

# AI1103: Assignment 2

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Download all python codes from

[https://github.com/tanmaygar/AI-Course/blob/main/Assignment2/codes/problem\\_47.py](https://github.com/tanmaygar/AI-Course/blob/main/Assignment2/codes/problem_47.py)

and latex-tikz codes from

<https://github.com/tanmaygar/AI-Course/blob/main/Assignment2/Assignment2.tex>

## PROBLEM GATE-EC-2019-Q47:

A random variable  $X$  takes values  $-1$  and  $+1$  with probabilities  $0.2$  and  $0.8$ , respectively. It is transmitted across a channel which adds noise  $N$ , so that the random variable at the channel output is  $Y = X + N$ . The noise  $N$  is independent of  $X$ , and is uniformly distributed over the interval  $[-2, 2]$ . The receiver makes a decision

$$\hat{X} = \begin{cases} -1, & \text{if } Y \leq \theta \\ +1, & \text{if } Y \geq \theta \end{cases}$$

where the threshold  $\theta \in [-1, 1]$  is chosen so as to minimize the probability of error  $\Pr(\hat{X} \neq X)$ . The minimum probability of error, rounded off to 1 decimal place, is?

SOLUTION:

We know that

$$X \in \{-1, +1\} \quad (0.0.1)$$

$$N \in [-2, 2] \quad (0.0.2)$$

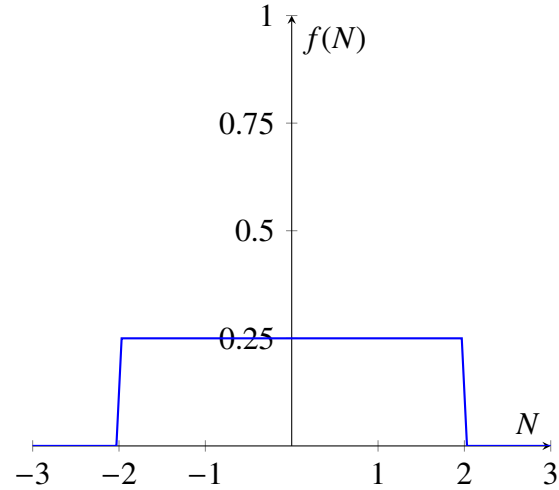
$$Y = X + N \quad (0.0.3)$$

$$\Pr(X = -1) = 0.2 \quad (0.0.4)$$

$$\Pr(X = +1) = 0.8 \quad (0.0.5)$$

Since  $N$  is uniformly distributed

$\therefore$  the probability distribution function of  $N$  is:



For  $X \neq \hat{X}$  we need to check for each case. Integrating the p.d.f to find the probability for each case

$$\therefore \Pr(\theta < -1 + N) = \int_{\theta+1}^2 \frac{1}{4} dN \quad (0.0.6)$$

$$= \frac{1}{4}(1 - \theta) \quad (0.0.7)$$

$$\therefore \Pr(\theta > N + 1) = \int_{-2}^{\theta-1} \frac{1}{4} dN \quad (0.0.8)$$

$$= \frac{1}{4}(1 + \theta) \quad (0.0.9)$$

The probability of error:

$$\Pr(\hat{X} \neq X) = P(-1) \cdot P(\theta < -1 + N) + P(1) \cdot P(\theta > N + 1) \quad (0.0.10)$$

Substituting (0.0.7) and (0.0.9) in (0.0.10). We get:

$$\Pr(\hat{X} \neq X) = 0.2 \cdot \frac{1}{4}(1 - \theta) + 0.8 \cdot \frac{1}{4}(1 + \theta) \quad (0.0.11)$$

On simplifying the equation we get

$$\Pr(\hat{X} \neq X) = \frac{1}{4} + \frac{3}{20}\theta \quad (0.0.12)$$

Since this is a linear equation in  $\theta$ , the minimum will occur at boundary points. Putting  $\theta = +1$ , we

get

$$\Pr(\hat{X} \neq X) = 0.4 \quad (0.0.13)$$

but on putting  $\theta = -1$ , we get

$$\Pr(\hat{X} \neq X) = 0.1 \quad (0.0.14)$$

$$(0.0.15)$$

Hence the value of probability of error is:

$$\therefore \Pr(\hat{X} \neq X) = 0.1 \quad (0.0.16)$$