

# CSE 551: Foundations of Algorithms

## Quiz 2 Solutions

Q1. Design an  $O(\log n)$  complexity algorithm to find the peak number in a unimodal sequence. Analyze your algorithm and show that it finds the peak in  $O(\log n)$  time. Show all your work.

A1. Let mid be the middle element of the sequence. Compare the value to the left of the mid and to the right of the mid, with mid. If the right value  $>$  mid  $>$  left value, then your new sequence starts from mid and ends at n, else your new sequence starts from 0 to mid. Calculate the mid of the new sequence again. And do the neighbor comparisons to determine the new sequence. You will eventually arrive at the peak value in  $O(\log n)$  time. This is very similar to binary search.

Q2\*. Find the optimal solution to the TSP for the following data set:

0	2	5	9
10	0	2	8
3	8	0	6
9	2	6	0

A2\*. Tour cost = 16. Node ordering 1->4->2->3->1

Find the optimal solution to the TSP for the following data set:

0	12	15	19
20	0	12	8
13	18	0	6
9	12	6	0

A2\*\*. Tour cost = 39. Node ordering 1->2->3->4->1

Find the optimal solution to the TSP for the following data set:

0	20	25	30
15	0	19	20
16	23	0	22
18	18	19	0

A2\*\*\*. Tour cost = 75. Node ordering 1->2->4->3->1

Find the optimal solution to the TSP for the following data set:

0	30	35	40
25	0	29	30
6	3	0	2
8	8	9	0

A2\*\*\*. Tour cost = 69. Node ordering 1->2->3->4->1

Find the optimal solution to the TSP for the following data set:

0	5	10	15
1	0	8	6
5	12	0	16
7	12	5	0

A2\*\*\*\*. Tour cost = 21. Node ordering 1->2->4->3->1

Find the optimal solution to the TSP for the following data set:

0	10	15	20
5	0	9	10
6	13	0	12
8	8	9	0

A2\*\*\*. Tour cost = 35. Node ordering 1->2->4->3->1

Find the optimal solution to the TSP for the following data set:

0	20	25	30
15	0	19	20
16	23	0	22
18	18	19	0

A2\*\*\*\*. Tour cost = 75. Node ordering 1->2->4->3->1

Solutions to OBST problem:

1. Set with  $a_1=0, a_2=19 \dots$  :      Root R1:F, L(R1):B, R(R1):[Root RL:J, R(RL):P], 348.
2. Set with  $a_1=0, a_2=29 \dots$  :      Root R1:F, L(R1):B, R(R1):[Root RL:J, R(RL):P], 452.
3. Set with  $a_1=0, a_2=35 \dots$  :      Root R1:F, L(R1):B, R(R1):[Root RL:P, L(RL):J], 466.

Q3. Optimally triangulate a polygon with the following vertex coordinates:  $P_1 = (11,5)$ ,  $P_2 = (13, 2)$ ,  $P_3 = (15,4)$ ,  $P_4 = (17, 7)$ . The weight of a triangle is equal to the sum of the length of the three sides of the triangle. Show all your work. Also show the structure of the optimally triangulated polygon.

A3. 24.58.

Q3\*. Optimally triangulate a polygon with the following vertex coordinates:  $P_1 = (2,5)$ ,  $P_2 = (4, 2)$ ,  $P_3 = (6,4)$ ,  $P_4 = (8, 7)$ . The weight of a triangle is equal to the sum of the length of the three sides of the triangle. Show all your work. Also show the structure of the optimally triangulated polygon.

A3. 24.608.

Q3\*\*. Optimally triangulate a polygon with the following vertex coordinates:  $P_1 = (1,5)$ ,  $P_2 = (3, 2)$ ,  $P_3 = (5,4)$ ,  $P_4 = (7, 7)$ . The weight of a triangle is equal to the sum of the length of the three sides of the triangle. Show all your work. Also show the structure of the optimally triangulated polygon.

A3. 24.608.

Q3\*\*\*. Optimally triangulate a polygon with the following vertex coordinates:  $P_1 = (0, 5)$ ,  $P_2 = (2, 2)$ ,  $P_3 = (1, 4)$ ,  $P_4 = (6, 7)$ . The weight of a triangle is equal to the sum of the length of the three sides of the triangle. Show all your work. Also show the structure of the optimally triangulated polygon.

A3. 20.823.

Q4. Problem 3 of ASP.

A4. No. You can come up with multiple ways to denote start and end times. We will be checking the example to see if you can indeed prove that the answer is “no”.