Revision Notes

Class - 10 Maths

Chapter 4 - Quadratic Equation

Definition of quadratic equation:

- **A quadratic equation** in the variable x is an equation of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers, $a \neq 0$.
- For example, $2x^2 + x - 300 = 0$ is a quadratic equation

Standard form of quadratic equation:

- Any equation of the form $p(x) = 0$, where $p(x)$ is a polynomial of degree 2, is a quadratic equation.
- When we write the terms of $p(x)$ in descending order of their degrees, then we get the standard form of the equation.
- That is, $ax^2 + bx + c,\ a \neq 0$ is called the **standard form of a quadratic equation.**

Roots of quadratic equation:

- **A solution** of the equation $p(x) = ax^2 + bx + c = 0$, with $a \neq 0$ is called a root of the quadratic equation.
- A real number α is called a root of the quadratic equation $ax^2 + bx + c = 0,\ a \neq 0$ if $a\alpha^2 + b\alpha + c = 0$.
- It means $x = \alpha$ satisfies the quadratic equation or $x = \alpha$ is the root of quadratic equation.
- **The zeroes** of the **quadratic polynomial** $ax^2 + bx + c$ and **the roots** of the **quadratic equation** $ax^2 + bx + c = 0$ are the **same.**

Method of solving a quadratic equation:

1. Factorization method

a. Factorize the quadratic equation by splitting the middle term.

b. After splitting the middle term, convert the equation into linear factors by taking common terms out.
c. Then on equating each factor to zero the roots are determined.
d. For example:

$\Rightarrow 2x^2 - 5x + 3$ (Split the middle term)

$\Rightarrow 2x^2 - 2x - 3x + 3$ (Take out common terms to determine linear factors)

$\Rightarrow 2x(x-1) - 3(x-1)$

$\Rightarrow (x-1)(2x-3)$ (Equate to zero)

$\Rightarrow (x-1)(2x-3) = 0$

When $(x-1) = 0$, $x = 1$

When $(2x-3) = 0$, $x = \dfrac{3}{2}$

So, the roots of $2x^2 - 5x + 3$ are 1 and $\dfrac{3}{2}$

2. Method of completing the square

a. The solution of quadratic equation can be found by converting any quadratic equation to perfect square of the form $(x+a)^2 - b^2 = 0$.
b. To convert quadratic equation $x^2 + ax + b = 0$ to perfect square equate b i.e., the constant term to the right side of equal sign then add square of half of a i.e., square of half of coefficient of x both sides.
c. To convert quadratic equation of form $ax^2 + bx + c = 0, a \neq 0$ to perfect square first divide the equation by a i.e., the coefficient of x^2 then follow the above-mentioned steps.
d. For example:

$\Rightarrow x^2 + 4x - 5 = 0$ (Equate constant term 5 to the right of equal sign)

$\Rightarrow x^2 + 4x = 5$ (Add square of half of 4 both sides)

$\Rightarrow x^2 + 4x + \left(\dfrac{4}{2}\right)^2 = 5 + \left(\dfrac{4}{2}\right)^2$

$\Rightarrow x^2 + 4x + 4 = 9$

$\Rightarrow (x+2)^2 = 9$

$\Rightarrow (x+2)^2 - (3)^2 = 0$

It is of the form $(x+a)^2 - b^2 = 0$

Now,

$$\Rightarrow (x+2)^2 - (3)^2 = 0$$
$$\Rightarrow (x+2)^2 = 9$$
$$\Rightarrow (x+2) = \pm 3$$
$\Rightarrow x = 1$ and $x = -5$

So, the roots of $x^2 + 4x - 5 = 0$ are 1 and -5

3. By using quadratic formula

a. The root of a quadratic equation $ax^2 + bx + c = 0$ is given by formula

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $\sqrt{b^2 - 4ac}$ is known as **discriminant.**

b. If $\sqrt{b^2 - 4ac} \ge 0$ then only the root of quadratic equation is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

c. For example:

$\Rightarrow x^2 + 4x + 3$

On using quadratic formula, we get

$$\Rightarrow x = \frac{-4 \pm \sqrt{(4)^2 - 4 \times 1 \times 3}}{2 \times 1}$$
$$\Rightarrow x = \frac{-4 \pm \sqrt{16 - 12}}{2}$$
$$\Rightarrow x = \frac{-4 \pm \sqrt{4}}{2}$$
$$\Rightarrow x = \frac{-4 \pm 2}{2}$$

$\Rightarrow x = \dfrac{-4+2}{2}$, $x = -1$

$\Rightarrow x = \dfrac{-4-2}{2}$, $x = -3$

So, the roots of $x^2 + 4x + 3 = 0$ are -1 and -3

Nature of roots based on discriminant:

a. If $\sqrt{b^2 - 4ac} = 0$ then the roots are **real and equal**

b. If $\sqrt{b^2 - 4ac} > 0$ then the roots are **real and distinct**

c. If $\sqrt{b^2 - 4ac} < 0$ then the roots are **imaginary**