Multi-channel Communications Fall 2022

Lecture 6
Introduction to Beamforming

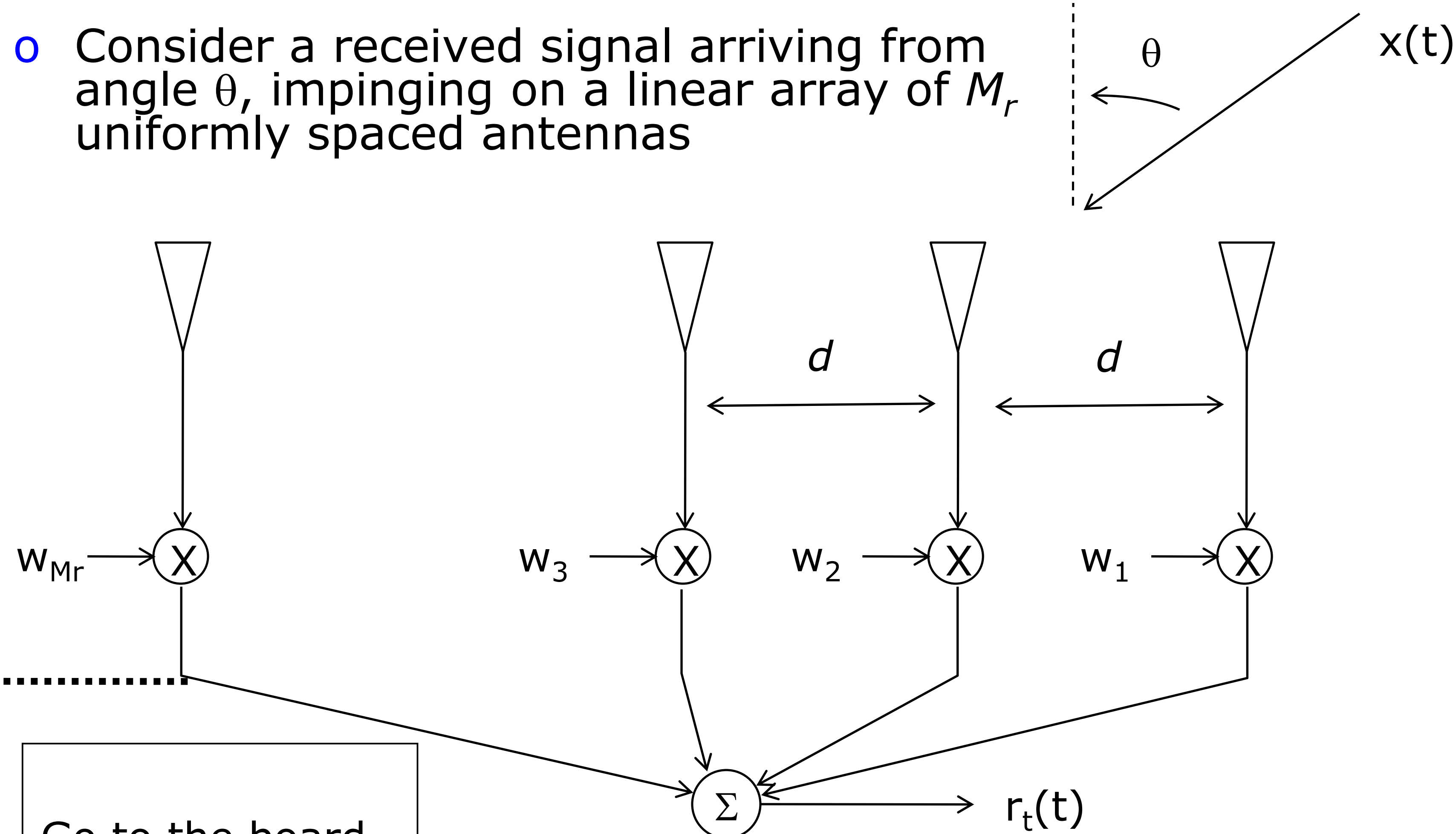
Dr. R. M. Buehrer

Introduction

- In our first discussion about MIMO communications we will assume that there is **no multipath** present near the array.
  - This is often referred to as the *plane wave assumption*
  - Channels seen by each antenna are *perfectly correlated*
- This use of multiple antennas provides **an SNR improvement**. Known as
  - **Beamforming gain** or
  - **Array gain**
- Classically this use of multiple antennas is known as **beamforming**

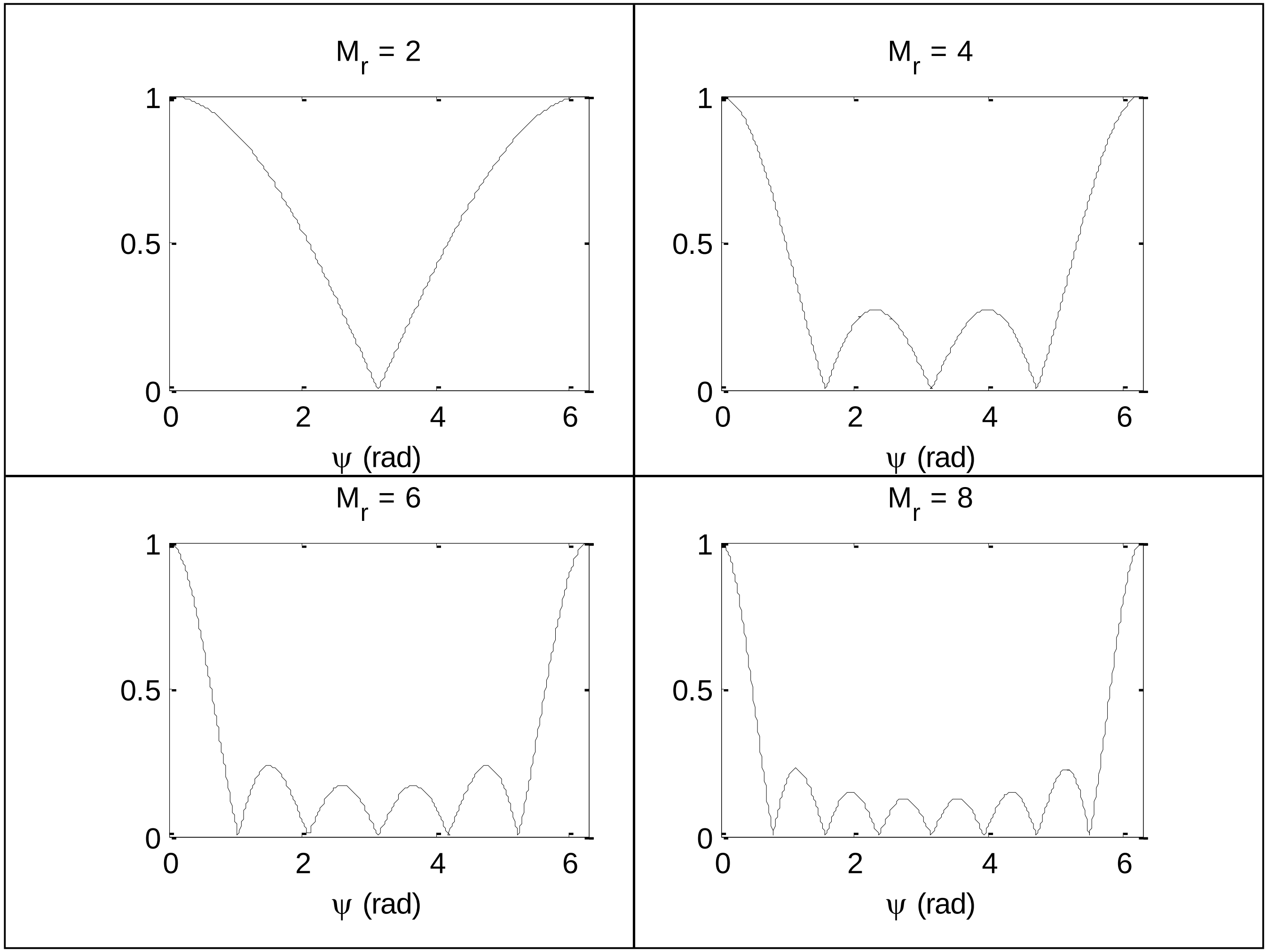
Beamforming

- Consider a received signal arriving from angle θ, impinging on a linear array of M_r uniformly spaced antennas

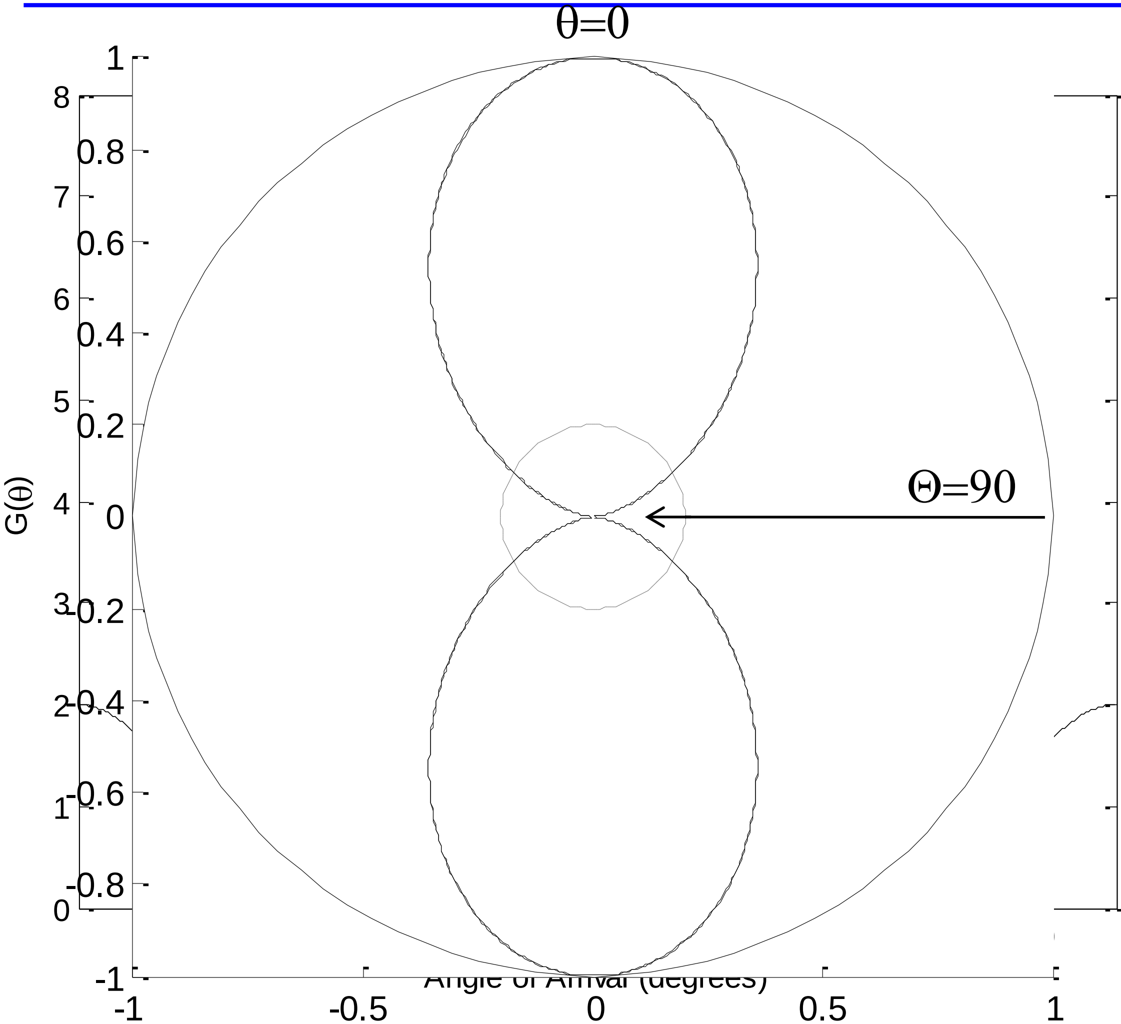


Go to the board…

$|f(\psi)|/M_r$

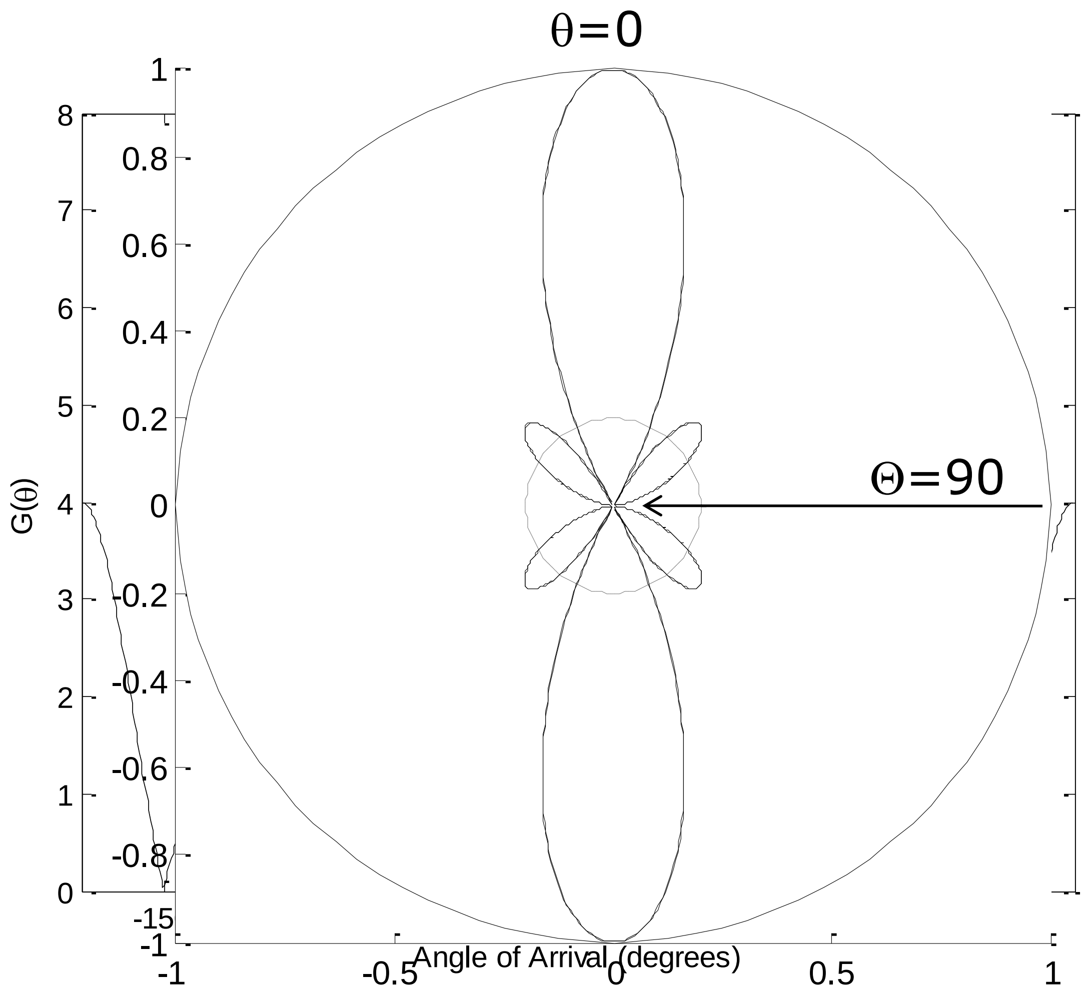


Beamforming Plots



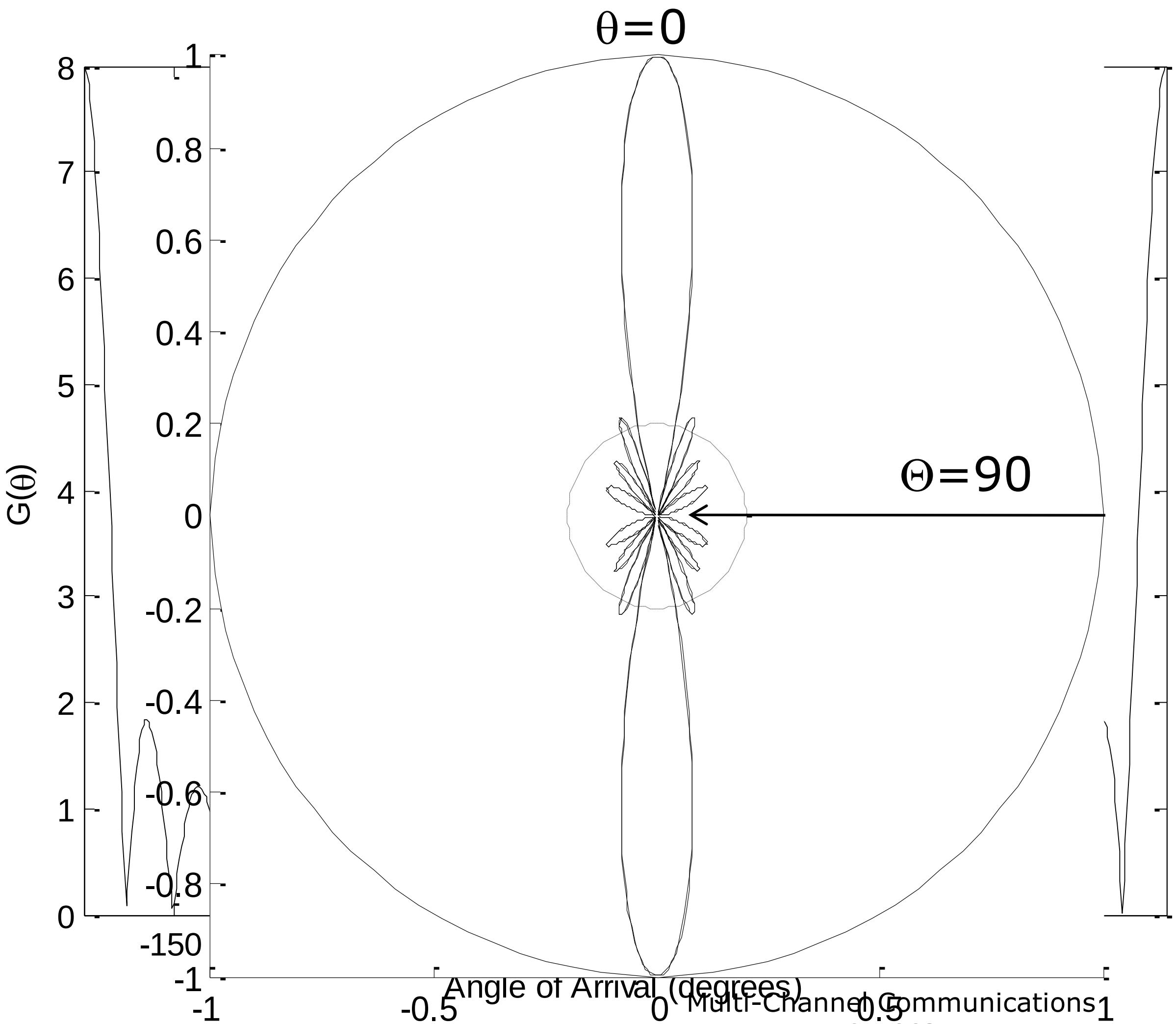
- Two elements
- d = 0.5λ
- **w** = [1,1]
- Polar Plot

Beamforming Plots



- Four elements
- d = 0.5λ
- **w** = [1,1,1,1]

Beamforming Plots



- Eight elements
- d = 0.5λ
- **w** = [1,1,1,1,1,1,1,1]

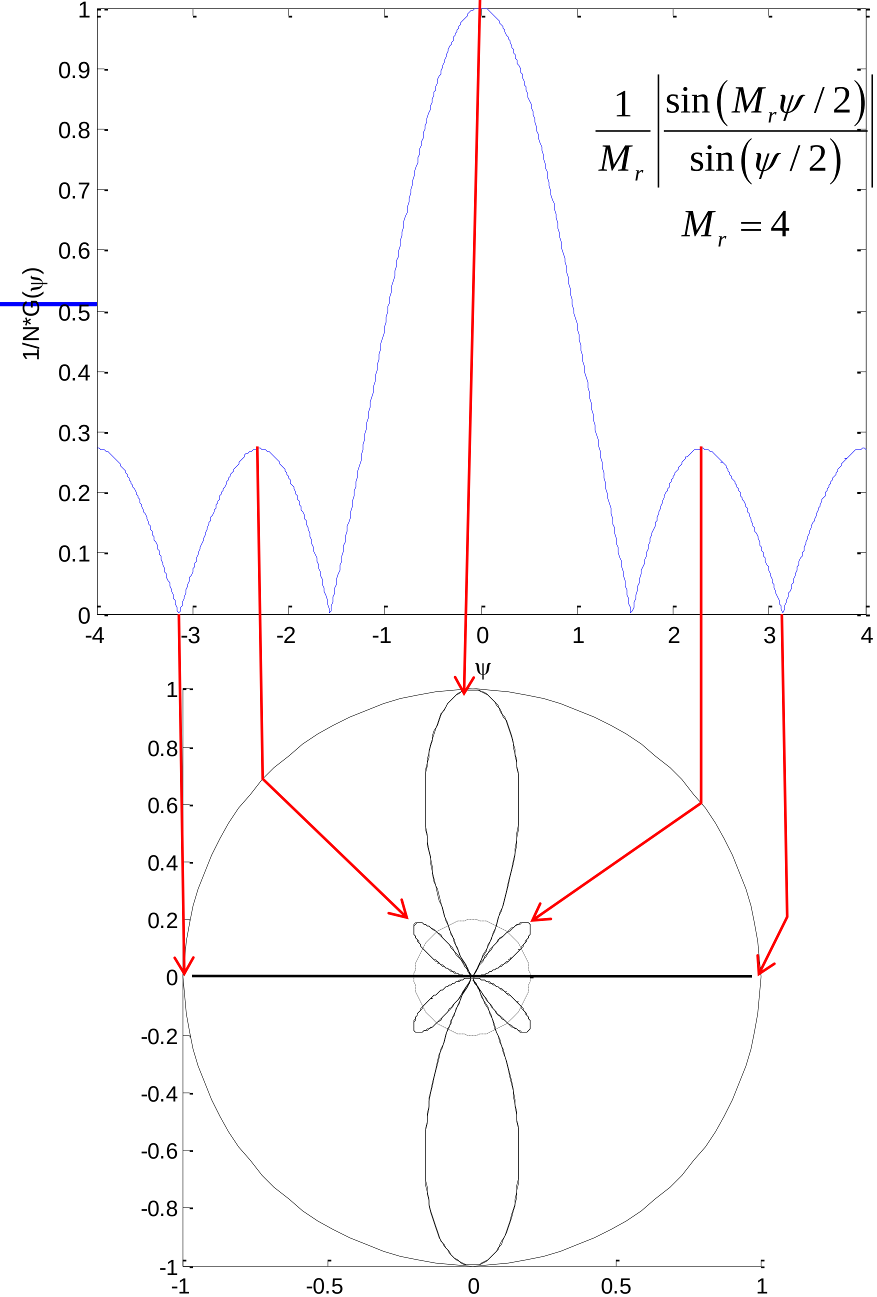
Impact of Antenna Spacing

- The relationship between ψ and θ determines the amount of the pattern that is *visible*
- In general

$$\psi = \frac{2\pi d}{\lambda}\sin\theta$$

- For the example to the right, *d* = λ/2, thus

$$\psi = \pi\sin\theta$$



Impact of Antenna Spacing

- The array gain of a uniformly spaced linear array can be written as

$$\left|f(\psi)\right| = \left|\frac{\sin\left(M_r\psi/2\right)}{\sin\left(\psi/2\right)}\right|$$

- The range of values can be found by considering the range of θ

$$-\pi \leq \theta \leq \pi$$

$$-\frac{2\pi d}{\lambda} \leq \underbrace{\psi}_{=\frac{2\pi d}{\lambda}\sin\theta} \leq \frac{2\pi d}{\lambda}$$

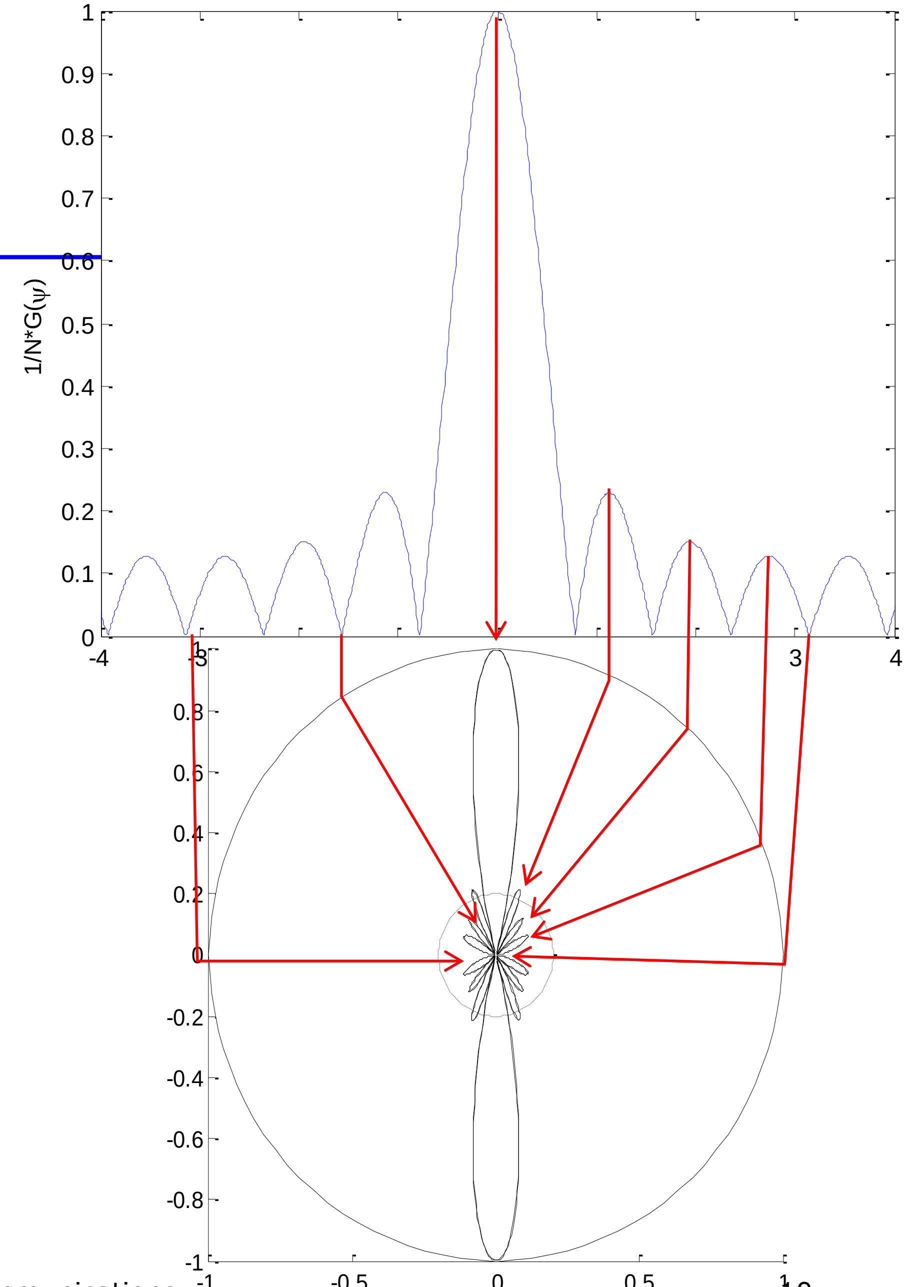
- The total range of ψ is then $\frac{4\pi d}{\lambda}$
- In order for one cycle of θ to correspond to one cycle to ψ, we must have

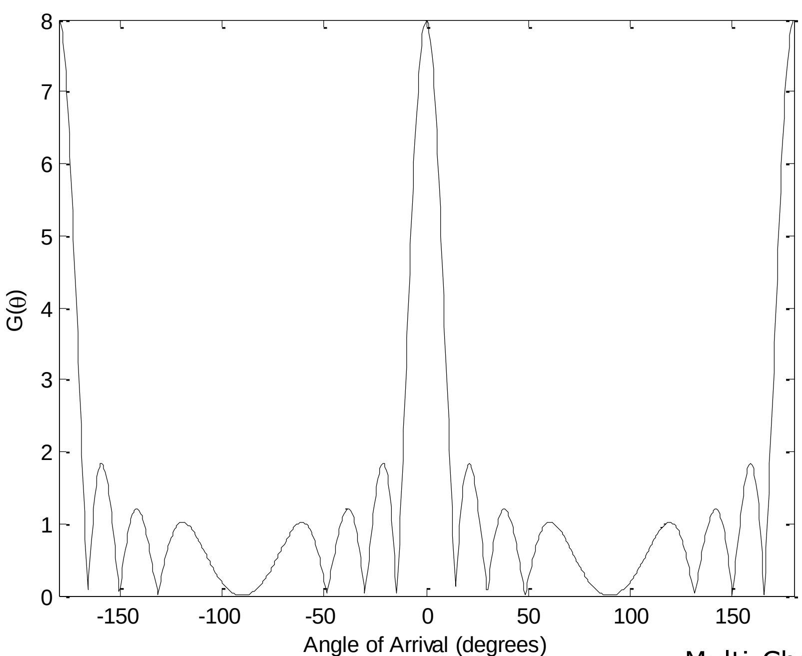
$$\frac{4\pi d}{\lambda} = 2\pi \quad \Rightarrow \quad \frac{d}{\lambda} = \frac{1}{2}$$

- If d/λ < ½ the visible pattern will be less than what is available, and if d/λ is greater than ½, the pattern will begin to repeat

Spacing = λ/2

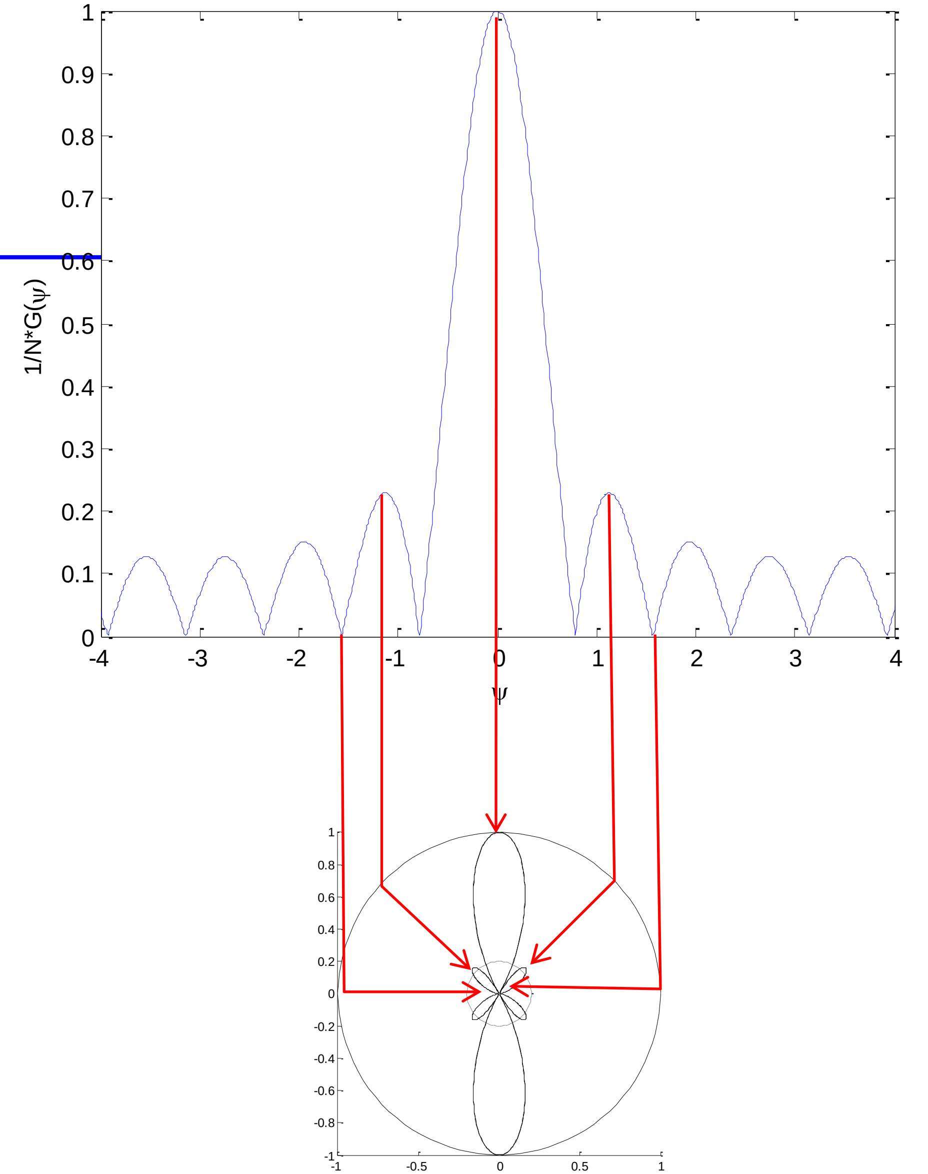
- $M_r = 8$
- $d = 0.5\lambda$
- $\psi = \pi \sin\theta$

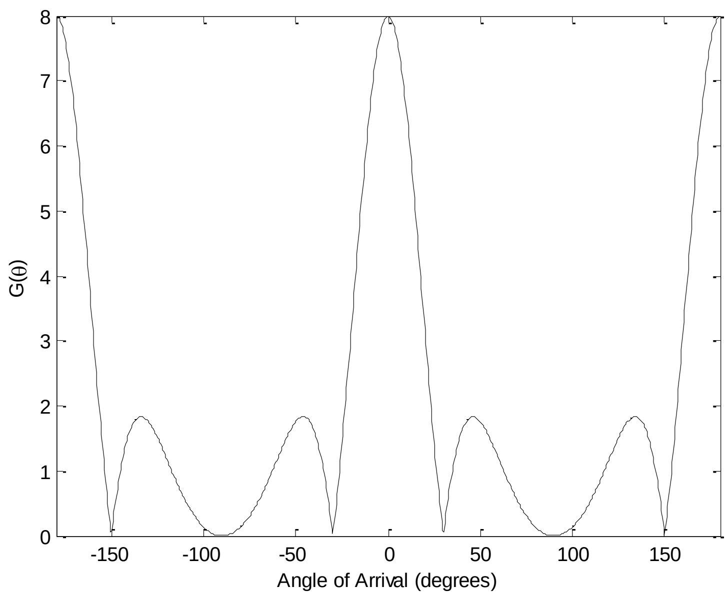




Smaller than λ/2 spacing

- $M_r = 8$
- d = 0.25λ
- $\psi = \frac{\pi}{2}\sin\theta$

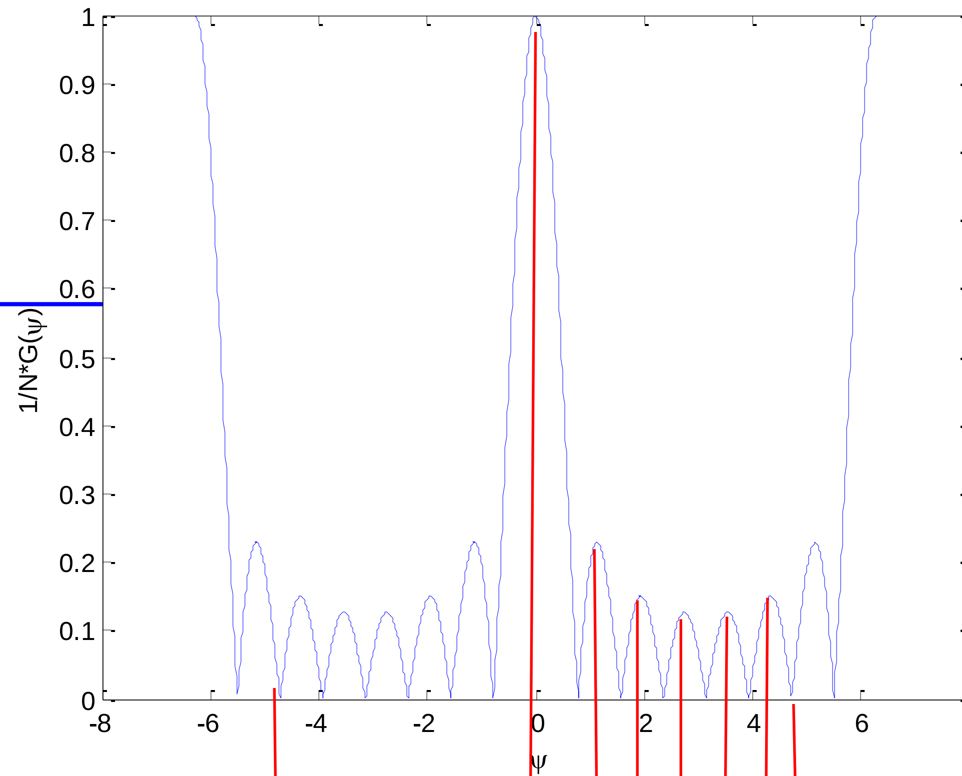


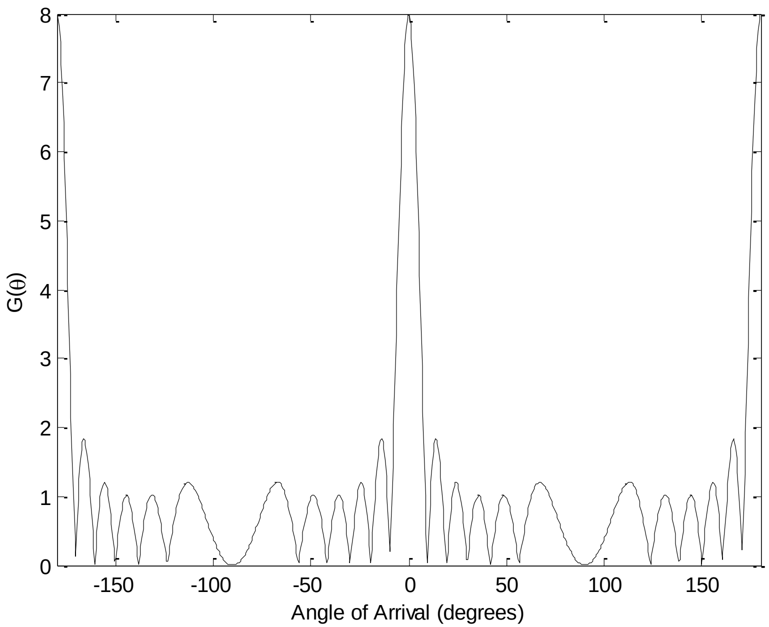


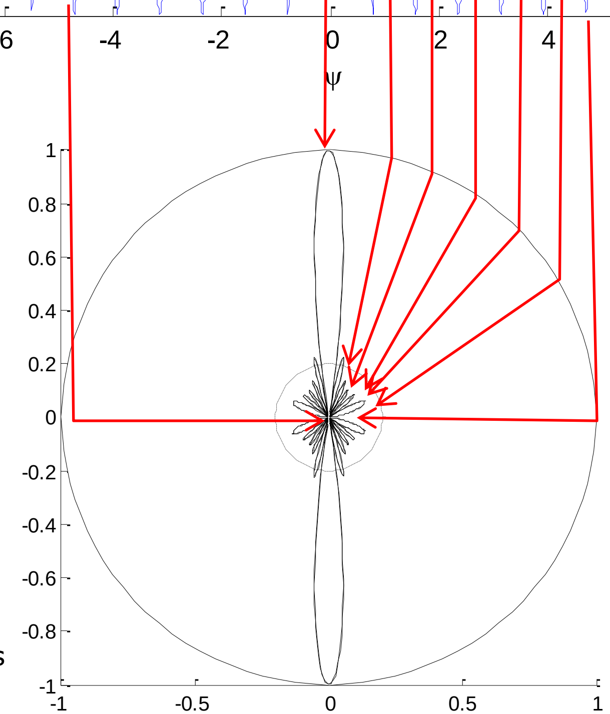
Smaller aperture → wider main beam

Greater than λ/2 spacing

- $M_r = 8$
- $d = 0.75\lambda$
- $\psi = \dfrac{3\pi}{2}\sin\theta$

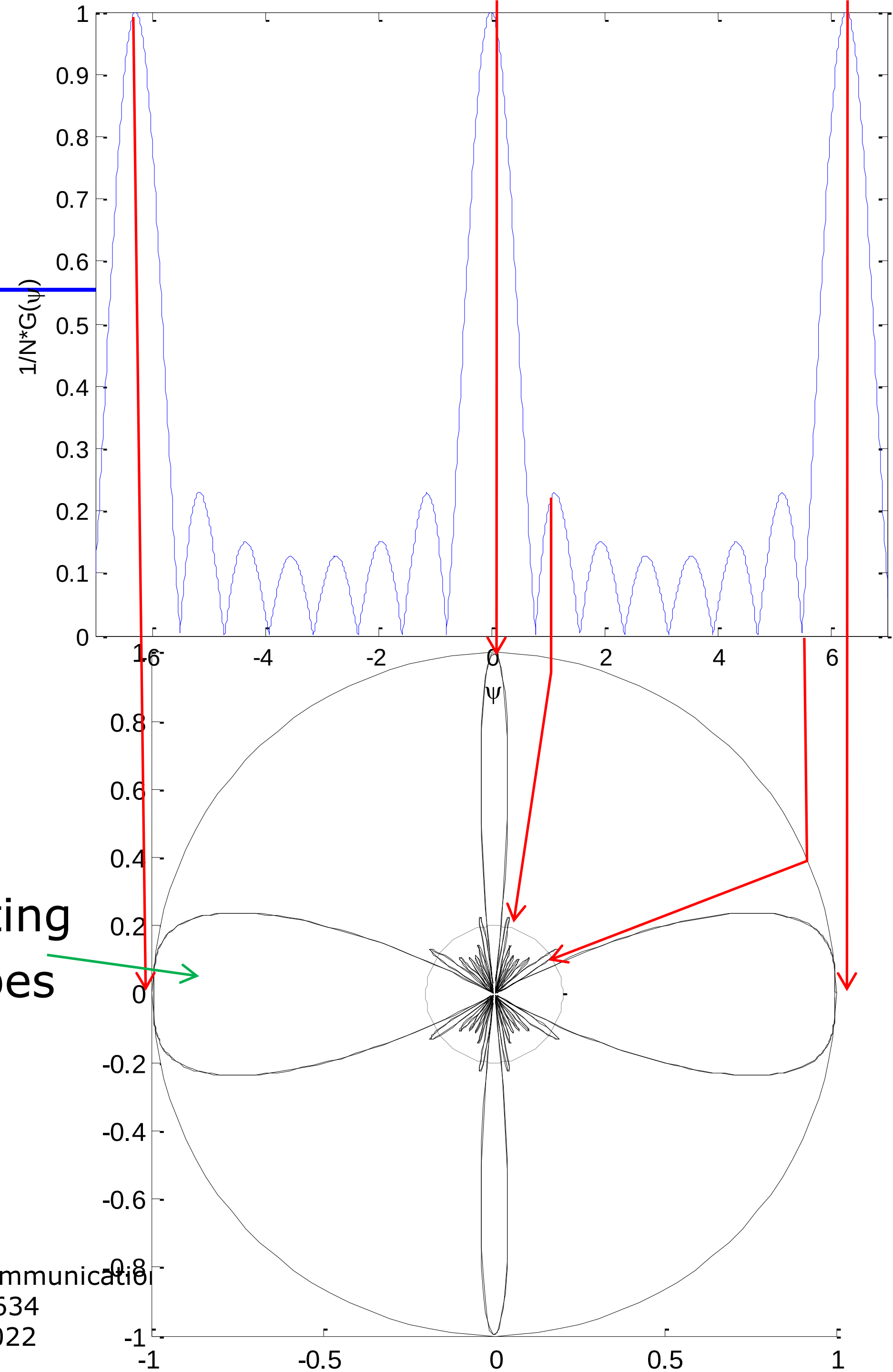


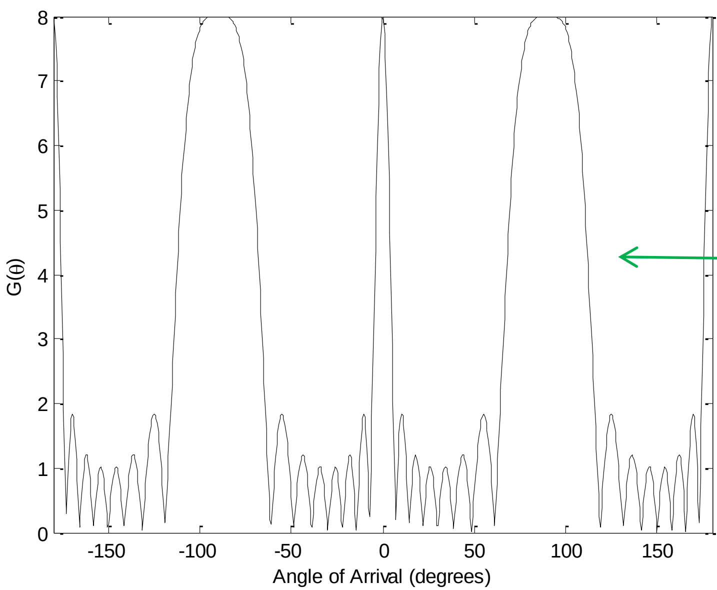




Greater than $\lambda/2$ spacing

- $M_r = 8$
- $d = \lambda$
- $\psi = 2\pi \sin\theta$





Grating lobes

Observations

- As the number of antennas M_r is increased,
  - The main beamwidth narrows
    - Beamwidth = $4\pi/M_r$ in ψ
  - The number of sidelobes increases
    - Number of lobes in $f(\psi)$ is M_r-1 (M_r-2 sidelobes)
  - The gain increases (Gain = M_r)
- As the distance between antennas increases
  - The main beamwidth narrows
  - More of the pattern is *visible*
  - Grating lobes often appear for $d/\lambda > ½$

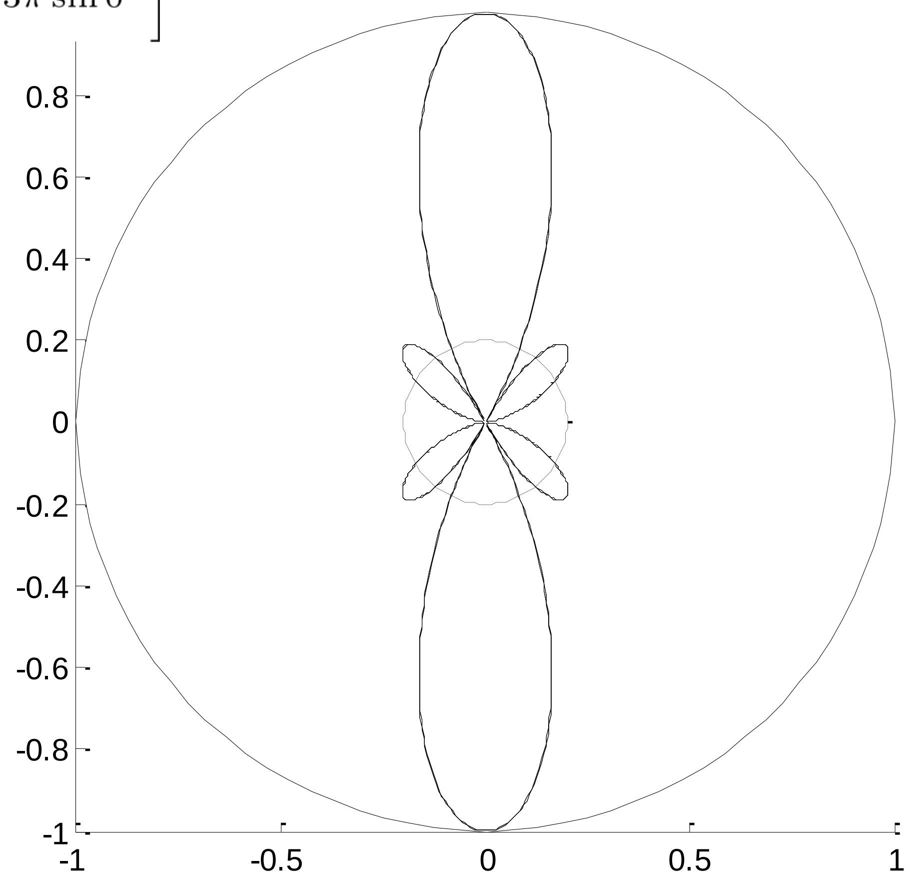
Beam-steering (The Phased Array)

- The previous discussion assumed that the weights were all unity, **w** = [1,1,1…1]
- This places the maximum gain at an AOA of 0^o. What if signal has an AOA other than 0^o?

Go to the board…

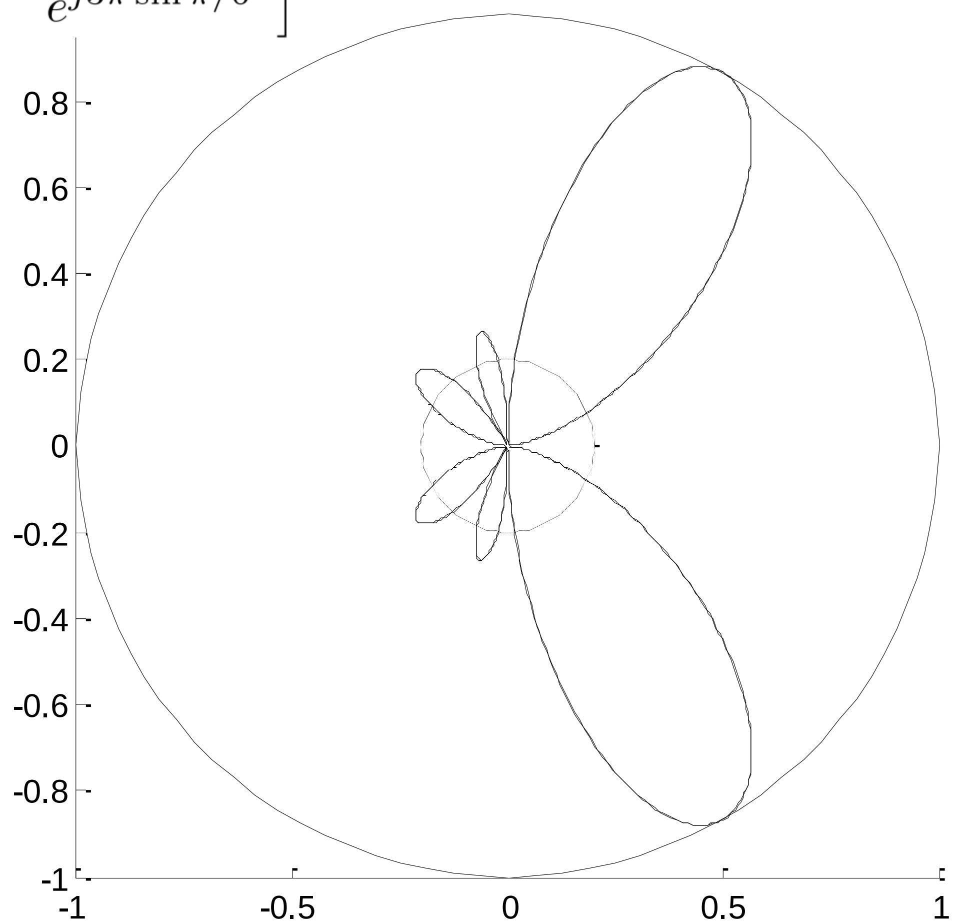
Beam-steering Example

- $\mathbf{w} = \begin{bmatrix} 1 & e^{j\pi \sin 0} & e^{j2\pi \sin 0} & e^{j3\pi \sin 0} \end{bmatrix}$
- $d = \lambda/2$
- $M_r = 4$
- $\theta_o = 0^o$



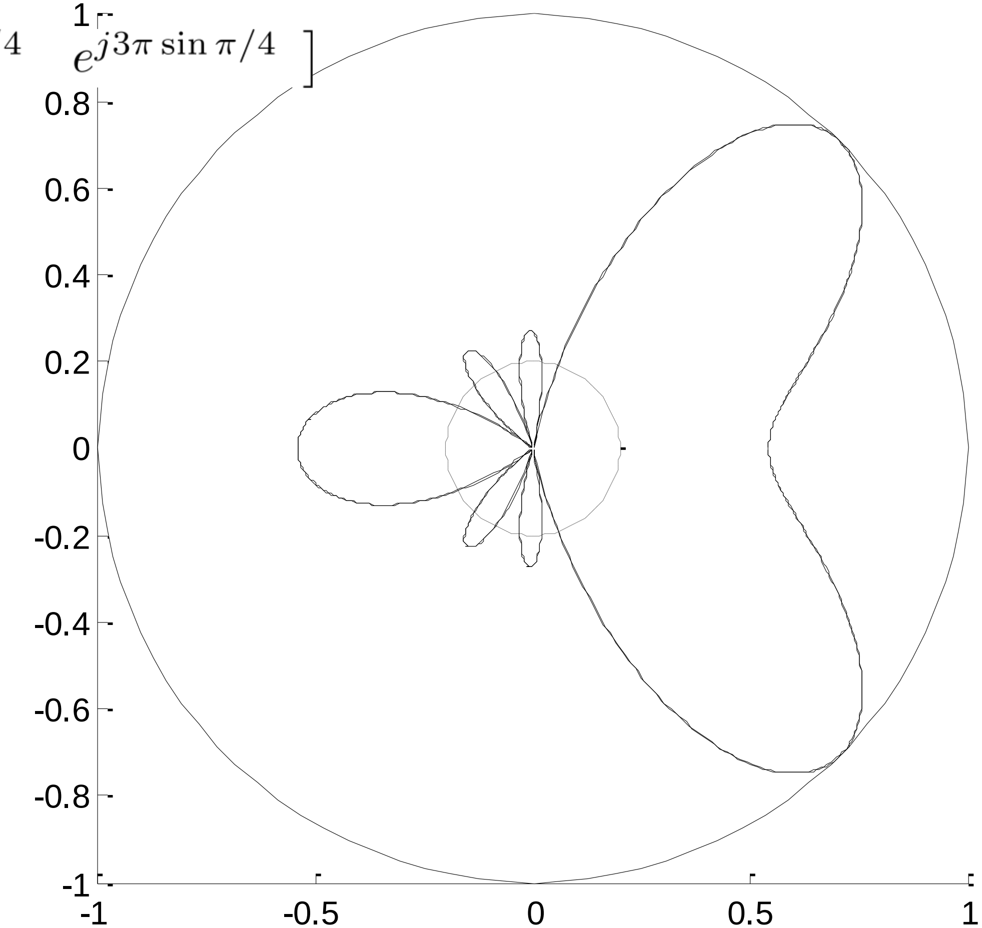
Beam-steering Example

- $\mathbf{w} = \begin{bmatrix} 1 & e^{j\pi \sin \pi/6} & e^{j2\pi \sin \pi/6} & e^{j3\pi \sin \pi/6} \end{bmatrix}$
- $d = \lambda/2$
- $M_r = 4$
- $\theta_o = 30^o$



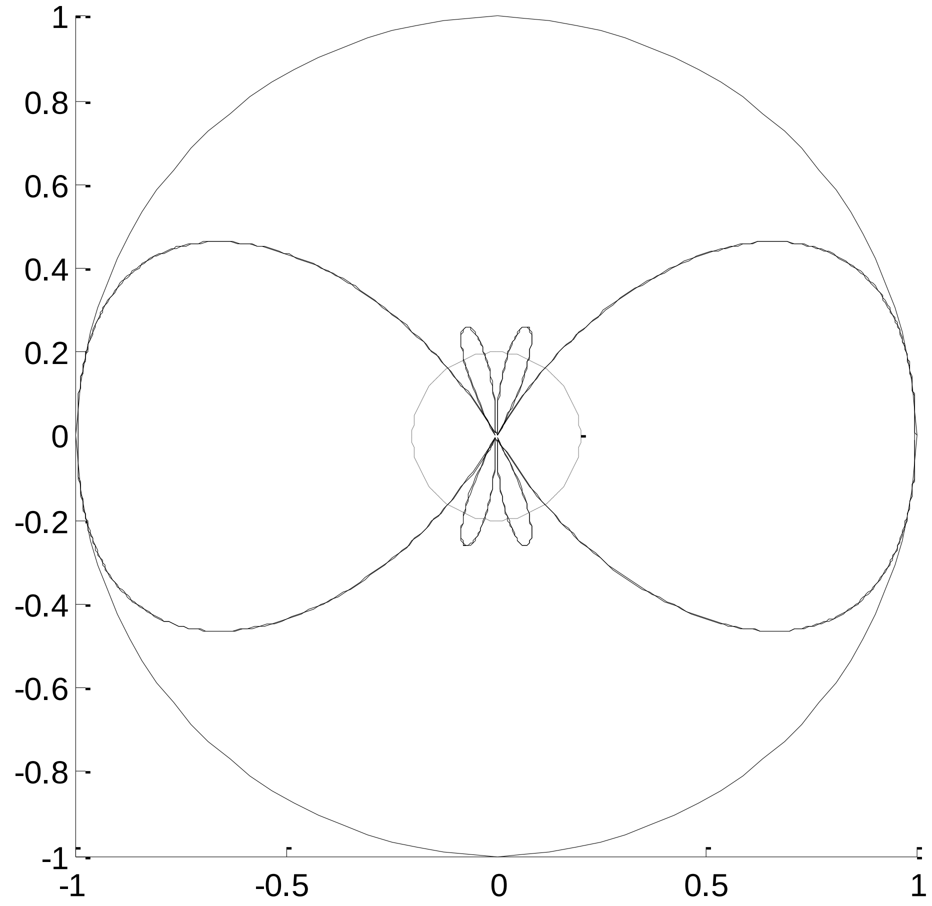
Beam-steering Example

- $\mathbf{w} = \begin{bmatrix} 1 & e^{j\pi \sin \pi/4} & e^{j2\pi \sin \pi/4} & e^{j3\pi \sin \pi/4} \end{bmatrix}$
- $d = \lambda/2$
- $M_r = 4$
- $\theta_o = 45^o$



Beam-steering Example

- $\mathbf{w} = \begin{bmatrix} 1 & e^{j\pi} & e^{j2\pi} & e^{j3\pi} \end{bmatrix}$
- $d = \lambda/2$
- $M_r = 4$
- $\theta_o = 90^o$



Observations

- Changing the phase progression across the array steers the beam

$$\mathbf{w} = \left[\begin{array}{cccc} 1 & \underbrace{e^{j\pi\sin\theta} & e^{j2\pi\sin\theta}}_{} & e^{j3\pi\sin\theta} \end{array}\right]$$

Change in phase of $\pi \sin\theta$

- As the steering angle approaches 90, the beam widens
- Steering the beam towards θ and $\pi - \theta$ provides the same beam pattern

Pattern Multiplication

- What we have examined so far is termed the *array factor*.
- The overall antenna pattern of the antenna array obeys the law of pattern multiplication which states

$$G_{tot}(\theta) = g_A(\theta)\,G(\theta)$$

Array factor

Overall Gain Pattern

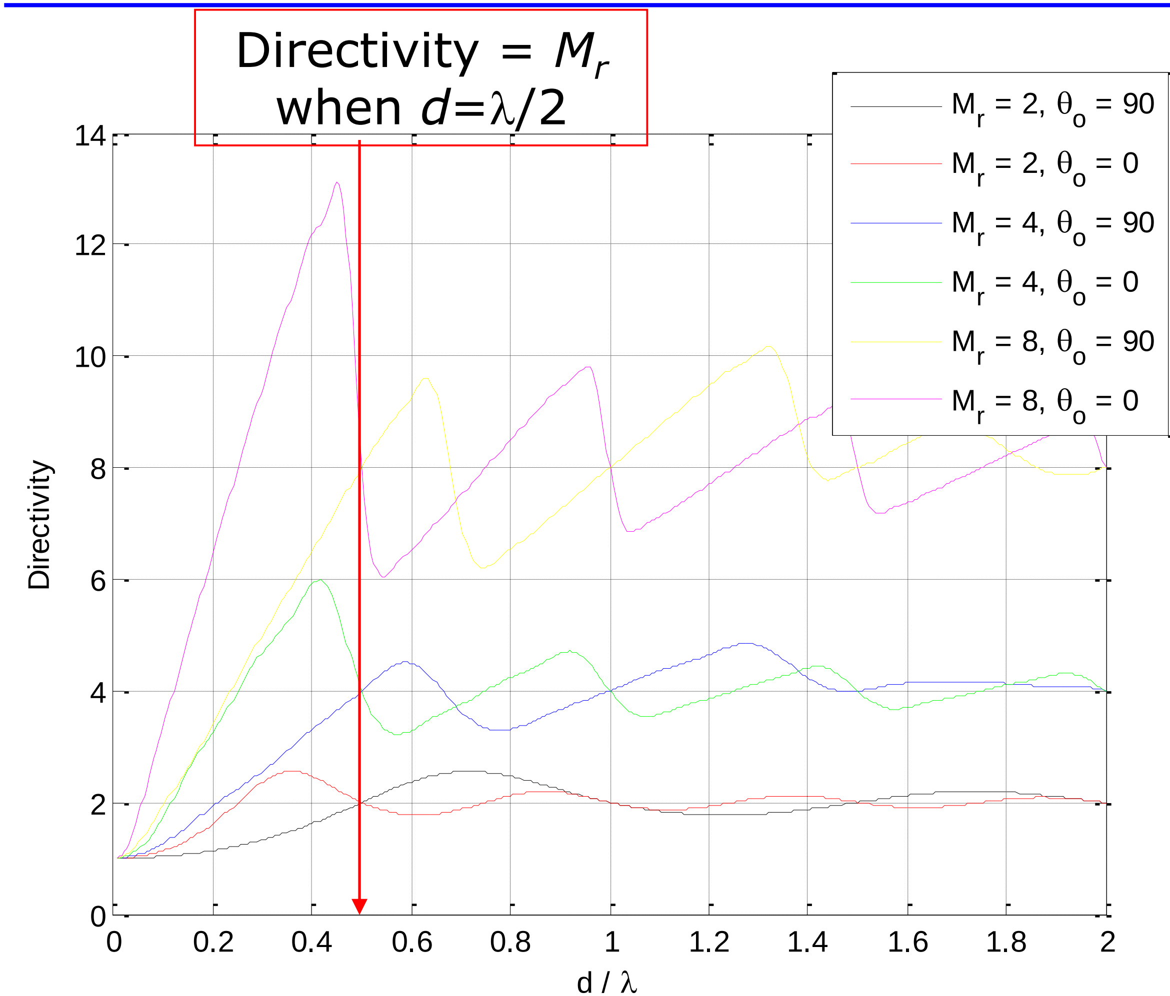
Individual element pattern

Directivity

- Directivity measures an antenna’s (or array of antennas) ability to focus energy in a particular direction relative to a point source
- We can measure directivity for a linear array

Go to the board…

Directivity



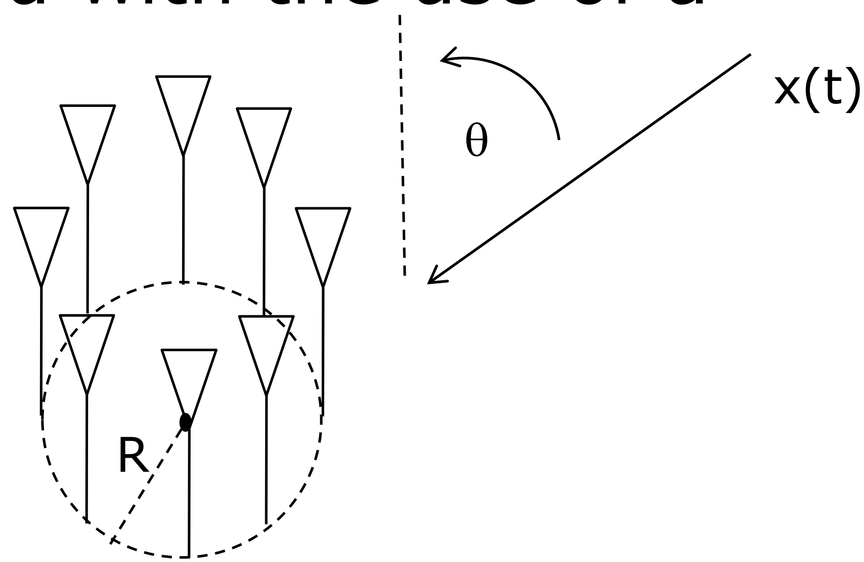
- Uniform linear array
- M_r = 2, 4, 8
- AOA = 0° and 90°

Transmit vs Receive Beamforming

- Although we have derived the characteristics of a beamforming array in *receive* mode, the exact same array can be used in *transmit* mode
- In transmit mode, the transmit antenna pattern is adjusted by changing the phases on the various antennas.
- The gain in power in the main beam direction is equal to N_t the number of transmit antennas
  - This is the same gain as seen in receive beamforming
  - However this requires prior knowledge of the proper transmit angle to achieve the gain – in receive beamforming the receiver can determine the angle through post processing before applying the appropriate weights

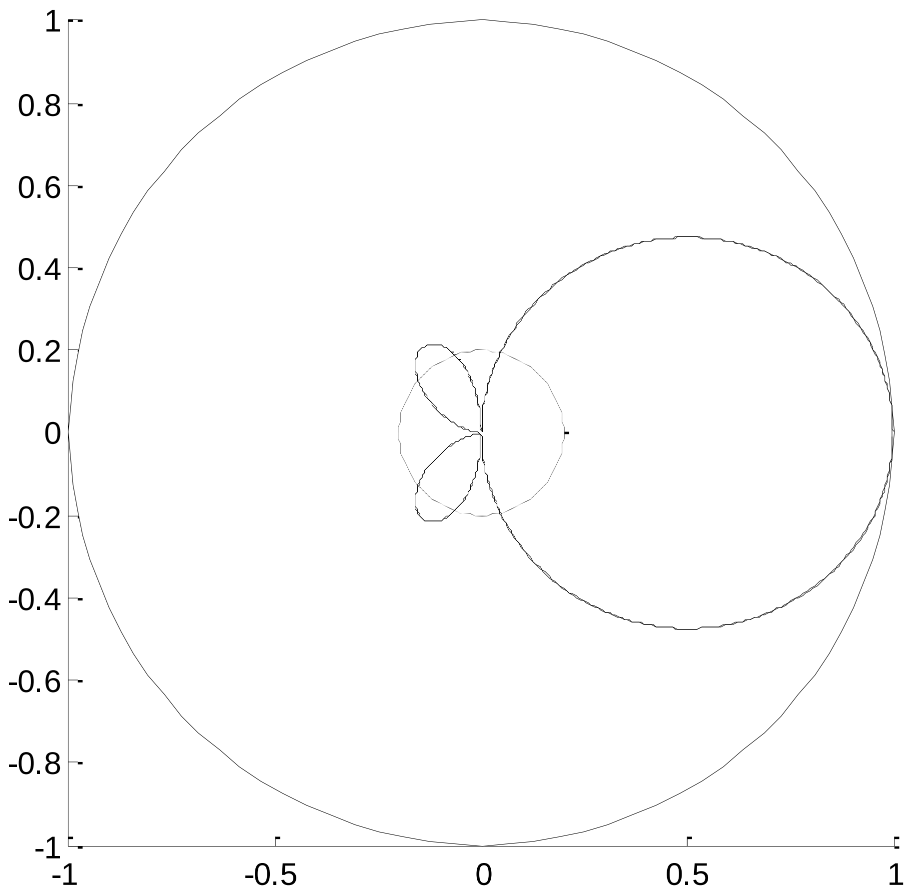
Circular Arrays

- A major disadvantage of linear arrays is that they have a line of symmetry (i.e., we cannot distinguish θ from $\pi-\theta$ with a linear array)
- This can be mitigated with the use of a *circular array*



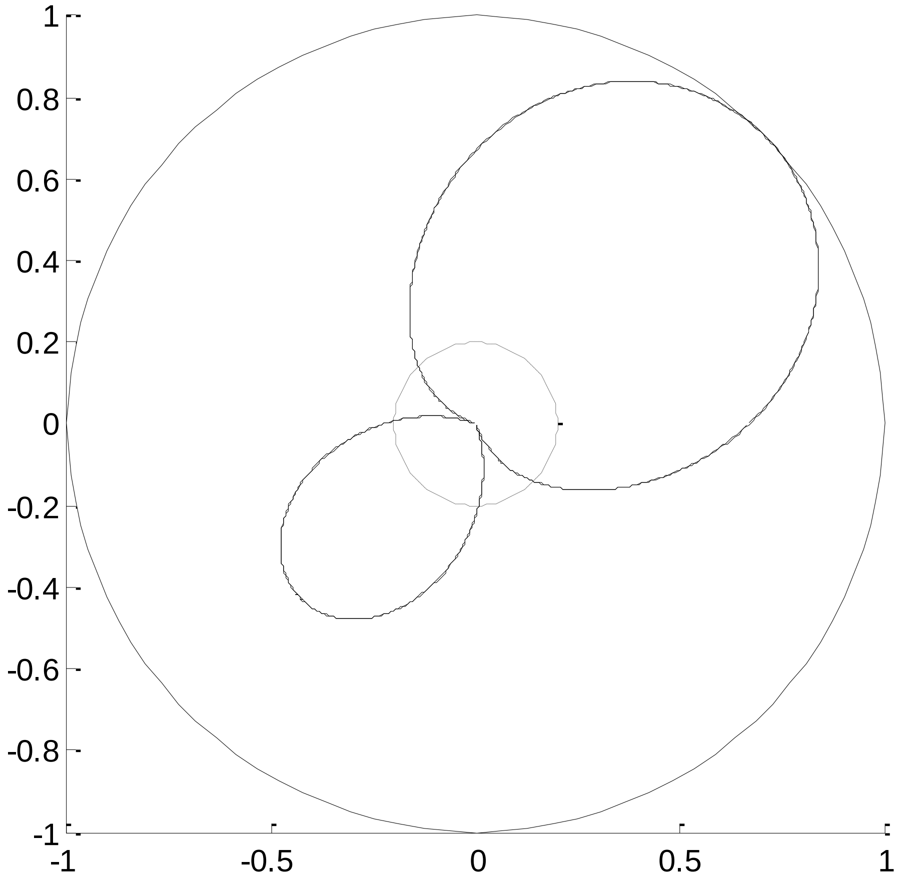
Go to the board…

Circular Array



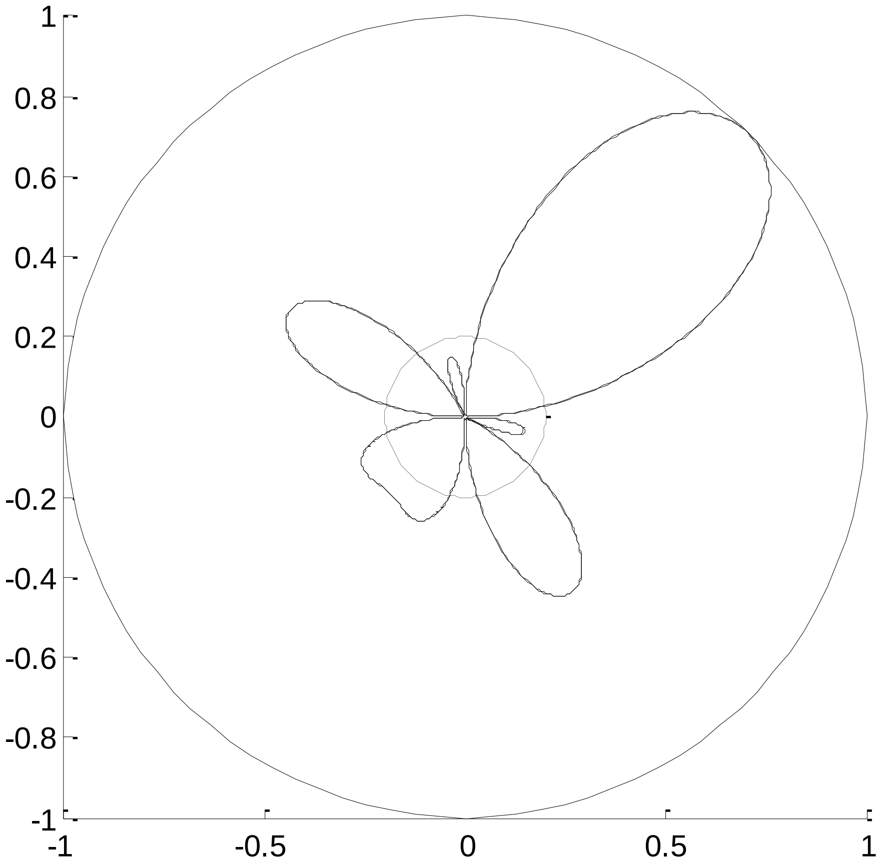
- $M_r = 4$
- $\theta_o = 90^o$
- $R/\lambda = 1/4$

Circular Array



- $M_r = 4$
- $\theta_o = 45^o$
- $R/\lambda = 1/4$

Circular Array



- $M_r = 4$
- $\theta_o = 45^o$
- $R/\lambda = 1/2$

Conclusions

- In this lecture we have investigated a use of multiple antenna systems known as **beamforming**
- Beamforming arrays (classically) assume plane wave propagation (i.e., perfectly correlated channels) unlike diversity arrays
- Linear arrays have good properties in terms of beamwidth and directivity, but suffer from direction ambiguity
- Circular arrays can overcome this limitation