

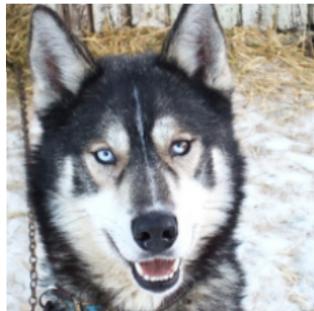
Lecture 4: Model Selection

Can I trust you?

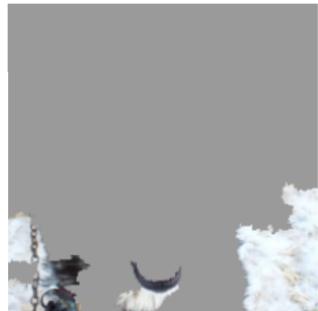
Joaquin Vanschoren

Evaluation

- To know whether we can *trust* our method or system, we need to evaluate it.
- Model selection: choose between different models in a data-driven way.
 - If you cannot measure it, you cannot improve it.
- Convince others that your work is meaningful
 - Peers, leadership, clients, yourself(!)
- When possible, try to *interpret* what your model has learned
 - The signal your model found may just be an artifact of your biased data
 - See 'Why Should I Trust You?' by Marco Ribeiro et al.



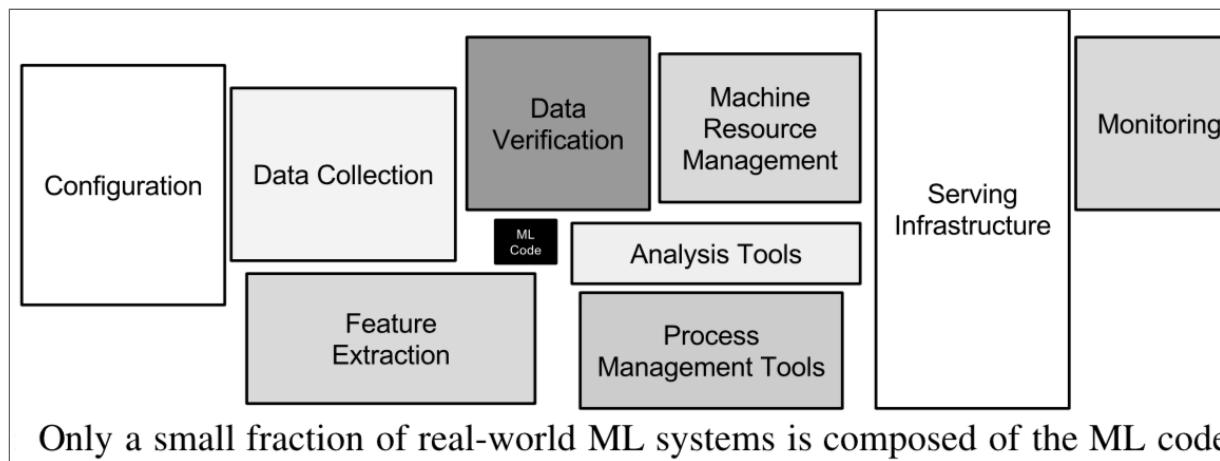
(a) Husky classified as wolf



(b) Explanation

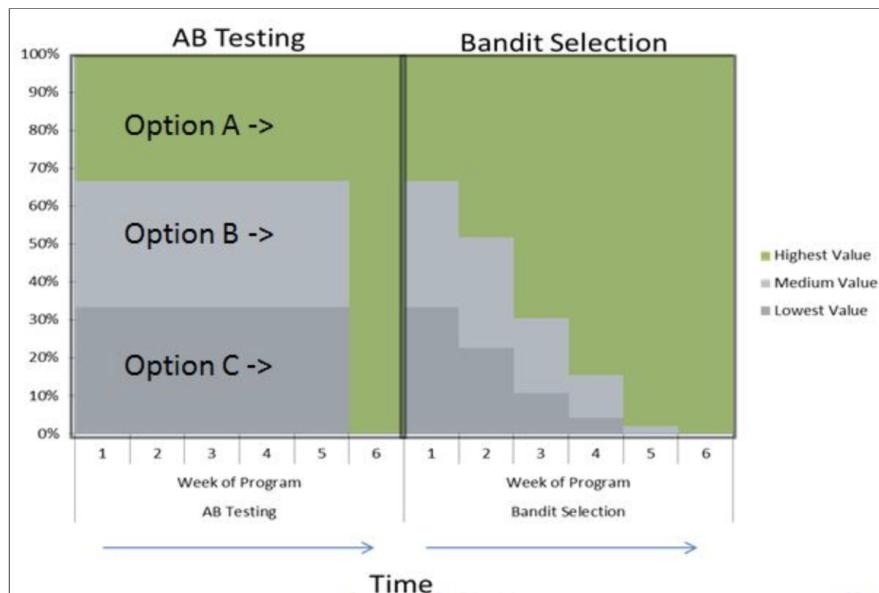
Designing Machine Learning systems

- Just running your favourite algorithm is usually not a great way to start
- Consider the problem: How to measure success? Are there costs involved?
 - Do you want to understand phenomena or do black box modelling?
- Analyze your model's mistakes. Don't just finetune endlessly.
 - Build early prototypes. Should you collect more, or additional data?
 - Should the task be reformulated?
- Overly complex machine learning systems are hard to maintain
 - See 'Machine Learning: The High Interest Credit Card of Technical Debt'



Real world evaluations

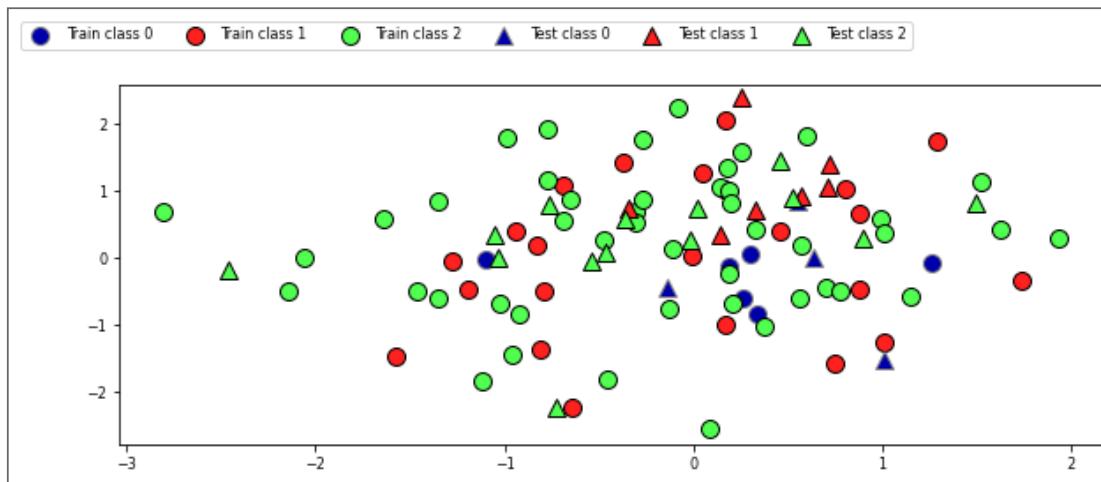
- Evaluate predictions, but also how outcomes improve *because of them*
- Beware of feedback loops: predictions can influence future input data means here data we see
 - Medical recommendations, spam filtering, trading algorithms,...
- Evaluate algorithms *in the wild.*
 - 1 ■ A/B testing: split users in groups, test different models in parallel
 - 2 ■ Bandit testing: gradually direct more users to the winning system



Example:
At least 10 links should be interesting->Recall
Not missing anything->Precision

Performance estimation techniques

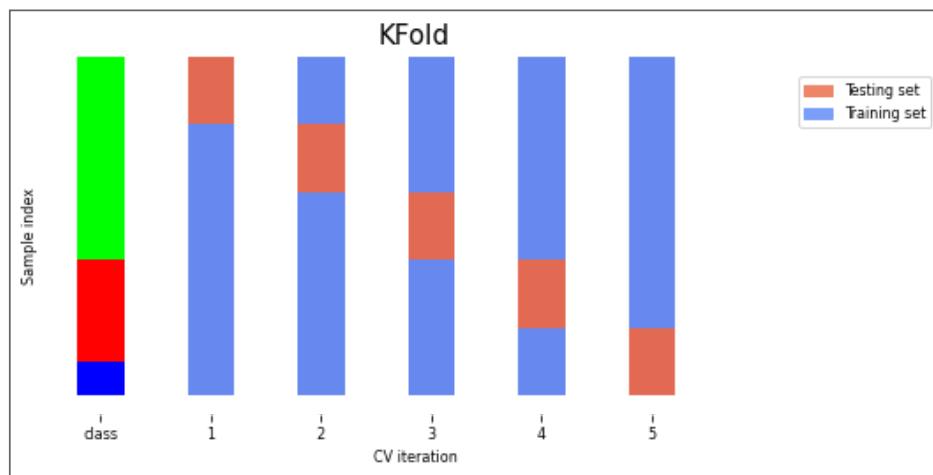
- Always evaluate models as *if they are predicting future data*
- We do not have access to future data, so we pretend that some data is hidden
- Simplest way: the *holdout* (simple train-test split)
 - Randomly split data (and corresponding labels) into training and test set (e.g. 75%-25%) one of the pitfall is that it introduces bias, and may sometimes mix up train test data
 - Train (fit) a model on the training data, score on the test data



K-fold Cross-validation

since the splits are completely independent so the ques of data independence does not arrise

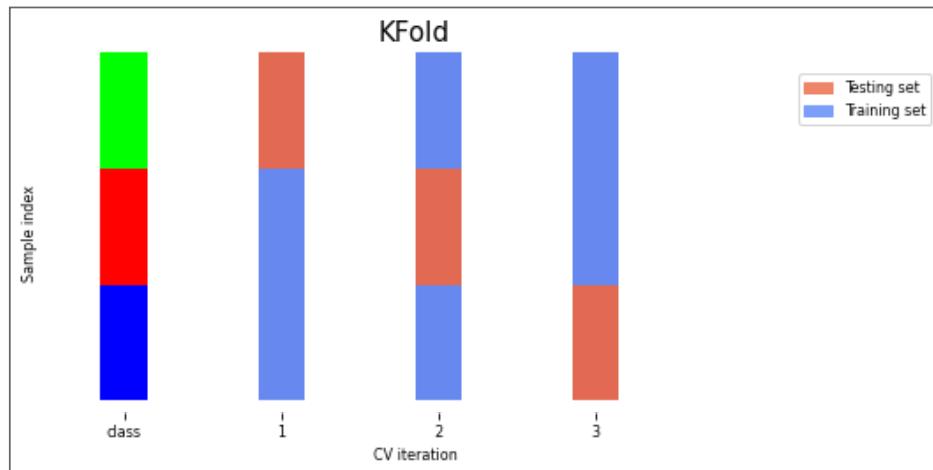
- Each random split can yield very different models (and scores)
 - e.g. all easy (of hard) examples could end up in the test set
- Split data into k equal-sized parts, called *folds*
 - Create k splits, each time using a different fold as the test set
- Compute k evaluation scores, aggregate afterwards (e.g. take the mean)
- Examine the score variance to see how *sensitive* (unstable) models are
- Large k gives better estimates (more training data), but is expensive



Can you explain this result?

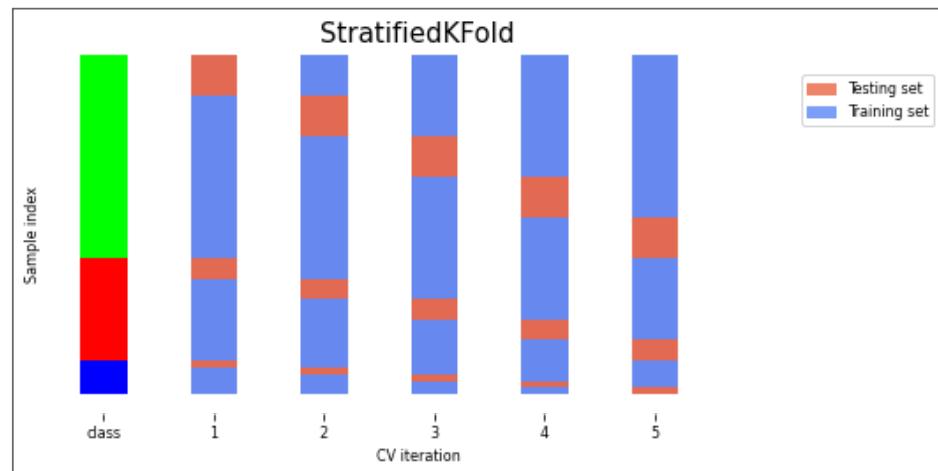
```
kfold = KFold(n_splits=3)
cross_val_score(logistic_regression, iris.data, iris.target, cv=kfold)
```

```
Cross-validation scores KFold(n_splits=3):
[0. 0. 0.]
```



STRATIFIED K-FOLD CROSS-VALIDATION

- If the data is unbalanced, some classes have only few samples
- Likely that some classes are not present in the test set
- Stratification: *proportions* between classes are conserved in each fold
 - Order examples per class
 - Separate the samples of each class in k sets (strata)
 - Combine corresponding strata into folds

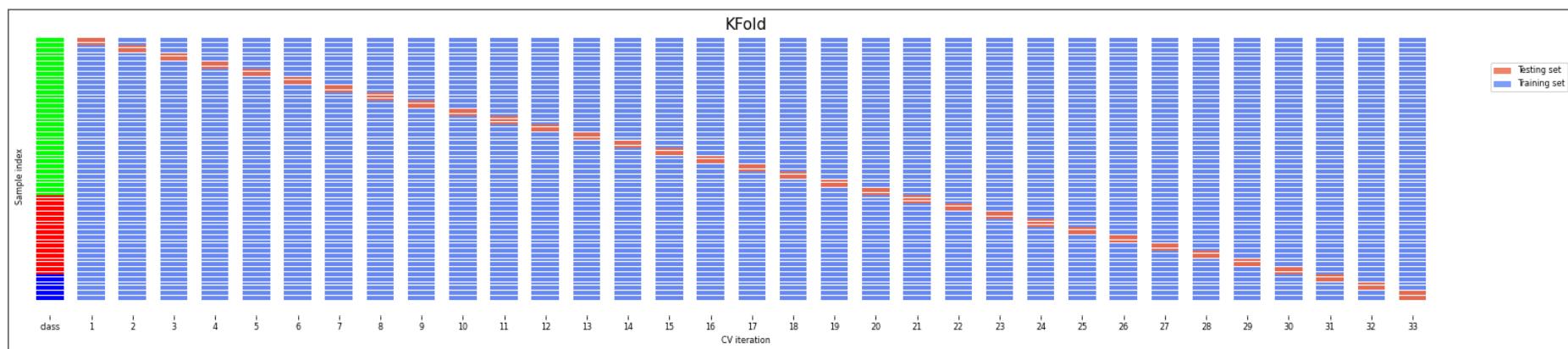


it is likely for every set same proportion of blue, red and green classes is distributed across the five. Blue->1/5. Red->1/5, Green ->1/5

also in this retraining five times does not occur but individual training occurs for 5 models independently

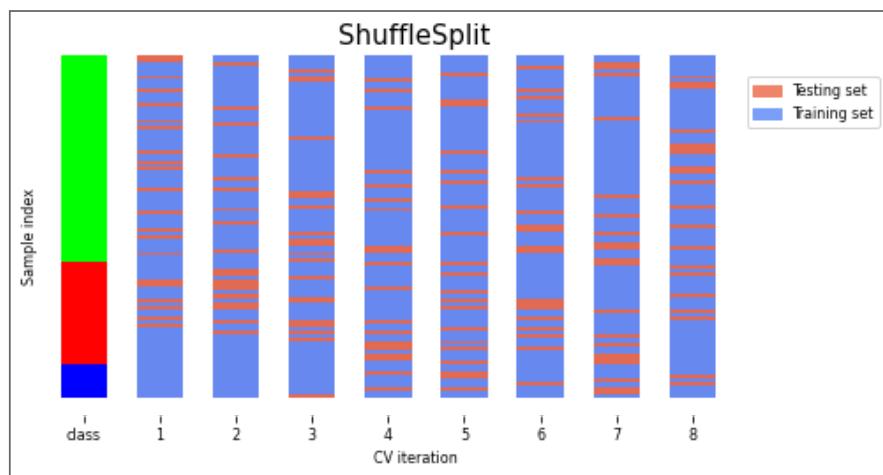
LEAVE-ONE-OUT CROSS-VALIDATION

- k fold cross-validation with k equal to the number of samples
- Completely unbiased (in terms of data splits), but computationally expensive
- Actually generalizes less well towards unseen data
 - The training sets are correlated (overlap heavily)
 - Overfits on the data used for (the entire) evaluation
 - A different sample of the data can yield different results
- Recommended only for small datasets



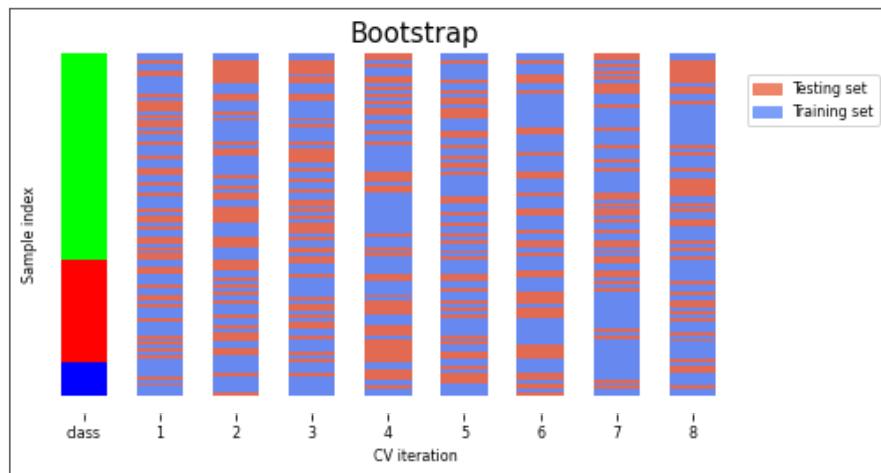
SHUFFLE-SPLIT CROSS-VALIDATION

- Shuffles the data, samples (`train_size`) points randomly as the training set
 - Can also use a smaller (`test_size`), handy with very large datasets
 - Never use if the data is ordered (e.g. time series)



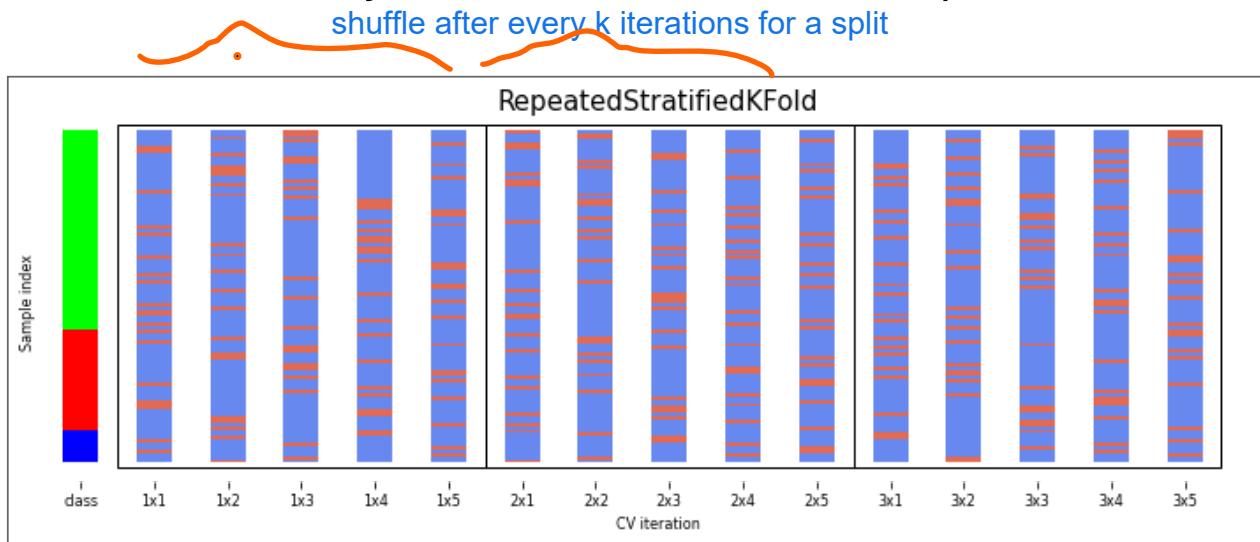
The Bootstrap possible to take same point twice

- Sample n (dataset size) data points, with replacement, as training set (the bootstrap)
 - On average, bootstraps include 66% of all data points (some are duplicates)
- Use the unsampled (out-of-bootstrap) samples as the test set
- Repeat k times to obtain k scores
- Similar to Shuffle-Split with `train_size=0.66`, `test_size=0.34` but without duplicates



Repeated cross-validation

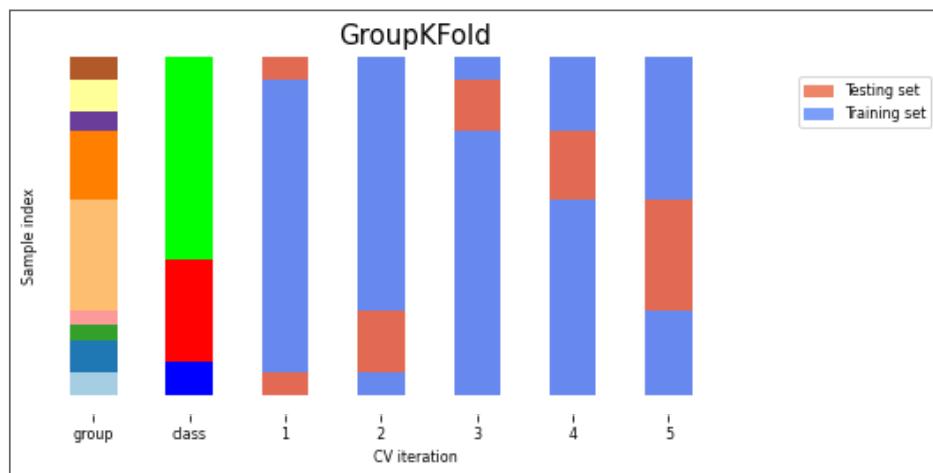
- Cross-validation is still biased in that the initial split can be made in many ways
- Repeated, or n-times-k-fold cross-validation:
 - Shuffle data randomly, do k-fold cross-validation
 - Repeat n times, yields n times k scores
- Unbiased, very robust, but n times more expensive



Cross-validation with groups

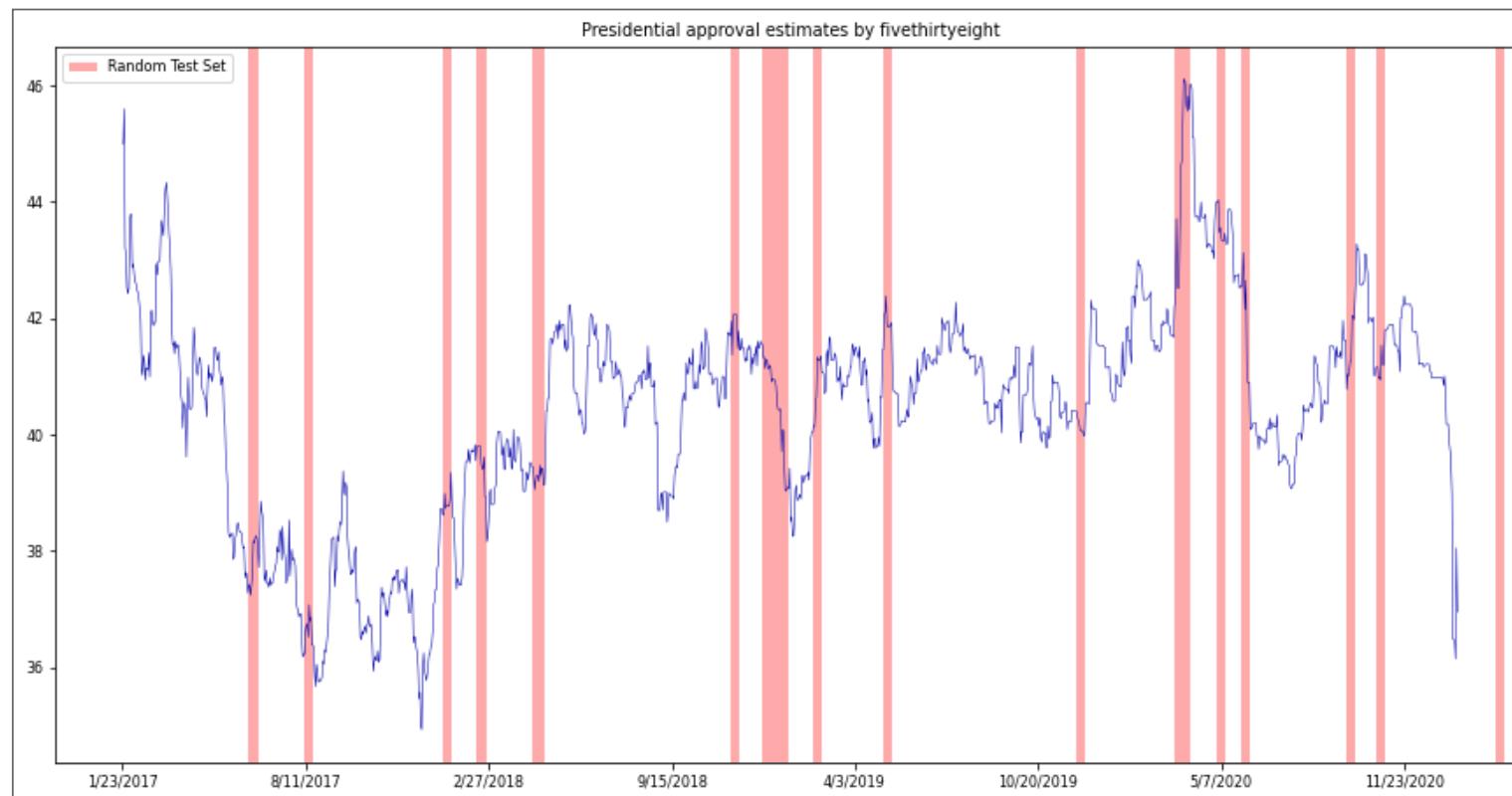
- Sometimes the data contains inherent groups:
 - Multiple samples from same patient, images from same person,...
- Data from the same person may end up in the training *and* test set
- We want to measure how well the model generalizes to *other* people
- Make sure that data from one person are in *either* the train or test set
 - This is called *grouping or blocking* ?
 - Leave-one-subject-out cross-validation: test set for each subject/group

to avoid shortcuts in training
we do not want data of same person en
ing in test and train set; this may hamper
whether model generalises properly or not
.Ex-Model may pick upon weird ears



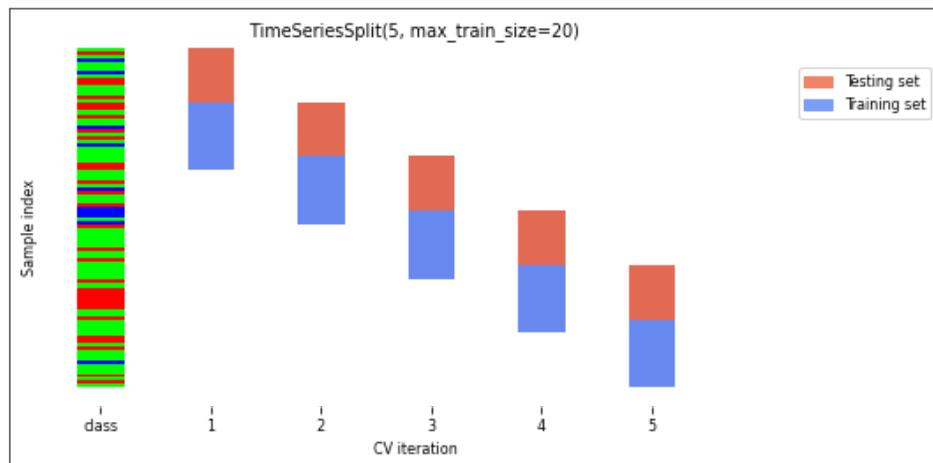
Time series

When the data is ordered, random test sets are not a good idea



TEST-THEN-TRAIN (PREQUENTIAL EVALUATION)

- Every new sample is evaluated only once, then added to the training set
 - Can also be done in batches (of n samples at a time)
- TimeSeriesSplit
 - In the k th split, the first k folds are the train set and the $(k+1)$ th fold as the test set
 - Often, a maximum training set size (or window) is used
 - more robust against concept drift (change in data over time)



Choosing a performance estimation procedure

No strict rules, only guidelines:

- Always use stratification for classification (sklearn does this by default)
- Use holdout for very large datasets (e.g. >1.000.000 examples)
 - Or when learners don't always converge (e.g. deep learning)
- Choose k depending on dataset size and resources
 - Use leave-one-out for very small datasets (e.g. <100 examples)
 - Use cross-validation otherwise
 - Most popular (and theoretically sound): 10-fold CV
 - Literature suggests 5x2-fold CV is better
- Use grouping or leave-one-subject-out for grouped data
- Use train-then-test for time series

Evaluation Metrics for Classification

Evaluation vs Optimization

we cannot do that job with only loss

maybe utilising only accuracy

- Each algorithm optimizes a given objective function (on the training data)
 - E.g. remember L2 loss in Ridge regression

$$\mathcal{L}_{Ridge} = \sum_{n=1}^N (y_n - (\mathbf{w}\mathbf{x}_n + w_0))^2 + \alpha \sum_{i=0}^p w_i^2$$

- The choice of function is limited by what can be efficiently optimized
- However, we *evaluate* the resulting model with a score that makes sense **in the real world**
 - Percentage of correct predictions (on a test set)
 - The actual cost of mistakes (e.g. in money, time, lives,...)
- We also tune the algorithm's hyperparameters to maximize that score

Binary classification

- We have a positive and a negative class
- 2 different kind of errors:
 - False Positive (type I error): model predicts positive while true label is negative
 - False Negative (type II error): model predicts negative while true label is positive
- They are not always equally important
 - Which side do you want to err on for a medical test?
fn more lethal, predicting no cancer when person has cancer



CONFUSION MATRICES

- We can represent all predictions (correct and incorrect) in a confusion matrix
 - n by n array (n is the number of classes)
 - Rows correspond to true classes, columns to predicted classes
 - Count how often samples belonging to a class C are classified as C or any other class.
 - For binary classification, we label these true negative (TN), true positive (TP), false negative (FN), false positive (FP)

	Predicted Neg	Predicted Pos
Actual Neg	TN	FP
Actual Pos	FN	TP

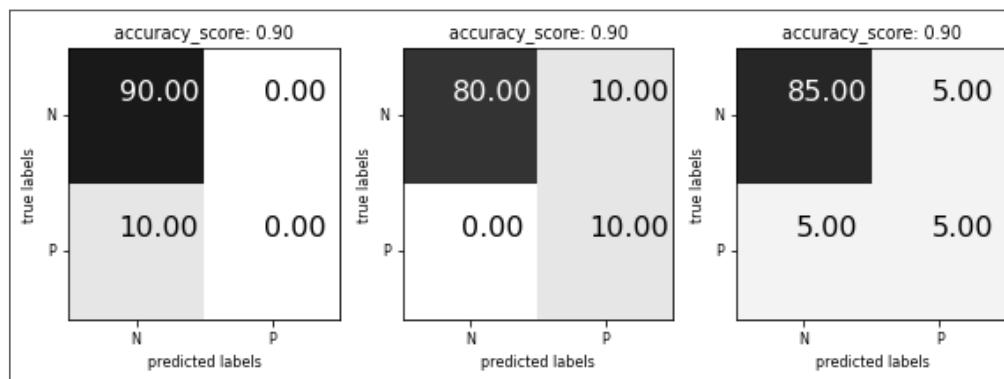
```
confusion_matrix(y_test, y_pred):
[[48  5]
 [ 5 85]]
```

PREDICTIVE ACCURACY

- Accuracy can be computed based on the confusion matrix
- Not useful if the dataset is very imbalanced
 - E.g. credit card fraud: is 99.99% accuracy good enough?

$$\text{Accuracy} = \frac{\text{TP} + \text{TN}}{\text{TP} + \text{TN} + \text{FP} + \text{FN}} \quad (1)$$

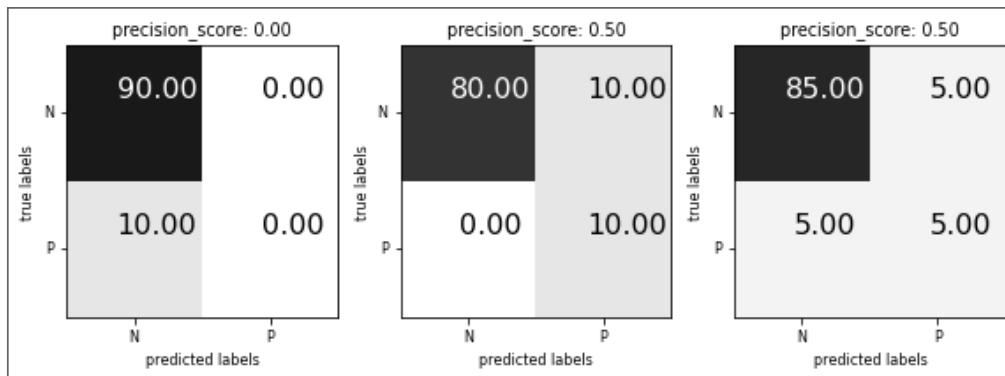
- 3 models: very different predictions, same accuracy:



PRECISION

- Use when the goal is to limit FPs
 - Clinical trials: you only want to test drugs that really work
 - Search engines: you want to avoid bad search results

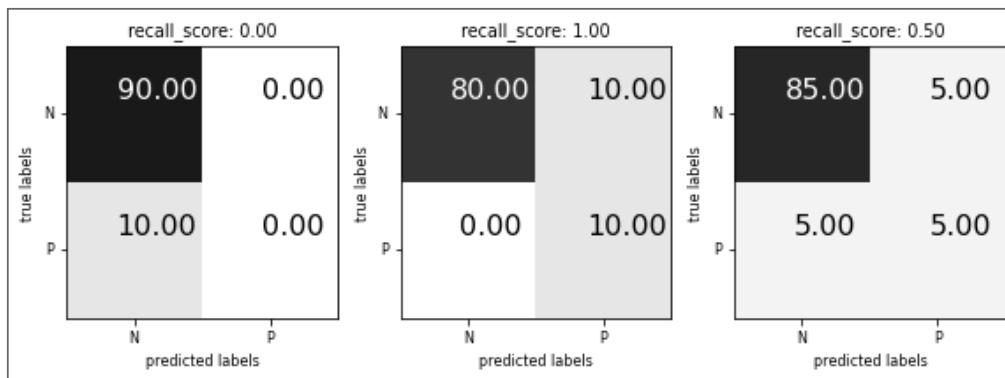
$$\text{Precision} = \frac{\text{TP}}{\text{TP} + \text{FP}} \quad (2)$$



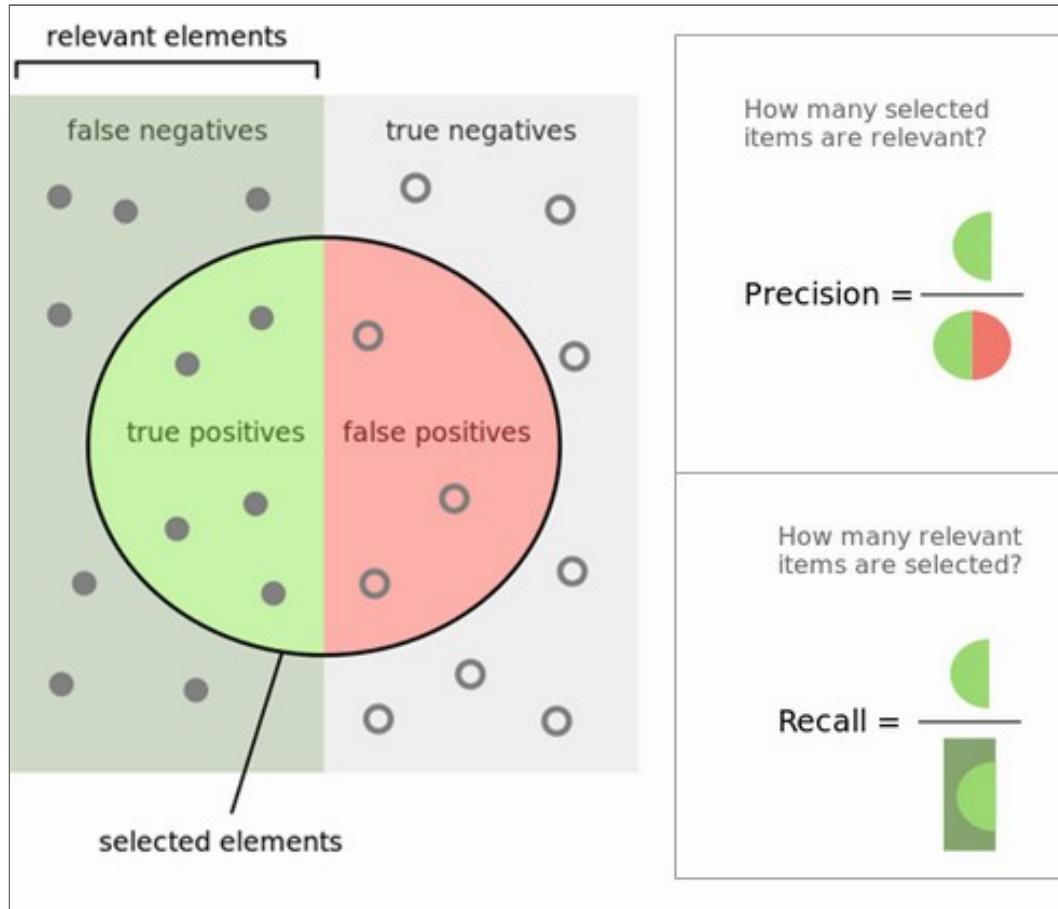
RECALL

- Use when the goal is to limit FNs
 - Cancer diagnosis: you don't want to miss a serious disease
 - Search engines: You don't want to omit important hits
- Also known as sensitivity, hit rate, true positive rate (TPR)

$$\text{Recall} = \frac{\text{TP}}{\text{TP} + \text{FN}} \quad (3)$$



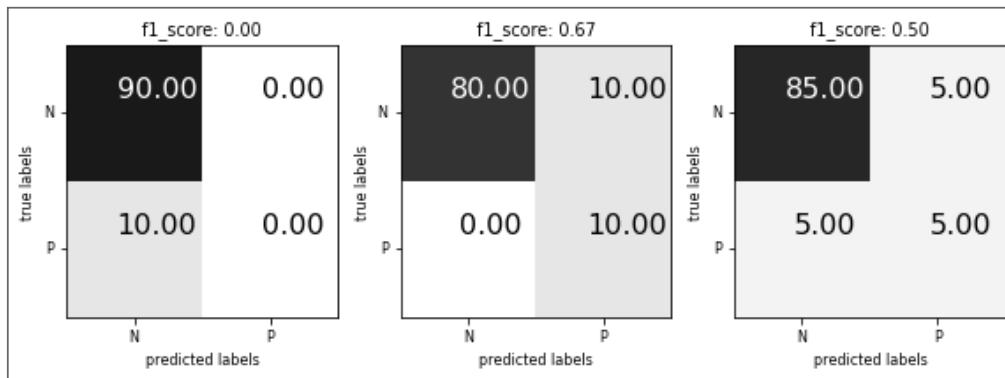
Comparison



F1-SCORE

- Trades off precision and recall:

$$F1 = 2 \cdot \frac{\text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}} \quad (4)$$



Classification measure Zoo

		True condition			
		Condition positive	Condition negative	Prevalence = $\frac{\sum \text{Condition positive}}{\sum \text{Total population}}$	Accuracy (ACC) = $\frac{\sum \text{True positive} + \sum \text{True negative}}{\sum \text{Total population}}$
Predicted condition	Predicted condition positive	True positive, Power	False positive, Type I error	Positive predictive value (PPV), Precision $= \frac{\sum \text{True positive}}{\sum \text{Predicted condition positive}}$	False discovery rate (FDR) = $\frac{\sum \text{False positive}}{\sum \text{Predicted condition positive}}$
	Predicted condition negative	False negative, Type II error	True negative	False omission rate (FOR) = $\frac{\sum \text{False negative}}{\sum \text{Predicted condition negative}}$	Negative predictive value (NPV) = $\frac{\sum \text{True negative}}{\sum \text{Predicted condition negative}}$
		True positive rate (TPR), Recall, Sensitivity, probability of detection = $\frac{\sum \text{True positive}}{\sum \text{Condition positive}}$	False positive rate (FPR), Fall-out, probability of false alarm = $\frac{\sum \text{False positive}}{\sum \text{Condition negative}}$	Positive likelihood ratio (LR+) = $\frac{\text{TPR}}{\text{FPR}}$	Diagnostic odds ratio (DOR) = $\frac{\text{LR}^+}{\text{LR}^-}$
		False negative rate (FNR), Miss rate $= \frac{\sum \text{False negative}}{\sum \text{Condition positive}}$	Specificity (SPC), Selectivity, True negative rate (TNR) = $\frac{\sum \text{True negative}}{\sum \text{Condition negative}}$	Negative likelihood ratio (LR-) = $\frac{\text{FNR}}{\text{TNR}}$	$F_1 \text{ score} = \frac{2}{\frac{1}{\text{Recall}} + \frac{1}{\text{Precision}}}$

https://en.wikipedia.org/wiki/Precision_and_recall

Multi-class classification

- Train models *per class*: one class viewed as positive, other(s) als negative, then average
 - micro-averaging: count total TP, FP, TN, FN (every sample equally important) ?
example theft ○ micro-precision, micro-recall, micro-F1, accuracy are all the same

$$\text{Precision: } \frac{\sum_{c=1}^C \text{TP}_c}{\sum_{c=1}^C \text{TP}_c + \sum_{c=1}^C \text{FP}_c} \xrightarrow{c=2} \frac{\text{TP} + \text{TN}}{\text{TP} + \text{TN} + \text{FP} + \text{FN}}$$

- macro-averaging: average of scores $R(y_c, \hat{y}_c)$ obtained on each class
recommend books ○ Preferable for imbalanced classes (if all classes are equally important) ?
○ macro-averaged recall is also called *balanced accuracy*

$$\frac{1}{C} \sum_{c=1}^C R(y_c, \hat{y}_c)$$

- weighted averaging (w_c : ratio of examples of class c , aka support):
 $\sum_{c=1}^C w_c R(y_c, \hat{y}_c)$

true labels			precision	recall	f1-score	support
	N	P				
N	90.00	0.00	0	0.90	1.00	0.95
	10.00	0.00		0.00	0.00	10
P	accuracy				0.90	100
	macro avg		0.45	0.50	0.47	100
P	weighted avg		0.81	0.90	0.85	100
	N	p				
true labels			precision	recall	f1-score	support
	N	P				
N	80.00	10.00	0	1.00	0.89	0.94
	0.00	10.00		0.50	1.00	0.67
P	accuracy				0.90	100
	macro avg		0.75	0.94	0.80	100
P	weighted avg		0.95	0.90	0.91	100
	N	p				
true labels			precision	recall	f1-score	support
	N	P				
N	85.00	5.00	0	0.94	0.94	0.94
	5.00	5.00		0.50	0.50	0.50
P	accuracy				0.90	100
	macro avg		0.72	0.72	0.72	100
P	weighted avg		0.90	0.90	0.90	100
	N	p				

Other useful classification metrics

- Cohen's Kappa
 - Measures 'agreement' between different models (aka inter-rater agreement)
 - To evaluate a single model, compare it against a model that does random guessing
 - Similar to accuracy, but taking into account the possibility of predicting the right class by chance
 - Can be weighted: different misclassifications given different weights
 - 1: perfect prediction, 0: random prediction, negative: worse than random
 - With p_0 = accuracy, and p_e = accuracy of random classifier:

$$\kappa = \frac{p_0 - p_e}{1 - p_e}$$

Type text here

- Matthews correlation coefficient
 - Corrects for imbalanced data, alternative for balanced accuracy or AUROC
 - 1: perfect prediction, 0: random prediction, -1: inverse prediction

$$MCC = \frac{tp \times tn - fp \times fn}{\sqrt{(tp + fp)(tp + fn)(tn + fp)(tn + fn)}}$$

Probabilistic evaluation

- Classifiers can often provide uncertainty estimates of predictions.
- Remember that linear models actually return a numeric value.
 - When $\hat{y} < 0$, predict class -1, otherwise predict class +1

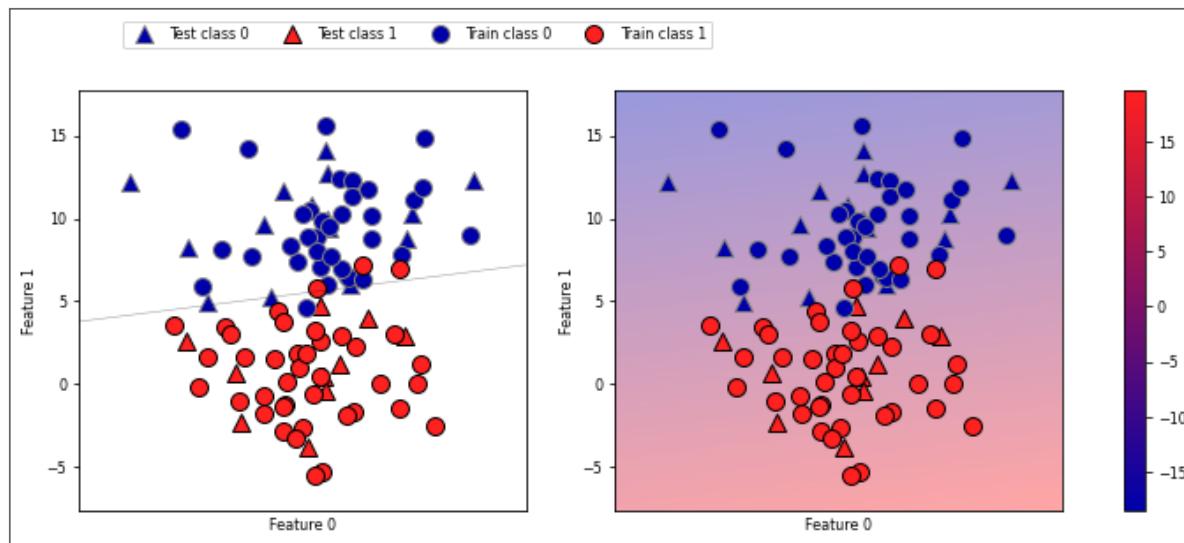
$$\hat{y} = w_0 * x_0 + w_1 * x_1 + \dots + w_p * x_p + b$$

- In practice, you are often interested in how certain a classifier is about each class prediction (e.g. cancer treatments).
- Most learning methods can return at least one measure of *confidence* in their predictions.
 - Decision function: floating point value for each sample (higher: more confident)
 - Probability: estimated probability for each class

The decision function

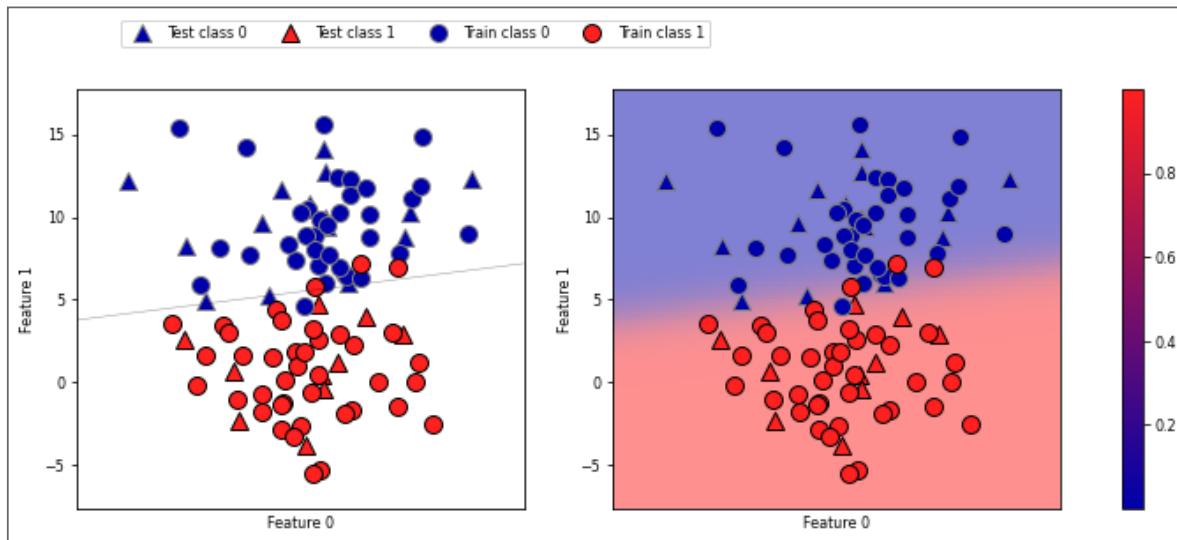
In the binary classification case, the return value of the decision function encodes how strongly the model believes a data point belongs to the “positive” class.

- Positive values indicate preference for the positive class.
- The range can be arbitrary, and can be affected by hyperparameters. Hard to interpret.



Predicting probabilities

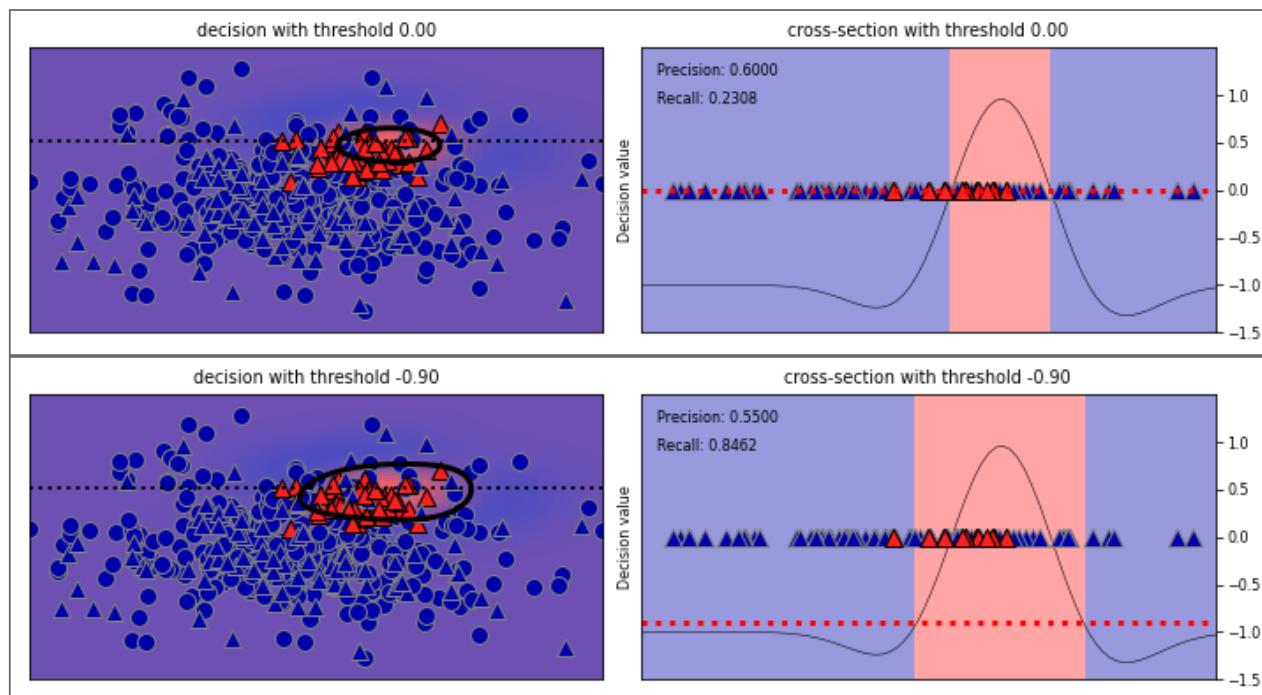
Some models can also return a *probability* for each class with every prediction. These sum up to 1. We can visualize them again. Note that the gradient looks different now.



Threshold calibration

- By default, we threshold at 0 for `decision_function` and 0.5 for `predict_proba`
- Depending on the application, you may want to threshold differently
 - Lower threshold yields fewer FN (better recall), more FP (worse precision), and vice-versa

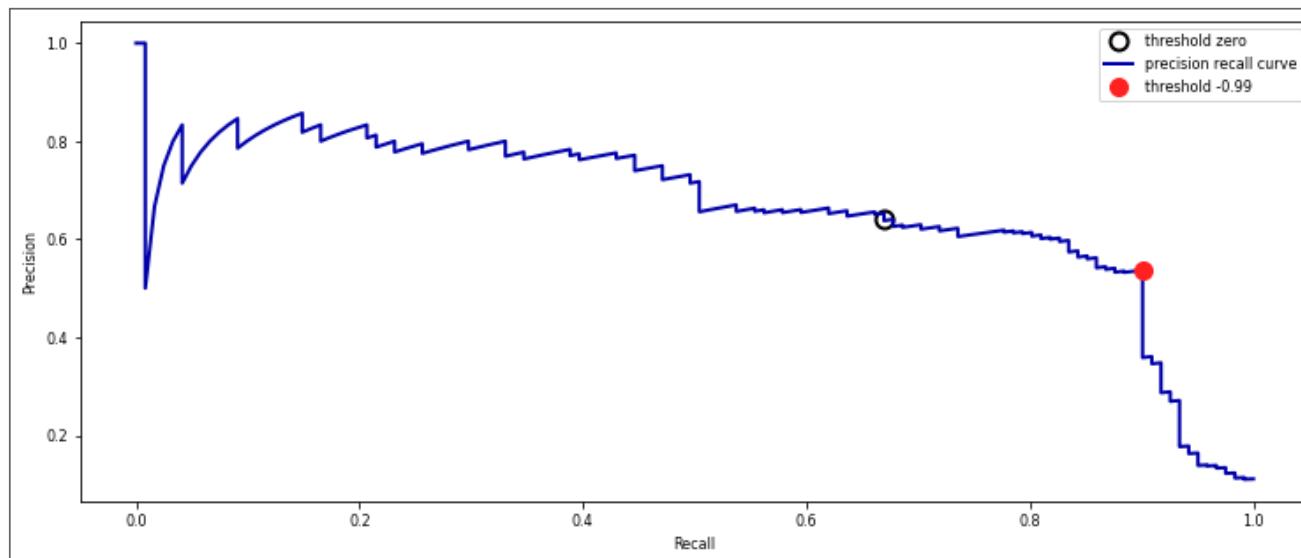
inside points red outside points blue



5yc

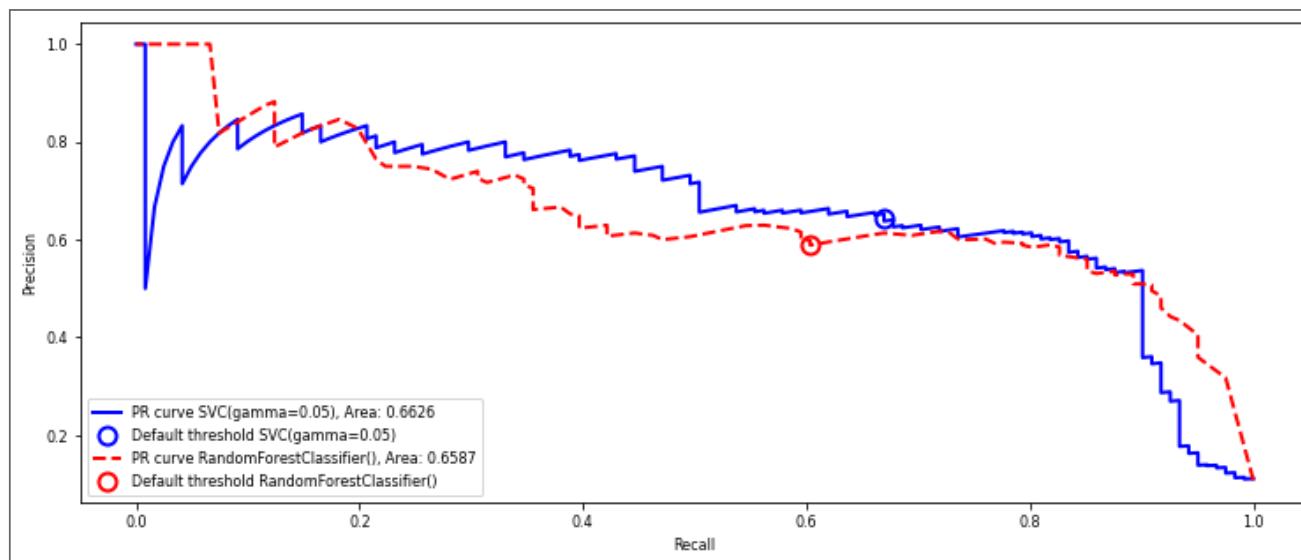
Precision-Recall curve

- The best trade-off between precision and recall depends on your application
 - You can have arbitrary high recall, but you often want reasonable precision, too.
- Plotting precision against recall *for all possible thresholds* yields a **precision-recall curve**
 - Change the threshold until you find a sweet spot in the precision-recall trade-off
 - Often jagged at high thresholds, when there are few positive examples left



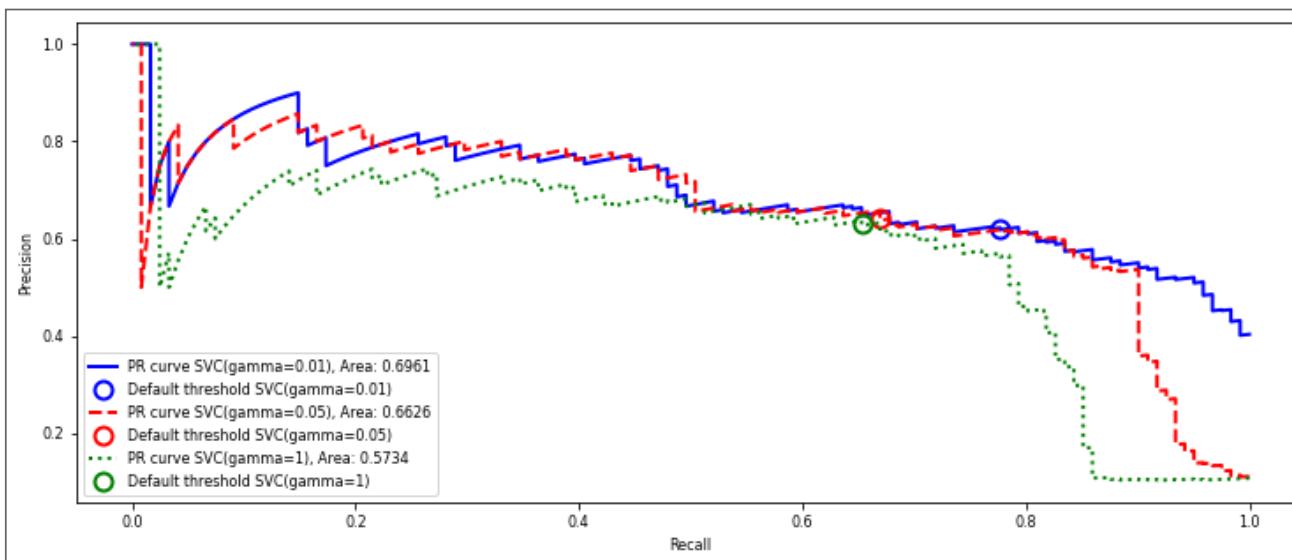
MODEL SELECTION

- Some models can achieve trade-offs that others can't
- Your application may require very high recall (or very high precision)
 - Choose the model that offers the best trade-off, given your application
- The area under the PR curve (AUPRC) gives the *best overall* model



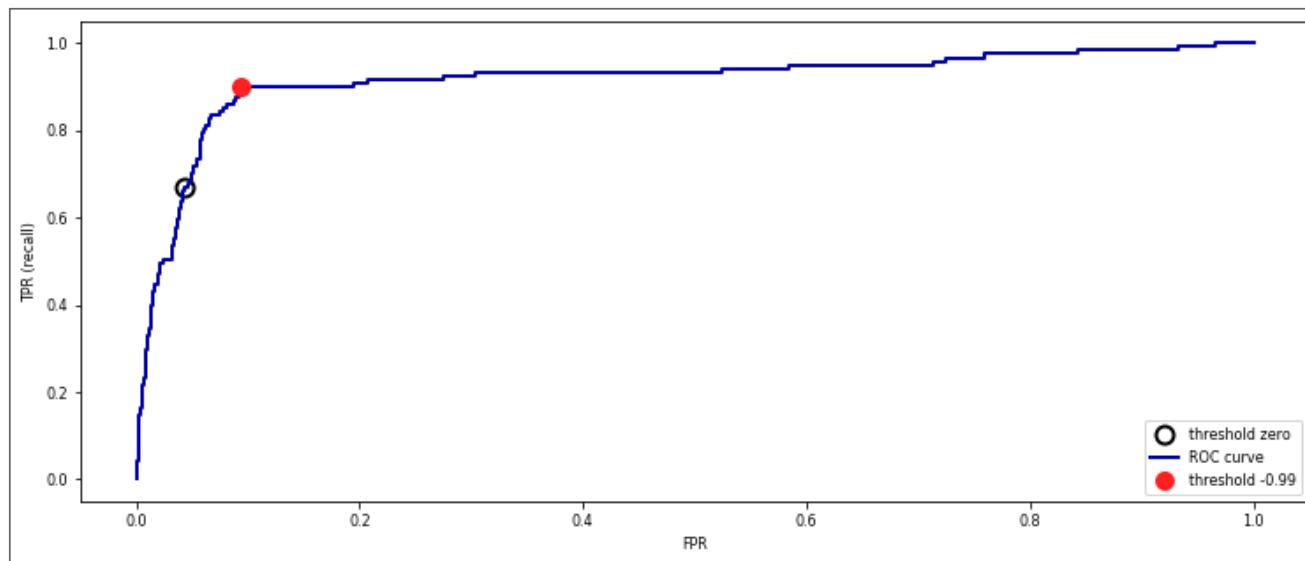
HYPERPARAMETER EFFECTS

Of course, hyperparameters affect predictions and hence also the shape of the curve



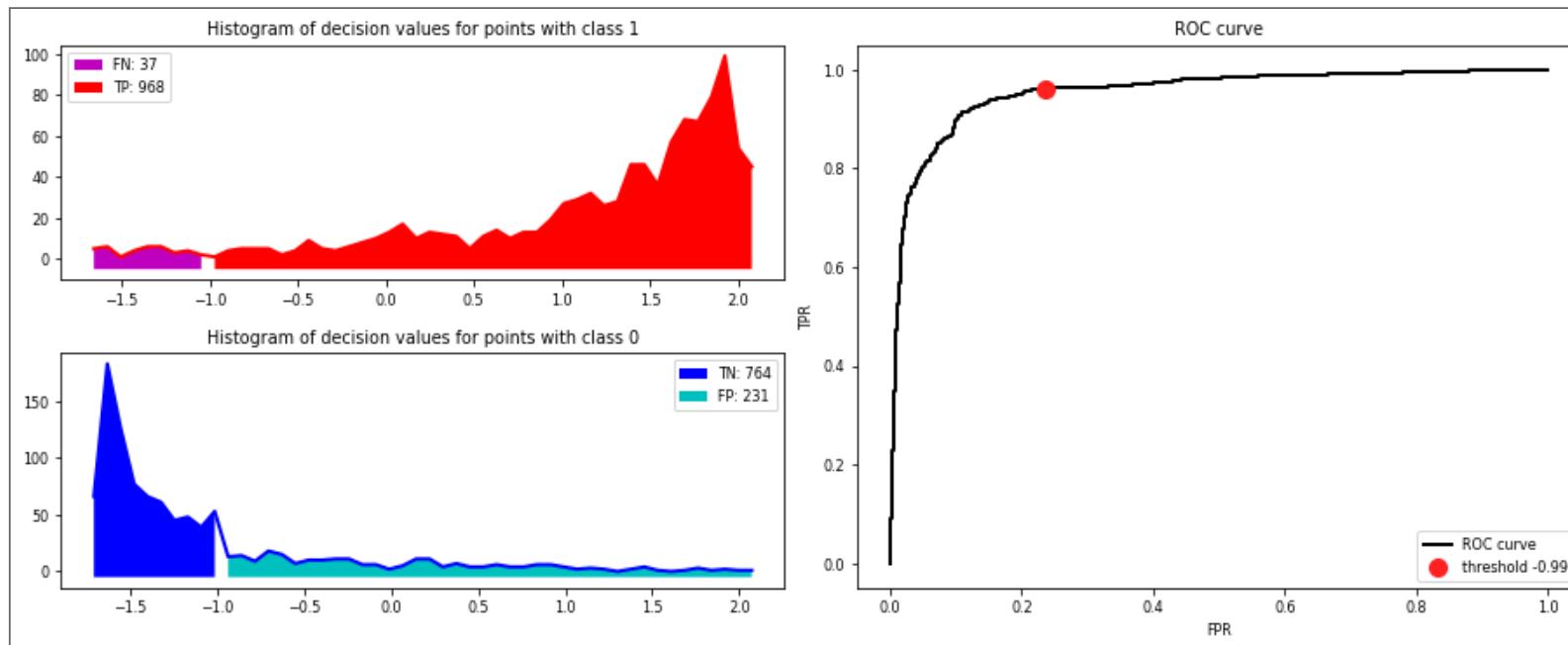
Receiver Operating Characteristics (ROC)

- Trade off *true positive rate* $TPR = \frac{TP}{TP+FN}$ with *false positive rate* $FPR = \frac{FP}{FP+TN}$
- Plotting TPR against FPR for all possible thresholds yields a *Receiver Operating Characteristics curve*
 - Change the threshold until you find a sweet spot in the TPR-FPR trade-off
 - Lower thresholds yield higher TPR (recall), higher FPR, and vice versa



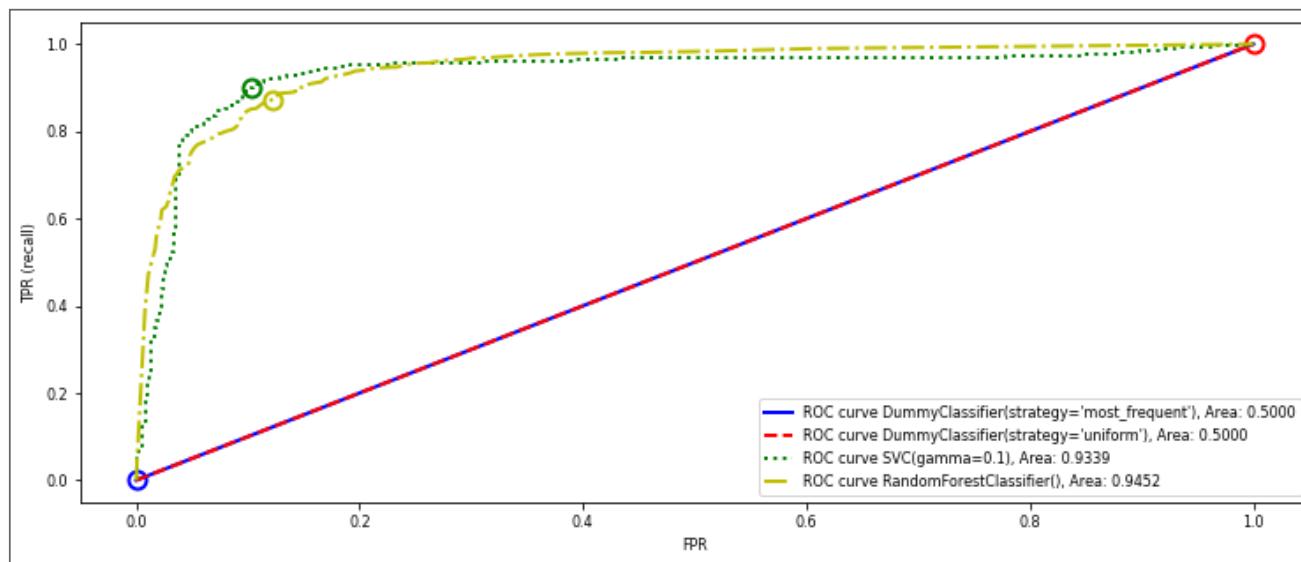
VISUALIZATION

- Histograms show the amount of points with a certain decision value (for each class)
- $TPR = \frac{TP}{TP+FN}$ can be seen from the positive predictions (top histogram)
- $FPR = \frac{FP}{FP+TN}$ can be seen from the negative predictions (bottom histogram)



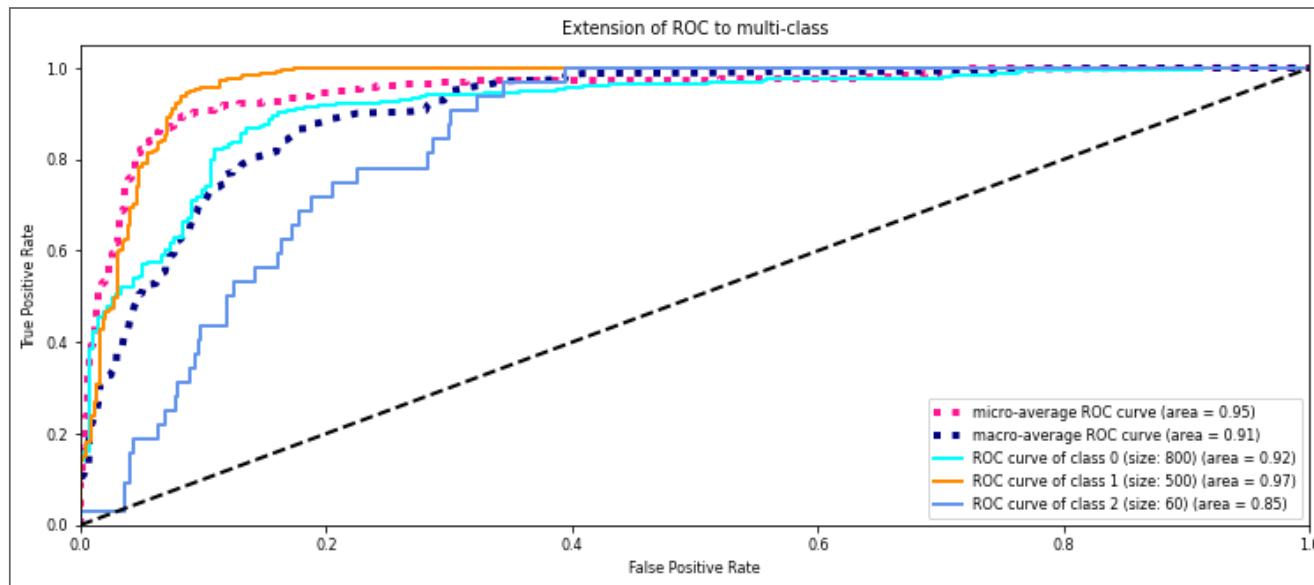
MODEL SELECTION

- Again, some models can achieve trade-offs that others can't
- Your application may require minimizing FPR (low FP), or maximizing TPR (low FN)
- The area under the ROC curve (AUROC or AUC) gives the *best overall* model
 - Frequently used for evaluating models on imbalanced data
 - Random guessing ($\text{TPR}=\text{FPR}$) or predicting majority class ($\text{TPR}=\text{FPR}=1$): 0.5 AUC



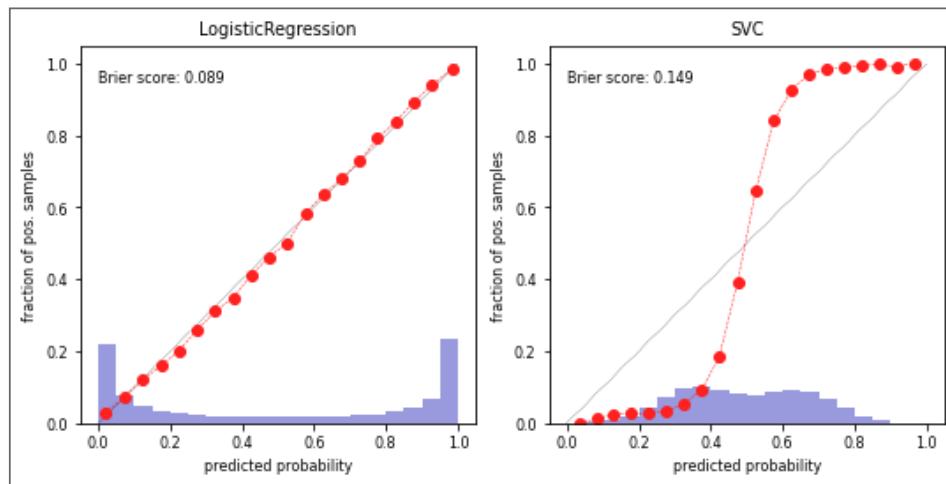
MULTI-CLASS AUROC (OR AUPRC)

- We again need to choose between micro- or macro averaging TPR and FPR.
 - Micro-average if every sample is equally important (irrespective of class)
 - Macro-average if every class is equally important, especially for imbalanced data



Model calibration

- For some models, the *predicted* uncertainty does not reflect the *actual* uncertainty
 - If a model is 90% sure that samples are positive, is it also 90% accurate on these?
- A model is called *calibrated* if the reported uncertainty actually matches how correct it is
 - Overfitted models also tend to be over-confident
 - LogisticRegression models are well calibrated since they learn probabilities
 - SVMs are not well calibrated. *Biased* towards points close to the decision boundary.



BRIER SCORE

- You may want to select models based on how accurate the class confidences are.
- The **Brier score loss**: squared loss between predicted probability \hat{p} and actual outcome y
 - Lower is better

$$\mathcal{L}_{Brier} = \frac{1}{n} \sum_{i=1}^n (\hat{p}_i - y_i)^2$$

```
Logistic Regression Brier score loss: 0.0322
SVM Brier score loss: 0.0795
```

MODEL CALIBRATION TECHNIQUES

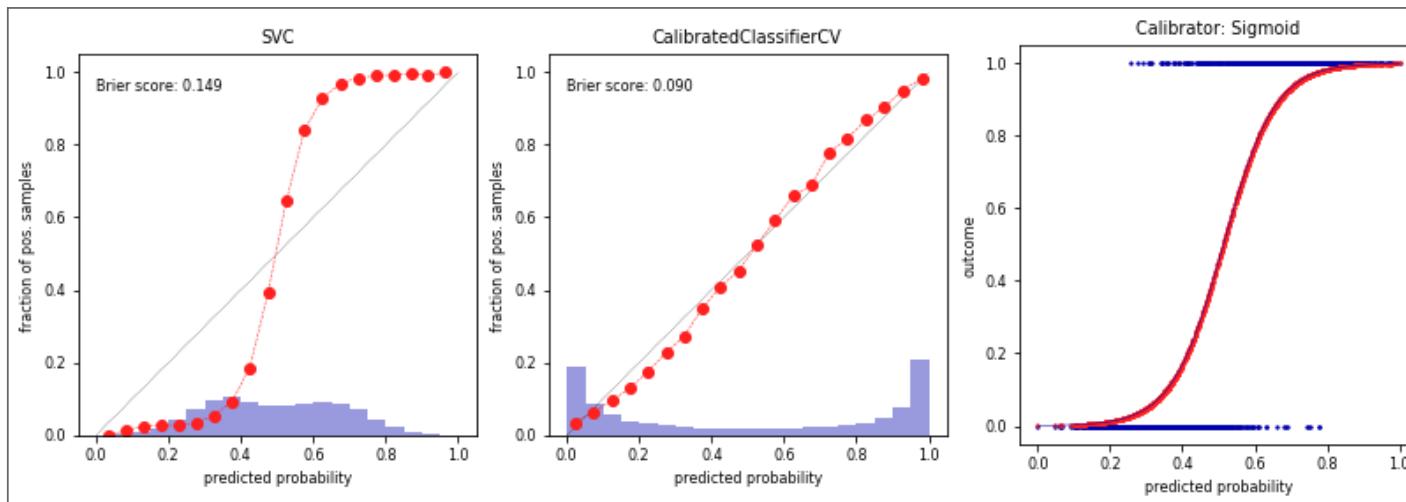
- We can post-process trained models to make them more calibrated.
- Fit a regression model (a calibrator) to map the model's outcomes $f(x)$ to a calibrated probability in [0,1]
 - $f(x)$ returns the decision values or probability estimates
 - f_{calib} is fitted on the training data to map these to the correct outcome
 - Often an internal cross-validation with few folds is used
 - Multi-class models require one calibrator per class

$$f_{calib}(f(x)) \approx p(y)$$

Platt Scaling

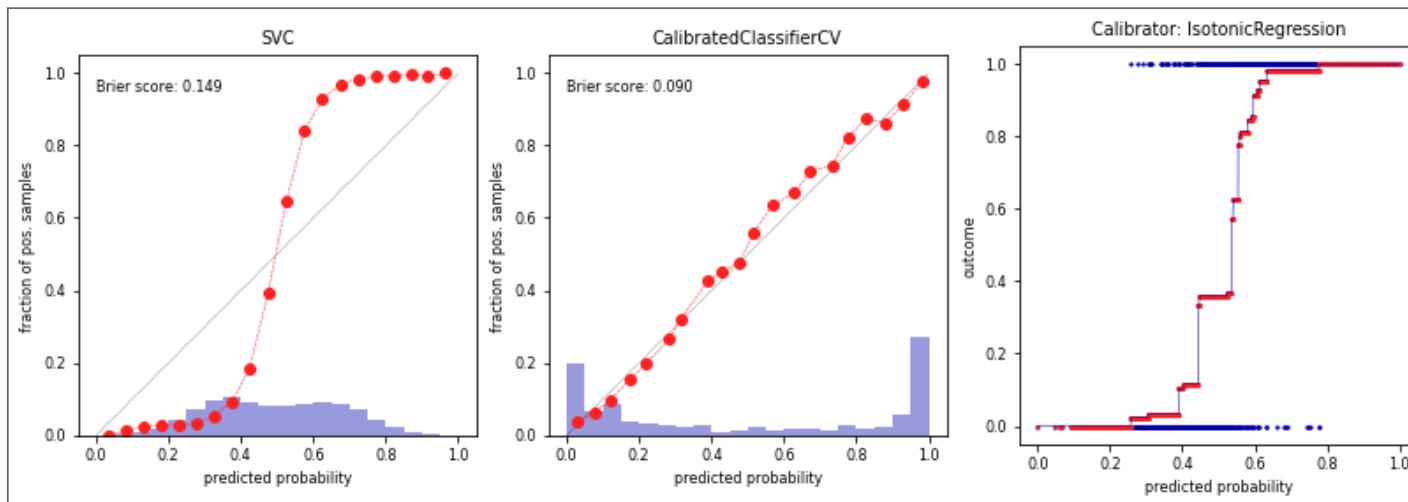
- Calibrator is a logistic (sigmoid) function:
 - Learn the weight w_1 and bias w_0 from data

$$f_{platt} = \frac{1}{1 + \exp(-w_1 f(x) - w_0)}$$



Isotonic regression

- Maps input x_i to an output \hat{y}_i so that \hat{y}_i increases monotonically with x_i and minimizes loss $\sum_i^n (y_i - \hat{y}_i)$
 - Predictions are made by interpolating the predicted \hat{y}_i
- Fit to minimize the loss between the uncalibrated predictions $f(x)$ and the actual labels
- Corrects any monotonic distortion, but tends to overfit on small samples



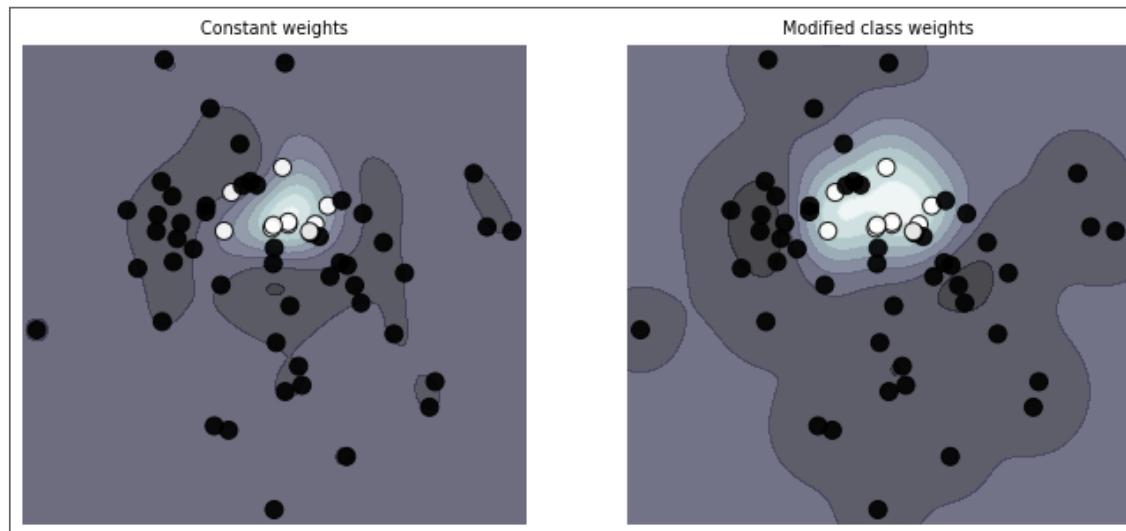
Cost-sensitive classification (dealing with imbalance)

- In the real worlds, different kinds of misclassification can have different costs
 - Misclassifying certain classes can be more costly than others
 - Misclassifying certain samples can be more costly than others
- Cost-sensitive resampling: resample (or reweight) the data to represent real-world expectations
 - oversample minority classes (or undersample majority) to 'correct' imbalance
 - increase weight of misclassified samples (e.g. in boosting)
 - decrease weight of misclassified (noisy) samples (e.g. in model compression)

Class weighting

- If some classes are more important than others, we can give them more weight
 - E.g. for imbalanced data, we can give more weight to minority classes
- Most classification models can include it in their loss function and optimize for it
 - E.g. Logistic regression: add a class weight w_c in the log loss function

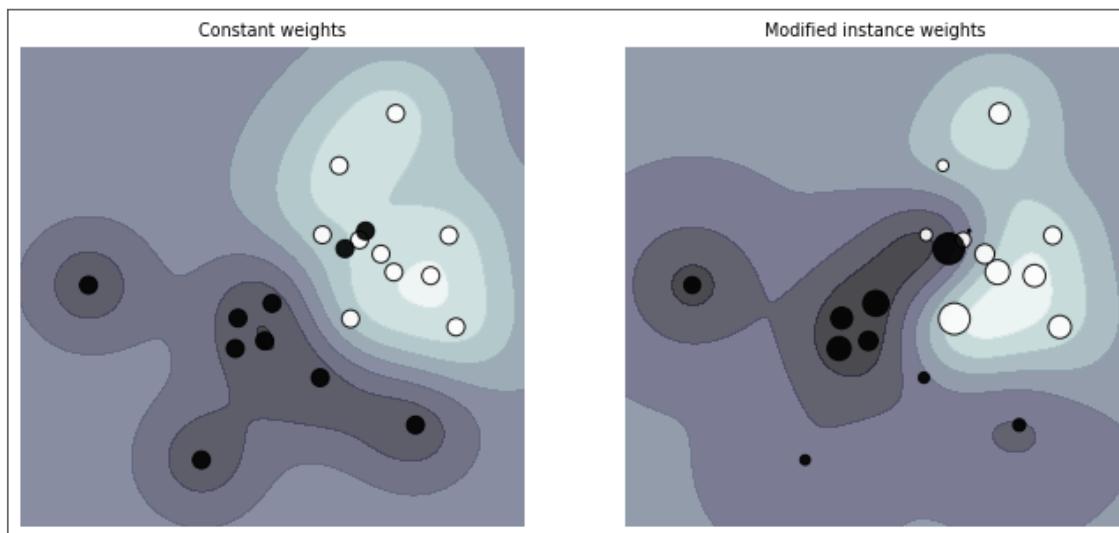
$$\mathcal{L}_{log}(\mathbf{w}) = - \sum_{c=1}^C \textcolor{red}{w_c} \sum_{n=1}^N p_{n,c} \log(q_{n,c})$$



Instance weighting

- If some *training instances* are important to get right, we can give them more weight
 - E.g. when some examples are from groups underrepresented in the data
- These are passed during training (fit), and included in the loss function
 - E.g. Logistic regression: add a instance weight w_n in the log loss function

$$\mathcal{L}_{log}(\mathbf{w}) = - \sum_{c=1}^C \sum_{n=1}^N \textcolor{red}{w}_n p_{n,c} \log(q_{n,c})$$



Cost-sensitive algorithms

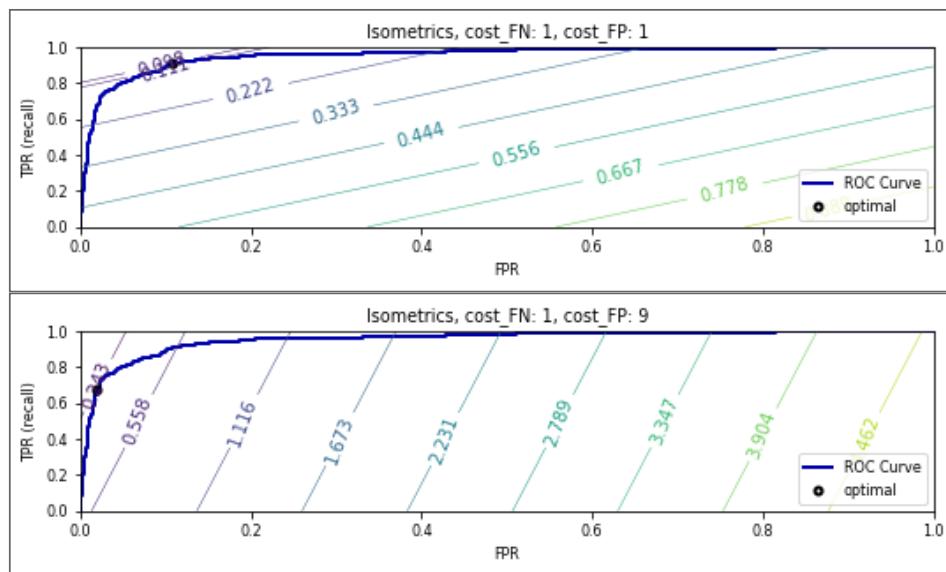
- Cost-sensitive algorithms
 - If misclassification cost of some classes is higher, we can give them higher weights
 - Some support cost matrix C : costs $c_{i,j}$ for every possible type of error
- Cost-sensitive ensembles: convert cost-insensitive classifiers into cost-sensitive ones
 - MetaCost: Build a model (ensemble) to learn the class probabilities $P(j|x)$
 - Relabel training data to minimize expected cost: $\operatorname{argmin}_i \sum_j P_j(x)c_{i,j}$
 - Accuracy may decrease but cost decreases as well.
 - AdaCost: Boosting with reweighting instances to reduce costs

Tuning the decision threshold

- If every FP or FN has a certain cost, we can compute the total cost for a given model:

$$\text{total cost} = \text{FPR} * \text{cost}_{FP} * \text{ratio}_{pos} + (1 - \text{TPR}) * \text{cost}_{FN} * (1 - \text{ratio}_{pos})$$

- This yields different *isometrics* (lines of equal cost) in ROC space
- Optimal threshold is the point on the ROC curve where cost is minimal (line search)



Regression metrics

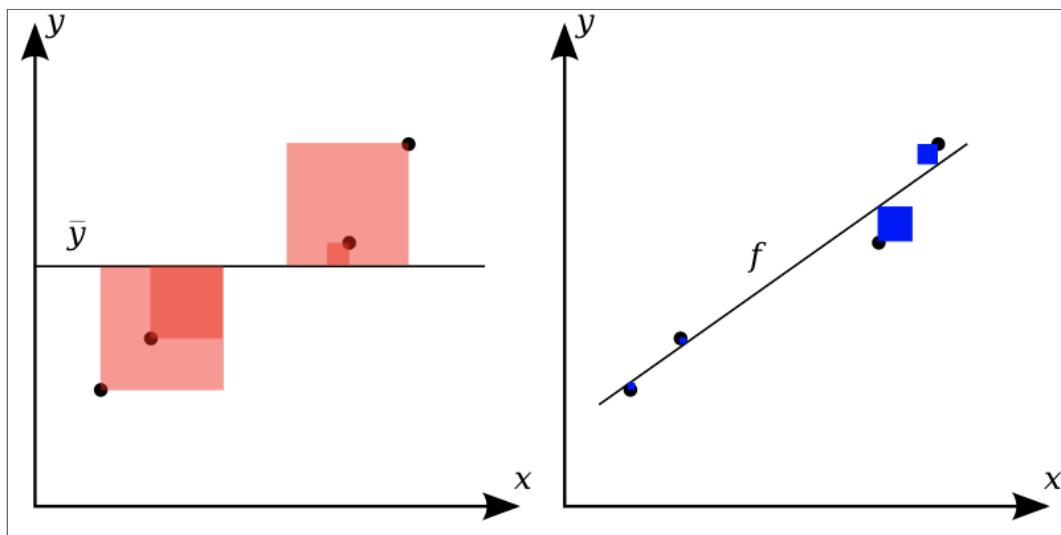
Most commonly used are

- mean squared error:
$$\frac{\sum_i (y_{pred_i} - y_{actual_i})^2}{n}$$
 - root mean squared error (RMSE) often used as well
- mean absolute error:
$$\frac{\sum_i |y_{pred_i} - y_{actual_i}|}{n}$$
 - Less sensitive to outliers and large errors



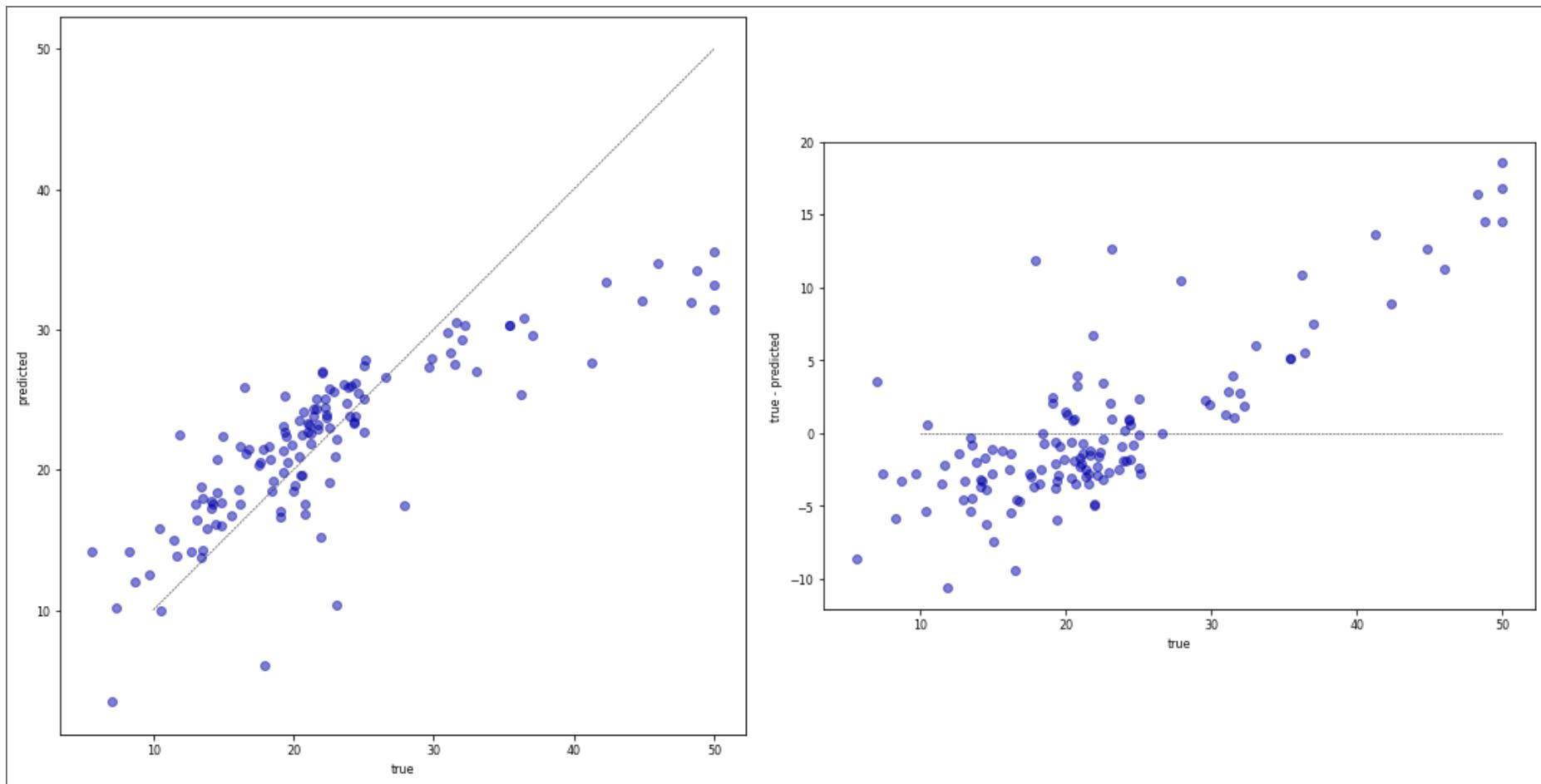
R squared

- $R^2 = 1 - \frac{\sum_i (y_{predicted_i} - y_{actual_i})^2}{\sum_i (y_{mean} - y_{actual_i})^2}$
 - Ratio of variation explained by the model / total variation
 - Between 0 and 1, but *negative* if the model is worse than just predicting the mean
 - Easier to interpret (higher is better).



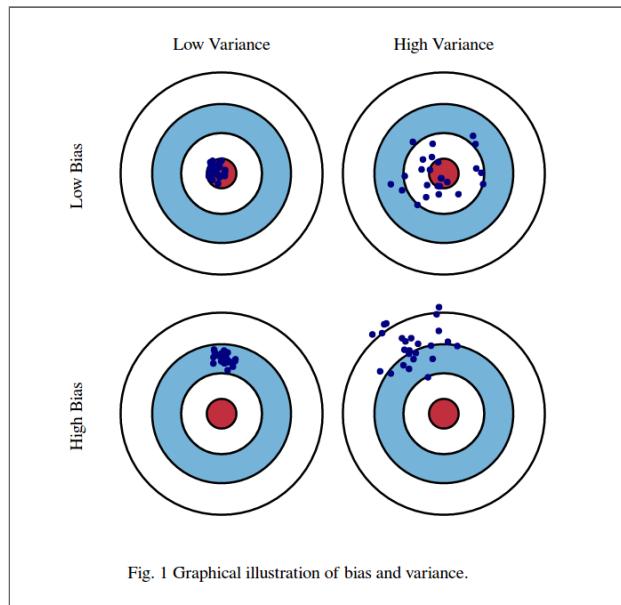
Visualizing regression errors

- Prediction plot (left): predicted vs actual target values
- Residual plot (right): residuals vs actual target values
 - Over- and underpredictions can be given different costs



Bias-Variance decomposition

- Evaluate the same algorithm multiple times on different random samples of the data
- Two types of errors can be observed:
 - Bias error: systematic error, independent of the training sample
 - These points are predicted (equally) wrong every time
 - Variance error: error due to variability of the model w.r.t. the training sample
 - These points are sometimes predicted accurately, sometimes inaccurately

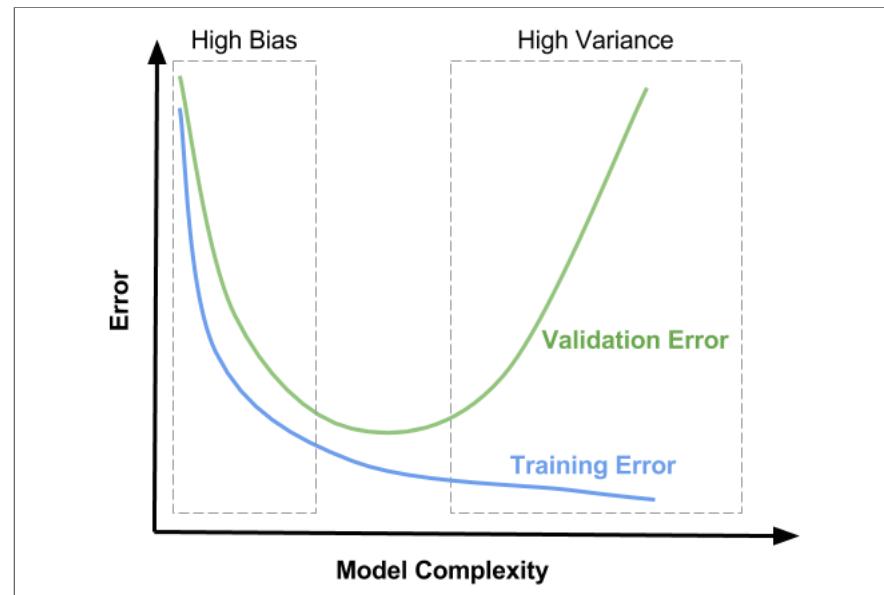


Computing bias and variance error

- Take 100 or more bootstraps (or shuffle-splits)
- Regression: for each data point x :
 - $bias(x)^2 = (x_{true} - mean(x_{predicted}))^2$
 - $variance(x) = var(x_{predicted})$
- Classification: for each data point x :
 - $bias(x)$ = misclassification ratio
 - $variance(x)$
 $= (1 - (P(class_1)^2 + P(class_2)^2))/2$
 - $P(class_i)$ is ratio of class i predictions
- Total bias: $\sum_x bias(x)^2 * w_x$ w_x : the percentage of times x occurs in the test sets
- Total variance: $\sum_x variance(x) * w_x$

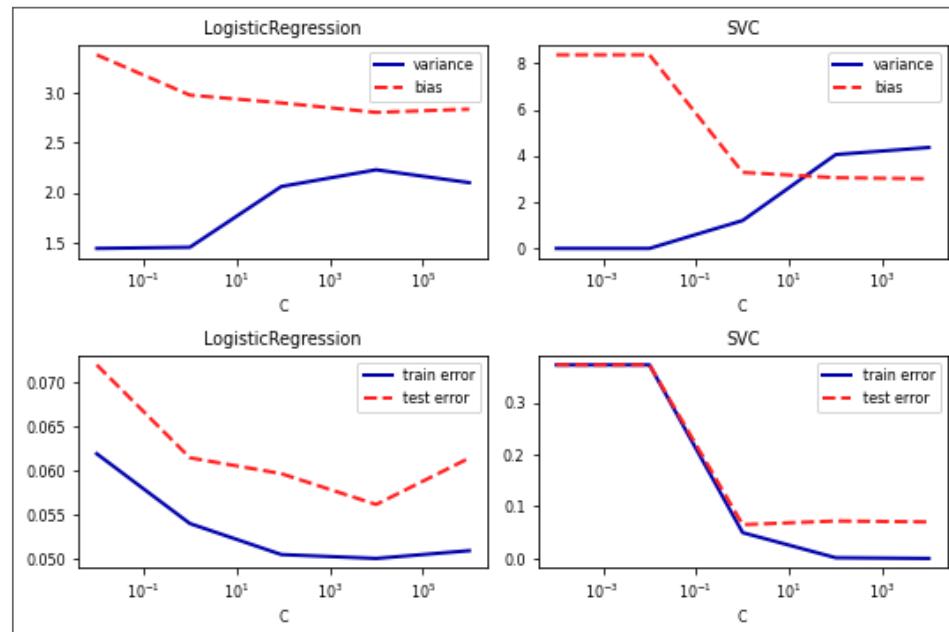
Bias and variance, underfitting and overfitting

- High variance means that you are likely overfitting
 - Use more regularization or use a simpler model
- High bias means that you are likely underfitting
 - Do less regularization or use a more flexible/complex model
- Ensembling techniques (see later) reduce bias or variance directly
 - Bagging (e.g. RandomForests) reduces variance, Boosting reduces bias

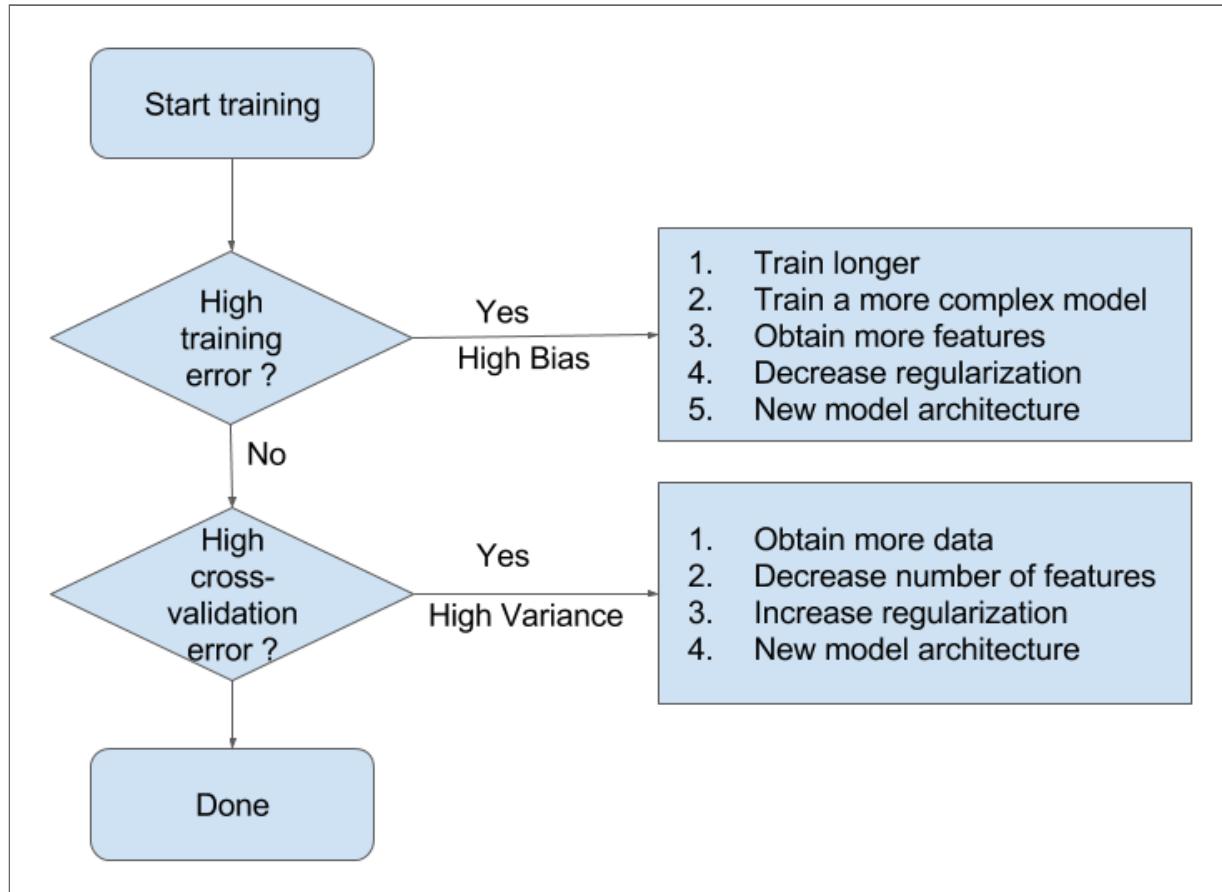


Understanding under- and overfitting

- Regularization reduces variance error (increases stability of predictions)
 - But too much increases bias error (inability to learn 'harder' points)
- High regularization (left side): Underfitting, high bias error, low variance error
 - High training error and high test error
- Low regularization (right side): Overfitting, low bias error, high variance error
 - Low training error and higher test error

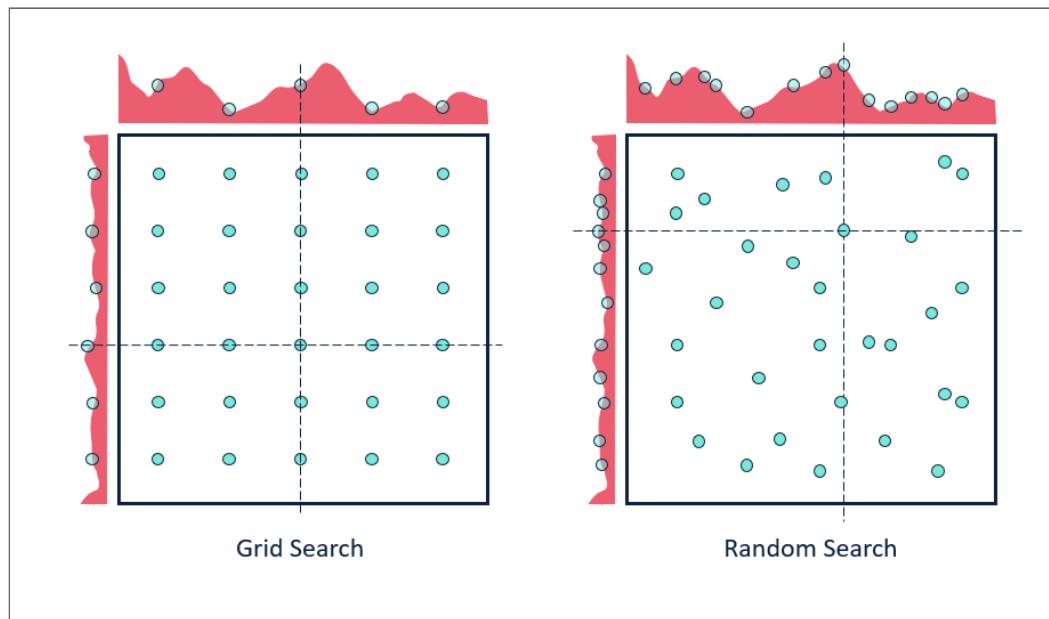


Summary Flowchart (by Andrew Ng)

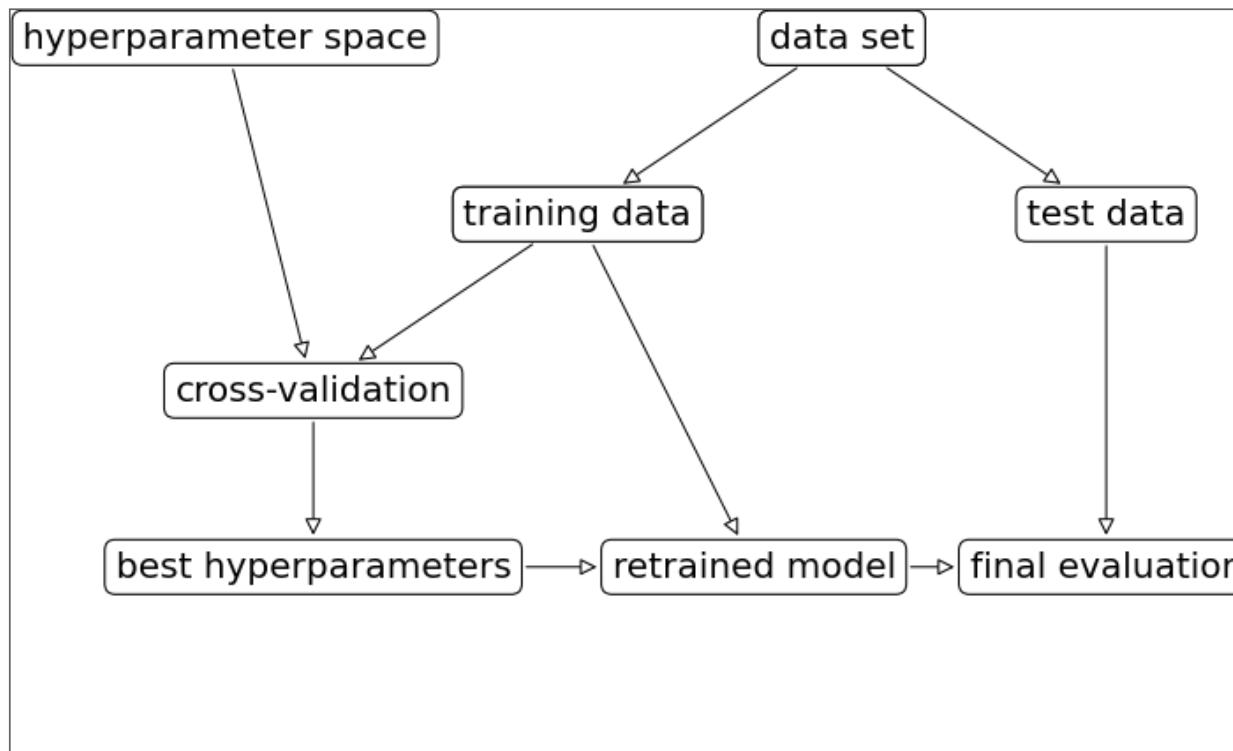


Hyperparameter tuning

- There exists a huge range of techniques to tune hyperparameters. The simplest:
 - Grid search: Choose a range of values for every hyperparameter, try every combination
 - Doesn't scale to many hyperparameters (combinatorial explosion)
 - Random search: Choose random values for all hyperparameters, iterate n times
 - Better, especially when some hyperparameters are less important
- Many more advanced techniques exist, see lecture on Automated Machine Learning



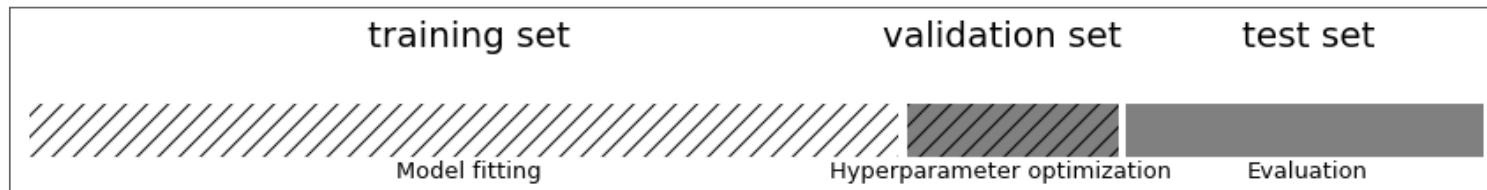
- First, split the data in training and test sets (outer split)
- Split up the training data again (inner cross-validation)
 - Generate hyperparameter configurations (e.g. random/grid search)
 - Evaluate all configurations on all inner splits, select the best one (on average)
- Retrain best configurations on full training set, evaluate on held-out test data



Nested cross-validation

- Simplest approach: single outer split and single inner split (shown below)
- Risk of over-tuning hyperparameters on specific train-test split
 - Only recommended for very large datasets
- Nested cross-validation:
 - Outer loop: split full dataset in k_1 training and test splits
 - Inner loop: split training data into k_2 train and validation sets
- This yields k_1 scores for k_1 possibly different hyperparameter settings
 - Average score is the expected performance of the tuned model
- To use the model in practice, retune on the **entire** dataset

```
hps = {'C': expon(scale=100), 'gamma': expon(scale=.1)}
scores = cross_val_score(RandomizedSearchCV(SVC(), hps, cv=3), X, y, cv=5)
```



Summary

- Split the data into training and test sets according to the application
 - Holdout only for large datasets, cross-validation for smaller ones
 - For classification, always use stratification
 - Grouped or ordered data requires special splitting
- Choose a metric that fits your application
 - E.g. precision to avoid false positives, recall to avoid false negatives
- Calibrate the decision threshold to fit your application
 - ROC curves or Precision-Recall curves can help to find a good tradeoff
- If possible, include the actual or relative costs of misclassifications
 - Class weighting, instance weighting, ROC isometrics can help
 - Be careful with imbalanced or unrepresentative datasets
- When using the predicted probabilities in applications, calibrate the models
- Always tune the most important hyperparameters
 - Manual tuning: Use insight and train-test scores for guidance
 - Hyperparameter optimization: be careful not to over-tune