Exercise – 3.1

1. Simplify each of the following:

(i) $\sqrt[3]{4}\times\sqrt[3]{16}$

(ii) $\dfrac{\sqrt[4]{1250}}{\sqrt[4]{2}}$

Sol:

(i) $\sqrt[3]{4}\times\sqrt[3]{16}$

$\Rightarrow \sqrt[3]{4\times16}$ $\boxed{\because \sqrt[n]{a}\times\sqrt[n]{b}=\sqrt[n]{ab}}$

$\Rightarrow \sqrt[3]{64}$

$\Rightarrow \sqrt[3]{4^3} \Rightarrow \left(4^3\right)^{\frac{1}{3}} \Rightarrow 4^{3\times\frac{1}{3}} \Rightarrow 4^1 \Rightarrow 4$

$\therefore \sqrt[3]{4}\times\sqrt[3]{16}=4$

(ii) $\dfrac{\sqrt[4]{1250}}{\sqrt[4]{2}}$

$\Rightarrow \sqrt[4]{\dfrac{1250}{2}}$ $\boxed{\because \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\dfrac{a}{b}}}$

$\Rightarrow \sqrt[4]{\dfrac{625\times2}{2}} \Rightarrow \sqrt[4]{625} \Rightarrow \sqrt[4]{5^4} \Rightarrow \left(5^4\right)^{\frac{1}{4}}$

$\Rightarrow 5^{4\times\frac{1}{4}} \Rightarrow 5^1=5$

$\therefore \sqrt[4]{\dfrac{1250}{2}}=5$

2. Simplify the following expressions:

(i) $\left(4+\sqrt{7}\right)\left(3+\sqrt{2}\right)$

(ii) $\left(3+\sqrt{3}\right)\left(5-\sqrt{2}\right)$

(iii) $\left(\sqrt{5}-2\right)\left(\sqrt{3}-\sqrt{5}\right)$

Sol:

(i) We have

$$\left(4+\sqrt{7}\right)\left(3+\sqrt{2}\right)=4\times3+4\times\sqrt{2}+\sqrt{7}\times3+\sqrt{7}\times\sqrt{2}$$

$$=12+4\sqrt{2}+3\sqrt{7}+\sqrt{7\times2}$$

$$= 12 + 4\sqrt{2} + 3\sqrt{7} + \sqrt{14} \qquad \boxed{\because \sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}}$$

$$\therefore \left(4+\sqrt{7}\right)\left(3+\sqrt{2}\right) = 12 + 4\sqrt{2} + 3\sqrt{7} + \sqrt{14}$$

(ii) We have,

$$\left(3+\sqrt{3}\right)\left(5-\sqrt{2}\right) = 3\times 5 + 3\times\left(-\sqrt{2}\right) + \sqrt{3}\times 5 + \sqrt{3}\times\left(-\sqrt{2}\right)$$

$$= 15 - 3\sqrt{2} + 5\sqrt{3} - \sqrt{3\times 2}$$

$$= 15 - 3\sqrt{2} + 5\sqrt{3} - \sqrt{6} \qquad \boxed{\because \sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}}$$

$$\therefore \left(3+\sqrt{3}\right)\left(5-\sqrt{2}\right) = 15 - 3\sqrt{2} + 5\sqrt{3} - \sqrt{6}$$

(iii) We have

$$\left(\sqrt{5}-2\right)\left(\sqrt{3}-\sqrt{5}\right) = \sqrt{5}\times\sqrt{3} + \sqrt{5}\times\left(-\sqrt{5}\right) + \left(-2\right)\times\sqrt{3}$$

$$= \sqrt{5\times 3} - \sqrt{5\times 5} - 2\sqrt{3} + 2\sqrt{5}$$

$$= \sqrt{15} - \sqrt{5^2} - 2\sqrt{3} + 2\sqrt{5} \qquad \boxed{\because \sqrt[n]{a} + \sqrt[n]{b} = \sqrt[n]{ab}}$$

$$= \sqrt{15} - \sqrt{5^2} - 2\sqrt{3} + 2\sqrt{5}$$

$$= \sqrt{15} - 5 - 2\sqrt{3} + 2\sqrt{5}$$

$$\therefore \left(\sqrt{5}-2\right)\left(\sqrt{3}-\sqrt{5}\right) = \sqrt{15} - 2\sqrt{3} + 2\sqrt{5} - 5$$

3. Simplify the following expressions:

(i) $\left(11+\sqrt{11}\right)\left(11-\sqrt{11}\right)$

(ii) $\left(5+\sqrt{7}\right)\left(5-\sqrt{7}\right)$

(iii) $\left(\sqrt{8}-\sqrt{2}\right)\left(\sqrt{8}+\sqrt{2}\right)$

(iv) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

(v) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$

Sol:

(i) We have,

$$\left(11+\sqrt{11}\right)\left(11-\sqrt{11}\right) = \left(11\right)^2 - \left(\sqrt{11}\right)^2 \qquad \boxed{\because \left(a+b\right)\left(a-b\right) = a^2 - b^2}$$

$$= 121 - 11$$

$$= 110$$

$$\therefore \left(11+\sqrt{11}\right)\left(11-\sqrt{11}\right) = 110$$

(ii) We have,

$$\left(5+\sqrt{7}\right)\left(5-\sqrt{7}\right) = 5^2 - \left(\sqrt{7}\right)^2 \qquad \boxed{\because \left(a+b\right)\left(a-b\right) = a^2 - b^2}$$

$= 25 - 7 = 18$

$\therefore \left(5+\sqrt{7}\right)\left(5-\sqrt{7}\right) = 18$

(iii) We have,

$\left(\sqrt{8}-\sqrt{2}\right)\left(\sqrt{8}+\sqrt{2}\right) = \left(\sqrt{8}\right)^2 - \left(\sqrt{2}\right)^2$ $\boxed{\because (a+b)(a-b) = a^2 - b^2}$

$= 8 - 2 = 6$

$\therefore \left(\sqrt{8}-\sqrt{2}\right)\left(\sqrt{8}+\sqrt{2}\right) = 6$

(iv) We have,

$\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right) = (3)^2 - \left(\sqrt{3}\right)^2$ $\boxed{\because (a+b)(a-b) = a^2 - b^2}$

$= 9 - 3 = 6$

$\therefore \left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right) = 6$

(v) We have,

$\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right) = \left(\sqrt{5}\right)^2 - \left(\sqrt{2}\right)^2$ $\boxed{\because (a-b)(a+b) = a^2 - b^2}$

$= 5 - 2 = 3$

$\therefore \left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right) = 3$

4. Simplify the following expressions:

(i) $\left(\sqrt{3}+\sqrt{7}\right)^2$

(ii) $\left(\sqrt{5}-\sqrt{3}\right)^2$

(iii) $\left(2\sqrt{5}+3\sqrt{2}\right)^2$

Sol:

(i) $\left(\sqrt{3}+\sqrt{7}\right)^2 = \left(\sqrt{3}\right)^2 + 2\times\sqrt{3}\times\sqrt{7} + \left(\sqrt{7}\right)^2$ $\boxed{\because (a+b)^2 = a^2 + 2ab + b^2}$

$= 3 + 2\sqrt{3\times 7} + 7$ $\boxed{\because \sqrt[n]{a}\times\sqrt[n]{b} = \sqrt[n]{ab}}$

$= 10 + 2\sqrt{21}$

$\therefore \left(\sqrt{3}+\sqrt{7}\right)^2 = 10 + 2\sqrt{21}$

(ii) We have

$\left(\sqrt{5}-\sqrt{3}\right)^2 = \left(\sqrt{5}\right)^2 - 2\times\sqrt{5}\times\sqrt{3} + \left(\sqrt{3}\right)^2$

$= 5 - 2\sqrt{5\times 3} + 3$ $\boxed{\because (a-b)^2 = a^2 - 2ab + b^2}$

$= 8 - 2\sqrt{15}$ $\boxed{\because \sqrt[n]{a}\times\sqrt[n]{b} = \sqrt[n]{ab}}$

$\therefore \left(\sqrt{5}-\sqrt{3}\right)^2 = 8-2\sqrt{15}$

(iii) We have

$$\left(2\sqrt{5}+3\sqrt{2}\right)^2 = \left(2\sqrt{5}\right)^2 + 2\times\left(2\sqrt{5}\right)\times\left(3\sqrt{2}\right)+\left(3\sqrt{2}\right)^2 \quad \left[\because \left(a+b\right)^2 = a^2+2ab+b^2\right]$$

$$= 2^2\times\left(\sqrt{5}\right)^2 + \left(2\times2\times3\right)\times\sqrt{5\times2}+3^2\left(\sqrt{2}\right)^2$$

$$= 4\times5+12\times\sqrt{10}+9\times2 \quad \boxed{\because \left(ab\right)^n = a^n\times b^n \text{ and}}$$

$$= 20+12\sqrt{10}+18 \quad \boxed{\sqrt[n]{a}\times\sqrt[n]{b}=\sqrt[n]{ab}}$$

$$= 38+12\sqrt{10}$$

$\therefore \left(2\sqrt{5}+3\sqrt{2}\right)^2 = 38+12\sqrt{10}$

Exercise – 3.2

1. Rationalise the denominator of each of the following (i – vii) :

(i) $\frac{3}{\sqrt{5}}$

(ii) $\frac{3}{2\sqrt{5}}$

(iii) $\frac{1}{\sqrt{12}}$

(iv) $\frac{\sqrt{2}}{\sqrt{5}}$

(v) $\frac{\sqrt{3}+1}{\sqrt{2}}$

(vi) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

(vii) $\frac{3\sqrt{2}}{\sqrt{5}}$

Sol:

(i) $\frac{3}{5}\sqrt{5}$

(ii) $\frac{3}{10}\sqrt{5}$

(iii) $\frac{\sqrt{3}}{6}$

(iv) $\frac{1}{5}\sqrt{10}$

(v) $\frac{\sqrt{6}+\sqrt{2}}{2}$

(vi) $\frac{\sqrt{6}+\sqrt{5}}{3}$

(vii) $\frac{3\sqrt{10}}{5}$

2. Find the value to three places of decimals of each of the following. It is given that $\sqrt{2}=1.414, \sqrt{3}=1.732, \sqrt{5}=2.236 \text{ and } \sqrt{10}=3.162.$

(i) $\dfrac{2}{\sqrt{3}}$

(ii) $\dfrac{3}{\sqrt{10}}$

(iii) $\dfrac{\sqrt{5}+1}{\sqrt{2}}$

(iv) $\dfrac{\sqrt{10}+\sqrt{15}}{\sqrt{2}}$

(v) $\dfrac{2+\sqrt{3}}{3}$

(vi) $\dfrac{\sqrt{2}-1}{\sqrt{5}}$

Sol:

Given that $\sqrt{2}=1.414, \sqrt{3}=1.732, \sqrt{5}=2.236$ and $\sqrt{10}=3.162$.

(i) We have $\frac{2}{\sqrt{3}}$

Rationalising factor of denominator is $\sqrt{3}$

$$\therefore \frac{2}{\sqrt{3}}=\frac{2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{2\sqrt{3}}{\left(\sqrt{3}\right)^2}=\frac{2\sqrt{3}}{3}=\frac{2\times1.732}{3}=\frac{3.464}{3}$$

$=1.15466667$

$=1.154$

(ii) We have $\frac{3}{\sqrt{10}}$

Rationalising factor of denominator is $\sqrt{10}$

$$\therefore \frac{3}{\sqrt{10}}=\frac{3}{\sqrt{10}}\times\frac{\sqrt{10}}{\sqrt{10}}=\frac{3\sqrt{10}}{\left(\sqrt{10}\right)^2}=\frac{3\sqrt{10}}{10}=\frac{3\times3.162}{10}=\frac{9.486}{10}$$

$=0.9486$

$=0.948$

$$\therefore \frac{3}{\sqrt{10}}=0.948$$

(iii) We have $\frac{\sqrt{5}+1}{\sqrt{2}}$

Rationalising factor of denominator is $\sqrt{2}$.

$$\therefore \frac{\sqrt{5}+1}{\sqrt{2}}=\frac{\sqrt{5}+1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\frac{\left(\sqrt{5}+1\right)\sqrt{2}}{\left(\sqrt{2}\right)^2}$$

$$=\frac{\sqrt{5}\times\sqrt{2}+1\times\sqrt{2}}{2}$$

$$=\frac{\sqrt{5\times2}+\sqrt{2}}{2}=\frac{\sqrt{10}+\sqrt{2}}{2}$$

$$=\frac{3.162+1.414}{2}=\frac{4.576}{2}=2.288$$

$$\therefore \frac{\sqrt{5}+1}{\sqrt{2}}=2.288$$

(iv) We have $\frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}}$

Rationalising factor of denominator is $\sqrt{2}$

$$\therefore \frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}} = \frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\left(\sqrt{10}+\sqrt{15}\right)\sqrt{2}}{\left(\sqrt{2}\right)^2}$$

$$= \frac{\sqrt{10}\times\sqrt{2}+\sqrt{15}\times\sqrt{2}}{2}$$

$$= \frac{\sqrt{10\times 2}+\sqrt{15\times 2}}{2}$$

$$= \frac{\sqrt{20}+\sqrt{30}}{2} = \frac{\sqrt{2\times 10}+\sqrt{3\times 10}}{2}$$

$$= \frac{\sqrt{2}\times\sqrt{10}+\sqrt{3}\times\sqrt{10}}{2}$$

$$= \frac{(1\cdot 414\times 3\cdot 162)+(1\cdot 732\times 3\cdot 162)}{2}$$

$$= \frac{4\cdot 471068+5\cdot 476584}{2}$$

$$\Rightarrow \frac{4\cdot 471068+5\cdot 476584}{2}$$

$$\Rightarrow \frac{9\cdot 947652}{2} = 4.973826 \simeq 4.973$$

(v) We have $\frac{2+\sqrt{3}}{3}$

$$\Rightarrow \frac{2+1\cdot 732}{3} = \frac{3\cdot 732}{3} = 1\cdot 244$$

(vi) We have $\frac{\sqrt{2}-1}{\sqrt{5}}$

Rationalising factor for $\frac{1}{\sqrt{5}}$ is $\sqrt{5}$

$$\Rightarrow \frac{\sqrt{2}-1}{\sqrt{5}} = \frac{\sqrt{2}-1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\left(\sqrt{2}\times\sqrt{5}\right)-\left(1\times\sqrt{5}\right)}{\left(\sqrt{5}\right)^2}$$

$$= \frac{\sqrt{2\times 5}-\sqrt{5}}{5}$$

$$= \frac{\sqrt{10}-\sqrt{5}}{5}$$

$$= \frac{3\cdot 162-2\cdot 236}{5} = \frac{0\cdot 926}{5} = 0\cdot 1852$$

$$\simeq 0.185$$

$$\therefore \frac{\sqrt{2}-1}{\sqrt{5}} = 0\cdot 185$$

3. Express each one of the following with rational denominator:

(i) $\dfrac{1}{3+\sqrt{2}}$

(ii) $\dfrac{1}{\sqrt{6}-\sqrt{5}}$

(iii) $\dfrac{16}{\sqrt{41}-5}$

(iv) $\dfrac{30}{5\sqrt{3}-3\sqrt{5}}$

(v) $\dfrac{1}{2\sqrt{5}-\sqrt{3}}$

(vi) $\dfrac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}$

(vii) $\dfrac{6-4\sqrt{2}}{6+4\sqrt{2}}$

(viii) $\dfrac{3\sqrt{2}+1}{2\sqrt{5}-3}$

(ix) $\dfrac{b^2}{\sqrt{a^2+b^2}+a}$

Sol:

(i) We have $\frac{1}{3+\sqrt{2}}$

Rationalising factor for $\frac{1}{3+\sqrt{2}}$ is $3-\sqrt{2}$

$$\Rightarrow \frac{1}{3+\sqrt{2}}=\frac{1}{3+\sqrt{2}}\times\frac{3-\sqrt{2}}{3-\sqrt{2}}$$

$$=\frac{3-\sqrt{2}}{(3)^2-\left(\sqrt{2}\right)^2} \qquad \boxed{\because (a+b)(a-b)=a^2-b^2}$$

$$=\frac{3-\sqrt{2}}{9-2}=\frac{3-\sqrt{2}}{7} \qquad \therefore \frac{1}{3+\sqrt{2}}=\frac{3-\sqrt{2}}{7}$$

(ii) We have $\frac{1}{\sqrt{6}-\sqrt{5}}$

Rationalising factor for $\frac{1}{\sqrt{6}-\sqrt{5}}$ is $\sqrt{6}+\sqrt{5}$

$$\Rightarrow \frac{1}{\sqrt{6}-\sqrt{5}}=\frac{1}{\sqrt{6}-\sqrt{5}}\times\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}$$

$$=\frac{\sqrt{6}+\sqrt{5}}{\left(\sqrt{6}-\sqrt{5}\right)\left(\sqrt{6}+\sqrt{5}\right)}$$

$$=\frac{\sqrt{6}+\sqrt{5}}{\left(\sqrt{6}\right)^2-\left(\sqrt{5}\right)^2} \qquad \boxed{\because (a+b)(a-b)=a^2-b^2}$$

$$=\frac{\sqrt{6}+\sqrt{5}}{6-5}$$

$$=\frac{\sqrt{6}+\sqrt{5}}{1}=\sqrt{6}+\sqrt{5}$$

$$\therefore \frac{1}{\sqrt{6}-\sqrt{5}}=\sqrt{6}+\sqrt{5}$$

(iii) We have $\frac{16}{\sqrt{41}-5}$

Rationalisation factor for $\frac{1}{\sqrt{41}-5}$ is $\left(\sqrt{41}-5\right)$

$$\Rightarrow \frac{16}{\sqrt{41}-5}=\frac{16}{\sqrt{41}-5}\times\frac{\sqrt{41}+5}{\sqrt{41}+5}$$

$$=\frac{16\left(\sqrt{41}+5\right)}{\left(\sqrt{41}-5\right)\left(\sqrt{41}+5\right)}$$

$$= \frac{16\left(\sqrt{41}+5\right)}{\left(\sqrt{41}\right)^2-(5)^2} \qquad \boxed{\because (a-b)(a+b)=a^2-b^2}$$

$$= \frac{16\left(\sqrt{41}+5\right)}{41-25} = \frac{16\left(\sqrt{41}+5\right)}{16} = \sqrt{41}+5$$

$$\therefore \frac{16}{41-5} = \sqrt{41}+5$$

(iv) We have $\frac{30}{5\sqrt{3}-3\sqrt{5}}$

Rationalisation factor for $\frac{1}{5\sqrt{3}-3\sqrt{5}}$ is $5\sqrt{3}+3\sqrt{5}$

$$\Rightarrow \frac{30}{5\sqrt{3}-3\sqrt{5}} = \frac{30}{5\sqrt{3}-3\sqrt{5}} \times \frac{5\sqrt{3}+3\sqrt{5}}{5\sqrt{3}+3\sqrt{5}}$$

$$= \frac{30\left(5\sqrt{3}+3\sqrt{5}\right)}{\left(5\sqrt{3}-3\sqrt{5}\right)\left(5\sqrt{3}+3\sqrt{5}\right)}$$

$$= \frac{30\left(5\sqrt{3}+3\sqrt{5}\right)}{\left(5\sqrt{3}\right)^2-\left(3\sqrt{5}\right)^2} \qquad \boxed{\because (a-b)(a-b)=a^2-b^2}$$

$$= \frac{30\left(5\sqrt{3}+3\sqrt{5}\right)}{5^2\left(\sqrt{3}\right)^2-3^2\left(\sqrt{5}\right)^2}$$

$$= \frac{30\left(5\sqrt{3}+3\sqrt{5}\right)}{25\times3-9\times5} = \frac{30\left(5\sqrt{3}+3\sqrt{5}\right)}{75-45} = \frac{30\left(5\sqrt{3}+3\sqrt{5}\right)}{30}$$

$$= 5\sqrt{3}+3\sqrt{5}$$

$$\therefore \frac{30}{5\sqrt{3}-3\sqrt{5}} = 5\sqrt{3}+3\sqrt{5}$$

(v) We have $\frac{1}{2\sqrt{5}-\sqrt{3}}$

Rationalisation factor for $\frac{1}{2\sqrt{5}-\sqrt{3}}$ is $2\sqrt{5}+\sqrt{3}$

$$\Rightarrow \frac{1}{2\sqrt{5}-\sqrt{3}} = \frac{1}{2\sqrt{5}-\sqrt{3}} \times \frac{2\sqrt{5}+\sqrt{3}}{2\sqrt{5}+\sqrt{3}}$$

$$= \frac{2\sqrt{5}+\sqrt{3}}{\left(2\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2} \qquad \boxed{\because (a-b)(a+b)=a^2-b^2}$$

$$= \frac{2\sqrt{5}+\sqrt{3}}{2^2\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{2\sqrt{5}+\sqrt{3}}{4\times5-3} = \frac{2\sqrt{5}+\sqrt{3}}{20-3} = \frac{2\sqrt{5}+\sqrt{3}}{17}$$

$$\therefore \frac{1}{2\sqrt{5}-\sqrt{3}} = \frac{2\sqrt{5}+\sqrt{3}}{17}$$

(vi) We have $\frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}$

Rationalisation factor for $\frac{1}{2\sqrt{2}-\sqrt{3}}$ is $2\sqrt{2}+\sqrt{3}$

$$\Rightarrow \frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}} = \frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}\times\frac{2\sqrt{2}+\sqrt{3}}{2\sqrt{2}+\sqrt{3}}$$

$$= \frac{\left(\sqrt{3}+1\right)\left(2\sqrt{2}+\sqrt{3}\right)}{\left(2\sqrt{2}-\sqrt{3}\right)\left(2\sqrt{2}+\sqrt{3}\right)}$$

$$= \frac{\sqrt{3}\times2\sqrt{2}+\sqrt{3}\times\sqrt{3}+1\times2\sqrt{2}+1\times\sqrt{3}}{\left(2\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2}$$

$$\boxed{\because (a-b)(a+b)=a^2-b^2}$$

$$= \frac{2\sqrt{2\times3}+\sqrt{3\times3}+2\sqrt{2}+\sqrt{3}}{2^2\left(\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{2\sqrt{6}+3+2\sqrt{2}+\sqrt{3}}{4\times2-3} = \frac{2\sqrt{6}+3+2\sqrt{2}+\sqrt{3}}{8-3}$$

$$= \frac{2\sqrt{6}+3+2\sqrt{2}+\sqrt{3}}{5}$$

$$\therefore \frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}} = \frac{2\sqrt{6}+3+2\sqrt{2}+\sqrt{3}}{5}$$

(vii) We have $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$

Rationalisation factor for $\frac{1}{6+4\sqrt{2}}$ is $6-4\sqrt{2}$

$$\Rightarrow \frac{6-4\sqrt{2}}{6+4\sqrt{2}} = \frac{6-4\sqrt{2}}{6+4\sqrt{2}}\times\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$$

$$= \frac{\left(6-4\sqrt{2}\right)^2}{(6)^2-\left(4\sqrt{2}\right)^2}$$

$$\boxed{\begin{array}{l}\because (a+b)(a-b)=a^2-b^2\\ \quad (a-b)(a-b)=(a-b)^2\end{array}}$$

$$= \frac{6^2 - 2\times 6\times 4\sqrt{2} + \left(4\sqrt{2}\right)^2}{36 - 4^2\left(\sqrt{2}\right)^2} \qquad \boxed{(a-b)^2 = a^2 - 2ab + b^2}$$

$$= \frac{36 - 48\sqrt{2} + 32}{36 - 32}$$

$$= \frac{68 - 48\sqrt{2}}{4} = \frac{4\left(17 - 12\sqrt{2}\right)}{4} = 17 - 12\sqrt{2}$$

$$\therefore \frac{6 - 4\sqrt{2}}{6 + 4\sqrt{2}} = 17 - 12\sqrt{2}$$

(viii) We have $\frac{3\sqrt{2}+1}{2\sqrt{5}-3}$.

Rationalising factor for $\frac{1}{2\sqrt{5}-3}$ is $2\sqrt{5}+3$

$$\Rightarrow \frac{3\sqrt{2}+1}{2\sqrt{5}-3} = \frac{3\sqrt{2}+1}{2\sqrt{5}-3} \times \frac{2\sqrt{5}+3}{2\sqrt{5}+3}$$

$$= \frac{\left(3\sqrt{2}+1\right)\left(2\sqrt{5}+3\right)}{\left(2\sqrt{5}-3\right)\left(2\sqrt{5}+3\right)}$$

$$= \frac{3\sqrt{2}\times 2\sqrt{5} + 3\sqrt{2}\times 3 + 1\times 2\sqrt{5} + 1\times 3}{\left(2\sqrt{5}\right)^2 - (3)^2} \qquad \boxed{(a-b)(a+b) = a^2 - b^2}$$

$$= \frac{6\sqrt{10} + 9\sqrt{2} + 2\sqrt{5} + 3}{20 - 9} = \frac{6\sqrt{10} + 9\sqrt{2} + 2\sqrt{5} + 3}{11}$$

$$\therefore \frac{3\sqrt{2}+1}{2\sqrt{5}-3} = \frac{6\sqrt{10} + 9\sqrt{2} + 2\sqrt{5} + 3}{11}$$

(ix) We have $\frac{b^2}{\sqrt{a^2+b^2}+a}$

Rationalisation factor for $\frac{1}{\sqrt{a^2+b^2}+a}$ is $\sqrt{a^2+b^2}-a$

$$\Rightarrow \frac{b^2}{\sqrt{a^2+b^2}+a} = \frac{b^2}{\sqrt{a^2+b^2}+a} \times \frac{\sqrt{a^2+b^2}-a}{\sqrt{a^2+b^2}-a}$$

$$= \frac{b^2\left(\sqrt{a^2+b^2}-a\right)}{\left(\sqrt{a^2+b^2}\right)^2 - (a)^2} \qquad \boxed{(x+y)(x-y) = x^2 - y^2}$$

$$= \frac{b^2\left(\sqrt{a^2+b^2}-a\right)}{a^2+b^2-a^2}$$

$$= \frac{b^2\left(\sqrt{a^2+b^2}-a\right)}{b^2}$$

$$= \left(\sqrt{a^2+b^2}-a\right)$$

$$\therefore \frac{b^2}{\sqrt{a^2+b^2}+a} = \sqrt{a^2+b^2}-a$$

4. Rationalize the denominator and simplify:

(i) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(ii) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}$

(iii) $\frac{1+\sqrt{2}}{3-2\sqrt{2}}$

(iv) $\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$

(v) $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

(vi) $\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$

Sol:

(i) We have $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

Rationalisation factor for $\frac{1}{\sqrt{a}+\sqrt{b}}$ is $\sqrt{a}-\sqrt{b}$

$\Rightarrow$ for $\frac{1}{\sqrt{3}+\sqrt{2}}$ it is $\sqrt{3}-\sqrt{2}$

$$\therefore \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{\left(\sqrt{3}-\sqrt{2}\right)^2}{\left(\sqrt{3}\right)^2+\left(\sqrt{2}\right)^2}$$

$\because (a-b)(a-b)=(a-b)^2$
and $(a+b)(a-b)=a^2-b^2$

$$= \frac{\left(\sqrt{3}\right)^2 - 2\times\sqrt{3}\times\sqrt{2} + \left(\sqrt{2}\right)^2}{3-2} \qquad \boxed{\because \left(a-b\right)^2 = a^2 - 2ab + b^2}$$

$$= \frac{3-2\sqrt{6}+2}{1} = 5-2\sqrt{6}$$

$$\therefore \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} = 5-2\sqrt{6}$$

(ii) We have $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}$

Rationalising factor for $\frac{1}{a+b\sqrt{c}}$ is $a-b\sqrt{c}$

$\Rightarrow$ for $\frac{1}{7+4\sqrt{3}}$ is $7-4\sqrt{3}$

$$\therefore \frac{5+2\sqrt{3}}{7+4\sqrt{3}} = \frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$$

$$= \frac{5\times7+5\times\left(-4\sqrt{3}\right)+2\sqrt{3}\times7+2\sqrt{3}\times\left(-4\sqrt{3}\right)}{7^2-\left(4\sqrt{3}\right)^2} \qquad \boxed{\because \left(a+b\right)\left(a-b\right) = a^2 - b^2}$$

$$= \frac{35-20\sqrt{3}+14\sqrt{3}-8\times3}{49-48}$$

$$= \frac{35-24-6\sqrt{3}}{1} = 11-6\sqrt{3}$$

$$\therefore \frac{5+2\sqrt{3}}{7+4\sqrt{3}} = 11-6\sqrt{3}$$

(iii) We have $\frac{1+\sqrt{2}}{3-2\sqrt{2}}$

Rationalisation factor for $\frac{1}{a-b\sqrt{c}}$ is $a+b\sqrt{c}$

$\Rightarrow$ for $\frac{1}{3-2\sqrt{2}}$ is $3+2\sqrt{2}$

$$\therefore \frac{1+\sqrt{2}}{3-2\sqrt{2}} = \frac{1+\sqrt{2}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$$

$$= \frac{1\times3+1\times2\sqrt{2}+\sqrt{2}\times3+\sqrt{2}\times2\sqrt{2}}{3^2-\left(2\sqrt{2}\right)^2} \qquad \boxed{\because \left(a-b\right)\left(a+b\right) = a^2 - b^2}$$

$$= \frac{3+2\sqrt{2}+3\sqrt{2}+2\times 2}{9-8}$$

$$= \frac{3+4+5\sqrt{2}}{1} = \frac{7+5\sqrt{2}}{1} = 7+5\sqrt{2}$$

$$\therefore \frac{1+\sqrt{2}}{3-2\sqrt{2}} = 7+5\sqrt{2}$$

(iv) We have $\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$

Rationalisation factor for $\frac{1}{a\sqrt{b}-c\sqrt{d}}$ is $a\sqrt{b}+c\sqrt{d}$

$\Rightarrow$ for $\frac{1}{3\sqrt{5}-2\sqrt{6}}$ is $3\sqrt{5}+2\sqrt{6}$

$$\therefore \frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}} = \frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}} \times \frac{3\sqrt{5}+2\sqrt{6}}{3\sqrt{5}+2\sqrt{6}}$$

$$= \frac{2\sqrt{6}\times 3\sqrt{5}+2\sqrt{6}\times 2\sqrt{6}+\left(-\sqrt{5}\right)\left(3\sqrt{5}\right)+\left(-\sqrt{5}\right)\left(2\sqrt{6}\right)}{\left(3\sqrt{5}\right)^2-\left(2\sqrt{6}\right)^2}$$

$$\boxed{\because (a-b)(a+b) = a^2-b^2}$$

$$= \frac{6\sqrt{30}+4\times 6-3\times 5-2\sqrt{5\times 6}}{45-24}$$

$$= \frac{6\sqrt{30}+24-15-2\sqrt{30}}{21}$$

$$= \frac{9+4\sqrt{30}}{21}$$

$$\therefore \frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}} = \frac{9+4\sqrt{30}}{21}$$

(v) We have $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

Rationalisation factor for $\frac{1}{\sqrt{a}+\sqrt{b}}$ is $\sqrt{a}-\sqrt{b}$

$\Rightarrow$ for $\frac{1}{\sqrt{48}+\sqrt{18}}$ is $\sqrt{48}-\sqrt{18}$

$$\therefore \frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}} = \frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}} \times \frac{\sqrt{48}-\sqrt{18}}{\sqrt{48}-\sqrt{18}}$$

$$= \frac{4\sqrt{3}\times\sqrt{48}+4\sqrt{3}\times\left(-\sqrt{18}\right)+5\sqrt{2}\times\sqrt{48}+5\sqrt{2}\times\left(-\sqrt{18}\right)}{\left(\sqrt{48}\right)^2-\left(\sqrt{18}\right)^2} \qquad \boxed{\because (a+b)(a-b)=a^2-b^2}$$

$$= \frac{4\sqrt{3}\times\sqrt{3\times16}-4\sqrt{3}\times\sqrt{2\times9}+5\sqrt{2}\times\sqrt{3\times16}-5\sqrt{2}\times\sqrt{2\times9}}{48-18}$$

$$= \frac{4\sqrt{3}\times4\sqrt{3}-4\sqrt{3}\times3\sqrt{2}+5\sqrt{2}\times4\sqrt{3}-5\sqrt{2}\times3\sqrt{2}}{30}$$

$$= \frac{48-12\sqrt{6}+20\sqrt{6}-30}{30}$$

$$= \frac{18+8\sqrt{6}}{30} = \frac{2\left(9+4\sqrt{6}\right)}{30} = \frac{9+4\sqrt{6}}{15}$$

$$\therefore \frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}} = \frac{9+4\sqrt{6}}{\sqrt{15}}$$

(vi) We have $\dfrac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$

Rationalisation factor for $\dfrac{1}{a\sqrt{b}+c\sqrt{d}}$ is $a\sqrt{b}-c\sqrt{d}$

$\Rightarrow$ for $\dfrac{1}{2\sqrt{2}+3\sqrt{3}}$ is $2\sqrt{2}-3\sqrt{3}$

$$\therefore \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}} = \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}\times\frac{2\sqrt{2}-3\sqrt{3}}{2\sqrt{2}+3\sqrt{3}}$$

$$= \frac{2\sqrt{3}\times2\sqrt{2}+2\sqrt{3}\times\left(-3\sqrt{3}\right)+\left(-\sqrt{5}\right)\left(2\sqrt{2}\right)+\left(-\sqrt{5}\right)\left(-3\sqrt{3}\right)}{\left(2\sqrt{2}\right)^2-\left(3\sqrt{3}\right)^2} \qquad \boxed{\because (a+b)(a-b)=a^2-b^2}$$

$$= \frac{4\sqrt{6}-6\sqrt{3^2}-2\sqrt{10}+3\sqrt{15}}{8-27}$$

$$= \frac{4\sqrt{6}-18-2\sqrt{10}+3\sqrt{15}}{-19}$$

$$= \frac{-\left(18-3\sqrt{15}+2\sqrt{10}-4\sqrt{6}\right)}{-19} = \frac{18-3\sqrt{15}+2\sqrt{10}-4\sqrt{6}}{19}$$

$$\therefore \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}} = \frac{18-3\sqrt{15}+2\sqrt{10}-4\sqrt{6}}{19}$$

5. Simplify:

(i) $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$

(ii) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

(iii) $\frac{7+3\sqrt{5}}{3+\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}$

(iv) $\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2-\sqrt{5}}$

(v) $\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}$

Sol:

(i) We have $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$

Rationalisation factor for $\frac{1}{3\sqrt{2}+2\sqrt{3}}$ is $3\sqrt{2}-2\sqrt{3}$ and for $\frac{1}{\sqrt{3}-\sqrt{2}}$ is $\sqrt{3}+\sqrt{2}$

$$\Rightarrow \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}\times\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}+\frac{\sqrt{4\times 3}}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$\Rightarrow \frac{\left(3\sqrt{2}-2\sqrt{3}\right)^2}{\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2}+\frac{\left(\sqrt{4}\times\sqrt{3}\right)\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$$

$$\boxed{\begin{array}{l}\because (a-b)(a-b)=(a-b)^2\\ \quad (a+b)(a-b)=a^2-b^2\end{array}}$$

$$\Rightarrow \frac{\left(3\sqrt{2}\right)^2-2\times 3\sqrt{2}\times 2\sqrt{3}+\left(2\sqrt{3}\right)^2}{3^2\left(\sqrt{2}\right)^2-(2)^2\left(\sqrt{3}\right)^2}+\frac{2\sqrt{3}\times\sqrt{3}+2\sqrt{3}\times\sqrt{2}}{3-2}$$

$$\Rightarrow \frac{18-12\sqrt{6}+12}{18-12}+\frac{6+2\sqrt{6}}{1}$$

$$\Rightarrow \frac{30-12\sqrt{6}}{6}+6+2\sqrt{6}$$

$$\Rightarrow \frac{6\left(5-2\sqrt{6}\right)}{6}+6+2\sqrt{6}$$

$$\Rightarrow 5-2\sqrt{6}+6+2\sqrt{6}\Rightarrow 5+6\Rightarrow 11$$

$$\therefore \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}=11$$

(ii) we have $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

Rationalisation factor for $\frac{1}{\sqrt{a}\pm\sqrt{b}}$ is $\sqrt{a}\mp\sqrt{b}$

$\Rightarrow$ for $\frac{1}{\sqrt{5}-\sqrt{3}}$ it is $\sqrt{5}+\sqrt{3}$ and for $\frac{1}{\sqrt{5}+\sqrt{3}}$ it is $\sqrt{5}-\sqrt{3}$

$$\Rightarrow \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}+\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$\Rightarrow \frac{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}+\frac{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)}$$

$$\Rightarrow \frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}+\frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$\boxed{\begin{aligned}\because (a+b)(a+b)&=(a+b)^2\\(a-b)(a-b)&=(a-b)^2\\(a+b)(a-b)&=a^2-b^2\end{aligned}}$$

$$\Rightarrow \frac{\left(\sqrt{5}\right)^2+2\sqrt{5}\sqrt{3}+\left(\sqrt{3}\right)^2}{5-3}+\frac{\left(\sqrt{5}\right)^2-2\sqrt{5}\sqrt{3}+\left(\sqrt{3}\right)^2}{5-3}$$

$$\Rightarrow \frac{5+2\sqrt{15}+3}{2}+\frac{5-2\sqrt{15}+3}{2}$$

$$\Rightarrow \frac{8+2\sqrt{15}}{2}+\frac{8-2\sqrt{15}}{2}$$

$$\Rightarrow \frac{2\left(4+\sqrt{15}\right)}{2}+\frac{2\left(4-\sqrt{15}\right)}{2}$$

$$\Rightarrow 4+\sqrt{15}+4-\sqrt{15}$$

$$\Rightarrow 4+4\Rightarrow 8$$

$$\therefore \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}=8$$

(iii) We have $\frac{7+3\sqrt{5}}{3+\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}$

Rationalisation factor for $\frac{1}{a\pm\sqrt{b}}$ is $a\mp\sqrt{b}$

$\Rightarrow$ for $\frac{1}{3+\sqrt{5}}$ it is $3-\sqrt{5}$ and for $\frac{1}{3-\sqrt{5}}$ it is $3+\sqrt{5}$

$$\Rightarrow \frac{7+3\sqrt{5}}{3+\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}$$

$$\Rightarrow \frac{7+3\sqrt{5}}{3+\sqrt{5}}\times\frac{3-\sqrt{5}}{3-\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}\times\frac{3+\sqrt{5}}{3+\sqrt{5}}$$

$$\Rightarrow \frac{\left(7+3\sqrt{5}\right)\left(3-\sqrt{5}\right)}{\left(3+\sqrt{5}\right)\left(3-\sqrt{5}\right)}-\frac{\left(7-3\sqrt{5}\right)\left(3+\sqrt{5}\right)}{\left(3-\sqrt{5}\right)\left(3+\sqrt{5}\right)}$$

$$\Rightarrow \frac{7\times3+7\times\left(-\sqrt{5}\right)+3\sqrt{5}\times3+3\sqrt{5}\times\left(-\sqrt{5}\right)}{3^2-\left(\sqrt{5}\right)^2}$$

$$-\frac{7\times3+7\times\sqrt{5}+\left(-3\sqrt{5}\right)\times3+\left(-3\sqrt{5}\right)\times\sqrt{5}}{3^2-\left(\sqrt{5}\right)^2} \qquad \boxed{\because (a-b)(a+b)=a^2-b^2}$$

$$\Rightarrow \frac{21-7\sqrt{5}+9\sqrt{5}-3\times5}{9-5}-\frac{21+7\sqrt{5}-9\sqrt{5}-3\times5}{9-5}$$

$$\Rightarrow \frac{21-15+2\sqrt{5}}{4}-\frac{21-15+2\sqrt{5}}{4}$$

$$\Rightarrow \frac{6+2\sqrt{5}}{4}-\frac{6-2\sqrt{5}}{4}$$

$$\Rightarrow \frac{6+2\sqrt{5}-\left(6-2\sqrt{5}\right)}{4}\Rightarrow\frac{6+2\sqrt{5}-6+2\sqrt{5}}{4}\Rightarrow\frac{4\sqrt{5}}{4}\Rightarrow\sqrt{5}$$

$$\therefore \frac{7+3\sqrt{5}}{3+\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}=\sqrt{5}$$

(iv) We have $\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2-\sqrt{5}}$

Rationalisation factor for $\frac{1}{a\pm\sqrt{b}}$ is $a\mp\sqrt{b}$

$\Rightarrow$ for $\frac{1}{2+\sqrt{3}}$ it is $2-\sqrt{3}$ and for $\frac{1}{2-\sqrt{5}}$ it is $2+\sqrt{5}$

And also, rationalisation factor for $\frac{1}{\sqrt{a}\pm\sqrt{b}}$ is $\sqrt{a}\mp\sqrt{b}\Rightarrow$ for $\frac{1}{\sqrt{5}-\sqrt{3}}$ it is $\sqrt{5}+\sqrt{3}$

$$\Rightarrow \frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2-\sqrt{5}}$$

$$\Rightarrow \frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}+\frac{1}{2-\sqrt{5}}\times\frac{2+\sqrt{5}}{2+\sqrt{5}}$$

$$\Rightarrow \frac{2-\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2}+\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}+\frac{2+\sqrt{5}}{2^2-\left(\sqrt{5}\right)^2} \qquad \boxed{\because (a+b)(a-b)=a^2-b^2}$$

$$\Rightarrow \frac{2-\sqrt{3}}{4-3}+\frac{2\sqrt{5}+2\sqrt{3}}{5-3}+\frac{2+\sqrt{5}}{4-5}$$

$$\Rightarrow \frac{2-\sqrt{3}}{1}+\frac{2\sqrt{5}+2\sqrt{3}}{2}+\frac{2+\sqrt{5}}{-1}$$
$$\Rightarrow 2-\sqrt{3}+\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{2}-\left(2+\sqrt{3}\right)$$
$$\Rightarrow 2-\sqrt{3}+\sqrt{5}+\sqrt{3}-2-\sqrt{3}$$
$$\Rightarrow 0$$
$$\therefore \frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2-\sqrt{5}}=0$$

(v) We have,

$$\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}$$

Rationalisation factor for $\frac{1}{\sqrt{a}\pm\sqrt{b}}$ is $\sqrt{a}\mp\sqrt{b} \Rightarrow$ for $\frac{1}{\sqrt{5}+\sqrt{3}}$ it is $\sqrt{5}-\sqrt{3}$

$\Rightarrow$ for $\frac{1}{\sqrt{3}+\sqrt{2}}$ it is $\sqrt{3}-\sqrt{2} \Rightarrow$ for $\frac{1}{\sqrt{5}+\sqrt{2}}$ it is $\sqrt{5}-\sqrt{2}$

$$\Rightarrow \frac{2}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$
$$\Rightarrow \frac{2\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}+\frac{\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}-\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$
$$\Rightarrow \frac{2\left(\sqrt{5}-\sqrt{3}\right)}{5-3}+\frac{\sqrt{3}-\sqrt{2}}{3-2}-\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{5-2}$$
$$\Rightarrow \frac{2\left(\sqrt{5}-\sqrt{3}\right)}{2}+\frac{\sqrt{3}-\sqrt{2}}{1}-\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{3}$$
$$\Rightarrow \sqrt{5}-\sqrt{3}+\sqrt{3}-\sqrt{2}-\sqrt{5}+\sqrt{2}$$
$$\Rightarrow 0$$
$$\therefore \frac{2}{5+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}=0$$

6. In each of the following determine rational numbers a and b:

(i) $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a - b\sqrt{3}$

(ii) $\frac{4+\sqrt{2}}{2+\sqrt{2}} = a - \sqrt{b}$

(iii) $\frac{3+\sqrt{2}}{3-\sqrt{2}} = a + b\sqrt{2}$

(iv) $\frac{5+3\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$

(v) $\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = a - b\sqrt{77}$

(vi) $\frac{4+3\sqrt{5}}{4-3\sqrt{5}} = a + b\sqrt{5}$

Sol:

(i) Given $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a - b\sqrt{3}$

Rationalisation factor for $\frac{1}{\sqrt{x}+y}$ is $\sqrt{x}-y \Rightarrow$ for $\frac{1}{\sqrt{3}+1}$ is $\sqrt{3}-1$

$$\Rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}\right)^2 - (1)^2} \qquad \boxed{\begin{aligned} &\because (a-b)(a-b) = (a-b)^2 \\ &(a+b)(a-b) = a^2 - b^2 \end{aligned}}$$

$$= \frac{\left(\sqrt{3}\right)^2 - 2\sqrt{3}\times 1 + (1)^2}{3-1} \qquad \boxed{\because (a-b)^2 = a^2 - 2ab + b^2}$$

$$= \frac{3 - 2\sqrt{3} + 1}{2}$$

$$= \frac{4 - 2\sqrt{3}}{2} = \frac{2\left(2-\sqrt{3}\right)}{2} = 2 - \sqrt{3}$$

We have

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} = a - b\sqrt{3}$$

$$\Rightarrow 2 - \sqrt{3} = a - b\sqrt{3} \Rightarrow 2 - (1)\sqrt{3} = a - b\sqrt{3}$$

On equating rational and irrational parts, we get $a = 2$ and $b = 1$

(ii) Given that

$$\frac{4+\sqrt{2}}{2+\sqrt{2}} = a-\sqrt{b}$$

Rationalisation factor for $\frac{1}{a \pm b}$ is $a \mp \sqrt{b} \Rightarrow$ for $\frac{1}{2+\sqrt{2}}$ it is $2-\sqrt{2}$

$$\Rightarrow \frac{4+\sqrt{2}}{2+\sqrt{2}} = \frac{4+\sqrt{2}}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}}$$

$$= \frac{4\times 2+\sqrt{2}\times 2+4\times\left(-\sqrt{2}\right)+\sqrt{2}\times\left(-\sqrt{2}\right)}{2^2-\left(\sqrt{2}\right)^2} \qquad \boxed{\because (a+b)(a-b) = a^2-b^2}$$

$$= \frac{8+2\sqrt{2}-4\sqrt{2}-2}{4-2}$$

$$= \frac{6-2\sqrt{2}}{2} = \frac{2\left(3-\sqrt{2}\right)}{2} = 3-\sqrt{2}$$

We have,

$$\frac{4+\sqrt{2}}{2+\sqrt{2}} = a-\sqrt{b}$$

$$\Rightarrow 3-\sqrt{2} = a-\sqrt{b}$$

On equating rational and irrational parts

We get

$a = 3$ *and* $b = 2$

(iii) Given that

$$\frac{3+\sqrt{2}}{3-\sqrt{2}} = a+b\sqrt{2}$$

The rationalisation factor for $\frac{1}{a-\sqrt{b}}$ is $a+\sqrt{b} \Rightarrow$ for $\frac{1}{3-\sqrt{2}}$ it is $3+\sqrt{2}$

$$\Rightarrow \frac{3+\sqrt{2}}{3-\sqrt{2}} = \frac{3+\sqrt{2}}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$$

$$= \frac{\left(3+\sqrt{2}\right)^2}{3^2-\left(\sqrt{2}\right)^2} \qquad \boxed{\begin{array}{l}\because (a+b)(a+b) = (a+b)^2 \\ (a+b)(a-b) = a^2-b^2\end{array}}$$

$$= \frac{3^2+2\times 3\sqrt{2}+\left(\sqrt{2}\right)^2}{9-2} \qquad \boxed{(a+b)^2 = a^2+2ab+b^2}$$

$$= \frac{9+6\sqrt{2}+2}{7} = \frac{11+6\sqrt{2}}{7} = \frac{11}{7}+\frac{6}{7}\sqrt{2}$$

We have

$$\frac{3+\sqrt{2}}{3-\sqrt{2}}=a+b\sqrt{2}$$

$$\Rightarrow \frac{11}{7}+\frac{6}{7}\sqrt{2}=a+b\sqrt{2}$$

On equating rational and irrational parts

We get

$$a=\frac{11}{7} \text{ and } b=\frac{6}{7}$$

(iv) Given that

$$\frac{5+3\sqrt{3}}{7+4\sqrt{3}}=a+b\sqrt{3}$$

Rationalisation factor for $\frac{1}{a+b\sqrt{c}}$ is $a-b\sqrt{c} \Rightarrow$ for $\frac{1}{7+4\sqrt{3}}$ it is $7-4\sqrt{3}$

$$\Rightarrow \frac{5+3\sqrt{3}}{7+4\sqrt{3}}=\frac{5+3\sqrt{3}}{7+4\sqrt{3}}\times\frac{7-4\sqrt{3}}{7-4\sqrt{3}}$$

$$=\frac{5\times 7+5\times\left(-4\sqrt{3}\right)+3\sqrt{3}\times 7+3\sqrt{3}\left(-4\sqrt{3}\right)}{(7)^2-\left(4\sqrt{3}\right)^2} \qquad \boxed{\because (a+b)(a-b)=a^2-b^2}$$

$$=\frac{35-20\sqrt{3}+21\sqrt{3}-12\times 3}{49-48}$$

$$=\frac{35-36+\sqrt{3}}{1}=\frac{\sqrt{3}-1}{1}=\sqrt{3}-1$$

We have

$$\frac{5+3\sqrt{3}}{7+4\sqrt{3}}=a+b\sqrt{3}$$

$$\Rightarrow \sqrt{3}-1=a+b\sqrt{3}$$

$$\Rightarrow -1+(1)\sqrt{3}=a+b\sqrt{3}$$

On equating the rational and irrational parts

We get

$a=-1$ and $b=1$

(v) Given that,

$$\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}}=a-b\sqrt{77}$$

We know that rationalisation factor for $\frac{1}{\sqrt{a}+\sqrt{b}}$ is $\sqrt{a}-\sqrt{b} \Rightarrow$ for $\frac{1}{\sqrt{11}+\sqrt{7}}$ it is $\sqrt{11}-\sqrt{7}$

$$\Rightarrow \frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = \frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} \times \frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}-\sqrt{7}}$$

$$= \frac{\left(\sqrt{11}-\sqrt{7}\right)^2}{\left(\sqrt{11}\right)^2+\left(\sqrt{7}\right)^2} \qquad \boxed{\begin{array}{l}\because (a-b)(a-b)=(a-b)^2\\ (a-b)(a+b)=a^2-b^2\end{array}}$$

$$= \frac{\left(\sqrt{11}\right)^2 - 2\sqrt{11}\times\sqrt{7}+\left(\sqrt{7}\right)^2}{11-2} \qquad \boxed{\because (a-b)^2 = a^2-2ab+b^2}$$

$$= \frac{11-2\sqrt{11\times 7}+7}{4}$$

$$= \frac{18-2\sqrt{77}}{4} = \frac{2\left(9-\sqrt{77}\right)}{4} = \frac{9-\sqrt{27}}{2} = \frac{9}{2}-\frac{\sqrt{77}}{2}$$

We have,

$$\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = a-b\sqrt{77}$$

$$\Rightarrow \frac{9}{2}-\frac{\sqrt{77}}{2} = a-b\sqrt{77}$$

$$\Rightarrow \frac{9}{2}-\frac{1}{2}\sqrt{77} = a-b\sqrt{77}$$

On equating the rational and irrational posts

We have

$a = \frac{9}{2}$ and $b = \frac{1}{2}$

(vi) Given that

$$\frac{4+3\sqrt{5}}{4-3\sqrt{5}} = a+b\sqrt{5}$$

Rationalisation factor for $\frac{1}{a-b\sqrt{c}}$ is $a+b\sqrt{c} \Rightarrow$ for $\frac{1}{4-3\sqrt{5}}$ it is $4+3\sqrt{5}$

$$\Rightarrow \frac{4+3\sqrt{5}}{4-3\sqrt{5}} = \frac{4+3\sqrt{5}}{4-3\sqrt{5}} \times \frac{4+3\sqrt{5}}{4+3\sqrt{5}}$$

$$= \frac{\left(4+3\sqrt{5}\right)^2}{4^2-\left(3\sqrt{5}\right)^2} \qquad \boxed{\begin{array}{l}\because (a+b)(a+b)=(a+b)^2\\ (a-b)(a+b)=a^2-b^2\end{array}}$$

$$= \frac{4^2\times 2\times 4\times 3\sqrt{5}+\left(3\sqrt{5}\right)^2}{16-3^2\left(\sqrt{5}\right)^2} \qquad \boxed{\because (a+b)^2 = a^2+2ab+b^2}$$

$$= \frac{16+24\sqrt{5}+45}{16-45} = \frac{61+24\sqrt{5}}{-29} = \frac{-\left(61+24\sqrt{5}\right)}{29}$$
$$= \frac{-61}{29} - \frac{24}{29}\sqrt{5}$$

7. If x = 2 + $\sqrt{3}$, find the value of $x^3 + \frac{1}{x^3}$

Sol:

Given $x = 2+\sqrt{3}$ and given to find the value of $x^3 + \frac{1}{x^3}$

We have $x = 2+\sqrt{3}$

$$\Rightarrow \frac{1}{x} = \frac{1}{2+\sqrt{3}}$$

Rationalization factor for $\frac{1}{a+\sqrt{b}}$ is $a-\sqrt{b}$

$$\Rightarrow \frac{1}{x} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$
$$= \frac{2-\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2} \qquad \boxed{\because (a-b)(a+b) = a^2-b^2}$$
$$= \frac{2-\sqrt{3}}{4-3} = \frac{2-\sqrt{3}}{1} = 2-\sqrt{3}$$
$$\therefore \frac{1}{x} = 2-\sqrt{3}$$

And also, $\left(x+\frac{1}{x}\right) = 2+\sqrt{3}+2-\sqrt{3} = 2+2 = 4$

$$\therefore \boxed{\left(x+\frac{1}{x}\right) = 4} \qquad ....(1)$$

We know that

$$x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)\left(x^2 - x\times\frac{1}{x}+\frac{1}{x^2}\right)$$
$$= \left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)$$
$$= \left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+2-2-1\right)$$
$$= \left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+2.x.\frac{1}{x}-3\right)$$

$$\Rightarrow x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+2.x.\frac{1}{x}-3\right)$$

$$=\left(x+\frac{1}{x}\right)\left(\left(x+\frac{1}{x}\right)^2-3\right) \qquad \left[\because (a+b)^2=a^2+2ab+b^2\right]$$

By putting $\left(x+\frac{1}{x}\right)=4,$ we get

$$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left[\left(x+\frac{1}{x}\right)^2-3\right]$$

$$=(4)\left(4^2-3\right)$$

$$=4(16-3)$$

$$=4(13)$$

$$=52$$

$\therefore$ The value off $x^3+\frac{1}{x^3}$ is 52

8. If x = 3 + $\sqrt{8}$, *find the value of* $x^2+\frac{1}{x^2}$

Sol:

$$\Rightarrow \frac{1}{x}=\frac{1}{3+\sqrt{8}}\times\frac{3-\sqrt{8}}{3-\sqrt{8}}$$

$$=\frac{3-\sqrt{8}}{3^2-\left(\sqrt{8}\right)^2}=\frac{3-\sqrt{8}}{9-8}=\frac{3-\sqrt{8}}{1}=3-\sqrt{8}$$

$$\therefore \boxed{\frac{1}{x}=3-\sqrt{8}}$$

And also,

$$\left(x+\frac{1}{x}\right)=3+\sqrt{8}+3-\sqrt{8}=3+3=6$$

$$\therefore \boxed{\left(x+\frac{1}{x}\right)=6}$$

We know that

$$x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2$$

By putting $x+\frac{1}{x}=6$ in the above

We get,

$x^2+\frac{1}{x^2}=(6)^2-2$

$=36-2$

$=34$

$\therefore$ The value of $x^2+\frac{1}{x^2}$ is 34.

Given that $x=3+\sqrt{8}$ and given to find the value of $x^2+\frac{1}{x^2}$

We have $x=3+\sqrt{8}$

$\Rightarrow \frac{1}{x}=\frac{1}{3+\sqrt{8}}$

The rationalization factor for $\frac{1}{a+\sqrt{b}}$ is $a-\sqrt{b}$

$\Rightarrow$ For $\frac{1}{3+\sqrt{8}}$ is $3-\sqrt{8}$

9. Find the value of $\frac{6}{\sqrt{5}-\sqrt{3}}$, it being given that $\sqrt{3}=1.732$ and $\sqrt{5}=2.236$.

Sol:

Given to find the value of $\frac{6}{\sqrt{5}-\sqrt{3}}$

Rationalisation factor for $\frac{1}{\sqrt{a}-\sqrt{b}}$ is $\sqrt{a}+\sqrt{b} \Rightarrow$ for $\frac{1}{\sqrt{5}-\sqrt{3}}$ is $\sqrt{5}+\sqrt{3}$

$\Rightarrow \frac{6}{\sqrt{5}-\sqrt{3}}=\frac{6}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$=\frac{6\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$ $\quad\left[\because (a-b)(a+b)=a^2-b^2\right]$

$=\frac{6\left(\sqrt{5}+\sqrt{3}\right)}{5-3}$

$=\frac{6\left(\sqrt{5}+\sqrt{3}\right)}{2}$

$=3\left(\sqrt{5}+\sqrt{3}\right)$

We have $\sqrt{3}=1\cdot 732, \sqrt{5}=2\cdot 236$

$\Rightarrow \frac{6}{5-\sqrt{3}} = 3(2\cdot 236 + 1\cdot 732)$

$= 3(3\cdot 968)$

$= 11.904$

$\therefore$ Value of $\frac{6}{5-\sqrt{3}}$ is $11\cdot 904$

10. Find the values of each of the following correct to three places of decimals, it being given that $\sqrt{2} = 1.4142$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.2360$, $\sqrt{6} = 2.4495$ and $\sqrt{10} = 3.162$

(i) $\frac{3-\sqrt{5}}{3+2\sqrt{5}}$

Sol:

(i) We have $\frac{3-\sqrt{5}}{3+2\sqrt{5}}$

Rationalization factor for $\frac{1}{a+b\sqrt{c}}$ is $a-b\sqrt{c} \Rightarrow$ for $\frac{1}{3+2\sqrt{5}}$ it is $3-2\sqrt{5}$

$\Rightarrow \frac{3-\sqrt{5}}{3+2\sqrt{5}} = \frac{3-\sqrt{5}}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$

$= \frac{3\times 3 + 3\times(-2\sqrt{5}) + (-\sqrt{5})(3) + (-\sqrt{5})(-2\sqrt{5})}{(3)^2 - (2\sqrt{5})^2}$ $\boxed{\because (a+b)(a-b) = a^2 - b^2}$

$= \frac{9 - 6\sqrt{5} - 3\sqrt{5} + 2\times 5}{9-20}$

$= \frac{9+10-9\sqrt{5}}{-11} = \frac{19-9\sqrt{5}}{-11} = \frac{9\sqrt{5}-19}{11}$

We have $\sqrt{5} = 2.2360$

$\Rightarrow \frac{3-\sqrt{5}}{3+2\sqrt{5}} = \frac{9(2.2360)-19}{11}$

$= \frac{20\cdot 124 - 19}{11}$

$= \frac{1.124}{11}$

$= 0.102181818$

$\simeq 0.102$ (upto 3 decimals)

$\therefore$ The value of $\frac{3-\sqrt{5}}{3+2\sqrt{5}} = 0.102$

11. If $x=\frac{\sqrt{3}+1}{2}$, find the value of $4x^3+2x^2-8x+7$.

Sol:

Given $x=\frac{\sqrt{3}+1}{2}$ and given to find the value of $4x^3+2x^2-8x+7$

Now, $x=\frac{\sqrt{3}+1}{2}$

$\Rightarrow 2x=\sqrt{3}+1 \Rightarrow (2x-1)=\sqrt{3}$

Squaring on both sides we get

$(2x-1)^2=(\sqrt{3})^2$

$\Rightarrow (2x)^2-2.2x.1+(1)^2=3$ $\boxed{\because (a-b)^2=a^2-2ab+b^2}$

$\Rightarrow 4x^2-4x+1=3$

$\Rightarrow 4x^2-4x+1-3=0$

$\Rightarrow 4x^2-4x-2=0$

$\Rightarrow 2(2x^2-2x-1)=0$

$\Rightarrow \boxed{2x^2-2x-1=0}$

Now take $4x^3+2x^2-8x+7$

$\Rightarrow 2x(2x^2-2x-1)+4x^2+2x+2x^2-8x+7$

$\Rightarrow 2x(2x^2-2x-1)+6x^2-6x+7$

$\Rightarrow 2x(0)+3(2x^2-2x-1)+7+3$

$\Rightarrow 0+3(0)+10$

$\Rightarrow 10$

$\therefore$ The value of $4x^3+2x^2-8x+7$ is 10.