Financial Times Series

Lecture 6

Extensions of the GARCH

- There are numerous extensions of the GARCH
- Among the more well known are EGARCH (Nelson 1991) and GJR (Glosten et al 1993)
- Both models allow for volatility skewness or leverage effects and are available in matlab econometrics

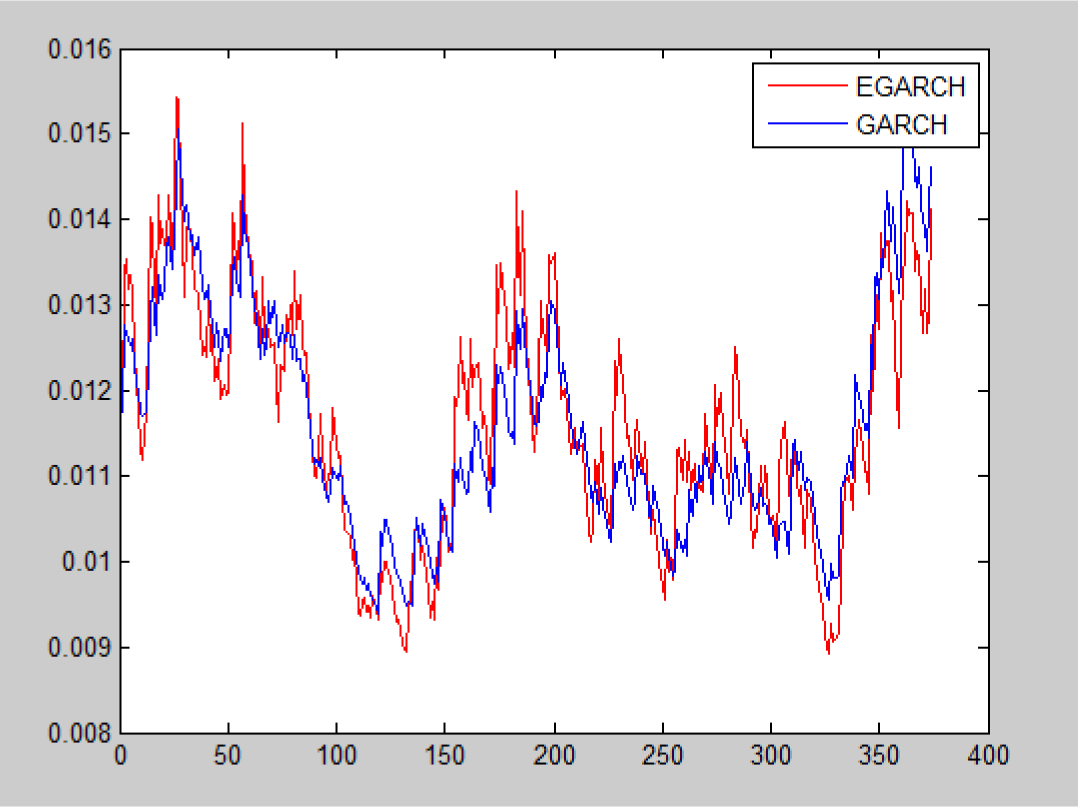
EGARCH

- E means exponential and the model for the conditional variance may be written

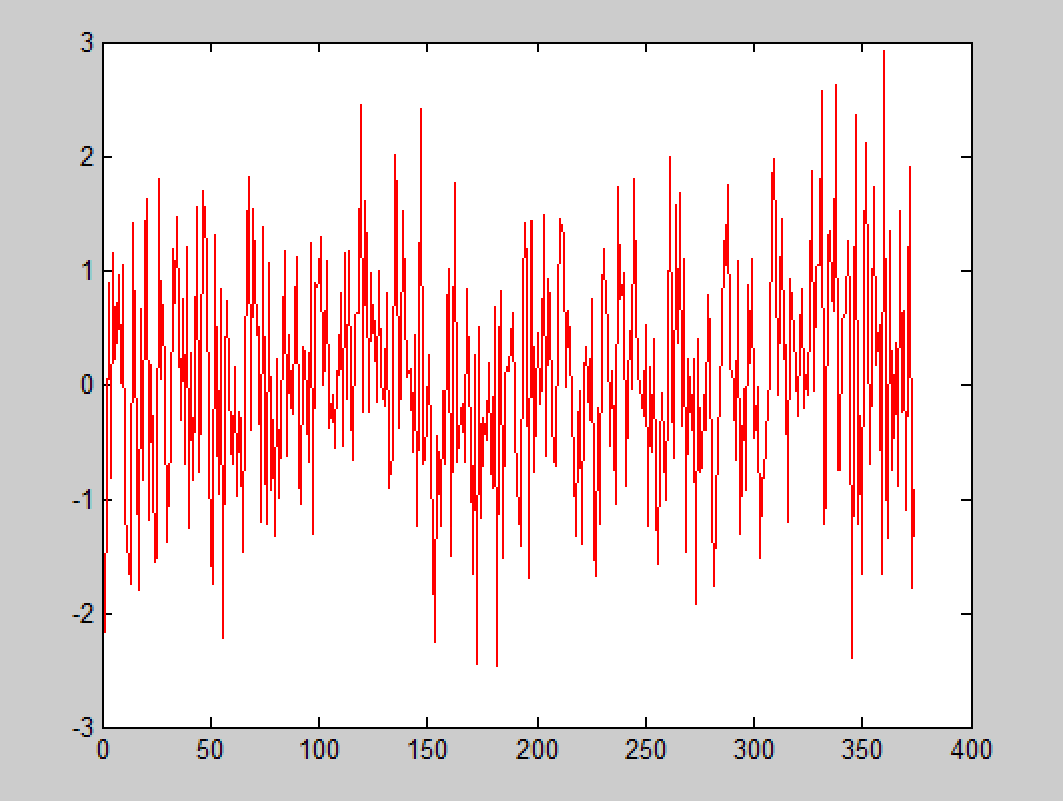
$$ln\sigma_t{}^2 = \omega + \alpha\frac{|r_{t-1}| + \gamma r_{t-1}}{\sigma_{t-1}} + \beta ln\sigma_{t-1}{}^2$$

- The parameter γ accounts for skewness
- We fit the model to the N225 data without the extreme event

EGARCH vs. GARCH



Devolatization with EGARCH



- p-value of Ljung-Box is 0.3106

GJR

- The Glosten-Jagannathan-Runkle GARCH may be written as

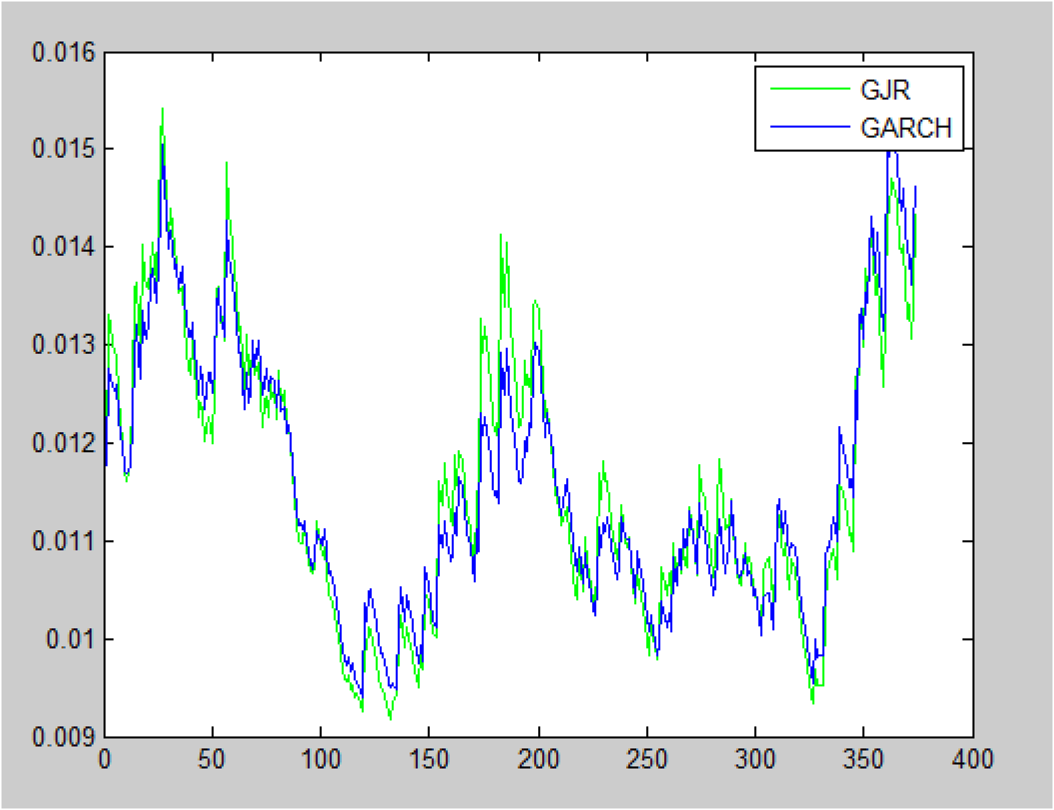
$$\sigma_t{}^2 = \omega + (\alpha + \varphi I_{t-1})r_{t-1}{}^2 + \beta\sigma_{t-1}{}^2$$

where $I_{t-1} = 0$ if $r_{t-1} \geq 0$ and $I_{t-1} = 1$ if $r_{t-1} < 0$, so that the parameter φ accounts for skewness

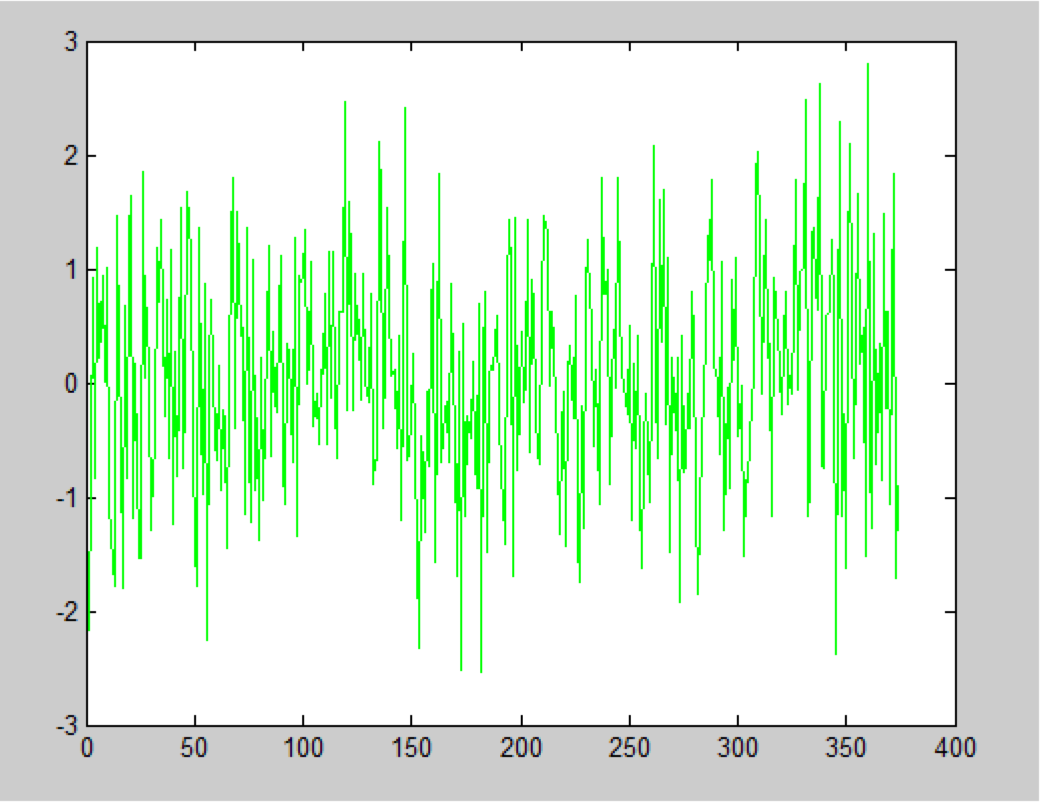
GJR philosophy

- Bad news gives higher volatility than good news
- Captures leverage effect

GJR fit (N225 without extreme event)



Devolatization with GJR

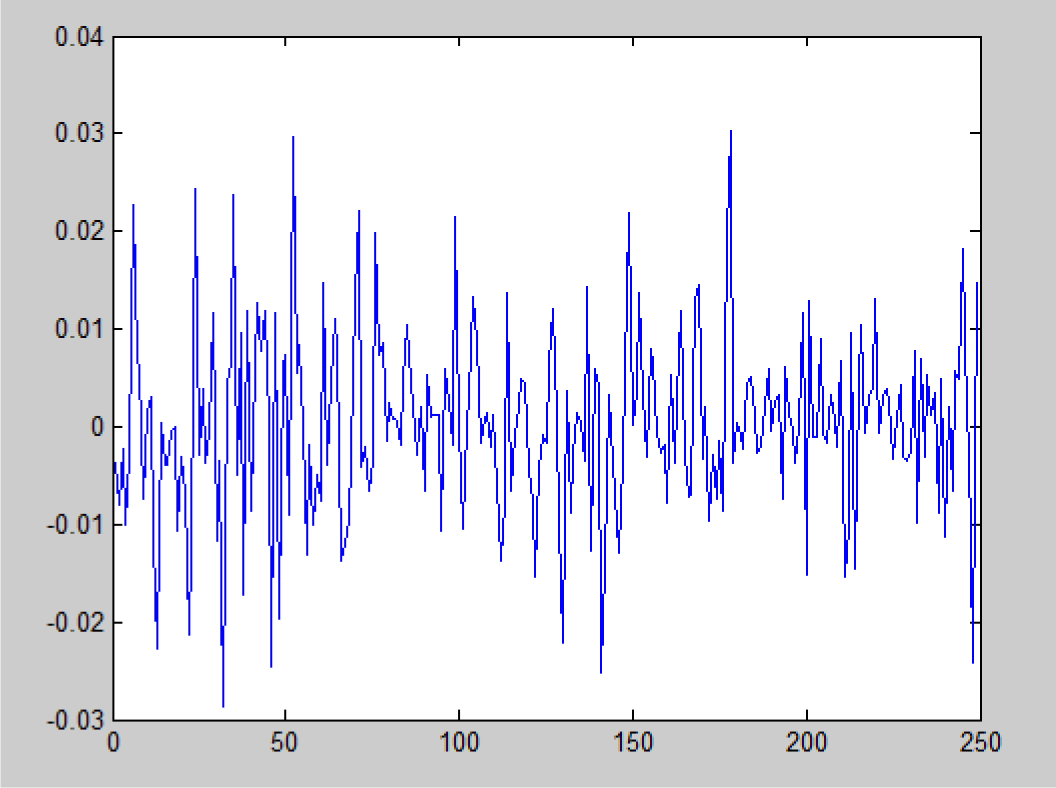


- p-value for Ljung-Box is 0.3838

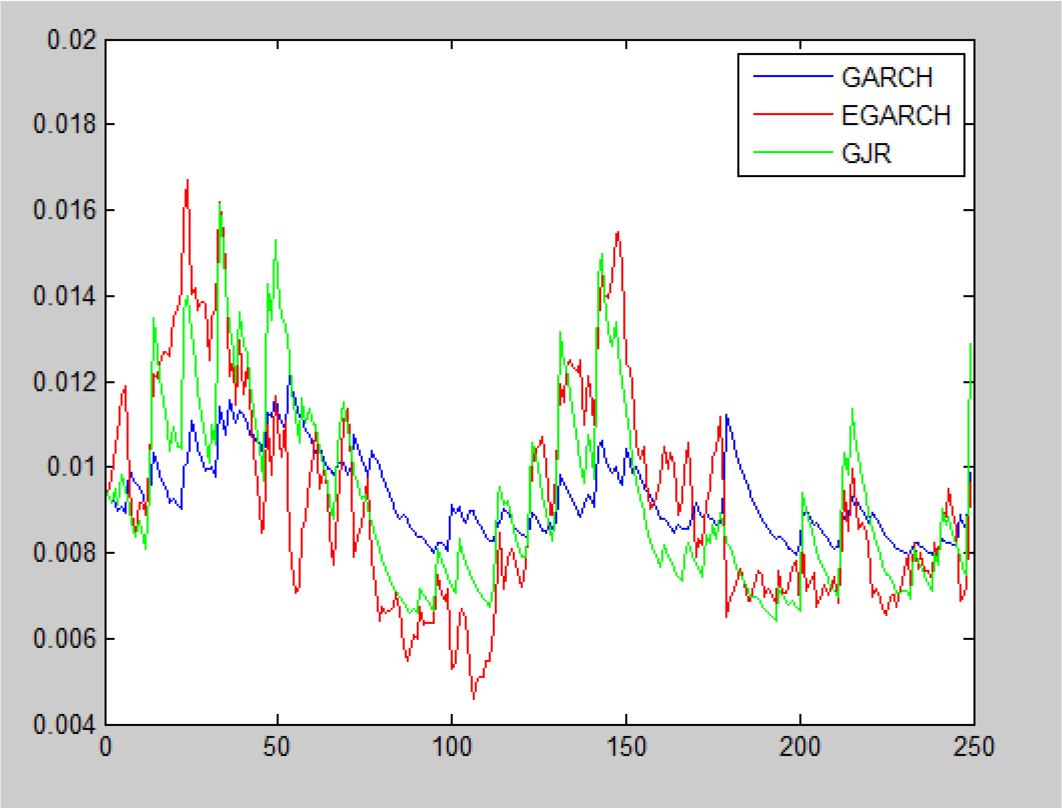
Comparison

- For the N225 ”without the tsunami” there does not seem to be an improvement, at least not in ”devolatizing properties”, using the more advanced models
- On the other hand, we have not yet used a statistical test procedure to compare the models…
- Below we try the three models for NASDAQ data and look at a statistical test for comparing the models

NASDAQ returns



Volatility fits



Devolatization of NASDAQ

- Ljung-Box p-value for GARCH is 0.4827

- Ljung-Box p-value for EGARCH is 0.4623

- Ljung-Box p-value for GJR is 0.5378

Evaluating predictions

- We may us squared returns as a proxy and compute MSE:s as

$$\frac{1}{T}\sum_{t=1}^{T}\left({r_t}^2 - {\hat{\sigma}_t}^2\right)^2$$

- For the NASDAQ data, we get $2.32 \cdot 10^{-8}$, $2.23 \cdot 10^{-8}$ and $2.29 \cdot 10^{-8}$ for the GARCH, EGARCH and GJR respectively

Evaluating predictions

- Another way of evaluating predictions, again with squared returns as proxy, is to regress squared returns on squared volatility predictions and hope for a slope close to one and $R^2 \approx 1$
- For the GARCH, EGARCH and GJR we have slopes 0.7975, 0.7052 and 0.6032 and R-squares 0.0095, 0.0568 and 0.0312 which is not so satistisfactory, however it can be shown theoretically that for a GARCH(1,1) that R-squares close to one are highly unlikely

Squared returns is a noisy proxy

- What if we instead use realized variance over 30 days and compare to 30 day squared volatility forecasts?

- The 30 day realized variance for is given by

$$\sum_{i=t}^{t+29} {r_i}^2$$

Squared returns is a noisy proxy

- Our 30 day volatility predictions will just be the sums of the daily volatility estimates of the past 30 days

- Using the 30 day framework, we get, for the GARCH, EGARCH and GJR slopes 2.37(!), 0.7489 and 1.06 and R-squares 0.8289, 0.6000 and 0.8399 which is more satisfactory, but the slope for the GARCH is not reasonable…

Diebold-Mariano

- If we choose a loss function and a proxy, there is a test proposed by Diebold and Mariano (1995) for evaluating if one prediction method is significantly better than another

- The null hypothesis is that both methods have the same accuracy

Diebold-Mariano

- Define $d_t = L(\varepsilon_{At}) - L(\varepsilon_{Bt})$ where L denotes the loss function ε_{At} and ε_{Bt} denote the prediction errors from method A and B, respectively

- The test statistic is

$$\frac{\bar{d}}{\sqrt{\widehat{LRV}/T}} \sim N(0,1)$$

where $LRV = Var(d_t) + 2\sum_{j=1}^{\infty} Cov\left(d_t, d_{t-j}\right)$

Diebold-Mariano

- Note that you have to keep track of which error is to the left and to the right of the minus sign in order to tell which method is better

- A DM test using $L(x) = x^2$, i.e. squared loss, is available at matlab central

- It also accounts for the length of the forecast horizon

Diebold-Mariano

- For our three models of 30 day NASDAQ volatility, the observed values of test statistic are -1.8547 for GJR vs. EGARCH, -1.2456 for GJR vs. GARCH and -0.1204 for EGARCH vs. GARCH.

- So, p-values are 0.0636, 0.2129 and 0.9042

- At 0.05 signicance level, no model is significantly better than the other, but of course this ”decision” depends on the choice of loss function…

Applications of volatility models

- Depending on their predictive ability, volatility models may be useful in option pricing and risk management

- We may replace the constant volatility in B-S with time series volatility and simulate price trajectories of underlying assets in order to price options

Option pricing

- Under a GARCH model we simulate stock prices starting from P_0 up to a terminal price P_T using

$$P_t = P_{t-1} exp\{r - 0.5{\sigma_t}^2 + \varepsilon_t\}$$

where r is the risk-free interest rate, $\varepsilon_t = \sigma_t z_t$ for i.i.d. $z_t \sim N(0,1)$ and

$${\sigma_t}^2 = \omega + \alpha {\varepsilon_{t-1}}^2 + \beta {\sigma_{t-1}}^2$$

Option pricing

- Simulating lots of paths we may price options using averages of pay-off functions

- Assume, for example, that we want to price an asian call option with pay-off

$$max\left\{\frac{1}{T}\sum_{t=1}^{T} P_t - K, 0\right\}$$

Option pricing

- Using N price paths, the simulated price will be given by

$$e^{-rT}\left(\frac{1}{N}\sum_{j=1}^{N} max\left\{\frac{1}{T}\sum_{t=1}^{T} P_{jt} - K, 0\right\}\right)$$

GARCH vs. B-S prices

- Setting $\omega = 0.0005, \alpha =, 0.05\ \beta = 0.85, P_0 = 50, T = 30, r = 0.015/365$, the B-S variance equal to the unconditional variance of the GARCH and using 10000 paths yields

Strike K	Black-Scholes	GARCH
45	7.04	7.10
50	4.34	4.31
55	2.47	2.48

For more on GARCH in pricing

- Check out the work of Jin-Chuan Duan
- http://www.rmi.nus.edu.sg/duanjc/
- Matlab codes available!

VaR

- One of the most common notions in financial risk management is that of Value at Risk (VaR)

- VaR may be used to determine the amount of regulatory capital to set aside for different types of risks

- For a given collection of assets we may define the loss variable L and VaR as

$$VaR_{\alpha} = inf\{x : P(L > x) \leq 1 - \alpha\}$$

VaR

- Typically $\alpha = 0.95$ or $\alpha = 0.99$

- The distribution function of the loss variable is typically not known, but we could simulate losses under some assumptions or use time-series models

- We will focus on VaR for log-returns r_{t+1} starting from the information available at time t

VaR and B-S

- In the Black-Scholes framework, we have $r_{t+1} \sim N(\mu_{t+1}, \sigma_{t+1})$ so that

$$VaR_\alpha = \mu_{t+1} + \sigma_{t+1} z_\alpha$$

- Note that VaR expressed in this way is an (approximate) percentage and to state VaR in dollar amount the percentage should be multiplied with dollar amount outstanding

VaR and RiskMetrics

- In the RiskMetrics framework it is assumed that $r_t | F_{t-1} \sim N(0, \sigma_t)$ with (IGARCH)

$$\sigma_t{}^2 = (1 - \lambda) r_{t-1}{}^2 + \lambda \sigma_{t-1}{}^2$$

- This gives the time scaling property that the k-day VaR is $\sqrt{k}$ times the one-day VaR

RiskMetrics example

- Under the RiskMetrics model the $\alpha = 0.95$ one-day VaR at time t is just $-1.65\sigma_{t+1}$
- Using the scaling property the $\alpha = 0.95$ k-day VaR is $-1.65\sqrt{k}\sigma_{t+1}$
- If the zero mean property or IGARCH assumption does not hold the time-scaling property will also fail to hold

RiskMetrics

- Another appealing property of the RiskMetrics framework is that if VaR_1 and VaR_2 are the values at risk for two positions under the special IGARCH model, it holds that the total value at risk is

$$VaR = \sqrt{{VaR_1}^2 + {VaR_2}^2 + 2\rho_{12} VaR_1 VaR_2}$$

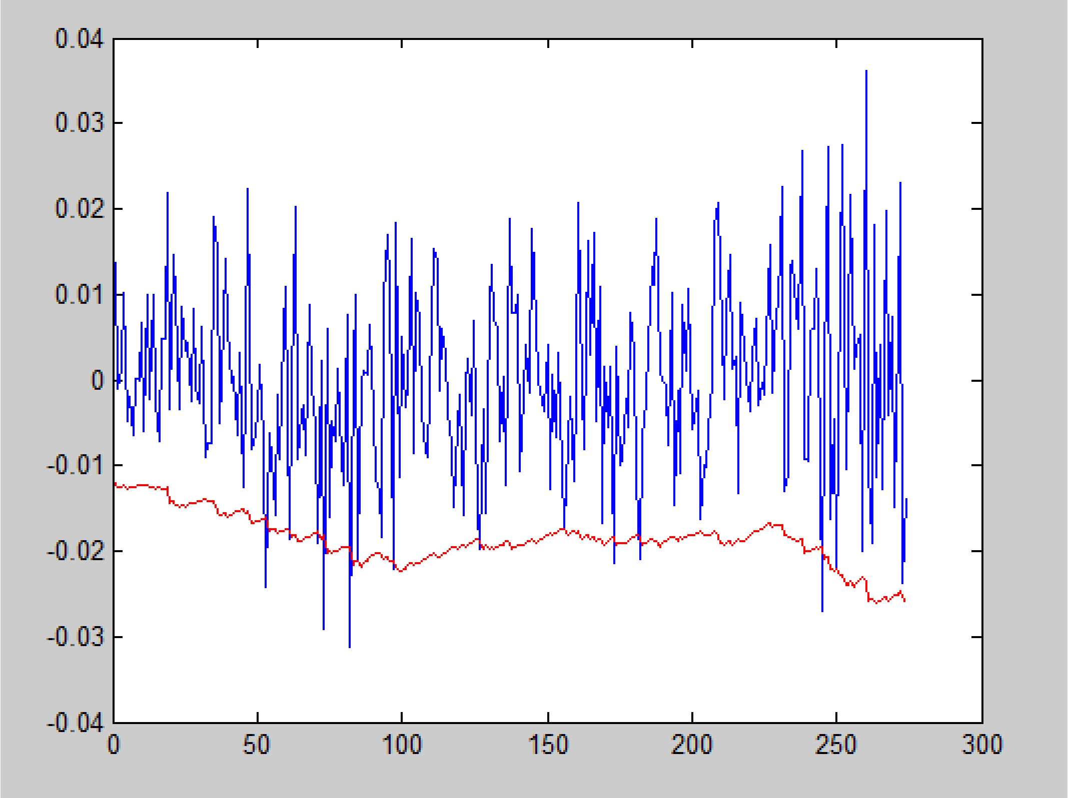
where ρ_{12} is the correlation between the returns

Example

- Fitting the RiskMetrics model to OMXS30 data (last year) yields a L-B p-value of 0.0024 for devolatized returns

- For N225 (last year) we get a L-B p-value of 0.0675, so we try compute VaR for these data

RiskMetrics one day 95% VaR



VaR time series model

- Assuming a general time series model, we have

$$r_t = \varphi_0 + \sum_{i=1}^{p} \varphi_i r_{t-i} + a_t - \sum_{j=1}^{q} \theta_j a_{t-j}$$

$$a_t = \sigma_t z_t$$

$${\sigma_t}^2 = \omega + \sum_{i=1}^{u} \alpha_i {a_{t-i}}^2 + \sum_{i=1}^{v} \beta_i {\sigma_{t-i}}^2$$

VaR time series model

- We get one-day ahead predictions as

$$\hat{r}_t(1) = \varphi_0 + \sum_{i=1}^{p} \varphi_i r_{t+1-i} - \sum_{j=1}^{q} \theta_j a_{t+1-j}$$

$$\hat{\sigma}_t{}^2(1) = \omega + \sum_{i=1}^{u} \alpha_i a_{t+1-i}{}^2 + \sum_{i=1}^{v} \beta_i \sigma_{t+1-i}{}^2$$

VaR time series model

- If we assume $\{z_t\} \sim N(0,1)$ and hence $r_{t+1}|F_t \sim N(\hat{r}_t(1), \hat{\sigma}_t(1))$ we may get the $\alpha = 0.95$ one-day VaR at time t as

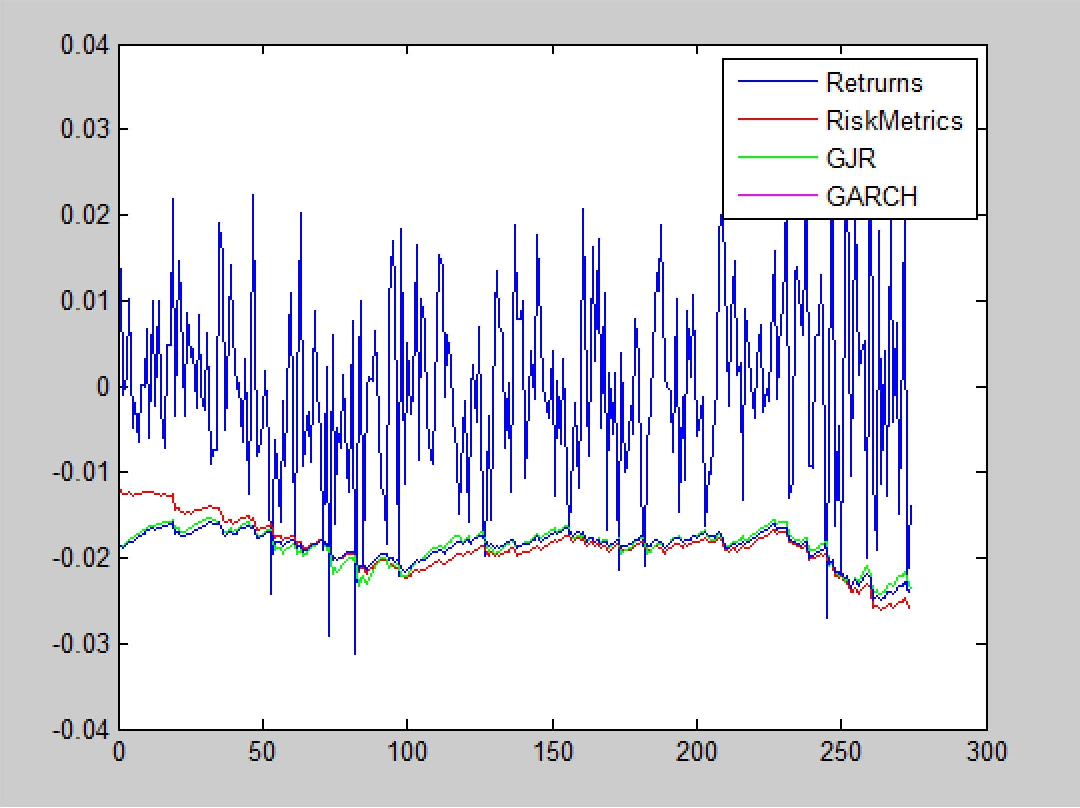
$$\hat{r}_t(1) - 1.65\hat{\sigma}_t(1)$$

- For an arbitrary WN-distribution, F, we have

$$\hat{r}_t(1) + F^{-1}(0.05)\hat{\sigma}_t(1)$$

VaR time series model

- With $\{z_t\} \sim N(0,1)$, we get (OMXS30 data)



Expected Shortfall

- Given that the VaR is exceeded, one may wonder how bad this can be

- The average of $VaR_{\alpha'}$:s, where $0 < \alpha' < \alpha$ is the expected shortfall corresponding to VaR_α

$$ES_\alpha = \frac{1}{\alpha}\int_{\alpha}^{1} VaR_{\alpha'} d\alpha'$$

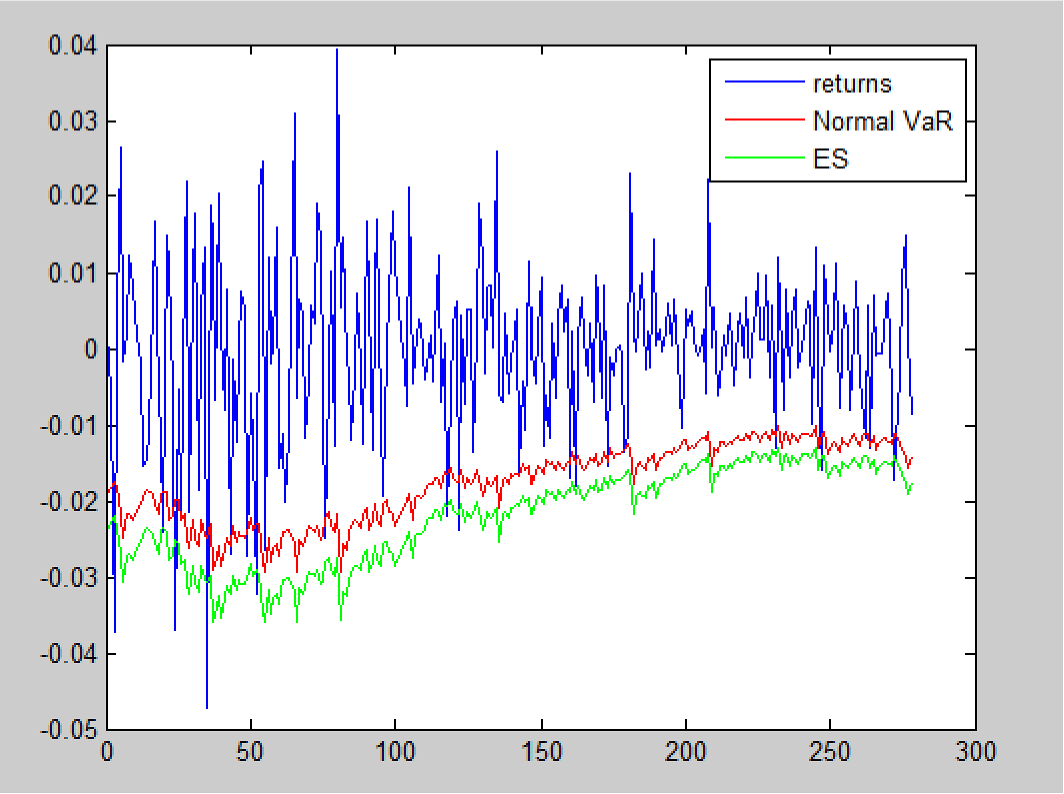
Numerical approximation

- For an arbitrary α we may approximate ES with e.g.,

$$\frac{1}{N}\sum_{i=1}^{N} VaR_{\alpha+(1-\alpha)(i-1)/N}$$

ES for GARCH based VaR with normal WN

- Here $\alpha = 0{,}95$ and $N = 5000$



ES for GARCH based VaR with t_8 WN

- Here $\alpha = 0,95$ and $N = 5000$