Assignment 3

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Download all python codes from

https://github.com/gaureeshk/assignment3/blob/main/Codes/assignment3.py

and latex-tikz codes from

https://github.com/gaureeshk/assignment3/blob/main/assignment3.tex

1 Problem

A random variable X has probability density function f(x) as shown below:

$$f(x)=\begin{cases} a+bx & \text{for } 0<x<1 \\ 0 & \text{otherwise} \end{cases}$$

If the expected value $E[X] = 2/3$, then $\Pr(X < 0.5)$ is ________

2 Solution

Since the total probabilty is 1,

$$\int_{-\infty}^{\infty} f(x)dx = 1 \tag{2.0.1}$$

$$\implies \int_{0}^{1} f(x)dx = 1 \tag{2.0.2}$$

$$\implies a + \frac{b}{2} = 1 \tag{2.0.3}$$

Also,

$$E[X] = \int_{-\infty}^{\infty} xf(x)dx \tag{2.0.4}$$

$$\frac{2}{3} = \int_{0}^{1} (ax + bx^2)dx \tag{2.0.5}$$

$$\implies \frac{a}{2} + \frac{b}{3} = \frac{2}{3} \tag{2.0.6}$$

using (2.0.3) and (2.0.6) we get,

$$\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{2}{3} \end{bmatrix} \tag{2.0.7}$$

$$\implies \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ \frac{2}{3} \end{bmatrix} \tag{2.0.8}$$

$$= \frac{\operatorname{adj}\left(\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix}\right) \begin{bmatrix} 1 \\ \frac{2}{3} \end{bmatrix}}{\begin{vmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{vmatrix}} \tag{2.0.9}$$

$$= \frac{\begin{bmatrix} \frac{1}{3} & \frac{-1}{2} \\ \frac{-1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 \\ \frac{2}{3} \end{bmatrix}}{\left(\frac{1}{12}\right)} \tag{2.0.10}$$

$$= \begin{bmatrix} 0 \\ 2 \end{bmatrix} \tag{2.0.11}$$

Hence a=0 and b=2.

$$F_X(x) = \int_{-\infty}^{x} f(x)dx. \tag{2.0.12}$$

$$= \begin{cases} 0 & \text{for } x \le 0 \\ x^2 & \text{for } 0 < x < 1 \\ 1 & \text{for } x \ge 1 \end{cases} \tag{2.0.13}$$

$$\Pr(X < 0.5) = F_X(0.5) = 0.25 \tag{2.0.14}$$

Hence the required probability is 0.25.