

# Assignment 6

Gaureesha Kajampady - EP20BTECH11005

Download latex-tikz codes from

[https://github.com/gaureeshk/assignment6\\_2/blob/main/assignment6.tex](https://github.com/gaureeshk/assignment6_2/blob/main/assignment6.tex)

respectively.

## 1 PROBLEM

gov/stats/2018/STATISTICS-PAPER-2 , Q.58

Let  $X_1$  and  $X_2$  be i.i.d random variables with poisson. Then  $(X_1 + 2X_2)$  is not sufficient because

- 1)  $\Pr(X_1 = 1, X_2 = 1 | T = 3)$  depends on  $\lambda$
- 2)  $X_1 + 2X_2$  is poisson
- 3)  $X_1 + 2X_2$  is not poisson
- 4)  $\Pr(X_1 = 1, X_2 = 1 | T = 3)$  is Poisson with parameter one

Where  $T = (X_1 + 2X_2)$

## 2 SOLUTION

**Definition 1.** *Statistic* : A statistic is a function  $T = r(X_1, X_2, \dots, X_n)$  of the random sample  $X_1, X_2, \dots, X_n$ .

**Definition 2.** *Sufficient Statistics* : A statistic  $t = T(X)$  is sufficient for  $\theta$  if the conditional probability distribution of data  $X$ , given the statistic  $t = T(X)$ , doesn't depend on the parameter  $\theta$ .

**Lemma 2.1.** Suppose that  $X_1$  and  $X_2$  are i.i.d poisson random variables with parameter  $\lambda$ ,  $T = X_1 + 2X_2$  does not follow poisson distribution.

*Proof.* Let  $\Phi_{X_1}(\omega)$ ,  $\Phi_{2X_2}(\omega)$  and  $\Phi_T(\omega)$  be the characteristic functions of probability density function of random variables  $X_1$ ,  $2X_2$  and  $T$

$$\Phi_{X_1}(\omega) = E(e^{i\omega X_1})$$

$$= \sum_{x=0}^{\infty} \Pr(X_1 = x) e^{i\omega x}$$

$$= \sum_{x=0}^{\infty} \frac{e^{i\omega x - \lambda} \lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(e^{i\omega} \lambda)^x}{x!}$$

$$= e^{\lambda(e^{i\omega} - 1)} \quad (2.0.1)$$

Similarly,

$$\Phi_{2X_2}(\omega) = E(e^{i\omega 2X_2})$$

$$= \sum_{x=0}^{\infty} \Pr\left(X_2 = \frac{x}{2}\right) e^{i\omega x}$$

$$= \sum_{x=0,2,4,\dots}^{\infty} \frac{e^{i\omega x - \lambda} \lambda^{\frac{x}{2}}}{(\frac{x}{2})!}$$

$$= e^{-\lambda} \sum_{\frac{x}{2}=0,1,2,\dots}^{\infty} \frac{(e^{2i\omega} \lambda)^{\frac{x}{2}}}{(\frac{x}{2})!}$$

$$= e^{\lambda(e^{2i\omega} - 1)}$$

$$(2.0.2)$$

$$\Phi_T(\omega) = \Phi_{X_1}(\omega) \times \Phi_{2X_2}(\omega) \quad (2.0.3)$$

$$= e^{\lambda(e^{i\omega} + e^{2i\omega} - 2)} \quad (2.0.4)$$

$$\neq e^{\mu(e^{i\omega} - 1)} \quad (2.0.5)$$

Hence the characteristic function of  $T$  ( $\Phi_T(\omega)$ ) is not in the form of the characteristic function of

a poisson random variable (for any value of the parameter  $\mu$ ).

□

- 1) We know  $T=3$  when  $(X_1, X_2)$  have values  $(1,1)$  and  $(3,0)$

$$\begin{aligned}
 & \Pr(X_1 = 1, X_2 = 1 | T = 3) \\
 &= \frac{\Pr(X_1 = 1, X_2 = 1 \cap T = 3)}{\Pr(T = 3)} \\
 &= \frac{\Pr(X_1 = 1, X_2 = 1)}{\Pr(X_1 = 1, X_2 = 1) + \Pr(X_1 = 3, X_2 = 0)} \\
 &= \frac{e^{-2\lambda} \lambda^2}{e^{-2\lambda} \lambda^2 + \frac{e^{-3\lambda} \lambda^3}{6}} \\
 &= \frac{6}{6 + \lambda} \neq \frac{e^{-1} 1^\lambda}{\lambda!} \quad (2.0.6)
 \end{aligned}$$

Hence  $\Pr(X_1 = 1, X_2 = 1 | T = 3)$  depends on  $\lambda$  but is not poisson with parameter 1.  
 $\Rightarrow$  option 4 is incorrect and option 1 is correct.

- 2) Using lemma 2.1,  $T = X_1 + 2X_2$  is not poisson.  
 $\Rightarrow$  option 2 is incorrect, option 3 is correct  
Hence options 1 and 3 are correct.