Multi-channel Communications Fall 2022

Lecture 19
The Performance of OFDM with Timing/Frequency Error & Multipath

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Introduction

- Impact of Synchronization Errors
  - Timing
  - Frequency
  - ICI
- Static Multipath
  - Zero-padding
  - Cyclic Prefix
- Time-Varying Multipath
  - Introduction of ICI
  - Receiver Structures
    - Matched Filter
    - Zero-Forcing
    - MMSE
    - MMSE/ZF - SIC

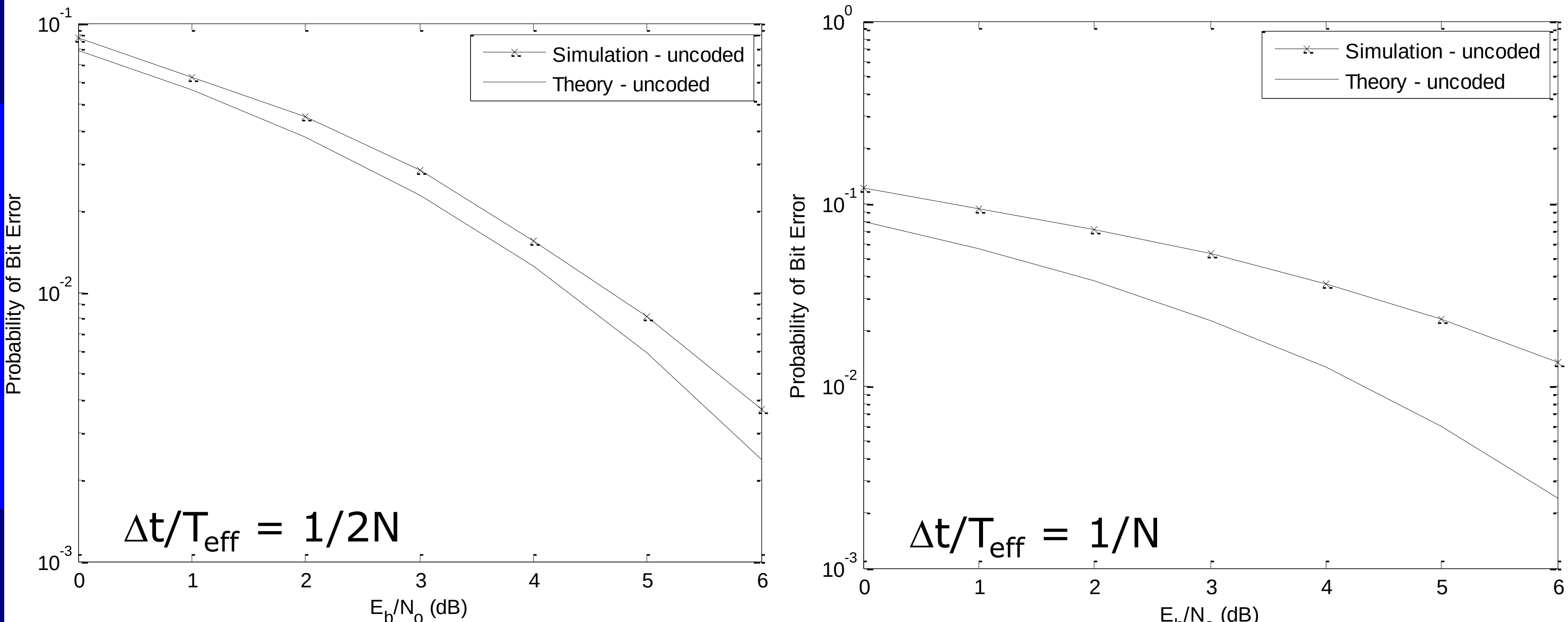
Performance of OFDM

- AWGN channel
  - OFDM has the same BER performance as single-carrier approaches
- Flat Rayleigh fading
  - OFDM has the same performance as single-carrier approaches
- Frequency selective Rayleigh fading
  - OFDM has an advantage over single-carrier, especially in coded systems
- These statements assume
  - Perfect synchronization
  - Delay spread < T_{cp}
  - Channel is fixed over OFDM symbol period

Impact of Timing Error

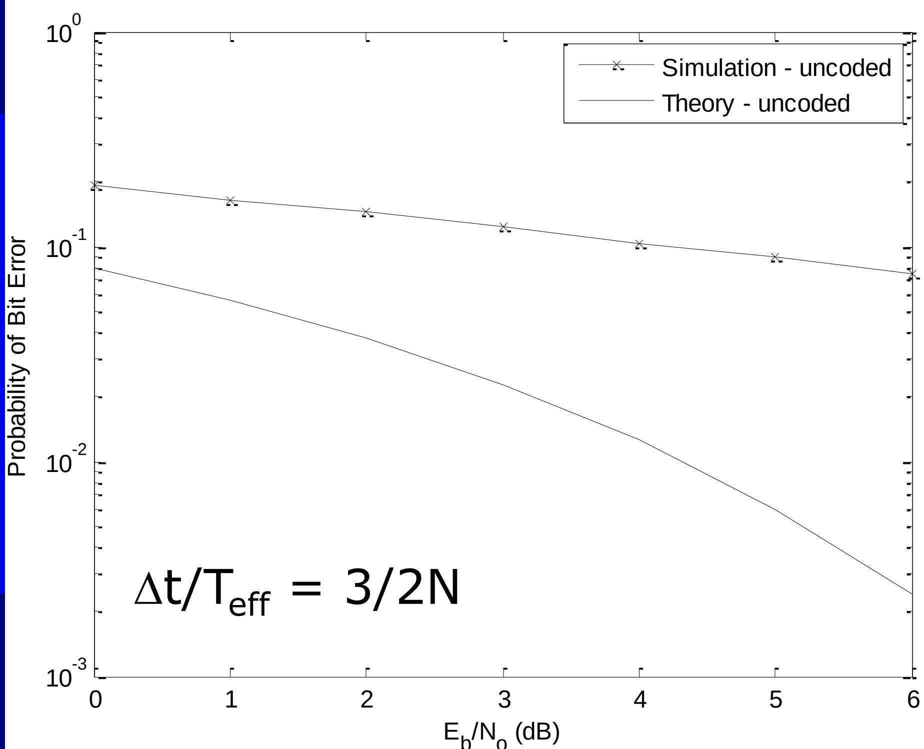
- Go to the board…

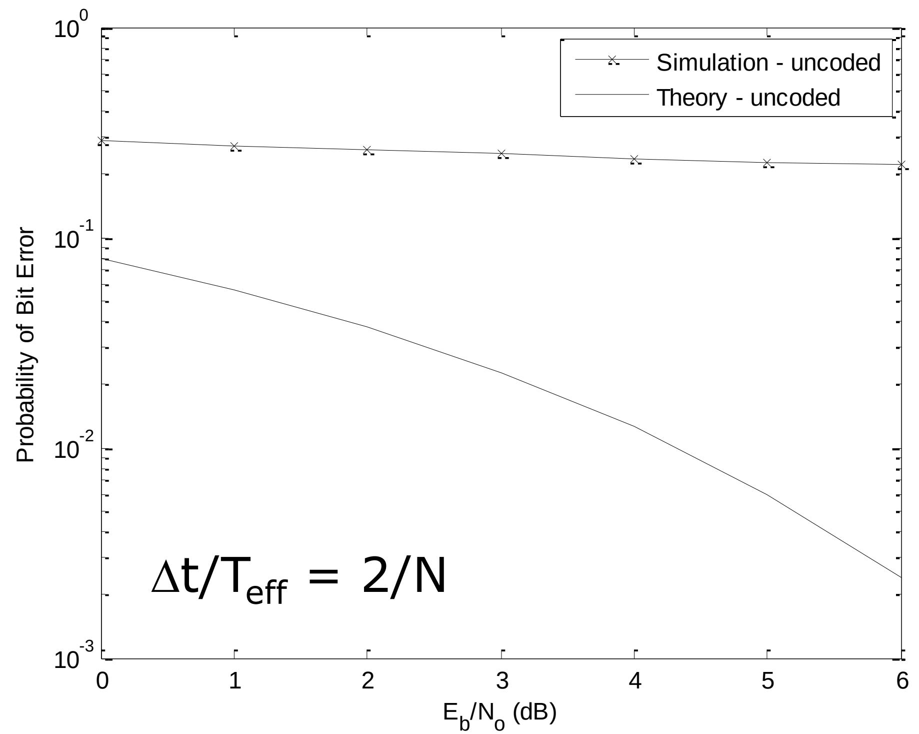
Performance- Timing Error



- AWGN, N=64, No ISI
- **Uncompensated** Phase rotation (with pilots, this phase rotation would be assumed to be part of the channel and removed)

Performance- Timing Error





- AWGN, N = 64, No ISI
- **Uncompensated** Phase rotation (with pilots, this phase rotation would be assumed to be part of the channel and removed)

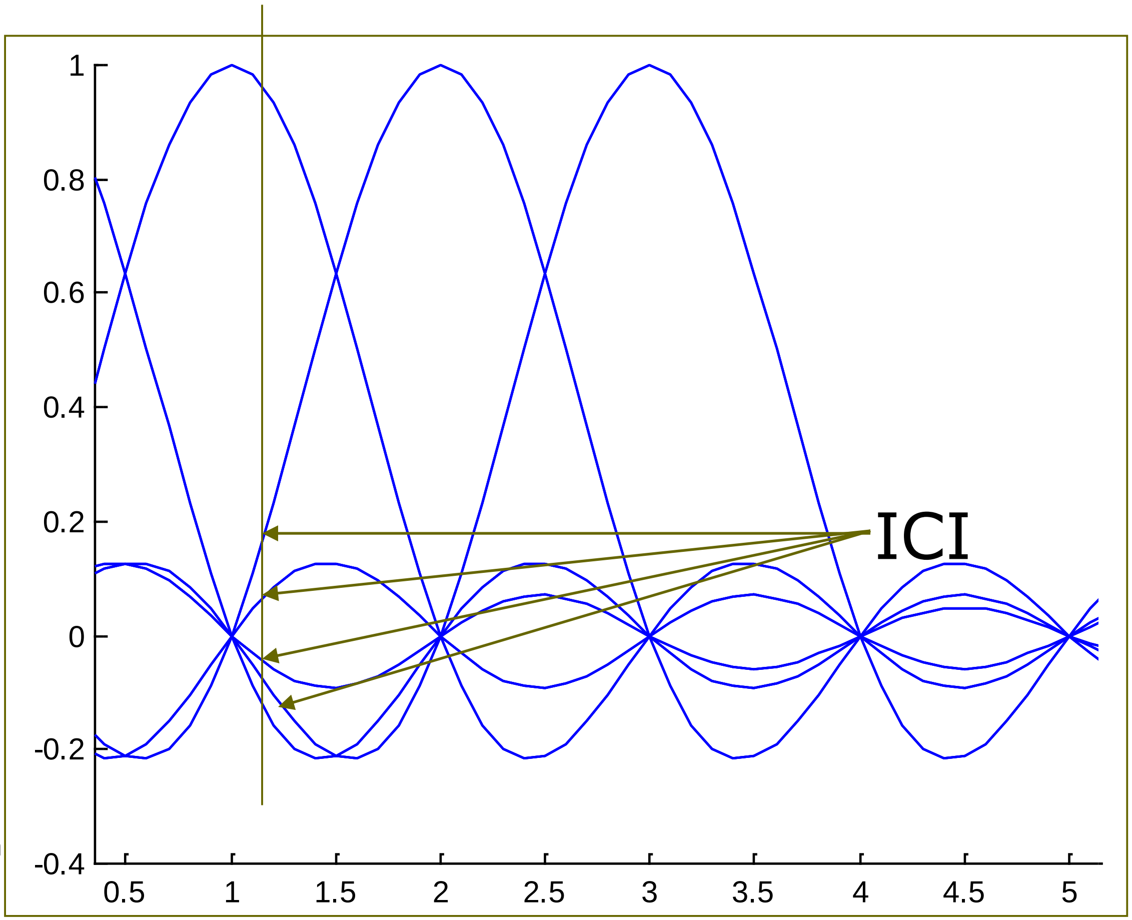
Impact of Frequency Shift

- Go to the board…

Impact of Frequency Shift

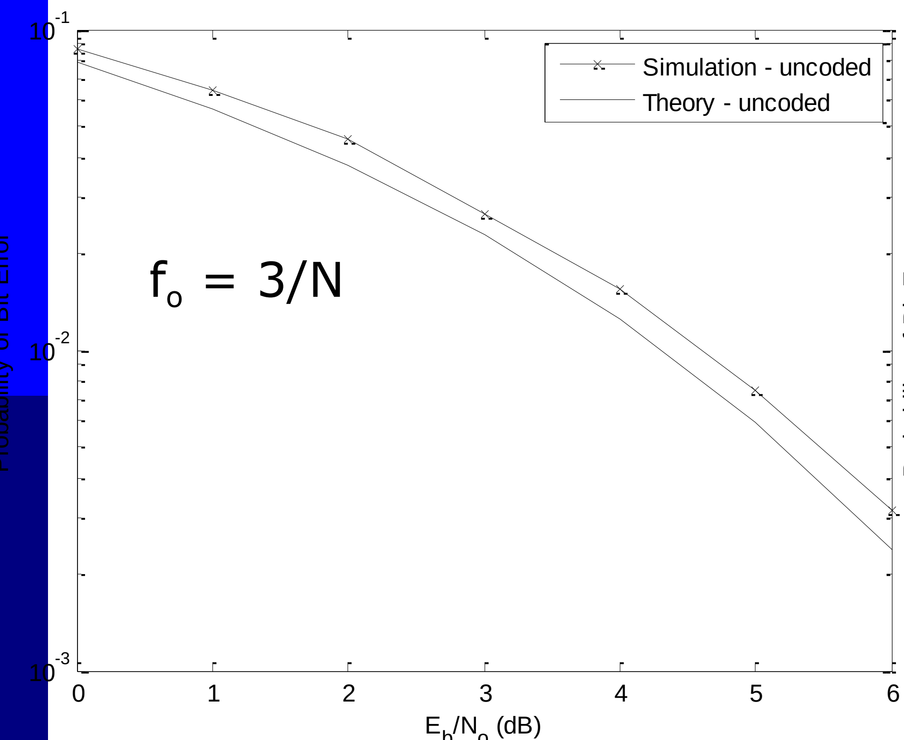
- Frequency offset causes sampling away from peak

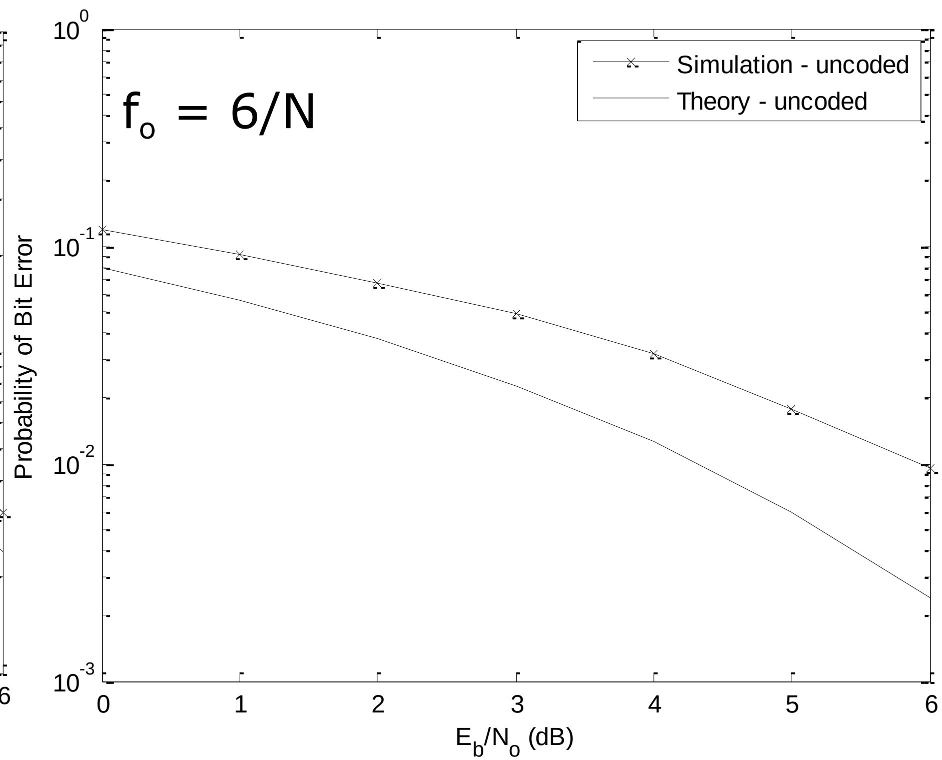
Sampling point in frequency (FFT locations)



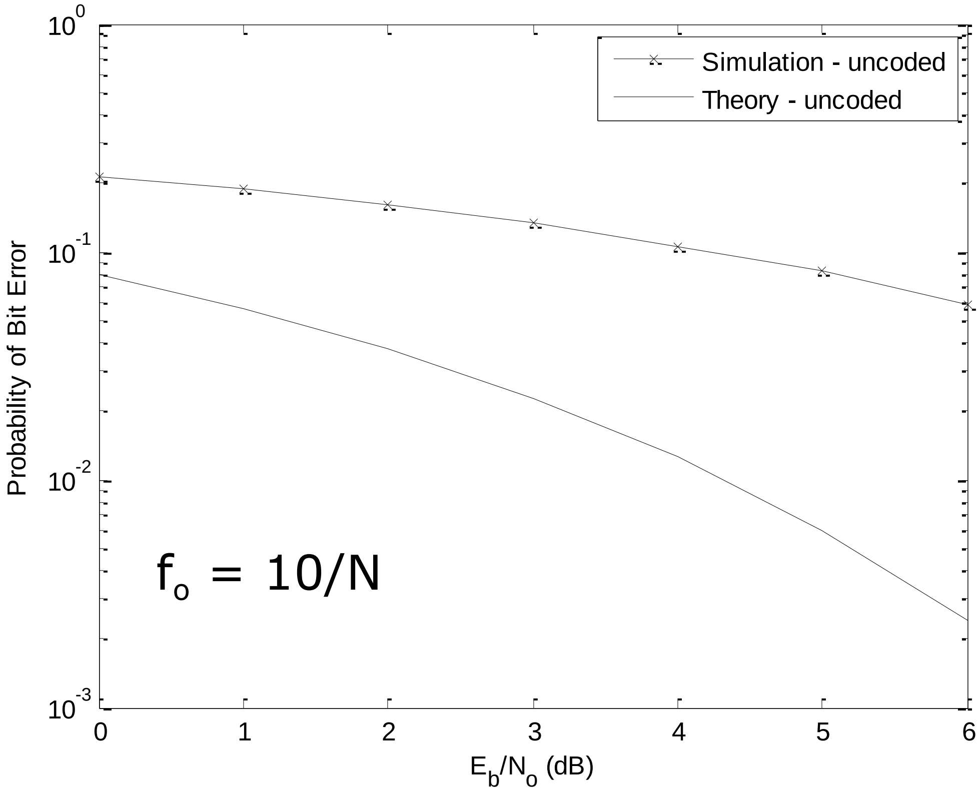
Example Performance

- AWGN
- $N = 64$
- No Compensation





Example Performance

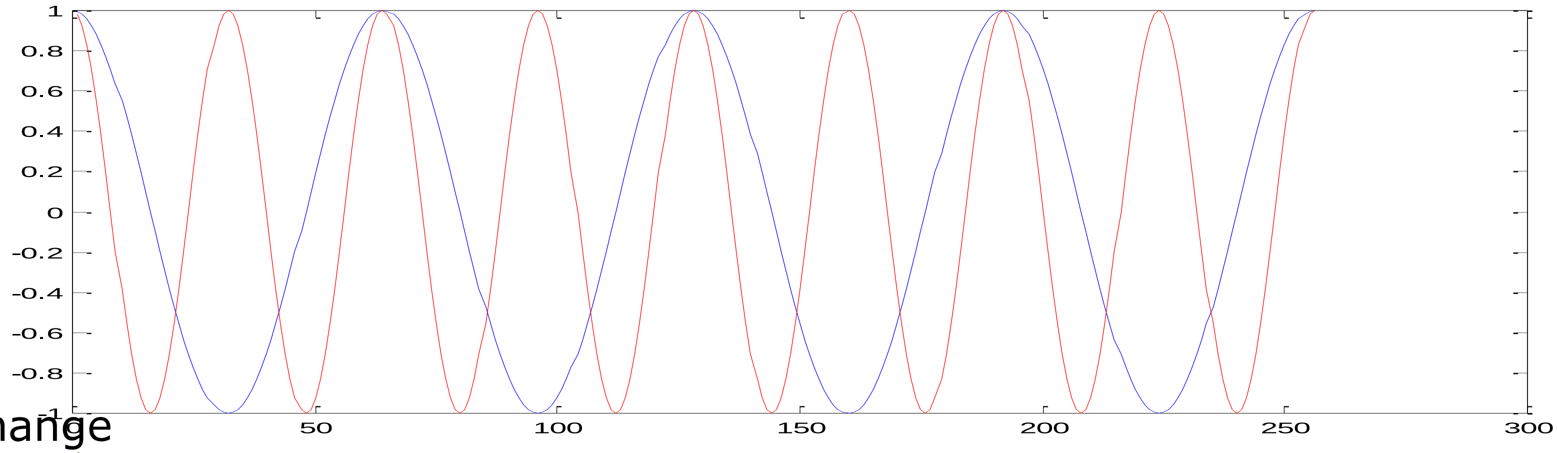


- AWGN
- N = 64
- No compensation

Multipath

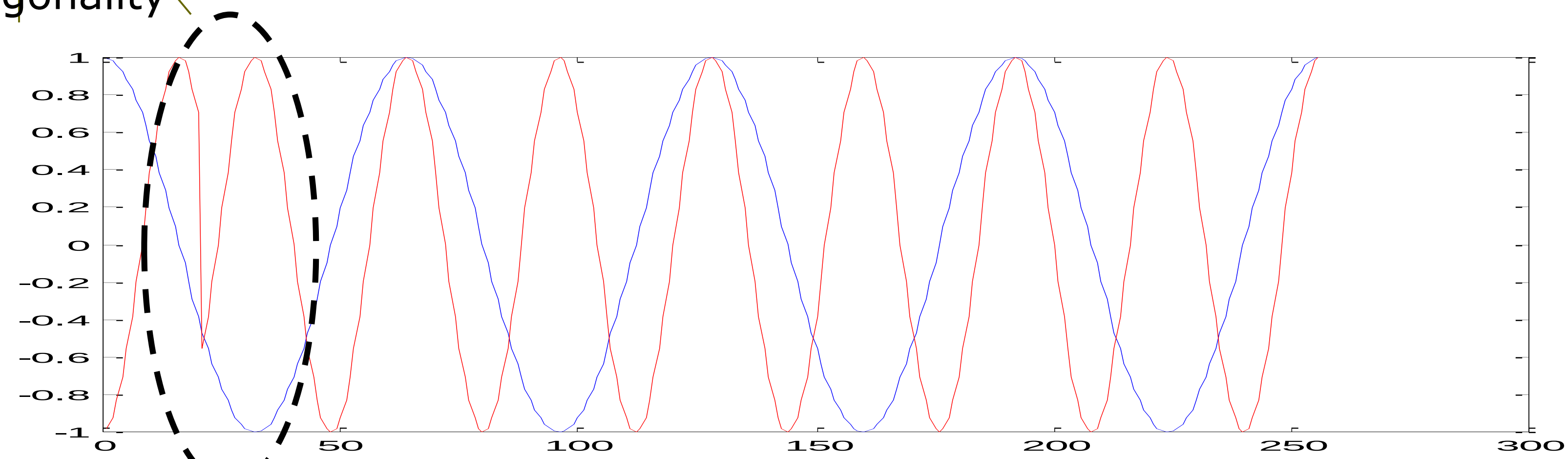
- When two versions of the signal (i.e., multipath) arrive at the receiver with different delays, the two paths will not be orthogonal.
- Symbol changes in consecutive intervals will destroy the orthogonality between subcarriers
- This can be remedied by extending the length of the symbol using a cyclic prefix
- The cyclic prefix is disregarded at the receiver maintaining orthogonality at the expense of slight time inefficiency

Impact of delay on orthogonality



Sign change destroys orthogonality

Subcarriers 4 and 8 out of 256 with no delay difference

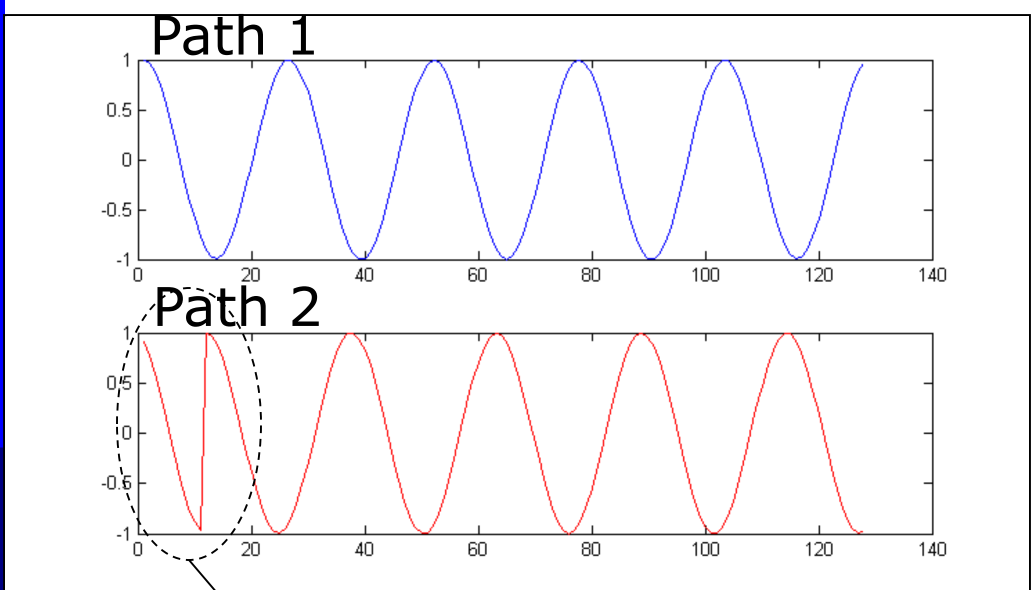


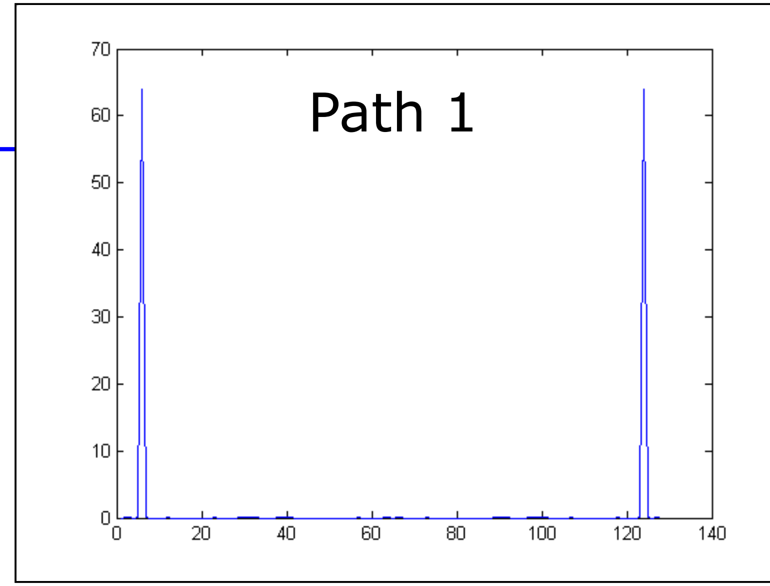
Subcarriers 4 and 8 out of 256 with 20 sample delay difference

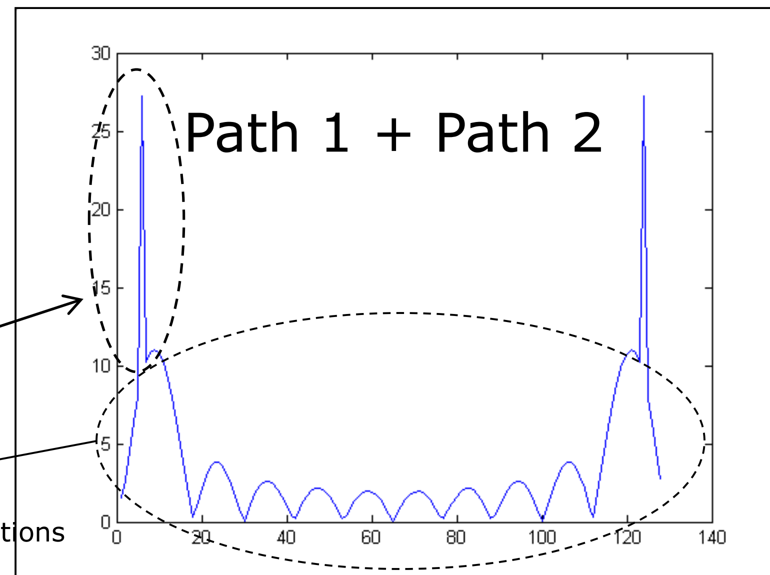
Frequency Domain

Multipath

- Time Domain





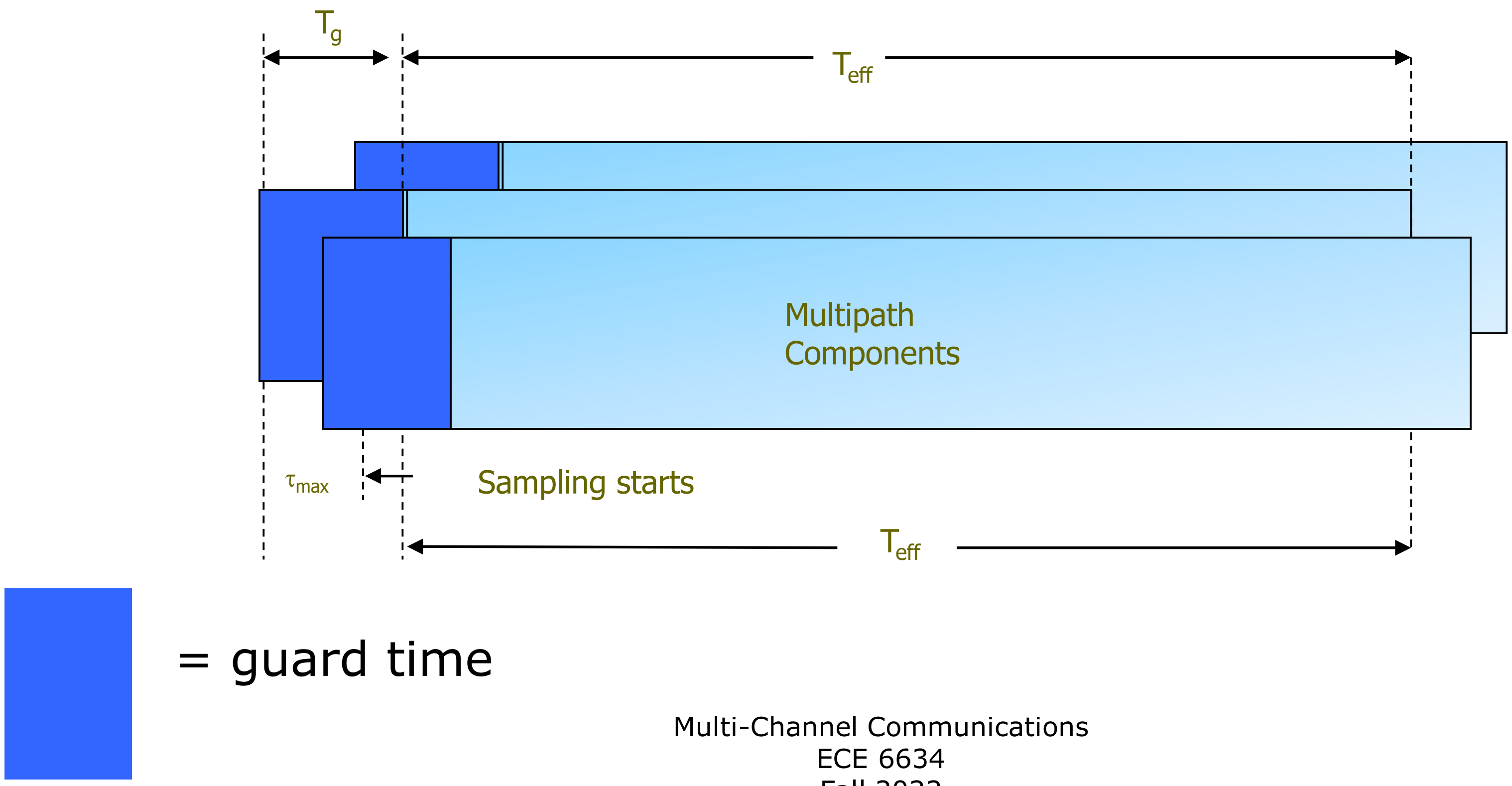


ISI – Loss in desired energy

ICI – Leakage in other subcarriers

Guard Time – Zero Padding

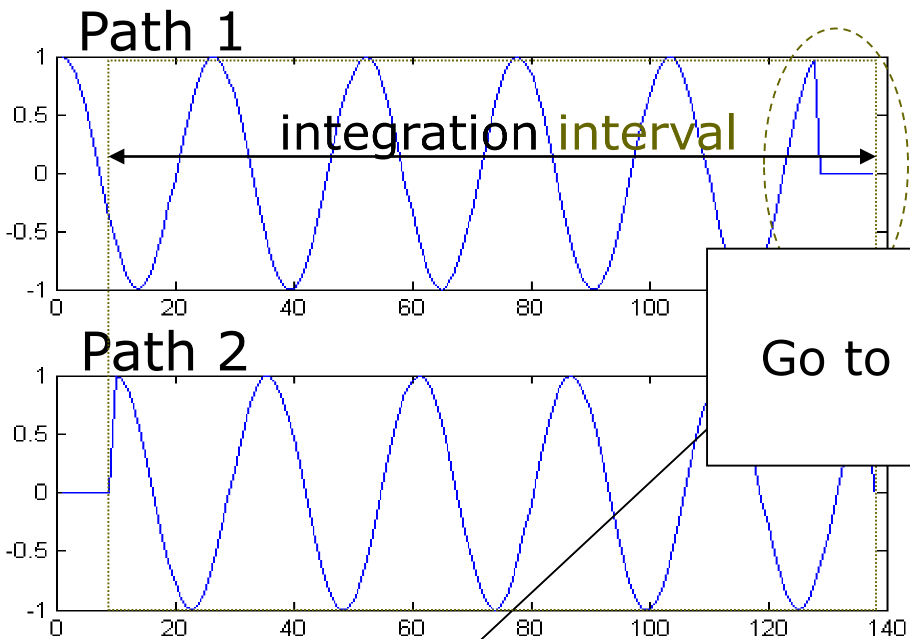
- One solution is to simply add guard time between symbols by zero-padding

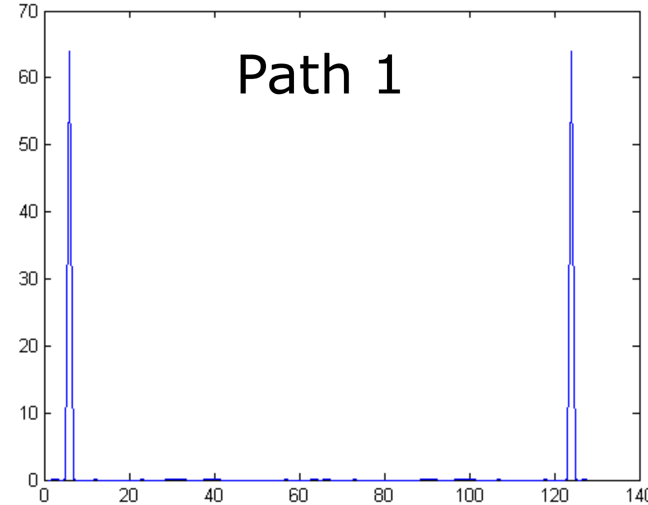


Frequency Domain

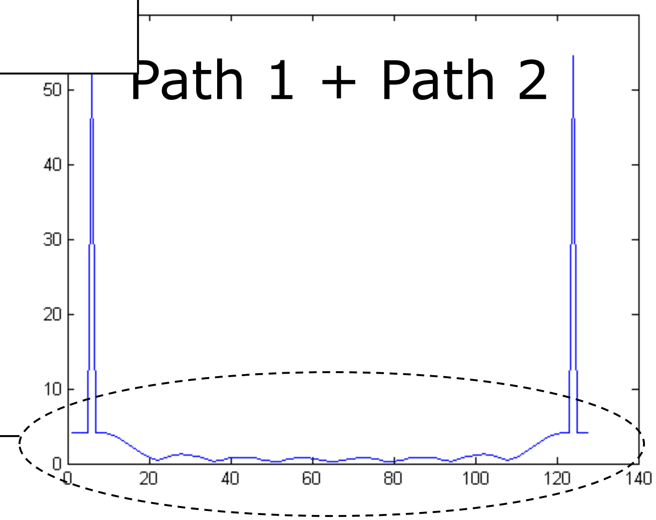
Multipath

- Time Domain





Go to the Board



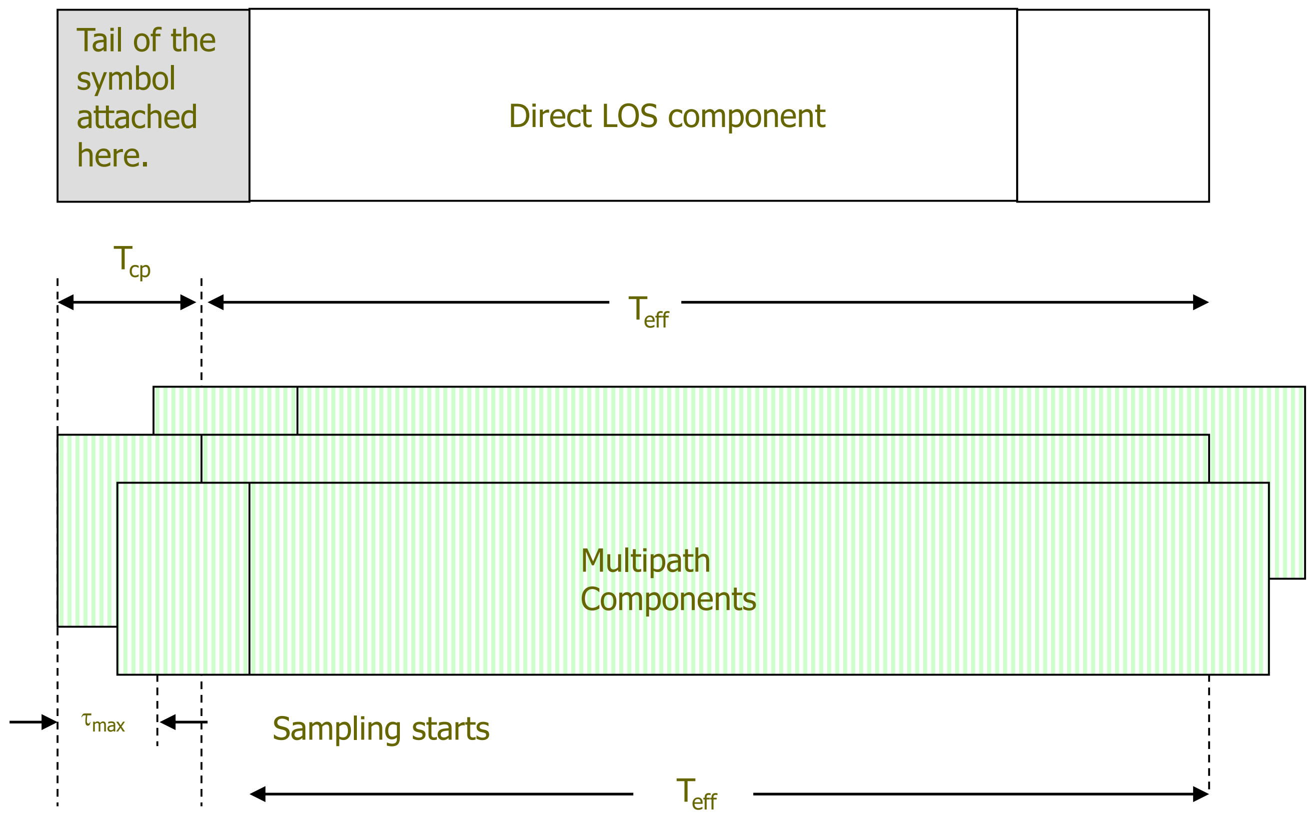
No ISI However, orthogonality still lost

Cyclic Prefix in OFDM

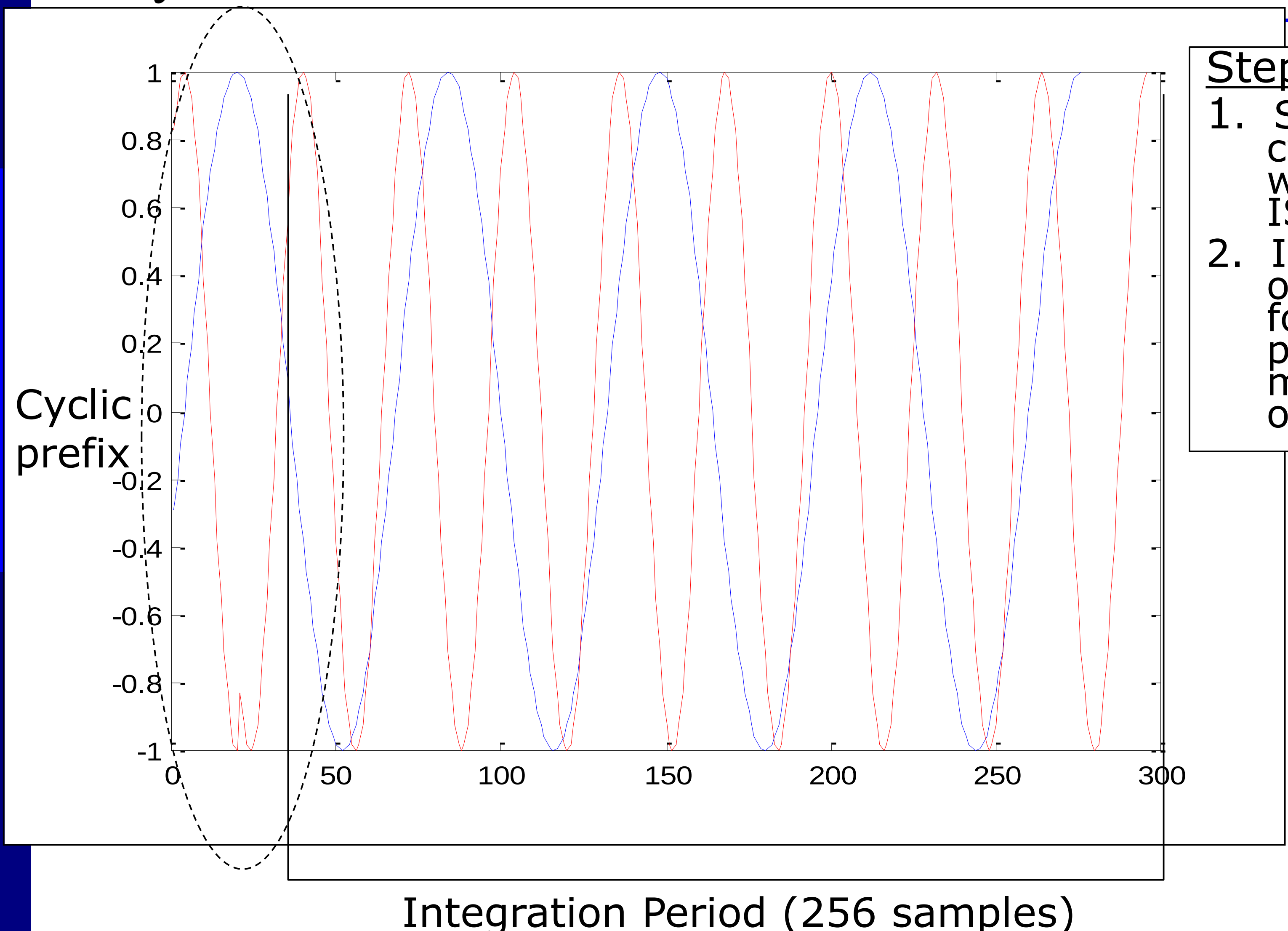
- By dividing the input data stream into *N* subcarriers, the symbol duration is made *N* times larger, which reduces the multipath delay spread relative to the symbol time, by the same factor.
- However, ISI can still occur if delay spread is large.
- To remove ISI completely, a guard time is inserted in each OFDM symbol. This guard time is always chosen to be larger than the maximum delay spread due to the channel.
- The OFDM symbol is cyclically extended in the guard time. This ensures that the delayed replicas always have an integer number of cycles during the FFT interval
- The guard time T_g should be always greater than the worst case delay spread (τ_{max}) of the channel.

Cyclic Prefix – Contd.

During the FFT interval, the OFDM receiver sees a sum of pure sine waves which does not destroy the orthogonality between the subcarriers.



Cyclic Prefix

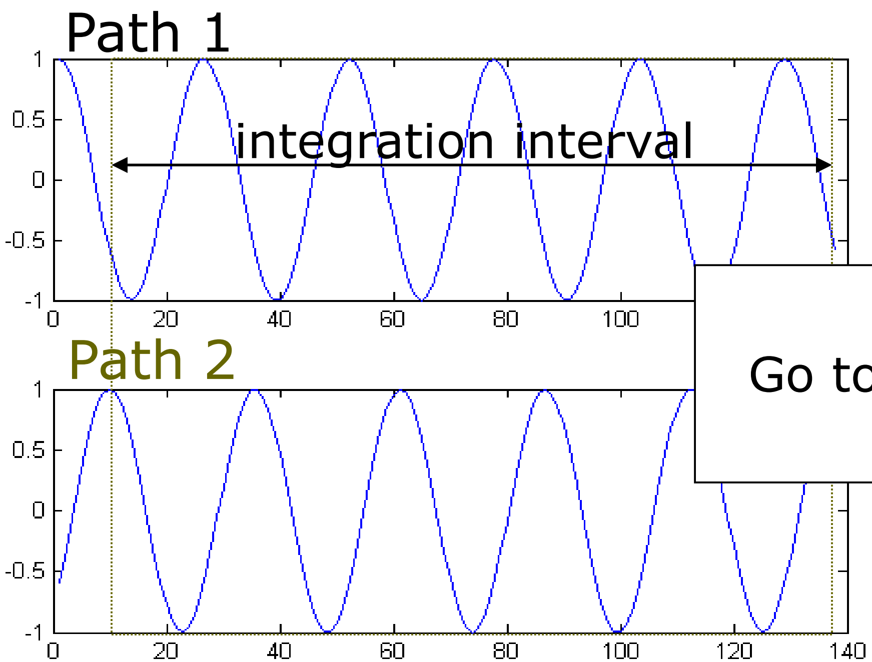


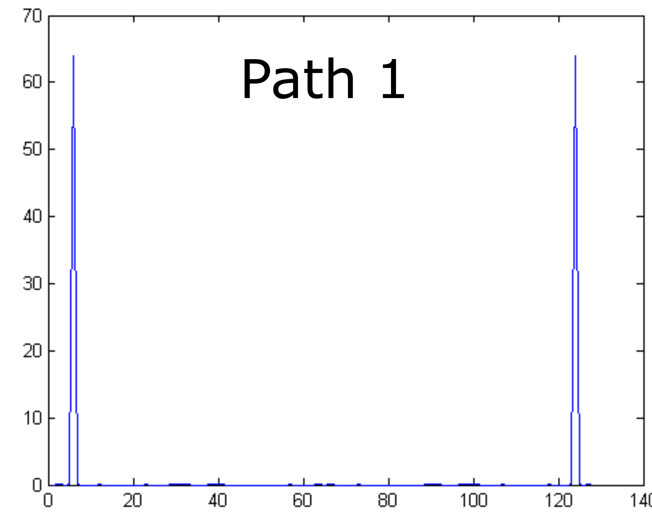
<u>Steps</u>

1. Strip off the cyclic prefix which contains ISI
2. Integrate over one period following cyclic prefix to maintain orthogonality

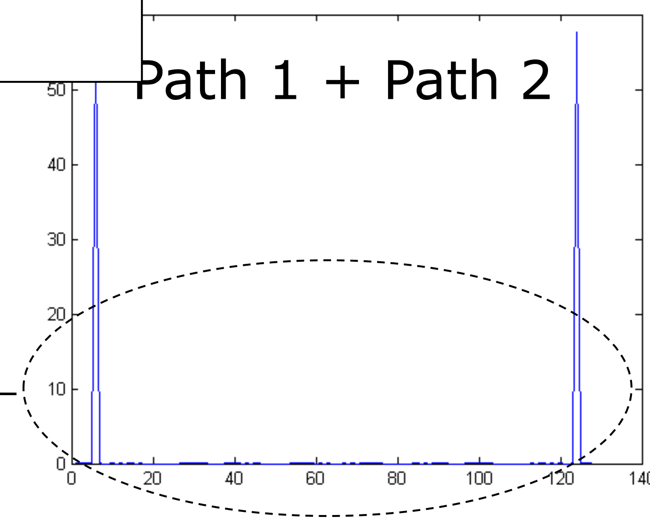
Multipath

- Time Domain





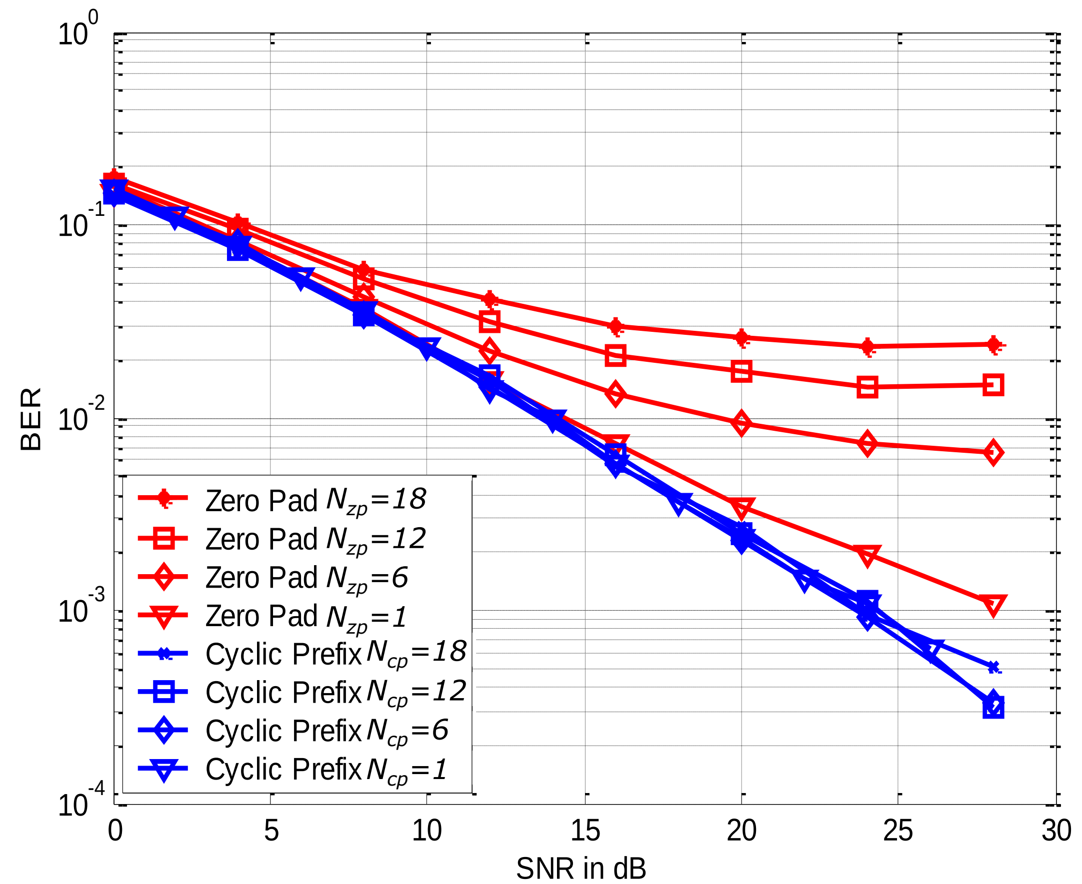
Go to the Board



No ISI and no loss of orthogonality

Benefit of Cyclic Prefix

- OFDM makes a frequency selective channel a flat fading channel on each subcarrier
- Insertion of Cyclic Prefix eliminates the inter-carrier interference



Degradation due to CP

- In order to eliminate effects of multipath we must transmit

$$\frac{T_{cp}}{T_{eff}}$$

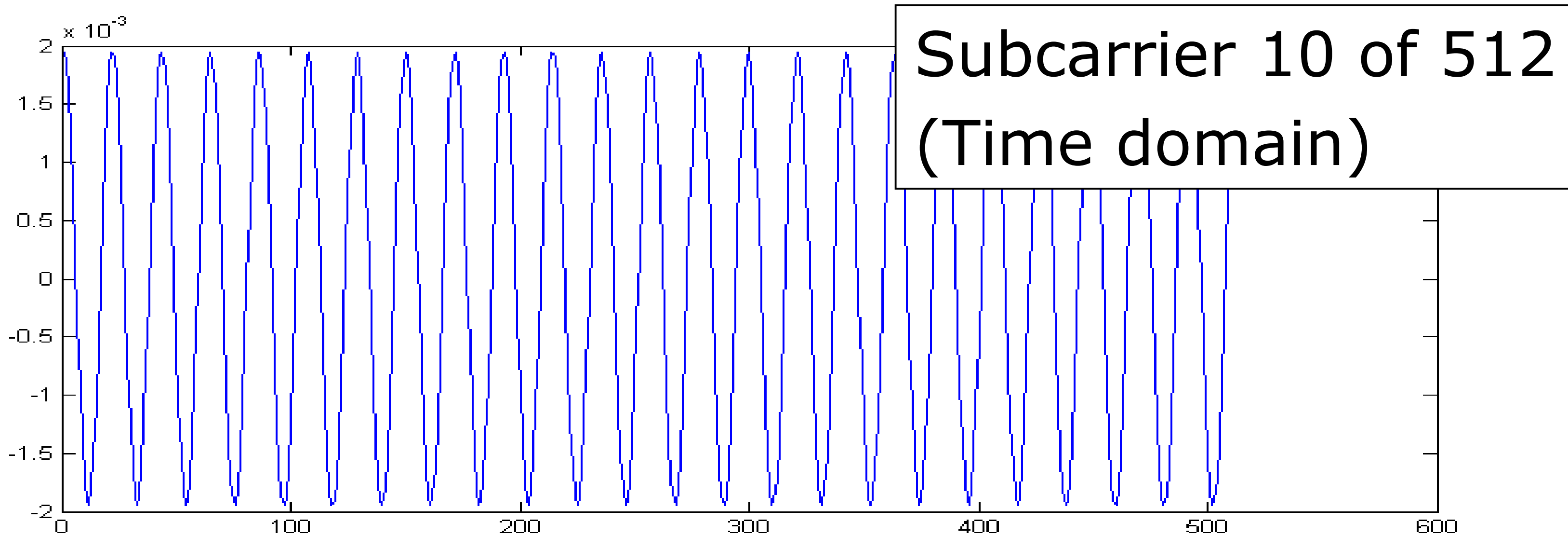
more energy than necessary

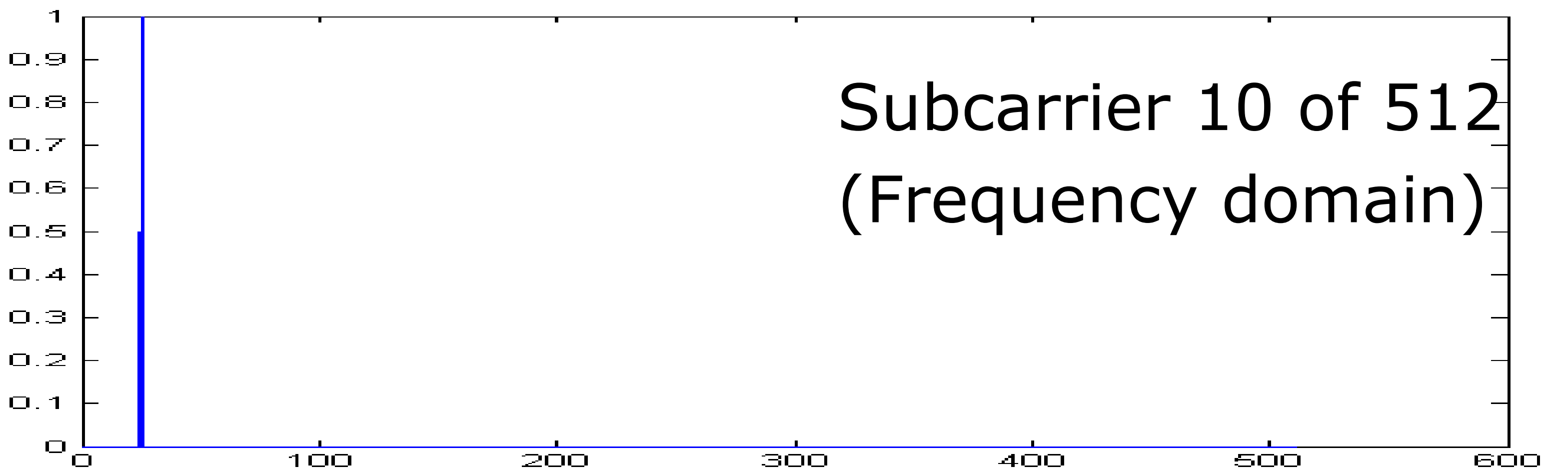
- This can be thought of as a loss in energy by same amount
  - 20% increase ($T_{cp} = T_{eff}/5$) results in less than 1dB loss
  - Small price to pay compared to benefit

Time-Varying Multipath (Doppler Spread)

- The subcarriers only remain orthogonal provided that the channel is constant over the duration of the OFDM symbol
- This becomes more unlikely since the OFDM symbol is N times longer than the original symbol
- Doppler effects cause the signal to change in time and possibly over a symbol duration causing inter-carrier interference (symbols are no longer orthogonal)
- In the frequency domain we can envision a frequency shift or frequency smearing

Impact of Doppler Spread





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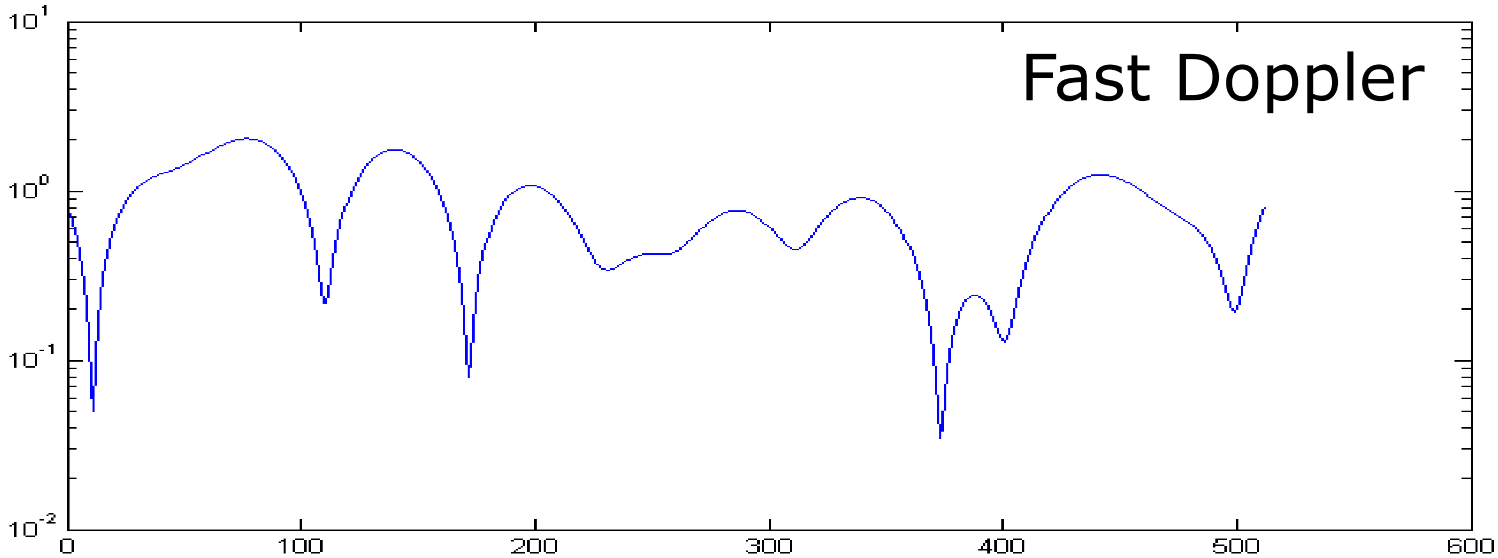

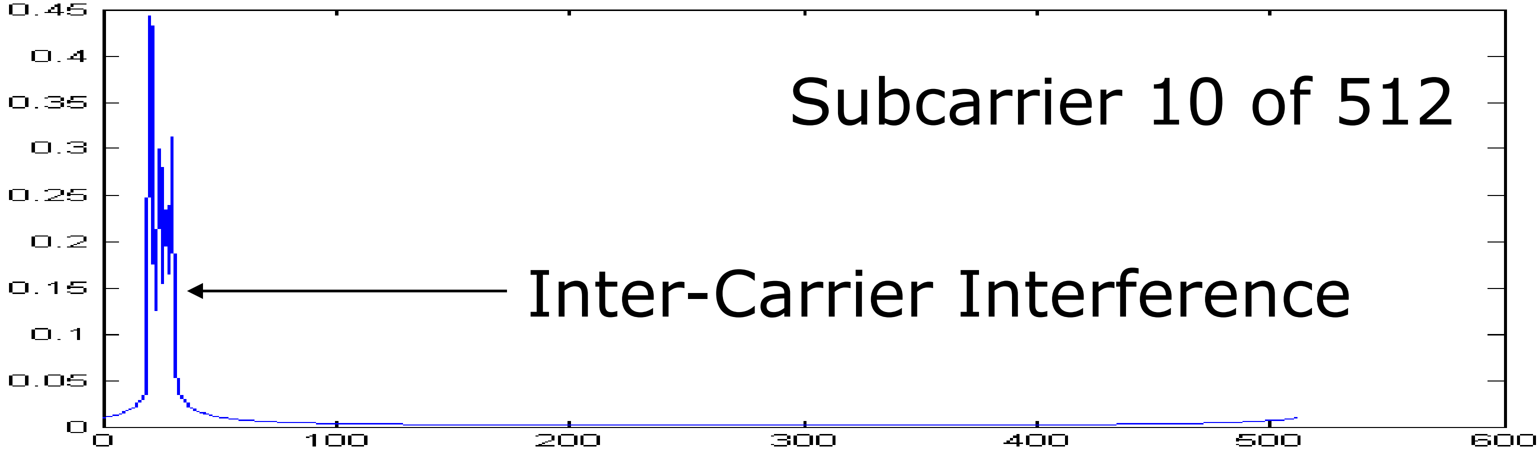

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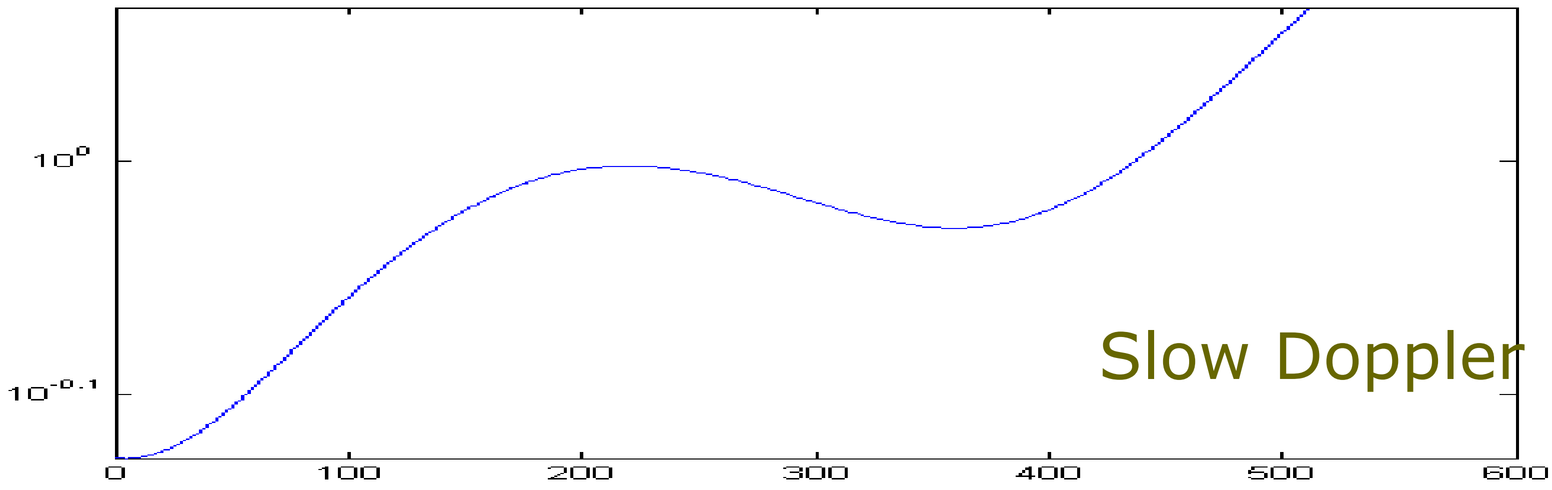
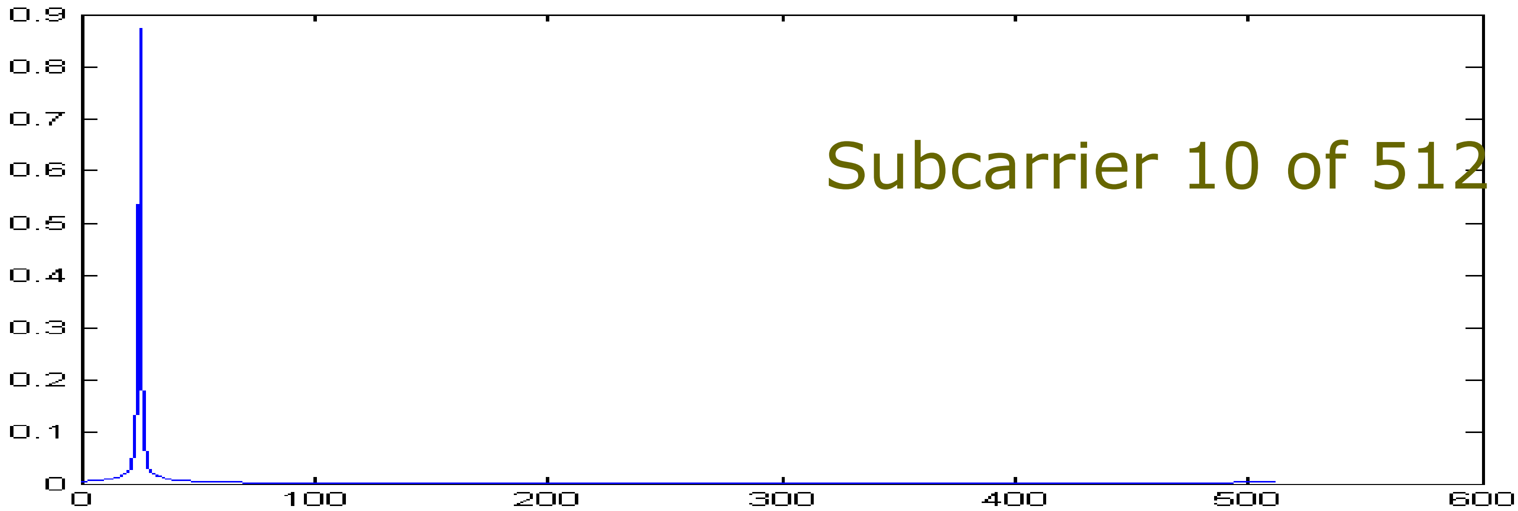
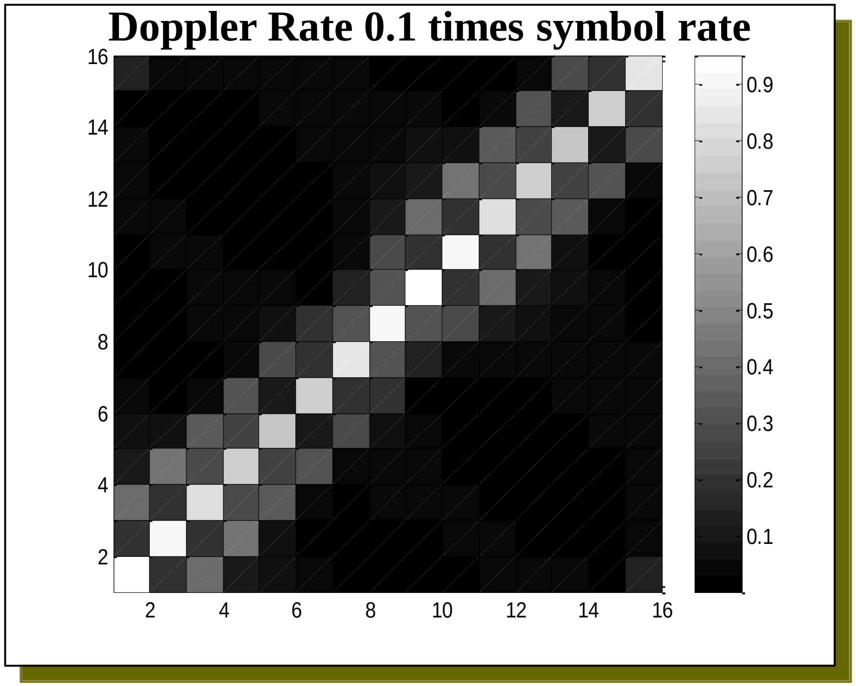
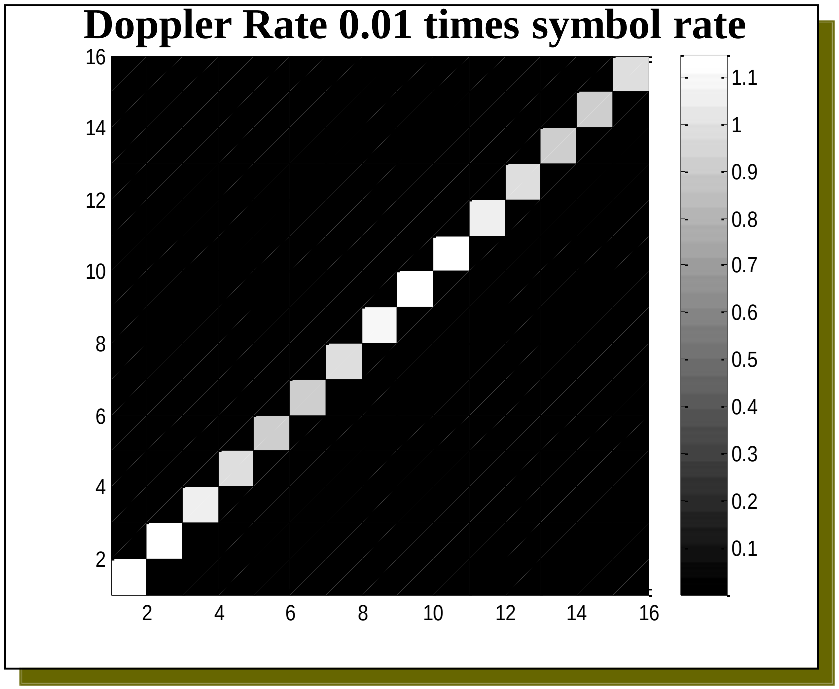




The effect of Doppler

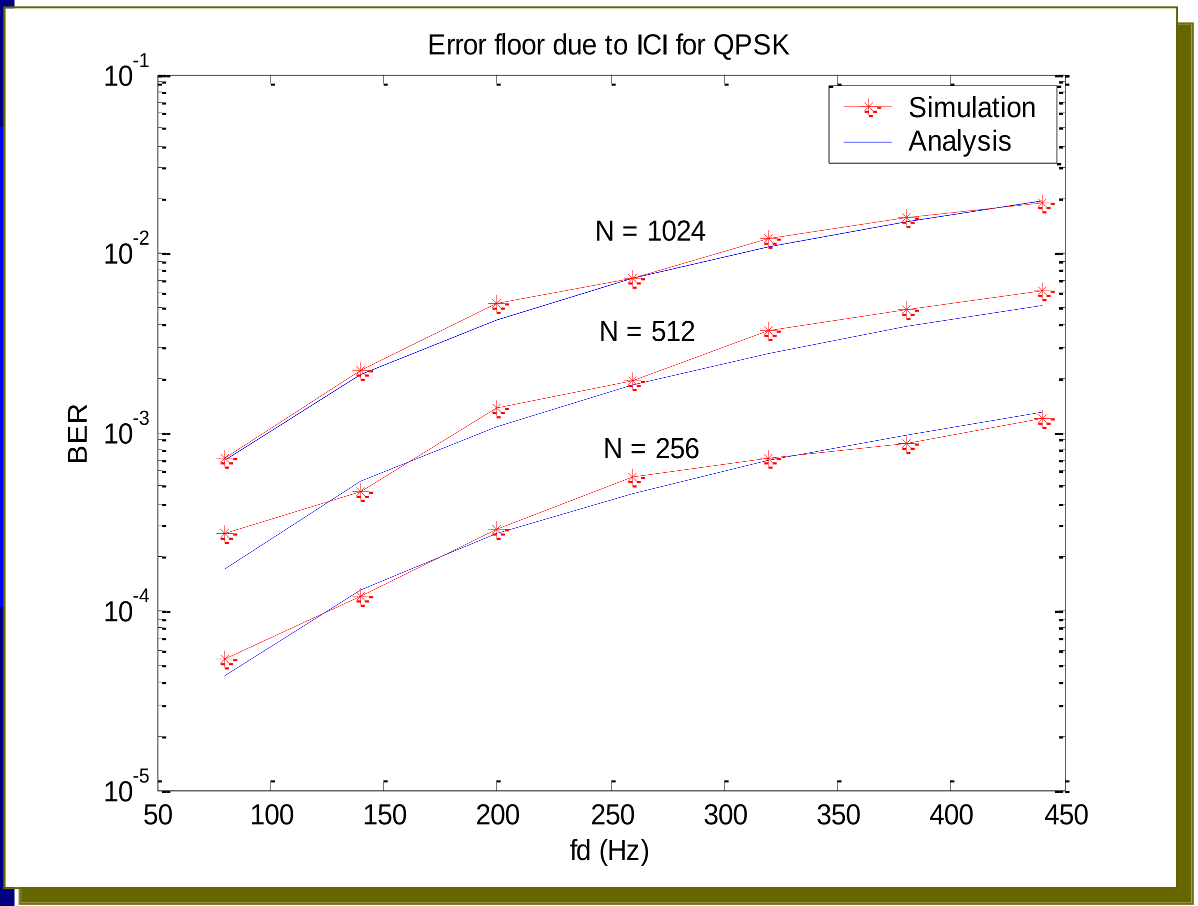
- Correlation between channel outputs





- Ideally the correlation matrix is a diagonal matrix
- At a Doppler rate of 0.01 times the symbol rate this is nearly accomplished
- At a Doppler rate of 0.1 times the symbol rate, significant inter-carrier interference occurs

Effect of Doppler-induced ICI



- Time varying channels disrupt the orthogonality between subcarriers (spread in frequency) resulting in Inter-Carrier Interference (ICI)
- Note: BW is fixed
- ICI increases with the number of subcarriers
- ICI can be reduced by proper detection technique like MMSE instead of a simple FFT

Detection Techniques

FFT Detection

$$\begin{aligned} \eta &= \mathbf{D}_N \tilde{\mathbf{r}} \\ &= \Gamma \mathbf{s} + \nu \\ \mathbf{z} &= \Gamma^H \eta \end{aligned}$$

LS Detection

$$\mathbf{z} = (\Gamma^H \Gamma)^{-1} \Gamma^H \eta$$

$$\ldots \mathbf{s} + (\Gamma^H \Gamma)^{-1} \Gamma^H \nu$$

Go to the Board

MMSE Detection

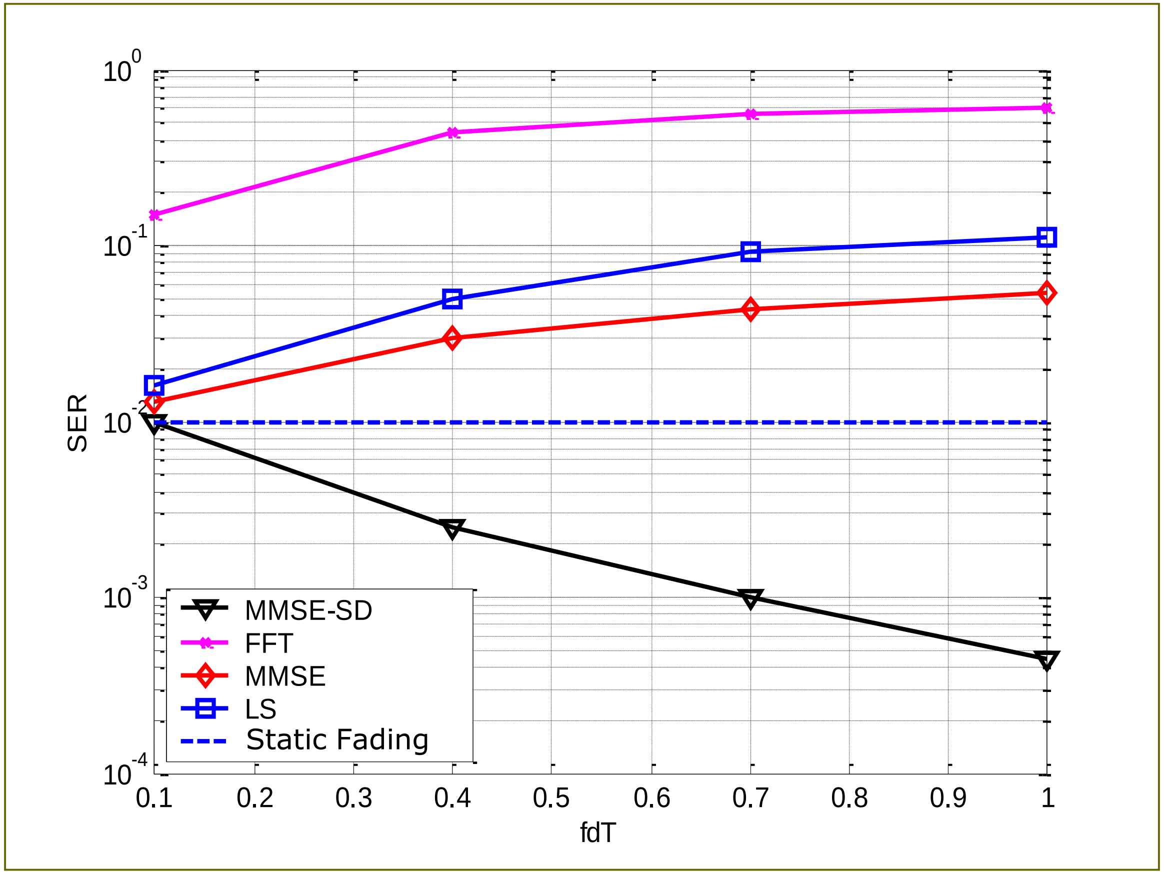
$$\mathbf{z} = (\ldots$$

$$= \left(\Gamma^H \Gamma + \frac{1}{SNR} \mathbf{I}_N \right)^{-1} \Gamma^H \Gamma \mathbf{s} + \left(\Gamma^H \Gamma + \frac{1}{SNR} \mathbf{I}_N \right)^{-1} \Gamma^H \nu$$

MMSE with Successive Detection

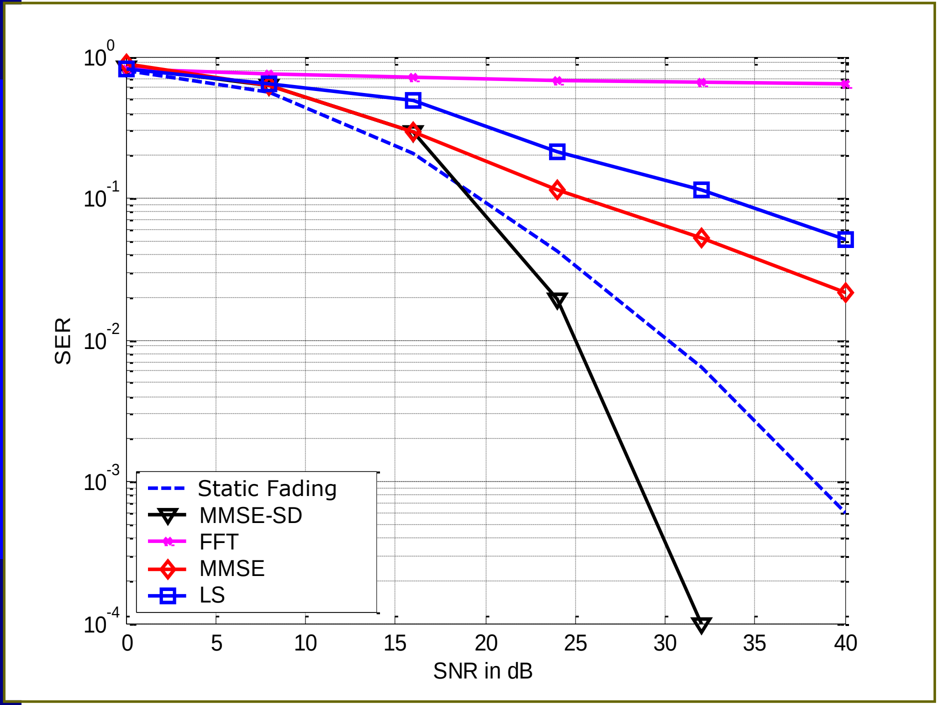
Detection techniques vs. Doppler Spread

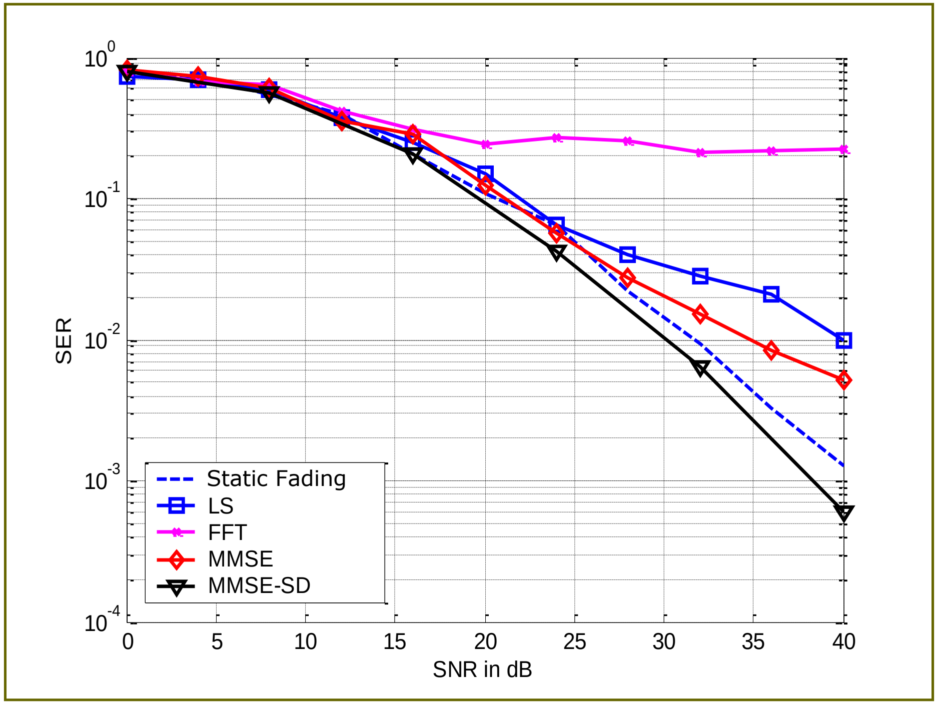
- Performance of FFT detection the worst – Matched filter
- MMSE and LS performance is better than simple FFT detection
- MMSE with SD (SIC) performs the best with the performance improving with increase in normalized Doppler. SD exploits time diversity provided by the channel.



Performance using 16PSK modulation for varying $f_d T_o$. SNR= 30dB

Detection techniques- contd.





Performance of different detection techniques_ $f_dT_o = 0.1$, N = 1024

Performance of different detection techniques_ $f_dT_o = 0.01$, N = 1024

Conclusions

- In this lecture we have examined the performance of OFDM
  - With timing error
  - With frequency error
  - In the presence of multipath
- These results show the limitations of OFDM as it requires good frequency synchronization and relatively low Doppler spreads (unless more sophisticated receivers are used)