

CS 6375 - Machine Learning

① given, $x = 4.2$

$x = 4$ is the nearest sample from the training data.

Therefore, the test sample with $x = 4.2$ has been classified into

Label B.

② According to 2-NN

$$x = 4.2$$

$$n=4, |x_4 - x_i| = 0.2$$

$$n=5, |x - x_i| = 0.8$$

As they are the closest two ^{neighbors}, according to the distances the

$x = 4.2$ has Label = B.

③ $x = 4.2$, 3-NN.

Using the euclidean norm.

$$x=3, |3-4.2| = 1.2$$

$$n=4, |4-4.2| = 0.2$$

$$n=5, |5-4.2| = 0.8$$

$$n=6, |6-4.2| = 1.8$$

The closest neighbors are $x=3, 4, 5$, There are two labels with A and one B. Therefore, for $x=4.2$, Label is A.

④ Estimating error for 1-NN by leave-one-out cross validation.

Leaving out $x=0$

The closest neighbor to $x=0$ is $x=1$, label A
classification of $x=0$ is A.

Leaving out $x=1$

The closest neighbors $x=1, 2$
classification of $x=1$ is A

Leaving out $x=2$

The closest neighbor is $x=1, 3$
classification of $x=2$ is A.

Leaving out $x=3$

The closest neighbor is $x=2, 4$
classification of $x=3$ is B which is error

Leaving out $x=4$

The closest neighbor is $x=3, 5$
classification of $x=4$ is A which is an error

Leaving out $x=5$

closest neighbors is $x=4, 6$
classification of $x=5$ is B which is an error

Leaving out $x=6$

closest neighbor is $x=5, 7$
classification A.

Leaving out $x=7$

closest neighbor $x=6, 8$
classification A

Leaving out $x=8$

closest neighbors $x = 7, 9$.
classification B. it is an error.

Leaving out $x = 9$

closest neighbors $x = 8, 10$
classification A. it is an error.

Leaving out $x = 10$

closest neighbor $x = 9, 11$
classification B.

Leaving out $x = 11$

closest neighbor $x = 10, 12$
classification B.

Leaving out $x = 12$

closest neighbor $x = 11, 13$
classification A. It is an error.

Leaving out $x = 13$

closest neighbor $x = 12, 14$
classification B. It is an error.

Leaving out $x = 14$

closest neighbor $x = 13, 15$
classification A. It is an error.

Leaving out $x = 15$

closest neighbor $x = 14, 16$
classification B.

Leaving out $x = 16$

closest neighbor $x = 15, 17$.
classification B.

Leaving out $x = 17$

closest neighbor $x = 16$
classification A.

$$\boxed{\text{Estimation of error} = \frac{8}{16}}$$

⑤ Leave one out cross validation for ANN

Leave $x=0$

closest neighbors $x=1, 2$

classification A

Leave $x=1$

closest neighbors $x=0, 2$

classification A

Leave $x=2$

closest neighbors $x=1, 3$

classification A

Leave $x=3$

closest neighbors $x=2, 4$

classification: B. It is an error

Leave $x=4$

closest neighbors $x=3, 5$

classification **A**. It is an error

Leave $x=5$

closest neighbors $x=4, 6$

classification B. It is an error

Leave $x=6$

closest neighbors $x=5, 7$

classification A

Leave $x=7$

closest neighbors $x=6, 8$

classification A

Leave $x=8$

closest neighbors $x=7, 9$ classification B. It is an error

Leave $x=9$

closest neighbors $x=8, 10$: classification A. It is an error.

Leave $x=10$

closest neighbors $x=9, 11$: classification B.

Leave $x=11$

closest neighbors $x=10, 12$: classification B.

Leave $x=12$

closest neighbors $x=11, 13$: classification A. It is an error.

Leave $x=13$

closest neighbors $x=12, 14$: classification B. It is an error.

Leave $x=14$

closest neighbors $x=13, 15$: classification A. It is an error.

Leave $x=15$

closest neighbors $x=14, 16$: classification B.

Leave $x=16$

classification B

Leave $x=17$

closest neighbors $x=15, 16$

classification B

No. of errors = $\frac{8}{18}$

⑥ For a 3-NN.

Leave $x=0$

closest neighbors = 1, 2, 3

classification : A.

Leave $x=1$

closest neighbors = 0, 2, 3

classification = A.

Leave $x=2$

closest neighbor = ~~0~~, 1, 3, 4

Since 0, 4 are equally distant. Taking 0 so error is maximized.

classification: A.

Leave 3

closest neighbor 0, 4, 1, 6

classification n.

Leave out $x=4$

Neighbors: 3, 5, 2

classification n. It is an error

Leave out $x=5$

Neighbors: 4, 6, 3

classification: n.

Leave $x=6$

Neighbors 3, 7, 4

classification n.

Leave $x=7$

Neighbors: 6, 8, 9

classification n.

Leave $x=8$

Neighbors: 7, 9, 10

classification B. It is an error.

Leave $x=9$

Neighbors = 8, 10, 7

classification n. It is an error.

Leave $x=10$

Neighbors: 9, 11, 8. Classification B

Leave out $x=11$

Neighbors: 10, 12, 13 · classification B.

Leave out $x=12$

Neighbors: 11, 13, 14 · classification B.

Leave out $x=13$

Neighbors: 12, 14, 11 · classification B. it is an error

Leave out $x=14$

Neighbors: 13, 15, 12 · classification B.

Leave out $x=15$

Neighbors: 14, 16, 13 · classification B.

Leave out $x=16$

Neighbors: 15, 17, 14 · classification B.

Leave out $x=17$

Neighbors: 14, 15, 16 · classification B.

Estimation of error = $\frac{4}{18}$

⑦ 4-NN

Leave out $x=0$

Neighbors: 1, 2, 3, 4 · classification: A.

Leave out $x=1$

Neighbors: 0, 2, 3, 4 · classification A

Leave out $x=2$

classification A

Leave out $x=3$

classification A.

Leave out $x=4$
Neighbors: 3, 5, 6, 2. classification ~~A~~ B. It is an error

Leave out $x=5$
Neighbors: 4, 6, 7, 3. classification A

Leave out $x=6$
Neighbors: 5, 7, 8, 4. classification D

Leave out $x=7$
Neighbors: 6, 8, 9, 5. classification D.

Leave out $x=8$
Neighbors: 7, 9, 10, 6. classification B. It is an error

Leave out $x=9$
Neighbors: 8, 10, 7, 11. classification A. It is an error.

Leave out $x=10$
Neighbors: 9, 11, 12, 8. classification B

Leave out $x=11$
Neighbors: 10, 12, 9, 13. classification B.

Leave out $x=12$
Neighbors: 11, 13, 14, 10. classification B

Leave out $x=13$
Neighbors: 12, 14, 11, 15. classification: B. It's an error

Leave out $x=14$
Neighbors: 13, 15, 16, 12. classification B

Leave out $x=15$
Neighbors: 14, 16, 13, 12. classification B.

Leave out $x=16$
classification B

leave out $x=17$
classification B.

Estimated error $\frac{4}{18}$

⑧ 17. NN

Leave out $x=0$

classification B. Its an error.

Leave out $x=1$

classification B. Its an error.

leave out $x=2$

classification B. Its an error.

Leave out $x=3$

classification B. Its an error.

Leave out $x=4$

classification A. Its an error.

Leave out $x=5$

classification B. Its an error.

Leave out $x=6$

classification B. Its an error.

Its the same case for $x=7$ to $x=17$

$$\therefore \boxed{\text{Estimated error} = \frac{18}{18}}$$