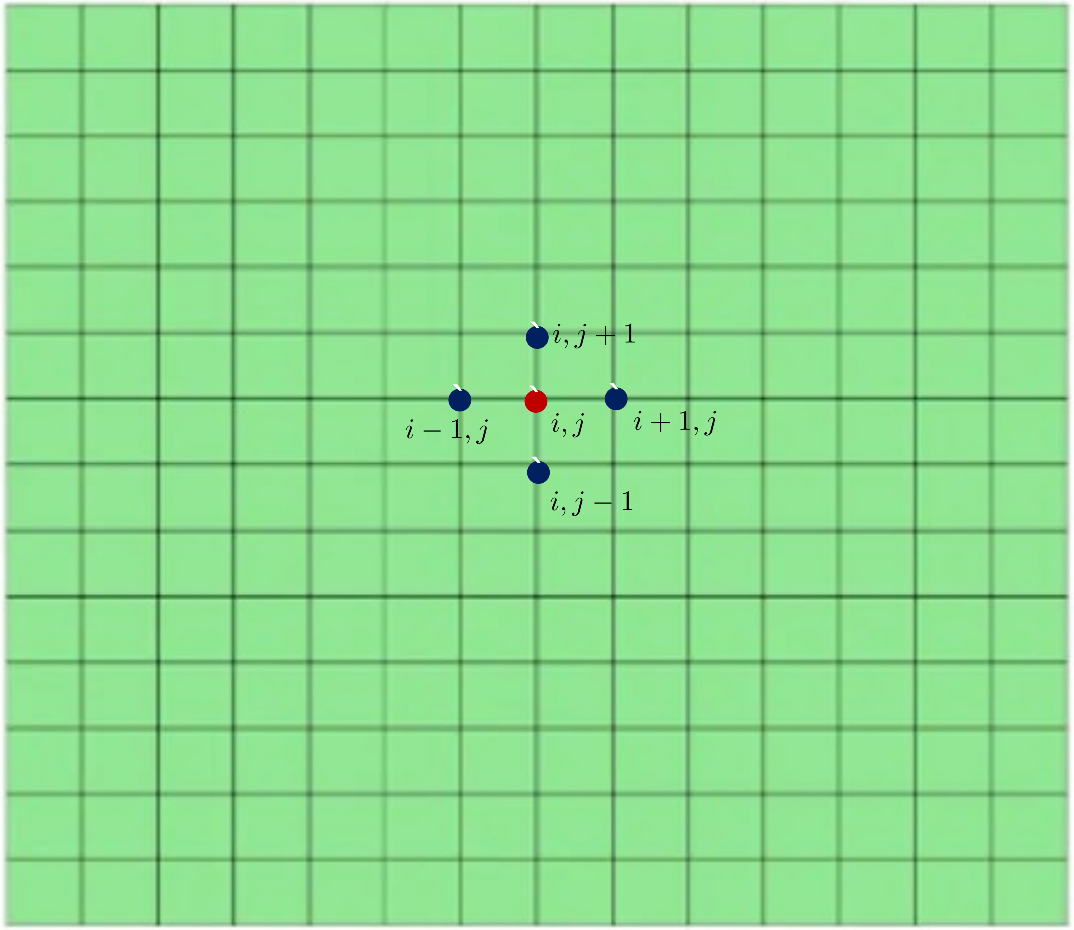
Finite Difference method

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_x}{\partial x} + \frac{\partial \rho u_y}{\partial y} = 0$$

$$\frac{\rho_{ij}^{n+1} - \rho_{ij}^{n}}{\Delta t} + \left[\frac{\rho_{i+1,j}^{n+1} u_{i+1,j}^{n+1} - \rho_{i-1,j}^{n+1} u_{i-1,j}^{n+1}}{2\Delta x} \right] + \left[\frac{\rho_{i,j+1}^{n+1} u_{i,j+1}^{n+1} - \rho_{i,j-1}^{n+1} u_{i,j-1}^{n+1}}{2\Delta y} \right] = 0$$



Finite Volume method

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0$$

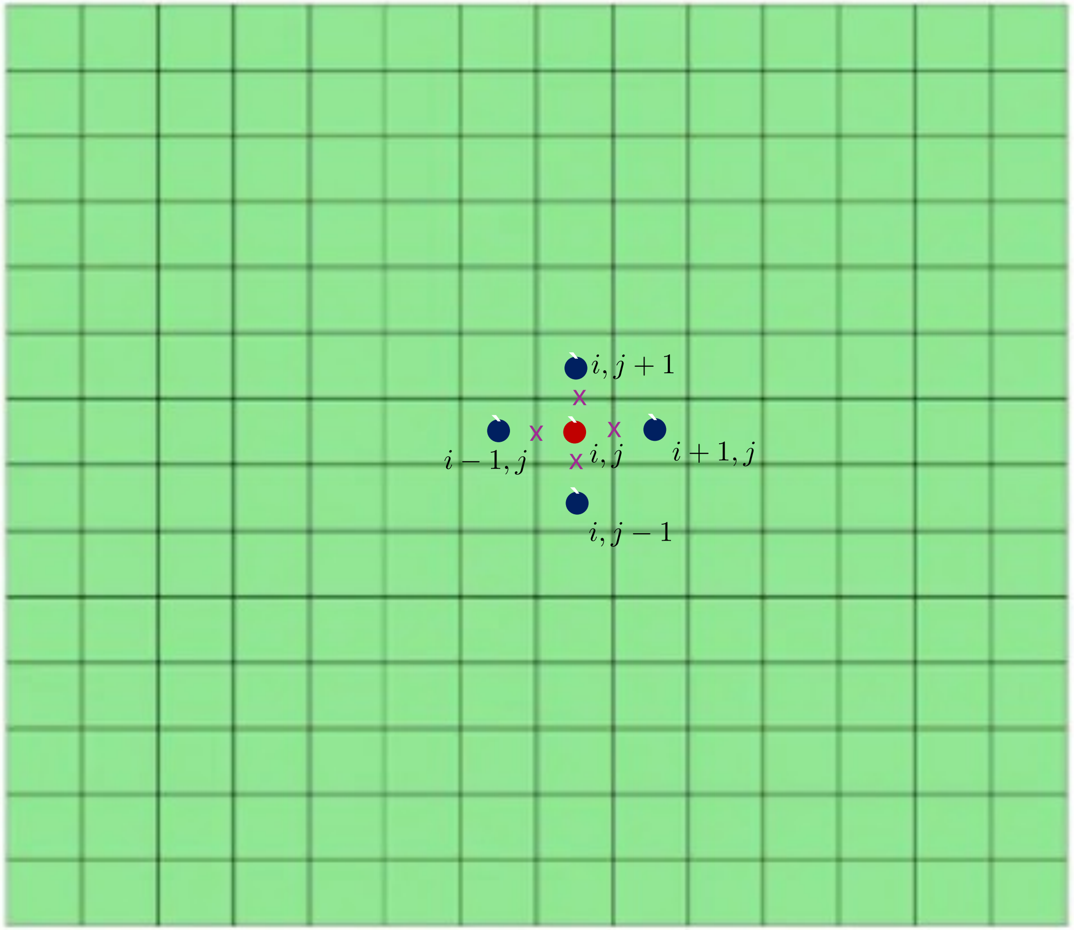
$$\iiint_{dV} \left[\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) \right] = 0 \quad \Longrightarrow \frac{\partial (\rho \Delta V)}{\partial t} + \iiint_{dV} \nabla \cdot (\rho \boldsymbol{u}) = 0$$

Divergence theorem: $\iiint_{dV} (\nabla \cdot \boldsymbol{F}) dV = \oiint_{dS} \boldsymbol{F} \cdot d\boldsymbol{S}$

$$\frac{\partial (\rho \Delta V)}{\partial t} + \oiint_{dS} (\rho \boldsymbol{u}) \cdot d\boldsymbol{S} = 0$$

$$\frac{\partial (\rho \Delta V)}{\partial t} + \sum_f [\rho \boldsymbol{u}]_f \cdot d\boldsymbol{S}_f = 0$$

$$\boxed{\begin{gathered} \frac{\partial (\rho \Delta V)}{\partial t} + \sum_f \rho_f \phi_f = 0, \quad \phi_f = \boldsymbol{u}_f . d\boldsymbol{S}_f \\ \psi_{i+1/2,j} = \alpha \psi_{i,j} + (1-\alpha)\psi_{i+1,j}, \quad \psi \in \{\rho, \boldsymbol{u}\} \end{gathered}}$$



Finite Volume method

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0$$

$$\frac{\partial (\rho \Delta V)}{\partial t} + \sum_f \rho_f \phi_f = 0, \quad \phi_f = \boldsymbol{u}_f . d\boldsymbol{S}_f$$

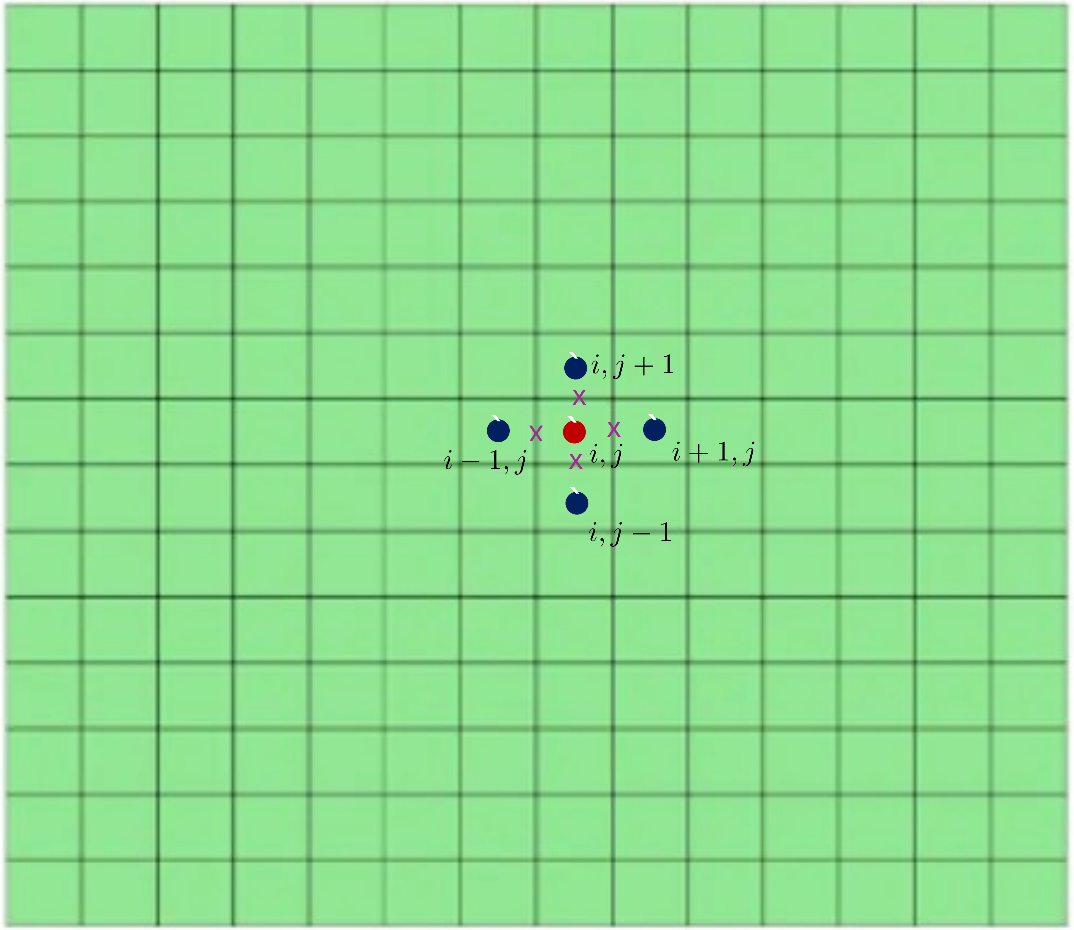
$$\psi_{i+1/2,j} = \alpha \psi_{i,j} + (1-\alpha)\psi_{i+1,j}, \quad \psi \in \{\rho, \boldsymbol{u}\}$$

$$\frac{\rho_{ij}^{n+1} - \rho_{ij}^{n}}{\Delta t} + \left[\frac{\rho_{i+1,j}^{n+1} u_{i+1,j}^{n+1} - \rho_{i-1,j}^{n+1} u_{i-1,j}^{n+1}}{2\Delta x}\right] + \left[\frac{\rho_{i,j+1}^{n+1} u_{i,j+1}^{n+1} - \rho_{i,j-1}^{n+1} u_{i,j-1}^{n+1}}{2\Delta y}\right] = 0$$

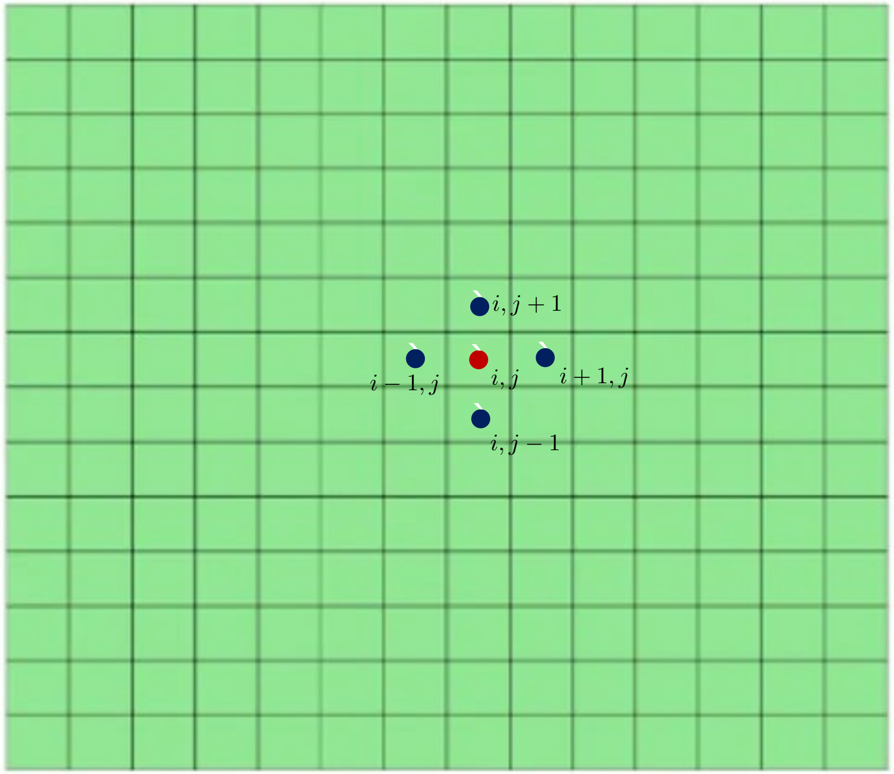
$\alpha = 1$: First order accurate, Forward

$\alpha = \frac{1}{2}$: Second order accurate, Central

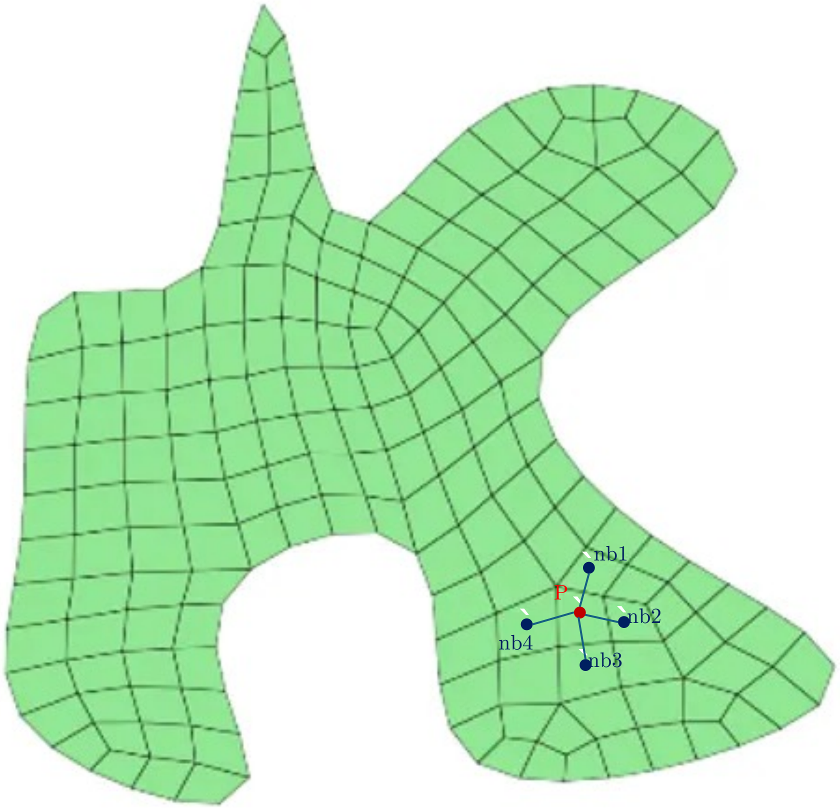
$\alpha = 0$: First order accurate, Backward



Grid



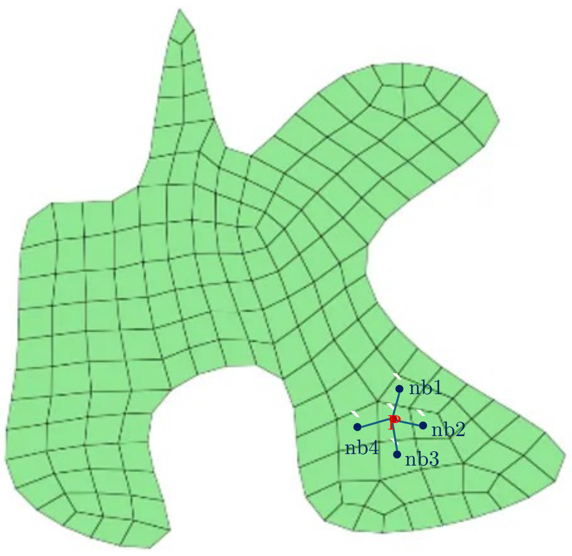
Structured grid

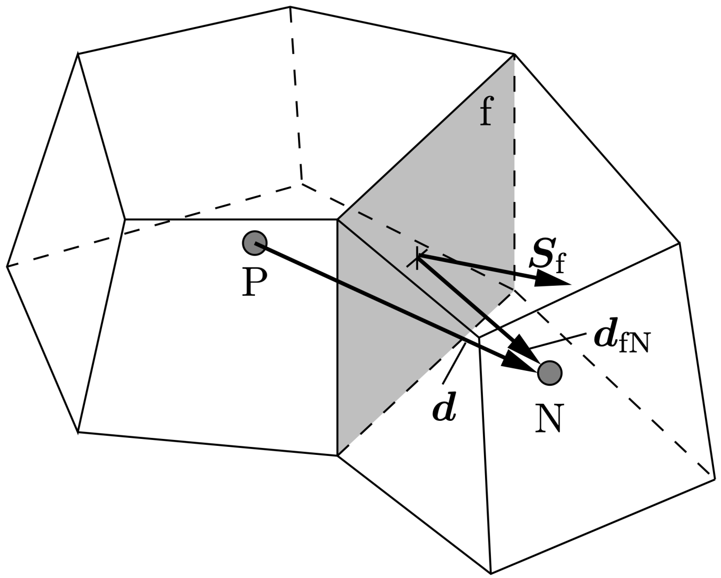


Unstructured grid

Finite Volume method

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0$$





$$\frac{\partial \left(\rho_P \Delta V_P\right)}{\partial t} + \sum_{f \in nb} \rho_f \phi_f = 0, \quad \phi_f = \boldsymbol{u}_f . d\boldsymbol{S}_f$$

$$\psi_f = \alpha \psi_P + (1-\alpha)\psi_{nb}, \quad \psi \in \{\rho, \boldsymbol{u}\}$$

$$\iiint_{dV} \Gamma \nabla^2 \rho = \iiint_{dV} \nabla \cdot (\Gamma \nabla \rho) = \oiint_{dS} \Gamma_f (\nabla \rho)_f \cdot d\boldsymbol{S}$$

Gauss-Green theorem: $(\nabla \rho)_p = \dfrac{1}{\Delta V_p} \sum_{f \in nb} [\rho_f d\boldsymbol{S}]$

Least-squared method: ?