Low Complexity User Selection Algorithms for Multiuser-MIMO systems

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Under guidance of

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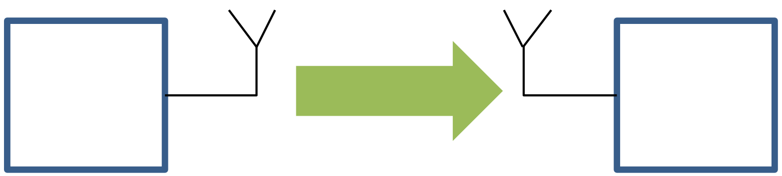
Why MIMO?

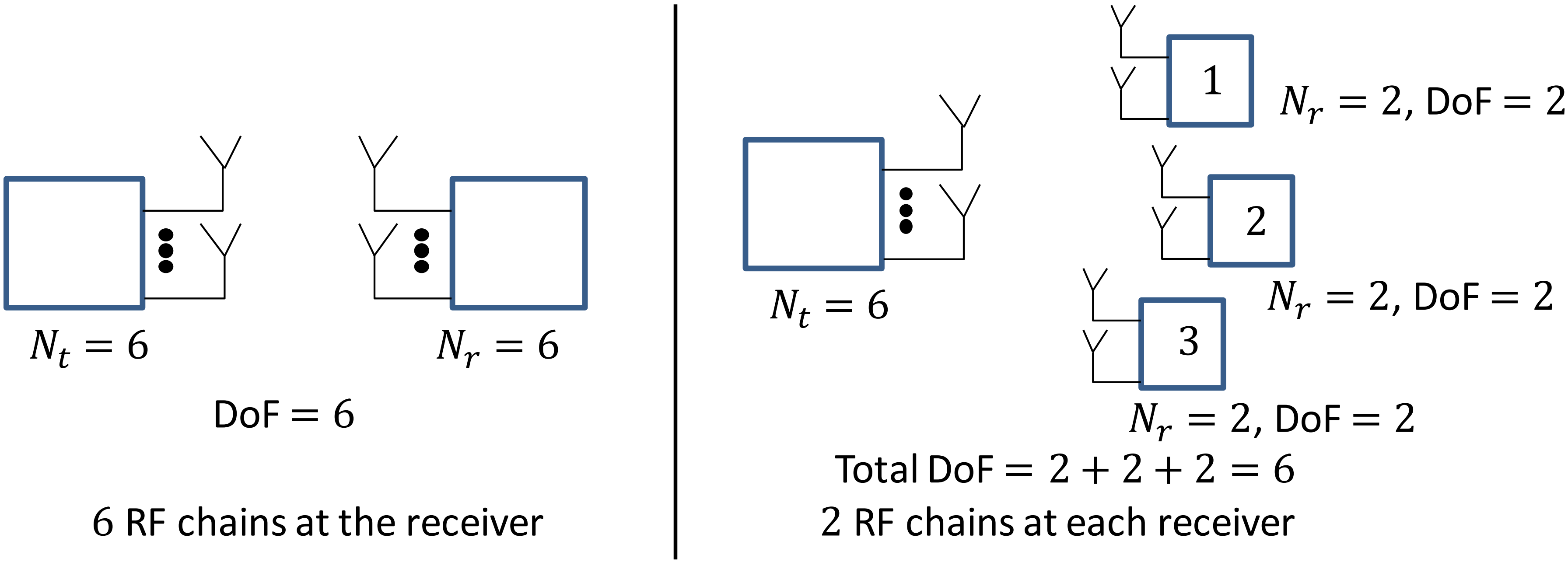
	SNR	Capacity
	10	$\log_2(1+10) = 3.5\ bits/\mathrm{sec}$
	2×10^6	$\log_2(1+2\times 10^6) = 21\ bits/\mathrm{sec}$
	10	$6\times\log_2(1+10) = 21\ bits/\mathrm{sec}$

6 Transmitter Antennas

6 Receiver Antennas

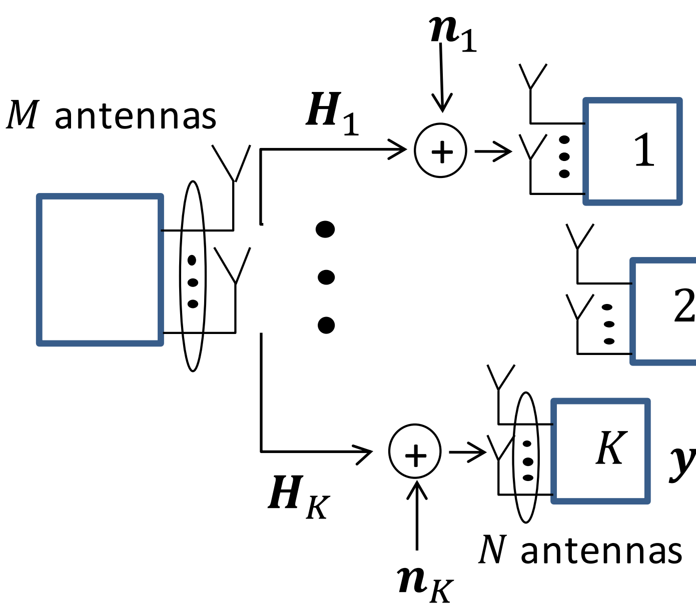
From SU-MIMO to MU-MIMO

Spectral Efficiency : serves more than one users in the same frequency band



MU-MIMO allows spatial multiplexing gain at the base station to be obtained without the need of larger receiver antennas at the receiver, thereby allowing development of small and cheap terminals [1]

(M,(K,N)) Broadcast channel (BC)



$$y_1 = H_1 x + n_1$$

$$y_2 = H_2 x + n_2$$

$$y_K = H_K x + n_K$$

$$H_k \in \mathbb{C}^{N\times M}, n_k \in \mathbb{C}^{N\times 1}$$

$$x = \sum_{i=1}^{K} T_i s_i$$

$$y_k = H_k T_k s_k + \overbrace{H_k \sum_{i=1, i\neq k}^{K} T_i s_i}^{\text{inter-user interference}} + n_k$$

Required signal

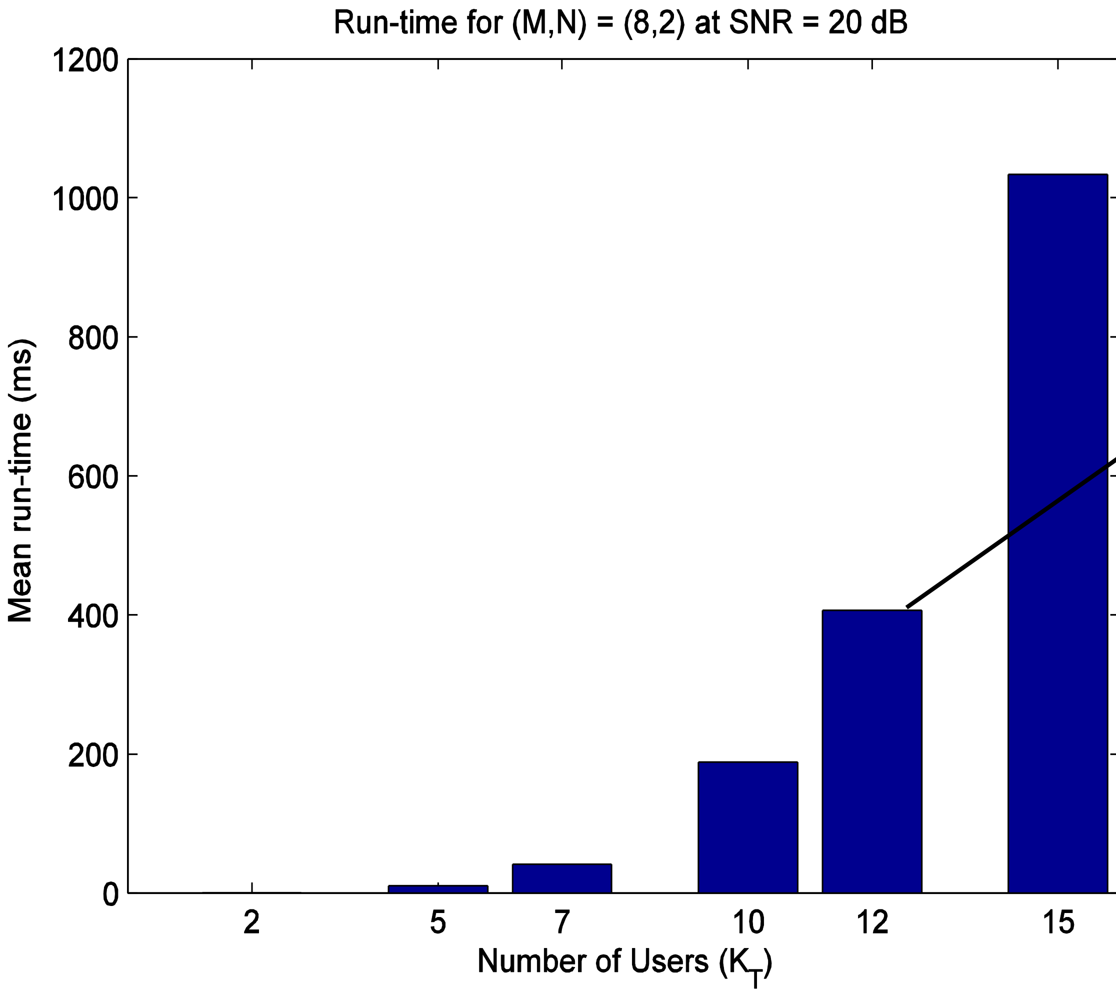
- Involves Precoding(T_i) at the transmitter to eliminate inter-user interference at the receiver
- Cannot take any large value of K ? , but restricted by the precoding scheme.

Precoding schemes

- Block Diagonalization (BD) [2]
  - Completely eliminates inter-user interference at the receiver, i.e.
$$\boldsymbol{H}_k \boldsymbol{T}_{\boldsymbol{j}} = 0, \qquad \forall\, k \neq j$$
$$\boldsymbol{H}_{eff} \boldsymbol{T}_j = 0, \text{ where } \boldsymbol{H}_{eff} = \begin{bmatrix} \boldsymbol{H}_1 \\ \vdots \\ \boldsymbol{H}_{j-1} \\ \boldsymbol{H}_{j+1} \\ \vdots \\ \boldsymbol{H}_K \end{bmatrix}$$
  - Interference elimination comes at the cost of restriction on simultaneous supported users due to non-zero dimension of null($\boldsymbol{H}_{eff}$),
$$M > (K-1) \times N$$
$$K = \left\lceil \frac{M}{N} \right\rceil$$
  - Total possible subsets of users = $\sum_{i=1}^{K} \binom{K_T}{i}$, where K_T is total users

User selection

- In BD scheme, total $\sum_{i=1}^{K}\binom{K_T}{i}$ searches are to be made for obtaining optimal solution using brute-force method whose complexity order is $O(K_T^K . K^{-K+\frac{1}{2}} . M^3)$ [2]



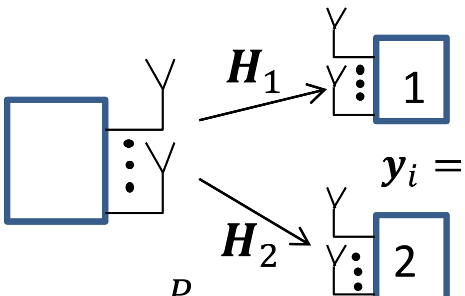
1. Exponential growth in run-time
2. Impractical in real scenarios where K_T is of order of 100's
3. This sets a need for a practical low complexity user search algorithm

Conditional Entropy based Algorithm

Key observations :

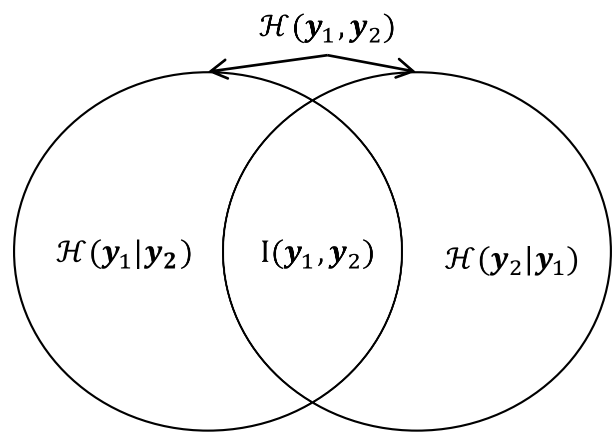
- Multiplication of precoding matrix at the receiver removes the subspace which is common to other channels space
- More common subspace, less is the channel space available after precoding

Therefore, we need a **metric** to select users with channels close to orthogonal, and in addition we want channels with maximum energy to increase sum-rate



$$\boldsymbol{y}_i = \boldsymbol{H}_i \boldsymbol{x} + \boldsymbol{n}_i, i = 1,2$$

$$\boldsymbol{x} : E\{\boldsymbol{x}\boldsymbol{x}^{\boldsymbol{H}}\} = \frac{P}{M} \boldsymbol{I}_M$$



We want $\sum_{i=1,2} I(\boldsymbol{y}_i, \boldsymbol{x})$ to be maximum and $I(\boldsymbol{y}_1, \boldsymbol{y}_2)$ to be minimum

$$I(\boldsymbol{y}_i, \boldsymbol{x}) = \mathcal{H}(\boldsymbol{y}_i) - \mathcal{H}(\boldsymbol{n}_i) \longrightarrow \text{constant}$$

Maximize $\mathcal{H}(\boldsymbol{y}_1|\boldsymbol{y}_2) + \mathcal{H}(\boldsymbol{y}_2|\boldsymbol{y}_1)$: Sum conditional Entropy

Conditional Entropy based Algorithm

Suppose at kth step, $\mathcal{S} = \{s_1, \dots, s_k\}$ has been selected. After adding $(k+1)$th user with channel matrix $\boldsymbol{H}_t$, $s_{k+1} \notin \mathcal{S}$, the sum conditional entropy is given as

$$\mathcal{H}_{SC}(\mathcal{S}, t) = \mathcal{H}\left(\begin{bmatrix} \boldsymbol{H}(\mathcal{S}) \\ \boldsymbol{H}_t \end{bmatrix}\right) - \mathcal{H}(\mathcal{S}) + \sum_{i=1}^{k}\left(\mathcal{H}\left(\begin{bmatrix} \boldsymbol{H}(\mathcal{S}) \\ \boldsymbol{H}_t \end{bmatrix}\right) - \mathcal{H}(\mathcal{S}_i)\right)$$

where , $\mathcal{S}_i = \mathcal{S} + \{t\} - \{s_i\}$.

$\mathcal{H}_{SC}(\mathcal{S}, t)$ becomes

$$\mathcal{H}_{SC}(\mathcal{S}, t) = \log_2 \det\left(\boldsymbol{I}_N + \boldsymbol{H}_t\left(\frac{M}{P}\boldsymbol{I}_M + \boldsymbol{H}(\mathcal{S})^H\boldsymbol{H}(\mathcal{S})\right)^{-1}\boldsymbol{H}_t^H\right) + \sum_{i=1}^{k}\log_2 \det\left(\boldsymbol{I}_N + \boldsymbol{H}_{s_i}\left(\frac{M}{P}\boldsymbol{I}_M + \boldsymbol{H}(\mathcal{S}_i)^H\boldsymbol{H}(\mathcal{S}_i)\right)^{-1}\boldsymbol{H}_{s_i}^H\right)$$

The $\mathcal{H}_{SC}(\mathcal{S}, t)$ is calculated using matrix recursion formula [3]

$$(\boldsymbol{A} + \boldsymbol{B}^H\boldsymbol{B})^{-1} = \boldsymbol{A}^{-1} + \boldsymbol{A}^{-1}\boldsymbol{B}^H(\boldsymbol{I}_N + \boldsymbol{B}\boldsymbol{A}^{-1}\boldsymbol{B}^H)^{-1}\boldsymbol{B}\boldsymbol{A}^{-1}$$

where $\boldsymbol{A}$ is $M \times N$ and $\boldsymbol{B}$ is $N \times M$ positive definite matrix.

Conditional Entropy based Algorithm

Initialize : $\mathcal{S} = \phi, \Gamma = \{1,2,\ldots,K_T\}, \mathcal{R}_{temp} = 0$

Step-1 : Select user with maximum entropy, $\mathcal{S} = \{s_1\}$

Step-2 : for $i = 2: K$

select user which maximizes sum conditional entropy with previously selected users

$s_i^* = \arg\max_{t \notin \mathcal{S}} \mathcal{H}_{SC}(\mathcal{S}, t)$

$\mathcal{R} = \mathcal{R}(\mathcal{S}_{temp}); \mathcal{S}_{temp} = \mathcal{S} + \{s_i^*\};$

if $\mathcal{R} < \mathcal{R}_{temp}$

break;

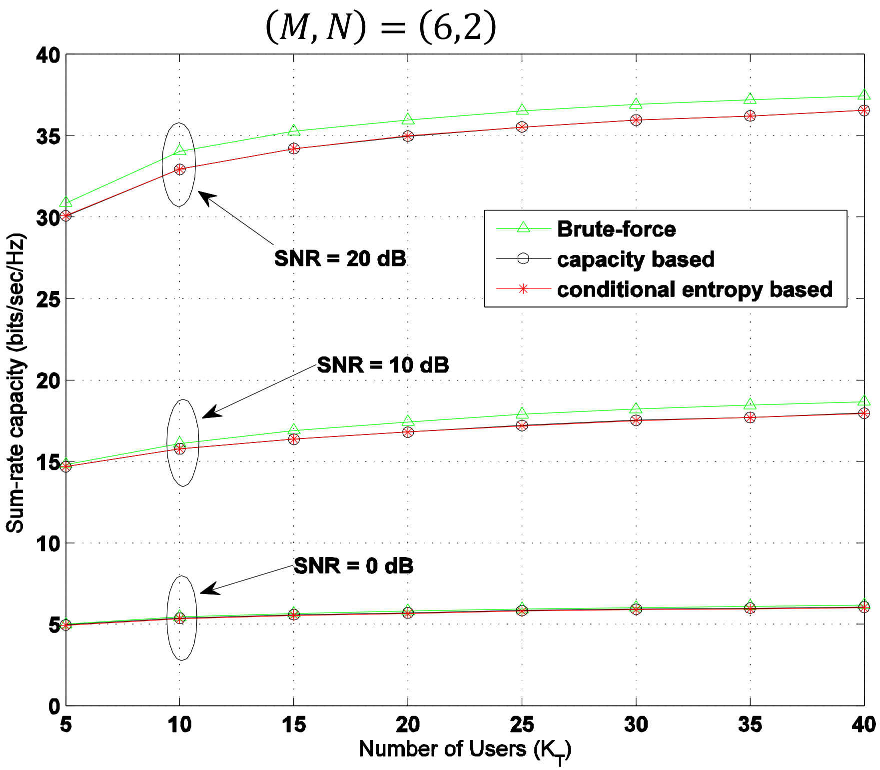
else

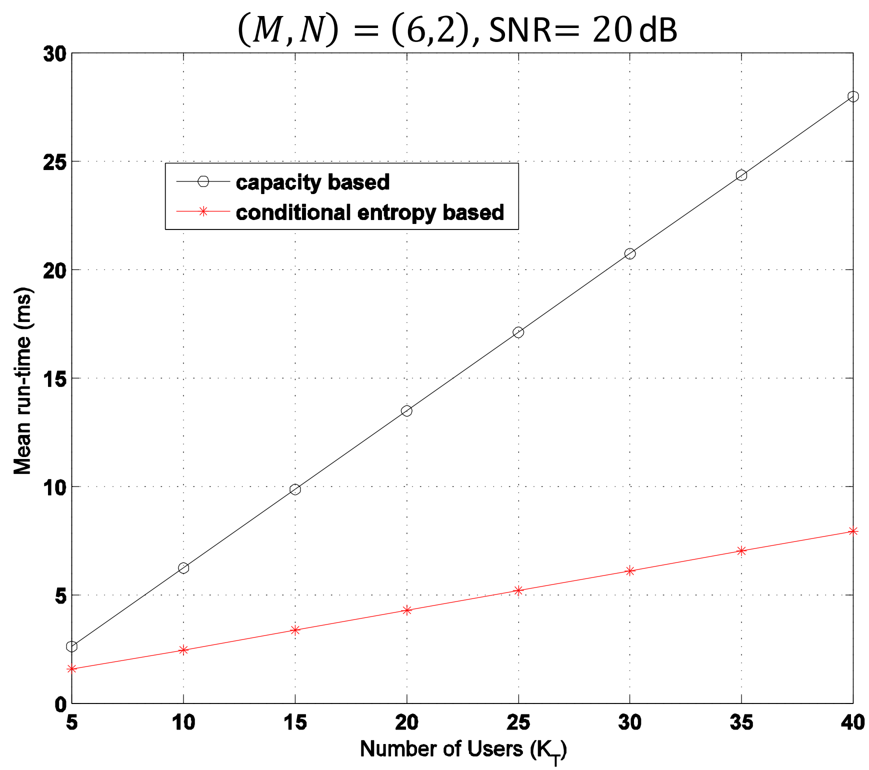
$\mathcal{S} = \mathcal{S}_{temp}\ ; \mathcal{R}_{temp} = \mathcal{R};$

end-if

end-for

Simulation Results





- Run time reduction from 118s to **8ms** with sum capacity within 3% of optimal solution
- Significant reduction in run-time (~ 70 %) as compared to capacity based algorithm, with which sum capacity plot is overlapping
- Conditional entropy algorithm is able to achieve capacity bound using greedy approach

Joint Antenna and User selection

- Multimode diversity can significantly increase system sum capacity
- Use of receiver combination matrix (R_k^H) at the receiver achieves multimode diversity [4]

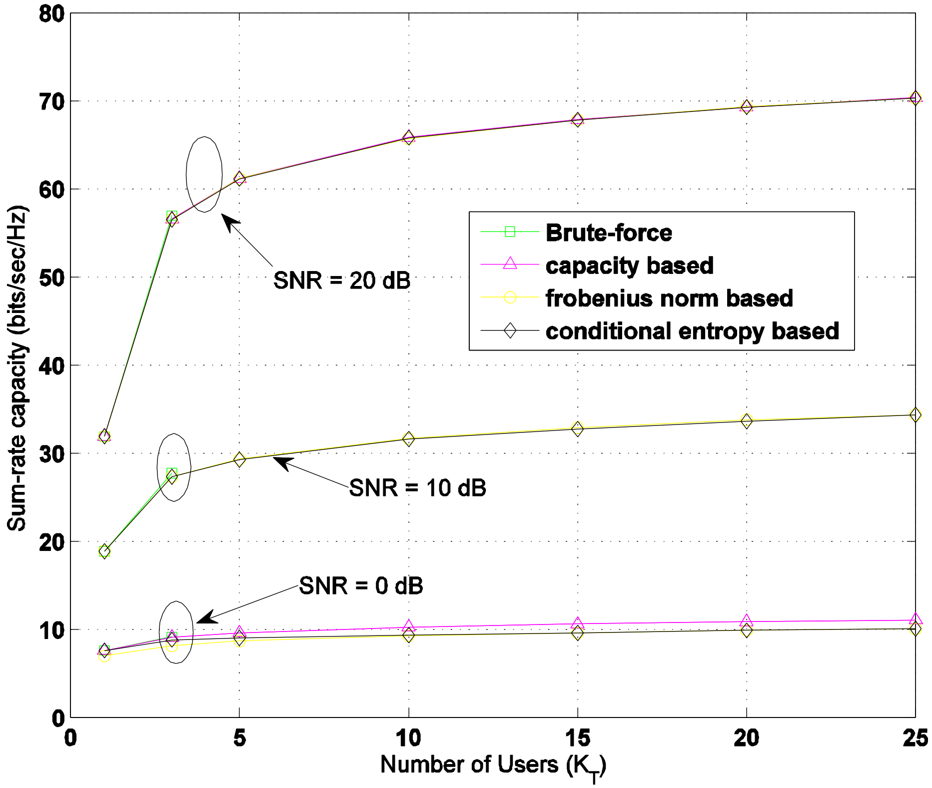
$$\boldsymbol{y}_k = \boldsymbol{R}_k^H \boldsymbol{H}_k \boldsymbol{T}_k \boldsymbol{s}_k + \boldsymbol{R}_k^H \boldsymbol{H}_k \sum_{i=1, i \neq k}^{K} \boldsymbol{T}_i \boldsymbol{s}_i + \boldsymbol{R}_k^H \boldsymbol{n}_k$$

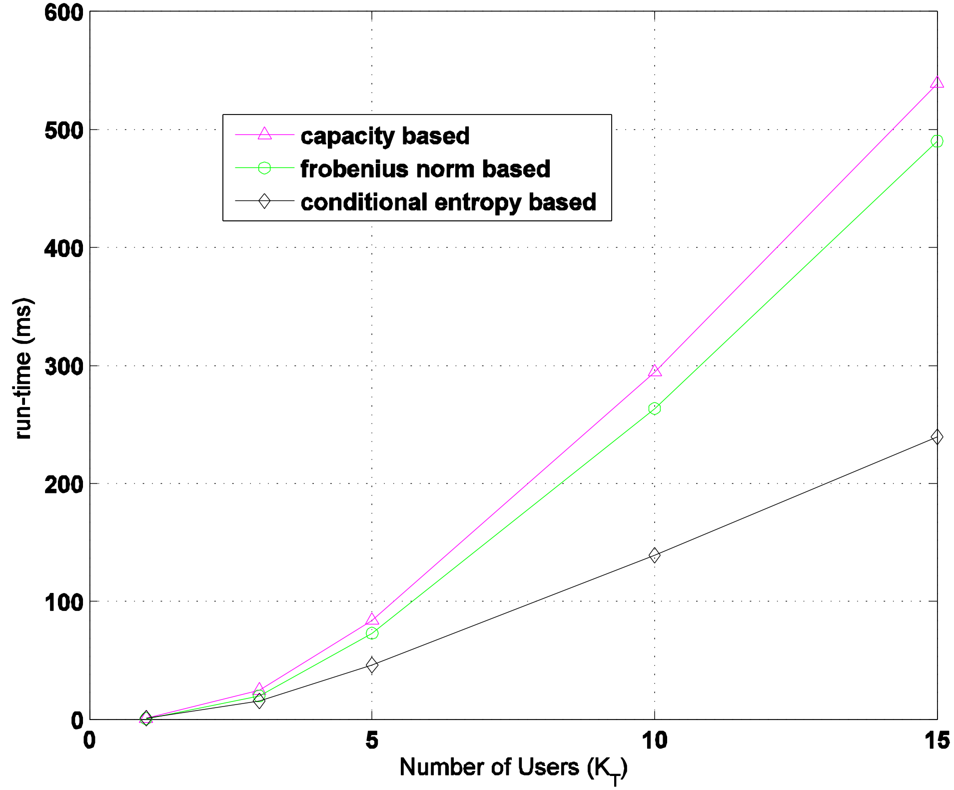
- Antenna selection can be viewed as a special case of multimode diversity [5]

$\boldsymbol{H}_k \in \mathbb{C}^{3\times 6}$ then $\boldsymbol{R}_k^H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ will select the antenna 1 and 2 from $\boldsymbol{H}_k$

- Use sum conditional entropy as a metric to select antennas, until maximum limit is reached (N_t) [5]
- Algorithm is initialized with selection of antenna having maximum channel energy
- Next user will be selected whose sum conditional entropy with the selected antenna is maximum
- Terminate algorithm when maximum antenna selection limit reached.

Simulation Results





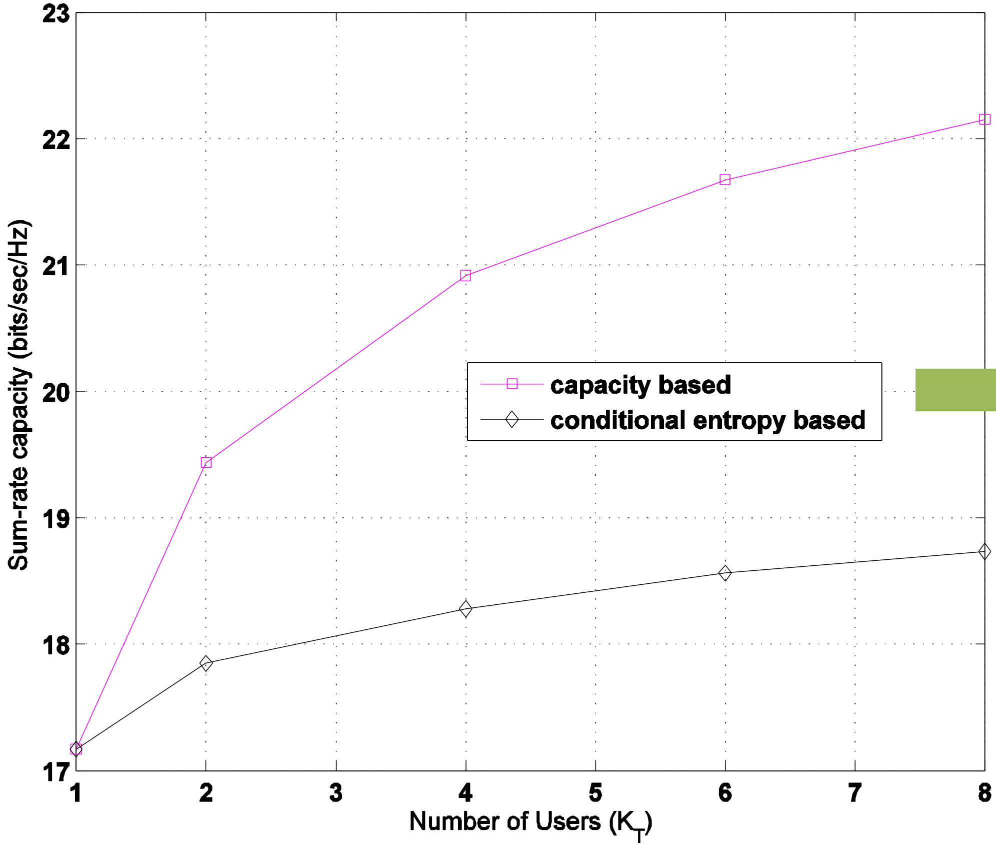
- Sum capacity plots are overlapping for all three algorithms
- Run-time reduction of more than 50% as compared to previous schemes, and this difference increases with number of users (K_T)

Conclusions

- Proposed conditional entropy based algorithm is lower in complexity and achieves greater or equal sum capacity as existing suboptimal algorithms for BC model
- Conditional entropy algorithm can be used in joint antenna and user selection system and offers more than 50% reduction in run-time as compared to existing suboptimal algorithms

Interference Alignment scheme

- The authors of [6] proposed a Interference Alignment Technique which achieves maximum DoF in interference system
- It was proved in [7] that DoF achieved in a 3-cell system is independent of number of users in each cell
- This gives an opportunity to use User scheduling to achieve multiuser diversity in addition with Spatial Multiplexing Gain



Performance loss as compared to capacity based algorithm

Future Work

- To study the effects of changing interference channels of user in ith cell on the selected users in the other two cells.
- To modify existing conditional entropy based algorithm to take care of above effects without computation of precoding matrices, or to come up with a new low computationally complex metric which could quantize the effects

References

[1] R.W. Heath et.al., "From Single User to Multiuser Communications: Shifting the MIMO Paradigm"

[2] Z. Shen, R. Chen, J. G. Andrews, R. W. Heath, and B. L. Evans, "Low complexity user selection algorithms for multiuser MIMO systems with block diagonalization," IEEE Trans. Signal Process., vol. 54, no. 9, pp. 3658–3663, Sep. 2006.

[3] A. Ben-Israel and T. N. E. Greville, Generalized Inverses: Theory and Applications. New York: Wiley, 1977.

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[5] R. Chen, Z. Shen, J. G. Andrews, and R. W. Heath, "Multimode transmission for multiuser mimo systems with block diagonalization," IEEE Trans. Signal Process., vol. 56, no. 7, p. 32943302, Jul. 2008.

[6] V. R. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the k-user interference channel," IEEE Trans. Inf. Theory, vol. 54, pp. 3425–3441, Aug. 2008.

[7] H. Park, S.-H. Park, H. Sung, and I. Lee, "Scheduling methods with mimo interference alignment for mutually interfering broadcast channels," in Proc. IEEE Globecom, Dec. 2010, pp. 1–5.

Questions