

Assignment 6

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https://github.com/gaureeshk/assignment6_2/blob/main/assignment6.tex

will depend on the observed value x only through the value of the statistic $T(x)$.

- 1) We know $T=3$ when (X_1, X_2) have values $(1,1)$ and $(3,0)$

1 PROBLEM

gov/stats/2018/STATISTICS-PAPER-2 , Q.58

Let X_1 and X_2 be i.i.d random variables with poisson. Then $(X_1 + 2X_2)$ is not sufficient because

- 1) $\Pr(X_1 = 1, X_2 = 1 | T = 3)$ depends on λ
- 2) $X_1 + 2X_2$ is poisson
- 3) $X_1 + 2X_2$ is not poisson
- 4) $\Pr(X_1 = 1, X_2 = 1 | T = 3)$ is Poisson with parameter one

Where $T=(X_1 + 2X_2)$

2 SOLUTION

Definition 1. *Statistic* : A statistic is a function $T = r(X_1, X_2, \dots, X_n)$ of the random sample X_1, X_2, \dots, X_n .

Definition 2. *Sufficient Statistics* : A statistic $t = T(X)$ is sufficient for θ if the conditional probability distribution of data X , given the statistic $t = T(X)$, doesn't depend on the parameter θ .

Theorem 2.1 (Factorization theorem). : Let X_1, X_2, X_n form a random sample from either a continuous distribution or a discrete distribution for which the pdf or the point mass function is $f(x|\theta)$, where the value of θ is unknown and belongs to a given parameter space Θ . A statistic $T(X_1, X_2, X_n)$ is a sufficient statistic for θ if and only if the joint pdf or the joint point mass function $f_n(x|\theta)$ X_1, X_2, X_n can be factorized as follows for all values of $x = (X_1, X_2, X_n) \rightarrow R^n$ and all values of $\theta \in \Theta$: $f_n(x|\theta) = u(x)v[T(x), \theta]$.

Here the function u may depend on x but does not depend on θ , and the function v depends on θ but

$$\Pr(X_1 = 1, X_2 = 1 | T = 3)$$

$$= \frac{\Pr(X_1 = 1, X_2 = 1 \cap T = 3)}{\Pr(T = 3)}$$

$$= \frac{\Pr(X_1 = 1, X_2 = 1)}{\Pr(X_1 = 1, X_2 = 1) + \Pr(X_1 = 3, X_2 = 0)}$$

$$= \frac{e^{-2\lambda} \lambda^2}{e^{-2\lambda} \lambda^2 + \frac{e^{-3\lambda} \lambda^3}{6}}$$

$$= \frac{6}{6 + \lambda} \neq \frac{e^{-1} 1^1}{1!} \quad (2.0.1)$$

Hence $\Pr(X_1 = 1, X_2 = 1 | T = 3)$ depends on λ but is not poisson with parameter 1. \Rightarrow option 4 is incorrect and option 1 is correct.

- 2) Now to find characteristic function of T ,
Let $\Phi_{X_1}(\omega)$, $\Phi_{2X_2}(\omega)$ and $\Phi_T(\omega)$ be the characteristic functions of probability density function of random variables X_1 , $2X_2$ and T respectively.

$$\begin{aligned}
\Phi_{X_1}(\omega) &= E(e^{i\omega X_1}) \\
&= \sum_{x=0}^{\infty} \Pr(X_1 = x) e^{i\omega x} \\
&= \sum_{x=0}^{\infty} \frac{e^{i\omega x} \lambda^x}{x!} \\
&= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(e^{i\omega} \lambda)^x}{x!} \\
&= e^{\lambda(e^{i\omega} - 1)} \quad (2.0.2)
\end{aligned}$$

Similarly,

$$\begin{aligned}
\Phi_{2X_2}(\omega) &= E(e^{i\omega 2X_2}) \\
&= \sum_{x=0}^{\infty} \Pr\left(X_2 = \frac{x}{2}\right) e^{i\omega x} \\
&= \sum_{x=0,2,4,\dots}^{\infty} \frac{e^{i\omega x} \lambda^{\frac{x}{2}}}{(\frac{x}{2})!} \\
&= e^{-\lambda} \sum_{\frac{x}{2}=0,1,2,\dots}^{\infty} \frac{(e^{2i\omega} \lambda)^{\frac{x}{2}}}{(\frac{x}{2})!} \\
&= e^{\lambda(e^{2i\omega} - 1)} \\
&\quad (2.0.3)
\end{aligned}$$

$$\Phi_T(\omega) = \Phi_{X_1}(\omega) \times \Phi_{2X_2}(\omega) \quad (2.0.4)$$

$$= e^{\lambda(e^{i\omega} + e^{2i\omega} - 2)} \quad (2.0.5)$$

$$\neq e^{\mu(e^{i\omega} - 1)} \quad (2.0.6)$$

Hence the characteristic function of T ($\Phi_T(\omega)$) is not in the form of the characteristic function of a poisson random variable (for any value of the parameter μ).

Hence $T = X_1 + 2X_2$ is not poisson distribution.

\implies option 2 is incorrect, option 3 is correct

Hence options 1 and 3 are correct.