Set 4: Fibonacci Number

$\underline{\underline{F_n}} \leftarrow$ # Additions $= A(n) <$ n^2 ✗, 2^n ✓

xyz1(n) — A(n)

xyz1(n-1) + xyz1(n-2)

P Q

$A(n) = \underline{P} + \underline{Q} + 1$

$= A(n-1) + A(n-2) +$

Difference Eqns

$$\boxed{A(n) = A(n-1) + A(n-2) + 1}$$

Recurrence Relation

$A(n) = O(1.7^n) \simeq O(2^n)$

Q.

$$A(n) = C_1 A(n-1) + C_2 A(n-2) + C_3 A(n-3) + \cdots + C_n$$

Linear Recurrence Relation with constant coefficients.

$$2^n \rightarrow 2^{n-1} \rightarrow n \rightarrow \lg n$$

$F_n \leftarrow A(n)$

xyz1(n) $\leftarrow$ xyz1$(n-1)$ + xyz1$(n-2)$

Additions = $(n-2)$

$\leftarrow O(n)$ 5 ms

$A(n) <$ $O(n)$ ✗ / $O(2^n)$ ✓

36 year

$R = P + Q + 1$

✓ $\boxed{A(n) = 1 \cdot A(n-1) + 1 \cdot A(n-2) + 1}$

Recurrence Relation

✗ $\boxed{A(n) = A^2(n-1) + A^3(n-2) + \cdots}$

Difference Eqn.

Linear Recurrence Relation with Const Coff

$A(n) \Rightarrow O(1.7^n) \Rightarrow O(2^n)$

$i = 0$

$j = 1$

$A(0) = 0$

$A(1) = 1$

$i = 2$ to n

$F_n \leftarrow n$

recursive way

$n \rightarrow F_n$

we correct.

$n = 50$

computation Time

$O(2^n)$ correct.

$F_0 \quad F_1 \quad F_2 \quad F_3 \quad F_4 \quad F_5 \quad F_6 \quad F_7 \quad F_8 \quad F_9$

$F_0 \to 0$, $F_1 \to 1$

$O(\lg n) \leftarrow O(n) \leftarrow O(2^n)$

$\boxed{i \quad j \quad \cdots \quad k}$

$\boxed{i \quad j \quad k}$

$k = i + j$

$\boxed{i \quad j \quad k}$

x

$O(\lg n)$

$F_n \to O(n)$

$F_n \to O(2^n)$

5 ms

36 years

$\boxed{i \quad j \quad k}$

$\underline{A_1 * A_2}$

$n \times n$ $\quad$ $n \times n$

$\underline{\underline{C}} = A * B$

n^2

$$[\ \ ]_A * [\ \ ]_B$$

$a_{11} \quad a_{12} \quad a_{13}$ $\qquad$ b_{11}, b_{21}, b_{31}

c_{11}

$O(8)$

n^2 $\qquad$ $n = 2$

$\underline{O(n^3)}$

$$\underline{\underline{c_{11}}} = \underline{a_{11} b_{11} + a_{12} b_{21} + a_{13} b_{31}}$$

n

$O(1)$

$A = A_1 * A_2 * \cdots * A_n$

$O(\lg n)$

$A_1 = A_2 = A_3 = \cdots = A_n = 16$

$A_1' \quad A_2' \cdots A_8$

$c^3 n \quad O(\lg n) \quad O(n) \quad O(2^n)$

$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} [\ ,\ ][\ ,\ ]$

c^{3n}

8^n

$10^n \Rightarrow O(2^n)$

$A_1 \cdots A_n$

$1 < A_1' \cdots A_8'$

$1 < A_1'' \cdots A_4'' \quad O(\lg n)$

$A = \boxed{A_1 * A_2 * \ldots \cdot * A_n}$

$\underline{\underline{O(n)}}$

$O(\underline{\lg n})$

$\boxed{A_1 = A_2 = A_3 = \ldots = A_n}$

3x3

5x5

2x2

$A_1 * A_2$

$A_{12} = \begin{bmatrix} - & - \\ - & - \end{bmatrix} \begin{bmatrix} - & - \\ - & - \end{bmatrix}$

$\boxed{n = 2}$

$O(\cancel{2n})$?

$O(\cancel{\;})$

$O(?)$

$\boxed{O(1)}$

$O(m^m)$

$O(m!)$

$O(m^2)$

$O(m^n)$

$O(2^2)$

$O(4)$

$A_1 \quad A_2 \quad \cdots \quad A_n$

$O(1)$

$O(1)$

A

$\boxed{\lg n}$

F_n

$$\underbrace{\begin{bmatrix} F_{n-1} & F_n \end{bmatrix}}_{1 \times 2} = \begin{bmatrix} F_{n-1} & F_{n-1} + F_{n-2} \end{bmatrix}$$

$$= \begin{bmatrix} F_{n-2} & F_{n-1} \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} F_{n-3} & F_{n-2} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} F_{n-4} & F_{n-3} \end{bmatrix} \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}}_{A}$$

$$\vdots$$

$$= \begin{bmatrix} F_0 = 0 & F_1 = 1 \end{bmatrix} \cdot A^{n-1} \ldots$$

$$\begin{bmatrix} F_{n-1} & F_n \end{bmatrix} = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}^{n-1} \leftarrow A$$

$$= \begin{bmatrix} 0 & 1 \end{bmatrix} A^{n-1}$$

$$= \begin{bmatrix} X & Y \end{bmatrix} \quad I$$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \end{bmatrix} A^{-1} \cdot A \cdot A^{n-1}$$

$$= \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix} A^{-1}} \cdot A^{n}$$

$$= \begin{bmatrix} P & Q \end{bmatrix} \cdot A^{n}$$

$$\begin{matrix} \uparrow & \uparrow \\ 1 & 0 \end{matrix}$$