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MIMO HW3

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1.1 Transmit and Receive Diversity Techniques

2 Description

2.1 Input

The inputs are summarised below

- M: Number of transmit antennas.
- N: Number of receive antennas.
- Ntrials: Number of trials of the experiment
- Ns: Number of tx symbols
- NPilots: Number of pilot symbols

2.2 Output

We have various BER for the different scenarios outlined in the questions.

3 Validation

3.1 BER for AWGN channel

3.1.1 Q1 BER for BPSK, QPSK, 16-QAM in AWGN

For the simulation we assume the symbol error is approximated by assuming the symbol error can happen only when a symbol is decoded as its neighbouring symbol in the constellation. Hence we need to select the symbol to bits mapping as gray code. This sets the neighbouring codewords to be different in only 1 bit. Hence the symbol error causes only 1 bit error and the bit error ate is $\frac{1}{k}$ the symbol error rate. For this section, we follow the steps outlined below:

- Create N data bits from a uniform distribution.
- Pick k bits at a time where k is the number of bits in 1 symbol. k= 1,2,3 for BPSK, QPSK and 16QAM.

- Interpret the k bits as a codeword using gray code and convert to its corresponding decimal symbol number.
- The decimal symbol number is converted to its complex equivalent using the constellation. This is the transmitted complex symbol. IMP: the constellation should have average symbol energy as 1 unit.
- The effect of addition of gaussian complex noise is to shift the transmitted symbol along I and Q depending on the SNR and this shifted complex value is the received complex symbol. Note the SNR is in terms of $\frac{E_b}{N_0}$. Symbol energy is k times bit energy.
- The decoder computes the distance of the received symbol with the constellation and converts the received complex symbol to its equivalent received decimal symbol using minimum distance. This is the maximum likelihood algorithm.
- The received decimal symbol is converted to bits using gray mapping
- The Bit Error rate is calculated by comparing the decoded received bits with the transmitted bits.

The conversion from codewords to decimal symbol number to complex and back to codewords was achieved using lookup tables generated based on the constellation shown in fig.1.

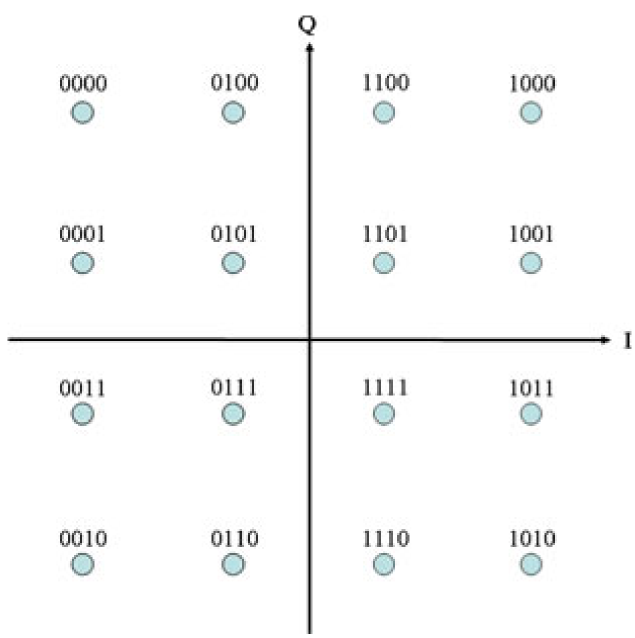

Figure 1: Symbol to gray code mapping for 16QAM

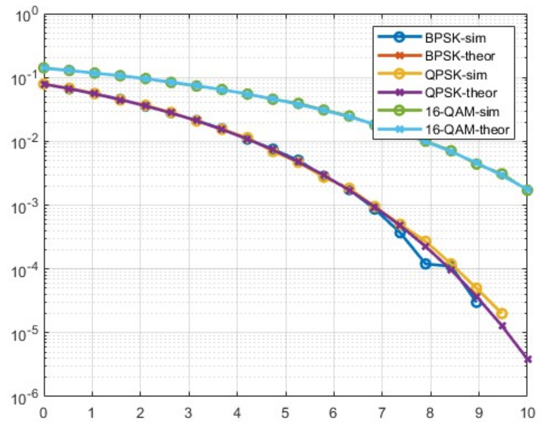

Figure 2: BER for 16-QAM, QPSK and BPSK simulation and closed-form expression

The BER for QPSK and BPSK is the same in the simulation and matches the theory. For 16-QAM we note the BER is much worse for a given SNR as expected and this matches with the theoretical BER shown in eq.1,2,3.

$$BER_{bpsk} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{1}$$

$$BER_{qpsk} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{2}$$

$$BER_{16-qam} = \frac{3}{2k} erfc\left(\sqrt{\frac{kE_b}{10N_0}}\right) \tag{3}$$

3.2 Fading channels receive diversity

3.2.1 Q2a). Nakagami fading derivation

In this section I guess there is typo in the question, m=1 and not 0.5. The BER for BPSK in an L branch fading channel is given by:

$$P_b = \frac{1}{\pi}\int_0^{\pi/2} \Phi_{MRC}\left(-\frac{1}{\sin^2(\theta)}\right) d\theta \tag{4}$$

Φ_{MRC} is the moment generating function of the resultant pdf of the SNR after Maximal Ratio combining. This BER equation assumes the channel is stationary for a block with gain specified by the SNR sampled from the resultant gain from the channel after MRC at the receiver. The BER for the instantiation of the channel is calculated assuming it is AWGN. We average this BER over the resultant SNR distribution.

For MRC the net SNR

$$\gamma_{mrc} = \sum_{i=1}^{L} \gamma_i \tag{5}$$

γ_i for nakagami fading are exponentially distributed. Assuming they are iid then the pdf of snr on each branch is

$$f_\gamma(\gamma) = \frac{1}{\Gamma(m)} \frac{m^m}{\bar{\gamma}^m} \gamma^{m-1} e^{-m\gamma/\bar{\gamma}} \tag{6}$$

and the Moment generating function is:

$$\Phi_\gamma(s) = \left(\frac{m/\bar{\gamma}}{m/\bar{\gamma} - s} \right)^m \tag{7}$$

The moment generating function for L branch MRC assuming all branches are independent is

$$\begin{aligned} \Phi_{\gamma_{mrc}}(s) &= \prod_{i=1}^{L} \Phi_{\gamma_i}(s) \\ &= \prod_{i=1}^{L} \left(\frac{m/\bar{\gamma}}{m/\bar{\gamma} - s} \right)^m \\ &= \left(\frac{m/\bar{\gamma}}{m/\bar{\gamma} - s} \right)^{mL} \end{aligned} \tag{8}$$

Substituting this moment generating function in eq.4.

$$P_b = \frac{1}{\pi} \int_0^{\pi/2} \left(\frac{m}{m + \frac{\bar{\gamma}}{\sin^2(\theta)}} \right)^{mL} d\theta \tag{9}$$

$$= \frac{1}{\pi} \int_0^{\pi/2} \left(\frac{m \sin^2(\theta)}{m \sin^2(\theta) + \bar{\gamma}} \right)^{mL} d\theta \tag{10}$$

Using equation 6.92 from the book

$$P_b = \frac{1}{2} \left[1 - \sqrt{\frac{\frac{\bar{\gamma}}{m}}{1 + \frac{\bar{\gamma}}{m}}} \sum_{k=0}^{mL-1} \binom{2k}{k} \left(\frac{1}{4} \frac{1}{1 + \frac{\bar{\gamma}}{m}} \right)^k \right] \tag{11}$$

For m = 1 this matches the BER for L branch MRC for uncorrelated Rayleigh Channel.

3.2.2 Q2b).Q2c).BER for MRC in Rayleigh fading channel - with channel estimation

Rayleigh fading happens when there is no direct path between the transmitter and receiver. The received signal is a sum of multipath that arrive within one symbol duration. This results in the channel coefficient being a complex gaussian random variable. Equivalently, the channel amplitude is distributed according to the Rayleigh distribution and the phase is uniformly distributed. Assume there are N symbols in a packet. We assume the channel is constant for the packet duration. Since the symbol rate is given to be 1Msps and maximum Doppler is 50 Hz, we can calculate the coherence time in terms of number of samples.

$$T_c = \frac{0.423}{f_d} \tag{12}$$

This corresponds to 8460 symbols, so the channel is stationary for 8460 samples.

For every transmit symbol s we have

$$\mathbf{r} = \mathbf{H}\, s + \mathbf{n} \tag{13}$$

where $\mathbf{r}$ is the received symbol on 4 antennas and is 4×1 vector, $\mathbf{H}$ is sampled from a complex Gaussian distribution with an identity covariance matrix and is 4×1, $\mathbf{n}$ is the uncorrelated Gaussian noise and is also 4×1. The channel is a complex At the receiver, we need to estimate the channel $\mathbf{H}$ using pilot symbols. Pilots are known transmitted symbols at the receiver. The channel estimate is given in eq.14. We assume that the average symbol power is 1 unit.

$$\hat{\mathbf{H}} = \frac{\mathbb{E}\{\mathbf{r}\, s\}}{\mathbb{E}\{|s|^2\}} = \sum_{i=0}^{N_{pilots}} \mathbf{r_i}\, s_i \tag{14}$$

Once we have the estimated channel, we perform MRC and then decode the resultant decision statistic by comparing the received sample to constellation to get the closest symbol in the constellation. From the decoded symbol we can get the decoded bits and hence calculate BER by comparing with the transmitted bits.

Using 30 pilots for channel estimation we see the following plots:

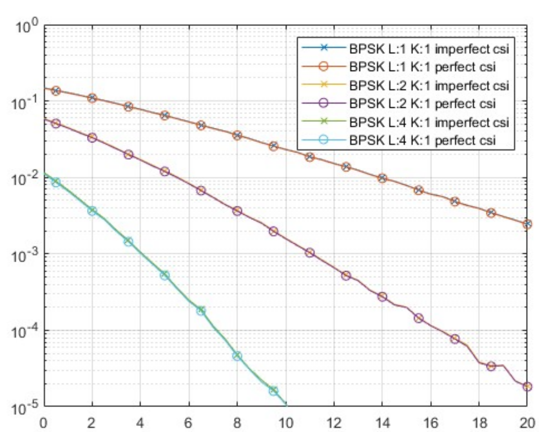

Figure 3: BER for BPSK simulation imperfect CSI (estimated using 30 pilots) vs perfect csi

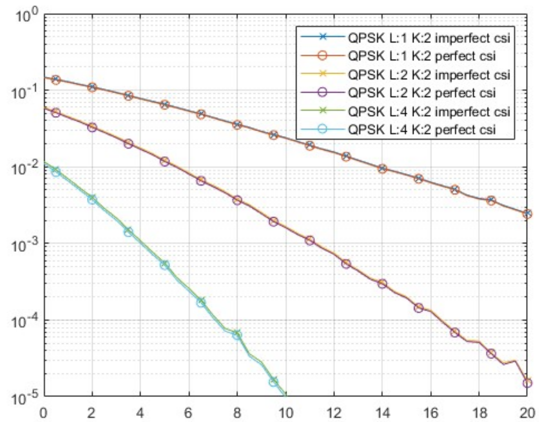

Figure 4: BER for QPSK simulation imperfect CSI (estimated using 30 pilots) vs perfect csi

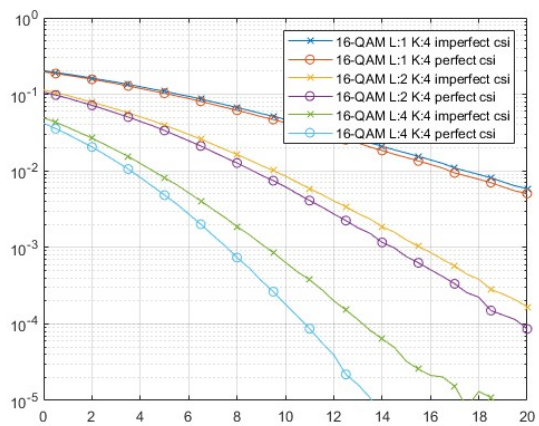

Figure 5: BER for 16-QAM simulation imperfect CSI (estimated using 30 pilots) vs perfect csi

In the 16-QAM we note that the BER after MRC for 4 chains for estimated channel and perfect channel estimation is not matching. This is because the error is dominated by the imperfect channel estimation that was done using 30 pilot symbols.

Now using 100 pilots we show the BER for only 16-QAM

3.2.3 Q2d). Correlated channel generation trick and BER results

Let the desired correlation matrix be C and decomposing it using cholesky decomposition to:

$$C = LL^T \tag{15}$$

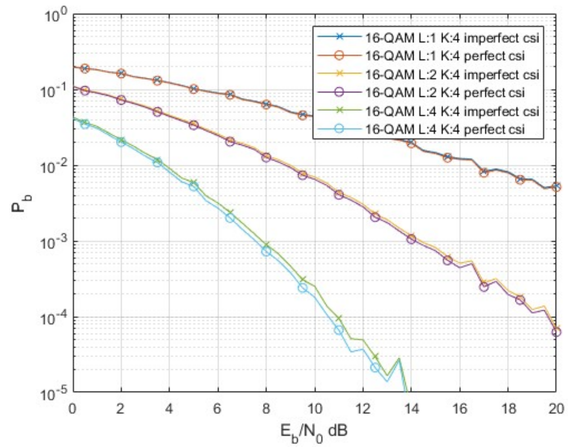

Figure 6: BER for 16-QAM simulation imperfect CSI (estimated using 100 pilots) vs perfect csi

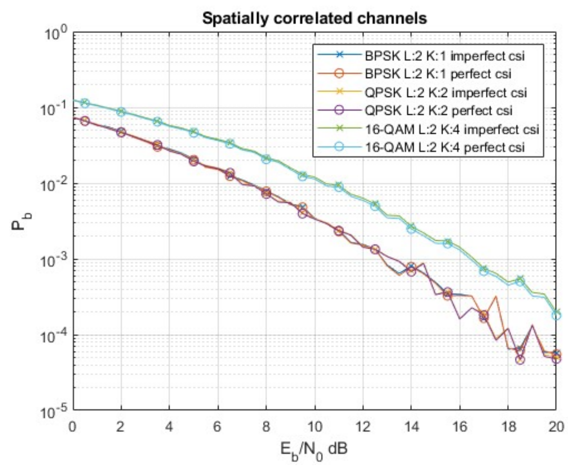

Figure 7: BER for BPSK,QPSK,16-QAM simulation using MRC with imperfect CSI (estimated using 100 pilots) vs perfect csi for spatially correlated channels

Here L is triangular (upper or lower). Let the uncorrelated samples be in matrix X then the covariance of $Z = LX$ is given by:

$$\begin{aligned} E\left[ZZ^T\right] &= E\left[(LX)(LX)^T\right] \\ &= E\left[LXX^T L^T\right] \\ &= LE\left[XX^T\right] L^T \\ &= LIL^T = LL^T = C \end{aligned} \tag{16}$$

Hence to generate a correlated channel, we generate an uncorrelated channel matrix and then premultiply it by the cholesky decomposition matrix of the desired covariance matrix.

3.2.4 Q2e).Nakagami channel BER results

As expected for m=4 the BER performance is much better as compared to to m=1 Nakagami fading.

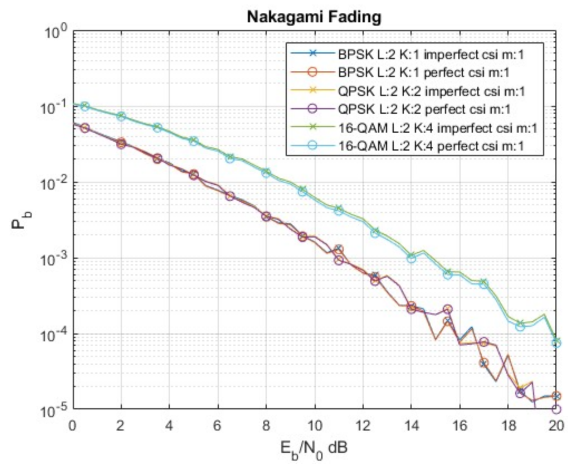

Figure 8: Nakagami m=1

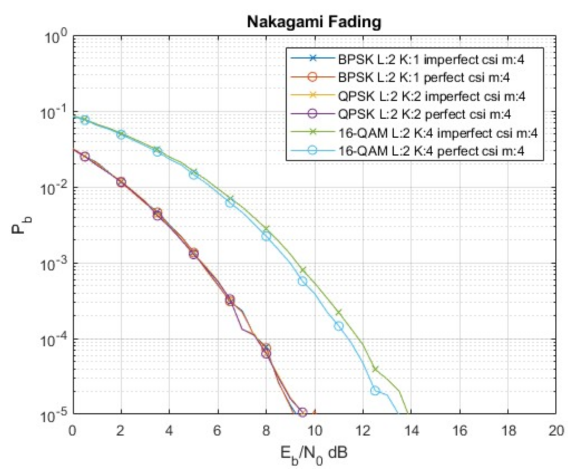

Figure 9: Nakagami m=4

3.3 Transmit Diversity

3.3.1 Alamouti transmit Scheme for TX diversity

This is an open loop transmit diversity scheme. Channel estimation for Alamouti transmit scheme needs to be performed at each received antenna. We have the measurements at the i^{th} rx antenna over two consecutive time slots r_{i1} and r_{i2}. The channel is assumed to be stationary for a block much greater than the symbol duration. The transmitters transmit s_1 and s_2 from tx1 and tx2 in the first time slot, in the second slot the transmitters transmit $-s_2^*$ and s_1^* The channel estimate between 1st transmit antenna and i^{th} antenna is given by

$$\hat{h_{i1}} = \frac{\mathbb{E}\left\{r_{i1}\, s_1^* \,-\, r_{i2}\, s_2\right\}}{2} \tag{17}$$

and the channel estimate between 2^{nd} transmit antenna and i^{th} receiver is given by

$$\hat{h_{i2}} = \frac{\mathbb{E}\left\{r_{i1}\, s_2^* \,+\, r_{i2}\, s_1\right\}}{2} \tag{18}$$

Now we can use these channel estimates to generate decision metrics for the transmitted symbols s_1 and s_2 using the below equations.

$$z_{i1} = \frac{\hat{h_{i1}}^*\, r_{i1} \,+\, \hat{h_{i2}} r_{i2}^*}{|\hat{h_{i1}}|^2 + |\hat{h_{i2}}|^2} \tag{19}$$

$$z_{i2} = \frac{\hat{h_{i2}}^*\, r_{i1} \,-\, \hat{h_{i1}} r_{i2}^*}{|\hat{h_{i1}}|^2 + |\hat{h_{i2}}|^2} \tag{20}$$

The decision metrics can be averaged over the received antennas i and MLE can be performed to get the decode the symbols s_1,s_2.

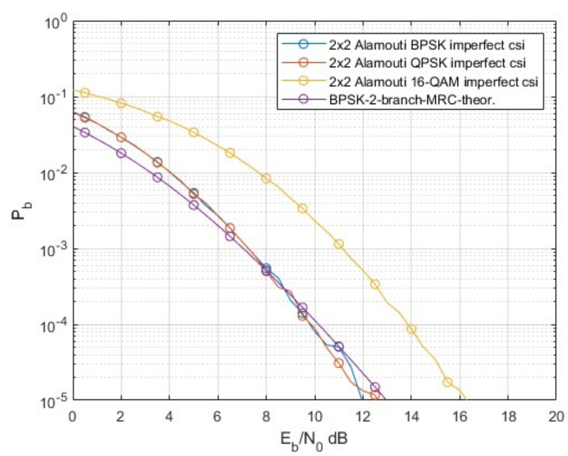

Figure 10: Q3a). BER for Alamouti 2x2 scheme using Alamouti based channel estimation

Note: For theoretical comparison, an L-branch MRC bit error expression was used with 3dB offset on the bit snr axis.

3.4 TX Antenna Selection

In order to select the best TX antenna, the channel gain at each of the receivers from the individual transmitters is evaluated and the transmit antenna with the higher overall net gain is sent as feedback

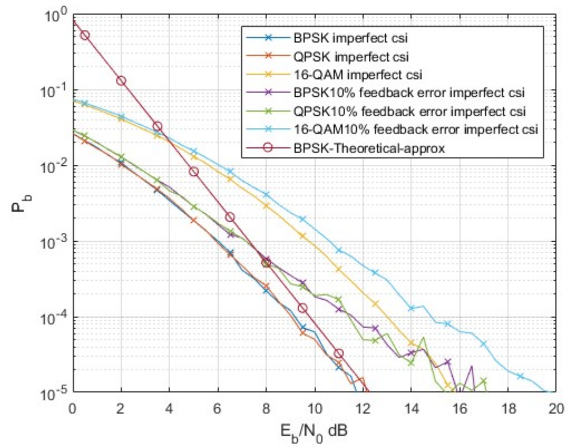

Figure 11: Q3b). BER for TX antenna selection based on 1 bit feedback in a 2x2 system

from receiver to the transmitter. Hence the transmitter uses that particular transmit antenna to transmit the symbol. The resultant gain G_i due to the i^{th} transmit antenna is given by

$$G_i = \sum_{j=0}^{N_r} |h_{ij}|^2 \tag{21}$$

The one bit feedback about the channel with the better overall gain is relayed to the transmitter and only the channel corresponding to the higher Gain is used by the transmitter. Also simulated is the BER with a 10% error in the feedback about the better transmit antenna.

NOTE: For the theoretical validation, the approximation in eq. 33 in [1] for BER for BPSK in Rayleigh fading with transmit antenna selection was used. This is valid for high SNR $\gamma >> 1$ and we can see the BER for BPSK simulation and theoretical converge for high SNR.

$$P_b \approx \frac{(4N_t - 1)!}{2^{5N_t} \cdot (2N_t - 1)!} \left(\frac{1}{\gamma}\right)^{2N_t} \tag{22}$$

3.5 Maximal Ratio Transmission

In this scheme we transmit a scalar symbol across the transmit side antennas. We apply weights at the transmit side $\mathbf{v}$ as well as the receive side $\mathbf{w}$ such that the received SNR is maximised. MRT assumes the transmit side weights $\mathbf{v}$ are a linear transformation of the channel and is given by

$$\mathbf{v} = \frac{(\mathbf{gH})^H}{|\mathbf{gH}|} \tag{23}$$

here $\mathbf{g}$ is set to be the eigenvector corresponding to the maximum eigenvalue of $\mathbf{HH^H}$ where H is the $M_r \times N_t$ channel matrix. The receiver side weights are set to $\mathbf{w} = \mathbf{g^H}$. The problem is we have no knowledge of the channel at the transmitter. In a real system we estimate the channel and calculate the transmit side weights to give as feedback to the transmitter. Then the transmitter uses these weights to transmit the symbol. On application with the received side weights the SNR is maximised.

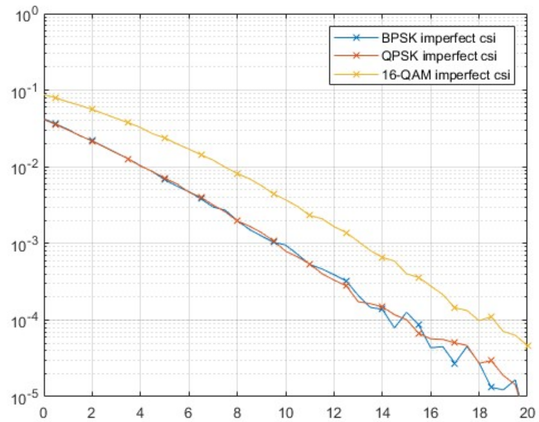

Figure 12: Q3b). BER for Maximal Ratio transmission

Note: BER for maximal ratio transmission for BPSK with 2 rx antennas and L transmit antennas is given by [2]

$$\begin{aligned} P_b = &Lg(c, L+1, 1) - 2(L-1)g(c, L, 1) \\ &+ Lg(c, L-1, 1) \\ &- L\sum_{i=0}^{L-2} g(c, i+L+1, 2)\begin{pmatrix} i+L \\ L \end{pmatrix} \\ &+ 2(L-1)\sum_{i=0}^{L-1} g(c, i+L, 2)\begin{pmatrix} i+L-1 \\ L-1 \end{pmatrix} \\ &- L\sum_{i=0}^{L} g(c, i+L-1, 2)\begin{pmatrix} i+L-2 \\ L-2 \end{pmatrix}, \end{aligned} \tag{24}$$

where

$$g(c, n, a) = \frac{1}{2a^n}\left[1 - \mu\sum_{k=0}^{n-1}\begin{pmatrix} 2k \\ k \end{pmatrix}\left(\frac{1-\mu^2}{4}\right)^k\right], \\ ; \mu = \sqrt{\frac{c}{c+2a}} \text{ and } c = \frac{2E_b}{N_0} \tag{25}$$

3.6 Q3e).Transmit Diversity

Since in a rayleigh fading channel the channels between the transmitter and receiver fluctuate (fade) temporally which gives rise to fluctuations of the SNR at the receivers. This directly affects performance in terms of Bit error rate at the receiver. Transmit diversity aims to improve the BER performance by exploiting diversity in the channel. In general the challenge is that the transmitter has no knowledge of the channel between the transmitter and receiver. This gives rise to two broad ideas through which we can exploit transmit diversity.

3.6.1 Open Loop - Transmit diversity

In these techniques, there is no feedback of information from the receiver to the transmitter and hence these techniques attempt to convert spatial diversity through the transmit antennas to temporal diversity which can exploited by error control coding.

- Antenna Hopping: Every symbol is transmitted from each widely separated transmit antenna and using coding we decode the received symbol. The hope is that at-least one of the channels from the transmitter to the receiver is not experiencing a deep fade. We need to use channel coding to exploit this transmit diversity that is ultimately converted to temporal diversity.

- Space time transmit diversity: In this technique we transmit symbols across all the transmit antennas and over a time block. At the receiver we take temporal measurements across the time block. Each measurement contains a linear combination of the transmitted symbols. Using space-time encoding we can form decision metrics on each transmitted symbols to pull apart the inter symbol interference symbol. The limitation is this is usually at the cost of loss in bandwidth since we want the space time code to be orthogonal. The exceptions to this are Alamouti code that exists only for a 2 transmitter scheme unless we use real symbols for which we can form any space time code. However the gain in performance due to being able to use QAM constellations outweighs the gain due to using space time codes for 1-D constellations.

3.6.2 Closed loop diversity

In these techniques we have a feedback information through which we can send information about the channel / performance so that the transmitter can optimise its transmission about the symbols to increase performance at the receiver. The disadvantage of this is limitation of the feedback mechanism in terms of either channel uses which results in reduction of capacity or error in feedback information can affect performance drastically. This also might require more sophisticated receivers.

- Antenna Selection: The idea in this is at the receiver after we perform channel estimation between each transmitter and receiver antenna, feedback information about the best few channels is sent to the transmitter. In the scheme we implemented for this assignment, for a 2×2 system, we sent information from the receiver to the transmitter about which transmitter offers the higher channel gain using 1 bit of feedback. The idea is to surf the peak of the channel gains and hence achieve better performance.

- Maximal ratio Transmission (generalisation of generalised beamforming) In this we require multiple received antennas which is a luxury that we may not always have. The transmitter and receiver have weights multiplied at their respective ends to maximise the received SNR hence maximising performance. The weights are chosen such that the effect of the channel is removed and this needs to be calculated at the receiver by using the eigen vector decomposition of the channel matrix to calculate the maximum eigenvector of the channel. This can be a computationally costly operation and can increase system complexity. Additionally we need a feedback mechanism between the receiver and transmitter to inform the transmitter of the weights that need to be used.

4 Conclusion

- We learnt about receive side diversity techniques in rayleigh and nakagami faded channels.
- Channel estimation is crucial to use diversity techniques since ideally we want to remove its effect.
- The effect of increase in receive diversity is to change the slope of the BER to reduce the BER.
- Transmit diversity is in general of two types - open loop and closed loop
- Alamouti space time block code can change spatial diversity to temporal diversity without a feedback mechanism, without complex coding schemes that other open loop coding schemes rely on and also without loss in the transmitted data rate. The limitation is only one such space time code exists for 2x2 systems with complex symbols.
- closed loop transmit diversity techniques can provide multiple kinds of feedback at the transmitter to improve BER performance at the receiver.

References

[1] Z. Chen, J. Yuan, and B. Vucetic, “Analysis of transmit antenna selection/maximal-ratio combining in rayleigh fading channels,” *IEEE Transactions on Vehicular Technology*, vol. 54, no. 4, pp. 1312–1321, 2005.

[2] B. D. Rao and M. Yan, “Performance of maximal ratio transmission with two receive antennas,” in *Conference Record of Thirty-Fifth Asilomar Conference on Signals, Systems and Computers (Cat. No. 01CH37256)*, vol. 2. IEEE, 2001, pp. 980–983.