



## Year 2 MetriX Mathematical Olympiad Day I

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✔ **Problem 1.** There are 2022 different lamps spaced evenly around a circle. On the first day, a subset of the lamps are lit. On any day after this, a lamp is lit if and only if exactly one of its two neighbors was lit yesterday. Otherwise, it is unlit.

Find the number of possible subsets of lamps that can be lit on the first day, such that on any day after the first, the subset of lit lamps on that day will not coincide with the subset of lit lamps on the first day.

✔ **Problem 2.** Find all functions  $f: \mathbb{N} \rightarrow \mathbb{N}$  such that

$$\frac{4f(x)f(y-3)}{f(y)f(x-2) + f(x)f(y-2)}$$

is an integer for all integers  $x > 2$  and  $y > 3$ .

*Remark.*  $\mathbb{N}$  denotes the set of positive integers.

✔ **Problem 3.** Let  $ABC$  be a scalene triangle. Suppose that a circle centered on line  $BC$  passing through  $A$ , a circle centered on line  $CA$  passing through  $B$ , and a circle centered on line  $AB$  passing through  $C$  share a common radical axis. Prove that the circumcenter of  $\triangle ABC$  lies on this radical axis.

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**Submission Process I.** You can submit your solutions on AoPS PM to [Aritra12](#), [EpicNumberTheory](#), [CANBANKAN](#), [k12byda5h](#), [Orestis.Lignos](#) or other specified members if you have an account at AoPS. This is the most preferred way of submission and it is also beneficial to participants because on AoPS PM you are allowed to send solutions one by one in that single PM however you are not allowed so for the other process but obviously you can send day 1 and day 2 separately.

**Submission Process II.** If you are unable to do the other process above then just simply mail your solutions pdf to us on our mail [gaussiancurv180@gmail.com](mailto:gaussiancurv180@gmail.com)

**Day I Time Limit.** The time limit is 4 hours and 30 minutes only.

**Day I Paper Language.** The language of the paper is English and no copy of this paper is available in other languages except for English.