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Homework-8

Question 1.

$$\mu_1 = \left(\frac{1+2+1}{3}, \frac{1+1+2}{3}\right) = \left(\frac{4}{3}, \frac{4}{3}\right) = (1.333, 1.33)$$

$$\mu_2 = \left(\frac{0+1+2+3+0+3}{6}, \frac{0+0+0+0+3+3}{6}\right) = \left(\frac{3}{2}, 1\right)$$

$$= (1.5, 1)$$

$$P(\omega+) = \frac{3}{9} = \frac{1}{3}$$

$$P(\omega-) = \frac{6}{9} = \frac{2}{3}.$$

Discrimant function

$$g_i(x) = \frac{|x - \mu_i|^2}{2\sigma^2} + \log P(\omega_i)$$

$$\text{for } g_-(x) = \frac{1}{2\sigma^2}\left[(x_1 - 1.5)^2 + (x_2 - 1.)^2\right] + \log 2/3$$

$$g_+(x) = \frac{1}{2\sigma^2}\left[(x_1 - 1.33)^2 + (x_2 - 1.33)^2\right] + \log \frac{1}{3}$$

$$= \frac{1}{2\sigma^2}\left[(x_1 - 1.5)^2 + (x_2 - 1.0)^2 - (x_1 - 1.33)^2 - (x_2 - 1.33)^2\right] > \log \frac{2}{3} - \log \frac{1}{3}$$

$$= \frac{1}{2\sigma^2}(0.34x_1 + 0.66x_2 - 0.7322) > \log 2$$

Discrimnant boundary

$$0.34x_1 + 0.66x_2 > 0.7322 + 2\sigma^2 \log 2$$

2 already solved $\mu_1 = \begin{bmatrix} \frac{1}{3} \\ \frac{4}{3} \end{bmatrix}$ $\mu_2 = \begin{bmatrix} 3/2 \\ 1 \end{bmatrix}$

$$C = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)(x_i - \mu)^T$$

$x_i - \mu$			
1 $\frac{-1}{3}$,	$1-\frac{1}{3}$	-0.3333 ,	-0.3333
$2-\frac{4}{3}$,	$1-\frac{4}{3}$	0.6666	-0.3333
$1-\frac{4}{3}$,	$2-\frac{4}{3}$	-0.3333	0.6666
$0-3/2$,	$6-1$	-1.5	-1
$1-3/2$,	$0-1$	-0.5	-1
$2-3/2$,	$0-1$	0.5	-1
$3-3/2$,	$0-1$	1.5	-1
$0-3/2$,	$3-1$	-1.5	2
$3-3/2$,	$3-1$	1.5	2

$$\frac{1}{9}\left(\begin{bmatrix} -0.3333 & 0.3333 \end{bmatrix} \begin{bmatrix} -0.3333 \\ 0.3333 \end{bmatrix} + \begin{bmatrix} 0.6666 & -0.3333 \end{bmatrix} \begin{bmatrix} 0.6666 \\ -0.3333 \end{bmatrix} + \begin{bmatrix} -0.3333 & 0.6666 \end{bmatrix} \begin{bmatrix} -0.3333 \\ 0.6666 \end{bmatrix} + \begin{bmatrix} -1.5 & -1 \end{bmatrix} \begin{bmatrix} -1.5 \\ -1 \end{bmatrix} + \begin{bmatrix} -0.5 \\ -1 \end{bmatrix} \begin{bmatrix} -0.5 & -1 \end{bmatrix} \right)$$

+ .

$$+\begin{bmatrix}-0.5 & -1\end{bmatrix}\begin{bmatrix}-0.5\\-1\end{bmatrix}+\begin{bmatrix}0.5 & -1\end{bmatrix}\begin{bmatrix}0.5\\-1\end{bmatrix}+\begin{bmatrix}1.5 & -1\end{bmatrix}\begin{bmatrix}1.5\\-1\end{bmatrix}$$

$$+\begin{bmatrix}-1.5 & 2\end{bmatrix}\begin{bmatrix}-1.5\\2\end{bmatrix}+\begin{bmatrix}1.5 & 2\end{bmatrix}\begin{bmatrix}1.5\\2\end{bmatrix}$$

$$=\frac{1}{9}\left(\begin{bmatrix}0.1111\\+\\0.1111\end{bmatrix}+\begin{bmatrix}0.4444\\+\\0.1111\end{bmatrix}+\begin{bmatrix}0.1111\\+\\0.4444\end{bmatrix}+\begin{bmatrix}2.25\\+\\1\end{bmatrix}+\right.$$

$$\begin{bmatrix}0.25+1\end{bmatrix}+\begin{bmatrix}0.25+1\end{bmatrix}+\begin{bmatrix}0.25+1\end{bmatrix}+$$

$$\begin{bmatrix}2.25+4\end{bmatrix}+\begin{bmatrix}2.25+4\end{bmatrix}$$

$$\frac{1}{9}\left(0.2222+0.5555+0.5555+3.25+1.25+1.25+\right.$$

$$\left.3.25+\ 6.25+6.25\right)$$

$$\frac{1}{9}\times(22.8332)=2.537\ //.$$

$$c=2.537$$

$$w=c^{-1}\left[\mu_1-\mu_2\right]=\frac{1}{2.537}\begin{bmatrix}4/3\\4/3\end{bmatrix}-\begin{bmatrix}3/2\\4\end{bmatrix}$$

$$\frac{1}{2.537}\begin{bmatrix}-0.1666\\0.3333\end{bmatrix}$$

$$w=\begin{bmatrix}-0.06566\\20\\0.1313\end{bmatrix}$$

$$b = \frac{1}{2}(\mu_2^T C^{-1} \mu_2 - \mu_1^T C^{-1} \mu_2) + (\ln P(h_1) - \ln P(h_2))$$

$$\mu_2^T C^{-1} \mu_2 = \begin{bmatrix} 3/2 & 1 \end{bmatrix} \times \frac{1}{2.537} \times \begin{bmatrix} 3/2 \\ 1 \end{bmatrix} - \begin{bmatrix} 4/3 & 4/3 \end{bmatrix} \frac{1}{2.537} \begin{bmatrix} 4/3 \\ 4/3 \end{bmatrix}$$

$$\frac{1}{2 \times 2.537} \left[[2.25 + 1) - [1.7777 + 1.7777] \right]$$

$$\frac{1}{5.074} \times \left[3.25 - 3.5554 \right]$$

$$\frac{1}{5.074} \times -0.3054$$

$$\boxed{b = -0.0601}$$

$$d(x) = w^T x + b$$

$$d(x) = \begin{bmatrix} 0.6566 & 0.1313 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - 0.0601 \text{ is positive.}$$

If $d(x) > 0$ and negative if $d(x) < 0$

3. Case 3 :-

$$d(x) = (x-\mu_2)^T C_2^{-1} (x-\mu_2) - (x-\mu_1)^T C_1^{-1} (x-\mu_1) + b_0$$

given, $\mu_1 = \begin{bmatrix} 4/3 \\ 4/3 \end{bmatrix}$ $\mu_2 = \begin{bmatrix} 3/2 \\ 1 \end{bmatrix}$

$$C_1 = \frac{1}{3}\left(\begin{bmatrix} 0.1111 & 0.1111 \\ 0.1111 & 0.1111 \end{bmatrix} \begin{bmatrix} 0.4444 & -0.2222 \\ -0.2222 & 0.1111 \end{bmatrix} + \begin{bmatrix} 0.1111 & -0.2222 \\ -0.2222 & 0.4444 \end{bmatrix} \right)$$

$$C_1 = \frac{1}{3}\begin{bmatrix} 0.6666 & -0.3333 \\ -0.3333 & 0.6666 \end{bmatrix} = \begin{bmatrix} 0.2222 & -0.1111 \\ -0.1111 & 0.2222 \end{bmatrix}$$

$$C_2 = \frac{1}{6}\left[\begin{bmatrix} 2.25 & 1.5 \\ 1.5 & 1 \end{bmatrix} + \begin{bmatrix} 0.25 & 0.5 \\ 0.5 & 1 \end{bmatrix} + \begin{bmatrix} 0.25 & -0.5 \\ -0.5 & 1 \end{bmatrix} + \begin{bmatrix} 2.25 & -1.5 \\ -1.5 & 1.0 \end{bmatrix} + \begin{bmatrix} 2.25 & -3.0 \\ -3.0 & 4 \end{bmatrix} + \begin{bmatrix} 2.25 & 3.0 \\ 3.0 & 4 \end{bmatrix} \right]$$

$$\frac{1}{6}\begin{bmatrix} 9.5 & 0 \\ 0 & 12.0 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 1.583 & 0 \\ 0 & 2 \end{bmatrix}$$

$$C_1^{-1} = \begin{bmatrix} 0.222 & 0 \\ 0 & 0.222 \end{bmatrix} \quad C_2 = \begin{bmatrix} 1.583 & 0 \\ 0 & 2 \end{bmatrix}$$

$$C_1^{-1} = \begin{bmatrix} 4.504 & 0 \\ 0 & 4.504 \end{bmatrix} \quad C_2^{-1} = \begin{bmatrix} 0.632 & 0 \\ 0 & 0.5 \end{bmatrix}$$

$$|C_1| = 0.222 \times 0.222 = 0.049$$

$$|C_2| = 1.583 \times 2.0 = 3.166$$

$$d(x) = \frac{-1}{2} \ln(0.049) + \frac{1}{2} (x-\mu_1)^T C_1^{-1} (x-\mu_1) + \frac{1}{2} (x-\mu_2)^T \Sigma_2^{-1} (x-\mu_2)$$

$$d(x) = 1.51 + \frac{1}{2}\left[4.504 \times (x_1 - 1.333)^2 + 4.502 (x_2 - 1.333)^2\right] - \frac{1}{2}\left[0.632 \cdot (x_1 - 1.5)^2 + 0.5 (x_2 - 1.0)^2\right] + 0.575$$