NCERT Solutions for Class 10

Maths

Chapter 3 – Pair of Linear Equations in Two Variables

Exercise 3.1

1. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.

Ans: Assuming that the present age of Aftab and his daughter are x and y respectively.

Their age seven years ago:

Aftab's age: $x-7$

Aftab's daughter's age: $y-7$

Therefore,

$(x-7)=7(y-7)$

$x-7=7y-49$

$x-7y=-42$ …… (i)

Their age after three years:

Aftab's age: $x+3$

Aftab's daughter's age: $y+3$

Therefore,

$(x+3)=3(y+3)$

$x+3=3y+9$

$x-3y=6$ …… (ii)

Representing equation (i) in algebraic form:

$x=7y-42$

Solution table:

x	–7	0	7
y	5	6	7

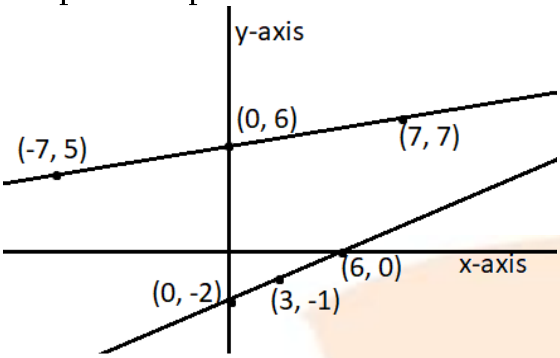
Representing equation (ii) in algebraic form:

$x=3y+6$

Solution table:

x	6	3	0
y	0	–1	–2

Graphical representation:



2. The coach of a cricket team buys 3 bats and 6 balls for $\text{Rs } 3900$. Later, she buys another bat and 2 more balls of the same kind for $\text{Rs } 1300$. Represent this situation algebraically and geometrically.

Ans: Assuming that the cost of bat and ball be x and y respectively.

Writing the algebraic representation using the information given in the question:

$3x+6y=3900$

$x+2y=1300$

Solution table for $3x+6y=3900$:

$$x=\frac{3900-6y}{3}$$

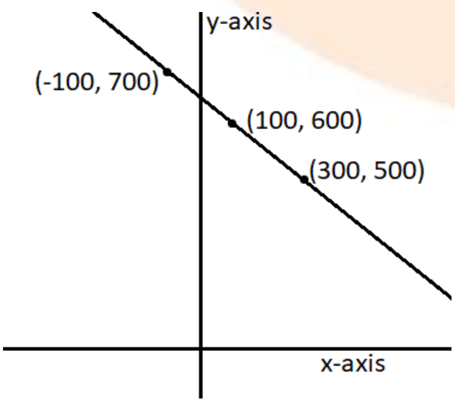
x	300	100	–100
y	500	600	700

Solution table for $x+2y=1300$:

$x=1300-2y$

x	300	100	–100
y	500	600	700

Graphical representation:



3. The cost of 2 kg of apples and 1 kg of grapes on a day was found to be Rs 160. After a month, the cost of 4 kg of apples and 2 kg of grapes is Rs 300. Represent the situation algebraically and geometrically.

Ans: Assuming that the cost of 1kg of apples and 1kg of grapes be x and y respectively.

Writing the algebraic representation using the information given in the question:

$2x + y = 160$

$4x + 2y = 300$

Solution table for $2x + y = 160$:

$y = 160 - 2x$

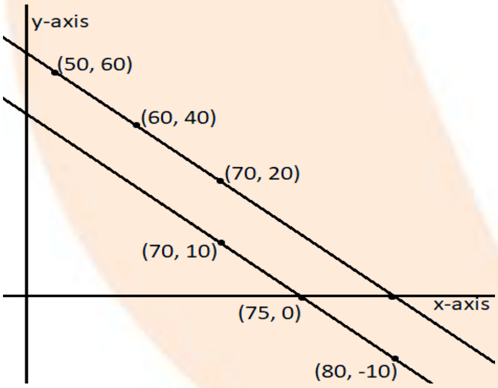
x	50	60	70
y	60	40	20

Solution table for $4x + 2y = 300$:

$y = 150 - 2x$

x	70	75	80
y	10	0	-10

Graphical representation:



Exercise 3.2

1. Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

Ans: Assuming that the number of girls and boys be x and y respectively.

Writing the algebraic representation using the information given in the question:

$x+y=10$

$x-y=4$

Solution table for $x+y=10$:

$x=10-y$

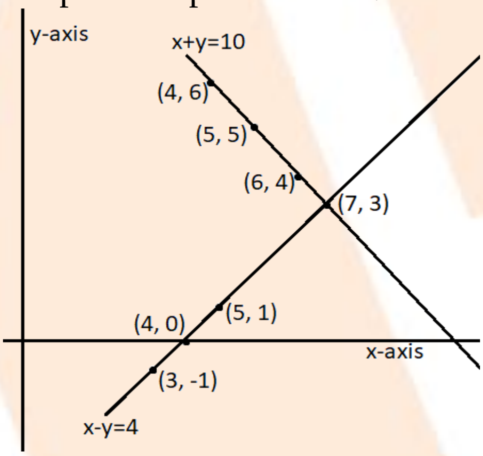
x	5	4	6
y	5	6	4

Solution table for $x-y=4$:

$x=4+y$

x	5	4	3
y	1	0	–1

Graphical representation:



As we can see from the graph above, the point of intersection for the lines is $(7,3)$. Therefore, we can say that there are 7 girls and 3 boys in the class.

(ii) 5 pencils and 7 pens together cost Rs 50, whereas 7 pencils and 5 pens together cost Rs 46. Find the cost of one pencil and that of one pen.

Ans: Assuming that the cost of 1 pencil and 1 pen be x and y respectively.

Writing the algebraic representation using the information given in the question:

$5x+7y=50$

$7x+5y=46$

Solution table for $5x+7y=50$:

$x = \frac{50 - 7y}{5}$

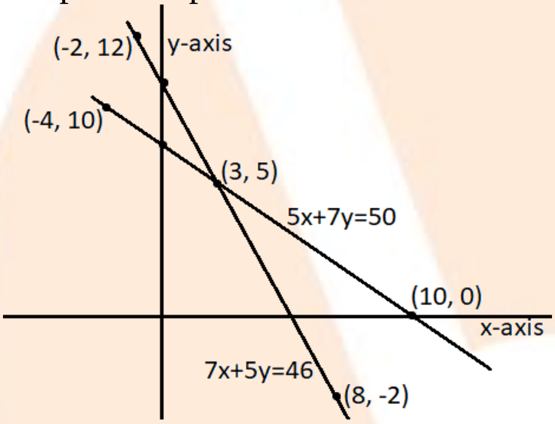
x	3	10	–4
y	5	0	10

Solution table for $7x + 5y = 46$:

$x = \frac{46 - 5y}{7}$

x	8	3	–2
y	–2	5	12

Graphical representation:



As we can see from the graph above, the point of intersection for the lines is $(3,5)$. Therefore, we can say that the cost of a pencil is Rs 3 and the cost of a pen is Rs 5.

2. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations at a point, are parallel or coincident:

(i) $5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

Ans: $5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

Calculating the values of a_1, b_1, c_1, a_2, b_2 and c_2 by comparing the above equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

$a_1 = 5$, $b_1 = -4$, $c_1 = 8$

$a_2 = 7$, $b_2 = 6$, $c_2 = -9$

$$\frac{a_1}{a_2} = \frac{5}{7}$$

$$\frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

Therefore, the given pair of equations have a unique solution, that is, the lines intersect at exactly one point.

(ii) $9x + 3y + 12 = 0$
$18x + 6y + 24 = 0$

Ans: Calculating the values of a_1, b_1, c_1, a_2, b_2 and c_2 by comparing the above equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

$a_1 = 9$, $b_1 = 3$, $c_1 = 12$

$a_2 = 18$, $b_2 = 6$, $c_2 = 24$

$$\frac{a_1}{a_2} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{1}{2}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Therefore, the lines representing the given pair of equations have infinite solutions as they are coincident.

(iii) $6x - 3y + 10 = 0$
$2x - y + 9 = 0$

Ans: Calculating the values of a_1, b_1, c_1, a_2, b_2 and c_2 by comparing the above equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

$a_1 = 6$, $b_1 = -3$, $c_1 = 10$

$a_2 = 2$, $b_2 = -1$, $c_2 = 9$

$$\frac{a_1}{a_2} = \frac{3}{1}$$

$$\frac{b_1}{b_2} = \frac{3}{1}$$

$$\frac{c_1}{c_2} = \frac{10}{9}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

Therefore, the lines representing the given pair of equations have no solutions as they are parallel to each other.

3. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pairs of linear equations are consistent, or inconsistent.

(i) $3x + 2y = 5; 2x - 3y = 7$

Ans: For the given equations**:**

$$\frac{a_1}{a_2} = \frac{3}{2}$$

$$\frac{b_1}{b_2} = \frac{-2}{3}$$

$$\frac{c_1}{c_2} = \frac{5}{7}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

So, the given pair of equations have a unique solution, that is, the lines intersect at exactly one point.

Therefore, the given pair of lines is consistent.

(ii) $2x - 3y = 8; 4x - 6y = 9$

Ans: For the given equations**:**

$$\frac{a_1}{a_2} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{8}{9}$$

Since $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$.

So, the lines representing the given pair of equations have no solutions as they are parallel to each other.

Therefore, the given pair of lines is inconsistent.

(iii) $\mathbf{\frac{3}{2}x+\frac{5}{3}y=7;9x-10y=14}$

Ans: For the given equations**:**

$$\frac{a_1}{a_2}=\frac{1}{6}$$

$$\frac{b_1}{b_2}=\frac{-1}{6}$$

$$\frac{c_1}{c_2}=\frac{1}{4}$$

Since $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$.

So, the given pair of equations have a unique solution, that is, the lines intersect at exactly one point.

Therefore, the given pair of lines is consistent.

(iv) $\mathbf{5x-3y=11;-10x+6y=-22}$

Ans: For the given equations**:**

$$\frac{a_1}{a_2}=\frac{-1}{2}$$

$$\frac{b_1}{b_2}=\frac{-1}{2}$$

$$\frac{c_1}{c_2}=\frac{-1}{2}$$

Since $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$.

So, the lines representing the given pair of equations have infinite number of solutions as they are coincident.

Therefore, the given pair of lines is consistent.

(v) $\frac{4}{3}x+2y=8; 2x+3y=12$

Ans: For the given equations**:**

$$\frac{a_1}{a_2}=\frac{2}{3}$$

$$\frac{b_1}{b_2}=\frac{2}{3}$$

$$\frac{c_1}{c_2}=\frac{2}{3}$$

Since $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$.

So, the lines representing the given pair of equations have infinite number of solutions as they are coincident.

Therefore, the given pair of lines is consistent.

4. Which of the following pairs of linear equations are consistent/ inconsistent? If consistent, obtain the solution graphically:

(i) $x+y=5; 2x+2y=10$

Ans: For the given equations**:**

$$\frac{a_1}{a_2}=\frac{1}{2}$$

$$\frac{b_1}{b_2}=\frac{1}{2}$$

$$\frac{c_1}{c_2}=\frac{1}{2}$$

Since $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$.

So, the lines representing the given pair of equations have infinite number of solutions as they are coincident.

Therefore, the given pair of lines is consistent.

Solution table for $x+y=5$:

$x=5-y$

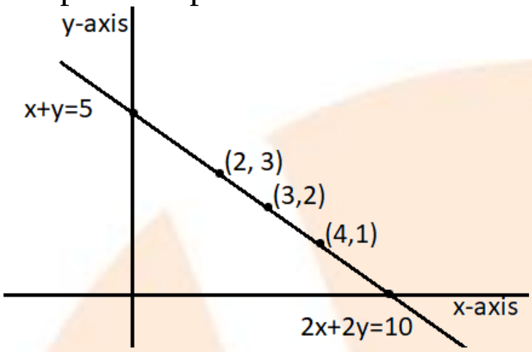
x	4	3	2
y	1	2	3

Solution table for $2x+2y=10$:

$x = \dfrac{10-2y}{2}$

x	4	3	2
y	1	2	3

Graphical representation:



As shown in the graph above, the two lines are overlapping each other. Hence, they have infinite solutions.

(ii) $\mathbf{x - y = 8; 3x - 3y = 16}$

Ans: For the given equations**:**

$\dfrac{a_1}{a_2} = \dfrac{1}{3}$

$\dfrac{b_1}{b_2} = \dfrac{1}{3}$

$\dfrac{c_1}{c_2} = \dfrac{1}{2}$

Since $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$.

So, the lines representing the given pair of equations have no solutions as they are parallel to each other.
Therefore, the given pair of lines is inconsistent.

(iii) $\mathbf{2x + y - 6 = 0; 4x - 2y - 4 = 0}$

Ans: For the given equations**:**

$\dfrac{a_1}{a_2} = \dfrac{1}{2}$

$\dfrac{b_1}{b_2} = \dfrac{-1}{2}$

$$\frac{c_1}{c_2} = \frac{3}{2}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

So, the given pair of equations have a unique solution, that is, the lines intersect at exactly one point.
Therefore, the given pair of lines is consistent.
Solution table for $2x + y - 6 = 0$:

$$y = 6 - 2x$$

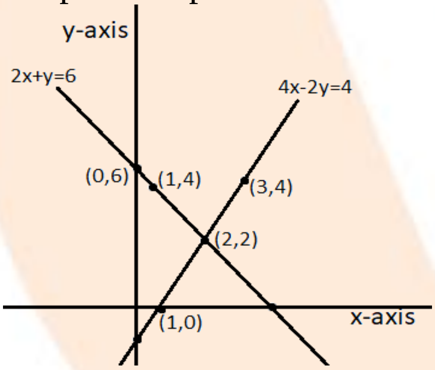
x	0	1	2
y	6	4	2

Solution table for $4x - 2y - 4 = 0$:

$$y = \frac{4x - 4}{2}$$

x	1	2	3
y	0	2	4

Graphical representation:



As shown in the graph above, the two lines intersect each other at only one point $(2,2)$.

(iv) $2x - 2y - 2 = 0; 4x - 4y - 5 = 0$

Ans: For the given equations**:**

$$\frac{a_1}{a_2} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{1}{2}$$

$$\frac{c_1}{c_2}=\frac{2}{5}$$

Since $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$.

So, the lines representing the given pair of equations have no solutions as they are parallel to each other.

Therefore, the given pair of lines is inconsistent.

5. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Ans: Assuming that the width and length of the garden be x and y respectively.

Writing the algebraic representation using the information given in the question:

$y-x=4$

$x+y=36$

Solution table for $y-x=4$:

$y=x+4$

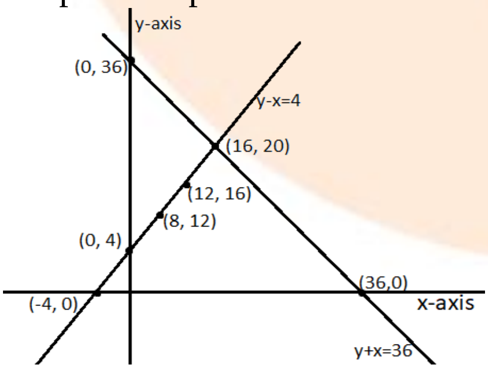
x	0	80	12
y	4	12	16

Solution table for $x+y=36$:

$y=36-x$

x	0	36	16
y	36	0	20

Graphical representation:



As shown in the graph above, the two lines intersect each other at only one point $(16,20)$. Therefore the length of the garden is 20 m and its breadth is 16 m.

6. Given the linear equation $2x+3y-8=0$, write another linear equations in two variables such that the geometrical representation of the pair so formed is:

(i) Intersecting lines

Ans: If two lines are intersecting then:

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, taking the second line as $2x+4y-6=0$,

Now,

$$\frac{a_1}{a_2} = 1$$

$$\frac{b_1}{b_2} = \frac{3}{4}$$

As $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the two lines are interesting each other.

(ii) Parallel lines

Ans: If two lines are intersecting then:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, taking the second line as $4x+6y-8=0$,

Now,

$$\frac{a_1}{a_2} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = 1$$

As $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the two lines are parallel to each other.

(iii) Coincident lines

Ans: If two lines are intersecting then:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, taking the second line as $6x+9y-24=0$, Now,

$\frac{a_1}{a_2} = \frac{1}{3}$

$\frac{b_1}{b_2} = \frac{1}{3}$

$\frac{c_1}{c_2} = \frac{1}{3}$

As $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the two lines are coincident.

7. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x – axis, and shade the triangular region.

Ans: Solution table for $x - y + 1 = 0$:

$x = y - 1$

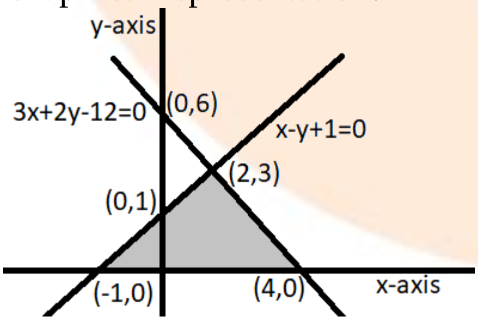
x	0	1	2
y	1	2	3

Solution table for $3x + 2y - 12 = 0$:

$x = \frac{12 - 2y}{3}$

x	4	2	0
y	0	3	6

Graphical representation:



As shown in the graph above, the lines are intersecting each other at point $(2,3)$ and x – axis at $(-1,0)$ and $(4,0)$. So the obtained triangle has vertices $(2,3)$, $(-1,0)$ and $(4,0)$.

Exercise 3.3

1. Solve the following pair of linear equations by the substitution method.

(i) $x+y=14; x-y=4$

Ans: The given equations are:

$x+y=14$ ...... (i)

$x-y=4$ ...... (ii)

From equation (i):

$x=14-y$ ...... (iii)

Substituting (iii) in equation (ii), we get

$(14-y)-y=4$

$14-2y=4$

$10=2y$

$y=5$ ...... (iv)

Substituting (iv) in (iii), we get

$x=9$

Therefore, $x=9$ and $y=5$.

(ii) $s-t=3; \frac{s}{3}+\frac{t}{2}=6$

Ans: The given equations are:

$s-t=3$ ...... (i)

$\frac{s}{3}+\frac{t}{2}=6$ ...... (ii)

From equation (i):

$s=t+3$ ...... (iii)

Substituting (iii) in equation (ii), we get

$\frac{t+3}{3}+\frac{t}{2}=6$

$2t+6+3t=36$

$5t=30$

$t=6$ ...... (iv)

Substituting (iv) in (iii), we get

$s=9$

Therefore, $s=9$ and $t=6$.

(iii) $3x-y=3; 9x-3y=9$

Ans: The given equations are:

$3x-y=3 \quad \ldots\ldots\text{ (i)}$

$9x-3y=9 \quad \ldots\ldots\text{ (ii)}$

From equation (i):

$y=3x-3 \quad \ldots\ldots\text{ (iii)}$

Substituting (iii) in equation (ii), we get

$9x-3(3x-3)=9$

$9x-9x+9=9$

$9=9$

For all x and y.

Therefore, the given equations have infinite solutions. One of the solution is $x=1, y=0$.

(iv) $0.2x-0.3y=1.3; 0.4x+0.5y=2.3$

Ans: The given equations are:

$0.2x-0.3y=1.3 \quad \ldots\ldots\text{ (i)}$

$0.4x+0.5y=2.3 \quad \ldots\ldots\text{ (ii)}$

From equation (i):

$x=\dfrac{1.3-0.3y}{0.2} \quad \ldots\ldots\text{ (iii)}$

Substituting (iii) in equation (ii), we get

$0.4\left(\dfrac{1.3-0.3y}{0.2}\right)-0.5y=2.3$

$2.6-0.6y+0.5y=2.3$

$2.6-2.3=0.1y$

$y=3 \quad \ldots\ldots\text{ (iv)}$

Substituting (iv) in (iii), we get

$x=\dfrac{1.3-0.3(3)}{0.2}$

$x=2$

Therefore, $x=2$ and $y=3$.

(v) $\sqrt{2}x-\sqrt{3}y=0; \sqrt{3}x-\sqrt{8}y=0$

Ans: The given equations are:

$\sqrt{2}x-\sqrt{3}y=0 \quad \ldots\ldots\text{ (i)}$

$\sqrt{3}x-\sqrt{8}y=0 \quad \ldots\ldots\text{ (ii)}$

From equation (i):

$$x = \frac{-\sqrt{3}y}{\sqrt{2}} \qquad \ldots\ldots \text{(iii)}$$

Substituting (iii) in equation (ii), we get

$$\sqrt{3}\left(\frac{-\sqrt{3}y}{\sqrt{2}}\right) - \sqrt{8}y = 0$$

$$\frac{-\sqrt{3}y}{\sqrt{2}} - 2\sqrt{2}y = 0$$

$$y\left(\frac{-\sqrt{3}}{\sqrt{2}} - 2\sqrt{2}\right) = 0$$

$$y = 0 \qquad \ldots\ldots \text{(iv)}$$

Substituting (iv) in (iii), we get

$x = 0$

Therefore, $x = 0$ and $y = 0$.

(vi) $\mathbf{\frac{3x}{2} - \frac{5y}{3} = -2; \frac{x}{3} + \frac{y}{2} = \frac{13}{6}}$

Ans: The given equations are:

$$\frac{3x}{2} - \frac{5y}{3} = -2 \qquad \ldots\ldots \text{(i)}$$

$$\frac{x}{3} + \frac{y}{2} = \frac{13}{6} \qquad \ldots\ldots \text{(ii)}$$

From equation (i):

$$x = \frac{-12 + 10y}{9} \qquad \ldots\ldots \text{(iii)}$$

Substituting (iii) in equation (ii), we get

$$\frac{\left(\frac{-12 + 10y}{9}\right)}{3} + \frac{y}{2} = \frac{13}{6}$$

$$\frac{-12 + 10y}{27} + \frac{y}{2} = \frac{13}{6}$$

$$\frac{-24 + 20y + 27y}{54} = \frac{13}{6}$$

$$47y = 141$$

$$y = 3 \qquad \ldots\ldots \text{(iv)}$$

Substituting (iv) in (iii), we get

$x = 2$

Therefore, $x = 0$ and $y = 3$.

2. Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$.

Ans: The given equations are:

$2x + 3y = 11 \quad \ldots\ldots \text{(i)}$

$2x - 4y = -24 \quad \ldots\ldots \text{(ii)}$

From equation (i):

$x = \dfrac{11 - 3y}{2} \quad \ldots\ldots \text{(iii)}$

Substituting (iii) in equation (ii), we get

$2\left(\dfrac{11 - 3y}{2}\right) - 4y = -24$

$11 - 3y - 4y = -24$

$-7y = -35$

$y = 5 \quad \ldots\ldots \text{(iv)}$

Substituting (iv) in (iii), we get

$x = -2$

Therefore, $x = -2$ and $y = 5$.

Calculating the value of m:

$y = mx + 3$

$5 = -2m + 3$

$m = -1$

3. Form the pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

Ans: Assuming one number be x and another number be y such that $y > x$,

Writing the algebraic representation using the information given in the question:

$y = 3x \quad \ldots\ldots \text{(i)}$

$y - x = 26 \quad \ldots\ldots \text{(ii)}$

Substituting the value of y from equation (i) in equation (ii), we get

$3x - x = 26$

$2x = 26$

$x = 13 \quad \ldots\ldots \text{(iii)}$

Substituting (iii) in (i), we get

$y=39$

Therefore, $x=13$ and $y=39$.

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

Ans: Assuming the larger angle be x and smaller angle be y.

The sum of a pair of supplementary angles is always 180°.

Writing the algebraic representation using the information given in the question:

$x+y=180$ …… (i)

$x-y=18$ …… (ii)

Substituting the value of x from equation (i) in equation (ii), we get

$180-y-y=18$

$162=2y$

$y=81$ …… (iii)

Substituting (iii) in (i), we get

$x=99$

Therefore, the two angles are $x=99^{\circ}$ and $y=81^{\circ}$.

(iii) The coach of a cricket team buys 7 bats and 6 balls for Rs 3800. Later, she buys 3 bats and 5 balls for Rs 1750. Find the cost of each bat and each ball.

Ans: Assuming the cost of a bat is x and the cost of a ball is y.

Writing the algebraic representation using the information given in the question:

$7x+6y=3800$ …… (i)

$3x+5y=1750$ …… (ii)

From equation (i):

$$y=\frac{3800-7x}{6} \quad \text{…… (iii)}$$

Substituting (iii) in equation (ii):

$$3x+5\left(\frac{3800-7x}{6}\right)=1750$$

$$3x+\frac{9500}{3}-\frac{35x}{6}=1750$$

$$3x-\frac{35x}{6}=1750-\frac{9500}{3}$$

$$\frac{18x-35x}{6}=\frac{5250-9500}{3}$$

$$\frac{17x}{6}=\frac{-4250}{3}$$

$x=500$ …… (iv)

Substituting (iv) in (iii), we get

$$y=\frac{3800-7(500)}{6}$$

$y=50$

Therefore, the bat costs $\text{Rs } 500$ and the ball costs $\text{Rs } 50$.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is $\text{Rs } 105$ and for a journey of 15 km, the charge paid is $\text{Rs } 155$. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km.

Ans: Assuming the fixed charge be $\text{Rs } x$ and the per km charge be $\text{Rs } y$.

Writing the algebraic representation using the information given in the question:

$x+10y=105$ …… (i)

$x+15y=155$ …… (ii)

From equation (i):

$x=105-10y$ …… (iii)

Substituting (iii) in equation (ii):

$105-10y+15y=155$

$5y=50$

$y=10$ …… (iv)

Substituting (iv) in (iii), we get

$x=105-10(10)$

$x=5$

Therefore, the fixed charge is $\text{Rs } 5$ and the per km charge is $\text{Rs } 10$.

So, charge for 25 km will be:

$=\text{Rs } (x+25y)$

$=\text{Rs } 255$

(v) A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator

it becomes $\frac{5}{6}$. Find the fraction.

Ans: Assuming the fraction be $\frac{x}{y}$.

Writing the algebraic representation using the information given in the question:

$$\frac{x+2}{y+2}=\frac{9}{11}$$

$$11x+22=9y+18$$

$$11x-9y=-4 \qquad \ldots\ldots \text{(i)}$$

$$\frac{x+3}{y+3}=\frac{5}{6}$$

$$6x+18=5y+15$$

$$6x-5y=-3 \quad \ldots\ldots \text{(ii)}$$

From equation (i):

$$x=\frac{-4+9y}{11} \qquad \ldots\ldots \text{(iii)}$$

Substituting (iii) in equation (ii):

$$6\left(\frac{-4+9y}{11}\right)-5y=-3$$

$$-24+54y-55y=-33$$

$$y=9 \qquad \ldots\ldots \text{(iv)}$$

Substituting (iv) in (iii), we get

$$x=\frac{-4+9(9)}{11}$$

$$x=7$$

Therefore, the fraction is $\frac{7}{9}$.

(vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Ans: Assuming the age of Jacob be x and the age of his son be y.

Writing the algebraic representation using the information given in the question:

$$(x+5)=3(y+5)$$

$$x-3y=10 \qquad \ldots\ldots \text{(i)}$$

$(x-5)=7(y-5)$

$x-7y=-30 \quad \ldots\ldots \text{(ii)}$

From equation (i):

$x=3y+10 \quad \ldots\ldots \text{(iii)}$

Substituting (iii) in equation (ii):

$3y+10-7y=-30$

$-4y=-40$

$y=10 \quad \ldots\ldots \text{(iv)}$

Substituting (iv) in (iii), we get

$x=3(10)+10$

$x=40$

Therefore, Jacob's present age is 40 years and his son's present age is 10 years.

Exercise 3.4

1. Solve the following pair of linear equations by the elimination method and the substitution method:

(i) $x+y=5$ and $2x-3y=4$

Ans: Elimination method

The given equations are:

$x+y=5 \quad \ldots\ldots \text{(i)}$

$2x-3y=4 \quad \ldots\ldots \text{(ii)}$

Multiplying equation (ii) by 2, we get

$2x+2y=10 \quad \ldots\ldots \text{(iii)}$

Subtracting equation (ii) from equation (iii), we obtain

$5y=6$

$y=\frac{6}{5} \quad \ldots\ldots \text{(iv)}$

Substituting the value of (iv) in equation (i), we get

$x=5-\frac{6}{5}$

$x=\frac{19}{5}$

Therefore, $x=\frac{19}{5}$ and $y=\frac{6}{5}$.

Substitution method:

From equation (i) we get

$x = 5 - y$ ...... (v)

Substituting (v) in equation (ii), we get

$2(5 - y) - 3y = 4$

$-5y = -6$

$y = \frac{6}{5}$ ...... (vi)

Substituting (vi) in equation (v), we obtain

$x = 5 - \frac{6}{5}$

$x = \frac{19}{5}$

Therefore, $x = \frac{19}{5}$ and $y = \frac{6}{5}$.

(ii) $3x + 4y = 10$ and $2x - 2y = 2$

Ans: Elimination method

The given equations are:

$3x + 4y = 10$ ...... (i)

$2x - 2y = 2$ ...... (ii)

Multiplying equation (ii) by 2, we get

$4x - 4y = 4$ ...... (iii)

Adding equation (ii) and (iii), we obtain

$7x = 14$

$x = 2$ ...... (iv)

Substituting the value of (iv) in equation (i), we get

$6 + 4y = 10$

$4y = 4$

$y = 1$

Therefore, $x = 2$ and $y = 1$.

Substitution method:

From equation (ii) we get

$x = 1 + y$ ...... (v)

Substituting (v) in equation (i), we get

$3(1 + y) + 4y = 10$

$7y = 7$

$y = 1$ ...... (vi)

Substituting (vi) in equation (v), we obtain

$x = 1 + 1$

$x = 2$

Therefore, $x = 2$ and $y = 1$.

(iii) $3x - 5y - 4 = 0$ and $9x = 2y + 7$

Ans: Elimination method

The given equations are:

$3x - 5y - 4 = 0 \quad \ldots\ldots \text{(i)}$

$9x = 2y + 7$

$9x - 2y = 7 \quad \ldots\ldots \text{(ii)}$

Multiplying equation (i) by 3, we get

$9x - 15y - 12 = 0 \quad \ldots\ldots \text{(iii)}$

Subtracting equation (iii) from equation (ii), we obtain

$13y = -5$

$$y = -\frac{5}{13} \quad \ldots\ldots \text{(iv)}$$

Substituting the value of (iv) in equation (i), we get

$$3x + \frac{25}{13} - 4 = 0$$

$$3x = \frac{27}{13}$$

$$x = \frac{9}{13}$$

Therefore, $x = \frac{9}{13}$ and $y = -\frac{5}{13}$.

Substitution method:

From equation (i) we get

$$x = \frac{5y + 4}{3} \quad \ldots\ldots \text{(v)}$$

Substituting (v) in equation (ii), we get

$$9\left(\frac{5y + 4}{3}\right) - 2y - 7 = 0$$

$13y = -5$

$$y = \frac{-5}{13} \quad \ldots\ldots \text{(vi)}$$

Substituting (vi) in equation (v), we obtain

$$x = \frac{5\left(\frac{-5}{13}\right) + 4}{3}$$

$$x = \frac{9}{13}$$

Therefore, $x = \frac{9}{13}$ and $y = \frac{-5}{13}$.

(iv) $\mathbf{\frac{x}{2} + \frac{2y}{3} = -1}$ **and** $\mathbf{x - \frac{y}{3} = 3}$

Ans: Elimination method

The given equations are:

$$\frac{x}{2} + \frac{2y}{3} = -1$$

$$3x + 4y = -6 \qquad \ldots\ldots \text{(i)}$$

$$x - \frac{y}{3} = 3$$

$$3x - y = 9 \quad \ldots\ldots \text{(ii)}$$

Subtracting equation (ii) from equation (i), we obtain

$$5y = -15$$

$$y = -3 \qquad \ldots\ldots \text{(iv)}$$

Substituting the value of (iv) in equation (i), we get

$$3x + 4(-3) = -6$$

$$3x = 6$$

$$x = 2$$

Therefore, $x = 2$ and $y = -3$.

Substitution method:

From equation (ii) we get

$$x = \frac{y+9}{3} \qquad \ldots\ldots \text{(v)}$$

Substituting (v) in equation (i), we get

$$3\left(\frac{y+9}{3}\right) + 4y = -6$$

$$5y = -15$$

$$y = -3 \qquad \ldots\ldots \text{(vi)}$$

Substituting (vi) in equation (v), we obtain

$$x=\frac{-3+9}{3}$$
$$x=2$$
Therefore, $x=2$ and $y=-3$.

2. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method:

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

Ans: Assuming the fraction be $\frac{x}{y}$.

Writing the algebraic representation using the information given in the question:
$$\frac{x+1}{y-1}=1$$
$$x-y=-2 \qquad \ldots\ldots \text{(i)}$$
$$\frac{x}{y+1}=1$$
$$2x-y=1 \qquad \ldots\ldots \text{(ii)}$$
Subtracting equation (i) from equation (ii), we obtain
$$x=3 \qquad \ldots\ldots \text{(iii)}$$
Substituting the value of (iii) in equation (i), we get
$$3-y=-2$$
$$y=5$$
Therefore, $x=2$ and $y=-3$.

Hence the fraction is $\frac{3}{5}$.

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

Ans: Assuming the present age of Nuri be x and present age of Sonu be y.

Writing the algebraic representation using the information given in the question:
$$(x-5)=3(y-5)$$
$$x-3y=-10 \qquad \ldots\ldots \text{(i)}$$

$(x+10)=2(y+10)$

$x-2y=10 \quad \ldots\ldots$ (ii)

Subtracting equation (i) from equation (ii), we obtain

$y=20 \quad \ldots\ldots$ (iii)

Substituting the value of (iii) in equation (i), we get

$x-60=-10$

$x=50$

Therefore, $x=50$ and $y=20$.

Hence Nuri's present age is 50 years and Sonu's present age is 20 years.

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

Ans: Assuming the unit digit of the number be x and the tens digit be y.

Therefore, the number is $10y+x$

The number after reversing the digits is $10x+y$.

Writing the algebraic representation using the information given in the question:

$x+y=9 \quad \ldots\ldots$ (i)

$9(10y+x)=2(10x+y)$

$-x+8y=0 \quad \ldots\ldots$ (ii)

Adding equation (i) and (ii), we obtain

$9y=9$

$y=1 \quad \ldots\ldots$ (iii)

Substituting the value of (iii) in equation (i), we get

$x=8$

Therefore, $x=8$ and $y=1$.

Hence the number is $10y+x=18$.

(iv) Meena went to bank to withdraw $\text{Rs } 2000$. She asked the cashier to give her $\text{Rs } 50$ and $\text{Rs } 100$ notes only. Meena got 25 notes in all. Find how many notes of $\text{Rs } 50$ and $\text{Rs } 100$ she received.

Ans: Assuming the number of $\text{Rs } 50$ notes be x and the number of $\text{Rs } 100$ be y.

Writing the algebraic representation using the information given in the question:

$x+y=25 \quad \ldots\ldots$ (i)

$50x+100y=2000$ …… (ii)

Multiplying equation (i) by 50, we obtain

$50x+50y=1250$ …… (iii)

Subtracting equation (iii) from equation (ii), we obtain

$50y=750$

$y=15$ …… (iv)

Substituting the value of (iv) in equation (i), we get

$x=10$

Therefore, $x=10$ and $y=15$.

Hence Meena has 10 notes of Rs 50 and 15 notes of Rs 100.

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Ans: Assuming that the charge for first three days is Rs x and the charge for each day thereafter is Rs y.

Writing the algebraic representation using the information given in the question:

$x+4y=27$ …… (i)

$x+2y=21$ …… (ii)

Subtracting equation (ii) from equation (i), we obtain

$2y=6$

$y=3$ …… (iii)

Subtracting equation (iii) from equation (i), we obtain

$x+12=27$

$x=15$ …… (iv)

Therefore, $x=15$ and $y=3$.

Hence, fixed charges are Rs 15 and charges per day are Rs 3.

Exercise 3.5

1. Which of the following pairs of linear equations has unique solution, no solution or infinitely many solutions? In case there is a unique solution, find it by using cross multiplication method.

(i) $x-3y-3=0$; $3x-9y-2=0$

Ans: Calculating $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$,

$\frac{a_1}{a_2} = \frac{1}{3}$

$\frac{b_1}{b_2} = \frac{1}{3}$

$\frac{c_1}{c_2} = \frac{3}{2}$

Hence, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

Therefore there are no solution for the given pair of linear equation as the given lines are parallel to each other and do not intersect.

(ii) $2x + y = 5;\ 3x + 2y = 8$

Ans: Calculating $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$,

$\frac{a_1}{a_2} = \frac{2}{3}$

$\frac{b_1}{b_2} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{-5}{-8}$

Hence, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

Therefore the given pair of linear equations have a unique solution as the given lines intersect each other at a unique point.

Using cross-multiplication method,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{x}{-8 - (-10)} = \frac{y}{15 + 16} = \frac{1}{4 - 3}$$

$$\frac{x}{2} = \frac{y}{1} = 1$$

$x = 2, y = 1$

Therefore, $x = 2$ and $y = 1$.

(iii) $3x - 5y = 20;\ 6x - 10y = 40$

Ans: Calculating $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$,

$\frac{a_1}{a_2} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{1}{2}$

Hence, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Therefore there are infinite number of solution for the given pair of linear equation as the given lines are overall lapping each other.

(iv) $\mathbf{x - 3y - 7 = 0;\ 3x - 3y - 15 = 0}$

Ans: Calculating $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$,

$\frac{a_1}{a_2} = \frac{1}{3}$

$\frac{b_1}{b_2} = 1$

$\frac{c_1}{c_2} = \frac{7}{15}$

Hence, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

Therefore the given pair of linear equations have a unique solution as the given lines intersect each other at a unique point.

Using cross-multiplication method,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{x}{45 - (21)} = \frac{y}{-21 - (-15)} = \frac{1}{-3 - (9)}$$

$$\frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$$

$x = 4, y = -1$

Therefore, $x = 4$ and $y = -1$.

3.

(i) For which values of a and b will the following pair of linear equations have an infinite number of solutions?

$\mathbf{2x+3y=7;\ (a - b)x+(a + b)y=3a+b-2}$

Ans: Calculating $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$,

$$\frac{a_1}{a_2}=\frac{2}{a-b}$$

$$\frac{b_1}{b_2}=\frac{3}{a+b}$$

$$\frac{c_1}{c_2}=\frac{-7}{-(3a+b-2)}$$

Condition for infinitely many solutions,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\frac{2}{a-b}=\frac{7}{(3a+b-2)}$$

$$6a+2b-4=7a-7b$$

$$a-9b=-4 \qquad \ldots\ldots \text{(i)}$$

$$\frac{2}{a-b}=\frac{3}{a+b}$$

$$2a+2b=3a-3b$$

$$a-5b=0 \qquad \ldots\ldots \text{(ii)}$$

Subtracting equation (i) from equation (ii), we get

$$4b=4$$

$$b=1 \qquad \ldots\ldots \text{(iii)}$$

Substituting (iii) in equation (ii), we get

$$a-5=0$$

$$a=5$$

Therefore, the given equations will have infinite number of solutions for $a=5$ and $b=1$.

(ii) For which value of k will the following pair of linear equations have no solution? $\mathbf{3x+y=1;\ (2k - 1)x+(k - 1)y=2k+1}$.

Ans: Calculating $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$,

$$\frac{a_1}{a_2} = \frac{3}{2k-1}$$
$$\frac{b_1}{b_2} = \frac{1}{k-1}$$
$$\frac{c_1}{c_2} = \frac{1}{2k+1}$$

Condition for no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$
$$\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$$
$$\frac{3}{2k-1} = \frac{1}{k-1}$$
$$3k-3 = 2k-1$$
$$k = 2$$

Therefore, the given equations will not have any solutions for $k = 2$.

3. Solve the following pair of linear equations by the substitution and cross multiplication methods:

$\mathbf{8x + 5y = 9;\ 3x + 2y = 4}$

Ans: Using substitution method,

$8x + 5y = 9$ …… (i)

$3x + 2y = 4$ …… (ii)

From equation (ii) we get

$x = \frac{4-2y}{3}$ …… (iii)

Substituting (iii) in equation (i), we get

$$8\left(\frac{4-2y}{3}\right) + 5y = 9$$
$$32 - 16y + 15y = 27$$

$y = 5$ …… (iv)

Substituting (vi) in equation (ii), we obtain

$$3x + 10 = 4$$
$$x = -2$$

Therefore, $x = -2$ and $y = 5$.

Using cross multiplication method:

$$8x + 5y = 9$$

$3x+2y=4$

$$\frac{x}{-20-(-18)}=\frac{y}{-27-(-32)}=\frac{1}{16-15}$$

$$\frac{x}{-2}=\frac{y}{5}=1$$

$x=-2, y=5$

Therefore, $x=-2$ and $y=5$.

4. Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:

(i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay $Rs\ 1000$ as hostel charges whereas a student B, who takes food for 26 days, pays $Rs\ 1180$ as hostel charges. Find the fixed charges and the cost of food per day.

Ans: Assuming that the fixed charge of the food is $Rs\ x$ and the charge for food per day is $Rs\ y$.

Writing the algebraic representation using the information given in the question:

$x+20y=1000 \quad \ldots\ldots \text{(i)}$

$x+26y=1180 \quad \ldots\ldots \text{(ii)}$

Subtracting equation (i) from equation (ii), we obtain

$6y=180$

$y=30 \quad \ldots\ldots \text{(iii)}$

Substituting (iii) in equation (i), we obtain

$x+20(30)=1000$

$x=400 \quad \ldots\ldots \text{(iv)}$

Therefore, $x=15$ and $y=3$.

Hence, fixed charges are $Rs\ 400$ and charges per day are $Rs\ 30$.

(ii) A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

Ans: Assuming that the fraction is $\frac{x}{y}$.

Writing the algebraic representation using the information given in the question:

$$\frac{x-1}{y}=\frac{1}{3}$$

$3x-y=3$ …… (i)

$$\frac{x}{y+8}=\frac{1}{4}$$

$4x-y=8$ …… (ii)

Subtracting equation (i) from equation (ii), we obtain

$x=5$ …… (iii)

Substituting (iii) in equation (i), we obtain

$15-y=3$

$y=12$ …… (iv)

Therefore, $x=5$ and $y=12$.

Hence, the fraction is $\frac{5}{12}$.

(iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

Ans: Assuming that the fixed charge of the food is $\text{Rs } x$ and the charge for food per day is $\text{Rs } y$.

Writing the algebraic representation using the information given in the question:

$x+20y=1000$ …… (i)

$x+26y=1180$ …… (ii)

Subtracting equation (i) from equation (ii), we obtain

$6y=180$

$y=30$ …… (iii)

Substituting (iii) in equation (i), we obtain

$x+20(30)=1000$

$x=400$ …… (iv)

Therefore, $x=15$ and $y=3$.

Hence, fixed charges are $\text{Rs } 400$ and charges per day are $\text{Rs } 30$.

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

Ans: Assuming that the speed of first car is $u\ km/h$ and the speed of second car is $v\ km/h$.

Respective speed of both cars when both cars are travelling in same direction $=(u-v)\ km/h$

Respective speed of both cars when both cars are travelling in opposite direction $=(u+v)\ km/h$

Writing the algebraic representation using the information given in the question:

$5(u-v)=100$

$u-v=20 \quad \ldots\ldots \text{(i)}$

$1(u+v)=100 \quad \ldots\ldots \text{(ii)}$

Adding equation (i) and (ii), we get

$2u=120$

$u=60 \quad \ldots\ldots \text{(iii)}$

Substituting (iii) in equation (ii), we obtain

$v=40$

Hence, speed of one car is $60\ km/h$ and speed of the other car is $40\ km/h$.

(v) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Ans: Assuming length of the rectangle be x and the breadth be y.

Writing the algebraic representation using the information given in the question:

$(x-5)(y+3)=xy-9$

$3x-5y-6=0 \quad \ldots\ldots \text{(i)}$

$(x+3)(y+2)=xy+67 \quad \ldots\ldots \text{(ii)}$

Using cross multiplication method:

$$\frac{x}{305-(-18)}=\frac{y}{-12-(-183)}=\frac{1}{9-(-10)}$$

$$\frac{x}{323}=\frac{y}{171}=\frac{1}{19}$$

$x=17, y=9$

Hence, the rectangle has length of 17 units and breadth of 9 units.

Exercise 3.6

1. Solve the following pairs of equations by reducing them to a pair of linear equations:

(i) $\frac{1}{2x}+\frac{1}{3y}=2;\ \frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

Ans: Taking $\frac{1}{x}=p$ and $\frac{1}{y}=q$,

Now,

$\frac{p}{2}+\frac{q}{3}=2$

$3p+2q-12=0$ …… (i)

$\frac{p}{3}+\frac{q}{2}=\frac{13}{6}$

$2p+3q-13=0$ …… (ii)

Using cross multiplication method:

$\frac{p}{-26-(-36)}=\frac{q}{-24-(-39)}=\frac{1}{9-4}$

$\frac{p}{10}=\frac{q}{15}=\frac{1}{5}$

$p=2, q=3$

$\frac{1}{x}=2, \frac{1}{y}=3$

Therefore, $x=\frac{1}{2}$ and $y=\frac{1}{3}$.

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2;\ \frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

Ans: Taking $\frac{1}{\sqrt{x}}=p$ and $\frac{1}{\sqrt{y}}=q$,

Now,

$2p+3q=2$ …… (i)

$4p-9q=-1$ …… (ii)

Multiplying equation (i) by 3, we get

$6p+9q=6$ …… (iii)

Adding equation (ii) and (iii), we get

$10p = 5$

$p = \frac{1}{2}$ …… (iv)

Substituting (iv) in equation (i), we get

$1 + 3q = 2$

$q = \frac{1}{3}$

Now,

$\frac{1}{\sqrt{x}} = \frac{1}{2}$

$x = 4$

And, $\frac{1}{\sqrt{y}} = \frac{1}{3}$

$y = 9$

Therefore, $x = 4$ and $y = 9$.

(iii) $\mathbf{\frac{4}{x} + 3x = 14; \ \frac{3}{x} - 4y = 23}$

Ans: Taking $\frac{1}{x} = p$,

Now,

$4p + 3y - 14 = 0$ …… (i)

$3p - 4y - 23 = 0$ …… (ii)

Using cross multiplication method:

$$\frac{p}{-69-56} = \frac{q}{-42-(-92)} = \frac{1}{-16-9}$$

$$\frac{p}{-125} = \frac{y}{50} = \frac{-1}{25}$$

$p = 5, y = -2$

$\frac{1}{x} = 5, y = -2$

Therefore, $x = \frac{1}{5}$ and $y = -2$.

(iv) $\mathbf{\frac{5}{x-1} + \frac{1}{y-2} = 2; \ \frac{6}{x-1} - \frac{3}{y-2} = 1}$

Ans: Taking $\frac{1}{x-1}=p$ and $\frac{1}{y-2}=q$,

Now,

$5p+q=2$ ...... (i)

$6p-3q=1$ ...... (ii)

Multiplying equation (i) by 3, we get

$15p+3q=6$ ...... (iii)

Adding equation (ii) and (iii), we get

$21p=7$

$p=\frac{1}{3}$ ...... (iv)

Substituting (iv) in equation (i), we get

$\frac{5}{3}+q=2$

$q=\frac{1}{3}$

Now,

$\frac{1}{x-1}=\frac{1}{3}$

$x=4$

And, $\frac{1}{y-2}=\frac{1}{3}$

$y=5$

Therefore, $x=4$ and $y=5$.

(v) $\mathbf{\frac{7x-2y}{xy}=5;\ \frac{8x+7y}{xy}=15}$

Ans: $\frac{7x-2y}{xy}=5$

$\frac{7}{y}-\frac{2}{x}=5$ ...... (i)

$\frac{8x+7y}{xy}=15$

$\frac{8}{y}+\frac{7}{x}=15$ ...... (ii)

Taking $\frac{1}{x}=p$ and $\frac{1}{y}=q$,

Now,

$-2p+7q-5=0$ ...... (iii)

$7p+8q-15=0$ ...... (iv)

Using cross multiplication method:

$$\frac{p}{-105-(-40)}=\frac{q}{-35-30}=\frac{1}{-16-49}$$

$$\frac{p}{-65}=\frac{q}{-65}=\frac{1}{-65}$$

$p=1, q=1$

$\frac{1}{x}=1, \frac{1}{y}=1$

Therefore, $x=1$ and $y=1$.

(vi) $6x+3y=6xy$; $2x+4y=5xy$

Ans: $6x+3y=6xy$

$\frac{6}{y}+\frac{3}{x}=6$ ...... (i)

$2x+4y=5xy$

$\frac{2}{y}+\frac{4}{x}=6$ ...... (ii)

Taking $\frac{1}{x}=p$ and $\frac{1}{y}=q$,

Now,

$3p+6q-6=0$ ...... (iii)

$4p+2q-5=0$ ...... (iv)

Using cross multiplication method:

$$\frac{p}{-30-(-12)}=\frac{q}{-24-(-15)}=\frac{1}{6-24}$$

$$\frac{p}{-18}=\frac{q}{-9}=\frac{1}{-18}$$

$p=1, q=\frac{1}{2}$

$\frac{1}{x}=1, \frac{1}{y}=\frac{1}{2}$

Therefore, $x=1$ and $y=2$.

(vii) $\mathbf{\frac{10}{x+y}+\frac{2}{x-y}=4; \ \frac{15}{x+y}-\frac{5}{x-y}=-2}$

Ans: Taking $\frac{1}{x+y}=p$ and $\frac{1}{x-y}=q$,

Now,

$10p+2q-4=0$ ...... (i)

$15p-5q+2=0$ ...... (ii)

Using cross multiplication method:

$\frac{p}{4-20}=\frac{q}{-60-20}=\frac{1}{-50-30}$

$\frac{p}{-16}=\frac{q}{-80}=\frac{1}{-80}$

$p=\frac{1}{5}, q=1$

Now,

$\frac{1}{x+y}=\frac{1}{5}$

$x+y=5$ ...... (iii)

And, $\frac{1}{x-y}=1$

$x-y=1$ ...... (iv)

Adding equation (iii) and (iv), we get

$2x=6$

$x=3$ ...... (v)

Substituting (v) in equation (iii), we get

$y=2$

Therefore, $x=3$ and $y=2$.

(viii) $\mathbf{\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}; \ \frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}}$

Ans: Taking $\frac{1}{3x+y}=p$ and $\frac{1}{3x-y}=q$, Now,

$p+q=\frac{3}{4}$ …… (i)

$\frac{p}{2}-\frac{q}{2}=\frac{-1}{8}$

$p-q=\frac{-1}{4}$ …… (ii)

Adding equation (i) and (ii), we get

$2p=\frac{1}{2}$

$p=\frac{1}{4}$ …… (iii)

Substituting (iii) in equation (ii), we get

$\frac{1}{4}-q=\frac{-1}{4}$

$p=\frac{1}{2}$

Now,

$\frac{1}{3x+y}=\frac{1}{4}$

$3x+y=4$ …… (iv)

And, $\frac{1}{3x-y}=\frac{1}{2}$

$3x-y=2$ …… (v)

Adding equation (iv) and (v), we get

$6x=6$

$x=1$ …… (vi)

Substituting (vi) in equation (iii), we get

$y=1$

Therefore, $x=1$ and $y=1$.

2. Formulate the following problems as a pair of equations, and hence find their solutions:

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

Ans: Assuming that speed of Ritu in still water is $x\ \text{km/h}$ and the speed of stream be $y\ \text{km/h}$.

Speed of Ritu while rowing upstream will be $=(x-y)$ km / h

Speed of Ritu while rowing downstream will be $=(x+y)$ km / h

Writing the algebraic representation using the information given in the question:

$2(x+y)=20$

$x+y=10 \qquad \ldots\ldots$ (i)

$2(x-y)=4$

$x-y=2 \qquad \ldots\ldots$ (ii)

Adding equation (i) and (ii), we get

$2x=12$

$x=6 \qquad \ldots\ldots$ (iii)

Substituting (iii) in equation (i), we obtain

$y=4$

Hence, speed of Ritu in still water is 6 km / h and speed of stream is 4 km / h.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

Ans: Assuming a woman takes x number of days and a man takes y number of days.

So, work done by a women in one day $=\frac{1}{x}$

Work done by a man in one day $=\frac{1}{y}$

Writing the algebraic representation using the information given in the question:

$4\left(\frac{2}{x}+\frac{5}{y}\right)=1$

$\frac{2}{x}+\frac{5}{y}=\frac{1}{4} \qquad \ldots\ldots$ (i)

$3\left(\frac{3}{x}+\frac{6}{y}\right)=1$

$\frac{3}{x}+\frac{6}{y}=\frac{1}{3} \qquad \ldots\ldots$ (ii)

Taking $\frac{1}{x}=p$ and $\frac{1}{y}=q$,

Now,

$$2p+5q=\frac{1}{4}$$

$$8p+20q=1 \qquad \ldots\ldots \text{(iii)}$$

$$3p+6q=\frac{1}{3}$$

$$9p+18q=1 \qquad \ldots\ldots \text{(iv)}$$

Using cross multiplication method:

$$\frac{p}{-20-(-18)}=\frac{q}{-9-(-8)}=\frac{1}{144-180}$$

$$\frac{p}{-2}=\frac{q}{-1}=\frac{1}{-36}$$

$$p=\frac{1}{18},y=\frac{1}{36}$$

$$\frac{1}{x}=\frac{1}{18},\frac{1}{y}=\frac{1}{36}$$

Therefore, $x=18$ and $y=36$.

Hence, a woman takes 18 days to finish a work while a man takes 36 days to finish a work.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Ans: Assuming that the speed of train is u km/h and the speed of bus is v km/h.

Writing the algebraic representation using the information given in the question:

$$\frac{60}{u}+\frac{240}{v}=4 \qquad \ldots\ldots \text{(i)}$$

$$\frac{100}{u}+\frac{200}{v}=\frac{25}{6} \qquad \ldots\ldots \text{(ii)}$$

Taking $\frac{1}{u}=p$ and $\frac{1}{v}=q$, we get

$$60p+240q=4 \qquad \ldots\ldots \text{(iii)}$$

$100p+200q=\frac{25}{6}$

$600p+1200q=25 \qquad \ldots\ldots \text{(iv)}$

Multiplying equation (iii) by 10, we get

$600p+2400q=40 \qquad \ldots\ldots \text{(v)}$

Subtracting equation (iv) from equation (v), we get

$1200q=15$

$q=\frac{1}{80} \qquad \ldots\ldots \text{(vi)}$

Substituting (vi) in equation (iii), we get

$60p=1$

$p=\frac{1}{60}$

So, $\frac{1}{u}=\frac{1}{60}, \frac{1}{v}=\frac{1}{80}$

$u=60$ and $v=80$.

Hence, speed of train is is $60\ \text{km}/\text{h}$ and speed of bus is $80\ \text{km}/\text{h}$.

Exercise 3.7

1. The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differs by 30 years. Find the ages of Ani and Biju.

Ans : We know that Ani can be three years older than Biju or Biju can be three years older than Ani. However, in any case, Ani's father's age will be 30 years more than that of Cathy's age.

Assuming the age of Ani be x and age of Biju be y.

Age of Dharam $=2x$ years.

Age of Cathy $=\frac{y}{2}$ years.

Case 1: Ani is three years older than Biju,

Writing the algebraic representation using the information given in the question:

$x-y=3 \qquad \ldots\ldots \text{(i)}$

$4x-y=60 \qquad \ldots\ldots \text{(ii)}$

Subtracting equation (i) from equation (ii), we obtain

$3x=57$

$x=19 \qquad \ldots\ldots \text{(iii)}$

Therefore, Ani's age is 19 years
And Biju's age is $=19-3=16$ years.
Case 2: Biju is three years older than Ani,
Writing the algebraic representation using the information given in the question:
$y-x=3$ …… (iv)
$4x-y=60$ …… (v)
Adding equation (iv) and equation (v), we get
$3x=63$
$x=21$ …… (iii)
Therefore, Ani's age is 21 years
And Biju's age is $=21+3=24$ years.

2. One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what isthe amount of their (respective) capital? [From the Bijaganita of Bhaskara II]
[Hint: x + 100 = 2 (y − 100), y + 10 = 6(x − 10)]
Ans: Assuming that those friends were having Rs x and Rs y with them.
Writing the algebraic representation using the information given in the question:
$x+100=2(y-100)$
$x+100=2y-200$
$x-2y=-300$ …… (i)
$6(x-10)=(y+10)$
$6x-y=70$ …… (ii)
Multiplying equation (ii) by 2, we obtain
$12x-2y=140$ …… (iii)
Subtracting equation (i) from equation (iii), we get
$11x=140+300$
$x=40$ …… (iv)
Substituting (iv) in equation (i), we get
$40-2y=-300$
$2y=340$
$y=170$
Therefore, those friends has Rs 40 and Rs 170.

3. A train covered a certain distance at a uniform speed. If the train would have been $10\ km/h$ faster, it would have taken 2 hours less than the scheduled time. And if the train were slower by $10\ km/h$; it would have

taken 3 hours more than the scheduled time. Find the distance covered by the train.

Ans : Assuming that the speed of train is $x\ \text{km}/\text{h}$ and it takes t hours to travel the given distance.

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken to travel the distance}}$$

$$x = \frac{d}{t}$$

$$d = xt \qquad \ldots\ldots \text{(i)}$$

Writing the algebraic representation using the information given in the question:

$$(x+10) = \frac{d}{(t-2)}$$

$$(x+10)(t-2) = d$$

$$xt + 10t - 2x - 20 = d$$

From equation (i):

$$-2x + 10t = 20 \qquad \ldots\ldots \text{(ii)}$$

$$(x-10) = \frac{d}{(t+3)}$$

$$(x-10)(t+3) = d$$

$$xt - 10t + 3x - 30 = d$$

From equation (i):

$$3x - 10t = 30 \qquad \ldots\ldots \text{(iii)}$$

Adding equations (ii) and (iii), we get

$$x = 50$$

Using equation (ii):

$$-100 + 10t = 20$$

$$t = 12 \text{ hours}$$

From equation (i), we get

Distance to travel $= d = xt$

$$= 50 \times 12$$

$$= 600 \text{ km}$$

So, the distance covered by the train is 600 km.

4. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Ans: Assuming that the number of rows be x and number of students be y.

Total number of students in the class

$= \text{Number of Rows} \times \text{Number of students in a row}$

$= xy$

Writing the algebraic representation using the information given in the question:

Condition 1:

Total number of students $= (x-1)(y+3)$

$xy = xy - y + 3x - 3$

$3x - y = 3$ …… (i)

Condition 2:

Total number of students $= (x+2)(y-3)$

$xy = xy + 2y - 3x - 6$

$3x - 2y = -6$ …… (ii)

Subtracting equation (ii) from equation (i),

$(3x - y) - (3x - 2y) = 3 - (-6)$

$-y + 2y = 3 + 6$

$y = 9$

Using equation (i), we get

$3x - 9 = 3$

$x = 4$

Therefore, the number of rows are 4, number of students in a row are 9 and total students in a class $= 4 \times 9 = 36$.

5. In a $\Delta ABC, \angle C = 3, \angle B = 2(\angle A + \angle B)$. Find the three angles.

Ans: $\angle C = 3\angle B = 2(\angle A + \angle B)$

$3\angle B = 2(\angle A + \angle B)$

$3\angle B = 2\angle A + 2\angle B$

$\angle B = 2\angle A$

$2\angle A - \angle B = 0$ …… (i)

The sum of all the angles of a triangle is 180°.

Therefore,

$\angle A + \angle B + \angle C = 180^\circ$

$\angle A + \angle B + 3\angle B = 180^\circ$

$\angle A + 4\angle B = 180^\circ$ …… (ii)

Multiplying equation (i) by 4, we obtain

$8\angle A - 4\angle B = 0$ …… (iii)

Adding equation (ii) and (iii), we obtain

$9\angle A = 180^\circ$

$\angle A = 20^\circ$

From equation (ii), we get

$20^\circ + 4\angle B = 180^\circ$

$4\angle B = 160°$
$\angle B = 40°$
$\angle C = 3\angle B$
$= 3 \times 40° = 120°$
Hence, the three angles of the triangle are $\angle A = 20^{\circ}$, $\angle B = 40°$ and $\angle C = 120°$.

6. Draw the graphs of the equations $5x - y = 5$ and $3x - y = 3$. Determine the coordinates of the vertices of the triangle formed by these lines and the $y -$ axis.

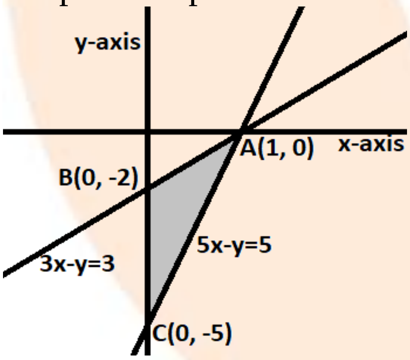
Ans: Solution table for $5x - y = 5$:
$y = 5x - 5$

x	0	1	2
y	–5	0	5

Solution table for $3x - y = 3$:
$y = 3x - 3$

x	0	1	2
y	–3	0	3

Graphical representation:



From the figure above we can observe that the required triangle is ΔABC formed by the given lines and $y -$ aixs.

7. Solve the following pair of linear equations.
(i) $px + qy = p - q$; $qx - py = p + q$
Ans: $px + qy = p - q$ …… (i)
$qx - py = p + q$ …… (ii)
Multiplying equation (i) by p and equation (ii) by q, we obtain

$p^2x + pqy = p^2 - pq$ ...... (iii)

$q^2x - qpy = pq + q^2$ ...... (iv)

Adding equation (iii) and equation (iv), we get

$p^2x + q^2x = p^2 + q^2$

$\left(p^2 + q^2\right)x = p^2 + q^2$

$x = \dfrac{p^2 + q^2}{p^2 + q^2} = 1$

From equation (i), we get

$p(1) + qy = p - q$

$qy = -q$

$y = -1$

(ii) $ax + by = c$; $bx + ay = 1 + c$

Ans: $ax + by = c$ ...... (i)

$bx + ay = 1 + c$ ...... (ii)

Multiplying equation (i) by a and equation (ii) by b, we get

$a^2x + aby = ac$ ...... (iii)

$b^2x + aby = b + bc$ ...... (iv)

Subtracting equation (iv) from equation (iii),

$\left(a^2 - b^2\right)x = ac - bc - b$

$x = \dfrac{ac - bc - b}{\left(a^2 - b^2\right)}$

From equation (i) we get

$ax + by = c$

$a\left\{\dfrac{c(a-b) - b}{a^2 - b^2}\right\} + by = c$

$\dfrac{ac(a-b) - ab}{a^2 - b^2} + by = c$

$by = c - \dfrac{ac(a-b) - ab}{a^2 - b^2}$

$by = \dfrac{a^2c - b^2c - a^2c + abc + ab}{a^2 - b^2}$

$by = \dfrac{abc - b^2c + ab}{a^2 - b^2}$

$$by = \frac{bc(a-b)+ab}{a^2-b^2}$$

$$y = \frac{c(a-b)+a}{a^2-b^2}$$

(iii) $\mathbf{\frac{x}{a}-\frac{y}{b}=0,\ ax+by=a^2+b^2}$

Ans: $\frac{x}{a}-\frac{y}{b}=0$

Or, $bx-ay=0$ …… (i)

$ax+by=a^2+b^2$ …… (ii)

Multiplying equation (i) by b and equation (ii) by a, we get

$b^2x-aby=0$ …… (iii)

$a^2x+aby=a^3+ab^2$ …… (iv)

Adding equation (iii) and equation (iv), we get

$b^2x+a^2x=a^3+ab^2$

$x\left(b^2+a^2\right)=a\left(a^2+b^2\right)$

$x=a$

$b(a)-ay=0$

$ab-ay=0$

$ay=ab$

$y=b$

(iv) $\mathbf{\left(a-b\right)x+\left(a+b\right)y=a^2-2ab-b^2;\ \left(a+b\right)\left(x+y\right)=a^2+b^2}$

Ans: $(a-b)x+(a+b)y=a^2-2ab-b^2$ …… (i)

$(a+b)(x+y)=a^2+b^2$

$(a+b)x+(a+b)y=a^2+b^2$ …… (ii)

Subtracting equation (ii) from (i), we get

$\left(a-b\right)x-\left(a+b\right)x=\left(a^2-2ab-b^2\right)-\left(a^2+b^2\right)$

$(a-b-a-b)x=-2ab-2b^2$

$-2bx=-2b(a+b)$

$x=a+b$

Using equation (i), we get

$(a-b)(a+b)+(a+b)y=a^2-2ab-b^2$

$a^2-b^2+(a+b)y=a^2-2ab-b^2$

$$(a+b)y=-2ab$$
$$y=\frac{-2ab}{a+b}$$

(v) $152x-378y=-74,\ \ -378x+152y=-604$

Ans: $152x-378y=-74$

$$76x-189y=-37$$
$$x=\frac{189y-37}{76} \qquad \ldots\ldots\ (i)$$
$$-378x+152y=-604$$
$$-189x+76y=-302 \qquad \ldots\ldots\ (ii)$$

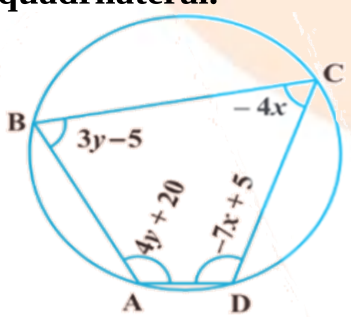
Substituting thje value of x in equation (ii), we get

$$-(189)^2y+189\times 37+(76)^2y=-302\times 76$$
$$189\times 37+302\times 76=(189)^2y-(76)^2y$$
$$6993+22952=(189-76)(189+76)y$$
$$29945=(113)(265)y$$
$$y=1$$

From equation (i), we get

$$x=\frac{189(1)-37}{76}$$
$$x=\frac{189-37}{76}$$
$$x=2$$

8. ABCD is a cyclic quadrilateral. Find the angles of the cyclic quadrilateral.is a cyclic quadrilateral finds the angles of the cyclic quadrilateral.



Ans: The sum of all the angles of a cyclic quadrilateral is 180°.

Therefore,

$$\angle A+\angle C=180$$

$4y+20-4x=180$
$-4x+4y=160$
$x-y=-40$ …… (i)
Also,
$\angle B+\angle D=180$
$3y-5-7x+5=180$
$-7x+3y=180$ …… (ii)
Multiplying equation (i) by 3, we get
$3x-3y=-120$ …… (iii)
Adding equation (ii) and equation (iii), we get
$-7x+3x=180-120$
$-4x=60$
$x=-15$
By using equation (i), we get
$x-y=-40$
$-15-y=-40$
$y=-15+40=25$
$\angle A=4y+20=4(25)+20=120^{\circ}$
$\angle B=3y-5=3(25)-5=70^{\circ}$
$\angle C=-4x=-4(-15)=60^{\circ}$
$\angle D=-7x+5=-7(-15)+5=110^{\circ}$