8-7-24 Omega Contest

Sample Test 3

Correct answers are worth 1 point. Incorrect and blank answers are worth 0 points. You are given 30 minutes to complete the test and no calculators are allowed.

1. _____ How many integers less than 100 have a sum of digits divisible by 3?

2. _____ What is the remainder when 2^{101} is divided by 3?

3. _____ A donut shop has donuts with sprinkles, frosting, or both. 20 of the donuts have sprinkles, 30 have frosting, and 15 have both. How many donuts does the shop have?

4. _____ Solve for x:

$$\log_4(\log_3(\log_2(x))) = 1$$

5. _____ Alice is hosting a Super Bowl party. She doesn't know how many people will come, but that at least 1 person will attend, and at most 4 people. If she has 5 cans of soda, how many ways are there for her to distribute the cans for any amount of people coming? For instance, if three people attend, she can distribute them 3-1-1. If four people attend, she can distribute them 2-2-1-0. Assume that the people are indistinguishable and she isn't including herself.

6. _____ Circle O has radius 5. Two chords, AB and AC, are drawn parallel to each other. If EC = 3 and the distance between the two chords is 7, then what is the length of FB?