Exercise 2.3 — Page No: 2.57

1. Apply division algorithm to find the quotient q(x) and remainder r(x) on dividing f(x) by g(x) in each of the following:

(i) $f(x) = x^3 - 6x^2 + 11x - 6$, $g(x) = x^2 + x + 1$

Solution:

Given,

$f(x) = x^3 - 6x^2 + 11x - 6$, $g(x) = x^2 + x + 1$

$$
\begin{array}{r|rrrr}
 & x & -7 & & \\
\hline
x^2+x+1 & x^3 & -6x^2 & +11x & -6 \\
- & x^3 & +x^2 & +x & \\
\hline
 & & -7x^2 & +10x & -6 \\
 & - & -7x^2 & -7x & -7 \\
\hline
 & & & 17x & +1
\end{array}
$$

Thus,

$q(x) = x - 7$ and $r(x) = 17x + 1$

(ii) $f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 3$, $g(x) = 2x^2 + 7x + 1$

Solution:

Given,

$f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 3$ and $g(x) = 2x^2 + 7x + 1$

$$
\begin{array}{r|rrrrr}
 & 5x^2 & -9x & -2 & & \\
\hline
2x^2+7x+1 & 10x^4 & +17x^3 & -62x^2 & +30x & -3 \\
- & 10x^4 & +35x^3 & +5x^2 & & \\
\hline
 & & -18x^3 & -67x^2 & +30x & -3 \\
 & - & -18x^3 & -63x^2 & -9x & \\
\hline
 & & & -4x^2 & +39x & -3 \\
 & & - & -4x^2 & -14x & -2 \\
\hline
 & & & & 53x & -1
\end{array}
$$

Thus,

q(x) = $5x^2 – 9x - 2$ and r(x) = $53x - 1$

(iii) f(x) = $4x^3 + 8x^2 + 8x + 7$, g(x)= $2x^2 – x + 1$

Solution:

Given,

f(x) = $4x^3 + 8x^2 + 8x + 7$ and g(x)= $2x^2 – x + 1$

$$
\begin{array}{r|rrrr}
 & 2x & +5 & & \\
\hline
2x^2 - x + 1 & 4x^3 & +8x^2 & +8x & +7 \\
- & & & & \\
 & 4x^3 & -2x^2 & +2x & \\
\hline
 & & 10x^2 & +6x & +7 \\
 & & - & & \\
 & & 10x^2 & -5x & +5 \\
\hline
 & & & 11x & +2
\end{array}
$$

Thus,

q(x) = $2x + 5$ and r(x) = $11x + 2$

(iv) f(x) = $15x^3 – 20x^2 + 13x – 12$, g(x) = $x^2 – 2x + 2$

Solution:

Given,
f(x) = $15x^3 – 20x^2 + 13x – 12$ and g(x) = $x^2 – 2x + 2$

$$
\begin{array}{r|rrrr}
 & 15x & +10 & & \\
\hline
x^2 - 2x + 2 & 15x^3 & -20x^2 & +13x & -12 \\
- & & & & \\
 & 15x^3 & -30x^2 & +30x & \\
\hline
 & & 10x^2 & -17x & -12 \\
 & & - & & \\
 & & 10x^2 & -20x & +20 \\
\hline
 & & & 3x & -32
\end{array}
$$

Thus,
q(x) = $15x + 10$ and r(x) = $3x - 32$

2. Check whether the first polynomial is a factor of the second polynomial by applying the division algorithm:

(i) $g(t) = t^2 – 3$; $f(t)=2t^4 + 3t^3 – 2t^2 – 9t – 12$
Solution:

Given,
$g(t) = t^2 – 3$; $f(t) = 2t^4 + 3t^3 – 2t^2 – 9t – 12$

$$
\begin{array}{r|rrrrr}
 & 2t^2 & +3t & +4 & & \\
\hline
t^2-3 & 2t^4 & +3t^3 & -2t^2 & -9t & -12 \\
- & & & & & \\
 & 2t^4 & +0t^3 & -6t^2 & & \\
\hline
 & & 3t^3 & +4t^2 & -9t & -12 \\
 & - & & & & \\
 & & 3t^3 & +0t^2 & -9t & \\
\hline
 & & & 4t^2 & +0t & -12 \\
 & & - & & & \\
 & & & 4t^2 & +0t & -12 \\
\hline
 & & & & & 0
\end{array}
$$

Since, the remainder $r(t) = 0$ we can say that the first polynomial is a factor of the second polynomial.

(ii) $g(x) = x^3 – 3x + 1$; $f(x) = x^5 – 4x^3 + x^2 + 3x + 1$
Solution:

Given,
$g(x) = x^3 – 3x + 1$; $f(x) = x^5 – 4x^3 + x^2 + 3x + 1$

$$
\begin{array}{r|rrrrrr}
 & x^2 & & & & & \\
\hline
x^3-3x+1 & x^5 & +0x^4 & -4x^3 & +x^2 & +3x & +1 \\
- & & & & & & \\
 & x^5 & +0x^4 & -3x^3 & +x^2 & & \\
\hline
 & & & & +0x^2 & +3x & +1 \\
 & & & - & & & \\
 & & & -x^3 & +0x^2 & +3x & -1 \\
\hline
 & & & & & & 2
\end{array}
$$

Since, the remainder $r(x) = 2$ and not equal to zero we can say that the first polynomial is not a factor of the second polynomial.

(iii) $g(x) = 2x^2 – x + 3$; $f(x) = 6x^5 − x^4 + 4x^3 – 5x^2 – x –15$
Solution:

Given,

g(x) = $2x^2 – x + 3$; f(x)=$6x^5 − x^4 + 4x^3 – 5x^2 – x –15$

$$
\begin{array}{r|rrrrrr}
 & 3x^3 & +x^2 & -2x & -5 & & \\
\hline
2x^2 - x + 3 & 6x^5 & -x^4 & +4x^3 & -5x^2 & -x & -15 \\
- & 6x^5 & -3x^4 & +9x^3 & & & \\
\hline
 & & 2x^4 & -5x^3 & -5x^2 & -x & -15 \\
- & & 2x^4 & -x^3 & +3x^2 & & \\
\hline
 & & & -4x^3 & -8x^2 & -x & -15 \\
- & & & -4x^3 & +2x^2 & -6x & \\
\hline
 & & & & -10x^2 & +5x & -15 \\
- & & & & -10x^2 & +5x & -15 \\
\hline
 & & & & & & 0
\end{array}
$$

Since, the remainder r(x) = 0 we can say that the first polynomial is not a factor of the second polynomial.

3. Obtain all zeroes of the polynomial f(x)= $2x^4 + x^3 – 14x^2 – 19x–6$, if two of its zeroes are -2 and -1.
Solution:

Given,

f(x)= $2x^4 + x^3 – 14x^2 – 19x – 6$

If the two zeros of the polynomial are -2 and -1, then its factors are (x + 2) and (x + 1)

⇒ $(x+2)(x+1) = x^2 + x + 2x + 2 = x^2 + 3x +2$ …… (i)

This means that (i) is a factor of f(x). So, performing division algorithm we get,

$$
\begin{array}{l|rrrrr}
 & 2x^2 & -5x & -3 & & \\
\hline
x^2+3x+2 & 2x^4 & +x^3 & -14x^2 & -19x & -6 \\
 & - & & & & \\
 & 2x^4 & +6x^3 & +4x^2 & & \\
\hline
 & & -5x^3 & -18x^2 & -19x & -6 \\
 & & - & & & \\
 & & -5x^3 & -15x^2 & -10x & \\
\hline
 & & & -3x^2 & -9x & -6 \\
 & & & - & & \\
 & & & -3x^2 & -9x & -6 \\
\hline
 & & & & & 0
\end{array}
$$

The quotient is $2x^2 - 5x – 3$.

⇒ $f(x)= (2x^2 - 5x – 3)(x^2 + 3x +2)$

For obtaining the other 2 zeros of the polynomial

We put,

$2x^2 - 5x – 3 = 0$

⇒ $(2x + 1)(x – 3) = 0$

∴ $x = -1/2$ or 3

Hence, all the zeros of the polynomial are -2, -1, -1/2 and 3.

4. Obtain all zeroes of $f(x) = x^3 + 13x^2 + 32x + 20$, if one of its zeros is -2.
Solution:

Given,
$f(x)= x^3 + 13x^2 + 32x + 20$
And, -2 is one of the zeros. So, $(x + 2)$ is a factor of $f(x)$,
Performing division algorithm, we get

$$\begin{array}{r|llll} & x^2 & +11x & +10 & \\ \hline x+2 & x^3 & +13x^2 & +32x & +20 \\ & - & & & \\ & x^3 & +2x^2 & & \\ \hline & & 11x^2 & +32x & +20 \\ & & - & & \\ & & 11x^2 & +22x & \\ \hline & & & 10x & +20 \\ & & & - & \\ & & & 10x & +20 \\ \hline & & & & 0 \end{array}$$

⇒ $f(x) = (x^2 + 11x + 10)(x + 2)$

So, putting $x^2 + 11x + 10 = 0$ we can get the other 2 zeros.

⇒ $(x + 10)(x + 1) = 0$

∴ $x = -10$ or -1

Hence, all the zeros of the polynomial are -10, -2 and -1.

5. Obtain all zeroes of the polynomial $f(x) = x^4 – 3x^3 – x^2 + 9x – 6$, if the two of its zeroes are $-\sqrt{3}$ and $\sqrt{3}$.

Solution:

Given,

$f(x) = x^4 – 3x^3 – x^2 + 9x – 6$

Since, two of the zeroes of polynomial are $-\sqrt{3}$ and $\sqrt{3}$ so, $(x + \sqrt{3})$ and $(x – \sqrt{3})$ are factors of f(x).

⇒ $x^2 - 3$ is a factor of f(x). Hence, performing division algorithm, we get

$$
\begin{array}{r|rrrrr}
 & x^2 & -3x & +2 & & \\
\hline
x^2 - 3 & x^4 & -3x^3 & -x^2 & +9x & -6 \\
- & x^4 & +0x^3 & -3x^2 & & \\
\hline
 & & -3x^3 & +2x^2 & +9x & -6 \\
 & - & -3x^3 & +0x^2 & +9x & \\
\hline
 & & & 2x^2 & +0x & -6 \\
 & & - & 2x^2 & +0x & -6 \\
\hline
 & & & & & 0
\end{array}
$$

⇒ f(x)= $(x^2 - 3x + 2)(x^2 - 3)$

So, putting $x^2 - 3x + 2 = 0$ we can get the other 2 zeros.

⇒ (x - 2)(x - 1) = 0

∴ x = 2 or 1

Hence, all the zeros of the polynomial are −√3, 1, √3 and 2.

6. Obtain all zeroes of the polynomial f(x)= $2x^4 - 2x^3 - 7x^2 + 3x + 6$, if the two of its zeroes are −√(3/2) and √(3/2).

Solution:

Given,

f(x)= $2x^4 - 2x^3 - 7x^2 + 3x + 6$

Since, two of the zeroes of polynomial are −√(3/2) and √(3/2) so, (x + √(3/2)) and (x – √(3/2)) are factors of f(x).

⇒ $x^2 - (3/2)$ is a factor of f(x). Hence, performing division algorithm, we get

$$
\begin{array}{r|rrrrr}
 & 2x^2 & -2x & -4 & & \\
\hline
x^2 - \frac{3}{2} & 2x^4 & -2x^3 & -7x^2 & +3x & +6 \\
 & - & & & & \\
 & 2x^4 & +0x^3 & -3x^2 & & \\
\hline
 & & -2x^3 & -4x^2 & +3x & +6 \\
 & & - & & & \\
 & & -2x^3 & +0x^2 & +3x & \\
\hline
 & & & -4x^2 & +0x & +6 \\
 & & & - & & \\
 & & & -4x^2 & +0x & +6 \\
\hline
 & & & & & 0
\end{array}
$$

⇒ f(x)= $(2x^2 - 2x - 4)(x^2 – 3/2)= 2(x^2 - x - 2)(x^2 – 3/2)$

So, putting $x^2 - x - 2 = 0$ we can get the other 2 zeros.

⇒ $(x - 2)(x + 1) = 0$

∴ $x = 2$ or -1

Hence, all the zeros of the polynomial are $-\sqrt{(3/2)}$, -1, $\sqrt{(3/2)}$ and 2.

7. Find all the zeroes of the polynomial $x^4 + x^3 – 34x^2 – 4x + 120$, if the two of its zeros are 2 and -2.

Solution:

Let,

$f(x) = x^4 + x^3 – 34x^2 - 4x + 120$

Since, two of the zeroes of polynomial are −2 and 2 so, (x + 2) and (x – 2) are factors of f(x).

⇒ $x^2 – 4$ is a factor of f(x). Hence, performing division algorithm, we get

$$
\begin{array}{r|rrrrr}
 & x^2 & +x & -30 & & \\
\hline
x^2 - 4 & x^4 & +x^3 & -34x^2 & -4x & +120 \\
 & - & & & & \\
 & x^4 & +0x^3 & -4x^2 & & \\
\hline
 & & x^3 & -30x^2 & -4x & +120 \\
 & & - & & & \\
 & & x^3 & +0x^2 & -4x & \\
\hline
 & & & -30x^2 & +0x & +120 \\
 & & & - & & \\
 & & & -30x^2 & +0x & +120 \\
\hline
 & & & & & 0
\end{array}
$$

$\Rightarrow$ f(x)= $(x^2 + x - 30)(x^2 – 4)$

So, putting $x^2 + x - 30 = 0$ we can get the other 2 zeros.

$\Rightarrow (x + 6)(x - 5) = 0$

$\therefore$ x = -6 or 5

Hence, all the zeros of the polynomial are 5, -2, 2 and -6.