Difficulty 1

1. Jack chose nine different integers from 1 through 19 and found their sum. From the remaining ten integers, Jill chose nine and found their sum. If the ratio of Jack's sum to Jill's sum was $7:15$, which of the nineteen integers was chosen by neither Jack nor Jill?

2. Regular hexagon $ABCDEF$ is given in the plane. If the area of the triangle whose vertices are the midpoints of AB, CD, and EF is 225, what is the area of $ABCDEF$?

3. Suppose x and y are positive real numbers such that $x+y=x^2y$ and $x-y=xy^2$. Find xy.

4. A random pizza is made by flipping a fair coin to decide whether to include pepperoni, then doing the same for sausage, mushrooms, and onions. The probability that two random pizzas have at least one topping in common can be written in the form $\frac{m}{n}$ where m and n are positive integers. Find $m+n$.

5. Suppose a and b are two distinct real roots of the quadratic $ax^2+x+b=0$. Compute ab.

6. Rectangle $ABCD$ has $AB=4$ and $BC=3$. Diagonal AC is extended past C to point E such that triangle ABE is isosceles. Compute CE.

7. How many two-digit positive integers have exactly four factors?

8. Suppose that P is the polynomial of least degree with integral coefficients for which $P(\sqrt{3}+\sqrt{2})=\sqrt{3}-\sqrt{2}$. Compute $P(2)$.

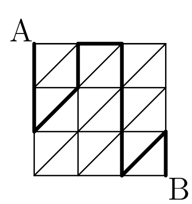
Difficulty 2

1. Determine the largest prime factor of $3^{15}+3^{11}+3^{6}+1$.

2. Let S be the set of twelve points $(\pm 1,\pm 1),(\pm 2,\pm 2),(\pm 3,\pm 3)$. How many circles pass through at least three of these points?

3. Two circles, ω_1 and ω_2, have radii of 5 and 12 respectively, and their centers are 13 units apart. The circles intersect at two different points P and Q. A line l is drawn through P and intersects the circle ω_1 at $X \neq P$ and ω_2 at $Y \neq P$. Find the maximum value of $PX \cdot PY$.

4. How many paths are there from A to B through the network shown if you may only move up, down, right, and up-right? A path also may not traverse any portion of the network more than once. A sample path is highlighted.



5. Let $f(x)$ be a function such that for all non-zero values x, we have $2f(x)+f(\frac{1}{x})=5x+4$. Let S denote the sum of all of the values of x for which f(x) = 2004. Compute the integer nearest to S.

6. An ant starts at the origin of a coordinate plane. Each minute, it either walks one unit to the right or one unit up, but it will never move in the same direction more than twice in the row. In how many different ways can it get to the point $(5, 5)$?

Difficulty 3

1. Consider every pair of positive integers (m, n) such that $m|n$ and $n|10^{10}$. Let P be the product of the values of $\frac{n}{m}$ as we go over every such pair (m, n). Compute $\log_{10} P$.

2. Let ΔABC be an isosceles triangle with $AB = AC$, and denote by ω the unique circle inscribed inside the triangle. Suppose the orthocenter of ΔABC lies on ω. Then there exist relatively prime positive integers m and n such that $cos(\angle BAC) = \frac{m}{n}$ Find $m + n$.

3. $ABCDEFG$ is a regular heptagon inscribed in a unit circle centered at O. ℓ is the line tangent to the circumcircle of $ABCDEFG$ at A, and P is a point on ℓ such that ΔAOP is isosceles. Let p denote value of $AP \cdot BP \cdot CP \cdot DP \cdot EP \cdot FP \cdot GP$. Determine the value of p^2.

4. If you flip a fair coin 1000 times, let P be the expected value of the product of the number of heads and the number of tails. What are the first three digits of P?

5. (Bonus!) Six chess players are ranked first through sixth. After a tournament, the rankings of these players are adjusted in such a way that three players ranked lower and three players ranked higher. In how many ways could this have happened?

Answers 1

1. 14
2. 600
3. $\sqrt{2}$
4. 31
5. -2
6. $7/5$
7. 32
8. 12

Answers 2

1. 61
2. 99
3. 120
4. 168
5. 601
6. 162

Answers 3

1. 12100
2. 10
3. 113
4. 250
5. 161