

Assignment 1

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Download all python codes from

<https://github.com/gaureeshk/AI1103/blob/main/Codes/assignment1.py>

and latex-tikz codes from

<https://github.com/gaureeshk/AI1103/blob/main/assignment1.tex>

1 PROBLEM

A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

2 SOLUTION

Let $\mathbf{X} \in \{0, 1\}$ be a random variable where 0 represents a diamond card getting lost and 1 represent a card which is not a diamond becoming lost.

Let $\mathbf{Y} \in \{0, 1\}$ be a random variable where 0 represents both cards drawn being diamonds and 1 represents the case where atleast 1 of the 2 cards drawn is not a diamond.

The required probability is $\Pr(X=0|Y=0)$.

Since there are 13 diamond cards,

$$\Pr(X = 0) = \frac{13}{52} = \frac{1}{4} \quad (2.0.1)$$

$$\Pr(X = 1) = \frac{39}{52} = \frac{3}{4} \quad (2.0.2)$$

$(X = 0 \cap Y = 0)$ is the event of a diamond card getting lost and getting 2 diamond cards in the 2 draws.

Hence,

$$\Pr(X = 0 \cap Y = 0) = \frac{{}^{13}C_3}{{}^{52}C_3} = \frac{13! 49!}{10! 52!} \quad (2.0.3)$$

Using Total probability theorem,

$$\Pr(F) = \sum_{i=1}^n \Pr(F|E_i) \Pr(E_i) \quad (2.0.4)$$

$\Pr(Y = 0|X = 0)$ is probability of selecting 2 diamond cards given that one diamond card is lost.

$$\Rightarrow \Pr(Y = 0|X = 0) = \frac{{}^{12}C_2}{{}^{51}C_2} = \frac{12! 49!}{10! 51!} \quad (2.0.5)$$

$\Pr(Y = 0|X = 1)$ is probability of selecting 2 diamond cards given that the card lost is not a diamond.

$$\Rightarrow \Pr(Y = 0|X = 1) = \frac{{}^{13}C_2}{{}^{51}C_2} = \frac{13! 49!}{11! 51!} \quad (2.0.6)$$

by using equation (2.0.4),

$$\Pr(Y = 0) = \frac{12! 49!}{10! 51! 4} + \frac{13! 49! 3}{11! 51! 4} \quad (2.0.7)$$

$$= \frac{12! 49! 50}{11! 51! 4}$$

by definition,

$$\begin{aligned} \Pr(X = 0|Y = 0) &= \frac{\Pr(X = 0 \cap Y = 0)}{\Pr(Y = 0)} \\ &= \frac{11}{50} \\ &= 0.22 \end{aligned} \quad (2.0.8)$$

Hence the probability of the lost card being a diamond (given that the 2 cards drawn are diamonds) is **0.22**.