Supplementary Textual Material

in

Physics

for

Class XI

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SUPPLEMENTARY TEXTUAL MATERIAL IN PHYSICS
CLASS XI

ERRORS IN MEASUREMENT

UNIT - I

The term 'Error' in Physics, has a meaning quite different from 'mistake'. An experimentalist can commit a mistake due to carelessness or 'casualness' on his/her part. We do not expect such a behavior from any serious experimentalist. 'Mistakes', therefore, do not get any consideration, or quantitative classification, in the observations of a 'well planned' and 'well executed' experiment.

No matter how well planned, and carefully executed our experiment is, we still cannot avoid **'errors'** in our observations. This is because no measurement is ever perfect. Errors can arise from causes like: a built-in fault in the 'design' or 'graduations' of a measuring instrument, **or** a faulty way of carrying out the measurement on the part of the observer. The more important errors, in Physics, however, are what are known as (i) random errors; and (ii) the errors caused by the limitations of the measuring devices used in a given situation. The 'limitation', of any measuring instrument, is quantitatively specified through its **'least count'** – the minimum magnitude of the relevant quantity that it can measure.

Random errors are errors which cannot be associated with any systematic or constant cause or with any definite 'law of action'. These errors, are usually assumed to follow the well known **'Gaussian law of Normal Distribution.**

In simple words, this law implies that the probability of an error $(+\Delta x)$ in a measurement is the same as the probability of an error $(-\Delta x)$ in that very measurement. Also, in a carefully carried out experiment, small magnitudes of 'error' are more likely than larger magnitudes of 'error'. The 'Normal Distribution law' is graphically represented by a curve of the type shown here (Figure (i))

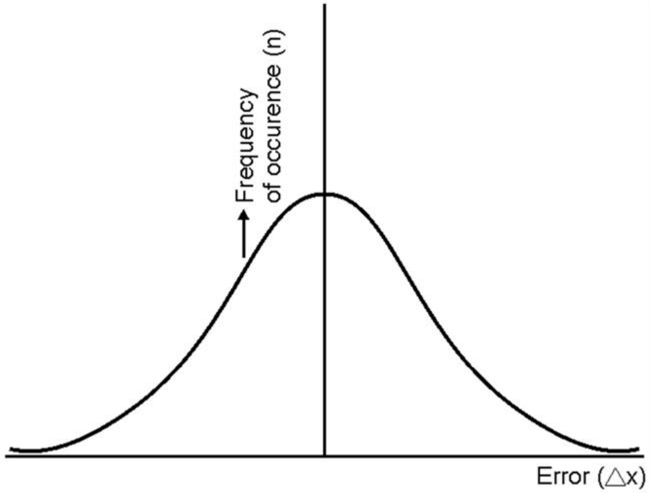


Fig. (i) **Illustrating Gaussian law of Normal Distribution**

A very significant ‘conclusion’, from this law, is that the ‘arithmetic mean’ of a large number of observations, is likely to be much closer to its ‘true value’ than any of the individual observations.

It is for this reason that we are always advised to take a large number of observations, and use their arithmetic mean, for doing, our ‘calculations’ **or** for drawing our ‘conclusions’ or ‘inferences’.

The ‘least counts’, of the measuring instruments, used in an experiment, play a very significant role in the ‘precision’ associated with that experiment. Scientists are, therefore, constantly striving to design instruments, and measuring techniques, that have better (smaller) values for their ‘least count’!

We can appreciate the importance of ‘least count’ by taking the simple example of the ‘measurement of a length’. When we use a meter scale for this purpose, we can rely on this measurement only up to a ‘mm’. This is because the least count of a meter scale is 1mm only. However, the use of a (simple) vernier caliper, pushes up this reliability to $\left({}^{1}\!/_{10}\right)^{th}$ of a mm or 0.1mm while the use of (usual) ‘screw gauge’ would take this reliability to $\left({}^{1}\!/_{10}\right)^{th}$ of a mm or 0.01mm.

Most of our experiments require us to use the measured values, of a number of different physical quantities , and put then in the appropriate ‘formula’ , to calculate the required quantity. We then calculate the 'percentage reliability , or 'maximum error', in our final result:

1. by associating a 'relative error'-equal to the ratio of the least count (of the measuring instrument used) to the measured value, with each of the quantities involved in our formula.
2. by using the standard 'rules' for finding the error in a 'sum or difference' , 'product or quotient', or 'power' , of different quantities, involved in a given formula.

We illustrate these ideas – for calculating the maximum error--through a few examples.

Example 1 : Suppose we use a physical balance to measure the mass of an object and find the mean value of our observations to be 156.28g.

Since we are somewhat uncertain, about the measurement, due to our instrument's imperfections, we need to express this in our result. Let the least count of physical balance be 0.1g: This implies that the uncertainly, of any measurement made with this instrument, is $\pm$0.1g. Therefore, we would report the mass of this object to be (156.3g $\pm$ 0.1g). This implies that we can only say that the mass of the object is somewhere between 156.2g and 156.4g.

Example 2 : It is required to find the volume of a rectangular block. A vernier caliper is used to measure the length, width and height of the block. The measured values are found to be 1.37cm, 4.11cm, and 2.56cm, respectively.

Solution : The measured (nominal) volume of the block is, therefore,

$V = \ell \times w \times h$

$= (1.37 \times 4.11 \times 2.56)\text{cm}^3.$

$= \quad 14.41\text{cm}^3$

However, each of these measurements has an uncertainty of $\pm$ 0.01cm, the least count of the vernier caliper. We can say that the values of length, width, and height should be written as

$\ell = (1.37\text{cm} \pm 0.01\text{cm})$

$w = (4.11\text{cm} \pm 0.01\text{cm})$

$h = (2.56\text{cm} \pm 0.01\text{cm})$

We thus find that the lower limit, of the volume of the block, is given by

V_{min} = 1.36cm x 4.10cm x 2.55cm

= 14.22 cm^3

This is 0.19cm^3 **lower** than the (nominal) measured value.

The upper limit can also be calculated:

V_{max} = 1.38cm x 4.12cm x 2.57cm

= 14.61cm^3

This is 0.20cm^3 **higher** than the measured value.

As a practical rule, we choose the higher of these two deviations (from the measured value) as the uncertainty, in our result. We, therefore, should report the volume of the block as (14.41cm^3 ± 0.20cm^3).

Example 3 : In an experiment, on determining the density of a rectangular block, the dimensions of the block are measured with a vernier caliper (with a least count of 0.01cm) and its mass is measured with a beam balance of least count 0.1g. How do we report our result for the density of the block?

Solution : Let the measured values be:

Mass of block (m) = 39.3g

Length of block (ℓ) = 5.12cm

Breadth of block (b) = 2.56cm

Thickness of block (t) = 0.37cm

The density of the block is given by

$$\boldsymbol{\rho} = \frac{\text{mass}}{\text{volume}} = \frac{m}{\ell \times b \times t}$$

$$= \; = \frac{39.3\text{g}}{5.12\text{cm} \times 2.56\text{cm} \times 0.37\text{cm}} = 8.1037\text{g cm}^{-3}$$

Now uncertainty in m = $\pm$ 0.01g

uncertainty in ℓ = $\pm$ 0.01cm

uncertainty in b = $\pm$ 0.01cm

uncertainty in t = $\pm$ 0.01cm

Maximum relative error, in the density value is, therefore, given by

$$\frac{\Delta\rho}{\rho} = \frac{\Delta\ell}{\ell} + \frac{\Delta b}{b} + \frac{\Delta t}{t} + \frac{\Delta m}{m}$$

$$= \frac{0.01}{5.12} + \frac{0.01}{2.56} + \frac{0.01}{0.37} + \frac{0.1}{39.3}$$

= 0.0019 + 0.0039 + 0.027 + 0.0024

= 0.0358

Hence $\Delta\rho$ = 0.0358 x 8.1037g cm^{-3} = $\simeq$ 0.3g cm^{-3}

We cannot, therefore, report the calculated value of ρ (= 8.1037 gm^{-3}) up to the fourth decimal place. Since $\Delta\rho$ = 0.3g cm^{-3} the value of ρ can be regarded as accurate up to the first decimal place only. Hence the value of ρ must be rounded off as 8.1 g cm^{-3} and the result of measurements should be reported as

ρ = (8.1 $\pm$ 0.3) g cm^{-3}.

A careful look, at the calculations done above, the main contribution to this (large) error in the measurement of ρ, is contributed by the (large) relative error (0.027) in the measurement of t, the smallest of the quantities measured. Hence the precision of the reported value of ρ could be increased by measuring t with an instrument having a least count smaller than 0.01cm. Thus if a micrometer screw gauge (least count = 0.001cm), (rather) than a vernier caliper were to be used, for measuring t, we would be reporting our result for ρ with a considerably lower degree of uncertainty. Experimentalists keep such facts in mind while designing their 'plan' for carrying out different measurements in a given experiment.

EXERCISES

1. The radius of a sphere is measured as (2.1 ± 0.5)cm

   Calculate its surface area with error limits.

   [Ans. {$(55.4 \pm 26.4)cm^2$}]

2. The voltage across a lamp is (6.0 ± 0.1) volt and the current passing through it is (4.0 ± 0.2) ampere. Find the power consumed by the lamp.

   [Ans. {(24.0 ± 1.6) watt}]

3. The length and breadth of a rectangular block are 25.2cm and 16.8cm, which have both been measured to an accuracy of 0.1cm. Find the area of the rectangular block.

   [Ans. {(423.4 ± 4.2) cm^2}]

4. A force of (2500 + 5) N is applied over an area of (0.32 ± 0.02) m^2. Calculate the pressure exerted over the area.

   [Ans. {(7812.5 ± 503.9) N/m^2}]

5. To find the value of 'g', by using a simple pendulum, the following observations were made :

   Length of the thread $\ell = (100 \pm 0.1)$ cm

   Time period of oscillation $T = (2 \pm 0.1)$ s

   Calculate the maximum permissible error in measurement of 'g'. Which quantity should be measured more accurately & why?

6. For a glass prism, of refracting angle 60^o, the minimum angle of deviation, D_m, is found to be 36^o, with a maximum error of 1.05^o, when a beam of parallel light is incident on the prism. Find the range of experimental value of refractive index

'μ' It is known that the refractive index 'μ' of the material of the prism is given by:

$$\mu = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin(A/2)}$$

[Ans. ($1.46 \leq \mu \leq 1.51$, with a mean value of 1.49)]

7. The radius of curvature of a concave mirror, measured by a spherometer, is given by

$$R = \frac{\ell^2}{6h} + \frac{h}{2}$$

The value of ℓ and h are 4.0cm and 0.065cm respectively, where ℓ is measured by a meter scale and h by a spherometer. Find the relative error in the measurement of R.

[Ans. (0.08)]

8. In Searle's experiment, the diameter of the wire, as measured by a screw gauge, of least count 0.001cm, is 0.500cm. The length, measured by a scale of least count 0.1cm, is 110.0cm. When a weight of 40N is suspended from the wire, its extension is measured to be 0.125cm by a micrometer of least count 0.001cm. Find the Young's modulus of the material of the wire from this data.

[Ans. {($2.2 \times 10^{11} \pm 10.758 \times 10^{9}$) N/m^2)]

9. A small error in the measurement of the quantity having the highest power (in a given formula), will contribute maximum percentage error in the value of the physical quantity to whom it is related. Explain why?

10. The two specific heat capacities of a gas are measured as $C_p = (12.28 \pm 0.2)$ units and $C_v = (3.97 + 0.3)$ units. Find the value of the gas constant R.

[Ans. {(8.31 ± 0.5) units}].

MOTION IN A VERTICAL CIRCLE

UNIT – IV

Consider a particle P suspended in a vertical plane, by a massless, inextensible string from a fixed point O. In equilibrium, the string is vertical with P vertically below the point of suspension O, as shown in Figure (i) (a).

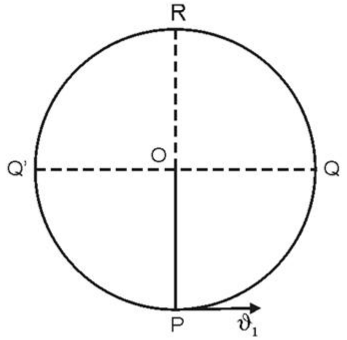


Figure (i) (a) **Figure (i) (b)**

Let the particle P be imparted an initial velocity $\overrightarrow{\vartheta_1}$ in a horizontal direction, as shown in Figure (i) (b). Under the tension in the string, the particle starts moving along a vertical circular path of radius equal to the length of the string. The point of suspension O is the center of this circle. It turns out that the initial velocity , $\overrightarrow{\vartheta_1}$ has to be more than a certain minimum critical value so that the particle may describe a vertical circular motion around point O.

The motion of a particle in a vertical circle, differs from that in a horizontal circle. In a horizontal circular motion, the force of gravity plays no role in the motion of the particle. However, in a vertical circular motion, gravity plays a very important role. It is easy to realize that a vertical circular motion has to be a **non-uniform circular motion**. In this case, the **velocity of the particle** varies both in **magnitude** and **direction**. In other words, in a vertical circular motion even the speed of the particle does not remain constant. As the particle moves up the circle, from its lowest position P, its speed continuously decreases till it reaches the highest point of its circular path. This is due to the work done against the force of gravity. When the particle moves down the circle, i.e., from R $\rightarrow$ Q' $\rightarrow$ P, its speed would keep on increasing. This is because of the work done, by the force of gravity, on the particle.

To obtain the basic characteristics of a vertical circular motion, consider an instantaneous position of particle, say at L. In this position, let the string make an angle θ with the vertical line OP, as shown in figure (ii)

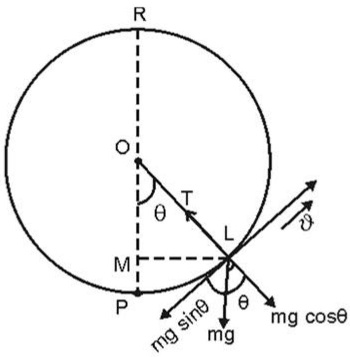


Figure (ii)

The forces acting on the particle (of mass m) at this position, L, are

(i) its weight = mg ; acting vertically downwards
(ii) the tension; T; in the string acting along LO.

The instantaneous velocity $\overline{\vartheta}$ of the particle is along the direction of the tangent to the circle at L. The corresponding instantaneous centripetal force, force, on the particle, equals $\frac{m\vartheta^2}{r}$ where r (= length of string l) is the radius of the particle's circular path. This force must act along $\overline{LO}$. We must, therefore, have

$$\frac{m\vartheta^2}{r} = T - mg\cos\theta$$
$$\therefore T = \frac{m\vartheta^2}{r} + mg\cos\theta \qquad \text{(i)}$$

We can take the horizontal direction, at the lowest point P, as the position of zero gravitational potential energy. Now, as per the law of conservation of energy;

Total energy at P = Total energy at L

$$\therefore \frac{1}{2} m\vartheta_1^{\,2} + 0 = \frac{1}{2} m\vartheta^2 + mgh \qquad \text{(ii)}$$

where MP = h, is the vertical height by which particle has risen above P. From right angled triangle OML,

$$OM = OL\cos\theta = r\cos\theta$$

$$\therefore MP = h = OP - OM$$

$$= r - r\cos\theta$$

$$= r\,(1 - \cos\theta) \qquad \text{(iii)}$$

From Eqns. (ii) and (iii), we get

$$\vartheta_1^{\,2} = \vartheta^2 + 2gr\,(1 - \cos\theta) \qquad \text{(iv)}$$

We now substitute the value of ϑ^2 , from Eqn (iv), in Eqn (i). Hence

$$T = \frac{m}{r}\left[\vartheta_1^{\,2} - 2gr\,(1 - \cos\theta)\right] + mg\cos\theta$$

$$= \frac{m\vartheta_1^{\,2}}{r} - 2mg\,(1 - \cos\theta) + mg\cos\theta$$

$$= \frac{m\vartheta_1^{\,2}}{r} - 2mg + 3mg\cos\theta \qquad \text{(v)}$$

This relation gives the tension T, in the string as a function of θ. We now use this relation to see the details of the particle when it is at the **(i)** lowest **(ii)** mid – way (horizontal) and **(iii)** highest position of its circular path.

When the particle is at the lowest point P of its vertical circular path, we have $\theta = 0°$. The tension T_p in the string in this position, from Eqn (V) ,is

$$T_p = \frac{m\vartheta_1^2}{r} - 2mg + 3mg\cos 0°$$

$$= \frac{m\vartheta_1^2}{r} + mg \qquad \text{(vi)}$$

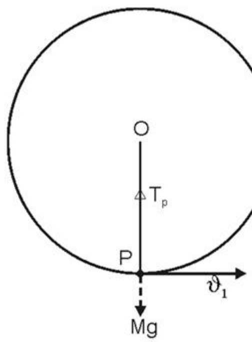


Figure (iii)

Consider next the case, when the particle is in position Q, where the string is momentarily) in its horizontal position. Clearly $\theta = \pi/2$ here. Let $\overrightarrow{\vartheta_2}$ be the instantaneous velocity of the particle here. Let T_Q be the instantaneous tension in the string here. Using Eqn (v), we have

$$T_Q = \frac{m\vartheta_1^2}{r} - 2mg + 3mg\cos\left(\pi/2\right)$$

$$= \frac{m\vartheta_1^2}{r} - 2mg \qquad \text{(vii)}$$

The change in the tension, as the particle moves from P to Q, equals ($T_P - T_Q$). We have ($T_P - T_Q$)

$$= \left(\frac{m\vartheta_1^2}{r} + mg\right) - \left(\frac{m\vartheta_1^2}{r} + 2mg\right) = 3mg$$

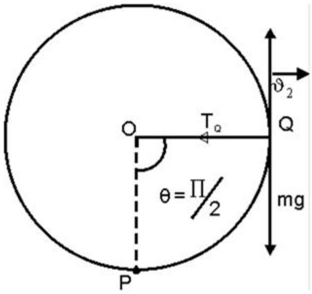


Figure (iv)

We next consider the particle P to be at the highest point R of its circular path. Let $\overrightarrow{\vartheta_3}$ be the instantaneous velocity of the particle here. In this position, $\theta = \pi$. If T_R denotes the tension in string in this position, we have, from eqn(v),

$$T_R = \frac{m\vartheta_1^2}{r} - 2mg + 3mg\cos(\pi)$$

$$= \frac{m\vartheta_1^2}{r} - 5mg \qquad \text{(viii)}$$

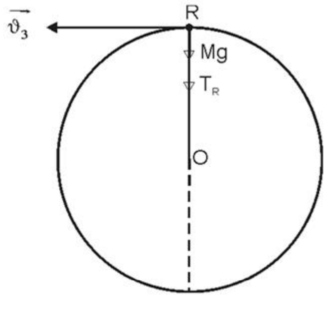


Figure (V)

Hence the change in the tension in the string, as the particle moves from P to R, along the vertical circle, is $(T_P - T_R) = \left(\frac{m\vartheta_1^2}{r} + mg\right) - \left(\frac{m\vartheta_1^2}{r} + 5mg\right) = 6mg$

It is thus seen that, the tension in the string is **maximum** when the particle is at **lowest point** P and is **minimum** at the **highest point** R of its vertical circular path. This is so because at the highest point, a part of the centeripetal force, needed to keep the particle moving in its circular path, is provided by the weight (mg) of the particle.

From Eqn (viii), it is easy to realize that T_R can be **(a)** positive; **(b)** negative or **(c)** zero depending on the value of ϑ_1 If T_R becomes a negative number, the **string would get slackened,** and the particle will be unable to continue moving along its vertical circular path. It will fall down before it is able to complete its circular path. Hence for completing the vertical circle, the minimum value of T_R has to be zero, We, therefore, have

$$\left(T_R\right)_{min} = \frac{m\ (\vartheta_1)^2{}_{min}}{r} - 5mg = o$$

$$\therefore\ \left(\vartheta_1\right)_{min} = \sqrt{5gr} \qquad \text{(ix)}$$

Using Eqn (iv) the minimum speed, which the particle must have at the highest point R, so that it is able to complete the vertical circle, is given by

$$\left(\vartheta_1\right)^2{}_{min} = \left(\vartheta_3\right)^2{}_{min} + 2gr\ (1-\cos\ \Pi\ )$$

$$\therefore\ 5gr = \left(\vartheta_3\right)^2{}_{min} + 4gr$$

$$\text{or } \left(\vartheta_3\right)_{min} = \sqrt{gr} \qquad \text{(x)}$$

When the particle completes its motion along the vertical circle it is referred to as **"looping the loop".** For this to be possible, **the minimum speed at the lowest point** , must be $\sqrt{5gr}$.

The values of the tension in the string, when the particle is just able to do 'looping the loop', correspond to $\vartheta_1 = \left(\vartheta_1\right)_{min} = \sqrt{5gr}$

Hence , in this case,

T_P = 6mg [From Eqn (vi)]

and T_R = mg [From Eqn (viii)]

The results obtained above (for 'looping the loop') are put to many practical applications. We list below some of these applications.

(i) The pilot of an air craft can successfully loop a vertical circle (of radius r), if the velocity of the air-craft, at the lowest point of its vertical circle, is more than $\sqrt{5gr}$

(ii) Consider a bucket full of water being rotated in a vertical circle. The water, in the bucket, would not spill-over (even when the bucket is at its highest position along the vertical circle, i.e. when it is upside down), if the starting speed of the bucket, at the lowest point of its path, is **more** than $\sqrt{5gr}$. Under these conditions, the **centerifugal force,** on the water, inside the bucket, is more than the weight mg of the water. Hence the water does not spill over. If, however, the starting speed, at the lowest point of the vertical circular path, is **less** than $\sqrt{5gr}$, water will spillover when the bucket is upside down, i.e. at the highest point of its circular path. It is thus obvious that the **"trick"** really lies in whirling the bucket **"fast enough".**

(iii) A circus acrobat, performing in the **"circle of death",** speeds up his motor cycle, inside the circular cage, before going into a **vertical loop.** When he acquire a speed **more** than $\sqrt{5gr}$, at the lowest point of the intended vertical circular path, he would not fall down, even when he is "upside down" (i.e. at the highest point of his circular path). This is again because, in such a case, the centerifugal force on the motor cyclist, when he is "up-side-down, is more than his weight.

Example 1 : A small stone, of mass 200g, is tied to one end of a string of length 80cm. Holding the other end in hand, the stone is whirled into a vertical circle. What is the minimum speed, that needs to be imparted, at the lowest point of the circular path, so that the stone is just able to complete the vertical circle? What would be the tension in the string at the lowest point of circular path? (Take g $\simeq$10ms^{-2})

Solution : We know that

ϑ_{min} = minimum speed needed at the lowest point, so that particle is just able to complete the vertical circle = $\sqrt{5g\ell}$

Hence $\vartheta_{min} = \sqrt{5 \times 10 \times 0.8}$ ms^{-1}

$\simeq$ 6.32 ms^{-1}

Also T_1 = Tension in the string at the lowest point of its circular path.

$$= \frac{m\vartheta^2{}_{min}}{\ell} + mg = 6mg = 6 \times 0.2 \times 10\,N = 12N$$

Example 2 : A massless string, of length 1.2m, has a breaking strength of 2kgwt. A stone of mass 0.4kg, tied to one end of the string, is made to move in a vertical circle, by holding the other end in the hand. Can the particle describe the vertical circle? (Take g $\simeq$10ms^{-2})

Solution : We are given that

T_{max} = maximum tension in the string so that it does not break

= 2 kgwt = 2x10N = 20N

Let T_1 be the tension in the string when the stone is in its lowest position of its circular path. We know that $T_1 = \frac{m\vartheta_1^2}{r} + mg$.

T_1 would have its minimum value when ϑ_1 equals its minimum value $\left(= \sqrt{5g\ell}\right)$, needed by the stone, to complete its vertical circular path.

Hence $(T_1)_{min} = \frac{m\vartheta_{min}^2}{\ell} + mg = 6mg$

= 6 x 0.4 x 10

= 24N

We thus see that $(T_1)_{min}$ is more than the breaking strength of the string. Hence the particle cannot describe the vertical circle.

Example 3 : A small stone, of mass 0.2kg, tied to a massless, inextensible string, is rotated in a vertical circle of radius 2m. if the particle is just able to complete the vertical circle, what is its speed at the highest point of its circular path? How would this speed get effected if the mass of the stone is increased by 50%? (Take g $\simeq$ $10ms^{-2}$)

Solution :

Let ϑ_1 be the speed of the stone at the lowest point of its vertical circle. Since the stone is just able to complete the vertical circle, we have

$\vartheta_1 = \sqrt{5gr}$
$= \sqrt{5 \times 10 \times 2}\ ms^{-1} = 10ms^{-1}$

Let ϑ_2 be the speed of the stone, at the highest point, on its circular path. Then

$\vartheta_2^2 = \vartheta_1^2 - 4gr$

$= (10)^2 - 4 \times 10 \times 2$

$= 100 - 80 = 20$

$\therefore \vartheta_2 = \sqrt{20}\ ms^{-1} \simeq 4.47ms^{-1}$

It is thus seen that the value of ϑ_2 , **does not depend** on the **mass** (m) of **the stone**. Hence ϑ_2 would **remains the same** when the mass of the stone increases by 50%.

Example 4 : A particle, of , mass 150g, is attached to one end of a massless, inextensible string. It is made to describe a vertical circle of radius 1m. When the string is making an angle of 48.2° with the vertical, its instantaneous speed is $2ms^{-1}$. What is the tension in the string in this position? Would this particle be able to complete its circular path?

(Take g $\simeq$ 10 ms^{-2})

Solution :

The tension T, in the string, when it makes an angle θ, with the vertical, is given by

$$T = \frac{m\vartheta^2}{\ell} + mg\cos\theta$$

where ϑ is the instantaneous speed of the particle.

Here $\vartheta = 2ms^{-1}$, ℓ = 1m, m = 0.15kg, and θ = 48.2°

$$\therefore T = \frac{0.15 \times (2)^2}{1} + (0.15 \times 10 \times \cos 48.2^\circ)$$

= (0.6 + 1.5 x 0.67) N

$\simeq$ 1.6N

Let ϑ_1 be the speed of the particle at the lowest point of its circular path. Then

$$\vartheta_1^2 = \vartheta^2 + 2gr(1 - \cos\theta)$$

$= (2)^2 + 2 \times 10 \times 1 \times (1 - \cos 48.2^\circ)$

$= (4 + 20 \times (1 - 0.67)$

$\simeq (4 + 6.6) = 10.6$

$\therefore\ \vartheta_1 = \sqrt{10.6}\ ms^{-1} \simeq 3.25\ ms^{-1}$

The minimum value of ϑ_1 , so that the particle is able to complete its vertical circle, is $\sqrt{5gr}$.

$\therefore\ (\vartheta_1)_{min} = \sqrt{5 \times 10 \times 1}\ ms^{-1} \simeq\ 7.07\ ms^{-1}$

The value of ϑ_1, obtained above, is **less** than this minimum speed. The particle, in the given case, would not be able to complete its vertical circular path.

Example 5 : A bucket, containg 4kg of water, is tied to a rope of length 2.5m and rotated in a vertical circle in such a way that the water in it just does not spill over when the bucket is in its 'upside down' position. What is the speed of bucket at the

(a) highest and (b) lowest point of its circular path? (Take g $\simeq$ 10ms^{-2})

Solution :

Let ϑ_1 be the speed of bucket at the lowest point of its circular path. Then

$\vartheta_1 = \sqrt{5gr}$
$= \sqrt{5 \times 10 \times 2.5}\ ms^{-1}$
$= \sqrt{125}\ ms^{-1} \simeq 11.18\ ms^{-1}$

Let ϑ_2 be the speed of the bucket at the highest point of its circular path. Then

$\vartheta_2 = \sqrt{gr} \in$
$= \sqrt{10 \times 2.5}\ ms^{-1}$
$= 5ms^{-1}$

Example 6 : The figure here shows a smooth 'looping-the-loop' track. A particle, of mass **m**, is released from point A, as shown. If H=3r, would the particle 'loop the loop'?

What is the force on the circular track when the particle is at point (i) B (ii) C ?

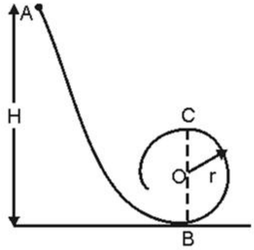


Figure (vi)

Solution :

Let ϑ_B be the speed acquired by the particle at the (lowest) point B. From law of conservation of energy, we have

Total energy at A = Total energy at B

$$\therefore (0 + mgH) = \frac{1}{2} m\vartheta_B^2 + 0$$

$$\therefore \vartheta_B = \sqrt{2gh} = \sqrt{2g \times 3r} = \sqrt{6gr}$$

The minimum, speed, needed by the particle at B, so that it can 'loop the loop' is $\sqrt{5gr}$. Since ϑ_B is more than $\sqrt{5gr}$; the particle would 'loop the loop'.

The forces, acting on particle at B, are as shown here, Let N_1 be the force exerted on the particle by the track. According to Newton's third law, the force exerted by the particle on the track, is equal and opposite to N_1. Now

$$N_1 = mg + \frac{m\vartheta_B^2}{r}$$

$$= mg + \frac{m \times 6gr}{r} = 7mg$$

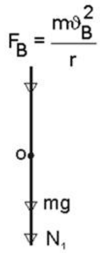


Figure (vi) (a)

Hence the force, exerted, by the particle, on the track, equals 7mg, directed vertically downwards.

The forces, acting on the particle in position C, are as shown in the figure here. The speed, ϑ_c , of the particle, at C, is given by $\vartheta_c^2 = \vartheta_B^2$ - 4gr = 6gr - 4gr = 2gr

$$\therefore\ N_2 = \frac{m\vartheta_c^2}{r} - mg$$

= 2mg – mg = mg

Hence the particle exerts a force mg, directed radially outwards, on the track, at point C.

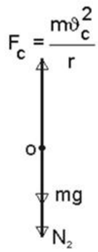


Figure (vi) (b)

EXERCISES

1. A stone of mass 0.2kg, is tied to one end of a string of length 80cm. Holding the other end, the stone is whirled into a vertical circle. What is the minimum speed of the stone at the lowest point so that it just completes the circle. What is the tension in string at the lowest point of the circular path? ($g=10ms^{-2}$).

[Ans: $6.32ms^{-1}$; 12N]

2. A particle, of mass 100g, is moving in a vertical circle of radius 2m. The particle is just 'looping the loop'. What is the speed of particle and the tension in string at the highest point of the circular path? ($g=10ms^{-2}$).

[Ans. $4.47ms^{-1}$, zero]

3. A particle, of mass 0.2kg, attached to a massless string is moving in a vertical circle of radius 1.2m. It is imparted a speed of $8ms^{-1}$ at the lowest point of its circular path. Does the particle complete the vertical circle? What is the change in tension in the string when the particle moves from the position, where the string is vertical, to the position where the string is horizontal?

[Ans. Yes, 6N]

4. A particle, of mass 200g, is whirled into a vertical circle of radius 80cm using a massless string. The speed of particle, when the string makes an angle of 60^{o} with the vertical line, is $1.5ms^{-1}$. What is the tension in the string in this position?

[Ans. 1.56N]

ELASTIC AND INELASTIC COLLISIONS IN TWO DIMENSIONS

UNIT – IV

Consider two particles A and B moving in a plane. If these two particles collide, and still continue moving in the same plane, the collision is referred to as a **two dimensional (or an oblique) collision.** A collision, between two billiard balls is an example of such a two dimensional collision.

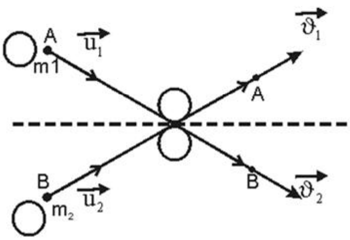
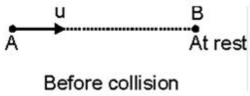


Figure (i)

To analyze the basic details of a two dimensional collision, consider a system of two particles A and B, moving, as shown, before and after the collision.

Since the forces, they exert on each other (during collision) are **internal forces**, and there are no other forces, (i.e.there is no external force), the **linear momentum , of the system is conserved.** When the collision is elastic, the kinetic energy of the system is also conserved. (The collision is inelastic if **kinetic energy of system is not conserved).**

To illustrate, how calculations can be carried out, we now, consider a simple two dimensional elastic collision, taking place , as shown below.



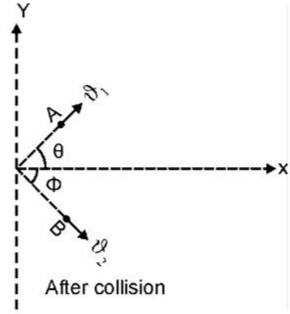


Figure (ii) (a) **Figure (ii) (b)**

Figure ((ii) (a)) shows a particle A, of mass m_1, moving along x-axis, in the x-y plane, with an initial speed u. Particle B, of mass m_2, is initially at rest. When particle A collides with B, the two particles move with speeds ϑ_1 and ϑ_2, in the x-y plane, after the collision (Figure (ii) (b))

After the collision, let particle A move in a direction inclined at an angle θ , with its initial direction of motion. Angle θ is known as the **angle of scattering.**

After the collision, let particle B move along a direction, making an angle ϕ, with the initial direction of motion of A. Angle Φ is known as the **angle of recoil.**

Knowing m_1, m_2, and u , we have to do the needed calculations. The law of conservation of linear momentum, used for the x and y components separately, gives us the two equations:

$$m_1u + 0 = m_1\vartheta_1 \cos\theta + m_2\, \vartheta_2 \, \cos\phi \qquad \text{(i)}$$

and $$0 = m_1\vartheta_1 \sin\theta - m_2\, \vartheta_2 \, \sin\phi \qquad \text{(ii)}$$

We have assumed the collision to be **perfectly elastic**. Hence total K.E. before collision = total K.E. after collision

$$\therefore \frac{1}{2}m_1u^2 + 0 = \frac{1}{2}m_1\vartheta_1^2 + \frac{1}{2}m_2\,\vartheta_2^2$$
$$\text{or } m_1u^2 = m_1\vartheta_1^2 + m_2\,\vartheta_2^2 \qquad \text{(iii)}$$

We thus have **three equations** in all but need to find four unknown parameters. A **complete solution,** is therefore, **NOT POSSIBLE.** However if the value of any one of the four unknowns, i.e, $\vartheta_1, \vartheta_2, \theta$ or ϕ, is **given,** the remaining three can be calculated, using Eqns (i), (ii) and (iii).

[For an **inelastic** two dimensional collision, we only have two equations i.e Eqn (i) and (ii). Hence, in this case, if two of the four unknowns, say θ and ϕ are given, we can calculate the remaining two unknowns (i e ϑ_1, and ϑ_2) using Eqns (i) and (ii)].

Special case : We now consider the special case of two dimensional collision of two particles of equal mass. Eqns (i), (ii) and (iii), in this case, reduce to

$$u = \vartheta_1 \cos\theta + \vartheta_2 \cos\phi \qquad \text{(iv)}$$

$$o = \vartheta_1 \sin\theta - \vartheta_2 \sin\phi \qquad \text{(v)}$$

$$u^2 = \vartheta_1^2 + \vartheta_2^2 \qquad \text{(vi)}$$

From Eqns (iv) and (vi)

$$\left(\vartheta_1 \cos\theta + \vartheta_2 \cos\phi\right)^2 = \vartheta_1^2 + \vartheta_2^2$$

$$\therefore\ \vartheta_1^2 \cos^2\theta + \vartheta_2^2 \cos^2\phi + 2\,\vartheta_1\vartheta_2 \cos\theta\cos\phi = \vartheta_1^2 + \vartheta_2^2$$

$$\begin{aligned} \text{or } 2\,\vartheta_1\,\vartheta_2 \cos\theta\cos\phi &= \vartheta_1^2\,(1-\cos^2\theta) + \vartheta_2^2\,(1-\cos^2\phi) \\ &= \vartheta_1^2 \sin^2\theta + \vartheta_2^2 \sin^2\phi \end{aligned} \qquad \text{(vii)}$$

Using Eqn (v), we can rewrite Eqn (vii) as

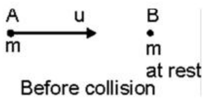
$$2\,\vartheta_1\,\vartheta_2 \cos\theta\cos\phi = 2\,\vartheta_1^2 \sin^2\theta$$

$$\text{or } \cos\theta = \left(\frac{\vartheta_1}{\vartheta_2}\right)\frac{\sin^2\theta}{\cos\phi} \qquad \text{(viii)}$$

$$\begin{aligned} \text{Now } \cos(\theta+\phi) &= \cos\theta\cos\phi - \sin\theta\sin\phi \\ &= \left(\frac{\vartheta_1}{\vartheta_2}\right)\frac{\sin^2\theta}{\cos\phi}\cos\phi - \frac{\vartheta_1}{\vartheta_2}\sin^2\theta \quad (\text{from Eqn (V) and (viii)}) \\ &= \frac{\vartheta_1}{\vartheta_2}\sin^2\theta - \frac{\vartheta_1}{\vartheta_2}\sin^2\theta \\ &= 0 \end{aligned}$$

$$\therefore \theta + \phi = \pi/2$$

We thus see that in the special case, of a perfectly elastic (two-dimensional) collision, between two particles, of the same mass, the two particles move along mutually perpendicular directions after the collision. This is illustrated in the figure below.



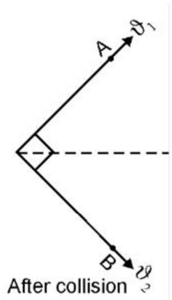


Figure (iii)(a)

Figure (iii)(b)

Example 1: A and B are two particles having the same, mass **m**. A is moving along x-axis with a speed of $10m^{-1}$, and B is at rest. After undergoing a perfectly elastic collision, with B, particle A gets scattered through an angle of 30^{o}. What is the direction of motion of B, and the speeds of A and B, after this collision?

Solution:

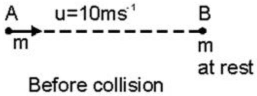


Figure (iv)(a)

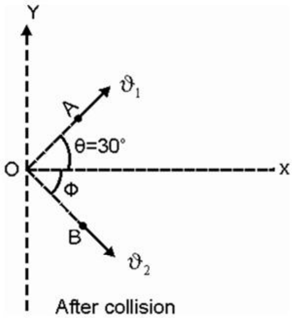


Figure (iv)(b)

Figure (iv), (a) and (b), show the particles A and B, before and after the collision. Since A and B have the same mass, and the collision is perfectly elastic, we would have

$$\theta + \phi = 90^\circ$$
$$\therefore \phi = 90^\circ - 30^\circ = 60^\circ \qquad \text{(i)}$$

Using law of conservation of linear momentum, we get

(i) for the x-components,

$$u = 10 = \vartheta_1 \cos 30^\circ + \vartheta_2 \cos 60^\circ$$

$$\text{or} \quad 10 = \frac{\sqrt{3}}{2}\vartheta_1 + \frac{\vartheta_2}{2}$$

$$\therefore 20 = \sqrt{3}\,\vartheta_1 + \vartheta_2 \qquad \text{(ii)}$$

and (ii) for the y – components,

$$o = \vartheta, \sin 30^\circ - \vartheta_2 \sin 60^\circ$$

$$\therefore \vartheta_1 \frac{1}{2} = \frac{\sqrt{3}}{2}\vartheta_2 \text{ or } \vartheta_1 = \sqrt{3}\,\vartheta_2 \qquad \text{(iii)}$$

From Eqn (ii) and (iii), we get

$$20 = 3\vartheta_2 + \vartheta_2 \text{ or } \vartheta_2 = 5\text{ms}^{-1}$$

$$\text{and } \vartheta_1 = \sqrt{3}\,\vartheta_2$$

$$= 1.732 \times 5\text{ms}^{-1} = 8.66\text{ms}^{-1}$$

Example 2: Two particles, A and B, of masses **m** and **2m**, are moving along the X and Y- axis, respectively, with the same speed ϑ.

They collide, at the origin, and coalesce into one body, after the collision. What is the velocity of this coalesced mass? What is the loss of energy during this collision?

Solution :

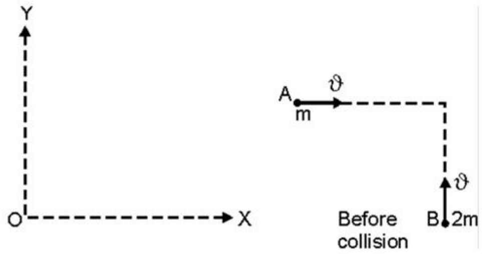


Figure (v) (a)

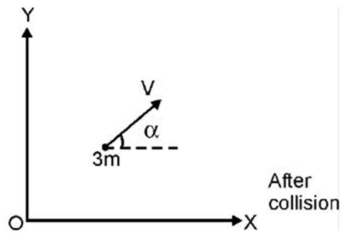


Figure (v) (b)

Figures (v) (a), and (v) (b), show the two particles before and after the collision. Let V be the speed of the combined mass and let the direction of $\vec{V}$ be making an angle α with the positive x-axis, after the collision. Using law of conservation of linear momentum, we have

For the x-components,

$$m\vartheta = 3m\,V\cos\alpha \qquad \text{(i)}$$

and for the y-components

$$2m\vartheta = 3m\,V\sin\alpha \qquad \text{(ii)}$$

From Eqns (i) and (ii), we get

$$\tan\alpha = \frac{2m\vartheta}{m\vartheta} = 2$$

$\therefore \alpha = \tan^{-1}(2) \simeq 63.4^{\circ}$ (iii)

Also $V^2 = \left(\frac{\vartheta}{3}\right)^2 + \left(\frac{2\vartheta}{3}\right)^2 = \frac{5}{9}\vartheta^2$

$\therefore \quad V = \left(\frac{\sqrt{5}}{3}\right)\vartheta$ (iv)

Now K_i = Total K.E. before collision = $\frac{1}{2} m\vartheta^2 + \frac{1}{2}(2m)\vartheta^2 = \frac{3m\vartheta^2}{2}$

and K_f = Total K.E. after collision = $\frac{1}{2}(3m)V^2 = \left(\frac{3}{2}\right)\left(\frac{5}{9}\right)m\vartheta^2$

$$= \frac{5}{6} m\vartheta^2$$

Hence ΔK = Loss of kinetic energy during the collision = $K_i - K_f$

$$= \left(\frac{3}{2} - \frac{5}{6}\right)m\vartheta^2 = \frac{2}{3}m\vartheta^2$$

EXERCISES

1. A billiard ball A moving with an initial speed of $1 ms^{-1}$, undergoes a perfectly elastic collision with another identical ball B at rest. A is scattered through an angle of 30^{o}. What is the angle of recoil of B? What is the speed of ball A after the collision?

$$\left[\text{Ans. } 60^{o};\ \frac{\sqrt{3}}{2} ms^{-1}\right]$$

2. Two identical balls, A and B, undergo a perfectly elastic two dimensional collision. Initially A is moving with a speed of $10 ms^{-1}$ and B is at rest. Due to collision, A is scattered through an angle of 30^{o}. What are the speeds of A and B after the collision?

$\left[\text{Ans. } \vartheta_A = 5\sqrt{3}\ \text{ms}^{-1}, \vartheta_B = 5\ \text{ms}^{-1}\right]$

3. A and B are two identical balls. A, moving with a speed of 6ms^{-1}, along the positive x-axis, undergoes a collision with B, initially at rest. After collision, each ball moves along directions making angles of $\pm 30^{o}$ with the x-axis. What are the speeds of A and B after the collision? Is this collision perfectly elastic?

$\left[\text{Ans. } \vartheta_A = \vartheta_B = 2\sqrt{3}\ \text{ms}^{-1}, \text{No}\right]$

NON – CONSERVATIVE FORCES

UNIT – IV

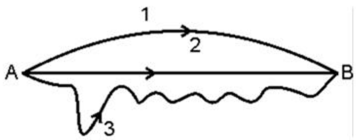


Figure (i)(a)

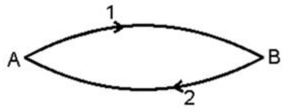


Figure (i)(b)

A force is non-conservative; if **work done** by, or against the force, in moving from one point A, in space, to another point B **DEPENDS ON THE PATH FOLLOWED IN MOVING FROM A to B**. In Figure (i) (a), let W_1, W_2 and W_3 denote the works done in moving a body, from A to B, along three different paths 1, 2 and 3 respectively. For a non-conservative force $W_1 \neq W_2 \neq W_3$

Figure (i) (b) shows a particle moving along a **closed path** $A \to 1 \to B \to 2 \to A$. Let W_1 be the work done along the path $A \to 1 \to B$ and let W_2 be the work done along the path $B \to 2 \to A$. For a non-conservative force $|W_1| \neq |W_2|$. **Therefore, in such a case, the net work done, along the closed path is not-zero.** Expressed mathematically.

$\oint \vec{F}.\vec{ds} \neq o$, i.e. for a non-conservative force, the work done along a closed path is not zero.

Two of the common examples, of non-conservative forces, are

(i) **Force of friction**
(ii) **Viscous force**

Non- Conservative forces are usually velocity **dependent.** Consider a particle moving from A to B, on a horizontal, rough surface.

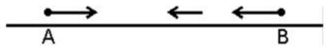


Figure (ii)

The force of friction coming into play, is a non-conservative force. Let W be the work done against the force of friction in moving from A to B. When the body moves from B to A, W would again be the work done against the force of friction. The net work done, against friction, in the round trip is therefore, 2W. If this work is being done at the cost of kinetic energy, the loss of Kinetic energy, in the round trip, is numerically equal to 2W.

Consider next, a body having both a kinetic energy (K), and a potential energy (U), moving in a non-conservative force field. The total energy E (=K+U), of the body, does not remain constant. Let E_i and E_f represent the total values of the initial and final energy. If W is the work done, against the non-conservative force, we would have

$E_i - E_f = W$

The work done, by non-conservative forces, appears in some other form of energy like heat, sound, light etc. When we take into account all forms of energy, the general **law of conservation of energy still holds good both for conservative as well as non-conservative forces.**

Example 1 : A particle, of mass 0.1kg, has an initial speed of $4ms^{-1}$ at a point A on a rough horizontal road. The coefficient of friction, between the object and the road is 0.15. The particle moves to a point B, at a distance of 2m from A. What is the speed of the particle at B?

(Take g $\simeq$ $10ms^{-2}$)