Assignment 4

Gaureesha Kajampady - EP20BTECH11005

Download all python codes from

https://github.com/gaureeshk/assignment4/blob/main/Codes/assignment4.py

and latex-tikz codes from

https://github.com/gaureeshk/assignment4/blob/main/assignment4.tex

1 Problem

Arrivals at a telephone booth are considered to be Poisson, with an average time of 10 minutes between successive arrivals.The length of a phone call is distributed exponentially with mean 3 minutes.The probability that an arrival does not have to wait before service is

1) 0.3
2) 0.5
3) 0.7
4) 0.9

2 Solution

Let X be a random variable with values equal to time between successive calls(in minutes) which is a Poisson distribution with mean of 10.

$$\implies \Pr(X = x) = \frac{e^{-10} \times 10^x}{x!} \quad (x = 1, 2, 3, ...) \tag{2.0.1}$$

Let Y be a random variable with values equal to length of a phone call which is an exponential distribution with mean 3.

$$\implies f_Y(y) = \begin{cases} \frac{e^{\frac{-y}{3}}}{3} & \text{for } x \geq 0 \\ 0 & \text{for } x < 0 \end{cases} \tag{2.0.2}$$

$$\implies \Pr(Y \leq y) = F_Y(y) = \int_{-\infty}^{y} f_Y(y) dy \tag{2.0.3}$$

$$= \int_0^y \frac{e^{\frac{-y}{3}}}{3} \tag{2.0.4}$$

$$= 1 - e^{\frac{-y}{3}} \tag{2.0.5}$$

Probability that an arrival does not have to wait is $\Pr(Y \leq X)$

$$\Pr(Y \leq X) = \sum_{x=0}^{x=\infty} \Pr(Y \leq x) \Pr(X = x) \tag{2.0.6}$$

$$= \sum_{x=0}^{x=\infty} (1 - e^{\frac{-x}{3}}) \times \left(\frac{e^{-10} \times 10^x}{x!} \right) \tag{2.0.7}$$

$$= e^{-10} \left(\sum_{x=0}^{x=\infty} \frac{10^x}{x!} - \sum_{x=0}^{x=\infty} \frac{10^x e^{\frac{-x}{3}}}{x!} \right) \tag{2.0.8}$$

$$= e^{-10} \left(e^{10} - \sum_{x=0}^{x=\infty} \frac{e^{\left(log_e 10 - \frac{1}{3}\right)x}}{x!} \right) \tag{2.0.9}$$

$$= 1 - e^{-10} \left(e^{e^{\left(log_e 10 - \frac{1}{3}\right)}} \right) \tag{2.0.10}$$

$$= 0.941 \tag{2.0.11}$$

Hence option (4) is correct.