Multi-channel Communications Fall 2022

Lecture 10

Generalized Diversity and Generalized Fading

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Overview

- In this lecture we will look at three items related to diversity
  - Generalized selection combining
  - Generalized fading
  - Impact of spatial correlation

- Again, although the analysis is targeted towards spatial diversity, it applies equally well to any form of diversity

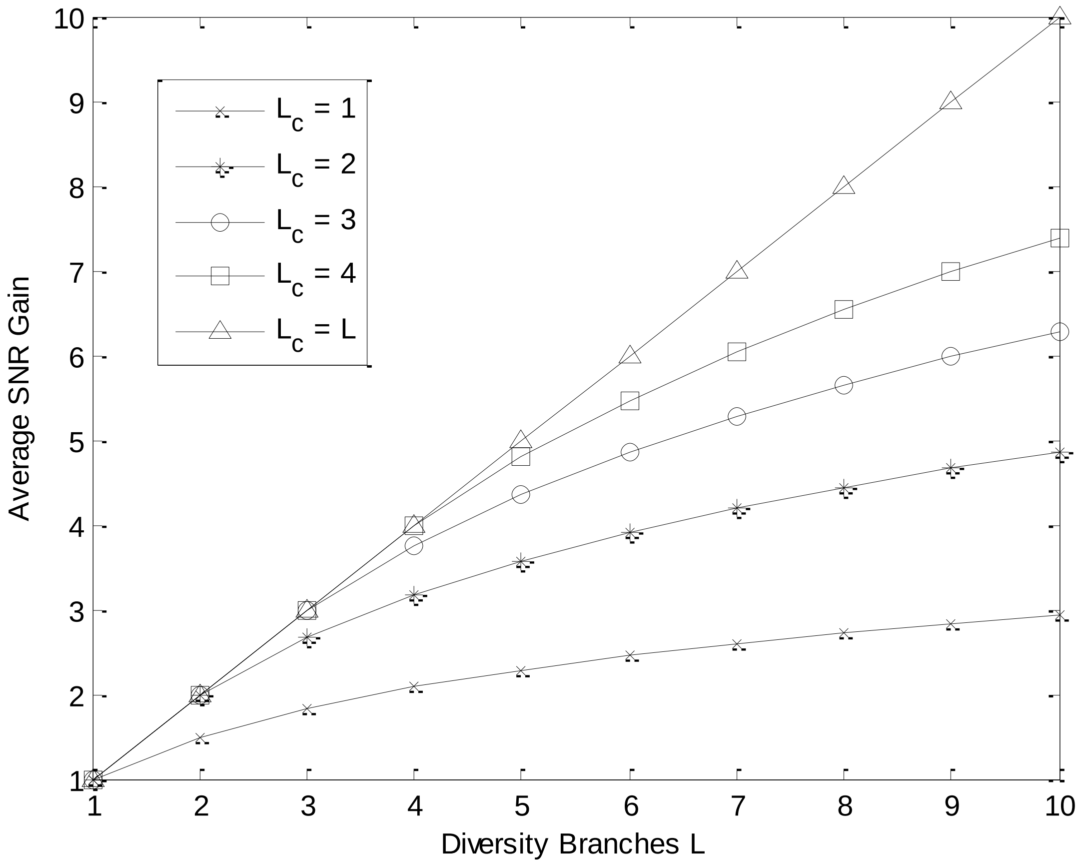
Generalized Selection Combining

- The most general form of diversity combining
  - Given L diversity branches, select the strongest L_c and combine them using MRC
  - If $L = L_c$, we have standard MRC
  - If $L_c = 1$, we have standard selection diversity

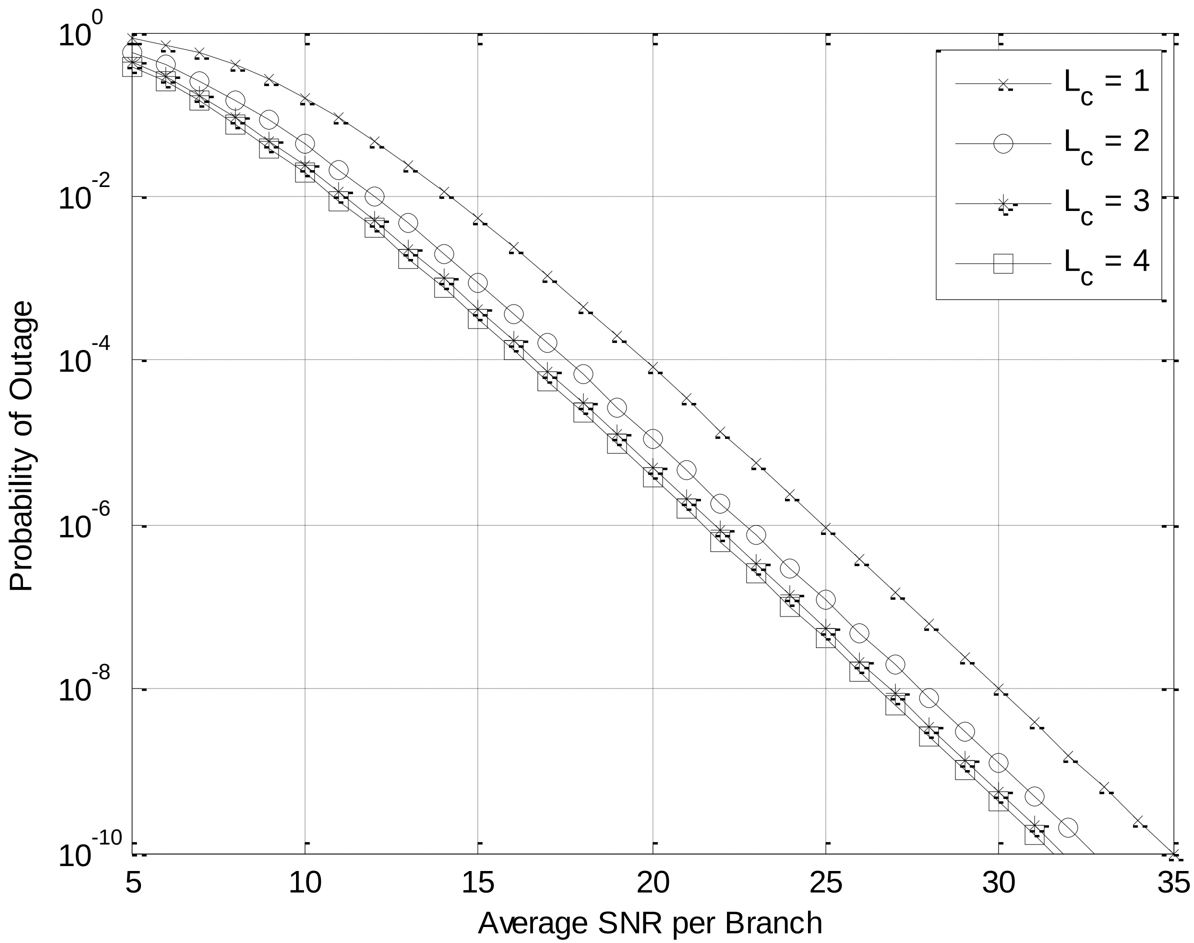
Go to the board….

Generalized Selection Diversity

- Average SNR Gain

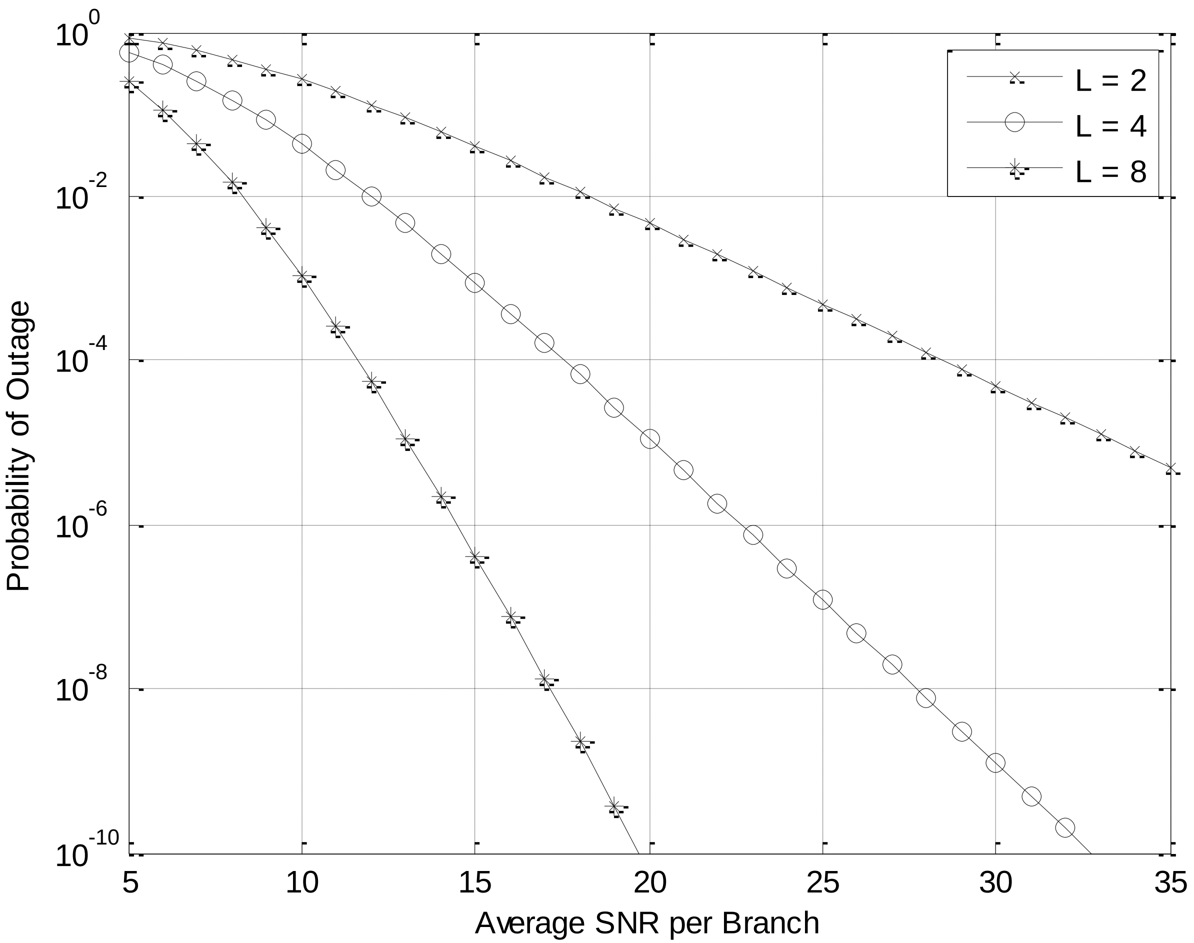


Generalized Selection Diversity



- γ_{th} = 10dB
- *L* = 4
- Diversity gain remains the same
- Avg. SNR gain improves with L_c

Generalized Selection Diversity



- γ_{th} = 10dB
- $L_c = 2$
- Diversity gain increases with *L*

Generalized Fading

- To this point we have assumed Rayleigh fading – a fairly severe type of fading
- A more general distribution for fading signal amplitude is the Nakagami distribution

$$f_\alpha(\alpha) = \frac{2}{\Gamma(m)}\left(\frac{m}{2\sigma^2}\right)^m \alpha^{2m-1} e^{-m\alpha^2 / 2\sigma^2}$$

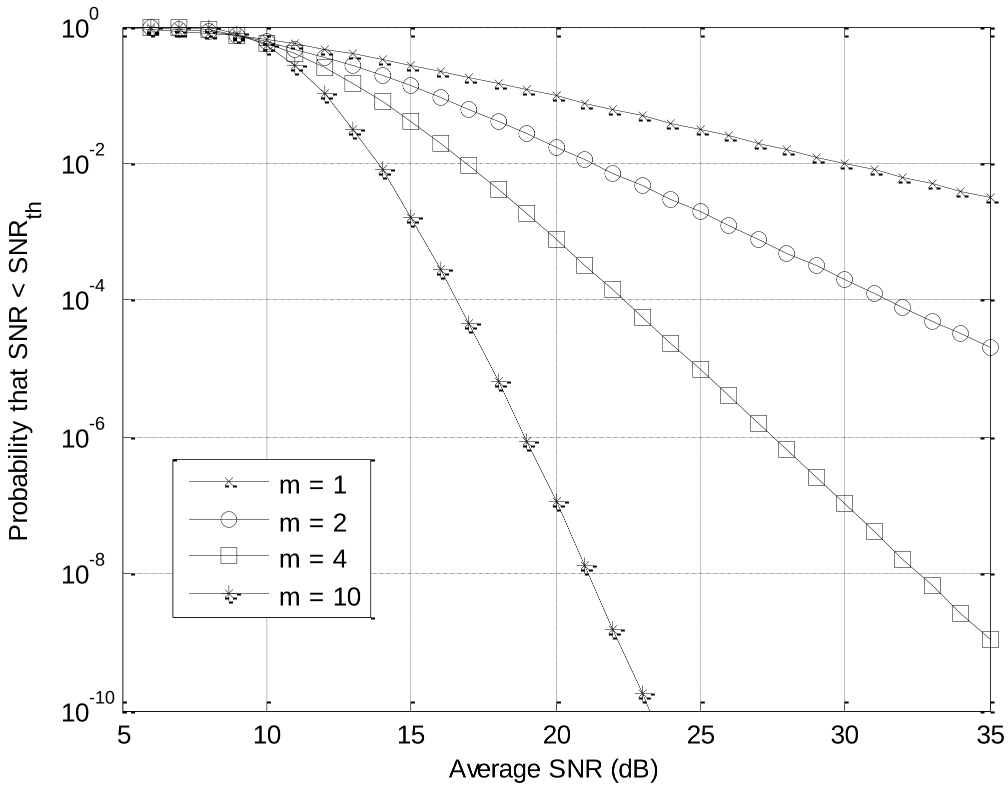
where $E\left(\alpha^2\right) = 2\sigma^2$

and *m* is the fading parameter defined as

$$m = \frac{2\sigma^2}{E\left(\left(\alpha^2 - \sqrt{2\sigma^2}\right)^2\right)}$$

Go to the board….

Outage Probability versus *m*



- $L = 1$
- γ_{th} = 10dB
- P_{out} = 1%

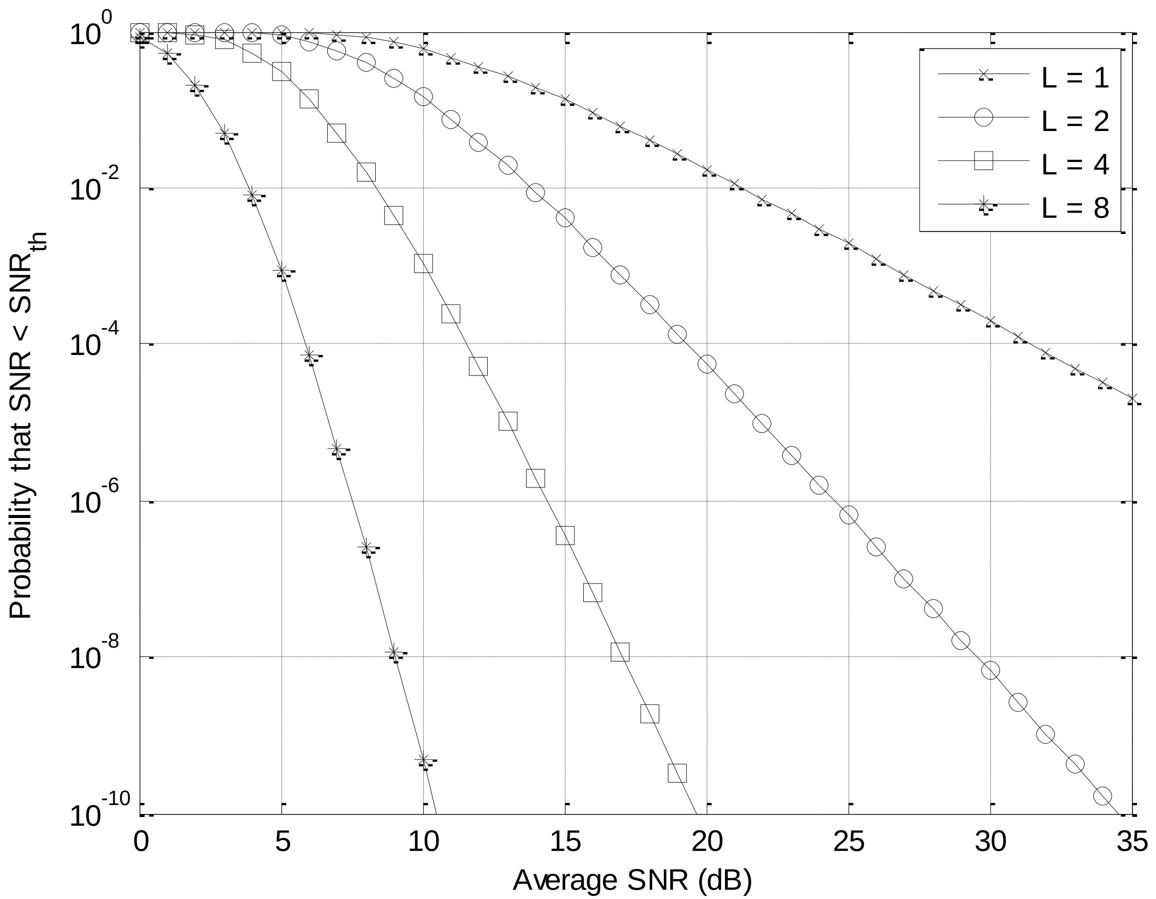
m	$\bar{\gamma}$
1	$30dB$
2	$22dB$
4	$17dB$
10	$14dB$

Diversity with Nakagami Fading

- Like with Rayleigh fading, the best performance is achieved with Maximal Ratio Combining

Go to the board….

MRC Diversity Gain

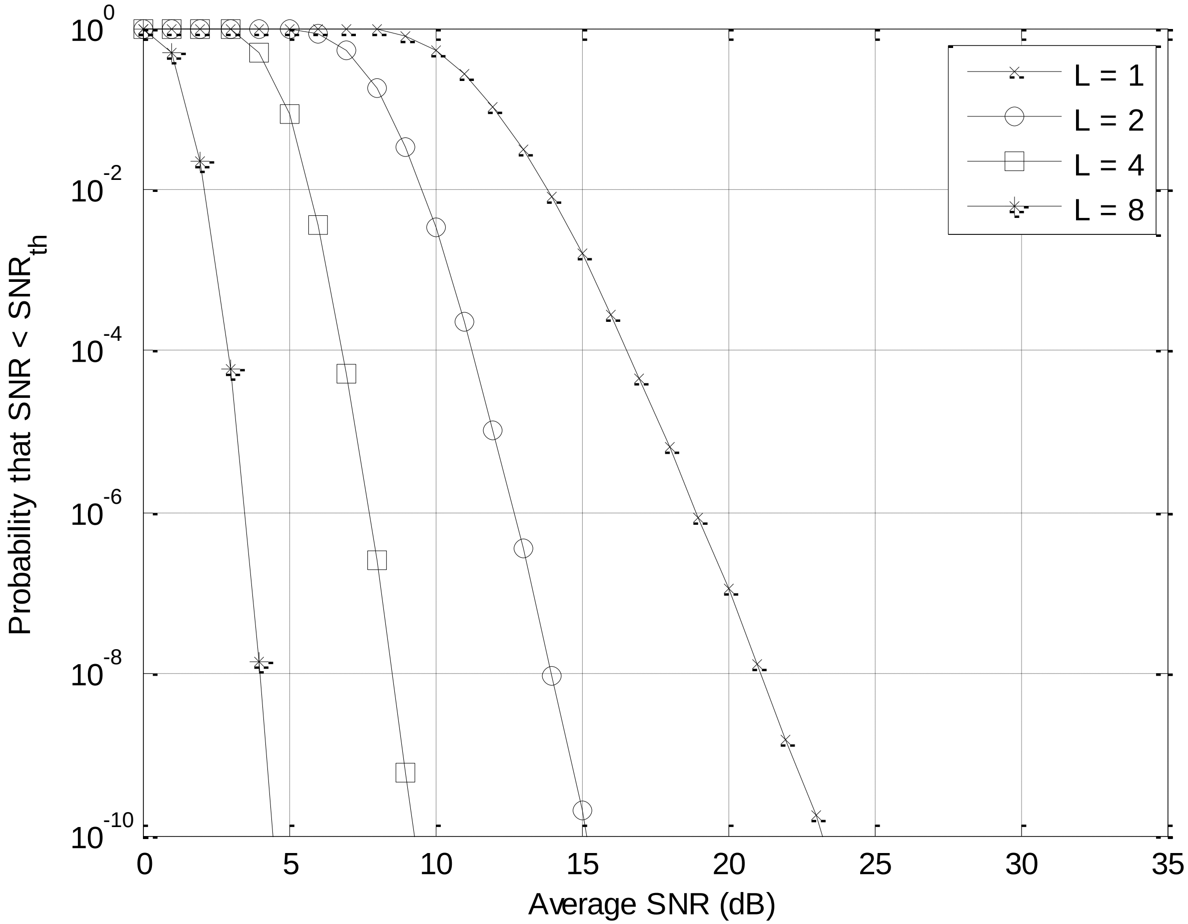


- $m = 2$
- $\gamma_{th} = 10\text{dB}$
- $P_{out} = 1\%$

L	$\bar{\gamma}$
1	$21dB$
2	$14dB$
4	$8dB$
8	$4dB$

Gains not quite as big as with Rayleigh fading

MRC Diversity Gain



- $m = 10$
- $\gamma_{th} = 10\text{dB}$
- $P_{out} = 1\%$

L	$\bar{\gamma}$
1	$14dB$
2	$9.5dB$
4	$5.5dB$
8	$2.5dB$

Gains reduced due to nearly the array gain due to lack of fading

Bit Error Rate for BPSK

- The bit error rate can be found to be

$$
\begin{aligned}
P_b &= \frac{1}{2}\sqrt{\frac{\overline{\gamma}/m}{\pi(1+\overline{\gamma}/m)}}\frac{\Gamma(Lm+\frac{1}{2})}{\Gamma(Lm+1)}\left(\frac{1}{1+\overline{\gamma}/m}\right)^{Lm} \times \ldots \\
& {}_2F_1\left(1, \Gamma\left(Lm+\frac{1}{2}\right); \Gamma\left(Lm+\frac{1}{2}\right), \frac{1}{1+\overline{\gamma}/m}\right)
\end{aligned}
$$

- Which simplifies to a form identical to Rayleigh fading with *Lm* diversity branches but γ/m SNR per branch when Lm is an integer.

Nakagami vs. Ricean

- Nakagami is more general than Ricean
  - Ricean with K = 0 → Rayleigh
    - Fading can't be worse than Rayleigh
  - Nakagami with m = 1 → Rayleigh
    - m = 0.5 → One-sided Gaussian (worse than Rayleigh)

- Although there is no direct conversion from Nakagami to Ricean (or vice versa) *nearly* equivalent conversion can be obtained with the relationships

$$m = \frac{(K+1)^2}{2K+1}$$

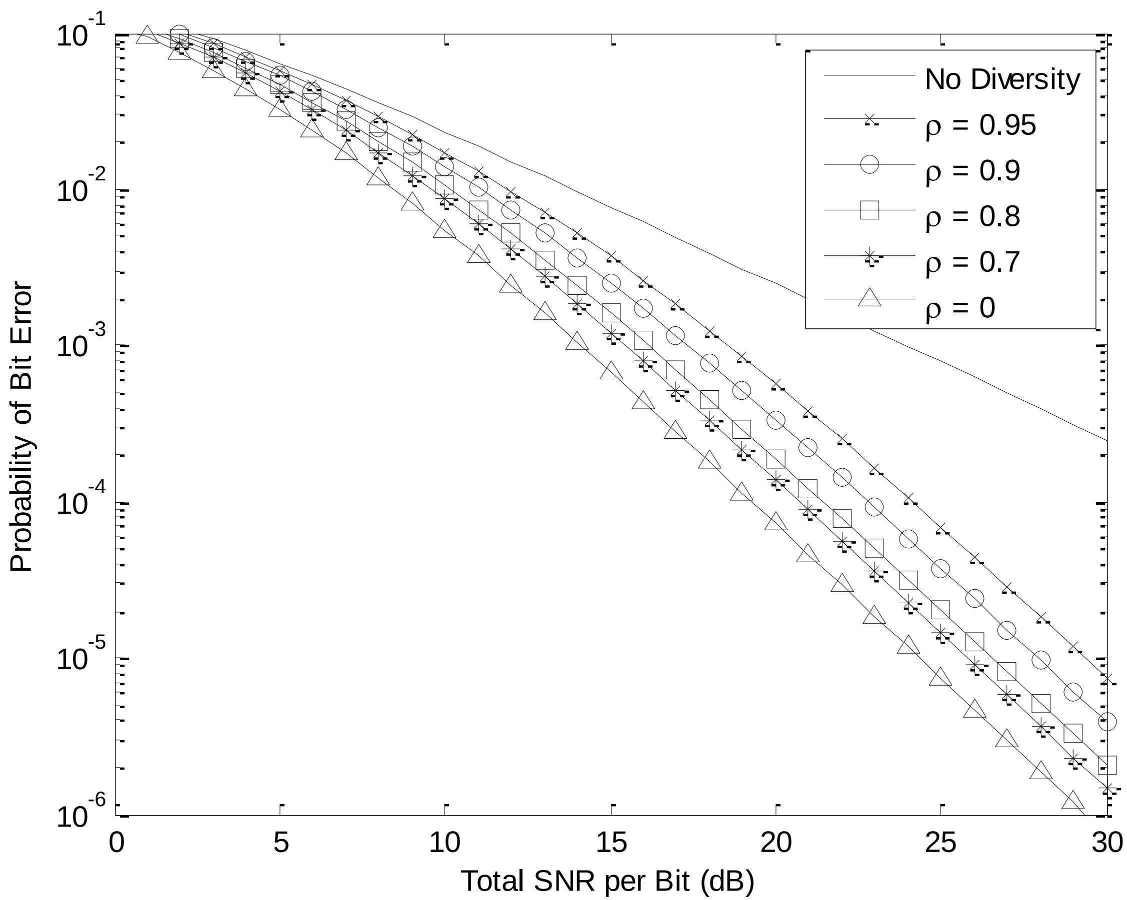
$$K = \frac{\sqrt{m^2 - m}}{m - \sqrt{m^2 - m}}$$

Correlated Rayleigh Fading

- To this point we have assumed that all branches observe independent signals
- Clearly if the signals are correlated, the usefulness of diversity is reduced
  - Consider the case where the branches are perfectly correlated → No diversity!

Go to the board….

Correlated Rayleigh Fading – Two Antennas



- BER of BPSK
- Diversity gain diminishes with increasing correlation
- Even with a correlation of 0.95, there is substantial diversity gain at 10^{-3} BER
- Correlation of 0.7 is very close to independent branches

$\rho_{12}=\rho$, $\beta_{12}=0$

Conclusions

- We considered three new topics in the area of diversity
  - Generalized selection combining
  - Generalized fading distribution
  - Correlated Rayleigh fading