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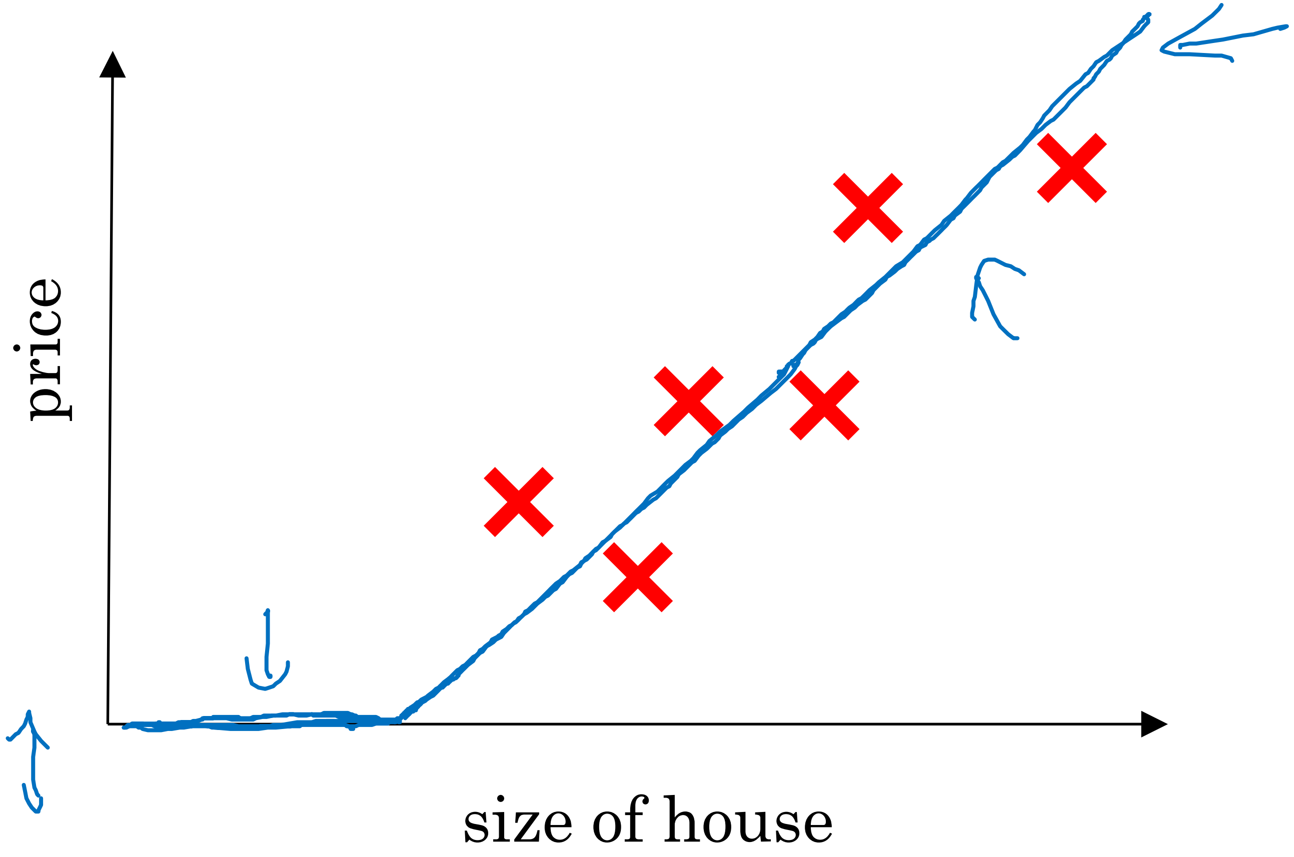
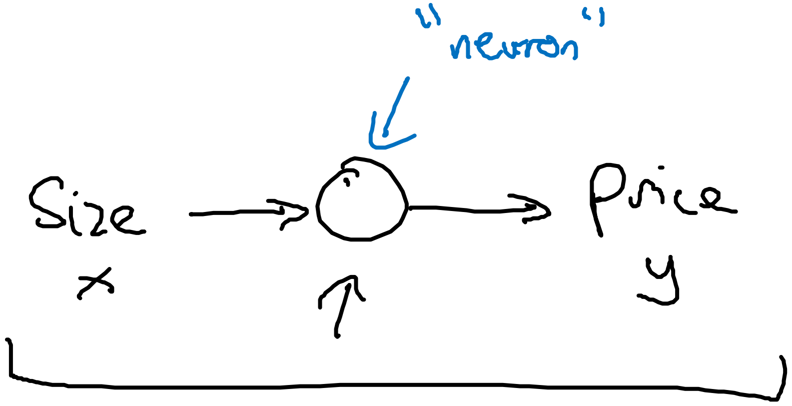
Introduction to Deep Learning

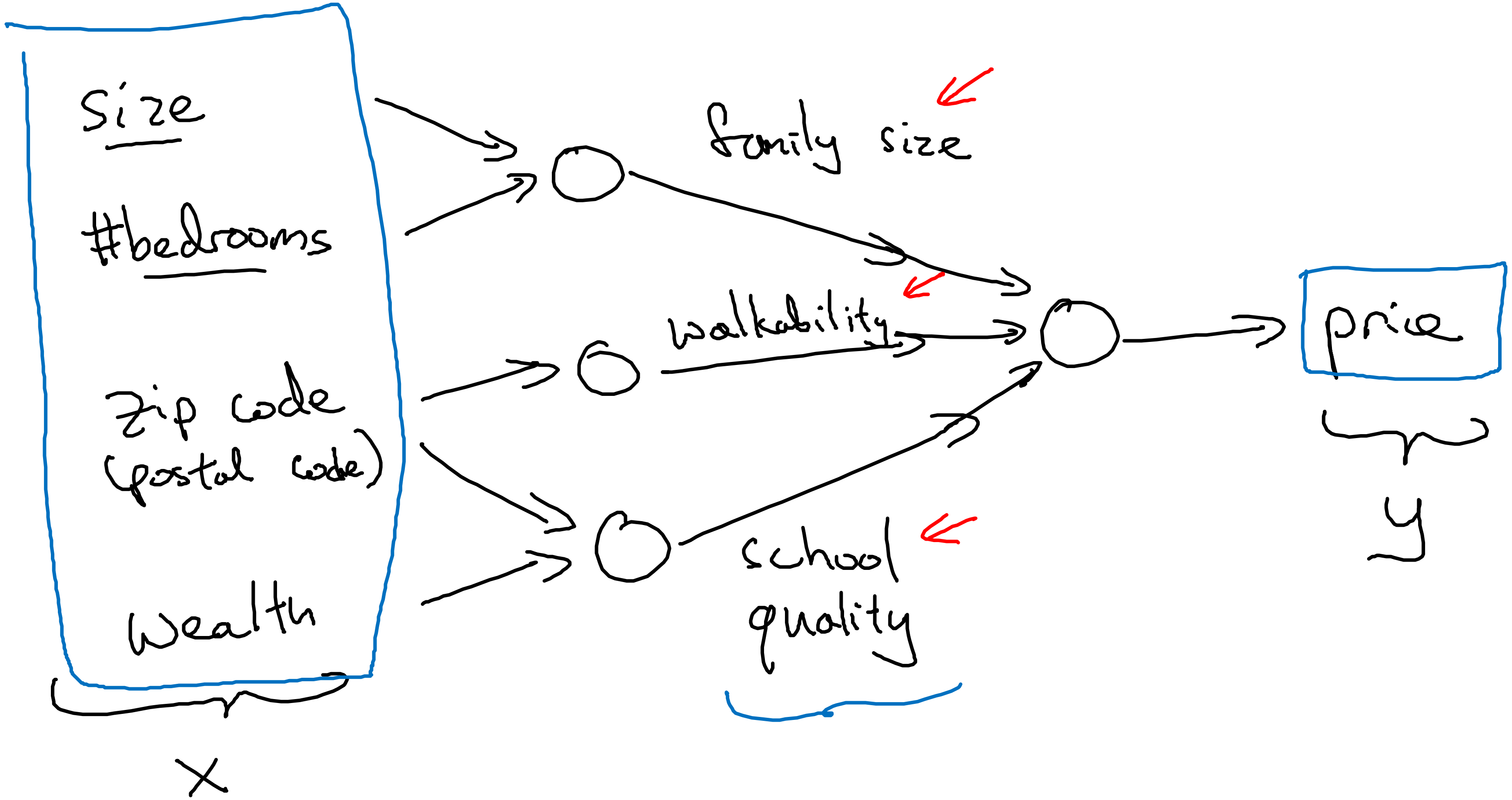
What is a Neural Network?

Housing Price Prediction



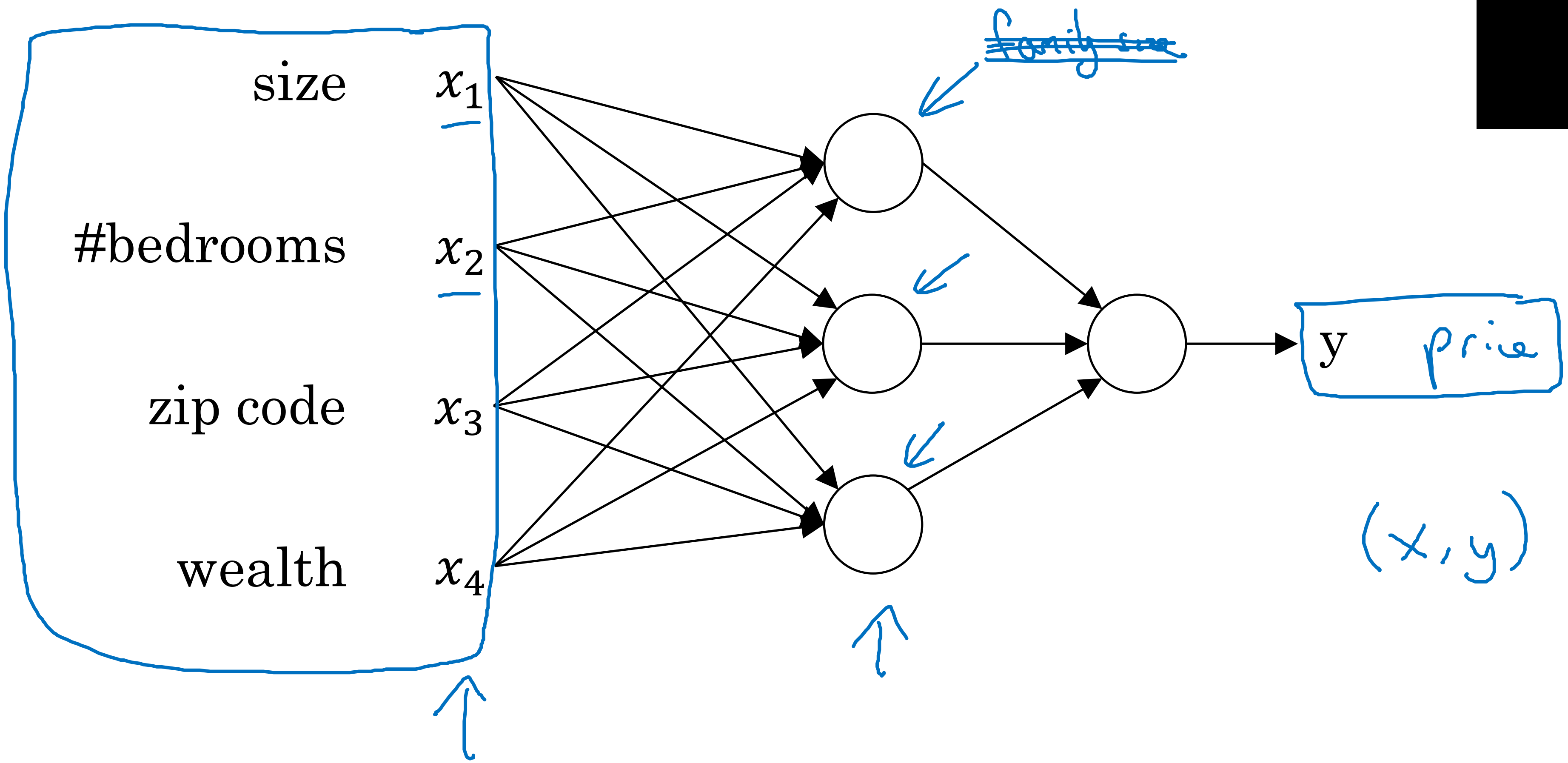


Housing Price Prediction



Housing Price Prediction

Drawing of previous Image



size x_1

#bedrooms x_2

zip code x_3

wealth x_4

y

~~family size~~

price

(x, y)

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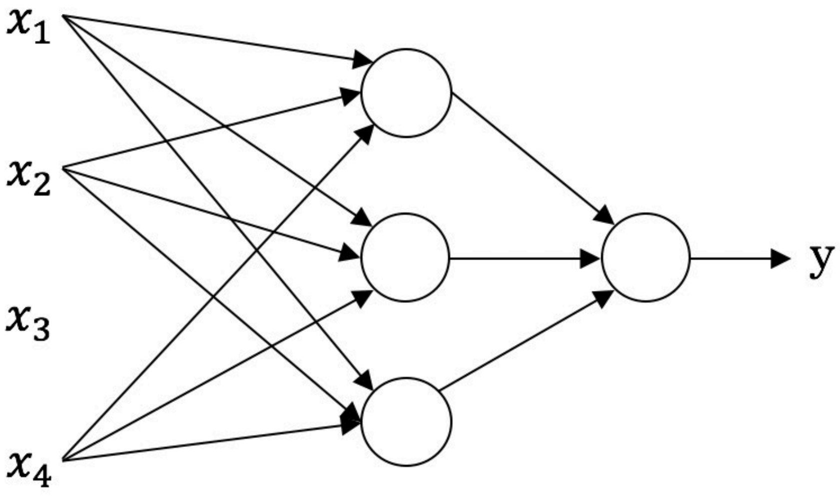
Introduction to Deep Learning

Supervised Learning with Neural Networks

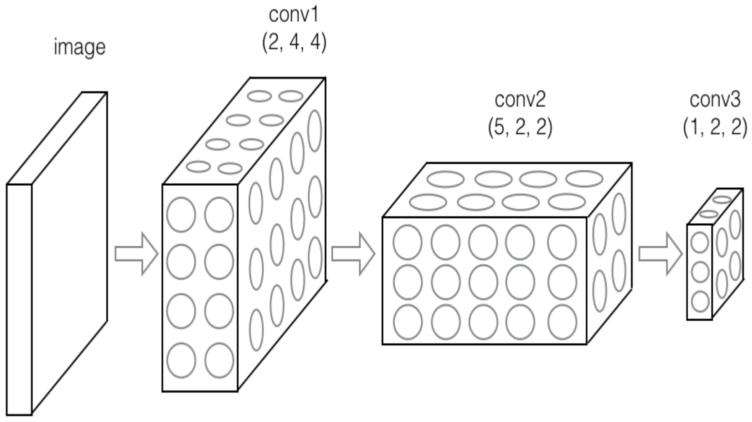
Supervised Learning

Input(x)	Output (y)	Application	
Home features	Price	Real Estate	Standard NN
Ad, user info	Click on ad? (0/1)	Online Advertising	
Image	Object (1,…,1000)	Photo tagging	CNN
Audio	Text transcript	Speech recognition	RNN
English	Chinese	Machine translation	
Image, Radar info	Position of other cars	Autonomous driving	Custom/Hybrid

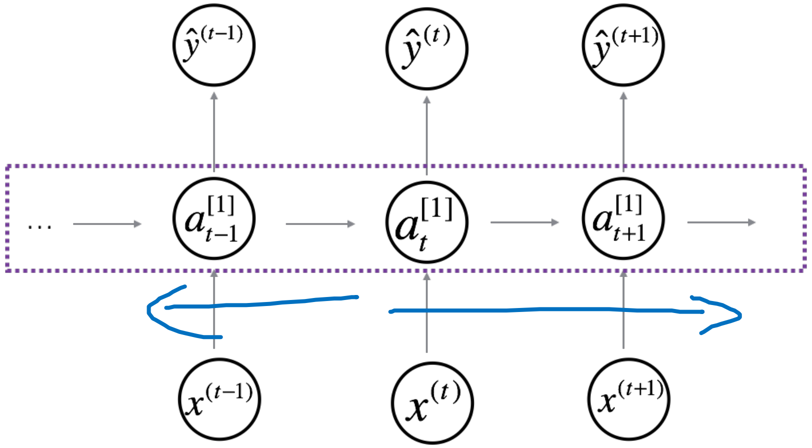
Neural Network examples



Standard NN



Convolutional NN



Recurrent NN

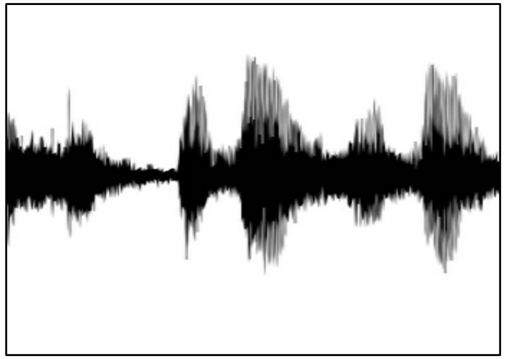
Supervised Learning

Structured Data

Size	#bedrooms	⋯	Price (1000$s)
2104	3		400
1600	3		330
2400	3		369
⋮	⋮		⋮
3000	4		540

User Age	Ad Id	⋯	Click
41	93242		1
80	93287		0
18	87312		1
⋮	⋮		⋮
27	71244		1

Unstructured Data



Audio

Image

Four scores and seven years ago…

Text

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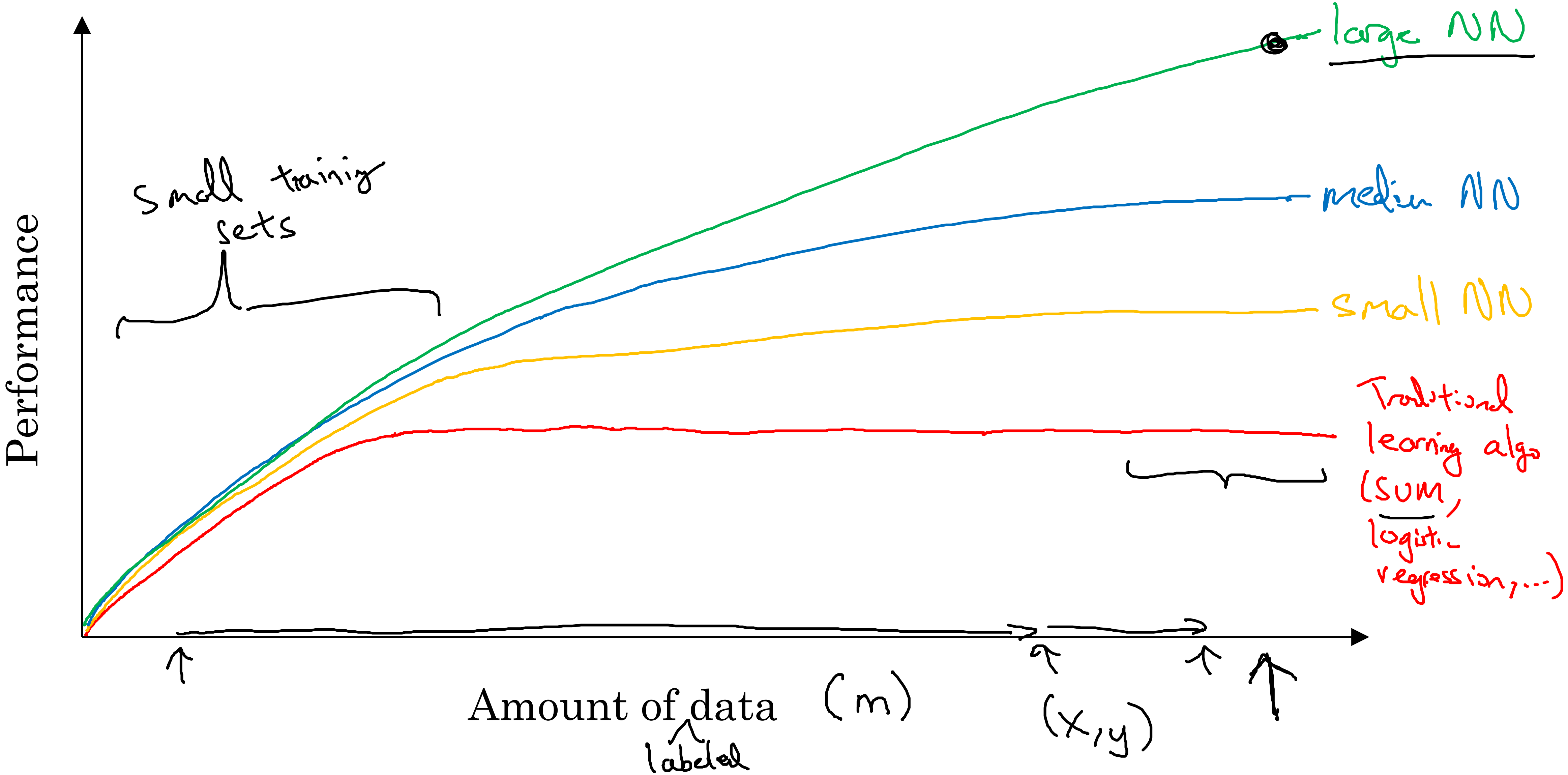
Introduction to Neural Networks

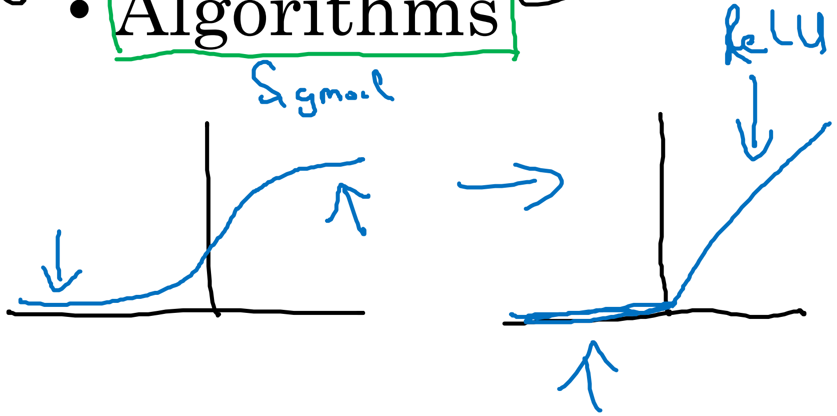
Why is Deep Learning taking off?

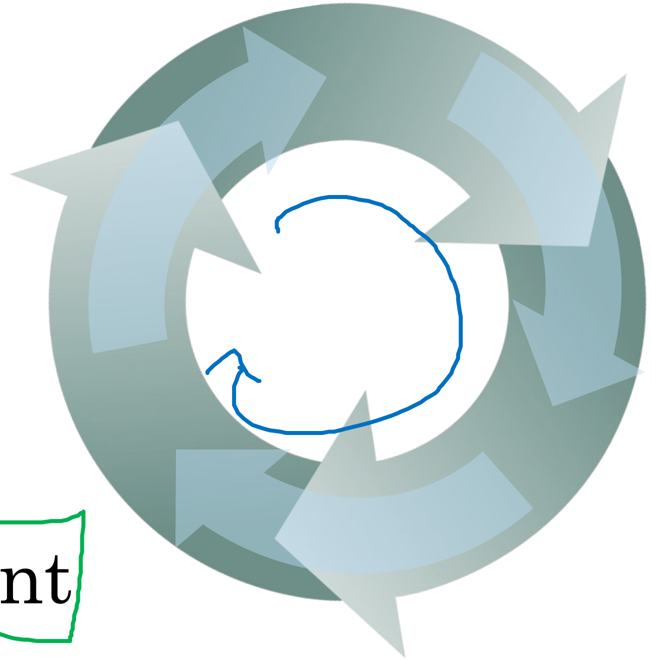
Scale drives deep learning progress



Scale drives deep learning progress

- Data
- Computation
- Algorithms





Idea

Code

Experiment

10 min ←
1 day ←
1 month

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Introduction to Neural Networks

About this Course

Courses in this Specialization

1. Neural Networks and Deep Learning ←
2. Improving Deep Neural Networks: Hyperparameter tuning, Regularization and Optimization
3. Structuring your Machine Learning project
4. Convolutional Neural Networks
5. Natural Language Processing: Building sequence models

Outline of this Course

Week 1: Introduction

Week 2: Basics of Neural Network programming

Week 3: One hidden layer Neural Networks

Week 4: Deep Neural Networks

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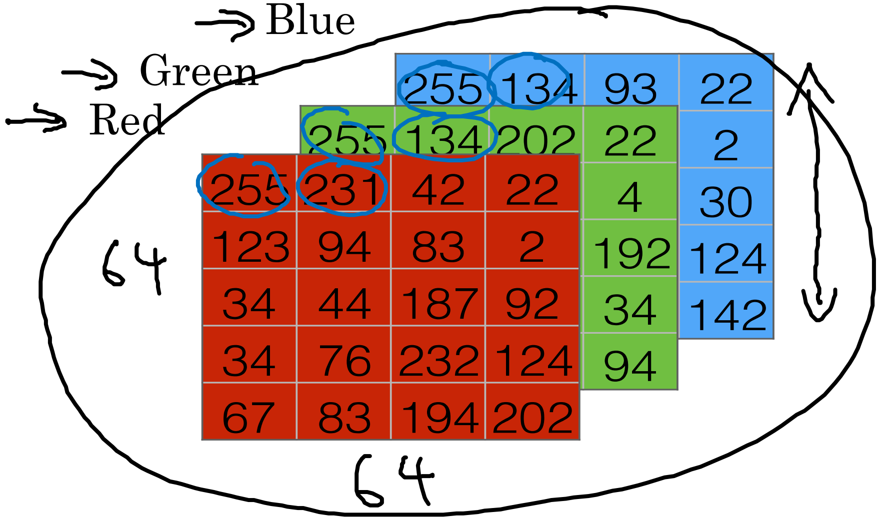
Basics of Neural Network Programming

Binary Classification

Binary Classification

$\underline{1}$ (cat) vs $\underline{0}$ (non cat)

y



→ Blue

255	134	93	22
			2
			30
			124
			142

→ Green

255	134	202	22
			4
			192
			34
			94

→ Red

255	231	42	22
123	94	83	2
34	44	187	92
34	76	232	124
67	83	194	202

$$x = \begin{bmatrix} 255 \\ 231 \\ \vdots \\ \vdots \\ 255 \\ 134 \\ \vdots \end{bmatrix}$$

$$64 \times 64 \times 3 = 12288$$

$$n = n_x = 12288$$

$$x \longrightarrow y$$

Notation

(x, y) $\quad x \in \mathbb{R}^{n_x}, \; y \in \{0, 1\}$

m training examples : $\{(\underline{x^{(1)}}, \underline{y^{(1)}}), (\underline{x^{(2)}}, \underline{y^{(2)}}), \ldots, (\underline{x^{(m)}}, \underline{y^{(m)}})\}$

$M = M_{train}$ $\qquad M_{test}$ = #test examples.

$$X = \underbrace{\begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}}_{m} \Big\updownarrow n_x \qquad \cancel{\begin{bmatrix} — & x^{(1)T} & — \\ & \vdots & \\ — & x^{(m)T} & — \end{bmatrix}}$$

$X \in \mathbb{R}^{n_x \times m}$ $\qquad X.\text{shape} = (n_x, m)$

$Y = [y^{(1)} \; y^{(2)} \; \ldots, y^{(m)}]$

$Y \in \mathbb{R}^{1 \times m}$

$Y.\text{shape} = (1, m)$

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Basics of Neural Network Programming

Logistic Regression

Logistic Regression

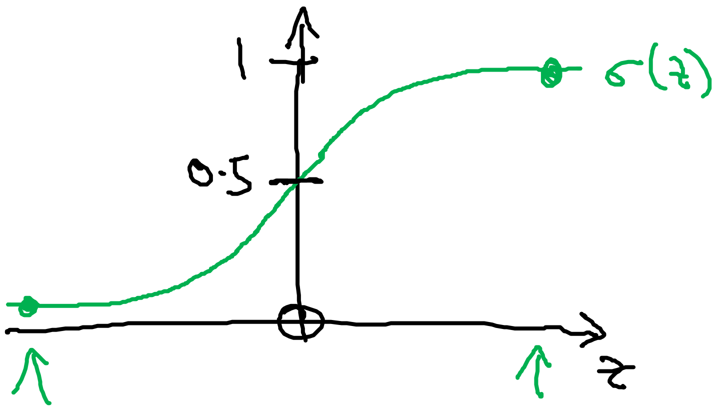
Given x, want $\underline{\hat{y}} = \underline{P(y=1 \mid x)}$

$\underline{0 \le \hat{y} \le 1}$

$x \in \mathbb{R}^{n_x}$

Parameters: $\boxed{\underline{w}} \in \mathbb{R}^{n_x}$, $\boxed{b} \in \mathbb{R}$.

Output $\hat{y} = \sigma(\underbrace{w^T x + b}_{z})$



$x_0 = 1$, $x \in \mathbb{R}^{n_x+1}$

$\hat{y} = \sigma(\theta^T x)$

$$\underbrace{\theta = \begin{bmatrix}\theta_0\\ \theta_1\\ \theta_2\\ \vdots\\ \theta_{n_x}\end{bmatrix} \begin{array}{l} \}\, b \leftarrow \\ \left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} w \leftarrow \end{array}}$$

$\sigma(z) = \dfrac{1}{1+\underline{e^{-z}}}$

If z large $\sigma(z) \approx \dfrac{1}{1+0} = 1$

If z large negative number

$\sigma(z) = \dfrac{1}{1+\underline{e^{-z}}} \approx \dfrac{1}{1+\text{Bignum}} \approx 0$

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Basics of Neural Network Programming

Logistic Regression cost function

Logistic Regression cost function

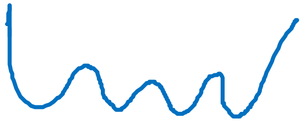
→ $\hat{y}^{(i)} = \sigma(w^T x^{(i)} + b)$, where $\sigma(z^{(i)}) = \frac{1}{1+e^{-z^{(i)}}}$ $z^{(i)} = w^T x^{(i)} + b$

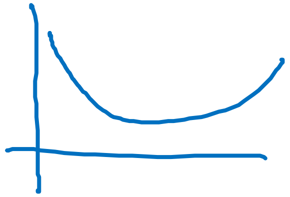
Given $\{(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})\}$, want $\hat{y}^{(i)} \approx y^{(i)}$.

$x^{(i)}$
$y^{(i)}$
$z^{(i)}$

i-th example.

Loss (error) function: $\mathcal{L}(\hat{y}, y) = \frac{1}{2}(\hat{y} - y)^2$





$\boxed{\mathcal{L}(\hat{y}, y)} = -(\boxed{y \log \hat{y}} + (1-y)\log(1-\hat{y}))$ ←

If $y=1$: $\mathcal{L}(\hat{y}, y) = -\log \hat{y}$ ← Want $\log \hat{y}$ large, want $\hat{y}$ large.

If $y=0$: $\mathcal{L}(\hat{y}, y) = -\log(1-\hat{y})$ ← Want $\log 1-\hat{y}$ large .... want $\hat{y}$ small

Cost function: $J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\hat{y}^{(i)} + (1-y^{(i)})\log(1-\hat{y}^{(i)})\right]$

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Basics of Neural Network Programming

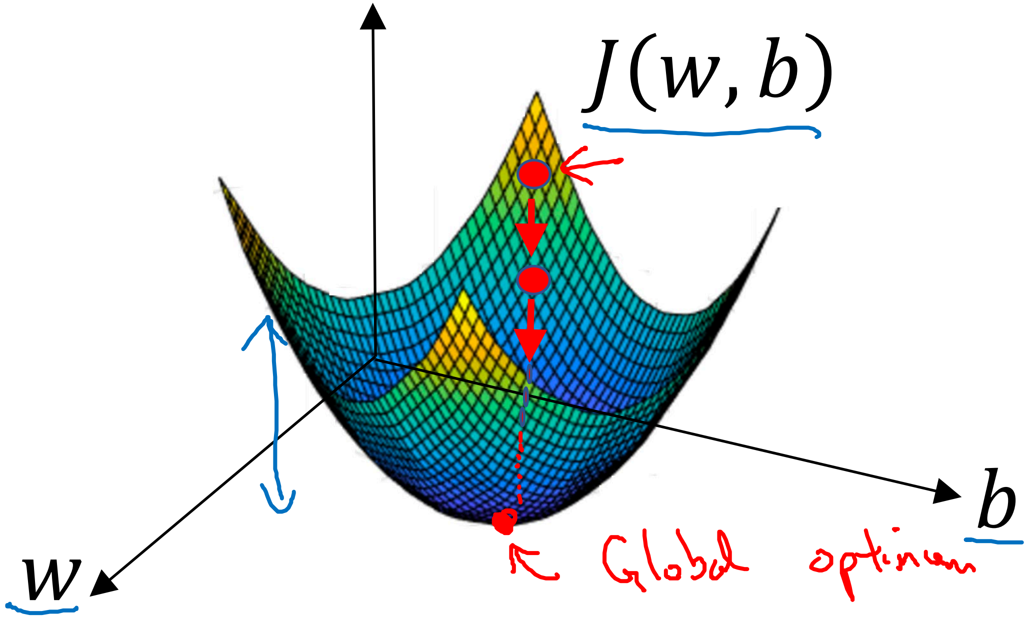
Gradient Descent

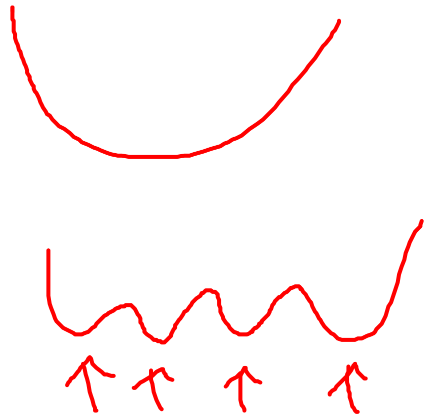
Gradient Descent

Recap: $\hat{y} = \sigma(w^T x + b)$, $\sigma(z) = \frac{1}{1+e^{-z}}$ $\leftarrow$

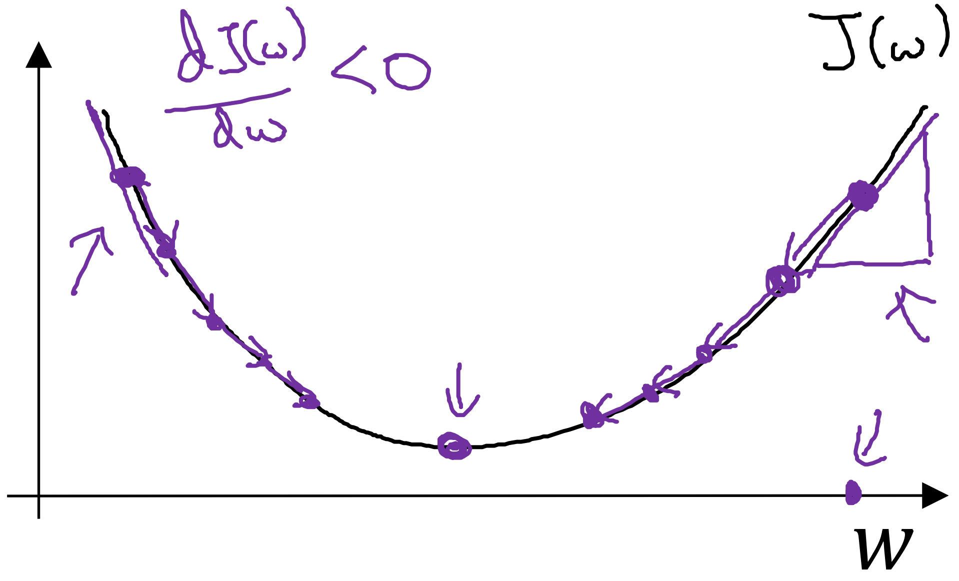
$$\underline{J(w,b)} = \frac{1}{m}\sum_{i=1}^{m} \underline{\mathcal{L}(\boxed{\hat{y}^{(i)}}, \boxed{y^{(i)}})} = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)} \log \hat{y}^{(i)} + (1-y^{(i)}) \log(1-\hat{y}^{(i)})$$

Want to find w, b that minimize $J(w,b)$





Gradient Descent



learning rate

Repeat {

$$w := w - \alpha \boxed{\frac{dJ(w)}{dw}}$$

}

"dw"

$$w := w - \alpha \underline{dw}$$

$$\underbrace{\frac{dJ(w)}{dw}} = ?$$

$J(w, b)$

$$w := w - \alpha \boxed{\frac{dJ(w,b)}{dw}} \qquad \boxed{\frac{\partial J(w,b)}{\partial w}} \rightarrow dw$$

$$b := b - \alpha \boxed{\frac{dJ(w,b)}{db}} \qquad \boxed{\frac{\partial J(w,b)}{\partial b}} \rightarrow db$$

∂ "partial derivative"

d J

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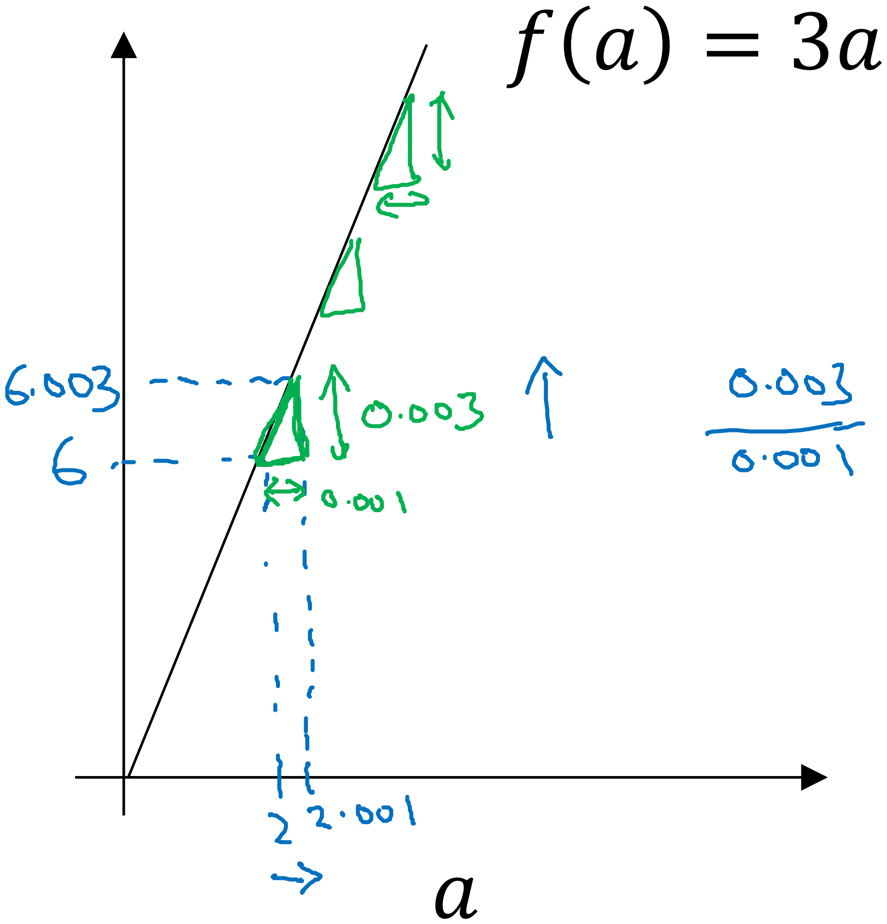
Basics of Neural Network Programming

Derivatives

Intuition about derivatives



$\frac{0.003}{0.001}$ $\frac{\text{height}}{\text{width}}$

→ $a = 2$ $f(a) = 6$

$a = 2.\underline{001}$ $f(a) = 6.\underline{003}$

slope (derivative) of $f(a)$ at $a=2$ is 3

→ $a = 5$ $f(a) = 15$

$a = 5.\underline{001}$ $f(a) = 15.\underline{003}$

slope at $a=5$ is also 3

$$\frac{d\,f(a)}{da} = 3 = \frac{d}{da}\,\underline{f(a)}$$

0.001

0.00000001

0.0000000001

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Basics of Neural Network Programming

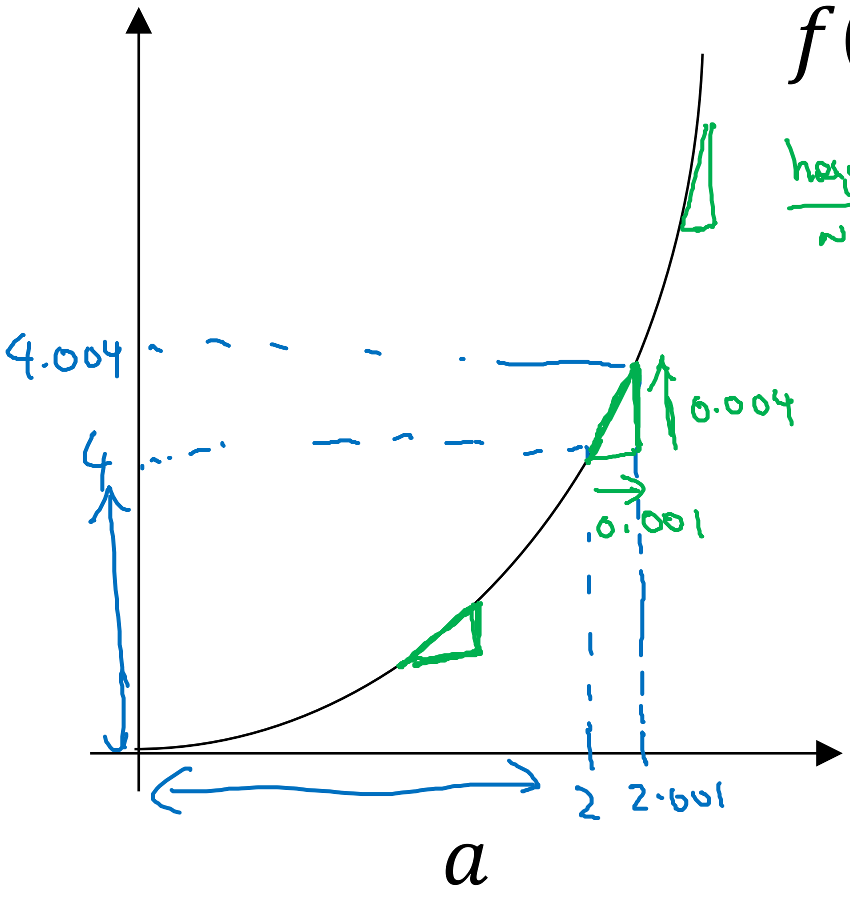
More derivatives examples

Intuition about derivatives

0.001 ←

0.00000.....01 ←

$$f(a) = a^2$$



$\frac{\text{height}}{\text{width}}$

$$\frac{d}{da} a^2 = 2a$$

0.001

$(2a) \times 0.001$

$a = 2$ $f(a) = 4$

$a = 2.001$ $f(a) \approx 4.004$

$(4.004\boxed{001})$

slope (derivative) of f(a) at $a=2$ is 4.

$\boxed{\frac{d}{da} f(a) = 4}$ when $\boxed{a=2}$.

$a = 5$ $f(a) = 25$

$a = 5.001$ $f(a) \approx 25.010$

$\boxed{\frac{d}{da} f(a) = 10}$ when $\boxed{a=5}$

$$\frac{d}{da} f(a) = \frac{d}{da} a^2 = \boxed{2a}$$

More derivative examples

$f(a) = a^2$

$\frac{d}{da} f(a) = \underbrace{2a}_{4}$

$a = 2$ $\quad f(a) = 4$

$a = 2.001$ $\quad f(a) \approx 4.004$

$f(a) = a^3$

$\frac{d}{da} f(a) = \underbrace{3a^2}_{3 \times 2^2 = 12}$

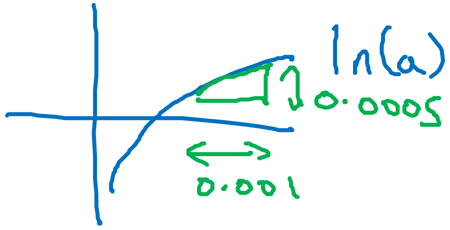
$a = 2$ $\quad f(a) = 8$

$a = \underline{2.001}$ $\quad f(a) \approx 8.\underline{012}$

$f(a) = \log_e(a)$

$\ln(a)$

$\frac{d}{da} f(a) = \underline{\frac{1}{a}}$



$\frac{d}{da} f(a) = \boxed{\frac{1}{2}}$

$a = 2$ $\quad f(a) \approx 0.69315$

$a = 2.\underline{001}$ $\quad \underline{f(a)} \approx 0.69365$

$0.0005 \longleftrightarrow \underline{0.0005}$

Basics of Neural Network Programming

Computation Graph

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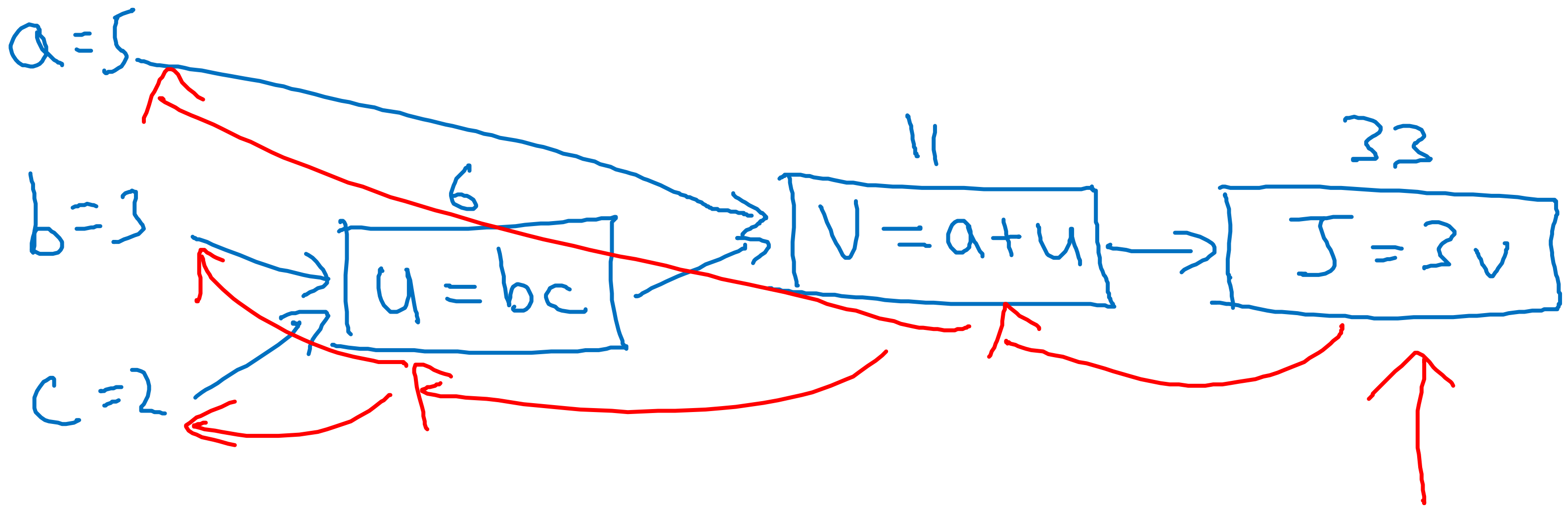
Computation Graph

$$J(a,b,c) = \underbrace{3(\underbrace{a+\underbrace{bc}_{u}}_{v})}_{J} = 3(5+3\times 2) = 33$$

$u = bc$

$v = a + u$

$J = 3v$



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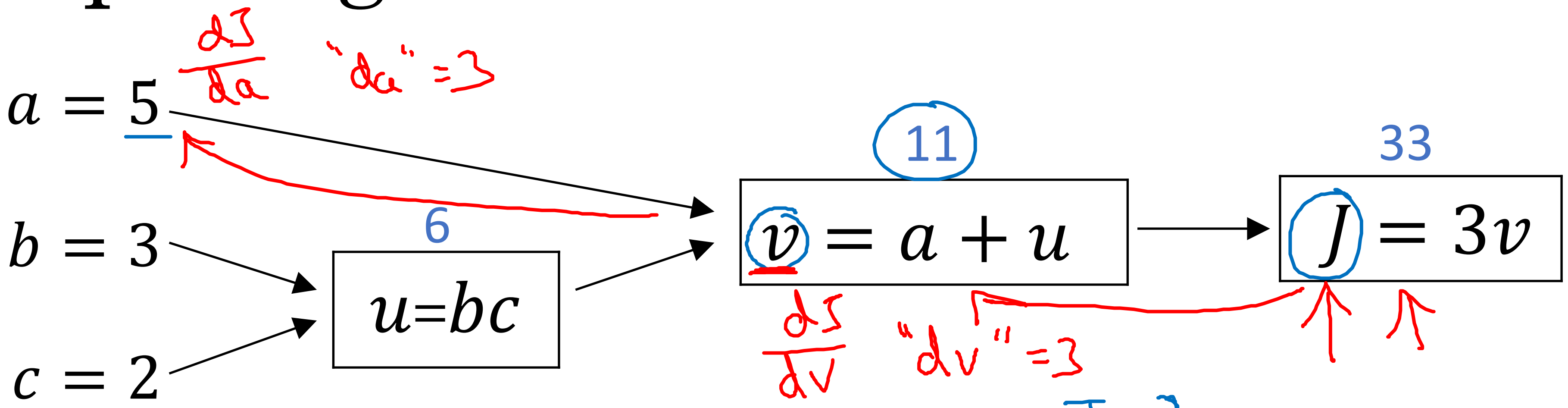
Basics of Neural Network Programming

Derivatives with a Computation Graph

Computing derivatives



$$\boxed{\frac{dJ}{dv} = ? = 3}$$

$$\frac{dJ}{da} = 3 = \frac{dJ}{dv}\frac{dv}{da}$$

3×1

$$\boxed{\frac{dv}{da} = 1}$$

$a \rightarrow v \rightarrow J$

$J = 3v$

$v = 11 \rightarrow 11.001$

$J = 33 \rightarrow 33.003$

$a = 5 \rightarrow 5.001$

$\rightarrow v = 11 \rightarrow 11.001$

$J = 33 \rightarrow 33.003$

dJdvar

$$\boxed{\frac{d\,FinalOutputVar}{d\,var}}$$

→ "dvar"

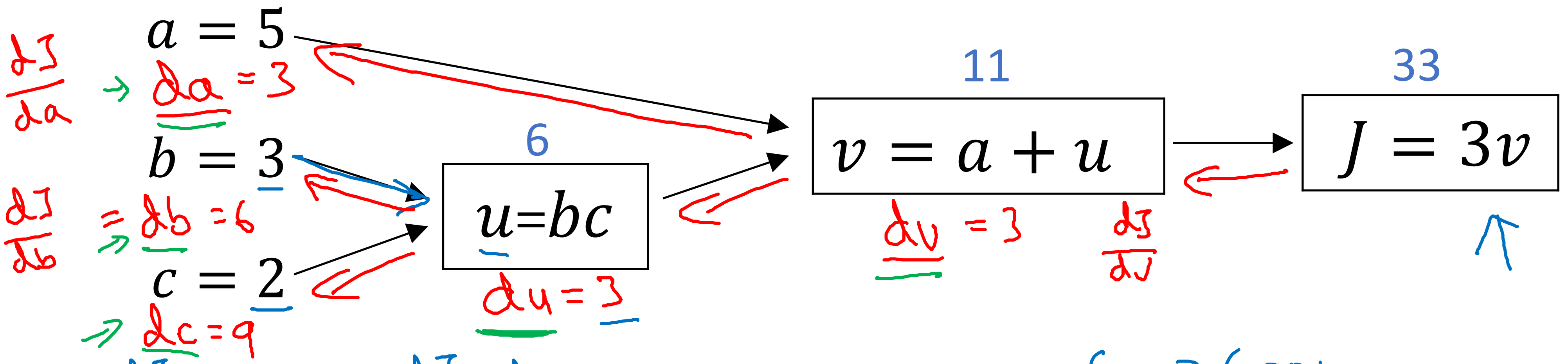
$$\boxed{f(a) = 3a}$$

$$\frac{df(a)}{da} = \frac{df}{da} = 3$$

$$\boxed{J = 3v}$$

$$\frac{dJ}{dv} = 3$$

Computing derivatives



$a = 5$

$\frac{dJ}{da} \to da = 3$

$b = 3$

$\frac{dJ}{db} = db = 6$

$c = 2$

$\to dc = 9$

6

$u=bc$

$du = 3$

11

$v = a + u$

$dv = 3$

$\frac{dJ}{dJ}$

33

$J = 3v$

$$\frac{dJ}{du} = 3 = \underbrace{\frac{dJ}{dv}}_{3} \cdot \underbrace{\frac{dv}{du}}_{1}$$

$$\frac{dJ}{db} = \underbrace{\boxed{\frac{dJ}{du}}}_{\to 3} \cdot \underbrace{\frac{du}{db}}_{=2} = 6$$

$$\frac{dJ}{da} = \underbrace{\boxed{\frac{dJ}{du}}}_{3} \cdot \underbrace{\frac{du}{da}}_{\times 3} = 9$$

$u = 6 \to 6.001$

$v = 11 \to 11.001$

$J = 33 \to 33.003$

$b = 3 \to 3.001$

$u = b \cdot c = 6 \to 6.002$

$c = 2$

$J = 33.006$

$.006$

$v = 11.002$

$J = 3v$

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Basics of Neural Network Programming

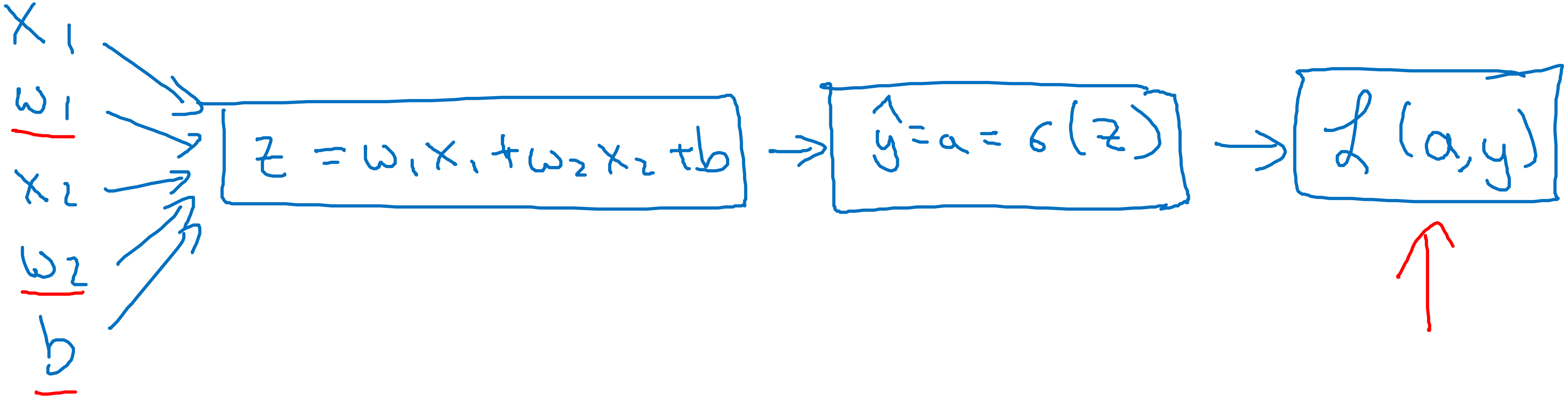
Logistic Regression Gradient descent

Logistic regression recap

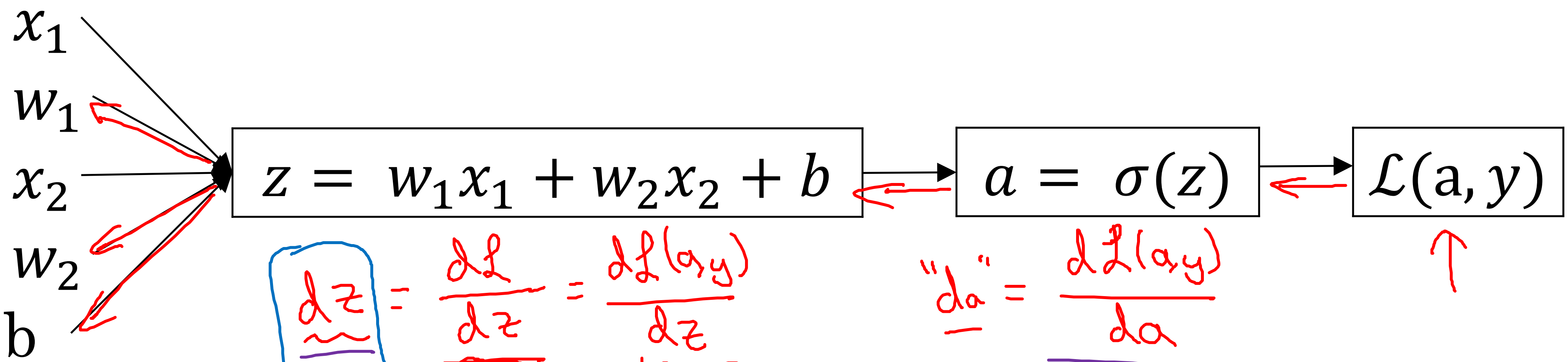
$$\rightarrow z = w^T x + b$$

$$\rightarrow \hat{y} = a = \sigma(\underline{z})$$

$$\rightarrow \mathcal{L}(a, y) = -(y \log(a) + (1 - y) \log(1 - a))$$



Logistic regression derivatives



x_1

w_1

x_2

w_2

b

$z = w_1 x_1 + w_2 x_2 + b$

$a = \sigma(z)$

$\mathcal{L}(a, y)$

$$dz = \frac{dL}{dz} = \frac{dL(a,y)}{dz}$$

$$= a - y$$

$$= \frac{dL}{da} \cdot \frac{da}{dz}$$

$$a(1-a)$$

$$\text{"}da\text{"} = \frac{dL(a,y)}{da}$$

$$= -\frac{y}{a} + \frac{1-y}{1-a}$$

$$\frac{\partial L}{\partial w_1} = \text{"}dw_1\text{"} = x_1 \cdot dz.$$

$$dw_2 = x_2 \cdot dz. \quad db = dz.$$

$$w_1 := w_1 - \alpha\, dw_1$$

$$w_2 := w_2 - \alpha\, dw_2$$

$$b := b - \alpha\, db.$$

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Basics of Neural Network Programming

Gradient descent on m examples

Logistic regression on m examples

$$\underline{J(w,b)} = \underbrace{\frac{1}{m}\sum_{i=1}^{m}\mathcal{L}(a^{(i)}, y^{(i)})}$$

$$(x^{(i)}, y^{(i)})$$

$$\rightarrow a^{(i)} = \hat{y}^{(i)} = \sigma(z^{(i)}) = \sigma(w^T x^{(i)} + b)$$

$$\underline{dw_1^{(i)}}, \underline{dw_2^{(i)}}, \underline{db^{(i)}}$$

$$\underline{\frac{\partial}{\partial w_1} J(w,b)} = \frac{1}{m}\sum_{i=1}^{m}\underbrace{\frac{\partial}{\partial w_1}\mathcal{L}(a^{(i)}, y^{(i)})}_{\underline{dw_1^{(i)}} \;-\; (x^{(i)}, y^{(i)})}$$

Logistic regression on m examples

```
$J=0$; $\underline{dw_1}=0$; $\underline{dw_2}=0$; $\underline{db}=0$
→ For $i=1$ to $m$
    $z^{(i)} = w^T x^{(i)} + b$
    $a^{(i)} = \sigma(z^{(i)})$
    $J += -[y^{(i)} \log a^{(i)} + (1-y^{(i)}) \log(1-a^{(i)})]$
    $\underline{dz^{(i)}} = a^{(i)} - y^{(i)}$
    $dw_1 += x_1^{(i)} dz^{(i)}$
    $dw_2 += x_2^{(i)} dz^{(i)}$
    $db += dz^{(i)}$
$J /= m$ ←
$dw_1 /= m$; $dw_2 /= m$; $db /= m$. ←
```

$\underline{n} = 2$

dw_3
$\vdots$
dw_n

$$dw_1 = \frac{\partial J}{\partial w_1}$$

$$w_1 := w_1 - \alpha \underline{dw_1}$$

$$w_2 := w_2 - \alpha \underline{dw_2}$$

$$b := b - \alpha \underline{db}.$$

Vectorization

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Basics of Neural Network Programming

Vectorization

What is vectorization?

$$z = \underbrace{w^T x} + b$$

$$w = \begin{bmatrix} \vdots \\ \\ \vdots \end{bmatrix} \qquad x = \begin{bmatrix} \vdots \\ \\ \vdots \end{bmatrix} \qquad \begin{array}{l} w \in \mathbb{R}^{n_x} \\ x \in \mathbb{R}^{n_x} \end{array}$$

Non-vectorized:

```
z = 0
for i in range(n-x):
    z += w[i] * x[i]
z += b
```

Vectorized

$$z = \underbrace{np.\underline{dot}(w, x)}_{w^T x} + b$$

$\rightarrow$ GPU
$\rightarrow$ CPU
$\}$ SIMD - single instruction multiple data.

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Basics of Neural Network Programming

More vectorization examples

Neural network programming guideline

Whenever possible, avoid explicit for-loops.

$u = Av$

$u_i = \sum_i \sum_j A_{ij} v_j$

↑ ↑

```
u = np.zeros((n,1))
for i ...          ←
    for j ....     ←
        u[i] += A[i][j]*v[j]
```

```
u = np.dot(A,v)
```

Vectors and matrix valued functions

Say you need to apply the exponential operation on every element of a matrix/vector.

$$v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} \rightarrow u = \begin{bmatrix} e^{v_1} \\ e^{v_2} \\ \vdots \\ e^{v_n} \end{bmatrix}$$

```
u = np.zeros((n,1))
for i in range(n):
    u[i]=math.exp(v[i])
```

```
import numpy as np
u = np.exp(v)

np.log(v)
np.abs(v)
np.maximum(v, 0)
v**2        1/v
```

Logistic regression derivatives

```
J = 0, ~~dw1 = 0, dw2 = 0~~, db = 0
for i = 1 to n:
    $z^{(i)} = w^T x^{(i)} + b$
    $a^{(i)} = \sigma(z^{(i)})$
    $J \mathrel{+}= -[y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})]$
    $dz^{(i)} = a^{(i)}(1 - a^{(i)})$
    ~~$dw_1 \mathrel{+}= x_1^{(i)} dz^{(i)}$~~
    ~~$dw_2 \mathrel{+}= x_2^{(i)} dz^{(i)}$~~
    db += $dz^{(i)}$
J = J/m, ~~$dw_1 = dw_1/m$, $dw_2 = dw_2/m$~~, db = db/m
```

dw = np.zeros((n_x, 1))

for $j = 1 \ldots n_x$

$dw_j \mathrel{+}= \ldots$

$n_x = 2$

$dw \mathrel{+}= x^{(i)} dz^{(i)}$

$dw \mathrel{/}= m.$

Basics of Neural Network Programming

Vectorizing Logistic Regression

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Vectorizing Logistic Regression

$z^{(1)} = w^T x^{(1)} + b$

$a^{(1)} = \sigma(z^{(1)})$

$z^{(2)} = w^T x^{(2)} + b$

$a^{(2)} = \sigma(z^{(2)})$

$z^{(3)} = w^T x^{(3)} + b$

$a^{(3)} = \sigma(z^{(3)})$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

(n_x, m)

$\mathbb{R}^{n_x \times m}$

$$[\text{——}] \; w^T \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

$$Z = [z^{(1)} \; z^{(2)} \; \cdots \; z^{(m)}] = w^T X + \underbrace{[b \; b \cdots b]}_{1 \times m} = \underbrace{[w^T x^{(1)} + b \quad w^T x^{(2)} + b \quad \cdots \quad w^T x^{(m)} + b]}_{1 \times m}$$

$Z = \text{np.dot}(w.T, X) + b$

$(1,1) \quad \mathbb{R}$

"Broadcasting"

$A = [a^{(1)} \; a^{(2)} \; \cdots \; a^{(m)}] = \sigma(Z)$

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Basics of Neural Network Programming

Vectorizing Logistic Regression’s Gradient Computation

Vectorizing Logistic Regression

$dz^{(1)} = a^{(1)} - y^{(1)}$ $\quad dz^{(2)} = a^{(2)} - y^{(2)}$ $\quad \dots$

$$\boxed{dZ = [dz^{(1)}\ \ \underset{1\times m}{dz^{(2)}} \dots dz^{(m)}]} \leftarrow$$

$A = [a^{(1)} \dots a^{(m)}]$. $\quad Y = [y^{(1)} \dots y^{(m)}]$

$\rightarrow dZ = A - Y = [a^{(1)} - y^{(1)} \quad a^{(2)} - y^{(2)} \quad \dots]$

$\rightarrow dw = 0$

$dw \mathrel{+}= x^{(1)} dz^{(1)}$

$dw \mathrel{+}= x^{(2)} dz^{(2)}$

$\vdots$

$dw \mathrel{/}= m$

$\cancel{dw_1}$

$\cancel{dw_2}$

$\vdots$

$db = 0$

$db \mathrel{+}= dz^{(1)}$

$db \mathrel{+}= dz^{(2)}$

$\vdots$

$db \mathrel{+}= dz^{(m)}$

$db \mathrel{/}= m.$

$$db = \frac{1}{m} \sum_{i=1}^{m} dz^{(i)}$$

$$= \frac{1}{m}\, np.sum(dZ)$$

$$dw = \frac{1}{m} X dZ^T$$

$$= \frac{1}{m} \begin{bmatrix} | & & | \\ x^{(1)} & \dots & x^{(m)} \\ | & & | \end{bmatrix} \begin{bmatrix} dz^{(1)} \\ \vdots \\ dz^{(m)} \end{bmatrix}$$

$$= \frac{1}{m} \left[x^{(1)} dz^{(1)} + \dots + x^{(m)} dz^{(m)} \right]$$

$n \times 1$

Implementing Logistic Regression

```
$J = 0,\ dw_1 = 0,\ dw_2 = 0,\ db = 0$
for i = 1 to m:
    $z^{(i)} = w^T x^{(i)} + b$
    $a^{(i)} = \sigma(z^{(i)})$
    $J \mathrel{+}= -[y^{(i)} \log a^{(i)} + (1 - y^{(i)}) \log(1 - a^{(i)})]$
    $dz^{(i)} = a^{(i)} - y^{(i)}$
    $dw_1 \mathrel{+}= x_1^{(i)} dz^{(i)}$        $dw \mathrel{+}= x^{(i)} * dz^{(i)}$
    $dw_2 \mathrel{+}= x_2^{(i)} dz^{(i)}$
    $db \mathrel{+}= dz^{(i)}$
$J = J/m,\ dw_1 = dw_1/m,\ dw_2 = dw_2/m$
$db = db/m$
```

```
for iter in range(1000):
    $Z = w^T X + b$
      $= \text{np.dot}(w.T, X) + b$
    $A = \sigma(Z)$
    $dZ = A - Y$
    $dw = \frac{1}{m} X dZ^T$
    $db = \frac{1}{m} \text{np.sum}(dZ)$

    $w := w - \alpha dw$
    $b := b - \alpha db$
```

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Basics of Neural Network Programming

Broadcasting in Python

Broadcasting example

Calories from Carbs, Proteins, Fats in 100g of different foods:

$$\begin{array}{lcccc} & \text{Apples} & \text{Beef} & \text{Eggs} & \text{Potatoes} \\ \text{Carb} & \boxed{56.0} & 0.0 & 4.4 & 68.0 \\ \text{Protein} & 1.2 & 104.0 & 52.0 & 8.0 \\ \text{Fat} & 1.8 & 135.0 & 99.0 & 0.9 \end{array} = \underset{(3,4)}{A}$$

59cal

$\frac{56}{59} \approx 94.9\%$

Calculate % of calories from Carb, Protein, Fat. Can you do this without explicit for-loop?

`cal = A.sum(axis = 0)`

percentage = 100*A/(cal~~.reshape(1,4)~~)

$(3,4) \;/\; (1,4)$

Broadcasting example

$$\begin{bmatrix}1\\2\\3\\4\end{bmatrix} + \begin{bmatrix}100\\100\\100\\100\end{bmatrix} \cancel{100}$$

$\downarrow \quad \downarrow \quad \downarrow$

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} +^{\swarrow} \begin{bmatrix}100 & 200 & 300\\100 & 200 & 300\end{bmatrix}$$

(m, n) $(2,3)$ $(1, n) \rightsquigarrow (m, n)$ $(2,3)$

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} + \begin{bmatrix}100 & 100 & 100\\200 & 200 & 200\end{bmatrix} = \quad \leftarrow \leftarrow$$

(m, n) $(m, 1) \downarrow (m, n)$

General Principle

$$\begin{array}{cccc} (m,n) & + & (1,n) & \rightsquigarrow (m,n) \\ \underline{\text{matrix}} & - & & \\ & * & (m,1) & \rightsquigarrow (m,n) \\ & / & & \end{array}$$

$$(m,1) \quad + \quad \mathbb{R}$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 100 = \begin{bmatrix} 101 \\ 102 \\ 103 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} + 100 = \begin{bmatrix} 101 & 102 & 103 \end{bmatrix}$$

Matlab/Octave: $\underline{\text{bsxfun}}$

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Basics of Neural Network Programming

A note on python/ numpy vectors

Python / numpy vectors

```
import numpy as np

a = np.random.randn(5)

a = np.random.randn((5,1))

a = np.random.randn((1,5))

assert(a.shape = (5,1))
```

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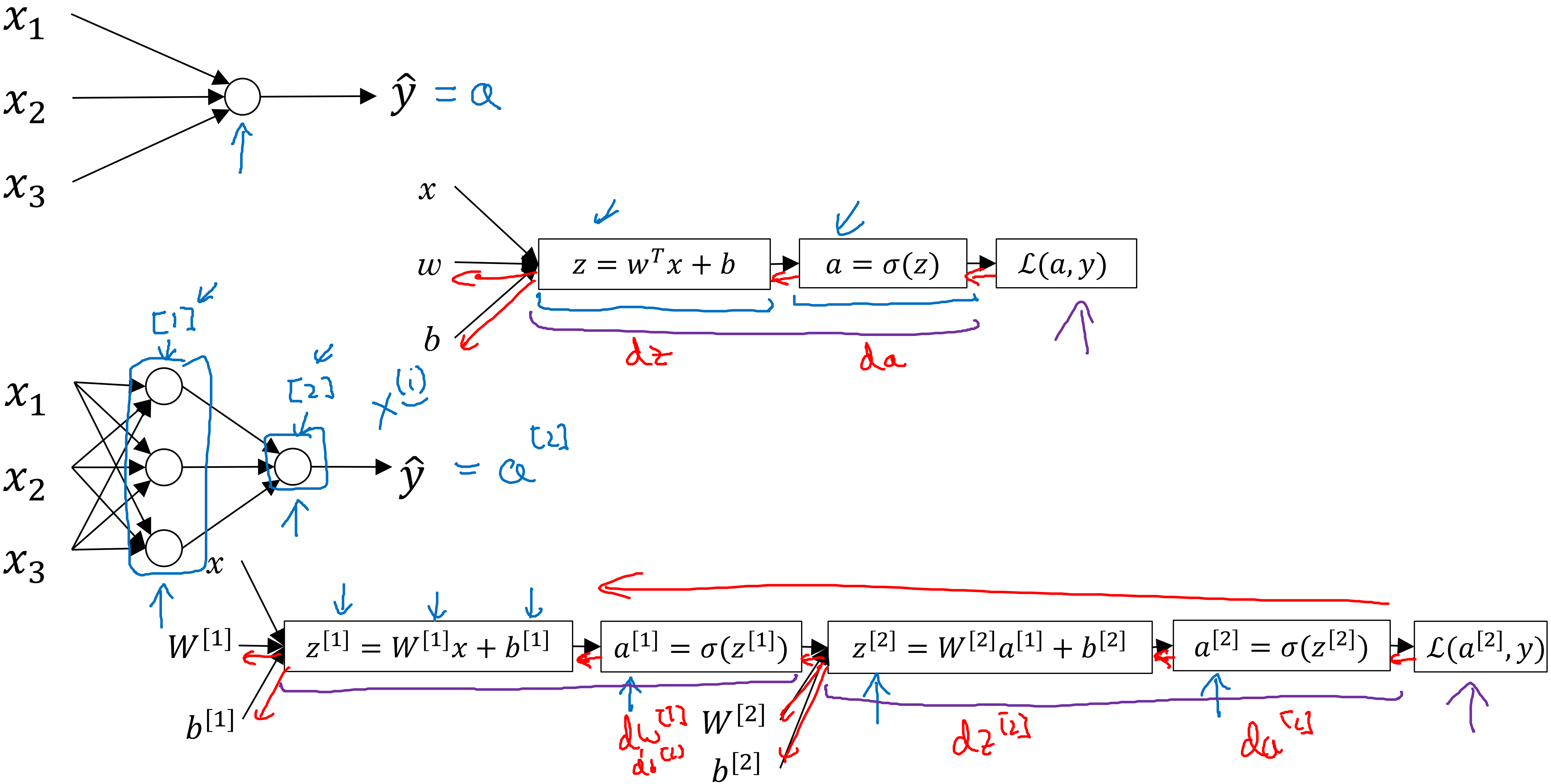
One hidden layer Neural Network

Neural Networks Overview

What is a Neural Network?



$z = w^T x + b$

$a = \sigma(z)$

$\mathcal{L}(a, y)$

$z^{[1]} = W^{[1]}x + b^{[1]}$

$a^{[1]} = \sigma(z^{[1]})$

$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$

$a^{[2]} = \sigma(z^{[2]})$

$\mathcal{L}(a^{[2]}, y)$

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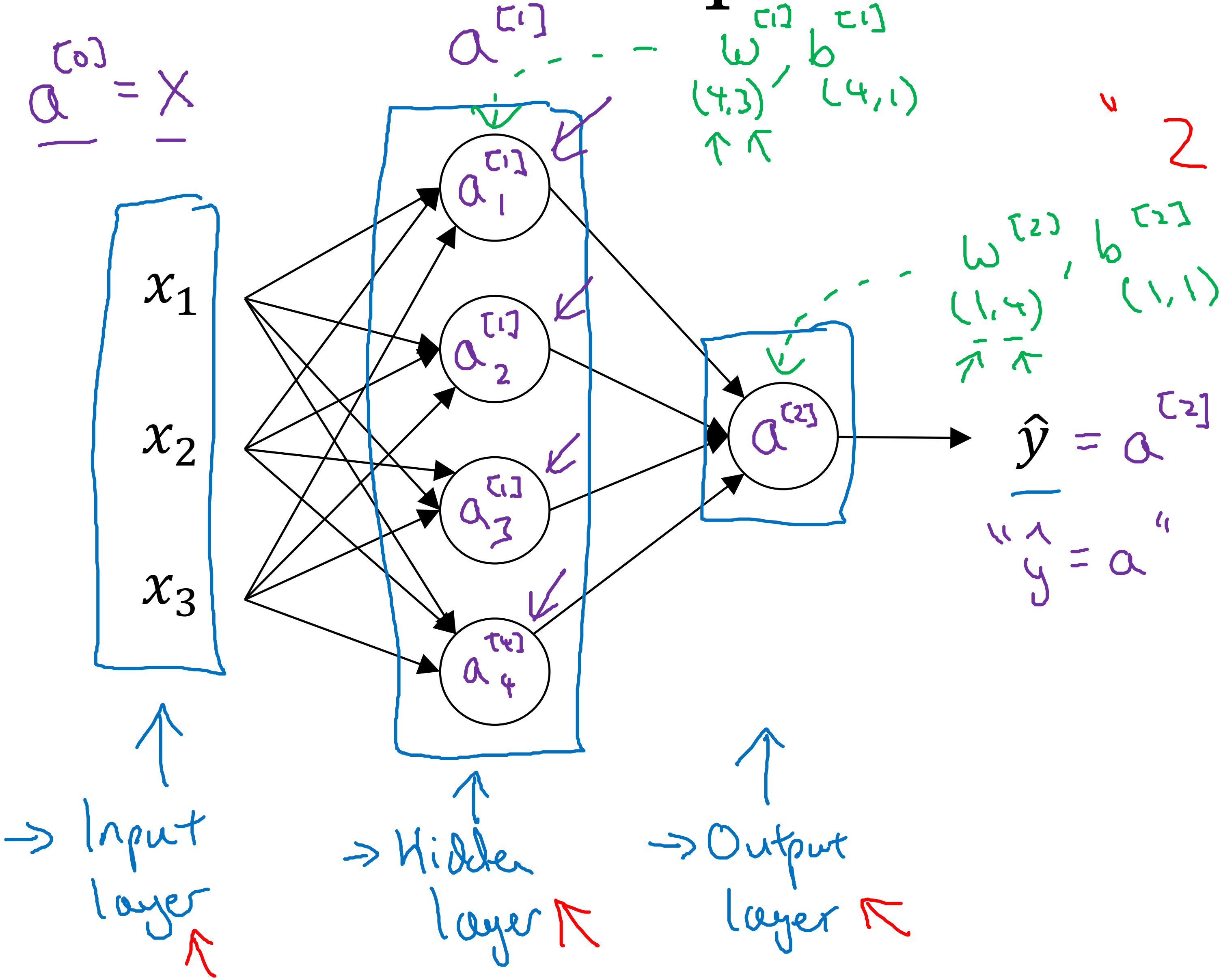
One hidden layer Neural Network

Neural Network Representation

Neural Network Representation



$\hat{y} = a^{[2]}$

"$\hat{y} = a$"

" 2 layer NN "

$$a^{[1]} = \begin{bmatrix} a^{[1]}_1 \\ a^{[1]}_2 \\ \vdots \\ a^{[1]}_4 \end{bmatrix}$$

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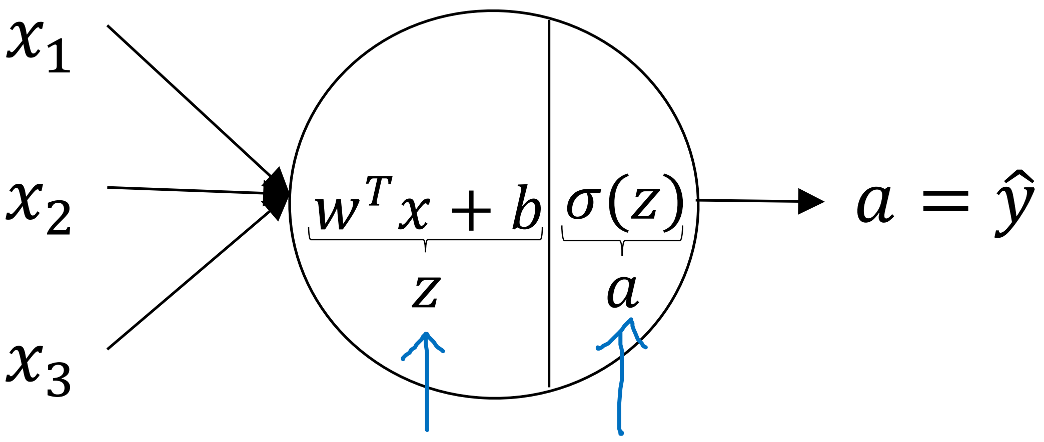
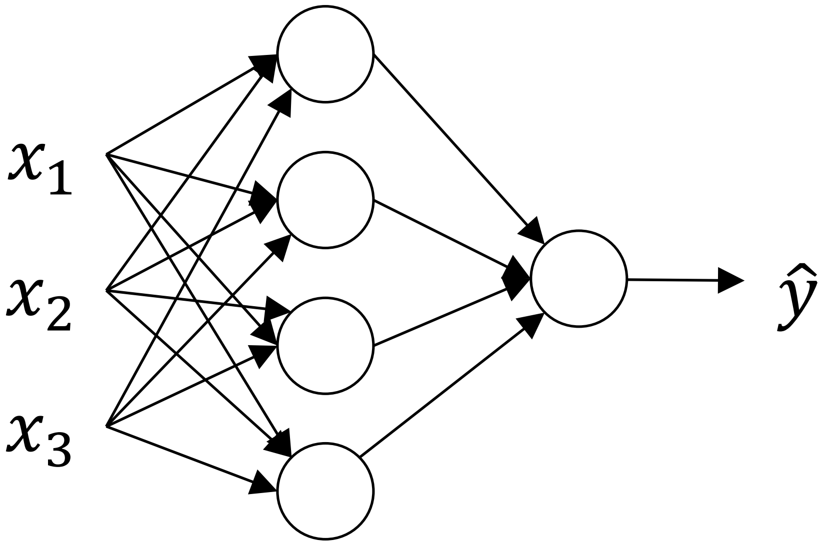
One hidden layer Neural Network

Computing a Neural Network’s Output

Neural Network Representation

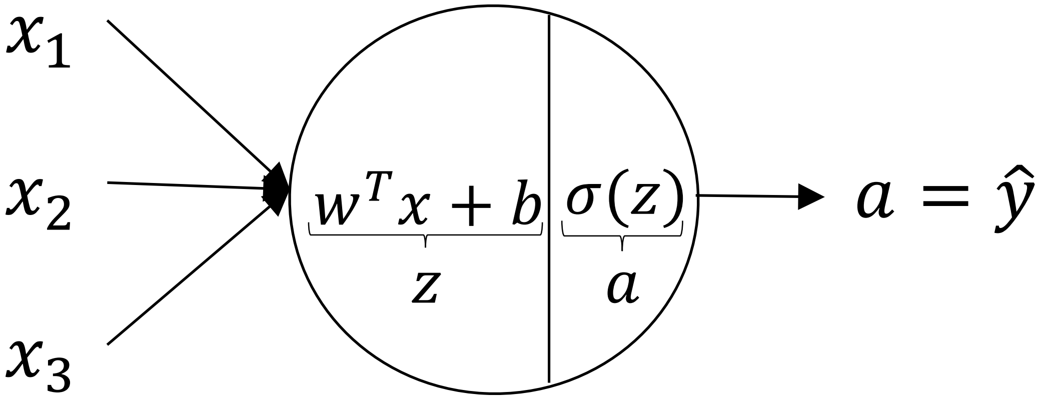




$$z = w^T x + b$$

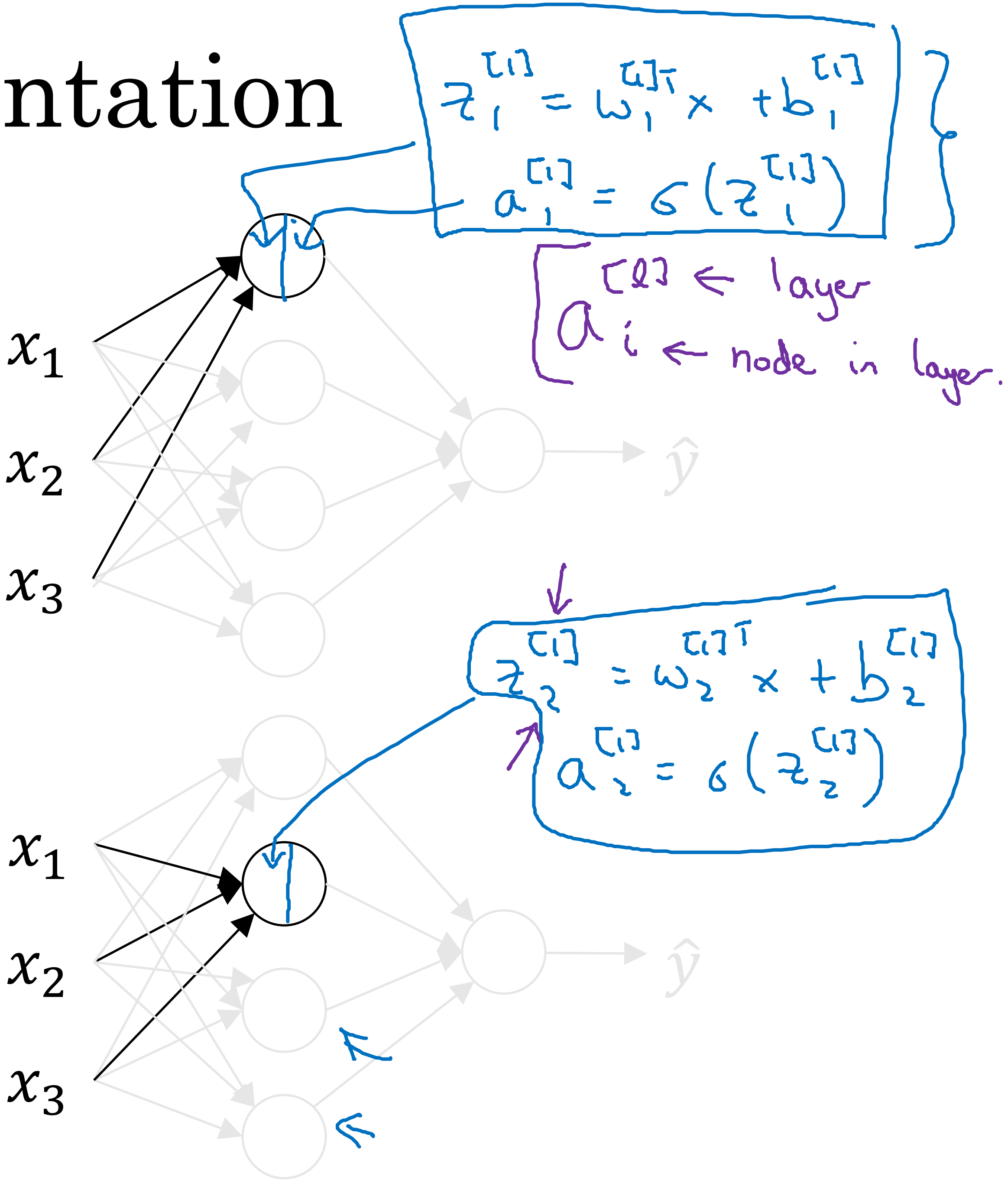
$$a = \sigma(z)$$

Neural Network Representation



$$z = w^T x + b$$

$$a = \sigma(z)$$



$$z_1^{[1]} = w_1^{[1]T} x + b_1^{[1]}$$

$$a_1^{[1]} = \sigma(z_1^{[1]})$$

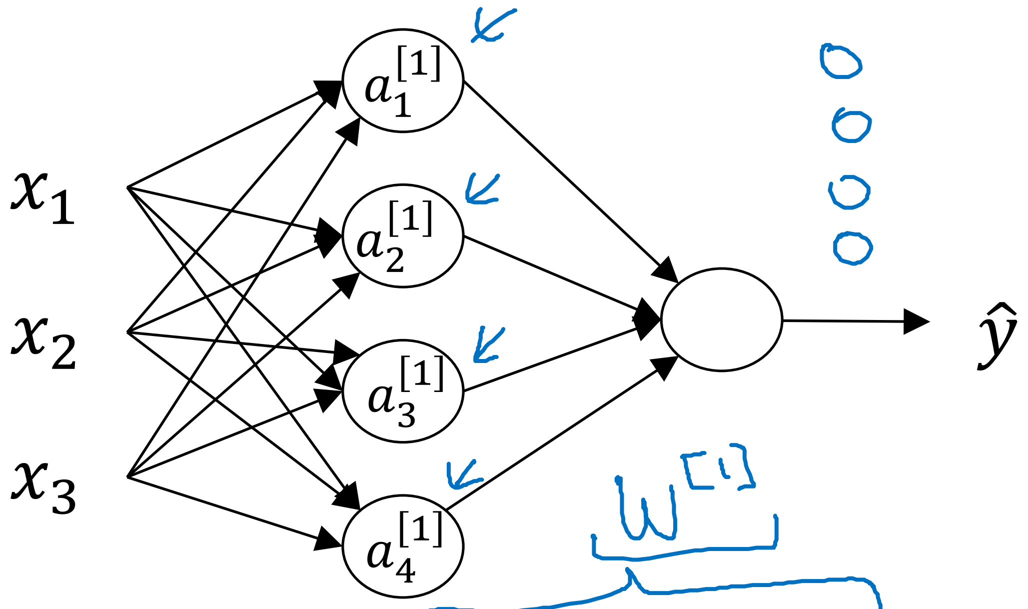
$a_i^{[l]}$ ← layer

← node in layer.

$$z_2^{[1]} = w_2^{[1]T} x + b_2^{[1]}$$

$$a_2^{[1]} = \sigma(z_2^{[1]})$$

Neural Network Representation



$$(w_1^{[1]})^T x \qquad a^{[1]}$$

$$z_1^{[1]} = w_1^{[1]T}x + b_1^{[1]}, \quad a_1^{[1]} = \sigma(z_1^{[1]})$$

$$z_2^{[1]} = w_2^{[1]T}x + b_2^{[1]}, \quad a_2^{[1]} = \sigma(z_2^{[1]})$$

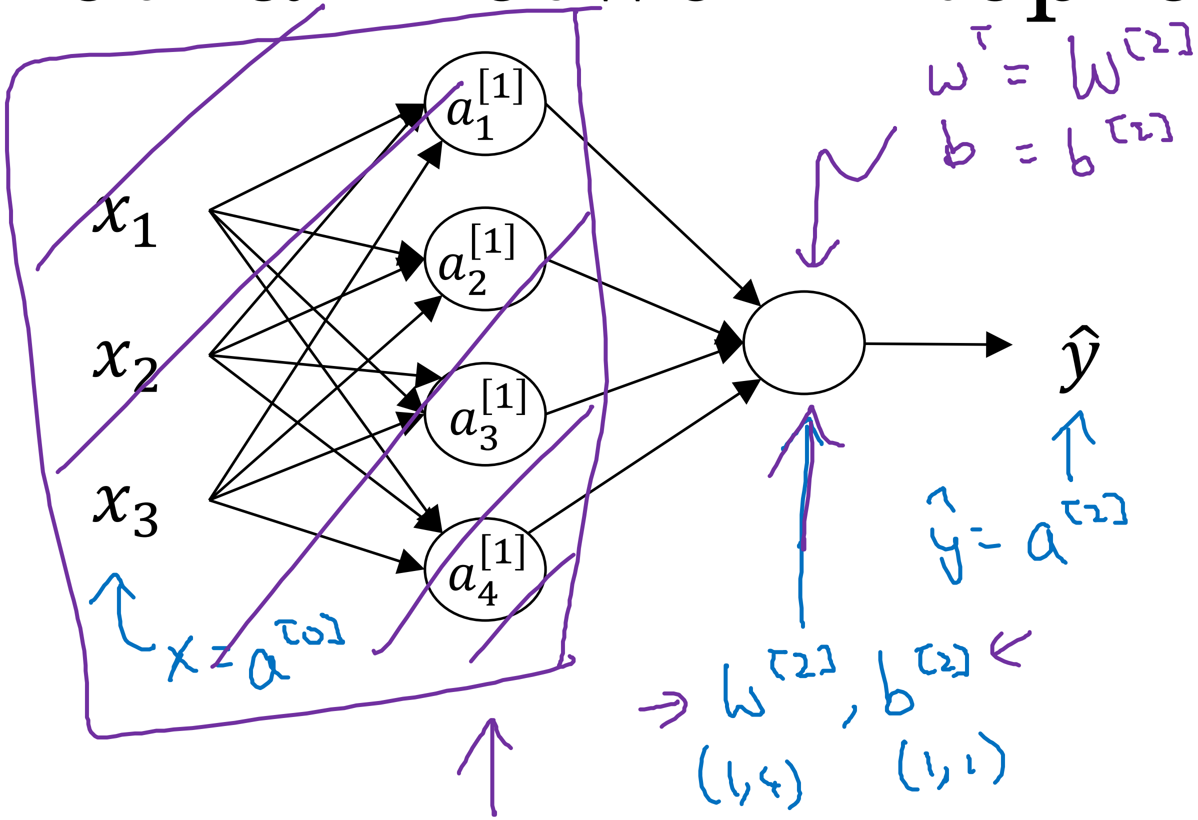
$$z_3^{[1]} = w_3^{[1]T}x + b_3^{[1]}, \quad a_3^{[1]} = \sigma(z_3^{[1]})$$

$$z_4^{[1]} = w_4^{[1]T}x + b_4^{[1]}, \quad a_4^{[1]} = \sigma(z_4^{[1]})$$

$$\rightarrow z^{[1]} = \underset{(4,3)}{\overbrace{\begin{bmatrix} \text{---}\, w_1^{[1]T} \,\text{---} \\ \text{---}\, w_2^{[1]T} \,\text{---} \\ \text{---}\, w_3^{[1]T} \,\text{---} \\ \text{---}\, w_4^{[1]T} \,\text{---} \end{bmatrix}}^{W^{[1]}}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \\ b_4^{[1]} \end{bmatrix}}_{b^{[1]}\ (4,1)} = \begin{bmatrix} \rightarrow w_1^{[1]T}x + b_1^{[1]} \\ \rightarrow w_2^{[1]T}x + b_2^{[1]} \\ \rightarrow w_3^{[1]T}x + b_3^{[1]} \\ \rightarrow w_4^{[1]T}x + b_4^{[1]} \end{bmatrix} = \begin{bmatrix} z_1^{[1]} \\ z_2^{[1]} \\ z_3^{[1]} \\ z_4^{[1]} \end{bmatrix}$$

$$\rightarrow a^{[1]} = \begin{bmatrix} a_1^{[1]} \\ \vdots \\ a_4^{[1]} \end{bmatrix} = \sigma(z^{[1]})$$

Neural Network Representation learning



$z = w^T x + b$

$\hat{y} = a = \sigma(z)$

Given input x:

$$\underset{(4,1)}{z^{[1]}} = \underset{(4,3)}{W^{[1]}} \underset{(3,1)}{\overset{a^{[0]}}{\cancel{x}}} + \underset{(4,1)}{b^{[1]}}$$

$$\underset{(4,1)}{a^{[1]}} = \sigma(\underset{(4,1)}{z^{[1]}})$$

$$\underset{(1,1)}{z^{[2]}} = \underset{(1,4)}{W^{[2]}} \underset{(4,1)}{a^{[1]}} + \underset{(1,1)}{b^{[2]}}$$

$$\underset{(1,1)}{a^{[2]}} = \sigma(\underset{(1,1)}{z^{[2]}})$$

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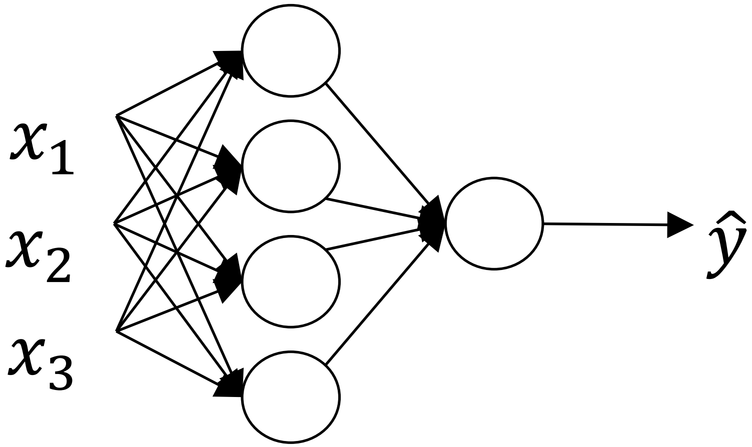
One hidden layer Neural Network

Vectorizing across multiple examples

Vectorizing across multiple examples



$$z^{[1]} = W^{[1]}x + b^{[1]}$$

$$a^{[1]} = \sigma(z^{[1]})$$

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = \sigma(z^{[2]})$$

$x \longrightarrow a^{[2]} = \hat{y}$

$x^{(1)} \longrightarrow a^{[2](1)} = \hat{y}^{(1)}$

$x^{(2)} \longrightarrow a^{[2](2)} \quad \hat{y}^{(2)}$

$\vdots$

$x^{(m)} \longrightarrow a^{[2](m)} \quad \hat{y}^{(m)}$

$a^{[2](i)}$

example i

layer 2

```
for i = 1 to m,
    $z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}$
    $a^{[1](i)} = \sigma(z^{[1](i)})$
    $z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$
    $a^{[2](i)} = \sigma(z^{[2](i)})$
```

Vectorizing across multiple examples

```
for i = 1 to m:
    $\boxed{z^{[1](i)}} = W^{[1]}x^{(i)} + b^{[1]}$
    $\boxed{a^{[1](i)}} = \sigma(z^{[1](i)})$
    $\boxed{z^{[2](i)}} = W^{[2]}a^{[1](i)} + b^{[2]}$
    $\boxed{a^{[2](i)}} = \sigma(z^{[2](i)})$
```

$$\boxed{\begin{aligned} Z^{[1]} &= W^{[1]}X + b^{[1]} \\ \rightarrow\ A^{[1]} &= \sigma(Z^{[1]}) \\ \rightarrow\ Z^{[2]} &= W^{[2]}A^{[1]} + b^{[2]} \\ \rightarrow\ A^{[2]} &= \sigma(Z^{[2]}) \end{aligned}}$$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix} \quad (n_x, m)$$

$$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

training examples ⟷

hidden units ↕

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

hidden units

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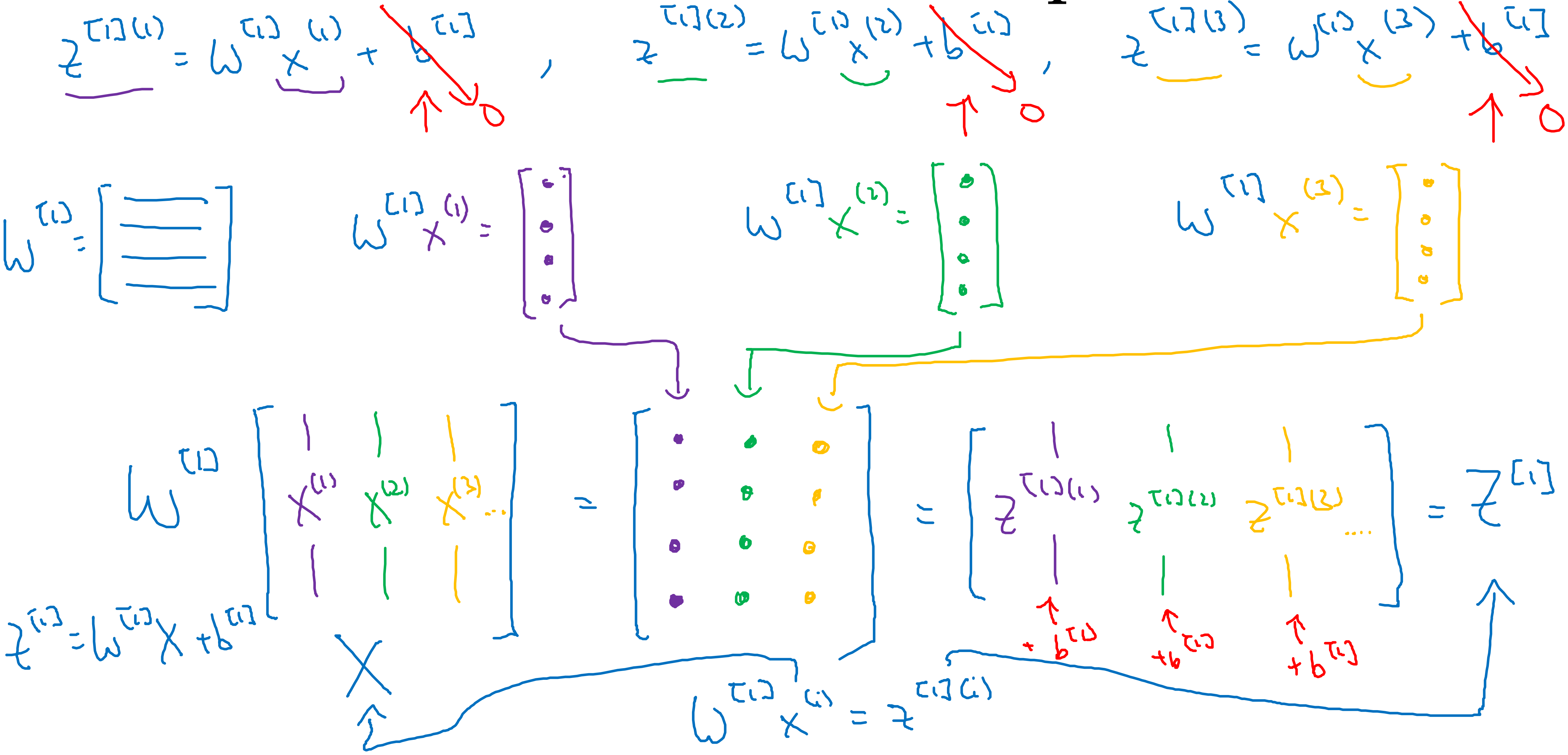
One hidden layer Neural Network

Explanation for vectorized implementation

Justification for vectorized implementation



$$\underline{z^{[1](1)}} = W^{[1]}\underline{x^{(1)}} + \cancel{b^{[1]}} \to 0, \quad \underline{z^{[1](2)}} = W^{[1]}\underline{x^{(2)}} + \cancel{b^{[1]}} \to 0, \quad \underline{z^{[1](3)}} = W^{[1]}\underline{x^{(3)}} + \cancel{b^{[1]}} \to 0$$

$$W^{[1]} = \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} \qquad W^{[1]}x^{(1)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \qquad W^{[1]}x^{(2)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \qquad W^{[1]}x^{(3)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

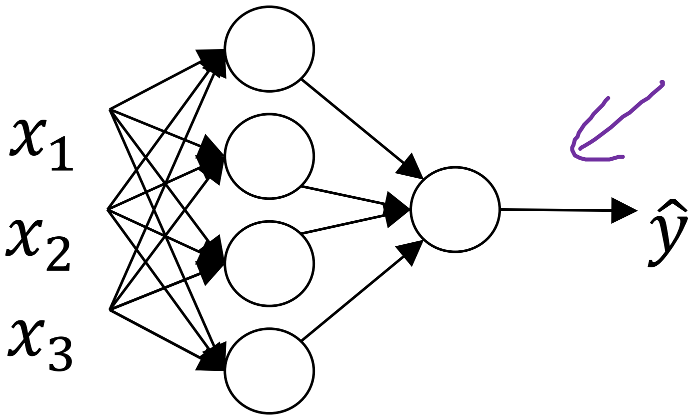
$$W^{[1]} \underbrace{\begin{bmatrix} | & | & | & \\ x^{(1)} & x^{(2)} & x^{(3)} & \cdots \\ | & | & | & \end{bmatrix}}_{X} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} = \begin{bmatrix} | & | & | & \\ z^{[1](1)} & z^{[1](2)} & z^{[1](3)} & \cdots \\ | & | & | & \end{bmatrix} = Z^{[1]}$$

$+b^{[1]}$ $\quad +b^{[1]}$ $\quad +b^{[1]}$

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$

$$W^{[1]}x^{(i)} = z^{[1](i)}$$

Recap of vectorizing across multiple examples



$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

$$\underline{A^{[1]}} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

```
for i = 1 to m
    $\boxed{z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}}$
    $a^{[1](i)} = \sigma(z^{[1](i)})$
    $z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$
    $a^{[2](i)} = \sigma(z^{[2](i)})$
```

$X = a^{[0]}$

$x^{(i)} = a^{[0](i)}$

$$Z^{[1]} = W^{[1]}\overset{A^{[0]}}{\boxed{X}} + b^{[1]} \leftarrow W^{[1]}A^{[0]} + b^{[1]}$$

$$A^{[1]} = \sigma(Z^{[1]})$$

$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$

$$A^{[2]} = \sigma(Z^{[2]})$$

One hidden layer Neural Network

Activation functions

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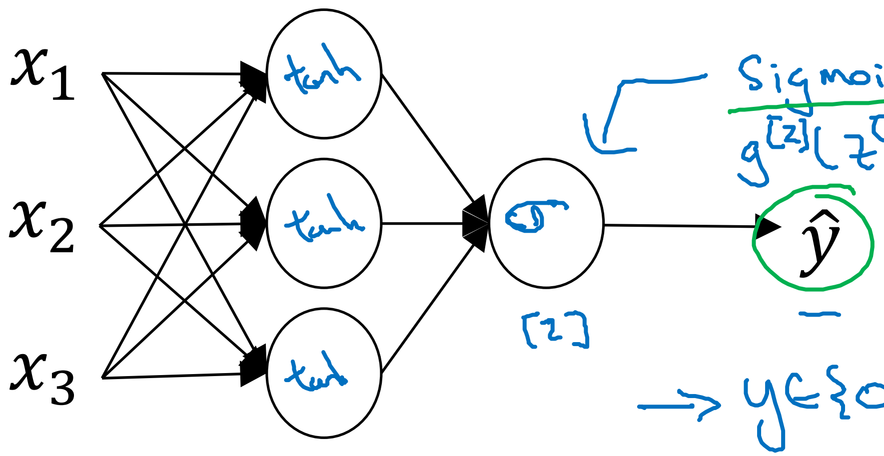
Activation functions

$g^{[1]}(z^{[1]}) = \tanh(z^{[1]})$



[1]

Sigmoid

$g^{[2]}(z^{[2]}) = \sigma(z^{[2]})$

→ $y \in \{0,1\}$

$0 \le \hat{y} \le 1$

-1 1

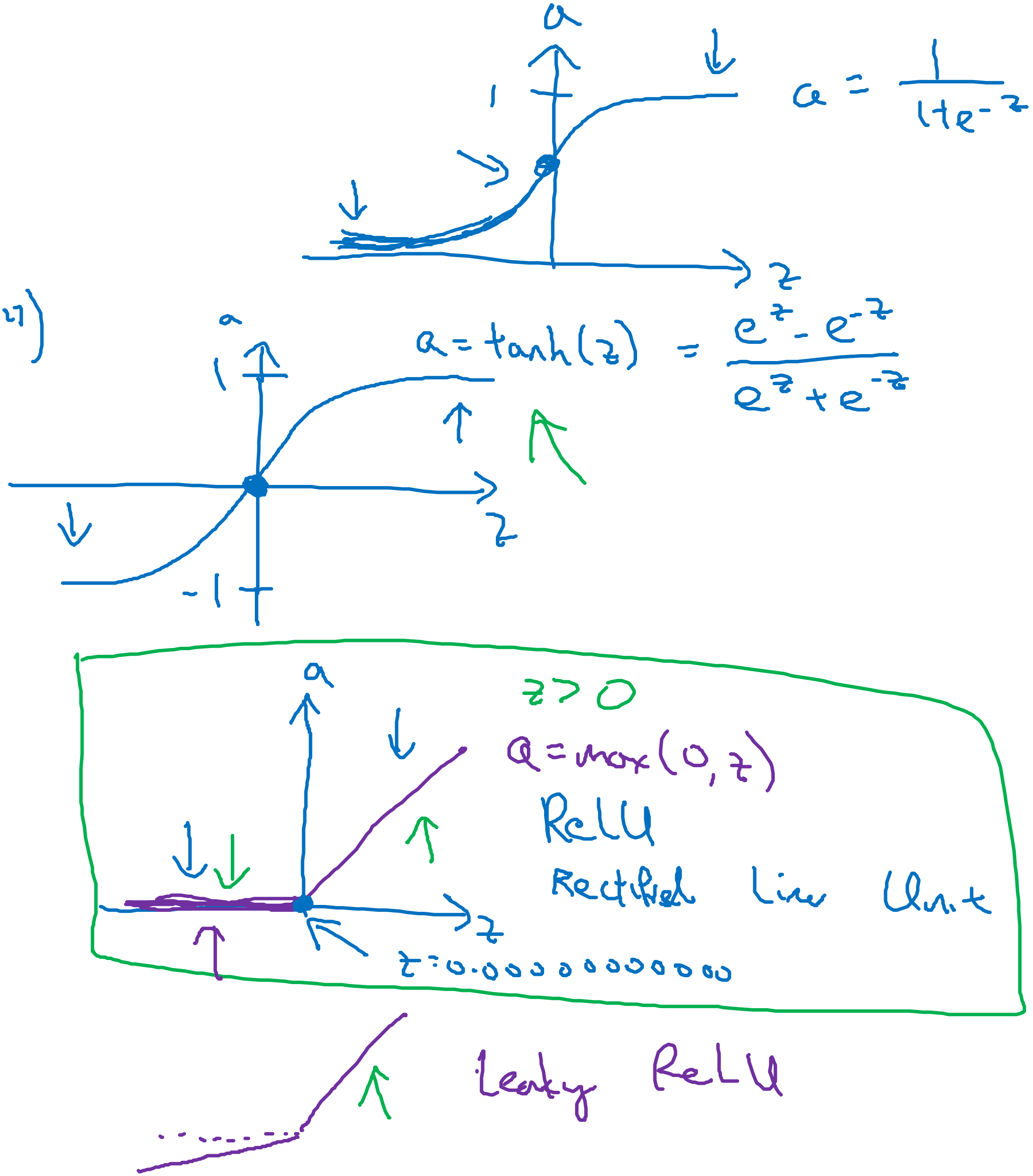
Given x:

$$z^{[1]} = W^{[1]}x + b^{[1]}$$

$$\rightarrow a^{[1]} = \cancel{\sigma(z^{[1]})}\ g^{[1]}(z^{[1]})$$

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$\rightarrow a^{[2]} = \cancel{\sigma(z^{[2]})}\ g^{[2]}(z^{[2]})$$



$a = \frac{1}{1+e^{-z}}$

$a = \tanh(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$

$z > 0$

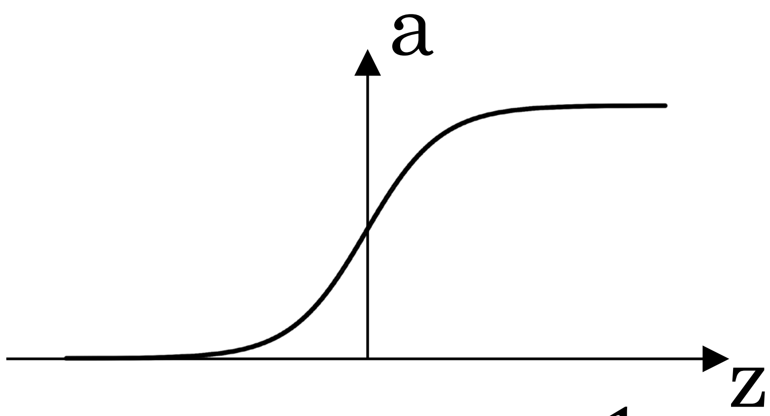
$a = \max(0, z)$

ReLU

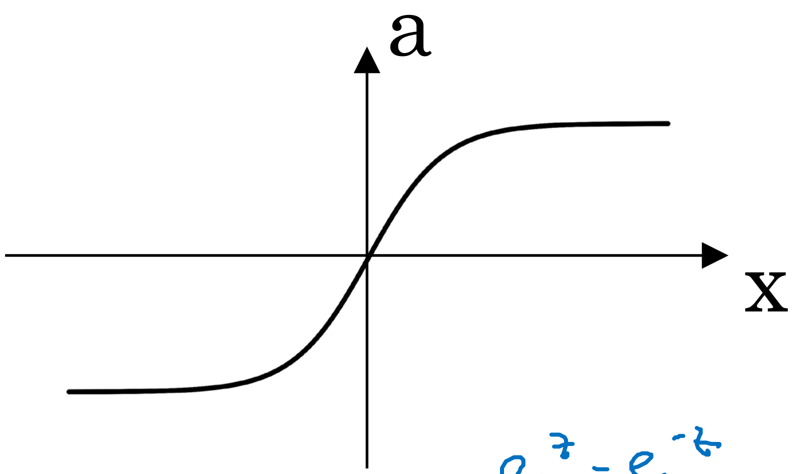
Rectified Linear Unit

Leaky ReLU

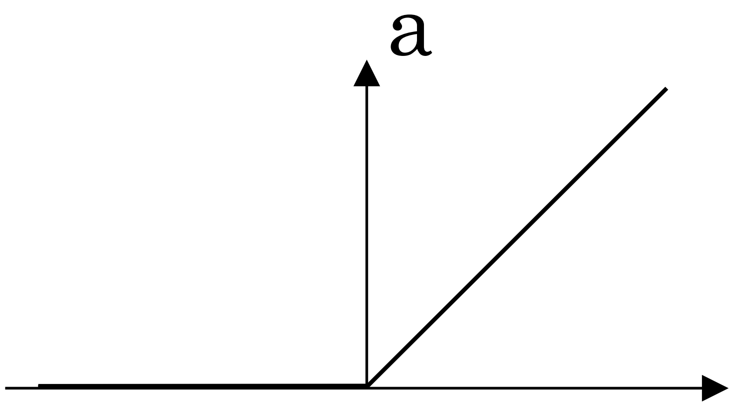
Pros and cons of activation functions



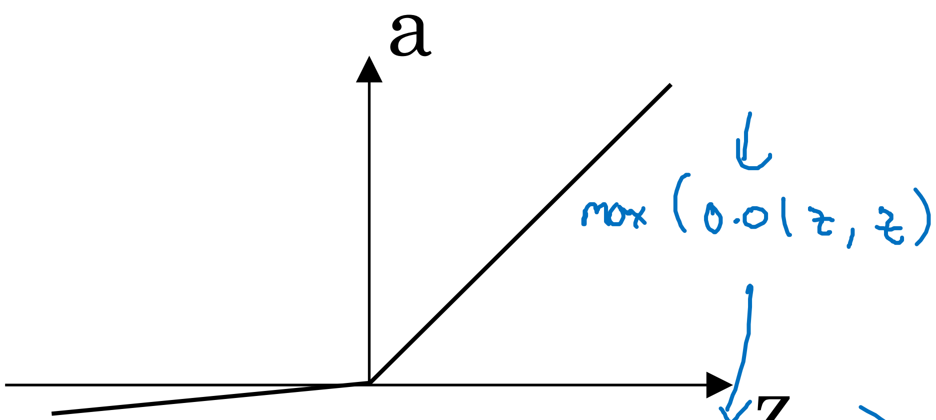
sigmoid: $a = \dfrac{1}{1+e^{-z}}$



tanh: $a = \dfrac{e^{z} - e^{-z}}{e^{z} + e^{z}}$



ReLU $a = \max(0, z)$



$\max(0.01z, z)$

Leaky ReLU $a = \max(0.01z, z)$

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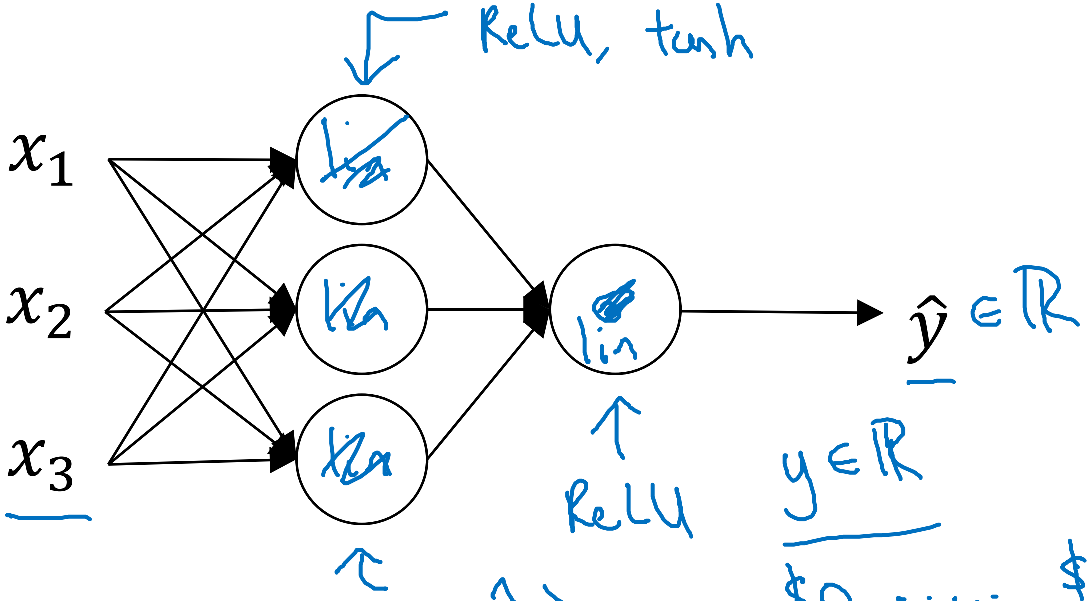
One hidden layer Neural Network

Why do you need non-linear activation functions?

Activation function



$y \in \mathbb{R}$

$\hat{y} \geq 0$.

$\$0 \ \ldots\ldots \ \$1{,}000{,}000s$

Given x:

$$\rightarrow \ z^{[1]} = W^{[1]}x + b^{[1]}$$

$$\rightarrow \ a^{[1]} = \cancel{g^{[1]}(z^{[1]})} \ z^{[1]}$$

$$\rightarrow \ z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$\rightarrow \ a^{[2]} = \cancel{g^{[2]}(z^{[2]})} \ z^{[2]}$$

$g(z) = z$

"linear activation function"

$$\underbrace{a^{[1]} = z^{[1]}} = \underbrace{W^{[1]}x + b^{[1]}}$$

$$a^{[2]} = z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = W^{[2]}(\underbrace{W^{[1]}x + b^{[1]}}_{a^{[1]}}) + b^{[2]}$$

$$= \underbrace{(W^{[2]}W^{[1]})}_{W'}x + \underbrace{(W^{[2]}b^{[1]} + b^{[2]})}_{b'}$$

$$= W'x + b'$$

$g(z) = z$

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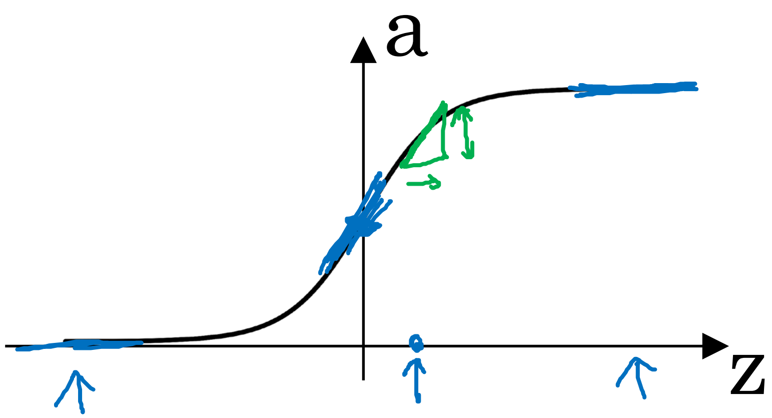
One hidden layer Neural Network

Derivatives of activation functions

Sigmoid activation function



$$\underline{g(z)} = \frac{1}{\underline{1 + e^{-z}}}$$

$$a = g(z) = \frac{1}{1+e^{-z}}$$

$$\boxed{g'(z)} = \boxed{\frac{d}{dz} g(z)} = \text{slope of } g(x) \text{ at } z$$

$$= \frac{1}{1+e^{-z}}\left(1 - \frac{1}{1+e^{-z}}\right)$$

$$= \underbrace{g(z)\left(1 - g(z)\right)}_{} \leftarrow$$

$$= \boxed{a\,(1-a)}$$

$$\left|\, g'(z) = \underset{\uparrow}{a}(1-a)\right.$$

$z = 10$. $g(z) \approx 1$

$$\frac{d}{dz} g(z) \approx 1\,(1-1) \approx 0$$

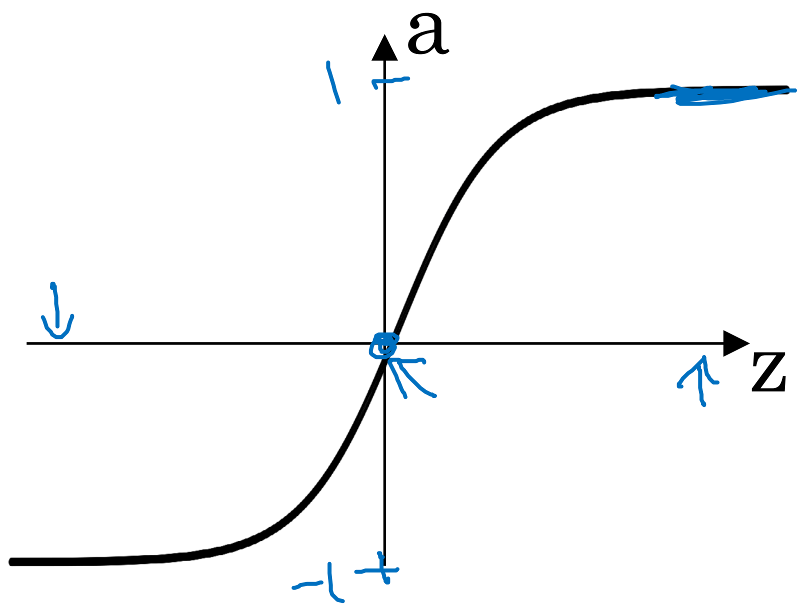
$z = -10$ $g(z) \approx 0$

$$\frac{d}{dz} g(z) \approx 0 \cdot (1-0) \approx 0$$

$z = 0$ $g(z) = \frac{1}{2}$

$$\frac{d}{dz} g(z) = \frac{1}{2}\left(1 - \frac{1}{2}\right) = \frac{1}{4}$$

Tanh activation function



$$g(z) = \tanh(z)$$

$$= \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$$

$$g'(z) = \frac{d}{dz} g(z) = \text{slope of } g(z) \text{ at } z$$

$$= \underline{1 - (\tanh(z))^2} \leftarrow$$

$z = 10 \quad \tanh(z) \approx 1$

$g'(z) \approx 0$

$z = -10 \quad \tanh(z) \approx -1$

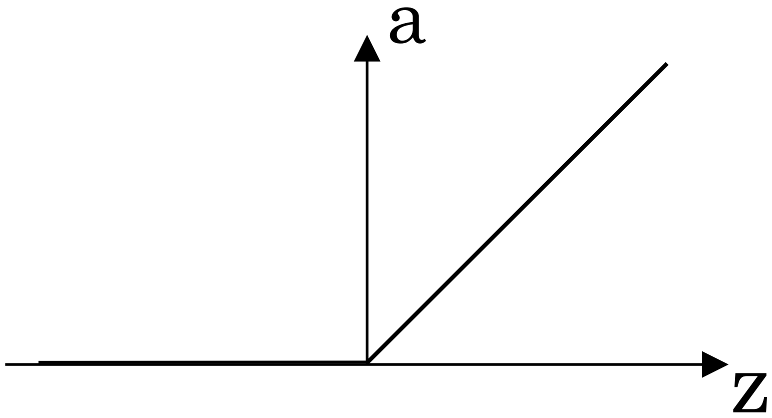
$g'(z) \approx 0$

$z = 0 \quad \tanh(z) = 0$

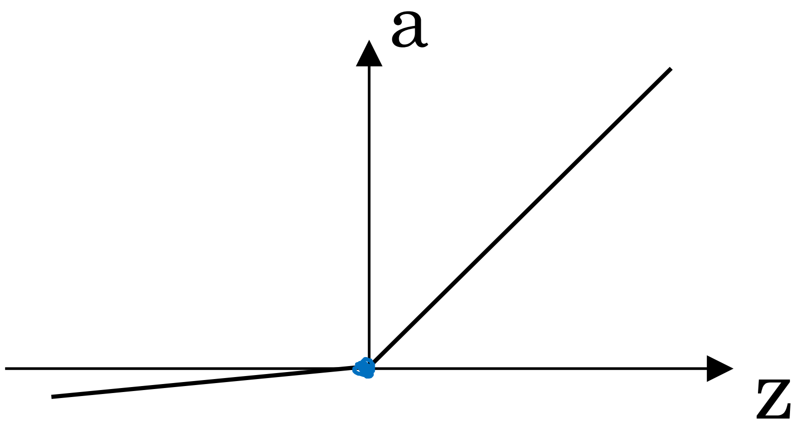
$g'(z) = 1$

$$a = g(z), \qquad \underset{\uparrow}{g'}(z) = 1 - \underset{\uparrow}{a}^2$$

ReLU and Leaky ReLU



ReLU



Leaky ReLU

$$g(z) = \max(0, z)$$

$$\rightarrow g'(z) = \begin{cases} 0 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}$$

~~undefined if $z = 0$~~

$$z = 0.00000 \cdots\cdots 0$$

$$g(z) = \max(0.01z, z)$$

$$g'(z) = \begin{cases} 0.01 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}$$

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One hidden layer Neural Network

Gradient descent for neural networks

Gradient descent for neural networks

Parameters: $W^{[1]}, b^{[1]}, W^{[2]}, b^{[2]}$

$(n^{[1]}, n^{[0]})$ $(n^{[1]}, 1)$ $(n^{[2]}, n^{[1]})$ $(n^{[2]}, 1)$

$n_x = n^{[0]}, n^{[1]}, \underline{n^{[2]} = 1}$

Cost function: $J(W^{[1]}, b^{[1]}, \underset{\uparrow}{\underline{W^{[2]}}}, \underset{\uparrow}{\underline{b^{[2]}}}) = \frac{1}{m} \sum_{i=1}^{n} \mathcal{L}(\underset{\uparrow a^{[2]}}{\hat{y}}, y)$

Gradient descent:

→ Repeat {

→ Compute predicts $(\hat{y}^{(i)}, i = 1 \dots, m)$

$\underline{dW^{[1]}} = \frac{\partial J}{\partial W^{[1]}}$, $\underline{db^{[1]}} = \frac{\partial J}{\partial b^{[1]}}$, ...

$W^{[1]} := W^{[1]} - \alpha \, dW^{[1]}$

$b^{[1]} := b^{[1]} - \alpha \, db^{[1]}$

$W^{[2]} := \dots$ $b^{[2]} := \dots$

}

Formulas for computing derivatives

Forward propagation:

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$

$$A^{[1]} = g^{[1]}(Z^{[1]}) \leftarrow$$

$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$

$$A^{[2]} = g^{[2]}(Z^{[2]}) = \underline{\sigma}(Z^{[2]})$$

Back propagation:

$$dZ^{[2]} = A^{[2]} - Y \swarrow \qquad Y = [y^{(1)}\ y^{(2)} \dots, y^{(m)}]$$

$$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]T}$$

$$(n^{[2]},) \leftarrow$$

$$\swarrow (n^{[2]}, 1) \leftarrow$$

$$db^{[2]} = \frac{1}{m} \underline{np.sum}(dZ^{[2]}, \underline{axis=1}, \underline{keepdims=True})$$

$$dZ^{[1]} = \underbrace{W^{[2]T} dZ^{[2]}}_{(n^{[1]}, m)} \ast \underbrace{g^{[1]'}(Z^{[1]})}_{(n^{[1]}, m)}$$

↑ element-wise product

$$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$$

$$db^{[1]} = \frac{1}{m} np.sum(dZ^{[1]}, axis=1, \underline{keepdims=True})$$

$$\underline{(n^{[1]}, 1)} \qquad (n^{[1]},) \qquad \text{reshape} \uparrow$$

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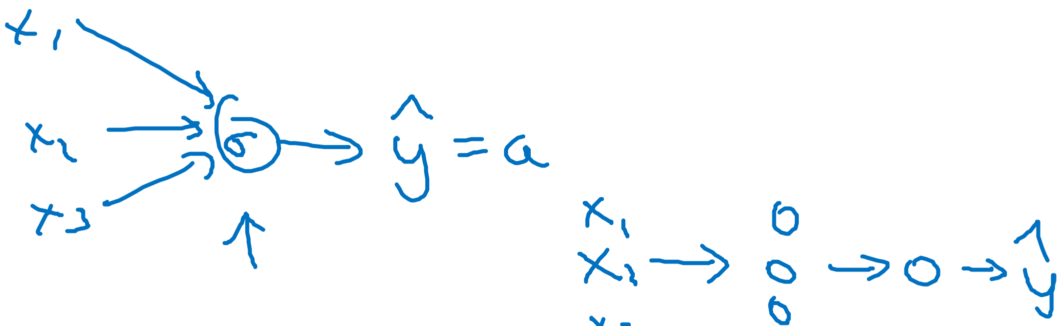
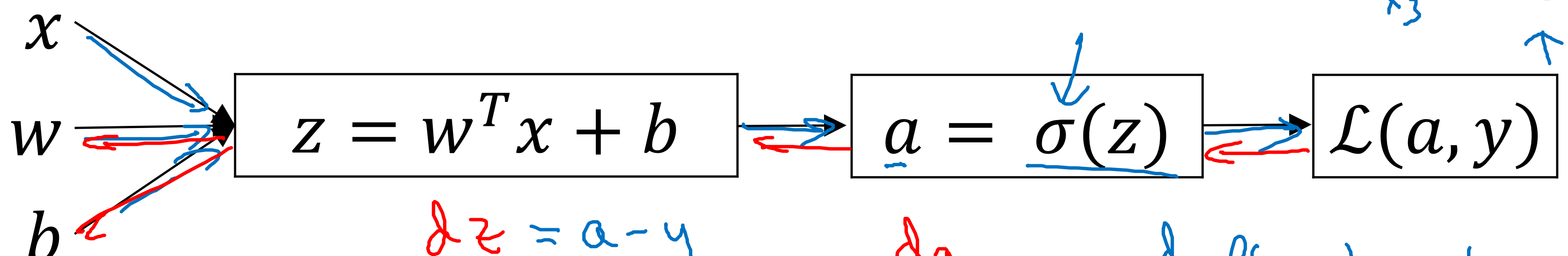
One hidden layer Neural Network

Backpropagation intuition (Optional)

Computing gradients

Logistic regression





$z = w^T x + b$ → $a = \sigma(z)$ → $\mathcal{L}(a, y)$

$dz = a - y$

$dw = dz \cdot x$

$db = dz$

$$\boxed{dz = da \cdot g'(z)}$$

$g(z) = \sigma(z)$

$$\boxed{\frac{\partial \mathcal{L}}{\partial z} = \frac{\partial \mathcal{L}}{\partial a} \cdot \boxed{\frac{da}{dz}}}$$

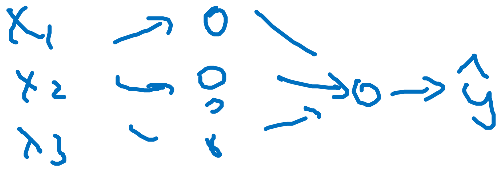
"dz" = "da"

$\frac{d}{dz} g(z) = g'(z)$

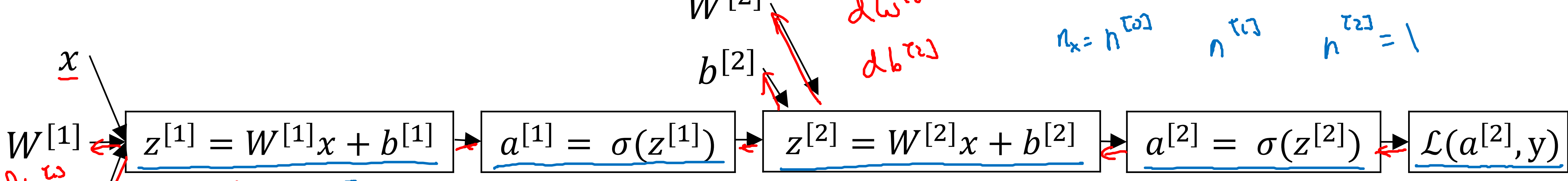
$da = \frac{d}{da}\mathcal{L}(a,y) = -y \log a - (1-y)\log(1-a)$

$= -\frac{y}{a} + \frac{1-y}{1-a}$

Neural network gradients



$n_x = n^{[0]}$ $n^{[1]}$ $n^{[2]} = 1$



$W^{[2]}$ $dW^{[2]}$

$b^{[2]}$ $db^{[2]}$

x

$W^{[1]}$ $dw^{[1]}$

$b^{[1]}$ $db^{[1]}$

$z^{[1]} = W^{[1]}x + b^{[1]}$ → $a^{[1]} = \sigma(z^{[1]})$ → $z^{[2]} = W^{[2]}x + b^{[2]}$ → $a^{[2]} = \sigma(z^{[2]})$ → $\mathcal{L}(a^{[2]},y)$

$dz^{[1]} = w^{[2]T} dz^{[2]}$ $da^{[1]} = \ldots$ $dz^{[2]} = a^{[2]} - y$ $da^{[2]}$

$* \, g^{[1]'}(z^{[1]})$

element wise product

$dW^{[2]} = dz^{[2]} a^{[1]T}$

$db^{[2]} = dz^{[2]}$

"$dw = dz \cdot x$"

$w^{[1]}$: [______]

$w^{[2]}$ $(n^{[2]}, n^{[1]})$

$z^{[2]}, dz^{[2]}$ $(n^{[2]}, 1)$ $- (1,1)$

$z^{[1]}, dz^{[1]}$ $(n^{[1]}, 1)$

foo $W^{[1]}$

dfoo $dW^{[2]}$

$dW^{[1]} = dz^{[1]} \cdot x^T$

$db^{[1]} = dz^{[1]}$

$a^{[0]T}$

$$dz^{[1]} = \boxed{w^{[2]T} dz^{[2]}} * \boxed{g^{[1]'}(z^{[1]})}$$

$$(n^{[1]},1) = (n^{[1]}, n^{[2]}) \; (n^{[2]},1) * (n^{[1]},1)$$

Summary of gradient descent

$$dz^{[2]} = a^{[2]} - y$$

$$dW^{[2]} = dz^{[2]} a^{[1]^T}$$

$$db^{[2]} = dz^{[2]}$$

$$dz^{[1]} = W^{[2]T} dz^{[2]} * g^{[1]\prime}(z^{[1]})$$

$$dW^{[1]} = dz^{[1]} x^T$$

$$db^{[1]} = dz^{[1]}$$

Vectorized Implementation:

$$z^{[1]} = w^{[1]} x + b^{[1]}$$

$$a^{[1]} = g^{[1]}(z^{[1]})$$

$$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$$Z^{[1]} = W^{[1]} X + b^{[1]}$$

$$A^{[1]} = g^{[1]}(Z^{[1]}).$$

Summary of gradient descent

$$\underline{dz^{[2]}} = \underline{a^{[2]}} - \underline{y}$$

$$dW^{[2]} = dz^{[2]} a^{[1]^T}$$

$$db^{[2]} = dz^{[2]}$$

$$\underbrace{dz^{[1]}}_{(n^{[1]},1)} = W^{[2]T} dz^{[2]} * g^{[1]\prime}(\mathrm{z}^{[1]})$$

$$dW^{[1]} = dz^{[1]} x^T$$

$$db^{[1]} = dz^{[1]}$$

$$\underline{dZ^{[2]} = A^{[2]} - Y}$$

$$J(\ldots) = \frac{1}{m} \sum_{i=1}^{n} \mathcal{L}(\hat{y}, y)$$

$$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]^T}$$

$$db^{[2]} = \frac{1}{m} np.sum(dZ^{[2]}, axis = 1, keepdims = True)$$

elementwise product

$$\underbrace{dZ^{[1]}}_{(n^{[1]},m)} = \underbrace{W^{[2]T} dZ^{[2]}}_{(n^{[1]},m)} * \underbrace{g^{[1]\prime}(\mathrm{Z}^{[1]})}_{(n^{[1]},m)}$$

$$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$$

$$db^{[1]} = \frac{1}{m} np.sum(dZ^{[1]}, axis = 1, keepdims = True)$$

One hidden layer Neural Network

Random Initialization

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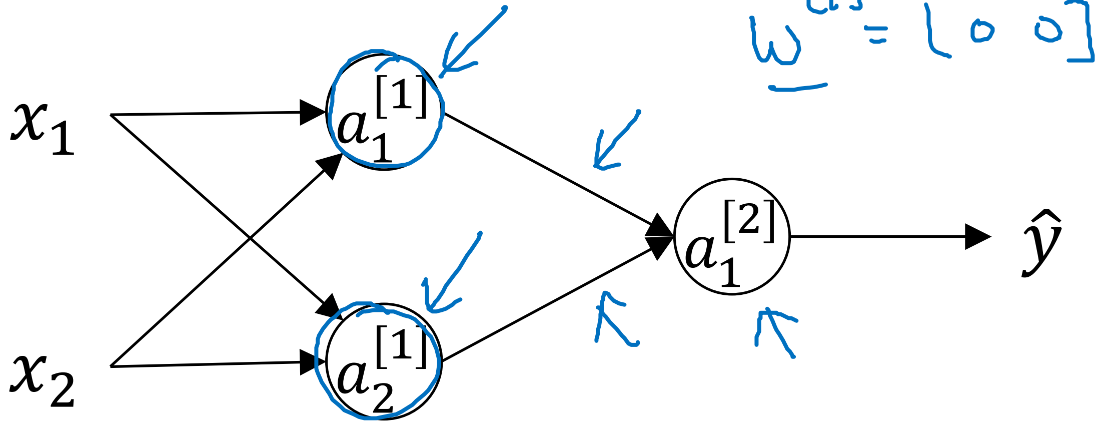
What happens if you initialize weights to zero?



$W^{[2]} = [0 \quad 0]$

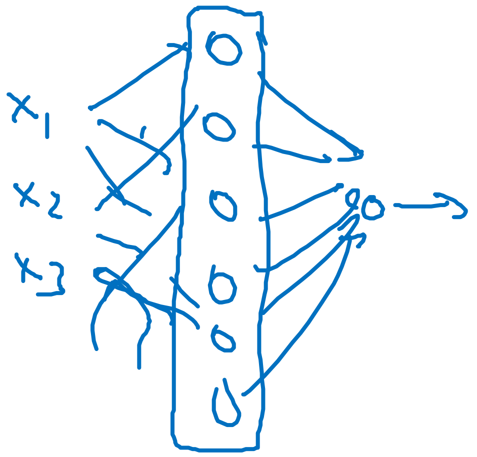
$n^{[0]} = 2$ $n^{[1]} = 2$

$W^{[1]} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $b^{[1]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$a_1^{[1]} = a_2^{[1]}$ $dz_1^{[1]} = dz_2^{[1]}$

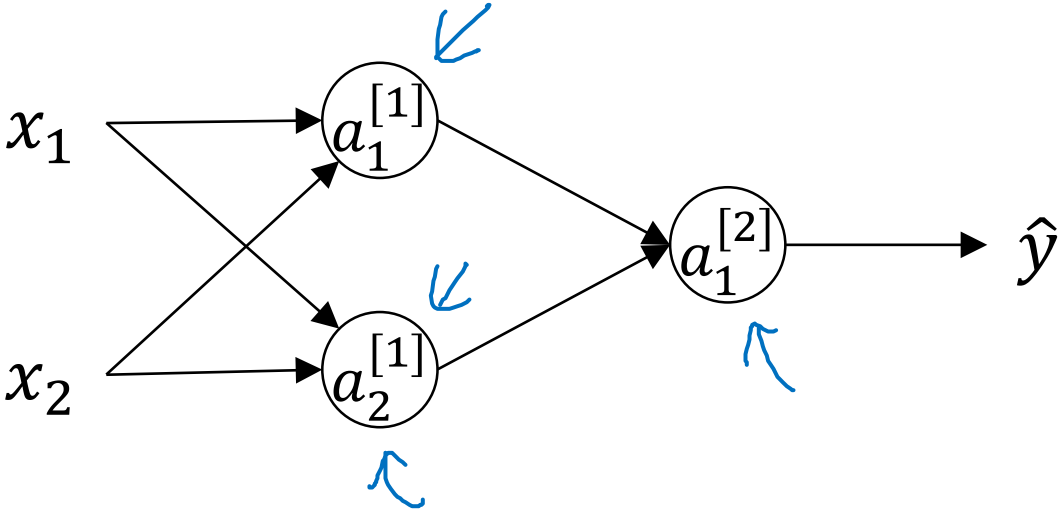
$dW = \begin{bmatrix} u & v \\ u & v \end{bmatrix}$ $W^{[1]} = W^{[1]} - \alpha dW$

Symmetric



$W^{[1]} = \begin{bmatrix} - - - - - \\ - - - - - - \end{bmatrix}$

Random initialization



→ $W^{[1]} =$ np.random.randn((2,2)) * 0.01

100?

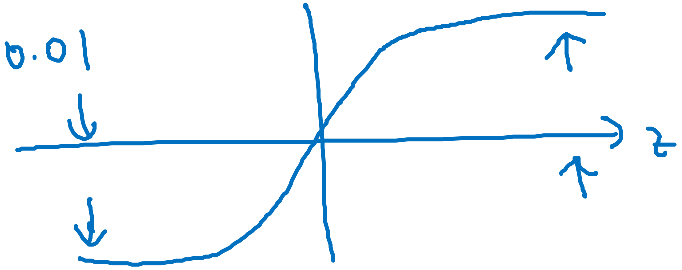
$b^{[1]} =$ np.zeros((2,1))

$W^{[2]} =$ np.random.randn((1,2)) * 0.01

$b^{[2]} = 0$

→ $z^{[1]} = W^{[1]}x + b^{[1]}$

$a^{[1]} = g^{[1]}(z^{[1]})$



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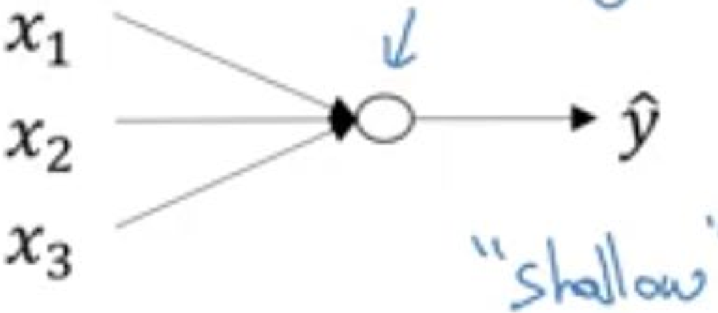
Deep Neural Networks

Deep L-layer Neural network

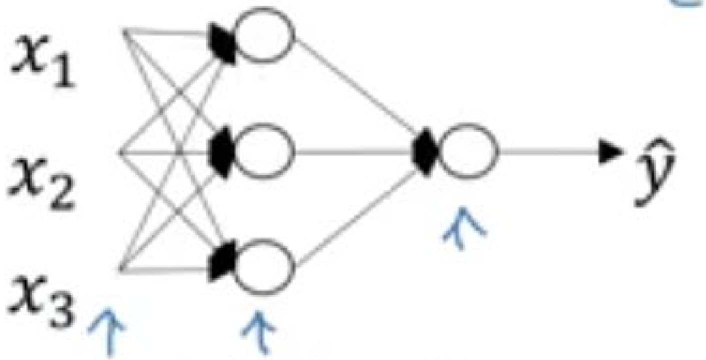
What is a deep neural network?



"1 layer NN"

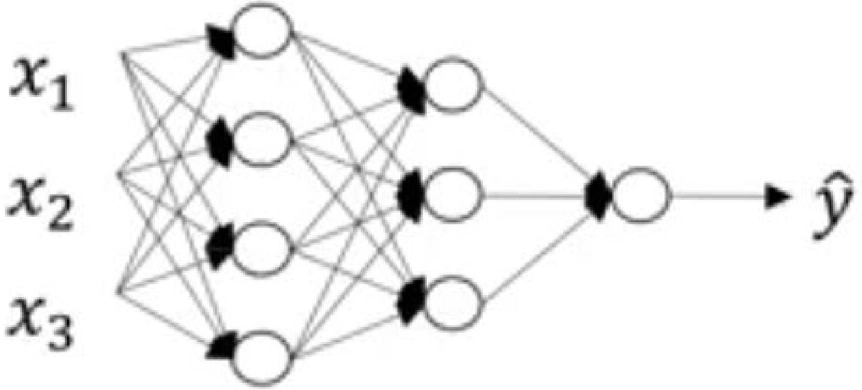
"shallow"

logistic regression

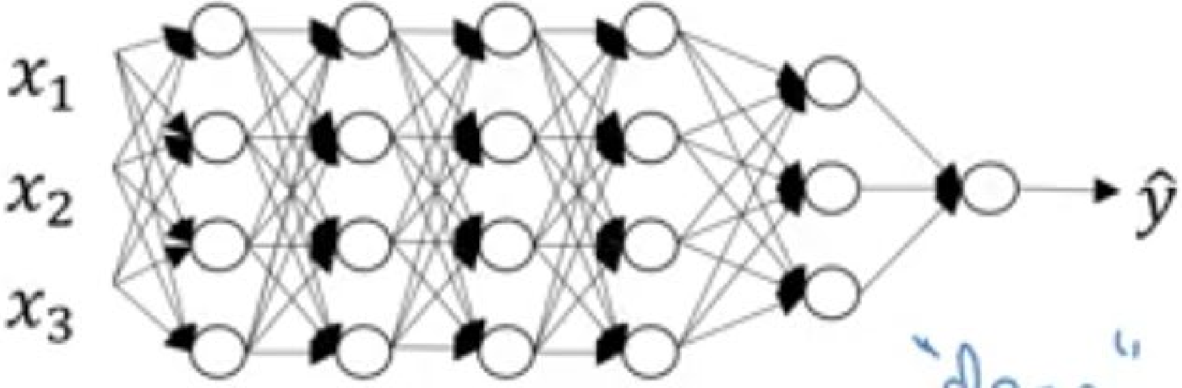


"2 layer" NN

1 hidden layer



2 hidden layers

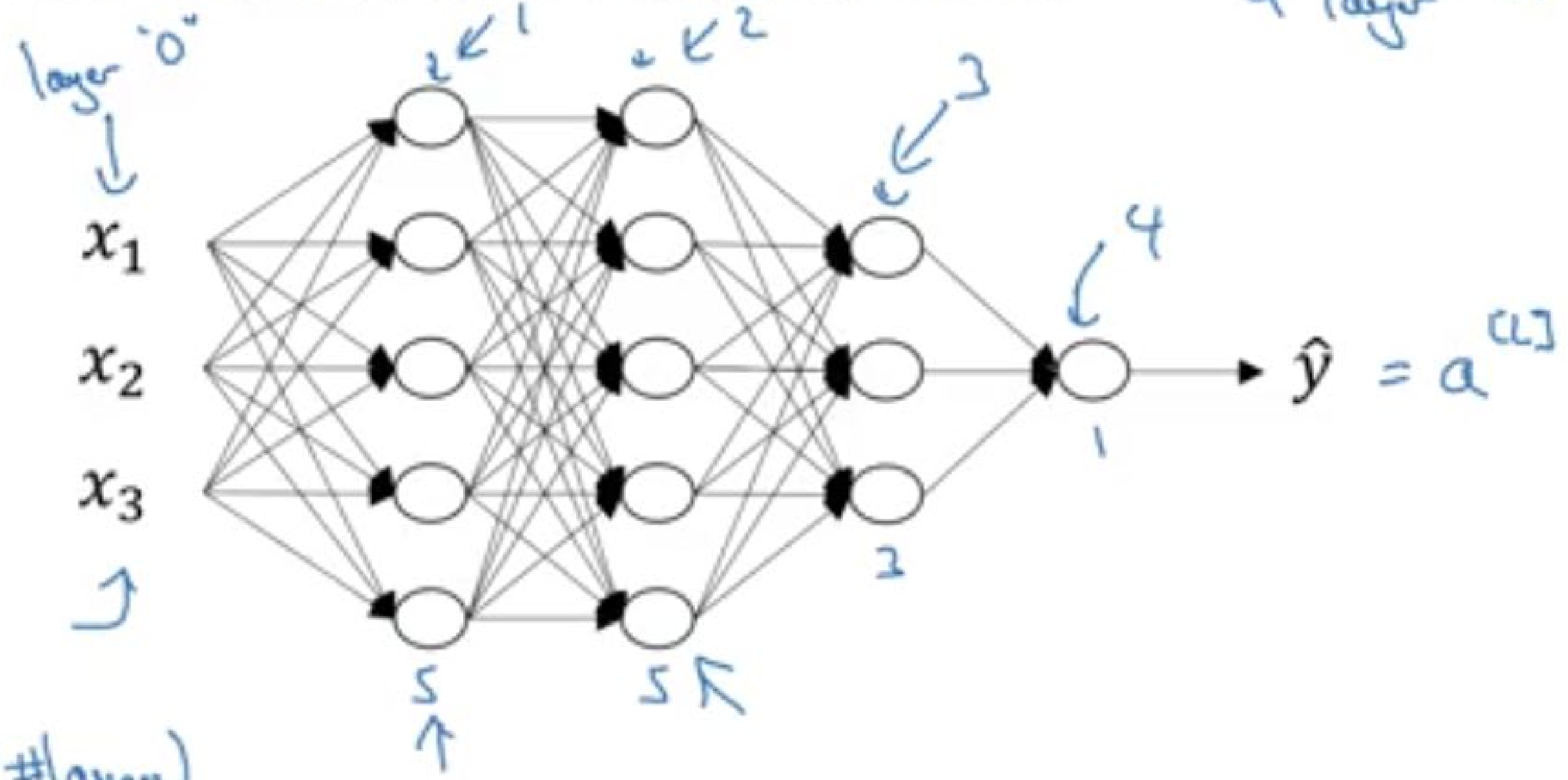


"deep"

5 hidden layers

Deep neural network notation

4 layer NN



$X = a^{[0]}$

$L = 4$ (#layers)

$n^{[l]}$ = #units in layer l

$n^{[1]} = 5$, $n^{[2]} = 5$, $n^{[3]} = 3$, $n^{[4]} = n^{[L]} = 1$

$n^{[0]} = n_x = 3$

$a^{[l]}$ = activations in layer l

$a^{[l]} = g^{[l]}(z^{[l]})$, $W^{[l]}$ = weights for $\underline{z^{[l]}}$

$b^{[l]}$

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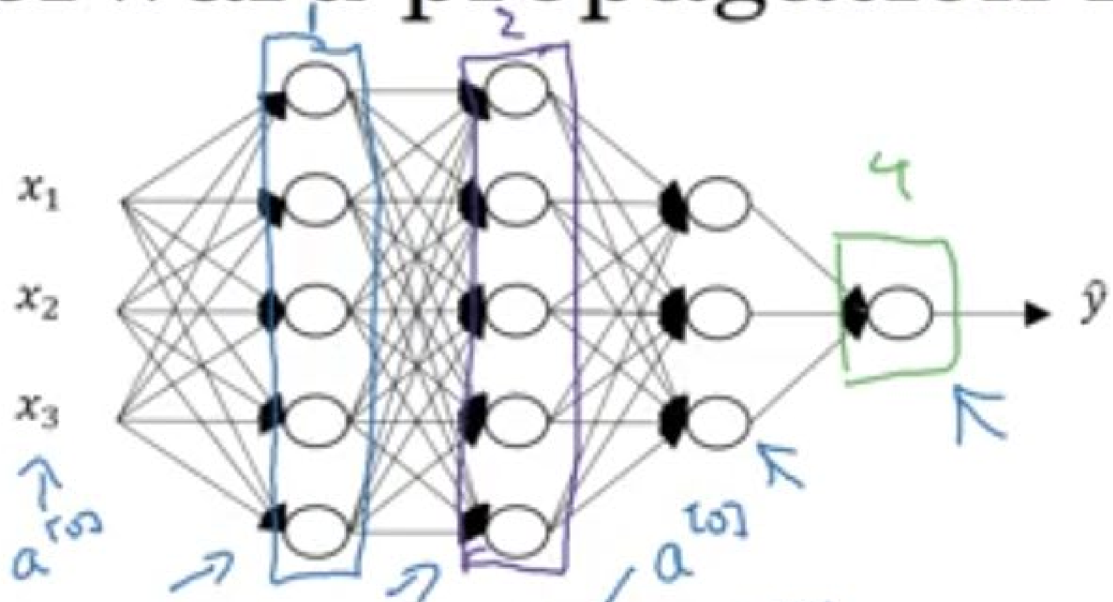
Deep Neural Networks

Forward Propagation in a Deep Network

Forward propagation in a deep network



$A^{[0]} = X$

$$Z^{[l]} = W^{[l]} A^{[l-1]} + b^{[l]}$$
$$A^{[l]} = g^{[l]}(Z^{[l]})$$

x: $z^{[1]} = W^{[1]} a^{[0]} + b^{[1]}$

$a^{[1]} = g^{[1]}(z^{[1]})$

$z^{[2]} = W^{[2]} a^{[1]} + b^{[2]}$

$a^{[2]} = g^{[2]}(z^{[2]})$

$z^{[4]} = W^{[4]} a^{[3]} + b^{[4]}$, $a^{[4]} = g^{[4]}(z^{[4]}) = \hat{y}$

$$\begin{bmatrix} z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

Vectorized:

$X = A^{[0]}$

$Z^{[1]} = W^{[1]} A^{[0]} + b^{[1]}$

$A^{[1]} = g^{[1]}(Z^{[1]})$

$Z^{[2]} = W^{[2]} A^{[1]} + b^{[2]}$

$A^{[2]} = g^{[2]}(Z^{[2]})$

$\hat{Y} = g(Z^{[4]}) = A^{[4]}$

for $l = 1..4$

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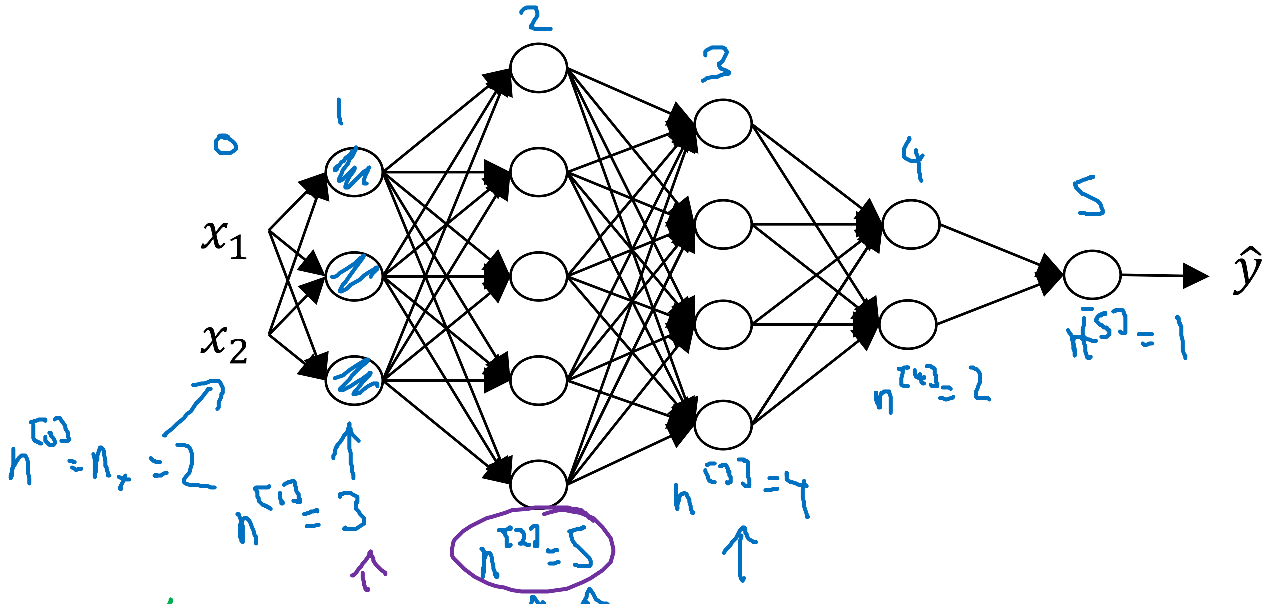
Deep Neural Networks

Getting your matrix dimensions right

Parameters $W^{[l]}$ and $b^{[l]}$

$L = 5$



$z^{[l]} = g^{[l]}(a^{[l]})$

$a^{[l]}$

$z^{[1]} = \boxed{W^{[1]} \cdot x} + \boxed{b^{[1]}}$

$(3,1) \leftarrow (3,2) \quad (2,1) \quad (3,1)$

$(n^{[1]},1) \quad (n^{[1]},n^{[0]}) \quad (n^{[0]},1) \quad (n^{[1]},1)$

$$\begin{bmatrix}\cdot\\\cdot\\\cdot\end{bmatrix} = \begin{bmatrix}\cdot&\cdot\\\cdot&\cdot\\\cdot&\cdot\end{bmatrix}\begin{bmatrix}\cdot\\\cdot\end{bmatrix}$$

$W^{[1]}: (n^{[1]}, n^{[0]})$

$W^{[2]}: (5,3) \quad (n^{[2]}, n^{[1]})$

$z^{[2]} = \boxed{W^{[2]} \cdot a^{[1]}} + \boxed{b^{[2]}}$

$\rightarrow (5,1) \quad (5,3) \quad (3,1) \quad (5,1)$

$(n^{[2]},1)$

$W^{[3]}: (4,5)$

$W^{[4]}: (2,4)$, $W^{[5]}: (1,2)$

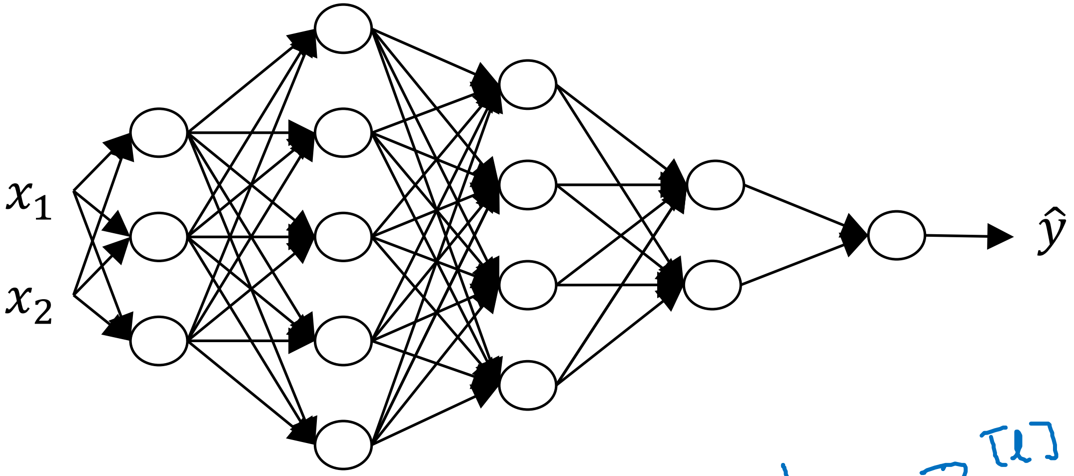
$\rightarrow W^{[l]}: (n^{[l]}, n^{[l-1]})$

$\rightarrow b^{[l]}: (n^{[l]}, 1)$

$dW^{[l]}: (n^{[l]}, n^{[l-1]})$

$db^{[l]}: (n^{[l]}, 1)$

Vectorized implementation



$z^{[1]} = W^{[1]} \cdot x + b^{[1]}$

$(n^{[1]},1)$ $(n^{[1]},n^{[0]})$ $(n^{[0]},1)$ $(n^{[1]},1)$

$[z^{[1](1)} \; z^{[1](2)} \; \dots \; z^{[1](m)}]$

$Z^{[1]} = W^{[1]} \cdot X + b^{[1]}$

$(n^{[1]},m)$ $(n^{[1]},n^{[0]})$ $(n^{[0]},m)$ $(n^{[1]},1)$

$(n^{[1]},m)$

$z^{[l]}, a^{[l]} : (n^{[l]},1)$

$Z^{[l]}, A^{[l]} : (n^{[l]},m)$

$l=0 \quad A^{[0]} = X = (n^{[0]},m)$

$dZ^{[l]}, dA^{[l]} : (n^{[l]},m)$

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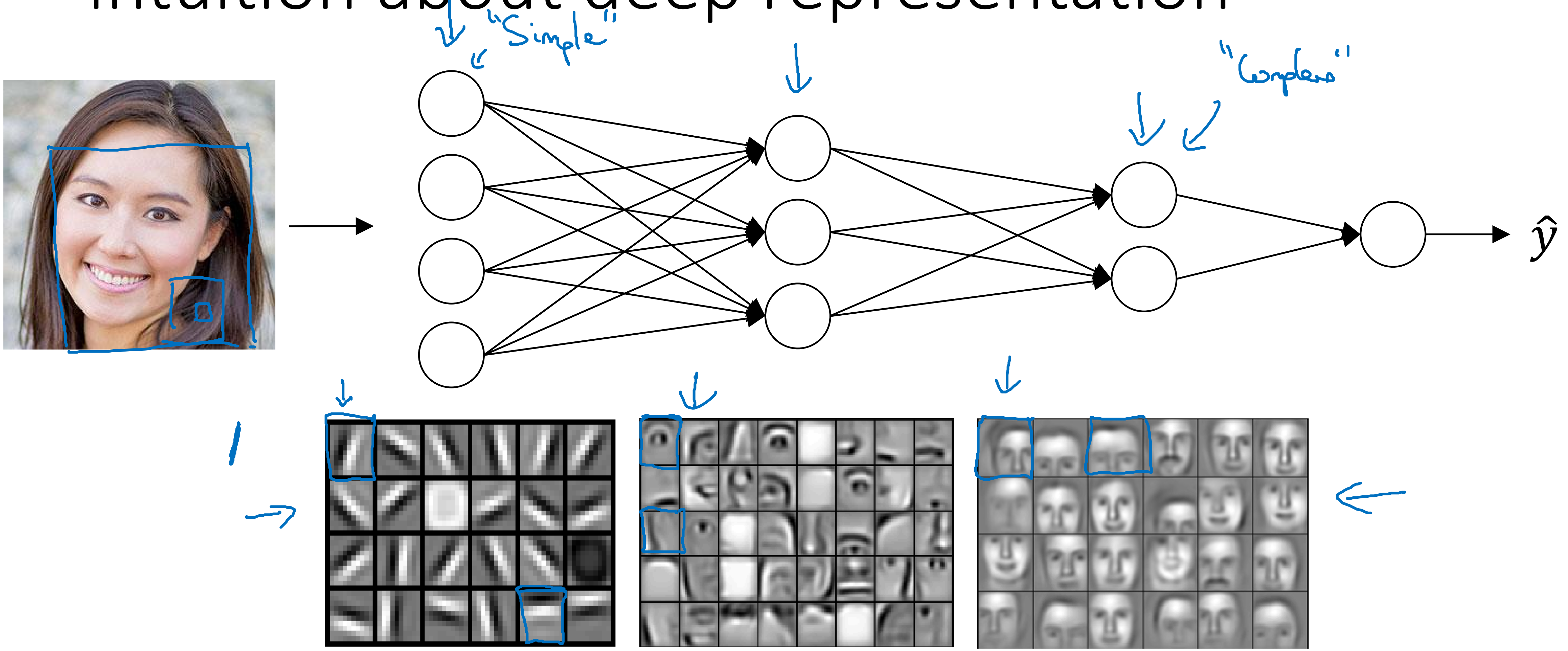
Deep Neural Networks

Why deep representations?

Intuition about deep representation

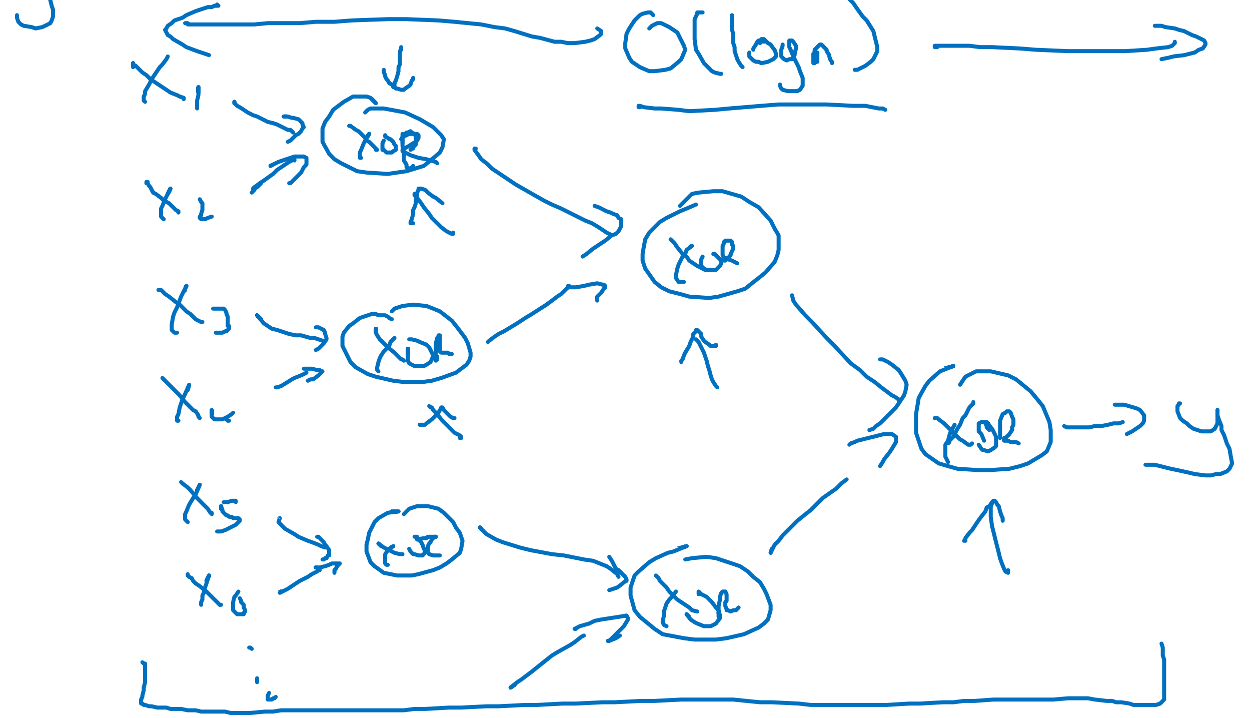


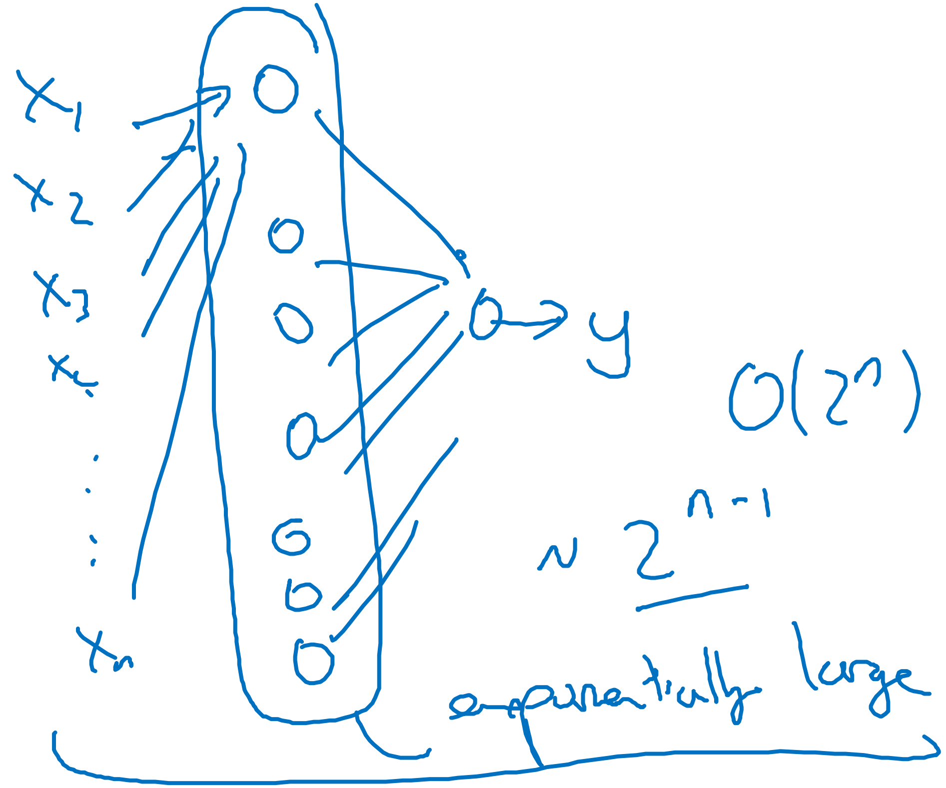
Audio → low level audio waveform → Phonemes C A T → Words → Sentence phrase

Circuit theory and deep learning

Informally: There are functions you can compute with a "small" L-layer deep neural network that shallower networks require exponentially more hidden units to compute.

$y = x_1 \text{ XOR } x_2 \text{ XOR } x_3 \text{ XOR} \dots \text{ XOR } x_n$





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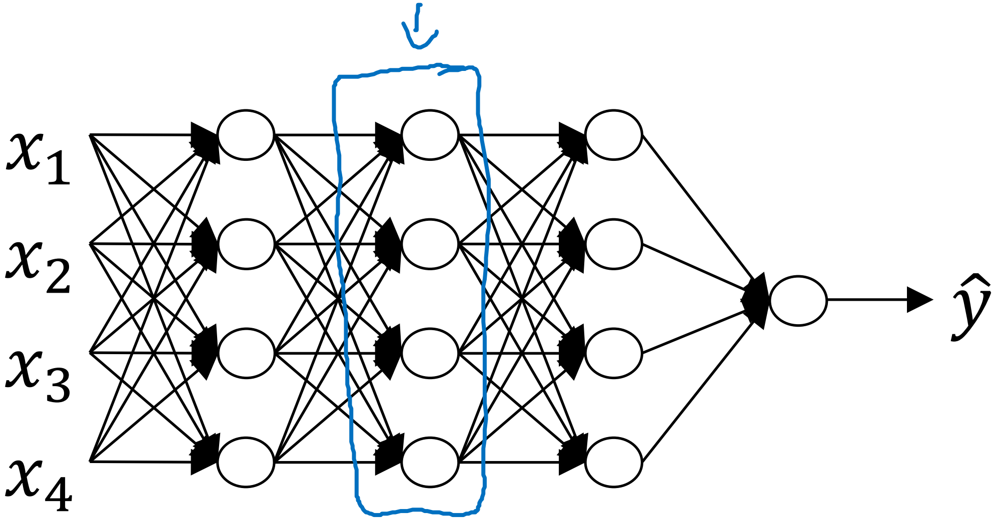
Deep Neural Networks

Building blocks of deep neural networks

Forward and backward functions



layer l: $W^{[l]}, b^{[l]}$

→ Forward: Input $a^{[l-1]}$, output $a^{[l]}$

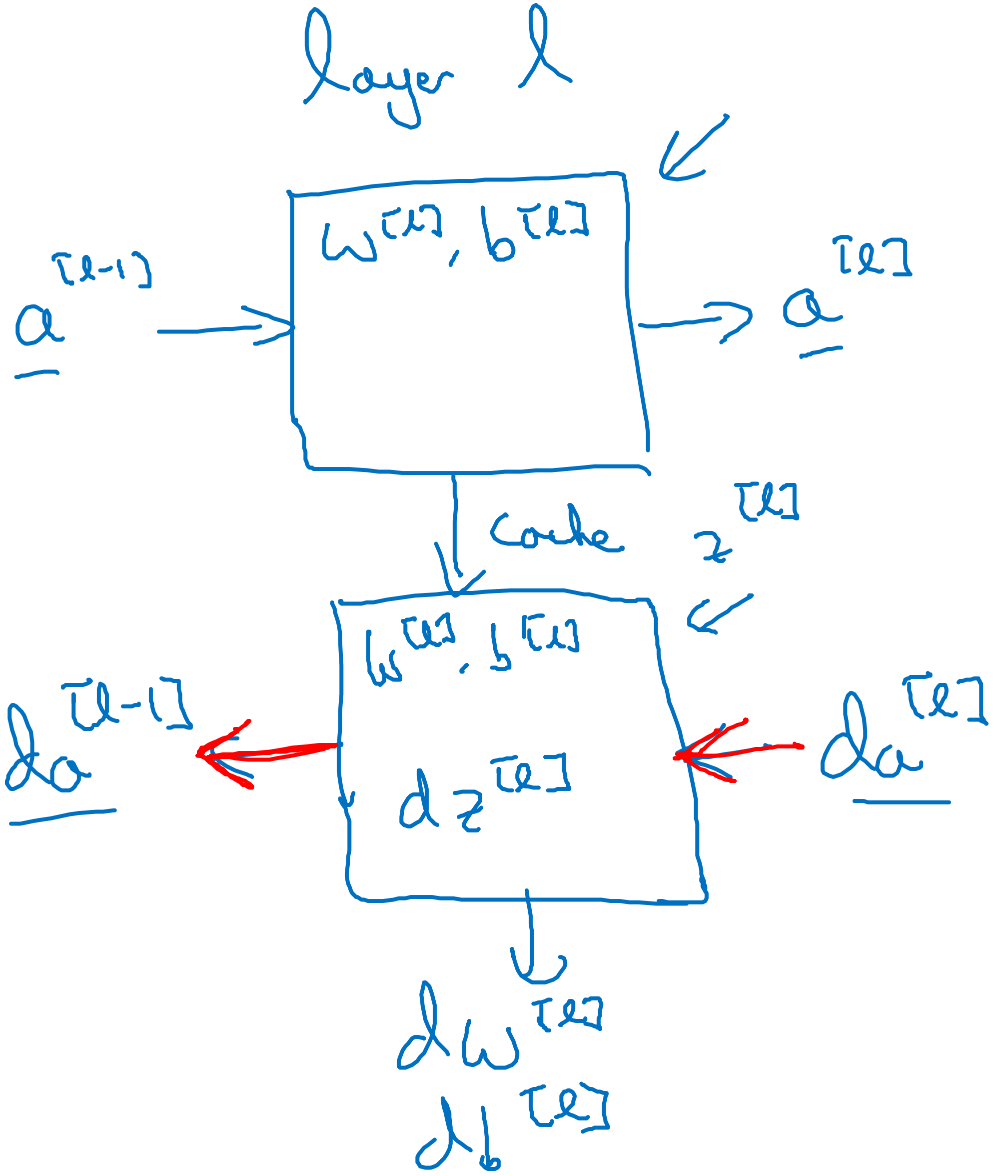
$\underline{z^{[l]}} = W^{[l]} \underline{a^{[l-1]}} + b^{[l]}$ cache $\underline{z^{[l]}}$

$\underline{a^{[l]}} = g^{[l]}(z^{[l]})$

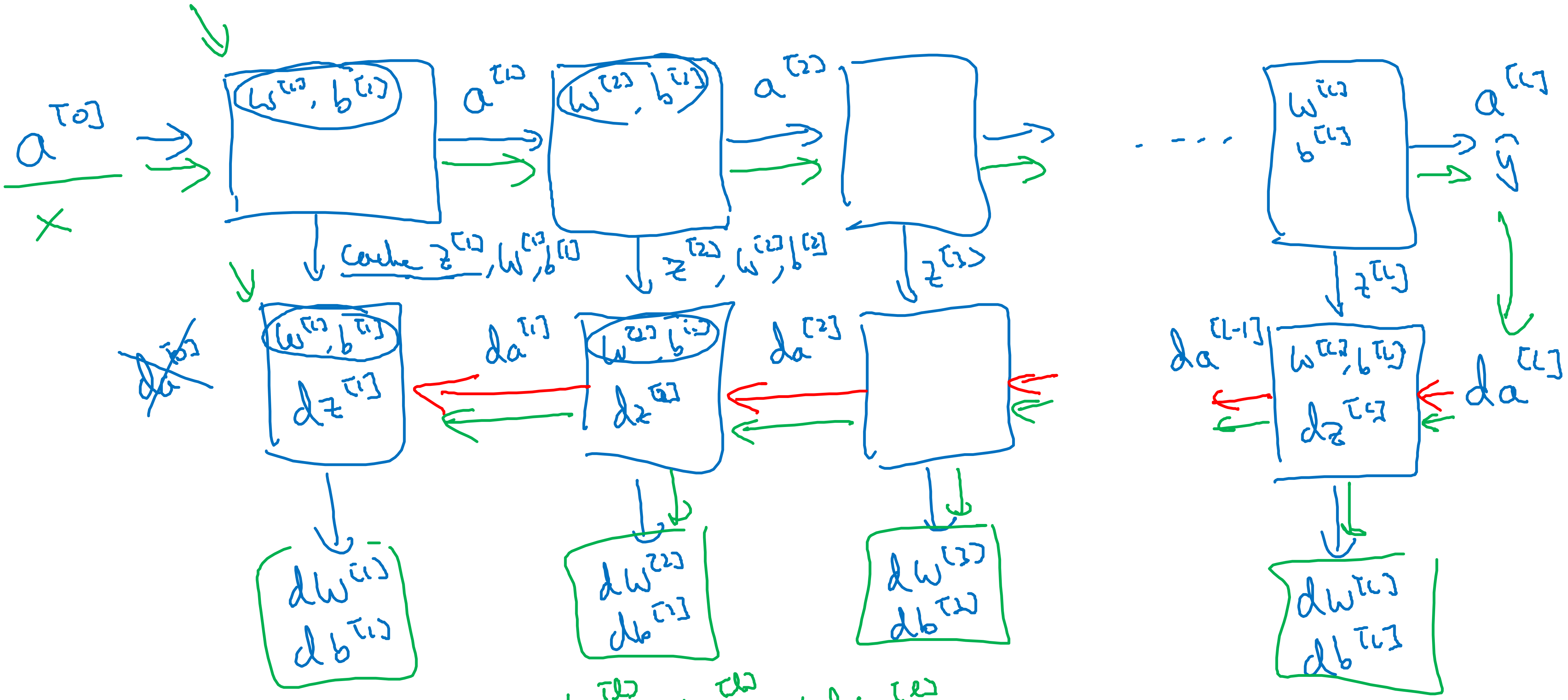
→ Backward: Input $da^{[l]}$, cache $(z^{[l]})$ output $da^{[l-1]}$

$dW^{[l]}$

$db^{[l]}$



Forward and backward functions



$$W^{[l]} := W^{[l]} - \alpha \, dW^{[l]}$$

$$b^{[l]} := b^{[l]} - \alpha \, db^{[l]}$$

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Deep Neural Networks

Forward and backward propagation

Forward propagation for layer l

→ Input $a^{[l-1]}$ ←

$W^{[l]}, b^{[l]}$

→ Output $a^{[l]}$, cache ($\underline{z^{[l]}}$)

$$z^{[l]} = W^{[l]} \cdot a^{[l-1]} + b^{[l]}$$

$$a^{[l]} = g^{[l]}(z^{[l]})$$

Vectorized:

$$Z^{[l]} = W^{[l]} \cdot A^{[l-1]} + b^{[l]}$$

$$A^{[l]} = g^{[l]}(Z^{[l]})$$

$a^{[0]}$

$A^{[0]}$

$$X = \underset{\uparrow}{A^{[0]}} \rightarrow \square \rightarrow \square \rightarrow \square \rightarrow$$

$$\xrightarrow{\hspace{4cm}}$$

Backward propagation for layer *l*

- → Input $\underline{da^{[l]}}$
- → Output $\boxed{da^{[l-1]}}, \underline{dW^{[l]}}, \underline{db^{[l]}}$

$$dz^{[l]} = da^{[l]} * g^{[l]'}(z^{[l]})$$

$$dW^{[l]} = dz^{[l]} \cdot \underline{a^{[l-1]}}$$

$$db^{[l]} = dz^{[l]}$$

$$\boxed{da^{[l-1]}} = W^{[l]T} \cdot dz^{[l]}$$

$$\underline{dz^{[l]} = W^{[l+1]T} dz^{[l+1]} * g^{[l]'}(z^{[l]})}$$

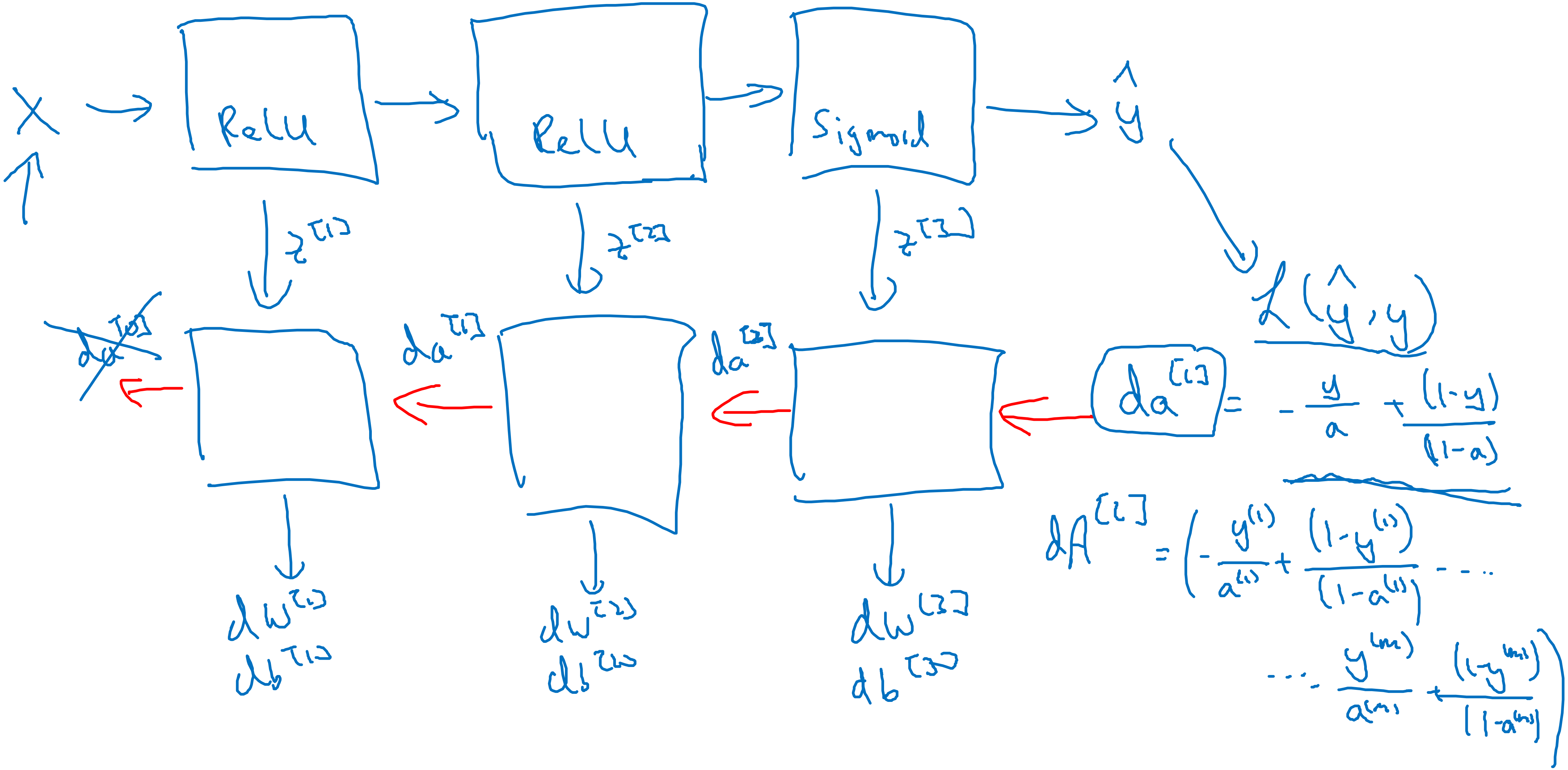
$$dZ^{[l]} = dA^{[l]} * g^{[l]'}(Z^{[l]})$$

$$dW^{[l]} = \frac{1}{m} dZ^{[l]} \cdot A^{[l-1]T}$$

$$db^{[l]} = \frac{1}{m} \text{np.sum}(dZ^{[l]}, \text{axis}=1, \text{keepdims}=\text{True})$$

$$\boxed{dA^{[l-1]}} = W^{[l]T} \cdot dZ^{[l]}$$

Summary



$$\boxed{da^{[l]}} = -\frac{y}{a} + \frac{(1-y)}{(1-a)}$$

$$dA^{[l]} = \left(-\frac{y^{(1)}}{a^{(1)}} + \frac{(1-y^{(1)})}{(1-a^{(1)})} \cdots \cdots -\frac{y^{(m)}}{a^{(m)}} + \frac{(1-y^{(m)})}{(1-a^{(m)})}\right)$$

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Deep Neural Networks

Parameters vs Hyperparameters

What are hyperparameters?

Parameters: $W^{[1]}, b^{[1]}, W^{[2]}, b^{[2]}, W^{[3]}, b^{[3]}$ …

Hyperparameters: learning rate α

#iterations

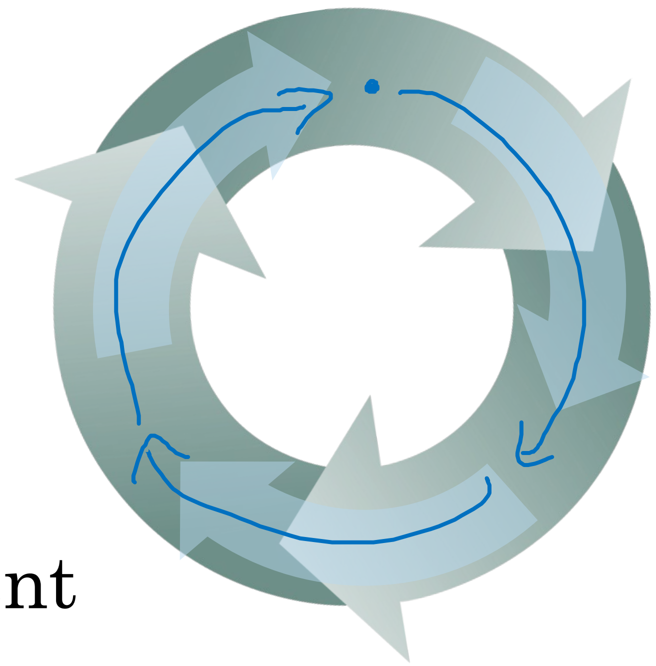
#hidden layers L

#hidden units $n^{[1]}, n^{[2]}, \ldots$

choice of activation function

Later: Momentum, mini-batch size, regularizations, …

Applied deep learning is a very empirical process

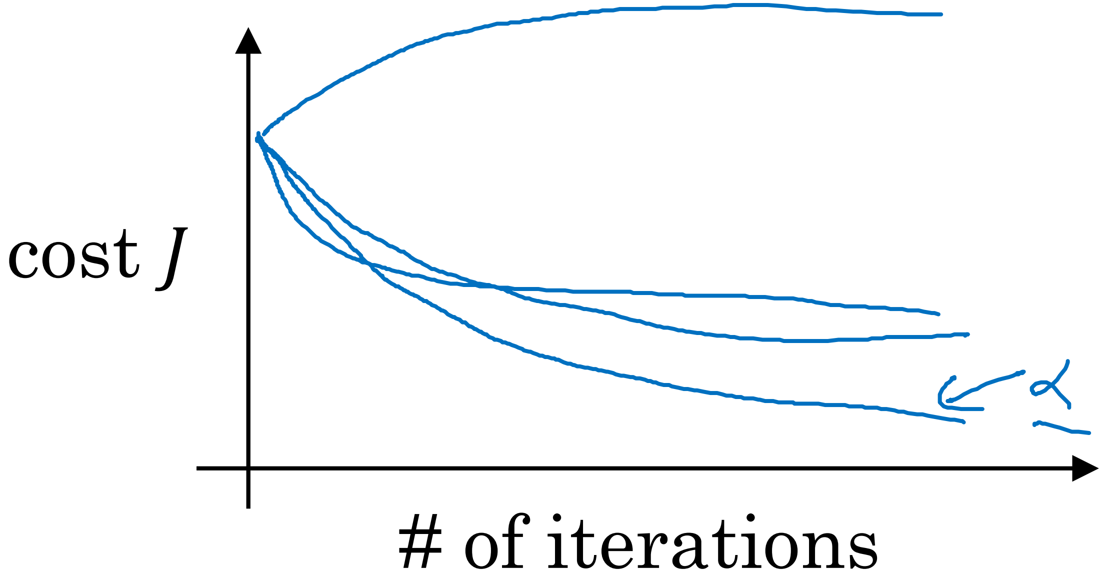


Idea

$\alpha = \cancel{0.01}$ 0.05

Experiment

Code



Vision, Speech, NLP, Ad, Search, Recommendation.

deeplearning.ai

Deep Neural Networks

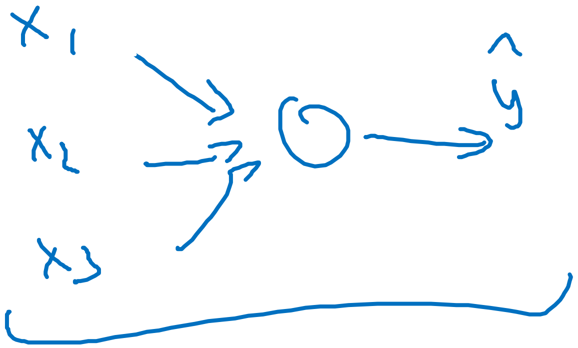
What does this have to do with the brain?

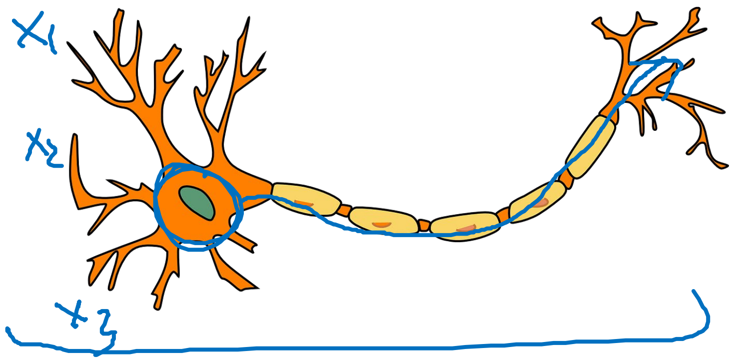
Forward and backward propagation

$$
\begin{aligned}
Z^{[1]} &= W^{[1]}X + b^{[1]} \\
A^{[1]} &= g^{[1]}(Z^{[1]}) \\
Z^{[2]} &= W^{[2]}A^{[1]} + b^{[2]} \\
A^{[2]} &= g^{[2]}(Z^{[2]}) \\
&\vdots \\
A^{[L]} &= g^{[L]}\left(Z^{[L]}\right) = \hat{Y}
\end{aligned}
$$

$$
\begin{aligned}
dZ^{[L]} &= A^{[L]} - Y \\
dW^{[L]} &= \frac{1}{m} dZ^{[L]} A^{[L]^T} \\
db^{[L]} &= \frac{1}{m} np.\mathrm{sum}(\mathrm{d}Z^{[L]}, axis = 1, keepdims = True) \\
dZ^{[L-1]} &= dW^{[L]^T} dZ^{[L]} g'^{[L]}(Z^{[L-1]}) \\
&\vdots \\
dZ^{[1]} &= dW^{[L]^T} dZ^{[2]} g'^{[1]}(Z^{[1]}) \\
dW^{[1]} &= \frac{1}{m} dZ^{[1]} A^{[1]^T} \\
db^{[1]} &= \frac{1}{m} np.\mathrm{sum}(\mathrm{d}Z^{[1]}, axis = 1, keepdims = True)
\end{aligned}
$$

"It's like the brain"





$X \longrightarrow y$