ASSIGNMENT No. 1

Dated:10-01-2024

Problems on Matrix

1.Find the inverse of the given problems

$$\begin{pmatrix} 2 & 2 & -3 \\ -3 & 2 & 2 \\ 2 & -3 & 2 \end{pmatrix}$$

2.Find the rank of the following matrices.

$$\begin{pmatrix} 0 & 1 & 2 & 1 \\ 2 & -3 & 0 & -1 \\ -1 & 1 & -1 & 0 \end{pmatrix}$$

3.Prove that the following matrix is orthogonal:

$$\begin{pmatrix} -2/3 & 1/3 & 2/3 \\ 2/3 & 2/3 & 1/3 \\ 1/3 & -2/3 & 2/3 \end{pmatrix}$$

4. i) Test whether the following system of linear equations is consistent.
If consistent solve: $x+y+z = 3$: $2x+y-z = 2$; $4x-y+2z = 5$
Ans $x = 1, y = 1, z = 1$
ii) Test whether the equations $2x-3y+7z = 5$; $3x+y-3z=13$; $2x+19y-47z = 32$ are consistent or not.

5. Show that the non singular matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$

satisfies the equation $A^2-2A-5I = 0$. Hence evaluate A^{-1}.

6. Find the inverse of matrix using cayley- Hamilton theorem.

$$\begin{pmatrix} 3 & 3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$$

7. Find the characteristics equation of the following matrices.

i) $\begin{matrix} -b & -c \\ 1 & 0 \end{matrix}$

ii) $\begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$

iii) $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -8 \\ 2 & -4 & 3 \end{pmatrix}$

iv) $\begin{pmatrix} -b & -c & -d \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

8. Find the eigen values and eigen vectors of the matrix

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$$

9. Show that

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{pmatrix} \begin{pmatrix} 1 & \tan\theta/2 \\ -\tan\theta/2 & 1 \end{pmatrix}$$

10. Find the eigen values of A^5 when

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{pmatrix}$$

11. Find the values of λ for which the equations

$(\lambda-1)x + (3\lambda+1)y + 2\lambda z = 0$

$(\lambda-1)x + (4\lambda-2)y + (\lambda+3)z = 0$

$2x + (3\lambda+1)y + 3(\lambda-1)z = 0$

are consistent, and find the ratio of x:y:z when λ has the smallest of these values. What happens when λ has the greatest of these values?

12. Verify that the following matrix is orthogonal:

$$\begin{pmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{pmatrix}$$

13. Prove that the inverse of an orthogonal matrix is orthogonal and its transpose is also Orthogonal.

14. Find the eigen values and eigen vectors of the matrix.

$$\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$$

15. Find the characteristic equation of the matrix A , hence find its inverse.

$$\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$$

16. Find the characteristic roots and characteristic vectors of the matrices:

i) $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 2 \\ 1 & 0 & 2 \end{pmatrix}$ ii) $\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$

iii) $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

17. Verify the Cayley- Hamilton theorem for the matrix A and find its inverse.

i) $\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ ii) $\begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$

iii). Using Cayley- Hamilton theorem , find A^8, if

$$\begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$$

Problems on Eigen values, Eigen vectors and Vectors Space.

18. Find the eigenvalues and eigenvectors of the matrix.

$$\begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}$$

19. Determine the largest eigenvalues and the corresponding eigenvector of the matrix

$$\begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

20. Examine the following vectors for linear dependence and find the relation if it exits.
$X_1 = (1,2,4)$, $X_2 = (2,-1,3)$, $X_3 = (0,1,2)$, $X_4 = (-3,7,2)$

21. Define linear dependence and independence of vectors.
Examine for linear dependence
$[1,0,2,11]$, $[3,1,2,11]$, $[4,6,2,-4]$, $[-6,0,-3,-4]$
and find the relation between them, if possible.

22. Is the system of vectors
$X_1 = (2,2,1)^T$, $X_2 = (1,3,1)^T$, $X_3 = (1,2,2)^T$ linearly independent.

23. Examine the following system of vectors for linearly dependence. If dependent, find the relation between them.
i) $X_1 = (1,-1,1)$, $X_2 = (2,1,1)$, $X_3 = (3,0,2)$.
ii) $X_1 = (1,2,3)$, $X_2 = (2,-2,6)$.
iii) $X_1 = (1,1,1,3)$, $X_2 = (1,2,3,4)$, $X_3 = (2,3,4,7)$.
iv) $X_1 = (1,-1,2,0)$, $X_2 = (2,1,1,1)$, $X_3 = (3,-1,2,-1)$, $X_4 = (3,0,3,1)$

24. Are the vectors $X_1 = (1,3,4,2)$, $X_2 = (3,-5,2,2)$, $X_3 = (2,-,1,3,2)$ linearly dependent? If so express one of these as a linear combination of others.

25. The vectors
$X_1 = (1,2,3)$, $X_2 = (1,0,0)$, $X_3 = (0,1,0)$, $X_4 = (0,0,1)$ are L.D.

26. To show that the vectors
$x = (1+I, 2i)$, $y = (1, 1+i)$ in V+2(C) are L.D. but $V_2(C)$ are L.I.

27. If x,y,z are L.I. vectors in a vector space V(F), then show that the vectors
i) $x + y$, $y + z$, $z + x$,
ii)) $x + y$, $x - y$, $x - 2y + z$
are also L.I.