Non Linear Effects in Optical Fibers

Gaurav Milind Vaidya

May 2020

1 Introduction

Optical fibers are an important aspect of modern communication technology. Despite silica having extremely low non-linearity coefficients, non-linear effects have a substantial impact on propagation of light through optical fibers. This report presents an analysis of the non-linear effects in Silica (which exhibits third order non linearity as $\chi^{(2)} = 0$ due to inversion symmetry) caused due to the modulation of its refractive index and their potential benefits in laser physics.

2 Why are Non-Linear Effects in Optical Fibers Important?

The extent of the non linear effect in a medium depends not only on the intensity of light, but also on the length in which the intense beam interacts with the non-linear medium. For a standard Gaussian beam with a spot size of a_o and power P, we have

$$IL_{eff} = \frac{P}{\pi a_o^2}\frac{\pi a_o^2}{\lambda} = \frac{P}{\lambda}$$

where the interaction length can be taken as the Rayleigh length, the length at which the intensity remains constant. In case of an optical fiber, the decrease in intensity happens only due to the attenuation. If the dissipation coefficient is γ, then we can say that the length over which the intensity remains constant is of the order of γ^-1. Hence, we have

$$IL_{eff} = \frac{P}{\pi a_o^2 \gamma}$$

For standard single mode core diameters, the parameter IL_{eff} is almost 10^9 times bigger for fibers.

3 Self Phase Modulation and Instability

For an electric field $Ecos(\omega t)$ the third order non-linear polarization will be given by

$$P_{NL}^{(3)} = \frac{\chi^{(3)}}{4}(3cos(\omega t) + cos(3\omega t))|(E)|^2 E$$

Since the third harmonic generation requires a phase matching condition, we assume that it is absent in the most general case. Hence we obtain the expression for the total polarization which is given by

$$P = Ecos(\omega t)(\chi^{(1)} + \frac{3}{4}\chi^{(3)}|E|^2)$$

We obtain an expression for the non-linear refractive index

$$n = n_o + n_2 I$$

where

$$n_2 = \frac{3}{4}\frac{\chi^{(3)}}{cn_o^2\epsilon_o}$$

Self Phase Modulation (SPM) essentially entails the modulation of the refractive index of the material based on the intensity of light. In a pulsed laser beam, the phenomena of SPM leads to the generation of new frequencies whereas in a CW pulse, it leads to the addition of an time invariant additional phase.

However, the continuous wave input pulse is not stable under small perturbations in the anomalous dispersion regime. The perturbations grow at an exponential rate and the CW is transformed into a pulse train. The perturbation grows at an exponential rate at the expense of the pump signal and as the pump signal weakens in the process, the growth is slowed down. Despite this, from a practical standpoint, this method allows for the generation of ultra-short pulse trains with a controlled repetition rate as high as $2Thz$. Another factor to consider is that the pulse train is generated at approximately five times the dispersion length of the fiber medium after which it deforms back into its original CW form. Section 6 discusses the generation of ultra-short pulses by exploiting Modulation Instability.

4 The Non Linear Schrodinger Equation

The fundamental wave equation describes the propagation of an EM wave in space. To study the propagation of light in optical fibers, certain modifications have to be made to the fundamental equation so as to take into account the non-linear polarization terms. We have, for the case of a non-linear polarization

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}(n_o^2(\omega)E + \frac{P_{NL}}{\epsilon_o}) = 0$$

Simplifying this, we get

$$\nabla^2 E - \frac{n^2(\omega, I)}{c^2}\frac{\partial^2 E}{\partial t} = 0$$

where,

$$n^2(\omega, I) = n_o^2(\omega) + \frac{3\chi^{(3)} I}{4\epsilon_o c n_o(\omega)}$$

This equation can be solved by transforming the field in the Fourier domain as follows

$$\tilde{E}(r, \phi, z, \omega) = F(r, \phi)\widetilde{A}(z, w)e^{i\beta_o z}$$

where $\tilde{F}$ is the transverse field distribution and $\tilde{A}$ is the envelope function. By using the technique of separation of variables, we obtain two equations; for the field function and for the envelope function respectively. At this stage, we are interested only in the evolution of the envelope function and hence. The evolution of the envelope is given by

$$2i\beta_o \frac{\partial \tilde{A}}{\partial z} + \tilde{A}(\beta^2 - \beta_o^2) = 0$$

where β is the propagation constant In this equation, the second derivative of $\tilde{A}$ w.r.t z has been neglected as we are talking the slowly varying envelope approximation. We can further simplify this by considering β is approximately equal to β_o (The spread of frequencies around the central frequency is not comparable to the order of the central frequency). We can then proceed to perform a Taylor Series expansion of β around ω_o and apply certain transformations to obtain

the Non Linear Schrodinger Equation or NLSE (which is called so due to its similarity to the Schrodinger equation for a non-linear potential) given by

$$i\frac{\partial U}{\partial z} - \frac{\beta_2}{2}\frac{\partial^2 U}{\partial T^2} + \gamma(\omega)|U|^2 U \ = 0$$

such that

$$U(z, t) = \sqrt{K} A(z, t)$$

$$|U(z, t)|^2 = P$$

The analytical solutions to this equation can be obtained by advanced mathematical tools such as the Inverse Scattering Method. However, particular solutions of interest can be studied using numerical analysis. The subsequent sections discuss the particular case of Solitons and Perturbed Continuous Waves.

5 Soliton Solutions to the NLSE

A particular solution of the NLSE are named Solitons. Solitons are formed when the broadening effect of dispersion (whether positive or negative) perfectly balances out the temporal narrowing effects of the non-linearity through self focusing. These pulses propagate in the medium with their intensity envelope unchanged or evolving periodically. There are two kinds of Solitons that are possible depending on whether the fiber has a positive or a negative GVD. In the case of a negative GVD (often referred to as anomalous dispersion), we obtain a Bright Soliton. These pulses are void of a chirp as dispersion pushes the higher frequencies to the leading edge while SPM causes the shift oh higher frequencies to the trailing end. The simplest of these is the Fundamental Bright Soliton which propagates unchanged and consequently finds many applications in optical communication and is given by

$$U(z,t) = \sqrt{P_o} e^{-\frac{iz}{L_D}} sech(\frac{t}{T_o})$$

The peak input power P_o of the Fundamental Bright Soliton depends on the Dispersion length. Here we define a quantity knows as the Soliton order, which equals 1 for the Fundamental Soliton.

$$N^2 = \frac{L_d}{L_{NL}} = \frac{T_o^2 \gamma P_o}{|\beta_2|}$$

where L_D is the dispersion length and T_o is the pulse duration. Hence, we see that for a Soliton of order 1, the input power that is required to perfectly balance dispersion and non-linearity turns out to be

$$P_o = \frac{N^2 |\beta_2|}{T_o^2 \gamma}$$

The propagation of a Fundamental Soliton over a fiber length of $z = 0.5Km$ is shown in Figure 1.

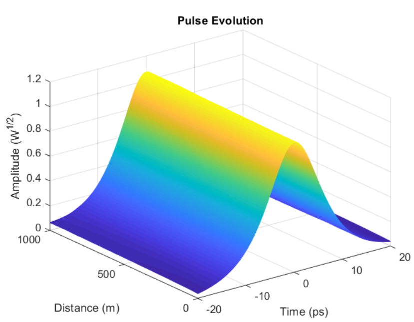

Figure 1: Propagation of Soliton in fiber with characteristics - $P_o = 1.1W, \beta_2 = -25ps^2/km, \gamma = 1W^{-1}/km, T_o = 5ps$

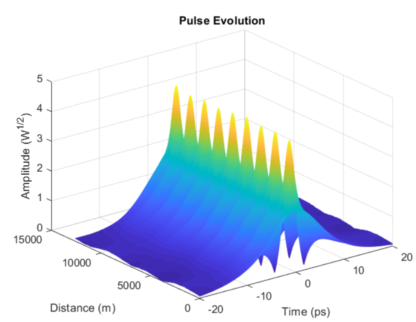

Figure 2: Propagation of Second Order Soliton in fiber with characteristics - $P_o = 4.8W, \beta_2 = -25ps^2/km, \gamma = 1W^{-1}/km, T_o = 5ps$

If we consider an input pulse that is the superposition of several (N) Fundamental Solitons, we obtain higher order Solitons whose power fluctuates periodically. A second order Soliton generated from 2 Solitons in Figure 1 is shown in Figure 2. The period of fluctuation (z_o) can be shown to be equal to $\frac{\pi T_o^2}{2|\beta_2|}$

Now, in the case of a positive group velocity dispersion, we get a Dark Soliton which has similar properties as the Bright Soliton in terms of the peak input power and order, and is given by

$$U(z,t) = \sqrt{P_o} e^{-\frac{iz}{L_D}} tanh(\frac{t}{T_o})$$

Since the envolope is a hyperbolic tangent function, the SPM is able to counteract the effects of the positive dispersion. In this case, the time derivative of intensity has the exact opposite sign as that in a Bright Soliton and hence, the SPM pushes the higher frequencies to the leading edge and the positive GVD does the exact opposite of that.

6 Generation of Ultra Short Pulses

As discussed before, a CW of the form

$$U(z,t) = \sqrt{P_o} e^{i\gamma P_o z}$$

is stable under SPM. However, when a harmonic perturbation is introduced, the perturbations induce the generation of new frequencies (spectral side bands). Consider the perturbed CW as

$$U(z,t) = (\sqrt{P_o} + p(z,t))e^{i\gamma P_o z}$$

the solution for $p(z,t)$ can be obtained by substituting it in the NLSE. For the case of anomalous dispersion ($\beta_2 < 0$) we see that the generated frequencies see a gain and hence the perturbation grows exponentially. The generation of these new frequency side bands causes the pulse to shorten in the time domain with a period $\frac{2\pi}{\omega_m}$ where $\omega_o + \omega_m$ is the frequency that sees the highest gain. But as explained before, since the side bands are formed at the expense of the pump pulse, after a certain point, we see the pulse deforms periodically into a CW. The modulation instability was modelled for a pulse of the form

$$U(z,t) = \sqrt{Po}(1 + 0.01cos(\omega_m t))$$

and the pulse train was observed at a fiber length of $0.5km$. The plot obtained is shown in Figure 3.

7 Pulse Compression

Pulse compression consists broadly of two artefacts

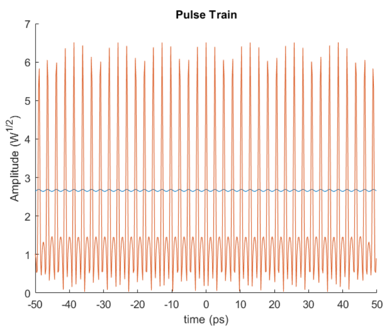

Figure 3: Generation of ultra-short pulses in a fiber with characteristics - $P_o = 7.1W, \beta_2 = -3ps^2/km, \gamma = 1.57W^{-1}/km, T_o = 11ps$. The plot in blue shows the modulated CW signal.

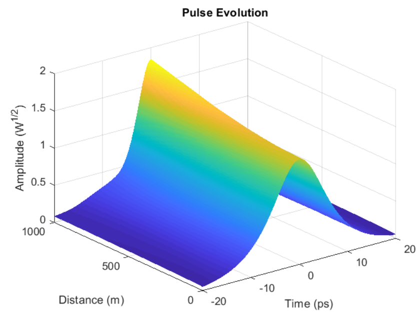

Figure 4: Propagation of a Soliton with characteristics - $P_o = 2W, \beta_2 = -25ps^2/km, \gamma = 1W^{-1}/km, T_o = 5ps$. The Soliton pulse can be seen narrowing to a Fundamental Soliton Asymptotically.

1. Linear Compression - When pulses have a frequency chirp, they can be shortened using a material that has a negative GVD so as to remove the chirp. This can be done using prism pairs, diffraction gratings and optical fibers.

2. Non-linear Compression - This involves two steps. The first of which is a spectral broadening which can be achieved through SPM. Following which a spectral broad pulse is obtained, but it has a significant amount of chirp which might make it longer in duration than the original pulse. Hence, to shorten the pulse, it is passed through a linear compressor as described above.

In this report, the compression technique described utilizes the anomalous dispersion and hence can be classified under linear compression. This effect is known as Soliton Like Pulse Compression. The degree of pulse compression achieved depends on N and the Soliton period Z_o.

z_o is the period of the higher order Soliton as is defined as

$$z_o = \frac{\pi}{2L_d}$$

As we have already seen, higher order Solitons are defined as

$$U(z,t) = \sqrt{P_o} e^{\frac{-iz}{L_d}} sech(\frac{t}{T_o})$$

where the order of the Soliton is given by the fundamental relation described in Section 6. An interesting point to note is that even if the Soliton order is not an integer as is of the form $N + \epsilon$ where $|\epsilon| < \frac{1}{2}$, the pulse evolves asymptotically with propagation distance into a Soliton of order N and continues to propagate. With higher order Solitons, this fact can be exploited in Soliton Like compression as the temporal profile of higher order Solitons varies periodically. Hence if the propagation length is set such that the pulse is the narrowest, optimum compression can be achieved. For Soliton like pulses of order $N > 10$, the optimum length can be given by

$$z = \frac{1.6z_o}{N}$$

where z_o is the period. Even pulses that do not have an envelope function of the form $sech(t)$, evolve asymptotically into Solitons (due to the competing influence of the GVD and the SPM) depending on their initial pulse shape. The evolution of these pulses can be studied numerically. The evolution and pulse profile of a Soliton with order $N = 1.4$ has been shown in Figure 4.

Beyond the optimum compression length, the pulse starts to develop pronounced auxiliary pedestal structures which causes some of the pulse energy to be contained outside its main structure. At high compression ratios, this becomes a serious problem as up to 80% of the pulse power is stored in the auxiliary pedestals. The pedestal can be removed by the means of a more advanced pulse compression technique known as Adiabatic Soliton Compression. In Adiabatic Soliton Compression, the dispersion of the fiber becomes weaker as the distance of propagation increases. The intensity birefringence of the optical fiber can also be used to diminish the peaks as this causes the peaks and the pedestal to become differently polarized and that way, the two can be separated. The plots shown below in Figures 5. 6, and 7 show the compression of a Gaussian input beam for different values of Soliton order $N = 11.3, 13$ and 15 respectively. The fiber characteristics used are $\beta_2 = -25ps^2/km, \gamma = 1W^{-1}/km, T_o = 10ps$ As

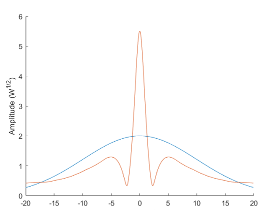

Figure 5: $z_{opt} = 0.89km$

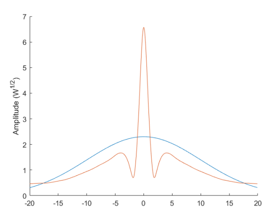

Figure 6: $z_{opt} = 0.77km$

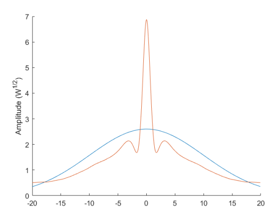

Figure 7: $z_{opt} = 0.63km$

can be clearly seen from the figures, Soliton Like Compression has the following characteristics.

1. As the Soliton order N increases, the optimum length z_{opt} required for the compression of the pulse decreases.

2. The extent of compression is higher for pulses with higher input power P_o which corresponds to a higher Soliton order.

3. As the order increases, the fraction of total pulse power stored in the pedestals also increases.

References

[1] Govind P. Agarwal. *Nonlinear Fiber Optics.* Elsevier, 5 edition, 2013.

[2] Bruno Crosignani. Non linear effects in optical fibers. *Integrated Fiber Optics*, 1992.

[3] Mario F. S. Ferreira. Nonlinear effects in optical fibers: Limitations and possibilities. *Journal of Nonlinear Optical Physics Materials*, 17, 2008.

[4] George F. R. Chen Doris K. T. Ng Ju Won Choi, Byoung-Uk Sohn and Dawn T. H. Tan. Soliton-effect optical pulse compression in cmos-compatible ultra-silicon-rich nitride waveguides. *APL Photonics*, 4, 2019.

[5] R. H. Stolen L. F. Mollenauer and J. P. Gordon. Extreme picosecond pulse narrowing by means of soliton effect in single-mode optical fibers. *Optics Letters*, 8(5), 1983.

[6] NPTEL. Introduction to nonlinear optics and applications. 2018.

[7] MIT OCW. Quantum time evolution using the split operator fourier transform algorithm. 2019.