



Perceptron





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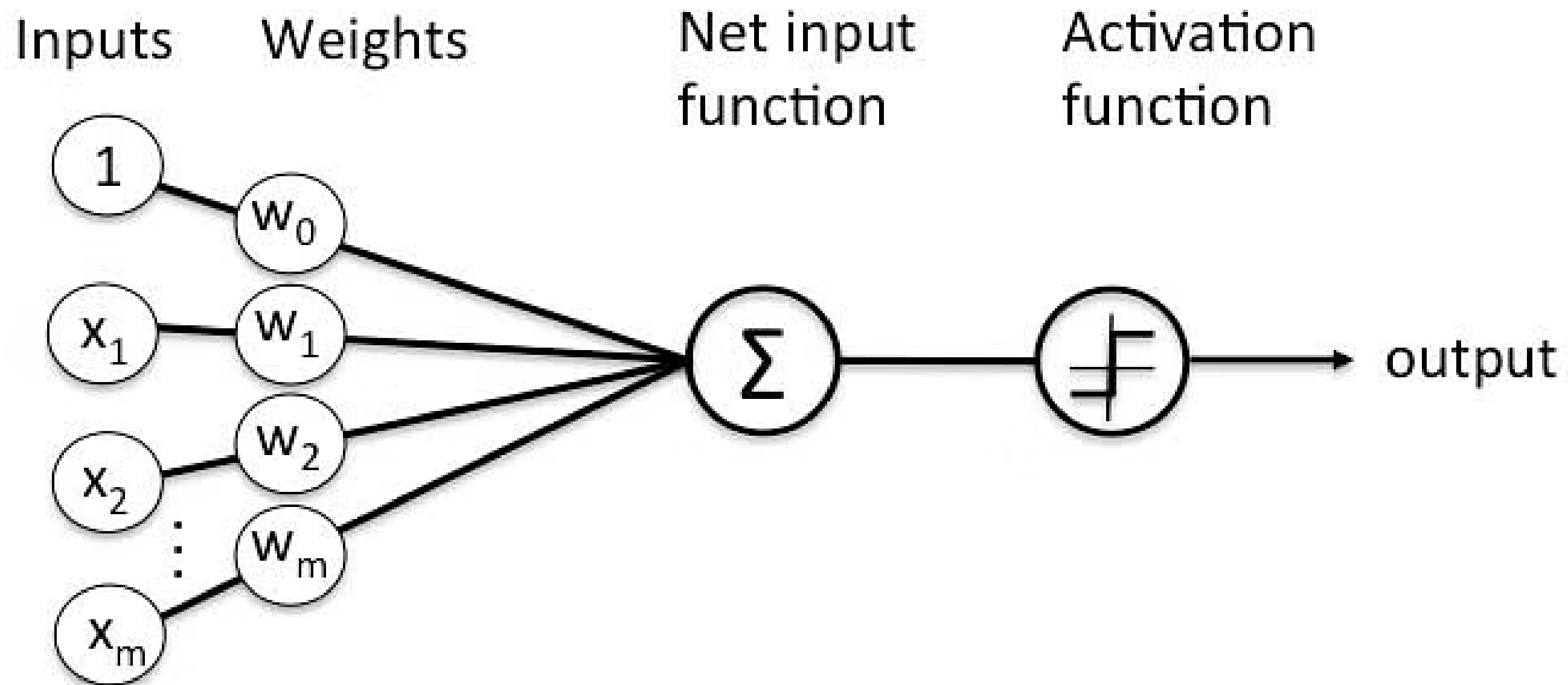
What is a Perceptron?

Mathematical Model of a Biological Neuron

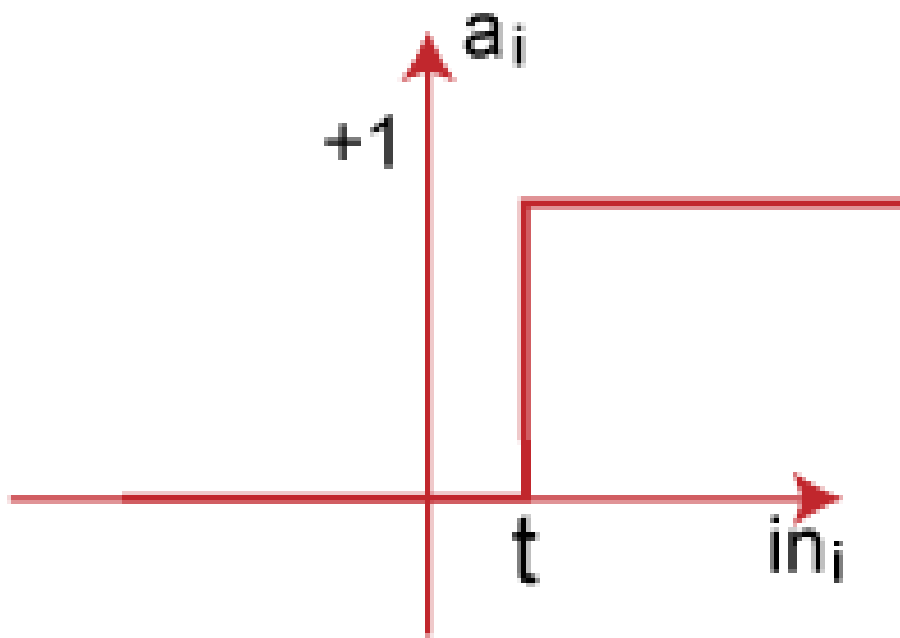
Building Block of an Artificial Neural Network.

Supervised Learning Algorithm for Binary Classifier.

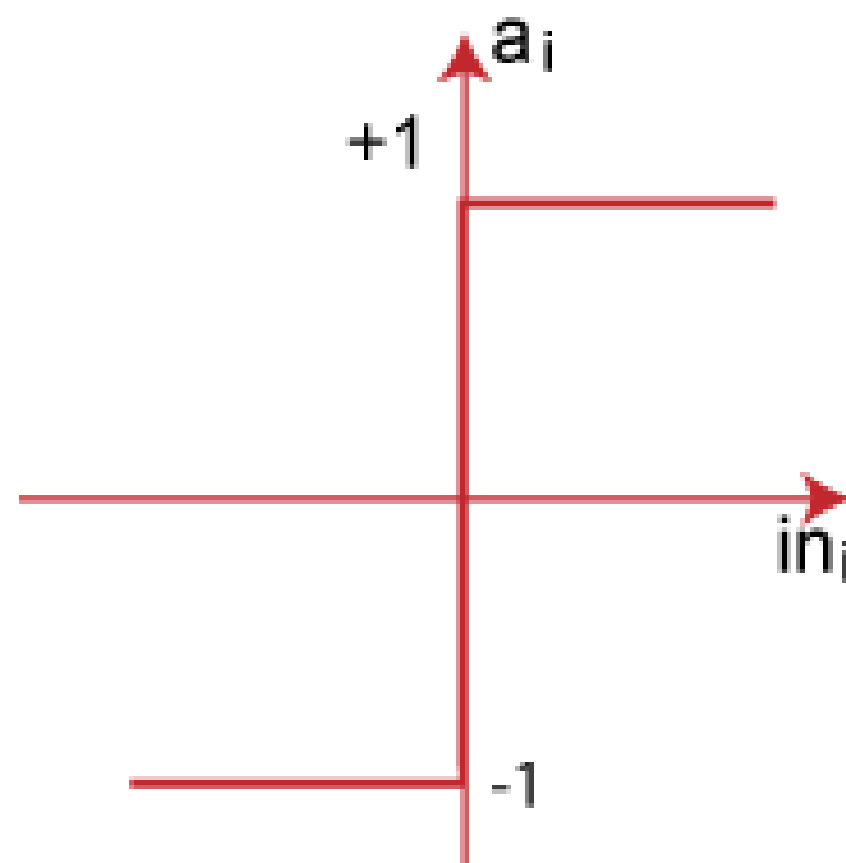
The Perceptron



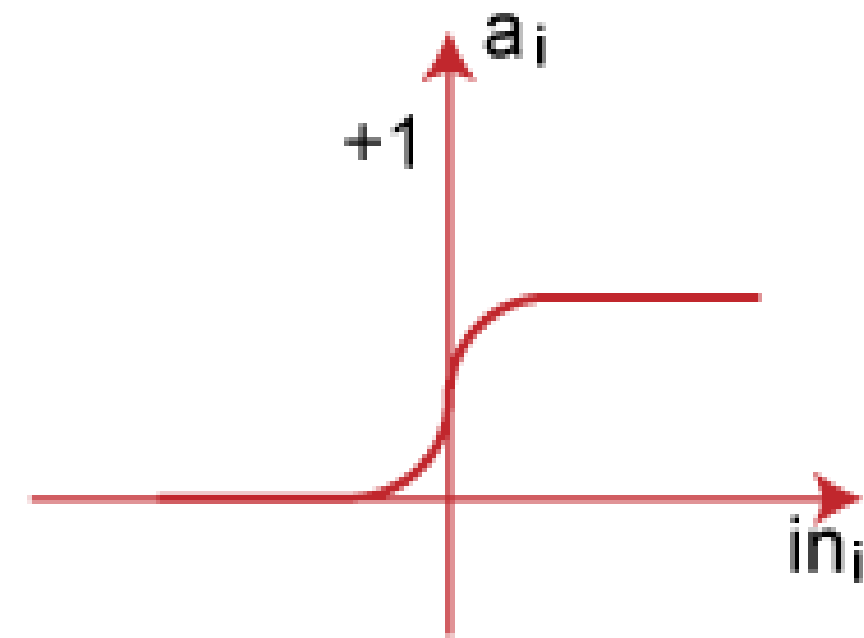
Activation Functions



Step Function

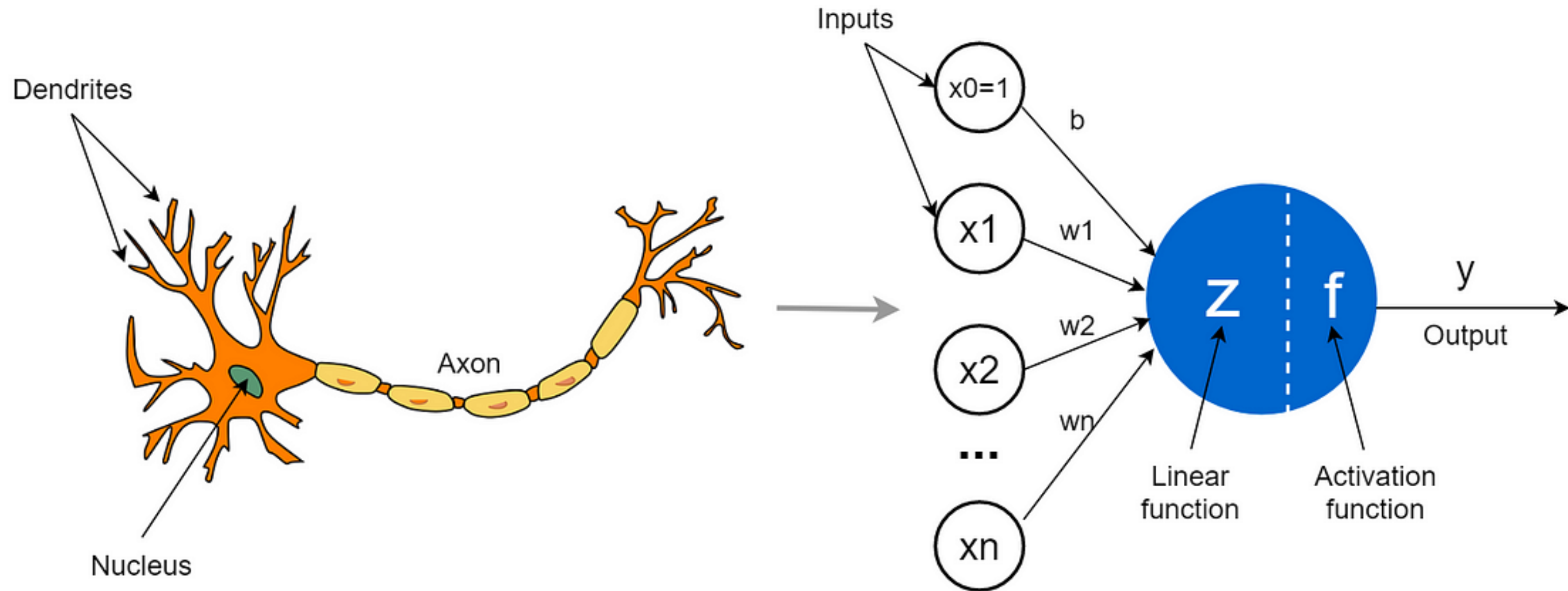


Sign Function

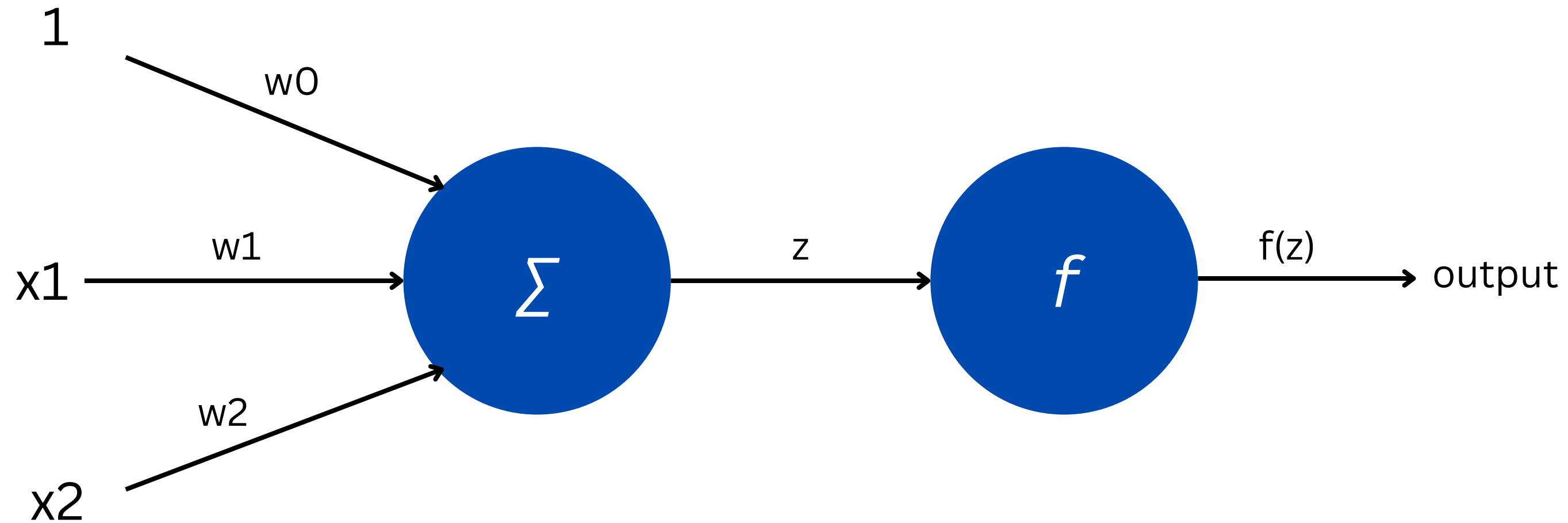


Sigmoid Function

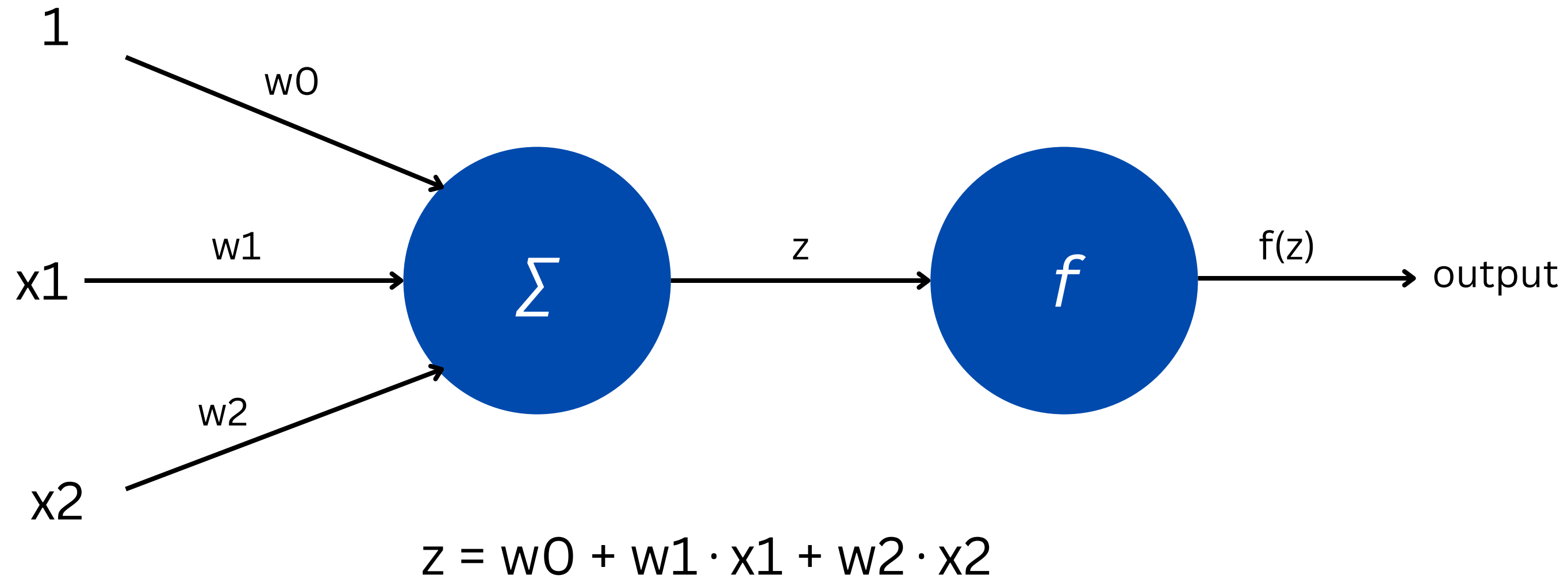
Biological Neuron vs Perceptron



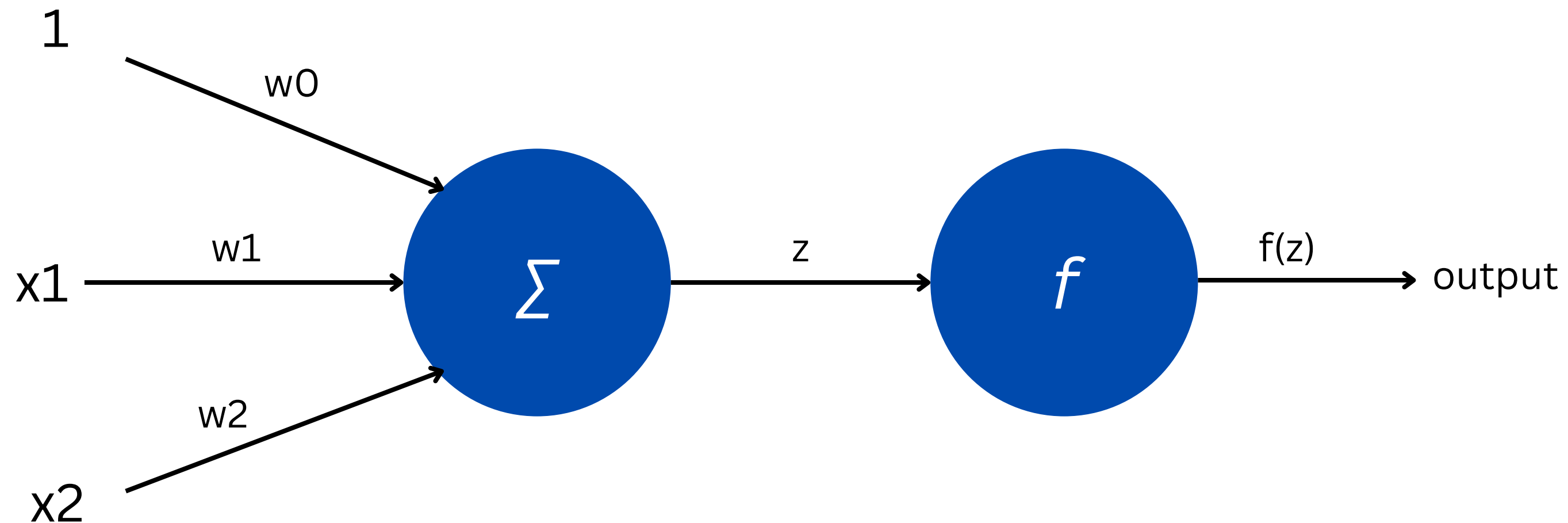
2 Input Perceptron



2 Input Perceptron



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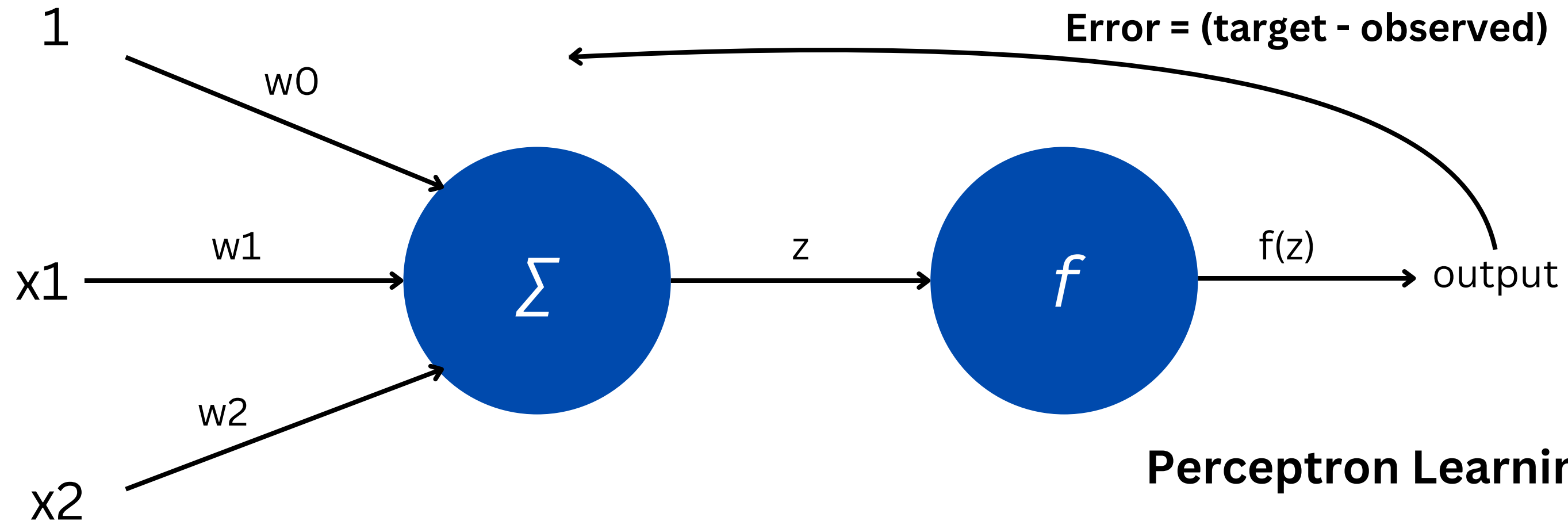


$$z = w_0 + w_1 \cdot x_1 + w_2 \cdot x_2$$

$$f(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{cases}$$

Error

Perceptron Training



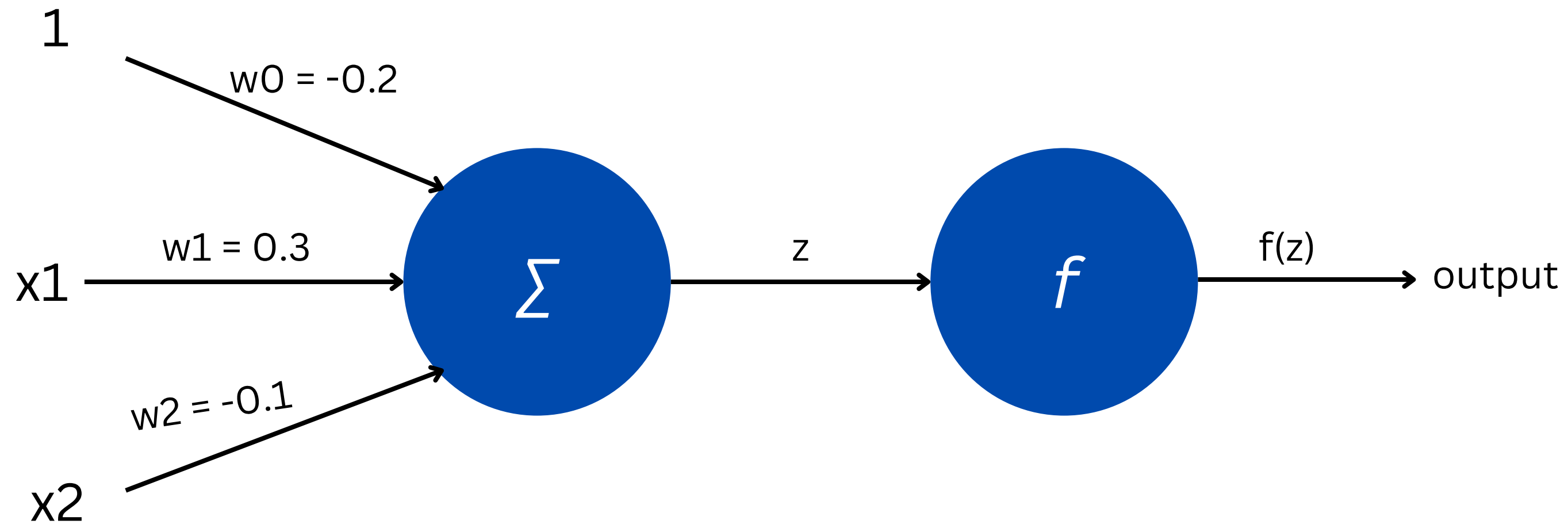
Perceptron Learning Rule

$$w_i \leftarrow w_i + \Delta w_i$$

$$\Delta w_i = \eta(t - o)x_i$$

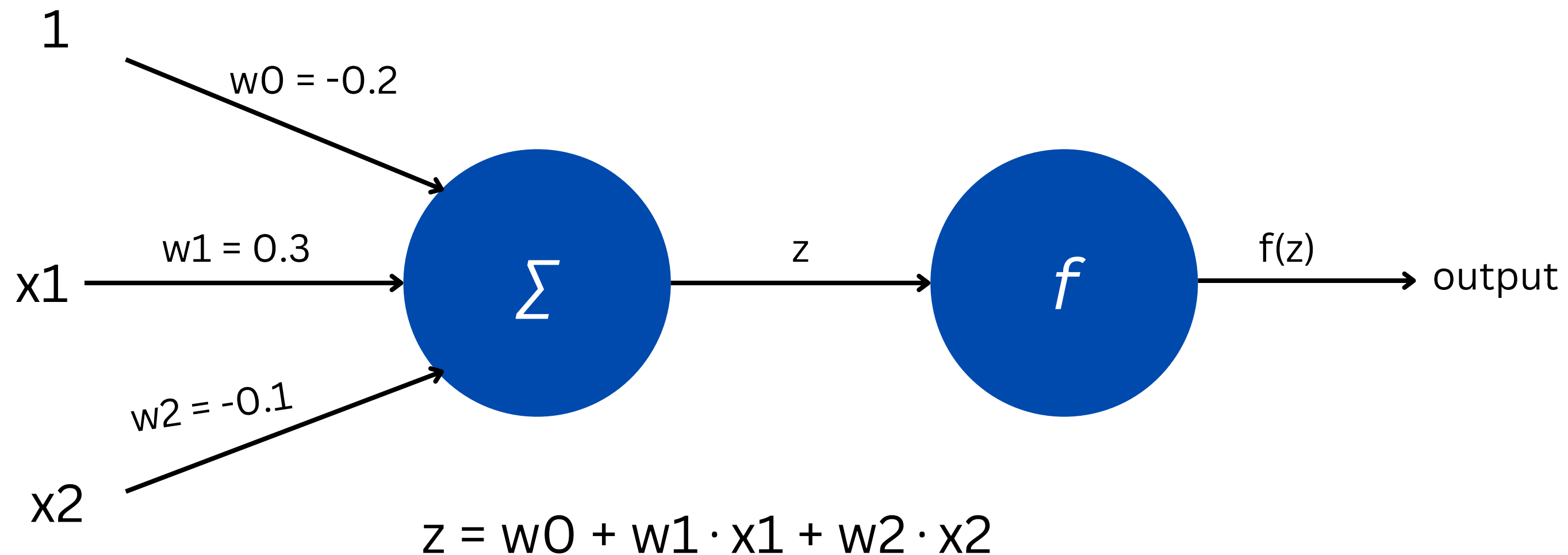
Training Algorithm

Step 1: Initialization



Training Algorithm

Step 2: Activation

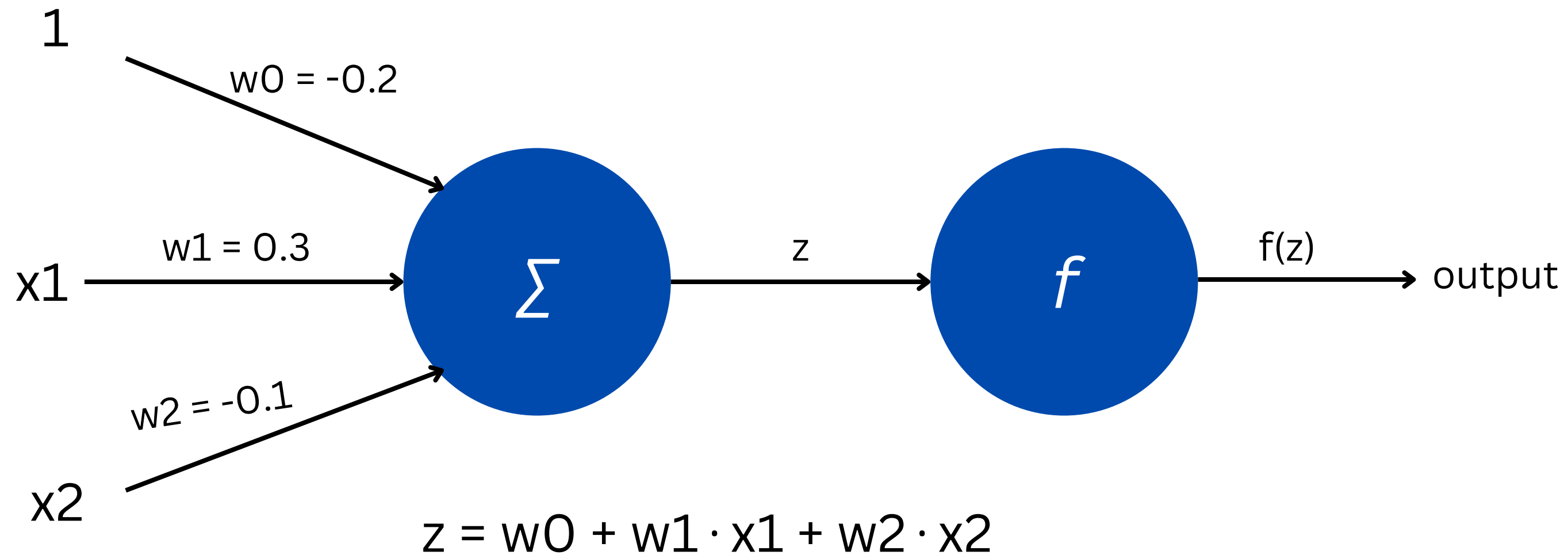


Error = target - observed

$$f(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{cases}$$

Training Algorithm

Step 3: Weight Update

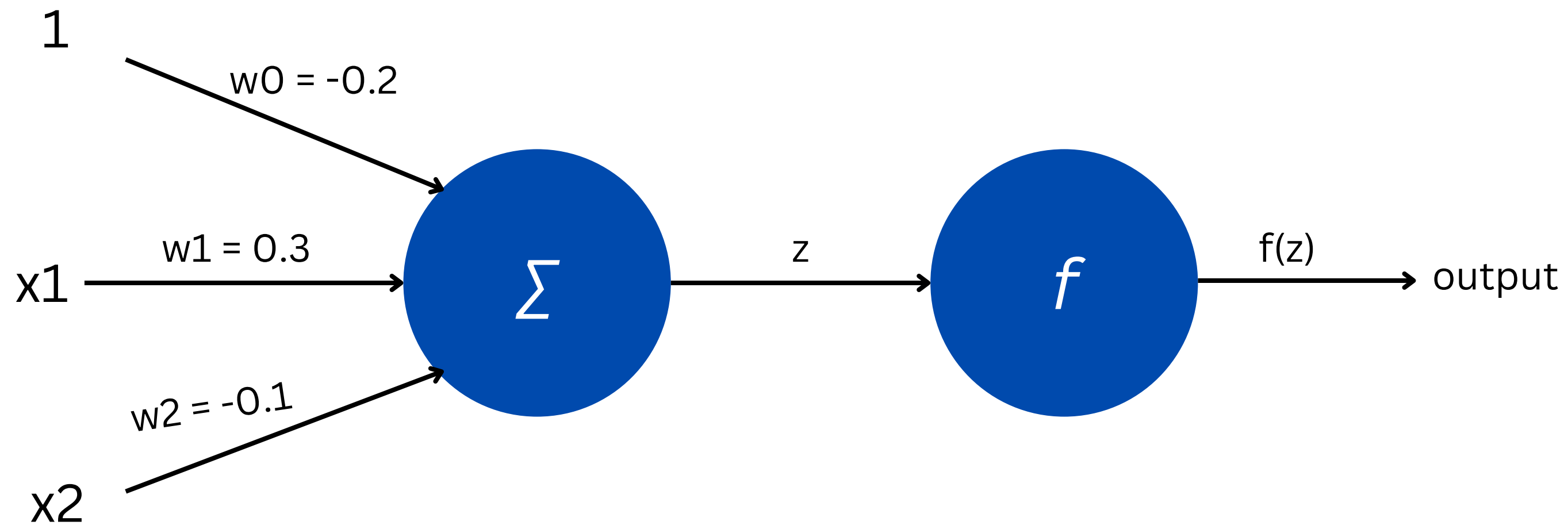


Error = target - observed

$$f(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{cases}$$

Training Algorithm


Step 4: Iteration



Building a Logical AND

Input		Output
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Building a Logical AND



Input		Output
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

We have:

$$w_0 = -0.2, w_1 = 0.3, w_2 = -0.1, lr = 0.1$$

1st Epoch:


For $x_1 = 0, x_2 = 0$

$$z = w_0 + w_1 * x_1 + w_2 * x_2 = -0.2 + 0.3 * 0 + (-0.1) * 0 = -0.2$$

$$\text{output} = f(z) = f(-0.2) = 0$$

target = 0 (No need for weight update)

Building a Logical AND



Input		Output
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

We have:

$$w_0 = -0.2, w_1 = 0.3, w_2 = -0.1, lr = 0.1$$

1st Epoch:

For $x_1 = 0, x_2 = 1$

$$z = w_0 + w_1 * x_1 + w_2 * x_2 = -0.2 + 0.3 * 0 + (-0.1) * 1 = -0.3$$

$$\text{output} = f(z) = f(-0.3) = 0$$

target = 0 (No need for weight update)

Building a Logical AND

We have:

$$w0 = -0.2, w1 = 0.3, w2 = -0.1, lr = 0.1$$

1st Epoch:

For $x1 = 1, x2 = 0$

$$z = w0 + w1*x1 + w2*x2 = -0.2 + 0.3 * 1 + (-0.1) * 0 = 0.1$$

$$\text{output} = f(z) = f(0.1) = 1$$


target = 0 (Weight Update Required)

Weight Update:

$$w0 = -0.2 + 0.1*(0-1)*1 = -0.3$$

$$w1 = 0.3 + 0.1*(0-1)*1 = 0.2$$

$$w2 = -0.1 + 0.1*(0-1)*0 = -0.1$$



Input		Output
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Building a Logical AND

We have:

$$w_0 = -0.3, w_1 = 0.2, w_2 = -0.1, lr = 0.1$$

1st Epoch:

For $x_1 = 1, x_2 = 1$

$$z = w_0 + w_1 * x_1 + w_2 * x_2 = -0.3 + 0.2 * 1 + (-0.1) * 1 = -0.2$$

$$\text{output} = f(z) = f(-0.2) = 0$$


target = 1 (Weight Update Required)

Weight Update:

$$w_0 = -0.3 + 0.1 * (1 - 0) * 1 = -0.2$$

$$w_1 = 0.2 + 0.1 * (1 - 0) * 1 = 0.3$$

$$w_2 = -0.1 + 0.1 * (1 - 0) * 1 = 0.0$$




Input		Output
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

$$w_i \leftarrow w_i + \Delta w_i$$

$$\Delta w_i = \eta(t - o)x_i$$

Building a Logical AND



Input		Output
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

We have:

$$w_0 = -0.2, w_1 = 0.3, w_2 = 0.0, lr = 0.1$$

$$w_i \leftarrow w_i + \Delta w_i$$

2nd Epoch:

$$\Delta w_i = \eta(t - o)x_i$$

For $x_1 = 0, x_2 = 0$

$$z = w_0 + w_1 * x_1 + w_2 * x_2 = -0.2 + 0.3 * 0 + 0.0 * 0 = -0.2$$

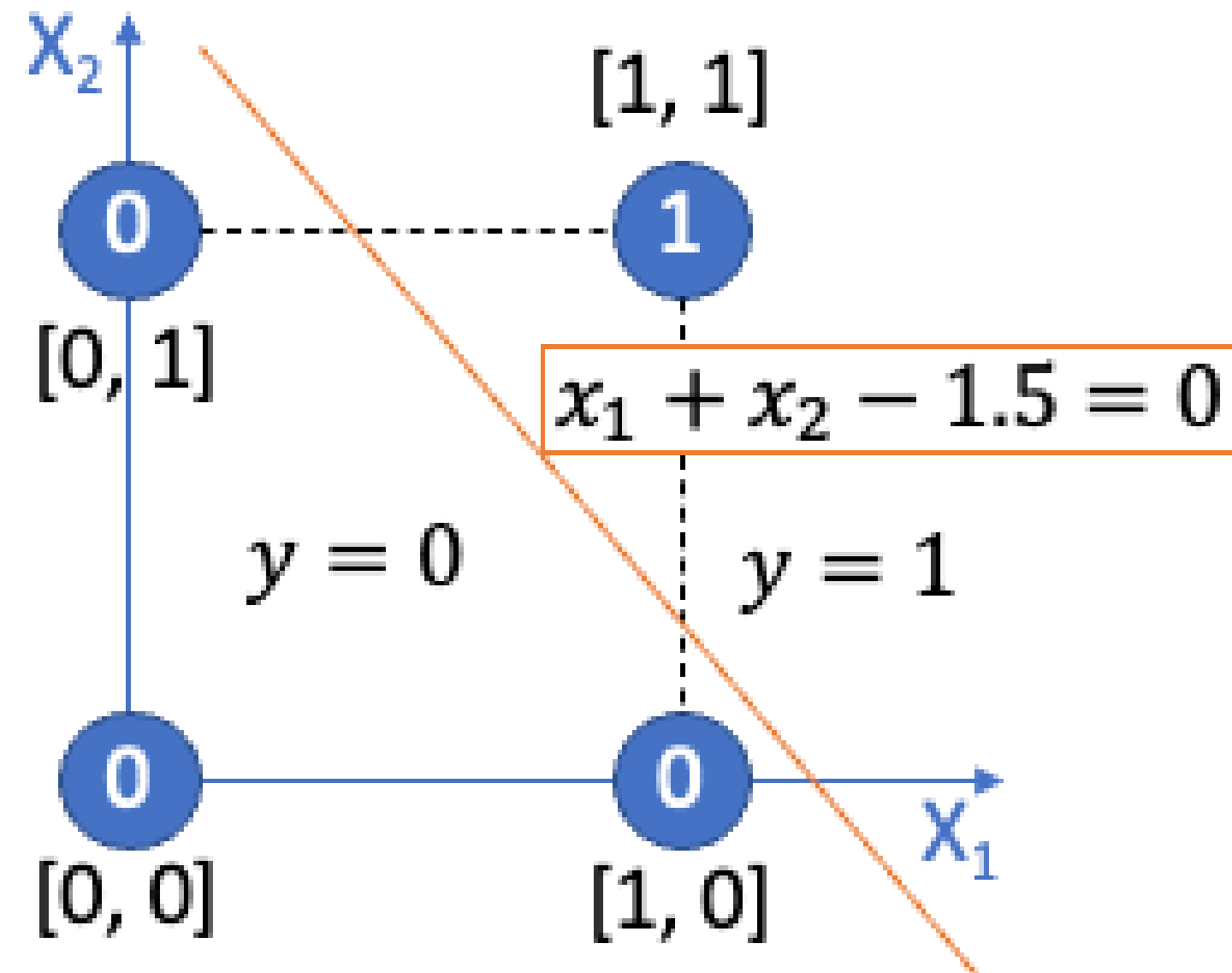
$$\text{output} = f(z) = f(-0.2) = 0$$

target = 0 (No weight update required)

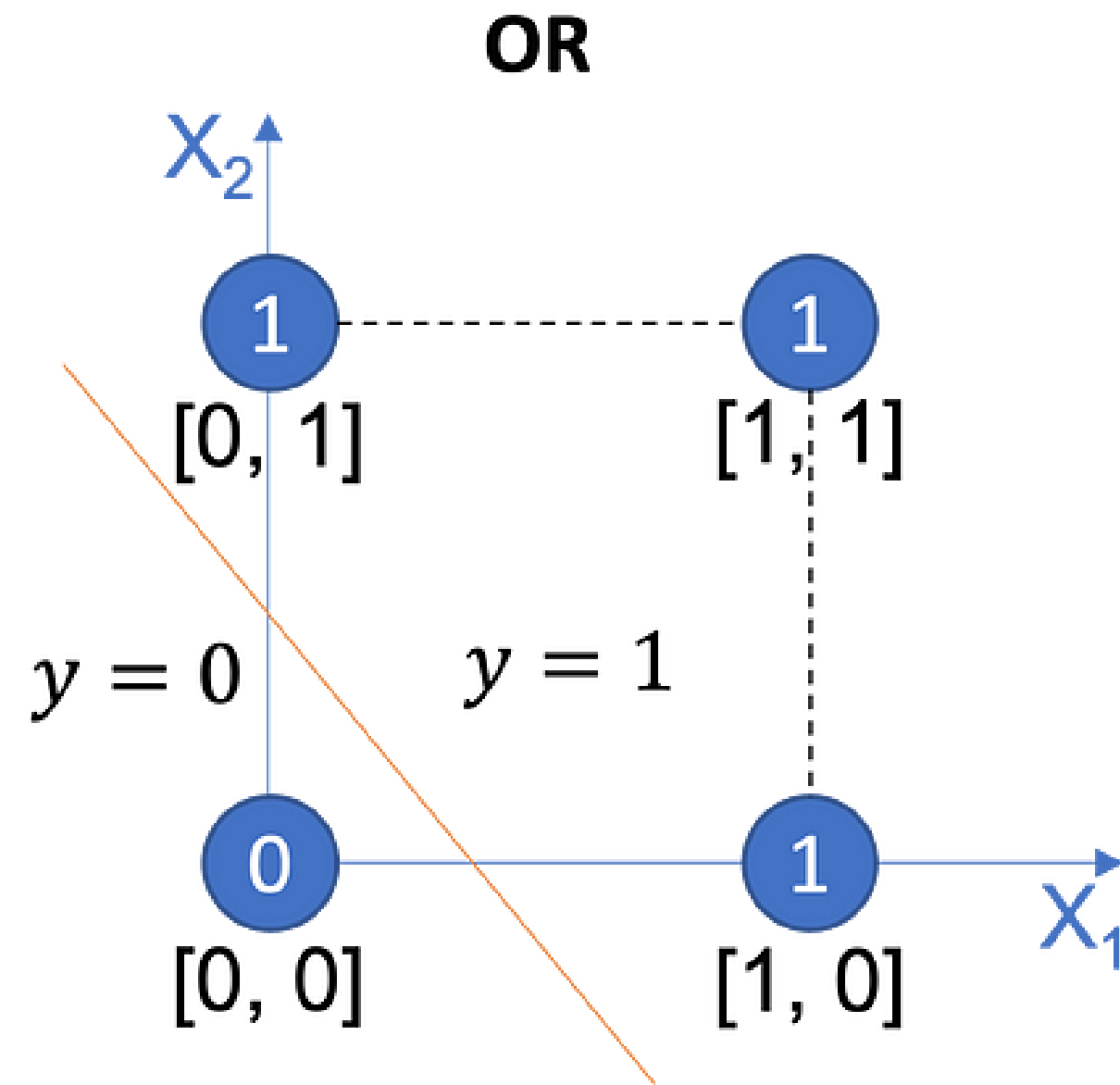
Iterate Until Convergence or Minimum Error

Building a Logical AND

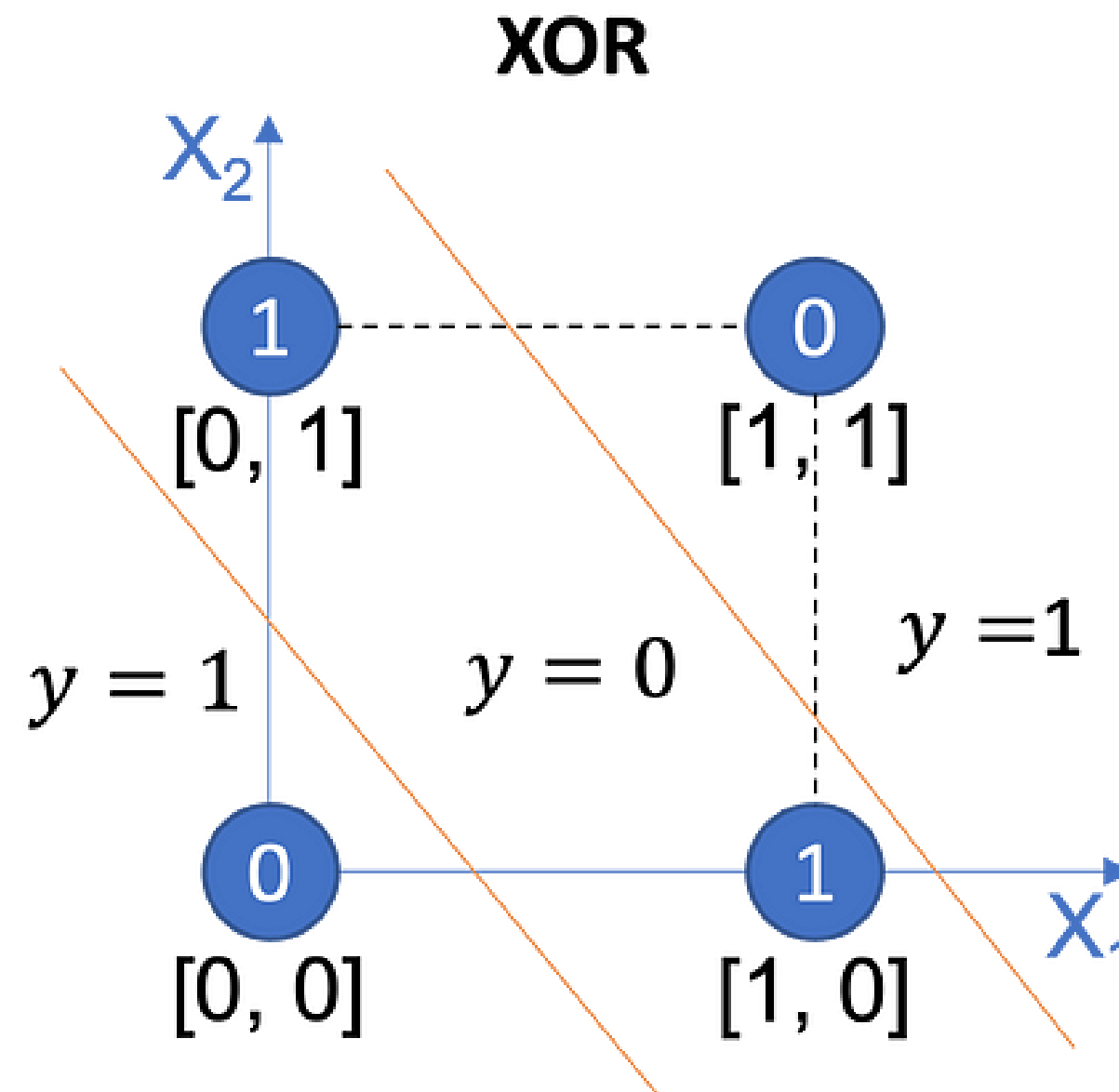
x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1



Logical OR

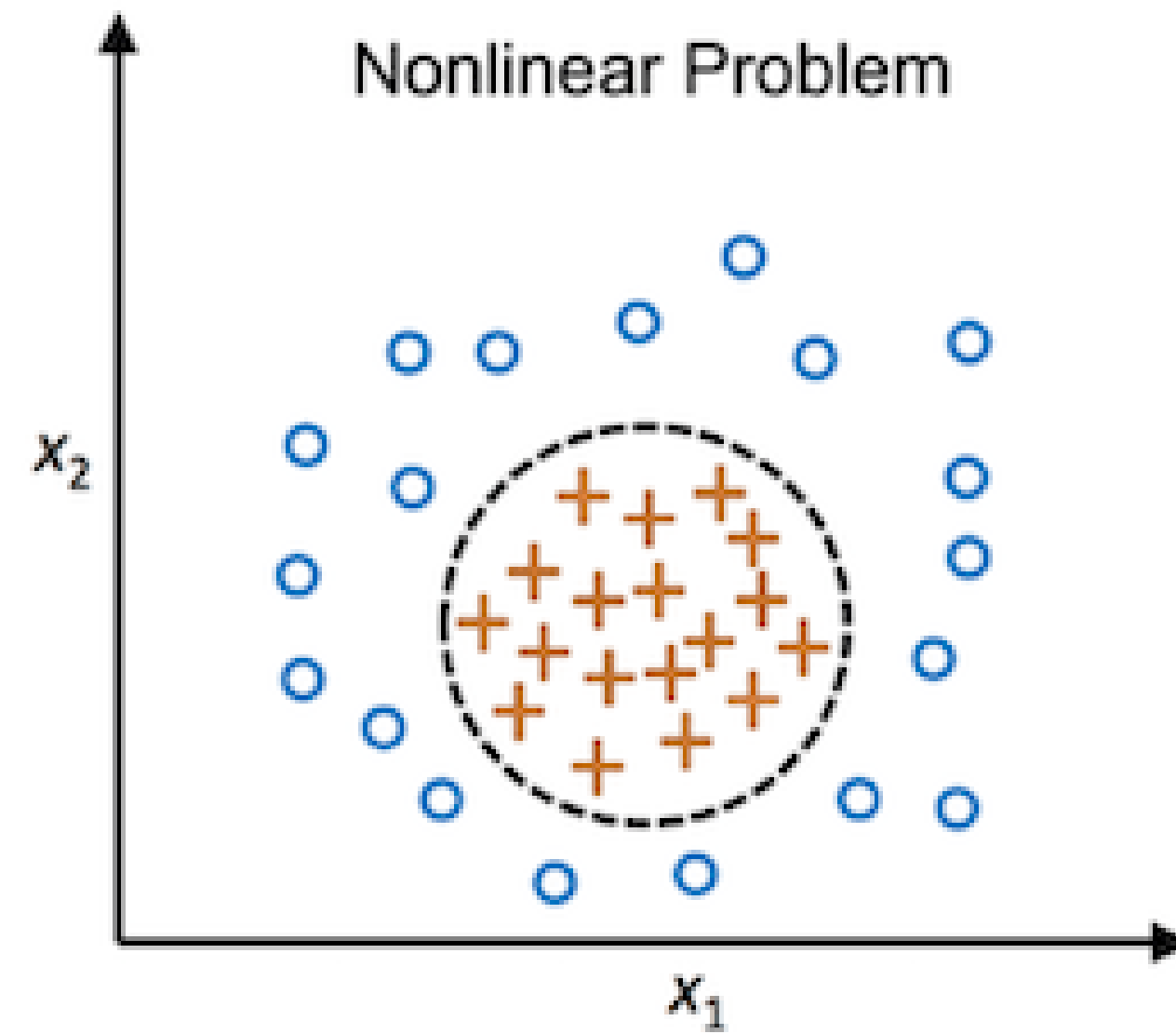
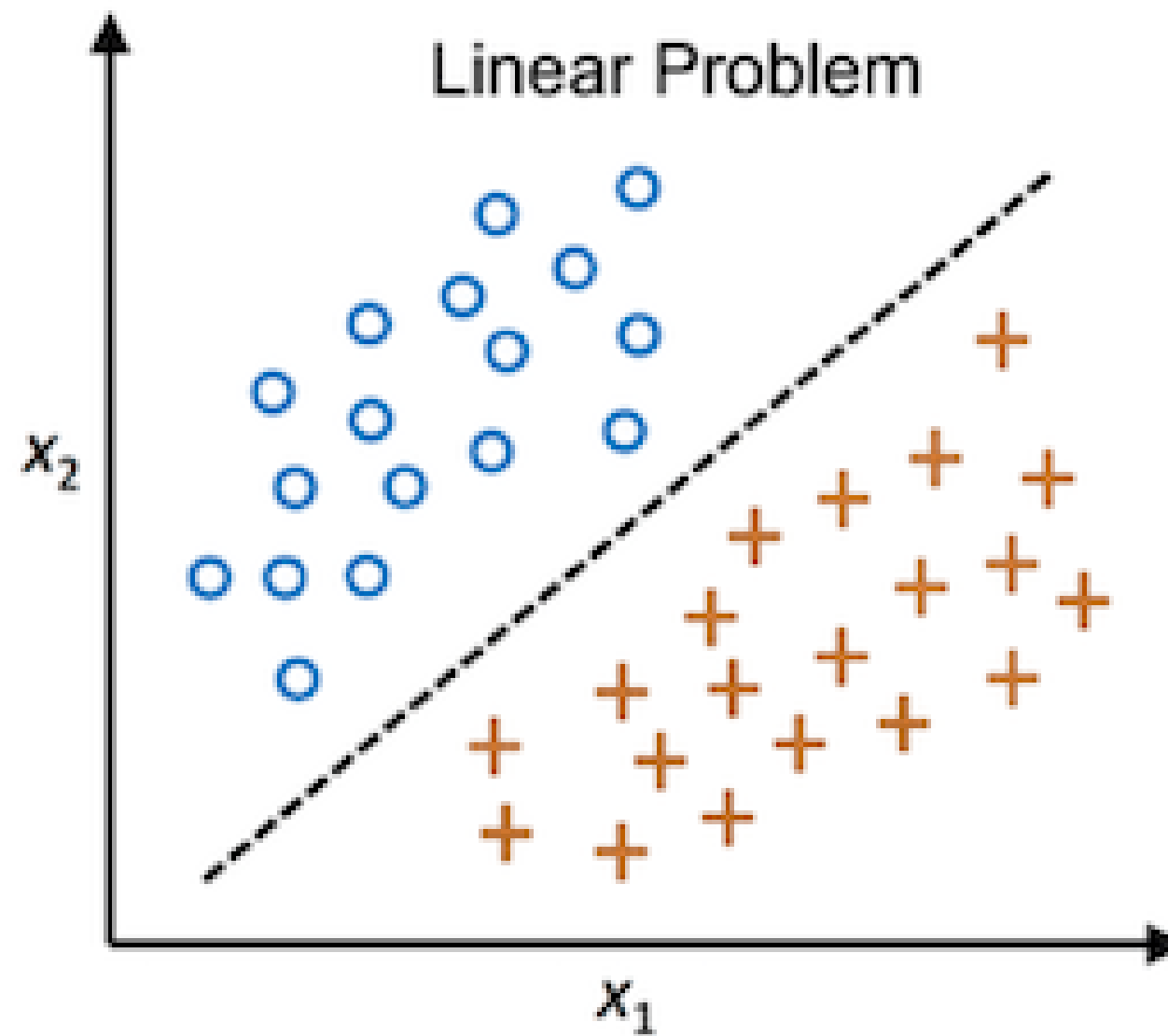


Logical XOR??

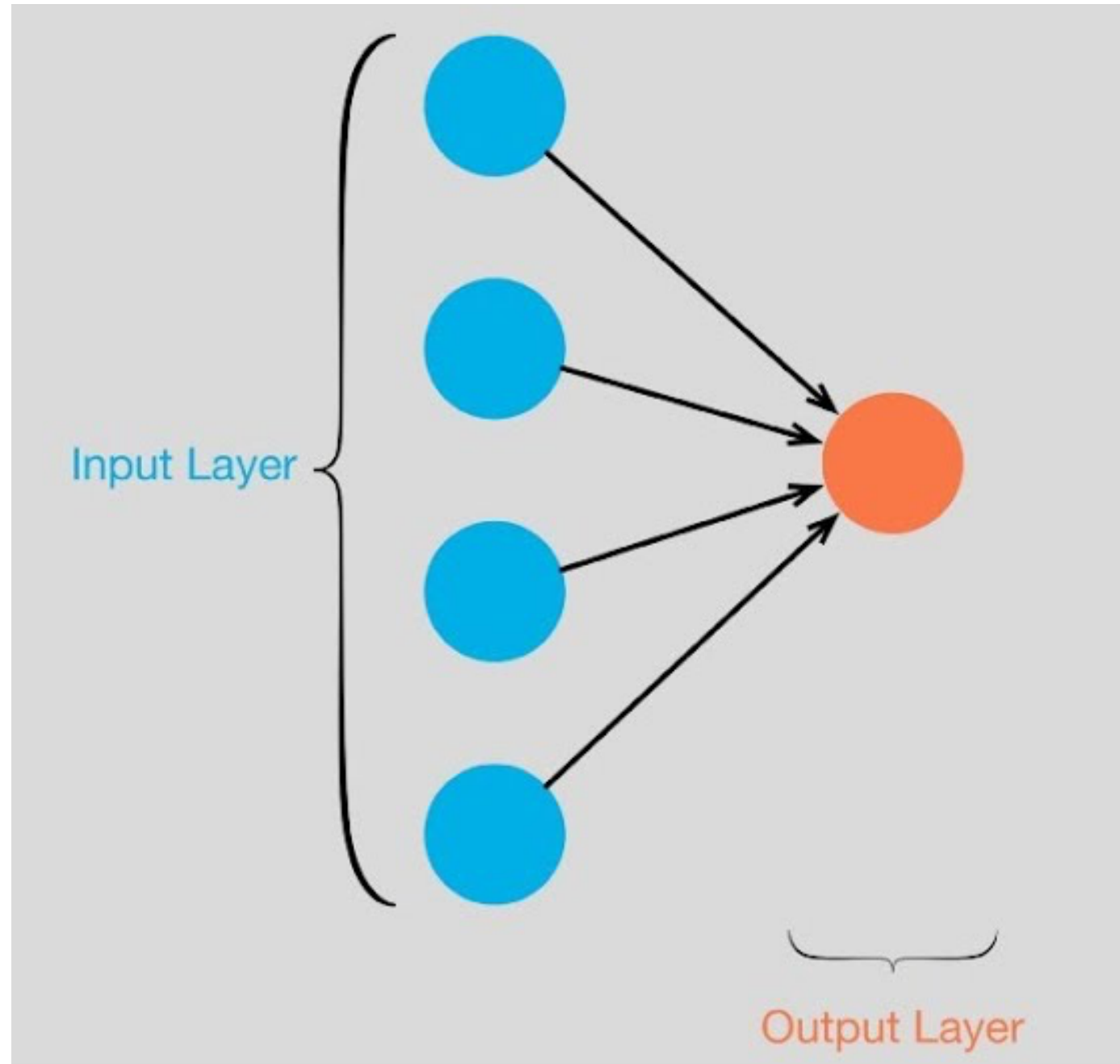


Not Linearly Separable!

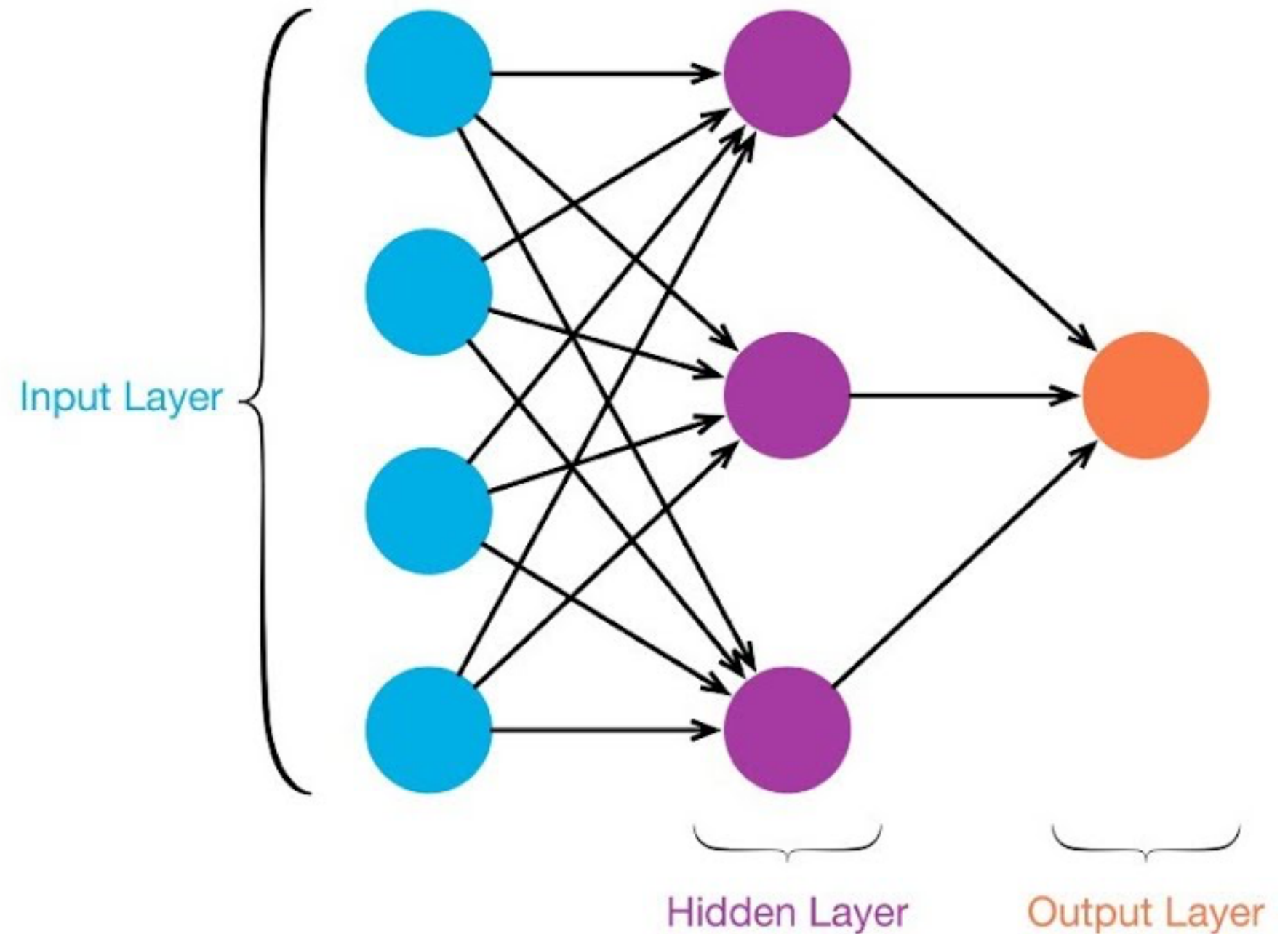
Linear vs Non-Linear Problem



Types of Perceptron



Single Layered Perceptron



Multi Layered Perceptron

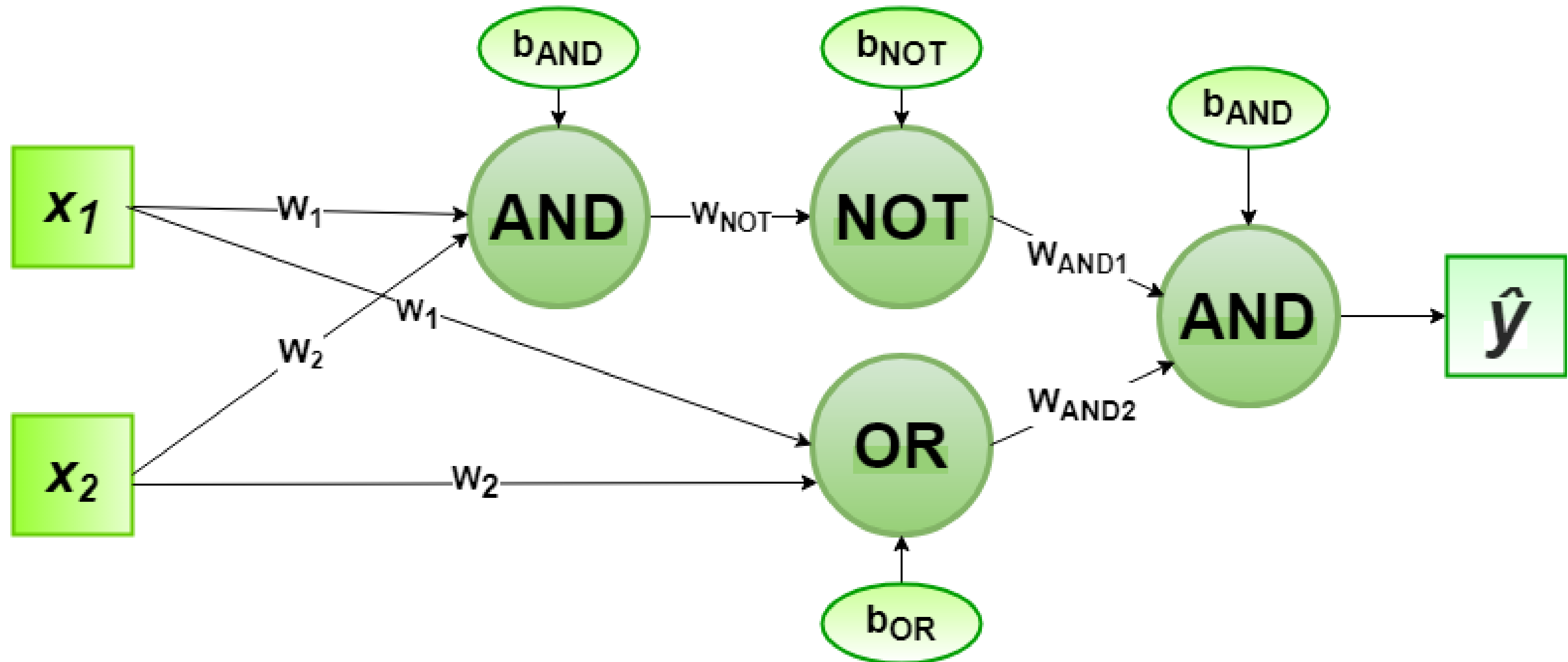
Lets Solve XOR Problem using MultiLayer Perceptron Approach!

Building a Logical XOR

Inputs		Output
A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

$$\text{XOR}(A,B) = (A \wedge \neg B) \vee (\neg A \wedge B)$$

Building a Logical XOR



$$XOR(A,B) = (A \wedge \neg B) \vee (\neg A \wedge B)$$



Thank you!

