Assignment No.II

Problems on Gamma Function and Beta Function

1.Prove that,

i) $\int_0^\infty x^{1/2} e^{-x^{1/3}} dx = 315/16 \sqrt{\pi}$

ii) $\int_0^\infty x^7 e^{-2x^2} dx = 3/16$

iii) $\int_0^\infty x^2 e^{-h^2 x^2} dx = \sqrt{\pi}/4h^3$

iv) $\int_0^\infty \sqrt{y}\, e^{-y^3} dy = \sqrt{\pi}/3$

v) $\int_0^\infty e^{-x^4} dx = 1/4 \ulcorner 1/4$

vi) $\int_0^1 dx/\sqrt{x \log 1/x} = \sqrt{2\pi}$

vii) $\int_0^1 dx/\sqrt{-\log x} = \sqrt{\pi}$

viii) $\int_0^\infty x^{n-1} e^{-h^2 x^2} dx = \ulcorner n/2 / 2h^n$

2.i) To show that

$\int_0^\infty x^4 /4^x dx = 24 /(\log 4)^5$

ii) $\int_0^1 x^m (\log x)^n dx = (-1)^n \ulcorner n+1/(m+1)^{n+1}$

3. Show that

$\int_0^\infty x^n e^{-\sqrt{ax}} dx = 2(2n+1)! / a^{n+1}$, (n is integer)

4. Show that

$\int_0^\infty a^{-4x^2} dx = \sqrt{\pi} / 4\sqrt{\log a}$

5. $\int x^{n-1} e^{-ax} \cos bx\, dx = \ulcorner n / (a^2+b^2)^{n/2} \cos (n\tan^{-1} b/a)$

6. $\int x^{n-1} e^{-ax} \sin x\, dx = \ulcorner n / (a^2+b^2)^{n/2} \sin (n\tan^{-1} b/a)$

7. $\int_0^1 (x\log x)^3 dx = -3/128$