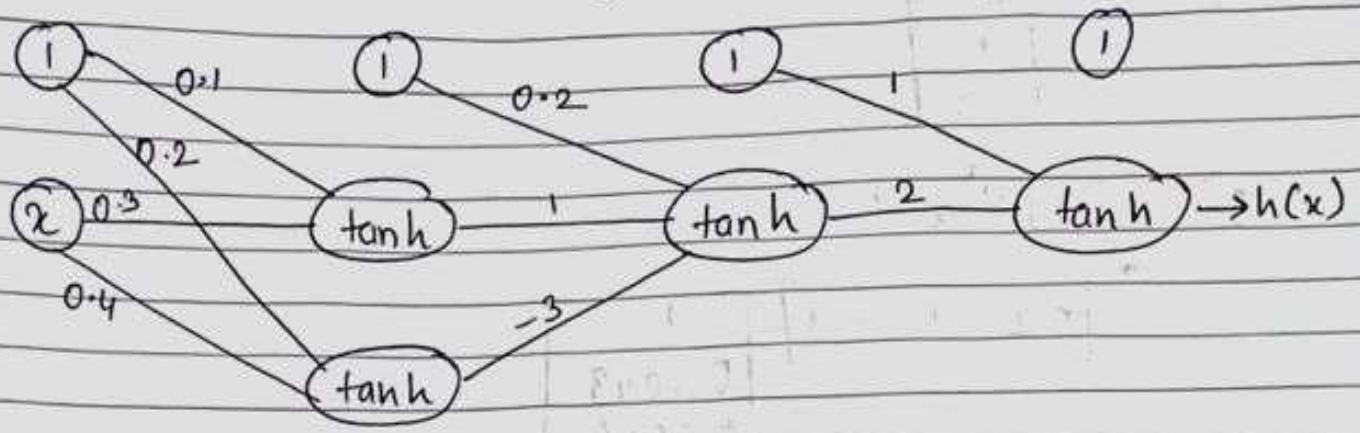
Class Activity 3:



$x = 2$

$y = 1$

Forward propagation

$$W^{(1)} = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{bmatrix} \qquad x^0 = \begin{bmatrix} 1 \\ x \end{bmatrix}$$

$$S^1 = W_1^T x^{(0)}$$

$$= \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & 0.4 \end{bmatrix} \begin{bmatrix} 1 \\ x \end{bmatrix} = \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & 0.4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 + 0.6 \\ 0.2 + 0.8 \end{bmatrix} = \begin{bmatrix} 0.7 \\ 1.0 \end{bmatrix}$$

$$\theta_1^{(1)} = \tanh(S^{(1)})$$

$$= \begin{bmatrix} \tanh(0.7) \\ \tanh(1.0) \end{bmatrix} = \begin{bmatrix} 0.6043 \\ 0.7616 \end{bmatrix}$$

$$x^{(1)} = \begin{bmatrix} 1 \\ 0.6043 \\ 0.7616 \end{bmatrix}$$

$$w^{(2)} = \begin{bmatrix} 0.2 \\ 1 \\ -3 \end{bmatrix}$$

$$s^{(2)} = w^{(2)T} x^{(1)}$$

$$= \begin{bmatrix} 0.2 & 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 0.6043 \\ 0.7616 \end{bmatrix}$$

$$= 0.2 + 0.6043 - 2.2848$$
$$= -1.4805$$
$$\approx \underline{-1.48}$$

$$x^{(2)} = \begin{bmatrix} 1 \\ \tanh(s^{(2)}) \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ \tanh(-1.48) \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -0.901 \end{bmatrix}$$

$$w^{(3)} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$s^{(3)} = w^{(3)T} x^{(2)}$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -0.901 \end{bmatrix}$$

$$= 1 - 1.082$$
$$= \underline{-0.802}$$

$$x^{(3)} = [\tanh(-0.82)]$$

$$= -0.664$$

Backward propagation

$$E(x^{(L)}) = (x^{(L)} - y)^2$$

$$\frac{d\,E(x^{(L)})}{dx^{(L)}} = 2(x^{(L)} - y)$$

$$G_3 = \frac{\partial e(x)}{\partial w^{(3)}} = \frac{\partial e(x)}{\partial s^{(3)}} \times \frac{\partial s^{(3)}}{\partial w^{(3)}}$$

$$\delta^{(3)} = \frac{\partial e(x)}{\partial s^{(3)}} = \frac{d(h(x)^{(3)} - y)^2}{d s^{(3)}}$$

$$= \frac{d(\theta(s^{(3)}) - y)^2}{d s^{(3)}}$$

where $\theta = \theta'(s^{(3)}) = 1 - \tanh^2(s^{(3)})$

when $\theta(s^{(3)}) = \tanh(s^{(3)})$

$$= 2(\theta(s^{(3)}) - y)\,\theta'(s^{(3)})$$

$$= 2[\tanh(s^{(3)}) - y][1 - \tanh^2(s^{(3)})]$$

$$= 2[\tanh(-0.8) - 1][1 - \tanh^2(-0.8)]$$

$$= 2[-0.664 - 1][1 - 0.44]$$

$$= 2[-1.664][0.56]$$

$$= -1.86368 \approx -1.86$$

$$\delta^{(2)} = \frac{\partial e(x)}{\partial s^{(2)}} = \frac{\partial e(x)}{\partial s^{(3)}} \cdot \frac{\partial s^{(3)}}{\partial s^{(2)}} \cdot \frac{\partial x^{(2)}}{\partial x^{(2)}}$$

Where $x^{(2)} = \theta(s^{(2)})$

$$\frac{\partial x^{(2)}}{\partial s^{(2)}} = \theta'(s^{(2)})$$

$$= \left(-1.86 \cdot 2\right) \times \left(1 - \tanh^2(s^{(2)})\right)$$

$$= \left(-1.86 \cdot 2\right)\left[1 - \tanh^2(-1.48)\right]$$

$$= (-1.86 \times 2)\,[0.187]$$

$$= -0.69$$

$$\delta^{(1)} = \frac{\partial e(x)}{\partial s^{(1)}} = \frac{\partial e(x)}{\partial s^{(2)}} \cdot \frac{\partial s^{(2)}}{\partial x^{(1)}} \cdot \frac{\partial x^{(1)}}{\partial s^{(1)}}$$

$$= -0.69 \begin{bmatrix} 1 \\ 3 \end{bmatrix} \begin{bmatrix} 1 - \tanh^2(0.7) \\ 1 - \tanh^2(1.0) \end{bmatrix}$$

$$= \begin{bmatrix} -0.483 \\ 0.869 \end{bmatrix}$$

$$G^3 = x^{(2)} \left(\delta^{(3)}\right)^T$$

$$= \begin{bmatrix} 1 \\ -0.9 \end{bmatrix} \begin{bmatrix} -1.86 \end{bmatrix}$$

$$= \begin{bmatrix} -1.86 \\ 1.67 \end{bmatrix}$$

$$G^{(2)} = x^1 \left(\delta^{(2)}\right)^T$$

$$= \begin{bmatrix} 1 \\ 0.6043 \\ 0.7616 \end{bmatrix} \begin{bmatrix} -0.69 \end{bmatrix}$$

$$= \begin{bmatrix} -0.69 \\ -0.416 \\ -0.524 \end{bmatrix}$$

$$G^{(1)} = x^0 \left(\delta^{(1)}\right)^T$$

$$= \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} -0.443 & 0.869 \end{bmatrix}$$

$$= \begin{bmatrix} -0.443 & 0.869 \\ -0.886 & 1.738 \end{bmatrix}$$

Given the gradient, we can update the weight as follows:

$$W_{new} = W_{old} - \eta G$$

where η is learning rate and G is the gradient

Lets take η as 1

$$W^1 = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{bmatrix} - \begin{bmatrix} -0.443 & 0.869 \\ -0.886 & 1.738 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 + 0.443 & 0.2 - 0.869 \\ 0.3 + 0.886 & 0.4 - 1.738 \end{bmatrix}$$

$$= \begin{bmatrix} 0.543 & -0.669 \\ 1.186 & -1.338 \end{bmatrix}$$

$$W^{(2)} = \begin{bmatrix} 0.2 \\ 1 \\ -3 \end{bmatrix} - 1 \begin{bmatrix} -0.69 \\ -0.416 \\ -0.524 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2 + 0.69 \\ 1 + 0.416 \\ -3 + 0.524 \end{bmatrix} = \begin{bmatrix} 0.89 \\ 1.416 \\ -2.476 \end{bmatrix}$$

$$W^{(3)} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} -0.443 & -1.86 \\ & 1.67 \end{bmatrix}$$

$$= \begin{bmatrix} 2.86 \\ 0.33 \end{bmatrix}$$

New Network is.