Unit - IV

Electromagnetism

The Del Operator (∇)

→ The differential vector ∇ called del or nabla is capable of differentiating both vector and scalar functions.

It is defined as:

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

where $\hat{i}$, $\hat{j}$ & $\hat{k}$ are the unit vectors along the cartesian co-ordinate axes.

Gradient of a Scalar Function →

→ If the vector differential operator ∇ is operated on a scalar function $\phi(x,y,z)$, this operation is known as gradient of a scalar function.

$$\text{Grad}\,\phi = \nabla\phi = \hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}$$

The grad ϕ is a vector quantity.

Ex → $[E = -\,\text{Grad}\,V = -\nabla V]$

Physical significance → Let scalar function ϕ represents temperature, then $\nabla\phi$ represents temperature gradient or rate of change of temperature with distance.

Divergence of a Vector →

→ It is the multiplication of del operator by another vector so that the resultant is the dot product or scalar product.

$$\text{Div } \bar{A} = \nabla \cdot \bar{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot \left(\hat{i}A_x + \hat{j}A_y + \hat{k}A_z\right)$$

$$\boxed{\nabla \cdot \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}}$$

where A_x, A_y & A_z are the components of vector A in the direction of x, y & z.

→ It defines the rate of flow of any fluid.

Curl of a Vector →

$$\text{Curl } \bar{A} = \nabla \times \bar{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

or

$$\boxed{\nabla \times \bar{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{i} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{j} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{k}}$$

→ Curl of a vector is a vector.

→ Curl represents the rotation of a vector quantity.

Integral Theorems →

[1] Gauss Theorem → If s is a closed surface surrounding the volume V, the divergence for any vector $\bar{A}$ is expressed as

$$\boxed{\int_S \bar{A}\cdot ds = \int_V (\nabla\cdot\bar{A})\, dV}$$

where $ds = \hat{n}\, ds$ an element of area on s & $\hat{n}$ the unit outward normal to s.

[3] Stokes Theorem → If C is the closed contour around the edge of the open surface S, then Stokes theorem states that

$$\boxed{\int_l \bar{A}\cdot d\bar{l} = \int_S (\nabla\times\bar{A})\, d\bar{s}}$$

Conduction and Displacement Currents

→ We are familiar with conduction currents.
→ But in electromagnetic theory, we have the magnetic field produced due to time-varying electric field leading to the displacement current.

[1] Conduction Current → Conduction current is due to drift of electric charges in a conductor when an electric field is applied.

Let '$\vec{E}$' be the electric field strength applied across the linear conductor of length 'l' and cross-sectional area 'A'. A current 'i' flows through the conductor and 'V' be the potential difference b/w the end points of the conductor.

Then,
$$R = \frac{l}{\sigma A} \qquad \text{——— (1)}$$

where R is the resistance b/w the points a & b, and σ the conductivity

Fig

Applying Ohm's law, we get
$$V = \vec{E}\, l = i R \qquad \text{——— (2)}$$

Replacing R using eqn. (1), we get

$$E = \frac{i}{\sigma A} = \frac{\bar{J}_1}{\sigma}$$

where, $$\boxed{\bar{J}_1 = \frac{i}{A} = \sigma \bar{E}} \qquad \text{———— (3)}$$

$\bar{J}_1$ is the conduction current per unit area referred to as the conduction current density and is directly proportional to the electric field intensity.

[2] Displacement current → Consider the case of a capacitor of Capacitance C, charged by applying a voltage V across its ends.

Then, the current through the capacitor is

$$\vec{i}_c = \frac{dQ}{dt} = C\frac{dV}{dt} \qquad \text{———— (4)}$$

where Q is the charge on the capacitor plates.

In case of a parallel plate capacitor,

$$C = \frac{\varepsilon A}{d} \qquad \text{———— (5)}$$

where 'A' is the crossectional area of the plates, 'd' the separation b/w the plates and 'ε' the dielectric constant of the medium.

Electric field strength

$$\bar{E} = \frac{V}{d} \qquad \text{———— (6)}$$

Substituting eqn. 5 & 6 in eqn. 4, we get

$$\bar{i}_c = \varepsilon A \frac{d\bar{E}}{dt}$$

$$\boxed{\bar{J}_2 = \frac{\bar{i}_c}{A} = \varepsilon \frac{dE}{dt} = \frac{d\bar{D}}{dt}} \quad \text{---} (7)$$

$$\& \ \boxed{\bar{D} = \varepsilon \bar{E}}$$

where $\bar{D}$ is the electric displacement density & J_2 is called displacement current density, representing the current which directly passes through the capacitor and is only an apparent current representing the rate at which flow of charge takes place from electrode to electrode in the external circuit.

Hence, it is known as displacement current.

→ In an electromagnetic field, both conduction current and displacement current are present, hence the expression for total current density is

$$\bar{J} = \bar{J}_1 + \bar{J}_2$$

$$\boxed{\bar{J} = \sigma \bar{E} + \frac{d\bar{D}}{dt}} \quad \text{---} (8)$$

Polarizability → When a dielectric is kept under the influence of electric field, the charge particles arrange their positions in such a way that electric dipole moment is established. This process is known as polarization.

The induced electric dipole moment (P) is directly proportional to the applied electric field ($\bar{E}$)

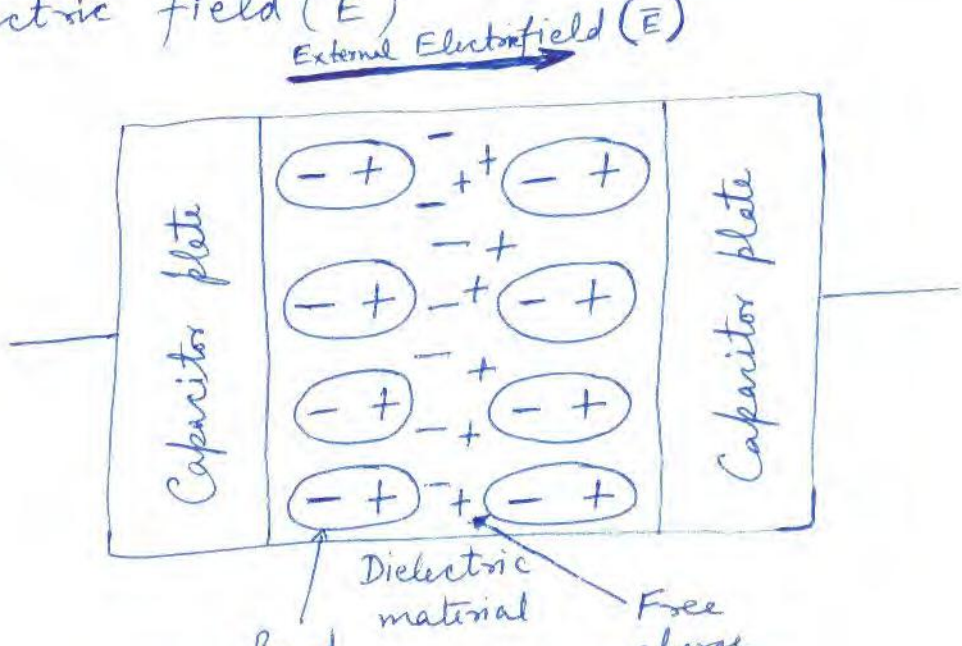

Fig. Capacitor

$$P \propto E_{loc} \quad \text{——— (1)}$$

$$P = \alpha E_{loc} \quad \text{——— (2)}$$

where E_{loc} is the electric field inside the dielectric which acts on the charged particles and α is the polarizability of dipole

If $E_{loc} = 1$, then $\boxed{P = \alpha}$

It means that polarizability is the dipole moment per unit electric field.

If total no. of dipoles be N, then

$$\boxed{P = N \alpha E_{loc}}$$

Three electric vectors →

Let us conside the polarization in a linear dielectric medium.

The total charge density is given as

$$\rho = \rho_b + \rho_f \quad \text{—— (1)}$$

Bound charge density

Free charge density

$$\text{or} \quad \rho = -\nabla.\bar{P} + \rho_f$$

Gauss's law can now be written as

$$\varepsilon_0 (\nabla.\bar{E}) = \rho_f - \nabla.\bar{P} \quad \text{—— (2)}$$

$$\text{or} \quad \nabla.(\varepsilon_0 \bar{E} + P) = \rho_f \quad \text{—— (3)}$$

$$\text{or} \quad \boxed{\nabla.\bar{D} = \rho_f} \quad \text{—— (4)}$$

$$\text{where} \quad \boxed{\bar{D} = \varepsilon_0 \bar{E} + P} \quad \text{—— (5)}$$

where $\bar{D}$ → Displacement vector
$\bar{P}$ → Polarization vector
$\bar{E}$ → Electric field vector

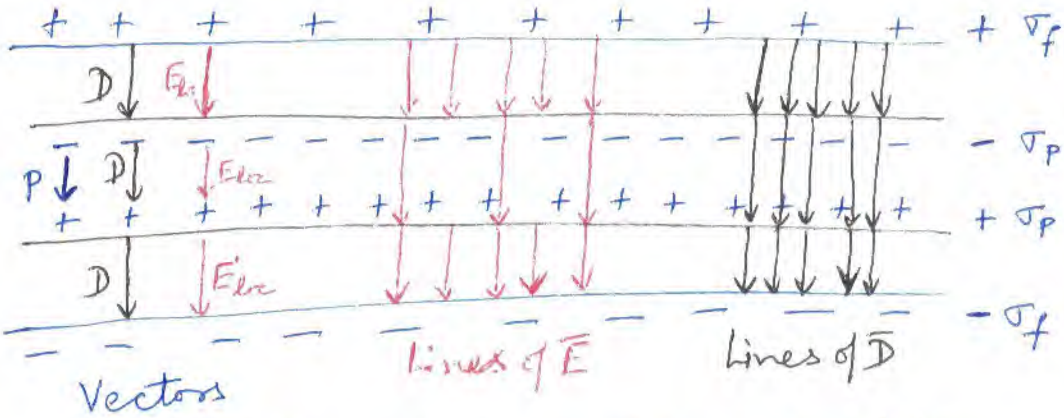

Fig. Vectors & their representation

Remarks →

→ Displacement vector ($\bar{D}$) is associated with the free charges only and constant throughout the capacitor.

→ Polarization vector ($\bar{P}$) is connected to polarized charges only and its value is nonzero inside the dielectric whereas it has zero value outside the dielectric or medium.

→ Electric field vector (E) is connected to all type of charges and reduces inside the dielectric.

→ In isotropic medium, the directions of $\bar{E}$ & $\bar{P}$ are in same direction.

Maxwell's Equations

→ When the charges are in motion, the electric and magnetic fields are associated with this motion which will have variations in both the space and time.

→ These electric and magnetic fields are inter related.

→ This phenomenon is called electromagnetism which is summarized by the set of equations, known as Maxwell's equations.

→ Maxwell's equations are the representation of the basic law of electromagnetism

Differential Form

1) $\nabla \cdot \bar{D} = \rho$ or $\nabla \cdot \bar{E} = \frac{\rho}{\varepsilon_0}$

2) $\nabla \cdot \bar{B} = 0$

} Gauss's Theorem

3) $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$ } Faraday's law of induction

4) $\nabla \times \bar{H} = J + \frac{\partial \bar{D}}{\partial t}$ } Ampere's Law

or $\boxed{\bar{\nabla} \times \bar{B} = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial \bar{E}}{\partial t}}$

→ $\bar{D} = \varepsilon_0 \bar{E}$

→ $\bar{B} = \mu_0 H$

→ $\bar{F} = q(\bar{E} + v \times \bar{B})$ Lorentz Force

Integral Form

1) $\oint_S \bar{D} \cdot d\bar{s} = q$

2) $\oint_S \bar{B} \cdot d\bar{s} = 0$

3) $\oint_c \bar{E} \cdot d\bar{l} = -\frac{\partial}{\partial t} \int_s \bar{B} \cdot d\bar{s}$

4) $\oint_c \bar{H} \cdot d\bar{l} = \int_s \left(\bar{J} + \frac{\partial \bar{D}}{\partial t} \right) \cdot d\bar{s}$

Derivation of Maxwell's eqn. in Differential Form

[1] Maxwell's First Equation →

→ Let us consider a surface S bounding a volume V in a dielectric medium, which is kept in the $\bar{E}$ field.

→ The application of external field $\bar{E}$ polarises the dielectric medium and charges are induced, called bound charges, or charges due to polarization.

→ The total charge density at a point in a small volume element dV would then be $(\rho + \rho_p)$, where ρ_p is the polarization charge density, given by $\rho_p = -\operatorname{div}\bar{P}$ and ρ is the free charge density at that point in the small volume element dV.

→ Thus the total charge density at that point will be $\rho - (\operatorname{div}\bar{P})$.

Then Gauss's theorem can be expressed as

$$\oint_S \bar{E}\cdot d\bar{s} = \int_V (\operatorname{div}\bar{E})\, dV$$

$$= \frac{1}{\varepsilon_0}\int_V (\rho - \operatorname{div}\bar{P})\, dV$$

$$\text{or} \quad \varepsilon_0 \int_V (\operatorname{div}\bar{E})\, dV = \int_V (\rho - \operatorname{div}\bar{P})\, dV$$

$$\int_V \operatorname{div}(\varepsilon_0 \bar{E} + \bar{P})\, dV = \int_V \rho\, dV$$

The quantity $(\varepsilon_0 \bar{E} + \bar{P})$ is denoted by a quantity $\bar{D}$, called the electric displacement.

Therefore,
$$\int_V (\text{div}\, \bar{D})\, dV = \int_V \rho\, dV$$

Since this eqn. is true for all the arbitrary volumes,

$$\therefore \boxed{\begin{array}{c} \text{div}\, \bar{D} = \rho \\ \text{or} \quad \nabla \cdot \bar{D} = \rho \end{array}}$$

This is the Maxwell's first equation.

When the medium is isotropic, the three vectors $\bar{D}$, E & $\bar{P}$ are in the same dir^n and for small field $\bar{E}$, $\boxed{\bar{D} = \varepsilon \bar{E}}$

where ε is called the permittivity of the dielectric medium.

The ratio $\varepsilon/\varepsilon_0$ is called the dielectric constant of the medium.

2 Derivation of Maxwell's Second Equation

The no. of magnetic lines of forces entering any arbitrary surface is exactly the same as leaving it. It means that the flux of magnetic induction $\bar{B}$ across any closed surface is always zero.

i.e. $\oint_S \bar{B} \cdot d\bar{S} = 0$

Using Gauss's divergence theorem $\oint_S \bar{B} \cdot d\bar{s} = \int_V (\text{div}\, \bar{B})\, dV = 0$

The integrand in the above eqn. should vanish at surface boundary. $\therefore \boxed{\text{div}\, \bar{B} = 0}$ or $\boxed{\nabla \cdot \bar{B} = 0}$

This is the Maxwell's second equation.

[3] Derivation of Maxwell's Third Equation

According to Faraday's law, the emf induced in a closed loop is given by

$$E_{emf} = -\frac{\partial \phi}{\partial t} = -\int_S \frac{\partial \bar{B}}{\partial t} \cdot d\bar{s} = -\frac{\partial}{\partial t} \oint_S \bar{B} \cdot d\bar{s} \quad \text{--- (1)}$$

Here the flux $\phi = \oint_c \bar{B} \cdot d\bar{s}$

where S is any closed surface having the loop as boundary.

The emf can also be found by calculating the work done in carrying a unit charge completely around the loop.

Thus,

$$E_{emf} = \oint_c \bar{E} \cdot d\bar{l} \quad \text{--- (2)}$$

Equating eqn. (1) & (2), we get

$$\oint_c \bar{E} \cdot dl = -\oint_S \frac{\partial \bar{B}}{\partial t} \cdot d\bar{s}$$

Acc. to Stokes' law $\oint_c \bar{E} \cdot d\bar{l} = \int_S (\nabla \times \bar{E}) \cdot d\bar{s}$

Therefore,

$$\int_S (\nabla \times \bar{E}) \cdot d\bar{s} = -\oint_S \frac{\partial \bar{B}}{\partial t} \cdot d\bar{s} \quad \text{--- (3)}$$

This equation must be true for any surface whether small or large in the field. So the two vectors in the integrand must be equal at every point, i.e.,

$$\boxed{\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}}$$

This is Maxwell's third equation.

4 Derivation of Maxwell's Fourth Equation

According to Ampere's law, the work done in carrying a unit magnetic pole once around a closed arbitrary path linked with the current is expressed by

$$\oint_C \bar{H}.d\bar{l} = I$$

$$\text{or} \quad \oint_C \bar{H}.d\bar{l} = \oint_C \bar{J}.d\bar{s} \quad \text{——— (1)}$$

As per Stokes' theorem

$$\oint_C \bar{H}.d\bar{l} = \int_S (\nabla \times \bar{H}).d\bar{s} \quad \text{— (2)}$$

Therefore, from eqn. (1) & (2)

$$\int_S (\nabla \times \bar{H}).d\bar{s} = \int_S \bar{J}.d\bar{s}$$

This give $\nabla \times \bar{H} = \bar{J}$ ——— (3)

The above eqn. (3) holds good only for the steady current however, for the changing electric fields, the current density should be modified.

Since $\operatorname{div}(\bar{\nabla} \times \bar{H}) = \operatorname{div}\bar{J}$

$$\Rightarrow \quad 0 = \operatorname{div}\bar{J} \quad [\text{Since div of a curl} = 0]$$

$$\Rightarrow \quad \boxed{\operatorname{div}\bar{J} = 0}$$

which conflicts with the continuity eqn. as

$$\left[\operatorname{div}\bar{J} = -\frac{\partial \rho}{\partial t}\right]$$

Therefore Maxwell realised that the definition of the total current density is incomplete and suggested to add another density $\bar{J}'$.

Therefore $\text{curl}\,\bar{H} = \bar{J} + \bar{J}'$ ——— (4)

Now, take the divergence of above eqn.

$$\text{div}\,(\text{curl}\,\bar{H}) = \text{div}\,\bar{J} + \text{div}\,\bar{J}'$$

or $$0 = \text{div}\,\bar{J} + \text{div}\,\bar{J}'$$

$$\text{div}\,\bar{J}' = -\text{div}\,\bar{J}$$

$$\text{div}\,\bar{J}' = \frac{\partial \rho}{\partial t} \quad \text{——— (5)}$$

Since,
$$\rho = \nabla \cdot \bar{D}$$

$$\text{div}\,\bar{J}' = \frac{\partial}{\partial t}(\nabla \cdot \bar{D}) \quad \text{——— (6)}$$

Hence, $$\bar{J}' = \frac{\partial \bar{D}}{\partial t} \quad \text{——— (7)}$$

Therefore, the Maxwell's fourth eqn. can be written as

$$\boxed{\bar{\nabla} \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}} \quad \text{——— (8)}$$

The last term of R.H.S. of this eqn. is called Maxwell's correction and is known as displacement current density

Eqn. (8) is called modified Ampere's law for unsteady or changing current which is responsible for electromagnetic fields.

Maxwell's equations in Integral Form

[1] Maxwell's First Equation →

As $\nabla . \bar{D} = \rho$ ——— (1)

Integrating eqn. (1) over a volume V, we have

$$\int_V (\bar{\nabla} . \bar{D})\, dV = \int_V \rho\, dV$$

Using Gauss's divergence theorem

$$\oint_S \bar{D} . d\bar{S} = \int_V \rho\, dV = q$$

or $$\boxed{\oint_S \bar{D} . d\bar{S} = q}$$

→ Here q is the net charge contained in volume V & S is the surface bounding the volume V.

→ It says that the total electric displacement through the surface S enclosing a volume V is equal to the total charge contained within this volume.

[2] Maxwell's Second Equation →

Differential form of the Maxwell's second eqn. is

$\nabla . \bar{B} = 0$ ——— (2)

In integral form

$$\boxed{\oint_S \bar{B} . d\bar{S} = 0}$$

which signifies that the total outward flux of magnetic induction $\bar{B}$ through any closed surface $\bar{S}$ is equal to zero.

3 Maxwell's Third Equation

In differential form

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \quad \text{——— (3)}$$

Integrating eqn. (3) over a surface $\bar{S}$ bounded by a closed path, we have

$$\int_S (\nabla \times \bar{E}) . d\bar{S} = -\int_S \frac{\partial \bar{B}}{\partial t} . d\bar{S}$$

Acc. to Stoke's theorem

$$\boxed{\oint_C \bar{E} . d\bar{l} = -\frac{\partial}{\partial t} \int_S \bar{B} . d\bar{S}}$$

which signifies that the electromotive force around a closed path is equal to the time derivative of the magnetic displacement through any closed surface bounded by that path.

4 Maxwell's Fourth Equation →

In differential form $\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$

Similarly, $\boxed{\oint_C \bar{H} . d\bar{l} = \int_S \left[\bar{J} + \frac{\partial \bar{D}}{\partial t} \right] . d\bar{S}}$

The above eqn. signifies that the electromotive force around a closed path is equal to the conduction current plus the time derivative of the electric displacement through any surface bounded by that path.

Electromagnetic waves in free space

Maxwell's eqns for free space are given as

$$\nabla \cdot \bar{E} = 0 \qquad \text{--- (1)}$$

$$\nabla \cdot \bar{H} = 0 \qquad \text{--- (2)}$$

$$\nabla \times \bar{E} = -\mu_0 \frac{\partial \bar{H}}{\partial t} \qquad \text{--- (3)}$$

$$\nabla \times \bar{H} = \varepsilon_0 \frac{\partial \bar{E}}{\partial t} \qquad \text{--- (4)}$$

Taking curl of eqn. (3), we get

$$\nabla \times (\nabla \times \bar{E}) = -\mu_0 \frac{\partial}{\partial t}(\nabla \times \bar{H})$$

$$\nabla(\nabla \cdot \bar{E}) - \nabla^2 \bar{E} = -\mu_0 \frac{\partial}{\partial t}\left[\varepsilon_0 \frac{\partial \bar{E}}{\partial t}\right] \quad \text{(from eqn (4))}$$

$$-\nabla^2 \bar{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \bar{E}}{\partial t^2} \quad \text{(from eqn. (1))}$$

$$\text{or} \quad \nabla^2 \bar{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \bar{E}}{\partial t^2} = 0 \qquad \text{--- (5)}$$

This is the wave equation governing the field $\bar{E}$.

As $v^2 = \frac{1}{\mu_0 \varepsilon_0}$, eqn. (5) can be written as

$$\nabla^2 \bar{E} - \frac{1}{v^2} \frac{\partial^2 \bar{E}}{\partial t^2} = 0 \qquad \text{--- (6)}$$

Similarly, the curl of eqn. (4) gives the wave eqn. for the field $\bar{H}$ as

$$\nabla^2 \bar{H} - \frac{1}{v^2} \frac{\partial^2 \bar{H}}{\partial t^2} = 0 \qquad \text{--- (7)}$$

The plane wave solutions of eqn. (6) & (7) may be written as

$$\bar{E}(\bar{r},t) = \bar{E}_0 \, e^{i(\bar{k}.\bar{r} - \omega t)} \quad \text{——— (8)}$$

$$\& \quad \bar{H}(\bar{r},t) = \bar{H}_0 \, e^{i(\bar{k}.\bar{r} - \omega t)} \quad \text{——— (9)}$$

where ω is the angular fequency of the variation of the fields $\bar{E}$ & $\bar{H}$ and $\bar{k}$ is the wave vector which tells the direction of propagation of the wave.

The ratio ω/k gives the phase velocity of the wave.

Now, $\bar{\nabla} = \left[\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right]$, $\bar{E}_0 = E_{0x}\hat{i} + E_{0y}\hat{j} + E_{0z}\hat{k}$

$$\bar{k} = k_x\hat{i} + k_y\hat{j} + k_z\hat{k}, \quad \bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\Rightarrow \bar{k}.\bar{r} = k_x x + k_y y + k_z z$$

$$\therefore \text{curl } \bar{E} = \bar{\nabla} \times \left[\bar{E}_0 \, e^{i(\bar{k}.\bar{r} - \omega t)}\right]$$

$$= \bar{\nabla} \times \left[\left(E_0 e^{i\bar{k}.\bar{r}}\right) e^{-i\omega t}\right]$$

$$\bar{\nabla} \times \bar{E} = \bar{\nabla} \times \left[\left\{(E_{0x}\hat{i} + E_{0y}\hat{j} + E_{0z}\hat{k}) e^{i(k_x x + k_y y + k_z z)}\right\} e^{-i\omega t}\right]$$

where $i = \sqrt{-1}$.

Solving the above eqn., we get

$$\bar{\nabla} \times \bar{E} = i\left[\hat{i}\{E_{0z}k_y - E_{0y}k_z\} + \hat{j}\{E_{0x}k_z - E_{0z}k_x\} + \hat{k}\{E_{0y}k_x - E_{0x}k_y\}\right]. \, e^{i(k_x x + k_y y + k_z z)} \, e^{-i\omega t}$$

$$\bar{\nabla} \times \bar{E} = i\left[\bar{k} \times \bar{E}_0\right] e^{i(\bar{k}.\bar{r} - \omega t)}$$

or $\quad \bar{\nabla} \times \bar{E} = i\,[\bar{k} \times \bar{E}]$ ———— (10)

Using eqn. (3) & (10), we get

$$-\mu_0 \frac{\partial \bar{H}}{\partial t} = i\,[\bar{k} \times \bar{E}] \qquad \text{———— (11)}$$

L.H.S. of eqn. (11) can be written as

$$-\mu_0 \frac{\partial}{\partial t}\left[\bar{H}_0\, e^{i(\bar{k}.\bar{r} - \omega t)}\right] = i\omega \mu_0 \bar{H}$$

Hence, $\boxed{\bar{k} \times \bar{E} = \omega \mu_0 \bar{H}}$ ———— (12)

Similarly, for eqn (4), it can be shown that

$$\boxed{\bar{k} \times \bar{H} = -\omega \varepsilon_0 \bar{E}} \qquad \text{———— (13)}$$

From eqn. (12), $\bar{H}$ is $\perp^r$ to both the propagation vector $\bar{k}$ & the electric field vector $\bar{E}$.

From eqn (13) $\bar{E}$ is $\perp^r$ to both $\bar{k}$ & $\bar{H}$.

Therefore, it may be concluded that the electric and magnetic vectors are normal to each other as well as to the dir^n of propagation of the wave or $\bar{E}$ & $\bar{H}$ and dir^n of wave propagation $\bar{k}$ form a set of orthogonal vectors.

Further,

It can be proved that the E.M wave travels at the speed of light 'c' in free space.

For this, the cross product of $\bar{k}$ with eqn. 12, gives

$$\bar{k} \times (\bar{k} \times \bar{E}) = \omega \mu_0 (\bar{k} \times \bar{H})$$

$$\bar{k}(\bar{k}.\bar{E}) - k^2 \bar{E} = \omega \mu_0 \left[-\omega \varepsilon_0 \bar{E} \right] \quad \text{(from eqn 13)}$$

Since $\bar{k}$ & $\bar{E}$ are $\perp^r$ to each other, hence

$$k^2 \bar{E} - \omega^2 \mu_0 \varepsilon_0 \bar{E} = 0$$

$$\text{or } (k^2 - \omega^2 \mu_0 \varepsilon_0) \bar{E} = 0$$

Since $\bar{E} \neq 0$

$$\therefore \quad k^2 - \omega^2 \mu_0 \varepsilon_0 = 0$$

$$\Rightarrow \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$= \frac{1}{\sqrt{[4\pi \times 10^{-7} \text{ weber/Amp. met.} \times 8.85 \times 10^{-12} \, C^2/Nm^2}}$$

$$\left\{ \because 1 \text{ Weber} = \frac{1 \text{ Nm}}{\text{Amp.}} \right]$$

$$= \frac{1}{\sqrt{4\pi \times 10^{-19} \times 8.85 \, \frac{Nm}{Amp^2 \, m} \, \frac{C^2}{Nm^2}}}$$

$$= \frac{1}{\sqrt{4 \times 3.14 \times 8.85 \times 10^{-19}}} \text{ m/s}$$

$$\boxed{v = \frac{\omega}{k} = 3 \times 10^8 \text{ m/s}}$$

Therefore, the phase velocity of the em wave is equal to the speed of light in free space or vacuum.

Q→1] If the magnitude of $\bar{H}$ in a plane wave is 1 A/m, find the magnitude of $\bar{E}$ for plane wave in free space.

Soln] We know that

$$\frac{E_0}{H_0} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$\text{or } E_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \; H_0$$

Here, $H_0 = 1$ A/m, $\mu_0 = 4\pi \times 10^{-7}$ Wb/A-m

& $\varepsilon_0 = 8.85 \times 10^{-12}$ C/N m²

$$\therefore \; E_0 = 1 \times \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}}$$

$$\boxed{E_0 = 376.72 \text{ V/m}}$$

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