Number Systems
Exercise 1.1

1. Every rational number is

(A) a natural number
(B) an integer
(C) a real number
(D) a whole number

Sol. **(C)** a real number
We know that rational and irrational numbers taken together are known as real numbers. Therefore, every real number is either a rational number or an irrational number. Hence, every rational number is a real number. Therefore, (c) is the correct answer.

2. Between two rational numbers

(A) there is no rational number
(B) there is exactly one rational number
(C) there are infinitely many rational numbers
(D) there are only rational numbers and no irrational numbers

Sol. **(C)** there are infinitely many rational numbers
Between two rational numbers there are infinitely many rational number. Hence, (C) is the correct answer.

3. Decimal representation of a rational number cannot be

(A) terminating
(B) non-terminating
(C) non-terminating repeating
(D) non-terminating non-repeating

Sol. **(D)** non-terminating non-repeating
The decimal representation of a rational number cannot be non-terminating and non-repeating.

4. The product of any two irrational numbers is

(A) always an irrational number
(B) always a rational number
(C) always an integer
(D) sometimes rational, sometimes irrational

Sol. **(D)** sometimes rational, sometimes irrational
The product of any two irrational numbers is sometimes rational and sometimes irrational. Hence, (d) is the correct answer.

5. The decimal expansion of the number $\sqrt{2}$ is

(A) a finite decimal
(B) 1.41421
(C) non-terminating recurring

(D) non-terminating non-recurring

Sol. The decimal expansion of the number $\sqrt{2}$ is 1.41421……

6. Which of the following is irrational?

(A) $\sqrt{\frac{4}{9}}$

(b) $\frac{\sqrt{12}}{\sqrt{3}}$

(c) $\sqrt{7}$

(d) $\sqrt{81}$

Sol. (a) $\sqrt{\frac{4}{9}}=\frac{3}{2}$, which is a rational number.

(b) $\frac{\sqrt{12}}{\sqrt{3}}=\frac{\sqrt{4\times 3}}{\sqrt{3}}=\frac{2\sqrt{3}}{\sqrt{3}}=2,$ Which is a rational number.

(c) $\sqrt{7}$ is a rational number.

(d) $\sqrt{81}=9,$ which is a rational number.

Hence, (C) is the correct answer.

7. Which of the following is irrational?

(A) 0.14

(B) $0.14\overline{16}$

(C) $0.\overline{1416}$

(D) 0.4014001400014...

Sol. A number is irrational if and only of its decimal representation is non-terminating and non-recurring.

(a) 0.14 is a terminating decimal and therefore cannot be an irrational number.

(b) $0.14\overline{16}$ is a non-terminating and recurring decimal and therefore cannot be irrational.

(c) $0.14\overline{16}$ is a non-terminating and recurring decimal and therefore cannot be irrational.

(d) 0.4014001400014... is a non-terminating and non-recurring decimal and therefore is an irrational number.

Hence, (d) is the correct answer.

8. A rational number between $\sqrt{2}$ and $\sqrt{3}$ is

(A) $\frac{\sqrt{2}+\sqrt{3}}{2}$

(B) $\frac{\sqrt{2}\cdot\sqrt{3}}{2}$

(C) 1.5

(D) 1.8

Sol. We know that

$\sqrt{2} = 1.4142135....$ and $\sqrt{3} = 1.732050807....$

We see that 1.5 is a rational number which lies between 1.4142135..... and 1.732050807....

Hence, (c) is the correct answer.

9. The value of 1.999.... in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is

(A) $\frac{19}{10}$

(B) $\frac{1999}{1000}$

(C) 2

(D) $\frac{1}{9}$

Sol. Let $x = 1.999... = 1.\overline{9}$. ....(1)

Then, $10x = 19.999... = 19.\overline{9}$ ....(2)

Subtracting (1) and (2), we get

$$9x = 18 \Rightarrow x = 18 \div 9 = 2$$

$\therefore$ The value of 1.999... in the form $\frac{p}{q}$ is 2 or $\frac{2}{1}$.

Hence, (c) is the correct answer.

10. $2\sqrt{3} + \sqrt{3}$ is equal to

(A) $2\sqrt{6}$

(B) 6

(C) $3\sqrt{3}$

(D) $4\sqrt{6}$

Sol. Given $2\sqrt{3} + \sqrt{3} = (2+1)\sqrt{3} = 3\sqrt{3}$

Hence, (c) is the correct answer.

11. $\sqrt{10} \times \sqrt{15}$ is equal to

(A) $6\sqrt{5}$

(B) $5\sqrt{6}$

(C) $\sqrt{25}$

(D) $10\sqrt{5}$

Sol. We have $\sqrt{10} \times \sqrt{15} = \sqrt{10 \times 15} = \sqrt{5 \times 2 \times 5 \times 3} = 5\sqrt{6}$

Hence, (b) is the correct answer.

12. The number obtained on rationalizing the denominator of $\frac{1}{\sqrt{7}-2}$ is

(A) $\frac{\sqrt{7}+2}{3}$

(B) $\frac{\sqrt{7}-2}{3}$

(C) $\frac{\sqrt{7}+2}{5}$

(D) $\frac{\sqrt{7}+2}{45}$

Sol. $\frac{1}{7-\sqrt{2}}=\frac{1}{7-\sqrt{2}}\times\frac{7+\sqrt{2}}{7+\sqrt{2}}=\frac{7+\sqrt{2}}{(7)^2-(\sqrt{2})^2}=\frac{7+\sqrt{2}}{49-2}=\frac{7+\sqrt{2}}{47}$

Hence, (d) is the correct answer.

13. $\frac{1}{\sqrt{9}-\sqrt{8}}$ **is equal to**

(A) $\frac{1}{2}(3-2\sqrt{2})$

(B) $\frac{1}{3+2\sqrt{2}}$

(C) $3-2\sqrt{2}$

(D) $3+2\sqrt{2}$

Sol. $\frac{1}{\sqrt{9}-\sqrt{8}}=\frac{1}{\sqrt{3\times3}-\sqrt{4\times2}}=\frac{1}{3-2\sqrt{2}}$

$=\frac{1}{3-2\sqrt{2}}\times\frac{3+2\sqrt{2}}{3+2\sqrt{2}}=\frac{3+2\sqrt{2}}{(3)^2-(2\sqrt{2})^2}$

$=\frac{3+2\sqrt{2}}{9-8}=\frac{3+2\sqrt{2}}{1}=3+2\sqrt{2}$

Hence, (d) is the correct answer.

14. After rationalizing the denominator of $\frac{7}{3\sqrt{3}-2\sqrt{2}}$, we get the denominator as

(A) 13

(B) 19

(C) 5

(D) 35

Sol. $\frac{7}{3\sqrt{3}-2\sqrt{2}}=\frac{7}{3\sqrt{3}-2\sqrt{2}}\times\frac{3\sqrt{3}+2\sqrt{2}}{3\sqrt{3}+2\sqrt{2}}$

$$= \frac{7(3\sqrt{3}+2\sqrt{2})}{(3\sqrt{3})^2-(2\sqrt{2})^2} = \frac{7(3\sqrt{3}+2\sqrt{2})}{27-8}$$

$$= \frac{7(3\sqrt{3}+2\sqrt{2})}{19}$$

Therefore, we get the denominator as 19.
Hence, (b) is the correct answer.

15. The value of $\frac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}}$ is equal to

(A) $\sqrt{2}$
(B) 2
(C) 4
(D) 8

Sol. $\frac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}} = \frac{\sqrt{16\times2}+\sqrt{16\times3}}{\sqrt{4\times2}+\sqrt{4\times3}}$

$$= \frac{4\sqrt{2}+4\sqrt{3}}{2\sqrt{2}+2\sqrt{3}} = \frac{4(\sqrt{2}+\sqrt{3})}{(2\sqrt{2}+\sqrt{3})} = \frac{4}{2} = 2$$

Hence, (b) is the correct answer.

16. If $\sqrt{2}=1.4142,$ then $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$ is equal to

(A) 2.4142
(B) 5.8282
(C) 0.4142
(D) 0.1718

Sol. $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} = \sqrt{\frac{(\sqrt{2}-1)\times(\sqrt{2}-1)}{(\sqrt{2}+1)\times(\sqrt{2}-1)}}$

$$= \sqrt{\frac{(\sqrt{2}-1)^2}{(\sqrt{2})^2-1^2}} = \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{1} = \sqrt{2}-1$$

= 1.4142 – 1 = 0.4142
Hence, (c) is the correct answer.

17. $\sqrt[4]{\sqrt[3]{2^2}}$ equal

(A) $2^{\frac{1}{6}}$
(B) 2^{-6}
(C) $2^{1/6}$
(D) 2^{6}

Sol. $\sqrt[4]{\sqrt[3]{2^2}} = \sqrt[4]{(2^2)^{\frac{1}{3}}} = \left(2^{\frac{2}{3}}\right)^{\frac{1}{4}} = 2^{\frac{2}{3}\times\frac{1}{4}} = 2^{\frac{1}{6}}$

Hence, (c) is the correct answer.

18. The product $\sqrt[3]{2}\cdot\sqrt[4]{2}\cdot\sqrt[12]{32}$ equals

(A) $\sqrt{2}$
(B) 2
(C) $\sqrt[12]{2}$
(D) $\sqrt[12]{32}$

Sol. We have,

$$\sqrt[3]{2}.\sqrt[4]{2}.\sqrt[12]{32} = 2^{\frac{1}{2}}\times 2^{\frac{1}{4}}\times(2^5)^{\frac{1}{12}} = 2^{\frac{1}{3}}\times 2^{\frac{1}{4}}\times 2^{\frac{5}{12}}$$

$$= 2^{\frac{1}{3}+\frac{1}{4}+\frac{5}{12}} = 2^{\frac{4+3+5}{12}} = 2^{\frac{12}{12}} = 2^1 = 2$$

Hence, (b) is the correct answer.

19. Value of $\sqrt[4]{(81)^{-2}}$ is

(A) $\frac{1}{9}$
(B) $\frac{1}{3}$
(C) 9
(D) $\frac{1}{81}$

Sol. $\sqrt[4]{(81)^{-2}} = \sqrt[4]{\left(\frac{1}{81}\right)^2} = \sqrt[4]{\left\{\left(\frac{1}{9}\right)^2\right\}^2} = \sqrt[4]{\left(\frac{1}{9}\right)^4} = \left(\frac{1}{9}\right)^{4\times\frac{1}{4}} = \frac{1}{9}$

Hence, (a) is the correct answer.

20. Value of $(256)^{0.16}\times(256)^{0.09}$ is

(A) 4
(B) 16
(C) 64
(D) 256.25

Sol. $(256)^{0.16}\times(256)^{0.09} = (256)^{0.16+0.09}$

$$= (256)^{0.25} = (256)^{\frac{1}{4}} = (4^4)^{\frac{1}{4}} = 4^{4\times\frac{1}{4}} = 4$$

Hence, (a) is the correct answer.

21. **Which of the following is equal to x?**

(A) $x^{\frac{12}{7}} - x^{\frac{5}{7}}$

(B) $\sqrt[12]{(x^4)^{\frac{1}{3}}}$

(C) $(\sqrt{x^3})^{\frac{2}{3}}$

(D) $x^{\frac{12}{7}} \times x^{\frac{7}{12}}$

Sol. (a) $x^{\frac{12}{7}} \times x^{\frac{5}{7}} \neq x$

(b) $\sqrt[12]{(x^4)^{\frac{1}{3}}} = \sqrt[12]{x^{4\times\frac{1}{3}}} = \left(x^{\frac{4}{3}}\right)^{\frac{1}{12}} = x^{\frac{4}{3}\times\frac{1}{12}} = x^{\frac{1}{9}} \neq x$

(c) $((x^3)^{\frac{1}{2}})^{\frac{2}{3}} = (x)^{\frac{3}{2}\times\frac{2}{3}} = x^1 = x$

(d) $x^{\frac{12}{7}} \times x^{\frac{7}{12}} = x^{\frac{12}{7}+\frac{7}{12}} = x^{\frac{193}{84}} \neq x$

Hence, (C) is the correct answer.

Number Systems
Exercise 1.2

1. Let x and y be rational and irrational numbers, respectively. Is x + y necessarily an irrational number? Give an example in support of your answer.

Sol. Yes, x+ y is necessary an irrational number.
Let x = 5 and y = $\sqrt{2}$.
Then, $x + y = 5 + \sqrt{2} = 5 + 1.4142..... = 6.4142.....$ which is non – terminating and non-repeating.
Hence, x + y is an irrational number.

2. Let x be rational and y be irrational. Is xy necessarily irrational? Justify your answer by an example.

Sol. Let x = 0 (a rational number) and $y = \sqrt{3}$ be an irrational number. Then,
$xy = 0(\sqrt{3}) = 0,$ which is not an irrational number.
Hence, xy is not necessarily an irrational number.

3. State whether the following statements are true or false? Justify your answer.

(i) $\frac{\sqrt{2}}{3}$ is a rational number.
(ii) There are infinitely many integers between any two integers.
(iii) Number of rational numbers between 15 and 18 is finite.
(iv) There are numbers which cannot be written in the form $\frac{p}{q}$, q ≠ 0, p, q both are integers.
(v) The square of an irrational number is always rational.
(vi) $\frac{\sqrt{12}}{\sqrt{3}}$ is not a rational number as $\sqrt{12}$ and $\sqrt{3}$ are not integers.
(vii) $\frac{\sqrt{15}}{\sqrt{3}}$ is written in the form $\frac{p}{q}$, where $q \neq 0$ so it is a rational number.

Sol. (i) The given statement is false. $\frac{\sqrt{2}}{3}$ is of the form $\frac{p}{q}$ but $p = \sqrt{2}$ is not an integer.

(ii) The given statement is false. Consider two integers 3 and 4. There is no integers between 3 and 4.

(iii) The given statement is false. There lies infinitely many rational numbers between any two rational number. Hence, number of rational numbers between 15 and 18 are infinite.

(iv) The given statement is true. For example, $\frac{\sqrt{3}}{\sqrt{5}}$ is of the form $\frac{p}{q}$ but $p = \sqrt{3}$ and $q = \sqrt{5}$ are not integers.

(v) The given statement is false. Consider an irrational number $\sqrt[4]{2}$. Then, its square $(\sqrt[4]{2})^2 = \sqrt{2}$, which is not a rational number.

(vi) The given statement is false. $\sqrt{\frac{12}{3}} = \sqrt{4} = 2$, Which is a rational number.

(vii) The given statement is false. $\frac{\sqrt{15}}{\sqrt{3}} = \sqrt{\frac{15}{3}} = \sqrt{5} = \frac{\sqrt{5}}{1}$, where $p = \sqrt{5}$ is irrational number.

4. Classify the following numbers as rational or irrational with justification:

(i) $\sqrt{196}$
(ii) $3\sqrt{18}$
(iii) $\sqrt{\frac{9}{27}}$
(iv) $\frac{\sqrt{28}}{\sqrt{343}}$
(v) $-\sqrt{0.4}$
(vi) $\frac{\sqrt{12}}{\sqrt{75}}$
(vii) 0.5918
(viii) $(1+\sqrt{5})-(4+\sqrt{5})$
(ix) 10.124124...
(x) 1.010010001....

Sol. (i) $\sqrt{196} = 14$, which is a rational number.

(ii) $3\sqrt{18} = 3\sqrt{9\times 2} = 3\times 3\sqrt{2}, = 9\sqrt{2}$, which is the product of a rational and an irrational number.
Hence, $3\sqrt{18}$ is an irrational number.

(iii) $\sqrt{\frac{9}{27}} = \frac{1}{\sqrt{3}}$, which of the quotient of a rational and an irrational number and therefore an irrational number.

(iv) $\frac{\sqrt{28}}{\sqrt{343}} = \sqrt{\frac{4}{49}} = \frac{2}{7}$, which is a rational number.

(v) $-\sqrt{0.4} = -\frac{2}{\sqrt{10}}$, which is a quotient of a rational and an irrational number and so it is an irrational number.

(vi) $\frac{\sqrt{12}}{\sqrt{75}} = \sqrt{\frac{4}{25}} = \frac{2}{5}$, which is a rational number.

(vii) 0.5918 is a terminating decimal expansion. Hence, it is rational number.

(viii) $(1+\sqrt{5})-(4+\sqrt{5})=-3,$ which is a rational number.

(ix) 10.124124... is a decimal expansion which non-terminating recurring. Hence, it is a rational number.

(x) 1.010010001... is a decimal expansion which is non-terminating non-recurring. Hence, it is an irrational number.

Number Systems
Exercise 1.3

1. Find which of the variables x, y, z and u represent rational numbers and which irrational numbers:

(i) $x^2 = 5$

(ii) $y^2 = 9$

(iii) $z^2 = 0.4$

(iv) $u^2 = \frac{17}{4}$

Sol. (i) $x^2 = 5 \Rightarrow x = \sqrt{5},$ which is an irrational number.

(ii) $y^2 = 9 \Rightarrow y = \sqrt{9} = 3,$ which is a rational number.

(iii) $z^2 = .04 \Rightarrow z = \sqrt{.04} = 0.2,$ which is a terminating decimal. Hence, it is rational number.

(iv) $u^2 = \frac{17}{4} \Rightarrow u = \sqrt{\frac{17}{4}} = \frac{\sqrt{17}}{2},$ which is of the form $\frac{p}{q},$ where $p = \sqrt{17}$ is not an integer.

Hence, u is an irrational number.

2. Find three rational numbers between

(i) –1 and –2

(ii) 0.1 and 0.11

(iii) $\frac{5}{6}$ and $\frac{6}{7}$

(iv) $\frac{1}{4}$ and $\frac{1}{5}$

Sol. (i) -1.1, -1.2, -1.3 (terminating decimals) are three rational numbers lying between – 1 and – 2.

(ii) 0.101, 0.102, 0.103 (terminating decimals) are three rational numbers which lie between 0.1 and 0.11.

(iii) $\frac{5}{7} = \frac{5}{7} \times \frac{10}{10} = \frac{50}{70}$ and $\frac{6}{7} = \frac{6}{7} \times \frac{10}{10} = \frac{60}{70}$

$\Rightarrow \frac{51}{70}, \frac{52}{70}, \frac{53}{70}$ are three rational numbers lying and between $\frac{50}{70}$ and $\frac{60}{70}$ and therefore lie between $\frac{5}{7}$ and $\frac{6}{7}$.

(iv) $\frac{1}{4} = \frac{1}{4} \times \frac{20}{20} = \frac{20}{80}$ and $\frac{1}{5} = \frac{1}{5} \times \frac{16}{16} = \frac{16}{80}$

Now, $\sqrt{2} \times \sqrt{3} \frac{18}{80} \left(= \frac{9}{40} \right), \frac{19}{80}$ are three rational numbers lying between $\frac{1}{4}$ and $\frac{1}{5}$.

3. Insert a rational number and an irrational number between the following:

(i) 2 and 3
(ii) 0 and 0.1
(iii) $\frac{1}{3}$ and $\frac{1}{2}$
(iv) $\frac{-2}{5}$ and $\frac{1}{2}$
(v) 0.15 and 0.16
(vi) $\sqrt{2}$ and $\sqrt{3}$
(vii) 2.357 and 3.121
(viii) 0.0001 and 0.001
(ix) 3.623623 and 0.484848
(x) 6.375289 and 6.375738

Sol. (i) A rational number between 2 and 3 is $\frac{2+3}{2}=\frac{5}{2}=2.5$.

Also, 2.1 (terminating decimal) is a rational between 2 and 3.
Again, 2.010010001… (a non-terminating and non-recurring decimal) is an irrational number between 2 and 3.

(ii) 0.04 is a terminating decimal and also it is lies between 0 and 0.1.
Hence, 0.04 is a rational number which lies between 0 and 0.1. Again 0.003000300003… is a non-terminating and non-recurring decimal which lies between 0 and 0.1.
Hence, 0.003000300003… is an irrational number between 0 and 0.1.

(iii) $\frac{1}{3}=\frac{1}{3}\times\frac{4}{4}=\frac{4}{12}$ and $\frac{1}{2}=\frac{1}{2}\times\frac{6}{6}=\frac{6}{12}$

Now, $\frac{5}{12}$ is a rational number between $\frac{4}{12}$ and $\frac{6}{12}$. So, $\frac{5}{12}$ is a rational number lying between $\frac{1}{3}$ and $\frac{1}{2}$.

Again, $\frac{1}{3}=0.33333\ldots$ and $\frac{1}{2}=0.5$.

Now, 0.414114111… is a non-terminating and non-recurring decimal.

Hence, 0.414114111… is an irrational number lying between $\frac{1}{3}$ and $\frac{1}{2}$.

(iv) $\frac{-2}{5}=-0.4$ and $\frac{1}{2}=0.5$

Now, 0 is a rational number between -0.4 and 0.5 i.e., 0 is a rational number lying between $\frac{-2}{5}$ and $\frac{1}{2}$.

Again, 0.131131113…. is a non – terminating and non – recurring decimal which lies between – 0.4 and 0.5.

Hence, 0.131131113… is an irrational number lying between $\frac{-2}{5}$ and $\frac{1}{2}$.

(v) 0.151 is a rational number between 0.15 and 0.16. Similarly, 0.153, 0.157, etc. are rational number lying between 0.15 and 0.16.
Again, 0.151151115… (a non-terminating and non-recurring decimal) is an irrational number between 0.15 and 0.16.

(vi) $\sqrt{2} = 1.4142135....$ and $\sqrt{3} = 1.732050807...$
Now, 1.5 (a terminating decimal) which lies between 1.4142135… and 1.732050807….
Hence, 1.5 is a rational number between $\sqrt{2}$ and $\sqrt{3}$.
Again, 1.575575557… (a non – terminating and non – recurring decimal) is an irrational number lying between $\sqrt{2}$ and $\sqrt{3}$.

(vii) 3 is a rational number between 2.357 and 3.121.
Again, 3.101101110… (a non-terminating and non-recurring decimal) is an irrational number lying between 2.357 and 3.121.

(viii) 0.00011 is a rational number 0.0001 and 0.001.
Again, 0.0001131331333…. (a non-terminating and non-recurring decimal) is an irrational number between 0.0001 and 0.001.

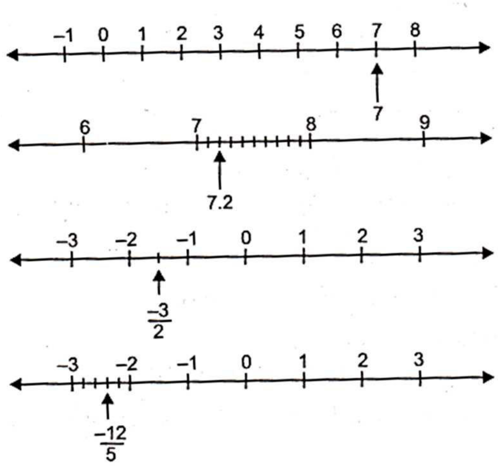
(ix) 1 is a rational number between 0.484848 and 3.623623.
Again, 1.909009000… (a non-terminating and non-recurring decimal) is an irrational number lying between 0.484848 and 3.623623.

(x) 6.3753 (a terminating decimal) is a rational number between 6.375289 and 6.375738.
Again, 6.375414114111… (a non-terminating and non-recurring decimal) is an irrational lying between 6.375289 and 6.375738.

4. Represent the following numbers on the number line:

$7, 7.2, \frac{-3}{2}, \frac{-12}{5}$

Sol.



5. Locate $\sqrt{5}, \sqrt{10}$ and $\sqrt{17}$ on the number line:

Sol. Presentation of $\sqrt{5}$ on number line:

We write 5 as the sum of the square of two natural numbers:

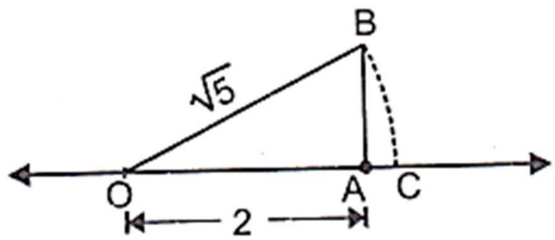
$$5 = 1 + 4 = 1^2 + 2^2$$

On the number line, take OA = 2 units.

Draw BA = 1 unit, perpendicular to OA. Join OB.

By Pythagoras theorem, $OB = \sqrt{5}$

Using a compass with centre O and radius OB, draw an arc which intersects the number line at the point C. Then, C corresponds to $\sqrt{5}$.



Presentation of $\sqrt{10}$ on the number line:

We write 10 as the sun of the square of two natural numbers:

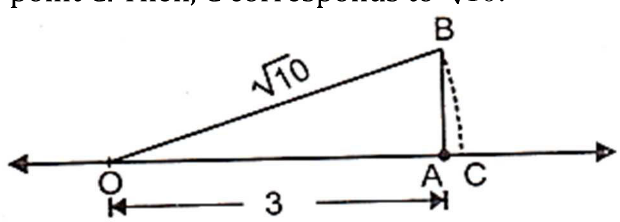
$$10 = 1 + 9 = 1^2 + 3^2$$

On the number line, taken OA = 3 units.

Draw BA = 1 unit, perpendicular to OA, Join OB.

By Pythagoras theorem, OB $= \sqrt{10}$

Using a compass with centre O and radius OB, draw an arc which intersects the number line at the point C. Then, C corresponds to $\sqrt{10}$.



Presentation of $\sqrt{17}$ on the number line:

We write 17 as the sum of the square of two natural numbers:

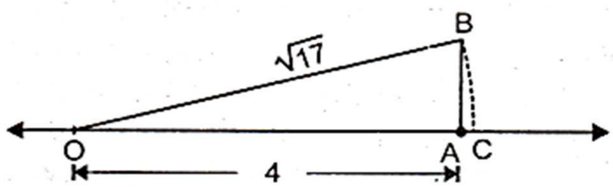
$$17 = 1 + 16 = 1^2 + 4^2$$

On the number line, take OA = 4 units.

Draw BA = 1 units, perpendicular to OA. Join OB.

By Pythagoras theorem, OB $= \sqrt{17}$

Using a compass with centre O and radius OB, draw an arc which intersects the number line at the point C. Then, C corresponds to $\sqrt{17}$.



6. **Represent geometrically the following numbers on the number line:**

(A) $\sqrt{4.5}$

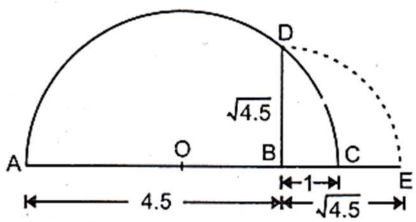
(B) $\sqrt{5.6}$

(C) $\sqrt{8.1}$

(D) $\sqrt{2.3}$

Sol. (i) $\sqrt{4.5}$

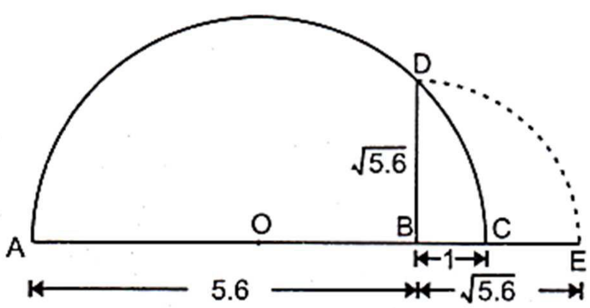
Presentation of $\sqrt{4.5}$ on number line:



Mark the distance 4.5 units from a fixed point A on a given line to obtain a point B such that AB = 4.5 units. From B, mark a distance of 1 units and mark the new points as C. Find the mid-point of AC and mark that points as O. Draw a semicircle with centre O and radius OC. Draw a line perpendicular to AC passing through B and intersecting the semicircle at D. Then, BD $=\sqrt{4.5}$.
Now, draw an arc with centre B and radius BD, which intersects the number line in E. Thus, E represent $\sqrt{4.5}$.

(ii) $\sqrt{5.6}$

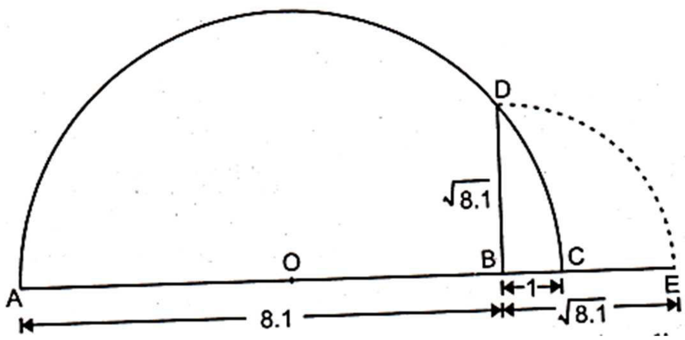
Presentation of $\sqrt{5.6}$ on number line:



Mark the distance 5.6 units from a fixed points A on a given line to obtain a point B such that AB = 5.6 units. From B, mark a distance of 1 unit and mark the new points as C. Find the mid-point of AC and mark the points as O. Draw a semicircle with centre O and radius

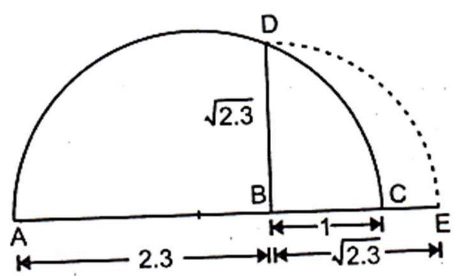
OC. Draw a line perpendicular to AC passing through B and intersecting the semicircle at D. Then BD $=\sqrt{5.6}$.
Now, draw an arc with centre B and radius BD, which intersects the number line in E.
Thus, E represent $\sqrt{5.6}$.

(iii) $\sqrt{8.1}$
Presentation of $\sqrt{8.1}$ on number line:



Mark the distance 8.1 units from a fixed point A on a given line to obtain a point B such that AB = 8.1 units. From B, mark a distance of 1 unit and mark the new points as C. Find the mid-point of AC and mark that point as O. Draw a semicircle with centre O and radius OC. Draw a line perpendicular to AC passing through B and intersecting the semicircle at D. Then, BD $=\sqrt{8.1}$.
Now, draw an arc with centre B and radius BD, which intersects the number line in E.
Thus, E represents $\sqrt{8.1}$.

(iv) $\sqrt{2.3}$
Presentation of $\sqrt{2.3}$ on number line:



Mark the distance 2.3 units from a fixed points A on a given line to obtain a point B such that AB = 2.3 units. From B mark, a distance of 1 unit and mark the new point as C. Find the mid-point of AC and mark that point as O. Draw a semicircle with centre O and radius OC. Draw a line perpendicular to AC passing through B and intersecting the semicircle at D. Then, BD $=\sqrt{2.3}$.
Now, draw an arc with centre B and radius BD, which intersects the number line in E.
Thus, E represents $\sqrt{2.3}$.

7. **Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.**

(i) 0.2
(ii) 0.888...
(iii) $5.\overline{2}$
(iv) $0.\overline{001}$
(v) 0.2555...
(vi) $0.1\overline{34}$
(vii) .00323232...
(viii) 0.404040...

Sol. (i) $0.2 = \frac{2}{10} = \frac{1}{5}.$

(ii) Let $x = 0.888\ldots = 0.\overline{8}$ ...(1)

$\therefore \quad 10x = 8.\overline{8}$ ...(2)

Subtracting (1) and (2), we get

9x = 8

Hence, $x = \frac{8}{9}$

(iii) let $x = 5.\overline{2} = 5.2222\ldots$ ...(1)

Multiplying both sides by 10, we get

$10 = 52.222\ldots = 52.\overline{2}$ ...(2)

If we subtract $5.\overline{2}$ from $52.\overline{2}$, the repeating portion the decimal cancels out.

$\therefore$ Subtracting (1) and (2), we get

$10x - x = 47 \Rightarrow 9x = 47 \Rightarrow x = \frac{47}{9}$

Hence, $5.\overline{2} = \frac{47}{9}.$

(iv) Let $x = 0.\overline{001} = 0.001001.$ ...(1)

$\therefore \quad 1000x = 1.00100\ldots$ ...(2)

Subtracting (1) from (2), we get

999x = 1

Hence, $x = \frac{1}{999}.$

(v) Let $x = 0.2555\ldots = 0.2\overline{5}.$ ...(1)

$\therefore \quad 10x = 2.\overline{5}\ldots$ ...(2)

And $\quad 100x = 25.\overline{5}$ ...(3)

Subtracting (2) from (3), we get

90x = 23

$\therefore \quad x = \frac{23}{90}$

(vi) Let $x = 0.1\overline{34} = 0.1343434\ldots$ ...(1)

Multiplying both sides by 100, we get

$$100x = 13.43434\ldots = 13.4\overline{34}$$ ...(2)

If we subtract $0.1\overline{34}$ from $13.4\overline{34}$, the represent portion of the decimal cancels out.

$\therefore$ subtracting (1) from (2), we get

$$100x - x = 13.3 \Rightarrow 99x = \frac{133}{10} \Rightarrow x = \frac{133}{990}$$

Hence, $0.1\overline{34} = \frac{133}{990}$.

(vii) Let $x = 0.00323232\ldots = 0.00\overline{32}$. ...(1)

$\therefore$ $100x = 0.\overline{32}$ ...(2)

And $10,000x = 32.\overline{32}$ ...(3)

Subtracting (2) from (3), we get

9900x = 32

$\therefore$ $x = \frac{32}{9900} = \frac{8}{2475}$

(viii) Let $x = 0.404040\ldots = 0.\overline{40}$. ...(1)

$\therefore$ $100x = 40.\overline{40}$ ...(2)

Subtracting (1) from (2), we get

$99x = 40$

$\therefore$ $x = \frac{40}{99}$

8. Show that $0.142857142857\ldots = \frac{1}{7}$.

Sol. Let $x = 0.142857142857\ldots$ ...(1)

$\therefore$ $1000000x = 142857.\overline{142857}$ ...(2)

Subtracting (1) from (2), we get

$999999x = 142857$ ...(3)

$\Rightarrow$ $x = \frac{142857}{999999} = \frac{1}{7}$

Hence, $0.142857142857\ldots = \frac{1}{7}$.

9. Simplify:

(i) $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$

(ii) $\frac{\sqrt{24}}{8} + \frac{\sqrt{54}}{9}$

(iii) $\sqrt[4]{12} \times \sqrt[6]{7}$

(iv) $4\sqrt{28} \div 3\sqrt{7}$

(v) $3\sqrt{3}+2\sqrt{27}+\dfrac{7}{\sqrt{3}}$

(vi) $(\sqrt{3}-\sqrt{2})^2$

(vii) $\sqrt[4]{81}-8\sqrt[3]{216}+15\sqrt[3]{32}+\sqrt{225}$

(viii) $\dfrac{3}{\sqrt{8}}+\dfrac{1}{\sqrt{2}}$

(ix) $\dfrac{2\sqrt{3}}{3}-\dfrac{\sqrt{3}}{6}$

Sol. (i) $\sqrt{45}-3\sqrt{20}+4\sqrt{5}=\sqrt{9\times5}-3\sqrt{4\times5}+4\sqrt{5}$

$=3\sqrt{5}-3\times2\sqrt{5}+4\sqrt{5}=(3-6+4)\sqrt{5}=\sqrt{5}$

(ii) $\dfrac{\sqrt{24}}{8}+\dfrac{\sqrt{54}}{9}=\dfrac{\sqrt{4\times5}}{8}+\dfrac{\sqrt{9\times6}}{9}=\dfrac{2\sqrt{6}}{8}+\dfrac{3\sqrt{6}}{9}=\dfrac{\sqrt{6}}{4}+\dfrac{\sqrt{6}}{3}$

$=\sqrt{6}\left(\dfrac{1}{4}+\dfrac{1}{3}\right)=\sqrt{6}\left(\dfrac{3+4}{12}\right)=\dfrac{7\sqrt{6}}{12}$

(iii) $4\sqrt{12}\times7\sqrt{6}=4\sqrt{2\times2\times3}\times7\sqrt{2\times3}$

$=8\sqrt{3}\times7\sqrt{2}\times\sqrt{3}$

$=24\times7\sqrt{2}=168\sqrt{2}$

(iv) $4\sqrt{28}\div3\sqrt{7}=4\sqrt{2\times2\times7}\times\dfrac{1}{3\sqrt{7}}$

$=\dfrac{8\sqrt{7}}{3\sqrt{7}}=\dfrac{8}{3}$

(v) $3\sqrt{3}+2\sqrt{27}+\dfrac{1}{\sqrt{3}}$

$=3\sqrt{3}+2\times3\sqrt{3}+\dfrac{7}{\sqrt{3}}=3\sqrt{3}+6\sqrt{3}+\dfrac{7}{\sqrt{3}}$

$=3\sqrt{3}+6\sqrt{3}+\dfrac{7\sqrt{3}}{3}$

$=\sqrt{3}\left(3+6+\dfrac{7}{3}\right)=\sqrt{3}\left(9+\dfrac{7}{3}\right)=\sqrt{3}\times\dfrac{34}{3}$

$=\dfrac{34}{3}\sqrt{3}$

(vi) $(\sqrt{3}-\sqrt{2})^2=(\sqrt{3})^2+(\sqrt{2})^2-2(\sqrt{3})(\sqrt{2})$

$=3+2-2\sqrt{3\times2}=5-2\sqrt{6}$

(vii) $\sqrt[4]{81}-8\sqrt[3]{216}+15\sqrt[5]{32}+\sqrt{225}$

$=\sqrt[4]{3^4}-8\sqrt[3]{6^3}+15\sqrt[5]{2^5}+\sqrt{(15)^2}$

$=3-(8\times6)+(15\times2)+15$

$$= 3 - 48 + 30 + 15 = 0$$

(viii) $\frac{3}{\sqrt{8}} + \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{4 \times 2}} + \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\frac{3}{2} + 1\right)$

$$= \frac{5}{2\sqrt{2}} = \frac{5}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{4}$$

(ix) $\frac{2\sqrt{3}}{3} - \frac{\sqrt{3}}{6} = \sqrt{3}\left(\frac{2}{3} - \frac{1}{6}\right) = \sqrt{3}\left(\frac{4-1}{6}\right) = \sqrt{3} \times \frac{3}{6} = \frac{\sqrt{3}}{2}$

10. Rationalize the denominator of the following:

(i) $\frac{2}{3\sqrt{3}}$

(ii) $\frac{\sqrt{40}}{\sqrt{3}}$

(iii) $\frac{3+\sqrt{2}}{4\sqrt{2}}$

(iv) $\frac{16}{\sqrt{41}-5}$

(v) $\frac{2+\sqrt{3}}{2-\sqrt{3}}$

(vi) $\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

(vii) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

(viii) $\frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

(ix) $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

Sol. (i) $\frac{2}{3\sqrt{3}} = \frac{2}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{9}$

(ii) $\frac{\sqrt{40}}{\sqrt{3}} = \frac{\sqrt{40}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{30}}{3}$

(iii) $\frac{3+\sqrt{2}}{4\sqrt{2}} = \frac{3+\sqrt{2}}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}(3+\sqrt{2})}{4 \times 2} = \frac{3\sqrt{2}+2}{8}$

(iv) $\frac{16}{\sqrt{41}-5} = \frac{16}{\sqrt{41}-5} \times \frac{\sqrt{41}+5}{\sqrt{41}+5}$

$$= \frac{16(\sqrt{41}+5)}{41-25} = \frac{16(\sqrt{41}+5)}{16}$$
$$= \sqrt{41}+5$$

(v) $\dfrac{2+\sqrt{3}}{2-\sqrt{3}} = \dfrac{2+\sqrt{3}}{2-\sqrt{3}} \times \dfrac{2+\sqrt{3}}{2+\sqrt{3}}$

$$= \frac{(2+\sqrt{3})^2}{(2)^2-(\sqrt{3})^2} = \frac{4+3+4\sqrt{3}}{4-3}$$
$$= \frac{7+4\sqrt{3}}{1} = 7+4\sqrt{3}$$

(vi) $\dfrac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} = \dfrac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} \times \dfrac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$

$$= \frac{\sqrt{6}(\sqrt{2}-\sqrt{3})}{(\sqrt{2})^2-(\sqrt{3})^2} = \frac{\sqrt{6}(\sqrt{2}-\sqrt{3})}{2-3}$$
$$= \sqrt{6}(\sqrt{3}-\sqrt{2}) = \sqrt{6\times 3} - \sqrt{6\times 2}$$
$$= \sqrt{18}-\sqrt{12} = \sqrt{9\times 2} - \sqrt{4\times 3} = 3\sqrt{2}-2\sqrt{3}$$

(vii) $\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$= \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{3+2+2\sqrt{3}\times\sqrt{2}}{3-2}$$
$$= \frac{5+2\sqrt{6}}{1} = 5+2\sqrt{6}$$

(viii) $\dfrac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \dfrac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$$= \frac{15+3\sqrt{15}+\sqrt{15}+3}{(\sqrt{15})^2-(\sqrt{3})^2}$$
$$= \frac{18+4\sqrt{15}}{5-3} = \frac{2(9+2\sqrt{15})}{2} = 9+2\sqrt{15}$$

(ix) $\dfrac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}} = \dfrac{4\sqrt{3}+5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}}$

$$= \frac{4\sqrt{3}+5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}} \times \frac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}}$$
$$= \frac{48-12\sqrt{6}+20\sqrt{6}-30}{(4\sqrt{3})^2-(3\sqrt{2})^2}$$
$$= \frac{18+8\sqrt{5}}{48-18} = \frac{18+8\sqrt{6}}{30} = \frac{9+4\sqrt{6}}{15}$$

11. Find the value of a and b in each of the following:

(i) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}=a-6\sqrt{3}$

(ii) $\frac{3-\sqrt{5}}{3+2\sqrt{5}}=a\sqrt{5}-\frac{19}{11}$

(iii) $\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}=2-b\sqrt{6}$

(iv) $\frac{7+\sqrt{5}}{7-\sqrt{5}}-\frac{7-\sqrt{5}}{7+\sqrt{5}}=a+\frac{7}{11}\sqrt{5}b$

Sol. (i) LHS $=\frac{5+2\sqrt{3}}{7+4\sqrt{3}}=\frac{5+2\sqrt{3}}{7+4\sqrt{3}}\times\frac{7-4\sqrt{3}}{7-4\sqrt{3}}$

$=\frac{(5+2\sqrt{3})(7-4\sqrt{3})}{(7)^2-(4\sqrt{3})^2}$

$=\frac{35-20\sqrt{3}+14\sqrt{3}-24}{49-48}$

$=\frac{11-6\sqrt{3}}{1}=11-6\sqrt{3}$

Now, $11-6\sqrt{3}=a-6\sqrt{3}$

$\Rightarrow \quad a=11$

(ii) LHS $=\frac{3-\sqrt{5}}{3+2\sqrt{5}}=\frac{3-\sqrt{5}}{3+2\sqrt{5}}\times\frac{3-2\sqrt{5}}{3-2\sqrt{5}}$

$=\frac{(3-\sqrt{5})(3-2\sqrt{5})}{(3)^2(2\sqrt{5})^2}$

$=\frac{9-6\sqrt{5}-3\sqrt{5}+10}{9-20}=\frac{19-9\sqrt{5}}{-11}$

Now, $\frac{19-9\sqrt{5}}{-11}=a\sqrt{5}-\frac{19}{11}$

$\Rightarrow \frac{-19}{11}+\frac{9}{11}\sqrt{5}=a\sqrt{5}-\frac{19}{11}$

$\Rightarrow \frac{9}{11}\sqrt{5}-\frac{19}{11}=a\sqrt{5}-\frac{19}{11}$

Hence, $a=\frac{19}{11}$.

(iii) LHS $=\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}=\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}\times\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$

$$= \frac{(\sqrt{2}+\sqrt{3})(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2})^2-(2\sqrt{3})^2}$$

$$= \frac{6+2\sqrt{6}+3\sqrt{6}+6}{18-12}$$

$$= \frac{12+5\sqrt{6}}{6} = 2+\frac{5\sqrt{6}}{6}$$

Now, $2-b\sqrt{6} = 2+\frac{5}{6}\sqrt{6} \Rightarrow b = -\frac{5}{6}$

(iv) $\frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} = a+\frac{7}{11}\sqrt{5}b$

$$\frac{7+\sqrt{5}}{7-\sqrt{5}} \times \frac{7+\sqrt{5}}{7+\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} \times \frac{7-\sqrt{5}}{7-\sqrt{5}} = a+\frac{7}{11}\sqrt{5}b$$

$$\frac{(7+\sqrt{5})^2}{(7)^2-(\sqrt{5})^2} - \frac{(7-\sqrt{5})^2}{(7)^2-(\sqrt{5})^2} = a+\frac{7}{11}\sqrt{5}b$$

$$\frac{49+5+14\sqrt{5}}{49-5} - \frac{49+5-14\sqrt{5}}{49-5} = a+\frac{7}{11}\sqrt{5}b$$

$$= \frac{54+14\sqrt{5}}{44} - \frac{54-14\sqrt{5}}{44} = a+\frac{7}{11}\sqrt{5}b$$

$$= \frac{54+14\sqrt{5}-54+14\sqrt{5}}{44} = a+\frac{7}{11}\sqrt{5}b = \frac{28\sqrt{5}}{44}$$

$$\Rightarrow \frac{7\sqrt{5}}{11} = a+\frac{7}{11}\sqrt{5}b$$

$$\Rightarrow 0+\frac{7\sqrt{5}}{11} = a+\frac{7}{11}\sqrt{5}b$$

Thus, a = 0 and b = 1.

12. If $a = 2+\sqrt{3}$, then find the value of $a-\frac{1}{a}$.

Sol. We have $a = 2+\sqrt{3}$

$\therefore \quad \frac{1}{a} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}$

$$= \frac{2-\sqrt{3}}{4-3} = \frac{2-\sqrt{3}}{1} = 2-\sqrt{3}$$

$\therefore \quad a-\frac{1}{a} = 2+\sqrt{3}-2+\sqrt{3} = 2\sqrt{3}$

13. Rationalize the denominator in each of the following and hence evaluate by taking $\sqrt{2}=1.414, \sqrt{3}=1.732$ and $\sqrt{5}=2.236,$ upto three places of decimal.

(i) $\dfrac{4}{\sqrt{3}}$

(ii) $\dfrac{6}{\sqrt{6}}$

(iii) $\dfrac{10-\sqrt{5}}{2}$

(iv) $\dfrac{\sqrt{2}}{2+\sqrt{2}}$

(v) $\dfrac{1}{\sqrt{3}+\sqrt{2}}$

Sol. (i) $\dfrac{4}{\sqrt{3}}=\dfrac{4}{\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{4\sqrt{3}}{3}=\dfrac{4\times1.732}{3}=\dfrac{6.928}{3}=2.309$

(ii) $\dfrac{6}{\sqrt{6}}=\dfrac{6}{\sqrt{6}}\times\dfrac{\sqrt{6}}{\sqrt{6}}=\dfrac{6\sqrt{6}}{6}=\sqrt{6}=\sqrt{2\times3}=\sqrt{2}\times\sqrt{3}$

$=1.414\times1.732=2.44909=2.448$ (approx.)

(iii) $\dfrac{\sqrt{10}-\sqrt{5}}{2}=\dfrac{\sqrt{2}\times\sqrt{5}-\sqrt{5}}{2}=\dfrac{\sqrt{5}(\sqrt{2}-1)}{2}=\dfrac{2.236(1.414-1)}{2}$

$=1.118\times0.414=0.463$

(iv) $\dfrac{\sqrt{2}}{2+\sqrt{2}}=\dfrac{\sqrt{2}}{2+\sqrt{2}}\times\dfrac{2-\sqrt{2}}{2-\sqrt{2}}=\dfrac{\sqrt{2}(2-\sqrt{2})}{(2)^2-(\sqrt{2})^2}=\dfrac{\sqrt{2}(2-\sqrt{2})}{4-2}$

$=\dfrac{\sqrt{2}(2-\sqrt{2})}{2}=\dfrac{2\sqrt{2}-2}{2}$

$=\sqrt{2}-1=1.414-1=0.414$

(v) $\dfrac{1}{\sqrt{3}+\sqrt{2}}=\dfrac{1}{\sqrt{3}+\sqrt{2}}\times\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\dfrac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2}=\dfrac{\sqrt{3}-\sqrt{2}}{3-2}$

$=\dfrac{\sqrt{3}-\sqrt{2}}{1}=\sqrt{3}-\sqrt{2}$

$=1.732-1.414=0.318$

Number Systems
Exercise 1.4

1. **Express** $0.6+0.\overline{7}+0.4\overline{7}$ **in the form** $\frac{p}{q}$**, where p and q are integers and** $q \neq 0.$

Sol. We have $0.6=\frac{6}{10}$ ...(1)

Let $x=0.\overline{7}=0.777...$ ...(2)

Subtracting (1) from (2), we get

$9x=7 \Rightarrow x=\frac{7}{9}$ or $0.\overline{7}=\frac{7}{9}$

Now, let $y=0.\overline{47}=0.4777...$

$\therefore \quad 10y=4.\overline{7}$ ...(3)

And $\quad 100y=47.\overline{7}$ ...(4)

Subtracting (3) from (4), we get

$90y=43 \Rightarrow y=\frac{43}{90}$

$\therefore \quad 0.\overline{47}=\frac{43}{90}$

Now, $0.6+0.\overline{7}+0.\overline{47}=\frac{6}{10}+\frac{7}{9}+\frac{43}{90}=\frac{54+70+43}{90}=\frac{167}{90}$

So, $\frac{167}{90}$ is of the from $\frac{p}{q}$ and $q \neq 0.$

2. **Simplify:** $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}.$

Sol.

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

$$=\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}\times\frac{\sqrt{10}-\sqrt{3}}{\sqrt{10}-\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}\times\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}\times\frac{\sqrt{15}-3\sqrt{2}}{\sqrt{15}-3\sqrt{2}}$$

$$=\frac{7\sqrt{3}(\sqrt{10}-\sqrt{3})}{10-3}-\frac{2\sqrt{5}(\sqrt{6}-\sqrt{5})}{6-5}=\frac{3\sqrt{2}(\sqrt{15}-3\sqrt{2})}{15-18}$$

$$=\sqrt{3}(\sqrt{10}-\sqrt{3})-2\sqrt{5}(\sqrt{6}-\sqrt{5})+\sqrt{2}(\sqrt{15}-3\sqrt{2})$$

$$=\sqrt{30}-3-2\sqrt{30}+10+\sqrt{30}-6$$

$$=2\sqrt{30}-9-2\sqrt{30}+10=1$$

3. **If** $\sqrt{2}=1.414, \sqrt{3}=1.732,$ **then find the value of**

$$\frac{4}{3\sqrt{3}-2\sqrt{2}}+\frac{3}{3\sqrt{3}+2\sqrt{2}}.$$

Sol. We have $\frac{4}{3\sqrt{3}-2\sqrt{2}}+\frac{3}{3\sqrt{3}+2\sqrt{2}}.$

$$=\frac{4\sqrt{3}+2\sqrt{2}+3(3\sqrt{3}-2\sqrt{2})}{(3\sqrt{3}-2\sqrt{2})(3\sqrt{3}+2\sqrt{2})}$$

$$=\frac{12\sqrt{3}+8\sqrt{2}+9\sqrt{3}-6\sqrt{2}}{(3\sqrt{3})^2-(2\sqrt{2})^2}=\frac{21\sqrt{3}+2\sqrt{2}}{27-8}$$

$$=\frac{21\sqrt{3}+2\sqrt{2}}{19}=\frac{21(1.732)+2(1.414)}{19}$$

$$=\frac{36.372+2.828}{19}=2.063$$

4. If $a=\frac{3+\sqrt{5}}{2}$, then find the value of $a^2+\frac{1}{a^2}$.

Sol. We have, $a=\frac{3+\sqrt{5}}{2}$

$$\Rightarrow \quad a^2=\frac{(3+\sqrt{5})^2}{4}$$

$$=\frac{9+5+6\sqrt{5}}{4}=\frac{14+6\sqrt{5}}{4}=\frac{7+3\sqrt{5}}{2}$$

Now, $\frac{1}{a^2}=\frac{2}{7+3\sqrt{5}}=\frac{2}{7+3\sqrt{5}}\times\frac{7-3\sqrt{5}}{7-3\sqrt{5}}$

$$=\frac{2(7-3\sqrt{5})}{(7)^2-(3\sqrt{5})^2}$$

$$=\frac{2(7-3\sqrt{5})}{49-45}=\frac{2(7-3\sqrt{5})}{4}=\frac{7-3\sqrt{5}}{2}$$

$$\therefore a^2+\frac{1}{a^2}=\frac{7+3\sqrt{5}}{2}+\frac{7-3\sqrt{5}}{2}$$

$$=\frac{7+3\sqrt{5}+7-3\sqrt{5}}{2}=\frac{14}{2}=7$$

5. If $x=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, and $y=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then find the value of x^2+y^2.

Sol. $x=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$= \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{3+2+2\sqrt{3\times 2}}{3-2}$$

$$\Rightarrow \qquad x = \frac{5+2\sqrt{6}}{1} = 5+2\sqrt{6}$$

Similarly, $y = 5-2\sqrt{6}$

Now, $x+y = 5+2\sqrt{6}+5-2\sqrt{6} = 10$

And, $xy = \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})} \times \frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})} = 1$

$\therefore \qquad x^2+y^2 = (10)^2-(1)^2 = 100-2 = 98$

6. Simplify: $(256)^{-\left(4^{\frac{3}{2}}\right)}$.

Sol. $(256)^{-\left(4^{-\frac{3}{2}}\right)} = \left(2^8\right)^{-\left(4^{-\frac{3}{2}}\right)} = \left(2^8\right)^{-\left(2^{2\times-\frac{3}{2}}\right)} = \left(2^8\right)^{-(2^{-3})}$

$= \left(2^8\right)^{-\left(\frac{1}{8}\right)} = 2^{8\times\left(-\frac{1}{8}\right)} = 2^{-1} = \frac{1}{2}$

7. Find the value of $\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}$

Sol. We have,

$$\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}} = 4(216)^{\frac{2}{3}} + (256)^{\frac{3}{4}} + 2(243)^{\frac{1}{5}}$$

$$= 4(6^3)^{\frac{2}{3}} + (4^4)^{\frac{3}{4}} + 2(3^5)^{\frac{1}{5}}$$

$$= 4\times 6^{3\times\frac{3}{2}} + 4^{4\times\frac{3}{4}} + 2\times 3^{5\times\frac{1}{5}}$$

$$= 4\times 6^2 + 4^3 + 2\times 3$$

$$= 144+64+6 = 214$$