Ex 26.1

Scalar Triple Product Ex 26.1 Q1(i)

We have

$$\left[\hat{i}\ \hat{j}\ \hat{k}\right]+\left[\hat{j}\ \hat{k}\ \hat{i}\right]+\left[\hat{k}\ \hat{i}\ \hat{j}\right]=\left(\hat{i}\times\hat{j}\right)\cdot\hat{k}+\left(\hat{j}\times\hat{k}\right)\cdot\hat{i}+\left(\hat{k}\times\hat{i}\right)\cdot\hat{j}$$
$$=\hat{k}\cdot\hat{k}+\hat{i}\cdot\hat{i}+\hat{j}\cdot\hat{j}$$
$$=1+1+1$$
$$=3$$

Therefore, $\left[\hat{i}\ \hat{j}\ \hat{k}\right]+\left[\hat{j}\ \hat{k}\ \hat{i}\right]+\left[\hat{k}\ \hat{i}\ \hat{j}\right]=3$

Scalar Triple Product Ex 26.1 Q1(ii)

We have

$$\left[2\hat{i}\ \hat{j}\ \hat{k}\right]+\left[\hat{i}\ \hat{k}\ \hat{j}\right]+\left[\hat{k}\ \hat{j}\ 2\hat{i}\right]=\left(2\hat{i}\times\hat{j}\right)\cdot\hat{k}+\left(\hat{i}\times\hat{k}\right)\cdot\hat{j}+\left(\hat{k}\times\hat{j}\right)\cdot 2\hat{i}$$
$$=2\hat{k}\cdot\hat{k}+\left(-\hat{j}\right)\cdot\hat{j}+\left(-\hat{i}\right)\cdot 2\hat{i}$$
$$=2-1-2$$
$$=-1$$

Therefore, $\left[2\hat{i}\ \hat{j}\ \hat{k}\right]+\left[\hat{i}\ \hat{k}\ \hat{j}\right]+\left[\hat{k}\ \hat{j}\ 2\hat{i}\right]=-1$

Scalar Triple Product Ex 26.1 Q2(i)

We have

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}2 & -3 & 0\\1 & 1 & -1\\3 & 0 & -1\end{vmatrix}$$
$$=2(-1-0)+3(-1+3)$$
$$=-2+6$$
$$=4$$

Therefore, $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=4$

Scalar Triple Product Ex 26.1 Q2(ii)

We have

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}1 & -2 & 3\\2 & 1 & -1\\0 & 1 & 1\end{vmatrix}$$
$$=1(1+1)+2(2+0)+3(2-0)$$
$$=2+4+6$$
$$=12$$

Therefore, $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=12$

Scalar Triple Product Ex 26.1 Q3(i)

We know that the volume of a parallelepiped whose three adjacent edges are $\vec{a},\vec{b},\vec{c}$ is equal to $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|$.

We have

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}2 & 3 & 4\\1 & 2 & -1\\3 & -1 & 2\end{vmatrix}$$
$$=2(4-1)-3(2+3)+4(-1-6)$$
$$=6-15-28$$
$$=-9-28$$
$$=-37$$

Therefore, the volume of the parallelepiped is $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|=|-37|=37$ cubic unit.

Scalar Triple Product Ex 26.1 Q3(ii)

Let $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k},\vec{b}=\hat{i}+2\hat{j}-\hat{k},\vec{c}=3\hat{i}-\hat{j}-2\hat{k}$

We know that the volume of a parallelepiped whose three adjacent edges are $\vec{a},\vec{b},\vec{c}$ is equal to $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|$.

We have

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right]=\begin{vmatrix}2 & -3 & 4\\1 & 2 & -1\\3 & -1 & -2\end{vmatrix}$$
$$=2(-4-1)+3(-2+3)+4(-1-6)$$
$$=-10+3-28$$
$$=-10-25$$
$$=-35$$

Therefore, the volume of the parallelepiped is $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|=|-35|=35$ cubic unit.

Scalar Triple Product Ex 26.1 Q3(iii)

Let $\vec{a}=11\hat{i}, \vec{b}=2\hat{j}, \vec{c}=13\hat{k}$

We know that the volume of a parallelepiped whose three adjacent edges are $\vec{a},\vec{b},\vec{c}$ is equal to $\left|[\vec{a}\,\vec{b}\,\vec{c}]\right|$.

We have

$$[\vec{a}\,\vec{b}\,\vec{c}]=\begin{vmatrix}11 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 13\end{vmatrix}$$
$$=11(26-0)+0+0$$
$$=286$$

Therefore, the volume of the parallelepiped is $\left|[\vec{a}\,\vec{b}\,\vec{c}]\right|=|286|=286$ cubic unit.

Scalar Triple Product Ex 26.1 Q1(iv)

Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}, \vec{c}=\hat{i}+2\hat{j}-\hat{k}$

We know that the volume of a parallelepiped whose three adjacent edges are $\vec{a},\vec{b},\vec{c}$ is equal to $\left|[\vec{a}\,\vec{b}\,\vec{c}]\right|$.

We have

$$[\vec{a}\,\vec{b}\,\vec{c}]=\begin{vmatrix}1 & 1 & 1\\ 1 & -1 & 1\\ 1 & 2 & -1\end{vmatrix}$$
$$=1(1-2)-1(-1-1)+1(2+1)$$
$$=-1+2+3$$
$$=4$$

Therefore, the volume of the parallelepiped is $\left|[\vec{a}\,\vec{b}\,\vec{c}]\right|=|4|=4$ cubic unit.

Scalar Triple Product Ex 26.1 Q4(i)

We know that three vectors $\vec{a},\vec{b},\vec{c}$ are coplanar iff their scalar triple product is zero i.e. $[\vec{a}\,\vec{b}\,\vec{c}]=0$.

Here,

$$[\vec{a}\,\vec{b}\,\vec{c}]=\begin{vmatrix}1 & 2 & -1\\ 3 & 2 & 7\\ 5 & 6 & 5\end{vmatrix}$$
$$=1(10-42)-2(15-35)-1(18-10)$$
$$=-32+40-8$$
$$=0$$

Hence, the given vectors are coplanar.

Scalar Triple Product Ex 26.1 Q4(ii)

We know that three vectors $\vec{a},\vec{b},\vec{c}$ are coplanar iff their scalar triple product is zero i.e. $[\vec{a}\,\vec{b}\,\vec{c}]=0$.

Here,

$$[\vec{a}\,\vec{b}\,\vec{c}]=\begin{vmatrix}-4 & -6 & -2\\ -1 & 4 & 3\\ -8 & -1 & 3\end{vmatrix}$$
$$=-4(12+3)+6(-3+24)-2(1+32)$$
$$=-60+126-66$$
$$=0$$

Hence, the given vectors are coplanar.

Scalar Triple Product Ex 26.1 Q5(i)

We know that vectors $\vec{a},\vec{b},\vec{c}$ are coplanar iff $[\vec{a}\,\vec{b}\,\vec{c}]=0$.

$\therefore$ $\vec{a},\vec{b},\vec{c}$ are coplanar

$\Rightarrow [\vec{a}\,\vec{b}\,\vec{c}]=0$

$$\begin{vmatrix}1 & -1 & 1\\ 2 & 1 & -1\\ \lambda & -1 & \lambda\end{vmatrix}=0$$
$$=1(\lambda-1)+1(2\lambda+\lambda)+1(-2-\lambda)$$
$$=\lambda-1+3\lambda-2-\lambda$$
$$3=3\lambda$$
$$1=\lambda$$

Scalar Triple Product Ex 26.1 Q5(ii)

We know that vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$.

$\therefore \vec{a}, \vec{b}, \vec{c}$ are coplanar

$\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ \lambda & \lambda & 5 \end{vmatrix}=0$$

$$=2(10+3\lambda)+1(5+3\lambda)+1(\lambda-2\lambda)$$
$$=20+6\lambda+5+3\lambda-\lambda$$
$$-25=8\lambda$$
$$\lambda=-\frac{25}{8}$$

Scalar Triple Product Ex 26.1 Q5(iii)

We know that vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$.

$\therefore \vec{a}, \vec{b}, \vec{c}$ are coplanar

$\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$

$$\begin{vmatrix} 1 & 2 & -3 \\ 3 & \lambda & 1 \\ 1 & 2 & 2 \end{vmatrix}=0$$

$$=1(2\lambda-2)-2(6-1)-3(6-\lambda)$$
$$=2\lambda-2-12+2-18+3\lambda$$
$$=5\lambda-30$$
$$30=5\lambda$$
$$\lambda=6$$

Scalar Triple Product Ex 26.1 Q5(iv)

We know that vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$.

$\therefore \vec{a}, \vec{b}, \vec{c}$ are coplanar

$\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$

$$\begin{vmatrix} 1 & 3 & 0 \\ 0 & 0 & 5 \\ \lambda & -1 & 0 \end{vmatrix}=0$$

$$=1(0+5)-3(0-5\lambda)+0$$
$$=5+15\lambda$$
$$-5=15\lambda$$
$$\lambda=-\frac{1}{3}$$

Scalar Triple Product Ex 26.1 Q6

Let

$OA=6\hat{i}-7\hat{j}, OB=16\hat{i}-19\hat{j}-4\hat{k}, OC=3\hat{j}-6\hat{k}, OD=2\hat{i}+5\hat{j}+10\hat{k}$

$AB=OB-OA=16\hat{i}-25\hat{j}-4\hat{k}$

$AC=OC-OA=-16\hat{i}-16\hat{j}+2\hat{k}$

$CD=OD-OC=2\hat{i}+2\hat{j}+16\hat{k}$

$AD=OD-OA=-4\hat{i}+12\hat{j}+10\hat{k}$

The four points are co-planer if vectors $\overline{AB}, \overline{AC}, \overline{AD}$ are co-planer.

$$\begin{vmatrix} 16 & -25 & -4 \\ -16 & -16 & 2 \\ -4 & 12 & 10 \end{vmatrix}=16(-160-24)+25(-160+8)-4(-144+64)$$
$$\neq 0$$

Hence the points are not coplanar.

Scalar Triple Product Ex 26.1 Q7

$AB = \text{position vector of } B - \text{position vector of } A$

$= 4\hat{i}-2\hat{j}-2\hat{k}$

$AC = \text{position vector of } C - \text{position vector of } A$

$= -2\hat{i}+4\hat{j}-2\hat{k}$

$AD = \text{position vector of } D - \text{position vector of } A$

$= -2\hat{i}-2\hat{j}+4\hat{k}$

The four points are co – planar if the vectors are co – planar.

$$\text{Thus, } \begin{vmatrix} 4 & -2 & -2 \\ -2 & 4 & -2 \\ -2 & -2 & 4 \end{vmatrix}=4[16-4]+2[-8-4]-2[4+8]=48-24-24=0$$

Hence proved.

Scalar Triple Product Ex 26.1 Q8

$Let\ OA = 6\hat{i} - 7\hat{j},\ OB = 16\hat{i} - 19\hat{j} - 4\hat{k},\ OC = 3\hat{i} - 6\hat{k},\ OD = 2\hat{i} - 5\hat{j} + 10\hat{k}$

$Thus,$

$AB = OB - OA = 10\hat{i} - 12\hat{j} - 4\hat{k}$

$AC = OC - OA = -3\hat{i} + 7\hat{j} - 6\hat{k}$

$AD = OD - OA = -4\hat{i} + 2\hat{j} + 10\hat{k}$

The four points are co – planar if vectors AB, AC and AD are co – planar.

Thus, we have

$$\begin{vmatrix} 10 & -12 & -4 \\ -3 & 7 & -6 \\ -4 & 2 & 10 \end{vmatrix} = 10(70+12) + 12(-30-24) - 4(-6+28) = 820 - 648 - 88$$

Scalar Triple Product Ex 26.1 Q9

Let

Position vector of $A = -\hat{j} - \hat{k}$

Position vector of $B = 4\hat{i} + 5\hat{j} + \lambda\hat{k}$

Position vector of $C = 3\hat{i} + 9\hat{j} + 4\hat{k}$

Position vector of $D = -4\hat{i} + 4\hat{j} + 4\hat{k}$

The four points are coplanar if the vectors $\overline{AB}, \overline{AC}, \overline{AD}$ are coplanar.

$\overline{AB} = 4\hat{i} + 6\hat{j} + (\lambda + 1)\hat{k}$

$\overline{AC} = 3\hat{i} + 10\hat{j} + 5\hat{k}$

$\overline{AD} = -4\hat{i} + 5\hat{j} + 5\hat{k}$

$$\begin{vmatrix} 4 & 6 & (\lambda+1) \\ 3 & 10 & 5 \\ -4 & 5 & 5 \end{vmatrix} = 0$$

$$4(50-25) - 6(15+20) + (\lambda+1)(15+40) = 0$$

$$100 - 210 + 55 + 55\lambda = 0$$

$$55\lambda = 55$$

$$\lambda = 1$$

Scalar Triple Product Ex 26.1 Q10

$$(\vec{a}-\vec{b}).\{(\vec{b}-\vec{c})\times(\vec{c}-\vec{a})\}$$

$$= [(\vec{a}-\vec{b}) \quad (\vec{b}-\vec{c}) \quad (\vec{c}-\vec{a})]$$

$$= [a \quad (\vec{b}-\vec{c}) \quad (\vec{c}-\vec{a})] + [-b \quad (\vec{b}-\vec{c}) \quad (\vec{c}-\vec{a})]$$

$$= 6[a \quad b \quad c] - 6[a \quad b \quad c]$$

$$= 0$$

Scalar Triple Product Ex 26.1 Q11

If $\bar{a}$ represents the sides AB , If $\bar{b}$ represents the sides BC, If $\bar{c}$ represents the sides AC of the triangle ABC.

$\bar{a} \times \bar{b}$ is perpendicular to the plane of the triangle ABC.

$\bar{b} \times \bar{c}$ is perpendicular to the plane of the triangle ABC.

$\bar{c} \times \bar{a}$ is perpendicular to the plane of the triangle ABC.

Hence $\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}$ is a vector perpendicular to the plane of the triangle ABC.

Scalar Triple Product Ex 26.1 Q12(i)

$\bar{a}, \bar{b}, \bar{c}$ are coplanar if

$$[a \quad b \quad c] = 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 2 & c_3 \end{vmatrix} = 0$$

$$0 - 1(c_3) + 1(2) = 0$$

$$c_3 = 2$$

Scalar Triple Product Ex 26.1 Q12(ii)

$\bar{a}, \bar{b}, \bar{c}$ are coplanar if

$$[a \quad b \quad c] = 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ -1 & c_2 & 1 \end{vmatrix} = 0$$

$$0 - 1 + 1(c_2) = 0$$

$$c_2 = 1$$

Scalar Triple Product Ex 26.1 Q13

Let

Position vector of $OA = 3\hat{i} + 2\hat{j} + \hat{k}$

Position vector of $OB = 4\hat{i} + \lambda\hat{j} + 5\hat{k}$

Position vector of $OC = 4\hat{i} + 2\hat{j} - 2\hat{k}$

Position vector of $OD = 6\hat{i} + 5\hat{j} - \hat{k}$

The four points are coplanar if the vectors $\overline{AB}, \overline{AC}, \overline{AD}$ are coplanar.

$$\overline{AB} = \hat{i} + (\lambda - 2)\hat{j} + 4\hat{k}$$
$$\overline{AC} = \hat{i} + 0\hat{j} - 3\hat{k}$$
$$\overline{AD} = 3\hat{i} + 3\hat{j} - 2\hat{k}$$

$$\begin{vmatrix} 1 & (\lambda - 2) & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$$

$$1(9) - (\lambda - 2)(-2 + 9) + 4(3 - 0) = 0$$
$$9 - 7\lambda + 14 + 12 = 0$$
$$7\lambda = 35$$
$$\lambda = 5$$