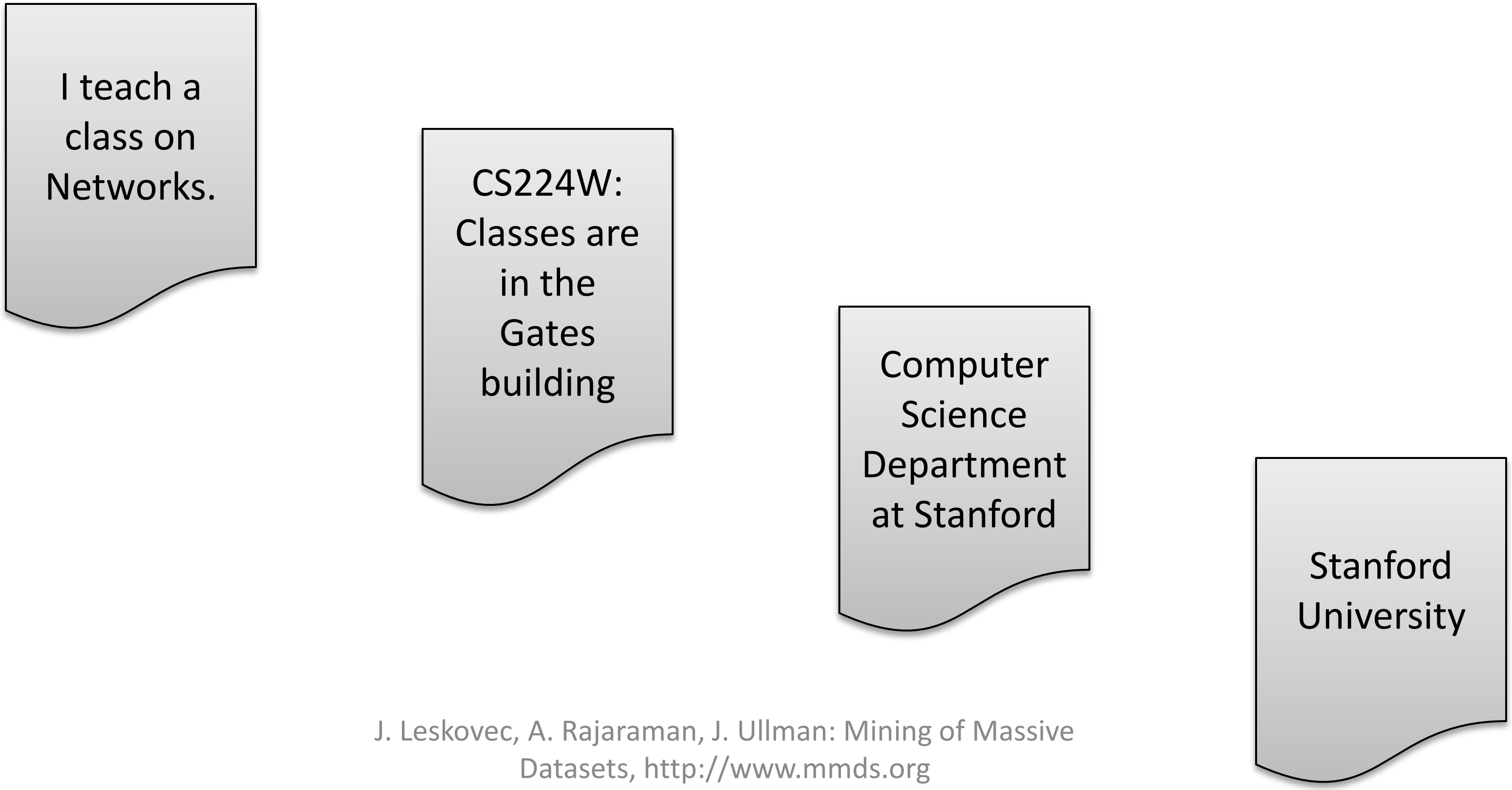
Unit II

Web as a Graph

- **Web as a directed graph:**
  - **Nodes: Webpages**
  - **Edges: Hyperlinks**



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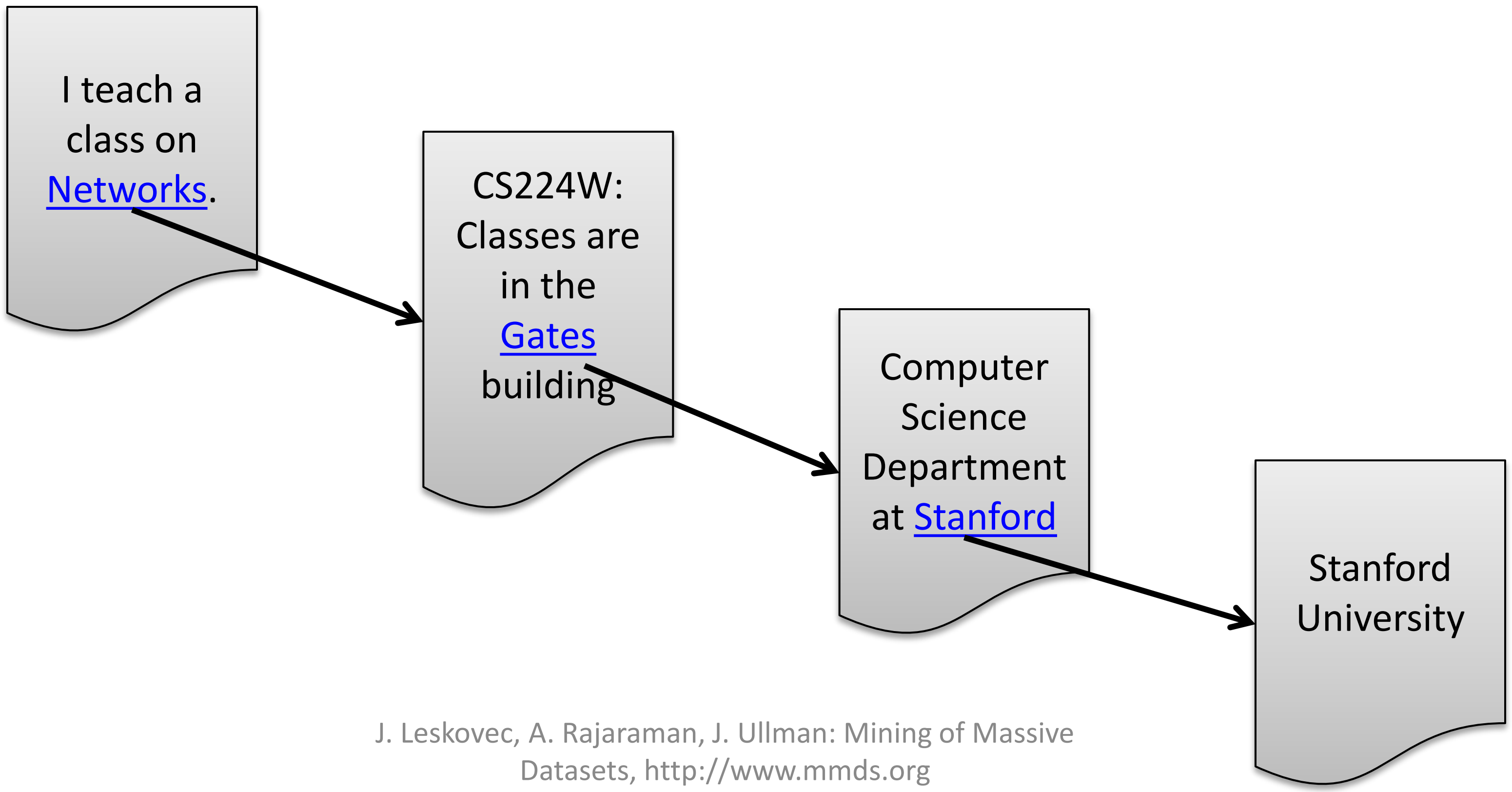
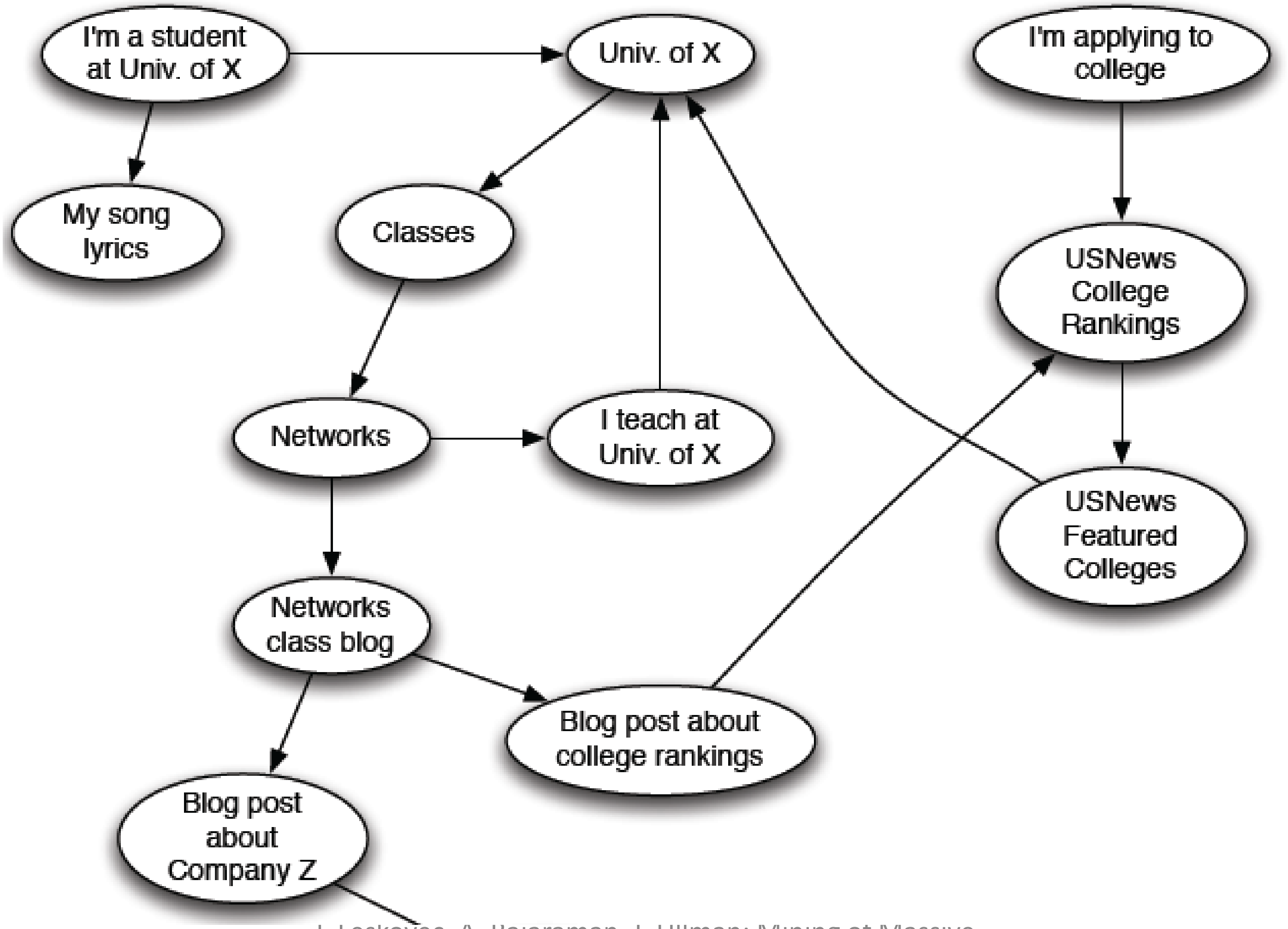
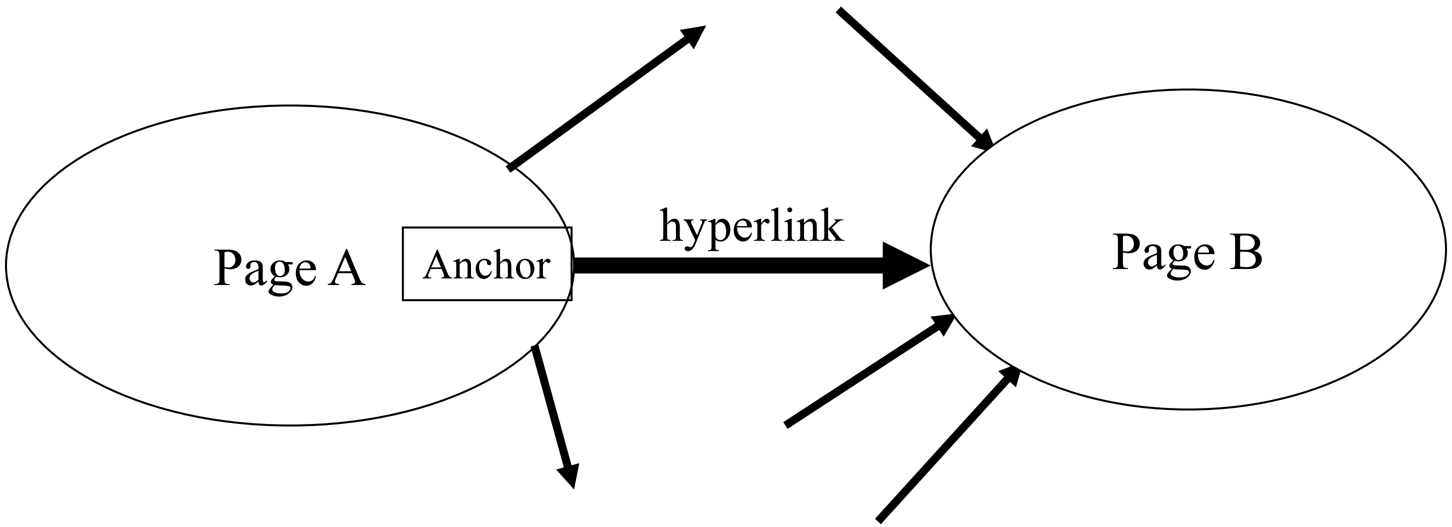


Web as a Directed Graph



The Web as a Directed Graph



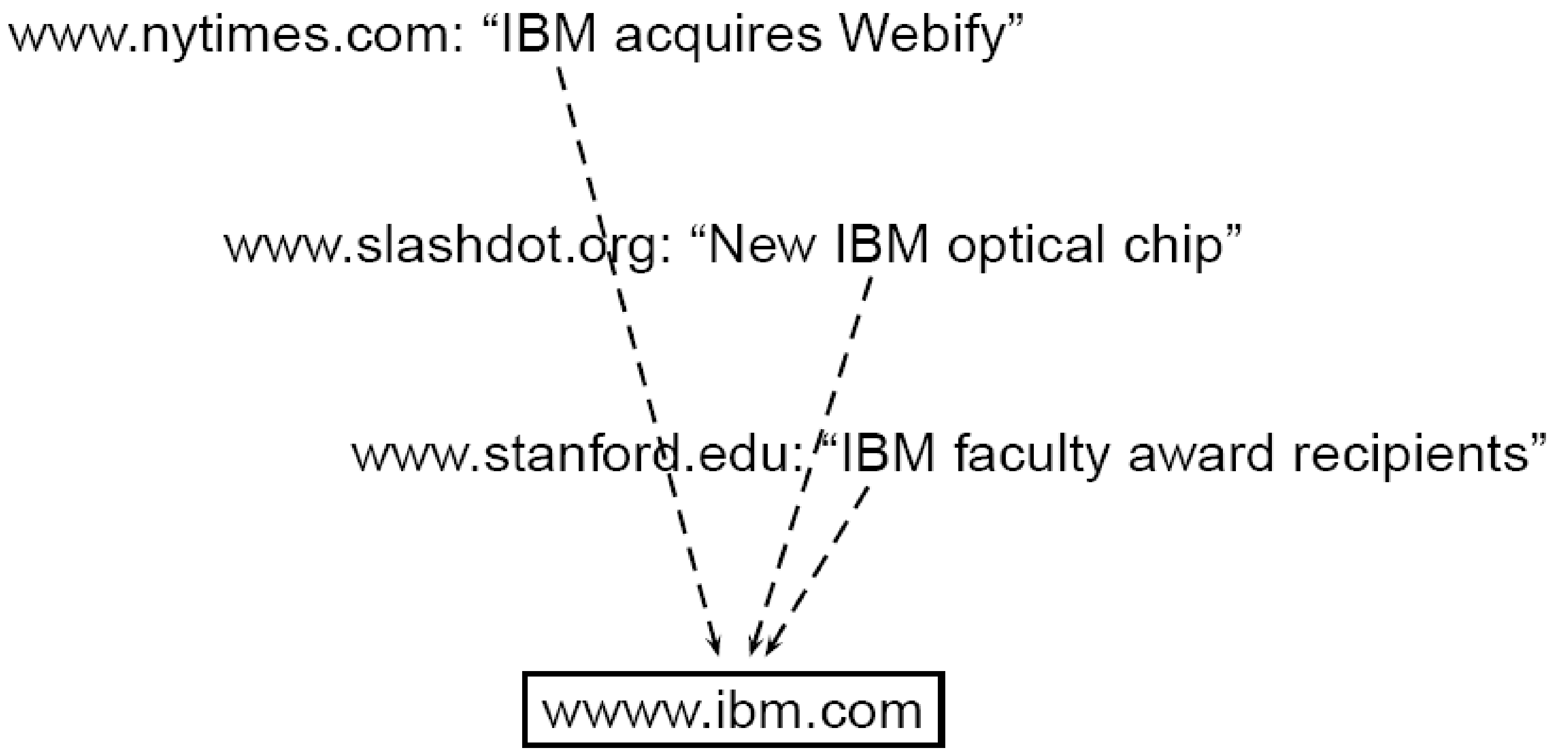
Assumption 1: A hyperlink between pages denotes author perceived relevance (quality signal)

Assumption 2: The text in the anchor of the hyperlink describes the target page (textual context)

[document text only] vs. [document text + anchor text]

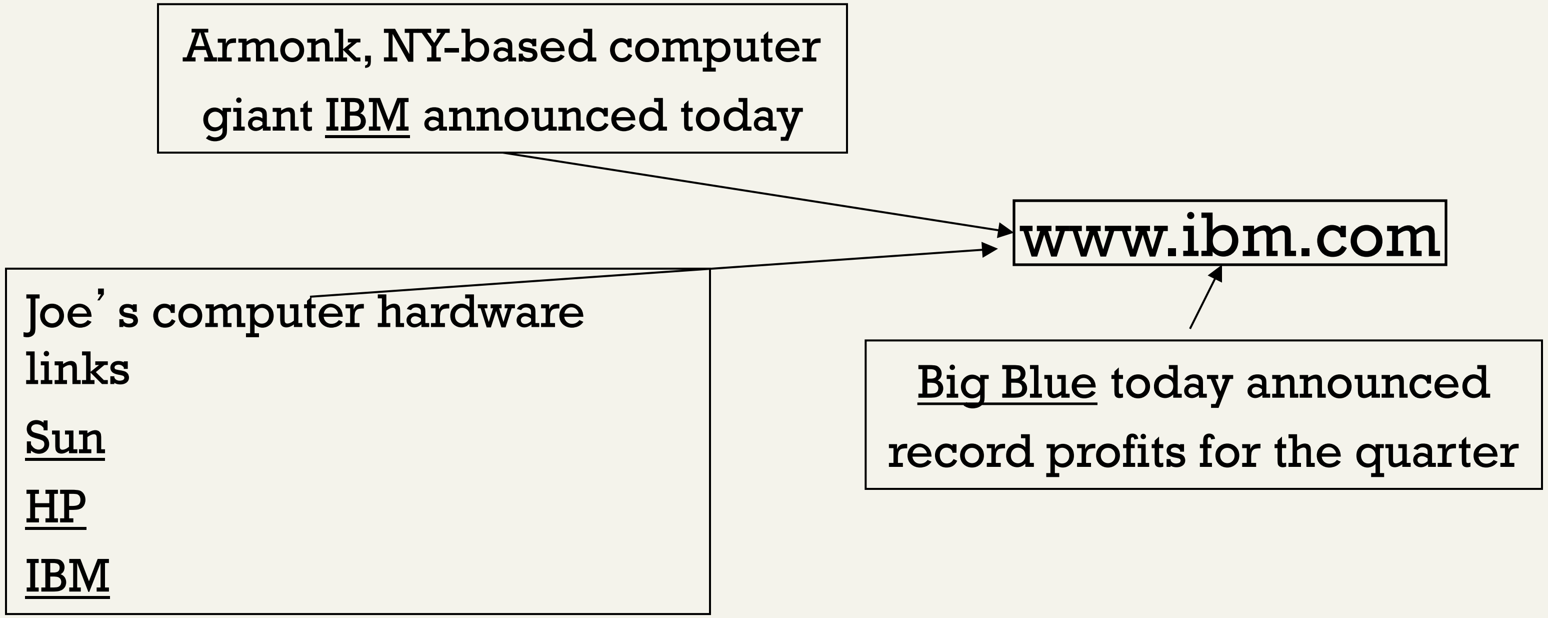
- Searching on [document text + anchor text] is often more effective than searching on [document text only].
- Example: Query *IBM*
  - Matches IBM's copyright page
  - Matches many spam pages
  - Matches IBM wikipedia article
  - May not match IBM home page! (if IBM home page is mostly graphical)
- Searching on anchor text is better for the query *IBM*.
- Represent each page by all the anchor text pointing to it.
- In this representation, the page with the most occurrences of *IBM* is www.ibm.com.

Anchor text containing *IBM* pointing to www.ibm.com



Indexing anchor text

- Thus: anchor text is often a better description of a page's content than the page itself
- Anchor text can be weighted more highly than document text (based on Assumptions 1 & 2)
- When indexing a document *D*, include anchor text from links pointing to *D*.



Using Anchor Text

- Finding summary of web pages
  - Many page authors do not provide a good summary of the page
  - Anchor text often describes the purpose of the links which provide good summary of the target pages
  - Retrieval: the frequent terms/patterns in the anchor text of links to a target page
- Finding aspects/facets of web pages
  - A page may contain information in many aspects
  - Anchor text of a link is often specific
  - Retrieval: clustering anchor text – each significant cluster is a candidate aspect
- (Abusing) anchor text for spamming
  - Many commercial search engines use anchor text in computing the relevance of a page to a query
  - Creating many links with specific anchor text may boost ranking of a target page in some target queries

Google bombs

- Indexing anchor text can have unexpected side effects: Google bombs.
  - whatelse does not have side effects?
- A Google bomb is a search with “bad” results due to maliciously manipulated anchor text
- Google introduced a new weighting function in January 2007 that fixed many Google bombs

Origins of PageRank: Citation analysis (1)

- Citation analysis: analysis of citations in the scientific literature
- Example citation: "Miller (2001) has shown that physical activity alters the metabolism of estrogens."
- "Miller (2001)" is a hyperlink linking two scientific articles.
- One application of these "hyperlinks" in the scientific literature:
  - Measure the similarity of two articles by the overlap of other articles citing them.
  - This is called cocitation similarity.
- Cocitation similarity on the web?

Cocitation similarity on Google: similar pages

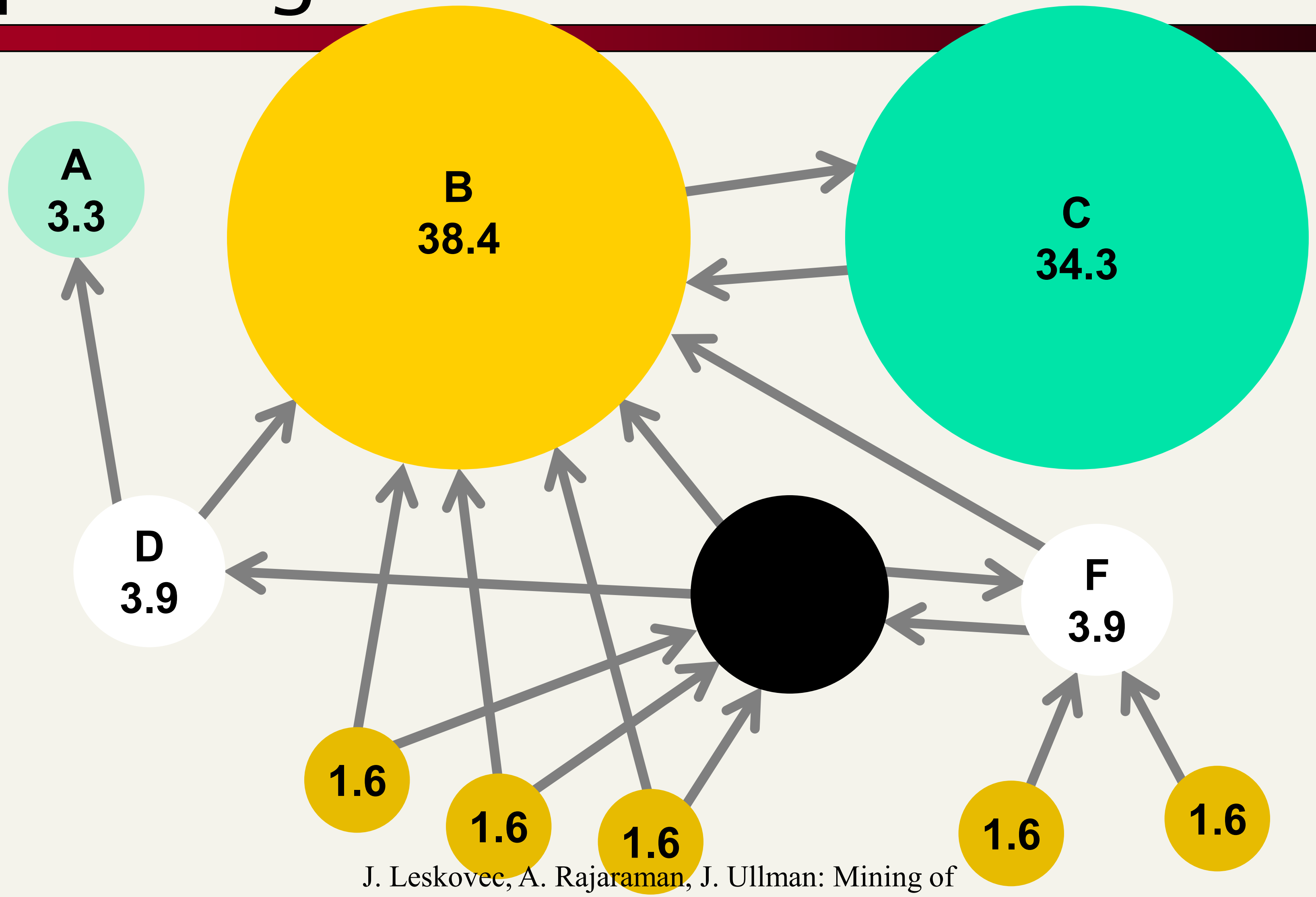
Origins of PageRank: Citation analysis (2)

- Citation frequency can be used to measure the impact of an article.
  - Each article gets one vote.
  - Not a very accurate measure
- Better measure: weighted citation frequency / citation rank
  - An article's vote is weighted according to its citation impact.
  - Circular? No: can be formalized in a well-defined way.
  - This is basically PageRank.
  - PageRank was invented in the context of citation analysis by Pinsker and Narin in the 1960s.

Links as Votes

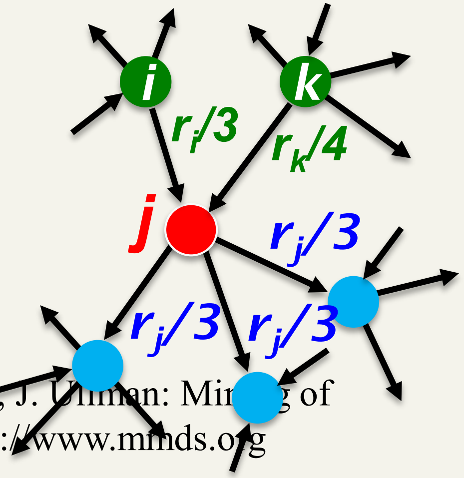
- **Idea: Links as votes**
  - **Page is more important if it has more links**
    - In-coming links? Out-going links?
- **Think of in-links as votes:**
  - www.stanford.edu has 23,400 in-links
  - www.joe-schmoe.com has 1 in-link
- **Are all in-links are equal?**
  - **Links from important pages count more**
  - Recursive question!

Example: PageRank Scores



Simple Recursive Formulation

- Each link's vote is proportional to the **importance** of its source page
- If page j with importance r_j has n out-links, each link gets r_j / n votes
- Page j's own importance is the sum of the votes on its in-links



$$r_j = r_i/3 + r_k/4$$

Eigenvector Formulation

- **The flow equations can be written**

$$r = M \cdot r$$

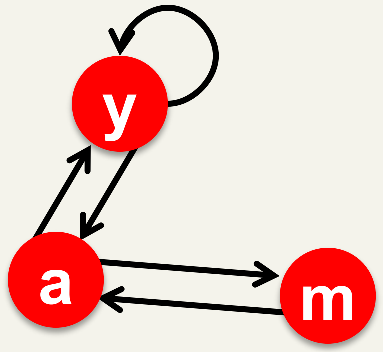
- So the **rank vector** r is an **eigenvector** of the stochastic web matrix M
  - In fact, its first or principal eigenvector, with corresponding eigenvalue 1
    - Largest eigenvalue of M is 1 since M is column stochastic (with non-negative entries)
      - *We know* $\boldsymbol{r}$ *is unit length and each column of* $\boldsymbol{M}$ *sums to one, so* $\boldsymbol{Mr} \leq \mathbf{1}$

NOTE: x is an eigenvector with the corresponding eigenvalue **λ** if:

$$Ax = \lambda x$$

- **We can now efficiently solve for r!**
  The method is called Power iteration

Example: Flow Equations & M



	y	a	m
y	½	½	0
a	½	0	1
m	0	½	0

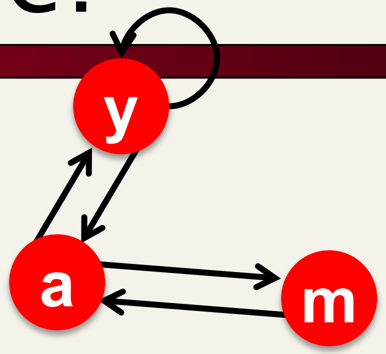
$$\boldsymbol{r = M \cdot r}$$

$$r_y = r_y/2 + r_a/2$$
$$r_a = r_y/2 + r_m$$
$$r_m = r_a/2$$

$$\begin{bmatrix} y \\ a \\ m \end{bmatrix} = \begin{bmatrix} ½ & ½ & 0 \\ ½ & 0 & 1 \\ 0 & ½ & 0 \end{bmatrix} \begin{bmatrix} y \\ a \\ m \end{bmatrix}$$

PageRank: How to solve?

- **Power Iteration:**
  - Set $r_j = 1/N$
  - 1: $r'_j = \sum_{i \to j} \frac{r_i}{d_i}$
  - 2: $r = r'$
  - Goto 1
- **Example:**



	y	a	m
y	½	½	0
a	½	0	1
m	0	½	0

$\mathbf{r_y} = \mathbf{r_y}/2 + \mathbf{r_a}/2$

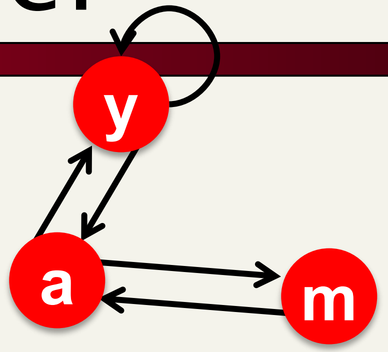
$\mathbf{r_a} = \mathbf{r_y}/2 + \mathbf{r_m}$

$\mathbf{r_m} = \mathbf{r_a}/2$

$$\begin{pmatrix} r_y \\ r_a \\ r_m \end{pmatrix} = \begin{matrix} 1/3 \\ 1/3 \\ 1/3 \end{matrix} \quad \begin{matrix} 1/3 \\ 3/6 \\ 1/6 \end{matrix} \quad \begin{matrix} 5 \\ 1 \\ 3 \end{matrix} \quad \begin{matrix} 6 \\ 6 \\ 3/15 \end{matrix}$$

Iteration 0, 1, 2, …

PageRank: How to solve?



	y	a	m
y	½	½	0
a	½	0	1
m	0	½	0

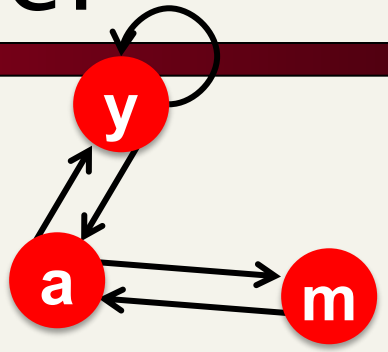
$$\mathbf{r_y} = \mathbf{r_y}/2 + \mathbf{r_a}/2$$
$$\mathbf{r_a} = \mathbf{r_y}/2 + \mathbf{r_m}$$
$$\mathbf{r_m} = \mathbf{r_a}/2$$

- **Power Iteration:**
  - Set $r_j = 1/N$
  - 1: $r'_j = \sum_{i \to j} \frac{r_i}{d_i}$
  - 2: $r = r'$
  - Goto 1
- **Example:**

$$\begin{bmatrix} r_y \\ r_a \\ r_m \end{bmatrix} = \begin{array}{cccccc} 1/3 & 1/3 & 5/12 & 9/24 & & 6/15 \\ 1/3 & 3/6 & 1/3 & 11/24 & \dots & 6/15 \\ 1/3 & 1/6 & 3/12 & 1/6 & & 3/15 \end{array}$$

Iteration 0, 1, 2, …

PageRank: How to solve?



	y	a	m
y	½	½	0
a	½	0	1
m	0	½	0

$r_y = r_y/2 + r_a/2$
$r_a = r_y/2 + r_m$
$r_m = r_a/2$

- **Power Iteration:**
  - Set $r_j = 1/N$
  - 1: $r'_j = \sum_{i \to j} \frac{r_i}{d_i}$
  - 2: $r = r'$
  - Goto 1
- **Example:**

$$\begin{bmatrix} r_y \\ r_a \\ r_m \end{bmatrix} = \begin{matrix} 1/3 & 1/3 & 5/12 & 9/24 & & 6/15 \\ 1/3 & 3/6 & 1/3 & 11/24 & \ldots & 6/15 \\ 1/3 & 1/6 & 3/12 & 1/6 & & 3/15 \end{matrix}$$

Iteration 0, 1, 2, …

Details!

Why Power Iteration works? (1)

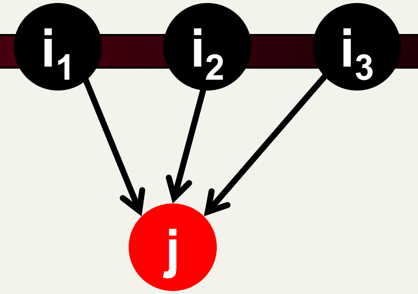
- **Power iteration:**
  A method for finding dominant eigenvector (the vector corresponding to the largest eigenvalue)
  - $\boldsymbol{r}^{(1)} = \boldsymbol{M} \cdot \boldsymbol{r}^{(0)}$
  - $\boldsymbol{r}^{(2)} = \boldsymbol{M} \cdot \boldsymbol{r}^{(1)} = \boldsymbol{M}\left(\boldsymbol{M}\boldsymbol{r}^{(1)}\right) = \boldsymbol{M}^{2} \cdot \boldsymbol{r}^{(0)}$
  - $\boldsymbol{r}^{(3)} = \boldsymbol{M} \cdot \boldsymbol{r}^{(2)} = \boldsymbol{M}\left(\boldsymbol{M}^{2}\boldsymbol{r}^{(0)}\right) = \boldsymbol{M}^{3} \cdot \boldsymbol{r}^{(0)}$
- **Claim:**
  Sequence $\boldsymbol{M} \cdot \boldsymbol{r}^{(0)}, \boldsymbol{M}^{2} \cdot \boldsymbol{r}^{(0)}, \ldots \boldsymbol{M}^{k} \cdot \boldsymbol{r}^{(0)}, \ldots$ approaches the dominant eigenvector of $\boldsymbol{M}$

Details!

Why Power Iteration works? (3)

- **Claim:** Sequence $\boldsymbol{M} \cdot \boldsymbol{r}^{(0)}, \boldsymbol{M}^2 \cdot \boldsymbol{r}^{(0)}, \ldots \boldsymbol{M}^k \cdot \boldsymbol{r}^{(0)}, \ldots$ approaches the dominant eigenvector of $\boldsymbol{M}$
- **Proof (continued):**
  - Repeated multiplication on both sides produces
    $$M^k r^{(0)} = c_1(\lambda_1^k x_1) + c_2(\lambda_2^k x_2) + \cdots + c_n(\lambda_n^k x_n)$$
  - $$M^k r^{(0)} = \lambda_1^k \left[c_1 x_1 + c_2 \left(\frac{\lambda_2}{\lambda_1}\right)^k x_2 + \cdots + c_n \left(\frac{\lambda_2}{\lambda_1}\right)^k x_n \right]$$
  - Since $\lambda_1 > \lambda_2$ then fractions $\frac{\lambda_2}{\lambda_1}, \frac{\lambda_3}{\lambda_1} \ldots < 1$
    and so $\left(\frac{\lambda_i}{\lambda_1}\right)^k = 0$ as $k \to \infty$ (for all $i = 2 \ldots n$).
  - **Thus:** $\boldsymbol{M^k r^{(0)} \approx c_1\left(\lambda_1^k x_1\right)}$
    - Note if $c_1 = 0$ then the method won't converge

The Stationary Distribution



$$p(t+1) = \mathrm{M} \cdot p(t)$$

- **Where is the surfer at time $t+1$?**
  - Follows a link uniformly at random

    $$\boldsymbol{p}(\boldsymbol{t}+\mathbf{1}) = \boldsymbol{M} \cdot \boldsymbol{p}(\boldsymbol{t})$$
- Suppose the random walk reaches a state

  $$\boldsymbol{p}(\boldsymbol{t}+\mathbf{1}) = \boldsymbol{M} \cdot \boldsymbol{p}(\boldsymbol{t}) = \boldsymbol{p}(\boldsymbol{t})$$

  then $\boldsymbol{p}(\boldsymbol{t})$ is **stationary distribution** of a random walk
- **Our original rank vector** $\boldsymbol{r}$ satisfies $\boldsymbol{r} = \boldsymbol{M} \cdot \boldsymbol{r}$
  - **So, $\boldsymbol{r}$ is a stationary distribution for the random walk**

Existence and Uniqueness

- **A central result from the theory of random walks (a.k.a. Markov processes):**

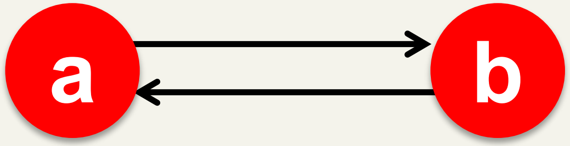
For graphs that satisfy **certain conditions**, the **stationary distribution is unique** and eventually will be reached no matter what the initial probability distribution at time **t = 0**

PageRank: Three Questions

$$r_j^{(t+1)} = \sum_{i \to j} \frac{r_i^{(t)}}{\mathrm{d_i}} \quad \text{or equivalently} \quad r = Mr$$

- **Does this converge?**
- **Does it converge to what we want?**
- **Are results reasonable?**

Does this converge?



$$r_j^{(t+1)} = \sum_{i \to j} \frac{r_i^{(t)}}{\mathrm{d_i}}$$

- **Example:**

$$\begin{array}{ccccc} r_a & & 1 & 0 & 1 & 0 \\ & = & & & & \\ r_b & & 0 & 1 & 0 & 1 \end{array}$$

Iteration 0, 1, 2, …

Does it converge to what we want?

$$r_j^{(t+1)} = \sum_{i \to j} \frac{r_i^{(t)}}{\mathrm{d_i}}$$

- **Example:**

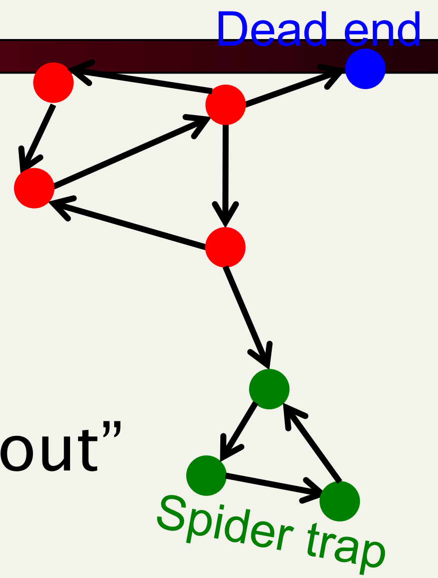
r_a	=	1	0	0	0
r_b		0	1	0	0

Iteration 0, 1, 2, …

PageRank: Problems

2 problems:

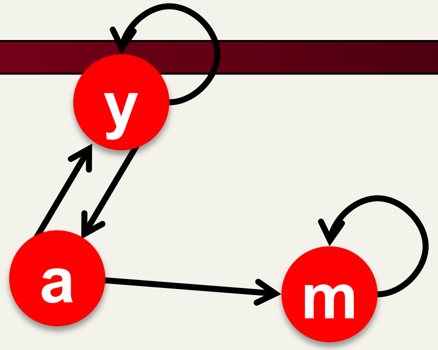
- **(1)** Some pages are **dead ends** (have no out-links)
  - Random walk has "nowhere" to go to
  - Such pages cause importance to "leak out"



- **(2) Spider traps:** (all out-links are within the group)
  - Random walked gets "stuck" in a trap
  - And eventually spider traps absorb all importance

Problem: Spider Traps

- **Power Iteration:**
  - Set $r_j = 1$
  - $r_j = \sum_{i \to j} \frac{r_i}{d_i}$
    - And iterate



m is a spider trap

	y	a	m
y	½	½	0
a	½	0	0
m	0	½	1

$\mathbf{r_y} = \mathbf{r_y/2 + r_a/2}$

$\mathbf{r_a} = \mathbf{r_y/2}$

$\mathbf{r_m} = \mathbf{r_a/2 + r_m}$

- **Example:**

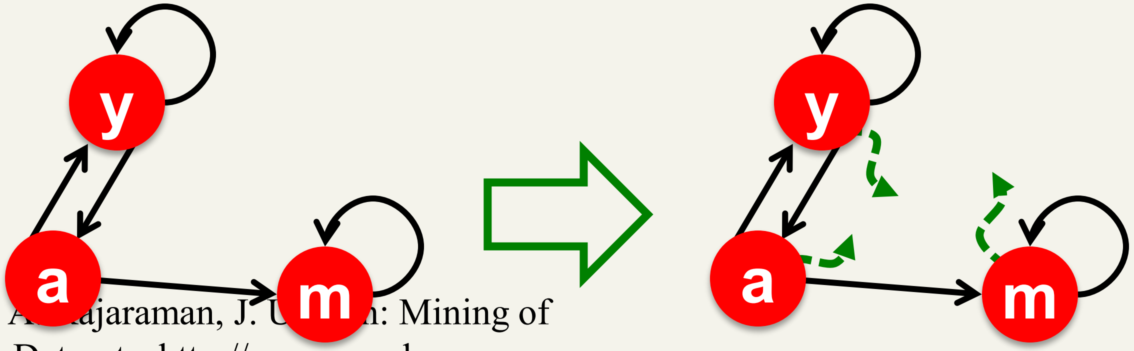
$$\begin{pmatrix} r_y \\ r_a \\ r_m \end{pmatrix} = \begin{matrix} 1/3 & 2/6 & 3/12 & 5/24 & & 0 \\ 1/3 & 1/6 & 2/12 & 3/24 & \ldots & 0 \\ 1/3 & 3/6 & 7/12 & 16/24 & & 1 \end{matrix}$$

Iteration 0, 1, 2, …

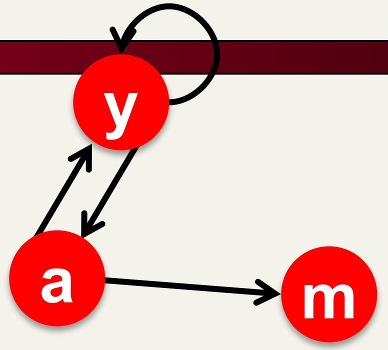
All the PageRank score gets "trapped" in node m.

Solution: Teleports!

- **The Google solution for spider traps: At each time step, the random surfer has two options**
  - With prob. β, follow a link at random
  - With prob. **1**-β, jump to some random page
  - Common values for β are in the range 0.8 to 0.9
- **Surfer will teleport out of spider trap within a few time steps**



Problem: Dead Ends



	y	a	m
y	½	½	0
a	½	0	0
m	0	½	0

- **Power Iteration:**
  - Set $r_j = 1$
  - $r_j = \sum_{i \to j} \frac{r_i}{d_i}$
    - And iterate

$$\mathbf{r_y} = \mathbf{r_y}/2 + \mathbf{r_a}/2$$
$$\mathbf{r_a} = \mathbf{r_y}/2$$
$$\mathbf{r_m} = \mathbf{r_a}/2$$

- **Example:**

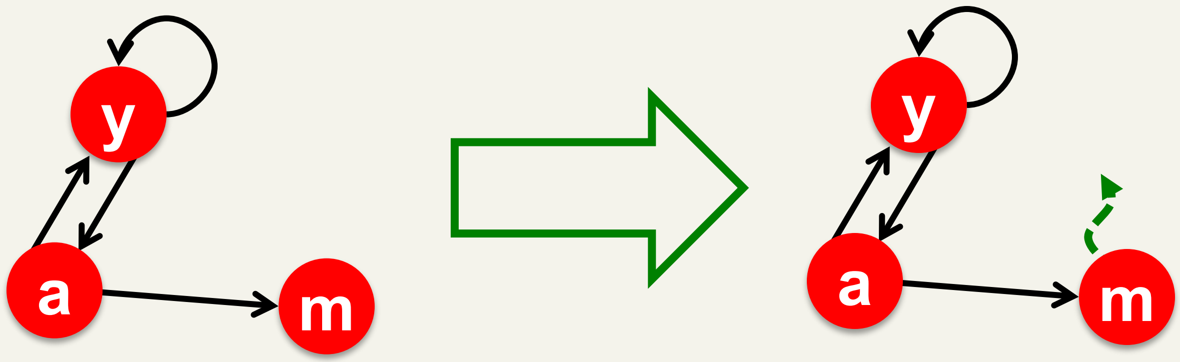
$$\begin{pmatrix} r_y \\ r_a \\ r_m \end{pmatrix} = \begin{matrix} 1/3 & 2/6 & 3/12 & 5/24 & & 0 \\ 1/3 & 1/6 & 2/12 & 3/24 & \ldots & 0 \\ 1/3 & 1/6 & 1/12 & 2/24 & & 0 \end{matrix}$$

Iteration 0, 1, 2, …

Here the PageRank “leaks” out since the matrix is not stochastic.

Solution: Always Teleport!

- **Teleports:** Follow random teleport links with probability 1.0 from dead-ends
  - Adjust matrix accordingly



	y	a	m
y	½	½	0
a	½	0	0
m	0	½	0

	y	a	m
y	½	½	⅓
a	½	0	⅓
m	0	½	⅓

Why Teleports Solve the Problem?

Why are dead-ends and spider traps a problem and why do teleports solve the problem?

- **Spider-traps** are not a problem, but with traps PageRank scores are **not** what we want
  - **Solution:** Never get stuck in a spider trap by teleporting out of it in a finite number of steps
- **Dead-ends** are a problem
  - The matrix is not column stochastic so our initial assumptions are not met
  - **Solution:** Make matrix column stochastic by always teleporting when there is nowhere else to go

Solution: Random Teleports

- **Google's solution that does it all:**
  At each step, random surfer has two options:
  - With probability β, follow a link at random
  - With probability $1\text{-}\beta$, jump to some random page

- **PageRank equation** [Brin-Page, 98]

$$r_j = \sum_{i \to j} \beta \frac{r_i}{d_i} + (1-\beta)\frac{1}{N}$$

d_i … out-degree of node i

This formulation assumes that M has no dead ends. We can either preprocess matrix M to remove all dead ends or explicitly follow random teleport links with probability 1.0 from dead-ends.

The Google Matrix

- **PageRank equation** [Brin-Page, '98]

$$r_j = \sum_{i \to j} \beta \frac{r_i}{d_i} + (1-\beta)\frac{1}{N}$$

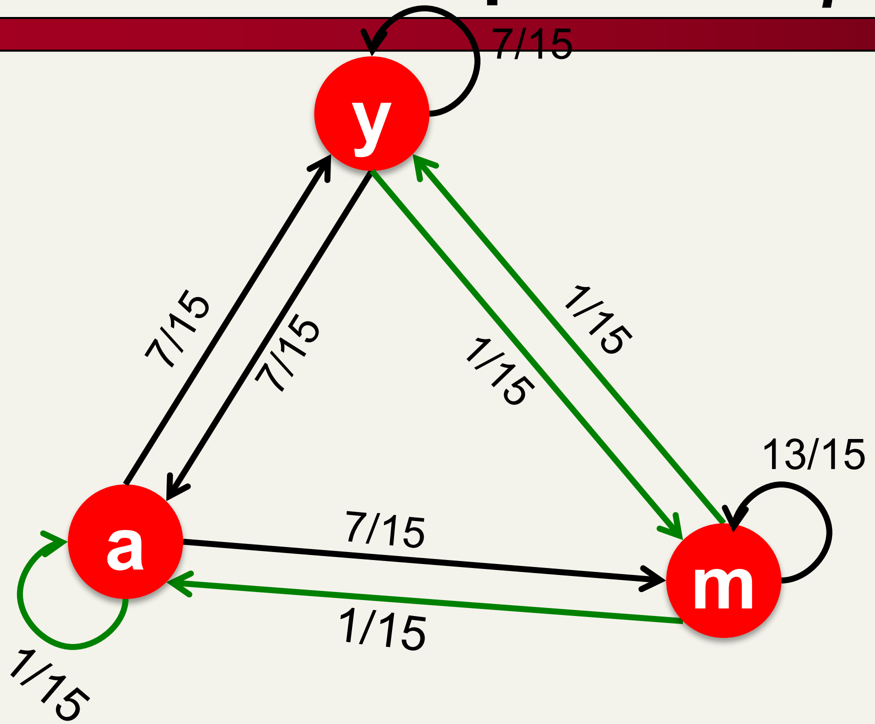
- **The Google Matrix *A*:**

$$A = \beta\, M + (1-\beta)\left[\frac{1}{N}\right]_{N \times N}$$

$[1/N]_{NxN}$…N by N matrix where all entries are 1/N

- **We have a recursive problem: $\boldsymbol{r = A \cdot r}$**
  And the Power method still works!
- **What is β?**
  - In practice $\beta = 0.8, 0.9$ (make 5 steps on avg., jump)

Random Teleports ($\beta = 0.8$)



$$0.8 \underbrace{\begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 1 \end{bmatrix}}_{\mathbf{M}} + 0.2 \underbrace{\begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}}_{[1/N]_{NxN}}$$

$$\mathbf{A} = \begin{array}{c} y \\ a \\ m \end{array} \begin{bmatrix} 7/15 & 7/15 & 1/15 \\ 7/15 & 1/15 & 1/15 \\ 1/15 & 7/15 & 13/15 \end{bmatrix}$$

$$\begin{array}{c} y \\ a \\ m \end{array} = \begin{array}{c} 1/3 \\ 1/3 \\ 1/3 \end{array} \quad \begin{array}{c} 0.33 \\ 0.20 \\ 0.46 \end{array} \quad \begin{array}{c} 0.24 \\ 0.20 \\ 0.52 \end{array} \quad \begin{array}{c} 0.26 \\ 0.18 \\ 0.56 \end{array} \quad \dots \quad \begin{array}{c} 7/33 \\ 5/33 \\ 21/33 \end{array}$$

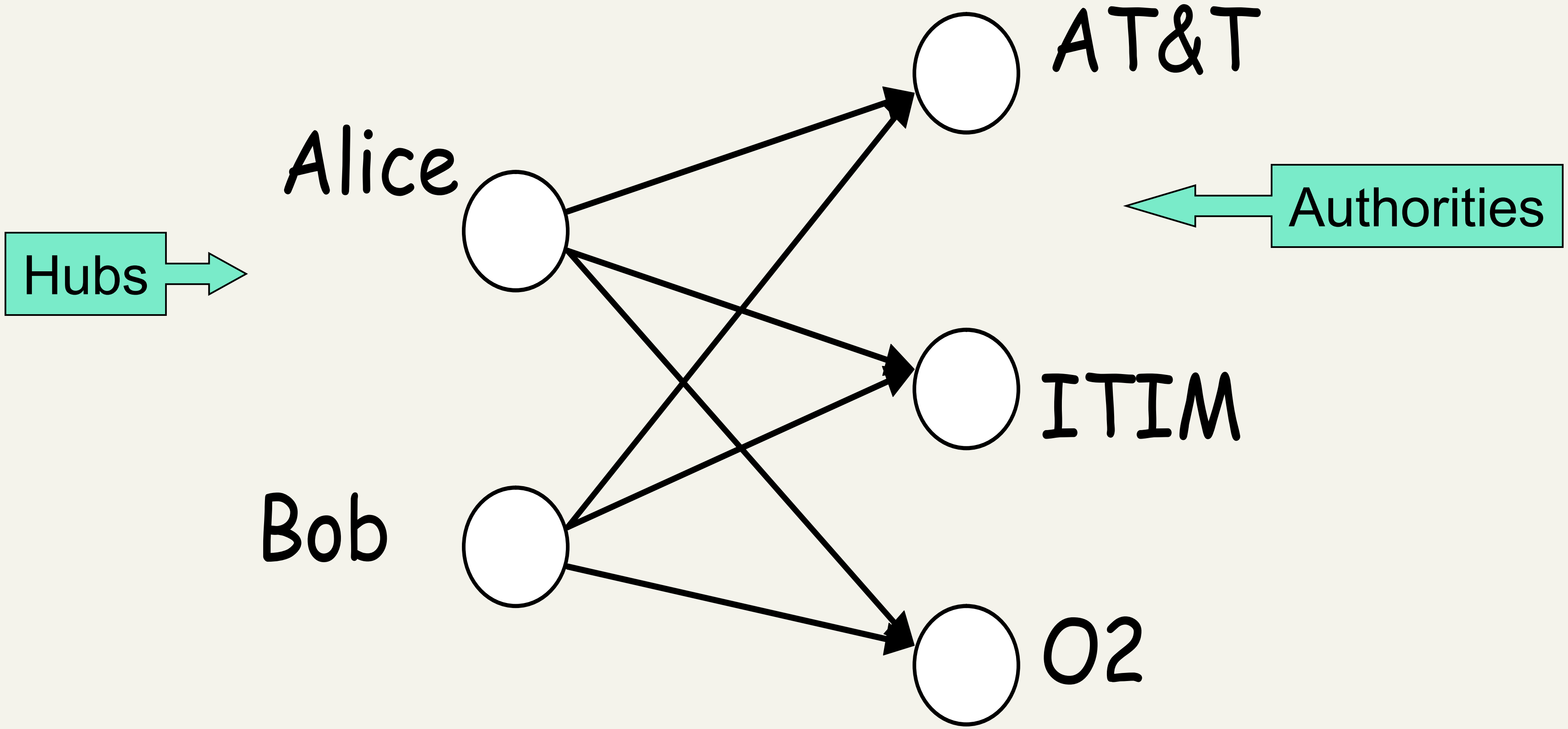
Hyperlink-Induced Topic Search (HITS)

- In response to a query, instead of an ordered list of pages each meeting the query, find two sets of inter-related pages:
  - *Hub pages* are good lists of links on a subject.
    - e.g., "Bob's list of cancer-related links."
  - *Authority pages* occur recurrently on good hubs for the subject.
- Best suited for "broad topic" queries rather than for page-finding queries.
- Gets at a broader slice of common *opinion*.

Hubs and Authorities

- Thus, a good hub page for a topic *points* to many authoritative pages for that topic.
- A good authority page for a topic is *pointed* to by many good hubs for that topic.
- Circular definition - will turn this into an iterative computation.

The hope



Mobile telecom companies

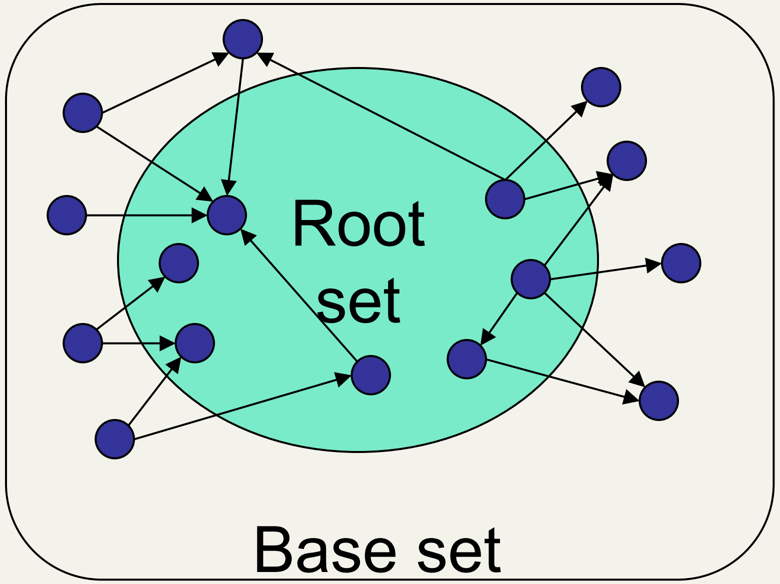
High-level scheme

- Extract from the web a base set of pages that *could* be good hubs or authorities.
- From these, identify a small set of top hub and authority pages;
  - →iterative algorithm.

Base set

- Given text query (say ***browser***), use a text index to get all pages containing ***browser.***
  - Call this the root set of pages.
- Add in any page that either
  - points to a page in the root set, or
  - is pointed to by a page in the root set.
- Call this the base set.

Visualization



Assembling the base set

- Root set typically 200-1000 nodes.
- Base set may have thousands of nodes
  - Topic-dependent
- How do you find the base set nodes?
  - Follow out-links by parsing root set pages.
  - Get in-links (and out-links) from a *connectivity server*