Jasper Math Club 9/22/16: Extra Geometry Problems

Part I: AMC problems with medians or angle bisectors. Kind of in increasing difficulty. **Half of these** are from the powerpoint. All solutions may be found here. If these look easy just go to part II.

1. (2011 AMC12A #13) Triangle ABC has AB=12, BC=24, AC=18. The line through the incenter of ABC parallel to BC meets AB,AC at M,N. What's the perimeter of AMN?

$(A)\ 27 \qquad (B)\ 30 \qquad (C)\ 33 \qquad (D)\ 36 \qquad (E)\ 42$

2. (2013 AMC10B #16) In triangle ABC, medians AD,CE meet at P. PE=1.5, PD=2, and DE=2.5 What's the area of AEDC?

$(A)\ 13 \qquad (B)\ 13.5 \qquad (C)\ 14 \qquad (D)\ 14.5 \qquad (E) 15$

3. (2010 AMC10A #16) Nondegenerate triangle ABC has integer side lengths, BD is an angle bisector, AD = 3, and DC=8. What is the smallest possible value of the perimeter?

$(A)\ 30 \qquad (B)\ 33 \qquad (C)\ 35 \qquad (D)\ 36 \qquad (E)\ 37$

4. (2016 AMC12A #12) In triangle ABC, AB=6, BC=7, CA=8. D and E lie on BC, AC with AD bisecting angle BAC and BE bisecting angle ABC. If AD,BE meet at F, find AF:FD.

$(A)\ 3:2 \qquad (B)\ 5:3 \qquad (C)\ 2:1 \qquad (D)\ 7:3 \qquad (E)\ 5:2$

5. (2002 AMC12A #23) In triangle ABC , side AC and the perpendicular bisector of BC meet in point D, and BD bisects angle ABC. If AD=9 and DC=7, what is the area of triangle ABD?

$(A)\ 14 \qquad (B)\ 21 \qquad (C)\ 28 \qquad (D)\ 14\sqrt{5} \qquad (E)\ 28\sqrt{5}$

6. (2000 AMC12A #19) In triangle ABC, AB = 13, BC = 14, AC = 15. Let D denote the midpoint of BC and let E denote the intersection of BC with the angle bisector of angle BAC. Which of the following is closest to the area of the triangle ADE?

$(A)\ 2 \qquad (B)\ 2.5 \qquad (C)\ 3 \qquad (D)\ 3.5 \qquad (E)\ 4$

7. (2009 AMC12A Problem 14) A triangle has vertices (0,0), (1,1), and (6m,0), and the line y=mx divides the triangle into two triangles of equal area. What's the sum of all values of m?

$(A)\ -\frac{1}{3} \qquad (B)\ -\frac{1}{6} \qquad (C) \frac{1}{6} \qquad (D)\ \frac{1}{3} \qquad (E)\ \frac{1}{2}$

8. (2004 AMC10B Problem 24) In triangle ABC we have AB=7, AC=8, BC=9. D is on circle ABC with AD bisecting angle BAC. What's AD:CD?

$(A)\ 9:8 \qquad (B)\ 5:3 \qquad (C)\ 2:1 \qquad (D)\ 17:7 \qquad (E)\ 5:2$

Part II: AIME level geometry problems involving Stewart's Theorem. These don't have sources so if you need solutions to the problems, ask smart friends or post in the facebook group. If you're still bored, go to part III.

Background Info: Stewart's Theorem is a very useful theorem which tells us the following: In triangle ABC, if D is on segment BC, we have $AB^2 * CD + AC^2 * BD = AD^2 * BC + BC * CD * BC$. We will be using Stewart's Theorem in the problems below without actually proving the theorem until problem 8.

1. If ABC is a triangle with M being the midpoint of BC, and AB=4,AC=5,BC=6, find the value of AM^2.
2. Let ABC be a triangle with M being the midpoint of BC. Prove that $4AM^2 = 2AB^2 + 2AC^2 - BC^2$.
3. If ABC is a triangle with AB=3, AC=4, BC=6, and G is the centroid of triangle ABC, compute the length of AG^2.
4. Let ABC be a triangle and let D be the point on BC so angles DAB, DAC are equal. If AB=3, AC=4, and BC=5, find the length of AD^2.
5. Let ABC be a triangle and P be a point on side BC. In terms of the sides and angles of triangle ABC, find the minimal value of AP^2.

6. Let ABC be a triangle and let P,Q be points on segment BC with BP=CQ. If $AB = 4, AC = 5$, find the value of $BP * PC + BQ * QC + AP^2 + AQ^2$.
7. Let ABC be a triangle and let M,N be the midpoints of AB, AC. If AB=3, AC=4, and CM,BN are perpendicular, find the length of BC^2.
8. Prove Stewart's Theorem. (Hint: Law of Cosines!)

Part III: Olympiad geometry problems involving Miquel's Theorem. These mostly don't have sources so if you need solutions ask a smart friend or post in the facebook group.

Background Info: Miquel's Theorem states that in any triangle ABC, if D,E,F are points on lines BC,CA,AB, then circles (AEF), (BDF), (CDE) concur.

1. Prove Miquel's Theorem. Note that D,E,F **don't** necessarily need to be on line **segments** BC,CA,AB.
2. Show with Miquel's Theorem that the circle through A tangent to BC at C, the circle through C tangent to AB at B, and the circle through B tangent to AC at A concur at a point P. Deduce the existence of the Brokard point P in triangle ABC which satisfies angle PAC = angle PCB = angle PBA.
4. Let ABC be a triangle with D,E,F on sides BC,CA,AB. Let O_1, O_2, O_3 be the circumcenters of triangles AEF, BFD, CDE. Prove that triangles $ABC,\ O_1O_2O_3$ are similar.
5. Let ABC be a triangle with circumcenter O. Points P,Q lie on AB,AC with A,P,O,Q lying on a common circle. If R is the orthocenter of triangle POQ, show that R lies on BC.
6. Let D be a variable point on side BC of triangle ABC. Let the circle through B and D tangent to AB and the circle through C and D tangent to AC meet again at point E. Prove that as D varies, line DE passes through a fixed point.
7. Let ABC be a triangle. Let D,E,F be on BC,CA,AB with AD,BE,CF concurring at P. Given that quadrilaterals AFDC and AEDB are cyclic, show that P is the orthocenter of triangle ABC.
8. Let O,H be the circumcenter and orthocenter of the acute triangle ABC. Suppose points A',B',C' are on sides BC,CA and AB such that circumcircles of triangles BC'A'$ and CA'B' pass through O. Let l_a be the radical axis of the circle with center B' and radius B'C and circle with center C' and radius C'B. Define l_b, l_c similarly. Prove that lines l_a, l_b, l_c form a triangle with orthocenter H.