SOLUTIONS TO CONCEPTS
CHAPTER – 18

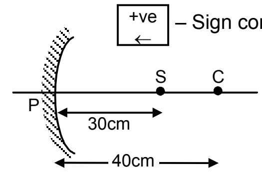
SIGN CONVENTION :

1) The direction of incident ray (from object to the mirror or lens) is taken as positive direction.
2) All measurements are taken from pole (mirror) or optical centre (lens) as the case may be.

1. $u = –30$ cm, $R = – 40$ cm

From the mirror equation,

+ve ← – Sign convertion



$$\frac{1}{v}+\frac{1}{u}=\frac{2}{R}$$

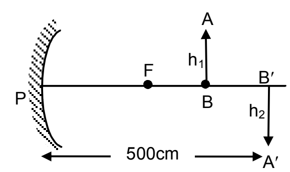
$$\Rightarrow \frac{1}{v}=\frac{2}{R}-\frac{1}{u}=\frac{2}{-40}-\frac{1}{-30} = -\frac{1}{60}$$

or, $v = –60$ cm

So, the image will be formed at a distance of 60 cm in front of the mirror.

2. Given that,

$H_1 = 20$ cm, $v = –5$ m $= –500$ cm, $h_2 = 50$ cm



+ve ← – Sign convertion

Since, $\frac{-v}{u}=\frac{h_2}{h_1}$

or $\frac{500}{u}=-\frac{50}{20}$ (because the image in inverted)

or $u = -\frac{500\times 2}{5} = –200$ cm $= – 2$ m

$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ or $\frac{1}{-5}+\frac{1}{-2}=\frac{1}{f}$

or $f = \frac{-10}{7} = –1.44$ m

So, the focal length is 1.44 m.

3. For the concave mirror, $f = –20$ cm, $M = –v/u = 2$

$\Rightarrow v = –2u$

<u>1st case</u>

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \frac{1}{2u}-\frac{1}{u}=-\frac{1}{f}$$

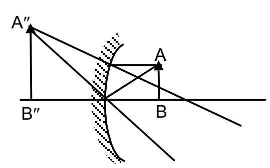
$\Rightarrow u = f/2 = 10$ cm

<u>2nd case</u>

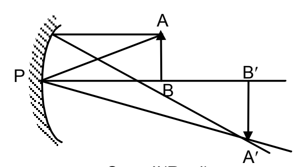
$$\frac{-1}{2u}-\frac{1}{u}=-\frac{1}{f}$$

$$\Rightarrow \frac{3}{2u}=\frac{1}{f}$$

$\Rightarrow u = 3f/2 = 30$ cm



Case I (Virtual)



Case II(Real)

∴ The positions are 10 cm or 30 cm from the concave mirror.

4. $m = –v/u = 0.6$ and $f = 7.5$ cm $= 15/2$ cm

From mirror equation,

$$\Rightarrow \frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{0.6u}-\frac{1}{u}=\frac{1}{f}$$

$\Rightarrow u = 5$ cm

5. Height of the object AB = 1.6 cm

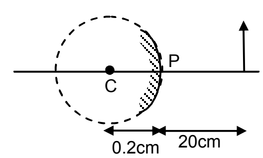
Diameter of the ball bearing = d = 0.4 cm

$\Rightarrow R = 0.2$ cm

Given, $u = 20$ cm

We know, $\frac{1}{u}+\frac{1}{v}=\frac{2}{R}$

+ve ← – Sign convertion



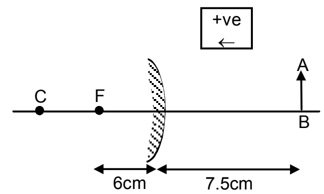
Putting the values according to sign conventions $\frac{1}{-20}+\frac{1}{v}=\frac{2}{0.2}$

$\Rightarrow \frac{1}{v}=\frac{1}{20}+10=\frac{201}{20} \Rightarrow$ v = 0.1 cm = 1 mm inside the ball bearing.

Magnification = m = $\frac{A'B'}{AB}=-\frac{v}{u}=-\frac{0.1}{-20}=\frac{1}{200}$

$\Rightarrow$ A′B′ = $\frac{AB}{200}=\frac{16}{200}$ = +0.008 cm = +0.8 mm.

6. Given AB = 3 cm, u = –7.5 cm, f = 6 cm.



Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}=\frac{1}{f}-\frac{1}{u}$

Putting values according to sign conventions,

$\frac{1}{v}=\frac{1}{6}-\frac{1}{-7.5}=\frac{3}{10}$

$\Rightarrow$ v = 10/3 cm

$\therefore$ magnification = m = $-\frac{v}{u}=\frac{10}{7.5\times 3}$

$\Rightarrow \frac{A'B'}{AB}=\frac{10}{7.5\times 3} \Rightarrow A'B'=\frac{100}{72}=\frac{4}{3}=1.33$ cm.

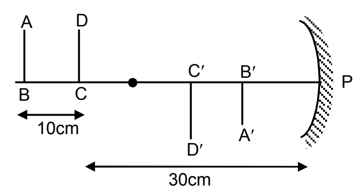
$\therefore$ Image will form at a distance of 10/3 cm. From the pole and image is 1.33 cm (virtual and erect).

7. R = 20 cm, f = R/2 = –10 cm

For part AB, PB = 30 + 10 = 40 cm

So, u = –40 cm $\Rightarrow \frac{1}{v}=\frac{1}{f}-\frac{1}{u}=-\frac{1}{10}-\left(\frac{1}{-40}\right)=-\frac{3}{40}$

$\Rightarrow$ v = $-\frac{40}{3}$ = –13.3 cm.



So, PB′ = 13.3 cm

m = $\frac{A'B'}{AB}=-\left(\frac{v}{u}\right)=-\left(\frac{-13.3}{-40}\right)=-\frac{1}{3}$

$\Rightarrow$ A′B′ = –10/3 = –3.33 cm

For part CD, PC = 30, So, u = –30 cm

$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=-\frac{1}{10}-\left(-\frac{1}{30}\right)=-\frac{1}{15} \Rightarrow$ v = –15 cm = PC′

So, m = $\frac{C'D'}{CD}=-\frac{v}{u}=-\left(\frac{-15}{-30}\right)=-\frac{1}{2}$

$\Rightarrow$ C′D′ = 5 cm

B′C′ = PC′ – PB′ = 15 – 13.3 = 17 cm

So, total length A′B′ + B′C′ + C′D′ = 3.3 + 1.7 + 5 = 10 cm.

8. u = –25 cm

m = $\frac{A'B'}{AB}=-\frac{v}{u} \Rightarrow 1.4=-\left(\frac{v}{-25}\right) \Rightarrow \frac{14}{10}=\frac{v}{25}$

$\Rightarrow$ v = $\frac{25\times 14}{10}$ = 35 cm.

Now, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

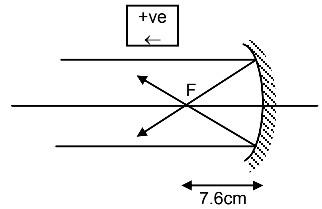
$\Rightarrow \frac{1}{f}=\frac{1}{35}-\left(\frac{1}{-25}\right)=\frac{5-7}{175}=-\frac{2}{175} \Rightarrow$ f = –87.5 cm.

So, focal length of the concave mirror is 87.5 cm.

9. $u = -3.8 \times 10^5$ km

diameter of moon = 3450 km ; f = –7.6 m

$\therefore \frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}+\left(-\frac{1}{3.8\times10^5}\right)=\left(-\frac{1}{7.6}\right)$



Since, distance of moon from earth is very large as compared to focal length it can be taken as ∞.

⇒ Image will be formed at focus, which is inverted.

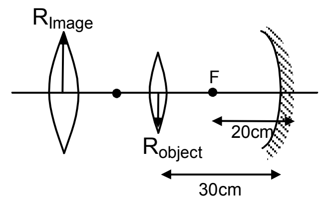
$\Rightarrow \frac{1}{v}=-\left(\frac{1}{7.6}\right) \Rightarrow v=-7.6$ m.

$m = -\frac{v}{u}=\frac{d_{image}}{d_{object}} \Rightarrow \frac{-(-7.6)}{(-3.8\times10^8)}=\frac{d_{image}}{3450\times10^3}$

$d_{image} = \frac{3450\times7.6\times10^3}{3.8\times10^8}$ = 0.069 m = 6.9 cm.

10. u = –30 cm, f = –20 cm

We know, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$



$\Rightarrow \frac{1}{v}+\left(-\frac{1}{30}\right)=\left(-\frac{1}{20}\right) \Rightarrow v = -60$ cm.

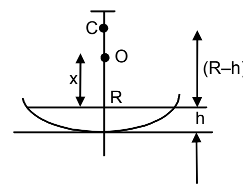
Image of the circle is formed at a distance 60 cm in front of the mirror.

$\therefore m = -\frac{v}{u}=\frac{R_{image}}{R_{object}} \Rightarrow -\frac{-60}{-30}=\frac{R_{image}}{2}$

$\Rightarrow R_{image}$ = 4 cm

Radius of image of the circle is 4 cm.

11. Let the object be placed at a height x above the surface of the water.



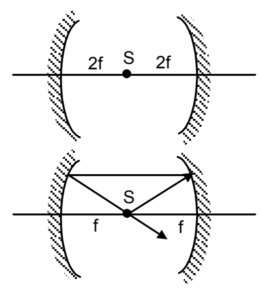
The apparent position of the object with respect to mirror should be at the centre of curvature so that the image is formed at the same position.

Since, $\frac{\text{Real depth}}{\text{Apparent depth}}=\frac{1}{\mu}$ (with respect to mirror)

Now, $\frac{x}{R-h}=\frac{1}{\mu} \Rightarrow x=\frac{R-h}{\mu}$.

12. Both the mirrors have equal focal length f.

They will produce one image under two conditions.



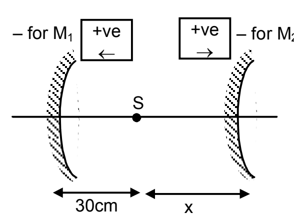
Case I : When the source is at distance '2f' from each mirror i.e. the source is at centre of curvature of the mirrors, the image will be produced at the same point S. So, d = 2f + 2f = 4f.

Case II : When the source S is at distance 'f' from each mirror, the rays from the source after reflecting from one mirror will become parallel and so these parallel rays after the reflection from the other mirror the object itself. So, only sine image is formed.

Here, d = f + f = 2f.

13. As shown in figure, for 1st reflection in M_1, u = –30 cm, f = –20 cm

$\Rightarrow \frac{1}{v}+\frac{1}{-30}=-\frac{1}{20} \Rightarrow v = -60$ cm.



So, for 2nd reflection in M_2

u = 60 – (30 + x) = 30 – x

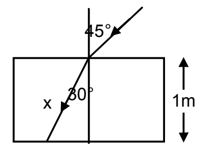
v = –x ; f = 20 cm

$\Rightarrow \frac{1}{30-x}-\frac{1}{x}=\frac{1}{20} \Rightarrow x^2+10x-600=0$

$\Rightarrow$ x = $\dfrac{10 \pm 50}{2} = \dfrac{40}{2}$ = 20 cm or –30 cm

$\therefore$ Total distance between the two lines is 20 + 30 = 50 cm.

14. We know, $\dfrac{\sin i}{\sin r} = \dfrac{3 \times 10^8}{v} = \dfrac{\sin 45°}{\sin 30°} = \sqrt{2}$



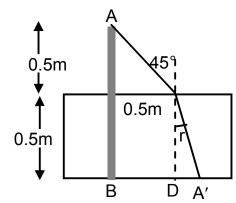
$\Rightarrow$ v = $\dfrac{3 \times 10^8}{\sqrt{2}}$ m/sec.

Distance travelled by light in the slab is,

x = $\dfrac{1\,\text{m}}{\cos 30°} = \dfrac{2}{\sqrt{3}}$ m

So, time taken = $\dfrac{2 \times \sqrt{2}}{\sqrt{3} \times 3 \times 10^8}$ = 0.54 × 10^{-8} = 5.4 × 10^{-9} sec.

15. Shadow length = BA′ = BD + A′D = 0.5 + 0.5 tan r



Now, 1.33 = $\dfrac{\sin 45°}{\sin r}$ $\Rightarrow$ sin r = 0.53.

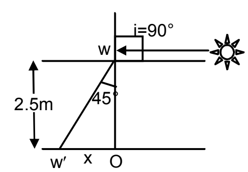
$\Rightarrow$ cos r = $\sqrt{1 - \sin^2 r} = \sqrt{1 - (0.53)^2}$ = 0.85

So, tan r = 0.6235

So, shadow length = (0.5) (1 + 0.6235) = 81.2 cm.

16. Height of the lake = 2.5 m

When the sun is just setting, θ is approximately = 90°



$\therefore$ $\dfrac{\sin i}{\sin r} = \dfrac{\mu_2}{\mu_1} \Rightarrow \dfrac{1}{\sin r} = \dfrac{4/3}{1} \Rightarrow \sin r = \dfrac{3}{4} \Rightarrow$ r = 49°

As shown in the figure, x/2.5 = tan r = 1.15

$\Rightarrow$ x = 2.5 × 1.15 = 2.8 m.

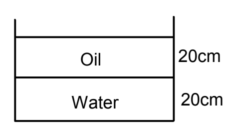
17. The thickness of the glass is d = 2.1 cm and μ =1.5

Shift due to the glass slab

ΔT = $\left(1 - \dfrac{1}{\mu}\right)d = \left(1 - \dfrac{1}{1.5}\right)2.1$ = 0.7 CM

So, the microscope should be shifted 0.70 cm to focus the object again.

18. Shift due to water Δt_w = $\left(1 - \dfrac{1}{\mu}\right)d = \left(1 - \dfrac{1}{1.33}\right)20$ = 5 cm



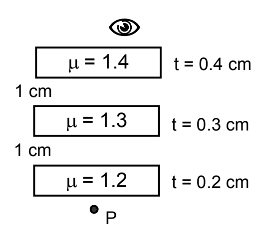
Shift due to oil, Δt_o = $\left(1 - \dfrac{1}{1.3}\right)20$ = 4.6 cm

Total shift Δt = 5 + 4.6 = 9.6 cm

Apparent depth = 40 – (9.6) = 30.4 cm below the surface.

19. The presence of air medium in between the sheets does not affect the shift.

The shift will be due to 3 sheets of different refractive index other than air.



= $\left(1 - \dfrac{1}{1.2}\right)(0.2) + \left(1 - \dfrac{1}{13}\right)(0.3) + \left(1 - \dfrac{1}{14}\right)(0.4)$

= 0.2 cm above point P.

20. Total no. of slabs = k, thickness = t_1, t_2, t_3 … t_k

Refractive index = μ_1, μ_2, μ_3, μ_4,… μ_k

$\therefore$ The shift Δt = $\left(1 - \dfrac{1}{\mu_1}\right)t_1 + \left(1 - \dfrac{1}{\mu_2}\right)t_2 + \ldots\ldots + \left(1 - \dfrac{1}{\mu_k}\right)t_k$ …(1)

If, $\mu \to$ refractive index of combination of slabs and image is formed at same place,

Δt = $\left(1 - \dfrac{1}{\mu}\right)(t_1 + t_2 + \ldots + t_k)$ …(2)

Equation (1) and (2), we get,

$$\left(1-\frac{1}{\mu}\right)(t_1+t_2+\ldots+t_k) = \left(1-\frac{1}{\mu_1}\right)t_1+\left(1-\frac{1}{\mu_2}\right)t_2+\ldots\ldots+\left(1-\frac{1}{\mu_k}\right)t_k$$

$$= (t_1+t_2+\ldots+t_k)-\left(\frac{t_1}{\mu_1}+\frac{t_2}{\mu_2}+\ldots+\frac{t_k}{\mu_k}\right)$$

$$= -\frac{1}{\mu}\sum_{i=1}^{k} t_1 = -\sum_{i=1}^{k}\left(\frac{t_1}{\mu_1}\right) \Rightarrow \mu = \frac{\sum_{i=1}^{k} t_i}{\sum_{i=1}^{k}(t_1/\mu_1)}.$$

21. Given r = 6 cm, r_1 = 4 cm, h_1 = 8 cm

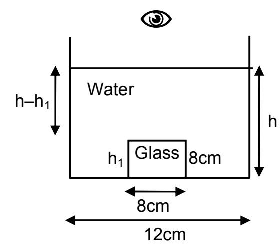
Let, h = final height of water column.

The volume of the cylindrical water column after the glass piece is put will be,

$\pi r^2 h = 800\pi + \pi r_1^2 h_1$

or $r^2 h = 800 + r_1^2 h_1$

or $6^2 h = 800 + 4^2 \times 8 = 25.7$ cm



There are two shifts due to glass block as well as water.

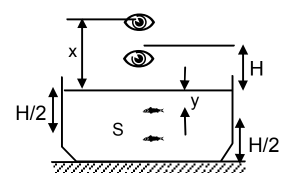
So, $\Delta t_1 = \left(1-\frac{1}{\mu_0}\right)t_0 = \left(1-\frac{1}{3/2}\right)8 = 2.26$ cm

And, $\Delta t_2 = \left(1-\frac{1}{\mu_w}\right)t_w = \left(1-\frac{1}{4/3}\right)(25.7-8) = 4.44$ cm.

Total shift = (2.66 + 4.44) cm = 7.1 cm above the bottom.

22. a) Let x = distance of the image of the eye formed above the surface as seen by the fish

So, $\frac{H}{x} = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{1}{\mu}$ or $x = \mu H$



So, distance of the direct image = $\frac{H}{2}+\mu H = H(\mu+\frac{1}{2})$

Similarly, image through mirror = $\frac{H}{2}+(H+x) = \frac{3H}{2}+\mu H = H(\mu+\frac{3}{2})$

b) Here, $\frac{H/2}{y} = \mu$, so, $y = \frac{H}{2\mu}$

Where, y = distance of the image of fish below the surface as seen by eye.

So, Direct image = H + y = $H+\frac{H}{2\mu} = H\left(1+\frac{1}{2\mu}\right)$

Again another image of fish will be formed H/2 below the mirror.

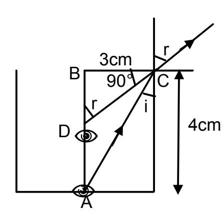
So, the real depth for that image of fish becomes H + H/2 = 3H/2

So, Apparent depth from the surface of water = $3H/2\mu$

So, distance of the image from the eye = $H+\frac{3H}{2\mu} = H(1+\frac{3}{2\mu})$.

23. According to the figure, x/3 = cot r …(1)

Again, $\frac{\sin i}{\sin r} = \frac{1}{1.33} = \frac{3}{4}$



$\Rightarrow \sin r = \frac{4}{3}\sin i = \frac{4}{3}\times\frac{3}{5} = \frac{4}{5}$ (because $\sin i = \frac{BC}{AC} = \frac{3}{5}$)

$\Rightarrow$ cot r = 3/4 …(2)

From (1) and (2) $\Rightarrow$ x/3 = ¾

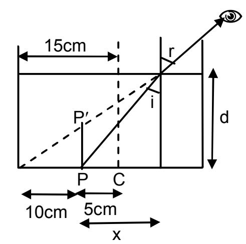
$\Rightarrow$ x = 9/4 = 2.25 cm.

$\therefore$ Ratio of real and apparent depth = 4 : (2.25) = 1.78.

24. For the given cylindrical vessel, dimetre = 30 cm

$\Rightarrow$ r = 15 cm and h = 30 cm

Now, $\frac{\sin i}{\sin r} = \frac{3}{4}\left[\mu_w = 1.33 = \frac{4}{3}\right]$



$\Rightarrow$ sin i = $3/4\sqrt{2}$ [because r = 45°]

The point P will be visible when the refracted ray makes angle 45° at point of refraction.

Let x = distance of point P from X.

Now, tan 45° = $\frac{x+10}{d}$

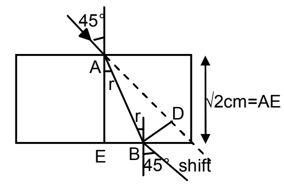
$\Rightarrow$ d = x + 10 …(1)

Again, tan i = x/d

$\Rightarrow \frac{3}{\sqrt{23}} = \frac{d-10}{d}$ $\left[\text{since, } \sin i = \frac{3}{4\sqrt{2}} \Rightarrow \tan i = \frac{3}{\sqrt{23}}\right]$

$\Rightarrow \frac{3}{\sqrt{23}} - 1 = -\frac{10}{d} \Rightarrow d = \frac{\sqrt{23}\times 10}{\sqrt{23}-3}$ = 26.7 cm.

25. As shown in the figure,



$\frac{\sin 45°}{\sin r} = \frac{2}{1} \Rightarrow \sin r = \frac{\sin 45°}{2} = \frac{1}{2\sqrt{2}} \Rightarrow r = 21°$

Therefore, θ = (45° – 21°) = 24°

Here, BD = shift in path = AB sin 24°

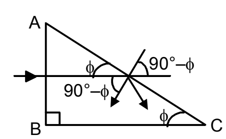
= $0.406 \times AB = \frac{AE}{\cos 21°} \times 0.406$ = 0.62 cm.

26. For calculation of critical angle,

$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \Rightarrow \frac{\sin C}{\sin 90} = \frac{15}{1.72} = \frac{75}{86}$

$\Rightarrow$ C = $\sin^{-1}\left(\frac{75}{26}\right)$.

27. Let θ_c be the critical angle for the glass



$\frac{\sin\theta_c}{\sin 90°} = \frac{1}{x} \Rightarrow \sin\theta_c = \frac{1}{1.5} = \frac{2}{3} \Rightarrow \theta_c = \sin^{-1}\left(\frac{2}{3}\right)$

From figure, for total internal reflection, 90° – $\phi > \theta_c$

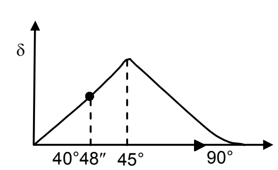
$\Rightarrow \phi < 90° - \theta_c \Rightarrow \phi < \cos^{-1}(2/3)$

So, the largest angle for which light is totally reflected at the surface is $\cos^{-1}(2/3)$.

28. From the definition of critical angle, if refracted angle is more than 90°, then reflection occurs, which is known as total internal reflection.

So, maximum angle of refraction is 90°.

29. Refractive index of glass μ_g = 1.5



Given, 0° < i < 90°

Let, C $\rightarrow$ Critical angle.

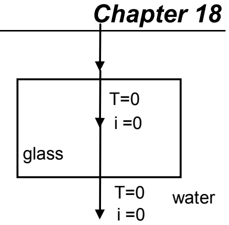
$\frac{\sin C}{\sin r} = \frac{\mu_a}{\mu_g} \Rightarrow \frac{\sin C}{\sin 90°} = \frac{1}{15}$ = 0.66

$\Rightarrow$ C = 40°48″

The angle of deviation due to refraction from glass to air increases as the angle of incidence increases from 0° to 40°48″. The angle of deviation due to total internal reflection further increases for 40°48″ to 45° and then it decreases.

30. μ_g = 1.5 = 3/2 ; μ_w = 1.33 = 4/3

For two angles of incidence,



1) When light passes straight through normal,

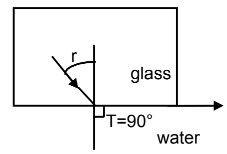
$\Rightarrow$ Angle of incidence = 0°, angle of refraction = 0°, angle of deviation = 0

2) When light is incident at critical angle,

$$\frac{\sin C}{\sin r} = \frac{\mu_w}{\mu_g} \quad \text{(since light passing from glass to water)}$$

$\Rightarrow$ sin C = 8/9 $\Rightarrow$ C = $\sin^{-1}(8/9)$ = 62.73°.

$\therefore$ Angle of deviation = 90° – C = 90° – $\sin^{-1}(8/9)$ = $\cos^{-1}(8/9)$ = 37.27°



Here, if the angle of incidence is increased beyond critical angle, total internal reflection occurs and deviation decreases. So, the range of deviation is 0 to $\cos^{-1}(8/9)$.

31. Since, μ = 1.5, Critial angle = $\sin^{-1}(1/\mu)$ = $\sin^{-1}$ (1/1.5) = 41.8°

We know, the maximum attainable deviation in refraction is (90° – 41.8°) = 47.2°

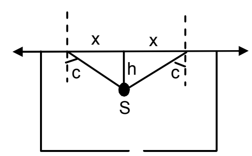
So, in this case, total internal reflection must have taken place.

In reflection,

Deviation = 180° – 2i = 90° $\Rightarrow$ 2i = 90° $\Rightarrow$ i = 45°.

32. a) Let, x = radius of the circular area

$\frac{x}{h} = \tan C$ (where C is the critical angle)



$$\Rightarrow \frac{x}{h} = \frac{\sin C}{\sqrt{1-\sin^2 C}} = \frac{1/\mu}{\sqrt{1-\frac{1}{\mu^2}}} \qquad \text{(because } \sin C = 1/\mu\text{)}$$

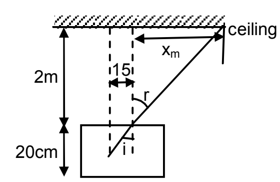
$$\Rightarrow \frac{x}{h} = \frac{1}{\sqrt{\mu^2-1}} \text{ or } x = \frac{h}{\sqrt{\mu^2-1}}$$

So, light escapes through a circular area on the water surface directly above the point source.

b) Angle subtained by a radius of the area on the source, C = $\sin^{-1}(1/\mu)$.

33. a) As shown in the figure, sin i = 15/25

So, $\frac{\sin i}{\sin r} = \frac{1}{\mu} = \frac{3}{4}$



$\Rightarrow$ sin r = 4/5

Again, x/2 = tan r (from figure)

So, sin r = $\frac{\tan r}{\sqrt{1+\tan^2 r}} = \frac{x/2}{\sqrt{1-x^2/4}}$

$$\Rightarrow \frac{x}{\sqrt{4+x^2}} = \frac{4}{5}$$

$\Rightarrow$ $25x^2 = 16(4 + x^2) \Rightarrow 9x^2 = 64 \Rightarrow x = 8/3$ m

$\therefore$ Total radius of shadow = 8/3 + 0.15 = 2.81 m

b) For maximum size of the ring, i = critical angle = C

Let, R = maximum radius

$\Rightarrow$ sin C = $\frac{\sin C}{\sin r} = \frac{R}{\sqrt{20^2+R^2}} = \frac{3}{4}$ (since, sin r = 1)

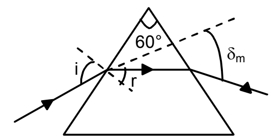
$\Rightarrow$ $16R^2 = 9R^2 + 9 \times 400$

$\Rightarrow$ $7R^2 = 9 \times 400$

$\Rightarrow$ R = 22.67 cm.

34. Given, A = 60°, μ = 1.732

Since, angle of minimum deviation is given by,



$\mu = \dfrac{\sin\left(\dfrac{A+\delta m}{2}\right)}{\sin A/2} \Rightarrow 1.732 \times ½ = \sin(30 + \delta m/2)$

$\Rightarrow \sin^{-1}(0.866) = 30 + \delta m/2 \Rightarrow 60° = 30\ \delta m/2 \Rightarrow \delta m = 60°$

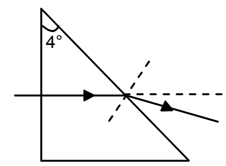
Now, $\delta m = i + i' - A$

$\Rightarrow$ 60° = i + i′ – 60° (δ = 60° minimum deviation)

$\Rightarrow$ i = 60°. So, the angle of incidence must be 60°.

35. Given μ = 1.5

And angle of prism = 4°



$\therefore\ \mu = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin A/2} = \dfrac{(A+\delta_m)/2}{(A/2)}$ (for small angle $\sin\theta = \theta$)

$\Rightarrow\ \mu = \dfrac{A+\delta_m}{2} \Rightarrow 1.5 = \dfrac{4°+\delta_m}{4°} \Rightarrow \delta_m = 4° \times (1.5) - 4° = 2°.$

36. Given A = 60° and δ = 30°

We know that,

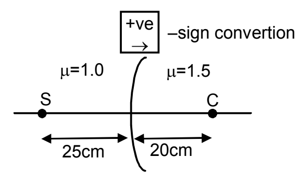
$\mu = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin A/2} = \dfrac{\sin\dfrac{60°+\delta_m}{2}}{\sin 30°} = 2\sin\dfrac{60°+\delta_m}{2}$

Since, one ray has been found out which has deviated by 30°, the angle of minimum deviation should be either equal or less than 30°. (It can not be more than 30°).

So, $\mu \le 2\sin\dfrac{60°+\delta_m}{2}$ (because μ will be more if δ_m will be more)

or, $\mu \le 2 \times 1/\sqrt{2}$ or, $\mu \le \sqrt{2}$.

37. $\mu_1 = 1$, $\mu_2 = 1.5$, R = 20 cm (Radius of curvature), u = –25 cm



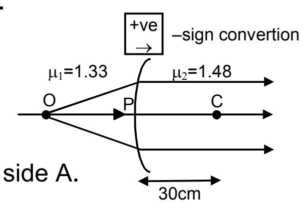
$\therefore\ \dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2-\mu_1}{R} \Rightarrow \dfrac{1.5}{v} = \dfrac{0.5}{20} - \dfrac{1}{25} = \dfrac{1}{40} - \dfrac{1}{25} = \dfrac{-3}{200}$

$\Rightarrow v = -200 \times 0.5 = -100$ cm.

So, the image is 100 cm from (P) the surface on the side of S.

38. Since, paraxial rays become parallel after refraction i.e. image is formed at ∞.

$v = \infty$, $\mu_1 = 1.33$, u = ?, $\mu_2 = 1.48$, R = 30 cm



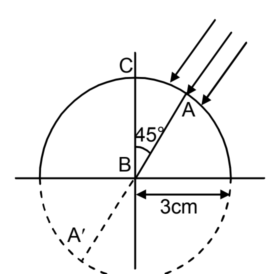
$\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2-\mu_1}{R} \Rightarrow \dfrac{1.48}{\infty} - \dfrac{1.33}{u} = \dfrac{1.48-1.33}{30} \Rightarrow -\dfrac{1.33}{u} - \dfrac{0.15}{30}$

$\Rightarrow u = -266.0$ cm

$\therefore$ Object should be placed at a distance of 266 cm from surface (convex) on side A.

39. Given, $\mu_2 = 2.0$

So, critical angle = $\sin^{-1}\left(\dfrac{1}{\mu_2}\right) = \sin^{-1}\left(\dfrac{1}{2}\right)$ = 30°



a) As angle of incidence is greater than the critical angle, the rays are totally reflected internally.

b) Here, $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2-\mu_1}{R}$

$\Rightarrow \dfrac{2}{v} - \left(-\dfrac{1}{\infty}\right) = \dfrac{2-1}{3}$ [For parallel rays, $u = \infty$]

$\Rightarrow \dfrac{2}{v} = \dfrac{1}{3} \Rightarrow v = 6$ cm

$\Rightarrow$ If the sphere is completed, image is formed diametrically opposite of A.

c) Image is formed at the mirror in front of A by internal reflection.

40. a) Image seen from left :

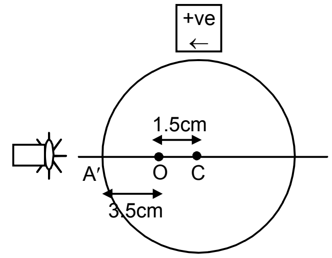
$u = (5 - 15) = -3.5$ cm

$R = -5$ cm

$\therefore \dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R} \Rightarrow \dfrac{1}{v} + \dfrac{1.5}{3.5} = -\dfrac{1-1.5}{5}$

$\Rightarrow \dfrac{1}{v} = \dfrac{1}{10} - \dfrac{3}{7} \Rightarrow v = \dfrac{-70}{23} = -3$ cm (inside the sphere).

$\Rightarrow$ Image will be formed, 2 cm left to centre.



b) Image seen from right :

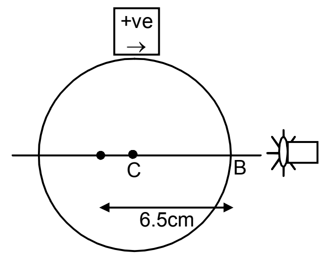
$u = -(5 + 1.5) = -6.5$ cm

$R = -5$ cm

$\therefore \dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R} \Rightarrow \dfrac{1}{v} + \dfrac{1.5}{6.5} = \dfrac{1-1.5}{-5}$

$\Rightarrow \dfrac{1}{v} = \dfrac{1}{10} - \dfrac{3}{13} \Rightarrow v = -\dfrac{130}{17} = -7.65$ cm (inside the sphere).

$\Rightarrow$ Image will be formed, 2.65 cm left to centre.



41. $R_1 = R_2 = 10$ cm, $t = 5$ cm, $u = -\infty$

For the first refraction, (at A)

$\dfrac{\mu_g}{v} - \dfrac{\mu_a}{u} = \dfrac{\mu_g - \mu_a}{R_1}$ or $\dfrac{1.5}{v} - 0 = \dfrac{1.5}{10}$

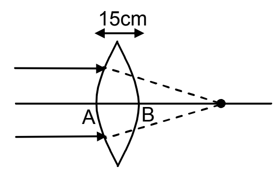
$\Rightarrow v = 30$ cm.

Again, for 2nd surface, $u = (30 - 5) = 25$ cm (virtual object)

$R_2 = -10$ cm

So, $\dfrac{1}{v} - \dfrac{15}{25} = \dfrac{-0.5}{-10} \Rightarrow v = 9.1$ cm.

So, the image is formed 9.1 cm further from the 2nd surface of the lens.



+ve → –Sign convention for both surfaces

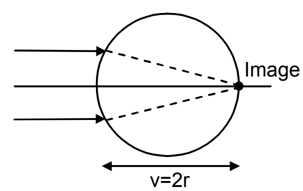
42. For the refraction at convex surface A.

$\mu = -\infty$, $\mu_1 = 1$, $\mu_2 = ?$

a) When focused on the surface, $v = 2r$, $R = r$

So, $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

$\Rightarrow \dfrac{\mu_2}{2r} = \dfrac{\mu_2 - 1}{r} \Rightarrow \mu_2 = 2\mu_2 - 2 \Rightarrow \mu_2 = 2$



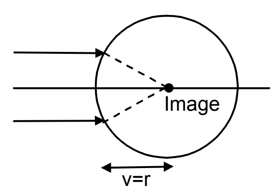
b) When focused at centre, $u = r_1$, $R = r$

So, $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

$\Rightarrow \dfrac{\mu_2}{R} = \dfrac{\mu_2 - 1}{r} \Rightarrow \mu_2 = \mu_2 - 1.$

This is not possible.

So, it cannot focus at the centre.



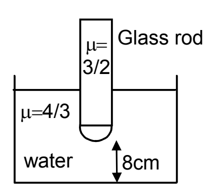
43. Radius of the cylindrical glass tube = 1 cm

We know, $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

Here, $u = -8$ cm, $\mu_2 = 3/2$, $\mu_1 = 4/3$, $R = +1$ cm

So, $\dfrac{3}{2v} + \dfrac{4}{3 \times 8} \Rightarrow \dfrac{3}{2v} + \dfrac{1}{6} = \dfrac{1}{6}$ $v = \infty$

$\therefore$ The image will be formed at infinity.

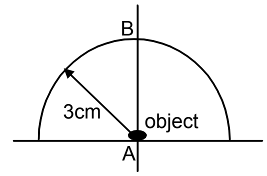


44. In the first refraction at A.

$\mu_2 = 3/2$, $\mu_1 = 1$, $u = 0$, $R = \infty$

So, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\Rightarrow$ v = 0 since ($R \Rightarrow \infty$ and u = 0)



$\therefore$ The image will be formed at the point, Now for the second refraction at B,

u = –3 cm, R = –3 cm, $\mu_1 = 3/2$, $\mu_2 = 1$

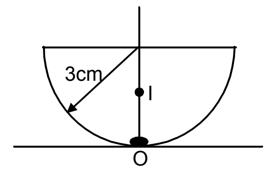
So, $\frac{1}{v} + \frac{3}{2 \times 3} = \frac{1 - 1.5}{-3} = \frac{1}{6}$

$\Rightarrow \frac{1}{v} = \frac{1}{6} - \frac{1}{2} = -\frac{1}{3}$

$\Rightarrow$ v = –3 cm, $\therefore$ There will be no shift in the final image.

45. Thickness of glass = 3 cm, $\mu_g = 1.5$

Image shift = $3\left(1 - \frac{1}{1.5}\right)$



[Treating it as a simple refraction problem because the upper surface is flat and the spherical surface is in contact with the object]

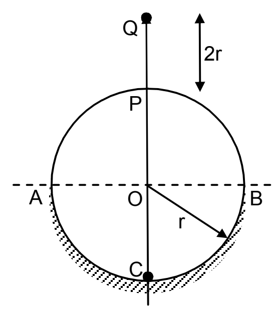
= $3 \times \frac{0.5}{1.5}$ = 1 cm.

The image will appear 1 cm above the point P.

46. As shown in the figure, OQ = 3r, OP = r

So, PQ = 2r

For refraction at APB



We know, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\Rightarrow \frac{1.5}{v} - \frac{1}{-2r} = \frac{0.5}{r} = \frac{1}{2r}$ [because u = –2r]

$\Rightarrow v = \infty$

For the reflection in concave mirror

$u = \infty$

So, v = focal length of mirror = r/2

For the refraction of APB of the reflected image.

Here, u = –3r/2

$\frac{1}{v} - \frac{1.5}{-3r/2} = \frac{-0.5}{-r}$ [Here, $\mu_1 = 1.5$ and $\mu_2 = 1$ and R = –r]

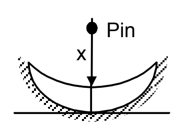
$\Rightarrow$ v = –2r

As, negative sign indicates images are formed inside APB. So, image should be at C.

So, the final image is formed on the reflecting surface of the sphere.

47. a) Let the pin is at a distance of x from the lens.

Then for 1st refraction, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$



Here $\mu_2 = 1.5$, $\mu_1 = 1$, u = –x, R = –60 cm

$\therefore \frac{1.5}{v} - \frac{1}{-x} = \frac{0.5}{-60}$

$\Rightarrow$ 120(1.5x + v) = –vx ...(1)

$\Rightarrow$ v(120 + x) = –180x

$\Rightarrow v = \frac{-180x}{120 + x}$

This image distance is again object distance for the concave mirror.

$u = \dfrac{-180x}{120+x}$, f = –10 cm ($\therefore$ f = R/2)

$\therefore\ \dfrac{1}{v}+\dfrac{1}{u}=\dfrac{1}{f} \Rightarrow \dfrac{1}{v_1}=\dfrac{1}{-10}-\dfrac{-(120+x)}{180x}$

$\Rightarrow\ \dfrac{1}{v_1}=\dfrac{120+x-18x}{180x} \Rightarrow v_1 = \dfrac{180x}{120-17x}$

Again the image formed is refracted through the lens so that the image is formed on the object taken in the 1st refraction. So, for 2nd refraction.

According to sign conversion v = –x, μ_2 = 1, μ_1 = 1.5, R = –60

Now, $\dfrac{\mu_2}{v}-\dfrac{\mu_1}{u}=\dfrac{\mu_2-\mu_1}{R}$ $\quad$ [$u = \dfrac{180x}{120-17x}$]

$\Rightarrow\ \dfrac{1}{-x}-\dfrac{1.5}{180x}(120-17x)=\dfrac{-0.5}{-60}$

$\Rightarrow\ \dfrac{1}{x}+\dfrac{120-17x}{120x}=\dfrac{-1}{120}$

Multiplying both sides with 120 m, we get

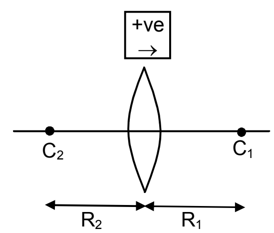
120 + 120 – 17x = –x

$\Rightarrow$ 16x = 240 $\Rightarrow$ x = 15 cm

$\therefore$ Object should be placed at 15 cm from the lens on the axis.

48. For the double convex lens

f = 25 cm, R_1 = R and R_2 = –2R (sign convention)



$\dfrac{1}{f}=(\mu-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)$

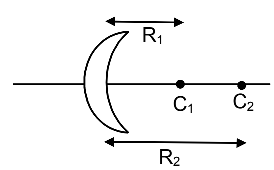
$\Rightarrow\ \dfrac{1}{25}=(15-1)\left(\dfrac{1}{R}-\dfrac{1}{-2R}\right) = 0.5\left(\dfrac{3R}{2}\right)$

$\Rightarrow\ \dfrac{1}{25}=\dfrac{3}{4}\dfrac{1}{R} \Rightarrow$ R = 18.75 cm

R_1 = 18.75 cm, R_2 = 2R = 37.5 cm.

49. R_1 = +20 cm ; R_2 = +30 cm ; μ = 1.6

a) If placed in air :



$\dfrac{1}{f}=(\mu_g-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right) = \left(\dfrac{1.6}{1}-1\right)\left(\dfrac{1}{20}-\dfrac{1}{30}\right)$

$\Rightarrow$ f = 60/6 = 100 cm

b) If placed in water :

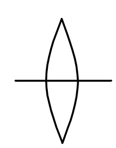
$\dfrac{1}{f}=(\mu_w-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right) = \left(\dfrac{1.6}{1.33}-1\right)\left(\dfrac{1}{20}-\dfrac{1}{30}\right)$

$\Rightarrow$ f = 300 cm

50. Given μ = 1.5

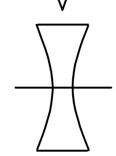
Magnitude of radii of curvatures = 20 cm and 30 cm

The 4types of possible lens are as below.



$\dfrac{1}{f}=(\mu-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)$

Case (1) : (Double convex) [R_1 = +ve, R_2 = –ve]



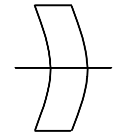
$\dfrac{1}{f}=(15-1)\left(\dfrac{1}{20}-\dfrac{1}{-30}\right) \Rightarrow$ f = 24 cm

Case (2) : (Double concave) [R_1 = –ve, R_2 = +ve]

$\dfrac{1}{f}=(15-1)\left(\dfrac{-1}{20}-\dfrac{1}{30}\right) \Rightarrow$ f = –24 cm

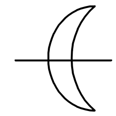
Case (3) : (Concave concave) [R_1 = –ve, R_2 = –ve]

$$\frac{1}{f} = (15-1)\left(\frac{1}{-20} - \frac{1}{-30}\right) \Rightarrow f = -120 \text{ cm}$$



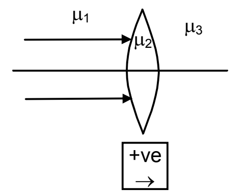
Case (4) : (Concave convex) [R_1 = +ve, R_2 = +ve]

$$\frac{1}{f} = (15-1)\left(\frac{1}{20} - \frac{1}{30}\right) \Rightarrow f = +120 \text{ cm}$$



51. a) When the beam is incident on the lens from medium μ_1.

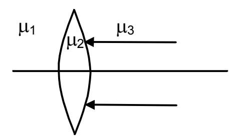
Then $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ or $\frac{\mu_2}{v} - \frac{\mu_1}{(-\infty)} = \frac{\mu_2 - \mu_1}{R}$



or $\frac{1}{v} = \frac{\mu_2 - \mu_1}{\mu_2 R}$ or $v = \frac{\mu_2 R}{\mu_2 - \mu_1}$

Again, for 2nd refraction, $\frac{\mu_3}{v} - \frac{\mu_2}{u} = \frac{\mu_3 - \mu_2}{R}$

or, $\frac{\mu_3}{v} = -\left[\frac{\mu_3 - \mu_2}{R} - \frac{\mu_2}{\mu_2 R}(\mu_2 - \mu_1)\right] \Rightarrow -\left[\frac{\mu_3 - \mu_2 - \mu_2 + \mu_1}{R}\right]$



or, $v = -\left[\frac{\mu_3 R}{\mu_3 - 2\mu_2 + \mu_1}\right]$

So, the image will be formed at = $\frac{\mu_3 R}{2\mu_2 - \mu_1 - \mu_3}$

b) Similarly for the beam from μ_3 medium the image is formed at $\frac{\mu_1 R}{2\mu_2 - \mu_1 - \mu_3}$.

52. Given that, f = 10 cm

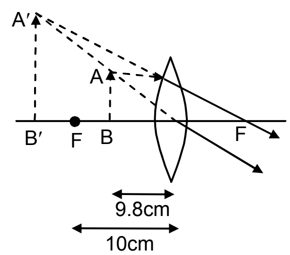
a) When u = –9.5 cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{9.8} = \frac{-0.2}{98}$$

$\Rightarrow$ v = – 490 cm

So, $\Rightarrow$ m = $\frac{v}{u} = \frac{-490}{-9.8}$ = 50 cm

So, the image is erect and virtual.



(Virtual image)

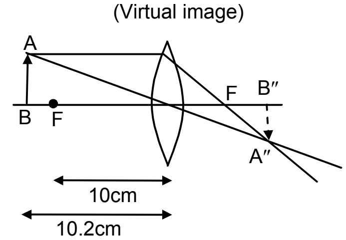
b) When u = –10.2 cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{-10.2} = \frac{102}{0.2}$$

$\Rightarrow$ v = 510 cm

So, m = $\frac{v}{u} = \frac{510}{-9.8}$

The image is real and inverted.



(Real image)

53. For the projector the magnification required is given by

m = $\frac{v}{u} = \frac{200}{3.5} \Rightarrow$ u = 17.5 cm

[35 mm > 23 mm, so the magnification is calculated taking object size 35 mm]

Now, from lens formula,

$$\Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{v} - \frac{1}{-u} = \frac{1}{f} \Rightarrow \frac{1}{1000} + \frac{1}{17.5} = \frac{1}{f}$$

$\Rightarrow$ f = 17.19 cm.

54. When the object is at 19 cm from the lens, let the image will be at, v_1.

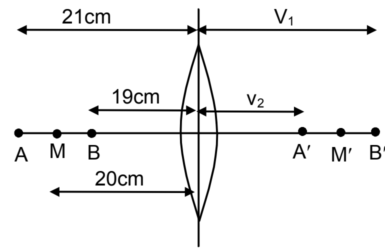
$\Rightarrow \frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f} \Rightarrow \frac{1}{v_1} - \frac{1}{-19} = \frac{1}{12}$

$\Rightarrow v_1 = 32.57$ cm

Again, when the object is at 21 cm from the lens, let the image will be at, v_2

$\Rightarrow \frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f} \Rightarrow \frac{1}{v_2} + \frac{1}{21} = \frac{1}{12}$

$\Rightarrow v_2 = 28$ cm



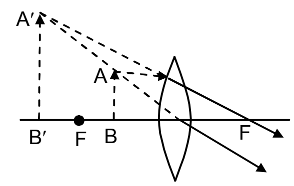
$\therefore$ Amplitude of vibration of the image is $A = \frac{A'B'}{2} = \frac{v_1 - v_2}{2}$

$\Rightarrow A = \frac{32.57 - 28}{2}$ = 2.285 cm.

55. Given, u = –5 cm, f = 8 cm

So, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{8} - \frac{1}{5} = \frac{-3}{40}$

$\Rightarrow$ v = –13.3 cm (virtual image).



56. Given that,

(–u) + v = 40 cm = distance between object and image

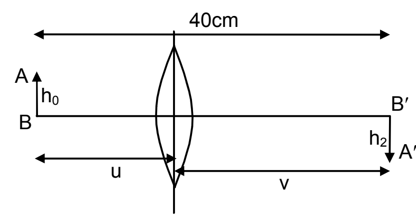
$h_o = 2$ cm, $h_i = 1$ cm

Since $\frac{h_i}{h_o} = \frac{v}{-u}$ = magnification

$\Rightarrow \frac{1}{2} = \frac{v}{-u} \Rightarrow u = -2v$ …(1)

Now, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{2v} = \frac{1}{f}$

$\Rightarrow \frac{3}{2v} = \frac{1}{f} \Rightarrow f = \frac{2v}{3}$ …(2)



Again, (–u) + v = 40

$\Rightarrow$ 3v = 40 $\Rightarrow$ v = 40/3 cm

$\therefore f = \frac{2 \times 40}{3 \times 3}$ = 8.89 cm = focal length

From eqn. (1) and (2)

u = –2v = –3f = –3(8.89) = 26.7 cm = object distance.

57. A real image is formed. So, magnification m = –2 (inverted image)

$\therefore \frac{v}{u} = -2 \Rightarrow v = -2u = (-2)(-18) = 36$

From lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{36} - \frac{1}{-18} = \frac{1}{f}$

$\Rightarrow$ f = 12 cm

Now, for triple sized image m = –3 = (v/u)

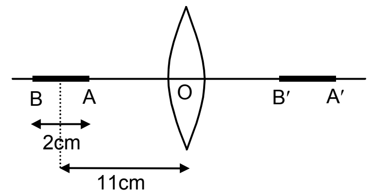
$\therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{-3u} - \frac{1}{u} = \frac{1}{12}$

$\Rightarrow$ 3u = –48 $\Rightarrow$ u = –16 cm

So, object should be placed 16 cm from lens.

58. Now we have to calculate the image of A and B. Let the images be A′, B′. So, length of A′ B′ = size of image.

For A, u = –10 cm, f = 6 cm



Since, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}-\frac{1}{-10}=\frac{1}{6}$

$\Rightarrow$ v = 15 cm = OA′

For B, u = –12 cm, f = 6 cm

Again, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}=\frac{1}{6}-\frac{1}{12}$

$\Rightarrow$ v = 12 cm = OB′

$\therefore$A′B′ = OA′ – OB′ = 15 – 12 = 3 cm.

So, size of image = 3 cm.

59. $u = -1.5 \times 10^{11}$ m ; $f = +20 \times 10^{-2}$ m

Since, f is very small compared to u, distance is taken as ∞. So, image will be formed at focus.

$\Rightarrow v = +20 \times 10^{-2}$ m

$\therefore$ We know, $m = \frac{v}{u} = \frac{h_{image}}{h_{object}}$

$\Rightarrow \frac{20 \times 10^{-2}}{1.5 \times 10^{11}} = \frac{D_{image}}{1.4 \times 10^{9}}$

$\Rightarrow D_{image}$ = 1.86 mm

So, radius = $\frac{D_{image}}{2}$ = 0.93 mm.

60. Given, P = 5 diopter (convex lens)

$\Rightarrow$ f = 1/5 m = 20 cm

Since, a virtual image is formed, u and v both are negative.

Given, v/u = 4

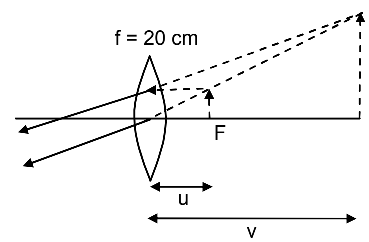
$\Rightarrow$ v = 4u ...(1)

From lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$\Rightarrow \frac{1}{f}=\frac{1}{4u}-\frac{1}{u} \Rightarrow \frac{1}{20}=\frac{1-4}{4u}=-\frac{3}{4u}$

$\Rightarrow$ u = –15 cm

$\therefore$ Object is placed 15 cm away from the lens.



61. Let the object to placed at a distance x from the lens further away from the mirror.

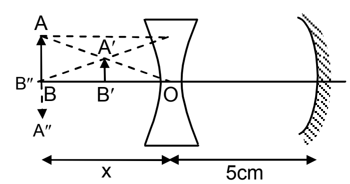
For the concave lens (1st refraction)

u = –x, f = –20 cm

From lens formula,

$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}=\frac{1}{-20}+\frac{1}{-x}$

$\Rightarrow v = -\left(\frac{20x}{x+20}\right)$



So, the virtual image due to fist refraction lies on the same side as that of object. (A′B′)

This image becomes the object for the concave mirror.

For the mirror,

$u = -\left(5+\frac{20x}{x+20}\right) = -\left(\frac{25x+100}{x+20}\right)$

f = –10 cm

From mirror equation,

$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}=\frac{1}{-10}+\frac{x+20}{25x+100}$

$\Rightarrow v = \frac{50(x+4)}{3x-20}$

So, this image is formed towards left of the mirror.

Again for second refraction in concave lens,

$u = -\left[5 - \frac{50(x+4)}{3x-20}\right]$ (assuming that image of mirror is formed between the lens and mirro)

v = +x (Since, the final image is produced on the object)

Using lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{x} + \frac{1}{5 - \frac{50(x+4}{3x-20}} = \frac{1}{-20}$$

$\Rightarrow$ x = 60 cm

The object should be placed at a distance 60 cm from the lens further away from the mirror.

So that the final image is formed on itself.

62. It can be solved in a similar manner like question no.61, by using the sign conversions properly. Left as an exercise for the student.

63. If the image in the mirror will form at the focus of the converging lens, then after transmission through the lens the rays of light will go parallel.

Let the object is at a distance x cm from the mirror

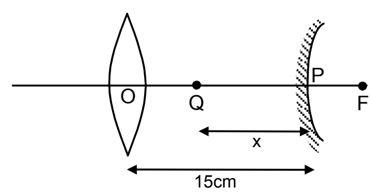
$\therefore$ u = –x cm ; v = 25 – 15 = 10 cm (because focal length of lens = 25 cm)

f = 40 cm

$$\Rightarrow \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{x} = \frac{1}{10} - \frac{1}{40}$$

$\Rightarrow$ x = 400/30 = 40/3

$\therefore$ The object is at distance $\left(15 - \frac{40}{3}\right) = \frac{5}{3}$ = 1.67 cm from the lens.



64. The object is placed in the focus of the converging mirror.

There will be two images.

a) One due to direct transmission of light through lens.

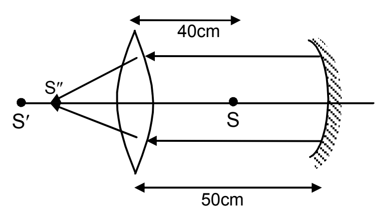
b) One due to reflection and then transmission of the rays through lens.

Case I : (S′) For the image by direct transmission,

u = –40 cm, f = 15 cm

$$\Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{15} + \frac{1}{-40}$$

$\Rightarrow$ v = 24 cm (left of lens)



Case II : (S″) Since, the object is placed on the focus of mirror, after reflection the rays become parallel for the lens.

So, u = ∞

$\Rightarrow$ f = 15 cm

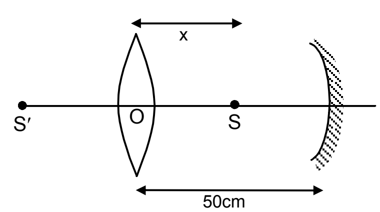
$$\Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow v = 15 \text{ cm (left of lens)}$$

65. Let the source be placed at a distance 'x' from the lens as shown, so that images formed by both coincide.

For the lens, $\frac{1}{v_\ell} - \frac{1}{-x} = \frac{1}{15} \Rightarrow v_\ell = \frac{15x}{x-15}$ …(1)

Fro the mirror, u = –(50 – x), f = –10 cm

So, $\frac{1}{v_m} + \frac{1}{-(50-x)} = -\frac{1}{10}$



$\Rightarrow \dfrac{1}{v_m} = \dfrac{1}{-(50-x)} - \dfrac{1}{10}$

So, $v_m = \dfrac{10(50-x)}{x-40}$ …(2)

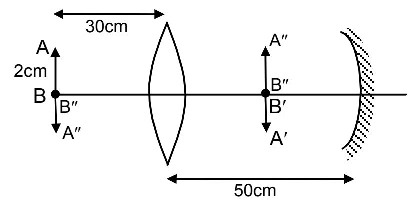
Since the lens and mirror are 50 cm apart,

$v_\ell - v_m = 50 \Rightarrow \dfrac{15x}{x-15} - \dfrac{10(50-x)}{(x-40)} = 50$

$\Rightarrow x = 30$ cm.

So, the source should be placed 30 cm from the lens.

66. Given that, $f_1 = 15$ cm, $F_m = 10$ cm, $h_o = 2$ cm



The object is placed 30 cm from lens $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$.

$\Rightarrow v = \dfrac{uf}{u+f}$

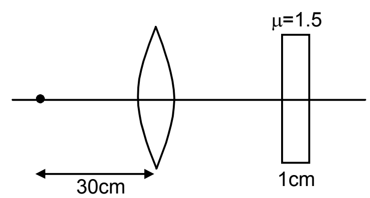
Since, $u = -30$ cm and $f = 15$ cm

So, $v = 30$ cm

So, real and inverted image (A′B′) will be formed at 30 cm from the lens and it will be of same size as the object. Now, this real image is at a distance 20 cm from the concave mirror. Since, $f_m = 10$ cm, this real image is at the centre of curvature of the mirror. So, the mirror will form an inverted image A″B″ at the same place of same size.

Again, due to refraction in the lens the final image will be formed at AB and will be of same size as that of object. (A‴B‴)

67. For the lens, $f = 15$ cm, $u = -30$ cm



From lens formula, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

$\Rightarrow \dfrac{1}{v} = \dfrac{1}{15} - \dfrac{1}{30} = \dfrac{1}{30} \Rightarrow v = 30$ cm

The image is formed at 30 cm of right side due to lens only.

Again, shift due to glass slab is,

$= \Delta t = \left(1 - \dfrac{1}{15}\right)1$ [since, $\mu_g = 1.5$ and $t = 1$ cm]

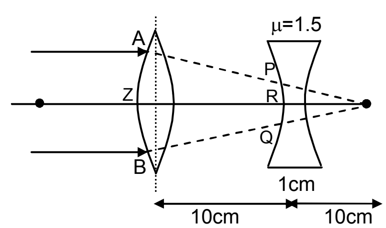
$= 1 - (2/3) = 0.33$ cm

$\therefore$ The image will be formed at 30 + 0.33 = 30.33 cm from the lens on right side.

68. Let, the parallel beam is first incident on convex lens.

d = diameter of the beam = 5 mm

Now, the image due to the convex lens should be formed on its focus (point B)



So, for the concave lens,

u = +10 cm (since, the virtual object is on the right of concave lens)

f = –10 cm

So, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f} \Rightarrow \dfrac{1}{v} = \dfrac{1}{-10} + \dfrac{1}{10} = 0 \Rightarrow v = \infty$

So, the emergent beam becomes parallel after refraction in concave lens.

As shown from the triangles XYB and PQB,

$\dfrac{PQ}{XY} = \dfrac{RB}{ZB} = \dfrac{10}{20} = \dfrac{1}{2}$

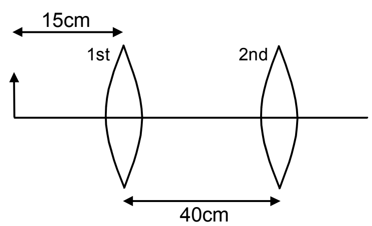
So, PQ = ½ × 5 = 25 mm

So, the beam diameter becomes 2.5 mm.

Similarly, it can be proved that if the light is incident of the concave side, the beam diameter will be 1cm.

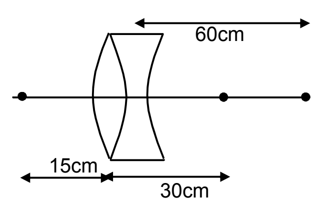
69. Given that, f_1 = focal length of converging lens = 30 cm
f_2 = focal length of diverging lens = –20 cm
and d = distance between them = 15 cm
Let, F = equivalent focal length
So, $\therefore \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \Rightarrow \frac{1}{30} + \left(-\frac{1}{20}\right) - \left(\frac{15}{30(-200}\right) = \frac{1}{120}$
$\Rightarrow$ F = 120 cm
$\Rightarrow$ The equivalent lens is a converging one.
Distance from diverging lens so that emergent beam is parallel (image at infinity),
$d_1 = \frac{dF}{f_1} = \frac{15 \times 120}{30}$ = 60 cm
It should be placed 60 cm left to diverging lens
$\Rightarrow$ Object should be placed (120 – 60) = 60 cm from diverging lens.
Similarly, $d_2 = \frac{dF}{f_2} = \frac{15 \times 120}{20}$ = 90 cm
So, it should be placed 90 cm right to converging lens.
$\Rightarrow$ Object should be placed (120 + 90) = 210 cm right to converging lens.

70. **a) First lens** :
u = –15 cm, f = 10 cm
$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \left(-\frac{1}{15}\right) = -\frac{1}{10}$
$\Rightarrow$ v = 30 cm
So, the final image is formed 10 cm right of second lens.



b) m for 1st lens :
$\frac{v}{u} = \frac{h_{image}}{h_{object}} \Rightarrow \left(\frac{30}{-15}\right) = \frac{h_{image}}{5mm}$
$\Rightarrow h_{image}$ = –10 mm (inverted)
Second lens :
u = –(40 – 30) = –10 cm ; f = 5 cm
[since, the image of 1st lens becomes the object for the second lens].
$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \left(-\frac{1}{10}\right) = \frac{1}{5}$
$\Rightarrow$ v = 10 cm
m for 2nd lens :
$\frac{v}{u} = \frac{h_{image}}{h_{object}} \Rightarrow \left(\frac{10}{10}\right) = \frac{h_{image}}{-10}$
$\Rightarrow h_{image}$ = 10 mm (erect, real).
c) So, size of final image = 10 mm

71. Let u = object distance from convex lens = –15 cm
v_1 = image distance from convex lens when alone = 30 cm
f_1 = focal length of convex lens



Now, $\therefore \frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}$
or, $\frac{1}{f_1} = \frac{1}{30} - \frac{1}{-15} = \frac{1}{30} + \frac{1}{15}$
or f_1 = 10 cm
Again, Let v = image (final) distance from concave lens = +(30 + 30) = 60 cm
v_1 = object distance from concave lens = +30 m

f_2 = focal length of concave lens

Now, $\therefore\ \frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_1}$

or, $\frac{1}{f_1} = \frac{1}{60} - \frac{1}{30}$ $\Rightarrow f_2 = -60$ cm.

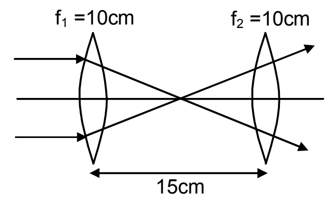
So, the focal length of convex lens is 10 cm and that of concave lens is 60 cm.

72. a) The beam will diverge after coming out of the two convex lens system because, the image formed by the first lens lies within the focal length of the second lens.

b) For 1st convex lens, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{10}$ (since, $u = -\infty$)

or, v = 10 cm

for 2nd convex lens, $\frac{1}{v'} = \frac{1}{f} + \frac{1}{u}$



or, $\frac{1}{v'} = \frac{1}{10} + \frac{1}{-(15-10)} = \frac{-1}{10}$

or, v′ = –10 cm

So, the virtual image will be at 5 cm from 1st convex lens.

c) If, F be the focal length of equivalent lens,

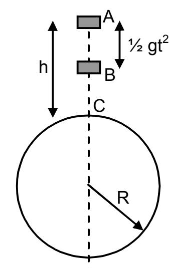
Then, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \Rightarrow \frac{1}{10} + \frac{1}{10} - \frac{15}{100} = \frac{1}{20}$

$\Rightarrow$ F = 20 cm.

73. Let us assume that it has taken time ‘t’ from A to B.

$\therefore$ AB = $\frac{1}{2}gt^2$

$\therefore$ BC = h – $\frac{1}{2}gt^2$



This is the distance of the object from the lens at any time ‘t’.

Here, u = – (h – $\frac{1}{2}gt^2$)

$\mu_2 = \mu$(given) and μ_1 = i (air)

So, $\Rightarrow \frac{\mu}{v} - \frac{1}{-(h - \frac{1}{2}gf^2)} = \frac{\mu - 1}{R}$

$$\Rightarrow \frac{\mu}{v} = \frac{\mu-1}{R} - \frac{1}{(h - \frac{1}{2}gt^2)} = \frac{(\mu-1)(h - \frac{1}{2}gt^2) - R}{R(h - \frac{1}{2}gt^2)}$$

So, v = image distance at any time ‘t’ = $\frac{\mu R(h - \frac{1}{2}gt^2)}{(\mu-1)(h - \frac{1}{2}gt^2) - R}$

So, velocity of the image = V = $\frac{dv}{dt} = \frac{d}{dt}\left[\frac{\mu R(h - \frac{1}{2}gt^2)}{(\mu-1)(h - \frac{1}{2}gt^2) - R}\right] = \frac{\mu R^2 gt}{(\mu-1)(h - \frac{1}{2}gt^2) - R}$ (can be found out).

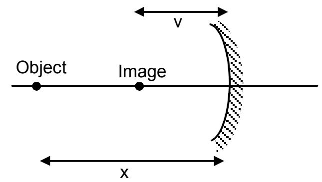
74. Given that, u = distance of the object = –x

f = focal length = –R/2

and, V = velocity of object = dx/dt

From mirror equation, $\frac{1}{-x} + \frac{1}{v} = -\frac{2}{R}$

$\frac{1}{v} = -\frac{2}{R} + \frac{1}{x} = \frac{R-2x}{Rx} \Rightarrow v = \frac{Rx}{R-2x}$ = Image distance



So, velocity of the image is given by,

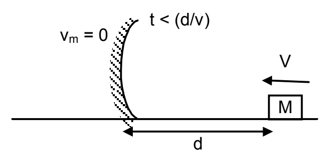
$$V_1 = \frac{dv}{dt} = \frac{[\frac{d}{dt}(xR)(R-2x)] - [\frac{d}{dt}(R-2x)][xR]}{(R-2x)^2}$$

$$= \frac{R[\frac{dx}{dt}(R-2x)] - [-2\frac{dx}{dt}x]}{(R-2x)^2} = \frac{R[v(R-2x)+2vx0}{(R-2x)^2}$$

$$= \frac{VR^2}{(2x-R)^2} = \frac{R[VR-2xV+2xV}{(R-2x)^2}.$$

75. a) When t < d/V, the object is approaching the mirror

As derived in the previous question,



$$V_{image} = \frac{\text{Velocity of object} \times R^2}{[2 \times \text{distance between them } - R]^2}$$

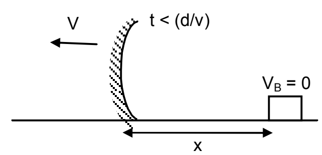
$$\Rightarrow V_{image} = \frac{VR^2}{[2(d-Vt)-R]^2} \quad \text{[At any time, x = d – Vt]}$$

b) After a time t > d/V, there will be a collision between the mirror and the mass.

As the collision is perfectly elastic, the object (mass) will come to rest and the mirror starts to move away with same velocity V.

At any time t > d/V, the distance of the mirror from the mass will be

$$x = V\left(t - \frac{d}{V}\right) = Vt - d$$



Here, u = –(Vt – d) = d – Vt ; f = –R/2

So, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = -\frac{1}{d-Vt} + \frac{1}{(-R/2)} = -\left[\frac{R+2(d-Vt)}{R(d-Vt)}\right]$

$\Rightarrow v = -\left[\frac{R(d-Vt)}{R-2(d-Vt)}\right]$ = Image distance

So, Velocity of the image will be,

$$V_{image} = \frac{d}{dt}(\text{Image distance}) = \frac{d}{dt}\left[\frac{R(d-Vt)}{R+2(d-Vt)}\right]$$

Let, y = (d – Vt)

$$\Rightarrow \frac{dy}{dt} = -V$$

So, $V_{image} = \frac{d}{dt}\left[\frac{Ry}{R+2y}\right] = \frac{(R+2y)R(-V) - Ry(+2)(-V)}{(R+2y)^2}$

$$= -Vr\left[\frac{R+2y-2y}{(R+2y)^2}\right] = \frac{-VR^2}{(R+2y)^2}$$

Since, the mirror itself moving with velocity V,

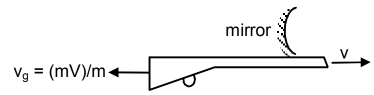
Absolute velocity of image = $V\left[1 - \frac{R^2}{(R+2y)^2}\right]$ (since, V = $V_{mirror} + V_{image}$)

$$= V\left[1 - \frac{R^2}{[2(Vt-d)-R^2}\right].$$

76. Recoil velocity of gun = $V_g = \frac{mV}{M}$.

At any time 't', position of the bullet w.r.t. mirror = $Vt + \frac{mV}{M}t = \left(1+\frac{m}{M}\right)Vt$

For the mirror, u = $-\left(1+\frac{m}{M}\right)Vt = kVt$



v = position of the image

From lens formula,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} \Rightarrow \frac{1}{v} = \frac{1}{-f} + \frac{1}{kVt} = \frac{1}{kVt} - \frac{1}{f} = \frac{f - kVt}{kVtf}$$

Let $\left(1+\frac{m}{M} = k\right)$,

So, v = $\frac{kVft}{-kVt+f} = \left(\frac{kVtf}{f-kVt}\right)$

So, velocity of the image with respect to mirror will be,

$$v_1 = \frac{dv}{dt} = \frac{d}{dt}\left[\frac{kVtf}{f-kVt}\right] = \frac{(f-kVt)kVf - kVtf(-kV)}{(f-kVt)^2} = \frac{kVt^2}{(f-kVt)^2}$$

Since, the mirror itself is moving at a speed of mV/M and the object is moving at 'V', the velocity of separation between the image and object at any time 't' will be,

$$v_s = V + \frac{mV}{M} + \frac{kVf^2}{(f-kVt)^2}$$

When, t = 0 (just after the gun is fired),

$$v_s = V + \frac{mV}{M} + kV = V + \frac{m}{M}V + \left(1+\frac{m}{M}\right)V = 2\left(1+\frac{m}{M}\right)V$$

77. Due to weight of the body suppose the spring is compressed by which is the mean position of oscillation.

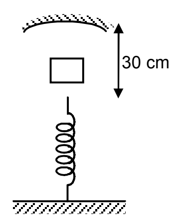


Fig-A

$m = 50 \times 10^{-3}$ kg, g = 10 ms^{-2}, k = 500 Nm^{-2}, h = 10 cm = 0.1 m

For equilibrium, mg = kx $\Rightarrow$ x = mg/k = 10^{-3}m = 0.1 cm

So, the mean position is at 30 + 0.1 = 30.1 cm from P (mirror).

Suppose, maximum compression in spring is δ.

Since, E.K.E. – I.K.E. = Work done

$\Rightarrow 0 - 0 = mg(h + \delta) - \frac{1}{2}k\delta^2$ (work energy principle)

$\Rightarrow mg(h + \delta) = \frac{1}{2}k\delta^2 \Rightarrow 50 \times 10^{-3} \times 10(0.1 + \delta) = \frac{1}{2}\,500\,\delta^2$

So, $\delta = \frac{0.5 \pm \sqrt{0.25+50}}{2 \times 250}$ = 0.015 m = 1.5 cm.

From figure B,

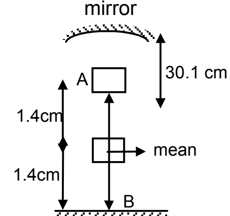


Fig-B

Position of B is 30 + 1.5 = 31.5 cm from pole.

Amplitude of the vibration = 31.5 – 30.1 – 1.4.

Position A is 30.1 – 1.4 = 28.7 cm from pole.

For A u = –31.5, f = –12 cm

$$\therefore \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{12} + \frac{1}{31.5}$$

$\Rightarrow v_A = -19.38$ cm

For B f = –12 cm, u = –28.7 cm

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{12} + \frac{1}{28.7}$$

$\Rightarrow v_B = -20.62$ cm

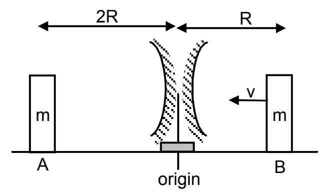
The image vibrates in length (20.62 – 19.38) = 1.24 cm.

78. a) In time, t = R/V the mass B must have moved (v × R/v) = R closer to the mirror stand

So, For the block B :

u = –R, f = –R/2

$\therefore \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{2}{R} + \frac{1}{R} = -\frac{1}{R}$



⇒ v = –R at the same place.

For the block A : u = –2R, f = –R/2

$\therefore \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{-2}{R} + \frac{1}{2R} = \frac{-3}{2R}$

⇒ v = $\frac{-2R}{3}$ image of A at $\frac{2R}{3}$ from PQ in the x-direction.

So, with respect to the given coordinate system,

∴ Position of A and B are $\frac{-2R}{3}$, R respectively from origin.

b) When t = 3R/v, the block B after colliding with mirror stand must have come to rest (elastic collision) and the mirror have travelled a distance R towards left form its initial position.

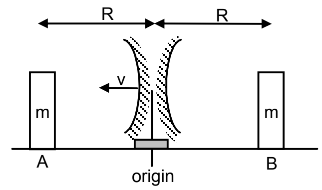
So, at this point of time,

For block A :

u = –R, f = –R/2

Using lens formula, v = –R (from the mirror),

So, position x_A = –2R (from origin of coordinate system)

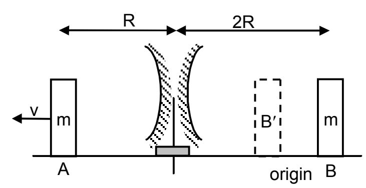


For block B :

Image is at the same place as it is R distance from mirror. Hence, position of image is ‘0’.

Distance from PQ (coordinate system)

∴ positions of images of A and B are = –2R, 0 from origin.



c) Similarly, it can be proved that at time t = 5R/v,

the position of the blocks will be –3R and –4R/3 respectively.

79. Let a = acceleration of the masses A and B (w.r.t. elevator). From the freebody diagrams,

T – mg + ma – 2m = 0 …(1)

Similarly, T – ma = 0 …(2)

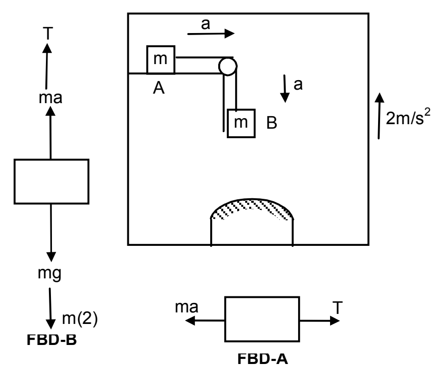
From (1) and (2), 2ma – mg – 2m = 0

⇒ 2ma = m(g + 2)

⇒ a = $\frac{10+2}{2} = \frac{12}{2}$ = 6 ms^{-2}

so, distance travelled by B in t = 0.2 sec is,

s = $\frac{1}{2}at^2 = \frac{1}{2} \times 6 \times (0.2)^2$ = 0.12 m = 12 cm.



So, Distance from mirror, u = –(42 – 12) = –30 cm ; f = +12 cm

From mirror equation, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \left(-\frac{1}{30}\right) = \frac{1}{12}$

⇒ v = 8.57 cm

Distance between image of block B and mirror = 8.57 cm.

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