Virginia Polytechnic Institute and State University

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MIMO HW2

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Contents

1 Introduction

1.1 Maximum SNR beamforming

In this assignment, we analyse various Beamforming algorithms. Assuming we have a desired signal impinging on an array with N antenna elements and uncorrelated thermal noise at every antenna, the received signal model at baseband is given by:

$$\tilde{\mathbf{r}} = \mathbf{a}(\theta) \cdot s + \mathbf{n} \tag{1}$$

s is a complex scalar representing the transmitted symbol, $a(\theta)$ is the received array response, n is the uncorrelated Gaussian noise at all the N antennas. $\tilde{\mathbf{r}}$ is the overall received signal.

The combiner, combines the received signals at each antenna and the output z is given by:

$$\begin{aligned} z &= \mathbf{w}^H \tilde{\mathbf{r}} \\ &= z_s + z_n \\ &= \mathbf{w}^H \mathbf{a} s + \mathbf{w}^H \mathbf{n} \end{aligned} \tag{2}$$

Here $\mathbf{w}$ are the weights used to combine and to maximise the SNR at the output of the combiner the weights are given by $\mathbf{w} = \mathbf{a}(\theta)$

1.2 MMSE

The MMSE signal model after matched filtering is given below:

$$\mathbf{r} = \mathbf{a}_0 s + \sum_{k=1}^{N_I} \mathbf{a}_k i_k + \mathbf{n} \tag{3}$$

Here a_k is the array response for the k^{th} interferer and a_0 is the signal of interest. The cost function $J(\mathbf{w})$ to minimise to get the MMSE weights $\mathbf{w}$ is given below

$$\begin{aligned} J(\mathbf{w}) &= \mathrm{E}\left\{\left|\mathbf{w}^H \mathbf{r} - s\right|^2\right\} \\ &= \mathbf{w}^H \mathrm{E}\left\{\tilde{\mathbf{r}}\tilde{\mathbf{r}}^H\right\} \mathbf{w} - \mathrm{E}\left\{s^* \tilde{\mathbf{r}}^H\right\} \mathbf{w} - \mathbf{w}^H \mathrm{E}\left\{\tilde{\mathbf{r}} s^*\right\} + \mathrm{E}\left\{s s^*\right\} \end{aligned} \tag{4}$$

On minimising the cost function by setting the gradient to zero, the optimal weights are given by:

$$\mathbf{w}_{mmse} = \mathbf{R}_{\mathbf{rr}}^{-1} \mathbf{a} \tag{5}$$

Here $\mathbf{R}_{\mathbf{rr}}$ is the received covariance matrix of the measurements given in eq.3. This equation is assuming perfect Channel estimation. In case the channel also needs to be estimated the MMSE weights are given by:

$$\mathbf{w}_{mmse} = \mathbf{R}_{\mathbf{rr}}^{-1} \cdot \mathrm{E}\left\{\tilde{\mathbf{r}} \cdot (s_{pilots})^*\right\} \tag{6}$$

2 Description

2.1 Input

The inputs are summarised below

- M: Number of transmit antennas.
- N: Number of receive antennas.
- c: speed of light.
- fc: carrier frequency.
- Aoa: Angle of Arrival of signal of interest at rx array.
- AoD: Angle of departure at tx array.
- AoAint: Angle of arrival of interferer 1
- AoAint2: Angle of arrival of interferer 2
- Ntrials: Number of trials of the experiment
- Ns: Number of tx symbols
- NPilots: Number of pilot symbols

2.2 Output

The out are the beampatterns and bit error rate plots for the various scenarios described.

3 Validation

3.1 Max SNR Beamforming and Array Factors

3.1.1 Q1a), Q1b) Max SNR Beamforming

For 1 transmitting antenna (M=1), we transmit in all directions. Using max SNR beamforming weights the SNR is maximised. The optimal SNR is γ_{opt} at the receive side is given by:

$$
\begin{aligned}
\gamma_{opt} &= \frac{E_s}{\sigma_n^2} \frac{\left|\mathbf{a}^H \mathbf{a}\right|^2}{\mathbf{a}^H \mathbf{a}} \\
&= \mathbf{a}^H \mathbf{a} \frac{E_s}{\sigma_n^2} \\
&= N \frac{E_s}{\sigma_n^2}
\end{aligned} \tag{7}
$$

To plot the BER curve vs SNR for each SNR, 100 uniformly sampled QPSK symbols were transmitted from the tx side. At the receive side the max SNR combining weights were used to combine the signals received across the 4 antennas. The QPSK symbols were then decoded and compared with the transmitted symbols to calculate the BER curve. 100 trials were performed at each SNR.

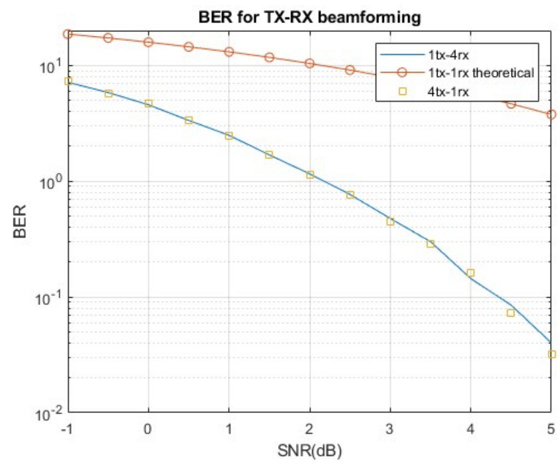


Figure 1: Q1(a). BER curve for 1TX-4RX and 4TX-1RX system for SNR ranging from 5dB/rx antenna to -1dB/rx antenna and theoretical BER for a QPSK system

In fig. 1 for a 1tx-4rx and 4tx-1rx antenna system the Bit error rate (BER) is lower compared to a 1tx-1rx system. This is as expected since we have used maximum SNR beamforming at the rx side using 4 antennas to boost the SNR hence reducing the BER. The BER of a 4tx-1rx system is the same as for a 4rx-1tx system. This is only valid if we have knowledge of the channel at the transmitter which is assumed in this case. Maximum SNR beamforming coherently adds the signals received at the rx antennas so the there is no effect of changing the Angle of Arrival (AoA) and Angle of Departure (AoD).

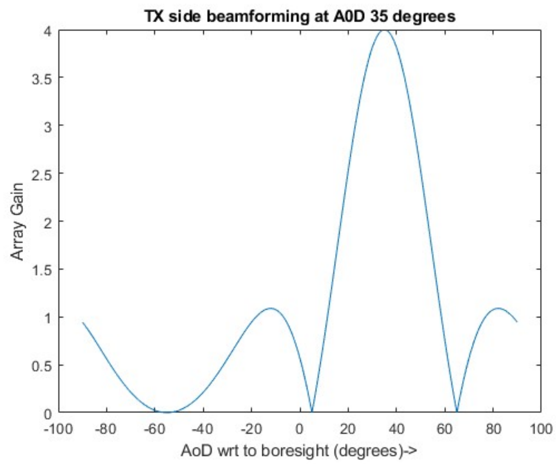


Figure 2: Q1(c). Array Factor for 4 Tx antenna beampattern with main lobe at 35°

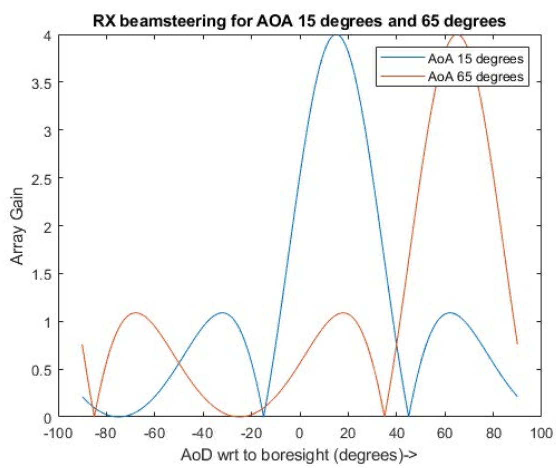


Figure 3: Q1(d). Receive beampatterns for received angles 15°and 65°

3.1.2 Q1c), Q1d) Array Factor and Beamsteering

3.2 Q2. MMSE Beamforming results

3.2.1 Q2a). MMSE vs MaxSNR with perfect CSI and channel estimation - BER vs NPilots

In fig.7, MMSE-perfectCSI needs lesser pilots than MMSE-estCSI as expected. The MaxSNR technique does not vary with number of pilots used in this case since the optimal weight is assumed to be known apriori. This is essentially same as saying assuming perfect channel estimation. For further analysis, Npilots = 50 is assumed.

Since for maxSNR the optimal weights are assumed, the observation at low SNR is maxSNR works better. However as SNR increases, the MMSE weights should give the same performance. This is observed in fig.5 as the SNR is increased the BER for the two beamformers converge

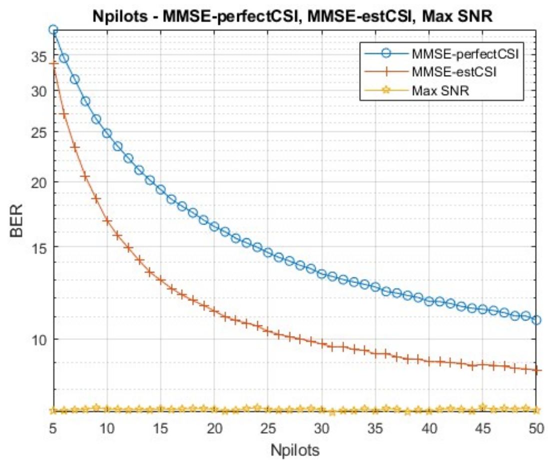


Figure 4: Q2a). The BER curves for MMSE-perfectCSI, MMSE-imperfectCSI and maxSNR plotted with number of pilots at SNR = -1dB/rxantenna

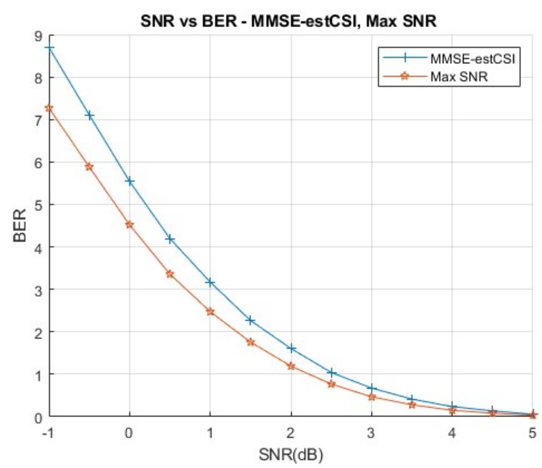


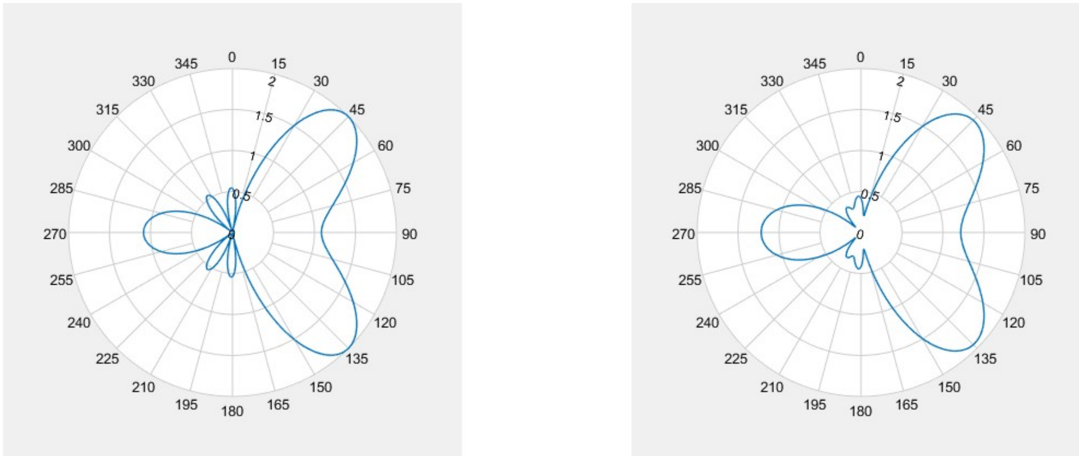
Figure 5: Q2a). The BER curves for MMSE-imperfectCSI and maxSNR plotted vs SNR

3.2.2 Q2b). Beam-pattern

The beam-pattern for MMSE with channel estimation with Npilots=50 with Signal of Interest (SOI) at Angle of Arrival (AoA) at 45 degrees in an AWGN channel. The Beam pattern is shown in fig.6. The SNR per rx antenna is 5db, and both the beampatterns look very similar. They would match exactly at higher SNR's or if more pilots were used.

3.2.3 Q2c),Q2d) MMSE and maxSNR with interferers

For this section, two beamformers - maxSNR and MMSE are used for three scenarios - No interferers, 1 interferer at 0 degrees, 2 interferers at 0,25 degrees in AWGN channel. The average interference power is considered to be the same as the signal of interest power (SIR = 0dB) whereas



(a) maxSNR beampattern with SOI at AoA = 45 deg (b) MMSE beampattern with SOI at AoA = 45 deg

Figure 6: Q2b). max SNR vs MMSE beampatterns with no interference

the noise power is varied with the SNR. The signal of interest is at AoA 45 degrees.

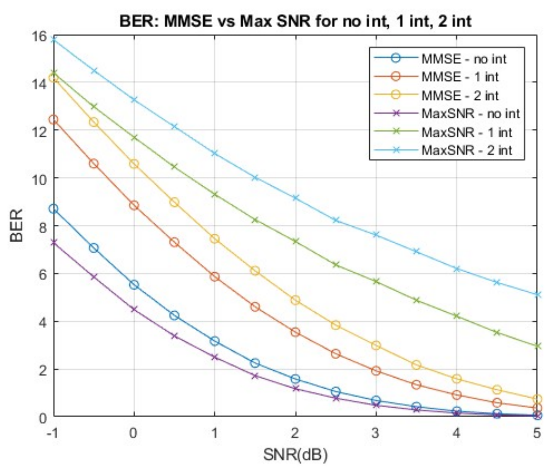
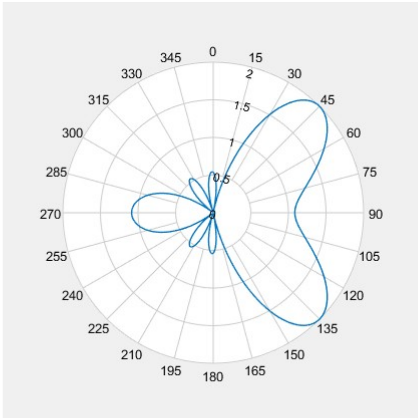
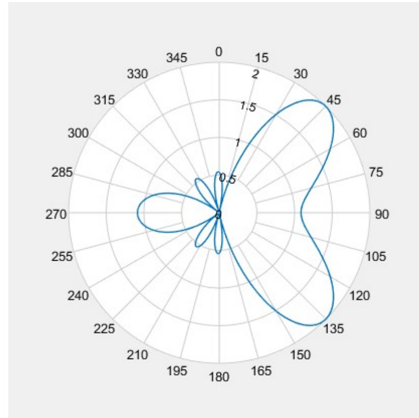


Figure 7: Q2a). The BER curves for MMSE-imperfectCSI and maxSNR plotted vs SNR

The maxSNR with no interference has the same performance as MMSE (assuming adequate SNR). As soon as interference signals are added at different AoA, MMSE outperforms maxSNR since it maximises the Signal to Interference+Noise Ratio and not just SNR. The MMSE weights try to suppress the interference as well as maximise the signal gain and this is observed in the beampattern plots in fig.8 and fig.9. As we introduce interferers at 0 degrees and 25 degrees, MMSE weights reduces the gain in the direction of the interferers.

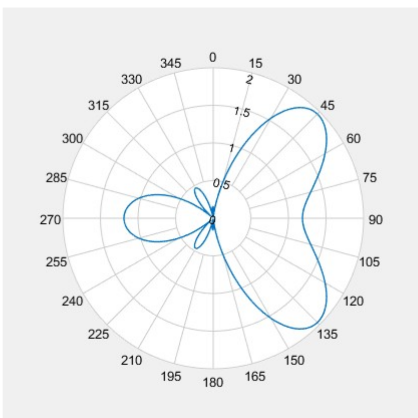


(a) Beampattern MMSE with 0 int

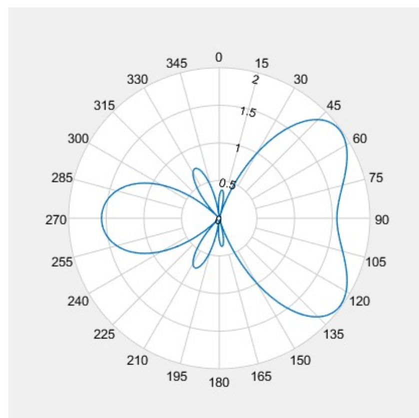


(b) Beampattern Max SNR with 0 int

Figure 8: Q2c),Q2d) Beam Pattern plots (a) MMSE with 0 interference, (b) maxSNR with 0 interference



(a) Beampattern MMSE with 1 int at 0 deg



(b) Beampattern Max SNR with 2 int at 0,25 deg

Figure 9: Q2c),Q2d) Beam Pattern plots (a) MMSE with 1 interferer, (b) MMSE with 2 interferers

3.2.4 Q2e) MMSE vs RLS

Recursive Least Squares is an adaptive algorithm that adapts the beamforming weights by minimising the squared error by weighting the current measurements more than the older measurements. In case the channel is changing i.e. for example the AoA of the signal of interest is changing, the weights need to adapt accordingly to track it. MMSE works by collecting a set of measurements and then minimising the mean squared error over all of the collected samples. Whereas, RLS minimises the squared error within a small window (controlled by the parameter χ) by weighting current samples more than historical samples and that is the adaptive nature of the algorithm. For this section, two interferers were placed at AoA 0,25 degrees and the AoA of the signal of interest was varied according to a sine wave centred around 45 degrees.

In the fig.10 , as time progresses, the signal's AoA changes, the MMSE algorithm minimises the mean squared error over the collected samples. The RLS algorithm minimises the weighted error and is able to track the changing AoA of the signal of interest hence resulting in low BER, whereas MMSE settles to an average low BER which remains relatively unchanging as we include more symbols.

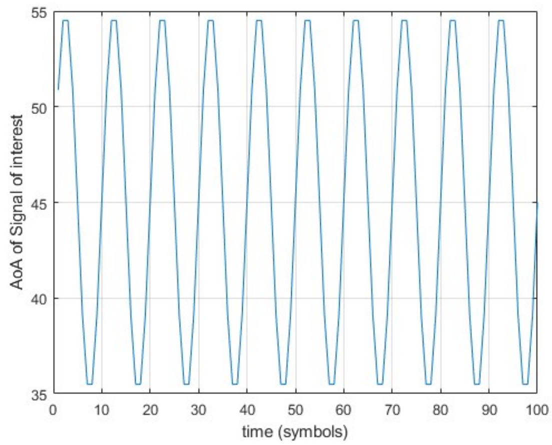


Figure 10: Q2e). The signal of interest's AoA is varied as shown

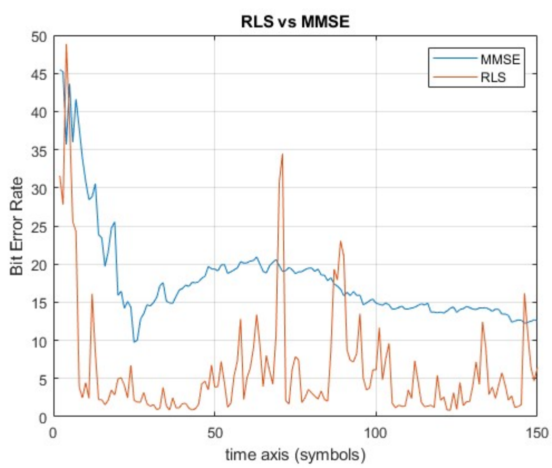


Figure 11: Q2e). BER for RLS vs MMSE with time

3.2.5 Q2f).Beamforming in Rayleigh Fading channel

The signal model for beamforming in rayleigh fading channel is shown as below

$$\mathbf{r} = \mathbf{h}\left(\mathbf{a}_0 \cdot s + \sum_{k=1}^{N_I} \mathbf{a}_k i_k + \mathbf{n}\right) \tag{8}$$

Here $\mathbf{h}$ is the $N \times 1$ Rayleigh fading channel coefficient. N is the number of receive antennas. The MMSE weights are given by eq.6. The plot fig.12 shows that in a Rayleigh fading channel, the

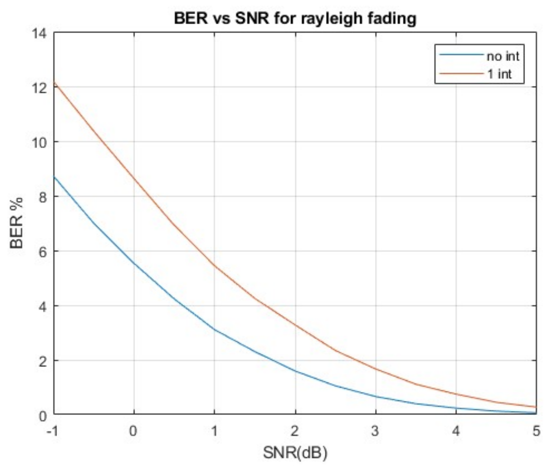


Figure 12: Q2f). BER for RLS vs MMSE with time

BER varies linearly with SNR. On addition of an interferer, as expected the BER is worse due to increased interference. At higher SNR, due to better estimation of the channel and covariance matrix, the two converge.

4 Conclusion

- maxSNR beamforming and MMSE beamforming was simulated.
- MMSE weights are same as maxSNR weights in an AWGN channel provided the channel estimation is perfect
- MMSE maximises the signal to interference+noise ratio and performs better than maxSNR beamforming if there are interferers present.
- RLS is an adaptive algorithm that minimises the squared error in a weighted window of measurements and is able to track the changing channel. MMSE minimises the mean squared error across all measurements.

- in a Rayleigh fading channel, the BER is linearly dependent with the SNR. MMSE can be employed here to minimise the mean squared error.