Assignment 6

Gaureesha Kajampady - EP20BTECH11005

Download latex-tikz codes from

https://github.com/gaureeshk/assignment6_2/blob/main/assignment6.tex

1 Problem

gov/stats/2018/STATISTICS-PAPER-2 , Q.58

Let X_1 and X_2 be i.i.d random variables with poisson. Then $(X_1 + 2X_2)$ is not sufficient because

1) $\Pr(X_1 = 1, X_2 = 1|T = 3)$ depends on λ
2) $X_1 + 2X_2$ is poisson
3) $X_1 + 2X_2$ is not poisson
4) $\Pr(X_1 = 1, X_2 = 1|T = 3)$ is Poisson with parameter one

Where T=$(X_1 + 2X_2)$

2 Solution

Definition 1. *Statistic : A statistic is a function $T = r(X_1, X_2, \dots, X_n)$ of the random sample $X_1, X_2, \dots, X_n$.*

Definition 2. *Sufficient Statistics : A statistic $t = T(X)$ is sufficient for θ if the conditional probability distribution of data X, given the statistic $t = T(X)$, doesn't depend on the parameter θ.*

1) We know T=3 when (X_1, X_2) have values (1,1) and (3,0)

$$\Pr(X_1 = 1, X_2 = 1|T = 3)$$

$$= \frac{\Pr(X_1 = 1, X_2 = 1 \cap T = 3)}{\Pr(T = 3)}$$

$$= \frac{\Pr(X_1 = 1, X_2 = 1)}{\Pr(X_1 = 1, X_2 = 1) + \Pr(X_1 = 3, X_2 = 0)}$$

$$= \frac{e^{-2\lambda}\lambda^2}{e^{-2\lambda}\lambda^2 + \frac{e^{-3\lambda}\lambda^3}{6}}$$

$$= \frac{6}{6 + \lambda} \neq \frac{e^{-1}1^{\lambda}}{\lambda!} \quad (2.0.1)$$

Hence $\Pr(X_1 = 1, X_2 = 1|T = 3)$ depends on λ but is not poisson with parameter 1. $\implies$ option 4 is incorrect and option 1 is correct.

2)

Lemma 2.1. *Suppose that X_1 and X_2 are i.i.d poisson random variables with parameter λ, $T = X_1 + 2X_2$ does not follow poisson distribution.*

Proof. Let $\Phi_{X_1}(\omega)$, $\Phi_{2X_2}(\omega)$ and $\Phi_T(\omega)$be the characteristic functions of probability density function of random variables X_1, $2X_2$ and T respectively.

$$\Phi_{X_1}(\omega)=\mathrm{E}(\mathrm{e}^{i\omega X_1})$$
$$= \sum_{x=0}^{\infty} \Pr(X_1 = x)\, e^{i\omega x}$$
$$= \sum_{x=0}^{\infty} \frac{e^{i\omega x-\lambda}\lambda^x}{x!}$$
$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(e^{i\omega}\lambda)^x}{x!}$$
$$= e^{\lambda(e^{i\omega}-1)} \quad (2.0.2)$$

Similarly,

$$\Phi_{2X_2}(\omega)=\mathrm{E}(\mathrm{e}^{i\omega 2X_2})$$
$$= \sum_{x=0}^{\infty} \Pr\left(X_2 = \frac{x}{2}\right) e^{i\omega x}$$
$$= \sum_{x=0,2,4....}^{\infty} \frac{e^{i\omega x-\lambda}\lambda^{\frac{x}{2}}}{(\frac{x}{2})!}$$
$$= e^{-\lambda} \sum_{\frac{x}{2}=0,1,2..}^{\infty} \frac{(e^{2i\omega}\lambda)^{\frac{x}{2}}}{(\frac{x}{2})!}$$
$$= e^{\lambda(e^{2i\omega}-1)} \quad (2.0.3)$$

$$\Phi_T(\omega) = \Phi_{X_1}(\omega) \times \Phi_{2X_2}(\omega) \quad (2.0.4)$$
$$= e^{\lambda(e^{i\omega}+e^{2i\omega}-2)} \quad (2.0.5)$$
$$\neq e^{\mu(e^{i\omega}-1)} \quad (2.0.6)$$

Hence the characteristic function of T ($\Phi_T(\omega)$) is not in the form of the characteristic function of a poisson random variable (for any value of the parameter μ).

□

Hence using lemma 2.1, $T = X_1 + 2X_2$ is not poisson.
$\implies$ option 2 is incorrect, option 3 is correct
Hence options 1 and 3 are correct.