Exercise 3.1

1. Akhila went to a fair in her village. She wanted to enjoy rides on the Giant Wheel and play Hoopla (a game in which you throw a rig on the items kept in the stall, and if the ring covers any object completely you get it). The number of times she played Hoopla is half the number of rides she had on the Giant Wheel. Each ride costs Rs 3, and a game of Hoopla costs Rs 4. If she spent Rs 20 in the fair, represent this situation algebraically and graphically.

Sol:

The pair of equations formed is:

$y - \frac{1}{2}x$

i.e., $x - 2y = 0$ ……..(1)

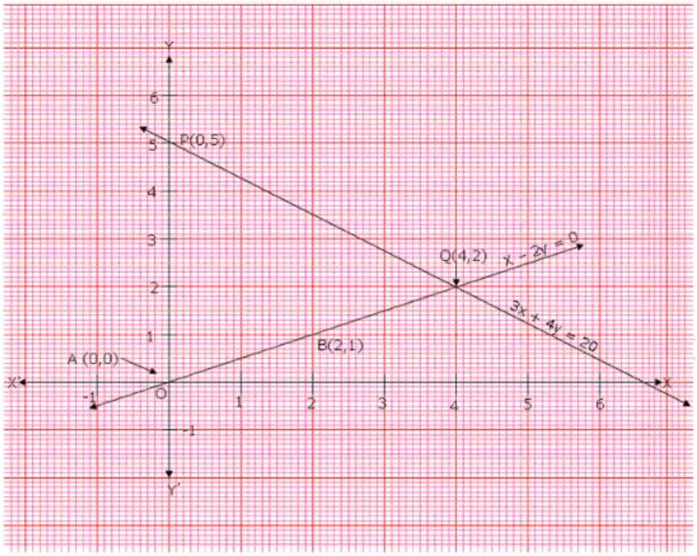
$3x + 4y = 20$ ………(2)

Let us represent these equations graphically. For this, we need at least two solutions for each equation. We give these solutions in Table

x	0	2
$y - \frac{x}{2}$	0	1

x	0	2	4
$y = \frac{20-3x}{4}$	5	0	2

Recall from Class IX that there are infinitely many solutions of each linear equation. So each of you choose any two values, which may not be the ones we have chosen. Can you guess why we have chosen $x = O$ in the first equation and in the second equation? When one of the variables is zero, the equation reduces to a linear equation is one variable, which can be solved easily. For instance, putting $x = O$ in Equation (2), we get 4y = 20 i.e., $y = 5$. Similarly, putting $y = O$ in Equation (2), we get $3x = 20$ *i.e.*, $x = \frac{20}{3}$. But as $\frac{20}{3}$ is not an integer, it will not be easy to plot exactly on the graph paper. So, we choose y = 2 which gives x = 4, an integral value.



Plot the points $A(O,O), B(2,1)$ and $P(O,5)$, $Q(412)$, corresponding to the draw the lines AB and PQ, representing the equations $x-2y=O$ and $3x+4y=20$, as shown in figure

In fig., observe that the two lines representing the two equations are intersecting at the point (4,2),

2. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." Is not this interesting? Represent this situation algebraically and graphically.

Sol:

Let the present age of Aftab and his daughter be x and y respectively. Seven years ago.

Age of Ahab $=x-7$

Age of his daughter $y-7$

According to the given condition.

$(x-7)=7(y-7)$

$\Rightarrow x-7=7y-49$

$\Rightarrow x-7y=-42$

Three years hence

Age of Aftab $=x+3$

Age of his daughter $=y+3$

According to the given condition,

$(x+3)=3(y+3)$

$\Rightarrow x+3=3y+9$

$\Rightarrow x - 3y = 6$

Thus, the given condition can be algebraically represented as

$x - 7y = -42$

$x - 3y = 6$

$x - 7y = -42 \Rightarrow x = -42 + 7y$

Three solution of this equation can be written in a table as follows:

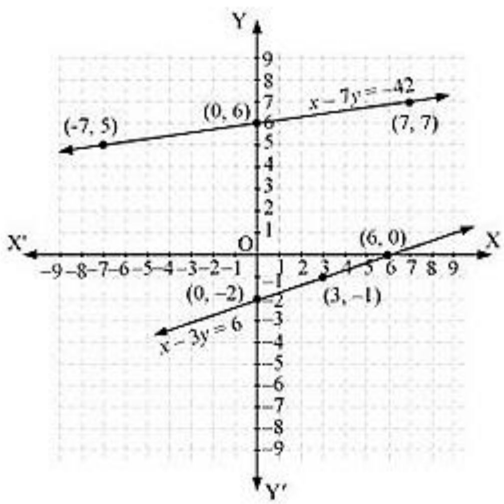
x	–7	0	7
y	5	6	7

$x - 3y = 6 \Rightarrow x = 6 + 3y$

Three solution of this equation can be written in a table as follows:

x	6	3	0
y	0	-1	-2

The graphical representation is as follows:



Concept insight In order to represent a given situation mathematically, first see what we need to find out in the problem. Here. Aftab and his daughters present age needs to be found so, so the ages will be represented by variables z and y. The problem talks about their ages seven years ago and three years from now. Here, the words 'seven years ago' means we have to subtract 7 from their present ages. and 'three years from now' or three years hence means we have to add 3 to their present ages. Remember in order to represent the algebraic equations graphically the solution set of equations must be taken as whole numbers only for the accuracy. Graph of the two linear equations will be represented by a straight line.

3. The path of a train A is given by the equation 3x + 4y − 12 = 0 and the path of another train B is given by the equation 6x + 8y − 48 = 0. Represent this situation graphically.

Sol:

The paths of two trains are giver by the following pair of linear equations.

$3x+4y-12=0 \qquad ...(1)$

$6x+8y-48=0 \qquad ...(2)$

In order to represent the above pair of linear equations graphically. We need two points on the line representing each equation. That is, we find two solutions of each equation as given below:

We have,

$3x+4y-12=0$

Putting $y=0$, we get

$3x+4\times0-12=0$

$\Rightarrow \quad 3x=12$

$\Rightarrow \quad x=\frac{12}{3}=4$

Putting $x=0$, we get

$3\times0+4y-12=0$

$\Rightarrow \quad 4y=12$

$\Rightarrow \quad y=\frac{12}{4}=3$

Thus, two solution of equation $3x+4y-12=0$ are $(0,3)$ *and* $(4,0)$

We have,

$6x+8y-48=0$

Putting $x=0$, we get

$6\times0+8y-48=0$

$\Rightarrow \quad 8y=48$

$\Rightarrow \quad y=\frac{48}{8}$

$\Rightarrow \quad y=6$

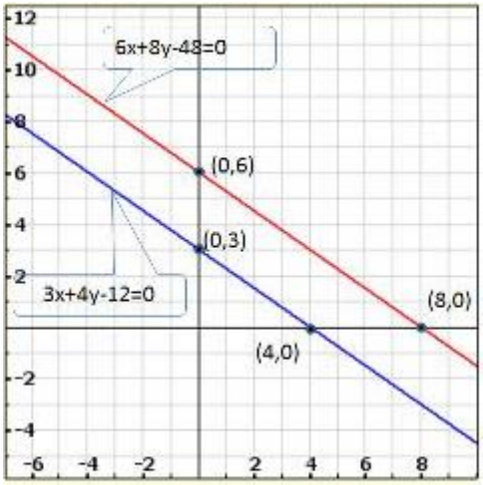
Putting $y=0$, we get

$6x+8\times0=48=0$

$\Rightarrow \quad 6x=48$

$\Rightarrow \quad x=\frac{48}{6}=8$

Thus, two solution of equation $6x+8y-48=0$ are $(0,6)$ *and* $(8,0)$



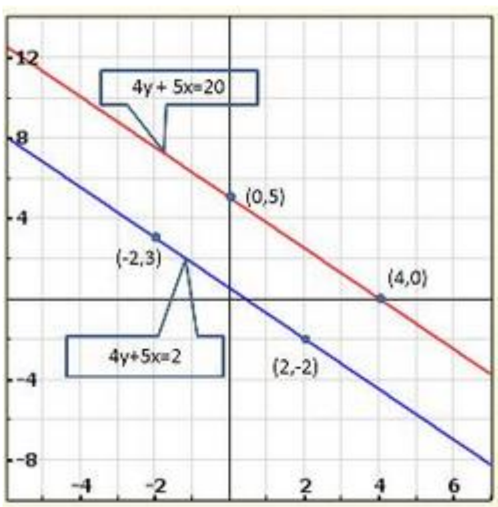
Clearly, two lines intersect at $(-1,2)$

Hence, $x=-1, y=2$ is the solution of the given system of equations.

4. Gloria is walking along the path joining (− 2, 3) and (2, − 2), while Suresh is walking along the path joining (0, 5) and (4, 0). Represent this situation graphically.

Sol:

It is given that Gloria is walking along the path Joining $(-2,3)$ *and* $(2,-2)$, while Suresh is walking along the path joining $(0,5)$ *and* $(4,0)$.



We observe that the lines are parallel and they do not intersect anywhere.

5. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ *and* $\frac{c_1}{c_2}$ and and without drawing them, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincide:

(i) 5x− 4y + 8 = 0
7x + 6y − 9 = 0

(ii) 9x + 3y + 12 = 0
18x + 6y + 24 = 0

(iii) 6x − 3y + 10 = 0
2x − y + 9 = 0

Sol:

We have,

$$5x-4y+8=0$$
$$7x+6y-9=0$$

Here,

$$a_1=5, b_1=-4, c_1=8$$
$$a_2=7, b_2=6, c_2=-9$$

We have,

$$\frac{a_1}{a_2}=\frac{5}{7}, \frac{b_1}{b_2}=\frac{-4}{6}=\frac{-2}{3} \text{ and } \frac{c_1}{c_2}=\frac{8}{-9}=\frac{-8}{9}$$

$\therefore$ $$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

$\therefore$ Two lines are intersecting with each other at a point.

We have,

$$9x+3y+12=0$$
$$18+6y+24=0$$

Here,

$$a_1=9, b_1=3, c_1=12$$
$$a_2=18, b_2=6, c_2=24$$

Now,

$$\frac{a_1}{a_2}=\frac{9}{18}=\frac{1}{2},$$
$$\frac{b_1}{b_2}=\frac{3}{6}=\frac{1}{2}$$

And $$\frac{c_1}{c_2}=\frac{12}{24}=\frac{1}{2}$$

$\therefore$ $$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$\therefore$ Both the lines coincide.

We have,

$$6x-3y+10=0$$
$$2x-y+9=0$$

Here,

$$a_1=6, b_1=-3, c_1=10$$
$$a_2=2, b_2=-1, c_2=9$$

Now,

$$\frac{a_1}{a_2}=\frac{6}{2}=\frac{3}{1},$$
$$\frac{b_1}{b_2}=\frac{-3}{-1}=\frac{3}{1},$$
And $$\frac{c_1}{c_2}=\frac{10}{9}$$
$\therefore$ $$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$
$\therefore$ The lines are parallel

6. Given the linear equation 2x + 3y - 8 = 0, write another linear equation in two variables such that the geometrical representation of the pair so formed is:
(i) intersecting lines (ii) parallel lines (iii) coincident lines.

Sol:

We have,
$$2x+3y-8=0$$
Let another equation of line is:
$$4x+9y-4=0$$
Here,
$$a_1=2, b_1=3, c_1=-8$$
$$a_2=4, b_2=9, c_2=-4$$

Now,
$$\frac{a_1}{a_2}=\frac{2}{4}=\frac{1}{2},$$
$$\frac{b_1}{b_2}=\frac{3}{9}=\frac{1}{3},$$
And $$\frac{c_1}{c_2}=\frac{-8}{-4}=\frac{2}{1}$$
$\therefore$ $$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$
$\therefore$ $2x+3y-8=0$ *and* $4x+9y-4=0$ intersect each other at one point.

Hence, required equation of line is $4x+9y-4=0$

We have,
$$2x+3y-8=0$$
Let another equation of line is:
$$4x+6y-4=0$$

Here,

$$a_1 = 2, b_1 = 3, c_1 = -8$$
$$a_2 = 4, b_2 = 6, c_2 = -4$$

Now,

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2},$$
$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2},$$

And $\frac{c_1}{c_2} = \frac{-8}{-4} = \frac{2}{1}$

$\therefore$ $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\therefore$ Lines are parallel to each other.

Hence, required equation of line is $4x + 6y - 4 = 0$.

7. The cost of 2kg of apples and 1 kg of grapes on a day was found to be Rs 160. After a month, the cost of 4kg of apples and 2kg of grapes is Rs 300. Represent the situation algebraically and geometrically.

Sol:

Let the cost of 1 kg of apples and 1 kg grapes be Rs x and Rs y.

The given conditions can be algebraically represented as:

$2x + y = 160$

$4x + 2y = 300$

$2x + y = 160 \Rightarrow y = 160 - 2x$

Three solutions of this equation cab be written in a table as follows:

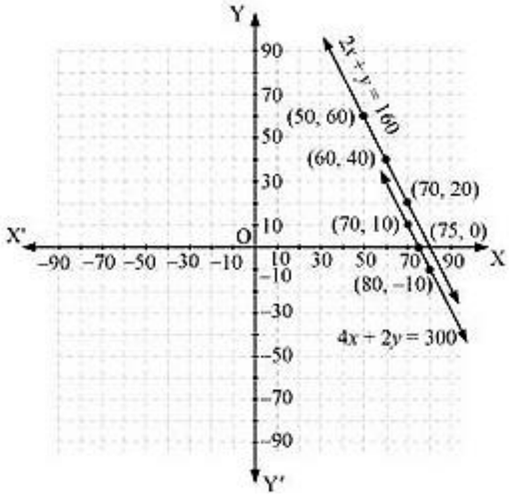
x	50	60	70
y	60	40	20

$4x + 2y = 300 \Rightarrow y = \frac{300 - 4x}{2}$

Three solutions of this equation cab be written in a table as follows:

x	70	80	75
y	10	-10	0

The graphical representation is as follows:



Concept insight: cost of apples and grapes needs to be found so the cost of 1 kg apples and 1kg grapes will be taken as the variables from the given condition of collective cost of apples and grapes, a pair of linear equations in two variables will be obtained. Then In order to represent the obtained equations graphically, take the values of variables as whole numbers only. Since these values are Large so take the suitable scale.

Exercise 3.2

Solve the following systems of equations graphically:

1. $x+y=3$
$2x+5y=12$

Sol:

We have

$x+y=3$

$2x+5y=12$

Now,

$x+y=3$

When $y=0,$ we have

$x=3$

When $x=0,$ we have

$y=3$

Thus, we have the following table giving points on the line $x+y=3$

x	0	3
y	3	0

Now,

$2+5y=12$

$\Rightarrow y=\dfrac{12-2x}{5}$

When $x=1,$ we have

$y=\dfrac{12-1(1)}{5}=2$

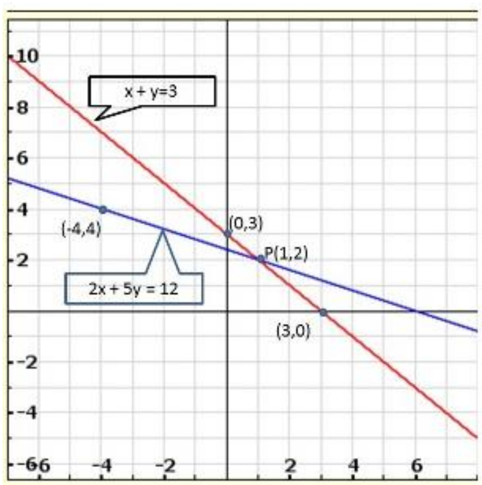
When $x=-4,$ we have

$y=\dfrac{12-1(4)}{5}=4$

Thus, we have the following table giving points on the line $2x+5y=12$

x	1	-4
y	2	4

Graph of the equation $x+y=3$ and $2x+5y=12$:



Clearly, two lines intersect at $P(1,2)$.

Hence, $x=1, y=2$ is the solution of the given system of equations.

2. $x-2y=5$

$2x+3y=10$

Sol:

We have

$x-2y=5$

$2x+3y=10$

Now,

$x-2y=5$

$\Rightarrow \quad x=5+2y$

When $y = 0$, we have

$x = 5 + 2 \times 0 = 5$

When $y = -2$, we have

$x = 5 + 2 \times (-2) = 1$

Thus, we have the following table giving points on the line $x - 2y = 5$

x	5	1
y	0	-2

Now,

$$2x + 3y = 10$$

$$\Rightarrow \quad 2x = 10 - 3y$$

$$\Rightarrow \quad x = \frac{10 - 3y}{2}$$

When $y = 0$, we have

$x = \frac{10}{2} = 5$

When $y = 0$, we have

$x = \frac{10}{2} = 5$

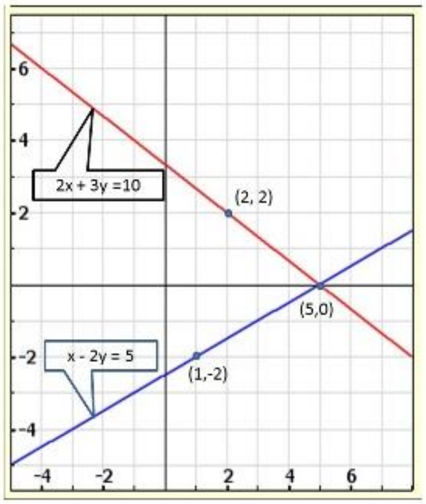
When $y = 2$, we have

$x = \frac{10 - 3 \times 2}{2} = 2$

Thus, we have the following table giving points on the line $2x + 3y = 10$

x	5	2
y	0	2

Graph of the equation $x - 2y = 5$ *and* $2x + 3y = 10$:



Clearly, two lines intersect at (5,0).
Hence, $x=5$, $y=0$ is the solution of the given system of equations.

3. $$3x+y+1=0$$ $$2x-3y+8=0$$

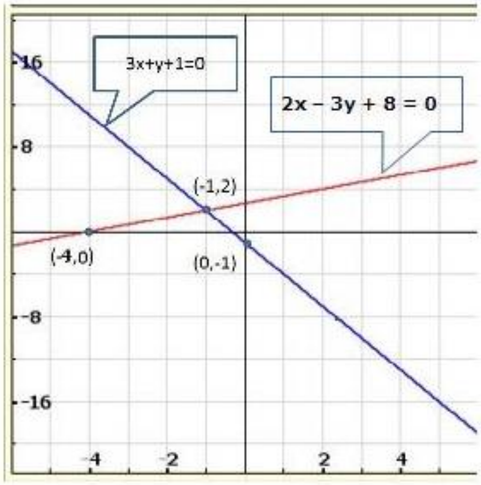
Sol:
We have,
$$3x+y+1=0$$
$$2x-3y+8=0$$
Now,
$$3x+y+1=0$$
$$\Rightarrow \quad y=-1-3x$$
When $x=0$, we have
$y=-1$
When $x=-1$, we have
$y=-1-3\times(-1)=2$
Thus, we have the following table giving points on the line $3x+y+1=0$

x	-1	0
y	2	-1

Now,
$$2x-3y+8=0$$
$$\Rightarrow \quad 2x=3y-8$$
$$\Rightarrow \quad x=\frac{3y-8}{2}$$
When $y=0$, we have
$$x=\frac{3\times 0-8}{2}=-4$$
When $y=2$, we have
$$x=\frac{3\times 2-8}{2}=-1$$
Thus, we have the following table giving points on the line $2x-3y+8=0$

x	-4	-1
y	0	-2

Graph of the equation are:



Clearly, two lines intersect at $(-1,2)$.

Hence, $x=-1, y=2$ is the solution of the given system of equations.

4. $$2x+y-3=0$$
$$2x-3y-7=0$$

Sol:

We have

$$2x+y-3=0$$
$$2x-3y-7=0$$

Now,

$$2x+y-3=0$$
$$\Rightarrow \quad y=3-2x$$

When $x=0$, we have

$y=3$

When $x=1$, we have

$y=1$

Thus, we have the following table giving points on the line $2x+y-3=0$

x	0	1
y	3	1

Now,

$$2x-3y-7=0$$
$$\Rightarrow \quad 3y=2x-7$$
$$\Rightarrow \quad y=\frac{2\times5-7}{3}=1$$

When $x=5$, we have

$$y = \frac{2\times 5-7}{3} = 1$$

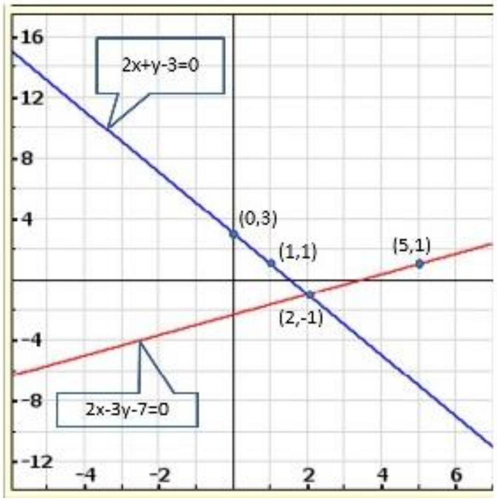
When $x = 2$, we have

$$y = \frac{2\times 2-7}{3} = -1$$

Thus, we have the following table giving points on the line $2x-3y-7=0$

x	2	5
y	-1	1

Graph of the given equation are



Clearly, two lines intersect at $(2,-1)$.

Hence, $x=2, y=-1$ is the solution of the given system of equations.

5. $$x+y=6$$
$$x-y=2$$

Sol:

We have.

$$x+y=6$$
$$x-y=2$$

Now,

$$x+y=6$$

$\Rightarrow$ $y=6-x$

When $x=2$, we have

$y=4$

When $x=3$, we have

$y = 3$

Thus, we have the following table giving points on the line $x + y = 6$

x	2	3
y	4	3

Now,

$$x - y = 2$$

$$\Rightarrow \quad y = x - 2$$

When $x = 0$, we have

$y = -2$

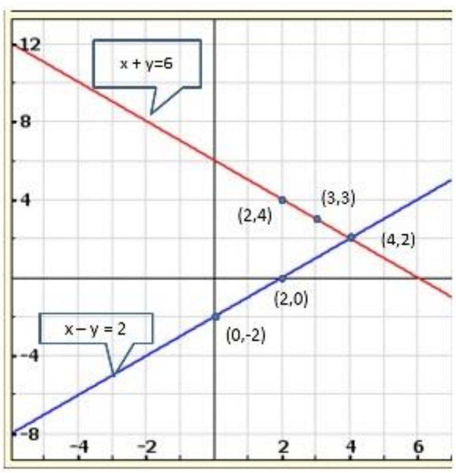
When $x = 2$, we have

$y = 0$

Thus, we have the following table giving points on the line $x - y = 6$

x	0	2
y	-2	0

Graph of the given equation are



Clearly, two lines intersect at (4,2).

Hence, $x = 4$, $y = 2$ is the solution of the given system of equations.

6. $$x - 2y = 6$$
$$3x - 6y = 0$$

Sol:

We have.

$$x - 2y = 6$$

$$3x - 6y = 0$$

Now,

$$x-2y=6$$

$$\Rightarrow \quad x=6+2y$$

When $y=-2$, we have

$x=6+2\times-2=2$

When $y=-3$, we have

$x=6+2\times-3=0$

Thus, we have the following table giving points on the line $x-2y=6$

x	2	0
y	-2	-3

Now,

$$3x-6y=0$$

$$\Rightarrow \quad 3x=6y$$

$$\Rightarrow \quad x=2y$$

When $y=0$, we have

$x=0$

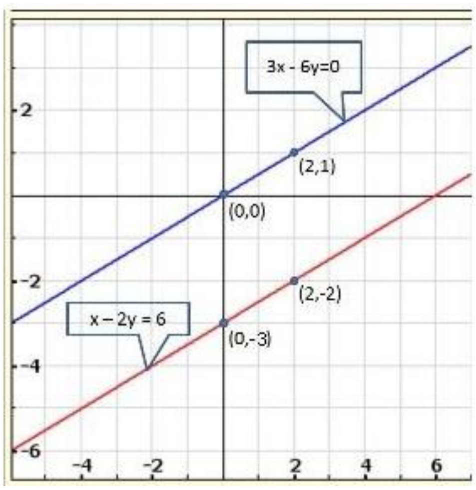
When $y=1$, we have

$x=2$

Thus, we have the following table giving points on the line $3x-6y=0$

x	0	2
y	0	1

Graph of the given equation are



Clearly, two lines are parallel to each other. So, the two lines have no common point

Hence, the given system of equations has no solution.

7. $x + y = 4$
$2x - 3y = 3$

Sol:

We have.

$x + y = 4$
$2x - 3y = 3$

Now,

$x + y = 4$

$\Rightarrow \quad x = 4 - y$

When $y = 0$, we have

$x = 4$

When $y = 2$, we have

$x = 2$

Thus, we have the following table giving points on the line $x + y = 4$

x	4	2
y	0	2

Now,

$2x - 3y = 3$

$\Rightarrow \quad 2x = 3y + 3$

$\Rightarrow \quad x = \frac{3y+3}{2}$

When $y = 1$, we have

$x = 3$

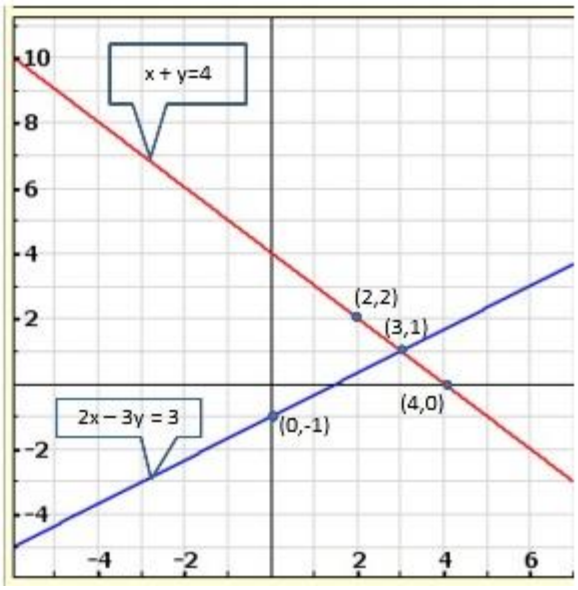
When $y = -1$, we have

$x = 0$

Thus, we have the following table giving points on the line $2x - 3y = 3$

x	3	0
y	1	-1

Graph of the given equation are



Clearly, two lines intersect at (3, 1).
Hence, $x=3,\ y=1$ is the solution of the given system of equations.

8. $$2x+3y=4$$
$$x-y+3=0$$

Sol:
We have.
$$2x+3y=4$$
$$x-y+3=0$$
Now,
$$2x+3y=4$$
$$\Rightarrow \quad 2x=4-3y$$
$$\Rightarrow \quad x=\frac{4-3y}{2}$$
When $y=0,$ we have
$$x=\frac{4-3\times2}{2}=-1$$
When $y=2,$ we have
$$x=\frac{4-3\times2}{2}=-1$$
Thus, we have the following table giving points on the line $2x+3y=4$

x	-1	2
y	2	0

Now,

$$x - y + 3 = 0$$

$$\Rightarrow \quad x = y - 3$$

When $y = 3$, we have

$x = 0$

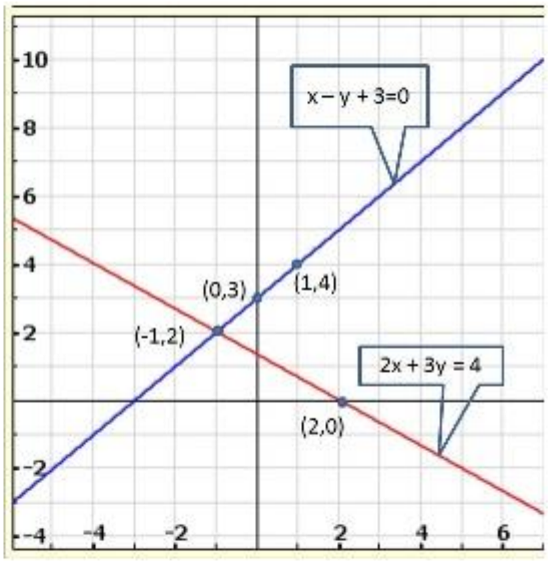
When $y = 4$, we have

$x = 1$

Thus, we have the following table giving points on the line $x - y + 3 = 0$

x	0	1
y	3	4

Graph of the given equation are



Clearly, two lines intersect at $(-1, 2)$.

Hence, $x = -1, y = 2$ is the solution of the given system of equations.

9. $$2x - 3y + 13 = 0$$
$$3x - 2y + 12 = 0$$

Sol:

We have,

$$2x - 3y + 13 = 0$$

$$3x - 2y + 12 = 0$$

Now,

$$2x - 3y + 13 = 0$$

$$\Rightarrow \quad 2x = 3y - 13$$

$$\Rightarrow \quad x = \frac{3y - 13}{2}$$

When $y = 1$, we have

$$x = \frac{3 \times 1 - 13}{2} = -5$$

When $y = 3$, we have

$$x = \frac{3 \times 3 - 13}{2} = -2$$

Thus, we have the following table giving points on the line $2x - 3y + 13 = 0$

x	-5	-2
y	1	3

Now,

$$3x - 2y + 12 = 0$$

$$\Rightarrow \quad 3x = 2y - 12$$

$$\Rightarrow \quad x = \frac{2y - 12}{3}$$

When $y = 0$, we have

$$x = \frac{2 \times 0 - 12}{3} = -14$$

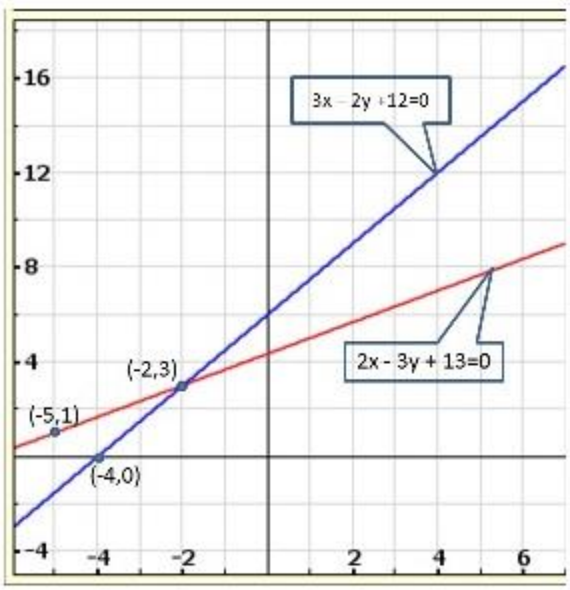
When $y = 3$, we have

$$x = \frac{2 \times 3 - 12}{3} = -2$$

Thus, we have the following table giving points on the line $3y - 2y + 12 = 0$

x	-4	-2
y	0	3

Graph of the given equations are:



Clearly, two lines intersect at (-2, 3)

Hence, $x=-2,\ y=3$ is the solution of the given system of equations.

10. $$2x+3y+5=0$$
$$3x+2y-12=0$$

Sol:

We have,

$$2x+3y+5=0$$
$$3x+2y-12=0$$

Now,

$$2x+3y+5=0$$
$$\Rightarrow \quad 2x=-3y-5$$
$$\Rightarrow \quad x=\frac{-3y-5}{2}$$

When $y=1$, we have

$$x=\frac{-3\times1-5}{2}=-4$$

When $y=-1$, we have

$$x=\frac{-3\times(-1)-5}{2}=-1$$

Thus, we have the following table giving points on the line $2x+3y+5=0$

x	-4	-1
y	1	-1

Now,

$$3x-2y-12=0$$

$$\Rightarrow \quad 3x=2y+12$$

$$\Rightarrow \quad x=\frac{2y+12}{3}$$

When $y=0$, we have

$$x=\frac{2\times 0+12}{3}=4$$

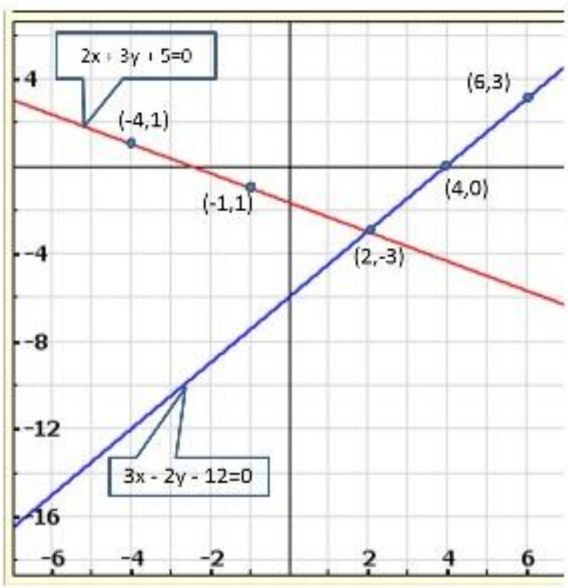
When $y=3$, we have

$$x=\frac{2\times 3+12}{3}=6$$

Thus we have the following table giving points on the line $3x-2y-12=0$

x	4	6
y	0	3

Graph of the given equations are:



Clearly, two lines intersect at $(2,-3)$.

Hence, $x=2,\ y=-3$ is the solution of the given system of equations.

Show graphically that each one of the following systems of equations has infinitely many solutions:

11. $$2x+3y=6$$ $$4x+6y=12$$

Sol:

We have,

$$2x+3y=6$$
$$4x+6y=12$$

Now,

$$2x+3y=6$$
$$\Rightarrow \quad 2x=6-3y$$
$$\Rightarrow \quad x=\frac{6-3y}{2}$$

When $y=0,$ we have

$x=3$

When $y=2,$ we have

$$x=\frac{6-3\times2}{2}=0$$

Thus, we have the following table giving points on the line $2x+3y=6$

x	0	3
y	2	0

Now,

$$4x+6y=12$$
$$\Rightarrow \quad 4x=12-6y$$
$$\Rightarrow \quad x=\frac{12-6y}{4}$$

When $y=0,$ we have

$x=3$

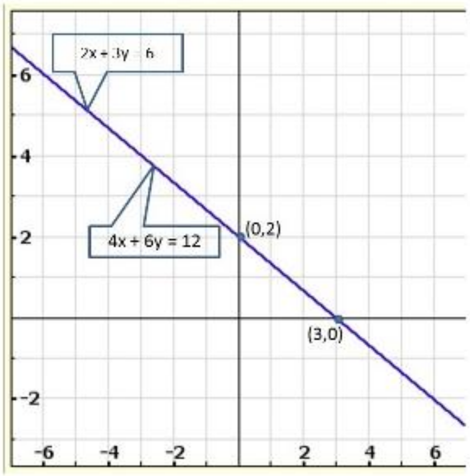
When $y=2,$ we have

$$x=\frac{12-6\times2}{3}=0$$

Thus, we have the following table giving points on the line $4x+6y=12$

x	0	3
y	2	0

Graph of the given equations:



Thus, the graphs of the two equations are coincident.
Hence, the system of equations has infinitely many solutions.

12. $$x-2y=5$$
$$3x-6y=15$$

Sol:
We have,
$$x-2y=5$$
$$3x-6y=15$$
Now,
$$x-2y=5$$
$$\Rightarrow \quad x=2y+5$$
When $y=-1,$ we have
$$x=2(-1)+5=3$$
When $y=0,$ we have
$$x=2\times0+5=5$$
Thus, we have the following table giving points on the line $x-2y=5$

x	3	5
y	1	0

Now,
$$3x-6y=15$$
$$\Rightarrow \quad 3x=15+6y$$
$$\Rightarrow \quad x=\frac{15+6y}{3}$$
When $y=-2,$ we have

$x = \frac{15+6(-2)}{3} = 1$

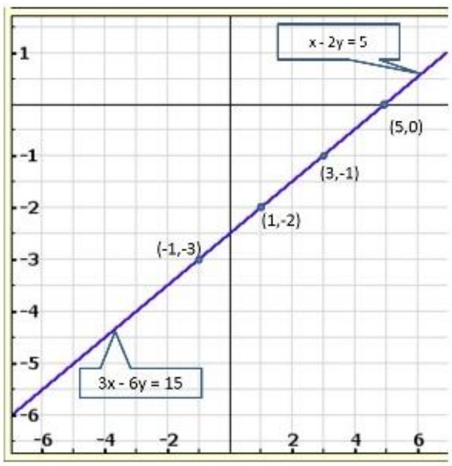
When $y = -3$, we have

$x = \frac{15+6(-3)}{3} = -1$

Thus, we have the following table giving points on the line $3x - 6y = 15$

x	1	-1
y	-2	-3

Graph of the given equations:



13. $3x + y = 8$
$6x + 2y = 16$

Sol:

We have,

$3x + y = 8$

$6x + 2y = 16$

Now,

$3x + y = 8$

$\Rightarrow \quad y = 8 - 3x$

When $x = 2$, we have

$y = 8, -3 \times 2 = 2$

When $x = 3$, we have

$y = 8, -3 \times 3 = -1$

Thus we have the following table giving points on the line $3x + y = 8$

x	2	3
y	2	-1

Now,

$$6x+2y=16$$

$$\Rightarrow \quad 2y=16-6x$$

$$\Rightarrow \quad y=\frac{16-6x}{2}$$

When $x=1,$ we have

$$y=\frac{16-6\times 1}{2}=5$$

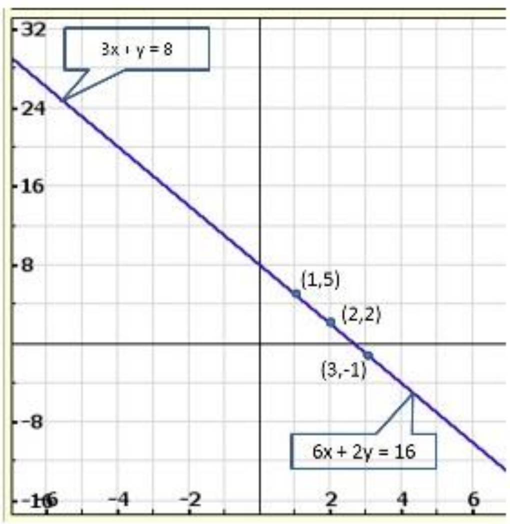
When $x=3,$ we have

$$y=\frac{16-6\times 3}{2}=-1$$

Thus we have the following table giving points on the line $6x+2y=16$

x	1	3
y	5	-1

Graph of the given equations:



Thus, the graphs of the two equations are coincident.

Hence, the system of equations has infinitely many solutions,

14. $$x+2y+11=0$$
$$3x+6y+33=0$$

Sol:

We have,

$$x+2y+11=0$$

$$3x+6y+33=0$$

Now,

$$x-2y+11=0$$
$$\Rightarrow \quad x=2y-11$$

When $y=5$, we have

$x=2\times5-11=-1$

When $x=4$, we have

$x=2\times4-11=-3$

Thus we have the following table giving points on the line $x-2y+11=0$

x	-1	-3
y	5	4

Now,

$$3x-6y+33=0$$
$$\Rightarrow \quad 3x=6y-33$$
$$\Rightarrow \quad x=\frac{6y-33}{3}=1$$

When $y=6$, we have

$$x=\frac{6\times6-33}{3}=-1$$

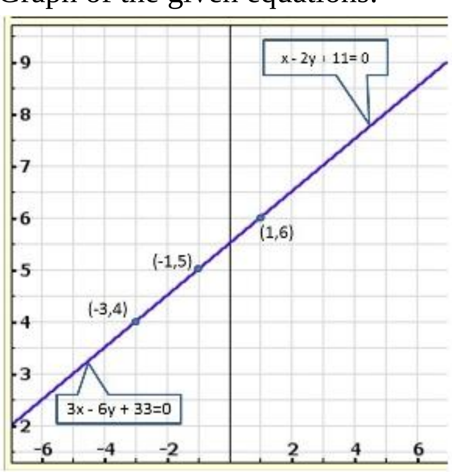
When $y=5$, we have

$$x=\frac{6\times5\times-33}{2}=-1$$

Thus we have the following table giving points on the line $3x+6y+33=0$

x	1	-1
y	6	5

Graph of the given equations:



Thus, the graphs of the two equations are coincident,

Hence, the system of equations has infinitely many solutions,

Show graphically that each one of the following systems of equations is in-consistent (i.e., has no solution)

15. $$3x-5y=20$$
$$6x-10y=-40$$

Sol:

We have,

$$3x-5y=20$$
$$6x-10y=-40$$

Now

$\Rightarrow \quad 3x-5y=20$

$\Rightarrow \quad x=\frac{5y+20}{3}$

When $y=-1$, we have

$$x=\frac{5(-1)+20}{3}=5$$

When $y=-4$, we have

$$x=\frac{5(-4)+20}{3}=0$$

Thus we have the following table giving points on the line $3x-5y=20$

x	5	0
y	-1	-4

Now

$$6x-10y=-40$$

$\Rightarrow \quad 6x=-40+10y$

$\Rightarrow \quad x=\frac{-40+10y}{6}$

When $y=4$, we have

$$x=\frac{-40+10\times 4}{6}=0$$

When $y=1$, we have

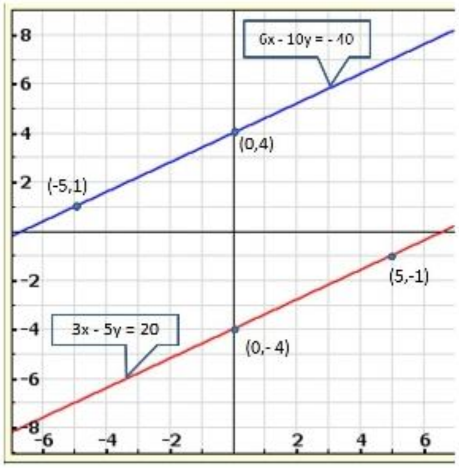
$$x=\frac{-40+10\times 1}{6}=-5$$

Thus we have the following table giving points on the line $6x-10y=-40$

x	0	-5

y	4	1

Graph of the given equations:



Clearly, there is no common point between these two lines
Hence, given system of equations is in-consistent.

16. $$x-2y=6$$
$$3x-6y=0$$

Sol:

We have

$$x-2y=6$$
$$3x-6y=0$$

Now,

$$x-2y=6$$
$$\Rightarrow \quad x=6+2y$$

When $y=0,$ we have

$x=6+2\times0=6$

When $y=-2,$ we have

$x=6+2\times(-2)=2$

Thus, we have the following table giving points on the line $x-2y=6$

x	6	2
y	0	-2

Now,

$$3x-6y=0$$
$\Rightarrow$ $3x=6y$

$\Rightarrow$ $x=\dfrac{6y}{3}$

$\Rightarrow$ $x=2y$

When $y=0$, we have

$x=2\times 0=0$

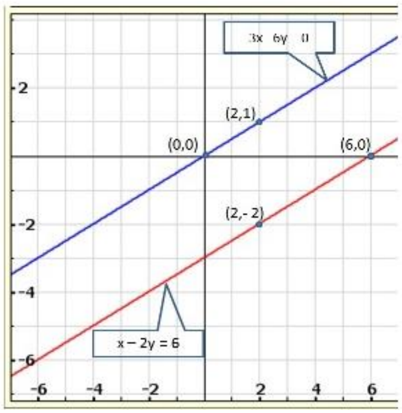
When $y=1$, we have

$x=2\times 1=2$

Thus, we have the following table giving points on the line $3x-6y=0$

x	0	2
y	0	1

Graph of the given equations:



We find the lines represented by equations $x-2y=6$ and $3x-6y=0$ are parallel. So, the two lines have no common point.

Hence, the given system of equations is in-consistent.

17. $2y-x=9$
$6y-3x=21$

Sol:

We have

$$2y-x=9$$
$$6y-3x=21$$

Now,

$$2y-x=9$$
$\Rightarrow$ $2y-9=x$

$\Rightarrow$ $x=2y-9$

When $y=3$, we have

$x = 2\times 3 - 9 = -3$

When $y = 4$, we have

$x = 2\times 4 - 9 = -1$

Thus, we have the following table giving points on the line $2x - x = 9$

x	-3	-1
y	3	4

Now,

$$6y - 3x = 21$$

$$\Rightarrow \quad 6y - 21 = 3x$$

$$\Rightarrow \quad 3x = 6y - 21$$

$$\Rightarrow \quad x = \frac{3(2y-7)}{3}$$

$$\Rightarrow \quad x = 2y - 7$$

When $y = 2$, we have

$x = 2\times 2 - 7 = -3$

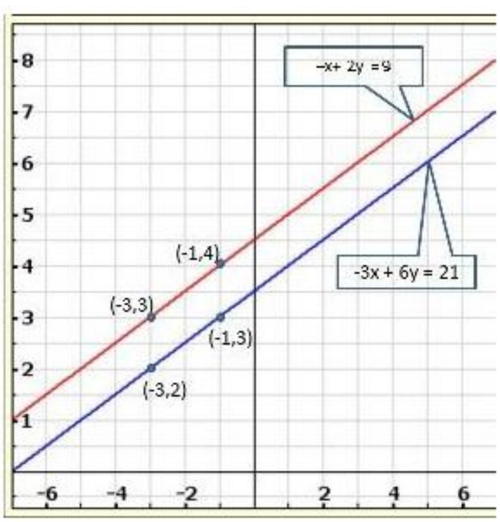
When $y = 3$, we have

$x = 2\times 3 - 7 = -1$

Thus, we have the following table giving points on the line $6y - 3x = 21$.

x	-3	-1
y	2	3

Graph of the given equations:



We find the lines represented by equations $2y - x = 9$ and $6y - 3x = 21$ are parallel. So, the two lines have no common point.

Hence, the given system of equations is in-consistent.

18.
$$3x-4y-1=0$$
$$2x-\frac{8}{3}y+5=0$$

Sol:

We have
$$3x-4y-1=0$$
$$2x-\frac{8}{3}y+5=0$$

Now,
$$3x-4y-1=0$$
$$\Rightarrow \quad 3x=1+4y$$
$$\Rightarrow \quad x=\frac{1+4y}{3}$$

When $y=2$, we have
$$x=\frac{1+4\times 2}{3}=3$$

When $y=-1$, we have
$$x=\frac{1+4\times(-1)}{3}=-1$$

Thus, we have the following table giving points on the line $3x-4y-1=0$.

x	-1	3
y	-1	2

Now,
$$2x-\frac{8}{3}y+5=0$$
$$\Rightarrow \quad \frac{6x-8y+15}{3}=0$$
$$\Rightarrow \quad 6x-8y+15=0$$
$$\Rightarrow \quad 6x=8y-15$$
$$\Rightarrow \quad x=\frac{8y-15}{6}$$

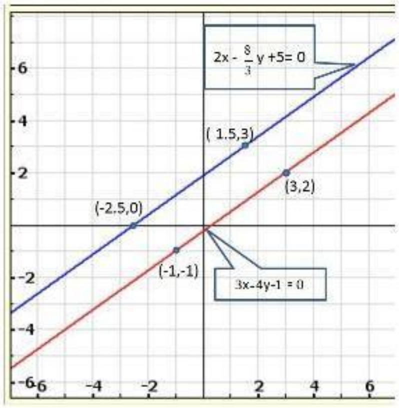
When $y=0$, we have
$$x=\frac{8\times 0-15}{6}=-2.5$$

When $y=3$, we have
$$x=\frac{8\times 3-15}{6}=1.5$$

Thus, we have the following table giving points on the line $2x-\frac{8}{3}y+5=0.$

x	-2.5	1.5
y	0	3

Graph of the given equations:



We find the lines represented by equations $3x-4y-1=0$ *and* $2x-\frac{8}{3}y+5=0$ are parallel. So, the two lines have no common point.
Hence, the given system of equations is in-consistent.

19. Determine graphically the vertices of the triangle, the equations of whose sides are given below:

(i)
$$2y-x=8$$
$$5y-x=14$$
$$y-2x=1$$

(ii)
$$y=x$$
$$y=0$$
$$3x+3y=10$$

Sol:

We have

$$2y-x=8$$
$$5y-x=14$$
$$y-2x=1$$

Now,

$$2y-x=8$$
$$\Rightarrow \quad 2y=8=x$$
$$\Rightarrow \quad x=2y-8$$

When $y=2$, we have

$x=2\times 2-8=-4$

When $y=4$, we have

$x=2\times 4-8=0$

Thus, we have the following table giving points on the line $2y-x=8$.

x	-4	0
y	2	4

Now,

$$5y-x=14$$

$$\Rightarrow \quad 5y-14=x$$

$$\Rightarrow \quad x=5y-14$$

When $y=2$, we have

$x=5\times 2-14=1$

When $y=3$, we have

$x=5\times 3-14=1$

Thus, we have the following table giving points on the line $5y-x=14$.

x	-4	1
y	2	3

We have

$$y-2x=1$$

$$\Rightarrow \quad y-1=2x$$

$$\Rightarrow \quad x=\frac{y-1}{2}$$

When $y=3$, we have

$$x=\frac{3-1}{2}=1$$

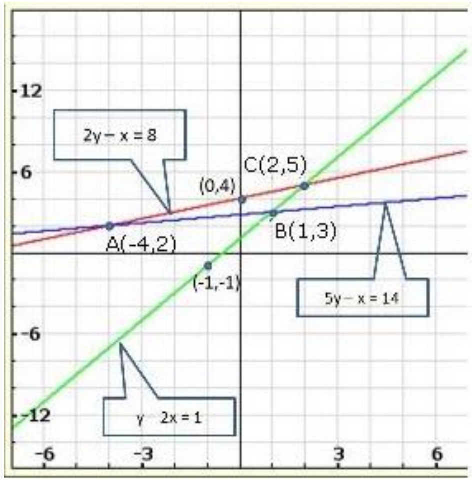
When $y=-1$, we have

$$x=\frac{-1-1}{2}=1$$

Thus, we have the following table giving points on the line $y-2x=1$.

x	-1	1
y	1	3

Graph of the given equations:



From the graph of the lines represented by the given equations, we observe that the lines taken in pairs intersect each other at points $A(-4,2), B(1,3)$ *and* $C(2,5)$

Hence, the vertices of the triangle are $A(-4,2)$, $B(1,3)$ *and* $C(2,5)$.

The given system of equations is

$y = x$

$y = 0$

$3x + 3y = 10$

We have,

$y = x$

When $x = 1,$ we have

$y = 1$

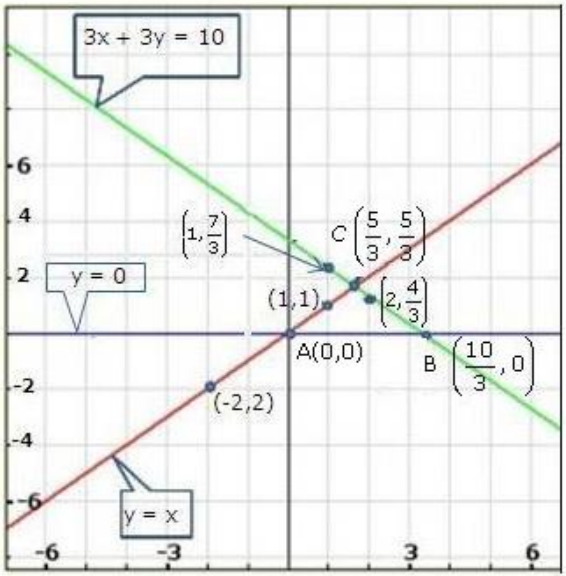
When $x = -2,$ we have

$y = -2$

Thus, we have the following table points on the line $y = x$

x	1	-2
y	7/3	4/3

Graph of the given equation:



From the graph of the lines represented by the given equations, we observe that the lines taken in pairs intersect each other at points $A(0,0), B\left(\frac{10}{3},0\right)$ *and* $C\left(\frac{5}{3},\frac{5}{3}\right)$

Hence, the required vertices of the triangle are $A(0,0), B\left(\frac{10}{0},0\right)$ *and* $C\left(\frac{5}{3},\frac{5}{3}\right)$.

20. Determine, graphically whether the system of equations x − 2y = 2, 4x − 2y = 5 is consistent or in-consistent.

Sol:

We have

$$x-2y=2$$
$$4x-2y=5$$

Now

$$x-2y=2$$
$$\Rightarrow \quad x=2+2y$$

When $y=0,$ we have

$x=2+2\times 0=2$

When $y=-1,$ we have

$x=2+2\times(-1)=0$

Thus, we have the following table giving points on the line $x-2y=2$

x	2	0
y	0	-1

Now,

$$4x-2y=5$$
$$\Rightarrow \quad 4x=5+2y$$
$$\Rightarrow \quad x=\frac{5+2y}{4}$$

When $y=0$, we have

$$x=\frac{5+2\times 0}{4}=\frac{5}{4}$$

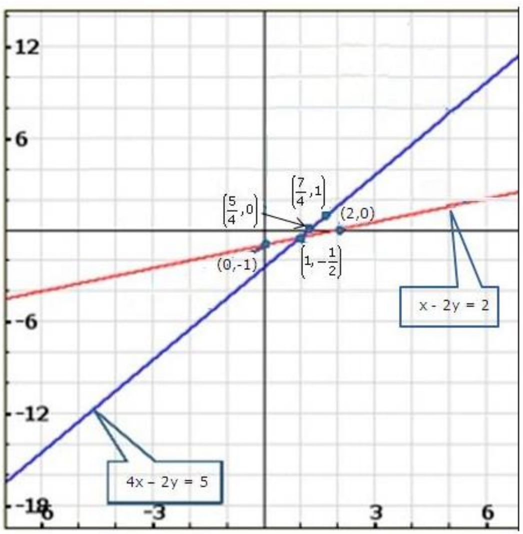
When $y=1$, we have

$$x=\frac{5+2\times 1}{4}=\frac{7}{4}$$

Thus, we have the following table giving points on the line $4x-2y=5$

x	5/4	7/4
y	0	1

Graph of the given equations:



Clearly, the two lines intersect at (i!).

Hence, the system of equations is consistent.

21. Determine, by drawing graphs, whether the following system of linear equations has a unique solution or not:

(i) 2x − 3y = 6, x + y = l (ii) 2y = 4x − 6, 2x = y + 3

Sol:

We have

$$2x-3y=6$$
$$x+y=1$$

Now

$$2x - 3y = 6$$
$$\Rightarrow \quad 2x = 6 + 3y$$

When $y = 0$, we have

$$x = \frac{6+3y}{2}$$

When $y = -2$, we have

$$x = \frac{6+3\times(-2)}{2} = 0$$

Thus, we have the following table giving points on the line $2x - 3y = 6$

x	3	0
y	0	-2

Now,

$$x + y = 1$$
$$\Rightarrow \quad x = 1 - y$$

When $y = 1$, we have

$x = 1 - 1 = 0$

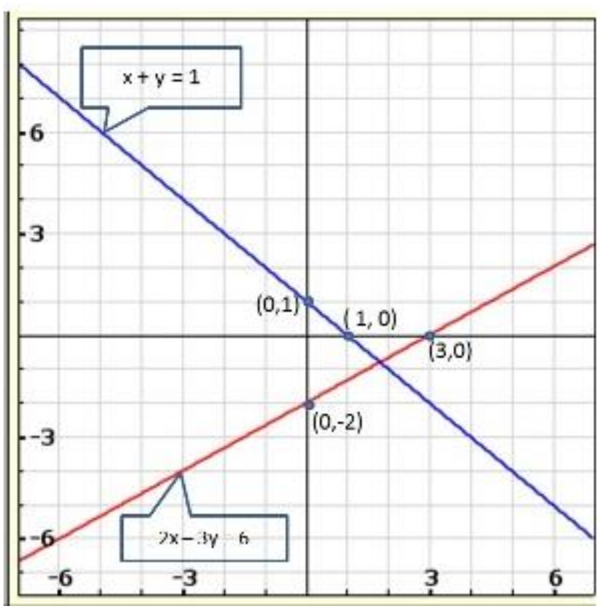
When $y = 0$, we have

$x = 1 - 0 = 1$

Thus, we have the following table giving points on the line $x + y = 1$

x	0	1
y	1	0

Graph of the given equations:



We have,

$$2y = 4x - 6$$
$$2x = y + 3$$

Now,

$$2y = 4x - 6$$
$$\Rightarrow \quad 2y + 6 = 4x$$
$$\Rightarrow \quad 4x = 2y + 6$$
$$\Rightarrow \quad x = \frac{2y+6}{4}$$

When $y = -1$, we have

$$x = \frac{2\times(-1)+6}{4} = 1$$

When $y = 5$, we have

$$x = \frac{2\times5+6}{4} = 4$$

Thus, we have the following table giving points on the line $2y = 4x - 6$

x	1	4
y	-1	5

Now,

$$2x = y + 3$$
$$\Rightarrow \quad x = \frac{y+3}{2}$$

When $y = 1$, we have

$$x = \frac{1+3}{2} = 2$$

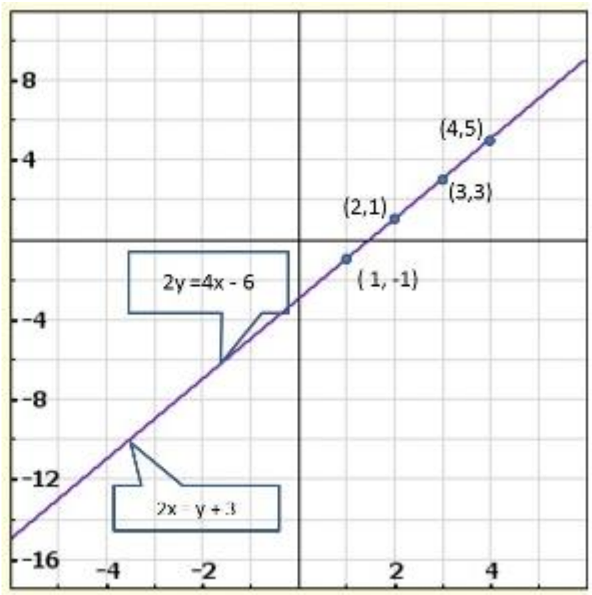
When $y = 3$, we have

$$x = \frac{3+3}{2} = 3$$

Thus, we have the following table giving points on the line $2x = y + 3$

x	2	3
y	1	3

Graph of the given equations:



We find the graphs of the two equations are coincident,
$\therefore$ Hence, the system of equations has infinity many solutions

22. Solve graphically each of the following systems of linear equations. Also find the coordinates of the points where the lines meet axis of y.

(i) $2x-5y+4=0$
$2x+y-8=0$

(ii) $3x+2y=12$
$5x-2y=4$

(iii) $2x+y-11=0$
$x-y-1=0$

(iv) $x+2y-7=0$
$2x-y-4=0$

(v) $3x+y-5=0$
$2x-y-5=0$

(vi) $2x-y-5=0$
$x-y-3=0$

Sol:
We have

$$2x-5y+4=0$$
$$2x+y-8=0$$

Now,

$$2x-5y+4=0$$

$\Rightarrow \quad 2x=5y-4$

$\Rightarrow \quad x=\frac{5y-4}{2}$

When $y=2,$ we have

$$x=\frac{5\times2-4}{2}=3$$

When $y=4,$ we have

$$x=\frac{5\times4-4}{2}=8$$

Thus, we have the following table giving points on the line $2x-5y+4=0$

x	3	8
y	2	4

Now,

$$2x+y-8=0$$

$\Rightarrow \quad 2x=8-y$

$\Rightarrow \quad x=\frac{8-y}{2}$

When $y=4,$ we have

$$x=\frac{8-4}{2}=2$$

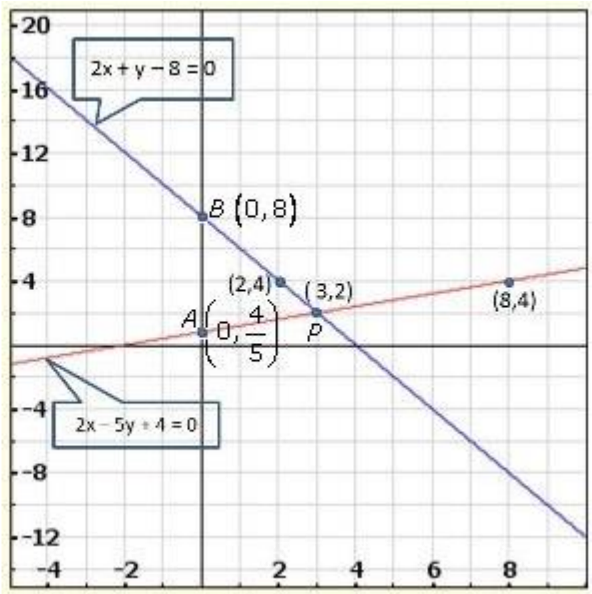
When $y=2,$ we have

$$x=\frac{8-2}{2}=3$$

Thus, we have the following table giving points on the line $2x-5y+4=0$

x	3	8
y	2	4

Graph of the given equations:



Clearly, two intersect at $P(3,2)$.

Hence, $x=2,\ y=3$ is the solution of the given system of equations.

We also observe that the lines represented by $2X-5y+4=0$ *and* $2x+y-8=0$ meet y-axis at $A\left(0,\frac{4}{5}\right)$ *and* $B(0,8)$ respectively.

We have,

$$3x+2y=12$$
$$5x-2y=4$$

Now,

$$3x+2y=12$$
$$\Rightarrow \quad 3x=12-2y$$
$$\Rightarrow \quad x=\frac{12-2y}{3}$$

When $y=3$, we have

$$x=\frac{12-2\times 3}{3}=2$$

When $y=-3$, we have

$$x=\frac{12-2\times(-3)}{3}=6$$

Thus, we have the following table giving points on the line $3x+2y=12$

x	2	6
y	3	-3

Now,

$$5x - 2y = 4$$

$$\Rightarrow \quad 5x = 4 + 2y$$

$$\Rightarrow \quad x = \frac{4+2y}{5}$$

When $y = 3$, we have

$$x = \frac{4+2\times3}{5} = 2$$

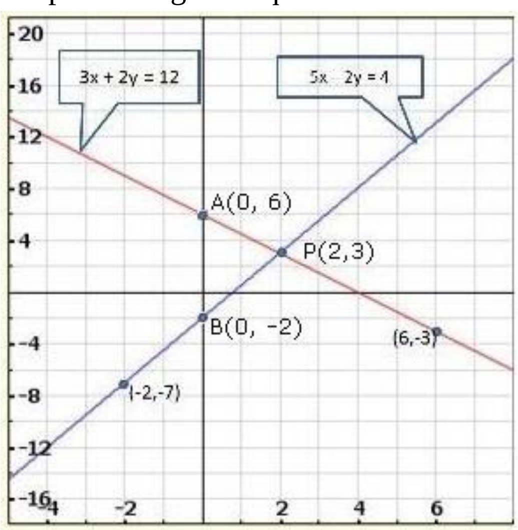
When $y = -7$, we have

$$x = \frac{4+2\times(-7)}{5} = -2$$

Thus, we have the following table giving points on the line $5x - 2y = 4$

x	2	-2
y	3	-7

Graph of the given equation



Clearly, two intersect at $p(2,3)$.

Hence, $x = 2, y = 3$ is the solution of the given system of equations.

We also observe that the lines represented by $3x + 2y = 12$ *and* $5x - 2y = 4$ meet y-axis at $A(0,6)$ *and* $B(0,-2)$ respectively.

We have,

$$2x + y - 11 = 0$$

$$x - y - 1 = 0$$

Now,

$$2x + y - 11 = 0$$

$\Rightarrow \quad y = 11 - 2x$

When $x = 4$, we have

$y = 11 - 2 \times 4 = 3$

When $x = 5$, we have

$y = 11 - 2 \times 5 = 1$

Thus, we have the following table giving points on the line $2x + y - 11 = 0$

x	4	5
y	3	1

Now,

$$x - y - 1 = 0$$

$\Rightarrow \quad x - 1 = y$

$\Rightarrow \quad y = x - 1$

When $x = 2$, we have

$y = 2 - 1 = 1$

When $x = 3$, we have

$y = 3 - 1 = 2$

Thus, we have the following table giving points on the line $x - y - 1 = 0$

x	2	3
y	1	2

Graph of the given equation

We have,

$$2x + y - 11 = 0$$

$$x - y - 1 = 0$$

Now,

$$2x + y - 11 = 0$$

$\Rightarrow \quad y = 11 - 2x$

When $x = 4$, we have

$y = 11 - 2 \times 4 = 3$

When $x = 5$, we have

$y = 11 - 2 \times 5 = 1$

Thus, we have the following table giving points on the line $2x + y - 11 = 0$

x	4	5
y	3	1

Now,

$$x-y-1=0$$
$$\Rightarrow \quad x-1=y$$
$$\Rightarrow \quad y=x-1$$

When $x=2$, we have

$y=2-1=1$

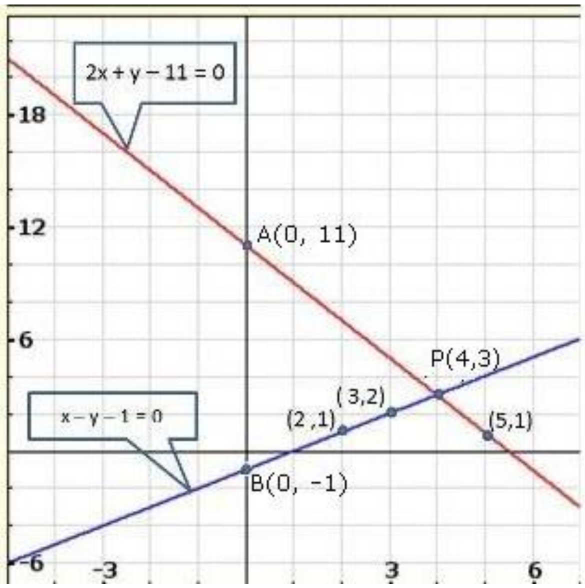
When $x=3$, we have

$y=3-1=2$

Thus, we have the following table giving points on the line $x-y-1=0$

x	2	3
y	1	2

Graph of the given equations:



Clearly, two intersect at $P(4,3)$.

Hence, $x=4,\ y=3$ is the solution of the given system of equations.

We also observe that the lines represented by $2x+y-11=0$ *and* $x-y-1=0$ meet y-axis at, $A(0,11)$ *and* $B(0,-1)$ respectively.

We have, $x+2y-7=0$

Now,

$2x-y-4=0$

$x+2y-7=0$

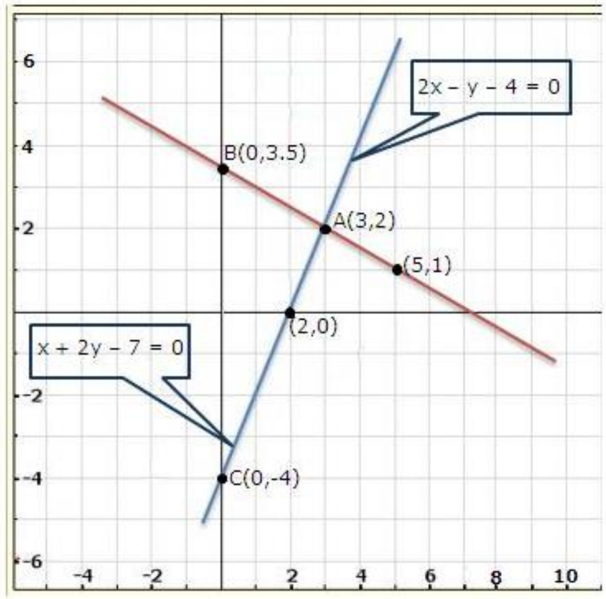
$x=7-2y$

When $\begin{matrix} y=1, x=5 \\ y=2, x=3 \end{matrix}$

x	5	3
y	1	2

Also, $2x-y-4=0$
$y=2x-4$

x	2	0
y	0	-4



From the graph, the solution is $A(3,2)$.

Also, the coordinates of the points where the lines meet the y-axis are $B(0,3.5)$ *and* $C(0,-4)$.

We have

$$3x+y-5=0$$
$$2x-y-5=0$$

Now,

$$3x+y-5=0$$
$$\Rightarrow \quad y=5-3x$$

When $x=1$, we have

$y=5,-3\times1=2$

When $x=2$, we have

$y=5,-3\times2=-1$

Thus, we have the following table giving points on the line $3x+y-5=0$

x	1	2

y	2	-1

Now,

$$2x - y - 5 = 0$$

$\Rightarrow \quad 2x - 5 = y$

$\Rightarrow \quad y = 2x - 5$

When $x = 0$, we have

$y = -5$

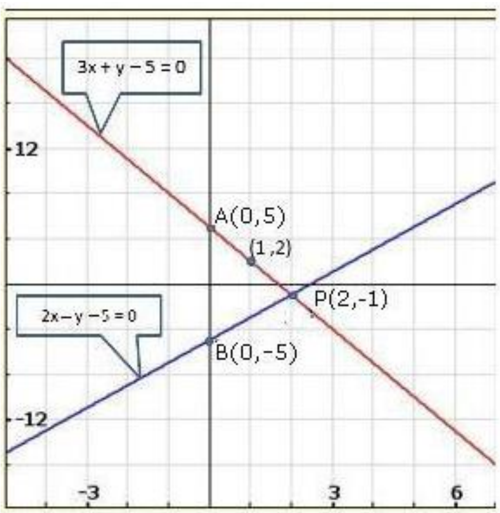
When $x = 2$, we have

$y = 2 \times 2 - 5 = -1$

Thus, we have the following table giving points on the line $2x - y - 5 = 0$

x	0	2
y	-5	-1

Graph of the given equations:



Clearly, two intersect at $P(2, -1)$.

Hence, $x = 2,\ y = -1$ is the solution of the given system of equations.

We also observe that the lines represented by $3x + y - 5 = 0$ *and* $2x - y - 5 = 0$ meet y-axis at $A(0,5)$ *and* $8(0,-5)$ respectively.

We have,

$$2x - y - 5 = 0$$

$$x - y - 3 = 0$$

Now,

$$2x - y - 5 = 0$$

$\Rightarrow \quad 2x - 5 = y$

$\Rightarrow \quad y = 2x - 5$

When $x = 1$, we have

$y = 2 \times 1 - 5 = -3$

When $x = 2$, we have

$y = 2 \times 2 - 5 = -1$

Thus, we have the following table giving points on the line $2x - y - 5 = 0$

x	1	2
y	-3	-1

Now,

$$x - y - 3 = 0$$

$$\Rightarrow \quad x - 3 = y$$

$$\Rightarrow \quad y = x - 3$$

When $x = 3$, we have

$y = 3 - 3 = 0$

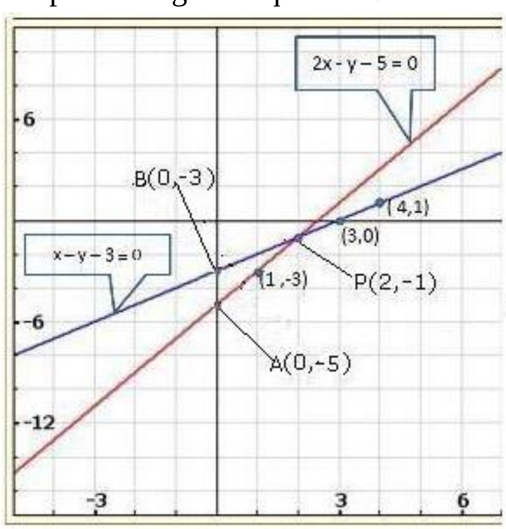
When $x = 4$, we have

$y = 4 - 3 = 1$

Thus, we have the following table giving points on the line $x - y - 3 = 0$

x	3	4
y	0	1

Graph of the given equations:



Clearly, two intersect at $P(2, -1)$.

Hence, $x = 2$, $y = -1$ is the solution of the given system of equations?

We also observe that the lines represented by $2x - y - 5 = 0$ *and* $x - y - 3 = 0$ meet y-axis at $A(0, -5)$ *and* $8(0, -3)$ respectively.

23. Determine graphically the coordinates of the vertices of a triangle, the equations of whose sides are:

(i) $y = x$
$y = 2x$
$y + x = 6$

(ii) $y = x$
$3y = x$
$x + y = 8$

Sol:

The system of the given equations is,

$y = x$

$y = 2x$

$y + x = 6$

Now,

$y = x$

When $x = 0$, we have

$y = 0$

When $x = -1$, we have

$y = -1$

Thus, we have the following table:

x	0	-1
y	0	-2

We have

$y = 2x$

When $x = 0$, we have

$y = 2 \times 0 = 0$

When $x = -1$, we have

$y = 2(-1) = -2$

Thus, we have the following table:

x	0	-1
y	0	-2

We have

$y + x = 6$

$\Rightarrow \quad y = 6 - x$

When $x = 2$, we have

$y = 6 - 2 = 4$

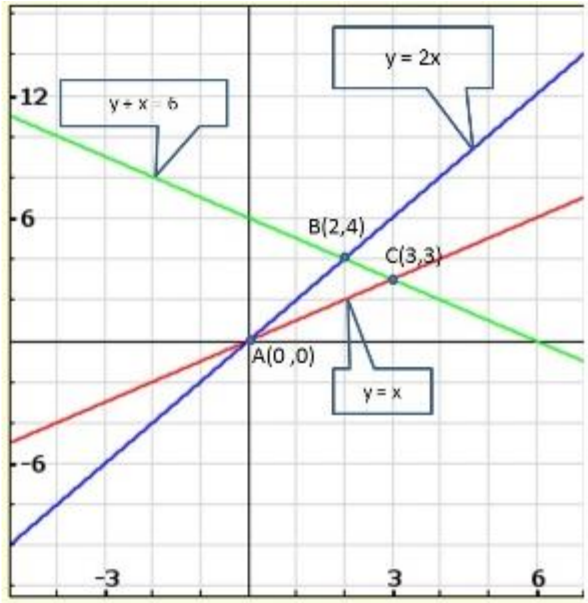
When $x=4$, we have

$y=6-4=2$

Thus, we have the following table:

x	2	4
y	4	2

Graph of the given system of equations:



From the graph of the three equations, we find that the three lines taken in pairs intersect each other at points $A(0,0), B(2,4)$ *and* $C(3,3)$.

Hence, the vertices of the required triangle are $(0,0),(2,4)$ *and* $(3,3)$.

The system of the given equations is,

$y=x$

$3y=x$

$x+y=8$

Now,

$y=x$

$\Rightarrow \quad x=y$

When $y=0$, we have

$x=0$

When $y=-3$, we have

$x=-3$

Thus, we have the following table.

x	0	-3
y	0	-3

We have

$$3y = x$$

$$\Rightarrow \quad x = 3y$$

When $y = 0$, we have

$x = 3 \times 0 = 0$

When $y = -1$, we have

$y = 3 \times (-1) = -3$

Thus, we have the following table:

x	0	-3
y	0	-1

We have

$$x + y = 8$$

$$\Rightarrow \quad x = 8 - y$$

When $y = 4$, we have

$x = 8 - 4 = 4$

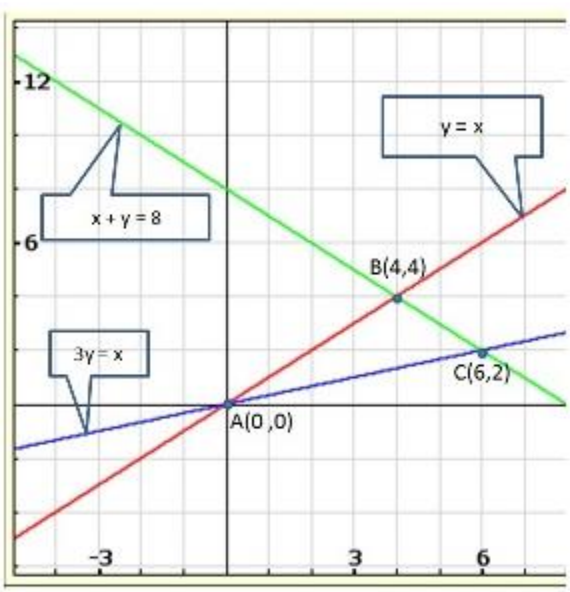
When $y = 5$, we have

$x = 8 - 5 = 3$

Thus, we have the following table:

x	4	5
y	4	3

Graph of the given system of equations:



From the graph of the three equations, we find that the three lines taken in pairs intersect each other at points $A(0,0)$, $B(4,4)$ *and* $C(6,2)$.

Hence, the vertices of the required triangle are $(0,0),(44)$ *and* $(6,2)$.

24. Solve the following system of linear equations graphically and shade the region between the two lines and x-axis:

(i) $2x+3y=12$
$x-y=1$

(ii) $3x+2y-4=0$
$2x-3y-7=0$

(iii) $3x+2y-11=0$
$2x-3y+10=0$

Sol:

The system of given equations is

$$2x+3y=12$$
$$x-y=1$$

Now,

$$2x+3y=12$$

$\Rightarrow$ $2x=12-3y$

$\Rightarrow$ $x=\frac{12-3\times 2}{2}=3$

When $y=2,$ we have

$$x=\frac{12-3\times 2}{2}=3$$

When $y=4,$ we have

$$x=\frac{12-3\times 4}{2}=0$$

Thus, we have the following table:

x	0	3
y	4	2

We have,

$$x-y=1$$

$\Rightarrow$ $x=1+y$

When $y=0,$ we have

$x=1$

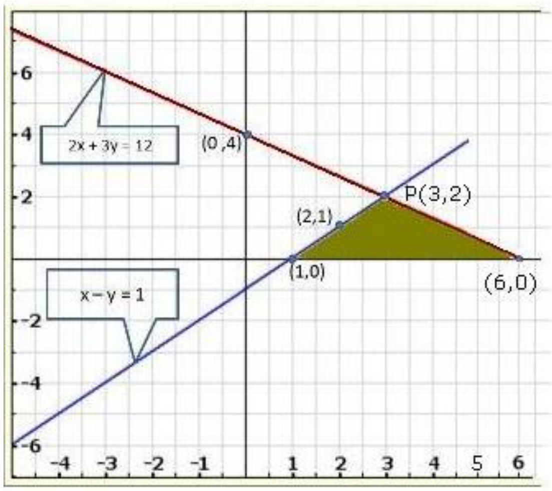
When $y=1,$ we have

$x=1+1=2$

Thus, we have the following table:

x	1	2
y	0	1

Graph of the given system of equations:



Clearly, the two lines intersect at $P(3,2)$.

Hence, $x=3,\ \ y=2$ is the solution of the given system of equations. The system of the given equations is,

$$3x+2y-4=0$$
$$2x-3y-7=0$$

Now,

$$3x+2y-4=0$$
$$\Rightarrow \quad 3x=4-2y$$
$$\Rightarrow \quad x=\frac{4-2y}{3}$$

When $y=5$, we have

$$x=\frac{4-2\times 5}{3}=-2$$

When $y=8$, we have

$$x=\frac{4-2\times 8}{3}=-4$$

Thus, we have the following table:

x	-2	-4
y	5	8

We have,

$$2x-3y-7=0$$
$$\Rightarrow \quad 2x=3y+7$$
$$\Rightarrow \quad x=\frac{3y+7}{2}$$

When $y=1$, we have

$$x=\frac{3\times1+7}{2}=5$$

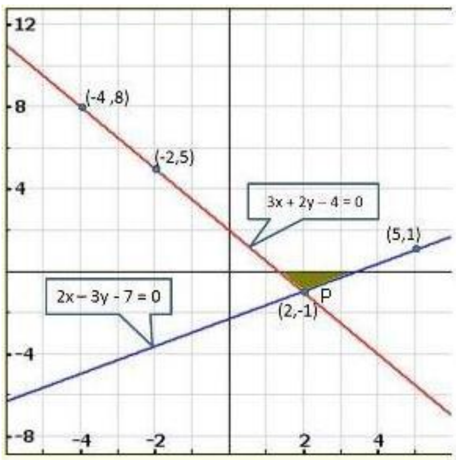
When $y=-1$, we have

$$x=\frac{3\times(-1)+7}{2}=2$$

Thus, we have the following table:

x	5	2
y	1	-1

Graph of the given system of equations:



Clearly, the two lines intersect at $P(2,-1)$.

Hence, $x=2,\ y=-1$ is the solution of the given system of equations.

The system of the given equations is,

$$3x+2y-11=0$$
$$2x-3y+10=0$$

Now,

$$3x+2y-11=0$$
$$\Rightarrow \quad 3x=11-2y$$
$$\Rightarrow \quad x=\frac{11-2y}{3}$$

When $y=1$, we have

$$x = \frac{11-2\times1}{3} = 3$$

When $y = 4$, we have

$$x = \frac{11-2\times4}{3} = 1$$

Thus, we have the following table:

x	3	1
y	1	4

We have,

$$2x - 3y + 10 = 0$$

$$\Rightarrow \quad 2x = 3y - 10$$

$$\Rightarrow \quad x = \frac{3y-10}{2}$$

When $y = 0$, we have

$$x = \frac{3\times0-10}{2} = -5$$

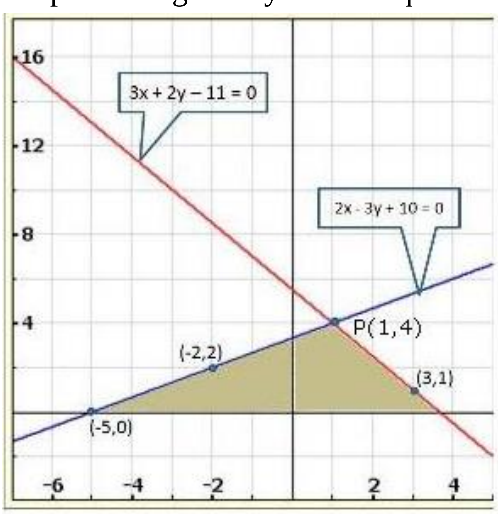
When $y = 2$, we have

$$x = \frac{3\times2-10}{2} = -2$$

Thus, we have the following table:

x	-5	-2
y	0	2

Graph of the given system of equations:



Clearly, the two lines intersect at $P(1,4)$.

Hence, $x = 1$, $y = 4$ is the solution of the given system of equations

25. Draw the graphs of the following equations on the same graph paper:

$$2x+3y=12$$
$$x-y=1$$

Sol:

The system of the given equations is

$$2x+3y=12$$
$$x-y=1$$

Now,

$$2x+3y=12$$
$$\Rightarrow \quad 2x=12-3y$$
$$\Rightarrow \quad x=\frac{12-3y}{2}$$

When $y=0$, we have

$$x=\frac{12-3\times 0}{2}=6$$

When $y=2$, we have

$$x=\frac{12-3\times 2}{2}=3$$

Thus, we have the following table:

x	6	3
y	0	2

We have

$$x-y=1$$
$$\Rightarrow \quad x=1+y$$

When $y=0$, we have

$x=1$

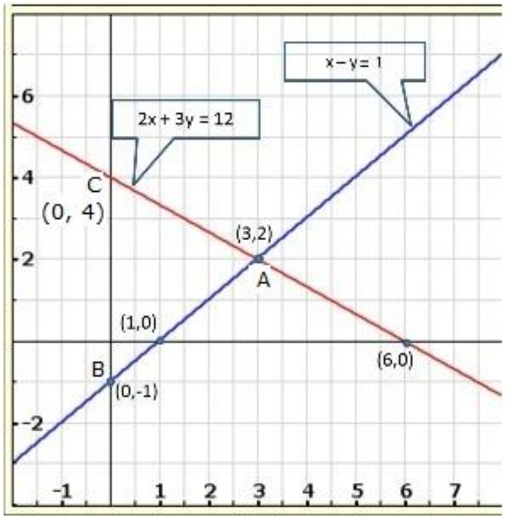
When $y=-1$, we have

$x=1-1=0$

Thus, we have the following table:

x	1	0
y	0	-1

Graph of the given system of equations:



Clearly, the two lines intersect at $A(3,2)$.

We also observe that the lines represented by the equations $2x+3y=12$ *and* $x-y=-1$ meet y-axis at $B(0,-1)$ *and* $C(0,4)$.

Hence, the vertices of the required triangle are $A(3,2)$, $B(0,-1)$ *and* $C(0,4)$.

26. Draw the graphs of x − y + 1 = 0 and 3x + 2y − 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and x- axis and shade the triangular area. Calculate the area bounded by these lines and x-axis.

Sol:

The given system of equations is

$$x-y+1=0$$

$$3x+2y-12=0$$

Now,

$$x-y+1=0$$

$$\Rightarrow \quad x=y-1$$

When $y=3$, we have

$x=3-1=2$

When $y=-1$, we have

$x=-1-1=-2$

Thus, we have the following table:

x	2	-2
y	3	-1

We have

$$3x+2y-12=0$$
$$\Rightarrow \quad 3x=12-2y$$
$$\Rightarrow \quad x=\frac{12-2y}{3}$$

When $y=6$, we have

$$x=\frac{12-2\times6}{3}=0$$

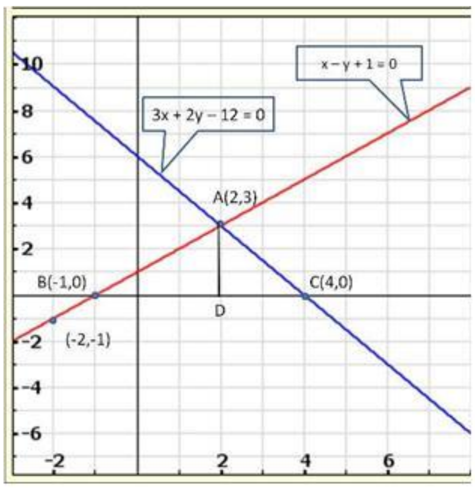
When $y=3$, we have

$$x=\frac{12-2\times3}{3}=2$$

Thus, we have the following table:

x	0	2
y	6	3

Graph of the given system of equations:



Clearly, the two lines intersect at $A(2,3)$.

We also observe that the lines represented by the equations $x-y+1=0$ *and* $3x+2y-12=0$ meet x-axis at $B(-1,0)$ *and* $C(4,0)$ respectively.

Thus, $x=2,\ y=3$ is the solution of the given system of equations.

Draw AD perpendicular from A on x-axis.

Clearly, we have

$$AD = y-\text{coordinate of point } A(2,3)$$
$$\Rightarrow \quad AD=3 \text{ and, } BC=4-(-1)=4+1=5$$

27. Solve graphically the system of linear equations:

$$4x-3y+4=0$$
$$4x+3y-20=0$$

Find the area bounded by these lines and x-axis.

Sol:

The given system of equation is

$$4x-3y+4=0$$
$$4x+3y-20=0$$

Now,

$$4x-3y+4=0$$
$$\Rightarrow \quad 4x=3y-4$$
$$\Rightarrow \quad x=\frac{3y-4}{4}$$

When $y=0,$ we have

$$x=\frac{3\times 0-4}{4}=-1$$

When $y=4,$ we have

$$x=\frac{3\times 4-4}{4}=2$$

Thus, we have the following table:

x	2	-1
y	4	0

We have

$$4x+3y-20=0$$
$$\Rightarrow \quad 4x=20-3y$$
$$\Rightarrow \quad x=\frac{20-3y}{4}$$

When $y=0,$ we have

$$x=\frac{20-3\times 0}{4}=5$$

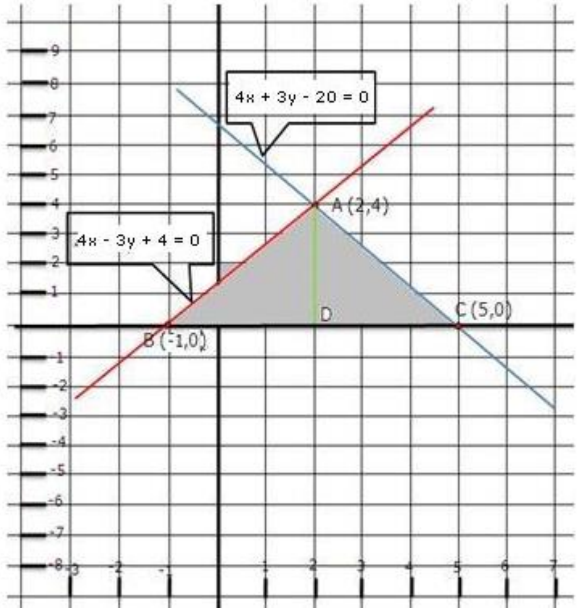
When $y=4,$ we have

$$x=\frac{20-3\times 4}{4}=2$$

Thus, we have the following table:

x	5	2
y	0	4

Graph of the given system of equation:



Clearly, the two lines intersect at $A(2,4)$. Hence $x=2,\ y=4$ is the solution of the given system of equations.

We also observe that the lines represented by the equations

$4x-3y+4=0$ *and* $4x+3y-20=0$ meet x-axis at $B(-1,0)$ *and* $C(5,0)$ respectively.

Thus, $x=2,\ y=4$ is the solution of the given system of equations.

Draw AD perpendicular from A on x-axis.

Clearly, we have

$AD = y-$coordinate of point $A(2,4)$

$\Rightarrow \quad AD = 4$ and, $BC = 5-(-1) = 5+1 = 6$

$\therefore$ Area of the shaded region = Area of ΔABC

$\Rightarrow$ Area of the shaded region $= \frac{1}{2}(Base \times Height)$

$= \frac{1}{2} \times (BC \times AD)$

$= \frac{1}{2} \times 6 \times 4$

$= 6 \times 2$

$= 12$ sq. units

$\therefore$ Area of shaded region $= 12$ sq. units

28. Solve the following system of linear equations graphically:

$$3x+y-11=0$$
$$x-y-1=0$$

Shade the region bounded by these lines and y -axis. Also, find the area of the region bounded by these lines and y-axis.

Sol:

The given system of equation is

$$3x+y-11=0$$
$$x-y-1=0$$

Now,

$$3x+y-11=0$$
$$\Rightarrow \quad y=11-3x$$

When $x=0$, we have

$y=11-3\times 0=11$

When $x=3$ we have

$y=11-3\times 3=2$

Thus, we have the following table:

x	0	3
y	11	2

We have

$$x-y-1=0$$
$$\Rightarrow \quad x-1=y$$
$$\Rightarrow \quad y=x-1$$

When $x=0$, we have

$y=0-1=-1$

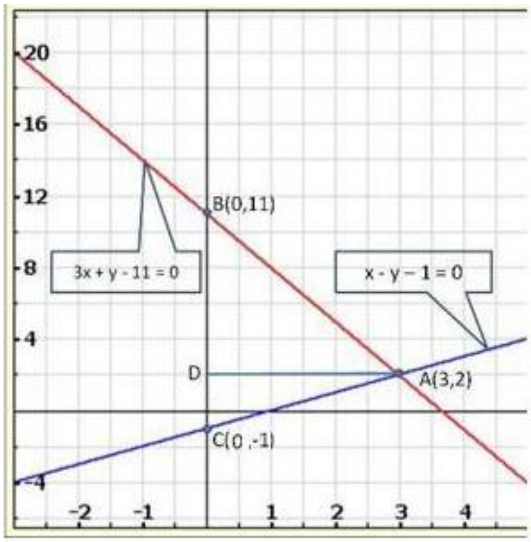
When $x=3$, we have

$y=3-1=2$

Thus, we have the following table:

x	0	3
y	-1	2

Graph of the given system of equations:



Clearly, the two lines intersect at $A(3,2)$. Hence $x=3,\ y=2$ is the solution of the given system of equations.

We so observe that the lines represented by the equations $3x+y-11=0$ *and* $x-y-1=0$ meet y-axis at $B(0,11)$ *and* $C(0,-1)$ respectively.

Thus, $x=3,\ y=2$ is the solution of the given system of equations.

Draw AD perpendicular from A on y-axis.

Clearly, we have

$AD = x-$ coordinate of point $A(3,2)$

$\Rightarrow \quad AD=3$ and, $BC=11-(-1)=11+1=12$

$\therefore$ Area of the shaded region = Area of ΔABC

$\Rightarrow$ Area of the shaded region $=\frac{1}{2}(Base\times Height)$

$=\frac{1}{2}\times(BC\times AD)$

$=\frac{1}{2}\times 12\times 3$

$=6\times 3$

$=18$ sq. units

$\therefore$ Area of the shaded region $=18$ sq. units

29. Solve graphically each of the following systems of linear equations. Also, find the coordinates of the points where the lines meet the axis of x in each system:

(i)
$$2x-y=2$$
$$4x-y=8$$

(ii) $2x - y = 2$
$4x - y = 8$

(iii) $x + 2y = 5$
$2x - 3y = -4$

(iv) $2x + 3y = 8$
$x - 2y = -3$

Sol:

The given system of equation is

$$2x - y = 2$$
$$4x - y = 8$$

Now,

$$2x + y = 2$$
$$\Rightarrow \quad 2x = y + 2$$
$$\Rightarrow \quad x = \frac{y+2}{2}$$

When $y = 0,$ we have

$$x = \frac{0+2}{2} = 1$$

When $y = 2,$ we have

$$x = \frac{2+2}{2} = 2$$

Thus, we have the following table:

x	1	2
y	0	2

We have,

$$4x - y = 8$$
$$\Rightarrow \quad 4x = y + 8$$
$$\Rightarrow \quad x = \frac{y+8}{4}$$

When $y = 0,$ we have

$$x = \frac{0+8}{4} = 2$$

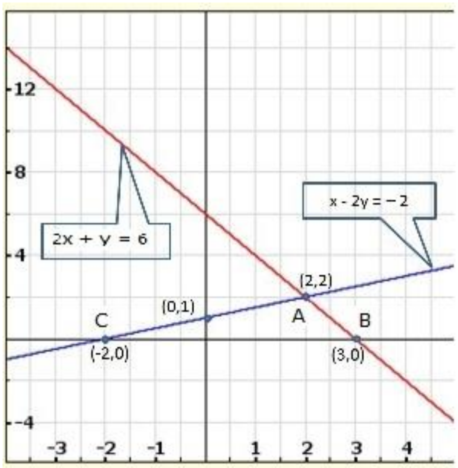
When $y = -4$ we have

$$x = \frac{-4+8}{4} = 1$$

Thus, we have the following table:

x	2	1
y	0	-4

Graph of the given system of equations:



Clearly, the two lines intersect at $A(2,2)$. Hence $x=2,\ y=2$ is the solution of the given system of equations.

We so observe that the lines represented by the equations $2x+y=6$ *and* $x-2y=-2$ meet x-axis at $B(3,0)$ *and* $C(-2,0)$ respectively.

The system of the given equations is

$$2x+y=6$$
$$x-2y=-2$$

Now,

$$2x+y=6$$

$$\Rightarrow \quad x=\frac{6-y}{2}$$

When $y=0$, we have

$$x=\frac{6-0}{2}=3$$

When $y=2$, we have

$$x=\frac{6-2}{2}=2$$

Thus, we have the following table:

x	3	2
y	0	2

We have,

$$x - 2y = -2$$

$\Rightarrow \quad y - 2y - 2$

When $y = 0$, we have

$x = 2\times 0 - 2 = -2$

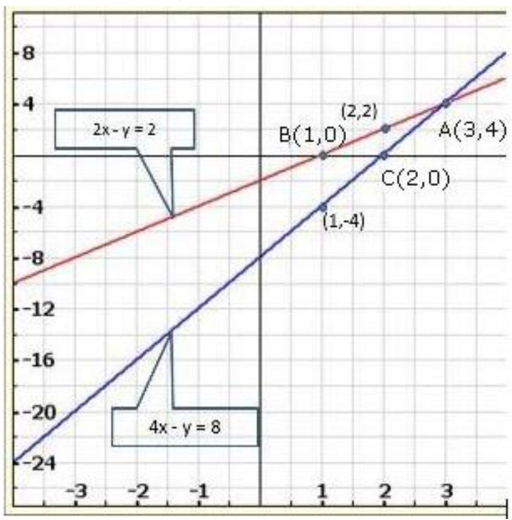
When $y = 1$, we have

$x = 2\times 1 - 2 = 0$

Thus, we have the following table:

x	-2	0
y	0	1

Graph of the given system of equations:



Clearly the two lines intersect at $A(3,4)$. Hence $x = 3,\ y = 4$ is the solution of the given system of equations.

We so observe that the lines represented by the equations $2x - y = 2$ *and* $4x - y = 8$ meet x-axis at $B(1,0)$ *and* $C(2,0)$ respectively

The system of the given equations is

$$x + 2y = 5$$
$$2x - 3y = -4$$

Now,

$$x + 2y = 5$$

$\Rightarrow \quad x = 5 - 2y$

When $y = 2$, we have

$x = 5 - 2\times 2 = 1$

When $y = 3$, we have

$x = 5 - 2\times 3 = -1$

Thus, we have the following table:

x	1	-1
y	2	3

We have,

$$2x - 3y = -4$$

$$\Rightarrow \quad 2x = 3y - 4$$

$$\Rightarrow \quad x = \frac{3y-4}{2}$$

When $y = 0$, we have

$$x = \frac{3\times 0 - 4}{2} = -2$$

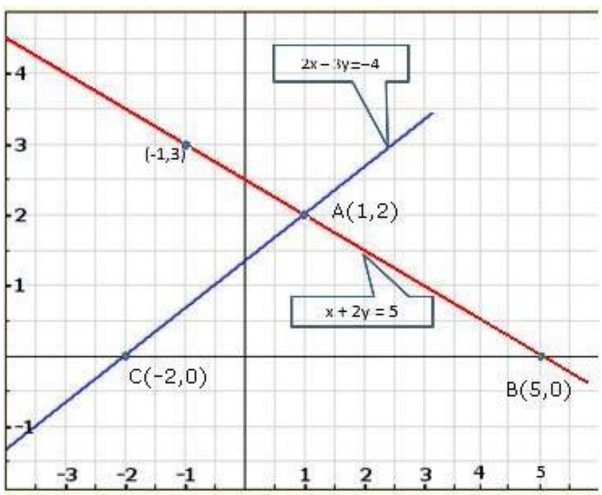
When $y = 2$, we have

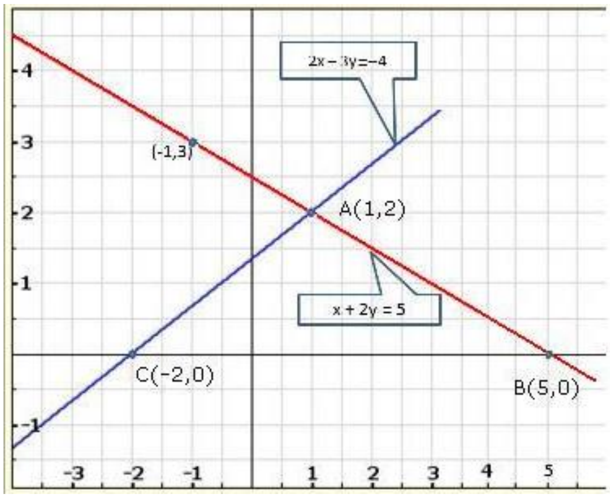
$$x = \frac{3\times 2 - 4}{2} = 1$$

Thus, we have the following table:

x	-2	1
y	0	2

Graph of the given system of equations:





The given system of equation is

$$2x+3y=8$$
$$x-2y=-3$$

Now,

$$2x+3y=8$$
$$\Rightarrow \quad 2x=8-3y$$
$$\Rightarrow \quad x=\frac{8-3y}{2}$$

When $y=2,$ we have

$$x=\frac{8-3\times 4}{2}=1$$

When $y=4,$ we have

$$x=\frac{8-3\times 4}{2}=-2$$

Thus, we have the following table:

x	1	-2
y	2	4

We have,

$$x-2y=-3$$
$$\Rightarrow \quad x=2y-3$$

When $y=0,$ we have

$$x=2\times 0-3=-3$$

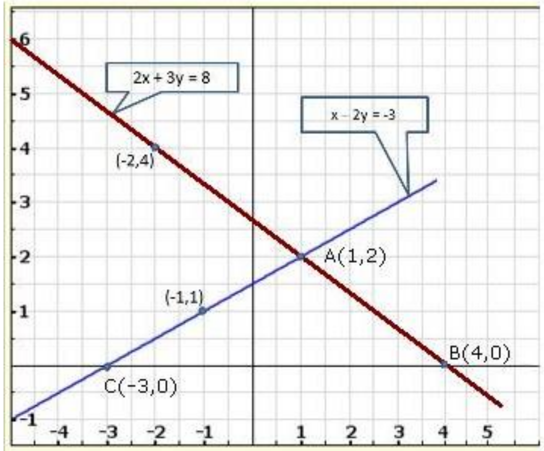
When $y=1,$ we have

$$x=2\times 1-3=-1$$

Thus, we have the following table:

x	-3	-1
y	0	1

Graph of the given system of equations:



Clearly, the two lines intersect at $A(1,2)$. Hence $x=1,\ y=2$ is the solution of the given system of equations.

We also observe that the lines represented by the equations $2x+3y=8$ *and* $x-2y=-3$ meet x-axis at $B(4,0)$ *and* $C(-3,0)$ respectively.

30. Draw the graphs of the following equations:

$$2x-3y+6=0$$
$$2x+3y-18=0$$
$$y-2=0$$

Find the vertices of the triangle so obtained. Also, find the area of the triangle.

Sol:

The given system of equation is

$$2x-3y+6=0$$
$$2x+3y-18=0$$
$$y-2=0$$

Now,

$$2x-3y+6=0$$
$$\Rightarrow \quad 2x=3y-6$$
$$\Rightarrow \quad x=\frac{3y-6}{2}$$

When $y=0$, we have

$$x=\frac{3\times0-6}{2}=-3$$

When $y = 2,$ we have

$$x = \frac{3 \times 2 - 6}{2} = 0$$

Thus, we have the following table:

x	-3	0
y	0	2

We have,

$$2x + 3y - 18 = 0$$

$$\Rightarrow \quad 2x = 18 - 3y$$

$$\Rightarrow \quad x = \frac{18 - 3y}{2}$$

When $y = 2,$ we have

$$x = \frac{18 - 3 \times 2}{2} = 6$$

When $y = 6,$ we have

$$x = \frac{18 - 3 \times 6}{2} = 0$$

Thus, we have the following table:

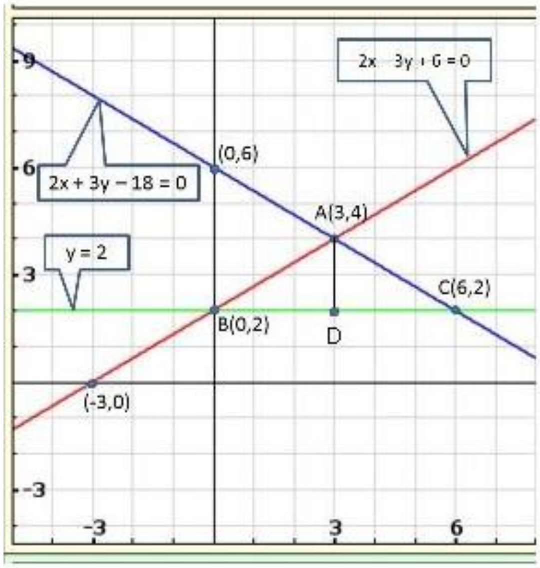
x	6	0
y	2	6

We have

$$y - 2 = 0$$

$$\Rightarrow \quad y = -2$$

Graph of the given system of equations:



From the graph of the three equations, we find that the three lines taken in pairs intersect each other at points $A(3,4),\ B(0,2)\ \textit{and}\ C(6,2)$.

Hence, the vertices of the required triangle are $(3,4),(0,2)\ \textit{and}\ (6,2)$.

From graph, we have

$AD = 4 - 2 = 2$

$BC = 6 - 0 = 6$

Area of $\Delta ABC = \frac{1}{2}(Base \times Height)$

$= \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 6 \times 2$

$= 6\ sq.\ units$

$\therefore$ Area of $\Delta ABC = 6\ sq.units$

31. Solve the following system of equations graphically:

$$2x - 3y + 6 = 0$$
$$2x + 3y - 18 == 0$$

Also, find the area of the region bounded by these two lines and y-axis.

Sol:

The given system of equation is

$$2x - 3y + 6 = 0$$
$$2x + 3y - 18 == 0$$

Now,

$$2x - 3y + 6 = 0$$

$$\Rightarrow \quad 2x + 6 = 3y$$

$$\Rightarrow \quad 3y = 2x + 6$$

$$\Rightarrow \quad y = \frac{2x+6}{3}$$

When $x = 0$, we have

$$y = \frac{2 \times 0 + 6}{3} = 2$$

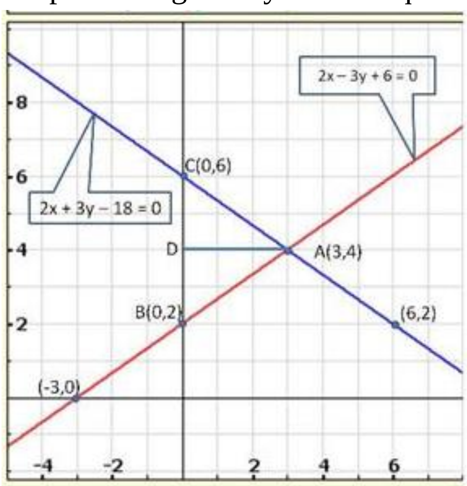
When $x = -3$, we have

$$y = \frac{2 \times (-3) + 6}{3} = 0$$

Thus, we have the following table:

x	0	-3
y	2	6

Graph of the given system of equations:



Clearly, the two lines intersect at $A(3,4)$. Hence, $x = 3,\ y = 4$ is the solution of the given system of equations.

We also observe that the lines represented by the equations

$2x - 3y + 6 = 0$ *and* $2x + 3y - 18 = 0$ meet y-axis at $B(0,2)$ *and* $C(0,6)$ respectively.

Thus, $x = 3,\ y = 4$ is the solution of the given system of equations.

Draw AD perpendicular from A on y-axis.

Clearly, we have,

$$AD = x - \text{coordinate of point } A(3,4)$$

$$\Rightarrow \quad AD = 3 \text{ and, } BC = 6 - 2 = 4$$

Area of the shaded region = Area of ΔABC

Area of the shaded region $= \frac{1}{2}(Base \times Height)$

$= \frac{1}{2}(BC \times AD)$

$= \frac{1}{2} \times 4 \times 3$

$= 2 \times 3$

$= 6 \ sq. \ units$

$\therefore$ Area of the region bounded by these two lines and y-axis is 6 sq. units.

32. Solve the following system of linear equations graphically:

$$4x - 5y - 20 = 0$$
$$3x + 5y - 15 = 0$$

Determine the vertices of the triangle formed by the lines representing the above equation and the y-axis.

Sol:

The given system of equation is

$$4x - 5y - 20 = 0$$
$$3x + 5y - 15 = 0$$

Now,

$$4x - 5y - 20 = 0$$

$\Rightarrow \quad 4x = 5y + 20$

$\Rightarrow \quad x = \frac{5y + 20}{4} = 5$

When $y = 0,$ we have

$x = \frac{5 \times 0 + 20}{4} = 5$

When $y = -4,$ we have

$x = \frac{5 \times (-4) + 20}{4} = 0$

Thus, we have the following table:

x	5	0
y	0	-4

We have,

$$3x + 5y - 15 = 0$$

$\Rightarrow \quad 3x = 15 - 5y$

$\Rightarrow \quad x = \frac{15 - 5y}{3}$

When $y=0$, we have

$$x=\frac{15-5\times 3}{3}=0$$

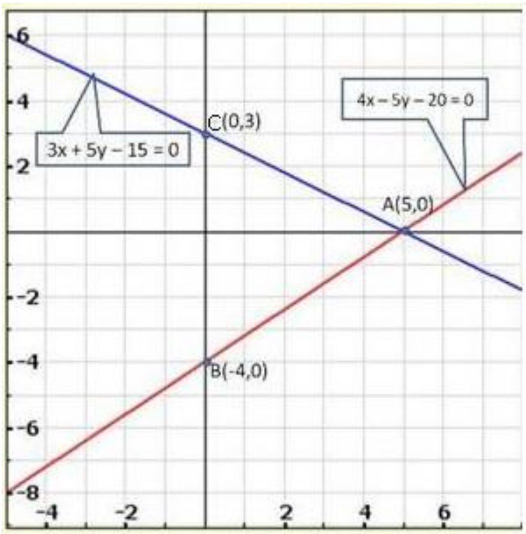
When $y=3$, we have

$$x=\frac{15-5\times 3}{3}=0$$

Thus, we have the following table:

x	5	0
y	0	3

Graph of the given system of equations:



Clearly, the two lines intersect at $4(5,0)$. Hence, $x-5,\ y-0$ is the solution of the given system of equations.

We also find that the two lines represented by the equations

$4x-5y-20=0$ *and* $3x+5y-15=0$ meet y-axis at $B(0,-4)$ *and* $C(0,3)$ respectively,

$\therefore$ The vertices of the required triangle are $(5,0)$, $(0,-4)$ *and* $(0,3)$.

33. Draw the graphs of the equations 5x − y = 5 and 3x − y = 3. Determine the co-ordinates of the vertices of the triangle formed by these lines and y-axis. Calculate the area of the triangle so formed.

Sol:

$5x-y=5 \Rightarrow y=5x-5$

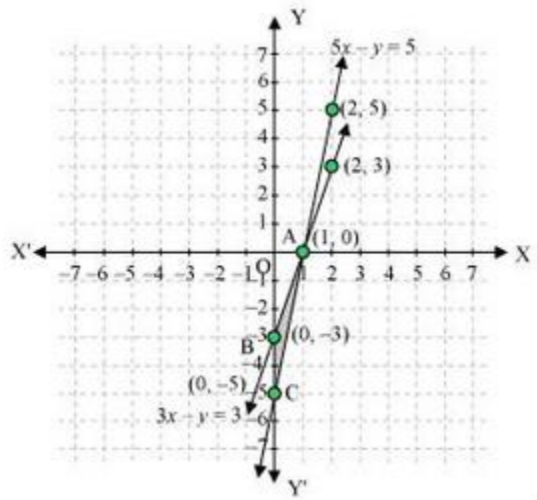
Three solutions of this equation can be written in a table as follows:

x	0	1	2
y	-5	0	5

$3x - y = 3 \Rightarrow y = 3x - 3$

x	0	1	2
y	-3	0	3

The graphical representation of the two lines will be as follows:



It can be observed that the required triangle is ΔABC.

The coordinates of its vertices are $A(1,0)$, $B(0, \ -3)$, $C(0,-5)$.

Concept insight: In order to find the coordinates of the vertices of the triangle so formed. Find the points where the two lines intersects the y-axis and also where the two lines intersect each other. Here, note that the coordinates of the intersection of lines with y-axis is taken and not with x-axis, this is became the question says to find the triangle formed by the two lines and the y-axis.

34. Form the pair of linear equations in the following problems, and find their solution graphically:

(i) 10 students of class X took part in Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost Rs 50, whereas 7 pencils and 5 pens together cost Rs 46. Find the cost of one pencil and a pen.

(iii)Champa went to a 'sale' to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, "The number of skirts is two less than twice the number of pants purchased. Also, the number of skirts is four less than four times the number of pants purchased." Help her friends to find how many pants and skirts Champa bought.

Sol:

(i) Let the number of girls and boys in the class be x and y respectively.

According to the given conditions, we have:

$x+y=10$

$x-y=4$

$x+y=10 \Rightarrow x=10-y$

Three solutions of this equation can be written in a table as follows:

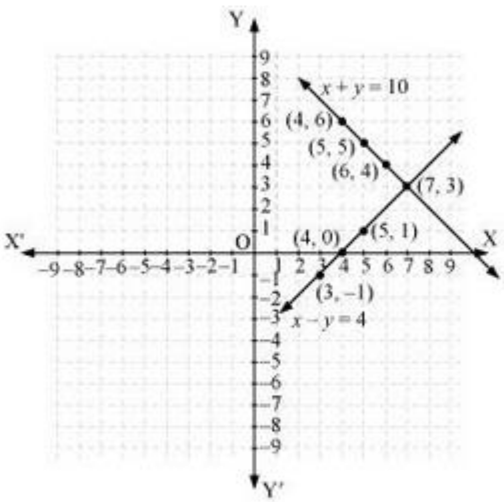
x	4	5	6
y	6	5	4

$x-y=4 \Rightarrow x=4+y$

Three solutions of this equation can be written in a table as follows:

x	5	4	3
y	1	0	-1

The graphical representation is as follows:



From the graph, it can be observed that the two lines intersect each other at the point $(7,3)$.

So. $x=7$ *and* $y=3.$

Thus, the number of girls and boys in the class are 7 and 3 respectively.

(ii) Let the cost of one pencil and one pen be Rs x and Rs y respectively.

According to the given conditions, we have:

$5x+7y=50$

$7x+5y=46$

$5x+7y=50 \Rightarrow x=\frac{50-7y}{5}$

Three solutions of this equation can be written in a table as follows:

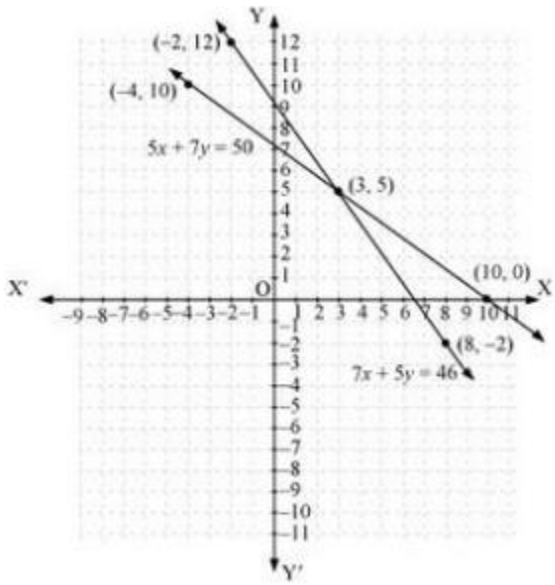
x	3	10	-4
y	5	0	10

$$7x+5y=46 \Rightarrow x=\frac{46-5y}{7}$$

Three solutions of this equation can be written in a table as follows:

x	8	3	-2
y	-2	5	12

The graphical representation is as follows:



From the graph. It can be observed that the two lines intersect each other at the point $(3,5)$.

So. $x=3$ *and* $y=5$.

Therefore, the cost of one pencil and one pen are Rs 3 and Rs 5 respectively.

(iii) Let us denote the number of pants by x and the number of skirts by y. Then the equations formed are:

$y=2x?2......(1)$ and $y=4x?4......(2)$

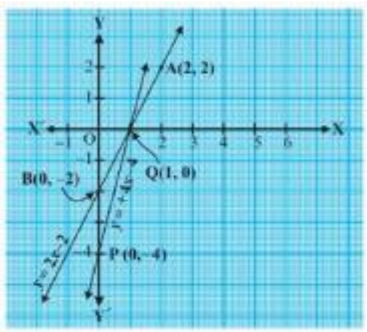
Let us draw the graphs of Equations (1) and (2) by finding two solutions for each of the equations.

They are given in Table

They are giving table

x	2	0
$y-2x?2$	2	-2

x	0	1
$y-2x?2$	-4	0



Plot the point and draw the lines passing through them to represent the equation, as shown in fig.

The t lines intersect at the point $(1,0)$. So. $x-1, y=0$ is the required solution of the pair of linear equations, i.e, the number of pants she purchased island she did not buy any skirt

Concept insight: Read the question carefully and examine what are the unknowns. Represent the given conditions with the help of equations by taking the unknowns quantities as variables. Also carefully state the variables as whole solution is based on it on the graph paper, mark the points accurately and neatly using a sharp pencil. Also take at least three points satisfying the two equations in order to obtain the correct straight line of the equation. Since joining any two points gives a straight line and if one of the points is computed incorrect will give a wrong line and taking third point will give a correct line. The point where the two straight lines will intersect will give the values of the two variables, i.e., the solution of the two linear equations. State the solution point.

35. Solve the following system of equations graphically:
Shade the region between the lines and the y-axis

(i)
$$3x-4y=7$$
$$5x+2y=3$$

(ii)
$$4x-y=4$$
$$3x+2y=14$$

Sol:

The given system of equations is

$$3x-4y=7$$
$$5x+2y=3$$

Now,

$$3x-4y=7$$
$$\Rightarrow \quad 3x-7=4y$$
$$\Rightarrow \quad 4y=3x-7$$
$$\Rightarrow \quad y=\frac{3x-7}{4}$$

When $x=1,$ we have

$$y=\frac{3\times1-7}{4}=-1$$

When $x=-3,$ we have

$$y=\frac{3\times(-3)-7}{4}=-4$$

Thus, we have the following table:

x	1	-3
y	-1	-4

We have,

$$5x+2y=3$$
$$\Rightarrow \quad 2y=3-5x$$
$$\Rightarrow \quad y=\frac{3-5x}{2}$$

When $x=1,$ we have

$$y=\frac{3-5\times1}{2}=-1$$

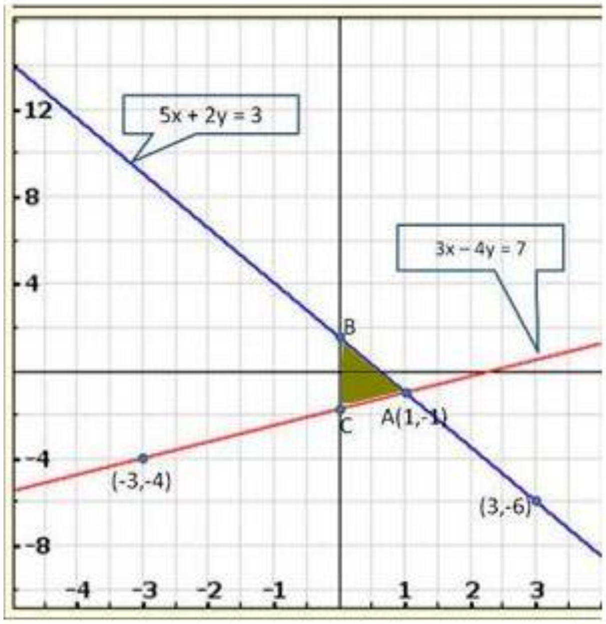
When $x=3,$ we have

$$y=\frac{3-5\times3}{2}=-6$$

Thus, we have the following table:

x	1	3
y	-1	-6

Graph of the given system of equations:



Clearly, the two lines intersect at $A(1,-1)$ Hence, $x=1, y=-1$ is the solution of the given system of equations.

We also observe that the required shaded region is ΔABC

The given system of equations is

$$4x-y=4$$
$$3x+2y=14$$

Now,

$$4x-y=4$$
$$\Rightarrow \quad 4x-4=y$$
$$\Rightarrow \quad y=4x-4$$

When $x=0$, we have

$y=4\times 0-4=-4$

When $x=-1$, we have

$y=4\times(-1)-4=-8$

Thus, we have the following table:

x	0	-1
y	-4	-8

We have,

$$3x+2y=14$$
$$\Rightarrow \quad 2y=14-3x$$
$$\Rightarrow \quad y=\frac{14-3x}{2}$$

When $x = 0$, we have

$$y = \frac{14 - 3\times 0}{2} = 7$$

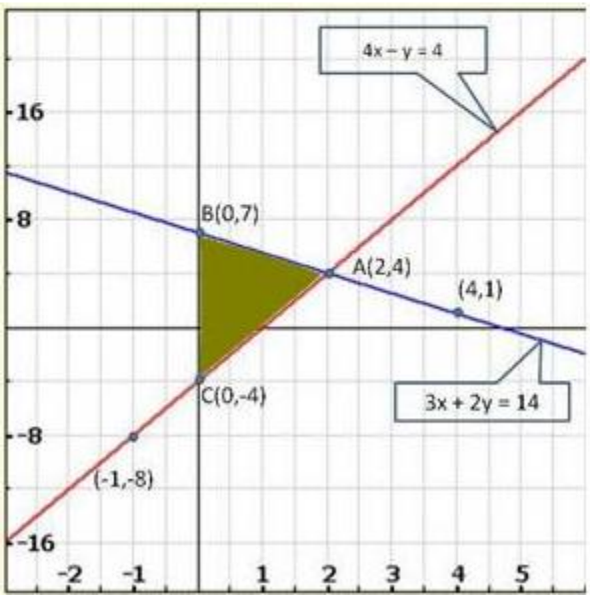
When $x = 0$, we have

$$y = \frac{14 - 3\times 4}{2} = 1$$

Thus, we have the following table:

x	0	4
y	7	1

Graph of the given system of equations:



Clearly, the two lines intersect at $A(2,4)$. Hence, $x = 2,\ y = 4$ is the solution of the given system of equations.

We also observe ΔABC is the required shaded region.

36. Represent the following pair of equations graphically and write the coordinates of points where the lines intersects y-axis

$$x + 3y = 6$$
$$2x - 3y = 12$$

Sol:

The given system of equations is

$$x + 3y = 6$$
$$2x - 3y = 12$$

Now,

$$x+3y=6$$

$$\Rightarrow \quad 3y=6-x$$

$$\Rightarrow \quad y=\frac{6-x}{3}$$

When $x=0$, we have

$$y=\frac{6-0}{3}=2$$

When $x=3$, we have

$$y=\frac{6-3}{3}=1$$

Thus, we have the following table:

x	0	3
y	2	1

We have,

$$2x+3y=12$$

$$\Rightarrow \quad 2x-12-3x$$

$$\Rightarrow \quad 3y=2x-12$$

$$\Rightarrow \quad y=\frac{2x-12}{3}$$

When $x=0$, we have

$$y=\frac{2\times 0-12}{3}=-4$$

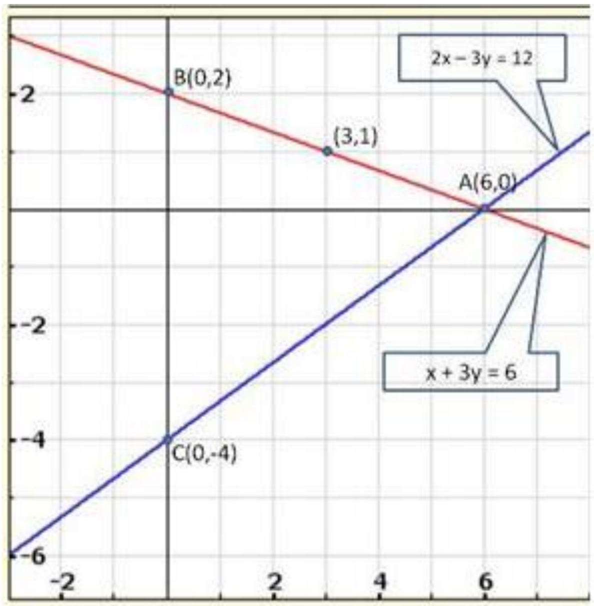
When $x=6$, we have

$$y=\frac{2\times 6-12}{3}=0$$

Thus, we have the following table:

x	0	6
y	-4	0

Graph of the given system of equations:



We observe that the lines represented by the equations $x+3y-6$ *and* $2x-3y-12$ meet y-axis at $B(0,2)$ *and* $C(0,-4)$ respectively.

Hence, the required co-ordinates are $(0,2)$ *and* $(0,-4)$.

37. Given the linear equation 2x + 3y − 8 = 0, write another linear equation in two variables such that the geometrical representation of the pair so formed is (i) intersecting lines (ii) Parallel lines (iii) coincident lines

Sol:

(i) For the two lines $a_1x+b_1x+c_1=0$ and $a_2x+b_2x+c_2=0$, to be intersecting, we must have

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

So, the other linear equation can be $5x+6y-16=0$

As $\frac{a_1}{a_2}=\frac{2}{5},\frac{b_1}{b_2}=\frac{3}{6}=\frac{1}{2},\frac{c_1}{c_2}=\frac{-8}{-16}=\frac{1}{2}$

(ii) For the two lines $a_1x+b_1x+c_1=0$ and $a_2x+b_2x+c_2=0$, to be parallel we must have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

So, the other linear equation can be $6x+9y+24=0,$

As $\frac{a_1}{a_2}=\frac{2}{6}=\frac{1}{3},\frac{b_1}{b_2}=\frac{3}{9}=\frac{1}{3},\frac{c_1}{c_2}=\frac{-8}{24}=\frac{-1}{3}$

(iii) For the two lines $a_1x+b_1x+c_1=0$ and $a_2x+b_2x+c_2=0,$ to be coincident, we must have $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

So, the other linear equation can be $6x+9y+24=0,$

As $\frac{a_1}{a_2}=\frac{2}{8}=\frac{1}{4},\frac{b_1}{b_2}=\frac{3}{12}=\frac{1}{4},\frac{c_1}{c_2}=\frac{-8}{-32}=\frac{1}{4}$

Concept insight: In orders to answer such type of problems, just remember the conditions for two lines to be intersecting parallel, and coincident

This problem will have multiple answers as their can be marry equations satisfying the required conditions.

Exercise 3.3

Solve the following systems of equations:

1. 11x + 15y + 23 = 0

7x – 2y – 20 = 0

Sol:

The given system of equation is

$11x+15y+23=0 \quad ...(i)$

$7x-2y-20=0 \quad ...(ii)$

From (ii), we get

$2y=7x-20$

$\Rightarrow \quad y=\frac{7x-20}{2}$

Substituting $y=\frac{7x-20}{2}$ in (i) we get

$11x+15\left(\frac{7x-20}{2}\right)+23=0$

$\Rightarrow \quad 11x+\frac{105x-300}{2}+23=0$

$\Rightarrow \quad \frac{22x+105x-300+46}{2}=0$

$\Rightarrow \quad 127x-254=0$

$\Rightarrow \quad 127x=254$

$\Rightarrow \quad x=\frac{254}{127}=2$

Putting $x=2$ in $y=\frac{7x-20}{2}$ we get

$\Rightarrow \quad y=\frac{7\times 2-20}{2}$

$=\frac{14-20}{2}$

$=\frac{-6}{2}$

$=-3$

Hence, the solution of the given system of equations is $x=2, y=-3.$

2. 3x – 7y + 10 = 0
y – 2x – 3 = 0
Sol:
The given system of equation is

$3x-7y+10=0 \quad ...(i)$

$y-2x-3=0 \quad ...(ii)$

From (ii), we get

$y=2x+3$

Substituting $y=2x+3$ in (i) we get

$3x-7(2x+3)+10=0$

$\Rightarrow \quad 3x+14x-21+10=0$

$\Rightarrow \quad -11x=11$

$\Rightarrow \quad x=\frac{11}{-11}=-1$

Putting $x=-1$ in $y=2x+3$, we get

$\Rightarrow \quad y=2\times(-1)+3$

$=-2+3$

$=1$

$\Rightarrow \quad y=1$

Hence, the solution of the given system of equations is $x=-1, y=1.$

3. 0.4x + 0.3y = 1.7
0.7x + 0.2y = 0.8
Sol:

The given system of equation is

$0.4x+0.3y=1.7 \quad ...(i)$

$0.7x-0.2y=0.8 \quad ...(ii)$

Multiplying both sides of (i) and (ii), by 10, we get

$4x+3y=17 \quad ...(iii)$

$7x-2y=8 \quad ...(iv)$

From (iv), we get

$$7x=8+2y$$

$$\Rightarrow \quad 7x\frac{8+2y}{7}$$

Substituting $x=\frac{8+2y}{7}$ in (iii), we get

$$4\left(\frac{8+2y}{7}\right)+3y=17$$

$$\Rightarrow \quad \frac{32+8y}{7}+3y=17$$

$$\Rightarrow \quad 32+29y=17\times 7$$

$$\Rightarrow \quad 29y=119-32$$

$$\Rightarrow \quad 29y=87$$

$$\Rightarrow \quad y=\frac{87}{29}=3$$

Putting $y=3$ in $x=\frac{8+2y}{7}$, we get

$$x=\frac{8+2\times 3}{7}$$

$$=\frac{8+6}{7}$$

$$=\frac{14}{7}$$

$$=2$$

Hence, the solution of the given system of equation is $x=2, y=3$.

4. $\frac{x}{2}+y=0.8$

Sol:

$$\frac{x}{2}+y=0.8$$

And $\dfrac{7}{x+\dfrac{y}{2}}=10$

$\therefore x+2y=1.6 \ \ and \ \ \dfrac{7\times 2}{2x+y}=10$

$x+2y=1.6 \ and \ 7=10x+5y$

Multiply first equation by 10

$10x+20y=16 \ \ and \ \ 10x+5y=7$

Subtracting the two equations

$15y=9$

$y=\dfrac{9}{15}=\dfrac{3}{5}$

$x=1.6-2\left(\dfrac{3}{5}\right)=1.6-\dfrac{6}{5}=\dfrac{2}{5}$

Solution is $\left(\dfrac{2}{5},\dfrac{3}{5}\right)$

5. 7(y + 3) – 2 (x + 3) = 14
4(y – 2) + 3 (x – 3) = 2

Sol:

The given system of equations id

$7(y+3)-2(x+3)=14 \qquad ...(i)$

$4(y-2)+3(x-3)=2 \qquad ...(ii)$

From (i), we get

$7x+21-2x-4=14$

$\Rightarrow \quad 7y=14+4-21+2x$

$\Rightarrow \quad y=\dfrac{2x-3}{7}$

From (ii), we get

$4y-8+3x-9=2$

$\Rightarrow \quad 4y+3x-17-2=0$

$\Rightarrow \quad 4y+3x-19=0 \qquad ...(iii)$

Substituting $y=\dfrac{2x-3}{7}$ in (iii), we get

$$4\left(\frac{2x-3}{7}\right)+3x-19=0$$

$\Rightarrow \quad \frac{8x-12}{7}+3x-19=0$

$\Rightarrow \quad 8x-12+21x-133=0$

$\Rightarrow \quad 29x-145=0$

$\Rightarrow \quad 29x=145$

$\Rightarrow \quad x=\frac{145}{29}=5$

Putting $x=5$ in $y=\frac{2x-3}{7}$, we get

$y=\frac{2\times5-7}{7}$

$=\frac{10-3}{7}$

$=\frac{7}{7}$

$=1$

$\Rightarrow y=1$

Hence, the solution of the given system of equations is $x=5, y=1$.

6. $\frac{x}{7}+\frac{y}{3}=5$
$\frac{x}{2}-\frac{y}{9}=6$

Sol:

The given system of equation is

$\frac{x}{7}+\frac{y}{3}=5 \quad ...(i)$

$\frac{x}{2}-\frac{y}{9}=6 \quad ...(ii)$

From (i), we get

$\frac{3x+7y}{21}=5$

$\Rightarrow \quad 3x+7y=105$

$\Rightarrow \quad 3x=105-7y$

$\Rightarrow \quad x=\frac{105-7y}{3}$

From (ii), we get

$$\frac{9x-2y}{18}=6$$

$\Rightarrow$ $9x-2y=108$ ...(iii)

Substituting $x=\frac{105-7y}{3}$ in (iii), we get

$$9\left(\frac{105-7y}{3}\right)-2y=108$$

$\Rightarrow$ $\frac{948-63y}{3}-2y=108$

$\Rightarrow$ $945-63y-6y=108\times 3$

$\Rightarrow$ $945-69y=324$

$\Rightarrow$ $945-324=69y$

$\Rightarrow$ $69y=621$

$\Rightarrow$ $y=\frac{621}{69}=9$

Putting $y=9$ in $x=\frac{1105-7y}{3}$, we get

$$x=\frac{105-7\times 9}{3}=\frac{105-63}{3}$$

$\Rightarrow$ $x=\frac{42}{3}=14$

Hence, the solution of thee given system of equations is $x=14, y=9$.

7. $\frac{x}{3}+\frac{y}{4}=11$
$\frac{5x}{6}-\frac{y}{3}=7$

Sol:

The given system of equations is

$\frac{x}{3}+\frac{y}{4}=11$ ...(i)

$\frac{5x}{6}-\frac{y}{3}=7$ ...(ii)

From (i), we get

$$\frac{4x+3y}{12}=11$$

$\Rightarrow$ $4x+3y=132$ ...(iii)

From (ii), we get

$$\frac{5x+2y}{6}=-7$$

$\Rightarrow \quad 5x-2y=-42 \qquad ...(iv)$

Let us eliminate y from the given equations. The coefficients of y in the equations(iii) and (iv) are 3 and 2 respectively. The L.C.M of 3 and 2 is 6. So, we make the coefficient of y equal to 6 in the two equations.

Multiplying (iii) by 2 and (iv) by 3, we get

$$8x+6y=264 \qquad ...(v)$$

$$15x-6x=-126 \qquad ...(vi)$$

Adding (v) and (vi), we get

$$8x+15x=264-126$$

$\Rightarrow \quad 23x=138$

$\Rightarrow \quad x=\frac{138}{23}=6$

Substituting $x=6$ in (iii), we get

$$4\times6+3y=132$$

$\Rightarrow \quad 3y=132-24$

$\Rightarrow \quad 3y=108$

$\Rightarrow \quad y=\frac{108}{3}=36$

Hence, the solution of the given system of equations is $x=6, y=36.$

8. $4u+3y=8$

$6u-4y=-5$

Sol:

Taking $\frac{1}{x}=u,$ then given equations become

$$4u+3y=8 \qquad ...(i)$$

$$6u-4y=-5 \qquad ...(ii)$$

From (i), we get

$$4u=8-3y$$

$\Rightarrow \quad u=\frac{8-3y}{4}$

Substituting $u=\frac{8-3y}{4}$ in (ii), we get

From (ii), we get

$$6\left(\frac{8-3y}{4}\right)-4y=-5$$

$\Rightarrow$ $\frac{3(8-3y)}{2}-4y=-5$

$\Rightarrow$ $\frac{24-9y}{2}-4y=-5$

$\Rightarrow$ $\frac{24-9y-8y}{2}=-5$

$\Rightarrow$ $24-17y=-10$

$\Rightarrow$ $-17y=-10-24$

$\Rightarrow$ $-17y=-34$

$\Rightarrow$ $y=\frac{-34}{-17}=2$

Putting $y=2,$ in $u=\frac{8-3y}{4}$, we get

$u=\frac{8-3\times 2}{4}=\frac{8-6}{4}=\frac{2}{4}=\frac{1}{2}$

Hence, $x=\frac{1}{u}=2$

So, the solution of the given system of equation is $x=2, y=2.$

9. $x+\frac{y}{2}=4$

$\frac{x}{3}+2y=5$

Sol:

The given system of equation is

$x+\frac{y}{2}=4$ ..(i)

$\frac{x}{3}+2y=5$ ..(ii)

From (i), we get

$\frac{2x+y}{2}=4$

$2x+y=8$

$y=8-2x$

From (ii), we get

$x+6y=15$ ..(iii)

Substituting $y=8-2x$ in (iii), we get

$$x+6(8-2x)=15$$

$\Rightarrow$ $x+48-12x=15$

$\Rightarrow$ $-11x=15-48$

$\Rightarrow$ $-11x=-33$

$\Rightarrow$ $x=\frac{-33}{-11}=3$

Putting $x=3$, in $y=8-2x$, we get

$$y=8-2\times3$$
$$=8-6$$
$$=2$$

$\Rightarrow$ $y=2$

Hence, solution of the given system of equation is $x=3, y=2$.

10. $x+2y=\frac{3}{2}$

$2x+y=\frac{3}{2}$

Sol:

The given system of equation is

$$x+2y=\frac{3}{2} \quad ..(i)$$

$$2x+y=\frac{3}{2} \quad ..(ii)$$

Let us eliminate y from the given equations. The Coefficients of y in the given equations are 2 and 1 respectively. The L.C.M of 2 and 1 is 2. So, we make the coefficient of y equal to 2 in the two equations.

Multiplying (i) by 1 and (ii) by 2, we get

$$x+2y=\frac{3}{2} \quad ..(iii)$$

$$4x+2y=3 \quad ..(iv)$$

Subtracting (iii) from (iv), we get

$$4x - x + 2y - 2y = 3 - \frac{3}{2}$$

$\Rightarrow \quad 3x = \frac{6-3}{2}$

$\Rightarrow \quad 3x = \frac{3}{2}$

$\Rightarrow \quad x = \frac{3}{2\times 3}$

$\Rightarrow \quad x = \frac{1}{2}$

Putting $x = \frac{1}{2}$, in equation (iv), we get

$$4\times\frac{1}{2} + 2y = 3$$

$\Rightarrow \quad 2 + 2y = 3$

$\Rightarrow \quad 2y = 3 - 2$

$\Rightarrow \quad y = \frac{1}{2}$

Hence, solution of the given system of equation is $x = \frac{1}{2}, y = \frac{1}{2}$.

11. $\sqrt{2}x + \sqrt{3}y = 0$
$\sqrt{3}x - \sqrt{8}y = 0$

Sol:

$\sqrt{2}x + \sqrt{3}y = 0 \qquad ...(i)$

$\sqrt{3}x - \sqrt{8}y = 0 \qquad ...(ii)$

From equation (i), we obtain:

$x = \frac{-\sqrt{3}y}{\sqrt{2}} \qquad ...(iii)$

Substituting this value in equation (ii), we obtain:

$$\sqrt{3}\left(-\frac{\sqrt{3}y}{\sqrt{2}}\right) - \sqrt{8}y = 0$$

$$-\frac{3y}{\sqrt{2}} - 2\sqrt{2}y = 0$$

$$y\left(-\frac{3}{\sqrt{2}} - 2\sqrt{2}\right) = 0$$

$$y = 0$$

Substituting the value of y in equation (iii), we obtain:

$x = 0$

$\therefore x = 0, y = 0$

12. $$3x - \frac{y+7}{11} + 2 = 10$$
$$2y + \frac{x+11}{7} = 10$$

Sol:

The given systems of equation is

$$3x - \frac{y+7}{11} + 2 = 10 \qquad ...(i)$$

$$2y + \frac{x+11}{7} = 10 \qquad ...(ii)$$

From (i), we get

$$\frac{33x - y - 7 + 22}{11} = 10$$

$\Rightarrow \quad 33x - y + 15 = 10 \times 11$

$\Rightarrow \quad 33x + 15 - 110 = y$

$\Rightarrow \quad y = 33x - 95$

From (ii) we get

$$\frac{14y + x + 11}{7} = 109$$

$\Rightarrow \quad 14y + x + 11 = 10 \times 7$

$\Rightarrow \quad 14y + x + 11 = 70$

$\Rightarrow \quad 14y + x = 70 - 11$

$\Rightarrow \quad 14y + x = 59 \qquad ...(iii)$

Substituting $y = 33x - 95$ in (iii), we get

$$14(33x - 95) + x = 59$$

$\Rightarrow \quad 462x - 1330 + x = 59$

$\Rightarrow \quad 463x = 59 + 1330$

$\Rightarrow \quad 463x = 1389$

$\Rightarrow \quad x = \frac{1389}{463} = 3$

Putting $x = 3,$ in $y = 33x - 95,$ we get

$y = 33 \times 3 - 95$

$\Rightarrow \quad y = 99 - 95$

$= 4$

$\Rightarrow \quad y = 4$

Hence, solution of the given system of equation is $x = 3, y = 4$.

13. $2x - \frac{3}{y} = 9$

$3x + \frac{7}{y} = 2, y \neq 0$

Sol:

The given systems of equation is

$2x - \frac{3}{y} = 9 \qquad ...(i)$

$3x + \frac{7}{y} = 2, y \neq 0 \qquad ...(ii)$

Taking $\frac{1}{y} = u$, the given equations becomes

$2x - 3u = 9 \qquad ...(iii)$

$3x + 7u = 2 \qquad ...(iv)$

From (iii), we get

$2x = 9 + 3u$

$\Rightarrow \quad x = \frac{9 + 3u}{2}$

Substituting $x = \frac{9 + 3u}{2}$ in (iv), we get

$3\left(\frac{9 + 3u}{2}\right) + 7u = 2$

$\Rightarrow \quad \frac{27 + 9u + 14u}{2} = 2$

$\Rightarrow \quad 27 + 23u = 2 \times 2$

$\Rightarrow \quad 23u = 4 - 27$

$\Rightarrow \quad u = \frac{-23}{23} = -1$

Hence, $y = \frac{1}{u} = \frac{1}{-1} = -1$

Putting $u=-1$ in $x=\frac{9+3u}{2}$, we get

$$x=\frac{9+3(-1)}{2}=\frac{9-3}{2}=\frac{6}{2}=3$$

$\Rightarrow \quad x=3$

Hence, solution of the given system of equation is $x=3, y=-1$.

14. $0.5x+0.7y=0.74$
$0.3x+0.5y=0.5$

Sol:

The given systems of equations is

$0.5x+0.7y=0.74 \qquad (i)$

$0.3x+0.5y=0.5 \qquad (ii)$

Multiplying (i) and (ii) by 100, we get

$50x+70y=74 \qquad ...(iii)$

$30x+50y=50 \qquad ...(iv)$

From (iii), we get

$$50x=74-70y$$

$$\Rightarrow \quad x=\frac{74-70y}{50}$$

Substituting $x=\frac{74-70y}{50}$ in equation (iv), we get

$$30\left(\frac{74-70y}{50}\right)+50y=50$$

$$\Rightarrow \quad \frac{3(74-70y)}{5}+50y=50$$

$$\Rightarrow \quad \frac{222-210y}{5}+50y=50$$

$$\Rightarrow \quad 222-210y+250y=250$$

$$\Rightarrow \quad 40y=250-222$$

$$\Rightarrow \quad 40y=28$$

$$\Rightarrow \quad y=\frac{28}{40}=\frac{14}{20}=\frac{7}{10}=0.7$$

Putting $y=0.7$ in $x=\frac{74-70y}{50}$, we get

$$x = \frac{74 - 70 \times 0 \cdot 7}{50}$$
$$= \frac{74 - 49}{50}$$
$$= \frac{25}{50}$$
$$= \frac{1}{2}$$
$$= 0.5$$

Hence, solution of the given system of equation is $x = 0 \cdot 5, y = 0.7$

15.
$$\frac{1}{7x} + \frac{1}{6y} = 3$$
$$\frac{1}{2x} - \frac{1}{3y} = 5$$

Sol:

$$\frac{1}{7x} + \frac{1}{6y} = 3 \qquad ....(1)$$
$$\frac{1}{2x} - \frac{1}{3y} = 5 \qquad ....(2)$$

Multiplying (2) by $\frac{1}{2}$, we get

$$\frac{1}{4x} - \frac{1}{6y} = \frac{5}{2} \qquad ....(3)$$

Solving (1) and (3), we get

$$\frac{1}{7x} + \frac{1}{6y} = 3$$
$$\frac{\frac{1}{4x} - \frac{1}{6y} = \frac{5}{2}}{\frac{1}{7x} + \frac{1}{4x} = 3 + \frac{5}{2}} \qquad \text{(Adding the equations)}$$
$$\Rightarrow \frac{4+7}{28x} = \frac{6+5}{2}$$
$$\Rightarrow \frac{11}{28x} = \frac{11}{2}$$
$$\Rightarrow x = \frac{11 \times 2}{28 \times 11} = \frac{1}{14}$$

When $x=\frac{1}{14}$, we get

$$\frac{1}{7\left(\frac{1}{14}\right)}+\frac{1}{6y}=3 \qquad \text{(Using (1))}$$

$$\Rightarrow 2+\frac{1}{6y}=3$$

$$\Rightarrow \frac{1}{6y}=3-2=1$$

$$\Rightarrow y=\frac{1}{6}$$

Thus, the solution of given equation is $x=\frac{1}{14}$ *and* $y=\frac{1}{6}$.

16. $\frac{1}{2x}+\frac{1}{3y}=2$

$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

Sol:

Let $\frac{1}{x}=u$ *and* $\frac{1}{y}=v$, the given equations become

$$\frac{u}{2}+\frac{v}{3}=2$$

$$\Rightarrow \frac{3u+2v}{6}=2$$

$$\Rightarrow 3u+2v=12 \qquad ....(i)$$

And, $\frac{u}{3}+\frac{v}{2}=\frac{13}{6}$

$$\Rightarrow \frac{2u+3v}{6}=\frac{13}{6}$$

$$\Rightarrow v=\frac{6}{2}=3$$

Hence, $x=\frac{1}{u}=\frac{1}{2}$ *and* $y=\frac{1}{v}=\frac{1}{3}$

So, the solution of the given system o equation is $x=\frac{1}{2}, y=\frac{1}{3}$.

17. $$\frac{x+y}{xy}=2$$
$$\frac{x-y}{xy}=6$$

Sol:

The given system of equation is

$$\frac{x+y}{xy}=2$$

$$\Rightarrow \quad \frac{x}{xy}+\frac{y}{xy}=2$$

$$\Rightarrow \quad \frac{1}{y}+\frac{1}{x}=2 \qquad .....(i)$$

And, $\frac{x-y}{xy}=6$

$$\Rightarrow \frac{x}{xy}-\frac{y}{xy}=6$$

$$\Rightarrow \frac{1}{y}-\frac{y}{x}=6 \qquad ...(ii)$$

Taking $\frac{1}{y}=v$ and $\frac{1}{x}=u$, the above equations become

$$v+u=2 \qquad .....(iii)$$
$$v-u=6 \qquad ......(iv)$$

Adding equation (iii) and equation (iv), we get

$$v+u+v-u=2+6$$

$$\Rightarrow \quad 2v=8$$

$$\Rightarrow \quad v=\frac{8}{2}=4$$

Putting $v=4$ in equation (iii), we get

$$4+u=2$$

$$\Rightarrow \quad u=2-4=-2$$

Hence, $x=\frac{1}{u}=\frac{1}{-2}=\frac{-1}{2}$ *and* $y=\frac{1}{v}=\frac{1}{4}$

So, the solution of the given system of equation is $x=\frac{-1}{2}, y=\frac{1}{4}$

18. $\frac{15}{u}+\frac{2}{v}=17$

Sol:

Let $\frac{1}{u}=x$ *and* $\frac{1}{v}=y$, then, the given system of equations become

$15x+2y=17 \qquad ...(i)$

$x+y=\frac{36}{5} \qquad ...(ii)$

From (i), we get

$2y=17-15x$

$\Rightarrow \quad y=\frac{17-15x}{2}$

Substituting $y=\frac{17-15x}{2}$ in equation (ii), we get

$x+\frac{17-15x}{2}=\frac{36}{5}$

$\Rightarrow \quad \frac{2x+17-15x}{2}=\frac{36}{5}$

$\Rightarrow \quad \frac{-13x+17}{2}=\frac{36}{5}$

$\Rightarrow \quad 5(-13x+17)=36\times 2$

$\Rightarrow \quad -65x+85=72$

$\Rightarrow \quad -65x=72-85$

$\Rightarrow \quad -65x=-13$

$\Rightarrow \quad 65x=\frac{-13}{-65}=\frac{1}{5}$

Putting $x=\frac{1}{5}$ in equation (ii), we get

$\frac{1}{5}+y=\frac{36}{5}$

$\Rightarrow \quad y=\frac{36}{5}-\frac{1}{5}$

$=\frac{36-1}{5}=\frac{35}{5}=7$

Hence, $u=\frac{1}{x}=5$ *and* $v=\frac{1}{y}=\frac{1}{7}$.

So, the solution off the given system of equation is $u=5, v=\frac{1}{7}$.

19. $\frac{3}{x} - \frac{1}{y} = -9$
$\frac{2}{x} + \frac{3}{y} = 5$

Sol:

Let $\frac{1}{x} = u$ *and* $\frac{1}{y} = v$, Then, the given system of equations becomes

$3u - v = -9$ ......(i)

$2u + 3v = 5$ ......(ii)

Multiplying equation (i) by 3 an equation (ii) by 1, we get

$9u - 3v = -27$ ....(iii)

$2u + 3v = 5$ ....(iv)

Adding equation (i) and equation (ii), we get

$9u + 2u - 3v + 3v = -27 + 5$

$\Rightarrow \quad 11u = -22$

$\Rightarrow \quad u = \frac{-22}{11} = -2$

Putting $u = -2$ in equation (iv), we get

$2 \times (-2) + 3v = 5$

$\Rightarrow \quad -4 + 3v = 5$

$\Rightarrow \quad 3v = 5 + 4$

$\Rightarrow \quad v = \frac{9}{3} = 3$

Hence, $x = \frac{1}{u} = \frac{1}{-2} = \frac{-1}{2}$ and $y = \frac{1}{v} = \frac{1}{3}$.

So, the solution of the given system of equation is $x = \frac{-1}{2}, y = \frac{1}{3}$.

20. $\frac{2}{x} + \frac{5}{y} = 1$
$\frac{60}{x} + \frac{40}{y} = 19, x \neq 0, y \neq 0$

Sol:

Taking $\frac{1}{x} = u$ *and* $\frac{1}{y} = v$, the given becomes

$2u + 5v = 1$ ......(i)

$60u + 40u = 19$ ......(ii)

Let us eliminate ‘u’ from equation (i) and (ii), multiplying equation (i) by 60 and equation (ii) by 2, we get

$120u+300v=60$ ......(iii)

$120u+80v=38$ ......(iv)

Subtracting (iv) from (iii), we get

$300v-80v=60-38$

$\Rightarrow$ $220v=22$

$\Rightarrow$ $v=\frac{22}{220}=\frac{1}{10}$

Putting $v=\frac{1}{10}$ in equation (i), we get

$2u+5\times\frac{1}{10}=1$

$\Rightarrow$ $2u+\frac{1}{2}=1$

$\Rightarrow$ $2u=1-\frac{1}{2}$

$\Rightarrow$ $2u=\frac{2-1}{2}=\frac{1}{2}$

$\Rightarrow$ $2u=\frac{1}{2}$

$\Rightarrow$ $u=\frac{1}{4}$

Hence, $x=\frac{1}{u}=4$ *and* $y=\frac{1}{v}=10$

So, the solution of the given system of equation is $x=4, y=10$.

21. $\frac{1}{5x}+\frac{1}{6y}=12$

$\frac{1}{3x}-\frac{3}{7y}=8, x\neq 0, y\neq 0$

Sol:

Taking $\frac{1}{x}=u$ *and* $\frac{1}{y}=v$, the given equations become\

$\frac{u}{5}+\frac{v}{6}=12$

$\Rightarrow$ $\frac{6u+5v}{30}=12$

$\Rightarrow$ $6u+5v=360$ .....(i)

And, $\frac{u}{3}-\frac{3v}{7}=8$

$\Rightarrow \quad \frac{7u+9v}{21}=8$

$\Rightarrow \quad 7u-9v=168 \qquad ......(ii)$

Let us eliminate 'v' from equation (i) and (ii), Multiplying equation (i) by 9 and equation (ii) by 5, we get

$54u+45v=3240 \qquad ......(iii)$

$35u-45v=840 \qquad ......(iv)$

Adding equation (i) adding equation (ii), we get

$54u+35u=3240+840$

$\Rightarrow \quad 89u=4080$

$\Rightarrow \quad u=\frac{4080}{89}$

Putting $u=\frac{4080}{89}$ in equation (i), we get

$6\times\frac{4080}{89}+5v=360$

$\Rightarrow \quad \frac{24480}{89}+5v=360$

$\Rightarrow \quad 5v=360-\frac{24480}{89}$

$\Rightarrow \quad 5v=\frac{32040-24480}{89}$

$\Rightarrow \quad 5v=\frac{7560}{89}$

$\Rightarrow \quad v=\frac{7560}{5\times 89}$

$\Rightarrow \quad v=\frac{1512}{89}$

Hence, $x=\frac{1}{u}=\frac{89}{4080}$ *and* $y=\frac{1}{v}=\frac{89}{1512}$

So, the solution of the given system of equation is $x=\frac{89}{4080}, y=\frac{89}{1512}$.

22.
$$\frac{2}{x}+\frac{3}{y}=\frac{9}{xy}$$
$$\frac{4}{x}+\frac{9}{y}=\frac{21}{xy}, where\ x\neq 0, y\neq 0$$

Sol:

The system of given equation is

$$\frac{2}{x}+\frac{3}{y}=\frac{9}{xy} \qquad .....(i)$$
$$\frac{4}{x}+\frac{9}{y}=\frac{21}{xy}, where\ x\neq 0, y\neq 0 \qquad .....(ii)$$

Multiplying equation (i) adding equation (ii) by xy, we get

$$2y+3x=9 \qquad ....(iii)$$
$$4y+9x=21 \qquad ....(iv)$$

From (iii), we get

$$3x=9-2y$$
$$\Rightarrow \quad x=\frac{9-2y}{3}$$

Substituting $x=\frac{9-2y}{3}$ in equation (iv), we get

$$4x+9\left(\frac{9-2y}{3}\right)=21$$
$$\Rightarrow \quad 4y+3(9-2y)=21$$
$$\Rightarrow \quad 4y+27-6y=21$$
$$\Rightarrow \quad -2y=21-27$$
$$\Rightarrow \quad -2y=-6$$
$$\Rightarrow \quad y=\frac{-6}{-2}=3$$

Putting $y=3$ in $x=\frac{9-2y}{3}$, we get

$$x=\frac{9-2\times 3}{3}$$
$$=\frac{9-6}{3}$$
$$=\frac{3}{3}$$
$$=1$$

Hence, solution of the system of equation is $x=1, y=3$

23. $\frac{6}{x+y} = \frac{7}{x-y} + 3$

$\frac{1}{2(x+y)} = \frac{1}{3(x-y)}$, where x + y ≠ 0 and x – y ≠ 0

Sol:

Let $\frac{1}{x+y} = u$ *and* $\frac{1}{x-y} = v$. Then, the given system of equation becomes

$$6u = 7v + 3$$

$$\Rightarrow \quad 6u - 7v = 3 \quad ....(i)$$

And, $\frac{u}{2} = \frac{v}{3}$

$$\Rightarrow \quad 3u = 2v$$

$$\Rightarrow \quad 3u - 2v = 0 \quad ......(ii)$$

Multiplying equation (ii) by 2, and equation (i) by 1, we get

$$6u - 7v = 3 \quad ....(iii)$$

$$6u - 4v = 0 \quad ....(iv)$$

Subtracting equation (iv) from equation (iii), we get

$$-7 + 4v = 3$$

$$\Rightarrow \quad -3v = 3$$

$$\Rightarrow \quad v = -1$$

Putting $v = -1$ in equation (ii), we get

$$3u - 2 \times (-1) = 0$$

$$\Rightarrow \quad 3u + 2 = 0$$

$$\Rightarrow \quad 3u = -2$$

$$\Rightarrow \quad u = \frac{-2}{3}$$

Now,

$$u = \frac{-2}{3}$$

$$\Rightarrow \quad \frac{1}{x+2} = \frac{-2}{3}$$

$$\Rightarrow \quad x + y = \frac{-3}{2} \quad ....(v)$$

And, $v = -1$

$$\Rightarrow \quad \frac{1}{x-y} = -1$$

$$\Rightarrow \quad x-y=-1 \qquad ......(vi)$$

Adding equation (v) and equation (vi), we get

$$2x=\frac{-3}{2}-1$$

$$\Rightarrow \quad 2x=\frac{-3-2}{2}$$

$$\Rightarrow \quad 2x=\frac{-5}{2}$$

$$\Rightarrow \quad x=\frac{-5}{4}$$

Putting $x=\frac{-5}{4}$ in equation (vi), we get

$$\frac{-5}{4}-y=-1$$

$$\Rightarrow \quad \frac{-5}{4}+1=y$$

$$\Rightarrow \quad \frac{-5+4}{4}=y$$

$$\Rightarrow \quad \frac{-1}{4}=y$$

$$\Rightarrow \quad y=\frac{-1}{4}$$

Hence, solution of the system of equation is $x=\frac{-5}{4}, y=\frac{-1}{4}$.

24. $$\frac{xy}{x+y}=\frac{6}{5}$$

$$\frac{xy}{y-x}=6$$

Sol:

The given system of equation is

$$\frac{xy}{x+y}=\frac{6}{5}$$

$$\Rightarrow \quad 5xy=6(x+y)$$

$$\Rightarrow \quad 5xy=6x+6y \qquad ....(i)$$

And, $\frac{xy}{y-x}=6$

$\Rightarrow \quad xy = 6(y - x)$

$\Rightarrow \quad xy = 6y - 6x \quad .....(ii)$

Adding equation (i) and equation (ii), we get

$6xy = 6y + 6y$

$\Rightarrow \quad 6xy = 12y$

$\Rightarrow \quad x = \frac{12y}{6y} = 2$

Putting $x = 2$ in equation (i), we get

$5 \times 2 \times y = 6 \times 2 + 6y$

$\Rightarrow \quad 10y = 12 + 6y$

$\Rightarrow \quad 10y - 6y = 12$

$\Rightarrow \quad 4y = 12$

$\Rightarrow \quad y = \frac{12}{4} = 3$

Hence, solution of the given system of equation is $x = 2, y = 3.$

25. $\frac{22}{x+y} + \frac{15}{x-y} = 5$

$\frac{55}{x+y} + \frac{45}{x-y} = 14$

Sol:

Let $\frac{1}{x+y} = u$ *and* $\frac{1}{x-y} = v.$ Then, the given system of equation becomes

$22u + 15v = 5 \quad ...(i)$

$55u + 45v = 14 \quad ...(ii)$

Multiplying equation (i) by 3, and equation (ii) by 1, we get

$66u + 45v = 15 \quad .....(iii)$

$55u + 45v = 14 \quad ......(iv)$

Subtracting equation (iv) from equation (iii), we get

$66u - 55u = 15 - 4$

$\Rightarrow \quad 11u = 1$

$\Rightarrow \quad u = \frac{1}{11}$

Putting $u = \frac{1}{11}$ in equation (i), we get

$22 \times \frac{1}{11} + 15v = 5$

$\Rightarrow \quad 2 + 15v = 5$

$\Rightarrow \quad 15v = 5 - 2$

$\Rightarrow \quad 15v = 3$

$\Rightarrow \quad v = \frac{3}{15} = \frac{1}{5}$

Now, $u = \frac{1}{x+y}$

$\Rightarrow \quad \frac{1}{x+y} = \frac{1}{11}$

$\Rightarrow \quad x + y = 11 \quad \text{....}(v)$

And $\quad v = \frac{1}{x-y}$

$\Rightarrow \quad \frac{1}{x-y} = \frac{1}{5}$

$\Rightarrow \quad x - y = 5 \quad \text{.....}(vi)$

Adding equation (v) and equation (vi), we get

$2x = 11 + 5$

$\Rightarrow \quad 2x = 16$

$\Rightarrow \quad x = \frac{16}{2} = 8$

Putting $x = 8$ in equation (v), we get

$8 + y = 11$

$\Rightarrow \quad y = 11 - 8 = 3$

Hence, solution of the given system of equation is $x = 8, y = 3$.

26. $\frac{5}{x+y} - \frac{2}{x-y} = -1$

$\frac{15}{x+y} + \frac{7}{x-y} = 10$

Sol:

Let $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v$. Then, the given system off equations becomes

$5u - 2v = -1 \quad \text{......}(i)$

$15u + 7v = 10 \quad \text{.....}(ii)$

Multiplying equation (i) by 7, and equation (ii) by 2, we get

$35u - 14v = -7$ .....(iii)

$30u + 14v = 20$ ......(iv)

Adding equation (iii) and equation (iv), we get

$\Rightarrow \quad 35u + 30u = -7 + 20$

$\Rightarrow \quad 65u = 13$

$\Rightarrow \quad u = \frac{13}{65} = \frac{1}{5}$

Putting $u = \frac{1}{5}$ in equation (i), we get

$5 \times \frac{1}{5} - 2v = -1$

$\Rightarrow \quad 1 - 2v = -1$

$\Rightarrow \quad -2v = -1 - 1$

$\Rightarrow \quad -2v = -2$

$\Rightarrow \quad v = \frac{-2}{-2} = 1$

Now, $u = \frac{1}{x+y}$

$\Rightarrow \quad \frac{1}{x+y} = \frac{1}{5}$

$\Rightarrow \quad x + y = 5$ ......(v)

and, $v = \frac{1}{x-y} = 1$

$\Rightarrow \quad x - y = 1$ ....(vi)

Adding equation (v) and equation (vi), we get

$2x = 5 + 1$

$\Rightarrow \quad 2x = 6$

$\Rightarrow \quad x = \frac{6}{2} = 3$

Putting $x = 3$ in equation (v), we get

$3 + y = 5$

$\Rightarrow \quad y = 5 - 3 = 2$

Hence, solution of the given system of equation is $x = 3, y = 2$.

27. $\frac{3}{x+y}+\frac{2}{x-y}=2$
$\frac{9}{x+y}-\frac{4}{x-y}=1$
Sol:
Let $\frac{1}{x+y}=u$ *and* $\frac{1}{x-y}=v.$ Then, the given system of equation becomes

$$3u+2v=2 \qquad ...(i)$$

$$9u+4v=1 \qquad ...(ii)$$

Multiplying equation (i) by 3, and equation (ii) by 1, we get

$$6u+4v=4 \qquad .....(iii)$$

$$9u-4v=1 \qquad ......(iv)$$

Adding equation (iii) and equation (iv), we get

$$6u+9u=4+1$$

$\Rightarrow \quad 15u=5$

$\Rightarrow \quad u=\frac{5}{15}=\frac{1}{3}$

Putting $u=\frac{1}{3}$ in equation (i), we get

$$3\times\frac{1}{3}+2v=2$$

$\Rightarrow \quad 1+2v=2$

$\Rightarrow \quad 2v=2-1$

$\Rightarrow \quad v=\frac{1}{2}$

Now, $u=\frac{1}{x+y}$

$\Rightarrow \quad \frac{1}{x+y}=\frac{1}{3}$

$\Rightarrow \quad x+y=3 \qquad ......(v)$

And, $v=\frac{1}{x-y}$

$\Rightarrow \quad \frac{1}{x-y}=\frac{1}{2}$

$\Rightarrow \quad x-y=2 \qquad ......(vi)$

Adding equation (v) and equation (vi), we get

$$2x = 3 + 2$$

$\Rightarrow \quad x = \frac{5}{2}$

Putting $x = \frac{5}{2}$ in equation (v), we get

$$\frac{5}{2} + y = 3$$

$\Rightarrow \quad y = 3 - \frac{5}{2}$

$\Rightarrow \quad y = \frac{6-5}{2} = \frac{1}{2}$

Hence, solution of the given system of equation is $x = \frac{5}{2}, y = \frac{1}{2}$.

28. $\frac{1}{2(x+2y)} + \frac{5}{3(3x-2y)} = \frac{-3}{2}$ $\qquad \frac{5}{4(x+2y)} - \frac{3}{5(3x-2y)} = \frac{61}{60}$

Sol:

Let $\frac{1}{x+2y} = u \ \ and \ \ \frac{1}{3x-2y} = v.$

Then, the given system of equation becomes

$$\frac{u}{2} + \frac{5v}{3} = \frac{-3}{2}$$

$\Rightarrow \quad \frac{3u+10v}{6} = \frac{-3}{2}$

$\Rightarrow \quad 3u + 10v = \frac{-3 \times 6}{2}$

$\Rightarrow \quad 3u + 10v = -9 \qquad .....(i)$

And, $\frac{5u}{4} - \frac{3v}{5} = \frac{61}{60}$

$\Rightarrow \quad \frac{25u - 12v}{20} = \frac{61}{60}$

$\Rightarrow \quad 25u - 12v = \frac{61}{3} \qquad ....(ii)$

Multiplying equation (i) by 12, and equation (ii) by 10, we get

$$36u + 120v = -108 \qquad ......(iii)$$

$$250u - 120v = \frac{610}{3} \qquad ......(iv)$$

Adding equation (iii) and equation (iv), we get

$$36u + 250u = \frac{610}{3} - 108$$

$$\Rightarrow \quad 286u = \frac{610 - 324}{3}$$

$$\Rightarrow \quad 286u = \frac{286}{3}$$

$$\Rightarrow \quad u = \frac{1}{3}$$

Putting $u = \frac{1}{3}$ in equation (i), we get

$$3 \times \frac{1}{3} + 10v = -9$$

$$\Rightarrow \quad 1 + 10v = -9$$

$$\Rightarrow \quad 10v = -9 - 1$$

$$\Rightarrow \quad v = \frac{-10}{10} = -1$$

Now, $u = \frac{1}{x + 2y}$

$$\Rightarrow \quad \frac{1}{x + y} = \frac{1}{3}$$

$$\Rightarrow \quad x + 2y = 3 \quad \text{..........}(v)$$

And, $v = \frac{1}{3x - 2y}$

$$\Rightarrow \quad \frac{1}{3x - 2y} = -1$$

$$\Rightarrow \quad 3x - 2y = -1 \quad \text{.......}(vi)$$

Putting $x = \frac{1}{2}$ in equation (v), we get

$$\frac{1}{2} + 2y = 3$$

$$\Rightarrow \quad 2y = 3 - \frac{1}{2}$$

$$\Rightarrow \quad 2y = \frac{6 - 1}{2}$$

$$\Rightarrow \quad y = \frac{5}{4}$$

Hence, solution of the given system of equations is $x = \frac{1}{2}, y = \frac{5}{4}$.

29. $\frac{5}{x+1} - \frac{2}{y-1} = \frac{1}{2}$

$\frac{10}{x+1} + \frac{2}{y-1} = \frac{5}{2}, where\ x \neq -1\ and\ y \neq 1$

Sol:

Let $\frac{1}{x+1} = u\ and\ \frac{1}{y-1} = v.$

Then, the given system of equations becomes

$\Rightarrow \quad 5u - 2v = \frac{1}{2} \quad ....(i)$

$\Rightarrow \quad 10u + 2y = \frac{5}{2} \quad ....(ii)$

Adding equation (i) equation (ii), we get

$5u + 10u = \frac{1}{2} + \frac{5}{2}$

$\Rightarrow \quad 15u = \frac{1+5}{2}$

$\Rightarrow \quad 15u = \frac{6}{2} = 3$

$\Rightarrow \quad u = \frac{3}{15} = \frac{1}{5}$

Putting $u = \frac{1}{5}$ in equation (i), we get

$5 \times \frac{1}{5} - 2v = \frac{1}{2}$

$\Rightarrow \quad 1 - 2v = \frac{1}{2}$

$\Rightarrow \quad -2v = \frac{1}{2} - 1$

$\Rightarrow \quad -2v = \frac{1-2}{2}$

$\Rightarrow \quad -2v = \frac{-1}{2}$

$\Rightarrow \quad v = \frac{-1}{-4} = \frac{1}{4}$

Now, $u = \frac{1}{x+1}$

$\Rightarrow \quad \frac{1}{x+1} = \frac{1}{5}$

$\Rightarrow \quad x+1 = 5$

$\Rightarrow \quad x = 5-1 = 4$

And, $v = \frac{1}{y-1}$

$\Rightarrow \quad \frac{1}{y-1} = \frac{1}{4}$

$\Rightarrow \quad y-1 = 4$

$\Rightarrow \quad y = 4+1 = 5$

Hence, solution of the give system of equation is $x = 4, y = 5.$

30. $x + y = 5xy$
$3x + 2y = 13xy$

Sol:

The give system of equation is

$x + y = 5xy \qquad ......(i)$

$3x + 2y = 13xy \qquad .....(ii)$

Multiplying equation (i) by 2 and equation (ii) by , we get

$2x + 2y = 10xy \qquad ....(iii)$

$3x + 2y = 13xy \qquad ....(iv)$

Subtracting equation (iii) from equation (iv), we get

$3x - 2x = 13xy - 10xy$

$\Rightarrow \quad x = 3xy$

$\Rightarrow \quad \frac{x}{3x} = y$

$\Rightarrow \quad y = \frac{1}{3}$

Putting $y = \frac{1}{3}$ in equation (i), we get

$$x+y=5\times x\times\frac{1}{3}$$

$$x+\frac{1}{3}=\frac{5x}{3}$$

$$\Rightarrow \quad \frac{1}{3}=\frac{5x}{3}-x$$

$$\Rightarrow \quad \frac{1}{3}=\frac{5x-3x}{3}$$

$$\Rightarrow \quad 1=2x$$

$$\Rightarrow \quad 2x=1$$

$$\Rightarrow \quad x=\frac{1}{2}$$

Hence, solution of the given system of equations is $x=\frac{1}{2}, y=\frac{1}{3}$.

31. $x+y=2xy$

$$\frac{x-y}{xy}=6 \ x\neq 0,\ y\neq 0$$

Sol:

The system of the given equation is

$$x+y=2xy \qquad ....(i)$$

And, $\frac{x-y}{xy}=6$

$$x-y=6xy \qquad ....(ii)$$

Adding equation (i) and equation (ii), we get

$$2x=2xy+6xy$$

$$\Rightarrow \quad 2x=8xy$$

$$\Rightarrow \quad \frac{2x}{8x}=y$$

$$\Rightarrow \quad y=\frac{1}{4}$$

Putting $y=\frac{1}{4}$ in equation (i), we get

$$x+\frac{1}{4}=2x\times\frac{1}{4}$$

$$\Rightarrow \quad x+\frac{1}{4}=\frac{x}{2}$$

$$\Rightarrow \quad x-\frac{x}{2}=\frac{-1}{4}$$

$$\Rightarrow \quad \frac{2x-x}{2}=\frac{-1}{4}$$

$$\Rightarrow \quad x=\frac{-2}{4}=\frac{-1}{2}$$

Hence, solution of the given system of equation is $x=\frac{-1}{2}, y=\frac{1}{4}$,

32.
$$2(3u-v)=5uv$$
$$2(u+3v)=5uv$$

Sol:

The system of the given equation is

$$2(3u-v)=5uv$$

$$\Rightarrow \quad 6u-2v=5uv \qquad ....(i)$$

And, $2(u+3v)=5uv$

$$\Rightarrow \quad 2u+6v=5uv \qquad ....(ii)$$

Multiplying equation (i) by 3 and equation (ii) by 1, we get

$$18u-6v=15uv \qquad ....(iii)$$

$$2u+6v=5uv \qquad ....(iv)$$

Adding equation (iii) and equation (iv), we get

$$18u+2u=15uv+5uv$$

$$\Rightarrow \quad 20u=20uv$$

$$\Rightarrow \quad \frac{20u}{20u}=v$$

$$\Rightarrow \quad v=1$$

Putting $v=1$ in equation (i), we get

$$6u-2\times1=5u\times1$$

$$\Rightarrow \quad 6u-2=5u$$

$$\Rightarrow \quad 6u-5u=2$$

$$\Rightarrow \quad u=2$$

Hence, solution of the given system of equation is $u=2, v=1$.

33. $\frac{2}{3x+2y}+\frac{3}{3x-2y}=\frac{17}{5}$

$\frac{5}{3x+2y}+\frac{1}{3x-2y}=2$

Sol:

Let $\frac{1}{3x+2y}=u$ and $\frac{1}{3x-2y}=v.$ Then, the given system of equation becomes

$$2u+3v=\frac{17}{5} \quad ....(i)$$

$$5u+v=2 \quad ....(ii)$$

Multiplying equation (ii) by 3, we get

$$15u-2u=6-\frac{17}{5}$$

$\Rightarrow \quad 13u=\frac{30-17}{5}$

$\Rightarrow \quad 13u=\frac{13}{5}$

$\Rightarrow \quad u=\frac{13}{5\times 13}=\frac{1}{5}$

Putting $u=\frac{1}{5}$ in equation (ii), we get

$$5\times\frac{1}{5}+v=2$$

$\Rightarrow \quad 1+v=2$

$\Rightarrow \quad v=2-1$

$\Rightarrow \quad v=1$

Now, $u=\frac{1}{3x+2y}$

$\Rightarrow \quad \frac{1}{3x+2y}=\frac{1}{5}$

$\Rightarrow \quad 3x+2y=5 \quad .....(iv)$

And, $v=\frac{1}{3x+2y}$

$\Rightarrow \quad \frac{1}{3x-2y}=1$

$\Rightarrow \quad 3x-2y=1 \quad ....(v)$

Adding equation (iv) and (v), we get

$6x = 1+5$

$\Rightarrow \quad 6x = 6$

$\Rightarrow \quad x = 1$

Putting $x = 1$ in equation (v), we get

$3\times 1 + 2y = 5$

$\Rightarrow \quad 2y = 5-3$

$\Rightarrow \quad 2y = 2$

$\Rightarrow \quad y = \frac{2}{2} = 1$

Hence, solution of the given system of equation is $x = 1, y = 1$.

34. $\frac{4}{x} + 3y = 14$

$\frac{3}{x} - 4y = 23$

Sol:

$\frac{4}{x} + 3y = 14$

$\frac{3}{x} - 4y = 23$

Let $\frac{1}{x} = p$

The given equations reduce to:

$4p + 3y = 14$

$\Rightarrow \quad 4p + 3y - 14 = 0 \qquad .....(1)$

$3p - 4y = 23$

$\Rightarrow \quad 3p - 4y - 23 = 0 \qquad .....(2)$

Using cross-multiplication method, we obtain

$$\frac{p}{-69-56} = \frac{y}{-42-(-92)} = \frac{1}{-16-9}$$

$$\frac{p}{-125} = \frac{y}{50} = \frac{-1}{25}$$

$$\frac{p}{-125} = \frac{-1}{25}, \frac{y}{50} = \frac{-1}{25}$$

$p = 5, y = -2$

$\therefore p = \frac{1}{x} = 5$

$x = \frac{1}{5}$

35. $$99x+101y=499$$
$$101x+99y=501$$

Sol:

The given system of equation is

$$99x+101y=499 \qquad .....(i)$$

$$101x+99y=501 \qquad ....(ii)$$

Adding equation (i) and equation (ii), we get

$$99x+101x+101y+99y=499+501$$

$\Rightarrow$ $200x+200y=1000$

$\Rightarrow$ $200(x+y)=1000$

$\Rightarrow$ $x+y=\frac{1000}{200}=5$

$\Rightarrow$ $x+y=5 \qquad .....(iii)$

Subtracting equation (i) by equation (ii), we get

$$101x-99x+99y-101y=501-499$$

$\Rightarrow$ $2x-2y=2$

$\Rightarrow$ $2(x-y)=2$

$\Rightarrow$ $x-y=\frac{2}{2}$

$\Rightarrow$ $x-y=1 \qquad ....(iv)$

Adding equation (iii) and equation (iv), we get

$$2x=5+1$$

$\Rightarrow$ $x=\frac{6}{2}=3$

Putting $x=3$ in equation (iii), we get

$$3+y=5$$

$\Rightarrow$ $y=5-3=2$

Hence, solution of the given system of equation is $x=3, y=2$.

36. $$23x-29y=98$$
$$29x-23y=110$$

Sol:

The given system of equation is

$$23x-29y=98 \qquad .....(i)$$

$$29x-23y=110 \qquad .....(ii)$$

Adding equation (i) and equation (ii), we get

$$23x+29x-29y-23y=98+110$$

$\Rightarrow$ $52x-52y=208$

$\Rightarrow$ $52(x-y)=208$

$\Rightarrow$ $x-y=\frac{208}{52}=4$

$\Rightarrow$ $x-y-4$ ....(iii)

Subtracting equation (i) by equation (ii), we get

$$29x-23x-23y+29y=110-98$$

$\Rightarrow$ $6x+6y=12$

$\Rightarrow$ $6(x+y)=12$

$\Rightarrow$ $x+y=\frac{12}{6}=2$

$\Rightarrow$ $x+y=2$ ......(iv)

Adding equation (iii) and equation (iv), we get

$$2x=2+4=6$$

$\Rightarrow$ $x=\frac{6}{2}=3$

Putting $x=3$ in equation (iv), we get

$$3+y=2$$

$\Rightarrow$ $y=2-3=-1$

Hence, solution of the given system of equation is $x=3, y=-1$.

37. $x-y+z=4$
$x-2y-2z=9$
$2x+y+3z=1$

Sol:

We have,

$x-y+z=4$ ......(i)

$x-2y-2z=9$ ......(ii)

$2x+y+3z=1$ .....(iii)

From equation (i), we get

$$z=4-x+y$$

$\Rightarrow$ $z=-x+y+4$

Subtracting the value of z in equation (ii), we get

$x-2y-2(-x+y+4)=9$

$\Rightarrow \quad x-2y+2x-2y-8=8$

$\Rightarrow \quad 3x-4y=9+8$

$\Rightarrow \quad 3x-4y=17 \quad .....(iv)$

Subtracting the value of z in equation (iii), we get

$2x+y+3(-x+y+4)=1$

$\Rightarrow \quad 2x+y+3x+3y+12=1$

$\Rightarrow \quad -x+4y=1-12$

$\Rightarrow \quad -x+4y=-11 \quad ......(v)$

Adding equations (iv) and (v), we get

$3x-x-4y+4y=17-11$

$\Rightarrow \quad 2x=6$

$\Rightarrow \quad x=\frac{6}{2}=3$

Putting $x=3$ in equation (iv), we get

$3\times3-4y=17$

$\Rightarrow \quad 9-4y=17$

$\Rightarrow \quad -4y=17-9$

$\Rightarrow \quad -4y=8$

$\Rightarrow \quad y=\frac{8}{-4}=-2$

Putting $x=3$ and $y=-2$ in $z=-x+y+4$, we get

$z=-3-2+4$

$\Rightarrow \quad z=-5+4$

$\Rightarrow \quad z=-1$

Hence, solution of the giving system of equation is $x=3, y=-2, z=-1.$

38. $x-y+z=4$
$x+y+z=2$
$2x+y-3z=0$

Sol:

We have,

$x-y+z=4 \quad ......(i)$

$x+y+z=2 \quad ......(ii)$

$2x+y-3z=0 \quad ......(iii)$

From equation (i), we get

$z = 4 - x + y$

$\Rightarrow \quad z = -x + y + 4$

Substituting $z = -x + y + 4$ in equation (ii), we get

$x + y + (-x + y + 4) = 2$

$\Rightarrow \quad x + y - x + y + 4 = 2$

$\Rightarrow \quad 2y + 4 = 2$

$\Rightarrow \quad 2y = 2 - 4 = -2$

$\Rightarrow \quad 2y = -2$

$\Rightarrow \quad y = \frac{-2}{2} = -1$

Substituting the value of z in equation (iii), we get

$2x + y - 3(-x + y + 4) = 0$

$\Rightarrow \quad 2x + y + 3x - 3y - 12 = 0$

$\Rightarrow \quad 5x - 2y - 12 = 0$

$\Rightarrow \quad 5x - 2y = 12 \qquad ....(iv)$

Putting $y = -1$ in equation (iv), we get

$5x - 2 \times (-1) = 12$

$\Rightarrow \quad 5x + 2 = 12$

$\Rightarrow \quad 5x = 12 - 2 = 10$

$\Rightarrow \quad x = \frac{10}{5} = 2$

Putting $x = 2$ and $y = -1$ in $z = -x + y + 4$, we get

$z = -2 + (-1) + 4$

$= -2 - 1 + 4$

$= -3 + 4$

$= 1$

Hence, solution of the giving system of equation is $x = 2, y = -1, z = 1$.

39. $\frac{44}{x+y} + \frac{30}{x-y} = 4$

$\frac{55}{x+y} + \frac{40}{x-y} = 13$

Sol:

Let $\frac{1}{x+y} = u$ *and* $\frac{1}{x-y} = v$.

Then, the system of the given equations becomes

$44u+30v=10 \quad .....(i)$

$55u+40v=13 \quad .....(ii)$

Multiplying equation (i) by 4 and equation (ii) by 3, we get

$176u+120v=40 \quad ....(iii)$

$165u+120v=39 \quad ....(iv)$

Subtracting equation (iv) by equation (iii), we get

$176-165u=40-39$

$\Rightarrow \quad 11u=1$

$\Rightarrow \quad u=\frac{1}{11}$

Putting $u=\frac{1}{11}$ in equation (i), we get

$44\times\frac{1}{11}+30v=10$

$4+30v=10$

$\Rightarrow \quad 30v=10-4$

$\Rightarrow \quad 30v=6$

$\Rightarrow \quad v=\frac{6}{30}=\frac{1}{5}$

Now, $u=\frac{1}{x+y}$

$\Rightarrow \quad \frac{1}{x+y}=\frac{1}{11}$

$\Rightarrow \quad x+y=11 \quad ....(v)$

Adding equation (v) and (vi), we get

$2x=11+5$

$\Rightarrow \quad 2x=16$

$\Rightarrow \quad x=\frac{16}{2}=8$

Putting $x=8$ in equation (v),, we get

$8+y=11$

$\Rightarrow \quad y=11-8-3$

Hence, solution of the given system of equations is $x=8, y=3.$

40. $\frac{4}{x}+15y=21$

$\frac{3}{x}+4y=5$

Sol:

The given system of equation is

$\frac{4}{x}+15y=21$ ......(i)

$\frac{3}{x}+4y=5$ ......(ii)

Multiplying equation (i) by 3 and equation (ii) by 4, we get

$\frac{12}{x}+15y=21$ ......(iii)

$\frac{12}{x}+16y=20$ ......(iv)

Subtracting equation (iii) from equation (iv), we get

$$\frac{12}{x}-\frac{12}{x}+16y-15y=20-21$$

$\Rightarrow \quad y=-1$

Putting $y=-1$ in equation (i), we get

$$\frac{4}{x}+5\times(-1)=7$$

$\Rightarrow \quad \frac{4}{x}-5=7$

$\Rightarrow \quad \frac{4}{x}=7+5$

$\Rightarrow \quad \frac{4}{x}=12$

$\Rightarrow \quad 4=12x$

$\Rightarrow \quad \frac{4}{12}=x$

$\Rightarrow \quad x=\frac{4}{12}$

$\Rightarrow \quad x=\frac{1}{3}$

Hence, solution of the given system of equation is $x=\frac{1}{3}, y=-1.$

41. $2\left(\frac{1}{x}\right)+3\left(\frac{1}{y}\right)=13$

$5\left(\frac{1}{x}\right)-4\left(\frac{1}{y}\right)=-2$

Sol:

Let us write the given pair of equation as

$$2\left(\frac{1}{x}\right)+3\left(\frac{1}{y}\right)=13 \qquad (1)$$

$$5\left(\frac{1}{x}\right)-4\left(\frac{1}{y}\right)=-2 \qquad (2)$$

These equation are not in the form $ax+by+c=0.$ However, if we substitute

$\frac{1}{x}=p$ *and* $\frac{1}{y}=q$ in equations (1) and (2), we get

$2p+3q=13$

$5p-4q=-2$

So, we have expressed the equations as a pair of linear equations. Now, you can use any method to solve these equations, and get $p=2, q=3$

You know that $p=\frac{1}{x}$ *and* $q=\frac{1}{y}.$

Substitute the values of p and q to get

$\frac{1}{x}=2,$ *i.e.*, $x=\frac{1}{2}$ *and* $\frac{1}{y}=3$ *i.e.*, $y=\frac{1}{3}.$

42. $\frac{5}{x-1}+\frac{1}{y-2}=2$

Sol:

x = 4, y = 5

Detailed answer not given in website

43. $\frac{10}{x+y}+\frac{2}{x-y}=4$

$\frac{15}{x+y}-\frac{5}{x-y}=-2$

Sol:

$\frac{10}{x+y}+\frac{2}{x-y}=4$

$$\frac{15}{x+y}-\frac{5}{x-y}=-2$$

Let $\frac{1}{x+y}=p$ *and* $\frac{1}{x-y}=q$

The given equations reduce to:

$$10p+2q=4$$

$$\Rightarrow \quad 10p+2q-4=0 \quad .....(1)$$

$$15p-5q=-2$$

$$\Rightarrow \quad 15p-5q+2=0 \quad .....(2)$$

Using cross-multiplication method, we obtain:

$$\frac{p}{4-20}=\frac{q}{-60-20}=\frac{1}{-50-30}$$

$$\frac{p}{-16}=\frac{q}{-80}=\frac{1}{-80}$$

$$\frac{p}{-16}=\frac{1}{-80} \text{ and } \frac{q}{-80}=\frac{1}{-80}$$

$$p=\frac{1}{5} \text{ and } q=1$$

$$p=\frac{1}{x+y}=\frac{1}{5} \text{ and } q=\frac{1}{x-y}=1$$

$$x+y=5 \quad ......(3)$$

$$x-y=1 \quad ......(4)$$

Adding equation (3) and (4), we obtain:

$$2x=6$$

$$x=3$$

Substituting the value of x in equation (3), we obtain:

$$y=2$$

$$\therefore x=3, y=2$$

44. $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$

$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=-\frac{1}{8}$

Sol:

Let us put $\frac{1}{x-1}=p$ *and* $\frac{1}{y-2}=q.$ Then the given equations

$5\left(\frac{1}{x-1}\right)+\frac{1}{y-2}=2$ ......(1)

$6\left(\frac{1}{x-1}\right)-3\left(\frac{1}{y-2}\right)=1$ ......(2)

Can be written as: $5p+q=2$ ....(3)

$6p-3q=1$ ......(4)

Equations (3) and (4) from a pair of linear equations in the general form. Now, you can use any method to solve these equations. We get $p=\frac{1}{3}$ *and* $q=\frac{1}{3}$.

Substituting $\frac{1}{x-1}$ for p, we have

$\frac{1}{x-1}=\frac{1}{3}$,

i.e., $x-1=3$, *i.e.*, $x=4$.

Similarly, substituting $\frac{1}{y-2}$ for q, we get

$\frac{1}{y-2}=\frac{1}{3}$

i.e., $x-1=3$, *i.e.*, $x=4$

Similarly, substituting $\frac{1}{y-2}$ for q, we get

$\frac{1}{y-2}=\frac{1}{3}$

i.e., $3=y-2$, *i.e.*, $y=5$

Hence, $x=4, y=5$ is the required solution of the given pair of equations.

45. $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$

$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

Sol:

$\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$

$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

Let $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$

The given equations reduce to:

$2p + 3q = 2 \qquad .....(1)$

$4p - 9q = -1 \qquad .....(2)$

Multiplying equation (1) by (3), we obtain:

$6p + 9q = 6 \qquad .....(3)$

Adding equation (2) and (3), we obtain:

$10p = 5$

$p = \frac{1}{2}$

Putting the value of p in equation (1), we obtain:

$2 \times \frac{1}{2} + 3q = 2$

$3q = 1$

$q = \frac{1}{3}$

$\therefore p = \frac{1}{\sqrt{x}} = \frac{1}{2}$

$\sqrt{x} = 2$

$x = 4$

$q = \frac{1}{\sqrt{y}} = \frac{1}{3}$

$\sqrt{y} = 3$

$y = 9$

$\therefore x = 4, y = 9$

46. $$\frac{7x - 2y}{xy} = 5$$
$$\frac{8x + 7y}{xy} = 15$$

Sol:

$$\frac{7x-2y}{xy}=5$$

$$\Rightarrow \quad \frac{7}{y}-\frac{2}{x}=5 \quad ...(1)$$

$$\frac{8x+7y}{xy}=15$$

$$\Rightarrow \quad \frac{8}{y}+\frac{7}{x}=15 \quad .....(2)$$

Let $\frac{1}{x}=p$ and $\frac{1}{y}=q$

The given equations reduce to:

$$-2p+7q=5$$

$$\Rightarrow \quad -2p+7q-5=0 \quad ....(3)$$

$$7p+8q=15$$

$$\Rightarrow \quad 7p+8q-15=0 \quad ....(4)$$

Using cross multiplication method, we obtain:

$$\frac{p}{-105-(-40)}=\frac{q}{-35-30}=\frac{1}{-16-49}$$

$$\frac{p}{-65}=\frac{1}{-65},\frac{q}{-65}=\frac{1}{-65}$$

$$p=1,q=1$$

$$p=\frac{1}{x}=1,q=\frac{1}{y}=1$$

$$x=1,y=1$$

47. $152x-378y=-74$
$-378x+152y=-604$

Sol:

$$152x-378y=-74 \quad ....(1)$$

$$-378x+152y=-604 \quad ....(2)$$

Adding the equations (1) and (2), we obtain:

$$-226x-226y=-678$$

$$\Rightarrow x+y=3 \quad .....(3)$$

Subtracting the equation (2) from equation (1), we obtain

$$530x-530y=530$$

$\Rightarrow x-y=1 \qquad .....(4)$

Adding equations (3) and (4), we obtain:

$2x=4$

$x=2$

Substituting the value of x in equation (3), we obtain:

$y=1$

Exercise 3.4

Solve each of the following systems of equations by the method of cross-multiplication:

1. $x+2y+1=0$
$2x-3y-12=0$

Sol:

The given system of equation is

$x+2y+1=0$

$2x-3y-12=0$

Here,

$a_1=1, b_1=2, c_1=1$

$a_2=2, b_2=-3 \ and \ c_2=-12$

By cross-multiplication, we get

$$\Rightarrow \frac{x}{2\times(-12)-1\times(-3)}=\frac{-y}{1\times(-12)-1\times 2}=\frac{1}{1\times(-3)-2\times 2}$$

$$\Rightarrow \frac{x}{-24+3}=\frac{-y}{-12-2}=\frac{1}{-3-4}$$

$$\Rightarrow \frac{x}{-21}=\frac{-y}{-14}=\frac{1}{-7}$$

Now,

$$\frac{x}{-21}=\frac{1}{-7}$$

$$\Rightarrow \qquad x=\frac{-21}{-7}=3$$

And,

$$\frac{-y}{-14} = \frac{1}{-7}$$

$$\Rightarrow \quad \frac{y}{14} = \frac{-1}{7}$$

$$\Rightarrow \quad y = \frac{-14}{7} = -2$$

Hence, the solution of the given system of equations is $x = 3, y = -2$.

2. $3x + 2y + 25 = 0$
$2x + y + 10 = 0$

Sol:

The given system of equation is

$3x + 2y + 25 = 0$

$2x + y + 10 = 0$

Here,

$a_1 = 3, b_1 = 2, c_1 = 25$

$a_2 = 2, b_2 = 1$ *and* $c_2 = 10$

By cross-multiplication, we have

$$\Rightarrow \frac{x}{2\times10 - 25\times1} = \frac{-y}{3\times10 - 25\times2} = \frac{1}{3\times1 - 2\times2}$$

$$\Rightarrow \frac{x}{20 - 25} = \frac{-y}{30 - 50} = \frac{1}{3 - 4}$$

$$\Rightarrow \frac{x}{-5} = \frac{-y}{-20} = \frac{1}{-1}$$

Now, $\frac{x}{-5} = \frac{1}{-1}$

$$\Rightarrow x = \frac{-5}{-1} = 5$$

And,

$$\frac{-y}{-20} = \frac{1}{-1}$$

$$\Rightarrow \quad \frac{y}{20} = 1$$

$$\Rightarrow \quad y = -20$$

Hence, $x = 5, y = -20$ is the solution of the given system of equations.

3. $2x + y - 35 = 0$
$3x + 4y - 65 = 0$

Sol:

The given system of equations may be written as

$2x+y-35=0$

$3x+4y-65=0$

Here,

$a_1=2, b_1=1, c_1=-35$

$a_2=3, b_2=4 \ and \ c_2=-65$

By cross multiplication, we have

$$\Rightarrow \frac{x}{1\times(-65)-(-35)\times 4}=\frac{-y}{2\times(-65)-(-35)\times 3}=\frac{1}{2\times 4-1\times 3}$$

$$\Rightarrow \frac{x}{-65+140}=\frac{-y}{-130+105}=\frac{1}{8-3}$$

$$\Rightarrow \frac{x}{75}=\frac{-y}{-25}=\frac{1}{5}$$

$$\Rightarrow \frac{x}{75}=\frac{y}{25}=\frac{1}{5}$$

Now,

$$\frac{y}{25}=\frac{1}{5}$$

$$\Rightarrow \quad y=\frac{25}{5}=5$$

Hence, $x=15, y=5$ is the solution of the given system of equations.

4. $2x-y-6=0$

$x-y-2=0$

Sol:

The given system of equations may be written as

$2x-y-6=0$

$x-y-2=0$

Here,

$a_1=2, b_1=-1, c_1=-6$

$a_2=1, b_2=-1 \ and \ c_2=-2$

By cross multiplication, we get

$$\Rightarrow \frac{x}{(-1)\times(-2)-(-6)\times(-1)} = \frac{-y}{2\times(-2)-(-6)\times 1} = \frac{1}{2\times(-1)-(-1)\times 1}$$

$$\Rightarrow \frac{x}{2-6} = \frac{-y}{-4+6} = \frac{1}{-2+1}$$

$$\Rightarrow \frac{x}{-4} = \frac{-y}{2} = \frac{1}{-1}$$

$$\Rightarrow \frac{x}{-4} = \frac{-y}{2} = -1$$

Now,

$$\frac{x}{-4} = -1$$

$$\Rightarrow \quad x = (-4)\times(-1) = 4$$

And,

$$\frac{-y}{2} = -1$$

$$\Rightarrow \quad -y = (-1)\times 2$$

$$\Rightarrow \quad -y = -2$$

$$\Rightarrow \quad y = 2$$

Hence, $x = 4, y = 2$ is the solution of the given system of the equations.

5. $\frac{x+y}{xy} = 2$

$\frac{x-y}{xy} = 6$

Sol:

The given system of equations is

$$\frac{x+y}{xy} = 2$$

$$\Rightarrow \quad \frac{x}{xy} + \frac{y}{xy} = 2$$

$$\Rightarrow \quad \frac{1}{y} + \frac{1}{x} = 2$$

$$\Rightarrow \quad \frac{1}{x} + \frac{1}{y} = 2 \qquad ......(i)$$

And,

$$\frac{x-y}{xy}=6$$

$$\Rightarrow \quad \frac{x}{xy}-\frac{y}{xy}=6$$

$$\Rightarrow \quad \frac{1}{y}-\frac{1}{x}=6$$

$$\Rightarrow \quad \frac{1}{x}-\frac{1}{y}=6 \qquad ......(ii)$$

Taking $u=\frac{1}{x}$ *and* $v=\frac{1}{y}$, we get

$u+v=2 \Rightarrow u+v-2=0 \qquad .....(iii)$

And, $u-v=-6 \Rightarrow u-v+6=0 \qquad .....(iv)$

Here,

$a_1=1, b_1=1, c_1=-2$

$a_2=1, b_2=-1$ *and* $c_2=6$

By cross multiplication

$$\Rightarrow \frac{u}{1\times 6-(-2)\times(-1)}=\frac{v}{1\times 6-(-2)\times 1}=\frac{1}{1\times(-1)-1\times 1}$$

$$\Rightarrow \frac{u}{6-2}=\frac{-v}{6+2}=\frac{1}{-1-1}$$

$$\Rightarrow \frac{u}{4}=\frac{-v}{8}=\frac{1}{-2}$$

Now, $\frac{u}{4}=\frac{1}{-2}$

$$\Rightarrow u=\frac{4}{-2}=-2$$

And, $\frac{-v}{8}=\frac{1}{-2}$

$$\Rightarrow -v=\frac{8}{-2}=-4$$

$$\Rightarrow -v=-4$$

$$\Rightarrow v=4$$

Now, $x=\frac{1}{u}=\frac{-1}{2}$ *and* $y=\frac{1}{v}=\frac{1}{4}$

Hence, $x=\frac{-1}{2}, y=\frac{1}{4}$ is the solution of the given system of equations.

6. $ax + by = a - b$
$bx - ay = a + b$

Sol:

The given system of equations is

$ax + by = a - b \quad ....(i)$

$bx - ay = a + b \quad ....(ii)$

Here,

$a_1 = a, b_1 = b, c_1 = b - a$

$a_2 = b, b_2 = -a \text{ and } c_2 = -a - b$

By cross multiplication, we get

$$\Rightarrow \frac{x}{(-a-b)\times(b)-(b-a)\times(-a)} = \frac{-y}{(-a-b)\times(a)-(b-a)\times(-b)} = \frac{1}{-a\times a - b\times b}$$

$$\Rightarrow \frac{x}{-ab-b^2+ab-a^2} = \frac{-y}{-a^2-ab-b^2+ab} = \frac{1}{-a^2-b^2}$$

$$\Rightarrow \frac{x}{-b^2-a^2} = \frac{-y}{-a^2-b^2} = \frac{1}{-a^2-b^2}$$

Now,

$$\frac{x}{-b^2-a^2} = \frac{1}{-a^2-b^2}$$

$$\Rightarrow \quad x = \frac{-b^2-a^2}{-a^2-b^2}$$

$$= \frac{-(b^2+a^2)}{(a^2+b^2)}$$

$$= \frac{(a^2+b^2)}{(a^2+b^2)}$$

$$\Rightarrow \quad x = 1$$

And,

$$\frac{-y}{-a^2-b^2} = \frac{1}{-a^2-b^2}$$

$$\Rightarrow \quad -y = \frac{-(a^2+b^2)}{-(a^2+b^2)}$$

$$\Rightarrow \quad -y = 1$$

$$\Rightarrow \quad y = -1$$

Hence, $x = 1, y = -1$ is the solution of the given system of the equations.

7. $x+ay-b=0$
$ax-by-c=0$

Sol:

The given system of equations may be written as

$x+ay-b=0$

$ax-by-c=0$

Here,

$a_1=1, b_1=a, c_1=-b$

$a_2=a, b_2=-b \text{ and } c_2=-c$

By cross multiplication, we get

$$\Rightarrow \frac{x}{(a)\times(-c)-(-b)\times(-b)}=\frac{-y}{1\times(-c)-(-b)\times a}=\frac{1}{1\times(-b)-a\times a}$$

$$\Rightarrow \frac{x}{-ac-b^2}=\frac{-y}{-c+ab}=\frac{1}{-b-a^2}$$

Now,

$$\frac{x}{-ac-b^2}=\frac{1}{-b-a^2}$$

$$\Rightarrow \quad x=\frac{-ac-b^2}{-b-a^2}$$

$$\Rightarrow \quad x=\frac{-\left(b^2+ac\right)}{-\left(a^2+b\right)}$$

$$=\frac{b^2+ac}{a^2+b}$$

And

$$\frac{-y}{-c+ab}=\frac{1}{-b-a^2}$$

$$\Rightarrow \quad -y=\frac{ab-c}{-\left(a^2+b\right)}$$

$$\Rightarrow \quad y=\frac{ab-c}{a^2+b}$$

Hence, $x=\dfrac{ac+b^2}{a^2+b}, y=\dfrac{ab-c}{a^2+b}$ is the solution of the given system of the equations.

8. $ax+by=a^2$
$bx+ay=b^2$

Sol:

The system of the given equations may be written as

$ax+by-a^2=0$

$bx+ay-b^2=0$

Here,

$a_1=a, b_1=b, c_1=-a^2$

$a_2=b, b_2=a$ and $c_2=-b^2$

By cross multiplication, we get

$$\Rightarrow \frac{x}{b\times\left(-b^2\right)-\left(-a^2\right)\times a}=\frac{-y}{a\times\left(-b^2\right)-\left(-a^2\right)\times b}=\frac{1}{a\times a-b\times b}$$

$$\Rightarrow \frac{x}{-b^3+a^3}=\frac{-y}{-ab^2+a^2b}=\frac{1}{a^2-b^2}$$

Now,

$$\frac{x}{-b^3+a^3}=\frac{1}{a^2-b^2}$$

$$\Rightarrow \quad x=\frac{a^3-b^3}{a^2-b^2}$$

$$=\frac{(a-b)\left(a^2+ab+b^2\right)}{(a-b)(a+b)}$$

$$=\frac{a^2+ab+b^2}{a+b}$$

And,

$$\frac{-y}{-ab^2+a^2b}=\frac{1}{a^2-b^2}$$

$$\Rightarrow \quad -y=\frac{a^2b-ab^2}{a^2-b^2}$$

$$\Rightarrow \quad y=\frac{ab^2-a^2b}{a^2-b^2}$$

$$=\frac{ab(b-a)}{(a-b)(a+b)}$$

$$=\frac{-ab(a-b)}{(a-b)(a+b)}$$

$$=\frac{-ab}{a+b}$$

Hence, $x=\dfrac{a^2+ab+b^2}{a+b}, y=\dfrac{-ab}{a+b}$ is the solution of the given system of the equations.

9. $\frac{x}{a}+\frac{y}{b}=2$

$ax-by=a^2-b^2$

Sol:

The system of the given equations may be written as

$$\frac{1}{a}x\times+\frac{1}{b}\times y-2=0$$

$$ax-by+b^2-a^2=0$$

Here,

$$a_1=\frac{1}{a},b_1=\frac{1}{b},c_1=-2$$

$$a_2=a,b_2=-b \text{ and } c_2=b^2-a^2$$

By cross multiplication, we get

$$\Rightarrow \frac{x}{\frac{1}{b}\times\left(b^2-a^2\right)-(-2)\times(-b)}=\frac{-y}{\frac{1}{a}\times\left(b^2-a^2\right)-(-2)\times a}=\frac{1}{\frac{-b\times 1}{a}-\frac{a\times 1}{b}}$$

$$\Rightarrow \frac{x}{\frac{b^2-a^2}{b}-2b}=\frac{-y}{\frac{b^2-a^2}{b}+2b}=\frac{1}{\frac{-b}{a}-\frac{a}{b}}$$

$$\Rightarrow \frac{x}{\frac{b^2-a^2-2b^2}{b}}=\frac{-y}{\frac{b^2-a^2+2b^2}{a}}=\frac{1}{\frac{-b^2-a^2}{ab}}$$

$$\Rightarrow \frac{x}{\frac{-a^2-b^2}{b}}=\frac{-y}{\frac{b^2+a^2}{a}}=\frac{1}{\frac{-b^2-a^2}{ab}}$$

Now,

$$\frac{x}{\frac{-a^2-b^2}{b}}=\frac{1}{\frac{-b^2-a^2}{ab}}$$

$$\Rightarrow \quad x=\frac{-a^2-b^2}{b}\times\frac{ab}{-b^2-a^2}$$

And,

$$\frac{-y}{\frac{b^2+a^2}{a}}=\frac{1}{\frac{-b^2-a^2}{ab}}$$

$$\Rightarrow \quad -y=\frac{b^2+a^2}{a}\times\frac{ab}{-b^2-a^2}$$

$$\Rightarrow \quad -y = \frac{(b^2+a^2)\times b}{-(b^2+a^2)}$$

$$\Rightarrow \quad y = b$$

Hence, $x = a, y = b$ is the solution of the given system of the equations.

10. $\frac{x}{a} + \frac{y}{b} = a + b$

Sol:

The given system of equation may be written as

$$\frac{1}{a}x\times+\frac{1}{b}\times y-(a+b)=0$$

$$\frac{1}{a^2}x\times+\frac{1}{b^2}\times y-2=0$$

Here,

$$a_1=\frac{1}{a}, b_2=\frac{1}{b}, c_1=-(a+b)$$

$$a_2=\frac{1}{a^2}, b_2=\frac{1}{b^2}, \textit{ and } c_2=-2$$

By cross multiplication, we get

$$\Rightarrow \frac{x}{\frac{1}{b}\times(-2)-\frac{1}{b^2}x-(a+b)} = \frac{-y}{\frac{1}{a}\times-2-\frac{1}{a^2}x-(a+b)} = \frac{1}{\frac{1}{a}\times\frac{1}{b^2}-\frac{1}{a^2}\times\frac{1}{b}}$$

$$\Rightarrow \frac{x}{-\frac{2}{b}+\frac{a}{b^2}+\frac{1}{b}} = \frac{-y}{-\frac{2}{a}+\frac{1}{a}+\frac{b}{a^2}} = \frac{1}{-\frac{1}{ab^2}-\frac{1}{a^2b}}$$

$$\Rightarrow \frac{x}{\frac{a}{b^2}-\frac{1}{b}} = \frac{-y}{-\frac{1}{a}+\frac{b}{a^2}} = \frac{1}{\frac{1}{ab^2}-\frac{1}{a^2b}}$$

$$\Rightarrow \frac{x}{\frac{a-b}{b^2}} = \frac{y}{\frac{a-b}{a^2}} = \frac{1}{\frac{a-b}{a^2b^2}}$$

$$\Rightarrow x = \frac{a-b}{b^2}\times\frac{1}{\frac{a-b}{a^2b^2}} = a^2 \textit{ and } y = \frac{a-b}{a^2}\times\frac{1}{\frac{a-b}{a^2b^2}} = b^2$$

Hence, $x = a^2, y = b^2$ is the solution of the given system of the equtaions.

11. $\frac{x}{a} = \frac{y}{b}$

$ax + by = a^2 + b^2$

Sol:

$\frac{x}{a}=\frac{y}{b}$

$ax+by=a^2+b^2$

Here $a_1=\frac{1}{a}, b_1=\frac{-1}{b}, c_1=0$

$a_2=a, b_2=b, c_2=-\left(a^2+b^2\right)$

By cross multiplication, we get

$$\frac{x}{-\frac{1}{b}\left(-\left(a^2+b^2\right)\right)-b(0)}=\frac{-y}{\frac{1}{a}\left(-\left(a^2+b^2\right)\right)-a(0)}=\frac{1}{\frac{1}{a}(b)-a\times\left(\frac{-1}{b}\right)}$$

$$\frac{x}{\frac{a^2+b^2}{b}}=\frac{y}{\frac{a^2+b^2}{a}}=\frac{1}{\frac{b}{a}+\frac{a}{b}}$$

$$x=\frac{\frac{a^2+b^2}{b}}{\frac{b}{a}+\frac{a}{b}}=\frac{\frac{a^2+b^2}{b}}{\frac{b^2+a^2}{ab}}=a$$

$$y=\frac{\frac{a^2+b^2}{a}}{\frac{b}{a}+\frac{a}{b}}=\frac{\frac{a^2+b^2}{b}}{\frac{b^2+a^2}{ab}}=b$$

Solution is (a, b)

12. $\frac{5}{x+y}-\frac{2}{x-y}=-1$

$\frac{15}{x+y}+\frac{7}{x-y}=10, where\ x\ \neq 0\ and\ y\ \neq 0$

Sol:

Let $\frac{1}{x+y}=u$ and $\frac{1}{x-y}=v$. Then, the given system of equations becomes

$5u-2v=-1$

$15u+7v=10$

Here

$a_1=5, b_1=-2, c_1=1$

$a_2=15, b_2=7$ and $c_2=-10$

By cross multiplication, we get

$$\Rightarrow\frac{u}{(-2)\times(-10)-1\times 7}=\frac{u}{5\times(-10)-1\times 15}=\frac{1}{5\times 7-(-2)\times 15}$$

$$\Rightarrow \frac{u}{20-7} = \frac{-v}{-50-15} = \frac{1}{35+30}$$

$$\Rightarrow \frac{u}{13} = \frac{-v}{-65} = \frac{1}{65}$$

$$\Rightarrow \frac{u}{13} = \frac{v}{65} = \frac{1}{65}$$

Now,

$$\frac{u}{13} = \frac{1}{65}$$

$$\Rightarrow \quad u = \frac{13}{65} = \frac{1}{5}$$

And,

$$\frac{v}{65} = \frac{1}{65}$$

$$\Rightarrow \quad v = \frac{65}{65} = 1$$

Now,

$$u = \frac{1}{x+y}$$

$$\Rightarrow \quad \frac{1}{x+y} = \frac{1}{5} \qquad ......(i)$$

And,

$$v = \frac{1}{x-y}$$

$$\Rightarrow \quad \frac{1}{x-y} = 1$$

$$\Rightarrow \quad x-y = 1 \qquad ......(ii)$$

Adding equation (i) and (ii), we get

$$2x = 5+1$$

$$\Rightarrow \quad 2x = 6$$

$$\Rightarrow \quad x = \frac{6}{2} = 3$$

13. $\frac{2}{x} + \frac{3}{y} = 13$

$\frac{5}{x} - \frac{4}{y} = -2$, where $x \neq 0$ *and* $y \neq 0$

Sol:

The given system of equation is

$$\frac{2}{x}+\frac{3}{y}=13$$

$$\frac{5}{x}-\frac{4}{y}=-2, \text{where } x\neq 0 \text{ } and \text{ } y\neq 0$$

Let $\frac{1}{x}=u$ *and* $\frac{1}{y}=v$, Then, the given system of equations becomes

$$2u+3v=13$$

$$5u-4v=-2$$

Here,

$$a_1=2, b_1=3, c_1=-13$$

$$a_2=5, b_2=-4 \text{ } and \text{ } c_2=2$$

By cross multiplication, we have

$$\Rightarrow \frac{u}{3\times 2-(-13)\times(-4)}=\frac{-v}{2\times 2-(-13)\times 5}=\frac{1}{2\times(-4)-3\times 5}$$

$$\Rightarrow \frac{u}{6-52}=\frac{-v}{4+65}=\frac{1}{-8-15}$$

$$\Rightarrow \frac{u}{-46}=\frac{-v}{69}=\frac{1}{-23}$$

Now,

$$\frac{u}{-46}=\frac{1}{-23}$$

$$\Rightarrow \quad u=\frac{-46}{-23}=2$$

And

$$\frac{-v}{69}=\frac{1}{-23}$$

$$\Rightarrow \quad v=\frac{-69}{-23}=3$$

Now,

$$x=\frac{1}{u}=\frac{1}{2}$$

And,

$$y=\frac{1}{v}=\frac{1}{3}$$

Hence, $x=\frac{1}{2}, y=\frac{1}{3}$ is the solution of the given system of equations.

14. $ax+by=\frac{a+b}{2}$

$3x+5y=4$

Sol:

The given system of equation is

$$ax+by=\frac{a+b}{2} \qquad ......(i)$$

$$3x+5y=4 \qquad ......(ii)$$

From (i), we get

$$2(ax+by)=a+b$$

$$\Rightarrow 2ax+2by-(a+b)=0 \qquad .....(iii)$$

From (ii), we get

$$3x+5y-4=0$$

Here,

$$a_1=2a, b_1=2b, c_1=-(a+b)$$

$$a_2=3, b_2=5, c_2=-4$$

By cross multiplication, we have

$$\Rightarrow \frac{x}{2b\times(-4)-[-(a+b)]\times 5}=\frac{-y}{2a\times(-4)-[-(a+b)]\times 3}=\frac{1}{2a\times 5-2b\times 3}$$

$$\Rightarrow \frac{x}{-8b+5(a+b)}=\frac{-y}{-8a+3(a+b)}=\frac{1}{10a-6b}$$

$$\Rightarrow \frac{x}{-8b+5a+5b}=\frac{-y}{-8a+3a+3b}=\frac{1}{10a-6b}$$

$$\Rightarrow \frac{x}{5a-3b}=\frac{-y}{-5a+3b}=\frac{1}{10a-6b}$$

Now,

$$\frac{x}{5a-3b}=\frac{-y}{-5a+3b}=\frac{1}{10a-6b}$$

$$\Rightarrow \qquad x=\frac{5a-3b}{10a-6b}=\frac{5a-3b}{2(5a-3b)}=\frac{1}{2}$$

And,

$$\frac{-y}{-5a+3b}=\frac{1}{10a-6b}$$

$$\Rightarrow \qquad -y=\frac{-5a+3b}{2(5a-3b)}$$

$$\Rightarrow \quad y=\frac{-(-5a+3b)}{2(5a-3b)}$$

$$=\frac{5a-3b}{2(5a-3b)}$$

$$\Rightarrow \quad y=\frac{1}{2}$$

Hence, $x=\frac{1}{2}, y=\frac{1}{2}$ is the solution of the given system of equations.

15. $2ax+3by=a+2b$
$3ax+2by=2a+b$

Sol:

The given system of equations is

$2ax+3by=a+2b \qquad ....(i)$

$3ax+2by=2a+b \qquad ...(ii)$

Here,

$a_1=2a, b_1=3b, c_1=-(a+2b)$

$a_2=3a, b_2=2b, c_2=-(2a+b)$

By cross multiplication we have

$$\Rightarrow \frac{x}{-3b\times(2a+b)-[-(a+2b)]\times 2b}=\frac{-y}{-2a\times(2a+b)-[-(a+2b)]\times 3a}=\frac{1}{2a\times 2b-3b\times 3a}$$

$$\Rightarrow \frac{x}{-3b+(2a+b)+2b(a+2b)}=\frac{-y}{-2a(2a+b)+3a(a+2b)}=\frac{1}{4ab-9ab}$$

$$\Rightarrow \frac{x}{-6ab-3b^2+2ab+4b^2}=\frac{-y}{-4a^2-2ab+3a^2+6ab}=\frac{1}{4ab-9ab}$$

$$\Rightarrow \frac{x}{-4ab+b^2}=\frac{-y}{-a^2+4ab}=\frac{1}{-5ab}$$

Now,

$$\frac{x}{-4ab+b^2}=\frac{1}{-5ab}$$

$$\Rightarrow x=\frac{-4ab+b^2}{-5ab}$$

$$=\frac{-b(4a-b)}{-5ab}$$

$$=\frac{4a-b}{5a}$$

And, $\frac{-y}{-a^2+4ab}=\frac{1}{-5ab}$

$\Rightarrow \quad -y=\frac{-a^2+4ab}{-5ab}$

$\Rightarrow \quad -y=\frac{-a(a-4b)}{-5ab}$

$\Rightarrow \quad -y=\frac{a-4b}{5b}$

$\Rightarrow \quad y=\frac{4b-a}{5b}$

Hence, $x=\frac{4a-b}{5a}, y=\frac{4b-a}{5b}$ is the solution of the given system of equation.

16. $5ax+6by=28$
$3ax+4by-18=0$

Sol:

The given system of equation is

$5ax+6by=28$

$\Rightarrow \quad 5ax+6by-28=0 \quad .....(i)$

and, $3ax+4by-18=0$

$\Rightarrow \quad 3ax+4by-18=0 \quad .....(ii)$

Here,

$a_1=5a, b_1=6b, c_1=-28$

$a_2=3a, b_2=4b$ *and* $c_2=-18$

By cross multiplication we have

$\Rightarrow \frac{x}{6b\times(-18)-(-28)\times 4b}=\frac{-y}{5a\times(-18)-(-28)\times 3a}=\frac{1}{5a\times 4b-6b\times 3a}$

$\Rightarrow \frac{x}{-108b+112b}=\frac{-y}{-90a+84a}=\frac{1}{20ab-18ab}$

$\Rightarrow \frac{x}{4b}=\frac{-y}{-6a}=\frac{1}{2ab}$

Now,

$\frac{x}{4b}=\frac{1}{2ab}$

$\Rightarrow \quad x=\frac{5b-2a}{10ab}$

And,

$$\frac{-y}{-6a}=\frac{1}{2ab}$$

$$\Rightarrow \quad y=\frac{6a}{2ab}=\frac{3}{b}$$

Hence, $x=\frac{2}{a}, y=\frac{3}{b}$ is the solution of the given system of equations.

17. $(a+2b)x+(2a-b)y=2$
$(a-2b)x+(2a+b)y=3$

Sol:

The given system of equations may be written as

$(a+2b)x+(2a-b)y-2=0$

$(a-2b)x+(2a+b)y-3=0$

Here,

$a_1=a+2b, b_1=2a-b, c_1=-2$

$a_2=a-2b, b_2=2a+b$ and $c_2=-3$

By cross multiplication, we have

$$\Rightarrow \frac{x}{-3(2a-b)-(-2)(2a+b)}=\frac{-y}{3(a+2b)-(-2)(a-2b)}=\frac{1}{(a+2b)(2a+b)-(2a-b)(a-2b)}$$

$$\Rightarrow \frac{x}{-6a+3b+4a+2b}=\frac{-y}{-3a-6b+2a-4b}=\frac{1}{2a^2+ab+4ab+2b^2-(2a^2-4ab-ab+2b^2)}$$

$$\Rightarrow \frac{x}{-2a+5b}=\frac{-y}{-a-10b}=\frac{1}{2a^2+ab+4ab+2b^2-(2a^2-4ab-ab+2b^2)}$$

$$\Rightarrow \frac{x}{-2a+5b}=\frac{-y}{-(a+10b)}=\frac{1}{10ab}$$

$$\Rightarrow \frac{x}{-2a+5b}=\frac{y}{a+10b}=\frac{1}{10ab}$$

Now,

$$\frac{x}{-2a+5b}=\frac{1}{10ab}$$

$$\Rightarrow y=\frac{a+10b}{10ab}$$

And,

$$\frac{y}{a+10b}=\frac{1}{10ab}$$

$$\Rightarrow \quad y=\frac{a+10b}{10ab}$$

Hence, $x=\frac{5b-2a}{10ab}, y=\frac{a+10b}{10ab}$ is the solution of the given system of equations.

18. $x\left(a-b+\frac{ab}{a-b}\right)=y\left(a+b-\frac{ab}{a+b}\right)$

$x+y=2a^2$

Sol:

The given system of equation is

$$x\left(a-b+\frac{ab}{a-b}\right)=y\left(a+b-\frac{ab}{a+b}\right) \quad \text{......(i)}$$

$$x+y=2a^2 \quad \text{......(ii)}$$

From equation (i), we get

$$x\left(a-b+\frac{ab}{a-b}\right)-y\left(a+b-\frac{ab}{a+b}\right)=0$$

$$\Rightarrow \quad x\left(\frac{(a-b)^2+ab}{a-b}\right)-y\left(\frac{(a+b)^2-ab}{a+b}\right)=0$$

$$\Rightarrow \quad x\left(\frac{a^2+b^2-2ab+ab}{a-b}\right)-y\left(\frac{a^2+b^2+2ab-ab}{a+b}\right)=0$$

$$\Rightarrow \quad x\left(\frac{a^2+b^2-ab}{a-b}\right)-y\left(\frac{a^2+b^2+ab}{a+b}\right)=0 \quad \text{.....(iii)}$$

From equation (ii), we get

$x+y-2a^2=0$

Here,

$a_1=\frac{a^2+b^2-ab}{a-b}, b_1=-\left(\frac{a^2+b^2+ab}{a+b}\right), c_1=0$

$a_2=1, b_2=1$ *and* $c_2=-2a^2$

By cross multiplication, we get

$$\Rightarrow \frac{x}{(-2a^2)\left[-\left(\frac{a^2+b^2+ab}{a+b}\right)\right]-0\times 1}=\frac{-y}{(-2a^2)\left[-\left(\frac{a^2+b^2-ab}{a-b}\right)\right]-0\times 1}=\frac{1}{\frac{a^2+b^2-ab}{a-b}\left[-\frac{(a^2+b^2+ab)}{a+b}\right]}$$

$$\Rightarrow \frac{x}{2a^2\left(\frac{a^2+b^2+ab}{a+b}\right)}=\frac{y}{(2a^2)\left(\frac{a^2+b^2-ab}{a-b}\right)}=\frac{1}{\frac{a^2+b^2-ab}{a-b}+\frac{a^2+b^2-ab}{a+b}}$$

$$\Rightarrow \frac{x}{2a^2\left(\frac{a^2+b^2+ab}{a+b}\right)} = \frac{y}{\left(2a^2\right)\left(\frac{a^2+b^2-ab}{a-b}\right)} = \frac{1}{\frac{(a+b)\left(a^2+b^2-ab\right)+(a-b)\left(a^2+b^2+ab\right)}{(a-b)(a+b)}}$$

$$\Rightarrow \frac{x}{2a^2\left(\frac{a^2+b^2+ab}{a+b}\right)} = \frac{y}{2a^2\left(\frac{a^2+b^2-ab}{a-b}\right)} = \frac{1}{\frac{a^3+b^3+a^3-b^3}{(a-b)(a+b)}}$$

$$\Rightarrow \frac{x}{2a^2\left(\frac{a^2+b^2+ab}{a+b}\right)} = \frac{y}{2a^2\left(\frac{a^2+b^2-ab}{a-b}\right)} = \frac{1}{\frac{2a^3}{(a-b)(a+b)}}$$

Now,

$$\frac{x}{2a^2\left(\frac{a^2+b^2+ab}{a+b}\right)} = \frac{1}{\frac{2a^3}{(a-b)(a+b)}}$$

$$\Rightarrow \quad x = \frac{2a^2\left(a^2+b^2+ab\right)}{a+b} \times \frac{(a-b)(a+b)}{2a^3}$$

$$= \frac{(a-b)\left(a^2+b^2+ab\right)}{a}$$

$$= \frac{a^3-b^3}{a} \qquad \left[\because a^3-b^3=(a-b)\left(a^2+b^2+ab\right)\right]$$

And,

$$\frac{y}{2a^2\left(\frac{a^2+b^2-ab}{a-b}\right)} = \frac{1}{\frac{2a^3}{(a-b)(a+b)}}$$

$$\Rightarrow \quad y = \frac{2a^2\left(a^2+b^2-ab\right)}{a-b} \times \frac{(a-b)(a+b)}{2a^3}$$

$$= \frac{(a+b)\left(a^2+b^2-ab\right)}{a}$$

$$= \frac{a^3+b^3}{a} \qquad \left[\because a^3+b^3-(a-b)\left(a^2+b^2-ab\right)\right]$$

Hence, $x = \frac{a^3-b^3}{a}, y = \frac{a^2+b^2}{a}$ is the solution of the given system of equaatiions.

The given system of equation id

$$x\left(a-b+\frac{ab}{a-b}\right) = y\left(a+b-\frac{ab}{a+b}\right) \qquad \ldots\text{(i)}$$

$$x+y=2a^2 \qquad \ldots\text{(ii)}$$

From equation (i), w get

$$x\left(a-b+\frac{ab}{a-b}\right)-y\left(a+b+\frac{ab}{a+b}\right)=0$$

$$\Rightarrow \quad x\left(\frac{(a-b)^2+ab}{a-b}\right)-y\left(\frac{(a+b)^2-ab}{a+b}\right)=0$$

$$\Rightarrow \quad x\left(\frac{a^2+b^2-2ab+ab}{a-b}\right)-y\left(\frac{a^2+b^2+2ab-ab}{a+b}\right)=0$$

$$\Rightarrow \quad x\left(\frac{a^2+b^2-ab}{a-b}\right)-y\left(\frac{a^2+b^2-ab}{a+b}\right)=0 \qquad \text{.....(iii)}$$

From equation (ii), we get

$$x+y-2a^2=0 \qquad \text{.....(iv)}$$

Here,

$$a_1=\frac{a^2+b^2-ab}{a-b}, b_1=-\left(\frac{a^2+b^2+ab}{a+b}\right), c_1=0$$

$$a_2=1, b_2=1 \; and \; c_2=-2a^2$$

By cross multiplication we get

$$\Rightarrow \frac{x}{\left(-2a^2\right)\left[-\left(\frac{a^2+b^2+ab}{a+b}\right)\right]-0\times 1}=\frac{-y}{\left(-2a^2\right)\left(\frac{a^2+b^2-ab}{a-b}\right)-0\times 1}=\frac{1}{\frac{a^2+b^2-ab}{a-b}-\left[-\frac{a^2+b^2-ab}{a-b}\right]}$$

$$\Rightarrow \frac{x}{2a^2\left(\frac{a^2+b^2+ab}{a+b}\right)}=\frac{y}{\left(2a^2\right)\left(\frac{a^2+b^2-ab}{a-b}\right)}=\frac{1}{\frac{a^2+b^2-ab}{a-b}+\frac{a^2+b^2+ab}{a+b}}$$

$$\Rightarrow \frac{x}{2a^2\left(\frac{a^2+b^2+ab}{a+b}\right)}=\frac{y}{\left(2a^2\right)\left(\frac{a^2+b^2-ab}{a-b}\right)}=\frac{1}{\frac{a^2+b^2-ab}{a-b}+\frac{a^2+b^2+ab}{a+b}}$$

$$\Rightarrow \frac{x}{2a^2\left(\frac{a^2+b^2+ab}{a+b}\right)}=\frac{y}{2a^2\left(\frac{a^2+b^2-ab}{a-b}\right)}=\frac{1}{\frac{a^3+b^3+a^3-b^3}{(a-b)(a+b)}}$$

$$\Rightarrow \frac{x}{2a^2\left(\frac{a^2+b^2+ab}{a+b}\right)}=\frac{y}{2a^2\left(\frac{a^2+b^2-ab}{a-b}\right)}=\frac{1}{\frac{2a^3}{(a-b)(a+b)}}$$

Now,

$$\frac{x}{2a^2\left(\frac{a^2+b^2+ab}{a+b}\right)} - \frac{1}{\frac{2a^3}{(a-b)(a+b)}}$$

$$\Rightarrow x = \frac{2a^2\left(a^2+b^2+ab\right)}{a+b} \times \frac{(a-b)(a+b)}{2a^3}$$

$$= \frac{(a-b)\left(a^2+b^2+ab\right)}{a}$$

$$= \frac{a^3-b^3}{a} \qquad \left[\because a^2-b^2 = (a-b)\left(a^2+b^2+ab\right)\right]$$

And,

$$\frac{y}{2a^2\left(\frac{a^2+b^2-ab}{a-b}\right)} = \frac{1}{\frac{2a^3}{(a-b)(a+b)}}$$

$$\Rightarrow y = \frac{2a^2\left(a^2+b^2-ab\right)}{a-b} \times \frac{(a-b)(a+b)}{2a^3}$$

$$= \frac{(a+b)\left(a^2+b^2-ab\right)}{a}$$

$$= \frac{a^3+b^3}{a} \qquad \left[\because a^3+b^3-(a+b)\left(a^2+b^2-ab\right)\right]$$

Hence, $x = \frac{a^2-b^2}{a}, y = \frac{a^3+b^3}{a}$ is the solution of the given system of equation.

19. $bx + cy = a + b$

$$ax\left(\frac{1}{a-b} - \frac{1}{a+b}\right) + cy\left(\frac{1}{b-a} - \frac{1}{b+a}\right) = \frac{2a}{a+b}$$

Sol:

The given system of equation is

$$bx + cy = a + b \qquad ....(i)$$

$$ax\left(\frac{1}{a-b} - \frac{1}{a+b}\right) + cy\left(\frac{1}{b-a} - \frac{1}{b+a}\right) = \frac{2a}{a+b} \qquad .....(ii)$$

From equation (ii), we get

$$bx + cy - (a+b) = 0 \qquad ....(iii)$$

From equation (ii), we get

$$ax\left[\frac{a+b-(a-b)}{(a-b)(a+b)}\right] + cy\left(\frac{b+a-(b-a)}{(b-a)(b+a)}\right) - \frac{2a}{a+b} = 0$$

$$\Rightarrow \quad ax\left[\frac{a+b-a+b}{(a-b)(a+b)}\right]+cy\left(\frac{b+a-b+a}{(b-a)(b+a)}\right)-\frac{2a}{a+b}=0$$

$$\Rightarrow \quad ax\left[\frac{2b}{(a-b)(a+b)}\right]+cy\left(\frac{2a}{(b-a)(b+a)}\right)-\frac{2a}{a+b}=0$$

$$\Rightarrow \quad x\left[\frac{2ab}{(a-b)(a+b)}\right]+y\left(\frac{2ac}{-(a-b)(a+b)}\right)-\frac{2a}{a+b}=0$$

$$\Rightarrow \quad x\left[\frac{2ab}{(a-b)(a+b)}\right]+y\left(\frac{2ac}{(a-b)(a+b)}\right)-\frac{2a}{a+b}=0$$

$$\Rightarrow \quad \frac{1}{a+b}\left[\frac{2abx}{a-b}-\frac{2acy}{a-b}-2a\right]=0$$

$$\Rightarrow \quad \frac{2abx}{a-b}-\frac{2acy}{a-b}-2a=0$$

$$\Rightarrow \quad \frac{2abx-2acy-2a(a-b)}{a-b}=0$$

$$\Rightarrow \quad 2abx-2acy-2a(a-b)=0 \qquad .....(iv)$$

From equation (i) and equation (ii), we get

$a_1=b, b_1=c, c_1=-(a+b)$

$a_2=2ab, b_2=-2ac \text{ and } c_2=-2a(a-b)$

By cross multiplication, we get

$$\Rightarrow \frac{x}{-2ac(a-b)-[-(a+b)][-2ac]}=\frac{-y}{-2ab(a-b)-[-(a+b)][2ab]}=\frac{1}{-2abc-2abc}$$

$$\Rightarrow \frac{x}{-2a^2c+2abc-[2a^2c+2abc]}=\frac{-y}{-2a^2b+2ab^2+[2a^2b+2ab^2]}=\frac{1}{-4abc}$$

$$\Rightarrow \frac{x}{-2a^2c+2abc-2a^2c-2abc}=\frac{-y}{-2a^2b+2ab^2+2a^2b-2ab^2}=\frac{-1}{4abc}$$

$$\Rightarrow \frac{x}{-4a^2c}=\frac{-y}{4ab^2}=\frac{-1}{4abc}$$

Now,

$$\frac{x}{-4a^2c}=\frac{-1}{4abc}$$

$$\Rightarrow \quad x=\frac{4a^2c}{4abc}=\frac{a}{b}$$

And,

$$\frac{-y}{4ab^2}=\frac{-1}{4abc}$$

$$\Rightarrow \quad y=\frac{4ab^2}{4abc}=\frac{b}{c}$$

Hence, $x=\frac{a}{b}, y=\frac{b}{c}$ is the solution of the given system of the equations.

20. $$(a-b)x+(a+b)y=2a^2-2b^2$$
$$(a+b)(x+y)=4ab$$

Sol:

The given system of equation is

$$(a-b)x+(a+b)y=2a^2-2b^2 \qquad \ldots\ldots(i)$$

$$(a+b)(x+y)=4ab \qquad \ldots\ldots(ii)$$

From equation (i), we get

$$(a-b)x+(a+b)y-\left(2a^2-2b^2\right)=0$$

$$\Rightarrow \quad (a-b)x+(a-b)y-2\left(a^2-b^2\right)=0 \qquad \ldots\ldots(iii)$$

From equation (ii), we get

$$(a+b)x+(a+b)y-4ab=0 \qquad \ldots\ldots(iv)$$

Here,

$$a_1=a-b, b_1=a+b, c_1=-2\left(a^2-b^2\right)$$

$$a_2=a+b, b_2=a+b \text{ and } c_2=-4ab$$

By cross multiplication, we get

$$\Rightarrow \frac{x}{-4ab(a+b)+2\left(a^2-b^2\right)(a+b)}=\frac{-y}{-4ab(a-b)+2\left(a^2-b^2\right)(a+b)}=\frac{1}{(a-b)(a+b)-(a+b)(a+b)}$$

$$\Rightarrow \frac{x}{2(a+b)\left[-2ab+a^2-b^2\right]}=\frac{-y}{-4ab(a-b)+2\left[(a-b)(a+b)\right](a+b)}=\frac{1}{(a+b)\left[(a-b)-(a+b)\right]}$$

$$\Rightarrow \frac{x}{2(a+b)\left(a^2-b^2-2ab\right)}=\frac{-y}{2(a-b)\left[-2ab+(a+b)(a+b)\right]}=\frac{1}{(a+b)[a-b-a-b]}$$

$$\Rightarrow \frac{x}{2(a+b)\left(a^2-b^2-2ab\right)}=\frac{-y}{2(a-b)\left[-2ab+\left(a^2+b^2+2ab\right)\right]}=\frac{1}{(a+b)(-2b)}$$

$$\Rightarrow \frac{x}{2(a+b)\left(a^2-b^2-2ab\right)}=\frac{-y}{2(a-b)\left(a^2+b^2\right)}=\frac{1}{-2b(a+b)}$$

Now,

$$\frac{x}{2(a+b)(a^2-b^2-2ab)}=\frac{1}{-2b(a+b)}$$

$$\Rightarrow x=\frac{2(a+b)(a^2-b^2-2ab)}{-2b(a+b)}$$

$$\Rightarrow x=\frac{a^2-b^2-2ab}{-b}$$

$$\Rightarrow x=\frac{-a^2+b^2+2ab}{b}$$

$$=\frac{2ab-a^2+b^2}{b}$$

Now,

$$\frac{-y}{2(a-b)(a^2+b^2)}=\frac{1}{-2ab(a+b)}$$

$$\Rightarrow \quad -y=\frac{2(a-b)(a^2+b^2)}{-2b(a+b)}$$

$$\Rightarrow \quad y=\frac{(a-b)(a^2+b^2)}{b(a+b)}$$

Hence, $x=\frac{2ab-a^2+b^2}{b}, y=\frac{(a-b)(a^2+b^2)}{b(a+b)}$ is the solution of the given system of equations.

$$\frac{-y}{-a^2d^2+b^2c^2}=\frac{1}{a^4-b^4}$$

$$\Rightarrow \quad -y=\frac{-a^2d^2+b^2c^2}{a^4-b^4}$$

$$\Rightarrow \quad y=\frac{a^2d^2-b^2c^2}{a^4-b^4}$$

21. $a^2x+b^2y=c^2$
$b^2x+a^2y=d^2$

Sol:

The given system of equations may be written as

$a^2x+b^2y-c^2=0$

$b^2x+a^2y-d^2=0$

Here,

$a_1 = a^2, b_1 = b^2, c_1 = -c^2$

$a_2 = b^2, b_2 = a^2 \text{ and } c_2 = -d^2$

By cross multiplication, we have

$$\Rightarrow \frac{x}{-b^2d^2 + a^2c^2} = \frac{-y}{-a^2d^2 + b^2c^2} = \frac{1}{a^4 - b^4}$$

Now,

$$\frac{x}{-b^2d^2 + a^2c^2} = \frac{1}{a^4 - b^4}$$

$$\Rightarrow \quad x = \frac{a^2c^2 - b^2d^2}{a^4 - b^4}$$

And,

$$\frac{-y}{-a^2d^2 + b^2c^2} = \frac{1}{a^4 - b^4}$$

$$\Rightarrow \quad -y = \frac{-a^2d^2 + b^2c^2}{a^4 - b^4}$$

$$\Rightarrow \quad y = \frac{a^2d^2 - b^2c^2}{a^4 - b^4}$$

Hence, $x = \frac{a^2c^2 - b^2d^2}{a^4 - b^4}, y \frac{a^2d^2 - b^2c^2}{a^4 - b^4}$ is the solution of the given system of the equations.

22. $\frac{57}{x+y} + \frac{6}{x-y} = 5$

$\frac{38}{x+y} + \frac{21}{x-y} = 9$

Sol:

Let $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v.$ Then, the given system of equations become

$$57u + 6v = 5 \Rightarrow 57u + 6v - 5 = 0$$

$$38u + 21v = 9 \Rightarrow 38u + 21v - 9 = 0$$

Here,

$a_1 = 57, b_1 = 6, c_1 = -5$

$a_2 = 38, b_2 = 21, \text{ and } c_2 = -9$

By cross multiplication, we have

$$\Rightarrow \quad \frac{u}{-54 + 105} = \frac{-v}{-513 + 190} = \frac{1}{1193 - 228}$$

$$\Rightarrow \quad \frac{u}{51} = \frac{-v}{-323} = \frac{1}{969}$$

$$\Rightarrow \quad \frac{u}{51} = \frac{v}{323} = \frac{1}{969}$$

Now,

$$\frac{u}{51}=\frac{1}{969}$$

$$\Rightarrow \quad u=\frac{51}{969}$$

$$\Rightarrow \quad u=\frac{1}{19}$$

And,

$$\frac{v}{323}=\frac{1}{969}$$

$$\Rightarrow \quad v=\frac{323}{969}$$

$$\Rightarrow \quad v=\frac{1}{3}$$

Now,

$$u=\frac{1}{x+y}$$

$$\Rightarrow \quad \frac{1}{x+y}=\frac{1}{19}$$

$$\Rightarrow \quad x+y=19 \qquad ....(i)$$

And,

$$v=\frac{1}{x-y}$$

$$\Rightarrow \quad \frac{1}{x-y}=\frac{1}{3}$$

$$\Rightarrow \quad x-y=3 \qquad ...(ii)$$

23. 2(ax – by) + a + 4b = 0
2(bx + ay) + b – 4a = 0

Sol:

The given system of equation may be written as

$$2ax-2by+a+4b=0$$

$$2bx+2ay+b-4a=0$$

Here,

$$a_1=2a, b_1=-2b, c_1=a+4b$$

$$a_2=2b, b_2=2a, c_2=b-4a$$

By cross multiplication, we have

$$\Rightarrow \frac{x}{(-2b)(b-4a)-(2a)(a+4b)}=\frac{-y}{(2b)(b-4a)-(2a)(a+4b)}=\frac{1}{4a^2+4b^2}$$

$$\Rightarrow \frac{x}{-2b^2+8ab-2a^2-8ab}=\frac{-y}{2ab-8a^2-2ab-8b^2}=\frac{1}{4a^2+4b^2}$$

$$\Rightarrow \frac{x}{-2a^2-2b^2}=\frac{-y}{-8a^2-8b^2}=\frac{1}{4a^2+4b^2}$$

Now,

$$\frac{x}{-2a^2-2b^2}=\frac{1}{4a^2+4b^2}$$

$$\Rightarrow \quad x=\frac{-2a^2-2b^2}{4a^2+4b^2}$$

$$=\frac{-2\left(a^2-b^2\right)}{4\left(a^2+b^2\right)}$$

$$=\frac{-1}{2}$$

And,

$$\frac{-y}{-8a^2-8b^2}=\frac{1}{4a^2+4b^2}$$

$$\Rightarrow \quad -y=\frac{-8a^2-8b^2}{4a^2+4b^2}$$

$$\Rightarrow \quad -y=\frac{-8\left(a^2-b^2\right)}{4\left(a^2+b^2\right)}$$

$$\Rightarrow \quad -y=\frac{-8}{4}$$

$$\Rightarrow \quad y=2$$

Hence, $x=\frac{-1}{2}, y=2$ is the solution of the given system of the equations.

The given system of equations may be written as

$$2ax-2by+a+4b=0$$

$$2bx+2ay+b-4a=0$$

Here,

$a_1=2a, b_1=-2b, c_1=a+4b$

$a_2=2b, b_2=2a, c_2=b-4a$

By cross multiplication, we have

$$\Rightarrow \frac{x}{(-2b)(b-4a)-(2a)(a+4b)}=\frac{-y}{(2a)(b-4a)-(2b)(a+4b)}=\frac{1}{4a^2+4b^2}$$

$$\Rightarrow \frac{x}{-2b^2+8ab-2a^2-8ab}=\frac{1}{4a^2+4b^2}$$

$$\Rightarrow \frac{x}{-2a^2-2b^2}=\frac{-y}{-8a^2-8b^2}=\frac{1}{4a^2+4b^2}$$

Now,

$$\frac{x}{-2a^2-2b^2}=\frac{1}{4a^2+4b^2}$$

$$\Rightarrow \quad x=\frac{-2a-2b^2}{4a^2+4b^2}$$

$$=\frac{-2\left(a^2-b^2\right)}{4a^2+4b^2}$$

$$=\frac{-1}{2}$$

And,

$$\frac{-y}{-8a^2-8b^2}=\frac{1}{4a^2+4b^2}$$

$$\Rightarrow \quad -y=\frac{-8a^2-8b^2}{4a^2+4b^2}$$

$$\Rightarrow \quad -y=\frac{-8\left(a^2-b^2\right)}{4\left(a^2+b^2\right)}$$

$$\Rightarrow \quad -y=\frac{-8}{4}$$

$$\Rightarrow \quad y=2$$

Hence, $x=\frac{-1}{2}, y=2$ is the solution of the given system of the equations.

24. $6\left(ax+by\right)=3a+2b$

$6\left(bx-ay\right)=3b-2a$

Sol:

The given system of equation is

$$6\left(ax+by\right)=3a+2b \qquad .....(i)$$

$$6\left(bx-ay\right)=3b-2a \qquad .....(ii)$$

From equation (i), we get

$$6ax+6by-\left(3a+2b\right)=0 \qquad ......(iii)$$

From equation (ii), we get

$$6bx-6ay-\left(3b-2a\right)=0 \qquad ......(iv)$$

Here,

$a_1 = 6a, b_1 = 6b, c_1 = -(3a+2b)$

$a_2 = 6b, b_2 = -6a$ *and* $c_2 = -(3b-2a)$

By cross multiplication, we have

$$\frac{x}{-6b(3b-2a)-6a(3a+2b)} = \frac{-y}{-6a(3b-2a)+6b(3a+2b)} = \frac{1}{-36a^2-36b^2}$$

$$\Rightarrow \quad \frac{x}{-18b^2+12ab-18a^2-12ab} = \frac{-y}{-18ab+12a^2+18ab+12b^2} = \frac{1}{-36(a^2+b^2)}$$

$$\Rightarrow \quad \frac{x}{-18a^2-18b^2} = \frac{-y}{12a^2+12b^2} = \frac{1}{-36(a^2+b^2)}$$

$$\Rightarrow \quad \frac{x}{-18(a^2+b^2)} = \frac{-y}{12(a^2+b^2)} = \frac{-1}{36(a^2+b^2)}$$

Now,

$$\frac{x}{-18(a^2+b^2)} = \frac{-1}{36(a^2+b^2)}$$

$$\Rightarrow \quad x = \frac{18(a^2+b^2)}{36(a^2+b^2)}$$

$$= \frac{1}{2}$$

And,

$$\frac{-y}{12(a^2+b^2)} = \frac{-1}{36(a^2+b^2)}$$

$$\Rightarrow \quad y = \frac{12(a^2+b^2)}{36(a^2+b^2)}$$

$$\Rightarrow \quad y = \frac{1}{3}$$

Hence, $x = \frac{1}{2}, y = \frac{1}{3}$ is the solution of the given system of equations.

25. $\frac{a^2}{x} - \frac{b^2}{y} = 0$

$\frac{a^2b}{x} + \frac{b^2a}{y} = a + b, x, y \neq 0$

Sol:

Taking $\frac{1}{x} = u$ *and* $\frac{1}{y} = v.$ Then, the given system of equations become

$a^2u - b^2v = 0$

$a^2bu + b^2av - (a+b) = 0$

Here,

$a_1 = a^2, b_1 = -b^2, c_1 = 0$

$a_2 = a^2b, b_2 = b^2a,\ and\ c_2 = -(a+b)$

By cross multiplication, we have

$$\Rightarrow \frac{u}{b^2(a+b) - 0 \times b^2a} = \frac{-v}{-a^2(a+b) - 0 \times a^2b} = \frac{1}{a^3b^2 + a^2b^3}$$

$$\Rightarrow \frac{u}{b^2(a+b)} = \frac{v}{a^2(a+b)} = \frac{1}{a^2b^2(a+b)}$$

Now,

$$\frac{u}{b^2(a+b)} = \frac{1}{a^2b^2(a+b)}$$

$$\Rightarrow \quad u = \frac{b^2(a+b)}{a^2b^2(a+b)}$$

$$\Rightarrow \quad u = \frac{1}{a^2}$$

And,

$$\frac{v}{a^2(a+b)} = \frac{1}{a^2b^2(a+b)}$$

$$\Rightarrow \quad v = \frac{a^2(a+b)}{a^2b^2(a+b)}$$

$$\Rightarrow \quad v = \frac{1}{b^2}$$

Now,

$$x = \frac{1}{u} = a^2$$

And,

$$y = \frac{1}{v} = b^2$$

Hence, $x = a^2, y = b^2$ is the solution of the given system of equations.

26. mx – my = $m^2 + n^2$
x + y = 2m
Sol:
The given system of equations may be written as

$mx - ny - \left(m^2 + n^2\right) = 0$

$x + y - 2m = 0$

Here,

$a_1 = m, b_1 = -n, c_1 = -\left(m^2 + n^2\right)$

$a_2 = 1, b_2 = 1,\ and\ c_2 = -2m$

By cross multiplication, we have

$$\frac{x}{2mn + \left(m^2 + n^2\right)} = \frac{-y}{-2m^2 + \left(m^2 + n^2\right)} = \frac{1}{m + n}$$

$$\Rightarrow \quad \frac{x}{2mn + m^2 + n^2} = \frac{-y}{-m^2 + n^2} = \frac{1}{m + n}$$

$$\Rightarrow \quad \frac{x}{(m + n)^2} = \frac{-y}{-m^2 + n^2} = \frac{1}{m + n}$$

Now,

$$\frac{x}{(m + n)^2} = \frac{1}{m + n}$$

$$\Rightarrow \quad x = \frac{\left(m + n^2\right)}{m + n}$$

$$\Rightarrow \quad x = m + n$$

And,

$$\frac{-y}{-m^2 + n^2} = \frac{1}{m + n}$$

$$\Rightarrow \quad -y = \frac{-m^2 + n^2}{m + n}$$

$$\Rightarrow \quad y = \frac{m^2 - n^2}{m + n}$$

$$\Rightarrow \quad y = \frac{(m - n)(m + n)}{m + n}$$

$$\Rightarrow \quad y = m - n$$

Hence, $x = m + n, y = m - n$ is the solution of the given system of equation.

27. $\frac{ax}{b} - \frac{by}{a} = a + b$

$ax - by = 2ab$

Sol:

The given system of equation may be written as

$$\frac{a}{b}x\times-\frac{b}{a}\times y-(a+b)=0$$

$$ax-by-2ab=0$$

Here,

$$a_1=\frac{a}{b},b_1=-\frac{b}{a},c_1=-(a+b)$$

$$\Rightarrow \quad \frac{x}{2b^2-ab-b^2}=\frac{-y}{-2a^2+a^2+ab}=\frac{1}{-a+b}$$

$$\Rightarrow \quad \frac{x}{b^2-ab}=\frac{-y}{-a^2+ab}=\frac{1}{-a+b}$$

$$\Rightarrow \quad \frac{x}{b(b-a)}=\frac{-y}{a(-a+b)}=\frac{1}{b-a}$$

Now,

$$\frac{x}{b(b-a)}=\frac{1}{b-a}$$

$$\Rightarrow \quad x=\frac{b(b-a)}{b-a}=b$$

And,

$$\frac{-y}{a(b-a)}=\frac{1}{b-a}$$

$$\Rightarrow \quad -y=\frac{a(b-a)}{b-a}$$

$$\Rightarrow \quad -y=a$$

$$\Rightarrow \quad y=-a$$

Hence, $x=b, y=-a$ is the solution of the given system of equations.

28. $\frac{b}{a}x+\frac{a}{b}y-(a^2+b^2)=0$

$x+y-2ab=0$

Sol:

The given system of equation may be written as

$$\frac{b}{a}x+\frac{a}{b}y-(a^2+b^2)=0$$

$$x+y-2ab=0$$

Here,

$$a_1=\frac{b}{a},b_1=\frac{a}{b},c_1=-(a^2+b^2)$$

$$a_2=1,b_2=1,\ and\ c_2=-2ab$$

By cross multiplication, we have

$$\frac{x}{-2ab\times\frac{a}{b}+a^2+b^2}=\frac{-y}{-2ab\times\frac{a}{b}+a^2+b^2}=\frac{1}{\frac{b}{a}-\frac{a}{b}}$$

$$\Rightarrow \quad \frac{x}{-2a^2+a^2+b^2}=\frac{-y}{-2b^2+a^2+b^2}=\frac{1}{\frac{b^2-a^2}{ab}}$$

$$\Rightarrow \quad \frac{x}{b^2-a^2}=\frac{-y}{-b^2+a^2}=\frac{1}{\frac{b^2-a^2}{ab}}$$

Now,

$$\frac{x}{b^2-a^2}=\frac{1}{\frac{b^2-a^2}{ab}}$$

$$\Rightarrow \quad x=b^2-a^2\times\frac{ab}{b^2-a^2}$$

$$\Rightarrow \quad x=ab$$

And,

$$\frac{-y}{-b^2+a^2}=\frac{1}{\frac{b^2-a^2}{ab}}$$

$$\Rightarrow \quad -y=-b^2+a^2\times\frac{ab}{b^2-a^2}$$

$$\Rightarrow \quad -y=-\left(b^2-a^2\right)\times\frac{ab}{b^2-a^2}$$

$$\Rightarrow \quad -y=-ab$$

$$\Rightarrow \quad y=ab$$

Hence, $x=ab, y=ab$ is the solution of the given system of equations.

Exercise 3.5

In each of the following systems of equations determine whether the system has a unique solution, no solution or infinitely many solutions. In case there is a unique solution, find it: (1 −4)

1. $x-3y-3=0$
   $3x-9y-2=0$

Sol:

The given system of equations may be written as

$x-3y-3=0$

$3x-9y-2=0$

The given system of equations is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=1, b_1=-3, c_1=-3$

And $a_2=3, b_2=-9, c_2=-2$

We have,

$\frac{a_1}{a_2}=\frac{1}{3}$

$\frac{b_1}{b_2}=\frac{-3}{-9}=\frac{1}{3}$

And, $\frac{c_1}{c_2}=\frac{-3}{-2}=\frac{3}{2}$

Clearly, $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

So, the given system of equation has no solutions.

2. $2x+y-5=0$

$4x+2y-10=0$

Sol:

The given system of equation may be written as

$2x+y-5=0$

$4x+2y-10=0$

The given system of equations is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=2, b_1=1, c_1=-5$

And $a_2=4, b_2=2, c_2=-10$

We have,

$\frac{a_1}{a_2}=\frac{2}{4}=\frac{1}{2}$

$\frac{b_1}{b_2}=\frac{1}{2}$

And, $\frac{c_1}{c_2}=\frac{-5}{-10}=\frac{1}{2}$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, the given system of equation has infinity many solutions.

3. $3x - 5y = 20$
$6x - 10y = 40$

Sol:

$3x - 5y = 20$

$6x - 10y = 40$

Compare it with

$a_1 x + by_1 + c_1 = 0$

$a_1 x + by_2 + c_2 = 0$

We get

$a1 = 3, b1 = -5 \ and \ c1 = -20$

$a2 = 6, b2 = -10 \ and \ c2 = -40$

$\frac{a_1}{a_2} = \frac{3}{6}, \frac{b_1}{b_2} = \frac{-5}{-10} \ and \ \frac{c_1}{c_2} = \frac{-20}{-40}$

Simplifying it we get

$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{2} \ and \ \frac{c_1}{c_2} = \frac{1}{2}$

Hence

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So both lines are coincident and overlap with each other
So, it will have infinite or many solutions

4. $x - 2y - 8 = 0$
$5x - 10y - 10 = 0$

Sol:

The given system of equation may be written as

$x - 2y - 8 = 0$

$5x - 10y - 10 = 0$

The given system if equation is of the form

$a_1 x + b_1 y + c_1 = 0$

$a_2 x + b_2 y + c_2 = 0$

Where, $a_1 = 1, b_1 = -2, c_1 = -8$

And, $a_2 = 5, b_2 = -10, c_2 = -10$

We have,

$\frac{a_1}{a_2} = \frac{1}{5}$

$\frac{b_1}{b_2} = \frac{-2}{-10} = \frac{1}{5}$

And, $\frac{c_1}{c_2} = \frac{-8}{-10} = \frac{4}{5}$

Clearly, $\frac{a_1}{a_2} = \frac{b_2}{b_2} \neq \frac{c_1}{c_2}$

So, the given system of equation has no solution.

5. $kx + 2y - 5 = 0$
$3x + y - 1 = 0$

Sol:

The given system of equation is

$kx + 2y - 5 = 0$

$3x + y - 1 = 0$

The system of equation is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = k, b_1 = 2, c_1 = -5$

And, $a_2 = 3, b_2 = 1, c_2 = -1$

For a unique solution, we must have

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\therefore \quad \frac{k}{3} \neq \frac{2}{1}$

$\Rightarrow \quad k \neq 6$

So, the given system of equations will have a unique solution for all real values of k other than 6.

6. 4x + ky + 8 = 0
2x + 2y + 2 = 0

Sol:

Here $a_1 = 4, a_2 = k, b_1 = 2, b_2 = 2$

Now for the given pair to have a unique solution: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

i.e., $\frac{4}{2} \neq \frac{k}{2}$

i.e., $k \neq 4$

Therefore, for all values of k, except 4, the given pair of equations will have a unique solution.

7. $4x - 5y = k$
$2x - 3y = 12$

Sol:

The given system of equation is

$4x - 5y - k = 0$

$2x - 3y - 12 = 0$

The system of equation is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 4, b_1 = -5, c_1 = -k$

And, $a_2 = 2, b_2 = -3, c_2 = -12$

For a unique solution, we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\therefore$ $\frac{4}{2} \neq \frac{-5}{-3}$

$\Rightarrow$ k is any real number.

So, the given system of equations will have a unique solution for all real values of k.

8. $x + 2y = 3$
$5x + ky + 7 = 0$

Sol:

The given system of equation is

$x + 2y - 3 = 0$

$5x + ky + 7 = 0$

The system of equation is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 1, b_1 = 2, c_1 = -3$

And, $a_2 = 5, b_2 = k, c_2 = 7$

For a unique solution, we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\therefore \quad \frac{1}{5} \neq \frac{2}{k}$

$\Rightarrow \quad k \neq 10$

So, the given system of equations will have a unique solution for all real values of k other than 10.

Find the value of k for which each of the following systems of equations have definitely many solution: (9-19)

9. $2x+3y-5=0$
$6x-ky-15=0$

Sol:

The given system of equation is

$2x+3y-5=0$

$6x-ky-15=0$

The system of equation is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=2, b_1=3, c_1=-5$

And, $a_2=6, b_2=k, c_2=-15$

For a unique solution, we must have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{2}{6}=\frac{3}{k}$

$\Rightarrow \quad 2k=18$

$\Rightarrow \quad k=\frac{18}{2}=9$

Hence, the given system of equations will have infinitely many solutions, if $k=9$.

10. $4x+5y=3$
$kx+15y=9$

Sol:

The given system of equation is

$4x+5y-3=0$

$kx+15y-9=0$

The system of equation is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 4, b_1 = 5, c_1 = -3$

And, $a_2 = k, b_2 = 15, c_2 = -9$

For a unique solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{4}{k} = \frac{5}{15} = \frac{-3}{-9}$$

Now,

$$\frac{4}{k} = \frac{5}{15}$$

$$\Rightarrow \quad \frac{4}{k} = \frac{1}{3}$$

$$\Rightarrow \quad k = 12$$

Hence, the given system of equations will have infinitely many solutions, if $k = 12$.

11. $kx - 2y + 6 = 0$

$4x + 3y + 9 = 0$

Sol:

The given system of equation is

$kx - 2y + 6 = 0$

$4x + 3y + 9 = 0$

The system of equation is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = k, b_1 = -2, c_1 = 6$

And, $a_2 = 4, b_2 = -3, c_2 = 9$

For a unique solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{k}{4} = \frac{-2}{-3} = \frac{6}{9}$$

Now,

$$\frac{k}{4}=\frac{6}{9}$$

$\Rightarrow \quad \frac{k}{4}=\frac{2}{3}$

$\Rightarrow \quad k=\frac{2\times4}{3}$

$\Rightarrow \quad k=\frac{8}{3}$

Hence, the given system of equations will have infinitely many solutions, if $k=\frac{8}{3}$.

12. $8x+5y=9$
$kx+10y=18$

Sol:

The given system of equation is

$8x+5y-9=0$

$kx+10y-18=0$

The system of equation is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=8, b_1=5, c_1=-9$

And, $a_2=k, b_2=10, c_2=-18$

For a unique solution, we must have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{8}{k}=\frac{5}{10}=\frac{-9}{-18}$

Now,

$$\frac{8}{k}=\frac{5}{10}$$

$\Rightarrow \quad 8\times10=5\times k$

$\Rightarrow \quad \frac{8\times10}{5}=k$

$\Rightarrow \quad k=8\times2=16$

Hence, the given system of equations will have infinitely many solutions, if $k=16$.

13. $2x-3y=7$
$(k+2)x-(2k+1)y-3(2k-1)$

Sol:

The given system of equation may be written as

$2x-3y-7=0$

$(k+2)x-(2k+1)y-3(2k-1)=0$

The system of equation is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=2, b_1=-3, c_1=-7$

And, $a_2=k, b_2=-(2k+1), c_2=-3(2k-1)$

For a unique solution, we must have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{2}{k+2}=\frac{3}{-(2k+1)}=\frac{-7}{-3(2k-1)}$$

$$\Rightarrow \quad \frac{2}{k+2}=\frac{-3}{-(2k+1)} \text{ and } \frac{-3}{-(2k+1)}=\frac{-7}{-3(2k-1)}$$

$$\Rightarrow \quad 2(2k+1)=3(k+2) \text{ and } 3\times3(2k-1)=7(2k+1)$$

$$\Rightarrow \quad 4k+2=3k+6 \text{ and } 18k-9=14k+7$$

$$\Rightarrow \quad 4k-3k=6-2 \text{ and } 18k-14k=7+9$$

$$\Rightarrow \quad k=4 \text{ and } 4k=16 \Rightarrow k=4$$

$$\Rightarrow \quad k=4 \text{ and } k=4$$

Hence, the given system of equations will have infinitely many solutions, if $k=4$.

14. $2x+3y=2$
$(k+2)x+(2k+1)y-2(k-1)$

Sol:

The given system of equation may be written as

$2x+3y-2=0$

$(k+2)x+(2k+1)y-2(k-1)=0$

The system of equation is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=2, b_1=3, c_1=-2$

And, $a_2 = k+2, b_2 = (2k+1), c_2 = -2(k-1)$

For a unique solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{2}{k+2} = \frac{3}{(2k+1)} = \frac{-2}{-2(k-1)}$

$\Rightarrow \quad \frac{2}{k+2} = \frac{3}{(2k+1)}$ *and* $\frac{3}{(2k+1)} = \frac{2}{2(k-1)}$

$\Rightarrow \quad 2(2k+1) = 3(k+2)$ *and* $3(k-1) = (2k+1)$

$\Rightarrow \quad 4k+2 = 3k+6$ *and* $3k-3 = 2k+1$

$\Rightarrow \quad 4k-3k = 6-2$ *and* $3k-2k = 1+3$

$\Rightarrow \quad k = 4$ *and* $k = 4$

Hence, the given system of equations will have infinitely many solutions, if $k = 4$.

15. $x+(k+1)y = 4$

$(k+1)x+9y-(5k+2)$

Sol:

The given system of equation may be written as

$x+(k+1)y-4 = 0$

$(k+1)x+9y-(5k+2) = 0$

The system of equation is of the form

$a_1x+b_1y+c_1 = 0$

$a_2x+b_2y+c_2 = 0$

Where, $a_1 = 1, b_1 = k+1, c_1 = -4$

And, $a_2 = k+1, b_2 = 9, c_2 = -(5k+2)$

For a unique solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{1}{k+1} = \frac{k+1}{9} = \frac{-4}{-(5k+2)}$

$\Rightarrow \quad \frac{1}{k+1} = \frac{k+1}{9}$ *and* $\frac{k+1}{9} = \frac{4}{5k+2}$

$\Rightarrow \quad 9 = (k+1)^2$ *and* $(k+1)(5k+2) = 36$

$\Rightarrow \quad 9 = k^2+1+2k$ *and* $5k^2+2k+5k+2 = 36$

$\Rightarrow \quad k^2+2k+1-9 = 0$ *and* $5k^2+7k+2-36 = 0$

$\Rightarrow \quad k^2+2k-8=0 \text{ and } 5k^2+7k-34=0$

$\Rightarrow \quad k^2+4k-2k-8=0 \text{ and } 5k^2+17k-10k-34=0$

$\Rightarrow \quad k(k+4)-2(k+4)=0 \text{ and } (5k+17)-2(5k+17)=0$

$\Rightarrow \quad (k+4)(k-2)=0 \text{ and } (5k+17)(k-2)=0$

$\Rightarrow \quad (k=-4 \text{ or } k=2) \text{ and } \left(k=\frac{-17}{5} \text{ or } k=2\right)$

$\Rightarrow \quad k=2$ satisfies both the conditions

Hence, the given system of equations will have infinitely many solutions, if $k=2$.

16. $kx+3y-2k+1$
$2(k+1)x+9y-(7k+1)$

Sol:

The given system of equation may be written as

$kx+3y-(2k+1)=0$

$2(k+1)x+9y-(7k+1)=0$

The system of equation is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=k, b_1=3, c_1=-(2k+1)$

And, $a_2=2(k+1), b_2=9, c_2=-(7k+1)$

For a unique solution, we must have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{1}{2(k+1)}=\frac{3}{9}=\frac{-(2k+1)}{-(7k+1)}$

$\Rightarrow \quad \frac{k}{2(k+1)}=\frac{3}{9} \text{ and } \frac{3}{9}=\frac{2k+1}{7k+1}$

$\Rightarrow \quad 9k=3\times 2(k+1) \text{ and } 3(7k+1)=9(2k+1)$

$\Rightarrow \quad 9k=6(k+1) \text{ and } 21k+3=18k+9$

$\Rightarrow \quad 9k-6k=6 \text{ and } 21k-18k=9-3$

$\Rightarrow \quad 3k=6 \text{ and } 3k=6$

$\Rightarrow \quad k=\frac{6}{3} \text{ and } k=\frac{6}{3}$

$\Rightarrow \quad k=2 \text{ and } k=2$

$\Rightarrow \quad k=2$ satisfies both the conditions

Hence, the given system of equations will have infinitely many solutions, if $k = 2$.

17. $2x + (k-2)y = k$
$6x + (2k-1)y - (2k+5)$

Sol:

The given system of equation may be written as

$2x + (k-2)y - k = 0$

$6x + (2k-1)y - (2k+5) = 0$

The system of equation is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 2, b_1 = k-2, c_1 = -k$

And, $a_2 = 6, b_2 = 2k-1, c_2 = -(2k+5)$

For a unique solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{2}{6} = \frac{k-2}{2k-1} = \frac{-k}{-2(2k+5)}$

$\Rightarrow \quad \frac{2}{6} = \frac{k-2}{2k-1} \text{ and } \frac{k-2}{2k-1} = \frac{k}{2k+5}$

$\Rightarrow \quad \frac{1}{3} = \frac{k-2}{2k-1} \text{ and } (k-2)(2k+5) = k(2k-1)$

$\Rightarrow \quad 2k-1 = 3(k-2) \text{ and } 2k^2 + 5k - 4k - 10 = 2k^2 - k$

$\Rightarrow \quad 2k - 3k - 6 \text{ and } k - 10 = -k$

$\Rightarrow \quad 2k - 3k = -6 + 1 \text{ and } k + k = 10$

$\Rightarrow \quad -k = -5 \text{ and } 2k = 10$

$\Rightarrow \quad k = \frac{-5}{-1} \text{ and } k = \frac{10}{2}$

$\Rightarrow \quad k = 5 \text{ and } k = 5$

$\Rightarrow \quad k = 5$ satisfies both the conditions

Hence, the given system of equations will have infinitely many solutions, if $k = 5$.

18. $2x + 3y = 7$
$(k+1)x + (2k-1)y - (4k+1)$

Sol:

The given system of equation may be written as

$2x+3y-7=0$

$(k+1)x+(2k-1)y-(4k+1)=0$

The system of equation is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=2, b_1=3, c_1=-7$

And, $a_2=k+1, b_2=2k-1, c_2=-(4k+1)$

For a unique solution, we must have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{2}{k+1}=\frac{3}{2k-1}=\frac{-7}{-(4k+1)}$$

$$\Rightarrow \quad \frac{2}{k+1}=\frac{3}{2k-1} \text{ and } \frac{3}{2k-1}=\frac{7}{4k+1}$$

$$\Rightarrow \quad 2(2k-1)=3(k+1) \text{ and } 3(4k+1)=7(2k-1)$$

$$\Rightarrow \quad 4k-2=3k+3 \text{ and } 12k+3=14k-7$$

$$\Rightarrow \quad 4k-3k=3+2 \text{ and } 12k-14k=-7-3$$

$$\Rightarrow \quad k=5 \text{ and } -2k=-10$$

$$\Rightarrow \quad k=5 \text{ and } k=\frac{10}{2}=5$$

$\Rightarrow \quad k=5$ satisfies both the conditions

Hence, the given system of equations will have infinitely many solutions, if $k=5$.

19. $2x+3y=k$

$(k-1)x+(k+2)y-3k$

Sol:

The given system of equation may be written as

$2x+3y-k=0$

$(k-1)x+(k+2)y-3k=0$

The system of equation is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=2, b_1=3, c_1=-k$

And, $a_2=k-1, b_2=k+2, c_2=3k$

For a unique solution, we must have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{2}{k-1}=\frac{3}{k+1}=\frac{-k}{-3k}$

$\Rightarrow \quad \frac{2}{k-1}=\frac{3}{k+1}$ *and* $\frac{3}{k+1}=\frac{-k}{-3k}$

$\Rightarrow \quad 2(k+2)=3(k-1)$ *and* $3\times 3=k+2$

$\Rightarrow \quad 2k+4=3k-3$ *and* $9=k+2$

$\Rightarrow \quad 4+3=3k-2k$ *and* $9-2=k$

$\Rightarrow \quad 7=k$ *and* $7=k$

$\Rightarrow \quad k=7$ *and* $k=7$

$\Rightarrow \quad k=7$ satisfies both the conditions

Hence, the given system of equations will have infinitely many solutions, if $k=7$.

Find the value of k for which the following system of equations has no solution: (20 – 25)

20. $kx-5y=2$
$6x+2y=7$

Sol:

Given

$kx-5y=2$

$6x+2y=7$

Condition for system of equations having no solution

$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

$\Rightarrow \frac{k}{6}=\frac{-5}{2}\neq\frac{2}{7}$

$\Rightarrow 2k=-30$

$\Rightarrow k=-15$

21. $x+2y=0$
$2x+ky-5=0$

Sol:

The given system of equation may be written as

$x+2y=0$

$2x+ky-5=0$

The system of equation is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=1, b_1=2, c_1=0$

And, $a_2=2, b_2=k, c_2=-5$

For a unique solution, we must have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

We have,

$$\frac{a_1}{a_2}=\frac{1}{2}$$

$$\frac{b_1}{b_2}=\frac{2}{k}$$

And, $\frac{c_1}{c_2}=\frac{0}{-5}$

Now, $\frac{a_1}{a_2}=\frac{b_1}{b_2}$

$\Rightarrow \quad \frac{1}{2}=\frac{2}{k}$

$\Rightarrow \quad k=4$

Hence, the given system of equations has no solutions, when $k=4$.

22. $3x-4y+7=0$
$kx+3y-5=0$

Sol:

The given system of equation may be written as

$3x-4y+7=0$

$kx+3y-5=0$

The system of equation is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=3, b_1=-4, c_1=7$

And, $a_2=k, b_2=3, c_2=-5$

For a unique solution, we must have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

We have,

$$\frac{b_1}{b_2} = \frac{-4}{3}$$

$$and, \quad \frac{c_1}{c_2} = \frac{-7}{5}$$

Clearly, $\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given system will have no solution.

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \Rightarrow \frac{3}{k} = \frac{-4}{3} \Rightarrow k = \frac{-9}{4}$

Clearly, for this value of k, we have $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the given system of equations has no solutions, when $k = \frac{-9}{4}$.

23. $2x - ky + 3 = 0$
$3x + 2y - 1 = 0$

Sol:

The given system of equation may be written as

$2x - ky + 3 = 0$

$3x + 2y - 1 = 0$

The system of equation is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 2, b_1 = -k, c_1 = 3$

And, $a_2 = 3, b_2 = 2, c_2 = -1$

For a unique solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

We have,

$$\frac{a_1}{a_2} = \frac{2}{3}$$

$$and, \quad \frac{c_1}{c_2} = \frac{3}{-1}$$

Clearly, $\frac{a_1}{a_2} \neq \frac{c_1}{c_2}$

So, the given system will have no solution. If

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$ *i.e.*, $\frac{2}{k} = \frac{-k}{2} \Rightarrow k = \frac{-4}{3}$

Hence, the given system of equations has no solutions, $k = \frac{-4}{3}$.

24. $2x + ky - 11 = 0$
$5x - 7y - 5 = 0$

Sol:

The given system of equation is

$2x + ky - 11 = 0$

$5x - 7y - 5 = 0$

The system of equation is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 2, b_1 = k, c_1 = -11$

And, $a_2 = 5, b_2 = -7, c_2 = -5$

For a unique solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{2}{5} = \frac{k}{-7} \neq \frac{-11}{-5}$$

$$\Rightarrow \quad \frac{2}{5} = \frac{k}{-7} \text{ and } \frac{k}{-7} \neq \frac{-11}{-5}$$

Now,

$$\frac{2}{5} = \frac{k}{-7}$$

$$\Rightarrow \quad 2 \times (-7) = 5k$$

$$\Rightarrow \quad 5k = -14$$

$$\Rightarrow \quad k = \frac{-14}{5}$$

Clearly, for $\frac{-14}{5}$ we have $\frac{k}{-7} \neq \frac{-11}{-5}$

Hence, the given system of equation will have no solution, if $k = \frac{-14}{5}$

25. $kx + 3y = 3$
$12x + ky = 6$

Sol:

$kx+3y=3$

$12x+ky=6$

For no solution $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

$\Rightarrow \quad \frac{k}{12}=\frac{2}{k}\neq\frac{3}{6}$

$\frac{k}{12}=\frac{3}{k}$

$k^2=36$

$k=\pm 6\, i.e., \quad k=6,-6$

Also,

$\frac{3}{k}\neq\frac{3}{6}$

$\frac{3\times 6}{3}\neq k$

$k\neq 6$

$k=-6$ satisfies both the condition

Hence, $k=-6$

26. For what value of α, the following system of equations will be inconsistent?

$4x+6y-11=0$

$2x+ky-7=0$

Sol:

The given system of equation may be written as

$4x+6y-11=0$

$2x+ky-7=0$

The system of equation is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=4, b_1=6, c_1=-11$

And, $a_2=2, b_2=k, c_2=-7$

For a unique solution, we must have

$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

Now,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$\Rightarrow \quad \frac{4}{2} = \frac{6}{k}$

$\Rightarrow \quad 4k = 12$

$\Rightarrow \quad k = \frac{12}{4} = 3$

Clearly, for this value of k, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the given system of equation is inconsistent, when $k = 3$.

27. For what value of α, the system of equations

$\alpha x + 3y = \alpha - 3$

$12x + \alpha y = \alpha$

will have no solution?

Sol:

The given system of equation may be written as

$\alpha x + 3y - (\alpha - 3) = 0$

$12x + \alpha y - \alpha = 0$

The system of equation is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = \alpha, b_1 = 3, c_1 = -(\alpha - 3)$

And, $a_2 = 12, b_2 = \alpha, c_2 = -\alpha$

For a unique solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{\alpha}{12} = \frac{3}{\alpha} \neq \frac{-(\alpha - 3)}{-\alpha}$

Now,

$$\frac{3}{\alpha} \neq \frac{-(\alpha-3)}{-\alpha}$$

$\Rightarrow \quad \frac{3}{\alpha} \neq \frac{\alpha-3}{\alpha}$

$\Rightarrow \quad 3 \neq \alpha - 3$

$\Rightarrow \quad 3+3 \neq \alpha$

$\Rightarrow \quad 6 \neq \alpha$

$\Rightarrow \quad \alpha \neq 6$

And,

$$\frac{\alpha}{12} = \frac{3}{\alpha}$$

$\Rightarrow \quad \alpha^2 = 36$

$\Rightarrow \quad \alpha = \pm 6$

$\Rightarrow \quad \alpha = -6 \qquad [\because \alpha \neq 6]$

Hence, the given system of equation will have no solution, if $\alpha = -6$.

28. Find the value of k for which the system

kx + 2y = 5

3x + y = 1

has (i) a unique solution, and (ii) no solution.

Sol:

The given system of equation may be written as

$kx + 2y - 5 = 0$

$3x + y - 1 = 0$

It is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = k, b_1 = 2, c_1 = -5$

And, $a_2 = 3, b_2 = 1, c_2 = -1$

(i) The given system will have a unique solution, if

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\Rightarrow \quad \frac{k}{3} \neq \frac{2}{1}$

$\Rightarrow \quad k \neq 6$

So, the given system of equations will have a unique solution, if $k \neq 6$

(ii) The given system will have no solution, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

We have

$\Rightarrow \quad \frac{b_1}{b_2} = \frac{2}{1}$ *and* $\frac{c_1}{c_2} = \frac{-5}{-1} = \frac{5}{1}$

Clearly, $\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given system of equations will have no solution, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$\Rightarrow \quad \frac{k}{3} = \frac{2}{1}$

$\Rightarrow \quad k = 6$

Hence, the given system of equations will have no solution, if $k = 6$.

29. Prove that there is a value of c ($\neq 0$) for which the system

$6x + 3y = c - 3$

$12x + cy = c$

has infinitely many solutions. Find this value.

Sol:

The given system of equation may be written as

$6x + 3y - (c - 3) = 0$

$12x + cy - c = 0$

This is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 6, b_1 = 3, c_1 = -(c - 3)$

And, $a_2 = 12, b_2 = c, c_2 = -c$

For infinitely many solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{6}{12} = \frac{13}{c} = \frac{-(c-3)}{-c}$

$\Rightarrow \quad \frac{6}{12} = \frac{13}{c} \text{ and } \frac{3}{c} = \frac{c-3}{c}$

$\Rightarrow \quad 6c = 12 \times 3 \text{ and } 3 = (c-3)$

$\Rightarrow \quad c = \frac{36}{6} \text{ and } c - 3 = 3$

$\Rightarrow \quad c = 6 \text{ and } c = 6$

Now,

$$\frac{a_1}{a_2} = \frac{6}{12} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{-(6-3)}{-6} = \frac{1}{2}$$

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Clearly, for this value of c, we have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the given system of equations has infinitely many solutions, if $c = 6$.

30. Find the values of k for which the system
2x + ky = 1
3x – 5y = 7
will have (i) a unique solution, and (ii) no solution. Is there a value of k for which the system has infinitely many solutions?

Sol:

The given system of equation may be written as

$2x + ky - 1 = 0$

$3x - 5y - 7 = 0$

It is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 2, b_1 = k, c_1 = -1$

And, $a_2 = 3, b_2 = -5, c_2 = -7$

(i) The given system will have a unique solution, if

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\Rightarrow \quad \frac{2}{3} \neq \frac{k}{-5}$

$\Rightarrow \quad -10 \neq 3k$

$\Rightarrow \quad 3k \neq -10$

$\Rightarrow \quad k \neq \frac{-10}{3}$

So, the given system of equations will have a unique solution, if $k = \frac{-10}{3}$.

(ii) The given system will have no solution, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

We have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$\Rightarrow \quad \frac{2}{3} = \frac{k}{-5}$

$\Rightarrow \quad -10 = 3k$

$\Rightarrow \quad 3k = -10$

$\Rightarrow \quad k = \frac{-10}{3}$

We have

$$\frac{b_1}{b_2} = \frac{k}{-5} = \frac{-10}{3 \times -5} = \frac{2}{3}$$

And, $\frac{c_1}{c_2} = \frac{-1}{-7} = \frac{1}{7}$

Clearly, $\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given system of equations will have no solution, if $k = \frac{-10}{3}$

For the given system to have infinite number of solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

We have,

$$\frac{a_1}{a_2}=\frac{2}{3},\frac{b_1}{b_2}=\frac{k}{-5}$$

And, $\frac{c_1}{c_2}=\frac{-1}{-7}=\frac{1}{7}$

Clearly, $\frac{a_1}{a_2}\neq\frac{c_1}{c_2}$

So, whatever be the value of *k*, we cannot have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Hence, there is no value of k, for which the given system of equations has infinitely many solutions.

31. For what value of k, the following system of equations will represent the coincident lines?

$x+2y+7=0$

$2x+ky+14=0$

Sol:

The given system of equations may be written as

$x+2y+7=0$

$2x+ky+14=0$

The given system of equations is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=1,b_1=2,c_1=7$

And $a_2=2,b_2=k,c_2=14$

The given equations will represent coincident lines if they have infinitely many solutions, The condition for which is

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}\Rightarrow\frac{1}{2}=\frac{2}{k}=\frac{7}{14}\Rightarrow k=4$$

Hence, the given system of equations will represent coincident lines, if $k=4$.

32. Obtain the condition for the following system of linear equations to have a unique solution

$ax+by=c$

$lx+my=n$

Sol:

The given system of equations may be written as

$ax+by-c=0$

$lx+my-n=0$

It is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 1, b_1 = 2, c_1 = -c$

And $a_2 = l, b_2 = m, c_2 = -n$

For unique solution, we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\Rightarrow \quad \frac{a}{l} \neq \frac{b}{m}$

$\Rightarrow \quad am \neq bl$

Hence, $am \neq bl$ is the required condition.

33. Determine the values of a and b so that the following system of linear equations have infinitely many solutions:

$(2a-1)x + 3y - 5 = 0$

$3x + (b-1)y - 2 = 0$

Sol:

The given system of equations may be written as

$(2a-1)x + 3y - 5 = 0$

$3x + (b-1)y - 2 = 0$

It is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 2a, b_1 = 3, c_1 = -5$

And $a_2 = 3, b_2 = b-1, c_2 = -2$

The given system of equations will have infinite number of solutions, if

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{2a-1}{3}=\frac{3}{b-1}=\frac{-5}{-2}$

$\Rightarrow \quad 2(2a-1)=\frac{-5}{-2}\ and\ \frac{3}{b-1}=\frac{-5}{-2}$

$\Rightarrow \quad 2(2a-1)=5\times 3\ and\ 3\times 2=5(b-1)$

$\Rightarrow \quad 4a-2=15\ and\ 6=5b-5$

$\Rightarrow \quad 4a=15+2\ and\ 6+5=5b$

$\Rightarrow \quad a=\frac{17}{4}\ and\ \frac{11}{5}=b$

$\Rightarrow \quad a=\frac{17}{4}\ and\ b=\frac{11}{5}$

Hence, the given system of equations will have infinitely many solutions,

If $a=\frac{17}{4}\ and\ b=\frac{11}{5}$.

34. Find the values of a and b for which the following system of linear equations has infinite number of solutions:

$2x-3y=7$

$(a+b)x-(a+b-3)y=4a+b$

Sol:

The given system of equations may be written as

$2x-3y-7=0$

$(a+b)x-(a+b-3)y-(4a+b)=0$

It is of the form

$a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

Where, $a_1=2, b_1=-3, c_1=-7$

And $a_2=a+b, b_2=-(a+b-3), c_2=-(4a+b)$

The given system of equations will have infinite number of solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \dfrac{2}{a+b} = \dfrac{-3}{-(a+b-3)} = \dfrac{-7}{-(4a+b)}$

$\Rightarrow \quad \dfrac{2}{a+b} = \dfrac{3}{(a+b-3)} \text{ and } \dfrac{3}{a+b-3} = \dfrac{7}{4a+b}$

$\Rightarrow \quad 2(a+b-3) = 3(a+b) \text{ and } 3(4a+b) = 7(a+b-3)$

$\Rightarrow \quad -6 = 3a - 2a + 3b - 2b \text{ and } 12a - 7a + 3b - 7b = 21$

$\Rightarrow \quad -6 = a + b \text{ and } 5a - 4b = -21$

Now,

$a + b = -6$

$\Rightarrow \quad a = -6 - b$

Substituting the value of 'a' in $5a - 4b = -2$, we get

$5(-b-6) - 4b = -21$

$\Rightarrow \quad -5b - 30 - 4b = -21$

$\Rightarrow \quad -9b = -21 + 30$

$\Rightarrow \quad -9b = 9$

$\Rightarrow \quad b = \dfrac{9}{-9} = -1$

Putting $b = -1$ in $a = -b - 6$, we get

$a = -(-1) - 6 = 1 - 6 = -5$

Hence, the given system of equations will have infinitely many solutions,

If $a = -5$ and $b = -1$.

35. Find the values of p and q for which the following system of linear equations has infinite number of solutions:

$2x - 3y = 9$

$(p+q)x + (2p-q)y = 3(p+q+1)$

Sol:

The given system of equations may be written as

$2x - 3y - 9 = 0$

$(p+q)x + (2p-q)y - 3(p+q+1) = 0$

It is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 2, b_1 = 3, c_1 = -9$

And $a_2 = p+q, b_2 = 2p-q, c_2 = -3(p+q+1)$

The given system of equations will have infinite number of solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{2}{p+q} = \frac{2}{2p-q} = \frac{-9}{-3(p+q+1)}$

$\Rightarrow \quad \frac{2}{p+q} = \frac{3}{2p-q} = \frac{3}{p+q+1}$

$\Rightarrow \quad \frac{2}{p+q} = \frac{3}{2p-q} \text{ and } \frac{3}{2p-q} = \frac{3}{p+q+1}$

$\Rightarrow \quad 2(2p-q) = 3(p+q) \text{ and } p+q+1 = 2p-q$

$\Rightarrow \quad 4p-2q = 3p+3q \text{ and } -2p+p+q+q = -1$

$\Rightarrow \quad p = 5q = 0 \text{ and } -p+2q = -1$

$\Rightarrow \quad p-5q-p+2q = -1$ [On adding]

$\Rightarrow \quad -3q = -1$

$\Rightarrow \quad q = \frac{1}{3}$

Putting $q = \frac{1}{3}$ in $p-5q$, we get

$$p - 5\left(\frac{1}{3}\right) = 0$$

$\Rightarrow \quad p = \frac{5}{3}$

Hence, the given system of equations will have infinitely many solutions,

If $p = \frac{5}{3.} \text{ and } q = \frac{1}{3}$

36. Find the values of a and b for which the following system of equations has infinitely many solutions:

$2x+3y=7$

$(a-b)x+(a+b)y = 3a+b-2$

Sol:

$2x+3y-7=0$

$(a-b)x+(a+b)y-(3a+b-2)=0$

Here, $a_1 = 2, b_1 = 3, c_1 = -7$

$a_2 = (a-b), b_2 = (a+b), c_2 = -(3a+b-2)$

$$\frac{a_1}{a_2}=\frac{2}{a-b},\frac{b_1}{b_2}=\frac{3}{a+b},\frac{c_1}{c_2}=\frac{-7}{-(3a+b-2)}=\frac{-7}{(3a+b-2)}$$

For the equation to have infinitely many solutions, we have:

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\frac{2}{a-b}=\frac{7}{3a+b-2}$$

$$6a+2b-4=7a-7b$$

$$a-9b=-4 \qquad ....(1)$$

$$\frac{2}{a-b}=\frac{3}{a+b}$$

$$2a+2b=3a-3b$$

$$a-5b=0 \qquad ....(2)$$

Subtracting (1) from (2), we obtain:

$$4b=4$$

$$b=1$$

Substituting the value of b in equation (2), we obtain

$$a-5\times1=0$$

$$a=5$$

Thus, the values of a and b are 5 and 1 respectively.

(i)

$$(2a-1)x-3y=5$$

$$3x+(b-2)y=3$$

Sol:

The given system of equations is

$$(2a-1)x-3y-5=0$$

$$3x+(b-2)y-3=0$$

It is of the form

$$a_1x+b_1y+c_1=0$$

$$a_2x+b_2y+c_2=0$$

Where, $a_1=2a-1,b_1=-3,c_1=-5$

And, $a_2=3,b_2=b-2,c_2=-3$

The given system of equations will have infinite number of solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{2a-1}{3} = \frac{-3}{b-2} = \frac{-5}{-3}$

$\Rightarrow \quad \frac{2a-1}{3} = \frac{-3}{b-2} = \frac{5}{3}$

$\Rightarrow \quad \frac{2a-1}{3} = \frac{5}{3} \; and \; \frac{-3}{b-2} = \frac{5}{3}$

$\Rightarrow \quad \frac{3(2a-1)}{3} = 5 \; and \; -9 = 5(b-2)$

$\Rightarrow \quad 2a-1 = 5 \; and \; -9 = 5b(b-2)$

$\Rightarrow \quad 2a = 5+1 \; and \; -9+10 = 5b$

$\Rightarrow \quad a = \frac{6}{2} \; and \; 1 = 5b$

$\Rightarrow \quad a = 3 \; and \; \frac{1}{5} = b$

$\Rightarrow \quad a = 3 \; and \; b = \frac{1}{5}$

Hence, the given system of equations will have infinitely many solutions,

If $a = 3$ and $b = \frac{1}{5}$

(ii)

$2x - (2a+5)y = 5$

$(2b+1)x - 9y = 15$

Sol:

The given system of equations is

$2x - (2a+5)y - 5 = 0$

$(2b+1)x - 9y - 15 = 0$

It is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = x = b_1 = -(2a+5), c_1 = -5$

And, $a_2 = (2b+1), b_2 = -9, c_2 = -15$

The given system of equations will be have infinite number of solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{2}{2b+1} = \frac{-(2a+5)}{-9} = \frac{-5}{-15}$$

$$\Rightarrow \quad \frac{2}{2b+1} = \frac{2a+5}{9} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{2}{2b+1} = \frac{1}{3} \ and \ \frac{2a+5}{9} = \frac{1}{3}$$

$$\Rightarrow \quad 6 = 2b+1 \ and \ \frac{3(2a+5)}{9} = 1$$

$$\Rightarrow \quad 6-1 = 2b \ and \ 2a+5 = 3$$

$$\Rightarrow \quad 5 = 2b \ and \ 2a = -2$$

$$\Rightarrow \quad \frac{5}{2} = b \ and \ a = \frac{-2}{2} = -1$$

Hence, the given system of equations will have infinitely many solutions,

If $a = -1 \ and \ b = \frac{5}{2}$.

(iii)

$(a-1)x + 3y = 2$

$6x + (1+2b)y = 6$

Sol:

The given system of equations is

$(a-1)x + 3y - 2 = 0$

$6x + (1+2b)y - 6 = 0$

It is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = a-1, b_1 = 3, c_1 = -2$

And, $a_2 = 6, b_2 = 1-2b, c_2 = -6$

The given system of equations will be have infinite number of solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{a-1}{6} = \frac{3}{1-2b} = \frac{-2}{-6}$

$\Rightarrow \quad \frac{a-1}{6} = \frac{3}{1-2b} = \frac{1}{3}$

$\Rightarrow \quad \frac{a-1}{b} = \frac{1}{3} \text{ and } \frac{3}{1-2b} = \frac{1}{3}$

$\Rightarrow \quad 3(a-1) = 6 \text{ and } 3\times 3 = 1-2b$

$\Rightarrow \quad a-1 = 2 \text{ and } 9 = 1-2b$

$\Rightarrow \quad a = 2+1 \text{ and } 2b = 1-9$

$\Rightarrow \quad a = 3 \text{ and } 2b = -8$

$\Rightarrow \quad a = 3 \text{ and } b = \frac{-8}{2} = -4$

Hence, the given system of equations will have infinitely many solutions,
If $a = 3$ *and* $b = -4$.

(iv)
$3x + 4y = 12$
$(a+b)x + 2(a-b)y = 5a-1$

Sol:
The given system of equations is
$3x + 4y - 12 = 0$
$(a+b)x + 2(a-b)y - (5a-1) = 0$
It is of the form
$a_1x + b_1y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$
Where, $a_1 = 3, b_1 = 4, c_1 = -12$
And, $a_2 = a+b, b_2 = 2(a-b), c_2 = -(5a-1)$
The given system of equations will be have infinite number of solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{3}{a+b} = \frac{4}{2(a-b)} = \frac{12}{5a-1}$

$\Rightarrow \quad \frac{3}{a+b} = \frac{2}{a-b} \text{ and } \frac{2}{a-b} = \frac{12}{5a-1}$

$\Rightarrow \quad 3(a-b) = 2(a+b) \text{ and } 2(5a-1) = 12(a-b)$

$\Rightarrow \quad 3a-3b = 2a+2b \text{ and } 10a-2-12a-12b$

$\Rightarrow \quad 3a-2a = 2b+3b \text{ and } 10a-12a = -12b+2$

$\Rightarrow \quad a = 5b \text{ and } -2a = -12b+2$

Substituting $a = 5b$ in $-2a = -12b+2,$ we get

$-2(5b) = -12b+2$

$\Rightarrow \quad -10b = -12b+2$

$\Rightarrow \quad 12b-10b = 2$

$\Rightarrow \quad 2b = 2$

$\Rightarrow \quad b = 1$

Putting $b = 1$ in $a = 5b,$ we get

$a = 5\times 1 = 5$

Hence, the given system of equations will have infinitely many solutions,
If $a = 5 \text{ and } b = 1.$

(v)
$2x+3y = 7$
$(a-1)x+(a+1)y = (3a-1)$

Sol:
The given system of equations is
$2x+3y-7 = 0$
$(a-1)x+(a+1)y-(3a-1) = 0$

It is of the form
$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$

Where, $a_1 = 2, b_1 = 3, c_1 = -7$

And, $a_2 = a-1, b_2 = a+1, c_2 = -(3a-1)$

The given system of equations will be have infinite number of solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{2}{a-b} = \frac{3}{a+1} = \frac{-7}{-(3a-1)}$

$\Rightarrow \quad \frac{2}{a-1} = \frac{3}{a+1} = \frac{-7}{3a-1}$

$\Rightarrow \quad \frac{3}{a-1} = \frac{3}{a+1}\ and\ \frac{3}{a+1} = \frac{7}{3a-1}$

$\Rightarrow \quad 2(a+1) = 3(a-1)\ and\ 3(3a-1) = 7(a+1)$

$\Rightarrow \quad 2a+2 = 3a-3\ and\ 9a-3 = 7a+7$

$\Rightarrow \quad 2a-3a = -3\ and\ 9a-3 = 7a+7$

$\Rightarrow \quad -a = -5\ and\ 2a = 10$

$\Rightarrow \quad a = 5\ and\ a = \frac{10}{2} = 5$

$\Rightarrow \quad a = 5$

Hence, the given system of equations will have infinitely many solutions,
If $a = 5$.

(vi)
$2x+3y = 7$
$(a-1)x+(a+2)y = 3a$

Sol:
The given system of equations is
$2x+3y-7 = 0$
$(a-1)x+(a+2)y-3a = 0$
It is of the form
$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$
Where, $a_1 = 2, b_1 = 3, c_1 = -7$
And, $a_2 = a-1, b_2 = a+1, c_2 = -3a$
The given system of equations will be have infinite number of solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{2}{a-b} = \frac{3}{a+1} = \frac{-7}{-3a}$

$\Rightarrow \quad \frac{2}{a-1} = \frac{3}{a+2} = \frac{7}{3a}$

$\Rightarrow \quad \frac{2}{a-1} = \frac{3}{a+2}$ *and* $\frac{3}{a+2} = \frac{7}{3a}$

$\Rightarrow \quad 2(a+2) = 3(a-1)$ *and* $3 \times 3a = 7(a+2)$

$\Rightarrow \quad 2a - 4a = -3$ *and* $9a = 7a + 14$

$\Rightarrow \quad 2a - 3a = -3$ *and* $9a - 7a = 14$

$\Rightarrow \quad -a = -7$ *and* $2a = 14$

$\Rightarrow \quad a = 7$ *and* $a = \frac{14}{2} = 7$

$\Rightarrow \quad a = 7$

Hence, the given system of equations will have infinitely many solutions,
If $a = 7$.

Exercise 3.6

1. 5 pens and 6 pencils together cost Rs 9 and 3 pens and 2 pencils cost Rs 5. Find the cost of 1 pen and 1 pencil.

Sol:

Let the cost of a pen be Rs x and that of a pencil be Rs y. Then,

$$5x + 6y = 9 \qquad ....(i)$$

and $$3x + 2y = 5 \qquad ......(ii)$$

Multiplying equation (i) by 2 and equation (ii) by 6, we get

$$10x + 12y = 18 \qquad ......(iii)$$

$$18x + 12y = 30 \qquad ......(iv)$$

Subtracting equation (iii) by equation (iv), we get

$$18x - 10x + 12y - 12y = 30 - 18$$

$\Rightarrow \quad 8x = 12$

$\Rightarrow \quad x = \frac{12}{8} = \frac{3}{2} = 1.5$

Substituting $x = 1.5$ in equation (i), we get

$$5 \times 1.5 + 6y = 9$$

$\Rightarrow \quad 7.5 + 6y = 9$

$\Rightarrow \quad 6y = 9 - 7.5$

$\Rightarrow \quad 6y = 1.5$

$\Rightarrow \quad y = \frac{1.5}{6} = \frac{1}{4} = 0.25$

Hence, cost of one pen $= Rs\ 1.50$ and cost of one pencil $= Rs\ 0.25$

2. 7 audio cassettes and 3 video cassettes cost Rs 1110, while 5 audio cassettes and 4 video cassettes cost Rs 1350. Find the cost of an audio cassette and a video cassette.

Sol:

Let the cost of a audio cassette be $Rs\ x$ and that of a video cassette be $Rs\ y$. Then,

$$7x+3y=1110 \qquad ....(i)$$

and $$5x+4y=1350 \qquad ....(ii)$$

Multiplying equation (i) by 4 and equation (ii) by 3, we get

$$28x+12y=4440 \qquad ......(iii)$$

$$15x+12y=4050 \qquad ......(iv)$$

Subtracting equation (iv) from equation (iii), we get

$$28x-15x+12y-12y=4440-4050$$

$$\Rightarrow \quad 13x=390$$

$$\Rightarrow \quad x=\frac{390}{13}=30$$

Substituting equation (iv) from equation (iii), we get

$$28x-15x+12y-12y=4440-4050$$

$$\Rightarrow \quad 13x=390$$

$$\Rightarrow \quad x=\frac{390}{13}=30$$

Substituting $x=30$ in equation (i), w get

$$7\times30+3y=1110$$

$$\Rightarrow \quad 210+3y=1110$$

$$\Rightarrow \quad 3y=1110-210$$

$$\Rightarrow \quad 3y=900$$

$$\Rightarrow \quad y=\frac{900}{3}=300$$

Hence, cost of one audio cassette $=Rs\,30$ and cost of one video cassette $=Rs\,300$

3. Reena has pens and pencils which together are 40 in number. If she has 5 more pencils and 5 less pens, then nuimber of pencils would become 4 times the number of pens. Find the original number of pens and pencils.

Sol:

Let the number of pens be x and that of pencil be y. then,

$$x+y=40 \qquad ....(i)$$

and $$(y+5)=4(x-5)$$

$\Rightarrow \quad y+5=4x-20$

$\Rightarrow \quad 5+20=4x-y$

$\Rightarrow \quad 4x-y=25 \quad ......(ii)$

Adding equation (i) and equation (ii), we get

$x+4x=40+25$

$\Rightarrow \quad 5x=65$

$\Rightarrow \quad x=\frac{65}{5}=13$

Putting $x=13$ in equation (i), we get

$13+y=40$

$\Rightarrow \quad y=40-13-27$

Hence, Reena has 13 pens 27 pencils.

4. 4 tables and 3 chairs, together, cost Rs 2,250 and 3 tables and 4 chairs cost Rs 1950. Find the cost of 2 chairs and 1 table.

Sol:

Let the cost of a table be $Rs\,x$ and that of a chairs be $Rs\ y.$ Then,

$4x+3y=2,250 \quad .....(i)$

$and, \quad 3x+4y=1950 \quad .....(ii)$

Multiplying equation (i) by 4 and equation (ii) by 3, we get

$16x+12y=9000 \quad ......(iii)$

$9x+12y=5850 \quad ....(iv)$

Subtracting equation (iv) by equation (iii), we get

$16x-9x=9000-5850$

$\Rightarrow \quad 7x=3150$

$\Rightarrow \quad x=\frac{3150}{7}=450$

Putting $x=450$ in equation (i), we get

$4\times 450+3y=2,250$

$\Rightarrow \quad 1800+3y=2250$

$\Rightarrow \quad 3y=2250-1800$

$\Rightarrow \quad 3y=450$

$\Rightarrow \quad y=\frac{450}{3}=150$

$\Rightarrow \quad 2y=2\times 150=300$

Cost of 2 chairs $=$ $Rs\ 300$ and cost of 1 table $=$ $Rs\ 450$

$\therefore$ The cost of 2 chairs and 1 table $=300+450=Rs\ 750$

5. 3 bags and 4 pens together cost Rs 257 whereas 4 bags and 3 pens together cost R 324. Find the total cost of 1 bag and 10 pens.

Sol:

Let the cost of a bag be $Rs\ x$ and that of a pen be $Rs\ y$. Then,

$$3x+4y=257 \quad .....(i)$$

$$and, \quad 4x+3y=324 \quad .....(ii)$$

Multiplying equation (i) by 3 and equation (ii) by 4, we get

$$9x+12y=770 \quad ....(iii)$$

$$16x+12y=1296 \quad ......(iv)$$

Subtracting equation (iii) by equation (iv), we get

$$16x-9x=1296-771$$

$$\Rightarrow \quad 7x=525$$

$$\Rightarrow \quad x=\frac{525}{7}=75$$

Cost of a bag $=Rs\,75$

Putting $x=75$ in equation (i), we get

$$3\times 75+4y=257$$

$$\Rightarrow \quad 225+4y=257$$

$$\Rightarrow \quad 4y=257-225$$

$$\Rightarrow \quad 4y=32$$

$$\Rightarrow \quad y=\frac{32}{4}=8$$

$\therefore$ Cost of a pen $=Rs\ 8$

$\therefore$ Cost of 10 pens $=8\times 10=Rs\ 80$

Hence, the total cost of 1 bag and 10 pens $=75+80=Rs$ 155.

6. 5 books and 7 pens together cost Rs 79 whereas 7 books and 5 pens together cost Rs 77. Find the total cost of 1 book and 2 pens.

Sol:

Let the cost of a book be Rs x and that of a pen be Rs y. Then,

$$5x+7y=79 \quad .....(i)$$

$$and, \quad 7x+5y=77 \quad ....(ii)$$

Multiplying equation (i) by 5 and equation (ii) by 7, we get

$$25+35y=395 \quad ......(iii)$$

$$49x+35y=539 \quad ....(iv)$$

Subtracting equation (iii) by equation (iv), we get

$$49x - 25x = 539 - 395$$

$\Rightarrow \quad 24x = 144$

$\Rightarrow \quad x = \frac{144}{24} = 6$

$\therefore$ Cost of a book $= Rs\ 6$

Putting $x = 6$ in equation (i), we get

$$5 \times 6 + 7y = 79$$

$\Rightarrow \quad 30 + 7y = 79$

$\Rightarrow \quad 7y = 79 - 30$

$\Rightarrow \quad 7y = 49$

$\Rightarrow \quad y = \frac{79}{7} = 7$

$\therefore$ Cost of a pen $= Rs\ 7$

$\therefore$ Cost of 2 pens $= 2 \times 7 = Rs\ 14$

Hence, the total cost of 1 book and 2 pens $= 6 + 14 = Rs\ 20$

7. A and B each have a certain number of mangoes. A says to B, "if you give 30 of your mangoes, I will have twice as many as left with you." B replies, "if you give me 10, I will have thrice as many as left with you." How many mangoes does each have?

Sol:

Suppose A has x mangoes and B has y mangoes

According to the given conditions, we have

$$x + 30 = 2(y - 30)$$

$\Rightarrow \quad x + 30 = 2y - 60$

$\Rightarrow \quad x - 2y = -60 - 30$

$\Rightarrow \quad x - 2y = -90 \qquad ....(i)$

And, $y + 10 = 3(x - 10)$

$\Rightarrow \quad y + 10 = 3x - 30$

$\Rightarrow \quad 10 + 30 = 3x - y$

$\Rightarrow \quad 3x - y = -40 \qquad ....(ii)$

Multiplying equation (i) by 3 and equation (ii) by 1, we get

$$3x - 6y = -270 \qquad .....(iii)$$

$$3x - y = 40 \qquad ......(iv)$$

Subtracting equation (iv) by equation (iii), we get

$$-6y-(-y)=-270-40$$
$\Rightarrow$ $-6y+y=-310$

$\Rightarrow$ $-5y=-310$

$\Rightarrow$ $y=\frac{310}{5}=62$

Putting $x=62$ in equation (i), we get

$$x-2\times 62=-90$$
$\Rightarrow$ $x-124=-90$

$\Rightarrow$ $x=-90+124$

$\Rightarrow$ $x=34$

Hence, *A* has 34 mangoes and *B* has 62 mangoes

8. On selling a T.V. at 5%gain and a fridge at 10% gain, a shopkeeper gains Rs 2000. But if he sells the T.V. at 10% gain and the fridge at 5% loss. He gains Rs 1500 on the transaction. Find the actual prices of T.V. and fridge.

Sol:

Let the price of a T.V. be Rs x and that of a fridge be Rs y. Then, we have

$$\frac{5x}{100}+\frac{10y}{100}=2000$$
$\Rightarrow$ $5x+10y=200000$

$\Rightarrow$ $5(x+2y)=200000$

$\Rightarrow$ $x+2y=400000 \quad ...(i)$

And, $\frac{10x}{100}-\frac{5y}{100}=1500$

$\Rightarrow 10x-5y=150000$

$\Rightarrow 5(2x-y)=150000$

$\Rightarrow 2x-y=30000$

Multiplying equation (ii) by 2, we get

$4x-2y=6000 \quad .....(iii)$

Adding equation (i) and equation (iii), we get

$$x+4x=40000+60000$$
$\Rightarrow$ $5x=100000$

$\Rightarrow$ $x=20000$

Putting $x=20000$ in equation (i), we get

$20000+2y=40000$

$\Rightarrow \quad 2y=40000-20000$

$\Rightarrow \quad y=\dfrac{20000}{2}=10000$

Hence, the actual price of T.V $= Rs\ 20,000$ and, the actual price of fridge $= Rs\ 10,000$

9. The coach of a cricket team buys 7 bats and 6 balls for Rs 3800. Later, he buys 3 bats and 5 balls for Rs 1750. Find the cost of each bat and each ball.

Sol:

Let the cost of bat and a ball be x and y respectively

According to the given information

$7x+6y=3800$ …….(1)

$3x+5y=1750$ ……..(2)

From (1), we obtain

$y=\dfrac{3800-7x}{6}$ ……..(3)

Substituting this value in equation (2), we obtain

$3x+5\left(\dfrac{3800-7x}{6}\right)=1750$

$3x+\dfrac{9500}{3}-\dfrac{35x}{6}=1750$

$3x-\dfrac{35x}{6}=1750=\dfrac{9500}{3}$

$3x-\dfrac{35x}{6}=\dfrac{5250-9500}{3}$

$-\dfrac{17x}{6}=\dfrac{-4250}{3}$

$-17x=-8500$

$x=500 \qquad (4)$

Substituting this in equation (3), we obtain

$y=\dfrac{3800-7\times 500}{6}$

$=\dfrac{300}{6}=50$

Hence, the cost of a bat is Rs 500 and that of a ball is Rs 50.

Concept Insight: Cost of bats and balls needs to be found so the cost of a ball and bat will be taken as the variables. Applying the conditions of total cost of bats and balls algebraic equations will be obtained. The pair of equations can then be solved by suitable substitution.

10. One says, "Give me a hundred, friend! I shall then become twice as rich as you." The other replies, "If you give me ten, I shall be six times as rich as you." Tell me what is the amount of their respective capital?

Sol:

Let the money with the first person and second person be Rs x and Rs y respectively.

According to the question

$x+100=2(y-100)$

$x+100=2y-200$

$x-2y=-300 \quad \ldots\ldots(1)$

$6(x-10)=(y+10)$

$6x-30=y+10$

$6x-y=70 \quad \ldots\ldots.(2)$

Multiplying equation (2) by 2, we obtain

$12x-2y=140 \quad \ldots\ldots.(3)$

Subtracting equation (1) from equation (3), we obtain:

$11x=140+300$

$11x=440$

$x=40$

Putting the value of x in equation (1), we obtain

$40-2y=-300$

$40+300=2y$

$2y=340$

$y=170$

Thus, the two friends had $Rs\,40$ and $Rs\,170$ with them.

11. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Sol:

Let the fixed charge for first three days and each day charge thereafter be Rs x and Rs y respectively.

According to the question,

$x+4y=27 \quad ...(1)$

$x+2y=21 \quad ...(2)$

Subtracting equation (2) from equation (1), we obtain:

$2y=6$

$y=3$

Subtracting the value of y in equation (1), we obtain

$x+12=27$

$x=15$

Hence, the fixed charge is Rs 15 and the charge per day is Rs 3.