Multi-channel Communications Fall 2022

Lecture 17
The Wireless Channel Revisited – Frequency Selective Fading

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Introduction

- The wireless channel (and our modeling of it) is a very important aspect of studying/analyzing wireless communications
- Previously we examined the wireless channel with an emphasis on temporal characteristics and spatial characteristics
- Today we emphasize the frequency domain aspects of the wireless channel

Basic Channel Effects

- Three general (second order) measures of the channel
  - Angle spread – dispersion of the received signal in angle (departure from a plane wave)
  - Doppler spread – dispersion of the received signal in frequency (transmitted tone is not received as a tone)
  - Delay spread – dispersion of the signal in time (resolvable multipath)

- These measures correspond directly to
  - Spatial fading (measured by spatial correlation)
  - Temporal fading (measured by temporal correlation)
  - Frequency fading (measured by frequency correlation)

Simplified Channels

- In classic cellular/mobile communication analysis, the bandwidth of the channel is small relative to the coherence bandwidth of the channel
  - Emphasis on temporal correlation, *i.e.*, classic fading
  - Spatial aspects greatly simplified (independent or perfectly correlated)
- In MIMO system analysis we typically ignore or simplify temporal and frequency dependent effects and focus on the spatial channel
- For OFDM, we will simplify temporal and spatial effects and focus on frequency-selective fading

The General Wireless Channel

- In general, when we transmit a sinusoidal signal we observe at the receiver a sum of sinusoids (i.e., multipath) where each sinusoid is
  - Scaled
  - Phase-shifted
  - Frequency-shifted
- The channel can be represented as a time-varying linear filter that has an impulse response

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Discrete Model

- Let the channel be time-invariant. The sampled channel impulse response can be written as an *L x 1* channel vector

$$\mathbf{h} = \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{L-1} \end{bmatrix} \qquad h_i = h(\tau)\big|_{\tau = iT_s}$$

Where $f_s = 1/T_s$ is the sampling frequency.

Definitions

- Define the DFT matrix as

$$\mathbf{D}_N = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & e^{-j2\pi/N} & e^{-j2\pi 2/N} & \cdots & e^{-j2\pi(N-1)/N} \\ 1 & \vdots & & \cdots & e^{-j2\pi 2(N-1)/N} \\ \vdots & & & & \vdots \\ 1 & e^{-j2\pi(N-1)/N} & e^{-j2\pi 2(N-1)/N} & \cdots & e^{-j2\pi(N-1)(N-1)/N} \end{bmatrix}$$

and the first *L* columns as

$$\mathbf{D}_{NL} = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & e^{-j2\pi/N} & e^{-j2\pi 2/N} & \cdots & e^{-j2\pi(L-1)/N} \\ 1 & \vdots & & \cdots & e^{-j2\pi 2(L-1)/N} \\ \vdots & & & & \vdots \\ 1 & e^{-j2\pi(N-1)/N} & e^{-j2\pi 2(N-1)/N} & \cdots & e^{-j2\pi(L-1)(N-1)/N} \end{bmatrix}$$

Frequency domain channel

- The *N x 1* frequency domain channel vector is

$$
\begin{aligned}
\mathbf{H} &= \mathbf{D}_{NL}\mathbf{h} \\
&= \begin{bmatrix} H_0 \\ H_1 \\ \vdots \\ H_{N-1} \end{bmatrix}
\end{aligned}
$$

$$H_i = H(f)\big|_{f=i\frac{f_s}{N}}$$

Continuous Model for PDP

- One common model for the static multipath channel is to model the power delay profile as being continuous
- The diffuse multipath amplitudes/phases are random following an exponential decay in average power:

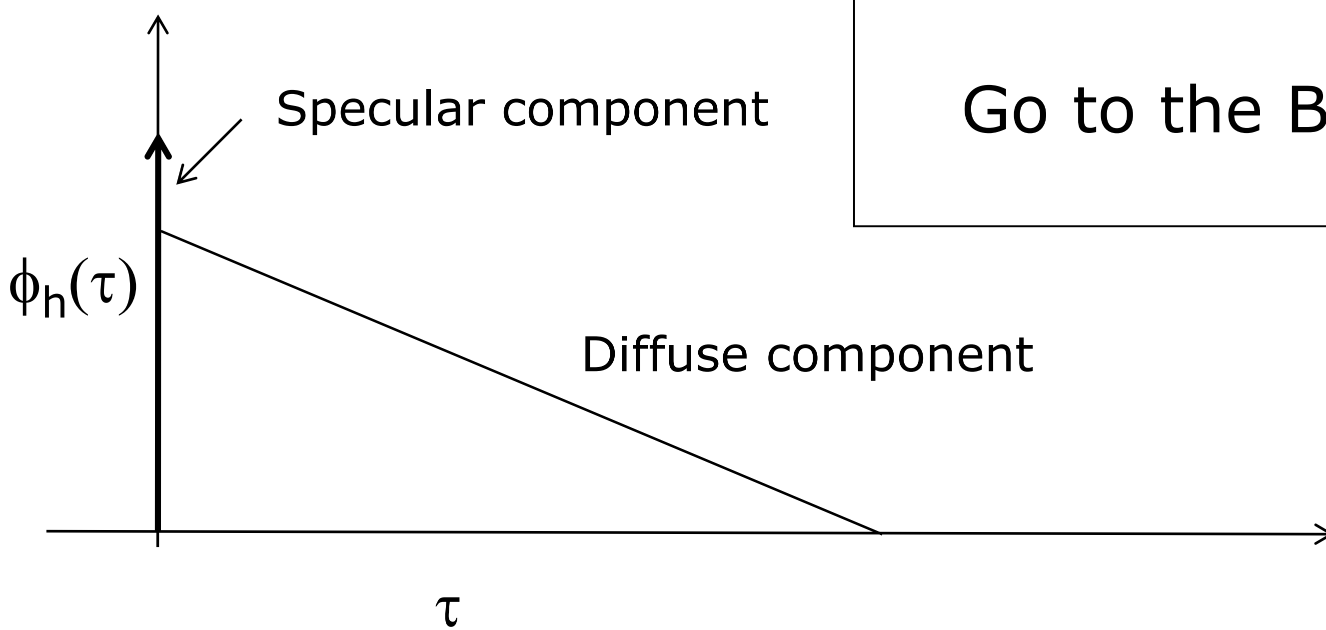
$$\varphi_h(\tau) = E\left\{h(\tau)h^*(\tau)\right\}$$
$$= \underbrace{\frac{P_T\gamma}{K+1}\exp(-\gamma\tau)u(\tau)}_{\text{Diffuse component}} + \underbrace{\frac{P_T K}{K+1}\delta(\tau)}_{\text{Specular LOS component}}$$

Diffuse component

Specular LOS component

Continuous PDP Model

- Power Delay Spectrum



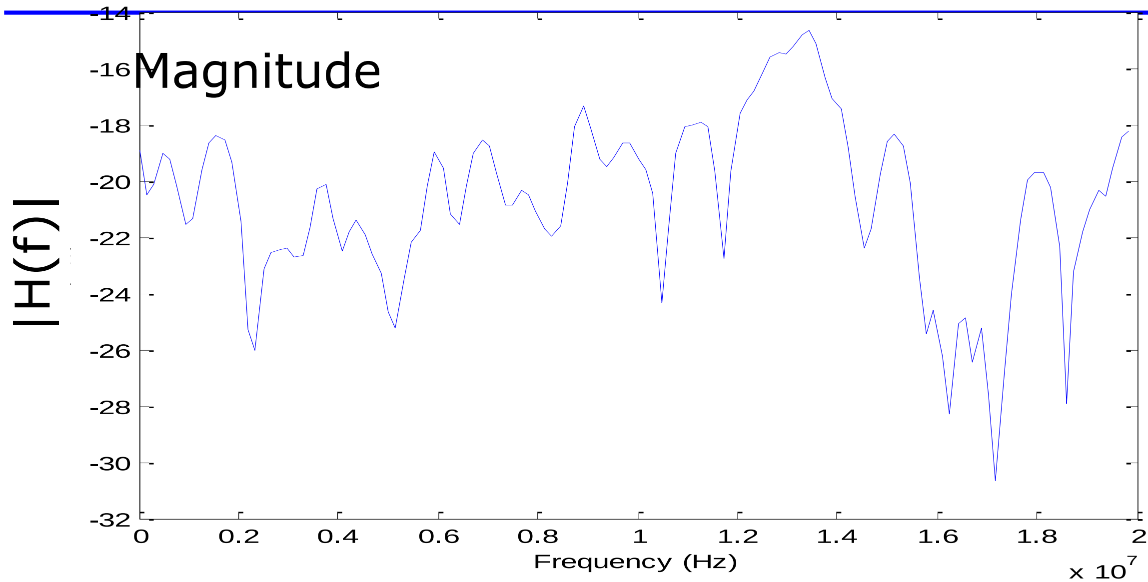
Go to the Board….

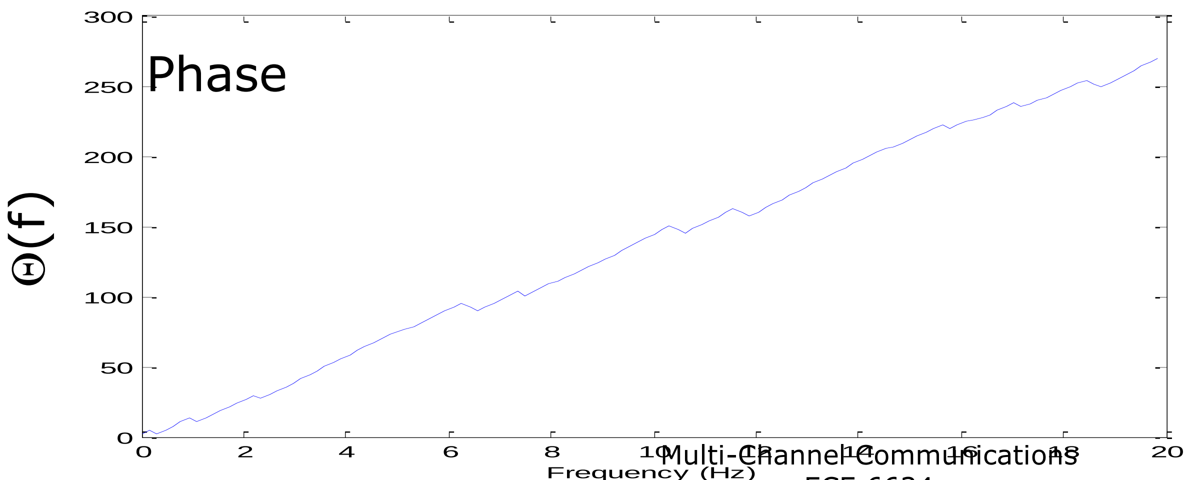
Frequency Domain Model

- A frequency domain channel with the appropriate channel correlation can be created using
  - Random GRV in frequency domain (white noise)
  - Desired power delay profile
  - Fourier Transform
  - Hilbert Tr

Go to the Board….

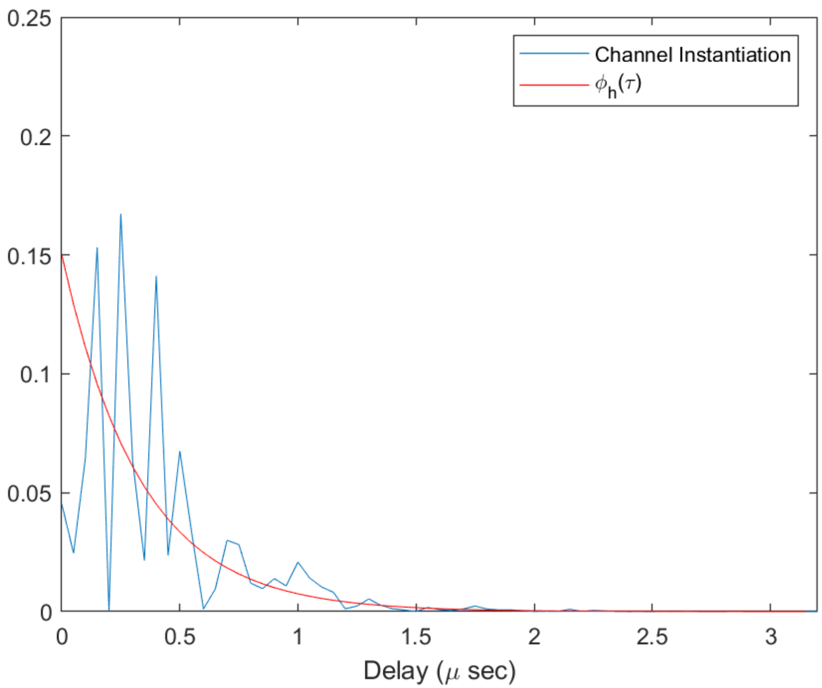
Example 1

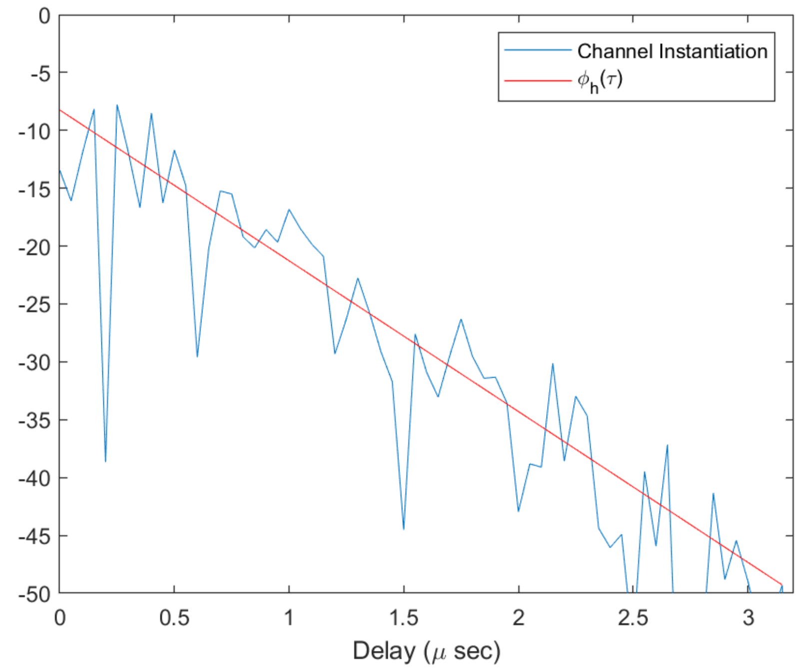




- $K = 0$
- $\tau_{rms} = 333\text{ns}$
- $\gamma = 3/\mu\text{s}$
- $P_T = 1$

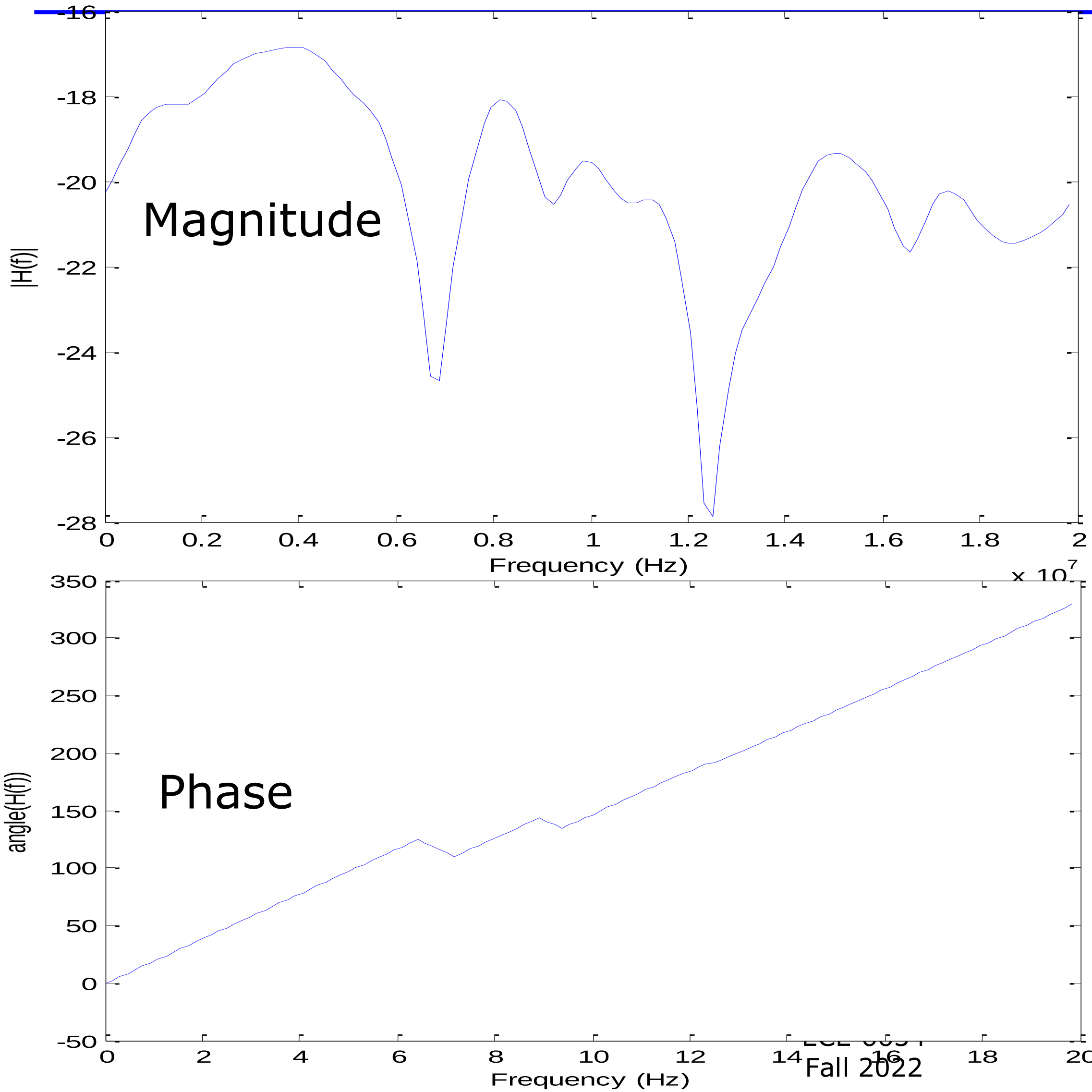
Example 1 (cont.)





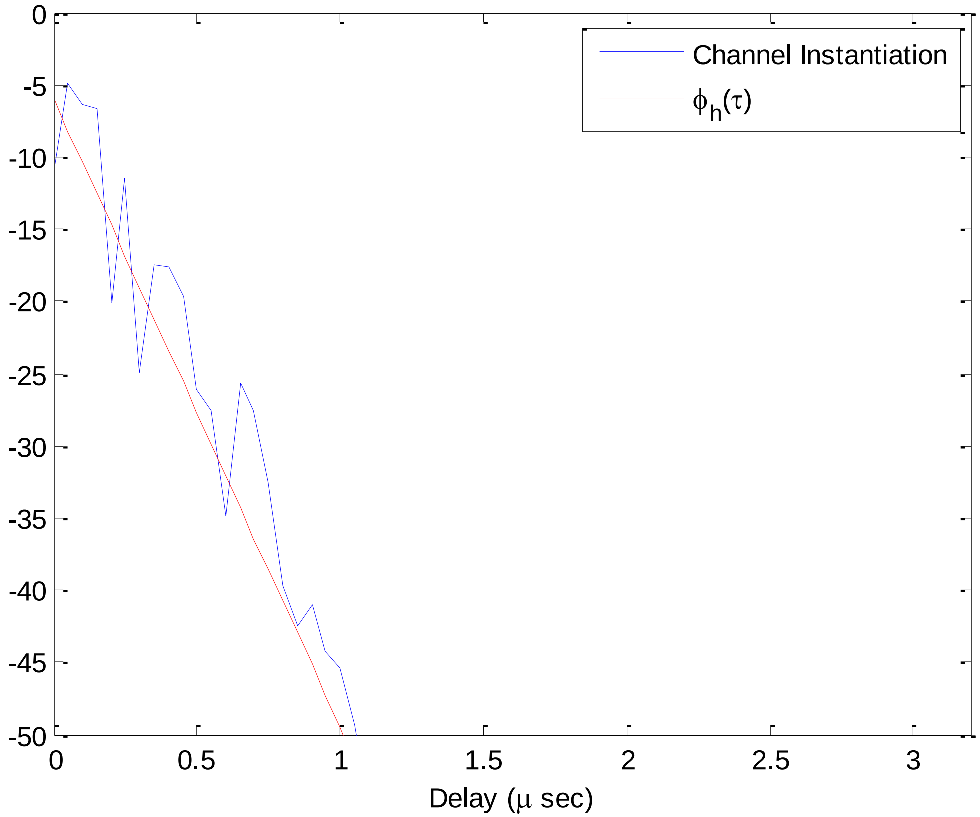
- K = 0
- Desired τ_{rms} = 333ns
- Measured τ_{rms} = 332ns
- γ = 3/µs
- P_T = 1

Example 2



- K = 0
- τ_{rms} = 100ns
- γ = 10/μs
- P_T = 1

Example 2 (cont.)



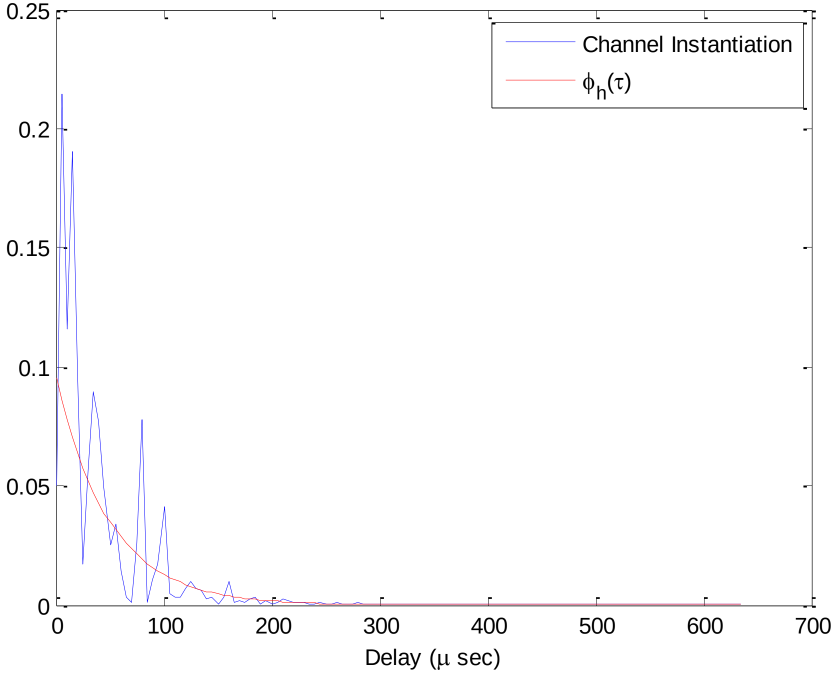
- K = 0
- Desired τ_{rms} = 100ns
- Measured τ_{rms} = 104ns
- γ = 10/μs
- P_T = 1

Time Domain Model

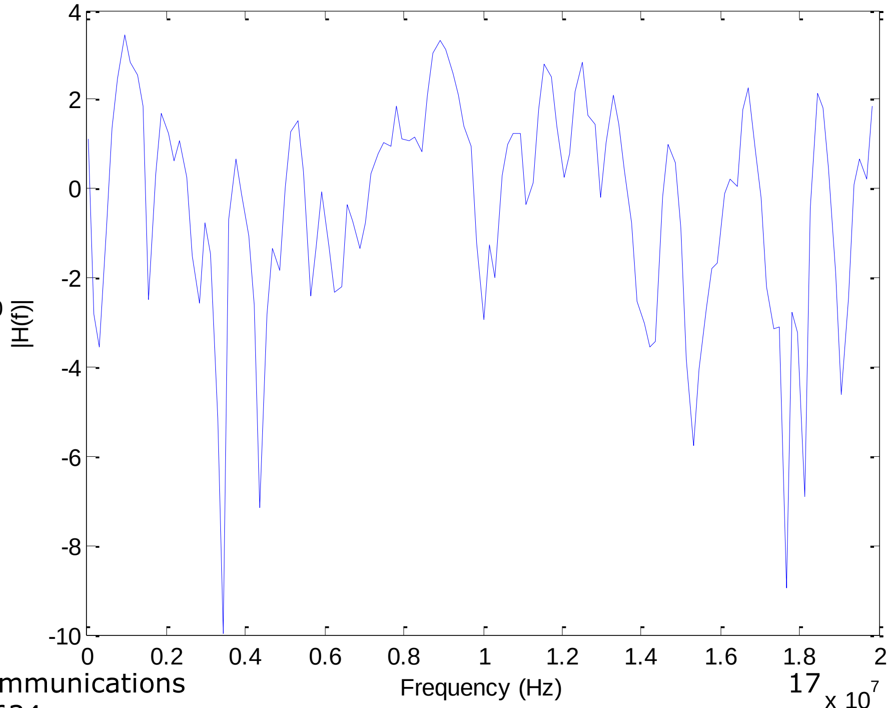
- We can also start with a time domain model and transform into the frequency domain

Go to the Board….

Example



- $T_s = 5\mu s$
- $\tau_{rms} = 50\mu s$



Standard Models

- ITU
  - Indoor
  - Pedestrian
  - Vehicular
- 3GPP
  - Extended Pedestrian A (EPA)
  - Extended Vehicular A (EVA)
  - Extended Typical Urban (ETU)

ITU Wideband Model

$$y(t) = \sum_{i=1}^{N_p} \sqrt{p_i}\gamma_i(t)x(t-\tau_i)$$

- p_i – power of the *i*th path
- τ_i – delay of the *i*th path
- N_p – number of discrete paths
- $\gamma_i(t)$ – independent (unit power) complex Gaussian random processes
- $y(t)$ – output signal
- $x(t)$ – input signal

Indoor Channel - Tap Values

Tap	Channel A		Channel B		Doppler Spectrum
	Relative Delay (ns)	Avg. Power (dB)	Relative Delay (ns)	Avg. Power (dB)	
1	0	0	0	0	Flat
2	50	-3.0	100	-3.6	Flat
3	110	-10.0	200	-7.2	Flat
4	170	-18.0	300	-10.8	Flat
5	290	-26.0	500	-18.0	Flat
6	310	-32.0	700	-25.2	Flat

Outdoor – to- Indoor Channel

Tap	Channel A		Channel B		Doppler Spectrum
	Relative Delay (ns)	Avg. Power (dB)	Relative Delay (ns)	Avg. Power (dB)	
1	0	0	0	0	Classic
2	110	-9.7	200	-0.9	Classic
3	190	-19.2	800	-4.9	Classic
4	410	-22.8	1200	-8.0	Classic
5			2300	-7.8	Classic
6			3700	-23.9	Classic

Vehicular Channel

Tap	Channel A		Channel B		Doppler Spectrum
	Relative Delay (ns)	Avg. Power (dB)	Relative Delay (ns)	Avg. Power (dB)	
1	0	0	0	-2.5	Classic
2	310	-1.0	300	0	Classic
3	710	-9.0	8900	-12.8	Classic
4	1090	-10.0	12900	-10.0	Classic
5	1730	-15.0	17100	-25.2	Classic
6	2510	-20.0	20000	-16.0	Classic

Doppler Spectra

- Assumption (a) – for outdoor channels a very large number of receive rays arrive uniform distributed in azimuth at the mobile station and at zero elevation for each delay interval. Antenna is uniform in azimuth. Results in the Clarke/Jakes "classic" Doppler spectrum:

$$S(f) \quad = \frac{1}{\pi}\frac{1}{\sqrt{(V/\lambda)^2 - f^2}} \quad |f| < V/\lambda$$

- Assumption (b) – for indoor channels a very large number of receive rays arrive uniformly distributed in elevation and azimuth for each delay interval at the base station. Results in "flat" Doppler spectrum (not same as flat fading):

$$S(f) \quad = \frac{\lambda}{2V} \quad |f| < V/\lambda$$

V=velocity

λ = wavelength

Claimed Occurrence and Delay Spread

Model	Channel A		Channel B	
	Delay Spread	P %	Delay Spread	P%
Indoor Office	35	50	100	45
Pedestrian	45	40	750	55
Vehicular	370	40	4000	55

- Channel A – low delay spread channel which happens frequently
- Channel B – medium delay spread channel which happens frequently
- Undefined high delay spread channel happens infrequently (5%)

EPA and EVA

EPA

Delay (ns)	Relative Power (dB)
0	0.0
30	-1.0
70	-2.0
90	-3.0
110	-8.0
190	-17.2
410	-20.8

EVA

Delay (ns)	Relative Power (dB)
0	0.0
30	-1.5
150	-1.4
310	-3.6
370	-0.6
710	-9.1
1090	-7.0
1730	-12.0
2510	-16.9

- All taps have classic Doppler spectra
- Max Doppler is either 5Hz, 70Hz or 300Hz

ETU Model

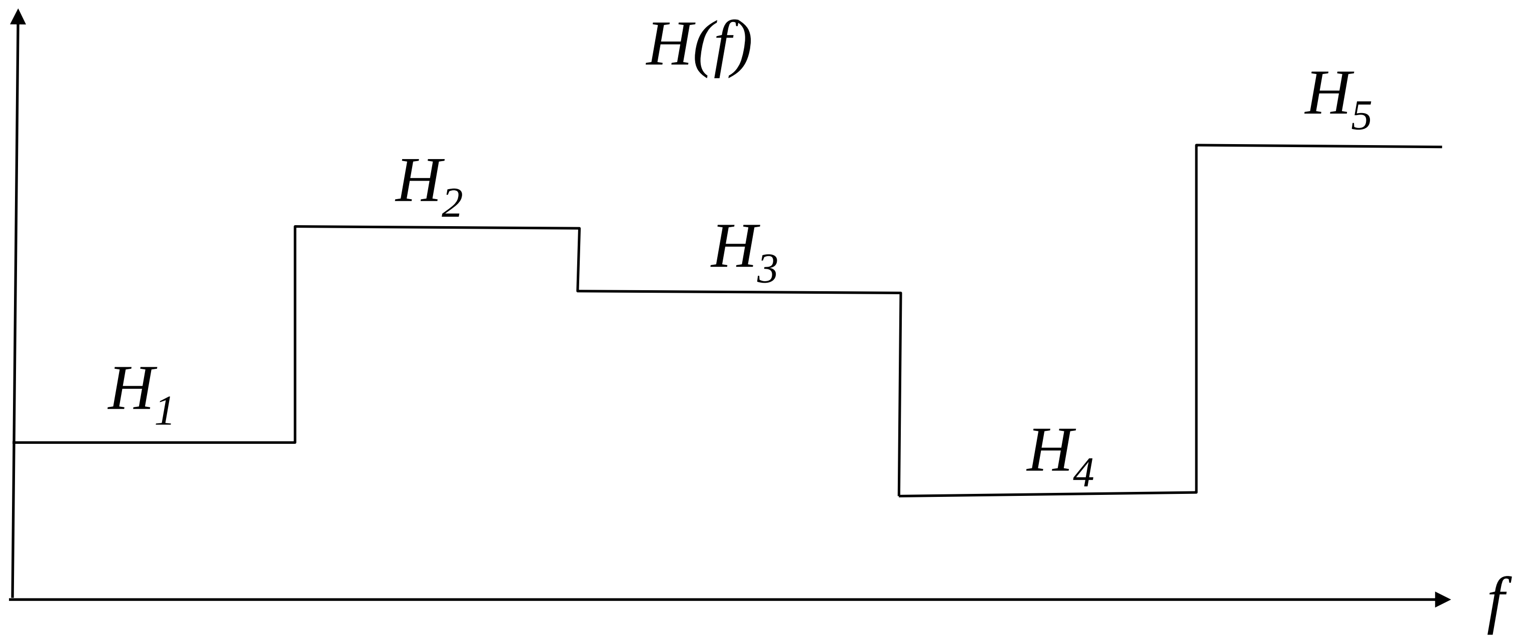
Delay (ns)	Relative Power (dB)
0	-1.0
50	-1.0
120	-1.0
200	0.0
230	0.0
500	0.0
1600	-3.0
2300	-5.0
5000	-7.0

Challenge of Frequency Selective Fading

- Frequency selectivity is relative to signal bandwidth
- As signal bandwidth increases, the probability of the channel being frequency selective also goes up.
- Increasing data rates requires increasing signal bandwidth
- Frequency selective fading is difficult to mitigate for classic digital modulation and requires high complexity
- Conclusion: High data rates are difficult to achieve with classic modulation in wireless channels (esp. indoors)

Capacity of Frequency Selective Fading

- Consider a time-invariant block fading channel with N_b sub-bands of bandwidth B



Capacity

- The capacity of this parallel set of channels is the sum of rates on each channel with power optimally allocated over all channels:

$$C = \max_{P_j : \sum_j P_j \leq P} \sum_{j=1}^{N_b} B \log_2 \left(1 + \frac{|H_j|^2 P_j}{N_o B}\right)$$

- The optimal power allocation is found via water-filling

Water-filling (in frequency)

- The water-filling power allocation is

$$\frac{P_j}{P} = \begin{cases} \frac{1}{\gamma_o} - \frac{1}{\gamma_j} & \gamma_j \geq \gamma_o \\ 0 & \gamma_j < \gamma_0 \end{cases}$$
$$= \left(\frac{1}{\gamma_o} - \frac{1}{\gamma_j}\right)^+$$

where $\gamma_j = \frac{|H_j|^2 P}{N_o B}$ is the SNR of the jth channel assuming full power, and the cut-off value γ_o is chosen such that

$$\sum_j \left(\frac{1}{\gamma_o} - \frac{1}{\gamma_j}\right)^+ = 1$$

$$\boxed{(x)^+ = \begin{cases} x & x \geq 0 \\ 0 & \text{else} \end{cases}}$$

Final Capacity Equation

- Thus, the final capacity equation becomes

$$C = \sum_{j:\gamma_j \geq \gamma_o} B \log_2 \left(\frac{\gamma_j}{\gamma_o} \right)$$

- **Thus, information theory suggests that to achieve capacity we must place different power in different sub-bands – something which is difficult (if not impossible) to do with classic digital modulation.**

Continuous Frequency Function

- For continuous function H(f)

$$C = \max_{P(f):\int P(f)df \leq P} \int \log_2 \left(1 + \frac{|H(f)|^2 P(f)}{N_o}\right)$$

$$\frac{P(f)}{P} = \begin{cases} \frac{1}{\gamma_o} - \frac{1}{\gamma(f)} & \gamma(f) \geq \gamma_o \\ 0 & \gamma(f) < \gamma_0 \end{cases}$$

$$\gamma(f) = \frac{|H(f)|^2 P}{N_o}$$

$$C = \int_{f:\gamma(f)\geq\gamma_o} \log_2 \left(\frac{\gamma(f)}{\gamma_o}\right) df$$

Conclusions

- Today we have examine the frequency selective channel
- Frequency selective channels are challenging in two respects:
  1. Channel compensation requires complex equalizers as the bandwidth of the signal exceeds the channel coherence bandwidth. This is an impediment to high data rates.
  2. Information theory tells us that maximizing rate requires dividing power across the band depending on the channel gain in that band