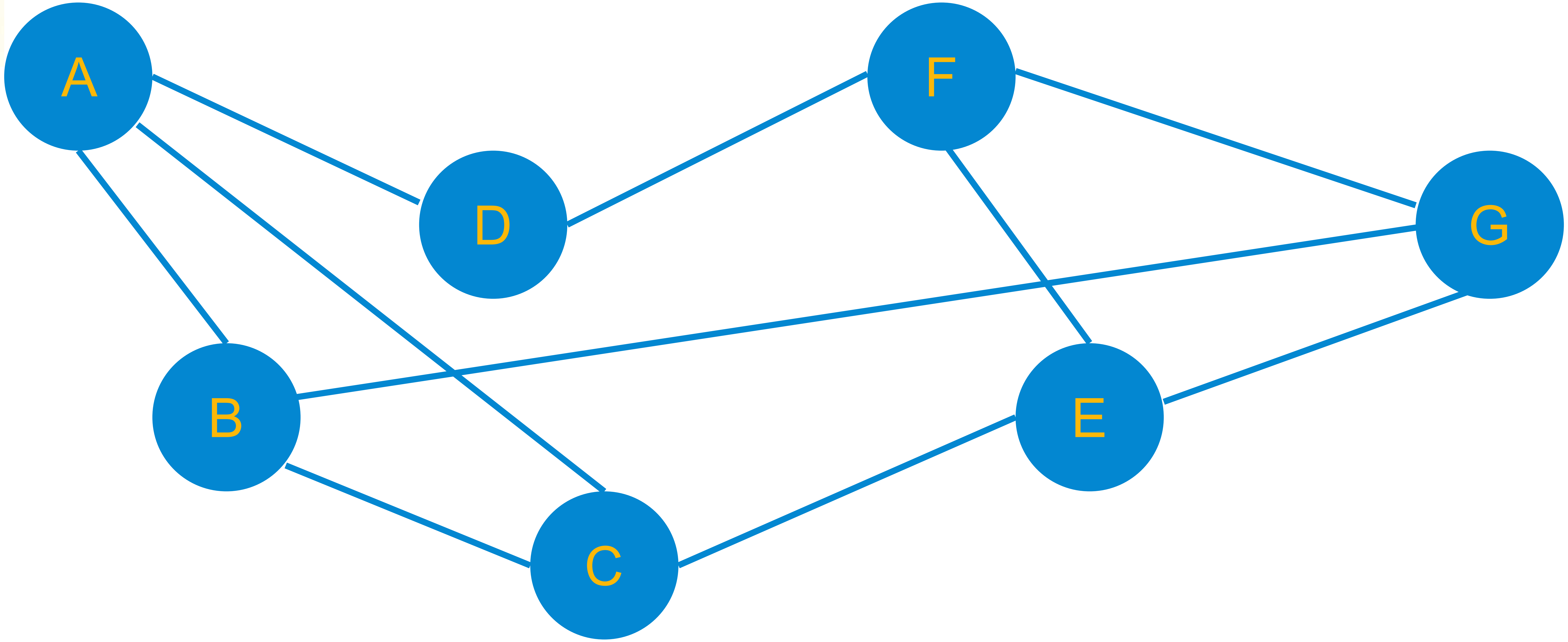
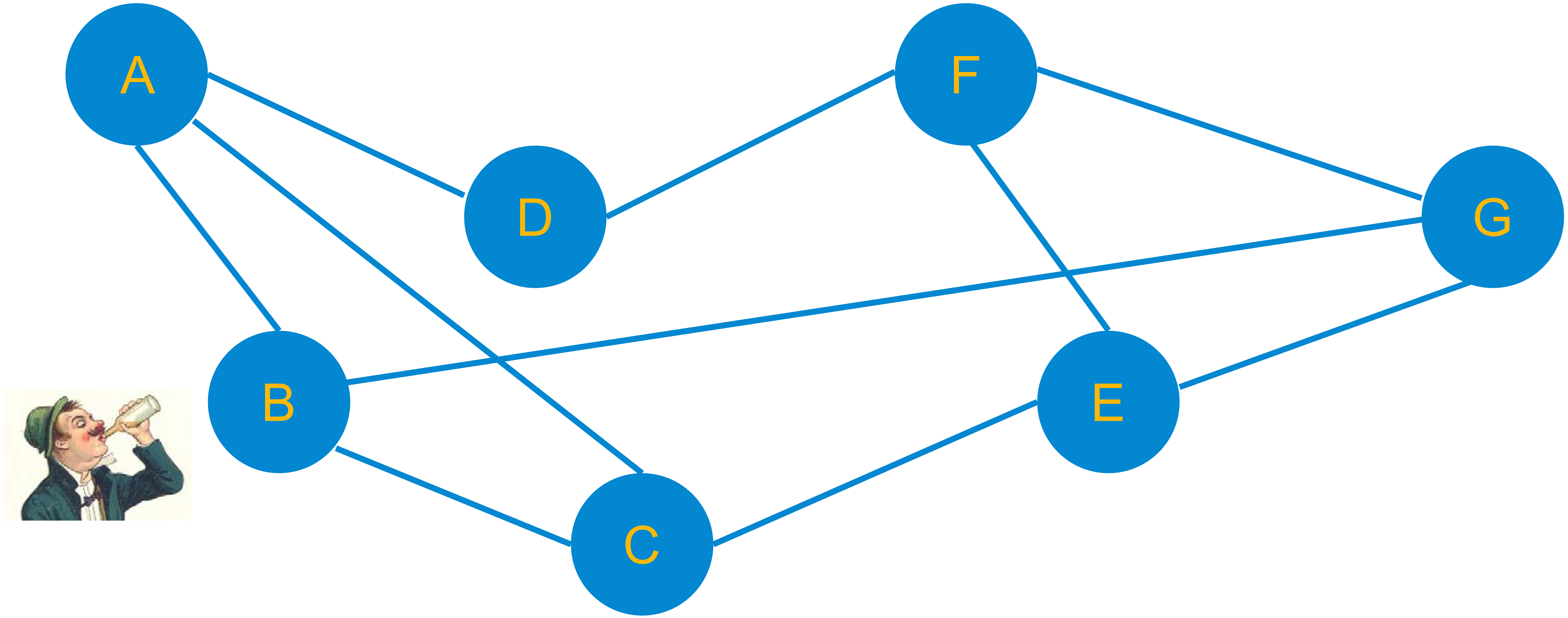
Random walk

Shizuo Kakutani : “A drunk man will find his way home, but a drunk bird may get lost forever.”

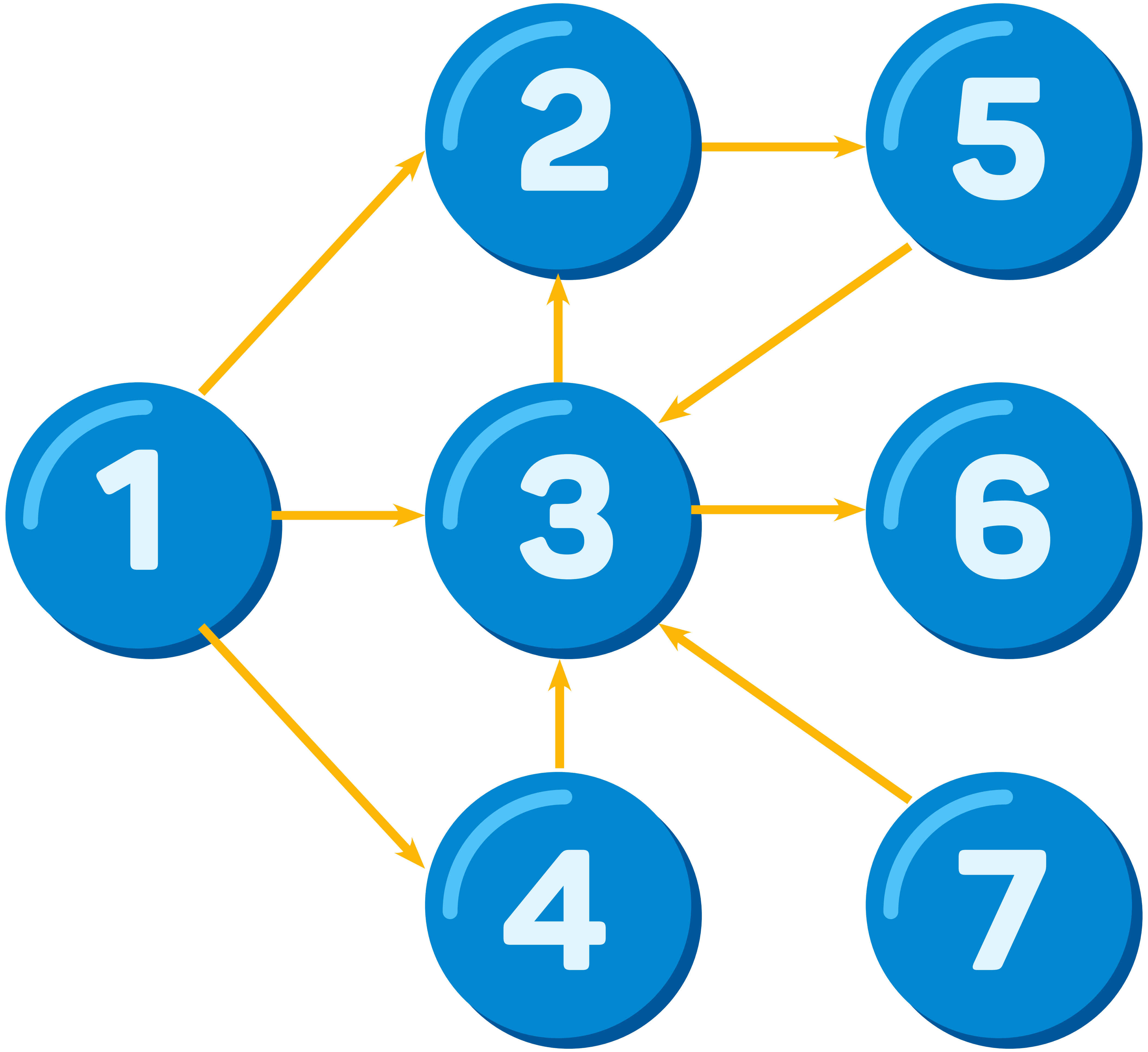
Random Walk on Graphs



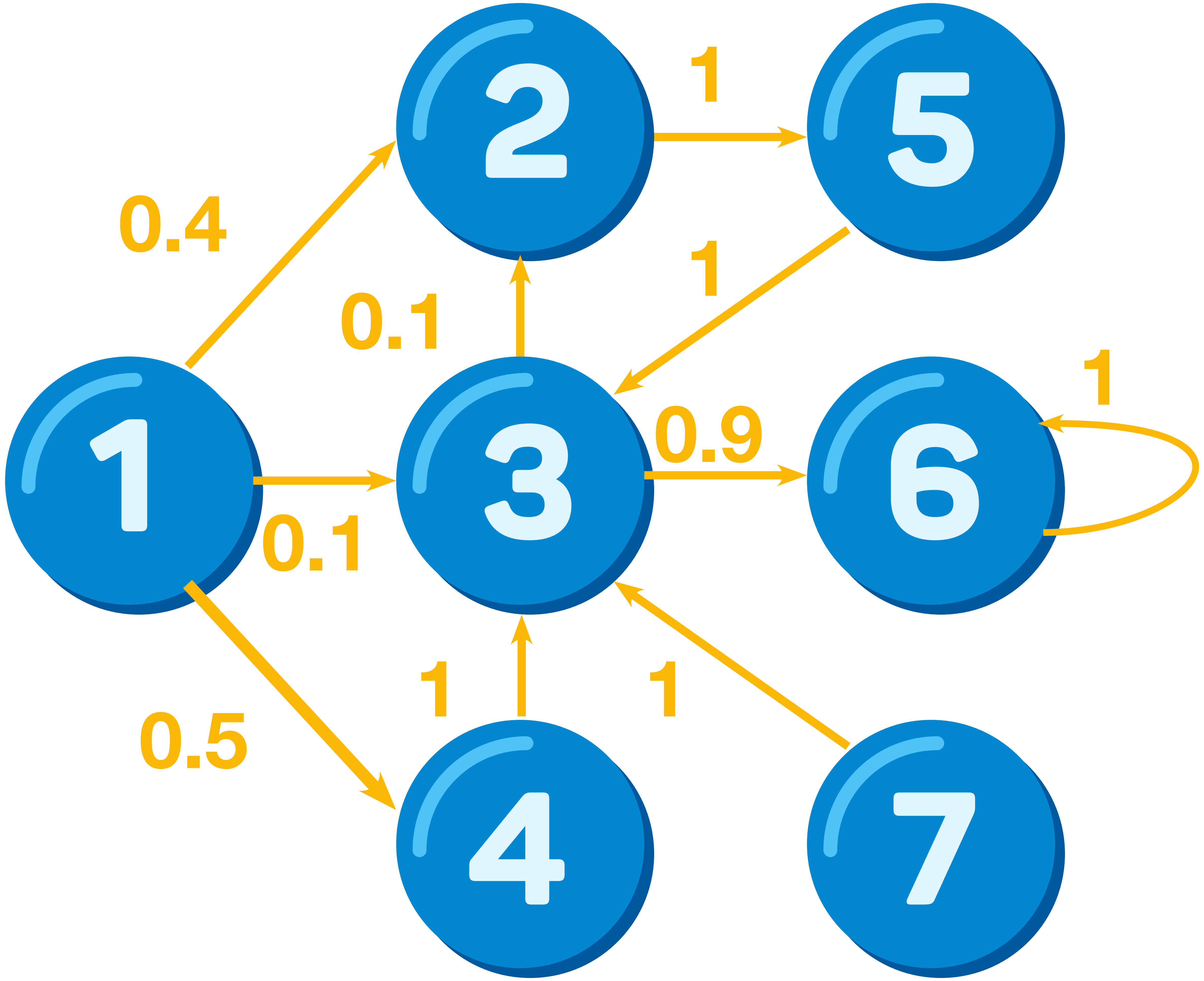
Random Walk on Graphs



Adjacency matrix

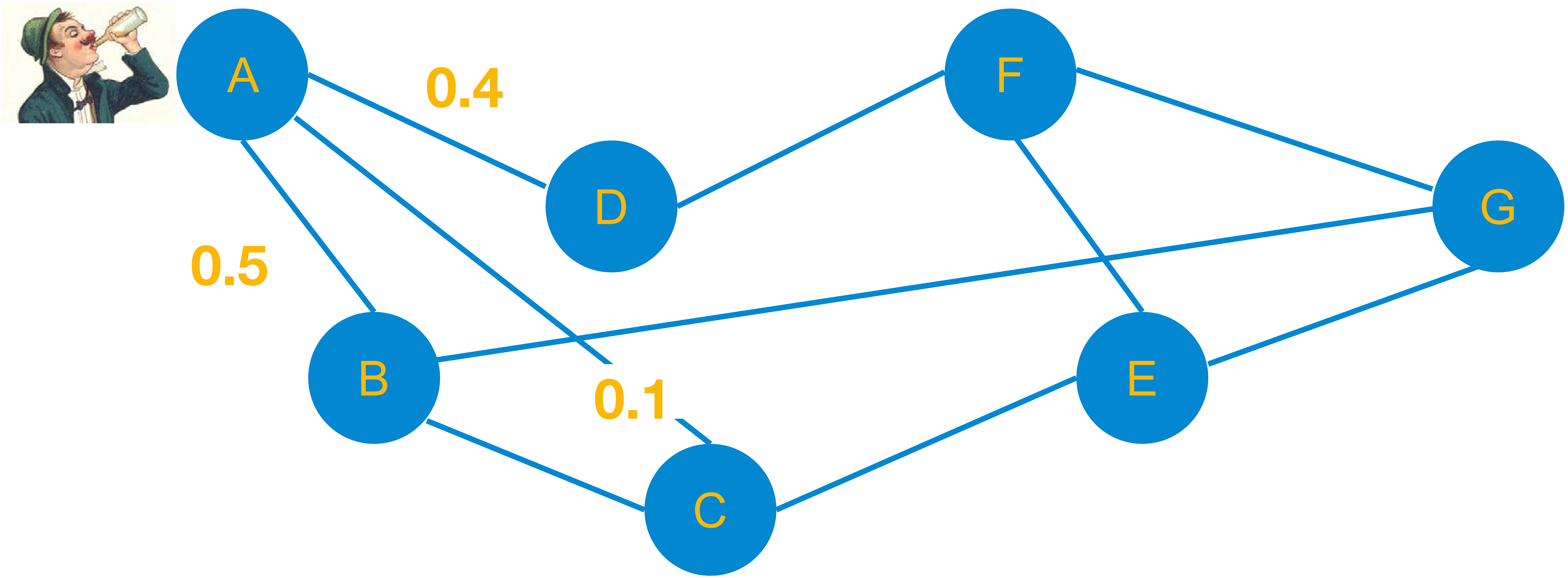



```
[ [0, 1, 1, 1, 0, 0, 0],
  [1, 0, 1, 0, 1, 0, 0],
  [1, 1, 0, 1, 1, 1, 1],
  [1, 0, 1, 0, 0, 0, 0],
  [0, 1, 1, 0, 0, 0, 0],
  [0, 0, 1, 0, 0, 0, 0],
  [0, 0, 1, 0, 0, 0, 0]]
```

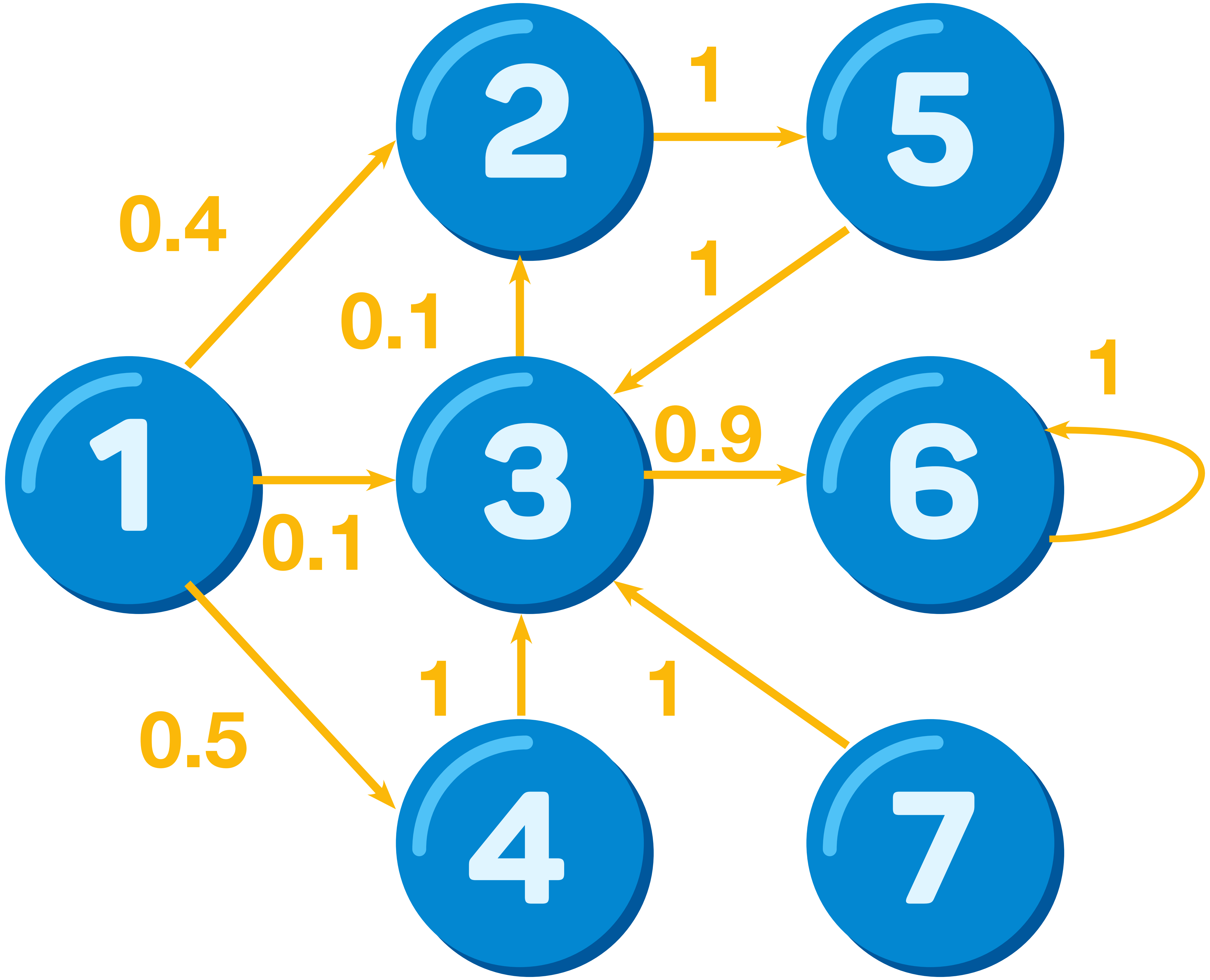


[[0, 0.4, 0.1, 0.5, 0, 0, 0],
[0, 0, 0, 0, 1, 0, 0],
[0, 0.1, 0, 0, 0, 0.9, 0],
[0, 0, 1, 0, 0, 0, 0],
[0, 0, 1, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 1, 0],
[0, 0, 1, 0, 0, 0, 0]]

Random Walk on Graphs



Stochastic graph



Transition matrix P

```
[ [0, 0.4, 0.1, 0.5, 0, 0, 0],
  [0, 0, 0, 0, 1, 0, 0],
  [0, 0.1, 0, 0, 0, 0.9, 0],
  [0, 0, 1, 0, 0, 0, 0],
  [0, 0, 1, 0, 0, 0, 0],
  [0, 0, 0, 0, 0, 1, 0],
  [0, 0, 1, 0, 0, 0, 0]]
```

Notations

- $x_t(i)$ = probability that the surfer is at node i at time t

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- $x_{t+1}(i) = \sum_j$(Probability of being at node j)*Pr(j->i) $=\sum_j x_t(j) * P(j,i)$
- $x_{t+1} = x_t P = x_{t-1} * P * P = x_{t-2} * P * P * P = \ldots = x_0 \, P^t$

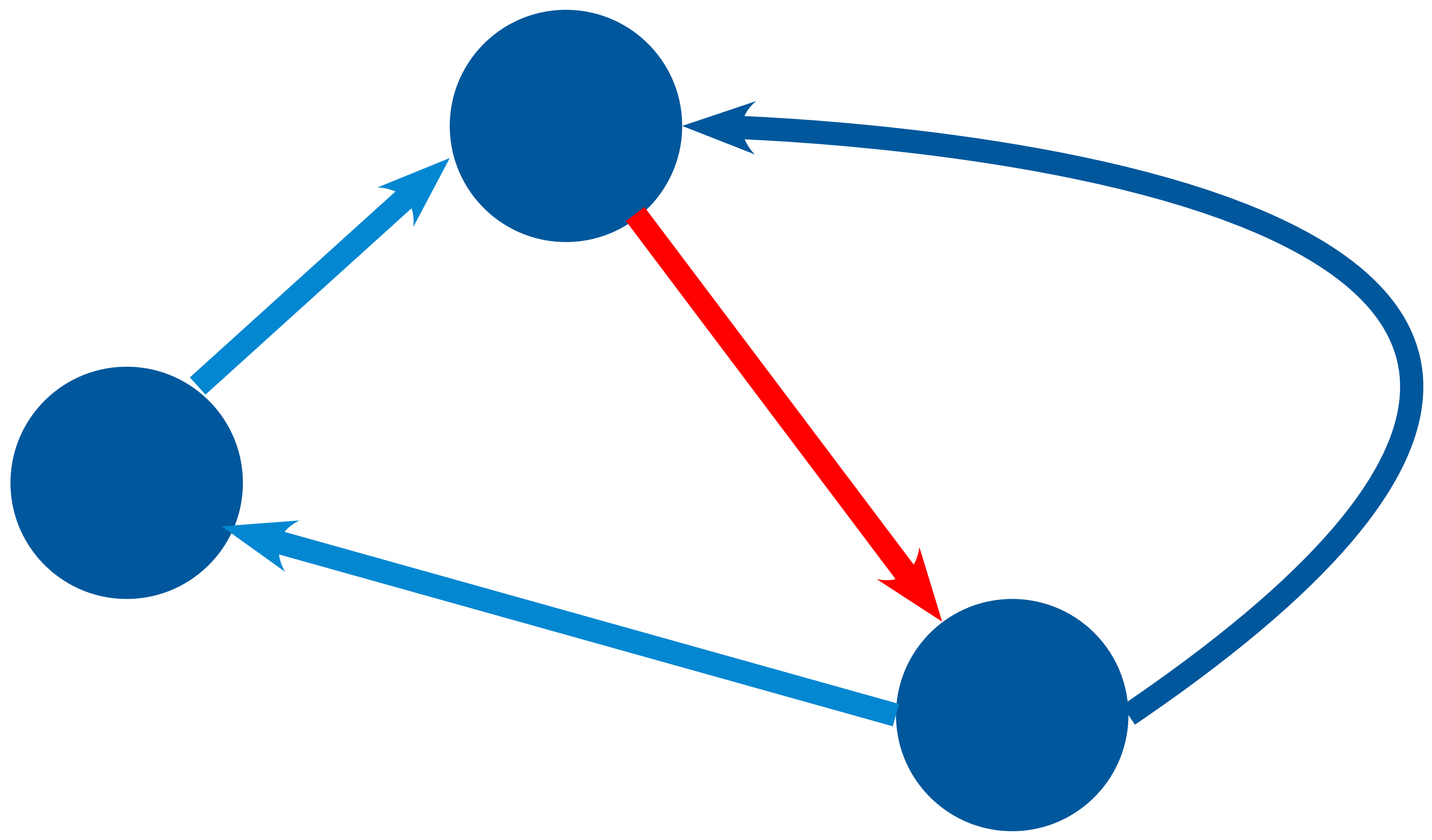
Stationary distribution

$$x^* = (x^*)^T P$$

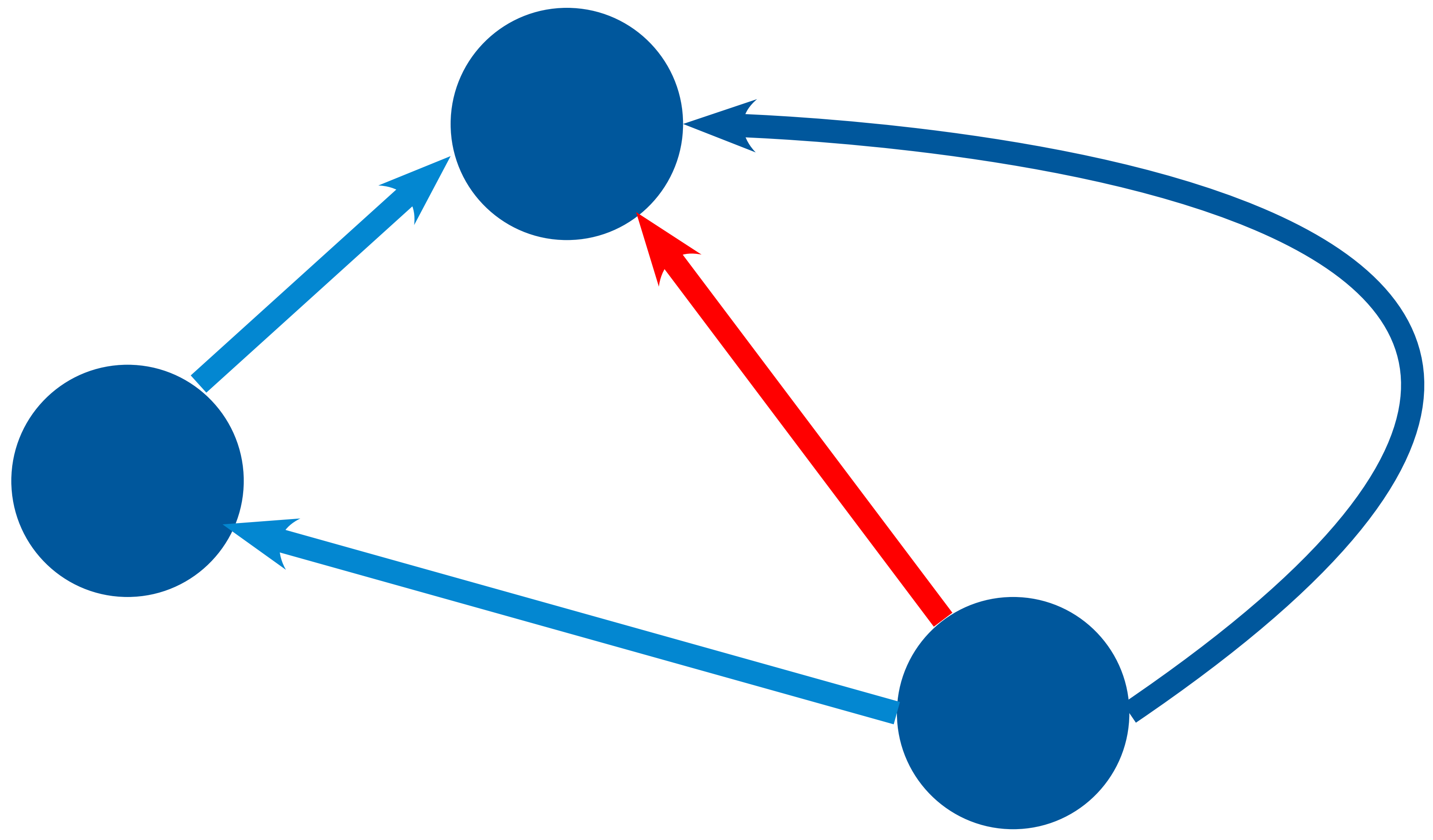
Theorem
asserting that if a stochastic graph satisfies two conditions:

1. There is a path from every node to every node

There is a path from every node to every other node.



YES



NO

Stationary distribution

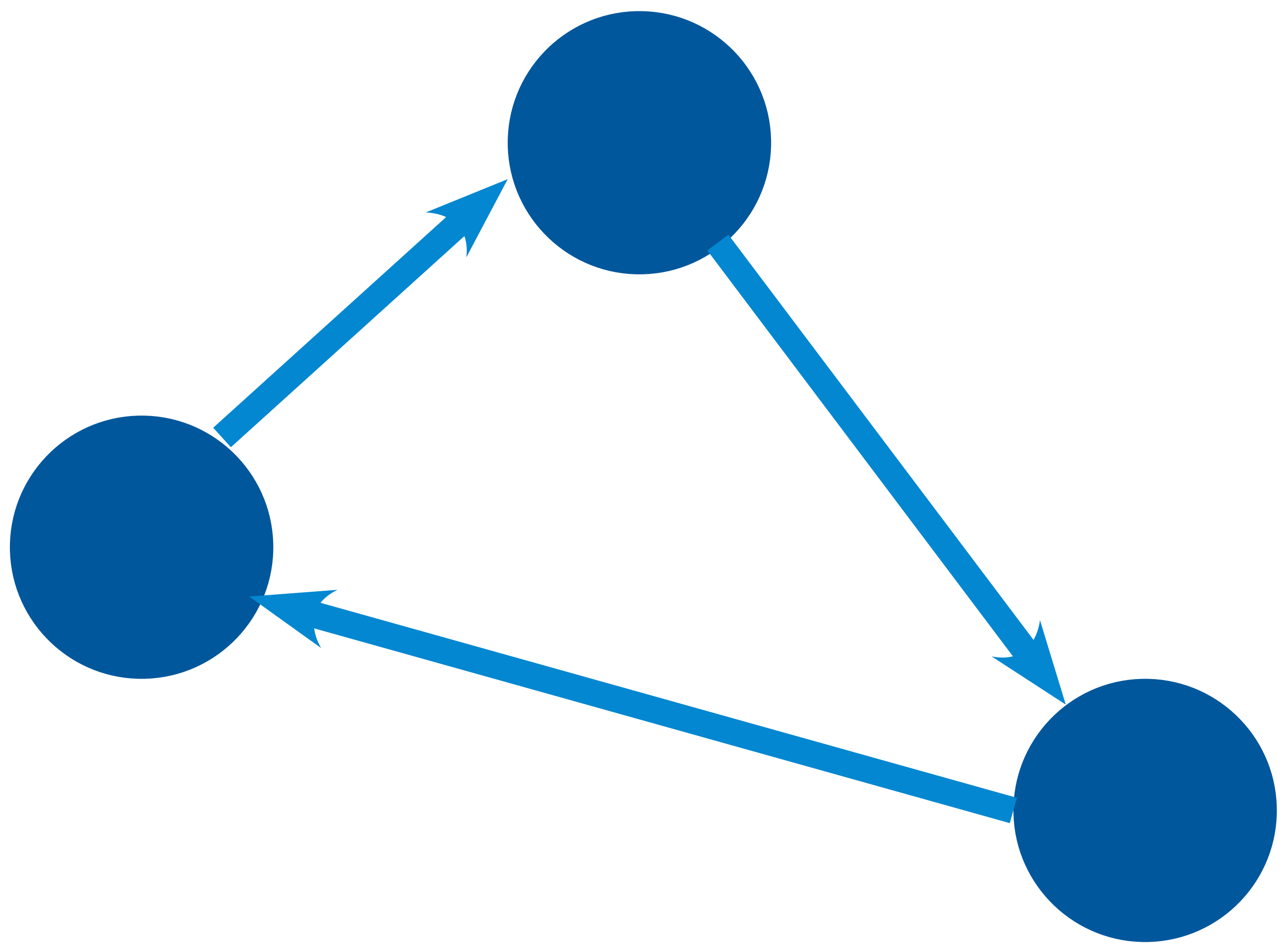
$$\mathbf{x}^* = (\mathbf{x}^*)^T \mathbf{P}$$

Theorem
asserting that if a stochastic graph satisfies two conditions:

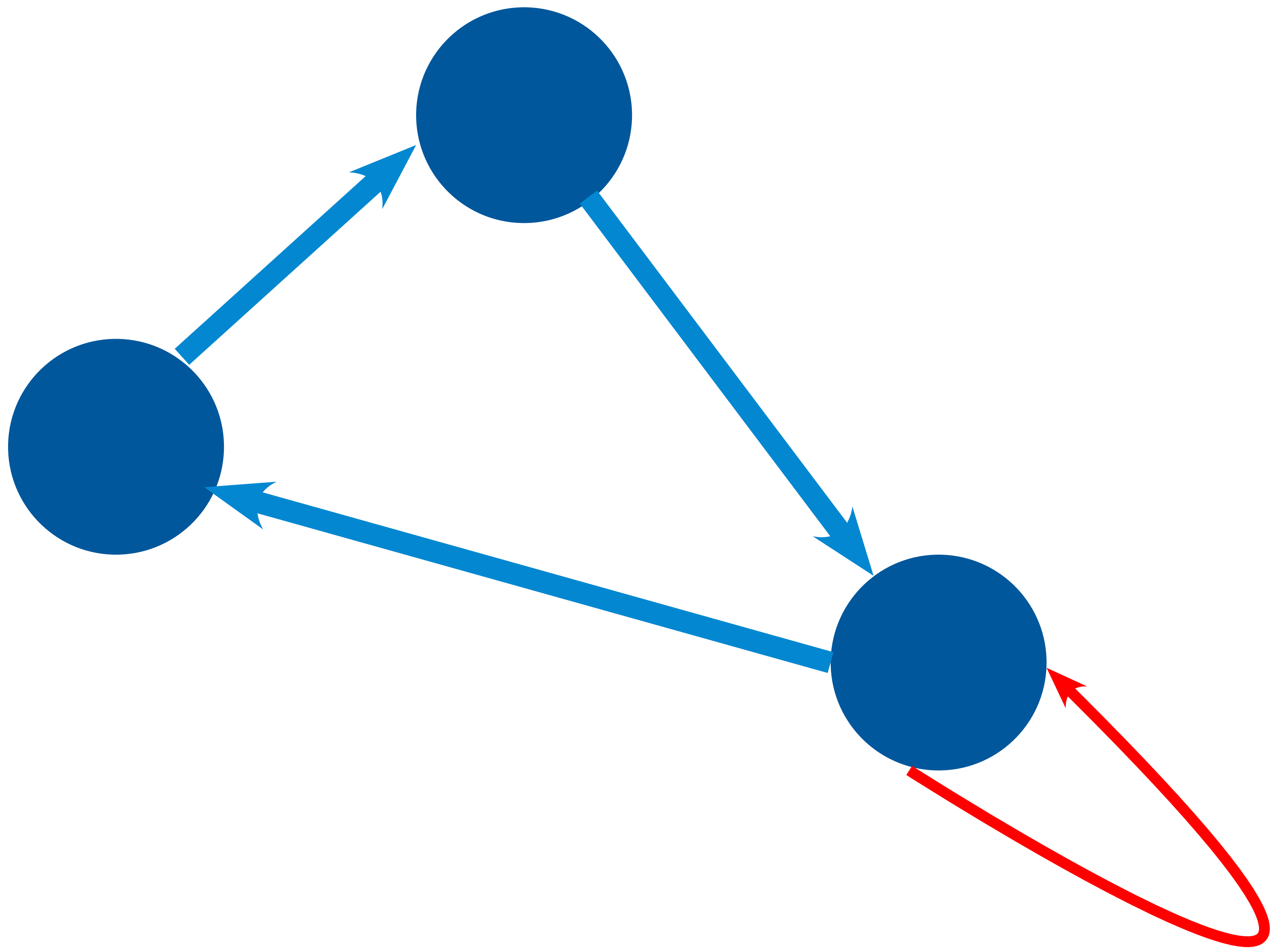
1. There is a path from every node to every node

2. The greatest common divider of all the cycle lengths id 1

The GCD of all cycle lengths is 1. The GCD is also called period.



Periodicity is 3



Aperiodic

Stationary distribution

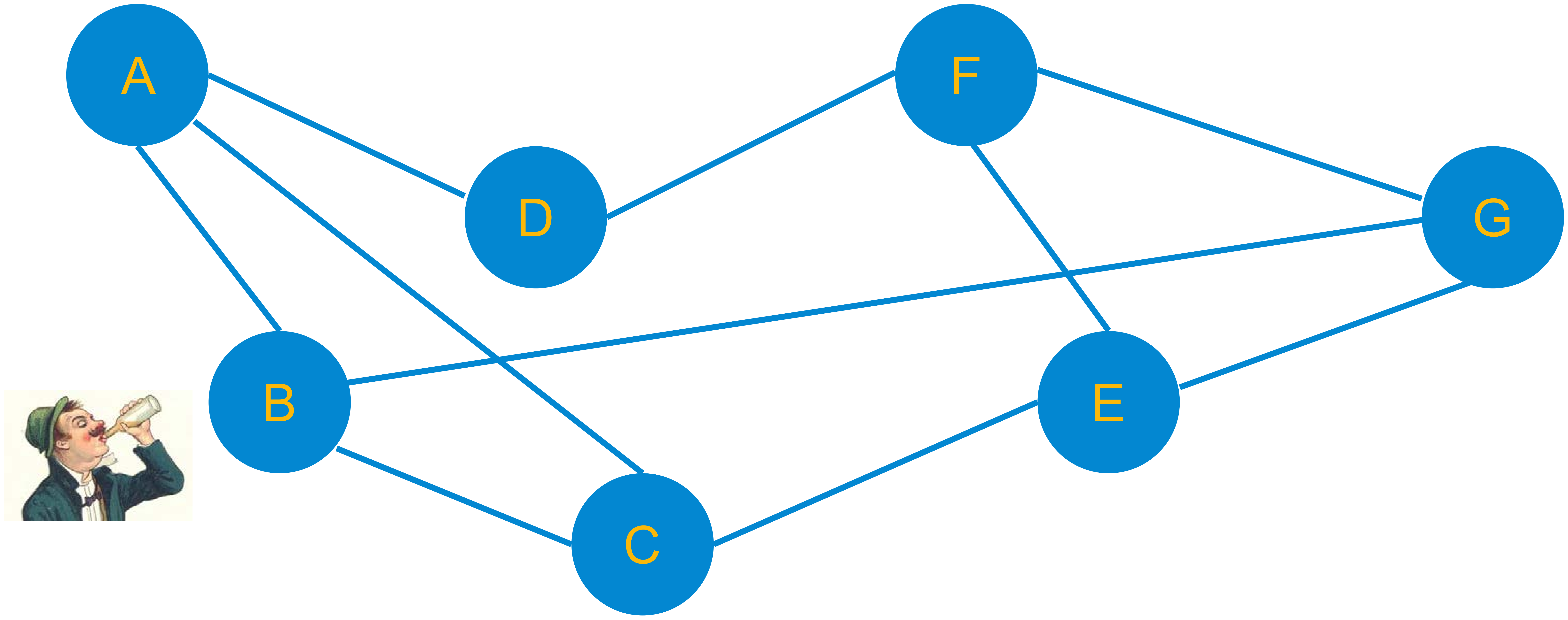
$$\mathbf{x}^* = (\mathbf{x}^*)^T \mathbf{P}$$

Theorem
asserting that if a stochastic graph satisfies two conditions:

1. There is a path from every node to every node
2. The greatest common divider of all the cycle lengths id 1

then there is a unique stationary probability distribution

Random Walk on Graphs



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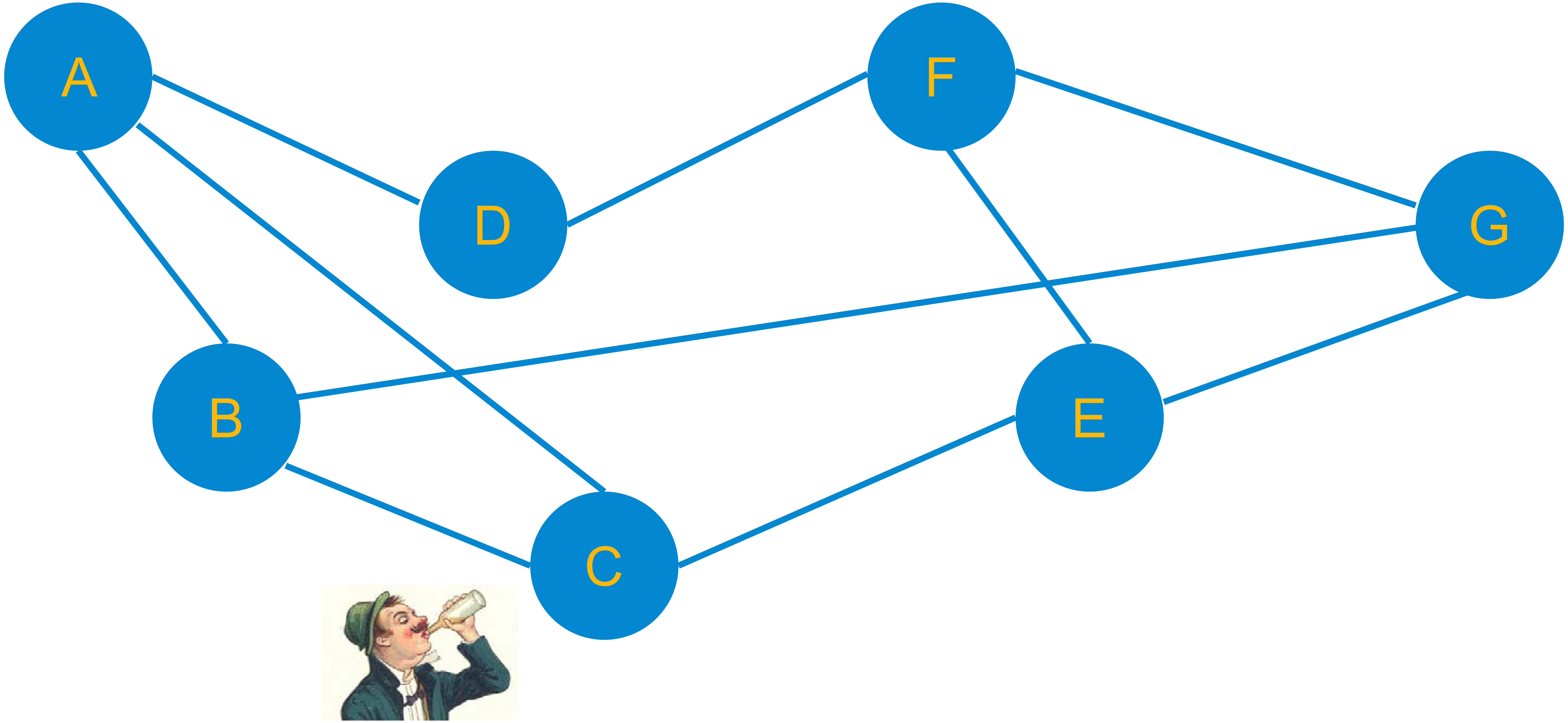
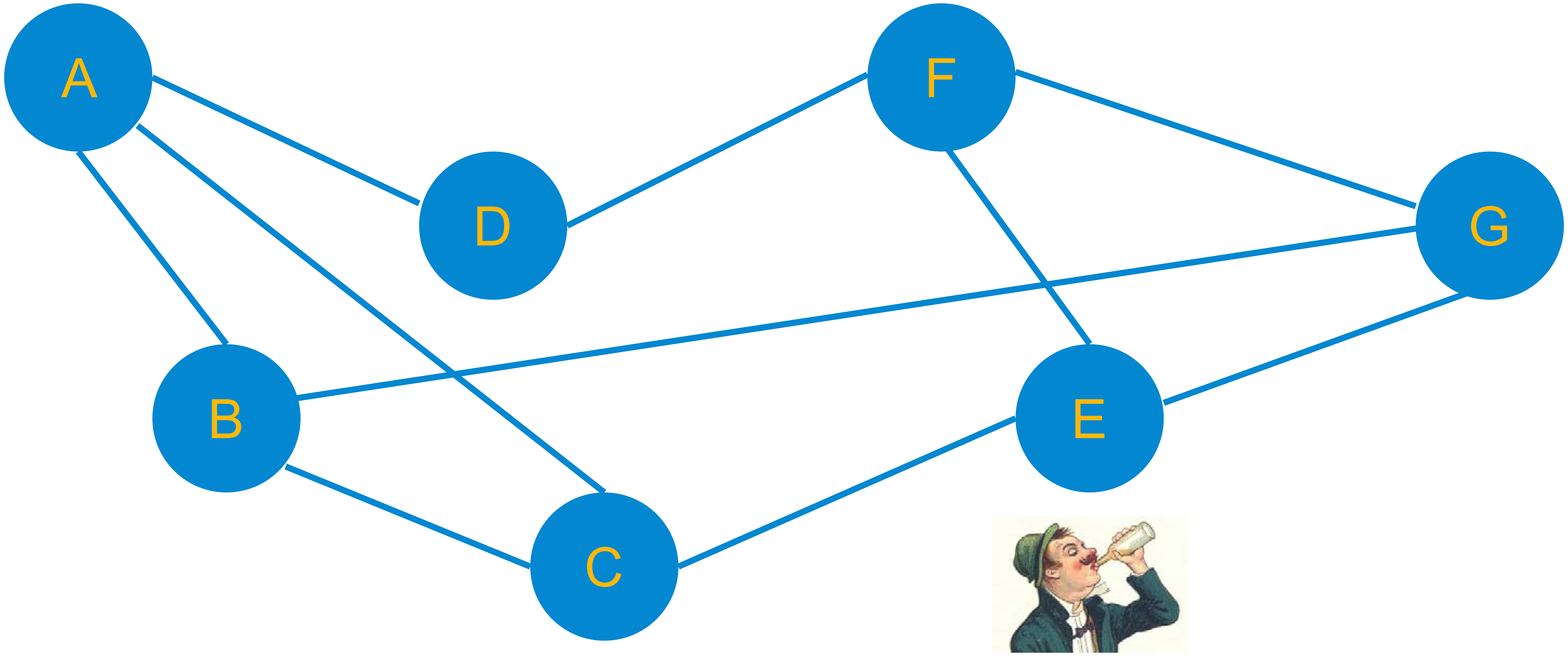
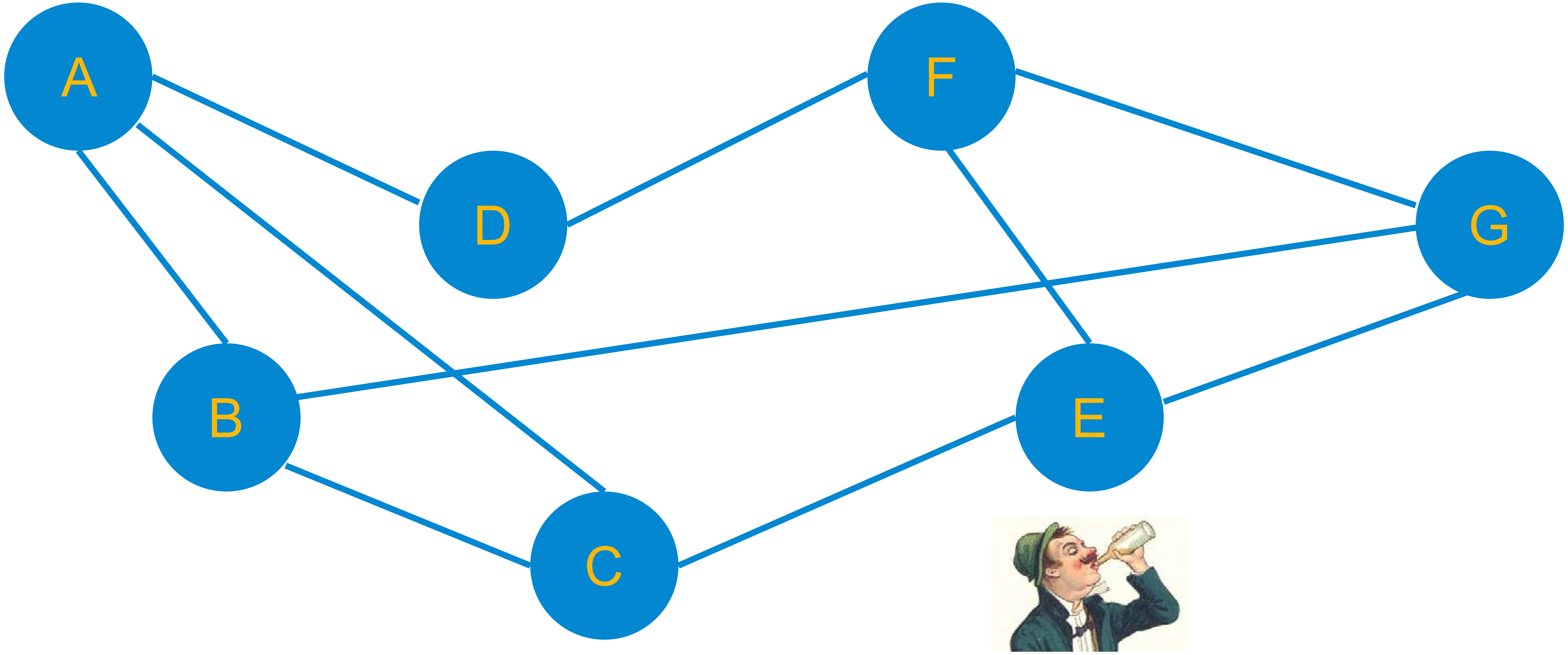


Random Walk on Graphs



Random Walk on Graphs



$$x_{t+1} = x_t P = x_{t-1} * P * P = x_{t-2} * P * P * P = \ldots = x_0 P^t$$

Summary

Asserting that if a stochastic graph satisfies two conditions:

1. There is a path from every node to every node
2. The greatest common divider of all the cycle lengths id 1

then there is a unique stationary probability distribution