EC 303 (2021): Stochastic and Spatial Dynamics in Biology
Homework #1

Instructions

- **Deadline:** 18^{th} **August 2021 by 11:00 am** via Canvas (pdfs only).
- Discussion with friends is absolutely fine if they haven't yet completed the assignment. Final solution must be written independently, in your own words. Plagiarism will attract severe penalties.
- **Auditing students** are also required to submit the assignment, since these questions are important to understand further concepts.

Questions

1. **Galton Board**: There is intricate connection between random walk, binomial distribution, normal distribution and central limit theorem! This is very nicely demonstrated via Galton board. For example, see this: `https://galtonboard.com` (and use Google search for more). You don't need to understand all of it, but just explain in words the following: How does the Galton board capture random walk? (**10 Marks**.)

2. Consider a **biased random walk in one-dimension**, defined as follows: Let $x(t)$ be the location of organism at time t, with $x(0) = 0$. At every discrete time unit, assume that the probability of the organism turning right (p) is more than that of turning left (q, such that $p+q=1$). Analytically calculate the mean displacement and mean squared displacement as a function of time. (**10 Marks**).

3. **The two dimensional unbiased random walk** can also be defined as follows: An organism at location $\mathbf{r}(t)$ at time t makes two 'moves': (i) turns a random angle $\theta(t)$ where $\theta \sim \text{Uni}(-\pi, \pi)$ and (ii) walks a random distance $d(t)$ where d, also called jump length, are drawn from iids.

   Therefore,
   $$\mathbf{r}(t+1) = \mathbf{r}(t) + d(t)(\cos(\theta(t))\hat{\mathbf{x}} + \sin(\theta(t))\hat{\mathbf{y}})$$

   Now, consider the following jump length distributions:

   (a) $d \sim \text{Uni}(0, a)$: Uniform distribution between 0 and a.

   (b) $d \sim \text{Power}(x_{min}, \mu)$: The Power-law distribution with parameter μ is given by $f(d) = \frac{1}{N} d^{-\mu}$ where x_{min} is the minimum value of random variable, μ is called the power-law exponent, and N is the normalisation constant.

(A) Modify (and upload with your submission) the random walk code given so that it simulates a power-law jump length distribution, with uniform random turning angles as described above.[1] (**10 Marks.**)

(B) Assume $x_{min} = 1$. Show representative plots for $\mu = 1.5, 2.0, 2.5$ and 3.0. Comment on the qualitative nature of trajectories and how they change with μ. Likewise, produce representative plots for different values of a if the jump length distribution was uniform random distribution, for a few values of a. Finally, comment on whether trajectories of uniform distribution jumps yield qualitatively similar plots as power-law jumps. Explain why? (**10 Marks**).

(C) Compute MSD exponent α as a function of a, for the uniform distribution jumps. Consider a from 5 to 100, at interval of five. For each a, generate a long enough time series so that you can compute α accurately! (**20 Marks**).

(D) Compute MSD exponent α as a function of power-law exponent μ. Assume $x_{min} = 1$. Consider μ from 1.5 to 3.5, at interval of 0.1. Is the exponent $\alpha = 2$. For each μ, generate a long enough time series so that you can compute α accurately! (**20 Marks**).

(E) Compare and comment on results of C and D. (**10 Marks**).

4. Formulate a (discrete time, discrete space) random walk model for **an animal with home range**: to make it simple, assume that origin is the home of the animal. Do not add unnecessary complications, but just enough to capture home range. (No need to calculate any properties – just state the model very clearly, defining all symbols). (**10 Marks**).

5. Formulate a (discrete time, discrete space) random walk model for **an animal that has 'memory' in its movement**. Do not add unnecessary complications, but just enough to capture a simple form of memory. (No need to calculate any properties – just state the model very clearly, defining all symbols). (**10 Marks**).

[1] Note that you can use the R package called poweRlaw (`https://rdrr.io/cran/poweRlaw/man/dplcon.html`) to draw random numbers from a power-law. First, install the package by typing install.packages("poweRlaw") in the command prompt of RStudio. Then, load the library by typing 'library(poweRlaw)' in the command again. Then you can, for example, draw 10 random numbers with $x_{min} = 1$ and $\mu = 2$ by the command: rplcon(10,1,2)