Counterparty Credit Risk
–Coding Assignment 4–

Professor L. Andersen

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We continue with the interest rate setup from Coding Assignments 2 and 3. That is, we consider a market where we have established at time 0 (e.g., by looking at quoted swap and futures prices) that the risk-free forward interest rate curve is given by

$$f(0,t) = a + b \cdot t$$

where $a = 3\%$ and $b = 0.2\%$. Assume also that we have established that the floating index curve is related to the risk-free curve through the relation

$$f_L(t,T) = f(t,T) + s$$

for any t and T (with $T \geq t$). Assume that $s = 0.5\%$. The dynamics of the forward curve $f(t,T)$ is given by a 2-factor Gaussian model with $\sigma_r = 2\%$, $c = 40\%$, $\kappa_2 = 5\%$, $\rho_\infty = 40\%$.

As before, we consider a 10-year semi-annual interest rate swap. Let the notional of the swap be USD50 million, and let its coupon be h. In Coding Assignment 3, you established the expected exposure (EE) profile for the payer and receiver swaps, on a weekly grid spanning [0, 10yrs]. You did this for coupons of $h = 2\%$, $h = 4\%$, $h = 6\%$, $h = 8\%$. You also established the unilateral and bilateral CVA, for a bank trading with a counterparty A with a flat risk-neutral intensity $\lambda_A = 4\%$.

Part 1: Computation of Capital

Q1 : Assume that the swap is uncollateralized. Compute the EPE(1), EEPE(1), EAD, and M for both payer and receiver swaps, at all coupons $h = 2\%$, $h = 4\%$, $h = 6\%$, $h = 8\%$. Show the results in neat tabular form.

Q2 : Assume that the historical one-year historical (NOT risk-neutral) default probability for A is PD = 0.5%. Also assume that the loss-given-default is LGD = 60%. Compute the transition function $k(M, \text{PD})$ and compute the regulatory AIRB credit risk capital for the 8 different swaps in Q1. Show the results in neat tabular form.

Q3 : Repeat Q1 and Q2, but this time for a swap with full variation margin. Assume that the MPoR is 2 weeks.

Q4 : The results in Q3 will depend on how you treat the cash-flow spikes in the EE profiles, as this will materially affect the "running-max" effective exposure profile EE*. Repeat Q3 but this time remove the cash-flow spikes for the purpose of computing EE* and EEPE(1).

Part 2: Wrong-Way Risk

For the remainder, assume that there is no collateral agreement in place, and also assume that there is no MPoR, i.e., $\delta = 0$. We recall from Lecture 7 that the "full-blown" definition of unilateral CVA is (Equation (3) in the lecture notes)

$$\mathrm{CVA}_U = (1-R)\int_0^{T_{\max}} \mathbb{E}\left(E(t)\lambda_A(t) e^{-\int_0^{T_{\max}}(r(u)+\lambda_A(u))\,du}\right) dt \tag{1}$$

where R is recovery and $E(t)$ is the portfolio exposure, as seen from the bank's perspective. If we assume that exposure and interest rates are approximately independent of λ_B, then this expression simplifies to the more common expression

$$\begin{aligned}\mathrm{CVA}_U &\approx (1-R)\int_0^{T_{\max}} \mathrm{PVEE}(t)\mathbb{E}\left(\lambda_A(t)e^{-\int_0^{T_{\max}}\lambda_A(u)\,du}\right) dt \\ &= -(1-R)\int_0^{T_{\max}} \mathrm{PVEE}(t)\frac{\partial X_A(0,t)}{\partial t}\,dt, \end{aligned} \tag{2}$$

where $X_A(0,t)$ is the risk-neutral survival probability function for A. Expression (2) is the one that you have used (after time discretization) in Coding Assignment 3.

Here in Coding Assignment 4, we are interested in using the bigger expression (1), as this can help us understand the effect of correlation between default intensity and exposure – the so-called Wrong-Way Risk (WWR). For this purpose, assume now that A's default intensity is no longer constant at 2%, but is stochastic and driven by the model you coded up in Coding Assignment 1. Specifically, we assume that the time 0 forward intensity curve for A is flat at 4% (so $\lambda_{f,A}(0,t) = 4\%$ for all t), and that the Gaussian model parameters are $\kappa = 3\%$, $\sigma = 1\%$ (i.e., the same as in Coding Assignment 1).

The stochastic credit model in Coding Assignment 1 has a single Brownian motion, call it $W_A(t)$. The stochastic rates model in Coding Assignment 2 has two Brownian motions, so a combined model that has both stochastic credit and stochastic rates has three Brownian motions and will need a 3x3 correlation matrix. In particular, we need to specify in a meaningful way the 2 correlations between $W_A(t)$ and the two rates Brownian motions. As described in your reading assignment (Section 2.4, with Z here being W_A), this specification can be done by measuring the correlations between credit spreads and the short and long ends of the interest rate curve (ρ_l and ρ_s, respectively).

Q5 : First set $\rho_l = \rho_s = 0$, such that interest rates are independent of credit spreads. Compute (1) by Monte Carlo simulation (50,000 paths, as before) for par-valued receiver and payer swaps, after suitable discretization of the various integrals. (Note that we are here only interested in a single coupon, namely h_{par}).

Q6 : Now set $\rho_l = 0.2$ and $\rho_s = 0.4$. Compute (1) again by Monte Carlo simulation, taking into account the non-zero correlation structure between rates and credit spreads (e.g., by using Cholesky decomposition). You should see that the two unilateral CVA numbers (one for payer, one for receiver swap) have now changed from Q5. Comment on the differences, and provide the intuition for why one number goes up and the other number goes down.