MATHEMATICS - II
BT - 202

Department of Engineering Mathematics

FM 1001 ***Course Completion Unit Plan***

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CCUP- (BT-202) ENGINEERING MATHEMATICS-II

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UNIT- 1

ORDINARY DIFF EQn

→ Order and degree of differential eqn

- Order :- The order of differential equation is the order of the highest derivative appearing in the equation

- Degree :- The degree of differential eqn is the power of the highest order derivative appearing in the differential equation, when the equation is made free from radical sign and fraction.

- Ex :-

① $\frac{d^2y}{dx^2} + \frac{dy}{dx} + 6y = 0$

order = 2
Degree = 1

② $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 + 6y = 0.$

order = 2
degree = 3

③ $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^4 + \frac{dy}{dx} = 0$

order = 3
degree = 2

$$\frac{dy}{dx} + \frac{1}{\left(\frac{dy}{dx}\right)} + y = 0$$

$$\left(\frac{dy}{dx}\right)^2 + 1 + y\frac{dy}{dx} = 0$$

Order = 1
Degree = 2

$$\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)\right]^{3/2}$$

$$\left(\frac{d^2y}{dx^2}\right)^2 = \left[1 + \left(\frac{dy}{dx}\right)\right]^3$$

Order = 2
Degree = 2

Formation of Differential equation.

Elimanate A and B from the given curve $y = A\cos \log x + B\sin \log x$ and forms a diff. eqn. where A & B are constants

Now given curve

$$y = A\cos \log x + B\sin \log x \quad \text{———} \quad (1)$$

Now eq. (1) diff. Both sides w.r.to x.

जितने que. में variable होंगे ... eqn. होगी और उतने ही बार diff. करना है।

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$$\frac{dy}{dx} = A\left[-\sin \mu x\right]\mu + B.\left[\cos \mu x\right]\mu \quad \text{---} (2)$$

Now, Again diff. w.r.to. x.

$$\frac{d^2y}{dx^2} = -A\mu^2 \cos \mu x - B\mu^2 \sin \mu x \quad \text{---} (3)$$

$$= -\mu^2 \left[A \cos \mu x + B \sin \mu x\right]$$

then by eqn (1)

$$= -\mu^2 . y$$

$$\therefore \boxed{\frac{d^2y}{dx^2} + \mu^2 . y = 0}$$

Q. Find the diff. eqn of the curve $y = A\left[\cos(\log x)\right] + B \sin(\log x)$

Solve- Now given curve.

$$y = A\left[\cos(\log x)\right] + B \sin(\log x) \quad \text{---} (1)$$

diff. eq. (1) w.r.to x

$$\frac{dy}{dx} = \frac{A}{x}\left[-\sin(\log x)\right] + \frac{B}{x} \cos(\log x) \quad \text{---} (2)$$

~~diff. eq. (2) w.r.to. x.~~

$$\cancel{\frac{d^2y}{dx^2} = +A \frac{\cos(\log x)}{x^2 \times x} + \frac{B}{x^2 \times x}(-\sin(\log x))}$$

$$\frac{d^2y}{dx^2} = \frac{A}{x^3}\cos(\log x) + \frac{B}{x^3}\sin(\log x)$$

$$\frac{d^2y}{dx^2} = \frac{1}{x^3}\, y$$

$$\frac{d^2y}{dx^2} - \frac{1}{x^3}\, y = 0$$

① $x \cdot \frac{dy}{dx} = -A(\sin(\log x)) + B\cos(\log x)$ — ②

diff. eq. ② w.r.to. x

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot 1 = -\frac{A\cos(\log x)}{x} - \frac{B\sin(\log x)}{x}$$

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y$$

$$\therefore \boxed{x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0}$$

Q. Find the diff. eq. of the curve
$y = A\cdot e^{2x} + B\cdot e^{x} + C$.

Solve= Now given curve

$$y = A\cdot e^{2x} + B\cdot e^{x} + C \quad — ①$$

diff. eq. ① w.r.to. x

(11)

$$\frac{dy}{dx} = A e^{2x}\times 2 + B e^{x} \quad — (11)$$

last में constant
हटा कर y तथा $\frac{dy}{dx}$ में Ans. लाना है।

diff ② w.r.to. x.

$$\frac{d^2y}{dx^2} = 4A \cdot e^{2x} + Be^{x} \cancel{+C} \quad \text{—— (iii)}$$

diff. eq. ③ w.r.to. x.

$$\frac{d^3y}{dx^3} = 8A \cdot e^{2x} + Be^{x} \cancel{+C} \quad \text{—— ④}$$

→ eq. ④ is subtract by eq. ③

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} = 4Ae^{2x} \quad \text{—— ⑤}$$

→ eq. ③ is subtract by eq. ②.

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = 2Ae^{2x} \quad \text{—— ⑥}$$

then by eq. ⑤

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} = 2\left[2Ae^{2x}\right]$$

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} = 2\left[\frac{d^2y}{dx^2} - \frac{dy}{dx}\right] \quad \text{by eq. ⑥}$$

$$\boxed{\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0}$$

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Q. Find the diff eq. of the circle whose Centres are H and K and radius is a where H and K are constant

Solve ⇒ Now eq. of circle whose centres are H and K and centre is a.

$$(x-h)^2 + (y-k)^2 = a^2 \quad \text{--- (1)}$$

$$\cancel{x^2 + h^2 - 2xh + y^2 + k^2 - 2yk = a^2} \quad \text{--- (1)}$$

diff eq. (1) w.r.to. x

$$2x + 0 - 2h + 2y\frac{dy}{dx} + 0 - 2k = a^2$$

$$2x + 2y\frac{dy}{dx} - 2h - 2k = a^2 \quad \text{--- (2)}$$

~~diff eq (2) w.r.to x.~~

diff. eq. (1) w.r. to x

$$2(x-h) + 2(y-k)\frac{dy}{dx} = 0 \quad \text{--- (2)}$$

$$\cancel{2x - 2h + 2y - 2k = 0}$$

diff eq. (2) w.r.to. x.

$$\cancel{2(1) + 2\left(\frac{dy}{dx}\right)\frac{dy}{dx} = 0}$$

$$2 + 2\frac{d^2y}{dx^2} = 0$$

$$1 + \frac{d^2y}{dx^2} = 0 \quad \text{--- (3)}$$

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by parts

$$1 + (y-k)\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{dy}{dx} = 0$$

$$(y-k) = -\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]}{d^2y/dx^2}$$

then by eq^n (2)

$$(x-h) = -(y-k)\frac{dy}{dx}$$

$$(x-h) = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]\cdot\frac{dy}{dx}}{d^2y/dx^2}$$

Now sub. the value of $(x-h)$ and $(y-k)$ in eq. (1)

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^2\left(\frac{dy}{dx}\right)^2}{d^2y/dx^2} + \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^2}{d^2y/dx^2} = a^2$$

$$\frac{\left[1+(dy/dx)^2\right]^2\left[\left(\frac{dy}{dx}\right)^2+1\right]}{\left(d^2y/dx^2\right)^2} = a^2$$

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$$

Ans

Q.5 $y = e^x [A \cos x + B \sin x]$

11) $y = ax^3 + bx^2$

Q.6 $y = A \cdot \cos x^2 + B \sin x^2$

Q.7 $y = a \sin(x+b)$

// constant A & B

Q.8 $y = A \cdot e^{3x} + B \cdot e^{5x}$

a & b

Q.9 $y = ax + bx^2$

Q.10 $y = \frac{A}{r} + B$

Solve-7- Given curve is

$y = a \sin(x+b)$ —— ①

Now eq. ① diff. w.r.to. x

$$\frac{dy}{dx} = a \cos(x+b)$$

again diff. w.r.to. x.

$$\frac{d^2y}{dx^2} = -a \sin(x+b)$$

by eq. ①

$$\therefore \frac{d^2y}{dx^2} = -y$$

$$\boxed{\frac{d^2y}{dx^2} + y = 0}$$

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Solve - 10: Given curve is

$$u = \frac{A}{r} + B \quad \text{—— (1)}$$

eq. (1) diff. w.r.to r

$$\frac{du}{dr} = \frac{-A}{r^2} \quad \text{—— (2)} \qquad \boxed{A = -r^2 \frac{du}{dr}}$$

again diff. w.r.to r

$$\frac{d^2u}{dr^2} = \frac{+2A}{r^3} \quad \text{—— (3)}$$

put value of A in eq. (3)

$$\frac{d^2u}{dr^2} = \frac{2}{r^3}\left[-r^2 \frac{du}{dr}\right]$$

$$\frac{d^2u}{dr^2} = -\frac{2}{r}\left(\frac{du}{dr}\right)$$

$$\boxed{\frac{d^2u}{dr^2} + \frac{2}{r}\frac{du}{dr} = 0}$$

Solve - 11 ⇒ Given curve is

$$y = ax^3 + bx^2 \quad \text{—— (1)}$$

eq. (1) diff. w.r.to x

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$$\frac{dy}{dx} = 3ax^2 + 2bx \longrightarrow (2)$$

eq. (2) diff. w.r.to. x.

$$\frac{d^2y}{dx^2} = 6ax + 2b \longrightarrow (3)$$

Now eq. (3) multiplying both side by "x" and subtract eq. (2)

$$x\frac{d^2y}{dx^2} - \frac{dy}{dx} = 3ax^2 \longrightarrow (4)$$

Now eq. (2) multiplying both side by "x" and eq. (1) multiplying both side by "2" and the subtract

$$x\frac{dy}{dx} - 2y = ax^3 \longrightarrow (5)$$

$$\therefore \quad a = \frac{1}{x^2}\frac{dy}{dx} - \frac{2y}{x^3}$$

Now ~~sub.~~ Put the value of 'a' in eq. (4)

$$x\frac{d^2y}{dx^2} - \frac{dy}{dx} = 3x^2\left[\frac{1}{x^2}\frac{dy}{dx} - \frac{2y}{x^3}\right]$$

$$x\frac{d^2y}{dx^2} - \frac{dy}{dx} = 3\frac{dy}{dx} - \frac{6y}{x}$$

$$\boxed{x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = 0.}$$

★ Linear differential equation. / Leibnitz's linear differential equation

Now given diff eqⁿ is of the form.

① $\frac{dy}{dx} + Py = Q.$

→ then the given equation linear in y
→ where P and Q are constant or function of x only.
→ Intigrating factor.

$$\underline{I.F.} = e^{\int P dx}$$

then the solution of the equation

$$y\,(I.F.) = \int Q\,(I.F.)\,dx + c$$

OR

$$y \cdot [e^{\int P dx}] = \int Q \cdot (e^{\int P dx})\,dx + c$$

(ii) $\frac{dx}{dy} + Px = Q.$

→ then the given eqⁿ linear in x
→ where P and Q are constant or function of y only.

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Q 6 $y = A.\cos x^2 + B\sin x^2$ —— ①

eq. ① diff w.r.to x

$$\frac{dy}{dx} = -A\sin x^2 . 2x + B\cos x^2 . 2x \longrightarrow ②$$

eq. ② diff w. r. to x.

$$\frac{d^2y}{dx^2} = -A\left[2\sin x^2 + 2x\cos x^2\, 2x\right] + B\left[2\cos x^2 - 2x\sin x^2\, 2x\right]$$

$$= -2A\sin x^2 - 4x^2 A\cos x^2 + 2B\cos x^2 - 4x^2 B\sin x^2$$

$$= \left[-2A\sin x^2 + 2B\cos x^2\right] - 4x^2\left[A\cos x^2 + B\sin x^2\right]$$

$$= 2\left[-A\sin x^2 + B\cos x^2\right] - 4x^2 y.$$

by eq. ②

$$-A\sin x^2 + B\cos x^2 = \frac{dy}{dx} \times \frac{1}{2x}$$

$$\frac{d^2y}{dx^2} = 2 \times \frac{dy}{dx} \times \frac{1}{2x} - 4x^2 y.$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} . \frac{1}{x} + 4x^2 y = 0.$$

Ans

Q. 8 $y = A.e^{3x} + B.e^{5x}$ — ①

diff eq. ① w.r. to x.

$\frac{dy}{dx} = 3Ae^{3x} + 5Be^{5x}$ — ②

diff eq. ② w.r. to. x

$\frac{d^2y}{dx^2} = 9Ae^{3x} + 25Be^{5x}$ — ③

elinimate eq. ② & ③

$\frac{dy}{dx} = 3Ae^{3x} + 5Be^{5x}$ × 5

$\frac{d^2y}{dx^2} = 9Ae^{3x} + 25Be^{5x}$

$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} = -6Ae^{3x}$ — ④

elinimate eq. ① & ②.

$y = A.e^{3x} + B.e^{5x}$ × 5

$\frac{dy}{dx} = 3Ae^{3x} + 5Be^{5x}$

$\frac{dy}{dx} - 5y = -2Ae^{3x}$ — ⑤

$\frac{-1}{2e^{3x}}\frac{dy}{dx} + \frac{5y}{2e^{3x}} = A.$

put the Value of A in eq. ④

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$$\frac{d^2y}{dx^2} - \frac{5dy}{dx} = 6e^{3x}\left[\frac{1}{2e^{3x}}\frac{dy}{dx} + \frac{5y}{2e^{3x}}\right].$$

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} = \frac{6e^{3x}}{2e^{3x}}\cdot\frac{dy}{dx} + \left(\frac{-6e^{3x}}{2\cdot e^{3x}}\cdot 5y\right)$$

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 3\frac{dy}{dx} + 15y = 0$$

$$\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0$$

Ans

Q. 9 $y = ax + bx^2$ — ①

diff eq. ① w.r.to. x

$$\frac{dy}{dx} = a + 2bx \quad \text{— ②}$$

diff w.r.to. x.

$$\frac{d^2y}{dx^2} = 2b \quad \text{— ③}$$

eliniminate eq. ① and ②

$$y = ax + bx^2$$

$$\frac{dy}{dx} = a + 2bx \quad \times x$$

$$x\frac{dy}{dx} - y = bx^2$$

$$b = \frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2}$$ put value of 'b' in eq. ③

$$\frac{d^2y}{dx^2} = \frac{2}{x}\frac{dy}{dx} - \frac{2y}{x^2}$$

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$\frac{d^2y}{dx^2} - \frac{2}{x}\frac{dy}{dx} + \frac{2y}{x^2} = 0$ **Ans**

Q.1 $(x+1)\frac{dy}{dx} - y = e^x (x+1)^2$

multiply both sides by $\frac{1}{1+x}$

$\frac{dy}{dx} - y \cdot \frac{1}{1+x} = e^x \cdot (1+x)$

then the given eqn linear in 'y'

where $P = -\frac{1}{1+x}$, $Q = e^x(1+x)$

Now,

$I.F. = e^{\int P dx} = e^{\int -\frac{1}{1+x} dx}$

$= e^{-\log(1+x)} = e^{\log(1+x)^{-1}}$

$= (1+x)^{-1} = \frac{1}{1+x}$

then the soln of the eqn.

$y.(I.F.) = \int Q \, (I.F.) \, dx + c$

$y . \frac{1}{1+x} = \int e^x \cancel{(1+x)} \times \frac{1}{\cancel{(1+x)}} + c$

$y \times \frac{1}{1+x} = e^x + c$

$y = (e^x + c)(1+x)$

Ans

Q.2 $\frac{dy}{dx} + 2y \tan x = \sin x$

Q.3 $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$

IMP Q.4 $(1+y^2)\,dx = (\tan^{-1}y - x)\,dy$

IMP Q.5 $(1+y^2) + (x - e^{\tan^{-1}y})\frac{dy}{dx} = 0$

Q.6 $\cos^2 x \frac{dy}{dx} + y = \tan x$

Q.7 $(x+y+1)\cdot\frac{dy}{dx} = 1.$

Solve-4 Given diff. equation

$$(1+y^2)\,dx = (\tan^{-1}y - x)\,dy$$

$$\frac{dx}{dy} = \left[\frac{\tan^{-1}y - x}{1+y^2}\right] \Rightarrow \frac{\tan^{-1}y}{1+y^2} - \frac{x}{1+y^2}$$

$$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$$

then the given equation linear in 'x'

where $P = \frac{1}{1+y^2}$, $Q = \frac{\tan^{-1}y}{1+y^2}$

Now, $I.F. = e^{\int P\,dy} = e^{\int \frac{1}{1+y^2}dy}$

$I.F. = e^{\tan^{-1}y}$

then the sol^n of the eq^n

$$x.(I.F.) = \int Q\,(I.F.)\,dy + C$$

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$$x \cdot e^{\tan^{-1}y} = \int \frac{\tan^{-1}y}{1+y^2} \cdot e^{\tan^{-1}y} \cdot dy + c$$

Putting $\tan^{-1}y = t$

$$\frac{1}{1+y^2} dy = dt$$

$$= \int t \cdot e^t \cdot dt + c$$

$$\left\{ \int I \cdot II \, dx = I \int II \, dx - \int \left[\frac{d}{dx} I \cdot \int II \, dx\right] dx \right.$$

$$x \cdot e^{\tan^{-1}y} = t \cdot e^t - \int 1 \cdot e^t \cdot dt + c$$

$$= t \cdot e^t - e^t + c$$

$$x \cdot e^{\tan^{-1}y} = \tan^{-1}y \cdot e^{\tan^{-1}y} - e^{\tan^{-1}y} + c$$

$$x = \frac{e^{\tan^{-1}y}\left[\tan^{-1}y - 1\right]}{e^{\tan^{-1}y}} + \frac{c}{e^{\tan^{-1}y}}$$

$$x = \tan^{-1}y - 1 + c\, e^{-\tan^{-1}y}$$

Ans

solve (5) $(1+y^2) + (x - e^{-\tan^{-1}y}) \frac{dy}{dx} = 0$

$$(1+y^2) = -\left[x - e^{-\tan^{-1}y}\right] \frac{dy}{dx}$$

$$\frac{dx}{dy} = -\frac{\left[x - e^{-\tan^{-1}y}\right]}{1+y^2}$$

$$\frac{dx}{dy} = \frac{-x}{1+y^2} + \frac{e^{tan^{-1}y}}{1+y^2}$$

$$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{e^{-tan^{-1}y}}{1+y^2}$$

then the given equation is linear in 'x'
where

$$P = \frac{1}{1+y^2}, \quad Q = \frac{e^{-tan^{-1}y}}{1+y^2}$$

Now,

$$IF = e^{\int P dy} = e^{\int \frac{1}{1+y^2} dy} = e^{tan^{-1}y}$$

then the sol^n of the eq^n.

$$x.(I.F.) = \int Q \, (I.F.) \, dy + c$$

$$x. e^{tan^{-1}y} = \int \frac{\cancel{e^{-tan^{-1}y}}}{1+y^2} . \cancel{e^{tan^{-1}y}} \, dy + c$$

~~put~~

$$x. e^{tan^{-1}y} = tan^{-1}y + c$$

$$x = tan^{-1}y \; e^{-tan^{-1}y} + c . e^{-tan^{-1}y}$$

solve (4) $(x+y+1)\frac{dy}{dx} = 1$

$$\frac{dy}{dx} = \frac{1}{x+y+1}, \quad \frac{dx}{dy} = x+y+1$$

$$\cancel{\frac{dy}{dx} = \frac{1}{x} +} \quad \frac{dx}{dy} - x = y+1$$

* $\frac{dy}{dx} + Py = Q$

* $\frac{dx}{dy} + Px = Q$

then the eqⁿ is linear in 'x'
where $p = -1$ and $Q = y+1$

NOW $IF = e^{\int P dy} = e^{\int -1\, dy} = e^{-y}$

then the solⁿ of equation is
$$x \cdot (IF) = \int Q \cdot (IF)\, dy + C$$

$$x\, e^{-y} = \int (y+1)(e^{-y})\, dy + C$$

$$x \cdot e^{-y} = (y+1)(-e^{-y}) \int [1 \cdot (-e^{-y})]\, dy + C$$

$$x \cdot e^{-y} = (y+1)(-e^{-y})(e^{-y}) + C$$

$$x = \frac{(y+1)(-e^{-y})\cancel{(e^{-y})}}{\cancel{(e^{-y})}} + C \cdot e^{y}$$

$$x = (y+1)(-e^{-y}) + C \cdot e^{y}$$

Ans

Q.6 $\cos^2 x \frac{dy}{dx} + y = \tan x$

$$\frac{dy}{dx} + \frac{y}{\cos^2 x} = \frac{\tan x}{\cos^2 x}$$

$$\frac{dy}{dx} + \frac{y}{\cos^2 x} = \frac{\tan x}{\cos^2 x}$$

then eqⁿ is linear in 'y'
where $p = \sec^2 x$ ~~$\frac{1}{\cos^2 x}$~~, $Q = \tan x \sec^2 x$ ~~$\frac{\tan x}{\cos^2 x}$~~

Now $IF = e^{\int P dx} = e^{\int \frac{1}{\cos^2 x} dx} = e^{\int \sec^2 x\, dx}$

~~$IF = e^{-2\sin x \log(\cos^2 x)}$~~ $IF = e^{\tan x}$

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then the soln of eqn is

$$y \cdot I.F. = \int Q\,(I.F.)\,dx + c.$$

$$\cancel{y \cdot e^{-2\sin x \cdot \log(\cos^2 x)} = \int \frac{\tan x}{\cos^2 x}\, e^{-2\sin x \log(\cos^2 x)}\, dx + c}$$

$$\cancel{y \cdot e^{-2\sin x \cdot \log(\cos^2 x)} =}$$

$$y \cdot e^{\tan x} = \int \tan x \sec^2 x\, e^{\tan x}\, dx + c.$$

then putting $\tan x = t$

$\sec^2 x\, dx = dt$

$$y \cdot e^{\tan x} = \int t \cdot e^t\, dt + c$$

$$y \cdot e^{\tan x} = t e^t - 1 e^t + c$$

$$y \cdot e^{\tan x} = \tan x\, e^{\tan x} - e^{\tan x} + c$$

$$y = \tan x - 1 + c e^{-\tan x}$$

Ans

Q.9. $x\cos x \frac{dy}{dx} + y(x\sin x + \cos x) = 1.$

dividing by $x\cos x$.

$$\frac{dy}{dx} + y\frac{(x\sin x + \cos x)}{x\cos x} = \frac{1}{x\cos x}$$

$$\frac{dy}{dx} + y\left[\tan x + \frac{1}{x}\right] = \frac{1}{x}\sec x$$

then the given eqⁿ is linear in 'y'

where,

$$P = \tan x + \frac{1}{x} \qquad Q = \frac{1}{x}\sec x$$

$$\text{I.F} = e^{\int P\,dx} = e^{\int[\tan x + \frac{1}{x}]}$$

$$= e^{\log\sec x + \log x} = e^{\log \sec x \cdot x} = x\cdot\sec x$$

then the solⁿ of the eqⁿ $\{\log m\cdot n = \log m + \log n$

$$y.(\text{IF}) = \int Q\,(\text{IF})\,dx + c$$

$$y.(x.\sec x) = \int \frac{1}{x}\sec x\,(x.\sec x)\,dx + c$$

$$= \int \sec^2 x\,dx + c$$

$$\boxed{y.x.\sec x = \tan x + c}$$

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Q.12 $\sin x \frac{dy}{dx} + 2y = \tan^3\left(\frac{x}{2}\right)$

dividing by '$\sin x$' both the sides.

$$\frac{dy}{dx} + \frac{2y}{\sin x} = \frac{\tan^3(x/2)}{\sin x}$$

~~then the given eqn is~~

$$\frac{dy}{dx} + 2y\ \text{cosec}\, x = \frac{\sin^3 x/2}{\cos^3 x/2\,(2\sin x/2 \times \cos x/2)}$$

$$[\sin x = 2\sin x/2 \cdot \cos x/2]$$

$$\frac{dy}{dx} + 2\,\text{cosec}\, x\, y = \frac{1}{2}\tan^2(x/2)\cdot\sec^2(x/2)$$

then the given eqn is linear in 'y' where,

$$P = 2\,\text{cosec}\, x \quad , \quad Q = \frac{1}{2}\tan^2(x/2)\cdot\sec^2(x/2)$$

$$\text{I.F.} = e^{\int P dx} = e^{\int 2\,\text{cosec}\, x\, dx} = e^{2\log(\tan x/2)}$$

$$\text{IF} = e^{\log\tan^2(x/2)} \quad \therefore \text{I.F} = \tan^2(x/2)$$

then the soln of eqn

$$y\cdot(\text{IF}) = \int Q\cdot(\text{IF})\,dx + C$$

$$y\cdot\tan^2(x/2) = \int\frac{1}{2}\tan^2(x/2)\cdot\sec^2(x/2)\cdot\tan^2(x/2)\,dx + C$$

$$y\cdot\tan^2(x/2) = \frac{1}{2}\int\tan^4(x/2)\cdot\sec^2(x/2)\,dx + C$$

Equations reducible to linear diff eqn form

{Bernoulli's Diff eqn form}

the given eqn is of the form

$$\frac{dy}{dx} + Py = Q\,y^n \quad ①$$

Now multiplying both side by $\left(\frac{1}{y^n}\right) / (y^{-n})$

$$y^{-n}\frac{dy}{dx} + P.\,y^{-n+1} = Q$$

putting $y^{-n+1} = t$

$$(-n+1)\,y^{-n}\frac{dy}{dx} = \frac{dt}{dx}$$

$$y^{-n}\frac{dy}{dx} = \frac{1}{(-n+1)}\frac{dt}{dx}$$

Substitute in eqn ①

$$\frac{1}{(-n+1)}\cdot\frac{dt}{dx} + P.\,t = Q$$

$$\frac{dt}{dx} + (-n+1)\,P.\,t = (-n+1)\,Q$$

then the given eqn linear in 't'

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Q.1 $x\frac{dy}{dx} + y \log y = x y e^{x}$

devide by 'x'

$$\frac{dy}{dx} + \frac{y}{x}\log y = y e^{x}$$

deviding both side by 'y'

$$\frac{1}{y}\frac{dy}{dx} + \frac{1}{x}\log y = e^{x} \quad \text{--- (1)}$$

Now putting $\log y = t$.

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{dt}{dx}$$

Substitute in eq. (1).

$$\frac{dt}{dx} + \frac{1}{x}t = e^{x}$$

then the given eq^n linear in 't'
where,

$$P = \frac{1}{x}, \quad Q = e^{x}$$

$$IF = e^{\int P dx} = e^{\log x} = x$$

$$\boxed{I.F. = x}$$

then the sol^n of eq^n is

ILATE → inverse, Log, algebric, trignometric, exponential

For intigration by parts

जो पहले आएगा उसको difrential करेंगे

$t(I.F.) = \int \phi (I.F)\, dx + c$

$t \cdot x = \int e^x \cdot x\, dx + c$

$t \cdot x = x \cdot e^x - \int 1 \cdot e^x dx + c$

$t \cdot x = x e^x - e^x + c$

$\log y \cdot x = x \cdot e^x - e^x + c$

Q.2 $(\sec x \cdot \tan x \cdot \tan y - e^x)dx + \sec x \cdot \sec^2 y\, dy = 0$

Q.3 $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x) e^x \cdot \sec y$

Q.4 $y \log y \frac{dx}{dy} + x - \log y = 0$

Q.5 $xy - \frac{dy}{dx} = y^3 \cdot e^{-x^2}$

Q.6 $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$

Q.7 $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$

Solve⇒2 $(\sec x \cdot \tan x \cdot \tan y - e^x)dx + \sec x \cdot \sec^2 y\, dy = 0$

$\frac{\sec x \cdot \tan x \tan y - e^x}{\sec x \sec^2 y} = -\frac{dy}{dx}$

$$\frac{\tan x \tan y \times \sec x - e^x}{\sec x \cdot \sec^2 y} = -\frac{dy}{dx}$$

$$\tan x \tan y - \frac{e^x}{\sec x} = -\frac{dy}{dx} \cdot \sec^2 y$$

$$\sec^2 y \frac{dy}{dx} + \tan x \tan y = \frac{e^x}{\sec x} \quad \text{--- (1)}$$

putting $\tan y = t$

$$\sec^2 y \frac{dy}{dx} = \frac{dt}{dx} \quad \text{--- (2)}$$

Sub. eq. (1) by (2)

$$\frac{dt}{dx} + \tan x \cdot t = \frac{e^x}{\sec x}$$

then the eq^n is linear in 't'

where.

$P = \tan x$, $Q = \frac{e^x}{\sec x}$

Now

$$I.F = e^{\int P dx} = e^{\int \tan x \, dx} = e^{\sec^2 x}$$

$$t(I.F) = \int Q \cdot I.F \, dx + c$$

$$t \, e^{\sec^2 x} = \int \frac{e^x e^{\sec x}}{\sec x} dx + c$$

* Exact differential equation.

The necessary and sufficient condition that the ordinary diff. eq^n $Mdx + Ndy = 0$ be exact if satisfy the following condition.

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

then the given eq^n is exact.
then the solution of the eq^n

$$\int M\,dx + \int N\,dy = c$$

where y = costant
(taking only those terms of N which do not contain 'x')

Questions.

1 $y.e^{xy}.dx + [x.e^{xy} + 2y]\,dy = 0$

Solve :- the given eq^n is of the form
$Mdx + N\,dy = 0.$

$M = y.e^{xy}$, $N = x.e^{xy} + 2y.$

then for exactness,

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}[y.e^{xy}]$$

$$= e^{xy}.1 + y.e^{xy}.x$$

$$\frac{\partial M}{\partial y} = e^{xy}[1 + y.x]$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left[x \cdot e^{xy} + 2y\right]$$

$$= x \cdot e^{xy} \cdot y + e^{xy} \cdot 1 + 0$$

$$\frac{\partial N}{\partial x} = e^{xy}\left[xy + 1\right]$$

$$\therefore \frac{\partial N}{\partial x} = \frac{\partial m}{\partial y}$$

then the given equn is equal
then the soln of eqn

$$\int_{y=\text{const}} M\,dx + \int N\,dy = c$$

(taking only those term of N which do not contain x.)

$$\int y \cdot e^{xy}\,dx + \int 2y\,dy = c \qquad \left\{ N = \cancel{x \cdot e^{xy}} + 2y \right.$$

इसको term नहीं लेना है

$$y \cdot \frac{e^{xy}}{y} + 2\frac{y^2}{2} = c$$

$$\boxed{e^{xy} + y^2 = c}$$

Ans

Q Imp

$$\left[1 + e^{x/y}\right] dx + e^{x/y}\left[1 - \frac{x}{y}\right] dy = 0$$

Solve ⇒ the eqn is in the form of

$$M\,dx + N\,dy = 0$$

$$M = \left(1 + e^{x/y}\right), \qquad N = e^{x/y}\left[1 - \frac{x}{y}\right]$$

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then for exactness

$$\frac{\partial m}{\partial y} = \frac{\partial}{\partial y}\left[1+e^{x/y}\right]$$

$$= -e^{x/y}\, x y^{-2} \Rightarrow -\frac{x}{y^2} e^{x/y}$$

$$\cancel{\frac{\partial N}{\partial x} = \frac{d}{dx}\left[e^{x/y}\left[1-\frac{x}{y}\right]\right]}$$

$$\cancel{e^{x/y}}$$

$$\frac{\partial N}{\partial x} = \frac{d}{dx}\left[e^{x/y} - e^{x/y}\cdot\frac{x}{y}\right]$$

$$= e^{x/y}\frac{1}{y} - \left[e^{x/y}\frac{1}{y} + \frac{x}{y}e^{x/y}\frac{1}{y}\right]$$

$$\frac{\partial N}{\partial x} = -e^{x/y}\frac{x}{y^2}$$

$$\therefore \frac{\partial m}{\partial y} = \frac{\partial n}{\partial x}$$

then the given eqn is equal
then the soln of eqn

$$\int M dx + \int N dy = C.$$

$y = $ costa. (taking only those terms of N which do not contain (x))

$$\int\left[1+e^{x/y}\right]dx + \int 0\, dy = c$$

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$$x + \frac{e^{x/y}}{1/y} = c$$

$$\boxed{x + y \cdot e^{x/y} = c}$$

Ans

Questions :-

Q.3. $y \cdot \sin 2x \, dx - (y^2 + \cos^2 x) dy = 0$

Q.4. $(5x^4 + 3x^2y^2 - 2xy^3) dx + (2x^3y - 3x^2y^2 - 5y^4) dy = 0$

Q.5. $(2xy \cdot \cos x^2 - 2xy + 1) dx + [\sin x^2 - x^2 + 3] dy = 0$

Q.6. $(y^2 e^{xy^2} + 4x^3) dx + (2xye^{xy^2} - 3y^2) dy = 0$

Q.7. $[1 + \log(xy)] dx + \left(1 + \frac{x}{y}\right) dy = 0$

Q.8 $(2xy - 3x^2) dx + (x^2 - 2y) dy = 0$

Solve-5 The given eq^n is in the form

$$M\,dx + N\,dy = 0$$

$$M = (2xy \cdot \cos x^2 - 2xy + 1)$$

$$N = (\sin x^2 - x^2 + 3)$$

then for exactness

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$$\frac{\partial m}{\partial y} = \frac{\partial}{\partial y}\left[2xy \cos x^2 - 2xy + 1\right]$$

$$= 2x \cos x^2 - 2x$$

$$\frac{\partial n}{\partial x} = \frac{\partial}{\partial x}\left[\sin x^2 - x^2 + 3\right]$$

$$= \cos x^2 \cdot 2x - 2x.$$

$$\therefore \frac{\partial m}{\partial y} = \frac{\partial N}{\partial x}$$

then the given eq^n is exact

then the soln of eq^n:

$$\underset{y = \text{costant}}{\int M\,dx} + \int N\,dy = C$$

(tonking only those term of N which donot contain x)

$$\int \left[2xy \cdot \cos x^2 - 2xy + 1\right] dx + \int 3\,dy = C$$

Int. by parts

$$\cancel{\left[\frac{2x^2 y}{2}\left(-\sin x^2\right)\frac{x^3}{3} - \frac{2x^2 y}{2} + x\right] + 3y = C}$$

$$\cancel{-\frac{x^5 y}{3}\sin x^2}$$

$$2xy \frac{\sin x^2}{2x} - \int\left[2y \times \frac{\sin x^2}{2x}\right] dx + 3y = C$$

$$y \sin x^2 -$$

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Q. 7 $[1+\log xy]\,dx + [1+\frac{x}{y}]\cdot dy = 0.$

the given eq. is of form of

$$M dx + N dy = 0$$

where, $M = 1+\log xy$, $N = 1+\frac{x}{y}$

then for exactness.

$$\frac{\partial m}{\partial y} = \frac{\partial}{\partial y}(1+\log xy) = 0 + \frac{1}{xy}\cdot x = \frac{1}{y}.$$

$$\frac{\partial n}{\partial x} = \frac{\partial}{\partial x}\left[1+\frac{x}{y}\right] = \frac{1}{y}$$

$$\therefore \frac{\partial m}{\partial y} = \frac{\partial n}{\partial x}$$

then the given eq^n is exact

then the sol^n of the eq^n

$$\int m\,dx + \int N\,dy = c$$

y = const. (taking only those terms of N which do not contain x)

$$\int [1+\log xy]\,dx + \int 1\,dy = c$$

{int. by part

$$x + \log xy \cdot x - \int \frac{1}{xy}\cdot y \cdot x\,dx + y = c$$

$$\cancel{x} + \log xy \cdot x - \cancel{x} + y = c$$

$$\boxed{\log xy \cdot x + y = c}$$

* Equation reducible to Exact diff. eqⁿ form :-

A linear diff. eqⁿ.

$$M\,dx + N\,dy = 0$$

which is not exact but can be made exact by multiplying some function of 'x' and 'y' (the function is called integrating factor of diff. eqⁿ)

Rule-1- Integrating factor (I.F.) formed by <u>inspection</u> method :-

(i) $d(xy) = x\,dy + y\,dx$

(ii) $d\left[\dfrac{x}{y}\right] = \dfrac{y\,dx - x\,dy}{y^2}$

(iii) $d\left[\dfrac{y}{x}\right] = \dfrac{x\,dy - y\,dx}{x^2}$

(iv) $d\left[\tan^{-1}\dfrac{x}{y}\right] = \dfrac{y\,dx - x\,dy}{x^2+y^2}$

(v) $d\left[\tan^{-1}\dfrac{y}{x}\right] = \dfrac{x\,dy - y\,dx}{x^2+y^2}$

(vi) $d[\log(xy)] = \dfrac{x\,dy + y\,dx}{xy}$

(vii) $d[\log(x^2+y^2)] = \dfrac{2(x\,dx + y\,dy)}{x^2+y^2}$

(viii) $d\left[\log\left(\dfrac{x}{y}\right)\right] = \dfrac{y\,dx - x\,dy}{xy}$

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Questions :-

(2) $(x^2+y^2+x)\,dx - (2x^2+2y^2-y)\,dy = 0$

(3) $(1+xy)\,y\,dx + (1-xy)\,x\,dy = 0$

(4) $y\,(2x^2y+e^x)\,dx - (e^x+y^3)\,dy = 0$

Solve-4- Now Given eqn is of the form

$$M\,dx + N\,dy = 0.$$

$M = 2y^2x^2 + ye^x \qquad N = -e^x - y^3$

then for exactness

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\left[2x^2y^2 + ye^x\right] = 4x^2y + e^x$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left[-e^x - y^3\right] = -e^x$$

$$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

then the given eqn is not exact.

Now given eqn will be exact if we have multiplying I.F. by the Inspection method.

$$2x^2y^2\,dx + ye^x\,dx - e^x\,dy - y^3\,dy = 0$$

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$$y^2(2x^2\,dx - y\,dy) + y e^x\,dx - e^x\,dy = 0$$

Now multip. I.F. $\left(\frac{1}{y^2}\right)$

$$2x^2\,dx - y\,dy + \frac{[y e^x\,dx - e^x\,dy]}{y^2} = 0$$

$$\frac{2}{3}d[x^3] - \frac{1}{2}d[y^2] + d\left[\frac{e^x}{y}\right] = 0$$

then the Int. both side

$$\frac{2}{3}x^3 - \frac{1}{2}y^2 + \frac{e^x}{y} = 0$$

Ans

Q.2 Now given eq^n is of the form
Solve =>
Check

$$M\,dx + N\,dy = 0$$

$M = (x^2+y^2+x)$, $N = -(2x^2+2y^2-y)$

then for exactness

$$\frac{dM}{dy} = \frac{d}{dy}[x^2+y^2+x] = 2y$$

$$\frac{dN}{dx} = \frac{d}{dx}[-(2x^2+2y^2-y)] = -4x$$

$$\therefore \frac{dM}{dy} \neq \frac{dN}{dx}$$

then the given eq^n is not exact

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$(x^2+y^2+x)\,dx-(2x^2+2y^2-y)\,dy=0$

$\Rightarrow (x^2+y^2)\,dx+x\,dx-2(x^2+y^2)\cdot dy+y\,dy=0$

$\Rightarrow (x^2+y^2)\,[dx-2dy)+x\,dx+y\,dy=0$

the IF $\left(\frac{1}{x^2+y^2}\right)$ multi.p inthe egiven eq^n.

$$dx-2dy+\frac{x\,dx+y\,dy}{x^2+y^2}=0$$

$$dx-2\,d(y)+d\left(\frac{\log(x^2+y^2)}{2}\right)=0$$

the integ. both side

$$x-2y+\frac{\log(x^2+y^2)}{2}=c$$

Ans

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Rule-2 :

when $Mx + Ny \neq 0$
and the eqn is homogeneous, then
the diff. eqn $M dx + N dy = 0$ is

$$\frac{1}{Mx + Ny}$$

Rule-3 :

* If the diff. eqn $M dx + N dy = 0$ has the form $f_1(xy)\, y\, dx + f_2(xy)\, x\, dy = 0$
and $Mx - Ny \neq 0$ then the I.F. is

$$\frac{1}{Mx - Ny}$$

Rule-4 :

If the eqn $M dx + N dy = 0$ and we have $\frac{1}{N}\left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right] = f(x)$ (is a func of 'x' only)

then integrating factor is $IF = e^{\int f(x)dx}$

Rule-5 :-

The given eqn $M dx + N dy = 0$ and
we have $\frac{1}{M}\left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right] = f(y)$ (is a function of 'y' only)

then IF is $= e^{\int f(y)dy}$

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Q.1 $x^2y\,dx - (x^3+y^3)\,dy = 0$

The given eq^n is of the form
$$M\,dx + N\,dy = 0$$
where, $M = x^2y$, $N = -x^3-y^3$
then for exactness
$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(x^2y) = x^2$$
$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(-x^3-y^3) = -3x^2$$
$$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$
then the given eq^n is not exact.

→ Now the given eqn will be exact. If we have multiplying I.f.

Now I.F. $= \dfrac{1}{Mx+Ny} = \dfrac{1}{\cancel{x^3y}-\cancel{x^3y}-y^4} = \dfrac{-1}{y^4}$

Now, multiplying I.f. $\left(-\dfrac{1}{y^4}\right)$ in given diff. eq^n:
$$\left[-\frac{x^2}{y^3}\right]dx + \left[\frac{x^3}{y^4}+\frac{1}{y}\right]dy = 0$$
where,
$$M = -\frac{x^2}{y^3}, \quad N = \frac{x^3}{y^4}+\frac{1}{y}$$
then for exactness.
$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(-x^2\cdot y^{-3}) = 3x^2y^{-4} = \frac{3x^2}{y^4}$$

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$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left[x^3 y^{-4} + y^{-1}\right]$$

$$= \frac{3x^2}{y^4}$$

the given eqn is equal and exact then the soln of the eqn

$$\int_{y=const.} M\,dx + \int N\,dy = c$$

(taking only those terms of N which do not contain x)

$$\int\left(\frac{-x^2}{y^3}\right)dx + \int \left[\frac{1}{y}\right] dy = c$$

$$\frac{-1}{y^3}\cdot\frac{x^3}{3} + \log y = c$$

Ans

Q.2 $y(1+xy)\,dx + x(1-xy)\,dy = 0$

Now given eqn is of the form

$$M\,dx + N\,dy = 0$$

where

$$M = y(1+xy), \quad N = x(1-xy)$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\left[y[1+xy]\right] = \frac{\partial}{\partial y}\left[y + xy^2\right]$$

$$\frac{\partial M}{\partial y} = 1 + 2xy$$

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$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}[x(1-xy)] = \frac{\partial}{\partial x}(x - x^2y)$$

$$\frac{\partial N}{\partial x} = 1 - 2xy$$

$$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

then this eq^n is not exact.

Now $I.F. = \frac{1}{Mx - Ny} = \frac{1}{\cancel{xy} + x^2y^2 - \cancel{xy} + x^2y^2}$

$$= \frac{1}{2x^2y^2} = \frac{1}{x^2y^2} \quad \left\{ \text{ignore} \left(\frac{1}{2}\right) \right.$$

Now multiplying IF $\left(\frac{1}{x^2y^2}\right)$ in given eq^n

$$\left[\frac{1}{x^2y} + \frac{1}{x}\right]dx + \left[\frac{1}{xy^2} - \frac{1}{y}\right]dy = 0$$

Now

$$M = \frac{1}{x^2y} + \frac{1}{x}, \qquad N = \frac{1}{xy^2} - \frac{1}{y}$$

$$\frac{\partial M}{\partial y} = \frac{-1}{x^2y^2}, \qquad \frac{\partial N}{\partial x} = \frac{-1}{x^2y^2}$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

then the given eq^n is exact.

then the sol^n of the eq^n

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$$\int_{y=const} M\,dx + \int N\,dy = 0 = c$$

(taking only those terms of N which do not contain x)

$$\Rightarrow \int\left[\frac{1}{x^2 y} + \frac{1}{x}\right]dx + \int\left(-\frac{1}{y}\right)dy = c$$

$$\Rightarrow \frac{1}{y}\left(-\frac{1}{x}\right) + \log x - \log y = c$$

Ans

Questions

3) $(3x^2 y^4 + 2xy)\,dx + (2x^3 \cdot y^3 - x^2)\,dy = 0$

Now given eqⁿ is in form of

$M\,dx + N\,dy = 0$

where

$M = 3x^2y^4 + 2xy, \quad N = 2x^3 \cdot y^3 - x^2$

then for exactness

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(3x^2y^4 + 2xy) = 12x^2y^3 + 2x$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left[2x^3 \cdot y^3 - x^2\right] = 2x^3 \times 0 + y^3 \cdot 6x^2 - 2x$$

$$= 6y^3 x^2 - 2x$$

$$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

the the given eqⁿ is not exact

Rule 5

Now ~~If~~ we have

$$I.F. = \frac{1}{M}\left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right]$$

$$= \frac{1}{3x^2y^4 + 2xy}\left[\frac{-12x^2y^3 + 2x + 6x^2y^3 + 2x}{}\right]$$

$$= \frac{-6x^2y^3 + 4x}{xy(3xy^3+2)} = \frac{-2x(3xy^3+2)}{xy(3xy^3+2)}$$

$$= \frac{-2}{y}$$

the $I.F. = e^{\int f(y)dy} = e^{\int -\frac{2}{y}dy}$

$$= e^{-2\log y} = e^{\log y^{-2}} = y^{-2} = \frac{1}{y^2}$$

Now mul I.F. $\left(\frac{1}{y^2}\right)$ in given diff. eqn

$$(3x^2y^4 + 2xy)\,dx + (2x^3y^3 - x^2)\,dy = 0$$

$$\left[3x^2y^2 + \frac{2x}{y}\right]dx + \left[2x^3y - \frac{x^2}{y^2}\right]dy = 0$$

Now $M = 3x^2y^2 + \frac{2x}{y}$, $N = 2x^3y - \frac{x^2}{y^2}$

Now ~~find~~ solve for exactness.

Eqⁿ soluable for 'y'

The given eqⁿ soluable for 'y' in the form of

$$y = f(x, p) \quad \text{—— (1)}$$

Now eqⁿ (1) diff. both side w.r.to. x

$$\frac{dy}{dx} = p = \phi\left(x . p . \frac{dp}{dx}\right) \quad \text{—— (2)}$$

Now solving eqⁿ (2) in the two variable 'x' and 'p' (suppose it is possible to solve an diff. eqⁿ (2) in factors involving $\frac{dp}{dx}$ is a part of solⁿ)

Let we have $f(x, P, c) = 0$ —— (3)

Now eliminating 'p' in eqⁿ (1) and (3)

~~No~~ or

~~eqⁿ~~ If we have not eliminate 'p' in eqⁿ (1) and (3) the eqⁿ (1) and (3) together solⁿ of the eqⁿ.

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eq^n soluable for 'x'

Let the given diff. eq^n soluable for 'x'

$$x = f(y, p) \quad \text{--- (1)}$$

Now eq (1) diff. w.r. to 'y'

$$\frac{dx}{dy} = \frac{1}{p} = \phi\left(y, p, \frac{dp}{dy}\right) \quad \text{--- (2)}$$

→ which is the diff. eq^n in two variables

~~Now solving~~ 'y' and 'P'

→ (suppose it is possible to solve the diff. eq. (2) in the factor involving $\frac{dP}{dy}$)

→ then we have $f(y, P, c) = 0$ --- (3)

Now we have eliminating 'P' from eq^n (1) and (3) then we have find the required sol^n.

Q. 1 $y = (x-a)P - P^2$ --- (1)

The given eq^n is soluable for 'y'

Now eq^n (1) diff both side w.r.to. x

$$\frac{dy}{dx} = \cancel{P} = (x-a)\frac{dP}{dx} + \cancel{P \cdot 1} - 2P\frac{dP}{dx}$$

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$$(x-a)\frac{dp}{dx} - 2p\frac{dp}{dx} = 0$$

$$\left[\frac{dp}{dx}\right][x-a-2p] = 0 \quad \text{--- (2)}$$

Now given eqn (2) in the variable x and p

the suppose soln of eqn (2)

$$\therefore \frac{dp}{dx} = 0$$

$dp = 0$ the integrate

$$\boxed{p = c} \quad \text{--- (3)}$$

Now we have eliminating 'p',
Now substitute the value of 'p' in eq (1)

$$y = (x-a)c - c^2$$

Q. 2 $y = 2px + p^4 x^2$

Q. 3 $y = 2px + p^n$

Q. 4 $y - 2px = \tan^{-1}(xp^2)$

Q. 5 $y = 2px + y^2 p^3$

Q. 6 $y^2 \log y = xyp + p^2$

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Solve → 1

$$y = 2px + \tan^{-1}(xp^2) \quad \text{--- (1)}$$

then the given eqn solvable for 'y'

Now eqn (1) diff. w.r.t. x.

$$\frac{dy}{dx} = p = 2\left[p.1 + x\frac{dp}{dx}\right] + \frac{1}{1+x^2.p^4}\left[x.2p\frac{dp}{dx} + p^2\right]$$

$$p = 2p + 2x\frac{dp}{dx} + \frac{p}{1+x^2p^4}\left[2x\frac{dp}{dx} + \right.$$

common liya hai yaha term ka

$$\Rightarrow \left[p + 2x\frac{dp}{dx}\right]\left[1 + \frac{p}{1+x^2p^4}\right] = 0$$

which is the soln of two variable of x and p then we have.

$$\therefore \quad p + 2x\frac{dp}{dx} = 0$$

$$-p = 2x\frac{dp}{dx}$$

$$-\frac{2}{p}dp = \frac{1}{x}dx$$

then int. both side.

$$-2\log p = \log x + \log c$$

$$\log p^{-2} = \log xc$$

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$$p^{-2} = xc$$

$$p^{2} = \frac{1}{xc}$$

$$\boxed{p = \frac{1}{\sqrt{xc}}} \quad \text{---- (3)}$$

Now we have eliminating 'p'

Now substitute the value of p in eq (1)

$$y = \frac{2x}{\sqrt{xc}} + \tan^{-1}\left(x \times \frac{1}{xc}\right)$$

$$y = \frac{2x}{\sqrt{xc}} + \tan^{-1}\left(\frac{1}{c}\right)$$

Solve-3

$$y = 2px + p^{n} \quad \text{---- (1)}$$

then the given eqn soluable for 'y'
Now given eqn (1) diff w.r.to 'x'

$$\frac{dy}{dx} = p = 2\left[x\frac{dp}{dx} + p\right] + n\,p^{n-1}\frac{dp}{dx}$$

$$= 2x\frac{dp}{dx} + p + n\,p^{n-1}\frac{dp}{dx} = 0$$

$$\frac{dp}{dx}\left[2x + n\,p^{n-1}\right] = -p$$

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$$-p\frac{dx}{dp} = 2x + n\cdot p^{n-1}$$

$$\frac{dx}{dp} + \frac{2x}{p} = -n\cdot p^{n-2}$$

then the given eqn linear 'x'

$$I.F. = e^{\int P\,dp}, = e^{\int \frac{2}{p}dp} = e^{2\log p} = e^{\log p^2}$$

$$I.F. = p^2$$

$$x(I.F.) = \int Q\,(I.F.)\,dp + c$$

$$x\,p^2 = -\int n\cdot p^{n-2}\,(p^2)\,dp + c$$

$$x\cdot p^2 = -\int n\cdot p^n\,dp + c$$

$$x\,p^2 = -n\,\frac{p^{n+1}}{n+1} + c$$

Ans

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Q.6 $y^2 \log y = xyp + p^2$

$xyp = y^2 \log y - p^2$

$$x = \frac{y^2 \log y}{yp} - \frac{p^2}{yp}$$

$$x = \frac{y \log y}{p} - \frac{p}{y} \quad \text{--- (1)}$$

then the given eq^n solucable for 'x'

Now eq. (1) diff. both side. w. r. to. 'y'

$$\cancel{\frac{dx}{dy} = \frac{1}{p} \frac{1}{yp} + \frac{p}{y^2}}$$

$$\Rightarrow \frac{dx}{dy} = \frac{1}{p} = y \cdot \log y \cdot \left(\frac{-1}{p^2}\right) \frac{dp}{dy} + \frac{\log y}{p} \cdot 1 + \frac{y}{p} \cdot \frac{1}{y} - \left[p\left(\frac{-1}{y^2}\right) \cdot 1 + \frac{1}{y} \frac{dp}{dy}\right]$$

$$\Rightarrow -\frac{y \log y}{p^2} \cdot \frac{dp}{dy} \times \frac{\log y}{p} + \frac{p}{y^2} - \frac{1}{y} \frac{dp}{dy} = 0$$

$$\Rightarrow \frac{\log y}{p}\left[-\frac{y}{p} \cdot \frac{dp}{dy} + 1\right] + \frac{p}{y^2}\left[1 - \frac{y}{p} \cdot \frac{dp}{dy}\right] = 0$$

$$\Rightarrow \left[1 - \frac{y}{p} \cdot \frac{dp}{dy}\right] \cdot \left[\frac{\log y}{p} + \frac{p}{y^2}\right] = 0 \quad \text{--- (2)}$$

→ which is the solution of two variab. 'y' and 'p' then

→ we have solution

$$1-\frac{y}{p}\cdot\frac{dp}{dy}=0$$

$$\frac{y}{p}\cdot\frac{dp}{dy}=1$$

$$\frac{1}{p}\,dp=\frac{1}{y}\,dy$$

then int. both side

$$\log p=\log y+\log c$$

$$\log p=\log yc$$

$$\boxed{p=yc}$$

Now, we have eliminating p'

Now, substitude the value of p in eq. ①

$$x=\frac{y\log y}{yc}-\frac{yc}{y}$$

$$x=\frac{\log y}{c}-c$$

Ans

इस method में अगर हमने पहचान लिया method ① को तब फिर इसे directque. में p की जगह c put करके Ans. likh dena है।

Method - 4

Clairaut's equation :-

① → The given equation is of the form

→ $y = px + f(p)$ is called clairaut's eqn.

$$y = px + f(p) \longrightarrow ①$$

Now eqn soluable for 'y'

Now eqn ① diffg w.r.to. 'x'

$$\frac{dy}{dx} = p = p \cdot 1 + x \cdot \frac{dp}{dx} + f'(p)\frac{dp}{dx}$$

$$\Rightarrow x\frac{dp}{dx} + f'(p)\frac{dp}{dx} = 0$$

$$\Rightarrow \left[\frac{dp}{dx}\right](x + f'(p)) = 0 \longrightarrow ②$$

which is the soln of two variable 'x' and 'p'.

then we have soln of the eqn.

$$\frac{dp}{dx} = 0$$

$$dp = 0$$

then Int.

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$\boxed{p = c}$ ——— ③

Now,

we have eliminating p

Now, subt. the value of p in eq ①

$y = cx + f(c)$

Q.1 $xp^2 - py + a = 0$

$py = xp^2 + a$

$y = xp + \frac{a}{p}$ ——— ①

then the given eq^n ① is Clairaut form

$y = xp + f(p)$

then putting $p = c$

$y = cx + \frac{a}{c}$

Ans

Q.2 $p = \sin(y - px)$

(Q.6) $yp^2 - 2px + y = 0$

Q.3 $(y - px)(p - 1) = p$

Q.4 $y = 2px - p^3$

Q.5 $p = \tan\left(x - \frac{p}{1+p^2}\right)$

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Solve-2- $p = \sin(y - px)$

$\sin^{-1}(p) = y - px$

$y = px + \sin^{-1}(p)$ —— ①

then the given eqn ① is clairaut form

$y = xp + f(p)$ then putting $p = c$

$y = cx + \sin^{-1}(c)$ Ans

Solve-3- $(y - px)(p - 1) = p$

$y - px = \dfrac{p}{p-1}$

$y = px + \dfrac{p}{p-1}$ —— ①

then the given eqn ① is Chairaut form

$y = xp + f(p)$

putting $p = c$

$y = cx + \dfrac{c}{c-1}$ Ans

Solve-4 $y = 2px - p^3$

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Q. 5 solve $p = \tan\left(x - \frac{p}{1+p^2}\right)$

$\tan^{-1}(p) = x - \frac{p}{1+p^2}$ ① $\Rightarrow \cancel{\tan^{-1}(p) = \frac{x + xp^2 - p}{(1+p^2)}}$

$\cancel{\tan^{-1}(p) \times (1+p^2) = x + xp^2 - p}$

~~devide by p in above eqn both the sides~~

$\cancel{\frac{(1+p^2)}{p} \tan^{-1}(p) = \frac{x}{p} + xp - 1}$

$x = \tan^{-1} p + \frac{p}{1+p^2}$ ——— ①

then the given eqn solvable for 'x' now eqn ① diff. 'y'

$\frac{dx}{dy} = \frac{1}{p} = \frac{1}{1+p^2}\frac{dp}{dy} + p\left(\frac{-1}{(1+p^2)^2}\right) 2p \frac{dp}{dy} + \frac{1}{1+p^2}\frac{dp}{dy}$

$\frac{1}{p} = \frac{2}{1+p^2}\frac{dp}{dy} - \frac{2p^2}{(1+p^2)^2}\frac{dp}{dy}$

$\frac{1}{p} = \frac{2}{1+p^2}\frac{dp}{dy}\left[1 - \frac{p^2}{1+p^2}\right]$

$\frac{1}{p} = \frac{2}{1+p^2}\frac{dp}{dy}\left[\frac{1}{1+p^2}\right]$

$dy = \frac{2p}{(1+p^2)^2} dp$

then int. both

$\int dy = \int \frac{2p}{(1+p^2)^2} dp$

putting $1+p^2 = t$

$2p\,dp = dt$

$\int dy = \int \frac{1}{t^2} dt$

$y = \frac{-1}{t} + c$

$y = \frac{-1}{1+p^2} + c$ —— ⑪

Now we have eliminating P in eqn the eqn ① ② together all the solⁿ of eqn.

Q. 6 solve $y p^2 - 2px + y = 0$

$y = 2px - yp^2$

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Higher order diff. eqⁿ with constant coefficient

→ A linear differential equation is an equation in which a dependent variable 'y' and its differential coefficient appear only in 1st degree

→ A linear differential eqⁿ of Order 'n' is of the form

$$\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}} \text{------} P_n y = Q$$

where $P_1, P_2, P_3, \text{---} P_n \Rightarrow$ constant

$Q \Rightarrow$ any function of 'x' and 'y'

Now change in operator form

$$D = \frac{d}{dx}, \quad D^2 = \frac{d^2}{dx^2}, \quad D^3 = \frac{d^3}{dx^3}$$

$$D^n = \frac{d^n}{dx^n}$$

$$[D^n + P_1 D^{n-1} + P_2 D^{n-2} + \text{----} P_n]. y = Q$$

$$f(D) \cdot y = Q$$

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$f(D).y = \phi$

Therefore, the method of solving a linear diff. eqn. is devide into two parts,

Ist Part

complimentry function

$C.F. \Rightarrow F(D).y = 0$

IInd Part

perticular integral.

$P.I. = \frac{1}{F(D)} \phi$

then the solution of the equation

$y = C.F. + P.I.$

* To find complimentry function.

$F(D).y = 0$

where, F(D) is a lin. diff. eqn of order 'n'.

$[D^n + P_1 D^{n-1} + P_2 D^{n-2} + \cdots + P_n] y = \phi$

Now its Auxilary equation is given by (putting $D = m$)

$[m^n + P_1 m^{n-1} + P_2 m^{n-2} + \cdots + P_n] = 0$

Now, solving this eqn. and find the roots of $[m]$

Case I If all the roots are real and distinct

$m = m_1, m_2, m_3$

$$C.F. = C_1 e^{m_1 x} + C_2 e^{m_2 x} + C_3 e^{m_3 x}$$

Ex:— like, $m = 1, -2, 3$

$$C.F. = C_1 e^{x} + C_2 e^{-2x} + C_3 e^{3x}.$$

Case-II If the roots are equal and real.

$m = m_1, m_1, m_2$

$$C.F. = (C_1 + C_2 x) e^{m_1 x} + C_3 e^{m_2 x}$$

$m = m_1, m_1, m_1, m_2$

$$C.F. = (C_1 + C_2 x + C_3 x^2) e^{m_1 x} + C x e^{m_2 x}$$

Case III If the roots are imaginary.

$m = \alpha \pm i\beta$

$$C.F. = e^{\alpha x} [A \cos \beta x + B \sin \beta x]$$

ex:— $m = 2 \pm 3i$

$$C.F. = e^{2x} [A \cos 3x + B \sin 3x]$$

repeated terms हो तो

ex:— $m = 1 \pm 3i, 1 \pm 3i$

$$C.F. = e^{x} [(A + Bx) \cos x + (C + Dx) \sin 3x]$$

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Case-4. If the roots are irrational number.

$$m = \alpha \pm \sqrt{\beta}$$

$$C.F. = e^{\alpha x}\left[A \cosh\sqrt{\beta}\, x + B \sinh\sqrt{\beta}\cdot x\right]$$

Ex:- $m = 2 \pm \sqrt{5}$

$$C.F. = e^{2x}\left[A \cdot \cosh\sqrt{5}\, x + B \sinh\sqrt{5}\, x\right]$$

Questions.

Q1 $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 0$

Now change in operator form

$$D = \frac{d}{dx}$$

$$[D^2 - 3D - 4]y = 0 \quad \text{—— (1)}$$

Now [its Auxiliary] eqn is given by

(putting $D = m$)

$$m^2 + 3m - 4 = 0$$

$$m^2 - 4m + m - 4 = 0$$

$$m(m-4) + 1(m-4) = 0$$

$$\boxed{m = -1}, \quad \boxed{m = 4}$$

the C.F. $= c_1 e^{4x} + c_2 e^{-x}$

then the sol^n of the eq^n

$y = c_1 e^{4x} + c_2 e^{-x}$

Q.② $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0$

Now change in operator form

$D = \frac{d}{dx}$

$[D^2 - 4D + 1].y = 0$

Now its Auxilary eq^n is given by (putting $D = m$)

$[m^2 - 4m + 1] = 0$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{16 - 4}}{2}$$

$$m = \frac{4 \pm \sqrt{12}}{2}$$

$$m = \frac{4 \pm 2\sqrt{3}}{2}$$

$$\boxed{m = 2 \pm \sqrt{3}}$$

then C.F. $= e^{2x}[A\cosh\sqrt{3}x + B\sinh\sqrt{3}x]$

or

C.F $= e^{2x}[c_1 e^{\sqrt{3}x} + c_2 e^{-\sqrt{3}x}]$

Ans

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Q.3 $(D^3-8)y=0$ —— (1)

Now its Auxiliary eqn. is given by (putting D = m)

$m^3-8=0$

$\cancel{m=\pm2\sqrt{2}}$

$(a^3-b^3)=(a-b)(a^2+ab+b^2)$

$\cancel{m=2}$

(m^3-2^3)

$\Rightarrow (m-2)(m^2+2m+4)=0$

$\boxed{m=2}$

$m^2+2m+4=0$

$m=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\boxed{m=-1\pm\sqrt{3}\,i}$

then $C.F. = C_1e^{2x}+e^{-x}(A\cos\sqrt{3}x+B\sin\sqrt{3}x)$

then the solⁿ of eqⁿ.

$y = c_1e^{2x}+e^{-x}[A\cos\sqrt{3}x+B\sin\sqrt{3}x]$

Ans

Q.4

Q.5

Q.6

Q.7

Q.8

Q.9

Q.10

Solve=

Q.4 $[D^4 - 1D^2 + 1)\, y = 0$

Q.5 $\dfrac{d^4y}{dx^4} - m^4 y = 0$

Q.6 $\dfrac{d^4y}{dx^4} + m^4 y = 0$

Q.7 $(D^2+D+1)^2 (D-2)\, y = 0$

Q.8 $(D^3+1)y = 0$

Q.9 $(D^3 - 2D^2 + 4D - 8).y = 0$

Q.10 $(D^4 + 13D^2 + 36).y = 0$

Solve-6 $\dfrac{d^4y}{dx^4} + m^4 y = 0.$

Now change in operator form

$$D = \frac{d}{dx}$$

$$[D^4 + m^4]y = 0 \quad \text{——} \quad (1)$$

Now its Auxilary ~~form~~ eqⁿ is given by [putting $D = M$]

$$[M^4 + m^4] = 0$$

$$[M^4 + m^4 + 2M^2m^2] - 2M^2m^2 = 0$$

$$(M^2 + m^2)^2 = 2M^2m^2$$

$$M^2 + m^2 = \pm\sqrt{2}\, M.m$$

for $(+)$ for $(-)$

$$[M^2 + m^2 + \sqrt{2}\, M.m]\,[M^2 + m^2 - \sqrt{2}\, Mm] = 0$$

$$M^2 + \sqrt{2}\, M.m + m^2 = 0$$

$a = 1$, $b = \sqrt{2}\, m$, $c = m^2$

$$M = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$M = \frac{-\sqrt{2}\, m \pm \sqrt{2m^2 - 4m^2}}{2}$$

$$M = \frac{-\sqrt{2}\, m \pm \sqrt{-2m^2}}{2}$$

$$M = \frac{-\sqrt{2}\, m \pm 2 m i}{2}$$

Discarted root of signs

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Q.10 $(D^4 + 13D^2 + 36)y = 0$

solve = $(D^4 + 13D^2 + 36)y = 0$

Now its Auxilary eq. is given by (putting $D = M$)

$m^4 + 13m^2 + 36 = 0$

$m^4 + 9m^2 + 4m^2 + 36 = 0$

$m^2(m^2+9) + 4(m^2+9) = 0$

$(m^2+9)\ (m^2+4) = 0$

NOW,

$\rightarrow\ m^2 = 9,\ m = \pm 3$

$\rightarrow\ m^2 = 4,\ m = \pm 2$

then the C.F.

$= e^{0x}.[A\cos 3x + B\sin 3x] + e^{0x}.[C\cos 2x + D\sin 2x]$

then the complete sol^n.

$y = \cancel{A\cos 3x}[A\cos 3x + B\sin 3x] + [C\cos 2x + D\sin 2x]$

perticular integral

To find ~~complementry function~~ : [PI]

Case I

if $\phi = e^{ax}$

$\Rightarrow PI = \frac{1}{f(D)} \cdot \phi = \frac{1}{f(D)} \cdot e^{ax} = \frac{1}{f(a)} \cdot e^{ax}$

Putting $(D = a)$

$\Rightarrow$ if $f(D) = f(a) \neq 0$

$PI = \frac{1}{f(a)} \cdot e^{ax}$

II

$\frac{1}{f(D)} \cdot \phi = \frac{1}{f(D)} e^{ax} = x \cdot \frac{1}{f'(a)} e^{ax}$

if $(F(D) = f(a) = 0$

$\frac{\partial}{\partial D} [F(D)] = f'(D) = f'(a) \neq 0.$

III

$\frac{1}{f(D)} \cdot \phi = \frac{1}{f(D)} e^{ax} = x^2 \cdot \frac{1}{f''(a)} e^{ax}$

if $f(D) = f(a) = 0$

$\frac{\partial}{\partial D} [f(D)] = f'(D) = f'(a) = 0$

$\frac{\partial}{\partial D} [f'(D)] = f''(D) = f''(a) \neq 0$

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IV $\frac{1}{f(D)} Q = \frac{1}{(D-a)^n} e^{ax} = \frac{x^n}{n!} e^{ax}$

$f(D) = f(a) = 0$

up to n times

Questions:-

1. $\frac{d^2y}{dx^2} - \frac{3dy}{dx} + 2y = e^{2x}$

Now change in operator form

$D = \frac{d}{dx}$

$[D^2 - 3D + 2]y = e^{2x}$ —— (1)

Now its Auxilary eq^n is given by

C.F. [puttin $D = m$]

$m^2 - 3m + 2 = 0$

$m^2 - 2m - 1m + 2 = 0$

$m(m-2) - 1(m-2) = 0$

$(m-1)(m-2) = 0$

$m = 1, m = 2$

$C.F. = c_1 e^{2x} + c_2 e^{x}$

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Now $(P.I) = \frac{1}{f(D)} \cdot Q$

$= \frac{1}{D^2-3D+2} e^{2x}$

$f(D) = D^2-3D+2$

Now $D = a = 2$

$f(D) = f(a) = f(2)$

put

$= 4-6+2 = 0$

Now $\frac{\partial}{\partial p} (f[D]) = \frac{\partial}{\partial D} [D^2-3D+2]$

$f'(D) = 2D-3$

$f'(D) = f'(a) = f'(2) = 4-3 = 1 \neq 0$

$\Rightarrow x \cdot \frac{1}{1} \cdot e^{2x}$

$PI = x e^{2x}$

then the complete solⁿ of the eqⁿ is

$y = C.f. + P.I$

$y = c_1 e^{2x} + c_2 e^{x} + x.e^{2x}$

OR

$\frac{1}{(D-1)(D-2)} \cdot e^{2x}$

$PI = \frac{1}{(D-2)} \left[\frac{1}{(D-1)} e^{2x} \right]$

$f(D) = D-1$, $D = a = 2$

$f(D) = f(a) = f(2) = 1 \neq 0$

$\Rightarrow \frac{1}{D-2} \left(\frac{1}{1} \cdot e^{2x} \right)$

$= \frac{1}{(D-2)} \cdot e^{2x} \Rightarrow \frac{x}{1!} e^{2x}$

$\Rightarrow x \cdot e^{2x}$

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(4) $(D+2)(D-1)^3 . y = e^x$

Solve ⇒ then for C.F.

$F(D) . y = 0$

$(D+2)(D-1)^2 . y = 0$

Now its Auxilary eq is given by (Putting $D=m$)

$(m+2)(m-1)^3 = 0$

~~m = -2 . m~~

$m = -2, 1, 1, 1$

~~the C.F. = $C_1 e^{-2x} + C_2 e^x + C_3 e^x + C_4 e^x$~~

the $C.F. = C_1 e^{-2x} + (C_2 + C_3 x + C_4 x^2) . e^x$

then for $P.I. = \frac{1}{F(D)} . Q$

$= \frac{1}{(D+2)(D-1)^3} . e^x$

$= \frac{1}{(D-1)^3} \boxed{\frac{1}{D+2} e^x}$

$f(D) = D+2$, $D = a = 1$

$f(D) = f(a) = f(1) = 1+2 = 3 \neq 0$

$\frac{1}{f(D)} e^{ax} = \frac{1}{f(a)}$

$f(D) = f($

$\neq$

$$\frac{1}{(D-1)^3}\left[\frac{1}{3}e^{x}\right]$$

$$= \frac{1}{3}\,\frac{1}{(D-1)^3}\,e^{x}$$

$D = a = 1$

$f(D) = f(a) = 1 - 1 = 0$

$$\boxed{\frac{1}{(D-a)^n}\,e^{ax} = \frac{x^n}{n!}\,e^{ax}}$$

if $f(D) = f(a) = 0$

$$P.I. = \frac{1}{3}\left[\frac{x^3}{3!}e^{x}\right]$$

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then the complete sol. of the eqn

$$y = C.F. + P.I.$$

~~Case II~~

Case – II if $Q = \sin ax$ or $\cos ax$

(I) $P.I. = \frac{1}{F(D^2)}\sin ax = \frac{1}{F(-a)^2}\sin ax$

if $\boxed{D^2 = -a^2}$ $f(D^2) = f(-a)^2 \neq 0$

(II) ~~$P.I. \frac{1}{F(D^2)}$~~

$P.I. = \frac{1}{F(D^2)}\cos ax = \frac{1}{f(-a)^2}\cos ax$

if $f(D^2) = f(-a)^2 \neq 0$

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Q.1 $(D^2-3D+2)y = \cos 2x$

Now for C.F.

$$f(D).\,y = 0$$

$$(D^2-3D+2).\,y = 0$$

Now its Auxilary eq^n is given by (putting $D=m$)

$$m^2-3m+2=0$$

$$m^2-2m-m+2=0$$

$$\cancel{m(1-2m)-1(1-m)=0}$$

$$\cancel{m=}$$

$$m(m-2)-1(m-2)=0$$

$$\boxed{m=1,2}$$

$$\boxed{\text{the C.F.} = c_1e^x + c_2e^{2x}}$$

then for P.I. $= \dfrac{1}{f(D)}.\,\phi$

$$= \frac{1}{D^2-3D+2}\cos 2x$$

$$D^2 = -a^2 = -4$$

$$= \frac{1}{-4-3D+2}\cos 2x$$

$$= \frac{1}{-2-3D}.\cos 2x$$

rationalise

$$= -1 \times \frac{(2-3D)}{(2+3D)(2-3D)}.\cos 2x$$

$$\frac{-(2-3D)}{(4-9D^2)} \cdot \cos 2x$$

$\{D^2 = -a^2 = -4$

$$= \frac{-(2-3D)}{4-9\times(-4)} \cos 2x$$

$$= \frac{-1}{40} (2-3D) \cos 2x$$

$$= \frac{-1}{40} [2 \cos 2x - 3D(\cos 2x)]$$

$D = \frac{d}{dx}$

$\frac{1}{D} = \int dx$

$$= \frac{-1}{40} [2 \cos 2x + 6 \sin 2x]$$

$(D^2 - 4) y = e^x + \sin 3x$

$(D^3 + 1) y = \sin^2 x$

$$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = \cos 2x + e^{-x} + e^{2x}$$

4. ~~Now for C.F.~~

~~$f(D) y = 0$~~

Now change in operator form

$\left(\frac{d}{dx} = D\right)$

$D^3 y + D^2 y - Dy - y = \cos 2x + e^{-x} + e^{2x}$

Now for C.F.

$(D^3 + D^2 - D - 1).y = 0$;

Now for A.E. is given by (putting $D = m$)

$$m^3 + m^2 - m - 1 = 0.$$

$$\boxed{m = 1, -1, -1}$$

C.F. $= c_1 e^{x} + (c_2 + c_3 x) e^{-x}$

then for P.I.

$$P.I = \frac{1}{f(D)} \cdot \phi.$$

$$= \frac{1}{D^3 + D^2 - D - 1} \cdot (\cos 2x + e^{-x} + e^{2x})$$

$$\Rightarrow \frac{1}{D^3 + D^2 - D - 1} \cdot \cos 2x + \frac{1}{D^3 + D^2 - D - 1} e^{-x} + \frac{1}{D^3 + D^2 - D - 1} e^{2x}$$

$$I_1 + I_2 + I_3$$

Now $I_1 = \frac{1}{D^3 + D^2 + D - 1} \cdot \cos 2x$

$\boxed{D^2 = -a^2 = -4}$

$$= \frac{1}{(-4).D + (-4) - D - 1} \cdot \cos 2x$$

$$= \frac{-1}{5(D+1)} \cdot \cos 2x \quad \{\text{rationalise}$$

$$= \frac{-1}{5}\left[\frac{1}{(D+1)} \cdot \frac{(D-1)}{(D-1)}\right]\cos 2x$$

$$= \frac{-1}{5} \cdot \frac{(D-1)}{(D^2-1)} \cdot \cos 2x$$

$D^2 = -a^2 = -4$

$$= \frac{1}{25}\left[D-1\right] \cdot \cos 2x$$

$$= \frac{1}{25}\left[-2\sin 2x - \cos 2x\right]$$

Ans

Now $I_2 = \dfrac{1}{D^3+D^2-D-1} \cdot e^{-x}$

$f(D) = D^3 + D^2 - D - 1$

$D = a = -1$

$\cancel{\dfrac{1}{-1+1+1-1} e^{-x}}$

$f(D) = f(a) = f(-1) = 0$

$-1+1+1-1 = 0$

$\cancel{f'(D)}$

$f'(D) = 3D^2 + 2D - 1$

$D = a = -1$

$f'(D) = f'(a) = f(-1)$

$[+3-2-1] = 0$

$f''(D) = 6D + 2$

$D = a = -1$

$f''(D) = f''(a) = f(-1)$

$= -6+2 = -4 \neq 0$

$\Rightarrow I_2 = x^2 \cdot \dfrac{1}{-4} e^{-x}$

Ans

Now $I_3 = \frac{1}{D^3+D^2-D-1} e^{2x}$

$F(D) = D^3+D^2-D-1$ $\quad f(D) = f(a) = f(2)$

$8+4-2-1 = 9 \neq 0$

$I_3 = \frac{1}{9} \cdot e^{2x}$

Ans

$PI = \frac{1}{25}[-2\sin 2x - \cos 2x] + \frac{1}{-4}x^2 e^{-x}$

$+ \frac{1}{9} \cdot e^{2x}$

Ans

Q3 $(D^3+1)y = \sin^2 x$

then for C.F.

$f(D) \cdot y = 0$

$(D^3+1)y = 0$

Now its A.E. is given by

[Putting $D = m$]

$m^3+1=0$

$m = -1, \frac{1 \pm \sqrt{3}i}{2}$

then C.F. $= C_1 e^{-x} + e^{\frac{1}{2}x}\left[A\cos\frac{\sqrt{3}}{2}x + B\sin\frac{\sqrt{3}}{2}x\right.$

for P.I

$$= \frac{1}{f(D)} \cdot \phi$$

$$= \frac{1}{D^3+1}\left[\sin^2 x\right]$$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

$\sin^2 x = \frac{1 - \cos 2x}{2}$

$$= \frac{1}{D^3+1}\left[\frac{1 - \cos 2x}{2}\right]$$

$$= \frac{1}{2}\left[\frac{1}{D^3+1}\, 1 - \frac{1}{D^3+1}\cos 2x\right]$$

$$= \frac{1}{2}\left[I_1 - I_2\right]$$

Now $I_1 = \frac{1}{D^3+1} e^{0x}$

$D = a = 0$

$f(D) = f(a) = f(0) = 1 \neq 0$

$$\boxed{I_1 = 1 \cdot e^{ax} = 1}$$

$$I_1 = \frac{1}{D^3+1}\cos 2x \qquad D^2 = a^2 = -4$$

$$= \frac{1}{(-4)D + 1}\cos 2x$$

$$= \frac{1}{(1-4D)}\cos 2x$$

rationa

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$$= \frac{(1+4D)}{(1+4D)(1-4D)} \cos 2x$$

$$= \frac{(1+4D)}{1+16D^2} \cos 2x$$

$D^2 = -a^2 = -4$

$$\Rightarrow \frac{1}{65}(1+4D)\cos 2x$$

$$I_2 = \frac{1}{65}\left[\cos 2x + 4D\cos 2x\right]$$

$$I_2 = \frac{1}{65}\left[\cos 2x + 8\sin 2x\right] \quad \boxed{D = \frac{d}{dx}}$$

$$P.I = \frac{1}{2}\left[1 - \frac{1}{65}(\cos 2x - 8\sin 2x)\right]$$

Ans

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Case III

If $\phi = x^n$

$$PI = \frac{1}{f(D)} \cdot \phi = \frac{1}{f(D)} \cdot x^n = [f(D)]^{-1} \cdot x^n$$

$$(1+D)^{-1} = 1 - D + D^2 + D^3 + \cdots$$

$$(1-D)^{-1} = 1 + D + D^2 + D^3 + \cdots$$

$$(1+D)^{-2} = 1 - 2D + 3D^2 - 4D^3 + \cdots$$

$$(1-D)^{-2} = 1 + 2D + 3D^2 + 4D^3 + \cdots$$

Case IV

If $\phi = e^{ax} \cdot V$ where V is any function of 'x'

$$PI = \frac{1}{f(D)} e^{ax} \cdot V = e^{ax} \cdot \frac{1}{f(D+a)} \cdot V$$

Case V

if $\phi = \cos ax$ or $\sin ax$

$f(D^2) = f(-a)^2 = 0$

① $\frac{1}{D^2+a^2} \cos ax = \frac{x}{2a} \sin ax$

if $f(D)^2 = f(-a^2) = 0$

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(2) $$\boxed{\frac{1}{D^2+a^2}\sin ax = \frac{-x}{2a}\cos ax \quad \text{if } f(D^2) = f(a^2) = 0}$$

Q.1 $(D^2+4)y = x^2 + \cos 2x$

then for C.F.

$$= f(D).\, y = 0$$

$$(D^2+4).\, y = 0$$

Now its A.E is given by (Putting D = m)

$$m^2+4 = 0$$

$$m^2 = -4$$

$$m = \pm 2i$$

$$\boxed{C.F. = e^{0x}[A\cos 2x + B\sin 2x]}$$

then for P.I.

$$P.I = \frac{1}{f(D)}.\, Q.$$

$$= \frac{1}{(D^2+4)}.\,(x^2+\cos 2x)$$

$$= \frac{1}{D^2+4}x^2 + \frac{1}{D^2+4}\cos 2x$$

$$= I_1 + I_2$$

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then for I_1

$$I_1 = \frac{1}{D^2+4} x^2$$

$$\cancel{I_1 = (D^2+4)^{-1} x^2}$$

$$I_1 = \frac{1}{4}\left[1+\frac{D^2}{4}\right]^{-1} \cdot x^2$$

$$\left\{ (1+D)^{-1} = 1 - D + D^2 - D^3 + - - \right.$$

$$= \frac{1}{4}\left[1 - \frac{D^2}{4} + \frac{D^4}{16} - - - \right] x^2$$

$$\cancel{= \frac{1}{4}\left[x^2 - \frac{1}{4} x^2\right.}$$

$$= \frac{1}{4}\left[x^2 - \frac{D^2}{4} x^2 + \frac{D^4}{16} x^2 - - - \right]$$

$$\boxed{D = \frac{d}{dx}}$$

$$= \frac{1}{4}\left[x^2 - \frac{2}{4} + 0\right]$$

$$= \frac{1}{4}\left[x^2 - \frac{1}{2}\right]$$

Ans

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then for I_2

$$I_2 = \frac{1}{D^2+4} \cos 2x$$

$\cancel{D^2 = (-a^2) = -4}$

$\cancel{f(D) = f(-a^2) = 20}$

$D^2 = (-a)^2 = -4$

$f(D^2) = D^2 + 4 = -4$

$$I_2 = \frac{x}{2\times 2} . \sin 2x \quad \left[\frac{1}{D^2+a^2} \cos ax = \frac{x}{2a} \sin \ldots \right.$$

if $F(D^2) = f(a^2)$

$$I_2 = \frac{x}{4} . \sin 2x$$

$P.I = I_1 + I_2$

$$P.I = \frac{1}{4}\left[x^2 - \frac{1}{2}\right] + \frac{x}{4} \sin 2x$$

Ans

Q.2 $(D^3 + 3D^2 + 2D)y = x^2$

Q.3 $(D^2 - 4D + 4)y = x^2 + e^x + \sin 2x$

Q.4 $(D^2 - 2D + 1)y = x^2 . e^{3x}$

Q.5 $(D^2 - 1)y = \cosh x . \cos x$

Q.6 $(D^2 - 2D + 4)y = e^x . \cos x$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

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Q5 $(D^2 - 1)y = \cosh x \cdot \cos x$

Solve then for C.F.

$$f(D).\, y = 0$$

$$(D^2 - 1)\, y = 0$$

Now its A.E is given by (put $D = m$)

$$m^2 - 1 = 0$$

$$m = \pm 1$$

$$\boxed{C.F. = C_1 e^x + C_2 e^{-x}}$$

then for P.I.

$$P.I = \frac{1}{f(D)} \cdot Q$$

$$= \frac{1}{(D^2 - 1)} \cosh x \cdot \cos x$$

$$= \frac{1}{D^2 - 1} \left[\frac{e^x + e^{-x}}{2}\right] \cos x \qquad \boxed{\cosh x = \frac{e^x + e^{-x}}{2}}$$

$$= \frac{1}{2}\left[\frac{1}{D^2 - 1} e^x \cos x + \frac{1}{D^2 - 1} e^{-x} \cos x\right]$$

$$P.I = \frac{1}{2}\left[I_1 + I_2\right]$$

then for I_1

$$I_1 = \frac{1}{D^2-1} e^x \cos x$$

$$\left[\frac{1}{f(D)} e^{ax} \cdot V = e^{ax} \frac{1}{f(D+a)} V \right.$$

where V is any function of x

$$I_1 = e^x \frac{1}{(D+1)^2-1} \cdot \cos x$$

$$I_1 = e^x \cdot \frac{1}{D^2+2D+\cancel{1}-\cancel{1}} \cdot \cos x$$

$$D^2 = -a^2 = -1$$

$$= e^x \cdot \frac{1}{-1+2D} \cos x$$

$$= e^x \cdot \frac{(2D+1)}{(2D-1)(2D+1)} \cos x.$$

$$= e^x \frac{(2D+1)}{(4D^2-1)} \cos x$$

$$D^2 = -a^2 = -1$$

$$= \frac{e^x}{-5} (2D+1) \cos x$$

$$= \frac{-e^x}{5} \left[2D(\cos x) + \cos x \right]$$

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$$\boxed{I_1 = \frac{-e^{x}}{5}\left[2\sin x + \cos x\right]}$$

then for I_2

$$I_2 = \frac{1}{D^2-1} e^{-x} \cos x$$

$$\frac{1}{f(D)} e^{ax} \cdot V = e^{ax} \frac{1}{f(D+a)} \cdot V$$

where V is any function of 'x'

Q.4 $(D^2 - 2D+1) y = x^2 \cdot e^{3x}$

solve = then for P.I. $= \frac{1}{f(D)}$

$$= \frac{1}{D^2-2D+1} x^2 \cdot e^{3x}$$

$$= \frac{1}{(D-1)^2} e^{3x} \cdot x^2$$

$$\frac{1}{f(D)} e^{ax} \cdot V = e^{ax} \frac{1}{f(D+a)} \cdot V$$

where V is fun of 'x'

$$= e^{3x} \frac{1}{((D+3)-1)^2} \cdot x^2$$

$$= e^{3x} \frac{1}{(D+2)^2} x^2$$

$$= e^{3x} \frac{1}{4}\left[1+\frac{D}{2}\right]^{-2} x^2$$

$$\left[(1+D)^{-2} = 1 - 2D + 3D^2 - 4D^3 + \cdots\right.$$

$$= \frac{e^{3x}}{4}\left[1 - \frac{2D}{2} + \frac{3D^2}{4} - \frac{4D^3}{8} + \cdots\right] x^2$$

$$= \frac{e^{3x}}{4}\left[x^2 - D(x^2) + \frac{3}{4} D^2(x^2) - \frac{1}{2} D^3(x^2)\right.$$

$$= \frac{e^{3x}}{4}\left[x^2 - 2x + \frac{3}{4} \times 2 - 0\right]$$

$$P.I. = \frac{e^{3x}}{4}\left[x^2 - 2x + \frac{3}{2}\right]$$

Ans

Case VI If $\phi = x.V$

where V is any function of x

$$\boxed{\frac{1}{f(D)} . x.V = x.\frac{1}{f(D)} V - \frac{f'(D)}{f(D)^2} . V}$$

Q.1. $(D^2 - 2D + 1)y = x \cdot \sin x$

solve⇒ then for c.f.

$$f(D)y = 0$$

$$(D^2 - 2D + 1)y = 0$$

Now its A.E is given by (putting D=m)

$$m^2 - 2m + 1 = 0$$

$$\boxed{m = 1, 1}$$

$$c.f. = \cancel{c_1 e^{x} +} (c_1 + c_2 x)e^x$$

then for P.I. =

$$\frac{1}{f(D)} \cdot q$$

$$= \frac{1}{(D^2 - 2D + 1)} \cdot x \cdot \sin x$$

$$= \frac{1}{(D-1)^2} \cdot x \cdot \sin x$$

$$\left[\frac{1}{f(D)} \cdot x \cdot V = x \cdot \frac{1}{f(D)} \cdot V - \frac{f'(D)}{[f(D)]^2} \cdot V \right.$$

where V = function of x

$$= x \cdot \frac{1}{\cancel{(D-1)^2}\; D^2 - 2D + 1} \sin x - \frac{(2D-2)}{(D^2 - 2D + 1)^2} \cdot \sin x$$

$$= x \cdot \frac{1}{D^2-2D+1}\sin x - \frac{(2D-2)}{(D^2-2D+1)^2}\sin x$$

$$[D^2 = -a^2 = -1]$$

$$\Rightarrow x \cdot \frac{1}{-1-2D+1}\sin x - \frac{2(D-1)}{[-1-2D+1]^2}\sin x$$

$$= \frac{-x}{2}\cdot\frac{1}{D}\sin x - \frac{2(D-1)}{4D^2}\sin x$$

$$\left[\frac{1}{D} = \int dx\right]$$

$$= \frac{-x}{2}(-\cos x) - \frac{2(D-1)}{4(-1)}\sin x$$

$$= \frac{x}{2}\cos x + \frac{1}{2}[D(\sin x) - \sin x)$$

$$\boxed{PI = \frac{x}{2}\cos x + \frac{1}{2}(\cos x - \sin x)}$$

Ans

Q.2 $(D^2+2D+1)\, y = x \cdot \cos x$

Q.3 $\frac{d^2y}{dx^2} + y = x^2 \cdot \sin 2x$

Q.4 $(D^2+1)\, y = e^{-x} + \cos x + x^3 + e^x \cdot \sin x$

Q.5 $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x \cdot e^x \cdot \sin x$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x \cdot e^x \cdot \sin x$$

Now change it in operator form $\left(\frac{d}{dx} = D\right)$

$$(D^2 - 2D + 1)y = x\, e^x \sin x$$

Solve-5 then for C.f. $= f(D) \cdot y = 0$

$$(D^2 - 2D + 1) \cdot y = 0$$

Now its A·E is given by. (Putting $D = m$)

$$(m^2 - 2m + 1) = 0.$$

$$m = 1, 1,$$

$$\boxed{C.f. = (c_1 + c_2 x)e^x}$$

then for P.I.

$$\frac{1}{f(D)}$$

$$= \frac{1}{D^2 - 2D + 1} \cdot x \cdot e^x \cdot \sin x$$

$$\left[\frac{1}{f(D)} e^{ax} \cdot V = e^{ax} \frac{1}{f(D+a)} V\right]$$

$$\left[\frac{1}{f(D)} x \cdot V = x \cdot \frac{1}{f(D)} V - \frac{f'(D)}{f(D)^2} V\right]$$

$$= \frac{1}{(D-1)^2} \cdot e^x \cdot (x \sin x)$$

$$= e^x \frac{1}{[D+1-1]^2} (x \sin x)$$

$$\frac{1}{D} = \int dx$$

$$= e^x \cdot \frac{1}{D^2} (x \sin x)$$

$$= e^x \cdot \frac{1}{D}\left[\int x \sin x \, dx\right]$$

$$= e^x \cdot \frac{1}{D}\left[x \cdot (-\cos x) - 1(-\sin x)\right]$$

$$= e^x \cdot \left[\int \left[x \cos x + \sin x\right] dx\right]$$

int. w.r. to x

$$= e^x \left[-x \sin x - 2\cos x\right]$$

$$\boxed{P.I. = -e^x \left[x \sin x + 2\cos x\right]}$$

Ans

Case VII

General method to find P.I. :-

Now $\boxed{\frac{1}{D - \alpha} \cdot Q = e^{+\alpha x} \int Q \cdot e^{-\alpha x} \, dx}$

$$\boxed{\frac{1}{D + \alpha} Q = e^{-\alpha x} \int Q \cdot e^{\alpha x} \, dx}$$

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Solve-3 ⇒ $$\frac{d^2y}{dx^2} + y = x^2 \cdot \sin 2x$$

Now change it in operator form
$$\left\{\frac{d}{dx} = D\right\}$$

for C.F. $$(D^2 + 1)\, y = x^2 \cdot \sin 2x \quad \text{—(1)}$$

Now its Auxilary eq^n is given by
(putting $D = m$)

$$m^2 + 1 = 0$$
$$m = \pm 1$$

$$C.F. = c_1 e^{x} + c_2 e^{-x}$$

Then for $P.I. = \dfrac{1}{f(D)}$

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Solve-1 $\frac{d^2y}{dx^2} + a^2y = \sec ax$

Solve:- Now change in operator form

$\{D = \frac{d}{dx}\}$

$(D^2 + a^2)\, y = \sec ax$ ---- ①

then for C.f.

$f(D) \cdot y = 0$

$(D^2 + a^2)y = 0$

Now its A.E is given by (Putting $D = m$)

$m^2 + a^2 = 0$

$m^2 = -a^2$

$m = \pm ai$

then $C.f. = e^{0x}\,[A\cos ax + B\sin ax]$

then for $P.I = \frac{1}{F(D)} \cdot Q$

$= \frac{1}{D^2 + a^2} \cdot \sec ax$

$= \frac{1}{(D+ai)(D-ai)} \cdot \sec ax$

$= \frac{1}{2ai}\left[\frac{1}{D-ai} - \frac{1}{D+ai}\right] \sec ax$

$$\frac{1}{D+\alpha}\cdot \varphi = e^{-\alpha x}\int \varphi \cdot e^{\alpha x}\cdot dx$$

$$\frac{1}{D-\alpha}\cdot \varphi = e^{\alpha x}\int x e^{-\alpha x}\cdot dx$$

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$$= \frac{1}{2ai}\left[\frac{1}{D-ai}\sec ax - \frac{1}{D+ai}\sec ax\right]$$

$$= \frac{1}{2ai}\left[I_1 - I_2\right]$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$
$$e^{-i\theta} = \cos\theta + i\sin\theta$$
$$e^{i\theta} + e^{-i\theta} = 2i\cos\theta$$
$$e^{i\theta} - e^{-i\theta} = 2i\sin\theta$$

Now $I_1 = \frac{1}{D-ai}\sec ax$

$$= e^{aix}\int \sec ax \cdot e^{-ax}\, dx$$

$$= e^{iax}\int \frac{1}{\cos ax}\left[\cos ax - i\sin ax\right]\cdot dx$$

$$= e^{iax}\int [1 - i\tan ax]\, dx$$

$$I_1 = e^{iax}\left[x - \frac{i\log\cdot\sec ax}{a}\right]$$

$$I_2 = \frac{1}{D+ai}\sec ax$$

$$= e^{-aix}\int \sec ax \cdot e^{aix}\, dx$$

$$= e^{-iax}\int \frac{1}{\cos ax}\left[\cos ax + i\sin ax\right] dx$$

$$= e^{-iax}\int [1 + i\tan ax]\, dx$$

$$I_2 = e^{-iax}\left[x + \frac{i\log\sec ax}{a}\right]$$

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Now substitute the values of I_1 & I_2

$$P.I = \frac{1}{2ai}\left[I_1 - I_2\right]$$

$$P.I = \frac{1}{2ai}\left[e^{iax}\left(x - \frac{i\log\sec ax}{a}\right)\right] - \left[e^{-iax}\left(x + \frac{i\log\sec ax}{a}\right)\right]$$

$$= \frac{1}{2ai}\left[x\left(e^{iax} - e^{-iax}\right] - \frac{i\log\sec ax}{a}\left(e^{iax} + e^{-iax}\right)\right.$$

$$= \frac{1}{2ai}\left[x(2i\sin ax) - \frac{i\log\sec ax}{a}(2\cos ax)\right.$$

$$= \frac{1}{2ai}\left[x\,2i\sin ax - \frac{2i\log\sec ax.\cos ax}{a}\right.$$

$$P.I = \frac{2i}{2ai}\left[x\sin ax - \frac{\cos ax.\log\sec ax}{a}\right]$$

then the complete sol^n of eq^n is

$$y = C.f. + P.I$$

$$y =$$

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$$= e^{x} \cdot \frac{1}{D^2-2D+2}(x\cdot \sin x)$$

* Cauchy (Homogenous) Linear diff. eq.

→ A homogenous linear diff. eqⁿ. is of the form

$$x^n \frac{d^n y}{dx^n} + P_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + P_2 x^{n-2} \frac{d^{n-2} y}{dx^{n-2}} + \dots + P_{n-1} x \frac{dy}{dx} + P_n y = \phi \quad \text{①}$$

Now Change in operator form $D = \frac{d}{dx}$

$$[x^n D^n + P_1 x^{n-1} D^{n-1} + P_2 x^{n-2} D^{n-2} \dots + P_{n-1} x D + P_n] y = \phi.$$

→ Where, $P_1, P_2, P_3 \dots P_n$ all are constants

ϕ is function of x.

→ Now we have introduce new independent variable 'z' such that $\boxed{x = e^z}$

$\boxed{z = \log x}$ $\quad \frac{dz}{dx} = \frac{1}{x}$

Now, $\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = \frac{1}{x} \cdot \frac{dy}{dz}$

$\boxed{x \cdot Dy = D'y}$ $\quad \left\{ D'y = \frac{dy}{dz} \right\}$

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& $\boxed{x^2 D^2 y = D'(D'-1)y}$

$\boxed{x^3 D^3 y = D'(D'-1)(D'-2)y}$

Now, Substitute all these values in eqn ① then eqn ① is reduced to linear diff. eqn with constant coefficient form.

Questions:-

$(x^2 D^2 - 4xD + 6)y =$

① $x^2 \frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = x$ —— ①

→ Now given eqn ① homogenous linear diff. eqn is of the form

then we have putting $\boxed{x = e^z}$

$\boxed{z = \log x}$

and $x \cdot \frac{dy}{dx} = x \cdot Dy = D'y$ where $\left\{ D = \frac{d}{dx} \quad D' = \frac{d}{dz} \right.$

& $x^2 \cdot \frac{d^2y}{dx^2} = \cancel{D^2y}\ x^2D^2y = D'(D'-1)y$

Now substitute all these values in eq. ①

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$= [D'(D'-1) - 4D' + 6] \cdot y = e^z$

$= [D'^2 - 5D' + 6] \cdot y = e^z$ —— (11)

Now, given eqn (11) linear diff. eqn. with constant coff. form

$(D'^2 - 5D' + 6) y = 0$

Now its A.E is given by (putting $D = m$)

$m^2 - 5m + 6 = 0$

$m^2 - 3m - 2m + 6 = 0$

$m(m-3) - 2(m-3) = 0$

$m = 2, 3$

$m - 2 = 0$ $m - 3 = 0$

$m = 2$ $m = 3$

$C.F. = c_1 e^{2z} + c_2 e^{3z}$

$(x = e^z)$

$\boxed{C.f = c_1 x^2 + c_2 x^3}$

then for $P.I. = \frac{1}{f(D)} \cdot Q$

$= \frac{e^z}{D'^2 - 5D' + 6}$

$D = a = 1$

$f(D) = D'^2 - 5D' + 6$

$f(D) = f(a) = f(1) =$

$1 - 5 + 6 = 2 \neq 0$

$PI = \frac{1}{2} e^z$

$\boxed{PI = \frac{1}{2} x}$

Simultaneous Linear diff. eqn

Q. Solve the Simultaneous eqn

$$\frac{dx}{dt} - 7x + y = 0$$

$$\frac{dy}{dt} - 2x - 5y = 0$$

Solve Now change in operator form $D = \frac{d}{dt}$

$$Dx - 7x + y = 0$$
$$-5y - 2x + Dy = 0$$

$$(D-7)x + y = 0 \times (D-5)$$
$$-2x + (D-5)y = 0$$

$$(D-7)(D-5)x + \cancel{(D-5)y} = 0$$
$$-2x + \cancel{(D-5)y} = 0$$
$$(+) \qquad (-)$$

$$(D-7)(D-5) + 2)x = 0$$
$$(D^2x - 7Dx + 35x - 5Dx + 2)x = 0$$
$$(D^2 - 12D + 33)x = 0$$

then for C.F. $f(D) \cdot x = 0$

Now its A.E. is given by (Putting $D = m$)

$$m^2 - 12m + 33 = 0$$

UNIT-2

Second order differential equation with variables coefficient

Is also called linear diff. eqn. of second order with varible coefficient

The general (standard) eqn is of the form.

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R \quad \text{——} \; (A)$$

where,

P = function of x or may be const.

Q = —— " ——

R = —— " ——

(A) there is no general method to solve such types of diff. eqn

Now we shall discuss following [basic methods for solving] the such types of diff. eqn.

Method-I - (A)

When an intregral of the complementary function (C.F) is known

[By the ~~integral~~ Inspection method]

Method-2 - ~~Resolve~~ Removal of the first derivative or change of dependent variable or Normal form.

Method-3 - change of Independent variable.

Method-4 - variation of parameter.

Method-1

→ To find complete sol^n when an integral of C.F. is known.

→ A linear diff eqn of second order

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R \quad \text{(A)}$$

→ Let $y = u$ be a known integral of C.F. [By the inspection method]

→ then $y = u.v$ be the complete sol. of the eqn. (A)

→ To find one integral of C.F. by inspection method

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0.$$

case-1 If $y = e^{mx}$ is a part of c.f. of eq. ①

$$\frac{dy}{dx} = me^{mx}, \quad \frac{d^2y}{dx^2} = m^2 e^{mx}$$

sub. all these values in eq. ①

$$m^2 e^{mx} + P.me^{mx} + Qe^{mx} = 0.$$

$$[m^2 + Pm + Q]e^{mx} = 0$$

$\therefore$ $m^2 + Pm + Q = 0$. then $y = e^{mx}$ is a part of c.f.

① $m = 1$

$\boxed{1 + P + Q = 0 \text{ then } y = e^{x} \text{ is a Part of c.f.}}$

② $m = -1$

$\boxed{1 - P + Q = 0 \text{ then } y = e^{-x} \text{ is a part of c.f}}$

③ $m = 2$

$\boxed{4 + 2P + Q = 0 \text{ then } y = e^{2x} \text{ is a part of c.f.}}$

④ $m = -2$

$\boxed{4 - 2P + Q = 0 \text{ then } y = e^{-2x} \text{ is a part of c.f.}}$

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Case = II If $y = x^m$ is a part of C.F.

(i) if $\boxed{P + Qx = 0}$ then $y = x$ is a Part of C.F.

(ii) if $\boxed{2 + 2Px + Q^2x^2 = 0}$ then $y = x^2$ is ---- C.F.

Let $y = u$ is a known int. of a C.F.

Let $y = u \cdot v$ be a complete sol^n of eq. (A) (ii)

Now eq^n (ii) diff w.r.to x and find $\frac{dy}{dx}, \frac{d^2y}{dx^2}$

substit all these values in eq^n (A) then eq. (A) is reduced to $\frac{d^2V}{dx^2}$

$$\boxed{\frac{d^2V}{dx^2} + \left[\frac{2}{u} \cdot \frac{du}{dx} + P\right] \cdot \frac{dV}{dx} = \frac{R}{u}} \quad (II)$$

Now sol. these eq^n (Putting $\frac{dV}{dx} = P$ ⇒ $\frac{d^2V}{dx^2} = \frac{dP}{dx}$

Now sub. in eq. (3)

$$\frac{dP}{dx} + \left[\frac{2}{u}\frac{du}{dx} + P\right] \cdot P = \frac{R}{u} \qquad (4)$$

given eq^n (4) first order first degree form (variable separable form

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linear form and solution)

and the find the value of V

$V = ?$

then the comp. soln of the soln

$$\boxed{y = u.v}$$

Questions

Q. 1 $(3-x)\dfrac{d^2y}{dx^2} - (9-4x)\dfrac{dy}{dx} + (6-3x)y = 0$

$$\frac{d^2y}{dx^2} - \left(\frac{9-4x}{3-x}\right)\frac{dy}{dx} + \left(\frac{6-3x}{3-x}\right)y = 0 \quad \text{——} (1)$$

Now, given eq^n (1) comparing with st. eq^n

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

where, $P = -\left[\dfrac{9-4x}{3-x}\right]$; $Q = \left[\dfrac{6-3x}{3-x}\right]$.

$R = 0$

Now, we have find one integral of c.f. by the inspection method

Now $\boxed{1+P+Q=0}$

$$1 - \left(\frac{9-4x}{3-x}\right) + \left[\frac{6-3x}{3-x}\right] = 0$$

$$\frac{3-x+9+4x+6-3x}{3-x}=0$$

Here

$1+P+Q=0$ then $y=e^x$ is a part of C.f.

Let $\boxed{y=u=e^x}$ $\left\{\frac{dy}{dx}=e^x\right.$

Let complete soln of eq. (1)

$$\boxed{y=u.v}$$

$$\boxed{y=e^x.v} \quad \text{———} \quad (2)$$

Now eqn (2) diff. w.r.t. x & find $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$, sub. all these value in eqn (1) then eqn reduced to

$$\frac{d^2v}{dx^2}+\left[\frac{2}{u}.\frac{du}{dx}+P\right]\frac{dv}{dx}=\frac{R}{u}$$

$$\frac{d^2v}{dx^2}+\left[\frac{2}{e^x}.e^x-\left(\frac{9-4x}{3-x}\right)\right].\frac{dv}{dx}=0$$

$$\frac{d^2v}{dx^2}+\left[\frac{6-2x-9+4x}{3-x}\right]\frac{dv}{dx}=0$$

$$\frac{d^2v}{dx^2}+\left[\frac{2x-3}{3-x}\right]\frac{dv}{dx}=0 \quad \text{———} \quad (3)$$

$\int u.v = u.\int v\,du - \int \frac{d}{du}$

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Putting $\frac{dv}{du} = p \Rightarrow \frac{d^2v}{du^2} = \frac{dp}{du}$

sub in eq. (3)

$$\frac{dp}{dx} + \left[\frac{2x-3}{3-x}\right]\cdot p = 0$$

$$\frac{dp}{dx} = \left[\frac{2x-3}{x-3}\right]\cdot p$$

$$\frac{1}{p}\,dp = \left[2 + \frac{x}{x-3}\right]dx$$

$$x-3 \overline{)\,2x-3\,}\;2 \qquad 2x-6 \qquad 3$$

then int. both sides

$$\int \frac{1}{p}\,dp = \int \left[\left(2 + \frac{3}{x-3}\right)\right]dx$$

$$\log p = 2x + 3\log(x-3) + \log c$$

$$\log p = \log e^{2x}\cdot(x-3)^3\cdot c$$

$$p = e^{2x}\cdot(x-3)^3\cdot c \qquad \left\{ p = \frac{dv}{dx} \right.$$

Now, $p = \frac{dv}{du} = e^{2x}\cdot(x-3)^3\cdot c$

$$dv = \left[e^{2x}\cdot(x-3)^3\cdot c\right]dx$$

then int. both side,

$$\int dv = \int\left[e^{2x}\cdot(x-3)^3\cdot c\right]dx$$

$$v = (x-3)^3\frac{e^{2x}}{2} - 3(x-3)^2\frac{e^{2x}}{4} +$$

$$+ c(x-3)\frac{e^{2x}}{8} - 6\frac{e^{2x}}{16}$$

then the complete sol^n is

$\boxed{y = u.v}$

Ans

Q2 $x \cdot \frac{d^2y}{dx^2} - (2x-1)\frac{dy}{dx} + (x+1)y = e^x$

3 $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$

given that $\left(x+\frac{1}{x}\right)$ is one integral

4 $\frac{d^2y}{dx^2} - \cot x \frac{dy}{dx} - (1-\cot x)y = e^x \cdot \sin x$

5 $x^2 \cdot \frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = x$

Q4 solve ⇒

$$\frac{d^2y}{dx^2} - \cot x\frac{dy}{dx} - (1-\cot x)y = e^x \cdot \sin x$$

Now given eq. is comp. with st. eq^n

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

$P = -\cot x$, $Q = (-1+\cot x)$,

$R = e^x \cdot \sin x$

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Now we have find one integral of C.F. by the inspection method.

Now $\boxed{1+P+Q=0}$

$$1+P+Q = \cancel{1} - \cot x - \cancel{1} + \cot x = 0$$

Here $1+P+Q=0$ then $y=e^x$ is a part of C.F.

~~Here~~ Let $y = u = e^x$ $\left\{ \frac{dy}{dx} = e^x \right.$

Let complete soln of eqn (1)

$$\boxed{y = u.v}$$

$$\boxed{y = e^x.v} \quad \text{—— (2)}$$

Now eqn (2) diff. w.r.to x & find $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$, sub. all these values in eqn (1) then eqn reduced to

$$\frac{d^2v}{dx^2} + \left[\frac{2}{u}\frac{du}{dx} + P\right]\frac{dv}{dx} = \frac{R}{u}$$

$$\frac{d^2v}{dx^2} + \left[\frac{2}{\cancel{e^x}}\cdot \cancel{e^x} + (-\cot x)\right]\frac{dv}{dx} = \frac{\cancel{e^x}\sin x}{\cancel{e^x}}$$

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$$\frac{d^2v}{dx^2} + [2 - \cot x]\frac{dv}{dx} = \sin x \quad \text{--- (3)}$$

Let $\frac{dv}{dx} = p \Rightarrow \frac{d^2v}{dx^2} = \frac{dp}{dx}$

sub, in eq. (3)

$$\frac{dp}{dx} + [2 - \cot x] p = \sin x \quad \text{--- (4)}$$

then the given eq. (4) linear in p

where $\boxed{P = 2 - \cot x}$

$$I.F. = e^{\int P dx} = e^{\int(2 - \cot x) dx}$$

$$= e^{2x - \log \sin x} = e^{2x} \cdot e^{\log(\sin x)^{-1}}$$

$$\boxed{I.F = \frac{e^{2x}}{\sin x}}$$

then the soln of the eqn

$$p.(I.f.) = \int Q.(I.f.)\, dx + C_1$$

$$p.\left(\frac{e^{2x}}{\sin x}\right) = \int \cancel{\sin x} \cdot \frac{e^{2x}}{\cancel{\sin x}}\, dx + c$$

$$2 - \cot x\left(\frac{e^{2x}}{\sin x}\right) = \frac{e^{2x}}{2} + c$$

method-2

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Removal of first derivative:-

The given eq^n is of the form:-

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R \quad \text{---} (1)$$

then the complete soln of give eq^n (1)

$$\boxed{y = u.v} \quad \text{---} (2)$$

Now eq^n (2) diff. w.r.to x and find $\frac{dy}{dx}$, and $\frac{d^2y}{dx^2}$, sub. all these values in eq (1)

$$\cancel{\frac{d^2y}{dx^2} + \left[\frac{2}{u}\frac{du}{dx}\right.}$$

$$\frac{d^2v}{dx^2} + \left[\frac{2}{u}\frac{du}{dx} + P\right]\frac{dv}{dx} + v\left[\frac{1}{u}\frac{d^2u}{dx^2} + \frac{P}{u}\frac{du}{dx} + Q\right] = \frac{R}{u} \quad \text{---} (3)$$

Now removal of first derivative.

$$\frac{2}{u}\frac{du}{dx} + P = 0$$

$$\boxed{u = e^{-\int \frac{P}{2} \cdot dx}} \quad \text{---} (4)$$

Now eq^n (4) diff. w.r.to. x and find $\frac{du}{dx}$, and $\frac{d^2u}{dx^2}$, sub. all these values in eq^n (3) then eq^n is reduced to Normal form.

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$$\boxed{\frac{d^2V}{dx^2} + I.V. = \frac{R}{u}} \quad \text{———} (5)$$

where, $\boxed{I = Q - \frac{1}{2}\frac{dP}{dx} - \frac{P^2}{4}}$

Now solving eq. (5) and find value of V

then the complete sol^n of the eq^n.

$$\boxed{y = u.v} \quad \text{Ans}$$

Questions:-

Q.1 solve $x^2 \frac{d^2y}{dx^2} - 2(x^2+x)\frac{dy}{dx} + (x^2+2x+2)y = 0$

Now dividing by x^2

$$\frac{d^2y}{dx^2} - 2\left[1+\frac{1}{x}\right]\frac{dy}{dx} + \left[1+\frac{2}{x}+\frac{2}{x^2}\right]y = 0$$

Now comp. the standard eq^n.

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Q\cdot y = R$$

where, $P = -2\left[1+\frac{1}{x}\right]$, $Q = \left[1+\frac{2}{x}+\frac{2}{x^2}\right]$

$R = 0$

$P + Qx \neq 0$ ~~$1+P+Q = 0 = 1-2$~~ $\therefore 1+P+Q \neq 0,\ 1-P+Q \neq 0$

first method is not applicable

then removal of the first derivative

$$\therefore \quad u = e^{-\int \frac{P}{2} dx}$$

$$u = e^{+\int 2 \cdot \frac{1}{2}\left[1+\frac{1}{x}\right] dx}$$

$$u = e^{x + \log x}$$

$$u = e^{x} \cdot e^{\log x}$$

$$\boxed{u = x \cdot e^{x}}$$

then the complete solⁿ of eqⁿ ①

$$y = u \cdot v.$$

$$\boxed{y = e^{x} \cdot x \cdot v} \text{ ——— ②}$$

Now eqⁿ ② diff w.r.to x and find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, sub. all these values in eqⁿ ① then eqⁿ ① reduced to normal form

$$\frac{d^2v}{dx^2} + I \cdot v = \frac{R}{u}$$

where, $I = Q - \frac{1}{2}\frac{dP}{dx} - \frac{P^2}{4}$

$$I = \left(1 + \frac{2}{x} + \frac{2}{x^2}\right) - \frac{1}{2}(-2)\left(-\frac{1}{x^2}\right) - \frac{4\left(1+\frac{1}{x}\right)^2}{4}$$

$$I = 1 + \frac{2}{x} + \frac{2}{x^2} - \frac{1}{x^2} - \frac{4\left(+1+\frac{1}{x}\right)^2}{4}$$

$$I = 1 + \frac{2}{x} + \frac{1}{x^2} - \frac{\left(1 + \frac{1}{x^2} + 2\frac{1}{x}\right)}{4}$$

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$$\cancel{1 + \frac{2}{x} + \frac{1}{x^2}} - \cancel{1 - \frac{2}{x}} - \cancel{\frac{1}{x^2}}$$

4

$\boxed{I = 0}$

$$\therefore \frac{d^2V}{dx^2} + 0 = 0.$$

$$\frac{d^2V}{dx^2} = 0$$

int. the eqn both side w.r to x

$$\frac{dV}{dx} = C_1$$

again the int.

$$\boxed{V = C_1 x + C_2}$$

then the complete soln of the eqn

$$y = u.v.$$

$$\boxed{y = e^x . x (C_1 x + C_2)}$$

$$\left(\frac{d^2y}{dx^2}\right) - 2 \tan x \frac{dy}{}$$

Method - 3

Change of independent variable:

The given eqn is of the form

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R \quad \text{---} (1)$$

Now we have introduce new independent variable 'z'

Now $\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx}$,

$$\cancel{\frac{d^2y}{dx^2} = \frac{d^2y}{dz^2} \cdot \frac{dz}{}} \; \cancel{\frac{d}{dx}\left(\frac{dy}{dx}\right) = \left[\frac{d}{dz}\left(\frac{dy}{dz}\right.\right.}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left[\frac{dy}{dz} \cdot \frac{dz}{dx}\right]$$

$$= \frac{dy}{dz} \cdot \frac{d^2z}{dx^2} + \frac{dz}{dx} \cdot \frac{d}{dx}\left(\frac{dy}{dz}\right)$$

$$= \frac{dy}{dz} \cdot \frac{d^2z}{dx^2} + \frac{dz}{dx} \cdot \frac{d}{dz}\left(\frac{dy}{dz}\right)\frac{dz}{dx}$$

$$= \frac{dy}{dz} \cdot \frac{d^2z}{dx^2} + \frac{d^2y}{dz^2}\left(\frac{dz}{dx}\right)^2$$

Now sub all these values in eqn (1) then eqn (1) is reduced to

$$\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1 y = R_1 \quad \text{---} (2)$$

where, $$\boxed{P_1 = \frac{\frac{d^2z}{dx^2} + P\cdot\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2}}$$

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$$\boxed{Q_1 = \frac{Q}{(dz/dx)^2}}, \quad \boxed{R_1 = \frac{R}{(dz/dx)^2}}$$

→ Here P_1, Q_1, R_1, are the function of 'x' which can be transformed into function of 'z'

→ Now we have choose the value of 'z' such that Q_1

$$Q_1 = \frac{Q}{(dz/dx)^2} = \text{constant}$$

(if we have Q is (+)ve then we have consider positive (+1). or if Q is negative then we have consider constant minus one (C-1))

And then we have find the value of 'z' and then we have find the value of P_1 and R_1

(1) Now sub. all these values P_1, Q_1, R_1 in eqn (2)

(2) and then find the solution.

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Q 1 $x\frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3y = 8x^3 \sin x^2$

devide by x both side

$$\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} - (4x^2 y) = 8x^2 \sin x^2 \quad \text{—} (1)$$

Now comp. eq (1) with st. eqn.

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R.$$

where,

$P = -\frac{1}{x}$, $Q = -4x^2$, $R = 8x^2 \sin x^2$

method (1) and (2) are not applicable

Now changing the independent variable x to z.

Now transform eqn is.

$$\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1 y = R_1 \quad \text{—} (2)$$

where, $P_1 = \frac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{(dz/dx)^2}$

$Q_1 = \frac{Q}{(dz/dx)^2}$, $R_1 = \frac{R}{(dz/dx)^2}$

Now we have choose the value of z such that

$$P_1 = \frac{P}{(dz/dx)^2} = \text{constant}$$

$$\frac{-4x^2}{(dz/dx)^2} = -1$$

$$\left(\frac{dz}{dx}\right)^2 = 4x^2$$

$$\boxed{\frac{dz}{dx} = 2x} \rightarrow \boxed{\frac{d^2z}{dx^2} = 2}$$

$$dz = 2x\,dx$$

Int. both side.

$$\boxed{z = x^2}$$

Now we have find the value of P_1 and R_1

$$P_1 = \frac{2 + \left(\frac{-1}{x}\right) \cdot 2x}{4x^2} = 0.$$

$$\boxed{P_1 = 0}$$

$$R_1 = \frac{8x^2 \cdot \sin x^2}{4x^2} = 2\sin x^2$$

$$= 2\sin z$$

$$\boxed{R_1 = 2\sin z}$$

Sub. the value P, Q, R, in eq (11)

$$\frac{d^2y}{dz^2} + 0 - 1y = 2\sin z$$

$$\frac{d^2y}{dz^2} - y = 2\sin z$$

$$\boxed{D = \frac{d}{dz}}$$

$$(D^2-1)y = 2\sin z$$

Now for C.F. $f(D)\cdot y = 0$.

$$(D^2-1)\cdot y = 0$$

Now its A.E. is given by (Put D=m)

$$\cancel{(D^2-1)=0}$$

$$m^2-1=0$$

$$\boxed{m = \pm 1}$$

$$C.F. = c_1 e^{z} + c_2 e^{-z}$$

$$= c_1 e^{x^2} + c_2 e^{-x^2}$$

then for P.I. $= \frac{1}{f(D)}\cdot Q$.

$$= \frac{1}{D^2-1}\, 2\sin z$$

$$D^2 = -a^2 = -1$$

$$f(D) = D^2-1 = -1-1 = -2 \neq 0$$

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$$P.I. = \frac{1}{-2}(2\sin z) = -\sin z = -\sin x^2$$

the comp sol^n of the eq^n

$$y = C.F + P.I.$$
$$y = c_1 e^{x^2} + c_2 e^{-x^2} - \sin x^2$$

Q2 $\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} + \frac{a^2}{x^4}y = 0$

Q3 $\cos x \frac{d^2y}{dx^2} + \sin \frac{dy}{dx} - 2y\cos^3 x = 2\cos^5 x$

Q4 $x\frac{d^2y}{dx^2} + (4x^2-1)\frac{dy}{dx} + 4x^2 y = 2x^3$

Q5 $(1+x^2)\frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} + 4y = 0$

Solve

3 $\cos x \frac{d^2y}{dx^2} + \sin x \frac{dy}{dx} - 2y\cos^3 x = 2\cos^5 x$

devide by $\cos x$ in bot

$$\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} - 2y\cos^2 x = 2\cos^4 x$$

Now com. eq^n ① by std eq

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

$p = \tan x$, $Q = -2y\cos^2 x$, $R = 2\cos^4 x$

Now changing the Ind. vari. x to z.

Now transform eq. 11

$$\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1 y = R_1 \quad \text{(11)}$$

where , $P_1 = \dfrac{\dfrac{d^2z}{dx^2} + p\dfrac{dz}{dx}}{(dz/dx)^2}$

$$Q_1 = \frac{Q}{(dz/dx)^2} , \quad R_1 = \frac{R}{(dz/dx)^2}$$

Now we have choose the value of z

such that

$$Q_1 = \frac{Q}{(dz/dx)^2} = \text{cons.}$$

$$= \frac{-2y\cos^2 x}{(dz/dx)^2} = -1$$

$$\left(\frac{dz}{dx}\right)^2 = 2y\cos^2 x$$

$$\frac{dz}{dx} = \sqrt{2y}\cos x \quad \boxed{\frac{d^2z}{dx^2} = -\sqrt{2}\sin x}$$

Int. w.r.to. x

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$$\boxed{z = \sqrt{2}\sin x}$$

Now we have find the value P_1 and R_1

$$P_1 = \frac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2}$$

$$P_1 = \frac{-\sqrt{2}\sin x + \tan x\,\sqrt{2}\cos x}{2\cos^2 x}$$

$$P_1 = \frac{-\sqrt{2}\sin x + \frac{\sin x}{\cos x}\sqrt{2}\cos x}{2\cos^2 x}$$

$$\boxed{P_1 = 0}$$

$$R_1 = \frac{R}{(dz/dx)^2} = \frac{2\cos^4 x}{2\cos^2 x}$$

$$\boxed{R_1 = \cos^2 x}$$

$$\boxed{R_1 = 1 - \sin^2 x}$$

Sub. all these P_1, Q_1, R_1 in eq

$$\frac{d^2y}{dz^2} + 0 + (-1)y = \cos^2 x$$

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$$\frac{d^2y}{dz^2} - y = 1 - \sin^2 x$$

$$(D^2-1)y = \left(1 - \frac{z^2}{2}\right)$$

then for C.F.

$$(D^2-1)y = 0.$$

Now its A.E is given by (Put D = m)

$$m^2 - 1 = 0$$

$$m = \pm 1$$

$$C.F = c_1 e^{z} + c_2 e^{-z}$$

$$C.f. = c_1 e^{\sqrt{2}\sin x} + c_2 e^{-\sqrt{2}\sin x}$$

$$P.I. = \frac{1}{(D^2-1)}\left(1 - \frac{z^2}{2}\right)$$

$$= \cancel{(D^2-1)^{-1}\left(1 - \frac{z^2}{2}\right)}$$

$$= -(1-D^2)^{-1}\left(1 - \frac{z^2}{2}\right)$$

$$(1-D^2)^{-1} = 1 + D + D^2 + D^3 - - -$$

$$= -\left[1 + D + D^2 + D^3 - - -\right]\left(1 - \frac{z^2}{2}\right)$$

$$= -\left[\cancel{1} - \frac{z^2}{2} - \cancel{1}\right] = +\frac{z^2}{2}$$

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$$\cancel{1+\frac{z^2}{2}} \quad \cancel{+1} \quad \cancel{=\frac{z^2}{2}}$$

Change in ~~x~~

$z=\sqrt{2}\sin x$

$$\rightarrow \boxed{\frac{2\sin^2 x}{2}} \quad \cancel{\frac{2\sin^2 x}{2}}$$

$\Rightarrow \sin^2 x$

Ans

Method 4

Method of variation of parameter.

The given eqn is of the form

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R \quad \text{(1)}$$

then for C.F.

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0.$$

Let $\boxed{C.F. = Ay_1 + By_2}$

then for $\boxed{P.I = u.y_1 + v y_1}$

where, $$\boxed{u = \int \frac{-y_2 R}{y_1 y_2' - y_1' y_2} dx}$$

$$\boxed{v = \int \frac{+y_1 R}{y_1 y_2' - y_1' y_2} dx}$$

then the complete soln of eqn

$$\boxed{y = C.F. + P.I.}$$

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Q. Solve by the method of variation of parameter.

(1) $\frac{d^2y}{dx^2} + a^2 \cdot y = \sec ax$

(2) $\frac{d^2y}{dx^2} + a^2 \cdot y = \tan ax$

Solve - 2 $\frac{d^2y}{dx^2} + a^2 y = \tan ax$

$D = \frac{d}{dx}$

$(D^2 + a^2) y = \tan ax$

then for C.F. $f(D) \cdot y = 0$

$(D^2 + a^2) \cdot y = 0$

Now its A.E is given by (Put — $D = m$)

$m^2 + a^2 = 0$

$m^2 = -a^2$

$m = \pm ai$

then C.F. = $A \cos ax + B \sin ax$

let $C.F = A\, y_1 + B\, y_2$

Let $y_1 = \cos ax$, $y_2 = \sin ax$

$y_1' = -a \sin ax$ $y_2' = a \cos ax$

then for P.I. $= u y_1 + v y_2$

where $u = \int \frac{-y_2 R}{y_1 y_2' - y_1' y_2} dx$

& $v = \int \frac{y_1 R}{y_1 y_2' - y_1' y_2} dx$

Now $u = \int \frac{-\sin ax \cdot \tan ax}{(\cos ax)(a\cos ax) - (-a\sin ax)(\sin ax)}$

$u = \frac{-1}{a} \int \sin ax . \tan ax \, dx$

$u = \frac{-1}{a} \int \sin ax \times \frac{\sin ax}{\cos ax} dx$

$u = \frac{-1}{a} \int \frac{\sin^2 ax}{\cos ax} dx$

$u = \frac{-1}{a} \int \frac{1-\cos^2 ax}{\cos ax} dx$

$u = \frac{-1}{a} \int \frac{1}{\cos ax} dx + \frac{1}{a} \int \cos ax . dx$

$u = \frac{-1}{a} \int \sec ax \, dx + \frac{1}{a} \int \cos ax \, dx$

$$\boxed{u = \frac{-1}{a} \left[\frac{\log(\sec ax + \tan ax)}{a} \right] + \frac{1}{a} \frac{\sin ax}{2}}$$

Series solution of diff. eqn

→ The soln of linear diff. eqn with variable coefficient can be determine in the form of an infinite conversant series. Arranged according to the power of independent variable.

→ A series of the form

$$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n + \cdots \infty = \sum_{r=0}^{\infty} a_r \cdot x^r$$

is called power series.
where, $a_0, a_1, a_2 \cdots \infty$ are constant and are called coefficient of the series.

→ A diff. eqn is of the form

$$\boxed{P_0(x)\frac{d^2y}{dx^2} + P_1(x)\frac{dy}{dx} + P_2(x)\,y = 0}$$

Where, $P_0(x)$, $P_1(x)$, $P_2(x)$ are the polynomial in x

① ordinary point :- If $P_0(x) \neq 0$ at $x = 0$

then $x = 0$ is called ordinary point of the given diff. eqn.

$\Rightarrow y = A_0 x^0 + A_1 x + A_2 x^2 + A_3 x^3 \ldots + A_n x^n + \ldots \infty$ —— (3)

Now eq^n (3) diff. w.r.t. x

$$\Rightarrow \frac{dy}{dx} = A_1 + 2A_2 x + 3A_3 x^2 + 4A_4 x^3 \ldots + A_n n x^{n-1} + \ldots$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2A_2 + 6A_3 x + 12A_4 x^2 + \ldots A_n n(n-1) x^{n-2} \ldots$$

$\Rightarrow$ Sub. all these value in eq^n (1).

$$\Rightarrow (1+x^2)\left[2A_2 + 6A_3 x + 12A_4 x^2 + \ldots + A_n \cdot n \cdot (n-1) x^{n-2} + \ldots\right]$$

$$+ x\left[A_1 + 2A_2 x + 3A_3 x^3 + 4A_4 x^3 + \ldots + A_n n x^{n-1} + \ldots\right]$$

$$- \left[A_0 + A_1 x + A_2 x^2 + A_3 x^3 + \ldots + A_n x^n + \ldots\right] = 0$$

$$\Rightarrow \left[2A_2 - A_0\right] + x\left[6A_3 + A_1 - A_1\right] + x^2\left[12A_4 + 2A_2 + 2A_2 - A_2\right] + \ldots + x^n\left[A_{n+2}(n+2)(n+1) + A_n n(n-1) + A_n n - A_n\right] + \ldots$$

$+ A_n (n^2 - \cancel{n} + \cancel{n} - 1)$ ——

$\ldots = 0$ —— (3)

$A_n x^n$

$A_{n+1} x^{n+1}$

$A_{n+2} x^{n+2}$ —— (A)

diff. eq. (A) w.r.to. x.

$A_{n+2} (n+2) x^{n+1}$

diff.

$\boxed{A_{n+2} (n+2)(n+1) x^n}$

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$$+ A_n n(n-1) x^{n-2} + A_{n+1}(n+1) n x^{n-1} + A_{n+2}(n+2)(n+1) + \dots$$

$$+ A_n n x^{n-1} + A_{n+1}(n+1) x^n + A_{n+2}(n+2) x^{n+1} + \dots$$

$$+ A_n x^n + A_{n+1} x^{n+1} + A_{n+2} x^{n+2} \dots = 0$$

Now eqⁿ ③ is an identity, Now equility to zero the coefficient of various power of 'x'.

$$2A_2 - A_0 = 0 \qquad \boxed{A_2 = \frac{A_0}{2}}$$

$$6A_3 = 0 \qquad \boxed{A_3 = 0}$$

$$12A_4 + 3A_2 = 0 \qquad \boxed{A_4 = -\frac{1}{4} A_2 = \frac{-A_0}{8}}$$

$$A_{n+2}(n+2)(n+1) + A_n(n^2-1) = 0$$

$$A_{n+2} = \frac{-(n^2-1)}{(n+1)(n+2)} A_n$$

$$A_{n+2} = -\frac{(n-1)}{n+2} A_n$$

putting $n=0$. $\qquad A_2 = +\frac{1}{2} A_0$

$n=1$, $A_3 = 0$

$n=2$, $A_4 = \frac{-1}{4} A_2 = \frac{-1}{8} A_0$

Now. sub. all these value in eqⁿ ⑪

$$y = \sum_{r=0}^{\infty} A_r . x^r = A_0 + A_1 x + A_2 x^2 + A_3 x^3 + \cdots$$

$$y = A_0 + A_1 x + \frac{A_0}{2} x^2 + 0 + \frac{-A_0}{8} x^4 + \cdots$$

$$y = A_0 \left[1 + \frac{1}{2} x^2 - \frac{1}{8} x^4 + \cdots \right] + A_1 x$$

~~y =~~

Case-II

When the roots are real and equal,

$$m = m_1, m_1$$

$$y = c_1 [y]_{m=m_1} + c_2 \left[\frac{\partial y}{\partial m}\right]_{m=m_1}$$

Case-III

When the roots are distinct, differ by integer & making a coefficient of 'y' infinite

Let m_1 & m_2 be the roots such that

$m_1 > m_2$ in this case if sum of the coefficient of y infinite when $m = m_2$, we modify the form of y by replacing A_0 by $\boxed{B_0 (m - m_2)}$

then the complete soln:

$$y = c_1 [y]_{m=m_1} + c_2 \left[\frac{\partial y}{\partial m}\right]_{m=m_2}$$

Legender's ~~method~~ equation:-

The diff eqn

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0 \quad \text{--- (1)}$$

is called legender's eqn. where n is positive integer.

solution of legenders eqn

Let the soln of eqn (1)

$$y = \sum_{r=0}^{\infty} A_r x^{m-r} \quad \text{--- (2)}$$

Now eqn (2) diff w.r.to. x.

$$\frac{dy}{dx} = \sum_{r=0}^{\infty} A_r (m-r) x^{m-r-1}$$

$$\frac{d^2y}{dx^2} = \sum_{r=0}^{\infty} A_r (m-r)(m-r-1) x^{m-r-2}$$

sub. all these values in eqn (1)

$$\Rightarrow \sum_{r=0}^{\infty} A_r \Big[(1-x^2)(m-r)(m-r-1)x^{m-r-2} - 2x(m-r)x^{m-r-1} + n(n+1)x^{m-r}\Big] = 0$$

$$\Rightarrow \sum_{r=0}^{\infty} A_r \left[(m-r)(m-r-1)\, x^{m-r-2} - (m-r)(m-r-1)\, x^{m-r} - 2(m-r)\, x^{m-r} + n(n+1)\, x^{m-r} \right] = 0$$

$$\Rightarrow \sum_{r=0}^{\infty} A_r \left[(m-r)(m-r-1)\, x^{m-r-2} - \left\{ (m-r)^2 + (m-r) - n(n+1) \right\} x^{m-r} \right] = 0 \quad \text{③}$$

Now eq^n ③ is an identity

Now equating to zero the coefficient of various power of x.

Now equating to zero the coeffient of highest power of x.

x, x^m

$$\Rightarrow A_0 \left\{ m^2 + m - n(n+1) \right\} = 0.$$

if $A_0 \neq 0 \Rightarrow m^2 + m - n^2 - n = 0$

$$\Rightarrow (m^2 - n^2) + (m - n) = 0$$

$$(m+n)(m-n) + (m-n) = 0$$

$$(m-n)(m+n+1) = 0$$

$$\boxed{m = n, -(n+1)}$$

Now equating to zero next higher power of x

$x, \quad x^{m-1}$

$= A_1 \{(m-1)^2 + (m-1) - n(n+1)\} = 0$

If $\boxed{A_1 = 0}$ then $(m-1)^2 + (m-1) - n(n+1) \neq 0$

Now equating the coefficient of General term.

x^{m-p-2}

$= A_p (m-p)(m-p-1) - A_{p+2} [(m-p-2)^2 + (m-p-2) - n(n+1)] = 0$

$n^2 + n - n(n+1) = 0$

where $\{x = m-p-2$

$$\boxed{A_{p+2} = \frac{(m-p)(m-p-1)}{[m-p-2-n][m-p-2+n+1]} A_p}$$

Now,

Putting $p=0$.

$$A_2 = \frac{m^2 - m}{(m-2-n)(m-2+n+1)} A_0$$

$$A_2 = \frac{m^2 - m}{(m-2-n)(m-1+n)} \cdot A_0$$

$p=1 \Rightarrow A_3 = 0$

$p=2 \Rightarrow A_4 = \dfrac{m(m-1)(m-2)(m-3)}{(m-4-n)(m-3+n)(m-2-n)(m-1+n)}$

Now sub. all these values in eq.ⁿ ②

$$y = \sum_{r=0}^{\infty} A_r x^{m-r}$$

$$y = A_0 x^m + A_1 x^{m-1} + A_2 x^{m-2} + A_3 x^{m-3} + \dots$$

$$y = x^m \left[A_0 + A_1 x^{-1} + A_2 x^{-2} + A_3 x^{-3} + \dots\right]$$

$$y = x^m \left[A_0 + 0 + \frac{(m^2-m)\, x^{-2} \cdot A_0}{(m-2-n)(m-1+n)} + 0 + \dots\right]$$

$$y = A_0 \left[x^m + \frac{(m^2-m) \cdot x^{m-2}}{(m-2-n)(m-1+n)} + \frac{m(m-1)(m-2)(m-3) \cdot x^{m-4}}{(m-4-n)(m-3+n)(m-2-n)(m-1+n)}\right]$$

Now putting $m=n$ in eq ④

$$[y]_{m=n} = A_0 \left[x^n - \frac{n(n-1)}{2(2n-1)} x^{n-2} + \frac{n(n-1)(n-2)(n-3)}{(2n-1)(2n-3)\,2\cdot 4}\right]$$

Now putting $m = -(n+1)$ in eqn ④

$$[y]_{m=-(n+1)} = A_0 \left[x^{-n-1} + \frac{(n+1)(n+2)}{2\cdot(2n+3)} x^{-n-3} + \frac{(n+1)(n+2)(n+3)}{2\cdot 4\,(2n+3)(2n+5)}\right]$$

then the complete sol^n of the eq^n :

$$y = c_1 [y]_{m=n} + c_2 [y]_{m=-(n+1)}$$

Ans

4

$x^{n-4} + \cdots]$

$3)(n+4)\, x^{-n-5} + \cdots$

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* Legender's functions of first kind:- $P_n(x)$.

If n is positive integer then

$$y = A_0\left[x^n - \frac{n(n-1)}{2(2n-1)}x^{n-2} + \frac{n(n-1)(n-2)(n-3)}{(2n-1)(2n-3)2.4}x^{n-4} + \cdots\right]$$

Let $A_0 = \dfrac{1.3.5\ldots(2n-1)}{n!}$

then

$$P_n(x) = y = \frac{1.3.5\ldots(2n-1)}{n!}\left[x^n - \frac{n(n-1)}{2(2n-1)} + \frac{n(n-1)(n-2)(n-3)x^{n-4}}{(2n-1)(2n-3)2.4}\right]$$

Proof ⇒

t kind:-

(n).

then

$\frac{(n-2)(n-3)}{(2n-3)\,2.4} x^{n-4} + \cdots$

$\frac{n(n-1)(n-2)(n-3)\,x^{n-4}}{(2n-1)(2n-3)\,2.4}$

Generating function for $P_n(x)$

$$\boxed{(1-2xt+t^2)^{-1/2} = \sum_{n=0}^{\infty} t^n\, P_n(x)}$$

Q1 Show that $P_n(1) = 1$

Proof ⇒ We know that by the generating function of Legendre's polynomial

$$(1-2xt+t^2)^{-1/2} = \sum_{n=0}^{\infty} t^n\, P_n(x)$$

Now putting $x = 1$

$$(1-2t+t^2)^{-1/2} = \sum_{n=0}^{\infty} t^n\, P_n(1)$$

$$= (1-t)^{-1} = \sum_{n=0}^{\infty} t^n\, P_n(1)$$

$$1+t+t^2+t^3+\cdots+t^n+\cdots\infty$$

$$= P_0(1) + t\,P_1(1) + t^2 P_2(1) + \cdots t^n P_n(1) \cdots$$

Now equating the coeff of 't^n'

$$1 = P_n(1)$$

$$\boxed{P_n(1) = 1}$$

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Q.2 Prove that $P_n(-x) = (-1)^n P_n(x)$

Proof = We know that

By the generating function of legendre's polynomial.

$$(1-2xt+t^2)^{-1/2} = \sum_{n=0}^{\infty} t^n . P_n(x) \quad \text{--- (1)}$$

Now putting $x = -x$

in eq^n (1)

$$[1+2xt+t^2]^{-1/2} = \sum_{n=0}^{\infty} t^n . P_n(-x) \quad \text{--- (2)}$$

Again putting $t = -t$ in eq^n (1)

$$(1+2xt+t^2)^{-1/2} = \sum_{n=0}^{\infty} (-t)^n \; P_n(x)$$

then by eq^n (2)

$$\sum_{n=0}^{\infty} t^n . P_n(-x) = \cancel{(-1)^n} \sum_{n=0}^{\infty} (-t)^n . P_n(x)$$

$\Rightarrow [P_0(-x) + tP_1(-x) + t^2 P_2(-x) + \cdots + t^n P_n(-x) + \cdots$

$\Rightarrow [P_0(x) + (-t)^1 . P_1(x) + (-t)^2 P_2(x) + \cdots + (-t)^n P_n(x) + \cdots$

Now equating the coeff. of "t^n"

$$P_n(-x) = (-1)^n \; P_n(x)$$

Ans