Arithmetic Progression

Introduction to AP

Sequences, Series and Progressions

- A **sequence** is a finite or infinite list of numbers following a certain pattern. For example - 1,2,3,4,5… is the sequence is infinite.sequence of natural numbers.
- A **series** is the sum of the elements in the corresponding sequence. For example - 1+2+3+4+5….is the series of natural numbers. Each number in a sequence or a series is called a term.
- A **progression** is a sequence in which the general term can be can be expressed using a mathematical formula.

Arithmetic Progression

An arithmetic progression (A.P) is a progression in which the **difference** between two **consecutive** terms is constant.
Example: 2,5,8,11,14…. is an arithmetic progression.

Common Difference

The difference between two consecutive terms in an AP, (*which is constant*) is the "**common difference**"**(d)** of an A.P. In the progression: *2,5,8,11,14* …the common difference is 3.
As it is the difference between any two consecutive terms. For any A.P, if the common difference is:

- **positive**, the AP is **increasing**.
- **zero**, the AP is **constant**.
- **negative**, the A.P is **decreasing.**

Finite and Infinite AP

- A finite AP is an A.P in which the number of terms is finite. For example: the A.P: *2,5,8……32,35,38*
- An **infinite** A.P is an A.P in which the **number of terms is infinite**. For example: *2,5,8,11…..*

  A finite A.P will have the last term, whereas an infinite A.P won't.

General Term of AP

nth term of an AP

The nth term of an A.P is given by $T_n = a + (n-1)d$, where **a** is the first term, **d** is the common difference and **n** is the number of terms.

General form of an AP

The general form of an A.P is: ***(a, a+d,a+2d,a+3d......)*** where **a** is the first term and **d** is the common difference. Here, d=0, OR d>0, OR d<0

Sum of Terms in an AP

Formula for sum to n terms of an AP

The sum to n terms of an A.P is given by:

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

Where **a** is the first term, **d** is the common difference and **n** is the number of terms.

The sum of n terms of an A.P is also given by

$$S_n = \frac{n}{2}(a + l)$$

Where **a** is the first term, **l** is the last term of the A.P. and **n** is the number of terms.

Arithmetic Mean (A.M)

The Arithmetic Mean is the simple average of a given set of numbers.The arithmetic mean of a set of numbers is given by:

$$A.M = \frac{Sum\ of\ terms}{Number\ of\ terms}$$

The arithmetic mean is defined for any set of numbers. The numbers need not necessarily be in an A.P.

Basic Adding Patterns in an AP

The sum of two terms that are equidistant from either end of an AP is constant.
For example: in an A.P: *2,5,8,11,14,17...*
$T_1 + T_6 = 2 + 17 = 19$

$T_2 + T_5 = 5 + 14 = 19$ and so on....
Algebraically, this can be represented as

$$T_r + T_{(n-r)+1} = constant$$

Sum of first n natural numbers

The **sum** of first **n** natural numbers is given by:

$$S_n = \frac{n(n+1)}{2}$$

This formula is derived by treating the sequence of natural numbers as an A.P where the first term (a) = 1 and the common difference (d) = 1.