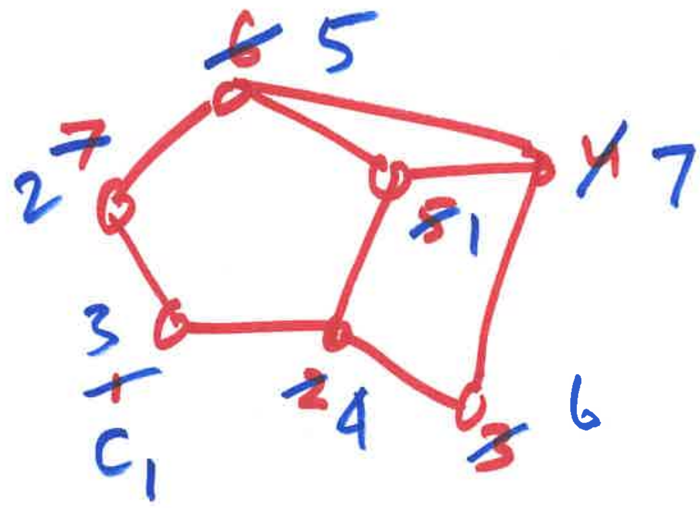
Set 7: GREEDY ALGORITHMS



5

						n
1	2	3	4	5	6	7
C_1	$\cancel{C_1}$ C_2	C_1			↑	

$\cancel{C_1}$

$\cancel{C_2}$

$\cancel{C_3}$

$\vdots$

C_6

$2^n / n^n / n!$ exp.

Bad high : ~~polynomial~~ $\Rightarrow O(n^2)$ ~~high.~~

Good low polynomial $\Rightarrow O(n)$ lg n

$n / n^2 / n^{10} / n^{100} \Rightarrow O(n^n)$ ✗ $O(2^n)$ ✗

$1 + 2 + \cdots + (n-1) \Rightarrow O(n^2)$

Alg < quality < high, low

cost ← Time / Space /

1. 9 - 10
2 15 - 16
3 21 - 23
4 7 - 12
5 6 - 8
6 11 - 18
7 17 - 22
8 13 - 14
9 19 - 20

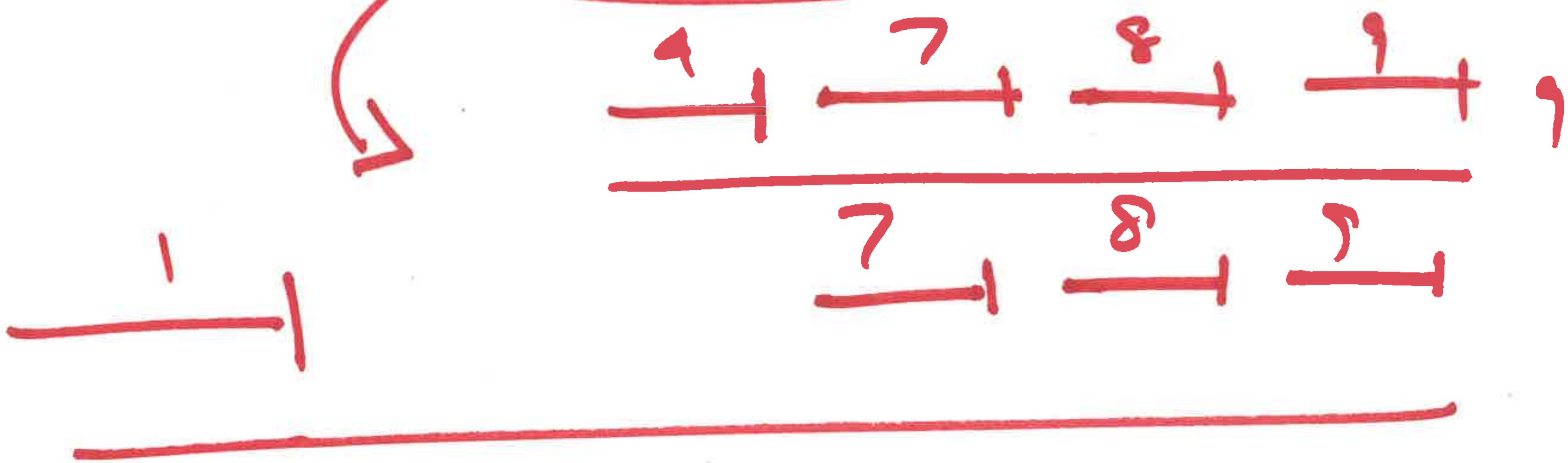
S	F
3	2
6	5
9	9
2	3
1	1
4	6
7	8
5	4
8	7

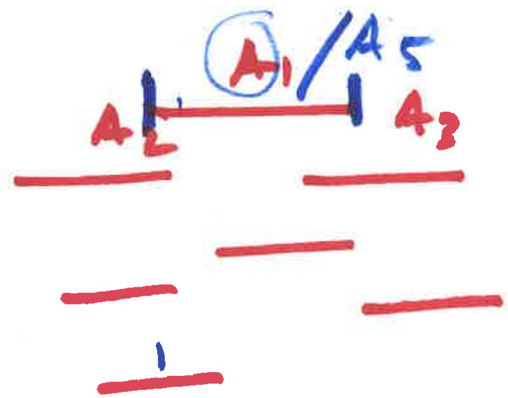
$$\frac{\{5, 1, 6, 9, 3\}}{\{4, 8, 2, 7\}}$$

$\{\underbrace{4, 7, 8, 9}\}$ Max Ind Set.

$$\Downarrow$$

$\{1, 7, 8, 9\}$





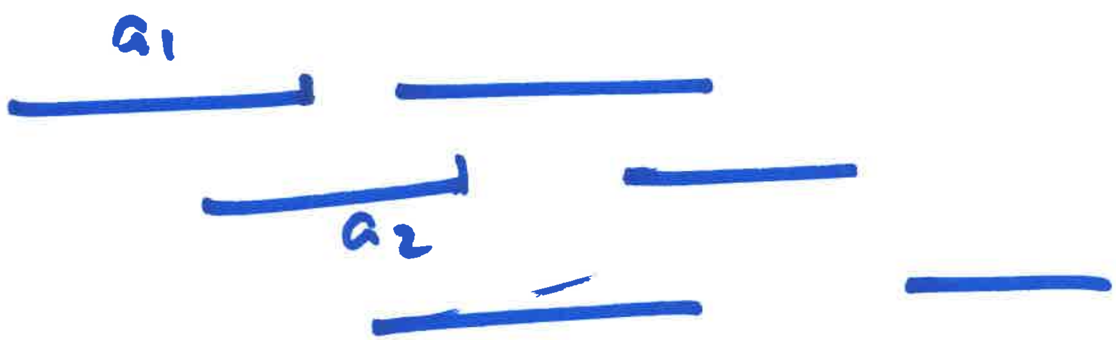
$A = \{A_1, A_2, \ldots, A_n\}$
$\quad I_1 \quad I_2 \quad \ldots I_n$

$A' \subseteq A$

Largest Ind. Set

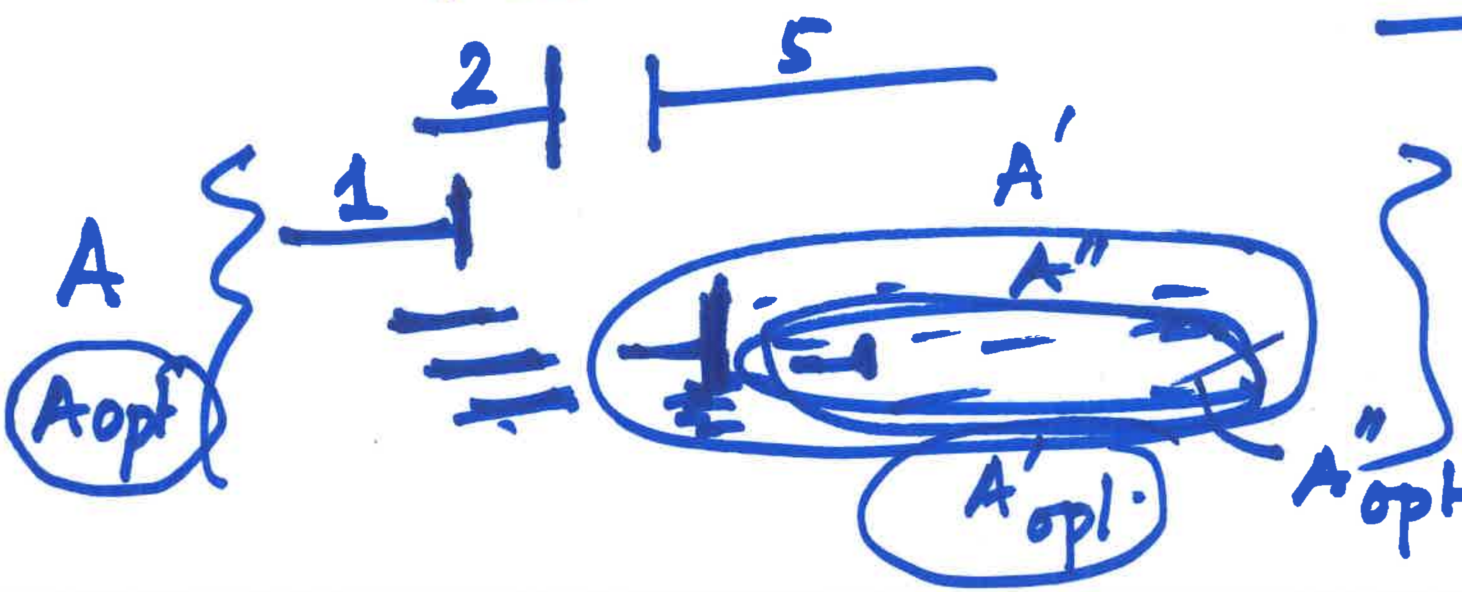
Largest Set of Compatible Activities

$A' = \{$

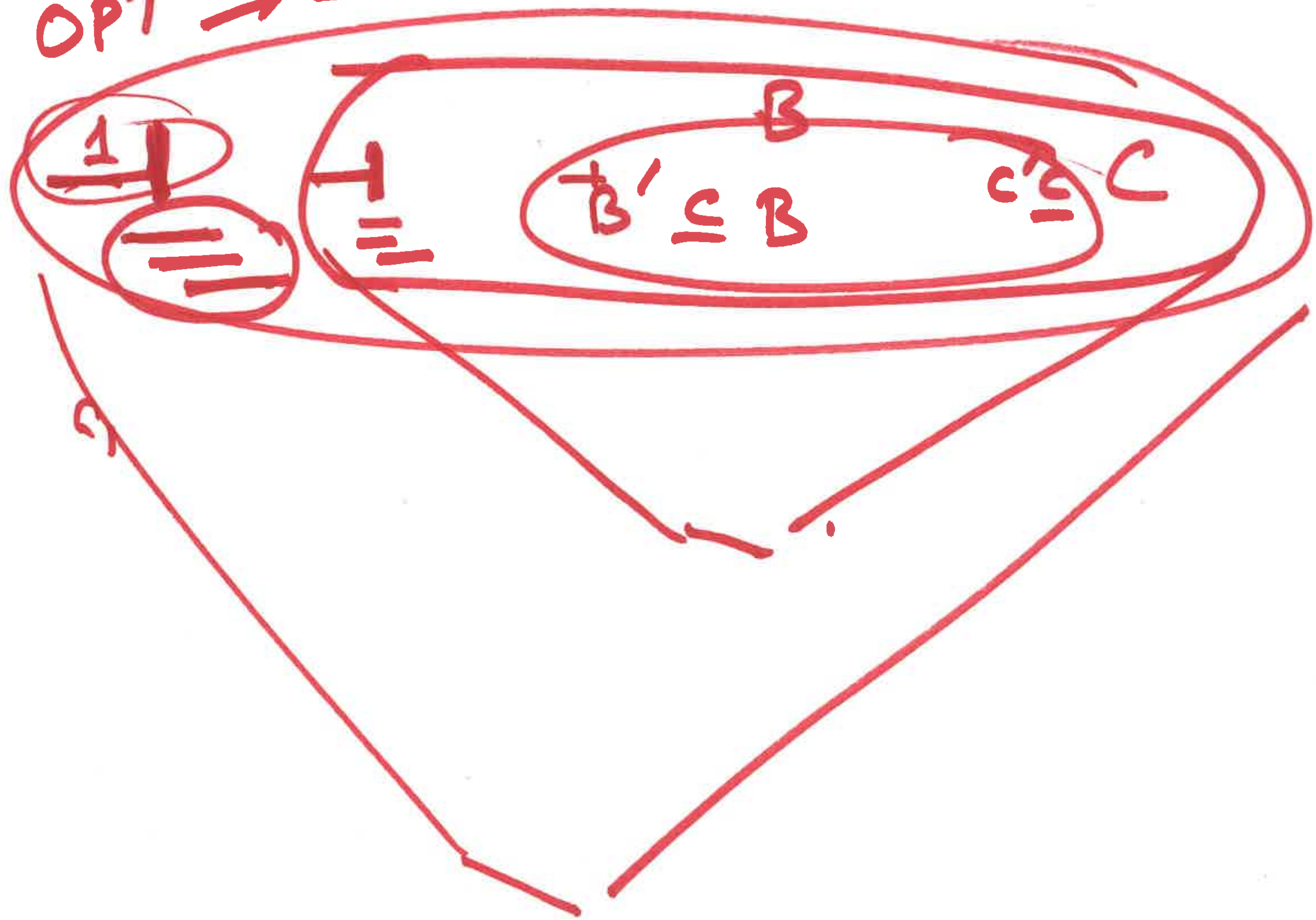


$$\underline{\underline{OPT}} \Rightarrow \underbrace{\{2, 5, 7, 9\}}$$

$$\{1, \underline{5, 7, 9}\} \Leftarrow \text{Valid } \underline{\underline{OPT}}$$



$$\frac{\text{LIS}}{\text{OPT}} \quad \begin{array}{l} \rightarrow A' \subseteq A \\ \rightarrow \equiv \end{array}$$



$$6 \quad 2 \quad 1$$

$$\underbrace{1.c_1 + 1.c_2 + \cdots + 1.c_9}_{} \geq \underbrace{6c_1 + 2c_2 + c_3}_{5c_1 + 4c_2}$$

$$c_1 \le c_2 \le c_3 \le \cdots$$

$$c_1 < c_2 < c_3 \cdots$$

$$\Downarrow$$

$$c_1 + c_2 + c_3 + \cdots + c_9$$

$$c_1 + c_1 + c_1 + c_1 + c_1 + c_1$$

$$+ c_2 + c_2 + c_3$$

$$9c_1 + 0.c_2 + 0.c_3$$

S_1: $\underline{6c_1 + 2c_2 + c_3} \rightarrow$ Max Ind Set based Soln — Alg1 Proc

$\underline{5c_1 + 4c_2} \rightarrow$ Min # Proc. based Soln — Alg2 Chromatic No based Soln

$6c_1 + 2c_2 + c_3 \leq 5c_1 + 4c_2$

$6c_1 + 2c_2 + c_3 : 5c_1 + 4c_2$ $\rightarrow$ Least Cost Soln. Alg3

$c_1 + c_3 : 2c_2$

$\boxed{c_1 + c_3 < 2c_2}$

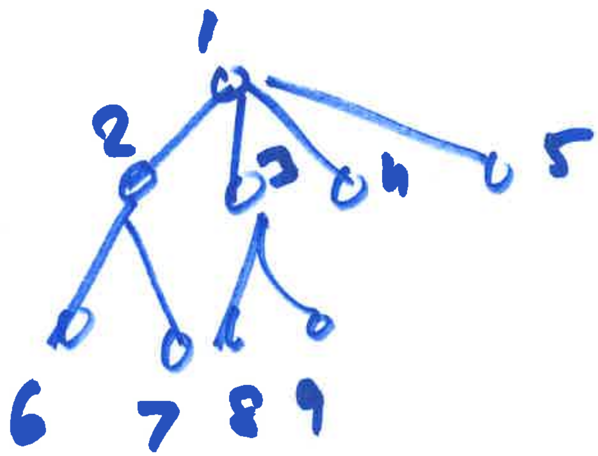
$c_1 + c_3 > 2c_2$

$c_1 \leq c_2 \leq c_3$

.4

5

A6



$$\Rightarrow \quad TC_1 = 6C_1 + 2C_2 + C_3 \; (A_1)$$

$$TC_2 = 5C_1 + 4C_2 \; (A_2)$$

Least Cost Soln

$$6C_1 + 2C_2 + C_3 \;:\; 5C_1 + 4C_2$$

$$C_1 + C_3 \;:\; 2C_2$$

$$C_1 + C_3 > 2C_2$$

$$C_1 + C_3 < 2C_2$$

$C_1 \le C_2 \le C_3$

$$\begin{array}{cccccccccccc} 1 & 2 & \cdot & \cdot & & 15 & & & 10 & & & n \\ i_1 & i_2 & & & i_{a-1} & i_a & & & i_b & & & i_n \\ \_ & \_ & \_ & \_ & \_ & \_ & \_ & \_ & \_ & \_ & \_ & \_ \\ & & & & & 3 & & & 7 & & & \end{array}$$

$$T(I) = \sum_{k=1}^{n} (n-k+1)\, t_{i_k} \quad \Big| \quad t_{i_a} > t_{i_b}$$

$$T(I) = \cancel{X} + (n-a+1)\, t_{i_a} + \cancel{Y} + (n-b+1)\, t_{i_b} + \cancel{Z}$$

$$T(I') = \cancel{X} + (n-a+1)\, t_{i_b} + \cancel{Y} + (n-b+1)\, t_{i_a} + \cancel{Z}$$

$$T(I) \; : \; T(I')$$

$$T(I) - T(I') = \begin{array}{l} > 0 \\ < 0 \end{array}$$

$$(n-a+1)\,t_{i_a} - (n-b+1)\,t_{i_a}$$

$$+\ (n-b+1)\,t_{i_b} - (n-a+1)\,t_{i_b}$$

$$= (\cancel{n}-a+\cancel{1}-\cancel{n}+b-\cancel{1})\,t_{i_a}$$

$$+\ (\cancel{n}-b+\cancel{1}-\cancel{n}+a-\cancel{1})\,t_{i_b}$$

$$= (b-a)\,t_{i_a} + (a-b)\,t_{i_b}$$

$$a : b \quad \Big| \quad t_{i_a} : t_{i_b}$$

$$a < b \qquad t_{i_a} > t_{i_b}$$

$$= (b-a)\,t_{i_a} - (b-a)(t_{i_b})$$

$$= (b-a)(t_{i_a} - t_{i_b})$$