Virginia Polytechnic Institute and State University

Bradley Department of Electrical and Computer Engineering

MIMO HW5

Author

Gaurav Duggal

Instructor

Dr. Michael Buehrer

November 16, 2022

Contents

1 Introduction

In this assignment, we simulate and understand what is a frequency-selective channel. We also implement OFDM and understand its advantages and disadvantages. Finally we can turn the frequency-selective channel into a flat fading channel by adding a cyclic prefix to the OFDM symbol. Further, on using error-correcting coding, we can move towards getting AWGN performance.

2 Description

2.1 Input

The inputs are summarised below

- N: number of subcarriers
- τ_{rms} Delay spread
- Ntrials: Number of trials of the experiment
- fs: Sampling frequency
- k: number of bits/symbol
- (N,K) BCH code

2.2 Output

We have various BER for the different scenarios outlined in the questions.

3 Validation

3.1 Q1. Frequency Selective channel

3.1.1 Introduction

In the Jakes model, we assumed that the delay due to multi-path i.e. the delay spread was much less than the symbol duration. Hence, the channel had just one tap and the amplitude of the channel was a Gaussian random variable. As a consequence of having a relatively wide-band signal, we have some resolvable multi-path. Therefore the channel has more than one channel taps with the resolving multi-path resulting in power at specific channel taps, whereas the non-resolvable multi-path leads to the amplitude at these channel taps being Gaussian distributed. The power-delay profile (PDP) shows the instantaneous power in each resolvable multi-path. While the instantaneous power fluctuates since it is a random variable, the channel itself is assumed to be static over the symbol duration. This essentially means that the delay taps themselves don't evolve within a symbol duration. The static channel is mathematically expressed as

$$h(\tau) = \sum_{i=1}^{N} \alpha_i e^{-j\phi_i} \delta(\tau - \tau_i). \tag{1}$$

3.1.2 Power Delay Profile

The power versus delay is expressed as

$$\varphi_h(\tau) = \sum_{i=1}^{N} \alpha_i^2 \delta(\tau - \tau_i). \tag{2}$$

The power per multi-path / delay decays exponentially with delay. Intuitively this makes sense since longer delays mean either that path undergoes multiple reflections or reflects off very far away objects. In either case, the power for these paths with high delay will reduce. The theoretical power delay profile for a scenario with Ricean K-factor is given by

$$\varphi_h(\tau) = \underbrace{\frac{P_T\gamma}{K+1} e^{-\gamma\tau} u(\tau)}_{\text{diffuse}} + \underbrace{\frac{P_T K}{K+1} \delta(\tau)}_{\text{specular}}. \tag{3}$$

In our case, $K = 0$ since we have assumed Rayleigh fading. In Fig. 1 we can see the theoretical and simulated power delay profile for a delay spread = 100 ns.

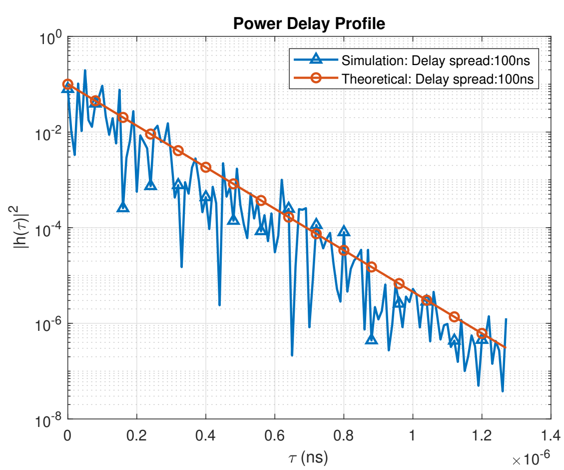

Figure 1: Power delay profile theory vs simulation for a delay spread = 100 ns

3.1.3 Frequency Selectivity

Now if we view the channel in the frequency domain, the amplitude and phase both vary with frequency as shown in Fig. 2, Fig. 3. In other words, the channel gain is frequency selective hence the name.

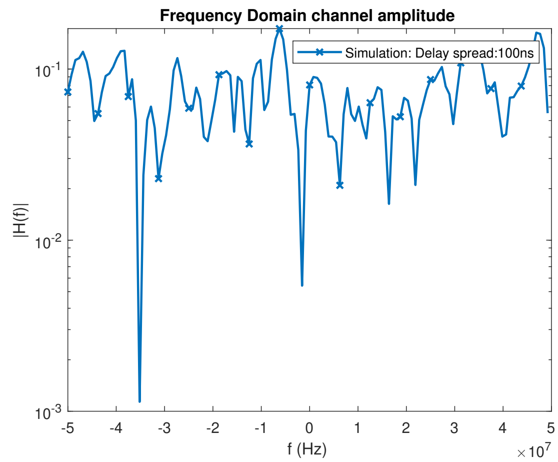

Figure 2: Frequency Domain amplitude of a frequency selective channel

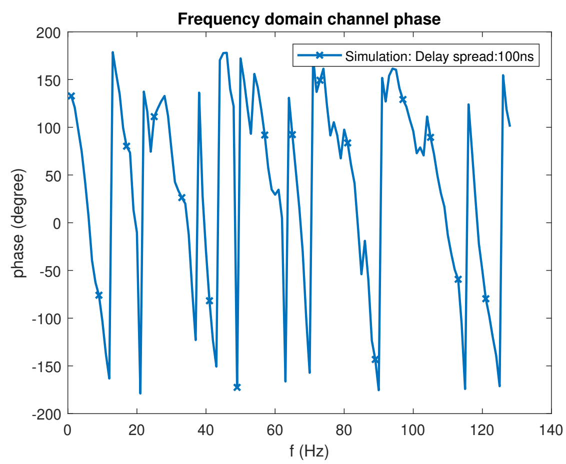

Figure 3: Frequency Domain phase of a frequency selective channel

3.1.4 Frequency correlation

The frequency correlation is the Fourier transform pair of the power delay spectrum and we can see the simulation matches the theoretical. The theoretical frequency correlation function was generated by taking the Fourier transform of the theoretical power delay profile in (3).

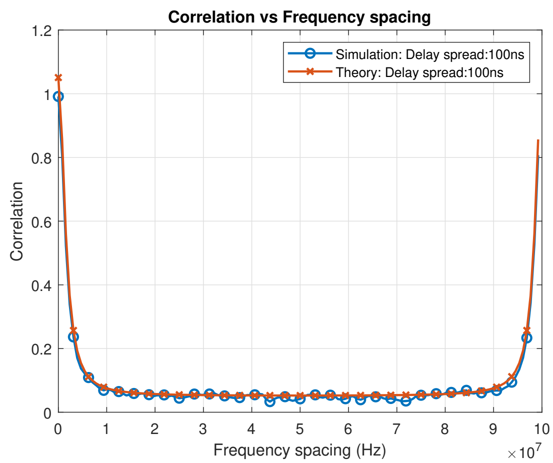

Figure 4: Frequency correlation vs frequency spacing for a frequency selective channel with delay spread = 100 *ns*

The histogram of the amplitude of a fixed frequency bin of the frequency selective channel follows Rayleigh distribution

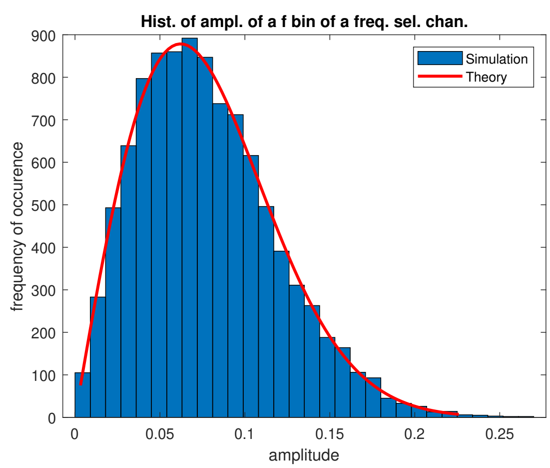

Figure 5: Histogram of the amplitude of a fixed frequency bin for a frequency selective channel with delay spread = 100 *ns*

.

3.1.5 Increasing Delay Spread

In increasing delay spread, we start seeing increasing channel gain for larger delays i.e. longer paths. This results in a faster variation in channel gain and phase across frequency. Also, the frequency correlation decays faster i.e. the channel gets more frequency selective.

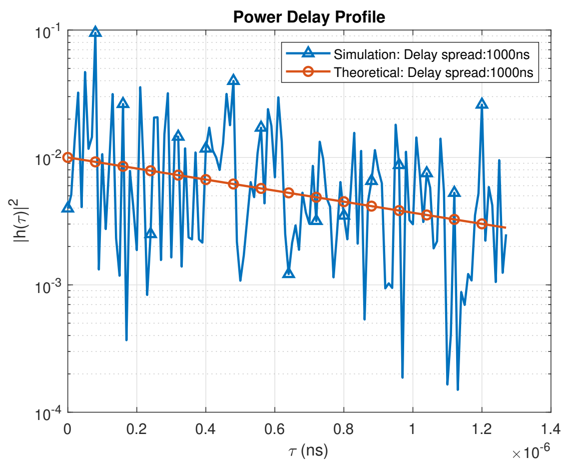

Figure 6: Power delay profile theory vs simulation for a delay spread = 1 μs

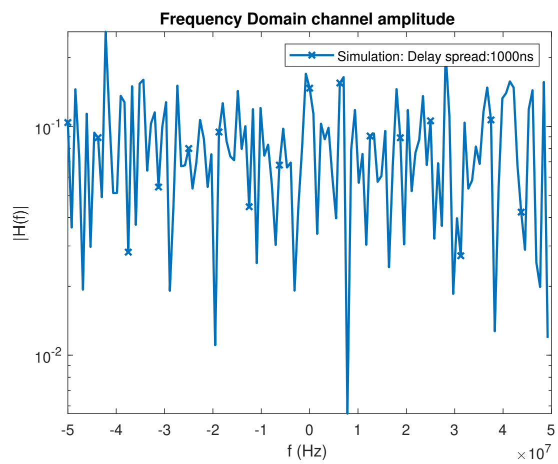

Figure 7: Frequency Domain amplitude starts varying faster vs frequency for higher delay spread. In this delay spread = 1μs

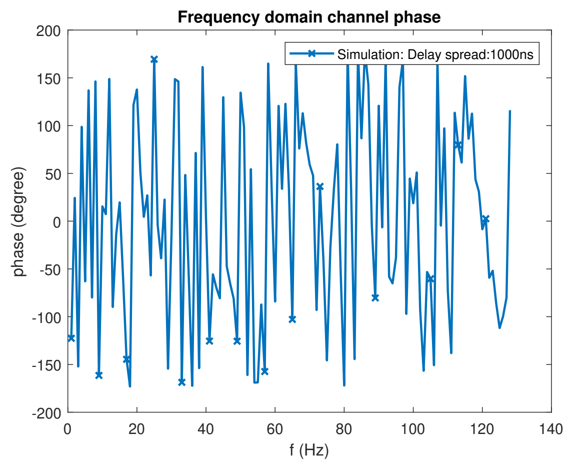

Figure 8: Frequency Domain phase starts varying faster vs frequency for higher delay spread. In this delay spread = $1\mu s$

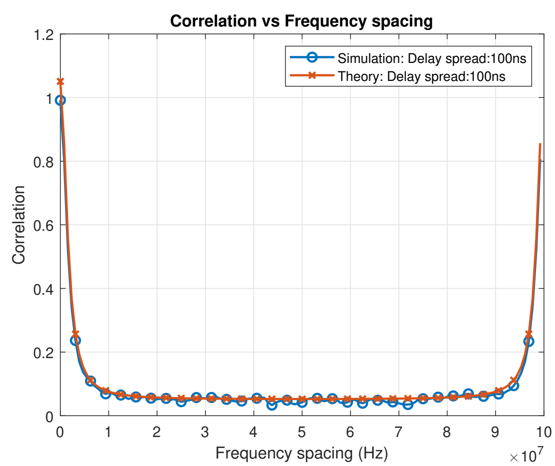

Figure 9: As we increase the delay spread, the frequency correlation reduces with increasing frequency spacing. In this case delay spread = $100\mu s$

3.2 OFDM

3.2.1 OFDM vs single carrier in AWGN channel

Comparing OFDM with 16-QAM modulation with a single carrier scheme with the same modulation order, we get the same performance in an AWGN channel.

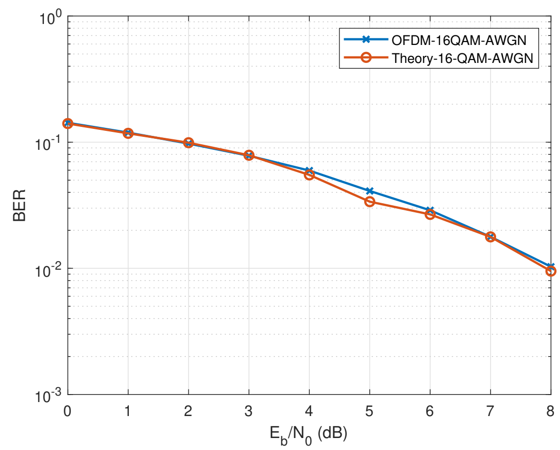

Figure 10: BER for OFDM compared with 16-QAM single carrier modulation in an AWGN channel

3.2.2 OFDM in a Rayleigh flat fading channel

In a flat fading channel, we get reduced performance compared to AWGN, since there are instances when the channel fades completely in which case we lose all bits.

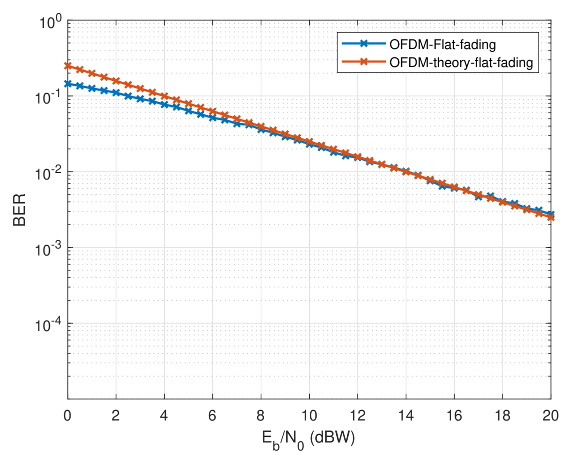

Figure 11: BER for OFDM theory vs simulation in a Rayleigh flat fading channel

3.2.3 OFDM with frequency Offset

If at the receiver we have a frequency offset in the receiver, we get a sudden increase in BER. This is due to inter-carrier-interference which is because the sub-carriers have lost orthogonality due to the frequency offset. This can be offset to some extent by channel estimation.

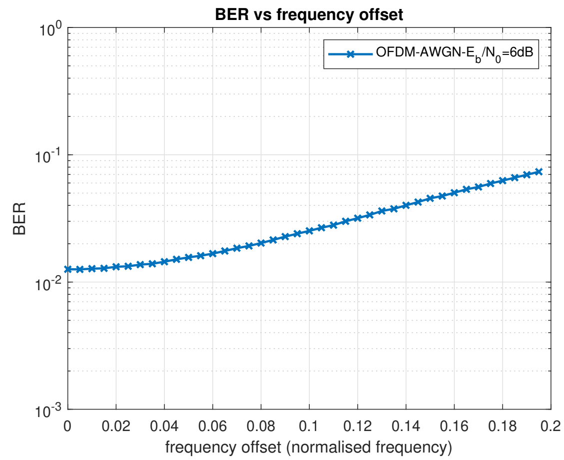

Figure 12: BER for OFDM with frequency offset for BPSK in AWGN channel with $\frac{E_b}{N_0} = 6dB$

3.2.4 Experiment 1: keeping delay spread fixed, increasing cyclic prefix, no coding

The effect of adding a cycling prefix greater than the delay spread, reduces Inter Symbol interference since the end of the previous symbol decays before the receiver starts receiving the current symbol. Additionally, compared to zero padding the cyclic prefix reduces the effect of losing orthogonality between symbols on adjacent sub-carriers. Alternatively, if we can use zero padding, which unfortunately leads to zero samples within the integration time which leads to loss of orthogonality of the sub-carriers. For a fixed delay spread, on increasing the cycling prefix above the delay spread, we convert the frequency selective channel into a flat fading channel. To estimate the channel in case of a frequency-selective channel is placing the Pilot evenly across the entire bandwidth with inter-pilot frequency spacing approximately equal to the coherence bandwidth or by using the frequency correlation function to obtain the range of subcarriers over which we can assume flat fading.

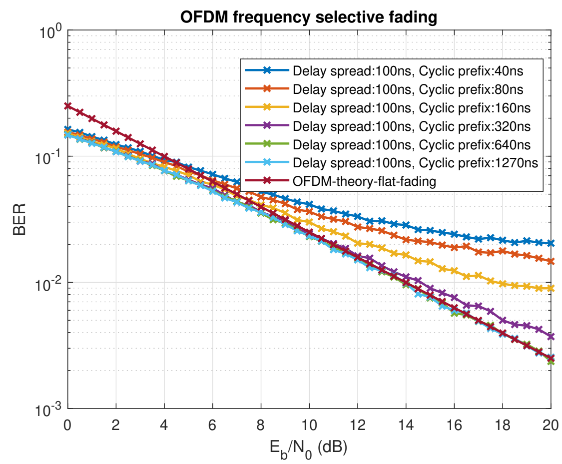

Figure 13: BER for OFDM in a frequency selective channel with delay spread 100ns with varying cyclic prefix compared with OFDM in a flat fading channel

3.2.5 Experiment 2: keeping cyclic prefix fixed, increasing delay spread with coding

For this simulation, some design changes were made to the parameters. I used a BCH code with codeword length M = 11 bits. This corresponds to 2047 possible codewords. For this using matlab command bchnumerr(2047) gave all possible BCH codes with different error correction capabilities. I chose a code with an error correction capability of 60 error bits. This corresponds to data bits = 1409. The following steps were implemented for the simulation.

- Generate 1409 data bits.
- Encode these data bits so we end up with 2047 bits.
- Append 1 dummy bit so we end up with 2048 bits.
- Since we are using QPSK, we effectively end up with 1024 QPSK symbols.
- 1024 symbols can be allocated over 8 OFDM symbols each with 128 sub-carriers with each sub-carrier having OFDM modulation
- At the receiver, drop the first dummy bit. The rest of the 2047 noisy bits can be decoded to 1409 error-corrected-data bits.
- error statistics can be generated using 1409 transmitted and received data bits.

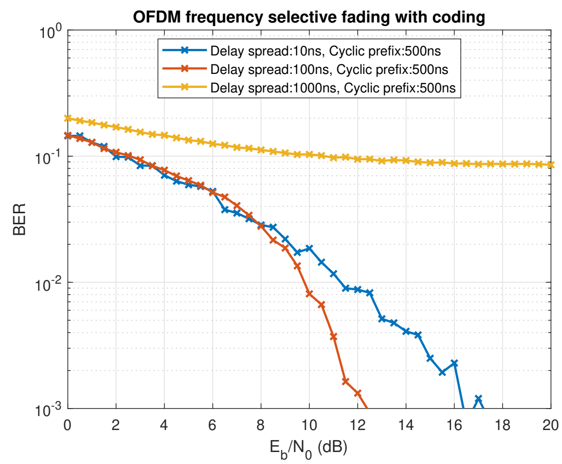

Figure 14: For cyclic prefix ≥ delay spread, error correcting codes are able to use the frequency diversity for higher delay spreads and give better BER results. For cyclic prefix much smaller than delay spread, we have too many errors due to ISI that even the error correcting code is not able to correct for those.

3.2.6 Experiment3: OFDM with coding in a flat fading channel

In this BCH-coding is not able to improve performance since we don't have frequency diversity. Over some blocks, we might have deep fades where we lose enough bits that the BCH-code is not able to recover the data bits. In this case if we use interleaving to spread our error bits over multiple blocks. This will increase performance at the cost of increased latency. In this case we will have to wait for all the interleaved blocks so as to recover the transmitted data bits.

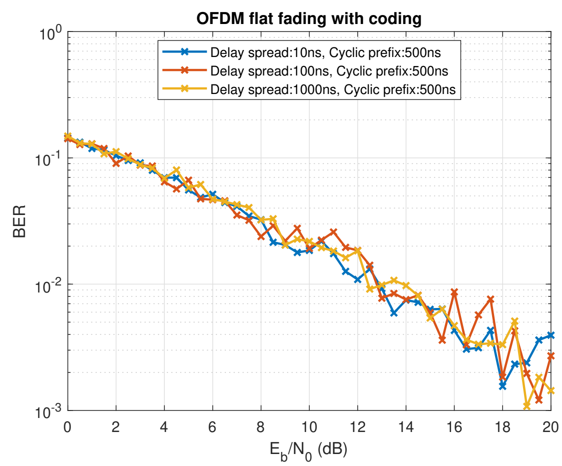

Figure 15: FOr flat fading BCH code is able to correct some errors but in case of deep fades all the data is lost. We can use interleaving over multiple blocks where one block is defined for a duration where the channel remains static

4 Conclusion

- Resolvable multi-path leads to a frequency selective channel
- higher delay spread means lower frequency correlation
- frequency correlation is the Fourier transform pair of the power delay profile/
- OFDM in AWGN channel provides the same performance as a single carrier modulation scheme
- OFDM in a flat fading channel offers worse performance than an AWGN channel.
- OFDM in a frequency-selective channel needs a cyclic prefix to prevent the effect of ISI.
- This also helps with the implementation, since the addition of the cyclic prefix turns the channel turns the linear convolution effect of the channel into a circular convolution. This means we can equalize the channel in the Frequency domain by an element-wise division.
- Addition of the cyclic prefix additionally helps reduce the effect of inter-carrier interference compared to using zero padding

References