

SVM-Example

Question 1

An SVM is trained with the following data:

| | | | |
|-------|------------|----------|----------|
| i | 1 | 2 | 3 |
| x_i | $(-1, -1)$ | $(1, 1)$ | $(0, 2)$ |
| y_i | -1 | 1 | 1 |

Let $\alpha_1, \alpha_2, \alpha_3$ be the Lagrangian multipliers associated with this data. (α_i is associated with (x_i, y_i) .)

A

Using the polynomial kernel of degree 2, what (dual) optimization problem needs to be solved in terms of the α_i in order to determine their values?

Reminder: the polynomial kernel of degree 2 is:

$$K(x_i, x_j) = (x_i'x_j + 1)^2$$

Answer

B

The solution to the optimization problem is:

$$\alpha_1 = 1/8, \quad \alpha_2 = 1/8, \quad \alpha_3 = 0$$

a. What are the indexes of the support vectors? Circle them below.

Answer: 1 2 3

b. This SVM classifies the example x according to the sign of $w'\phi(x) + b$, where the transformation ϕ is implicitly defined by the kernel. Compute the value of the constant b . (This can be done without explicit computation of ϕ or w .)

Answer: The value of b is:

.

c. What computation needs to be carried out to determine the classification of the point $x = (-1, 0)$ by this SVM?

Answer:

.

d. What computation needs to be carried out to determine the classification of the point $x = (1, 0)$ by this SVM?

Answer:

.

Question 2

An SVM is trained with the following data:

| i | 1 | 2 | 3 |
|-------|------------|----------|----------|
| x_i | $(-1, -1)$ | $(1, 1)$ | $(0, 2)$ |
| y_i | -1 | 1 | 1 |

Let $\alpha_1, \alpha_2, \alpha_3$ be the Lagrangian multipliers associated with this data. (α_i is associated with (x_i, y_i) .)

A

Using the polynomial kernel of degree 2, what (dual) optimization problem needs to be solved in terms of the α_i in order to determine their values?

Reminder: the polynomial kernel of degree 2 is:

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Answer

B

The solution to the optimization problem is:

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b. This SVM classifies the example x according to the sign of $w'\phi(x) + b$, where the transformation ϕ is implicitly defined by the kernel. Compute the value of the constant b . (This can be done without explicit computation of ϕ or w .)

Answer: The value of b is:

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c. What computation needs to be carried out to determine the classification of the point $x = (-1, 0)$ by this SVM?

Answer:

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d. What computation needs to be carried out to determine the classification of the point $x = (1, 0)$ by this SVM?

Answer:

.