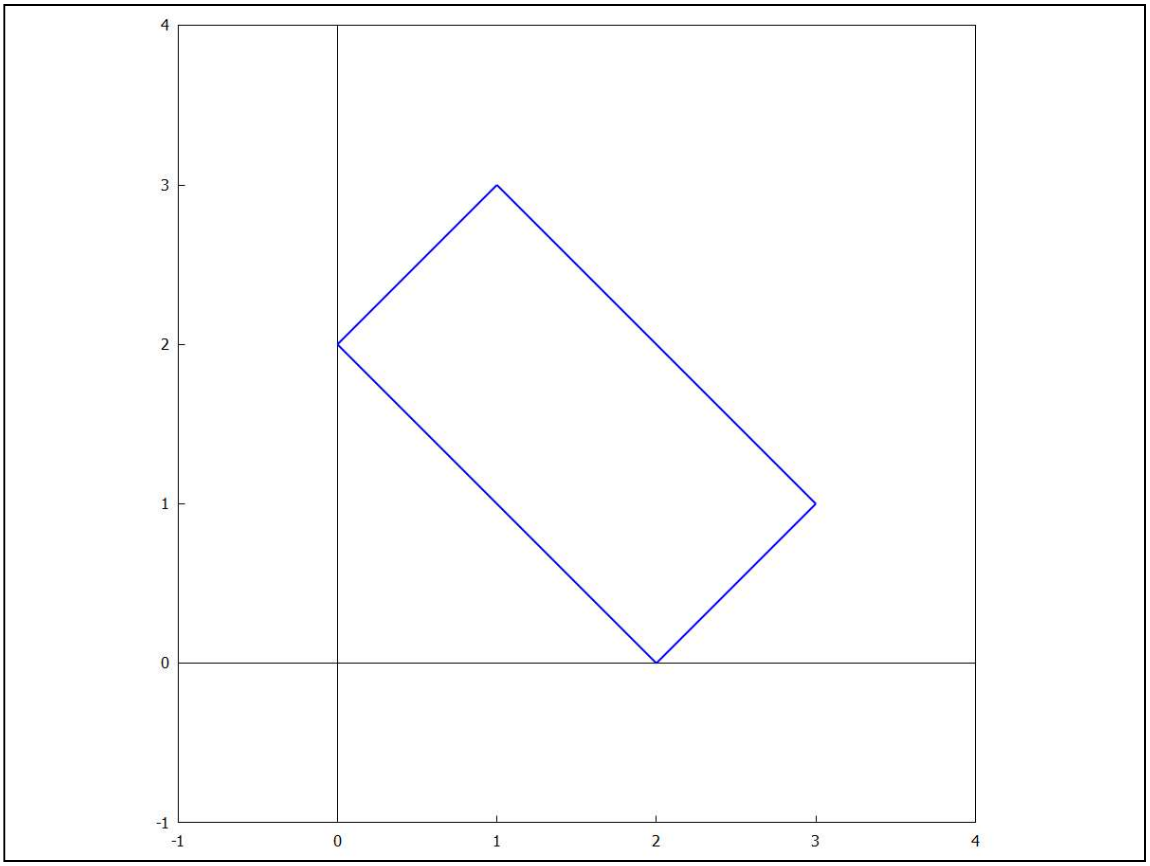



rotating about the origin by π/12

```
theta:%pi/12;
r1(t):=s1(t)·(1+%i)+2;
r2(t):=s2(t)·(1+%i)+2;
r3(t):=s3(t)·(1+%i)+2;
r4(t):=s4(t)·(1+%i)+2;
```

$\frac{\pi}{12}$ r1(t):= s1(t) (1 + %i) + 2 r2(t):= s2(t) (1 + %i) + 2 r3(t):= s3(t) (1 + %i) + 2 r4(t):= s4(t) (1 + %i) + 2

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−1,4],
yaxis=true,yaxis_type=solid,yrange=[−1,4],
proportional_axes=xy,
nticks=200,
line_width=2,
parametric(realpart(r1(t)),imagpart(r1(t)),t,0,1),
parametric(realpart(r2(t)),imagpart(r2(t)),t,0,2),
parametric(realpart(r3(t)),imagpart(r3(t)),t,0,1),
parametric(realpart(r4(t)),imagpart(r4(t)),t,0,2));
```




# 4 Practical 4
# image of disk,line,halfplane

Let w= f(z)=(3+4i)z - 2 + i
(a) Find the image of disk |z-1|<1
(b) Find the image of the line x=t , y=1-2t for -∞<t<∞
(c) Find the image of the half plane Im(z)>1
(d) For part 'a' and 'b' and 'c' sketch the mapping, identify the points z1=0, z2=1-i and z3 = 2, and indicate their images

```
kill(all)$
f(z):=block(
[x,y],
x:realpart(z),
y:imagpart(z),
w:rectform((3+4·%i)·(x+y·%i)+(−2+%i)));
```

f(z):= block([x,y],x:realpart(z),y:imagpart(z),w: rectform((3+4 %i) (x+y %i)+(−2+%i)))

```
c:f(1);
```

5 %i + 1

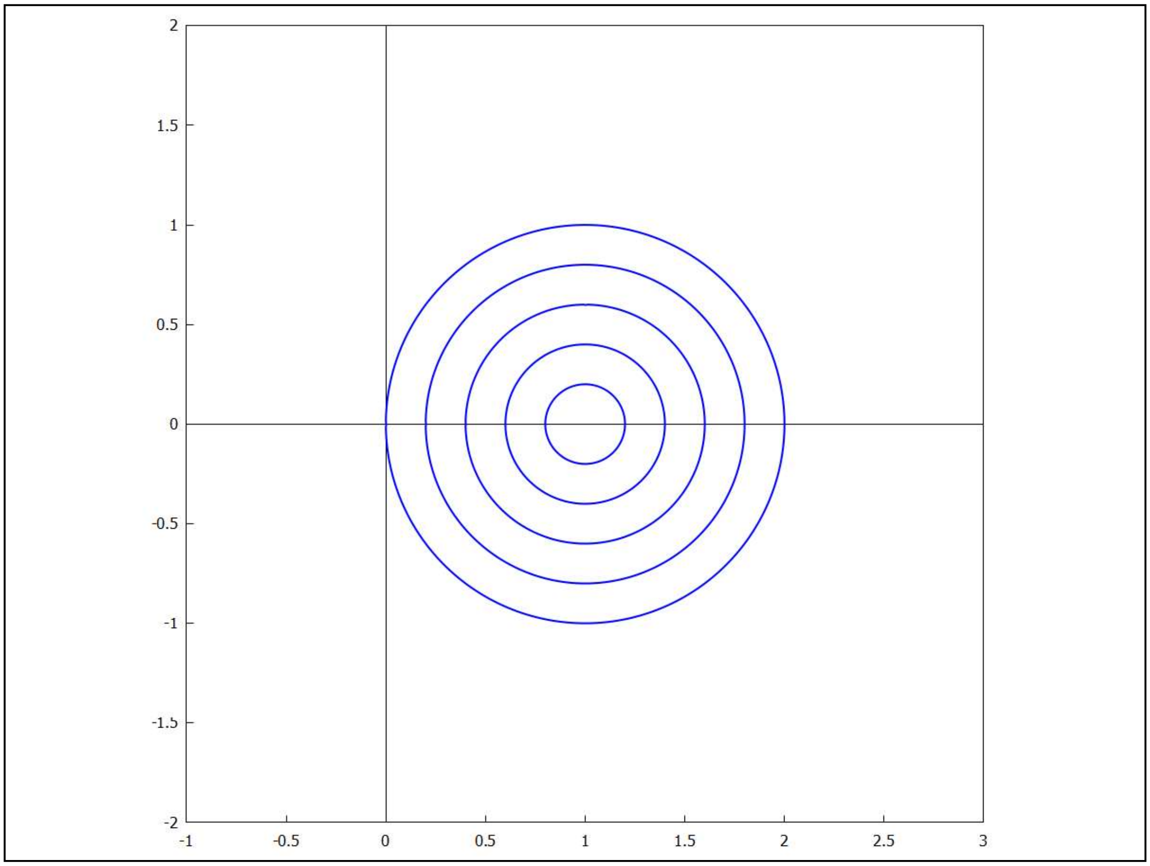
```
r(t,s):=1+s·(cos(t)+%i·sin(t));
```

$r(t,s):=1+s\,(\cos(t)+\%i\,\sin(t))$

```
zdomain:makelist(parametric(realpart(r(t,s)), imagpart(r(t,s)),t,0,2·%pi),s,0,1,1/5);
```

$$[\mathrm{parametric}(1,0,t,0,2\,\pi),$$
$$\mathrm{parametric}\left(\frac{\cos(t)}{5}+1,\frac{\sin(t)}{5},t,0,2\,\pi\right),$$
$$\mathrm{parametric}\left(\frac{2\,\cos(t)}{5}+1,\frac{2\,\sin(t)}{5},t,0,2\,\pi\right),$$
$$\mathrm{parametric}\left(\frac{3\,\cos(t)}{5}+1,\frac{3\,\sin(t)}{5},t,0,2\,\pi\right),$$
$$\mathrm{parametric}\left(\frac{4\,\cos(t)}{5}+1,\frac{4\,\sin(t)}{5},t,0,2\,\pi\right),$$
$$\mathrm{parametric}(\cos(t)+1,\sin(t),t,0,2\,\pi)]$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−1,3],
yaxis=true,yaxis_type=solid,yrange=[−2,2],
proportional_axes=xy,
line_width=2,
nticks=600,
zdomain);
```




```
w(t,s):=f(r(t,s));
```

$$w(t,s):=f(r(t,s))$$

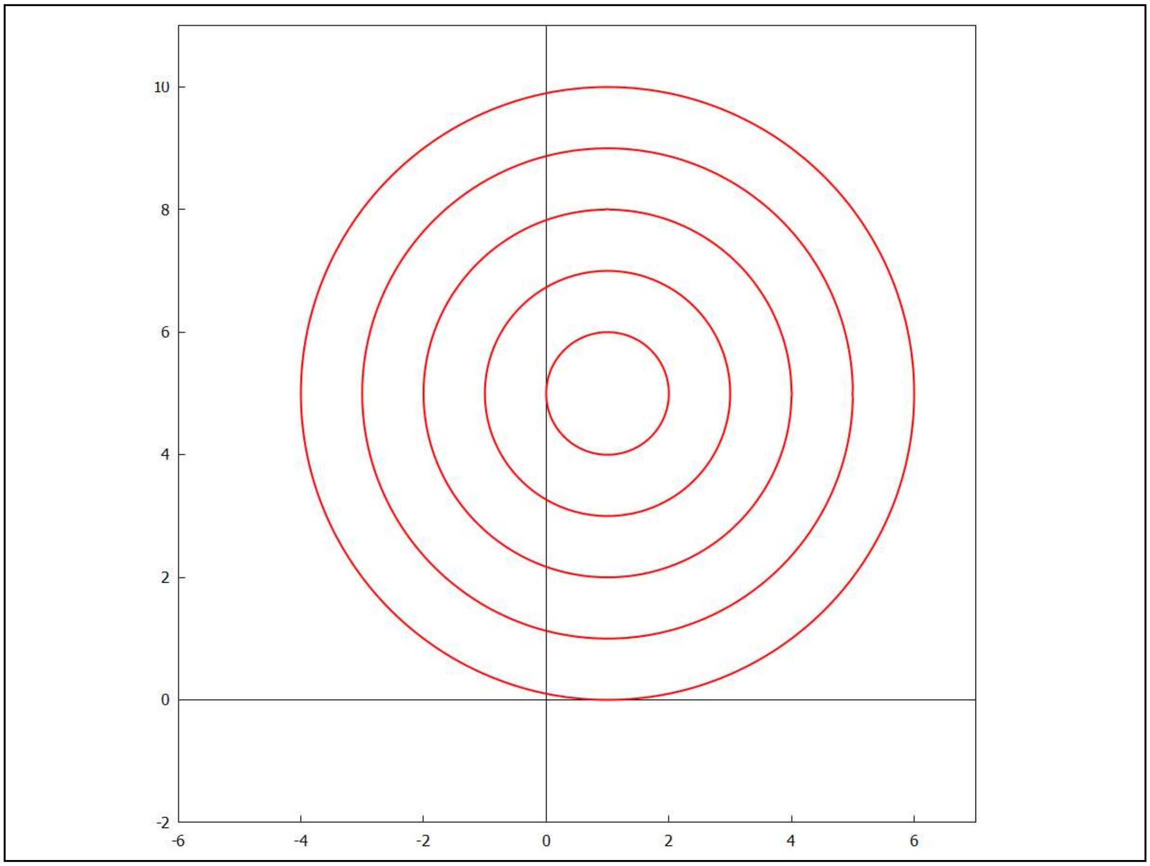
```
wdomain:makelist(parametric(realpart(w(t,s)),imagpart(w(t,s)),t,0,2·%pi),s,0,1,1/5);
```

$$[\mathrm{parametric}(1,5,t,0,2\pi),\mathrm{parametric}\left(-\left(\frac{4\sin(t)}{5}\right)+3\left(\frac{\cos(t)}{5}+1\right)-2,\frac{3\sin(t)}{5}+4\left(\frac{\cos(t)}{5}+1\right)+1,t,0,2\pi\right),\mathrm{parametric}\left(-\left(\frac{8\sin(t)}{5}\right)+3\left(\frac{2\cos(t)}{5}+1\right)-2,\frac{6\sin(t)}{5}+4\left(\frac{2\cos(t)}{5}+1\right)+1,t,0,2\pi\right),\mathrm{parametric}\left(-\left(\frac{12\sin(t)}{5}\right)+3\left(\frac{3\cos(t)}{5}+1\right)-2,\frac{9\sin(t)}{5}+4\left(\frac{3\cos(t)}{5}+1\right)+1,t,0,2\pi\right),\mathrm{parametric}\left(-\left(\frac{16\sin(t)}{5}\right)+3\left(\frac{4\cos(t)}{5}+1\right)-2,\frac{12\sin(t)}{5}+4\left(\frac{4\cos(t)}{5}+1\right)+1,t,0,2\pi\right),\mathrm{parametric}(-(4\sin(t))+3(\cos(t)+1)-2,3\sin(t)+4(\cos(t)+1)+1,t,0,2\pi)]$$

```
cabs(c−f(r(t,s)));
```

$$\sqrt{(4\ s\sin(t)-3\ s\cos(t))^2+(-(3\ s\sin(t))-4\ s\cos(t))^2}$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−6,7],
yaxis=true,yaxis_type=solid,yrange=[−2,11],
proportional_axes=xy,
nticks=600,
line_width=2,
color=red,
wdomain);
```




```
cabs(c−f(r(t,s)));
```

$$\sqrt{(4\ s\sin(t)-3\ s\cos(t))^2+(-(3\ s\sin(t))-4\ s\cos(t))^2}$$

```
trigsimp(%);
```

$$5\left|s\right|$$

```
cabs(c−f(r(t,1)));
```

$$\sqrt{(4\sin(t)-3\cos(t))^2+(-(3\sin(t))-4\cos(t))^2}$$

```
trigsimp(%);
```

$$5$$

```
makelist(cabs(c−f(r(t,s))),s,1/5,1,1/5);
```

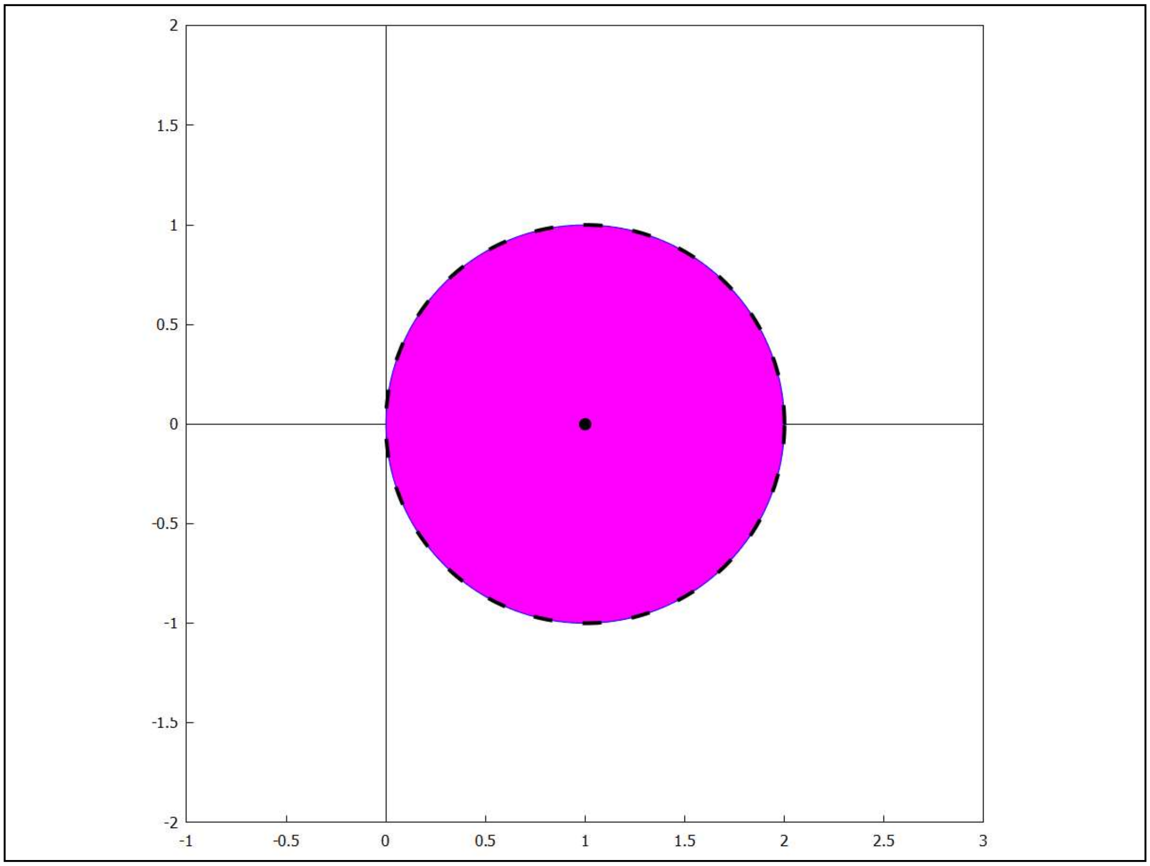
$$[\sqrt{\left(\frac{4\sin(t)}{5}-\frac{3\cos(t)}{5}\right)^2+\left(-\left(\frac{3\sin(t)}{5}\right)-\frac{4\cos(t)}{5}\right)^2},$$
$$\sqrt{\left(\frac{8\sin(t)}{5}-\frac{6\cos(t)}{5}\right)^2+\left(-\left(\frac{6\sin(t)}{5}\right)-\frac{8\cos(t)}{5}\right)^2},$$
$$\sqrt{\left(\frac{12\sin(t)}{5}-\frac{9\cos(t)}{5}\right)^2+\left(-\left(\frac{9\sin(t)}{5}\right)-\frac{12\cos(t)}{5}\right)^2},$$
$$\sqrt{\left(\frac{16\sin(t)}{5}-\frac{12\cos(t)}{5}\right)^2+\left(-\left(\frac{12\sin(t)}{5}\right)-\frac{16\cos(t)}{5}\right)^2},$$
$$\sqrt{(4\sin(t)-3\cos(t))^2+(-(3\sin(t))-4\cos(t))^2}]$$

```
trigsimp(%);
```

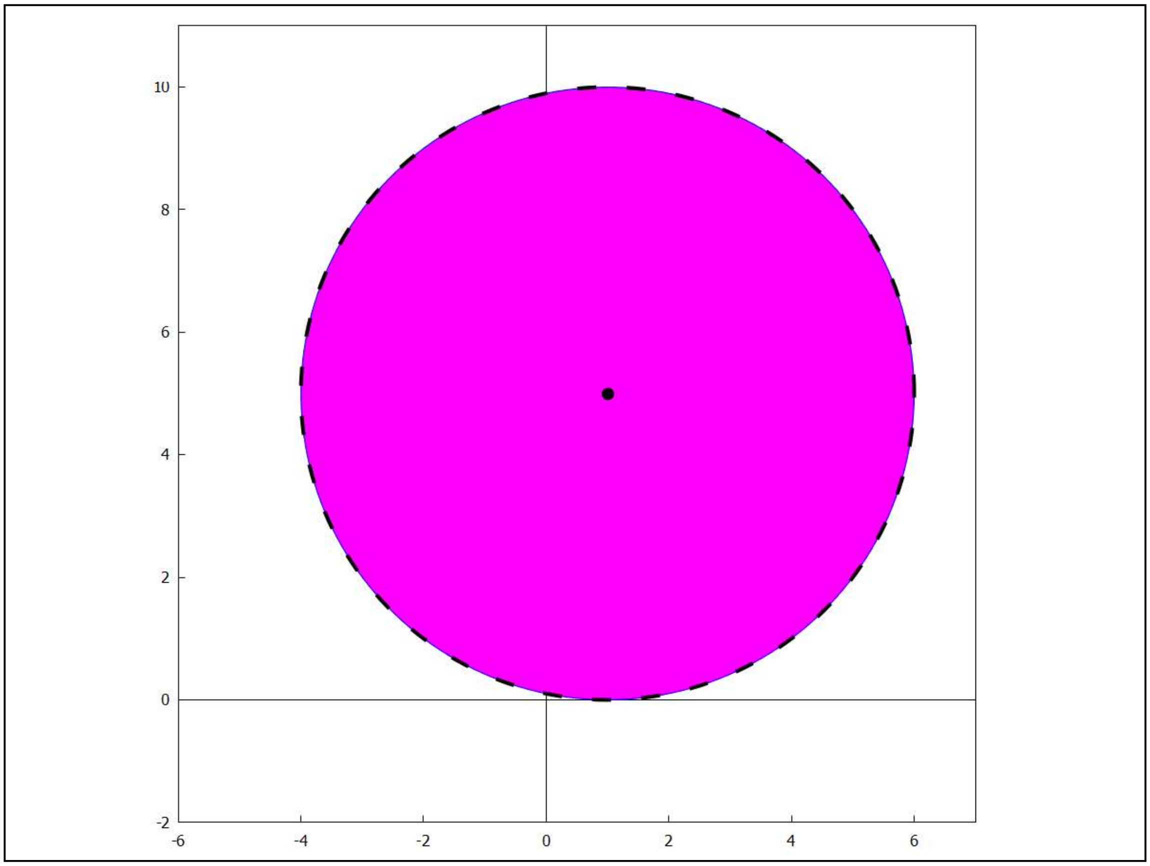
$$[1,2,3,4,5]$$

## 4.1 (a)

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−1,3],
yaxis=true,yaxis_type=solid,yrange=[−2,2],
proportional_axes=xy,
nticks=200,
fill_color=magenta,
ellipse(1,0,1,1,0,360),
color=black,
line_type=dashes,
line_width=4,
parametric(1+cos(t),sin(t),t,0,2·%pi),
point_size=2,
point_type=7,
points([[1,0]]));
```




```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−6,7],
yaxis=true,yaxis_type=solid,yrange=[−2,11],
proportional_axes=xy,
nticks=200,
fill_color=magenta,
ellipse(1,5,5,5,0,360),
color=black,
line_type=dashes,
line_width=4,
parametric(1+5·cos(t),5+5·sin(t),t,0,2·%pi),
point_size=2,
point_type=7,
points([[1,5]]));
```

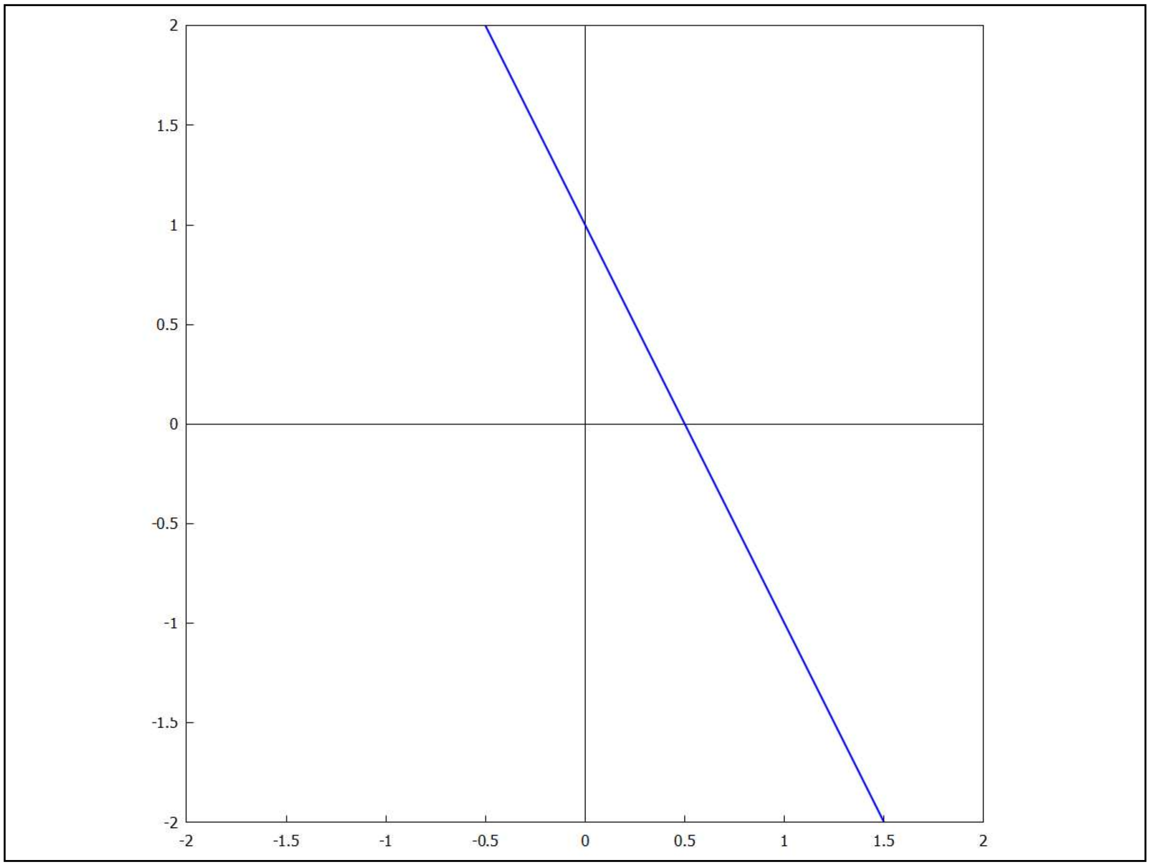



## 4.2 (b)

```
s(t):=t+%i·(1−2·t);
```

$$s(t):=t+\%i\,(1-2\,t)$$

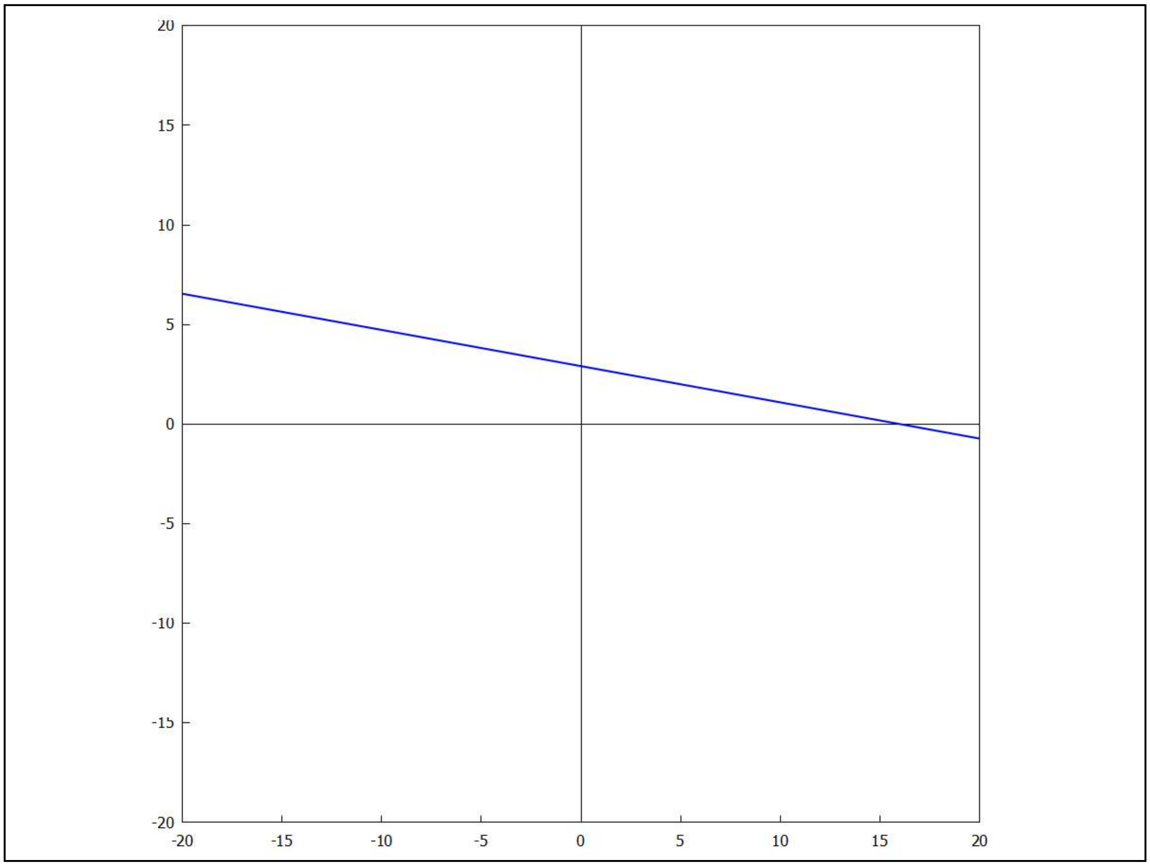
```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−2,2],
yaxis=true,yaxis_type=solid,yrange=[−2,2],
proportional_axes=xy,
nticks=200,
line_width=2,
parametric(realpart(s(t)),imagpart(s(t)),t,−2,2));
```




```
r(t):=s(t)·(3+4·%i)+(−2+%i);
```

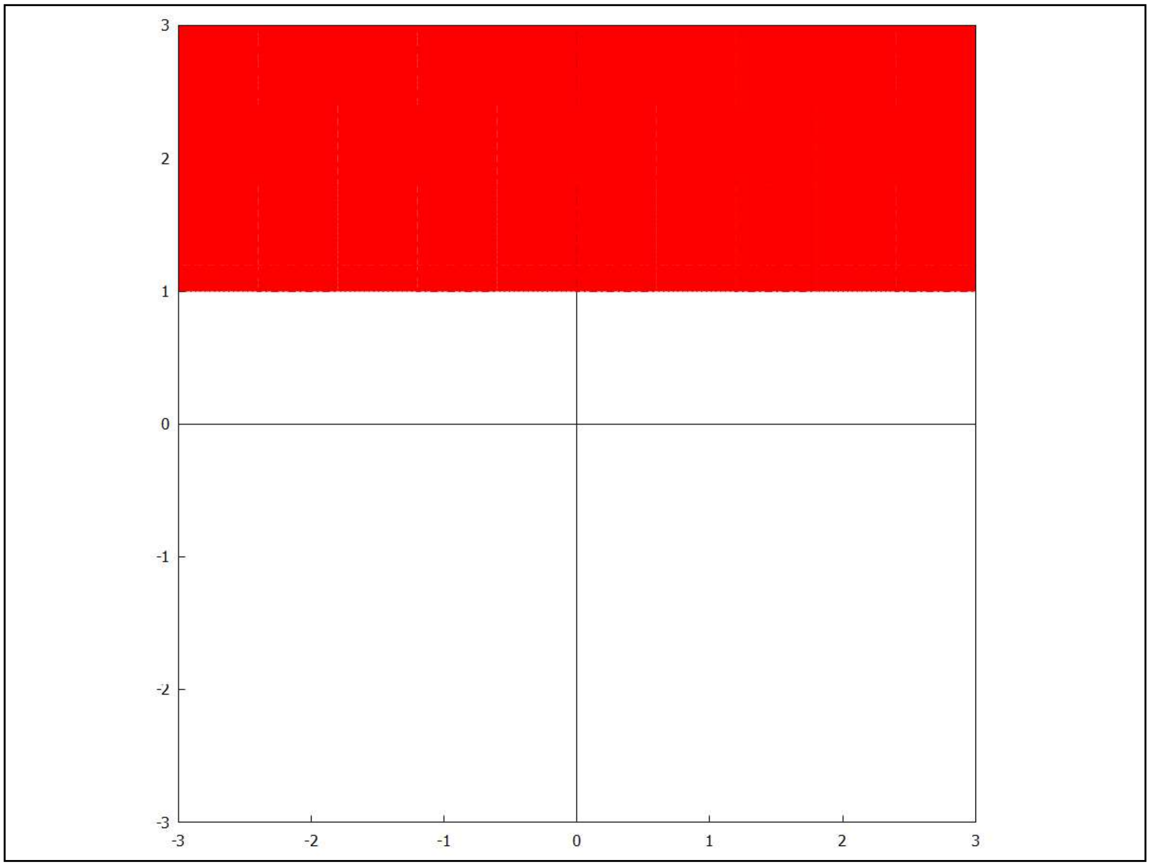
$$r(t):=s(t)\,(3+4\,\%i)+(-2+\%i)$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−20,20],
yaxis=true,yaxis_type=solid,yrange=[−20,20],
proportional_axes=xy,
nticks=200,
line_width=2,
parametric(realpart(r(t)),imagpart(r(t)),t,−3,3));
```

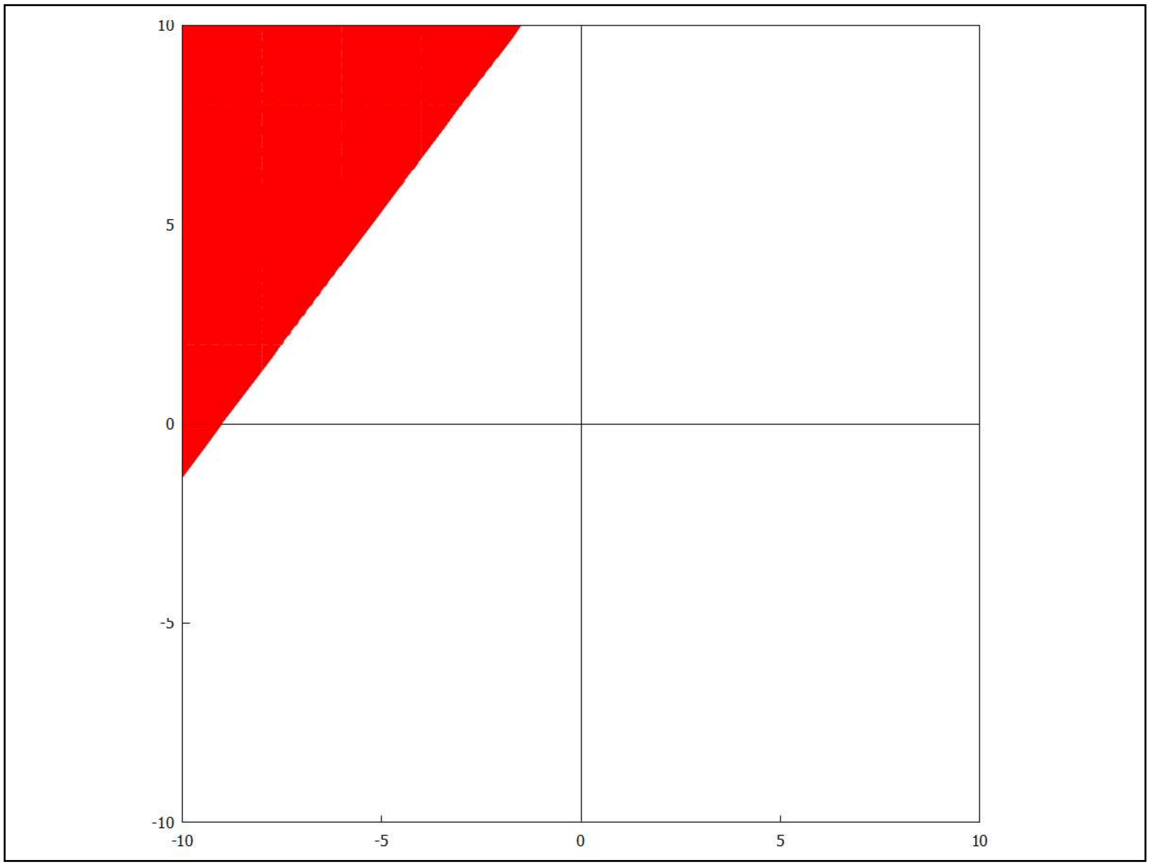



## 4.3 (c)

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−3,3],
yaxis=true,yaxis_type=solid,yrange=[−3,3],
proportional_axes=xy,
region(y>1,x,−3,3,y,−3,3));
```



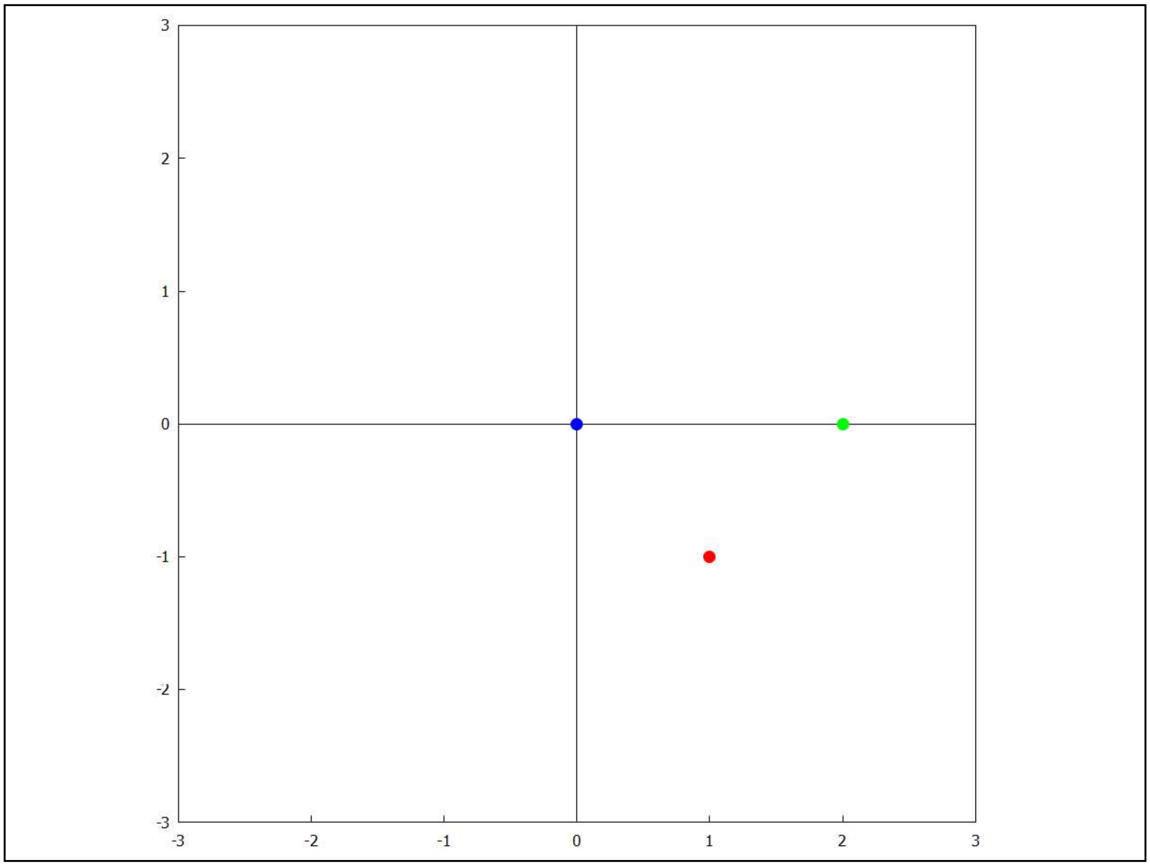

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−10,10],
yaxis=true,yaxis_type=solid,yrange=[−10,10],
proportional_axes=xy,
nticks=200,
line_width=2,
region(3·y>4·x+36,x,−10,10,y,−10,10));
```




```
z1:0;
z2:1−%i;
z3:2;
```

0 1−%i 2

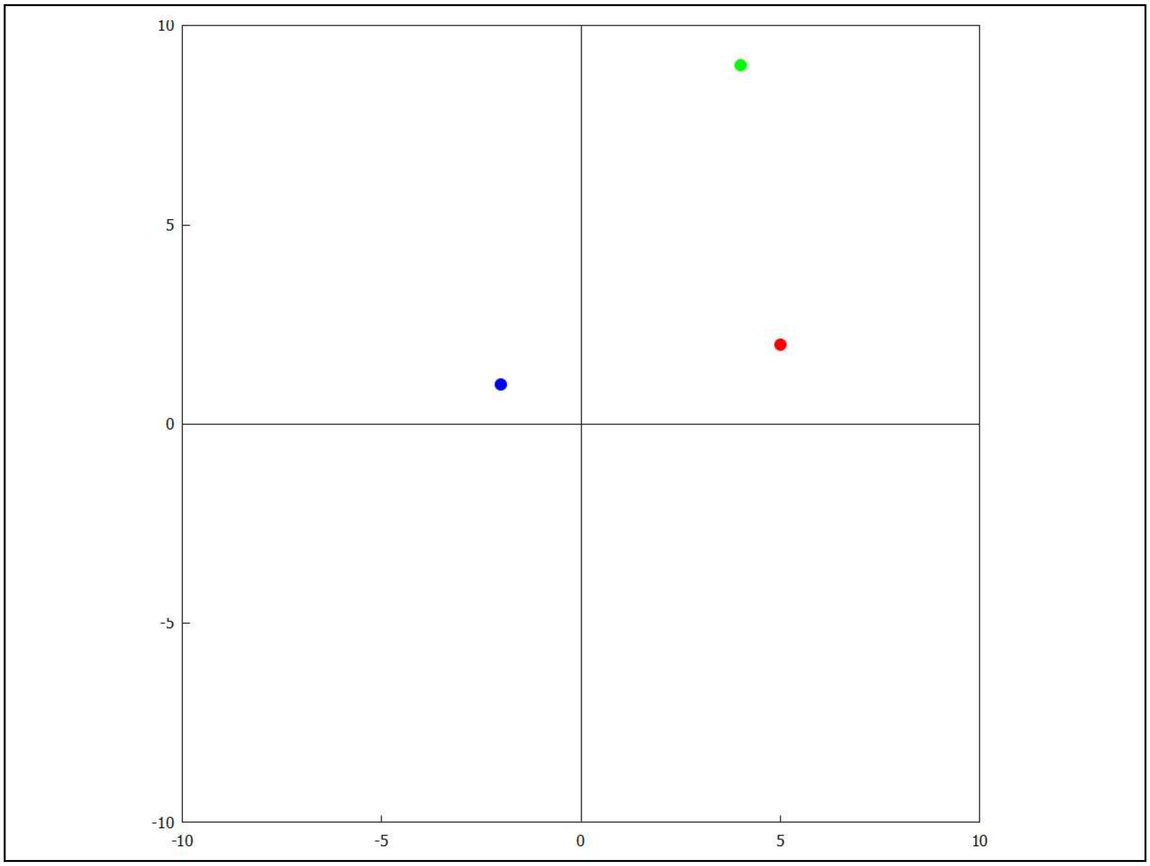
```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−3,3],
yaxis=true,yaxis_type=solid,yrange=[−3,3],
proportional_axes=xy,
point_size=2,
point_type=7,
points([[realpart(z1),imagpart(z1)]]),
color=red,
points([[realpart(z2),imagpart(z2)]]),
color=green,
points([[realpart(z3),imagpart(z3)]]));
```




```
f(z1);
f(z2);
f(z3);
```

$\%i-2$ $2\,\%i+5$ $9\,\%i+4$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−10,10],
yaxis=true,yaxis_type=solid,yrange=[−10,10],
proportional_axes=xy,
point_size=2,
point_type=7,
points([[realpart(f(z1)),imagpart(f(z1))]]),
color=red,
points([[realpart(f(z2)),imagpart(f(z2))]]),
color=green,
points([[realpart(f(z3)),imagpart(f(z3))]]));
```




# 5 Practical 5 image of half plane under linear trans. w=(-1+i)z-2+3i

Show that the linear transformation

w = iz+i

maps the right half plane

Re(z)>1

onto the upper half plane

Im(w)>2

Plot the map

```
kill(all)$
f(z):=block(
[x,y],
x:realpart(z),
y:imagpart(z),
w:rectform(%i·(x+y·%i)+(%i)));
```

$$f(z):=\mathrm{block}([x,y],x:\mathrm{realpart}(z),y:\mathrm{imagpart}(z),w:\mathrm{rectform}(\%i\,(x+y\,\%i)+\%i))$$

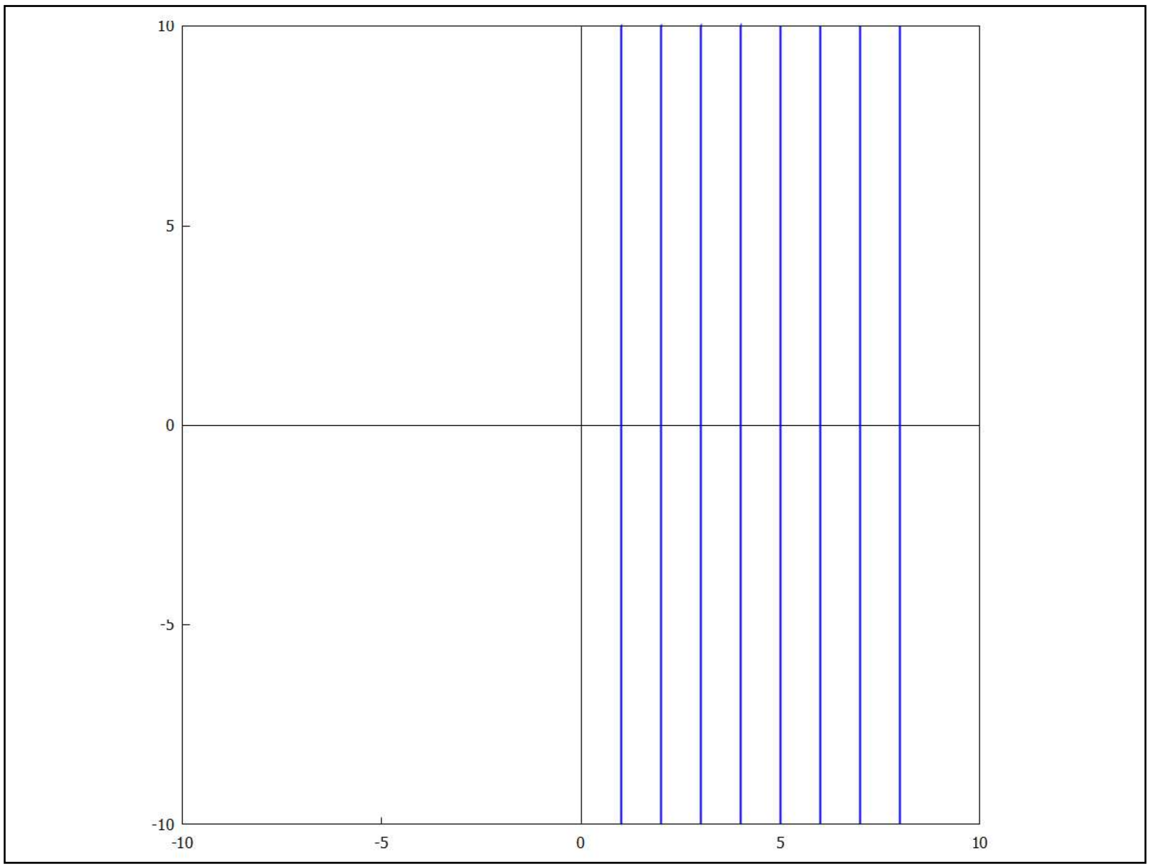
```
r(t,s):=(s+%i·t);
```

r(t,s):=s+ %i t

```
zdomain:makelist(parametric(realpart(r(t,s)), imagpart(r(t,s)),t,−10,10),s,1,8,1);
```

[parametric(1,t,t,−10,10),parametric(2,t,t,−10,10),
parametric(3,t,t,−10,10),parametric(4,t,t,−10,10),
parametric(5,t,t,−10,10),parametric(6,t,t,−10,10),
parametric(7,t,t,−10,10),parametric(8,t,t,−10,10)]

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−10,10],
yaxis=true,yaxis_type=solid,yrange=[−10,10],
proportional_axes=xy,
line_width=2,
nticks=600,
zdomain);
```




```
w(t, s):=f(r(t, s));
```

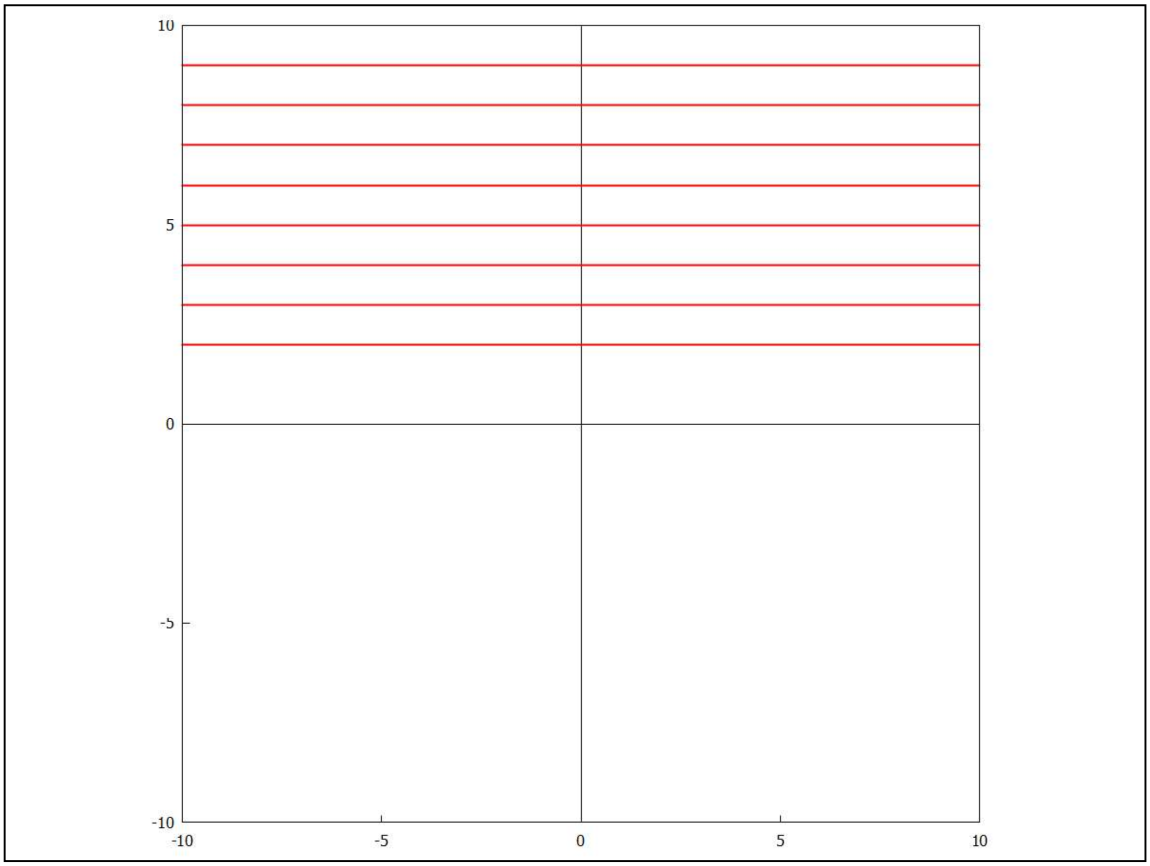
w(t,s):=f(r(t,s))

```
wdomain:makelist(parametric(realpart(w(t,s)),imagpart(w(t,s)),t,−10,10),s,1,8,1);
```

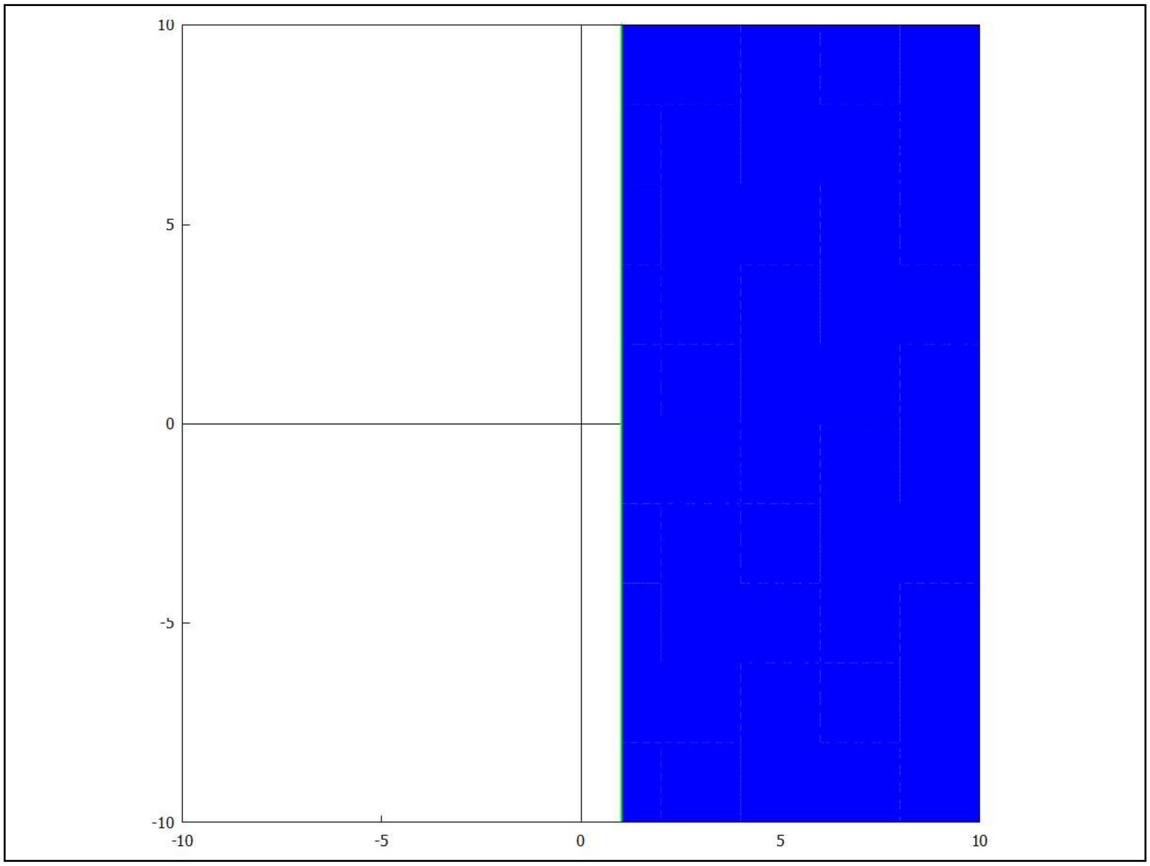
[parametric(−t,2,t,−10,10),parametric(−t,3,t,−10,10),

```
parametric(- t,4 ,t,- 10 ,10 ),parametric(- t,5 ,t,- 10 ,10 ),
parametric(- t,6 ,t,- 10 ,10 ),parametric(- t,7 ,t,- 10 ,10 ),
parametric(- t,8 ,t,- 10 ,10 ),parametric(- t,9 ,t,- 10 ,10 )]
```

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−10,10],
yaxis=true,yaxis_type=solid,yrange=[−10,10],
proportional_axes=xy,
nticks=600,
line_width=2,
color=red,
wdomain);
```



```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−10,10],
yaxis=true,yaxis_type=solid,yrange=[−10,10],
proportional_axes=xy,
fill_color=blue,
region(x>1,x,−10,10,y,−10,10),
line_width=2,
color=green,
parametric(1,t,t,−10,10));
```




```
W:u+%i·v;
```

$$\%i\ v+u$$

```
sol:solve(W=f(z),z);
```

$$[z=v-\%i\ u-1]$$

```
sol[1];
```

$$z=v-\%i\ u-1$$

```
q:rhs(sol[1]);
```

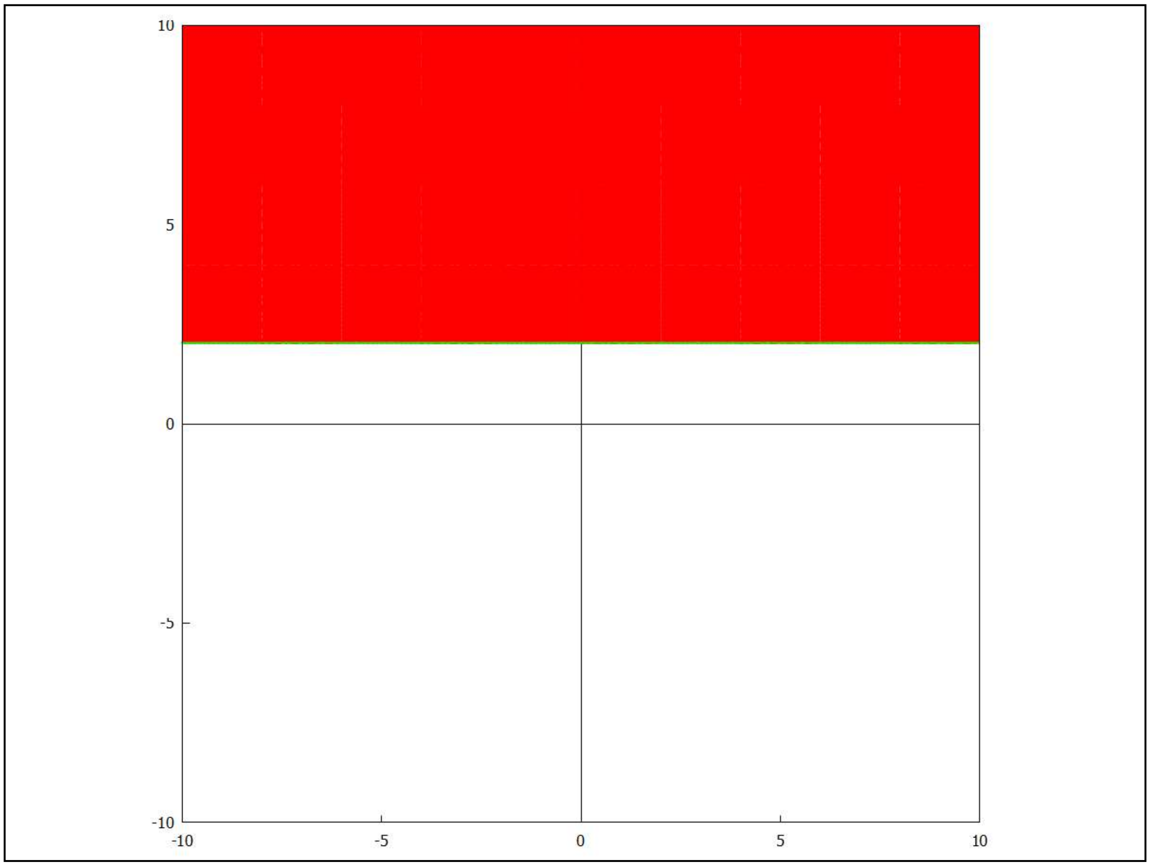
$$v-\%i\ u-1$$

```
realpart(q)>1;
```

$$v-1>1$$

```
eq:realpart(q)=1;
        v-1=1
```

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[-10,10],
yaxis=true,yaxis_type=solid,yrange=[-10,10],
proportional_axes=xy,
region(realpart(q)>1,u,-10,10,v,-10,10),
line_width=2,
color=green,
implicit(eq,u,-10,10,v,-10,10));
```




# 6 Practical 6

# Image of right half plane under w=f(z)=1/z:{Re z <-1/2}

```
kill(all)$
f(z):=block(
[x,y],
x:realpart(z),
y:imagpart(z),
w:rectform(1/(x+y·%i)));
```

f(z):=block

$$\left([x,y],x:\mathrm{realpart}(z),y:\mathrm{imagpart}(z),w:\mathrm{rectform}\left(\frac{1}{x+y\,\%i}\right)\right)$$

```
f(1);
```

$$1$$

```
f(%i);
```

$$-\%i$$

```
f(1+%i);
```

$$\frac{1}{2}-\frac{\%i}{2}$$

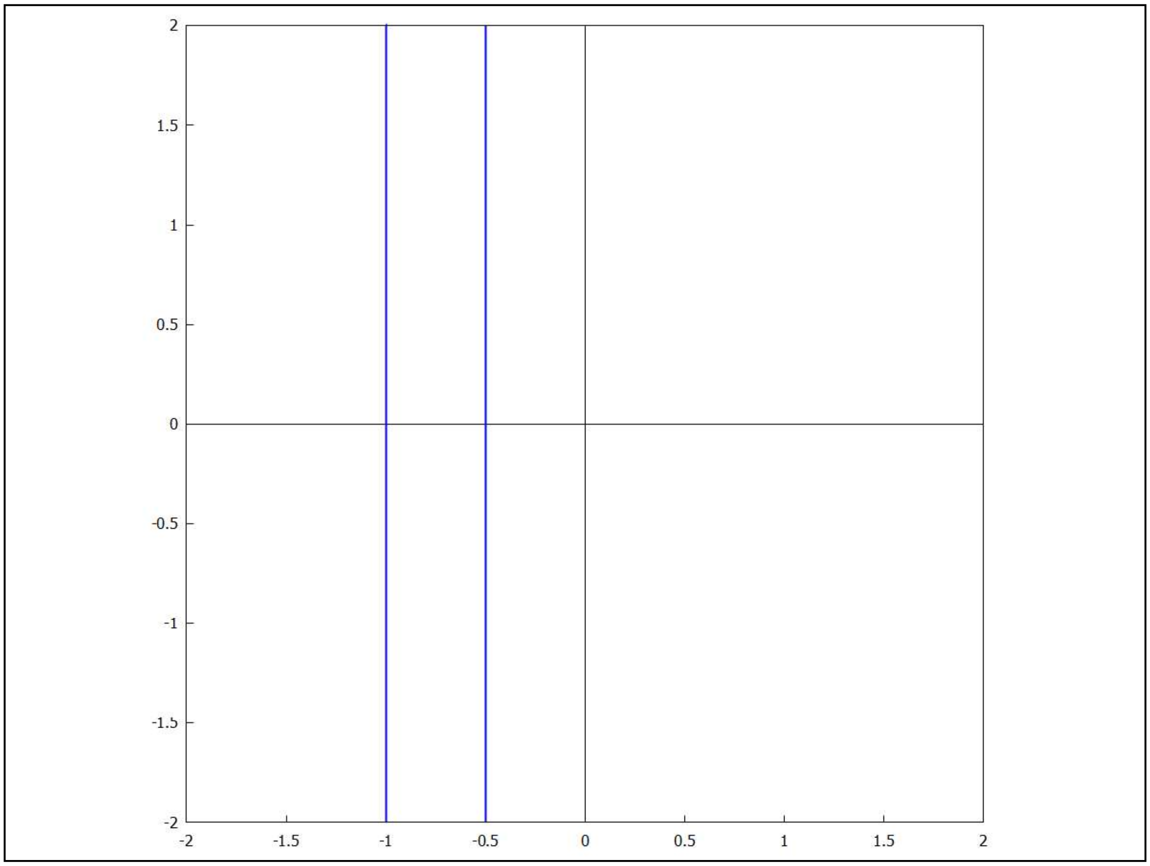
```
r(t,s):=(s+%i·t);
```

$$r(t,s):=s+\%i\ t$$

```
zdomain:makelist(parametric(realpart(r(t,s)),
   imagpart(r(t,s)),t,−2,2),s,−1,−0.5,0.5);
```

$$[\mathrm{parametric}(-1,t,t,-2,2),\mathrm{parametric}(-0.5,t,t,-2,2)]$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−2,2],
yaxis=true,yaxis_type=solid,yrange=[−2,2],
proportional_axes=xy,
line_width=2,
nticks=600,
zdomain);
```



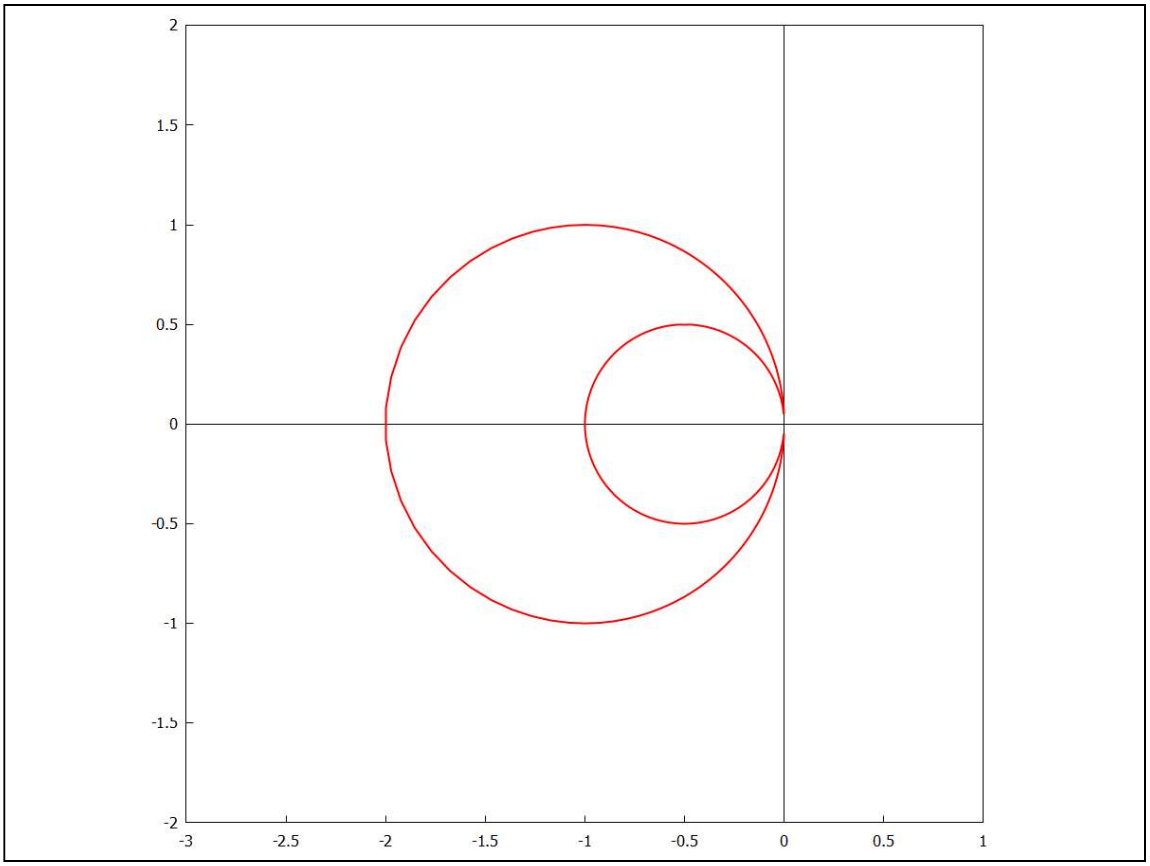

```
w(t,s):=f(r(t,s));
wdomain:makelist(parametric(realpart(w(t,s)),
  imagpart(w(t,s)),t,−20,20),s,−1,−0.5,0.5);
```

$$w(t,s):=f(r(t,s)) \qquad [$$

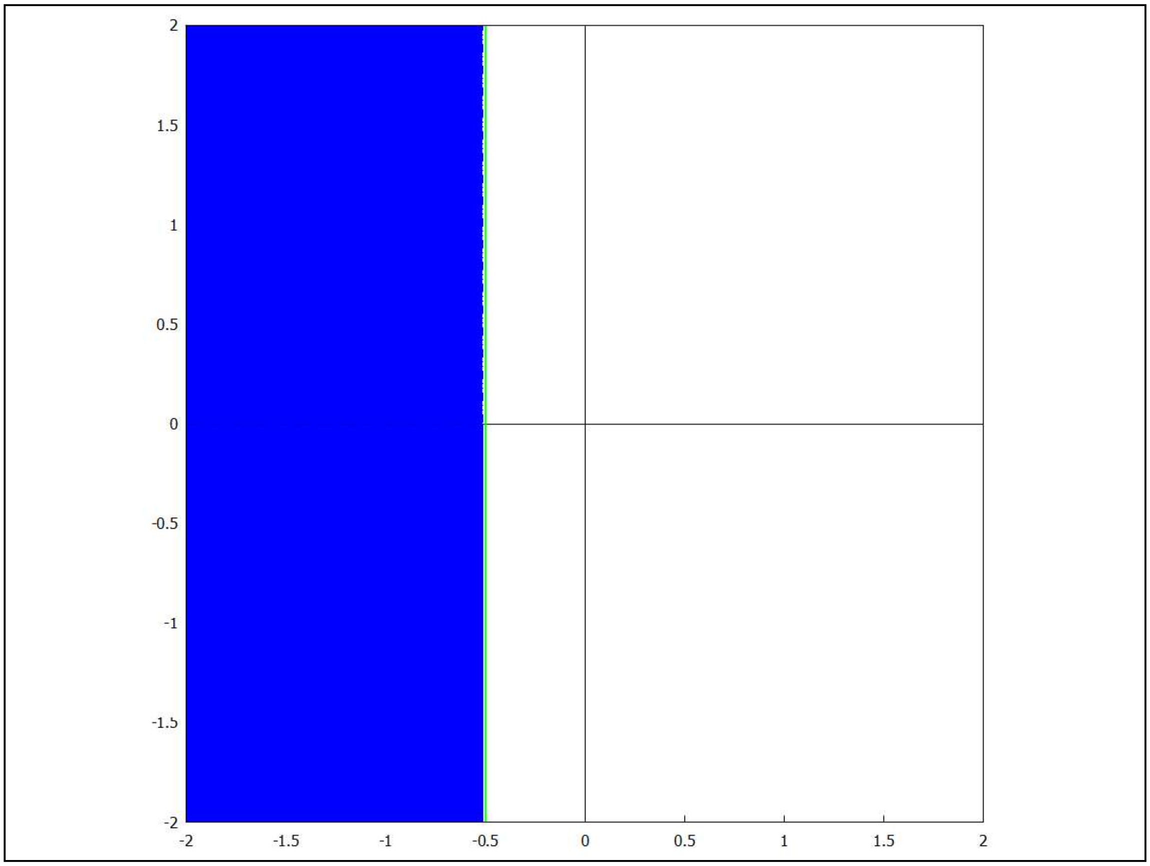
$$\mathrm{parametric}\left(-\left(\frac{1}{t^2+1}\right),-\left(\frac{t}{t^2+1}\right),t,-20,20\right),$$

$$\mathrm{parametric}\left(-\left(\frac{0.5}{t^2+0.25}\right),-\left(\frac{t}{t^2+0.25}\right),t,-20,20\right)]$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−3,1],
yaxis=true,yaxis_type=solid,yrange=[−2,2],
proportional_axes=xy,
line_width=2,
nticks=1000,
color=red,
wdomain);
```




```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−2,2],
yaxis=true,yaxis_type=solid,yrange=[−2,2],
proportional_axes=xy,
fill_color=blue,
region(x<−1/2,x,−10,10,y,−10,10),
line_width=2,
color=green,
parametric(−1/2,t,t,−10,10));
```




**W:u+%i·v;**

**sol:solve(W=f(z),z);**

$$\%i\ v+u \qquad \left[z=\frac{1}{\%i\ v+u}\right]$$

**sol[1];**

$$z=\frac{1}{\%i\ v+u}$$

**q:rhs(sol[1]);**

$$\frac{1}{\%i\ v+u}$$

**realpart(q)<−1/2;**

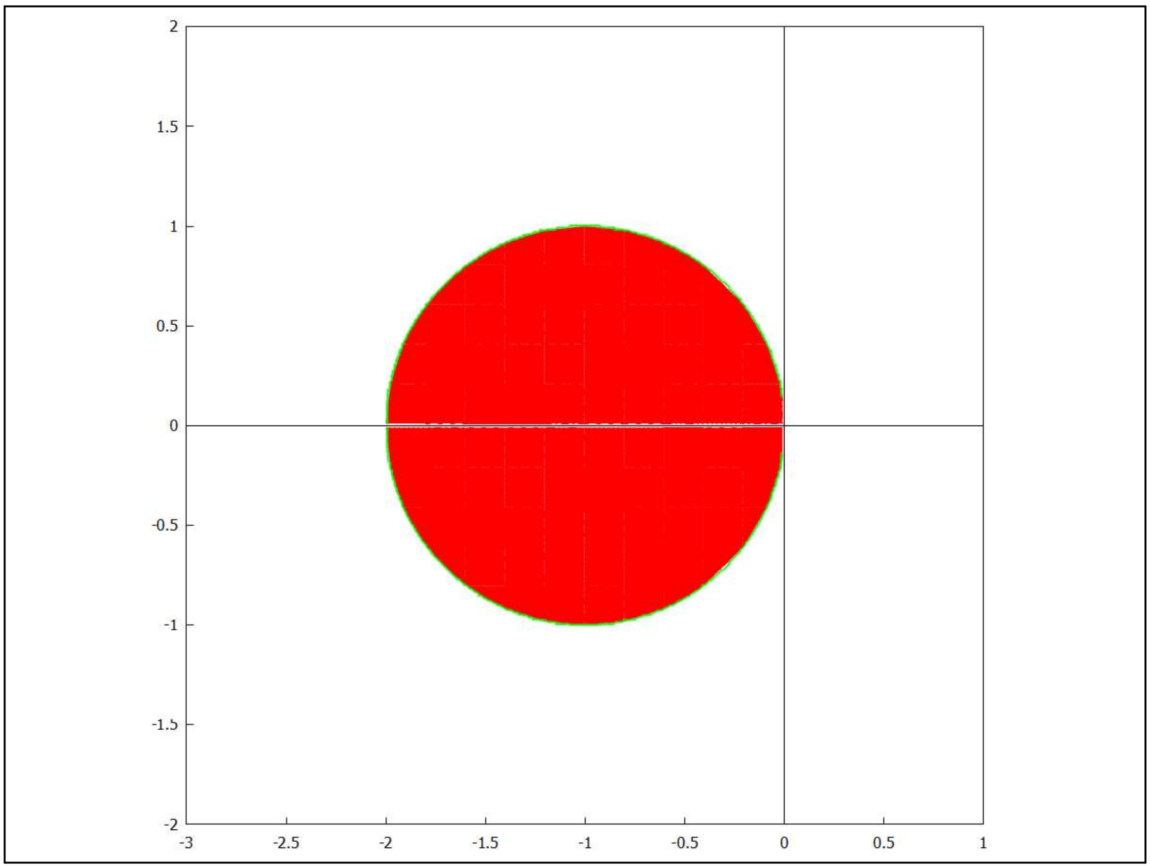
$$\frac{u}{v^2+u^2}<-\left(\frac{1}{2}\right)$$

**eq:realpart(q)=−1/2;**

$$\frac{u}{v^2+u^2}=-\left(\frac{1}{2}\right)$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−3,1],
yaxis=true,yaxis_type=solid,yrange=[−2,2],
proportional_axes=xy,

region(realpart(q)<−1/2,u,−2,−1/100,v,1/100,2),
region(realpart(q)<−1/2,u,−2,−1/100,v,−2,−1/100),
line_width=2,
color=green,
implicit(eq,u,−2,−1/100,v,1/100,2),
implicit(eq,u,−2,−1/100,v,−2,−1/100));
```




# 7 Practical 7

# Plot of vertical line x=a , a=-1,-1/2,1/2,1 and horizontal lines. Plot the grid under the map f(z)=1/z

```
kill(all)$
f(z):=block(
[x,y],
x:realpart(z),
y:imagpart(z),
w:rectform(1/(x+y·%i)));
```

$$f(z):=block\left([x,y],x:realpart(z),y:imagpart(z),w:rectform\left(\frac{1}{x+y\,\%i}\right)\right)$$

```
r(t,s):=(t+%i·s);
```

$$r(t,s):=t+\%i\,s$$

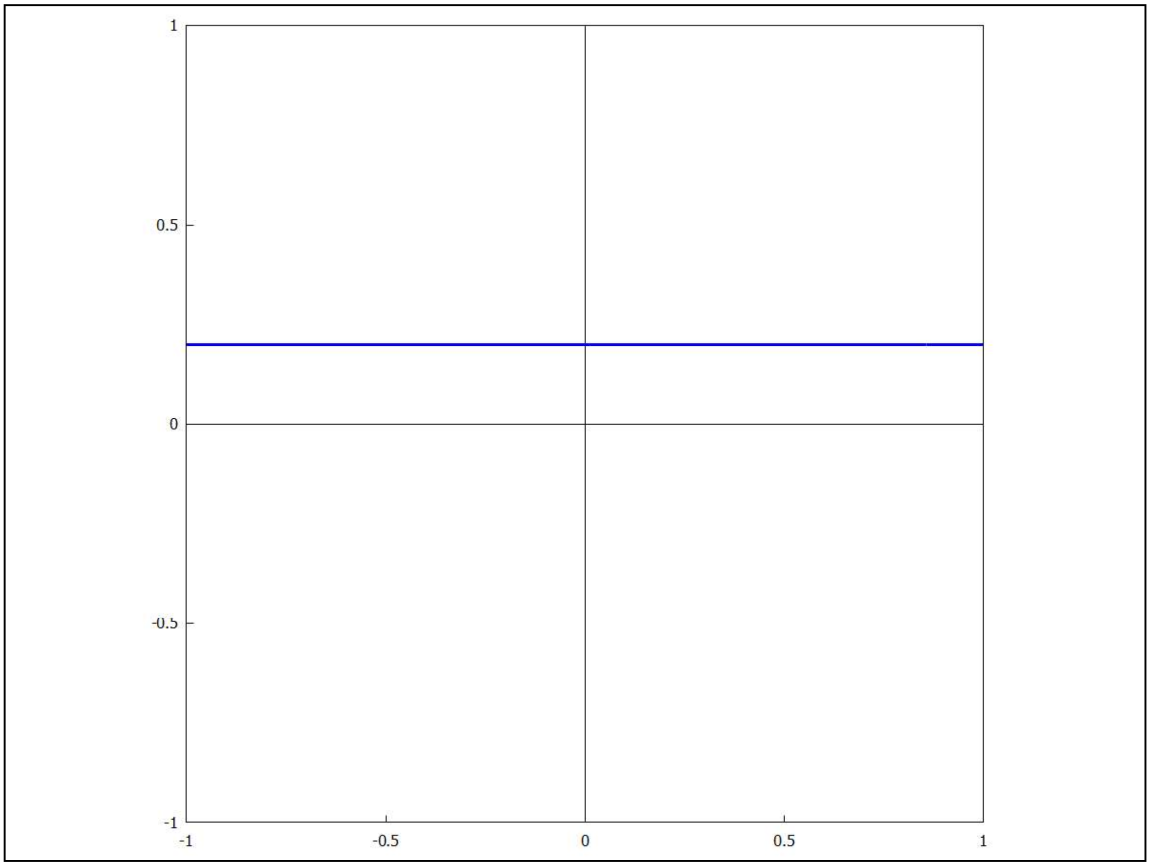
```
r(t,s):=(-%i/2)+s*(cos(t)+%i*sin(t));
```

```
zdomain:parametric(realpart(r(t,1/5)),imagpart(r(t,1/5)),t,−3,3);
```

$$parametric\left(t,\frac{1}{5},t,-3,3\right)$$

```
zdomain:parametric(realpart(r(t,1/2)),imagpart(r(t,1/2)),t,0,2*%pi);
```

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−1,1],
yaxis=true,yaxis_type=solid,yrange=[−1,1],
proportional_axes=xy,
line_width=3,
zdomain);
```



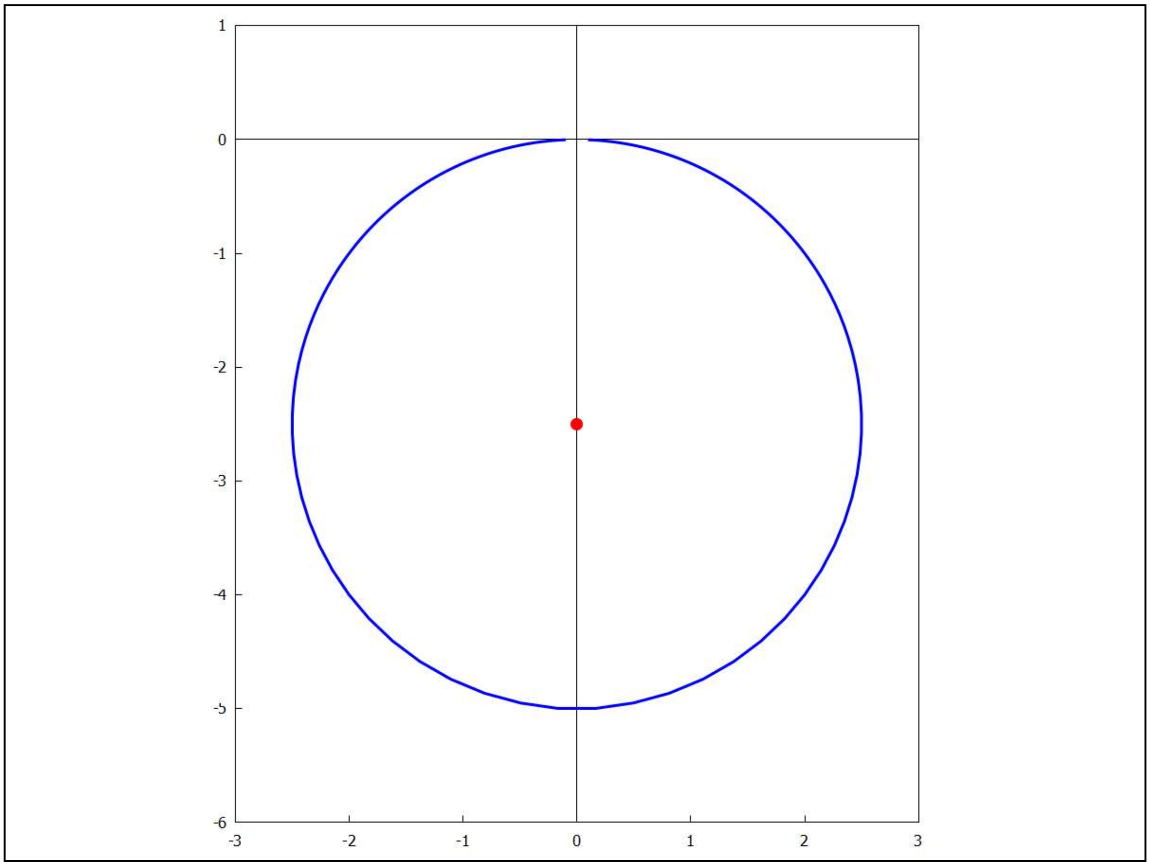

```
w(t,s):=f(r(t,s));
```

$$w(t,s):=f(r(t,s))$$

```
wdomain:parametric(realpart(w(t,1/5)),imagpart(w(t,1/5)),t,−10,10);
```

$$\mathrm{parametric}\left(\frac{t}{t^2+\frac{1}{25}},-\left(\frac{1}{5\left(t^2+\frac{1}{25}\right)}\right),t,-10,10\right)$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−3,3],
yaxis=true,yaxis_type=solid,yrange=[−6,1],
proportional_axes=xy,
nticks=1500,
line_width=3,
wdomain,
color=red,
point_type=7,
point_size=2,
points([[0,−5/2]]));
```




# 8 Practical -8 Parametrization of polygon path/ Contours

## 8.1

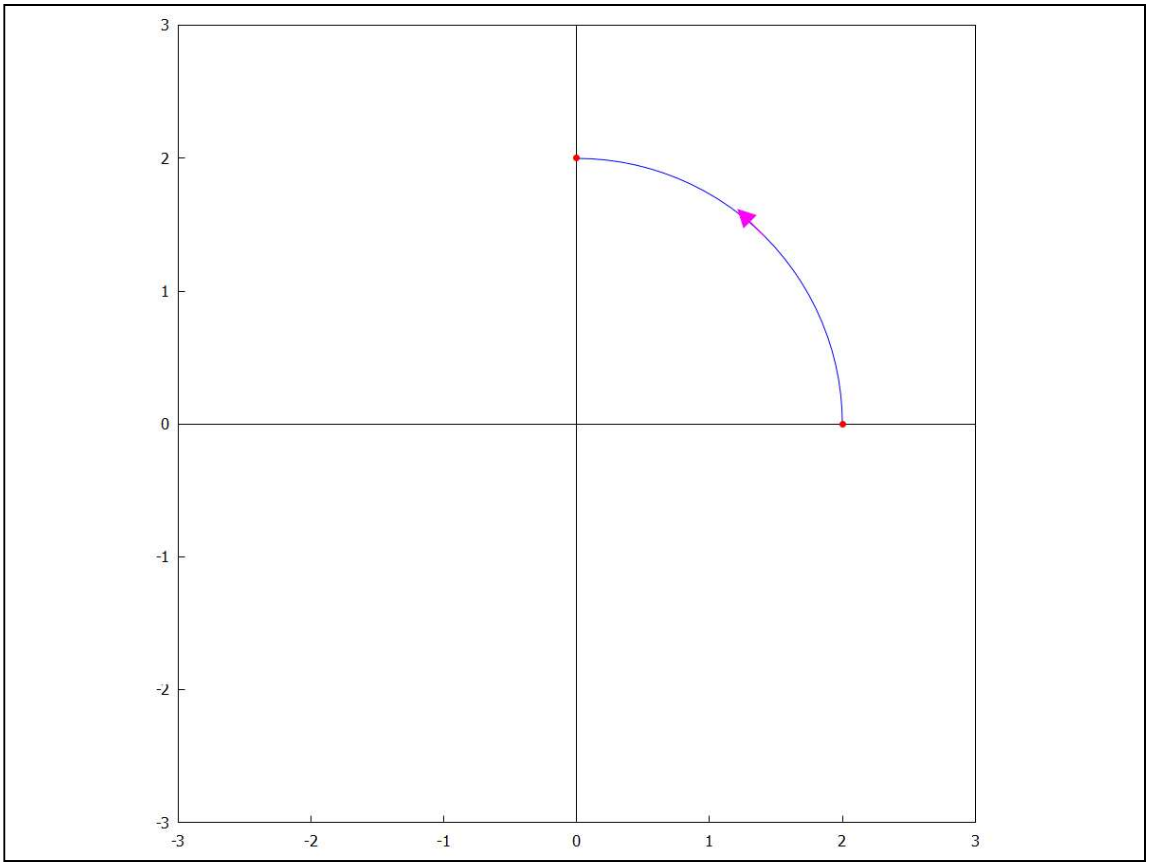
Give a parametrization of the contour C1+C2 and make a plot of this path.

### 8.1.1 C1

```
kill(all)$
z1(t):=2·cos(t)+%i·2·sin(t);
```

$$z1(t):=2\cos(t)+\%i\ 2\sin(t)$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−3,3],
yaxis=true,yaxis_type=solid,yrange=[−3,3],
proportional_axes=xy,
parametric(realpart(z1(t)),imagpart(z1(t)),t,0,%pi/2),
head_length=0.2,
head_angle=20,
color=magenta,
vector([2/sqrt(2),2/sqrt(2)],[−0.2,0.2]),
color=red,
point_type=7,
point_size=1,
points([[2,0],[0,2]]));
```

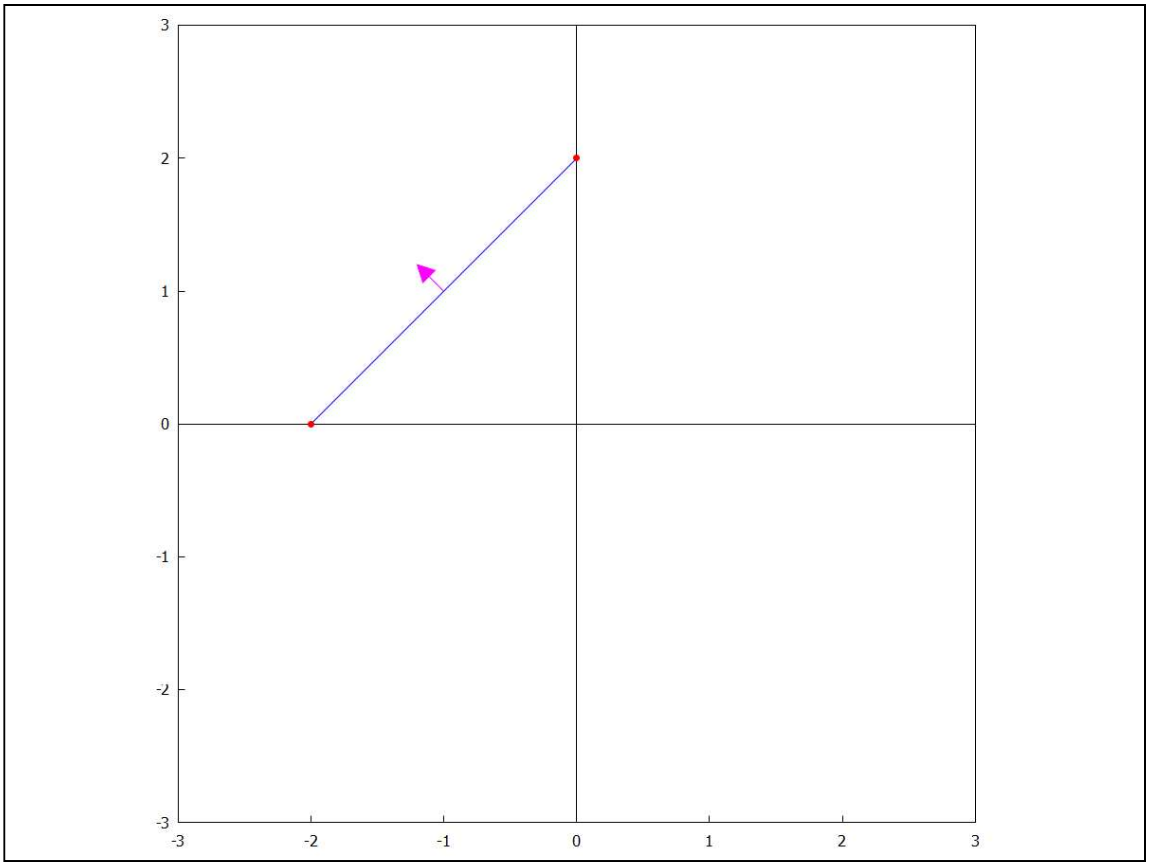



### 8.1.2 C2

```
z2(t):=(−2·t)+%i·((−2·t)+2);
```

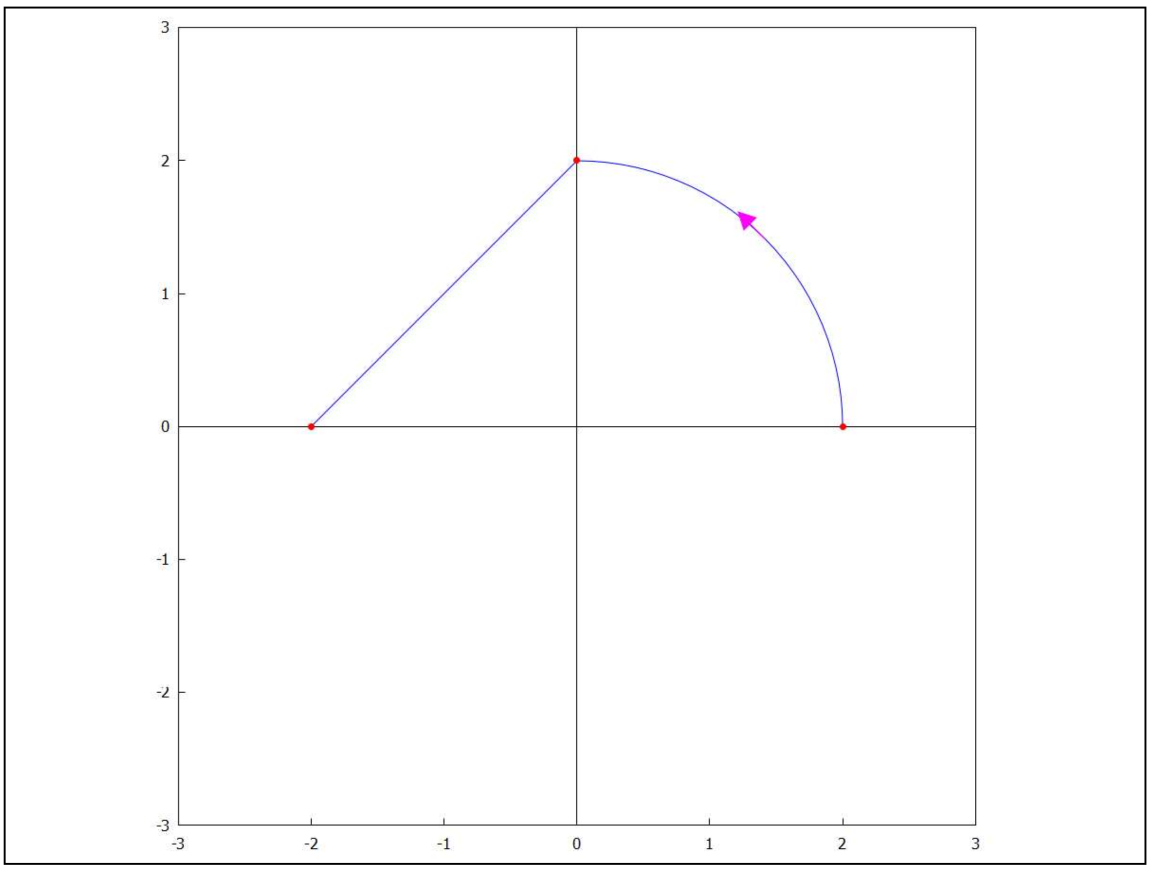
$$z2(t):=-2\,t+\%i\,(-2\,t+2)$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−3,3],
yaxis=true,yaxis_type=solid,yrange=[−3,3],
proportional_axes=xy,
parametric(realpart(z2(t)),imagpart(z2(t)),t,0,1),
head_length=0.2,
head_angle=20,
color=magenta,
vector([−1,1],[−0.2,0.2]),
color=red,
point_type=7,
point_size=1,
points([[−2,0],[0,2]]));
```




### 8.1.3 C1+C2

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−3,3],
yaxis=true,yaxis_type=solid,yrange=[−3,3],
proportional_axes=xy,
parametric(realpart(z1(t)),imagpart(z1(t)),t,0,%pi/2),
parametric(realpart(z2(t)),imagpart(z2(t)),t,0,1),
head_length=0.2,
head_angle=20,
color=magenta,
vector([2/sqrt(2),2/sqrt(2)],[−0.2,0.2]),
color=red,
point_type=7,
point_size=1,
points([[2,0],[0,2],[−2,0]]));
```




## 8.2

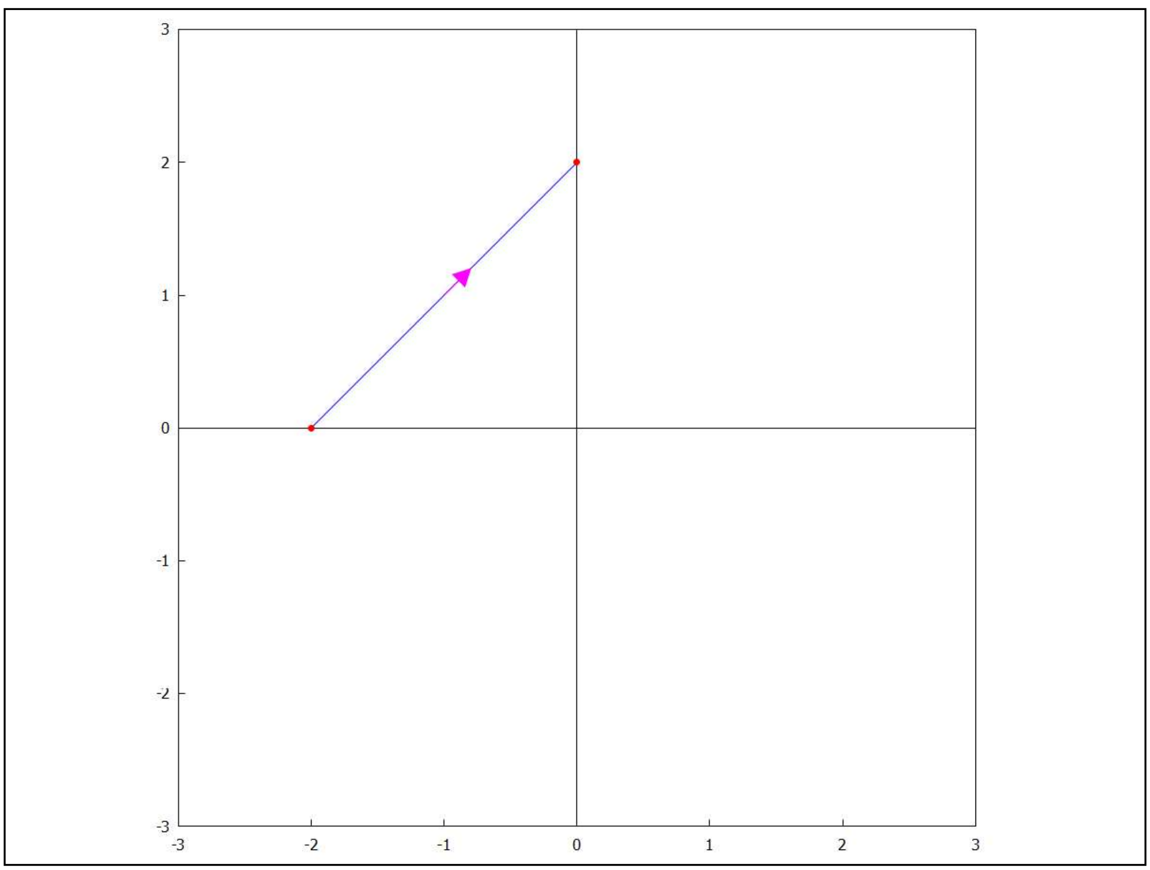
Give a parametrization of the contour C1+C2+C3 and make a plot of this path.

### 8.2.1 C1

```
kill(all)$
z1(t):=t+%i·(t+2);
```

$$z1\left(t\right):=t+\%i\left(t+2\right)$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−3,3],
yaxis=true,yaxis_type=solid,yrange=[−3,3],
proportional_axes=xy,
parametric(realpart(z1(t)),imagpart(z1(t)),t,−2,0),
head_length=0.2,
head_angle=20,
color=magenta,
vector([−1,1],[0.2,0.2]),
color=red,
point_type=7,
point_size=1,
points([[−2,0],[0,2]]));
```

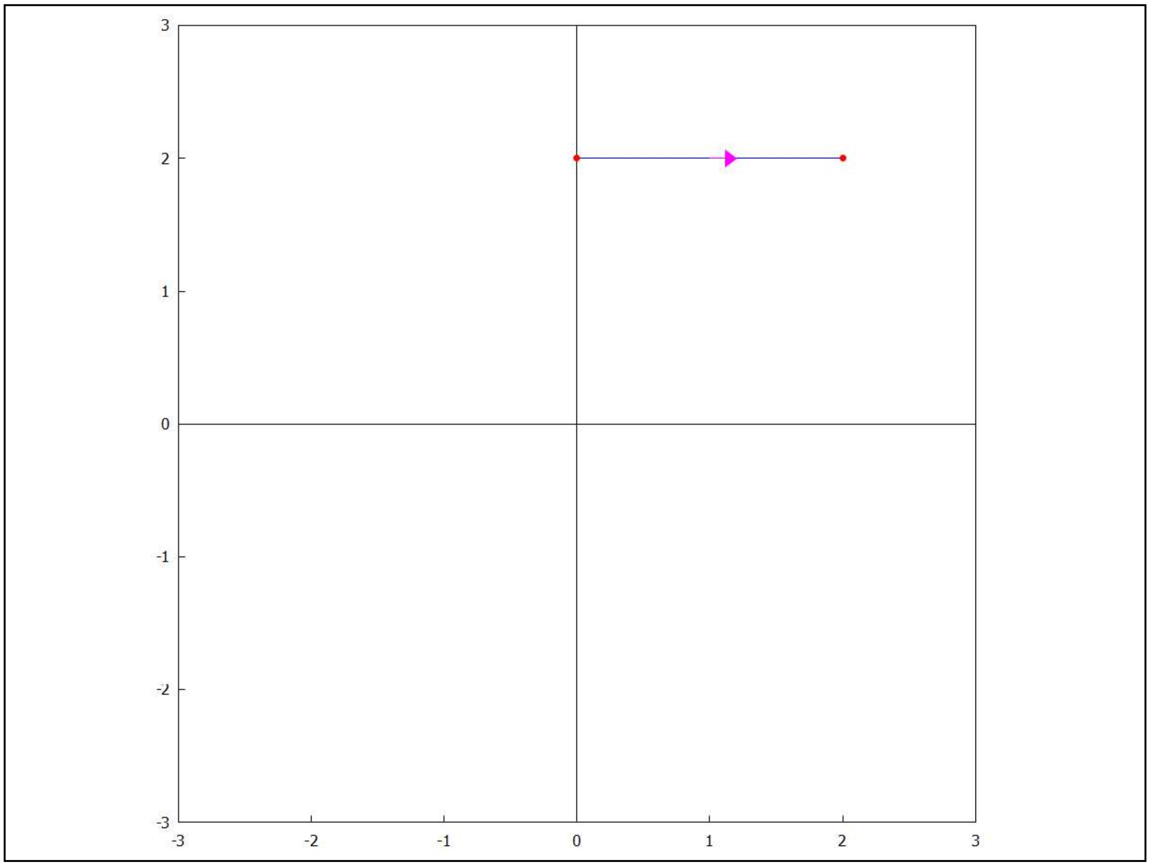



### 8.2.2 C2

```
z2(t):=t+%i·2;
```

$$z2\left(t\right):=t+\%i\,2$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−3,3],
yaxis=true,yaxis_type=solid,yrange=[−3,3],
proportional_axes=xy,
parametric(realpart(z2(t)),imagpart(z2(t)),t,0,2),
head_length=0.2,
head_angle=20,
color=magenta,
vector([1,2],[2/10,0]),
color=red,
point_type=7,
point_size=1,
points([[0,2],[2,2]]));
```

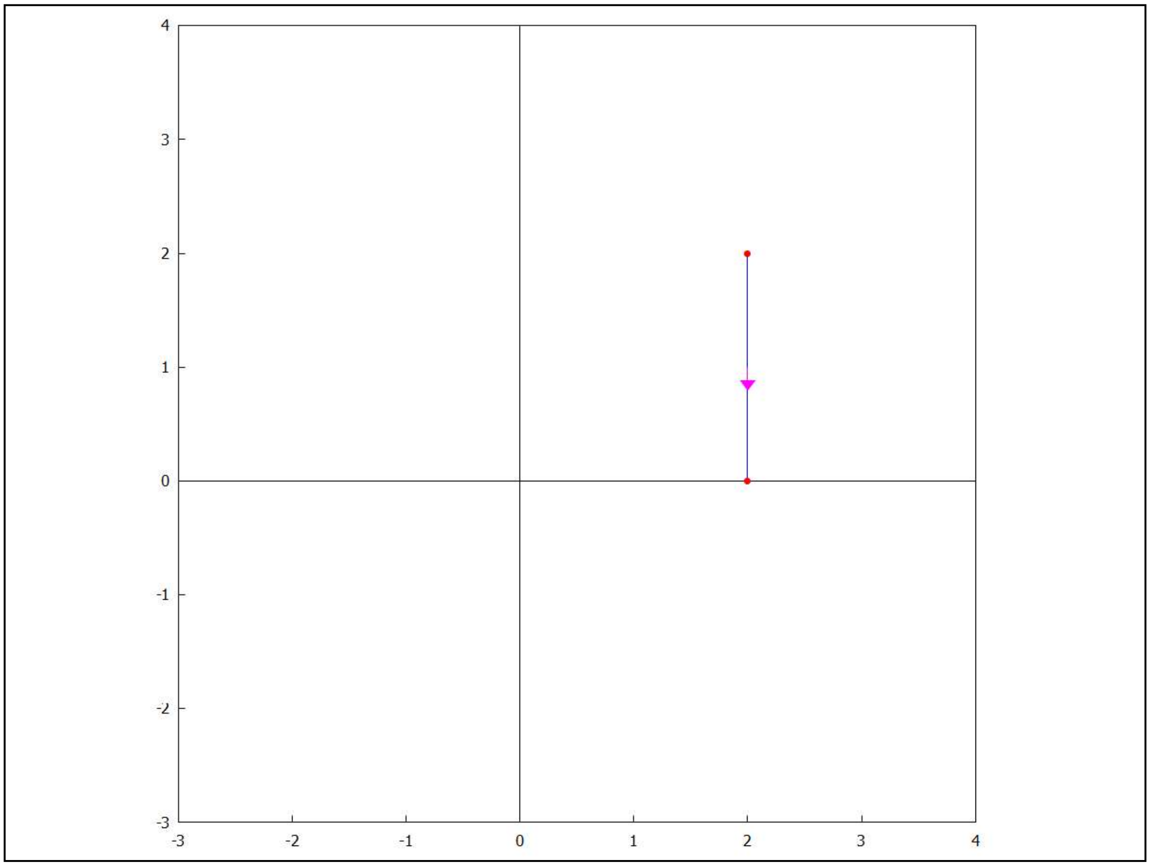



### 8.2.3 C3

```
z3(t):=2+%i·(t);
```

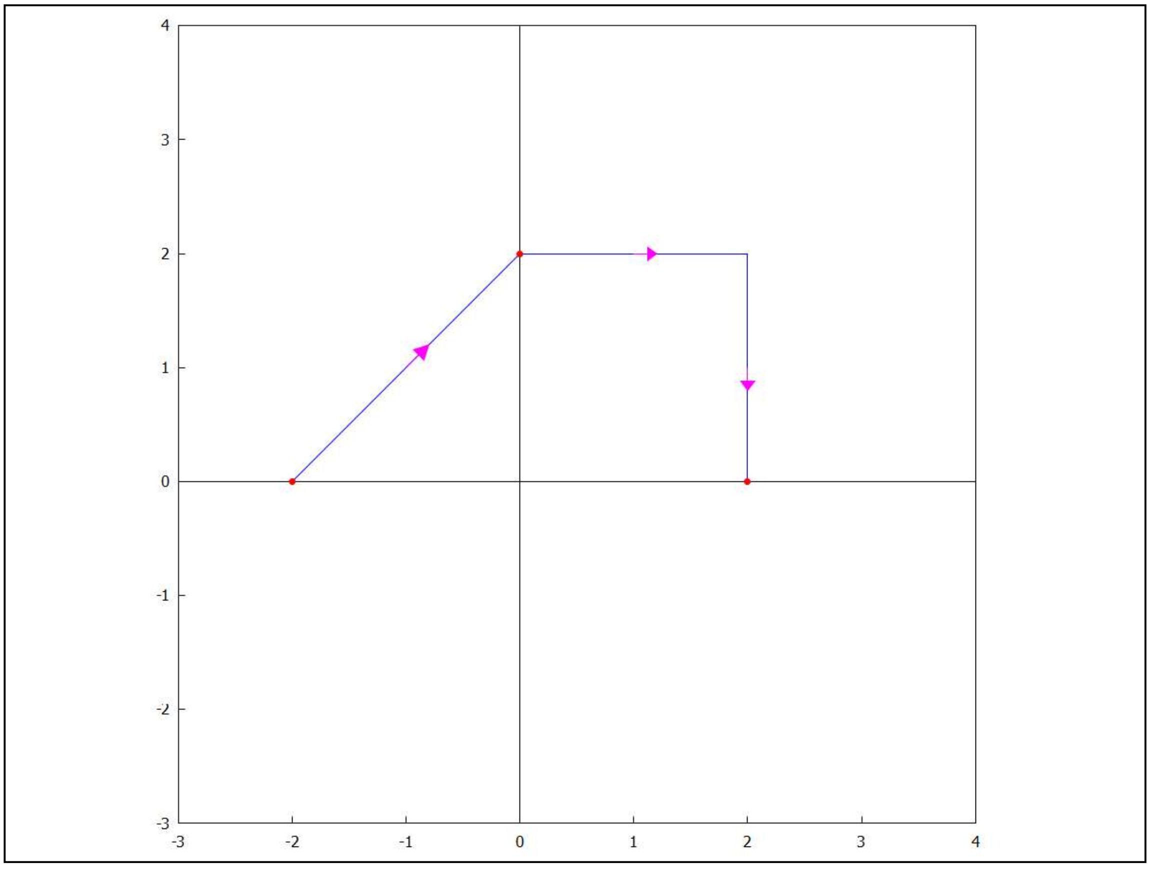
$$z3(t):=2+\%i\ t$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−3,4],
yaxis=true,yaxis_type=solid,yrange=[−3,4],
proportional_axes=xy,
parametric(realpart(z3(t)),imagpart(z3(t)),t,0,2),
head_length=0.2,
head_angle=20,
color=magenta,
vector([2,1],[0,−2/10]),
color=red,
point_type=7,
point_size=1,
points([[2,2],[2,0]]));
```




### 8.2.4 C1+C2+C3

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−3,4],
yaxis=true,yaxis_type=solid,yrange=[−3,4],
proportional_axes=xy,
parametric(realpart(z1(t)),imagpart(z1(t)),t,−2,0),
parametric(realpart(z2(t)),imagpart(z2(t)),t,0,2),
parametric(realpart(z3(t)),imagpart(z3(t)),t,0,2),
head_length=0.2,
head_angle=20,
color=magenta,
vector([−1,1],[0.2,0.2]),
vector([1,2],[2/10,0]),
vector([2,1],[0,−2/10]),
color=red,
point_type=7,
point_size=1,
points([[−2,0],[0,2],[2,0]]));
```



9 Practical-9

line segment 'L' joining the point

A=0 to B=2+(π/4)i and give an exact

value of ∫□^z dz over L

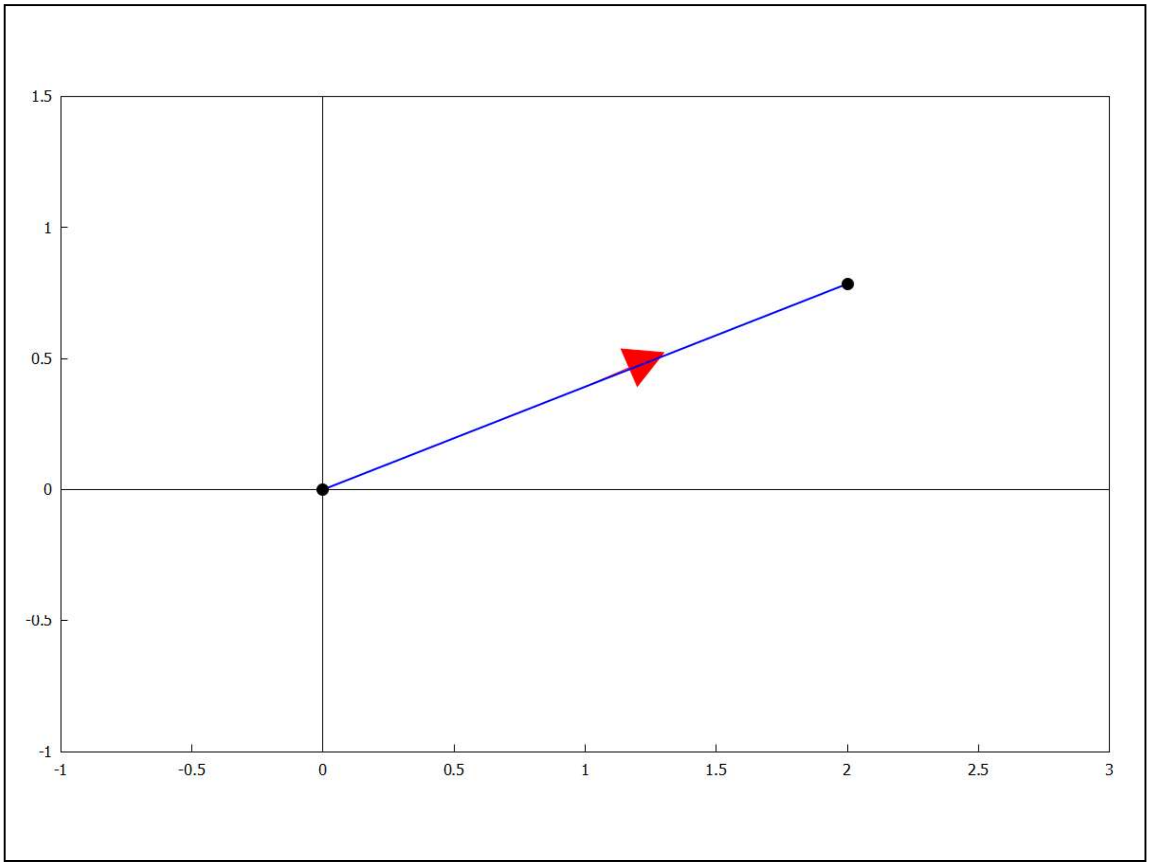
Plot the line segment 'L' joining the point
A=0 to B=2+(π/4)i and give an exact value of
∫ℯ^z dz over L

## 9.1 Plotting the line seg

```
kill(all)$
z(t):=(t)+%i·((%pi/8)·t);
```

$$z(t):=t+\%i\left(\frac{\pi}{8}t\right)$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−1,3],
yaxis=true,yaxis_type=solid,yrange=[−1,3/2],
proportional_axes=xy,
head_length=0.5,
head_angle=10,
color=red,
vector([1,%pi/8],[0.3,0.13]),
color=blue,
line_width=2,
parametric(realpart(z(t)),imagpart(z(t)),t,0,2),
color=black,
point_type=7,
point_size=2,
points([[realpart(z(0)),imagpart(z(0))],[realpart(z(2)),imagpart(z(2))]]));
```




## 9.2 Evaluating integral

```
kill(all)$
cIntegral(p,q,a,b):=block(
f(z):=exp(z),
g(t):=(p)+%i·(q),
rectform(integrate(rectform(f(g(t))·diff(g(t),t)),t,a,b)));
```

$$\mathrm{cIntegral}(p,q,a,b):=\mathrm{block}(f(z):=\exp(z),g(t):=p+\%i\ q,$$
$$\mathrm{rectform}\left(\int_a^b \mathrm{rectform}\left(f(g(t))\left(\frac{d}{d\,t}\,g(t)\right)\right)dt\right))$$

```
cIntegral(t,(%pi/8)·t,0,2);
```

$$\frac{\%e^2\ \%i}{\sqrt{2}}+\frac{\%e^2-\sqrt{2}}{\sqrt{2}}$$

# 10 Practical 10

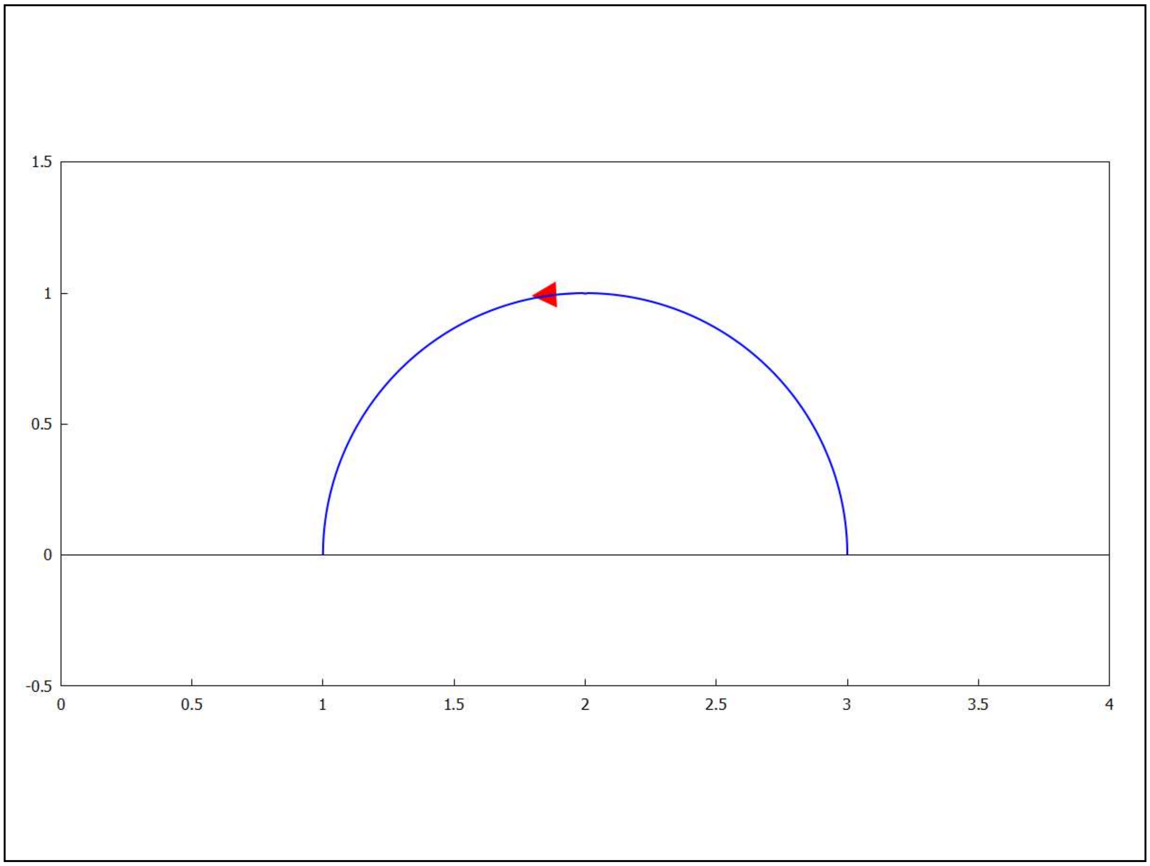
# Contour integral of semicircle

## 10.1 Cicle centred at z=2 with radius=1

```
kill(all)$
z(t):=(2+cos(t))+%i·(sin(t));
```

$$z(t):=2+\cos(t)+\%i\ \sin(t)$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[0,4],
yaxis=true,yaxis_type=solid,yrange=[−1/2,3/2],
proportional_axes=xy,
head_length=0.3,
head_angle=10,
color=red,
vector([2,1],[−0.2,−0.01]),
color=blue,
line_width=2,
nticks=500,
parametric(realpart(z(t)),imagpart(z(t)),t,0,%pi));
```




eval integral

```
kill(all)$
cIntegral(p,q,a,b):=block(
f(z):=1/(z−2),
g(t):=(p)+%i·(q),
rectform(integrate(rectform(f(g(t))·diff(g(t),t)),t,a,b)));
```

$$\mathrm{cIntegral}(p,q,a,b):=\mathrm{block}\left(f(z):=\frac{1}{z-2},g(t):=p+\%i\,q,\ \mathrm{rectform}\left(\int_a^b \mathrm{rectform}\left(f(g(t))\left(\frac{d}{d\,t}\,g(t)\right)\right)d\,t\right)\right)$$

```
cIntegral(2+cos(t),sin(t),0,%pi);
```

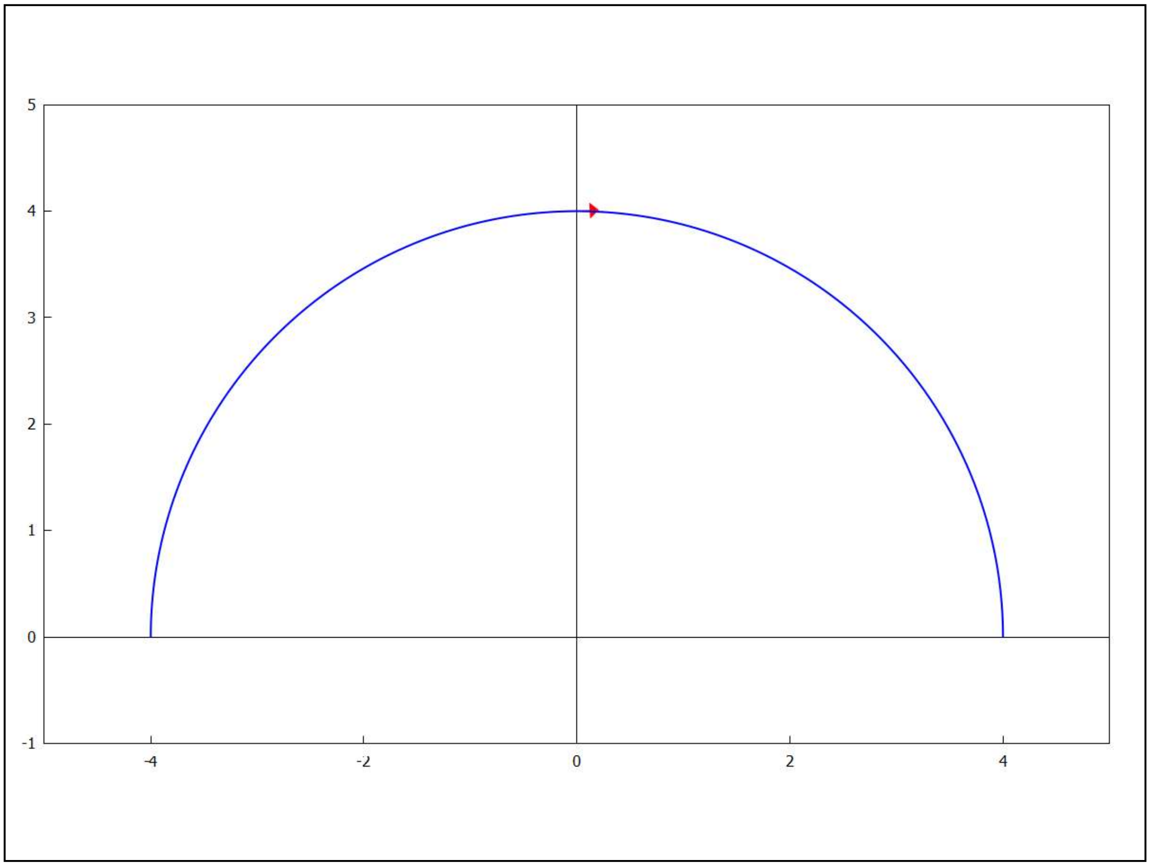
$$\%i\ \pi$$

## 10.2 Circle centred at origin with radius =4

```
kill(all)$
z(t):=(4·cos(−t))+%i·(4·sin(−t));
```

$$z(t):=4\cos(-t)+\%i\,(4\sin(-t))$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−5,5],
yaxis=true,yaxis_type=solid,yrange=[−1,5],
proportional_axes=xy,
head_length=0.5,
head_angle=10,
color=red,
vector([0,4],[0.2,0.01]),
color=blue,
line_width=2,
nticks=500,
parametric(realpart(z(t)),imagpart(z(t)),t,−%pi,0));
```




```
kill(all)$
cIntegral(p,q,a,b):=block(
f(z):=realpart(z),
g(t):=(p)+%i·(q),
rectform(integrate(rectform(f(g(t))·diff(g(t),t)),t,a,b)));
```

$$\mathrm{cIntegral}(p,q,a,b):=\mathrm{block}(f(z):=\mathrm{realpart}(z),g(t):=p+\%i\,q,\ \mathrm{rectform}\left(\int_a^b \mathrm{rectform}\left(f(g(t))\left(\frac{d}{d\,t}g(t)\right)\right)d\,t\right))$$

```
cIntegral(4·cos(−t),4·sin(−t),−%pi,0);
```

$$-(8\,\%i\,\pi)$$

11 Practical 11

contour integral of line seg and

parabola of two points is equal

∫z dz over c1 = ∫z dz over c2 (-1-i to

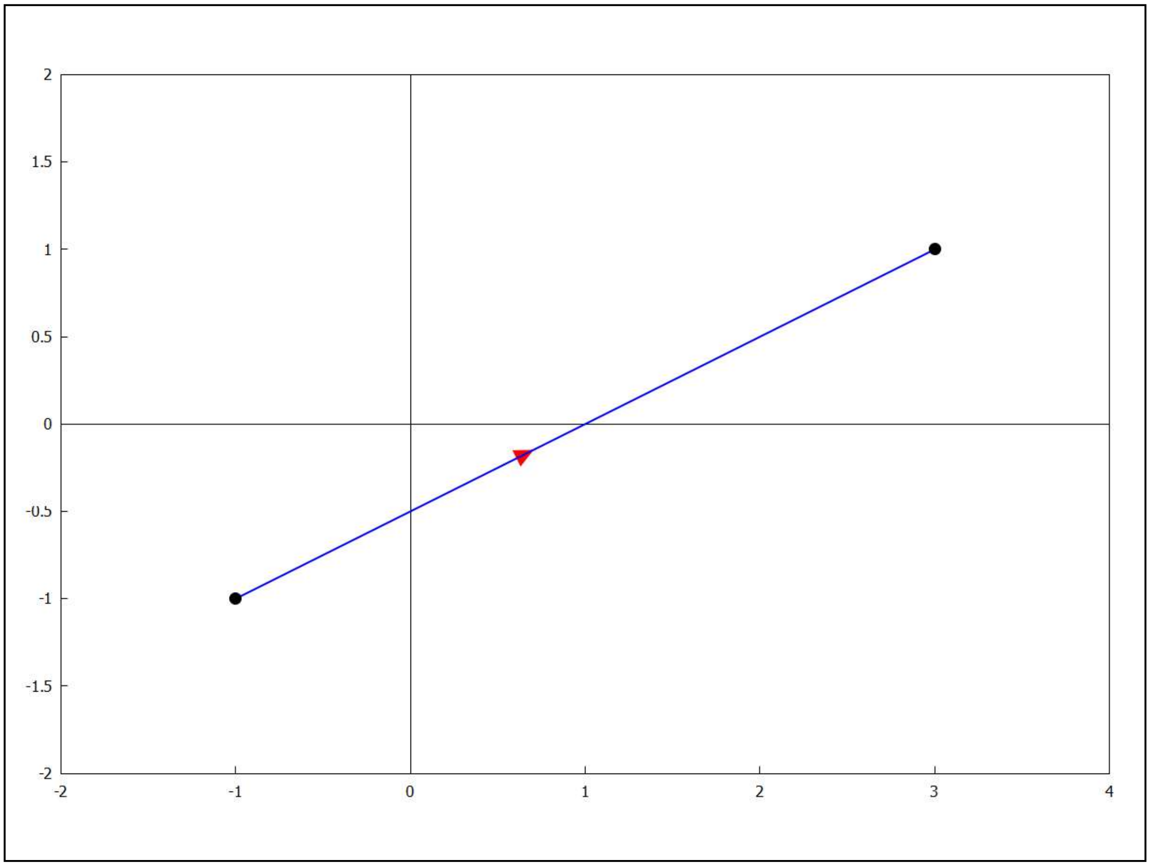
3+i)

C1

```
kill(all)$
z(t):=(t)+%i·(1+((1/2)·(t−3)));
```

$$z(t):=t+\%i\left(1+\frac{1}{2}(t-3)\right)$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−2,4],
yaxis=true,yaxis_type=solid,yrange=[−2,2],
proportional_axes=xy,
head_length=0.3,
head_angle=10,
color=red,
vector([1/2,−1/4],[1/5,1/10]),
nticks=500,
color=blue,
line_width=2,
parametric(realpart(z(t)),imagpart(z(t)),t,−1,3),
color=black,
point_type=7,
point_size=2,
points([[realpart(z(3)),imagpart(z(3))],[realpart(z(−1)),imagpart(z(−1))]]));
```




```
kill(all)$
cIntegral(p,q,a,b):=block(
f(z):=z,
g(t):=(p)+%i·(q),
rectform(integrate(rectform(f(g(t))·diff(g(t),t)),t,a,b)));
```

$$\mathrm{cIntegral}(p,q,a,b):=\mathrm{block}\left(f(z):=z,g(t):=p+\%i\,q,\mathrm{rectform}\left(\int_a^b \mathrm{rectform}\left(f(g(t))\left(\frac{d}{d\,t}g(t)\right)\right)d\,t\right)\right)$$

```
cIntegral(t,1+((1/2)·(t-3)),-1,3);
```

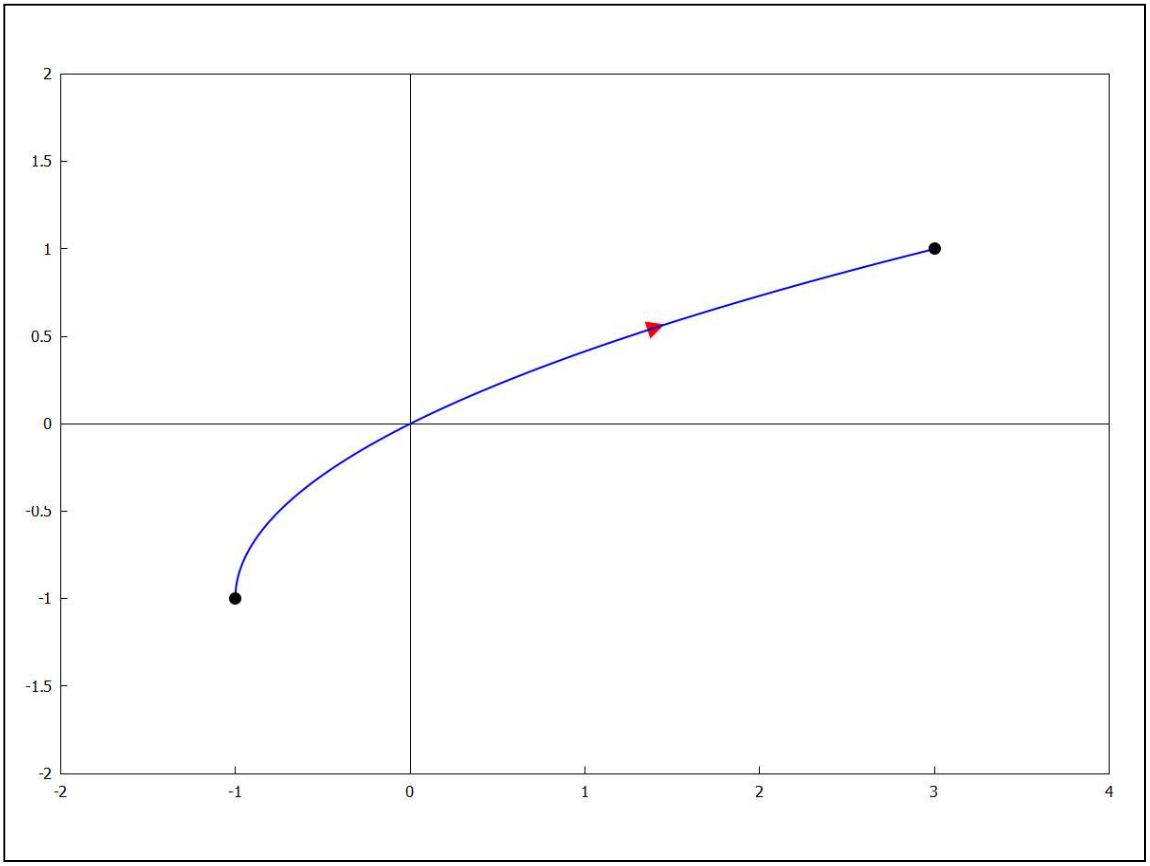
$$2\,\%i+4$$

C2

```
kill(all)$
z(t):=(t^2+2·t)+%i·(t);
```

$$z(t):=t^2+2\,t+\%i\,t$$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−2,4],
yaxis=true,yaxis_type=solid,yrange=[−2,2],
proportional_axes=xy,
head_length=0.3,
head_angle=10,
color=red,
vector([5/4,1/2],[1/5,1/15]),
nticks=500,
color=blue,
line_width=2,
parametric(realpart(z(t)),imagpart(z(t)),t,−1,1),
color=black,
point_type=7,
point_size=2,
points([[realpart(z(1)),imagpart(z(1))],[realpart(z(−1)),imagpart(z(−1))]]));
```



```
kill(all)$
cIntegral(p,q,a,b):=block(
f(z):=z,
g(t):=(p)+%i·(q),
rectform(integrate(rectform(f(g(t))·diff(g(t),t)),t,a,b)));
```

$$\mathrm{cIntegral}(p,q,a,b):=\mathrm{block}\left(f(z):=z,g(t):=p+\%i\,q,\ \mathrm{rectform}\left(\int_a^b \mathrm{rectform}\left(f(g(t))\left(\frac{d}{d\,t}\,g(t)\right)\right)d\,t\right)\right)$$

```
cIntegral((t^2+2·t),t,−1,1);
```

$$2\,\%i+4$$

## 12 Practical 12

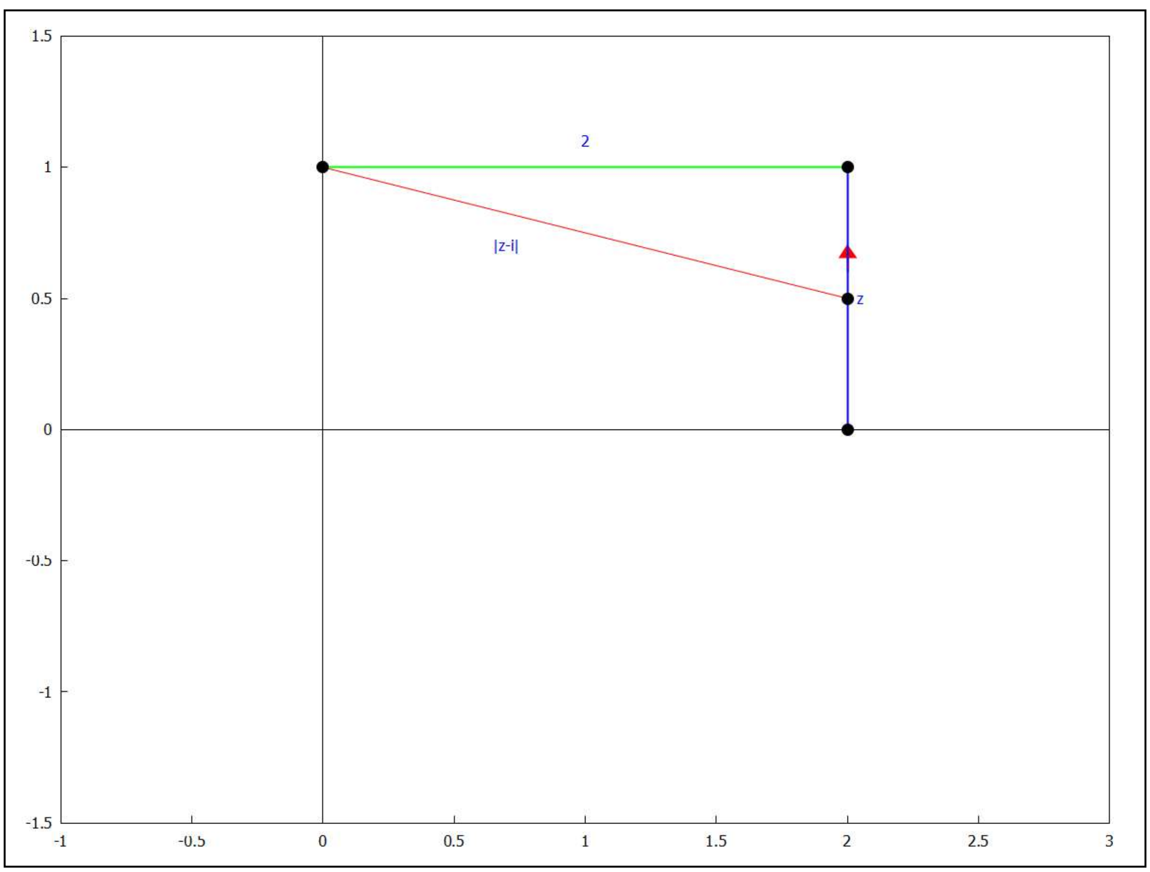
## Using ML inequality ∫1/($z^2$+1) dz ≤ 1/2√5

## over C - line segment from 2 to 2+ i

```
kill(all)$
```

note that $|z^2+1|=|z-i|*|z+i|$
a lower bound for $|z-i|$ on c

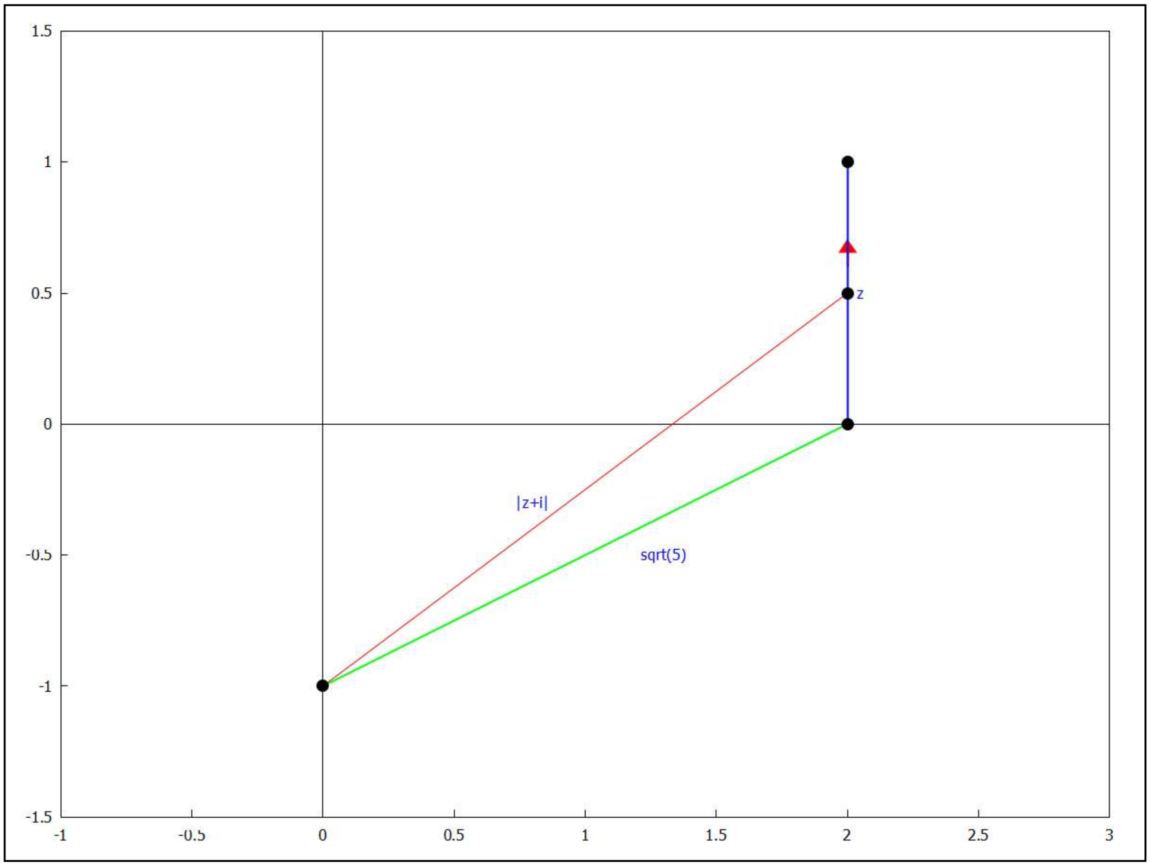
```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−1,3],
yaxis=true,yaxis_type=solid,yrange=[−3/2,3/2],
proportional_axes=xy,
color=red,
parametric(t,1+(−1/4)·(t),t,0,2),
line_width=2,
head_length=0.2,
head_angle=10,
vector([2,0.6],[0,0.1]),
color=green,
parametric(t,1,t,0,2),
color=blue,
parametric(2,t,t,0,1),
label(["2",1,1.1]),
label(["z",2.05,0.5]),
label(["|z-i|",0.7,0.7]),
color=black,
point_type=7,
point_size=2,
points([[2,0],[2,1],[0,1],[2,1/2]]));
```



from the figure |z-i|≥2 when z is on C

A lower bound for |z+i| on C

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−1,3],
yaxis=true,yaxis_type=solid,yrange=[−3/2,3/2],
proportional_axes=xy,
color=red,
parametric(t,−1+(3/4)·(t),t,0,2),
line_width=2,
head_length=0.2,
head_angle=10,
vector([2,0.6],[0,0.1]),
color=green,
parametric(t,−1+(1/2)·(t),t,0,2),
color=blue,
parametric(2,t,t,0,1),
label(["sqrt(5)",1.3,−0.5]),
label(["z",2.05,0.5]),
label(["|z+i|",0.8,−0.3]),
color=black,
point_type=7,
point_size=2,
points([[2,0],[2,1],[0,−1],[2,1/2]]));
```




from thw figure $|z+i| \geq \sqrt{5}$ when z is on C
now $|z^2+1|=|z-i|\ |z+i| \geq 2\sqrt{5}$ when z is on C
therefore $|1/(z^2+1)| \leq 1/2\sqrt{5}$ when z is on C
That is M= $1/2\sqrt{5}$

```
L:1;
M:1/(2·sqrt(5));
M·L;
```

$$1 \qquad \frac{1}{2\sqrt{5}} \qquad \frac{1}{2\sqrt{5}}$$

By MI inequlity $||| \leq 1/2\sqrt{5}$

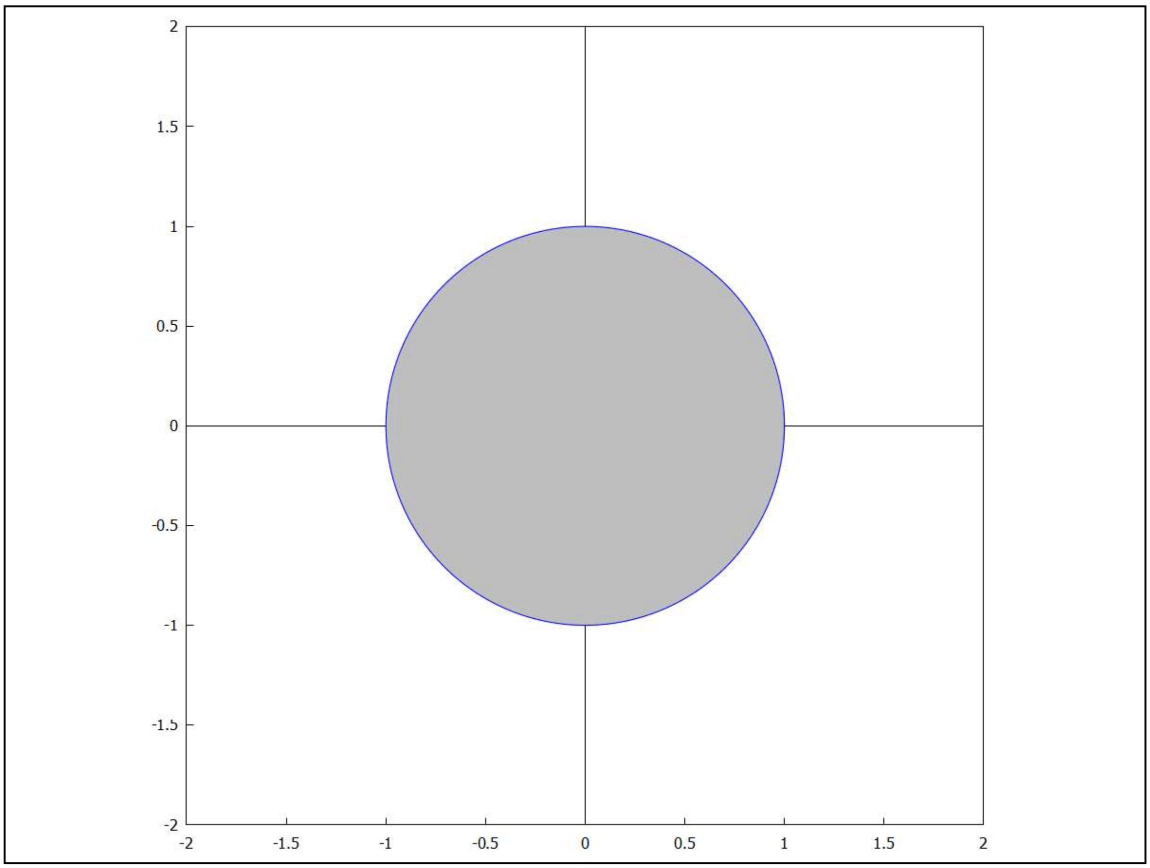
# 13 Practical 13 3 diff Laurent series represent for $f(z)=3/2+z\text{-}z^2$

```
kill(all)$
f(z):=3/(2+z−z^2);
g(z):=1/(1+z);
h(z):=(1/2)·(1/(1−(z/2)));
```

$$f(z):=\frac{3}{2+z-z^2} \qquad g(z):=\frac{1}{1+z} \qquad h(z):=\frac{1}{2}\frac{1}{1-\frac{z}{2}}$$

$|z|<1$

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−2,2],
yaxis=true,yaxis_type=solid,yrange=[−2,2],
proportional_axes=xy,
nticks=200,
fill_color=gray,
ellipse(0,0,1,1,0,360));
```




```
f(z):=3/(2+z−z^2);
g(z):=1/(1+z);
h(z):=(1/2)·(1/(1−(z/2)));
```

$$f(z):=\frac{3}{2+z-z^2} \qquad g(z):=\frac{1}{1+z} \qquad h(z):=\frac{1}{2}\frac{1}{1-\frac{z}{2}}$$

```
taylor(g(z),z,0,4);
```

$$1-z+z^2-z^3+z^4+\ldots$$

```
taylor(h(z),z,0,4);
```

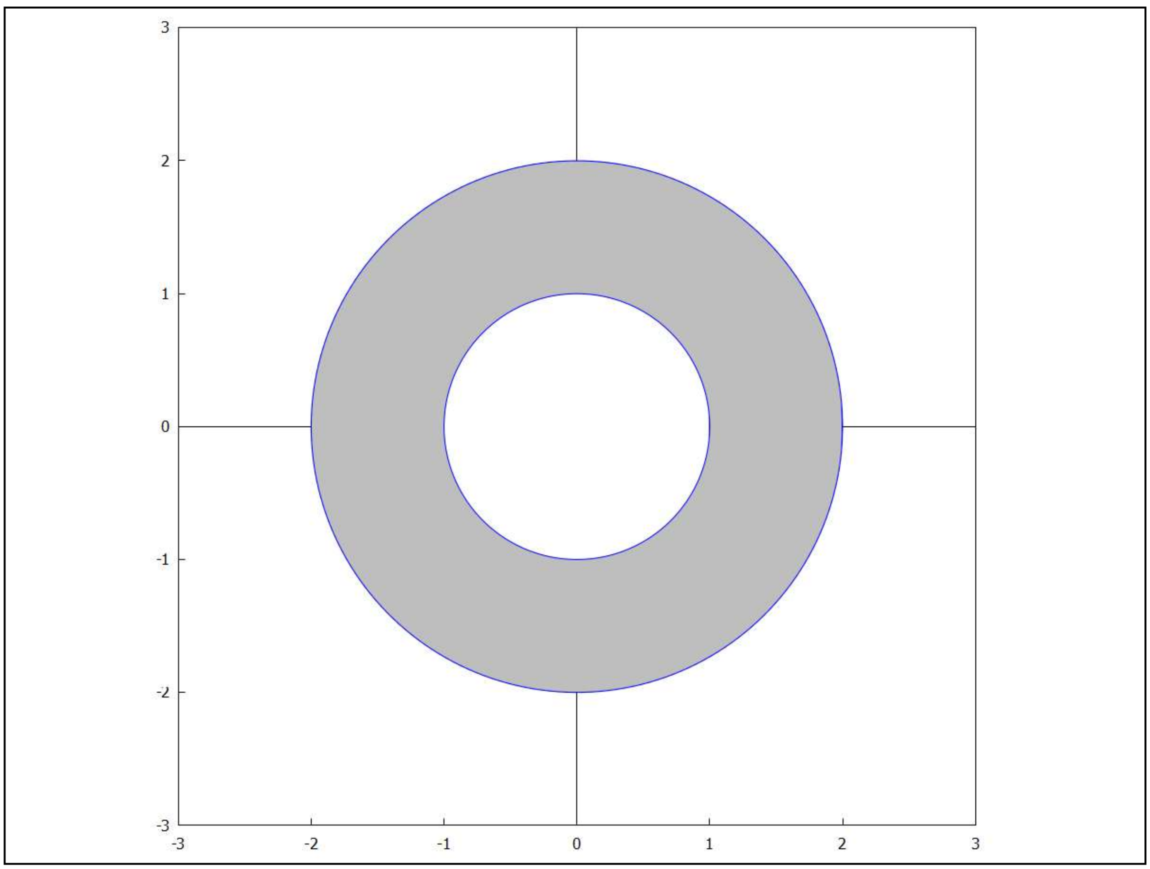
$$\frac{1}{2}+\frac{z}{4}+\frac{z^2}{8}+\frac{z^3}{16}+\frac{z^4}{32}+\ldots$$

```
taylor(g(z),z,0,4) + taylor(h(z),z,0,4) ;
```

$$\frac{3}{2}-\frac{3\,z}{4}+\frac{9\,z^2}{8}-\frac{15\,z^3}{16}+\frac{33\,z^4}{32}+\ldots$$

1<|z|<2

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[−3,3],
yaxis=true,yaxis_type=solid,yrange=[−3,3],
proportional_axes=xy,
nticks=200,
fill_color=gray,
ellipse(0,0,2,2,0,360),
fill_color=white,
ellipse(0,0,1,1,0,360));
```



```
f(z):=3/(2+z-z^2);
g(z):=1/(1+z);
h(z):=(1/2)·(1/(1-(z/2)));
```

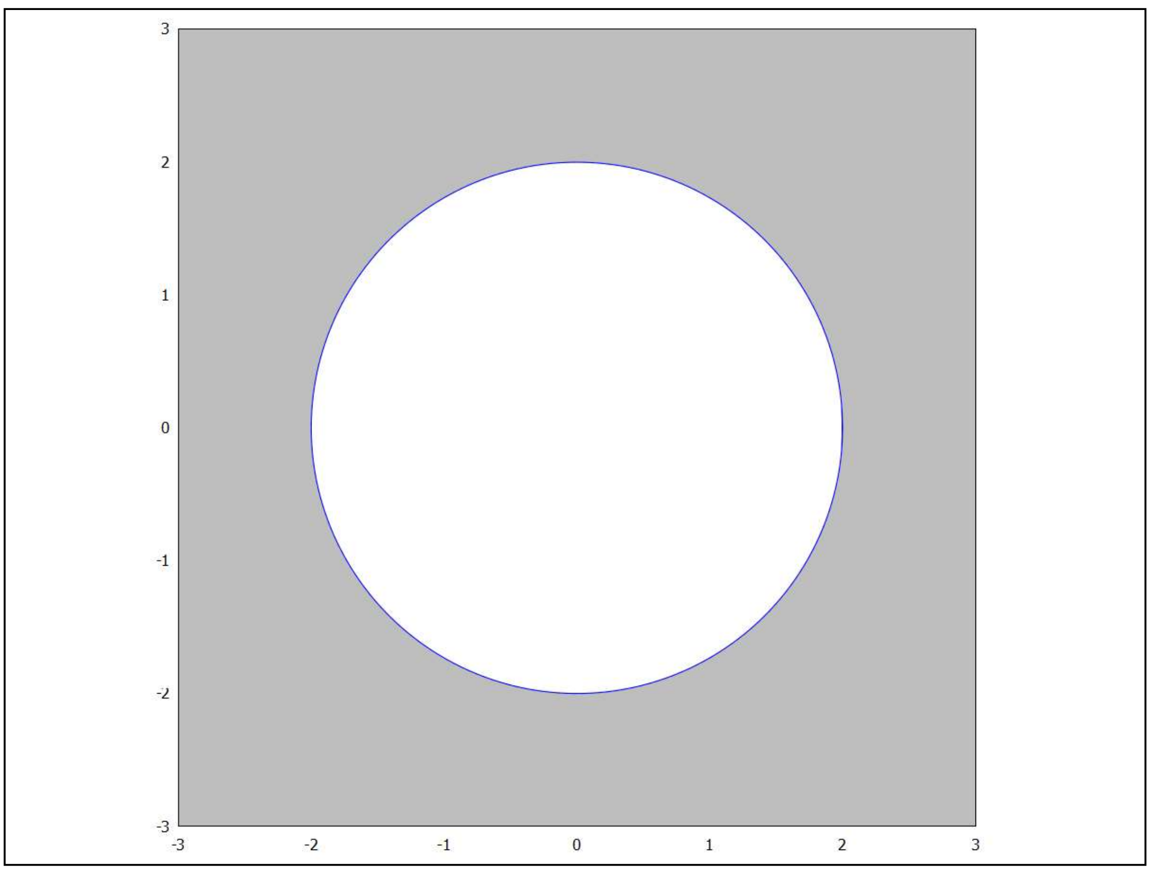
$$f(z):=\frac{3}{2+z-z^{2}} \qquad g(z):=\frac{1}{1+z} \qquad h(z):=\frac{1}{2}\,\frac{1}{1-\frac{z}{2}}$$

```
taylor(g(z),[z,0,4,'aymp]) + taylor(h(z),z,0,4);
```

$$\frac{3}{2}-\frac{3\,z}{4}+\frac{9\,z^{2}}{8}-\frac{15\,z^{3}}{16}+\frac{33\,z^{4}}{32}+\ldots$$

|z|>2

```
wxdraw2d(
xaxis=true,xaxis_type=solid,xrange=[-3,3],
yaxis=true,yaxis_type=solid,yrange=[-3,3],
proportional_axes=xy,
nticks=200,
fill_color=gray,
ellipse(0,0,6,6,0,360),
fill_color=white,
ellipse(0,0,2,2,0,360));
```



```
f(z):=3/(2+z-z^2);
g(z):=1/(1+z);
h(z):=(1/2)·(1/(1-(z/2)));
```

$$f(z):=\frac{3}{2+z-z^{2}} \qquad g(z):=\frac{1}{1+z} \qquad h(z):=\frac{1}{2}\,\frac{1}{1-\frac{z}{2}}$$

```
taylor(g(z),[z,0,4,'asymp]) + taylor(h(z),[z,0,4,'asymp]);
```

$$-\left(\frac{3}{z^{2}}\right)-\frac{3}{z^{3}}-\frac{9}{z^{4}}+\ldots$$

# 14 Practical 14

# Poles of f(z)=1/5s^4 + 26s^2 +5 and specify order

```
kill(all)$
load(coma);
```

coma v.2.1 (Wilhelm Haager, 2019−05−21)
D:/Desktop/Dyal Singh/maxima−5.47.0/share/maxima/5.47.0/share/contrib/coma/coma.mac

```
zeros(s^2-1);
```

[1.0, − 1.0]

```
zeros((s^4-1)/(s^4+1));
```

[1.0 %i, − 1.0, − (1.0 %i), 1.0]

```
zeros(5·s^4+26·s^2+5);
```

[0.44721 %i, − (0.44721 %i), − (2.2361 %i), 2.2361 %i]

```
f(s):=1/(5·s^4+26·s^2+5);
```

$$f(s):=\frac{1}{5\,s^{4}+26\,s^{2}+5}$$

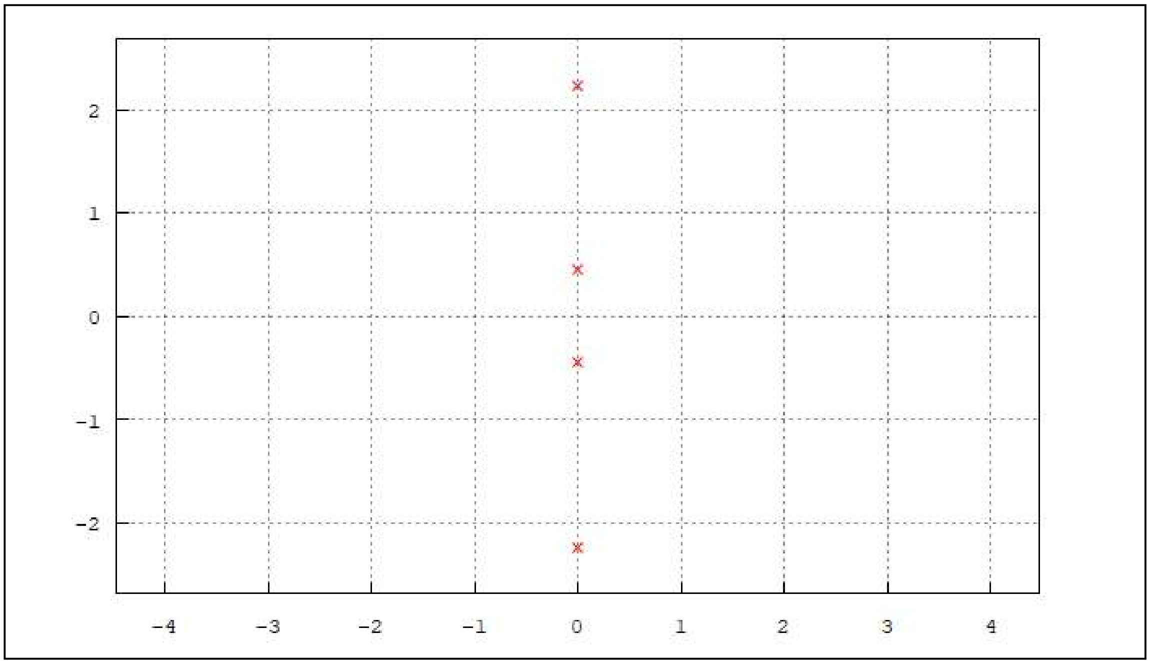
```
zeros(f(s));
```

[ ]

```
poles(f(s));
```

[0.44721 %i, − (0.44721 %i), − (2.2361 %i), 2.2361 %i]

```
poles_and_zeros(f(s));
```




```
float(sqrt(5));
```

2.2361

```
float(1/(sqrt(5)));
```

0.44721

# 15 Practical 15

## zeros and poles of g(z)=π cot(πz)/$z^2$ and detremine the order. Also res(g,0)=-$\pi^2$/3

```
kill(all)$
load(coma);
```

coma v.2.1 (Wilhelm Haager, 2019−05−21)
D:/Desktop/Dyal Singh/maxima−5.47.0/share/maxima/5.47.0/share/contrib/coma/coma.mac

```
g(s):=%pi·cot(%pi·s)/s^2;
```

$$g(s):=\frac{\pi \cot(\pi s)}{s^2}$$

```
zeros(g(s));
```

[ ]

```
poles(g(s));
```

[ ]

```
residue(g(s),s,0);
```

$$-\left(\frac{\pi^2}{3}\right)$$

# 16 Practical 16 Evaluate ∫e^(2/z) dz over c1+(0) where this denotes the circle {z:|z|=1} similairly eval ∫1/(z^4 + z³-2z²) dz

```
kill(all)$
g(s):=exp(2/s);
```

$$g(s):=\exp\left(\frac{2}{s}\right)$$

```
residue(g(s),s,0);
```

$$0$$

```
taylor(g(s),[s,0,3,'asymp]);
```

$$1+\frac{2}{s}+\frac{2}{s^2}+\frac{4}{3\,s^3}+\ldots$$

```
I:2·%pi·%i·2;
```

$$4\,\%i\,\pi$$

∫1/(z^4 + z³-2z²) dz

```
kill(all)$
g(s):=1/(s^4+s^3−2·s^2);
```

$$g(s):=\frac{1}{s^4+s^3+-2\,s^2}$$

```
load(coma);
```

coma v.2.1 (Wilhelm Haager, 2019−05−21)
D:/Desktop/Dyal Singh/maxima−5.47.0/share/maxima/5.47.0/share/contrib/coma/coma.mac

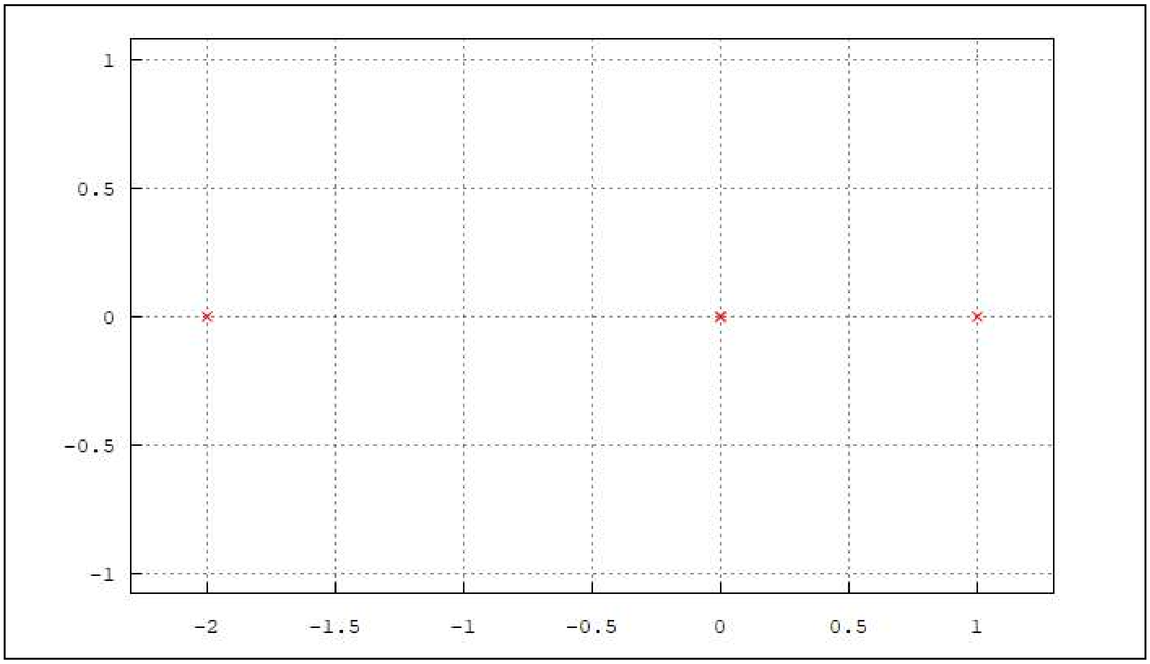
```
zeros(g(s));
```

$$[\,]$$

```
poles(g(s));
```

$$[0,0,1.0,-2.0]$$

```
poles_and_zeros(g(s));
```




```
r1:residue(g(s),s,0);
r2:residue(g(s),s,1);
r3:residue(g(s),s,−2);
```

$$-\left(\frac{1}{4}\right) \qquad \frac{1}{3} \qquad -\left(\frac{1}{12}\right)$$

```
I:2·%pi·%i·(r1+r2+r3);
```

$$0$$