

AI1103: Assignment 1

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Download all python codes from

https://github.com/tanmaygar/AI-Course/blob/main/Assignment1/codes/problem5_7.py

and latex-tikz codes from

<https://github.com/tanmaygar/AI-Course/blob/main/Assignment1/Assignment1.tex>

PROBLEM STATEMENT:

An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that:

- 1) all will bear 'X' mark.
- 2) not more than 2 will bear 'Y' mark.
- 3) at least one ball will bear 'Y' mark.
- 4) the number of balls with 'X' mark and 'Y' mark will be equal.

SOLUTION:

Let X be the number of balls which have 'X' mark on them

Using the expression of binomial distribution

$$P(X = r) = \binom{n}{r} p^r q^{n-r}$$

We have

$$n = 6$$

$$p = \frac{2}{5} = 0.4$$

$$q = \frac{3}{5} = 0.6$$

For (i) we need to find $P(X = 6)$

$$P(X = 6) = \binom{6}{6} p^6 q^0 = \left(\frac{2}{5}\right)^6 = 0.004096$$

For (ii) we need to find $P(X \geq 4)$

$$\begin{aligned} P(X \geq 4) &= \sum_{r=4}^6 \binom{6}{r} p^r q^{n-r} \\ &= \binom{6}{4} \left(\frac{2}{5}\right)^4 \left(\frac{3}{5}\right)^2 + \binom{6}{5} \left(\frac{2}{5}\right)^5 \left(\frac{3}{5}\right) + \binom{6}{6} \left(\frac{2}{5}\right)^6 \\ &= \frac{432}{3125} + \frac{576}{15625} + \frac{64}{15625} \\ &= \frac{112}{625} = 0.1792 \end{aligned}$$

For (iii) we need to find $P(X \leq 5)$

$$\begin{aligned} P(X \leq 5) &= \sum_{r=0}^5 \binom{6}{r} p^r q^{n-r} \\ &= \binom{6}{0} \left(\frac{3}{5}\right)^6 + \binom{6}{1} \left(\frac{2}{5}\right)^1 \left(\frac{3}{5}\right)^5 + \binom{6}{2} \left(\frac{2}{5}\right)^2 \left(\frac{3}{5}\right)^4 + \binom{6}{3} \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^3 \\ &\quad + \binom{6}{4} \left(\frac{2}{5}\right)^4 \left(\frac{3}{5}\right)^2 + \binom{6}{5} \left(\frac{2}{5}\right)^5 \left(\frac{3}{5}\right) \\ &= \frac{729}{15625} + \frac{2916}{15625} + \frac{972}{3125} + \frac{864}{3125} + \frac{432}{3125} + \frac{576}{15625} \\ &= \frac{15561}{15625} = 0.995904 \end{aligned}$$

For (iv) we need to find $P(X = 3)$

$$\begin{aligned} P(X = 3) &= \binom{6}{3} p^3 q^3 \\ &= \binom{6}{3} \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^3 = 0.27648 \end{aligned}$$