Assignment 6

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Download all python codes from

https://github.com/gaureeshk/assignment6_1/blob/main/Codes/assignment6.py

Download latex-tikz codes from

https://github.com/gaureeshk/assignment6_1/blob/main/assignment6.tex

1 Problem

(CSIR UGC NET EXAM (Dec 2012), Q.51)
Suppose X_1, X_2, X_3, X_4 are i.i.d random variables taking values 1 and -1 with probability $\frac{1}{2}$ each. Then $E(X_1 + X_2 + X_3 + X_4)^4$ equals

1) 40
2) 76
3) 16
4) 12

2 Solution

Using multinomial expansion for$(X_1+X_2+X_3+X_4)^4$,

$$(X_1+X_2+X_3+X_4)^4 = \frac{4!}{4!}\sum_{i=1}^{4} X_i^4 + \frac{4!}{3!1!}\sum_{i=1}^{4}\sum_{j=1(j\neq i)}^{4} X_i^3 X_j + \frac{4!}{2!2!}\sum_{i=1}^{4}\sum_{j=1(j\neq i)}^{4} X_i^2 X_j^2 \tag{2.0.1}$$

Since expectation is a linear function,

$$E(X_1+X_2+X_3+X_4)^4 = \sum_{i=1}^{4} E(X_i^4) + 4\sum_{i=1}^{4}\sum_{j=1(j\neq i)}^{4} E(X_i^3 X_j) + 6\sum_{i=1}^{4}\sum_{j=1(j\neq i)}^{4} E(X_i^2 X_j^2) \tag{2.0.2}$$

Since X_i can only be +1 or -1, X_i^4 is always 1. Hence

$$E(X_i^4) = 1 \tag{2.0.3}$$

Similarly for $X_i^3 X_j$ term we get the following possibilities

X_i	X_j	$X_i^3 Xj$
1	1	1
-1	-1	1
-1	1	-1
1	-1	-1

Hence $X_i^3 X_j$ has values 1 and -1 with probability $\frac{1}{2}$ each.

$$\implies E(X_i^3 X_j) = \frac{1}{2} \times 1 + \frac{1}{2} \times -1 = 0 \tag{2.0.4}$$

$X_i^2 X_j^2$ is always 1 hence $E(X_i^2 X_j^2)$=1
To find the number of $X_i^2 X_j^2$ terms,
There are 4 random variables and we need to select 2 (i and j) without any order (because $X_i^2 X_j^2 = X_j^2 X_i^2$)

$$\implies \text{total number of terms } = {}^4C_2 \tag{2.0.5}$$
$$= 6 \tag{2.0.6}$$
$$\implies E(X_1 + X_2 + X_3 + X_4)^4 = 4 \times 1 + 0 + 6 \times 6 \times 1 \tag{2.0.7}$$
$$= 40 \tag{2.0.8}$$

Hence option 1 is correct.