AI1103: Assignment 2

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Download all python codes from

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need
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and latex-tikz codes from

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https://github.com/tanmaygar/AI−Course/blob/main
    /Assignment3/Assignment3.tex
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Problem GATE-EC-2019-Q47:

Let X and Y be jointly distributed random variables having the joint probability density function

$$f(x,y) = \begin{cases} \frac{1}{\pi}, & \text{if} \quad x^2+y^2 \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Then $P(Y > \max(X, -X))$ is

Solution:

Since we require $Y > \max(X, -X)$

$$\max(X, -X) = |X| \tag{0.0.1}$$

We need to find

$$\Pr(Y > |X|) \tag{0.0.2}$$

Integrating the probability distribution function on our domain

$$\Pr(Y > |X|) = \iint_{\{y>|x|\}} f(x,y)\,dx\,dy \tag{0.0.3}$$

$$= \iint_{\{y>|x|,\ x^2+y^2\leq 1\}} \frac{1}{\pi}\,dx\,dy \tag{0.0.4}$$

$$+ \iint_{\{y>|x|,\ x^2+y^2>1\}} 0\,dx\,dy \tag{0.0.5}$$

$$= \frac{1}{\pi} \iint_{\{y>|x|,\ x^2+y^2\leq 1\}} dx\,dy \tag{0.0.6}$$

On converting to r and θ form, we get:

$$\Pr(Y > |X|) = \frac{1}{\pi} \int_{\pi/4}^{3\pi/4} \int_0^1 r\,dr\,d\theta \tag{0.0.7}$$

$$= \frac{1}{\pi} \int_{\pi/4}^{3\pi/4} \left(\frac{1}{2}r^2\right)\Big|_0^1 d\theta \tag{0.0.8}$$

$$= \frac{1}{\pi} \int_{\pi/4}^{3\pi/4} \frac{1}{2} d\theta \tag{0.0.9}$$

$$= \frac{1}{2\pi}\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) \tag{0.0.10}$$

$$= \frac{1}{2\pi} \cdot \frac{\pi}{2} \tag{0.0.11}$$

$$= \frac{1}{4} \tag{0.0.12}$$