Exam for the course "Options and Mathematics" (CTH[$MVE095$], GU[$MMA700$]). Period 4, 2013/14

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REMARK: No aids permitted

1. Assume that the stock price $S(t)$ follows a 1-period binomial model with parameters $u > d$ and that the interest rate of the bond is $r > 0$. Show that there exists no self-financing arbitrage portfolio invested in the stock and the bond in the interval $t \in [0, 1]$ if and only if $d < r < u$ (max 3 points). Show that any derivative on the stock expiring at time $t = 1$ can be hedged in this market (max 2 points).

2. Let $c(t)$ denote the Black-Scholes price at time t of a European call with strike $K > 0$ and maturity $T > 0$ on a stock with price $S(t)$ and volatility $\sigma > 0$. Let $r > 0$ denote the interest rate of the bond. Compute the following limits:

$$\lim_{K\to 0^+} c(t), \qquad \lim_{K\to +\infty} c(t), \qquad \lim_{T\to +\infty} c(t), \qquad \lim_{\sigma\to 0^+} c(t), \qquad \lim_{\sigma\to +\infty} c(t).$$

   Each limit gives 1 point if it is correct, 0 otherwise.

3. Consider an American put option with strike $K = 3/4$ at the maturity time $T = 2$. Let the price $S(t)$ of the underlying stock be given by the binomial model with parameters

$$e^u = \frac{7}{4}, \quad e^d = \frac{1}{2}, \quad e^r = \frac{9}{8}.$$

   Assume $S(0)$=1. Compute the fair price of the derivative (max 2 points) and the hedging portfolio (max 2 points) at each time $t = 0, 1, 2$. Verify if the put-call parity holds at all times (max 1 point).