*CSE 551 Foundations of Algorithms
Mid Term, Fall 2017
Closed Books, Closed Notes
Time: 1 hour
Answer any three questions
Each question carries 25 pts.

Problem 1: Define the notations Big-O, Big-Ω and Big-Θ. If a function $T(n)$ is of the *order* of another function $f(n)$, we denote it as $T(n) = O(f(n))$. Prove or disprove the following assertions:

(i) If $f(n) = O(g(n))$ then $log_2 f(n) = O(log_2 g(n))$
(ii) If $f(n) = O(g(n))$ then $2^{f(n)} = O(2^{g(n)})$
(iii) Let $f(n)$ and $g(n)$ be asymptotically positive functions. Prove of disprove the following conjectures:
(a) For $\alpha > 1$, $n^{\alpha log\ n}$ is not $O(n^{log\ n})$
(b) For $\alpha > 1$, $log\ n^{\alpha log\ n}$ is $O(log\ n^{log\ n})$

Problem 2: Suppose there is a set A of men and a set B of women. Each set contain n elements. There exist two $n \times n$ arrays P and Q such that $P(i,j)$ is the preference of man i for woman j and Q is the preference of woman i for man j . Give an algorithm which finds a pairing of men and women such that the following condition is not satisfied. There is an element $a_i \in A$ that has a higher preference for an element $b_k \in B$ over the element $b_j \in B$ with which a_i is paired, and $b_k \in B$ has a higher preference for $a_i \in A$ over the element $a_l \in A$ with which b_k is paired.

Prove the correctness of your algorithm (i.e., it ensures that the given condition isn't satisfied).

Problem 2: Compute the best case and worst case complexity of the following algorithm. Show all your work.
Algorithm XYZ(S)
if $|S| = 2$ then compare the two numbers and return (min,max)
else
begin
1. Pick an arbitrary element s_k of the sequence S.
2. Divide S into parts S_1, S_2,S_3 such that the elements of S_1, S_2, and S_3 are less than, equal to and greater than s_k respectively.
3. return (XYZ(S_1),S_2,XYZ(S_3))
end