# Multi-channel Communications Fall 2022

## Lecture 22
## Bit Loading

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# Introduction

- Information theory tells us that capacity is reached in a frequency selective fading channel by creating parallel channels and transmitting the optimal rate on each channel
- This can be accomplished by feeding back channel state information and changing the modulation (and coding) scheme on a sub-carrier basis

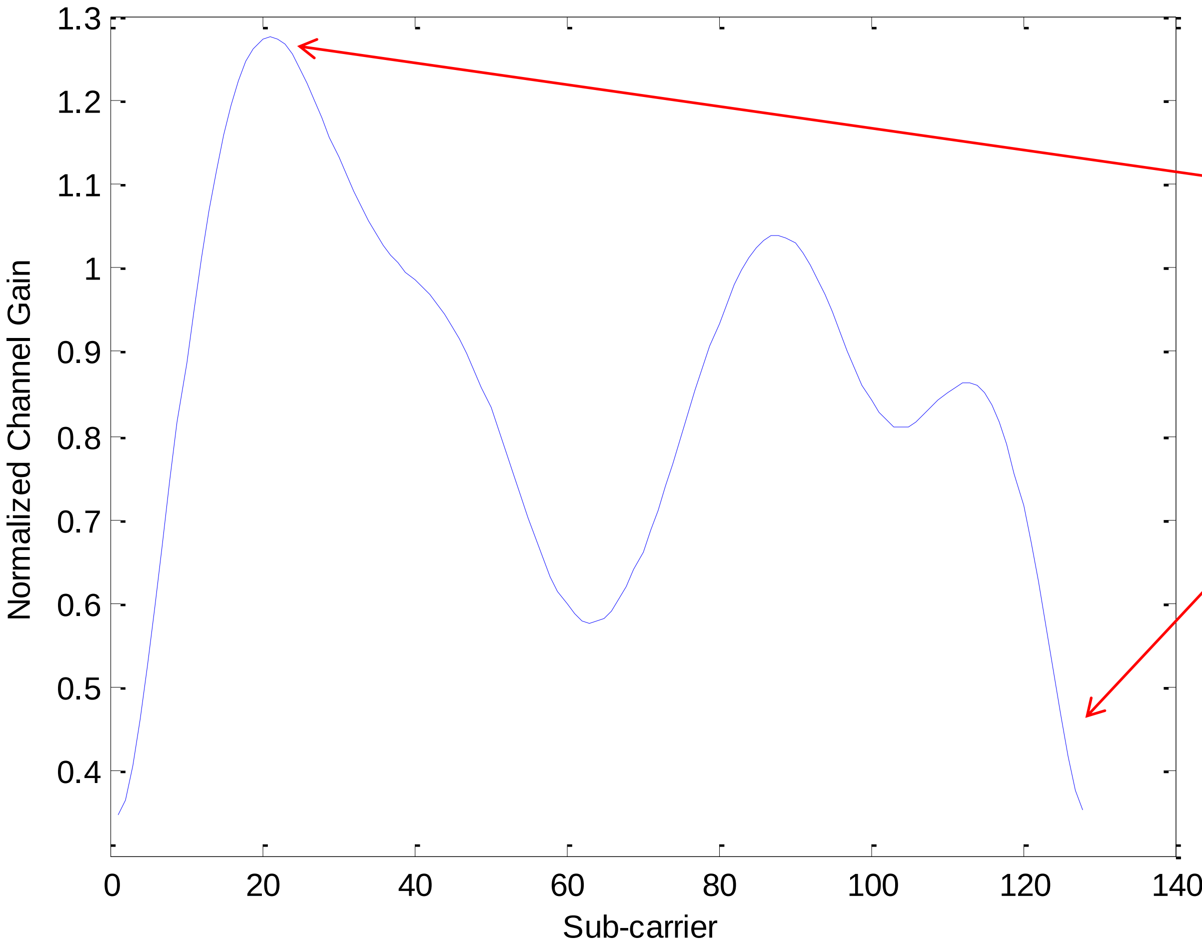
# Objective Functions

- Adaptive Bit Loading can be used to
  - Maximize throughput based on a power constraint
    - Transmit the largest constellation sizes that can be sent given the channel SNRs
    - Can adapt both power and rate allocation per carrier
  - Minimize energy based on a throughput constraint
    - Transmit only what is needed to achieve desired throughput
    - Can adapt both power and rate per carrier

# Algorithm Types

- Incremental / greedy algorithms
  - High complexity but good accuracy
- Bit-loading based on channel capacity approximation
  - Low complexity but poor accuracy
- Bit-loading based on probability of error expressions
  - Low complexity but poor accuracy
- Trade-off between closeness to optimal and complexity

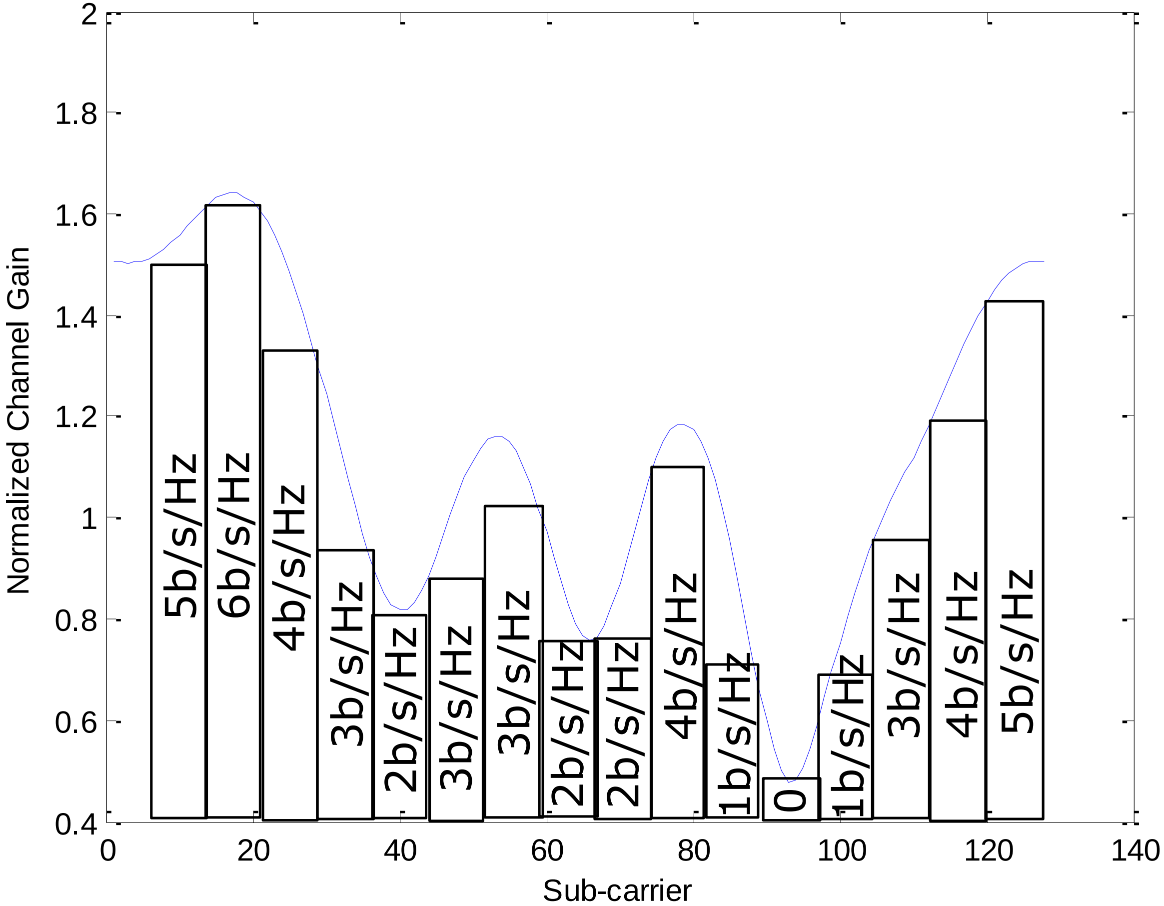
# Frequency Selective Channel




Use QAM at high channel values

Use BPSK or nothing at low channel values

# Second Example




- Higher selectivity provides more flexibility

# Incremental/Greedy Example

- Attempt to solve

$$\max_{b_i} \sum_{i=1}^{N} b_i \quad \text{subject to} \quad \overline{p} = \frac{\sum_{i=1}^{N} b_i p_i}{\sum_{i=1}^{N} b_i} \leq p_T$$

Where $b_i$ and $p_i$ are the number of bits and estimated BER for subcarrier $i$ respectively, and $p_T$ is the BER target.

NOTE: Tx power is constant

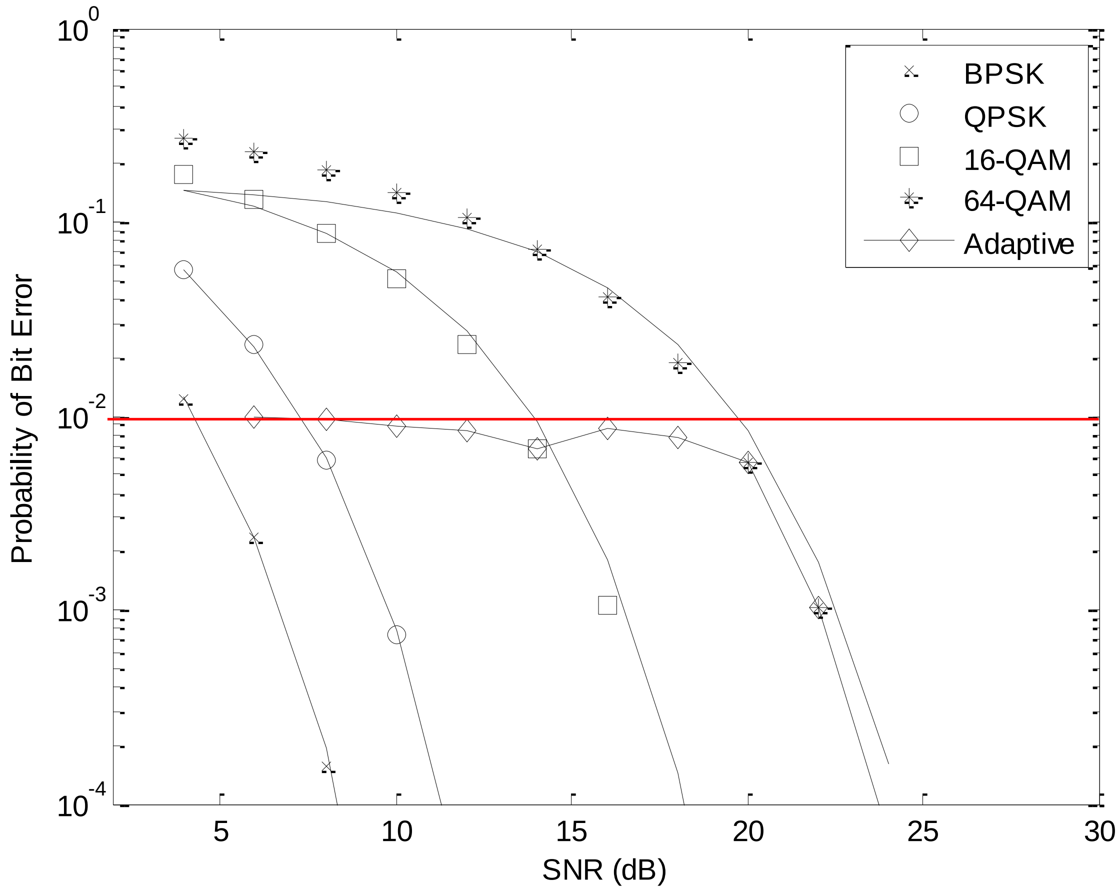
# Algorithm

1. Initialize all sub-carriers to highest modulation
2. Determine $p_i$ for all subcarriers using theoretical expressions and estimated channel values
3. Compare average BER with target. If predicted average is less than target stop.
4. Find the subcarrier with the worst $p_i$. Reduce the constellation size by one order. If $b_i$=1, null that subcarrier.
5. Recompute $p_i$ and goto step 3.

# Example – Theoretical Expressions

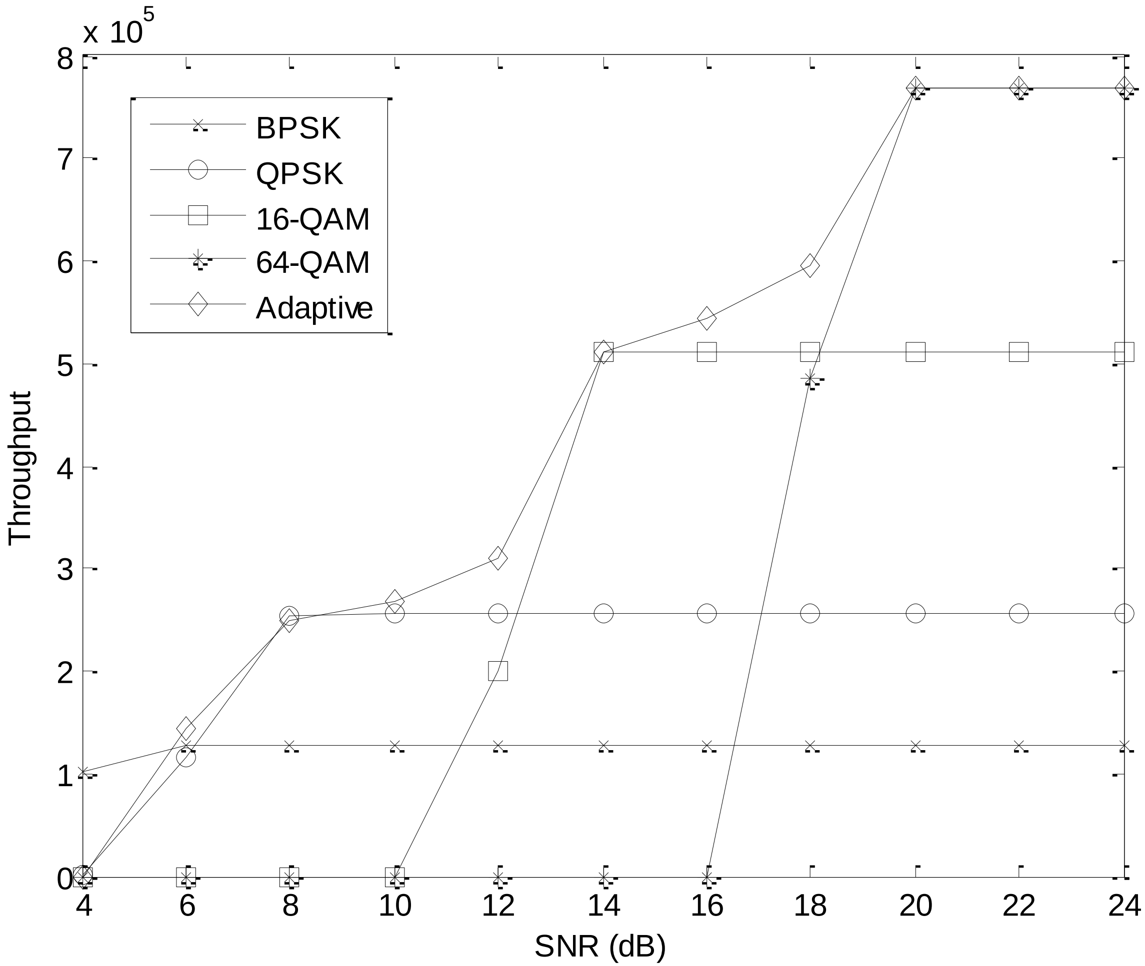
- $p_b$ = probability of bit error; $\gamma$ = SNR for carrier of interest
- BPSK
$$p_b = Q\left(\sqrt{2\gamma}\right)$$
- QPSK
$$p_b = Q\left(\sqrt{\gamma}\right)$$
- 16-QAM
$$p_b = 3Q\left(\sqrt{\frac{\gamma}{5}}\right)\left(1-\frac{3}{4}Q\left(\sqrt{\frac{\gamma}{5}}\right)\right)$$
- 64-QAM
$$p_b = \frac{7}{2}Q\left(\sqrt{\frac{\gamma}{21}}\right)\left(1-\frac{7}{8}Q\left(\sqrt{\frac{\gamma}{21}}\right)\right)$$

# Performance in AWGN




- N=128
- 1000 packets
- BER target = 1%
- For SNR>20dB power control needed to maintain target error rate

# Performance in AWGN



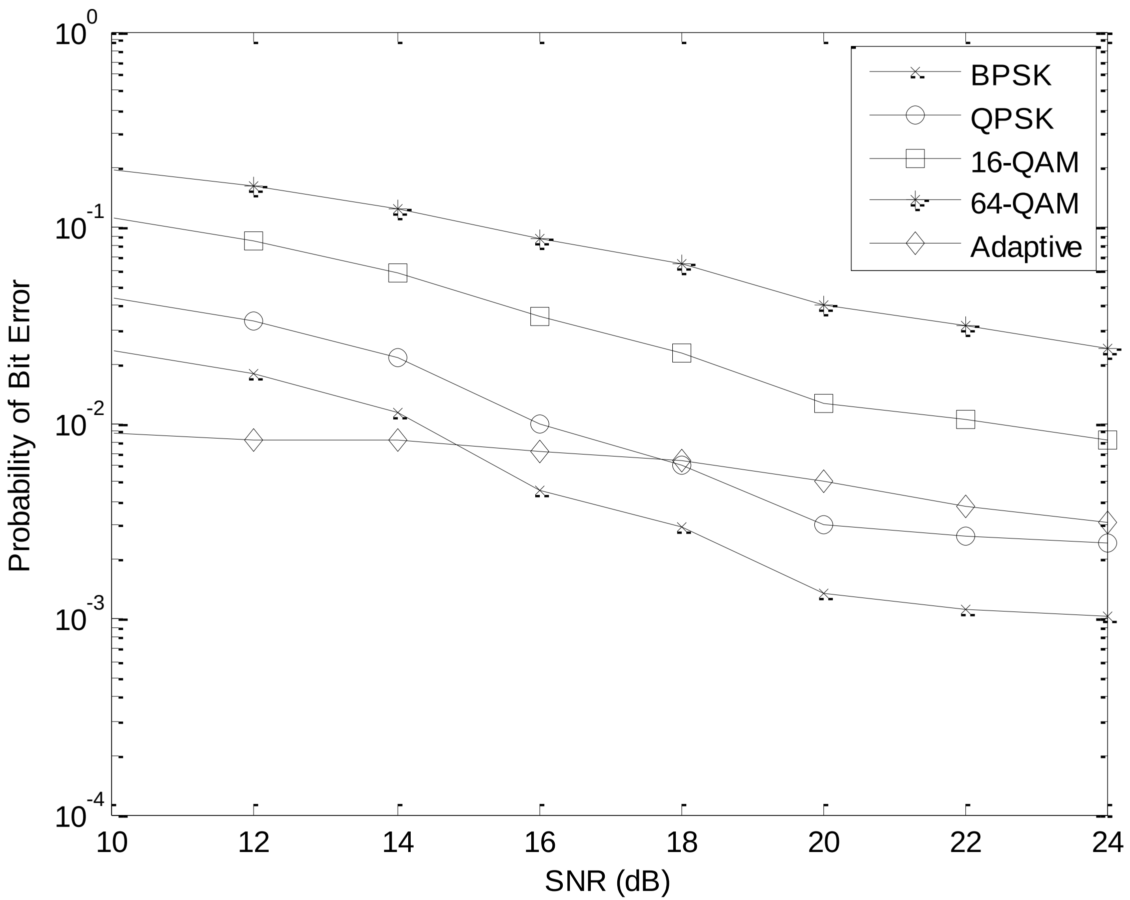

- N=128
- 1000 packets
- Througput = PER*packet size
- Algorithm fails at low SNR since theoretical expressions consider BER not PER

* - Packet error corresponds to number of errors > 0.02*packet size

# Performance in AWGN

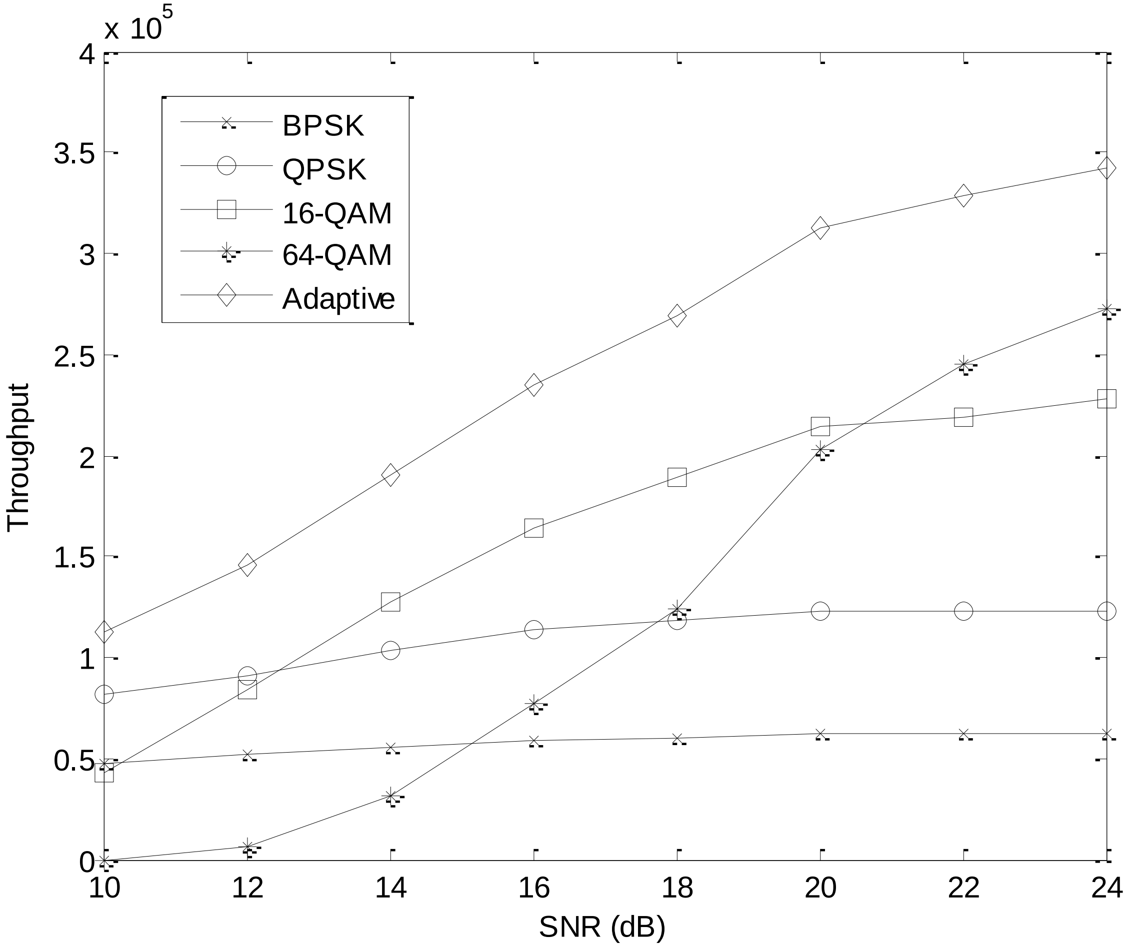
- In an AWGN channel, all sub-carriers experience the same SNR
  - Essentially all carriers use the same modulation
  - In some cases, a fraction of the carriers use one modulation while the rest of the carriers use the next higher/lower modulation
- Bit-loading behaves as single-carrier adaptive modulation
- BER meets threshold when possible
- Throughput "surfs" the maximum throughput for all modulation schemes

# Frequency Selective Fading



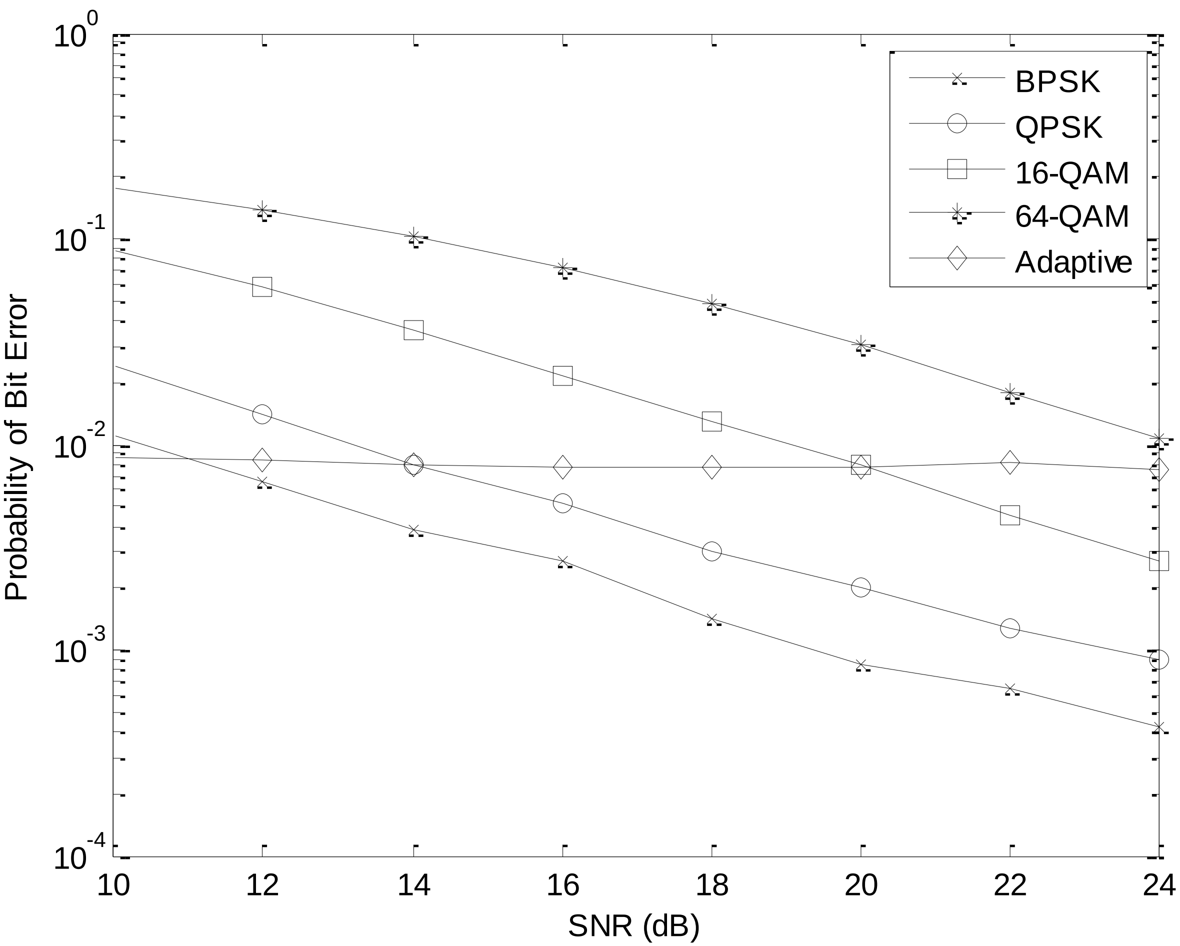

- N=128
- 1000 packets
- Multipath delay profile = [1 0.2]
- BER target = 1%

# Frequency Selective Fading



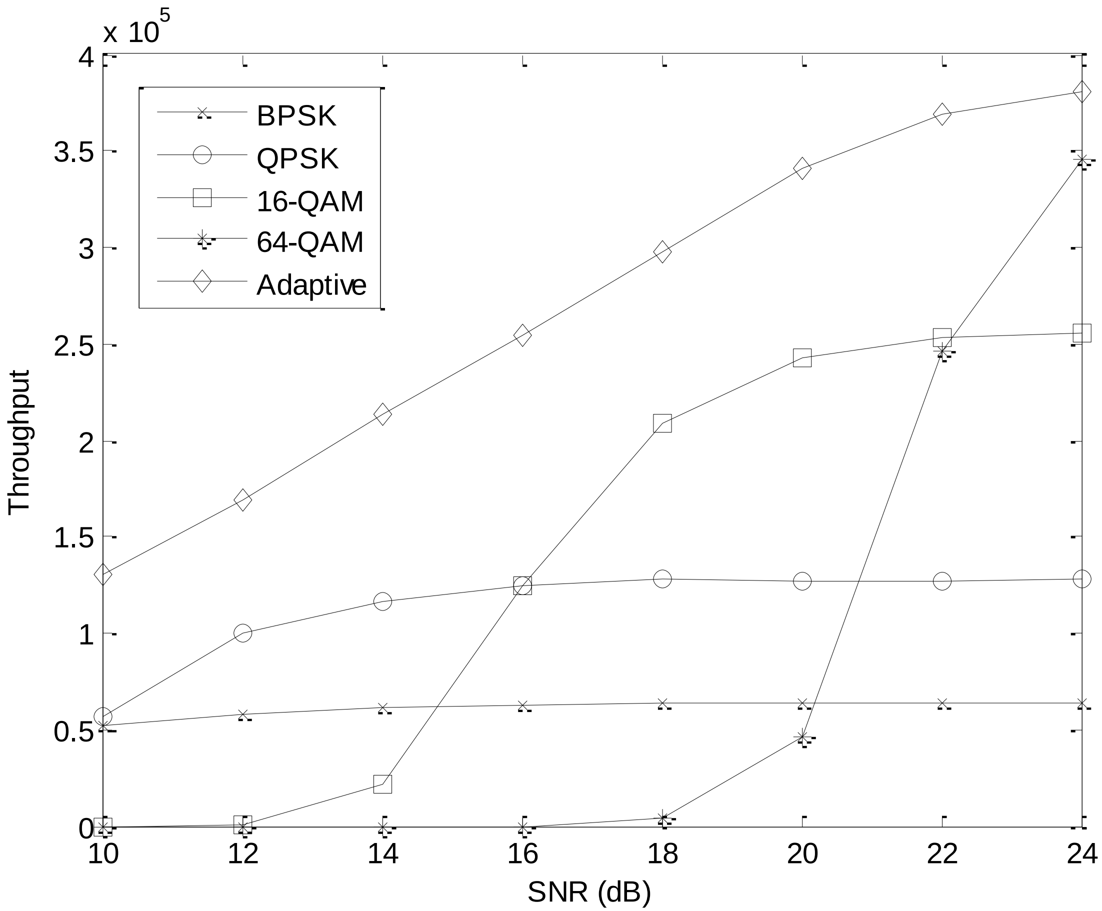

- N=128
- 1000 packets
- Multipath delay profile = [1 0.2]
- BER target = 1%
- Throughput = (1-PER)*packet size

# Frequency Selective Fading




- N=128
- 1000 packets
- Multipath delay profile = [1 0.5 0.35 0.15 0.05]
- BER target = 1%
- Performance much closer to target than previous case

# Frequency Selective Fading




- N=128
- 1000 packets
- Multipath delay profile = [1 0.2 0.35 0.15 0.05]
- BER target = 1%
- Throughput = PER*packet size
- Higher throughput than previous case

# Performance Notes

- In an frequency-selective fading channel, all sub-carriers experience different SNR values
- Aggressive modulation schemes behave poorly on low-SNR channels
- Low-rate modulation schemes perform well but do not provide much throughput
- Bit-loading allows for maximum throughput to be achieved
- BER meets threshold when possible
- Gains much more significant in frequency selective fading than in AWGN or flat fading
- Higher ‘selectivity’ means more flexibility
  - BER performance closer to target
  - Higher throughput

# Conclusions

- In this lecture we examined adaptive bit-loading which tailors the modulation scheme for each sub-carrier to the specific SNR
- In AWGN channels, gains are modest
- In Frequency-selective channels we see a large improvement
  - No one modulation scheme is best for all sub-carriers
  - The more selectivity the more flexibility and better performance