

Traffic Modeling

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Abstract

Introductory concepts for studying traffic modeling. Macroscopic and microscopic models are used to introduce modern techniques for understanding traffic flow. A microscopic model utilizing ordinary differential equations is explained in detail. Reasons for studying traffic flow and current applications in science are provided.

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1 Motivation for project

My primary interest in researching traffic models was to investigate a common commuting phenomenon. I would frequently be stopped at a traffic light while an empty lane received a green traffic signal. I wondered how an intelligent traffic system might control such a situation. What type of information would the system need to make complex traffic routing decisions? As my research uncovered, the problem is a unique and challenging one with no clear answers.

2 Why do we study traffic?

Traffic has been studied extensively since the beginning of the twentieth century. With the advent of the mass-produced automobile, increasing numbers of vehicles began to filter onto our road infrastructure. With the increase of vehicles came an increase in traffic congestion problems. From the 1920's through 1950's several researchers attempted to model this new phenomena. Based on these initial traffic models, a new field of research emerged and continues to this day.

One of the basic goals of studying traffic is to understand traffic congestion and look for ways to prevent it. The goal is to provide efficient movement of traffic while minimizing congestion problems.

Traffic modeling has become a multidisciplinary field. Engineers, city planners, computer scientists, and mathematicians among others study traffic and lend their expertise to understanding the topic. Each discipline lends a slightly different perspective on the study of traffic.

2.1 Engineering

Engineers are interested in traffic for as many reasons as there are engineering fields. Civil engineers need to be able to predict and model traffic flow to create effective and safe highway systems and intersections. Electrical engineers may develop intelligent electronic devices that monitor and adapt to traffic conditions. Environmental engineers may study how traffic congestion affects air pollution and fuel consumption.

2.2 City Planning

City planners study traffic to decide how to most effectively use and implement traffic systems. They may need to predict future traffic conditions, or determine if certain types of vehicles should be blocked from a particular roadway. A city planner may need to determine the affect of adding an on-ramp to a freeway or whether to build a bypass. Traffic models can help city planners and state governments determine speed limits.

2.3 Computer Science

Computer scientists approach the study of traffic from a modeling perspective. By creating accurate simulations of traffic systems, computer scientists allow engineers and city planners to quickly test new ideas. Simulations can determine how a proposed change in infrastructure will affect traffic before any construction is begun. Simulation is even used in video games to provide realistic traffic flow as a tool to immerse the player in a virtual world.

2.4 Mathematics

Mathematicians create new traffic models and study the efficiency of current models. They may find ways to calibrate models to empirical observational data or determine where a model has difficulty providing realistic results. Mathematicians may try to explain traffic phenomenon such as congestion waves and traffic jams.

2.5 Environmental reasons to study traffic

Traffic congestion problems are inefficient in a number of ways. As congestion worsens, velocity decreases and the flow of traffic grinds to a halt. This causes time delays, decreased fuel economy, and an increased risk of vehicle collision. The Texas Transportation Institute estimated that, in the year 2000:

“The 75 largest metropolitan areas experienced 3.6 billion vehicle-hours of delay, resulting in 5.7 billion U.S. gallons (21.6 billion liters) in wasted fuel and \$67.5 billion in lost productivity, or about 0.7 percent of the nation’s GDP. It also estimated that the annual cost of congestion for each driver was approximately \$1,000 in very large cities and \$200 in small cities.”

3 Current traffic model research

Current traffic model research is largely divided into two types of models, macroscopic and microscopic. Macroscopic models study traffic from an overall or average perspective, while microscopic models study the motion of individual vehicles. Both types of models have active research and modern, state-of-the-art equations. Traffic phenomenon (such as traffic jams) can be studied with both types of models.

While these are the most popular models circa 2011, there is no unified theory to describe traffic flow. Difficulty lies in modeling any type of phenomena when human elements are present. Humans change their driving style constantly, which may prevent any one model from ever describing complex traffic patterns with complete accuracy. That being said, these models can simulate general traffic behavior with some measure of accuracy.

3.1 Macroscopic Models

Macroscopic models were the first to be derived by scientists studying traffic in the 1950’s. Macroscopic models were chosen because traffic flow initially appeared to be similar to the flow of a fluid through a river or pipe system. These models attempt to classify the average behavior of a system instead of the behavior of a specific vehicle.

As an example, in Figure 1 we see an overhead shot of a very large cloverleaf-like freeway section. A macroscopic model would study this system in terms of the flow of vehicles into and out of the system (flux), average speed of vehicles, or total vehicles.



Figure 1: A complex freeway cloverleaf

Macroscopic models are typically modeled using partial differential equations. Modern macroscopic models utilize hyperbolic partial differential equations. Interpreting and solving partial differential equations is outside the scope of this paper, however the equations will be presented as reference.

3.1.1 Macroscopic Model Equation Examples

The first published model developed in the 1950's was the LWR model given by

$$\rho_t(x, t) + (\rho(x, t)V(\rho(x, t)))_x = 0 \quad (1)$$

The LWR model (named after its creators Lighthill, Whitham, and Richards) is considered a scalar, time-varying, non-linear, hyperbolic partial differential equation. One of its basic assumptions is that velocity depends on traffic density.

A more recent model is the AR Model (named after its creators Aw and Rascle). The AR model attempts to move away from a fluid-flow based model. The authors argue that the older macroscopic models have held too closely to the fluid dynamic approach. The AR model is described by

$$\begin{aligned} \rho_t + (\rho v)_x &= 0 \\ (v + P(\rho))_t + v(v + P(\rho))_x &= 0 \end{aligned} \quad (2)$$

The final model referenced is the Zhang Model (named after its creator H. Michael Zhang). This model moves completely away from fluid behavior. The Zhang Model implements a second equation derived from a microscopic model, which establishes a macro-micro link. The Zhang model is given by

$$\begin{aligned} \rho_t + (\rho v)_x &= 0 \\ v_t + vv_x + \rho V'(\rho)v_x &= 0 \end{aligned} \quad (3)$$

3.1.2 Advantages and Disadvantages of Macroscopic Models

The primary advantage of macroscopic models is they have relatively "simple" calculations when compared to microscopic models. Macroscopic models have fewer parameters than their microscopic counterparts. As the equations model density, velocity and flow, only a handful of different parameters are required. The disadvantage of a macroscopic model is the loss of small details or dynamics that can be modeled with microscopic models.

3.2 Microscopic Models

Microscopic models attempt to model the motion of individual vehicles within a system. They are typically functions of position, velocity, and acceleration. Microscopic models are typically created using ordinary differential equations, with each vehicle having its own equation. Because the behavior of these models is usually dictated by a lead vehicle, they are termed "car-following" models. Figure 2 demonstrates how microscopic models number vehicles in car-following situations.

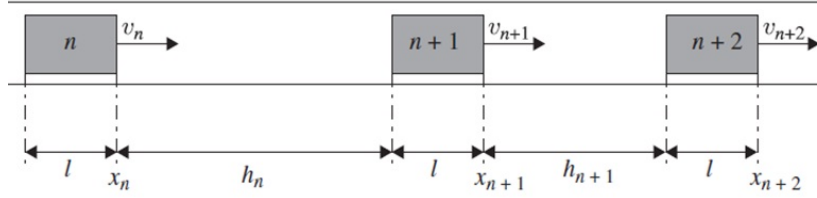


Figure 2: Numbering vehicles in car-following models

3.2.1 Driving States

Microscopic models were developed to try to emulate the way a human behaves in traffic situations. To accomplish this, the models contain different driving states to describe typical driving responses encountered.

The first driving state is the Free Traffic state. This situation is encountered with low vehicle density, and individual vehicles can accelerate to their desired velocity. No lead vehicle is present to influence vehicle position, velocity, or acceleration.

The second driving state is the Following state. The Following state is encountered in everyday traffic, with medium to high vehicle density. In this state, a vehicle's velocity and acceleration is largely determined by the vehicle in front of it. A driver attempts to maintain a minimum and maximum vehicle gap (or time gap) between themselves and the lead vehicle.

The final driving state is the Braking state. This state is sometimes referred to as an Emergency Response. This state becomes active if the current vehicle is approaching a stopped or significantly slower vehicle. The driver will attempt to stop using various degrees of braking force in an attempt to avoid colliding with the object in front of it.

3.2.2 Microscopic Model Equation Examples

The most basic Microscopic Model used is Gipp's Model. Developed in the 1970's, the model uses driving states to model traffic flow. The model is given by

$$\ddot{x}_n(t) = C \frac{\dot{x}_n(t) - \dot{x}_{n-1}(t)}{x_n(t) - x_{n-1}(t)} \quad (4)$$

With the n th car location denoted $x_n(t)$. This shows that the acceleration of the current car $\ddot{x}_n(t)$ depends on the speed and position of the car in front, with C being a sensitivity parameter.

One disadvantage of the earlier models was that certain parameters had unrealistic parameters or behaviors. As an example, they may allow unrealistic breaking behavior beyond the capabilities of physical vehicles. Modern models attempt to resolve these issues by utilizing multiple sensitivity parameters or other methods.

A current, state-of-the-art model is the Intelligent Driver Model (IDM). This equation was developed by Treiber, Hennecke, and Helbing to improve on previous models, and was published in 2000. The model contains an acceleration strategy with a braking strategy to cover the three driving states above. The IDM model is given by

$$\dot{v}_{\text{IDM}}(s, v, \Delta v) = a \left[1 - \left(\frac{v}{v_0} \right)^\delta - \left(\frac{s^*(v, \Delta v)}{s} \right)^2 \right] \quad (5)$$

$$s^*(v, \Delta v) = s_0 + vT + \frac{v\Delta v}{2\sqrt{ab}}$$

The s^* term below the main function is an expansion of s^* in the numerator of the main function.

We can see that the IDM model is an acceleration function of vehicle gap s , velocity v , and velocity difference Δv . Other parameters with their typical values are given below

Parameter	Typical value
Desired velocity v_0	120 km/h
Safe time headway T	1.6 s
Maximum acceleration a	0.73 m/s ²
Desired deceleration b	1.67 m/s ²
Acceleration exponent δ	4
Jam distance s_0	2 m
Jam distance s_1 (not shown)	0 m
Vehicle length $l = 1/\rho_{\max}$	5 m

The free traffic state of equation (5) dominates when s is very large, causing the interaction term to become negligible. The free traffic term is then

$$\dot{v}_{\text{free}}(v) = a \left[1 - \left(\frac{v}{v_0} \right)^\delta \right] \quad (6)$$

It is easily seen that as $v \rightarrow v_0$, the acceleration $\dot{v}_{\text{free}}(v) \rightarrow 0$. This models the tendency for a driver to gradually decrease their acceleration as they approach their desired velocity v_0 .

The braking or interaction term of equation (5) governs the braking and following driving states. The braking term is given by

$$\begin{aligned} \dot{v}_{\text{brake}}(s, v, \Delta v) &= -a \left(\frac{s^*}{s} \right)^2 \\ s^*(v, \Delta v) &= s_0 + vT + \frac{v\Delta v}{2\sqrt{ab}} \end{aligned} \quad (7)$$

In normal driving conditions, the vT term dominates. The vT term attempts to maintain a specific time gap T from the vehicle being followed. The $v\Delta v/2\sqrt{ab}$ term dominates when approaching an object at a high rate of speed. The model attempts to brake within the limit b , but will exceed b 's value if required to avoid a collision.

3.2.3 Microscopic Model Assumptions

We make several assumptions with the microscopic traffic models. Many of these assumptions can be accounted for by increasing the amount, complexity, or range of parameters.

The macroscopic models assume a homogeneous driver response. This indicates that a driver modeled by these equations reacts exactly the same way in all situations. It is also assumed by most models that no collisions or accidents can occur (vehicles occupying the same space at the same time). Homogenous vehicles are assumed (having the same acceleration and braking response, vehicle length, etc). *Ideal* driving conditions are also assumed.

To simplify matters further, models and simulations usually assume a single lane of traffic. Depending on what is being studied, this single lane may also be formed into a ring-road. This indicates that the first driver is following the last driver as in Figure 3.

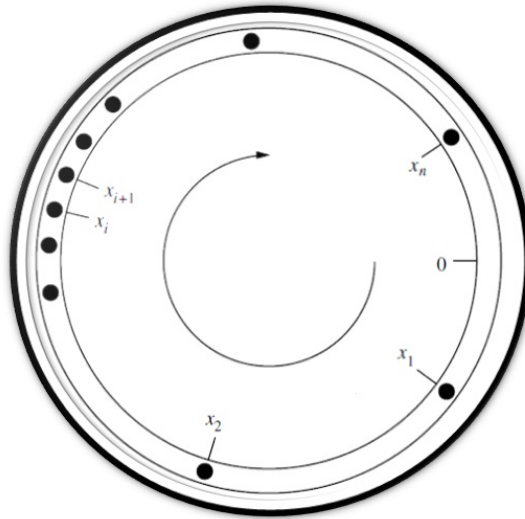


Figure 3: Ring-road

3.2.4 Advantages and Disadvantages of Microscopic Models

A primary advantage of microscopic models is the ability to study individual vehicle motion. Macroscopic ideas (flow and density for example) can also be studied with microscopic models.

The large disadvantage of microscopic models is that one ordinary differential equation is required for each vehicle. Microscopic models become very computationally expensive with large systems of equations, necessitating modern computer power to make them convenient. This is likely the reason microscopic models were not used in the 1950's. As computers increased in power and decreased in cost, the advantage of simple computations became less important.

Some microscopic models can suffer with extreme values, such as emergency braking reactions which defy physical possibility. Modern models have largely decreased this type of behavior.

4 Results

A common theme in all research materials reviewed is that vehicle velocity decreases as vehicle density increases. In order to visualize the effect of vehicle density on velocity, MATLAB can be used to solve traffic model systems of ordinary differential equations.

Three density states will be demonstrated, where vehicle parameters are kept constant. Density will be varied to provide low, medium, and high density systems. The emergence of traffic jams (areas of decreased vehicle velocity) demonstrate the dependence of velocity on vehicle density.

Figure 4 models a low density system. Although there are initial disturbances in the system, the system quickly allows for individual vehicles to reach their desired velocity. As seen in the velocity versus time plot, the velocity reaches a maximum level and allows the system to stay in that state.

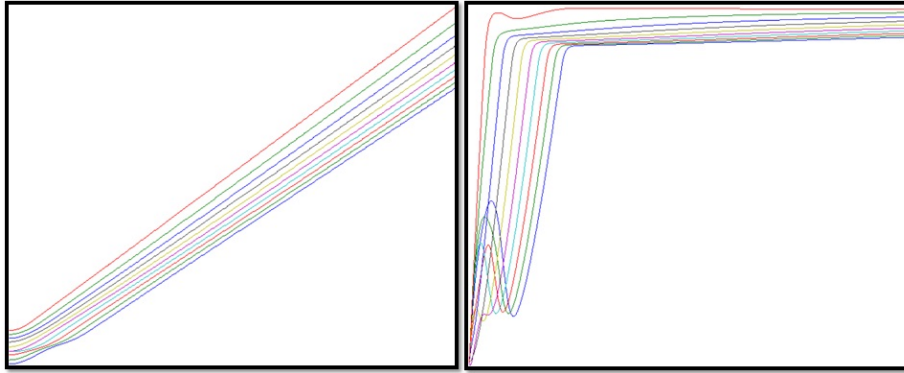


Figure 4: Distance vs Time and Velocity vs Time

Figure 5 models a medium density system. Here the slightly higher vehicle density is causing periodic traffic jams to propagate through the system. Vehicles approach their desired velocity, but must then decelerate as they reach the traffic jam.

Figure 6 models a high density system. In this system, the density is high enough to lower the maximum velocity to a narrow range as time progresses. This would be typical of stop-and-go traffic characterized by very high density and very low velocity.

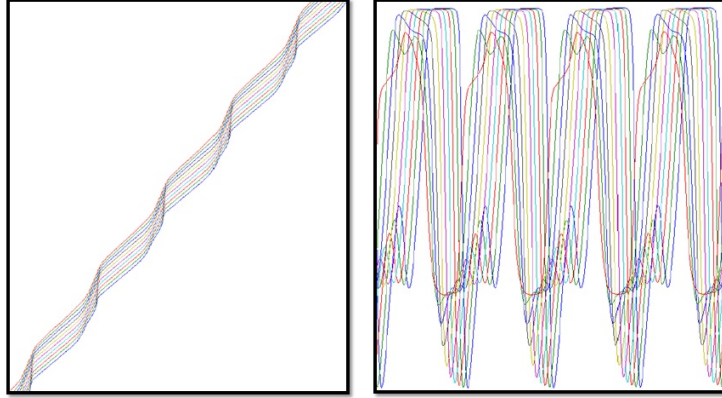


Figure 5: Distance vs Time and Velocity vs Time

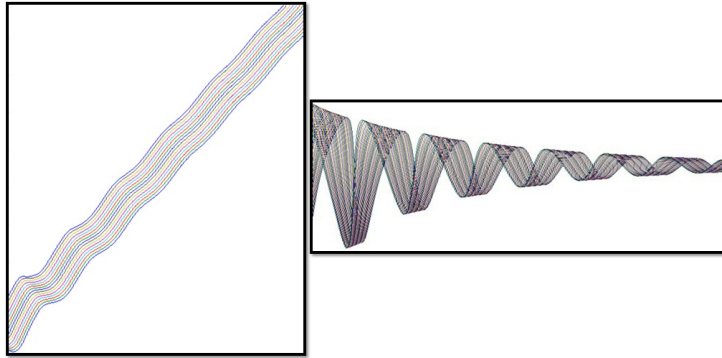


Figure 6: Distance vs Time and Velocity vs Time

4.1 Applications

Current applications of microscopic car-following models are only limited by the imagination. Traffic congestion is a continuing problem with no clear answer. Traffic models are being used in many ways to try to find solutions to the congestion problem. Several examples are listed below.

Car following algorithms are being used in technology known as adaptive-cruise-control. By utilizing various sensors, a vehicle equipped with an adaptive-cruise-control system can detect the velocity of lead vehicles. This allows the car to cruise at a set velocity until it encounters a slower vehicle. The car will then dynamically adapt its speed to match that of the slower vehicle automatically. Fleets of vehicles with this technology are being studied to determine if adaptive-cruise-control can increase traffic density without resulting in traffic congestion problems.

Several car manufacturers offer vehicles with advanced crash-avoidance features. These cars utilize various sensors to monitor for unavoidable collisions by using adapted car-following models. If the algorithm detects an unavoidable collision before the human driver reacts, it can instantaneous start applying braking force to lessen the crash impact.

Multiple commercial adaptations of traffic models are available for city planners and engineers to utilize. These systems generally have both microscopic and macroscopic components. Commercial solutions are much more complex than the models shown in this paper, with parameter lists that are pages long. These simulations can often be interfaced with intelligent traffic systems to dynamically control traffic flow throughout a city.

Intelligent Traffic Systems have been developed in several large cities. An intelligent traffic system can vary traffic signal timing, close or open lanes of traffic, or even dynamically change special speed limit signs. Monitoring systems can also notify traffic management departments if traffic patterns indicate a deviation from average levels. Coupled with commercial traffic modeling software these types of systems can increase vehicle density without decreasing velocity

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