Year 2 MetriX Mathematical Olympiad Day II

Problem 4. Define $\triangle ABC$ with circumcircle ω. Define point D on segment BC, and define points E and F on AC and AB respectively such that $ABDE$ and $ACDF$ are cyclic. Lines EF and BC meet at P. Define point Q on EF such that $QD \perp BC$. Let AQ meet ω again at R. Show that as D varies on BC, the intersection of PR and ω is fixed.

Problem 5. Let $P(x) = a_3x^3 + a_2x^2 + a_1x + a_0$ be a polynomial in $\mathbb{Z}[x]$ that doesn't have any integer roots. Let $P'(x) = 3a_3x^2 + 2a_2x + a_1$ be its derivative. Furthermore, there exist polynomials $U(x), V(x)$ such that

$$U(x)P(x) - V(x)P'(x) = 1$$

is a polynomial identity. Furthermore, $P(x)$ is odd for all $x \in \mathbb{Z}$. Prove that there exists infinitely many n such that $P(n)$ is not divisible by the squares of two primes.

Problem 6. Let p be a prime. Initially there are p^4 girls in a k-pop group. Each week, they go into p^2 groups of p^2 people and each group does a dance. Find an explicit construction of the group's dance schedule such that after $p^2 + 1$ weeks, each girl has danced with (in other words, been in the same group with) any other girl exactly once.

Submission Process I. You can submit your solutions on AoPS PM to Aritra12, EpicNumberTheory, CANBANKAN, k12byda5h, Orestis_Lignos or other specified members if you have an account at AoPS. This is the most preferred way of submission and it is also beneficial to participants because on AoPS PM you are allowed to send solutions one by one in that single PM however you are not allowed so for the other process but obviously you can send day 1 and day 2 separately.

Submission Process II. If you are unable to do the other process above then just simply mail your solutions pdf to us on our mail gaussiancurv180@gmail.com

Day I Time Limit. The time limit is 4 hours and 30 minutes only.

Day I Paper Language. The language of the paper is English and no copy of this paper is available in other languages except for english.