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IIIT, Vadodara
Autumn 2017-18
CS405 Computer Vision
Lab#2: Ill-posedness and ill-conditioning: Need for regularization

Q. 1: **Naive deblurring** $\widehat{X} = A^{-1} * Y_1$: Consider an image X. Assume that it is a focused image. Now, consider a blur matrix A (nonsingular, i.e., invertible, but with a high condition number). Find the condition number and the inverse of the blur matrix. Convolve the image with the blur matrix, i.e., generate blurred (defocused) version of the input image and display it, $Y_1 = A * X$. Such images are generated when your camera is out of focus. Let us try to deblur the blurred image using an algorithmic approach, i.e., estimate the deblurred (focused) image $\widehat{X}$. To this end, let us consider a naive approach, i.e., take the inverse of the blur matrix and convolve with the blurred image, $\widehat{X} = A^{-1} * Y_1$. Display, compare and comment on the results. Also calculate the root mean-squared error (RMSE) between input image and estimated deblurred image. Finally display the RMSE map showing mean and standard deviation values. Observe that deblurring is an ill-posed problem, why?. Note that in the real scenario we are not even aware of the blur matrix and/or the focused image. Hence, the problem becomes severely ill-posed, why?.

Q. 2: **Noise in image** X: Reconsider the Q1. Let us add a very small amount of Gaussian noise N, i.e., zero mean with say 0.0001 variance, to the blurred image. Note that now the resultant image is not only blurred but also noisy, i.e., $Y_2 = A * X + N$. Apply the same algorithm (Q1) and try to estimate the deblurred or focused image, i.e., $\widehat{X} = A^{-1} * Y_2$. Display the result, comment (ill-conditioning) and compare with Q1.

Q. 3: **Noise in blur matrix** A: Repeat the Q2 but add the same noise in the blur matrix A, i.e., $Y_3 = (A + N) * X$. Again apply the same deblurring algorithm (Q1) and try to recover the focused image, i.e., $\widehat{X} = (A + N)^{-1} * Y_3$. Now display the result, comment (ill-conditioning) and compare with Q1 and Q2.

Regularization*[1]*: To use additional information (from the given problem) explicitly, at the start, to construct families of approximate solutions to the inverse ill-posed problems (IIPs). **For example** (referring to our deblurring problem),

$$\widehat{X} = A^{-1} * Y + \lambda \left\| LX \right\|_2, \tag{1}$$

where, λ is the regularization parameter to be estimated and L is the matrix representative of derivative operator. Regularizations are now one of the most powerful tools for the solution of the IIPs.

[1] Not to be implemented in this Lab

Q. 4: The ability of human visual system to detect an object in a uniform background depends on its size (resolution) and the contrast ratio γ which is defined as

$$\gamma = \frac{\sigma}{\mu}, \tag{2}$$

where, μ is the average luminance of the object and σ is the standard deviation of the luminance of the object plus its surround. Now consider the *inverse contrast ratio*,

$$v(m,n) = \frac{\mu(m,n)}{\sigma(m,n)}, \tag{3}$$

where, $\mu(m,n)$ and $\sigma(m,n)$ are the local mean and local standard deviation, respectively, measured over a window W, within the given image $u(m,n)$. The local mean and local variance are defined as,

$$\mu(m,n) = \frac{1}{N_W} \sum_{(k,l)\in W} \sum u(m-k, n-l), \tag{4}$$

$$\sigma(m,n) = \left\{ \frac{1}{N_W} \sum_{(k,l)\in W} \sum \left[u(m-k, n-l) - \mu(m,n)\right]^2 \right\}^{\frac{1}{2}}. \tag{5}$$

Apply this transformation on an input image $u(m,n)$ and generate a scaled version of the $u(m,n)$. Observe the output image and comment on the result. A special case of equation (3)

$$v_s(m,n) = \frac{u(m,n)}{\sigma(m,n)} \tag{6}$$

is called *statistical scaling*. Now, apply the statistical scaling on the $u(m,n)$. Comment and compare the output images generated after applying the statistical scaling and inverse contrast scaling.

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