Exercise 3.11 Page No: 3.111

1. If in a rectangle, the length is increased and breadth reduced each by 2 units, the area is reduced by 28 square units. If, however the length is reduced by 1 unit and the breadth increased by 2 units, the area increases by 33 square units. Find the area of the rectangle.

Solution:

Let's assume the length and breadth of the rectangle be x units and y units respectively.
Hence, the area of rectangle = xy sq.units

From the question we have the following cases,

Case 1:
Length is increased by 2 units ⇒ now, the new length is x+2 units
Breadth is reduced by 2 units ⇒ now, the new breadth is y-2 units
And it's given that the area is reduced by 28 square units i.e. = xy - 28
So, the equation becomes

$(x+2)(y-2) = xy - 28$

⇒ $xy - 2x + 2y - 4 = xy - 28$

⇒ $-2x + 2y - 4 + 28 = 0$

⇒ $-2x + 2y + 24 = 0$

⇒ $2x - 2y - 24 = 0$ ……… (i)

Case 2:
Length is reduced by 1 unit ⇒ now, the new length is x-1 units
Breadth is increased by 2 units ⇒ now, the new breadth is y+2 units
And, it's given that the area is increased by 33 square units i.e. = i.e. = xy + 33
So, the equation becomes

$(x-1)(y+2) = xy + 33$

⇒ $xy + 2x - y - 2 = x + 33$

⇒ $2x - y - 2 - 33 = 0$

⇒ $2x - y - 35 = 0$ ……….. (ii)

Solving (i) and (ii),
By using cross multiplication, we get

$$\frac{x}{(-2*-35)-(-1*-24)} = \frac{y}{(2*-35)-(2*-24)} = \frac{1}{(2*-1)-(2*-2)}$$

$$\frac{x}{70-24} = \frac{-y}{-70+48} = \frac{1}{-2+4}$$

$$\frac{x}{46} = \frac{-y}{-22} = \frac{1}{2}$$

x = 46/2
x = 23

And,

y = 22/2
y = 11

Hence,
The length of the rectangle is 23 units.
The breadth of the rectangle is 11 units.
So, the area of the actual rectangle = length x breadth,
= x×y
= 23 x 11
= 253 sq. units
Therefore, the area of rectangle is 253 sq. units.

2. The area of a rectangle remains the same if the length is increased by 7 metres and the breadth is decreased by 3 metres. The area remains unaffected if the length is decreased by 7 metres and the breadth is increased by 5 metres. Find the dimensions of the rectangle.
Solution:

Let's assume the length and breadth of the rectangle be x units and y units respectively.
Hence, the area of rectangle = xy sq.units

From the question we have the following cases,

Case 1
Length is increased by 7 metres ⇒ now, the new length is x+7
Breadth is decreased by 3 metres ⇒ now, the new breadth is y-3
And it's given, the area of the rectangle remains same i.e. = xy.
So, the equation becomes

$xy = (x+7)(y-3)$
$xy = xy + 7y - 3x - 21$
$3x - 7y + 21 = 0$ ………. (i)

Case 2:
Length is decreased by 7 metres ⇒ now, the new length is x-7
Breadth is increased by 5 metres ⇒ now, the new breadth is y+5
And it's given that, the area of the rectangle still remains same i.e. = xy.
So, the equation becomes

$xy = (x-7)(y+5)$
$xy = xy - 7y + 5x - 35$
$5x - 7y - 35 = 0$ ………. (ii)

Solving (i) and (ii),
By using cross-multiplication, we get,

$$\frac{x}{(-7\times-5)-(-7\times21)}=\frac{y}{(3\times-35)-(5\times21)}=\frac{1}{(3\times-7)-(5\times-7)}$$

$$\frac{x}{245+147}=\frac{-y}{-105-105}=\frac{1}{-21+35}$$

$$\frac{x}{392}=\frac{-y}{-210}=\frac{1}{14}$$

x = 392/14
x = 28

And,

y = 210/14
y = 15

Therefore, the length of the rectangle is 28 m. and the breadth of the actual rectangle is 15 m.

3. In a rectangle, if the length is increased by 3 metres and breadth is decreased by 4 metres, the area of the triangle is reduced by 67 square metres. If length is reduced by 1 metre and breadth is increased by 4 metres, the area is increased by 89 sq. metres. Find the dimension of the rectangle.

Solution:

Let's assume the length and breadth of the rectangle be x units and y units respectively.
Hence, the area of rectangle = xy sq.units

From the question we have the following cases,
According to the question,

Case 1:
Length is increased by 3 metres ⇒ now, the new length is x+3
Breadth is reduced by 4 metres ⇒ now, the new breadth is y-4
And it's given, the area of the rectangle is reduced by 67 m^2 = xy - 67.
So, the equation becomes

xy – 67 = (x + 3)(y – 4)
xy – 67 = xy + 3y – 4x – 12
4xy – 3y – 67 + 12 = 0
4x – 3y – 55 = 0 —— (i)

Case 2:
Length is reduced by 1 m ⇒ now, the new length is x-1
Breadth is increased by 4 metre ⇒ now, the new breadth is y+4
And it's given, the area of the rectangle is increased by 89 m^2 = xy + 89.
Then, the equation becomes

xy + 89 = (x -1)(y + 4)
4x – y – 93 = 0 —— (ii)

Solving (i) and (ii),
Using cross multiplication, we get

$$\frac{x}{(-3\times-93)-(-1\times-55)}=\frac{-y}{(4\times-93)-(4\times-55)}=\frac{1}{(4\times-1)-(4\times-3)}$$

$$\frac{x}{279-55}=\frac{-y}{-372+220}=\frac{1}{-4+12}$$

$$\frac{x}{224}=\frac{-y}{-152}=\frac{1}{8}$$

x = 224/8

x = 28

And,

y = 152/8

y = 19

Therefore, the length of rectangle is 28 m and the breadth of rectangle is 19 m.

4. The income of X and Y are in the ratio of 8: 7 and their expenditures are in the ratio 19: 16. If each saves ₹ 1250, find their incomes.

Solution:

Let the income be denoted by x and the expenditure be denoted by y.
Then, from the question we have
The income of X is ₹ 8x and the expenditure of X is 19y.
The income of Y is ₹ 7x and the expenditure of Y is 16y.

So, on calculating the savings, we get
Saving of X = 8x – 19y = 1250
Saving of Y = 7x – 16y = 1250

Hence, the system of equations formed are

8x – 19y – 1250 = 0 —— (i)

7x – 16y – 1250 = 0 —— (ii)

Using cross-multiplication method, we have

$$\frac{x}{(-19\times-1250)-(-16\times-1250)}=\frac{-y}{(8\times-1250)-(7\times-1250)}=\frac{1}{(8\times-16)-(7\times-19)}$$

$$\frac{x}{23750-20000}=\frac{-y}{-10000+8750}=\frac{1}{-128+133}$$

$$\frac{x}{3750}=\frac{y}{1250}=\frac{1}{5}$$

x = 3750/5

x = 750

If, x = 750, then

The income of X = 8x

= 8 x 750

= 6000

The income of Y = 7x

$= 7 \times 750$
$= 5250$

Therefore, the income of X is ₹ 6000 and the income of Y is ₹ 5250

5. A and B each has some money. If A gives ₹ 30 to B, then B will have twice the money left with A. But, if B gives ₹ 10 to A, then A will have thrice as much as is left with B. How much money does each have?
Solution:

Let's assume the money with A be ₹ x and the money with B be ₹ y.
Then, from the question we have the following cases

Case 1: If A gives ₹ 30 to B, then B will have twice the money left with A.
So, the equation becomes
$y + 30 = 2(x - 30)$
$y + 30 = 2x - 60$
$2x - y - 60 - 30 = 0$
$2x - y - 90 = 0$ —— (i)

Case 2: If B gives ₹ 10 to A, then A will have thrice as much as is left with B.
$x + 10 = 3(y - 10)$
$x + 10 = 3y - 10$
$x - 3y + 10 + 30 = 0$
$x - 3y + 40 = 0$ —— (ii)

Solving (i) and (ii),

On multiplying equation (ii) with 2, we get,
$2x - 6y + 80 = 0$
Subtract equation (ii) from (i), we get,
$2x - y - 90 - (2x - 6y + 80) = 0$
$5y - 170 = 0$
$y = 34$
Now, on using y = 34 in equation (i), we find,
$x = 62$

Hence, the money with A is ₹ 62 and the money with B be ₹ 34

7. 2 men and 7 boys can do a piece of work in 4 days. The same work is done in 3 days by 4 men and 4 boys. How long would it take one man and one boy to do it?
Solution:

Assuming that the time required for a man alone to finish the work be x days and also the time required for a boy alone to finish the work be y days.
Then, we know

The work done by a man in one day = 1/x
The work done by a boy in one day = 1/y
Similarly,
The work done by 2 men in one day = 2/x
The work done by 7 boys in one day = 7/y

So, the condition given in the question states that,
2 men and 7 boys together can finish the work in 4 days

4(2/x + 7/y) = 1
8/x + 2/8y = 1 ———(i)

And, the second condition from the question states that,
4 men and 4 boys can finish the work in 3 days
For this, the equation so formed is

3(4/x + 4/y) = 1
12/x + 12/y = 1 ———(ii)

Hence, solving (i) and (ii) ⇒

Taking, 1/x = u and 1/y = v
So, the equations (i) and (ii) becomes,

8u + 28v = 1
12u + 12v = 1
8u + 28v – 1 = 0 —— (iii)
12u + 12v – 1 = 0 —— (iv)

By using cross multiplication, we get,

u = 1/15
1/x = 1/15
x = 15

And,

v = 1/60
1/y = 1/60
y = 60

Therefore,
The time required for a man alone to finish the work is 15 days and the time required for a boy alone to finish the work is 60 days.

8. In a △ABC, ∠A = x°, ∠B = (3x – 2)°, ∠C = y°. Also, ∠C – ∠B = 9°. Find the three angles.
Solution:

It's given that,

∠A = x°,
∠B = (3x – 2)°,
∠C = y°

Also given that,

$\angle C - \angle B = 9^o$

$\Rightarrow \quad \angle C = 9^\circ + \angle B$

$\Rightarrow \quad \angle C = 9^\circ + 3x^\circ - 2^\circ$

$\Rightarrow \quad \angle C = 7^\circ + 3x^\circ$

Substituting the value for

$\angle C = y^o$ in above equation we get,

$y^o = 7^o + 3x^o$

We know that, $\angle A + \angle B + \angle C = 180^o$ (Angle sum property of a triangle)

$\Rightarrow \quad x^\circ + (3x^\circ - 2^\circ) + (7^\circ + 3x^\circ) = 180^\circ$

$\Rightarrow \quad 7x^\circ + 5^\circ = 180^\circ$

$\Rightarrow \quad 7x^\circ = 175^\circ$

$\Rightarrow \quad x^\circ = 25^\circ$

Hence, calculating for the individual angles we get,

$\angle A = x^o = 25^o$

$\angle B = (3x - 2)^o = 73^o$

$\angle C = (7 + 3x)^o = 82^o$

Therefore,

$\angle A = 25^o$, $\angle B = 73^o$ and $\angle C = 82^o$.

9. In a cyclic quadrilateral ABCD, $\angle A = (2x + 4)^o$, $\angle B = (y + 3)^o$, $\angle C = (2y + 10)^o$, $\angle D = (4x - 5)^o$. Find the four angles.

Solution:

We know that,

The sum of the opposite angles of cyclic quadrilateral should be 180^o.

And, in the cyclic quadrilateral ABCD,

Angles $\angle A$ and $\angle C$ & angles $\angle B$ and $\angle D$ are the pairs of opposite angles.

So,

$\angle A + \angle C = 180^o$ and

$\angle B + \angle D = 180^o$

Substituting the values given to the above two equations, we have

For $\angle A + \angle C = 180^o$

$\Rightarrow \quad \angle A = (2x + 4)^o$ and $\angle C = (2y + 10)^o$

$2x + 4 + 2y + 10 = 180^o$

$2x + 2y + 14 = 180^o$

$2x + 2y = 180^o - 14^o$

$2x + 2y = 166$ —— (i)

And for, $\angle B + \angle D = 180^o$, we have

$\Rightarrow \quad \angle B = (y+3)^o$ and $\angle D = (4x - 5)^o$

$y + 3 + 4x - 5 = 180^o$

$4x + y – 5 + 3 = 180^o$
$4x + y – 2 = 180^o$
$4x + y = 180^o + 2^o$
$4x + y = 182^o$ ——- (ii)

Now for solving (i) and (ii), we perform

Multiplying equation (ii) by 2 to get,
$8x + 2y = 364$ —— (iii)
And now, subtract equation (iii) from (i) to get
$-6x = -198$
$x = −198/ −6$
$\Rightarrow$ $x = 33^o$

Now, substituting the value of $x = 33^o$ in equation (ii) to find y
$4x + y = 182$
$132 + y = 182$
$y = 182 – 132$
$\Rightarrow$ $y = 50$

Thus, calculating the angles of a cyclic quadrilateral we get:
$\angle A = 2x + 4$
$= 66 + 4$
$= 70^o$
$\angle B = y + 3$
$= 50 + 3$
$= 53^o$
$\angle C = 2y + 10$
$= 100 + 10$
$= 110^o$
$\angle D = 4x – 5$
$= 132 – 5$
$= 127^o$

Therefore, the angles of the cyclic quadrilateral ABCD are
$\angle A = 70^o$, $\angle B = 53^o$, $\angle C = 110^o$ and $\angle D = 127^o$

10. Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?
Solution:

Let's assume that the total number of correct answers be x and the total number of incorrect answers be y.
Hence, their sum will give the total number of questions in the test i.e. x + y

Further from the question, we have two type of marking scheme:

1) When 3 marks is awarded for every right answer and 1 mark deducted for every wrong answer.
According to this type, the total marks scored by Yash is 40. (Given)
So, the equation formed will be

$3x - 1y = 40$ ….. (i)

Next,
2) When 4 marks is awarded for every right answer and 2 marks deducted for every wrong answer.
According to this type, the total marks scored by Yash is 50. (Given)
So, the equation formed will be

$4x - 2y = 50$ …… (ii)

Thus, by solving (i) and (ii) we obtained the values of x and y.
From (i), we get

$y = 3x - 40$ …….. (iii)

Using (iii) in (ii) we get,

$4x – 2(3x – 40) = 50$
$4x – 6x + 80 = 50$
$2x = 30$
$x = 15$

Putting x = 14 in (iii) we get,

$y = 3(15) – 40$
$y = 5$

So, $x + y = 15 + 5 = 20$

Therefore, the number of questions in the test were 20.

11. In a ΔABC, ∠A = x°, ∠B = 3x°, ∠C = y°. If 3y – 5x = 30, prove that the triangle is right angled.
Solution:

We need to prove that ΔABC is right angled.

Given:
$\angle A = x^o$, $\angle B = 3x^o$ and $\angle C = y^o$
Sum of the three angles in a triangle is 180^o (Angle sum property of a triangle)
i.e., $\angle A + \angle B + \angle C = 180^o$

$x + 3x + y = 180^o$
$4x + y = 180$ —— (i)

From question it's given that, $3y – 5x = 30$ —– (ii)

To solve (i) and (ii), we perform
Multiplying equation (i) by 3 to get,

$12x + 36y = 540$ —— (iii)

Now, subtracting equation (ii) from equation (iii) we get

$17x = 510$

$x = 510/17$

⇒ $x = 30^o$

Substituting the value of $x = 30^o$ in equation (i) to find y

$4x + y = 180$

$120 + y = 180$

$y = 180 – 120$

⇒ $y = 60^o$

Thus the angles ∠A, ∠B and ∠C are calculated to be

$\angle A = x^o = 30^o$

$\angle B = 3x^o = 90^o$

$\angle C = y^o = 60^o$

A right angled triangle is a triangle with any one side right angled to other, i.e., 90^o to other.
And here we have,

$\angle B = 90^o$.

Therefore, the triangle ABC is right angled. Hence proved.

12. The car hire charges in a city comprise of a fixed charges together with the charge for the distance covered. For a journey of 12 km, the charge paid is ₹ 89 and for a journey of 20 km, the charge paid is ₹ 145. What will a person have to pay for travelling a distance of 30 km?
Solution:

Let the fixed charge of the car be ₹ x and,
Let the variable charges of the car be ₹ y per km.
So according to the question, we get 2 equations

$x + 12y = 89$ —— (i) and,

$x + 20y = 145$ —— (ii)

Now, by solving (i) and (ii) we can find the charges.

On subtraction of (i) from (ii), we get,

$-8y = -56$

$y = −56 − 8$

⇒ $y = 7$

So, substituting the value of y = 7 in equation (i) we get

$x + 12y = 89$

$x + 84 = 89$

$x = 89 – 84$

⇒ $x = 5$

Thus, the total charges for travelling a distance of 30 km can be calculated as: $x + 30y$

$\Rightarrow \quad x + 30y = 5 + 210 =$ ₹ 215

Therefore, a person has to pay ₹ 215 for travelling a distance of 30 km by the car.