Assignment 6

Gaureesha Kajampady - EP20BTECH11005

Download latex-tikz codes from

https://github.com/gaureeshk/assignment6/blob/main/assignment6.tex

1 Problem

(CSIR UGC NET EXAM (Dec 2012), Q.105)
X, Y, Z are independent random variables with N(0,1) (standard normal) distribution.
Let f:R→R be defined by f(x)=1, if x≥0 and f(x)=-1, if x<0.
Let U,V,W be defined by U=$|X|$f(Y), V=$|Y|$f(X), W=$|Z|$f(X), Then

1) U and V are independent each having N(0,1) distribution
2) U and W are independent each having N(0,1) distribution
3) V and W are independent each having N(0,1) distribution
4) U, V and W are independent random variables

2 Solution

Since N(0,1) distribution is symmetric about x=0,

$$f_X(-x) = f_X(x) \tag{2.0.1}$$

(2.0.2)

Finding the pdf of $|X|$,

$$\implies f_{|X|}(x) = \begin{cases} 2f_X(x) & \text{when } x \geq 0 \\ 0 & \text{when } x < 0 \end{cases}$$

Since Y is N(0,1) distribution ,

$$\Pr(Y \geq 0) = \frac{1}{2} \tag{2.0.3}$$

$$\Pr(Y < 0) = \frac{1}{2} \tag{2.0.4}$$

$$\implies \Pr(f(Y) = 1) = \frac{1}{2} \tag{2.0.5}$$

$$\Pr(f(Y) = -1) = \frac{1}{2} \tag{2.0.6}$$

Finding the pdf of $U = |X|\, f(Y)$,

$$\mathrm{f}_{|X|f(Y)}(x) = \begin{cases} 2f_X(x) \times \frac{1}{2} & \text{when } x \geq 0 \\ 2f_X(x) \times \frac{1}{2} & \text{when } x < 0 \end{cases}$$

$= f_X(x)$

$\implies f_U(x) = f_X(x)$

Hence U also has a N(0,1) distribution.
Using similar arguments for V=$|Y|$f(X) and W=$|Z|$f(X),
$\implies$ U, V and W all have N(0,1) distribution.