February 7, 2016

EEB 200B Ecology

Assignment 1: Analysis of invisibility and coexistence

1. Consider the Lotka-Volterra model for two competing species:

$$\begin{aligned}\frac{dN_1}{dt} &= r_1 N_1\left(1-\alpha_{11}N_1-\alpha_{12}N_2\right)\\ \frac{dN_2}{dt} &= r_2 N_2\left(1-\alpha_{22}N_2-\alpha_{21}N_1\right).\end{aligned} \tag{1}$$

(i) Solve Equation (1) for the coexistence equilibrium. Show algebraically that there are two situations under which the coexistence equilibrium is feasible. Also show that the two cases correspond to stable coexistence and priority effects respectively.

(ii) Construct the Jacobian matrix for the Lotka-Volterra model.

(iii) Derive the Routh-Hurwitz criteria for the priority effects case. Compare your results with those you obtained in class for the stable coexistence case. Point out the major differences and what they imply about long-term coexistence.

(iv) Derive an algebraic expression for the eigenvalues when competition involves priority effects. (Note: you need to express the eigenvalues in terms of r_i, α and $N_i^\star$, $i = (1, 2)$.) Show that the two eigenvalues have opposite signs.

(v) Using the algebraic expression you derived for the eigenvalues, show that local stability of the coexistence equilibrium requires both eigenvalues to be negative.

2. The following is a model for two species engaged in a mutualistic interaction:

$$\begin{aligned}\frac{dN_1}{dt} &= r_1 N_1\left(1-\alpha_{11}N_1+m_{12}N_2\right)\\ \frac{dN_2}{dt} &= r_2 N_2\left(1-\alpha_{22}N_2+m_{21}N_1\right).\end{aligned} \tag{2}$$

All state variables and parameters are the same as in the Lotka-Volterra competition model except for m_{ij} which describe the per capita mutualistic effect that species j has on species i.

(i) Solve for equilibria. Identify the boundary and interior (coexistence) equilibria.

(ii) Compute the invasion criterion for each species. Show that each species can always increase when rare when its mutualist is abundant.

(ii) Construct the Jacobian matrix. Use the trick the I showed you in class to simplify the elements.

(iii) Compare the Jacobian matrix with the one you constructed for the competition model. What major difference(s) do you see?

(iv) Using Routh-Hurwitz criteria, evaluate the local stability of the coexistence equilibrium.

(v) What is the biological interpretation of the stability criterion/criteria you derived in (iv)?

(vi) What is the general significance of the biological insights you mentioned in (v)? (Hint: think of the general principle about species coexistence I mentioned at the beginning of my section.)