

# Assignment 3

Gaureesha Kajampady - EP20BTECH11005

Download all python codes from

<https://github.com/gaureeshk/assignment3/blob/main/Codes/assignment3.py>

and latex-tikz codes from

<https://github.com/gaureeshk/assignment3/blob/main/assignment3.tex>

## 1 PROBLEM

A random variable  $X$  has probability density function  $f(x)$  as shown below:

$$f(x) = \begin{cases} a + bx & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

If the expected value  $E[X] = 2/3$ , then  $\Pr(X < 0.5)$  is \_\_\_\_\_

## 2 SOLUTION

Since the total probability is 1,

$$\int_{-\infty}^{\infty} f(x) dx = 1 \quad (2.0.1)$$

$$\Rightarrow \int_0^1 f(x) dx = 1 \quad (2.0.2)$$

$$\Rightarrow a + \frac{b}{2} = 1 \quad (2.0.3)$$

Also,

$$E[X] = \int_{-\infty}^{\infty} xf(x) dx \quad (2.0.4)$$

$$\frac{2}{3} = \int_0^1 (ax + bx^2) dx \quad (2.0.5)$$

$$\Rightarrow \frac{a}{2} + \frac{b}{3} = \frac{2}{3} \quad (2.0.6)$$

using (2.0.3) and (2.0.6) we get,

$$\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{2}{3} \end{bmatrix} \quad (2.0.7)$$

$$\Rightarrow \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ \frac{2}{3} \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \quad (2.0.8)$$

Hence  $a=0$  and  $b=2$ .

$$F_X(x) = \int_{-\infty}^x f(x) dx. \quad (2.0.9)$$

$$= \begin{cases} 0 & \text{for } x \leq 0 \\ x^2 & \text{for } 0 < x < 1 \\ 1 & \text{for } x \geq 1 \end{cases} \quad (2.0.10)$$

$$\Pr(X < 0.5) = F_X(0.5) = 0.25 \quad (2.0.11)$$

Hence the required probability is 0.25.

