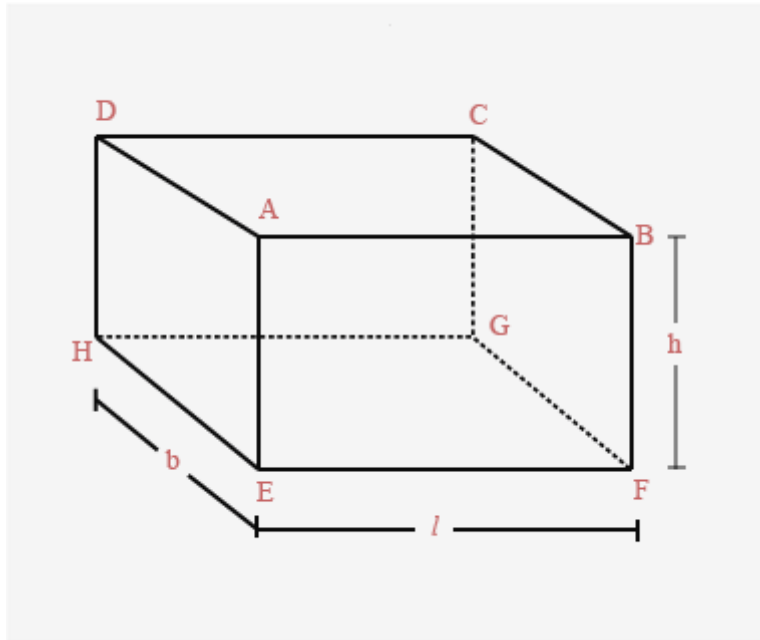# Surface Areas and Volumes

Basics

Cuboid and It's Surface Area

The surface area of a cuboid is equal to the sum of the areas of its six rectangular faces.
Consider a cuboid whose dimensions are $l \times b \times h$ respectively.

Cuboid with length l, breadth b and height h

Total surface area of cuboid(TSA) = Sum of the areas of all its six faces
TSA (cuboid) $= 2(l \times b) + 2(b \times h) + 2(l \times h) = 2(lb + bh + lh)$

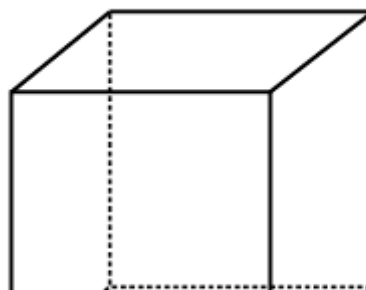Lateral surface area (LSA) is the area of all the sides apart from the top and bottom faces.
Lateral surface area of the cuboid = Area of face AEHD + Area of face BFGC + Area of face ABFE + Area of face DHGC
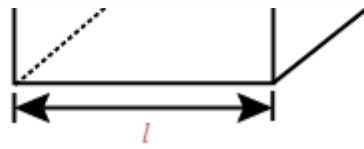LSA (cuboid) $= 2(b \times h) + 2(l \times h) = 2h(l + b)$

Length of diagonal of a cuboid = $\sqrt{l^2 + b^2 + h^2}$

Cube and Its Surface Area

For a cube length = breadth = height

Cube with length l

TSA (cube) $= 2 \times (3l^2) = 6l^2$
Similarly, the Lateral surface area of cube $= 2(l \times l + l \times l) = 4l^2$
Note: Diagonal of a cube $= \sqrt{3}l$

Cylinder and Its Surface Area

Take a cylinder of base radius r and height h units. The curved surface of this cylinder, if opened along the diameter ($d = 2r$) of the circular base can be transformed into a rectangle of length $2\pi r$ and height h units. Thus,
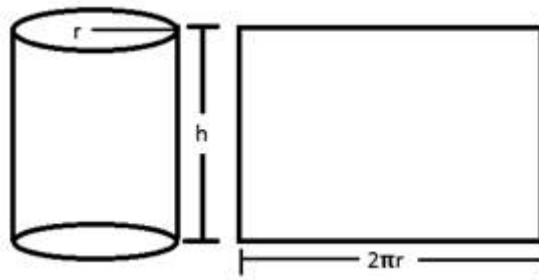
Transformation of a Cylinder into a rectangle.

CSA of a cylinder of base radius r and height $h = 2\pi \times r \times h$
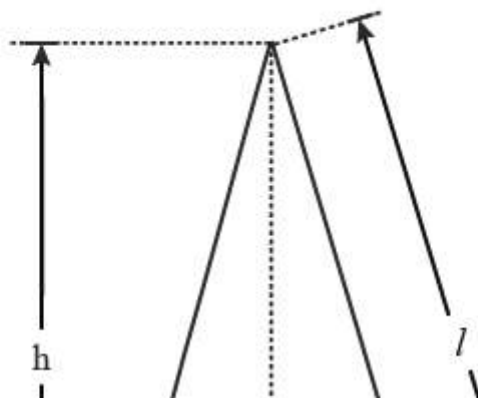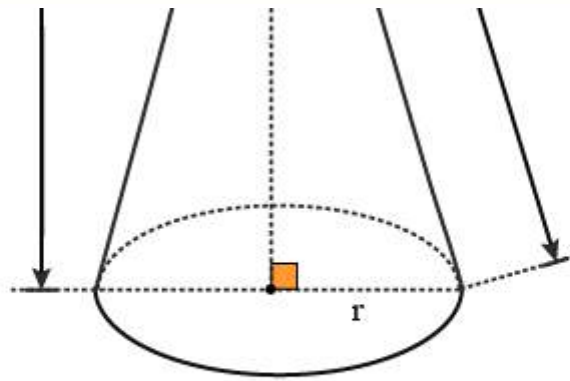TSA of a cylinder of base radius r and height $h = 2\pi \times r \times h$ + area of two circular bases
$= 2\pi \times r \times h + 2\pi r^2$
$= 2\pi r(h + r)$

Right Circular Cone and Its Surface Area

Consider a right circular cone with slant length l, radius r and height h.
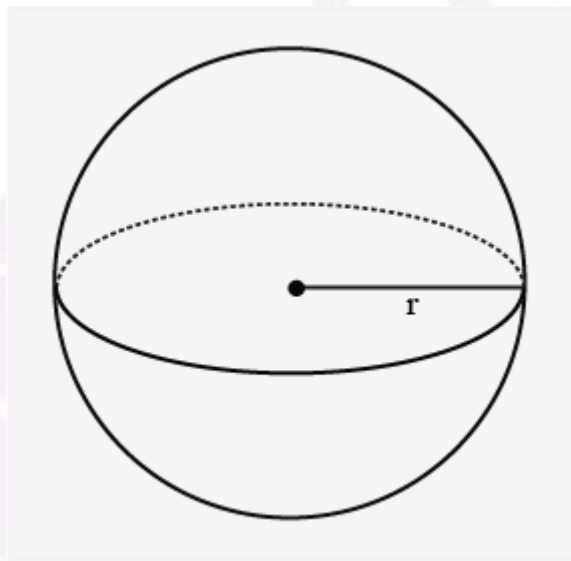
Cone with base radius r and height h

CSA of right circular cone $= \pi r l$

TSA = CSA + area of base $= \pi r l + \pi r^2 = \pi r (l + r)$

Sphere and Its Surface Area

For a sphere of radius r

Curved Surface Area (CSA) = Total Surface Area (TSA) $= 4\pi r^2$

Sphere with radius r

Volume of a Cuboid

Volume of a cuboid $= (base\ area) \times height = (lb)h = lbh$
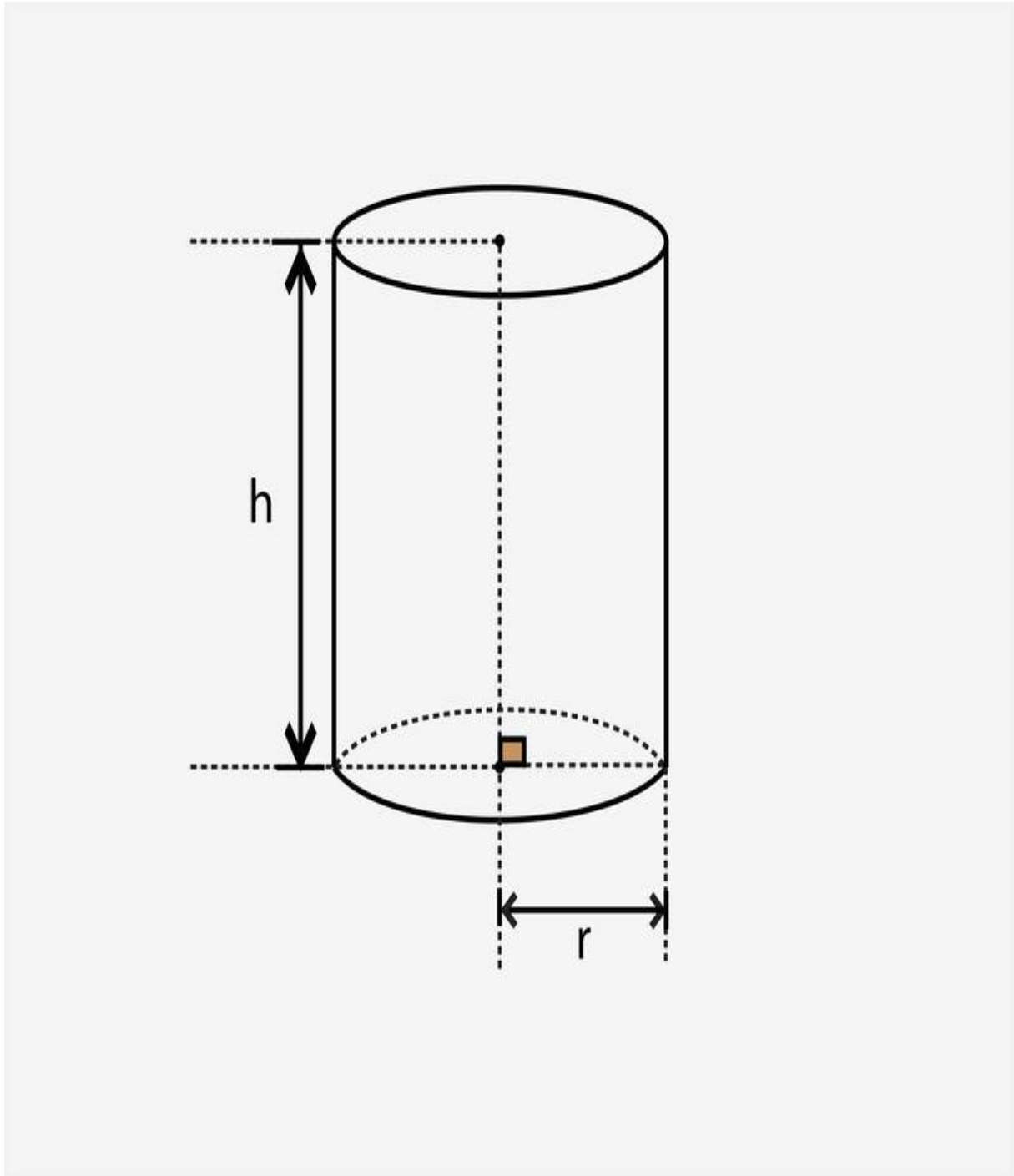
Volume of a Cube

Volume of a cube $= base\ area \times height$

Since all dimensions of a cube are identical, volume = l^3

Where l is the length of the edge of the cube.

Volume of a Cylinder

Volume of a cylinder = Base area × its height = $(\pi \times r^2) \times h = \pi r^2 h$

Cylinder with height h and base radius r

Volume of a Right Circular Cone

The volume of a Right circular cone is $\frac{1}{3}$ times that of a cylinder of same height and base.
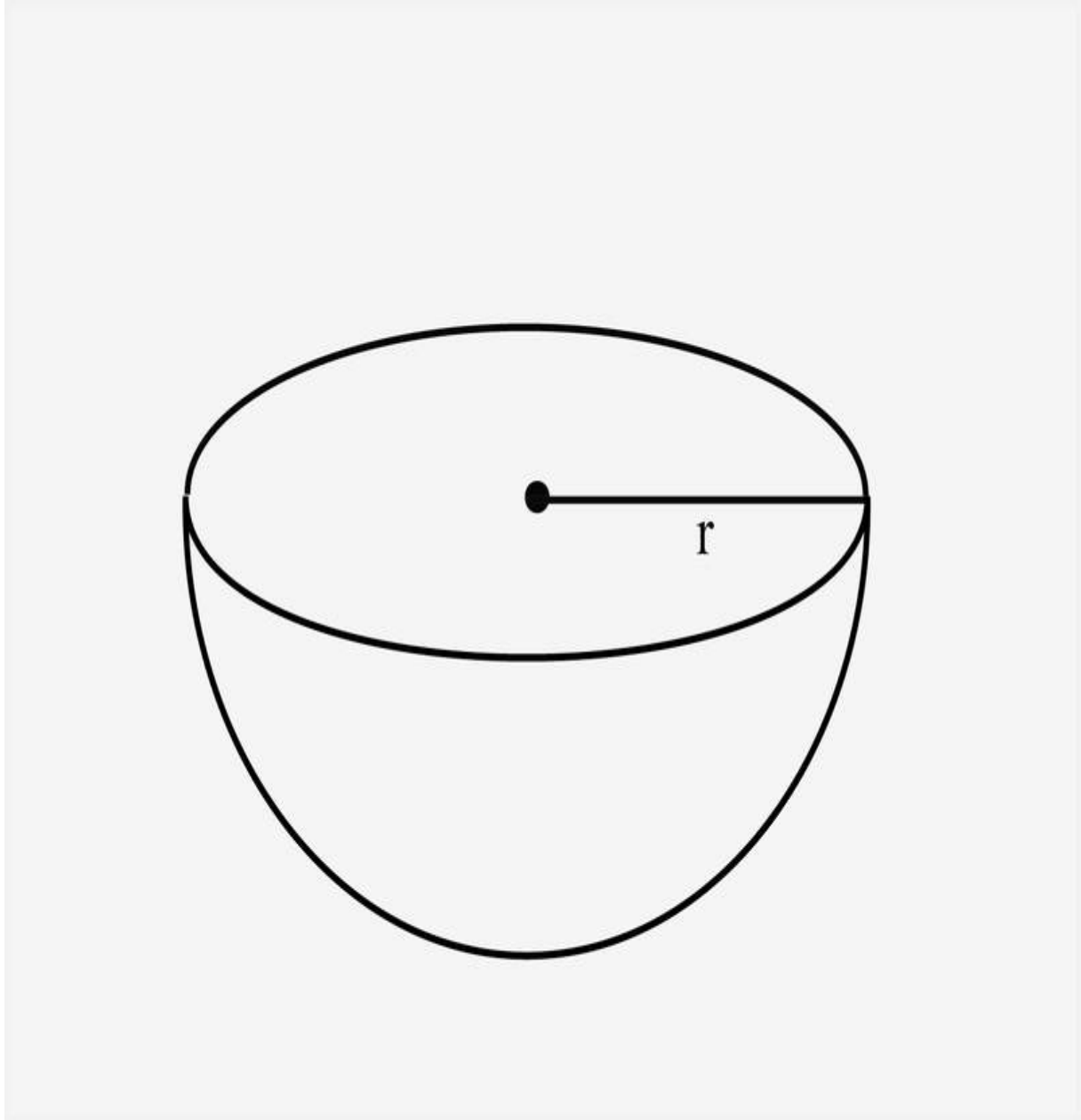In other words, 3 cones make a cylinder of the same height and base.

Volume of a Right circular cone $= \frac{1}{3}\pi r^2 h$

Where r is the radius of the cone and h is the height of the cone.

Volume of a Sphere

The volume of a sphere of radius $r = \frac{4}{3}\pi r^3$

Hemisphere and Its Surface Area

Hemisphere of radius r

We know that the CSA of a sphere $= 4\pi r^2$. A hemisphere is half of a sphere.

$\therefore$ CSA of a hemisphere of radius $r = 2\pi r^2$

Total Surface Area = curved surface area + area of the base circle

$\Rightarrow$ TSA $= 3\pi r^2$

Volume of Hemisphere

The volume (V) of a hemisphere will be half of that of a sphere.

∴ Volume of the hemisphere of radius $r = \frac{2}{3}\pi r^3$

Combination of Solids

Surface Area of Combined Figures

Areas of complex figures can be broken down and analysed as simpler known shapes. By finding the areas of these known shapes we can find out the required area of the unknown figure.

Example: 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Length of each cube $= 64^{\frac{1}{3}} = 4cm$

Since these cubes are joined adjacently, they form a cuboid whose length l = 8cm. But height and breadth will remain same = 4cm.
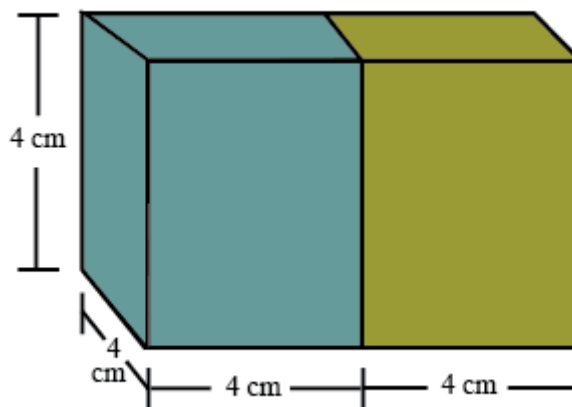
Combination of 2 equal cubes

∴ The new surface area

$= TSA$

$= 2(lb + bh + lh)$

$= 2(8 \times 4 + 4 \times 4 + 8 \times 4)$

$= 2(32 + 16 + 32)$

$= 2(80)$
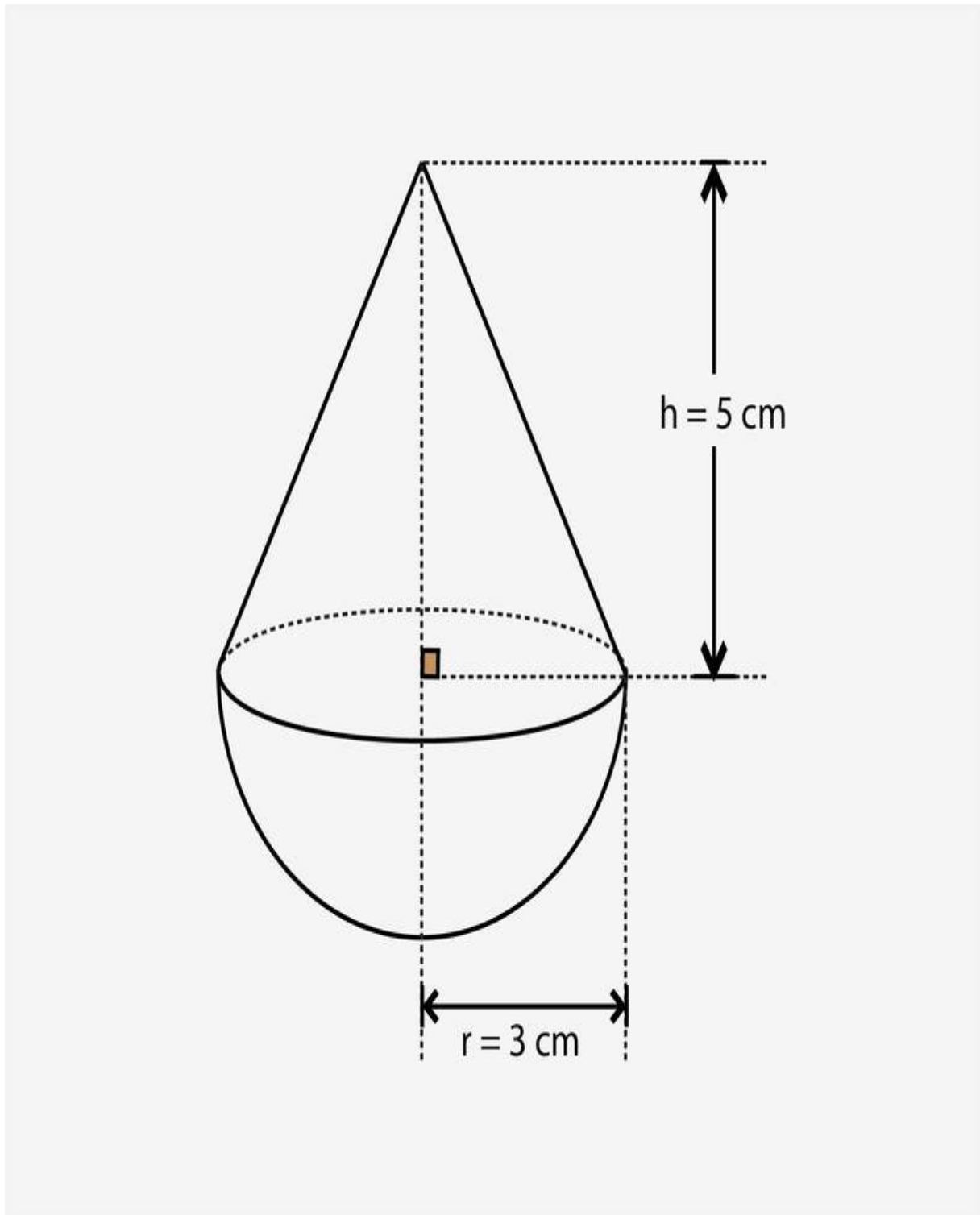
$= 160 \ cm^2$

Volume of Combined Solids

The volume of complex objects can be simplified by visualising it as a combination of shapes of known solids.

Example: A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius.
The above figure can be visualised as follows :

Volume of combined solids

V(solid) = V(Cone) + V(hemisphere)
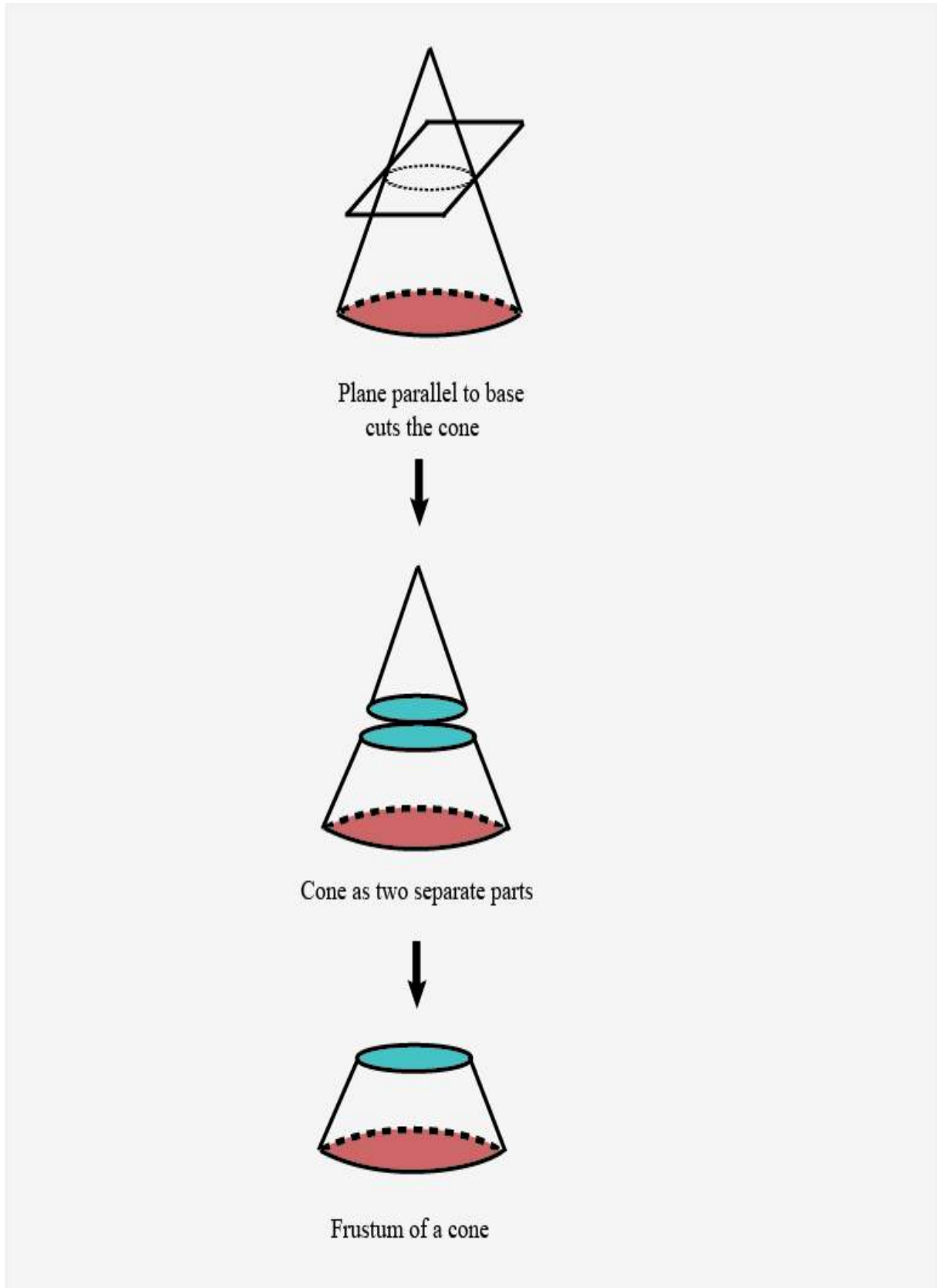
$\Rightarrow V(solid) = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$\Rightarrow V(solid) = \frac{1}{3}\pi(9)(5) + \frac{2}{3}\pi(27)$

$\Rightarrow V(solid) = 33\pi\ cm^3$
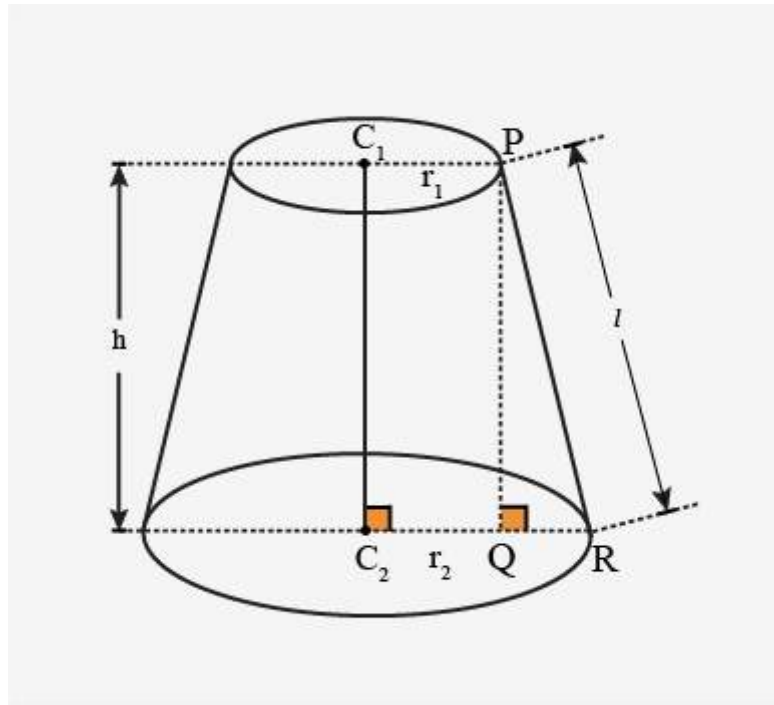
Shape Conversion of Solids

Frustum of a Cone

If a right circular cone is sliced by a plane parallel to its base then the part with the two circular bases is called a Frustum.

Plane parallel to base
cuts the cone

Cone as two separate parts

Frustum of a cone

Surface Area of a Frustum

Frustum with radius r_1 and r_2 and height h

CSA of frustum $= \pi(r_1 + r_2)l,$ where $l = \sqrt{h^2 + (r_1 - r_2)^2}$
TSA of the frustum is the CSA + the areas of the two circular faces $= \pi(r_1 + r_2)l + \pi(r_1^2 + r_2^2)$

Volume of a Frustum

Volume of frustum of a cone $= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

Shape Conversion of Solids

When a solid is converted into another solid of a different shape(by melting or casting), the volume remains constant.

Suppose a metallic sphere of radius 9 cm is melted and recast into the shape of a cylinder of radius 6 cm. Since the volume remains same after recast, therefore the volume of the cylinder will be equal to the volume of the sphere.

The radius of the cylinder is known however the height is not known. Let h be the height of the cylinder.
r_1 and r_2 be the radius of the sphere and cylinder respectively. Then,
V(sphere) = V(cylinder)

$\Rightarrow \frac{4}{3}\pi r_1^3 = \pi r_2^2 h$

$\Rightarrow \frac{4}{3}\pi(9^3) = \pi(6^2)h$ (On substituting the values)

$\Rightarrow h = 27cm$