Exercise 2.2 Page No: 2.43

1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also, verify the relationship between the zeros and coefficients in each of the following cases:

(i) $f(x) = 2x^3 + x^2 - 5x + 2$; 1/2, 1, -2

Solution:

Given, $f(x) = 2x^3 + x^2 - 5x + 2$, where a= 2, b= 1, c= -5 and d= 2

For x = 1/2

$f(1/2) = 2(1/2)^3 + (1/2)^2 - 5(1/2) + 2$

$= 1/4 + 1/4 - 5/2 + 2 = 0$

⇒ f(1/2) = 0, hence x = 1/2 is a root of the given polynomial.

For x = 1

$f(1) = 2(1)^3 + (1)^2 - 5(1) + 2$

$= 2 + 1 - 5 + 2 = 0$

⇒ f(1) = 0, hence x = 1 is also a root of the given polynomial.

For x = -2

$f(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$

$= -16 + 4 + 10 + 2 = 0$

⇒ f(-2) = 0, hence x = -2 is also a root of the given polynomial.

Now,

Sum of zeros = -b/a

1/2 + 1 – 2 = - (1)/2

-1/2 = -1/2

Sum of the products of the zeros taken two at a time = c/a

(1/2 x 1) + (1 x -2) + (1/2 x -2) = -5/ 2

1/2 - 2 + (-1) = -5/2

-5/2 = -5/2

Product of zeros = - d/a

1/2 x 1 x (– 2) = -(2)/2

-1 = -1

Hence, the relationship between the zeros and coefficients is verified.

(ii) $g(x) = x^3 - 4x^2 + 5x - 2$; 2, 1, 1

Solution:

Given, $g(x) = x^3 - 4x^2 + 5x - 2$, where a= 1, b= -4, c= 5 and d= -2

For x = 2

$g(2) = (2)^3 - 4(2)^2 + 5(2) - 2$

$= 8 - 16 + 10 - 2 = 0$

⇒ f(2) = 0, hence x = 2 is a root of the given polynomial.

For x = 1

$g(1) = (1)^3 - 4(1)^2 + 5(1) - 2$

$= 1 - 4 + 5 - 2 = 0$

⇒ g(1) = 0, hence x = 1 is also a root of the given polynomial.

Now,

Sum of zeros = -b/a

$1 + 1 + 2 = -(-4)/1$

$4 = 4$

Sum of the products of the zeros taken two at a time = c/a

$(1 \times 1) + (1 \times 2) + (2 \times 1) = 5/1$

$1 + 2 + 2 = 5$

$5 = 5$

Product of zeros = - d/a

$1 \times 1 \times 2 = -(-2)/1$

$2 = 2$

Hence, the relationship between the zeros and coefficients is verified.

2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and product of its zeros as 3, -1 and -3 respectively.

Solution:

Generally,

A cubic polynomial say, f(x) is of the form $ax^3 + bx^2 + cx + d$.

And, can be shown w.r.t its relationship between roots as.

⇒ f(x) = k [x^3 – (sum of roots)x^2 + (sum of products of roots taken two at a time)x – (product of roots)]

Where, k is any non-zero real number.

Here,

$f(x) = k [x^3 – (3)x^2 + (-1)x – (-3)]$

$\therefore f(x) = k [x^3 – 3x^2 - x + 3)]$

where, k is any non-zero real number.

3. If the zeros of the polynomial $f(x) = 2x^3 – 15x^2 + 37x – 30$ are in A.P., find them.

Solution:

Let the zeros of the given polynomial be α, β and γ. (3 zeros as it's a cubic polynomial)

And given, the zeros are in A.P.

So, let's consider the roots as

$\alpha = a – d$, $\beta = a$ and $\gamma = a + d$

Where, a is the first term and d is the common difference.

From given f(x), a= 2, b= -15, c= 37 and d= 30

⇒ Sum of roots = $\alpha + \beta + \gamma = (a - d) + a + (a + d) = 3a = (-b/a) = -(-15/2) = 15/2$

So, calculating for a, we get $3a = 15/2$ ⇒ $a = 5/2$

⇒ Product of roots = $(a - d) \times (a) \times (a + d) = a(a^2 – d^2) = -d/a = -(30)/2 = 15$

⇒ $a(a^2 – d^2) = 15$

Substituting the value of a, we get

⇒ $(5/2)[(5/2)^2 – d^2] = 15$

⇒ $5[(25/4) - d^2] = 30$
⇒ $(25/4) - d^2 = 6$
⇒ $25 - 4d^2 = 24$
⇒ $1 = 4d^2$
∴ $d = 1/2$ or $-1/2$

Taking $d = 1/2$ and $a = 5/2$
We get,
the zeros as 2, 5/2 and 3

Taking $d = -1/2$ and $a = 5/2$
We get,
the zeros as 3, 5/2 and 2