Exercise 4.5 Page No: 4.37

1. In fig. 4.136, ΔACB ~ ΔAPQ. If BC = 8 cm, PQ = 4 cm, BA = 6.5 cm and AP = 2.8 cm, find CA and AQ.

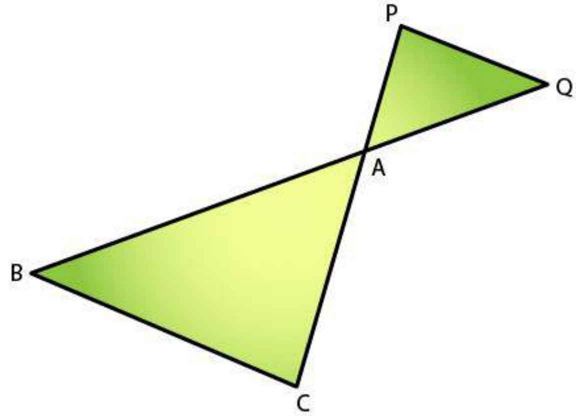
Solution:

Given,

ΔACB ~ ΔAPQ

BC = 8 cm, PQ = 4 cm, BA = 6.5 cm and AP = 2.8 cm

Required to find: CA and AQ



We know that,

ΔACB ~ ΔAPQ [given]

BA/ AQ = CA/ AP = BC/ PQ [Corresponding Parts of Similar Triangles]

So,

6.5/ AQ = 8/ 4

AQ = (6.5 x 4)/ 8

AQ = 3.25 cm

Similarly, as

CA/ AP = BC/ PQ

CA/ 2.8 = 8/ 4

CA = 2.8 x 2

CA = 5.6 cm

Hence, CA = 5.6 cm and AQ = 3.25 cm.

2. In fig.4.137, AB ∥ QR**, find the length of PB.**

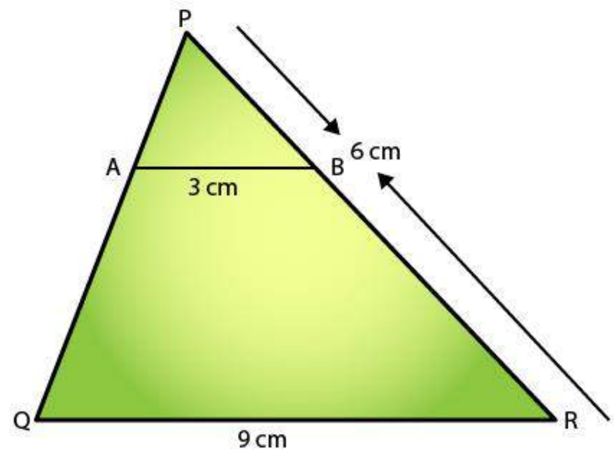
Solution:

Given,

ΔPQR, AB ∥ QR and

AB = 3 cm, QR = 9 cm and PR = 6 cm

Required to find: PB



In ΔPAB and ΔPQR

We have,

∠P = ∠P	[Common]
∠PAB = ∠PQR	[Corresponding angles as AB‖QR with PQ as the transversal]
⇒ ΔPAB ∼ ΔPQR	[By AA similarity criteria]

Hence,

AB/ QR = PB/ PR [Corresponding Parts of Similar Triangles are propositional]

⇒ 3/ 9 = PB/6

PB = 6/3

Therefore, PB = 2 cm

3. In fig. 4.138 given, XY‖BC**. Find the length of XY.**

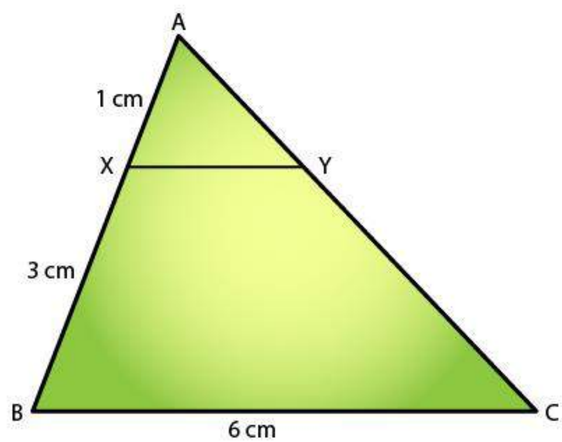
Solution:

Given,

XY‖BC

AX = 1 cm, XB = 3 cm and BC = 6 cm

Required to find: XY



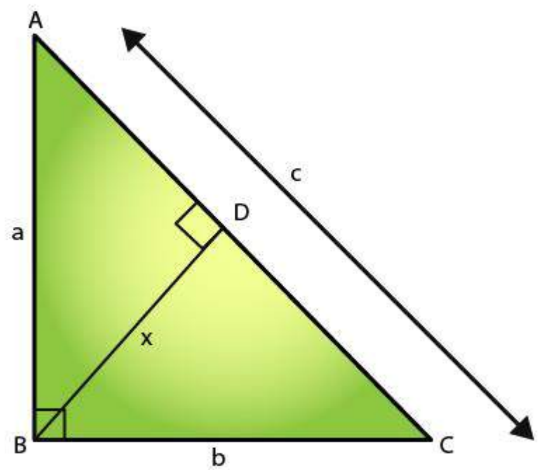
In ΔAXY and ΔABC
We have,
∠A = ∠A [Common]
∠AXY = ∠ABC [Corresponding angles as AB||QR with PQ as the transversal]
⇒ ΔAXY ~ ΔABC [By AA similarity criteria]

Hence,
XY/ BC = AX/ AB [Corresponding Parts of Similar Triangles are propositional]
We know that,
(AB = AX + XB = 1 + 3 = 4)
XY/6 = 1/4
XY/1 = 6/4

Therefore, XY = 1.5 cm

4. In a right-angled triangle with sides a and b and hypotenuse c, the altitude drawn on the hypotenuse is x. Prove that ab = cx.
Solution:

Consider ΔABC to be a right angle triangle having sides a and b and hypotenuse c. Let BD be the altitude drawn on the hypotenuse AC.
Required to prove: ab = cx



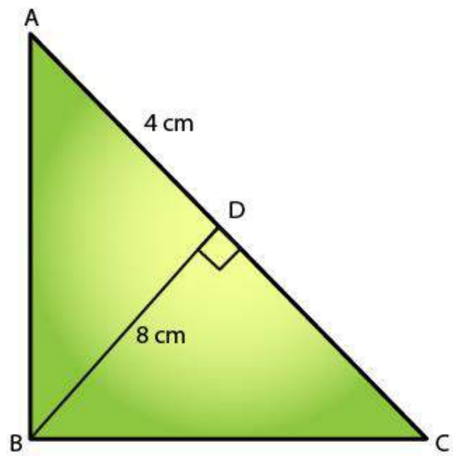
We know that,
In ΔACB and ΔCDB
∠B = ∠B [Common]
∠ACB = ∠CDB = 90°
⇒ ΔACB ~ ΔCDB [By AA similarity criteria]

Hence,
AB/ BD = AC/ BC [Corresponding Parts of Similar Triangles are propositional]
a/ x = c/ b
⇒ xc = ab

Therefore, ab = cx

5. In fig. 4.139, ∠ABC = 90 and BD⊥AC. If BD = 8 cm, and AD = 4 cm, find CD.
Solution:

Given,
∠ABC = 90º and BD⊥AC
BD = 8 cm
AD = 4 cm
Required to find: CD.

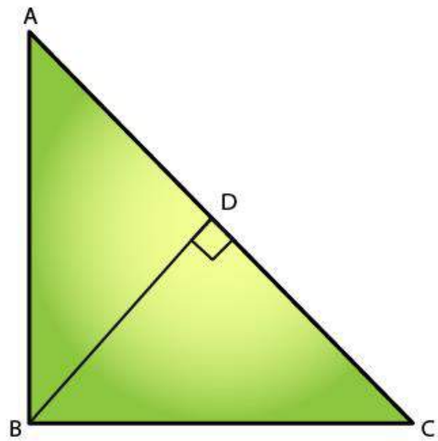


We know that,
ABC is a right angled triangle and BD⊥AC.
Then, ΔDBA~ΔDCB [By AA similarity]
BD/ CD = AD/ BD
BD^2 = AD x DC
$(8)^2$ = 4 x DC
DC = 64/4 = 16 cm

Therefore, CD = 16 cm

6. In fig.4.140, ∠ABC = 90º and BD ⊥ AC. If AC = 5.7 cm, BD = 3.8 cm and CD = 5.4 cm, Find BC.
Solution:

Given:
BD ⊥ AC
AC = 5.7 cm, BD = 3.8 cm and CD = 5.4 cm
∠ABC = 90º
Required to find: BC



We know that,

$\Delta ABC \sim \Delta BDC$ [By AA similarity]

$\angle BCA = \angle DCA = 90^o$

$\angle AXY = \angle ABC$ [Common]

Thus,

AB/ BD = BC/ CD [Corresponding Parts of Similar Triangles are propositional]

5.7/ 3.8 = BC/ 5.4

BC = (5.7 x 5.4)/ 3.8 = 8.1

Therefore, BC = 8.1 cm

7. In the fig.4.141 given, DE ∥ BC **such that AE = (1/4)AC. If AB = 6 cm, find AD.**
Solution:

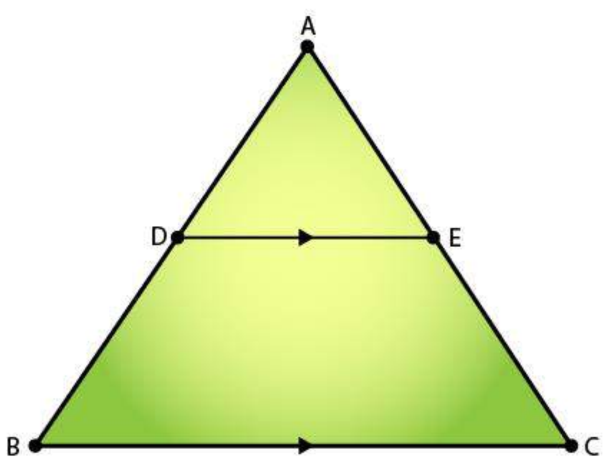
Given:

DE∥BC

AE = (1/4)AC

AB = 6 cm.

Required to find: AD.



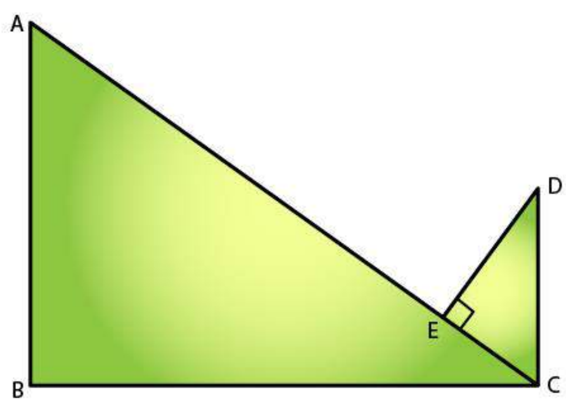
In ΔADE and ΔABC
We have,
∠A = ∠A [Common]
∠ADE = ∠ABC [Corresponding angles as AB||QR with PQ as the transversal]
⇒ ΔADE ~ ΔABC [By AA similarity criteria]

Then,
AD/AB = AE/ AC [Corresponding Parts of Similar Triangles are propositional]
AD/6 = 1/4
4 x AD = 6
AD = 6/4

Therefore, AD = 1.5 cm

8. In the fig.4.142 given, if AB ⊥ BC, DC ⊥ BC, and DE ⊥ AC, prove that ΔCED ~ ΔABC
Solution:

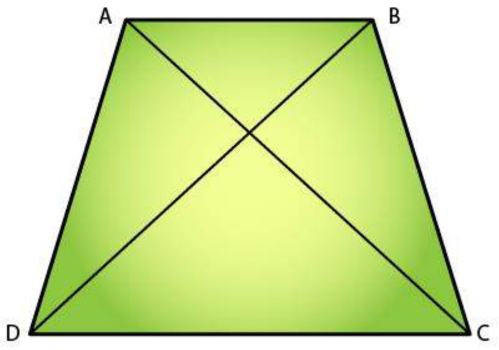
Given:
AB ⊥ BC,
DC ⊥ BC,
DE ⊥ AC
Required to prove: ΔCED~ΔABC



We know that,
From ΔABC and ΔCED
∠B = ∠E = 90º [given]
∠BAC = ∠ECD [alternate angles since, AB || CD with BC as transversal]
Therefore, ΔCED~ΔABC [AA similarity]

9. Diagonals AC and BD of a trapezium ABCD with AB ‖ DC **intersect each other at the point O. Using similarity criterion for two triangles, show that** OA/ OC = OB/ OD
Solution:

Given: OC is the point of intersection of AC and BD in the trapezium ABCD, with AB ‖ DC.
Required to prove: OA/ OC = OB/ OD



We know that,

In ΔAOB and ΔCOD

$\angle AOB = \angle COD$ [Vertically Opposite Angles]

$\angle OAB = \angle OCD$ [Alternate angles]

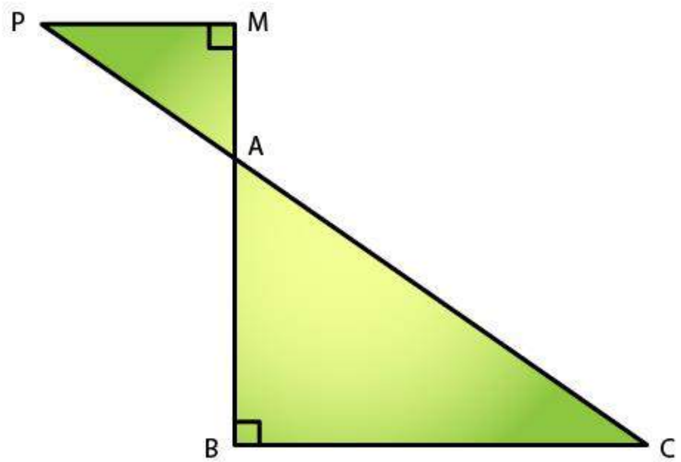
Then, $\Delta AOB \sim \Delta COD$

Therefore, OA/ OC = OB/ OD [Corresponding sides are proportional]

10. If Δ ABC and Δ AMP are two right triangles, right angled at B and M, respectively such that ∠MAP = ∠BAC. Prove that
(i) ΔABC ~ ΔAMP
(ii) CA/ PA = BC/ MP
Solution:

(i) Given:

Δ ABC and Δ AMP are the two right triangles.



We know that,

$\angle AMP = \angle B = 90^o$

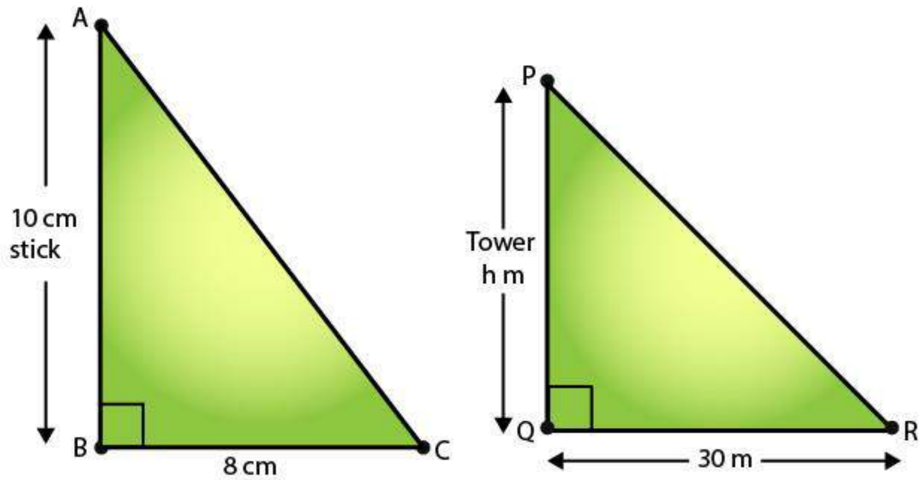
$\angle MAP = \angle BAC$ [Vertically Opposite Angles]

⇒ $\Delta ABC \sim \Delta AMP$ [AA similarity]

(ii) Since, $\Delta ABC \sim \Delta AMP$
$CA/PA = BC/MP$ [Corresponding sides are proportional]
Hence proved.

11. A vertical stick 10 cm long casts a shadow 8 cm long. At the same time, a tower casts a shadow 30 m long. Determine the height of the tower.
Solution:

Given:
Length of stick = 10cm
Length of the stick's shadow = 8cm
Length of the tower's shadow = 30m = 3000cm
Required to find: the height of the tower = PQ.



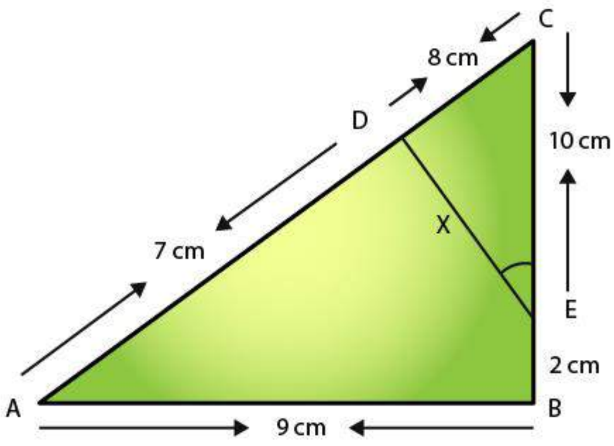
In $\Delta ABC \sim \Delta PQR$
$\angle ABC = \angle PQR = 90^o$
$\angle ACB = \angle PRQ$ [Angular Elevation of Sun is same for a particular instant of time]
⇒ $\Delta ABC \sim \Delta PQR$ [By AA similarity]

So, we have
$AB/BC = PQ/QR$ [Corresponding sides are proportional]
$10/8 = PQ/3000$
$PQ = (3000 \times 10)/8$
$PQ = 30000/8$
$PQ = 3750100$

Therefore, PQ = 37.5 m

12. In fig.4.143, ∠A = ∠CED, prove that ΔCAB ~ ΔCED. Also find the value of x.
Solution:

Given:
∠A = ∠CED
Required to prove: ΔCAB ~ ΔCED



In ΔCAB ~ ΔCED
∠C = ∠C [Common]
∠A = ∠CED [Given]

⇒ ΔCAB ~ ΔCED [By AA similarity]

Hence, we have
CA/ CE = AB/ ED [Corresponding sides are proportional]
15/10 = 9/x
x = (9 x 10)/ 15

Therefore, x = 6 cm