Exercise 4.7 Page No: 4.119

1. If the sides of a triangle are 3 cm, 4 cm, and 6 cm long, determine whether the triangle is a right-angled triangle.
Solution:

We have,
Sides of triangle as
AB = 3 cm
BC = 4 cm
AC = 6 cm
On finding their squares, we get
$AB^2 = 3^2 = 9$
$BC^2 = 4^2 = 16$
$AC^2 = 6^2 = 36$

Since, $AB^2 + BC^2 \neq AC^2$
So, by converse of Pythagoras theorem the given sides cannot be the sides of a right triangle.

2. The sides of certain triangles are given below. Determine which of them are right triangles.
(i) a = 7 cm, b = 24 cm and c = 25 cm
(ii) a = 9 cm, b = 16 cm and c = 18 cm
(iii) a = 1.6 cm, b = 3.8 cm and c = 4 cm
(iv) a = 8 cm, b = 10 cm and c = 6 cm
Solutions:

(i) Given,
a = 7 cm, b = 24 cm and c = 25 cm
$\therefore a^2 = 49$, $b^2 = 576$ and $c^2 = 625$
Since, $a^2 + b^2 = 49 + 576 = 625 = c^2$
Then, by converse of Pythagoras theorem
The given sides are of a right triangle.

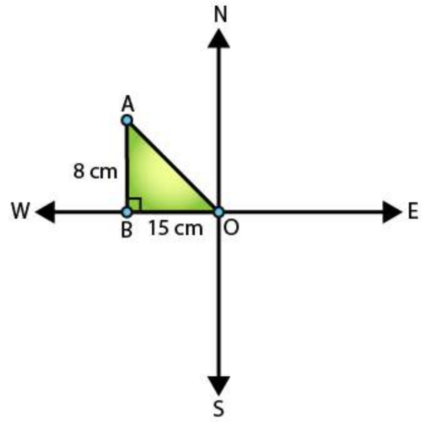
(ii) Given,
a = 9 cm, b = 16 cm and c = 18 cm
$\therefore a^2 = 81$, $b^2 = 256$ and $c^2 = 324$
Since, $a^2 + b^2 = 81 + 256 = 337 \neq c^2$
Then, by converse of Pythagoras theorem
The given sides cannot be of a right triangle.

(iii) Given,
a = 1.6 cm, b = 3.8 cm and C = 4 cm
$\therefore a^2 = 2.56$, $b^2 = 14.44$ and $c^2 = 16$
Since, $a^2 + b^2 = 2.56 + 14.44 = 17 \neq c^2$
Then, by converse of Pythagoras theorem
The given sides cannot be of a right triangle.

(iv) Given,
$a = 8$ cm, $b = 10$ cm and $C = 6$ cm
$\therefore a^2 = 64$, $b^2 = 100$ and $c^2 = 36$
Since, $a^2 + c^2 = 64 + 36 = 100 = b^2$
Then, by converse of Pythagoras theorem
The given sides are of a right triangle

3. A man goes 15 metres due west and then 8 metres due north. How far is he from the starting point?
Solution:

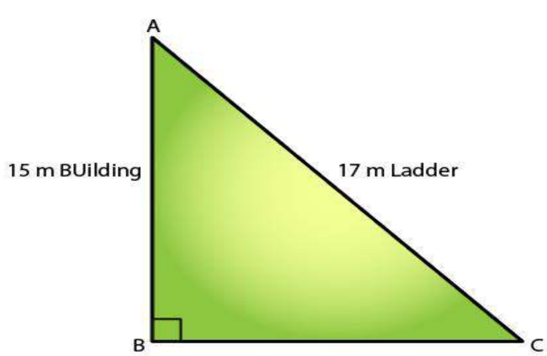
Let the starting point of the man be O and final point be A.



In ΔABO,
by Pythagoras theorem $AO^2 = AB^2 + BO^2$
$\Rightarrow \quad AO^2 = 8^2 + 15^2$
$\Rightarrow \quad AO^2 = 64 + 225 = 289$
$\Rightarrow \quad AO = \sqrt{289} = 17$m

$\therefore$ the man is 17m far from the starting point.

4. A ladder 17 m long reaches a window of a building 15 m above the ground. Find the distance of the foot of the ladder from the building.
Solution:

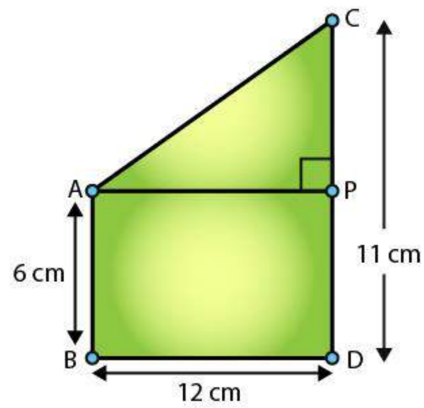


In ΔABC, by Pythagoras theorem
$AB^2 + BC^2 = AC^2$
$\Rightarrow \quad 15^2 + BC^2 = 17^2$
$225 + BC^2 = 17^2$
$BC^2 = 289 - 225$
$BC^2 = 64$
$\therefore BC = 8$ m

Therefore, the distance of the foot of the ladder from building = 8 m

5. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.
Solution:

Let CD and AB be the poles of height 11m and 6m.
Then, its seen that CP = 11 - 6 = 5m.
From the figure, AP should be 12m (given)

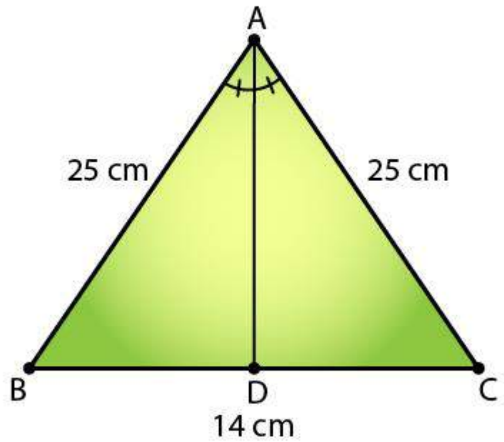


In triangle APC, by applying Pythagoras theorem, we have
$AP^2 + PC^2 = AC^2$
$12^2 + 5^2 = AC^2$
$AC^2 = 144 + 25 = 169$
$\therefore AC = 13$ (by taking sq. root on both sides)

Thus, the distance between their tops = 13 m.

6. In an isosceles triangle ABC, AB = AC = 25 cm, BC = 14 cm. Calculate the altitude from A on BC.
Solution:

Given,
ΔABC, AB = AC = 25 cm and BC = 14.



In ΔABD and ΔACD, we see that

$\angle ADB = \angle ADC$ [Each = 90°]

$AB = AC$ [Given]

$AD = AD$ [Common]

Then, $\Delta ABD \cong \Delta ACD$ [By RHS condition]

Thus, $BD = CD = 7$ cm [By corresponding parts of congruent triangles]

Finally,

In ΔADB, by Pythagoras theorem

$AD^2 + BD^2 = AB^2$

$\Rightarrow$ $AD^2 + 7^2 = 25^2$

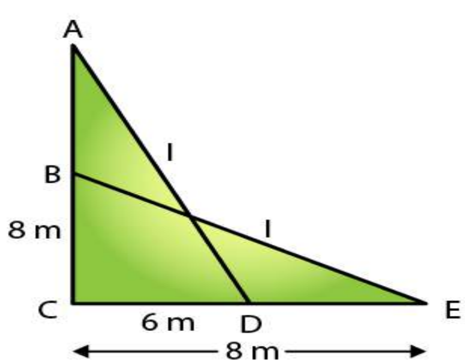
$AD^2 = 625 - 49 = 576$

$\therefore AD = \sqrt{576} = 24$ cm

7. The foot of a ladder is 6 m away from a wall and its top reaches a window 8 m above the ground. If the ladder is shifted in such a way that its foot is 8 m away from the wall, to what height does its tip reach?

Solution:

Let's assume the length of ladder to be, AD = BE = x m



So, in ΔACD, by Pythagoras theorem
We have,

$AD^2 = AC^2 + CD^2$

$\Rightarrow x^2 = 8^2 + 6^2$ ... (i)

Also, in ΔBCE, by Pythagoras theorem

$BE^2 = BC^2 + CE^2$

$\Rightarrow x^2 = BC^2 + 8^2$ ... (ii)

Compare (i) and (ii)

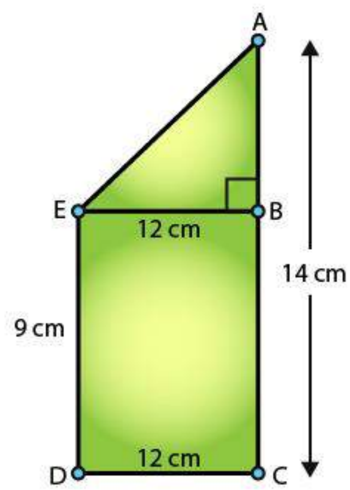
$BC^2 + 8^2 = 8^2 + 6^2$

$\Rightarrow$ $BC^2 + 6^2$

$\Rightarrow$ $BC = 6$ m

Therefore, the tip of the ladder reaches to a height od 6m.

8. Two poles of height 9 in and 14 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.
Solution:



Comparing with the figure, it's given that
AC = 14 m, DC = 12m and ED = BC = 9 m

Construction: Draw EB ⊥ AC
Now,
It's seen that AB = AC – BC = (14 – 9) = 5 m
And, EB = DC = 12m [distance between their feet]
Thus,
In ΔABE, by Pythagoras theorem, we have

$AE^2 = AB^2 + BE^2$

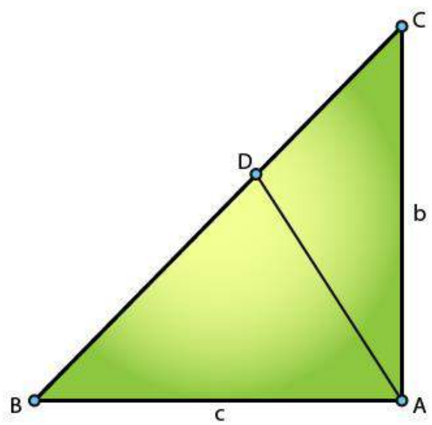
$AE^2 = 5^2 + 12^2$

$AE^2 = 25 + 144 = 169$

⇒ $AE = \sqrt{169} = 13$ m

Therefore, the distance between their tops = 13 m

9. Using Pythagoras theorem determine the length of AD in terms of b and c shown in Fig. 4.219
Solution:



We have,
In ΔBAC, by Pythagoras theorem, we have

$BC^2 = AB^2 + AC^2$

⇒ $BC^2 = c^2 + b^2$

⇒ $BC = \sqrt{(c^2 + b^2)}$

In ΔABD and ΔCBA

∠B = ∠B [Common]

∠ADB = ∠BAC [Each 90°]

Then, ΔABD ~ ΔCBA [By AA similarity]

Thus,

AB/ CB = AD/ CA [Corresponding parts of similar triangles are proportional]

$c/ \sqrt{(c^2 + b^2)} = AD/ b$

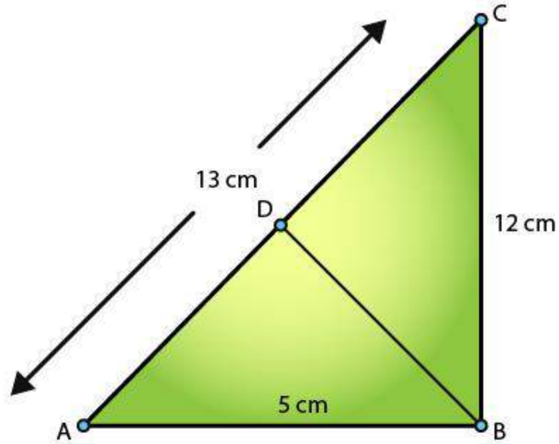
∴ $AD = bc/ \sqrt{(c^2 + b^2)}$

10. A triangle has sides 5 cm, 12 cm and 13 cm. Find the length to one decimal place, of the perpendicular from the opposite vertex to the side whose length is 13 cm.
Solution:

From the fig. AB = 5cm, BC = 12 cm and AC = 13 cm.
Then, $AC^2 = AB^2 + BC^2$.

⇒ $(13)^2 = (5)^2 + (12)^2 = 25 + 144 = 169 = 13^2$

This proves that ΔABC is a right triangle, right angled at B.
Let BD be the length of perpendicular from B on AC.



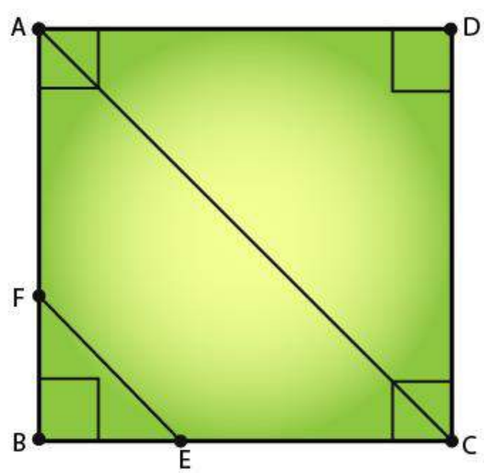
So, area of ΔABC = (BC x BA)/ 2 [Taking BC as the altitude]
= (12 x 5)/ 2
= 30 cm^2

Also, area of ΔABC = (AC x BD)/ 2 [Taking BD as the altitude]
= (13 x BD)/ 2

$\Rightarrow$ (13 x BD)/ 2 = 30
BD = 60/13 = 4.6 (to one decimal place)

11. ABCD is a square. F is the mid-point of AB. BE is one third of BC. If the area of Δ FBE = 108cm^2, find the length of AC.
Solution:

Given,
ABCD is a square. And, F is the mid-point of AB.
BE is one third of BC.
Area of Δ FBE = 108cm^2
Required to find: length of AC



Let's assume the sides of the square to be x.

⇒ $AB = BC = CD = DA = x$ cm

And, $AF = FB = x/2$ cm

So, $BE = x/3$ cm

Now, the area of Δ FBE = 1/2 x BE x FB

⇒ $108 = (1/2) \times (x/3) \times (x/2)$

⇒ $x^2 = 108 \times 2 \times 3 \times 2 = 1296$

⇒ $x = \sqrt{(1296)}$ [taking square roots of both the sides]

$\therefore x = 36$cm

Further in Δ ABC, by Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

⇒ $AC^2 = x^2 + x^2 = 2x^2$

⇒ $AC^2 = 2 \times (36)^2$

⇒ $AC = 36\sqrt{2} = 36 \times 1.414 = 50.904$ cm

Therefore, the length of AC is 50.904 cm.

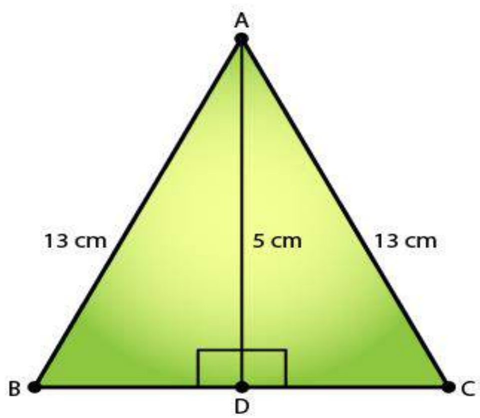
12. In an isosceles triangle ABC, if AB = AC = 13cm and the altitude from A on BC is 5cm, find BC.

Solution:

Given,

An isosceles triangle ABC, AB = AC = 13cm, AD = 5cm

Required to find: BC



In Δ ADB, by using Pythagoras theorem, we have

$AD^2 + BD^2 = 13^2$

$5^2 + BD^2 = 169$

$BD^2 = 169 – 25 = 144$

⇒$BD = \sqrt{144} = 12$ cm

Similarly, applying Pythagoras theorem is Δ ADC we can have,

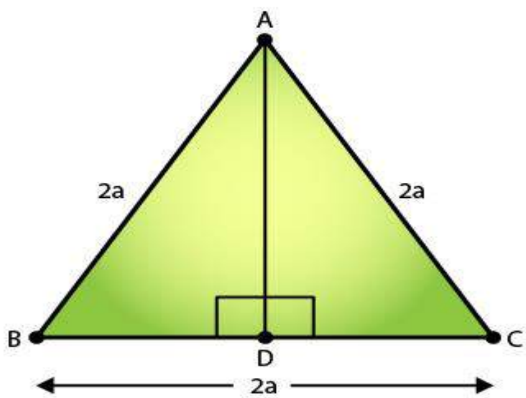
$AC^2 = AD^2 + DC^2$

$13^2 = 5^2 + DC^2$

⇒ $DC = \sqrt{144} = 12$ cm

Thus, BC = BD + DC = 12 + 12 = 24 cm

13. In a ΔABC, AB = BC = CA = 2a and AD ⊥ BC. Prove that
(i) AD = a√3 (ii) Area (ΔABC) = √3 a^2
Solution:



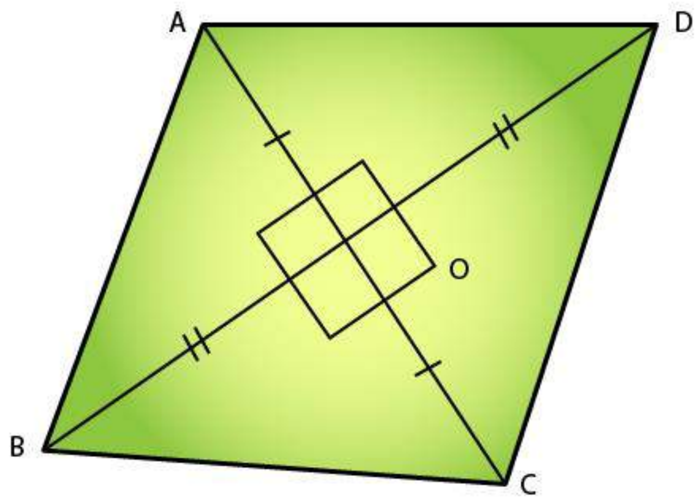
(i) In ΔABD and ΔACD, we have
∠ADB = ∠ADC = 90°
AB = AC [Given]
AD = AD [Common]
So, ΔABD ≅ ΔACD [By RHS condition]
Hence, BD = CD = a [By C.P.C.T]

Now, in ΔABD, by Pythagoras theorem
$AD^2 + BD^2 = AB^2$
$AD^2 + a^2 = 2a^2$
$AD^2 = 4a^2 – a^2 = 3a^2$
AD = a√3

(ii) Area (ΔABC) = 1/2 x BC x AD
= 1/2 x (2a) x (a√3)
= √3 a^2

14. The lengths of the diagonals of a rhombus is 24cm and 10cm. Find each side of the rhombus.
Solution:

Let ABCD be a rhombus and AC and BD be the diagonals of ABCD.
So, AC = 24cm and BD = 10cm



We know that diagonals of a rhombus bisect each other at right angle. (Perpendicular to each other)
So,
AO = OC = 12cm and BO = OD = 3cm
In ΔAOB, by Pythagoras theorem, we have

$AB^2 = AO^2 + BO^2$
$= 12^2 + 5^2$
$= 144 + 25$
$= 169$

$\Rightarrow AB = \sqrt{169} = 13cm$

Since, the sides of rhombus are all equal.
Therefore, AB = BC = CD = AD = 13cm.