Theory of Computation

Regular Expression

Lecture 7

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Operators

$\Sigma = \{a, b\}$

$*$: Kleene Star

$+$: Kleene +

$\Sigma^* = (a, b)^*$

$\{\underset{0}{\epsilon}, \underset{1}{a, b}, \underset{2}{aa, ab, ba, bb}$

. . . . . $\}$

$\Sigma^+ : (a, b)^+$

$\{a, b, aa, ab, ba, bb, aaa, ..\}$

Unary

$+$: OR

$L1 : \{ab, ba\}$, $L2 : \{ba, bb\}$

$L1 + L2 : \{ab, ba, bb\}$

$\cdot$: Concatenation

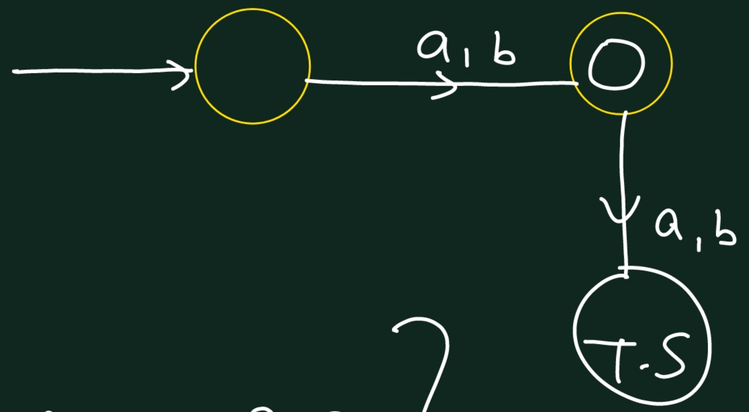
$ab \cdot b = abb$

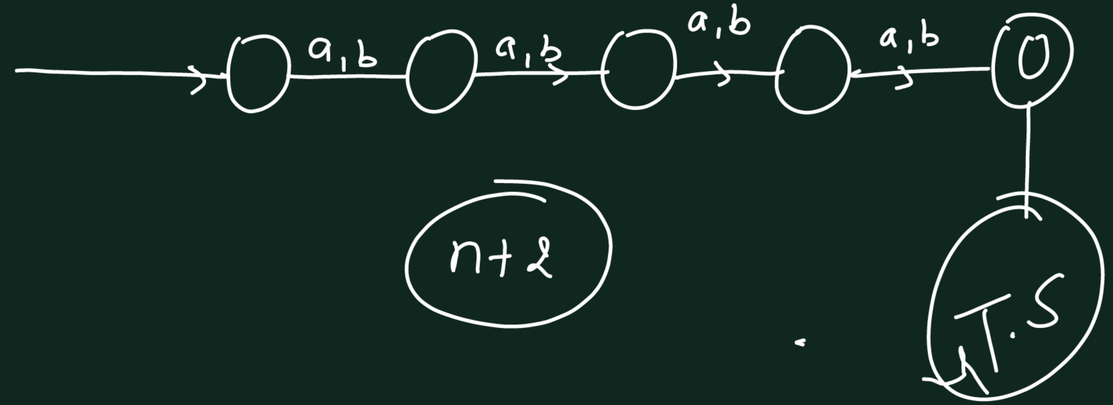
$a \cdot (b + a) = \{ab + aa\}$

Binary

$\Sigma = \{a, b\}$

L1: String length = 4





$L2 = a^3 b^3 \}$ $a^{111} b^{111}$ | $a^n b^n$

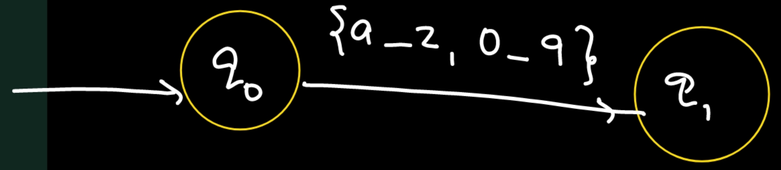
Qn $\{ab + ba\}$

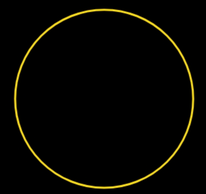
Regex : Node JS

Password Rule:

Password must:
1.Contain **only lowercase letters and digits**
2.Have **minimum length = 8**

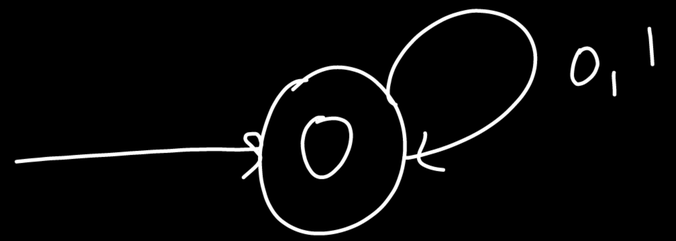
Alphabet:
Σ = { a–z, 0–9 }





min length = 1

Σ : {0, 1}



State	valid char [a-z0-9]	invalid
→ q0	q1	dead
q1	q2	dead
q2	q3	dead
q3	q4	dead
q4	q5	dead
q5	q6	dead
q6	q7	dead
q7	q8	dead
*q8	q8	dead
dead	dead	dead

} Real Scenario

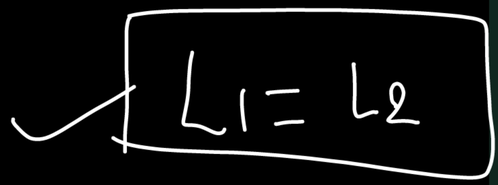
Symbols Reminder

- + → Union / OR
- . → Concatenation
- ε → Empty string
- Ø → Empty language
- R, S, T → Regular expressions

Properties of + (OR / Union)

Identity
Identity element does **not change** the expression
R+Ø=R

Explanation:
Union with an empty language gives the same language.



Commutative
Order does **not matter**
R+S=S+R

Example:
a + b = b + a

$L_1 = \{aa, ab\} \cup \{ba, bb\}$

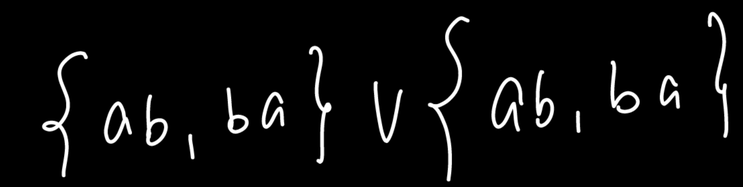
$L_2 = \{ba, bb\} \cup \{aa, ab\}$

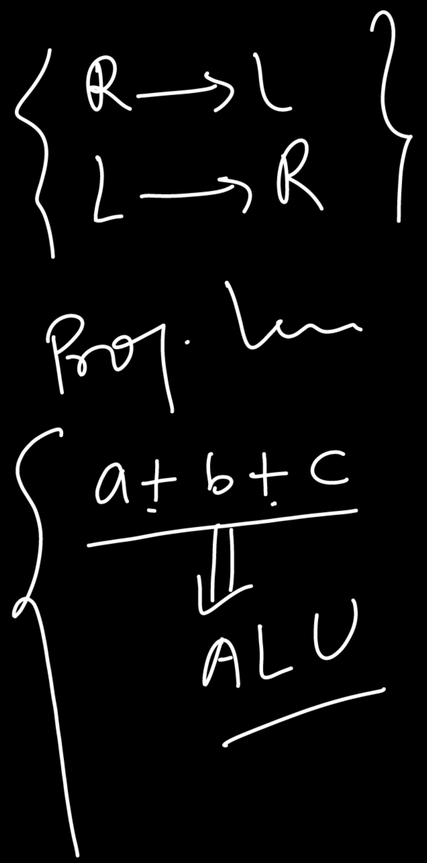
Associative
Grouping does **not matter**
(R+S)+T=R+(S+T)

Example:
(a + b) + c = a + (b + c)

Idempotent
Repeating does not change result
R+R=R

Example:
a + a = a





Dominator (Absorbing Element)

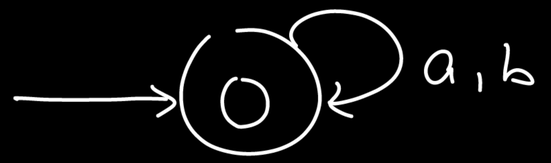
Σ* dominates union

$R+\Sigma^*=\Sigma^*$

Because Σ* already contains all strings.

$\Sigma = \{a, b\}$

$\Sigma^* = \{\epsilon, a, b, aa \ldots\}$



Annihilator

NO annihilator exists for +

Because nothing makes union empty except both being empty.

$L1 \cup \{\} = L1$

Properties of . (Concatenation)

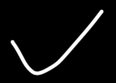
Identity
Identity element keeps expression same
R.ε=ε.R=R

$\{ab\}.\epsilon = ab$

Explanation:
Concatenating with empty string does nothing.

Associative
Grouping does **not matter**
(R.S).T=R.(S.T)

Example:
(ab)c = a(bc)



Commutative (NOT TRUE)

Order **matters**
R.S≠S.R

Example:
ab ≠ ba

$a.b \neq b.a$ ✗

$a+b = b+a$

R.L : Reg Exp : R.G : DFA INF.

Annihilator
Element that destroys everything
R.∅=∅.R=∅
Explanation:
No strings available to concatenate.

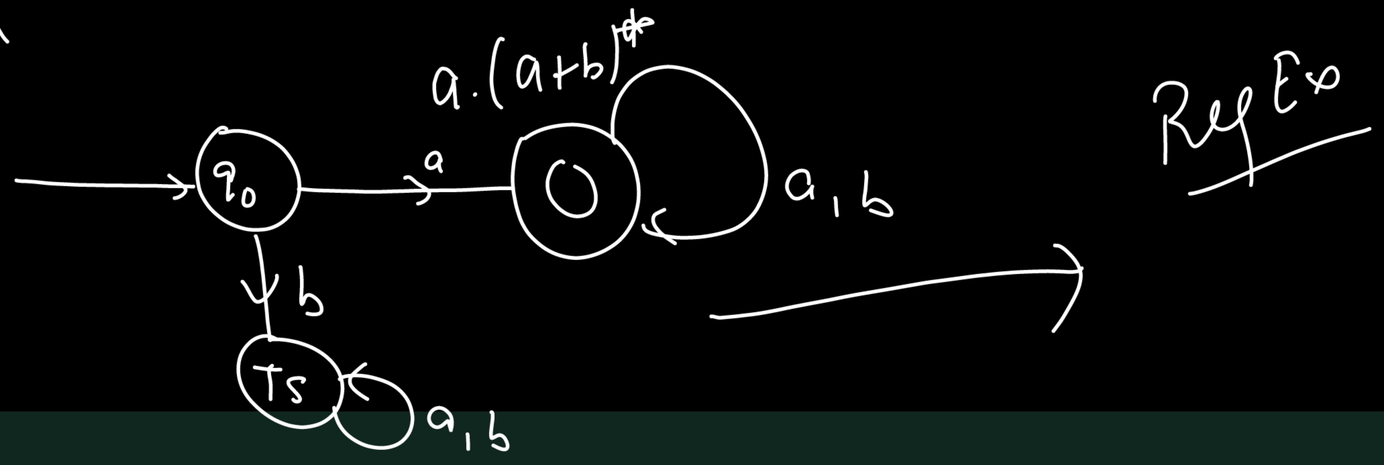
$a.\Sigma^*$

Dominator
No dominator for concatenation

Reg Ex: $a(a+b)^*$

$a.\{\in, a, b, aa, bb, ab, ba, \ldots\}$

$\{a, aa, ab, aaa \ldots\}$



Thank You