

Assignment 6

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https://github.com/gaureeshk/assignment6_2/blob/main/assignment6.tex

1 PROBLEM

gov/stats/2018/STATISTICS-PAPER-2 , Q.58

Let X_1 and X_2 be i.i.d random variables with poisson. Then $(X_1 + 2X_2)$ is not sufficient because

- 1) $\Pr(X_1 = 1, X_2 = 1 | T = 3)$ depends on λ
- 2) $X_1 + 2X_2$ is poisson
- 3) $X_1 + 2X_2$ is not poisson
- 4) $\Pr(X_1 = 1, X_2 = 1 | T = 3)$ is Poisson with parameter one

Where $T = (X_1 + 2X_2)$

2 SOLUTION

Definition 1. *Statistic* : A statistic is a function $T = r(X_1, X_2, \dots, X_n)$ of the random sample X_1, X_2, \dots, X_n .

Definition 2. *Sufficient Statistics* : A statistic $t = T(X)$ is sufficient for θ if the conditional probability distribution of data X , given the statistic $t = T(X)$, doesn't depend on the parameter θ .

Theorem 2.1 (Factorization theorem). : Let X_1, X_2, X_n form a random sample from either a continuous distribution or a discrete distribution for which the pdf or the point mass function is $f(x|\theta)$, where the value of θ is unknown and belongs to a given parameter space Θ . A statistic $T(X_1, X_2, X_n)$ is a sufficient statistic for θ if and only if the joint pdf or the joint point mass function $f_n(x|\theta)$ X_1, X_2, X_n can be factorized as follows for all values of $x = (X_1, X_2, X_n) \rightarrow R^n$ and all values of $\theta \in \Theta$: $f_n(x|\theta) = u(x)v[T(x), \theta]$.

Here the function u may depend on x but does not depend on θ , and the function v depends on θ but

will depend on the observed value x only through the value of the statistic $T(x)$.

We know $T=3$ when (X_1, X_2) have values $(1,1)$ and $(3,0)$

$$\Pr(X_1 = 1, X_2 = 1 | T = 3) \quad (2.0.1)$$

$$= \frac{\Pr(X_1 = 1, X_2 = 1 \cap T = 3)}{\Pr(T = 3)} \quad (2.0.2)$$

$$= \frac{\Pr(X_1 = 1, X_2 = 1)}{\Pr(X_1 = 1, X_2 = 1) + \Pr(X_1 = 3, X_2 = 0)} \quad (2.0.3)$$

$$= \frac{e^{-2\lambda}\lambda^2}{e^{-2\lambda}\lambda^2 + \frac{e^{-3\lambda}\lambda^3}{6}} \quad (2.0.4)$$

$$= \frac{6}{6 + \lambda} \quad (2.0.5)$$

\Rightarrow option 4 is incorrect and option 1 is correct.
Now to find pdf of T ,

$$\Pr(T = x) = \Pr(X_1 + 2X_2 = x) \quad (2.0.6)$$

$$= \sum_{i=0}^{\infty} \Pr(X_1 = x - 2i) \Pr(X_2 = i) \quad (2.0.7)$$

$$= \sum_{i=0}^{\infty} i = 0 \frac{e^{-\lambda}\lambda^{x-2i} \times e^{-\lambda}\lambda^i}{(x-2i)!i!} \quad (2.0.8)$$

$$= e^{-2\lambda}\lambda^x \sum_{i=0}^{\infty} \frac{\lambda^{-i}}{(x-2i)!i!} \quad (2.0.9)$$

We find that the above expression is not reducible to the form of $\frac{e^{-\mu}\mu^x}{x!}$

Hence $X_1 + 2X_2$ is not poisson distribution.

\Rightarrow option 2 is incorrect, option 3 is correct

Hence options 1 and 3 are correct.