

**Exercise 4.2**

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**1. In a  $\Delta ABC$ , D and E are points on the sides AB and AC respectively such that  $DE \parallel BC$ .****i) If  $AD = 6$  cm,  $DB = 9$  cm and  $AE = 8$  cm, Find AC.****Solution:**Given:  $\Delta ABC$ ,  $DE \parallel BC$ ,  $AD = 6$  cm,  $DB = 9$  cm and  $AE = 8$  cm.

Required to find AC.

By using Thales Theorem, [As  $DE \parallel BC$ ]

$$AD/BD = AE/CE$$

Let  $CE = x$ .

So then,

$$6/9 = 8/x$$

$$6x = 72 \text{ cm}$$

$$x = 72/6 \text{ cm}$$

$$x = 12 \text{ cm}$$

$$\therefore AC = AE + CE = 12 + 8 = 20.$$

**ii) If  $AD/DB = 3/4$  and  $AC = 15$  cm, Find AE.****Solution:**Given:  $AD/BD = 3/4$  and  $AC = 15$  cm [As  $DE \parallel BC$ ]

Required to find AE.

By using Thales Theorem, [As  $DE \parallel BC$ ]

$$AD/BD = AE/CE$$

Let,  $AE = x$ , then  $CE = 15 - x$ .

$$\Rightarrow \quad 3/4 = x / (15 - x)$$

$$45 - 3x = 4x$$

$$-3x - 4x = -45$$

$$7x = 45$$

$$x = 45/7$$

$$x = 6.43 \text{ cm}$$

$$\therefore AE = 6.43 \text{ cm}$$

**iii) If  $AD/DB = 2/3$  and  $AC = 18$  cm, Find AE.****Solution:**Given:  $AD/BD = 2/3$  and  $AC = 18$  cm

Required to find AE.

By using Thales Theorem, [As  $DE \parallel BC$ ]

$$AD/BD = AE/CE$$

Let,  $AE = x$  and  $CE = 18 - x$ 

$$\Rightarrow \quad 2/3 = x / (18 - x)$$

$$\begin{aligned}3x &= 36 - 2x \\5x &= 36 \text{ cm} \\x &= 36/5 \text{ cm} \\x &= 7.2 \text{ cm} \\\therefore AE &= 7.2 \text{ cm}\end{aligned}$$

**iv) If AD = 4 cm, AE = 8 cm, DB = x - 4 cm and EC = 3x - 19, find x.**

**Solution:**

Given: AD = 4 cm, AE = 8 cm, DB = x - 4 and EC = 3x - 19  
Required to find x.

By using Thales Theorem, [As DE || BC]

$$AD/BD = AE/CE$$

Then,  $4/(x - 4) = 8/(3x - 19)$

$$4(3x - 19) = 8(x - 4)$$

$$12x - 76 = 8(x - 4)$$

$$12x - 8x = -32 + 76$$

$$4x = 44 \text{ cm}$$

$$x = 11 \text{ cm}$$

**v) If AD = 8 cm, AB = 12 cm and AE = 12 cm, find CE.**

**Solution:**

Given: AD = 8 cm, AB = 12 cm, and AE = 12 cm.  
Required to find CE,

By using Thales Theorem, [As DE || BC]

$$AD/BD = AE/CE$$

$$8/4 = 12/CE$$

$$8 \times CE = 4 \times 12 \text{ cm}$$

$$CE = (4 \times 12)/8 \text{ cm}$$

$$CE = 48/8 \text{ cm}$$

$$\therefore CE = 6 \text{ cm}$$

**vi) If AD = 4 cm, DB = 4.5 cm and AE = 8 cm, find AC.**

**Solution:**

Given: AD = 4 cm, DB = 4.5 cm, AE = 8 cm  
Required to find AC.

By using Thales Theorem, [As DE || BC]

$$AD/BD = AE/CE$$

$$4/4.5 = 8/AC$$

$$AC = (4.5 \times 8)/4 \text{ cm}$$

$$\therefore AC = 9 \text{ cm}$$

**vii) If  $AD = 2$  cm,  $AB = 6$  cm and  $AC = 9$  cm, find  $AE$ .**

**Solution:**

Given:  $AD = 2$  cm,  $AB = 6$  cm and  $AC = 9$  cm

Required to find  $AE$ .

$$DB = AB - AD = 6 - 2 = 4 \text{ cm}$$

By using Thales Theorem, [As  $DE \parallel BC$ ]

$$AD/BD = AE/CE$$

$$2/4 = x/(9-x)$$

$$4x = 18 - 2x$$

$$6x = 18$$

$$x = 3 \text{ cm}$$

$$\therefore AE = 3 \text{ cm}$$

**viii) If  $AD/BD = 4/5$  and  $EC = 2.5$  cm, Find  $AE$ .**

**Solution:**

Given:  $AD/BD = 4/5$  and  $EC = 2.5$  cm

Required to find  $AE$ .

By using Thales Theorem, [As  $DE \parallel BC$ ]

$$AD/BD = AE/CE$$

$$\text{Then, } 4/5 = AE/2.5$$

$$\therefore AE = 4 \times 2.5 / 5 = 2 \text{ cm}$$

**ix) If  $AD = x$  cm,  $DB = x - 2$  cm,  $AE = x + 2$  cm, and  $EC = x - 1$  cm, find the value of  $x$ .**

**Solution:**

Given:  $AD = x$ ,  $DB = x - 2$ ,  $AE = x + 2$  and  $EC = x - 1$

Required to find the value of  $x$ .

By using Thales Theorem, [As  $DE \parallel BC$ ]

$$AD/BD = AE/CE$$

$$\text{So, } x/(x-2) = (x+2)/(x-1)$$

$$x(x-1) = (x-2)(x+2)$$

$$x^2 - x - x^2 + 4 = 0$$

$$x = 4$$

**x) If  $AD = 8x - 7$  cm,  $DB = 5x - 3$  cm,  $AE = 4x - 3$  cm, and  $EC = (3x - 1)$  cm, Find the value of  $x$ .**

**Solution:**

Given:  $AD = 8x - 7$ ,  $DB = 5x - 3$ ,  $AE = 4x - 3$  and  $EC = 3x - 1$

Required to find  $x$ .

By using Thales Theorem, [As  $DE \parallel BC$ ]

$$AD/BD = AE/CE$$

$$(8x-7)/(5x-3) = (4x-3)/(3x-1)$$

$$(8x-7)(3x-1) = (5x-3)(4x-3)$$

$$24x^2 - 29x + 7 = 20x^2 - 27x + 9$$

$$4x^2 - 2x - 2 = 0$$

$$2(2x^2 - x - 1) = 0$$

$$2x^2 - x - 1 = 0$$

$$2x^2 - 2x + x - 1 = 0$$

$$2x(x-1) + 1(x-1) = 0$$

$$(x-1)(2x+1) = 0$$

$$\Rightarrow x = 1 \text{ or } x = -1/2$$

We know that the side of triangle can never be negative. Therefore, we take the positive value.

$$\therefore x = 1.$$

**xi) If  $AD = 4x - 3$ ,  $AE = 8x - 7$ ,  $BD = 3x - 1$ , and  $CE = 5x - 3$ , find the value of  $x$ .**

**Solution:**

Given:  $AD = 4x - 3$ ,  $BD = 3x - 1$ ,  $AE = 8x - 7$  and  $EC = 5x - 3$

Required to find  $x$ .

By using Thales Theorem, [As  $DE \parallel BC$ ]

$$AD/BD = AE/CE$$

$$\text{So, } (4x-3)/(3x-1) = (8x-7)/(5x-3)$$

$$(4x-3)(5x-3) = (3x-1)(8x-7)$$

$$4x(5x-3) - 3(5x-3) = 3x(8x-7) - 1(8x-7)$$

$$20x^2 - 12x - 15x + 9 = 24x^2 - 29x + 7$$

$$20x^2 - 27x + 9 = 24x^2 - 29x + 7$$

$$\Rightarrow -4x^2 + 2x + 2 = 0$$

$$4x^2 - 2x - 2 = 0$$

$$4x^2 - 4x + 2x - 2 = 0$$

$$4x(x-1) + 2(x-1) = 0$$

$$(4x+2)(x-1) = 0$$

$$\Rightarrow x = 1 \text{ or } x = -2/4$$

We know that the side of triangle can never be negative. Therefore, we take the positive value.

$$\therefore x = 1$$

**xii) If  $AD = 2.5$  cm,  $BD = 3.0$  cm, and  $AE = 3.75$  cm, find the length of  $AC$ .**

**Solution:**

Given:  $AD = 2.5$  cm,  $AE = 3.75$  cm and  $BD = 3$  cm

Required to find  $AC$ .

By using Thales Theorem, [As  $DE \parallel BC$ ]

$$AD/BD = AE/CE$$

$$\begin{aligned}2.5/3 &= 3.75/CE \\2.5 \times CE &= 3.75 \times 3 \\CE &= 3.75 \times 32.5 \\CE &= 11.252.5 \\CE &= 4.5\end{aligned}$$

$$\begin{aligned}\text{Now, } AC &= 3.75 + 4.5 \\ \therefore AC &= 8.25 \text{ cm.}\end{aligned}$$

**2. In a  $\triangle ABC$ , D and E are points on the sides AB and AC respectively. For each of the following cases show that  $DE \parallel BC$ :**

**i)  $AB = 12$  cm,  $AD = 8$  cm,  $AE = 12$  cm, and  $AC = 18$  cm.**

**Solution:**

Required to prove  $DE \parallel BC$ .

We have,

$AB = 12$  cm,  $AD = 8$  cm,  $AE = 12$  cm, and  $AC = 18$  cm. (Given)

So,

$$BD = AB - AD = 12 - 8 = 4 \text{ cm}$$

And,

$$CE = AC - AE = 18 - 12 = 6 \text{ cm}$$

It's seen that,

$$AD/BD = 8/4 = 1/2$$

$$AE/CE = 12/6 = 1/2$$

Thus,

$$AD/BD = AE/CE$$

So, by the converse of Thale's Theorem

We have,

$$DE \parallel BC.$$

Hence Proved.

**ii)  $AB = 5.6$  cm,  $AD = 1.4$  cm,  $AC = 7.2$  cm, and  $AE = 1.8$  cm.**

**Solution:**

Required to prove  $DE \parallel BC$ .

We have,

$AB = 5.6$  cm,  $AD = 1.4$  cm,  $AC = 7.2$  cm, and  $AE = 1.8$  cm. (Given)

So,

$$BD = AB - AD = 5.6 - 1.4 = 4.2 \text{ cm}$$

And,

$$CE = AC - AE = 7.2 - 1.8 = 5.4 \text{ cm}$$

It's seen that,

$$AD/BD = 1.4/4.2 = 1/3$$

$$AE/CE = 1.8/5.4 = 1/3$$

Thus,

$$AD/BD = AE/CE$$

So, by the converse of Thale's Theorem

We have,

$$DE \parallel BC.$$

Hence Proved.

**iii)  $AB = 10.8$  cm,  $BD = 4.5$  cm,  $AC = 4.8$  cm, and  $AE = 2.8$  cm.**

**Solution:**

Required to prove  $DE \parallel BC$ .

We have

$AB = 10.8$  cm,  $BD = 4.5$  cm,  $AC = 4.8$  cm, and  $AE = 2.8$  cm.

So,

$$AD = AB - DB = 10.8 - 4.5 = 6.3$$

And,

$$CE = AC - AE = 4.8 - 2.8 = 2$$

It's seen that,

$$AD/BD = 6.3/4.5 = 2.8/2.0 = AE/CE = 7/5$$

So, by the converse of Thale's Theorem

We have,

$$DE \parallel BC.$$

Hence Proved.

**iv)  $AD = 5.7$  cm,  $BD = 9.5$  cm,  $AE = 3.3$  cm, and  $EC = 5.5$  cm.**

**Solution:**

Required to prove  $DE \parallel BC$ .

We have

$AD = 5.7$  cm,  $BD = 9.5$  cm,  $AE = 3.3$  cm, and  $EC = 5.5$  cm

Now,

$$AD/BD = 5.7/9.5 = 3/5$$

And,

$$AE/CE = 3.3/5.5 = 3/5$$

Thus,

$$AD/BD = AE/CE$$

So, by the converse of Thale's Theorem

We have,

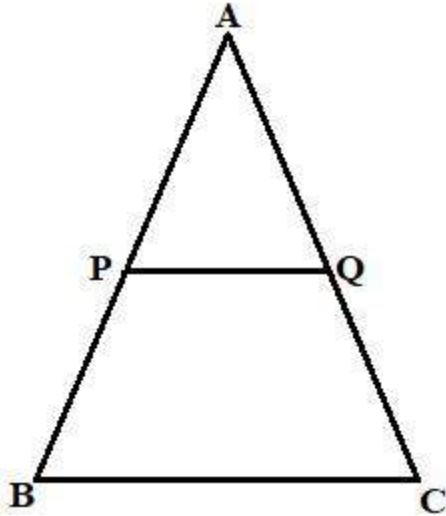
$$DE \parallel BC.$$

Hence Proved.

**3. In a  $\triangle ABC$ , P and Q are the points on sides AB and AC respectively, such that  $PQ \parallel BC$ . If  $AP = 2.4$  cm,  $AQ = 2$  cm,  $QC = 3$  cm and  $BC = 6$  cm. Find AB and PQ.**

## Solution:

Given:  $\triangle ABC$ ,  $AP = 2.4$  cm,  $AQ = 2$  cm,  $QC = 3$  cm, and  $BC = 6$  cm. Also,  $PQ \parallel BC$ .  
Required to find:  $AB$  and  $PQ$ .



By using Thales Theorem, we have [As it's given that  $PQ \parallel BC$ ]

$$\begin{aligned} \frac{AP}{PB} &= \frac{AQ}{QC} \\ \frac{2.4}{PB} &= \frac{2}{3} \\ 2 \times PB &= 2.4 \times 3 \\ PB &= \frac{(2.4 \times 3)}{2} \text{ cm} \\ \Rightarrow PB &= 3.6 \text{ cm} \end{aligned}$$

Now finding,  $AB = AP + PB$

$$\begin{aligned} AB &= 2.4 + 3.6 \\ \Rightarrow AB &= 6 \text{ cm} \end{aligned}$$

Now, considering  $\triangle APQ$  and  $\triangle ABC$

We have,

$$\angle A = \angle A$$

$$\angle APQ = \angle ABC \text{ (Corresponding angles are equal, } PQ \parallel BC \text{ and } AB \text{ being a transversal)}$$

Thus,  $\triangle APQ$  and  $\triangle ABC$  are similar to each other by AA criteria.

Now, we know that

Corresponding parts of similar triangles are proportional.

$$\begin{aligned} \Rightarrow \frac{AP}{AB} &= \frac{PQ}{BC} \\ \Rightarrow PQ &= \left(\frac{AP}{AB}\right) \times BC \\ &= \left(\frac{2.4}{6}\right) \times 6 = 2.4 \end{aligned}$$

$$\therefore PQ = 2.4 \text{ cm.}$$

**4. In a  $\triangle ABC$ , D and E are points on AB and AC respectively, such that  $DE \parallel BC$ . If  $AD = 2.4$  cm,  $AE = 3.2$  cm,  $DE = 2$  cm and  $BC = 5$  cm. Find BD and CE.**

**Solution:**



Given:  $\triangle ABC$  such that  $AD = 2.4$  cm,  $AE = 3.2$  cm,  $DE = 2$  cm and  $BE = 5$  cm. Also  $DE \parallel BC$ .  
Required to find:  $BD$  and  $CE$ .

As  $DE \parallel BC$ ,  $AB$  is transversal,  
 $\angle APQ = \angle ABC$  (corresponding angles)

As  $DE \parallel BC$ ,  $AC$  is transversal,  
 $\angle AED = \angle ACB$  (corresponding angles)

In  $\triangle ADE$  and  $\triangle ABC$ ,  
 $\angle ADE = \angle ABC$   
 $\angle AED = \angle ACB$   
 $\therefore \triangle ADE = \triangle ABC$  (AA similarity criteria)

Now, we know that  
Corresponding parts of similar triangles are proportional.

$$\begin{aligned}\Rightarrow \quad AD/AB &= AE/AC = DE/BC \\ AD/AB &= DE/BC \\ 2.4/(2.4 + DB) &= 2/5 \text{ [Since, } AB = AD + DB\text{]} \\ 2.4 + DB &= 6 \\ DB &= 6 - 2.4 \\ DB &= 3.6 \text{ cm}\end{aligned}$$

In the same way,

$$\begin{aligned}\Rightarrow \quad AE/AC &= DE/BC \\ 3.2/(3.2 + EC) &= 2/5 \text{ [Since } AC = AE + EC\text{]} \\ 3.2 + EC &= 8 \\ EC &= 8 - 3.2 \\ EC &= 4.8 \text{ cm}\end{aligned}$$

$\therefore BD = 3.6$  cm and  $CE = 4.8$  cm.