Name - Gaurav Bhandari ①
Tutorial Sheet - 6 Sec - F Roll no. - 58
U. Roll no. - 2016747

Sol:- Minimum spanning tree is a subset of the edges of a connected edge-weighted undirected graph that connects all the vertices together without any cycles & with the minimum possible total edge weighted.

Application :-

i) Consider n stations are to be linked using a communication network and lying of ~~comm~~ communication link between any two stations involves a cost.

The ideal ~~graph~~ solution would be to extract a subgraph termed as minimum cost spanning tree.

ii) Suppose you want to construct highways or ~~rio~~ railroads spanning several cities then we can use the concept of minimum spanning trees.

iii) Designing LAN.

iv) Laying pipelines connecting offshore drilling sites, refineries, & consumer markets.

v) Suppose you meant to apply a set of houses o with:-

→ electric power.
→ Water.
→ telephone lines.
→ Sewage lines.

Sol2. - Time complexity of prim's algo : $O(|E| \log |V|)$

Space complexity of prim's algo : $O(V)$

Time complexity of Kruskal's algo : $O(|E| \log |E|)$

Space complexity of Kruskal's algo : $O(|V|)$

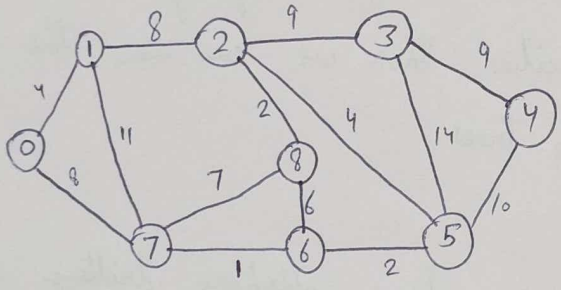
Time complexity of Dijstra's algo : $O(V^2)$

Space complexity of Dijkstra's algo : $O(V^2)$

Time complexity of Bellman ford's algo : $O(VE)$

Space complexity of Bellman ford's algo : $O(E)$.

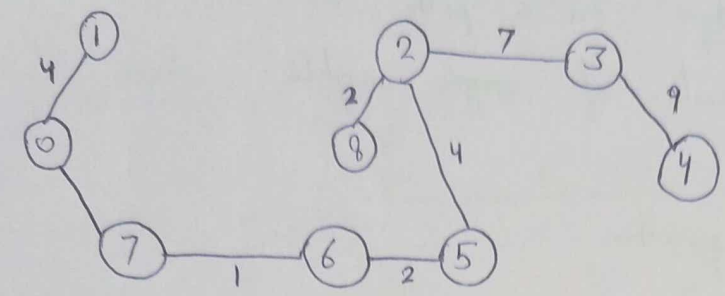
Sol 3 -



Kruskal's algo:

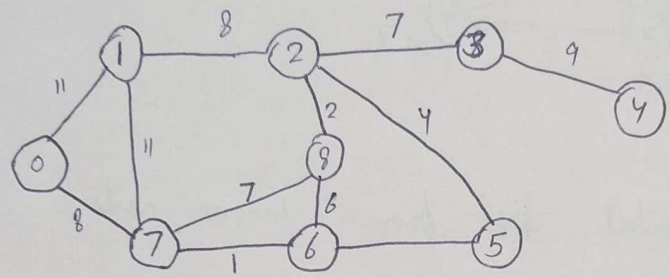
u	v	w	
6	7	1	✓
5	6	2	✓
2	8	2	✓
0	1	4	✓
2	5	4	✓
6	8	6	×
2	3	7	✓
7	8	7	×
0	7	8	✓
1	2	8	×

u	v	w	
4	3	9	✓
4	5	10	×
1	7	11	×
3	5	14	×



Weight = 1+ 2+2+ 4+ 4 +7+ 8+ 9 = 37

Prim's algorithm :-



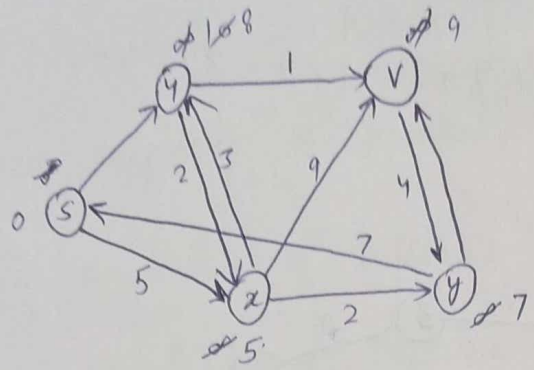
Weight = 4+ 8+ + 2 + 4 + 2+ 7+9 = 37.

Sol 4 = (i) The shortest path may change. The reason is there may be different number of ~~so~~ edges in different path from 's' to 't' for eg, let shortest path be of weight 15 and has edge 5 edges. Let there be another path with 2 edges and total weight 25. This weight of the shortest path is increased by $5V_{10}$ and it becomes $15 + 50$ weight of the other path is increased by $2^{V}{}_{10}$ & becomes $25 + 20$. So, the shortest path changes to the other path with weight ~~of~~ as 45.

(ii) If we multiply all edges weight by 60, the ~~sorted~~ shortest path doesn't change. The reason is simple, weights of all path from 's' to 't' get multiplied by same amount.

The number of edges on a path doesn't matter. It is like changing units of ~~weigh~~ weights.

Sol 5. Dijkstra Algorithm:



Node	Shortest dist from source node.
u	8
x	5
v	9
y	7

⇒ Bellman ford algo:

1st → s(0) u(10) v(∞) x(5) y(∞)

2nd → s(0) u(10) v(11) x(5) y(∞)

3rd → s(0) u(8) v(9) x(5) y(7)

4th → s(0) u(8) v(9) x(5) y(7)

graph doesn't have -ve cycle.

Final graph