# Counterparty Credit Risk
# –Coding Assignment 2–

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Consider a market where we have established at time 0 (e.g., by looking at quoted swap and futures prices) that the risk-free forward interest rate curve is given by

$$f(0,t) = a + b \cdot t$$

for two positive constants $a$ and $b$. Assume also that we have established that the floating index curve is related to the risk-free curve through the relation

$$f_L(t,T) = f(t,T) + s$$

for any $t$ and $T$ (with $T \geq t$). Assume that $s$ is a constant.

**Q1** : Derive an analytical expression for the time 0 discount functions $P(0,t)$ and $P_L(0,t)$, for any $t \geq 0$. Show your work.

We now introduce the 2-factor model covered in Lecture 6. We recall that the model is specified by parameters $\sigma_r$, $c$, $\kappa_2$, $\rho_\infty$.

**Q2** : Compute an analytical expression for formula (8) in Lecture 6, thereby establishing the reconstitution formula the relates $P(t,T)$ to the state of $x(t) = (x_1(t), x_2(t))^\top$. Show your work.

We now want to simulate the process for $x(t)$ on a discrete time grid $\{t_j\}$ that spans the horizon $[0, 10\text{yrs}]$. We also fix all our model constants to $a = 3\%$, $b = 0.2\%$, $s = 0.5\%$, $\sigma_r = 2\%$, $c = 40\%$, $\kappa_2 = 5\%$, $\rho_\infty = 40\%$.

**Q3** : Assume that the time grid has monthly spacing: $t_0 = 0$, $t_1 = 1/12$, $t_2 = 2/12$, ... etc. Document in detail the simulation algorithm you are using, and use it to draw 10 independent paths of $x(t)$ on the monthly grid, with each path ending at the 10-year point. Graph these 10 paths (one graph for $x_1(t)$ and one graph for $x_2(t)$).

We now introduce a 10-year swap with a fixed coupon $h$ and a floating coupon being computed of the floating index curve $P_L$, as explained in Lecture 6. We assume that the swap makes payments semi-annually. Also assume that the notional of the swap is USD50 million.

**Q4** : Graph the time 0 value of both the payer and receiver versions of the 10-yr swap as a function of $h$, with $h$ varying from 0% to 10%. There is a value of $h$ – known as the *par coupon* $h_{par}$ – that will make the time 0 value of both payer and receiver swaps equal to zero. What is the value of $h_{par}$?

**Q5** : Using the result of Q2, translate the 10 paths of $x(t)$ in Q3 into 10 paths of the payer and receiver swaps, with the coupon $h$ set equal to $h_{par}$ (such that the time 0 value of the swap paths all start at zero).

**Q6** : Using a weekly time grid, use Monte Carlo simulation with $M = 50,000$ paths to estimate the expected exposure profiles for the payer and receiver swaps (with par coupon). Assume that there is no collateral. Graph the expected exposure profiles on a weekly grid out to 10 years.