AI1103: Assignment 2

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Problem GATE-EC-2019-Q47:

A random variable X takes values -1 and +1 with probabilities 0.2 and 0.8, respectively. It is transmitted across a channel which adds noise N, so that the random variable at the channel output is $Y = X + N$. The noise N is independent of X, and is uniformly distributed over the interval [-2 , 2]. The receiver makes a decision

$$\hat{X} = \begin{cases} -1, & \text{if} \quad Y \le \theta \\ +1, & \text{if} \quad Y \ge \theta \end{cases}$$

where the threshold $\theta \in [-1, 1]$ is chosen so as to minimize the probability of error $\Pr\left(\hat{X} \neq X\right)$. The minimum probability of error, rounded off to 1 decimal place, is?

Solution:

We know that

$$X = -1, +1 \tag{0.0.1}$$

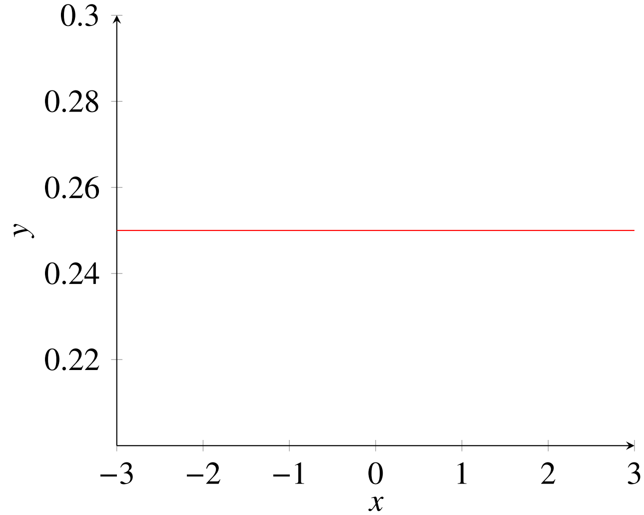
$$N \in [-2, 2] \tag{0.0.2}$$

$$Y = X + N \tag{0.0.3}$$

$$P(X = -1) = 0.2 \tag{0.0.4}$$

$$P(X = +1) = 0.8 \tag{0.0.5}$$

Since N is uniformly distributed
$\therefore$ the probability distribution function of N is:



For $X = \hat{X}$ we need to check for each case

$$\therefore P(\theta < -1 + N) = \int_{\theta+1}^{2} \frac{1}{4} dN \tag{0.0.6}$$

$$= \frac{1}{4}(1 - \theta) \tag{0.0.7}$$

$$\therefore P(\theta > N + 1) = \int_{-2}^{\theta-1} \frac{1}{4} dN \tag{0.0.8}$$

$$= \frac{1}{4}(1 + \theta) \tag{0.0.9}$$

The probability of error:

$$\Pr\left(\hat{X} \neq X\right) = P(-1) \cdot P(\theta < -1 + N) + P(1) \cdot P(\theta > N + 1) \tag{0.0.10}$$

Substituting (0.0.7) and (0.0.9) in (0.0.10). We get:

$$\Pr\left(\hat{X} \neq X\right) = 0.2 \cdot \frac{1}{4}(1 - \theta) + 0.8 \cdot \frac{1}{4}(1 + \theta) \tag{0.0.11}$$

Putting $\theta = -1$, we get

$$\Pr\left(\hat{X} \neq X\right) = 0.1 \tag{0.0.12}$$