

**Exercise 3.8****Page No: 3.88**

**1. The numerator of a fraction is 4 less than the denominator. If the numerator is decreased by 2 and denominator is increased by 1, then the denominator is eight times the numerator. Find the fraction.**

**Solution:**

Let's assume the numerator of the fraction to be  $x$  and the denominator of the fraction to be  $y$ .

So, the required fraction is  $x/y$ .

From the question it's given as,

The numerator of the fraction is 4 less the denominator.

Thus, the equation so formed is,

$$\begin{aligned}x &= y - 4 \\ \Rightarrow x - y &= -4 \dots\dots (i)\end{aligned}$$

And also it's given in the question as,

If the numerator is decreased by 2 and denominator is increased by 1, then the denominator is 8 times the numerator.

Putting the above condition in an equation, we get

$$\begin{aligned}y + 1 &= 8(x - 2) \\ \Rightarrow y + 1 &= 8x - 16 \\ \Rightarrow 8x - y &= 1 + 16 \\ \Rightarrow 8x - y &= 17 \dots\dots (ii)\end{aligned}$$

Solving (i) and (ii),

Subtracting the equation (ii) from (i), we get

$$\begin{aligned}(x - y) - (8x - y) &= -4 - 17 \\ \Rightarrow x - y - 8x + y &= -21 \\ \Rightarrow -7x &= -21 \\ \Rightarrow x &= 21/7 \\ \Rightarrow x &= 3\end{aligned}$$

Substituting the value of  $x = 3$  in the equation (i), we find  $y$

$$\begin{aligned}3 - y &= -4 \\ \Rightarrow y &= 3 + 4 \\ \Rightarrow y &= 7\end{aligned}$$

Therefore, the fraction is  $3/7$ .

**2. A fraction becomes  $9/11$  if 2 is added to both numerator and the denominator. If 3 is added to both the numerator and the denominator it becomes  $5/6$ . Find the fraction.**

**Solution:**

Let's assume the numerator of the fraction to be  $x$  and the denominator of the fraction to be  $y$ .

So, the required fraction is  $x/y$ .

From the question it's given as,

If 2 is added to both numerator and the denominator, the fraction becomes  $\frac{9}{11}$ .

Thus, the equation so formed is,

$$\begin{aligned}x + 2y + 2 &= \frac{9}{11} \\ \Rightarrow 11(x+2) &= 9(y+2) \\ \Rightarrow 11x + 22 &= 9y + 18 \\ \Rightarrow 11x - 9y &= 18 - 22 \\ \Rightarrow 11x - 9y + 4 &= 0 \dots\dots (i)\end{aligned}$$

And also it's given in the question as,

If 3 is added to both numerator and the denominator, the fraction becomes  $\frac{5}{6}$ ,

Expressing the above condition in an equation, we have

$$\begin{aligned}x + 3y + 3 &= \frac{5}{6} \\ \Rightarrow 6(x+3) &= 5(y+3) \\ \Rightarrow 6x + 18 &= 5y + 15 \\ \Rightarrow 6x - 5y &= 15 - 18 \\ \Rightarrow 6x - 5y + 3 &= 0 \dots\dots (ii)\end{aligned}$$

Solving (i) and (ii), to find the fraction

By using cross-multiplication method, we have

$$\begin{aligned}\frac{x}{-9 \times 3 - (-5) \times 4} &= \frac{-y}{11 \times 3 - 6 \times 4} = \frac{1}{11 \times (-5) - 6 \times (-9)} \\ \Rightarrow \frac{x}{-27 + 20} &= \Rightarrow \frac{-y}{33 - 24} = \frac{1}{-55 + 54} \\ \Rightarrow \frac{x}{-7} &= \frac{-y}{9} = \frac{1}{-1} \\ \Rightarrow \frac{x}{7} &= \frac{y}{9} = 1\end{aligned}$$

$$x = 7, y = 9$$

Hence, the required fraction is  $\frac{7}{9}$ .

**3. A fraction becomes  $\frac{1}{3}$  if 1 is subtracted from both its numerator and denominator. If 1 is added to both the numerator and denominator, it becomes  $\frac{1}{2}$ . Find the fraction.**

**Solution:**

Let's assume the numerator of the fraction to be  $x$  and the denominator of the fraction to be  $y$ .

So, the required fraction is  $\frac{x}{y}$ .

From the question it's given as,

If 1 is subtracted from both numerator and the denominator, the fraction becomes  $\frac{1}{3}$ .

Thus, the equation so formed is,

$$\begin{aligned}(x - 1)/(y - 1) &= \frac{1}{3} \\ \Rightarrow 3(x - 1) &= (y - 1) \\ \Rightarrow 3x - 3 &= y - 1 \\ \Rightarrow 3x - y - 2 &= 0 \dots (i)\end{aligned}$$

And also it's given in the question as,

If 1 is added to both numerator and the denominator, the fraction becomes 12. Expressing the above condition in an equation, we have

$$\begin{aligned} (x+1)/(y+1) &= 1/2 \\ \Rightarrow 2(x+1) &= (y+1) \\ \Rightarrow 2x + 2 &= y + 1 \\ \Rightarrow 2x - y + 1 &= 0 \dots\dots\dots (ii) \end{aligned}$$

Solving (i) and (ii), to find the fraction

By using cross-multiplication, we have

$$\begin{aligned} \frac{x}{(-1) \cdot 1 - (-1) \cdot (-2)} &= \frac{-y}{3 \cdot 1 - 2 \cdot (-2)} = \frac{1}{3 \cdot (-1) - 2 \cdot (-1)} \\ \Rightarrow \frac{x}{-1-2} &= \Rightarrow \frac{-y}{3+4} = \frac{1}{-3+2} \\ \Rightarrow \frac{x}{-3} &= \frac{-y}{7} = \frac{1}{-1} \\ \Rightarrow \frac{x}{3} &= \frac{y}{7} = 1 \end{aligned}$$

$$\Rightarrow x = 3, y = 7$$

Hence, the required fraction is 3/7.

**4. If we add 1 to the numerator and subtract 1 from the denominator, a fraction becomes 1. It also becomes 1/2 if we only add 1 to the denominator. What is the fraction?**

**Solution:**

Let's assume the numerator of the fraction to be x and the denominator of the fraction to be y.

So, the required fraction is x/y.

From the question it's given as,

If 1 is added to the numerator and 1 is subtracted from the denominator, the fraction becomes 1.

Thus, the equation so formed is,

$$\begin{aligned} (x+1)/(y-1) &= 1 \\ \Rightarrow (x+1) &= (y-1) \\ \Rightarrow x + 1 - y + 1 &= 0 \\ \Rightarrow x - y + 2 &= 0 \dots\dots\dots (i) \end{aligned}$$

And also it's given in the question as,

If 1 is added to the denominator, the fraction becomes 1/2.

Expressing the above condition in an equation, we have

$$\begin{aligned} x/(y+1) &= 1/2 \\ \Rightarrow 2x &= (y+1) \\ \Rightarrow 2x - y - 1 &= 0 \dots\dots\dots (ii) \end{aligned}$$

Solving (i) and (ii), to find the fraction

By using cross-multiplication, we have

$$\frac{x}{(-1)*(-1)-(-1)*2} = \frac{-y}{1*(-1)-2*2} = \frac{1}{1*(-1)-2*(-1)}$$

$$\Rightarrow \frac{x}{1+2} = \Rightarrow \frac{-y}{-1-4} = \frac{1}{-1+2}$$

$$\Rightarrow \frac{x}{3} = \frac{-y}{-5} = \frac{1}{1}$$

$$\Rightarrow \frac{x}{3} = \frac{y}{5} = 1$$

$$\Rightarrow x = 3, y = 5$$

Hence, the required fraction is  $\frac{3}{5}$ .

**5. The sum of the numerator and denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes  $\frac{1}{2}$ . Find the fraction.**

**Solution:**

Let's assume the numerator of the fraction to be  $x$  and the denominator of the fraction to be  $y$ .

So, the required fraction is  $\frac{x}{y}$ .

From the question it's given as,

The sum of the numerator and denominator of the fraction is 12.

Thus, the equation so formed is,

$$\begin{aligned} x + y &= 12 \\ \Rightarrow x + y - 12 &= 0 \end{aligned}$$

And also it's given in the question as,

If the denominator is increased by 3, the fraction becomes  $\frac{1}{2}$ .

Putting this as an equation, we get

$$\begin{aligned} \frac{x}{y+3} &= \frac{1}{2} \\ \Rightarrow 2x &= (y+3) \\ \Rightarrow 2x - y - 3 &= 0 \end{aligned}$$

The two equations are,

$$x + y - 12 = 0 \dots\dots (i)$$

$$2x - y - 3 = 0 \dots\dots\dots (ii)$$

Adding (i) and (ii), we get

$$\begin{aligned} x + y - 12 + (2x - y - 3) &= 0 \\ \Rightarrow 3x - 15 &= 0 \\ \Rightarrow x &= 5 \end{aligned}$$

Using  $x = 5$  in (i), we find  $y$

$$5 + y - 12 = 0$$

$$\Rightarrow y = 7$$

Therefore, the required fraction is  $5/7$ .

