Ex 7.1

Adjoint and Inverse of Matrix Ex 7.1 Q1(i)

Here, $A = \begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix}$

Cofactors of A are:

$C_{11} = 4$
$C_{12} = -2$
$C_{21} = -5$
$C_{22} = -3$

$\therefore \quad \text{adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

$\therefore \quad (\text{adj } A) = \begin{bmatrix} 4 & -2 \\ -5 & -3 \end{bmatrix}^T$

$= \begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix}$

Now, $(\text{adj } A)A = \begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$

And, $|A|.I = \begin{vmatrix} -3 & 5 \\ 2 & 4 \end{vmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (-22)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$

Also,

$A.(\text{adj } A) = \begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$

Therefore, $(\text{adj } A)A = |A|.I = A.(\text{adj } A)$

Adjoint and Inverse of Matrix Ex 7.1 Q1(ii)

Here, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Cofactors of A are:

$$C_{11} = d$$
$$C_{12} = -c$$
$$C_{21} = -b$$
$$C_{22} = a$$

$\therefore$ $\text{adj}A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

$\therefore$ $\text{adj}A = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}^T$
$= \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Now, $(\text{adj}A)(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ad-bc & bd-bd \\ -ac+ac & ad-bc \end{bmatrix} = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$

And, $|A|.I = \begin{vmatrix} a & b \\ c & d \end{vmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (ad-bc)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$

Also,

$$A(\text{adj}A) = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$$

$\therefore$ $(\text{adj}A)(A) = |A|.I = A(\text{adj}A)$

Adjoint and Inverse of Matrix Ex 7.1 Q1(iii)

Here, $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

Cofactors of A are:

$C_{11}=\cos\alpha$

$C_{12}=-\sin\alpha$

$C_{21}=-\sin\alpha$

$C_{22}=\cos\alpha$

$$\therefore\quad \text{Adj}\,A=\begin{bmatrix}C_{11} & C_{12}\\ C_{21} & C_{22}\end{bmatrix}^T$$

$$\text{Adj}\,A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}^T$$
$$=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

Now, $$(\text{adj}A).(A)=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$
$$=\begin{bmatrix}\cos^2\alpha-\sin^2\alpha & \cos\alpha\sin\alpha-\sin\alpha\cos\alpha\\ -\cos\alpha\sin\alpha+\sin\alpha\cos\alpha & -\sin^2\alpha+\cos^2\alpha\end{bmatrix}$$
$$=\begin{bmatrix}\cos2\alpha & 0\\ 0 & \cos2\alpha\end{bmatrix}$$

And, $$A(\text{adj}A)=\begin{bmatrix}\cos\alpha & \sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$
$$=\begin{bmatrix}\cos^2\alpha-\sin^2\alpha & -\cos\alpha\sin\alpha+\sin\alpha\cos\alpha\\ \sin\alpha\cos\alpha-\cos\alpha\sin\alpha & -\sin^2\alpha+\cos^2\alpha\end{bmatrix}$$
$$=\begin{bmatrix}\cos2\alpha & 0\\ 0 & \cos2\alpha\end{bmatrix}$$

Also,
$$|A|.I=\begin{vmatrix}\cos\alpha & \sin\alpha\\ \sin\alpha & \cos\alpha\end{vmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\left(\cos^2\alpha-\sin^2\alpha\right)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}\cos^2\alpha-\sin^2\alpha & 0\\ 0 & \cos^2\alpha-\sin^2\alpha\end{bmatrix}$$

$$=\begin{bmatrix}\cos2\alpha & 0\\ 0 & \cos2\alpha\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q1(iv)

We have,

$$A = \begin{bmatrix} 1 & \tan\alpha/2 \\ -\tan\alpha/2 & 1 \end{bmatrix}$$

Cofactors of A are:

$c_{11} = 1$, $\quad c_{12} = -\left(-\tan\alpha/2\right) = \tan\alpha/2$

$c_{21} = -\tan\alpha/2$, $\quad c_{22} = 1$

$$\therefore \text{ adj } A = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & \tan\alpha/2 \\ -\tan\alpha/2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -\tan\alpha/2 \\ \tan\alpha/2 & 1 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 1 & \tan\alpha/2 \\ -\tan\alpha/2 & 1 \end{vmatrix}$$

$$= 1 + \tan^2\alpha/2$$

$$= \sec^2\alpha/2$$

We have,

$$A = \begin{bmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{bmatrix}$$

Cofactors of A are:

$c_{11} = 1, \quad c_{12} = -\left(-\tan \alpha/2\right) = \tan \alpha/2$

$c_{21} = -\tan \alpha/2, \quad c_{22} = 1$

$$\therefore \text{ adj } A = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -\tan \alpha/2 \\ \tan \alpha/2 & 1 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{vmatrix}$$

$$= 1 + \tan^2 \alpha/2$$

$$= \sec^2 \alpha/2$$

Adjoint and Inverse of Matrix Ex 7.1 Q2(i)

Here $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

Cofactors of A are:

$C_{11} = -3 \qquad C_{21} = +2 \qquad C_{31} = 2$

$C_{12} = +2 \qquad C_{22} = -3 \qquad C_{32} = 2$

$C_{13} = 2 \qquad C_{23} = 2 \qquad C_{33} = -3$

$$\therefore \quad \text{adj}\, A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}^T$$

Therefore,

$$\text{adj}\, A = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

Now,

$$(\text{adj}\, A).A = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$|A|.I = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= (-3+4+4) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$A.(\text{adj}\, A) = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\therefore \quad (\text{adj}\, A).A = |A|.I = A.(\text{adj}\, A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q2(ii)

Here, $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

Cofactors of A are:

$C_{11} = 2 \qquad C_{21} = 3 \qquad C_{31} = -13$

$C_{12} = -3 \qquad C_{22} = 6 \qquad C_{32} = 9$

$C_{13} = 5 \qquad C_{23} = -3 \qquad C_{33} = -1$

$$\therefore \quad \operatorname{adj} A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & -3 & 5 \\ 3 & 6 & -3 \\ -13 & 9 & -1 \end{bmatrix}^T$$

Therefore,

$$\operatorname{adj} A = \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix}$$

Now,

$$(\operatorname{adj} A).A = \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}$$

$$|A|.I = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{vmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}$$

$$A.(\operatorname{adj} A) = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}$$

$$\therefore \quad (\operatorname{adj} A).A = |A|.I = A.(\operatorname{adj} A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q2(iii)

Here, $A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{bmatrix}$

Cofactors of A are:

$C_{11} = -22 \qquad C_{21} = 11 \qquad C_{31} = -11$
$C_{12} = 4 \qquad C_{22} = -2 \qquad C_{32} = 2$
$C_{13} = 16 \qquad C_{23} = -8 \qquad C_{33} = 8$

$$\therefore \quad \text{adj}A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$\therefore \quad = \begin{bmatrix} -22 & 4 & 16 \\ 11 & -2 & -8 \\ -11 & 2 & 8 \end{bmatrix}^T$$

Therefore,

$$\text{adj}A = \begin{bmatrix} -22 & 11 & -11 \\ 4 & -2 & 2 \\ 16 & -8 & 8 \end{bmatrix}$$

Now,

$$(\text{adj}A).A = \begin{bmatrix} -22 & 11 & -11 \\ 4 & -2 & 2 \\ 16 & -8 & 8 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$|A|.I = \begin{vmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{vmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= (44 - 4 + 48) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$A(\text{adj}A) = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{bmatrix} \begin{bmatrix} -22 & 11 & -11 \\ 4 & -2 & 2 \\ 16 & -8 & 8 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \quad (\text{adj}A).A = |A|I = A(\text{adj}A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q2(iv)

Here, $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$

Cofactors of A are:

$C_{11} = 3$ $\quad C_{21} = -1$ $\quad C_{31} = 1$

$C_{12} = -15$ $\quad C_{22} = 7$ $\quad C_{32} = -5$

$C_{13} = 4$ $\quad C_{23} = -2$ $\quad C_{33} = 2$

$$\therefore \quad \text{adj} A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 3 & -15 & 4 \\ -1 & 7 & -2 \\ 1 & -5 & 2 \end{bmatrix}^T$$

Therefore,

$$\text{adj} A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{bmatrix}$$

Now,

$$(\text{adj} A) A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$|A|.I = \begin{vmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{vmatrix} I_3$$

$$= (6-4) I_3 = 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A.(\text{adj} A) = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\therefore \quad (\text{adj} A) A = |A|.I = A.(\text{adj} A)$$

Here, $A=\begin{bmatrix}1 & 2 & 3\\ 0 & 5 & 0\\ 2 & 4 & 3\end{bmatrix}$

Cofactors of A are:

$C_{11} = 15$ $\quad C_{21} = 6$ $\quad C_{31} = -15$

$C_{12} = 0$ $\quad C_{22} = -3$ $\quad C_{32} = 0$

$C_{13} = -10$ $\quad C_{23} = 0$ $\quad C_{33} = 5$

$$\therefore \quad \operatorname{adj} A = \begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T$$

$$= \begin{bmatrix}15 & 0 & -10\\ 6 & -3 & 0\\ -15 & 0 & 5\end{bmatrix}^T$$

Therefore,

$$\operatorname{adj} A = \begin{bmatrix}15 & 6 & -15\\ 0 & -3 & 0\\ -10 & 0 & 5\end{bmatrix}$$

Now,

$$(\operatorname{adj} A) A = \begin{bmatrix}15 & 6 & -15\\ 0 & -3 & 0\\ -10 & 0 & 5\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 0 & 5 & 0\\ 2 & 4 & 3\end{bmatrix} = \begin{bmatrix}-15 & 0 & 0\\ 0 & -15 & 0\\ 0 & 0 & -15\end{bmatrix}$$

$$|A|.I = \begin{vmatrix}1 & 2 & 3\\ 0 & 5 & 0\\ 2 & 4 & 3\end{vmatrix} \quad I_3$$

$$= (\ -15) I_3 = \begin{bmatrix}-15 & 0 & 0\\ 0 & -15 & 0\\ 0 & 0 & -15\end{bmatrix}$$

$$A.(\operatorname{adj} A) = \begin{bmatrix}1 & 2 & 3\\ 0 & 5 & 0\\ 2 & 4 & 3\end{bmatrix}\begin{bmatrix}15 & 6 & -15\\ 0 & -3 & 0\\ -10 & 0 & 5\end{bmatrix} = \begin{bmatrix}-15 & 0 & 0\\ 0 & -15 & 0\\ 0 & 0 & -15\end{bmatrix}$$

$$\therefore \quad (\operatorname{adj} A) A = |A|.I = A.(\operatorname{adj} A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q3

Here

$$A=\begin{bmatrix}1 & -1 & 1\\ 2 & 3 & 0\\ 18 & 2 & 10\end{bmatrix}$$

Cofactors of A are

$$\begin{array}{lll} C_{11}=30 & C_{21}=12 & C_{31}=-3\\ C_{12}=-20 & C_{22}=-8 & C_{32}=2\\ C_{13}=-50 & C_{23}=-20 & C_{33}=5\end{array}$$

Therefore,

$$\operatorname{adj} A=\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T$$

$$=\begin{bmatrix}30 & -20 & -50\\ 12 & -8 & -20\\ -3 & 2 & 5\end{bmatrix}^T$$

So,

$$\operatorname{adj} A=\begin{bmatrix}30 & 12 & -3\\ -20 & -8 & 2\\ -50 & -20 & 5\end{bmatrix}$$

Now,

$$A(\operatorname{adj} A)=\begin{bmatrix}1 & -1 & 1\\ 2 & 3 & 0\\ 18 & 2 & 10\end{bmatrix}\begin{bmatrix}30 & 12 & -3\\ -20 & -8 & 2\\ -50 & -20 & 5\end{bmatrix}$$

$$=\begin{bmatrix}1 & -1 & 1\\ 2 & 3 & 0\\ 18 & 2 & 10\end{bmatrix}(0)$$

$$=0$$

Hence proved.

Adjoint and Inverse of Matrix Ex 7.1 Q4

Here, $A=\begin{bmatrix}-4 & -3 & -3\\ 1 & 0 & 1\\ 4 & 4 & 3\end{bmatrix}$

Cofactors of A are:

$$\begin{array}{lll} C_{11}=-4 & C_{21}=-3 & C_{31}=-3\\ C_{12}=1 & C_{22}=0 & C_{32}=1\\ C_{13}=4 & C_{23}=4 & C_{33}=3\end{array}$$

$$\therefore \quad \operatorname{adj} A=\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T$$

$$=\begin{bmatrix}-4 & 1 & 4\\ -3 & 0 & 4\\ -3 & 1 & 3\end{bmatrix}^T$$

Therefore, $\operatorname{adj} A=\begin{bmatrix}-4 & -3 & -3\\ 1 & 0 & 1\\ 4 & 4 & 3\end{bmatrix}$

So, $\operatorname{adj} A = A$

Adjoint and Inverse of Matrix Ex 7.1 Q5

Here $A=\begin{bmatrix}-1 & -2 & -2\\ 2 & 1 & -2\\ 2 & -2 & 1\end{bmatrix}$

Cofactors of A are:

$C_{11}=-3$ $C_{21}=6$ $C_{31}=6$

$C_{12}=-6$ $C_{22}=3$ $C_{32}=-6$

$C_{13}=-6$ $C_{23}=-6$ $C_{33}=3$

$$\therefore \quad \operatorname{adj}A=\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T$$

$$=\begin{bmatrix}-3 & -6 & -6\\ 6 & 3 & -6\\ 6 & -6 & 3\end{bmatrix}^T$$

Therefore, $\operatorname{adj}A=\begin{bmatrix}-3 & 6 & 6\\ -6 & 3 & -6\\ -6 & -6 & 3\end{bmatrix}$ ---(i)

Now, $3.A^T=3\begin{bmatrix}-1 & 2 & 2\\ -2 & 1 & -2\\ -2 & -2 & 1\end{bmatrix}=\begin{bmatrix}-3 & 6 & 6\\ -6 & 3 & -6\\ -6 & -6 & 3\end{bmatrix}$ ---(ii)

$$\therefore \quad \operatorname{adj}A=3.A^T$$

Adjoint and Inverse of Matrix Ex 7.1 Q6

Here, $A=\begin{bmatrix}1 & -2 & 3\\ 0 & 2 & -1\\ -4 & 5 & 2\end{bmatrix}$

Cofactors of A are:

$C_{11}=9$ $C_{21}=19$ $C_{31}=-4$

$C_{12}=4$ $C_{22}=14$ $C_{32}=1$

$C_{13}=8$ $C_{23}=3$ $C_{33}=2$

$$\therefore \quad \operatorname{adj}A=\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T$$

$$=\begin{bmatrix}9 & 4 & 8\\ 19 & 14 & 3\\ -4 & 1 & 2\end{bmatrix}^T$$

Therefore,

$$\operatorname{adj}A=\begin{bmatrix}9 & 19 & -4\\ 4 & 14 & 1\\ 8 & 3 & 2\end{bmatrix}$$

Now, $A\operatorname{adj}A=\begin{bmatrix}1 & -2 & 3\\ 0 & 2 & -1\\ -4 & 5 & 2\end{bmatrix}\begin{bmatrix}9 & 19 & -4\\ 4 & 14 & 1\\ 8 & 3 & 2\end{bmatrix}$

$$=\begin{bmatrix}25 & 0 & 0\\ 0 & 25 & 0\\ 0 & 0 & 25\end{bmatrix}$$

$$=25\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=25I_3$$

Adjoint and Inverse of Matrix Ex 7.1 Q7 (i)

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Now, $|A| = 1 \neq 0$

Hence A^{-1} exists.

Cofactors of A are:

$$C_{11} = \cos\theta \qquad C_{21} = -\sin\theta$$
$$C_{12} = \sin\theta \qquad C_{22} = \cos\theta$$

$$\therefore \quad \text{adj}A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$
$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\therefore \quad \text{adj}A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Now, $A^{-1} = \frac{1}{|A|} . \text{adj}A$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q7 (ii)

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Now, $|A| = -1 \neq 0$

Hence A^{-1} exists.

Cofactors of A are:

$$C_{11} = 0 \qquad C_{12} = -1$$
$$C_{21} = -1 \qquad C_{22} = 0$$

$$\therefore \quad \text{adj}A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$
$$= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

Also, $A^{-1} = \frac{1}{|A|}(\text{adj}A)$

$$A^{-1} = \frac{1}{(-1)}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q7 (iii)

$$A = \begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix}$$

Now, $|A| = \frac{a+abc}{a} - bc = \frac{a+abc-abc}{a} = 1 \neq 0$

Hence A^{-1} exists.

Now, cofactors of A are:

$C_{11} = \frac{1+bc}{a}$ $\quad C_{12} = -c$

$C_{21} = -b$ $\quad C_{22} = a$

$$\therefore \quad \operatorname{adj} A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$

$$= \begin{bmatrix} \frac{1+bc}{a} & -c \\ -b & a \end{bmatrix}^T$$

$$\therefore \quad \operatorname{adj} A = \begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$

Also, $A^{-1} = \frac{1}{|A|} . \operatorname{adj} A$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$

or $\quad A^{-1} = \begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q7 (iv)

$$A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$$

Now, $|A| = 2 + 15 = 17 \neq 0$

Hence A^{-1} exists.

Now, cofactors of A are:

$C_{11} = 1$ $\quad C_{12} = 3$

$C_{21} = -5$ $\quad C_{22} = 2$

$$\therefore \quad \operatorname{adj} A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & 3 \\ -5 & 2 \end{bmatrix}$$

$$\therefore \quad \operatorname{adj} A = \begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} . (\operatorname{adj} A)$$

$$A^{-1} = \frac{1}{17}\begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$$

Hence, $A^{-1} = \frac{1}{17}\begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q8(i)

Here, $A=\begin{bmatrix}1&2&3\\2&3&1\\3&1&2\end{bmatrix}$

Expanding using 1^{st} row, we get

$$|A|=1\begin{vmatrix}3&1\\1&2\end{vmatrix}-2\begin{vmatrix}2&1\\3&2\end{vmatrix}+3\begin{vmatrix}2&3\\3&1\end{vmatrix}$$
$$=1(6-1)-2(4-3)+3(2-9)$$
$$=5-2\times1+3\times(-7)$$
$$=5-2-21=-18\neq0$$

Therefore, A^{-1} exists.

Cofactors of A are:

$C_{11}=5$ $\quad C_{21}=-1$ $\quad C_{31}=-7$

$C_{12}=-1$ $\quad C_{22}=-7$ $\quad C_{32}=5$

$C_{13}=-7$ $\quad C_{23}=5$ $\quad C_{33}=-1$

$$\therefore \quad \text{adj}A=\begin{bmatrix}C_{11}&C_{12}&C_{13}\\C_{21}&C_{22}&C_{23}\\C_{31}&C_{32}&C_{33}\end{bmatrix}^T$$
$$=\begin{bmatrix}5&-1&-7\\-1&-7&5\\-7&5&-1\end{bmatrix}^T$$
$$=\begin{bmatrix}5&-1&-7\\-1&-7&5\\-7&5&-1\end{bmatrix}$$

Now,

$$A^{-1}=\frac{1}{|A|}.\text{adj}A$$

Hence, $A^{-1}=\dfrac{1}{(-18)}\begin{bmatrix}5&-1&-7\\-1&-7&5\\-7&5&-1\end{bmatrix}=\begin{bmatrix}\frac{-5}{18}&\frac{1}{18}&\frac{7}{18}\\\frac{1}{18}&\frac{7}{18}&\frac{-5}{18}\\\frac{7}{18}&\frac{-5}{18}&\frac{1}{18}\end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q8(ii)

Here, $A=\begin{bmatrix}1 & 2 & 5\\ 1 & -1 & -1\\ 2 & 3 & -1\end{bmatrix}$

Expanding using first column, we get

$$|A| = 1\begin{vmatrix}-1 & -1\\ 3 & -1\end{vmatrix} - 2\begin{vmatrix}1 & -1\\ 2 & -1\end{vmatrix} + 5\begin{vmatrix}1 & -1\\ 2 & 3\end{vmatrix}$$
$$= 1\times(1+3) - 2(-1+2) + 5(3+2)$$
$$= 4 - 2(1) + 5(5) = 27 \neq 0$$

Therefore, A^{-1} exists.

Cofactors of A are:

$C_{11} = 4$ $\quad C_{21} = +17$ $\quad C_{31} = 3$

$C_{12} = -1$ $\quad C_{22} = -11$ $\quad C_{32} = +6$

$C_{13} = 5$ $\quad C_{23} = +1$ $\quad C_{33} = -3$

$$\therefore \quad \text{adj}A = \begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T$$
$$= \begin{bmatrix}4 & -1 & 5\\ 17 & -11 & 1\\ 3 & 6 & -3\end{bmatrix}^T$$
$$= \begin{bmatrix}4 & 17 & 3\\ -1 & -11 & 6\\ 5 & 1 & -3\end{bmatrix}$$

Now, $A^{-1} = \frac{1}{|A|}.\text{adj}A$

$$\therefore \quad A^{-1} = \frac{1}{27}\begin{bmatrix}4 & 17 & 3\\ -1 & -11 & 6\\ 5 & 1 & -3\end{bmatrix} = \begin{bmatrix}\frac{4}{27} & \frac{17}{27} & \frac{3}{27}\\ \frac{-1}{27} & \frac{-11}{27} & \frac{6}{27}\\ \frac{5}{27} & \frac{1}{27} & \frac{-3}{27}\end{bmatrix} = \begin{bmatrix}\frac{4}{27} & \frac{17}{27} & \frac{1}{9}\\ \frac{-1}{27} & \frac{-11}{27} & \frac{2}{9}\\ \frac{5}{27} & \frac{1}{27} & \frac{-1}{9}\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q8(iii)

Here, $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

Expanding using first column, we get

$$|A| = 2\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + 1\begin{vmatrix} -1 & -1 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix}$$
$$= 2(4-1) + 1(-2+1) + 1(1-2)$$
$$= 2(3) + 1(-1) + 1(-1) = 6 - 2 = 4 \neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$C_{11} = 3$ $\quad C_{21} = +1$ $\quad C_{31} = -1$

$C_{12} = +1$ $\quad C_{22} = 3$ $\quad C_{32} = +1$

$C_{13} = -1$ $\quad C_{23} = +1$ $\quad C_{33} = 3$

$$\therefore \quad \text{adj}\,A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$
$$= \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}^T$$
$$= \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

Now, $A^{-1} = \frac{1}{|A|}.\text{adj}\,A$

$$\therefore \quad A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & \frac{-1}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} \\ \frac{-1}{4} & \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q8(iv)

Here, $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

Expanding using first column, we get

$$|A| = 2\begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} - 0\begin{vmatrix} 5 & 0 \\ 0 & 3 \end{vmatrix} - 1\begin{vmatrix} 5 & 1 \\ 0 & 1 \end{vmatrix}$$
$$= 2(3-0) - 0 - 1(5)$$
$$= 2(3) - 1(5) = 1 \neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 3 & C_{21} = -1 & C_{31} = 1 \\ C_{12} = -15 & C_{22} = 6 & C_{32} = -5 \\ C_{13} = 5 & C_{23} = -2 & C_{33} = 2 \end{array}$$

$$\therefore \quad \text{adj}A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$
$$= \begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & 2 \end{bmatrix}^T$$
$$= \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}.\text{adj}\,A$$
$$= \frac{1}{1}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

Hence, $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q8(v)

Here, $A=\begin{bmatrix}0 & 1 & -1\\ 4 & -3 & 4\\ 3 & -3 & 4\end{bmatrix}$

Expanding using first column, we get

$$|A|=0\begin{vmatrix}-3 & 4\\ -3 & 4\end{vmatrix}-1\begin{vmatrix}4 & 4\\ 3 & 4\end{vmatrix}-1\begin{vmatrix}4 & -3\\ 3 & -3\end{vmatrix}$$
$$=0-1(6-12)-1(-12+9)$$
$$=-1(4)-1(-3)=-1\neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$C_{11}=0$ $\quad C_{21}=-1$ $\quad C_{31}=1$
$C_{12}=-4$ $\quad C_{22}=3$ $\quad C_{32}=-4$
$C_{13}=-3$ $\quad C_{23}=+3$ $\quad C_{33}=-4$

$$\therefore \quad \operatorname{adj}A=\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T$$
$$=\begin{bmatrix}0 & -4 & -3\\ -1 & 3 & 3\\ 1 & -4 & -4\end{bmatrix}^T$$
$$=\begin{bmatrix}0 & -1 & 1\\ -4 & 3 & -4\\ -3 & 3 & -4\end{bmatrix}$$

Now, $A^{-1}=\frac{1}{|A|}.\operatorname{adj}A$

$$A^{-1}=\frac{1}{(-1)}\begin{bmatrix}0 & -1 & 1\\ -4 & 3 & -4\\ -3 & 3 & -4\end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}0 & 1 & -1\\ 4 & -3 & 4\\ 3 & -3 & 4\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q8(vi)

Here, $A = \begin{bmatrix} 0 & 0 & -1 \\ 3 & 4 & 5 \\ -2 & -4 & -7 \end{bmatrix}$

Expanding using first column, we get

$$|A| = 0\begin{vmatrix} 4 & 5 \\ -4 & -7 \end{vmatrix} - 0\begin{vmatrix} 3 & 5 \\ -2 & -7 \end{vmatrix} - 1\begin{vmatrix} 3 & 4 \\ -2 & -4 \end{vmatrix}$$
$$= 0 - 0 - 1(-12 + 8)$$
$$= -1(-4) = 4 \neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$C_{11} = -8$ $\quad C_{21} = +4$ $\quad C_{31} = 4$

$C_{12} = +11$ $\quad C_{22} = -2$ $\quad C_{32} = -3$

$C_{13} = -4$ $\quad C_{23} = +0$ $\quad C_{33} = 0$

$$\therefore \quad \operatorname{adj} A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$
$$= \begin{bmatrix} -8 & 11 & -4 \\ 4 & -2 & 0 \\ 4 & -3 & 0 \end{bmatrix}^T$$
$$= \begin{bmatrix} -8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}.\operatorname{adj} A$$
$$A^{-1} = \frac{1}{4}\begin{bmatrix} -8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix}$$

Hence, $A^{-1} = \frac{1}{4}\begin{bmatrix} -8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 1 \\ \frac{11}{4} & \frac{-1}{2} & \frac{-3}{4} \\ -1 & 0 & 0 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q8(vii)

Here, $A=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\alpha & \sin\alpha\\ 0 & \sin\alpha & -\cos\alpha\end{bmatrix}$

Expanding using first column, we get

$$|A| = 1\begin{vmatrix}\cos\alpha & \sin\alpha\\ \sin\alpha & -\cos\alpha\end{vmatrix} - 0 + 0$$
$$= -\cos^2\alpha + \sin^2\alpha$$
$$= -\left(\cos^2\alpha + \sin^2\alpha\right)$$
$$|A| = -1 \neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$C_{11} = -1$ $\quad C_{21} = 0$ $\quad C_{31} = 0$

$C_{12} = 0$ $\quad C_{22} = -\cos\alpha$ $\quad C_{32} = -\sin\alpha$

$C_{13} = 0$ $\quad C_{23} = -\sin\alpha$ $\quad C_{33} = \cos\alpha$

$$\therefore \quad \operatorname{adj} A = \begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T$$
$$= \begin{bmatrix}-1 & 0 & 0\\ 0 & -\cos\alpha & -\sin\alpha\\ 0 & -\sin\alpha & \cos\alpha\end{bmatrix}^T$$
$$= \begin{bmatrix}-1 & 0 & 0\\ 0 & -\cos\alpha & -\sin\alpha\\ 0 & -\sin\alpha & \cos\alpha\end{bmatrix}$$

Now, $A^{-1} = \frac{1}{|A|}.\operatorname{adj} A$

$$\therefore \quad A^{-1} = \frac{1}{(-1)}\begin{bmatrix}-1 & 0 & 0\\ 0 & -\cos\alpha & -\sin\alpha\\ 0 & -\sin\alpha & \cos\alpha\end{bmatrix}$$

Hence, $A^{-1} = \begin{bmatrix}1 & 0 & 0\\ 0 & \cos\alpha & \sin\alpha\\ 0 & \sin\alpha & -\cos\alpha\end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q9(i)

$$A=\begin{bmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{bmatrix}$$

Expanding using 1^{st} row, we get

$$|A| = 1\begin{vmatrix}4 & 3\\ 3 & 4\end{vmatrix} - 3\begin{vmatrix}1 & 3\\ 1 & 4\end{vmatrix} + 3\begin{vmatrix}1 & 4\\ 1 & 3\end{vmatrix}$$
$$= 1(16-9) - 3(4-3) + 3(3-4)$$
$$= 7 - 3(1) + 3(-1)$$
$$= 7 - 3 - 3 = +1 = 1 \neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 7 & C_{21} = -3 & C_{31} = -3 \\ C_{12} = -1 & C_{22} = 1 & C_{32} = -0 \\ C_{13} = -1 & C_{23} = -0 & C_{33} = 1 \end{array}$$

$$\therefore \quad \operatorname{adj} A = \begin{bmatrix} C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T$$
$$= \begin{bmatrix} 7 & -1 & -1\\ -3 & 1 & 0\\ -3 & 0 & 1\end{bmatrix}^T$$
$$= \begin{bmatrix} 7 & -3 & -3\\ -1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}$$

Now, $A^{-1} = \dfrac{1}{|A|}.\operatorname{adj} A$

$$\therefore \quad A^{-1} = \frac{1}{1}\begin{bmatrix} 7 & -3 & -3\\ -1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}$$

Also, $A^{-1}.A = \begin{bmatrix} 7 & -3 & -3\\ -1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q9(ii)

$$A=\begin{bmatrix}2&3&1\\3&4&1\\3&7&2\end{bmatrix}$$

Expanding 1st row, we get

$$|A|=2\begin{vmatrix}4&1\\7&2\end{vmatrix}-3\begin{vmatrix}3&1\\3&2\end{vmatrix}+1\begin{vmatrix}3&4\\3&7\end{vmatrix}$$
$$=2(8-7)-3(6-3)+1(21-12)$$
$$=2-3(3)+1(9)=2\neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll} C_{11}=1 & C_{21}=+1 & C_{31}=-1\\ C_{12}=-3 & C_{22}=1 & C_{32}=+1\\ C_{13}=9 & C_{23}=-5 & C_{33}=-1\end{array}$$

$$\therefore \quad \operatorname{adj}A=\begin{bmatrix}C_{11}&C_{12}&C_{13}\\C_{21}&C_{22}&C_{23}\\C_{31}&C_{32}&C_{33}\end{bmatrix}^T$$
$$=\begin{bmatrix}1&-3&9\\1&1&-5\\-1&1&-1\end{bmatrix}^T=\begin{bmatrix}1&1&-1\\-3&1&1\\9&-5&-1\end{bmatrix}$$

Now, $A^{-1}=\frac{1}{|A|}.\operatorname{adj}A\Rightarrow A^{-1}=\frac{1}{2}\begin{bmatrix}1&1&-1\\-3&1&1\\9&-5&-1\end{bmatrix}$

Also, $A^{-1}.A=\frac{1}{2}\begin{bmatrix}1&1&-1\\-3&1&1\\9&-5&-1\end{bmatrix}\begin{bmatrix}2&3&1\\3&4&1\\3&7&2\end{bmatrix}$

$$=\frac{1}{2}\begin{bmatrix}2+3-3&3+4-7&1+1-2\\-6+3+3&-9+4+7&-3+1+2\\18-15-3&27-20-7&9-5-2\end{bmatrix}$$
$$=\frac{1}{2}\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$\therefore A^{-1}.A=I_3$

Adjoint and Inverse of Matrix Ex 7.1 Q10(i)

$$A=\begin{bmatrix}3&2\\7&5\end{bmatrix} \quad \therefore |A|=1\neq 0$$

$$\text{adj}\,A=\begin{bmatrix}5&-2\\-7&3\end{bmatrix} \Rightarrow A^{-1}=\frac{\text{adj}\,A}{|A|}=\frac{1}{1}\begin{bmatrix}5&-2\\-7&3\end{bmatrix}$$

$$B=\begin{bmatrix}4&6\\3&2\end{bmatrix} \quad \therefore |B|=-10\neq 0$$

$$\text{adj}B=\begin{bmatrix}2&-6\\-3&4\end{bmatrix} \Rightarrow B^{-1}=\frac{\text{adj}B}{|B|}=\frac{1}{-10}\begin{bmatrix}2&-6\\-3&4\end{bmatrix}$$

$$\text{Also,} \quad AB=\begin{bmatrix}3&2\\7&5\end{bmatrix}\begin{bmatrix}4&6\\3&2\end{bmatrix}=\begin{bmatrix}18&22\\43&52\end{bmatrix}$$

$$|AB|=936-946=-10\neq 0$$

$$\text{adj}(AB)=\begin{bmatrix}52&-22\\-43&18\end{bmatrix}$$

$$(AB)^{-1}=\frac{\text{adj}(AB)}{|AB|}=\frac{1}{|AB|}\begin{bmatrix}52&-22\\-43&18\end{bmatrix}=\frac{1}{-10}\begin{bmatrix}+52&-22\\-43&+18\end{bmatrix}=\frac{1}{10}\begin{bmatrix}-52&22\\43&-18\end{bmatrix}$$

$$B^{-1}.A^{-1}=\frac{-1}{10}\begin{bmatrix}2&-6\\-3&4\end{bmatrix}\begin{bmatrix}5&-2\\-7&3\end{bmatrix}=\frac{-1}{10}\begin{bmatrix}52&-22\\-43&18\end{bmatrix}=\frac{1}{10}\begin{bmatrix}-52&22\\43&-18\end{bmatrix}$$

Hence, $(AB)^{-1}=B^{-1}.A^{-1}$

Adjoint and Inverse of Matrix Ex 7.1 Q10(ii)

$$A=\begin{bmatrix}2&1\\5&3\end{bmatrix} \quad \therefore |A|=1\neq 0 \text{ and adj}\,A=\begin{bmatrix}3&-1\\-5&2\end{bmatrix}$$

$$\therefore \quad A^{-1}=\frac{\text{adj}\,A}{|A|}=\frac{1}{1}\begin{bmatrix}3&-1\\-5&2\end{bmatrix}$$

$$B=\begin{bmatrix}4&5\\3&4\end{bmatrix} \quad \therefore |A|=-1 \neq 0 \text{ and adj}B=\begin{bmatrix}4&-5\\-3&4\end{bmatrix}$$

$$\therefore \quad B^{-1}=\frac{\text{adj}B}{|B|}=\frac{1}{1}\begin{bmatrix}4&-5\\-3&4\end{bmatrix}$$

$$\text{Also,} \quad A.B=\begin{bmatrix}2&1\\5&3\end{bmatrix}\begin{bmatrix}4&5\\3&4\end{bmatrix}=\begin{bmatrix}11&14\\29&27\end{bmatrix}$$

$$|AB|=407-406=1\neq 0$$

$$\text{and,} \quad \text{adj}(AB)=\begin{bmatrix}37&-14\\-29&11\end{bmatrix}$$

$$\therefore \quad (AB)^{-1}=\frac{1}{|AB|}.\text{adj}(AB)$$

$$=\frac{1}{1}\begin{bmatrix}37&-14\\-29&11\end{bmatrix}=\begin{bmatrix}37&-14\\-29&11\end{bmatrix}$$

$$\text{Again, } B^{-1}.A^{-1}=\begin{bmatrix}4&-5\\-3&4\end{bmatrix}\begin{bmatrix}3&-1\\-5&2\end{bmatrix}$$

$$=\begin{bmatrix}37&-14\\-29&11\end{bmatrix}$$

Hence, $(AB)^{-1}=B^{-1}.A^{-1}$

Adjoint and Inverse of Matrix Ex 7.1 Q11

$$A=\begin{bmatrix}3&2\\7&5\end{bmatrix}\quad \therefore |A|=1\neq 0 \text{ and adj} A=\begin{bmatrix}5&-2\\-7&3\end{bmatrix}$$

$$\therefore\ A^{-1}=\frac{\text{adj}A}{|A|}=\frac{1}{1}\begin{bmatrix}5&-2\\-7&3\end{bmatrix}$$

$$B=\begin{bmatrix}6&7\\7&9\end{bmatrix}\quad \therefore |B|=-2\neq 0 \text{ and adj} B=\begin{bmatrix}9&-7\\-8&6\end{bmatrix}$$

$$\therefore\ B^{-1}=\frac{\text{adj}B}{|B|}=\frac{1}{(-2)}\begin{bmatrix}9&-7\\-8&6\end{bmatrix}$$

Now, $(AB)^{-1}=B^{-1}.A^{-1}$

$$(AB)^{-1}=\frac{1}{(-2)}\begin{bmatrix}9&-7\\-8&6\end{bmatrix}\begin{bmatrix}5&-2\\-7&3\end{bmatrix}$$

$$(AB)^{-1}=-\frac{1}{2}\begin{bmatrix}94&-39\\-82&34\end{bmatrix}$$

$$(AB)^{-1}=\begin{bmatrix}-47&\frac{39}{2}\\41&-17\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q12

$$A=\begin{bmatrix}2&-3\\-4&7\end{bmatrix}\quad \therefore |A|=2\neq 0$$

$$\text{adj}\, A=\begin{bmatrix}7&3\\4&2\end{bmatrix}$$

$$\therefore\ A^{-1}=\frac{1}{2}\begin{bmatrix}7&3\\4&2\end{bmatrix}$$

To show: $2A^{-1}=9I-A$

LHS: $2A^{-1}=2.\frac{1}{2}\begin{bmatrix}7&3\\4&2\end{bmatrix}=\begin{bmatrix}7&3\\4&2\end{bmatrix}$

RHS: $9I-A=\begin{bmatrix}9&0\\0&9\end{bmatrix}-\begin{bmatrix}2&-3\\-4&7\end{bmatrix}=\begin{bmatrix}7&3\\4&2\end{bmatrix}$

Therefore, $2A^{-1}=9I-A$

Adjoint and Inverse of Matrix Ex 7.1 Q13

$$A=\begin{bmatrix}4&5\\2&1\end{bmatrix}\quad \therefore |A|=-6 \text{ and adj} A=\begin{bmatrix}1&-5\\-2&4\end{bmatrix}\quad \therefore A^{-1}=\frac{1}{-6}\begin{bmatrix}1&-5\\-2&4\end{bmatrix}$$

To show: $A-3I=2\left(I+3A^{-1}\right)$

$\therefore$ LHS$=A-3I=\begin{bmatrix}4&5\\2&1\end{bmatrix}-\begin{bmatrix}3&0\\0&3\end{bmatrix}=\begin{bmatrix}1&5\\2&-2\end{bmatrix}$

$$\text{RHS: } 2\left(I+3A^{-1}\right)=2I+2.3.A^{-1}=\begin{bmatrix}2&0\\0&2\end{bmatrix}+2.3.\frac{1}{6}\begin{bmatrix}-1&5\\2&-4\end{bmatrix}$$

$$=\begin{bmatrix}2&0\\0&2\end{bmatrix}+\begin{bmatrix}-1&5\\2&-4\end{bmatrix}$$

$$=\begin{bmatrix}1&5\\2&-2\end{bmatrix}$$

$$\therefore\ A-3I=2\left(I+3A^{-1}\right)$$

Adjoint and Inverse of Matrix Ex 7.1 Q14

$$A = \begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix}$$

$$\Rightarrow |A| = (1+bc) - bc = 1 \neq 0$$

$$A^{-1} = \frac{1}{|A|}.adjA = \frac{1}{1}\begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$

Now $aA^{-1} = (a^2+bc+1)I - aA$

$$LHS : aA^{-1} = a\begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} 1+bc & -ab \\ -ac & a^2 \end{bmatrix}$$

$$RHS : (a^2+bc+1)I - aA = \begin{bmatrix} a^2+bc+1 & 0 \\ 0 & a^2+bc+1 \end{bmatrix} - \begin{bmatrix} a^2 & ab \\ ac & 1+bc \end{bmatrix} = \begin{bmatrix} 1+bc & -ab \\ -ac & a^2 \end{bmatrix}$$

Since, $L.H.S = RHS$

Hence, proved

Adjoint and Inverse of Matrix Ex 7.1 Q15

Here

$$(AB)^{-1} = B^{-1}A^{-1}$$

Now we need to find A^{-1}.

We have

$$A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

So,

$$|A| = -5+4 = -1$$

Co-factors of A are

$$C_{11} = -1 \qquad C_{21} = 8 \qquad C_{31} = -12$$
$$C_{12} = 0 \qquad C_{22} = 1 \qquad C_{32} = -2$$
$$C_{13} = 1 \qquad C_{23} = -10 \qquad C_{33} = 15$$

Therefore,

$$adjA = \begin{bmatrix} -1 & 8 & -12 \\ 0 & 1 & -2 \\ 1 & -10 & 15 \end{bmatrix}$$

So,

$$A^{-1} = \frac{1}{|A|}adjA = \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$$

Hence,

$$(AB)^{-1} = B^{-1}A^{-1}$$
$$= \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$$
$$= \begin{bmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & -42 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q16(i)

$$F(\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix} \Rightarrow |F(\alpha)| = \cos^2\alpha+\sin^2\alpha = 1$$

$C_{11}=\cos\alpha \quad C_{21}=+\sin\alpha \quad C_{31}=0$
$C_{12}=-\sin\alpha \quad C_{22}=\cos\alpha \quad C_{32}=0$
$C_{13}=0 \quad C_{23}=0 \quad C_{33}=1$

$$[F(\alpha)]^{-1}=\frac{adj\,(F(\alpha))}{|F(\alpha)|}=\frac{1}{1}\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix} \qquad ---(1)$$

Now

$$F(-\alpha)=\begin{bmatrix}\cos(-\alpha) & -\sin(-\alpha) & 0\\ \sin(-\alpha) & \cos(-\alpha) & 0\\ 0 & 0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix} \qquad ---(2)$$

From (1) & (2) $F(-\alpha)=[F(\alpha)]^{-1}$

Hence, proved

Adjoint and Inverse of Matrix Ex 7.1 Q16(ii)

$$G(\beta)=\begin{bmatrix}\cos\beta & 0 & \sin\beta\\ 0 & 1 & 0\\ -\sin\beta & 0 & \cos\beta\end{bmatrix} \Rightarrow |G(\beta)| = \cos^2\beta+\sin^2$$

$C_{11}=\cos\beta \quad C_{21}=0 \quad C_{31}=\sin\beta$
$C_{12}=+0 \quad C_{22}=1 \quad C_{32}=0$
$C_{13}=\sin\beta \quad C_{23}=0 \quad C_{33}=\cos\beta$

$$[G(\beta)]^{1}=\frac{adj\,(G(\beta))}{|G(\beta)|}=\frac{1}{1}\begin{bmatrix}\cos\beta & 0 & -\sin\beta\\ 0 & 1 & 0\\ \sin\beta & 0 & \cos\beta\end{bmatrix} \qquad ---(1)$$

Now

$$G(-\beta)=\begin{bmatrix}\cos(-\beta) & 0 & \sin(-\beta)\\ 0 & 1 & 0\\ -\sin(-\beta) & 0 & \cos(-\beta)\end{bmatrix}$$
$$=\begin{bmatrix}\cos\beta & 0 & -\sin\beta\\ 0 & 1 & 0\\ \sin\beta & 0 & \cos\beta\end{bmatrix} \qquad ---(2)$$

From (1) & (2)

$[G(\beta)]^{1}=G(-\beta)$

Adjoint and Inverse of Matrix Ex 7.1 Q16(iii)

We have to show that

$$[F(\alpha)G(\beta)]^{-1}=G(-\beta)F(-\alpha)$$

We have already shown that

$$G(-\beta)=[G(\beta)]^{-1}$$

and $F(-\beta)=[F(\beta)]^{-1}$

$\therefore$ $\text{LHS}=[F(\alpha)G(\beta)]^{1}$

$=[G(\beta)]^{-1}[F(\alpha)]^{-1} \qquad \left[\because (AB)^{-1}=B^{-1}A^{-1}\right]$

$=G(-\beta)\times F(-\alpha)$

$=\text{RHS}$

Adjoint and Inverse of Matrix Ex 7.1 Q17

We have $A^2 = A.A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$

Hence $A^2 - 4A + I = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 7-8+1 & 12-12+0 \\ 4-4+0 & 7-8+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

Now, $A^2 - 4A + I = O$

$\Rightarrow A.A - 4A = -I$

Post multiplying both sides by A^{-1}, since $|A| \neq 0$

$AA\left(A^{-1}\right) - 4AA^{-1} = -IA^{-1}$

$\Rightarrow A\left(AA^{-1}\right) - 4I = -A^{-1}$

$\Rightarrow AI - 4I = -A^{-1}$

or $A^{-1} = 4I - A = 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4-2 & 0-3 \\ 0-1 & 4-2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q18

$A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$

Now $A^2 + 4A - 42I = 0$

For this $A^2 = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix}$

Hence,

$A^2 + 4A - 42I = \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix} + \begin{bmatrix} -32 & 20 \\ 8 & 16 \end{bmatrix} - \begin{bmatrix} 42 & 0 \\ 0 & 42 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Hence proved.

Now, $A^2 + 4A - 42I = 0$

$\Rightarrow \quad A^{-1}.A.A + 4A^{-1}.A - 42A^{-1}.I = 0$

$\Rightarrow \quad IA + 4I - 42A^{-1} = 0$

$\Rightarrow \quad 42A^{-1} = A + 4I$

$\Rightarrow \quad A^{-1} = \frac{1}{42}\left[A + 4I\right] = \frac{1}{42}\left\{\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}\right\} = \frac{1}{42}\begin{bmatrix} -4 & 5 \\ 2 & 8 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q19

Here

$$A=\begin{bmatrix}3&1\\-1&2\end{bmatrix}$$

$$A^2=\begin{bmatrix}3&1\\-1&2\end{bmatrix}\begin{bmatrix}3&1\\-1&2\end{bmatrix}$$

$$=\begin{bmatrix}8&5\\-5&3\end{bmatrix}$$

Now,

$$A^2-5A+7I=0$$

$$=\begin{bmatrix}8&5\\-5&3\end{bmatrix}-5\begin{bmatrix}3&1\\-1&2\end{bmatrix}+7\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

So,

$$A^2-5A+7I=0$$

Px-multipling with A^{-1}

$$A^{-1}A^2-5A^{-1}A+7IA^{-1}=0$$

$$A-5I+7A^{-1}=0$$

$$A^{-1}=\frac{1}{7}[5I-A]$$

$$=\frac{1}{7}\left\{\begin{bmatrix}5&0\\0&5\end{bmatrix}-\begin{bmatrix}3&1\\-1&2\end{bmatrix}\right\}$$

$$=\frac{1}{7}\begin{bmatrix}2&-1\\1&3\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q20

$$A=\begin{bmatrix}4&3\\2&5\end{bmatrix}$$

$$\therefore\ A^2=\begin{bmatrix}4&3\\2&5\end{bmatrix}\begin{bmatrix}4&3\\2&5\end{bmatrix}=\begin{bmatrix}22&27\\18&31\end{bmatrix}$$

Now $A^2-xA+yI=0$

$$\Rightarrow\quad \begin{bmatrix}22&27\\18&31\end{bmatrix}-\begin{bmatrix}4x&3x\\2x&5x\end{bmatrix}+\begin{bmatrix}y&0\\0&y\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

$\Rightarrow\quad 22-4x+y=0 \qquad$ or $\qquad 4x-y=22$

$\Rightarrow\quad 18-2x=0 \qquad$ or $\qquad x=9$

$\therefore\quad y=14$

Again,

$A^2-9A+14I=0$

$\Rightarrow\quad 9A=A^2+14I=0$

$\Rightarrow\quad 9A^{-1}A=A^{-1}.A.A+14A^{-1}$

$\Rightarrow\quad 9I=IA+14A^{-1}$

$$\Rightarrow\quad A^{-1}=\frac{1}{14}\{9I-A\}=\frac{1}{14}\left\{\begin{bmatrix}9&0\\0&9\end{bmatrix}-\begin{bmatrix}4&3\\2&5\end{bmatrix}\right\}=\frac{1}{14}\left\{\begin{bmatrix}5&-3\\-2&4\end{bmatrix}\right\}$$

Adjoint and Inverse of Matrix Ex 7.1 Q21

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

If $A^2 = \lambda A - 2I$

$$\lambda A = A^2 + 2I$$
$$= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\lambda \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$
$$\begin{bmatrix} 3\lambda & -2\lambda \\ 4\lambda & -2\lambda \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$
$$3\lambda = 3$$
$$\lambda = 1 \qquad \text{Ans } \lambda = 1$$

$$A^2 = A - 2I$$

Px multiplying by A^{-1}

$$A^{-1}.AA = A^{-1}.A - 2A^{-1}.I$$
$$A = I - 2A^{-1}$$
$$2A^{-1} = I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ -4 & 3 \end{bmatrix}$$

$$\therefore \qquad A^{-1} = \frac{1}{2}\begin{bmatrix} -2 & 2 \\ -4 & 3 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q22

We have $\qquad A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$

To prove: $\qquad A^2 - 3A - 7 = 0$

Now, $\quad A^2 = AA = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix}$

So, $\quad A^2 - 3A - 7 = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Now, $\quad A^2 - 3A - 7 = 0$

$\Rightarrow \quad AA^{-1}.A - 3A^{-1}.A - 7A^{-1} = 0$

$\Rightarrow \quad A - 3I - 7A^{-1} = 0$

$\Rightarrow \quad A - 3I - 7A^{-1} = 0$

$\Rightarrow \quad 7A^{-1} = A - 3I$

$\Rightarrow \quad A^{-1} = \frac{1}{7}\left(\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right) = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix} = \begin{bmatrix} \frac{2}{7} & \frac{3}{7} \\ \frac{-1}{7} & \frac{-5}{7} \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q23

Show that $A = \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}$ satisfies the equation $x^2 - 12x + I = 0$. Thus, find A^{-1}.

$$A = \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}$$

$$\therefore\ A^2 = \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}\begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix} = \begin{bmatrix} 71 & 60 \\ 84 & 71 \end{bmatrix}$$

Now $A^2 - 12A + I = \begin{bmatrix} 71 & 60 \\ 84 & 71 \end{bmatrix} - 12\begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$A^2 - 12A + I = 0$$

$$\Rightarrow \quad A - 12I + A^{-1} = 0$$

$$\Rightarrow \quad A^{-1} = 12I - A = \left\{\begin{bmatrix} 12 & 0 \\ 0 & 12 \end{bmatrix} - \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}\right\} = \left\{\begin{bmatrix} 6 & -5 \\ -7 & 6 \end{bmatrix}\right\}$$

Adjoint and Inverse of Matrix Ex 7.1 Q24

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+2 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -3-24-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

$\therefore A^3-6A^2+5A+11I$

$$=\begin{bmatrix}8&7&1\\-23&27&-69\\32&-13&58\end{bmatrix}-6\begin{bmatrix}4&2&1\\-3&8&-14\\7&-3&14\end{bmatrix}+5\begin{bmatrix}1&1&1\\1&2&-3\\2&-1&3\end{bmatrix}+11\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}8&7&1\\-23&27&-69\\32&-13&58\end{bmatrix}-\begin{bmatrix}24&12&6\\-18&48&-84\\42&-18&84\end{bmatrix}+\begin{bmatrix}5&5&5\\5&10&-15\\10&-5&15\end{bmatrix}+\begin{bmatrix}11&0&0\\0&11&0\\0&0&11\end{bmatrix}$$

$$=\begin{bmatrix}24&12&6\\-18&48&-84\\42&-18&84\end{bmatrix}-\begin{bmatrix}24&12&6\\-18&48&-84\\42&-18&84\end{bmatrix}$$

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}=O$$

Thus, $A^3-6A^2+5A+11I=O.$

Now,

$A^3-6A^2+5A+11I=O$

$\Rightarrow (AAA)A^{-1}-6(AA)A^{-1}+5AA^{-1}+11IA^{-1}=0$ $\quad$ [Post-multiplying by A^{-1} as $|A|\neq 0$]

$\Rightarrow AA(AA^{-1})-6A(AA^{-1})+5(AA^{-1})=-11(IA^{-1})$

$\Rightarrow A^2-6A+5I=-11A^{-1}$

$\Rightarrow A^{-1}=-\frac{1}{11}(A^2-6A+5I)$ $\quad$ ...(1)

Now,

$A^2-6A+5I$

$$=\begin{bmatrix}4&2&1\\-3&8&-14\\7&-3&14\end{bmatrix}-6\begin{bmatrix}1&1&1\\1&2&-3\\2&-1&3\end{bmatrix}+5\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}4&2&1\\-3&8&-14\\7&-3&14\end{bmatrix}-\begin{bmatrix}6&6&6\\6&12&-18\\12&-6&18\end{bmatrix}+\begin{bmatrix}5&0&0\\0&5&0\\0&0&5\end{bmatrix}$$

$$=\begin{bmatrix}9&2&1\\-3&13&-14\\7&-3&19\end{bmatrix}-\begin{bmatrix}6&6&6\\6&12&-18\\12&-6&18\end{bmatrix}$$

$$=\begin{bmatrix}3&-4&-5\\-9&1&4\\-5&3&1\end{bmatrix}$$

From equation (1), we have:

$$A^{-1}=-\frac{1}{11}\begin{bmatrix}3&-4&-5\\-9&1&4\\-5&3&1\end{bmatrix}=\frac{1}{11}\begin{bmatrix}-3&4&5\\9&-1&-4\\5&-3&-1\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q25

$$A=\begin{bmatrix}1 & 0 & -2\\ -2 & -1 & 2\\ 3 & 4 & 1\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & 0 & -2\\ -2 & -1 & 2\\ 3 & 4 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & -2\\ -2 & -1 & 2\\ 3 & 4 & 1\end{bmatrix}=\begin{bmatrix}-5 & -8 & -4\\ 6 & 9 & 4\\ -2 & 0 & 3\end{bmatrix}$$

$$A^3=A^2.A=\begin{bmatrix}-5 & -8 & -4\\ 6 & 9 & 4\\ -2 & 0 & 3\end{bmatrix}\begin{bmatrix}1 & 0 & -2\\ -2 & -1 & 2\\ 3 & 4 & 1\end{bmatrix}=\begin{bmatrix}-1 & -8 & -10\\ 0 & 7 & 10\\ 7 & 12 & 7\end{bmatrix}$$

$$\text{Now } A^3-A^2-3A-I_3=\begin{bmatrix}-1 & -8 & -10\\ 0 & 7 & 10\\ 7 & 12 & 7\end{bmatrix}-\begin{bmatrix}-5 & -8 & -4\\ 6 & 9 & 4\\ -2 & 0 & 3\end{bmatrix}-\begin{bmatrix}3 & 0 & -6\\ -6 & -3 & 6\\ 9 & 12 & 3\end{bmatrix}-\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$\therefore \quad A^3-A^2-3A-I_3=0$

$\Rightarrow \quad A^2-A-3I-A^{-1}=0$

$$\Rightarrow \quad A^{-1}=A^2-A-3I=\begin{bmatrix}-5 & -8 & -4\\ 6 & 9 & 4\\ -2 & 0 & 3\end{bmatrix}-\begin{bmatrix}1 & 0 & -2\\ -2 & -1 & 2\\ 3 & 4 & 1\end{bmatrix}-\begin{bmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{bmatrix}=\begin{bmatrix}-9 & -8 & -2\\ 8 & 7 & 2\\ -5 & -4 & -1\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q26

$$A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$$

$$A^2=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}4+1+1 & -2-2-1 & 2+1+2\\ -2-2-1 & 1+4+1 & -1-2-2\\ 2+1+2 & -1-2-2 & 1+1+4\end{bmatrix}$$

$$=\begin{bmatrix}6 & -5 & 5\\ -5 & 6 & -5\\ 5 & -5 & 6\end{bmatrix}$$

$$A^3=A^2A=\begin{bmatrix}6 & -5 & 5\\ -5 & 6 & -5\\ 5 & -5 & 6\end{bmatrix}\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}12+5+5 & -6-10-5 & 6+5+10\\ -10-6-5 & 5+12+5 & -5-6-10\\ 10+5+6 & -5-10-6 & 5+5+12\end{bmatrix}$$

$$=\begin{bmatrix}22 & -21 & 21\\ -21 & 22 & -21\\ 21 & -21 & 22\end{bmatrix}$$

Now,

$A^3-6A^2+9A-4I$

$$=\begin{bmatrix}22&-21&21\\-21&22&-21\\21&-21&22\end{bmatrix}-6\begin{bmatrix}6&-5&5\\-5&6&-5\\5&-5&6\end{bmatrix}+9\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}-4\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}22&-21&21\\-21&22&-21\\21&-21&22\end{bmatrix}-\begin{bmatrix}36&-30&30\\-30&36&-30\\30&-30&36\end{bmatrix}+\begin{bmatrix}18&-9&9\\-9&18&-9\\9&-9&18\end{bmatrix}-\begin{bmatrix}4&0&0\\0&4&0\\0&0&4\end{bmatrix}$$

$$=\begin{bmatrix}40&-30&30\\-30&40&-30\\30&-30&40\end{bmatrix}-\begin{bmatrix}40&-30&30\\-30&40&-30\\30&-30&40\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$\therefore A^3-6A^2+9A-4I=O$

Now,

$A^3-6A^2+9A-4I=O$

$\Rightarrow (AAA)A^{-1}-6(AA)A^{-1}+9AA^{-1}-4IA^{-1}=O$ $\quad\quad$ $\left[\text{Post-multiplying by } A^{-1} \text{ as } |A|\neq 0\right]$

$\Rightarrow AA\left(AA^{-1}\right)-6A\left(AA^{-1}\right)+9\left(AA^{-1}\right)=4\left(IA^{-1}\right)$

$\Rightarrow AAI-6AI+9I=4A^{-1}$

$\Rightarrow A^2-6A+9I=4A^{-1}$

$\Rightarrow A^{-1}=\frac{1}{4}\left(A^2-6A+9I\right) \quad\quad ...(1)$

$A^2-6A+9I$

$$=\begin{bmatrix}6&-5&5\\-5&6&-5\\5&-5&6\end{bmatrix}-6\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}+9\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$=\begin{bmatrix}6&-5&5\\-5&6&-5\\5&-5&6\end{bmatrix}-\begin{bmatrix}12&-6&6\\-6&12&-6\\6&-6&12\end{bmatrix}+\begin{bmatrix}9&0&0\\0&9&0\\0&0&9\end{bmatrix}$$

$$=\begin{bmatrix}3&1&-1\\1&3&1\\-1&1&3\end{bmatrix}$$

From equation (1), we have:

$$A^{-1}=\frac{1}{4}\begin{bmatrix}3&1&-1\\1&3&1\\-1&1&3\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q27

$A=\frac{1}{9}\begin{bmatrix}-8&1&4\\4&4&7\\1&-8&4\end{bmatrix}$ and $A^T=\frac{1}{9}\begin{bmatrix}-8&4&1\\1&4&-8\\4&7&4\end{bmatrix}$

$|A|=\frac{1}{9}\left[-8(16+56)-1(9)+4(-36)\right]=-81$

$C_{11}=72 \quad C_{21}=-36 \quad C_{31}=-9$

$C_{12}=-9 \quad C_{22}=-36 \quad C_{32}=+72$

$C_{13}=-36 \quad C_{23}=-63 \quad C_{33}=-36$

$$A^{-1}=\frac{1}{-81}\begin{bmatrix}72&-36&-9\\-9&-36&72\\-36&-63&-36\end{bmatrix}=\frac{1}{9}\begin{bmatrix}-8&4&1\\1&4&-8\\4&7&4\end{bmatrix}=A^T$$

Hence proved.

Adjoint and Inverse of Matrix Ex 7.1 Q28

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \text{ and } |A| = 3 + 6 - 8 = 1$$

$C_{11} = 1 \qquad C_{21} = -1 \qquad C_{31} = 0$
$C_{12} = -2 \qquad C_{22} = 3 \qquad C_{32} = -4$
$C_{13} = -2 \qquad C_{23} = +3 \qquad C_{33} = -3$

$$A^{-1} = \frac{1}{|A|}.adjA = \frac{1}{1}\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \qquad ---(1)$$

Now

$$A^2 = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A^3 = A^2.A = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \qquad ---(2)$$

From (1) and (2)
$A^{-1} = A^3$

Hence proved.

Adjoint and Inverse of Matrix Ex 7.1 Q29

$$A = \begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Expanding using 1st row, we get

$$|A| = -1\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} - 2\begin{vmatrix} -1 & 1 \\ 0 & 0 \end{vmatrix} + 0$$
$$= -1(0-1) - 2(0) + 0$$
$$= 1 - 0 + 0$$
$$|A| = 1$$

$$A^2 = AA = \begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

Cofactors of A are:

$C_{11} = -1 \qquad C_{21} = 0 \qquad C_{31} = 2$
$C_{12} = 0 \qquad C_{22} = 0 \qquad C_{32} = 1$
$C_{13} = -1 \qquad C_{23} = +1 \qquad C_{33} = 1$

$$\therefore \quad \text{adj}A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T = \begin{bmatrix} -1 & 0 & -1 \\ 0 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}^T = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

Now, $A^{-1} = \frac{1}{|A|}.\text{adj}A$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix} = A^2$$

Hence, $A^2 = A^{-1}$

Adjoint and Inverse of Matrix Ex 7.1 Q30

$$\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$$

Let $A = \begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$

So, $AX = B$

or $X = A^{-1}.B$ $\quad ---(i)$

$$|A| = 1 \neq 0$$

Cofactors of A are:

$C_{11} = 1 \qquad C_{12} = -1$

$C_{21} = -4 \qquad C_{22} = 5$

$$\therefore \quad \text{adj}\, A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -1 \\ -4 & 5 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}$$

Now, $A^{-1} = \frac{1}{|A|}.\text{adj}\, A$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}$$

So from (i)

$$X = \begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$$

$$X = \begin{bmatrix} -3 & -14 \\ 4 & 17 \end{bmatrix}$$

Ans.

Adjoint and Inverse of Matrix Ex 7.1 Q31

$$X\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}=\begin{bmatrix}14 & 7\\ 7 & 7\end{bmatrix}$$

Let $B=\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}$ and $C=\begin{bmatrix}14 & 7\\ 7 & 7\end{bmatrix}$

So, $XB=C$

$$XBB^{-1}=CB^{-1}$$

$$XI=C.B^{-1}$$

$$X=C.B^{-1} \qquad ---(i)$$

Now, $|B|=-7\neq 0$

Cofactors of B are:

$$C_{11}=-2 \qquad C_{12}=1$$

$$C_{21}=-3 \qquad C_{22}=5$$

$$\therefore \quad \operatorname{adj}A=\begin{bmatrix}C_{11} & C_{12}\\ C_{21} & C_{22}\end{bmatrix}^T$$

$$=\begin{bmatrix}-2 & 1\\ -3 & 5\end{bmatrix}^T$$

$$=\begin{bmatrix}-2 & -3\\ 1 & 5\end{bmatrix}$$

$$B^{-1}=\frac{1}{|B|}.\operatorname{adj}(B)$$

$$=\frac{1}{(-7)}\begin{bmatrix}-2 & -3\\ 1 & 5\end{bmatrix} \quad \frac{-1}{7}\begin{bmatrix}2 & 3\\ -1 & -5\end{bmatrix}$$

Now from (i)

$$X=\begin{bmatrix}14 & 7\\ 7 & 7\end{bmatrix}.\frac{1}{7}.\begin{bmatrix}2 & 3\\ -1 & -5\end{bmatrix}$$

$$=\frac{7}{7}\begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}2 & 3\\ -1 & -5\end{bmatrix}$$

$$=\begin{bmatrix}3 & 1\\ 1 & -2\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q32

Let $A=\begin{bmatrix}3 & 2\\ 7 & 5\end{bmatrix}$

$$B=\begin{bmatrix}-1 & 1\\ -2 & 1\end{bmatrix}$$

$$C=\begin{bmatrix}2 & -1\\ 0 & 4\end{bmatrix}$$

Then the given equation becomes

$$A\times B=C$$

$$\Rightarrow \quad X=A^{-1}CB^{-1}$$

Now $|A|=35-14=21$

$$|B|=-1+2=1$$

$$A^{-1}=\frac{adj\,(A)}{|A|}=\frac{1}{21}\begin{bmatrix}5 & -2\\ -7 & 3\end{bmatrix}$$

$$B^{-1}=\frac{adj\,(B)}{|B|}=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$$

$$\therefore \quad X=A^{-1}CB^{-1}=\frac{1}{21}\begin{bmatrix}5 & -2\\ -7 & 3\end{bmatrix}\begin{bmatrix}2 & -1\\ 0 & 4\end{bmatrix}\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$$

$$=\begin{bmatrix}-16 & 3\\ 24 & -5\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q33

Let $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$

$B = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$

Then the given equation can be written as

$A \times B = I$

$\Rightarrow \quad X = A^{-1}B^{-1}$

Now $|A| = 6 - 5 = 1$

$|B| = 10 - 9 = 1$

$$A^{-1} = \frac{adj\,(A)}{|A|} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

$$B^{-1} = \frac{adj\,(B)}{|B|} = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$$

$$\therefore \quad X = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 9 & -14 \\ -16 & 25 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q34

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

Now, $A^2 + 4A - 5I$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Also, $A^2 - 4A + 5I = O$

$A^{-1}.AA - 4A^{-1}.A - 5A^{-1}.I = 0$

$A - 4I - 5A^{-1} = 0$

$$A^{-1} = \frac{1}{5}[A - 4I]$$

$$= \frac{1}{5}\left\{\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}\right\}$$

$$= \frac{1}{5}\left(\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}\right) = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q35

$|A \text{ adj} A| = |A|^n$

LHS $= |A \text{ Adj} A|$

$= |A| \cdot |\text{Adj} A|$

$= |A| \cdot |A|^{n-1}$

$= |A|^{n-1+1}$

$= |A|^n$

$= \text{RHS}$

Adjoint and Inverse of Matrix Ex 7.1 Q36

Here

$$B^{-1} = \frac{1}{|B|} adj\,|B|$$

Co-factors of B are

$$\begin{array}{lll} C_{11} = 3 & C_{21} = 2 & C_{31} = 6 \\ C_{12} = 1 & C_{22} = 1 & C_{32} = 2 \\ C_{13} = 2 & C_{33} = 2 & C_{33} = 5 \end{array}$$

Therefore,

$$adj\,B = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}^T$$

$$= \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

Therefore,

$$B^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

Now,

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$= \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 61 & -24 & 22 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q37

$$\text{Let } B = A^T = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{vmatrix} = (-1-8) - 0 - 2(-8+3) = -9 + 10 = 1 \neq 0$$

So, B is invertible matrix.

$$B_{11} = (-1)^{1+1}(-9) = -9;\ B_{12} = (-1)^{1+2}(-8) = 8;\ B_{13} = (-1)^{1+3}(-5) = -5$$

$$B_{21} = (-1)^{2+1}(8) = -8;\ B_{22} = (-1)^{2+2}(7) = 7;\ B_{23} = (-1)^{2+3}(4) = -4$$

$$B_{31} = (-1)^{3+1}(-2) = -2;\ B_{32} = (-1)^{3+2}(-2) = 2;\ B_{33} = (-1)^{3+3}(-1) = -1$$

$$\text{adj } B = \begin{bmatrix} -9 & 8 & -5 \\ -8 & 7 & -4 \\ -2 & 2 & -1 \end{bmatrix}^T = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|}\text{adj}B$$

$$\Rightarrow B^{-1} = \frac{1}{1}\begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

$$\Rightarrow (A^T)^{-1} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q38

$$|A| = \begin{vmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{vmatrix} = -1(1-4) + 2(2+4) - 2(-4-2) = 3 + 12 + 12 = 27$$

$A_{11} = (-1)^{1+1}(-3) = -3$; $A_{12} = (-1)^{1+2}(6) = -6$; $A_{13} = (-1)^{1+3}(-6) = -6$

$A_{21} = (-1)^{2+1}(-6) = 6$; $A_{22} = (-1)^{2+2}(3) = 3$; $A_{23} = (-1)^{2+3}(6) = -6$

$A_{31} = (-1)^{3+1}(6) = 6$; $A_{32} = (-1)^{3+2}(6) = -6$; $A_{33} = (-1)^{3+3}(3) = 3$

$$\text{adj } A = \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}^T = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$A(\text{adj}A) = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}\begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$\Rightarrow A(\text{adj}A) = \begin{bmatrix} 27 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 27 \end{bmatrix}$$

$$\Rightarrow A(\text{adj}A) = 27\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A(\text{adj}A) = |A| I_3$$

Adjoint and Inverse of Matrix Ex 7.1 Q39

$$|A| = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 0 - 1(0-1) + 1(1-0) = 0 + 1 + 1 = 2 \neq 0$$

So, A is invertible matrix.

$A_{11} = (-1)^{1+1}(-1) = -1$; $A_{12} = (-1)^{1+2}(-1) = 1$; $A_{13} = (-1)^{1+3}(1) = 1$

$A_{21} = (-1)^{2+1}(-1) = 1$; $A_{22} = (-1)^{2+2}(-1) = -1$; $A_{23} = (-1)^{2+3}(-1) = 1$

$A_{31} = (-1)^{3+1}(1) = 1$; $A_{32} = (-1)^{3+2}(-1) = 1$; $A_{33} = (-1)^{3+3}(-1) = -1$

$$\text{adj } A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \text{..........(i)}$$

$$A^2 - 3I = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^2 - 3I = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$A^2 - 3I = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \text{.................(ii)}$$

From (i) and (ii) we can see that,

$$A^{-1} = \frac{1}{2}(A^2 - 3I)$$

Ex 7.2

Adjoint and Inverse of Matrix Ex 7.2 Q1

$$A=\begin{bmatrix}7 & 1\\ 4 & -3\end{bmatrix}$$

For row transformations $A = IA$

$$\begin{bmatrix}7 & 1\\ 4 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A$$

Applying $R_1 \to \frac{1}{7}R_1$

$$\begin{bmatrix}1 & \frac{1}{7}\\ 4 & -3\end{bmatrix}=\begin{bmatrix}\frac{1}{7} & 0\\ 0 & 1\end{bmatrix}A$$

Applying $R_2 \to R_2 - 4R_1$

$$\begin{bmatrix}1 & \frac{1}{7}\\ 0 & \frac{-25}{7}\end{bmatrix}=\begin{bmatrix}\frac{1}{7} & 0\\ \frac{-4}{7} & 1\end{bmatrix}A$$

Applying $R_2 \to \left(-\frac{7}{25}\right)R_2$

$$\begin{bmatrix}1 & \frac{1}{7}\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{7} & 0\\ \frac{4}{25} & \frac{-7}{25}\end{bmatrix}A$$

Applying $R_1 \to R_1 - \frac{1}{7}R_2$

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{21}{175} & \frac{1}{25}\\ \frac{4}{25} & \frac{-7}{25}\end{bmatrix}A$$

Hence, $I = B\,A$

So, $B=\begin{bmatrix}\frac{3}{25} & \frac{1}{25}\\ \frac{4}{25} & -\frac{7}{25}\end{bmatrix}$ is the inverse of A.

Adjoint and Inverse of Matrix Ex 7.2 Q2

$$A=\begin{bmatrix}5 & 2\\ 2 & 1\end{bmatrix}$$

For row- transformation $A = IA$

$$\begin{bmatrix}5 & 2\\ 2 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A$$

Applying $R_1 \to \frac{1}{5}R_1$

$$\begin{bmatrix}1 & \frac{2}{5}\\ 2 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{5} & 0\\ 0 & 1\end{bmatrix}A$$

Applying $R_1 \to R_2 - 2R_1$

$$\begin{bmatrix}1 & \frac{2}{5}\\ 0 & \frac{1}{5}\end{bmatrix}=\begin{bmatrix}\frac{1}{5} & 0\\ -\frac{2}{5} & 1\end{bmatrix}A$$

Applying $R_2 \to 5.R_2$

$$\begin{bmatrix}1 & \frac{2}{5}\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{5} & 0\\ -2 & 5\end{bmatrix}A$$

Applying $R_1 \to R_1 - \frac{2}{5}R_2$

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & -2\\ -2 & 5\end{bmatrix}A$$

$$I = B.A$$

Hence, $B=\begin{bmatrix}1 & -2\\ -2 & 5\end{bmatrix}$ is the inverse of A.

Adjoint and Inverse of Matrix Ex 7.2 Q3

Let $A=\begin{bmatrix}1 & 6\\ -3 & 5\end{bmatrix}$

For row transformations $A = IA$

$$\begin{bmatrix}1 & 6\\ -3 & 5\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A$$

Applying $R_2 \to R_2 + 3R_1$

$$\begin{bmatrix}1 & 6\\ 0 & 23\end{bmatrix}=\begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}A$$

Applying $R_2 \to \frac{1}{23}.R_2$

$$\begin{bmatrix}1 & 6\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ \frac{3}{23} & \frac{1}{23}\end{bmatrix}A$$

Applying $R_1 \to R_1 - 6R_2$

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{5}{23} & \frac{-6}{23}\\ \frac{3}{23} & \frac{1}{23}\end{bmatrix}A$$

$$I = B.A$$

Hence, $B=\frac{1}{23}\begin{bmatrix}5 & -6\\ 3 & 1\end{bmatrix}$ is the inverse of A.

Adjoint and Inverse of Matrix Ex 7.2 Q4

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

Now, $A = I\,A$

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to \frac{1}{2} R_1$

$$\begin{bmatrix} 1 & \frac{5}{2} \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & \frac{5}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ \frac{-1}{2} & 1 \end{bmatrix} A$$

Applying $R_2 \to 2.R_2$

$$\begin{bmatrix} 1 & \frac{5}{2} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -1 & 2 \end{bmatrix} A$$

Applying $R_1 \to R_1 - \frac{5}{2} R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} .A$$

or $\quad I = B.A$

Hence, B is the inv. of A.

Adjoint and Inverse of Matrix Ex 7.2 Q5

$$A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$$

Now, $A = I\,A$

$$\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to \frac{1}{3} R_1$

$$\begin{bmatrix} 1 & \frac{10}{3} \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & \frac{10}{3} \\ 0 & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ \frac{-2}{3} & 1 \end{bmatrix} A$$

Applying $R_2 \to 3R_2$

$$\begin{bmatrix} 1 & \frac{10}{3} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ -2 & 3 \end{bmatrix} A$$

Applying $R_1 \to R_1 - \frac{10}{3} R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} .A$$

$I = B.A$

Hence, B is the inv. of A.

Adjoint and Inverse of Matrix Ex 7.2 Q6

$$A=\begin{bmatrix}0&1&2\\1&2&3\\3&1&1\end{bmatrix}$$

$$A = I\,A$$

$$\begin{bmatrix}0&1&2\\1&2&3\\3&1&1\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}.A$$

Applying $R_1 \leftrightarrow R_2$

$$\begin{bmatrix}1&2&3\\0&1&2\\3&1&1\end{bmatrix}=\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}.A$$

Applying $R_3 \to R_3 - 3R_1$

$$\begin{bmatrix}1&2&3\\0&1&2\\0&-5&-8\end{bmatrix}=\begin{bmatrix}0&1&0\\1&0&0\\0&-3&1\end{bmatrix}.A$$

Applying $R_1 \to R_1 - 2R_2,\ R_3 \to R_3 + 5R_2$

$$\begin{bmatrix}1&0&-1\\0&1&2\\0&0&2\end{bmatrix}=\begin{bmatrix}-2&1&0\\1&0&0\\5&-3&1\end{bmatrix}.A$$

Applying $R_3 \to \dfrac{R_3}{2}$

$$\begin{bmatrix}1&0&-1\\0&1&2\\0&0&1\end{bmatrix}=\begin{bmatrix}-2&1&0\\1&0&0\\\frac{5}{2}&\frac{-3}{2}&\frac{1}{2}\end{bmatrix}.A$$

Applying $R_1 \to R_1 + R_3,\ R_2 \to R_2 - 2R_3$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}\frac{1}{2}&\frac{-1}{2}&\frac{1}{2}\\-4&3&1\\\frac{5}{2}&\frac{-3}{2}&\frac{1}{2}\end{bmatrix}.A$$

$$I = B.A$$

Hence, $A^{-1}=\begin{bmatrix}\frac{1}{2}&\frac{-1}{2}&\frac{1}{2}\\-4&3&1\\\frac{5}{2}&\frac{-3}{2}&\frac{1}{2}\end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.2 Q7

$$A = I\,A$$

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to \frac{R_1}{2}$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 5R_1$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ \frac{5}{2} & -1 & 1 \end{bmatrix} A$$

Applying $R_3 \to 2R_3$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} A$$

Applying $R_1 \to R_1 + \frac{1}{2}R_3,\ R_2 \to R_2 - \frac{5}{2}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$I = B.A$$

Adjoint and Inverse of Matrix Ex 7.2 Q8

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$$

Here, $A = I\,A$

$$\begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to \frac{1}{2}R_1$

$$\begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & \frac{5}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -1 & 1 & 0 \\ \frac{-3}{2} & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - \frac{3}{2}R_2,\ R_3 \to R_3 - \frac{5}{2}R_2$

$$\begin{bmatrix}1&0&\frac{1}{2}\\0&1&0\\0&0&\frac{1}{2}\end{bmatrix}=\begin{bmatrix}2&\frac{-3}{2}&0\\-1&1&0\\1&\frac{-5}{2}&1\end{bmatrix}A$$

Applying $R_3 \to 2R_3$

$$\begin{bmatrix}1&0&\frac{1}{2}\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}2&\frac{-3}{2}&0\\-1&1&0\\2&-5&2\end{bmatrix}A$$

Applying $R_1 \to R_1 - \frac{1}{2}R_3$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}1&1&-1\\-1&1&0\\2&-5&2\end{bmatrix}A$$

Hence, $A^{-1}=\begin{bmatrix}1&1&-1\\-1&1&0\\2&-5&2\end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.2 Q9

$$A=\begin{bmatrix}3&-3&4\\2&-3&4\\0&-1&1\end{bmatrix}$$

Now, $A = I\,A$

$$\begin{bmatrix}3&-3&4\\2&-3&4\\0&-1&1\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}.A$$

Applying $R_1 \to \frac{1}{3}R_1$

$$\begin{bmatrix}1&-1&\frac{4}{3}\\2&-3&4\\0&-1&1\end{bmatrix}=\begin{bmatrix}\frac{1}{3}&0&0\\0&1&0\\0&0&1\end{bmatrix}.A$$

Applying $R_2 \to R_2 - 2R_1$

$$\begin{bmatrix}1&-1&\frac{4}{3}\\0&-1&\frac{4}{3}\\0&-1&1\end{bmatrix}=\begin{bmatrix}\frac{1}{3}&0&0\\\frac{-2}{3}&1&0\\0&0&1\end{bmatrix}.A$$

Applying $R_2 \to (-1)R_2$

$$\begin{bmatrix}1&-1&\frac{4}{3}\\0&1&\frac{-4}{3}\\0&-1&1\end{bmatrix}=\begin{bmatrix}\frac{1}{3}&0&0\\\frac{2}{3}&-1&0\\0&0&1\end{bmatrix}.A$$

Applying $R_1 \to R_1 + R_2,\ R_3 \to R_3 + R_2$

$$\begin{bmatrix}1&0&0\\0&1&\frac{-4}{3}\\0&0&-\frac{1}{3}\end{bmatrix}=\begin{bmatrix}1&-1&0\\\frac{2}{3}&-1&0\\\frac{2}{3}&-1&1\end{bmatrix}.A$$

Applying $R_3 \to (-3)R_3$

$$\begin{bmatrix}1&0&0\\0&1&\frac{-4}{3}\\0&0&1\end{bmatrix}=\begin{bmatrix}1&-1&0\\\frac{2}{3}&-1&0\\-2&3&-3\end{bmatrix}.A$$

$R_2 \to R_2 + \frac{4}{3}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} . A$$

$$I = B.A$$

Hence, B is the inv. of A.

Adjoint and Inverse of Matrix Ex 7.2 Q10

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

$$A = I\,A$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & -1 \\ 0 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} . A$$

Applying $R_2 \to (-1)R_2$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix} . A$$

Applying $R_1 \to R_1 - 2R_2,\ R_3 \to R_3 + 3R_2$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 6 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 0 \\ 2 & -1 & 0 \\ 5 & -3 & 1 \end{bmatrix} . A$$

Applying $R_3 \to \frac{R_3}{6}$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 0 \\ 2 & -1 & 0 \\ \frac{5}{6} & \frac{-1}{2} & \frac{1}{6} \end{bmatrix} . A$$

Applying $R_1 \to R_1 + 2R_3,\ R_2 \to R_2 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-4}{3} & 1 & \frac{1}{3} \\ \frac{7}{6} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{5}{6} & \frac{-1}{2} & \frac{1}{6} \end{bmatrix} . A$$

Hence, $A^{-1} = \begin{bmatrix} \frac{-4}{3} & 1 & \frac{1}{3} \\ \frac{7}{6} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{5}{6} & \frac{-1}{2} & \frac{1}{6} \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.2 Q11

$$A=\begin{bmatrix}2 & -1 & 3\\ 1 & 2 & 4\\ 3 & 1 & 1\end{bmatrix}$$

$$A = I\,A$$

$$\begin{bmatrix}2 & -1 & 3\\ 1 & 2 & 4\\ 3 & 1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.A$$

Applying $R_1 \to \frac{R_1}{2}$

$$\begin{bmatrix}1 & \frac{-1}{2} & \frac{3}{2}\\ 1 & 2 & 4\\ 3 & 1 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.A$$

Applying $R_2 \to R_2 - R_1,\ R_3 \to R_3 - 3R_1$

$$\begin{bmatrix}1 & \frac{-1}{2} & \frac{3}{2}\\ 0 & \frac{5}{2} & \frac{5}{2}\\ 0 & \frac{5}{2} & \frac{-7}{2}\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & 0 & 0\\ \frac{-1}{2} & 1 & 0\\ \frac{-3}{2} & 0 & 1\end{bmatrix}.A$$

Applying $R_2 \to \left(\frac{2}{5}\right)R_2$

$$\begin{bmatrix}1 & \frac{-1}{2} & \frac{3}{2}\\ 0 & 1 & 1\\ 0 & \frac{5}{2} & \frac{-7}{2}\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & 0 & 0\\ \frac{-1}{5} & \frac{2}{5} & 0\\ \frac{-3}{2} & -1 & 1\end{bmatrix}.A$$

Applying $R_1 \to R_1 + \frac{1}{2}R_2,\ R_3 \to R_3 - \frac{5}{2}R_2$

$$\begin{bmatrix}1 & 0 & 2\\ 0 & 1 & 1\\ 0 & 0 & -6\end{bmatrix}=\begin{bmatrix}\frac{2}{5} & \frac{1}{5} & 0\\ \frac{-1}{5} & \frac{2}{5} & 0\\ -1 & -1 & 1\end{bmatrix}.A$$

Applying $R_3 \to \frac{R_3}{-6}$

$$\begin{bmatrix}1 & 0 & 2\\ 0 & 1 & 1\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{2}{5} & \frac{1}{5} & 0\\ \frac{-1}{5} & \frac{2}{5} & 0\\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6}\end{bmatrix}.A$$

Applying $R_2 \to R_2 - R_3,\ R_1 \to R_1 - 2R_3$

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{15} & \frac{-2}{15} & \frac{-1}{3}\\ \frac{-11}{30} & \frac{7}{30} & \frac{1}{6}\\ \frac{1}{6} & \frac{1}{6} & \frac{-1}{6}\end{bmatrix}.A \qquad \left[\because I = A^{-1}.A\right]$$

Ans.

Adjoint and Inverse of Matrix Ex 7.2 Q12

$$A=\begin{bmatrix}1&1&2\\3&1&1\\2&3&1\end{bmatrix}$$

$$A = I\,A$$

$$\begin{bmatrix}1&1&2\\3&1&1\\2&3&1\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}.A$$

Applying $R_2 \to R_2 - 3R_1,\ R_3 \to R_3 - 2R_1$

$$\begin{bmatrix}1&1&2\\0&-2&-5\\0&1&-3\end{bmatrix}=\begin{bmatrix}1&0&0\\-3&1&0\\-2&0&1\end{bmatrix}.A$$

Applying $R_2 \to \dfrac{R_2}{(-2)}$

$$\begin{bmatrix}1&1&2\\0&1&\frac{5}{2}\\0&1&-3\end{bmatrix}=\begin{bmatrix}1&0&0\\\frac{3}{2}&\frac{-1}{2}&0\\-2&0&1\end{bmatrix}.A$$

Applying $R_1 \to R_1 - R_2,\ R_3 \to R_3 - R_2$

$$\begin{bmatrix}1&0&\frac{-1}{2}\\0&1&\frac{5}{2}\\0&0&\frac{-11}{2}\end{bmatrix}=\begin{bmatrix}\frac{-1}{2}&\frac{1}{2}&0\\\frac{3}{2}&\frac{-1}{2}&0\\\frac{-7}{2}&\frac{1}{2}&\frac{-2}{11}\end{bmatrix}.A$$

Applying $R_3 \to R_3.\left(\dfrac{-2}{11}\right)$

$$\begin{bmatrix}1&0&\frac{-1}{2}\\0&1&\frac{5}{2}\\0&0&1\end{bmatrix}=\begin{bmatrix}\frac{-1}{2}&\frac{1}{2}&0\\\frac{3}{2}&\frac{-1}{2}&0\\\frac{7}{11}&\frac{-1}{11}&\frac{-2}{11}\end{bmatrix}.A$$

Applying $R_1 \to R_1 + \frac{1}{2}R_3,\ R_2 \to R_2 - \frac{5}{2}R_3$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}\frac{-2}{11}&\frac{5}{11}&\frac{-1}{11}\\\frac{-1}{11}&\frac{-3}{11}&\frac{5}{11}\\\frac{7}{11}&\frac{-1}{11}&\frac{-2}{11}\end{bmatrix}.A$$

Adjoint and Inverse of Matrix Ex 7.2 Q13

$$A=\begin{bmatrix}2&-1&4\\4&0&2\\3&-2&7\end{bmatrix}$$

Now, $A = I\,A$

$$\begin{bmatrix}2&-1&4\\4&0&2\\3&-2&7\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}.A$$

Applying $R_1 \to \frac{1}{2}R_1$

$$\begin{bmatrix}1&\frac{-1}{2}&2\\4&0&2\\3&-2&7\end{bmatrix}=\begin{bmatrix}\frac{1}{2}&0&0\\0&1&0\\0&0&1\end{bmatrix}.A$$

Applying $R_2 \to R_2 - 4R_1,\ R_3 \to R_3 - 3R_1$

$$\begin{bmatrix}1&\frac{-1}{2}&2\\0&2&-6\\0&\frac{-1}{2}&1\end{bmatrix}=\begin{bmatrix}\frac{1}{2}&0&0\\-2&1&0\\\frac{-3}{2}&0&1\end{bmatrix}A$$

Applying $R_2 \to \frac{1}{2}.R_2$

$$\begin{bmatrix}1&\frac{-1}{2}&2\\0&1&-3\\0&\frac{-1}{2}&1\end{bmatrix}=\begin{bmatrix}\frac{1}{2}&0&0\\-1&\frac{1}{2}&0\\\frac{-3}{2}&0&1\end{bmatrix}A$$

Applying $R_1 \to R_1 + \frac{1}{2}R_2,\ R_3 \to R_3 + \frac{1}{2}R_2$

$$\begin{bmatrix}1&0&\frac{1}{2}\\0&1&-3\\0&0&\frac{-1}{2}\end{bmatrix}=\begin{bmatrix}0&\frac{1}{4}&0\\-1&\frac{1}{2}&0\\-2&\frac{1}{4}&1\end{bmatrix}.A$$

Applying $R_3 \to (-2).R_3$

$$\begin{bmatrix}1&0&\frac{1}{2}\\0&1&-3\\0&0&1\end{bmatrix}=\begin{bmatrix}0&\frac{1}{4}&0\\-1&\frac{1}{2}&0\\4&\frac{-1}{2}&-2\end{bmatrix}.A$$

Applying $R_1 \to R_1 - \frac{1}{2}R_3,\ R_2 \to R_2 + 3R_3$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}-2&\frac{1}{2}&1\\11&-1&-6\\4&\frac{-1}{2}&-2\end{bmatrix}.A$$

$I = B.A$

Hence, B is the inv. of A.

Adjoint and Inverse of Matrix Ex 7.2 Q14

$$A=\begin{bmatrix}3 & 0 & -1\\ 2 & 3 & 0\\ 0 & 4 & 1\end{bmatrix}$$

Now, $A = I\,A$

$$\begin{bmatrix}3 & 0 & -1\\ 2 & 3 & 0\\ 0 & 4 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.A$$

Applying $R_1 \to \frac{1}{3}R_1$

$$\begin{bmatrix}1 & 0 & \frac{-1}{3}\\ 2 & 3 & 0\\ 0 & 4 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{3} & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.A$$

Applying $R_2 \to R_2 - 2R_1$

$$\begin{bmatrix}1 & 0 & \frac{-1}{3}\\ 0 & 3 & \frac{2}{3}\\ 0 & 4 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{3} & 0 & 0\\ \frac{-2}{3} & 1 & 0\\ 0 & 0 & 1\end{bmatrix}A$$

Applying $R_2 \to \frac{1}{3}R_2$

$$\begin{bmatrix}1 & 0 & \frac{-1}{3}\\ 0 & 1 & \frac{2}{9}\\ 0 & 4 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{3} & 0 & 0\\ \frac{-2}{9} & \frac{1}{3} & 0\\ 0 & 0 & 1\end{bmatrix}A$$

Applying $R_3 \to R_3 - 4R_2$

$$\begin{bmatrix}1 & 0 & \frac{-1}{3}\\ 0 & 1 & \frac{2}{9}\\ 0 & 0 & \frac{1}{9}\end{bmatrix}=\begin{bmatrix}\frac{1}{3} & 0 & 0\\ \frac{-2}{9} & \frac{1}{3} & 0\\ \frac{8}{9} & \frac{-4}{3} & 1\end{bmatrix}A$$

Applying $R_3 \to 9.R_3$

$$\begin{bmatrix}1 & 0 & \frac{-1}{3}\\ 0 & 1 & \frac{2}{9}\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{3} & 0 & 0\\ \frac{-2}{9} & \frac{1}{3} & 0\\ 8 & -12 & 9\end{bmatrix}A$$

Applying $R_1 \to R_1 + \frac{1}{3}R_3,\ R_2 \to R_2 - \frac{2}{9}R_3$

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}3 & -4 & 3\\ -2 & 3 & -2\\ 8 & -12 & 9\end{bmatrix}.A$$

or $\quad I = B.A$

Hence, B is the inv. of A.

Adjoint and Inverse of Matrix Ex 7.2 Q15

Consider the given matrix:

$$\text{Let } A=\begin{bmatrix}1 & 3 & -2\\ -3 & 0 & 1\\ 2 & 1 & 0\end{bmatrix}$$

We know that $A = IA$

Thus, we have,

$$\begin{bmatrix}1 & 3 & -2\\ -3 & 0 & 1\\ 2 & 1 & 0\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}A$$

Applying $R_2 \to 3R_1 + R_2$ *and* $R_3 \to R_3 - 2R_1$, *we have,*

$$\begin{bmatrix}1 & 3 & -2\\ 0 & 9 & -5\\ 0 & -5 & 4\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 3 & 1 & 0\\ -2 & 0 & 1\end{bmatrix}A$$

Applying $R_1 \to R_1 - 3R_2$ *and* $R_3 \to R_3 + 5R_2$, *we have,*

$$\begin{bmatrix}1 & 0 & \frac{-1}{3}\\ 0 & 1 & \frac{-5}{9}\\ 0 & 0 & \frac{11}{9}\end{bmatrix}=\begin{bmatrix}0 & \frac{-1}{3} & 0\\ \frac{1}{3} & \frac{1}{9} & 0\\ \frac{-1}{3} & \frac{5}{9} & 1\end{bmatrix}A$$

Applying $R_1 \to \frac{R_1}{\frac{11}{9}}$ *we have,*

$$\begin{bmatrix}1 & 0 & \frac{-1}{3}\\ 0 & 1 & \frac{-5}{9}\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}0 & \frac{-1}{3} & 0\\ \frac{1}{3} & \frac{1}{9} & 0\\ \frac{-3}{11} & \frac{5}{11} & \frac{9}{11}\end{bmatrix}A$$

Applying $R_2 \to R_2 + \frac{5}{9}R_3$ *and* $R_1 \to R_1 + \frac{1}{3}R_3$, *we have,*

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{-1}{11} & \frac{-2}{11} & \frac{3}{11}\\ \frac{2}{11} & \frac{4}{11} & \frac{5}{11}\\ \frac{-3}{11} & \frac{5}{11} & \frac{9}{11}\end{bmatrix}A$$

$\Rightarrow$ *Inverse of the given matrix is* $\begin{bmatrix}\frac{-1}{11} & \frac{-2}{11} & \frac{3}{11}\\ \frac{2}{11} & \frac{4}{11} & \frac{5}{11}\\ \frac{-3}{11} & \frac{5}{11} & \frac{9}{11}\end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.2 Q16

Consider the given matrix $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

Let $A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

We know that $A = IA$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to (-1)R_1$, we have

$$\begin{bmatrix} 1 & -1 & -2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - 3R_1$, we have

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & 3 & 5 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to \frac{R_2}{3}$, we have,

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & \frac{5}{3} \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 + R_2$ and $R_3 \to R_3 - 4R_2$, we have

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{5}{3} & \frac{-4}{3} & 1 \end{bmatrix} A$$

Applying $R_3 \to \frac{R_3}{3}$, we have

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ 5 & -4 & 3 \end{bmatrix} A$$

Applying $R_1 \to R_1 + \frac{1}{3}R_3$, $R_2 \to R_2 - \frac{5}{3}R_3$, we have,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} A$$

Thus, the inverse of the given matrix is $\begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$.