AI1103: Assignment 4

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Download all python codes from

https://github.com/tanmaygar/AI−Course/blob/main/Assignment4/codes/GATE−2015(CS−SET−3)−Q37.py

and latex-tikz codes from

https://github.com/tanmaygar/AI−Course/blob/main/Assignment4/Assignment4.tex

Problem GATE 2015(CS-SET 3), Q.37:

Suppose X_i for $i = 1,2,3$ are independent and identically distributed random variables whose probability mass functions are $\Pr(X_i = 0) = \Pr(X_i = 1) = \frac{1}{2}$ for $i = 1,2,3$. Define another random variable $Y = X_1X_2 \oplus X_3$, where $\oplus$ denotes XOR. Then $\Pr(Y = 0|X_3 = 0) =$

Solution:

We know that

$$\Pr(Y = 0|X_3 = 0) = \frac{\Pr(Y = 0, X_3 = 0)}{\Pr(X_3 = 0)} \tag{0.0.1}$$

$$\Pr(X_3 = 0) = \frac{1}{2} \tag{0.0.2}$$

For

$$Y = 0 \tag{0.0.3}$$

$$X_1X_2 \oplus X_3 = 0 \tag{0.0.4}$$

$$\because X_3 = 0, \quad \therefore X_1X_2 = 0 \tag{0.0.5}$$

Since the random variables are independent of each other

$$\Pr\left(X_i = a, X_j = b\right) = \Pr(X_i = a) \cdot \Pr\left(X_j = b\right) \tag{0.0.6}$$

$$i \neq j \tag{0.0.7}$$

$$a, b \in \{0, 1\} \tag{0.0.8}$$

$$i, j \in \{1, 2, 3\} \tag{0.0.9}$$

$\Pr(X_1 = 0, X_2 = 0)$	$\Pr(X_1 = 0) \cdot \Pr(X_2 = 0)$	0.25
$\Pr(X_1 = 1, X_2 = 0)$	$\Pr(X_1 = 1) \cdot \Pr(X_2 = 0)$	0.25
$\Pr(X_1 = 0, X_2 = 1)$	$\Pr(X_1 = 0) \cdot \Pr(X_2 = 1)$	0.25

TABLE 0: Probabilities

$$\begin{aligned}\Pr(X_1X_2 = 0) = \Pr(X_1 = 0, X_2 = 0) \\ + \Pr(X_1 = 0, X_2 = 1) \\ + \Pr(X_1 = 1, X_2 = 0)\end{aligned} \tag{0.0.10}$$

$$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4} \tag{0.0.11}$$

$$\Pr(Y = 0, X_3 = 0) = \Pr(X_1X_2 = 0) \cdot \Pr(X_3 = 0) \tag{0.0.12}$$

$$= \frac{3}{4} \cdot \frac{1}{2} \tag{0.0.13}$$

$$= \frac{3}{8} \tag{0.0.14}$$

Upon substituting (0.0.14) and (0.0.2) in (0.0.1)

$$\Pr(Y = 0|X_3 = 0) = \frac{3}{4} = 0.75 \tag{0.0.15}$$