Ex - 30.1

Linear Programming Ex 30.1 Q1

The given data may be put in the following tabular form: –

Gadget	Foundry	Machine-shop	Profit
A	**10**	**5**	**Rs 30**
B	**6**	**4**	**Rs 20**
Firm's capacity per week	1000	600	

Let required weekly production of gadgets A and B be x and y respectively.

Given that, profit on each gadget A is Rs 30
So, profit on x gadget of type $A = 30x$
Profit on each gadget of type B = Rs 20
So, profit on y gadget of type $B = 20y$

Let Z denote the total profit, so

$$Z = 30x + 20y$$

Given, production of one gadget A requires 10 hours per week for foundry and gadget B requires 6 hours per week for foundry.

So, x units of gadget A requires $10x$ hours per week and y units of gadget B requires by hours per week, But the maximum capacity of foundry per week is 1000 hours, so

$$10x + 6y \le 1000$$

This is first constraint.

Given, production of one unit gadget A requires 5 hours per week of machine shop and production of one unit of gadget B requires 4 hours per week of machine shop.

So, x units of gadget A requires $5x$ hours per week and y units of gadget B requires $4y$ hours per week, but the maximum capacity of machine shop is 600 hours per week
So, $5x + 4y \le 600$

This is second constraint.
Hence, mathematical formulation of LPP is:
Find x and y which
Maximize $Z = 30x + 2y$

Subject to constraints,

$$10x + 6y \le 1000$$
$$5x + 4y \le 600$$

And, $x, y \ge 0$ [Since production cannot be less than zero]

Linear Programming Ex 30.1 Q2

The given information can be written in tabular form as below:

Product	Machine hours	Labour hours	Profit
A	**1**	**1**	**Rs 60**
B	**–**	**1**	**Rs 80**
Total capacity	400 for A	500	
Minimum supply of product B is 200 units.			

Let production of product A be x units and production of product B be y units.

Given, profit on one unit of product A = Rs 60
So, profit on x unit of product A = Rs $60x$
Given, profit on one unit of product B = Rs 80
So, profit on y units of product B = Rs $80y$

Let Z denote the total profit, so

$$Z = 60x + 80y$$

Given, minimum supply of product B is 200
So, $y \ge 200$ (First constraint)

Given that, production of one unit of product A requires 1 hour of machince hours, so x units of product A requires x hours but given total machine hours available for product A is 400 hours, so

$x \le 400$ (Second constraint)

Given, each unit of product A and B requires one hour of labour hour, so x units of product A require x hours and y units of product B require y hours of labour hours but total labour hours available are 500, so

$x + y \le 500$ (Third constraint)

Hence, mathematical formulation of LPP is,
Find x and y which

Minimize $Z = 60x + 80y$

Subject to constraints,

$y \ge 200$
$x \le 400$
$x + y \le 500$
$x, y \ge 0$ [Since production of product cannot be less than zero]

Linear Programming Ex 30.1 Q3

Product	Machine (M_1)	Machine (M_2)	Profit
A	**4**	**2**	**3**
B	**3**	**2**	**2**
C	**5**	**4**	**4**
Capacity maximum	2000	2500	

Let required production of product A, B and C be x, y and z units respectively.

Given, profit on one unit of product A, B and C are Rs 3, Rs 2, Rs 4, so
Profit on x unit of A, y unit of B and z unit of C are given by Rs. $3x$, Rs $2y$, Rs $4z$.

Let U be the total profit, so

$$U = 3x + 2y + 4z$$

Given, one unit of product A, B and C requires 4,3 and 5 minutes on machine M_1. So, x units of product A, y units of B and z units of product C need $4x$, $3y$ and $5z$ minutes on machine M_1 is 2000 minutes, so

$$4x + 3y + 5z \le 200 \quad \text{(First constraint)}$$

Given, one unit of product A, B and C requires 2,2 and 4 minutes on machine M_2. So, x units of A, y units of B and z units of C require $2x$, $2y$ and $4z$ minutes on machine M_2 is 2500 minutes, so

$$2x + 2y + 4z \le 2500 \quad \text{(Second constraint)}$$

Also, given that firm must manufacture 100 A's, 200 B's and 50 C's but not more than 150 A's.

$$100 \le x \le 150$$
$$y \ge 200 \qquad \text{(Other constraints)}$$
$$z \ge 50$$

Hence, mathematical formulation of LPP is :–
Find x, y and z which

$$\text{maximize } U = 3x + 2y + 4z$$

Subject to constraints,

$$4x + 3y + 5z \le 2000$$
$$2x + 2y + 4z \le 2500$$
$$100 \le x \le 150$$
$$y \ge 200$$
$$z \ge 50$$

And, $x, y, z \ge 0$ [Since, x, y, z are non-negative]

Linear Programming Ex 30.1 Q4

Given information can be written in tabular form as below:

Product	M_1	M_2	Profit
A	**1**	**2**	**2**
B	**1**	**1**	**3**
Capacity	6 hours 40 min $= 400$ min.	10 hours $= 600$ min.	

Let required production of product A be x units and product B be y units.

Given, profit on one unit of product A and B are Rs 2 and Rs 3 respectively, so profits on x units of product A and y units of product B will be Rs $2x$ and Rs $3y$ respectively.

Let total profit be Z, so

$$Z = 2x + 3y$$

Given, production of one unit of product A and B require 1 and 1 minute on machine M_1 respectively, so production of x units of product A and y units of product B require x minutes and y minutes on machine M_1 but total time available on machine M_1 is 600 minutes, so

$$x + y \le 400 \quad \text{(First constraint)}$$

Given, production of one unit of product A and B require 2 minutes and 1 minutes on machine M_2 respectively. So production of x units of product A and y units of product B require $2x$ minutes and y minutes respectively on machine M_2 but machine M_2 is available for 600 minutes, so

$$2x + y \le 600 \quad \text{(Second constraint)}$$

Hence, mathematical formulation of LPP is:-

Find x and y which

maximize $Z = 2x + 3y$

Subject to constraints,

$$x + y \le 400$$
$$2x + y \le 600$$

and, $x, y \ge 0$ $\quad$ [Since production of product can not be less than zero]

Linear Programming Ex 30.1 Q5

Plant	A	B	C	Cost
I	**50**	**100**	**100**	**2500**
II	**60**	**60**	**200**	**3500**
Monthly demand	2500	3000	7000	

Let plant I requires x days and plant II requires y days per month to minimize cost.

Given, plant I and II costs Rs 2500 perday and Rs 3500 perday respectively, so cost to run plant I and II is Rs $2500x$ and Rs $3500y$ per month.

Let Z be the total cost per month, so

$$Z = 2500x + 3500y$$

Given, production of tyre A from plant I and II is 50 and 60 respectively, so production of tyre A from plant I and II will be $50x$ and $60y$ respectively per month but the maximum demand of tyre A is 2500 per month so,

$$50x + 60y \geq 2500 \quad \text{[First constraint]}$$

Given, production of tyre B from plant I and II is 100 and 60 respectively, so production of tyre B from plant I and II will be $100x$ and $60y$ per month respectively but the maximum demand of tyre B is 3000 per month, so

$$100x + 60y \geq 3000 \quad \text{[Second constraint]}$$

Given, production of tyre C from plant I and II is 100 and 200 respectively. So production of tyre B from plant I and II will be $100x$ and $200y$ per month respectively but the maximum demand of tyre C is 7000 per day, so

$$100x + 200y \geq 7000 \quad \text{[Third constraint]}$$

Hence, mathematical formulation of LPP is..

Find x and y which

Minimize $Z = 2500x + 3500y$

Subject to constraint,

$$50x + 60y \geq 2500$$
$$100x + 60y \geq 3000$$
$$100x + 200y \geq 7000$$

And, $x, y \geq 0$ [Since number of days can not be less than zero]

Linear Programming Ex 30.1 Q6

Product	Man hours	Maximum demand	Profit
A	**5**	**7000**	**60**
B	**3**	**10000**	**40**
Total capacity	45000		

Let required production of product A be x units and production of product B be y units.

Given, profits on one unit of product A and B are Rs 60 and Rs 40 respectively, so profits on x units of product A and y units of product B are Rs $60x$ and Rs $40y$.

Let Z be the total profit, so

$$Z = 60x + 40y$$

Given, production of one unit of product A and B require 5 hours and 3 hours respectively man hours, so x unit of product A and y units of product B require $5x$ hours and $3y$ hours of man hours respectively but total man hours available are 45000 hours, so

$$5x + 3y \le 45000 \qquad \text{(First constraint)}$$

Given, demand for product A is maximum 7000, so

$$x \le 7000 \qquad \text{(Second constraint)}$$

Hence, mathematical formulation of LPP is,
Find x and y which

$$\text{maximize } Z = 60x + 40y$$

Subject to constraints,

$$5x + 3y \le 45000$$
$$x \le 7000$$
$$y \le 10000$$

$$x, y \ge 0 \qquad \left[\text{Since production can not be less than zero}\right]$$

Linear Programming Ex 30.1 Q7

Let x and y be the packets of 25 gm of Food I and Food II purchased. Let Z be the price paid. Obviously price has to be minimized.

Take a mass balance on the nutrients from Food I and II,

Calcium $10x + 4y \geq 20$

$5x + 2y \geq 10$ ....(i)

Protein $5x + 5y \geq 20$

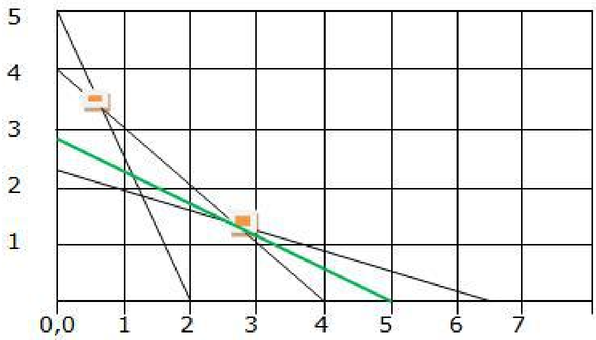
$x + y \geq 4$ ....(ii)

Calories $2x + 6y \geq 13$ ......(iii)

These become the constraints for the cost function, Z to be minimized ie., $0.6x + y = Z$, given cost of Food I is Rs 0.6/- and Rs 1/- per lb

From (i), (ii) & (iii) we get points on the X & Y-axis as [0, 5] & [2, 0] ; [0, 4] & [4, 0] ; [0, 13/6] & [6.5, 0]

Plotting these



The smallest value of Z is 2.9 at the point (2.75, 1.25). We cannot say that the minimum value of Z is 2.9 as the feasible region is unbounded.

Therefore, we have to draw the graph of the inequality $0.6x + y < 2.9$

Plotting this to see if the resulting line (in green) has any point common with the feasible region. Since there are no common points this is the minimum value of the function Z and the mix is

Food I = 2.75 lb; Food II = 1.25 lb; Price = Rs 2.9

When Z has an optimal value (maximum or minimum), where the variables *x* and *y* are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.

Here the feasible region is the unbounded region A-B-C-D

Computing the value of Z at the corner points of the feasible region ABHG

Point	Corner point	Value of Z = 0.6x + y
A	2, 5	6.2
B	0.67, 3.33	3.73
C	2.75, 1.25	2.9
D	6.5, 2.16	6.06

Linear Programming Ex 30.1 Q8

Given information can be tabulated as:-

Product	Grinding	Turning	Assembling	Testing	Profit
A B	1 2	3 1	6 3	5 4	2 3
Maximum capacity	30 hours	60 hours	200 hours	200 hours	

Let required production of product A and B be x and y respectively

Given, profits on one unit of product A and B are Rs 2 and Rs 3 respectively, so profits on x units of product A and y units of product B are given by $2x$ and $3y$ respectively. Let Z be total profit, so

$$Z = 2x + 3y$$

Given, production of 1 unit of product A and B require 1 hour and 2 hours of grinding respectively, so, production of x units of product A and y units of of product B require x hours and $2y$ hours of grinding respectively but maximum time available for grinding is 3 hours, so

$$x + 2y \le 30 \quad \text{(First constraint)}$$

Given, production of 1 unit of product A and B require 3 hours and 1 hours of turning respectively, so x units of product A and y units of of product B require $3x$ hours and y hours of turning respectively but total time available for turning is 60 hours, so

$$3x + y \le 60 \quad \text{(Second constraint)}$$

Given, production of 1 unit of product A and B require 6 hour and 3 hours of assembling respectively, so productinon of x units of product A and y units of product B require $6x$ hours and $3y$ hours of assembling respectively but total time available for assembling is 200 hours, so

$$6x + 3y \le 200 \quad \text{(Third constraint)}$$

Given, production of 1 unit of product A and B require 5 hours and 4 hours of testing respectively, so productinon of x units of product A and y units of product B require $5x$ hours and $4y$ hours of testing respectively but total time available for testing is 200 hours, so

$$5x + 4y \le 200 \quad \text{(Fourth constraint)}$$

Hence, mathematical formulation of LPP is,
Find x and y which

$$\text{maximize } Z = 2x + 3y$$

Subject to constraints,

$$x + 2y \le 30$$
$$3x + y \le 60$$
$$6x + 3y \le 200$$
$$5x + 4y \le 200$$

and, $x, y \ge 0$ [Since production can not be negative]

Linear Programming Ex 30.1 Q9

Given information can be tabulated as below:

Foods	Vitamin A	Vitamin B	Cost
F_1	**2**	**3**	**5**
F_2	**4**	**2**	**2.5**
Minimum daily requirement	40	50	

Let required quantity of food F_1 be x units and quantity of food F_2 be y units.

Given, costs of one unit of food F_1 and F_2 are Rs 5 and Rs 2.5 respectively, so costs of x units of food F_1 and y units of food F_2 are Rs $5x$ and Rs $2.5y$ respectively.
Let Z be the total cost, so

$$Z = 5x + 2.5y$$

Given, one unit of food F_1 and food F_2 contain 2 and 4 units of vitamin A respectively, so x unit of Food F_1 and y units of food F_2 contain $2x$ and $4y$ units of vitamin A respectively, but minimum requirement of vitamin A is 40 unit, so

$$2x + 4y \geq 40 \qquad \text{(First constraint)}$$

Given, one unit of food F_1 and food F_2 contain 3 and 2 units of vitamin B respectively, so x unit of Food F_1 and y units of food F_2 contain $3x$ and $2y$ units of vitamin B respectively, but minimum daily requirement of vitamin B is 40 unit, so

$$3x + 2y \geq 50 \qquad \text{(Second constraint)}$$

Hence, mathematical formulation of LPP is,
Find x and y which

Minimize $Z = 5x + 2.5y$

Subject to constraint,

$$2x + 4y \geq 40$$
$$3x + 2y \geq 50$$

$x, y \geq 0$ [Since requirement of food F_1 and F_2 can not be less than zero.]

Linear Programming Ex 30.1 Q10

Let the number of automobiles produced be x and let the number of trucks produced be y.

Let Z be the profit function to be maximized.
Z = 2,000x + 30,000y

The constraints are on the man hours worked

Shop A	$2x + 5y \le 180$	(i)	assembly
Shop B	$3x + 3y \le 135$	(ii)	finishing
	$x \ge 0$; $y \ge 0$		

Corner points can be obtained from
$2x + 5y = 180 \Rightarrow x=0;\ y=36$ and $x=90;\ y=0$
$3x + 3y \le 135 \Rightarrow x=0;\ y=45$ and $x=45;\ y=0$
Solving (i) & (ii) gives x = 15 & y = 30

Corner point	Value of Z =2,000x + 30,000y
0,0	0
0, 36	10,80,000
15, 30	9,30,000
45, 0	90,000

0 automobiles and 36 trucks will give max profit of 10,80,000/-

Linear Programming Ex 30.1 Q11

	Taylor A		Taylor B	Limit
Variable	x		y	
Shirts	6x	+	10y	≥ 60
Pants	4x	+	4y	≥ 32
Earn Rs.	150	+	200	Z

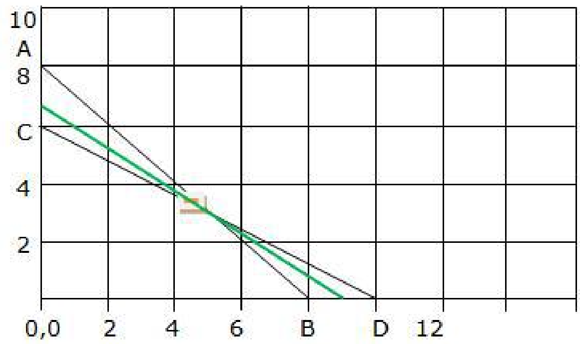
The above LPP can be presented in a table above.

To minimize labour cost means to assume minimize the earnings i.e. Min Z = 150x + 200y
s.t. the constraints

$x \ge 0;\ y \ge 0$	at least 1 shirt & pant is required
$6x + 10y \ge 60$	require at least 60 shirts
$4x + 4y \ge 32$	require at least 32 pants

Solving the above inequalities as equations we get,
x = 5 and y = 3

other corner points obtained are [0, 6] & [10, 0]
[0, 8] & [8,0]



The feasible region is the open unbounded region A-E-D

Point E(5, 3) may not be the minimal value. So, plot 150x + 200y < 1350 to see if there is a common region with A-E-D

The green line has no common point , therefore

Corner point	Value of Z = 150x + 200y
0,8	0
10, 0	1500
5, 3	1350

Stitching 5 shirts and 3 pants minimizes labour cost to Rs.1350/-

	Model 314		Model 535	Limit
Variable	x		y	
F class	20x	+	20y	≥ 160
T class	30x	+	60y	≥ 300
Cost	1.x lakh	+	1.5y lakh	Z

The above LPP can be presented in a table above.

The flight cost is to be minimized i.e, Min $Z = x + 1.5y$
s.t. the constraints

$x \geq 2$	at least 2 planes of model 314 must be used
$y \geq 0$	at least 1 plane of model 535 must be used
$20x + 20y \geq 160$	require at least 160 F class seats
$30x + 60y \geq 300$	require at least 300 T class seats

Solving the above inequalities as equations we get,
When x=0, y=8 and when y=0, x=8
When x=0, y=5 and when y=0, x=10

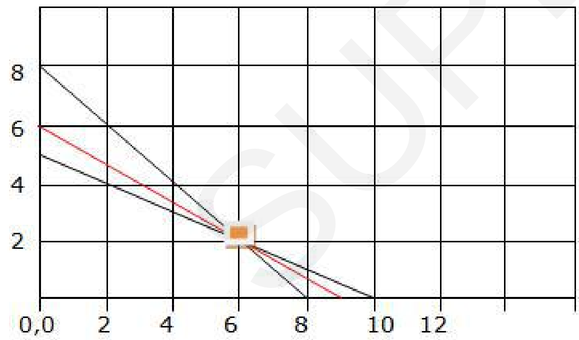
We get an unbounded region 8-E-10 as a feasible solution. Plotting the corner points and evaluating we have,

Corner point	Value of $Z = x + 1.5y$
10, 0	10
0, 8	12
6, 2	9

Since we obtained an unbounded region as the feasible solution a plot of Z (x+1.5 < 9) is plotted.

Since there are no common points point E is the point that gives a minimum value.

Using 6 planes of model 314 & 2 of model 535 gives minimum cost of 9 lakh rupees.



Linear Programming Ex 30.1 Q13

Given information can be tabulated as below

Sets	Time requirement	Points
I	**3**	**5**
II	**2**	
III	**4**	**6**

Time for all three sets $= 3\frac{1}{2}$ hours

Time for Set I and Set II $= 2\frac{1}{2}$ hours

Number of questions maximum 100

Let he should x, y, z questions from set I, II and III respectively.

Given, each question from set I, II, III earn $5, 4, 6$ points respectively, so x questions of set I, y questions of set II and z questions of set III earn $5x$, $4y$ and $6z$ points, let total point credit be U

So, $U = 5x + 4y + 6z$

Given, each question of set I, II and III require 3,2 and 4 minutes respectively, so x questions of set I, y questions of set II and z questions of set III require $3x$, $2y$ and $4z$ mimutes respectively but given that total time to devote in all three sets is $3\frac{1}{2}$ hours $= 210$ minutes and first two sets is $2\frac{1}{2}$ hours $= 150$ minutes

So,

$$3x + 2y + 4z \le 210 \quad \text{(First constraint)}$$

$$3x + 2y \le 150 \quad \text{(Second constraint)}$$

Given, total number of questions cannot exceed 100

So, $x + y + z \le 100$ (Third constraint)

Hence, mathematical formulation of LPP is

Find x and y which

maximize $U = 5x + 4y + 6z$

Subject to constraint,

$$3x + 2y + 4z \le 210$$

$$3x + 2y \le 150$$

$$x + y + z \le 100$$

$$x, y, z \ge 0$$

[Since number of questions to solve from each set cannot be less than zero]

Linear Programming Ex 30.1 Q14

Given information can be tabulated as below

Product	Yield	Cultivation	Price	Fertilizers
Tomatoes	2000 kg	5 days	1	100 kg
Lettuce	3000 kg	6 days	0.75	100 kg
Radishes	1000 kg	5 days	2	50 kg
Average 2000 kg/per acre Total land $=100$ Acre Cost g fertilizers $=$ Rs 0.50 per kg. A total of 400 days of cultivation labour with Rs 20 per day				

Let required quantity of field for tomatoes, lettuce and radishes be x, y and z Acre respectively.

Given, costs of cultivation and harveshing of tomatoes, lettuce and radishes are 5×20 = Rs 100, 6×20 = Rs 120, 5×20 = Rs 100 respectively per acre. Cost of fertilizers for tomatoes, lettuce and radishes 100×0.50 = Rs 50, 100×0.50 = Rs 50 and 50×0.50 = Rs 25 respectively per acre.

So, total costs of production of tomatoes, lettuce and radishes are Rs $100 + 50$ = Rs $150x$, Rs $120 + 50$ = Rs $170y$ and radishes are Rs $100 + 25$ = Rs $125z$ respectively total selling price of tomatoes, lettuce and radishes, according to yield are 2000×1 = Rs $2000x$, 3000×0.75 = Rs $2250y$ and 100×2 = Rs $2000z$ respectively.

Let U be the total profit,
So,

$$U = (2000x - 150x) + (2250y - 170y) + (2000z - 125z)$$
$$U = 1850x + 2080y + 1875z$$

Given, farmer has 100 acre form
So, $x + y + z \le 100$ (First constraint)

Number of cultivation and harvesting days are 400
So, $5x + 6y + 5z \le 400$

Hence, mathematical formulation of LPP is
Find x, y, z which
maximize $U = 1850x + 2080y + 1875z$

Subject to constraint,

$$x + y + z \le 100$$
$$5x + 6y + 5z \le 400$$

$$x, y, z \ge 0$$

[Since from used for cultivation cannot be less than zero.]

Linear Programming Ex 30.1 Q15

Given information can be tabulated as below:

Product	Department 1	Department 2	Selling price	Labour cost	Raw material cost
A	**3**	**4**	**25**	**16**	**4**
B	**2**	**6**	**30**	**20**	**4**
Capacity	130	260			

Let the required product of product A and B be x and y units respectively.

Given, labour cost and raw material cost of one unit of product A is Rs 16 and Rs 4, so total cost of product A is Rs 16 + Rs 4 = Rs 20
And given selling price of 1 unit of product A is Rs 25,
So, profit on one unit of product

$A = 25 - 20 = \text{Rs } 5$

Again, given labour cost and raw material cost of one unit of product B is Rs 20 and Rs 4
So, that cost of product B is Rs 20 + Rs 4 = Rs 24
And given selling price of 1 unit of product B is Rs 30
So, profit on one unit of product $B = 30 - 24 = \text{Rs } 6$

Hence, profits on x unit of product A and y units of product B are Rs $5x$ and Rs $6y$ respectively.
Let Z be the total profit , so $Z = 5x + 6y$

Given, production of one unit of product A and B need to process for 3 and 4 hours respectively in department 1, so production of x units of product A and y units of product B need to process for $3x$ and $4y$ hours respectively in Department 1. But total capacity of Department 1 is 130 hour ,
So, $3x + 2y \le 130$ (First constraint)

Given, production of one unit of product A and B need to process for 4 and 6 hours respectively in department 2, so production of x units of product A and y units of product B need to process for $4x$ and $6y$ hours respectively in Department 2 but total capacity of Department 2 is 260 hours
So, $4x + 6y \le 260$ (Second constraint)

Hence, mathematical formulation of LPP is,
Find x and y which
Maximize $Z = 5x + 6y$

Subject to constraint,

$3x + 2y \le 130$
$4x + 6y \le 260$

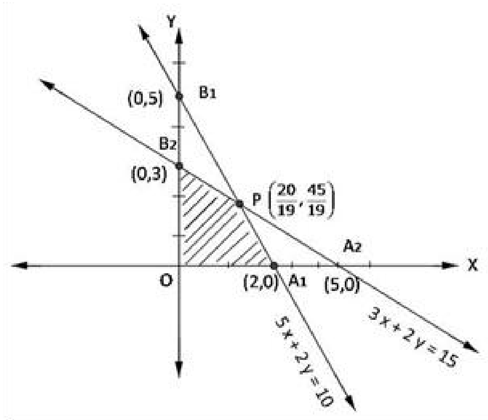
$x, y \ge 0$ [Since production cannot be less than zero]

Ex 30.2

Linear Programming Ex 30.2 Q1

Converting the given inequations into equations, we get

$$3x + 5y = 15,\ 5x + 2y = 10,\ x = 0,\ y = 0$$



Region represented by $5x + 2y \le 10$: The line meets coordinate axes at $A_1(2,0)$ and $B_1(0,5)$ respectively. Join these points to obtain the line $5x + 2y = 10$, clearly, $(0,0)$ satisfies the in eqation $5x + 2y \le 10$, so, the region in xy-plane that contains the origin represents the solution set if the given in equation.

Region represented by $3x + 5y \le 10$: The line meets coordinate axes at $A_2(5,0)$ and $B_2(0,3)$ respectively. Join these points to obtain the line $3x + 5y = 15$, clearly, $(0,0)$ satisfies the in eqation $3x + 5y \le 15$, so, the region in xy-plane contains the origin represents the solution set if the given in equation.

Region represented by $x \ge 0,\ y \ge 0$: It clearly represents first quadrant of xy-plane. Common region to regions represented by above in equalities.

The coordinates of the corner points of the shaded region are $0(0,0)$, $A(2,0)$, $P\left(\frac{20}{19}, \frac{45}{19}\right)$, $B_2(0,3)$.

The value of $Z = 5x + 3y$ at

$0(0,0) \quad = 5 \times +3 \times 0$

$A(2,0) \quad = 5 \times 2 + 3 \times 0 = 10$

$P\left(\frac{20}{19}, \frac{45}{19}\right) \quad = 5\left(\frac{20}{19}\right) + 3\left(\frac{45}{19}\right) = \frac{235}{19}$

$B_2(0,3) \quad = 5 \times 0 + 3 \times 3 = 9$

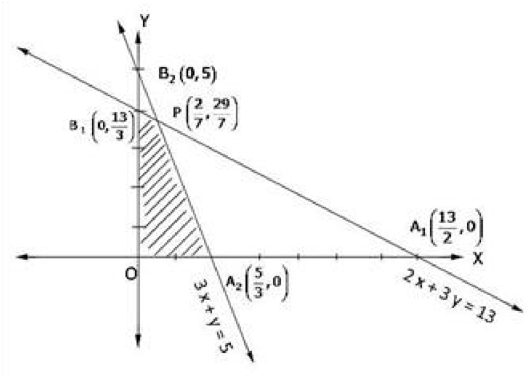
Clearly, Z is maximum at $P\left(\frac{20}{19}, \frac{45}{19}\right)$

So, $x = \frac{20}{19},\ y = \frac{45}{19},$ maximum $Z = \frac{235}{19}$

Linear Programming Ex 30.2 Q3

Converting the given inequations into equations, we get

$$2x + 3y = 13,\ 3x + y = 5,\ \text{and } x = 0,\ y = 0$$



Region represented by $2x + 3y \le 13$: The line meets coordinate axes at $A_1\left(\frac{13}{2}, 0\right)$ and $B_1\left(0, \frac{13}{3}\right)$ respectively. Join these points to obtain the line $2x + 3y = 13$, clearly, $(0,0)$ satisfies the in eqation $2x + 3y \le 13$, so, the region in xy-plane that contains origin represents the solution set of $2x + 3y \le 13$.

Region represented by $3x + y \le 5$: The line meets coordinate axes at $A_2\left(\frac{5}{3}, 0\right)$ and $B_2(0,5)$ respectively. Join these points to obtain the line $3x + y = 5$, clearly, $(0,0)$ satisfies the in eqation $3x + y \le 5$, so, the region in xy-plane that contains origin represents the solution set of $3x + y \le 5$.

Region represented by $x, y \ge 0$: It clearly represent first quadrant of xy-plane. The common region to regions represented by above in equalities.

The coordinates of the corner points of the shaded region are $O(0,0)$, $A\left(\frac{5}{3}, 0\right)$, $P\left(\frac{2}{7}, \frac{29}{7}\right)$, $B_2\left(0, \frac{13}{3}\right)$.

The value of $Z = 9x + 3y$ at

$O(0,0)$	$= 9(0) + 3(0) = 0$
$A_1\left(\frac{5}{3}, 0\right)$	$= 9\left(\frac{5}{3}\right) + 3(0) = 15$
$P\left(\frac{2}{7}, \frac{29}{7}\right)$	$= 9\left(\frac{2}{7}\right) + 3\left(\frac{29}{7}\right) = 15$
$B_2\left(0, \frac{13}{3}\right)$	$= 9(0) + 3\left(\frac{13}{3}\right) = 13$

Clearly, Z is maximum at at every point on the line joining A_1 and P, so

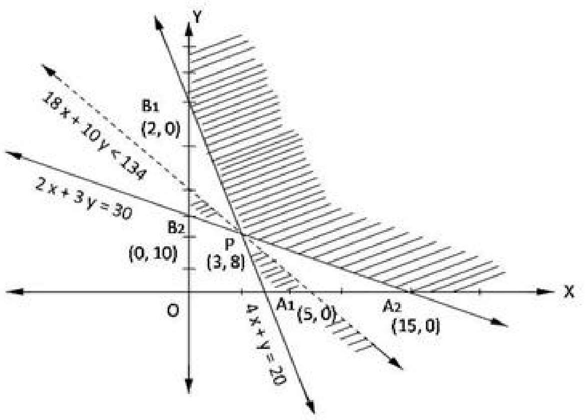
$x = \frac{5}{3}$ or $\frac{2}{7}$, $y = 0$ or $\frac{29}{7}$

and maximum $Z = 15$.

Linear Programming Ex 30.2 Q3

Converting given inequations into equations as

$$4x + y = 20,\ 2x + 3y = 30,\ x = 0,\ y = 0$$



Region represented by in equation $4x + y \geq 20$: The line $4x + y = 20$ meets the coordinate axes at $A_1(5,0)$ and $B_1(0,20)$. Joining A_1B_1 we get $4x + y = 20$. Clearly, $(0,0)$, also does not satisfies the in eqation, so the region does not containing the origin represents the in equality $4x + y \geq 20$ in the xy-plane.

Region represented by in equation $2x + 3y \geq 30$: The line $2x + 3y = 30$ meets the coordinate axes at $A_2(15,0)$ and $B_2(0,20)$. Obtain line $2x + 3y = 30$ by joining A_2 and B_2. Clearly, $(0,0)$, does not satisfies the in eqation $2x + 3y \geq 30$, so the region does not containing the origin represents the in equality $2x + 3y \geq 30$ in the xy-plane.

Region represented by $x, y \geq 0$: $x, y \geq 0$ represents the first quadrant of xy-plane.

The shaded region is the feasible region with corner points $A_2(15,0), P(3,8), B_1(0,20)$ where P is obtained by solving $2x + 3y = 30$ and $4x + y = 20$ simultaneously.

The value of $Z = 18x + 10y$ at

$A_2(15,0)$ $= 18(15) + 10(8) = 270$

$P(3,8)$ $= 18(3) + 10(8) = 134$

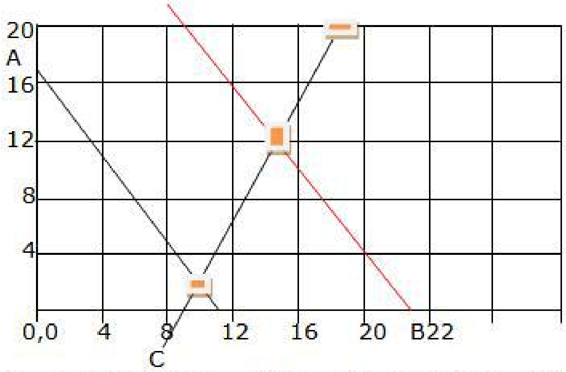
$B_1(0,20)$ $= 18(0) + 10(20) = 200$

Clearly, Z is manimum at $x = 3$ and $y = 8$. The minimum value of Z is 134.

We observe that open half plane represented by $18x + 10y < 134$ does not have points in common with the solution region. So Z has

Minimum value $= 134$ at $x = 3,\ y = 8$

Linear Programming Ex 30.2 Q4



2x-y ≥ 18 ; when x = 12, y = 6 & when y=0, x=9
3x+2y ≤ 34 ; when x = 0, y = 17 & when y=0, x=34/3

Plotting these points gives line AB and CD
The feasible area is the unbounded area D-E-12

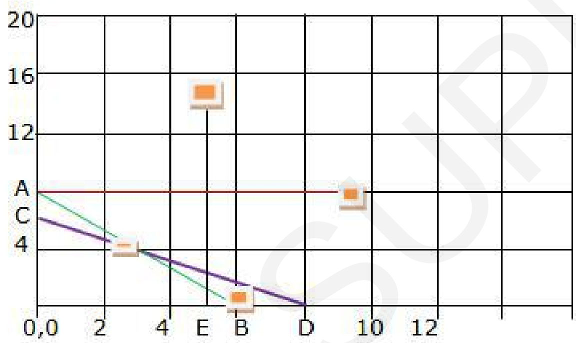
Corner point	Value of Z = 50x + 30y
10, 2	560
11.3, 17	1076.66

The maximize value of Z = 50x+30y, occurs at x = 34/3, y = 17

Since we have an unbounded region as the feasible area plot 50x + 30y > 1076.66

Since the region D-F-B has common points with region D-E-12 the problem has no optimal maximum value.

Linear Programming Ex 30.2 Q5



3x+4y ≤ 24 ; when x = 0, y = 6 & when y=0, x=8, line AB

8x+6y ≤ 48 ; when x = 0, y = 8 & when y=0, x=6, line CD

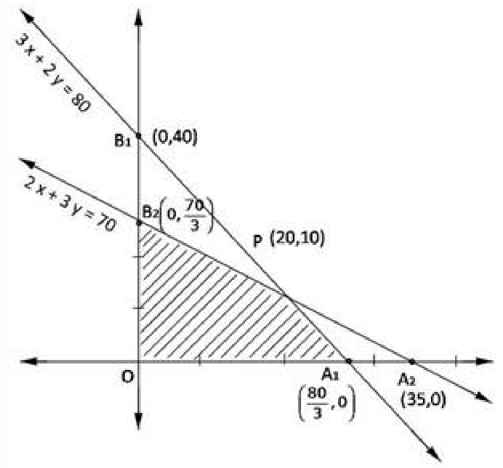
Plotting x≤5 gives line EF; Plotting y≤6 gives line AG
The feasible area is 0,0-C-H-G-E

Corner point	Value of Z = 4x + 3y
0, 0	0
0, 6	18
3.4, 3.4	24
5, 1	23
5,0	20

Linear Programming Ex 30.2 Q6

Converting the inequations into equations as

$3x + 2y = 80,\ 2x + 3y = 70,\ x = y = 0$



Region represented by $3x + 2y \le 80$: Line $3x + 2y = 80$ meets coordinate axes at $A_1\left(\frac{80}{3}, 0\right)$ and $B_1(0, 40)$, clearly, $(0,0)$ satisfies the $3x + 2y \le 80$, so, region containing the origin represents by $3x + 2y \le 80$ in xy-plane

Region represented by $2x + 3y \le 70$: Line $2x + 3y = 70$ meets the coordinate axes at $A_2(35,0)$ and $B_2\left(0, \frac{70}{3}\right)$, clearly, $(0,0)$ satisfies the $2x + 3y \le 70$ so, the region containing the origin represents by $2x + 3y \le 70$ in xy-plane

Region represented by $x, y \ge 0$: It represent the first quadrant in xy-plane

So, shaded area OA_1PB_2 represents the feasible region.

Coordinate of $P(20,10)$ can be obtained by solving $3x + 2y = 80$ and $2x + 3y = 70$

Now, the value of $Z = 15x + 10y$ at

$$
\begin{aligned}
&O(0,0) && = 15(0) + 10(0) = 0 \\
&A_1\left(\frac{80}{3}, 0\right) && = 15\left(\frac{80}{3}\right) + 10(0) = 400 \\
&P(20,10) && = 15(20) + 10(10) = 400 \\
&B_2\left(0, \frac{70}{3}\right) && = 15(0) + 10\left(\frac{70}{3}\right) = \frac{700}{3}
\end{aligned}
$$

So, maximum $Z = 400$ is on each and every point on the line joining A_1P, so we can have,

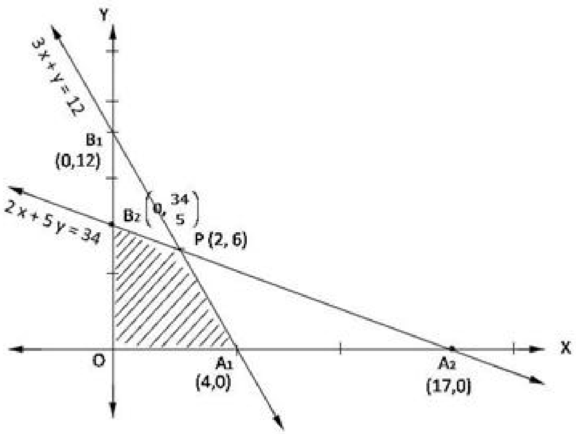
maximum $Z = 400$ at $x = \frac{80}{3}$ and $y = 0$

maximum $Z = 400$ at $x = 20$ and $y = 10$

Linear Programming Ex 30.2 Q7

Converting the given inequations into equations

$$3x + y = 12,\ 2x + 5y = 34,\ x = y = 0$$



Region represented by $3x + y \le 12$: Line $3x + y = 12$ meets the coordinate axes at $A_1(4,0)$ and $B_1(0,12)$, clearly, $(0,0)$ satisfies $3x + y \le 12$, so, region containing origin is represented by $3x + y \le 12$ in xy-plane

Region represented by $2x + 5y \le 34$: Line $2x + y = 34$ meets coordinate axes at $A_2(17,0)$ and $B_2\left(0,\frac{34}{5}\right)$, clearly, $(0,0)$ satisfies the $2x + 5y \le 34$ so, region containing origin represents $2x + 5y \le 34$ in xy-plane

Region represented by $x, y \ge 0$: It represent the first quadrant in xy-plane

Therefore, shaded area OA_1PB_2 is the feasible region.

The coordinate of $P(2,6)$ is obtained by solving $2x + 5y = 34$ and $3x + y = 12$

The value of $Z = 10x + 6y$ at

$O(0,0) \quad = 10(0) + 6(0) = 0$

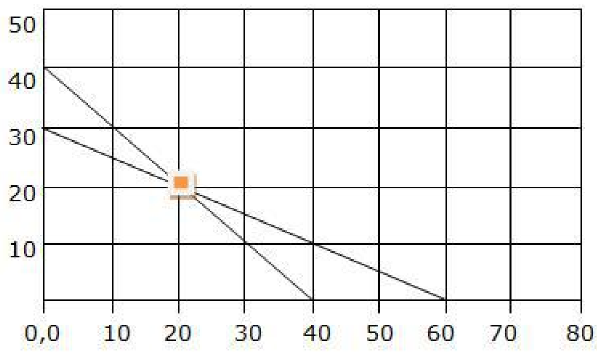
$A_1(4,0) \quad = 10(4) + 6(0) = 40$

$P(2,6) \quad = 10(2) + 6(6) = 56$

$B_2\left(0,\frac{34}{5}\right) \quad = 10(0) + 6\left(\frac{34}{5}\right) = \frac{204}{5} = 40\frac{4}{5}$

Hence, maximum $Z = 56$ at $x = 2,\ y = 6$

Linear Programming Ex 30.2 Q8



$2x+2y\leq 80$; when $x=0$, $y=40$ and when $y=0$, $x=40$
$2x+4y\leq 120$; when $x=0$, $y=30$ and when $y=0$, $x=60$

The intersection of the two plotted lines gives (20, 20)
Feasible area is 30-C-40

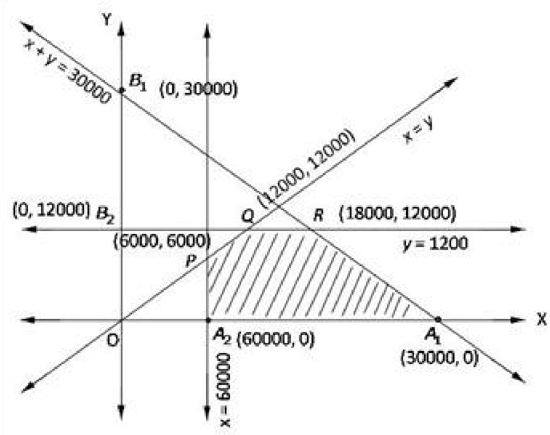
Corner point	Value of $Z = 3x + 4y$
0, 0	0
0, 30	120
20, 20	140
40, 0	120

The maxima is obtained at $x=20$, $y=20$ and is 140

Linear Programming Ex 30.2 Q9

Converting the given inequations into equations,

$x + y = 30000$, $y = 12000$, $x = 6000$, $x = y$, $x = y = 0$



Region represented by $x + y \le 30000$: Line $x + y = 30000$ meets the coordinate axes at $A_1(30000,0)$ and $B_1(0,30000)$, clearly $(0,0)$ satisfies $x + y \le 30000$, so, region containing the origin represents $x + y \le 30000$ in xy-plane

Region represented by $y \le 12000$: Line $y = 12000$ is parallel to x-axis and meets y-axis at $B_2(0,12000)$. Clearly $(0,0)$ satisfies $y \le 12000$, so, region containing origin represents $y \le 12000$ in xy-plane.

Region represented by $x \le 6000$: Line $x = 6000$ is parallel to y-axis and meets x-axis at $A_2(6000,0)$. Clearly $(0,0)$ satisfies $x \le 6000$, so, region containing origin represents $x \le 6000$ in xy-plane.

Region represented by $x \ge y$: Line $x = y$ passes through origin and point $Q(12000,12000)$. Clearly, $A_2(6000,0)$ satisfies $x \ge y$, so, region containing $A_2(6000,0)$ represents $x \ge y$ in xy-plane.

Region represented by $x, y \ge 0$: It represents the first quadrant in xy-plane.

Shaded region A_2A_1QP represents the feasible region.

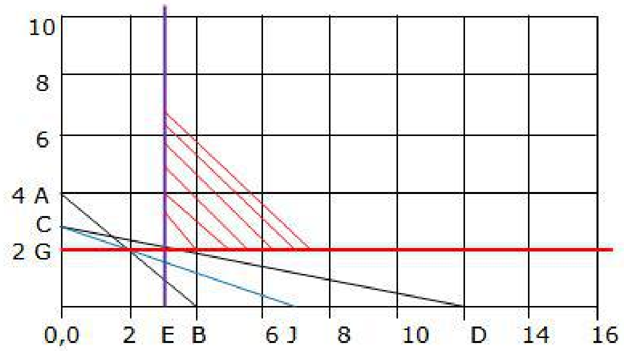
Coordinates of $R(18000,12000)$ is obtained by solving $x + y = 30000$ and $y = 12000$, $Q(12000,12000)$ is obtained by solving $x = y$ and $y = 12000$, $P(6000,6000)$ is obtained by solving $x = y$ and $x = 6000$.

The value of $Z = 7x + 10y$ at

$A_2(6000,0)$	$= 7(6000) + 10(0) = 42000$
$A_1(30000,0)$	$= 7(30000) + 10(0) = 210000$
$R(18000,12000)$	$= 7(18000) + 10(12000) = 246000$
$Q(12000,12000)$	$= 7(12000) + 10(12000) = 204000$
$P(6000,6000)$	$= 7(6000) + 10(6000) = 102000$

So, maximum $Z = 246000$ at $x = 18000$, $y = 12000$

Linear Programming Ex 30.2 Q10



2x+2y≥8 ;When x=0, y=4 & when y=0, x=4 line AB
x+4y≥12; When x=0, y=3 & when y=0, x=12 line CD
x≥3, y≥2 are the lines parallel to Y-axis and X-axis resp.

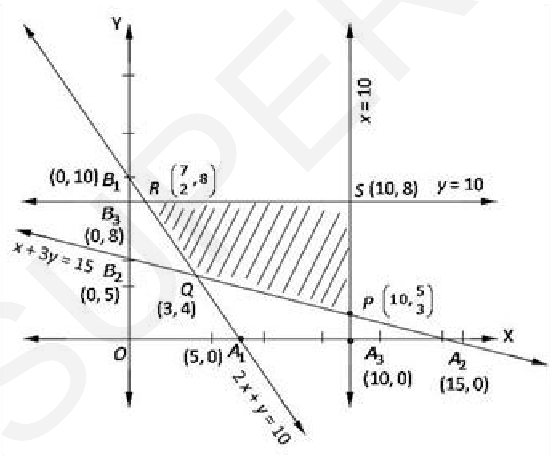
The diverging shaded area in red lines is the area of feasible solution. This area is unbounded.
Z = 2x+4y @ (3,2) = 14.

Plot 2x+4y > 14 line CJ to see if there is any common region. There is no common region so there is no optimal solution.

Linear Programming Ex 30.2 Q11

Converting the given inequations into equations,

$$2x + y = 10,\ x + 3y = 15,\ x = 10,\ y = 8,\ x = y = 0$$



Region represented by $2x + y \ge 10$: Line $2x + y = 10$ meets coordinate axes at $A_1(5,0)$ and $B_1(0,10)$. Clearly, $(0,0)$ does not satisfy $2x + y \ge 10$, so, region not containing origin represents $2x + y \ge 10$ in xy - plane.

Region represented by $x + 3y \ge 15$: Line $x + 3y = 15$ meets coordinate axes at $A_2(15,0)$ and $B_2(0,5)$. Clearly, $(0,0)$ does not satisfy $x + 3y \ge 15$, so, region not containing origin represents $x + 3y \ge 15$ in xy - plane.

Region represented by $x \le 10$: Line $x = 10$ is parallel to y-axis and meet x-axis at $A_3(10,0)$. Clearly $(0,0)$ satisfies $x \le 10$, so region containing origin represent $x \le 10$ in xy-plane.

Region represented by $y \le 8$: Line $y = 8$ is parallel to x-axis and meet y-axis at $B_3(0,8)$, clearly $(0,0)$ satisfies $y \le 8$, so region containing origin represent $y \le 8$ in xy-plane.

Region represented by $x, y \ge 0$: It represent the first quadrant in xy-plane.

Shaded region $QPSR$ is the feasible region. $Q(3,4)$ is obtained by solving $2x + y = 10$ and $x + 3y = 15$, $P\left(10,\frac{5}{3}\right)$ is obtained by solving $x + 3y = 15$ and $x = 10$, $R\left(\frac{7}{2},8\right)$ is obtained by $2x + y = 10$ and $y = 8$.

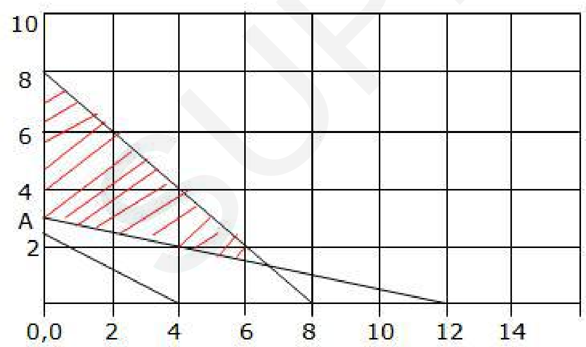
The value of $Z = 5x + 3y$ at

$$P\left(10,\frac{5}{3}\right) = 5(10) + 3\left(\frac{5}{3}\right) = 55$$
$$Q(3,4) = 5(3) + 3(4) = 27$$
$$R\left(\frac{7}{2},8\right) = 5\left(\frac{7}{2}\right) + 3(8) = \frac{83}{2} = 41\frac{1}{2}$$
$$S(10,8) = 5(10) + 3(8) = 74$$

So,

Minimum $Z = 27$ at $x = 3$, $y = 4$

Linear Programming Ex 30.2 Q12



x + y≤8 ; when x=0, y=8 & when y=0, x=8, line 8-8
x + 4y≥12; when x=0, y=3 & when y=0, x=12 line A-12
5x+8y=20; when x=0, y=5/2 & when y=0, x=4

The shaded area in red is the area of feasible solution.

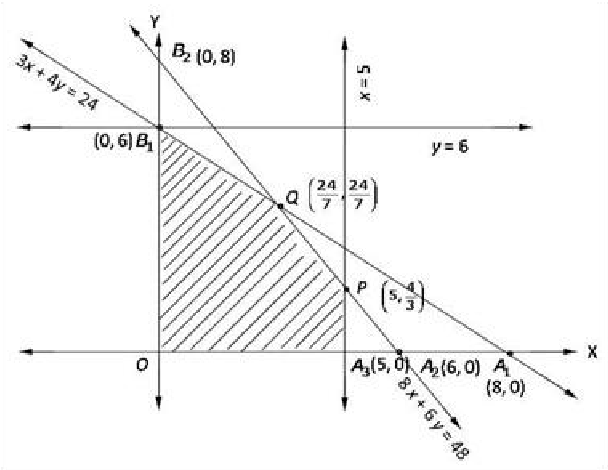
Corner point	Value of Z = 30x + 20y
0, 3	60
0, 8	160
6.66, 1.33	226.66

The maxima is obtained at x=6.66, y=1.33 and is 226.66

Linear Programming Ex 30.2 Q13

Converting the given inequations into equations,

$3x + 4y = 24,\ 8x + 6y = 48,\ x = 5,\ y = 6,\ x = y = 0$



Region represented by $3x + 4y \le 24$: Line $3x + 4y = 24$ meets coordinate axes at $A_1\,(8,0)$ and $B_1\,(0,6)$, clearly $(0,0)$ satisfies $3x + 4y \le 24$, so region containing origin represents $3x + 4y \le 24$ in xy -plane.

Region represented by $8x + 6y \le 48$: Line $8x + 6y = 48$ meets coordinate axes at $A_2\,(6,0)$ and $B_2(0,8)$. Clearly, $(0,0)$ satisfies $8x + 6y \le 48$, so region containing origin represents $8x + 6y \le 48$ in xy -plane.

Region represented $x \le 5$: Line $x = 5$ is parallel to y-axis and meets x-axis at $A_3\,(5,0)$. Clearly $(0,0)$ satisfies $x \le 5$, so region containing origin represent $x \le 5$ in xy-plane.

Region represented by $y \le 6$: Line $y = 6$ is parallel to x-axis and meets y-axis at $B_1(0,6)$. Clearly $(0,0)$ satisfies $y \le 6$, so, region containing origin represents $y \le 6$ in xy-plane.

Region represented by $x, y \ge 0$: It represents the first quadrant in xy-plane.

So, shaded region QA_3PQB represents feasible region.

Coordinate of $P\left(5,\frac{4}{3}\right)$ is obtained by solving $8x + 6y = 48$ and $x = 5$, coordinate of $Q\left(\frac{24}{7},\frac{24}{7}\right)$ is obtained by solving $3x + 4y = 24$ and $8x + 6y = 48$.

The value of $Z = 4x + 3y$ at

$0\,(0,0) \quad = 4(0) + 3(0) = 0$

$A_3\,(5,0) \quad = 4(5) + 3(0) = 20$

$P\left(5,\frac{4}{3}\right) \quad = 4(5) + 3\left(\frac{4}{3}\right) = 24$

$Q\left(\frac{24}{7},\frac{24}{7}\right) \quad = 4\left(\frac{24}{7}\right) + 3\left(\frac{24}{7}\right) = 24$

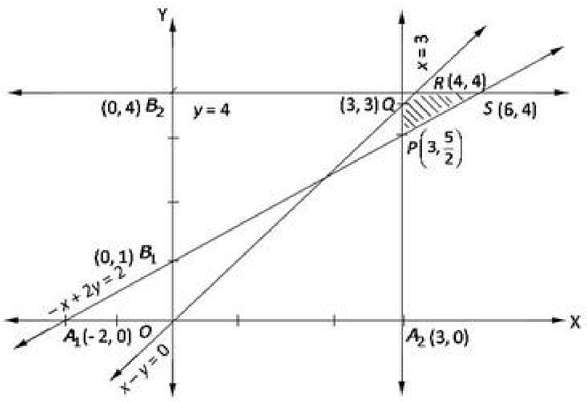
$B_1\,(0,6) \quad = 4(0) + 3(6) = 18$

So, maximum $Z = 24$ at $x = 5,\ y = \frac{4}{3}$ or $x = \frac{24}{7},\ y = \frac{24}{7}$ or at every point joining PQ.

Linear Programming Ex 30.2 Q14

Converting the given inequations into equations,

$$x - y = 0,\ -x + 2y = 2,\ x = 3,\ y = 4,\ x = y = 0$$



Region represented by $x - y \geq 0$: $x - y = 0$ is a line passing through origin and $R(4,4)$. Clearly, $(3,0)$ satisfies $x - y \geq 0$, so, region containing $(3,0)$ represents $x - y \geq 0$ in xy-plane.

Region represented by $-x + 2y \geq 2$: Line $-x + 2y = 2$ meets coordinate axes at $A_1(-2,0)$ and $B_1(0,1)$. Clearly, $(0,0)$ does not satisfy $-x + 2y \geq 2$, so, region not containing origin represents $-x + 2y \geq 2$ in xy-plane.

Region represented $x \geq 3$: Line $x = 3$ is parallel to y-axis and meets x-axis at $A_2(3,0)$. Clearly, $(0,0)$ does not satisfy $x \geq 3$, so region not containing origin represent $x \geq 3$ in xy-plane.

Region represented by $y \leq 4$: Line $y = 4$ is parallel to x-axis and meets y-axis at $B_2(0,4)$. Clearly $(0,0)$ satisfies $y \leq 4$, so region containing origin represents $y \leq 4$ in xy-plane.

Region represented by $x, y \geq 0$: It represent the first quadrant in xy-plane.

So, shaded region $PQRS$ represents feasible region.

The coordinate of $P\left(3, \frac{5}{2}\right)$ is obtained by solving $x = 3$ and $-x + 2y = 2$, $Q(3,3)$ by solving $x = 3$ and $x - y = 0$, $R(4,4)$ by solving $x = 4$ and $x - y = 0$, $S(6,4)$ by solving $y = 4$ and $-x + 2y = 2$

The value of $Z = x - 5y + 20$ at

$$P\left(3, \frac{5}{2}\right) = 3 - 5\left(\frac{5}{2}\right) + 20 = \frac{21}{2} = 11\frac{1}{2}$$

$$Q(3,3) = 3 - 5(3) + 20 = 8$$

$$R(4,4) = 4 - 5(4) + 20 = 4$$

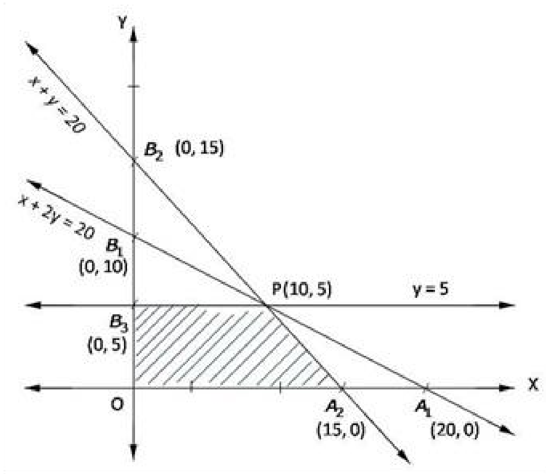
$$S(6,4) = 6 - 5(4) + 20 = 6$$

Hence,

Minimum $Z = 4$ at $x = 4$ and $y = 4$

Converting the given inequations into equations:-

$$x + 2y = 20,\ x + y = 15,\ y = 5,\ x = y = 0$$



Region represented by $x + 2y \le 20$: Line $x + 2y = 20$ meets coordinate axes at $A_1\,(20, 0)$ and $B_1\,(0, 10)$, clearly, $(0,0)$ satisfies $x + 2y \le 20$, so region containing origin represents $x + 2y \le 20$ in xy -plane.

Region represented by $x + y \le 15$: Line $x + y = 15$ meets coordinate axes at $A_2\,(15, 0)$ and $B_2\,(0, 15)$, clearly, $(0,0)$ satisfies $x + y \le 15$, so region containing origin represents $x + y \le 15$ in xy - plane.

Region represented by $y \le 5$: Line $y = 5$ is parallel to x-axis and meets at $B_3\,(0, 5)$ on y-axis. Clearly $(0,0)$ satisfies $y \le 5$, so region containing origin represents $y \le 5$ in xy-plane.

Region represented by $x, y \ge 0$: It represent the first quadrant in xy-plane.

So, shaded region OA_2PB_3 represents the feasible region.

Coordinate of $P\,(10, 5)$ is obtained by solving $x + 2y = 20$ and $y = 5$.

The value of $Z = 3x + 5y$ at

$O\,(0,0) \quad = 3(0) + 5(0) = 0$

$A_2\,(15, 0) = 3(15) + 5(0) = 45$

$P\,(10, 5) \quad = 3(10) + 5(5) = 55$

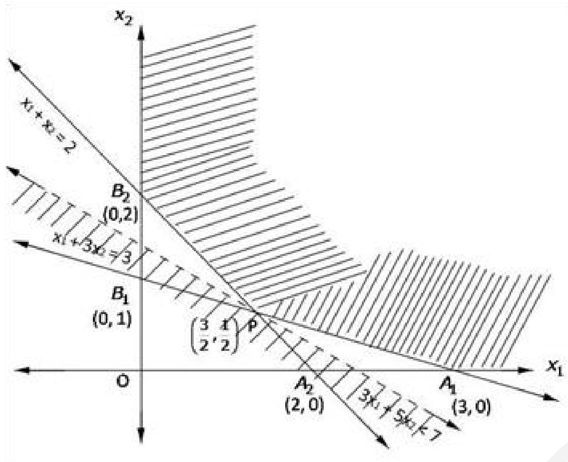
$B_3\,(0, 5) \quad = 3(0) + 5(5) = 25$

Hence, maximum $Z = 55$ at $x = 10$ and $y = 5$

Linear Programming Ex 30.2 Q16

Converting the given inequations into equations,

$x_1 + 3x_2 = 3,\ x_1 + x_2 = 2,\ x_1 = x_2 = 0$



Region represented by $x_1 + 3x_2 \geq 3$: Line $x_1 + 3x_2 = 3$ meets the coordinate axes at $A_1(3,0)$ and $B_1(0,1)$, clearly, $(0,0)$ does not satisfy $x_1 + 3x_2 \geq 3$, so, region not containing $(3,0)$ represents $x_1 + 3x_2 \geq 3$ in x_1x_2 -plane.

Region represented by $x_1 + x_2 \geq 2$: Line $x_1 + x_2 = 2$ meets the coordinate axes at $A_2(2,0)$ and $B_2(0,2)$, clearly, $(0,0)$ does not satisfy $x_1 + x_2 \geq 2$, so, region not containing origin represents $x_1 + x_2 \geq 2$ in x_1x_2 -plane.

Region represented $x_1, x_2 \geq 0$: It represents the first quadrant in x_1x_2-plane.

The unbounded shaded region with corner points $A_1(3,0)$, $B_2(0,2)$, and $P\left(\frac{3}{2}, \frac{1}{2}\right)$.

$P\left(\frac{3}{2}, \frac{1}{2}\right)$ is obtained by $x_1 + x_2 = 2$ and $x_1 + 3x_2 = 3$.

The value of $Z = 3x_1 + 5x_2$ at

$A_1(3,0) \quad = 3(3) + 5(0) = 9$

$P\left(\frac{3}{2}, \frac{1}{2}\right) \quad = 3\left(\frac{3}{2}\right) + 5\left(\frac{1}{2}\right) = 7$

$B_2(0,2) \quad = 3(0) + 5(2) = 10$

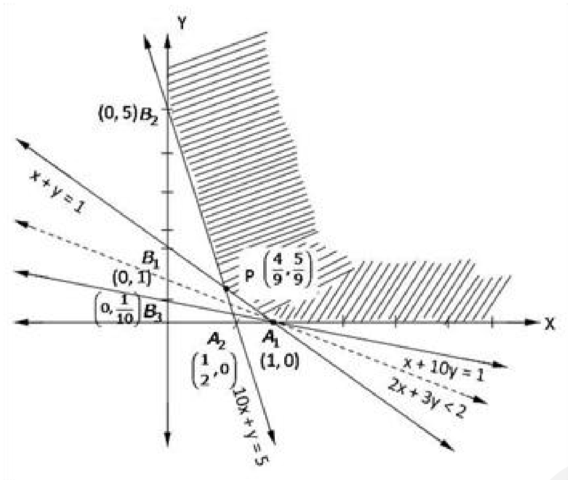
The smallest value of $Z = 7$,
region has no point in common, so smallest value is the minimum value.

Hence, minimum $Z = 7$ at $x = \frac{3}{2}$ and $y = \frac{1}{2}$

Linear Programming Ex 30.2 Q17

Converting the given inequations into equations

$$x + y = 1,\ 10x + y = 5,\ x + 10y = 1,\ x = y = 0$$



Region represented by $x + y \geq 1$: Line $x + y = 1$ meets coordinate axes at $A_1(1,0)$ and $B_1(0,1)$, dearly, $(0,0)$ does not satisfy $x + y \geq 1$, so region not containing origin represents $x + y \geq 1$ in xy -plane.

Region represented by $10x + y \geq 5$: Line $10x + y = 5$ meets coordinate axes at $A_2\left(\frac{1}{2}, 0\right)$ and $B_2(0,5)$. Clearly, $(0,0)$ does not satisfy $10x + y \geq 5$, so region not containing origin represents $10x + y \geq 5$ in xy -plane.

Region represented by $x + 10y \geq 1$: Line $x + 10y = 1$ meets coordinate axes $A_1(1,0)$ and $B_3\left(0, \frac{1}{10}\right)$. Clearly, $(0,0)$ does not satisfy $x + 10y \geq 1$, so, region not containing origin represents $x + 10y \geq 1$ in xy-plane.

Region represented by $x, y \geq 0$: It represents first quadrant in xy-plane.

So, unbounded shaded represents feasible region. Its corner points are $A_1(1,0)$, $P\left(\frac{4}{9}, \frac{5}{9}\right)$ and $B_2(0,5)$.

The coordinate of $P\left(\frac{4}{9}, \frac{5}{9}\right)$ is obtained by solving $10x + y = 5$ and $x + y = 1$.

The value of $Z = 2x + 3y$ at

$$A_1(1,0) \quad = 2(1) + 3(0) = 2$$

$$P\left(\frac{4}{9}, \frac{5}{9}\right) \quad = 2\left(\frac{4}{8}\right) + 3\left(\frac{5}{9}\right) = \frac{23}{9} = 2\frac{5}{9}$$

$$B_2(0,5) \quad = 2(0) + 3(5) = 15$$

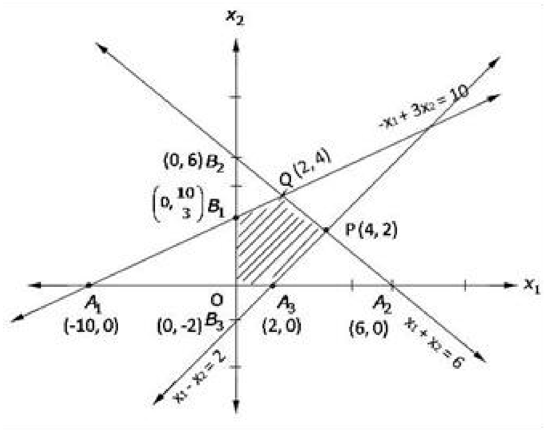
The smallest value of Z is 2. Now, open half plane $2x + 3y < 2$ has no point in common with feasible region so, smallest value of Z is the minimum value.

Hence, maximum $Z = 2$ at $x = 1$ and $y = 0$

Linear Programming Ex 30.2 Q18

Converting the given inequations into equations,

$$-x_1 + 3x_2 = 10,\ x_1 + x_2 = 6,\ x_1 = x_2 = 2,\ x_1 = x_2 = 0$$



Region represented by $-x_1 + 3x_2 \leq 10$: Line $-x_1 + 3x_2 = 10$ meets coordinate axes at $A_1(-10, 0)$ and $B_1\left(0, \frac{10}{3}\right)$, clearly, $(0,0)$ satisfies $-x_1 + 3x_2 \leq 10$, so region containing origin represents $-x_1 + 3x_2 \leq 10$ in x_1x_2-plane.

Region represented by $x_1 + x_2 \leq 6$: Line $x_1 + x_2 = 6$ meets coordinate axes at $A_2(6, 0)$ and $B_2(0, 6)$. Clearly, $(0,0)$ satisfies $x_1 + x_2 \leq 6$, so region containing origin represents $x_1 + x_2 \leq 6$ in x_1x_2-plane.

Region represented by $x_1 - x_2 \leq 2$: Line $x_1 - x_2 = 2$ meets coordinate axes at $A_3(2, 0)$ and $B_3(0, -2)$. Clearly, $(0,0)$ satisfies $x_1 - x_2 \leq 2$, so, region containing origin represents $x_1 - x_2 \leq 2$ in x_1x_2-plane.

Region represented $x_1, x_2 \geq 0$: It represents first quadrant in x_1x_2-plane.

So, shaded region OA_3PQB_1 represents feasible region.

Coordinate of $P(4, 2)$ is obtained by solving $x_1 + x_2 = 6$ and $x_1 - x_2 = 2$, $Q(2, 4)$ by solving $x_1 + x_2 = 6$ and $-x_1 + 3x_2 = 10$

The value of $Z = -x_1 + 2x_2$ at

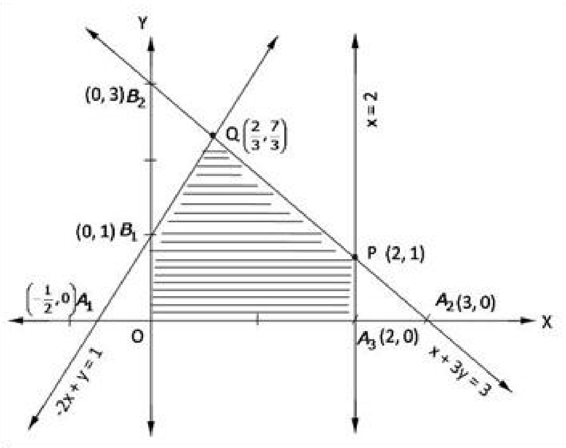
$$O(0,0) = -(0) + 2(0) = 0$$
$$A_3(2,0) = -(2) + 2(0) = -2$$
$$P(4,2) = -(4) + 2(2) = 0$$
$$Q(2,4) = -(2) + 2(4) = 6$$
$$B_1\left(0, \frac{10}{3}\right) = -(0) + 2\left(\frac{10}{3}\right) = \frac{20}{3} = 6\frac{2}{3}$$

Hence, maximum $Z = \frac{20}{3}$ at $x = 0$ and $y = \frac{10}{3}$

Linear Programming Ex 30.2 Q19

Converting the given inequations into equations,

$$-2x + y = 1,\ x = 2,\ x + y = 3,\ x = y = 0$$



Region represented by $-2x + y \le 1$: Line $-2x + y = 1$ meets coordinate axes at $A_1\left(\frac{-1}{2}, 0\right)$ and $B_1(0,1)$, clearly, $(0,0)$ satisfies $-2x + y \le 1$, so region containing origin represents $-2x + y \le 1$ in xy -plane.

Region represented by $x \le 2$: Line $x = 2$ is parallel to y-axis and meets x-axis at $A_3(2,0)$. Clearly, $(0,0)$ satisfies $x \le 2$, so region containing origin represents $x \le 2$ in xy-plane.

Region represented by $x + y \le 3$: Line $x + y = 3$ meets coordinate axes at $A_2(3,0)$ and $B_2(0,3)$. Clearly, $(0,0)$ satisfies $x + y \le 3$, so region containing origin represents $x + y \le 3$ in xy -plane.

Region represented by $x, y \ge 0$: It represents first quadrant in xy-plane.

So, shaded region OA_3PQB, represents the feasible region.

Coordinates of $P(2,1)$ is obtained by solving $x + y = 3$ and $x = 2$, $Q\left(\frac{2}{3}, \frac{7}{3}\right)$ by solving $-2x + y = 1$ and $x + y = 3$.

The value of $Z = x + y$ at

$$O(0,0) = 0 + 0 = 0$$
$$A_3(2,0) = 2 + 0 = 2$$
$$P(2,1) = 2 + 1 = 2$$
$$Q\left(\frac{2}{3}, \frac{7}{3}\right) = \frac{2}{3} + \frac{7}{3} = 3$$
$$B_1(0,1) = 0 + 1 = 1$$

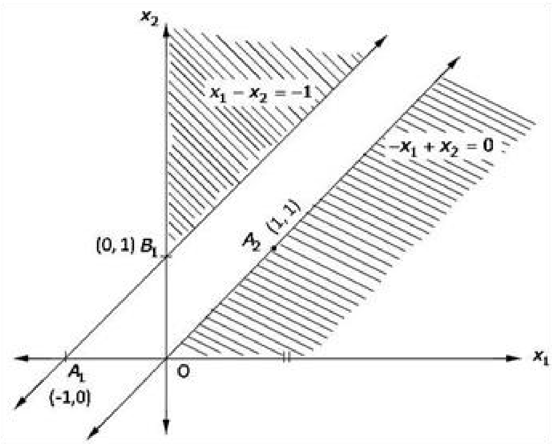
So, maximum $Z = 3$ is at every point on the line joining PQ.

Hence, maximum $Z = 3$ at $x = 2$ and $y = 1$ Or $x = \frac{2}{3}$ and $y = \frac{7}{3}$

Linear Programming Ex 30.2 Q20

Converting the given inequations into equations,

$$x_1 - x_2 = -1,\ -x_1 + x_2 = 0,\ x_1 = x_2 = 0$$



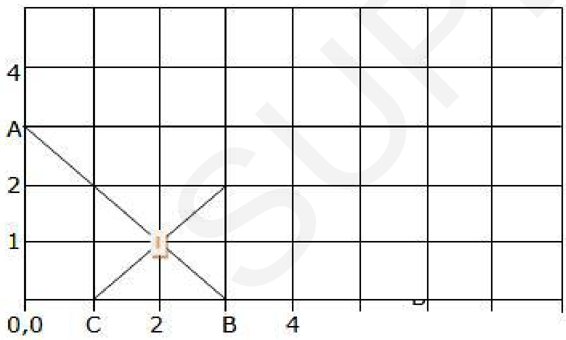
Region represented by $x_1 - x_2 \le -1$: Line $x_1 - x_2 = -1$ meets coordinate axes at $A_1(-1, 0)$ and $B_1(0, 1)$, clearly, $(0,0)$ does not satisfy $x_1 - x_2 \le -1$, so region not containing origin represents $x_1 - x_2 \le -1$ in x_1x_2 -plane.

Region represented by $-x_1 + x_2 \le 0$: Line $-x_1 + x_2 = 0$ passes through origin and $A_2(1,1)$. Clearly, $(0,0)$ does not satisfy $-x_1 + x_2 \le 0$, so, region not containing $(0,1)$ represents $-x_1 + x_2 \le 0$ in x_1x_2 -plane.

Since, there is not common shaded region represented by $x_1 - x_2 \le -1$ and $-x_1 + x_2 \le 0$ which can form feasible region.

Hence, maximum $Z = 3x_1 + 4x_2$ does not exists.

Linear Programming Ex 30.2 Q21



x-y≤1; when x = 0, y=1 & when y=0, x=2
x + y≥3; when x = 0, y=3 & when y=0, x=3, line AB
a unbounded region A-C-D is obtained using the constraints.

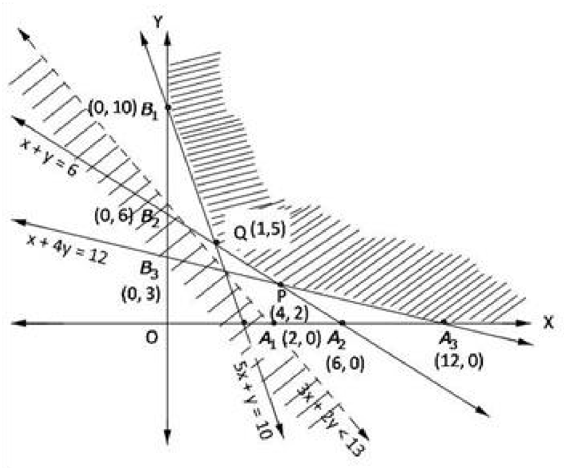
Corner point	Value of Z = 3x + 3y
0, 3	9
2, 1	9

So an optimal solution does not exist.

Linear Programming Ex 30.2 Q22

Converting the given inequations into equations

$5x + y = 10,\ x + y = 6,\ x + 4y = 12,\ x = y = 0$



Region represented by $5x + y \geq 10$: Line $5x + y = 10$ meets coordinate axes at $A_1(2,0)$ and $B_1(0,10)$. Clearly, $(0,0)$ does not satisfy $5x + y \geq 10$, so region not containing origin represents $5x + y \geq 10$ in xy -plane.

Region represented by $x + y \geq 6$: Line $x + y = 6$ meets coordinate axes at $A_2(6,0)$ and $B_2(0,6)$. Clearly, $(0,0)$ does not satisfy $x + y \geq 6$, so region not containing origin represents $x + y \geq 6$ in xy -plane.

Region represented by $x + 4y \geq 12$: Line $x + 4y = 12$ meets coordinate axes at $A_3(12,0)$ and $B_3(0,3)$. Clearly, $(0,0)$ does not satisfy $x + 4y \geq 12$, so, region not containing origin $x + 4y \geq 12$ in xy -plane.

Region represented by $x, y \geq 0$: It represents first quadrant in xy-plane.

The unbounded shaded region with corner points $A_3(12,0)$, $P(4,2)$, $Q(1,5)$, $B_1(0,10)$ represents feasible region. Point P is obtained by solving $x + 4y = 12$ and $x + y = 6$, Q by solving $x + y = 6$ and $5x + y = 10$.

The value of $Z = 3x + 2y$ at

$A_3(12,0) = 3(12) + 2(0) = 36$

$P(4,2) = 3(4) + 2(2) = 16$

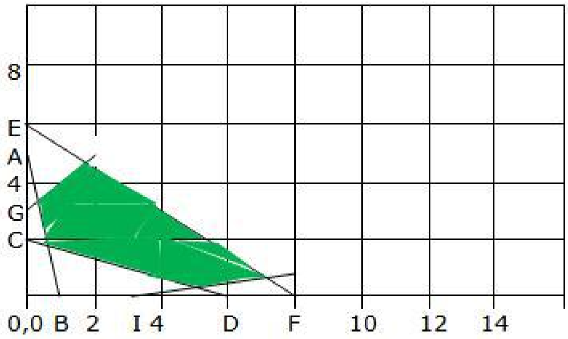
$Q(1,5) = 3(1) + 2(5) = 13$

$B(0,10) = 3(0) + 2(10) = 20$

Smallest value of $Z = 13$, Now open half plane $3x + 2y < 13$ has no point in common with feasible region, so, smallest value is the minimum value of Z, Hence

Minimum $Z = 13$ at $x = 1$, $y = 5$

Linear Programming Ex 30.2 Q23



$x+3y\geq 6$; or $y=-0.333x+2$; when $x=0$, $y=2$ & when $y=0$, $x=6$; line CD
$x-3y\leq 3$; or $y=0.333x-1$; when $x=0$, $y=-1$ & when $y=0$, $x=3$; line IJ
$3x+4y\leq 24$; or $y=-0.75x+6$; when $x=0$, $y=6$ & when $y=0$, $x=8$; line EF

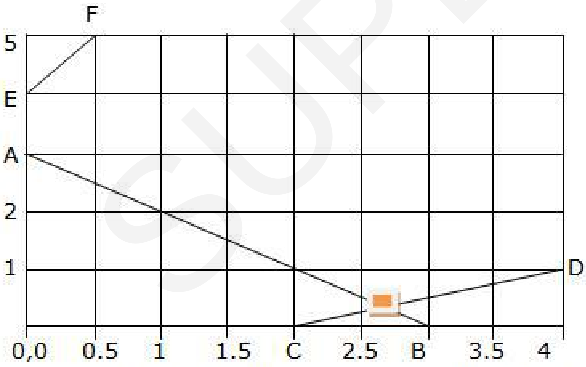
$-3x+2y\leq 6$; or $y= 1.5x+3$; when $x=0$, $y=3$ & when $y=0$, $x=-2$;line GH
$5x+y\geq 5$; or $y=-5x+5$; when $x=0$, $y=5$ & when $y=0$, $x=1$; line AB

The feasible area is shaded in green

Corner point	Value of Z = 2x + y
4.5, 0.5	9.5
0.64, 1.78	3.07
6.46, 1.15	Maximum 14.07
1.33, 5	7.6667
0.30, 3.46	4.0769

Maximum value is 14.07 at the point (6.46, 1.15)
Minimum value is 3.07 at the point (0.64, 1.78)

Linear Programming Ex 30.2 Q24



$-2x+y\leq 4$; or $y=2x+4$; when $x=0$, $y=4$ & when $y=0$, $x=-2$ line EF x+y≥3; or y=-x+3; when x=0, y=3 & when y=0, x=3; line AB

x-2y≤2; or y=0.5x-1; when x=0, y=-1 & when y=0,

x=2 line CD

The feasible solution is the unbounded area with F-E-A-G-D

Corner point	Value of Z = 3x + 5y	
(2.67, 0.33)	Minimum	9.66
(0, 3)		15
(0, 4)		20

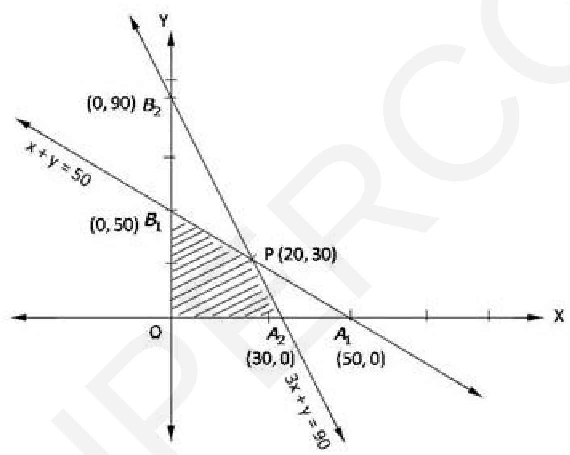
To check whether it is the minimal value plot the objective function with a value less than 9.66 or y=-0.6x-1.932

it can be seen that the values of x and y are always negative. So there is no optimal solution.

Linear Programming Ex 30.2 Q25

Converting the given inequations into equations,

$x + y = 50,\ 3x + y = 90,\ x = y = 0$



Region represented by $x + y \le 50$: Line $x + y = 50$ meets coordinate axes at $A_1(50, 0)$ and $B_1(0, 50)$. Clearly, $(0, 0)$ satisfies $x + y \le 50$, so, region containing origin represents $x + y \le 50$ in xy-plane.

Region represented by $3x + y \le 90$: Line $3x + y = 90$ meets coordinate axes at $A_2(30, 0)$ and $B_2(0, 90)$. Clearly, $(0, 0)$ satisfies $3x + y \le 90$, so, region containing origin represents $3x + y \le 90$ in xy-plane.

Region represented by $x, y \ge 0$: It represents first quadrant in xy-plane.

Shaded region OA_2PB_1 represents the feasible region. $P(20, 30)$ can be obtained by solving $x + y = 50$ and $3x + y = 90$.

The value of $Z = 60x + 15y$ at

$$\begin{aligned}
&O(0,0) && = 60(0) + 15(0) = 0\\
&A_2(30,0) && = 60(30) + 15(0) = 1800\\
&P(20,30) && = 60(20) + 15(30) = 1650\\
&B_1(0,50) && = 60(0) + 15(50) = 750
\end{aligned}$$

Hence,

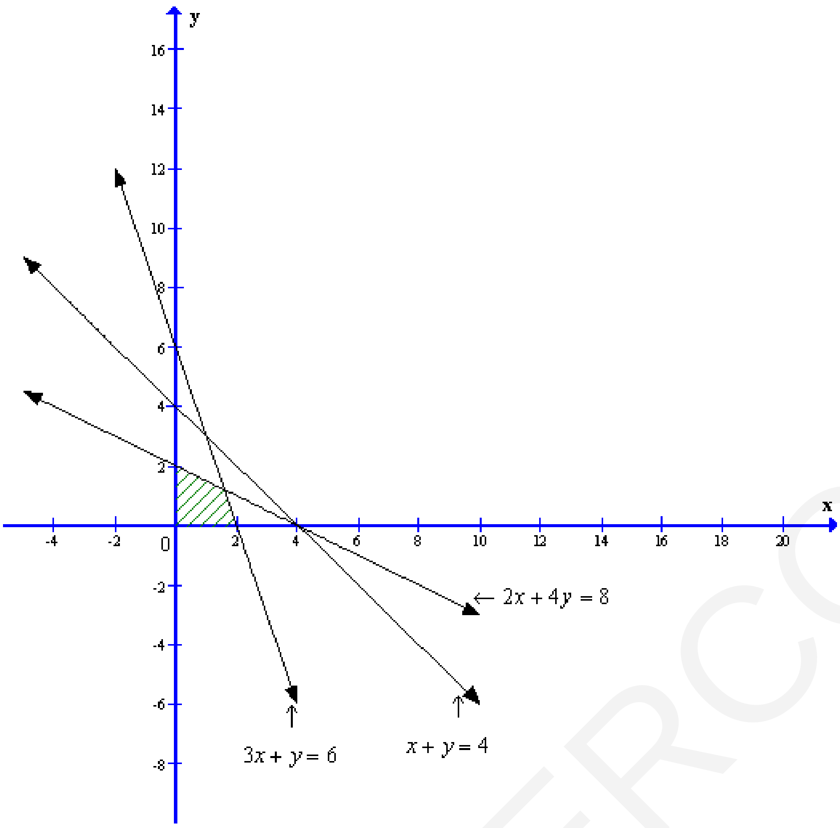
maximum Z is 1800 at $x = 30$ and $y = 0$.

Linear Programming Ex 30.2 Q26

Converting the inequations into equations, we obtain the lines

$2x + 4y = 8$, $3x + y = 6$, $x + y = 4$, $x = 0$, $y = 0$.

These lines are drawn on a suitable scale and the feasible region of the LPP is shaded in the graph.



From the graph we can see the corner points as (0, 2) and (2, 0).

Now solving the equations $3x + y = 6$ and $2x + 4y = 8$ we get the values of x and y as $x = \frac{8}{5}$ and $y = \frac{6}{5}$.

Substituting $x = \frac{8}{5}$ and $y = \frac{6}{5}$ in $Z = 2x + 5y$ we get,

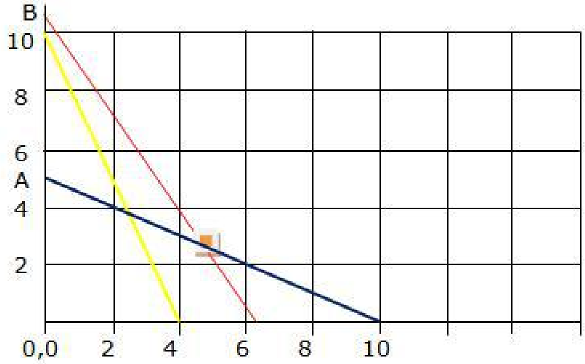
$$Z = 2\left(\frac{8}{5}\right) + 5\left(\frac{6}{5}\right)$$

$$Z = \frac{46}{5}$$

Hence maximum value of Z is $\frac{46}{5}$ at $x = \frac{8}{5}$ and $y = \frac{6}{5}$.

Ex 30.3

Linear Programming Ex 30.3 Q1



Let x and y be the No. of 25 gm packets of foods F_1 and F_2

Minimum cost of diet $Z = 0.20x + 0.15y$

The constraints are
$0.25x + 0.1y \geq 1$; when $x=0$, $y=10$ & $y=0$, $x=4$ 10-4
$0.75x + 1.5y \geq 7.5$; when $x=0$, $y=5$ & $y=0$, $x=10$ A-10
$1.6x + 0.8y \geq 10$; when $x=0$, $y=25/2$ & $y=0$, $x=25/4$

The feasible region is the open region B-E-10

The minimum cost of the diet can be checked by finding the value of Z at corner points B, E & 10

Corner point	Value of Z = 20x + 15y
0, 12.5	187.5
10, 0	200
5, 2.5	137.5

Since the feasible region is an open region so we plot $20x + 15y < 137.5$, to check whether the resulting open half plane has any point common with the feasible region. Since it has common points $Z = 20x + 15y$

There is no optimal minimum value subject to the given constraints.

Linear Programming Ex 30.3 Q2

Let required quantity of food A and B be x and y units respectively.

Costs of one unit of food A and B are Rs 4 and Rs 3 per unit respectively, so, costs of x unit of food A and y unit of food B are $4x$ and $3y$ respectively. Let Z be minimum total cost, so

$$Z = 4x + 3y$$

Since one unit of food A and B contain 200 and 100 units of vitamin respectively. So, x units of food A and y units of food B contain $200x$ and $100y$ units of vitamin but minimum requirement of vitamin is 4000 units, so

$$200x + 100y \geq 4000$$

$$\Rightarrow \quad 2x + y \geq 40 \quad \text{(first constraint)}$$

Since one unit of food A and B contain 1 unit and 2 unit of minerals, so x units of food A and y units of food B contain x and $2y$ units of minerals respectively but minimum requirement of minerals is 50 units, so

$$x + 2y \geq 50 \quad \text{(second constraint)}$$

Since one unit of food A and B contain 40 calories each, so x units of food A and y units of food B contain $40x$ and $40y$ calories respectively but minimum requirement of calories is 1400, so

$$40x + 40y \geq 1400$$

$$\Rightarrow \quad 2x + 2y \geq 70$$

$$\Rightarrow \quad x + y \geq 35 \quad \text{(third constraint)}$$

So, mathematical formulation of LPP is find x and y which

$$\text{minimize} \quad Z = 4x + 3y$$

Subject to constraint,

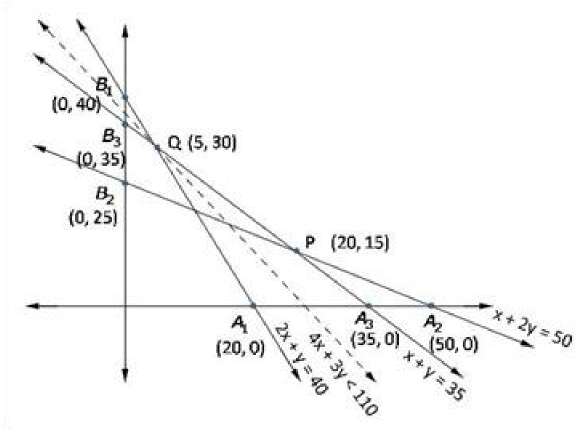
$$2x + y \geq 40$$
$$x + 2y \geq 50$$
$$x + y \geq 35$$
$$x, y \geq 0 \quad \text{[Since quantity of food can not be less than zero]}$$

Region $2x + y \geq 40$: Line $2x + y = 40$ meets axes at $A_1(20, 0)$, $B_1(0, 40)$ region not containing origin represents $2x + y \geq 40$ as $(0,0)$ does not satisfy $2x + y \geq 40$.

Region $x + 2y \geq 50$: Line $x + 2y = 50$ meets axes at $A_2(50,0)$, $B_2(0,25)$. Region not containing origin represents $x + 2y \geq 50$ as $(0,0)$ does not satisfy $x + 2y \geq 50$.

Region $x+y \geq 35$: Line $x+y=35$ meets axes at $A_3(35,0)$, $B_3(0,35)$. Region not containing origin represents $x+y \geq 35$ as $(0,0)$ does not satisfy $x+y \geq 35$.

Region $x, y \geq 0$: It represent first quadrant in xy-plane.



Unbounded shaded region A_2PQB_1 represents feasible region with corner points $A_2(50,0)$, $P(20,15)$, $Q(5,30)$, $B_1(0,40)$

The value of $Z=4x+3y$ at

$$A_2(50,0) = 4(50)+3(0) = 2000$$
$$P(20,15) = 4(20)+3(15) = 125$$
$$Q(5,30) = 4(5)+3(30) = 110$$
$$B_1(0,40) = 4(0)+3(40) = 110$$

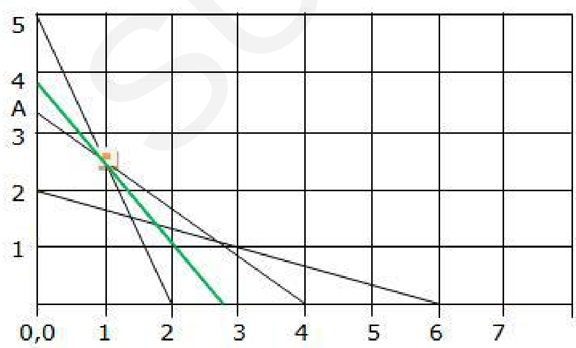
Smallest value of $Z = 110$

Open half plane $4x+3y<110$ has no point in common with feasible region, so, smallest value is the minimum value.

Hence,

quantity of food $A = x = 5$ unit
quantity of food $B = y = 30$ unit
minimum cost = Rs 110

Linear Programming Ex 30.3 Q3



Let x & y be the units of Food I and Food II resptly.

The objective function is to minimize the function
Z = 0.6x + y such that
10x + 4y ≥ 20 requirement of calcium, line 5-2
5x + 6y ≥ 20 requirement of protein, line A-4
2x + 6y ≥ 12 requirement of calories, line 2-6

These when plotted give 5-F-E-6 an open unbounded region.

The function $20x + 15y < 57.5$ needs to be plotted to check if there are any common points. The green line shows that there are no common points. So

Corner point	Value of $Z = 0.6x + y$
0, 5	5
F(1, 2.5)	3.1
E(2.67, 1.11)	2.71
6, 0	3.6

The minimum cost occurs when Food I is 1 unit and Food II is 2.5 units. Since it is an unbounded region plotting $Z<3.1$ gives the green line which has no common points, so (1, 2.5) can be said to be a minimum point.

Linear Programming Ex 30.3 Q4

Let required quantity of food A and food B be x and y units.

Given, costs of one unit of food A and B are 10 paise per unit each, so costs of x unit of food A and y unit of food B are $10x$ and $10y$ respectively, let Z be total cost of foods, so

$$Z = 10x + 10y$$

Since one unit of food A and B contain 0.12 mg and 0.10 mg of Thiamin respectively, so, x units of food A and y units of food B contain $0.12x$ mg and $0.10y$ mg of Thiamin respectively but minimum requirement of Thiamin is 0.4 mg, so

$$0.12x + 0.10y \ge 0.5$$

$$\Rightarrow \quad 12x + 10y \ge 50$$

$$\Rightarrow \quad 6x + 5y \ge 25 \qquad \text{(first constraint)}$$

Since one unit of food A and B contain 100 and 150 Calories respectively, so x units of food A and y units of food B contain $100x$ and $150y$ units of Calories but minimum requirement of Calories is 600, so

$$100x + 150y \ge 600$$

$$\Rightarrow \quad 2x + 3y \ge 12 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

minimize $Z = 10x + 10y$

Subject to constraint,

$$6x + 5y \ge 25$$

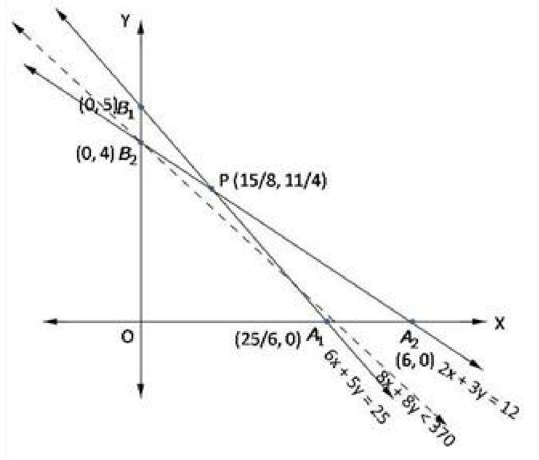
$$2x + 3y \ge 12$$

$$x, y \ge 0 \qquad \left[\text{Since quantity of food } A \text{ and } B \text{ can not be less than zero}\right]$$

Region $6x + 5y \ge 25$: $6x + 5y = 25$ meets axes at $A_1\left(\frac{25}{6}, 0\right)$, $B_1(0,5)$. Region not containing origin represents $6x + 5y \ge 25$ as $(0,0)$ does not satisfy $6x + 5y \ge 25$.

Region $2x + 3y \ge 12$: Line $2x + 3y = 12$ meets axes at $A_2(6,0)$, $B_2(0,4)$. Region not containing origin represents $2x + 3y \ge 12$ as $(0,0)$ does not satisfy $2x + 3y \ge 12$.

Region $x, y \ge 0$ represent first quadrant in xy-plane.



Unbounded shaded region $A_2 P B_1$ represents feasible region with corner points $A_2(6,0)$, $P\left(\frac{15}{8},\frac{11}{4}\right)$, $B_1(0,5)$

The value of $Z = 10x + 10y$ at

$$A_2(6,0) \quad = 10(6) + 10(0) = 60$$

$$P\left(\frac{15}{8},\frac{11}{4}\right) \quad = 10\left(\frac{15}{8}\right) + 10\left(\frac{11}{4}\right) = \frac{370}{8} = 46\frac{1}{4}$$

$$B_1(0,5) \quad = 10(0) + 10(5) = 50$$

Smallest value of Z is $46\frac{1}{4}$.

Now open half plane $10x + 10y < \frac{370}{8}$

$\Rightarrow$ $8x + 8y < 370$ has no point in common with feasible region, so smallest value is the minimum value.

Hence,

Required quantity of food $A = \frac{15}{8}$ units, food $B = \frac{11}{4}$ units

minimum cost $=$ Rs 46.25

Linear Programming Ex 30.3 Q5

Let required quantity of food X and food Y be x kg and y kg.

Since costs of food X and Y are Rs 5 and Rs 8 per kg., So, costs of food X and food Y are Rs. $5x$ and Rs. $8y$ respectively. Let Z be the total cost of food, then

$$Z = 5x + 8y$$

Since one kg of food X and Y contain 1 and 2 unit of vitamin A, so, x kg of food X and y kg of food Y contain x and $2y$ units of vitamin A respectively but minimum requirement of vitamin A is 6 units, so

$$x + 2y \geq 6 \qquad \text{(first constraint)}$$

Since one kg of food X and Y contain 1 unit of vitamin B each, so x kg of food X and y kg of food Y contain x and y units of vitamin B but minimum requirement of vitamin B is 7 units, so

$$x + y \geq 7 \qquad \text{(second constraint)}$$

Since one kg of food X and food Y contain 1 unit and 3 units of vitamin C respectively, so x kg of food X and y kg of food Y contain x and $3y$ units of vitamin C respectively but minimum requirement of vitamin C is 11 units, so

$$x + 3y \geq 11 \qquad \text{(third constraint)}$$

Since 1 kg of food X and food Y contain 2 units and 1 unit of vitamin D respectively, so, x kg of food X and y kg of food Y contain $2x$ and y units of vitamin D respectively but minimum requirement of vitamin D is 9 units, so

$$2x + y \geq 9 \qquad \text{(fourth constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

minimize $Z = 5x + 8y$

Subject to constraints,

$$x + 2y \geq 6$$
$$x + y \geq 7$$
$$x + 3y \geq 11$$
$$2x + y \geq 9$$
$$x, y \geq 0 \qquad [\text{Since quantity of food } X \text{ and } Y \text{ can not be less than zero}]$$

Region $x + 2y \geq 6$: Line $x + 2y = 6$ meets axes at $A_1(6,0)$, $B_1(0,3)$. Region not containing origin represents $x + 2y \geq 6$ as $(0,0)$ does not satisfy $x + 2y \geq 6$.

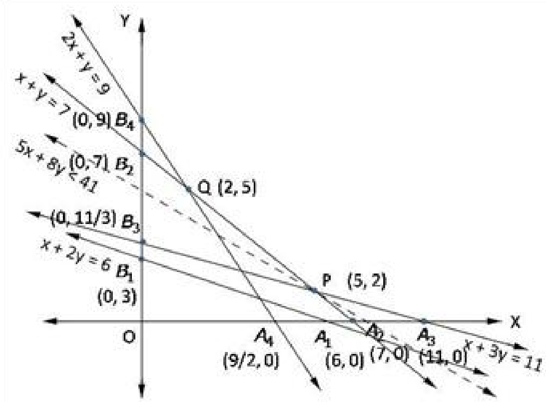
Region $x + y \geq 7$: Line $x + y = 7$ meets axes at $A_2(7,0)$, $B_2(0,7)$ respectively. Region not containing origin represents $x + y \geq 7$ as $(0,0)$ does not satisfy $x + y \geq 7$.

Region $x + 3y \geq 11$: Line $x + 3y = 11$ meets axes at $A_3(11, 0)$, $B_3\left(0, \frac{11}{3}\right)$ respectively.

Region not containing origin represents $x + 3y \geq 11$ as $(0,0)$ does not satisfy $x + 3y \geq 11$.

Region $2x + y \geq 9$: Line $2x + y = 9$ meets axes at $A_4\left(\frac{9}{2}, 0\right)$, $B_4(0, 9)$ respectively. Region not containing origin represents $2x + y \geq 9$ as $(0,0)$ does not satisfy $2x + y \geq 9$.

Region $x, y \geq 0$ it represent first quadrant.



Unbounded shaded region $A_2PQ\ B_4$ is the feasible region with corner points $A_3(11, 0)$, $P(5, 2)$, $Q(2,5)$, $B_4(0, 9)$

The value of $Z = 5x + 8y$ at

$A_3(11, 0)$	$= 5(11) + 8(0) = 55$
$P(5, 2)$	$= 5(5) + 8(2) = 41$
$Q(2,5)$	$= 5(2) + 8(5) = 50$
$B_4(0, 9)$	$= 5(0) + 8(9) = 72$

Smallest value of Z is 41.

Now open half plane $5x + 8y < 41$ has no point is common with feasible region, os,
smallest value of is the minimum value.
hence
last cost of mixture= Rs 41

Linear Programming Ex 30.3 Q6

Let quantity of food F_1 and F_2 be x and y units.
respectively.
Given, costs of one unit of food F_1 and F_2 be Rs 4 and Rs 6 per unit, So, costs of X unit of food $F_1$and Y units of food F_2 be $4x$ and $6y$ respectively,
Let Z be the total cost , so

$$Z = 4x + 8y$$

Since one unit of food F_1 and F_2 contain 3 and 6 unit of vitamin A respectively, so, x units of food F_1 and y units of food F_2 contain $3x$ and $6y$ units of vitamin A respectively, but minimum requirement of vitamin A is 80 units, so

$$3x + 6y \geq 80 \qquad \text{(first constraint)}$$

Since one unit of food F_1 and F_2 contain 4 unit and 3 unit of mineral, so x unit of food F_1 and y unit of food F_2 contain $4x$ and $3y$ units of mineral respectively but minimum requirement of minerals be 100 units, so

$$4x + 3y \geq 100$$

$$\Rightarrow \quad 4x + 3y \geq 100 \qquad \text{(second constraint)}$$

mathematical formulation of LPP is, Find x and y which minimum

$$Z = 4x + 6y$$

Subject to constraints,

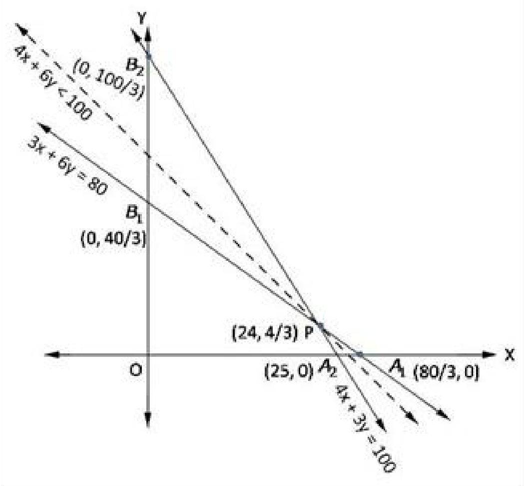
$$3x + 6y \geq 80$$
$$4x + 3y \geq 100$$
$$x, y \geq 0$$

[since quantity of food can not be less than zero]

Region $3x + 6y \geq 80$: line $3x + 6y = 80$ meets axes at $A_1\left(\frac{80}{3}, 0\right), B_1\left(0, \frac{40}{3}\right)$ respectively. Region not containing origin represents $3x + 6y \geq 80$ as $(0,0)$ does not satisfy $3x + 6y \geq 80$.

Region $4x + 3y \geq 100$ line $3x + 6y = 100$ meets axes at $A_2(25, 0), B_2\left(0, \frac{100}{3}\right)$ respectively. Region not containing origin represents $4x + 3y \geq 100$ as $(0,0)$ does not satisfy $4x + 3y \geq 100$.

Region $x, y \geq 0$ represents first quadrant



Unbounded shaded region $A_1 P\ B_2$ represents feasible region with corner points $A_1\left(\frac{80}{3}, 0\right)$, $P\left(24, \frac{4}{3}\right)$, $B_2\left(0, \frac{100}{3}\right)$.

The value of $Z = 4x + 6y$ at

$$A_1\left(\frac{80}{3}, 0\right) \quad = 4\left(\frac{80}{3}\right) + 6(0) = \frac{320}{3}$$

$$P\left(24, \frac{4}{3}\right) \quad = 4(24) + 6\left(\frac{4}{3}\right) = 104$$

$$B_2\left(0, \frac{100}{3}\right) \quad = 4(0) + 6\left(\frac{100}{3}\right) = 200$$

Smallest value of Z is 104 .Now open half plane $4x + 6y < 104$ has no point in common with feasible region so, smallest value is minimum value.
Hence,

Minimum cost of mixture = Rs 104

Linear Programming Ex 30.3 Q7

Let required quantity of bran and rice be x kg and y kg.
Given, costs of one kg of bran and rice are Rs 5 and Rs 4 per kg, So, costs of X unit of bran and Y kg of rice are $5x$ and Rs $4y$ respectively,
Let total cost of bran and rice be Z, so,

$$Z = 5x + 4y$$

Since one kg of bran and rice contain 80 and 100 mg of protien, so,
x kg of bran and y kg of rice contain $80x$ and $100y$ grms of protien respectively, but minimum requirement of protien for kelloggs is 88 gms, so

$$80x + 100y \geq 88$$

$$\Rightarrow \quad 20x + 25y \geq 22 \qquad \text{(first constraint)}$$

Since one kg of bran and rice contain 40 mg and 30 mg of iron, so,
x kg of bran and y kg of rice contain $40x$ and $30y$ mg of iron respectively, but minimum requirement of iron is 36 mg for kelloggs, so

$$40x + 30y \geq 36 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which minimize

$$Z = 5x + 4y$$

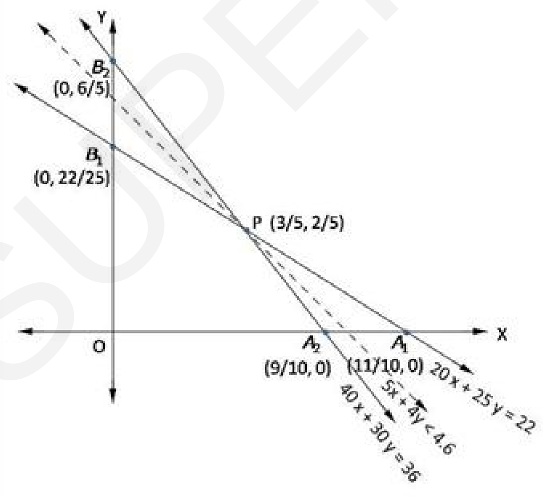
subject to constraints,

$$20x + 25y \geq 22$$
$$40x + 30y \geq 36$$
$$x, y \geq 0$$

[Since quantity of bran and rice can not be less than zero]

Region $20x + 25y \geq 22$: line $20x + 25y = 22$ meets axes at $A_1\left(\frac{11}{10}, 0\right), B_1\left(0, \frac{22}{25}\right)$ respectively. Region not containing origin represents $20x + 25y \geq 22$ as $(0,0)$ does not satisfy $20x + 25y \geq 22$.

Region $40x + 30y \geq 36$ line $40x + 30y = 36$ meets axes at $A_2\left(\frac{9}{10}, 0\right), B_2\left(0, \frac{6}{5}\right)$. Region not containing origin represents $40x + 30y \geq 36$ as $(0,0)$ does not satisfy $40x + 30y \geq 36$.



The value of $Z = 5x + 4y$ at

$$A_1\left(\frac{11}{10}, 0\right) \quad = 5\left(\frac{11}{10}\right) + 4(0) = 5.5$$

$$P\left(\frac{3}{5}, \frac{2}{5}\right) \quad = 5\left(\frac{3}{5}\right) + 4\left(\frac{2}{5}\right) = 4.6$$

$$B_2\left(0, \frac{6}{5}\right) \quad = 5(0) + 4\left(\frac{6}{5}\right) = 4.8$$

Smallest value of Z is 4.6. Now open half plane $5x + 4y < 4.6$ has no point in common with feasible region so, smallest value z is the minimum value.
Hence

Minimum cost of mixture = Rs 4.6

Linear Programming Ex 30.3 Q8

Let required number of bag A and bag B be x and y respectively.

Since, costs of each bag A and bag B are Rs 8 and Rs 12 per kg., So,
cost of x number of bag A and y number of bag B are Rs $8x$ and Rs $12y$ respectively,
Let Z be total cost of bags, so,

$$Z = 8x + 12y$$

Since, each bag A and B contain 60 and 30 gms. of almonds respectively. so,
x bags of A and y bags of B contain $60x$ and $30y$ gms. of almonds respectively but,
mixtures should contain at least 240 gms almonds, so,

$$60x + 30y \geq 240$$

$$\Rightarrow \quad 2x + y \geq 8 \quad \text{(first constraint)}$$

Since, each bag A and B contain 30 and 60 gms. of cashew nuts respectively. so,
x bags of A and y bags of B contain $30x$ and $60y$ gms. of cashew nuts respectively but,
mixtures should contain at least 300 gms of cashew nuts, so,

$$30x + 60y \geq 300$$

$$\Rightarrow \quad x + 2y \geq 10 \quad \text{(second constraint)}$$

Since, each bag A and B contain 30 and 180 gms. of hazel nuts respectively. so,
x bags of A and y bags of B contain $30x$ and $180y$ gms. of hazel nuts respectively but,
mixtures should contain at least 540 gms of hazel nuts, so,

$$30x + 180y \geq 540$$

$$\Rightarrow \quad x + 6y \geq 18 \quad \text{(third constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 8x + 12y$$

subject to constraints,

$$2x + y \geq 8$$
$$x + 2y \geq 10$$
$$x + 6y \geq 18$$
$$x, y \geq 0$$

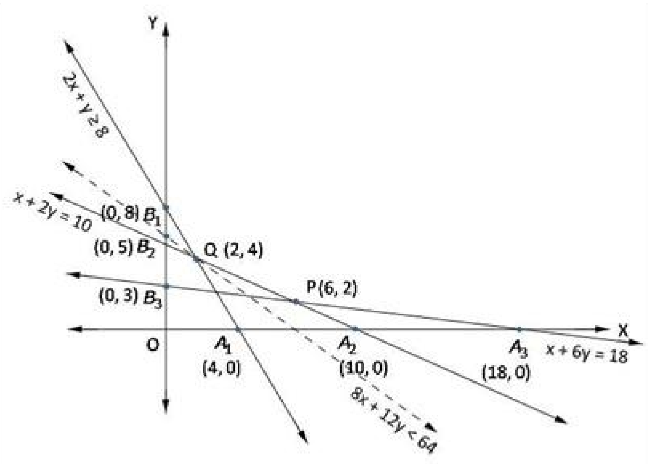
[Since quantity of bags can not be less than zero]

Region $2x + y \geq 8$: line $2x + y = 8$ meets axes at $A_1(4,0), B_1(0,8)$ respectively. Region not containing origin represents $2x + y \geq 8$ as $(0,0)$ does not satisfy $2x + y \geq 8$.

Region $x + 2y \geq 10$: line $x + 2y = 10$ meets axes at $A_2(10,0)$, $B_2(0,5)$ respectively. Region not containing origin represents $2x + y \geq 10$ as $(0,0)$ does not satisfy $x + 2y \geq 10$

Region $x + 6y \geq 18$: line $x + 6y = 18$ meets axes at $A_3(18,0)$, $B_3(0,3)$ respectively. Region not containing origin represents $x + 6y \geq 8$ as $(0,0)$ does not satisfy $x + 6y \geq 8$

Region $x, y \geq 0$: it represents first quadrant.



Unbouded shaded region A_3PQB_1 is feasible region with corner point $A_3(18,0)$, $P(6,2)$ $Q(2,4)$, $B_1(0,8)$. P is obtained by solving $x + 6y = 18$ and $x + 2y = 10$, Q is obtained by solving $2x + y = 8$ and $x + 2y = 10$

The value of $z = 8x + 12y$ at

$A_3(18,0) \quad = 8(18) + 12(0) = 144$

$P(6,2) \quad = 8(6) + 12(2) = 72$

$Q(2,4) \quad = 8(2) + 12(4) = 64$

$B_1(0,8) \quad = 8(0) + 12(8) = 96$

Smallest value of Z is 64, open half plane $8x + 12y \geq 64$ has no point is common with feasible region, so, smallest value is the minimum value

Minimum cost = Rs64
quantity of mixture A = 2 kg.
quantity of mixture B = 4kg

Linear Programming Ex 30.3 Q9

Let required number of cakes of type A and B are x and y respectively.

Let Z be total number of cakes ,so,

$$Z = x + y$$

Since one unit of cake of type A and B contain 300 gm and 150 gm flour respectively, so, x unit of cake of type A and y units of cake of type B require $300x$ and $150y$ gms of flour respectivley, but maximum flour available is $7.5 \times 1000 = 7500$ gm ,so

$$300x + 150y \le 7500$$

$$\Rightarrow \quad 2x + y \le 50 \qquad \text{(first constraint)}$$

Since one unit of cake of type A and B contain 15 and 30 gm fat respectively, so, x unit of cake of type A and y units of cake of type B contain $15x$ and $30y$ gms of fat respectivley, but maximum fat available is 600 gm ,so

$$15x + 30y \le 600$$

$$\Rightarrow \quad x + 2y \le 40 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = x + y$

Subject to constriants,

$$2x + y \le 50$$

$$x + 2y \le 40$$

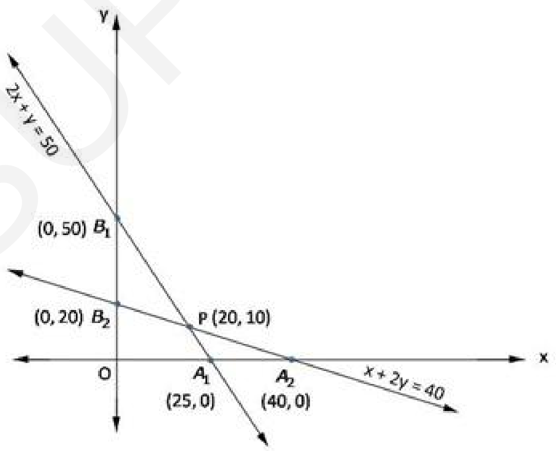
$$x, y \ge 0 \qquad \text{[Since number of cakes can not be less than zero]}$$

Region $2x + y \le 50$: line $2x + y = 50$ meets axes at $A_1(25,0)$, $B_1(0,50)$ respectively.
Region containing origin represents $2x + y \le 50$ as $(0,0)$ satisfies $2x + y \le 50$.

Region $x + 2y \le 40$: line $x + 2y = 40$ meets axes at $A_2(40,0)$, $B_2(0,20)$ respectively.
Region containing origin represents $x + 2y \le 40$ as $(0,0)$ satisfies $x + 2y \le 40$.

Region $x, y \ge 0$: it represent first quadrant

Shaded region OA_1PB_2 represents feasible region.
Point $P(20,10)$ is obtained by solving $x + 2y = 40$ and $2x + y = 50$



The value of $Z = x + y$ at

$O(0,0) \quad = 0 + 0 = 0$

$A_1(25,0) \quad = 25 + 0 = 25$

$P(20,10) \quad = 20 + 10 = 30$

$B_2(0,20) \quad = 0 + 20 = 20$

maximum $Z = 30$ at $x = 20$, $y = 10$

Number of books of type $A = 20$, type $B = 10$

Linear Programming Ex 30.3 Q10

Let x kg of food P and y kg of food Q are mixed together to make the mixture.

Then the mathematical model of the LPP is as follows:

Minimize $Z = 60x + 80y$

Subject to $3x + 4y \geq 8$,

$5x + 2y \geq 11$

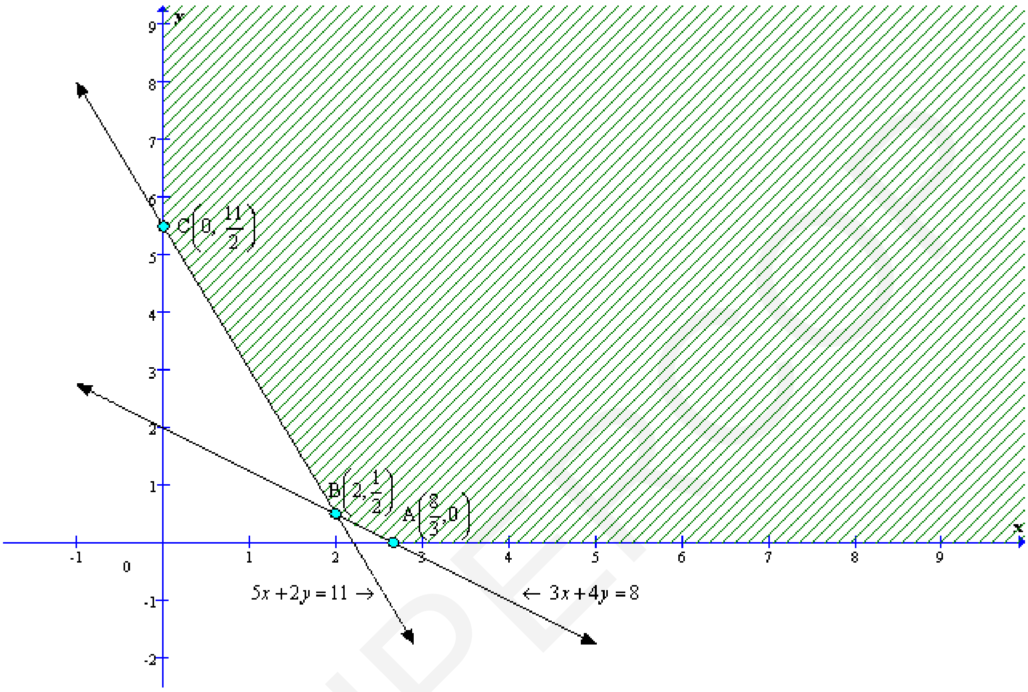
and $x \geq 0$, $y \geq 0$

To solve the LPP we draw the lines,

$3x + 4y = 8$,

$5x + 2y = 11$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are $A\left(\frac{8}{3}, 0\right)$, $B\left(2, \frac{1}{2}\right)$ and $C\left(0, \frac{11}{2}\right)$.

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 60x + 80y$
$A\left(\frac{8}{3}, 0\right)$	$Z = 160$
$B\left(2, \frac{1}{2}\right)$	$Z = 160$
$C\left(0, \frac{11}{2}\right)$	$Z = 440$

The minimum value of the mixture is Rs. 160 at all points on the line segment joining points $\left(\frac{8}{3}, 0\right)$ and $\left(2, \frac{1}{2}\right)$

Linear Programming Ex 30.3 Q11

Let x be the number of one kind of cake and
y be the number of second kind of cakes that are made.

Then the mathematical model of the LPP is as follows:

Maximize $Z = x + y$

Subject to $200x + 100y \leq 5000,$

$25x + 50y \leq 1000$

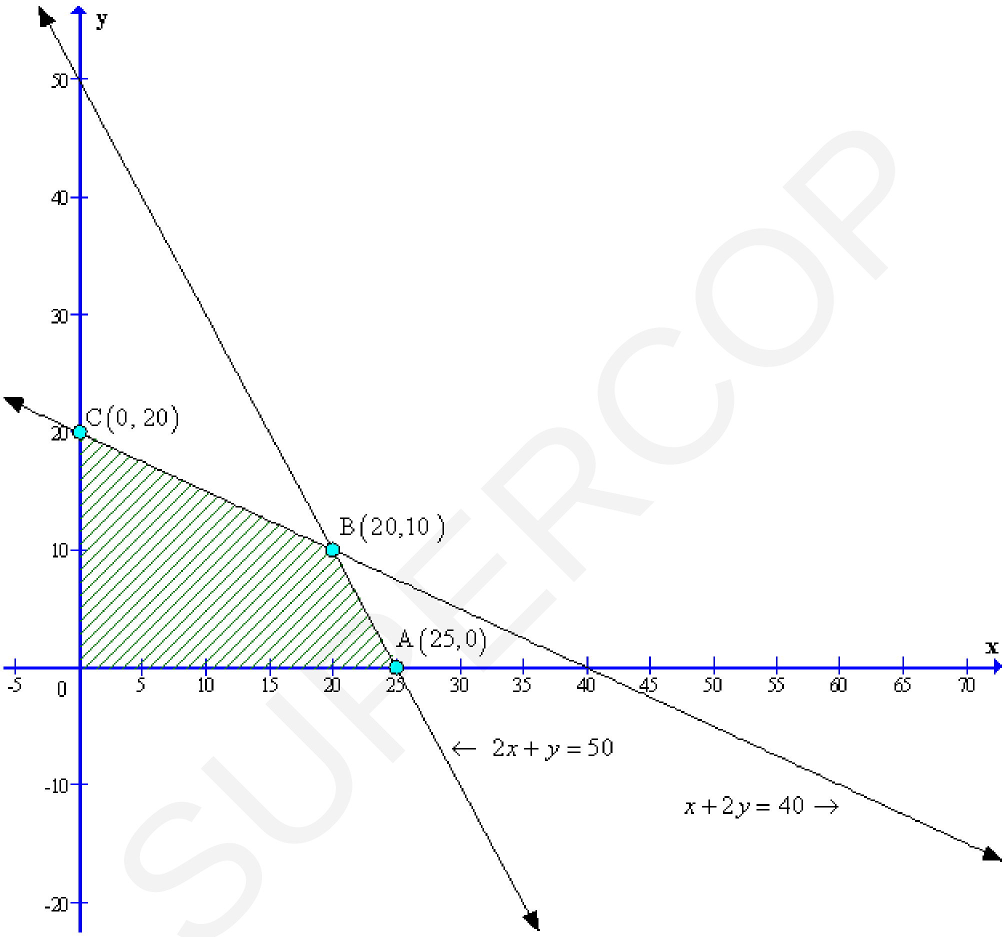
and $x \geq 0,\ y \geq 0$

To solve the LPP we draw the lines,

$2x + y = 50,$

$x + 2y = 40$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are $A(25, 0), B(20,10)$ and $C(0,20)$.

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = x + y$
A(25, 0)	Z = 25
B(20, 10)	Z = 30
C(0, 20)	Z = 20

The maximum of 30 cakes can be made.

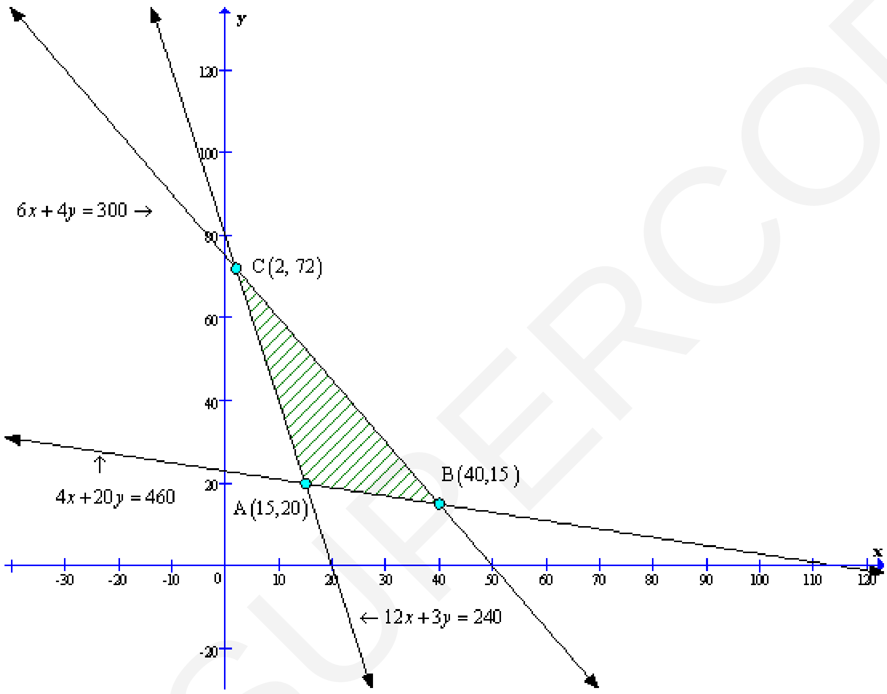
Linear Programming Ex 30.3 Q12

Let x be the number of packets of food P
y be the number of packets of food Q used to minimize vitamin A.

Then the mathematical model of the LPP is as follows:
Minimize $Z = 6x + 3y$
Subject to $12x + 3y \geq 240$,
$4x + 20y \geq 460$
$6x + 4y \leq 300$
and $x \geq 0$, $y \geq 0$

To solve the LPP we draw the lines,
$12x + 3y = 240$,
$4x + 20y = 460$,
$6x + 4y = 300$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(15, 20), B(40, 15) and C(2, 72).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 6x + 3y$
A(15, 20)	Z = 150
B(40, 15)	Z = 285
C(2, 72)	Z = 228

15 packets of food P and 20 packets of food Q should be used to minimise the amount of vitamin A.
The minimum amount of vitamin A is 150 units.

Linear Programming Ex 30.3 Q13

Let x be the number of bags of brand P
y be the number of bags of brand Q.

Then the mathematical model of the LPP is as follows:

Minimize $Z = 250x + 200y$

Subject to $3x + 1.5y \geq 18$,

$2.5x + 11.25y \geq 45$

$2x + 3y \geq 24$

and $x \geq 0,\ y \geq 0$

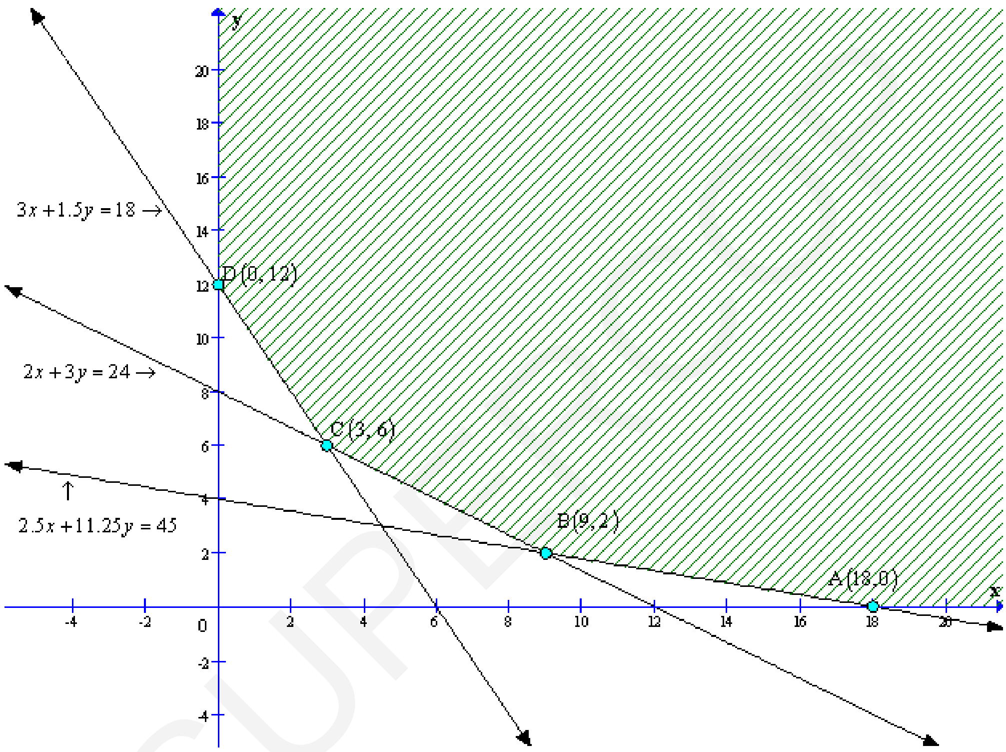
To solve the LPP we draw the lines,

$3x + 1.5y = 18$,

$2.5x + 11.25y = 45$

$2x + 3y = 24$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABCD are A(18, 0), B(9, 2), C(3, 6) and D(0, 12).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 250x + 200y$
A(18, 0)	Z = 4500
B(9, 2)	Z = 2650
C(3, 6)	Z = 1950
D(0,12)	Z = 2400

3 bags of brand P and 6 bags of brand Q should be mixed in order to prepare the mixture having a minumum cost per bag.

Mimum cost of the mixture per bag is $= \frac{1950}{9}$ = Rs. 216.67.

Note: Answer given in the book is incorrect.

Linear Programming Ex 30.3 Q14

Let x be the amount of food X and y be the amount of food Y that is to be mixed which will produce the required diet.

Then the mathematical model of the LPP is as follows:

Minimize $Z = 16x + 20y$

Subject to $x + 2y \geq 10$,

$2x + 2y \geq 12$

$3x + y \geq 8$

and $x \geq 0,\ y \geq 0$

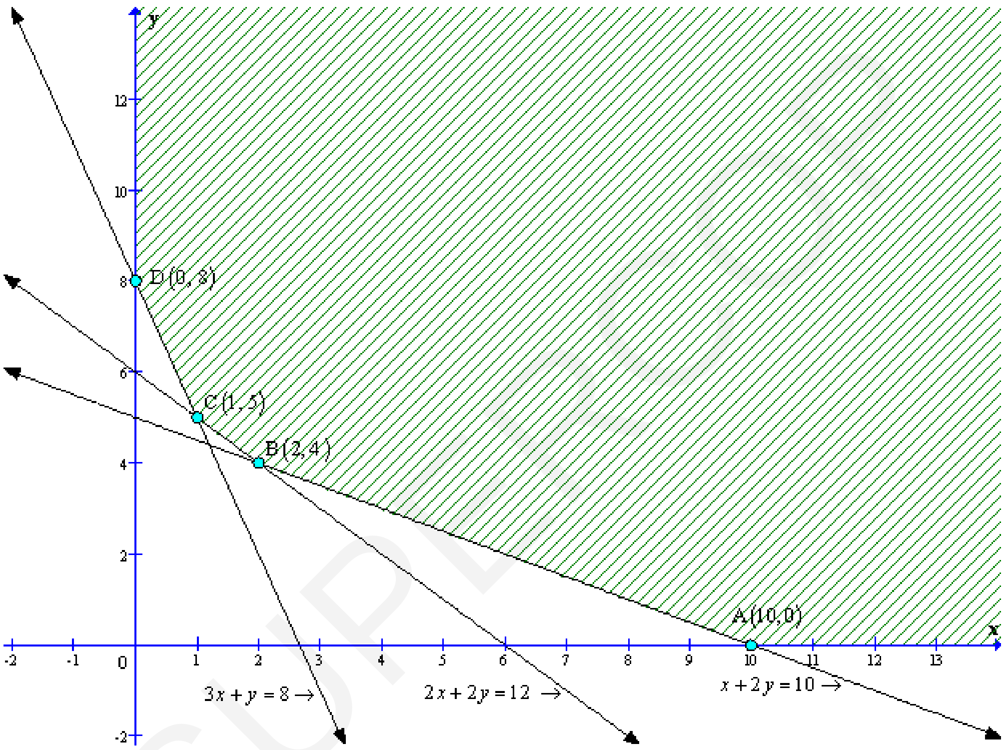
To solve the LPP we draw the lines,

$x + 2y = 10$,

$2x + 2y = 12$

$3x + y = 8$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABCD are A(10, 0), B(2, 4), C(1, 5) and D(0, 8).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 16x + 20y$
A(10, 0)	Z = 160
B(2, 4)	Z = 112
C(1, 5)	Z = 116
D(0, 8)	Z = 160

2 kg of food X and 4 kg of food y will be required to mimimize the cost of the diet. The least cost of the mixture is Rs. 112.

Linear Programming Ex 30.3 Q15

Let x bags of fertilizer P and y bags of fertilizer Q used in the garden to minimize the usage of nitrogen.

Then the mathematical model of the LPP is as follows:

Minimize $Z = 3x + 3.5y$

Subject to $x + 2y \geq 240$,

$3x + 1.5y \geq 270$

$1.5x + 2y \leq 310$

and $x \geq 0$, $y \geq 0$

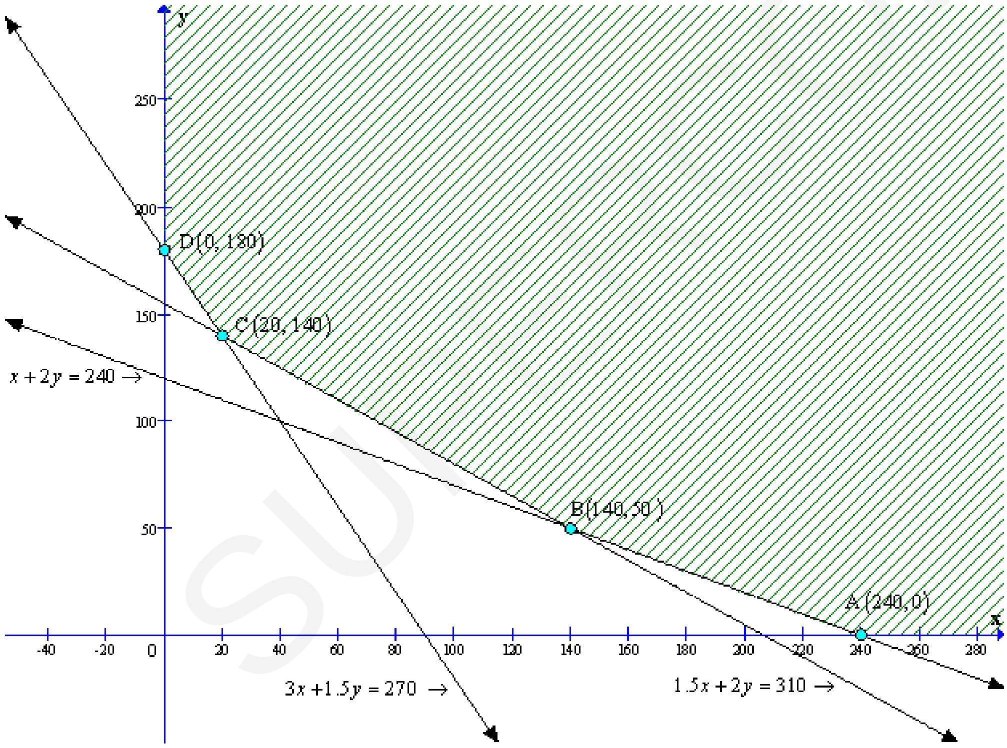
To solve the LPP we draw the lines,

$x + 2y = 240$,

$3x + 1.5y = 270$

$1.5x + 2y = 310$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(40, 100), B(140, 50) and C(20, 140).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 3x + 3.5y$
A(40, 100)	Z = 470
B(140, 50)	Z = 595
C(20, 140)	Z = 550

40 bags of brand P and 100 bags of brand Q should be used to minimize the amount of nitrogen added to the garden.

The minimum amount of nitrogen added in the garden is 470kg.

Ex 30.4

Linear Programming Ex 30.4 Q1

Let he drives x km at a speed f 25 km/hr and y km at a speed of 40 km/hr.
Let Z be total distance travelled by him, so,

$$Z = x + y$$

Since he spend Rs 2 per km on petrol when speed is 25 km/hr and Rs 5 per km on petrol when speed is 40 km/hr, so, expence on x km and y km are Rs $2x$ and Rs $5y$ respectivley, but he has only Rs 100.,so

$$2x + 5y \le 100 \qquad \text{(first constraint)}$$

$$\text{Time taken to travel } x \text{ km} = \frac{\text{Distance}}{\text{speed}} = \frac{x}{25} \text{ hr}$$

$$\text{Time taken to travel } y \text{ km} = \frac{y}{40} \text{ hr}$$

Given he has 1 hr to travel, so

$$\frac{x}{25} + \frac{y}{40} \le 1$$

$$\Rightarrow \quad 40x + 25y \le 1000$$

$$\Rightarrow \quad 8x + 5y \le 200 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

$$\text{maximize } Z = x + y$$

Subject to constriants,

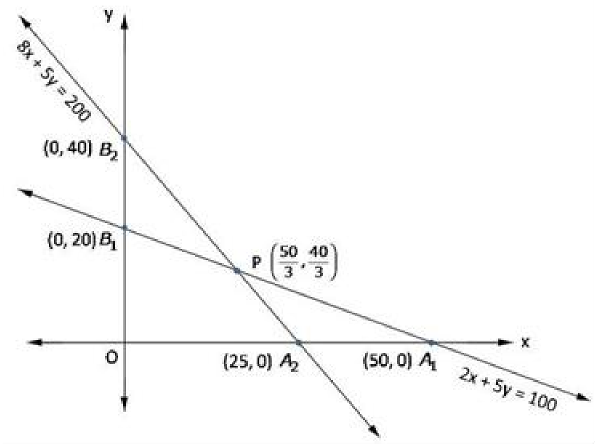
$$2x + 5y \le 100$$
$$8x + 5y \le 200$$
$$x, y \ge 0 \qquad \text{[Since distances can not be less than zero]}$$

Region $2x + 5y \le 100$: line $2x + 5y = 100$ meets axes at $A_1(50,0)$, $B_1(0,20)$ respectively.
Region containing origin represents $2x + 5y \le 100$ as $(0,0)$ satisfies $2x + 5y \le 100$.

Region $8x + 5y \le 200$: line $8x + 5y = 200$ meets axes at $A_2(25,0)$, $B_2(0,40)$ respectively.
Region containing origin represents $8x + 5y \le 200$ as $(0,0)$ satisfies $8x + 5y \le 200$.

Region $x, y \ge 0$: it represent first quandrant

Shaded region OA_2PB_1 represents feasible region.
Point $P\left(\frac{50}{3}, \frac{40}{3}\right)$ is obtained by solving $8x + 5y = 200$, $2x + 5y = 100$



The value of $Z = x + y$ at

$O(0,0) = 0 + 0 = 0$

$A_2(25,0) = 25 + 0 = 25$

$P\left(\frac{50}{3}, \frac{40}{3}\right) = \frac{50}{3} + \frac{40}{3} = 30$

$B_1(0,20) = 0 + 20 = 20$

maximum $Z = 30$ at $x = \frac{50}{3}$, $y = \frac{40}{3}$

Distance travelled at speed of 25 km/hr = $\frac{50}{3}$ km

and at speed of 40 km/hr = $\frac{40}{3}$ km

maximum distance = 30 km.

Linear Programming Ex 30.4 Q2

Let required quantity of items A and B.

Given, profits on one item A and B are Rs 6 and Rs 4 respectively So, profits on X items of type A and Y items of type B are $6x$ and Rs $4y$ respectively,
Let total profit be z, so,

$$Z = 6x + 4y$$

Given, machine I works 1 hour and 2 hours on item A and B respectively, so,
x number of item A and y number of item B need x hour and $2y$ hours on machine I respectively, but machine I works at most 12 hours, so

$$x + 2y \geq 12 \qquad \text{(first constraint)}$$

Given, machine II works 2 hours and 1 hours on item A and B respectively, so,
x number of item A and y number of item B need $2x$ hours and y hour on machine II , but machine II works maximum 12 hours, so

$$2x + y \geq 12 \qquad \text{(second constraint)}$$

Given, machine III works 1 hour and $\frac{5}{4}$ hour on one item A and B respectively, so,

x number of item A and y number of item B need x hour and $\frac{5}{4}y$ hours respectively on machine III , but machine III works at least 5 hours, so

$$x + \frac{5}{4}y \geq 5$$
$$4x + 5y \geq 20 \qquad \text{(third constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which miximize

$$z = 6x + 4y$$

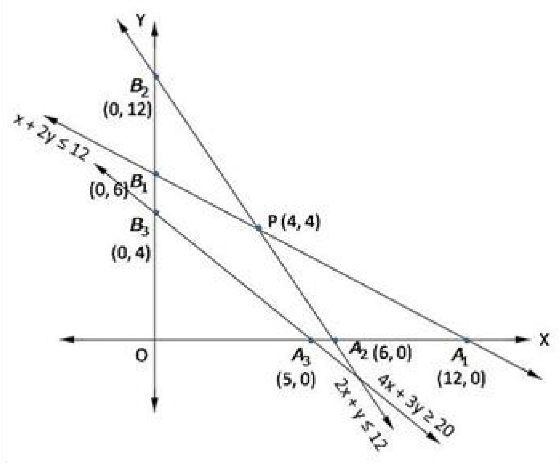
subject to constraints,

$$x + 2y \geq 12$$
$$2x + y \geq 12$$
$$4x + 5y \geq 20$$
$$x, y \geq 0$$
$$\text{[Since number of item A and B not be less than zero]}$$

Region $x + 2y \geq 12$: line $x + 2y = 12$ meets axes at $A_1(12,0), B_1(0,6)$ respectively. Region containing origin represents $x + 2y \geq 12$ as $(0,0)$ satisfies $2x + y \geq 12$.

Region $4x + 5y \geq 20$: line $4x + 5y = 20$ meets axes at $A_3(5,0), B_3(0,4)$ respectively. Region not containing origin represents $4x + 5y \geq 20$ as $(0,0)$ does not satisfy $4x + 5y \geq 20$.

Region $x, y \geq 0$: it represent first quadrant.



Shaded region $A_2A_3PB_3B_1$ represents feasible region.

The value of $Z = 6x + 4y$ at

$A_2(6,0)$	$= 6(6) + 4(0) = 36$
$A_3(5,0)$	$= 6(5) + 4(0) = 30$
$B_3(0,4)$	$= 6(0) + 4(4) = 16$
$B_2(0,6)$	$= 6(0) + 4(6) = 24$
$P(4,4)$	$= 6(4) + 4(4) = 40$

Hence, Z is maximum at $x = 4, Y = 4$

Required number of product $A = 4$, product $B = 4$

Miximum profit $=$ Rs 40

Linear Programming Ex 30.4 Q3

Suppose tailor A and B work for x and y days respectively.

Since, tailor A and B earn Rs 15 and Rs 20 respectively So, tailor A and B earn is X and Y days Rs $15x$ and $20y$ respectively, let Z denote maximum profit that gives minimum labour cost, so,

$$Z = 15x + 20y$$

Since, Tailor A and B stitch 6 and 10 shirts respectively in a day, so,
tailor A can stitch 6x and B can stitch $10y$ shirts in x and y days respectively, but it is desired to produce 60 shirts at least, so

$$6x + 10y \geq 60$$
$$3x + 5y \geq 30 \qquad \text{(first constraint)}$$

Since, Tailor A and B stitch 4 pants per day each, so,
tailor A can stitch 4x and B can stitch $4y$ pants in x and y days respectively, but it is desired to produce at least 32 pants, so

$$4x + 4y \geq 32$$
$$x + y \geq 8 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which minimize

$$Z = 15x + 20y$$

subject to constraints,

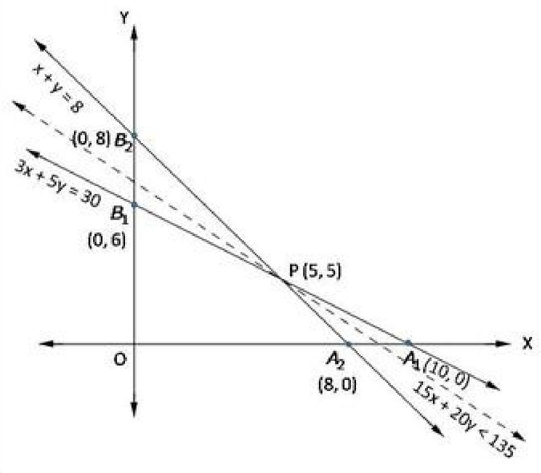
$$3x + 5y \geq 30$$
$$x + y \geq 8$$
$$x, y \geq 0$$
$$\left[\text{Since x and y not be less than zero}\right]$$

Region $3x + 5y \geq 30$: line $3x + 5y = 30$ meets axes at $A_1(10,0)$, $B_1(0,6)$ respectively. Region not containing origin represents $3x + 5y \geq 30$ as $(0,0)$ does not satisfy $3x + 5y \geq 30$.

Region $x + y \geq 8$: line $x + y = 8$ meets axes at $A_2(8,0)$, $B_2(0,8)$ respectively. Region not containing origin represents $x + y \geq 8$ as $(0,0)$ does not satisfy $x + y \geq 8$.

Region $x, y \geq 8$: it represent first quadrant.

Unbounded shaded region $A_1 P B_2$ represents feasible region with corner points $A_1(10,0)$, $P(5,3)$, $B_2(0,8)$.



The value of $Z = 15x + 20y$ at

$A_1(10, 0) = 15(10) + 20(0) = 150$

$P(5, 3) = 15(5) + 20(3) = 135$

$B_2(0, 8) = 15(0) + 20(8) = 160$

Smallest value of Z is 135 ,Now open half plane $15x + 20y < 135$ has no point in common with feasible region, so smallest value is the minimum value. So,

$Z = 135$, at $x = 5, y = 3$

Tailor A should work for 5 days and B should work for 3 days

Linear Programming Ex 30.4 Q4

Let the factory manufacture x screws of type A and y screws of type B on each day. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows.

	Screw A	**Screw B**	**Availability**
Automatic Machine (min)	4	6	$4 \times 60 = 120$
Hand Operated Machine (min)	6	3	$4 \times 60 = 120$

The profit on a package of screws A is Rs 7 and on the package of screws B is Rs 10. Therefore, the constraints are

$4x + 6y \leq 240$

$6x + 3y \leq 240$

Total profit, $Z = 7x + 10y$

The mathematical formulation of the given problem is

Maximize $Z = 7x + 10y$ ... (1)

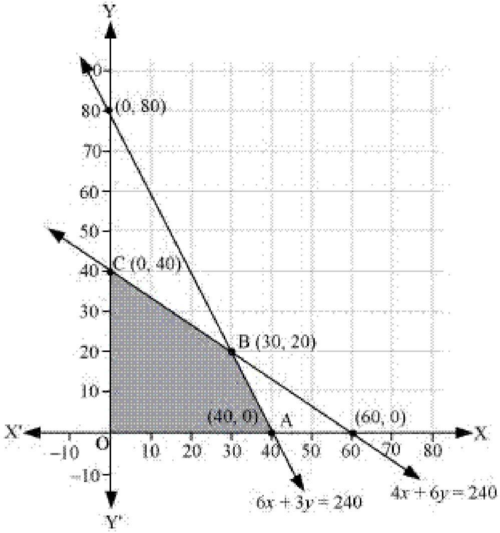
subject to the constraints,

$4x + 6y \leq 240$ ... (2)

$6x + 3y \leq 240$ ... (3)

$x, y \geq 0$ ... (4)

The feasible region determined by the system of constraints is



The corner points are A (40, 0), B (30, 20), and C (0, 40).

The values of Z at these corner points are as follows.

Corner point	**Z = 7*x* + 10*y***	
A(40, 0)	280	
B(30, 20)	410	→ Maximum
C(0, 40)	400	

The maximum value of Z is 410 at (30, 20).

Thus, the factory should produce 30 packages of screws A and 20 packages of screws B to get the maximum profit of Rs 410.

Linear Programming Ex 30.4 Q5

Let required number of belt A and B be x and y.
Given, profit on belt A and B be Rs 2 and Rs 1.50 per belt, So, profit on x belt of type A and Y belt fo type B be 2x and $1.5y$ respectively,
Let Z be total profit, so,

$$Z = 2x + 1.5y$$

Since, each belt of type A requires twice as much time as belt B. Let each belt B require 1 hour to make, so, A requires 2 hours. For x and y belts of type A and B. It required 2x and y hours to make but total time available is equal to procduction 1000 belt B that is 1000 hours, so,

$$2x + y \le 1000 \qquad \text{(first constraint)}$$

Given supply of leather only for 800 belts per day (both A and B combined), so

$$x + y \le 800 \qquad \text{(second constraint)}$$

Buckels available for A is only 400 and for B only 700, so,

$$x \le 400 \qquad \text{(third constraint)}$$

$$y \le 700 \qquad \text{(fourth constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which miximize

$$Z = 2x + 1.5y$$

subject to constraints,

$$2x + y \le 1000$$
$$x + y \le 800$$
$$x \le 400$$
$$y \le 700$$
$$x, y \ge 0$$

[Since number of belt can not be less than zero]

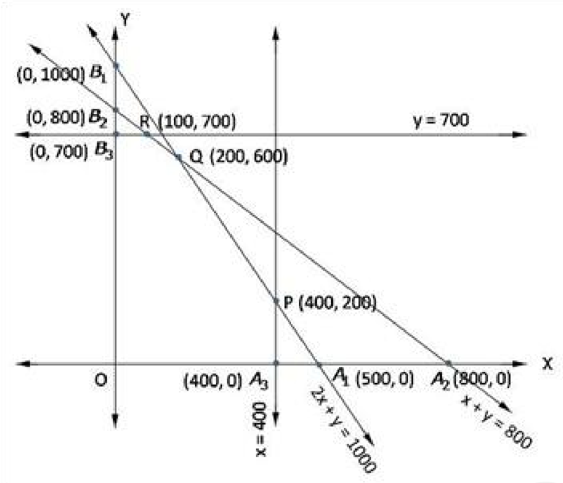
Region $2x + y \le 1000$: line $2x + y = 1000$ meets axes at $A_1(500,0), B_1(0,1000)$ respectively. Region containing origin represents $2x + y \le 1000$ as $(0,0)$ satisfies $2x + y \le 1000$.

Region $x + y \le 800$: line $x + y = 800$ meets axes at $A_2(800,0), B_2(0,800)$ respectively. Region containing origin represents $x + y \le 800$ as $(0,0)$ satisfies $x + y \le 800$.

Region Region $x \le 400$: line $x = 400$ meets axes is parallel to y axis and meet x – axis at $A_3(400,0)$. Region containing origin represents $x \le 400$ as $(0,0)$ satisfies $x \le 400$.

Region Region $y \le 400$: line $y = 700$ is parallel to x– axis and meet y – axis at $B_3(0,700)$. Region containing origin represents $y \le 700$ as $(0,0)$ satisfies $y \le 700$.

Region $x, y \ge 0$: it represent first quadrant.



Shaded region OA_3PQRB_3 is feasible region, P is points of intersections of $2x + y = 1000$ and $x = 400$, Q is the point of intersection of $x + y = 800$ and $2x + y = 1000$, R is not point of intersection of y=700, $x + y = 800$.

The value of $Z = 2x + 1.5y$ at

$O(0,0) \quad = 2(0) + 1.5(0) = 0$

$A_3(400,0) \quad = 2(400) + 1.5(0) = 800$

$P(400,200) \quad = 2(400) + 1.5(200) = 1100$

$Q(200,600) \quad = 2(200) + 1.5(600) = 1300$

$R(100,700) \quad = 2(100) + 1.5(700) = 1250$

$B_3(0,700) \quad = 2(0) + 1.5(700) = 1050$

Therefore, maximum $Z = 1300$, at $x = 200, y = 600$

Required number belt $A = 200$,belt $B = 600$
maximum profit = Rs 1300

Linear Programming Ex 30.4 Q6

Let required number of deluxe model and ordinary model be x and y respectively.

Since, profits on each model of deluxe and ordinary type model are Rs 15 and Rs 10 respectively. So, profits on x deluxe models and y ordinary models are $15x$ and $10y$

Let Z be total profit, then,

$$Z = 15x + 10y$$

Since, each deluxe and ordinary model require 2 and 1 hour of skilled men,so, x deluxe and y ordinary models required 2x and y hours of skilled men but time available by skilled men is $5 \times 8 = 40$ hours, So,

$$2x + y \le 40 \qquad \text{(first constraint)}$$

Since, each deluxe and ordinary model require 2 and 3 hours of semi-skilled men,so, x deluxe and y ordinary models require 2x and 3y hours of semi-skilled men respectively but total time available by semi-skilled men is $10 \times 8 = 80$ hours, So,

$$2x + 3y \le 80 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 15x + 10y$$

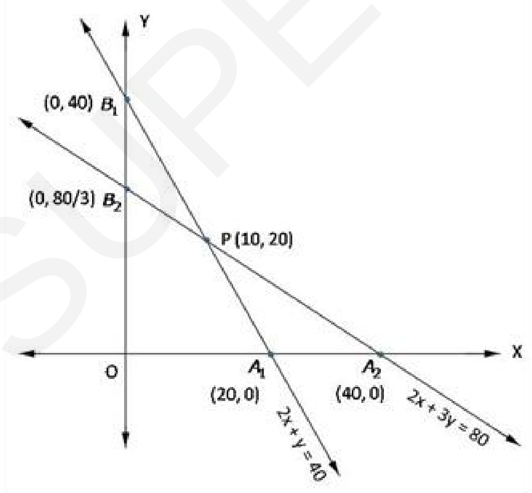
subject to constraints,

$$2x + y \le 40$$
$$2x + 3y \le 80$$
$$x, y \ge 0$$

[Since number of deluxe and ordinary models can not be less than zero]

Region $2x + y \le 40$: line $2x + y = 40$ meets axes at $A_1(20,0)$, $B_1(0,40)$ respectively. Region containing origin represents $2x + y \le 40$ as $(0,0)$ satisfies $2x + y \le 40$.

Region $2x + 3y \le 80$: line $2x + 3y = 80$ meets axes at $A_2(40,0)$, $B_2\left(0, \frac{80}{3}\right)$ respectively. Region containing origin represents $2x + 3y \le 80$.



The value of $Z = 15x + 10y$ at

$O(0,0) \qquad = 15(0) + 10(0) = 0$

$A_1(20,0) \qquad = 15(20) + 10(0) = 300$

$P(10,20) \qquad = 15(10) + 10(20) = 350$

$B_2\left(0, \frac{80}{3}\right) \qquad = 15(0) + 10\left(\frac{80}{3}\right) = \frac{800}{3}$

Therefore, maximum $Z = 350$, at $x = 10, y = 20$

Required number deluxe model = 10

number of ordinary model = 600

maximum profit =Rs 350

Linear Programming Ex 30.4 Q7

Let required number of tea-cups of type A and B are x and y respectively.

Since, profits on each tea-cups of type A and B are 75 paise and 50 paise So,
profits on x tea-cups of type A and y tea-cups of type B are $75x$ and $50y$
respectively, Let total profit on tea-cups be Z, so,

$$Z = 75x + 50y$$

Since, each tea-cup of type A and B require to work machine I for 12 and 6 minutes respectively so,
x tea cups of type B require to work on machine I for $12x$ and $6y$ minutes respectively .
Total time available on machine I is $6 \times 60 = 360$ minutes. so,

$$12x + 6y \geq 360 \qquad \text{(first constraint)}$$

Since, each tea-cup of type A and B require to work machine II for 18 and 0 minutes respectively so,
x tea cups of type A and y tea cups of B require to work on machine II for $18x$ and $0y$ minutes respectively .
but Total time available on machine II is $6 \times 60 = 360$ minutes. so,

$$18x + 0y \geq 360 \qquad \text{(second constraint)}$$

$$x \leq 20$$

Since, each tea-cup of type A and B require to work machine III for 6 and 9 minutes respectively so,
x tea cups of type A and y tea cups of B require to work on machine III for $6x$ and $9y$ minutes respectively .
Total time available on machine III is $6 \times 60 = 360$ minutes. so,

$$6x + 9y \geq 360 \qquad \text{(third constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 75x + 50y$$

subject to constraints,

$$12x + 6y \leq 360$$
$$x \leq 20$$
$$6x + 9y \leq 360$$
$$x, y \geq 0$$

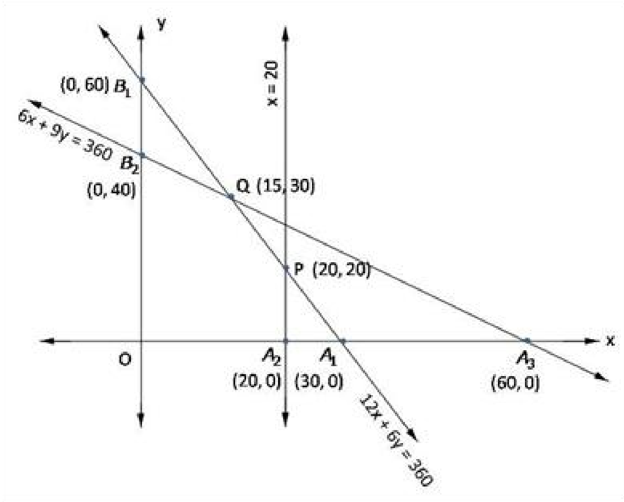
[Since production of tea cups can not be less than zero]

Region $12x + 6y \leq 360$: line $12x + 6y = 360$ meets axes at $A_1(30,0)$, $B_1(0,60)$ respectively. Region containing origin represents $12x + 6y \leq 360$ as $(0,0)$ satisfies $12x + 6y \geq 360$.

Region $x \leq 20$: line $x = 20$ is parallel to y – axes and meets x – axes at $A_2(20,0)$. Region containing origin represents $x \leq 20$ as $(0,0)$ satisfies $x \leq 20$.

Region $6x + 9y \leq 360$: line $8x + 9y = 360$ meets axes at $A_3(60,0)$, $B_2(0,40)$ respectively. Region containing origin represents $6x + 9y \leq 360$ as $(0,0)$ satisfies $6x + 9y \geq 360$.

Region $x, y \geq 0$: it represents first quadrant.



Shaded region OA_2PQB_2 is the feasible region. P is point obtained by solving $x = 20$ and $12x + 6y = 360$ and Q is point obtained by solving $12x + 6y = 360$ and $6x + 9y = 360$.

The value of $Z = 75x + 50y$ at

$O(0,0)$	$= 75(0) + 50(0) = 0$
$A_2(20,0)$	$= 75(20) + 50(0) = 1500$
$P(20,20)$	$= 75(20) + 50(20) = 2500$
$Q(15,30)$	$= 75(15) + 50(30) = 2624$
$B_2(0,40)$	$= 75(0) + 50(40) = 2000$

Hence, Z is maximum at $x = 15, Y = 30$

Therefore,

15 teacups of type A and 30 tea-cups f type B are needed to maximize profit

Linear Programming Ex 30.4 Q8

Let required number of machine A and B are x and y respectively.

Since, production of each machine A and B are 60 and 40 units daily respectively,So, productions by x number of machine A and y number of machine B are $60x$ and $40y$ respectively, Let Z denote total output daily, so,

$$Z = 60x + 40y$$

Since, each machine of type A and B require 1000 sq.m and 1200 sq.m area so, x machine of type A and y machine of type B require $100x$ and $1200y$ sq.m area but, Total area available for machine is 7600 sq.m. so,

$$1000x + 1200y \le 7600$$

$$5x + 6y \le 38 \qquad \text{(first constraint)}$$

Since, each machine of type A and B require 12 men and 8 men to work respectively so, x machine of type A and y machine of type B require $12x$ and $8y$ men to work respectively but, Total 72 men available for work so,

$$12x + 8y \le 72$$

$$3x + 2y \le 18 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 60x + 40y$$

subject to constraints,

$$5x + 6y \le 38$$

$$3x + 2y \le 18$$

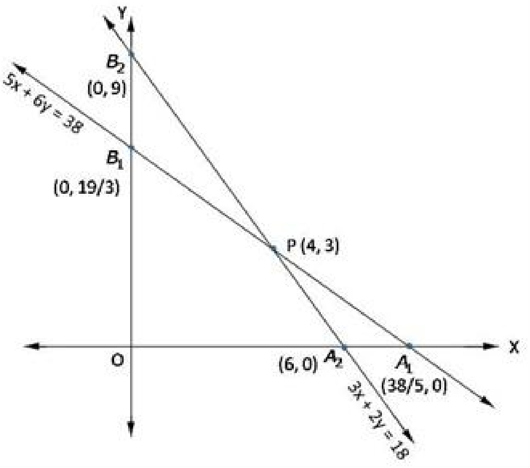
$$x, y \ge 0$$

$$\left[\text{Number of machines can not be less than zero}\right]$$

Region $5x + 6y \le 38$: line $5x + 6y = 38$ meets axes at $A_1\left(\frac{38}{5}, 0\right), B_1\left(0, \frac{19}{3}\right)$ respectively. Region containing origin represents $5x + 6y \le 38$ as origin satisfies $5x + 6y \ge 38$.

Region $3x + 2y \le 18$: line $3x + 2y = 18$ meets axes at $A_2(6,0)$, $B_2(0,9)$ respectively. Region containing origin represents $3x + 2y \le 18$ as $(0,0)$ satisfies $3x + 2y \le 18$.

Region $x, y \ge 0$: it represents first quadrant.



Shaded region OA_2PB_1 is the feasible region. $P(4, 3)$ is obtained by solving $3x + 2y = 18$ and $5x + 6y = 38$

The value of $Z = 60x + 40y$ at

$$
\begin{aligned}
&O(0,0) && = 60(0) + 40(0) = 0 \\
&A_2(6,0) && = 60(6) + 40(0) = 360 \\
&P(4,3) && = 60(4) + 40(3) = 360 \\
&B_1\left(0, \frac{19}{3}\right) && = 60(0) + 40\left(\frac{19}{3}\right) = \frac{760}{3}
\end{aligned}
$$

Therefore maximum $Z = 360$ at $x = 4, Y = 3$ or $x = 6, y = 0$

Output is maximum when 4 machines of type A and 3 machine of type B or 6 machines of type A and no machine of type B.

Linear Programming Ex 30.4 Q9

Let number of goods A and B are x and y respectively.

Since, profits on each A and B are Rs 40 and Rs 50 respectively.So,
profits on x of type A and y of type B are $40x$ and $50y$ respectively,
Let Z be total profit on A and B, so,

$$Z = 40x + 50y$$

Since, each A and B require 3 gm and 1 gm of silver respectively. so,
x of type A and y type B require $3x$ and y gm silver respectively but,
Total silver available is 9 gm. so,

$$3x + y \le 9 \qquad \text{(first constraint)}$$

Since, each A and B require 1 gm and 2 gm of gold respectively. so,
x of type A and y type B require x and $2y$ gm of gold respectively but,
Total gold available is 8 gm, so,

$$x + 2y \le 8 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 40x + 50y$$

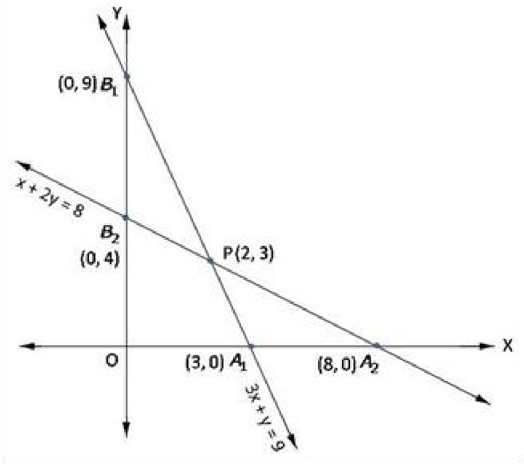
Subject to constraints,

$$3x + y \le 9$$
$$x + 2y \le 8$$
$$x, y \ge 0$$
$$\left[\text{Since production of } A \text{ and } B \text{ can not be less than zero}\right]$$

Region $3x + y \le 9$: line $3x + y = 9$ meets axes at $A_1(3,0), B_1(0,9)$ respectively. Region containing origin represents $3x + y \le 9$ as $(0,0)$ satisfies $3x + y \ge 9$.

Region $x + 2y \le 8$: line $x + 2y = 8$ meets axes at $A_2(8,0)$, $B_2(0,4)$ respectively. Region containing origin represents $x + 2y \le 8$ as $(0,0)$ satisfies $x + 2y \le 8$.

Region $x, y \ge 0$: it represents first quadrant.



Shaded region OA_2PB_2 is the feasible region.Point $P(2,3)$ is obtained by solving $3x+y=9$ and $x+2y=8$

The value of $Z=40x+50y$ at

$O(0,0) \quad = 40(0)+50(0)=0$

$A_1(3,0) \quad = 40(3)+50(0)=120$

$P(2,3) \quad = 40(2)+50(3)=230$

$B_2(0,4) \quad = 40(0)+50(4)=200$

Therefore maximum $Z=230$ at $x=2, Y=3$

Hence,

Maximum profit = Rs 230 number of goods of type $A=2$, type $B=3$

Linear Programming Ex 30.4 Q10

Let daily production of chairs and tables be x and y respectively.

Since, profits on each chair and table are Rs 3 and Rs 5. So,
profits on x number of chairs and y number of tables are Rs $3x$ and Rs $5y$ respectively,
Let Z be total profit on table and chair, so,

$$Z = 3x + 5y$$

Since, each chair and table require 2 hrs and 4 hrs on machine A respectively. so,
x number of chair and y number of table require $2x$ and $4y$ hrs on machine A respectively but,
maximum time available on machine A be 16 hrs, so,

$$2x + 4y \le 16$$
$$x + 2y \le 8 \qquad \text{(first constraint)}$$

Since, each chair and table require 6 hrs and 2 hrs on machine B. so,
x number of chair and y number of table require $6x$ and $2y$ hrs on machine B respectively but,
maximum time available on machine B be 30 hrs, so,

$$6x + 2y \le 30$$
$$3x + y \le 15 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 3x + 5y$$

subject to constraints,

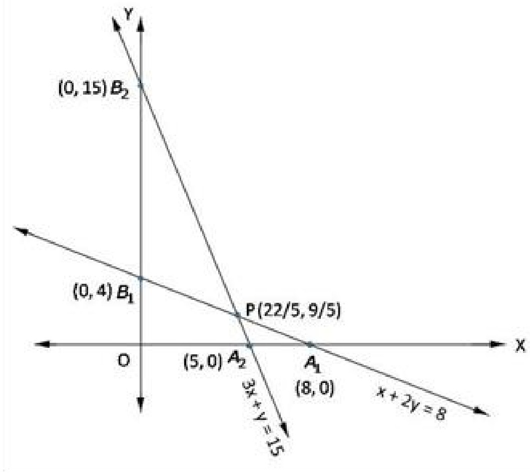
$$x + 2y \le 8$$
$$3x + y \le 15$$
$$x, y \ge 0$$

[Since production of chair and table can not be less than zero]

Region $x + 2y \le 8$: line $x + 2y = 8$ meets axes at $A_1(8, 0)$, $B_1(0, 4)$ respectively. Region containing origin represents $x + 2y \le 8$ as $(0,0)$ satisfies $x + 2y \le 8$.

Region $3x + y \le 15$: line $3x + y = 15$ meets axes at $A_2(5, 0)$, $B_2(0,15)$ respectively. Region containing origin represents $3x + y \le 15$ as $(0,0)$ satisfies $3x + y \le 15$.

Region $x, y \ge 0$: it represents first quadrant.



Shaded region $O\ A_2 P\ B_1$ representa feasible region.Point $P\left(\frac{22}{5}, \frac{9}{5}\right)$ is obtained by solving $x + 2y = 8$ and $3x + y = 15$

The value of $Z = 3x + 5y$ at

$$
\begin{aligned}
&O(0,0) && = 3(0) + 5(0) = 0 \\
&A_2(5,0) && = 3(5) + 5(0) = 15 \\
&P\left(\frac{22}{5}, \frac{9}{5}\right) && = 3\left(\frac{22}{5}\right) + 5\left(\frac{9}{5}\right) = \frac{111}{5} = 22.2 \\
&B_1(0,4) && = 3(0) + 5(4) = 20
\end{aligned}
$$

Maximum $Z = 22.2$ at $x = \frac{22}{5}, y = \frac{9}{5}$

Daily production of chair = $\frac{22}{5}$, table = $\frac{9}{5}$

maximum profit = Rs 22.2

Linear Programming Ex 30.4 Q11

Let required production of chairs and tables be x and y.

Since, profits on each chair and table are Rs 45 and Rs 80, So,
profits on x number of chairs and y number of tables are Rs $45x$ and Rs $80y$,
Let Z be total profit on tables and chairs, so,

$$Z = 45x + 80y$$

Since, each chair and table require 5 sq.ft. and 20 sq.ft. of wood respectively. so,
x number of chair and y number of table require $5x$ and $20y$ sq.ft. of wood respectively but,
400 sq.ft. of wood is available, so,

$$5x + 20y \le 400$$

$$\Rightarrow \quad x + 4y \le 80 \qquad \text{(first constraint)}$$

Since, each chair and table require 10 and 25 men-hrs respectively. so,
x number of chairs and y number of tables require $10x$ and $25y$ men-hrs
respectively but, only 450 men-hrs are available, so,

$$10x + 25y \le 450$$

$$\Rightarrow \quad 2x + 5y \le 90 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 45x + 80y$$

Subject to constraints,

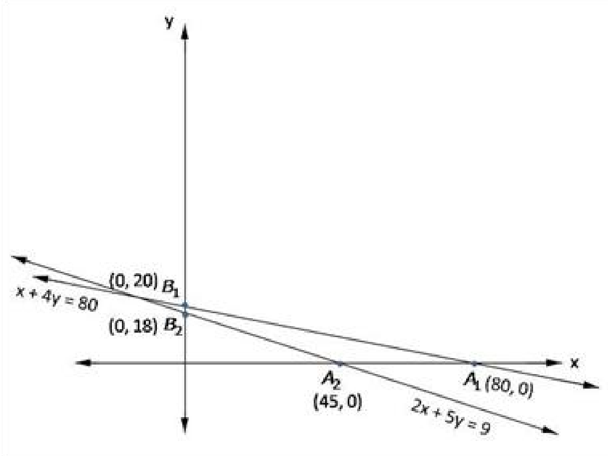
$$x + 4y \le 80$$
$$2x + 5y \le 90$$
$$x, y \ge 0$$

[Since production of tabel and chair can not be less than zero]

Region $x + 4y \le 80$: line $x + 4y = 80$ meets axes at $A_1(80,0)$, $B_1(0,20)$ respectively. Region containing origin represents $x + 4y \le 80$ as $(0,0)$ satisfies $x + 4y \le 80$.

Region $2x + 5y \le 90$: line $2x + 5y = 90$ meets axes at $A_2(45,0)$, $B_2(0,18)$ respectively. Region containing origin represents $2x + 5y \le 90$ as $(0,0)$ satisfies $2x + 5y \le 90$.

Region $x, y \ge 0$: it represents first quadrant.



Shaded region OA_2B_2 is the feasible region.

The value of $Z = 45x + 80y$ at

$O(0,0) \quad = 45(0) + 80(0) = 0$

$A_2(45,0) \quad = 45(45) + 80(0) = 2025$

$B_2(0,18) \quad = 45(0) + 80(18) = 1440$

Therefore,

Maximum $Z = 2025$ at $x = 45, y = 0$

Profit is maximum when number of chairs = 45, tables = 0

profit = Rs 2025

Linear Programming Ex 30.4 Q12

Let required production of product A and B be x and y respectively.

Since, profit on each product A and B are Rs 3 and Rs 4 respectively, So,
profit on x product A and y product B are Rs $3x$ and Rs $4y$ respectively,
Let Z be the total profit on product, so,

$$Z = 3x + 4y$$

Since, each product A and B requires 4 minutes each on machine M_1. so,
x product A and y product B require $4x$ and $4y$ minutes on machine M_1 respectively
but maximum available time on machine M_1 is 8 hrs 20 min.=500 min. so,

$$4x + 4y \le 500$$

$$\Rightarrow \quad x + y \le 125 \quad \text{(first constraint)}$$

Since, each product A and B requires 8 minutes and 4 min. on machine M_2 respectively. so,
x product A and y product B require $8x$ and $4y$ min. respectively on machine M_2
but, maximum available time on machine M_2 is 10 hrs =600 min. so,

$$8x + 4y \le 600$$

$$\Rightarrow \quad 2x + y \le 150 \quad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 3x + 4y$$

subject to constraints,

$$x + y \le 125$$
$$2x + y \le 150$$
$$x, y \ge 0$$

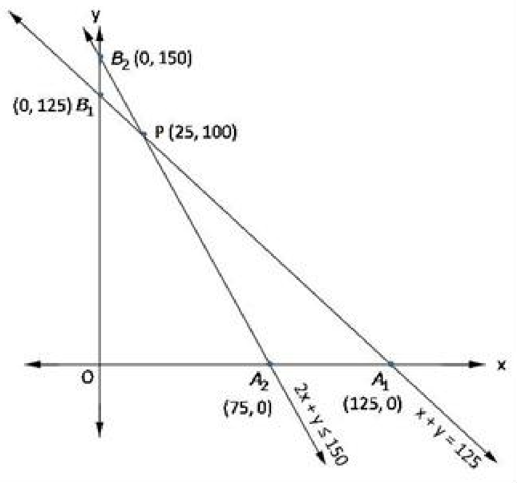
[Since number of product can not be less than zero]

Region $x + y \le 125$: line $x + y = 125$ meets axis at $A_1(125,0)$, $B_1(0,125)$ respectively. Region $x + y \le 125$ contains origin represents as $(0,0)$ satisfies $x + y \le 125$.

Region $2x + y \le 150$: line $2x + y = 150$ meets axis at $A_2(75,0)$, $B_2(0,150)$ respectively. Region containing origin represents $2x + y \le 150$ as $(0,0)$ satisfies $2x + y \le 150$

Region $x, y \ge 0$: it represents first quadrant.

Shaded region OA_2PB_1 is feasible region $P(25,100)$ is obtained by solving $x + y = 125$ and $2x + y = 150$



The value of $Z = 3x + 4y$ at

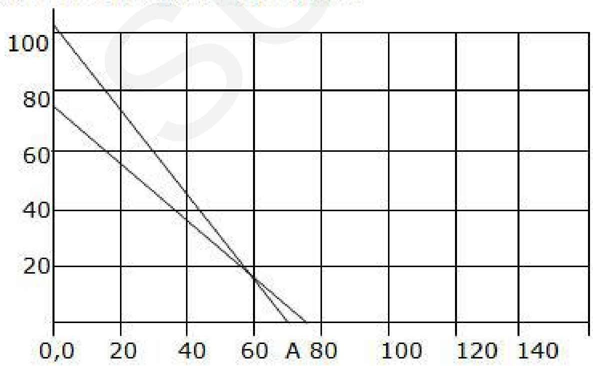
$$O(0,0) \quad = 3(0) + 4(0) = 0$$
$$A_2(75,0) \quad = 3(75) + 4(0) = 225$$
$$P(25,100) \quad = 3(25) + 4(100) = 475$$
$$B_1(0,125) \quad = 3(0) + 4(125) = 500$$

Maximum profit = Rs 500, product $A = 0$
product $B = 125$

Linear Programming Ex 30.4 Q13

	Item A	Item B	
	x	y	
Motors	3x	2y	≤ 210
Transformer	4x	4y	≤ 300
Profit Rs.	20x	30y	Maximize

The above LPP can be presented in a table above.
Aim is to find the values of x & y that maximize the function Z = 20x + 30y, subject to the conditions
3x + 2y ≤ 210; gives x=0, y= 105 & y=0, x=70
4x + 4y ≤ 300; gives x=0, y=75 & y=0, x=75
x, y ≥ 0. Plotting the constraints,



The feasible region is 80-B-A-0,0
Tabulating the value of Z at the corner points

Corner point	Value of Z = 20x + 30y
0, 0	0
0, 75	2250
70, 0	1400
60, 15	1650

The maximum occur with the production of 0 units of Item A and 75 units of Item B, with a value of Rs. 2250/-

Linear Programming Ex 30.4 Q14

Let number of I product and II product produced are x and y respectively.

Since, profits on each unit of product I and product II are 2 and 3 monetary unit, So,
profits on x units of product I and y units of product II are $2x$ and $3y$ monetary units respectively,
Let Z be total profit, so,

$$Z = 2x + 3y$$

Since, each product I and II require 2 and 4 units of resources A, so,
x units of product I and y units of product II require $2x$ and $4y$ units of resource A respectively, but maximum available quantity of resource A is 20 units. so,

$$2x + 4y \le 20$$
$$\Rightarrow \quad x + 2y \le 10 \qquad (\text{first constraint})$$

Since, each product I and II require 2 and 4 units of resource B each, so,
x units of product I and y units of product II require $2x$ and $2y$ units of resource B respectively, but maximum available quantity of resource B is 12 units. so,

$$2x + 2y \le 12$$
$$\Rightarrow \quad x + y \le 6 \qquad (\text{second constraint})$$

Since, each units of product I require 4 units of resource C. It is not required by product II , so, x units of product I require $4x$ units of resource C, but maximum available quantity of resource C is 16 units. so,

$$4x \le 16$$
$$\Rightarrow \quad x \le 6 \qquad (\text{Third constraint})$$

Hence, mathematical formulation of LPP is, Find x and y which maximize

$$Z = 2x + 3y$$

Subject to constraints,

$$x + 2y \le 10$$
$$x + y \le 6$$
$$x \le 4$$
$$x, y \ge 0$$

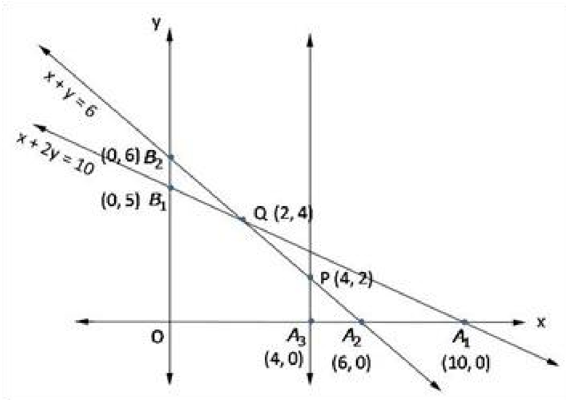
[Since production fo I and II can not be less than zero]

Region $x + 2y \le 10$: line $x + 2y = 10$ meets axes at $A_1(10,0)$, $B_1(0,5)$ respectively. Region containing origin represents $x + 2y \le 10$ as $(0,0)$ satisfies $x + 2y \le 10$.

Region $x + y \le 6$: line $x + y = 6$ meets axes at $A_2(6,0)$, $B_2(0,6)$ respectively. Region containing origin represents $x + y \le 6$ as $(0,0)$ satisfies $x + y \le 6$.

Region $x \le 4$: line $x = 4$ is parallel to y - axis and meets y-axis at $A_3(4,0)$. Region containing origin represents $x \le 4$ as $(0,0)$ satisfies $x \le 4$

Region $x, y \ge 0$: it represents first quadrant.



Shaded region OA_3PQB_1 represents feasible region $P(4,2)$ is obtained by solving $x = 4$ and $x + y = 6$, $Q(2,4)$ is obtained by solving $x + y = 6$ and $x + 2y = 10$.

The value of $Z = 2x + 3y$ at

$O(0,0)$	$= 2(0) + 3(0) = 0$
$A_3(4,0)$	$= 2(4) + 3(0) = 8$
$P(4,2)$	$= 2(4) + 3(2) = 14$
$Q(12,4)$	$= 2(12) + 3(4) = 16$
$B_1(0,5)$	$= 2(0) + 3(5) = 15$

Maximum $Z = 16$ at $x = 2, y = 4$

First product = 2 units, second product = 4 unit

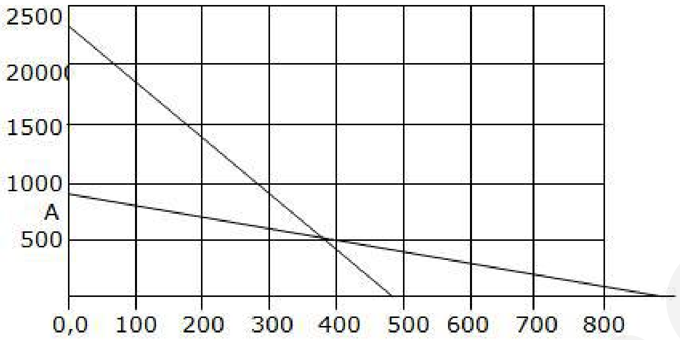
Maximum profit = 16 monetary units

Linear Programming Ex 30.4 Q15

	Hardcover	Paperback	
	x	y	
Printing time	5x	5y	≤ 4800
Binding time	10x	2y	≤ 4800
Selling price Rs.	72x	40y	Maximize

The above LPP can be presented in a table above.

Aim is to find the values of x & y that maximize the function Z = 72x + 40y, subject to the conditions
5x + 5y ≤ 4800; gives x=0, y= 960 & y=0, x=960
10x + 2y ≤ 4800; gives x=0, y=2400 & y=0, x=480
x, y ≥ 0. Plotting the constraints,



The feasible region is A-B-480-0,0
Tabulating the value of Z at the corner points

Corner point	Value of Z = 72x + 40y
0, 0	0
0, 480	19200
360, 600	49920
480, 0	34560

The maximum occurs with the production of 360 units of Hardcover books and 600 units of Paperback books, with a value of Rs. 49920/-. This the selling price.

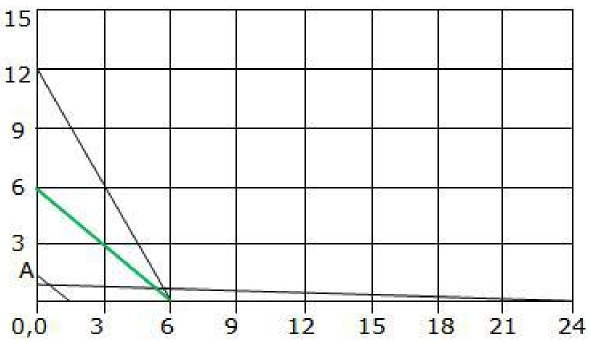
Cost price = fixed cost + variable cost
= 9600+56x360+28x600 = 46560

Profit = Selling price – cost price = 49920-46560
= Rs. 3360

Linear Programming Ex 30.4 Q16

	Pill size A	Pill size B	
	x	y	
Aspirin	2x	1.y	≥ 12
Bicarbonate	5x	8y	≥ 7.4
Codeine	1.x	66y	≥ 24
Relief	x	y	Minimize

The above LPP can be presented in a table above.
Aim is to find the values of x & y that minimize the function $Z = x + y$, subject to the conditions
$2x + y \geq 12$; gives $x=0,\ y= 12$ & $y=0,\ x=6$
$5x + 8y \geq 7.4$; gives $x=0,\ y=7.4/8$ & $y=0,\ x=7.4/5$
$x + 66y \geq 24$; gives $x=0,\ y=4/11$ & $y=0,\ x=24$
$x, y \geq 0$. Plotting the constraints,



The feasible region is 12-C-24
Tabulating the value of Z at the corner points

Corner point	Value of Z = x + y
0, 12	12
24, 0	24
5.86, 0.27	6.13

The minimum occurs with 5.86 pills of size A and 0.27 pills of size B. since the feasible region is unbounded plot $x+y < 6.13$. the green line shows here are no common points with the unbounded feasible region so the obtained point is the point that gives minimum pills to be consumed.

Linear Programming Ex 30.4 Q17

Let required quantity of compound A and B are x and y kg.

Since, cost of one kg of compound A and B are Rs 4 and Rs 6 per kg.So,
cost of x kg. of compound A and y kg. of compound B are Rs $4x$ and Rs $6y$
respectively, Let Z be the total cost of compounds, so,

$$Z = 4x + 6y$$

Since, compound A and B contain 1 and 2 units of ingredient C per kg. respectively, so,
x kg. of compound A and y kg. of compound B contain x and $2y$ units of ingredient C
respectively but minimum requirement of ingredient C is 80 units, so,

$$x + 2y \geq 80 \qquad \text{(first constraint)}$$

Since, compound A and B contain 3 and 1 unit of ingredient D per kg. respectively, so,
x kg. of compound A and y kg. of compound B contain $3x$ and y units of ingredient D
respectively but minimum requirement of ingredient D is 75 units, so,

$$3x + y \geq 75 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which minimize

$$Z = 4x + 6y$$

Subject to constraints,

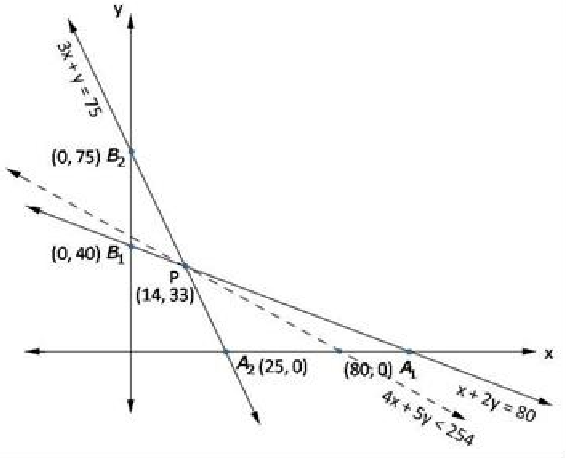
$$x + 2y \geq 80$$
$$3x + y \geq 75$$
$$x, y \geq 0$$

[Since production can not be less than zero]

Region $x + 2y \geq 80$: line $x + 2y = 80$ meets axes at $A_1(80, 0)$, $B_1(0, 40)$ respectively. Region not containing origin represents $x + 2y \geq 80$ as $(0,0)$ does not satisfy $x + 2y \geq 80$.

Region $3x + y \geq 75$: line $3x + y = 75$ meets axes at $A_2(25, 0)$, $B_2(0, 75)$ respectively. Region not containing origin represents $3x + y \geq 75$ as $(0,0)$ does not satisfy $3x + y \geq 75$.

Region $x, y \geq 0$: it represents first quadrant.



Unbouded shaded region $A_1 P B_2$ represents feasible region. point P is obtained by solving $x + 2y = 80$ and $3x + y = 75$

The value of $Z = 4x + 6y$ at

$A_1(80,0) \quad = 4(80) + 6(0) = 320$

$P(14,33) \quad = 4(14) + 6(33) = 254$

$B_2(0,75) \quad = 4(0) + 6(75) = 450$

Smallest value of $Z =$ 254 open half plane $4x + 6y < 254$ has no point in common with feasible region. so,

Smallest value is the minimum value.

Minimum cost=Rs 254
quantity of A = 14 kg
quantity of B = 33 kg

Linear Programming Ex 30.4 Q18

Let the company manufacture x souvenirs of type A and y souvenirs of type B. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be complied in a table as follows.

	Type A	Type B	Availability
Cutting (min)	5	8	$3 \times 60 + 20 = 200$
Assembling (min)	10	8	$4 \times 60 = 240$

The profit on type A souvenirs is Rs 5 and on type B souvenirs is Rs 6. Therefore, the constraints are

$5x + 8y \leq 200$

$10x + 8y \leq 240$ i.e., $5x + 4y \leq 120$

Total profit, $Z = 5x + 6y$

The mathematical formulation of the given problem is

Maximize $Z = 5x + 6y$ ... (1)

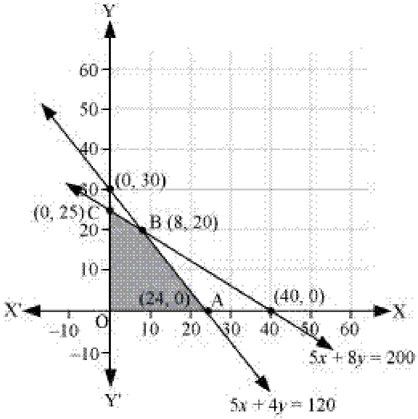
subject to the constraints,

$5x + 8y \leq 200$ ... (2)

$5x + 4y \leq 120$ ... (3)

$x, y \geq 0$ ... (4)

The feasible region determined by the system of constraints is as follows.



The corner points are A (24, 0), B (8, 20), and C (0, 25).

The values of Z at these corner points are as follows.

Corner point	**Z = 5*x* + 6*y***	
A(24, 0)	120	
B(8, 20)	160	→ Maximum
C(0, 25)	150	

The maximum value of Z is 200 at (8, 20).

Thus, 8 souvenirs of type A and 20 souvenirs of type B should be produced each day to get the maximum profit of Rs 160.

Linear Programming Ex 30.4 Q19

Let required number of product A and B be x and y respectively.

Since, profit on each product A and B are Rs 20 and Rs 30 respectively.So,
x number of product A and y number of product B gain profits of Rs $20x$ and Rs $30y$ respectively, Let Z be total profit then,

$$Z = 20x + 30y$$

Since, selling prices of each product A and B are Rs 200 and Rs 300 respectively, so, revenues earned by selling x units of product A and y units of product B are $200x$ and $300y$ respectively but weekly turnover must not be less than Rs 10000, so,

$$200x + 300y \geq 10000$$
$$2x + 3y \geq 100 \qquad \text{(first constraint)}$$

Since, each product A and B require $\frac{1}{2}$ and 1 hr.to make so, x units of product A and y units of product B are $\frac{1}{2}x$ and y hrs. to make respectively but working time available is 40 hrs maximum, so,

$$\frac{1}{2}x + y \leq 40$$
$$x + 2y \leq 80 \qquad \text{(second constraint)}$$

There is a permanent order of 14 and 16 of product A and B respectively, so,

$$x \geq 14$$
$$y \geq 16 \qquad \text{(third and fourth constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which miximize

$$Z = 20x + 30y$$

Subject to constraints,

$$2x + 3y \geq 100$$
$$x + 2y \leq 80$$
$$x \geq 40$$
$$y \geq 14$$
$$x, y \geq 0$$
$$\left[\text{Since production can not be less than zero}\right]$$

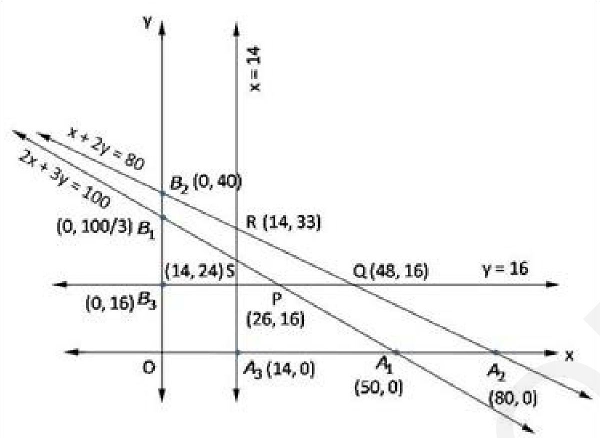
Region $2x + 3y \geq 100$: line $2x + 3y = 100$ meets axes at $A_1(50,0), B_1\left(0, \frac{100}{3}\right)$ respectively. Region not containing origin represents $2x + 3y \geq 100$ as $(0,0)$ does not satisfy $2x + 3y \geq 100$.

Region $x + 2y \le 80$: line $x + 2y = 80$ meets axes at $A_2(80,0), B_2(0,40)$ respectively. Region not containing origin represents $x + 2y \le 80$ as $(0,0)$ satisfies $x + 2y \le 80$.

Region $x \ge 14$: line $x = 14$ is parallel to y-axis and meets x-axis at $A_3(14,0)$. Region not containing origin represents $x \ge 14$ as $(0,0)$ does not satisfy $x \ge 14$.

Region $y \ge 16$: line $y = 16$ is parallel to x-axis and meets y-axis at $B_3(0,16)$. Region not containing origin represents $y \ge 16$ as $(0,0)$ does not satisfy $y \ge 16$.

Region $x, y \ge 0$: it represents first quadrant.



Shaded region $PQRS$ represents feasible region. Point $P(26,16)$ is obtained by solving $y = 16$ and $2x + 3y = 100$, $Q(48,16)$ is obtained by solving $y = 16$ and $x + 2y = 80$, $R(14,33)$ is obtained by solving $x = 14$ and $x + 2y = 80$, $S(14,24)$ is obtained by solving $x = 14$ and $2x + 3y = 100$

The value of $Z = 20x + 30y$ at

$P(26,16) \quad = 20(26) + 30(16) = 1000$

$Q(48,16) \quad = 20(48) + 3(16) = 1440$

$R(14,33) \quad = 20(14) + 3(33) = 1270$

$S(14,24) \quad = 20(14) + 3(24) = 1000$

maximum $Z = 1440$ at $x = 48, y = 16$

Number product $A = 48$, product $B = 16$

maximum profit = Rs 1440

Linear Programming Ex 30.4 Q20

Let required number of trunk I and trunk II be x and y respectively.

Since, profit on each trunk I and trunk II are Rs 30 and Rs 25 respectively.So, profit on x trunk of type I and y trunk of type II are Rs $30x$ and Rs $25y$ respectively, Let total profit on trunks be Z, so,

$$Z = 30x + 25y$$

Since, each trunk I and trunk II is reequired to work 3 hrs each on machine A, so, x trunk I and y trunk II is required $3x$ and $3y$ hrs respectively to work on machine A but machine A can work for at most 18 hrs, so,

$$3x + 3y \le 18$$

$$\Rightarrow \quad x + y \le 6 \qquad \text{(first constraint)}$$

Since, each trunk I and II is reequired to work 3 hrs and 2 hrs on machine B, so, x trunk I and y trunk II is required $3x$ and $2y$ hrs to work respectively on machine B but machine B can work for at most 15 hrs, so,

$$3x + 2y \le 15 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is, Find x and y which miximize

$$Z = 30x + 25y$$

Subject to constraints,

$$x + y \le 6$$
$$3x + 2y \le 15$$
$$x, y \ge 0$$

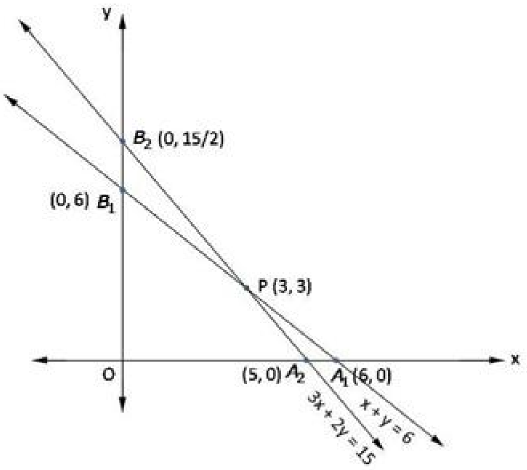
[Since production of trunk can not be less than zero]

Region $x + y \le 6$: line $x + y = 6$ meets axes at $A_1(6,0), B_1(0,6)$ respectively. Region containing origin represents $x + y \le 6$ as $(0,0)$ satisfies $x + y \le 6$.

Region $3x + 2y \le 15$: line $3x + 2y = 15$ meets axes at $A_2(5,0), B_2\left(0, \frac{15}{2}\right)$ respectively. Region containing origin represents $3x + 2y \le 15$ as $(0,0)$ satisfies $3x + 2y \le 15$.

Region $x, y \ge 0$: it represents first quadrant.

Shaded region A_2PB_1 represents feasible region. Point $P(3,3)$ is obtained by solving $x + y = 6$ and $3x + 2y = 15$,



The value of $Z = 30x + 25y$ at

$A_2(5,0) = 30(5) + 25(0) = 150$

$P(3,3) = 30(3) + 25(3) = 165$

$B_1(0,6) = 30(0) + 25(6) = 150$

$O(0,0) = 30(0) + 25(0) = 0$

maximum $Z = 165$ at $x = 3, y = 3$

Trunk of type $A = 3$, type $B = 3$

maximum profit = Rs 165

Linear Programming Ex 30.4 Q21

Let production of each bottle of A and B are x and y respectively.

Since, profits on each bottle of A and B are Rs 8 and Rs 7 per bottle respectively.So, profit on x bottles of A and y bottles of B are $8x$ and $7y$ respectively, Let Z be total profit on bottles so,

$$Z = 8x + 7y$$

Since, it takes 3 hrs and 1 hr to prepare enough material to fill 1000 bottlesof type A and B respectively, so, x bottles of A and y bottles of B are preparing is $\frac{3x}{1000}$hrs and $\frac{y}{100}$ hrs respectively but total 66 hrs are available, so,

$$\frac{3x}{1000} + \frac{y}{1000} \le 66$$

$$\Rightarrow \quad 3x + y \le 66000 \qquad \text{(first constraint)}$$

Since, row material available to make 2000 bottles of A and 4000 bottles of B but there are 45000 bottles into which either of medicines can be put so,

$$\Rightarrow \quad x \le 20000 \qquad \text{(second constraint)}$$
$$y \le 40000 \qquad \text{(third constraint)}$$
$$x + y \le 45000 \qquad \text{(fourth constraint)}$$
$$x, y \ge 0$$

[Since production of bottles can not be less than zero]

Hence, mathematical formulation of LPP is find x and y which

$$\text{maximize } Z = 8x + 7y$$

Subject to constriants,

$$3x + y \le 66000$$
$$x \le 20000$$
$$y \le 40000$$
$$x + y \le 45000$$
$$x, y \ge 0$$

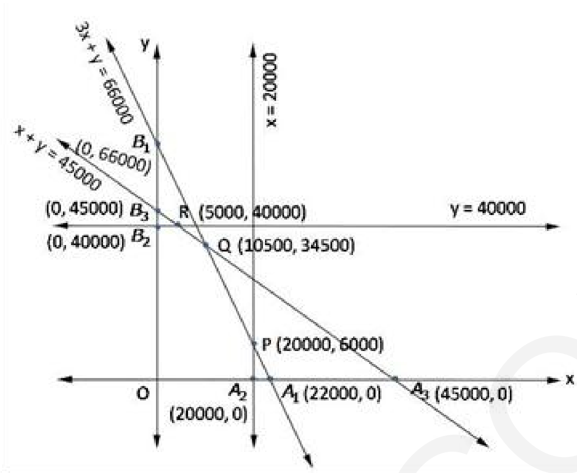
Region $3x + y \le 66000$: line $3x + y = 66000$ meets axes at $A_1(22000,0)$, $B_1(0,66000)$ respectively. Region containing origin represents $3x + y \le 66000$ as $(0,0)$ satisfies $3x + y \le 66000$.

Region $x \le 20000$: line $x = 20000$ is parallel to y-axis and meets x-axis at $A_2(20000, 0)$. Reg
containing origin represents $x \le 20000$ as $(0,0)$ satisfies $x \le 20000$.

Region $y \le 40000$: line $y = 40000$ is parallel to x-axis and meets y-axis at $B_2(0, 40000)$. Reg
containing origin represents $y \le 40000$ as $(0,0)$ satisfies $y \le 40000$.

Region $x + y \le 45000$: line $x + y = 45000$ meets axes at $A_3(45000,0)$, $B_3(0,45000)$ respectiv
containing origin represents $x + y \le 45000$ as $(0,0)$ satisfies $x + y \le 45000$.

Region $x, y \ge 0$: it represents first quadrant.



Shaded region OA_2PRB_2 represents feasible region. Point $P(20000,6000)$ is obtained by solving $x = 20000$ and $3x + y = 66000$, $Q(10500, 34500)$ is obtained by solving $x + y = 45000$ and $3x + y = 66000$, $R(15000,40000)$ is obtained by solving $x + y = 45000$, $y = 40000$

The value of $Z = 8x + 7y$ at

$O(0, 0)$	$= 8(0) + 7(0) = 0$
$A_2(20000, 0)$	$= 8(20000) + 7(0) = 160000$
$P(20000, 6000)$	$= 8(20000) + 7(6000) = 202000$
$Q(10500, 34500)$	$= 8(10500) + 7(34500) = 325500$
$R(5000, 4000)$	$= 8(5000) + 7(40000) = 32000$
$B_2(0, 40000)$	$= 8(0) + 7(40000) = 250000$

maximum $Z = 325500$ at $x = 10500, y = 34500$

Number bottles A type $= 10500$, B type $= 34500$

maximum profit = Rs 325500

Linear Programming Ex 30.4 Q22

Let required number of first class and economy class tickets be x and y respectively.

Each ticket of first class and economy class make profit of Rs 400 and Rs 600 respectively.So, x ticket of first class and y tickets of economy class make profits of Rs $400x$ and Rs $600y$ respectively, Let total profit be Z, so,

$$Z = 400x + 600y$$

Given, aeroplane can carry a miximum of 200 passengers,so,

$\Rightarrow \quad x + y \le 200 \qquad$ (first constraint)

Given, airline reservesa at least 20 seats for first class,so,

$\Rightarrow \quad x \ge 20 \qquad$ (second constraint)

Given, at least 4 times as mnay passengers prefer totravel by economy class to the first class ,so,

$y \ge 4x$

$\Rightarrow \quad 4x - y \le 0 \qquad$ (third constraint)

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 400x + 600y$

Subject to constriants,

$x + y \le 200$

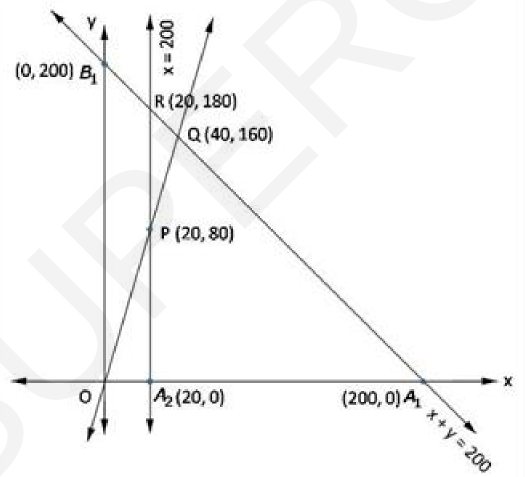
$x \ge 20$

$4x - y \le 0$

$x, y \ge 0$

[Since seats of both the classes can not be less than zero]

Region $x + y \le 200$: line $x + y = 200$ meets axes at $A_1(200,0)$, $B_1(0,200)$ respectively.
Region containing origin represents $x + y \le 200$ as $(0,0)$ satisfies $x + y \le 200$.



Shaded region PQR represents feasible region. $Q(40,160)$ is obtained by solving $x + y = 200$ and $4x - y = 0$, $R(20,180)$ is obtained by solving $x = 20$ and $x + y = 200$

The value of $Z = 400x + 600y$ at

$P(20,80) \quad = 400(20) + 600(80) = 56000$

$Q(40,160) \quad = 400(40) + 600(160) = 112000$

$R(20,180) \quad = 400(20) + 600(180) = 116000$

so,

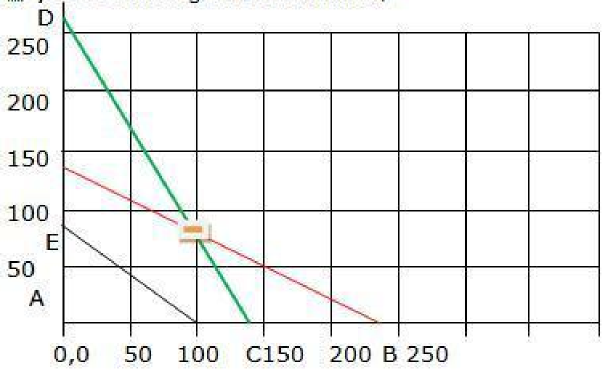
maximum $Z =$ Rs 116000 at $x = 20, y = 180$

Number of first class ticket = 20,
Number of economy class ticket= 180
maximum profit = Rs 116000

Linear Programming Ex 30.4 Q23

	Type I	Type II	
	x	y	
Nitrogen	0.1x	0.05y	≥ 14
Bicarbonate	0.06x	0.1y	≥ 14
Cost	0.6x	0.4y	Minimize

The above LPP can be presented in a table above.
Aim is to find the values of x & y that minimize the function $Z = 0.6x + 0.4y$, subject to the conditions
$0.1x + 0.05y \geq 14$; gives $x=0$, $y= 280$ & $y=0$, $x=140$
$0.06x + 0.1y \geq 14$; gives $x=0$, $y=140$ & $y=0$, $x=233.33$
$x, y \geq 0$. Plotting the constraints,



The feasible region is the unbounded region D-C-B

Corner point	Value of $Z = 0.6x + 0.4y$
0, 280	112
233.33, 0	140
100, 80	92

The minimum occurs at $x=100$, $y=80$ with a value of 92
Since the region is unbounded plot $0.6x + 0.4y \leq 92$
Plotting the points, we get line E-100.
There are no common points so $x=100$, $y=80$ with a value of 92 is the optimal minimum.

Linear Programming Ex 30.4 Q24

Let he invests Rs x and Rs y in saving certificate (sc) and National saving bond (NSB) respectively.

Since, rate of interest on SC is 8% annual and on NSB is 10% annual,So, interest on Rs x of SC is $\frac{8x}{100}$ and Rs y of NSB is $\frac{10x}{100}$ per annum.

Let Z be total interest earned so,

$$Z = \frac{8x}{100} + \frac{10y}{100}$$

Given he wants to invest Rs 12000 is total

$$x + y \le 12000 \qquad \text{(third constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

$$\text{maximize } Z = \frac{8x}{100} + \frac{10y}{100}$$

Subject to constriants,

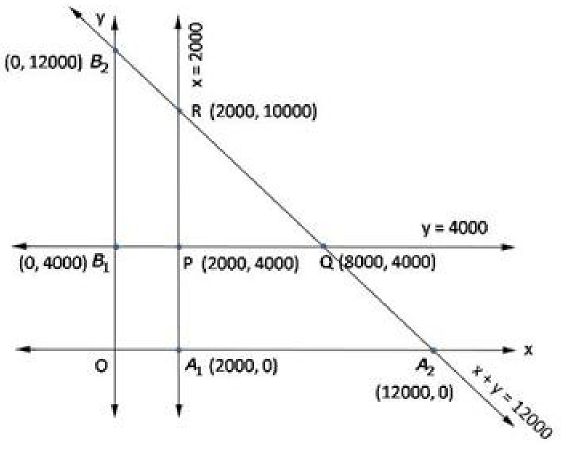
$$x \ge 2000$$
$$y \ge 4000$$
$$x + y \le 12000$$
$$x, y \ge 0 \qquad [\text{Since investment can not be less than zero}]$$

Region $x \ge 2000$: line $x = 2000$ is parallel to y-axis and meets x-axis at $A_1(2000,0)$.
Region not containing origin represents $x \ge 2000$ as $(0,0)$ does not satisfy $x \ge 2000$

Region $y \ge 4000$: line $y = 4000$ is parallel to x-axis and meets y-axis at $B_1(0,4000)$. Region not containing origin represents $y \ge 4000$ as $(0,0)$ does not satisfy $y \ge 4000$.

Region $x + y \le 12000$: line $x + y = 12000$ meets axes at $A_2(12000,0)$, $b_2(0,1200)$ respectively.
Region containing represents $x + y \le 12000$ as $(0,0)$ satisfies $x + y \le 12000$.

Region $x, y \ge 0$: it represents first quadrant.



Shaded region PQR represents feasible region. $P(2000, 4000)$ is obtained by solving $x = 2000$ and $y = 4000$, $Q(8000, 4000)$ is obtained by solving $x + y = 12000$ and $y = 4000$ $R(2000, 10000)$ is obtained by solving $x = 2000$ and $y + x = 1200$

The value of $Z = \frac{8x}{100} + \frac{10y}{100}$ at

$P(2000, 4000)$ $= \frac{8}{100}(2000) + \frac{10}{100}(4000) = 560$

$Q(8000, 4000)$ $= \frac{8}{100}(8000) + \frac{10}{100}(4000) = 1040$

$R(2000, 10000)$ $= \frac{8}{1000}(2000) + \frac{10}{100}(10000) = 1160$

so,

maximum Z = Rs 1160 at $x = 2000, y = 10000$

He should invest Rs 2000 in Saving Certificates and 1000 in National Saving scheme, maximum Interest = Rs 1160

Linear Programming Ex 30.4 Q25

Let required number of trees of type A and B be Rs x and Rs y respectively.

Since, selling price of 1 kg of type A is Rs 2 and growth is 20 kg per tree, so, revenue from type A is Rs 40x , selling price of 1 kg of type B is Rs 1.5 and growth 40 kg per tree, so, revenue from type B is Rs $60y$. Total revenue is $(40x+60y)$. Costs of each tree of type A and B are Rs 20 and Rs 25, so,costs of x trees of type A and y trees of type B are Rs $20x$ and $25y$ respectively.
Total cost is Rs $(20x+25y)$

Let Z be total profit so,

$$Z = (40x - 60y) - (20x + 25y)$$
$$Z = 20x + 35y$$

Since he has Rs 1400 to invest so,

$$\text{cost} \le 1400$$

$$\Rightarrow \quad 20x + 35y \le 1400$$
$$\Rightarrow \quad 4x + 5y \le 280 \qquad (\text{ first constraint})$$

Since each tree of type A and B needs 10 sq. m and 20 sq. m of ground respectively so, x trees of type A and y trees of type B need $10x$ sq. m and $20y$ sq. m of ground respectively. but total ground available is 1000 sq. m so,

$$10x + 20y \le 1000$$

$$\Rightarrow \quad x + 2y \le 100 \qquad (\text{second constraint})$$
$$x, y \ge 0$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 20x + 35y$

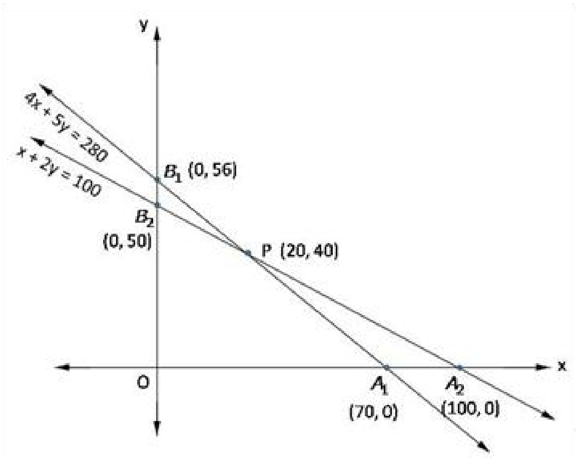
Subject to constriants,

$$4x + 5y \le 280$$
$$\Rightarrow \quad x + 2y \le 100$$
$$x, y \ge 0 \qquad [\text{Since number of trees can not be less than zero}]$$

Region $4x + 5y \le 280$: line $4x + 5y = 280$ meets axes at $A_1(70,0)$, $B_1(0,56)$ respectively.
Region containing origin represents $4x + 5y \le 280$ as $(0,0)$ satisfies $4x + 5y \le 280$.

Region $x + 2y \le 100$: line $x + 2y = 100$ meets axes at $A_2(100,0)$, $B_2(0,50)$ respectively.
Region containing origin represents $x + 2y \le 100$ as $(0,0)$ satisfies $x + 2y \le 100$.

Region $x, y \ge 0$: it represents first quadrant.



Shaded region OA_1PB_2 the feasible region. $P(20,40)$ is obtained
by solving $x + 2y = 100$ and $4x + 5y = 280$,

The value of $Z = 20x + 35y$ at

$O(0,0)$	$= 20(0) + 35(0) = 0$
$A_1(70,0)$	$= 20(0) + 35(0) = 1400$
$P(20,40)$	$= 20(20) + 35(40) = 1800$
$B_2(0,50)$	$= 20(0) + 35(50) = 1750$

maximum $Z = 1800$ at $x = 20, y = 40$

20 trees of type A , 40 trees of type B, profit = Rs 1800

Linear Programming Ex 30.4 Q26

Let the cottage industry manufacture x pedestal lamps and y wooden shades.
Therefore,

$x \ge 0$ and $y \ge 0$

The given information can be compiled in a table as follows.

	Lamps	Shades	Availability
Grinding/Cutting Machine (h)	2	1	12
Sprayer (h)	3	2	20

The profit on a lamp is Rs 5 and on the shades is Rs 3. Therefore, the constraints are

$2x + y \le 12$

$3x + 2y \le 20$

Total profit, $Z = 5x + 3y$

The mathematical formulation of the given problem is

Maximize $Z = 5x + 3y$ ... (1)

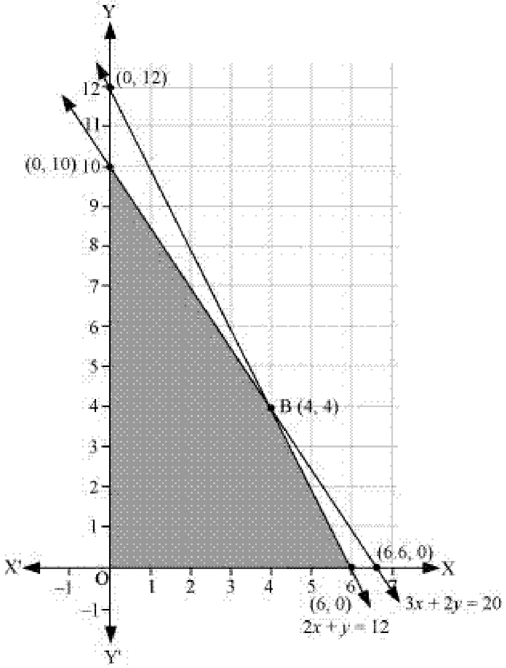
subject to the constraints,

$2x + y \le 12$ ... (2)

$3x + 2y \le 20$ ... (3)

$x, y \ge 0$ ... (4)

The feasible region determined by the system of constraints is as follows.



The corner points are A (6, 0), B (4, 4), and C (0, 10).

The values of Z at these corner points are as follows

Corner point	$Z = 5x + 3y$	
A(6, 0)	30	
B(4, 4)	32	→ Maximum
C(0, 10)	30	

The maximum value of Z is 32 at (4, 4).

Thus, the manufacturer should produce 4 pedestal lamps and 4 wooden shades to maximize his profits.

Linear Programming Ex 30.4 Q27

Let required number of goods of type x and y be x_1 and x_2 respectively.

Since, selling prices of each goods of type x and y are Rs 100 and Rs 120 respectively, so, selling price of x_1 units of goods of type x and x_2 units of goods of type y are Rs $100x$ and Rs $120y$ respective respectively

Let Z be total revenue, so

$$Z = 100x_1 + 120x_2.$$

Since each unit of goods x and y require 2 and 3 units of labour, so, x_1 unit of x and x_2 unit of y require $2x_1$ and $3x_2$ units of labour units but maximum labour units available is 30 units, so,

$$2x_1 + 3x_2 \le 30 \qquad \text{(first constraint)}$$

Since each unit of goods x and y require 3 and 1 unit of capital so, x_1 unit of x and x_2 unit of y require $3x_1$ and x_2 units of capital respectively but maximum units available for capital is 17 , so,

$$3x_1 + x_2 \le 17 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 20x + 35y$

Subject to constriants,

$$2x_1 + 3x_2 \le 30$$

$$\Rightarrow \quad 3x_1 + x_2 \le 17$$

$$x_1, x_2 \ge 0 \qquad \text{[Since production of goods can not be less than zero]}$$

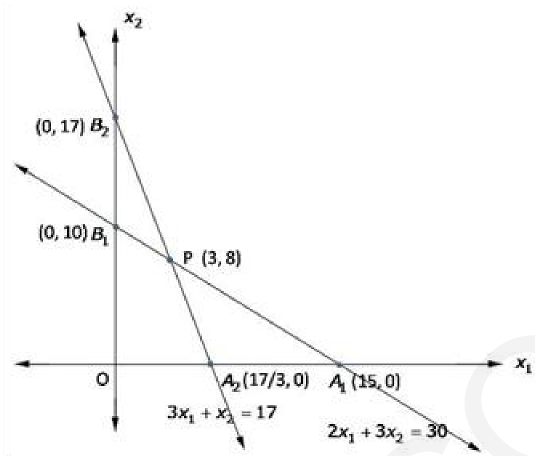
Region $2x_1 + 3x_2 \le 30$: line $2x + 3y = 30$ meets axes at $A_1(15,0)$, $B_1(0,10)$ respectively.
Region containing origin represents $2x_1 + 3x_2 \le 30$ as $(0,0)$ satisfies $2x_1 + 3x_2 = 30$.

Region $3x_1 + x_2 \le 17$: line $3x_1 + x_2 \le 17$ meets axes at $A_2\left(\frac{17}{3},0\right)$, $B_2(0,17)$ respectively.

Region containing origin represents $3x_1 + x_2 \le 17$ as $(0,0)$ satisfies $3x_1 + x_2 \le 17$.

Region $x_1, x_2 \ge 0$: it represent first quandrant shaded region OA_2PB_1 represents feasible region. Point $P(3,8)$ is obtained by solving

$2x_1 + 3x_2 = 30$ and $3x_1 + x_2 = 17$



The value of $Z = 100x_1 + 120x_2$ at

$O(0,0) \quad = 100(0) + 120(0) = 0$

$A_2\left(\frac{17}{3},0\right) \quad = 100\left(\frac{17}{3}\right) + 120(0) = \frac{1700}{3} = 566\frac{2}{3}$

$P(3,8) \quad = 100(3) + 120(8) = 1260$

$B_1(0,10) \quad = 100(0) + 120(10) = 1200$

maximum $Z = 1260$ at $x = 3, y = 8$

goods of type $x = 3$, type $y = 8$
maximum profit = Rs 12160

Linear Programming Ex 30.4 Q28

Let required number of product A and B be x and y respectively.

Since,profit on each product A and B are Rs 5 and Rs 3 respectively, so,
profits on x product A and y product B are Rs $5x$ and Rs $3y$ respectively
Let Z be total profit so

$$Z = 5x + 3y$$

Since each unit of product A and B require one min. each on machine M_1 so,
x unit of product A and y units of product B require x and y min. respectivley
on machine M_1 but M_1 can work at most $5 \times 60 = 300$ min.,so

$$x + y \leq 300 \qquad \text{(first constraint)}$$

Since each unit of product A and B require 2 and one min.respectively on machine
M_2 so, x unit of product A and y units of product B require $2x$ and y min. respectivley
on machine M_2 but M_2 can work at most $6 \times 60 = 360$ min.,so

$$2x + y \leq 360 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

$$\text{maximize } Z = 5x + 3y$$

Subject to constriants,

$$x + y \leq 300$$

$$\Rightarrow \quad 2x + y \leq 360$$

$$x, y \geq 0 \qquad \left[\text{Since production can not be less than zero}\right]$$

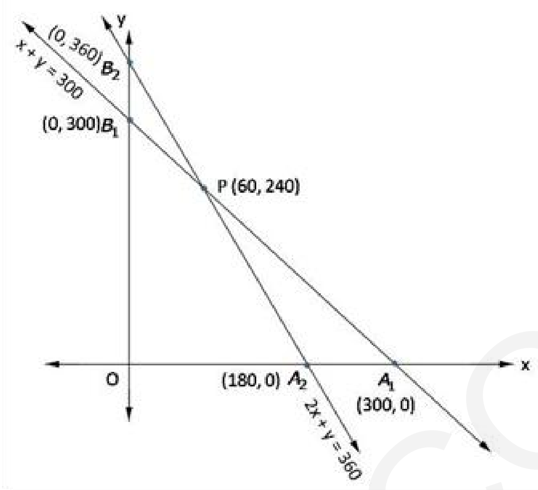
Region $x+y \le 300$: line $x+y=300$ meets axes at $A_1(300,0)$, $B_1(0,300)$ respectively.
Region containing origin represents $x+y \le 300$ as $(0,0)$ satisfies $x+y=300$.

Region $2x+y \le 360$: line $2x+y=360$ meets axes at $A_2(180,0)$, $B_2(0,360)$ respectively.
Region containing origin represents $2x+y \le 360$ as $(0,0)$ satisfies $2x+y \le 360$.

Region $x, y \ge 0$: it represent first quandrant

Shaded region OA_2PB_1 represents feasible region.
Point $P(60,240)$ is obtained by solving

$x+y=300$ and $2x+y=360$



The value of $Z = 5x+3y$ at

$O(0,0) = 5(0)+3(0)=0$

$A_2(180,0) = 5(180)+3(0)=900$

$P(60,240) = 5(60)+3(240)=1020$

$B_1(0,300) = 5(0)+3(300)=900$

maximum $Z=1020$ at $x=60, y=240$

Number of product $A=60$, product $B=240$
maximum profit = Rs 1020

Linear Programming Ex 30.4 Q29

Let required quantity of item A and B produced be x and y respectively.

Since,profits on each item A and B are Rs 300 and Rs 160 respectively, so, profits on x unit of item A and y units of item B are Rs $300x$ and Rs $160y$ respectively
Let Z be total profit so

$$Z = 300x + 160y$$

Since one unit of item A and B require one and $\frac{1}{2}$ hr respectively, so, x units of item A and y units of item B require x and $\frac{1}{2}y$ hr. respectivley but maximum time available is 16 hours.,so

$$x + \frac{1}{2}y \le 16$$

$$\Rightarrow \quad 2x + y \le 32 \qquad \text{(first constraint)}$$

Given, manufacturer can produce at most 24 items, so,

$$\Rightarrow \quad x + y \le 24 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

$$\text{maximize } Z = 300x + 160y$$

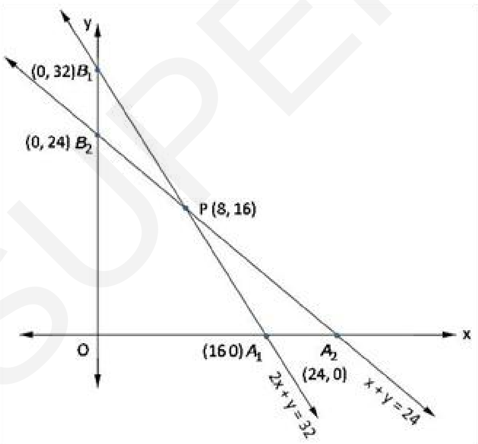
Subject to constriants,

$$2x + y \le 32$$
$$x + y \le 24$$
$$x, y \ge 0 \qquad \text{[Since production can not be less than zero]}$$

Region $2x + y \le 32$: line $2x + y = 32$ meets axes at $A_1(16,0)$, $B_1(0,32)$ respectively.
Region containing origin represents $2x + y \le 32$ as $(0,0)$ satisfies $2x + y \le 32$.

Region $x + y \le 24$: line $x + y = 24$ meets axes at $A_2(24,0)$, $B_2(0,24)$ respectively.
Region containing origin represents $x + y \le 24$ as $(0,0)$ satisfies $x + y \le 24$.

Region $x, y \ge 0$: it represent first quandrant



Shaded region OA_1PB_2 represents feasible region.
Point P is obtained by solving

$$x + y = 24 \text{ and } 2x + y = 32$$

The value of $Z = 300x + 160y$ at

$$O(0,0) \qquad = 300(0) + 160(0) = 0$$
$$A_1(16,0) \qquad = 300(180) + 160(0) = 4800$$
$$P(8,16) \qquad = 300(8) + 160(16) = 4960$$
$$B_2(0,24) \qquad = 300(0) + 160(24) = 3640$$

maximum $Z = 4960$

Number of item $A = 8$, item $B = 16$
maximum profit = Rs 4960

Linear Programming Ex 30.4 Q30

Let number of toys of type A and B produced are x and y respectively.

Since, profits on each unit of toys A and B are Rs 50 and Rs 60 respectively, so, profits on x units of toys A and y units of toy B are Rs $50x$ and Rs $60y$ respectively
Let Z be total profit so

$$Z = 50x + 60y$$

Since each unit of toy A and toy B require 5 min. and 8 min. on cutting, so, x units of toy A and y units of toy B require $5x$ and $8y$ min. respectivley but maximum time available for cutting $3 \times 60 = 180$ min., so

$$5x + 8y \le 180 \qquad \text{(first constraint)}$$

Since each unit of toy A and toy B require 10 min. and 8 min. for assembling, so, x units of toy A and y units of toy B require $10x$ and $8y$ min. for assembling respectivley but maximum time available for assembling is $4 \times 60 = 240$ min., so

$$10x + 8y \le 240$$
$$\Rightarrow \quad 5x + 4y \le 120 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 50x + 60y$

Subject to constriants,

$$5x + 8y \le 180$$
$$5x + 4y \le 120$$
$$x, y \ge 0 \qquad \text{[Since production can not be less than zero]}$$

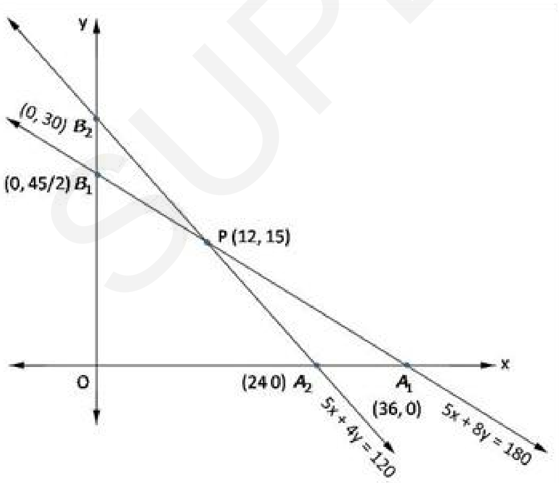
Region $5x + 8y \le 180$: line $5x + 8y = 180$ meets axes at $A_1(36,0)$, $B_1\left(0, \frac{45}{2}\right)$ respectively.
Region containing origin represents $5x + 8y \le 180$ as $(0,0)$ satisfies $5x + 8y \le 180$.

Region $5x + 4y \le 120$: line $5x + 4y = 120$ meets axes at $A_2(24,0)$, $B_2(0,30)$ respectively.
Region containing origin represents $5x + 4y \le 120$ as $(0,0)$ satisfies $5x + 4y \le 120$.

Region $x, y \ge 0$: it represent first quandrant

Shaded region OA_2PB_1 represents feasible region.
Point $P(12, 15)$ is obtained by solving

$$5x + 8y = 180 \text{ and } 5x + 4y = 120$$



The value of $Z = 50x + 60y$ at

$$O(0,0) \qquad = 50(0) + 60(0) = 0$$
$$A_2(24,0) \qquad = 50(24) + 60(0) = 1200$$
$$P(12,15) \qquad = 50(12) + 60(15) = 1500$$
$$B_1\left(0, \frac{45}{2}\right) \qquad = 50(0) + 60\left(\frac{45}{2}\right) = 1350$$

Maximum $Z = 1500$ at $x = 12$, $y = 15$

Number of toys $A = 12$, toys $B = 15$
maximum profit = Rs 1500

Linear Programming Ex 30.4 Q31

Let required number of product A and B are x and y respectively.

Since,profits on each unit of product A and product B are Rs 6 and Rs 8 respectively, so, profits on x units of product A and y units of product B are Rs $6x$ and Rs $8y$ respectively
Let Z be total profit so

$$Z = 6x + 8y$$

Since each unit of product A and B require 4 and 2 hrs for assembling respectively, so, x units of product A and y units of product B require $4x$ and $2y$ hrs for assembling respectivley but maximum time available for assembling is 60 hrs.,so

$$4x + 2y \le 60$$
$$2x + y \le 30 \qquad \text{(first constraint)}$$

Since each unit of product A and B require 2 and 4 hrs for finishing, so, x units of product A and y units of product B require $2x$ and $4y$ hrs for finishing respectivley but maximum time available for finishing is 48 hrs.,so

$$2x + 4y \le 48$$
$$x + 2y \le 24 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 6x + 8y$

Subject to constriants,

$$2x + y \le 30$$
$$x + 2y \le 24$$
$$x, y \ge 0 \qquad [\text{Since production of both can not be less than zero}]$$

Region $2x + y \le 30$: line $2x + y = 24$ meets axes at $A_1(15,0)$, $B_1(0,30)$ respectively.
Region containing origin represents $2x + y \le 30$ as $(0,0)$ satisfies $2x + y \le 30$.

Region $x + 2y \le 24$: line $x + 2y = 24$ meets axes at $A_2(24,0)$, $B_2(0,12)$ respectively.
Region containing origin represents $x + 2y \le 24$ as $(0,0)$ satisfies $x + 2y \le 24$.

Region $x, y \ge 0$: it represent first quandrant

Shaded region OA_1PB_2 represents feasible region.
Point $P(12,6)$ is obtained by solving

$$x + 2y = 24 \text{ and } x + 2y = 30$$

The value of $Z = 6x + 8y$ at

$O(0,0) \quad = 6(0) + 8(0) = 0$
$A_1(15,0) \quad = 6(15) + 8(0) = 90$
$P(12,6) \quad = 6(12) + 8(6) = 120$
$B_2(0,12) \quad = 6(0) + 8(12) = 96$

maximum $Z = 120$ at $x = 12$, $y = 6$

Number of product $A = 12$, product $B = 6$
maximum profit = Rs 120

Linear Programming Ex 30.4 Q32

Let x & y be the No. of items of A & B respectively.

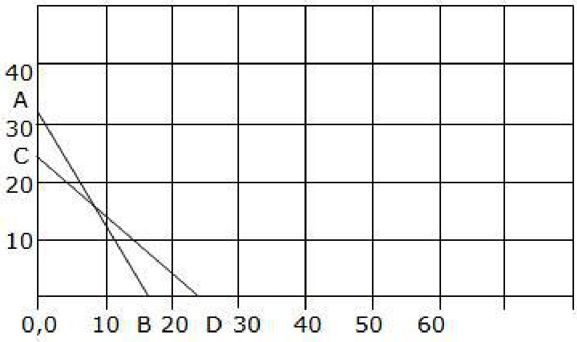
$x + y = 24$ (total No. of items constraint)

$x + 0.5y \le 16$ (time constraint)

$x, y \ge 0$

$Z = 300x + 160y$ (profit function to be maximized)

Plotting the inequalities gives,



The feasible region is 0,0-C-F-B

Corner point	Value of Z = 300x + 160y
0, 0	0
0, 24	3840
16, 0	4800
8 ,16	4960

The firm must produce 8 items of A and 16 items of B to maximize the profit at Rs. 4960/-

Linear Programming Ex 30.4 Q33

Let required number of product A and B are x and y respectively.

Since,profits on each unit of product A and product B are Rs 20 and Rs 15 respectively, so, x units of product A and y units of product B give profit of Rs $20x$ and Rs $15y$ respectively

Let Z be total profit so

$$Z = 20x + 15y$$

Since each unit of product A and B require 5 and 3 man-hrs respectively, so, x units of product A and y units of product B require $5x$ and $3y$ man-hrs respectivley but maximum time available for is 500 man-hrs., so

$5x + 3y \le 500$ (first constraint)

Since maximum number that product A and B can be sold is 70 and 125 respectively, so,

$x \le 70$ (second constraint)

$y \le 125$ (third constraint)

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 20x + 15y$

Subject to constriants,

$5x + 3y \le 500$

$x \le 70$

$y \le 70$

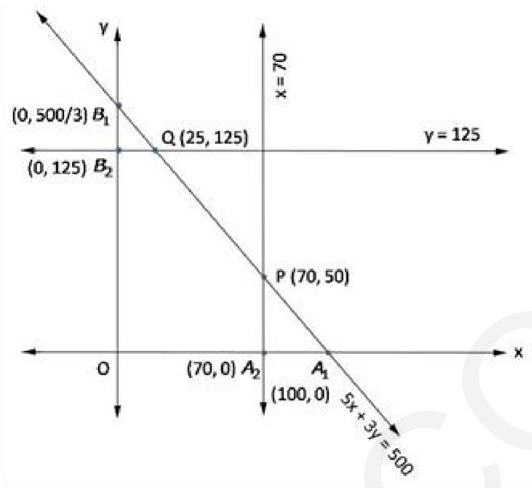
$x, y \ge 0$ [Since production of both can not be less than zero]

Region $5x + 3y \le 500$: line $5x + 3y = 500$ meets axes at $A_1(100,0)$, $B_1\left(0, \frac{500}{3}\right)$ respectively.
Region containing origin represents $5x + 3y \le 500$ as $(0,0)$ satisfies $5x + 3y \le 500$.

Region $x \le 70$: line $x = 70$ is parallel to y – axis meets x–axes at $A_2(70,0)$. Region containing origin represents $x \le 70$ as $(0,0)$ satisfies $x \le 70$.

Region $y \le 125$: line $y = 125$ is parallel to x – axis meets y–axes at $B_2(0,125)$. with y – axis.
Region containing origin represents $y \le 125$ as $(0,0)$ satisfies $y \le 125$.

Region $x, y \ge 0$: it represent first quandrant.



Shaded region OA_2PQB_2 represents feasible region.
Point $P(70,50)$ is obtained by solving $x = 70$
Point $Q(25,125)$ is obtained by solving $y = 125$ and $5x + 3y = 500$.

The value of $Z = 20x + 15y$ at

$O(0,0)$	$= 20(0) + 15(0) = 0$
$A_2(70,0)$	$= 20(70) + 15(0) = 1400$
$P(70,50)$	$= 20(70) + 15(50) = 2150$
$Q(25,125)$	$= 20(25) + 15(125) = 2375$
$B_2(0,125)$	$= 20(0) + 15(125) = 1875$

maximum $Z = 2375$ at $x = 25$, $y = 125$

Number of product $A = 25$, product $B = 125$
maximum profit = Rs 2375

Linear Programming Ex 30.4 Q34

Let required quantity of large and small boxes are x and y respectively.

Since,profits on each unit of large and small boxes are Rs 3 and Rs 2 respectively, so,
profit on x units of large and y units of small boxes are Rs $3x$ and Rs $2y$ respectively
Let Z be total profit so

$$Z = 3x + 2y$$

Since each large and small box require 4 sq. m. and 3 sq. m. cardboard respectively, so,
x units of large and y units of small boxes require $4x$ and $3y$ sq.m. cardboard respectivley
but only 60 sq. m. of cardboard is available,so

$$4x + 3y \le 60 \qquad \text{(first constraint)}$$

Since manufacturer is required to make at least three large boxes,so,

$$x \ge 3 \qquad \text{(second constraint)}$$

Since manufacturer is required to make at least twice as many small boxes
as large boxes ,so,

$$y \ge 2x \qquad \text{(third constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 20x + 15y$

Subject to constriants,

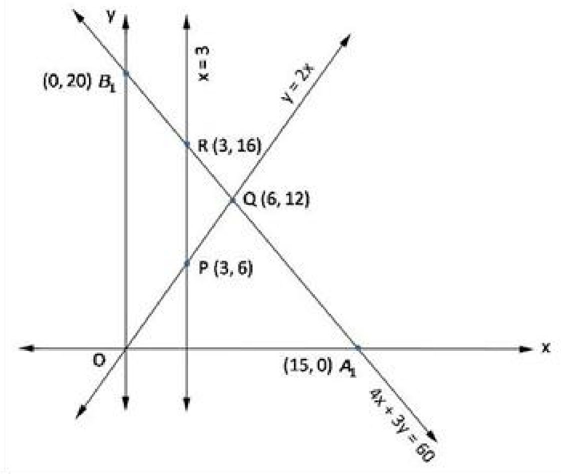
$$4x + 3y \le 60$$
$$x \ge 3$$
$$y \ge 2x$$
$$x, y \ge 0 \qquad \left[\text{Since production can not be less than zero}\right]$$

Region $4x + 3y \le 60$: line $4x + 3y = 60$ meets axes at $A_1(15,0)$, $B_1(0,20)$ respectively.
Region containing origin represents $4x + 3y \le 60$ as $(0,0)$ satisfies $4x + 3y \le 60$.

Region $x \ge 3$: line $x = 3$ is parallel to y - axis meets x-axes at $A_2(3,0)$. Region containing
origin represents $x \ge 70$ as $(0,0)$ satisfies $x \ge 3$.

Region $y \ge 2x$: line $y = 2x$ is passes throgh origin and $P(3,6)$. Region containging $B_1(0,20)$
represents $y \ge 2x$ as $(0,20)$ satisfies $y \ge 2x$.

Region $x, y \ge 0$: it represent first quandrant.



Shaded region PQR represents feasible region.
Point $Q(6,12)$ is obtained by solving $y = 2x$ and $4x + 3y = 60$
Point $R(3,16)$ is obtained by solving $x = 3$ and $4x + 3y = 60$.

The value of $Z = 3x + 2y$ at

$P(3,6) \quad = 3(3) + 2(6) = 21$

$Q(6,12) \quad = 3(6) + 2(12) = 42$

$R(3,16) \quad = 3(3) + 2(16) = 41$

maximum $Z = 42$ at $x = 6$, $y = 12$

Number of large box = 6, small box =12
maximum profit = Rs 42

Linear Programming Ex 30.4 Q35

The given data can be written in the tabular form as follows:

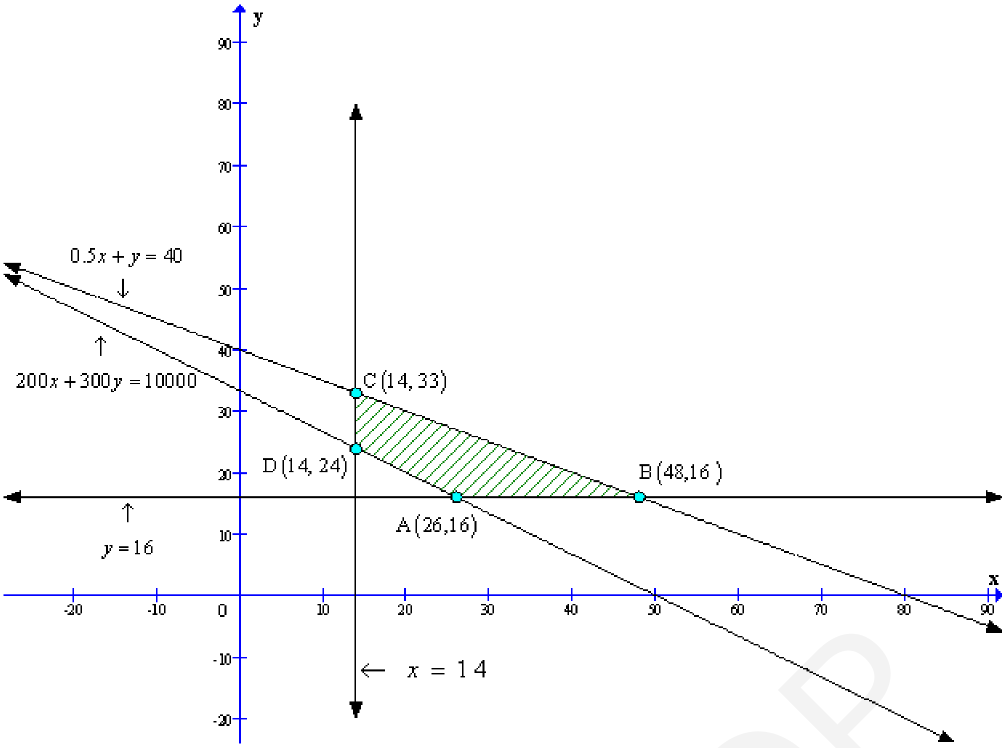
Product	A	B	Working week	Turn over
Time	0.5	1	40	
Prise	200	300		10000
Profit	20	30		
Permanent order	14	16		

Let x be the number of units of A and y be the number of units of B produced to earn the maximum profit.

Then the mathematical model of the LPP is as follows:
Maximize $Z = 20x + 30y$
Subject to $0.5x + y \le 40$,
$200x + 300y \ge 10000$

and $x \ge 14$, $y \ge 16$

To solve the LPP we draw the lines,
$0.5x + y = 40$,
$200x + 300y = 10000$
$x = 14$
$y = 16$



The coordinates of the vertices (Corner - points) of shaded feasible region ABCD are A(26, 16), B(48, 16), C(14, 33) and D(14, 24).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function Z = 20x + 30y
A(26, 16)	Z = 1000
B(48, 16)	Z = 1440
C(14, 33)	Z = 1270
D(14, 24)	Z = 600

48 units of product A and 16 units of product B should be produced to earn the maximum profit of Rs. 1440.

Linear Programming Ex 30.4 Q36

Let the distance covered with the speed of 25 km/hr be x.
Let the distance covered with the speed of 40 km/hr be y.

Then the mathematical model of the LPP is as follows:
Maximize Z = x + y
Subject to $2x + 5y \le 100$,

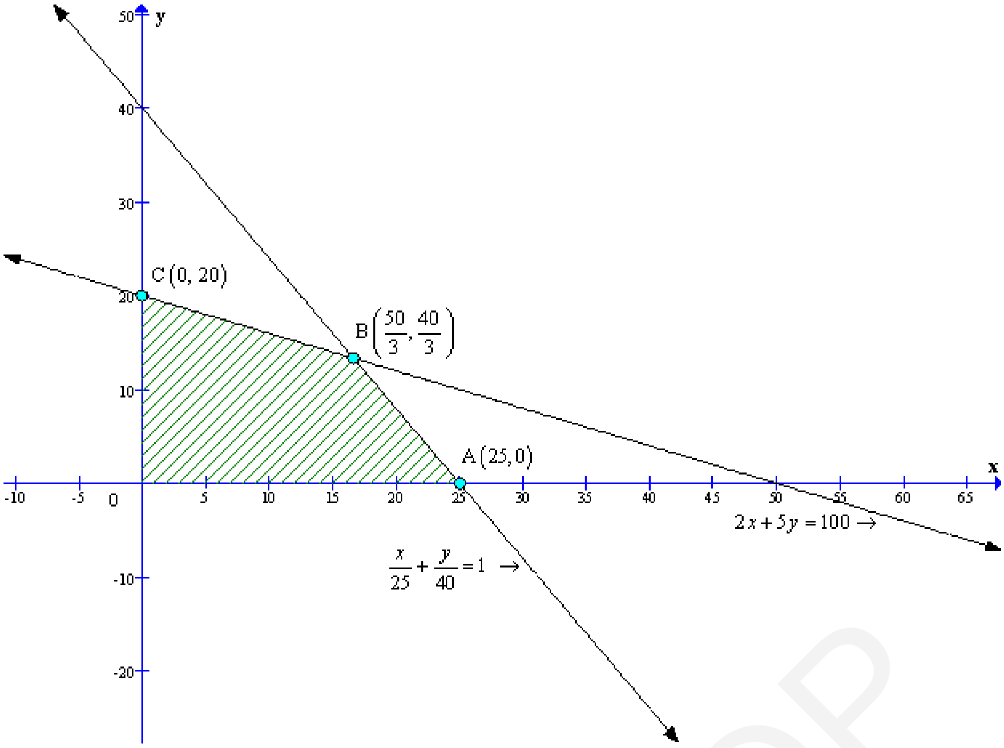
$$\frac{x}{25} + \frac{y}{40} \le 1$$

and $x \ge 0$, $y \ge 0$

To solve the LPP we draw the lines,
$2x + 5y = 100$,

$$\frac{x}{25} + \frac{y}{40} = 1$$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are $A(25, 0), B\left(\frac{50}{3}, \frac{40}{3}\right)$ and $C(0, 20)$.

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = x + y$
$A(25, 0)$	$Z = 25$
$B\left(\frac{50}{3}, \frac{40}{3}\right)$	$Z = 30$
$C(0, 20)$	$Z = 20$

The distance covered at the speed of 25km/hr is $\frac{50}{3}$km and

The distance covered at the speed of 40km/hr is $\frac{40}{3}$km.

Maximum distance travelled is 30 km.

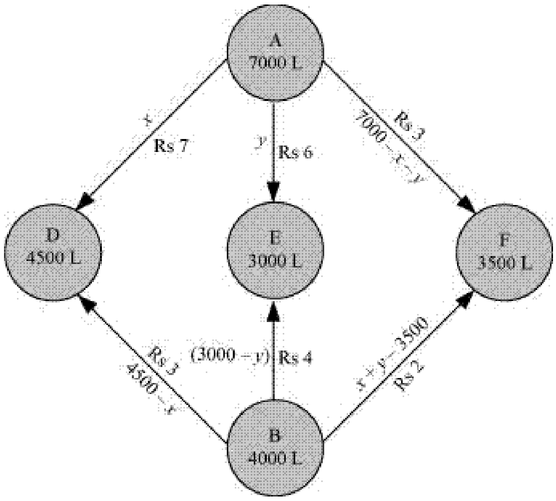
Linear Programming Ex 30.4 Q37

Let x and y litres of oil be supplied from A to the petrol pumps, D and E. Then, $(7000 - x - y)$ will be supplied from A to petrol pump F.

The requirement at petrol pump D is 4500 L. Since x L are transported from depot A, the remaining $(4500 - x)$ L will be transported from petrol pump B.

Similarly, $(3000 - y)$ L and $3500 - (7000 - x - y) = (x + y - 3500)$ L will be transported from depot B to petrol pump E and F respectively.

The given problem can be represented diagrammatically as follows.



$x \geq 0,\ y \geq 0,$ and $(7000 - x - y) \geq 0$

$\Rightarrow x \geq 0,\ y \geq 0,$ and $x + y \leq 7000$

$4500 - x \geq 0,\ 3000 - y \geq 0,$ and $x + y - 3500 \geq 0$

$\Rightarrow x \leq 4500,\ y \leq 3000,$ and $x + y \geq 3500$

Cost of transporting 10 L of petrol = Re 1

Cost of transporting 1 L of petrol = Rs $\frac{1}{10}$

Therefore, total transportation cost is given by,

$$z = \frac{7}{10} \times x + \frac{6}{10} y + \frac{3}{10}(7000 - x - y) + \frac{3}{10}(4500 - x) + \frac{4}{10}(3000 - y) + \frac{2}{10}(x + y - 3500)$$
$$= 0.3x + 0.1y + 3950$$

The problem can be formulated as follows.

Minimize $z = 0.3x + 0.1y + 3950$ ... (1)

subject to the constraints,

$x + y \leq 7000$ ...(2)

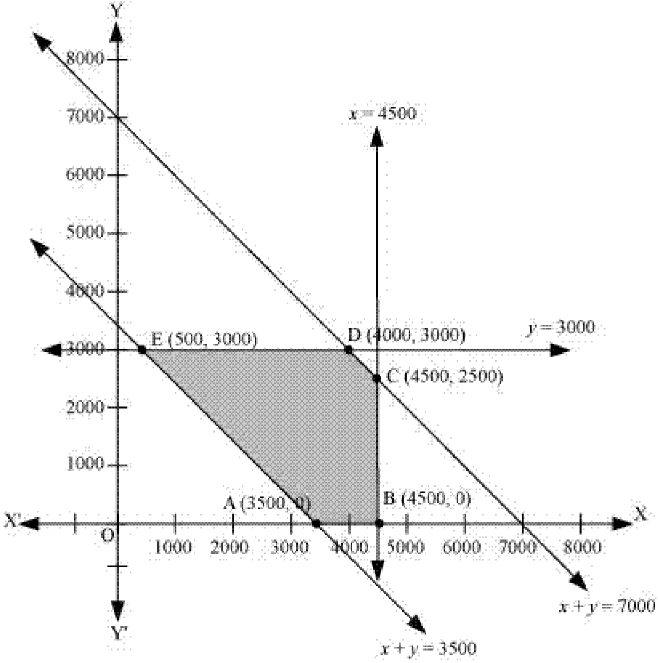
$x \leq 4500$ ...(3)

$y \leq 3000$ ...(4)

$x + y \geq 3500$ ...(5)

$x,\ y \geq 0$ ...(6)

The feasible region determined by the constraints is as follows.



The corner points of the feasible region are A (3500, 0), B (4500, 0), C (4500, 2500), D (4000, 3000), and E (500, 3000).

The values of z at these corner points are as follows.

Corner point	**z = 0.3x + 0.1y + 3950**	
A (3500, 0)	5000	
B (4500, 0)	5300	
C (4500, 2500)	5550	
D (4000, 3000)	5450	
E (500, 3000)	4400	→ Minimum

The minimum value of z is 4400 at (500, 3000).

Thus, the oil supplied from depot A is 500 L, 3000 L, and 3500 L and from depot B is 4000 L, 0 L, and 0 L to petrol pumps D, E, and F respectively.

The minimum transportation cost is Rs 4400.

Linear Programming Ex 30.4 Q38

Let required number of gold rings and chains are x and y respectively.

Since,profits on each ring and chains are Rs 300 and Rs 190 respectively, so,
profit on x units of ring and y units of chains are Rs $300x$ and Rs $190y$ respectively
Let Z be total profit so

$$Z = 300x + 190y$$

Since each unit of ring and chain require 1 hr and 30 min. to make respectively, so,
x units of rings and y units of rings require $60x$ and $30y$ min. to make respectivley,
but total time available to make is $16 \times 60 = 960$, so

$$60x + 30y \le 960$$

$$\Rightarrow \quad 2x + y \le 32 \qquad \text{(first constraint)}$$

Given, total number of rings and chains manufactured is at most 24, so,

$$x + y \le 24 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 300x + 160y$

Subject to constriants,

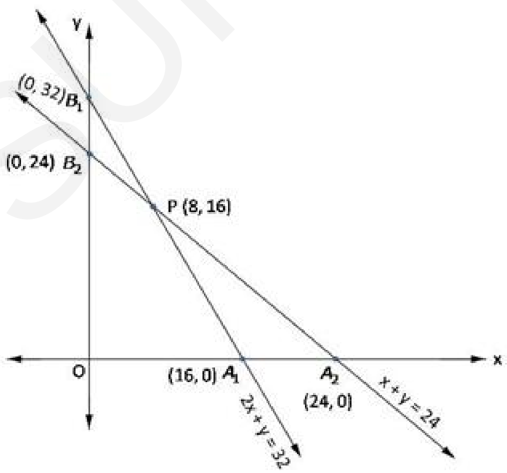
$$2x + y \le 32$$
$$x + y \le 24$$
$$x, y \ge 0 \qquad \text{[Since production can not be less than zero]}$$

Region $2x + y \le 32$: line $2x + y = 32$ meets axes at $A_1(16,0)$, $B_1(0,32)$ respectively.
Region containing origin represents $2x + y \le 32$ as $(0,0)$ satisfies $2x + y \le 32$.

Region $x + y \le 24$: line $x + y = 24$ meets axes at $A_2(24,0)$, $B_2(0,24)$ respectively.
Region containing origin represents $x + y \le 24$ as $(0,0)$ satisfies $x + y \le 24$.

Region $x, y \ge 0$: it represent first quandrant

Shaded region OA_1PB_2 represents feasible region.
Point $P(8,16)$ is obtained by solving $2x + y = 32$ and $x + y = 24$.



The value of $Z = 300x + 160y$ at

$O(0,0)$	$= 300(0) + 160(0) = 0$
$A_1(16,0)$	$= 300(16) + 160(0) = 4800$
$P(8,16)$	$= 300(8) + 160(16) = 4960$
$B_2(0,24)$	$= 300(0) + 160(24) = 3840$

maximum $Z = 4960$ at $x = 8$, $y = 16$

Number of rings = 8, chains = 16
maximum profit = Rs 4960

Linear Programming Ex 30.4 Q39

Let required number of books of type I and II be x and y respectively.

Let Z be total number of books in the shelf ,so,

$$Z = x + y$$

Since 1 book of type I and II 6 cm and 4 cm. thick respectively, so, x books of type I and y books of type II has thickness of $6x$ and $4y$ cm. respectivley, but shelf is 96 cm. long ,so

$$6x + 4y \le 96$$

$$\Rightarrow \quad 3x + 2y \le 48 \qquad \text{(first constraint)}$$

Since 1 book of type I and II weight 1 kg and $1\frac{1}{2}$ kg respectively, so, x books of type I and y books of type II weight x kg and $\frac{3}{2}y$ kg respectivley, but shelf can support at most 21 kg,so

$$x + \frac{3}{2}y \le 21$$

$$\Rightarrow \quad 2x + 3y \le 42 \qquad \text{(second constraint)}$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = x + y$

Subject to constriants,

$$3x + 2y \le 48$$

$$2x + 3y \le 42$$

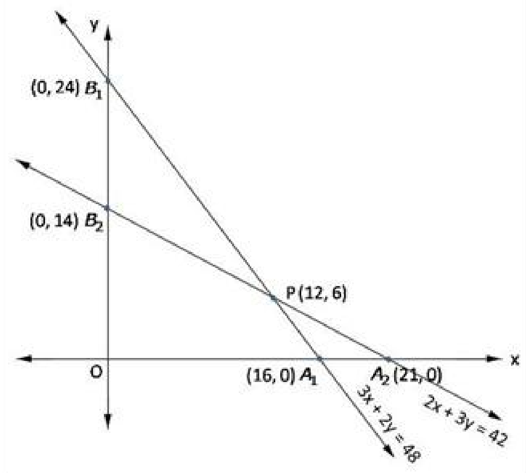
$$x, y \ge 0 \qquad [\text{Since number of books can not be less than zero}]$$

Region $3x + 2y \le 48$: line $3x + 2y = 48$ meets axes at $A_1(16,0)$, $B_1(0,24)$ respectively.
Region containing origin represents $3x + 2y \le 48$ as $(0,0)$ satisfies $3x + 2y \le 48$.

Region $2x + 3y \le 42$: line $2x + 3y = 42$ meets axes at $A_2(21,0)$, $B_2(0,14)$ respectively.
Region containing origin represents $2x + 3y \le 42$ as $(0,0)$ satisfies $2x + 3y \le 42$.

Region $x, y \ge 0$: it represent first quandrant

Shaded region OA_1PB_2 represents feasible region.
Point $P(12,6)$ is obtained by solving $2x + 3y = 42$ and $3x + 2y = 48$



The value of $Z = x + y$ at

$O(0,0) \quad = 0 + 0 = 0$

$A_1(16,0) \quad = 16 + 0 = 16$

$P(12,6) \quad = 12 + 6 = 18$

$B_2(0,14) \quad = 0 + 14 = 14$

maximum $Z = 18$ at $x = 12$, $y = 6$

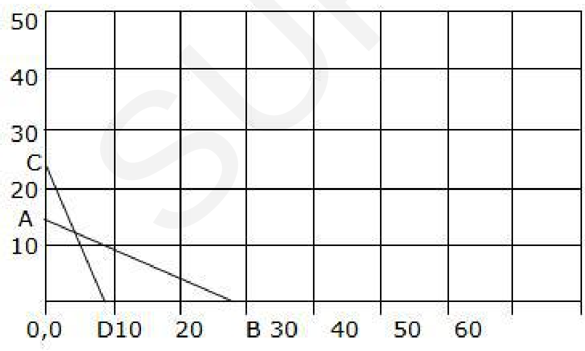
Number of books of type I = 12, type II = 6

Linear Programming Ex 30.4 Q40

Let x & y be the No. of tennis rackets and cricket bats produced.

$1.5x + 3y \leq 42$ (constraint on machine time)
$3x + y \leq 24$ (constraint on craftsman's time)
$Z = 20x + 10y$ (Maximize profit)
$x, y \geq 0$
plotting the inequalities we have,

when x=0, y= 14 and when y=0, x=28 and
when x=0, y= 24 and when y=0, x=8



The feasible region is given by 0,0-A-F-D
Tabulating Z and corner points we have

Corner point	Value of Z = 20x + 10y
0, 0	0
0, 14	140
4, 12	200
8, 0	160

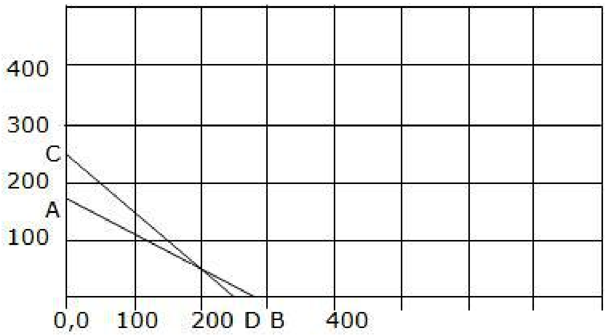
The factory must manufacture 4 tennis rackets and 12 cricket bats to earn the maximum profit of Rs. 200/-

Linear Programming Ex 30.4 Q41

Let x & y be the No. of desktop model and portable model of personal computers stocked.

$x + y \le 250$ (constraint on total demand of computers)
$25000x + 40000y \le 70,00,000$ (constraint on cost)
$Z = 4500x + 5000y$ (Maximize profit)
$x, y \ge 0$

plotting the inequalities we have,
when x=0, y= 250 and when y=0, x=250 and line CD
when x=0, y= 175 and when y=0, x=280



The feasible region is given by 0,0-A-E-D-0,0

Tabulating Z and corner points we have

Corner point	Value of Z = 4500x + 5000y
0, 0	0
0, 175	8,75,000
250, 0	11,25,000
200, 50	11,50,000

The merchant must stock 200 desktop models and 50 portable models to earn a maximum profit of Rs. 11,50,000/-

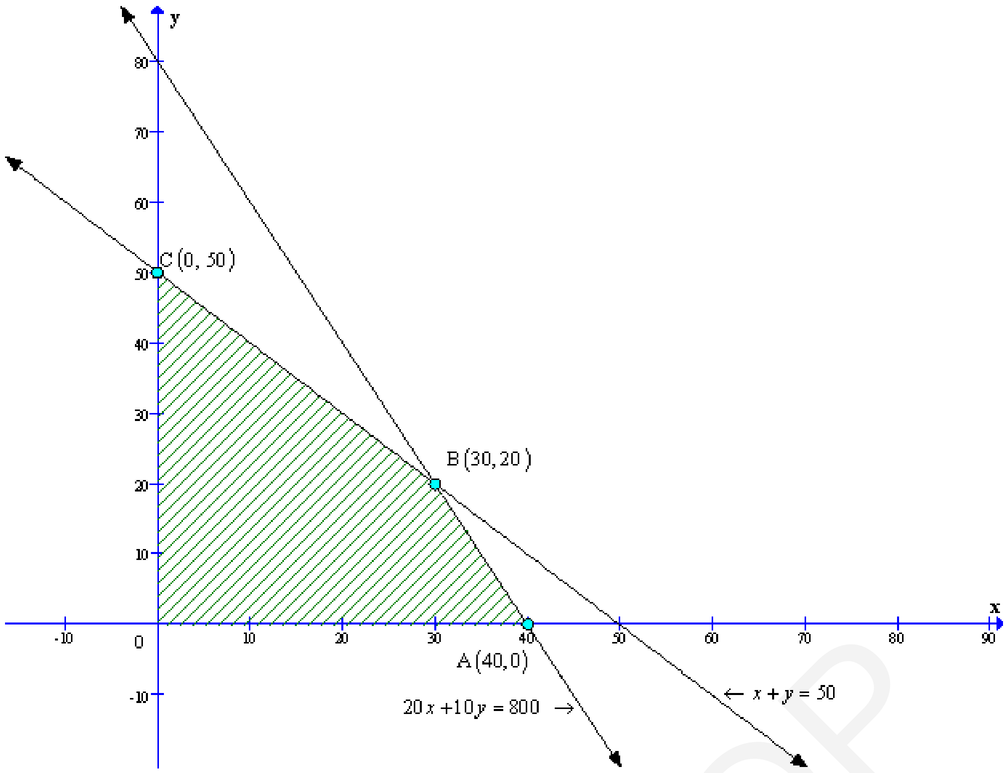
Linear Programming Ex 30.4 Q42

Let x hectores of land grows crop X.
Let y hectores of land grows crop Y.

Then the mathematical model of the LPP is as follows:
Maximize $Z = 10{,}500x + 9{,}000y$
Subject to $x + y \le 50$,
$20x + 10y \le 800$
and $x \ge 0$, $y \ge 0$

To solve the LPP we draw the lines,
$x + y = 50$,
$20x + 10y = 800$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(40, 0), B(30, 20) and C(0, 50).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function Z = 10,500x + 9,000y
A(40, 0)	Z = 4,20,000
B(30, 20)	Z = 4,95,000
C(0, 50)	Z = 4,50,000

30 hectors of land should be allocated to crop X and
20 hectors of land should be allocated to crop Y to maximize the profit.
The maximum profit that can be eared is Rs. 4,95,000.

Linear Programming Ex 30.4 Q43

The given data can be written in the tabular form as follows:

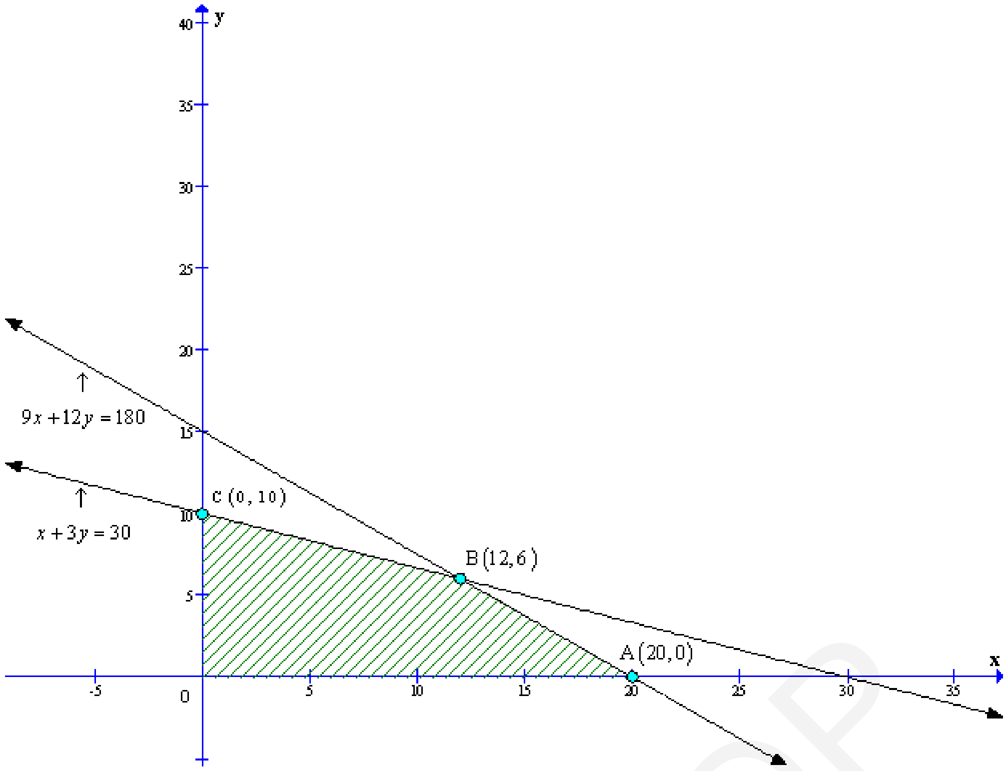
Model	A	B	Maximum hours
Fabricating	9	12	180
Finishing	1	3	30
Profit	8000	12000	

Let x be the number of pieces of A and y be the number of pieces of B manufactured to earn the maximum profit.

Then the mathematical model of the LPP is as follows:
Maximize $Z = 8000x + 12000y$
Subject to $9x + 12y \le 180$,
$x + 3y \le 30$
and $x \ge 0$, $y \ge 0$

To solve the LPP we draw the lines,
$9x + 12y = 180$,
$x + 3y = 30$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(20, 0), B(12,6) and C(0, 10).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function Z = 8,000x + 12,000y
A(20, 0)	Z = 1,60,000
B(12, 6)	Z = 1,68,000
C(0, 10)	Z = 1,20,000

12 pieces of Model A and 6 pieces of Model B should be eaned maximize the profit.
The maximum profit that can be eared is Rs. 1,68,000.

Linear Programming Ex 30.4 Q44

The given data can be written in the tabular form as follows:

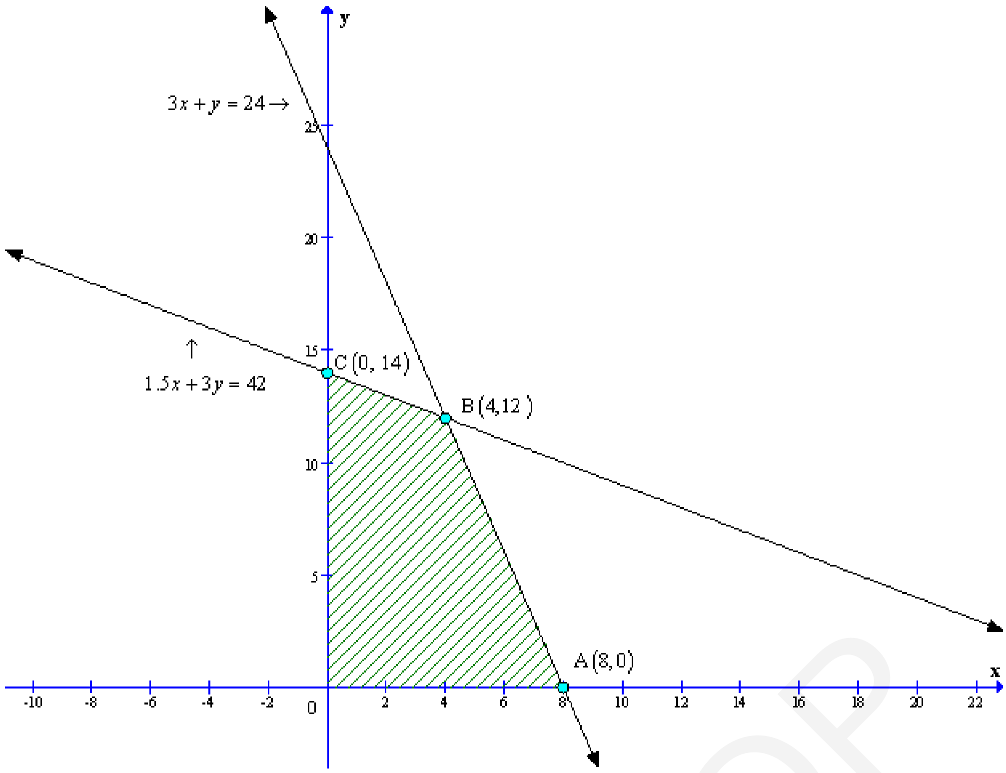
Product	Racket	Bat	Maximum hours
Machine	1.5	3	42
Craftman	3	1	24
Profit	20	10	

Let x be the number of rackets and y be the number of bats made to earn the maximum profit.

Then the mathematical model of the LPP is as follows:
Maximize Z = 20x + 10y
Subject to $1.5x + 3y \leq 42$,
$3x + y \leq 24$
and $x \geq 0$, $y \geq 0$

To solve the LPP we draw the lines,
$1.5x + 3y = 42$,
$3x + y = 24$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(8, 0),B(4,12) and C(0, 14).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 20x + 10y$
A(8, 0)	Z = 160
B(4, 12)	Z = 200
C(0, 14)	Z = 140

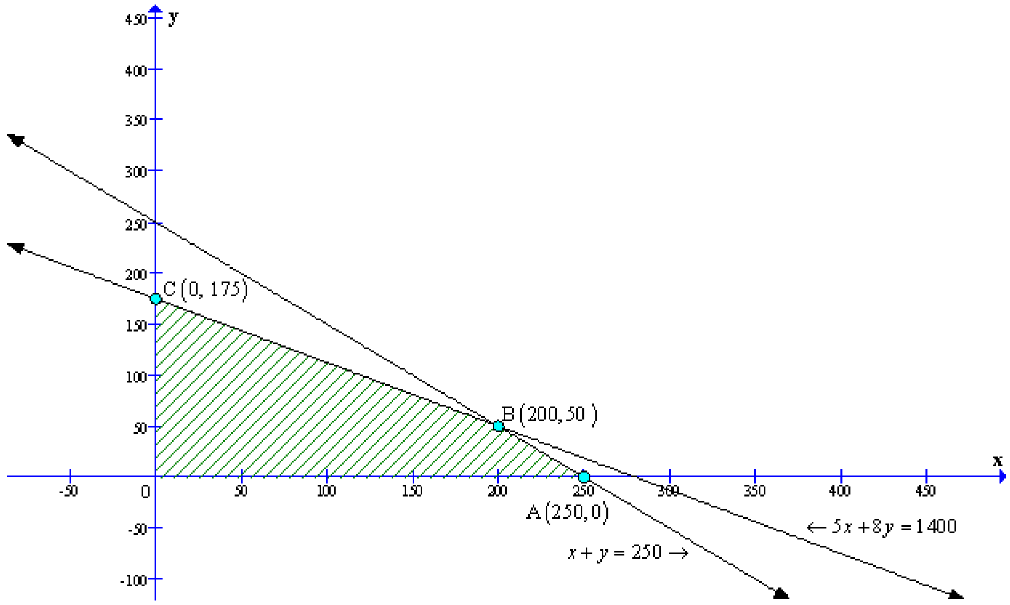
4 rackets and 12 bats must be made if the factory is to work at full capacity.
The maximum profit that can be eared is Rs. 200.

Linear Programming Ex 30.4 Q45

Let x be the number of desktop computers and y be the number of portable computers which merchant should stock to earn the maximum profit.

Then the mathematical model of the LPP is as follows:
Maximize $Z = 4500x + 5000y$
Subject to $25000x + 40000y \leq 70,00,000$
$x + y \leq 250$
and $x \geq 0,\ y \geq 0$

To solve the LPP we draw the lines,
$5x + 8y = 1,400$
$x + y = 250$
The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(250, 0), B(200, 50) and C(0, 175).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function Z = 4500x + 5000y
A(250, 0)	Z = 11,25,000
B(200, 50)	Z = 11,50,000
C(0, 175)	Z = 8,75,000

The merchant should stock 200 personal computer and 50 portable computers to earn maximum profit. The maximum profit that can be eared is Rs. 11,50,000.

Linear Programming Ex 30.4 Q46

Let x be the number of dolls of type A and y be the number of dolls of type B should be produced to earn the maximum profit.

Then the mathematical model of the LPP is as follows:

Maximize $Z = 12x + 16y$

Subject to $x + y \le 1200$

$\frac{1}{2}x - y \ge 0$

$x - 3y \le 600$

and $x \ge 0,\ y \ge 0$

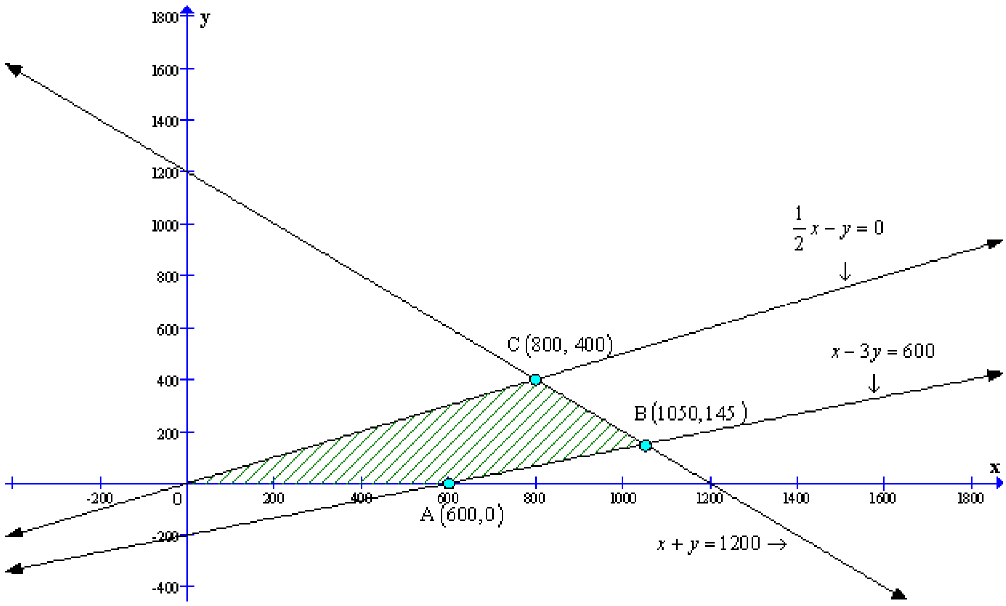
To solve the LPP we draw the lines,

$x + y = 1200$

$\frac{1}{2}x - y = 0$

$x - 3y = 600$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(600, 0), B(1050, 145) and C(800, 400).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function Z = 12x + 16y
A(600, 0)	Z = 7200
B(1050, 145)	Z = 14920
C(800, 400)	Z = 16000

The toy company should manufacture 800 dolls of type A and 400 dolls of type B to earn maximum profit.
The maximum profit that can be eared is Rs. 16,000.

Linear Programming Ex 30.4 Q47

Let x kg of fertiliser F_1 and y kg of fertiliser F_2 should be used to minimise the cost.

Then the mathematical model of the LPP is as follows:

Maximize Z = 6x + 5y

Subject to $10x + 5y \geq 1400$

$6x + 10y \geq 1400$

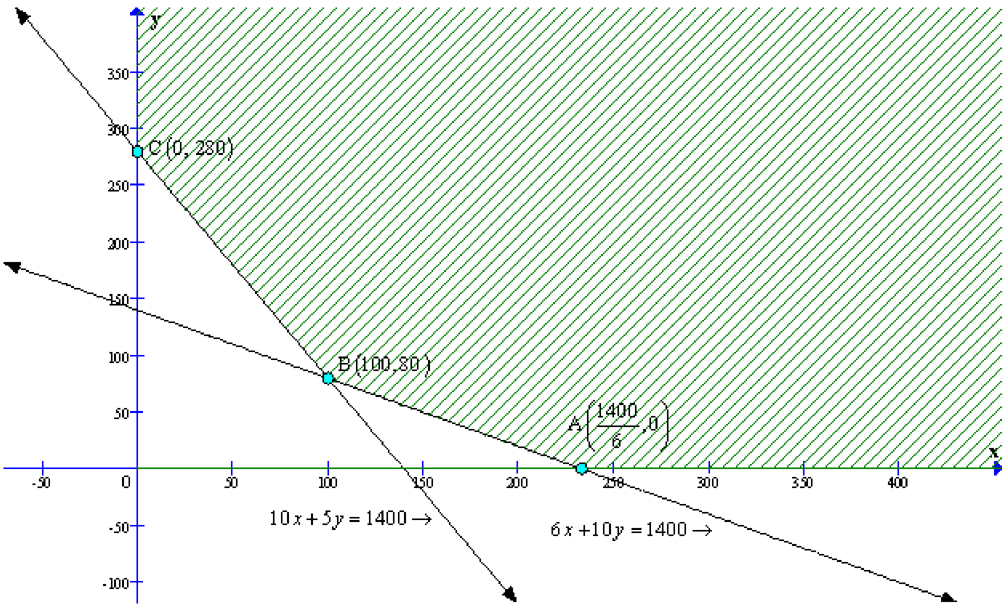
and $x \geq 0$, $y \geq 0$

To solve the LPP we draw the lines,

$10x + 5y = 1400$

$6x + 10y = 1400$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are $A\left(\frac{1400}{6},\ 0\right)$, $B(100, 80)$ and $C(0,\ 280)$.

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 6x + 5y$
$A\left(\frac{1400}{6},\ 0\right)$	$Z = 1400$
$B(100,\ 80)$	$Z = 1000$
$C(0,\ 280)$	$Z = 1400$

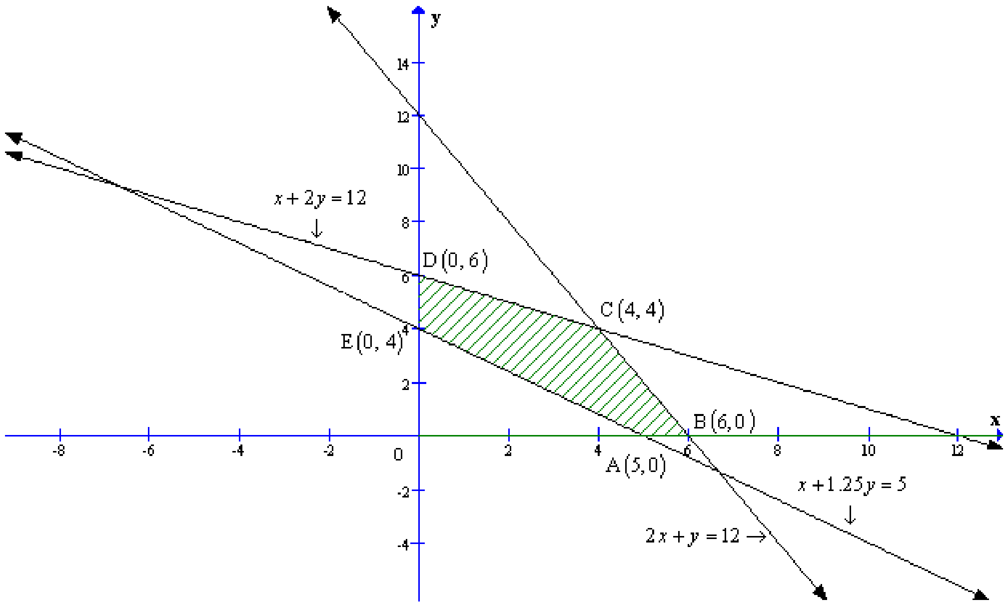
100 kg of fertiliser F_1 and 80 kg of fertiliser F_2 to earn minimise the cost.
The maximum cost Rs. 1,000.

Linear Programming Ex 30.4 Q48

Let x units of item M and y units of item N should be produced to maximise the cost.

Then the mathematical model of the LPP is as follows:
Maximize $Z = 600x + 400y$
Subject to $x + 2y \leq 12$
$2x + y \leq 12$
$x + 1.25y \geq 5$
and $x \geq 0,\ y \geq 0$

To solve the LPP we draw the lines,
$x + 2y = 12$
$2x + y = 12$
$x + 1.25y = 5$
The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABCDE are A(5, 0), B(6, 0), C(4, 4), D(0, 6) and E(0, 4).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function Z = 600x + 400y
A(5, 0)	Z = 3000
B(6, 0)	Z = 3600
C(4, 4)	Z = 4000
D(0, 6)	Z = 2400
E(0, 4)	Z = 1600

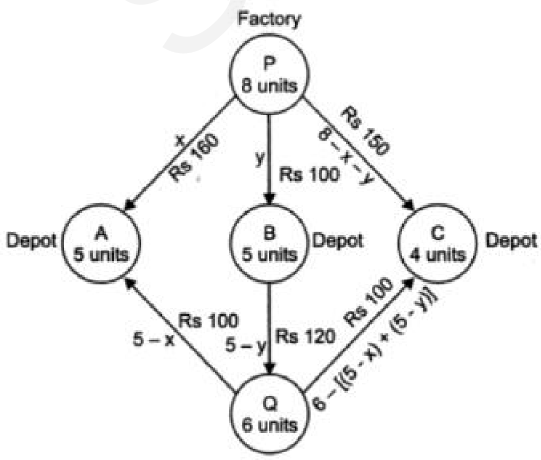
4 units of item M and 4 units of item N should be produced to maximise the profit.
The maximum profit is Rs. 4,000.

Linear Programming Ex 30.4 Q49

Let x and y units of commodity be transported from factory P to the depots at A and B respectively.

Then (8 - x - y) units will be transported to depot at C.

The flow is shown below.

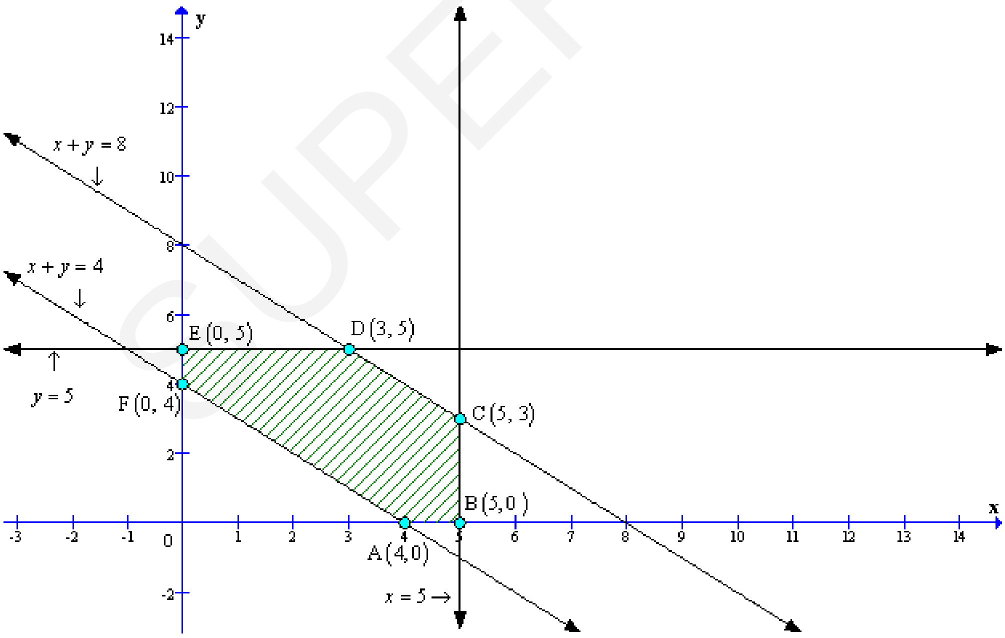


Hence we have, $x \ge 0$, $y \ge 0$ and $8 - x - y \ge 0$
i.e. $x \ge 0$, $y \ge 0$ and $x + y \ge 8$
Now, the weekly requirement of the depot at A is 5 units of the commodity.
Since x units are transported from the factory at P, remaimning $(5 - x)$ units need to be transported from the factory at Q.
$\therefore 5 - x \ge 0 \Rightarrow x \le 5$
Simillarly, $(5 - y)$ and $6 - (5 - x + 5 - y) = x + y - 4$ units are to be transported from the factory at Q to the depots at B and C respectively.
$\therefore 5 - y \ge 0$ and $x + y - 4 \ge 0$
$\Rightarrow y \le 5$ and $x + y \ge 4$

Total transportation cost Z is given by
$Z = 160x + 100y + 100(5 - x) + 120(5 - y) + 100(x + y - 4) + 150(8 - x - y)$
$Z = 10(x - 7y + 190)$

So the mathematical model of given LPP is as follows.
Minimize $Z = 10(x - 7y + 190)$
Subject to $x + y \le 8$
$x \le 5, y \le 5$
$x + y \ge 4$
$x \ge 0, y \ge 0$

To solve the LPP we draw the lines,
$x + y = 8$
$x = 5, y = 5$
$x + y = 4$
The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABCDEF are A(4, 0), B(5, 0), C(5, 3), D(3, 5), E(0, 5) and F(0, 4).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 10(x - 7y + 190)$
A(4, 0)	Z = 1940
B(5, 0)	Z = 1950
C(5, 3)	Z = 1740
D(3,5)	Z = 1580
E(0,5)	Z = 1550
F(0,4)	Z = 1620

Deliver 0, 5, 3 units from factory at P and 5, 0, 1 from the factory at Q to the depots at A, B and C respectively.
The minimum transportation cost is Rs. 1550.

Linear Programming Ex 30.4 Q50

Let the mixture contains x toys of type A and y toys of type B.

Type of toys	No. of toys	Machine I (in min)	Machine II (in min)	Machine III (in min)	Profit Rs.
A	x	12x	18x	6x	7.5x
B	y	6y	0	9y	5y
Total	x+y	12x + 6y	18x	6x + 9y	7.5x + 5y
Requirement		360	360	360	

Then the mathematical model of the LPP is as follows:

Maximize $Z = 7.5x + 5y$

Subject to $12x + 6y \le 360 \Rightarrow 2x + y \le 60$

$18x \le 360 \Rightarrow x \le 20$

$6x + 9y \le 360 \Rightarrow 2x + 3y \le 120$

and $x \ge 0,\ y \ge 0$

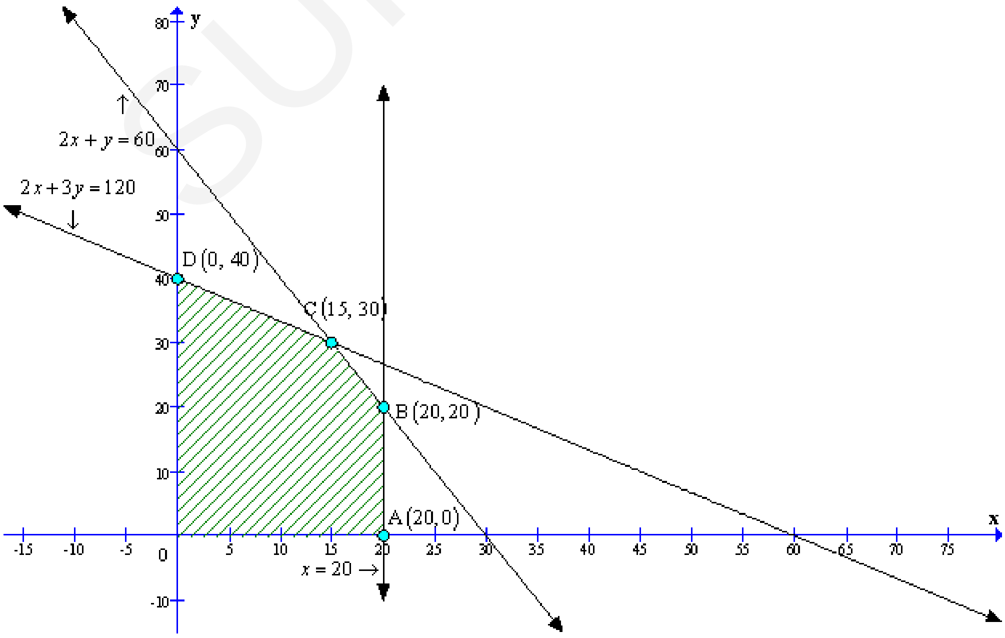
To solve the LPP we draw the lines,

$2x + y = 60$

$x = 20$

$2x + 3y = 120$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABCD are A(20, 0), B(20,20), C(15, 30) and D(0, 40).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 7.5x + 5y$
A(20, 0)	Z = 150
B(20, 20)	Z = 250
C(15, 30)	Z = 262.5
D(0, 40)	Z = 200

Manufacturer should make 15 toys of type A and 30 toys of type B to maximize the profit.
The maximum profit that can be earned is Rs. 262.5

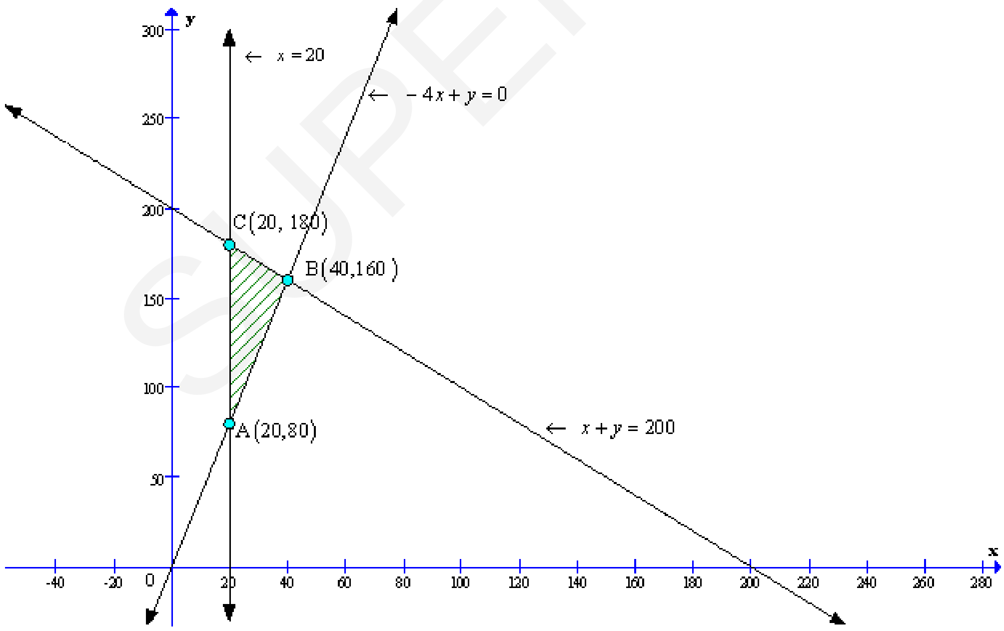
Linear Programming Ex 30.4 Q51

Let x be the number of executive class tickets and y be the number of economic class tickets.

Then the mathematical model of the LPP is as follows:
Maximize $Z = 1000x + 600y$
Subject to $x + y \leq 200$
$x \geq 20$
$y \geq 4x \Rightarrow -4x + y \geq 0$
and $x \geq 0, y \geq 0$

To solve the LPP we draw the lines,
$x + y = 200$
$x = 20$
$-4x + y = 0$

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are A(20, 80), B(40,160) and C(20, 180).

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 1000x + 600y$
A(20, 80)	Z = 68,000
B(40, 160)	Z = 1,36,000
C(20, 180)	Z = 1,28,000

40 tickets of executive class and 160 tickets of economic class must be sold to maximize the profit.
The maximum profit that can be earned is Rs. 1,36,000.

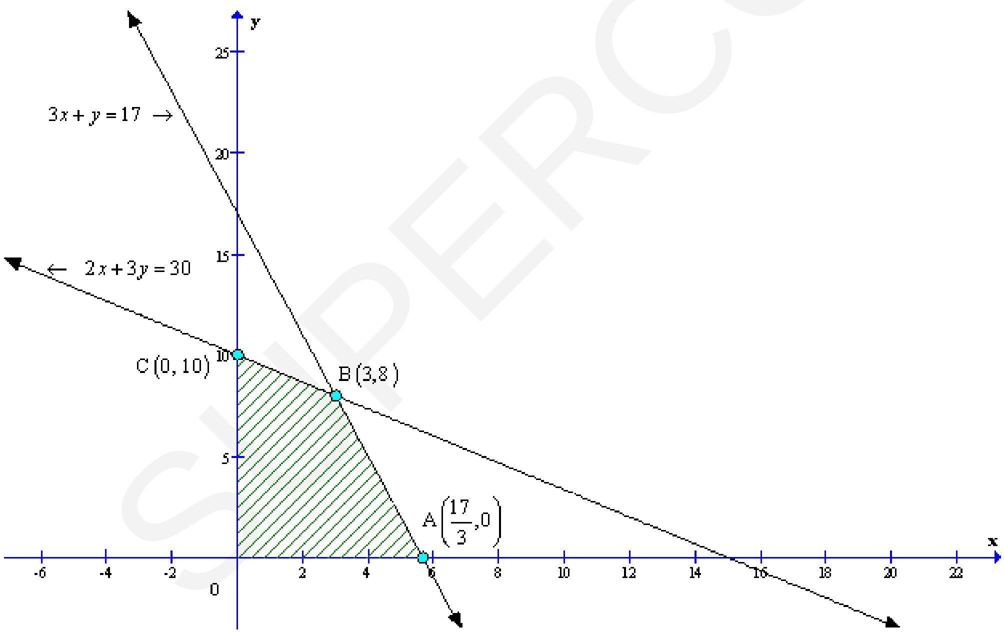
Linear Programming Ex 30.4 Q52

Then the mathematical model of the LPP is as follows:
Maximize $Z = 100x + 120y$
Subject to $2x + 3y \le 30$
$3x + y \le 17$
and $x \ge 0,\ y \ge 0$

To solve the LPP we draw the lines,
$2x + 3y = 30$
$3x + y = 17$

The feasible region of the LPP is shaded in graph.

The feasible region of the LPP is shaded in graph.



The coordinates of the vertices (Corner - points) of shaded feasible region ABC are $A\left(\frac{17}{3}, 0\right)$, $B(3, 8)$ and $C(0, 10)$.

The values of the objective of function at these points are given in the following table:

Point (x_1, x_2)	Value of objective function $Z = 100x + 120y$
$A\left(\frac{17}{3}, 0\right)$	$Z = 566.67$
$B(3, 8)$	$Z = 1260$
$C(0, 10)$	$Z = 1200$

3 units of workers and 8 units of capital must be used to maximize the profit.
The maximum profit that can be earned is Rs. 1260.

Yes, because efficiency of a person does not depend on sex (male or female).

Ex - 30.5

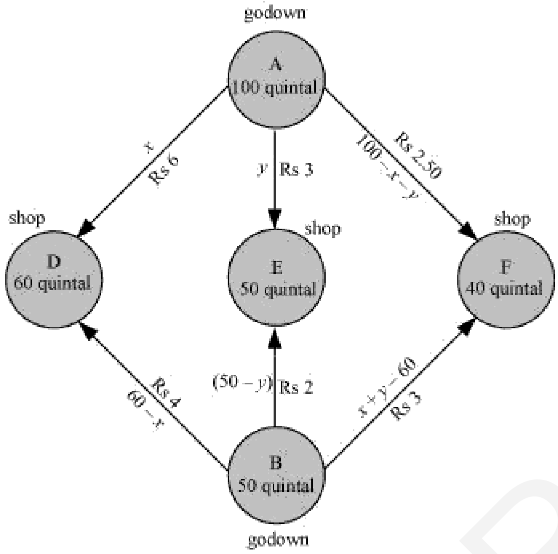
Linear Programming Ex 30.5 Q1

Let godown A supply x and y quintals of grain to the shops D and E respectively. Then, $(100 - x - y)$ will be supplied to shop F.

The requirement at shop D is 60 quintals since x quintals are transported from godown A. Therefore, the remaining $(60 - x)$ quintals will be transported from godown B.

Similarly, $(50 - y)$ quintals and $40 - (100 - x - y) = (x + y - 60)$ quintals will be transported from godown B to shop E and F respectively.

The given problem can be represented diagrammatically as follows.



$x \geq 0,\ y \geq 0,$ and $100 - x - y \geq 0$

$\Rightarrow x \geq 0,\ y \geq 0,$ and $x + y \leq 100$

$60 - x \geq 0,\ 50 - y \geq 0,$ and $x + y - 60 \geq 0$

$\Rightarrow x \leq 60,\ y \leq 50,$ and $x + y \geq 60$

Total transportation cost z is given by,

$$\begin{aligned} z &= 6x + 3y + 2.5(100 - x - y) + 4(60 - x) + 2(50 - y) + 3(x + y - 60) \\ &= 6x + 3y + 250 - 2.5x - 2.5y + 240 - 4x + 100 - 2y + 3x + 3y - 180 \\ &= 2.5x + 1.5y + 410 \end{aligned}$$

The given problem can be formulated as

Minimize $z = 2.5x + 1.5y + 410$ ... (1)

subject to the constraints,

$x + y \leq 100$ ...(2)

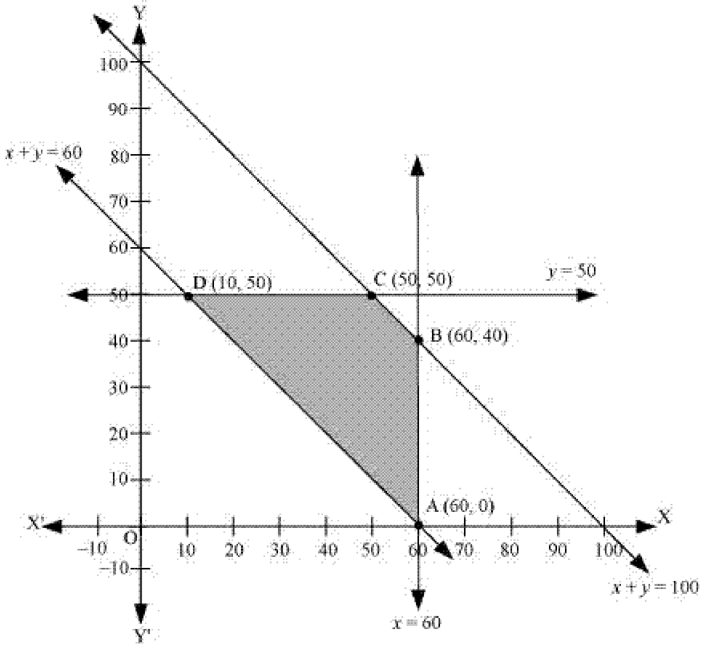
$x \leq 60$ ...(3)

$y \leq 50$ ...(4)

$x + y \geq 60$ ...(5)

$x,\ y \geq 0$ ...(6)

The feasible region determined by the system of constraints is as follows.



The corner points are A (60, 0), B (60, 40), C (50, 50), and D (10, 50).

The values of z at these corner points are as follows.

Corner point	$z = 2.5x + 1.5y + 410$	
A (60, 0)	560	
B (60, 40)	620	
C (50, 50)	610	
D (10, 50)	510	→ Minimum

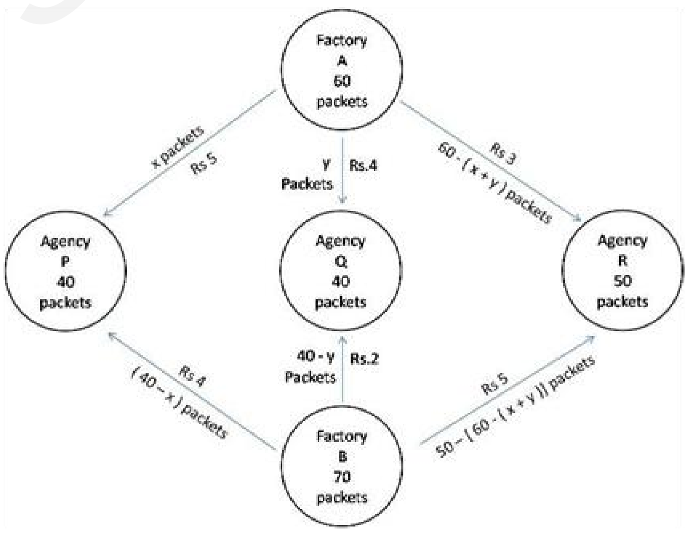
The minimum value of z is 510 at (10, 50).

Thus, the amount of grain transported from A to D, E, and F is 10 quintals, 50 quintals, and 40 quintals respectively and from B to D, E, and F is 50 quintals, 0 quintals, and 0 quintals respectively.

The minimum cost is Rs 510.

Linear Programming Ex 30.5 Q2

The given information can be exhibited diagramatically as below:



Let factory A transports x packets to agency P and y packet to agency Q. Since factory A has capacity of 60 packets so, rest $\left[60-(x+y)\right]$ packets transported to agency R.

Since requirements are always non negative so,

$\Rightarrow \quad x, y \geq 0 \qquad$ (first constraint)

and $\quad 60-(x+y) \geq 0$

$\quad (x+y) \leq 60 \qquad$ (second constraint)

Since requirement of agency P is 40 packet but it has recieved x packet, so $(40-x)$ packets are transported from factory B, requirement of agency Q is 40 packets but it has recieved y packets, so $(40-y)$ packets are transported from factory B. Requirement of agency R is 50 packets but it has recieved $(60-x-y)$ packets from factory A, so $50-\left[60-x-y\right]=(x+y-10)$ is transported from factory B, As the requirements of agencies P, Q, R are always non negative, so,

$$40-x \geq 0$$

$\Rightarrow \quad x \leq 40 \qquad$ (third constraint)

$\quad 40-y \geq 0$

$\Rightarrow \quad y \leq 40 \qquad$ (fourth constraint)

$\quad x+y-10 \geq 0$

$\Rightarrow \quad x+y \geq 10 \qquad$ (fifth constraint)

Costs of transportation of each packet from factory A to agency P, Q, R are Rs 5,4,3 respectively and costs of transportation of each packet from factory B to agency P, Q, R are Rs 4,2,5 respectively,

Let Z be total cost of transportation so,

$$Z = 5x+4y+3\left[60-x-y\right]+4(40-x)+2(40-y)+5(x+y-10)$$

$$= 5x+4y+180-3x-3y+160-4x+80-2y+5x+5y-50$$

$$= 3x+4y+370$$

Hence, mathematical formulation of LPP is find x and y which

maximize $Z = 3x+4y+370$

subject to constraints,

$x, y \geq 0$
$x + y \leq 60$
$x \leq 40$
$y \leq 0$
$x + y \geq 10$

Region $x, y \geq 0$: It is represents first quandrant.

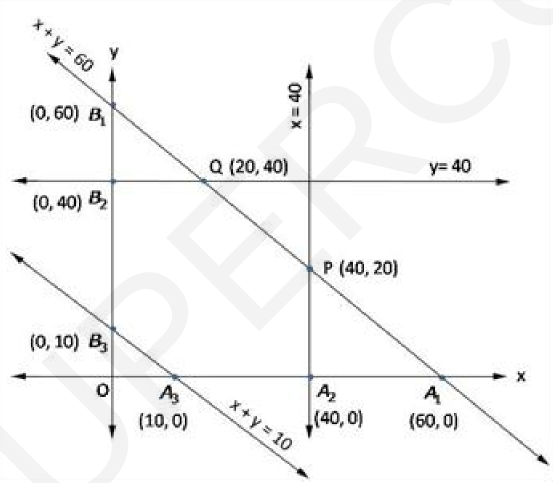
Region $x + y \leq 60$: line $x + y = 60$ meets axes at $A_1(60,0)$, $B_1(0,60)$ respectively.
Region containing origin represents $x + y \leq 60$ as $(0,0)$ satisfies $x + y \leq 60$.

Region $x \leq 40$: line $x = 40$ is parallel to y-axis and meets x-axis at $A_2(40,0)$.
Region containing origin represents $x \leq 40$ as $(0,0)$ satisfies $x \leq 40$.

Region $y \leq 40$: line $y = 40$ is parallel to x-axis and meets y-axis at $B_2(0,40)$.
Region containing origin represents $y \leq 40$ as $(0,0)$ satisfies $y \leq 40$.

Region $x + y \geq 10$: line $x + y = 10$ meets axes at $A_2(10,0)$, $B_3(0,10)$ respectively.
Region containing origin represents $x + y \geq 10$ as $(0,0)$ does not satisfy $x + y \geq 10$.

Shaded region $A_3A_2PQB_2B_3$ represents feasible region.
Point $P(40,20)$ is obtained by solving $x = 40$ and $x + y = 60$
Point $Q(20,40)$ is obtained by solving $y = 40$ and $x + y = 60$



The value of $Z = 3x + 4y + 370$ at

$A_3(10,0)$	$= 3(10) + 4(0) + 370 = 400$
$A_2(40,0)$	$= 3(40) + 4(0) + 370 = 490$
$P(40,20)$	$= 3(40) + 4(20) + 370 = 570$
$Q(20,40)$	$= 3(20) + 4(40) + 370 = 590$
$B_2(0,40)$	$= 3(0) + 4(40) + 370 = 530$
$B_3(0,10)$	$= 3(0) + 4(10) + 370 = 410$

minimum $Z = 400$ at $x = 10$, $y = 0$

From $A \to P =$ 10 packets
From $A \to Q =$ 0 packets
From $A \to R =$ 50 packets
From $B \to P =$ 30 packets
From $B \to Q =$ 40 packets
From $B \to R =$ 0 packets

minimum cost = Rs 400