

../ML/online/Q/svm05.stex

% q06 ho07 ho08 h08f h09 h09f h11 ho11f ho12 ho12f ho13s ho13f ho14 ho14s ho14f ho15 ho16 ho17 ho18 ho19

An SVM is trained with the following data:

$i$	1	2	3
$x_i$	$(-1, -1)$	$(1, 1)$	$(0, 2)$
$y_i$	$-1$	$1$	$1$

Let  $\alpha_1, \alpha_2, \alpha_3$  be the Lagrangian multipliers associated with this data. ( $\alpha_i$  is associated with  $(x_i, y_i)$ .)

**A**

Using the polynomial kernel of degree 2, what (dual) optimization problem needs to be solved in terms of the  $\alpha_i$  in order to determine their values?

Reminder: the polynomial kernel of degree 2 is:

$$K(x_i, x_j) = (x_i'x_j + 1)^2$$

**Answer**

The Gram matrix for the linear kernel:  $G = \begin{pmatrix} 2 & -2 & -2 \\ -2 & 2 & 2 \\ -2 & 2 & 4 \end{pmatrix}$

The Gram matrix for the specified kernel:  $G = \begin{pmatrix} 9 & 1 & 1 \\ 1 & 9 & 9 \\ 1 & 9 & 25 \end{pmatrix}$

$$\text{Maximize: } \alpha_1 + \alpha_2 + \alpha_3 - \frac{1}{2} (9\alpha_1^2 - 2\alpha_1\alpha_2 - 2\alpha_1\alpha_3 + 9\alpha_2^2 + 18\alpha_2\alpha_3 + 25\alpha_3^2)$$

$$\text{subject to: } \alpha_1 \geq 0, \alpha_2 \geq 0, \alpha_3 \geq 0, \quad -\alpha_1 + \alpha_2 + \alpha_3 = 0$$

**B**

The solution to the optimization problem is:

$$\alpha_1 = 1/8, \quad \alpha_2 = 1/8, \quad \alpha_3 = 0$$

**a.** What are the indexes of the support vectors? Circle them below.

**Answer:** 1 2

**b.** This SVM classifies the example  $x$  according to the sign of  $w'\phi(x) + b$ , where the transformation  $\phi$  is implicitly defined by the kernel. Compute the value of the constant  $b$ . (This can be done without explicit computation of  $\phi$  or  $w$ .)

**Answer:** Using the first support vector:

$$b = -1 - (1 * (-1) * 9 + 1 * 1 * 1) / 8 = -1 - (-9 + 8/8) / 8 = 0$$

**c.** What computation needs to be carried out to determine the classification of the point  $x = (-1, 0)$  by this SVM?

**Answer:**  $K(x_j, x) = (4, 0)$ .

$$-\frac{1}{8}(4) + \frac{1}{8}(0) < 0$$

Therefore the classification of  $x$  is  $-1$ .

- d. What computation needs to be carried out to determine the classification of the point  $x = (1, 0)$  by this SVM?

**Answer:**  $K(x_j, x) = (0, 4)$ .

$$-\frac{1}{8}(0) + \frac{1}{8}(4) > 0$$

Therefore the classification of  $x$  is +1.