Homework 2

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```r
library(R2jags)
library(lattice) # Needed for scatterplot matrix
# Set working directory
setwd("/home/gsk/grad/courses/UCLA/biost234/homework/lab2")
getwd()
```

```
## [1] "/home/gsk/grad/courses/UCLA/biost234/homework/lab2"
```

```r
# Save the data file to your working directory.

#READ IN DATA
housing=read.table("housingdata2.txt")
head(housing)   ## Always look at your data
```

```
##       V1   V2   V3 V4 V5
## 1 15.783 3.00 2.00  2  2
## 2 12.570 1.66 2.33  3  2
## 3 19.600 3.33 2.33  2  2
## 4  8.206 1.66 1.66  2  2
## 5 15.333 2.33 2.33  5  2
## 6 14.955 5.00 3.00  2  2
```

```r
## Anything funny about any of the columns?
## There are no column names!
str(housing)
```

```
## 'data.frame':    21 obs. of  5 variables:
##  $ V1: num  15.78 12.57 19.6 8.21 15.33 ...
##  $ V2: num  3 1.66 3.33 1.66 2.33 5 4.33 2.33 1.33 3 ...
##  $ V3: num  2 2.33 2.33 1.66 2.33 3 3 2.33 1.66 2.66 ...
##  $ V4: int  2 3 2 2 5 2 2 3 2 2 ...
##  $ V5: int  2 2 2 2 2 2 2 2 2 2 ...
```

```r
colnames(housing) <- c("cost", "eaves", "windows", "yard", "roof")

#SEPARATE X & Y
y <- housing[,1]
x <- as.matrix(housing[,2:5])

## Remember: look at your data.
y
```

```
##  [1] 15.783 12.570 19.600  8.206 15.333 14.955 13.710 11.388  4.802 12.547
## [11] 13.677  9.683 16.798 25.615 15.734 13.510 13.855  3.986  5.997  9.778
## [21] 10.152
```

```r
head(x)
```

```
##      eaves windows yard roof
## [1,]  3.00    2.00    2    2
## [2,]  1.66    2.33    3    2
## [3,]  3.33    2.33    2    2
## [4,]  1.66    1.66    2    2
## [5,]  2.33    2.33    5    2
## [6,]  5.00    3.00    2    2
```

Classical regression

Before beginning our Bayesian analysis, we can conduct classical multiple linear regression:

```r
reg = lm(y~x)
summary(reg) # classical regression
```

```
##
## Call:
## lm(formula = y ~ x)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.8827 -1.8219 -0.9953  1.3467  7.4674
##
## Coefficients: (1 not defined because of singularities)
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -7.5055     3.9087  -1.920  0.07177 .
## xeaves        1.8981     0.9868   1.923  0.07133 .
## xwindows      3.5310     1.6703   2.114  0.04960 *
## xyard         2.5450     0.8721   2.918  0.00958 **
## xroof             NA         NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.275 on 17 degrees of freedom
## Multiple R-squared:  0.6318, Adjusted R-squared:  0.5668
## F-statistic: 9.722 on 3 and 17 DF,  p-value: 0.0005765
```

The yard and window quality seem to correlate with the contractor's estimates. We see a row of NAs for coefficients of roof, since all of the roofs in this dataset had a quality of 2- so, there is no variation in that measurement.

Bayesian approach

First we write out our model

```r
# sink("housingmodel.txt")
cat("
model
{
```

```
   for(i in 1:N) {
         y[i] ~ dnorm(mu[i] , tau )
         mu[i] <- beta0 + inprod(x[i,] , beta[] )
        }

     beta0 ~ dnorm( mbeta0 , precbeta0) # mean and precision of beta0 (intercept) defined in R

for (j in 1:K) {
     beta[j] ~ dnorm( m[j] , prec[j] ) # the prior for the four different betas will be provided throug
                                     # in all models we will use mean beta values estimated in an old a

        }
       tau ~ dgamma( tau.a , tau.b )  # we tend to use gamma for tau because it is always non-negative
                                    # we will weigh the priors differently in a few different models by
                                    # fidgeting with the precision in prior
       sigma <- 1 / sqrt(tau)         # note that we use <- here since it's a simple calculation
    }
  ",fill = TRUE)
# sink()
```

Now, we set up three different models:

```
# define some variables
N = nrow(housing)
K = ncol(x) # number of coefficients to estimate betas for
m = c(1.6053, 1.2556, 2.3413, 3.6771)# mean betas from previous analysis

dataA <- list(N=N, K=4, m = m,
              prec = c(.2164, .1105, .2061, .1337), tau.a=17,
              tau.b = 1128, mbeta0= -5.682, precbeta0=.05464, x=x, y=y)

dataB <- list(N=N, K=4, m=m,
            prec=c(.02774, .014160, .02642, .01714), tau.a=2.1795,
            tau.b=144.6, mbeta0= -5.682, precbeta0=.007005, x=x, y=y)

dataC <- list(N=N, K=4, m=m,
            prec=c(.005549, .002832, .005284, .003428), tau.a=.4359,
            tau.b=28.92, mbeta0= -5.682, precbeta0=.00140, x=x, y=y)


inits <- rep(list(list(beta0=0, beta=c(1,1,1,1),tau=1)),5) # 5 equal to the n.chains in jags call

#DEFINE PARAMETERS TO MONITOR
parameters <- c("beta0", "beta" , "tau", "sigma")
```

Then we run jags:

```
#RUN THE JAGS PROGRAM, SAVING DATA TO LAB2.SIM
lab2.simA <- jags (dataA, inits, parameters, "housingmodel.txt", n.chains=5,
    n.iter=5100, n.burnin=100, n.thin=1, DIC=FALSE) # DIC = F - deviance not calcuated
```

```
## module glm loaded
```

```
## module dic loaded

## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 21
##    Unobserved stochastic nodes: 6
##    Total graph size: 191
##
## Initializing model
```

```r
lab2.simB <- jags (dataB, inits, parameters, "housingmodel.txt", n.chains=5,
    n.iter=5100, n.burnin=100, n.thin=1, DIC=FALSE)
```

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 21
##    Unobserved stochastic nodes: 6
##    Total graph size: 191
##
## Initializing model
```

```r
lab2.simC <- jags (dataC, inits, parameters, "housingmodel.txt", n.chains=5,
    n.iter=5100, n.burnin=100, n.thin=1, DIC=FALSE)
```

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 21
##    Unobserved stochastic nodes: 6
##    Total graph size: 191
##
## Initializing model
```

```r
lab2.sims = list(lab2.simA, lab2.simB, lab2.simC)

knitr::kable(lab2.simA$BUGSoutput$summary[,c("mean", "sd", "2.5%", "97.5%")], digits = 3,
             caption = "Parameter estimates from Model A (informative)")
```

Table 1: Parameter estimates from Model A (informative)

	mean	sd	2.5%	97.5%
beta[1]	1.779	1.359	-0.864	4.433
beta[2]	1.777	1.969	-2.119	5.604
beta[3]	2.235	1.315	-0.388	4.802
beta[4]	2.602	2.238	-1.771	6.990
beta0	-6.999	3.835	-14.504	0.479
sigma	6.947	0.694	5.744	8.459
tau	0.021	0.004	0.014	0.030

```
knitr::kable(lab2.simB$BUGSoutput$summary[,c("mean", "sd", "2.5%", "97.5%")], digits = 3,
             caption = "Parameter estimates from Model B (medium informative)")
```

Table 2: Parameter estimates from Model B (medium informative)

	mean	sd	2.5%	97.5%
beta[1]	1.926	1.406	-0.838	4.698
beta[2]	3.193	2.348	-1.402	7.788
beta[3]	2.489	1.274	-0.039	4.964
beta[4]	1.115	5.001	-8.644	11.006
beta0	-8.781	9.669	-27.653	10.225
sigma	4.849	0.777	3.620	6.624
tau	0.046	0.014	0.023	0.076

```
knitr::kable(lab2.simC$BUGSoutput$summary[,c("mean", "sd", "2.5%", "97.5%")], digits = 3,
             caption = "Parameter estimates from Model C (uninformative)")
```

Table 3: Parameter estimates from Model C (uninformative)

	mean	sd	2.5%	97.5%
beta[1]	1.881	1.164	-0.447	4.163
beta[2]	3.489	1.973	-0.397	7.397
beta[3]	2.532	1.035	0.495	4.578
beta[4]	0.827	10.736	-20.226	21.812
beta0	-8.980	21.333	-50.863	32.693
sigma	3.832	0.681	2.776	5.414
tau	0.074	0.025	0.034	0.130

Question 1

1. Summarize briefly the effects on all parameters of changing from prior A to B to C.

For all priors, changing from A to B to C brought the point estimate (mean of posterior) closer to the likelihood estimates reported at the beginning of this document and increased the magnitude of the credibility intervals.

Question 2

2. Give a table of inferences for the coefficient of roofs for the three priors. Briefly explain why it comes out as it does.

```
temp3 <- t(sapply(lab2.sims, function(x) x$BUGSoutput$summary["beta[4]",
                                                              c("mean", "sd", "2.5%", "97.5%")]))
rownames(temp3) <- c("Model A", "Model B", "Model C")
knitr::kable(temp3, caption = "Estimates of Roof coefficient under three priors")
```

Table 4: Estimates of Roof coefficient under three priors

	mean	sd	2.5%	97.5%
Model A	2.6015237	2.238464	-1.770660	6.990348
Model B	1.1145502	5.001253	-8.643814	11.005742
Model C	0.8265005	10.735822	-20.226143	21.812401

In our dataset there is no variation in the roof parameter across samples, so as we place less and less emphasis on our prior (moving from models A to B to C), our estimate of β_{roof} gets wider and more centered on zero.

Question 3

3. For one of the three priors:

b. Which house is in the worst condition? Calculate the three futurefit, futureobs and futuretail variables for this house and provide a formatted table.

First we write a new model:

```
# sink("housingmodelq3.txt", eval = F)
cat("
model
{
   for(i in 1:N) {
         y[i] ~ dnorm(mu[i] , tau )
         mu[i] <- beta0 + inprod(x[i,] , beta[] )
        }

     beta0 ~ dnorm( mbeta0 , precbeta0) # mean and precision of beta0 (intercept) defined in R
```

```
for (j in 1:K) {
     beta[j] ~ dnorm( m[j] , prec[j] ) # the prior for the four different betas will be provided throug
                                     # in all models we will use mean beta values estimated in an old a

        }
       tau ~ dgamma( tau.a , tau.b )  # we tend to use gamma for tau because it is always non-negative
                                    # we will weigh the priors differently in a few different models by
                                    # fidgeting with the precision in prior
       sigma <- 1 / sqrt(tau)         # note that we use <- here since it's a simple calculation
    futurefit <- beta0 + beta[1]*fut[1] + beta[2]*fut[2] + beta[3]*fut[3] + beta[4]*fut[4]
        # note that I made fut[] be an input vector so we don't need to rewrite
        # new models every time we want to fit future for a different house
    futureobs ~ dnorm(futurefit, tau)
    futuretail <- beta0 + beta[1]*fut[1] + beta[2]*fut[2] + beta[3]*fut[3] + beta[4]*fut[4] + 1.645*sig
    }
  ",fill = TRUE)
# sink()
```

I will run this with Model A (most informative prior)

```
inits <- rep(list(list(beta0=0, beta=c(1,1,1,1),tau=1, futureobs = 0)),5) # 5 equal to the n.chains in
parameters <- c("beta0", "beta" , "tau", "sigma", "futurefit", "futureobs", "futuretail")
dataA <- list(N=N, K=4, m = m,
              prec = c(.2164, .1105, .2061, .1337), tau.a=17,
              tau.b = 1128, mbeta0= -5.682, precbeta0=.05464, x=x, y=y,
              fut = c(1,1,2,2)) # added a futures line here for the model

lab2.simq3 <- jags (dataA, inits, parameters, "housingmodelq3.txt", n.chains=5,
    n.iter=5100, n.burnin=100, n.thin=1, DIC=FALSE) # DIC = F - deviance not calcuated
```

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 21
##    Unobserved stochastic nodes: 7
##    Total graph size: 206
##
## Initializing model
```

a. Show summaries of the futurefit, futureobs, futuretail in a properly formatted table for the house in perfect condition.

```
futures <- c("futurefit", "futureobs", "futuretail")
for_table <- c("mean", "sd", "2.5%", "97.5%")
knitr::kable(lab2.simq3$BUGSoutput$summary[futures, for_table],
             caption = "Summary of predictions for a perfect house")
```

Table 5: Summary of predictions for a perfect house

	mean	sd	2.5%	97.5%
futurefit	6.205960	2.922440	0.4966809	12.00728
futureobs	6.213878	7.553578	-8.6591682	20.96878
futuretail	17.609937	3.204356	11.5149895	24.11950

We see that the best house is predicte to cost ~$6000 to repair, unlike the "perfect" house in the dataset, which was estimated to cost $18000

b. Which house is in the worst condition? Calculate the three futurefit, futureobs and futuretail variables for this house and provide a formatted table.
   By one measure, house 14 is the "worst", as it has the highest average score. Its scores are:

```
x[14,]
```

```
##   eaves windows    yard    roof
##    3.00    3.33    4.00    2.00
```

```
dataA <- list(N=N, K=4, m = m,
              prec = c(.2164, .1105, .2061, .1337), tau.a=17,
              tau.b = 1128, mbeta0= -5.682, precbeta0=.05464, x=x, y=y,
              fut = x[14,]) # futures for house 14
lab2.simq3b <- jags (dataA, inits, parameters, "housingmodelq3.txt", n.chains=5,
    n.iter=5100, n.burnin=100, n.thin=1, DIC=FALSE) # DIC = F - deviance not calcuated
```

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 21
##    Unobserved stochastic nodes: 7
##    Total graph size: 206
##
## Initializing model
```

```
knitr::kable(lab2.simq3b$BUGSoutput$summary[futures, for_table],
             caption = "Summary of predictions for the worst house")
```

Table 6: Summary of predictions for the worst house

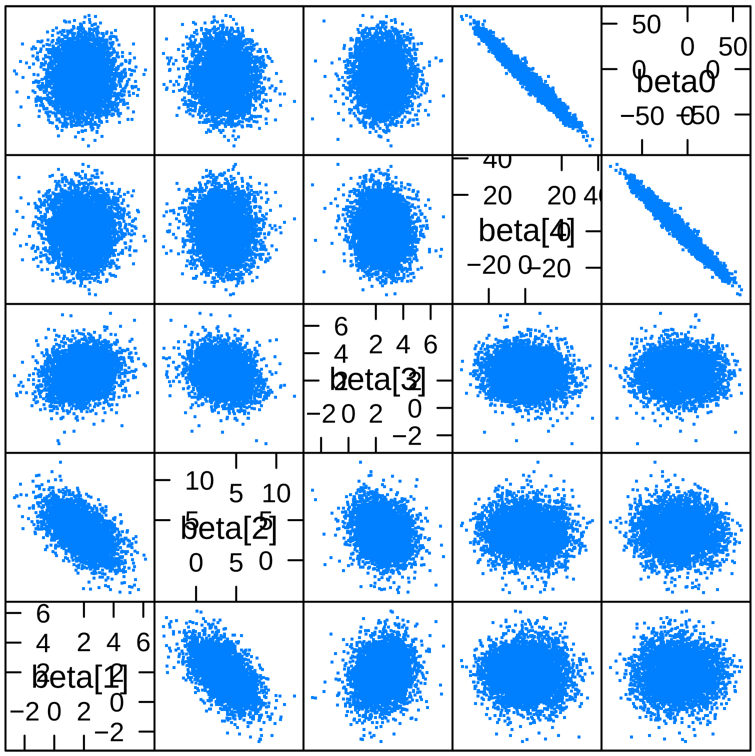
	mean	sd	2.5%	97.5%
futurefit	18.36606	2.702114	13.053340	23.63679
futureobs	18.36468	7.478619	3.545757	33.04996
futuretail	29.78087	2.896599	24.288192	35.69236

The worst house is predicted to cost ~$18000 to repair.

Question 4

4. For prior (C), what two coefficients (including the intercept) have the highest posterior correlation? Briefly explain why.

```
temp = apply(lab2.simC$BUGSoutput$sims.array, 3, unlist)
lattice::splom(temp[1:5000,1:5],pch=".")     #   Scatterplot matrix of correlation plots
```



Scatter Plot Matrix

```
            # 25000 takes a long time. "." is a better character to plot with
```

We can see in the plot above that beta0 and beta[4] (i.e. the estimates for intercept and coefficient for roof, respectively) are highly correlated with Prior C. Since we don't have any variation in roof quality in our dataset, that value is pretty much treated as a constant number in this analysis, so that any combination of $\beta_4 * roof + \beta_0$ yields the same constant in the regression model. We see the strong negative correlation between β_0 and β_4 for this reason.

Question 5

5. Briefly interpret the three variables futurefit, futureobs, futuretail in your own words.

`futurefit` is the mean of the distribution that is used to estimate the cost to fix some unknown house.
`futureobs` is a sample from the distribution of predictions for the cost estimate; the distribution has mean `futurefit` and precision τ.
`futuretail` is the upper limit of the credible interval around the value reported by `futureobs`; we can similarly calculate the lower bound of the interval.

Question 6

6. Suppose we pool the two data sets after the inflation correction. Also, the expert at the housing department told you he thought each unit increase in any rating scale ought to increase the cost by around $1000. You're not sure that all coefficients should be positive. Suggest priors (all regression coefficients and for sigma^2) to use now. Write one or two sentences justifying your priors.

- Intercept β_0 ~No (-5.682, .007005)
- EAVES: β_1 ~No (1, 1)
- WINDOWS: β_2 ~No (1, 1)
- YARDS: β_3 ~No (1, 1)
- ROOF: β_4 ~No (1, 1)
- Tau: τ ~Gamma(17, 1128)

Since the local housing expert thinks that each unit increase in any of the scales results in a ~$1000 increase in repair cost, I use 1 as the prior mean for the coefficients $beta_0$, $beta_1$, $beta_2$, and $beta_3$ in this scenario. I also set the precision to 1 so that ~85% of my prior distribution's density is greater than zero, but there is still a fair amount of density in the priors for negative coefficients. I am not very confident about my prior knowledge for an intercept, so I will stick with an uninformative prior with mean 0 and precision 0.001; this will let the data influence the posterior distribution. Assuming that we double our dataset to have 80 samples now, I am proposing a prior for tau of being gamma-distributed with shape 37 (= 76 degrees of freedom/2) and scale 2434 (so that the mean $\approx$ 66).