Set 6 : Divide & Conquer

Fine Grain Measure MaxMin

$\boxed{2n - 3}$ Comp $\Rightarrow$ $\boxed{O(n)}$

$\boxed{100n + 4}$ comp $\Rightarrow$ $\boxed{O(n)}$

Coarse Grain Measure

Sorting $\Rightarrow$ $\boxed{O(n \lg n)}$

10000 miles

50000 mi

100 miles $\Rightarrow$ 100 miles 576 ft 3 inches

900 miles

300

500

$a_1 \cdots a_i \quad a_n$ $\Rightarrow$ Sorting $\begin{cases} \text{Largest} \\ \text{Small} \end{cases}$

$\Downarrow$

$O(n \log n)$

Max $\longleftarrow$

$\left.\begin{array}{l}\text{Max} \leftarrow n-1 \\ \text{Min} \leftarrow \end{array}\right\}$

$(n-1)$

$(n-2)$

$\downarrow \qquad \downarrow$

2Max 3Max . . .

Max $(n-1)$

Min $(n-2)$

$2n-3$

$O(n)$

$n-1$

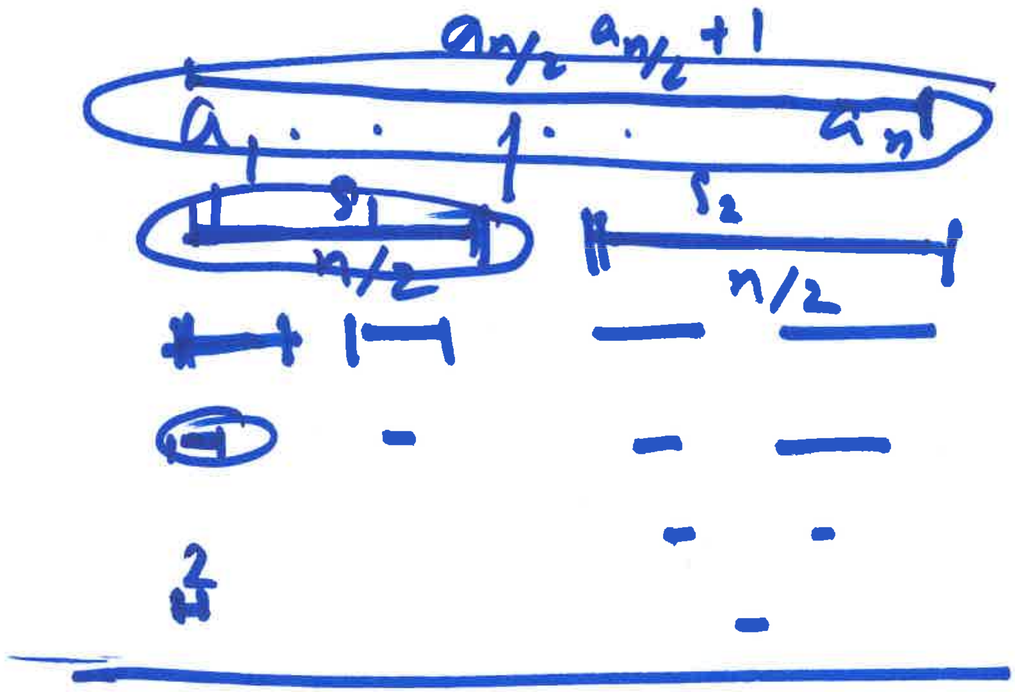
$n-2$

$2n-3 \Leftrightarrow n \log n \rightarrow 1000$

$\Downarrow$

$O(n)$

$\log n$



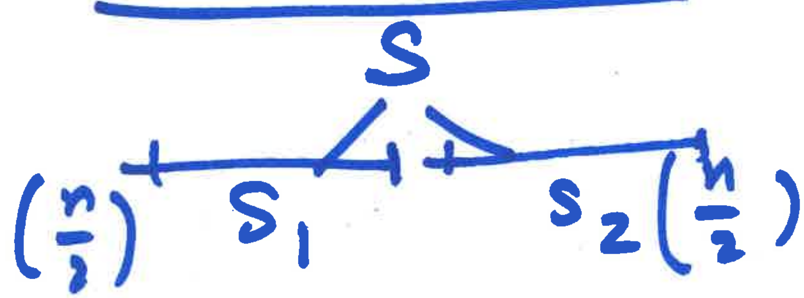
Max
Min

$n = 2^k$

$O(1\cancel{/n})$ / $O(n\cancel{/n})$

$(n = 2^m)$ / $O(n)$

$a_1 \cdots \cdots a_n$



Find Max Min $(\underline{S})$ — Max, Min

1. Split in two equal parts

Find Max Min $(S_1) \Rightarrow$ Max_1, Min_1

Find Max Min $(S_2) \Rightarrow$ Max_2, Min_2

$Max = max(Max_1, Max_2)$

$Min = min(Min_1, Min_2)$

$\underline{\underline{C(n)}}$ = # Comp needed to find Max & Min of n number(s)

$\boxed{1.5n-1}$ = $\underset{?}{\underline{2n-3}}$ / $\underset{?}{\underline{n+4}}$ / $\underset{?}{\underline{n \lg n}}$ / $\underset{?}{\lg n}$

$n/2 - 1$

$C(\frac{n}{2})$ $C(\frac{n}{2})$

10 10 + 2

$$\boxed{C(n) = 2C(\frac{n}{2}) + 2}$$

✓ | 2n-3 Y N

$$\boxed{C(n) = 2C\left(\frac{n}{2}\right) + 2}$$

$n = 64$

$\cancel{\quad}$

$$C(n) = \underline{\underline{5n+3}}$$

$$C(n) = 3C\left(\frac{n}{3}\right)$$

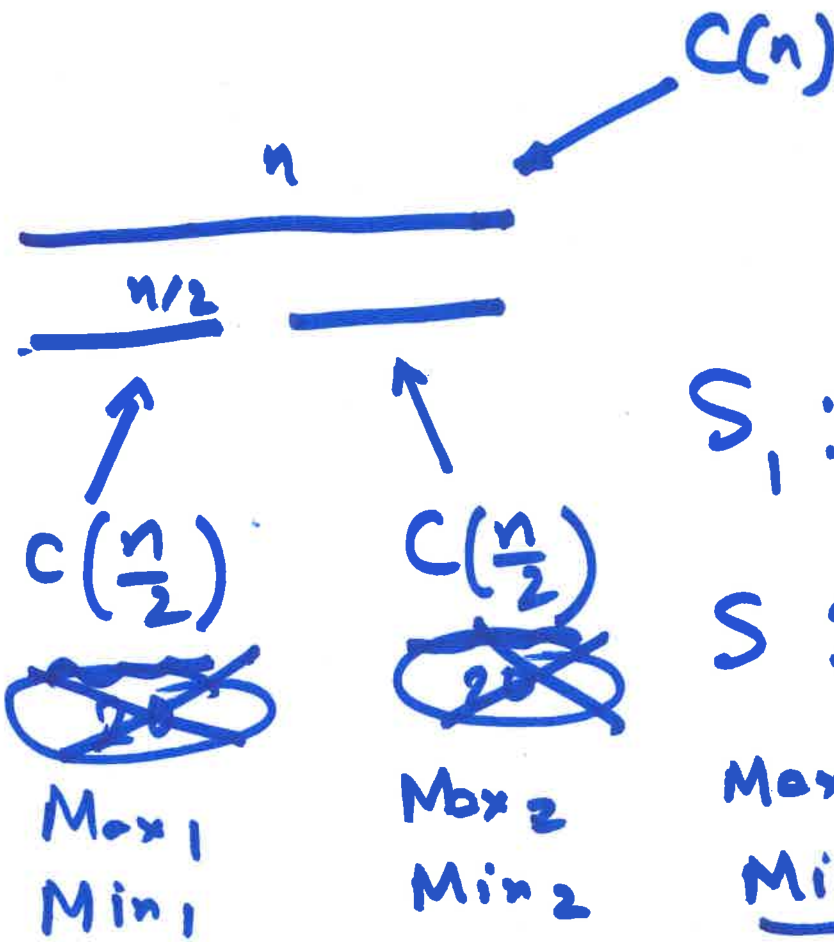
$$= \underset{\substack{\Downarrow \\ 2}}{a}\, C\left(\underset{\substack{\Downarrow \\ 2}}{\frac{n}{b}}\right)$$

$2 \leftarrow a$: no. of subproblems

$\frac{n}{b}$: size of the subproblem

$b \swarrow 2$

$C(n)$



$n = 64$

$C(64) = ?$

$C(32) =$

$\vdots$

$\boxed{C(2) = 1}$

$\Rightarrow \frac{n}{2} - 1$

$n - 1$

$S_1 : \frac{n}{2} \rightarrow \frac{n}{2} - 1$ cmp ✓

$S : n \rightarrow n - 1$ cmp ✗

$\Uparrow$ Comp

Max = max(1, 2)

Min = min(1, 2)

2

$C(n) = n - 1$ (?)

$$C\left(\frac{n}{2}\right) = 2C\left(\frac{n}{4}\right) + 2$$

$$\boxed{n = 2^K}$$

$$C(n) = 2\left[2C\left(\frac{n}{4}\right) + 2\right] + 2$$

$$= 2^2 C\left(\frac{n}{4}\right) + 2^2 + 2$$

$$= 2^2\left[2C\left(\frac{n}{8}\right) + 2\right] + 2^2 + 2$$

$$= 2^3 C\left(\frac{n}{2^3}\right) + 2^3 + 2^2 + 2$$

$$\vdots$$

$$\boxed{C(2)}$$

$$\frac{n}{2^{x = K-1}}$$

$$C(n) = 2^{k-1} C\left(\frac{n}{2^{k-1}}\right) + 2^{k-1} + 2^{k-2} + \cdots + 2$$

$$= 2^{k-1} C(2) + 2^{k-1} + 2^{k-2} + \cdots + 2$$

$$= 2^{k-1} \cdot 1 + 2\left[\underbrace{2^{k-2} + 2^{k-3} + \cdots + 1}\right]$$

$$\boxed{C(n) = 2^{k-1} + 2\left(2^{k-1} - 1\right)}$$

$$\Downarrow$$

$$\frac{n}{2} + 2^{k} - 2$$

$$\downarrow$$

$$C(n) = \frac{n}{2} + n - 2$$

$$= 1.5n - 2$$

$A_1 : 2n - 3 \Rightarrow O(n) \to O(n^2)$

$A_2 : 1.5n - 2 \Rightarrow O(n)$

$50n^2 \Rightarrow O(n^2)$

$5000n^2$

Fine Grain Measurement

Coarse grain measurement

$C_1 C_2 = 100$ mile

$C_3 C_4 = 200$ mile

$= 10^6$ mile

100 miles 376 yards
416 feet
3 " inch

Recurrence Relation for Computing Max only

$$C(n) = 2C\left(\frac{n}{2}\right) + 1$$

$$C\left(\frac{n}{2}\right) = 2C\left(\frac{n}{4}\right) + 1$$

$$\boxed{1.5n - 2}$$

$$= 2\left[2C\left(\frac{n}{4}\right) + 1\right] + 1$$

$$= 2^2 C\left(\frac{n}{4}\right) + 2 + 1$$

$$= 2^2 \left[2C\left(\frac{n}{8}\right) + 1\right] + 2 + 1$$

$n = 2^k$

$$= 2^3 C\left(\frac{n}{8}\right) + 2^2 + 2 + 1$$

$$\vdots$$

$$2^{k-1} \underbrace{C\left(\frac{n}{2^{k-1}}\right)}_{\substack{C(2) \\ 1}} + 2^{k-2} + 2^{k-3} + \cdots + 1$$

$$2^{k-1} \cdot 1 + 2^{k-2} + \cdots + 1$$

$$C(n) = 2^{k-1} + 2^{k-2} + \cdots + 1$$

$$C(n) = 2^{k} - 1 = n - 1$$

mov.

n-1 disks $\rightarrow$ (n-1) moves (Tru

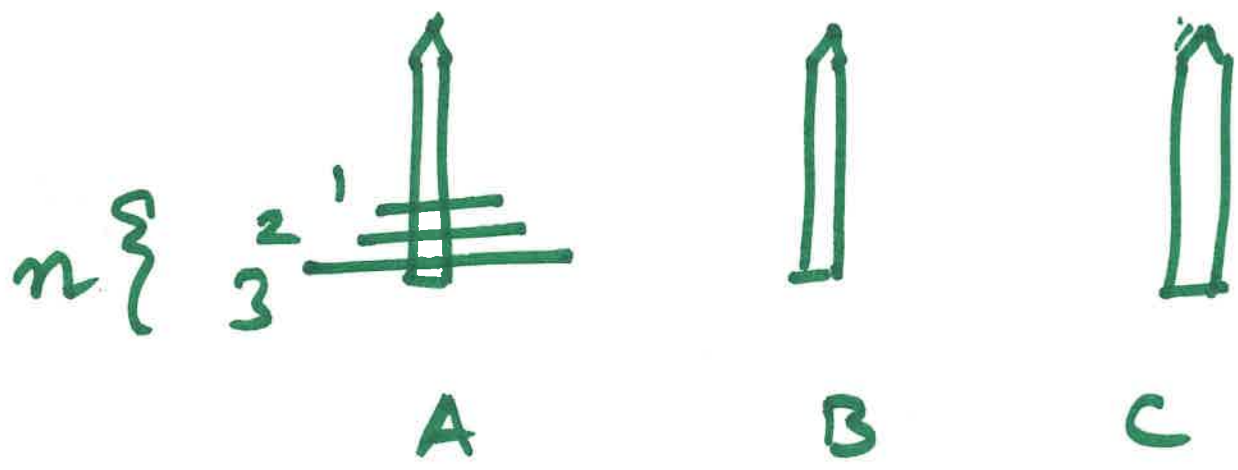
$\Downarrow$

n   n moves

X disks   X moves

$\downarrow$

n disk   n moves



$n! \,/\, 2^n - 1$

$M(n)$ = # moves needed to transfer n disks from one peg to another using the third as an intermediary.

$$\begin{aligned} M(n) &= 2\,M(n-1) + 1 \\ &= 2\left[2M(n-2)+1\right]+1 \\ &= 2^2\,M(n-2) + 2 + 1 \end{aligned}$$

$\Big[\; M(n-1) = 2M(n-2) + 1$

$$= \cancel{2^3 M(n-3) + 2 + 1}$$

$$= 2^2 \left[2 M(n-3) + 1 \right] + 2 + 1$$

$$= \cancel{2^3 + 8}$$

$$= 2^3 M(\underline{\underline{n-3}}) + 2^2 + 2 + 1$$

$$\vdots$$

$$= 2$$

$$= 2^{n-1} \underbrace{M(1)}_{1} + 2^{n-2} + 2^{n-3} + \cdots + 1$$

$$= 2^{n-1} + 2^{n-2} + 2^{n-3} + \cdots + 1$$

$$\boxed{M(n) = 2^n - 1}$$

$$S$$

$s_1 \quad s_2 \quad . \quad . \quad . \quad . \quad . \qquad s_n$

$\uparrow$

s_k

$$S$$

$0 \le S_1 \le n-1 \qquad S_2 \qquad 0 \le S_3 \le n-1$

$n/2 \qquad\qquad\qquad\qquad n/2$

$x \qquad y \qquad z$

$< s_k \qquad = s_k \qquad > s_k$

$x + y + z = n$

$\downarrow$

$x + z = n - 1$

$$\boxed{C(S) = C(S_1) + C(S_3) + (n-1)}$$

$\text{Sort}(S) \rightarrow S \leftarrow \underline{\underline{(n-1)}}$ Comp x

$|S| = n$

1 Max
2 Max
3 Max
0 Max

$n-1$

$n-1$

$x + y = n - 1$

Max Comp:

$(n-1)$

$= S_1$

S_n

$S_x \ \vdots \ S_2$

S_3

$S_1 \quad S_2 \quad S_3$

$x \quad y$

0

$(n-1)$

$\text{Sort}(S_1)$

$\text{Sort}(S_n)$ — – —

$S_{11} \quad S_{12} \quad S_{13}$

$S_{111} \quad S_{112} \quad S_{113}$

$\underbrace{x + y = n - 1}$

$0 \le x \le n - 1$

$0 \le y \le n - 1$

$\boxed{|S[i:j]| = 2}$

$\underline{C(n)} = \underbrace{C(x)}_{10} \quad \underbrace{C(y)}_{20} + (n-1)$

$(n-1) \times$

$(n^2 - 1)$?

$(n^3 - 1)$

$\boxed{C(n) = C(x) + C(y) + (n-1)}$

$n/2 \quad n/2$

Uneven Split

$C(n) = C(n-1) + C(0) + (n-1)$

$C(n) = C(n-1) + (n-1)$

$C(n) = C(n-1) + (n-1)$ (Uneven Case)

$= C(n-2) + (n-2) + (n-1)$

$= C(n-3) + \underline{(n-3) + (n-2) + (n-1)}$

$\vdots$

$= C(2) + (n-(n-2)) + \dots + (n-3) + (n-2) + (n-1)$

$= 1 + 2 + 3 + 4 + \dots + (n-1)$

$\boxed{C(n) = \frac{n(n-1)}{2}} \Rightarrow O(n^2)$ ← Uneven Split

$\Rightarrow O(n \lg n)$ ← Even Split.

$\Rightarrow O(n)$ ✗

Even Split

$$C(n) \le C\left(\frac{n}{2}\right) + C\left(\frac{n}{2}\right) + (n-1)$$

$$\boxed{C(n) \le 2C\left(\frac{n}{2}\right) + (n-1)} \qquad n = 2^k$$

$$= 2\left[2C\left(\frac{n}{4}\right) + \left(\frac{n}{2} - 1\right)\right] + (n-1)$$

$$= 2^2 C\left(\frac{n}{4}\right) + 2\left(\frac{n}{2} - 1\right) + (n-1)$$

$$= 2^2 C\left(\frac{n}{4}\right) + n - 2 + (n-1)$$

$$= 2^2\left[2C\left(\frac{n}{8}\right) + \left(\frac{n}{4} - 1\right)\right] + (n-2) + (n-1)$$

$$= 2^3 C\left(\frac{n}{8}\right) + 2^2\left(\frac{n}{4}\right) - 4 + (n-2) + (n-1)$$

=

$$C(n) \leq 2^{k-1}\underbrace{C(2)}_{\frac{n}{2^{k-1}}} + (n-2^{k-2}) \quad (n-4) + (n-2) + (n-1)$$

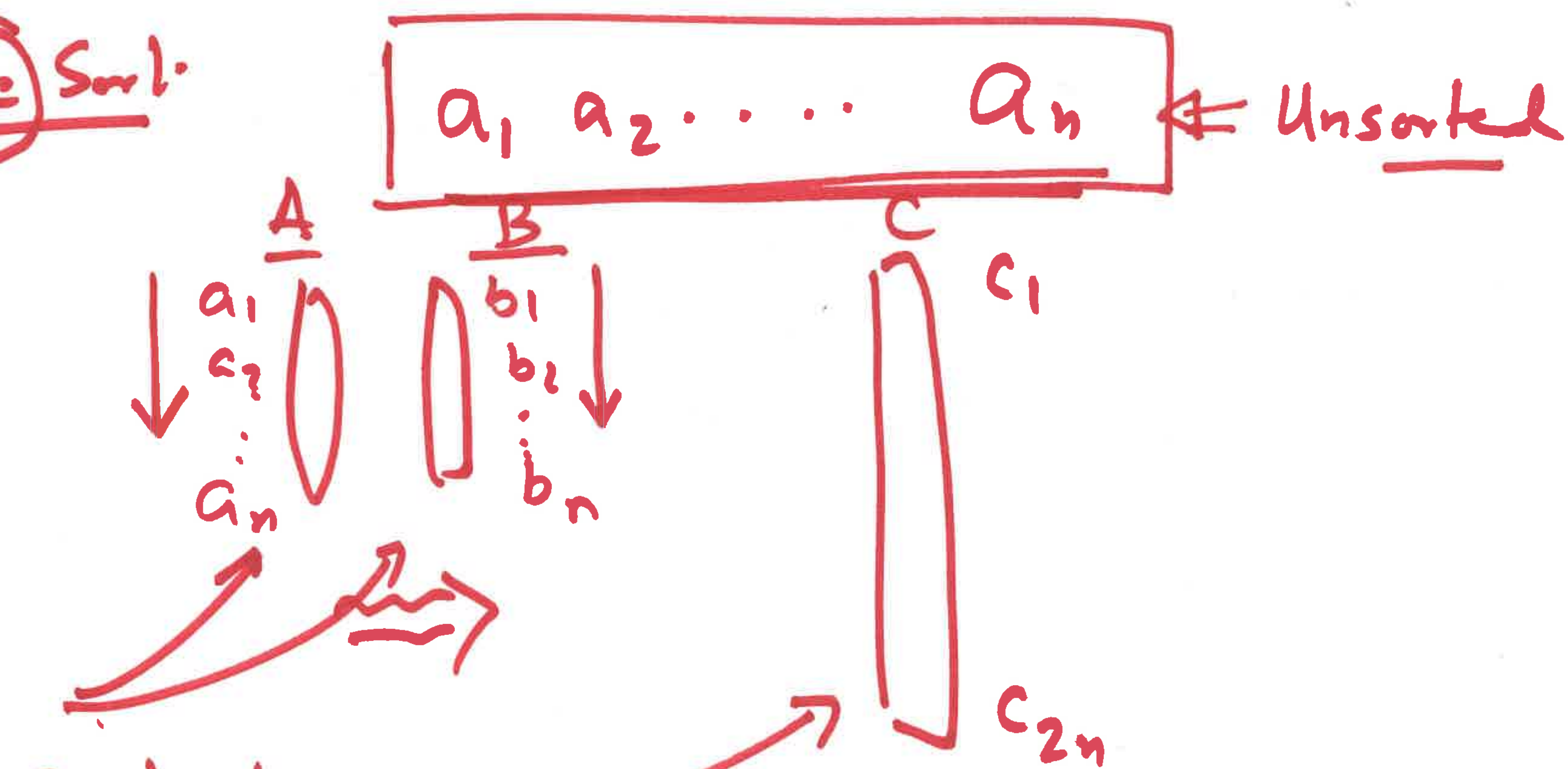
$$= 2^{k-1} \cdot 1$$

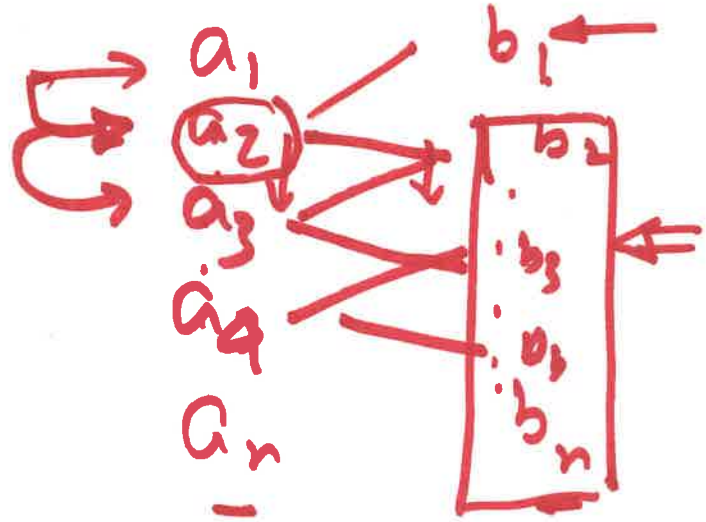
$$C(n) \leq 2^{k-1} \cdot 1 + \underbrace{n + n + \cdots + n}_{k-1}$$

$$\frac{n}{2^n} + \cdots \qquad C(n) \rightarrow O(n \log n)$$

$$C(n) \leq n \cdot k = \qquad k \qquad \boxed{C(n) \leq n \log n}$$

~~Merge~~ Sort.





<u>C</u>

$c_1 = a_1$

$c_2 = \cancel{a_2}\, b_1$

$c_3 \leftarrow \cancel{b}\, a_2$

$c_n \leftarrow a_n$

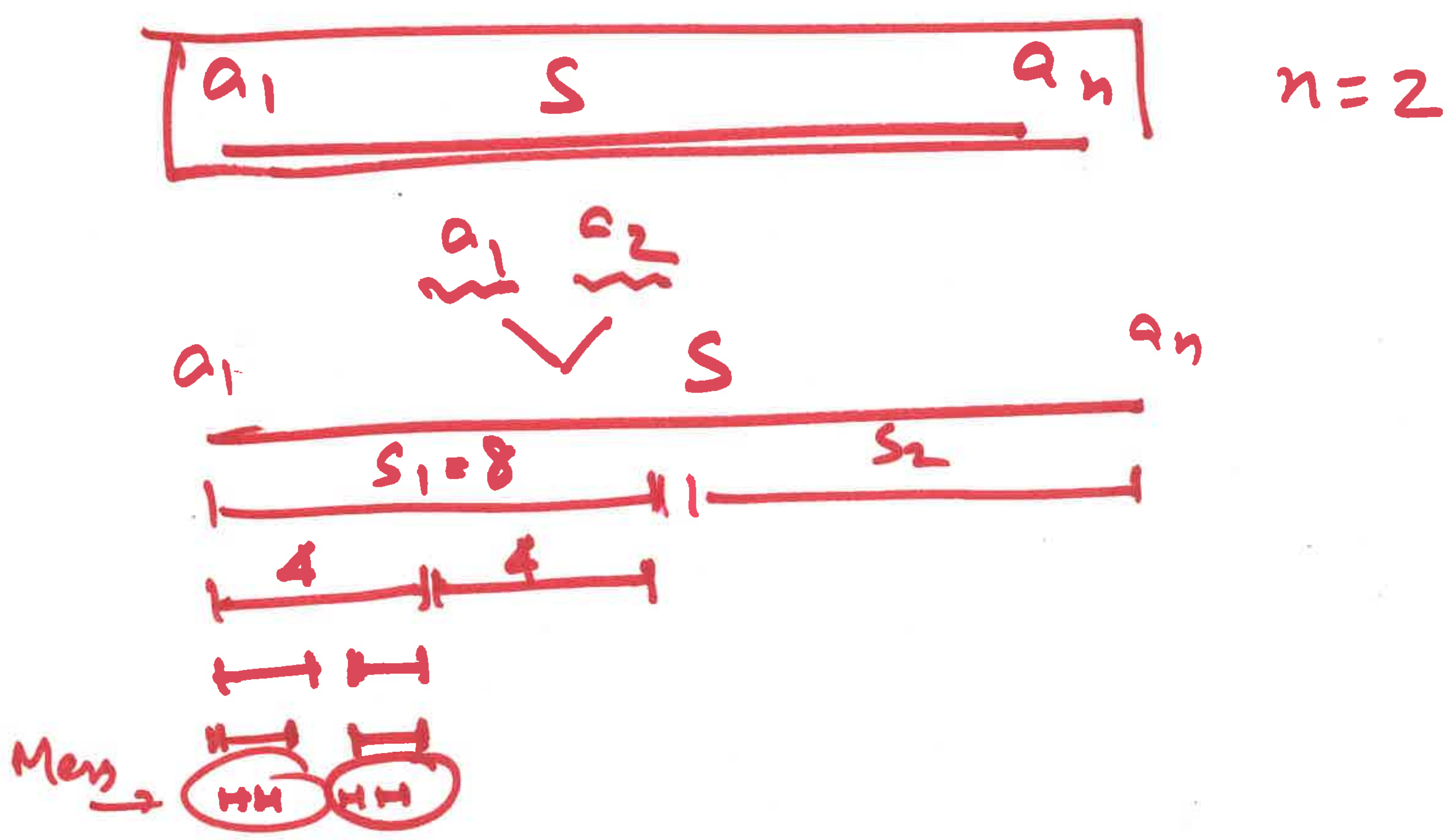
$\left.\begin{array}{l} c_{n+1} \\ \vdots \\ c_{2n} \end{array}\right\}$

Best Case $\rightarrow$ $\underline{n \text{ comparison}}$ $(a_n < b_1)$

$2(n-1)+1 = 2n-1$ comp

$a_1 < b_1 < a_2 < b_2 < a_3 < b_3 \cdots$

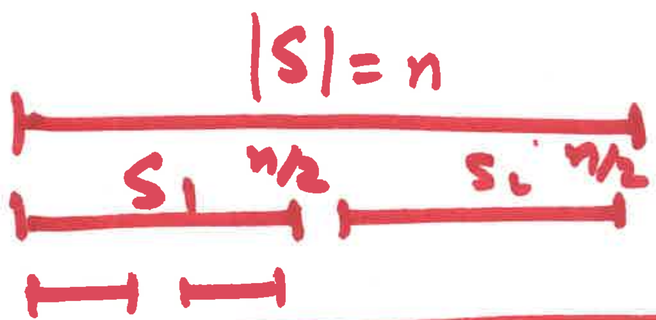
$S: \quad a_1 \cdots \cdots a_n$



$$\boxed{C(n) \le 2C\left(\frac{n}{2}\right) + n} \Leftarrow \text{Quicksort}$$

$$\hookrightarrow C(n) \Rightarrow O(n \lg n)$$

Merge Sort



$$\boxed{C(n) = 2C\left(\frac{n}{2}\right) + O(n)}$$

$$\hookrightarrow C(n) = O(n \log n)$$

		Q.S	M.S
Step 1	Divide into SP	$O(n)$	0
Step 2	Solve SP	$2C(\frac{n}{2})$	$2C(\frac{n}{2})$
Step 3	Combine SP	0	$O(n)$

$$C(n) = 2C\left(\frac{n}{2}\right) + O(n) \rightsquigarrow O(n \lg n)$$