Multi-channel Communications Fall 2022

Lecture 13
Spatial Multiplexing – No Transmit Channel Info

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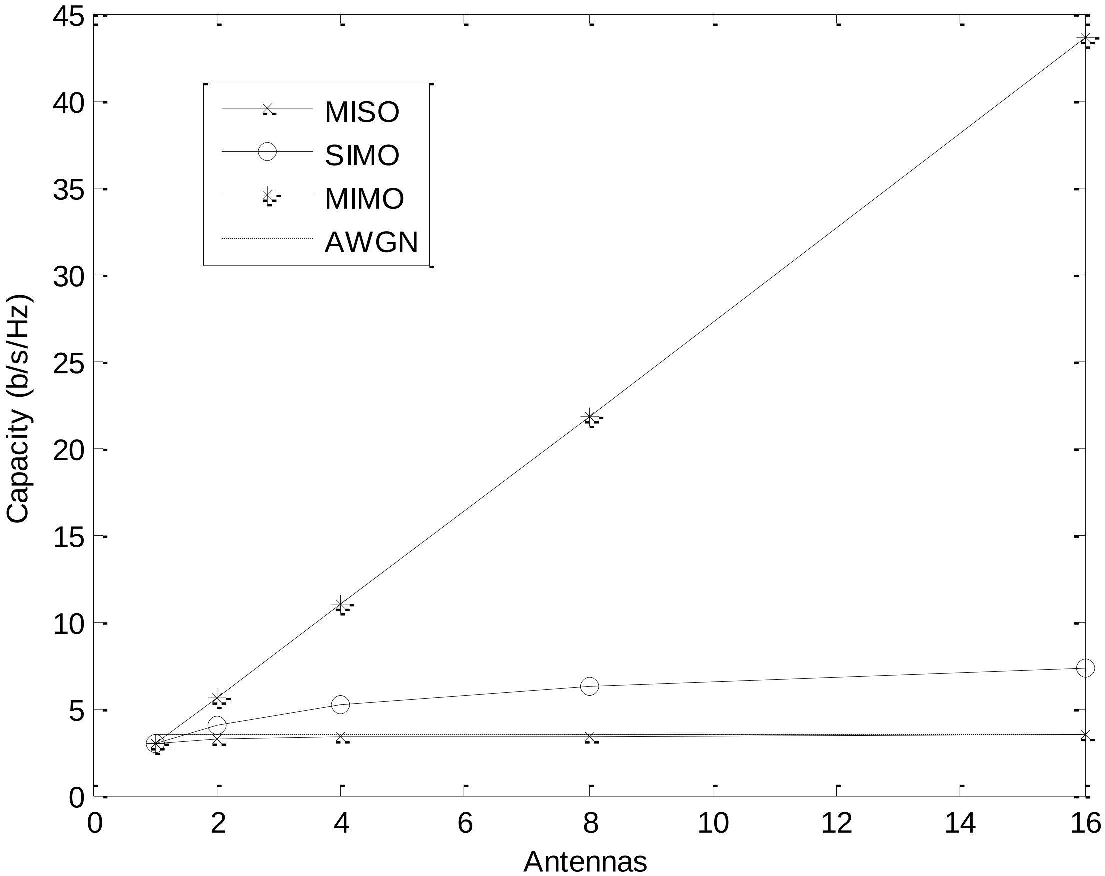
Introduction

- Multiple antennas are used for many purposes including
  - Mitigating fading
  - Mitigating interference
  - Increasing SNR
- Today we will show that multiple antennas can also be used to increase the capacity of a channel through spatial multiplexing
- Spatial multiplexing can be divided into three general areas
  - **Without channel knowledge at the transmitter**
  - With partial channel knowledge at the transmitter
  - With full channel knowledge at the transmitter

MIMO Ergodic Capacity



- MISO converges to AWGN
- SIMO achieves logarithmic increase with number of antennas
- MIMO achieves linear increase with number of antennas

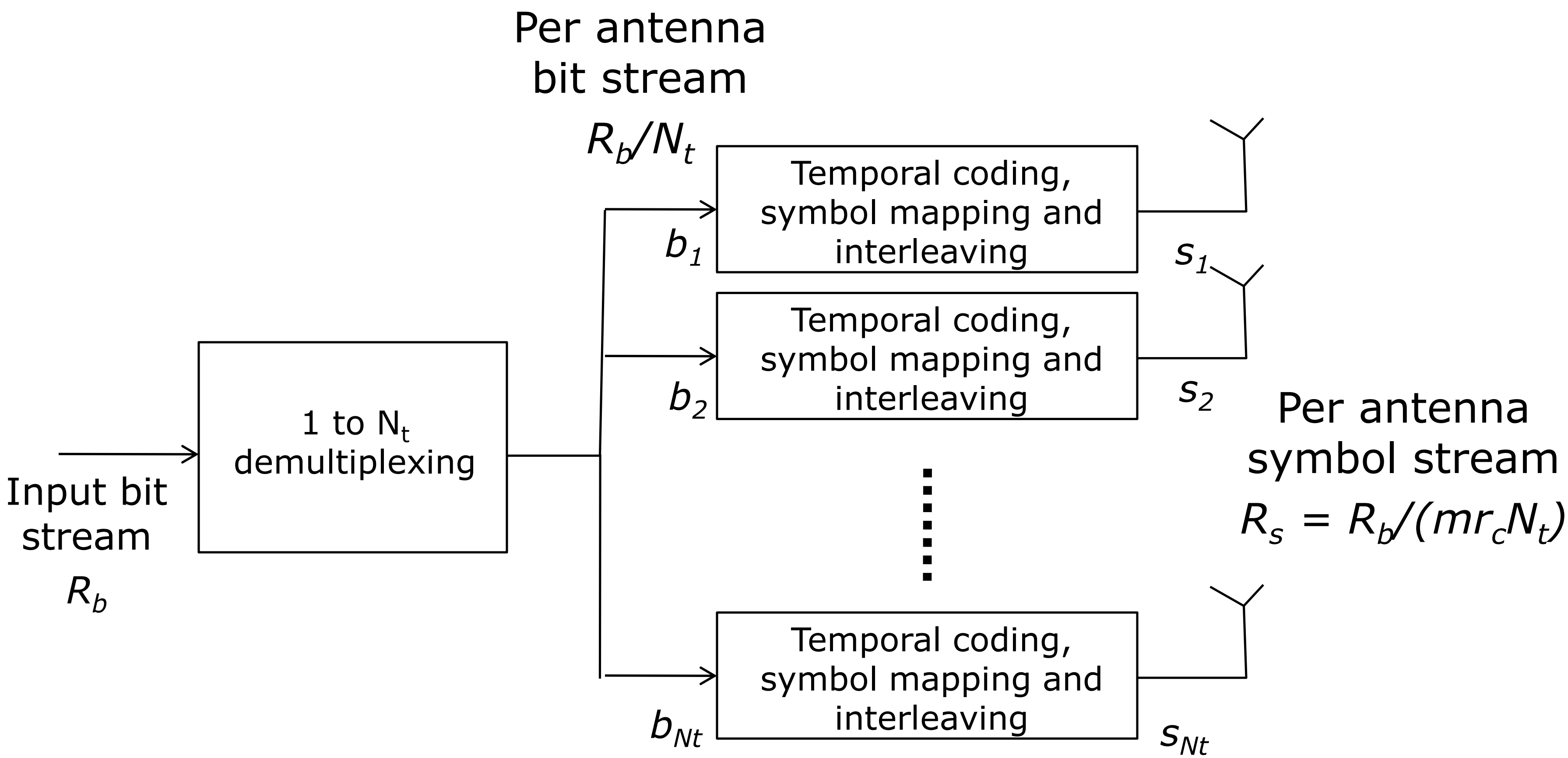
Spatial Multiplexing

- Spatial multiplexing refers to the use of multiple antennas to increase the data rate by transmitting different data streams on each antenna
  - Ideally this produces a linear increase in capacity with the number of antennas
- For STBC the spatial *rate* is less than or equal to unity whereas the spatial *rate* here is between unity and N_t
- There are three basic encoding approaches
  - Horizontal encoding
  - Vertical encoding
  - Diagonal encoding

Horizontal Encoding

- With *horizontal* encoding the information is multiplexed onto parallel antennas and each antenna is encoded and transmitted independently
- The advantage is that the receiver complexity is less than other encoding techniques
  - Data streams can be decoded independently
- The disadvantage is that the data does not go out over multiple antennas and thus does not gain full diversity advantage

Horizontal Encoding



r_c - *temporal code rate*

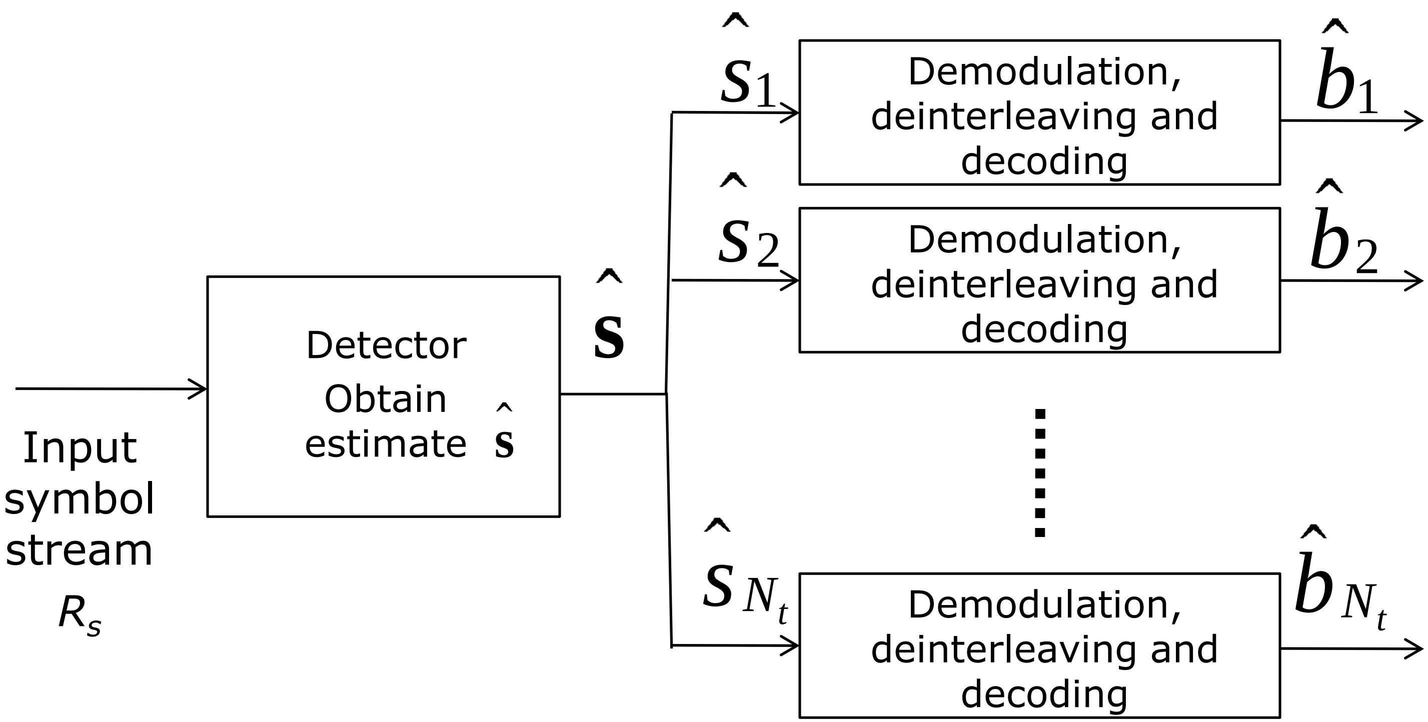
m – bits per modulation symbol

Receiver Structures

- With horizontal encoding per-stream detection and decoding is possible and near optimal
- In general we separate modulation symbol detection and FEC decoding
- Detector structures
  - Maximum Likelihood
  - Zero-forcing
  - MMSE
  - SIC
  - SIC w/MMSE/ZF
  - Sphere decoding

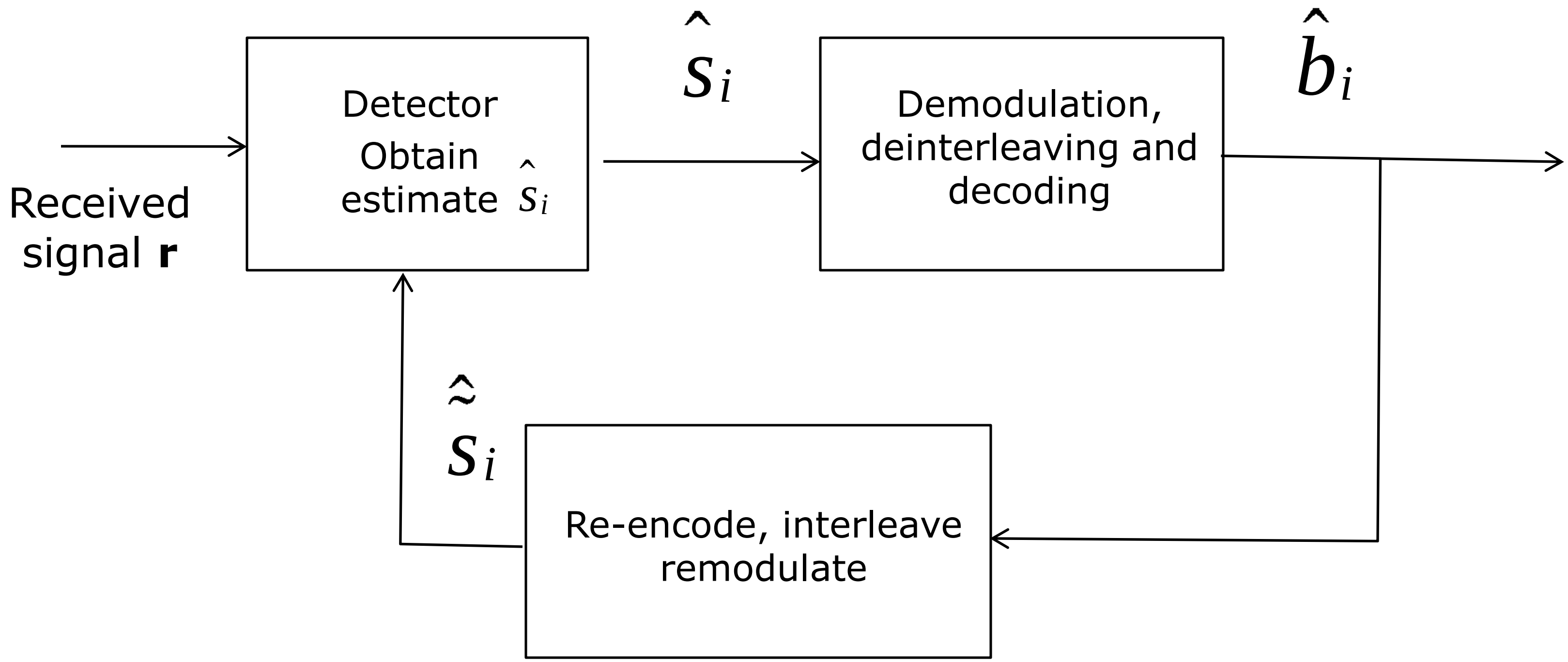
Detection and Decoding

- Detection and decoding can be done separately with HE
- For ML, ZF, MMSE detectors:



Detection and Decoding

- For SIC detectors:



Maximum Likelihood Detector

- Consider the received signal vector at an arbitrary time

$$\mathbf{r} = \sqrt{\frac{E_s}{N_t}}\mathbf{H}\mathbf{s} + \mathbf{n}$$

where

$$\mathbf{r} = M_r \times 1 \quad \text{received vector}$$
$$\mathbf{s} = N_t \times 1 \quad \text{transmitted vector}$$
$$\mathbf{H} = M_r \times N_t \quad \text{Channel Matrix}$$
$$\mathbf{n} = M_r \times 1 \quad \text{noise vector}$$

Maximum Likelihood

- For uncoded transmit symbols the maximum likelihood detector chooses the vector of symbols which is the closest in distance from the received signal

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}} \left\| \mathbf{r} - \sqrt{\frac{E_s}{N_t}} \mathbf{H}\mathbf{s} \right\|^2$$

- Requires knowledge of **H**
- For a modulation symbol alphabet of $M = 2^k$ requires a brute force search over M^{Nt} different possible symbol vectors

Maximum Likelihood – Diversity

- By transmitting different information (i.e., different streams) over each of the antennas with horizontal encoding no diversity is provided at the transmitter
- Since the same data is received on each receive antenna, M_r-fold diversity is achieved at the receiver
- Overall diversity gain is M_r
  - How does this compare to space-time block coding?

Zero-Forcing (ZF) Detector

- Decouples the received signal vectors into N_t parallel streams (i.e., creates a separate decision metric for each symbol stream)
  - Linear complexity as compared to exponential complexity for ML
- Each decision metric is used to make N_t independent symbol decisions or create soft-decision values for decoding
- Like all zero-forcing solutions this receiver can eliminate parallel stream interference, but at the cost of enhanced noise (i.e., reduced SNR)

Zero-forcing

- Decision metric

$$\boxed{\mathbf{T} = \mathbf{H}^{\dagger} = \left(\mathbf{H}^{H}\mathbf{H}\right)^{-1}\mathbf{H}^{H}}$$

$$\begin{aligned}
\mathbf{z} &= \mathbf{T}\mathbf{r} \\
&= \sqrt{\frac{N_t}{E_s}}\mathbf{H}^{\dagger}\mathbf{r} \\
&= \sqrt{\frac{N_t}{E_s}}\mathbf{H}^{\dagger}\left(\sqrt{\frac{E_s}{N_t}}\mathbf{H}\mathbf{s} + \mathbf{n}\right) \\
&= \mathbf{s} + \sqrt{\frac{N_t}{E_s}}\mathbf{H}^{\dagger}\mathbf{n}
\end{aligned}$$

Zero forcing

- For independent Rayleigh fading channels with average SNR per channel ρ, it can be shown that the SNR per antenna stream after processing is distributed as

$$f(x) = \frac{N_t}{\gamma(M_r - N_t)!} e^{-\frac{N_t}{\gamma}x} \left(\frac{N_t}{\gamma}x\right)^{M_r - N_t} \quad x > 0$$

- This is a Chi-Square random variable with $2(M_r\text{-}N_t+1)$ degrees of freedom
- Since the zero-forcing process must use $N_t\text{-}1$ degrees of freedom to separate the data streams, the diversity level is reduced from M_r (ML detection) to $M_r\text{-}N_t+1$

MMSE Detector

- Like with temporal equalization, beamforming or many other signal processing approaches, better performance can be obtained by choosing the linear transformation that balances cross-stream interference rejection and noise reduction
- This is accomplished via the MMSE solution

$$\boxed{\mathbf{T} = \sqrt{\frac{N_t}{E_s}} \left(\mathbf{H}^H \mathbf{H} + \frac{N_t N_o}{E_s} \mathbf{I} \right)^{-1} \mathbf{H}^H}$$

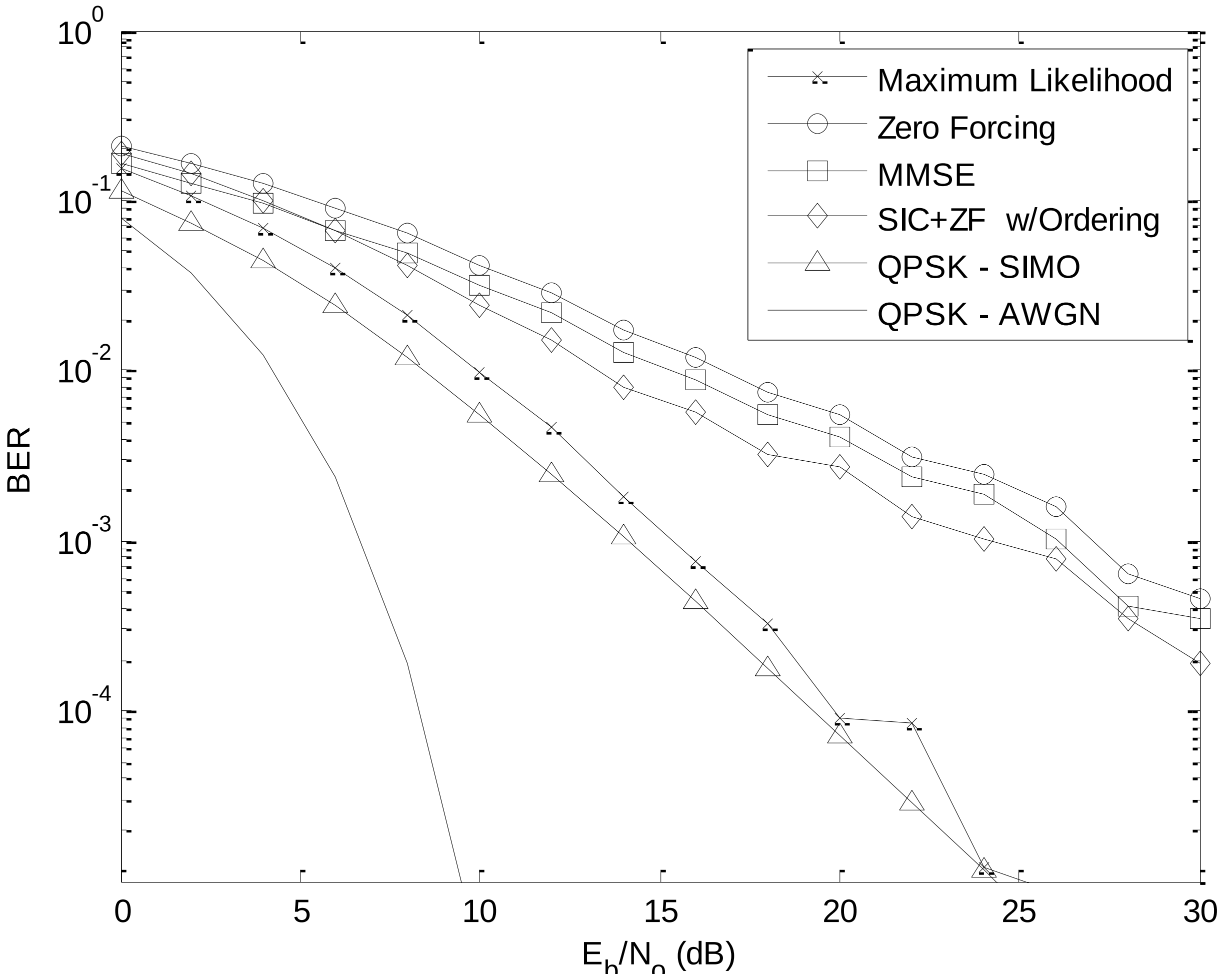
SIC Detector

- Successive Interference Cancellation (SIC) attempts to decode the strongest received stream in the presence of the other streams
- Once the stream is decoded, it is cancelled from the received signal
- Suffers from error propagation
  - The first decoded stream limits performance
- Error propagation can be reduced by ordering the streams in descending received energy before detection
- This technique has limited usefulness by itself, but is relatively powerful is combined with linear detection

SIC + Linear Detector

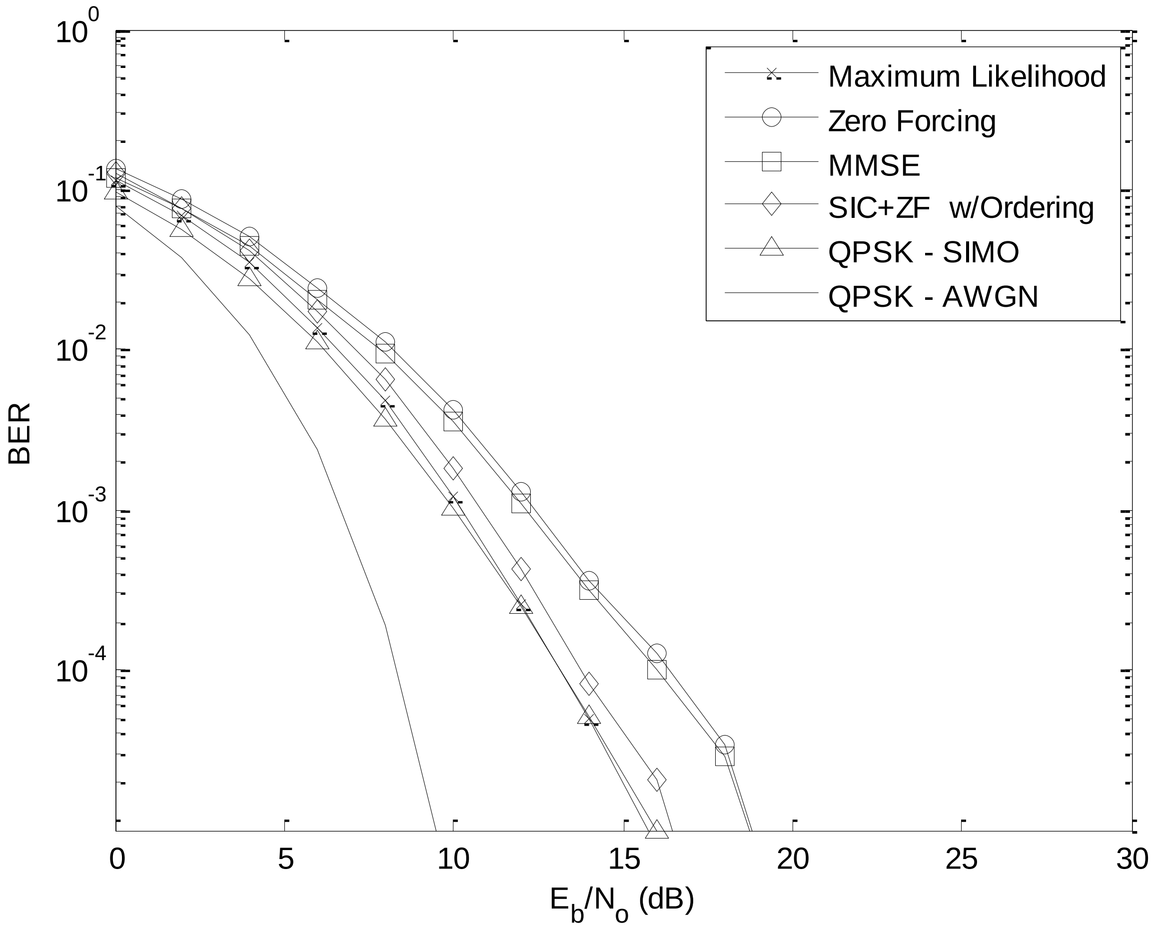
- One technique to improve SIC (esp. with ordering) is to combine with linear detection
- The strongest data stream is detected using the ZF or MMSE receiver and then cancelled from the received signal
- After cancellation an updated version of the ZF/MMSE receiver is constructed and used to detect the second strongest stream. This stream is then cancelled
- This continues until all streams are detected.
- This algorithm with horizontal encoding was the original V-BLAST algorithm

Simulated Performance



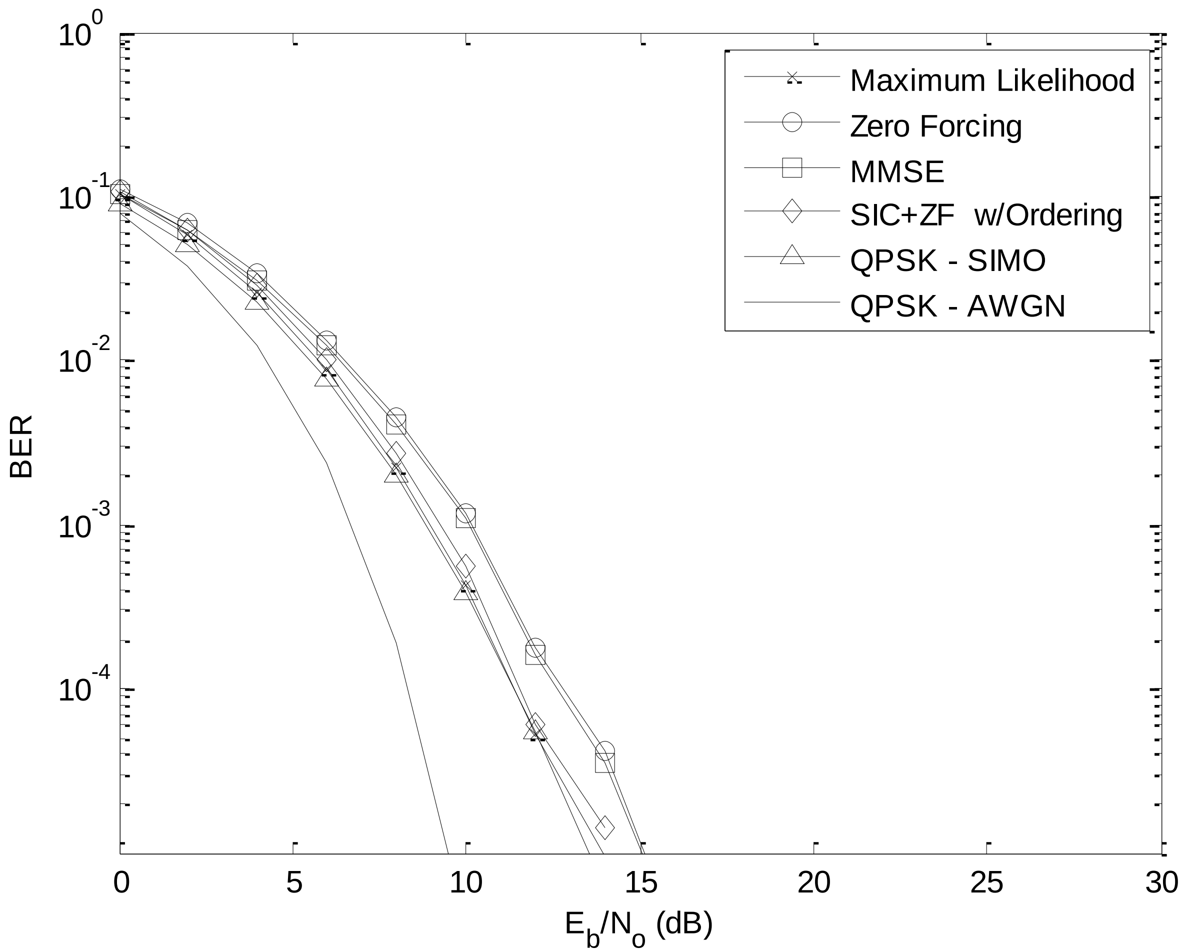
- $N_t = 2$
- $M_r = 2$
- Independent Rayleigh fading
- QPSK
- 4 bps/Hz
- No coding

Simulated Performance



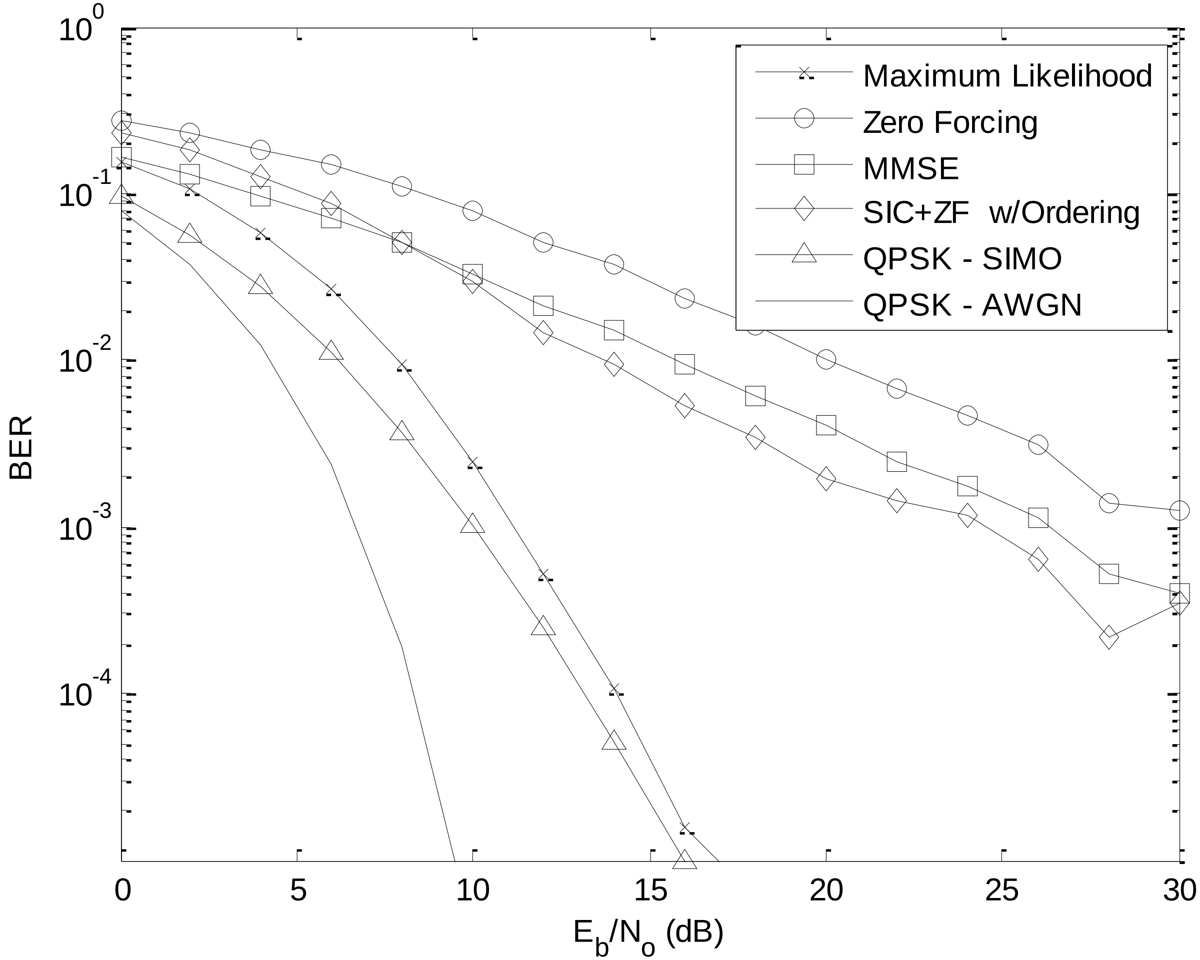
- $N_t = 2$
- $M_r = 4$
- Independent Rayleigh fading
- QPSK
- 4 bps/Hz

Simulated Performance



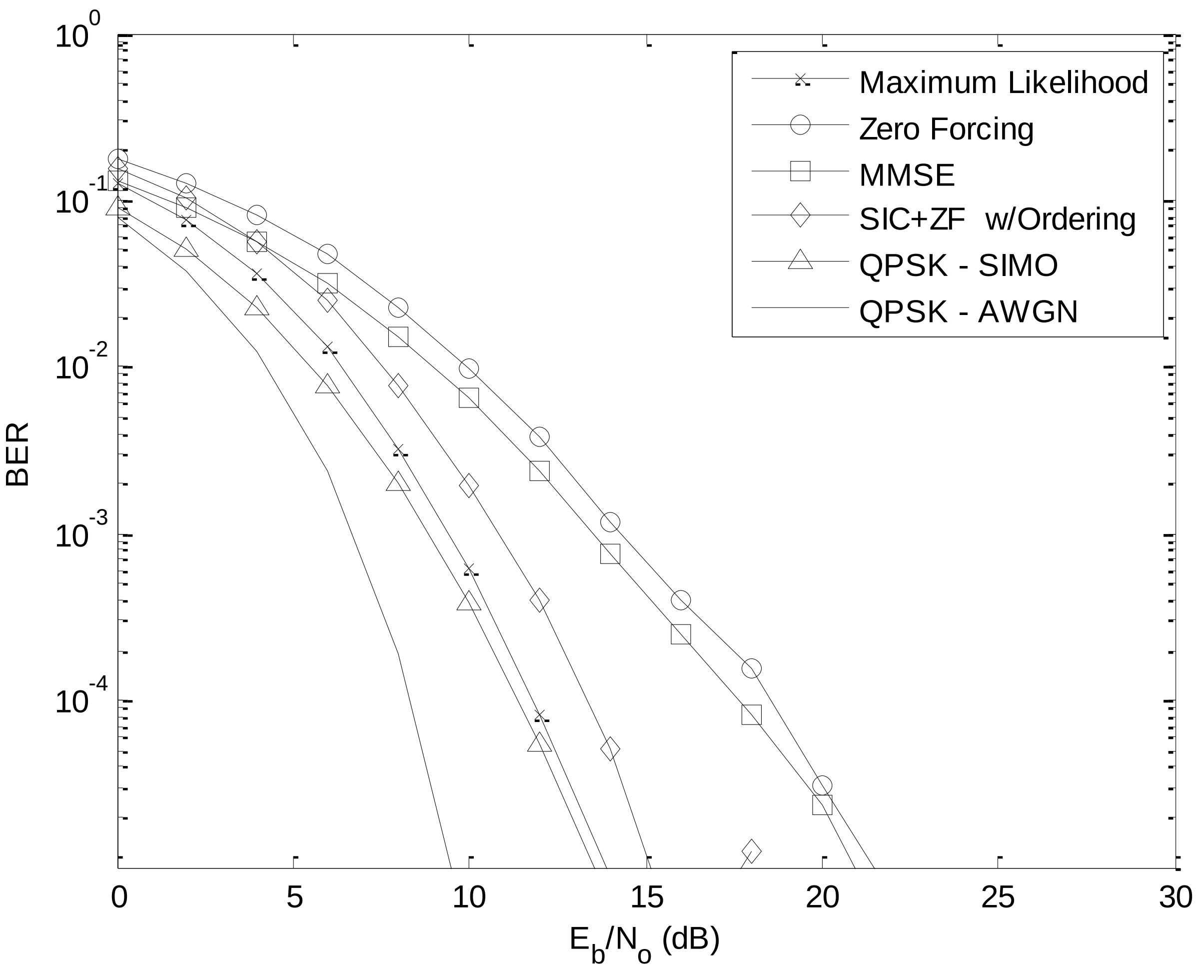
- $N_t = 2$
- $M_r = 6$
- Independent Rayleigh fading
- QPSK
- 4 bps/Hz

Simulated Performance



- $N_t = 4$
- $M_r = 4$
- Independent Rayleigh fading
- QPSK
- 8 bps/Hz

Simulated Performance

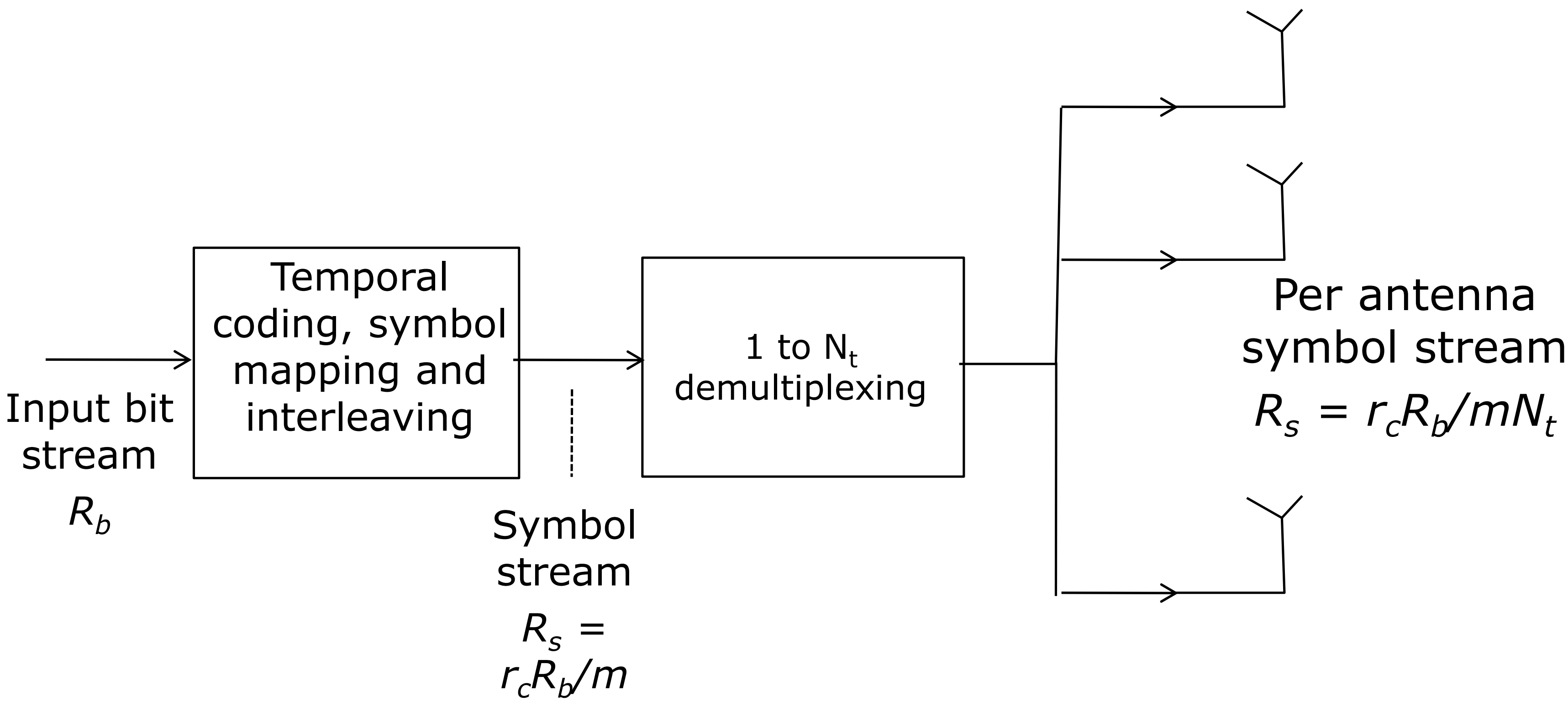


- N_t = 4
- M_r = 6
- Independent Rayleigh fading
- QPSK
- 8 bps/Hz

Vertical Encoding

- Horizontal encoding has limited diversity available since a symbol is only transmitted over a single antenna
  - M_r-fold diversity is maximum diversity gain available
- Vertical encoding performs encoding, modulation and interleaving *before* demulitplexing
- In this way a single coded stream is transmitted across all antennas
  - M_rN_t diversity is possible with proper interleaving
- Receiver complexity is substantial issue (streams should be detected/decoded jointly)

Vertical Encoding



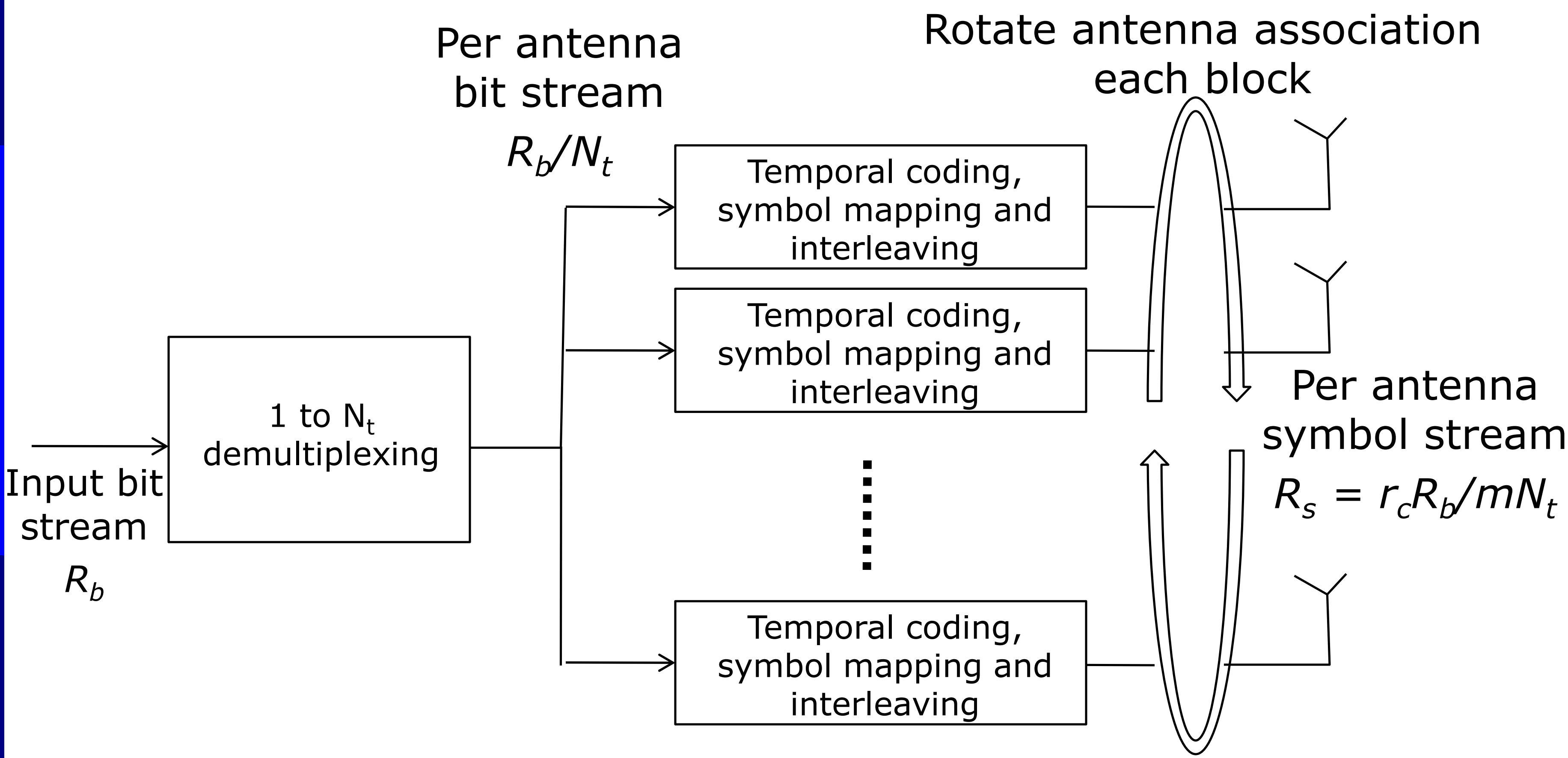
r_c - temporal code rate

m – bits per modulation symbol

Diagonal Encoding

- Diagonal encoding provides a means for achieving full diversity with a simple receiver structure
- Loss of throughput can result due to "wasted" space at the beginning and end of the horizontal blocks
- Involves horizontal encoding with antenna rotation

Diagonal Encoding

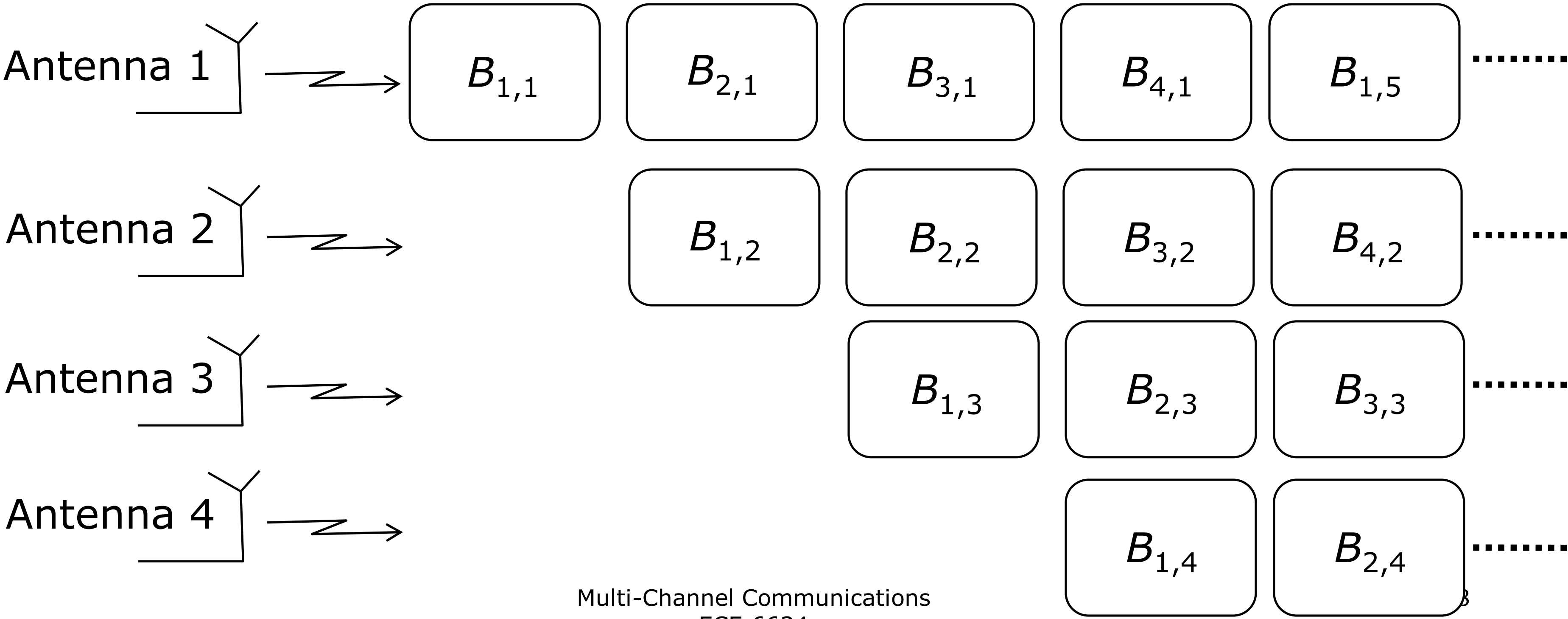


r_c - temporal code rate

m – bits per modulation symbol

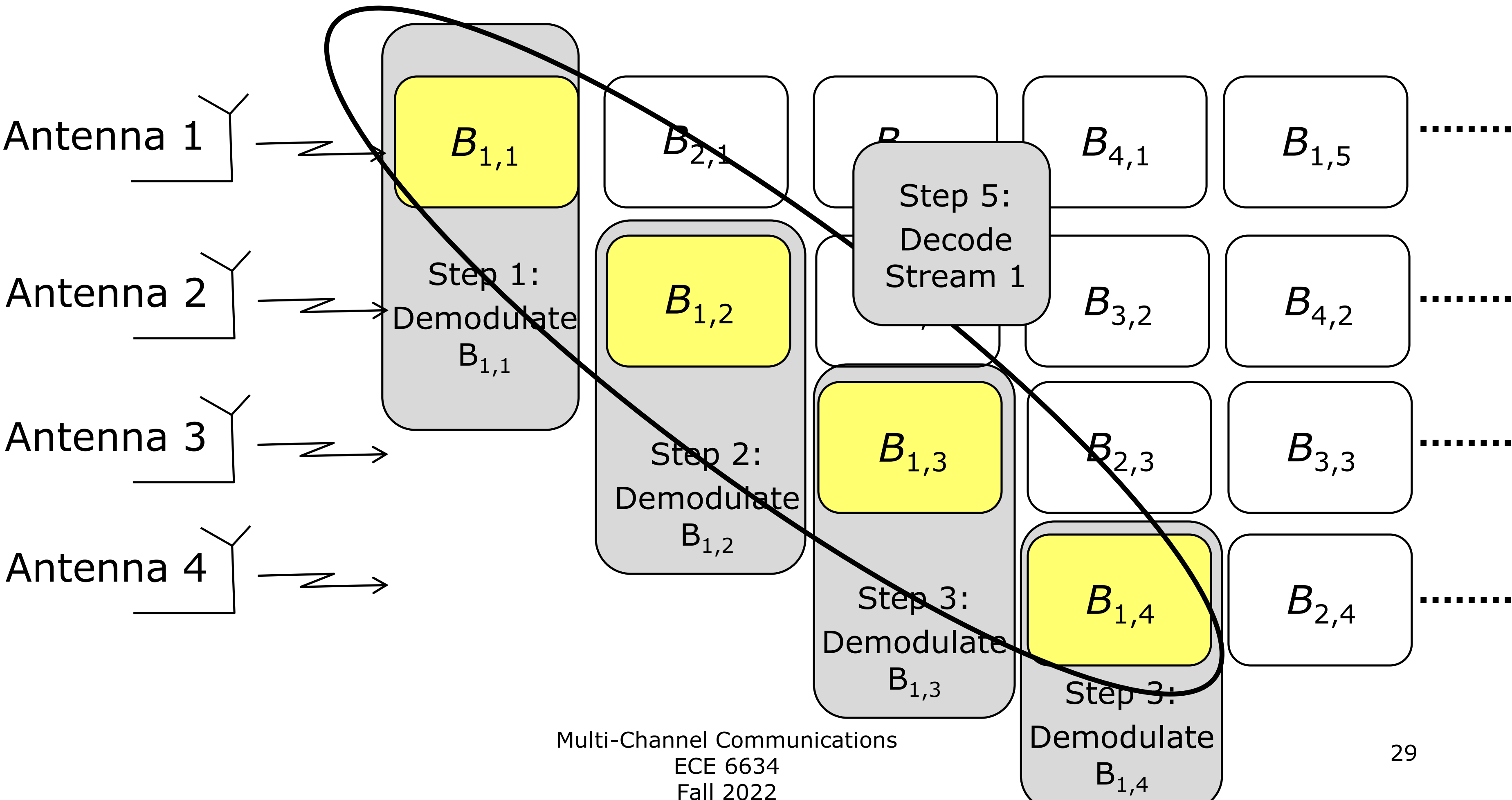
Diagonal Encoding (4 antenna example)

- Let data symbol block $B_{i,j}$ be the jth block generated by the ith encoder/modulator:



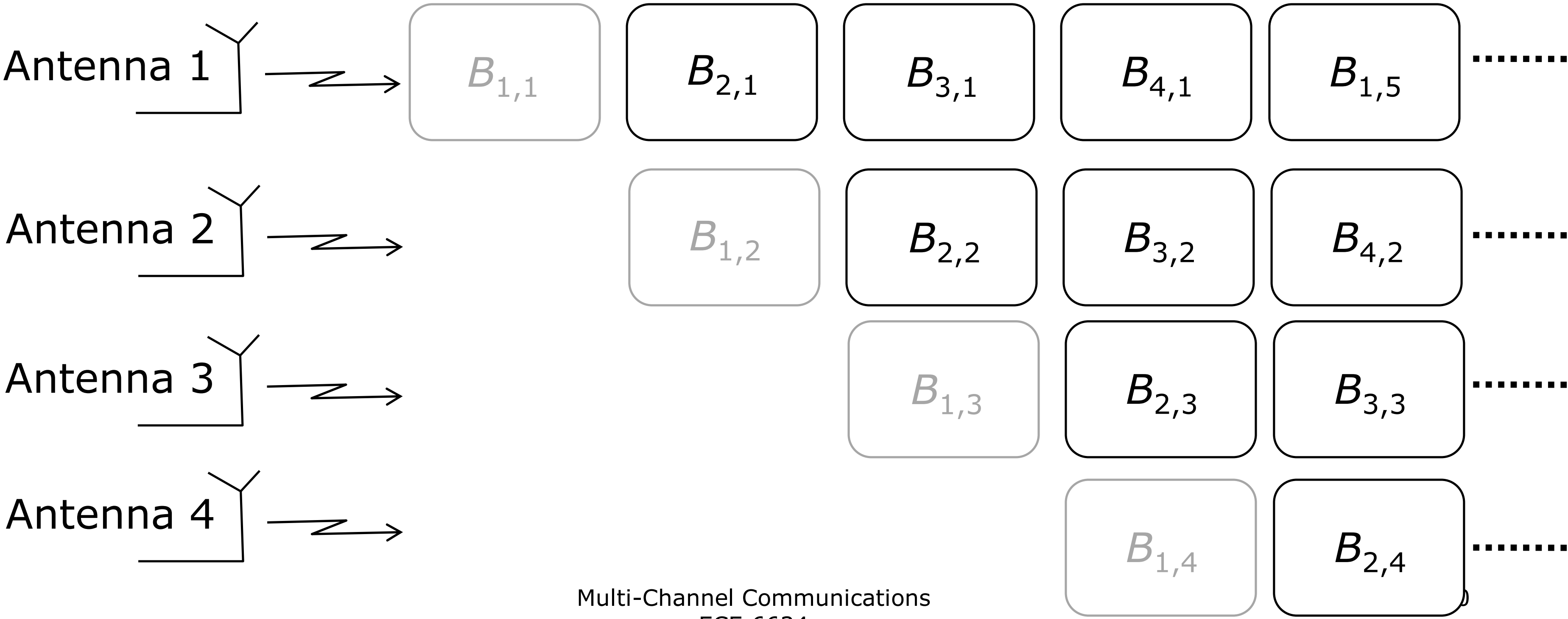
DE Decoding (4 antenna example)

- First Demodulate/Decode Stream 1



DE Decoding (4 antenna example)

- Next cancel stream 1 by re-encoding, remodulating and cancelling:



DE Decoding

- Demodulation/decoding continues with stream 2, repeating the steps
  - Demodulate Block $B_{2,1}$ with no interference
  - Demodulate Block $B_{2,2}$ with interference from Block $B_{3,1}$
  - Demodulate Block $B_{2,3}$ with interference from Blocks $B_{3,2}$ and $B_{4,1}$
  - Demodulate Block $B_{2,4}$ with interference from Blocks $B_{3,3}$,$B_{4,2}$ and $B_{1,5}$
  - Decode stream 2
- Cancel re-encoded Stream 2
- Repeat for streams 3 and 4

Conclusions

- Today we have examined techniques for spatial multiplexing when there is no channel knowledge available at the transmitter
- Three encoding structures are possible: Horizontal, Vertical, and Diagonal
- We also examined several detector structures which are primarily applicable to HE techniques
  - Some can be applied to DE or VE