AI1103: Assignment 1

Tanmay Garg
CS20BTECH11063 EE20BTECH11048

Download all python codes from

```
https://github.com/tanmaygar/AI−Course/blob/main
/Assignment1/codes/Assignment1.py
```

and latex-tikz codes from

```
https://github.com/tanmaygar/AI−Course/blob/main
/Assignment1/Assignment1.tex
```

Problem Statement:

An urn contains 25 balls of which 10 balls bear a mark ’X’ and the remaining 15 bear a mark ’Y’. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that:

1) all will bear ’X’ mark.
2) not more than 2 will bear ’Y’ mark.
3) at least one ball will bear ’Y’ mark.
4) the number of balls with ’X’ mark and ’Y’ mark will be equal.

Solution:

Let X be the number of balls which have ’X’ mark on them
Using the expression of binomial distribution

$$P(X = r) = \binom{n}{r} p^r q^{n-r}$$

We have

$$n = 6$$

$$p = \frac{2}{5} = 0.4$$

$$q = \frac{3}{5} = 0.6$$

For (i) we need to find $P(X = 6)$

$$P(X = 6) = \binom{6}{6} p^6 q^0 = \left(\frac{2}{5}\right)^6 = 0.004096$$

For (ii) we need to find $P(X \geq 4)$

$$P(X \geq 4) = \sum_{r=4}^{6} \binom{6}{r} p^r q^{n-r}$$

$$= \binom{6}{4}\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^2 + \binom{6}{5}\left(\frac{2}{5}\right)^5\left(\frac{3}{5}\right) + \binom{6}{6}\left(\frac{2}{5}\right)^6$$

$$= \frac{432}{3125} + \frac{576}{15625} + \frac{64}{15625}$$

$$= \frac{112}{625} = 0.1792$$

For (iii) we need to find $P(X \leq 5)$

$$P(X \leq 5) = \sum_{r=0}^{5} \binom{6}{r} p^r q^{n-r}$$

$$= \binom{6}{0}\left(\frac{3}{5}\right)^6 + \binom{6}{1}\left(\frac{2}{5}\right)^1\left(\frac{3}{5}\right)^5 + \binom{6}{2}\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)^4 + \binom{6}{3}\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^3$$

$$+ \binom{6}{4}\left(\frac{2}{5}\right)^4\left(\frac{3}{5}\right)^2 + \binom{6}{5}\left(\frac{2}{5}\right)^5\left(\frac{3}{5}\right)$$

$$= \frac{729}{15625} + \frac{2916}{15625} + \frac{972}{3125} + \frac{864}{3125} + \frac{432}{3125} + \frac{576}{15625}$$

$$= \frac{15561}{15625} = 0.995904$$

For (iv) we need to find $P(X = 3)$

$$P(X = 3) = \binom{6}{3} p^3 q^3$$

$$= \binom{6}{3}\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^3 = 0.27648$$