

Set 4: Fibonacci Number

$$F_n \leftarrow \# \text{ Additions} = A(n) < \frac{n^2 \cdot x}{2^n} \checkmark$$

$$\boxed{xy_{i+1}(n)} - A(n)$$

$$\boxed{xy_{i+1}(n-1)} + \boxed{xy_{i+1}(n-2)}$$

P

Q

$$A(n) = \underline{P} + \underline{Q} + 1$$

$$= A(n-1) + A(n-2) +$$

Difference

$$\boxed{A(n) = A(n-1) + A(n-2) + 1}$$

Recurr

$$A(n) = O(1.7^n) \approx O(2^n)$$

Relation

Q:

$$A(n) = C_1 A(n-1) + C_2 A(n-2) + C_3 A(n-3) + \dots + C_n$$

Linear Recurrence Relation
with constant coefficients.

$$\sum_{i=1}^n \rightarrow 2^{n-1} \rightarrow \textcircled{n} \rightarrow \log n$$

$$\underline{\underline{F_n}} \leftarrow A(n)$$

$$\underline{\underline{\text{Additions}}} = \underline{\underline{(n-2)}}$$

$$\textcircled{xy+1(n)} \leftarrow xy+1(n-1)$$

$$\sim O(n) \text{ ms}$$

$$\leftarrow xy+1(n-2)$$

$$A(n) = \underline{\underline{O(n)}} \times$$

$$\xrightarrow{36\text{-year}} \underline{\underline{O(2^n)}}$$

$$R = P + Q + 1$$

$$\downarrow \quad \downarrow$$

$$W \left[A(n) = 1 \cdot A(n-1) + 1 \cdot A(n-2) + 1 \right]$$

Recursive
Relaha
Difference
Eqn.

$$\times \left[A(n) = A^2(n-1) + A^3(n-2) + \dots \right]$$

Linear Recurrence Relation with Const Cof

$$A(n) \Rightarrow O(1.7^n) \Rightarrow O(2^n)$$

$$i = 0$$

$$j = 1$$

$$A(0) = 0$$

$$A(1) = 1$$

$$i = 2 \text{ to } n$$

...

$$F_n \leftarrow n$$

recursion way

$$i \rightarrow F_i$$

✓
correct.

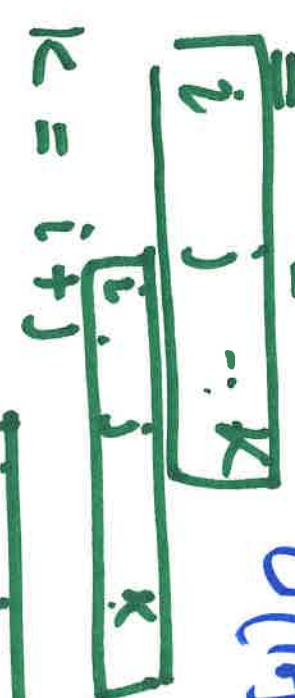
$$n = 5$$

computation
Time

$$O(2^n) \text{ correct.}$$

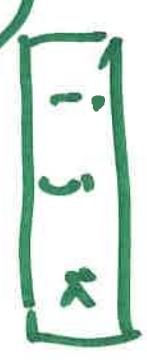
$F_0 \quad F_1 \quad F_2 \quad F_3 \quad F_4 \quad F_5 \quad F_6 \quad F_7 \quad F_8 \quad F_9$

$O(17n) \leftarrow O(n) \leftarrow O(2^n)$



$O(17n)$

5 μs



36 years

$F_n \rightarrow \frac{O(n)}{O(2^n)}$

$$\underline{\underline{A_1}} * A_2$$

$$\boxed{\begin{matrix} n \times n & n \times n \\ C = A & * B \\ \begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix} & \begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix} \end{matrix}}$$

$$O(8)$$

$$n=2$$

$$n^2$$

$$\underline{O(n^3)}$$

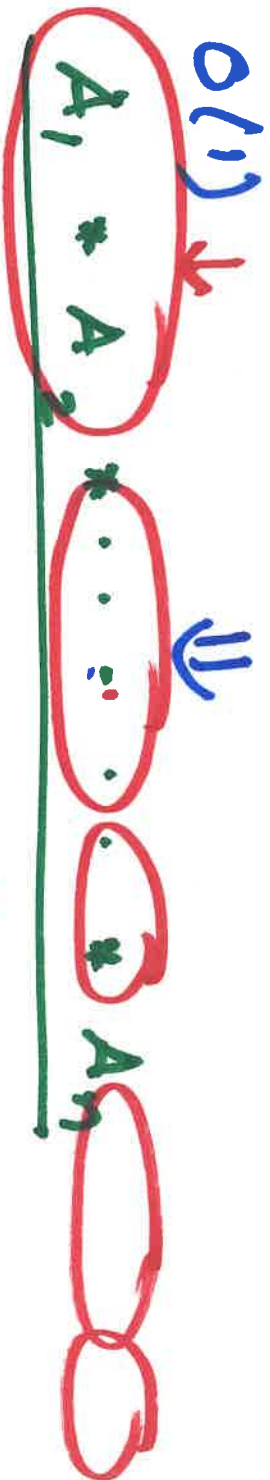
$$\begin{bmatrix} \end{bmatrix} * \begin{bmatrix} \end{bmatrix}$$

$$\begin{matrix} a_{11} & a_{12} & a_{13} \\ b_{11} & b_{21} & b_{31} \end{matrix}$$

$$C_{11} =$$

$$a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31}$$

$$\textcircled{n}$$



A =

$A_1 = A_2 = A_3 = \dots = A_n = 16$

$O(19^n)$

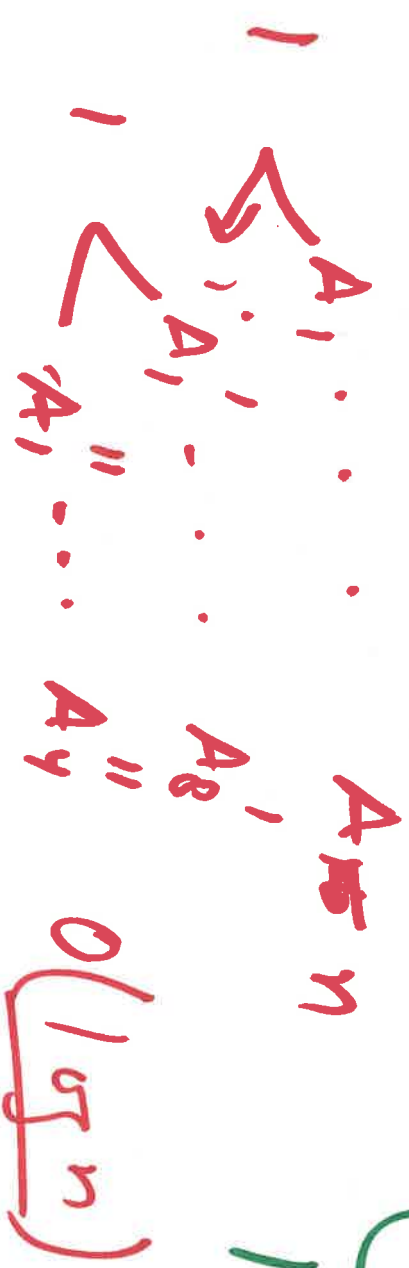
$C_3^n \quad O(19^n) \quad O(n)$

$O(2^n)$

$\begin{bmatrix} c_{11} & c_{12} \\ \vdots & \vdots \\ c_{n1} & c_{n2} \end{bmatrix} \begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix}$

$C_{3n} \quad 8^n$

$10^n \Rightarrow O(2^n)$



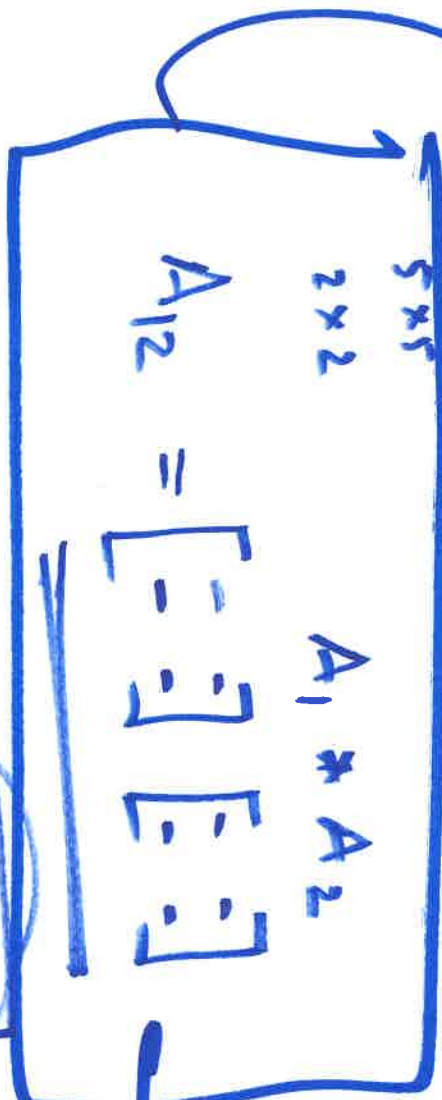
$O(19^n)$

$$A = \begin{bmatrix} A_1 * A_2 * \dots * A_n \end{bmatrix}$$

$$A_1 = A_2 = A_3 = \dots = A_n$$

$$\underline{\underline{O(n)}}$$

$$O(\sqrt{gn})$$



$$O(m^m)$$

$$O(m^1)$$

$$O(m^2)$$

$$O(\frac{n}{2})$$

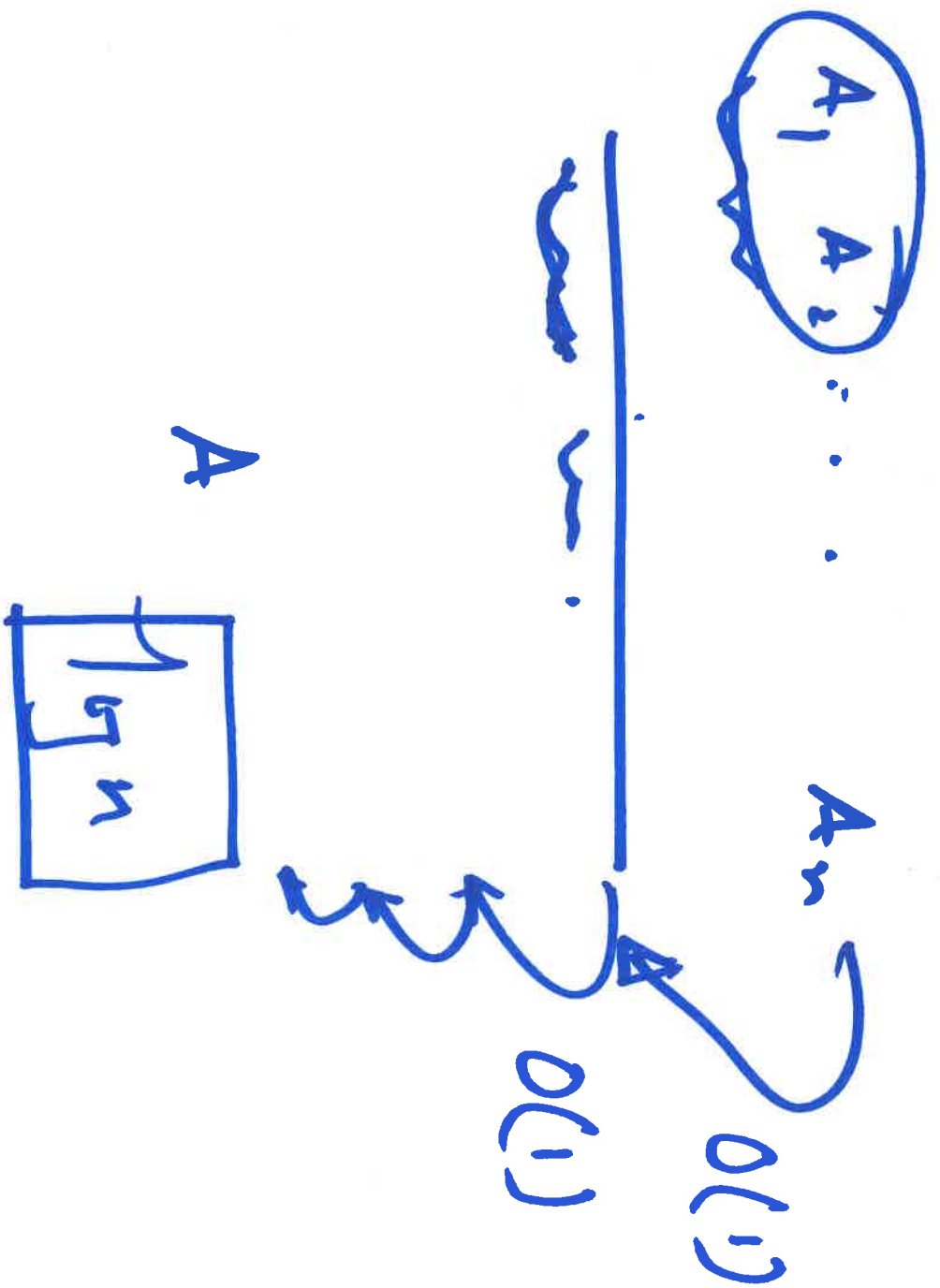
$$O(\frac{n}{2})$$

$$O(?) \rightarrow O(1)$$

$$O(n^n)$$

$$O(2^2)$$

$$O(4)$$



F_n

$$\frac{\begin{bmatrix} F_{n-1} & F_n \end{bmatrix}}{1 \times 2}$$

$$= \begin{bmatrix} F_{n-1} & F_{n-1} + F_{n-2} \end{bmatrix}$$

$$= \begin{bmatrix} F_{n-2} & F_{n-1} \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} F_{n-3} & F_{n-2} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} F_{n-4} & F_{n-3} \end{bmatrix} \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}}_{\tilde{A}} \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}}_{\tilde{A}} \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}}_{\tilde{A}}$$

$$\equiv \begin{bmatrix} F_0 & F_1 \end{bmatrix} \cdot \tilde{A}^{n-1} \dots$$

$$\begin{aligned}
 & \left[E_{n-1} \right] \left(\begin{array}{c} E_n \\ \vdots \\ E_n \end{array} \right) \xrightarrow{\text{blue arrow}} \\
 & = \left[\begin{array}{cc} 0 & 1 \end{array} \right] \left[\begin{array}{cc} 0 & 1 \\ -1 & -1 \end{array} \right]^{n-1} \xrightarrow{\text{blue arrow}} \\
 & = \left[\begin{array}{cc} 0 & 1 \end{array} \right] \underline{A^{n-1}}
 \end{aligned}$$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{aligned}
 & = \left[\begin{array}{cc} x & y \end{array} \right] I \\
 & = \left[\begin{array}{cc} 0 & 1 \end{array} \right] A^{-1} \cdot A \cdot A^{n-1} \\
 & = \left[\begin{array}{cc} 0 & 1 \end{array} \right] A^{-1} \cdot A^n \\
 & = \left[\begin{array}{cc} p & q \end{array} \right] \cdot \left(\begin{array}{c} A_n \\ \vdots \\ A_n \end{array} \right) \xrightarrow{\text{blue arrow}}
 \end{aligned}$$