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MIMO HW4

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1.1 Spatial Multiplexing Techniques

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2.1 Input

The inputs are summarised below

- M: Number of transmit antennas.
- N: Number of receive antennas.
- Ntrials: Number of trials of the experiment
- Ns: Number of tx symbols
- NPilots: Number of pilot symbols

2.2 Output

We have various BER for the different scenarios outlined in the questions.

3 Validation

3.1 Q1. SIC+MMSE vs MLE vs ZF

3.1.1 SIC + MMSE Nulling

SIC on its own does not perform well due to noise amplification as its very sensitive to the first symbol that is decoded. In case the first symbol decoded is in error it leads to noise amplification. The algorithm is outline below:

- 1. Initialize
  - $\mathbf{T}(1) = \mathbf{H}^{\dagger}$
  - $r_1 = \mathbf{r}$
  - Set $\mathcal{A} = \{1, 2, 3, \ldots N_t\}$. Set $\mathcal{B} = \{\}$.
- 2. $k_i = \arg\min_{j \notin B} \left\| [\mathbf{T}(i)]_{j,=} \right\|^2$

- 3. Remove k_i from $\mathcal{A}$. Add k_i to $\mathcal{B}$.
- 4. Create the nulling vector as $\mathbf{w}_{k_i} = [\mathbf{T}(i)]_{k_i,:}$
- 5. Apply the nulling vector to the k_i th stream to obtain the decision variable $z_{k_i} = \mathbf{w}_{k_i}\mathbf{r}_i$
- 6. Obtain a symbol estimate for the k_i th stream using an appropriate decision function: $\hat{s}_{k_i} = f\left(z_{k_i}\right)$
- 7. Cancel the k_i th stream: $\mathbf{r}_{i+1} = \mathbf{r}_i - \hat{s}_{k_i}\mathbf{H}_{i,k_i}$
- 8. Determine the new transform $\mathbf{T}(i) = \left(\mathbf{H}_{\bar{B}}\right)^{\dagger}$
- 9. Set $i = i+1$ and go to 2

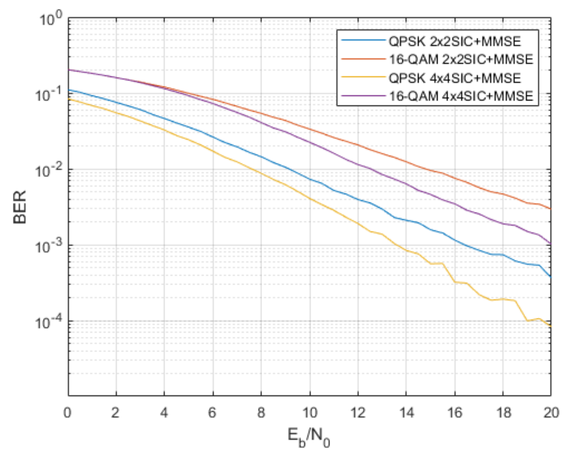

Figure 1: SIC with MMSE nulling

In Figure 1 the BER plots for SIC with MMSE nulling is shown SIC with MMSE works better than SIC with F since, ZF is faced with an issue of noise amplification. ZF works by forcing the interference from other transmitters to 0 however this has the effect of amplification of the noise in the symbol of interest. This is shown in the equation of the decision metric below. The noise term has the pseudo inverse of the channel multiplied. On calculating the new noise covariance matrix, we note on streams with eigen values > 1 the noise will be amplified compared to a SIMO case.

$$
\begin{aligned}
\mathbf{z} &= \sqrt{\frac{N_t}{E_s}}\mathbf{H}^{\dagger}\mathbf{r} \\
&= \sqrt{\frac{N_t}{E_s}}\mathbf{H}^{\dagger}\left(\sqrt{\frac{E_s}{N_t}}\mathbf{H}\mathbf{s}+\mathbf{n}\right) \\
&= \mathbf{s} + \sqrt{\frac{N_t}{E_s}}\mathbf{H}^{\dagger}\mathbf{n}
\end{aligned}
$$

3.1.2 Zero Forcing

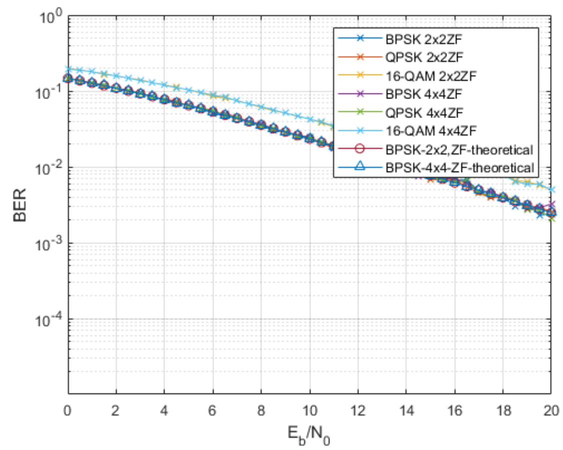

Figure 2: ZF receiver

ZF receiver performs much worse than the other receivers with the same spectral efficiency.

3.1.3 MLE

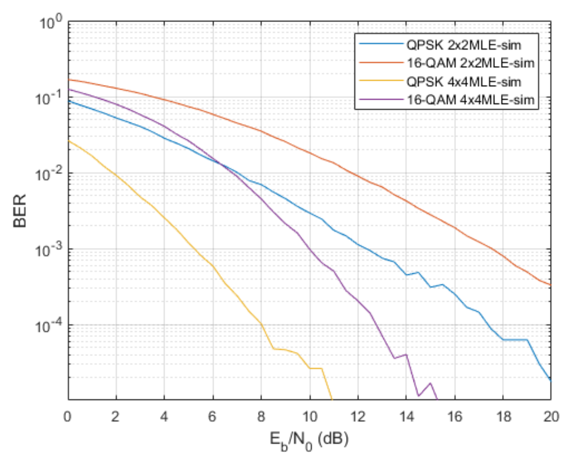

Figure 3: MLE receiver

The MLE receiver offers best performance however it has extremely high computation requirements. Specifically we do a brute force search over $(2^k)^{N_t}$ possible transmit symbols at the receiver, where k is bits per symbol and N_t is the number of transmit antennas. The reason for increase in performance when we increase tx antennas is because since we add more dimensions to the possible transmit signals, the distance between symbols increases. The logic is similar to FSK where each carrier frequency is a new dimension.

3.1.4 SIC+MMSE vs SIMO

For the same spectral efficiency and receiver antennas at high SNR's SIMO performs better since SIC+MMSE has to separate interfering symbols at the receiver in addition to dealing with noise, whereas SIMO only has to deal with noise though it has a larger constellation size.

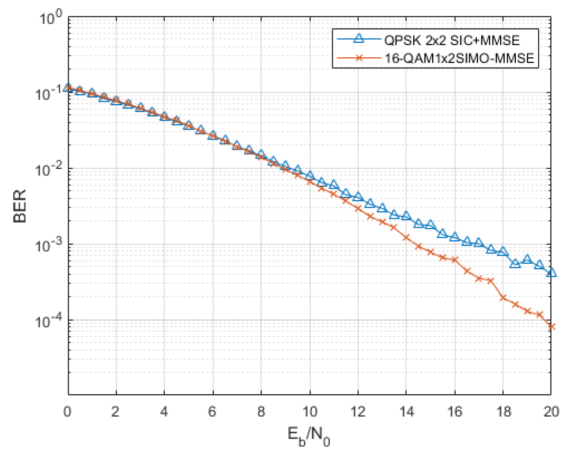

Figure 4: 16-QAM 1X2 MMSE system vs QPSK 2x2 SIC+MMSE

3.2 Q2. SIC with MMSE for correlated channels

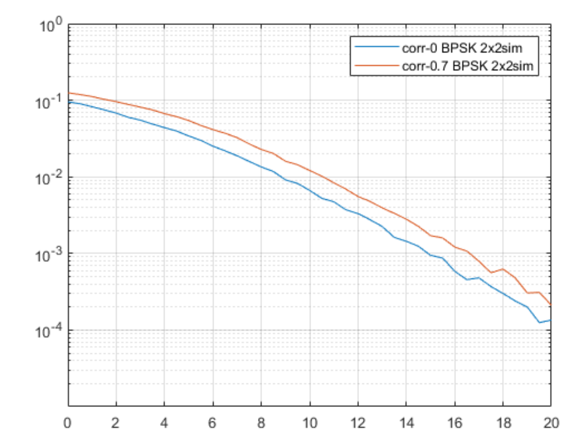

Figure 5: 2X2 BPSK, SIC + MMSE in a correlated channel

For correlated channels, we expect lower performance since if one of the channels is correlated, the other one is also likely to be in fade. Hence as compared to uncorrelated channels the performance for correlated channels will be lower. The simulation confirms this.

3.3 Q3. Full channel state information at the transmitter

Full channel state information can be exploited using Singular Value Decomposition. First the channel is estimated and then using svd we obtain three matrices U,S,V. U is $Nr \times Nr$, S is $Nr \times Nt$ and V is $Nt \times Nt$. U and V are the left and right orthogonal matrices and S is a diagonal matrix with the singular values. Note the singular values are the square root of the eigen values. If the matrix V is premultiplied at the transmitter side, and at the received signal is multiplied with U, the net effect is the symbols become separable at the receiver. Optimum power allocation is also done at the transmitter and that is achieved using waterfilling according to the algorithm shown below

- 1. Initialize $p = 1(r - p + 1$ modes will be used where r is the rank of the channel and is the number of available modes). Further, order the eigenvalues (or modes) from largest to smallest.

- 2. Calculate the "water level":

$$\mu = \frac{N_t}{r-p+1}\left[1 + \frac{N_o}{E_s}\sum_{i=1}^{r-p+1}\frac{1}{\lambda_i}\right]$$

  where λ_i are the eigenvalues of the $\mathbf{HH}^H$ and E_s/N_o is the average energy per symbol across all modes.

- 3. Determine the gain per mode:

$$\eta_i = \mu - \frac{N_t}{\lambda_i}\frac{N_o}{E_s} \quad i = 1, 2, \ldots r - p + 1$$

- 4. Determine if $\eta_i < 0$ for any mode. If not, stop. If so, $p = p + 1$, go to 2 .

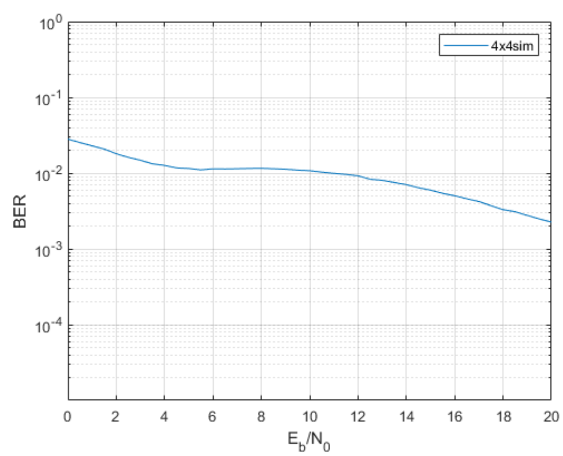

Figure 6: 4X4 QPSK, SIC + MMSE in an uncorrelated channel

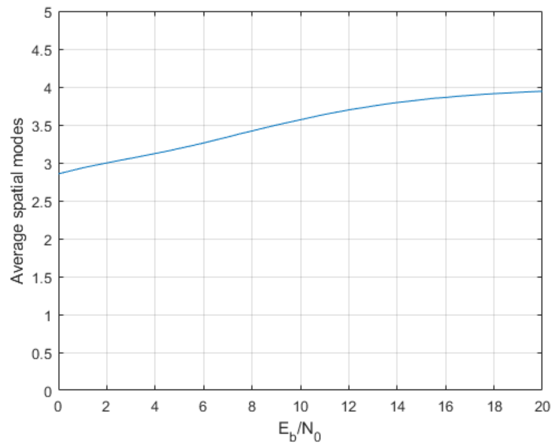

Figure 7: Average Spatial modes used vs SNR in a 4x4 uncorrelated channel

To show how the SVD based eigenbeamforming works, we plot the average spatial modes vs SNR for a 4X4 channel. in Figure 7 for low SNR we see on an average 3 spatial modes are assigned using waterfilling, however as the SNR goes up it goes upto the maximum 4 possible spatial modes. We now repeat the experiment after introducing correlation into the channels using the below idea. Let the desired correlation matrix be C and decomposing it using cholesky decomposition to:

$$C = LL^T \tag{1}$$

Here L is triangular (upper or lower). Let the uncorrelated samples be in matrix X then the covariance of $Z = LX$ is given by:

$$\begin{aligned} E\left[ZZ^T\right] &= E\left[(LX)(LX)^T\right] \\ &= E\left[LXX^TL^T\right] \\ &= LE\left[XX^T\right]L^T \\ &= LIL^T = LL^T = C \end{aligned} \tag{2}$$

Hence to generate a correlated channel, we generate an uncorrelated channel matrix and then premultiply it by the cholesky decomposition matrix of the desired covariance matrix. On examining the plots for correlation $\rho = 0.9$ at first glance the BER gets worse on increasing the SNR, however as we see the spatial modes at low SNR's the average spatial modes are just 1 and then on increasing the snr, the number of spatial modes rises up.

Equivalent Spectral efficient system: For equivalent spectral efficiency, assuming the channel is uncorrelated, at high snr, we can assume we have as many spatial modes as the number of transmit antennas. Now since we are using QPSK, we have spectral efficiency $N_t = 4 \times 2 = 8bps/Hz$. An equivalent 1×4 SIMO system with same spectral efficiency should have a 256-QAM modulation scheme. To theoretically compare the performance, I expect the SIMO system will behave worse. We know both systems have 4 spatial modes since in the SIMO case we have 4 rx antennas and an uncorrelated channel between the single tx and 4 rx. However in the SVD-QPSK MIMO system because we assign power using water-filling, we maximise the capacity. Hence the capcity will be better due to better distribution of power over the spatial modes.

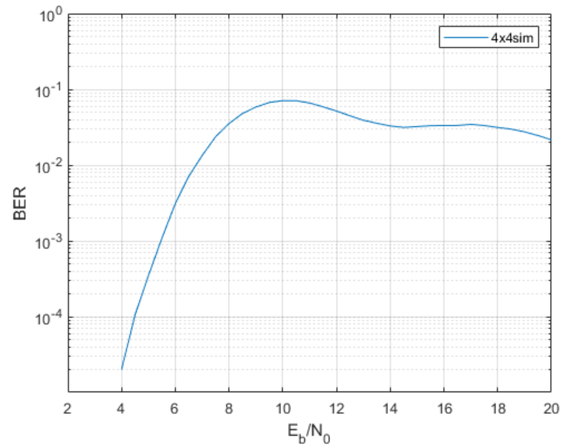

Figure 8: 4X4 QPSK, SIC + MMSE in a correlated channel $\rho = 0.99$

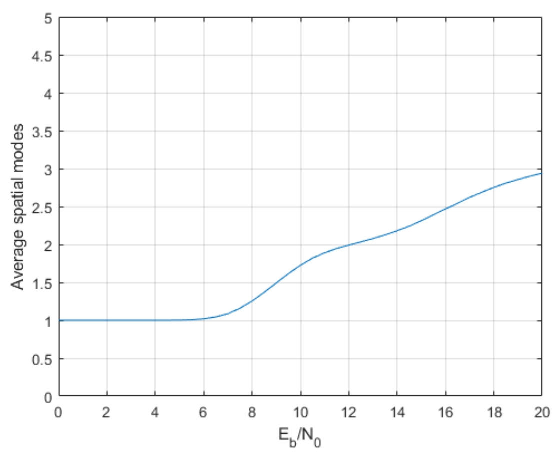

Figure 9: Spatial modes vs snr in a correlated channel $\rho = 0.99$

3.3.1 Q4. Channel estimation for Spatial Multiplexing

To estimate the $Nr \times Nt$ channel matrix, we send pilots on all the transmit antennas and using them we can estimate the channel at the receiver. One thing to note is the pilots should be orthogonal with each other since we need to be able to distinguish them at the receiver. We can make them orthogonal either by TDMA, or over frequency or using orthogonal codes. There are also spatial techniques like Alamouti that we have used for channels before.

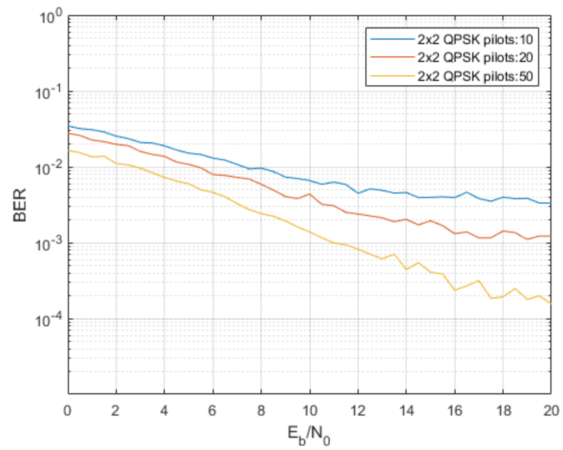

Figure 10: SIC-MMSE with 10,20 and 50 pilots.

As expected using more Pilots improves the BER performance especially at higher SNR's.

4 Conclusion

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- Spatial Multiplexing offers very good spectral efficiency for low order modulation schemes
- the challenge is to decode the symbols at the receiver which is computationally a hard problem to solve using MLE which offers the best performance
- Zero forcing receiver makes the problem computationally easy, however suffers from noise amplification issues
- MMSE minimises the mean squared error hence offers better performance
- SIC is a non linear way to separate the transmitted symbols however is sensitive to the error in the first symbol decoded so its usually paired with an MMSE/ZF nulling step first
- SVD based beamforming offers good performance by finding the number of orthogonal spatial modes that the channel can be spit into. The downside is it requires feedback to the transmitters. It is usually paired with with water-filling which maximises capacity by dividing power over all the spatial modes if there is enough power or finding the best number of modes to maximise capacity

References

[1] "Grcon22 - introduction to mimo and simple ways to use it in gnu radio by matt ettus 2022," https://www.youtube.com/watch?v=4IaoepVDv1wt=2138sab$_c hannel = GNURadio, Oct$2022.