AI1103: Assignment 4

Tanmay Garg
CS20BTECH11063 EE20BTECH11048

Download all python codes from

https://github.com/tanmaygar/AI−Course/blob/main/Assignment4/codes/GATE−2015(CS−SET−3)−Q37.py

and latex-tikz codes from

https://github.com/tanmaygar/AI−Course/blob/main/Assignment4/Assignment4.tex

PROBLEM GATE 2015(CS-SET 3), Q.37:

Suppose X_i for $i = 1, 2, 3$ are independent and identically distributed random variables whose probability mass functions are $\Pr(X_i = 0) = \Pr(X_i = 1) = \frac{1}{2}$ for $i = 1, 2, 3$. Define another random variable $Y = X_1X_2 \oplus X_3$, where $\oplus$ denotes XOR. Then $\Pr(Y = 0|X_3 = 0) =$

SOLUTION:

We know that

$$\Pr(Y = 0|X_3 = 0) = \frac{\Pr(Y = 0, X_3 = 0)}{\Pr(X_3 = 0)} \tag{0.0.1}$$

$$\Pr(X_3 = 0) = \frac{1}{2} \tag{0.0.2}$$

For

$$\because Y = (X_1X_2) \oplus X_3 = 0 \tag{0.0.3}$$

$$\implies X_1X_2 = X_3 \tag{0.0.4}$$

$$\because X_3 = 0, \quad \therefore X_1X_2 = 0 \tag{0.0.5}$$

The random variables are independent of each other:

$\Pr(X_1 = 0, X_2 = 0)$	$\Pr(X_1 = 0) \cdot \Pr(X_2 = 0)$	0.25
$\Pr(X_1 = 1, X_2 = 0)$	$\Pr(X_1 = 1) \cdot \Pr(X_2 = 0)$	0.25
$\Pr(X_1 = 0, X_2 = 1)$	$\Pr(X_1 = 0) \cdot \Pr(X_2 = 1)$	0.25

TABLE 0: Probabilities

$$\Pr(X_1X_2 = 0) = \Pr(X_1 = 0, X_2 = 0) + \Pr(X_1 = 0, X_2 = 1) + \Pr(X_1 = 1, X_2 = 0) \tag{0.0.6}$$

$$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4} \tag{0.0.7}$$

$$\Pr(Y = 0, X_3 = 0) = \Pr(X_1X_2 = X_3 = 0) \tag{0.0.8}$$

$$= \Pr(X_1X_2 = 0) \cdot \Pr(X_3 = 0) \tag{0.0.9}$$

$$= \frac{3}{4} \cdot \frac{1}{2} \tag{0.0.10}$$

$$= \frac{3}{8} \tag{0.0.11}$$

Upon substituting (0.0.11) and (0.0.2) in (0.0.1)

$$\Pr(Y = 0|X_3 = 0) = \frac{3}{4} = 0.75 \tag{0.0.12}$$