Ex 27.1

Direction Cosines and Direction Ratios Ex 27.1 Q1

Let l, m and n be the direction cosines of a line.
$l = \cos 90^\circ = 0$

$m = \cos 60^\circ = \frac{1}{2}$

$n = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$\therefore$ The direction cosines of the line are $0, \frac{1}{2}, \frac{\sqrt{3}}{2}$.

Direction Cosines and Direction Ratios Ex 27.1 Q2

Let the direction cosines of the line be l, m, n.
Here,
$a = 2, b = -1, c = -2$ are the direction ratios of the line.

$$l = \pm\frac{a}{\sqrt{a^2+b^2+c^2}}, m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}}, n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}}$$

$$l = \frac{2}{\sqrt{2^2+(-1)^2+(-2)^2}}, m = \frac{-1}{\sqrt{2^2+(-1)^2+(-2)^2}}, n = \frac{-2}{\sqrt{2^2+(-1)^2+(-2)^2}}$$

$$l = \frac{2}{\sqrt{9}}, m = \frac{-1}{\sqrt{9}}, n = \frac{-2}{\sqrt{9}}$$

$$l = \frac{2}{3}, m = -\frac{1}{3}, n = -\frac{2}{3}$$

$\therefore$ The direction ratios of the line are $\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}$.

Direction Cosines and Direction Ratios Ex 27.1 Q3

The direction ratios of the line joining $(-2, 4, -5)$ and $(1, 2, 3)$ are,
$(1+2, 2-4, 3+5) = (3, -2, 8)$
Here, $a = 3, b = -2, c = 8$
Direction cosines are

$$\frac{3}{\sqrt{3^2+(-2)^2+8^2}}, \frac{-2}{\sqrt{3^2+(-2)^2+8^2}}, \frac{8}{\sqrt{3^2+(-2)^2+8^2}}$$

$$= \frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$$

Direction Cosines and Direction Ratios Ex 27.1 Q4

Here $A(2, 3, -4)$, $B(1, -2, 3)$ and $C(3, 8, -11)$.
Direction ratios of $AB = (1-2, -2-3, 3+4) = (-1, -5, 7)$
Direction ratios of $BC = (3-1, 8+2, -11-3) = (2, 10, -14)$

Here, the respective direction cosines of AB and AC,
$\frac{-1}{2} = \frac{-5}{10} = \frac{7}{-14}$ are proportional.

Also, B is the common point between the two lines,
$\therefore$ The points $A(2, 3, -4)$, $B(1, -2, 3)$ and $C(3, 8, -11)$ are collinear.

Direction Cosines and Direction Ratios Ex 27.1 Q5

$A(3,5,-4), B(-1,1,2)$ and $C(-5,-5,-2)$

The direction ratios of the side $AB = (-1-3, 1-5, 2+4)$

$= (-4,-4,6)$

Direction cosines of AB will be

$$\frac{-4}{\sqrt{(-4)^2+(-4)^2+6^2}}, \frac{-4}{\sqrt{(-4)^2+(-4)^2+6^2}}, \frac{6}{\sqrt{(-4)^2+(-4)^2+6^2}}$$

$$= \frac{-4}{\sqrt{68}}, \frac{-4}{\sqrt{68}}, \frac{6}{\sqrt{68}}$$

$$= \frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{\sqrt{17}}$$

The direction ratios of the side $BC = (-5+1, -5-1, -2-2)$

$= (-4,-6,-4)$

Direction cosines of BC will be

$$\frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}, \frac{-6}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}, \frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}$$

$$= \frac{-4}{\sqrt{68}}, \frac{-6}{\sqrt{68}}, \frac{-4}{\sqrt{68}}$$

$$= \frac{-2}{\sqrt{17}}, \frac{-3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}$$

The direction ratios of the side $AC = (-5-3, -5-5, -2+4)$

$= (-8,-10,2)$

Direction cosines of AC will be

$$\frac{-8}{\sqrt{(-8)^2+(-10)^2+2^2}}, \frac{-10}{\sqrt{(-8)^2+(-10)^2+2^2}}, \frac{2}{\sqrt{(-8)^2+(-10)^2+2^2}}$$

$$= \frac{-8}{\sqrt{168}}, \frac{-10}{\sqrt{168}}, \frac{2}{\sqrt{168}}$$

$$= \frac{-4}{\sqrt{42}}, \frac{-5}{\sqrt{42}}, \frac{1}{\sqrt{42}}$$

Direction Cosines and Direction Ratios Ex 27.1 Q6

Let, θ be the angle between the vectors with direction ratios a, b, c and a_2, b_2, c_2 then.

$$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$= \frac{(1)(4)+(-2)(3)+(1)(2)}{\sqrt{(1)^2+(-2)^2+(1)^2}\sqrt{(4)^2+(3)^2+(2)^2}}$$

$$= \frac{4-6+2}{\sqrt{1+4+1}\sqrt{16+9+4}}$$

$$= \frac{6-6}{\sqrt{6}\sqrt{29}}$$

$$= \frac{0}{\sqrt{174}}$$

$\cos\theta = 0$

$\theta = \cos^{-1}(0)$

$\theta = \frac{\pi}{2}$

Direction Cosines and Direction Ratios Ex 27.1 Q7

Here, given that the direction cosines of the vectors are proportional to 2, 3, − 6 and 3, − 4, 5.

Therefore, 2, 3, − 6 and 3, − 4, 5 are the direction ratios of two vectors.

Let, θ be the angle between two vectors having direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{(2)(3) + (3)(-4) + (-6)(5)}{\sqrt{(2)^2 + (3)^2 + (-6)^2}\sqrt{(3)^2 + (-4)^2 + (5)^2}}$$

$$\cos\theta = \frac{6 - 12 - 30}{\sqrt{4 + 9 + 36}\sqrt{9 + 16 + 25}}$$

$$= \frac{6 - 42}{\sqrt{49}\sqrt{50}}$$

$$= \frac{-36 \times \sqrt{2}}{7 \times 5 \times \sqrt{2} \times \sqrt{2}} \qquad \text{(Rationalizing the denominator)}$$

$$= \frac{-36\sqrt{2}}{70}$$

$$\cos\theta = \frac{-18\sqrt{2}}{35}$$

$$\theta = \cos^{-1}\left(\frac{-18\sqrt{2}}{35}\right)$$

Direction Cosines and Direction Ratios Ex 27.1 Q8

The vectors, represented by these are

$\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

and $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$

Let, θ be the angle between the lines,
then,

$$\cos\theta = \frac{\vec{a} \times \vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{(2\hat{i} + 3\hat{j} + 6\hat{k})(\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{(2)^2 + (3)^2 + (6)^2}\sqrt{(1)^2 + (2)^2 + (2)^2}}$$

$$= \frac{(2)(1) + (3)(2) + (6)(2)}{\sqrt{4 + 9 + 36}\sqrt{1 + 4 + 4}}$$

$$= \frac{2 + 6 + 12}{\sqrt{49}\sqrt{9}}$$

$$= \frac{20}{7 \times 3}$$

$$\cos\theta = \frac{20}{21}$$

$$\theta = \cos^{-1}\left(\frac{20}{21}\right)$$

Angle between the lines $= \cos^{-1}\left(\frac{20}{21}\right)$

Direction Cosines and Direction Ratios Ex 27.1 Q9

We have, $(2, 3, 4), (-1, -2, 1)$ and $(5, 8, 7)$

Let the points are A, B, C respectively.

Position vector of $A = 2\hat{i} + 3\hat{j} + 4\hat{k}$
Position vector of $B = -\hat{i} - 2\hat{j} + \hat{k}$
Position vector of $C = 5\hat{i} + 8\hat{j} + 7\hat{k}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= \left(-\hat{i} - 2\hat{j} + \hat{k}\right) - \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$
$= -\hat{i} - 2\hat{j} + \hat{k} - 2\hat{i} - 3\hat{j} - 4\hat{k}$

$\overrightarrow{AB} = -3\hat{i} - 5\hat{j} - 3\hat{k}$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$= \left(5\hat{i} + 8\hat{j} + 7\hat{k}\right) - \left(-\hat{i} - 2\hat{j} + \hat{k}\right)$
$= 5\hat{i} + 8\hat{j} + 7\hat{k} + \hat{i} + 2\hat{j} - \hat{k}$

$\overrightarrow{BC} = 6\hat{i} + 10\hat{j} + 6\hat{k}$

Using $\overrightarrow{AB}$ and $\overrightarrow{BC}$, we get

$\overrightarrow{BC} = -2\ \overrightarrow{AB}$

So, $\overrightarrow{BC}$ is parallel to $\overrightarrow{AB}$ but $\vec{B}$ is the common vector,

Hence, A, B, C are collinear

Direction Cosines and Direction Ratios Ex 27.1 Q10

line throught points $(4,\ 7,\ 8)$ *and* $(2,\ 3,\ 4)$

$$\frac{x-4}{2} = \frac{y-7}{4} = \frac{z-8}{4} \rightarrow \frac{x-4}{1} = \frac{y-7}{2} = \frac{z-8}{2}$$

line through the points $(-1, -2, 1)$ and $(1, 2, 5)$

$$\frac{x+1}{-2} = \frac{y+2}{-4} = \frac{z-1}{-4} \rightarrow \frac{x+1}{1} = \frac{y+2}{2} = \frac{z-1}{2}$$

the direction ratios are same for both the lines
$\therefore$ they are parallel to each other

Direction Cosines and Direction Ratios Ex 27.1 Q11

Given,

$A(1, -1, 2)$ and $B(3, 4, -2)$
$C(0, 3, 2)$ and $D(3, 5, 6)$

Direction ratios of line AB
$a_1 = 2, \quad b_1 = 5, \quad c_1 = -4$

Direction ratios of line CD
$a_2 = 3, \quad b_2 = 2, \quad c_2 = 4$

We know that, lines are perpendicular if

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$\begin{aligned} LHS &= (2)(3) + (5)(2) + (-4)(4) \\ &6 + 10 - 16 \\ &16 - 16 \\ &0 \end{aligned}$$

$\therefore LHS = RHS$

Lines are perpendicular

Direction Cosines and Direction Ratios Ex 27.1 Q12

Here,

$A(0, 0, 0)$ and $B(2, 1, 1)$
$C(3, 5, -1)$ and $D(4, 3, -1)$

Direction ratios of line AB
$a_1 = 2, \quad b_1 = 1, \quad c_1 = 1$

Direction ratios of line CD
$a_2 = 1, \quad b_2 = -2, \quad c_2 = 0$

Now,

$a_1a_2 + b_1b_2 + c_1c_2$
$= (2)(1) + (1)(-2) + (1)(0)$
$= 2 - 2 + 0$

$= 0$

Since, $a_1a_2 + b_1b_2 + c_1c_2 = 0$, lines are perpendicular

Direction Cosines and Direction Ratios Ex 27.1 Q13

Given, that the direction ratios of lines are proportional to a, b, c and $b-c, c-a, a-b$.

Let, $\vec{x}$ and $\vec{y}$ be the vector parallel to these lines respectively, so

$\vec{x} = a\hat{i} + b\hat{j} + c\hat{k}$

And, $\vec{y} = (b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k}$

Let, θ be the angle between $\vec{x}$ and $\vec{y}$, so,

$$\cos\theta = \frac{\vec{x} \times \vec{y}}{|\vec{x}||\vec{y}|}$$

$$= \frac{(a\hat{i} + b\hat{j} + c\hat{k})\left[(b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k}\right]}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}}$$

$$= \frac{(a)(b-c) + b(c-a) + c(a-b)}{\sqrt{a^2+b^2+c^2}\sqrt{b^2+c^2-2bc+c^2+a^2-2ca+a^2+b^2-2ab}}$$

$$\cos\theta = \frac{ab-ac+bc-ba+ca-bc}{\sqrt{a^2+b^2+c^2}\sqrt{2a^2+2b^2+2c^2-2ab-2bc-2ca}}$$

$\cos\theta = 0$

$\theta = \cos^{-1}(0)$

$\theta = \frac{\pi}{2}$

Angle between the lines $= \frac{\pi}{2}$

Direction Cosines and Direction Ratios Ex 27.1 Q14

Here we have,

$A(1, 2, 3)$, $B(4, 5, 7)$, $C(-4, 3, -6)$ $D(2, 9, 2)$

Direction ratios of AB

$a_1 = 3$, $b_1 = 3$, $c_1 = 4$

Direction ratios of CD

$a_2 = 6$, $b_2 = 6$, $c_2 = 8$

Let, θ be the angle between AB and CD, so,

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\cos\theta = \frac{(3)(6) + (3)(6) + (4)(8)}{\sqrt{(3)^2+(3)^2+(4)^2}\sqrt{(6)^2+(6)^2+(8)^2}}$$

$$= \frac{18+18+32}{\sqrt{9+9+16}\sqrt{36+36+64}}$$

$$= \frac{68}{\sqrt{34}\sqrt{136}}$$

$$= \frac{68}{\sqrt{34}\ 2\sqrt{34}}$$

$$= \frac{68}{34 \times 2}$$

$\cos\theta = 1$

$\theta = \cos^{-1}(1)$

$\theta = 0°$

Therefore,

Angle between AB and CD = $0°$

Direction Cosines and Direction Ratios Ex 27.1 Q15

The given equations are

$2lm+2ln-mn=0$

$l+m+n=0$

$\rightarrow l=-(m+n)\ldots\ldots\ldots(1)$

$2l(m+n)=mn\rightarrow l=\dfrac{mn}{2(m+n)}\ldots\ldots\ldots\ldots(2)$

put $l=-(m+n)$ in (2)

$\rightarrow -(m+n)=\dfrac{mn}{2(m+n)}\rightarrow -2(m+n)^2=mn$

$\rightarrow -2\left(m^2+n^2+2mn\right)=mn\rightarrow\left(m^2+n^2+2mn\right)=-\dfrac{mn}{2}$

$\rightarrow\left(m^2+n^2+2mn+\dfrac{mn}{2}\right)=0\rightarrow\left(m^2+n^2+\dfrac{5mn}{2}\right)=0$

$\rightarrow\left(2m^2+2n^2+5mn\right)=0\rightarrow(2m+n)(m+2n)=0$

$\rightarrow m=-\dfrac{n}{2}\rightarrow l=-\left(n-\dfrac{n}{2}\right)=-\dfrac{n}{2}$

$\rightarrow m=-2n\rightarrow l=-(-2n+n)=n$

Thus the direction ratios of two lines are proportional to $-\dfrac{n}{2},-\dfrac{n}{2},n$

and $n,-2n,n$

i.e. $-\dfrac{1}{2},-\dfrac{1}{2},1$ and $1,-2,1$

Hence the direction cosines are

$-\dfrac{1}{\sqrt{3}},-\dfrac{1}{\sqrt{3}},\dfrac{2}{\sqrt{3}}$ and $1,-2,1$

Direction Cosines and Direction Ratios Ex 27.1 Q16(i)

Given that, $l+m+n=0$ --- (i)

$l^2+m^2-n^2=0$ --- (ii)

From equation (i),

$l=-(m+n)$

Put the value of l in equation (ii),

$[-(m+n)]^2+m^2-n^2=0$

$(m+n)^2+m^2-n^2=0$

$m^2+n^2+2mn+m^2-n^2=0$

$2m^2+2mn=0$

$2m(m+n)=0$

$m=0,\ m+n=0$

$m=-n$ and $m=0$

Put the value of $m=-n$ in equation (i)

$l=-(-n+n)$

$l=0$

Again put the value of $m=0$ in equation (i)

$l=-(m+n)$

$=-(0+n)$

$l=-n$

Thus the direction ratios are proportional to

$0, -n, n$ and $-n, 0, n$

$\Rightarrow\quad 0, -1, 1$ and $-1, 0, 1$

So, vectors parallel to these lines are

$\vec{a}=0\times\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=-\hat{i}+0\times\hat{j}+\hat{k}$ respectively.

Let, θ be the angle between the $\vec{a}$ and $\vec{b}$

So, $\cos\theta = \dfrac{\vec{a}\times\vec{b}}{|\vec{a}||\vec{b}|}$

$\vec{a} = 0\times\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = -\hat{i} + 0\times\hat{j} + \hat{k}$ respectively.

$$\cos\theta = \frac{\left(0\times\hat{i} - \hat{j} + \hat{k}\right)\times\left(-\hat{i} + 0\times\hat{j} + \hat{k}\right)}{\sqrt{0^2 + (-1)^2 + (1)^2}\sqrt{(-1)^2 + (0)^2 + (1)^2}}$$

$$= \frac{(0)(-1) + (-1)(0) + (1)(1)}{\sqrt{1+1}\sqrt{1+1}}$$

$$= \frac{0+0+1}{\sqrt{2}\times\sqrt{2}}$$

$$= \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\theta = \frac{\pi}{3}$$

So, angle between the lines $= \dfrac{\pi}{3}$

Direction Cosines and Direction Ratios Ex 27.1 Q16(ii)

Given that,

$2l - m + 2n = 0$ --- (i)

$mn + nl + lm = 0$ --- (ii)

From equation (i),

$2l - m + 2n = 0$
$m = 2l + 2n$

Put the value of m in equation (ii),

$mn + nl + lm = 0$
$(2l + 2n)n + nl + l(2l + 2n) = 0$
$2ln + 2n^2 + nl + 2l^2 + 2ln = 0$
$2l^2 + 5ln + 2n^2 = 0$
$2l^2 + 4ln + ln + 2n^2 = 0$
$2l(l + 2n) + n(l + 2n) = 0$
$(l + 2n)(2l + n) = 0$
$l + 2n = 0$ or $2l + n = 0$
$l = -2n$ or $l = -\frac{n}{2}$

Put the value of $l = -2n$ in equation (i)

$2l - m + 2n = 0$
$2(-2n) - m + 2n = 0$
$-4n - m + 2n = 0$
$-2n - m = 0$
$-2n = m$

$m = -2n$

Again, put the value of $l = -\frac{1}{2}n$ in equation (i)

$2l - m + 2n = 0$
$2\left(-\frac{1}{2}n\right) - m + 2n = 0$
$-n - m + 2n = 0$
$-m + n = 0$
$-m = -n$

$m = n$

So, direction cosines of the lines are given by,

$-2n, -2n, n$ or $-\frac{1}{2}n, n, n$

$-2, -2, 1$ or $-\frac{1}{2}, 1, 1$

So, vectors parallel to these lines

$\vec{a} = -2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = -\frac{1}{2}\hat{i} + \hat{j} + \hat{k}$ respectively.

Let, θ be the angle between $\vec{a}$ and $\vec{b}$,

$$\cos\theta = \frac{\vec{a} \times \vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{\left(-2\hat{i} - 2\hat{j} + \hat{k}\right) \times \left(-\frac{1}{2}\hat{i} + \hat{j} + \hat{k}\right)}{\sqrt{(-2)^2 + (-2)^2 + (1)^2}\sqrt{\left(-\frac{1}{2}\right)^2 + (1)^2 + (1)^2}}$$

$$= \frac{(-2)\left(-\frac{1}{2}\right) + (-2)(1) + (1)(1)}{\sqrt{4+4+1}\sqrt{\frac{1}{4} + 1 + 1}}$$

$$= \frac{1 - 2 + 1}{\sqrt{9}\sqrt{\frac{9}{4}}}$$

$$\cos\theta = \frac{0}{3 \times \frac{3}{2}}$$

$$\cos\theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$\theta = \frac{\pi}{2}$$

Angle between the lines $= \frac{\pi}{2}$

Direction Cosines and Direction Ratios Ex 27.1 Q16(iii)

Here,

$l + 2m + 3n = 0 \qquad ---(\text{i})$

$3lm - 4ln + mn = 0 \qquad ---(\text{ii})$

From equation (i),

$l + 2m + 3n = 0$

$l = -2m - 3n$

Put the value of l in equation (ii),

$3lm - 4ln + mn = 0$

$3(-2m - 3n)m - 4(-2m - 3n)n + mn = 0$

$-6m^2 - 9nm + 8mn + 12n^2 + mn = 0$

$-6m^2 + 12n^2 = 0$

$-6m^2 = -12n^2$

$m^2 = 2n^2$

$m = \pm\sqrt{2n^2}$

$m = n\sqrt{2}$ or $m = -n\sqrt{2}$

Put $m = n\sqrt{2}$ in equation (i)

$l + 2m + 3n = 0$

$l + 2(n\sqrt{2}) + 3n = 0$

$l + n(2\sqrt{2} + 3) = 0$

$l = -(2\sqrt{2} + 3)n$

Again, m $= -\sqrt{2}n$ in equation (i)

$l + 2m + 3n = 0$

$l + 2(-\sqrt{2}n) + 3n = 0$

$l - 2\sqrt{2}n + 3n = 0$

$l + n(-2\sqrt{2} + 3) = 0$

$l = (2\sqrt{2} - 3)n$

Thus, direction cosines of the lines are given by,

$$-\left(2\sqrt{2}+3\right)n,\ \sqrt{2}n,\ n \quad or \quad \left(2\sqrt{2}-3\right)n,\ -\sqrt{2}n,\ n$$
$$-\left(2\sqrt{2}+3\right),\ \sqrt{2},\ 1 \quad or \quad \left(2\sqrt{2}-3\right),\ -\sqrt{2},\ 1$$

So, vectors parallel to these lines are

$$\vec{a} = -\left(2\sqrt{2}+3\right)\hat{i} + \sqrt{2}\hat{j} + \hat{k} \text{ and } \vec{b} = \left(2\sqrt{2}-3\right)\hat{i} - \sqrt{2}\hat{j} + \hat{k}$$

Let, θ be the angle between the lines,
then,

$$\cos\theta = \frac{\vec{a}\times\vec{b}}{\left|\vec{a}\right|\times\left|\vec{b}\right|}$$
$$= \frac{-\left(2\sqrt{2}+3\right)\times\left(2\sqrt{2}-3\right)+\left(\sqrt{2}\right)\mathbf{\times}\left(-\sqrt{2}\right)+(1)(1)}{\sqrt{\left(2\sqrt{2}+3\right)^2+\left(\sqrt{2}\right)^2+(1)^2}\sqrt{\left(2\sqrt{2}-3\right)^2+\left(-\sqrt{2}\right)^2+(1)^2}}$$

$$= \frac{-(8-9)-2+1}{\sqrt{8+9+12\sqrt{2}+2+1}\sqrt{8+9-12\sqrt{2}+2+1}}$$

$$= \frac{-(-1)-2+1}{\sqrt{20+12\sqrt{2}}\sqrt{20-12\sqrt{2}}}$$
$$= \frac{1-2+1}{\sqrt{20+12\sqrt{2}}\sqrt{20-12\sqrt{2}}}$$

$\cos\theta = 0$
$\theta = \cos^{-1}(0)$

$\theta = \frac{\pi}{2}$

Angle between the lines $= \frac{\pi}{2}$

Direction Cosines and Direction Ratios Ex 27.1 Q16(iv)

The given equations are,
$2l + 2m - n = 0$.........(i)
$mn + ln + lm = 0$.........(ii)

From(i), we get $n = 2l + 2m$.
Putting $n = 2l + 2m$ in (ii),we get
$m(2l + 2m) + l(2l + 2m) + lm = 0$
$\Rightarrow 2lm + 2m^2 + 2l^2 + 2ml + lm = 0$
$\Rightarrow 2m^2 + 5lm + \ + 2l^2 = 0$
$\Rightarrow 2m^2 + 4lm + lm + 2l^2 = 0$
$\Rightarrow (2m + l)(m + 2l) = 0$
$\Rightarrow m = -\frac{l}{2}$ or $m = -2l$

By putting $m = -\frac{l}{2}$ in (i) we get $n = l$
By putting $m = -2l$ in (i) we get $n = -2l$

So direction ratios of two lines are proportional to
$l,\ -\frac{l}{2},\ l$ and $l,\ -2l,\ -2l$ or, $1,\ -\frac{1}{2},1$ and $1,\ -2,\ -2$

So, vectors parallel to these lines are

$\vec{a} = \hat{i} - \frac{1}{2}\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} - 2\hat{k}$

If θ is the angle between the lines, then θ is also the angle between $\vec{a}$ and $\vec{b}$.

$$\cos\theta = \frac{\vec{a} \bullet \vec{b}}{|\vec{a}||\vec{b}|} = \frac{1 + 1 - 2}{\sqrt{1 + \frac{1}{4} + 1}\sqrt{1 + 4 + 9}} = 0$$

$$\Rightarrow \theta = \cos^{-1}(0) = \frac{\pi}{2}$$