Statistical Methods for Discrete Response, Time Series, and Panel Data (W271): Lab 2

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```r
library(knitr)
opts_chunk$set(tidy.opts = list(width.cutoff = 60), tidy = TRUE)

# Start with a clean R environment
rm(list = ls())

# Set Fixed random seed to replicate the results
set.seed(28740)

#library(ggplot2)
library(Hmisc)
```

```
## Loading required package: lattice

## Loading required package: survival

## Loading required package: Formula

## Loading required package: ggplot2

##
## Attaching package: 'Hmisc'

## The following objects are masked from 'package:base':
##
##     format.pval, units
```

```r
#library(car)
library(gridExtra)
library(dplyr)
```

```
##
## Attaching package: 'dplyr'

## The following object is masked from 'package:gridExtra':
##
##     combine

## The following objects are masked from 'package:Hmisc':
##
##     src, summarize
```

```
## The following objects are masked from 'package:stats':
##
##     filter, lag
```

```
## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```r
#library(scales)
#library(GGally)
#library(MASS)
#library(purrr)
#library(tidyr)
#library(nnet)
library(purrr)
library(imputeTS)
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```r
library(tsibble)
```

```
##
## Attaching package: 'tsibble'
```

```
## The following object is masked from 'package:dplyr':
##
##     id
```

```r
library(fabletools)
library(feasts)
library(fable)
library(lubridate)
```

```
##
## Attaching package: 'lubridate'
```

```
## The following objects are masked from 'package:tsibble':
##
##     interval, new_interval
```

```
## The following object is masked from 'package:base':
##
##     date
```

```r
library(forecast)
```

```
##
## Attaching package: 'forecast'
```

```
## The following objects are masked from 'package:fabletools':
##
```

```
##     accuracy, forecast, GeomForecast, StatForecast
```

```
library(seasonal)
library(tsbox)
library(tibble)
```

```
##
## Attaching package: 'tibble'

## The following object is masked from 'package:seasonal':
##
##     view
```

```
library(svMisc)
```

```
##
## Attaching package: 'svMisc'

## The following object is masked from 'package:utils':
##
##     ?
```

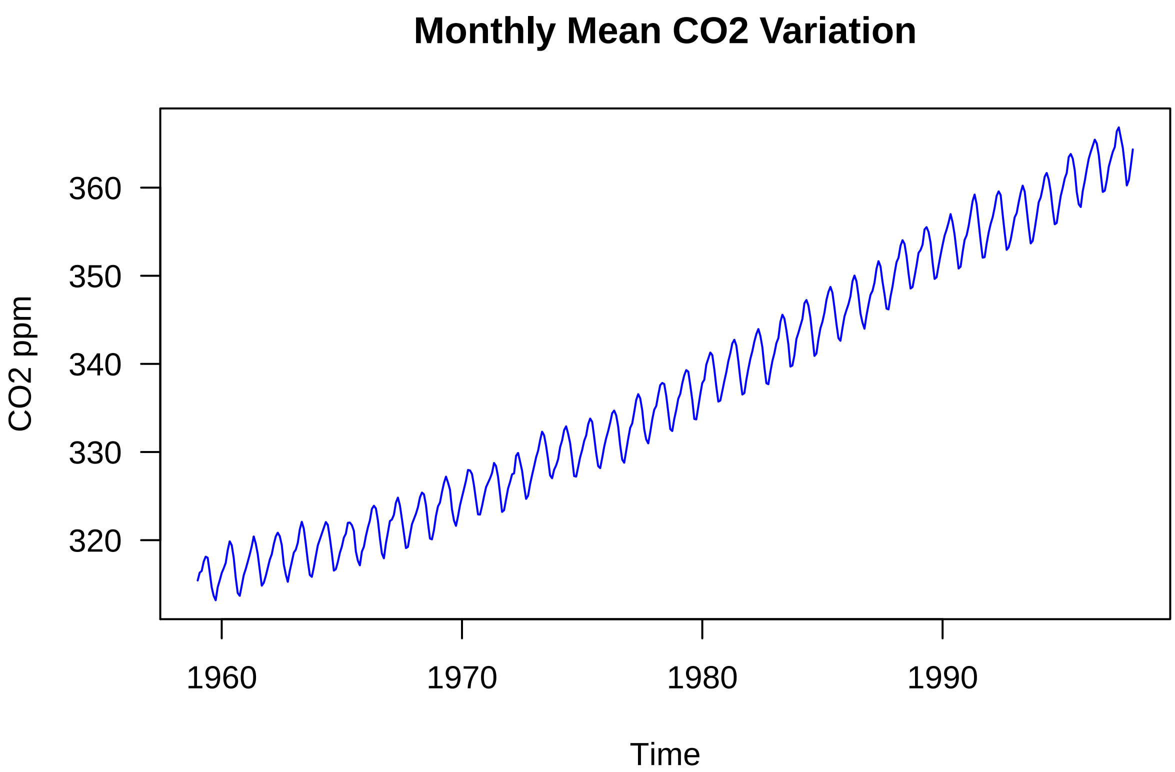
The Keeling Curve

In the 1950s, the geochemist Charles David Keeling observed a seasonal pattern in the amount of carbon dioxide present in air samples collected over the course of several years. He was able to attribute this pattern to the difference in the amount of land area and vegetation cover between the northern and southern hemispheres, and the resulting variation in global rates of photosynthesis as the hemispheres' seasons alternated throughout the year.

In 1958 Keeling began continuous monitoring of atmospheric carbon dioxide concentrations from the Mauna Loa Observatory in Hawaii and soon observed a trend increase carbon dioxide levels in addition to the seasonal cycle. He was able to attribute this trend increase to growth in global rates of fossil fuel combustion. This trend has continued to the present.

The `co2` data set in R's `datasets` package (automatically loaded with base R) is a monthly time series of atmospheric carbon dioxide concentrations measured in ppm (parts per million) at the Mauna Loa Observatory from 1959 to 1997. The curve graphed by this data is known as the 'Keeling Curve'.

```
plot(co2, ylab = expression("CO2 ppm"), col = "blue", las = 1)
title(main = "Monthly Mean CO2 Variation")
```


Monthly Mean CO2 Variation
CO2 ppm
320
330
340
350
360
1960
1970
1980
1990
Time

Part 1 (4 points)

Conduct a comprehensive Exploratory Data Analysis on the `co2` series. This should include thorough analyses of the trend, seasonal and irregular elements. Trends both in levels and growth rates should be discussed.

Answer Lets look at the data

```
head(co2)
```

```
##         Jan    Feb    Mar    Apr    May    Jun
## 1959 315.42 316.31 316.50 317.56 318.13 318.00
```

```
tail(co2)
```

```
##         Jul    Aug    Sep    Oct    Nov    Dec
## 1997 364.52 362.57 360.24 360.83 362.49 364.34
```

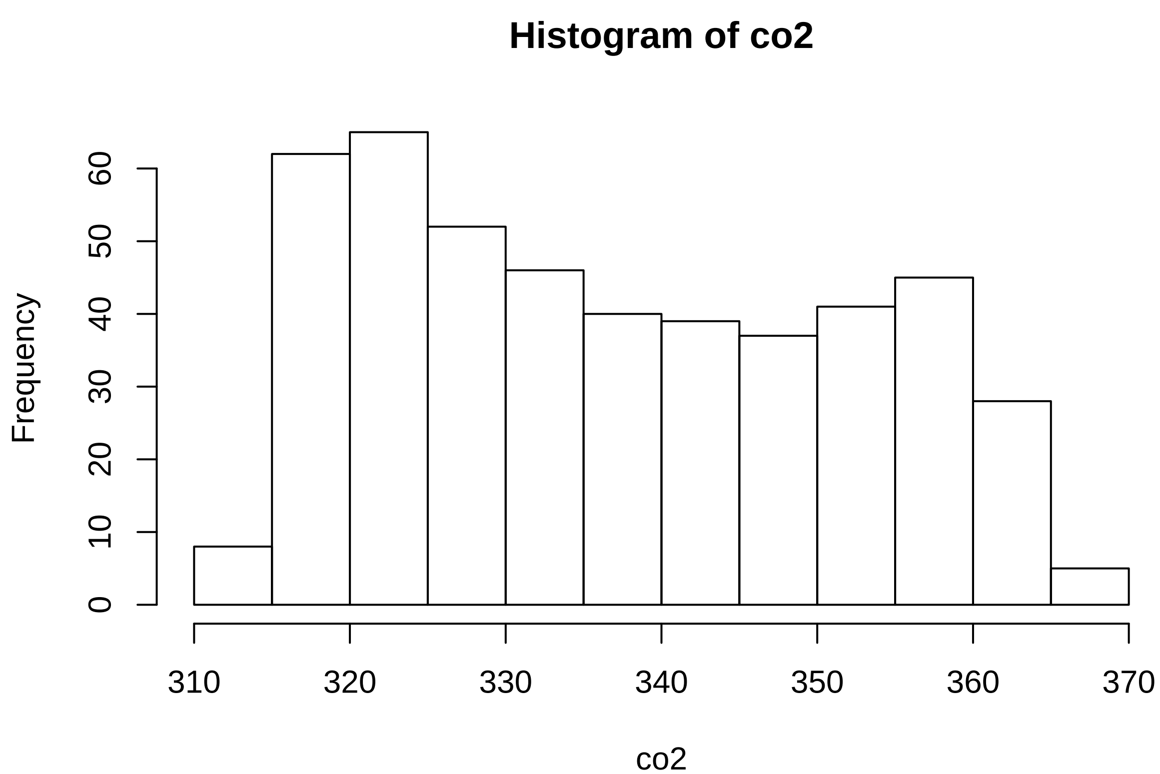
```
str(co2)
```

```
##  Time-Series [1:468] from 1959 to 1998: 315 316 316 318 318 ...
```

```
summary(co2)
```

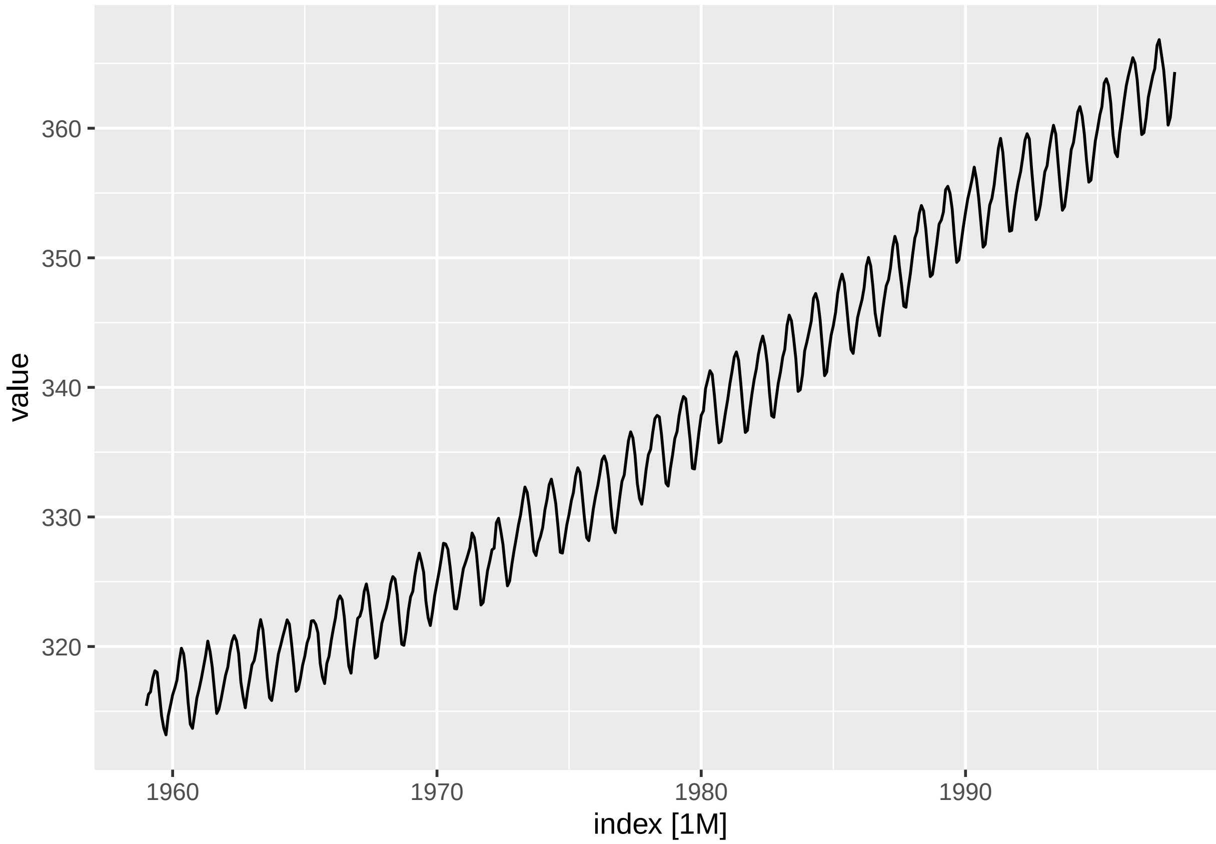
```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   313.2   323.5   335.2   337.1   350.3   366.8
```

```
hist(co2)
```



The co2 dataset is comprised of 468 observations of data from 1959 to 1998 where each datapoint is one month's data. The co2 levels in the dataset range from between 313.2 to 366.8. The histogram shows us that the most common co2 levels are between 320 and 325, and generally the frequency of co2 values decreases as co2 goes up. There is a small spike of frequencies between co2 level 355 and 360. There are no missing values in the dataset. Histogram of value is left skewed but that may not be correct representation of the co2 emission levels over time. So lets look at it using time dimension.

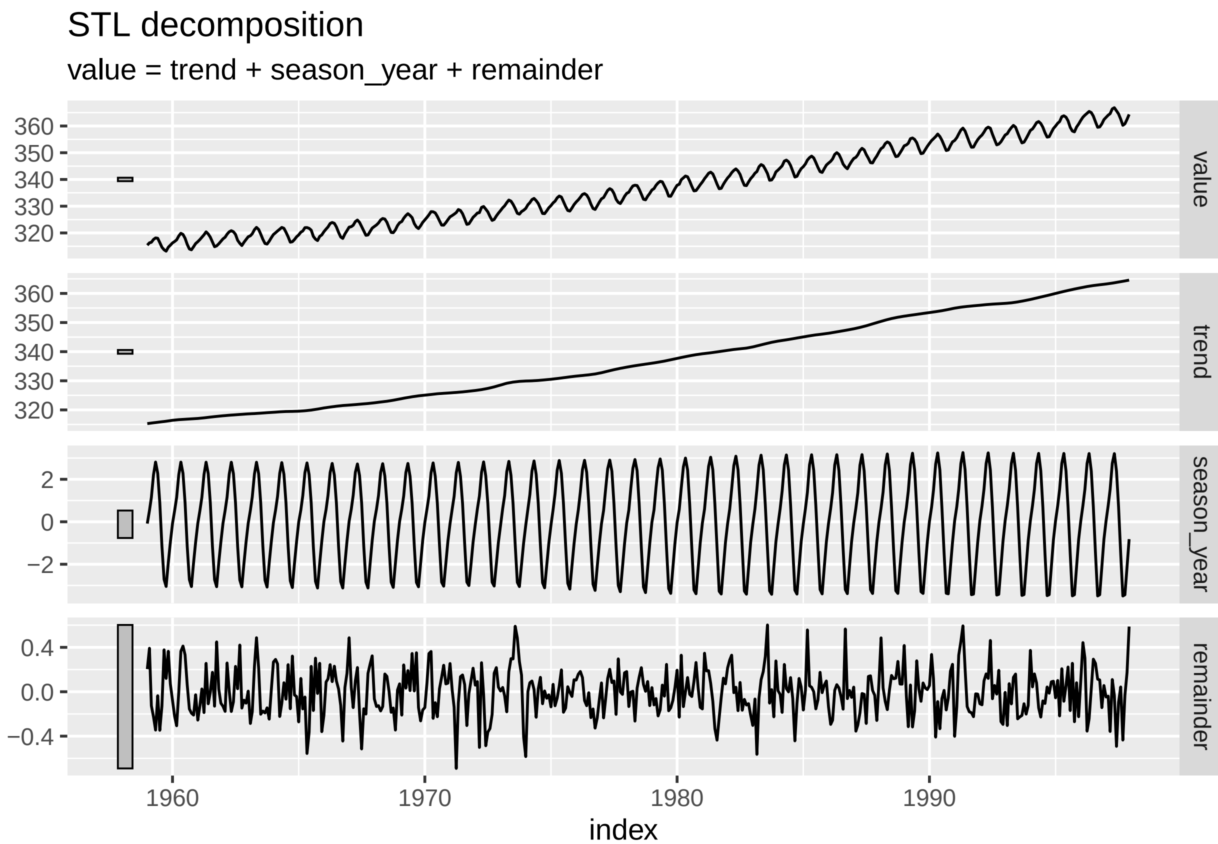
```
co2.ts <- as_tsibble(co2)
autoplot(co2.ts, .vars = value)
```



We observe two things 1. There is upward trend year on year in CO2 emissions 2. There is a seasonal fluctuation within a year

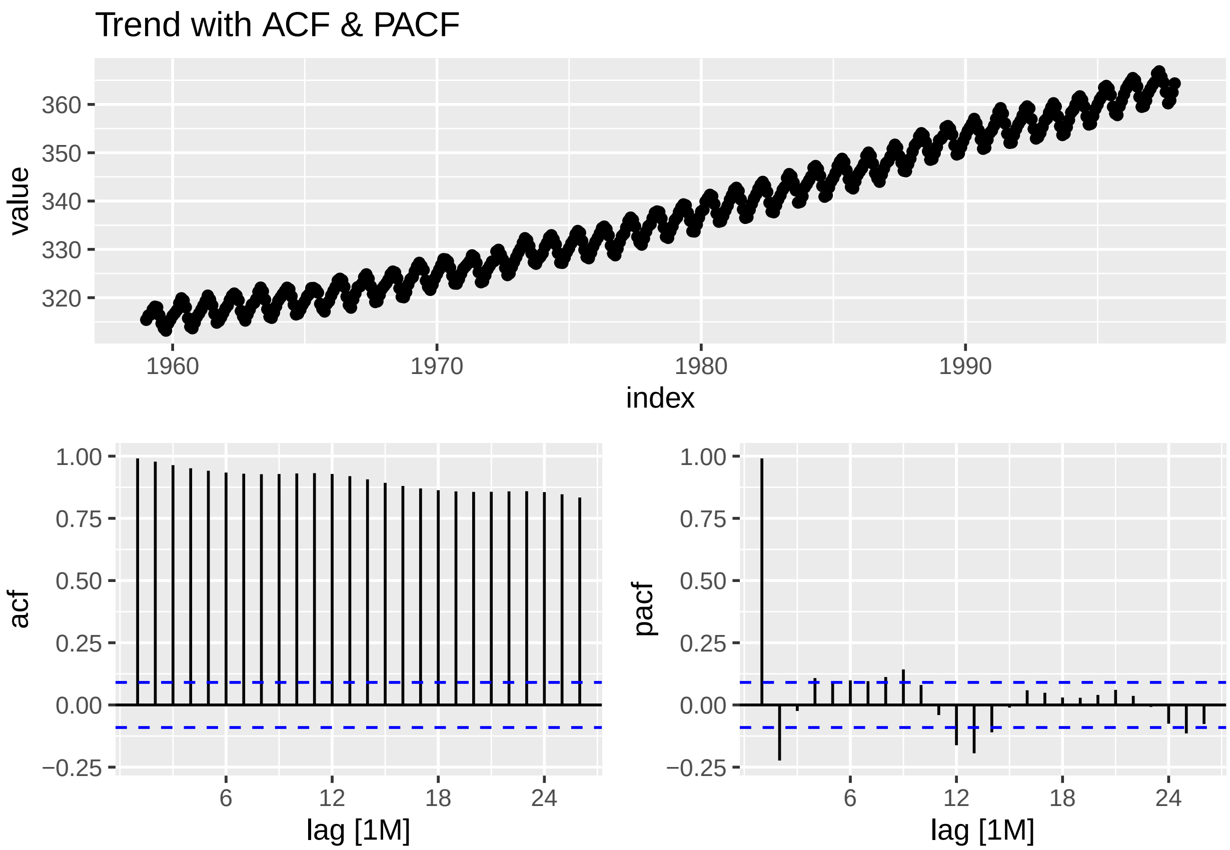
Lets try to decompose and see the individual components in more detail

```
co2.ts %>% model(STL(value)) %>% components() %>% autoplot()
```



When we decompose using STL model, we can see the upward trend separated from yearly seasonal cycles. We note that seasonal cycle ups and downs are growing with time, so we may need to stabilize the seasons for better predictions.

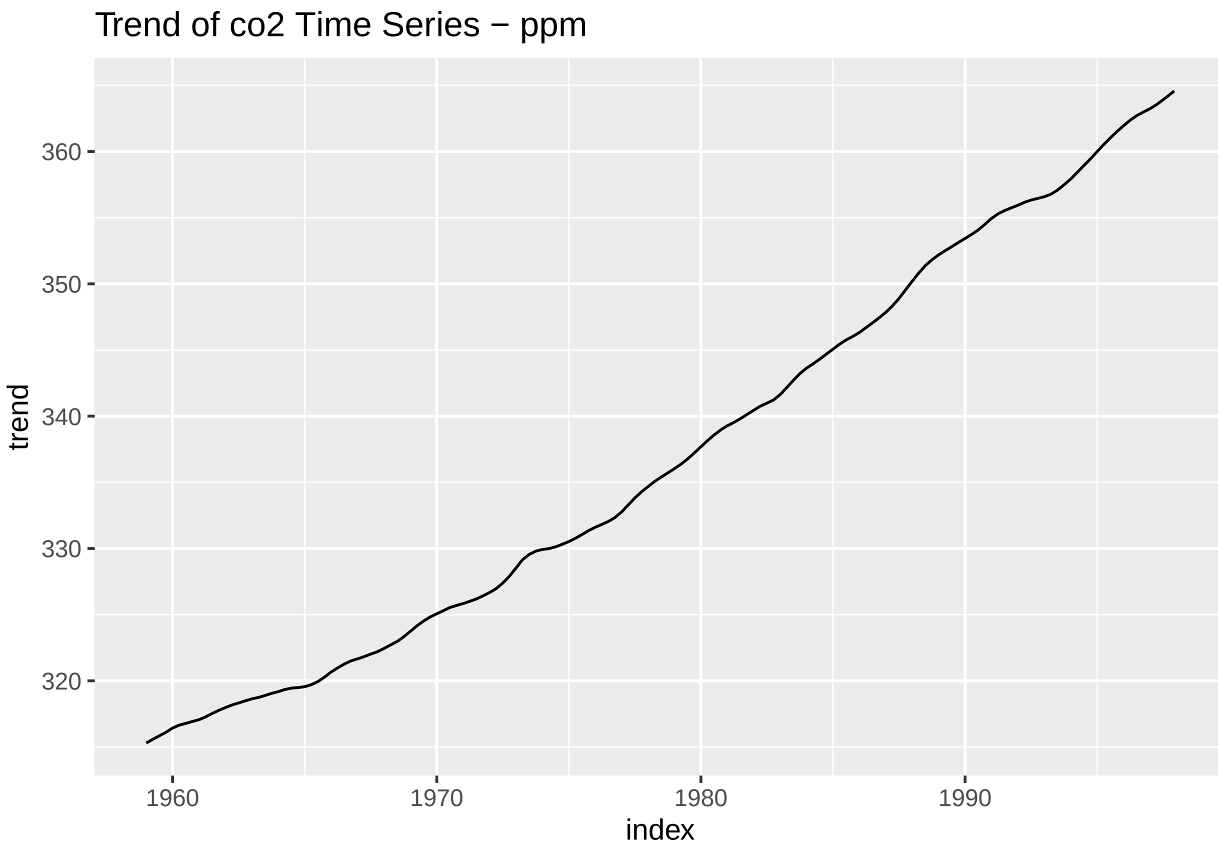
```
co2.ts %>% gg_tsdisplay(plot_type = "partial", y = value) + ggtitle("Trend with ACF & PACF")
```



The ACF tells us that there is autcorrelation even after 24 months. On the other hand PACF drops after 2 but again pops up at multples of 12 months. This indicates that the model could be AR model with 12 months seasonal cycles.

Now lets look at each component one by one starting with trend

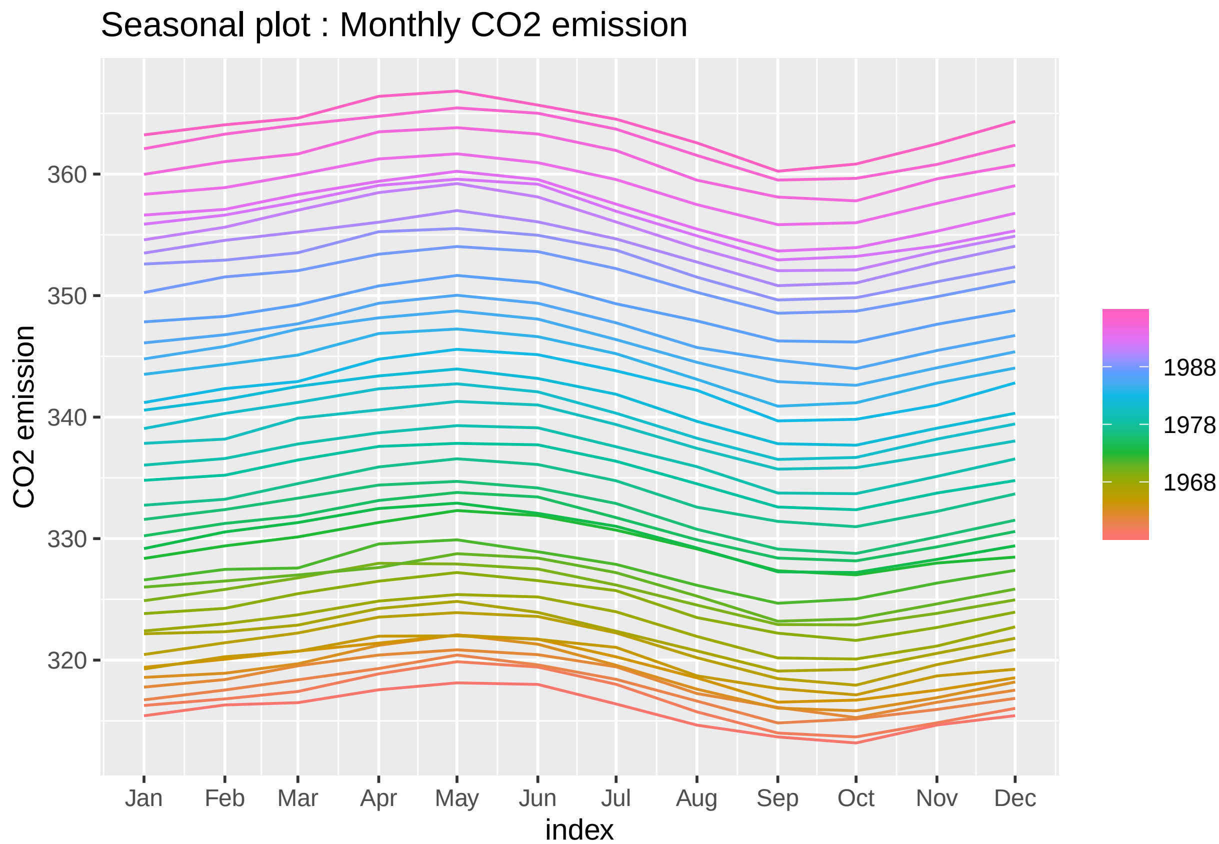
```
co2.ts.components <- co2.ts %>% model(STL(value)) %>% components()
ggplot(data = co2.ts.components) + geom_line(aes(x = index, y = trend)) +
    ggtitle("Trend of co2 Time Series - ppm")
```



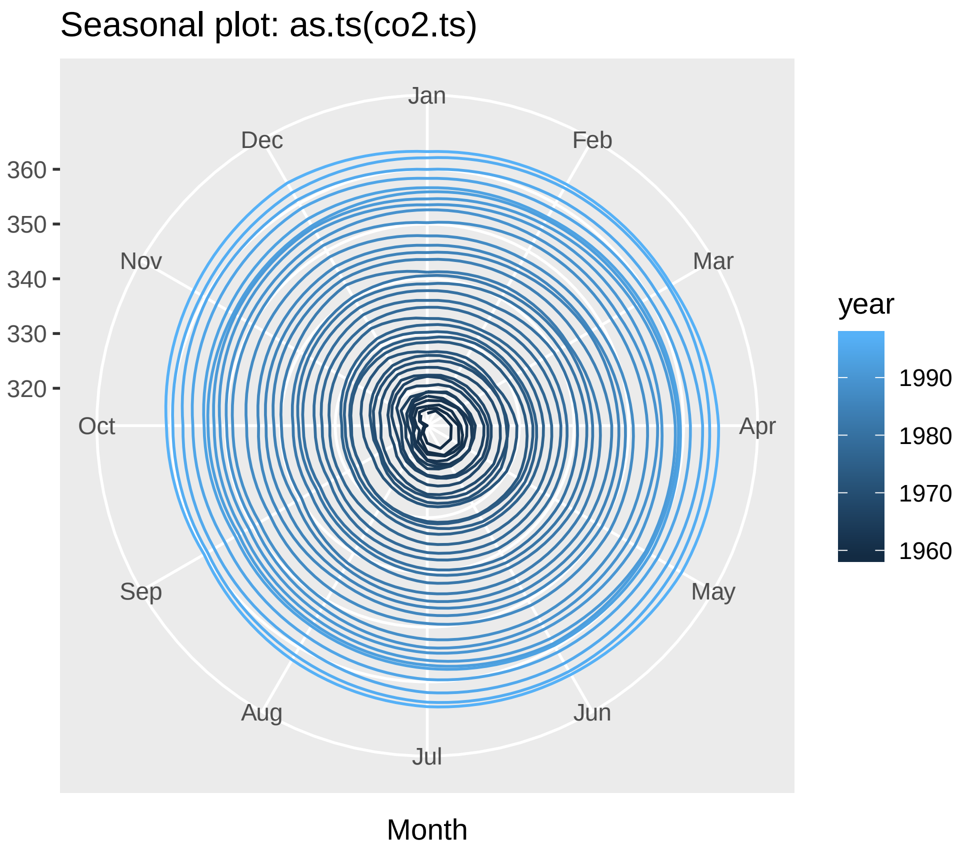
There are minor bumps and dips but there is no major shock or sudden movement in trend.

Now lets look at seasonal cycles.

```
co2.ts %>% gg_season(y = value, period = "year") + ylab("CO2 emission") +
    ggtitle("Seasonal plot : Monthly CO2 emission")
```



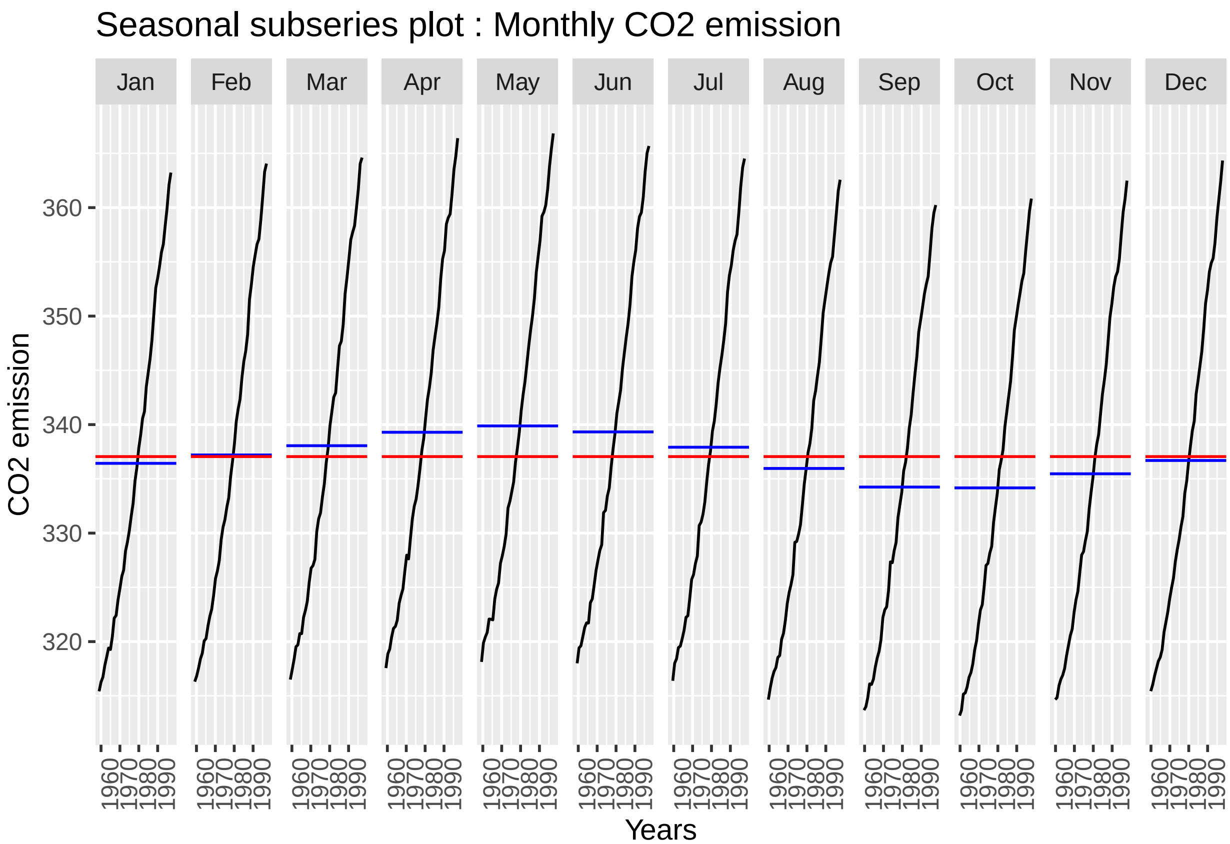

```
ggseasonplot(as.ts(co2.ts), year.labels = FALSE, continuous = TRUE,
    polar = TRUE)  #Looks good may be?
```



We can clearly see that CO2 emission is higher in April-May and goes down in September-October every year. This could be due to summer in Northern hemisphere vs southern hemisphere. Population, vegitation and other attributes of these two halves of earth are quite different which could potentially cause this seasonal effect.

Now lets look at same data from another angle.

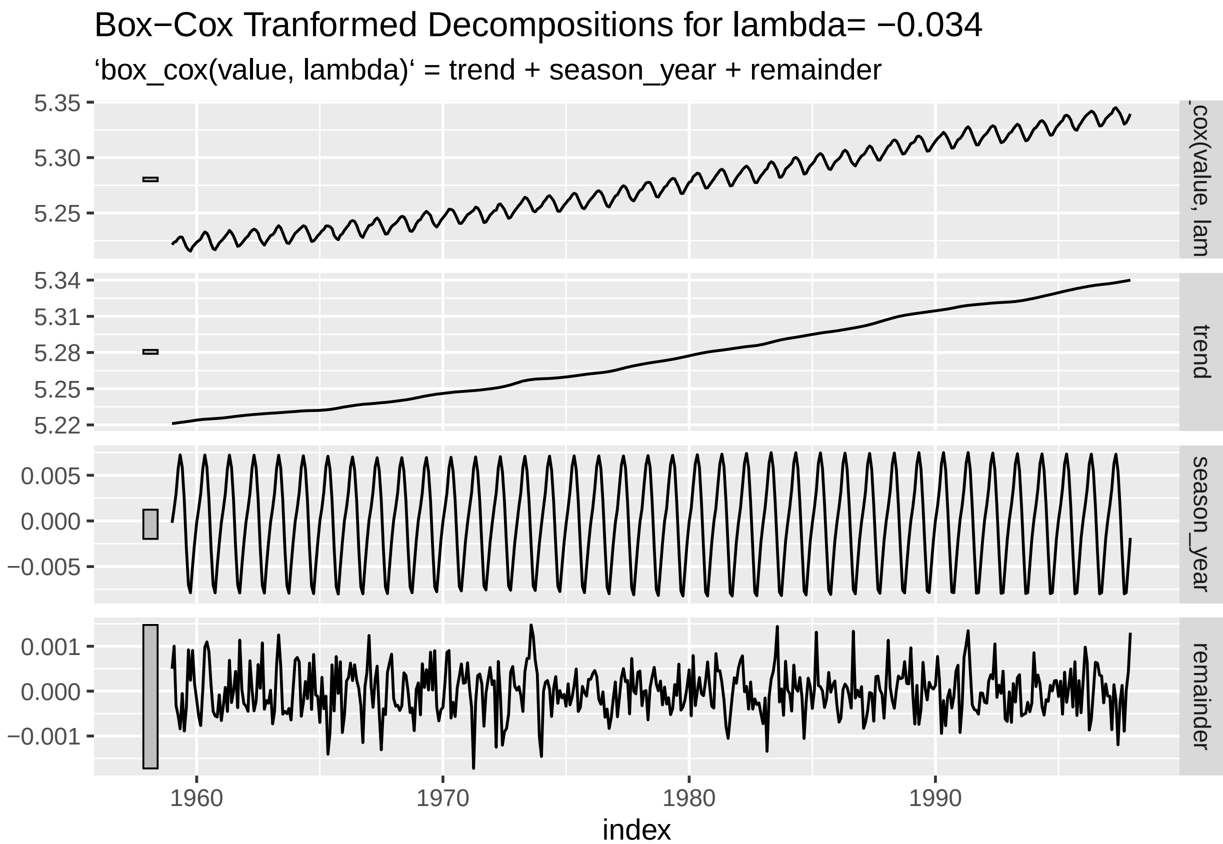
```
co2.ts %>% gg_subseries(y = value, period = "year") + geom_hline(aes(yintercept = mean(co2.ts$
    colour = "red") + ylab("CO2 emission") + xlab("Years") +
    ggtitle("Seasonal subseries plot : Monthly CO2 emission")
```



It is more or less same observation that northern summer increases CO2 emission and southern summer decreases CO2 eission within a year. But we can cearly see that CO2 emission is on the rise year on year for each and every month.

Since we saw earlier that seasonal patterns are are increasin in variance, we can try to transform it so the seasonal changes are uniform across time series. This is desirable property for finding a good model fit.

```
lambda <- co2.ts %>% features(value, features = guerrero) %>%
    pull(lambda_guerrero)

co2.ts.trans <- as_tsibble(ts(box_cox(co2.ts$value, lambda),
    start = c(1959, 1)))
co2.ts.trans.comp <- co2.ts %>% model(STL(box_cox(value, lambda)))

co2.ts.trans.comp %>% components() %>% autoplot() + ggtitle(paste("Box-Cox Tranformed Decomposi
    round(lambda, 3)))
```



After transformation we can see the seasonal pattern variance remains constant throughout.

Now lets look at residuals

```
# Lets look at residuals

# Before box-cox transformation
plot.co2.ts.components <- co2.ts.components %>% select("remainder") %>%
    ggplot(aes(x = remainder)) + geom_histogram() + ggtitle("Residuals Prior to Box-Cox")
```

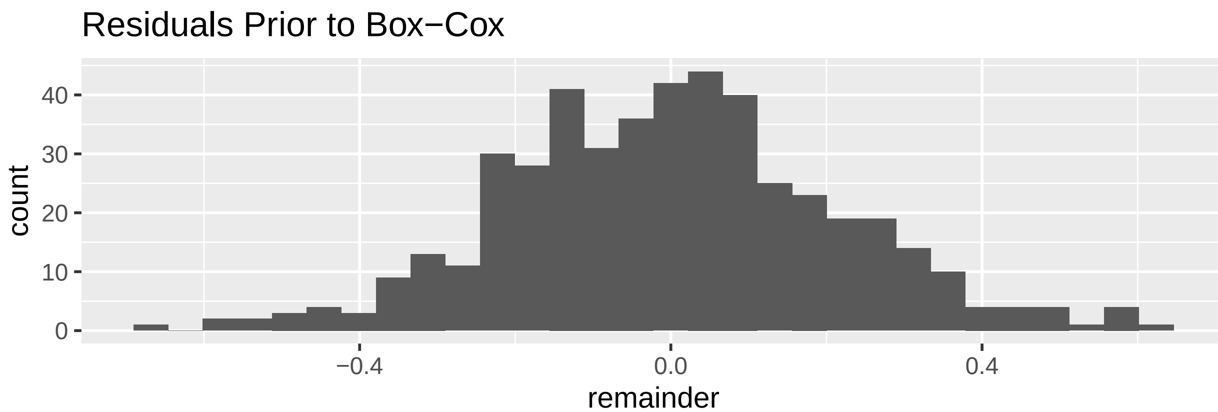
```
## Selecting index: "index"
```

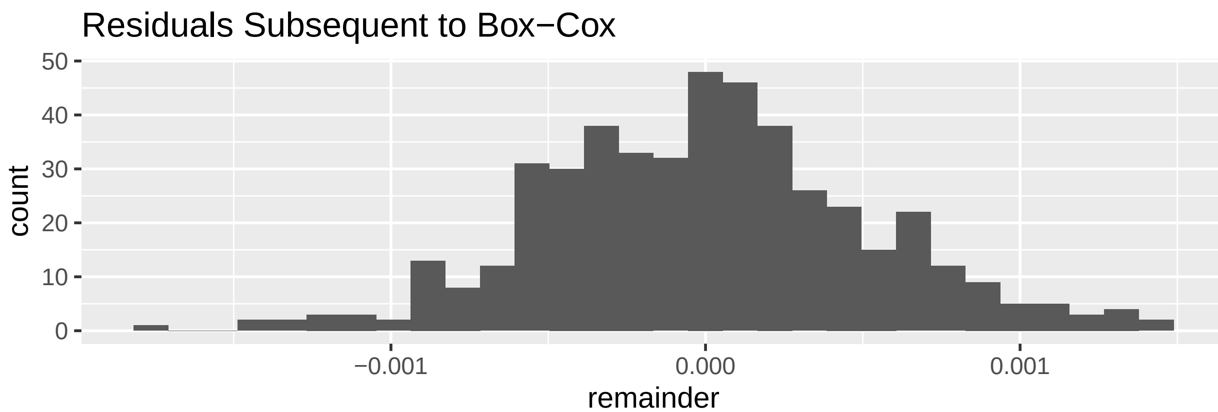
```
# After box-cox transformation
plot.co2.ts.trans.comp <- co2.ts.trans.comp %>% components() %>%
    select("remainder") %>% ggplot(aes(x = remainder)) + geom_histogram() +
    ggtitle("Residuals Subsequent to Box-Cox")
```

```
## Selecting index: "index"
```

```
grid.arrange(plot.co2.ts.components, plot.co2.ts.trans.comp,
    nrow = 2)
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.

## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```





Both before and after box-cox transformation residuals look fairly normally distributed

```
plot.resid.co2.ts.components <- co2.ts.components %>% select("remainder") %>%
    autoplot() + ggtitle("Residuals Prior to Box-Cox Transformation")
```

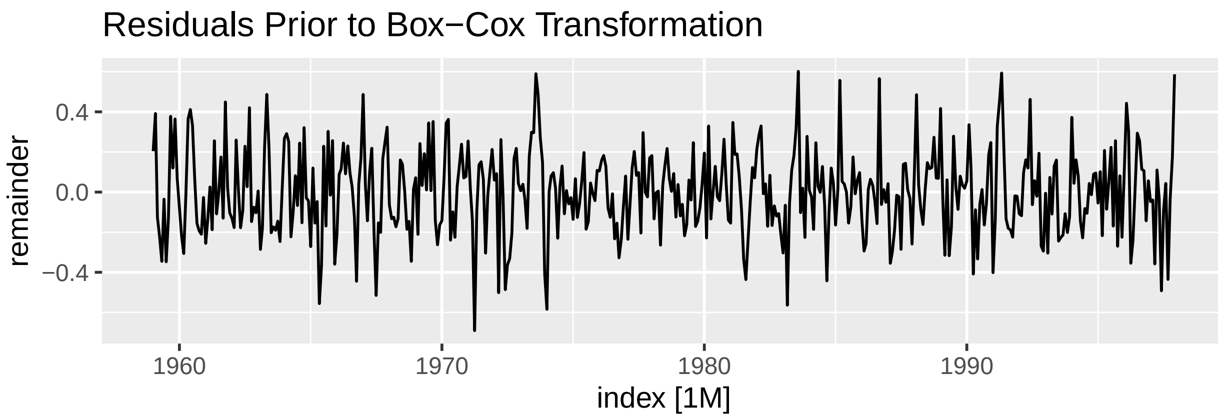
```
## Selecting index: "index"
```

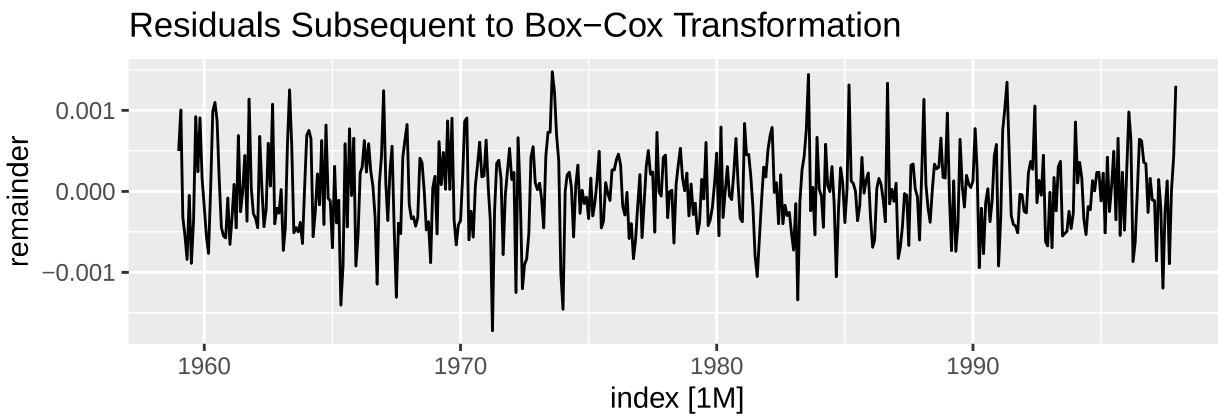
```
## Plot variable not specified, automatically selected `.vars = remainder`
```

```
plot.resid.co2.ts.trans.comp <- co2.ts.trans.comp %>% components() %>%
    select("remainder") %>% autoplot() + ggtitle("Residuals Subsequent to Box-Cox Transformati
```

```
## Selecting index: "index"
## Plot variable not specified, automatically selected `.vars = remainder`
```

```
grid.arrange(plot.resid.co2.ts.components, plot.resid.co2.ts.trans.comp,
    nrow = 2)
```



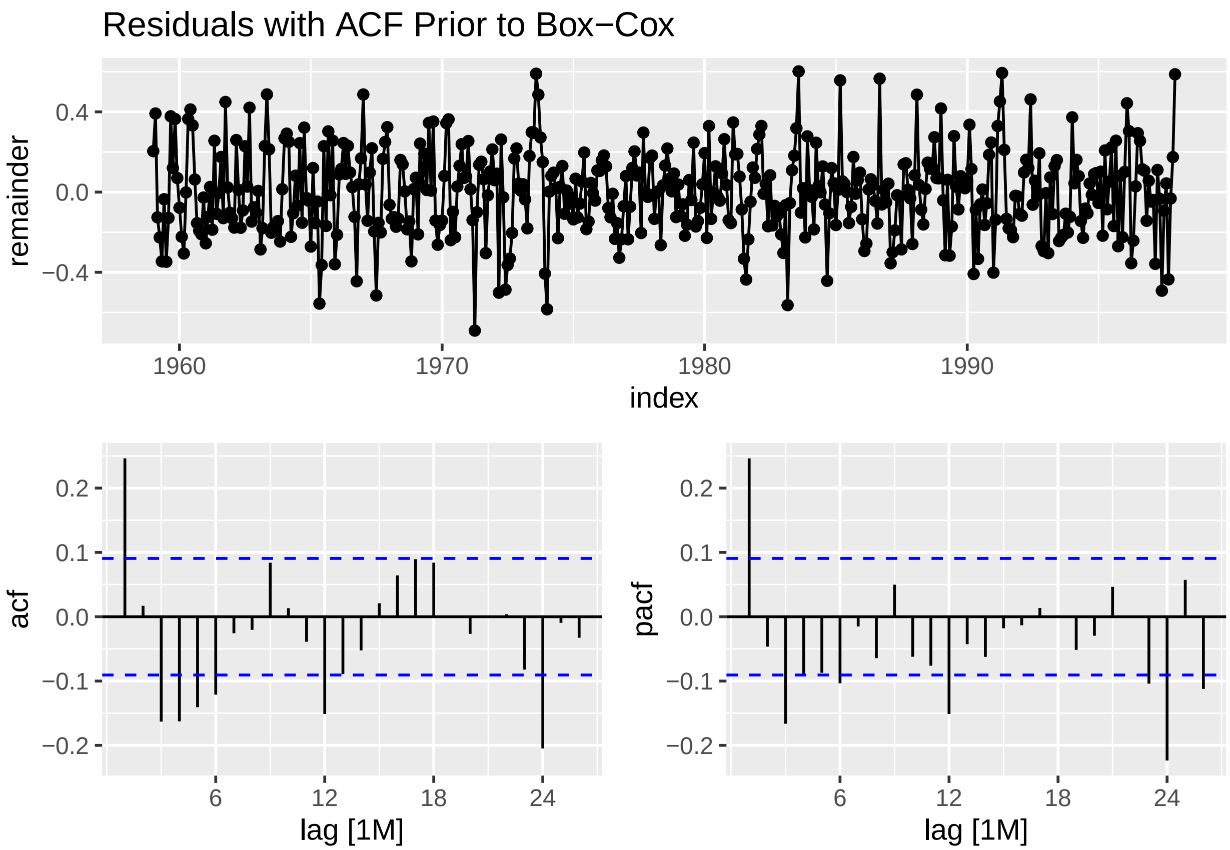


Residuals appear to be stationary and appear to be white noise with no trend or seasonal patterns.

```
co2.ts.components %>% select("remainder") %>% gg_tsdisplay(plot_type = "partial") +
    ggtitle("Residuals with ACF Prior to Box-Cox")
```

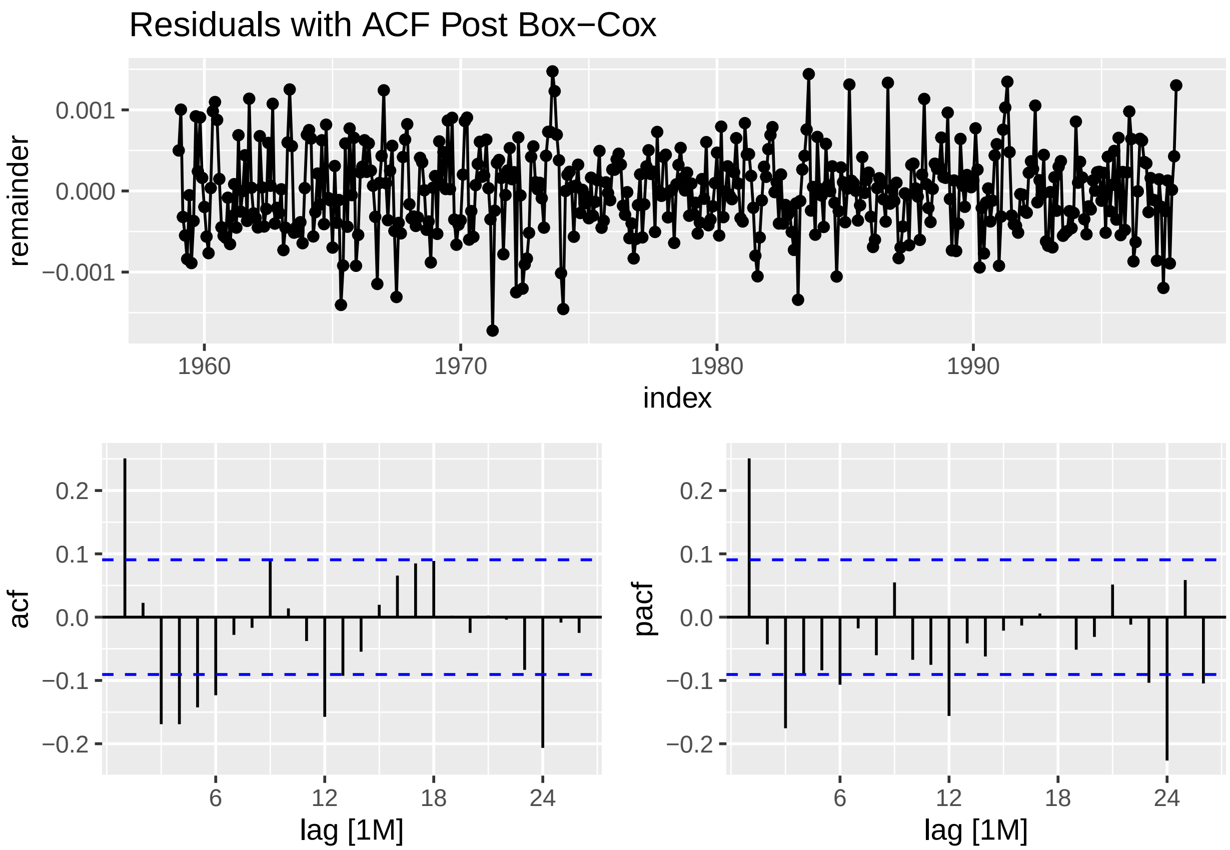
```
## Selecting index: "index"
```

```
## Plot variable not specified, automatically selected `y = remainder`
```




```
co2.ts.trans.comp %>% components() %>% select("remainder") %>%
    gg_tsdisplay(plot_type = "partial") + ggtitle("Residuals with ACF Post Box-Cox")
```

```
## Selecting index: "index"
## Plot variable not specified, automatically selected `y = remainder`
```



Part 2 (3 points)

Fit a linear time trend model to the `co2` series, and examine the characteristics of the residuals. Compare this to a quadratic time trend model. Discuss whether a logarithmic transformation of the data would be appropriate. Fit a suitable polynomial time trend model that incorporates seasonal dummy variables, and use this model to generate forecasts to the year 2020.

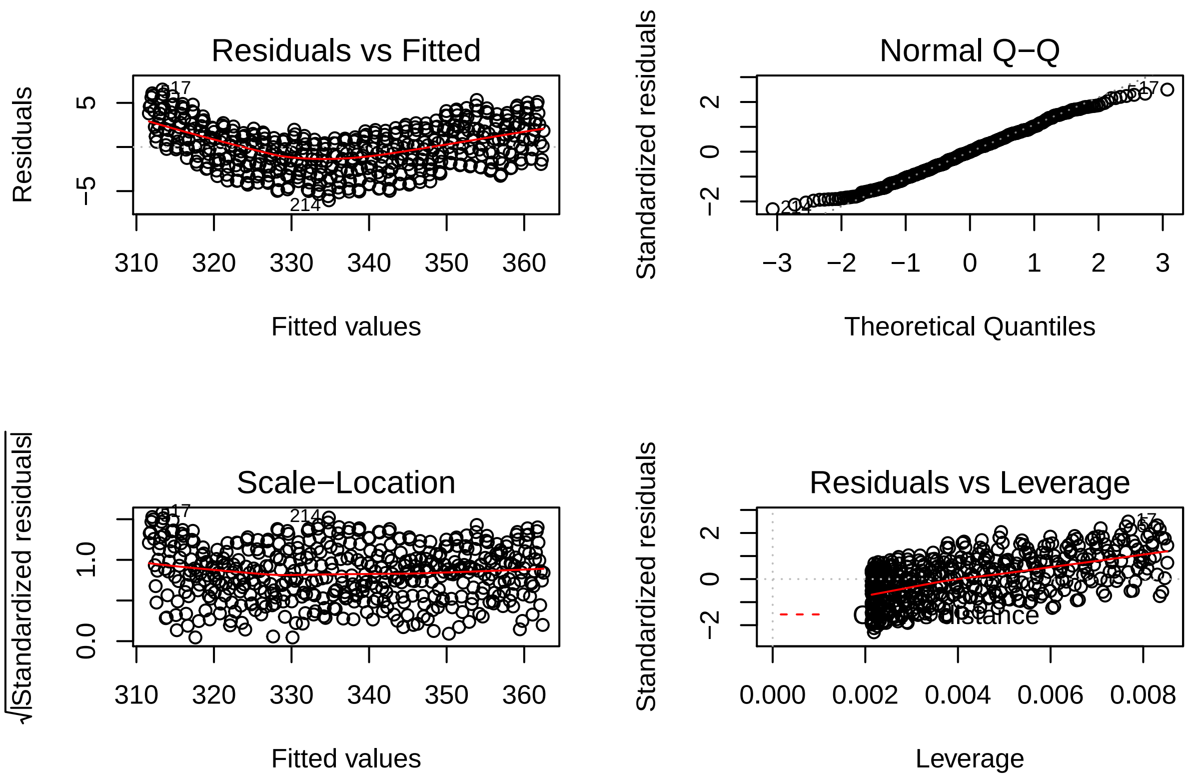
Lets first fit a linear time trend model

```
time.index <- 1:length(co2.ts$index)
mod.ln.1 <- lm(formula = value ~ time.index, data = co2.ts)
par(mfrow = c(2, 2))
summary(mod.ln.1)
```

```
##
## Call:
## lm(formula = value ~ time.index, data = co2.ts)
##
## Residuals:
##      Min       1Q  Median      3Q     Max
## -6.0399 -1.9476 -0.0017  1.9113  6.5149
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 3.115e+02  2.424e-01  1284.9   <2e-16 ***
```

```
## time.index  1.090e-01  8.958e-04   121.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.618 on 466 degrees of freedom
## Multiple R-squared:  0.9695, Adjusted R-squared:  0.9694
## F-statistic: 1.479e+04 on 1 and 466 DF,  p-value: < 2.2e-16
```

```
plot(mod.ln.1)
```



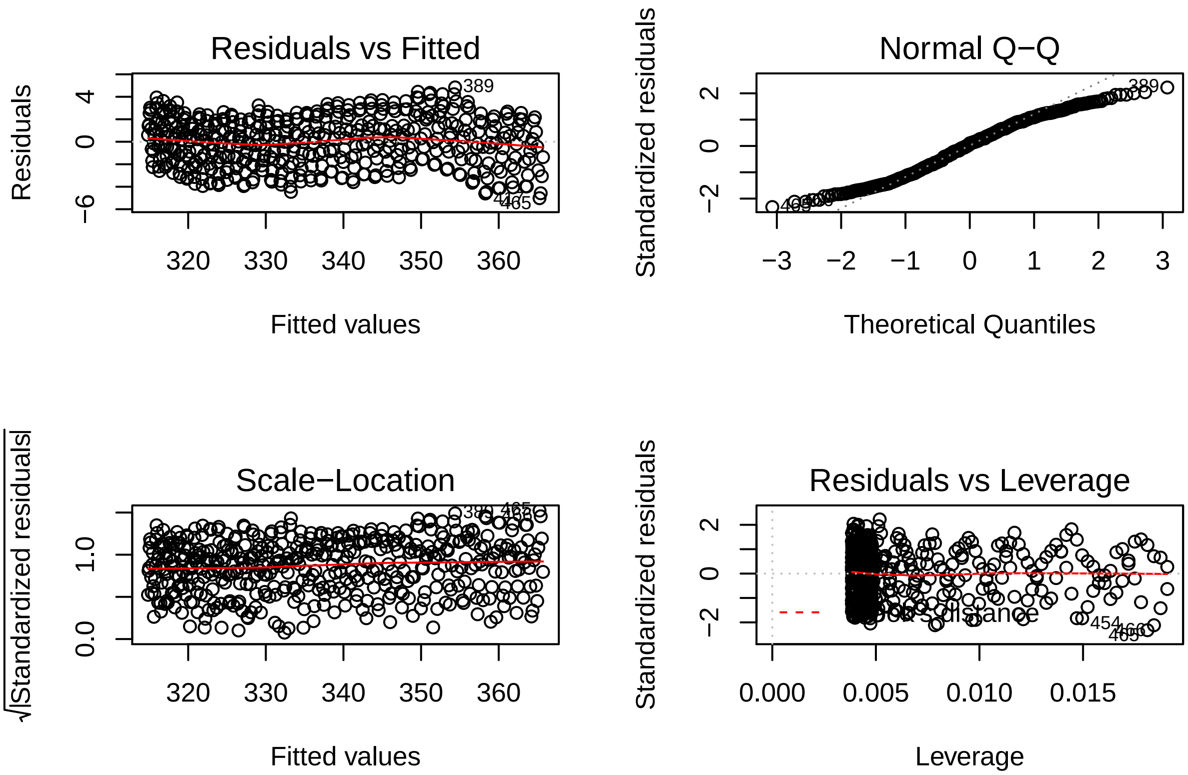
The residuals show a pattern indicating linear model is not able to capture the curvature of the trend. Lets try quadratic term

```
mod.ln.2 <- lm(formula = value ~ time.index + I(time.index^2),
    data = co2.ts)
summary(mod.ln.2)
```

```
##
## Call:
## lm(formula = value ~ time.index + I(time.index^2), data = co2.ts)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.0195 -1.7120  0.2144  1.7957  4.8345
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept)     3.148e+02  3.039e-01 1035.65   <2e-16 ***
## time.index      6.739e-02  2.993e-03   22.52   <2e-16 ***
## I(time.index^2) 8.862e-05  6.179e-06   14.34   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.182 on 465 degrees of freedom
## Multiple R-squared:  0.9788, Adjusted R-squared:  0.9787
## F-statistic: 1.075e+04 on 2 and 465 DF,  p-value: < 2.2e-16
```

```r
par(mfrow = c(2, 2))
plot(mod.ln.2)
```



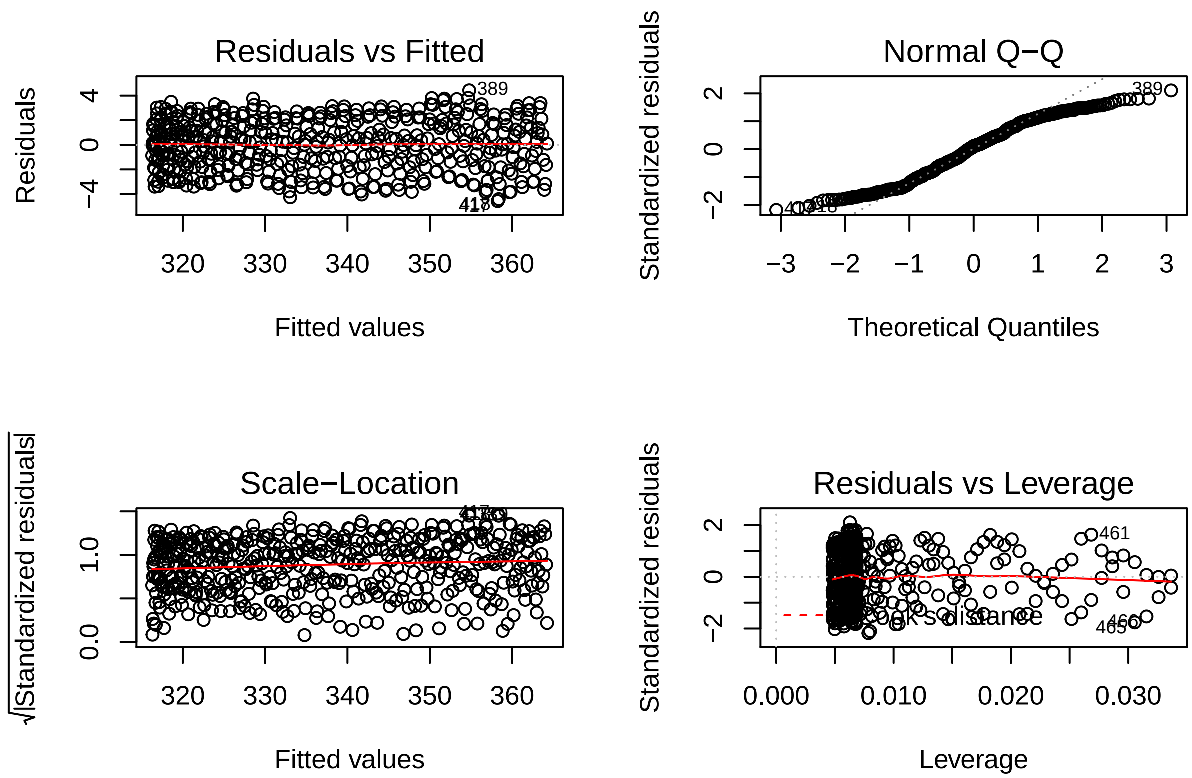
The residuals are showing a much better fit. Now lets try cubic term and see if we get any more improvements.

```r
mod.ln.3 <- lm(formula = value ~ time.index + I(time.index^2) +
    I(time.index^3), data = co2.ts)
summary(mod.ln.3)
```

```
##
## Call:
## lm(formula = value ~ time.index + I(time.index^2) + I(time.index^3),
##     data = co2.ts)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
```

```
## -4.5786 -1.7299  0.2279  1.8073  4.4318
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)        3.163e+02  3.934e-01 804.008  < 2e-16 ***
## time.index         2.905e-02  7.256e-03   4.004 7.25e-05 ***
## I(time.index^2)  2.928e-04  3.593e-05   8.149 3.44e-15 ***
## I(time.index^3) -2.902e-07  5.036e-08  -5.763 1.51e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.11 on 464 degrees of freedom
## Multiple R-squared:  0.9802, Adjusted R-squared:  0.9801
## F-statistic:  7674 on 3 and 464 DF,  p-value: < 2.2e-16
```

```
par(mfrow = c(2, 2))
plot(mod.ln.3)
```



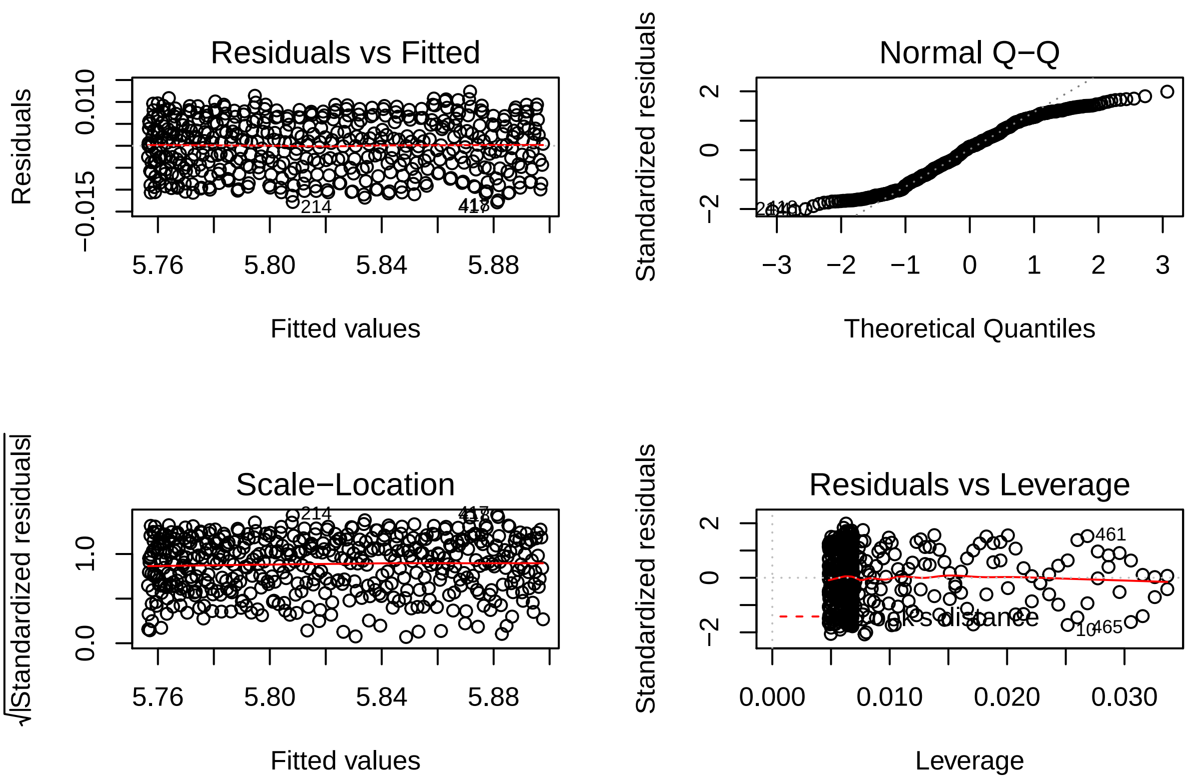
This model is even better fit, but Q-Q plot shows deviation at start and end. So lets try taking log of value and see if that scale fit any better.

```
mod.ln.4 <- lm(formula = log(value) ~ time.index + I(time.index^2) +
    I(time.index^3), data = co2.ts)
summary(mod.ln.4)
```

```
##
## Call:
## lm(formula = log(value) ~ time.index + I(time.index^2) + I(time.index^3),
```

```
##     data = co2.ts)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0128986 -0.0051482  0.0007055  0.0054841  0.0123625
##
## Coefficients:
##                   Estimate Std. Error  t value Pr(>|t|)
## (Intercept)      5.756e+00  1.163e-03 4948.350  < 2e-16 ***
## time.index       9.899e-05  2.146e-05    4.613 5.13e-06 ***
## I(time.index^2)  8.679e-07  1.063e-07    8.168 2.99e-15 ***
## I(time.index^3) -9.283e-10  1.489e-10   -6.233 1.03e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.006241 on 464 degrees of freedom
## Multiple R-squared:  0.9802, Adjusted R-squared:  0.9801
## F-statistic:  7661 on 3 and 464 DF,  p-value: < 2.2e-16
```

```
par(mfrow = c(2, 2))
plot(mod.ln.4)
```



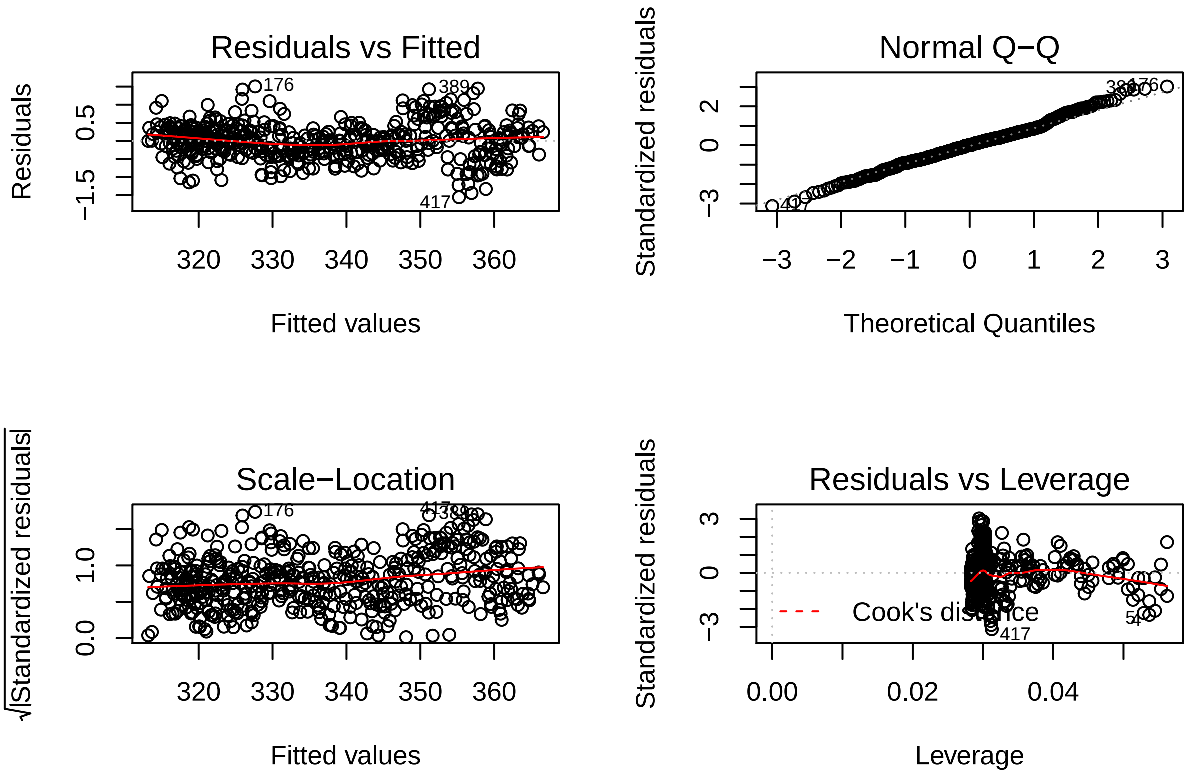
Taking log doesn't improve much.

So finally lets choose the best of the models, which so far is cubic model, and add seasonal dummy variables for Jan to December

```r
co2.df <- data.frame(value = co2.ts$value, time.index = time.index,
    season = factor(time.index%%12, ordered = F))
mod.ln.5 <- lm(formula = value ~ 0 + time.index + I(time.index^2) +
    I(time.index^3) + season, data = co2.df)
par(mfrow = c(2, 2))
summary(mod.ln.5)
```

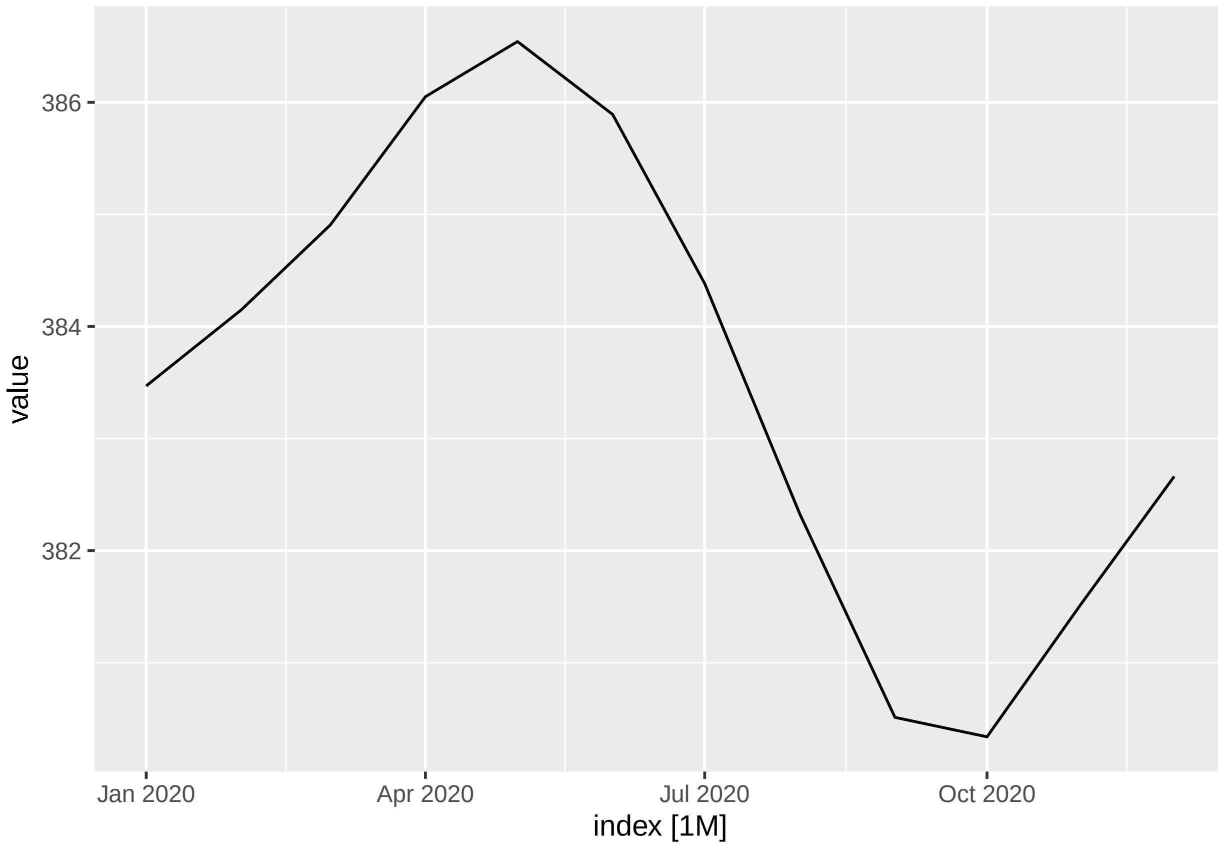
```
##
## Call:
## lm(formula = value ~ 0 + time.index + I(time.index^2) + I(time.index^3) +
##     season, data = co2.df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.5573 -0.3312  0.0008  0.2880  1.5040
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## time.index       3.275e-02  1.740e-03   18.83   <2e-16 ***
## I(time.index^2)  2.744e-04  8.614e-06   31.85   <2e-16 ***
## I(time.index^3) -2.640e-07  1.207e-08  -21.86   <2e-16 ***
## season0          3.151e+02  1.231e-01 2559.39   <2e-16 ***
## season1          3.160e+02  1.210e-01 2611.63   <2e-16 ***
## season2          3.167e+02  1.212e-01 2612.90   <2e-16 ***
## season3          3.174e+02  1.214e-01 2614.84   <2e-16 ***
## season4          3.186e+02  1.216e-01 2619.99   <2e-16 ***
## season5          3.191e+02  1.218e-01 2619.77   <2e-16 ***
## season6          3.184e+02  1.220e-01 2610.22   <2e-16 ***
## season7          3.169e+02  1.222e-01 2593.69   <2e-16 ***
## season8          3.148e+02  1.224e-01 2572.76   <2e-16 ***
## season9          3.130e+02  1.226e-01 2553.89   <2e-16 ***
## season10         3.128e+02  1.228e-01 2548.46   <2e-16 ***
## season11         3.140e+02  1.229e-01 2554.22   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5056 on 453 degrees of freedom
## Multiple R-squared:      1,  Adjusted R-squared:      1
## F-statistic: 1.389e+07 on 15 and 453 DF,  p-value: < 2.2e-16
```

```r
plot(mod.ln.5)
```



This model has better R-square than any of the previous models. Lets use this model to forecast for year 2020

```
time.index.2020 = seq(from = (2020 - 1959) * 12 + 1, length.out = 12)
co2.2020.df <- data.frame(time.index = time.index.2020, season = factor(time.index.2020%%12,
    ordered = F))

co2.2020.pred <- predict(object = mod.ln.5, newdata = co2.2020.df)

co2.2020.pred.ts <- as_tsibble(ts(data = co2.2020.pred, start = c(2020,
    1), frequency = 12))

co2.2020.pred.ts %>% autoplot(.vars = value)
```



Part 3 (3 points)

Following all appropriate steps, choose an ARIMA model to fit to the series. Discuss the characteristics of your model and how you selected between alternative ARIMA specifications. Write your model (or models) using backshift notation. Use your model (or models) to generate forecasts to the year 2020.

```
no.p = no.P = 2
no.q = no.Q = 2
no.d = no.D = 1
no.of.models <- (no.p + 1) * (no.d + 1) * (no.q + 1) * (no.P +
    1) * (no.D + 1) * (no.Q + 1)

print(paste("Number of models to fit =", no.of.models))
```

```
## [1] "Number of models to fit = 324"
```

```
params.df <- expand.grid(p = 0:no.p, d = 0:no.d, q = 0:no.q,
    P = 0:no.P, D = 0:no.D, Q = 0:no.Q)
i <- 1

funFitModel <- function(param.row) {
    progress(value = i, max.value = no.of.models, console = TRUE,
        progress.bar = TRUE)
    i <- i + 1
```

```
    p = param.row["p"]
    q = param.row["q"]
    d = param.row["d"]
    P = param.row["P"]
    Q = param.row["Q"]
    D = param.row["D"]

    # print(paste(p,q,d,P,Q,D))
    tryCatch({
        model.fit = Arima(y = as.ts(co2.ts), order = c(p, d,
            q), seasonal = c(P, D, Q), lambda = lambda, include.drift = FALSE)
        model.info = data.frame(p, q, d, P, Q, D, model.fit$aic,
            model.fit$aicc, model.fit$bic)

        return(model.info)
    }, error = function(e) {
        return(data.frame())
    })

}

model.fit.info.df <- do.call("rbind", (apply(params.df, 1, funFitModel)))
```

```
##           0----------------------------------------------------------------324
## Progress:
```

```
colnames(model.fit.info.df) <- c("p", "q", "d", "P", "Q", "D",
    "aic", "aicc", "bic")
print("Top 6 models by BIC")
```

```
## [1] "Top 6 models by BIC"
```

```
model.fit.info.df %>% arrange(bic) %>% head()
```

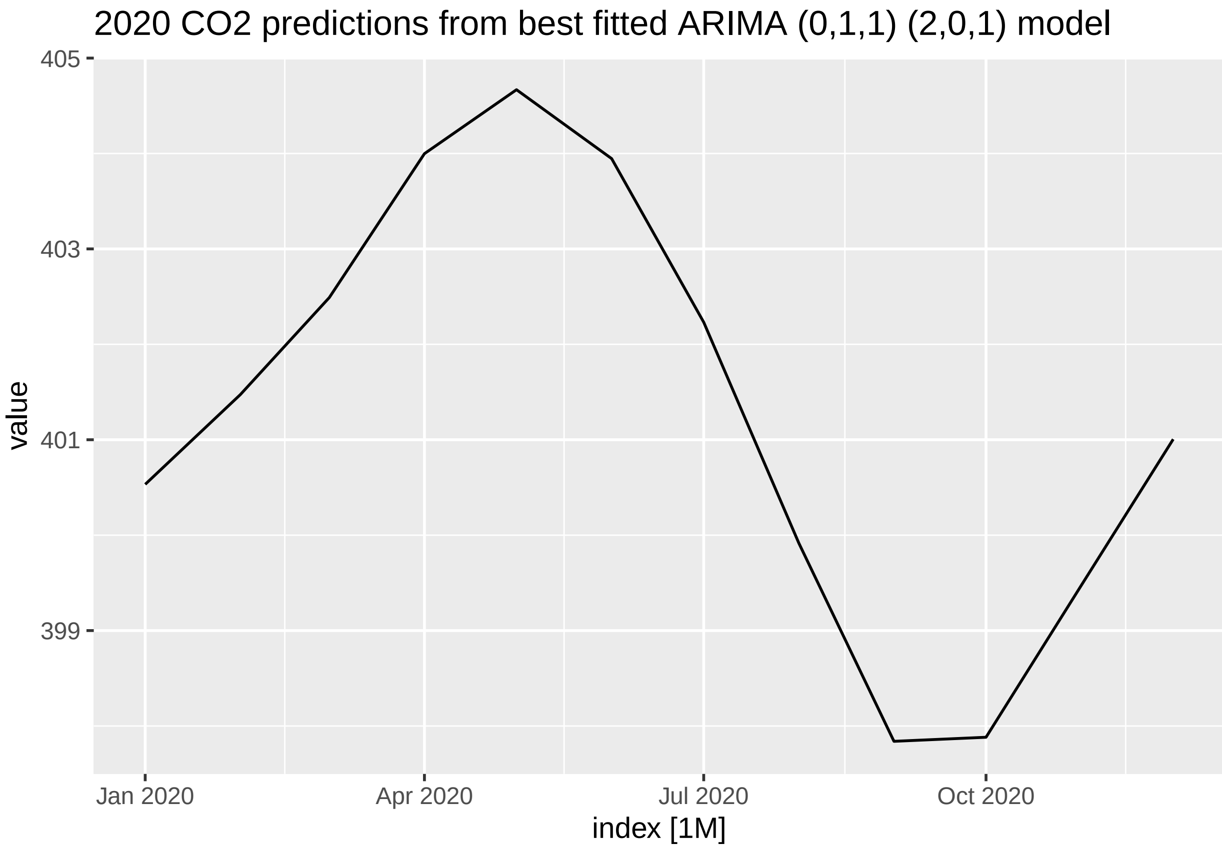
```
##   p q d P Q D       aic      aicc       bic
## 1 0 1 1 2 1 0 -5410.497 -5410.367 -5389.765
## 2 1 1 1 2 1 0 -5410.653 -5410.471 -5385.775
## 3 1 0 1 2 1 0 -5405.987 -5405.857 -5385.255
## 4 0 2 1 2 1 0 -5409.346 -5409.164 -5384.468
## 5 2 1 1 2 1 0 -5411.773 -5411.529 -5382.748
## 6 1 2 1 2 1 0 -5410.656 -5410.412 -5381.632
```

Using BIC as criteria we can see that ARIMA (0,1,1) with seasonality (2,0,1) as best model for CO2 time series. BIC for this model is least -5389.765. Note that this model uses lambda= -0.03432107 and no drift.

Lets just save the model to forecast for year 2020

```
model.arima.best.q3 = Arima(y = as.ts(co2.ts), order = c(0, 1,
    1), seasonal = c(2, 0, 1), lambda = lambda, include.drift = FALSE)
```

```
co2.2020.arima.pred.ts <- forecast(model.arima.best.q3, h = 276) %>%
    as_tibble() %>% dplyr::select("Point Forecast") %>% ts(start = c(1998,
    1), frequency = 12) %>% as_tsibble() %>% filter_index("2019-12-31" ~
    .)

co2.2020.arima.pred.ts %>% autoplot(.vars = value) + labs(title = "2020 CO2 predictions from be
```



Below, we inscribe the model using Latex and Backshift notation.

$$x_t(1-B) - \alpha B^{24} x_t(1-B) = B w_t$$

Part 4 (4 points)

The file `co2_weekly_mlo.txt` contains weekly observations of atmospheric carbon dioxide concentrations measured at the Mauna Loa Observatory from 1974 to 2020, published by the National Oceanic and Atmospheric Administration (NOAA). Convert these data into a suitable time series object, conduct a thorough EDA on the data, and address the problem of missing observations. Describe how the Keeling Curve evolved from 1997 to the present and compare current atmospheric CO2 levels to those predicted by your forecasts in Parts 2 and 3. Use the weekly data to generate a month-average series from 1997 to the present, and compare the overall forecasting performance of your models from Parts 2 and 3 over the entire period.

```
co2.noaa.weekly.df <- read.table("co2_weekly_mlo.txt", header = FALSE,
    comment.char = "#", na.strings = "-999.99")
colnames(co2.noaa.weekly.df) <- c("year", "month", "day", "decimal.day",
```

```r
    "record.value", "#days", "one.year.ago", "ten.years.ago",
    "since.1800.diff")
```

```r
head(co2.noaa.weekly.df)
```

```
##   year month day decimal.day record.value #days one.year.ago ten.years.ago
## 1 1974     5  19    1974.380       333.34     6           NA            NA
## 2 1974     5  26    1974.399       332.95     6           NA            NA
## 3 1974     6   2    1974.418       332.32     5           NA            NA
## 4 1974     6   9    1974.437       332.18     7           NA            NA
## 5 1974     6  16    1974.456       332.37     7           NA            NA
## 6 1974     6  23    1974.475       331.59     6           NA            NA
##   since.1800.diff
## 1           50.36
## 2           50.06
## 3           49.57
## 4           49.63
## 5           50.07
## 6           49.60
```

```r
tail(co2.noaa.weekly.df)
```

```
##      year month day decimal.day record.value #days one.year.ago ten.years.ago
## 2383 2020     1  12    2020.031       412.82     6       410.66        388.41
## 2384 2020     1  19    2020.051       413.65     7       412.19        388.27
## 2385 2020     1  26    2020.070       414.09     7       411.06        389.37
## 2386 2020     2   2    2020.089       414.33     7       411.11        390.67
## 2387 2020     2   9    2020.108       414.40     6       412.70        390.32
## 2388 2020     2  16    2020.127       414.01     7       411.22        390.45
##      since.1800.diff
## 2383          132.51
## 2384          133.17
## 2385          133.47
## 2386          133.61
## 2387          133.58
## 2388          133.10
```

```r
summary(co2.noaa.weekly.df)  # 20 readings are missing
```

```
##       year          month             day         decimal.day    record.value
##  Min.   :1974   Min.   : 1.000   Min.   : 1.00   Min.   :1974   Min.   :326.7
##  1st Qu.:1985   1st Qu.: 4.000   1st Qu.: 8.00   1st Qu.:1986   1st Qu.:346.9
##  Median :1997   Median : 7.000   Median :16.00   Median :1997   Median :364.3
##  Mean   :1997   Mean   : 6.539   Mean   :15.71   Mean   :1997   Mean   :366.8
##  3rd Qu.:2008   3rd Qu.:10.000   3rd Qu.:23.00   3rd Qu.:2009   3rd Qu.:386.2
##  Max.   :2020   Max.   :12.000   Max.   :31.00   Max.   :2020   Max.   :415.4
##                                                                 NA's   :20
##      #days        one.year.ago   ten.years.ago   since.1800.diff
##  Min.   :0.000   Min.   :326.8   Min.   :326.6   Min.   : 49.57
```

```
##  1st Qu.:5.000   1st Qu.:346.4   1st Qu.:342.9   1st Qu.: 66.72  
##  Median :6.000   Median :363.3   Median :356.2   Median : 83.49  
##  Mean   :5.858   Mean   :365.8   Mean   :357.3   Mean   : 86.86  
##  3rd Qu.:7.000   3rd Qu.:384.8   3rd Qu.:370.8   3rd Qu.:106.31  
##  Max.   :7.000   Max.   :412.7   Max.   :390.7   Max.   :133.61  
##                  NA's   :69      NA's   :542     NA's   :20
```

```
describe(co2.noaa.weekly.df)
```

```
## co2.noaa.weekly.df 
## 
##  9  Variables      2388  Observations
## --------------------------------------------------------------------------------
## year 
##        n  missing distinct     Info     Mean      Gmd      .05      .10 
##     2388        0       47        1     1997    15.26     1976     1978 
##      .25      .50      .75      .90      .95 
##     1985     1997     2008     2015     2017 
## 
## lowest : 1974 1975 1976 1977 1978, highest: 2016 2017 2018 2019 2020
## --------------------------------------------------------------------------------
## month 
##        n  missing distinct     Info     Mean      Gmd      .05      .10 
##     2388        0       12    0.993    6.539    3.971        1        2 
##      .25      .50      .75      .90      .95 
##        4        7       10       11       12 
## 
## lowest :  1  2  3  4  5, highest:  8  9 10 11 12
## 
## Value          1     2     3     4     5     6     7     8     9    10    11
## Frequency    203   185   199   193   201   198   204   203   198   203   197
## Proportion 0.085 0.077 0.083 0.081 0.084 0.083 0.085 0.085 0.083 0.085 0.082
## 
## Value         12
## Frequency    204
## Proportion 0.085
## --------------------------------------------------------------------------------
## day 
##        n  missing distinct     Info     Mean      Gmd      .05      .10 
##     2388        0       31    0.999    15.71    10.16        2        4 
##      .25      .50      .75      .90      .95 
##        8       16       23       28       29 
## 
## lowest :  1  2  3  4  5, highest: 27 28 29 30 31
## --------------------------------------------------------------------------------
## decimal.day 
##        n  missing distinct     Info     Mean      Gmd      .05      .10 
##     2388        0     2388        1     1997    15.26     1977     1979
```

```
##      .25      .50      .75      .90      .95
##     1986     1997     2009     2016     2018
##
## lowest : 1974.380 1974.399 1974.418 1974.437 1974.456
## highest: 2020.051 2020.070 2020.089 2020.108 2020.127
## ---------------------------------------------------------------------------------
## record.value
##        n  missing distinct     Info     Mean      Gmd      .05      .10
##     2368       20     2111        1    366.8    27.14    332.8    336.4
##      .25      .50      .75      .90      .95
##    346.9    364.3    386.2    401.6    407.4
##
## lowest : 326.73 326.96 327.07 327.23 327.31, highest: 414.37 414.40 414.41 414.74 415.39
## ---------------------------------------------------------------------------------
## #days
##        n  missing distinct     Info     Mean      Gmd
##     2388        0        8    0.899    5.858    1.384
##
## lowest : 0 1 2 3 4, highest: 3 4 5 6 7
##
## Value          0     1     2     3     4     5     6     7
## Frequency     20    12    39    92   178   382   653  1012
## Proportion 0.008 0.005 0.016 0.039 0.075 0.160 0.273 0.424
## ---------------------------------------------------------------------------------
## one.year.ago
##        n  missing distinct     Info     Mean      Gmd      .05      .10
##     2319       69     2028        1    365.8     26.3    332.5    336.3
##      .25      .50      .75      .90      .95
##    346.4    363.3    384.8    399.0    404.7
##
## lowest : 326.77 326.85 326.98 327.21 327.38, highest: 411.58 411.70 411.84 412.19 412.70
## ---------------------------------------------------------------------------------
## ten.years.ago
##        n  missing distinct     Info     Mean      Gmd      .05      .10
##     1846      542     1608        1    357.3    19.49    332.0    334.6
##      .25      .50      .75      .90      .95
##    342.9    356.2    370.8    382.0    385.2
##
## lowest : 326.64 327.06 327.10 327.26 327.27, highest: 390.32 390.36 390.45 390.53 390.67
## ---------------------------------------------------------------------------------
## since.1800.diff
##        n  missing distinct     Info     Mean      Gmd      .05      .10
##     2368       20     2011        1    86.86    27.02    52.39    56.09
##      .25      .50      .75      .90      .95
##    66.72    83.49   106.31   120.98   127.12
##
## lowest :  49.57  49.60  49.63  49.93  49.99, highest: 133.17 133.26 133.47 133.58 133.61
## ---------------------------------------------------------------------------------
```

```r
co2.noaa.weekly.df <- co2.noaa.weekly.df %>% mutate(record.date = ymd(paste(year,
    month, day)))
```

We can see in the dataset there are 2388 observations. The make the time series easier to work with, we create an index using all of the time columns and convert the data to a tsibble object.

```r
week.frequency = 365.25/7
co2.noaa.weekly.ts.raw <- ts(data = co2.noaa.weekly.df$record.value,
    start = 1974.3795, frequency = week.frequency)
co2.noaa.weekly.ts <- as_tsibble(ts(data = co2.noaa.weekly.df$record.value,
    start = 1974.3795, frequency = week.frequency), class = "matrix")
cbind(head(co2.noaa.weekly.ts), tail(co2.noaa.weekly.ts))
```

```
## # A tsibble: 6 x 4 [1W]
##      index value   index1 value1
##     <week> <dbl>   <week>  <dbl>
## 1 1974 W20  333. 2020 W02   413.
## 2 1974 W21  333. 2020 W03   414.
## 3 1974 W22  332. 2020 W04   414.
## 4 1974 W23  332. 2020 W05   414.
## 5 1974 W24  332. 2020 W06   414.
## 6 1974 W25  332. 2020 W07   414.
```

```r
summary(co2.noaa.weekly.ts)
```

```
##   index              value
##  NULL:1974 W20   Min.   :326.7
##  NULL:1985 W42   1st Qu.:346.9
##  NULL:1997 W13   Median :364.3
##  NULL:1997 W13   Mean   :366.8
##  NULL:2008 W36   3rd Qu.:386.2
##  NULL:2020 W07   Max.   :415.4
##                  NA's   :20
```

```r
class(co2.noaa.weekly.ts)
```

```
## [1] "tbl_ts"     "tbl_df"     "tbl"        "data.frame"
```

In the summary of the tsibble object, we can see that there are 20 missing values.

To fill in the missing data, we fill in the values with interpolation from neighbour data points. We can try and fit a model and use that model instead for interpolation. After the interpolation we can see that there are 0 rows with missing values.

```r
co2.noaa.weekly.ts %>% filter(is.na(value))
```

```
## # A tsibble: 20 x 2 [1W]
##       index value
##      <week> <dbl>
##  1 1975 W40    NA
##  2 1975 W49    NA
```

```
##   3 1975 W50    NA
##   4 1975 W51    NA
##   5 1975 W52    NA
##   6 1976 W26    NA
##   7 1979 W20    NA
##   8 1982 W11    NA
##   9 1982 W14    NA
## 10 1982 W15    NA
## 11 1983 W31    NA
## 12 1984 W13    NA
## 13 1984 W14    NA
## 14 1984 W15    NA
## 15 1984 W16    NA
## 16 1984 W48    NA
## 17 2005 W41    NA
## 18 2008 W26    NA
## 19 2008 W27    NA
## 20 2008 W28    NA
```

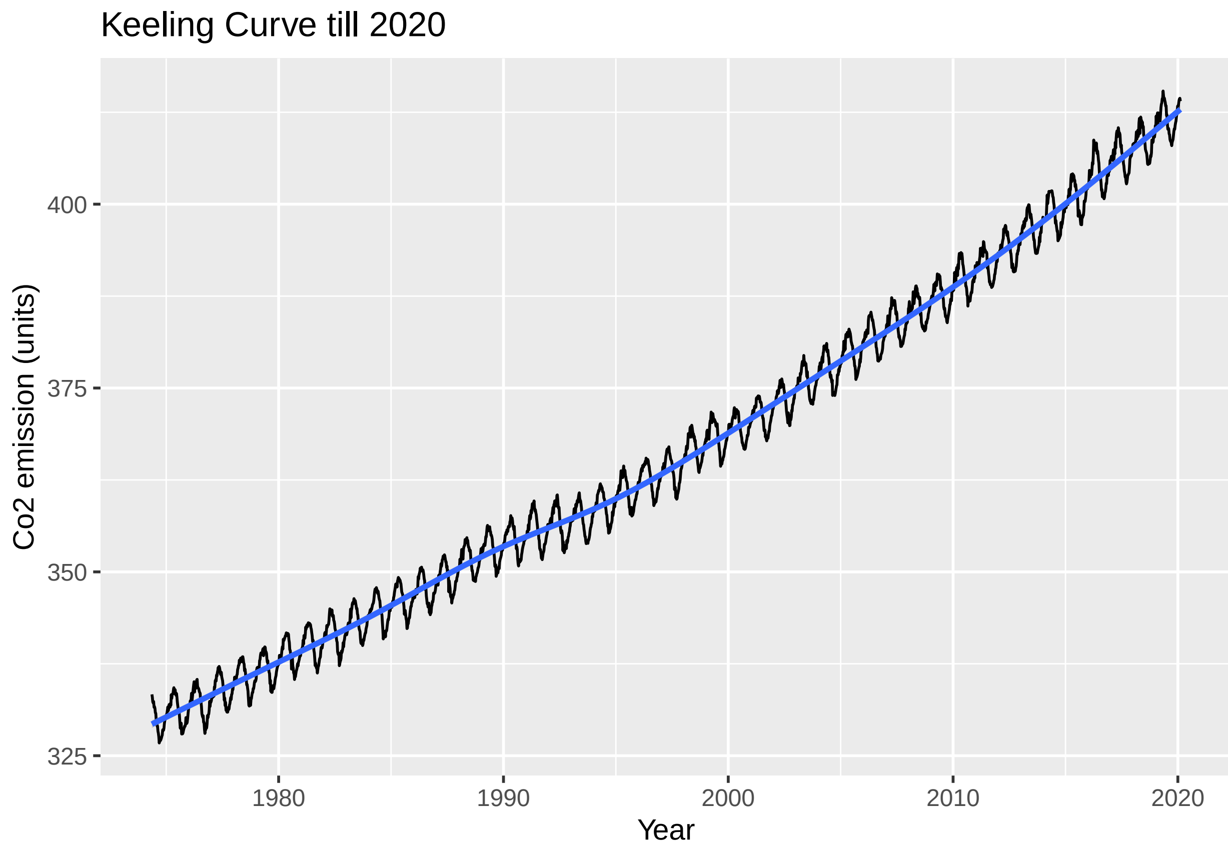
```
co2.noaa.weekly.ts <- na_interpolation(co2.noaa.weekly.ts, option = "spline")

co2.noaa.weekly.ts %>% filter(is.na(value))
```

```
## # A tsibble: 0 x 2 [?]
## # ... with 2 variables: index <week>, value <dbl>
```

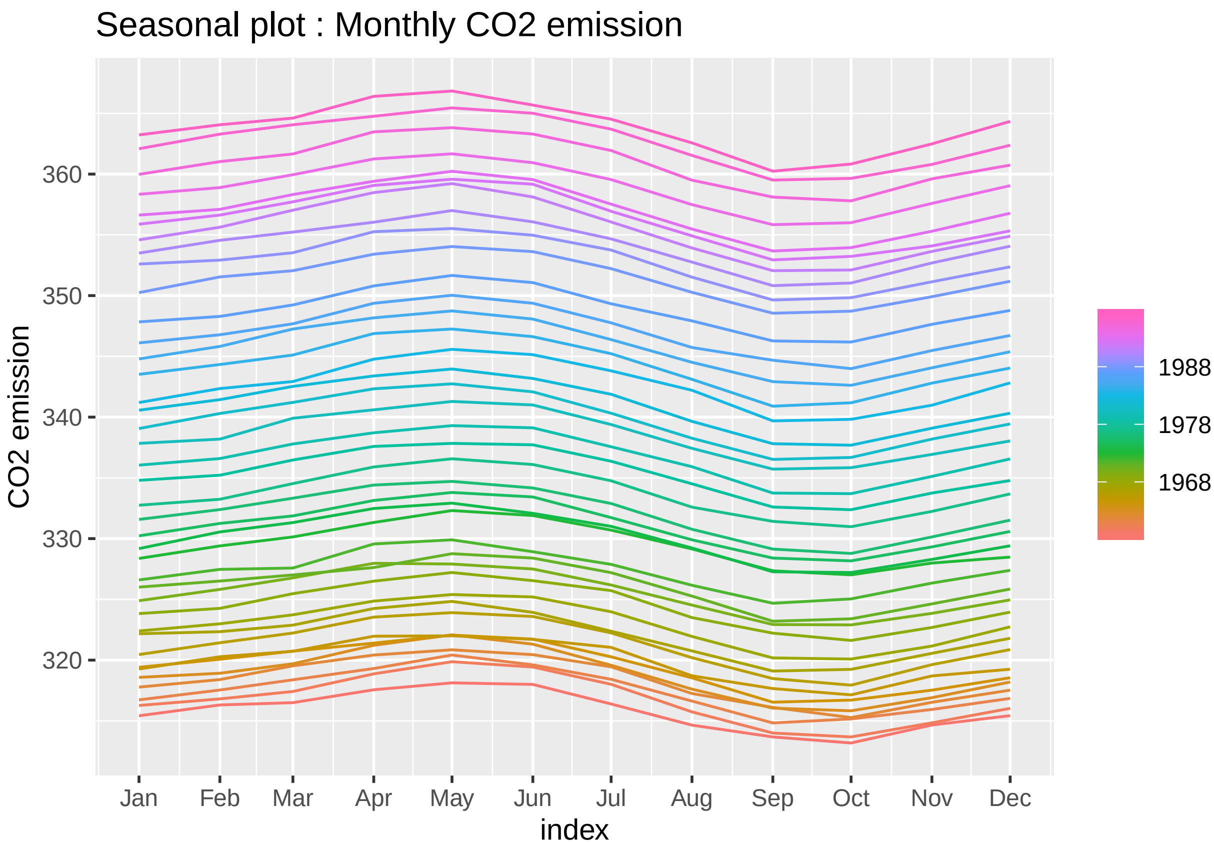
Plot Keeling curve

```
co2.noaa.weekly.ts %>% autoplot(.vars = value) + geom_smooth() +
    labs(title = "Keeling Curve till 2020", y = "Co2 emission (units)",
        x = "Year")
```

```
## `geom_smooth()` using method = 'gam' and formula 'y ~ s(x, bs = "cs")'
```

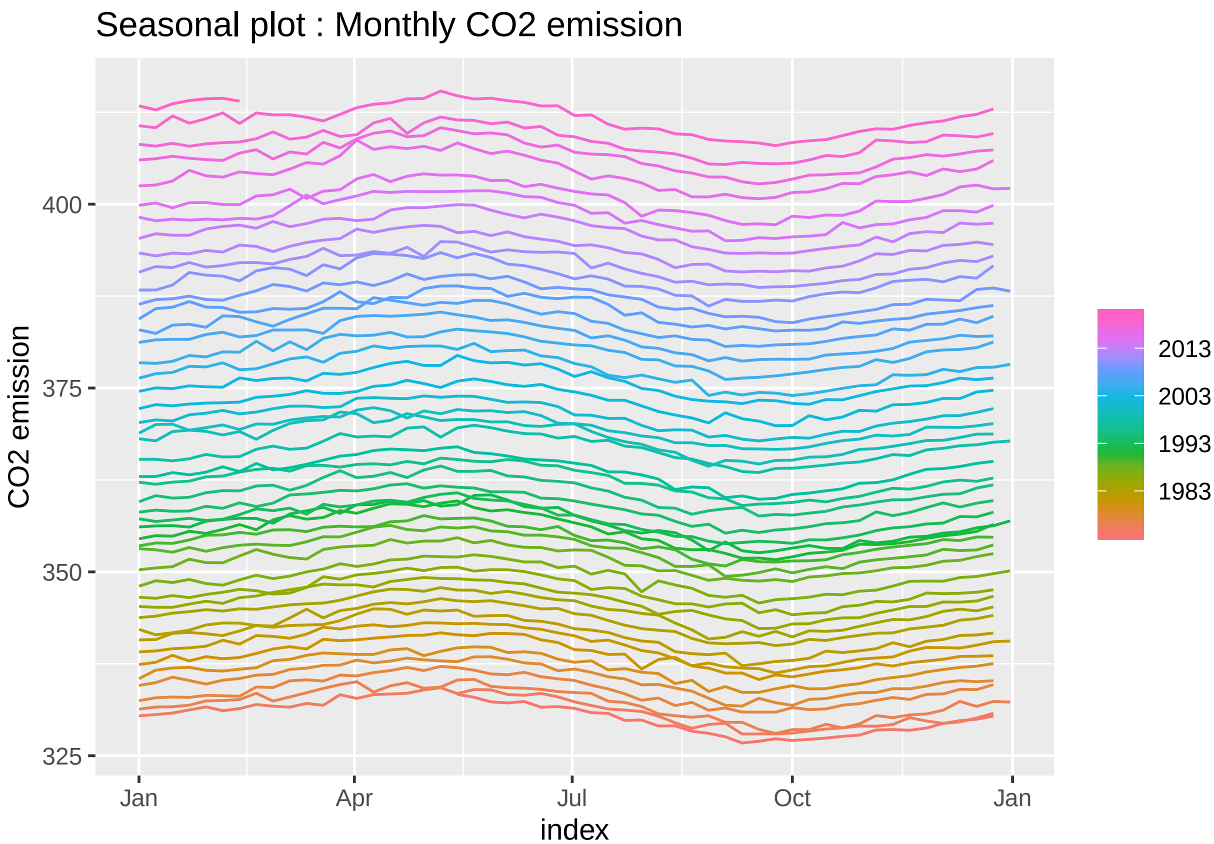



```r
co2.ts %>% gg_season(y = value, period = "year") + ylab("CO2 emission") +
    ggtitle("Seasonal plot : Monthly CO2 emission")
```



We can see that from about 1995 onwards the trend has picked up and increasing faster than pre 1995 trend.

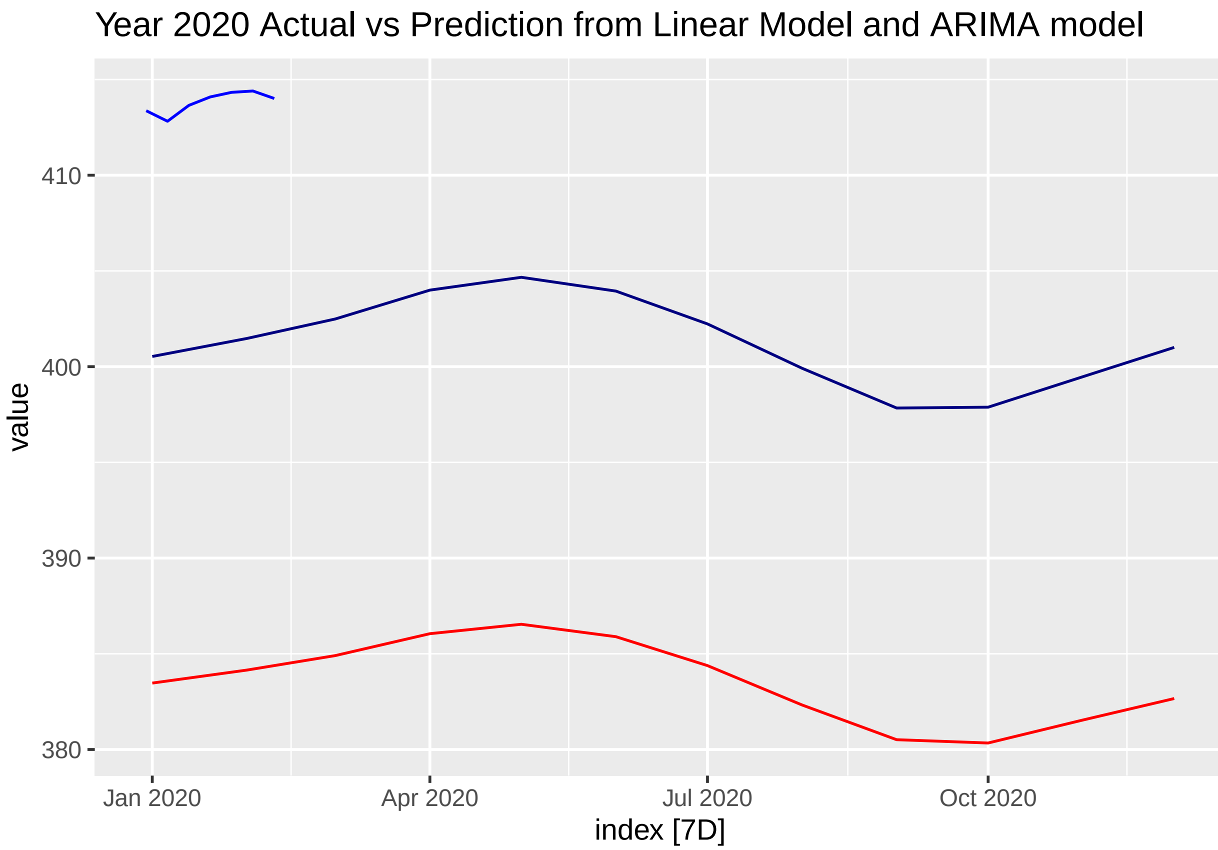
```
# Additional EDA
co2.noaa.weekly.ts %>% gg_season(y = value, period = "year") +
    ylab("CO2 emission") + ggtitle("Seasonal plot : Monthly CO2 emission")
```



The seasonal plot looks similar to the monthly CO2 data evaluated earlier, but we can now see the effects of the weekly data points by the variation in each line versus the smoother line in the monthly data. The seasonal monthly trends, however, are clearly the same.

Now lets compare the current CO2 levels to predicted from Q2 and 3

```
co2.noaa.weekly.ts %>% filter_index("2019-12-31" ~ .) %>% mutate(index = as.Date(index)) %>%
    autoplot(.vars = value, color = "blue") + autolayer(co2.2020.pred.ts,
    .vars = value, color = "red") + autolayer(co2.2020.arima.pred.ts,
    .vars = value, color = "navy") + labs(title = "Year 2020 Actual vs Prediction from Linear M
```



The predicted CO2 levels are lesser compared to actual levels. The red line is the predicted values from the cubic model with seasonal variables and the navy line is the predicted values from the ARIMA model.

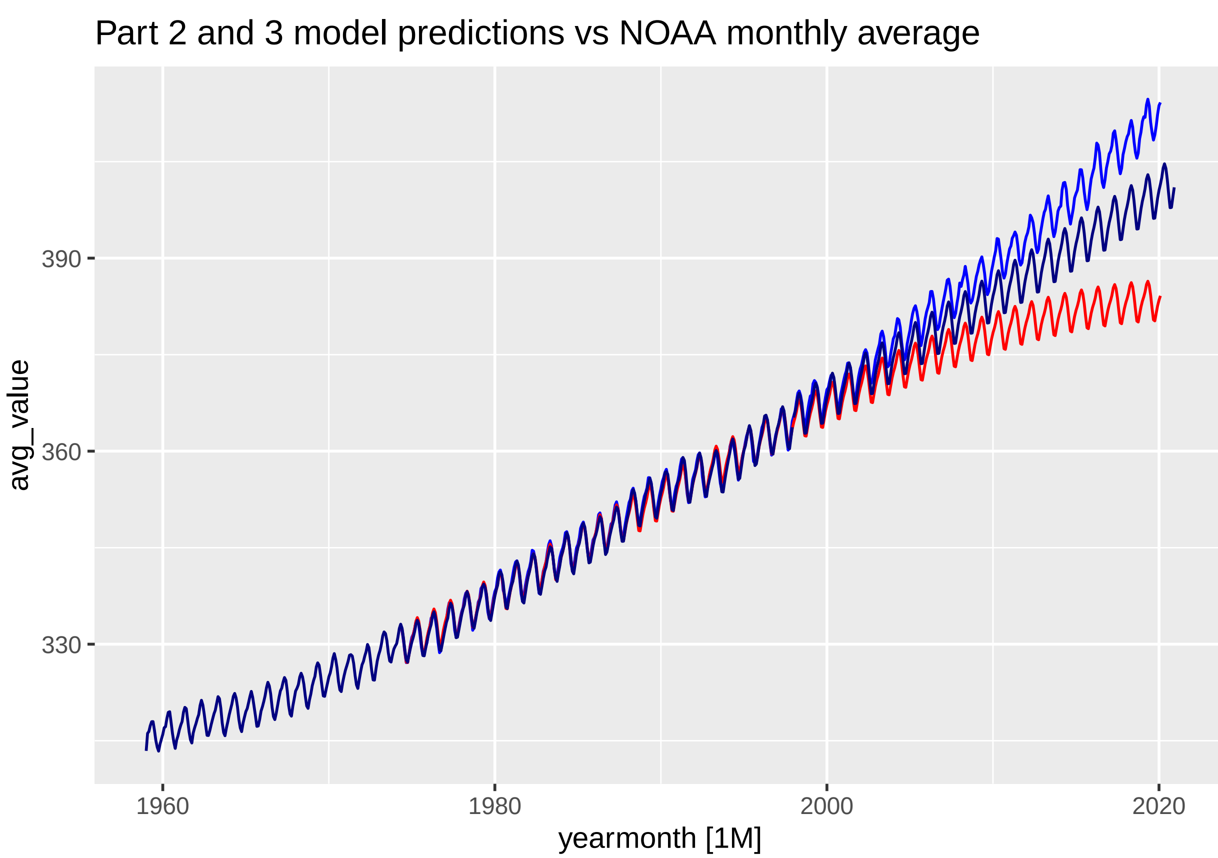
Now lets compare monthly average curve with model forecasts

```
# First lets convert to monthly time series
co2.noaa.monthly.ts <- co2.noaa.weekly.ts %>% as_tibble() %>%
    mutate(yearmonth = yearmonth(yearweek(index))) %>% group_by(yearmonth) %>%
    summarise(avg_value = mean(value)) %>% as_tsibble(index = yearmonth,
    value = avg_value)

summary(co2.noaa.monthly.ts)  #> 1974 May to 2020 Feb
```

```
##    yearmonth          avg_value
##  Min.   :1974 May   Min.   :327.1
##  1st Qu.:1985 Oct   1st Qu.:346.6
##  Median :1997 Mar   Median :364.0
##  Mean   :1997 Mar   Mean   :366.7
##  3rd Qu.:2008 Aug   3rd Qu.:386.1
##  Max.   :2020 Feb   Max.   :414.7
```

```
# Now predict using linear model for same time frame
time.index.1974.may = seq(from = (1974 - 1959) * 12 + 5, length.out = 550)
co2.noaa.like.df <- data.frame(time.index = time.index.1974.may,
    season = factor(time.index.1974.may%%12, ordered = F))
```

```r
co2.noaa.like.pred <- predict(object = mod.ln.5, newdata = co2.noaa.like.df)
co2.noaa.like.pred.ts <- as_tsibble(ts(data = co2.noaa.like.pred,
    start = c(1974, 5), frequency = 12))


# now predict using Arima model selected in Q3
co2.noaa.arima.pred.ts <- forecast(model.arima.best.q3, h = 276) %>%
    as_tibble() %>% dplyr::select("Point Forecast") %>% ts(start = c(1998,
    1), frequency = 12) %>% as_tsibble()

model.arima.best.q3.fitted.ts <- model.arima.best.q3$fitted %>%
    as_tsibble()


co2.noaa.monthly.ts %>% autoplot(.vars = avg_value, color = "blue") +
    autolayer(co2.noaa.like.pred.ts, .vars = value, color = "red") +
    autolayer(co2.noaa.arima.pred.ts, .vars = value, color = "navy") +
    autolayer(model.arima.best.q3.fitted.ts, .vars = value, color = "navy") +
    labs(title = "Part 2 and 3 model predictions vs NOAA monthly average")
```



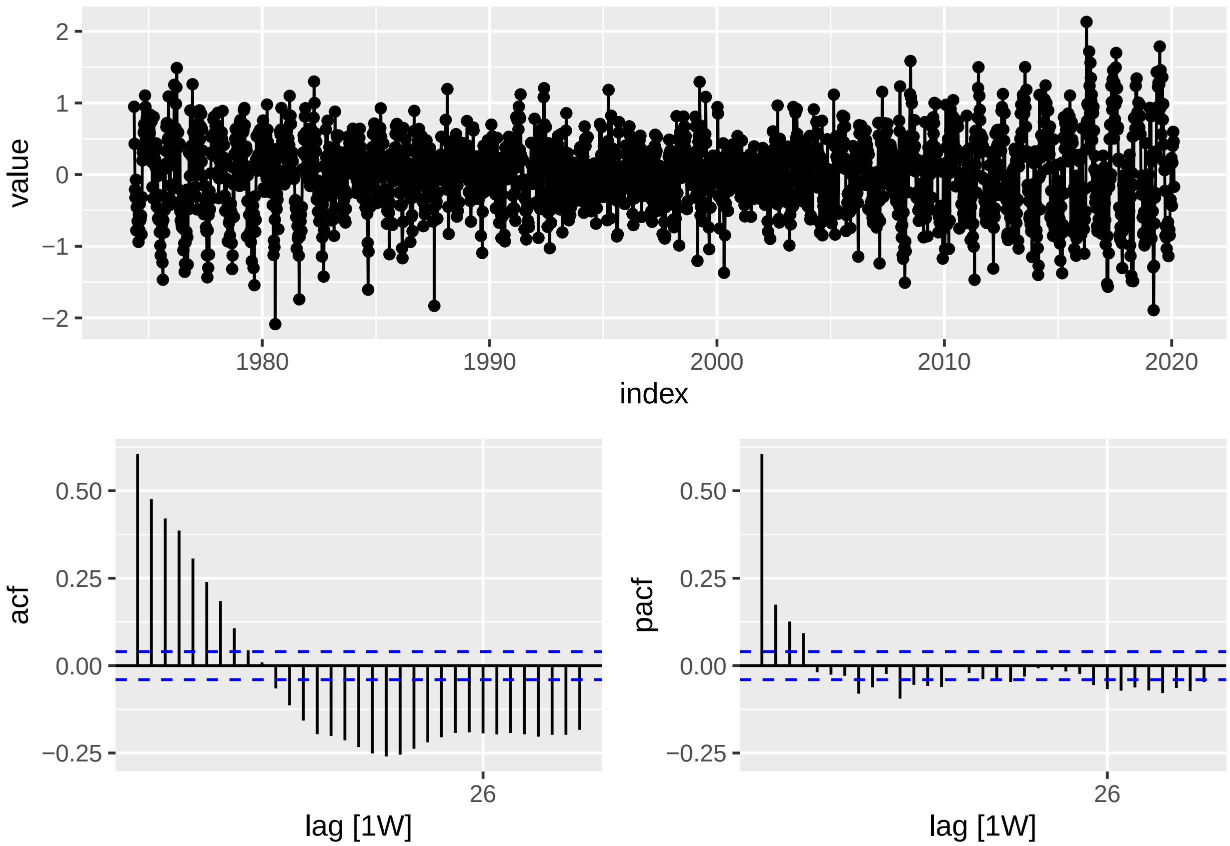
We see that up until about the year 2000, both models model the actual data closely. After, the models separate from the actuals with the cubic model showing the lowest values (red), and the ARIMA model (navy) also showing lower values than the actual values (blue).

Part 5 (3 points)

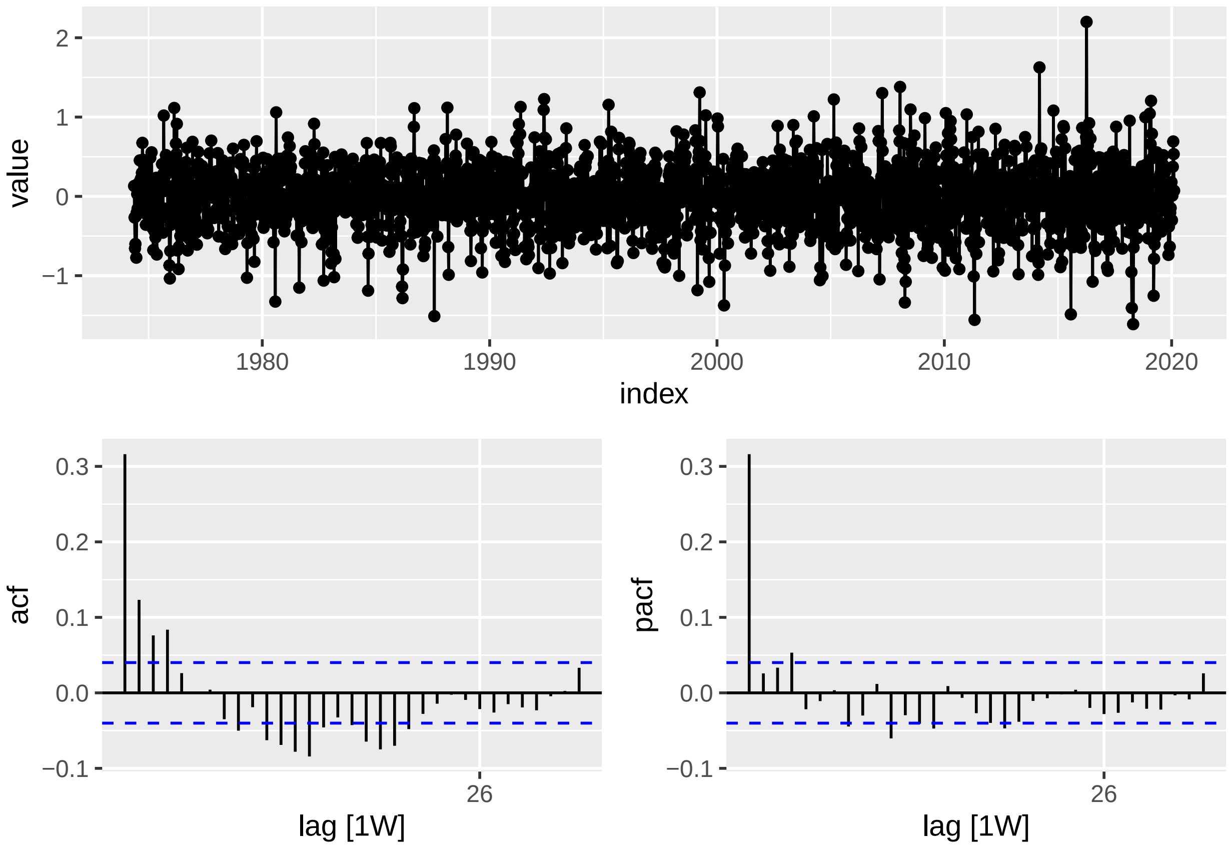
Seasonally adjust the weekly NOAA data, and split both seasonally-adjusted (SA) and non-seasonally-adjusted (NSA) series into training and test sets, using the last two years of observations as the test sets. For both SA and NSA series, fit ARIMA models using all appropriate steps. Measure and discuss how your models perform in-sample and (psuedo-) out-of-sample, comparing candidate models and explaining your choice. In addition, fit a polynomial time-trend model to the seasonally-adjusted series and compare its performance to that of your ARIMA model.

First lets try couple of ways in which we can decompose seasonality by analysing residual

```
plotRemainderFromSTL <- function(s.degree = 0) {
    stl.val <- co2.noaa.weekly.ts %>% stl(s.window = week.frequency,
        s.degree = s.degree)
    ts.obj <- as_tsibble(ts(stl.val$time.series[, "remainder"],
        start = 1974.3795, frequency = week.frequency))

    par(mfrow = c(1, 2))
    # p1 <- ts.obj %>% autoplot(.vars = value)
    ts.obj %>% gg_tsdisplay(y = value, plot_type = "partial")

}


plotRemainderFromSTL(s.degree = 0)
```



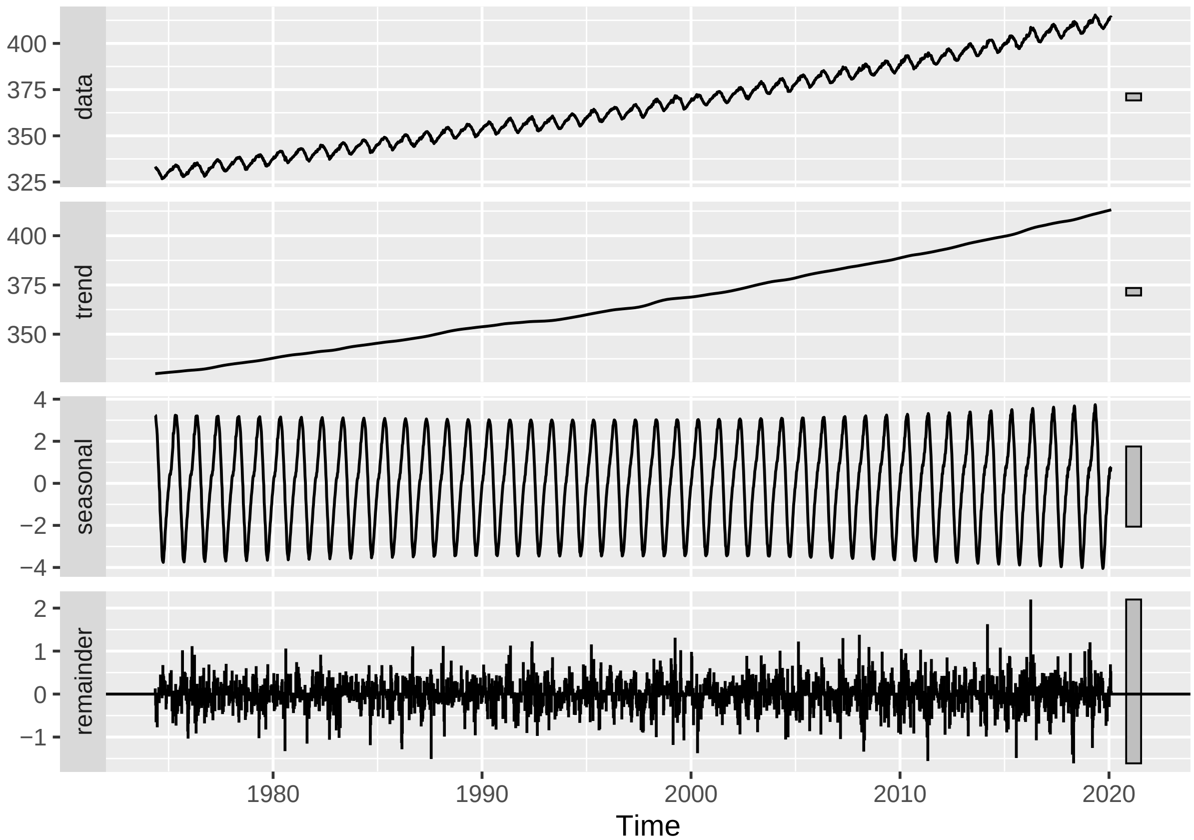

```
plotRemainderFromSTL(s.degree = 1)
```



We can see that with degree of locality 1 in seasonal extraction gives is much better residuals with dampening ACF and sharp decrease in PACF at lag 1. There is still some seasonality left but still this looks much better than degree 0.

Lets first see how seasonal decomposition looks

```
co2.noaa.weekly.ts %>% stl(s.window = week.frequency, s.degree = 1) %>%
    autoplot()
```



Now lets substract seasonal data from time series to get seasonally adjusted time series

```
co2.noaa.weekly.ts.components <- stl(co2.noaa.weekly.ts, s.window = week.frequency)$time.series
co2.noaa.weekly.ts.components.seasonal <- co2.noaa.weekly.ts.components[,
    "seasonal"]
co2.noaa.weekly.ts.SA <- as_tsibble(ts(co2.noaa.weekly.ts$value -
    co2.noaa.weekly.ts.components.seasonal, start = 1974.3795,
    frequency = week.frequency))
cbind(head(co2.noaa.weekly.ts), tail(co2.noaa.weekly.ts), head(co2.noaa.weekly.ts.SA),
    tail(co2.noaa.weekly.ts.SA))
```

```
## # A tsibble: 6 x 8 [1W]
##      index value   index1 value1   index2 value2   index3 value3
##     <week> <dbl>   <week>  <dbl>   <week>  <dbl>   <week>  <dbl>
## 1 1974 W20  333. 2020 W02   413. 1974 W20   330. 2020 W02   412.
## 2 1974 W21  333. 2020 W03   414. 1974 W21   330. 2020 W03   413.
## 3 1974 W22  332. 2020 W04   414. 1974 W22   329. 2020 W04   413.
## 4 1974 W23  332. 2020 W05   414. 1974 W23   329. 2020 W05   413.
## 5 1974 W24  332. 2020 W06   414. 1974 W24   330. 2020 W06   413.
## 6 1974 W25  332. 2020 W07   414. 1974 W25   329. 2020 W07   413.
```

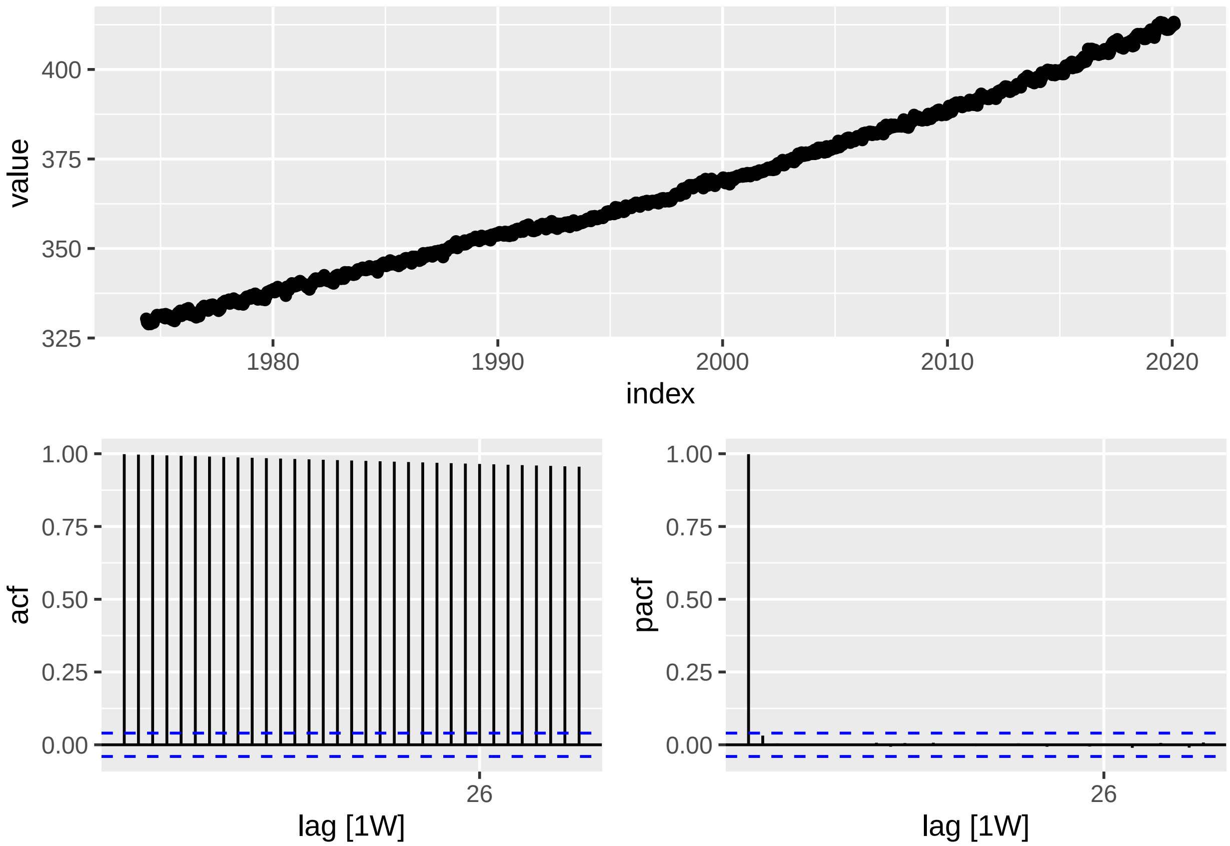
Lets check the SA time series

```
co2.noaa.weekly.ts.SA %>% autoplot()
```

```
## Plot variable not specified, automatically selected `.vars = value`
```


```
co2.noaa.weekly.ts.SA %>% gg_tsdisplay(plot_type = "partial")
```

```
## Plot variable not specified, automatically selected `y = value`
```



We can see a smoother trend line with the seasonally adjusted series. The acf chart shows a very slow decreasing lag for every 30+ lags. The pacf chart suggests a non-stationary random walk process.

Now we split the seasonally adjusted and non-seasonally adjusted time series into training and test sets. With 2388 obversations, we can take the last 2 years of the dataset by extracting the last 104 observations, since every observation is 1 week. The remaining 2284 observations are the training set.

```
trainTestSplit <- function(time.series) {
    time.series.test <- time.series %>% filter_index("2018-01-01" ~
        .)

    time.series.train <- time.series %>% filter_index(. ~ "2017-12-31")

    return(list(train = time.series.train, test = time.series.test))
}

# Split SA data into training and test

co2.noaa.weekly.ts.SA.train.test <- trainTestSplit(co2.noaa.weekly.ts.SA)
co2.noaa.weekly.ts.SA.test <- co2.noaa.weekly.ts.SA.train.test$test
co2.noaa.weekly.ts.SA.training <- co2.noaa.weekly.ts.SA.train.test$train

# Split NSA data into training and test
```

```
co2.noaa.weekly.ts.train.test <- trainTestSplit(co2.noaa.weekly.ts)
co2.noaa.weekly.ts.test <- co2.noaa.weekly.ts.train.test$test
co2.noaa.weekly.ts.training <- co2.noaa.weekly.ts.train.test$train
```

Now lets fit Arima model for SA and non SA time serieses.

First lets start with SA time sries

```
# Fit ARIMA model for SA data
no.p = 3
no.d = 2
no.q = 3

# P,D, Q are set to 0 because training data is already
# seasonally adjusted
no.P = 0
no.D = 0
no.Q = 0

no.of.models <- (no.p + 1) * (no.d + 1) * (no.q + 1) * (no.P +
    1) * (no.D + 1) * (no.Q + 1)

print(paste("Number of models to fit =", no.of.models))
```

```
## [1] "Number of models to fit = 48"
```

```
params.df <- expand.grid(p = 0:no.p, d = 0:no.d, q = 0:no.q,
    P = 0:no.P, D = 0:no.D, Q = 0:no.Q)
i <- 1

funFitModel <- function(param.row) {
    progress(value = i, max.value = no.of.models, console = TRUE,
        progress.bar = TRUE)
    i <- i + 1
    p = param.row["p"]
    d = param.row["d"]
    q = param.row["q"]
    P = param.row["P"]
    D = param.row["D"]
    Q = param.row["Q"]

    # print(paste(p,d,q,P,D,Q))
    tryCatch({
        model.fit = Arima(y = as.ts(co2.noaa.weekly.ts.SA.training),
            order = c(p, d, q), seasonal = c(0, 0, 0), include.drift = FALSE)
        model.info = data.frame(p, d, q, P, D, Q, model.fit$aic,
            model.fit$aicc, model.fit$bic)

        return(model.info)
```

```r
    }, error = function(e) {
        return(data.frame())
    })

}

model.fit.info.df.SA <- do.call("rbind", (apply(params.df, 1,
    funFitModel)))
colnames(model.fit.info.df.SA) <- c("p", "d", "q", "P", "D",
    "Q", "aic", "aicc", "bic")
```

```r
print("Top 6 models by BIC")
```

```
## [1] "Top 6 models by BIC"
```

```r
model.fit.info.df.SA %>% arrange(bic) %>% head()
```

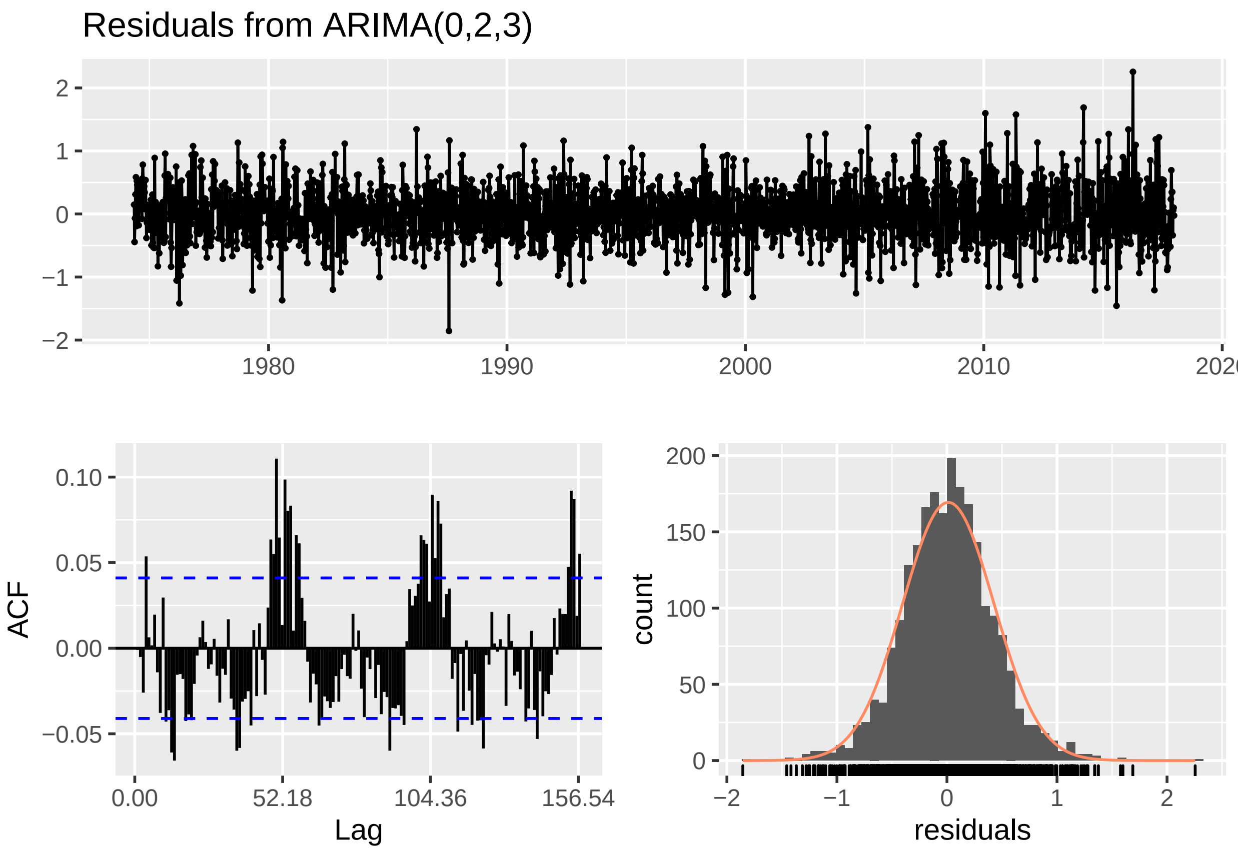
```
##   p d q P D Q      aic     aicc      bic
## 1 0 2 3 0 0 0 2485.557 2485.575 2508.476
## 2 1 2 2 0 0 0 2487.160 2487.178 2510.079
## 3 2 2 2 0 0 0 2486.372 2486.399 2515.021
## 4 3 2 2 0 0 0 2483.124 2483.161 2517.502
## 5 1 2 3 0 0 0 2491.976 2492.002 2520.624
## 6 3 2 1 0 0 0 2492.165 2492.191 2520.813
```

Arima(0,2,3) is in top 3 models by all 3 measres AIC, AICc and BIC. We pick this model as best model as this has least number of parameters compared to other top models.

```r
model.arima.best.SA = Arima(y = as.ts(co2.noaa.weekly.ts.SA.training),
    order = c(0, 2, 3), seasonal = c(0, 0, 0), include.drift = FALSE)
summary(model.arima.best.SA)
```

```
## Series: as.ts(co2.noaa.weekly.ts.SA.training)
## ARIMA(0,2,3)
##
## Coefficients:
##           ma1     ma2     ma3
##       -1.4632  0.3590  0.1046
## s.e.   0.0209  0.0358  0.0204
##
## sigma^2 estimated as 0.1737:  log likelihood=-1238.78
## AIC=2485.56   AICc=2485.57   BIC=2508.48
##
## Training set error measures:
##                      ME      RMSE       MAE         MPE       MAPE      MASE
## Training set 0.01205627 0.4162734 0.3200364 0.003176143 0.08773688 0.1791174
##                      ACF1
## Training set -0.001043418
```

```r
checkresiduals(model.arima.best.SA)
```




```
##
##  Ljung-Box test
##
## data:  Residuals from ARIMA(0,2,3)
## Q* = 338, df = 101.36, p-value < 2.2e-16
##
## Model df: 3.   Total lags used: 104.36
```

Now lets fit Arima model to non SA time series.

```r
# Fit ARIMA model for NSA data
no.p = no.P = 1
no.q = no.Q = 1
no.d = no.D = 1
no.of.models <- (no.p + 1) * (no.d + 1) * (no.q + 1) * (no.P +
    1) * (no.D + 1) * (no.Q + 1)

print(paste("Number of models to fit =", no.of.models))
```

```
## [1] "Number of models to fit = 64"
```

```r
params.df <- expand.grid(p = 0:no.p, d = 0:no.d, q = 0:no.q,
    P = 0:no.P, D = 0:no.D, Q = 0:no.Q)
```

```r
i <- 1

funFitModel <- function(param.row) {
    progress(value = i, max.value = no.of.models, console = TRUE,
        progress.bar = TRUE)
    i <- i + 1
    p = param.row["p"]
    d = param.row["d"]
    q = param.row["q"]
    P = param.row["P"]
    D = param.row["D"]
    Q = param.row["Q"]

    # print(paste(p,q,d,P,Q,D))
    tryCatch({
        model.fit = Arima(y = as.ts(co2.noaa.weekly.ts.training),
            order = c(p, d, q), seasonal = c(P, D, Q), include.drift = FALSE)
        model.info = data.frame(p, d, q, P, D, Q, model.fit$aic,
            model.fit$aicc, model.fit$bic)

        return(model.info)
    }, error = function(e) {
        return(data.frame())
    })

}

model.fit.info.df.NSA <- do.call("rbind", (apply(params.df, 1,
    funFitModel)))
colnames(model.fit.info.df.NSA) <- c("p", "d", "q", "P", "D",
    "Q", "aic", "aicc", "bic")
```

```r
print("Top 6 models by BIC")
```

```
## [1] "Top 6 models by BIC"
```

```r
model.fit.info.df.NSA %>% arrange(bic) %>% head()
```

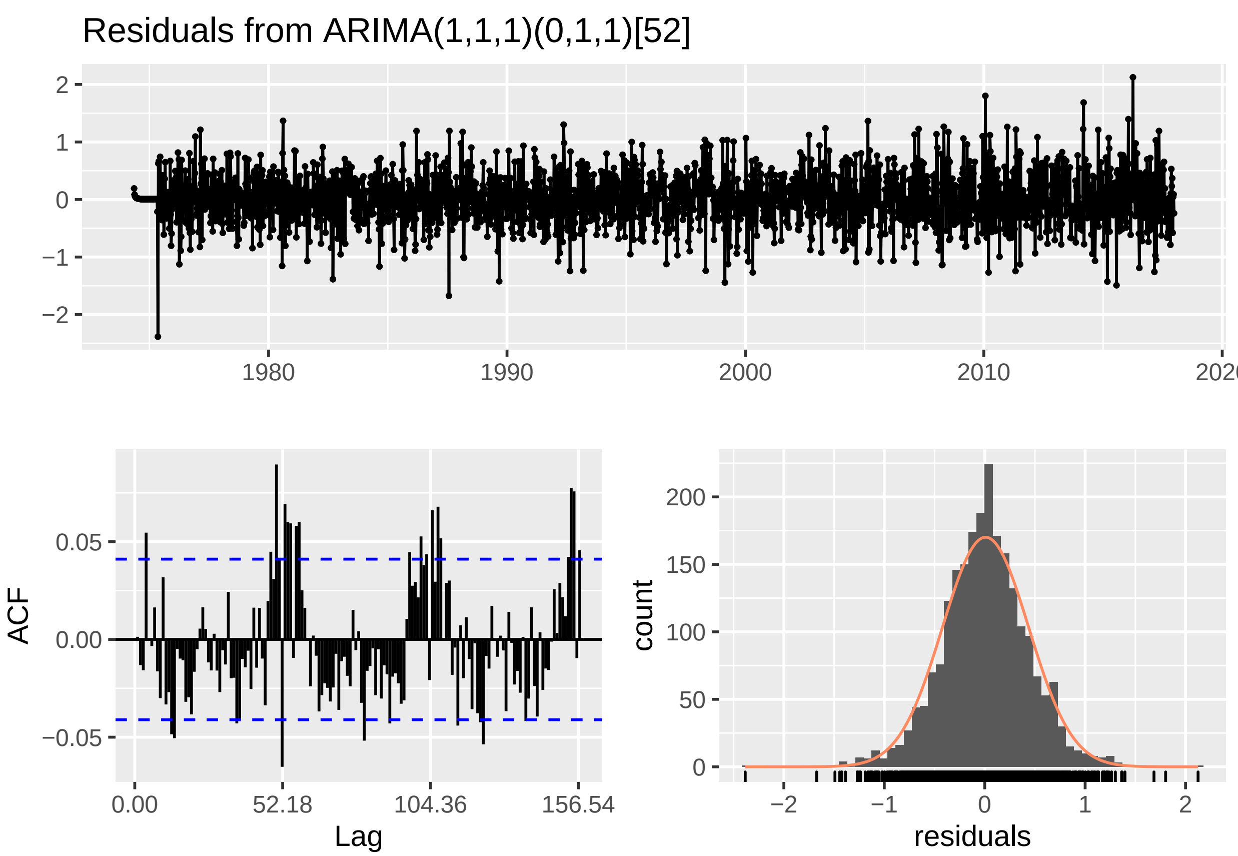
```
##   p d q P D Q      aic     aicc      bic
## 1 1 1 1 0 1 1 2641.192 2641.210 2664.020
## 2 0 1 1 1 1 1 2686.552 2686.570 2709.380
## 3 0 1 1 0 1 1 2694.043 2694.054 2711.164
## 4 0 1 1 1 0 1 2803.372 2803.390 2826.293
## 5 1 1 0 1 1 1 2910.521 2910.539 2933.349
## 6 1 1 0 0 1 1 2916.375 2916.386 2933.496
```

Best model for NSA time sries is Arima(1,1,1)(0,1,1). We selected this model because has lowest AIC, AICc and BIC so this is natural selection. We can try with higher values of p,d and q if but it is going to take much longer to estimate the models as number of parameters quickly increase exponentially.

```r
model.arima.best.NSA = Arima(y = as.ts(co2.noaa.weekly.ts.training),
    order = c(1, 1, 1), seasonal = c(0, 1, 1), include.drift = FALSE)
summary(model.arima.best.NSA)
```

```
## Series: as.ts(co2.noaa.weekly.ts.training)
## ARIMA(1,1,1)(0,1,1)[52]
##
## Coefficients:
##          ar1      ma1     sma1
##       0.2490  -0.7955  -0.8049
## s.e.  0.0326   0.0232   0.0130
##
## sigma^2 estimated as 0.1895:  log likelihood=-1316.6
## AIC=2641.19   AICc=2641.21   BIC=2664.02
##
## Training set error measures:
##                      ME      RMSE      MAE         MPE       MAPE      MASE
## Training set 0.00850045 0.4299258 0.327362 0.002071709 0.08946474 0.1832073
##                     ACF1
## Training set 0.001344668
```

```r
checkresiduals(model.arima.best.NSA)
```



```
##
##  Ljung-Box test
##
## data:  Residuals from ARIMA(1,1,1)(0,1,1)[52]
## Q* = 219.5, df = 101.36, p-value = 1.051e-10
##
## Model df: 3.   Total lags used: 104.36
```

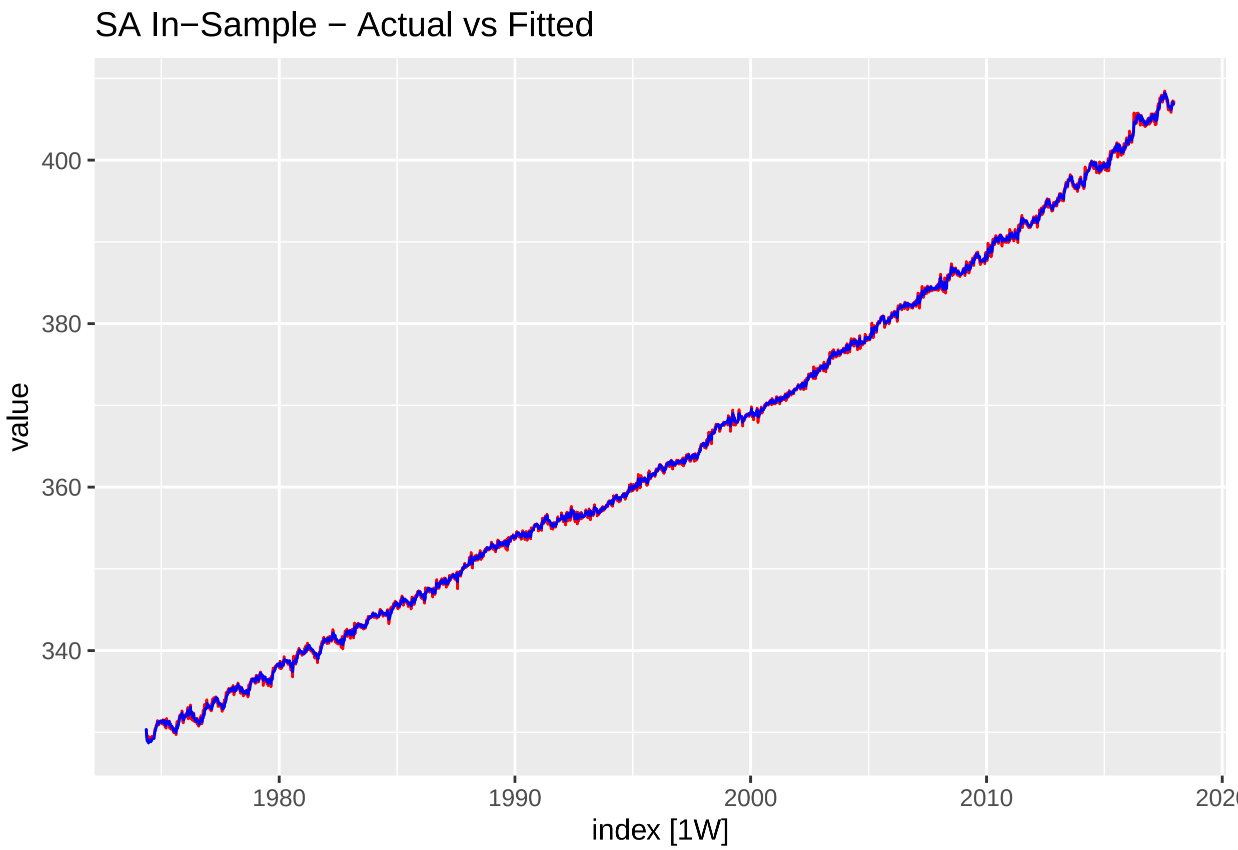
Now lets compare how these selected model perform in-sample and pseudo out-of-sample data

```
# in-sample
in.sample.accuracy = data.frame(rbind(accuracy(model.arima.best.SA),
    accuracy(model.arima.best.NSA)), row.names = c("SA", "NSA"))
in.sample.accuracy
```

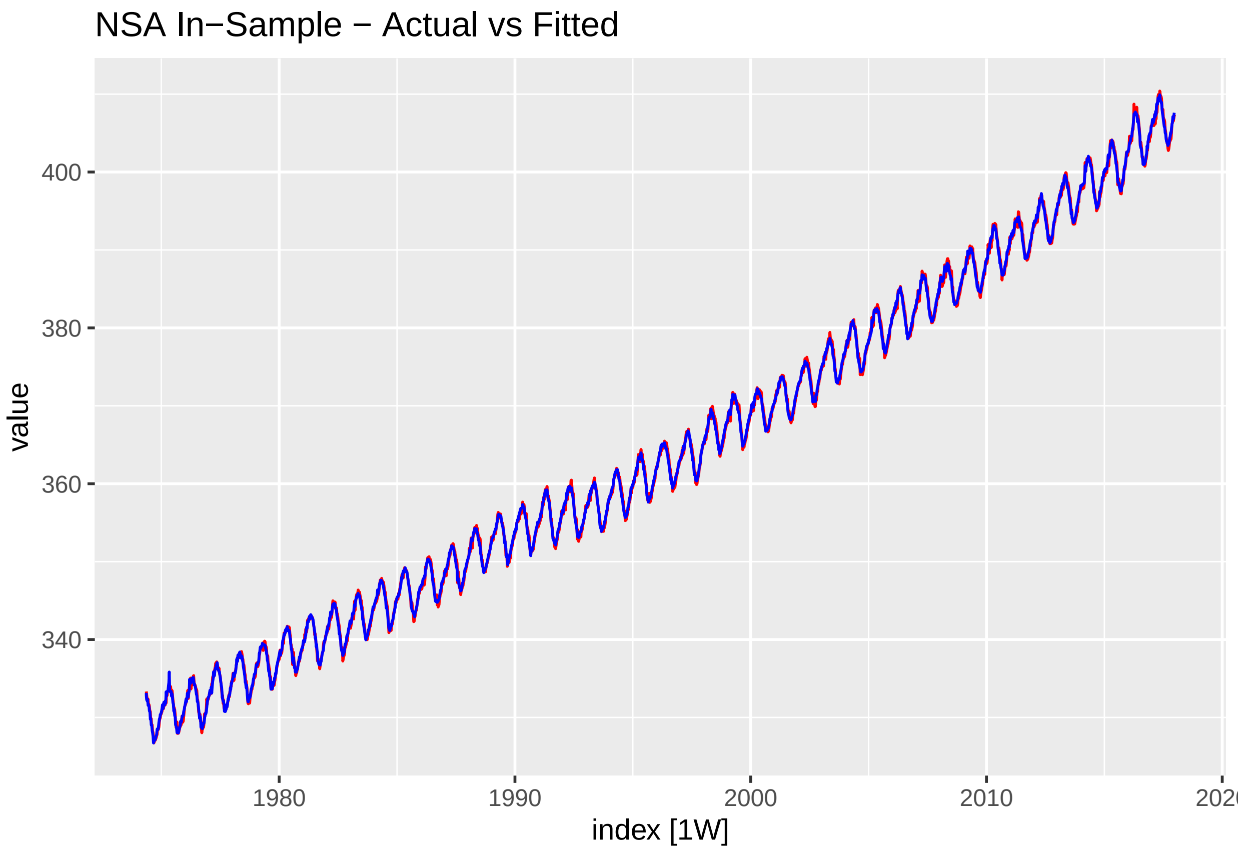
```
##              ME      RMSE       MAE         MPE       MAPE      MASE
## SA  0.01205627 0.4162734 0.3200364 0.003176143 0.08773688 0.1791174
## NSA 0.00850045 0.4299258 0.3273620 0.002071709 0.08946474 0.1832073
##             ACF1
## SA  -0.001043418
## NSA  0.001344668
```

If we look at the in-sample accuracy of our SA and NSA best models then SA model marginal does better than NSA but there is not much to choose. Lets look at actual vs fitted values in chart.

```
co2.noaa.weekly.ts.SA.training %>% autoplot(.vars = value, color = "red") +
    autolayer(as_tsibble(model.arima.best.SA$fitted), .vars = value,
        color = "blue") + labs(title = "SA In-Sample - Actual vs Fitted ")
```



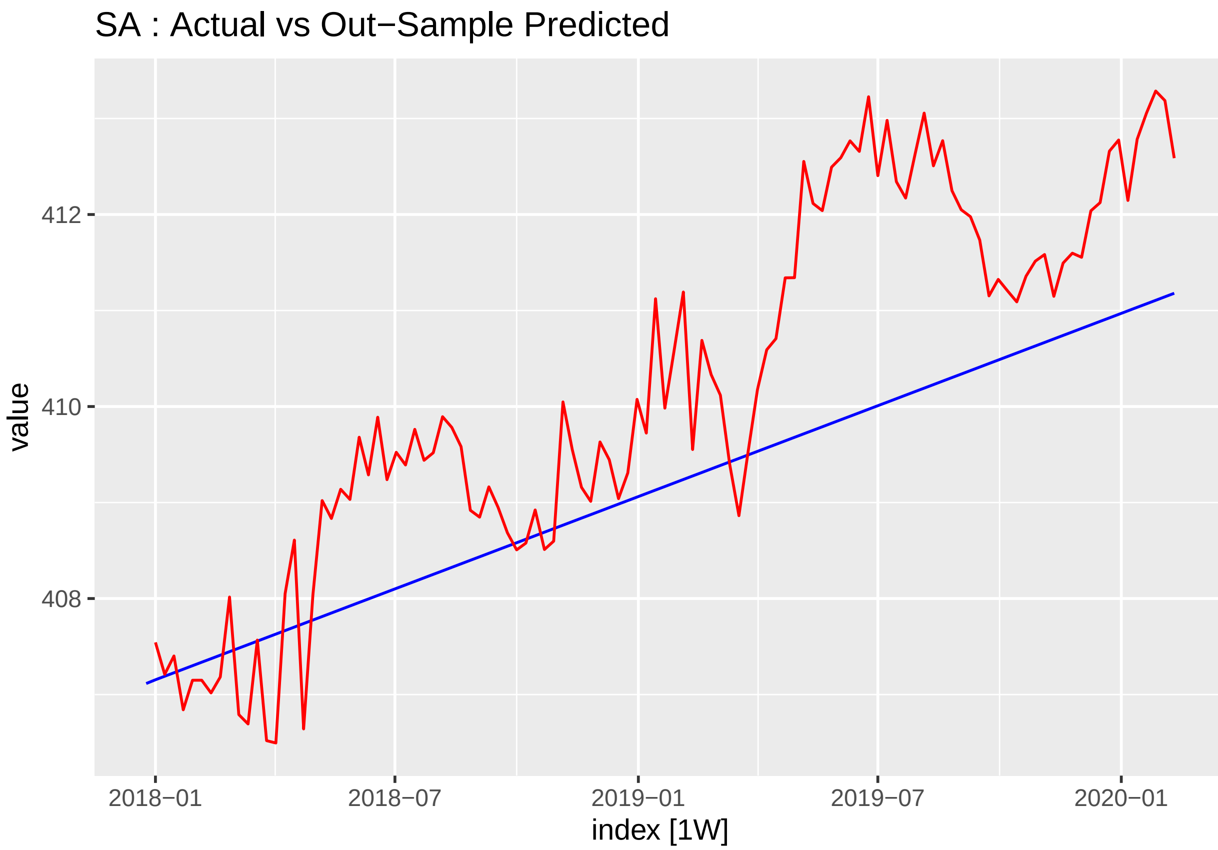

```r
co2.noaa.weekly.ts.training %>% autoplot(.vars = value, color = "red") +
    autolayer(as_tsibble(model.arima.best.NSA$fitted), .vars = value,
        color = "blue") + labs(title = "NSA In-Sample - Actual vs Fitted ")
```



Now lets look at Out-sample accuracy

For SA model

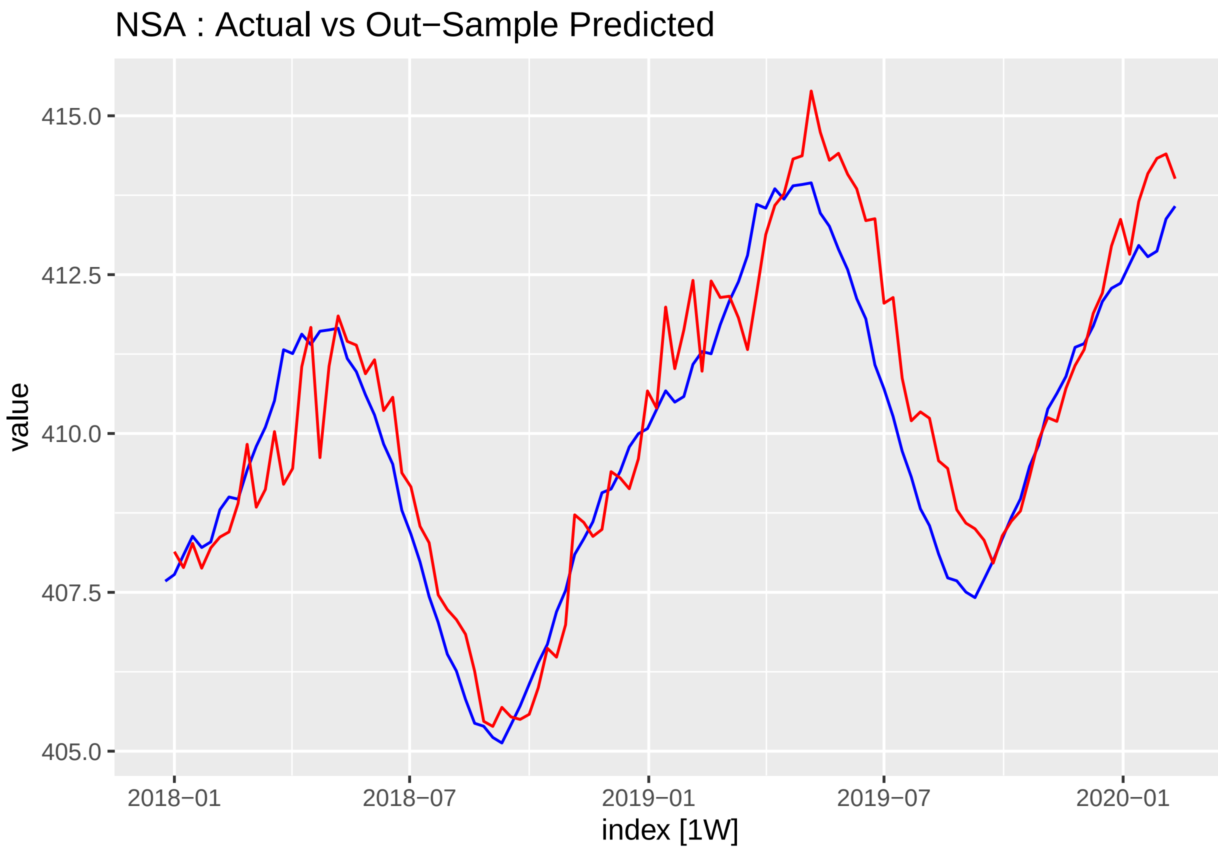
```
# SA -

forecast(model.arima.best.SA, h = length(co2.noaa.weekly.ts.SA.test$index) +
    1)$mean %>% as_tsibble() %>% autoplot(.vars = value, color = "blue") +
    autolayer(co2.noaa.weekly.ts.SA.test, .vars = value, color = "red") +
    labs(title = "SA : Actual vs Out-Sample Predicted")
```



Seasonally adjusted model predicts the point estimates as a linear model and predicts the pseudo out-saple as a trend line. This is expected as model is based on seasonally adjusted training dataset.

For NSA model

```
# NSA -

forecast(model.arima.best.NSA, h = length(co2.noaa.weekly.ts.test$index) +
    1)$mean %>% as_tsibble() %>% autoplot(.vars = value, color = "blue") +
    autolayer(co2.noaa.weekly.ts.test, .vars = value, color = "red") +
    labs(title = "NSA : Actual vs Out-Sample Predicted")
```



Non SA model follows the pseudo out-sample dataset much more closely than SA model. The predictions are smooth over weekly fluctuations but still follows the original time series curve quite closely.

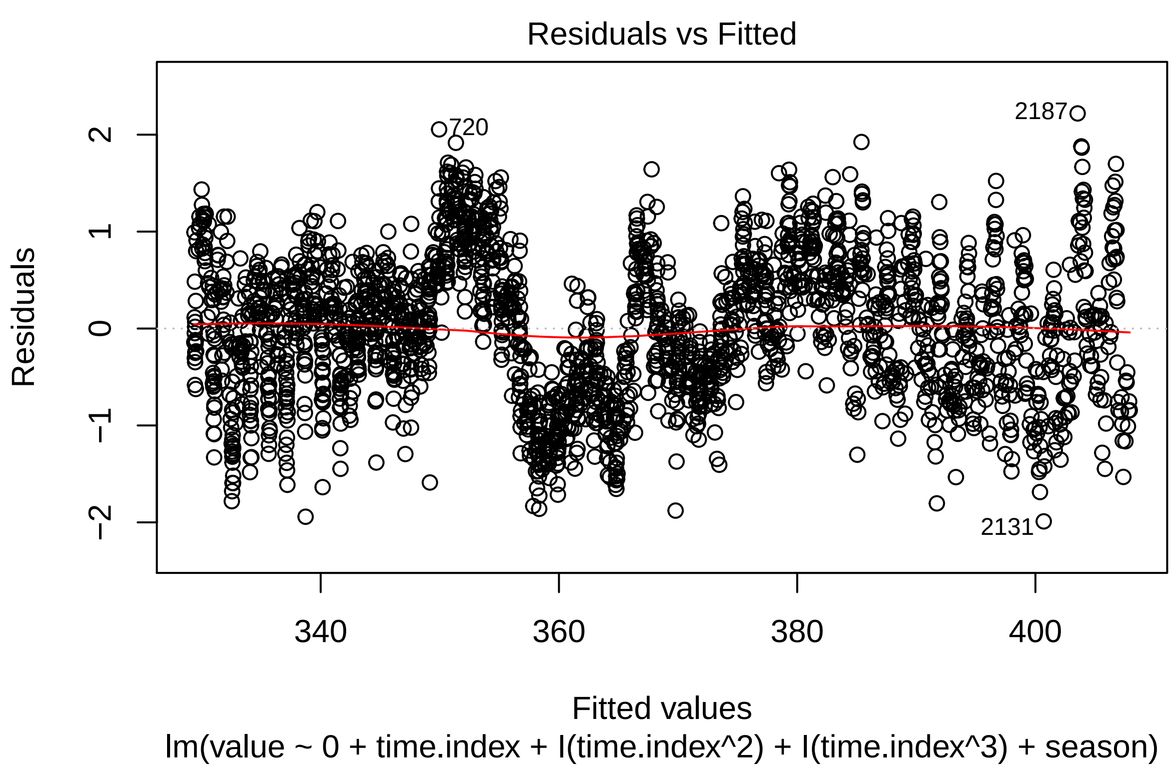
For our pplynomial model, we will use a cubic polynomial with dummy variables for each month since this is the polynomial model variation we previously had the most success with.

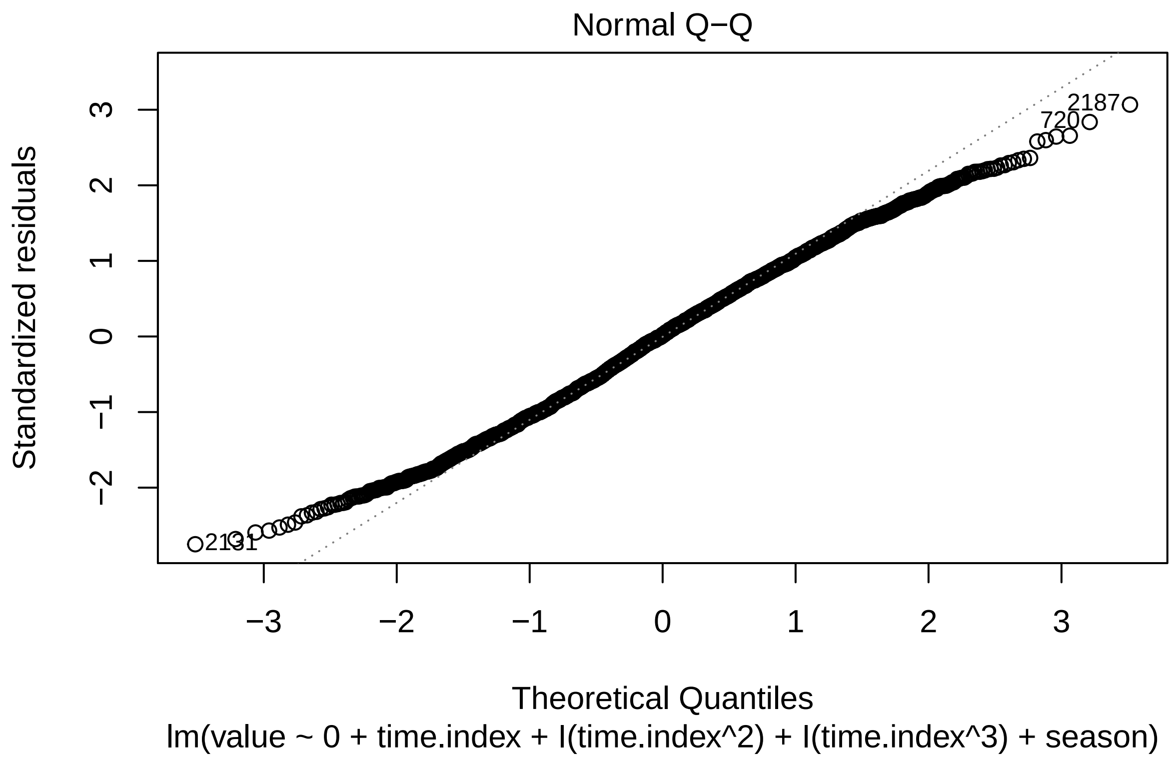
```
# Fit cubic model with month dummy variable

co2.noaa.weekly.ts.SA.training.df <- data.frame(value = co2.noaa.weekly.ts.SA.training$value,
    time.index = 1:length(co2.noaa.weekly.ts.SA.training$index),
    season = factor(month(co2.noaa.weekly.ts.SA.training$index),
        ordered = F))
mod.ln.noaa.sa <- lm(formula = value ~ 0 + time.index + I(time.index^2) +
    I(time.index^3) + season, data = co2.noaa.weekly.ts.SA.training.df)
summary(mod.ln.noaa.sa)
```

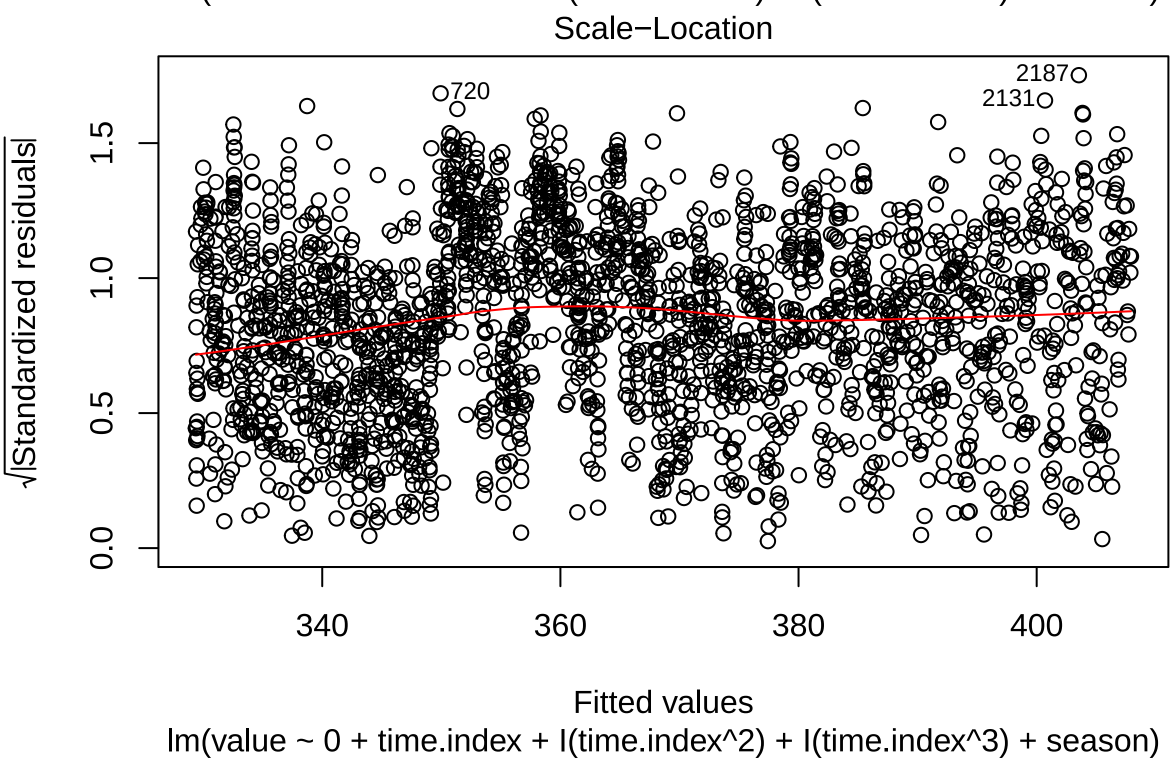
```
##
## Call:
## lm(formula = value ~ 0 + time.index + I(time.index^2) + I(time.index^3) +
##     season, data = co2.noaa.weekly.ts.SA.training.df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.9898 -0.5391  0.0125  0.5333  2.2192
```

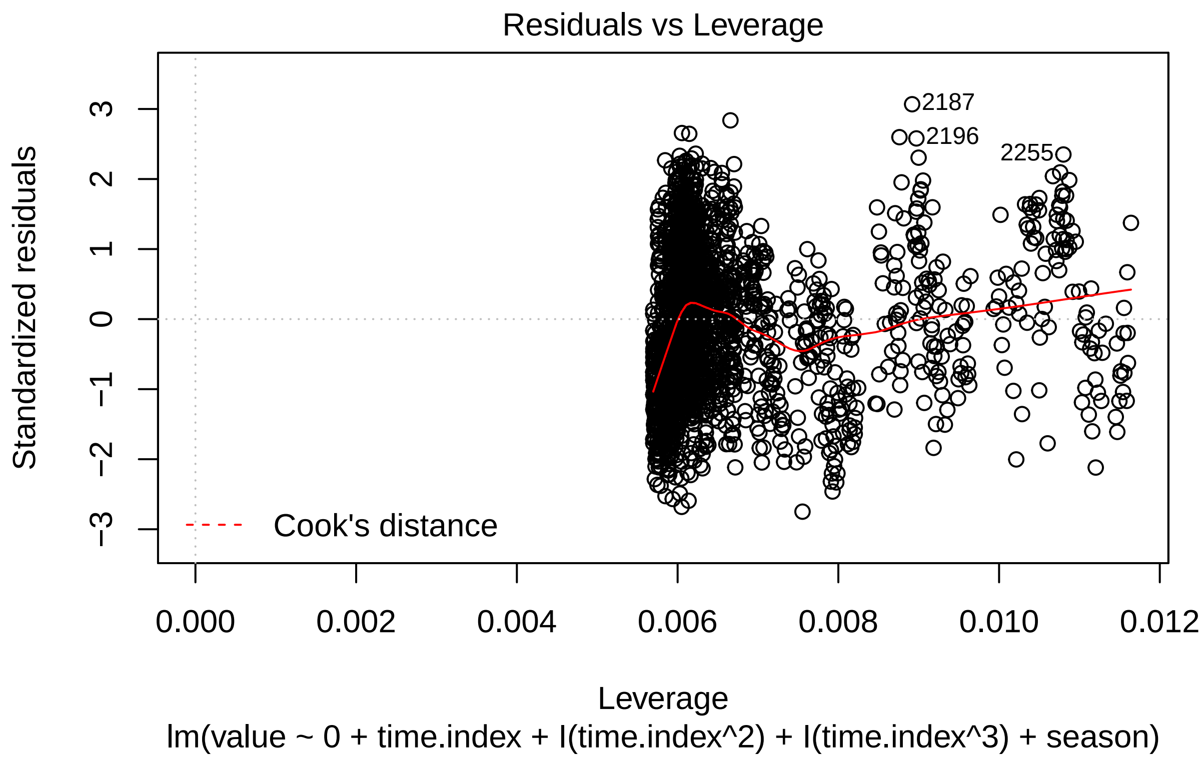
```
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## time.index       3.028e-02  2.319e-04  130.54   <2e-16 ***
## I(time.index^2) -3.561e-06  2.366e-07  -15.05   <2e-16 ***
## I(time.index^3)  2.390e-09  6.826e-11   35.01   <2e-16 ***
## season1          3.292e+02  7.924e-02 4154.45   <2e-16 ***
## season2          3.291e+02  8.104e-02 4061.02   <2e-16 ***
## season3          3.292e+02  7.917e-02 4158.17   <2e-16 ***
## season4          3.294e+02  8.035e-02 4099.21   <2e-16 ***
## season5          3.294e+02  7.855e-02 4192.95   <2e-16 ***
## season6          3.293e+02  7.887e-02 4174.85   <2e-16 ***
## season7          3.292e+02  7.856e-02 4190.34   <2e-16 ***
## season8          3.290e+02  7.864e-02 4183.98   <2e-16 ***
## season9          3.289e+02  7.901e-02 4163.52   <2e-16 ***
## season10         3.291e+02  7.894e-02 4169.13   <2e-16 ***
## season11         3.292e+02  7.958e-02 4136.85   <2e-16 ***
## season12         3.292e+02  7.884e-02 4175.99   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7266 on 2262 degrees of freedom
## Multiple R-squared:      1,  Adjusted R-squared:      1
## F-statistic: 3.835e+07 on 15 and 2262 DF,  p-value: < 2.2e-16
```

```r
plot(mod.ln.noaa.sa)
```




Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(value ~ 0 + time.index + I(time.index^2) + I(time.index^3) + season)
2187
720
2131


Scale−Location
√|Standardized residuals|
Fitted values
lm(value ~ 0 + time.index + I(time.index^2) + I(time.index^3) + season)
720
2187
2131



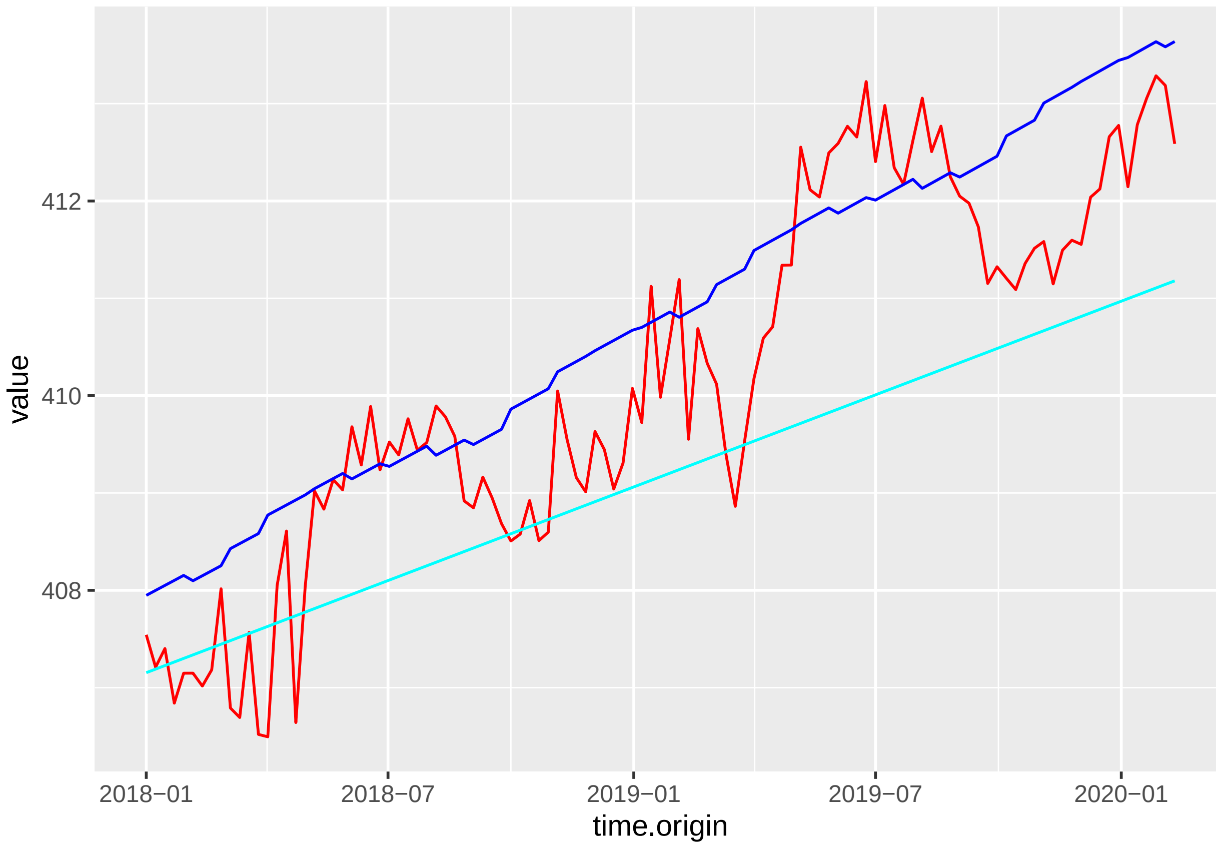
Linear model all the features as significant. Lets check the accuracy of this model

```
in.sample.accuracy.df = data.frame(RMSE = rbind(accuracy(model.arima.best.SA)[,
    "RMSE"], accuracy(mod.ln.noaa.sa)[, "RMSE"]), row.names = c("Arima-SA",
    "Linear-SA"))
in.sample.accuracy.df
```

```
##                  RMSE
## Arima-SA  0.4162734
## Linear-SA 0.7242178
```

Accuracy of polynomial model is much worse than Arima model. Lets see how this model performs to pseudo out-sample data

```
co2.noaa.weekly.ts.SA.test.df <- data.frame(value=co2.noaa.weekly.ts.SA.test$value, time.index=

co2.noaa.weekly.ts.SA.test.df$predicted = predict(mod.ln.noaa.sa, newdata = co2.noaa.weekly.ts
ggplot(co2.noaa.weekly.ts.SA.test.df, aes(x=time.origin))+
  geom_line(aes(y=value), color="red")+ #original
  geom_line(aes(y=predicted), color="blue")+ #prediected by polynomial model
  geom_line(aes(y=forecast(model.arima.best.SA, h=length(co2.noaa.weekly.ts.SA.test$index)+1 )$
```



The polynomial model follows the seasonally adjusted out sample data at the highs of seasons whereas Arima SA model passes through the mean.

Part 6 (3 points)

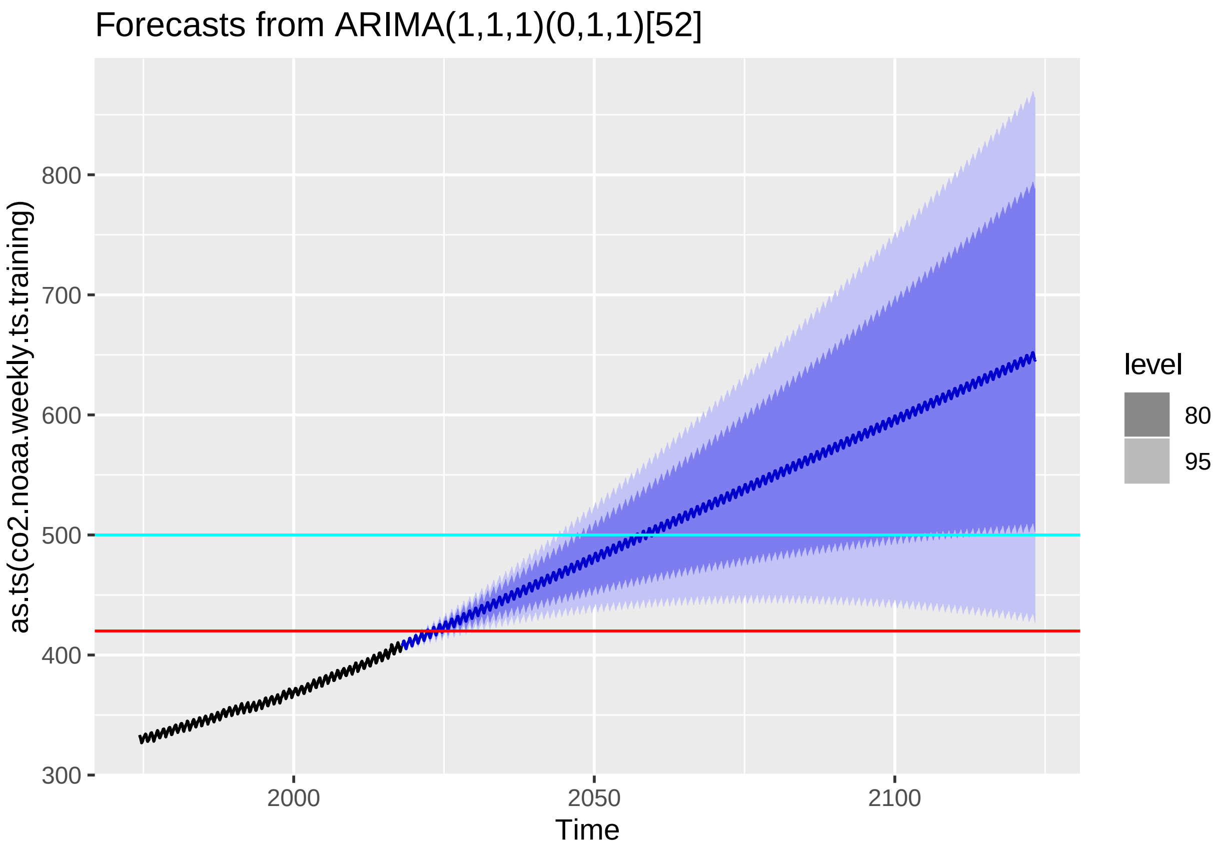
Generate predictions for when atmospheric CO2 is expected to be at 420 ppm and 500 ppm levels for the first and final times (consider prediction intervals as well as point estimates in your answer). Generate a prediction for atmospheric C02 levels in the year 2100. How confident are you that these are accurate predictions?

```
part.6 <- predict(model.arima.best.NSA, h = 500)
part.6$pred
```

```
## Time Series:
## Start = 2017.99905280457
## End = 2017.99905280457
## Frequency = 52.18
## [1] 407.6763
```

Lets first plot forecast for 5500 future intervals which would cover 420 and 500 ppm levels for irst and final times for 95% conf interval

```
forecast(model.arima.best.NSA, h = 5500) %>% autoplot() + geom_hline(yintercept = 420,
    color = "red") + geom_hline(yintercept = 500, color = "cyan")
```




```
# Get forcast for next 5500 weeks, suffucuent for lower 95%
# CI to pass 500 mark
model.arima.best.NSA.predict.ft <- forecast(model.arima.best.NSA,
    h = 5500)

# convert to ts object to find time points in future
model.arima.best.NSA.predict.ts <- as_tsibble(ts(model.arima.best.NSA.predict.ft$mean,
    start = 2017.99905280457, frequency = 52.18))

# find all data point which cross 420 or 500 ppm values
model.arima.best.NSA.predict.tbl <- model.arima.best.NSA.predict.ft %>%
    as_tibble() %>% mutate(PF.420 = `Point Forecast` >= 420,
    Lo95.420 = `Lo 95` >= 420, Hi95.420 = `Hi 95` >= 420, Lo80.420 = `Lo 80` >=
        420, Hi80.420 = `Hi 80` >= 420, PF.500 = `Point Forecast` >=
        500, Lo95.500 = `Lo 95` >= 500, Hi95.500 = `Hi 95` >=
        500, Lo80.500 = `Lo 80` >= 500, Hi80.500 = `Hi 80` >=
        500)

findForecastFirst <- function(val) {
    cnt <- model.arima.best.NSA.predict.tbl %>% mutate(count = row_number(),
        first.val = val & !lag(val)) %>% filter(first.val) %>%
        head(1) %>% select(count)

    idx <- model.arima.best.NSA.predict.ts %>% select(index) %>%
```

```
        filter(row_number() == cnt$count)

    idx$index
}

findForecastLast <- function(val) {
    cnt <- model.arima.best.NSA.predict.tbl %>% mutate(count = row_number(),
        first.val = val & !lead(val)) %>% filter(first.val) %>%
        tail(1) %>% select(count)

    idx <- model.arima.best.NSA.predict.ts %>% select(index) %>%
        filter(row_number() == cnt$count)

    idx$index
}

# 420
PF.420.first <- findForecastFirst(model.arima.best.NSA.predict.tbl$PF.420)
PF.420.last <- findForecastLast(model.arima.best.NSA.predict.tbl$PF.420)

Lo95.420.first <- findForecastFirst(model.arima.best.NSA.predict.tbl$Lo95.420)
Lo95.420.last <- findForecastLast(model.arima.best.NSA.predict.tbl$Lo95.420)

Hi95.420.first <- findForecastFirst(model.arima.best.NSA.predict.tbl$Hi95.420)
Hi95.420.last <- findForecastLast(model.arima.best.NSA.predict.tbl$Hi95.420)

Lo80.420.first <- findForecastFirst(model.arima.best.NSA.predict.tbl$Lo80.420)
Lo80.420.last <- findForecastLast(model.arima.best.NSA.predict.tbl$Lo80.420)

Hi80.420.first <- findForecastFirst(model.arima.best.NSA.predict.tbl$Hi80.420)
Hi80.420.last <- findForecastLast(model.arima.best.NSA.predict.tbl$Hi80.420)

# 500
PF.500.first <- findForecastFirst(model.arima.best.NSA.predict.tbl$PF.500)
PF.500.last <- findForecastLast(model.arima.best.NSA.predict.tbl$PF.500)

Lo95.500.first <- NULL  #findForecast(head, model.arima.best.NSA.predict.tbl$Lo95.500)
Lo95.500.last <- NULL  #findForecast(tail, model.arima.best.NSA.predict.tbl$Lo95.500)

Hi95.500.first <- findForecastFirst(model.arima.best.NSA.predict.tbl$Hi95.500)
Hi95.500.last <- findForecastLast(model.arima.best.NSA.predict.tbl$Hi95.500)

Lo80.500.first <- findForecastFirst(model.arima.best.NSA.predict.tbl$Lo80.500)
Lo80.500.last <- findForecastLast(model.arima.best.NSA.predict.tbl$Lo80.500)

Hi80.500.first <- findForecastFirst(model.arima.best.NSA.predict.tbl$Hi80.500)
Hi80.500.last <- findForecastLast(model.arima.best.NSA.predict.tbl$Hi80.500)
```

```r
predict.420 = list(Lo95.420.first, Lo95.420.last, Lo80.420.first,
    Lo80.420.last, PF.420.first, PF.420.last, Hi80.420.first,
    Hi80.420.last, Hi95.420.first, Hi95.420.last)
predict.500 = list(Lo95.500.first, Lo95.500.last, Lo80.500.first,
    Lo80.500.last, PF.500.first, PF.500.last, Hi80.500.first,
    Hi80.500.last, Hi95.500.first, Hi95.500.last)
col.names <- c("First time lower estimate with 95% CI", "Last time lower estimate with 95% CI"
    "First time lower estimate with 80% CI", "Last time lower estimate with 80% CI",
    "First time Point Estimate", "Last time Point Estimate",
    "First time higher estimate with 80% CI", "Last time higher estimate with 80% CI",
    "First time higher estimate with 95% CI", "Last time higher estimate with 95% CI")
results.df <- data.frame(predict.420)
results.df[2, ] = predict.500
colnames(results.df) <- col.names
rownames(results.df) <- c("Predictions for 420 ppm", "Predictions for 500 ppm")
results.df.t <- t(results.df)

results.df.t[order(results.df.t[, 1]), ]
```

```
##                                        Predictions for 420 ppm
## First time higher estimate with 95% CI "2021 W08"
## First time higher estimate with 80% CI "2021 W12"
## First time Point Estimate              "2022 W12"
## Last time higher estimate with 80% CI  "2022 W27"
## Last time higher estimate with 95% CI  "2022 W32"
## First time lower estimate with 80% CI  "2024 W12"
## Last time Point Estimate               "2024 W30"
## First time lower estimate with 95% CI  "2026 W11"
## Last time lower estimate with 80% CI   "2027 W30"
## Last time lower estimate with 95% CI   "2030 W30"
##                                        Predictions for 500 ppm
## First time higher estimate with 95% CI "2043 W51"
## First time higher estimate with 80% CI "2047 W04"
## First time Point Estimate              "2057 W06"
## Last time higher estimate with 80% CI  "2048 W29"
## Last time higher estimate with 95% CI  "2044 W23"
## First time lower estimate with 80% CI  "2101 W50"
## Last time Point Estimate               "2059 W25"
## First time lower estimate with 95% CI  NA
## Last time lower estimate with 80% CI   "2116 W17"
## Last time lower estimate with 95% CI   NA
```

Also note that if we keep forcasting any further, the 95% CI forecast turns downwards and may touch 420 ppm again, so above table would change if we take even wider forecast window.

Noe lets generate prediction for year 2100

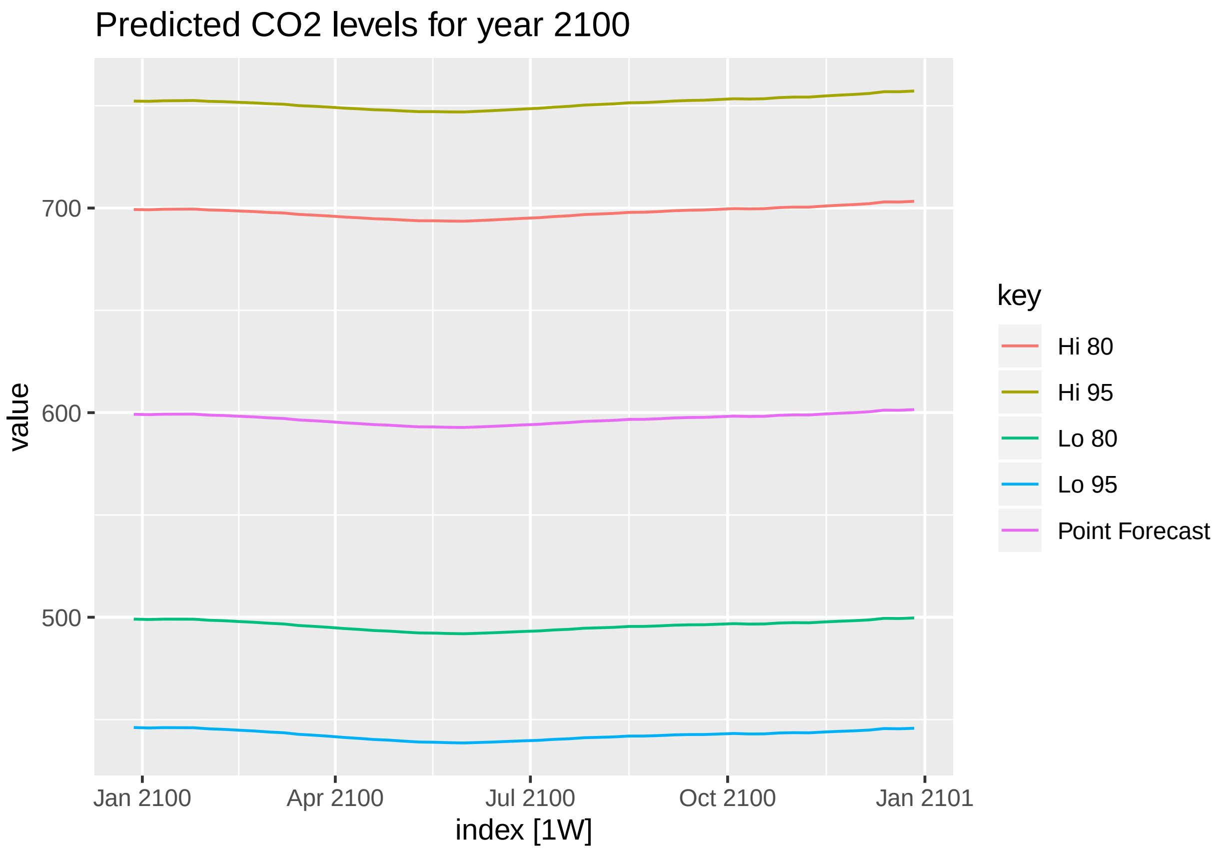
```r
model.arima.best.NSA
```

```
## Series: as.ts(co2.noaa.weekly.ts.training)
## ARIMA(1,1,1)(0,1,1)[52]
##
## Coefficients:
##          ar1      ma1     sma1
##       0.2490  -0.7955  -0.8049
## s.e.  0.0326   0.0232   0.0130
##
## sigma^2 estimated as 0.1895:  log likelihood=-1316.6
## AIC=2641.19   AICc=2641.21   BIC=2664.02
```

```r
co2.noaa.weekly.ts.training
```

```
## # A tsibble: 2,277 x 2 [1W]
##       index value
##      <week> <dbl>
##  1 1974 W20  333.
##  2 1974 W21  333.
##  3 1974 W22  332.
##  4 1974 W23  332.
##  5 1974 W24  332.
##  6 1974 W25  332.
##  7 1974 W26  332.
##  8 1974 W27  331.
##  9 1974 W28  331.
## 10 1974 W29  331.
## # ... with 2,267 more rows
```

```r
co2.noaa.weekly.ts.2100 = yearweek(seq(as.Date("2100-01-01"),
    as.Date("2100-12-31"), by = "1 week"))

model.arima.best.NSA.predict.ft %>% as_tibble() %>% ts(start = 2017.99905280457,
    frequency = 52.18) %>% as_tsibble() %>% filter_index("2100-01-01" ~
    .) %>% filter_index(. ~ "2100-12-31") %>% autoplot(.vars = value) +
    labs(title = "Predicted CO2 levels for year 2100")
```



Point estimate for CO2 levels in year 2100 is around 600 ppm. 80% CI CO2 levels are from 500 ppm to 700 ppm whereas 95 % CI CO2 levels are 450 ppm to 750 ppm.