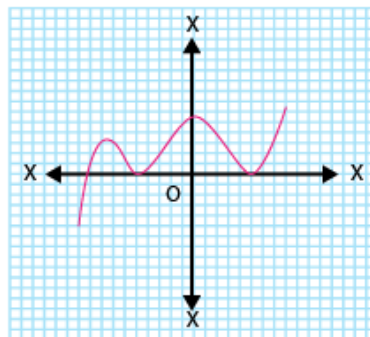
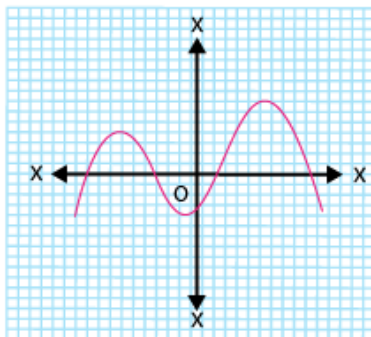
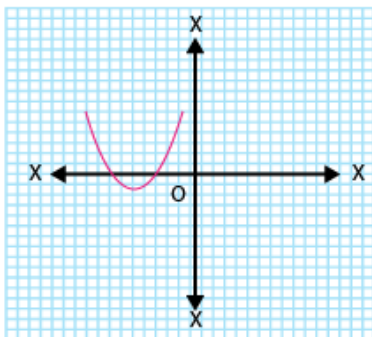
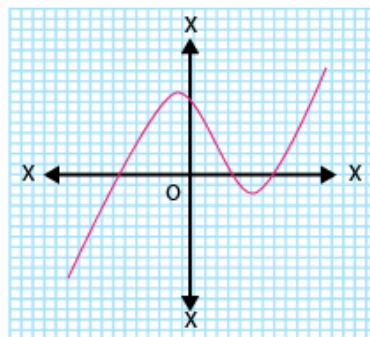
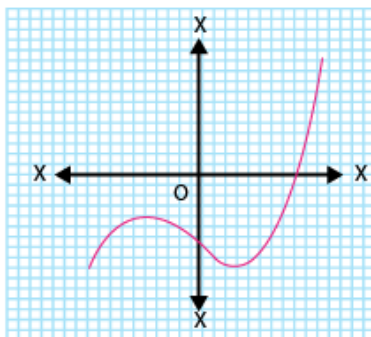
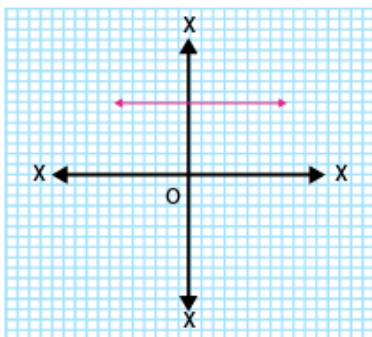


EXERCISE 2.1

PAGE: 28

1. The graphs of $y = p(x)$ are given in Fig. 2.10 below, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.



Solutions:

Graphical method to find zeroes:-

Total number of zeroes in any polynomial equation = total number of times the curve intersects x-axis.

- (i) In the given graph, the number of zeroes of $p(x)$ is 0 because the graph is parallel to x-axis does not cut it at any point.
- (ii) In the given graph, the number of zeroes of $p(x)$ is 1 because the graph intersects the x-axis at only one point.
- (iii) In the given graph, the number of zeroes of $p(x)$ is 3 because the graph intersects the x-axis at any three points.
- (iv) In the given graph, the number of zeroes of $p(x)$ is 2 because the graph intersects the x-axis at two points.
- (v) In the given graph, the number of zeroes of $p(x)$ is 4 because the graph intersects the x-axis at four points.
- (vi) In the given graph, the number of zeroes of $p(x)$ is 3 because the graph intersects the x-axis at three points.

EXERCISE 2.2

PAGE: 33

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

Solutions:

(i) $x^2 - 2x - 8$

$$\Rightarrow x^2 - 4x + 2x - 8 = x(x-4) + 2(x-4) = (x-4)(x+2)$$

Therefore, zeroes of polynomial equation $x^2 - 2x - 8$ are (4, -2)

$$\text{Sum of zeroes} = 4 - 2 = 2 = -(-2)/1 = -(\text{Coefficient of } x)/(\text{Coefficient of } x^2)$$

$$\text{Product of zeroes} = 4 \times (-2) = -8 = (-8)/1 = (\text{Constant term})/(\text{Coefficient of } x^2)$$

(ii) $4s^2 - 4s + 1$

$$\Rightarrow 4s^2 - 2s - 2s + 1 = 2s(2s-1) - 1(2s-1) = (2s-1)(2s-1)$$

Therefore, zeroes of polynomial equation $4s^2 - 4s + 1$ are (1/2, 1/2)

$$\text{Sum of zeroes} = (1/2) + (1/2) = 1 = -(-4)/4 = -(\text{Coefficient of } s)/(\text{Coefficient of } s^2)$$

$$\text{Product of zeroes} = (1/2) \times (1/2) = 1/4 = (1)/4 = (\text{Constant term})/(\text{Coefficient of } s^2)$$

(iii) $6x^2 - 3 - 7x$

$$\Rightarrow 6x^2 - 7x - 3 = 6x^2 - 9x + 2x - 3 = 3x(2x-3) + 1(2x-3) = (3x+1)(2x-3)$$

Therefore, zeroes of polynomial equation $6x^2 - 3 - 7x$ are (-1/3, 3/2)

$$\text{Sum of zeroes} = -(1/3) + (3/2) = (7/6) = -(\text{Coefficient of } x)/(\text{Coefficient of } x^2)$$

$$\text{Product of zeroes} = -(1/3) \times (3/2) = -(3/6) = (\text{Constant term})/(\text{Coefficient of } x^2)$$

(iv) $4u^2 + 8u$

$$\Rightarrow 4u(u+2)$$

Therefore, zeroes of polynomial equation $4u^2 + 8u$ are (0, -2).

$$\text{Sum of zeroes} = 0 + (-2) = -2 = -(8/4) = -(\text{Coefficient of } u)/(\text{Coefficient of } u^2)$$

$$\text{Product of zeroes} = 0 \times -2 = 0 = 0/4 = (\text{Constant term})/(\text{Coefficient of } u^2)$$

(v) $t^2 - 15$

$$\Rightarrow t^2 = 15 \text{ or } t = \pm\sqrt{15}$$

Therefore, zeroes of polynomial equation $t^2 - 15$ are ($\sqrt{15}$, $-\sqrt{15}$)

$$\text{Sum of zeroes} = \sqrt{15} + (-\sqrt{15}) = 0 = -(0/1) = -(\text{Coefficient of } t)/(\text{Coefficient of } t^2)$$

$$\text{Product of zeroes} = \sqrt{15} \times (-\sqrt{15}) = -15 = -15/1 = (\text{Constant term})/(\text{Coefficient of } t^2)$$

(vi) $3x^2 - x - 4$

$$\Rightarrow 3x^2 - 4x + 3x - 4 = x(3x-4) + 1(3x-4) = (3x-4)(x+1)$$

Therefore, zeroes of polynomial equation $3x^2 - x - 4$ are (4/3, -1)

$$\text{Sum of zeroes} = (4/3) + (-1) = (1/3) = -(-1/3) = -(\text{Coefficient of } x) / (\text{Coefficient of } x^2)$$

$$\text{Product of zeroes} = (4/3) \times (-1) = (-4/3) = (\text{Constant term}) / (\text{Coefficient of } x^2)$$

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes, respectively.

(i) $1/4, -1$

Solution:

From the formulas of sum and product of zeroes, we know,

$$\text{Sum of zeroes} = \alpha + \beta$$

$$\text{Product of zeroes} = \alpha \beta$$

$$\text{Sum of zeroes} = \alpha + \beta = 1/4$$

$$\text{Product of zeroes} = \alpha \beta = -1$$

\therefore If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (1/4)x + (-1) = 0$$

$$4x^2 - x - 4 = 0$$

Thus, $4x^2 - x - 4$ is the quadratic polynomial.

(ii) $\sqrt{2}, 1/3$

Solution:

$$\text{Sum of zeroes} = \alpha + \beta = \sqrt{2}$$

$$\text{Product of zeroes} = \alpha \beta = 1/3$$

\therefore If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (\sqrt{2})x + (1/3) = 0$$

$$3x^2 - 3\sqrt{2}x + 1 = 0$$

Thus, $3x^2 - 3\sqrt{2}x + 1$ is the quadratic polynomial.

(iii) $0, \sqrt{5}$

Solution:

Given,

$$\text{Sum of zeroes} = \alpha + \beta = 0$$

$$\text{Product of zeroes} = \alpha \beta = \sqrt{5}$$

\therefore If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (0)x + \sqrt{5} = 0$$

Thus, $x^2 + \sqrt{5}$ is the quadratic polynomial.

(iv) 1, 1

Solution:

Given,

$$\text{Sum of zeroes} = \alpha + \beta = 1$$

$$\text{Product of zeroes} = \alpha \beta = 1$$

\therefore If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - x + 1 = 0$$

Thus, $x^2 - x + 1$ is the quadratic polynomial.

(v) $-1/4$, $1/4$

Solution:

Given,

$$\text{Sum of zeroes} = \alpha + \beta = -1/4$$

$$\text{Product of zeroes} = \alpha \beta = 1/4$$

\therefore If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (-1/4)x + (1/4) = 0$$

$$4x^2 + x + 1 = 0$$

Thus, $4x^2 + x + 1$ is the quadratic polynomial.

(vi) 4, 1

Solution:

Given,

$$\text{Sum of zeroes} = \alpha + \beta = 4$$

$$\text{Product of zeroes} = \alpha\beta = 1$$

\therefore If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - 4x + 1 = 0$$

Thus, $x^2 - 4x + 1$ is the quadratic polynomial.

EXERCISE 2.3

PAGE: 36

1. Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

Solution:

Given,

Dividend = $p(x) = x^3 - 3x^2 + 5x - 3$

Divisor = $g(x) = x^2 - 2$

$$\begin{array}{r}
 \overline{x^3 - 3x^2 + 5x - 3} \\
 \underline{x^3 - 2x } \\
 + 7x - 3 \\
 \underline{ - 3x^2 + 0x + 6} \\
 7x - 9
 \end{array}$$

Therefore, upon division we get,

Quotient = $x - 3$

Remainder = $7x - 9$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

Solution:

Given,

Dividend = $p(x) = x^4 - 3x^2 + 4x + 5$

Divisor = $g(x) = x^2 + 1 - x$

$$\begin{array}{r}
 x^2 - x + 1 \overline{) x^4 + 0x^3 - 3x^2 + 4x + 5} \\
 \underline{x^4 - x^3 + x^2} \\
 x^3 - 4x^2 + 4x + 5 \\
 \underline{x^3 - x^2 + x} \\
 -3x^2 + 3x + 5 \\
 \underline{-3x^2 + 3x - 3} \\
 8
 \end{array}$$

Therefore, upon division we get,

$$\text{Quotient} = x^2 + x - 3$$

$$\text{Remainder} = 8$$

$$\text{(iii) } p(x) = x^4 - 5x + 6, g(x) = 2 - x^2$$

Solution:

Given,

$$\text{Dividend} = p(x) = x^4 - 5x + 6 = x^4 + 0x^3 + 0x^2 - 5x + 6$$

$$\text{Divisor} = g(x) = 2 - x^2 = -x^2 + 2$$

$$\begin{array}{r}
 -x^2 + 2 \overline{) x^4 + 0x^3 + 0x^2 - 5x + 6} \\
 \underline{x^4 + 0x^3 - 2x^2} \\
 2x^2 - 5x + 6 \\
 \underline{2x^2 + 0x - 4} \\
 -5x + 10
 \end{array}$$

Therefore, upon division we get,

Quotient = $-x^2 - 2$

Remainder = $-5x + 10$

2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2 - 3$, $2t^4 + 3t^3 - 2t^2 - 9t - 12$

Solutions:

Given,

First polynomial = $t^2 - 3$

Second polynomial = $2t^4 + 3t^3 - 2t^2 - 9t - 12$

$$\begin{array}{r}
 \begin{array}{ccccccc}
 & 2t^2 & +3t & +4 \\
 t^2 - 3 & \overline{) 2t^4 + 3t^3 - 2t^2 - 9t - 12} \\
 \underline{2t^4 + 0t^3 - 6t^2} & & & & & & \\
 & 3t^3 & +4t^2 & -9t & -12 & & \\
 & \underline{3t^3 + 0t^2 - 9t} & & & & & \\
 & & 4t^2 & +0t & -12 & & \\
 & & \underline{4t^2 + 0t - 12} & & & & \\
 & & & 0 & & &
 \end{array}
 \end{array}$$

As we can see, the remainder is left as 0. Therefore, we say that, $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

(ii) $x^2 + 3x + 1$, $3x^4 + 5x^3 - 7x^2 + 2x + 2$

Solutions:

Given,

First polynomial = $x^2 + 3x + 1$

Second polynomial = $3x^4 + 5x^3 - 7x^2 + 2x + 2$

$$\begin{array}{r}
 \overline{3x^2 - 4x + 2} \\
 x^2 + 3x + 1 \overline{) 3x^4 + 5x^3 - 7x^2 + 2x + 2} \\
 \underline{3x^4 + 9x^3 + 3x^2} \\
 -4x^3 - 10x^2 + 2x + 2 \\
 \underline{-4x^3 - 12x^2 - 4x} \\
 2x^2 + 6x + 2 \\
 \underline{2x^2 + 6x + 2} \\
 0
 \end{array}$$

As we can see, the remainder is left as 0. Therefore, we say that, $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

(iii) $x^3 - 3x + 1$, $x^5 - 4x^3 + x^2 + 3x + 1$

Solutions:

Given,

First polynomial = $x^3 - 3x + 1$

Second polynomial = $x^5 - 4x^3 + x^2 + 3x + 1$

$$\begin{array}{r}
 \overline{x^2 - 1} \\
 x^3 - 3x + 1 \overline{) x^5 + 0x^4 - 4x^3 + x^2 + 3x + 1} \\
 \underline{x^5 + 0x^4 - 3x^3 + x^2} \\
 -x^3 + 0x^2 + 3x + 1 \\
 \underline{-x^3 + 0x^2 + 3x - 1} \\
 2
 \end{array}$$

As we can see, the remainder is not equal to 0. Therefore, we say that, $x^3 - 3x + 1$ is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$.

3. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{5/3}$ and $-\sqrt{5/3}$.

Solutions:

Since this is a polynomial equation of degree 4, hence there will be total 4 roots.

$\sqrt{5/3}$ and $-\sqrt{5/3}$ are zeroes of polynomial $f(x)$.

$$\therefore (x - \sqrt{5/3})(x + \sqrt{5/3}) = x^2 - (5/3) = 0$$

$(3x^2 - 5) = 0$, is a factor of given polynomial $f(x)$.

Now, when we will divide $f(x)$ by $(3x^2 - 5)$ the quotient obtained will also be a factor of $f(x)$ and the remainder will be 0.

$$\begin{array}{r}
 \overline{) 3x^4 + 6x^3 - 2x^2 - 10x - 5} \\
 \underline{3x^4 - 5x^2} \\
 (-) (+) \\
 + 6x^3 + 3x^2 - 10x - 5 \\
 \underline{- 6x^3 - 10x} \\
 (+) (-) \\
 3x^2 - 5 \\
 \underline{3x^2 - 5} \\
 (-) (+) \\
 0
 \end{array}$$

$$\text{Therefore, } 3x^4 + 6x^3 - 2x^2 - 10x - 5 = (3x^2 - 5)(x^2 + 2x + 1)$$

Now, on further factorizing $(x^2 + 2x + 1)$ we get,

$$x^2 + 2x + 1 = x^2 + x + x + 1 = 0$$

$$x(x+1) + 1(x+1) = 0$$

$$(x+1)(x+1) = 0$$

So, its zeroes are given by: $x = -1$ and $x = -1$.

Therefore, all four zeroes of given polynomial equation are:

$$\sqrt{5/3}, -\sqrt{5/3}, -1 \text{ and } -1.$$

Hence, is the answer.

4. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

Solution:

Given,

$$\text{Dividend, } p(x) = x^3 - 3x^2 + x + 2$$

$$\text{Quotient} = x - 2$$

$$\text{Remainder} = -2x + 4$$

We have to find the value of Divisor, $g(x) = ?$

As we know,

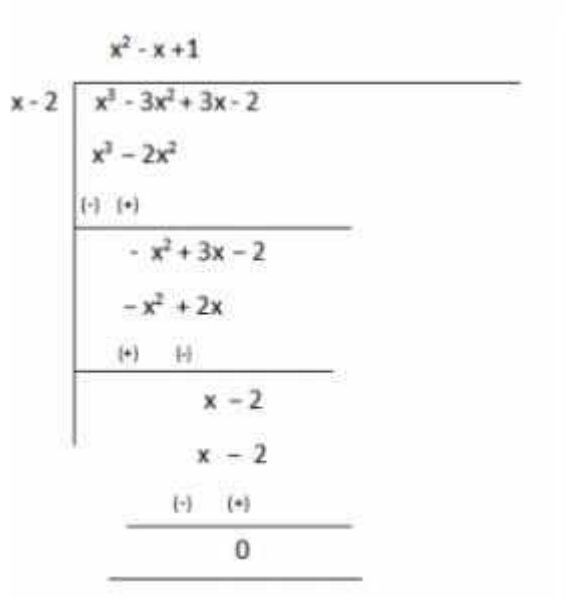
$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

$$\therefore x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$$

$$x^3 - 3x^2 + x + 2 - (-2x + 4) = g(x) \times (x - 2)$$

$$\text{Therefore, } g(x) \times (x - 2) = x^3 - 3x^2 + 3x - 2$$

Now, for finding $g(x)$ we will divide $x^3 - 3x^2 + 3x - 2$ with $(x - 2)$



$$\begin{array}{r}
 x^2 - x + 1 \\
 x - 2 \overline{) x^3 - 3x^2 + 3x - 2} \\
 \underline{x^3 - 2x^2} \\
 (-) (+) \\
 - x^2 + 3x - 2 \\
 \underline{- x^2 + 2x} \\
 (+) (-) \\
 x - 2 \\
 \underline{x - 2} \\
 (-) (+) \\
 0
 \end{array}$$

$$\text{Therefore, } g(x) = (x^2 - x + 1)$$

5. Give examples of polynomials $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) $\deg p(x) = \deg q(x)$

(ii) $\deg q(x) = \deg r(x)$

(iii) $\deg r(x) = 0$

Solutions:

According to the division algorithm, dividend $p(x)$ and divisor $g(x)$ are two polynomials, where $g(x) \neq 0$. Then we can find the value of quotient $q(x)$ and remainder $r(x)$, with the help of below given formula;

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

$$\therefore p(x) = g(x) \times q(x) + r(x)$$

Where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$.

Now let us proof the three given cases as per division algorithm by taking examples for each.

(i) $\deg p(x) = \deg q(x)$

Degree of dividend is equal to degree of quotient, only when the divisor is a constant term.

Let us take an example, $p(x) = 3x^2 + 3x + 3$ is a polynomial to be divided by $g(x) = 3$.

$$\text{So, } (3x^2 + 3x + 3)/3 = x^2 + x + 1 = q(x)$$

Thus, you can see, the degree of quotient $q(x) = 2$, which also equal to the degree of dividend $p(x)$.

Hence, division algorithm is satisfied here.

(ii) $\deg q(x) = \deg r(x)$

Let us take an example, $p(x) = x^2 + 3$ is a polynomial to be divided by $g(x) = x - 1$.

$$\text{So, } x^2 + 3 = (x - 1) \times (x) + (x + 3)$$

Hence, quotient $q(x) = x$

Also, remainder $r(x) = x + 3$

Thus, you can see, the degree of quotient $q(x) = 1$, which is also equal to the degree of remainder $r(x)$.

Hence, division algorithm is satisfied here.

(iii) $\deg r(x) = 0$

The degree of remainder is 0 only when the remainder left after division algorithm is constant.

Let us take an example, $p(x) = x^2 + 1$ is a polynomial to be divided by $g(x) = x$.

$$\text{So, } x^2 + 1 = (x) \times (x) + 1$$

Hence, quotient $q(x) = x$

And, remainder $r(x) = 1$

Clearly, the degree of remainder here is 0.

Hence, division algorithm is satisfied here.

EXERCISE 2.4

PAGE: 36

1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3+x^2-5x+2$; $-1/2, 1, -2$

Solution:

Given, $p(x) = 2x^3+x^2-5x+2$

And zeroes for $p(x)$ are $= 1/2, 1, -2$

$$\therefore p(1/2) = 2(1/2)^3 + (1/2)^2 - 5(1/2) + 2 = (1/4) + (1/4) - (5/2) + 2 = 0$$

$$p(1) = 2(1)^3 + (1)^2 - 5(1) + 2 = 0$$

$$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2 = 0$$

Hence, proved $1/2, 1, -2$ are the zeroes of $2x^3+x^2-5x+2$.

Now, comparing the given polynomial with general expression, we get;

$$\therefore ax^3+bx^2+cx+d = 2x^3+x^2-5x+2$$

$$a=2, b=1, c=-5 \text{ and } d=2$$

As we know, if α, β, γ are the zeroes of the cubic polynomial ax^3+bx^2+cx+d , then;

$$\alpha + \beta + \gamma = -b/a$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$$

$$\alpha\beta\gamma = -d/a.$$

Therefore, putting the values of zeroes of the polynomial,

$$\alpha + \beta + \gamma = 1/2 + 1 + (-2) = -1/2 = -b/a$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = (1/2 \times 1) + (1 \times -2) + (-2 \times 1/2) = -5/2 = c/a$$

$$\alpha\beta\gamma = 1/2 \times 1 \times (-2) = -2/2 = -d/a$$

Hence, the relationship between the zeroes and the coefficients are satisfied.

(ii) x^3-4x^2+5x-2 ; $2, 1, 1$

Solution:

Given, $p(x) = x^3-4x^2+5x-2$

And zeroes for $p(x)$ are $2, 1, 1$.

$$\therefore p(2) = 2^3 - 4(2)^2 + 5(2) - 2 = 0$$

$$p(1) = 1^3 - (4 \times 1^2) + (5 \times 1) - 2 = 0$$

Hence proved, $2, 1, 1$ are the zeroes of x^3-4x^2+5x-2

Now, comparing the given polynomial with general expression, we get;

$$\therefore ax^3+bx^2+cx+d = x^3-4x^2+5x-2$$

$$a = 1, b = -4, c = 5 \text{ and } d = -2$$

As we know, if α, β, γ are the zeroes of the cubic polynomial ax^3+bx^2+cx+d , then;

$$\alpha + \beta + \gamma = -b/a$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$$

$$\alpha\beta\gamma = -d/a.$$

Therefore, putting the values of zeroes of the polynomial,

$$\alpha + \beta + \gamma = 2+1+1 = 4 = -(-4)/1 = -b/a$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 2 \times 1 + 1 \times 1 + 1 \times 2 = 5 = 5/1 = c/a$$

$$\alpha\beta\gamma = 2 \times 1 \times 1 = 2 = -(-2)/1 = -d/a$$

Hence, the relationship between the zeroes and the coefficients are satisfied.

2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, -7, -14 respectively.

Solution:

Let us consider the cubic polynomial is ax^3+bx^2+cx+d and the values of the zeroes of the polynomials be α, β, γ .

As per the given question,

$$\alpha + \beta + \gamma = -b/a = 2/1$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = c/a = -7/1$$

$$\alpha\beta\gamma = -d/a = -14/1$$

Thus, from above three expressions we get the values of coefficient of polynomial.

$$a = 1, b = -2, c = -7, d = 14$$

Hence, the cubic polynomial is $x^3-2x^2-7x+14$

3. If the zeroes of the polynomial x^3-3x^2+x+1 are $a-b, a, a+b$, find a and b .

Solution:

We are given with the polynomial here,

$$p(x) = x^3-3x^2+x+1$$

And zeroes are given as $a-b, a, a+b$

Now, comparing the given polynomial with general expression, we get;

$$\therefore px^3+qx^2+rx+s = x^3-3x^2+x+1$$

$$p = 1, q = -3, r = 1 \text{ and } s = 1$$

$$\text{Sum of zeroes} = a-b + a + a+b$$

$$-q/p = 3a$$

Putting the values q and p .

$$-(-3)/1 = 3a$$

$$a=1$$

Thus, the zeroes are $1-b$, 1 , $1+b$.

Now, product of zeroes = $1(1-b)(1+b)$

$$-s/p = 1-b^2$$

$$-1/1 = 1-b^2$$

$$b^2 = 1+1 = 2$$

$$b = \pm\sqrt{2}$$

Hence, $1-\sqrt{2}$, 1 , $1+\sqrt{2}$ are the zeroes of x^3-3x^2+x+1 .

4. If two zeroes of the polynomial $x^4-6x^3-26x^2+138x-35$ are $2 \pm \sqrt{3}$, find other zeroes.

Solution:

Since this is a polynomial equation of degree 4, hence there will be total 4 roots.

$$\text{Let } f(x) = x^4-6x^3-26x^2+138x-35$$

Since $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are zeroes of given polynomial $f(x)$.

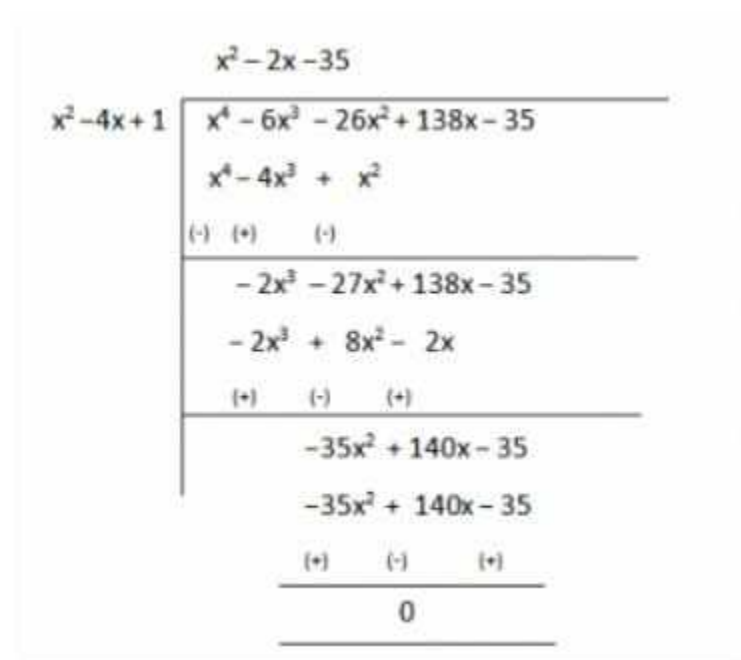
$$\therefore [x-(2+\sqrt{3})][x-(2-\sqrt{3})] = 0$$

$$(x-2-\sqrt{3})(x-2+\sqrt{3}) = 0$$

On multiplying the above equation we get,

x^2-4x+1 , this is a factor of a given polynomial $f(x)$.

Now, if we will divide $f(x)$ by $g(x)$, the quotient will also be a factor of $f(x)$ and the remainder will be 0.



$$\begin{array}{r}
 x^2-2x-35 \\
 x^2-4x+1 \overline{) x^4-6x^3-26x^2+138x-35} \\
 \underline{x^4-4x^3+x^2} \\
 (-) \quad (+) \quad (-) \\
 -2x^3-27x^2+138x-35 \\
 \underline{-2x^3+8x^2-2x} \\
 (+) \quad (-) \quad (+) \\
 -35x^2+140x-35 \\
 \underline{-35x^2+140x-35} \\
 (+) \quad (-) \quad (+) \\
 0
 \end{array}$$

$$\text{So, } x^4-6x^3-26x^2+138x-35 = (x^2-4x+1)(x^2-2x-35)$$

Now, on further factorizing $(x^2 - 2x - 35)$ we get,

$$x^2 - (7+5)x - 35 = x^2 - 7x + 5x + 35 = 0$$

$$x(x - 7) + 5(x - 7) = 0$$

$$(x+5)(x-7) = 0$$

So, its zeroes are given by:

$$x = -5 \text{ and } x = 7.$$

Therefore, all four zeroes of given polynomial equation are: $2+\sqrt{3}$, $2-\sqrt{3}$, -5 and 7 .

Q.5: If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a .

Solution:

Let's divide $x^4 - 6x^3 + 16x^2 - 25x + 10$ by $x^2 - 2x + k$.

$$\begin{array}{r}
 x^2 - 2x + k \overline{) x^4 - 6x^3 + 16x^2 - 25x + 10} \quad (x^2 - 4x + (8 - k)) \\
 \underline{x^4 - 2x^3 + kx^2} \\
 -4x^3 + (16 - k)x^2 - 25x \\
 \underline{-4x^3 + 8x^2 - 4kx} \\
 (8 - k)x^2 + (4k - 25)x + 10 \\
 \underline{(8 - k)x^2 - 2(8 - k)x + k(8 - k)} \\
 (4k - 25 + 16 - 2k)x + [10 - k(8 - k)]
 \end{array}$$

Given that the remainder of the polynomial division is $x + a$.

$$(4k - 25 + 16 - 2k)x + [10 - k(8 - k)] = x + a$$

$$(2k - 9)x + (10 - 8k + k^2) = x + a$$

Comparing the coefficients of the above equation, we get;

$$2k - 9 = 1$$

$$2k = 9 + 1 = 10$$

$$k = 10/2 = 5$$

And

$$10 - 8k + k^2 = a$$

$$10 - 8(5) + (5)^2 = a \text{ [since } k = 5]$$

$$10 - 40 + 25 = a$$

$$a = -5$$

Therefore, $k = 5$ and $a = -5$.

