Exercise 4.1 Page No: 4.3

1. Fill in the blanks using the correct word given in brackets:

(i) All circles are ____________ (congruent, similar).
(ii) All squares are ___________ (similar, congruent).
(iii) All ____________ triangles are similar (isosceles, equilaterals).
(iv) Two triangles are similar, if their corresponding angles are ____________ (proportional, equal)
(v) Two triangles are similar, if their corresponding sides are ____________ (proportional, equal)
(vi) Two polygons of the same number of sides are similar, if (a) ____________their corresponding angles are and their corresponding sides are (b)____________ (equal, proportional).

Solutions:

(i) All circles are similar.
(ii) All squares are similar.
(iii) All equilateral triangles are similar.
(iv) Two triangles are similar, if their corresponding angles are equal.
(v) Two triangles are similar, if their corresponding sides are proportional.
(vi) Two polygons of the same number of sides are similar, if (a) equal their corresponding angles are and their corresponding sides are (b) proportional.

Exercise 4.2 Page No: 4.19

1. In a Δ ABC, D and E are points on the sides AB and AC respectively such that DE || BC.
i) If AD = 6 cm, DB = 9 cm and AE = 8 cm, Find AC.
Solution:

Given: Δ ABC, DE ∥ BC, AD = 6 cm, DB = 9 cm and AE = 8 cm.
Required to find AC.

By using Thales Theorem, [As DE ∥ BC]
AD/BD = AE/CE
Let CE = x.
So then,
6/9 = 8/x
6x = 72 cm
x = 72/6 cm
x = 12 cm
∴ AC = AE + CE = 12 + 8 = 20.

ii) If AD/DB = 3/4 and AC = 15 cm, Find AE.
Solution:

Given: AD/BD = 3/4 and AC = 15 cm [As DE ∥ BC]
Required to find AE.

By using Thales Theorem, [As DE ∥ BC]
AD/BD = AE/CE
Let, AE = x, then CE = 15-x.
⇒ 3/4 = x/ (15–x)
45 – 3x = 4x
-3x – 4x = – 45
7x = 45
x = 45/7
x = 6.43 cm
∴ AE= 6.43cm

iii) If AD/DB = 2/3 and AC = 18 cm, Find AE.
Solution:

Given: AD/BD = 2/3 and AC = 18 cm
Required to find AE.

By using Thales Theorem, [As DE ∥ BC]
AD/BD = AE/CE
Let, AE = x and CE = 18 – x
⇒ 23 = x/ (18–x)

$3x = 36 - 2x$
$5x = 36$ cm
$x = 36/5$ cm
$x = 7.2$ cm
∴ AE = 7.2 cm

iv) If AD = 4 cm, AE = 8 cm, DB = x – 4 cm and EC = 3x – 19, find x.
Solution:

Given: AD = 4 cm, AE = 8 cm, DB = x – 4 and EC = 3x – 19
Required to find x.

By using Thales Theorem, [As DE ∥ BC]
AD/BD = AE/CE
Then, $4/(x - 4) = 8/(3x - 19)$
$4(3x - 19) = 8(x - 4)$
$12x - 76 = 8(x - 4)$
$12x - 8x = -32 + 76$
$4x = 44$ cm
$x = 11$ cm

v) If AD = 8 cm, AB = 12 cm and AE = 12 cm, find CE.
Solution:

Given: AD = 8 cm, AB = 12 cm, and AE = 12 cm.
Required to find CE,

By using Thales Theorem, [As DE ∥ BC]
AD/BD = AE/CE
8/4 = 12/CE
8 x CE = 4 x 12 cm
CE = (4 x 12)/8 cm
CE = 48/8 cm
∴ CE = 6 cm

vi) If AD = 4 cm, DB = 4.5 cm and AE = 8 cm, find AC.
Solution:

Given: AD = 4 cm, DB = 4.5 cm, AE = 8 cm
Required to find AC.

By using Thales Theorem, [As DE ∥ BC]
AD/BD = AE/CE
4/4.5 = 8/AC
AC = $(4.5 \times 8)/4$ cm
∴AC = 9 cm

vii) If AD = 2 cm, AB = 6 cm and AC = 9 cm, find AE.
Solution:

Given: AD = 2 cm, AB = 6 cm and AC = 9 cm
Required to find AE.

DB = AB – AD = 6 – 2 = 4 cm
By using Thales Theorem, [As DE ∥ BC]
AD/BD = AE/CE
2/4 = x/ (9–x)
4x = 18 – 2x
6x = 18
x = 3 cm
∴ AE= 3cm

viii) If AD/BD = 4/5 and EC = 2.5 cm, Find AE.
Solution:

Given: AD/BD = 4/5 and EC = 2.5 cm
Required to find AE.

By using Thales Theorem, [As DE ∥ BC]
AD/BD = AE/CE
Then, 4/5 = AE/2.5
∴ AE = 4 × 2.55 = 2 cm

ix) If AD = x cm, DB = x – 2 cm, AE = x + 2 cm, and EC = x – 1 cm, find the value of x.
Solution:

Given: AD = x, DB = x – 2, AE = x + 2 and EC = x – 1
Required to find the value of x.

By using Thales Theorem, [As DE ∥ BC]
AD/BD = AE/CE
So, x/ (x–2) = (x+2)/ (x–1)
x(x – 1) = (x – 2)(x + 2)
$x^2 – x – x^2 + 4 = 0$
x = 4

x) If AD = 8x – 7 cm, DB = 5x – 3 cm, AE = 4x – 3 cm, and EC = (3x – 1) cm, Find the value of x.
Solution:

Given: AD = 8x – 7, DB = 5x – 3, AER = 4x – 3 and EC = 3x -1
Required to find x.

By using Thales Theorem, [As DE ∥ BC]

$AD/BD = AE/CE$

$(8x–7)/ (5x–3) = (4x–3)/ (3x–1)$

$(8x – 7)(3x – 1) = (5x – 3)(4x – 3)$

$24x^2 – 29x + 7 = 20x^2 – 27x + 9$

$4x^2 – 2x – 2 = 0$

$2(2x^2 – x – 1) = 0$

$2x^2 – x – 1 = 0$

$2x^2 – 2x + x – 1 = 0$

$2x(x – 1) + 1(x – 1) = 0$

$(x – 1)(2x + 1) = 0$

⇒ $x = 1$ or $x = -1/2$

We know that the side of triangle can never be negative. Therefore, we take the positive value.

∴ $x = 1$.

xi) If AD = 4x – 3, AE = 8x – 7, BD = 3x – 1, and CE = 5x – 3, find the value of x.
Solution:

Given: AD = 4x – 3, BD = 3x – 1, AE = 8x – 7 and EC = 5x – 3
Required to find x.

By using Thales Theorem, [As DE ∥ BC]

$AD/BD = AE/CE$

So, $(4x–3)/ (3x–1) = (8x–7)/ (5x–3)$

$(4x – 3)(5x – 3) = (3x – 1)(8x – 7)$

$4x(5x – 3) -3(5x – 3) = 3x(8x – 7) -1(8x – 7)$

$20x^2 – 12x – 15x + 9 = 24x^2 – 29x + 7$

$20x^2 -27x + 9 = 24^2 -29x + 7$

⇒ $-4x^2 + 2x + 2 = 0$

$4x^2 – 2x – 2 = 0$

$4x^2 – 4x + 2x – 2 = 0$

$4x(x – 1) + 2(x – 1) = 0$

$(4x + 2)(x – 1) = 0$

⇒ $x = 1$ or $x = -2/4$

We know that the side of triangle can never be negative. Therefore, we take the positive value.

∴ $x = 1$

xii) If AD = 2.5 cm, BD = 3.0 cm, and AE = 3.75 cm, find the length of AC.
Solution:

Given: AD = 2.5 cm, AE = 3.75 cm and BD = 3 cm
Required to find AC.

By using Thales Theorem, [As DE ∥ BC]

$AD/BD = AE/CE$

2.5/ 3 = 3.75/ CE
2.5 x CE = 3.75 x 3
CE = 3.75×32.5
CE = 11.252.5
CE = 4.5

Now, AC = 3.75 + 4.5
∴ AC = 8.25 cm.

2. In a ∆ ABC, D and E are points on the sides AB and AC respectively. For each of the following cases show that DE ∥ BC:
i) AB = 12 cm, AD = 8 cm, AE = 12 cm, and AC = 18 cm.
Solution:

Required to prove DE ∥ BC.
We have,
AB = 12 cm, AD = 8 cm, AE = 12 cm, and AC = 18 cm. (Given)
So,
BD = AB – AD = 12 – 8 = 4 cm
And,
CE = AC – AE = 18 – 12 = 6 cm

It's seen that,
AD/BD = 8/4 = 1/2
AE/CE = 12/6 = 1/2
Thus,
AD/BD = AE/CE
So, by the converse of Thale's Theorem
We have,
DE ∥ BC.
Hence Proved.

ii) AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm, and AE = 1.8 cm.
Solution:

Required to prove DE ∥ BC.
We have,
AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm, and AE = 1.8 cm. (Given)
So,
BD = AB – AD = 5.6 – 1.4 = 4.2 cm
And,
CE = AC – AE = 7.2 – 1.8 = 5.4 cm

It's seen that,
AD/BD = 1.4/4.2 = 1/3
AE/CE = 1.8/5.4 =1/3

Thus,

AD/BD = AE/CE

So, by the converse of Thale's Theorem

We have,

DE ∥ BC.

Hence Proved.

iii) AB = 10.8 cm, BD = 4.5 cm, AC = 4.8 cm, and AE = 2.8 cm.

Solution:

Required to prove DE ∥ BC.

We have

AB = 10.8 cm, BD = 4.5 cm, AC = 4.8 cm, and AE = 2.8 cm.

So,

AD = AB – DB = 10.8 – 4.5 = 6.3

And,

CE = AC – AE = 4.8 – 2.8 = 2

It's seen that,

AD/BD = 6.3/ 4.5 = 2.8/ 2.0 = AE/CE = 7/5

So, by the converse of Thale's Theorem

We have,

DE ∥ BC.

Hence Proved.

iv) AD = 5.7 cm, BD = 9.5 cm, AE = 3.3 cm, and EC = 5.5 cm.

Solution:

Required to prove DE ∥ BC.

We have

AD = 5.7 cm, BD = 9.5 cm, AE = 3.3 cm, and EC = 5.5 cm

Now,

AD/BD = 5.7/9.5 =3/5

And,

AE/CE = 3.3/5.5 = 3/5

Thus,

AD/BD = AE/CE

So, by the converse of Thale's Theorem

We have,

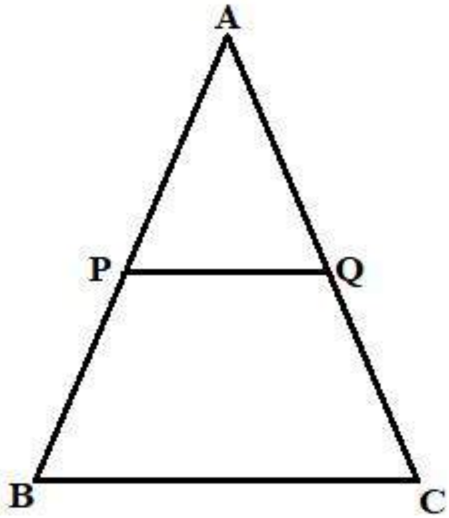
DE ∥ BC.

Hence Proved.

3. In a Δ ABC, P and Q are the points on sides AB and AC respectively, such that PQ ∥ BC. If AP = 2.4 cm, AQ = 2 cm, QC = 3 cm and BC = 6 cm. Find AB and PQ.

Solution:

Given: Δ ABC, AP = 2.4 cm, AQ = 2 cm, QC = 3 cm, and BC = 6 cm. Also, PQ ∥ BC.
Required to find: AB and PQ.



By using Thales Theorem, we have [As it's given that PQ ∥ BC]

AP/PB = AQ/ QC
2.4/PB = 2/3
2 x PB = 2.4 x 3
PB = (2.4 × 3)/2 cm
⇒ PB = 3.6 cm

Now finding, AB = AP + PB
AB = 2.4 + 3.6
⇒ AB = 6 cm

Now, considering Δ APQ and Δ ABC
We have,
∠A = ∠A
∠APQ = ∠ABC (Corresponding angles are equal, PQ||BC and AB being a transversal)
Thus, Δ APQ and Δ ABC are similar to each other by AA criteria.
Now, we know that
Corresponding parts of similar triangles are propositional.
⇒ AP/AB = PQ/ BC
⇒ PQ = (AP/AB) x BC
= (2.4/6) x 6 = 2.4

∴ PQ = 2.4 cm.

4. In a Δ ABC, D and E are points on AB and AC respectively, such that DE ∥ BC. If AD = 2.4 cm, AE = 3.2 cm, DE = 2 cm and BC = 5 cm. Find BD and CE.
Solution:

Given: Δ ABC such that AD = 2.4 cm, AE = 3.2 cm, DE = 2 cm and BE = 5 cm. Also DE ∥ BC.
Required to find: BD and CE.

As DE ∥ BC, AB is transversal,
$\angle APQ = \angle ABC$ (corresponding angles)
As DE ∥ BC, AC is transversal,
$\angle AED = \angle ACB$ (corresponding angles)

In Δ ADE and Δ ABC,
$\angle ADE = \angle ABC$
$\angle AED = \angle ACB$
∴ Δ ADE = Δ ABC (AA similarity criteria)

Now, we know that
Corresponding parts of similar triangles are propositional.
⇒ AD/AB = AE/AC = DE/BC
AD/AB = DE/BC
2.4/ (2.4 + DB) = 2/5 [Since, AB = AD + DB]
2.4 + DB = 6
DB = 6 – 2.4
DB = 3.6 cm

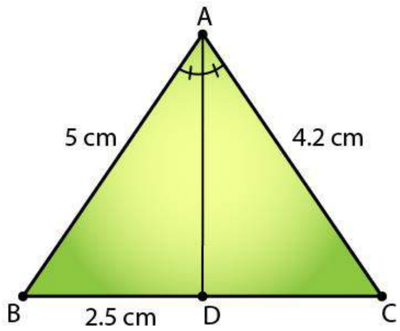
In the same way,
⇒ AE/AC = DE/BC
3.2/ (3.2 + EC) = 2/5 [Since AC = AE + EC]
3.2 + EC = 8
EC = 8 – 3.2
EC = 4.8 cm

∴ BD = 3.6 cm and CE = 4.8 cm.

Exercise 4.3 Page No: 4.31

1. In a Δ ABC, AD is the bisector of ∠ A, meeting side BC at D.
(i) if BD = 2.5 cm, AB = 5 cm, and AC = 4.2 cm, find DC.
Solution:

Given: Δ ABC and AD bisects ∠A, meeting side BC at D. And BD = 2.5 cm, AB = 5 cm, and AC = 4.2 cm.
Required to find: DC

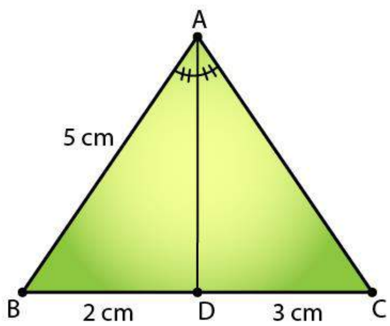


Since, AD is the bisector of ∠ A meeting side BC at D in Δ ABC

⇒ AB/ AC = BD/ DC
5/ 4.2 = 2.5/ DC
5DC = 2.5 x 4.2
∴ DC = 2.1 cm

(ii) if BD = 2 cm, AB = 5 cm, and DC = 3 cm, find AC.
Solution:

Given: Δ ABC and AD bisects ∠A, meeting side BC at D. And BD = 2 cm, AB = 5 cm, and DC = 3 cm.
Required to find: AC

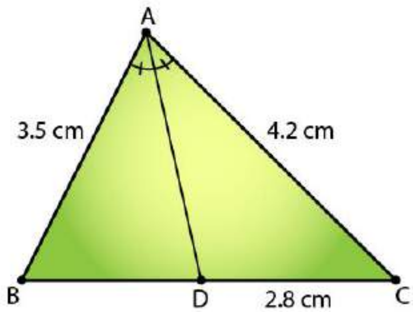


Since, AD is the bisector of ∠ A meeting side BC at D in Δ ABC

⇒ AB/ AC = BD/ DC
5/ AC = 2/ 3
2AC = 5 x 3
∴ AC = 7.5 cm

(iii) if AB = 3.5 cm, AC = 4.2 cm, and DC = 2.8 cm, find BD.
Solution:

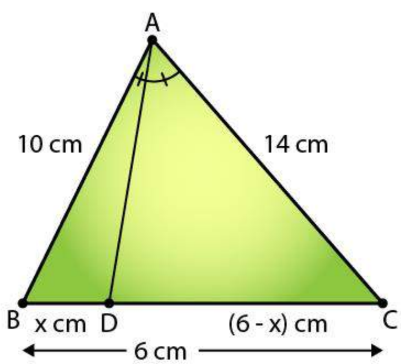
Given: Δ ABC and AD bisects ∠A, meeting side BC at D. And AB = 3.5 cm, AC = 4.2 cm, and DC = 2.8 cm.
Required to find: BD



Since, AD is the bisector of ∠ A meeting side BC at D in Δ ABC
⇒ AB/ AC = BD/ DC
3.5/ 4.2 = BD/ 2.8
4.2 x BD = 3.5 x 2.8
BD = 7/3
∴ BD = 2.3 cm

(iv) if AB = 10 cm, AC = 14 cm, and BC = 6 cm, find BD and DC.
Solution:

Given: In Δ ABC, AD is the bisector of ∠A meeting side BC at D. And, AB = 10 cm, AC = 14 cm, and BC = 6 cm
Required to find: BD and DC.

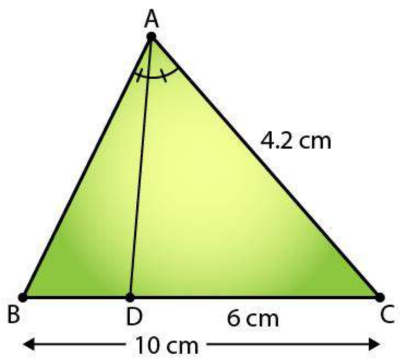


Since, AD is bisector of ∠A
We have,
AB/AC = BD/DC (AD is bisector of ∠ A and side BC)
Then, 10/ 14 = x/ (6 – x)
14x = 60 – 6x
20x = 60

x = 60/20
∴ BD = 3 cm and DC = (6 – 3) = 3 cm.

(v) if AC = 4.2 cm, DC = 6 cm, and BC = 10 cm, find AB.
Solution:

Given: Δ ABC and AD bisects ∠A, meeting side BC at D. And AC = 4.2 cm, DC = 6 cm, and BC = 10 cm.
Required to find: AB

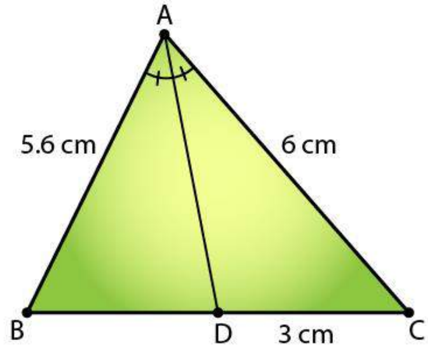


Since, AD is the bisector of ∠ A meeting side BC at D in Δ ABC
⇒ AB/ AC = BD/ DC
AB/ 4.2 = BD/ 6
We know that,
BD = BC – DC = 10 – 6 = 4 cm

⇒ AB/ 4.2 = 4/ 6
AB = (2 x 4.2)/ 3
∴ AB = 2.8 cm

(vi) if AB = 5.6 cm, AC = 6 cm, and DC = 3 cm, find BC.
Solution:

Given: Δ ABC and AD bisects ∠A, meeting side BC at D. And AB = 5.6 cm, AC = 6 cm, and DC = 3 cm.
Required to find: BC



Since, AD is the bisector of ∠ A meeting side BC at D in Δ ABC

⇒ AB/ AC = BD/ DC

5.6/ 6 = BD/ 3

BD = 5.6/ 2 = 2.8cm

And, we know that,

BD = BC – DC

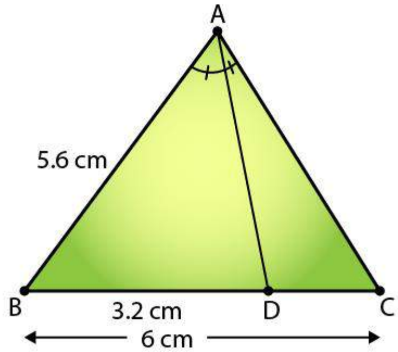
2.8 = BC – 3

∴ BC = 5.8 cm

(vii) if AB = 5.6 cm, BC = 6 cm, and BD = 3.2 cm, find AC.
Solution:

Given: Δ ABC and AD bisects ∠A, meeting side BC at D. And AB = 5.6 cm, BC = 6 cm, and BD = 3.2 cm.

Required to find: AC



Since, AD is the bisector of ∠ A meeting side BC at D in Δ ABC

⇒ AB/ AC = BD/ DC

5.6/ AC = 3.2/ DC

And, we know that

BD = BC – DC

3.2 = 6 – DC

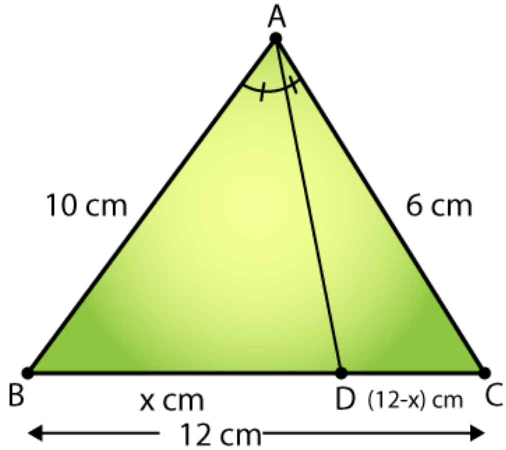
∴ DC = 2.8 cm

⇒ 5.6/ AC = 3.2/ 2.8

AC = (5.6 x 2.8)/ 3.2
∴ AC = 4.9 cm

(viii) if AB = 10 cm, AC = 6 cm, and BC = 12 cm, find BD and DC.
Solution:

Given: Δ ABC and AD bisects ∠A, meeting side BC at D. AB = 10 cm, AC = 6 cm, and BC = 12 cm.
Required to find: DC



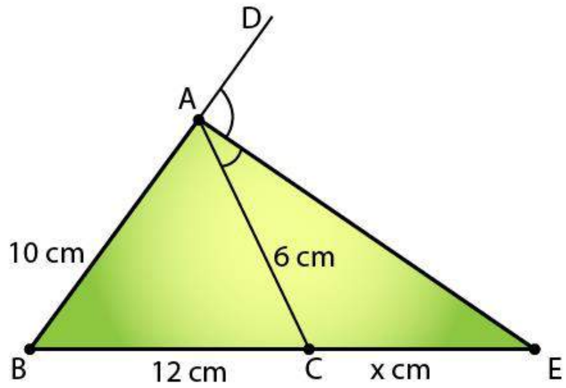
Since, AD is the bisector of ∠ A meeting side BC at D in Δ ABC
⇒ AB/ AC = BD/ DC
10/ 6 = BD/ DC …….. (i)
And, we know that
BD = BC – DC = 12 – DC
Let BD = x,
⇒ DC = 12 – x

Thus (i) becomes,
10/ 6 = x/ (12 - x)
5(12 – x) = 3x
60 -5x = 3x
∴ x = 60/8 = 7.5

Hence, DC = 12 – 7.5 = 4.5cm and BD = 7.5 cm

2. In figure 4.57, AE is the bisector of the exterior ∠CAD meeting BC produced in E. If AB = 10 cm, AC = 6 cm, and BC = 12 cm, find CE.
Solution:

Given: AE is the bisector of the exterior ∠CAD and AB = 10 cm, AC = 6 cm, and BC = 12 cm.
Required to find: CE



Since AE is the bisector of the exterior ∠CAD.

BE / CE = AB / AC

Let's take CE as x.

So, we have

BE/ CE = AB/ AC

(12+x)/ x = 10/ 6

6x + 72 = 10x

10x - 6x = 72

4x = 72

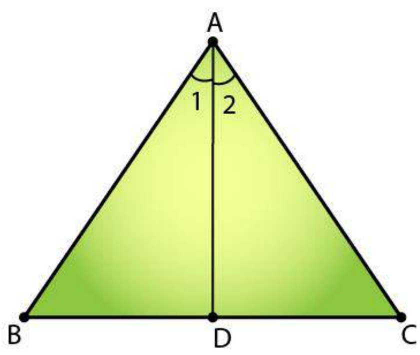
∴ x = 18

Therefore, CE = 18 cm.

3. In fig. 4.58, Δ ABC is a triangle such that AB/AC = BD/DC, ∠B=70^o, ∠C = 50^o, find ∠BAD.
Solution:

Given: Δ ABC such that AB/AC = BD/DC, ∠B = 70^o and ∠C = 50^o

Required to find: ∠BAD



We know that,

In ΔABC,

∠A = 180 – (70 + 50) [Angle sum property of a triangle]

= 180 – 120

= 60^o

Since,

AB/AC = BD/DC,

AD is the angle bisector of angle ∠A.

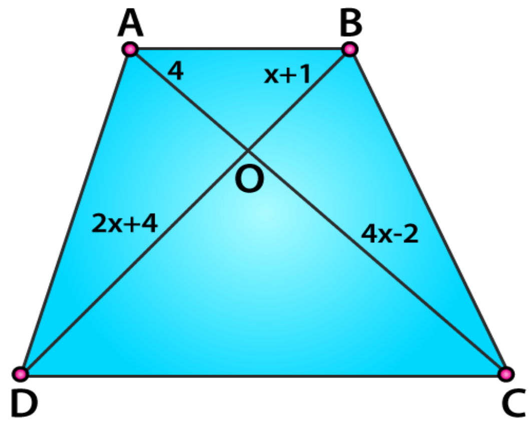
Thus,

$\angle BAD = \angle A/2 = 60/2 = 30^{o}$

Exercise 4.4

1. (i) In fig. 4.70, if AB∥CD, find the value of x.
Solution:

It's given that AB∥CD.
Required to find the value of x.



We know that,
Diagonals of a parallelogram bisect each other.
So,

AO/ CO = BO/ DO

⇒ 4/ (4x – 2) = (x +1)/ (2x + 4)

4(2x + 4) = (4x – 2)(x +1)

8x + 16 = x(4x – 2) + 1(4x – 2)

8x + 16 = $4x^2$ – 2x + 4x – 2

-$4x^2$ + 8x + 16 + 2 – 2x = 0

-$4x^2$ + 6x + 8 = 0

$4x^2$ – 6x – 18 = 0

$4x^2$ – 12x + 6x – 18 = 0

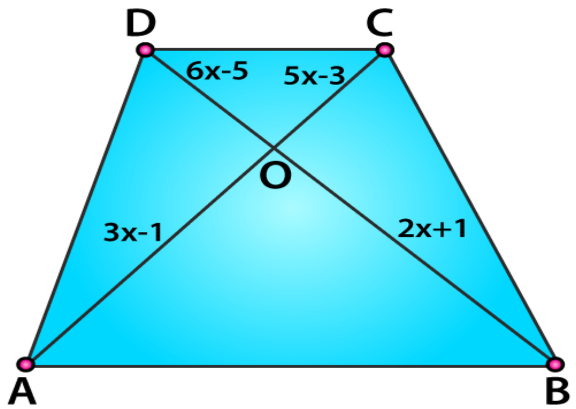
4x(x – 3) + 6(x – 3) = 0

(4x + 6) (x – 3) = 0

∴ x = – 6/4 or x = 3

(ii) In fig. 4.71, if AB∥CD, find the value of x.
Solution:

It's given that AB∥CD.
Required to find the value of x.



We know that,
Diagonals of a parallelogram bisect each other
So,

AO/ CO = BO/ DO

⇒ (6x – 5)/ (2x + 1) = (5x – 3)/ (3x – 1)

(6x – 5)(3x – 1) = (2x + 1)(5x – 3)

3x(6x – 5) – 1(6x – 5) = 2x(5x – 3) + 1(5x – 3)

$18x^2 – 10x^2 – 21x + 5 + x +3 = 0$

$8x^2 – 16x – 4x + 8 = 0$

8x(x – 2) – 4(x – 2) = 0

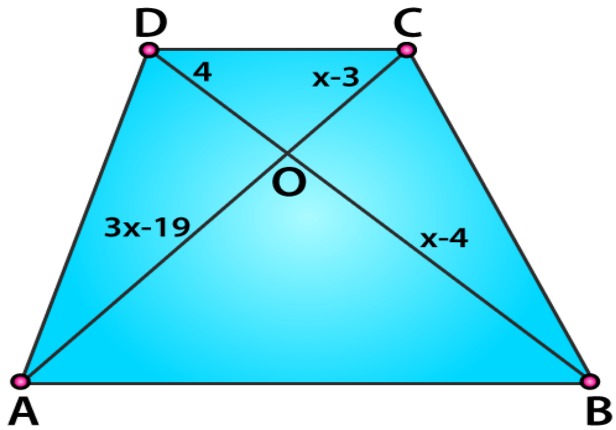
(8x – 4)(x – 2) = 0

x = 4/8 = 1/2 or x = -2

∴ **x= 1/2**

(iii) In fig. 4.72, if AB ∥ CD**. If OA = 3x – 19, OB = x – 4, OC = x- 3 and OD = 4, find x.**
Solution:

It's given that AB∥CD.
Required to find the value of x.



We know that,
Diagonals of a parallelogram bisect each other
So,

$AO/ CO = BO/ DO$
$(3x – 19)/ (x – 3) = (x–4)/ 4$
$4(3x – 19) = (x – 3) (x – 4)$
$12x – 76 = x(x – 4) -3(x – 4)$
$12x – 76 = x^2 – 4x – 3x + 12$
$-x^2 + 7x – 12 + 12x -76 = 0$
$-x^2 + 19x – 88 = 0$
$x^2 – 19x + 88 = 0$
$x^2 – 11x – 8x + 88 = 0$
$x(x – 11) – 8(x – 11) = 0$
$\therefore$ x = 11 or x = 8

Exercise 4.5 Page No: 4.37

1. In fig. 4.136, ΔACB ~ ΔAPQ. If BC = 8 cm, PQ = 4 cm, BA = 6.5 cm and AP = 2.8 cm, find CA and AQ.

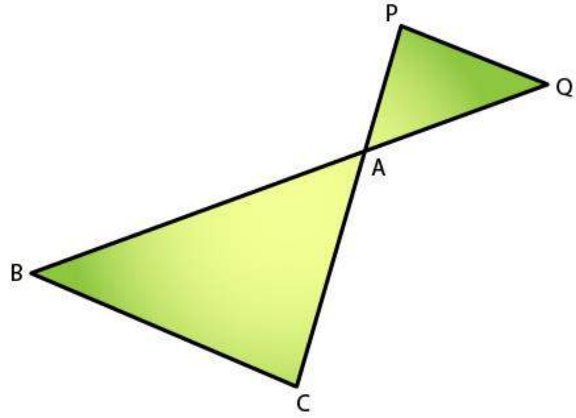
Solution:

Given,

ΔACB ~ ΔAPQ

BC = 8 cm, PQ = 4 cm, BA = 6.5 cm and AP = 2.8 cm

Required to find: CA and AQ



We know that,

ΔACB ~ ΔAPQ [given]

BA/ AQ = CA/ AP = BC/ PQ [Corresponding Parts of Similar Triangles]

So,

6.5/ AQ = 8/ 4

AQ = (6.5 x 4)/ 8

AQ = 3.25 cm

Similarly, as

CA/ AP = BC/ PQ

CA/ 2.8 = 8/ 4

CA = 2.8 x 2

CA = 5.6 cm

Hence, CA = 5.6 cm and AQ = 3.25 cm.

2. In fig.4.137, AB ∥ QR**, find the length of PB.**

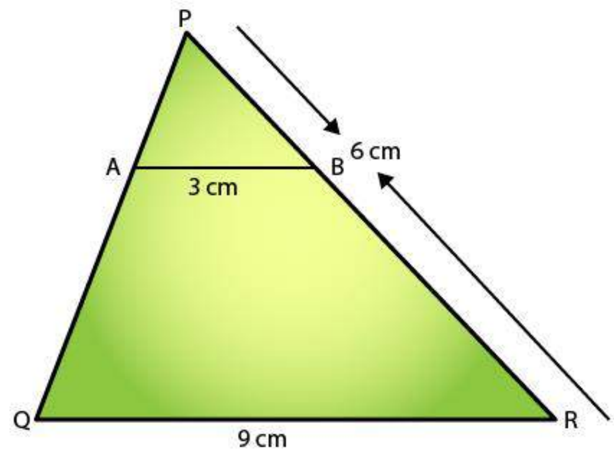
Solution:

Given,

ΔPQR, AB ∥ QR and

AB = 3 cm, QR = 9 cm and PR = 6 cm

Required to find: PB



In ΔPAB and ΔPQR

We have,

∠P = ∠P	[Common]
∠PAB = ∠PQR	[Corresponding angles as AB‖QR with PQ as the transversal]
⇒ ΔPAB ~ ΔPQR	[By AA similarity criteria]

Hence,

	AB/ QR = PB/ PR	[Corresponding Parts of Similar Triangles are propositional]
⇒	3/ 9 = PB/6	
	PB = 6/3	

Therefore, PB = 2 cm

3. In fig. 4.138 given, XY‖BC**. Find the length of XY.**

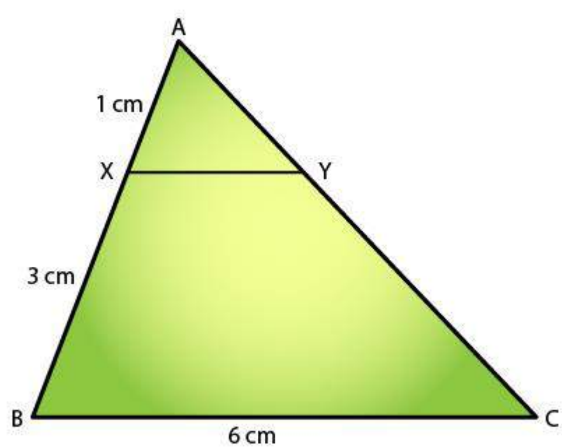
Solution:

Given,

XY‖BC

AX = 1 cm, XB = 3 cm and BC = 6 cm

Required to find: XY



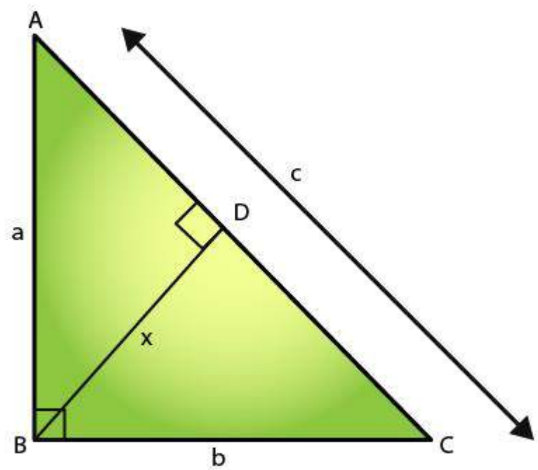
In ΔAXY and ΔABC
We have,
$\angle A = \angle A$ [Common]
$\angle AXY = \angle ABC$ [Corresponding angles as AB||QR with PQ as the transversal]
$\Rightarrow \Delta AXY \sim \Delta ABC$ [By AA similarity criteria]

Hence,
$XY/ BC = AX/ AB$ [Corresponding Parts of Similar Triangles are propositional]
We know that,
$(AB = AX + XB = 1 + 3 = 4)$
$XY/6 = 1/4$
$XY/1 = 6/4$

Therefore, XY = 1.5 cm

4. In a right-angled triangle with sides a and b and hypotenuse c, the altitude drawn on the hypotenuse is x. Prove that ab = cx.
Solution:

Consider ΔABC to be a right angle triangle having sides a and b and hypotenuse c. Let BD be the altitude drawn on the hypotenuse AC.
Required to prove: ab = cx



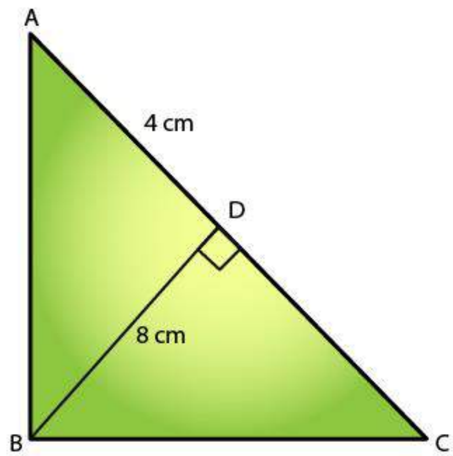
We know that,
In ΔACB and ΔCDB
$\angle B = \angle B$ [Common]
$\angle ACB = \angle CDB = 90^o$
$\Rightarrow \Delta ACB \sim \Delta CDB$ [By AA similarity criteria]

Hence,
$AB/ BD = AC/ BC$ [Corresponding Parts of Similar Triangles are propositional]
$a/ x = c/ b$
$\Rightarrow$ $xc = ab$

Therefore, ab = cx

5. In fig. 4.139, ∠ABC = 90 and BD⊥AC. If BD = 8 cm, and AD = 4 cm, find CD.
Solution:

Given,
∠ABC = 90º and BD⊥AC
BD = 8 cm
AD = 4 cm
Required to find: CD.

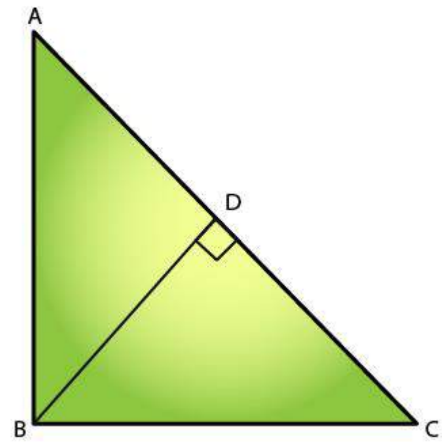


We know that,
ABC is a right angled triangle and BD⊥AC.
Then, ΔDBA~ΔDCB [By AA similarity]
BD/ CD = AD/ BD
BD^2 = AD x DC
$(8)^2$ = 4 x DC
DC = 64/4 = 16 cm

Therefore, CD = 16 cm

6. In fig.4.140, ∠ABC = 90º and BD ⊥ AC. If AC = 5.7 cm, BD = 3.8 cm and CD = 5.4 cm, Find BC.
Solution:

Given:
BD ⊥ AC
AC = 5.7 cm, BD = 3.8 cm and CD = 5.4 cm
∠ABC = 90º
Required to find: BC



We know that,

$\Delta ABC \sim \Delta BDC$ [By AA similarity]

$\angle BCA = \angle DCA = 90^o$

$\angle AXY = \angle ABC$ [Common]

Thus,

AB/ BD = BC/ CD [Corresponding Parts of Similar Triangles are propositional]

5.7/ 3.8 = BC/ 5.4

BC = (5.7 x 5.4)/ 3.8 = 8.1

Therefore, BC = 8.1 cm

7. In the fig.4.141 given, DE ∥ BC **such that AE = (1/4)AC. If AB = 6 cm, find AD.**
Solution:

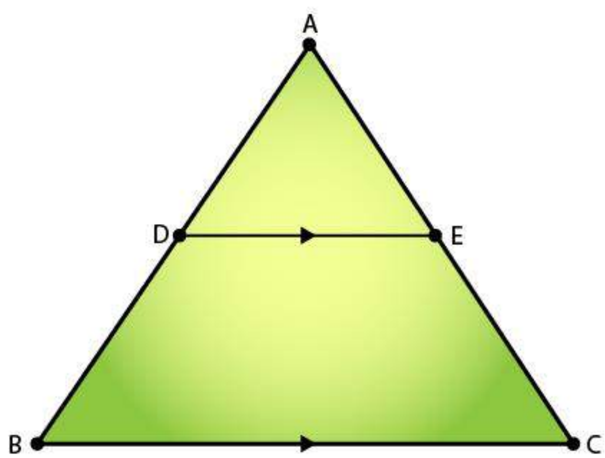
Given:

DE∥BC

AE = (1/4)AC

AB = 6 cm.

Required to find: AD.



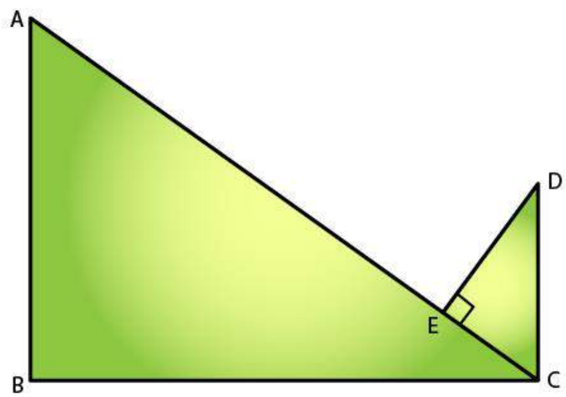
In ΔADE and ΔABC
We have,
∠A = ∠A [Common]
∠ADE = ∠ABC [Corresponding angles as AB||QR with PQ as the transversal]
⇒ ΔADE ~ ΔABC [By AA similarity criteria]

Then,
AD/AB = AE/ AC [Corresponding Parts of Similar Triangles are propositional]
AD/6 = 1/4
4 x AD = 6
AD = 6/4

Therefore, AD = 1.5 cm

8. In the fig.4.142 given, if AB ⊥ BC, DC ⊥ BC, and DE ⊥ AC, prove that ΔCED ~ ΔABC
Solution:

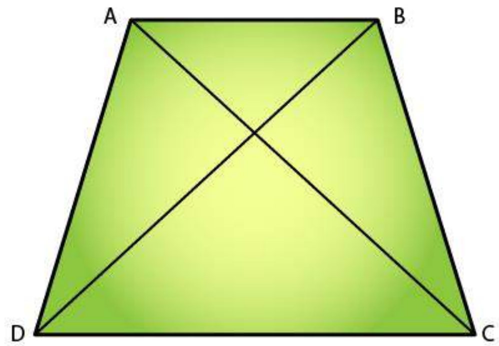
Given:
AB ⊥ BC,
DC ⊥ BC,
DE ⊥ AC
Required to prove: ΔCED~ΔABC



We know that,
From ΔABC and ΔCED
∠B = ∠E = 90º [given]
∠BAC = ∠ECD [alternate angles since, AB || CD with BC as transversal]
Therefore, ΔCED~ΔABC [AA similarity]

9. Diagonals AC and BD of a trapezium ABCD with AB ∥ DC **intersect each other at the point O. Using similarity criterion for two triangles, show that** OA/ OC = OB/ OD
Solution:

Given: OC is the point of intersection of AC and BD in the trapezium ABCD, with AB ∥ DC.
Required to prove: OA/ OC = OB/ OD



We know that,

In ΔAOB and ΔCOD

∠AOB = ∠COD [Vertically Opposite Angles]

∠OAB = ∠OCD [Alternate angles]

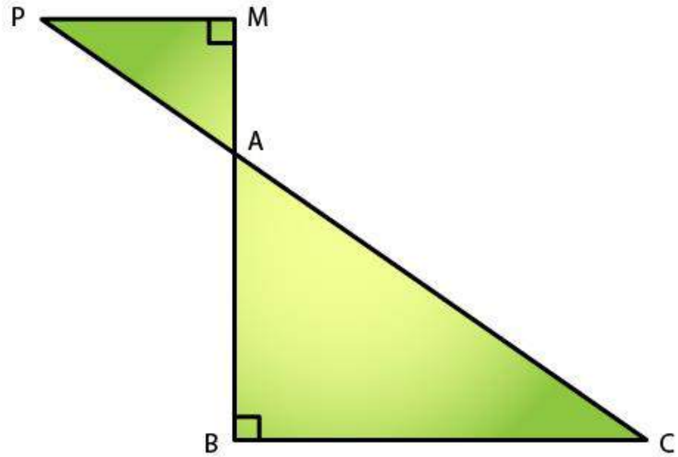
Then, ΔAOB ~ ΔCOD

Therefore, OA/ OC = OB/ OD [Corresponding sides are proportional]

10. If Δ ABC and Δ AMP are two right triangles, right angled at B and M, respectively such that ∠MAP = ∠BAC. Prove that
(i) ΔABC ~ ΔAMP
(ii) CA/ PA = BC/ MP
Solution:

(i) Given:

Δ ABC and Δ AMP are the two right triangles.



We know that,

∠AMP = ∠B = 90^{o}

∠MAP = ∠BAC [Vertically Opposite Angles]

⇒ $\Delta ABC \sim \Delta AMP$ [AA similarity]

(ii) Since, $\Delta ABC \sim \Delta AMP$

$CA/PA = BC/MP$ [Corresponding sides are proportional]

Hence proved.

11. A vertical stick 10 cm long casts a shadow 8 cm long. At the same time, a tower casts a shadow 30 m long. Determine the height of the tower.

Solution:

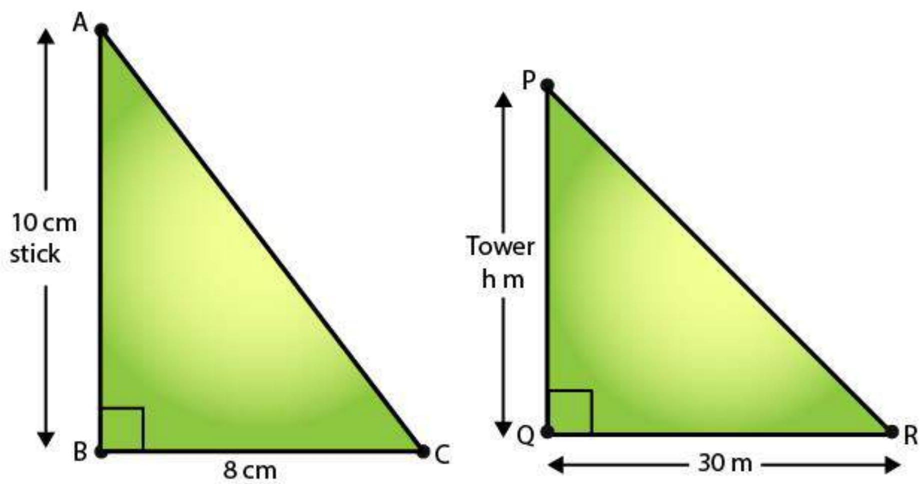
Given:

Length of stick = 10cm

Length of the stick's shadow = 8cm

Length of the tower's shadow = 30m = 3000cm

Required to find: the height of the tower = PQ.



In $\Delta ABC \sim \Delta PQR$

$\angle ABC = \angle PQR = 90^o$

$\angle ACB = \angle PRQ$ [Angular Elevation of Sun is same for a particular instant of time]

⇒ $\Delta ABC \sim \Delta PQR$ [By AA similarity]

So, we have

$AB/BC = PQ/QR$ [Corresponding sides are proportional]

$10/8 = PQ/3000$

$PQ = (3000 \times 10)/8$

$PQ = 30000/8$

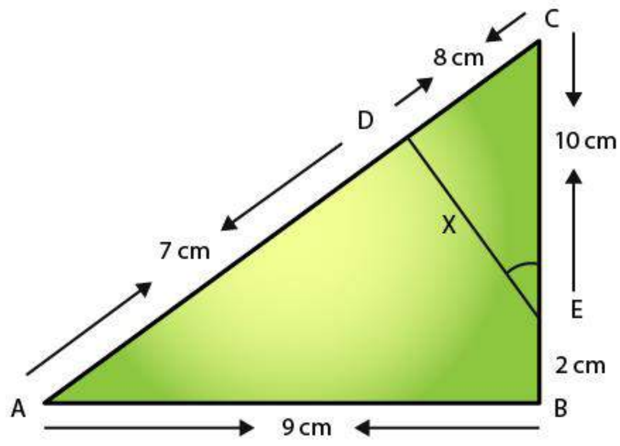
$PQ = 3750/100$

Therefore, PQ = 37.5 m

12. In fig.4.143, ∠A = ∠CED, prove that ΔCAB ~ ΔCED. Also find the value of x.

Solution:

Given:
∠A = ∠CED
Required to prove: ΔCAB ~ ΔCED



In ΔCAB ~ ΔCED
∠C = ∠C [Common]
∠A = ∠CED [Given]

⇒ ΔCAB ~ ΔCED [By AA similarity]

Hence, we have
CA/ CE = AB/ ED [Corresponding sides are proportional]
15/10 = 9/x
x = (9 x 10)/ 15

Therefore, x = 6 cm

Exercise 4.6 Page No: 4.94

1. Triangles ABC and DEF are similar.
(i) If area of (ΔABC) = 16 cm^2, area (ΔDEF) = 25 cm^2 and BC = 2.3 cm, find EF.
(ii) If area (ΔABC) = 9 cm^2, area (ΔDEF) = 64 cm^2 and DE = 5.1 cm, find AB.
(iii) If AC = 19 cm and DF = 8 cm, find the ratio of the area of two triangles.
(iv) If area of (ΔABC) = 36 cm^2, area (ΔDEF) = 64 cm^2 and DE = 6.2 cm, find AB.
(v) If AB = 1.2 cm and DE = 1.4 cm, find the ratio of the area of two triangles.
Solutions:

As we know that, the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides, we get

(i) $\frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{BC}{EF})^2$ $\frac{16}{25} = (\frac{2.3}{EF})^2$ $\frac{4}{5} = \frac{2.3}{EF}$

Therefore, EF = 2.875 cm

(ii) $\frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{AB}{DE})^2$ $\frac{9}{64} = (\frac{AB}{DE})^2$ $\frac{3}{8} = \frac{AB}{5.1}$

Therefore, AB = 1.9125 cm

(iii) $\frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{AC}{DF})^2$ $\frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{19}{8})^2$ $\frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{361}{64})$

Therefore, the ratio of the areas of the two triangles are 361: 64

(iv) $\frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{AB}{DE})^2$ $\frac{36}{64} = (\frac{AB}{DE})^2$ $\frac{6}{8} = \frac{AB}{6.2}$

Therefore, AB = 4.65 cm

(v) $\frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{AB}{DE})^2$ $\frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{1.2}{1.4})^2$ $\frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{36}{49})$

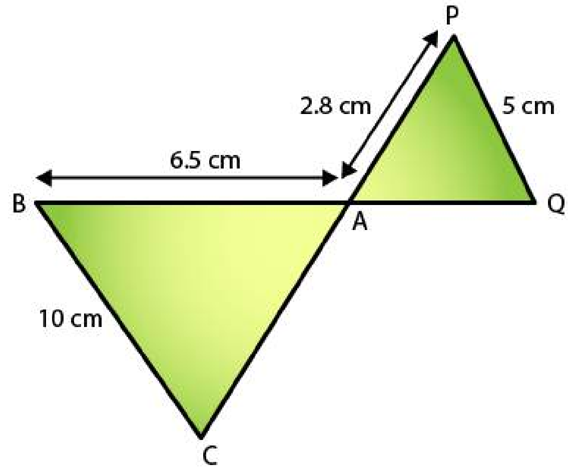
Therefore, the ratio of the areas of the two triangles are 36: 49

2. In the fig 4.178, ΔACB ~ ΔAPQ. If BC = 10 cm, PQ = 5 cm, BA = 6.5 cm, AP = 2.8 cm, find CA and AQ. Also, find the area (ΔACB): area (ΔAPQ).
Solution:

Given:

ΔACB is similar to ΔAPQ
BC = 10 cm
PQ = 5 cm
BA = 6.5 cm
AP = 2.8 cm

Required to Find: CA, AQ and that the area (ΔACB): area (ΔAPQ).



Since, ΔACB ~ ΔAPQ
We know that,

AB/ AQ = BC/ PQ = AC/ AP [Corresponding Parts of Similar Triangles]
AB/ AQ = BC/ PQ
6.5/ AQ = 10/5
⇒ AQ = 3.25 cm

Similarly,

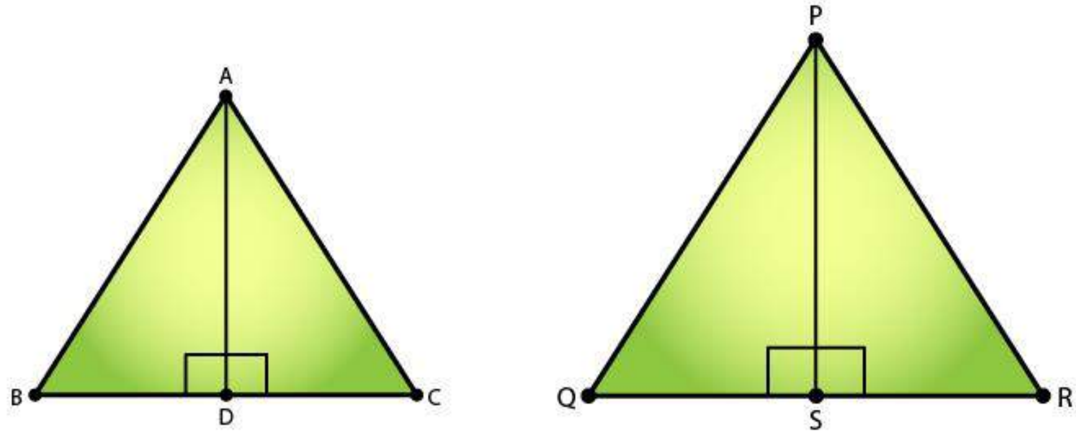
BC/ PQ = CA/ AP
CA/ 2.8 = 10/5
⇒ CA = 5.6 cm

Next,
Since the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides, we have,
ar(ΔACQ): ar(ΔAPQ) = (BC/ PQ)2
= (10/5)2
= (2/1)2
= 4/1

Therefore, the ratio is 4:1.

3. The areas of two similar triangles are 81 cm² and 49 cm² respectively. Find the ration of their corresponding heights. What is the ratio of their corresponding medians?
Solution:

Given: The areas of two similar triangles are $81cm^2$ and $49cm^2$.
Required to find: The ratio of their corresponding heights and the ratio of their corresponding medians.



Let's consider the two similar triangles as ΔABC and ΔPQR, AD and PS be the altitudes of ΔABC and ΔPQR respectively.
So,
By area of similar triangle theorem, we have

$ar(\Delta ABC)/ ar(\Delta PQR) = AB^2/ PQ^2$

⇒ $81/ 49 = AB^2/ PQ^2$

⇒ $9/ 7 = AB/ PQ$

In ΔABD and ΔPQS

∠B = ∠Q [Since ΔABC ~ ΔPQR]

∠ABD = ∠PSQ = 90°

⇒ ΔABD ~ ΔPQS [By AA similarity]

Hence, as the corresponding parts of similar triangles are proportional, we have

AB/ PQ = AD/ PS

Therefore,

AD/ PS = 9/7 (Ratio of altitudes)

Similarly,
The ratio of two similar triangles is equal to the ratio of the squares of their corresponding medians also.

Thus, ratio of altitudes = Ratio of medians = 9/7

4. The areas of two similar triangles are 169 cm^2 and 121 cm^2 respectively. If the longest side of the larger triangle is 26 cm, find the longest side of the smaller triangle.
Solution:

Given:

The area of two similar triangles is $169cm^2$ and $121cm^2$.
The longest side of the larger triangle is 26cm.
Required to find: the longest side of the smaller triangle

Let the longer side of the smaller triangle = x

We know that, the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides, we have
ar(larger triangle)/ ar(smaller triangle) = (side of the larger triangle/ side of the smaller triangle)^2
= 169/ 121
Taking square roots of LHS and RHS, we get
= 13/ 11
Since, sides of similar triangles are propositional, we can say
3/ 11 = (longer side of the larger triangle)/ (longer side of the smaller triangle)
⇒ 13/ 11 = 26/ x
x = 22

Therefore, the longest side of the smaller triangle is 22 cm.

5. The area of two similar triangles are 25 cm^2 and $36cm^2$ respectively. If the altitude of the first triangle is 2.4 cm, find the corresponding altitude of the other.
Solution:

Given: The area of two similar triangles are 25 cm^2 and $36cm^2$ respectively, the altitude of the first triangle is 2.4 cm
Required to find: the altitude of the second triangle

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes, we have
⇒ ar(triangle1)/ar(triangle2) = (altitude1/ altitude2)^2
⇒ 25/ 36 = $(2.4)^2$/ (altitude2)^2
Taking square roots of LHS and RHS, we get
5/ 6 = 2.4/ altitude2
⇒ altitude2 = (2.4 x 6)/5 = 2.88cm

Therefore, the altitude of the second triangle is 2.88cm.

6. The corresponding altitudes of two similar triangles are 6 cm and 9 cm respectively. Find the ratio of their areas.
Solution:

Given:
The corresponding altitudes of two similar triangles are 6 cm and 9 cm.
Required to find: Ratio of areas of the two similar triangles

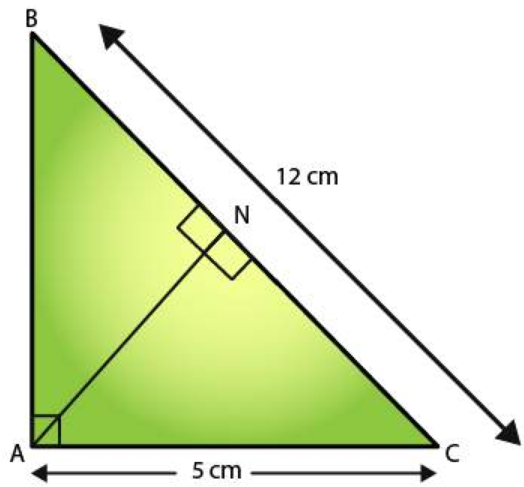
We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes, we have

ar(triangle1)/ar(triangle2) = (altitude1/ altitude2)^2 = (6/9)^2
= 36/ 81
= 4/9

Therefore, the ratio of the areas of two triangles = 4: 9.

7. ABC is a triangle in which ∠ A = 90°, AN ⊥ BC, BC = 12 cm and AC = 5 cm. Find the ratio of the areas of ΔANC and ΔABC.
Solution:
Given:

Given,
ΔABC, ∠A = 90°, AN ⊥ BC
BC= 12 cm
AC = 5 cm.
Required to find: ar(ΔANC)/ ar(ΔABC).



We have,
In ΔANC and ΔABC,
∠ACN = ∠ACB [Common]
∠A = ∠ANC [each 90°]
⇒ ΔANC ~ ΔABC [AA similarity]

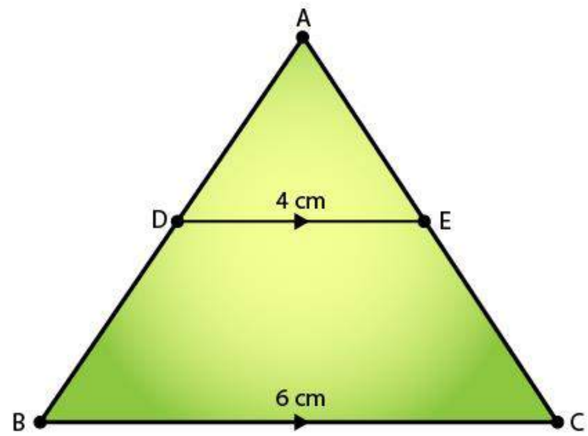
Since the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides, we get have
ar(ΔANC)/ ar(ΔABC) = (AC/ BC)^2 = (5/12)^2 = 25/ 144

Therefore, ar(ΔANC)/ ar(ΔABC) = 25:144

8. In Fig 4.179, DE || BC
(i) If DE = 4m, BC = 6 cm and Area (ΔADE) = 16cm^2, find the area of ΔABC.
(ii) If DE = 4cm, BC = 8 cm and Area (ΔADE) = 25cm^2, find the area of ΔABC.
(iii) If DE: BC = 3: 5. Calculate the ratio of the areas of ΔADE and the trapezium BCED.
Solution:

Given,
DE ∥ BC.
In ΔADE and ΔABC
We know that,
∠ADE = ∠B [Corresponding angles]
∠DAE = ∠BAC [Common]

Hence, ΔADE ~ ΔABC (AA Similarity)



(i) Since the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides, we have,
Ar(ΔADE)/ Ar(ΔABC) = DE^2/ BC^2
16/ Ar(ΔABC) = 4^2/ 6^2
⇒ Ar(ΔABC) = (6^2 × 16)/ 4^2
⇒ Ar(ΔABC) = 36 cm^2

(ii) Since the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides, we have,
Ar(ΔADE)/ Ar(ΔABC) = DE^2/ BC^2
25/ Ar(ΔABC) = 4^2/ 8^2
⇒ Ar(ΔABC) = (8^2 × 25)/ 4^2
⇒ Ar(ΔABC) = 100 cm^2

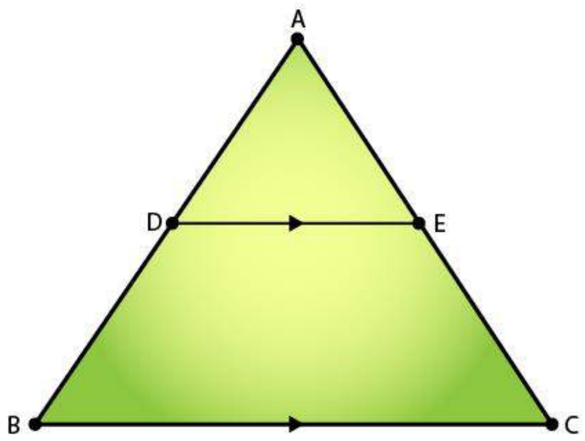
(iii) According to the question,
Ar(ΔADE)/ Ar(ΔABC) = DE^2/ BC^2
Ar(ΔADE)/ Ar(ΔABC) = 3^2/ 5^2
Ar(ΔADE)/ Ar(ΔABC) = 9/25

Assume that the area of ΔADE = 9x sq units
And, area of ΔABC = 25x sq units
So,
Area of trapezium BCED = Area of ΔABC – Area of ΔADE
= 25x – 9x
= 16x

Now, Ar(ΔADE)/ Ar(trap BCED) = 9x/ 16x
Ar(ΔADE)/ Ar(trapBCED) = 9/16

9. In ΔABC, D and E are the mid- points of AB and AC respectively. Find the ratio of the areas ΔADE and ΔABC.
Solution:

Given:
In ΔABC, D and E are the midpoints of AB and AC respectively.
Required to find: Ratio of the areas of ΔADE and ΔABC



Since, D and E are the midpoints of AB and AC respectively.
We can say,
DE || BC (By converse of mid-point theorem)
Also, DE = (1/2) BC

In ΔADE and ΔABC,
∠ADE = ∠B (Corresponding angles)
∠DAE = ∠BAC (common)

Thus, ΔADE ~ ΔABC (AA Similarity)

Now, we know that
The ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides, so

$Ar(\Delta ADE)/ Ar(\Delta ABC) = AD^2/ AB^2$
$Ar(\Delta ADE)/ Ar(\Delta ABC) = 1^2/ 2^2$
$Ar(\Delta ADE)/ Ar(\Delta ABC) = 1/4$

Therefore, the ratio of the areas ΔADE and ΔABC is 1:4

10. The areas of two similar triangles are 100 cm^2 and 49 cm^2 respectively. If the altitude of the bigger triangles is 5 cm, find the corresponding altitude of the other.
Solution:

Given: The area of the two similar triangles is 100cm^2 and 49cm^2. And the altitude of the bigger triangle is 5cm.
Required to find: The corresponding altitude of the other triangle

We know that,
The ratio of the areas of the two similar triangles is equal to the ratio of squares of their corresponding altitudes.

ar(bigger triangle)/ ar(smaller triangle) = (altitude of the bigger triangle/ altitude of the smaller triangle)^2

(100/ 49) = (5/ altitude of the smaller triangle)^2

Taking square root on LHS and RHS, we get

(10/ 7) = (5/ altitude of the smaller triangle) = 7/2

Therefore, altitude of the smaller triangle = 3.5cm

11. The areas of two similar triangles are 121 cm^2 and 64 cm^2 respectively. If the median of the first triangle is 12.1 cm, find the corresponding median of the other.
Solution:

Given: the area of the two triangles is 121cm^2 and 64cm^2 respectively and the median of the first triangle is 12.1cm
Required to find: the corresponding median of the other triangle

We know that,
The ratio of the areas of the two similar triangles are equal to the ratio of the squares of their medians.

ar(triangle1)/ ar(triangle2) = (median of triangle 1/median of triangle 2)^2

121/ 64 = (12.1/ median of triangle 2)^2

Taking the square roots on both LHS and RHS, we have

11/8 = (12.1/ median of triangle 2) = (12.1 x 8)/ 11

Therefore, Median of the other triangle = 8.8cm

Exercise 4.7 Page No: 4.119

1. If the sides of a triangle are 3 cm, 4 cm, and 6 cm long, determine whether the triangle is a right-angled triangle.

Solution:

We have,
Sides of triangle as
AB = 3 cm
BC = 4 cm
AC = 6 cm
On finding their squares, we get
$AB^2 = 3^2 = 9$
$BC^2 = 4^2 = 16$
$AC^2 = 6^2 = 36$

Since, $AB^2 + BC^2 \neq AC^2$
So, by converse of Pythagoras theorem the given sides cannot be the sides of a right triangle.

2. The sides of certain triangles are given below. Determine which of them are right triangles.
(i) a = 7 cm, b = 24 cm and c = 25 cm
(ii) a = 9 cm, b = 16 cm and c = 18 cm
(iii) a = 1.6 cm, b = 3.8 cm and c = 4 cm
(iv) a = 8 cm, b = 10 cm and c = 6 cm
Solutions:

(i) Given,
a = 7 cm, b = 24 cm and c = 25 cm
$\therefore a^2 = 49$, $b^2 = 576$ and $c^2 = 625$
Since, $a^2 + b^2 = 49 + 576 = 625 = c^2$
Then, by converse of Pythagoras theorem
The given sides are of a right triangle.

(ii) Given,
a = 9 cm, b = 16 cm and c = 18 cm
$\therefore a^2 = 81$, $b^2 = 256$ and $c^2 = 324$
Since, $a^2 + b^2 = 81 + 256 = 337 \neq c^2$
Then, by converse of Pythagoras theorem
The given sides cannot be of a right triangle.

(iii) Given,
a = 1.6 cm, b = 3.8 cm and C = 4 cm
$\therefore a^2 = 2.56$, $b^2 = 14.44$ and $c^2 = 16$
Since, $a^2 + b^2 = 2.56 + 14.44 = 17 \neq c^2$
Then, by converse of Pythagoras theorem
The given sides cannot be of a right triangle.

(iv) Given,

$a = 8$ cm, $b = 10$ cm and $C = 6$ cm

$\therefore a^2 = 64, b^2 = 100$ and $c^2 = 36$

Since, $a^2 + c^2 = 64 + 36 = 100 = b^2$

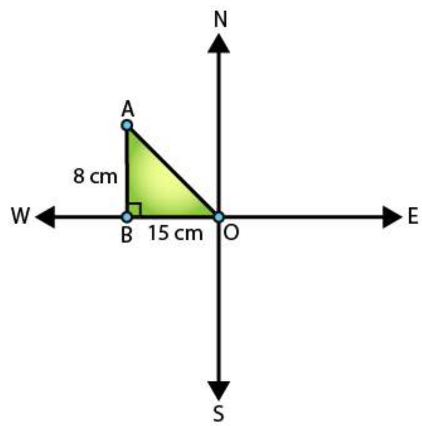
Then, by converse of Pythagoras theorem

The given sides are of a right triangle

3. A man goes 15 metres due west and then 8 metres due north. How far is he from the starting point?

Solution:

Let the starting point of the man be O and final point be A.



In ΔABO,

by Pythagoras theorem $AO^2 = AB^2 + BO^2$

$\Rightarrow \quad AO^2 = 8^2 + 15^2$

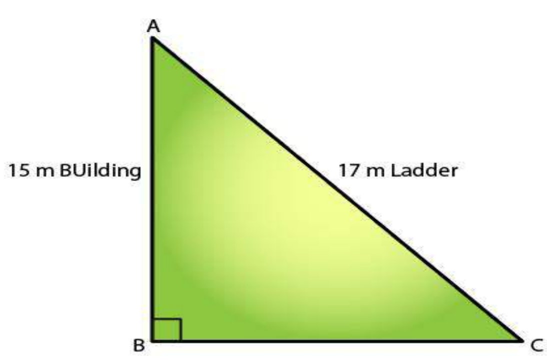
$\Rightarrow \quad AO^2 = 64 + 225 = 289$

$\Rightarrow \quad AO = \sqrt{289} = 17m$

$\therefore$ the man is 17m far from the starting point.

4. A ladder 17 m long reaches a window of a building 15 m above the ground. Find the distance of the foot of the ladder from the building.

Solution:



In ΔABC, by Pythagoras theorem

$AB^2 + BC^2 = AC^2$

$\Rightarrow \quad 15^2 + BC^2 = 17^2$

$225 + BC^2 = 17^2$

$BC^2 = 289 - 225$

$BC^2 = 64$

$\therefore BC = 8$ m

Therefore, the distance of the foot of the ladder from building = 8 m

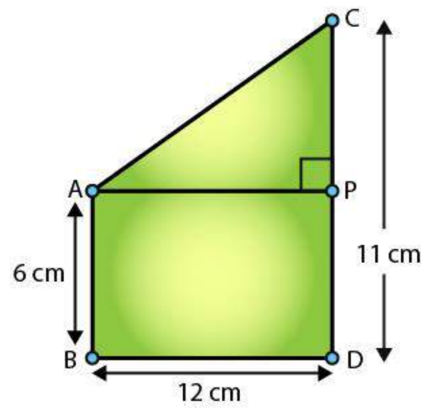
5. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

Solution:

Let CD and AB be the poles of height 11m and 6m.

Then, its seen that CP = 11 - 6 = 5m.

From the figure, AP should be 12m (given)



In triangle APC, by applying Pythagoras theorem, we have

$AP^2 + PC^2 = AC^2$

$12^2 + 5^2 = AC^2$

$AC^2 = 144 + 25 = 169$

$\therefore AC = 13$ (by taking sq. root on both sides)

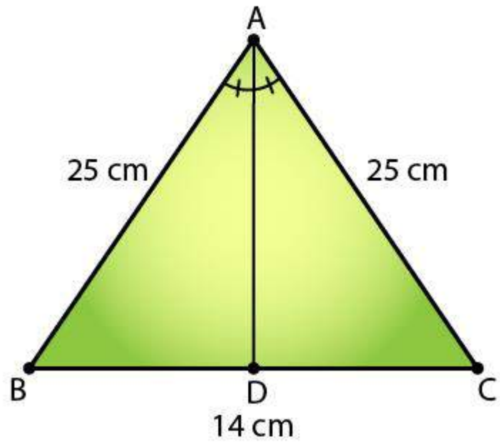
Thus, the distance between their tops = 13 m.

6. In an isosceles triangle ABC, AB = AC = 25 cm, BC = 14 cm. Calculate the altitude from A on BC.

Solution:

Given,

ΔABC, AB = AC = 25 cm and BC = 14.



In ΔABD and ΔACD, we see that

$\angle ADB = \angle ADC$ [Each = 90°]
$AB = AC$ [Given]
$AD = AD$ [Common]

Then, $\Delta ABD \cong \Delta ACD$ [By RHS condition]

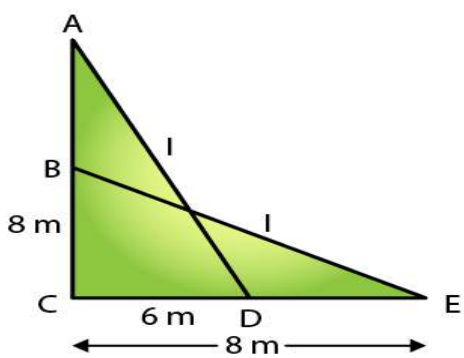
Thus, $BD = CD = 7$ cm [By corresponding parts of congruent triangles]

Finally,
In ΔADB, by Pythagoras theorem

$AD^2 + BD^2 = AB^2$
$\Rightarrow AD^2 + 7^2 = 25^2$
$AD^2 = 625 – 49 = 576$
$\therefore AD = \sqrt{576} = 24$ cm

7. The foot of a ladder is 6 m away from a wall and its top reaches a window 8 m above the ground. If the ladder is shifted in such a way that its foot is 8 m away from the wall, to what height does its tip reach?
Solution:

Let's assume the length of ladder to be, AD = BE = x m



So, in ΔACD, by Pythagoras theorem
We have,

$AD^2 = AC^2 + CD^2$

$\Rightarrow x^2 = 8^2 + 6^2$ ... (i)

Also, in ΔBCE, by Pythagoras theorem

$BE^2 = BC^2 + CE^2$

$\Rightarrow x^2 = BC^2 + 8^2$ ... (ii)

Compare (i) and (ii)

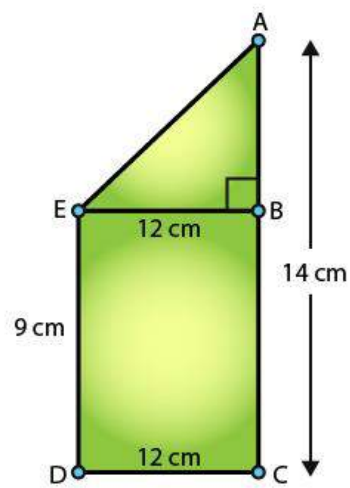
$BC^2 + 8^2 = 8^2 + 6^2$

$\Rightarrow \quad BC^2 + 6^2$

$\Rightarrow \quad BC = 6$ m

Therefore, the tip of the ladder reaches to a height od 6m.

8. Two poles of height 9 in and 14 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.
Solution:



Comparing with the figure, it's given that
AC = 14 m, DC = 12m and ED = BC = 9 m

Construction: Draw EB ⊥ AC
Now,
It's seen that AB = AC – BC = (14 – 9) = 5 m
And, EB = DC = 12m [distance between their feet]
Thus,
In ΔABE, by Pythagoras theorem, we have

$AE^2 = AB^2 + BE^2$

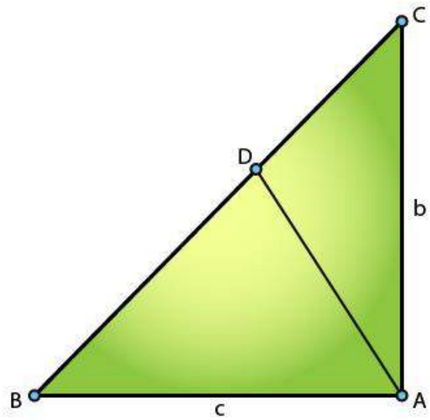
$AE^2 = 5^2 + 12^2$

$AE^2 = 25 + 144 = 169$

⇒ $AE = \sqrt{169} = 13$ m

Therefore, the distance between their tops = 13 m

9. Using Pythagoras theorem determine the length of AD in terms of b and c shown in Fig. 4.219
Solution:



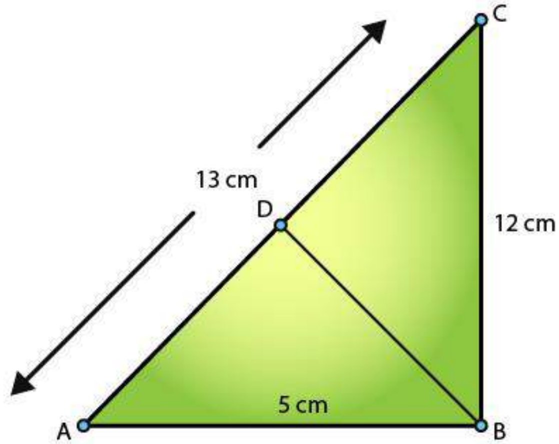
We have,
In ΔBAC, by Pythagoras theorem, we have
$BC^2 = AB^2 + AC^2$
⇒ $BC^2 = c^2 + b^2$
⇒ $BC = \sqrt{(c^2 + b^2)}$
In ΔABD and ΔCBA
∠B = ∠B [Common]
∠ADB = ∠BAC [Each 90°]

Then, ΔABD ~ ΔCBA [By AA similarity]

Thus,
AB/ CB = AD/ CA [Corresponding parts of similar triangles are proportional]
$c/ \sqrt{(c^2 + b^2)} = AD/ b$
∴ $AD = bc/ \sqrt{(c^2 + b^2)}$

10. A triangle has sides 5 cm, 12 cm and 13 cm. Find the length to one decimal place, of the perpendicular from the opposite vertex to the side whose length is 13 cm.
Solution:

From the fig. AB = 5cm, BC = 12 cm and AC = 13 cm.
Then, $AC^2 = AB^2 + BC^2$.
⇒ $(13)^2 = (5)^2 + (12)^2 = 25 + 144 = 169 = 13^2$
This proves that ΔABC is a right triangle, right angled at B.
Let BD be the length of perpendicular from B on AC.



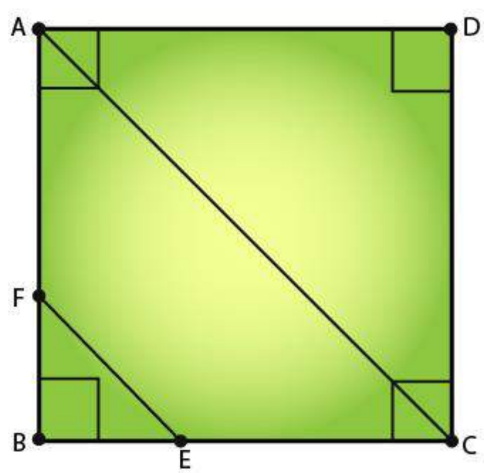
So, area of ΔABC = (BC x BA)/ 2 [Taking BC as the altitude]
= (12 x 5)/ 2
= 30 cm^2

Also, area of ΔABC = (AC x BD)/ 2 [Taking BD as the altitude]
= (13 x BD)/ 2

⇒ (13 x BD)/ 2 = 30
BD = 60/13 = 4.6 (to one decimal place)

11. ABCD is a square. F is the mid-point of AB. BE is one third of BC. If the area of Δ FBE = 108cm^2, find the length of AC.
Solution:

Given,
ABCD is a square. And, F is the mid-point of AB.
BE is one third of BC.
Area of Δ FBE = 108cm^2
Required to find: length of AC

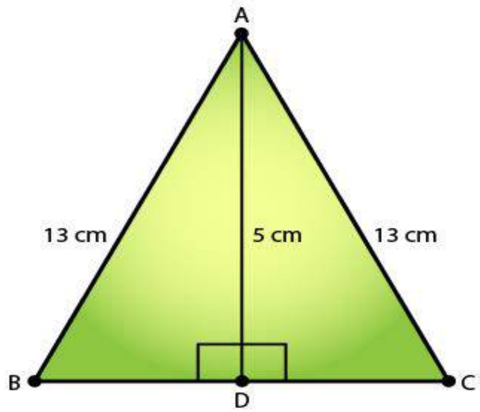


Let's assume the sides of the square to be x.
⇒ AB = BC = CD = DA = x cm
And, AF = FB = x/2 cm
So, BE = x/3 cm
Now, the area of Δ FBE = 1/2 x BE x FB
⇒ 108 = (1/2) x (x/3) x (x/2)
⇒ $x^2 = 108 \times 2 \times 3 \times 2 = 1296$
⇒ $x = \sqrt{(1296)}$ [taking square roots of both the sides]
∴ x = 36cm
Further in Δ ABC, by Pythagoras theorem, we have
$AC^2 = AB^2 + BC^2$
⇒ $AC^2 = x^2 + x^2 = 2x^2$
⇒ $AC^2 = 2 \times (36)^2$
⇒ $AC = 36\sqrt{2} = 36 \times 1.414 = 50.904$ cm

Therefore, the length of AC is 50.904 cm.

12. In an isosceles triangle ABC, if AB = AC = 13cm and the altitude from A on BC is 5cm, find BC.
Solution:

Given,
An isosceles triangle ABC, AB = AC = 13cm, AD = 5cm
Required to find: BC

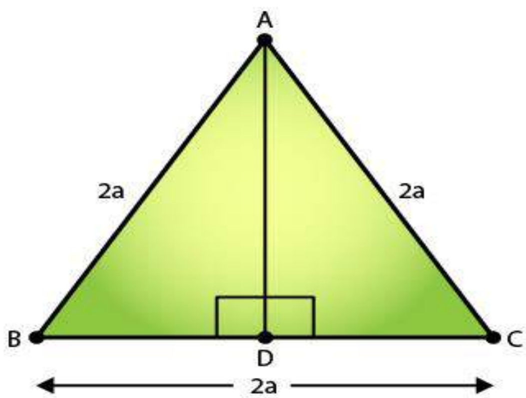


In Δ ADB, by using Pythagoras theorem, we have
$AD^2 + BD^2 = 13^2$
$5^2 + BD^2 = 169$
$BD^2 = 169 – 25 = 144$
⇒$BD = \sqrt{144} = 12$ cm

Similarly, applying Pythagoras theorem is Δ ADC we can have,
$AC^2 = AD^2 + DC^2$
$13^2 = 5^2 + DC^2$
⇒ $DC = \sqrt{144} = 12$ cm

Thus, BC = BD + DC = 12 + 12 = 24 cm

13. In a ΔABC, AB = BC = CA = 2a and AD ⊥ BC. Prove that
(i) AD = a√3 (ii) Area (ΔABC) = √3 a²
Solution:



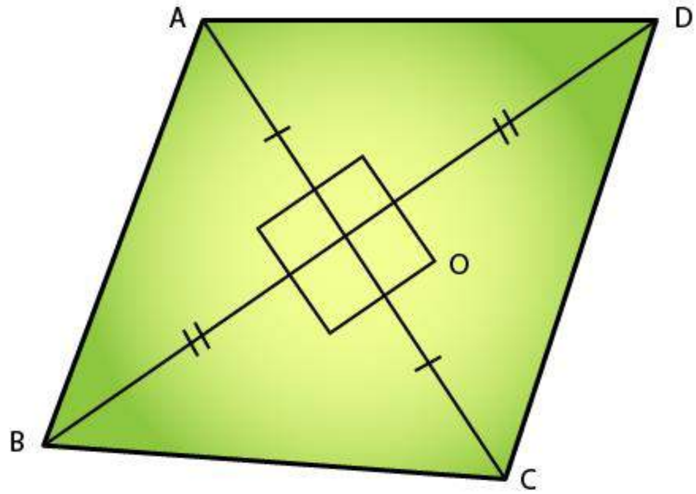
(i) In ΔABD and ΔACD, we have
∠ADB = ∠ADC = 90°
AB = AC [Given]
AD = AD [Common]
So, ΔABD ≅ ΔACD [By RHS condition]
Hence, BD = CD = a [By C.P.C.T]

Now, in ΔABD, by Pythagoras theorem
$AD^2 + BD^2 = AB^2$
$AD^2 + a^2 = 2a^2$
$AD^2 = 4a^2 – a^2 = 3a^2$
AD = a√3

(ii) Area (ΔABC) = 1/2 x BC x AD
= 1/2 x (2a) x (a√3)
= √3 a^2

14. The lengths of the diagonals of a rhombus is 24cm and 10cm. Find each side of the rhombus.
Solution:

Let ABCD be a rhombus and AC and BD be the diagonals of ABCD.
So, AC = 24cm and BD = 10cm



We know that diagonals of a rhombus bisect each other at right angle. (Perpendicular to each other)
So,
AO = OC = 12cm and BO = OD = 3cm
In ΔAOB, by Pythagoras theorem, we have

$AB^2 = AO^2 + BO^2$
$= 12^2 + 5^2$
$= 144 + 25$
$= 169$

$\Rightarrow AB = \sqrt{169} = 13cm$

Since, the sides of rhombus are all equal.
Therefore, AB = BC = CD = AD = 13cm.