

# AI1103: Assignment 2

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PROBLEM GATE-EC-2019-Q47:

A random variable  $X$  takes values  $-1$  and  $+1$  with probabilities  $0.2$  and  $0.8$ , respectively. It is transmitted across a channel which adds noise  $N$ , so that the random variable at the channel output is  $Y = X + N$ . The noise  $N$  is independent of  $X$ , and is uniformly distributed over the interval  $[-2, 2]$ . The receiver makes a decision

$$\hat{X} = \begin{cases} -1, & \text{if } Y \leq \theta \\ +1, & \text{if } Y \geq \theta \end{cases}$$

where the threshold  $\theta \in [-1, 1]$  is chosen so as to minimize the probability of error  $\Pr(\hat{X} \neq X)$ . The minimum probability of error, rounded off to 1 decimal place, is?

SOLUTION:

We know that

$$X = -1, +1 \quad (0.0.1)$$

$$N \in [-2, 2] \quad (0.0.2)$$

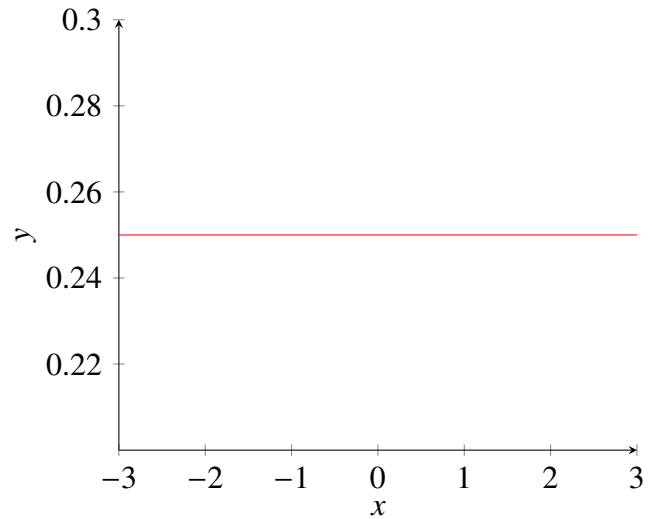
$$Y = X + N \quad (0.0.3)$$

$$P(X = -1) = 0.2 \quad (0.0.4)$$

$$P(X = +1) = 0.8 \quad (0.0.5)$$

Since  $N$  is uniformly distributed

$\therefore$  the probability distribution function of  $N$  is:



For  $X = \hat{X}$  we need to check for each case

$$\therefore P(\theta < -1 + N) = \int_{\theta+1}^{-2} \frac{1}{4} da \quad (0.0.6)$$

$$= \frac{1}{4}(1 - \theta) \quad (0.0.7)$$

$$\therefore P(\theta > N + 1) = \int_{-2}^{\theta-1} \frac{1}{4} da \quad (0.0.8)$$

$$= \frac{1}{4}(1 + \theta) \quad (0.0.9)$$

The probability of error:

$$P_e = P(-1) \cdot P(\theta < -1 + N) + P(1) \cdot P(\theta > N + 1) \quad (0.0.10)$$

Substituting (0.0.7) and (0.0.9) in (0.0.10). We get:

$$P_e = 0.2 \cdot \frac{1}{4}(1 - \theta) + 0.8 \cdot \frac{1}{4}(1 + \theta) \quad (0.0.11)$$

Putting  $\theta = -1$ , we get

$$P_e = 0.1 \quad (0.0.12)$$