

AVERAGE

The average of a given set of numbers usually refers to the sum of given numbers divided by the total number of terms. In other words, it is the mean value of a set of numbers. Therefore, the average of a set of n numbers is given by

$$\frac{(x_1 + x_2 + x_3 + \dots + x_n)}{n}$$

i.e. Average of given observations = $\frac{\text{Sum of all observations}}{\text{Number of observations}}$

EXAMPLE 1 If the age of four students are 20 yr, 22 yr, 18 yr and 24 yr, then what is the average age of the students?

(a) 21 yr

(b) 22 yr

(c) 24 yr

(d) 25 yr

Solution (a) Average age = $\frac{20 + 22 + 18 + 24}{4} = \frac{84}{4} = 21$ yr

Therefore, the average age of the four students is 21 yr.

EXAMPLE 2 The average of 20 numbers is zero. Of them, at the most how many may be greater than zero?

(a) 0

(b) 1

(c) 10

(d) 19

Solution (d) \because Average of 20 numbers = 0

\therefore Sum of 20 numbers = 0 (given)

Hence, it is possible that 19 of these numbers may be positive. Also, if their sum is a , then 20th number is $(-a)$.

Note

- If all the numbers get increased by ' a ', then their average also gets increased by ' a '.
- If all the numbers get decreased by ' a ', then their average also gets decreased by ' a '.
- If all the numbers are multiplied by ' a ', then their average also gets multiplied by ' a '.
- If all the numbers are divided by ' a ', then their average also gets divided by ' a '.

4.1 WEIGHTED AVERAGE

If the average of n_1 quantities is x and that of n_2 quantities is y , then the average of total

$(n_1 + n_2)$ quantities is given by $\frac{n_1 x + n_2 y}{n_1 + n_2}$.

EXAMPLE 3 In a class, there are 2 students of 20 yr, 3 students of 21 yr, 4 students of 22 yr and 5 students of 23 yr. Find their average age.

- (a) 380/7 yr (b) 426/9 yr
(c) 306/14 yr (d) 295/32 yr

Solution (c) Required average age

$$= \frac{(2 \times 20) + (3 \times 21) + (4 \times 22) + (5 \times 23)}{2 + 3 + 4 + 5}$$

$$= \frac{2}{14} \times 20 + \frac{3}{14} \times 21 + \frac{4}{14} \times 22 + \frac{5}{14} \times 23$$

$$= \frac{306}{14} \text{ yr}$$

EXAMPLE 4 A cricketer has a certain average of 9 innings. In the tenth inning, he scores 100 runs, thereby increasing his average by 8 runs. Calculate his new average.

- (a) 24 runs (b) 25 runs
(c) 32 runs (d) 28 runs

Solution (d) Let the average of 9 innings be x runs, then new average will be $(x + 8)$ runs.

Total runs scored for 9 innings = $9x$

Total runs scored for 10 innings = $(9x + 100)$

$$\therefore \text{Average for 10 innings} = \frac{\text{Total runs}}{10}$$

$$\Rightarrow (x + 8) = \frac{(9x + 100)}{10}$$

$$\Rightarrow x = 20$$

Therefore, new average = $20 + 8 = 28$ runs

EXAMPLE 5 Find out the average of consecutive even numbers till 20.

- (a) 12 (b) 11
(c) 13 (d) 20

Solution (b) Average of consecutive even numbers till $n = \frac{n+2}{2}$

$$= \frac{(20+2)}{2} = \frac{22}{2} = 11 \quad (\because n = 20)$$

4.2 AVERAGE SPEED

Average speed is defined as total distance divide by the total time.

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

Let the distance between two points A and B is d km and the speed in travelling from point A to B is x km/h and from point B to A is y km/h. Then, average speed = $\frac{2xy}{x+y}$

EXAMPLE 6 If a person travels two equal distances at 30 km/h and 10 km/h, then what is the average speed for the entire journey?

- (a) 15 km/h (b) 25 km/h
(c) 30 km/h (d) 20 km/h

Solution (a) Here, $x = 30$ km/h and $y = 10$ km/h

$$\therefore \text{Average speed} = \frac{2xy}{x+y} = \frac{2 \times 30 \times 10}{30+10} = \frac{600}{40} = 15 \text{ km/h}$$

EXAMPLE 7 Distance between two stations A and B is 778 km. A train covers the journey from A to B at 84 km/h and returns back to A with a uniform speed of 56 km/h. Find the average speed of train during the whole journey.

- (a) 32 km/h (b) 47.9 km/h
(c) 58.6 km/h (d) 67.2 km/h

Solution (d) Here, $x = 84$ km/h and $y = 56$ km/h

$$\therefore \text{Required average speed} = \frac{2xy}{x+y}$$

$$= \frac{2 \times 84 \times 56}{84+56} = \frac{9408}{140} = 67.2 \text{ km/h}$$

EXAMPLE 8 Joe drives 120 miles at 60 miles/h and then drives the next 120 miles at 40 miles/h. What is his average speed for the entire trip?

- (a) 42 (b) 48 (c) 50 (d) 54

Solution (b) Time spent during 60 miles/h = $\frac{120}{60} = 2$ h

$$\text{Time spent during 40 miles/h} = \frac{120}{40} = 3 \text{ h}$$

\therefore Total time = $2 + 3 = 5$ h
and total distance = $120 + 120 = 240$ miles

$$\therefore \text{Required average speed} = \frac{240 \text{ miles}}{5 \text{ h}} = 48 \text{ miles/h}$$

4.1.1 Average of Some Important Series of Numbers

1. Average of consecutive odd numbers

$$\text{from 1 to } n = \frac{(n+1)}{2}$$

where n = last odd number

2. Average of consecutive even numbers

$$\text{from 2 to } n = \frac{(n+2)}{2}$$

where n = last even number

3. Average of squares of natural numbers till n

$$= \frac{(n+1)(2n+1)}{6}$$

4. Average of cubes of natural numbers till n

$$= \left[\frac{n(n+1)^2}{4} \right]$$

5. Average of first n consecutive even numbers = $(n+1)$

6. Average of squares of first n consecutive even numbers

$$= \frac{2(n+1)(2n+1)}{3}$$



CHECK YOUR PREPARATION LEVEL

{EXERCISE 1}

1. The average of two numbers is M . If one number is N , then another number is
(a) $2N$ (b) $2M$ (c) $M - N$ (d) $2M - N$
2. The average of marks of a student in 7 subjects is 75. His average of marks in 6 subjects excluding Science is 72. How many marks did he get in Science?
(a) 72 (b) 90
(c) 93 (d) None of these
3. The mean temperature of Monday to Wednesday was 37°C and of Tuesday to Thursday was 38°C . If the temperature on Thursday was $\frac{4}{5}$ that of Monday, the temperature on Thursday was
(a) 38°C (b) 36°C (c) 40°C (d) Data inadequate
4. The average salary per month of 30 employees in a company is ₹ 4000. If the manager's salary is added, the average salary increases to ₹ 4300, what is the salary of the manager?
(a) ₹ 10000 (b) ₹ 13000 (c) ₹ 12000 (d) ₹ 13300
5. In seven given numbers, the average of first four numbers is 4 and that of last four numbers is also 4. If the average of these seven numbers is 3, the fourth number is
(a) 3 (b) 4 (c) 7 (d) 11
6. Mukesh has twice as much money as Sohan and Sohan has 50% more money than what Pankaj has. If the average money with them is ₹ 110, then Mukesh has
(a) ₹ 55 (b) ₹ 60 (c) ₹ 90 (d) ₹ 180
7. The average marks of a group of 20 students in a test is reduced by 4 when the topper who scored 90 marks is replaced by a new student. How many marks did the new student get?
(a) 12 (b) 15 (c) 10 (d) 20
8. The average weight of 24 students of section A of a class is 58 kg, whereas the average weight of 26 students of section B of the same class is 60.5 kg. Find the average weight of all the 50 students of the class.
(a) 62 kg (b) 59.3 kg
(c) 60 kg (d) 63.2 kg
9. The average of first 62 natural numbers is
(a) 31 (b) 31.5 (c) 31.2 (d) 32
10. The average of squares of natural numbers from 1 to 35 is
(a) 436 (b) 426 (c) 416 (d) 446
11. The average of cubes of natural numbers from 1 to 8 is
(a) 162 (b) 172 (c) 153 (d) 163
12. The average of first 52 consecutive even numbers is
(a) 52 (b) 53 (c) 51 (d) 50
13. The average of squares of first 11 consecutive even numbers is
(a) 184 (b) 148 (c) 186 (d) 174
14. A motorist travels to a place 150 km away at an average speed of 50 km/h and returns at 30 km/h. His average speed (in km/h) for the whole journey is
(a) 35 (b) 37 (c) 37.5 (d) 40
15. A person covers half his journey by train at 60 km/h, half the remainder by bus at 30 km/h and the rest by bicycle at 10 km/h. Find his average speed during the entire journey.
(a) 15 km/h (b) 24 km/h
(c) 20 km/h (d) 33.3 km/h
16. The average age of boys is twice the number of girls in a class. If the ratio of boys and girls in the class of 36 is 5 : 1, what is the total ages (in yr) of the boys in class?
(a) 490 (b) 196 (c) 420 (d) 360
17. The average of 100 observations was calculated as 40. It was found later on that one of the observations was misread as 83 instead of 53. The correct average is
(a) 39 (b) 39.7 (c) 40.3 (d) 42.7
18. The average of 50 numbers is 38. If two numbers, namely 45 and 55 are discarded, the average of remaining numbers is
(a) 36.5 (b) 37 (c) 37.5 (d) 37.52
19. A team of 8 persons joins a shooting competition. The best marksman scored 85 points. If he had scored 92 points, the average score for the team would have been 84. The number of points, the team scored was
(a) 672 (b) 665 (c) 645 (d) 588
20. Three years ago, the average age of a family of 5 members was 17 yr. A baby having been born, the average age of the family is the same today. The present age of the baby is
(a) 2 yr (b) 2.4 yr (c) 3 yr (d) 1.5 yr
21. The average weight of 3 men A, B and C is 84 kg. Another man D joins the group and the average now becomes 80 kg. If another man E, whose weight is 3 kg more than that of D, replaces A, then the average weight of B, C, D and E becomes 79 kg. The weight of men A is
(a) 70 kg (b) 72 kg (c) 75 kg (d) 80 kg
22. The average age of a committee of eight members is 40 yr. A member aged 55 yr retired and his place was taken by another member aged 39 yr. The average age of the present committee is
(a) 39 yr (b) 38 yr
(c) 36 yr (d) 35 yr

23. In a cricket team, the average age of eleven players is 28 yr. Out of these, the average ages of three groups of three players each are 25 yr, 28 yr and 30 yr, respectively. If in these groups, the captain and the youngest players are not included and the captain is eleven years older than the youngest player, what is the age of the captain?

- (a) 33 yr (b) 34 yr
(c) 35 yr (d) 36 yr

24. The average of 5 numbers is 7. When 3 new numbers are added, the average of the eight numbers is 8.5. The average of three new numbers is

- (a) 11 (b) 7.75
(c) 8.5 (d) 7

{EXERCISE 2}

1. The average marks scored by Ganesh in English, Science, Mathematics and History is less than 15 from that scored by him in English, History, Geography and Mathematics. What is the difference of marks in Science and Geography scored by him?

- (a) 40 (b) 50 (c) 60 (d) Data inadequate

2. Out of three numbers, the first number is twice the second number and is half of the third number. If the average of the three numbers is 56, the three numbers in order are

- (a) 48, 96 and 24 (b) 48, 24 and 96
(c) 96, 24 and 48 (d) 96, 48 and 24

3. The average weight of 15 students was calculated to be 52 kg and it was later discovered that one weight was misread as 21 kg instead of the correct one of 12 kg. The correct average weight is

- (a) 51.4 kg (b) 50.6 kg (c) 52.4 kg (d) 51.6 kg

4. The average age of a husband and wife was 23 when they were married 5 yr ago. The average age of the husband, the wife and a child who was born during the interval, is 20 yr now. How old is the child now?

- (a) 9 months (b) 1 yr
(c) 3 yr (d) 4 yr

5. 5 yr ago, the average age of A, B, C and D was 45 yr. By including X, the present average of all the five is 49 yr. The present age of X is

- (a) 64 yr (b) 48 yr
(c) 45 yr (d) 40 yr

6. 5 yr ago, the average of Ram's and Shyam's ages was 20 yr. Now, the average age of Ram, Shyam and Mohan is 30 yr. What will be Mohan's age 10 yr hence?

- (a) 45 yr (b) 50 yr (c) 49 yr (d) 60 yr

7. A batsman in his 20th innings makes a score of 110 and thereby increases his average by 4. What is his average after 20 innings?

- (a) 34 (b) 43
(c) 36 (d) 30

8. The average age of A and B is 20 yr. If C were to replace A, the average would be 19 and if C were to replace B, the average would be 21. What are the ages of A, B and C, respectively?

- (a) 22, 18 and 20 (b) 18, 22 and 20
(c) 22, 20 and 18 (d) 18, 20 and 22

9. There was one mess for 30 boarders in a certain hostel. If the number of boarders was increased by 10, the expenses of the mess increased by ₹ 40 per month, while the average expenditure per head diminished by ₹ 2. Find the original monthly expenses.

- (a) ₹ 390 (b) ₹ 360 (c) ₹ 410 (d) ₹ 480

10. The average age of a family of 6 members is 22 yr. If the age of the youngest member be 7 yr, what was the average age of the family at the birth of the youngest members?

- (a) 15 yr (b) 18 yr (c) 21 yr (d) 22 yr

{ANSWERS}

EXERCISE 1

1	d	2	c	3	b	4	d	5	d	6	d	7	c	8	b	9	b	10	b
11	a	12	b	13	a	14	c	15	b	16	d	17	b	18	c	19	b	20	a
21	c	22	b	23	c	24	a												

EXERCISE 2

1	c	2	b	3	a	4	d	5	c	6	b	7	a	8	a	9	b	10	b
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EXPLANATIONS

{EXERCISE 1}

1. Given, $M = \frac{N+x}{2}$, where x is the other number.

Then, $x = 2M - N$

2. \therefore Average marks

$$= \frac{\text{Sum of marks in all subjects}}{\text{Total number of subjects}}$$

$$\therefore \text{Sum of marks in 7 subjects} = 75 \times 7 = 525$$

$$\text{Sum of marks in 6 subjects (excluding Science)} = 72 \times 6 = 432$$

$$\therefore \text{Total marks in Science} = 525 - 432 = 93$$

3. Temperature of (Mon + Tue + Wed) = $37 \times 3 = 111^\circ \text{C}$... (i)

and temperature of (Tue + Wed + Thurs) = $34 \times 3 = 102^\circ \text{C}$... (ii)

Given, Thurs = $\frac{4}{5}$ Mon

\therefore Eq. (ii) becomes

$$\text{Tue} + \text{Wed} + \frac{4}{5} \text{Mon} = 102^\circ \text{C} \quad \dots \text{(iii)}$$

On subtracting Eq. (iii) from Eq. (i), we get $\frac{1}{5} \text{Mon} = 9^\circ \text{C} \Rightarrow \text{Mon} = 45^\circ \text{C}$

Hence, Thurs = $\frac{4}{5} \times 45 = 36^\circ \text{C}$

4. Salary of manager = $(4300 \times 31) - (4000 \times 30) = 133300 - 120000 = ₹13300$

5. Fourth number = $[(4 \times 4) + (4 \times 4)] - (7 \times 3) = 32 - 21 = 11$

6. Let Pankaj has ₹ x , then Sohan has ₹ $1.5x$ and Mukesh has ₹ $2 \times 1.5x$ i.e. ₹ $3x$.

Total money with all the three = ₹ $(x + 1.5x + 3x) = 5.5x$

Average money = $\left(\frac{5.5x}{3}\right) = ₹110$ (given)

$$\Rightarrow x = ₹60$$

$$\therefore \text{Mukesh has} = ₹60 \times 3 = ₹180$$

7. Total marks reduction of 20 students = $20 \times 4 = 80$
This reduction in marks takes place, because the topper with 90 marks is replaced by a student whose marks are 80 less than that of topper.
Hence, marks scored by the new student = $90 - 80 = 10$.

8. Here, $n_1 = 24, n_2 = 26, x = 58$ and $y = 60.5$

\therefore Average weight of all the 50 students

$$= \frac{n_1 x + n_2 y}{n_1 + n_2} = \frac{24 \times 58 + 26 \times 60.5}{24 + 26} = \frac{1392 + 1573}{50} = \frac{2965}{50} = 59.3 \text{ kg}$$

9. Average of first n natural numbers

$$= \left(\frac{n+1}{2}\right) = \left(\frac{62+1}{2}\right) = 31.5 \quad (\because n = 62)$$

10. Average of squares of natural numbers till

$$n = \frac{(n+1)(2n+1)}{6} = \frac{(35+1)(2 \times 35+1)}{6} = \frac{36 \times 71}{6} = 426 \quad (\because n = 35)$$

11. Average of cubes of natural

numbers from 1 to $n = \left[\frac{n(n+1)^2}{4}\right]$
 $= \left[\frac{8(8+1)^2}{4}\right] = \frac{8 \times 81}{4} = 162 \quad (\because n = 8)$

12. Average of first n consecutive even numbers = $(n+1)$

$$\therefore \text{Average} = 52 + 1 = 53 \quad (\because n = 52)$$

13. Average of squares of first n consecutive even numbers

$$= \frac{2(n+1)(2n+1)}{3} \therefore \text{Average} = \frac{2(11+1)(2 \times 11+1)}{3} = \frac{2 \times 12 \times 23}{3} = 184 \quad (\because n = 11)$$

14. Average speed = $\frac{\text{Total distance}}{\text{Total time}} = \frac{300}{\frac{150}{30} + \frac{150}{50}} = \frac{300}{8} = 37.5 \text{ km/h}$

Alternate Method

Here, $x = 50 \text{ km/h}$ and $y = 30 \text{ km/h}$

$$\text{Average speed} = \frac{2xy}{x+y} = \frac{2 \times 50 \times 30}{80} = 37.5 \text{ km/h}$$

15. Let the total distance travelled be $x \text{ km}$.

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{x}{\frac{x}{2 \times 60} + \frac{x}{4 \times 30} + \frac{x}{4 \times 10}} = \frac{120}{5} = 24 \text{ km/h}$$

16. \therefore Number of boys = 30 and number of girls = 6
Average age of boys = 12 yr
 \therefore Total age of boys = $12 \times 30 = 360 \text{ yr}$

17. Total of 100 observations = 4000
Correct total of 100 observations = $(4000 - 83 + 53) = 3970$
 \therefore Correct average of 100 observations = $\frac{3970}{100} = 39.7$

18. Average of remaining numbers = $\frac{(38 \times 50) - (45 + 55)}{48} = \frac{1800}{48} = 37.5$

19. Let the number of points scored by 7 persons be x .

$$\therefore \text{New average} = \frac{x+92}{8} = 84 \quad x = 580$$

\therefore Here, number of points scored by the team = $(580 + 85) = 665$

20. Total age of family 3 yr ago = $17 \times 5 = 85 \text{ yr}$
Total age of family today = $85 + 5 \times 3 = 100 \text{ yr}$

Let the age of the baby be $x \text{ yr}$.
 $\therefore \frac{100+x}{6} = 17$
 $\Rightarrow x = 2 \text{ yr}$

21. Total weight of A, B and C

$$= 84 \times 3 = 252 \text{ kg}$$

Total weight of A, B, C and D

$$= 80 \times 4 = 320 \text{ kg}$$

Weight of D = $320 - 252 = 68 \text{ kg}$

Weight of E = $68 + 3 = 71 \text{ kg}$

Total weight of B, C, D and E

$$= 79 \times 4 = 316 \text{ kg}$$

\therefore Total weight of B, C, D = $(316 - 71)$

$$= 245 \text{ kg}$$

\therefore Weight of A = $320 - 245$

$$= 75 \text{ kg}$$

22. Total age of 8 members

$$= 40 \times 8 = 320 \text{ yr}$$

New age of 8 members

$$= 320 - 55 + 39$$

$$= 304 \text{ yr}$$

New average age of 8 members

$$= \frac{304}{8} = 38 \text{ yr}$$

23. Total age of 11 players = 308 yr

Total age of 9 players in a group of 3

$$= (25 \times 3 + 28 \times 3 + 30 \times 3)$$

$$= 249 \text{ yr}$$

Total age of remaining 2 players

$$= 308 - 249 = 59 \text{ yr}$$

Let the age of youngest player be

x yr, then age of captain = $(x + 11)$

Now, $x + (x + 11) = 59$

$$\therefore x = 24 \text{ yr}$$

Therefore, age of captain = $24 + 11$

$$= 35 \text{ yr}$$

$$24. \text{ Average} = \frac{(8.5 \times 8) - (5 \times 7)}{3}$$

$$= \frac{68 - 35}{3} = \frac{33}{3} = 11$$

{EXERCISE 2}

1. Average marks

(English + Science + Maths

$$= \frac{\text{+ History}}{4}$$

English + Geography

$$= \frac{\text{+ Maths + History} - 15}{4}$$

\Rightarrow English + Science + Maths

+ History

= English + Geography + Maths

+ History - 60

\therefore Geography - Science = 60

2. Let the first number be x .

Then, second number = $\frac{x}{2}$

and third number = $2x$

$$\therefore \text{Average} = \frac{x + \frac{x}{2} + 2x}{3} = 56$$

$$\Rightarrow \frac{7x}{6} = 56$$

$$\Rightarrow x = 48$$

\therefore Numbers are 48, 24 and 96.

3. Total weight of 15 students

$$= 15 \times 52$$

$$= 780 \text{ kg}$$

Correct weight of 15 students

$$= 780 + 12 - 21$$

$$= 771$$

\therefore Average weight of 15 students

$$= \frac{771}{15} = 51.4 \text{ kg}$$

4. Present total age of husband and

wife = $(23 \times 2 + 5 \times 2) = 56 \text{ yr}$

Present total age of husband, wife

and child

$$= 20 \times 3 = 60 \text{ yr}$$

\therefore Age of child = $60 - 56 = 4 \text{ yr}$

5. Total present age of A, B, C and D

$$= (45 \times 4 + 5 \times 4) = 200 \text{ yr}$$

Total present age of A, B, C, D and X

$$= 49 \times 5 = 245 \text{ yr}$$

\therefore Present age of X = $245 - 200$

$$= 45 \text{ yr}$$

6. Present age of Ram and Shyam

$$= (20 \times 2 + 5 \times 2) = 50 \text{ yr}$$

Present age of Ram, Shyam and

Mohan = $30 \times 3 = 90 \text{ yr}$

Present age of Mohan = 40 yr

\therefore Age of Mohan 10 yr hence = 50 yr

7. Let the runs scored upto 19th

innings = x

$$\text{Then, average runs} = \frac{x}{19}$$

Now, total runs scored upto 20th

innings = $(x + 110)$ runs

$$\text{Given, } \frac{x}{19} + 4 = \frac{x + 110}{20}$$

$$\Rightarrow \frac{x + 76}{19} = \frac{x + 110}{20}$$

$$\Rightarrow 20x + 1520 = 19x + 2090$$

$$\Rightarrow x = 570$$

\therefore Average after 20 innings

$$= \frac{570 + 110}{20} = 34$$

8. Total age of A and B = $20 \times 2 = 40 \text{ yr}$

Total age of B and C = $19 \times 2 = 38$

Total age of A and C = $21 \times 2 = 42 \text{ yr}$

Total age of A, B and C

$$= \frac{40 + 38 + 42}{2} = 60 \text{ yr}$$

\therefore Age of A = $60 - 38 = 22 \text{ yr}$

Age of B = $60 - 42 = 18 \text{ yr}$

and age of C = $60 - 40 = 20 \text{ yr}$

9. Let the original average expenses be ₹ x .

Then, according to the question,

$$40(x - 2) - 30x = 40$$

$$\Rightarrow 10x = 120 \Rightarrow x = 12$$

\therefore Original monthly expenses

$$= 30 \times 12 = ₹ 360$$

10. Total present age of the family of 6 members = $6 \times 22 = 132 \text{ yr}$

Total age of the family of 6 members

7 yr ago = $(132 - 7 \times 6) = 90 \text{ yr}$

\therefore Average age of the family at the

$$\text{birth of the youngest member} = \frac{90}{5} = 18 \text{ yr}$$