Ex 29.1

The Plane 29.1 Q1(i)

Given three points are,
$(2,1,0),(3,-2,-2)$ and $(3,1,7)$

We know that, equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-1 & z-0 \\ 3-2 & -2-1 & -2-0 \\ 3-2 & 1-1 & 7-0 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-1 & z \\ 1 & -3 & -2 \\ 1 & 0 & 7 \end{vmatrix} = (x-2)(-21-0)-(y-1)(7+2)+z(0+3)=0$$

$$= -21x+42-9y+9+3z=0$$
$$= -21x-9y+3z+51=0$$

Dividing by -3, we get

Equation of plane, $7x+3y-z-17=0$

The Plane 29.1 Q1(ii)

Given points are,
$(-5,0,-6),(-3,10,-9)$ and $(-2,6,-6)$

We know that, equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x+5 & y-0 & z+6 \\ -3+5 & 10-0 & -9+6 \\ -2+5 & 6-0 & -6+6 \end{vmatrix} = 0$$

$$\begin{vmatrix} x+5 & y & z+6 \\ 2 & 10 & -3 \\ 3 & 6 & 0 \end{vmatrix} = 0$$

$(x+5)(0+18)-y(0+9)+(z+6)(12-30)=0$
$(x+5)(18)-y(9)+(z+6)(-18)=0$
$18x+90-9y-18z-108=0$

Dividing by 9, we get

Equation of plane, $2x-y-2z-2=0$

The Plane 29.1 Q1(iii)

Given three points are,

$(1,1,1),(1,-1,2)$ and $(-2,-2,2)$

We know that, equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 1-1 & -1-1 & 2-1 \\ -2-1 & -2-1 & 2-1 \end{vmatrix}=0$$

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 0 & -2 & 1 \\ -3 & -3 & 1 \end{vmatrix}=0$$

$(x-1)(-2+3)-(y-1)(0+3)+(z-1)(0-6)=0$

$(x-1)(1)-(y-1)(3)+(z-1)(-6)=0$

$x-1-3y+3-6z+6=0$

$x-3y-6z+8=0$

Equation of plane is, $x-3y-6z+8=0$

The Plane 29.1 Q1(iv)

Given points are,

$(2,3,4),(-3,5,1)$ and $(4,-1,2)$

We know that, equation of plane passing through three points are given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$

$$\begin{vmatrix} x-2 & y-3 & z-4 \\ -3-2 & 5-3 & 1-4 \\ 4-2 & -1-3 & 2-4 \end{vmatrix}=0$$

$$\begin{vmatrix} x-2 & y-3 & z-4 \\ -5 & 2 & -3 \\ 2 & -4 & -2 \end{vmatrix}=0$$

$(x-2)(-4-12)-(y-3)(10+6)+(z-4)(20-4)=0$

$(x-2)(-16)-(y-3)(16)+(z-4)(16)=0$

$-16x+32-16y+48+16z-64=0$

$-16x-16y+16z+16=0$

Dividing by (-16), we get,

Equation of plane, $x+y-z-1=0$

The Plane 29.1 Q1(v)

Given points are,
$(0,-1,0), (3,3,0)$ and $(1,1,1)$

We know that, equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-0 & y+1 & z-0 \\ 3-0 & 3+1 & 0-0 \\ 1-0 & 1+1 & 1-0 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y+1 & z \\ 3 & 4 & 0 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$x(4-0) - (y+1)(3-0) + z(6-4) = 0$$
$$4x - (y+1)(3) + z(2) = 0$$
$$4x - 3y - 3 + 2z = 0$$

$$4x - 3y + 2z - 3 = 0$$

Equation of plane is, $4x - 3y + 2z - 3 = 0$

The Plane 29.1 Q2

We have to prove that points
$(0,-1,-1), (4,5,1), (3,9,4)$ and $(-4,4,4)$ are coplanar.

First we shall find the equation of plane passing through three points:
$(0,-1,1), (4,5,1)$ and $(3,9,4)$

We know that equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-0 & y+1 & z+1 \\ 4-0 & 5+1 & 1+1 \\ 3-0 & 9+1 & 4+1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y+1 & z+1 \\ 4 & 6 & 2 \\ 3 & 10 & 5 \end{vmatrix} = 0$$

$$x(30-20) - (y+1)(20-6) + (z+1)(40-18) = 0$$
$$10x - (y+1)(14) + (z+1)(22) = 0$$
$$10x - 14y - 14 + 22z + 22 = 0$$

$$10x - 14y + 22z + 8 = 0$$

Dividing by 2, we get

$$5x - 7y + 11z + 4 = 0 \qquad ---(i)$$

Now, for the fourth point $(-4, 4, 4)$ put $x = -4$, $y = 4$, $z = 4$ in equation (i),

$$5(-4) - 7(4) + 11(4) + 4 = 0$$
$$-20 - 28 + 44 + 4 = 0$$
$$-48 + 48 = 0$$

$$0 = 0$$
$$LHS = RHS$$

Since, fourth point satisfies the equation of plane passing through three points
So, all four points are collinear

Equation of common plane is, $5x - 7y + 11z + 4 = 0$

The Plane 29.1 Q3(i)

Given, four points are
$(0, -1, 0), (2, 1, -1), (1, 1, 1)$ and $(3, 3, 0)$.

Now, first we find the equation of plane passing through three points:
$(0, -1, 0), (2, 1, -1), (1, 1, 1)$

We know that equation of plane passing through three points is given by

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x - 0 & y + 1 & z - 0 \\ 2 - 0 & 1 + 1 & -1 - 0 \\ 1 - 0 & 1 + 1 & 1 - 0 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y + 1 & z \\ 2 & 2 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$x(2 + 2) - (y + 1)(2 + 1) + z(4 - 2) = 0$$
$$x(4) - (y + 1)(3) + z(2) = 0$$
$$4x - 3y - 3 + 2z = 0$$

$$4x - 3y + 2z - 3 = 0 \qquad ---(i)$$

Put, $x = 3$, $y = 3$, $z = 0$ in equation (i), we get

$$4x - 3y + 2z - 3 = 0$$
$$4(3) - 3(3) + 2(0) - 3 = 0$$
$$12 - 9 + 0 - 3 = 0$$
$$12 - 12 = 0$$

$$0 = 0$$
$$LHS = RHS$$

Since, fourth point satisfies the equation of plane passing through three points,
Hence, four points are coplanar

The Plane 29.1 Q3(ii)

Given, four points are

$(0,4,3),(-1,-5,-3),(-2,-2,1)$ and $(1,1,-1)$

First we shall find the equation of plane passing through three points:

$(0,4,3),(-1,-5,-3),(-2,-2,1)$

We know that, equation of plane passing through three given points is,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-0 & y-4 & z-3 \\ -1-0 & -5-4 & -3-3 \\ -2-0 & -2-4 & 1-3 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y-4 & z-3 \\ -1 & -9 & -6 \\ -2 & -6 & -2 \end{vmatrix} = 0$$

$x(18-36)-(y-4)(2-12)+(z-3)(6-18)=0$

$x(-18)-(y-4)(-10)+(z-3)(-12)=0$

$-18x+10y-40-12z+36=0$

$-18x+10y-12z-4=0 \qquad ---(i)$

Put, $x=1$, $y=1$, $z=-1$ in equation (i),

$-18(1)+10(1)-12(-1)-4=0$

$-18+10+12-4=0$

$-22+22=0$

$0=0$

$LHS=RHS$

So, fourth point $(1,1,-1)$ satisfies the equation of plane passing through three points,

Hence, four points are coplanar

Ex 29.2

The Plane 29.2 Q1

Given, intercepts on the coordinate axes are $2, -3$ and 4

We know that,
The equation of a plane whose intercepts on the coordinate axes are a, b and c respectively, is given by

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad ---(i)$$

Here, $a=2$, $b=-3$, $c=4$

So,
Equation of required plane is

$$\frac{x}{2}+\frac{y}{-3}+\frac{z}{4}=1$$
$$\frac{6x-4y+3z}{12}=1$$

$$6x-4y+3z=12$$

The Plane 29.2 Q2(i)

Reduce the equation $4x+3y-6z-12=0$ in intercept form:

$$4x+3y-6z-12=0$$
$$4x+3y-6z=12$$

Divide by 12,

$$\frac{4x}{12}+\frac{3y}{12}-\frac{6z}{12}=\frac{12}{12}$$
$$\frac{x}{3}+\frac{y}{4}-\frac{z}{2}=1$$

$$\frac{x}{3}+\frac{y}{4}+\frac{z}{(-2)}=1 \qquad ---(i)$$

This is of the form,

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad ---(ii)$$

Comparing equation (i) and (ii),

$a=3$, $b=4$, $c=-2$

Intercepts on the coordinate axes are $3, 4, -2$

The Plane 29.2 Q2(ii)

Reduce $2x+3y-z=6$ in the intercept form:

$2x+3y-z=6$

Divide by 6,

$$\frac{2x}{6}+\frac{3y}{6}-\frac{z}{6}=\frac{6}{6}$$
$$\frac{x}{3}+\frac{y}{2}-\frac{z}{6}=1$$

$$\frac{x}{3}+\frac{y}{2}+\frac{z}{(-6)}=1 \qquad ---(\text{i})$$

We know intercept form of plane with a, b, c as intercepts on coordinate axes is,

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad ---(\text{ii})$$

Comparing equation (i) and (ii),

$a=3,\ b=2,\ c=-6$

So, intercepts on coordinate axes by the given plane are $3, 2, -6$

The Plane 29.2 Q2(iii)

We have to find intercepts on coordinate axes by plane $2x-y+z=5$

$2x-y+z=5$

Divide by 5,

$$\frac{2x}{5}-\frac{y}{5}+\frac{z}{5}=\frac{5}{5}$$
$$\frac{x}{\left(\frac{5}{2}\right)}+\frac{y}{(-5)}+\frac{z}{5}=1 \qquad ---(\text{i})$$

We know that if a, b, c are intercepts on coordinate axes by the plane, then equation of such plane is given by,

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad ---(\text{ii})$$

Comparing the equation (i) and (ii),

$a=\frac{5}{2},\ b=-5,\ c=5$

So, intercepts on coordinate axes by the plane are $\frac{5}{2}, -5, 5$.

The Plane 29.2 Q3

Here, it is given that the plane meets axes in A, B and C

Let, $A = (a,0,0)$, $B = (0,b,0)$, $C = (0,0,c)$

We have centroid of $\square ABC$ is (α, β, γ) we know that, centroid of $\square ABC$ is given by

$$\text{Centroid} = \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}$$

$$(\alpha, \beta, \gamma) = \left(\frac{a+0+0}{3}, \frac{0+b+0}{3}, \frac{0+0+c}{3}\right)$$

$$(\alpha, \beta, \gamma) = \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$$

So,

$$\frac{a}{3} = \alpha \quad \Rightarrow \quad a = 3\alpha \qquad ---(i)$$

$$\frac{b}{3} = \beta \quad \Rightarrow \quad b = 3\beta \qquad ---(ii)$$

$$\frac{c}{3} = \gamma \quad \Rightarrow \quad c = 3\gamma \qquad ---(iii)$$

We know that, if a, b, c are intercepts by plane on coordinate axes, then equation of the plane is given by

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Put a, b, c from equation (i), (ii) and (iii),

$$\frac{x}{3\alpha} + \frac{y}{3\beta} + \frac{z}{3\gamma} = 1$$

Multiplying by 3 on both the sides,

$$\frac{3x}{3\alpha} + \frac{3y}{3\beta} + \frac{3z}{3\gamma} = 3$$

$$\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$$

The Plane 29.2 Q4

Intercepts on the coordinate axes are equal.

We know that, if a, b, c are intercepts on coordinate axes by a plane, then equation of the plane is given by,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Here, it is givin that $a = b = c = p$ (Say)

$$\frac{x}{p} + \frac{y}{p} + \frac{z}{p} = 1$$

$$\frac{x+y+z}{p} = 1$$

$$x + y + z = p \qquad ---(i)$$

It is given that plane is passing through the point $(2,4,6)$, so, using equation (i)

$$x + y + z = p$$

$$2 + 4 + 6 = p$$

$$12 = p$$

Put, value of p in equation (i)

$$x + y + z = 12$$

So, the required equation of the plane is given by,

$$x + y + z = 12$$

The Plane 29.2 Q5

Here, it is given that plane meets the coordinate axes at A, B and C with centroid of $\triangle ABC$ is $(1, -2, 3)$

The equation of plane with intercepts a, b and c on the coordinate axes is given by,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad ---(i)$$

We know that, centroid of a triangle is given by

$$\text{Centroid} = \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}$$

$$(1, -2, 3) = \left(\frac{a + 0 + 0}{3}, \frac{0 + b + 0}{3}, \frac{0 + 0 + c}{3}\right)$$

$$(1, -2, 3) = \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$$

Comparing LHS and RHS,

$$\frac{a}{3} = 1 \Rightarrow a = 3 \qquad ---(i)$$

$$\frac{b}{3} = -2 \Rightarrow b = -6 \qquad ---(ii)$$

$$\frac{c}{3} = 3 \Rightarrow c = 9 \qquad ---(iii)$$

Put, a, b, c is equation (i), we get the equation of required plane

$$\frac{x}{3} + \frac{y}{-6} + \frac{z}{9} = 1$$

$$\frac{6x - 3x + 2z}{18} = 1$$

$$6x - 3x + 2z = 18$$

Ex - 29.3

The Plane 29.3 Q1

We know that, vector equation of a plane passing through a point $\vec{a}$ and normal to $\vec{n}$ is given by,

$$(\vec{r} - \vec{a}).\vec{n} = 0 \qquad ---(i)$$

Here,
$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$
$\vec{n} = 4\hat{i} + 2\hat{j} - 3\hat{k}$

Put, $\vec{a}$ and $\vec{n}$ in equation (i)

$$\left[\vec{r} - \left(2\hat{i} - \hat{j} + \hat{k}\right)\right].\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) = 0$$
$$\vec{r}.\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) - \left(2\hat{i} - \hat{j} + \hat{k}\right).\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) = 0$$
$$\vec{r}.\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) - \left[(2)(4) + (-1)(2) + (1)(-3)\right] = 0$$
$$\vec{r}\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) - [8 - 2 - 3] = 0$$

$$\vec{r}\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) - 3 = 0$$

So, equation of required plane is given by,

$$\vec{r}.\left(4\hat{i} + 2\hat{j} - 3\hat{k}\right) = 3$$

The Plane 29.3 Q2(i)

Given the vector equation of a plane,

$$\vec{r}.\left(12\hat{i} - 3\hat{j} + 4\hat{k}\right) + 5 = 0$$

let, $\vec{r} = \left(x\hat{i} + y\hat{j} + z\hat{k}\right)$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(12\hat{i} - 3\hat{j} + 4\hat{k}\right) + 5 = 0$$
$$(x)(12) + (y)(-3) + (z)(4) + 5 = 0$$

$$12x - 3y + 4z + 5 = 0$$

Cartesian form of the equation of the plane is given by

$$12x - 3y + 4z + 5 = 0$$

The Plane 29.3 Q2(ii)

Here, equation of the plane is,

$$\vec{r}.\left(-\hat{i} + \hat{j} + 2\hat{k}\right) = 9$$

let, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(-\hat{i} + \hat{j} + 2\hat{k}\right) = 9$$
$$(x)(-1) + (y)(1) + (z)(2) = 9$$

$$-x + y + 2z = 9$$

Cartesian form of the equation of plane is,

$$-x + y + 2z = 9$$

The Plane 29.3 Q3

We have to find vector equation of coordinate planes.
For xy-plane.

It passes through origin and is perpendicular to z-axis, so
Put $\vec{a} = 0.\hat{i} + 0.\hat{j} + 0\hat{k}$ and $\vec{n} = \hat{k}$ in the vector equation of plane passing through point $\vec{a}$ and perpendicular to vector $\vec{n}$

$$\left(\vec{r} - \vec{a}\right).\vec{n} = 0$$
$$\left(\vec{r} - 0.\hat{i} - 0.\hat{j} - 0\hat{k}\right).\hat{k} = 0$$

$$\vec{r}.\hat{k} = 0 \qquad ---(i)$$

For xz-plane,
It passes through origin and perpendicular to y-axis, so

$\vec{a} = 0.\hat{i} + 0.\hat{j} + 0\hat{k}$ and $\vec{n} = \hat{j}$

Equation of xz-plane is given by
$$\left(\vec{r} - \vec{a}\right).\vec{n} = 0$$
$$\left(\vec{r} - 0.\hat{i} - 0.\hat{j} - 0\hat{k}\right).\hat{j} = 0$$

$$\vec{r}.\hat{j} = 0$$

For yz-plane.

It passes through origin and is perpendicular to x-axis, so
$\vec{a} = 0.\hat{i} + 0.\hat{j} + 0\hat{k}$, $\vec{n} = \hat{i}$

$$\left(\vec{r} - \vec{a}\right).\vec{n} = 0$$
$$\left(\vec{r} - 0.\hat{i} - 0.\hat{j} - 0\hat{k}\right).\hat{i} = 0$$

$$\vec{r}.\hat{i} = 0$$

Hence, equation of xy, yz, zx-plane are given by

$$\vec{r}.\hat{k} = 0$$
$$\vec{r}.\hat{i} = 0$$
$$\vec{r}.\hat{j} = 0$$

The Plane 29.3 Q4(i)

Given, equation of plane is,

$$2x - y + 2z = 8$$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(2\hat{i} - \hat{j} + 2\hat{k}\right) = 8$$
$$\vec{r}.\left(2\hat{i} - \hat{j} + 2\hat{k}\right) = 8$$

So,
Vector equation of the plane is $\vec{r}.\left(2\hat{i} - \hat{j} + 2\hat{k}\right) = 8$

The Plane 29.3 Q4(ii)

Given, cartesian equation of the plane is,

$x + y - z = 5$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} + \hat{j} - \hat{k}\right) = 5$

$\vec{r}\left(\hat{i} + \hat{j} - \hat{k}\right) = 5$

So,

Vector equation of the plane is $\vec{r}\left(\hat{i} + \hat{j} - \hat{k}\right) = 5$

The Plane 29.3 Q4(iii)

Given, cartesian equation of plane is,

$x + y = 3$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} + \hat{j}\right) = 3$

$\vec{r}.\left(\hat{i} + \hat{j}\right) = 3$

So,

Vector equation of the plane is $\vec{r}.\left(\hat{i} + \hat{j}\right) = 3$

The Plane 29.3 Q5

We know that, vector equation of a plane passing through point $\vec{a}$ and perpendicular to the vector $\vec{n}$ is given by,

$\left(\vec{r} - \vec{a}\right).\vec{n} = 0 \qquad ---(i)$

The given plane is passing through the point $(1,-1,1)$ and normal to the line joining $A(1,2,5)$ and $B(-1,3,1)$. So,

$\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{n} = \overrightarrow{AB}$

= Position vector of $\vec{B}$ – Position vector of A

$= \left(-\hat{i} + 3\hat{j} + \hat{k}\right) - \left(\hat{i} + 2\hat{j} + 5\hat{k}\right)$

$= -\hat{i} + 3\hat{j} + \hat{k} - \hat{i} - 2\hat{j} - 5\hat{k}$

$= -2\hat{i} + \hat{j} - 4\hat{k}$

Put, $\vec{n}$ and $\vec{a}$ in equation (i),

$\left[\vec{r} - \left(\hat{i} - \hat{j} + \hat{k}\right)\right].\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) = 0$

$\vec{r}.\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) - \left(\hat{i} - \hat{j} + \hat{k}\right)\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) = 0$

$\vec{r}.\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) - \left[(1)(-2) + (-1)(1) + (1)(-4)\right] = 0$

$\vec{r}.\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) - \left[-2 - 1 - 4\right] = 0$

$\vec{r}\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) - \left[-7\right] = 0$

$\vec{r}.\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) + 7 = 0$

$\vec{r}.\left(-2\hat{i} + \hat{j} - 4\hat{k}\right) = -7$

Multiplying by (-1) on both the sides

$$\vec{r}\left(2\hat{i}-\hat{j}+4\hat{k}\right)=7$$

Put, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(2\hat{i}-\hat{j}+4\hat{k}\right)=7$$
$$(x)(2)+(y)(-1)+(2)(4)=7$$
$$2x-y+4z=7$$

So, vector and cartesian equation the plane is

$$\vec{r}.\left(2\hat{i}-\hat{j}+4\hat{k}\right)=7,\ 2x-y+4z=7$$

The Plane 29.3 Q6

Here, it is given that $\bar{n}=\sqrt{3}$ and $\bar{n}$ makes equal angle with coordinate axes.

Let, $\bar{n}$ has direction cosine as l, m and n and it makes angle of α,β and γ with the coordinate axes. So

Here, $\alpha=\beta=\gamma$

$\Rightarrow \qquad \cos\alpha=\cos\beta=\cos\gamma$

$\Rightarrow \qquad l=m=n=p(\text{Say})$

We know that,

$$l^2+m^2+n^2=1$$
$$p^2+p^2+p^2=1$$
$$3p^2=1$$
$$p^2=\frac{1}{3}$$

$$p=\pm\frac{1}{\sqrt{3}}$$

So,

$$l=\pm\frac{1}{\sqrt{3}}$$
$$\cos\alpha=\pm\frac{1}{\sqrt{3}}$$

Now, $\alpha=\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

It gives, α is an obtuse angle so, neglect it.

Again, $\alpha=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

It gives, α is an acute angle, so

$$\cos\alpha=\frac{1}{\sqrt{3}}$$

$\therefore \qquad l=m=n=\frac{1}{\sqrt{3}}$

So,

$\vec{n} = |\vec{n}|\left(l\hat{i} + m\hat{j} + n\hat{k}\right)$

$\quad = \sqrt{3}\left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)$

$\vec{n} = \hat{i} + \hat{j} + \hat{k}$

And, $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$

We know that, vector equation of a plane passing through the point $\vec{a}$ and perpendicular to the vector $\vec{n}$ is given by,

$\left(\vec{r} - \vec{a}\right).\vec{n} = 0$

$\left[\vec{r} - \left(2\hat{i} + \hat{j} - \hat{k}\right)\right].\left(\hat{i} + \hat{j} + \hat{k}\right) = 0$

$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) - \left(2\hat{i} + \hat{j} - \hat{k}\right)\left(\hat{i} + \hat{j} + \hat{k}\right) = 0$

$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) - \left[(2)(1) + (1)(1) + (-1)(1)\right] = 0$

$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) - \left[2 + 1 - 1\right] = 0$

$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) - 2 = 0$

$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 2$

Put, $\vec{r} = \left(x\hat{i} + y\hat{j} + z\hat{k}\right)$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} + \hat{j} + \hat{k}\right) = 2$

$(x)(1) + (y)(1) + (z)(1) = 2$

$x + y + z = 2$

So, vector and cartesian equation of the plane is,

$\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 2,\ x + y + z = 2$

The Plane 29.3 Q7

Here, it is given that foot of the perpendicular drawn from origin O to the plane is $P(12,-4,3)$

It means, the required plane is passing through $P(12,-4,3)$ and perpendicular to OP.

We know that, equation of a plane passing through $\vec{a}$ and perpendicular to $\vec{n}$ is given by,

$$(\vec{r}-\vec{a}).\vec{n}=0 \qquad ---(i)$$

Here, $\vec{a}=12\hat{i}-4\hat{j}+3\hat{k}$

And, $\vec{n}=\overrightarrow{OP}$

$=$ Position vector of P $-$ Position vector of O

$$=\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-\left(0\hat{i}+0\hat{j}+0\hat{k}\right)$$

$$\vec{n}=12\hat{i}-4\hat{j}+3\hat{k}$$

Put, value of $\vec{a}$ and $\vec{n}$ in equation (i),

$$\left[\vec{r}-\left(12\hat{i}-4\hat{j}+3\hat{k}\right)\right].\left(12\hat{i}-4\hat{j}+3\hat{k}\right)=0$$
$$\vec{r}.\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-\left(12\hat{i}-4\hat{j}+3\hat{k}\right)\left(12\hat{i}-4\hat{j}+3\hat{k}\right)=0$$
$$\vec{r}.\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-\left[(12)(12)+(-4)(-4)+(3)(3)\right]=0$$
$$\vec{r}.\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-\left[144+16+9\right]=0$$
$$\vec{r}.\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-169=0$$

Put, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(12\hat{i}-4\hat{j}+3\hat{k}\right)-169=0$$
$$(x)(12)+(y)(-4)+(z)(3)=169$$

$$12x-4y+3z=169$$

So, the vector and cartesian equation of the required plane is,

$$\vec{r}.\left(12\hat{i}-4\hat{j}+3\hat{k}\right)=169,\ 12x-4y+3z=169$$

The Plane 29.3 Q8

Given that, the plane is passing through $P(2,3,1)$ having 5,3,2 as the direction ratios of the normal to the plane.

We know that,
Equation of a plane passing through a point $\vec{a}$ and $\vec{n}$ is a vector normal to the plane, is given by,

$$(\vec{r}-\vec{a}).\vec{n}=0 \qquad ---(i)$$

So, $\vec{a}=2\hat{i}+3\hat{j}+\hat{k}$
$\vec{n}=5\hat{i}+3\hat{j}+2\hat{k}$

Put, $\vec{a}$ and $\vec{n}$ in equation (i),

$$\left[\vec{r}-\left(2\hat{i}+3\hat{j}+\hat{k}\right)\right]\left(5\hat{i}+3\hat{j}+2\hat{k}\right)=0$$
$$\vec{r}\left(5\hat{i}+3\hat{j}+2\hat{k}\right)-\left(2\hat{i}+3\hat{j}+\hat{k}\right)\left(5\hat{i}+3\hat{j}+2\hat{k}\right)=0$$
$$\vec{r}\left(5\hat{i}+3\hat{j}+2\hat{k}\right)-\left[(2)(5)+(3)(3)+(1)(2)\right]=0$$
$$\vec{r}\left(5\hat{i}+3\hat{j}+2\hat{k}\right)-\left[10+9+2\right]=0$$
$$\vec{r}\left(5\hat{i}+3\hat{j}+2\hat{k}\right)-21=0$$

Put, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(5\hat{i}+3\hat{j}+2\hat{k}\right)-21=0$$
$$(x)(5)+(y)(3)+(z)(2)=21$$

$$5x+3y+2z=21$$

The Plane 29.3 Q9

Here, given that P is the point $(2,3,-1)$ and required plane is passing through P at right angles to OP

It means, the plane is passing through P and OP is the vector normal to the plane.

We know that, equation of a plane, passing through a point $\vec{a}$ and $\vec{n}$ is vector normal to the plane, is given by,

$$(\vec{r}-\vec{a}).\vec{n}=0 \qquad ---(i)$$

Here, $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$
$\vec{n}=\overrightarrow{OP}$
$=$ Position vector of P − Position vector of O

$$=\left(2\hat{i}+3\hat{j}-\hat{k}\right)-\left(0\hat{i}+0\hat{j}+0\hat{k}\right)$$
$$\vec{n}=2\hat{i}+3\hat{j}-\hat{k}$$

Put, the value of $\vec{a}$ and $\vec{n}$ in equation (i),

$$\left[\vec{r}-\left(2\hat{i}+3\hat{j}-\hat{k}\right)\right].\left(2\hat{i}+3\hat{j}-\hat{k}\right)=0$$
$$\vec{r}\left(2\hat{i}+3\hat{j}-\hat{k}\right)-\left[\left(2\hat{i}+3\hat{j}-\hat{k}\right)\left(2\hat{i}+3\hat{j}-\hat{k}\right)\right]=0$$
$$\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)-\left[(2)(2)+(3)(3)+(-1)(-1)\right]=0$$
$$\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)-\left[4+9+1\right]=0$$
$$\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)-14=0$$
$$\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)=14$$

Put, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$
$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(2\hat{i}+3\hat{j}-\hat{k}\right)=14$$
$$(x)(2)+(y)(3)+(z)(-1)=14$$

$$2x+3y-z=14$$

Equation of required plane is,

$$2x+3y-z=14$$

The Plane 29.3 Q10

Here, given equation of plane is,

$2x + y - 2z = 3$

Dividing by 3 on both the sides,

$$\frac{2x}{3} + \frac{y}{3} - \frac{2z}{3} = \frac{3}{3}$$

$$\frac{x}{\frac{3}{2}} + \frac{y}{3} + \frac{z}{-\frac{3}{2}} = 1 \qquad ---(i)$$

We know that, if a, b, c are the intercepts by a plane on the coordinate axes, new equation of the plane is,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad ---(ii)$$

Comparing the equation (i) and (ii),

$a = \frac{3}{2}$, $b = 3$, $c = -\frac{3}{2}$

Again, given equation of plane is,

$$2x + y - 2z = 3$$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(2\hat{i} + \hat{j} - 2\hat{k}\right) = 3$$

$$\vec{r}\left(2\hat{i} + \hat{j} - 2\hat{k}\right) = 3$$

So, vector normal to the plane is given by

$$\vec{n} = 2\hat{i} + \hat{j} - 2\hat{k}$$

$$\left|\vec{n}\right| = \sqrt{(2)^2 + (1)^2 + (-2)^2}$$

$$= \sqrt{4 + 1 + 4}$$

$$= \sqrt{9}$$

$$\left|\vec{n}\right| = 3$$

Direction vector of $\vec{n} = 2, 1, -2$

Direction vector of $\vec{n} = \frac{2}{\left|\vec{n}\right|}, \frac{1}{\left|\vec{n}\right|}, \frac{-2}{\left|\vec{n}\right|}$

$$= \frac{2}{3}, \frac{1}{3}, -\frac{2}{3}$$

So,

Intercepts by the plane on coordinate axes are $= \frac{3}{2}, 3, -\frac{3}{2}$

Direction cosine of normal to the plane are $= \frac{2}{3}, \frac{1}{3}, -\frac{2}{3}$

The Plane 29.3 Q11

Here, given that, the required plane passes through the point $(1,-2,5)$ and is perpendicular to the line joining origin O to the point $P\left(3\hat{i}+\hat{j}-\hat{k}\right)$.

We know that, equation of a plane passing through a point $\vec{a}$ and perpendicular to a vector $\vec{n}$ is given by,

$$\left(\vec{r}-\vec{a}\right).\vec{n}=0 \qquad ---(i)$$

Here, $\vec{a}=\hat{i}-2\hat{j}+5\hat{k}$

$\vec{n}=\overrightarrow{OP}$

$=$ Position vector of P − Position vector of O

$=\left(3\hat{i}+\hat{j}-\hat{k}\right)-\left(0\hat{i}+0\hat{j}+0\hat{k}\right)$

$\vec{n}=3\hat{i}+\hat{j}-\hat{k}$

Put, the value of $\vec{a}$ and $\vec{n}$ in equation (i), we get,

$$\left[\vec{r}-\left(\hat{i}-2\hat{j}+5\hat{k}\right)\right]\left(3\hat{i}+\hat{j}-\hat{k}\right)=0$$
$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)-\left(\hat{i}-2\hat{j}+5\hat{k}\right)\left(3\hat{i}+\hat{j}-\hat{k}\right)=0$$
$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)-\left[(1)(3)+(-2)(1)+(5)(-1)\right]=0$$
$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)-\left[3-2-5\right]=0$$
$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)-\left[-4\right]=0$$
$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)+4=0$$

$$\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)=-4$$

The Plane 29.3 Q12

We have to find the equation of plane that bisects $A(1,2,3)$ and $B(3,4,5)$ perpendicularly

We know that, equation of a plane passing through the point $\vec{a}$ and perpendicular to vector $\vec{n}$ is given by,

$$\left(\vec{r}-\vec{a}\right).\vec{n}=0 \qquad ---(i)$$

Here, $\vec{a}=$ mid-point of AB

$$=\frac{\text{Position vector of } A+\text{Position vector of } B}{2}$$
$$=\frac{\hat{i}+2\hat{j}+3\hat{k}+3\hat{i}+4\hat{j}+5\hat{k}}{2}$$
$$\vec{a}=\frac{4\hat{i}+6\hat{j}+8\hat{k}}{2}$$

$$\vec{a}=2\hat{i}+3\hat{j}+4\hat{k}$$

And, $\vec{n}=\overrightarrow{AB}$

$=$ Position vector of B − Position vector of A

$=\left(3\hat{i}+4\hat{j}+5\hat{k}\right)-\left(\hat{i}+2\hat{j}+3\hat{k}\right)$

$=3\hat{i}+4\hat{j}+5\hat{k}-\hat{i}-2\hat{j}-3\hat{k}$

$\vec{n}=2\hat{i}+2\hat{j}+2\hat{k}$

Put, the value of $\vec{a}$ and $\vec{n}$ in equation (i),

$$\vec{r}-\left(2\hat{i}+3\hat{j}+4\hat{k}\right)\left(2\hat{i}+2\hat{j}+2\hat{k}\right)=0$$
$$\vec{r}\left(2\hat{i}+2\hat{j}+2\hat{k}\right)-\left[\left(2\hat{i}+3\hat{j}+4\hat{k}\right)\left(2\hat{i}+2\hat{j}+2\hat{k}\right)\right]=0$$
$$\vec{r}\left(2\hat{i}+2\hat{j}+2\hat{k}\right)-\left[(2)(2)+(3)(2)+(4)(2)\right]=0$$
$$\vec{r}\left(2\hat{i}+2\hat{j}+2\hat{k}\right)-\left[4+6+8\right]=0$$
$$\vec{r}\left(2\hat{i}+2\hat{j}+2\hat{k}\right)-18=0$$

$$\vec{r}\left(2\hat{i}+2\hat{j}+2\hat{k}\right)=18$$

The Plane 29.3 Q13(i)

Given, two equation of plane are,

$x - y + z - 2 = 0$ and
$3x + 2y - z + 4 = 0$

$x - y + z = 2$
$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} - \hat{j} + \hat{k}\right) = 2$
$\vec{r}.\overrightarrow{n_1} = 2 \qquad ---(i)$

$3x + 2y - z = -4$
$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(3\hat{i} + 2\hat{j} - \hat{k}\right) = -4$
$\vec{r}\left(3\hat{i} + 2\hat{j} - \hat{k}\right) = -4$
$\vec{r}.\overrightarrow{n_2} = -4 \qquad ---(i)$

From equation (i) and (ii), we get that
$\overrightarrow{n_1}$ is normal to equation (i) and
$\overrightarrow{n_2}$ is normal to equation (ii).

Now,
$\overrightarrow{n_1}.\overrightarrow{n_2} = \left(\hat{i} - \hat{j} + \hat{k}\right).\left(3\hat{i} + 2\hat{j} - \hat{k}\right)$
$= (1)(3) + (-1)(2) + (1)(-1)$
$= 3 - 2 - 1$
$= 3 - 3$

$= 0$

$\overrightarrow{n_1}.\overrightarrow{n_2} = 0$

So, $\overrightarrow{n_1}$ is perpendicular to $\overrightarrow{n_2}$

The Plane 29.3 Q13(ii)

Given, two vector equation of plane are,

$\vec{r}.\left(2\hat{i} - \hat{j} + 3\hat{k}\right) = 5$
$\vec{r}.\overrightarrow{n_1} = 5$

So, $\overrightarrow{n_1} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right)$
And, $\vec{r}.\left(2\hat{i} - 2\hat{j} - 2\hat{k}\right) = 5$
$\vec{r}.\overrightarrow{n_2} = 5$

So, $\overrightarrow{n_2} = 2\hat{i} - 2\hat{j} - 2\hat{k}$

Now, $\overrightarrow{n_1}.\overrightarrow{n_2}$
$= \left(2\hat{i} - \hat{j} + 3\hat{k}\right).\left(2\hat{i} - 2\hat{j} - 2\hat{k}\right)$
$= (2)(2) + (-1)(-2) + (3)(-2)$
$= 4 + 2 - 6$
$= 6 - 6$

$= 0$

$\overrightarrow{n_1}.\overrightarrow{n_2} = 0$

Hence, normals to planes $\overrightarrow{n_1}$ and $\overrightarrow{n_2}$ are perpendicular.

The Plane 29.3 Q14

Given, equation of plane is,

$$2x+2y+2z=3$$
$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(2\hat{i}+2\hat{j}+2\hat{k}\right)=3$$
$$\vec{r}.\left(2\hat{i}+2\hat{j}+2\hat{k}\right)=3$$

$$\vec{r}.\vec{n}=d$$

Normal to the plane $\vec{n}=2\hat{i}+2\hat{j}+2\hat{k}$

Direction ratio of $\vec{n}=2,2,2$

Direction cosine of $\vec{n}=\frac{2}{|\vec{n}|},\frac{2}{|\vec{n}|},\frac{2}{|\vec{n}|}$

$$|\vec{n}|=\sqrt{(2)^2+(2)^2+(2)^2}$$
$$=\sqrt{4+4+4}$$
$$=\sqrt{12}$$

$$|\vec{n}|=2\sqrt{3}$$

Direction cosine of $|\vec{n}|=\frac{2}{2\sqrt{3}},\frac{2}{2\sqrt{3}},\frac{2}{2\sqrt{3}}$

$$=\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$$

So, $l=\frac{1}{\sqrt{3}}$, $m=\frac{1}{\sqrt{3}}$, $n=\frac{1}{\sqrt{3}}$

Let, α,β,γ be the angle that normal $\vec{n}$ makes with the coordinate axes respectively.

$$l=\cos\alpha=\frac{1}{\sqrt{3}}$$
$$\alpha=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \qquad ---(i)$$

$$m=\cos\beta=\frac{1}{\sqrt{3}}$$
$$\beta=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \qquad ---(ii)$$

$$n=\cos\gamma=\frac{1}{\sqrt{3}}$$
$$\gamma=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \qquad ---(iii)$$

From equation (i), (ii) and (iii),

$$\alpha=\beta=\gamma$$

So, normal to the plane, $\vec{n}$ is equally inclined with the coordinate axes.

The Plane 29.3 Q15

Given, equation of plane is,

$$12x - 3y + 4z = 1$$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(12\hat{i} - 3\hat{j} + 4\hat{k}\right) = 1$$

$$\vec{r}.\vec{n} = 1$$

So, normal to the plane is

$$\vec{n} = 12\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\left|\vec{n}\right| = \sqrt{(12)^2 + (-3)^2 + (4)^2}$$

$$= \sqrt{144 + 9 + 16}$$

$$= \sqrt{144 + 25}$$

$$= 169 = 13$$

Unit vector $\hat{n} = \dfrac{12\hat{i} - 3\hat{j} + 4\hat{k}}{13}$

$$= \frac{12\hat{i}}{13} - \frac{3}{13}\hat{j} + 4\hat{k}$$

A vector normal to the plane with magnitude

$$26 = 26\hat{n}$$

$$= 26\left(\frac{12\hat{i}}{13} - 3\hat{j} + 4\hat{k}\right)$$

Required vector $= 24\hat{i} - 6\hat{j} + 8\hat{k}$

The Plane 29.3 Q16

Given that, line drawn from $A(4,-1,2)$ meets a plane at right angle, at the point $B(-10,5,4)$.

We know that,
Equation of a plane passing through the point $\vec{a}$ and perpendicular to $\vec{n}$ is given by,

$$\left(\vec{r} - \vec{a}\right).\vec{n} = 0 \qquad ---\text{(i)}$$

Here, $\vec{a}$ = Position vector B

$$\vec{a} = -10\hat{i} + 5\hat{j} + 4\hat{k}$$

$$\vec{n} = \overrightarrow{AB}$$

$= $ Position vector of B – Position vector of A

$$= \left(-10\hat{i} + 5\hat{j} + 4\hat{k}\right) - \left(4\hat{i} - \hat{j} + 2\hat{k}\right)$$

$$= -10\hat{i} + 5\hat{j} + 4\hat{k} - 4\hat{i} + \hat{j} - 2\hat{k}$$

$$\vec{n} = -14\hat{i} + 6\hat{j} + 2\hat{k}$$

Put, the value of $\vec{a}$ and $\vec{n}$ in equation (i),

$$\left[\vec{r} - \left(-10\hat{i} + 5\hat{j} + 4\hat{k}\right)\right].\left(-14\hat{i} + 6\hat{j} + 2\hat{k}\right) = 0$$

$$\vec{r}.\left(-14\hat{i} + 6\hat{j} + 2\hat{k}\right) - \left(-10\hat{i} + 5\hat{j} + 4\hat{k}\right)\left(-14\hat{i} + 6\hat{j} + 2\hat{k}\right) = 0$$

$$\vec{r}\left(-14\hat{i} + 6\hat{j} + 2\hat{k}\right) - \left[(-10)(-14) + (5)(6) + (4)(2)\right] = 0$$

$$\vec{r}\left(-14\hat{i} + 6\hat{j} + 2\hat{k}\right) - \left[140 + 30 + 8\right] = 0$$

$$\vec{r}\left(-14\hat{i} + 6\hat{j} + 2\hat{k}\right) - 178 = 0$$

$$\vec{r}\left(-14\hat{i} + 6\hat{j} + 2\hat{k}\right) = 178$$

The Plane 29.3 Q17

We have to find the equation of plane which bisects the line joining the points $A(-1,2,3)$ and $B(3,-5,6)$ at right angles.

Let, C be the mid-point of AB

We know that, equation of a plane passing through a point $\vec{a}$ and perpendicular to a vector $\vec{n}$ is given by,

$$(\vec{r}-\vec{a}).\vec{n}=0 \qquad ---(i)$$

Here, $\vec{a}$ = Position vector of C

= Mid-point of A and B

$$= \frac{\text{Position vector of } A + \text{Position vector of } B}{2}$$

$$\vec{a} = \frac{-\hat{i}+2\hat{j}+3\hat{k} + 3\hat{i}-5\hat{j}+6\hat{k}}{2}$$

$$= \frac{2\hat{i}}{2}-\frac{3\hat{j}}{2}+\frac{9\hat{k}}{2}$$

$$\vec{a} = \hat{i}-\frac{3}{2}\hat{j}+\frac{9}{2}\hat{k}$$

$\vec{n} = \overrightarrow{AB}$

= Position vector of B – Position vector of A

$$= \frac{(3\hat{i}-5\hat{j}+6\hat{k})-(-\hat{i}+2\hat{j}+3\hat{k})}{2}$$

$$= \frac{3\hat{i}-5\hat{j}+6\hat{k}+\hat{i}-2\hat{j}-3\hat{k}}{2}$$

$$= \frac{4\hat{i}-7\hat{j}+3\hat{k}}{2}$$

$$= \frac{4}{2}\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}$$

$$\vec{n} = 2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}$$

Put, the value of $\vec{a}$ and $\vec{n}$ in equation (i), we get,

$$\left[\vec{r}-\left(\hat{i}-\frac{3}{2}\hat{j}+\frac{9}{2}\hat{k}\right)\right]\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)=0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-\left(\hat{i}-\frac{3}{2}\hat{j}+\frac{9}{2}\hat{k}\right)\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)=0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-\left[(1)(2)+\left(-\frac{3}{2}\right)\left(-\frac{7}{2}\right)+\left(\frac{9}{2}\right)\left(+\frac{3}{2}\right)\right]=0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-\left[2+\frac{21}{4}+\frac{27}{4}\right]=0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-\left[\frac{29+27}{4}\right]=0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-\frac{56}{4}=0$$

$$\vec{r}.\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-14=0$$

Put, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$$(x\hat{i}+y\hat{j}+z\hat{k})\left(2\hat{i}-\frac{7}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-14=0$$

$$(x)(2)+(y)\left(-\frac{7}{2}\right)+(z)\left(+\frac{3}{2}\right)-14=0$$

$$2x-\frac{7y}{2}+\frac{3z}{2}-14=0$$

$$\frac{4x-7y+3z-28}{2}=0$$

$$4x-7y+3z=28$$

Equation of required plane is,

$$4x-7y+3z=28$$

Vector equation of the plane:

Given that the required plane passes through the point $(5, 2, -4)$ having the position vector

$\vec{a} = 5\hat{i} + 2\hat{j} - 4\hat{k}$

Also given that the required plane is perpendicular to the line with direction ratios 2, 3 and -1.

Thus the vector equation of the normal vector to the plane is $\vec{n} = 2\hat{i} + 3\hat{j} - \hat{k}$.

We know that the vector equation of the plane passing through a point having position vector $\vec{a}$ and normal to vector $\vec{n}$ is given by $(\vec{r} - \vec{a}) \cdot \vec{n} = 0$ or, $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$.

Thus the required equation of the required plane is

$$\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = (5\hat{i} + 2\hat{j} - 4\hat{k}) \cdot (2\hat{i} + 3\hat{j} - \hat{k})$$
$$\Rightarrow \vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 10 + 6 + 4$$
$$\Rightarrow \vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 20$$

The Cartesian equation of the plane is

$$\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 20$$
$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 2$$
$$\Rightarrow 2x + 3y - z = 20$$

The Plane Ex 29.3 Q19

Consider the point $P(1,2,-3)$.

Thus the position vector of the point P is

$\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$

Direction ratios of the line OP, where O is the origin, are 1,2 and -3

Thus the vector equation of the normal vector, OP, to the plane is $\vec{n} = \hat{i} + 2\hat{j} - 3\hat{k}$.

We know that the vector equation of the plane passing through a point having position vector $\vec{a}$ and normal to vector $\vec{n}$ is given by $(\vec{r} - \vec{a}) \cdot \vec{n} = 0$ or, $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$.

Thus the required equation of the required plane is

$$\vec{r} \cdot (\hat{i} + 2\hat{j} - 3\hat{k}) = (\hat{i} + 2\hat{j} - 3\hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})$$

$$\Rightarrow \vec{r} \cdot (\hat{i} + 2\hat{j} - 3\hat{k}) = 1 + 4 + 9$$

$$\Rightarrow \vec{r} \cdot (\hat{i} + 2\hat{j} - 3\hat{k}) = 14$$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k}) = 14$$

$$\Rightarrow x + 2y - 3z = 14$$

The Plane Ex 29.3 Q20

O is the origin and the coordinates of A are (a, b, c).

$\overline{OA} = a\hat{i} + b\hat{j} + c\hat{k}$

$\therefore$ The direction the direction ratios of OA are proportional to, a, b, c.

$\therefore$ Direction cosines are,

$$\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$$

The equation of the line passing through A(a, b, c) and perpendicular to $\overline{OA}$ is,

$$\left\{x\hat{i} + y\hat{j} + z\hat{k} - \left(a\hat{i} + b\hat{j} + c\hat{k}\right)\right\} \bullet a\hat{i} + b\hat{j} + c\hat{k} = 0$$

$$ax + by + cz = a^2 + b^2 + c^2$$

Ex - 29.4

The Plane Ex 29.4 Q1

Here, it is given that, the required plane is at a distanceof 3 unit from origin and k is unit vector normal to it. We know that, vector equation of a plane normal to unit vector $\hat{n}$ and at distance d from origin, is

$$\vec{r}.\hat{n} = d$$

So, here d = 3 unit
$\hat{n} = \hat{k}$

The equation of the required plane is,

$$\vec{r}.\hat{k} = 3$$

The Plane Ex 29.4 Q2

We know that, vector equation of a plane which is at a distance d unit from origin and normal to unit vector $\hat{n}$ is given by

$$\vec{r}.\hat{n} = d \qquad ---(\text{i})$$

Here, d = 5 unit
$\vec{n} = \hat{i} - 2\hat{j} - 2\hat{k}$

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|}$$

$$= \frac{\hat{i} - 2\hat{j} - 2\hat{k}}{\sqrt{(1)^2 + (-2)^2 + (-2)^2}}$$

$$= \frac{\hat{i} - 2\hat{j} - 2\hat{k}}{\sqrt{9}}$$

$$\hat{n} = \frac{1}{3}\left(\hat{i} - 2\hat{j} - 2\hat{k}\right)$$

Put, value of d and $\hat{n}$ in equation (i),
The equation of required plane is,

$$\vec{r}.\frac{1}{3}\left(\hat{i} - 2\hat{j} - 2\hat{k}\right) = 5$$

The Plane Ex 29.4 Q3

Given equation of plane is,

$$2x - 3y - 6z = 14$$
$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right).\left(2\hat{i} - 3\hat{j} - 6\hat{k}\right) = 14$$

Dividing the equation by $\sqrt{(2)^2 + (-3)^2 + (-6)^2}$

$$\vec{r}.\frac{\left(2\hat{i} - 3\hat{j} - 6\hat{k}\right)}{\sqrt{4+9+36}} = \frac{14}{\sqrt{4+9+36}}$$
$$\vec{r}.\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = \frac{14}{7}$$

$$\vec{r}.\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = 2 \qquad ---(i)$$

We know that the vector equation of a plane with distance d from origin and normal to unit vector $\hat{n}$ is given by

$$\vec{r}.\hat{n} = d \qquad ---(ii)$$

Comparing (i) and (ii),

$d = 2$ and

$$\hat{n} = \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}$$

So, distance of plane from origin = 2 unit

Direction cosine of normal to plane = $\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}$

The Plane Ex 29.4 Q4

Given equation of plane is,

$$\vec{r}.\left(\hat{i} - 2\hat{j} + 2\hat{k}\right) + 6 = 0$$
$$\vec{r}.\left(\hat{i} - 2\hat{j} + 2\hat{k}\right) = -6$$

Multiplying both the sides by (-1),

$$\vec{r}.\left(-\hat{i} + 2\hat{j} - 2\hat{k}\right) = 6$$
$$\vec{r}.\vec{n} = 6 \qquad ---(i)$$

Here, $\vec{n} = -\hat{i} + 2\hat{j} - 2\hat{k}$

$$|\vec{n}| = \sqrt{(-1)^2 + (2)^2 + (-2)^2}$$
$$= \sqrt{1+4+4}$$
$$= \sqrt{9}$$
$$= 3$$

Dividing equation (i) by $|\vec{n}| = 3$ both the sides,

$$\vec{r}.\frac{\vec{n}}{|\vec{n}|} = \frac{6}{|\vec{n}|}$$
$$\vec{r}.\frac{1}{3}\left(-\hat{i} + 2\hat{j} - 2\hat{k}\right) = \frac{6}{3}$$
$$\vec{r}\left(-\frac{\hat{i}}{3} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}\right) = 2 \qquad ---(ii)$$

We know that, equation of a plane at distance d from origin and normal to unit vector $\hat{n}$ is

$$\vec{r}.\hat{n} = d \qquad ---(iii)$$

Comparing equation (ii) and (iii),

$d = 2$

$$\hat{n} = -\frac{\hat{i}}{3} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}$$

The Plane Ex 29.4 Q5

Given equation of plane is,

$$2x - 3y + 6z + 14 = 0$$
$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right) = -14$$
$$\vec{r}.\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right) = -14$$

Multiplying by (-1) both the sides,

$$\vec{r}.\left(-2\hat{i} + 3\hat{j} - 6\hat{k}\right) = 14 \qquad ---(i)$$

So, $\vec{r}.\vec{n} = 14$

$$\vec{n} = -2\hat{i} + 3\hat{j} - 6\hat{k}$$
$$\left|\vec{n}\right| = \sqrt{(-2)^2 + (3)^2 + (-6)^2}$$
$$= \sqrt{4 + 9 + 36}$$
$$= \sqrt{49}$$

$$\left|\vec{n}\right| = 7$$

Dividing equation (i) by $\left|\vec{n}\right| \Rightarrow$ both the sides,

$$\vec{r}.\frac{\left(-2\hat{i} + 3\hat{j} - 6\hat{k}\right)}{7} = \frac{14}{7}$$

$$\vec{r}.\left(-\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = 2$$

$$-\frac{2}{7}x + \frac{3}{7}y - \frac{6}{7}z = 2$$

The Plane Ex 29.4 Q6

Given, direction ratios of perpendicular from origin to a plane is 12, − 3, 4

So,

Normal vector = $12\hat{i} - 3\hat{j} + 4\hat{k}$

$$\vec{n} = 12\hat{i} - 3\hat{j} + 4\hat{k}$$
$$\left|\vec{n}\right| = \sqrt{(12)^2 + (-3)^2 + (4)^2}$$
$$= \sqrt{144 + 9 + 16}$$
$$= \sqrt{169}$$

$$\left|\vec{n}\right| = 13$$

Normal unit vector $\hat{n} = \dfrac{\vec{n}}{\left|\vec{n}\right|}$

$$= \frac{1}{13}\left(12\hat{i} - 3\hat{j} + 4\hat{k}\right)$$

Given that, perpendicular distance of plane from origin is 5 unit.

$\Rightarrow \qquad d = 5$ unit

We know that, equation of a plane at a distance d from origin and normal unit vector $\hat{n}$ is

$$\vec{r}.\hat{n} = d$$

So, vector equation of required plane is

$$\vec{r}.\left(\frac{12}{13}\hat{i} - \frac{3}{13}\hat{j} + \frac{4}{13}\hat{k}\right) = 5$$

Put, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\frac{12}{13}\hat{i} - \frac{3}{13}\hat{j} + \frac{4}{13}\hat{k}\right) = 5$$
$$(x)\left(\frac{12}{13}\right) + (y)\left(-\frac{3}{13}\right) + (z)\left(\frac{4}{13}\right) = 5$$

$$\frac{12}{13}x - \frac{3}{13}y + \frac{4}{13}z = 5$$

The Plane Ex 29.4 Q7

Given equation of plane is

$$x + 2y + 3z - 6 = 0$$
$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - 6 = 0$$
$$\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 6 \qquad ---(i)$$

$$\vec{r}.\vec{n} = 6$$

So, $\vec{n} = \hat{i} + 2\hat{j} + 3\hat{k}$

$$\left|\vec{n}\right| = \sqrt{(1)^2 + (2)^2 + (3)^2}$$
$$= \sqrt{1 + 4 + 9}$$

$$\left|\vec{n}\right| = \sqrt{14}$$

Dividing equation (i) by $\sqrt{14}$, we get

$$\vec{r}.\left(\frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}\right) = \frac{6}{\sqrt{14}} \qquad ---(ii)$$

We know that, vector equation of a plane at distance d unit from origin and normal to unit vector $\hat{n}$ is

$$\vec{r}.\hat{n} = d \qquad ---(iii)$$

Comparing (ii) and (iii), we get

Normal unit vector $= \hat{n} = \frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}$

The Plane Ex 29.4 Q8

We know that, vector equation of a plane which is at a distance d from origin and normal to unit vector $\hat{n}$ is given by

$$\vec{r}.\hat{n} = d \qquad ---(i)$$

Here, given $\vec{d} = 3\sqrt{3}$ unit.

Let, $\vec{a} = \left(p\hat{i} + q\hat{j} + r\hat{k}\right)$

Where $\vec{a}$ is normal vector.

Given that, $\vec{a}$ is equally inclined to the coordinate axes

If l, m, n are direction cosines of $\vec{n}$,

Here, $l = m = n \qquad ---(ii)$

We know that,

$$l^2 + m^2 + n^2 = 1$$
$$l^2 + l^2 + l^2 = 1 \qquad \left[\text{Using } (ii)\right]$$
$$3l^2 = 1$$

$$l = \frac{1}{\sqrt{3}}$$

So, $l = m = n = \frac{1}{\sqrt{3}}$

Here,

$$l = \frac{p}{|\vec{a}|} = \frac{1}{\sqrt{3}}$$

$$m = \frac{q}{|\vec{a}|} = \frac{1}{\sqrt{3}}$$

$$n = \frac{r}{|\vec{a}|} = \frac{1}{\sqrt{3}}$$

Now,

$$\vec{a} = p\hat{i} + q\hat{j} + r\hat{k}$$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

$$= \frac{p}{|\vec{a}|}\hat{i} + \frac{q}{|\vec{a}|}\hat{j} + \frac{r}{|\vec{a}|}\hat{k}$$

$$\hat{a} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

Put the value of $d = 3\sqrt{3}$ unit and $\hat{n} = \hat{a} = \frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}$ in equation (i),
vector equation of the required plane is

$$\vec{r}.\frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right) = 3\sqrt{3}$$

$$\vec{r}\left(\hat{i} + \hat{j} + \hat{k}\right) = 9$$

$$x + y + z = 9$$

The Plane Ex 29.4 Q9

Here, we have to find equating a plane passing through $A(1,2,1)$ and perpendicular to line joining $B(1,4,2)$ and $C(2,3,5)$.

We know that, the vector equation of a plane passing through a point $\vec{a}$ and perpendicular to vector $\vec{n}$ is given by,

$$(\vec{r}-\vec{a}).\vec{n} = 0 \qquad ---(i)$$

Here, $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$

$\vec{n} = \overrightarrow{BC}$

= Position vector of C – Position vector of B

$= \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right) - \left(\hat{i} + 4\hat{j} + 2\hat{k}\right)$

$= 2\hat{i} + 3\hat{j} + 5\hat{k} - \hat{i} - 4\hat{j} - 2\hat{k}$

$\vec{n} = \hat{i} - \hat{j} + 3\hat{k}$

Put, $\vec{a}$ and $\vec{n}$ in equation (i),

Vector equation of plane is

$$\left[\vec{r} - \left(\hat{i} + 2\hat{j} + \hat{k}\right)\right].\left(\hat{i} - \hat{j} + 3\hat{k}\right) = 0$$
$$\vec{r}.\left(\hat{i} - \hat{j} + 3\hat{k}\right) - \left(\hat{i} + 2\hat{j} + \hat{k}\right)\left(\hat{i} - \hat{j} + 3\hat{k}\right) = 0$$
$$\vec{r}.\left(\hat{i} - \hat{j} + 3\hat{k}\right) - \left[(1)(1) + (2)(-1) + (1)(3)\right] = 0$$
$$\vec{r}.\left(\hat{i} - \hat{j} + 3\hat{k}\right) - \left[1 - 2 + 3\right] = 0$$
$$\vec{r}.\left(\hat{i} - \hat{j} + 3\hat{k}\right) - (4 - 2) = 0$$
$$\vec{r}.\left(\hat{i} - \hat{j} + 3\hat{k}\right) - 2 = 0$$

$$\vec{r}.\left(\hat{i} - \hat{j} + 3\hat{k}\right) = 2 \qquad ---(ii)$$

$$|\vec{n}| = \sqrt{(1)^2 + (-1)^2 + (3)^2}$$
$$= \sqrt{1+1+9}$$
$$= \sqrt{11}$$

Dividing equation (i) by $\sqrt{11}$,

$$\vec{r}.\left(\frac{1}{\sqrt{11}}\hat{i} - \frac{1}{\sqrt{11}}\hat{j} + \frac{3}{\sqrt{11}}\hat{k}\right) = \frac{2}{\sqrt{11}}$$

$$\vec{r}.\hat{n} = d$$

So, perpendicular distance of plane from origin = $\frac{2}{\sqrt{11}}$ units

Equation of plane, $\vec{r}.\left(\hat{i} - \hat{j} + 3\hat{k}\right) = 2$

Equation of plane, $x - y + 3z - 2 = 0$

The Plane 29.4 Q10

We know that the vector equation of a plane at a distance

'p' from the origin and normal to the unit vector $\hat{n}$ is $\vec{r}\cdot\hat{n}=p$

Vector normal to the plane is $\vec{n}=2\hat{i}-3\hat{j}+4\hat{k}$

The unit vector normal to the plane is

$$\hat{n}=\frac{2}{\sqrt{2^2+(-3)^2+4^2}}\hat{i}-\frac{3}{\sqrt{2^2+(-3)^2+4^2}}\hat{j}+\frac{4}{\sqrt{2^2+(-3)^2+4^2}}\hat{k}$$

$$\Rightarrow\hat{n}=\frac{2}{\sqrt{4+9+16}}\hat{i}-\frac{3}{\sqrt{4+9+16}}\hat{j}+\frac{4}{\sqrt{4+9+16}}\hat{k}$$

$$\Rightarrow\hat{n}=\frac{2}{\sqrt{29}}\hat{i}-\frac{3}{\sqrt{29}}\hat{j}+\frac{4}{\sqrt{29}}\hat{k}$$

Here, given that $p=\frac{6}{\sqrt{29}}$

Thus, the vector equation of the plane is

$$\vec{r}\cdot\left(\frac{2}{\sqrt{29}}\hat{i}-\frac{3}{\sqrt{29}}\hat{j}+\frac{4}{\sqrt{29}}\hat{k}\right)=\frac{6}{\sqrt{29}}$$

The Cartesian equation of the plane is

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\cdot\left(\frac{2}{\sqrt{29}}\hat{i}-\frac{3}{\sqrt{29}}\hat{j}+\frac{4}{\sqrt{29}}\hat{k}\right)=\frac{6}{\sqrt{29}}$$

$$\Rightarrow\left(\frac{2x}{\sqrt{29}}-\frac{3y}{\sqrt{29}}+\frac{4z}{\sqrt{29}}\right)=\frac{6}{\sqrt{29}}$$

$$\Rightarrow\left(\frac{2x-3y+4z}{\sqrt{29}}\right)=\frac{6}{\sqrt{29}}$$

$$\Rightarrow 2x-3y+4z=6$$

The Plane 29.4 Q11

The Cartesian equation of the given plane is

$2x-3y+4z-6=0.$

The above equation can be rewritten as

$2x-3y+4z=6$

Therefore, the vector equation of the plane is

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\cdot\left(2\hat{i}-3\hat{j}+4\hat{k}\right)=6$$

$$\Rightarrow\vec{r}\cdot\left(2\hat{i}-3\hat{j}+4\hat{k}\right)=6\ldots.(1)$$

We know that the vector equation of a plane at a distance

'p' from the origin and normal to unit vector $\hat{n}$ is $\vec{r}\cdot\hat{n}=p$

We have, $\vec{n}=2\hat{i}-3\hat{j}+4\hat{k}.$

Thus $|\vec{n}|=\sqrt{2^2+(-3)^2+4^2}=\sqrt{29}$

Dividing the equation (1) by $|\vec{n}|=\sqrt{29}$, we have,

$$\vec{r}\cdot\left(\frac{2\hat{i}-3\hat{j}+4\hat{k}}{\sqrt{29}}\right)=\frac{6}{\sqrt{29}}$$

Hence the normal form of the equation of the plane is

$$\vec{r}\cdot\left(\frac{2}{\sqrt{29}}\hat{i}-\frac{3}{\sqrt{29}}\hat{j}+\frac{4}{\sqrt{29}}\hat{k}\right)=\frac{6}{\sqrt{29}}$$

Hence the perpendicular distance of the

origin from the plane is $p=\frac{6}{\sqrt{29}}$.

Ex - 29.5

The Plane Ex 29.5 Q1

Given that, plane is passing through
$(1,1,1)$, $(1,-1,1)$ and $(-7,-3,-5)$

We know that, equation of plane passing through 3 points,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 1-1 & -1-1 & 1-1 \\ -7-1 & -3-1 & -5-1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 0 & -2 & 0 \\ -8 & -4 & -6 \end{vmatrix} = 0$$

$$(x-1)(12-0)-(y-1)(0-0)+(z-1)(0-16)=0$$
$$(x-1)(12)-(y-1)(0)+(z-1)(-16)=0$$
$$12x-12-0-16z+16=0$$

$$12x-16z+4=0$$

Dividing by 4,

Dividing by 4,

$$3x-4z+1=0$$
$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(3\hat{i}+0\hat{j}-4\hat{k}\right)+1=0$$

$$\vec{r}.\left(3\hat{i}-4\hat{k}\right)+1=0$$

Equation of the required plane,

$$\vec{r}.\left(3\hat{i}-4\hat{k}\right)+1=0$$

The Plane Ex 29.5 Q2

Let P(2,5, − 3), Q(− 2, − 3,5) and R(5,3, − 3) be the three points on a plane having position vectors $\vec{p}$, $\vec{q}$ and $\vec{s}$ respectively. Then the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are in the same plane.

Therefore, $\overrightarrow{PQ}\times\overrightarrow{PR}$ is a vector perpendicular to the plane.

Let $\vec{n}=\overrightarrow{PQ}\times\overrightarrow{PR}$

$\overrightarrow{PQ}=(-2-2)\hat{i}+(-3-5)\hat{j}+(5-(-3))\hat{k}$

$\Rightarrow \overrightarrow{PQ}=-4\hat{i}-8\hat{j}+8\hat{k}$

Similarly,

$\overrightarrow{PR}=(5-2)\hat{i}+(3-5)\hat{j}+(-3-(-3))\hat{k}$

$\Rightarrow \overrightarrow{PR}=3\hat{i}-2\hat{j}+0\hat{k}$

Thus

$\vec{n}=\overrightarrow{PQ}\times\overrightarrow{PR}$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -4 & -8 & 8\\ 3 & -2 & 0\end{vmatrix}$$

$=16\hat{i}+24\hat{j}+32\hat{k}$

The plane passes through the point P with position vector $\vec{p}=2\hat{i}+5\hat{j}-3\hat{k}$

Thus, its vector equation is

$\{\vec{r}-(2\hat{i}+5\hat{j}-3\hat{k})\}\cdot(16\hat{i}+24\hat{j}+32\hat{k})=0$

$\Rightarrow \vec{r}\cdot(16\hat{i}+24\hat{j}+32\hat{k})-(32+120-96)=0$

$\Rightarrow \vec{r}\cdot(16\hat{i}+24\hat{j}+32\hat{k})-56=0$

$\Rightarrow \vec{r}\cdot(16\hat{i}+24\hat{j}+32\hat{k})=56$

$\Rightarrow \vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k})=7$

The Plane Ex 29.5 Q3

Let A(a,0,0), B(0,b,0) and C(0,0,c) be three points on a plane having their position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively. Then vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are in the same plane.

Therefore, $\overrightarrow{AB}\times\overrightarrow{AC}$ is a vector perpendicular to the plane.

Let $\vec{n}=\overrightarrow{AB}\times\overrightarrow{AC}$

$\overrightarrow{AB}=(0-a)\hat{i}+(b-0)\hat{j}+(0-0)\hat{k}$

$\Rightarrow \overrightarrow{AB}=-a\hat{i}+b\hat{j}+0\hat{k}$

Similarly,

$\overrightarrow{AC}=(0-a)\hat{i}+(0-0)\hat{j}+(c-0)\hat{k}$

$\Rightarrow \overrightarrow{AC}=-a\hat{i}+0\hat{j}+c\hat{k}$

Thus

$\vec{n}=\overrightarrow{AB}\times\overrightarrow{AC}$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -a & b & 0\\ -a & 0 & c\end{vmatrix}$$

$\vec{n}=bc\hat{i}+ac\hat{j}+ab\hat{k}$

$\Rightarrow \hat{n}=\dfrac{bc\hat{i}+ac\hat{j}+ab\hat{k}}{\sqrt{b^2c^2+a^2c^2+a^2b^2}}$

The plane passes through the point P with position vector $\vec{a}=a\hat{i}+0\hat{j}+0\hat{k}$

Thus, the vector equation in the normal form is

$$\{\vec{r}-[(a\hat{i}+0\hat{j}+0\hat{k})]\}\cdot\left(\frac{bc\hat{i}+ac\hat{j}+ab\hat{k}}{\sqrt{b^2c^2+a^2c^2+a^2b^2}}\right)=0$$

$$\Rightarrow \vec{r}\cdot\frac{(bc\hat{i}+ac\hat{j}+ab\hat{k})}{\sqrt{b^2c^2+a^2c^2+a^2b^2}}=\frac{abc}{\sqrt{b^2c^2+a^2c^2+a^2b^2}}$$

$$\Rightarrow \vec{r}\cdot\frac{(bc\hat{i}+ac\hat{j}+ab\hat{k})}{\sqrt{b^2c^2+a^2c^2+a^2b^2}}=\frac{1}{\sqrt{\frac{b^2c^2+a^2c^2+a^2b^2}{a^2b^2c^2}}}$$

$$\Rightarrow \vec{r}\cdot\frac{(bc\hat{i}+ac\hat{j}+ab\hat{k})}{\sqrt{b^2c^2+a^2c^2+a^2b^2}}=\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\ldots(1)$$

The vector equation of a plane normal to the unit vector $\hat{n}$ and at a distance 'd' from the origin is $\vec{r}\cdot\hat{n}=d\ldots(2)$

Given that the plane is at a distance 'p' from the origin.

Comparing equations (1) and (2), we have,

$$d=p=\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}$$

$$\Rightarrow \frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$$

The Plane Ex 29.5 Q4

Let P(1, 1, − 1), Q(6, 4, − 5) and R(− 4, − 2, 3) be three points on a plane having position vectors $\vec{p}$, $\vec{q}$ and $\vec{s}$ respectively. Then the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are in the same plane.
Therefore, $\overrightarrow{PQ} \times \overrightarrow{PR}$ is a vector perpendicular to the plane.
Let $\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR}$

$\overrightarrow{PQ} = (6-1)\hat{i} + (4-1)\hat{j} + (-5-(-1))\hat{k}$
$\Rightarrow \overrightarrow{PQ} = 5\hat{i} + 3\hat{j} - 4\hat{k}$
Similarly,
$\overrightarrow{PR} = (-4-1)\hat{i} + (-2-1)\hat{j} + (3-(-1))\hat{k}$
$\Rightarrow \overrightarrow{PR} = -5\hat{i} - 3\hat{j} + 4\hat{k}$
Thus
Here, $\overrightarrow{PQ} = -\overrightarrow{PR}$
Therefore, the given points are collinear.
Thus, $\vec{n} = a\hat{i} + b\hat{j} + c\hat{k}$ where, $5a + 3b - 4c = 0$
The plane passes through the point P with position vector $\vec{p} = \hat{i} + \hat{j} - \hat{k}$
Thus, its vector equation is
$\{\vec{r} - (\hat{i} + \hat{j} - \hat{k})\} \cdot (a\hat{i} + b\hat{j} + c\hat{k}) = 0$, where, $5a + 3b - 4c = 0$

The Plane Ex 29.5 Q5

Let, A, B, C be the points with position vector $\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$, $\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$ and $\left(7\hat{i} + 6\hat{k}\right)$ respectively. Then

$\overrightarrow{AB}$ = Position vector of B – Poosition vector of A

$= \left(2\hat{i} - 2\hat{j} - \hat{k}\right) - \left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$

$= 2\hat{i} - 2\hat{j} - \hat{k} - 3\hat{i} - 4\hat{j} - 2\hat{k}$

$= -\hat{i} - 6\hat{j} - 3\hat{k}$

$\overrightarrow{BC}$ = Position vector of C – Poosition vector of B

$= \left(7\hat{i} + 6\hat{k}\right) - \left(2\hat{i} - 2\hat{j} - \hat{k}\right)$

$= 7\hat{i} + 6\hat{k} - 2\hat{i} + 2\hat{j} + \hat{k}$

$\overrightarrow{BC} = 5\hat{i} + 2\hat{j} + 7\hat{k}$

A vector normal to A, B, C is a vector perpendicular to $\overrightarrow{AB} \times \overrightarrow{BC}$

$\vec{n} = \overrightarrow{AB} \times \overrightarrow{BC}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -6 & -3 \\ 5 & 2 & 7 \end{vmatrix}$$

$\vec{n} = \hat{i}\left(-42 + 6\right) - \hat{j}\left(-7 + 15\right) + \hat{k}\left(-2 + 30\right)$

$= -36\hat{i} - 8\hat{j} + 28\hat{k}$

We know that, equation of a plane passing through vector $\vec{a}$ and perpendicular to vector $\vec{n}$ is given by,

$$\vec{r}.\vec{n} = \vec{a}.\vec{n} \qquad \text{---(i)}$$

Put $\vec{a}$ and $\vec{n}$ in equation (i),

$$\vec{r}.\left(-36\hat{i} - 8\hat{j} + 28\hat{k}\right) = \left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)\left(-36\hat{i} - 8\hat{j} + 28\hat{k}\right)$$

$$= (3)(-36) + (4)(-8) + (2)(28)$$
$$= -108 - 32 + 56$$
$$= -140 + 56$$

$$\vec{r}.\left(-36\hat{i} - 8\hat{j} + 28\hat{k}\right) = -84$$

Dividing by (-4), we get

$$\vec{r}.\left(9\hat{i} + 2\hat{j} - 7\hat{k}\right) = 21$$

Equation of required plane is,

$$\vec{r}.\left(9\hat{i} + 2\hat{j} - 7\hat{k}\right) = 21$$

Ex - 29.6

The Plane 29.6 Q1(i)

Given equation of two planes are

$\vec{r}.\left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) = 1$ --- (i)

$\vec{r}.\left(-\hat{i} + \hat{j}\right) = 4$ --- (ii)

We know that, angle between two planes

$\vec{r}.\vec{n_1} = d_1$ and $\vec{r}.\vec{n_2} = d_2$ is given by,

$$\cos\theta = \frac{\vec{n_1}.\vec{n_2}}{\left|\vec{n_1}\right|\left|\vec{n_2}\right|} \quad \text{--- (iii)}$$

Here, $\vec{n_1} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$\vec{n_2} = -\hat{i} + \hat{j}$

$$\cos\theta = \frac{\left(2\hat{i} - 3\hat{j} + 4\hat{k}\right)\left(-\hat{i} + \hat{j}\right)}{\sqrt{(2)^2 + (-3)^2 + (4)^2}\sqrt{(-1)^2 + (1)^2}}$$

$$= \frac{(2)(-1) + (-3)(1) + (4)(0)}{\sqrt{4 + 9 + 16}\sqrt{1 + 1}}$$

$$= \frac{-2 - 3 + 0}{\sqrt{29}\sqrt{2}}$$

$$\cos\theta = \frac{-5}{\sqrt{58}}$$

$$\theta = \cos^{-1}\left(\frac{-5}{\sqrt{58}}\right)$$

The Plane 29.6 Q1(ii)

Given equation of planes are

$\vec{r}.\left(2\hat{i} - \hat{j} + 2\hat{k}\right) = 6$ --- (i)

$\vec{r}.\left(3\hat{i} + 6\hat{j} - 2\hat{k}\right) = 9$ --- (ii)

We know that, angle between the planes

$\vec{r}.\vec{n_1} = d_1$ and $\vec{r}.\vec{n_2} = d_2$ is given by,

$$\cos\theta = \frac{\vec{n_1}.\vec{n_2}}{\left|\vec{n_1}\right|\left|\vec{n_2}\right|} \quad \text{--- (iii)}$$

Here, from equation (i) and (ii),

$\vec{n_1} = 2\hat{i} - \hat{j} + 2\hat{k}$

$\vec{n_2} = 3\hat{i} + 6\hat{j} - 2\hat{k}$

Put $\vec{n_1}$ and $\vec{n_2}$ in equation (iii),

$$\cos\theta = \frac{\left(2\hat{i} - \hat{j} + 2\hat{k}\right)\left(3\hat{i} + 6\hat{j} - 2\hat{k}\right)}{\sqrt{(2)^2 + (-1)^2 + (2)^2}\sqrt{(3)^2 + (6)^2 + (-2)^2}}$$

$$= \frac{(2)(3) + (-1)(6) + (2)(-2)}{\sqrt{4 + 1 + 4}\sqrt{9 + 36 + 4}}$$

$$\cos\theta = \frac{6 - 6 + 4}{\sqrt{9}\sqrt{49}}$$

$$= \frac{-4}{3.7}$$

$$= \frac{-4}{21}$$

$$\theta = \cos^{-1}\left(\frac{-4}{21}\right)$$

The Plane 29.6 Q1(iii)

Given equation of planes are

$$\vec{r}.\left(2\hat{i}+3\hat{j}-6\hat{k}\right)=5 \quad ---(i)$$
$$\vec{r}.\left(\hat{i}-2\hat{j}+2\hat{k}\right)=9 \quad ---(ii)$$

We know that, angle between equation of planes

$\vec{r}.\vec{n_1}=d_1$ and $\vec{r}.\vec{n_2}=d_2$ is given by,

$$\cos\theta=\frac{\vec{n_1}.\vec{n_2}}{\left|\vec{n_1}\right|\left|\vec{n_2}\right|} \quad ---(iii)$$

From equation (i) and (ii),

$\vec{n_1}=2\hat{i}+3\hat{j}-6\hat{k}$

$\vec{n_2}=\hat{i}-2\hat{j}+2\hat{k}$

Put $\vec{n_1}$ and $\vec{n_2}$ in equation (iii),

$$\cos\theta=\frac{\left(2\hat{i}+3\hat{j}-6\hat{k}\right)\left(\hat{i}-2\hat{j}+2\hat{k}\right)}{\sqrt{(2)^2+(3)^2+(-6)^2}\sqrt{(1)^2+(-2)^2+(2)^2}}$$
$$=\frac{(2)(1)+(3)(-2)+(-6)(2)}{\sqrt{4+9+36}\sqrt{1+4+4}}$$
$$=\frac{2-6-12}{\sqrt{49}\sqrt{9}}$$
$$=\frac{-16}{7.3}$$
$$=\frac{-16}{21}$$

$$\theta=\cos^{-1}\left(\frac{-16}{21}\right)$$

The Plane 29.6 Q2(i)

Given, equation of planes are,

$$2x-y+z=4 \quad ---(i)$$
$$x+y+2z=3 \quad ---(ii)$$

We know that, angle between two planes

$a_1x+b_1y+c_1z+d_1=0$ and

$a_2x+b_2y+c_2z+d_2=0$ is given by

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \quad ---(iii)$$

Here, from equation (i) and (ii),

$a_1=2,\ b_1=-1,\ c_1=1$

$a_2=1,\ b_2=1,\ c_2=2$

Put then values in equation (iii),

$$\cos\theta=\frac{(2)(1)+(-1)(1)+(1)(2)}{\sqrt{(2)^2+(-1)^2+(1)^2}\sqrt{(1)^2+(1)^2+(2)^2}}$$
$$=\frac{2-1+2}{\sqrt{4+1+1}\sqrt{1+1+4}}$$
$$\cos\theta=\frac{4-1}{\sqrt{6}\sqrt{6}}$$
$$=\frac{3}{6}$$
$$=\frac{1}{2}$$

$$\theta=\cos^{-1}\left(\frac{1}{2}\right)$$

$$\theta=\frac{\pi}{3}$$

The Plane 29.6 Q2(ii)

Given, equation of two planes are,

$x + y - 2z = 3$ --- (i)

$2x - 2y + z = 5$ --- (ii)

We know that, angle between two planes

$a_1x + b_1y + c_1z + d_1 = 0$ and

$a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Here, from equation (i) and (ii),

$a_1 = 1, b_1 = 1, c_1 = -2$

$a_2 = 2, b_2 = -2, c_2 = 1$

Put these values in equation (iii),

$$\cos\theta = \frac{(1)(2) + (1)(-2) + (-2)(1)}{\sqrt{(1)^2 + (1)^2 + (-2)^2}\sqrt{(2)^2 + (-2)^2 + (1)^2}}$$

$$\cos\theta = \frac{2 - 2 - 2}{\sqrt{1 + 1 + 4}\sqrt{4 + 4 + 1}}$$

$$= \frac{-2}{\sqrt{6}\sqrt{9}}$$

$$= \frac{-2}{3\sqrt{6}}$$

$$\theta = \cos^{-1}\left(\frac{-2}{3\sqrt{6}}\right)$$

The Plane 29.6 Q2(iii)

Given, equation of planes are,

$x - y + z = 5$ --- (i)

$x + 2y + z = 9$ --- (ii)

We know that, angle between the planes

$a_1x + b_1y + c_1z + d_1 = 0$ and

$a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \quad \text{--- (iii)}$$

From equation (i) and (ii),

$a_1 = 1, b_1 = -1, c_1 = 1$

$a_2 = 1, b_2 = 2, c_2 = 1$

Put these values in equation (iii),

$$\cos\theta = \frac{(1)(1) + (-1)(2) + (1)(1)}{\sqrt{(1)^2 + (-1)^2 + (1)^2}\sqrt{(1)^2 + (2)^2 + (1)^2}}$$

$$\cos\theta = \frac{1 - 2 + 1}{\sqrt{1 + 1 + 1}\sqrt{1 + 4 + 1}}$$

$$= \frac{0}{\sqrt{3}\sqrt{6}}$$

$$\cos\theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$\theta = \frac{\pi}{2}$$

The Plane 29.6 Q2(iv)

Given, equation of planes are,

$2x - 3y + 4z = 1$ --- (i)

$-x + y = 4$ --- (ii)

We know that, angle between two planes

$a_1x + b_1y + c_1z + d_1 = 0$ and
$a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Here, from equation (i) and (ii),
$a_1 = 2,\ b_1 = -3,\ c_1 = 4$
$a_2 = -1,\ b_2 = 1,\ c_2 = 0$

Put these values in equation (iii),

$$\cos\theta = \frac{(2)(-1) + (-3)(1) + (4)(0)}{\sqrt{(2)^2 + (-3)^2 + (4)^2}\sqrt{(-1)^2 + (1)^2 + (0)^2}}$$

$$\cos\theta = \frac{-2 - 3 + 0}{\sqrt{4 + 9 + 16}\sqrt{1 + 1 + 0}}$$

$$= \frac{-5}{\sqrt{29}\sqrt{2}}$$

$$\cos\theta = \frac{-5}{\sqrt{58}}$$

$$\theta = \cos^{-1}\left(\frac{-5}{\sqrt{58}}\right)$$

The Plane Ex 29.6 Q2(v)

We know that the angle between the planes
$a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\cdot\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Therefore, the angle between $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$

$$\cos\theta = \frac{2\times3 + 1\times(-6) + (-2)\times(-2)}{\sqrt{2^2 + 1^2 + (-2)^2}\cdot\sqrt{3^2 + (-6)^2 + (-2)^2}}$$

$$\Rightarrow \cos\theta = \frac{6 - 6 + 4}{\sqrt{9}\cdot\sqrt{9 + 36 + 4}}$$

$$\Rightarrow \cos\theta = \frac{4}{3\times7}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{4}{21}\right)$$

The Plane 29.6 Q3(i)

Given equation of planes are

$\vec{r}.\left(2\hat{i} - \hat{j} + \hat{k}\right) = 5$ --- (i)

$\vec{r}.\left(-\hat{i} - \hat{j} + \hat{k}\right) = 3$ --- (ii)

We know that, planes

$\vec{r}.\vec{n_1} = d_1$ and $\vec{r}.\vec{n_2} = d_2$ are perpendicular

if $\vec{n_1}.\vec{n_2} = 0$ --- (iii)

From equation (i) and (ii),
$\vec{n_1} = 2\hat{i} - \hat{j} + \hat{k}$
$\vec{n_2} = -\hat{i} - \hat{j} + \hat{k}$

Put $\vec{n_1}$ and $\vec{n_2}$ in equation (iii),

$\vec{n_1}.\vec{n_2} = 0$
$\left(2\hat{i} - \hat{j} + \hat{k}\right)\left(-\hat{i} - \hat{j} + \hat{k}\right) = 0$
$(2)(-1) + (-1)(-1) + (1)(1) = 0$
$-2 + 1 + 1 = 0$

$0 = 0$
$LHS = RHS$

Hence, planes are at right angle

The Plane 29.6 Q3(ii)

Given, equation of planes are,

$x - 2y + 4z = 10$
$18x + 17y + 4z = 49$

$\Rightarrow \quad x - 2y + 4z - 10 = 0 \qquad ---(i)$
$18x + 17y + 4z - 49 = 0 \qquad ---(ii)$

We know that, planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are at right angles if $a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iii)$

From (i) and (ii),
$a_1 = 1,\ b_1 = -2,\ c_1 = 4$
$a_2 = 18,\ b_2 = 17,\ c_2 = 4$

Put these in equation (iii),

$a_1a_2 + b_1b_2 + c_1c_2 = 0$
$(1)(18) + (-2)(17) + (4)(4) = 0$
$18 - 34 + 16 = 0$

$0 = 0$
$LHS = RHS$

Hence, planes are at right angles

The Plane 29.6 Q4(i)

Here, given equation of planes are

$\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 7 \qquad ---(i)$
$\vec{r}.\left(\lambda\hat{i} + 2\hat{j} - 7\hat{k}\right) = 26 \qquad ---(ii)$

We know that, planes $\vec{r}.\overrightarrow{n_1} = d_1$ and $\vec{r}.\overrightarrow{n_2} = d_2$ are perpendicular if

$\overrightarrow{n_1}.\overrightarrow{n_2} = 0 \qquad ---(iii)$

From equation (i) and (ii), we get
$\overrightarrow{n_1} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\overrightarrow{n_2} = \lambda\hat{i} + 2\hat{j} - 7\hat{k}$

Since, (i) and (ii) are perpendicular, so from (iii),

$\overrightarrow{n_1}.\overrightarrow{n_2} = 0$
$\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)\left(\lambda\hat{i} + 2\hat{j} - 7\hat{k}\right) = 0$
$(1)(\lambda) + (2)(2) + (3)(-7) = 0$
$\lambda + 4 - 21 = 0$

$\lambda - 17 = 0$
$\lambda = 17$

The Plane 29.6 Q4(ii)

Given, that plane $2x-4y+3z-5=0$ ---(i)

and $x+2y+\lambda z-5=0$ are ---(ii) perpendicular.

We know that, planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ are perpendicular if $a_1a_2+b_1b_2+c_1c_2=0$ ---(iii)

From equation (i) and (ii),

$a_1=2,\ b_1=-4,\ c_1=3$

$a_2=1,\ b_2=2,\ c_2=\lambda$

Put these in equation (iii),

$a_1a_2+b_1b_2+c_1c_2=0$

$(2)(1)+(-4)(2)+(3)(\lambda)=0$

$2-8+3\lambda=0$

$-6+3\lambda=0$

$3\lambda=6$

$\lambda=\frac{6}{3}$

$\lambda=2$

The Plane 29.6 Q4(iii)

Given, that planes

$3x-6y-2z-7=0$ ---(i)

and $2x+y-\lambda z-5=0$ ---(ii)

are perpendicular.

We know that, planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ are perpendicular if $a_1a_2+b_1b_2+c_1c_2=0$ ---(iii)

From (i) and (ii),

$a_1=3,\ b_1=-6,\ c_1=-2$

$a_2=2,\ b_2=1,\ c_2=-\lambda$

Put these in equation (iii),

$a_1a_2+b_1b_2+c_1c_2=0$

$(3)(2)+(-6)(1)+(-2)(-\lambda)=0$

$6-6+2\lambda=0$

$0+2\lambda=0$

$2\lambda=0$

$\lambda=0$

The Plane 29.6 Q5

We know that equation of a plane passing through (x_1, y_1, z_1) is given by

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ --- (i)

Given, plane is passing through $(-1,-1,2)$,

$a(x + 1) + b(y + 1) + c(z - 2) = 0$ --- (ii)

We know that plane $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ --- (iii)

Given, plane (ii) is perpendicular to plane
$3x + 2y - 3z = 1$ --- (iv)

So, using (ii),(iv) in (iii),
$a_1a_2 + b_1b_2 + c_1c_2 = 0$
$(a)(3) + (b)(2) + (c)(-3) = 0$

$3a + 2b - 3c = 0$ --- (v)

Also, plane (ii) is perpendicular to plane
$5x - 4y + z = 5$ --- (vi)

So, using (ii),(vi) in (iii),
$a_1a_2 + b_1b_2 + c_1c_2 = 0$
$(a)(5) + (b)(-4) + (c)(1) = 0$

$5a - 4b + c = 0$ --- (vii)

On solving (v) and (vii),

$$\frac{a}{(2)(1) - (-3)(-4)} = \frac{b}{(5)(-3) - (3)(1)} = \frac{c}{(3)(-4) - (2)(5)}$$

$$\frac{a}{2 - 12} = \frac{b}{-15 - 3} = \frac{c}{-12 - 10}$$

$$\frac{a}{-10} = \frac{b}{-18} = \frac{c}{-22} = \lambda \text{ (Say)}$$

$a = -10\lambda,\ b = -18\lambda,\ c = -22\lambda$

Put the value of a, b, c in equation (ii)

$a(x + 1) + b(y + 1) + c(z - 2) = 0$
$(-10\lambda)(x + 1) + (-18\lambda)(y + 1) + (-22\lambda)(z - 2) = 0$
$-10\lambda x - 10\lambda - 18\lambda y - 18\lambda - 22\lambda z + 44\lambda = 0$
$-10\lambda x - 18\lambda y - 22\lambda z + 16\lambda = 0$

Dividing by -2λ,
$5x + 9y + 11z - 8 = 0$

So, equation of required plane is,

$5x + 9y + 11z - 8 = 0$

The Plane 29.6 Q6

We know that equation of a plane passing through $(x_1, y_1 z_1)$ is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \qquad ---(i)$$

Now, equation of plane passing through $(1, -3, -2)$,

$$a(x - 1) + b(y + 3) + c(z + 2) = 0 \qquad ---(ii)$$

We know that planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ $\qquad ---(iii)$

Given, plane (ii) is perpendicular to plane

$x + 2y + 2z = 5 \qquad ---(iv)$

Using equation $(ii), (iv)$ in (iii),

$(a)(1) + (b)(2) + (c)(2) = 0$

$$a + 2b + 2c = 0 \qquad ---(v)$$

Also, plane (ii) is perpendicular to plane

$3x + 3y + 2z = 8 \qquad ---(vi)$

Using equation $(ii), (vi)$ in (iii),

$(a)(3) + (b)(3) + (c)(2) = 0$

$$3a + 3b + 2c = 0 \qquad ---(vii)$$

Solving (v) and (vii) by cross multiplication,

$$\frac{a}{(2)(2) - (3)(2)} = \frac{b}{(3)(2) - (1)(2)} = \frac{c}{(1)(3) - (2)(3)}$$

$$\frac{a}{4 - 6} = \frac{b}{6 - 2} = \frac{c}{3 - 6}$$

$$\frac{a}{-2} = \frac{b}{4} = \frac{c}{-3} = \lambda \text{ (Say)}$$

$a = -2\lambda$, $b = 4\lambda$, $c = -3\lambda$

Put a, b, c in equation (ii)

$$a(x - 1) + b(y + 3) + c(z + 2) = 0$$
$$(-2\lambda)(x - 1) + (4\lambda)(y + 3) + (-3\lambda)(z + 2) = 0$$
$$-2\lambda x + 2\lambda + 4\lambda y + 12\lambda - 3\lambda z - 6\lambda = 0$$
$$-2\lambda x + 4\lambda y - 3\lambda z + 8\lambda = 0$$

Dividing by $(-\lambda)$,

$2x - 4y + 3z - 8 = 0$

Equation of required plane is,

$$2x - 4y + 3z - 8 = 0$$

The Plane 29.6 Q7

We know that equation of a plane passing through a point (x_1, y_1, z_1) is

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \qquad ---(i)$$

Given that, plane is passing through origin, so

$$a(x - 0) + b(y - 0) + c(z - 0) = 0$$
$$ax + by + cz = 0 \qquad ---(ii)$$

We know that planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iii)$

Given that, plane (ii) is perpendicular to plane

$$x + 2y - z = 1 \qquad ---(iv)$$

Using $(ii), (iv)$ in equation (iii),

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(a)(1) + (b)(2) + (c)(-1) = 0$$

$$a + 2b - c = 0 \qquad ---(v)$$

Given, plane (ii) is perpendicular to plane

$$3x - 4y + z = 5 \qquad ---(vi)$$

Using equation $(ii), (vi)$ in (iii),

$$(a)(3) + (b)(-4) + (c)(1) = 0$$

$$3a - 4b + c = 0 \qquad ---(vii)$$

Solving (v) and (vii) by cross multiplication,

$$\frac{a}{(2)(1) - (-4)(-1)} = \frac{b}{(3)(-1) - (1)(1)} = \frac{c}{(1)(-4) - (2)(3)}$$
$$\frac{a}{2 - 4} = \frac{b}{-3 - 1} = \frac{c}{-4 - 6}$$
$$\frac{a}{-2} = \frac{b}{-4} = \frac{c}{-10} = \lambda \text{ (Say)}$$
$$a = -2\lambda,\ b = -4\lambda,\ c = -10\lambda$$

Put a, b, c in equation (ii)

$$ax + by + cz = 0$$
$$-2\lambda x - 4\lambda y - 10\lambda z = 0$$

Dividing by -2λ,

$$x + 2y + 5z = 0$$

Equation of required plane is,

$$x + 2y + 5z = 0$$

The Plane 29.6 Q8

We know that equation of a plane passing through (x_1, y_1, z_1) is given by
$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Given that, plane is passing through $(1, -1, 2)$, so

$a(x - 1) + b(y + 1) + c(z - 2) = 0$ ---(i)

Plane (i) is also passing through $(2, -2, 2)$, so $(2, -2, 2)$ must satisfy the equation (i),
$a(2 - 1) + b(-2 + 1) + c(2 - 2) = 0$
$a - b = 0$ ---(ii)

We know that planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ ---(iii)

Given that, plane (i) is perpendicular to plane
$6x - 2y + 2z - 9 = 0$ ---(iv)

Using plane (i), (iv) in equation (iii),
$a_1a_2 + b_1b_2 + c_1c_2 = 0$
$(a)(6) + (b)(-2) + (c)(2) = 0$

$6a - 2b + 2c = 0$ ---(v)
Solving (ii) and (v) by cross-multiplication,

$$\frac{a}{(-1)(2) - (-2)(0)} = \frac{b}{(6)(0) - (1)(2)} = \frac{c}{(1)(-2) - (6)(-1)}$$
$$\frac{a}{-2 + 0} = \frac{b}{0 - 2} = \frac{c}{-2 + 6}$$
$$\frac{a}{-2} = \frac{b}{-2} = \frac{c}{4} = \lambda \text{ (Say)}$$
$\Rightarrow \quad a = -2\lambda,\ b = -2\lambda,\ c = 4\lambda$

Put a, b, c in equation (i)

$a(x - 1) + b(y + 1) + c(z - 2) = 0$
$(-2\lambda)(x - 1) + (-2\lambda)(y + 1) + (4\lambda)(z - 2) = 0$
$-2\lambda x + 2\lambda - 2\lambda y - 2\lambda + 4\lambda z - 8\lambda = 0$

$-2\lambda x - 2\lambda y + 4\lambda z - 8\lambda = 0$

Dividing by (-2λ),
$x + y - 2z + 4 = 0$

Equation of required plane is,

$x + y - 2z + 4 = 0$

The Plane 29.6 Q9

We know that, equation of plane passing through the point (x_1, y_1, z_1) is given by,

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Here, the plane is passing through $(2, 2, 1)$

$a(x - 2) + b(y - 2) + c(z - 1) = 0 \qquad ---(i)$

It is also passing through $(9, 3, 6)$, so it must satisfy the equation (i),

$a(9 - 2) + b(3 - 2) + c(6 - 1) = 0$

$7a + b + 5c = 0 \qquad ---(ii)$

We know that, plane $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iii)$

Given that, plane (i) is perpendicular to plane

$2x + 6y + 6z = 1 \qquad ---(iv)$

Using plane $(i), (iv)$ in equation (iii),

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(a)(2) + (b)(6) + (c)(6) = 0$

$2a + 6b + 6c = 0 \qquad ---(v)$

Solving (ii) and (v) by cross-multiplication,

$$\frac{a}{(1)(6) - (5)(6)} = \frac{b}{(2)(5) - (7)(6)} = \frac{c}{(7)(6) - (2)(1)}$$

$$\frac{a}{6 - 30} = \frac{b}{10 - 42} = \frac{c}{42 - 2}$$

$$\frac{a}{-24} = \frac{b}{-32} = \frac{c}{40} = \lambda \text{ (Say)}$$

$\Rightarrow \quad a = -24\lambda,\ b = -32\lambda,\ c = 40\lambda$

The Plane 29.6 Q10

We know that, equation of plane passing through the point (x_1, y_1, z_1) is given by,

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Given, the required plane is passing through $(-1, 1, 1)$,

$a(x + 1) + b(y - 1) + c(z - 1) = 0 \qquad ---(i)$

It is also passing through $(1, -1, 1)$, so it must satisfy the equation (i),

$a(1 + 1) + b(-1 - 1) + c(1 - 1) = 0$

$2a - 2b = 0 \qquad ---(ii)$

We know that, plane $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iii)$

Given, plane (i) is perpendicular to plane

$x + 2y + 2z = 5 \qquad ---(iv)$

Using plane $(i), (iv)$ in equation (iii),

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(a)(1) + (b)(2) + (c)(2) = 0$

$a + 2b + 2c = 0 \qquad ---(v)$

Solving (ii) and (v) by cross-multiplication,

$$\frac{a}{(-2)(2) - (2)(0)} = \frac{b}{(1)(0) - (2)(2)} = \frac{c}{(2)(2) - (1)(-2)}$$

$$\frac{a}{-4 - 0} = \frac{b}{0 - 4} = \frac{c}{4 + 2}$$

$$\frac{a}{-4} = \frac{b}{-4} = \frac{c}{6} = \lambda \text{ (Say)}$$

$\Rightarrow \quad a = -4\lambda,\ b = -4\lambda,\ c = 6\lambda$

Put the value of a, b, c in equation (i),

$$a(x+1)+b(y-1)+c(z-1)=0$$
$$(-4\lambda)(x+1)+(-4\lambda)(y-1)+(6\lambda)(z-1)=0$$
$$-4\lambda x+4\lambda-4\lambda y+4\lambda+6\lambda z-6\lambda=0$$

$$-4\lambda x-4\lambda y+6\lambda z-6\lambda=0$$

Dividing by (-2λ), we get
$$2x+2y-3z+3=0$$

The equation of required plane is,

$$2x+2y-3z+3=0$$

The Plane Ex 29.6 Q11

The equation of the plane parallel to ZOX is $y = \text{constant}$.
Given that the $y-$intercept is 3.
Thus the equation of the plane is $y = 3$.

The Plane Ex 29.6 Q12

The equation of any plane passing through $(1, -1, 2)$
is $a(x-1)+b(y+1)+c(z-2)=0 \ldots(1)$
Given that, plane (1) is perpendicular to the planes
$2x+3y-2z=5$
and
$x+2y-3z=8$
Therefore, we have,
$2a+3b-2c=0 \ldots(2)$
and
$a+2b-3c=0 \ldots(3)$
Solving equations (2) and (3) by cross multiplication, we have,
$$\frac{a}{3\times(-3)-2\times(-2)}=\frac{b}{1\times(-2)-2\times(-3)}=\frac{c}{2\times2-1\times3}=\lambda(\text{say})$$
$$\Rightarrow \frac{a}{-9+4}=\frac{b}{-2+6}=\frac{c}{4-3}=\lambda$$
$$\Rightarrow \frac{a}{-5}=\frac{b}{4}=\frac{c}{1}=\lambda$$
Thus, we have,
$a=-5\lambda, b=4\lambda$ and $c=\lambda$
Substituting the above values in equation (1), we have,
$-5\lambda(x-1)+4\lambda(y+1)+\lambda(z-2)=0$
Since $\lambda \neq 0$, we have,
$-5(x-1)+4(y+1)+(z-2)=0$
$\Rightarrow -5x+5+4y+4+z-2=0$
$\Rightarrow -5x+4y+z+7=0$
$\Rightarrow 5x-4y-z-7=0$
$\Rightarrow 5x-4y-z=7$
Thus the required equation of the plane is $5x-4y-z=7$

The Plane Ex 29.6 Q13

Given that the equation of the required
plane is parallel to the plane
$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=2 \ldots(1)$
$\therefore$ Vector equation of any plane parallel to (1) is
$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=k \ldots(2)$
Since the given plane passes through (a, b, c), then
$(a\hat{i}+b\hat{j}+c\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k})=k$
$\Rightarrow a+b+c=k \ldots(3)$

Substituting the above value of k in equation (2), we have,
$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=a+b+c$
Thus the required equation of the plane is $x+y+z=a+b+c$

The Plane Ex 29.6 Q14

The equation of any plane passing through $(-1,3,2)$

is $a(x+1)+b(y-3)+c(z-2)=0 \ldots.(1)$

Given that, Plane (1) is perpendicular to the planes

$x+2y+3z=5$

and

$3x+3y+z=0$

Therefore, we have,

$a+2b+3c=0 \ldots(2)$

and

$3a+3b+c=0 \ldots(3)$

Solving equations (2) and (3) by cross multiplication, we have,

$$\frac{a}{2\times1-3\times3}=\frac{b}{3\times3-1\times1}=\frac{c}{1\times3-3\times2}=\lambda(\text{say})$$

$$\Rightarrow \frac{a}{2-9}=\frac{b}{9-1}=\frac{c}{3-6}=\lambda$$

$$\Rightarrow \frac{a}{-7}=\frac{b}{8}=\frac{c}{-3}=\lambda$$

Thus, we have,

$a=-7\lambda, b=8\lambda$ and $c=-3\lambda$

Substituting the above values in equation (1), we have,

$-7\lambda(x+1)+8\lambda(y-3)-3\lambda(z-2)=0$

Since $\lambda \neq 0$, we have,

$-7(x+1)+8(y-3)-3(z-2)=0$

$\Rightarrow -7x-7+8y-24-3z+6=0$

$\Rightarrow -7x+8y-3z-25=0$

$\Rightarrow 7x-8y+3z+25=0$

Thus the required equation of the plane is $7x-8y+3z+25=0$

The Plane Ex 29.6 Q15

The equation of any plane passing through $(2,1,-1)$

is $a(x-2)+b(y-1)+c(z+1)=0 \ldots.(1)$

Also, the above plane passes through the point $(-1,3,4)$.

Thus, equation (1), becomes,

$a(-1-2)+b(3-1)+c(4+1)=0$

$\Rightarrow -3a+2b+5c=0 \ldots(2)$

Given that, Plane (1) is perpendicular to the plane

$x-2y+4z=10$

Therefore, we have,

$a-2b+4c=0 \ldots(3)$

Solving equations (2) and (3) by cross multiplication, we have,

$$\frac{a}{2\times4-5\times(-2)}=\frac{b}{1\times5-(-3)\times4}=\frac{c}{(-3)\times(-2)-1\times2}=\lambda(\text{say})$$

$$\Rightarrow \frac{a}{8+10}=\frac{b}{5+12}=\frac{c}{6-2}=\lambda$$

$$\Rightarrow \frac{a}{18}=\frac{b}{17}=\frac{c}{4}=\lambda$$

Thus, we have,

$a=18\lambda, b=17\lambda$ and $c=4\lambda$

Substituting the above values in equation (1), we have,

$18\lambda(x-2)+17\lambda(y-1)+4\lambda(z+1)=0$

Since $\lambda \neq 0$, we have,

$18(x-2)+17(y-1)+4(z+1)=0$

$\Rightarrow 18x-36+17y-17+4z+4=0$

$\Rightarrow 18x+17y+4z-49=0$

Thus the required equation of the plane is $18x+17y+4z-49=0$

Ex - 29.7

The Plane 29.7 Q1(i)

Here, $\vec{r} = \left(2\hat{i} - \hat{k}\right) + \lambda\hat{i} + \mu\left(\hat{i} - 2\hat{j} - \hat{k}\right)$

We know that, $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represent a plane passing through a point having position vector $\vec{a}$ and parallel to vectors $\vec{b}$ and $\vec{c}$.

Here, $\vec{a} = 2\hat{i} - \hat{k}$, $\vec{b} = \hat{i}$, $\vec{c} = \hat{i} - 2\hat{j} - \hat{k}$

The given plane is perpendicular to a vector

$$\vec{n} = \vec{b} \times \vec{c}$$
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= \hat{i}\left(0 - 0\right) - \hat{j}\left(-1 - 0\right) + \hat{k}\left(-2 - 0\right)$$
$$= 0\hat{i} + \hat{j} - 2\hat{k}$$

$\vec{n} = \hat{j} - 2\hat{k}$.

We know that vector equation of plane in scalar product form is,
$\vec{r}.\vec{n} = \vec{a}.\vec{n}$ $\quad$ ---(i)

Put $\vec{n}$ and $\vec{a}$ in equation (i),

$$\vec{r}.\left(\hat{j} - 2\hat{k}\right) = \left(2\hat{i} - \hat{k}\right)\left(\hat{j} - 2\hat{k}\right)$$
$$\vec{r}.\left(\hat{j} - 2\hat{k}\right) = (2)(0) + (0)(1) + (-1)(-2)$$
$$= 0 + 0 + 2$$
$$\vec{r}.\left(\hat{j} - 2\hat{k}\right) = 2$$

The equation in required form is,

$$\vec{r}.\left(\hat{j} - 2\hat{k}\right) = 2$$

The Plane 29.7 Q1(ii)

Here, $\vec{r} = (1 + s - t)\hat{i} + (2 - s)\hat{j} + (3 - 2s + 2t)\hat{k}$

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + s\left(\hat{i} - \hat{j} - 2\hat{k}\right) + t\left(-\hat{i} + 2\hat{k}\right)$$

We know that, $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represent a plane passing through a point having position vector $\vec{a}$ and parallel to vectors $\vec{b}$ and $\vec{c}$.

Here, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} - \hat{j} - 2\hat{k}$, $\vec{c} = -\hat{i} + 2\hat{k}$

The given plane is perpendicular to a vector

$$\vec{n} = \vec{b} \times \vec{c}$$
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -2 \\ -1 & 0 & 2 \end{vmatrix}$$

$$= \hat{i}\left(-2 - 0\right) - \hat{j}\left(2 - 2\right) + \hat{k}\left(0 - 1\right)$$

$\vec{n} = -2\hat{i} - \hat{k}$

We know that, vector equation of a plane in scalar product form is,
$\vec{r}.\vec{n} = \vec{a}.\vec{n}$ $\quad$ ---(i)

Put value of $\vec{a}$ and $\vec{n}$ in equation (i),

$$\vec{r}.\left(-2\hat{i}-\hat{k}\right) = \left(-2\hat{i}-\hat{k}\right)\left(\hat{i}+2\hat{j}+3\hat{k}\right)$$

$$\vec{r}.\left(-2\hat{i}-\hat{k}\right) = (-2)(1)+(0)(2)+(-1)(3)$$

$$= -2+0-3$$

$$\vec{r}.\left(-2\hat{i}-\hat{k}\right) = -5$$

Multiplying both the sides by (-1),

$$\vec{r}.\left(2\hat{i}+\hat{k}\right) = 5$$

The equation in the required form,

$$\vec{r}.\left(2\hat{i}+\hat{k}\right) = 5$$

The Plane 29.7 Q1(iii)

Given, equation of plane,

$$\vec{r} = \left(\hat{i}+\hat{j}\right)+\lambda\left(\hat{i}+2\hat{j}-\hat{k}\right)+\mu\left(-\hat{i}+\hat{j}-2\hat{k}\right)$$

We know that, $\vec{r} = \vec{a}+\lambda\vec{b}+\mu\vec{c}$ is the equation of a plane passing through point $\vec{a}$ and parallel to $\vec{b}$ and $\vec{c}$.

Here, $\vec{a} = \hat{i}+\hat{j}$, $\vec{b} = \hat{i}+2\hat{j}-\hat{k}$, $\vec{c} = -\hat{i}+\hat{j}-2\hat{k}$

The given plane is perpendicular to a vector

$$\vec{n} = \vec{b}\times\vec{c}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix}$$

$$= \hat{i}(-4+1)-\hat{j}(-2-1)+\hat{k}(1+2)$$

$$= -3\hat{i}+3\hat{j}+3\hat{k}$$

We know that, the equation of plane in scalar product form is given by,
$\vec{r}.\vec{n} = \vec{a}\vec{n}$

$$\vec{r}.\left(-3\hat{i}+3\hat{j}+3\hat{k}\right) = \left(\hat{i}+\hat{j}\right)\left(-3\hat{i}+3\hat{j}+3\hat{k}\right)$$

$$= (1)(-3)+(1)(3)+(0)(3)$$

$$= -3+3$$

$$\vec{r}.\left(-3\hat{i}+3\hat{j}+3\hat{k}\right) = 0$$

Dividing by 3, we get

$$\vec{r}.\left(-\hat{i}+\hat{j}+\hat{k}\right) = 0$$

Equation in required form is,

$$\vec{r}.\left(-\hat{i}+\hat{j}+\hat{k}\right) = 0$$

The Plane 29.7 Q1(iv)

$\bar{r} = \hat{i} - \hat{j} + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(4i - 2\hat{j} + 3k)$

Plane is passing through $(\hat{i} - \hat{j})$and parallel to

$b(\hat{i} + \hat{j} + \hat{k})$ and $c(4i - 2\hat{j} + 3k)$

$$n = b \times c = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 4 & -2 & 3 \end{vmatrix}$$

$n = 5i + j - 6k$

$\bar{r}.n = (\hat{i} - \hat{j}).(5i + j - 6k)$=5-1=4

$\bar{r}.(5i + j - 6k) = 4$

The Plane 29.7 Q2(i)

Here, given equation of plane is,

$$\vec{r} = \left(\hat{i} - \hat{j}\right) + s\left(-\hat{i} + \hat{j} + 2\hat{k}\right) + t\left(\hat{i} + 2\hat{j} + \hat{k}\right)$$

We know that, $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represents the equation of a plane passing through a vector $\vec{a}$ and parallel to vector $\vec{b}$ and $\vec{c}$.

Here, $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = -\hat{i} + \hat{j} + 2\hat{k}$, $\vec{c} = \hat{i} + 2\hat{j} + \hat{k}$

Given plane is perpendicular to vector

$$\vec{n} = \vec{b} \times \vec{c}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}\,(1 - 4) - \hat{j}\,(-1 - 2) + \hat{k}\,(-2 - 1)$$

$$\vec{n} = -3\hat{i} + 3\hat{j} - 3\hat{k}$$

We know that, equation of plane in the scalar product form,

$$\vec{r}.\vec{n} = \vec{a}.\vec{n} \qquad ---(i)$$

Put the value of $\vec{a}$ and $\vec{n}$ in equation (i),

$$\vec{r}.\left(\hat{i} - \hat{j}\right) = \left(\hat{i} - \hat{j}\right)\left(-3\hat{i} + 3\hat{j} - 3\hat{k}\right)$$

$$\vec{r}.\left(-3\hat{i}+3\hat{j}-3\hat{k}\right)=(1)(-3)+(-1)(3)+(0)(-3)$$
$$=-3-3+0$$

$$\vec{r}.\left(-3\hat{i}+3\hat{j}-3\hat{k}\right)=-6$$

Put $\vec{r}.=x\hat{i}+y\hat{j}+z\hat{k}$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(-3\hat{i}+3\hat{j}-3\hat{k}\right)=-6$$
$$(x)(-3)+(y)(3)+(z)(-3)=-6$$

$$-3x+3y-3z=-6$$

Dividing by (-3), we get

$$x-y+z=2$$

Equation in required form is,

$$x-y+z=2$$

The Plane 29.7 Q2(ii)

Given, equation of plane,

$$\vec{r}=(1+s+t)\hat{i}+(2-s+t)\hat{j}+(3-2s+2t)\hat{k}$$
$$=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+s\left(\hat{i}-\hat{j}-2\hat{k}\right)+t\left(\hat{i}+\hat{j}+2\hat{k}\right)$$

We know that, $\vec{r}=\vec{a}+\lambda\vec{b}+\mu\vec{c}$ represents the equation of a plane passing through the vector $\vec{a}$ and parallel to vector $\vec{b}$ and $\vec{c}$.

Here, $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$
$\vec{b}=\hat{i}-\hat{j}-2\hat{k}$
$\vec{c}=\hat{i}+\hat{j}+2\hat{k}$

The given plane is perpendicular to vector

$$\vec{n}=\vec{b}\times\vec{c}$$
$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & -2\\ 1 & 1 & 2\end{vmatrix}$$

$$=\hat{i}(-2+2)-\hat{j}(2+2)+\hat{k}(1+1)$$
$$=0.\left(\hat{i}\right)-4\hat{j}+2\hat{k}$$
$$\vec{n}=-4\hat{j}+2\hat{k}$$

We know that, equation of plane in scalar product form is given by,

$$\vec{r}.\vec{n}=\vec{a}.\vec{n} \qquad ---(i)$$

Put, the value of $\vec{a}$ and $\vec{n}$ in (i),

$$\vec{r}.\left(-4\hat{j}+2\hat{k}\right)=\left(\hat{i}+2\hat{j}+3\hat{k}\right)\left(-4\hat{j}+2\hat{k}\right)$$

$$\vec{r}.\left(-4\hat{j}+2\hat{k}\right)=(1)(0)+(2)(-4)+(3)(2)$$
$$=0-8+6$$

$$\vec{r}.\left(-4\hat{j}+2\hat{k}\right)=-2$$

Put $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(-4\hat{j}+2\hat{k}\right)=-2$$
$$(x)(0)+(y)(-4)+(z)(2)=-2$$

$$-4y+2z=-2$$

Dividing by (-2), we get

$$2y-z=1$$

The equation in required form is,

$$2y-z=1$$

The Plane 29.7 Q3(i)

Given, equation of plane is,

$$\vec{r}=(\lambda-2\mu)\hat{i}+(3-\mu)\hat{j}+(2\lambda+\mu)\hat{k}$$
$$\vec{r}=\left(3\hat{j}\right)+\lambda\left(\hat{i}+2\hat{k}\right)+\mu\left(-2\hat{i}-\hat{j}+\hat{k}\right)$$

We know that, $\vec{r}=\vec{a}+\lambda\vec{b}+\mu\vec{c}$ represents the equation of a plane passing through a point $\vec{a}$ and parallel to vector $\vec{b}$ and $\vec{c}$.

Given, $\vec{a}=3\hat{j}$
$\vec{b}=\hat{i}+2\hat{k}$
$\vec{c}=-2\hat{i}-\hat{j}+\hat{k}$

The given plane is perpendicular to

$$\vec{n}=\vec{b}\times\vec{c}$$
$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 0 & 2\\ -2 & -1 & 1\end{vmatrix}$$

$$=\hat{i}(0+2)-\hat{j}(1+4)+\hat{k}(-1-0)$$

$$\vec{n}=2\hat{i}-5\hat{j}-\hat{k}$$

Vector equation of plane in non-parametric form is,

$\vec{r}.\vec{n} = \vec{a}.\vec{n}$

$\vec{r}.\left(2\hat{i} - 5\hat{j} - \hat{k}\right) = \left(3\hat{j}\right)\left(2\hat{i} - 5\hat{j} - \hat{k}\right)$

$= (0)(2) + (3)(-5) + (0)(-1)$

$= 0 - 15 + 0$

$\vec{r}.\left(2\hat{i} - 5\hat{j} - \hat{k}\right) = -15$

$\vec{r}.\left(2\hat{i} - 5\hat{j} - \hat{k}\right) + 15 = 0$

The required form of equation is,

$\vec{r}.\left(2\hat{i} - 5\hat{j} - \hat{k}\right) + 15 = 0$

The Plane 29.7 Q3(ii)

Given, equation of plane is,

$\vec{r} = \left(2\hat{i} + 2\hat{j} - \hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \mu\left(5\hat{i} - 2\hat{j} + 7\hat{k}\right)$

We know that, $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represents the equation of a plane passing through a vector $\vec{a}$ and parallel to vector $\vec{b}$ and $\vec{c}$.

Here, $\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{c} = 5\hat{i} - 2\hat{j} + 7\hat{k}$

The given plane is perpendicular to vector

$\vec{n} = \vec{b} \times \vec{c}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix}$

$= \hat{i}\,(14 + 6) - \hat{j}\,(7 - 15) + \hat{k}\,(-2 - 10)$

$\vec{n} = 20\hat{i} + 8\hat{j} - 12\hat{k}$

We know that, equation of a plane in non-parametric form is given by,

$\vec{r}.\vec{n} = \vec{a}.\vec{n}$

$\vec{r}.\left(20\hat{i} + 8\hat{j} - 12\hat{k}\right) = \left(2\hat{i} + 2\hat{j} - \hat{k}\right)\left(20\hat{i} + 8\hat{j} - 12\hat{k}\right)$

$= (2)(20) + (2)(8) + (-1)(-12)$

$= 40 + 16 + 12$

$\vec{r}.\left(20\hat{i} + 8\hat{j} - 12\hat{k}\right) = 68$

Dividing by 4,

$\vec{r}.\left(5\hat{i} + 2\hat{j} - 3\hat{k}\right) = 17$

Equation of plane in required form is,

$\vec{r}.\left(5\hat{i} + 2\hat{j} - 3\hat{k}\right) = 17$

Ex - 29.8

The Plane 29.8 Q1

Given, equation of plane is

$2x - 3y + z = 0$ --- (i)

We know that equation of a plane parallel the plane (i) is given by

$2x - 3y + z + \lambda = 0$ --- (ii)

Given that, plane (ii) is passing through the point $(1, -1, 2)$ so it must satisfy the equation (ii),

$2(1) - 3(-1) + (2) + \lambda = 0$
$2 + 3 + 2 + \lambda = 0$
$7 + \lambda = 0$

$\lambda = -7$

Put the value of λ in equation (ii),

$2x - 3y + z - 7 = 0$

So, equation of the required plane is,

$2x - 3y + z = 7$

The Plane 29.8 Q2

Given, equation of plane is

$\vec{r}.\left(2\hat{i} - 3\hat{j} + 5\hat{k}\right) + 2 = 0$ --- (i)

We know that equation of a plane parallel to the plane (i) is given by

$\vec{r}.\left(2\hat{i} - 3\hat{j} + 5\hat{k}\right) + \lambda = 0$ --- (ii)

Given that, plane (ii) is passing through vector $\left(3\hat{i} + 4\hat{j} - \hat{k}\right)$ so it must satisfy equation (ii),

$\left(3\hat{i} + 4\hat{j} - \hat{k}\right)\left(2\hat{i} - 3\hat{j} + 5\hat{k}\right) + \lambda = 0$
$(3)(2) + (4)(-3) + (-1)(5) + \lambda = 0$
$6 - 12 - 5 + \lambda = 0$
$-11 + \lambda = 0$

$\lambda = 11$

Put the value of λ in equation (ii),

$\vec{r}.\left(2\hat{i} - 3\hat{j} + 5\hat{k}\right) + 11 = 0$

Equation of required plane is,

$\vec{r}.\left(2\hat{i} - 3\hat{j} + 5\hat{k}\right) + 11 = 0$

The Plane 29.8 Q3

We know that, equation of a plane passing through the line of intersection of two planes

$a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0$

Given, equations of plane is,

$2x - 7y + 4z - 3 = 0$ and
$3x - 5y + 4z + 11 = 0$

So, equation of plane passing through the line of intersection of given two planes is

$(2x - 7y + 4z - 3) + \lambda(3x - 5y + 4z + 11) = 0$
$2x - 7y + 4z - 3 + 3\lambda x - 5\lambda y + 4\lambda z + 11\lambda = 0$
$x(2 + 3\lambda) + y(-7 - 5\lambda) + z(4 + 4\lambda) - 3 + 11\lambda = 0 \qquad ---(i)$

Plane (1) is passing through the points $(-2,1,3)$, so it satisfies the equation (i),

$(-2)(2 + 3\lambda) + (1)(-7 - 5\lambda) + (3)(4 + 4\lambda) - 3 + 11\lambda = 0$
$-4 - 6\lambda - 7 - 5\lambda + 12 + 12\lambda - 3 + 11\lambda = 0$
$-2 + 12\lambda = 0$
$12\lambda = 2$
$\lambda = \frac{2}{12}$

$\lambda = \frac{1}{6}$

Put λ in equation (i),

$x(2 + 3\lambda) + y(-7 - 5\lambda) + z(4 + 4\lambda) - 3 + 11\lambda = 0$
$x\left(2 + \frac{3}{6}\right) + y\left(-7 - \frac{5}{6}\right) + z\left(4 + \frac{4}{6}\right) - 3 + \frac{11}{6} = 0$
$x\left(\frac{12 + 3}{6}\right) + y\left(\frac{-42 - 5}{6}\right) + z\left(\frac{24 + 4}{6}\right) - \frac{18 + 11}{6} = 0$

$\frac{15}{6}x - \frac{47}{6}y + \frac{28}{6}z - \frac{7}{6} = 0$

Multiplying by 6, we get

$15x - 47y + 28z - 7 = 0$

Therefore, equation of required plane is,

$15x - 47y + 28z = 7$

The Plane 29.8 Q4

We know that, equation of a plane passing the line of intersection of planes

$\vec{r}.\vec{n_1} = d_1$ and $\vec{r}.\vec{n_2} = d_2$ is given by
$\vec{r}.\left(\vec{n_1} + \lambda \vec{n_2}\right) = d_1 + \lambda d_2$

So, equation of plane through the line of intersection of planes $\vec{r}.\left(\hat{i} + 3\hat{j} - \hat{k}\right) = 0$

and $\vec{r}.\left(\hat{j} + 2\hat{k}\right) = 0$ is given by

$$\vec{r}.\left[\left(\hat{i} + 3\hat{j} - \hat{k}\right) + \lambda\left(\hat{j} + 2\hat{k}\right)\right] = 0 \qquad ---(i)$$

Given that plane (i) is passing through the point $\left(2\hat{i} + \hat{j} - \hat{k}\right)$, so

$$\left(2\hat{i} + \hat{j} - \hat{k}\right)\left(\hat{i} + 3\hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} - \hat{k}\right)\left(\hat{j} + 2\hat{k}\right) = 0$$
$$(2)(1) + (1)(3) + (-1)(-1) + \lambda\left[(2)(0) + (1)(1) + (-1)(2)\right] = 0$$
$$(2 + 3 + 1) + \lambda(1 - 2) = 0$$
$$6 - \lambda = 0$$

$$\lambda = 6$$

Put λ in equation (i),

$$\vec{r}.\left[\left(\hat{i} + 3\hat{j} - \hat{k}\right) + \lambda\left(\hat{j} + 2\hat{k}\right)\right] = 0$$
$$\vec{r}.\left[\hat{i} + 3\hat{j} - \hat{k} + 6\left(\hat{j} + 2\hat{k}\right)\right] = 0$$
$$\vec{r}.\left[\hat{i} + 3\hat{j} - \hat{k} + 6\hat{j} + 12\hat{k}\right] = 0$$

$$\vec{r}.\left(\hat{i} + 9\hat{j} + 11\hat{k}\right) = 0$$

So, equation of required plane is,

$$\vec{r}.\left(\hat{i} + 9\hat{j} + 11\hat{k}\right) = 0$$

The Plane 29.8 Q5

We know that, equation of a plane passing through the line of intersection of $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by
$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$

So, equation of plane passing through the line of intersection of plane
$2x - y = 0$ and $3z - y = 0$ is
$$(2x - y) + \lambda(3z - y) = 0$$
$$2x - y + 3\lambda z - \lambda y = 0$$
$$x(2) + y(-1 - \lambda) + z(3\lambda) = 0 \qquad ---(i)$$

We know that, two planes are perpendicular if
$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(ii)$$

Given, plane (i) is perpendicular to plane

$$4x + 5y - 3z = 8 \qquad ---(iii)$$

Using (i) and (iii) in equation (ii),
$$(2)(4) + (-1 - \lambda)(5) + (3\lambda)(-3) = 0$$
$$8 - 5 - 5\lambda - 9\lambda = 0$$
$$3 - 14\lambda = 0$$
$$-14\lambda = -3$$

$$\lambda = \frac{3}{14}$$

Put the value of λ in equation (i),
$$2x + y(-1 - \lambda) + z(3\lambda) = 0$$
$$2x + y\left(-1 - \frac{3}{14}\right) + z.3\left(\frac{3}{14}\right) = 0$$
$$2x + y\left(\frac{-14 - 3}{14}\right) + \frac{9z}{14} = 0$$
$$2x + y\left(-\frac{17}{14}\right) + \frac{9z}{14} = 0$$

Multiplying with 14, we get

$$28x - 17y + 9z = 0$$

Equation of required plane is,

$$28x - 17y + 9z = 0$$

The Plane 29.8 Q6

We know that, the equation plane passing through the line of intersection of plane $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

Here, equation of plane passing through the intersection of plane $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ is given by,

$$\left(x + 2y + 3z - 4\right) + \lambda\left(2x + y - z + 5\right) = 0$$
$$x + 2y + 3z - 4 + 2\lambda x + \lambda y - \lambda z + 5\lambda = 0$$
$$x\left(1 + 2\lambda\right) + y\left(2 + \lambda\right) + z\left(3 - \lambda\right) - 4 + 5\lambda = 0 \qquad ---\text{(i)}$$

We know, that two planes are perpendicular if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---\text{(ii)}$$

Given that plane (i) is perpendicular to plane,

$$5x + 3y - 6z + 8 = 0 \qquad ---\text{(iii)}$$

Using plane (i) and (iii) in equation (ii),

$$\left(5\right)\left(1 + 2\lambda\right) + \left(3\right)\left(2 + \lambda\right) + \left(-6\right)\left(3 - \lambda\right) = 0$$
$$5 + 10\lambda + 6 + 3\lambda - 18 + 6\lambda = 0$$
$$-7 + 19\lambda = 0$$
$$19\lambda = 7$$

$$\lambda = \frac{7}{19}$$

Put value of λ in equation (i),

$$x\left(1 + 2\lambda\right) + y\left(2 + \lambda\right) + z\left(3 - \lambda\right) - 4 + 5\lambda = 0$$
$$x\left(1 + \frac{14}{19}\right) + y\left(2 + \frac{7}{19}\right) + z\left(3 - \frac{7}{19}\right) - 4 + \frac{35}{19} = 0$$
$$x\left(\frac{19 + 14}{19}\right) + y\left(\frac{38 + 7}{19}\right) + z\left(\frac{57 - 7}{19}\right)\frac{-76 + 35}{19} = 0$$

$$x\left(\frac{33}{19}\right) + y\left(\frac{45}{19}\right) + z\left(\frac{50}{19}\right) - \frac{41}{19} = 0$$

Multiplying by 19, we get

$$33x + 45y + 50z - 41 = 0$$

Equation of required plane is,

$$33x + 45y + 50z - 41 = 0$$

The Plane 29.8 Q7

We know that, equation of a plane passing through the line of intersection of planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by,

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

So, equation of plane passing through the line of intersection of planes $x + 2y + 3z + 4 = 0$ and $x - y + z + 3 = 0$ is

$$\left(x + 2y + 3z + 4\right) + \lambda\left(x - y + z + 3\right) = 0$$
$$x\left(1 + \lambda\right) + y\left(2 - \lambda\right) + z\left(3 + \lambda\right) + 4 + 3\lambda = 0 \qquad ---(i)$$

Equation (i) is passing through origin, so

$$(0)\left(1 + \lambda\right) + (0)\left(2 - \lambda\right) + (0)\left(3 + \lambda\right) + 4 + 3\left(\lambda\right) = 0$$
$$0 + 0 + 0 + 4 + 3\lambda = 0$$
$$3\lambda = -4$$

$$\lambda = -\frac{4}{3}$$

Put the value of λ in equation (i),

$$x\left(1 + \lambda\right) + y\left(2 - \lambda\right) + z\left(3 + \lambda\right) + 4 + 3\lambda = 0$$
$$x\left(1 - \frac{4}{3}\right) + y\left(2 + \frac{4}{3}\right) + z\left(3 - \frac{4}{3}\right) + 4 - \frac{12}{3} = 0$$
$$x\left(\frac{3-4}{3}\right) + y\left(\frac{6+4}{3}\right) + z\left(\frac{9-4}{3}\right) + 4 - 4 = 0$$

$$-\frac{x}{3} + \frac{10y}{3} + \frac{5z}{3} = 0$$

Multiplying by 3, we get

$$-x + 10y + 5z = 0$$
$$x - 10y - 5z = 0$$

The equation of required plane is,

$$x - 10y - 5z = 0$$

The Plane 29.8 Q8

We know that equation of plane passing through the line of intersection of planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by,

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

So, equation of plane passing through the line of intersection of planes $x - 3y + 2z - 5 = 0$ and $2x - y + 3z - 1 = 0$ is given by

$$\left(x - 3y + 2z - 5\right) + \lambda\left(2x - y + 3z - 1\right) = 0$$
$$x\left(1 + 2\lambda\right) + y\left(-3 - \lambda\right) + z\left(2 + 3\lambda\right) - 5 - \lambda = 0 \qquad ---(\text{i})$$

Plane (i) is passing through the point $(1, -2, 3)$ so,

$$(1)(1 + 2\lambda) + (-2)(-3 - \lambda) + (3)(2 + 3\lambda) - 5 - \lambda = 0$$
$$1 + 2\lambda + 6 + 2\lambda + 6 + 9\lambda - 5 - \lambda = 0$$
$$8 + 12\lambda = 0$$
$$12\lambda = -8$$
$$\lambda = -\frac{8}{12}$$
$$\lambda = -\frac{2}{3}$$

Put the value of λ in equation (i),

$$x\left(1 + 2\lambda\right) + y\left(-3 - \lambda\right) + z\left(2 + 3\lambda\right) - 5 - \lambda = 0$$
$$x\left(1 - \frac{4}{3}\right) + y\left(-3 + \frac{2}{3}\right) + z\left(2 - \frac{6}{3}\right) - 5 + \frac{2}{3} = 0$$
$$x\left(\frac{3 - 4}{3}\right) + y\left(\frac{-9 + 2}{3}\right) + z\left(\frac{6 - 6}{3}\right)\frac{-15 + 2}{3} = 0$$

$$-\frac{1}{3}x - \frac{7}{3}y + z(0) - \frac{13}{3} = 0$$

Multiplying by (-3),

$$x + 7y + 13 = 0$$
$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} + 7\hat{j}\right) + 13 = 0$$

$$\vec{r}\left(\hat{i} + 7\hat{j}\right) + 13 = 0$$

Equation of required plane is,

$$\vec{r}\left(\hat{i} + 7\hat{j}\right) + 13 = 0$$

The Plane 29.8 Q9

We know that, equation of plane passing through the line of intersection of planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

So, equation of plane passing through the line of intersection of planes is $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ is given by,

$$\left(x + 2y + 3z - 4\right) + \lambda\left(2x + y - z + 5\right) = 0$$
$$x\left(1 + 2\lambda\right) + y\left(2 + \lambda\right) + z\left(3 - \lambda\right) - 4 + 5\lambda = 0 \qquad ---(\text{i})$$

We know that two planes are perpendicular if
$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(\text{ii})$$

Given that plane (i) is perpendicular to plane,

$$5x + 3y + 6z + 8 = 0 \qquad ---(\text{iii})$$

Using (i) and (iii) in equation (ii),
$$\left(5\right)\left(1 + 2\lambda\right) + \left(3\right)\left(2 + \lambda\right) + \left(6\right)\left(3 - \lambda\right) = 0$$
$$5 + 10\lambda + 6 + 3\lambda + 18 - 6\lambda = 0$$
$$29 + 7\lambda = 0$$
$$7\lambda = -29$$

$$\lambda = -\frac{29}{7}$$

Put the value of λ in equation (i),
$$x\left(1 + 2\lambda\right) + y\left(2 + \lambda\right) + z\left(3 - \lambda\right) - 4 + 5\lambda = 0$$
$$x\left(1 - \frac{58}{7}\right) + y\left(2 - \frac{29}{7}\right) + z\left(3 + \frac{29}{7}\right) - 4 - \frac{145}{7} = 0$$
$$x\left(\frac{7 - 58}{7}\right) + y\left(\frac{14 - 29}{7}\right) + z\left(\frac{21 + 29}{7}\right)\frac{-28 - 145}{7} = 0$$

$$x\left(-\frac{51}{7}\right) + y\left(-\frac{15}{7}\right) + z\left(\frac{50}{7}\right) - \frac{173}{7} = 0$$

The Plane 29.8 Q10

$$\vec{r}.(\hat{i} + 3\hat{j}) + 6 = 0 \text{ and } \vec{r}.(3\hat{i} - \hat{j} - 4\hat{k}) = 0$$
$$x + 3y + 6 = 0;\ 3x - y - 4z = 0$$
$$x + 3y + 6 + \lambda(3x - y - 4z) = 0$$
$$x(1 + 3\lambda) + y(3 - \lambda) + -4z\lambda + 6 = 0$$
$$\text{Distance from origin to plane} = \left|\frac{6}{\sqrt{(1 + 3\lambda)^2 + (3 - \lambda)^2 + (4\lambda)^2}}\right| = 1$$
$$36 = (1 + 3\lambda)^2 + (3 - \lambda)^2 + (4\lambda)^2$$
$$36 = 1 + 6\lambda + 9\lambda^2 + 9 - 6\lambda + \lambda^2 + 16\lambda^2$$
$$26 = 26\lambda^2$$
$$\lambda^2 = 1$$
$$\lambda = \pm 1$$

Case : 1 $\lambda = 1$
$$x + 3y + 6 + 1(3x - y - 4z) = 0$$
$$4x + 2y - 4z + 6 = 0$$

Case : 2 $\lambda = -1$
$$x + 3y + 6 - 1(3x - y - 4z) = 0$$
$$2x - 4y - 4z - 6 = 0$$

The Plane 29.8 Q11

We know that equation of a plane passing through the line of intersection of two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

So, equation of plane passing through the planes $2x + 3y - z + 1 = 0$ and $x + y - 2z + 3 = 0$ is

$$\left(2x + 3y - z + 1\right) + \lambda\left(x + y - 2z + 3\right) = 0$$
$$x\left(2 + \lambda\right) + y\left(3 + \lambda\right) + z\left(-1 - 2\lambda\right) + 1 + 3\lambda = 0 \qquad ---(i)$$

We know that two planes are perpendicular if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(ii)$$

Given, plane (i) is perpendicular to the plane,

$$3x - y - 2z - 4 = 0 \qquad ---(iii)$$

Using (i) and (iii) in equation (ii),

$$(3)(2 + \lambda) + (-1)(3 + \lambda) + (-2)(-1 - 2\lambda) = 0$$
$$6 + 3\lambda - 3 - \lambda + 2 + 4\lambda = 0$$
$$6\lambda + 5 = 0$$
$$6\lambda = -5$$

$$\lambda = -\frac{5}{6}$$

Put the value of λ in equation (i),

$$x\left(2 + \lambda\right) + y\left(3 + \lambda\right) + z\left(-1 - 2\lambda\right) + 1 + 3\lambda = 0$$
$$x\left(2 - \frac{5}{6}\right) + y\left(3 - \frac{5}{6}\right) + z\left(-1 + \frac{10}{6}\right) + 1 - \frac{15}{6} = 0$$
$$x\left(\frac{12 - 5}{6}\right) + y\left(\frac{18 - 5}{6}\right) + z\left(\frac{-6 + 10}{6}\right) + \frac{6 - 15}{6} = 0$$

$$\frac{7x}{6} + \frac{13y}{6} + \frac{4z}{6} - \frac{9}{6} = 0$$

The Plane 29.8 Q12

We know that, equation of a plane passing through the line of intersection of plane

$\vec{r}.\vec{n_1} - d_1 = 0$ and $\vec{r}.\vec{n_2} - d_2 = 0$ is

$\left(\vec{r}.\vec{n_1} - d_1\right) + \lambda\left(\vec{r}.\vec{n_2} - d_2\right) = 0$

So, equation of plane passing through the line of intersection of plane $\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - 4 = 0$ and $\vec{r}.\left(2\hat{i} + \hat{j} - \hat{k}\right) + 5 = 0$ is given by

$$\left[\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - 4\right] + \lambda\left[\vec{r}.\left(2\hat{i} + \hat{j} - \hat{k}\right) + 5\right] = 0$$

$$\vec{r}.\left[\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} - \hat{k}\right)\right] - 4 + 5\lambda = 0 \qquad ---(i)$$

We know that two planes are perpendicular if

$$\vec{n_1}.\vec{n_2} = 0 \qquad ---(ii)$$

Given that plane (i) is perpendicular to plane

$$\vec{r}.\left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) + 8 = 0 \qquad ---(iii)$$

Using (i) and (iii) in equation (ii),

$$\left[\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} - \hat{k}\right)\right]\left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) = 0$$

$$\left[\hat{i}\left(1 + 2\lambda\right) + \hat{j}\left(2 + \lambda\right) + \hat{k}\left(3 - \lambda\right)\right]\left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) = 0$$

$$\left(1 + 2\lambda\right)\left(5\right) + \left(2 + \lambda\right)\left(3\right) + \left(3 - \lambda\right)\left(-6\right) = 0$$

$$5 + 10\lambda + 6 + 3\lambda - 18 + 6\lambda = 0$$

$$19\lambda - 7 = 0$$

$$\lambda = \frac{7}{19}$$

Put value of λ in equation (i),

$$\vec{r}.\left[\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} - \hat{k}\right)\right] - 4 + 5\lambda = 0$$

$$\vec{r}\left[\hat{i} + 2\hat{j} + 3\hat{k} + \frac{14}{19}\hat{i} + \frac{7}{19}\hat{j} - \frac{7}{19}\hat{k}\right] - 4 + 5\left(\frac{7}{19}\right) = 0$$

$$\vec{r}\left[\frac{33\hat{i}}{19} + \frac{45\hat{j}}{19} - \frac{50\hat{k}}{19}\right]\frac{-76 + 35}{19} = 0$$

$$\vec{r}\left(\frac{33\hat{i} + 45\hat{j} + 50\hat{k}}{19}\right) - \frac{41}{19} = 0$$

Multiplying by 19,

$$\vec{r}\left(33\hat{i} + 45\hat{j} + 50\hat{k}\right) - 41 = 0$$

Equation of required plane is,

$$\vec{r}\left(33\hat{i} + 45\hat{j} + 50\hat{k}\right) - 41 = 0$$

$$33x + 45y + 50z - 41 = 0$$

The Plane 29.8 Q13

The equation of a plane passing through the intersection of

$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=6$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k})=-5$ is

$[\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-6]+\lambda[\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k})+5]=0$

$\Rightarrow \vec{r}\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1+4\lambda)\hat{k}]=(6-5\lambda)$...(1)

$\Rightarrow [x\hat{i}+y\hat{j}+z\hat{k}]\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1+4\lambda)\hat{k}]=(6-5\lambda)$

$\Rightarrow [x(1+2\lambda)+y(1+3\lambda)+z(1+4\lambda)]=(6-5\lambda)$...(2)

The requried plane also passes through the point(1, 1, 1).

Substiuting $x=1, y=1, z=1$ in equation (2), we have,

$1\times(1+2\lambda)+1\times(1+3\lambda)+1\times(1+4\lambda)=(6-5\lambda)$

$\Rightarrow 1+2\lambda+1+3\lambda+1+4\lambda=6-5\lambda$

$\Rightarrow 3+9\lambda=6-5\lambda$

$\Rightarrow 14\lambda=6-3$

$\Rightarrow 14\lambda=3$

$\Rightarrow \lambda=\frac{3}{14}$

Substituting the value $\lambda=\frac{3}{14}$ in equation (1), we have,

$\vec{r}\cdot\left[\left(1+2\left(\frac{3}{14}\right)\right)\hat{i}+\left(1+3\left(\frac{3}{14}\right)\right)\hat{j}+\left(1+4\left(\frac{3}{14}\right)\right)\hat{k}\right]=\left(6-5\left(\frac{3}{14}\right)\right)$

$\Rightarrow \vec{r}\cdot\left[\frac{20}{14}\hat{i}+\frac{23}{14}\hat{j}+\frac{26}{14}\hat{k}\right]=\frac{69}{14}$

$\Rightarrow \vec{r}\cdot[20\hat{i}+23\hat{j}+26\hat{k}]=69$

The Plane 29.8 Q14

We know that, equation of the plane passing through the line of intersection of planes

$\vec{r}.\overrightarrow{n_1} - d_1 = 0$ and $\vec{r}.\overrightarrow{n_2} - d_2 = 0$ is

$\left(\vec{r}.\overrightarrow{n_1} - d_1\right) + \lambda\left(\vec{r}.\overrightarrow{n_2} - d_2\right) = 0$

So, equation of plane passing through the line of intersection of plane $\vec{r}.\left(2\hat{i} + \hat{j} + 3\hat{k}\right) - 7 = 0$ and $\vec{r}.\left(2\hat{i} + 5\hat{j} + 3\hat{k}\right) - 9 = 0$ is given by

$$\left[\vec{r}.\left(2\hat{i} + \hat{j} + 3\hat{k}\right) - 7\right] + \lambda\left[\vec{r}.\left(2\hat{i} + 5\hat{j} + 3\hat{k}\right) - 9\right] = 0$$

$$\vec{r}\left[\left(2\hat{i} + \hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 5\hat{j} + 3\hat{k}\right)\right] - 7 - 9\lambda = 0$$

$$\vec{r}\left[(2 + 2\lambda)\hat{i} + (1 + 5\lambda)\hat{j} + (3 + 3\lambda)\hat{k}\right] - 7 - 9\lambda = 0 \qquad ---(i)$$

Given that plane (i) is passing through $\left(2\hat{i} + \hat{j} + 3\hat{k}\right)$, so

$$\left(2\hat{i} + \hat{j} + 3\hat{k}\right)\left[(2 + 2\lambda)\hat{i} + (1 + 5\lambda)\hat{j} + (3 + 3\lambda)\hat{k}\right] - 7 - 9\lambda = 0$$

$$(2)(2 + 2\lambda) + (1)(1 + 5\lambda) + (3)(3 + 3\lambda) - 7 - 9\lambda = 0$$

$$4 + 4\lambda + 1 + 5\lambda + 9 + 9\lambda - 7 - 9\lambda = 0$$

$$9\lambda + 7 = 0$$

$$9\lambda = -7$$

$$\lambda = -\frac{7}{9}$$

Put value of λ in equation (i),

$$\vec{r}.\left[(2 + 2\lambda)\hat{i} + (1 + 5\lambda)\hat{j} + (3 + 3\lambda)\hat{k}\right] - 7 - 9\lambda = 0$$

$$\vec{r}.\left[\left(2 + \frac{14}{9}\right)\hat{i} + \left(1 - \frac{35}{9}\right)\hat{j} + \left(3 - \frac{21}{9}\right)\hat{k}\right] - 7 + \frac{63}{9} = 0$$

$$\vec{r}.\left[\left(\frac{18 - 14}{9}\right)\hat{i} + \left(\frac{9 - 35}{9}\right)\hat{j} + \left(\frac{27 - 21}{9}\right)\hat{k}\right] - 7 + 7 = 0$$

$$\vec{r}.\left[\left(\frac{4}{9}\right)\hat{i} - \frac{26}{9}\hat{j} + \frac{6\hat{k}}{9}\right] + 0 = 0$$

$$\vec{r}.\left[\frac{4}{9}\hat{i} - \frac{26}{9}\hat{j} + \frac{6}{9}\hat{k}\right] = 0$$

Multiplying by $\left(\frac{9}{2}\right)$, we get

$$\vec{r}\left[2\hat{i} - 13\hat{j} + 3\hat{k}\right] = 0$$

Equation of required plane is,

$$\vec{r}.\left(2\hat{i} - 13\hat{j} + 3\hat{k}\right) = 0$$

The Plane 29.8 Q15

The equation of the family of planes through the intersection of planes
$3x - y + 2z = 4$ and $x + y + z = 2$ is,
$(3x - y + 2z - 4) + \lambda(x + y + z - 2) = 0$..........(i)

If it passes through $(2, 2, 1)$, then
$(6 - 2 + 2 - 4) + \lambda(2 + 2 + 1 - 2) = 0$
$\Rightarrow \lambda = -\frac{2}{3}$

Substituting $\lambda = -\frac{2}{3}$ in (i) we get, $7x - 5y + 4z = 0$ as the equation of the required plane.

The Plane 29.8 Q16

The equation of the family of planes through the line of intersection of planes
$x + y + z = 1$ and $2x + 3y + 4z = 5$ is,
$(x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0$..........(i)
$(2\lambda + 1)x + (3\lambda + 1)y + (4\lambda + 1)z = 5\lambda + 1$

It is perpendicular to the plane $x - y + z = 0$.
$\therefore (2\lambda + 1)(1) + (3\lambda + 1)(-1) + (4\lambda + 1)(1) = 5\lambda + 1$
$\Rightarrow 2\lambda + 1 - 3\lambda - 1 + 4\lambda + 1 = 5\lambda + 1$
$\Rightarrow \lambda = -\frac{1}{3}$

Substituting $\lambda = -\frac{1}{3}$ in (i), we get, $x - z + 2 = 0$ as the equation of the required plane
and its vector equation is $\vec{r}\cdot(\hat{i} - \hat{k}) + 2 = 0$.

The equation of the family of planes parallel to $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 2$ is,
$\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = d$........(i)

If it passes through (a, b, c) then
$(a\hat{i} + b\hat{j} + c\hat{k})\cdot(\hat{i} + \hat{j} + \hat{k}) = d$
$\Rightarrow a + b + c = d$

Substituting $a + b + c = d$ in (i), we get,
$\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = a + b + c$

$x + y + z = a + b + c$ as the equation of the required plane.

Ex 29.9

The Plane Ex 29.9 Q1

We know that distance of a point $\vec{a}$ from a plane $\vec{r}.\vec{n} - d = 0$ is given by

$$D = \left|\frac{\vec{a}\vec{n} - d}{|\vec{n}|}\right| \text{ unit}$$

Here, $\vec{a} = 2\hat{i} - \hat{j} - 4\hat{k}$ and
plane $\vec{r}.\left(3\hat{i} - 4\hat{j} + 12\hat{k}\right) - 9 = 0$

$\vec{r}.\vec{n} - d = 0$

So, required distance

$$D = \left|\frac{\left(2\hat{i} - \hat{j} - 4\hat{k}\right)\left(3\hat{i} - 4\hat{j} + 12\hat{k}\right) - 9}{\sqrt{(3)^2 + (-4)^2 + (12)^2}}\right|$$
$$= \left|\frac{(2)(3) + (-1)(-4) + (-4)(12) - 9}{\sqrt{9 + 16 + 144}}\right|$$
$$= \left|\frac{6 + 4 - 48 - 9}{\sqrt{169}}\right|$$
$$= \left|-\frac{47}{13}\right|$$
$$= \frac{47}{13} \text{ units}$$

Required distance is $\frac{47}{13}$ units

The Plane Ex 29.9 Q2

We know that, distance of a point $\vec{a}$ to a plane $\vec{r}.\vec{n} - d = 0$ is given by

$$D = \left|\frac{\vec{a}\vec{n} - d}{|\vec{n}|}\right| \qquad ---(i)$$

Let D_1 be the distance of point $\left(\hat{i} - \hat{j} + 3\hat{k}\right)$
from the plane $\vec{r}.\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9 = 0$, then

$$D_1 = \left|\frac{\left(\hat{i} - \hat{j} + 3\hat{k}\right)\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9}{\sqrt{(5)^2 + (2)^2 + (-7)^2}}\right| \qquad \left[\text{Using equation (i)}\right]$$
$$= \left|\frac{(1)(5) + (-1)(2) + (3)(-7) + 9}{\sqrt{25 + 4 + 49}}\right|$$
$$= \left|\frac{5 - 2 - 21 + 9}{\sqrt{78}}\right|$$
$$= \left|-\frac{9}{\sqrt{78}}\right|$$

$$D_1 = \frac{9}{\sqrt{78}} \text{ units} \qquad ---(ii)$$

Again, let D_2 be the distance of point $\left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)$ from the plane $\vec{r}.\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9 = 0$, then, using equation (i), we get

$$D_2 = \left|\frac{\left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9}{\sqrt{(5)^2 + (2)^2 + (-7)^2}}\right|$$
$$= \left|\frac{(3)(5) + (3)(2) + (3)(-7) + 9}{\sqrt{25 + 4 + 49}}\right|$$
$$= \left|\frac{15 + 6 - 21 + 9}{\sqrt{78}}\right|$$
$$= \left|\frac{9}{\sqrt{78}}\right|$$

$$= \frac{9}{\sqrt{78}} \text{ units} \qquad ---(iii)$$

From equation (ii) and (iii)
$D_1 = D_2$

Distance of point $\left(\hat{i} - \hat{j} + 3\hat{k}\right)$ from plane $\vec{r}.\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9 = 0$

= Distance of point $\left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)$ from plane $\vec{r}.\left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9 = 0$

The Plane Ex 29.9 Q3

We know that, distance of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is given by

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right| \qquad ---(i)$$

So, distance of point $(2, 3, -5)$ from the plane $x + 2y - 2z - 9 = 0$ is given by

$$D = \left|\frac{2 + (2)(3) - 2(-5) - 9}{\sqrt{(1)^2 + (2)^2 + (-2)^2}}\right| \qquad \left[\text{Using equation } (i)\right]$$

$$= \left|\frac{2 + 6 + 10 - 9}{\sqrt{1 + 4 + 4}}\right|$$

$$= \left|\frac{9}{\sqrt{9}}\right|$$

$$= \left|\frac{9}{3}\right|$$

$D = 3$ units

The Plane Ex 29.9 Q4

Given equation of plane is

$$x + 2y - 2z + 8 = 0 \qquad ---(i)$$

We know that, equation of the plane parallel to plane (i) is given by

$$x + 2y - 2z + \lambda = 0 \qquad ---(ii)$$

We know that, distance (D) of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is given by

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right| \qquad ---(iii)$$

Given, $D = 2$ unit is the distance of the plane (ii) from the point $(2,1,1)$, so
Using (i),

$$2 = \left|\frac{2 + (2)(1) - 2(1) + \lambda}{\sqrt{(1)^2 + (2)^2 + (-2)^2}}\right|$$

$$2 = \left|\frac{2 + 2 - 2 + \lambda}{\sqrt{1 + 4 + 4}}\right|$$

$$2 = \left|\frac{2 + \lambda}{\sqrt{9}}\right|$$

Squaring both the sides, we get

$$4 = \frac{(2 + \lambda)^2}{9}$$

$$36 = (2 + \lambda)^2$$

$$2 + \lambda = \pm 6$$

$$\Rightarrow \quad 2 + \lambda = 6 \quad or \quad 2 + \lambda = -6$$

$$\Rightarrow \quad \lambda = 4 \quad or \quad \lambda = -8$$

Put $\lambda = 4$ in equation (ii),
$x + 2y - 2z + 4 = 0$

Put $\lambda = -8$ in equation (ii),
$x + 2y - 2z - 8 = 0$

Hence, equation of the required plane are
$x + 2y - 2z + 4 = 0$
$x + 2y - 2z - 8 = 0$

The Plane Ex 29.9 Q5

We know that distance (D) of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is given by

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{(a)^2 + (b)^2 + (c)^2}}\right| \qquad ---(i)$$

Let D_1 be the distance of the point $(1,1,1)$ from plane $3x + 4y - 12z + 13 = 0$, so using (i), we get

$$D_1 = \left|\frac{(3)(1) + (4)(1) - 12(1) + 13}{\sqrt{(3)^2 + (4)^2 + (-12)^2}}\right|$$
$$= \left|\frac{3 + 4 - 12 + 13}{\sqrt{9 + 16 + 144}}\right|$$
$$= \left|\frac{8}{\sqrt{169}}\right|$$

$$D = \frac{8}{13} \text{ units} \qquad ---(ii)$$

Let D_2 be the distane of a point $(-3,0,1)$ from the plane $3x + 4y - 12z + 13 = 0$, so using equation (i),

$$D_2 = \left|\frac{(3)(-3) + (4)(0) - 12(1) + 13}{\sqrt{(3)^2 + (4)^2 + (-12)^2}}\right|$$
$$= \left|\frac{-9 + 0 - 12 + 13}{\sqrt{9 + 4 + 144}}\right|$$
$$= \left|-\frac{8}{\sqrt{169}}\right|$$

$$D_2 = \frac{8}{13} \text{ units} \qquad ---(iii)$$

Hence, from equation (ii) and (iii)

$$D_1 = D_2$$

The Plane Ex 29.9 Q6

Given equation of plane is

$x - 2y + 2z - 3 = 0 \qquad ---(i)$

We know that, equation of a plane parallel to plane (i) is given by,

$x - 2y + 2z + \lambda = 0 \qquad ---(ii)$

We know that distance (D) of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is given by,

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{(a)^2 + b^2 + c^2}}\right| \qquad ---(iii)$$

Given that, distance of plane (ii) from a point $(1,1,1)$ is one unit, so using (iii),

$$1 = \left|\frac{(1) - 2(1) + 2(1) + \lambda}{\sqrt{(1)^2 + (-2)^2 + (2)^2}}\right|$$

$$= \left|\frac{1 - 2 + 2 + \lambda}{\sqrt{1 + 4 + 4}}\right|$$

$$1 = \left|\frac{1 + \lambda}{\sqrt{9}}\right|$$

$$1 = \left|\frac{1 + \lambda}{3}\right|$$

Squaring both the sides,

$$1 = \frac{(1 + \lambda)^2}{9}$$

$$9 = (1 + \lambda)^2$$

$$1 + \lambda = \pm 3$$

$\Rightarrow \quad 1 + \lambda = 3 \quad$ or $\quad 1 + \lambda = -3$

$\Rightarrow \quad \lambda = 2 \quad$ or $\quad \lambda = -4$

Put the value of λ in equation (ii) to get the equations of required planes,

$x - 2y + 2z + 2 = 0$

$x - 2y + 2z - 4 = 0$

The Plane Ex 29.9 Q7

We know that, distance (D) of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is given by,

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right| \qquad ---(i)$$

So, distance of point $(2,3,5)$ from xy-plane (we know that equation of xy-plane is $z = 0$) is

$$= \left|\frac{(2)(0) + (3)(0) + (5)(1) + 0}{\sqrt{(0)^2 + (0)^2 + (1)^2}}\right| \qquad [\text{Using } (i)]$$

$$= \left|\frac{0 + 0 + 5}{\sqrt{0 + 0 + 1}}\right|$$

$= 5$ unit

Distance of the point $(2, 3, 5)$ from xy-plane $= 5$ unit

The Plane Ex 29.9 Q8

We know that, distance (D) of a point $\vec{a}$ from a plane $\vec{r}.\vec{n} - d = 0$ is given by,

$$D = \left|\frac{\vec{a}.\vec{n} - d}{|\vec{n}|}\right| \qquad ---(i)$$

So, distance of point $\left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)$ from plane $\vec{r}.\left(5\hat{i} + 2\hat{j} + 3\hat{k}\right) + 9 = 0$ is

$$D = \left|\frac{\left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)\left(5\hat{i} + 2\hat{j} + 3\hat{k}\right) + 9}{\sqrt{(5)^2 + (2)^2 + (-7)^2}}\right|$$

$$= \left|\frac{(3)(5) + (3)(2) + (3)(-7) + 9}{\sqrt{25 + 4 + 49}}\right|$$

$$= \left|\frac{15 + 6 - 21 + 9}{\sqrt{78}}\right|$$

$$= \left|\frac{9}{\sqrt{78}}\right|$$

Therefore, required distance is

$$= \frac{9}{\sqrt{78}} \text{ units}$$

The Plane Ex 29.9 Q9

Distance of point $(1,1,1)$ from origin is $\sqrt{3}$

Distance of point $(1,1,1)$ from plane is $\left|\frac{1+\lambda}{\sqrt{3}}\right|$

$$\text{Product} = \left|\frac{1+\lambda}{\sqrt{3}}\right| \times \sqrt{3} = 5$$

$|1+\lambda| = 5$

so $\lambda = 4$ or -6

The Plane Ex 29.9 Q10

Consider

$$3x-4y+12z-6=0 \qquad \ldots\ldots (1)$$

$$4x+3z-7=0 \qquad \ldots\ldots (2)$$

The distance of a point (x_1, y_1, z_1) from the plane $3x-4y+12z-6=0$ is

$$D_1 = \left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$

$$= \left|\frac{3x_1-4y_1+12z_1-6}{\sqrt{3^2+(-4)^2+12^2}}\right|$$

$$= \left|\frac{3x_1-4y_1+12z_1-6}{\sqrt{169}}\right|$$

$$= \left|\frac{3x_1-4y_1+12z_1-6}{13}\right|$$

The distance of the point (x_1, y_1, z_1) from the plane $4x+3z-7=0$ is

$$D_2 = \left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$

$$= \left|\frac{4x_1+3z_1-7}{\sqrt{4^2+3^2}}\right|$$

$$= \left|\frac{4x_1+3z_1-7}{\sqrt{25}}\right|$$

$$= \left|\frac{4x_1+3z_1-7}{5}\right|$$

Since the point (x_1, y_1, z_1) are equidistant from the planes $3x-4y+12z-6=0$ and $4x+3z-7=0$

So

$$D_1 = D_2$$

$$\left|\frac{3x_1-4y_1+12z_1-6}{13}\right| = \left|\frac{4x_1+3z_1-7}{5}\right|$$

$$\frac{3x_1-4y_1+12z_1-6}{13} = \pm\frac{4x_1+3z_1-7}{5}$$

Taking positive sign

$$\frac{3x_1-4y_1+12z_1-6}{13} = \frac{4x_1+3z_1-7}{5}$$

$$15x_1-20y_1+60z_1-30 = 52x_1+39z_1-91$$

$$37x_1+20y_1-21z_1-61=0$$

Taking negative sign

$$\frac{3x_1-4y_1+12z_1-6}{13} = -\frac{4x_1+3z_1-7}{5}$$

$$15x_1-20y_1+60z_1-30 = -52x_1-39z_1+91$$

$$67x_1-20y_1+99z_1-121=0$$

The Plane Ex 29.9 Q11

The equation of any plane passing through $A(2,5,-3)$
is $a(x-2)+b(y-5)+c(z+3)=0$....(1)
The above plane passes through the point $B(-2,-3,5)$
and hence, we have,
$a(-2-2)+b(-3-5)+c(5+3)=0$
$\Rightarrow -4a-8b+8c=0$...(2)
Again the required plane passes through the point $C(5,3,-3)$
and hence, we have,
$a(5-2)+b(3-5)+c(-3+3)=0$
$\Rightarrow 3a-2b+0c=0$...(3)
Solving equations (2) and (3) by cross multiplication, we have,

$$\frac{a}{(-8)\times 0-(-2)\times 8}=\frac{b}{3\times 8-(-4)\times 0}=\frac{c}{(-4)\times(-2)-3\times(-8)}=\lambda(\text{say})$$

$$\Rightarrow \frac{a}{0+16}=\frac{b}{24+0}=\frac{c}{8+24}=\lambda$$

$$\Rightarrow \frac{a}{16}=\frac{b}{24}=\frac{c}{32}=\lambda$$

$$\Rightarrow \frac{a}{2}=\frac{b}{3}=\frac{c}{4}=\lambda$$

$\Rightarrow a=2\lambda, b=3\lambda$ and $c=4\lambda$
Substituting the above values in equation (1), we have,
$2\lambda(x-2)+3\lambda(y-5)+4\lambda(z+3)=0$
Since $\lambda \neq 0$, we have,
$2(x-2)+3(y-5)+4(z+3)=0$
$\Rightarrow 2x-4+3y-15+4z+12=0$
$\Rightarrow 2x+3y+4z-7=0$

Thus the equation of the plane is
$2x+3y+4z-7=0$
The distance from the point $P(7,2,4)$ to the plane is

$$d=\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$

$$\therefore \text{Distance, } d=\left|\frac{2x+3y+4z-7}{\sqrt{2^2+3^2+4^2}}\right|$$

$$\Rightarrow d_{(7,2,4)}=\left|\frac{2\times 7+3\times 2+4\times 4-7}{\sqrt{2^2+3^2+4^2}}\right|$$

$$\Rightarrow d_{(7,2,4)}=\left|\frac{29}{\sqrt{29}}\right|$$

$$\Rightarrow d_{(7,2,4)}=\sqrt{29}\text{ units}$$

The Plane Ex 29.9 Q12

Given that a plane is making intercepts $-6,3$ and 4
respectively on the coordinate axes.
Thus the equation of the plane is

$$\frac{x}{-6}+\frac{y}{3}+\frac{z}{4}=1 \quad ...(1)$$

We need to find the length of the perpendicular
from the origin on the plane.
If the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ is at a distance 'p', then

$$\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2} \quad ...(2)$$

Comparing equation (1) with the
general equation, we get,
$a=-6, b=3$ and $c=4$
Thus, equation (2) becomes,

$$\frac{1}{p^2}=\frac{1}{(-6)^2}+\frac{1}{3^2}+\frac{1}{4^2}$$

$$\Rightarrow \frac{1}{p^2}=\frac{4+16+9}{144}$$

$$\Rightarrow \frac{1}{p^2}=\frac{29}{144}$$

$$\Rightarrow p^2=\frac{144}{29}$$

$$\Rightarrow p=\frac{12}{\sqrt{29}}\text{ units}$$

Ex 29.10

The Plane Ex 29.10 Q1

Let $p\ (x_1, y_1, z_1)$ be any point as the plane $2x - y + 3z - 4 = 0$, then

$$2x_1 - y_1 + 3z_1 - 4 = 0 \qquad ---(i)$$

We know that distance (D) of a point (x_1, y_1, z_1) from a plane $ax + by + cz + d = 0$ is given by

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{(a)^2 + (b)^2 + (c)^2}}\right| \qquad ---(ii)$$

Length of perpendicular from $P\ (x_1, y_1, z_1)$ to the plane $6x - 3y + 9z + 13 = 0$ is

$$= \left|\frac{6x_1 - 3y_1 + 9z_1 + 13}{\sqrt{(6)^2 + (-3)^2 + (9)^2}}\right| \qquad [\text{Using (ii)}]$$

$$= \left|\frac{3(2x_1 - y_1 + 3z_1) + 13}{\sqrt{36 + 9 + 81}}\right| \qquad [\text{Using (i)}]$$

$$= \left|\frac{3(4) + 13}{\sqrt{126}}\right|$$

$$= \left|\frac{12 + 13}{\sqrt{126}}\right|$$

$$= \left|\frac{25}{\sqrt{126}}\right|$$

$$= \frac{25}{3\sqrt{14}} \quad \text{units}$$

Since P is the point on plane (i) and $\frac{25}{3\sqrt{14}}$ is the distance of P from plane

$6x - 3y + 9z + 13 = 0$, so

the distance between the parallel planes is $\frac{25}{3\sqrt{14}}$ units

The Plane Ex 29.10 Q2

Equation of plane which is parallel to 2x-3y+5z+7=0
is of the form 2x-3y+5z=d
Above plane is passing through (3, 4, -1)
So, substitute above point in the equation, we get
6-12-5=d
d=-11
So plane equation is 2x-3y+5z=-11
Distance between planes is given by

$$\left|\frac{-7+11}{\sqrt{4+9+25}}\right| = \frac{4}{\sqrt{38}}$$

The Plane Ex 29.10 Q3

Given equations of planes are

$$2x - 2y + z + 3 = 0 \qquad ---(i)$$

$$2x - 2y + z + 9 = 0 \qquad ---(ii)$$

Let equation of the mid parallel plane is

$$2x - 2y + z + \lambda = 0 \qquad ---(iii)$$

Let $P(x_1, y_1, z_1)$ be any point on plane (iii),

$$2x_1 - 2y_1 + z_1 + \lambda = 0 \qquad ---(iv)$$

We know that, distance of a plane $ax + by + cz + d = 0$ from a point (x_1, y_1, z_1) is given by

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right| \qquad ---(v)$$

Let D_1 be the distance of $P(x_1, y_1, z_1)$ from plane (i), so using equation (v),

$$D_1 = \left|\frac{2x_1 - 2y_1 + z_1 + 3}{\sqrt{(2)^2 + (-2)^2 + (1)^2}}\right|$$

$$D_1 = \left|\frac{-\lambda + 3}{\sqrt{4 + 4 + 1}}\right| \qquad [\text{Using equation (iv)}]$$

$$D_1 = \left|\frac{3 - \lambda}{3}\right| \qquad ---(vi)$$

Again, let D_2 be the distance of point (x_1, y_1, z_1) from plane (ii), so using equation (v),

$$D_2 = \left|\frac{2x_1 - 2y_1 + z_1 + 9}{\sqrt{(2)^2 + (-2)^2 + (1)^2}}\right|$$

$$D_2 = \left|\frac{-\lambda + 9}{\sqrt{9}}\right| \qquad [\text{Using equation (iv)}]$$

$$D_2 = \left|\frac{9 - \lambda}{3}\right| \qquad ---(vii)$$

Since, $P\left(x_1, y_1, z_1\right)$ is a point on mid parallel plane, so

$D_1 = D_2$

$\left|\frac{3-\lambda}{3}\right| = \left|\frac{9-\lambda}{3}\right|$ [Using (vi),(vii)]

Squaring both the sides,

$\frac{(3-\lambda)^2}{9} = \frac{(9-\lambda)^2}{9}$

$9 - 6\lambda + \lambda^2 = 81 - 18\lambda + \lambda^2$

$-6\lambda + 18\lambda = 81 - 9$

$12\lambda = 72$

$\lambda = \frac{72}{12}$

$\lambda = 6$

Put the value of λ in equation (iii),

$2x - 2y + z + 6 = 0$

So, equation of required plane is

$2x - 2y + z + 6 = 0$

The Plane Ex 29.10 Q4

Let the position vector of any point P on a plane $\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + 7$ is $\vec{a}$, so

$\vec{a}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + 7 = 0$ ---(i)

We know that distance (D) of a point $\vec{a}$ from a plane $\vec{r}.\vec{n} - d = 0$ is given by

$D = \left|\frac{\vec{a}.\vec{n} - d}{\vec{n}}\right|$ ---(ii)

Length of perpendicular from $P\left(\vec{a}\right)$ to plane $\vec{r}.\left(2\hat{i} + 4\hat{j} + 6\hat{k}\right) + 7 = 0$ is given by

$= \left|\frac{\vec{a}.\left(2\hat{i} + 4\hat{j} + 6\hat{k}\right) + 7}{\sqrt{(2)^2 + (4)^2 + (6)^2}}\right|$

$= \left|\frac{2\vec{a}\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + 7}{\sqrt{4 + 16 + 36}}\right|$

$= \left|\frac{2.(-7) + 7}{\sqrt{56}}\right|$ [Using equation (i)]

$= \left|\frac{-14 + 7}{\sqrt{56}}\right|$

$= \frac{7}{\sqrt{56}}$

Distance between two given plane

= Distance of $P\left(\vec{a}\right)$ from plane $\vec{r}.\left(2\hat{i} + 4\hat{j} + 6\hat{k}\right) + 7 = 0$

$= \frac{7}{\sqrt{56}}$

So, required distance $= \frac{7}{\sqrt{56}}$ units

Ex 29.11

The Plane Ex 29.11 Q1

$\vec{r} = (2\hat{i} + 3\hat{j} + 9\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$

$\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 5$

Angle between line and plane is given by

$$\cos\theta = \left|\frac{2+3+4}{\sqrt{(1+1+1)(4+9+16)}}\right| = \frac{9}{\sqrt{87}}$$

The Plane Ex 29.11 Q2

We know that the angle (θ) between the line $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and plane $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\sin\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \qquad ---(i)$$

Given, equation of line is

$$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z+1}{1}$$

So, $a_1 = 1$, $b_1 = -1$, $c_1 = 1$

Given equation of plane is $2x + y - z - 4 = 0$

So, $a_2 = 2$, $b_2 = 1$, $c_2 = -1$

Put these value in equation (i),

$$\sin\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{(1)(2) + (-1)(1) + (1)(-1)}{\sqrt{(1)^2 + (-1)^2 + (1)^2}\sqrt{(2)^2 + (1)^2 + (-1)^2}}$$

$$\sin\theta = \frac{2-1-1}{\sqrt{1+1+1}\sqrt{4+1+1}}$$

$$= \frac{0}{\sqrt{3}\sqrt{6}}$$

$\sin\theta = 0$

$\theta = 0°$

angle between plane and line = 0°

The Plane Ex 29.11 Q3

We know that angle (θ) between line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and plane $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\sin\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \qquad ---(i)$$

Given that, line is passing through

$A(3, -4, -2)$ and $B(12, 2, 0)$, so direction ratios of line AB
$= (12 - 3,\ 2 + 4,\ 0 + 2)$
$= (9,\ 6,\ +2)$

So, $a_1 = 9$, $b_1 = 6$, $c_1 = 2$ $\qquad ---(ii)$

Given equation of plane is $3x - y + z = 1$
$a_2 = 3$, $b_2 = -1$, $c_2 = 1$ $\qquad ---(iii)$

Using (ii) and (iii) in equation (i),
Angle (θ) between plane and line is

$$\sin\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$
$$= \frac{(9)(3) + (6)(-1) + (2)(1)}{\sqrt{(9)^2 + (6)^2 + (2)^2}\sqrt{(3)^2 + (-1)^2 + (1)^2}}$$
$$= \frac{27 - 6 + 2}{\sqrt{81 + 36 + 4}\sqrt{9 + 1 + 1}}$$
$$= \frac{23}{\sqrt{121}\sqrt{11}}$$
$$= \frac{23}{11\sqrt{11}}$$

$$\theta = \sin^{-1}\left(\frac{23}{11\sqrt{11}}\right)$$

so, required angle between plane and line is given by

$$\theta = \sin^{-1}\left(\frac{23}{11\sqrt{11}}\right)$$

The Plane Ex 29.11 Q4

We know that, line $\vec{r} = \vec{a} + \lambda\vec{b}$ is parallel to plane $\vec{r}.\vec{n} = d$ if

$$\vec{b}.\vec{n} = 0 \qquad ---(i)$$

Given, equation of line is $\vec{r} = \hat{i} + \lambda\left(2\hat{i} - m\hat{j} - 3\hat{k}\right)$ and equation of plane $\vec{r}.\left(m\hat{i} + 3\hat{j} + \hat{k}\right) = 4$

So $\quad \vec{b} = \left(2\hat{i} - m\hat{j} - 3\hat{k}\right)$
$\quad \vec{n} = \left(m\hat{i} + 3\hat{j} + \hat{k}\right)$

Put $\vec{b}$ and $\vec{n}$ in equation (i),

$\left(2\hat{i} - m\hat{j} - 3\hat{k}\right)\left(m\hat{i} + 3\hat{j} + \hat{k}\right) = 0$
$(2)(m) + (-m)(3) + (-3)(1) = 0$
$2m - 3m - 3 = 0$
$-m - 3 = 0$
$-m = 3$

$m = -3$

The Plane Ex 29.11 Q5

We know that, line $\vec{r} = \vec{a} + \lambda\vec{b}$ and plane $\vec{r}.\vec{n} = d$ is parallel if

$$\vec{b}.\vec{n} = 0 \qquad ---(i)$$

Given, equation of line $\vec{r} = \left(2\hat{i} + 5\hat{j} + 7\hat{k}\right) + \lambda\left(\hat{i} + 3\hat{j} + 4\hat{k}\right)$ and equation of plane $\vec{r}\left(\hat{i} + \hat{j} - \hat{k}\right) = 7$, so

$\vec{b} = \hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{n} = \hat{i} + \hat{j} - \hat{k}$

Now,

$$\begin{aligned}\vec{b}.\vec{n} &= \left(\hat{i} + 3\hat{j} + 4\hat{k}\right)\left(\hat{i} + \hat{j} - \hat{k}\right)\\ &= (1)(1) + (3)(1) + (4)(-1)\\ &= 1 + 3 - 4\\ &= 0\end{aligned}$$

Since $\vec{b}.\vec{n} = 0$ so using (i), we get
Given line and plane are parallel
We know that, distance (D) of a plane $\vec{r}\vec{n} - d = 0$ from a point $\vec{a}$ is given by,

$$D = \left|\frac{\vec{a}.\vec{n} - d}{|\vec{n}|}\right| \qquad ---(ii)$$

We have to find distance between line and plane which is equal to the distance between point $\vec{a} = \left(2\hat{i} + 5\hat{j} + 7\hat{k}\right)$ from plane, so

$$\begin{aligned}D &= \left|\frac{\left(2\hat{i} + 5\hat{j} + 7\hat{k}\right)\left(\hat{i} + \hat{j} - \hat{k}\right) - 7}{\sqrt{(1)^2 + (1)^2 + (-1)^2}}\right|\\ &= \left|\frac{(2)(1) + (5)(1) + (7)(-1) - 7}{\sqrt{1 + 1 + 1}}\right|\\ &= \left|\frac{2 + 5 - 7 - 7}{\sqrt{3}}\right|\\ &= \left|\frac{-7}{\sqrt{3}}\right|\end{aligned}$$

$$D = \frac{7}{\sqrt{3}}$$

So, required distance between plane and line is $D = \frac{7}{\sqrt{3}}$ unit

The Plane Ex 29.11 Q6

Required line is perpendicular to plane $\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 3$, so line is parallel to the normal vector $\vec{n} = \hat{i} + 2\hat{j} + 3\hat{k}$ of plane.

And it is passing through point $\vec{a} = 0\hat{i} + 0\hat{j} + 0\hat{k}$.

We know that equation of a line passing through $\vec{a}$ and parallel to vector $\vec{b}$ is

$$\vec{r} = \vec{a} + \lambda\vec{b} \qquad ---(i)$$

Here, $\vec{a} = 0\hat{i} + 0\hat{j} + 0.\hat{k}$ and $\vec{b} = \vec{n} = \hat{i} + 2\hat{j} + 3\hat{k}$

So, $\vec{r} = \left(0\hat{i} + 0\hat{j} + 0.\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$
Hence, equation required line is

$$\vec{r} = \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$

The Plane Ex 29.11 Q7

We know that equation of plane passing through (x_1, y_1, z_1) is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \qquad ---(i)$$

So, equation of plane passing through $(2, 3, -4)$ is

$$a(x - 2) + b(y - 3) + c(z + 4) = 0 \qquad ---(ii) \qquad [\text{Using } (i)]$$

It is also passing through $(1, -1, 3)$, so,

$$a(1 - 2) + b(-1 - 3) + c(3 + 4) = 0$$
$$-a - 4b + 7c = 0$$
$$a + 4b - 7c = 0 \qquad ---(iii)$$

We know that line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ is parallel to plane $a_2x + b_2y + c_2z + d_2 = 0$ if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iv)$$

Here, equation (ii) is parallel to x-axis

$$\frac{x}{1} = \frac{y}{0} = \frac{z}{0} \qquad ---(v)$$

Using (ii) and (v) in equation (iv),

$$(a)(1) + (b)(0) + c(0) = 0$$
$$a = 0 \qquad ---(vi)$$

Put the value of a in equation (iii),

$$a - 4b + 7c = 0$$
$$0 - 4b + 7c = 0$$
$$-4b = -7c$$
$$4b = 7c$$

$$b = \frac{7}{4}c$$

Put the value of a and b in equation (ii),

$$a(x - 2) + b(y - 3) + c(z + 4) = 0$$
$$0(x - 2) + \left(\frac{7}{4}c\right)(y - 3) + c(z + 4) = 0$$
$$0 + \frac{7cy}{4} - \frac{21c}{4} + \frac{cz}{1} + \frac{4c}{1} = 0$$

$$7cy - 21c + 4cz + 16c = 0$$

Dividing by c,

$$7y + 4z - 5 = 0$$

Equation of required plane is

$$7y + 4z - 5 = 0$$

The Plane Ex 29.11 Q8

We know that equation a plane passing through the point (x_1, y_1, z_1) is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \qquad ---(i)$$

Given the required plane is passing through $(0,0,0)$, so using (i),

$$a(x - 0) + b(y - 0) + c(z - 0) = 0$$
$$ax + by + cz = 0 \qquad ---(ii)$$

Plane (ii) is also passing through $(3,-1,2)$,

$$3a - b + 2c = 0 \qquad ---(iii)$$

We know that line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ is parallel to plane $a_2x + b_2y + c_2z + d_2 = 0$ if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iv)$$

Given that, plane (ii) is parallel to line

$\frac{x - 4}{1} = \frac{y + 3}{-4} = \frac{z + 1}{7}$, so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(a)(1) + (b)(-4) + (c)(7) = 0$$
$$a - 4b + 7c = 0 \qquad ---(ii)$$

Solving equation (iii) and (v) by cross-multiplication, we get

$$\frac{a}{(-1)(7) - (-4)(2)} = \frac{b}{(1)(2) - (3)(7)} = \frac{c}{(3)(-4) - (1)(-1)}$$
$$\frac{a}{-7 + 8} = \frac{b}{2 - 21} = \frac{c}{-12 + 1}$$
$$\frac{a}{1} = \frac{b}{-19} = \frac{c}{-11} = \lambda \text{ (Say)}$$

$\Rightarrow \qquad a = \lambda,\ b = -19\lambda,\ c = -11\lambda$

Put the value of a, b, c in equation (ii),

$$ax + by + cz = 0$$
$$\lambda x - 19\lambda y - 11\lambda z = 0$$

Dividing by λ, we get

$$x - 19y - 11z = 0$$

Equation of required plane is

$$x - 19y - 11z = 0$$

The Plane Ex 29.11 Q9

We know that equation of a line passing through (x_1, y_1, z_1) is given by

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1} \qquad ---(i)$$

Here, required line is passing through $(1,2,3)$, is given by, $\quad$ [Using (i)]

$$\frac{x - 1}{a_1} = \frac{y - 2}{b_1} = \frac{z - 3}{c_1} \qquad ---(ii)$$

We know that, line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ is parallel to plane $a_2x + b_2y + c_2z + d_2 = 0$ if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iii)$$

Given, line (ii) is parallel to plane $x - y + 2z = 5$

So,

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(a_1)(1) + (b_1)(-1) + (c_1)(2) = 0$

$$a_1 - b_1 + 2c_1 = 0 \qquad ---(iv)$$

Also, given line (ii) is parallel to plane $3x + y + z = 6$

So,

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$(a_1)(3) + (b_1)(1) + (c_1)(1) = 0$

$$3a_1 + b_1 + c_1 = 0 \qquad ---(v)$$

Solving (iv) and (v) by cross-multiplication,

$$\frac{a_1}{(-1)(1) - (1)(2)} = \frac{b_1}{(3)(2) - (1)(1)} = \frac{c_1}{(1)(1) - (3)(-1)}$$

$$\frac{a_1}{-1 - 2} = \frac{b_1}{6 - 1} = \frac{c_1}{1 + 3}$$

$$\frac{a_1}{-3} = \frac{b_1}{5} = \frac{c_1}{4} = \lambda \text{ (Say)}$$

$\Rightarrow \quad a_1 = -3\lambda,\ b_1 = 5\lambda,\ c_1 = 4\lambda$

Put a_1, b_1, c_1 in equation (ii),

$$\frac{x - 1}{-3\lambda} = \frac{y - 2}{5\lambda} = \frac{z - 3}{4\lambda}$$

Multiplying by λ,

$$\frac{x - 1}{-3} = \frac{y - 2}{5} = \frac{z - 3}{4}$$

Equation of required line is

$$\frac{x - 1}{-3} = \frac{y - 2}{5} = \frac{z - 3}{4}$$

The vector equation of the line is

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(-3\hat{i} + 5\hat{j} + 4\hat{k}\right)$$

The Plane Ex 29.11 Q10

Firstly we have to find the line of section of planes $5x+2y-4z+2=0$ and $2x+8y+2z-1=0$
Let a_1, b_1, c_1 be the direction ratios of the line $5x+2y-4z+2=0$ and $2x+8y+2z-1=0$

Since, line lies in both the planes, so it is perpendicular to both planes, so

$$5a_1+2b_1-4c_1=0 \qquad ---(i)$$
$$2a_1+8b_1+2c_1=0 \qquad ---(ii)$$

Solving equation (i) and (ii), by cross-multiplication

$$\frac{a_1}{(2)(2)-(-4)(8)}=\frac{b_1}{(2)(-4)-(5)(2)}=\frac{c_1}{(5)(8)-(2)(2)}$$
$$\frac{a_1}{4+32}=\frac{b_1}{-8-10}=\frac{c_1}{40-4}$$
$$\frac{a_1}{36}=\frac{b_1}{-18}=\frac{c_1}{36}$$
$$\frac{a_1}{2}=\frac{b_1}{-1}=\frac{c_1}{2}=\lambda\text{(Say)}$$

$$\Rightarrow \quad a_1=2\lambda,\ b_1=-\lambda,\ c_1=2\lambda$$

We know that, line $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ is parallel to plane $a_2x+b_2y+c_2z+d_2=0$ if
$$a_1a_2+b_1b_2+c_1c_2=0 \qquad ---(iii)$$

Here line with direction ratio a_1, b_1, c_1 is parallel to plane $4x-2y-5z-2=0$,

$$a_1a_2+b_1b_2+c_1c_2$$
$$=(2)(4)+(-1)(-2)+(2)(-5)$$
$$=8+2-10$$

$$=0$$

Therefore, line of section is parallel to the plane.

The Plane Ex 29.11 Q11

Equation of line passing through $\vec{a}$ and parallel to $\vec{b}$ is given by

$$\vec{r}=\vec{a}+\lambda\vec{b} \qquad ---(i)$$

Given that, required line is passing through $(1,-1,2)$ is,

$$\vec{r}=\left(\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\vec{b} \qquad ---(ii)$$

Since, line (i) is perpendicular to plane $2x-y+3z-5=0$, so normal to plane is parallel to the line.

In vector form,
$\vec{b}$ is parallel to $\vec{n}=2\hat{i}-\hat{j}+3\hat{k}$
So, $\vec{b}=\mu\left(2\hat{i}-\hat{j}+3\hat{k}\right)$ as μ is any scaler
Thus, equation of required line is,

$$\vec{r}=\left(\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\left(\mu\left(2\hat{i}-\hat{j}+3\hat{k}\right)\right)$$

$$\vec{r}=\left(\hat{i}-\hat{j}+2\hat{k}\right)+\delta\left(2\hat{i}-\hat{j}+3\hat{k}\right)$$

The Plane Ex 29.11 Q12

We know that, equation of plane passing through (x_1, y_1, z_1) is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \qquad ---(i)$$

Given that, required plane is passing through $(2, 2, -1)$, so using (i),

$$a(x - 2) + b(y - 2) + c(z + 1) = 0 \qquad ---(ii)$$

Given, plane (ii) is passing through $(3, 4, 2)$,

$$a(3 - 2) + b(4 - 2) + c(2 + 1) = 0$$
$$a + 2b + 3c = 0 \qquad ---(iii)$$

We know that plane $a_1x + b_1y + c_1z + d_1 = 0$ and line $\frac{x - x_1}{a_2} = \frac{y - y_1}{b_2} = \frac{z - z_1}{c_2}$ are parallel if $a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(iv)$

Given that, plane (ii) is parallel to a line whose direction ratios are 7, 0, 6 so using (iv), we get

$$(a)(7) + (b)(0) + (c)(6) = 0$$
$$7a + 0 + 6c = 0$$
$$7a + 6c = 0$$

$$a = -\frac{6c}{7}$$

Put the value of a in equation (iii),

$$a + 2b + 3c = 0$$
$$-\frac{6c}{7} + 2b + 3c = 0$$
$$-6c + 14b + 21c = 0$$
$$14b + 15c = 0$$

$$b = -\frac{15c}{14}$$

Put the value of a and b in equation (ii),

$$a(x - 2) + b(y - 2) + c(z + 1) = 0$$
$$\left(-\frac{6c}{7}\right)(x - 2) + \left(-\frac{15c}{14}\right)(y - 2) + c(z + 1) = 0$$
$$-\frac{6cx}{7} + \frac{12c}{7} - \frac{15cy}{14} + \frac{30c}{14} + cz + c = 0$$

Multiplying by $\left(\frac{14}{c}\right)$, we get

$$-12x + 24 - 15y + 30 + 14z + 14 = 0$$
$$-12x + 15y + 14z + 68 = 0$$

Multiplying by (-1),
Equation of required plane is,

$$12x + 15y - 14z - 68 = 0$$

The Plane Ex 29.11 Q13

We know that angle (θ) between line $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and plane $a_2x+b_2y+c_2z+d_2=0$ is given by

$$\sin\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \qquad ---(i)$$

Given line is $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-3}{2}$ and equation of plane is $3x+4y+z+5=0$, so angle between plane and line is,

$$\sin\theta=\frac{(3)(3)+(-1)(4)+(2)(1)}{\sqrt{(3)^2+(-1)^2+(2)^2}\sqrt{(3)^2+(4)^2+(1)^2}}$$
$$=\frac{9-4+2}{\sqrt{9+1+4}\sqrt{9+16+1}}$$
$$=\frac{7\times\sqrt{7}}{\sqrt{14}\sqrt{26}\times\sqrt{7}}$$

$$=\frac{7\sqrt{7}}{7\sqrt{52}}$$

$$\theta=\sin^{-1}\left(\sqrt{\frac{7}{52}}\right)$$

The Plane Ex 29.11 Q14

We know that equation of plane passing through the intersection of planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ is given by

$$(a_1x+b_1y+c_1z+d_1)+\lambda(a_2x+b_2y+c_2z+d_2)=0$$

So, equation of plane passing through the intersection of two planes $x-2y+z-1=0$ and $2x+y+z-8=0$ is given by

$$(x-2y+z-1)+\lambda(2x+y+z-8)=0$$
$$x-2y+z-1+2\lambda x+\lambda y+\lambda z-8\lambda=0$$
$$x(1+2\lambda)+y(-2+\lambda)+z(1+\lambda)-1-8\lambda=0 \qquad ---(i)$$

We know that line $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and plane $a_2x+b_2y+c_2z+d_2=0$ are parallel if $a_1a_2+b_1b_2+c_1c_2=0$ $\qquad ---(ii)$

Given that plane (i) is parallel to line with direction ratio 1,2,1, so

$$a_1a_2+b_1b_2+c_1c_2=0$$
$$(1)(1+2\lambda)+(2)(-2+\lambda)+(1)(1+\lambda)=0$$
$$1+2\lambda-4+2\lambda+1+\lambda=0$$
$$5\lambda-2=0$$

$$\lambda=\frac{2}{5}$$

Put the value of λ in equation (i),

$$x\left(1+\frac{4}{5}\right)+y\left(-2+\frac{2}{5}\right)+z\left(1+\frac{2}{5}\right)-1-\frac{16}{5}=0$$

Multiplying by 5,

$$x(5+4)+y(-10+2)+z(5+2)-5-16=0$$
$$9x-8y+7z-21=0$$

So, equation of required plane is

$$9x-8y+7z-21=0 \qquad ---(iii)$$

We know that distance (D) of a point (x_1, y_1, z_1) from plane $ax+by+cz+d=0$ is given by

$$D=\frac{ax_1+by_1+cz_1+d_1}{\sqrt{a^2+b^2+c^2}}$$

So, distance of point $(1,1,1)$ from plane (i) is given by

$$D=\left|\frac{(9)(1)+(-8)(1)+(7)(1)-21}{\sqrt{(9)^2+(-8)^2+(7)^2}}\right|$$
$$=\left|\frac{9-8+7-21}{\sqrt{81+64+49}}\right|$$
$$=\left|\frac{16-29}{\sqrt{194}}\right|$$
$$=\left|\frac{-13}{\sqrt{194}}\right|$$

$$D=\frac{13}{\sqrt{194}} \text{ units}$$

The Plane Ex 29.11 Q15

We know that line $\vec{r}=\vec{a}+\lambda\vec{b}$ is paralle to plane $\vec{r}.\vec{n}=d$ if

$$\vec{b}.\vec{n}=0$$

Given, line is $\vec{r}=(\hat{i}+\hat{j})+\lambda(3\hat{i}-\hat{j}+2\hat{k})$ and plane is $\vec{r}.(2\hat{j}+\hat{k})=3$, so

$\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$, $\vec{a}=(\hat{i}+\hat{j})$ and $\vec{n}=(2\hat{j}+\hat{k})$

Now, $\vec{b}\vec{n}=(3\hat{i}-\hat{j}+2\hat{k})(2\hat{j}+\hat{k})$
$$=(3)(0)+(-1)(2)+(2)(1)$$
$$=0-2+2$$
$$=0$$

Since, $\vec{b}.\vec{n}=0$, so line is parallel to plane

Distance between point $\vec{a}$ and plane $\vec{r}.\vec{n}-d=0$ is given by

$$D=\left|\frac{\vec{a}\vec{n}-d}{|\vec{n}|}\right| \qquad ---(i)$$

$\vec{a}$ is a point on the line. So distance between line and plane is equal to the distance between $\vec{a} = (\hat{i} + \hat{j})$ and plane $\vec{r}.(2\hat{j} + \hat{k}) = 3$, so using (i),

$$D = \left|\frac{(\hat{i} + \hat{j})(2\hat{j} + \hat{k}) - 3}{\sqrt{(2)^2 + (1)^2}}\right|$$

$$= \left|\frac{(1)(0) + (1)(2) + (0)(1) - 3}{\sqrt{4+1}}\right|$$

$$= \left|\frac{0 + 2 + 0 - 3}{\sqrt{5}}\right|$$

$$= \left|\frac{-1}{\sqrt{5}}\right|$$

$$= \frac{1}{\sqrt{5}} \text{ unit}$$

So, required distance $= \frac{1}{\sqrt{5}}$ unit

The Plane Ex 29.11 Q16

We know that line $\vec{r} = \vec{a} + \lambda\vec{b}$ and plane $\vec{r}.\vec{n} - d = 0$ are parallel if

$$\vec{b}.\vec{n} = 0 \qquad ---(i)$$

Given, line $\vec{r} = (-\hat{i} + \hat{j} + \hat{k}) + \lambda(2\hat{i} + \hat{j} + 4\hat{k})$ and plane is $\vec{r}.(\hat{i} + 2\hat{j} - \hat{k}) - 1 = 0$

So, $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + 4\hat{k}$, $\vec{n} = \hat{i} + 2\hat{j} - \hat{k}$

Now, $\vec{b}.\vec{n} = (2\hat{i} + \hat{j} + 4\hat{k})(\hat{i} + 2\hat{j} - \hat{k})$

$= (2)(1) + (1)(2) + (4)(-1)$

$= 2 + 2 - 4$

$= 0$

Since, $\vec{b}.\vec{n} = 0$, so by equation (i), line is parallel to plane

Distance (D) between point $\vec{a}$ and plane $\vec{r}\vec{n} - d = 0$ is given by

$$D = \left|\frac{\vec{a}\vec{n} - d}{|\vec{n}|}\right| \qquad ---(ii)$$

Distance between given line and plane

= Distance of point $\vec{a} = (-\hat{i} + \hat{j} + \hat{k})$ from $\vec{r}.(\hat{i} + 2\hat{j} - \hat{k}) - 1 = 0$

$$D = \left|\frac{\vec{a}\vec{n} - d}{|\vec{n}|}\right|$$

$$= \left|\frac{\left(-\hat{i}+\hat{j}+\hat{k}\right)\left(\hat{i}+2\hat{j}-\hat{k}\right)-1}{\sqrt{(1)^2+(2)^2+(-1)^2}}\right|$$
$$= \left|\frac{(-1)(1)+(1)(2)+(1)(-1)-1}{\sqrt{1+4+1}}\right|$$
$$= \left|\frac{-1+2-1-1}{\sqrt{6}}\right|$$
$$= \left|\frac{-1}{\sqrt{6}}\right|$$
$$= \frac{1}{\sqrt{6}}$$

So, required distance $= \frac{1}{\sqrt{6}}$ units

The Plane Ex 29.11 Q17

We know that equation of plane passing through the line of intersection of two planes
$a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ is given by,

$$\left(a_1x+b_1y+c_1z+d_1\right)+\lambda\left(a_2x+b_2y+c_2z+d_2\right)=0 \qquad ---(i)$$

So, equation of plane passing through the line of intersection of planes $3x-4y+5z-10=0$ and $2x+2y-3z-4=0$ is,

$$(3x-4y+5z-10)+\lambda(2x+2y-3z-4)=0$$
$$(3+2\lambda)x+(-4+2\lambda)y+(5-3\lambda)z-10-4\lambda=0 \qquad ---(ii)$$

We know that, line $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ parallel to plane
$a_2x+b_2y+c_2z+d_2=0$ if $a_1a_2+b_1b_2+c_1c_2=0$ $\qquad ---(iii)$

Given that, plane (ii) is parallel to line $x=2y=3z$ or $\frac{x}{6}=\frac{y}{3}=\frac{z}{2}$
So,

$$(6)(3+2\lambda)+(3)(-4+2\lambda)+(2)(5-3\lambda)=0$$
$$18+12\lambda-12+6\lambda+10-6\lambda=0$$
$$12\lambda+16=0$$
$$\lambda=-\frac{16}{12}$$
$$\lambda=-\frac{4}{3}$$

Put λ in equation (ii),
$$x(3+2\lambda)+y(-4+2\lambda)+z(5-3\lambda)-10-4\lambda=0$$
$$x\left(3-\frac{8}{3}\right)+y\left(-4-\frac{8}{3}\right)+z\left(5+\frac{12}{3}\right)-10+\frac{16}{3}=0$$

Multiplying by 3,

$$x(9-8)+y(-12-8)+z(15+12)-30+16=0$$
$$x-20y+27z-14=0$$

Equation of required plane is given by

$$x-20y+27z-14=0$$

The Plane Ex 29.11 Q18

The plane passes through the point $\vec{a}(1, 2, -4)$
A vector in a direction perpendicular to
$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = (\hat{i} - 3\hat{j} + 5\hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})$
is $\vec{n} = (2\hat{i} + 3\hat{j} + 6\hat{k}) \times (\hat{i} + \hat{j} - \hat{k})$

$$\Rightarrow \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{vmatrix} = -9\hat{i} + 8\hat{j} - \hat{k}$$

Equation of the plane is $(\vec{r} - \vec{a}).\vec{n} = 0$

$$(\vec{r} - (\hat{i} + 2\hat{j} - 4\hat{k})).(-9\hat{i} + 8\hat{j} - \hat{k}) = 0$$
$$\Rightarrow \vec{r}.(-9\hat{i} + 8\hat{j} - \hat{k}) = 11$$

Substituting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, we get the Cartesian form as
$-9x + 8y - z = 11$
The distance of the point $(9, -8, -10)$ from the plane

$$= \left| \frac{-9(9) + 8(-8) - (-10) - 11}{\sqrt{9^2 + 8^2 + 1^2}} \right| = \frac{146}{\sqrt{146}} = \sqrt{146}$$

The Plane Ex 29.11 Q19

We know that equation of plane passing through (x_1, y_1, z_1) is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) \qquad ---(i)$$

Given that, required equation of plane is passing through $(3, 4, 1)$, so

$$a(x - 3) + b(y - 4) + c(z - 1) = 0 \qquad ---(ii)$$

Plane (ii) is also passing through $(0, 1, 0)$, so

$$a(0 - 3) + b(1 - 4) + c(0 - 1) = 0$$
$$-3a - 3b - c = 0$$

$$3a + 3b + c = 0 \qquad ---(iii)$$

We know that, plane $a_1x + b_1y + c_1z + d_1 = 0$ and line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ are parallel if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Here, line $\frac{x + 3}{2} = \frac{y - 3}{7} = \frac{z - 2}{5}$ is parallel to plane (ii), so

$$(2)(a) + (7)(b) + (c)(5) = 0$$
$$2a + 7b + 5c = 0 \qquad ---(iv)$$

Solving (iii) and (iv) by cross-multiplication,

$$\frac{a}{(3)(5)-(7)(1)}=\frac{b}{(2)(1)-(3)(5)}=\frac{c}{(3)(7)-(2)(3)}$$
$$\frac{a}{15-7}=\frac{b}{2-15}=\frac{c}{21-6}$$

$$\frac{a}{8}=\frac{b}{-13}=\frac{c}{15}=\lambda\,(\text{Say})$$

$\Rightarrow \quad a = 8\lambda,\ b = -13\lambda,\ c = 15\lambda$

Put a, b, c in equation (ii),

$$a(x-3)+b(y-4)+c(z-1)=0$$
$$8\lambda(x-3)+(-13\lambda)(y-4)+(15\lambda)(z-1)=0$$
$$8\lambda x-24\lambda-13\lambda y+52\lambda+15\lambda z-15\lambda=0$$

$$8\lambda x-13\lambda y+15\lambda z+13\lambda=0$$

Dividing by λ, equation of required plane is,

$$8x-13y+15z+13=0$$

The Plane Ex 29.11 Q20

Find the coordinates of the point where the line
$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}=r$
$\Rightarrow x=3r+2, y=4r-1, z=2r+2$
Substituting in the equation of the plane $x-y+z-5=0$,
we get $(3r+2)-(4r-1)+(2r+2)-5=0$
$\Rightarrow r=0$
$\Rightarrow x=2, y=-1, z=2$
Direction ratios of the line are $3,4,2$
Direction ratios of a line perpendicular to the plane are $1,-1,1$
$\sin\theta=\frac{3\times1+4\times-1+2\times1}{\sqrt{9+16+4}\sqrt{1+1+1}}=\frac{1}{\sqrt{87}}$
$\theta=\sin^{-1}\frac{1}{\sqrt{87}}$

The Plane Ex 29.11 Q21

We know that equation of line passing through point $\vec{a}$ and parallel to vector $\vec{b}$ is given by

$$\vec{r}=\vec{a}+\lambda\vec{b} \qquad ---(i)$$

Given that, line is passing through $(1,2,3)$.
So, $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$

It is given that line is perpendicular to plane $\vec{r}.(\hat{i}+2\hat{j}-5\hat{k})+9=0$

So, normal to plane $(\vec{n})$ is parallel to $\vec{b}$.
So, let $\vec{b}=\mu\vec{n}=\mu(\hat{i}+2\hat{j}-5\hat{k})$

Put $\vec{a}$ and $\vec{b}$ in (i), equation of line is,

$$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda\left[\mu(\hat{i}+2\hat{j}-5\hat{k})\right]$$
$$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\delta(\hat{i}+2\hat{j}-5\hat{k}) \qquad [\text{As } \delta=\lambda\mu]$$

Equation of required line is,

$$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\delta(\hat{i}+2\hat{j}-5\hat{k})$$

The Plane Ex 29.11 Q22

Direction ratios of the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$

are (2,3,6)

Direction ratio of a line perpendicular to the plane

$10x+2y-11z = 3$ are $10, 2, -11$

If θ is the angle between the line and the plane, then

$$\sin\theta = \frac{2\times 10+3\times 2+6\times -11}{\sqrt{2^2+3^2+6^2}\sqrt{10^2+2^2+11^2}} = -\frac{40}{\sqrt{49}\sqrt{225}} = -\frac{40}{7\times 15} = -\frac{8}{21}$$

$$\Rightarrow \theta = \sin^{-1}\left(-\frac{8}{21}\right)$$

The Plane Ex 29.11 Q23

We know that, equation of line passing through (x_1, y_1, z_1) is given by

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \qquad ---(i)$$

Given that, required line is passing through $(1,2,3)$, so

$$\frac{x-1}{a_1}=\frac{y-2}{b_1}=\frac{z-3}{c_1} \qquad ---(ii)$$

We know that, line $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and plane $a_2x+b_2y+c_2z+d_2=0$

are parallel if $a_1a_2+b_1b_2+c_1c_2=0$ $\qquad ---(iii)$

Given line (ii) is parallel to

$$\vec{r}.\left(\hat{i}-\hat{j}+2\hat{k}\right)=5$$

$$\Rightarrow \qquad x-y+2z-5=0, \text{ so}$$

$$a_1a_2+b_1b_2+c_1c_2=0$$

$$(a_1)(1)+(b_1)(-1)+(c_1)(2)=0$$

$$a_1-b_1+2c_1=0 \qquad ---(iii)$$

Line (ii) is also parallel to plane

$$\vec{r}.\left(3\hat{i}+\hat{j}+\hat{k}\right)=6$$

$$\Rightarrow \qquad 3x+y+z-6=0, \text{ so}$$

$a_1a_2 + b_1b_2 + c_1c_2 = 0$
$(a_1)(3) + (b_1)(1) + (c_1)(1) = 0$

$3a_1 + b_1 + c_1 = 0 \qquad\qquad --- (iv)$

Solving equation (iii) and (iv) by cross-multiplication,

$$\frac{a_1}{(-1)(1)-(2)(1)} = \frac{b_1}{(3)(2)-(1)(1)} = \frac{c_1}{(1)(1)-(3)(-1)}$$
$$\frac{a_1}{-1-2} = \frac{b_1}{6-1} = \frac{c_1}{1+3}$$
$$\frac{a_1}{-3} = \frac{b_1}{5} = \frac{c_1}{4} = \lambda \text{ (Say)}$$

$\Rightarrow \quad a_1 = -3\lambda,\ b_1 = 5\lambda,\ c_1 = 4\lambda$

Put a_1, b_1, c_1 in equation (ii), so, equation line is given by

$$\frac{x-1}{-3\lambda} = \frac{y-2}{5\lambda} = \frac{z-3}{4\lambda}$$
$$\frac{x-1}{-3} = \frac{y-2}{5} = \frac{z-3}{4}$$

So, vector equation of required line is

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(-3\hat{i} + 5\hat{j} + 4\hat{k}\right)$$

The Plane Ex 29.11 Q24

Here, given mid line $\frac{x-2}{6} = \frac{y-1}{\lambda} = \frac{z+5}{-4}$ is perpendicular to plane $3x - y - 2z = 7$
So, normal vector of plane is parallel to line so,
Direction ratios of normal to plane are proparional to the direction ratios of line
Here,

$$\frac{6}{3} = \frac{\lambda}{-1} = \frac{-4}{-2}$$

cross multiplying the last two

$$-2\lambda = 4$$
$$\lambda = \frac{4}{-2}$$
$$\lambda = -2$$

The Plane Ex 29.11 Q25

The equation of a plane passing through (-1, 2, 0) is

$a(x+1)+b(y-2)+c(z-0)=0$.............(i)

This passes through (2, 2, -1)

$\therefore\ a(2+1)+b(2-2)+c(-1-0)=0$

$3a-c=0$............(ii)

The plane in (i) is parallel to $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{1}$.

Theefore normal to the plane is perpendicular to the line.

$\therefore\ a(1)+b(2)+c(1)=0$.........(iii)

Solving (ii) and (iii) by cross multiplication we get,

$$\frac{a}{0-(-1)(2)}=\frac{b}{(1)(-1)-(3)(1)}=\frac{c}{(3)(2)-0}$$

$$\Rightarrow \frac{a}{2}=-\frac{b}{4}=\frac{c}{6}$$

$$\Rightarrow a=-\frac{b}{2}=\frac{c}{3}=\lambda(\text{say})$$

$$\Rightarrow a=\lambda, b=-2\lambda, c=3\lambda$$

Substituting $a=\lambda, b=-2\lambda, c=3\lambda$ in (i) we get,

$\lambda(x+1)-2\lambda(y-2)+3\lambda(z-0)=0$

$x-2y+3z+5=0$

$\therefore$ The required equation of the plane is $x-2y+3z+5=0$.

Ex 29.12

The Plane Ex 29.12 Q1(i)

Direction ratios of the given line are

$(5-3, 1-4, 6-1) = (2, -3, 5)$

Hence, equation of the line is

$\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-6}{5} = r$

$\Rightarrow x = 2r+5, y = -3r+1, z = 5r+6$

For any point on the yz − plane x = 0

$\Rightarrow 2r+5 = 0 \Rightarrow r = -\frac{5}{2}$

$y = -3(-\frac{5}{2}) + 1 = \frac{17}{2}$

$z = 5(-\frac{5}{2}) + 6 = -\frac{13}{2}$

Hence, the coordinates of the point are $\left(0, \frac{17}{2}, -\frac{13}{2}\right)$.

The Plane Ex 29.12 Q1(ii)

Direction ratios of the given line are

$(5-3, 1-4, 6-1) = (2, -3, 5)$

Hence, equation of the line is

$\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-6}{5} = r$

$\Rightarrow x = 2r+5, y = -3r+1, z = 5r+6$

For any point on zx − plane y = 0

$\Rightarrow -3r+1 = 0 \Rightarrow r = \frac{1}{3}$

$x = 2\left(\frac{1}{3}\right) + 5 = \frac{17}{3}$

$z = 5\left(\frac{1}{3}\right) + 6 = \frac{23}{3}$

Hence, the coordinates of the point are $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$.

The Plane Ex 29.12 Q2

Let the coordinates of the points A and B be

$(3, -4, -5)$ *and* $(2, -3, 1)$ *repectively.*

Equation of the line joining the points (x_1, y_1, z_1) *and* (x_2, y_2, z_2) *is*

$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} = r$, *where r is some constant.*

Thus equation of AB is

$\frac{x-3}{2-3} = \frac{y-(-4)}{(-3)-(-4)} = \frac{z-(-5)}{1-(-5)} = r$

$\Rightarrow \frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = r$

Any point on the line AB is of the form

$-r+3, r-4, 6r-5$

Let P be the point of intersection of the line AB and the plane $2x+y+z=7$

Thus, we have,

$2(-r+3) + r-4+6r-5 = 7$

$\Rightarrow -2r+6+r-4+6r-5 = 7$

$\Rightarrow 5r = 10$

$\Rightarrow r = 2$

Substituting the value of r in $-r+3, r-4, 6r-5$, *the coordinates of P are:*

$(-2+3, 2-4, 6\times 2-5) = (1, -2, 7)$

The Plane Ex 29.12 Q3

The equation of the given line is

$$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right) \qquad ...(1)$$

The equation of the given plane is

$$\vec{r}.\left(\hat{i} - \hat{j} + \hat{k}\right) = 5 \qquad ...(2)$$

Substituting the value of $\vec{r}$ from equation (1) in equation (2), we obtain

$$\left[2\hat{i} - \hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)\right]\cdot\left(\hat{i} - \hat{j} + \hat{k}\right) = 5$$

$$\Rightarrow \left[(3\lambda+2)\hat{i} + (4\lambda-1)\hat{j} + (2\lambda+2)\hat{k}\right]\cdot\left(\hat{i} - \hat{j} + \hat{k}\right) = 5$$

$$\Rightarrow (3\lambda+2) - (4\lambda-1) + (2\lambda+2) = 5$$

$$\Rightarrow \lambda = 0$$

Substituting this value in equation (1), we obtain the equation of the line as

$$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}$$

This means that the position vector of the point of intersection of the line and the plane is $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}$

This shows that the point of intersection of the given line and plane is given by the coordinates, (2, −1, 2). The point is (−1, −5, −10).

The distance d between the points, (2, −1, 2) and (−1, −5, −10), is

$$d = \sqrt{(-1-2)^2 + (-5+1)^2 + (-10-2)^2} = \sqrt{9+16+144} = \sqrt{169} = 13$$

The Plane Ex 29.12 Q4

To find the point of intersection of the line

$\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r}.(\hat{i} - 2\hat{j} + \hat{k}) = 0$,

we substitute $\vec{r}$ of line in the plane.

$$[2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})].(\hat{i} - 2\hat{j} + \hat{k}) = 0$$

$$\Rightarrow [(2+3\lambda)\hat{i} + (-4+4\lambda)\hat{j} + (2+2\lambda)\hat{k}].(\hat{i} - 2\hat{j} + \hat{k}) = 0$$

$$\Rightarrow 2 + 3\lambda + 8 - 8\lambda + 2 + 2\lambda = 0$$

$$\Rightarrow 3\lambda = 12 \Rightarrow \lambda = 4$$

$$\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + 4(3\hat{i} + 4\hat{j} + 2\hat{k}) = 14\hat{i} + 12\hat{j} + 10\hat{k}$$

Hence, the distance of the point $2\hat{i} + 12\hat{j} + 5\hat{k}$ from $14\hat{i} + 12\hat{j} + 10\hat{k}$ is

$$\sqrt{(14-2)^2 + (12-12)^2 + (10-5)^2} = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$$

The Plane Ex 29.12 Q5

Equation of the line through the points A(2, −1,2)

and B(5,3,4) is $\frac{x-2}{5-2} = \frac{y+1}{3+1} = \frac{z-2}{4-2} = r$

$$\Rightarrow \frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} = r$$

$$\Rightarrow x = 3r+2, y = 4r-1, z = 2r+2$$

Substituting these in the plane equation we get

$$(3r+2) - (4r-1) + (2r+2) = 5$$

$$\Rightarrow r = 0$$

$$\Rightarrow x = 2, y = -1, z = 2$$

Distance of (2, −1,2) from (−1, −5, −10) is

$$= \sqrt{(2-(-1))^2 + (-1-(-5))^2 + (2-(-10))^2} = \sqrt{3^2 + 4^2 + 12^2}$$

$$= \sqrt{169} = 13$$

The Plane Ex 29.12 Q6

The equation of a line joining the points A(3, -4, -5) and B(2, -3, 1)is

$$\frac{x-3}{2-3} = \frac{y+4}{-3+4} = \frac{z+5}{1+5} = r$$

$$\Rightarrow x = 3-r, y = -4+r, z = -5+6r$$

Substituting this into the given plane equation we get,

$$2(3-r) + (-4+r) + (-5+6r) = 7$$

$$\Rightarrow r = 2$$

$$\Rightarrow x = 1,\ y = -2,\ z = 7$$

Distance of (1, -2, 7) from (3, 4, 4) is

$$= \sqrt{(3-1)^2 + (4+2)^2 + (4-7)^2}$$

$$= \sqrt{4 + 36 + 9}$$

$$= \sqrt{49}$$

$$= 7$$

Ex 29.13

The Plane Ex 29.13 Q1

$\vec{r} = (2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

$\vec{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$

We know that the lines,

$\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ are coplanar if

$\vec{a_1} \cdot (\vec{b_1} \times \vec{b_2}) = \vec{a_2} \cdot (\vec{b_1} \times \vec{b_2})$

and the equation of the plane containing them is

$\vec{r} \cdot (\vec{b_1} \times \vec{b_2}) = \vec{a_1} \cdot (\vec{b_1} \times \vec{b_2})$

Here

$\vec{a_1} = 2\hat{j} - 3\hat{k}, \vec{b_1} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{a_2} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ and $\vec{b_2} = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

Therefore, $\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} = \hat{i}(8-9) - \hat{j}(4-6) + \hat{k}(3-4)$

$\Rightarrow \vec{b_1} \times \vec{b_2} = -\hat{i} + 2\hat{j} - \hat{k}$

$\Rightarrow \vec{a_1} \cdot (\vec{b_1} \times \vec{b_2}) = (2\hat{j} - 3\hat{k}) \cdot (-\hat{i} + 2\hat{j} - \hat{k}) = 0 + 4 + 3 = 7$

and

$\vec{a_2} \cdot (\vec{b_1} \times \vec{b_2}) = (2\hat{i} + 6\hat{j} + 3\hat{k}) \cdot (-\hat{i} + 2\hat{j} - \hat{k}) = -2 + 12 - 3 = 7$

Since $\vec{a_1} \cdot (\vec{b_1} \times \vec{b_2}) = \vec{a_2} \cdot (\vec{b_1} \times \vec{b_2})$, the lines are coplanar.

Now the equation of the plane containing the given lines is

$\vec{r} \cdot (\vec{b_1} \times \vec{b_2}) = \vec{a_1} \cdot (\vec{b_1} \times \vec{b_2})$

$\vec{r} \cdot (-\hat{i} + 2\hat{j} - \hat{k}) = 7$

$\Rightarrow \vec{r} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = -7$

$\Rightarrow \vec{r} \cdot (\hat{i} - 2\hat{j} + \hat{k}) + 7 = 0$

The Plane Ex 29.13 Q2

We know that lines $\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$ and $\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$ are coplanar if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

And equation of plane containing them is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

Here, equation of lines are

$$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1} \text{ and } \frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$$

So, $x_1 = -1,\ y_1 = 3,\ z_1 = -2,\ l_1 = -3,\ m_1 = 2,\ n_1 = 1$

$x_2 = 0,\ y_2 = 7,\ z_2 = -7,\ l_2 = 1,\ m_2 = -3,\ n_2 = 2$

So,

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 0+1 & 7-3 & -7+2 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 4 & -5 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}$$

$= 1(4+3) - 4(-6-1) - 5(9-2)$
$= 7 + 28 - 35$

$= 0$

So, lines are coplanar.

Equation of plane containing line is

$$\begin{vmatrix} x+1 & y-3 & z+2 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix} = 0$$

$(x+1)(4+3) - (y-3)(-6-1) + (z+2)(9-2) = 0$
$7x + 7 + 7y - 21 + 7z + 14 = 0$
$7x + 7y + 7z = 0$

The Plane Ex 29.13 Q3

We know that the plane passing through (x_1, y_1, z_1) is given by

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ $\qquad$ ---(i)

Required plane is passing through $(0, 7, -7)$, so

$a(x - 0) + b(y - 7) + c(z + 7) = 0$
$ax + b(y - 7) + c(z + 7) = 0$ $\qquad$ ---(ii)

Plane (ii) also contains line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ so, it passes through point $(-1, 3, -2)$,

$a(-1) + b(3 - 7) + c(-2 + 7) = 0$
$-a - 4b + 5c = 0$ $\qquad$ ---(iii)

Also, plane (ii) will be parallel to line
so, $a_1a_2 + b_1b_2 + c_1c_2 = 0$
$(a)(-3) + (b)(2) + (c)(1) = 0$
$-3a + 2b + c = 0$ $\qquad$ ---(iv)

Solving (iii) and (iv) by cross-multiplication,

$$\frac{a}{(-4)(1) - (5)(2)} = \frac{b}{(-3)(5) - (-1)(1)} = \frac{c}{(-1)(2) - (-4)(-3)}$$
$$\frac{a}{-4 - 10} = \frac{b}{-15 + 1} = \frac{c}{-2 - 12}$$

$$\frac{a}{-14} = \frac{b}{-14} = \frac{c}{14} = \lambda \text{ (Say)}$$

$$\Rightarrow \quad a = -14\lambda,\ b = -14\lambda,\ c = -14\lambda$$

Put a, b, c in equation (ii),

$$ax + b(y-7) + c(z+7) = 0$$
$$(-14\lambda)x + (-14\lambda)(y-7) + (-14\lambda)(z+7) = 0$$

Dividing by (-14λ), we get

$$x + y - 7 + z + 7 = 0$$
$$x + y + z = 0$$

So, equation of plane containing the given point and line is $x + y + z = 0$

The other line is $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$

So, $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$(1)(1) + (1)(-3) + (1)(2) = 0$$
$$1 - 3 + 2 = 0$$

$$0 = 0$$

$$LHS = RHS$$

So, $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ lie on plane $x + y + z = 0$

The Plane Ex 29.13 Q4

We know that equation of plane passing through (x_1, y_1, z_1) is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \qquad \text{---(i)}$$

Since, required plane contain lines

$$\frac{x-4}{1} = \frac{y-3}{-4} = \frac{z-2}{5} \text{ and } \frac{x-3}{1} = \frac{y+2}{-4} = \frac{z}{5}$$

So, required plane passes through $(4,3,2)$ and $(3,-2,0)$, so, equation of required plane is,

$$a(x-4) + b(y-3) + c(z-2) = 0 \qquad \text{---(ii)}$$

Plane (ii) also passes through $(3,-2,0)$, so

$$a(3-4) + b(-2-3) + c(0-2) = 0$$
$$-a - 5b - 2c = 0$$
$$a + 5b + 2c = 0 \qquad \text{---(iii)}$$

Now plane (ii) is also parallel to line with direction ratios $1, -4, 5$, so,

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(a)(1) + (b)(-4) + (c)(5) = 0$$
$$a - 4b + 5c = 0 \qquad \text{---(iv)}$$

Solving equation (iii) and (iv) by cross-multiplication,

$$\frac{a}{(5)(5)-(-4)(2)} = \frac{b}{(1)(2)-(1)(5)} = \frac{c}{(1)(-4)-(1)(5)}$$
$$\frac{a}{25+8} = \frac{b}{2-5} = \frac{c}{-4-5}$$

$$\frac{a}{33} = \frac{b}{-3} = \frac{c}{-9}$$

Multiplying by 3,

$$\frac{a}{11} = \frac{b}{-1} = \frac{c}{-3} = \lambda \text{ (Say)}$$

$$\Rightarrow \quad a = 11\lambda,\ b = -\lambda,\ c = -3\lambda$$

Put a, b, c in equation (ii),

$$a(x-4) + b(y-3) + c(z-2) = 0$$
$$(11\lambda)(x-4) + (-\lambda)(y-3) + (-3\lambda)(z-2) = 0$$
$$11\lambda x - 44\lambda - \lambda y + 3\lambda - 3\lambda z + 6\lambda = 0$$

$$11\lambda x - \lambda y - 3\lambda z - 35\lambda = 0$$

Dividing by λ,

$$11x - y - 3z - 35 = 0$$

So, equation of required plane is,

$$11x - y - 3z - 35 = 0$$

The Plane Ex 29.13 Q5

We have, equation of line is

$$\frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2} = \lambda \text{ (Say)}$$

General point on this line is given by

$$(3\lambda - 4,\ 5\lambda - 6,\ -2\lambda + 1) \qquad ---\text{(i)}$$

Another equation of line is

$$3x - 2y + z + 5 = 0 = 2x + 3y + 4z - 4$$

Let a, b, c be the direction ratio of the line so, it will be perpendicular to normal of $3x - 2y + z + 5 = 0$ and $2x + 3y + 4z - 4 = 0$, so

Using $a_1a_2 + b_1b_2 + c_1c_2 = 0$
$$(3)(a) + (-2)(b) + (1)(c) = 0$$
$$3a - 2b + c = 0 \qquad ---\text{(ii)}$$

Again,

$$(2)(a) + (3)(b) + (4)(c) = 0$$
$$2a + 3b + 4c = 0 \qquad ---\text{(iii)}$$

Solving (ii) and (iii) by cross-multiplication,

$$\frac{a}{(-2)(4)-(3)(1)} = \frac{b}{(2)(1)-(3)(4)} = \frac{c}{(3)(3)-(-2)(2)}$$

$$\frac{a}{-8-3} = \frac{b}{2-12} = \frac{c}{9+4}$$

$$\frac{a}{-11} = \frac{b}{-10} = \frac{c}{13}$$

Direction ratios are proportional to $-11, -10, 13$

Let $z = 0$, so

$$3x - 2y = -5 \qquad ---(A)$$
$$2x + 3y = 4 \qquad ---(B)$$

Solving (A) and (B),

$$\begin{array}{r} 6x - 4y = -10 \\ 6x + 9y = \ \ 12 \\ - \quad - \qquad - \\ \hline -13y = -12 \end{array}$$

$$y = \frac{22}{13}$$

Put y in equation (A),

$$3x - 2\left(\frac{22}{13}\right) = -5$$

$$3x - \frac{44}{13} = -5$$

$$3x = -5 + \frac{44}{13}$$

$$3x = -\frac{21}{13}$$

$$x = -\frac{7}{13}$$

So, equation of line (2) in symmetrical form,

$$\frac{x + \frac{7}{13}}{-11} = \frac{y - \frac{22}{13}}{-10} = \frac{z-0}{13}$$

Put the general point of line (1) from equation (1)

$$\frac{3\lambda - 4 + \frac{7}{13}}{-11} = \frac{5\lambda - 6 - \frac{22}{13}}{-10} = \frac{-2\lambda + 1}{13}$$

$$\frac{39\lambda - 52 + 7}{-11 \times 13} = \frac{65\lambda - 78 - 22}{-10 \times 13} = \frac{-2\lambda + 1}{13}$$

$$\frac{39\lambda - 45}{-11} = \frac{65\lambda - 100}{-10} = \frac{-2\lambda + 1}{1}$$

The equation of the plane is $45x - 17y + 25z + 53 = 0$

Their point of intersection is $(2, 4, -3)$

The Plane Ex 29.13 Q6

We know that plane $\vec{r}.\vec{n} = d$ contains the line $\vec{r} = \vec{a} + \lambda \vec{b}$ if

(i) $\vec{b}.\vec{n} = 0$ (ii) $\vec{a}.\vec{n} = d$ - - - (i)

Given, equation of plane $\vec{r}.\left(\hat{i} + 2\hat{j} - \hat{k}\right) = 3$ and equation of line $\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(2\hat{i} + \hat{j} + 4\hat{k}\right)$

so, $\vec{n} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{a} = \hat{i} + \hat{j}$

$d = 3$ $\vec{b} = 2\hat{i} + \hat{j} + 4\hat{k}$

$$\vec{b}.\vec{n} = \left(2\hat{i} + \hat{j} + 4\hat{k}\right)\left(\hat{i} + 2\hat{j} - \hat{k}\right)$$
$$= (2)(1) + (1)(2) + (4)(-1)$$
$$= 2 + 2 - 4$$

$$\vec{b}.\vec{n} = 0$$

$$\vec{a}.\vec{n} = \left(\hat{i} + \hat{j}\right)\left(\hat{i} + 2\hat{j} - \hat{k}\right)$$
$$= (1)(1) + (1)(2) + (0)(-1)$$
$$= 1 + 2 - 0$$
$$= 3$$

$$= d$$

since, $\vec{b}.\vec{n} = 0$ and $\vec{a}.\vec{n} = d$, so, from (i)

Given line lie on the given plane.

The Plane Ex 29.13 Q7

Let $L_1 : \frac{x+3}{3} = \frac{y}{-2} = \frac{z-7}{6}$ and

$L_2 : \frac{x+6}{1} = \frac{y+5}{-3} = \frac{z-1}{2}$ be the equations of two lines

Let the plane be $ax + by + cz + d = 0$...(1)

Given that the required plane passes through the intersection of the lines L_1 and L_2.

Hence the normal to the plane is perpendicular to the lines L_1 and L_2.

$\therefore 3a - 2b + 6c = 0$

$a - 3b + 2c = 0$

Using cross-multiplication, we get,

$$\frac{a}{-4+18} = \frac{b}{6-6} = \frac{c}{-9+2}$$
$$\Rightarrow \frac{a}{14} = \frac{b}{0} = \frac{c}{-7}$$
$$\Rightarrow \frac{a}{2} = \frac{b}{0} = \frac{c}{-1}$$

The Plane Ex 29.13 Q8

Let the equation of the plane be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$........(i)

Plane is passing through (3,4,2) and (7,0,6)

$$\frac{3}{a}+\frac{4}{b}+\frac{2}{c}=1$$

$$\frac{7}{a}+\frac{0}{b}+\frac{6}{c}=1$$

Required plane is perpendicular to $2x-5y-15=0$

$$\frac{2}{a}+\frac{-5}{b}+\frac{0}{c}=0$$

$\Rightarrow 2b=5a$

$\therefore b=2.5a$

$$\frac{3}{a}+\frac{4}{2.5a}+\frac{2}{c}=1$$

$$\frac{7}{a}+\frac{6}{c}=1$$

Solving the above 2 equations,

$a=3.4=\frac{17}{5}$, $b=\ 8.5=\frac{17}{2}$ and $c=\frac{-34}{6}=-\frac{17}{3}$

Substituting the values in (i)

$$\frac{x}{\frac{17}{5}}+\frac{y}{\frac{17}{2}}+\frac{z}{-\frac{17}{3}}=1$$

$$\Rightarrow \frac{5x}{17}+\frac{2y}{17}-\frac{3z}{17}=1$$

$\Rightarrow 5x+2y-3z=17$

$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k}).(5\hat{i}+2\hat{j}-3\hat{k})=17$

$\Rightarrow \vec{r}.(5\hat{i}+2\hat{j}-3\hat{k})=17$

Vector equation of the plane is $\vec{r}.(5\hat{i}+2\hat{j}-3\hat{k})=17$.

The line passes through $B(1,3,-2)$.

$5(1)+2(3)-3(-2)=17$

The point B lies on the plane.

$\therefore$The line $\vec{r}=\hat{i}+3\hat{j}-2\hat{k}+\lambda(\hat{i}-\hat{j}+\hat{k})$ lies on the plane $\vec{r}.(5\hat{i}+2\hat{j}-3\hat{k})=17$.

The Plane Ex 29.13 Q9

The direction ratio of the line $\frac{x-1}{-3}=\frac{y-2}{-2k}=\frac{z-3}{2}$ is

$r_1=(-3,-2k,2)$

The direction ratio of the line $\frac{x-1}{k}=\frac{y-2}{1}=\frac{z-3}{5}$ is

$r_2=(k,1,5)$

Since the lines $\frac{x-1}{-3}=\frac{y-2}{-2k}=\frac{z-3}{2}$ and $\frac{x-1}{k}=\frac{y-2}{1}=\frac{z-3}{5}$ are perpendicular so

$$r_1\cdot r_2=0$$
$$(-3,-2k,2)\cdot(k,1,5)=0$$
$$-3k-2k+10=0$$
$$-5k=-10$$
$$k=2$$

Therefore the equation of the lines are $\frac{x-1}{-3}=\frac{y-2}{-4}=\frac{z-3}{2}$ and $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{5}$

The equation of the plane containing the perpendicular lines $\frac{x-1}{-3}=\frac{y-2}{-4}=\frac{z-3}{2}$ and $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{5}$ is

$$\begin{vmatrix} x & y & z \\ -3 & -4 & 2 \\ 2 & 1 & 5 \end{vmatrix}=0$$

$$\begin{vmatrix} x & y & z \\ -3 & -4 & 2 \\ 2 & 1 & 5 \end{vmatrix}=0$$

$$(-20-2)x-y(-15-4)+z(-3+8)+d=0$$
$$-22x+19y+5z+d=0$$

The line $\frac{x-1}{-3}=\frac{y-2}{-4}=\frac{z-3}{2}$ pass through the point $(1,2,3)$ so putting $x=1, y=2, z=3$ in the equation $-22x+19y+5z+d=0$ we get

$$-22(1)+19(2)+5(3)+d=0$$
$$d=22-38-15$$
$$d=-31$$

Therefore the equation of the plane containing the lines is

$-22x+19y+5z=31$

The Plane Ex 29.13 Q10

Any point on the line

$$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}=k$$

is of the form, $(3k+2,\ 4k-1,\ 2k+2)$.

If the point $P(3k+2,\ 4k-1,\ 2k+2)$ *lies in the plane* $x-y+z-5=0$, *we have*,

$(3k+2)-(4k-1)+(2k+2)-5=0$

$\Rightarrow 3k+2-4k+1+2k+2-5=0$

$\Rightarrow k=0$

Thus, the coordinates of the point of intersection of the line and the plane are: $P(3\times0+2,\ 4\times0-1,\ 2\times0+2)=P(2,-1,2)$

Let θ *be the angle between the line and the plane.*

Thus,

$$\sin\theta=\frac{al+bm+cn}{\sqrt{a^2+b^2+c^2}\sqrt{l^2+m^2+n^2}}$$, *where*, l,m *and* n *are the direction ratios of the line and* a,b *and* c *are the direction ratios of the normal to the plane.*

Here, $l=3$, $m=4$, $n=2$, $a=1$, $b=-1$, *and* $c=1$

Hence,

$$\sin\theta=\frac{1\times3+(-1)\times4+1\times2}{\sqrt{1^2+(-1)^2+1^2}\sqrt{3^2+4^2+2^2}}$$

$$\Rightarrow\sin\theta=\frac{1}{\sqrt{3}\sqrt{29}}$$

$$\Rightarrow\theta=\sin^{-1}\left(\frac{1}{\sqrt{3}\sqrt{29}}\right)$$

The Plane Ex 29.13 Q11

Let A, B *and* C *be three points with position vectors*

$\hat{i}+\hat{j}-2\hat{k}$, $2\hat{i}-\hat{j}+\hat{k}$ *and* $\hat{i}+2\hat{j}+\hat{k}$.

Thus, $\overrightarrow{AB}=\vec{b}-\vec{a}=\left(2\hat{i}-\hat{j}+\hat{k}\right)-\left(\hat{i}+\hat{j}-2\hat{k}\right)=\hat{i}-2\hat{j}+3\hat{k}$

$\overrightarrow{AC}=\vec{c}-\vec{a}=\left(\hat{i}+2\hat{j}+\hat{k}\right)-\left(\hat{i}+\hat{j}-2\hat{k}\right)=\hat{j}+3\hat{k}$

Now consider $\overrightarrow{AB}\times\overrightarrow{AC}$:

$$\vec{n}=\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-2&3\\0&1&3\end{vmatrix}$$

$\vec{n}=\hat{i}(-6-3)-3\hat{j}+\hat{k}=-9\hat{i}-3\hat{j}+\hat{k}$

So, the equation of the required plane is

$(\vec{r}-\vec{a})\cdot\vec{n}=0$

$\Rightarrow(\vec{r}\cdot\vec{n})=(\vec{a}\cdot\vec{n})$

$\Rightarrow\left(\vec{r}\cdot\left(-9\hat{i}-3\hat{j}+\hat{k}\right)\right)=\left(\hat{i}+\hat{j}-2\hat{k}\right)\cdot\left(-9\hat{i}-3\hat{j}+\hat{k}\right)$

$\Rightarrow\vec{r}\cdot\left(9\hat{i}+3\hat{j}-\hat{k}\right)=14$

Also, find the coordinates of the point of intersection of this plane and the line $\vec{r}=3\hat{i}-\hat{j}-\hat{k}+\lambda\left(2\hat{i}-2\hat{j}+\hat{k}\right)$

Any point on the line $\vec{r}=3\hat{i}-\hat{j}-\hat{k}+\lambda\left(2\hat{i}-2\hat{j}+\hat{k}\right)$ *is of the form*,

$(3+2\lambda,\ -1-2\lambda,\ -1+\lambda)$

If the point $P(3+2\lambda,\ -1-2\lambda,\ -1+\lambda)$ *lies in the plane,*

$\vec{r}\cdot\left(9\hat{i}+3\hat{j}-\hat{k}\right)=14$, *we have,*

$9(3+2\lambda)-3(1+2\lambda)-(-1+\lambda)=14$

$\Rightarrow 27+18\lambda-3-6\lambda+1-\lambda=14$

$\Rightarrow 11\lambda=-11$

$\Rightarrow \lambda=-1$

Thus, the required point of intersection is

$P(3+2\lambda,\ -1-2\lambda,\ -1+\lambda)$

$\Rightarrow P(3+2(-1),\ -1-2(-1),\ -1+(-1))$

$\Rightarrow P(1,\ 1,\ -2)$

The Plane Ex 29.13 Q12

$$\frac{5-x}{-4}=\frac{y-7}{4}=\frac{z+3}{-5}$$

$$\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}\text{.......(i)}$$

$$\frac{x-8}{7}=\frac{2y-8}{2}=\frac{z-5}{3}$$

$$\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}\text{.....(ii)}$$

Here,

$a_1=4, b_1=4, c_1=-5$

$a_2=7, b_2=1, c_2=3$

$x_1=5, y_1=7, z_1=-3$

$x_2=8, y_2=4, z_2=5$

Condition for two lines to be coplanar,

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

$$\therefore \begin{vmatrix} 8-5 & 4-7 & 5+3 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$

$$=\begin{vmatrix} 3 & -3 & 8 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$

$=3(12+5)+3(12+35)+8(4-28)$

$=3\times 17+3\times 47+8\times(-24)$

$=51+141-192$

$=192-192$

$=0$

$\therefore$ The lines are coplanar to each other.

The Plane Ex 29.13 Q13

Required equation of plane is passing through the point (3, 2, 0),

$\therefore\ a(x-3)+b(y-2)+c(z-0)=0$

$\Rightarrow a(x-3)+b(y-2)+cz=0$..........(i)

Required equation of plane also contains the line $\frac{x-3}{1}=\frac{y-6}{5}=\frac{z-4}{4}$,

so it passes through the point (3, 2, 0)

$\Rightarrow a(3-3)+b(6-2)+c4=0$

$\Rightarrow 4b+4c=0$..............................(ii)

Also plane will be parallel to,

$a(1)+b(5)+c(4)=0$

$a+5b+4c=0$.............(iii)

Solving (ii) and (iii) by cross multiplication,

$\frac{a}{16\text{-}20}=\frac{b}{4-0}=\frac{c}{0-4}=\lambda(\text{say})$

$-\frac{a}{4}=\frac{b}{4}=-\frac{c}{4}=\lambda(\text{say})$

$\Rightarrow a=-\lambda, b=\lambda,\ c=-\lambda$

Put $a=-\lambda, b=\lambda,\ c=-\lambda$ in equation (i) we get

$(-\lambda)(x-3)+(\lambda)(y-2)+(-\lambda)z=0$

$\Rightarrow -x+3+y-2-z=0$

$\Rightarrow x-y+z-1=0$

Ex 29.14

The Plane Ex 29.14 Q1

Consider

$$l_1 : \frac{x-2}{-1} = \frac{y-5}{2} = \frac{z-0}{3}$$
$$l_2 : \frac{x-0}{2} = \frac{y+1}{-1} = \frac{z-1}{2}$$

Clearly line l_1 passes through the point $P(2,5,0)$

The equation of a plane containing line l_2 is

$$a(x-0)+b(y+1)+c(z-1)=0 \qquad \text{...... (1)}$$

Where $2a-b+2c=0$

If it is parallel to line l_1 then

$$-a+2b+3c=0$$

Therefore

$$\frac{a}{-7} = \frac{b}{-8} = \frac{c}{3}$$

Substituting values of a, b, c in the equation (1) we obtain

$$a(x-0)+b(y+1)+c(z-1)=0$$
$$-7(x-0)-8(y+1)+3(z-1)=0$$
$$-7x-8y-8+3z-3=0$$
$$7x+8y-3z+11=0 \qquad \text{...... (2)}$$

This is the equation of the plane containing line l_2 and parallel to line l_1

Shortest distance between l_1 and l_2 = Distance between point $P(2,5,0)$ and plane (2)

$$= \left| \frac{14+40+11}{\sqrt{7^2+8^2+(-3)^2}} \right| = \frac{65}{\sqrt{122}}$$

The Plane Ex 29.14 Q2

$$l_1 : \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$
$$l_2 : \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

Let the equation of the plane containing l_1 be $a(x+1)+b(y+1)+c(z+1)=0$

Plane is parallel to l_1: $7a-6b+c=0$......(i)

Plane is parallel to l_2: $a-2b+c=0$........(ii)

Solving (i) and (ii),

$$\frac{a}{-6+2} = \frac{b}{1-7} = \frac{c}{-14+6}$$
$$\frac{a}{-4} = \frac{b}{-6} = \frac{c}{-8}$$

$\therefore$ Equation of the plane is $-4(x+1)-6(y+1)-8(z+1)=0$

$4(x+1)+6(y+1)+8(z+1)=0$ is the equation of the plane.

The Plane Ex 29.14 Q3

The equation of a plane containing the line 3x - y - 2z + 4 = 0 = 2x + y + z + 1 is
$x(2\lambda+3)+y(\lambda-1)+z(\lambda-2)+\lambda+4=0$............(i)

If it is parallel to the line then $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z+2}{1}$ then,
$2(2\lambda+3)+4(\lambda-1)+(\lambda-2)=0$
$\Rightarrow \lambda=0$

Putting $\lambda=0$ in (i) we get,
3x - y - 2z + 4 = 0............(ii)
As this equation of the plane containing the second line and paralle to the first line.

Clearly the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z+2}{1}$ passes through the point (1, 3, -2)
So, the shortest distance 'd' between the given lines is equal to the length of perpendicular from (1, 3, -2)on the plane (ii).
$d=\left|\frac{3-3+4+4}{\sqrt{1+9+4}}\right|=\frac{8}{\sqrt{14}}$

Ex 29.15

The Plane Ex 29.15 Q1

$3x + 4y - 6z + 1 = 0$

Line passing through orgin and perpendicular to plane is given by

$\frac{x}{3} = \frac{y}{4} = \frac{z}{-6} = r(say)$

So let the image of (0,0,0) is (3r, 4r, -6r)

Midpoint of (0,0,0) and (3r, 4r, -6r) lies on plane.

$3\left(\frac{3r}{2}\right) + 2(4r) - 3(-6r) + 1 = 0$

$30.5r = -1$

$r = \frac{-2}{61}$

So image is $\left(\frac{-6}{61}, \frac{-8}{61}, \frac{12}{61}\right)$

The Plane Ex 29.15 Q2

Here, we have to find reflection of the point $P(1,2,-1)$ in the plane $3x - 5y + 4z = 5$

Let Q be the reflection of the point P and R be the mid-point of PQ.
Then, R lies on the plane $3x - 5y + 4z = 5$.

Direction ratios of PQ are proportional to $3, -5, 4$ and PQ is passing through $(1,2,-1)$.
So, equation of PQ is given by,

$$\frac{x-1}{3} = \frac{y-2}{-5} = \frac{z+1}{4} = \lambda \text{ (Say)}$$

Let Q be $(3\lambda + 1, -5\lambda + 2, 4\lambda - 1)$

The coordinates of R are $\left(\frac{3\lambda+1+1}{2}, \frac{-5\lambda+2+2}{2}, \frac{4\lambda-1-1}{2}\right) = \left(\frac{3\lambda+2}{2}, \frac{-5\lambda+4}{2}, \frac{4\lambda-2}{2}\right)$

Since, R lies on the given plane $3x - 5y + 4z = 5$

$$\therefore \quad 3\left(\frac{3\lambda+2}{2}\right) - 5\left(-\frac{5\lambda+4}{2}\right) + 4\left(\frac{4\lambda-2}{2}\right) = 5$$

$\Rightarrow \quad 9\lambda + 6 + 25\lambda - 20 + 16\lambda - 8 = 10$

$\Rightarrow \quad 50\lambda - 22 = 10$

$\Rightarrow \quad 50\lambda = 10 + 22$

$\Rightarrow \quad 50\lambda = 32$

$\Rightarrow \quad \lambda = \frac{16}{25}$

So, $Q = (3\lambda + 1, -5\lambda + 2, 4\lambda - 1)$

$= \left(3\left(\frac{16}{25}\right) + 1, -5\left(\frac{16}{25}\right) + 2, 4\left(\frac{16}{25}\right) - 1\right)$

$= \left(\frac{48}{25} + 1, -\frac{16}{5} + 2, \frac{64}{25} - 1\right)$

$= \left(\frac{73}{25}, -\frac{6}{5}, \frac{39}{25}\right)$

So, reflection of $P(1,2,-1) = \left(\frac{73}{25}, -\frac{6}{5}, \frac{39}{25}\right)$

The Plane Ex 29.15 Q3

We have to find foot of the perpendicular, say Q, drawn from $P(5,4,2)$ to the line

$$\frac{x+1}{2}=\frac{y-3}{3}=\frac{z-1}{-1}=\lambda \quad \text{(say)}$$

Let Q be $(2\lambda-1, 3\lambda+3, -\lambda+1)$

Direction ratios of line PQ are $(2\lambda-1-5,\ 3\lambda+3-4, -\lambda+1-2)$ or $(2\lambda-6,\ 3\lambda-1, -\lambda-1)$

Here, line PQ is perpendicular to line given(AB).

So,

$$a_1a_2+b_1b_2+c_1c_2=0$$
$$(2\lambda-6)(2)+(3\lambda-1)(3)+(-\lambda-1)(-1)=0$$
$$4\lambda-12+9\lambda-3+\lambda+1=0$$
$$14\lambda-14=0$$
$$\lambda=\frac{14}{14}$$
$$\lambda=1$$

So, $Q=(2\lambda-1, 3\lambda+3, -\lambda+1)$

$$=(2(1)-1, 3(1)+3, -(1)+1)$$
$$=(2-1, 3+3, -1+1)$$
$$=(1,6,0)$$

Length of perpendicular PQ

$$=\sqrt{(5-1)^2+(4-6)^2+(2-0)^2} \qquad [\text{Using Distance formula}]$$
$$=\sqrt{16+4+4}$$
$$=\sqrt{24}$$
$$=2\sqrt{6}$$

So,

Foot of perpendicular is $(1,6,0)$

Length of the perpendicular is $2\sqrt{6}$ units

The Plane Ex 29.15 Q4

Here, we have to find image of the point $P(3,1,2)$ in the plane $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})=4$ or $2x-y+z=4$.

Let Q be the image of the point P.

So,

Direction ratios of normal to the point are $2,-1,1$

Direction ratios of line PQ perpendicular to $2,-1,1$ and PQ is passing through $(3,1,2)$.

So equation of PQ is

$$\frac{x-3}{2}=\frac{y-1}{-1}=\frac{z-2}{1}=\lambda \quad \text{(say)} \qquad \left[\begin{array}{l}\text{Using equation of line passing through }(x_1,y_1,z_1)\text{ is}\\ \dfrac{x-x_1}{a}=\dfrac{y-y_1}{b}=\dfrac{z-z_1}{c}\end{array}\right]$$

General point on the line PQ is $(2\lambda+3,\ -\lambda+1,\ \lambda+2)$

Let Q be $(2\lambda+3,\ -\lambda+1,\ \lambda+2)$

Let R be the mid point of PQ. Then,

coordinates of R are $\left(\frac{2\lambda+3+3}{2}, \frac{-\lambda+1+1}{2}, \frac{\lambda+2+2}{2}\right)=\left(\frac{2\lambda+6}{2}, \frac{-\lambda+2}{2}, \frac{\lambda+4}{2}\right)$

Since, R lies on the plane $2x - y + z = 4$, we have,

$$2\left(\frac{2\lambda+6}{2}\right) - \left(\frac{-\lambda+2}{2}\right) + \left(\frac{\lambda+4}{2}\right) = 4$$

$\Rightarrow \quad 4\lambda + 12 + \lambda - 2 + \lambda + 4 = 8$

$\Rightarrow \quad 6\lambda = 8 - 14$

$\Rightarrow \quad \lambda = \frac{-6}{6}$

$\Rightarrow \quad \lambda = -1$

So,

Image of $P = Q\left(2(-1) + 3, -(-1) + 1, -1 + 2\right)$

Image of $P = (1, 2, 1)$

The equation of the perpendicular line through $3\hat{i} + \hat{j} + 2\hat{k}$ is

$\bar{r} = 3\hat{i} + \hat{j} + 2\hat{k} + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$

The position vector of the image point is

$3\hat{i} + \hat{j} + 2\hat{k} + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right) = (3+2\lambda)\hat{i} + (1-\lambda)\hat{j} + (2+k)\hat{k}$

The position vector of the foot of the perpendicular is

$$\frac{\left[(3+2\lambda)\hat{i} + (1-\lambda)\hat{j} + (2+\lambda)\hat{k}\right] + \left[3\hat{i} + \hat{j} + 2\hat{k}\right]}{2}$$

$$= (3+\lambda)\hat{i} + \left(1 - \frac{\lambda}{2}\right)\hat{j} + \left(2 + \frac{\lambda}{2}\right)\hat{k}$$

Putting $\lambda = -1$ the position vector of the foot of the perpendicular is

$2\hat{i} + \frac{3}{2}\hat{j} + \frac{3}{2}\hat{k}$

The Plane Ex 29.15 Q5

$2x - 2y + 4z + 5 = 0$

$(1,1,2)$

$= \left|\frac{2-2+8+5}{\sqrt{1+1+4}}\right| = \frac{13}{\sqrt{6}}$

Let the foot of perpendicular be(x,y,z). So DR's are in proportional

$\frac{x-1}{2} = \frac{y-1}{-2} = \frac{z-2}{4} = k$

$x = 2k + 1$

$y = -2k + 1$

$z = 4k + 2$

Substitute (x,y,z)=$(2k+1, \ -2k+1, 4k+2)$ in plane equation

$2x - 2y + 4z + 5 = 0$

$4k + 2 + 4k - 2 + 16k + 8 + 5 = 0$

$24k = -13$

$k = \frac{-13}{24}$

$(x, y, z) = (\frac{-1}{12}, \frac{5}{3}, \frac{-1}{6})$

The Plane Ex 29.15 Q6

Here, we have to find distance of the point $P(1,-2,3)$ from the plane

$x-y+z=5$ measured parallel to line AB, $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$

Let Q be the mid point of the line joining P to plane.

Here, PQ is parallel to line AB

$\Rightarrow$ Direction ratios of line PQ are proportional to direction ratios of line AB

$\Rightarrow$ Direction ratios of line PQ are $2, 3, -6$ and PQ is passing through $P(1,-2,3)$.

So equation of PQ is given by

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

$$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6}=\lambda \quad \text{(say)}$$

General point on line PQ is $(2\lambda+1,\ 3\lambda-2, -6\lambda+3)$

Suppose coordinates of Q be $(2\lambda+1, 3\lambda-2, -6\lambda+3)$

General point on line PQ is $(2\lambda+1,\ 3\lambda-2, -6\lambda+3)$

Suppose coordinates of Q be $(2\lambda+1, 3\lambda-2, -6\lambda+3)$

Since Q lies on the plane $x-y+z=5$

$$(2\lambda+1)-(3\lambda-2)+(-6\lambda+3)=5$$

$$2\lambda+1-3\lambda+2-6\lambda+3=5$$

$$-7\lambda=5-6$$

$$-7\lambda=-1$$

$$\lambda=\frac{1}{7}$$

Coordinate of Q = $(2\lambda+1, 3\lambda-2, -6\lambda+3)=\left(\frac{9}{7},\frac{-11}{7},\frac{15}{7}\right)$

Distance between $(1,-2,3)$ and plane $= PQ$

$$=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

$$=\sqrt{\left(1-\frac{9}{7}\right)^2+\left(-2+\frac{11}{7}\right)^2+\left(3-\frac{15}{7}\right)^2}$$

$$=\sqrt{\frac{4}{49}+\frac{9}{49}+\frac{36}{49}}$$

$$=\sqrt{\frac{49}{49}}$$

$$=1$$

Required distance = 1 unit

The Plane Ex 29.15 Q7

Let Q be the foot of the perpendicular.

Here, Direction ratios of normal to plane is $3, -1, -1$

$\Rightarrow$ Line PQ is parallel to normal to plane

$\Rightarrow$ Direction ratios of PQ are proportional to $3, -1, -1$ and PQ is passing through $P(2,3,7)$.

So,

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$\frac{x - 2}{3} = \frac{y - 3}{-1} = \frac{z - 7}{-1} = \lambda \quad \text{(say)}$$

General point on line PQ

$$= (3\lambda + 2,\ -\lambda + 3, -\lambda + 7)$$

Coordinates of Q be $(3\lambda + 2, -\lambda + 3, -\lambda + 7)$

Point Q lies on the plane $3x - y - z = 7$.

So,

$$3(3\lambda + 2) - (-\lambda + 3) - (-\lambda + 7) = 7$$

$$9\lambda + 6 + \lambda - 3 + \lambda - 7 = 7$$

$$11\lambda = 7 + 4$$

$$11\lambda = 11$$

$$\lambda = \frac{11}{11}$$

$$\lambda = 1$$

$\therefore$ Coordinate of Q $= (3\lambda + 2, -\lambda + 3, -\lambda + 7)$

$$= (3(1) + 2, -(1) + 3, -(1) + 7)$$

$$= (5, 2, 6)$$

Length of the perpendicular PQ

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$= \sqrt{(2 - 5)^2 + (3 - 2)^2 + (7 - 6)^2}$$

$$= \sqrt{9 + 1 + 1}$$

$$= \sqrt{11}$$

The Plane Ex 29.15 Q8

Here, we have to find image of point $P(1,3,4)$ in the plane $2x - y + z + 3 = 0$

Let Q be the image of the point.

Here, Direction ratios of normal to plane are $2,-1,1$

$\Rightarrow$ Direction ratios of PQ which is parallel to normal to the plane is proportional to $2,-1,1$ and line PQ is passing through $P(1,3,4)$.

So, equation of line PQ is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$\frac{x - 1}{2} = \frac{y - 3}{-1} = \frac{z - 4}{1} = \lambda \quad (\text{say})$$

General point on line PQ

$$= (2\lambda + 1,\ -\lambda + 3, \lambda + 4)$$

Let Q be $(2\lambda + 1, -\lambda + 3, \lambda + 4)$

Q is image of P, so R is the mid point of PQ

Coordinates of R $\left(\frac{2\lambda + 1 + 1}{2}, \frac{-\lambda + 3 + 3}{2}, \frac{\lambda + 4 + 4}{2}\right)$

$$= \left(\frac{2\lambda + 2}{2}, \frac{-\lambda + 6}{2}, \frac{\lambda + 8}{2}\right)$$

$$= \left(\lambda + 1, \frac{-\lambda + 6}{2}, \frac{\lambda + 8}{2}\right)$$

Point R is on th plane $2x - y + z + 3 = 0$

$$= 2(\lambda + 1) - \left(\frac{-\lambda + 6}{2}\right) + \left(\frac{\lambda + 8}{2}\right) = 0$$

$$4\lambda + 4 + \lambda - 6 + \lambda + 8 + 6 = 0$$

$$6\lambda = -12$$

$$\lambda = -2$$

So,

$$\text{Image } Q = (2\lambda + 1, -\lambda + 3, \lambda + 4)$$

$$= (-4 + 1, 2 + 3, -2 + 4)$$

$$= (-3, 5, 2)$$

Image of P $(1,3,4)$ is $(-3,5,2)$

The Plane Ex 29.15 Q9

Here, we have to find distance of a point A with position vector $\left(-\hat{i}-5\hat{j}-10\hat{k}\right)$ from the point of intersection of line $\vec{r}=\left(2\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\left(3\hat{i}+4\hat{j}+12\hat{k}\right)$ with plane $\vec{r}\cdot\left(\hat{i}-\hat{j}+\hat{k}\right)=5$.

Let the point of intersection of line and plane be $B\left(\vec{b}\right)$

The line and the plane will intersect when,

$$\left[\left(2\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\left(3\hat{i}+4\hat{j}+12\hat{k}\right)\right]\left(\hat{i}-\hat{j}+\hat{k}\right)=5$$
$$\left[(2+3\lambda)\hat{i}+(-1+4\lambda)\hat{j}+(2+12\lambda)\hat{k}\right]\left(\hat{i}-\hat{j}+\hat{k}\right)=5$$
$$(2+3\lambda)(1)+(-1+4\lambda)(-1)+(2+12\lambda)(1)=5$$
$$2+3\lambda+1-4\lambda+2+12\lambda=5$$
$$11\lambda=5-5$$
$$\lambda=0$$

So, the point B is given by

$$\vec{b}=\left(2\hat{i}-\hat{j}+2\hat{k}\right)+(0)\left(3\hat{i}+4\hat{j}+12\hat{k}\right)$$
$$\vec{b}=\left(2\hat{i}-\hat{j}+2\hat{k}\right)$$

$$\overrightarrow{AB}=\vec{b}-\vec{a}$$
$$=\left(2\hat{i}-\hat{j}+2\hat{k}\right)-\left(-\hat{i}-5\hat{j}-10\hat{k}\right) \qquad =\left(2\hat{i}-\hat{j}+2\hat{k}+\hat{i}+5\hat{j}+10\hat{k}\right)=\left(3\hat{i}+4\hat{j}+12\hat{k}\right)$$
$$\left|\overrightarrow{AB}\right| \quad =\sqrt{(3)^2+(4)^2+(12)^2}=\sqrt{9+16+144}=\sqrt{169}=13$$

Required distance = 13 units

The Plane Ex 29.15 Q10

$$x-2y+4z+5=0$$
$$(1,1,2)$$
$$=\left|\frac{1-2+4+5}{\sqrt{1+4+16}}\right|=\frac{8}{\sqrt{21}}$$

Let the foot of perpendicular be (x,y,z). So DR's are in proportional

$$\frac{x-1}{1}=\frac{y-1}{-2}=\frac{z-2}{4}=k$$
$$x=k+1$$
$$y=-2k+1$$
$$z=4k+2$$

Substitute (x,y,z)=(k+1, -2k+1, 4k+2) in plane equation

$$x-2y+4z+5=0$$
$$k+1+4k-2+16k+8+5=0$$
$$21k=-12$$
$$k=\frac{-12}{21}=\frac{-4}{7}$$
$$(x,y,z)=\left(\frac{3}{7},\frac{15}{7},\frac{-2}{7}\right)$$

The Plane Ex 29.15 Q11

$$2x - y + z + 1 = 0$$
$$(3, 2, 1)$$
$$= \left|\frac{6-2+1+1}{\sqrt{4+1+1}}\right| = \frac{6}{\sqrt{6}} = \sqrt{6}$$

Let the foot of perpendicular be (x,y,z). So DR's are in proportional

$$\frac{x-3}{2} = \frac{y-2}{-1} = \frac{z-1}{1} = k$$
$$x = 2k + 3$$
$$y = -k + 2$$
$$z = k - 1$$

Substitute $(x,y,z) = (2k+3, -k+2, k-1)$ in plane equation

$$2x - y + z + 1 = 0$$
$$4k + 6 + k - 2 + k - 1 + 1 = 0$$
$$6k = -4$$
$$k = \frac{-4}{6} = \frac{-2}{3}$$
$$(x, y, z) = \left(\frac{5}{3}, \frac{8}{3}, \frac{-5}{3}\right)$$

The Plane Ex 29.15 Q12

Given equation of the plane $\vec{r} \cdot \left(6\hat{i} - 3\hat{j} - 2\hat{k}\right) + 1 = 0$

Thus, the direction ratios normal to the plane are $6, -3$ *and* -2

Hence the direction cosines to the normal to the plane are

$$\frac{6}{\sqrt{6^2+(-3)^2+(-2)^2}}, \frac{-3}{\sqrt{6^2+(-3)^2+(-2)^2}}, \frac{-2}{\sqrt{6^2+(-3)^2+(-2)^2}}$$
$$= \frac{6}{7}, \frac{-3}{7}, \frac{-2}{7}$$
$$= \frac{-6}{7}, \frac{3}{7}, \frac{2}{7}$$

The direction cosines of the unit vector perpendicular to the plane are same as the direction cosines of the normal to the plane.

Thus, the direction cosines of the unit vector perpendicular to the plane are: $\frac{-6}{7}, \frac{3}{7}, \frac{2}{7}$

The Plane Ex 29.15 Q13

Consider the given equation of the plane $2x-3y+4z-6=0$

The direction ratios of the normal to the plane are $2,\ -3$ *and* 4

Thus, the directio ratios of the line perpendicular to the plane are $2,\ -3$ *and* $4.$

The equation of the line passing (x_1,y_1,z_1) *having direction ratios a,b and c is*

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

Thus, the equation of the line passing through the origin with direction ratios $2,\ -3$ *and* 4 *is*

$$\frac{x-0}{2}=\frac{y-0}{-3}=\frac{z-0}{4}$$

$$\Rightarrow \frac{x}{2}=\frac{y}{-3}=\frac{z}{4}=r, \text{ where } r \text{ is some constant}$$

Any point on the line is of the form $2r,\ -3r$ *and* $4r$

If the point $P(2r,\ -3r,\ 4r)$ *lies on the plane* $2x-3y+4z-6=0,$

it should satisfies the equation, $2x-3y+4z-6=0$

Thus, we have,

$$2(2r)-3(-3r)+4(4r)-6=0$$

$$\Rightarrow 4r+9r+16r-6=0$$

$$\Rightarrow 29r=6$$

$$\Rightarrow r=\frac{6}{29}$$

Thus, the coordinates of the point of intersection of the perpendicular from the origin and the plane are:

$$P\left(2\times\frac{6}{29},\ -3\times\frac{6}{29},\ 4\times\frac{6}{29}\right)=P\left(\frac{12}{29},\ \frac{18}{29},\ \frac{24}{29}\right)$$

The Plane Ex 29.15 Q14

The length of perpendicular from the point $\left(1,\ \frac{3}{2},2\right)$ to the plane $2x - 2y + 4z + 5 = 0$.

$$d=\left|\frac{2-3+8+5}{\sqrt{4+4+16}}\right|=\frac{12}{2\sqrt{6}}=\sqrt{6}$$

Let the foot of perpendicular be $(x,\ y,\ z)$. So DR's are in proportional

$$\frac{x-1}{2}=\frac{y-\frac{3}{2}}{-2}=\frac{z-2}{4}=k$$

$$x=2k+1$$

$$y=-2k+\frac{3}{2}$$

$$z=4k+2$$

So using values of $x,\ y,\ z$ in equation of the plane we have,

$$2(2k+1)-2\left(-2k+\frac{3}{2}\right)+4(4k+2)+5=0$$

$$4k+2+4k-3+16k+8+5=0$$

$$24k=-12$$

$$k=-\frac{1}{2}$$

$$(x,\ y,\ z)=\left(0,\frac{5}{2},0\right)$$