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* UNIT - 5 (AUXILIARY FRACTIONS)

* Case - I : When the divisor ends in 9

Eg : $\frac{1}{19}, \frac{1}{29}, \frac{2}{39} \dots$

Q. Convert $\frac{1}{19}$ into decimal place.

Step 1: Its A.F. is $\frac{1}{19} \rightarrow \frac{1}{19+1} = \frac{1}{20}$

$$\therefore A.P. = \frac{0.1}{2}$$

$$S_1: \frac{0.1}{2}$$

$$G.D. = 1 \div 2 \rightarrow Q:0; R=1$$

$$= 0.10\overline{526}$$

$$G.D. = RQ$$

(Gross dividend)

$$\frac{0.1}{2} = 0.10\overline{5263115718}$$

$$S_2: G.D. = RQ = 10 \rightarrow 10 \div 2$$

$$Q=5; R=0$$

$$S_3: G.D. = 05 \div 2$$

$$Q=2; R=1$$

$$S_4 \rightarrow G.D. = 12 \div 2$$

$$Q=6; R=0$$

Q.1) $\frac{14}{79} \Rightarrow \frac{1.4}{8} = 0.\underline{15717124157186958273}$
 $= 0.177215189873$

2) $\frac{68}{129} \Rightarrow \frac{6.8}{13} = 0.\underline{527171317829457}$
 $= 0.5271317829457$

3) $\frac{18}{49} \Rightarrow \frac{1.8}{5} = 0.\underline{36734693877551}$
 $= 0.36734693877551$

4) $\frac{52}{99} \Rightarrow \frac{5.2}{10} = 0.\underline{52525252} \dots$
 $= 0.525252 \dots$

5) $\frac{35}{119} \Rightarrow \frac{3.5}{12} = 0.\underline{2932773109243}$
 $= 0.2932773109243$

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Case - II : When the divisor ends in 8 \Rightarrow

$$\text{Eg : } \frac{9}{38}$$

$$\text{A.P. } \Rightarrow \frac{9}{38+2} = \frac{9}{40} = 0.\overline{9}$$

$$= 0.\overset{+2}{1}\overset{+3}{2}\overset{+6}{3}\overset{+8}{2}\overset{+4}{0}\overset{+2}{8}\overset{+1}{0}\overset{+0}{2}\overset{+5}{0}\overset{+5}{5} \dots \boxed{S_1 : \text{G.D.} = 9 \div 4}$$

$$Q=2; R=1$$

$$= 0.236842105 \dots$$

$$\boxed{S_2 : \text{G.D.} = RQ + Q}$$

$$= 12+2=14 \div 4$$

$$Q=3; R=2$$

$$(Q.1) \quad \frac{57}{138}$$

$$\Rightarrow \frac{57}{140} = \frac{5.7}{14}$$

$$= 0.\overset{+4}{1}\overset{+1}{4}\overset{+3}{1}\overset{+0}{0}\overset{+4}{3}\overset{+3}{6}\overset{+4}{0}\overset{+4}{4}\overset{+7}{3}\overset{+4}{10}\overset{+7}{4}\overset{+2}{10}\overset{+8}{7}\overset{+2}{2}\overset{+6}{6}\overset{+6}{8}\overset{+2}{2}\overset{+6}{6}$$

$$= 0.4130434782$$

$$(2) \quad \frac{13}{38}$$

$$\Rightarrow \frac{13}{40} = \frac{1.3}{4}$$

$$= 0.\overset{+3}{1}\overset{+4}{3}\overset{+2}{0}\overset{+1}{2}\overset{+1}{0}\overset{+0}{1}\overset{+5}{2}\overset{+5}{0}\overset{+2}{2}\overset{+6}{0}\overset{+3}{6}\overset{+1}{0}\overset{+5}{2}\overset{+7}{2}\overset{+8}{7}\overset{+9}{2}\overset{+9}{0}$$

$$= 0.342105263157$$

3)

$$\begin{array}{r} 5 \\ \times 28 \\ \hline \end{array}$$

$$\Rightarrow \frac{5}{30} = \frac{0.5}{3}$$

$$= 0.1^{+1} 7^{+7} 0^{+8} 1^{+5} 6^{+6}$$

\downarrow

$$\begin{array}{r} 6 \\ 3) 20 \\ -18 \\ \hline 2 \end{array}$$

$$G.D. = 26 + 6 = 32 \div 3$$

$$Q = 10$$

$$\begin{array}{r} 7 \\ 3) 20 \\ -21 \\ \hline -1 \end{array}$$

$$\begin{aligned} G.D. &= 17 + 7 \\ &= -10 + 7 + 7 \\ &= 4 \div 3 \Rightarrow Q : 1; R = 1 \end{aligned}$$

$$\begin{array}{r} +7 \\ 17 \\ -7 \\ \hline \end{array}$$

$$17 + 7$$

$$-10 + 7 + 7 = 4$$

4)

$$\begin{array}{r} 63 \\ \times 88 \\ \hline \end{array}$$

$$\Rightarrow \frac{63}{90} \rightarrow \frac{6 \cdot 3}{9}$$

$$= 0.07^{+7} 81^{+1} 75^{+5} 88^{+8}$$

5) $\frac{29}{118}$

$$\Rightarrow \frac{29}{120} = \frac{2.9}{12}$$

$$= 0.\underline{5}^2\underline{2}^4\underline{4}^5\underline{8}^5$$

* Case - III : When the divisor ends in 1 \Rightarrow

Eg : $\frac{8}{21}$

$$A.F. = \frac{8-1}{21-1} = \frac{7}{20} = \frac{0.7}{2}$$

$$G.D. = R(9-0)$$

$$= 0.\underline{3}^6\underline{8}^1\underline{0}^9\underline{9}^0\underline{5}^4\underline{0}^2\underline{7}$$

$$Q.1) \frac{7}{61}$$

$$\Rightarrow \frac{6}{60} = \frac{0.6}{6}$$

$$= 0.\underset{0}{1}\underset{8}{2}\underset{1}{4}\underset{5}{3}\underset{2}{7}\underset{4}{5}\underset{0}{4}\underset{5}{0}\underset{9}{5}\underset{0}{8}\underset{1}{3}\underset{6}{6}$$
$$= 0.1147540983$$

$$2) \frac{9}{121}$$

$$\Rightarrow \frac{8}{120} = \frac{0.8}{12}$$

$$= 0.\underset{8}{0}\underset{5}{7}\underset{2}{4}\underset{5}{9}\underset{6}{3}\underset{1}{0}\underset{8}{1}\underset{0}{9}\underset{1}{6}\underset{3}{6}\underset{3}{5}\underset{4}{1}\underset{7}{8}\underset{1}{1}$$
$$= 0.8743801652$$

$$3) \frac{14}{91}$$

$$\Rightarrow \frac{13}{90} = \frac{1.3}{9}$$

$$= 0.\underset{4}{1}\underset{3}{5}\underset{4}{7}\underset{6}{3}\underset{1}{8}\underset{5}{4}\underset{5}{1}\underset{6}{6}\underset{3}{4}\underset{1}{8}$$

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Q.1) $\frac{4}{31} \rightarrow A.F. = \frac{3}{30} = \frac{0.3}{3}$

2) $\frac{8}{51} \rightarrow A.F. = \frac{7}{50} = \frac{0.7}{5}$

3) $\frac{11}{51} \rightarrow A.F. = \frac{10}{50} = \frac{1}{5}$

4) $\frac{16}{31} \rightarrow A.F. = \frac{15}{30} = \frac{1.5}{3}$

5) $\frac{19}{161} \rightarrow A.F. = \frac{18}{160} = \frac{1.8}{16}$

* Case IV : When the divisor ends in 2 :)

Eg : $\frac{7}{32}$

$A.F. \rightarrow \frac{7-1}{32-2} = \frac{6}{30}$

$= \frac{0.6}{3}$

$$G \cdot D \cdot = 6$$

$$G \cdot D \cdot = R(9-2Q)$$

$$\Rightarrow \frac{0.6}{3}$$

$$= 0.0\overline{210}\overline{9}$$

↓

$$3\overline{8} \overline{2} \overline{7} \overline{5} \overline{4} \overline{1} \dots$$

$$S_1 = 6 \div 3; Q=2; R=0$$

$$S_2 = G \cdot D \cdot = 5 \div 3$$

$$Q=1; R=2$$

$$S_3 = G \cdot D \cdot = 27 \div 3$$

$$Q=9; R=0$$

$$S_4 = \overline{09} = 0 - 9 = -9$$

Case of failure

$$S_4^* \quad G \cdot D \cdot = \overline{37}$$

$$= 30 - 7$$

$$= 23$$

$$Q \cdot 1) \quad \frac{9}{52}$$

$$A \cdot F \cdot = \frac{8}{50} \Rightarrow \frac{0.8}{5}$$

$$= 0.\overline{3} \overline{1} \overline{2} \overline{7} \overline{5} \overline{3} \overline{0} \overline{9} \overline{7} \overline{0} \overline{7} \overline{5}$$

↓

$$\overline{5} \overline{6} \overline{3} \overline{9} \overline{1} \overline{2} \overline{0} \overline{3} \overline{3}$$

$$= 0.173076923$$

2) $\frac{5}{22}$

$$A \cdot F. = \frac{4}{20} \Rightarrow \frac{0.4}{2}$$

$$= 0.\overline{2}, \overline{2}\overline{5}, \overline{7}\overline{5}, \overline{2}, \overline{7}\overline{2}, \dots$$

$$= 0.2\overline{272727} \dots$$

3) $\frac{3}{32}$

$$A \cdot F. = \frac{2}{30} = \frac{0.2}{3}$$

$$= 0.\overline{2}, \overline{9}\overline{9}, \overline{2}\overline{3}, \overline{2}\overline{7}, \overline{2}\overline{6}, \overline{3}\overline{5}, \overline{1}\overline{6}$$

4) $\frac{7}{42}$

$$A \cdot F. = \frac{6}{40} = \frac{0.6}{4}$$

$$= 0.\overline{1}, \overline{3}\overline{6}, \overline{2}\overline{7}, \overline{0}\overline{5}\overline{1}$$

↓
49¹

5) $\frac{5}{12}$

$$A \cdot F. = \frac{4}{10} = \underline{\underline{0.4}}$$

$$= 0.\overline{4}, \overline{1}\overline{7}, \overline{6}\overline{5}\overline{5}$$

$$= 0.4\overline{16}$$

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* Case - IV : Divisor ends in 3, 4, 6 & 7 :

Sub - Case - I : Divisor ends in 3 :

$$\text{Eg: } \frac{7}{23}$$

(either multiply by
3 or 7)

* Method - I : $\frac{7}{23} \times \frac{3}{3} = \frac{21}{69}$

$$A \cdot F \cdot \Rightarrow \frac{2 \cdot 1}{7}$$

$$= 0.\underline{3}\underline{0}\underline{4}\underline{3}\underline{4}\underline{7}\underline{8}\underline{4}\underline{2}\underline{6}\\ = 0.304347826$$

* Method - II : $\frac{7}{23} \times \frac{7}{7} = \frac{49}{161}$

$$A \cdot F \cdot \Rightarrow \frac{4 \cdot 8}{16}$$

$$= 0.\underline{3}\underline{6}\underline{0}\underline{9}\underline{4}\underline{5}\underline{7}\underline{3}\underline{6}\underline{4}\underline{5}\underline{2}\underline{1}\underline{8}\underline{9}\underline{2}\underline{6}\underline{7}\underline{3}$$

$$= 0.304347826$$

Q.1) $\frac{4}{17}$

$$\nexists \frac{4}{17} \times \frac{3}{3} = \frac{12}{51}$$

$$A \cdot F \cdot \Rightarrow \frac{11}{50} = \frac{1 \cdot 1}{5}$$

~~$$= 0.\underline{2}\underline{2}\underline{4}\underline{4}\underline{8}\underline{9}\underline{7}\underline{9}\underline{5}\underline{9}\underline{1}\underline{8}\underline{3}\underline{3}\underline{6}\underline{7}$$~~

$$= 0.235294117647$$

$$2) \frac{4}{13}$$

$$\Rightarrow \frac{4}{13} \times \frac{3}{3} = \frac{12}{39}$$

$$A \cdot F \cdot \Rightarrow \frac{1 \cdot 2}{4}$$

$$= 0.0\overset{3}{3}\overset{0}{2}\overset{7}{3}\overset{6}{0}\overset{9}{1}\overset{2}{2}\overset{3}{0}\overset{7}{3}\overset{6}{2}$$

$$= 0.307692$$

$$3) \frac{7}{27} \times \frac{7}{7} = \frac{49}{189}$$

$$A \cdot F \cdot = \frac{4 \cdot 9}{19}$$

$$= 0.\overset{2}{1}\overset{5}{9}\overset{1}{1}\overset{7}{5}\overset{4}{9}$$

$$= 0.259$$

Sub-Case - II : Divisor Ends in 4:

$$Eg: \frac{5}{14}$$

$$\frac{5}{14} \times \frac{2}{2} = \frac{10}{28}$$

$$A \cdot F \cdot \Rightarrow \frac{10}{30} = \frac{01}{3}$$

$$= 0.\overset{0}{1}\overset{3}{3}\overset{5}{1}\overset{6}{5}\overset{7}{2}\overset{6}{6}$$

$$= 0.357142 \overline{7}$$

$$Q.1) \frac{5}{56} \times \frac{2}{2} = \frac{10}{112}$$

$$A \cdot F \cdot = \frac{0.9}{11}$$

$$= 0.089285$$

~~03) 04) 93.~~
* UNIT- 4 (ALGEBRAIC DIVISIONS)

* Case-I: Eg: $(x^2 - 3x^2 + 2x + 5) \div (x-1)$

1	1	-3	2	5	(Horner Method)
x	+1	-2	0	5	
1	-1	-2	0	5	

$$Q = (1)x^2 - 2x + 0$$

$$R = 5$$

$$Q.1) (x^3 + 3x^2 - 2x - 7) \div (x+3)$$

-3	1	3	2	7	
x	-3	0	2	-6	
1	0	2	1		

$$Q = x^3 + 0x^2 + 2x ; R = 1$$

$$2) (x^4 - x^3 + 2x^2 - 9x + 8) \div (x+1)$$

-1	1	-1	2	-9	8
	-1	2	-4	13	
	1	-2	4	-13	21

$$Q = x^3 - 2x^2 + 4x^1 - 13x$$

$$R = 21$$

$$3) (2x^5 - x^4 + x^3 + x + 1) \div (x-1)$$

1	2	-1	1	1	1
	2	1	2	3	
	-2	1	2	3	4

$$Q = 2x^4 + x^3 + 2x^2 + 3x$$

$$R = 4$$

* Case - II : If divisor is near the base

I - Case : Base 10

Sub - Case - I : 721590152 ÷ 9

(10^{-9})

1	7	2	1	5	9	0	1	5	2
	7	9	10	15	24	24	25	30	
	7	9	10	15	24	24	25	30	32

$Q = 80176680$

$R = 32 > 9 \quad \uparrow +$

$32 \div 9 = Q = 3 ; R = 5$

$Q = 80176683 ; R = 5$

Q.1) $302 \div 9$

1	3	0	2
	3	3	
	3	3	5

$Q = 33 ; R = 5$

2) $2515 \div 8$

2	2	5	1	5
	4	18	38	
	2	9	9	43

$$Q = \underline{3}0\ 9 ; R = 43 > 8$$

\downarrow

$\curvearrowleft +$

$$\div 8 = Q = 5 ; R = 3$$

$$Q = 314 ; R = 3$$

3) $4125123 \div 9$

1	4	1	2	5	1	2	3
	4	5	7	12	13	15	
	4	5	7	12	13	15	18

$$Q = 458345 ; R = 18$$

$$Q = 458347 ; R = 0$$

4) $13525037 \div 9$

1	1	3	5	2	5	0	3	7
	1	4	9	9	11	16	16	19
	1	4	9	1	6	6	1	26

$$Q = 1502779 + 2 \Rightarrow 1502781 ; R = 8$$

5) $23721601 \div 8$

2	2	3	7	2	1	6	0	1
	4	14	42	88	179	370	740	

$$Q = 2965200 ; R = 1$$

Sub- Case II : Above the base 10

Eg : $2449 \div 12$

2	4	4	9
	4	0	8
2	0	4	1 < 12

$$Q = 204 ; R = 1$$

Q.1)

$$267 \div 11$$

1	2	6	7	
		2	4	
	2	4	3	

$$Q = 24; R = 3$$

2)

$$25477 \div 12$$

2	2	5	4	7	7
	4	2	4		6
	2	1	2	3	1

$$Q = 2123; R = 1$$

3)

$$39158 \div 13$$

3	3	9	1	5	8
	9	0	3		6
	3	0	1	2	2

$$Q = 3012; R = 2$$

4) $29694 \div 14$

<u>4</u>	2	9	6	9	4
	<u>8</u>	<u>4</u>	<u>8</u>	<u>4</u>	
	2	1	2	1	0

$$Q = 2121 ; R = 0$$

5) $121283151671 \div 12$

<u>2</u>	1	2	1	2	8	3	1	5	1	6	7	1
	<u>2</u>	0	<u>2</u>	0	<u>1</u> 6							
	1	0	1	0	8							

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* Case III : When divisor is in quadratic form

Eg : $(12x^4 - 3x^3 - 3x - 12) \div (x^2 - 3x - 12)$

\downarrow put = 0

$$x^2 - 3x - 12 = 0$$
$$x^2 = 3x + 12$$

3	12	12	-3	0	-3	-12
x	x	12	36	144	396	
x	x	12	33	99	729	2916
x	x	x	33	243	1122	2904

$$Q : 12x^2 + 33x + 243$$

$$R = 1122x + 2904$$

w/o Polynomial : $2403 \div 99$ 100

(compliment)		1	2	4	0	3
					0	2
					0	4
					2	4
					2	7 < 99 ✓

$$Q = 24 ; R = 27$$

$$Q.1) (x^3 - 3x^2 + 3x - 1) \div (x^2 - 2x + 1)$$

$$x^2 - 2x + 1 = 0$$

$$x^2 = -1 + 2x$$

2	-1	1	-3	3	-1
		2		-1	
			-2	1	
		1	-1	0	0

$$Q : x - 1 ; R = 0$$

$$2) (2x^3 + 9x^2 + 18x + 20) \div (x^2 + 2x + 4)$$

$$x^2 = -2x - 4$$

-2	-4	2	9	18	20
		-4		-8	
			-10	-20	
		2	5	0	0

$$Q : 2x + 5 ; R = 0$$

$$3) (x^4 + x^2 + 1) \div (x^2 - x + 1)$$

$$x^2 = x - 1$$

$$\begin{array}{c|ccc|cc} 1 & -1 & 1 & 0 & 1 & 0 & 1 \\ \hline & & 1 & -1 & & 1 & -1 \\ & & & 1 & -1 & & 1 & -1 \\ \hline & & 1 & 1 & 1 & 0 & 0 \end{array}$$

$$Q: x^2 + x + 1$$

$$R = 0$$

$$4) (12x^4 + 41x^3 + 81x^2 + 79x + 42) \div (4x^2 + 7x + 6)$$

$$4x^2 = -7x - 6$$

$$x^2 = -\frac{7}{4}x - \frac{3}{2}$$

Also, divide the dividend by the same no so as to make the base 100.

$$(12x^4 + 41x^3 + 81x^2 + 79x + 42) \div 4$$

$$= 3x^4 + \frac{41}{4}x^3 + \frac{81}{4}x^2 + \frac{79}{4}x + \frac{21}{2}$$

$$\begin{array}{r} -7 \\ 4 \end{array} \quad \begin{array}{r} -3 \\ 2 \end{array}$$

3

41

$$\begin{array}{r} 81 \\ 4 \end{array}$$

$$\begin{array}{r} 79 \\ 4 \end{array}$$

$$\begin{array}{r} 21 \\ 2 \end{array}$$

3

* CASE - II : Base 100

$$\text{Eg: } 2432 \div 89$$

1 1

2 4

3 2

2

2

6 6

2

6

11

8

789
⋮

$$Q = 1; R = 29$$

$$Q = 27; R = 29$$

$$Q.1) \quad 11023 \div 97$$

<u>0</u>	3	1	1	0	2	3
		0	3			
		0		3		
		1	1	3	0	9
					5	(12)

write in
carry form

\downarrow $Q = 113; R = 27$ \times

$$Q = 154; R = 27 \times \quad \therefore Q = 113 \\ R = 62$$

$$Q.2) \quad 53064 \div 93$$

<u>0</u>	7	5	3	0	6	4
		0	35			
		0		21		
				0	245	
		5	3	35	27	9
					24	

$$Q = 585; R = 519$$

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Q. $(2x^4 - 5x^3 + 7x^2 - 3x + 9) \div (2x^2 + x + 1)$

$\frac{-1}{2}$	$\frac{-1}{2}$	2	-5	7	-3	9
			-1	-1	3	3
		2	-6	9		

Sub-case - II : Above the base divisor

Eg: $1056 \div 103$

$100 - 103 = \overline{03}$

$\overline{03}$	10	5 6
	0	$\overline{3}$
	1 0	2 6 < 103

Q = 10 ; R = 26

Q.1) $25841 \div 112$

<u>1</u>	<u>2</u>	2	5	8	4	1
		<u>2</u>	<u>4</u>			
			<u>3</u>		<u>6</u>	
				<u>1</u>	<u>2</u>	
		<u>2</u>	<u>3</u>	<u>1</u>	<u>3</u>	<u>1</u>

$$Q = 231 - 1 = 230 ; R = \boxed{81} = -31 + 112$$

Q.2) $16453 \div 102$

<u>0</u>	<u>2</u>	1	6	4	5	3
		<u>0</u>	<u>2</u>			
			<u>0</u>		<u>1</u>	<u>2</u>
				<u>0</u>	<u>4</u>	
		<u>1</u>	<u>6</u>	<u>2</u>	<u>7</u>	<u>1</u>

$$Q = 161 ; R = 31$$

$$79999 \div 111$$

<u>11</u>	7 9 9	9 9
	7 7	2
	2	0 0
	7 2 0	7 9

$$Q = 720 ; R = 79$$

$$598502 \div 113$$

<u>13</u>	5 9 8 5	0 2
	5 15	4 12
	11	33
	5 4 8	4 29 10

$$\begin{aligned}
 Q &= 54\bar{1}4 \\
 &= 53\bar{1}4 \rightarrow \text{compliment} \\
 &= 5294
 \end{aligned}$$

$$\begin{aligned} R &= 2 \bar{9} \bar{7} \bar{0} \\ &= 280 > 113 \\ &= 280 - 226 \end{aligned}$$

$$R = 54$$

$$\begin{aligned} Q &= 5294 + 2 \\ Q &= 5296 \end{aligned}$$

Q.5 $2468186 \div 121$

$$\begin{array}{r|ccccc|cc} \overline{21} & 2 & 4 & 6 & 8 & 1 & 8 & 6 \\ \hline & 4 & 2 & & & & & \\ & 0 & 0 & & & & & \\ & 8 & 4 & & & & & \\ \hline 2 & 0 & 4 & 0 & \overline{3} & & 0 & 0 \\ & & & & & & 6 & 3 \\ & & & & & & 2 & 3 \end{array}$$

* Case - III : Base 1000

Sub - Case - I :

Eg: $13225 \div 888$

Below Base

112	1 3	2 2 5
	1	1 2
		4 4 8
	1 4	7 9 3
		:

Q = 14 ; R = 793

Q.1) $42301 \div 996$

0 0 4	4 2	3 0 1
	0	0 16
		0 0 8
	4 2	3 , 6 9

Q = 42 ; R = 469

2) $93678 \div 991$

<u>0 0 9</u>	9 3	6 <u>7 8</u>
	0	0 81
		0 0 27

9 3 6 88 35
15 1 5

$$Q = 94; \quad R = 24$$

3) $14367 \div 889$

<u>1 2 1</u>	1 4	3 6 7
	1	2 1
	1	

Q.1) $2152349 \div 992$

<u>0 0 8</u>	<u>2 1 5 2</u>	<u>3 4 9</u>
0 0 16		
0 0 8		
0 0 40		
2 1 5 ①8	0 0 144	
	1 49 153	

$$Q = 2169 ; R = 693 + 8 = 701$$

2) $3245398 \div 989$

<u>0 1 1</u>	<u>3 2 4 5</u>	<u>3 9 8</u>
0 3 3		
0 2		2
0		7 7
3 2 7 10	0 10 10	
	4 7 8	

$$Q = 3281 ; R = 409$$

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Case - III : Base is 1000 \Rightarrow

Sub - case - I : Above the base

Eg : $321987 \div 1003$

$$\begin{array}{c|cc|cc|cc} \overline{003} & 3 & 2 & 1 & 9 & 8 & 7 \\ \hline & 0 & 0 & & \overline{9} & & \\ & 0 & & & 0 & \overline{6} & \\ & & & & 0 & 0 & \overline{3} \\ \hline & 3 & 2 & 1 & 0 & 2 & 4 \end{array}$$

$$Q = 321 ; R = 024$$

Q.1) $13543 \div 1212$

$$\begin{array}{c|cc|cc|cc} \overline{212} & 1 & 3 & 5 & 4 & 3 \\ \hline & 2 & 1 & 2 & & \\ & 2 & 1 & 2 & & \\ \hline & 1 & 1 & 2 & 1 & 1 \end{array}$$

$$Q = 11 ; R = 211$$

$$2) \quad 3456 \div 1321$$

<u>3 2 1</u>	3	4 5 6
		9 6 3
3	5 - 1	3

$$\begin{aligned}
 R &= -5 \times 100 - 1 \times 10 + 3 \\
 Q &= 3 - 1 = 2 \\
 &\quad = -507 + 1321 \\
 R &= 814
 \end{aligned}$$

$$3) \quad 133586 \div 1133$$

<u>1 3 3</u>	1 3 3	5 8 6
	1 3	3
	2	6 6
1 2 2	2 6 6	2 8 12

$$Q = 12\bar{2} = 120 - 2 = 118 - 1 = 117$$

$$\begin{aligned}
 R &= \bar{2}92 \\
 &= -200 + 92 = -108 + 1133 \\
 &= 1025
 \end{aligned}$$

$$4) \quad 24681641 \div 1111$$

<u>111</u>	2 4 6 8 1	6 4 1
	2 2 2	
	2 2 2	
	2 2 2	2
2 2 2 2	5	7 7 6

$$Q = 22215 ; R = 776$$

$$5) \quad 42095312607 \div 1112$$

<u>112</u>	4 2 0 9 5 3 1 2	6 0 7

Q.1)

$$235897 \div 9898$$

<u>0</u> <u>1</u> <u>0</u> <u>2</u>	2 3	5 8 9 <u>7</u>	.
0	2 0 4		
0 3 0	6		

2 3 3 5 5 1

$$Q = 23 ; R = 3551$$

Q.2)

$$21505315 \div 99996$$

<u>0</u> <u>0</u> <u>0</u> <u>0</u> <u>4</u>	2 1 5	0 5 3 1 5	
0 0	0 0 8		
0 0	0 0 0 4		
0 0	0 0 0 0 20		

2 1 5 0 5, 1 5 25

$$Q = 215 ; R = 6175$$

3) $3231520178 \div 9999998$

<u>0000002</u>	323	1520178
	00	000006
	0	0000004
	00	000006
323	1520714	

$Q = 323 ; R = 1520824$

4) $598502 \div 11002$

<u>11002</u>	59	8502
	5	0010
	4	008
	54	45706

$Q = 54$

$$\begin{aligned} R &= 4406 = 4400 - 6 \\ &= 4394 \end{aligned}$$

4) $65295317 \div 100002$

<u>00002</u>	6 5 2	9 5 3 1 7
0 0	0 0	<u>1 2</u>
0	0 0	0 <u>1 0</u>
	0 0	0 0 <u>4</u>
	6 5 2	9 5 <u>9</u> 9 3

$$\begin{aligned} Q &= 652 ; R = 95 \bar{9} \bar{9} 3 \\ &= 95 - 900 + 90 + 3 \\ &= 94013 \end{aligned}$$

5) $123812045 \div 11113$

<u>1 1 1 1 3</u>	1 2 3 8	1 2 0 4 5
	1 1 1	<u>1 3</u>
	1 1	<u>1 1 3</u>
	1	<u>1 1 1 3</u>
		<u>5 5 5 5 1 5</u>
	1 1 1 5	<u>7 8 9 4 1 0</u>

$$\begin{aligned} Q &= 1115 ; R = -70000 - 8000 \\ &\quad - 900 - 50 \end{aligned}$$

$$R = 78950$$

~~11/4/23~~

6) $12023312607 \div 10001123$

$Q =$

Algebraic Division :)

$$(x^5 + x^3 - 7x^2 + 9) \div (x^3 - x^2 + 2x - 3)$$

$$x^3 - x^2 + 2x - 3 = 0$$

$$x^3 = x^2 - 2x + 3$$

1	-2	3	1	0	1	-7	0	9
			1	-2	3			
			1	-2	3			
			0	0	0			
			1	1	0	-6	3	9

$$Q = x^2 + x$$

$$R = -6x^2 + 3x + 9$$

Q. 1) $-2x^5 - 7x^4 + 2x^3 + 18x^2 - 3x - 8 \div x^3 - 2x^2 + 1$

$$x^3 - 2x^2 + 1 = 0$$

$$x^3 = 2x^2 - 1$$

2	0	1	-2	-7	2	18	-3	8
			-4	0		-2		
				-22		0	-11	
					-40	0	-20	
			-2	-11	-20	-24	-14	-12

$$Q = -2x^2 - 11x - 20$$

$$R = -24x^2 - 14x - 12$$

2) $2x^5 - 9x^4 + 5x^3 + 16x^2 - 16x + 36 \div$

$$x^4 + 3x^3 - 16x^2 + 13x + 1$$

-3	-16	3	1	2	-9	5	16	-16	36
				-6		-32	6	2	
				2					

UNIT-4 (FACTORIZATIONS)

$$\text{eg: } (x^2 + 9x + 18)$$

$$\begin{array}{c} \text{constant} \\ 1 : 3 = 6 : 18 \\ \text{coefficient of } x \end{array} \rightarrow \frac{(x+3)}{(x+6)}$$
$$\left(x^2 + \frac{18}{6}x + \frac{18}{6} \right)$$

$$\text{eg: } 6x^2 + 11x + 3$$

$$\begin{array}{c} 6 : 9 \\ \div 3 \end{array} \div = 2 : 3$$
$$(2x+3) (3x+1)$$

$$\text{Q. } 8x^2 - 22x + 5$$

$$8 : -20 = -2 : 5$$

$$(2x-5)(4x-1)$$

~~12/4/23~~

$$8.1 \quad 2x^2 + 3y^2 - z^2 + 5xy - 2yz - xz$$

putting $z=0$

$$2x^2 + 3y^2 + 5xy$$

$$2:3 = 2:3$$

$$(2x+3y)(x+y)$$

Putting $y=0$

$$2x^2 - xz - z^2$$

$$2: -2 = 1: -1$$

$$(x-z)(2x+z)$$

$$2x^2 + 3y^2 - z^2 + 5xy - 2yz - xz = (2x+3y+z)(x+y-z)$$

Q.2) $x^2 + 2y^2 + 3xy + 2xz + 3yz + z^2$

putting $z=0$

$$x^2 + 2y^2 + 3yx + 0$$

$$2 : 1 = 2 : 1$$

$$(2x+y) : (x+y)$$

putting $y=0$

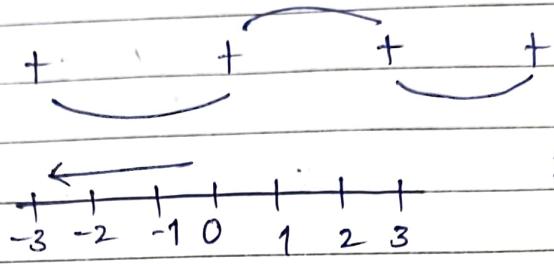
$$x^2 + 2xz + z^2$$

$$1 : 1 = 1 : 1$$

$$(x+z)(x+z)$$

Cubic Factorisation :

$$x^3 + 6x^2 + 11x + 6$$



If no sign change,
go in ← direction

putting $x = -1$

$$(-1)^3 + 6(-1)^2 + 11(-1) + 6 = 0$$

$x = -1$ is a factor

$$(x+1)^0 = 0$$

-1	1	6	11	6
	-1	-5		-6
	1	5	6	0

$$x^2 + 5x + 6$$

$$1 : 3 = 2 : 6$$

$$(x+3)(x+2)$$

$$x^3 + 6x^2 + 11x + 6 = (x+1)(x+3)(x+2)$$

UNIT-5 (OSCOLATOR)* Positive Osculator :-

Osculator of a number is considered positive when the number ends in 9. The formula used to find a positive osculator i.e. "one more than the one before" is that increasing the digit before 9 by one & take as positive osculator of the number.

$$\text{eg: } 29 \rightarrow 2+1=3$$

$$1) 89 \rightarrow 9$$

$$7) 69 \rightarrow 7$$

$$2) 7 \rightarrow 7 \times 7 = \cancel{49} \rightarrow \cancel{5}$$

$$8) 149 \rightarrow 15$$

$$3) 139 \rightarrow 14$$

$$9) 23 \rightarrow 69 \rightarrow 7$$

$$4) 43 \rightarrow 129 \rightarrow 13$$

$$10) 99 \rightarrow 10$$

$$5) 37 \rightarrow \cancel{429} \rightarrow \cancel{13} \ 259 \rightarrow 26$$

$$6) 31 \rightarrow 279 \rightarrow 28$$

Q. Check if 38479 is divisible by 7?

$$\text{Soln: } P.O. \Rightarrow 7 \rightarrow 5$$

← Step - I : Find P.O.
of divisor

$$\begin{array}{r}
 3 \quad 8 \quad 4 \quad 7 \\
 \swarrow \quad \searrow \\
 \textcircled{2} \quad \textcircled{8} \quad 54 \quad 19 \quad \underline{52} \\
 \checkmark
 \end{array}$$

①
 ↓ X P.O.
 45

$$\text{Step - II : } (9 \times P.O.)$$

$$\text{Step - III : } (2 \times P.O.) + 5$$

→ Yes divisible by 7

Q. Check if 27426 is divisible by 13?

Soln:

$$P.O. \text{ of } 13 \rightarrow 4$$

$$\begin{array}{r}
 2 \quad 7 \quad 4 \quad 2 \quad 6 \\
 \swarrow \quad \searrow \\
 \textcircled{3} \quad \underline{10} \quad \underline{30} \quad \underline{26} \\
 \times
 \end{array}$$

Not divisible

Q.1) 63094821 by 79

P.O. of 79 \rightarrow 8

6 3 0 9 4 8 2 (1)
x 8
~~(6)~~ 5 6 4 6 6 ~~4~~ 7 6 9 1 0 8
X Not divisible

2) 27745223 by 19

P.O. of 19 \rightarrow 2

2 7 7 4 5 2 2 (3)
x 2
~~(10)~~ 2 3 8 1 0 2 2 1 8 8 6
X Not divisible

3) 1406121947 by 17

P.O. of 17 \rightarrow 12

1 4 0 6 1 2 1 9 4 7

~~(43)~~ 63 114 69 35 102 48 113 88
X Not divisible

~~17/4/23~~

Page

Date

* Negative Osculator :

Osculator of a number is considered negative when the number ends in 1, to get the negative osculator just drop 1 and the remaining number is taken as the negative osculator.

$$\text{eg: } 21 \rightarrow 2$$

$$41 \rightarrow 4$$

$$3 \times 7 \rightarrow 21 \rightarrow 2$$

1) $51 \rightarrow 5$

10) $13 \rightarrow 9$

2) $17 \rightarrow 5$

3) $37 \rightarrow 11$

4) $71 \rightarrow 7$

5) $47 \rightarrow 14$

6) $23 \rightarrow 16$

7) $81 \rightarrow 8$

8) $111 \rightarrow 11$

9) $121 \rightarrow 12$

Q. Check the divisibility of 84651 by 21.

Soln

$$8 \overline{4} \ 6 \ \overline{5} \quad \begin{array}{l} (1) \xrightarrow{\times P.O.} \\ 2 \end{array}$$

(Always put a bar on the even place nos.)

$$\cancel{0} \overline{4} \ 0 \ \overline{3}$$

Yes, divisible

If two-digit no. is there,
the next digit should
be subtracted

Q. 1) 58321 by 27

$$P.O. \text{ of } 27 \rightarrow 8$$

$$5 \ 8 \ 3 \ 2 \ 1$$

$$\begin{array}{l} (9) 40 \ 4 \ 10 \\ \cancel{5} \quad \text{Not divisible} \end{array}$$

2) 43721 by 51

$$4 \ 3 \ 7 \ 2 \ 1$$

$$(40) 17 \ 42 \ 7$$

3) 404395 by 31

$$4 \ 0 \ 4 \ 3 \ 9 \ 5$$

$$(6) 20 \ 26 \ 17 \ 24$$

4) 442165 by 47

P.O. → 14

4 4 2 1 6 5

(57) 133 39 92 76

5) 38573 by 17

P.O. → 5

3 8 5 7 3

(27) 441722

6) 660311234 by 31

6 6 0 3 1 1 2 3 4

6 0 2 2 0 2 7 2 8 2 9 9

* Long Division in One-line :- (# FLAG Method)

Q. $10576 \div 83$

$$\begin{array}{r} 8^3 \\ \hline 10576 | 6000000 \\ \hline 127 \cdot 421686 \end{array}$$

$$S_1 \Rightarrow G.D = N.D = 10 \div 8 \\ Q=1; R=2$$

$$S_2 \Rightarrow G.D = 25;$$

$$N.D. = 25 - (3 \times 1) \\ = 22 \geq 0$$

Q.1) $5312 \div 94 = 56 \cdot 510638$

$$22 \div 8; Q=2 \\ R=6$$

2) $682 \div 31 = 22.00$

$$S_3 \Rightarrow N.D. = 61 > 0 \\ 61 \div 8; Q=7 \\ R=5$$

3) $70 \div 83 = 0.8433734$

4) $93562 \div 82 = 1141$

5) $72688 \div 61 = 1191.606557$

~~18/4/23~~

* Division of a number by a 3-digit number :-

$$25711 \div 724$$

$$\begin{array}{r|rr}
 724 & 257 & 1 \ 1 \ 0 \ 0 \ 0 \ 0 \\
 & \underline{-4} & 6 \ 4 \ 4 \ 4 \ 3 \\
 & 3 \ 5 & . \ 5 \ 1 \ 2 \ 4 \ 3
 \end{array}$$

$$\begin{aligned}
 S_1: GD &= 47; ND = 97 - \left(\frac{24}{724} \right) \\
 &= 47 - (6+0) \\
 &= 41 \div 7
 \end{aligned}$$

$$S_2 = GD = 61, ND = 61 - \left(\frac{24}{35} \right)$$

$$\begin{aligned}
 &= 61 - (10 + 12) \\
 &= 61 - 22 = 39
 \end{aligned}$$

$$39 \div 7; R = 4; Q = 5$$

$$S_3 = G.D. = 41, N.D. = 41 - \left(\frac{24}{55} \right)$$

$$\begin{aligned}
 &= 41 - (10 + 20) \\
 &= 11
 \end{aligned}$$

$$S_4 = G.D. = 40, N.D. = 40 - \left(\frac{24}{51} \right)$$

$$= 40 - 22 = 18$$

Q.1) $857.21 \div 632$

$$\begin{array}{r}
 6^{32} \\
 \hline
 8 \overline{) 857.21} \\
 \underline{-64} \\
 \hline
 21 \\
 \underline{-12} \\
 \hline
 9 \\
 \underline{-8} \\
 \hline
 1
 \end{array}$$

$1 \cdot \overset{5}{\underset{\wedge}{3}} 6344$

* 4-digit No. :-

Q. $703195 \div 8231$

$$\begin{array}{r}
 8^{231} \\
 \hline
 703 \overline{) 195} \\
 \underline{-16} \\
 \hline
 35 \\
 \underline{-32} \\
 \hline
 3
 \end{array}$$

$85 \cdot 43251$

$S_1 : GD = 63$

$ND = 63 - \left(\frac{231}{8231} \right)$

$= 63 - 16 = 47$

$S_2 = N.D. =$

$71 - \left(\frac{231}{854} \right)$

$= 71 - 34 = 37/8$

$C_3 = 59 - \left(\frac{231}{854} \right)$

$= 59 - 31$

$Q = 3, R = 4$

$$Q.1) \quad 73856 \div 1243$$

$$2) \quad 49362 \div 1632$$

$$3) \quad 25588 \div 7302$$

Sol: 1)

$$\begin{array}{r|rr|l} 12 & 7 & 3 & 8 \\ \hline & 5 & 6 \\ \hline & 6 & 5 \end{array}$$

$$4) \quad 5378942 \div 812059$$

$$5) \quad 7382462 \div 82359619$$

$$6) \quad 3972056 \div 7310568 \cdot 965$$

$$7) \quad 83.7425 \div 91624$$

$$8) \quad 0.0235896 \div 72.3578$$