Virginia Polytechnic Institute and State University

Bradley Department of Electrical and Computer Engineering

MIMO HW6

Author

Gaurav Duggal

Instructor

Dr. Michael Buehrer

December 5, 2022

Contents

1 Introduction

In this assignment, we learn about channel estimation for OFDM modulation. We also learn how OFDM can exploit frequency diversity in a frequency-selective channel to offer increased throughput using bit-loading adaptive modulation.

2 Description

2.1 Input

The inputs are summarised below

- N: number of sub-carriers
- τ_{rms} Delay spread
- Ntrials: Number of trials of the experiment
- fs: Sampling frequency
- k: number of bits/symbol

2.2 Output

We have channel estimation for OFDM discussed in the first section and then followed by various plots showing increased throughput for OFDM waveforms in AWGN, flat fading and frequency selective channels.

3 Validation

3.1 Q1. OFDM channel estimation

3.1.1 Introduction

Due to multi-path reception at the receiver, especially in urban and indoor scenarios, we observe that the channel becomes frequency-selective. This is further enforced by the relatively higher bandwidths seen in 4G and 5G where we use OFDM as a physical layer waveform. Assuming we have N sub-carriers for the OFDM waveform, the frequency-selective nature of the channel manifests as different gains on each of the N sub-carriers. Hence, to estimate the channel we need to appropriately place pilots on the time-frequency grid that represents a packet containing N sub-carriers and N_t OFDM symbols.

The coherence bandwidth is a statistical measure that depends on the delay spread of the channel and it quantifies the range of frequencies over which we can assume the channel to be flat-fading. This helps us decide on the pilot spacing in the frequency domain that is required to estimate the channel. We have the pilot spacing Δf_p less than or equal to the coherent bandwidth B_c of the channel which is less than or equal to the bandwidth of the signal B.

$$\Delta f_p \leq B_c \leq B \tag{1}$$

Note:

1. We assume that the signal bandwidth is greater than the coherence bandwidth of the channel.
2. Due to the time-varying nature of the channel which is a consequence of the Doppler spread of the channel, we also need to place pilots every few OFDM symbols. In this simulation, we have assumed block fading so the channel is assumed to be static for a few OFDM symbols and hence we only place pilots on the first OFDM symbol within a block.

3.1.2 System Model

Assuming the transmitted symbols $\mathbf{s_t}$ across N sub-carriers contain both data symbols $\mathbf{s}$ and pilots $\mathbf{p}$. Now, $\mathbf{s}_t = \mathbf{I}_{N\mathcal{S}}\mathbf{s} + \mathbf{I}_{N\mathcal{P}}\mathbf{p}$. Where, the matrices $\mathbf{I}_{N\mathcal{S}}$ and $\mathbf{I}_{N\mathcal{P}}$ place the pilots and data symbols appropriately across the N sub-carriers. The transmitted time domain signal with the cyclic prefix appended is given by

$$\begin{aligned} \mathbf{x}_{cp} &= \mathbf{T}_{cp}\mathbf{D}_N^H\mathbf{s}_t \\ &= \mathbf{D}_{N_{cp}}\mathbf{s}_t. \end{aligned} \tag{2}$$

The matrix $\tilde{\mathbf{H}}_g$ represents the $\mathbf{L}$ tap frequency selective channel and is given by

$$\tilde{\mathbf{H}}_g = \begin{bmatrix} h_0 & 0 & 0 & \dots & 0 \\ h_1 & h_0 & 0 & \dots & 0 \\ h_2 & h_1 & h_0 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ h_{L-1} & h_{L-2} & h_{L-3} & & 0 \\ 0 & h_{L-1} & h_{L-2} & & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & h_0 \end{bmatrix} \tag{3}$$

Note that the channel is assumed to be static for the packet duration and the **L** taps are given by:

$$h_i = \alpha_i e^{-j\phi_i} \delta(\tau - \tau_i), \; i \in (0, L-1) \tag{4}$$

where α_i, ϕ_i, and τ_i are assumed to come from an exponential Power Delay Profile with a given delay spread.

Now, the received signal after passing through the channel is given by

$$\mathbf{r}_{cp} = \tilde{\mathbf{H}}_g \mathbf{x}_{cp} + \mathbf{n}. \tag{5}$$

At the receiver, we first remove the cyclic prefix using

$$\tilde{\mathbf{r}} = \mathbf{R}_{cp} \mathbf{r}_{cp}. \tag{6}$$

Applying the DFT (or FFT) operation at the receiver side to obtain the noisy received symbols in the frequency domain.

$$\mathbf{e} = \mathbf{D}_N \tilde{\mathbf{r}} \tag{7}$$

We can pull apart the noise received pilot symbols from the received error vector **e** using

$$\mathbf{e}_p = \mathbf{I}_{N\mathcal{P}}^T \mathbf{e} \tag{8}$$

3.1.3 LS estimate of the channel

We have the Least Squares estimate of the channels given by

$$\hat{\mathbf{h}}^{(LS)} = \left(\mathbf{Q}_{N_pL}^H \mathbf{Q}_{N_pL}\right)^{-1} \mathbf{Q}_{N_pL}^H \mathbf{P}^{-1} \mathbf{e}_p, \tag{9}$$

where, $\mathbf{P} = diag(\mathbf{p})$ with $\mathbf{p}$ being the known pilots at the receiver.

Note:

1. $\mathbf{Q}_{N_pL}^H$ is the $N_p \times L$ IDFT matrix.
2. To invert $\mathbf{Q}_{N_pL}^H \mathbf{Q}_{N_pL}$ we must have $N_p \geq L$ which means **we must have at least as many pilots as the channel taps**.
3. Note the plots are in terms of snr per sub-carrier.

3.1.4 Channel estimation plots

The following assumptions are made in this section:

1. For this section, a packet is assumed to contain 10 OFDM symbols.

2. We assume the Doppler spread is sufficiently large so that the channel is stationary within these 10 OFDM symbols.

3. The first OFDM symbol in a packet contains N_p pilots use for channel estimation and the rest of the OFDM symbols contain data symbols.

4. Pilots are approximately evenly spaced along the sub-carriers on the first symbol in a packet.

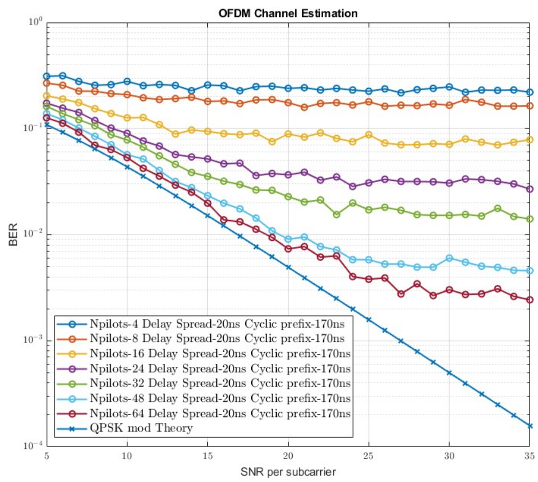

Figure 1: Channel Estimation using Least Squares method for various pilots for a delay spread $= 20ns$

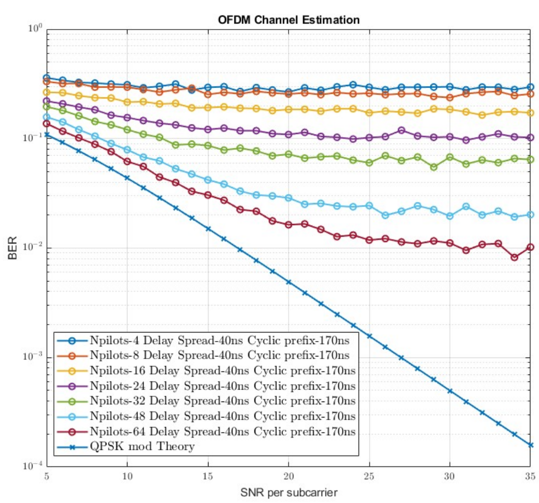

Figure 2: Channel Estimation using Least Squares method for various pilots for a delay spread $= 40ns$

We observe that for relatively low avg SNRs per subcarrier, for 32 pilots or more we are reasonably close to the theoretical BER for QPSK modulation. For higher SNRs and larger delay spreads we need to use more pilots. Additionally, we could also use pilots across different OFDM symbols.

3.2 Q2. Bit-Loading based adaptive modulation

3.2.1 Introduction

For practical communication systems, our goal is to achieve as high a throughput as possible. To achieve this, we first set a target Bit-Error-Rate (BER) so that the error-correcting code can correct the errors introduced into the data stream. Now as the received SNR is increased, the BER falls giving us some leeway into moving to a higher-order modulation scheme. This must be done keeping in mind that the resultant BER should not go above the target BER, or else the throughput will fall. The effect of moving to a higher-order modulation scheme with a suitable powerful error-correcting code is to increase the throughput.

3.2.2 Bit-loading for OFDM systems

We can visualize an N subcarrier OFDM waveform as N orthogonal single carrier modulation waveforms. Now, in the context of bit-loading, each of these sub-carriers carries an M-QAM symbol (or BPSK). Assuming a fixed transmit power of N units, we assign each of the sub-carriers 1 unit of power. At the receiver, we can measure the SNR across each sub-carrier and accordingly choose the modulation scheme to maximize the throughput. For this, we need to define the average bit error across N sub-carriers as

$$BER_{avg} = \frac{\sum\limits_{n \in N_e} k_n b_n}{\sum\limits_{n \in N_e} k_n}. \tag{10}$$

Here N_e is the number of enabled sub-carriers, k_n, and b_n are the number of bits per symbol and bit-error-rate on the n^{th} sub-carrier respectively. Assuming the palette of modulation schemes available are BPSK, QPSK, 16-QAM, 64-QAM hence $k \in (1, 2, 4, 6)$.

Algorithm for maximizing Throughput:

1. Set the maximum modulation scheme on all N sub-carriers.

2. Calculate the SNR on all the sub-carriers

3. Calculate the BER b_n based on the modulation scheme and SNR on the sub-carrier. Note b_n is obtained by using the theoretical BER expressions for M-QAM in an AWGN channel with known SNR and order M.

4. Calculate the Average BER using (10).

5. while the Average BER is above the target BER, keep looping

6. Select the sub-carrier index with the highest BER and reduce the modulation order. If the modulation order is BPSK, disable the sub-carrier i.e. transmit 0 on that sub-carrier.

7. Calculate the Avg BER using (10) and exit if below target else go back to step 5.

8. Note the plots are in terms of snr per sub-carrier.

3.2.3 Bit-loading for OFDM in AWGN channels

Key assumptions for this simulation:

1. Packet Error Rate is assumed to be the same as Bit Error Rate

2. The transmit power per OFDM symbol is N units.

3. For the theoretical single carrier throughput plot, when the BER rises above the target BER 10^{-2}, the throughput is made zero, since the error correcting code cannot handle the number of errors.

4. Note the plots are in terms of snr per sub-carrier.

Using the algorithm specified in the introduction, we can calculate the modulation scheme for each sub-carrier based on the SNR on that sub-carrier so that we achieve the target BER 10^{-2}.

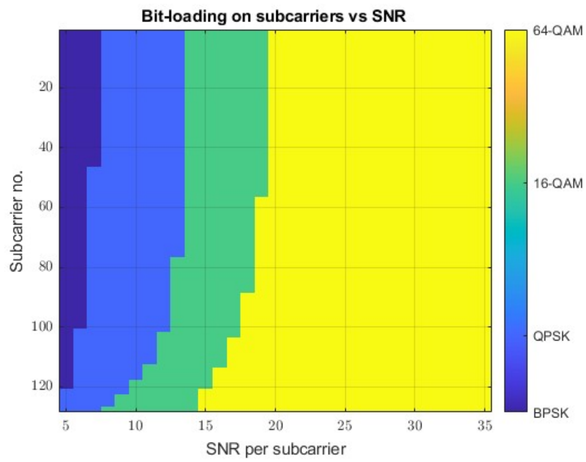

Figure 3: Bit-Loading for OFDM across different sub-carriers and SNRs in an AWGN channel for a target BER = 10^{-2}

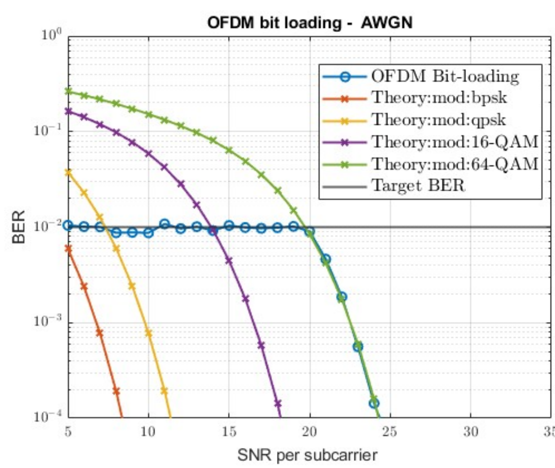

Figure 4: BER for Bit-Loading for OFDM compared to single carrier modulation schemes in an AWGN channel. Target BER = 10^{-2}

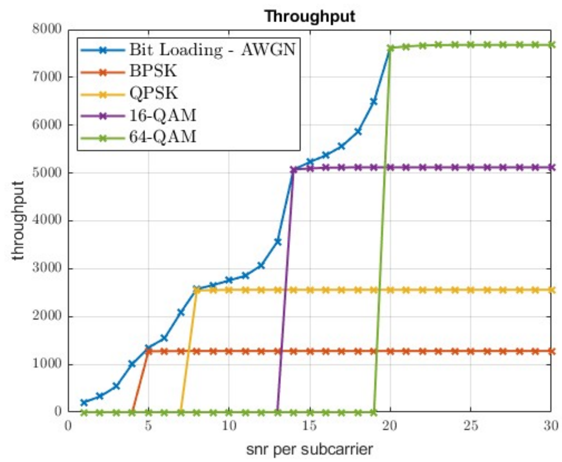

Figure 5: Bit-Loading for OFDM compared to single carrier modulation schemes in an AWGN channel. Target BER = 10^{-2}

We observe that the throughput of the adaptive OFDM modulation in an AWGN channel surfs the peaks obtained by single carrier modulation schemes. Essentially it offers a better granularity on the palette of possible throughput values.

3.2.4 OFDM bit modulation in a Rayleigh Flat-Fading channel

The following assumptions are made in this section

1. The multi-path arrives at the receiver at delays insignificant compared to the symbol duration. This is essentially the Clarkes model assumption and results in a single tap channel with magnitude rayleigh distributed

2. The SNR on the n^{th} sub-carrier snr_n is given by:

$$snr_n = N \times |H_n|^2 snr \tag{11}$$

   where N is the transmit power for the OFDM symbol, H_n is the frequency domain channel gain for the $n^t h$ sub-carrier and snr is the signal-to-noise ratio per sub-carrier.

3. The packet is assumed to contain $P = 1000$ OFDM symbols and throughput is simulated by assuming it is zero if the ber for a trial is above the target ber and $P \times Nbits$ where

$$Nbits = \sum_{n \in N_e} k_n. \tag{12}$$

   Here, k_n is the number of bits per symbol on the n^{th} sub-carrier and N_e is the set of the enabled sub-carriers.

4. The effect of the Rayleigh channel is that the gain offered to each sub-carrier in the frequency domain is the same and is sampled from a Rayleigh distribution. The gain changes between coherence times.

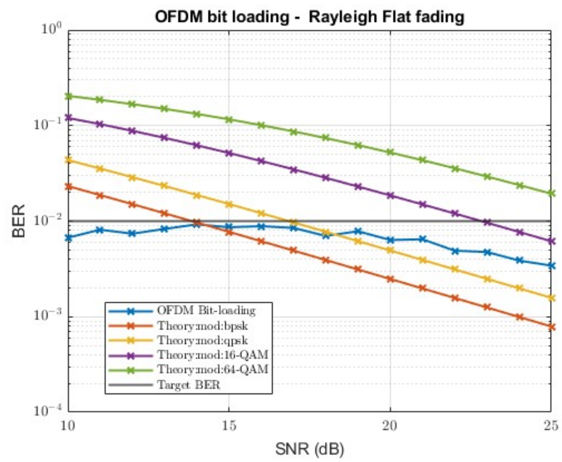

Figure 6: Bit-Loading for OFDM compared to single carrier modulation schemes in a flat fading Rayleigh channel. Target BER = 10^{-2}

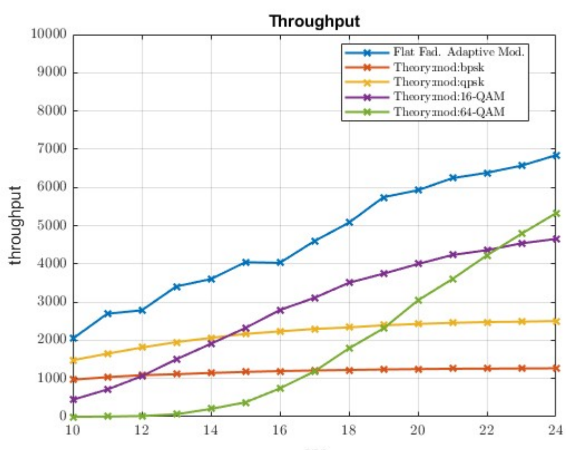

Figure 7: Bit-Loading for OFDM compared to single carrier modulation schemes in a flat fading Rayleigh channel. Target BER = 10^{-2}

3.2.5 OFDM bit modulation for frequency selective channel

1. The delay spread of the channel is sufficiently large such that the coherent bandwidth is less than the bandwidth of the signal

2. The Doppler spread of the channel is assumed to be sufficiently large such that the channel is assumed to be static over a packet containing 100 OFDM symbols.

3. The SNR for each sub-carrier changes significantly and bit-loading offers the most throughput compared to single-carrier modulation techniques.

4. Note the plots are in terms of snr per sub-carrier.

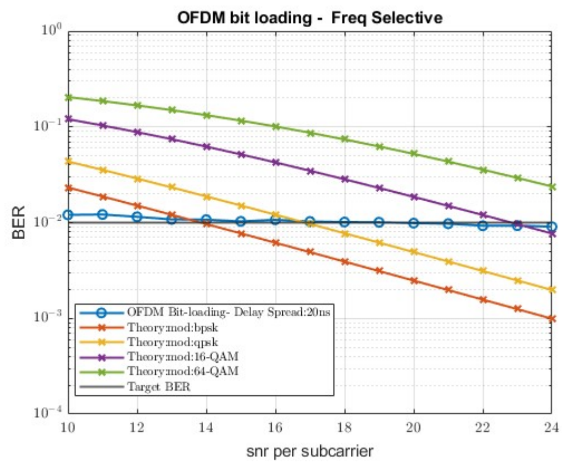

Figure 8: Bit-Loading for OFDM compared to single carrier modulation schemes in a frequency selective channel with delay spread = $20ns$. Target BER = 10^{-2}

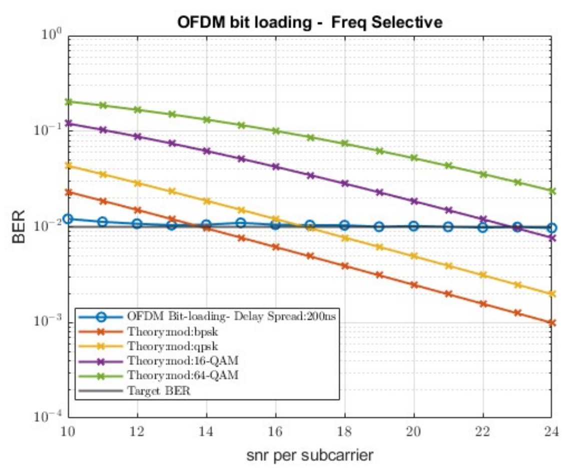

Figure 9: Bit-Loading for OFDM compared to single carrier modulation schemes in a frequency selective channel with delay spread = $200ns$. Target BER = 10^{-2}

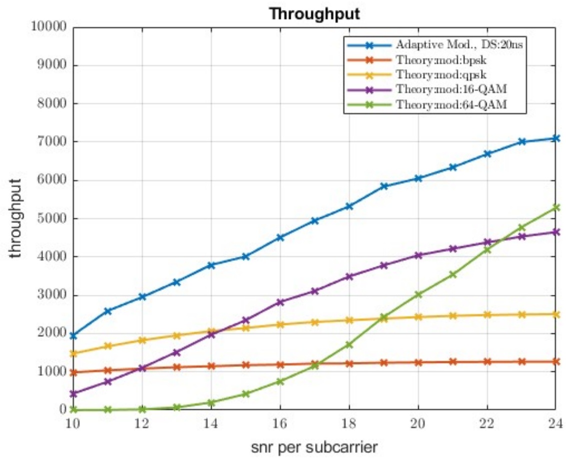

Figure 10: Bit-Loading for OFDM compared to single carrier modulation schemes in a frequency selective channel with delay spread = $20ns$. Target BER = 10^{-2}

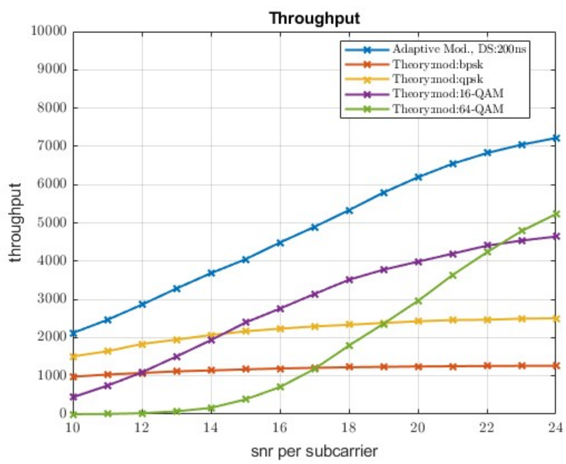

Figure 11: Bit-Loading for OFDM compared to single carrier modulation schemes in a frequency selective channel with delay spread = $200ns$. Target BER = 10^{-2}

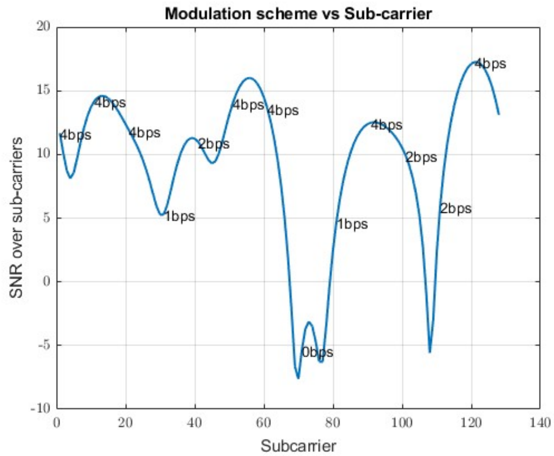

Figure 12: Bit-loading scheme allotment visualized to maximize throughput for an OFDM waveform in a frequency selective channel realization

We observe that the frequency selective channel offers the maximum advantage in terms of throughput over the single-carrier modulation schemes in the context of the bit-loading adaptive schemes. The higher the delay spread, we get a higher granularity over the throughput vs snr.

4 Conclusion

- OFDM channel estimation for a frequency selective channel requires evenly placed pilots over sub-carriers.
- The minimum number of pilots required is at least one per coherent bandwidth duration.
- The number of pilots should be more than the number of taps in a channel.
- Bitloading adaptive modulation in AWGN modulation offers small improvement in throughput over single carrier modulation schemes.
- Bitloading adaptive modulation in both flat fading, as well as frequency selective channel, achieves higher throughput compared to single carrier modulation schemes

References