# Homework-8

1. Training data set given ⇒ 9 samples (3+, 6−)

Prior probabilities $P(+) = 3/9$

$P(-) = 6/9$

$\mu_+ = (\frac{4}{3}, \frac{4}{3})^T \quad \mu_- = (\frac{3}{2}, 1)^T$

convariance matrix for the positive class:

$$\Sigma_+ = \begin{bmatrix} 1/3 \Sigma (x_{1i} - \mu_{+1})^2 & 1/3 \Sigma (x_{1i} - \mu_{+1})(x_{2i} - \mu_{+2}) \\ 1/3 \Sigma (x_{1i} - \mu_{+1})(x_{2i} - \mu_{+2}) & 1/3 \Sigma (x_{2i} - \mu_{+2})^2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.222 & -0.111 \\ -0.111 & 0.222 \end{bmatrix}$$

convariance matrix for the negative class:

$$\Sigma_- = \begin{bmatrix} 1/6 \Sigma (x_{1i} - \mu_{-1})^2 & 1/6 \Sigma (x_{1i} - \mu_{-1})(x_{2i} - \mu_{-2}) \\ 1/6 \Sigma (x_{1i} - \mu_{-1})(x_{2i} - \mu_{-2}) & 1/6 \Sigma (x_{2i} - \mu_{-2})^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1.25 & 0.75 \\ 0.75 & 1.5 \end{bmatrix}$$

$\sigma_+^2 = 1/2 (0.222 + 0.222) = 0.222$

$\sigma_-^2 = 1/2 (1.25 + 1.5) = 1.375$

$$g(x) = \ln(1/2) - 2/2 \ln\left(\frac{0.222}{1.375}\right) - \frac{1}{2(0.222)} \left[(x_1 - 4/3)^2 + (x_2 - 4/3)^2\right] + \frac{1}{2(1.375)} \left[(x_1 - 3/2)^2 + (x_2 - 1)^2\right]$$

$$= -0.693 + 1.824 - 2.252 \left[(x_1 - 4/3)^2 + (x_2 - 4/3)^2\right] + 0.364 \left[(x_1 - 3/2)^2 + (x_2 - 1)^2\right]$$

Pooled convariance matrix:

$$\Sigma = \frac{3}{9} \Sigma_+ + \frac{6}{9} \Sigma_- = \begin{bmatrix} 0.907 & 0.510 \\ 0.510 & 1.074 \end{bmatrix}$$

$$\Sigma^{-1} = \begin{bmatrix} 1.254 & -0.595 \\ -0.595 & 1.059 \end{bmatrix}$$

$$g(x) = \ln(1/2) + \left(\frac{4}{3} - \frac{3}{3} - \frac{4}{3} - 1\right) \begin{bmatrix} 1.254 & -0.595 \\ -0.595 & 1.059 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$- \frac{1}{2}\left(\frac{4}{3}, \frac{4}{3}\right) \begin{bmatrix} 1.254 & -0.595 \\ -0.595 & 1.059 \end{bmatrix} \begin{bmatrix} 4/3 \\ 4/3 \end{bmatrix}$$

$$+ \frac{1}{2}\left(\frac{3}{2}, 1\right) \begin{bmatrix} 1.254 & -0.595 \\ -0.595 & 1.059 \end{bmatrix} \begin{bmatrix} 3/2 \\ 1 \end{bmatrix}$$

General Guassian with equal priors:

$$\Sigma_+^{-1} = \begin{bmatrix} 5.143 & 2.571 \\ 2.571 & 5.143 \end{bmatrix} \quad \Sigma_-^{-1} = \begin{bmatrix} 1.034 & -0.517 \\ -0.517 & 0.862 \end{bmatrix}$$

$$g(x) = -1/2 \ln |0.222^2| + 1/2 \ln |1.375^2| - 1/2 (x - \mu_+)^T \Sigma_+^{-1} (x - \mu_5) + 1/2 (x - \mu_-)^T \Sigma^{-1} (x - \mu_-).$$