User Selection in MIMO Interference Broadcast Channels

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Outline

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Preview

In previous semester we have

- Studied user selection problem defined in the literature and existing suboptimal algorithms.
- Proposed a better suboptimal algorithm than all existing algorithms in terms of sum rate and/or complexity.
- Extended the proposed algorithm to joint user and antenna selection and obtained better results than all existing algorithms.
- Tried to apply the proposed algorithm in Multi - transmitter Interference systems.

Outline

Interference Alignment (IA)

Interference alignment [1] achieves maximum degrees of freedom in an interference channel which aims at:

- **Alignment**: Maximizing the overlap between the interference spaces received from multiple sources.
- Aligning the interference to exactly half of the signal space leaving other half for the desired signal.
- Degrees of freedom achieved are exactly half of the received signal space dimension.
- The capacity achieved employing interference alignment is therefore $\frac{1}{2}\log_2(\text{SNR}) + o(\text{SNR})$.

IA in MIMO systems

Precoding matrices are designed to achieve IA at the receiver

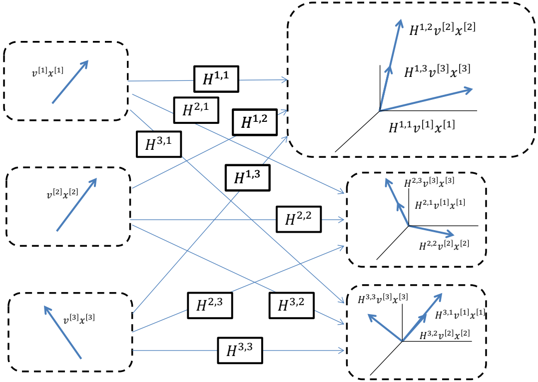


Figure: IA scheme for MIMO systems

IA in MIMO Broadcast systems

MIMO Interfering Broadcast (IFBC) channel is characterized as

- Base station (BS) broadcasts signal to users in home cell as well as in neighbouring cells.
- At each receiver their is inter-user interference and inter-cell interference.
- Receiver beamforming is used to deal with the interference such that following conditions are satisfied [2]

$$\begin{aligned}
&\mathbf{U}_k^{l\,H}\mathbf{H}_k^{l,l}\mathbf{V}_i^l = 0, \quad \forall i = 1, ..., K \neq k \\
&\mathbf{U}_k^{l\,H}\mathbf{H}_k^{l,j}\mathbf{V}_i^j = 0, \quad \forall i = 1, ..., K \text{ and } \forall j = 1, ..., L \neq l \\
&\text{rank}(\mathbf{U}_k^{l\,H}\mathbf{H}_k^{l,l}\mathbf{V}_k^l) = d_s
\end{aligned} \tag{1}$$

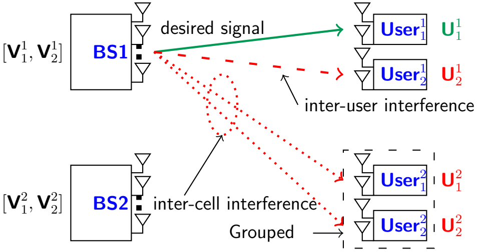
Grouping method in IFBC system

The receive matrices are required to satisfy (1), therefore

$$\mathbf{U}_k^l \in [\mathbf{H}_k^{l,l}\mathbf{V}_i^l \quad \mathbf{H}_k^{l,j}\mathbf{V}_m^j]^\perp \quad \forall i \neq k \text{ and } \forall j \neq l, m = 1, ..., K \tag{2}$$

which can be rewritten as [3]

$$\mathbf{V}_k^l \in null([(\mathbf{U}_i^{l\,H}\mathbf{H}_k^{l,l})^H \quad (\mathbf{U}_m^{n\,H}\mathbf{H}_k^{l,n})^H \quad (\mathbf{U}_m^{j\,H}\mathbf{H}_k^{l,j})^H]^H) \quad \forall i \neq k, j \neq l, n \text{ and } m = 1, ..., K \tag{3}$$



$$\mathbf{H}_1^{2,1\,H}\mathbf{U}_1^2 = \mathbf{H}_2^{2,1\,H}\mathbf{U}_2^2$$

Grouping method in IFBC system

To achieve IA in IFBC system following requirements must be satisfied

- Null space in computation of $\mathbf{V}$ must have non-zero dimension.
- Null space in computation of $\mathbf{U}$ [3] must have non-zero dimension.

Both these conditions can be written as [2]

$$\begin{aligned} M &\geq [K(L-1)+1] \times d_s \\ N &\geq [(K-1)(L-1)+1] \times d_s \end{aligned} \tag{4}$$

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User selection

The following problems need to be addressed before using IFBC system in practical situations

- Number of users supported by IFBC system (K) in each cell is small, can be for e.g. 3 or 4.
- Total potential users in each cell (K_T) are large say 100, so all users cannot be supported.
- Given BS and receiver configuration, how many users can be supported by the system to achieve IA using grouping method??

User selection

- The answer to the above problems is user selection.
- We cannot support all users in a cell to share same radio resource but can select some users from total users.
- The maximum number of users that can be supported can be found on reciprocating (4) as

$$K = \min\left\{ \left\lfloor \frac{1}{L-1}\left[\frac{M}{d_s} - 1\right] \right\rfloor, 1 + \left\lfloor \frac{1}{L-1}\left[\frac{N}{d_s} - 1\right] \right\rfloor \right\} \quad (5)$$

where $\lfloor x \rfloor$ is floor operation

User selection

- Given the maximum number of users that can be supported K, the optimal user subset can be found as

$$\mathcal{S}^l_{opt} = \text{arg} \max_{\mathcal{S}^l \subset \Gamma^l, |\mathcal{S}^l| \leq K} C(\mathcal{S}^1, \mathcal{S}^2, ..., \mathcal{S}^L) \quad \forall l = 1, ..., L \tag{6}$$

- The number of searches to be made are $\binom{K_T}{K}^L$ which is quite large even for small value of K_T, for e.g if $K_T = 50$ and $K = 2, L = 3$, number of searches to be made are 1.838×10^9.
- This sets a need for linear search algorithms with lower computational complexity.

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User selection algorithms

- Linear search is made using coordinate ascent approach [4] to update user subsets such that sum rate is maximized.

$$\mathcal{S}^l = \{s_1^l, ..., s_k^l, ..., s_K^l\}$$

$$s \in \Gamma^l - \{s_1^l, ..., s_{k-1}^l, s_{k+1}^l, ..., s_K^l\}$$

- Algorithm is initialized by taking the users with best channel energy (channel frobenius norm).
- Users are updated by evaluating capacity at each step and comparing with previous value.

Complexity Analysis

The total flops [5] required are

$$\begin{aligned}\psi_c &\leq K_T L \times 4MN + L \times f_U + \{KL \times \mathcal{F}(M, K(L-1)d_s) \\ &\quad + f_U + KL \times (8MNd_s + 8M^2 d_s + 8N^2 d_s)\} \\ &\qquad \times (K_T - K + 1)L \\ &\approx \begin{cases} \mathcal{O}\left(K_T K^3 M^2 N L\right) & \text{if } K \leq 3 \\ \mathcal{O}\left(K_T K M^3 L\right) & \text{if } K \geq 3 \end{cases}\end{aligned} \quad (7)$$

where,

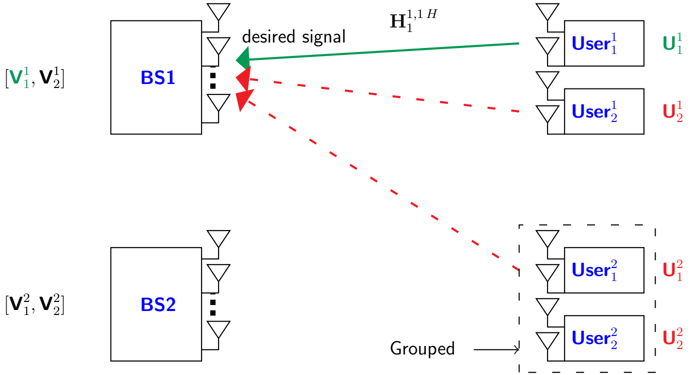
- $\mathcal{F}(N, M)$ are flops required to compute SVD of $N \times M$ matrix [6].
- f_U are flops required to compute receiver beamforming matrix [2].

User selection algorithms

- The complexity of linear search can further be reduced if we develop some deep insight to receive and transmit processing.
- Null space of $\mathbf{A}$ is orthogonal space of $\mathbf{A}^H$, therefore (3) can be written as

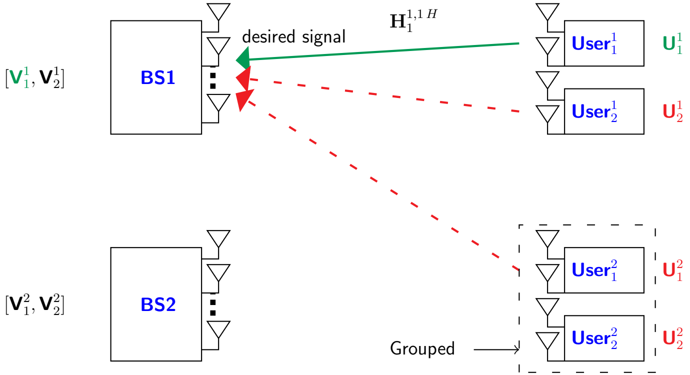
$$\mathbf{V}_k^l \in [\mathbf{H}_k^{l,l\,H}\mathbf{U}_i^l \quad \mathbf{H}_k^{l,n\,H}\mathbf{U}_m^n \quad \mathbf{H}_k^{l,j\,H}\mathbf{U}_m^j]^{\perp}$$
$$\forall i \neq k, j \neq l, n \text{ and } m = 1, ..., K \qquad (8)$$

User selection algorithms



- $\mathbf{V}_k^l$ now can be interpreted as receive matrix in the reciprocal system [7, 8] of original IFBC system.

User selection algorithms



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User selection algorithms

- $\mathbf{V}$ is projecting desired signal onto orthogonal space of the received interference.

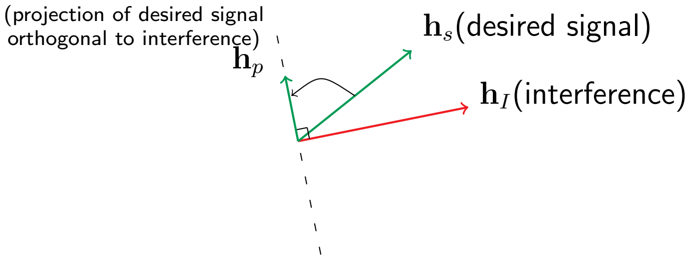


Figure: Receiver processing in reciprocal system

- User with desired signal close to orthogonal of interference should be selected.

Chordal distance

Chordal distance [9] can be used as a measure of orthogonality between subspaces.

- Generator matrix $\mathbf{A}_G$ is defined as matrix whose columns are orthonormal basis of of matrix $\mathbf{A}$.
- The chordal distance between subspaces defined by matrix $\mathbf{A}$ and $\mathbf{B}$ is

$$d_c\left(\mathbf{A},\mathbf{B}\right) = \frac{1}{\sqrt{2}}||\mathbf{A}_G\mathbf{A}_G^H - \mathbf{B}_G\mathbf{B}_G^H||_F \tag{9}$$

- Algorithm will be formulated such that it maximize chordal distance using coordinate ascent approach.

Complexity Analysis

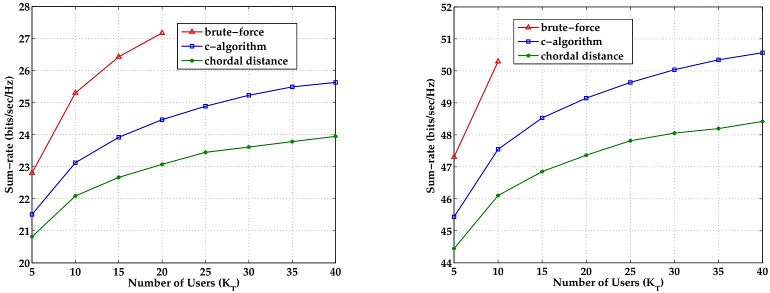
The total flops required are

$$\begin{aligned}\psi_{cho} &\leq K_T L \times 4MN + L \times f_U + \{f_U + 8Md_s^2 + 8M^2 d_s \\ &\quad +8M^2(K(L-1)d_s)^2 - 2Md_s - 2M(K(L-1)d_s) \\ &\quad +8M^3\} \times (K_T - K + 1)L \\ &\approx \mathcal{O}\left(K_T K L M^3\right) \qquad (10)\end{aligned}$$

where $K_T L \times 4MN + L \times f_U$ flops are required to initialize the algorithm.

Outline

Simulation Results



(a) SNR $= 10$ dB

(b) SNR $= 20$ dB

Figure: Sum rate versus total number of users when $M = 6, N = 4, K = 2, L = 2$

Simulation Results

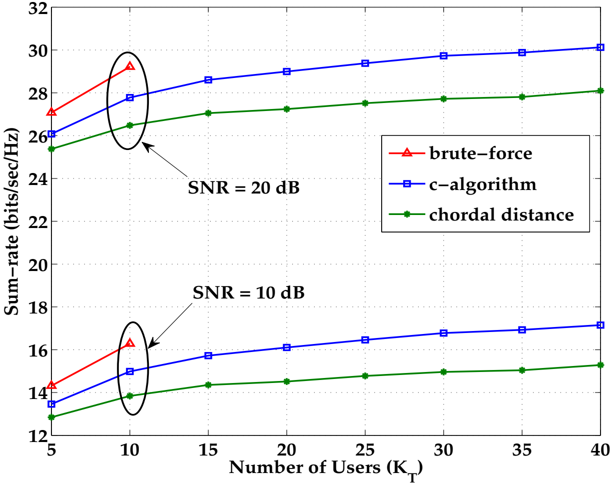
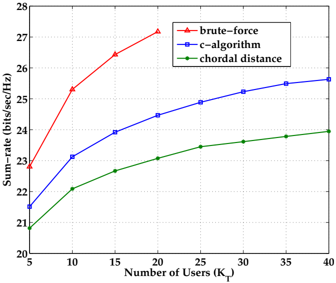
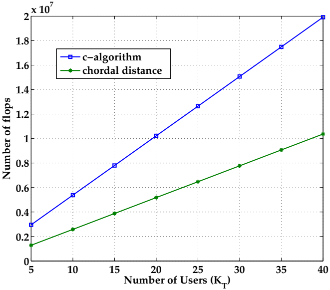


Figure: Sum rate versus total number of users when $M = 3, N = 2, K = 2, L = 2$.

Simulation Results



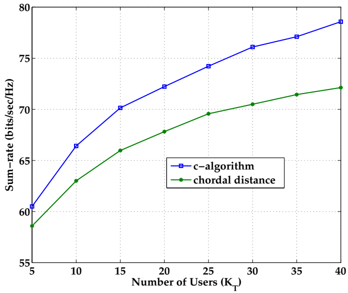
(a) sum rate at SNR $= 10$ dB



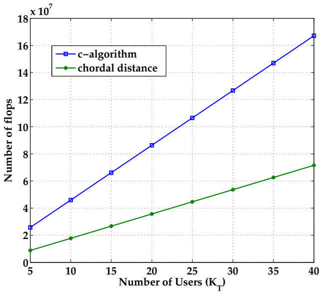
(b) Number of flops

Figure: Performance versus total number of users when $M = 6, N = 4, K = 2, L = 2$

Simulation Results



(a) sum rate at SNR $= 10$ dB



(b) Number of flops

Figure: Performance versus total number of users when $M = 10, N = 6, K = 2, L = 3$

Outline

Conclusions

- Formulated user selection problem in MIMO IFBC system using grouping method.
- Proposed two suboptimal linear user selection algorithms namely c-algorithm and chordal distance based algorithm.
- Performed detailed computational complexity analysis of the proposed algorithms.

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Thank You