Assignment 2

Gaureesha Kajampady - EP20BTECH11005

Download all python codes from

https://github.com/gaureeshk/assignment2/blob/ main/Codes/assignment2.py

and latex-tikz codes from

https://github.com/gaureeshk/assignment2/blob/ main/assignment2.tex

1 Problem

For each element in a set of size 2n, an unbiased coin is tossed. The 2n coin tosses are independent. An element is chosen if the corresponding coin toss were head. The probability that exactly n elements are chosen is:

(A)
$$\frac{2^n C_n}{4^n}$$

(B)
$$\frac{^{2n}C_n}{2^n}$$

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 (B) $\frac{^{2n}C_n}{2^n}$ (C) $\frac{1}{^{2n}C_n}$ (D) $\frac{1}{2}$

(D)
$$\frac{1}{2}$$

2 Solution

The number of elements chosen is equal to the number of heads obtained by 2n coin tosses. Let X be a random variable with value of X equal to the number of heads obtained.

Probability of getting a head, $p = \frac{1}{2}$

Probability of getting a tail, $q = \frac{1}{2}$

Probability that n elements are chosen out of 2n elements is Pr(X = n)

From binomial distribution we know that,

$$\Pr(X = r) = {}^{n}C_{r}p^{r}q^{n-r}$$
 (2.0.1)

$$\Pr(X = n) = {}^{2n}C_n \times \left(\frac{1}{2}\right)^n \times \left(\frac{1}{2}\right)^n \tag{2.0.2}$$

$$=\frac{^{2n}C_n}{4^n} \tag{2.0.3}$$

Hence option (A) is correct.