Assignment 6

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https://github.com/gaureeshk/assignment6_2/blob/main/assignment6.tex

1 Problem

gov/stats/2018/STATISTICS-PAPER-2 , Q.58

Let X_1 and X_2 be i.i.d random variables with poisson. Then $(X_1 + 2X_2)$ is not sufficient because

1) $\Pr(X_1 = 1, X_2 = 1 | T = 3)$ depends on λ
2) $X_1 + 2X_2$ is poisson
3) $X_1 + 2X_2$ is not poisson
4) $\Pr(X_1 = 1, X_2 = 1 | T = 3)$ is Poisson with parameter one

Where T=$(X_1 + 2X_2)$

2 Solution

Definition 1. *Statistic : A statistic is a function $T = r(X_1, X_2, \ldots, X_n)$ of the random sample $X_1, X_2, \ldots, X_n$.*

Definition 2. *Sufficient Statistics : A statistic t = T(X) is sufficient for θ if the conditional probability distribution of data X, given the statistic t = T(X), doesn't depend on the parameter θ.*

Theorem 2.1 (Factorization theorem)**.** *: Let X_1, X_2, X_n form a random sample from either a continuous distribution or a discrete distribution for which the pdf or the point mass function is $f(x|\theta)$, where the value of θ is unknown and belongs to a given parameter space Θ. A statistic T(X_1, X_2, X_n) is a sufficient statistic for θ if and only if the joint pdf or the joint point mass function $f_n(x|\theta)$ X_1, X_2, X_n can be factorized as follows for all values of $x = (X_1, X_2, X_n) \rightarrow R^n$ and all values of $\theta \in \Theta$: $f_n(x|\theta) = u(x)v[T(x), \theta]$.*
Here the function u may depend on x but does not depend on θ, and the function v depends on θ but will depend on the observed value x only through the value of the statistic T(x).

1) We know T=3 when (X_1, X_2) have values (1,1) and (3,0)

$$\Pr(X_1 = 1, X_2 = 1 | T = 3) \tag{2.0.1}$$
$$= \frac{\Pr(X_1 = 1, X_2 = 1 \cap T = 3)}{\Pr(T = 3)} \tag{2.0.2}$$
$$= \frac{\Pr(X_1 = 1, X_2 = 1)}{\Pr(X_1 = 1, X_2 = 1) + \Pr(X_1 = 3, X_2 = 0)} \tag{2.0.3}$$
$$= \frac{e^{-2\lambda}\lambda^2}{e^{-2\lambda}\lambda^2 + \frac{e^{-3\lambda}\lambda^3}{6}} \tag{2.0.4}$$
$$= \frac{6}{6 + \lambda} \tag{2.0.5}$$

$\implies$ option 4 is incorrect and option 1 is correct.

2) Now to find pmf of T,

$$\Pr(T = x) = \Pr(X_1 + 2X_2 = x) \tag{2.0.6}$$
$$= \sum_{i=0}^{\infty} \Pr(X_1 = x - 2i) \Pr(X_2 = i) \tag{2.0.7}$$
$$= \sum i = 0^{\infty} \frac{e^{-\lambda}\lambda^{x-2i} \times e^{-\lambda}\lambda^i}{(x - 2i)!i!} \tag{2.0.8}$$
$$= e^{-2\lambda}\lambda^x \sum_{i=0}^{\infty} \frac{\lambda^{-i}}{(x - 2i)!i!} \tag{2.0.9}$$

We find that the above expression is not reducible to the form of $\frac{e^{-\mu}\mu^x}{x!}$
Hence $X_1 + 2X_2$ is not poisson distribution.
$\implies$ option 2 is incorrect, option 3 is correct
Hence options 1 and 3 are correct.