Virginia Polytechnic Institute and State University

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MIMO HW1

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Contents

1 Introduction

The task is to create a narrow band MIMO channel simulator for multiple transmit and receive antennas. There is a channel between each transmit and receive antenna that varies over time. The simulator can control the temporal correlation along time as well spatial correlation between antennas. The effect of differing amounts of fading to this MIMO channel is also shown through different experiments.This simulator is based on the Clarke's sum of sinusoids model.

$$\begin{aligned}\mathbf{H}(n,m,t) &= 1\sqrt{\frac{1}{N}}\sum_{i=1}^{N}\mathbf{H}_i(t) \\ &= \sqrt{\frac{1}{N}}\sum_{i=1}^{N} e^{j(2\pi \mathrm{f_d} cos(\theta_i)t+\phi_i)} e^{-j\left(\frac{2\pi}{\lambda}\mathrm{d}_1(n-1)\sin(\theta_\mathrm{i})\right)} e^{-j\left(\frac{2\pi}{\lambda}d_2(m-1)\sin(\Phi_\mathrm{i})\right)}\end{aligned} \tag{1}$$

2 Description

2.1 Input

We have two channel functions: one that samples a uniform distribution and the other that samples a gaussian distribution to generate the AoA and AoD for each multipath component. The input to these channel functions are shown below:

- Nt: Number of transmit antennas.
- Nr: Number of receive antennas.
- t: time axis
- fd: maximum Doppler
- d1: transmit antenna spacing
- d2: receive antenna spacing
- N: number of multipath components
- l: wavelength
- K: Rician Factor
- u: mean of the Gaussian distribution used to model AoA and AoD
- sigma: standard deviation of the Gaussian distribution used to model AoA and AoD.

In addition to this we also require the below to generate inputs to the channel function

1. fs: sampling frequency of the time axis
2. T: time duration of the time axis
3. f: center frequency of the transmit signal.

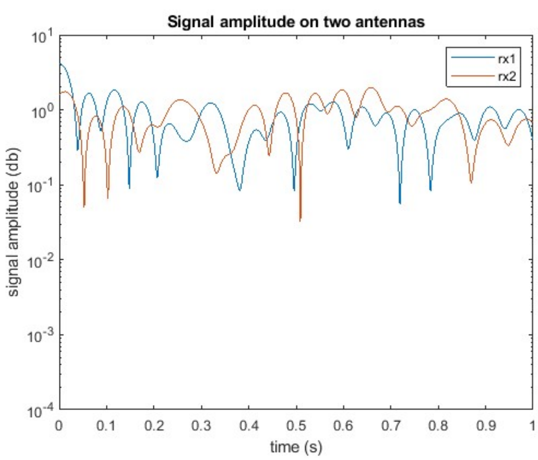
2.2 Output

The output of the channel functions is an $N_r \times N_t$ matrix for every time instant on the time axis.

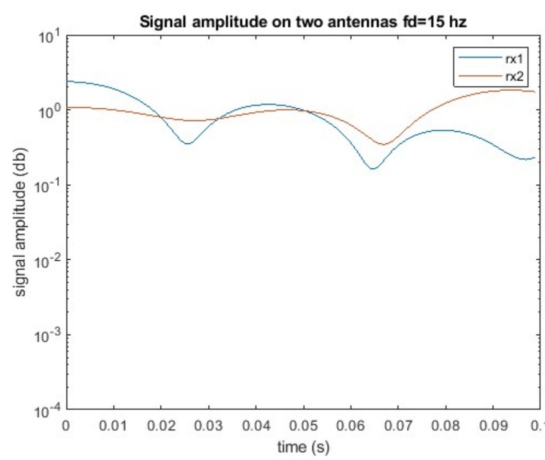
3 Validation

3.1 Q1. Magnitude, Temporal correlation and Average received Power

Assuming we are in a rich multipath environment, The magnitudes of the amplitude of the signal at the receiver changes depending on the max Doppler frequency fd. We can see the channel amplitude fading in the magnitude plots and this is further confirmed in the temporal autocorrelation plots with fd=100Hz and fd = 15Hz.



(a) Amplitude of channel with fd = 100 Hz



(b) Amplitude of channel with fd = 15 Hz

To visualise received power, the channel was simulated for 100 trials and for each trial we calculate the average received power for both receivers. The plot below is the received power in two rx seen over 100 trials.

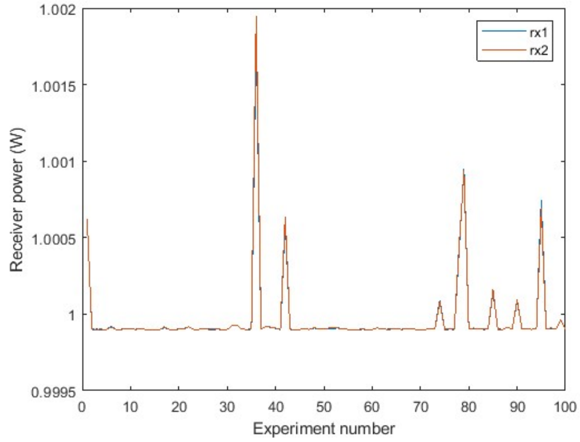
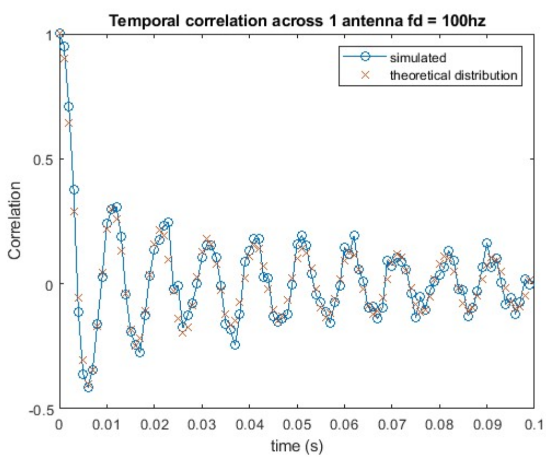
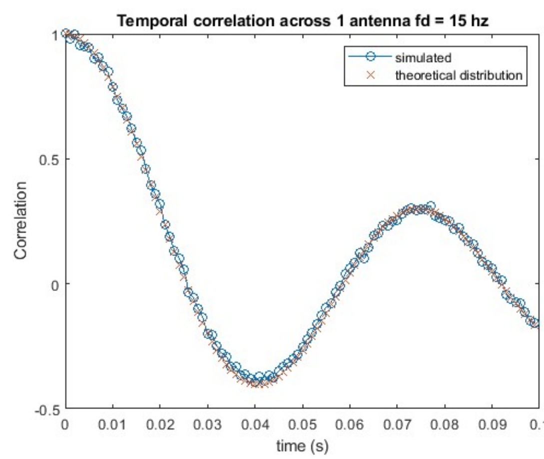


Figure 2: Q1(b). RX power on two receivers plotted vs experiment number.



(a) Temporal correlation with fd = 100Hz



(b) Temporal correlation with fd = 15Hz

Figure 3: Q1 (a),(c),(d)

3.2 Q2. Spatial correlation

For this part ergodicity was assumed and the channel was instantiated over 100 trials. The spatial correlation between two rx antennas was plotted for received antenna spacing d=$\frac{\lambda}{4}$ and d=$\frac{\lambda}{2}$ over the 100 trials. The angle spread of the DoA and AoD was sampled from a uniform distribution spanning $(0,2\pi)$. Observe that for antenna spacing d=$\frac{\lambda}{4}$ the spatial correlation is centered around 0.85 (high correlation) whereas for antenna spacing d=$\frac{\lambda}{2}$ it is centered around 0.2 (low correlation). The spatial correlation vs antenna spacing was also plotted and the simulation results closely follow the closed form expression given by:

$$\rho(d) = J_0\left(\frac{2\pi d}{\lambda}\right) \tag{2}$$

Here J_0 is the zeroth order bessel function of the first kind.

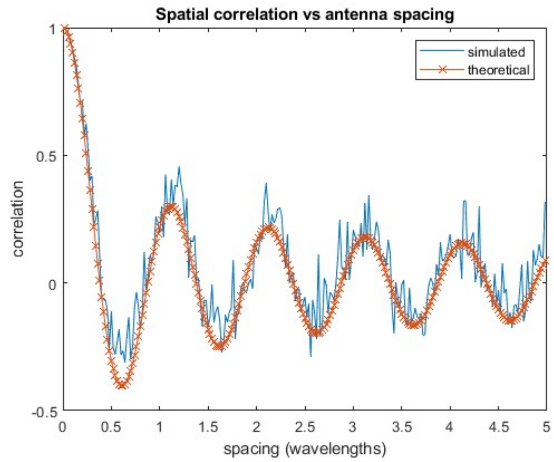


Figure 4: Q2. Spatial correlation vs antenna spacing for AoA,AoD ~Uniform$(0, 2\pi)$

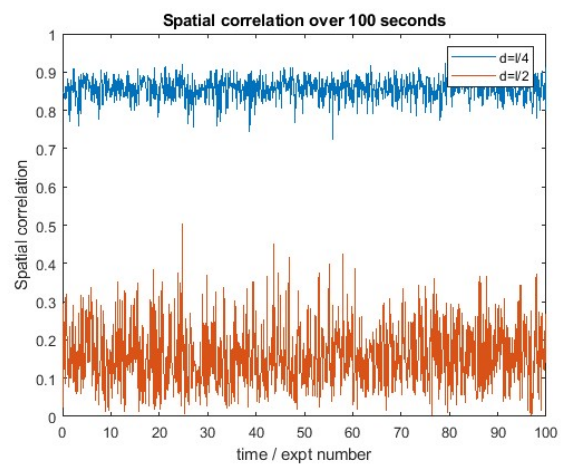


Figure 5: Q2. Spatial correlation between receivers plotted for d=$\frac{\lambda}{4}$ and d=$\frac{\lambda}{2}$ over 100 trials

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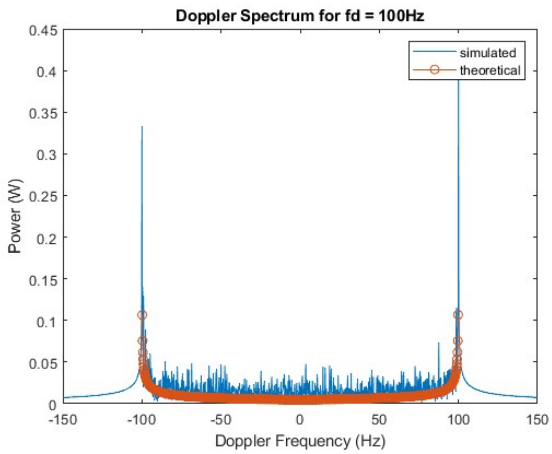
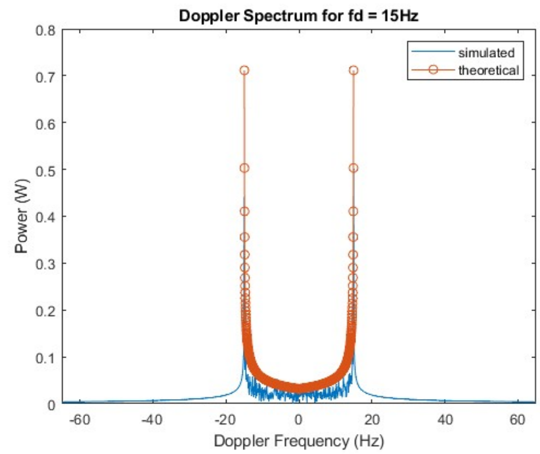
3.3 Q3. Doppler spectrum

The Doppler spectrum of the channel between 1rx and 1tx for fd=100Hz and fd = 15Hz is plotted. It matches the theoretical result plotted using the closed form expression and is bathtub shaped.



(a) Amplitude of channel with fd = 100 Hz



(b) Amplitude of channel with fd = 15 Hz

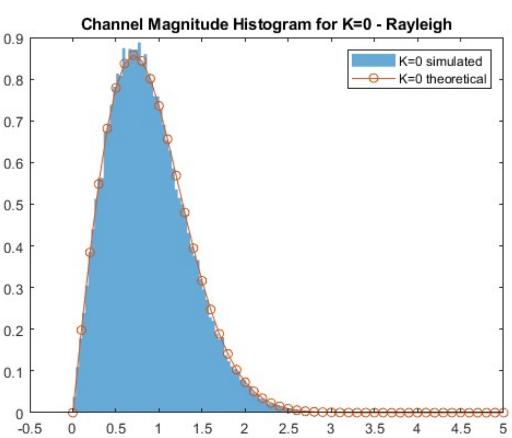
3.4 Q4. Fading and Maximal ratio combining

3.4.1 Q4 a),Q4) b). Rayleigh vs Ricean Fading

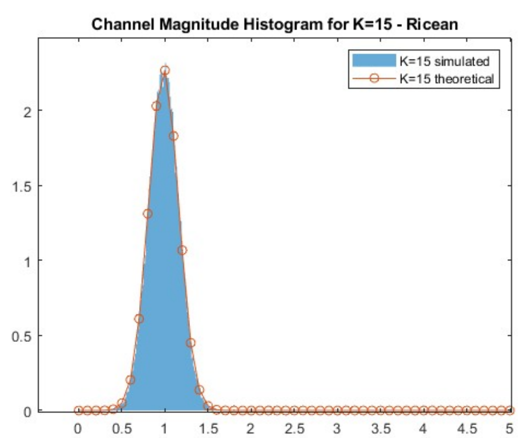
Rician Factor K=0 means we are in a fading environment with no Line of Sight or dominant path and the histogram of the magnitude is expected to follow the Rayleigh distribution. Rician Factor K=15 assumes the power of the dominant path is 15 times the power contained in the other multipath components. The Ricean distribution [1] for K=0 reduces to the Rayleigh distribution and is given by:

$$p_R(r) = \frac{2(K+1)r}{\Omega} \exp\left(-K - \frac{(K+1)r^2}{\Omega}\right) \cdot I_0\left(2r\sqrt{\frac{K(K+1)}{\Omega}}\right) \tag{3}$$

The histogram of the amplitudes for K=0 and K=15 matche the theoretical results as demonstrated in the figures below. Note: As K approaches ∞ the Ricean distribution approaches a Gaussian



(a) Amplitude of channel with fd = 100 Hz



(b) Amplitude of channel with fd = 15 Hz

distribution.

3.4.2 Q4 c).

Maximal ratio combining results in the maximisation of the SNR at the receiver. The formula governing this is:

$$\hat{r} = \sum_{i=1}^{N_r} h_i^* r_i \tag{4}$$

Since we are transmitting a single sine-wave this reduces to:

$$\hat{r} = \sum_{i=1}^{N_r} |h_i|^2 \tag{5}$$

The sum of N_r squared Rayleigh random variables is a Gamma distribution given by:

$$\hat{r} = \sum_{i=1}^{N_r} |h_i|^2 \sim \Gamma(N_r, \sigma^2) \tag{6}$$

We see the simulated and theoretical plots of the magnitude of the Maximal ratio combining for spatially uncorrelated channel below

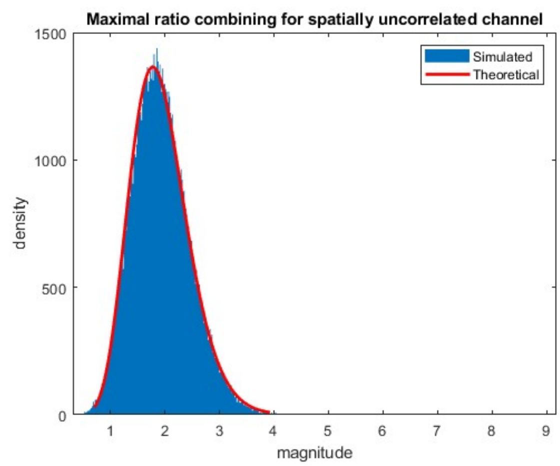


Figure 8: Q4 c). Maximal ratio combined magnitude plot for uncorrelated channels

We see the same plot for correlated channels below. The average power in the maximal ratio combined signal is calculated for both uncorrelated channels as well as correlated channels and is shown in the table below. Since the antennas are close together and the Angle spread is very low, we expect the channels between two receivers (or transmitters) to only differ in the phase introduced by the array geometry. Hence when we employ Maximal ratio combining we see a much higher received signal power in the correlated channel case.

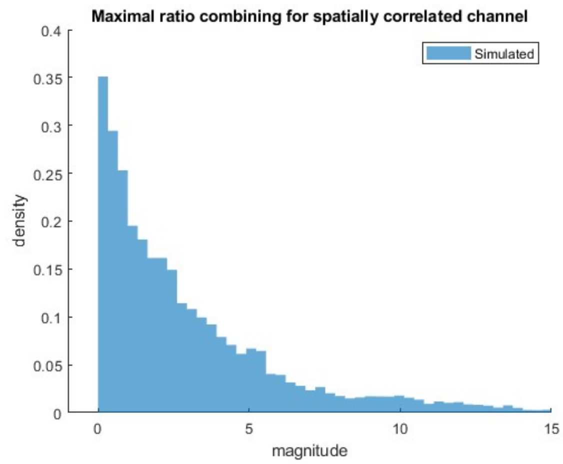


Figure 9: Q4 c). Maximal ratio combined magnitude plot for correlated channels

SNo.	Uncorrelated	Correlated
Power	4.003	73.220

3.5 Q5 b. Eigenvalues of correlated channels

For this part, the channel function with angular spread modelled by a Gaussian distribution was used. Looking at the The spatial correlation of two channels between 1 rx and 2 tx over a range of antenna spacing. The AoA and AoD spread of each multipath was modelled by a Gaussian distribution with parameters $\mu = 0, \sigma = 90$ degrees. Using this we find out the antenna spacing for which we see correlation values of 0.1,0.02 and 0.8. The antenna distances corresponding to the correlations required are 0.32,1.88 and 2 wavelengths. Hence we model three different channels using AoA, AoD modelled by a Gaussian distribution with mean 0 and standard deviation 90 degrees and the rx/tx side antenna spacing changed appropriately to get the desired correlation.

The eigenvalue decomposition for a given channel reveals the equivalent orthogonal channels with the gain represented by the eigenvalues. On plotting the max eigenvalue histograms for the three channels, the highly correlated channels are centred around a lower value compared to channels with a lower correlation. The latter are centred around higher values along the x-axis. This aligns with the intuition that correlated channels are worse compared to uncorrelated channels. The theoretical distribution of the largest eigenvalue of the channel matrix was first derived in [2] assuming the channel matrix is zero mean. The distribution of the largest eigenvalue of the channel matrix for non zero mean was derived in [3]

3.6 Q6. Capacity

The capacity of the channel at time instant indexed by i is given by:

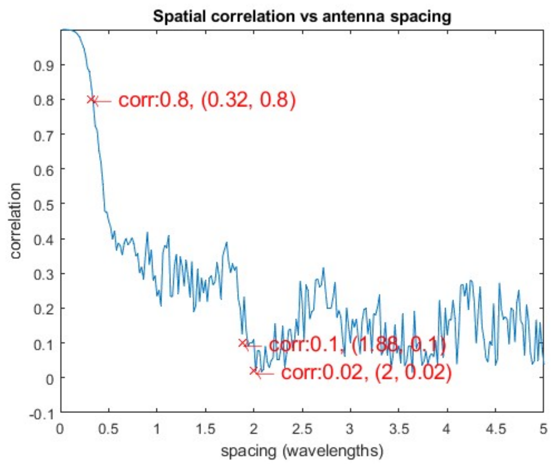


Figure 10: Q5. Spatial correlation for angle spread with $N(\mu = 0, \sigma = 90)$ for angle spacing from d=0 to d=5 wavelengths

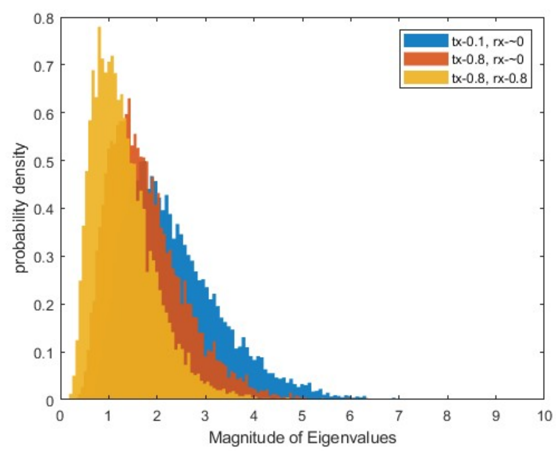


Figure 11: Max eigenvalue histograms for channels with different correlation

$$C_i = log_2\left(Nr \cdot \mathbf{I} + \sigma^2 \frac{\mathbf{H}_i \mathbf{H}_i^H}{N_t}\right) \tag{7}$$

Here $\mathbf{H}_i$ is the N_{tr} channel matrix at time instant indexed by i. σ^2 is the average SNR per channel, N_t and N_r are the tx side and rx side antennas. Assuming ergodicity a histogram of the instantaneous capacity for the three channels generated in Q5 is plotted below. For a given capacity C on the x axis, the y axis specifies the probability of achieving a capacity equal to or below the given capacity or outage capacity. The ergodic capacity for the channels is summarised below and follows the intution that higher correlation has lower cpacity, as well as lower capacity for lesser rx-tx antennas.

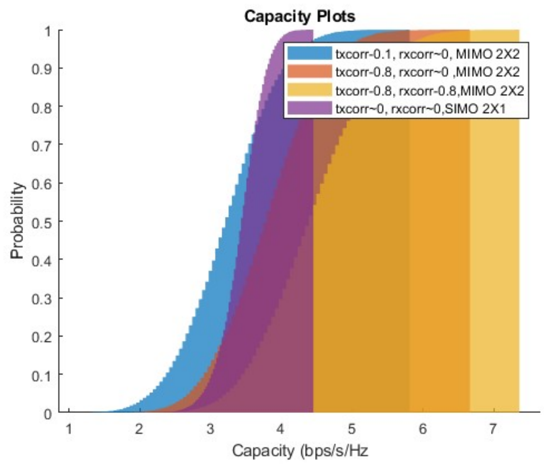


Figure 12: Q6. Capacity for various channels with low to high correlation and different MIMO configurations

SNo.	tx corr	rx corr	array	Capacity(bps/Hz)
1.	0.1	~ 0	2x2	4.966
2.	0.8	~ 0	2x2	4.652
3.	0.8	0.8	2x2	4.268
4.	~ 0	~ 0	2x1	3.715

4 Conclusion

- A narrowband MIMO channel simulator was created with the ability to add temporal correlation and spatial correlation due to angle spread as well as antenna spacing.
- The magnitude plots under different fading conditions were studied and the empirical plots matched the theoretical plots for purely NLOS rayleigh fading and ricean distributions for fading with one dominant path.
- Correlated channels affect ergodic capacity by reducing it
- Capacity scales better with increasing the number of antennas than a single channel with high SNR.
- Maximal ratio combining increases SNR significantly when the channels are spatially correlated as compared to the uncorrelated case.

References

[1] C. Tepedelenlioglu, A. Abdi, and G. B. Giannakis, “The ricean k factor: estimation and performance analysis,” *IEEE Transactions on Wireless Communications*, vol. 2, no. 4, pp. 799–810, 2003.

[2] C. Khatri, “Distribution of the largest or the smallest characteristic root under null hypothesis concerning complex multivariate normal populations,” *The Annals of Mathematical Statistics*, vol. 35, no. 4, pp. 1807–1810, 1964.

[3] M. Kang and M.-S. Alouini, “Largest eigenvalue of complex wishart matrices and performance analysis of mimo mrc systems,” *IEEE Journal on Selected Areas in Communications*, vol. 21, no. 3, pp. 418–426, 2003.