First OJIMC 2021

Online International Mathematical Cup

Day II Problems

Problem 4. Let ABC be a triangle such that $\angle A = 75°, \angle B = 60°,$ and $\angle C = 45°$. Let N_9 be the 9-point center of $\triangle ABC$, and let M be the midpoint of AC. What is the measure of the acute angle formed by the lines AN_9 and BM?

Problem 5. Prove that no matter how we tile a fixed grid polygon with (possibly rotated) 1×2021 tiles, the number of tiles whose bottom left vertex has the sum of coordinates divisible by 2021 is a constant (which may depend on the polygon).

Problem 6. Let a, b, c be positive real numbers. For any positive integer n, prove that

$$\left(\frac{4a^2+2ab}{3a+3b}\right)^n + \left(\frac{4b^2+2bc}{3b+3c}\right)^n + \left(\frac{4c^2+2ac}{3c+3a}\right)^n \geq a^n + b^n + c^n.$$

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Note. Each problem is worth 7 points. You have 4 hours and 30 minutes to solve the problems and write down your solutions. The use of calculators, engines and any other form of external help is forbidden. For more information, see the official guidelines. In order to receive full marks for problem 4, you must provide a diagram.