Lines and Angles
Exercise 6.1

Write the correct answer in each of the following:

1. In Fig. 6.1, if AB || CD || EF, PQ || RS, ∠RQD= 25° and ∠CQP = 60°, then ∠QRS is equal to

(A) 85°
(B) 135°
(C) 145°
(D) 110°

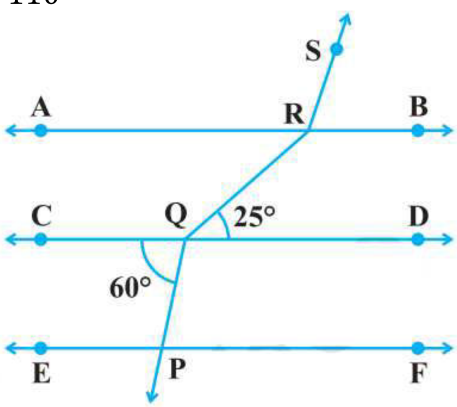


Fig. 6.1

Sol. We have PQ||RS. Produce PQ to M.

$\angle CQP = \angle MQD$ [Vertically opp. $\angle s$]

$\therefore \quad 60^0 = \angle 1 + 25^0$

$\Rightarrow \quad \angle 1 = 35^0$

Now, QM||RS and QR cuts them.

$\angle ARQ = \angle RQD = 25^0$ [Alt. $\angle s$]

$\therefore \quad \angle 1 + (\angle ARQ + \angle ARS) = 180^0$

$\Rightarrow \quad 35^0 (25^0 + \angle ARS) = 180^0$

$\Rightarrow \quad \angle ARS = 180^0 - 60^0 = 120^0$

$\therefore \quad \angle QRS = \angle ARQ + \angle ARS = 25^0 + 120^0 = 145^0$

Hence, (c) is the correct answer.

2. If one angle of a triangle is equal to the sum of the other two angles, then the triangle is

(A) an isosceles triangle
(B) an obtuse triangle
(C) an equilateral triangle
(D) a right triangle

Sol. Let the angles of ΔABC be $\angle A, \angle B$ and $\angle C$

Given that $\angle A = \angle B + \angle C$ ...(1)

But, in any ΔABC, $\angle A+\angle B+\angle C=180^0$ ...(2)

[Angles sum property of triangle]

From equations (1) and (2), we get

$$\angle A+\angle A=180^0 \Rightarrow 2\angle A=180^0 \Rightarrow \angle A=180^0/2=90^0$$

$\therefore$ $A=90^0$

Hence, the triangle is a right triangle and option (d) is correct.

3. An exterior angle of a triangle is 105° and its two interior opposite angles are equal. Each of these equal angles is

(a) $37\frac{1}{2}^0$

(b) $52\frac{1}{2}^0$

(c) $72\frac{1}{2}^0$

(d) 75^0

Sol. An exterior angle of triangle is 150^0.

Let each of the two interior opposite angles be x.

We know that exterior angle of a triangle is equal to the sum of two interior opposite angles.

$\therefore$ $150^0=x+x \Rightarrow 2x=150^0$

$\Rightarrow$ $x=\frac{1}{2}\times 150^0=52\frac{1}{2}^0$

So, each of equal angle is $52\frac{1}{2}^0$

Hence, (b) is the correct answer.

4. The angles of a triangle are in the ratio 5 : 3 : 7. The triangle is

(A) an acute angled triangle

(B) an obtuse angled triangle

(C) a right triangle

(D) an isosceles triangle

Sol. Let the angles of the triangle be 5x, 3x and 7x.

As the sum of the angles of a triangle is 180^0, then

$$5x+3x+7x=180^0$$

$\Rightarrow$ $15x=180^0 \Rightarrow x=180^0 \div 15=12^0$

Therefore, the angle of the triangle are

$$5\times 12^0, 3\times 12^0 \text{ and } 7\times 12^0, \text{ i.e., } 60^0, 36^0 \text{ and } 84^0$$

As the measure of each angle of the triangle is less than 90^0, so the angles of tangle are acute angles.

Therefore, the triangle is an acute angled triangle.

Hence, (a) is the correct answer.

5. If one of the angles of a triangle is 130°, then the angle between the bisectors of the other two angles can be

(A) 50°
(B) 65°
(C) 145°
(D) 155°

Sol. In ΔABC, we have $\angle A = 130^0$.

OB and OC are the bisectors of the angles B and C.

Now, $\angle BOC = 180^0 - (\angle OBC + \angle OCB)$

$= 180^0 - 25^0 = 155^0$

Hence, (d) is the correct answer.

6. In Fig. 6.2, POQ is a line. The value of x is

(A) 20°
(B) 25°
(C) 30°
(D) 35°

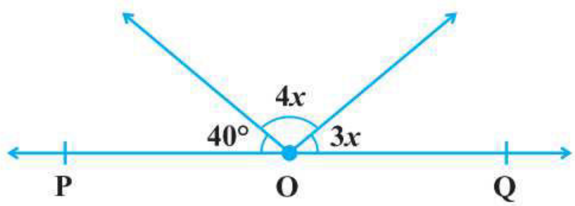


Fig. 6.2

Sol. We have $3x + 4x + 40^0 = 180^0$

$\Rightarrow$ $7x + 40^0 = 180^0 \Rightarrow 7x = 180^0 - 40^0 = 140^0$

$\Rightarrow$ $x = 140^0 \div 7 = 20^0$

Hence, (a) is the correct answer.

7. In Fig. 6.3, if OP||RS, ∠OPQ = 110° and ∠QRS = 130°, then ∠PQR is equal to

(A) 40°
(B) 50°
(C) 60°
(D) 70°

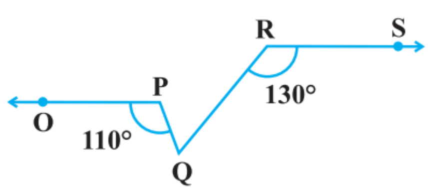


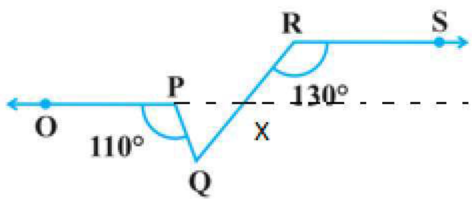
Fig. 6.3

Sol. In the given figure, producing OP, which intersect RQ at X.

Since, OP||RS and RS is a transversal.

So, $\angle RXP = \angle XRS$ [Alternate angles]

$\Rightarrow \quad \angle RXP = 130^0$ ...(1) [$\because \angle QRS = 130^0$]



Now, RQ is a line segment.

So, $\angle PXQ + \angle RXP = 180^0$

$\Rightarrow \quad \angle PXQ = 180^0 - \angle RXP = 180^0 - 130^0$ [From equation (1)]

$\Rightarrow \quad \angle PXQ = 50^0$

In $\Delta PQX, \angle OPQ$ is an exterior angle.

$\therefore \quad \angle OPQ = \angle PXQ + \angle PQX$

[$\because$ Exterior angle = sum of two opposite interior angles]

$\Rightarrow \quad 110^0 = 50^0 + \angle PQX$

$\Rightarrow \quad \angle PQX = 110^0 - 50^0$

$\Rightarrow \quad \angle PQX = 60^0$

$\therefore \quad \angle PQR = 60^0$ [$\because \angle PQX = \angle PQR$]

Hence, the option (c) is correct.

8. Angles of a triangle are in the ratio 2 : 4 : 3. The smallest angle of the triangle is

(A) 60°
(B) 40°
(C) 80°
(D) 20°

Sol. Given that: The ratio of angles of a triangle is 2 : 4 : 3.

Let the angles of the triangle be $\angle A, \angle B$ and $\angle C$

$\therefore \angle A = 2x, \angle B = 4x$ and $\angle C = 3x$

In $\angle ABC, \angle A + \angle B + \angle C = 180^0$

[$\because$ Sum of angles of a triangle is 180^0]

$\Rightarrow 2x + 4x + 3x = 180^0 \Rightarrow 9x = 180^0 \Rightarrow x = 180^0 / 9 = 20^0$

$\therefore \angle A = 2x = 2 \times 20^0 = 40^0$

$\angle B = 4x = 4 \times 20^0 = 80^0$

And $\angle C = 3x = 3 \times 20^0 = 60^0$

Hence, the smallest angles of a triangle is 40^0 and option (b) is correct answer.

Lines and Angles
Exercise 6.2

1. For what value of x + y in Fig. 6.4 will ABC be a line? Justify your answer.

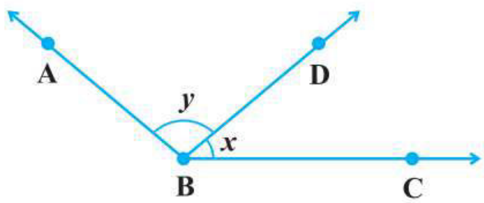


Fig. 6.4

Sol. In the given figure, x and y are two adjacent angles.
For ABC to be a straight line, the sum of two adjacent angles x and y must be 180^0.

2. Can a triangle have all angles less than 60°? Give reason for your answer.

Sol. A triangle cannot have all angle less than 60°. Then, sum of all the angles will be less than 180° whereas sum of all the angles of a triangle is always 180°.

3. Can a triangle have two obtuse angles? Give reason for your answer.

Sol. An angle whose measure is more than 90^o but less than 180^o is called an obtuse angle.
A triangle cannot have two obtuse angles because the sum of all the angles of it cannot be more than 180^o. It is always equal to 180^o.

4. How many triangles can be drawn having its angles as 45°, 64° and 72°? Give reason for your answer.

Sol. We cannot draw any triangle having its angles 45°, 64° and 72° because the sum of the angles (45° + 64° + 72° =181°) cannot be 181°.

5. How many triangles can be drawn having is angles as 53°, 64° and 63°? Give reason for your answer.

Sol. Sum of these angles = 53° + 64° + 63° = 180°. So, we can draw infinitely many triangles, sum of the angles of every triangle having its angles as 53°, 64° and 63° is 180°.

6. In Fig. 6.5, find the value of x for which the lines l and m are parallel.

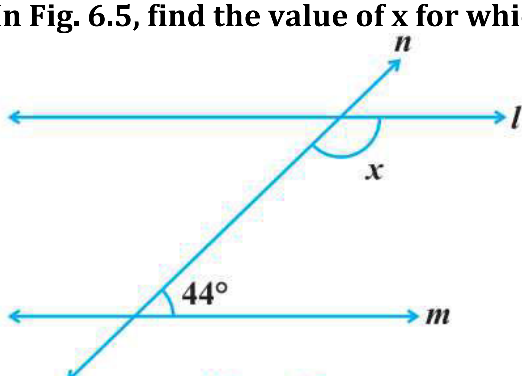


Fig. 6.5

Sol. If a transversal intersects two parallel lines, then each pair of consecutive interior angles are supplementary. Here, the two given lines l and m are parallel.
Angles x and 44°, are consecutive interior angles on the same side of the transversal.
Therefore, $x+44^0=180^0$
Hence, $x=180^0-44^0=136^0$

7. Two adjacent angles are equal. Is it necessary that each of these angles will be a right angle? Justify your answer.

Sol. No, each of these angles will be a right angle only when they form a linear pair, i.e., when the non-common arms of the given two adjacent angles are two opposite rays.

8. If one of the angles formed by two intersecting lines is a right angle, what can you say about the other three angles? Give reason for your answer.

Sol. If two intersect each other at a point, then four angles are formed. If one of these four angles is a right angle, then each of the other three angles will also be a right by linear pair axiom.

9. In Fig.6.6, which of the two lines are parallel and why?

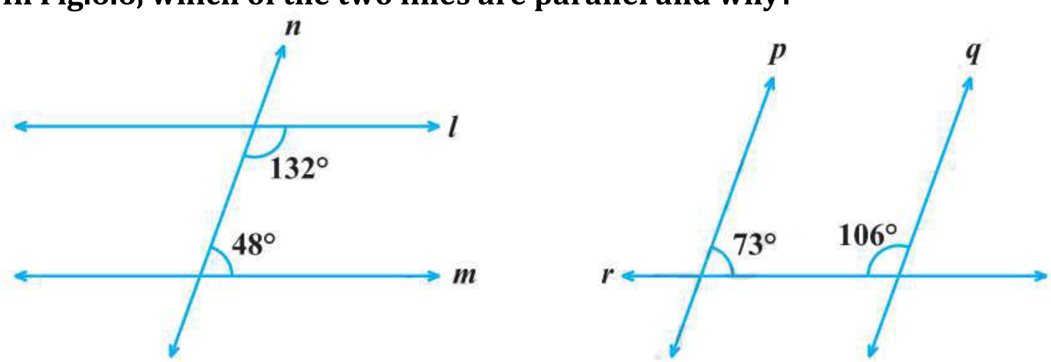


Fig. 6.6

Sol. For fig(i), a transversal intersects two lines such that the sum of interior angles on the same side on the same side of the transversal is $132^o + 48^o = 180^o$.
Therefore, the line l and m are parallel.
For fig. (ii), a transversal intersects two line such that the sum of interior angles on the same sides of the transversal is $73^o + 106^o = 179^o$.
Therefore, the lines p and q are not parallel.

10. Two lines l and m are perpendicular to the same line n. Are l and m perpendicular to each other? Give reason for your answer.

Sol. When two lines l and m are perpendicular to the same line n, each of the two corresponding angles formed by these lines l and m with the line n are equal (each is equal to 90^o). Hence, the line l and m are parallel.

Lines and Angles
Exercise 6.3

1. In Fig. 6.9, OD is the bisector of ∠AOC, OE is the bisector of ∠BOC and OD ⊥ OE. Show that the points A, O and B are collinear.

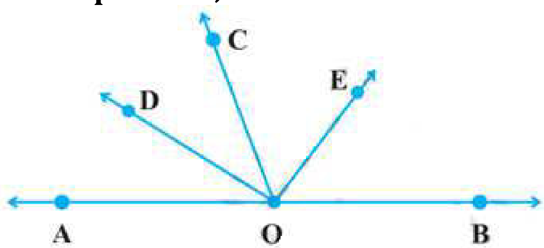


Fig. 6.9

Sol. Given: In figure, OD ⊥ OE, OD and OE are the bisector of ∠AOC and ∠BOC.
To prove: points A, O and B are collinear i.e., AOB is a straight line.
Proof: Since, OD and OE bisect angles ∠AOC and ∠BOC respectively.

∴ $\angle AOC = 2\angle DOC$ ...(1)

And $\angle COB = 2\angle COE$ ...(2)

On adding equations (1) and (2), we get

$\angle AOC = \angle COB = 2\angle DOC + 2\angle COE$

$\Rightarrow \angle AOC + \angle COB = 2(\angle DOC + \angle COE)$

$\Rightarrow \angle AOC + \angle COB = 2\angle DOE$

$\Rightarrow \angle AOC + \angle COB = 2\times 90^0$ $[\because OD \perp OE]$

$\Rightarrow \angle AOC + \angle COB = 180^0$

∴ $\angle AOB = 180^0$

So, $\angle AOC + \angle COB$ are forming linear pair or AOB is a straight line. Hence, points A, O and B are collinear.

2. In Fig. 6.10, ∠1 = 60° and ∠6 = 120°. Show that the lines m and n are parallel.

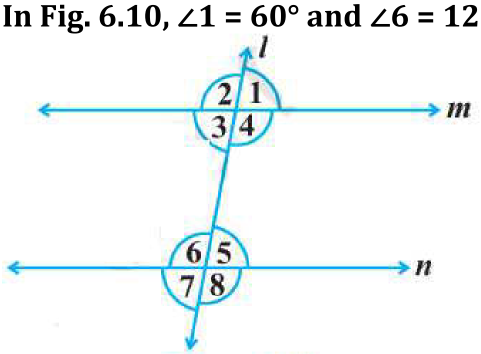


Fig. 6.10

Sol. We have,

$\angle 5 + \angle 6 = 180^0$ [Angles pf a linear pair]

$\Rightarrow \angle 5 + 120^0 = 180^0 \Rightarrow \angle 5 = 180^0 - 120^0 = 60^0$

Now, $\angle 1 = \angle 5$ [Each = 60^{o}]
But, these are corresponding angles.
Therefore, the lines m and n are parallel.

3. AP and BQ are the bisectors of the two alternate interior angles formed by intersection of a transversal t with parallel lines l and m (Fig. 6.1 1). Show that AP || BQ.

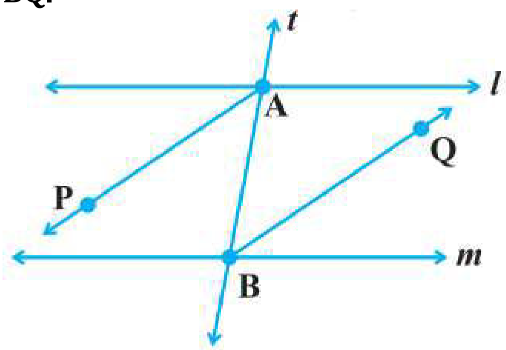


Fig. 6.11

Sol. $\because$ l || m and t is the transversal

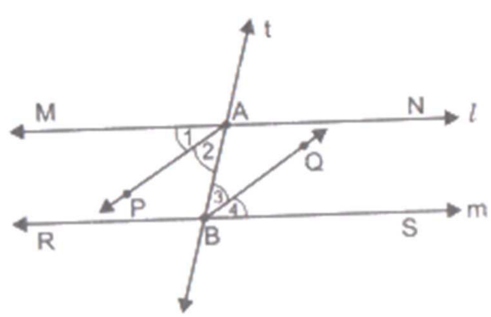
$\angle MAB = \angle SBA$ [Alt. $\angle s$]

$\Rightarrow \frac{1}{2}\angle MAB = \frac{1}{2}\angle SBA \Rightarrow \angle 2 = \angle 3$

But, $\angle 2$ and $\angle 3$ are alternate angles.
Hence, AP||BQ.

4. If in Fig. 6.1 1, bisectors AP and BQ of the alternate interior angles are parallel, then show that l ||m.

Sol.



AP is the bisector of $\angle MAB$ and BQ is the bisector of $\angle SBA$. We are given that AP||BQ.
As AP||BQ, So $\angle 2 = \angle 3$ [Alt. $\angle s$]

$\therefore \quad 2\angle 2 = 2\angle 3$

$\Rightarrow \quad \angle 2 + \angle 2 = \angle 3 + \angle 3$

$\Rightarrow \quad \angle 1 + \angle 2 = \angle 3 + \angle 4 \quad [\because \angle 1 = \angle 2 \text{ and } \angle 3 = \angle 4]$

$\Rightarrow \quad \angle MAB = \angle SBA$

But, these are alternate angles. Hence, the lines l and m are parallel, i.e., l ||m.

5. In Fig. 6.12, BA|| ED and BC || EF. Show that ∠ABC = ∠DEF [Hint: Produce DE to intersect BC at P (say)].

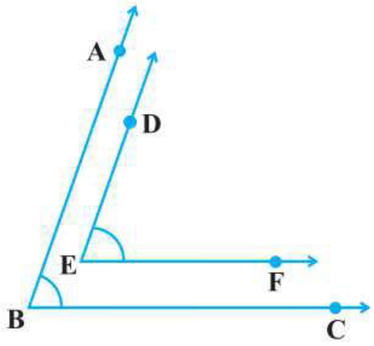
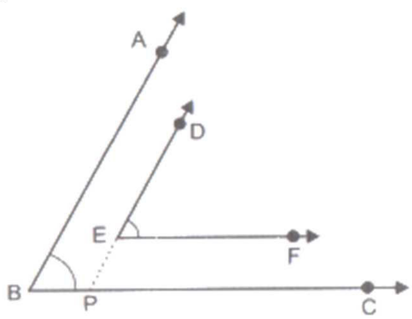


Fig. 6.12

Sol. Produce DE to intersect BC at P(say).
EF||BC and DP is the transversal,



$\therefore \quad \angle DEF = \angle DPC$ ...(1) [Corres. $\angle s$]
Now, AB||DP and BC is the transversal,
$\therefore \quad \angle DPC = \angle ABC$ ...(2) [Corres. $\angle s$]
From (1) and (2), we get
$\angle ABC = \angle DEF$
Hence, Proved.

6. In Fig. 6.13, BA || ED and BC || EF. Show that ∠ ABC + ∠ DEF = 180°

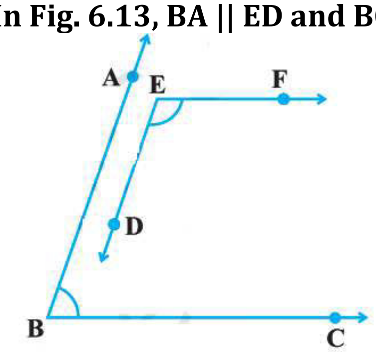
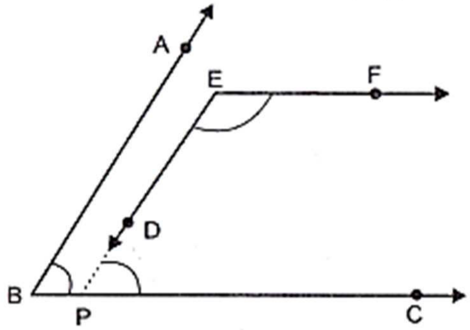


Fig. 6.13

Sol. Produce ED to meet BC at P(say)



Now, EF||BC and EP is the transversal.

$\therefore \quad \angle DEF = \angle EPC = 180^0$ ...(1)

Again, EP||AB and BC is the transversal.

$\therefore \quad \angle EPC = \angle ABC$ ...(2)[corresponding $\angle s$]

From (1) and (2), we get

$\angle DEF = \angle ABC = 180^0$

$\Rightarrow \angle ABC + \angle DEF = 180^0$

Hence, proved.

7. In Fig. 6.14, DE || QR and AP and BP are bisectors of ∠EAB and ∠RBA, respectively. Find ∠APB.

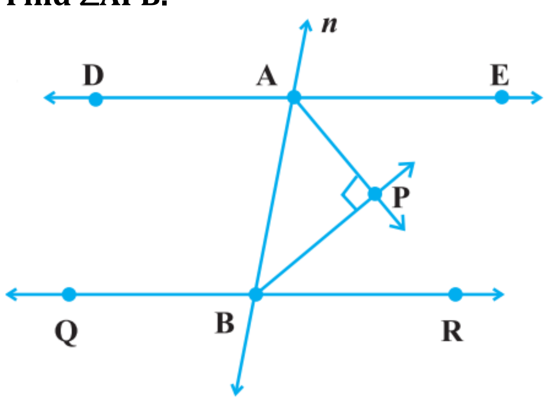


Fig. 6.14

Sol. DE||QR and the line n is the transversal line.

$\therefore \angle EAB + \angle RBA = 180^0$ ...(1)

[$\because$ If a transversal intersects two parallel lines, then each pair of consecutive interior angles are supplementary]

$\Rightarrow \quad \angle PAB + \angle PBA = 90^0$

[$\because$ AP is the bisector of $\angle EAB$ and BP is the bisector of $\angle RBA$]

Now, from ΔAPB, we have

$\angle APB = 180^0 - (\angle PAB + \angle PBA)$

$\Rightarrow \quad \angle APB = 180^0 - 90^0 = 90^0$

8. The angles of a triangle are in the ratio 2 : 3 : 4. Find the angles of the triangle.

Sol. Given: Ratio of angles is 2 : 3 : 4.

To find: Angles of triangle.

Proof: The ratio of angles of a triangle is 2 : 3 : 4.

Let the angles of a triangle be $\angle A, \angle B$ and $\angle C$

Therefore, $\angle A = 2x,$ then $\angle B = 3x$ and $\angle C = 4x.$

In $\Delta ABC, \angle A + \angle B + \angle C = 180^0$ [$\because$ Sum of angles of a triangle is 180°]

$\therefore \quad 2x + 3x + 4x = 180^0$

$\Rightarrow 9x = 180^0 \Rightarrow x = 180^0 / 9 = 20^0$

$\therefore \quad \angle A = 2x = 2 \times 20^0 = 40^0$

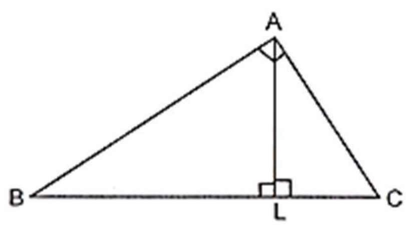
$\angle B = 3x = 3 \times 20^0 = 60^0$

And $\quad \angle C = 4x = 4 \times 20^0 = 80^0$

Hence, the angles of the triangles are 40°, 60° and 80°.

9. A triangle ABC is right angled at A. L is a point on BC such that AL ⊥ BC. Prove that ∠BAL = ∠ACB.

Sol.



Given: In ΔABC

$\angle A = 90^0$ and $AL \perp BC.$

To prove: $\angle BAL = \angle ACB.$

Proof: In ΔABC and $\Delta LAC,$

$\angle BAC = \angle ALC$ ...(1) [Each = 90°]

And $\quad \angle ABC = \angle ABL.$ ...(2) [Common angle]

Adding equations (1) and (2), we get

$\angle BAC + \angle ABC = \angle ALC + \angle ABC$ ...(3)

In $\Delta ABC, \angle BAC + \angle ACB + \angle ABC = 180^0$

[Sum of angles of triangle is 180°]

$\Rightarrow \angle BAC + \angle ABC = 180^0 - \angle ACB$ ...(4)

In $\Delta ABL, \angle ABL + \angle ALB + \angle BAL = 180^0$

[Sum of the angles of triangle is 180°]

$\Rightarrow \angle ABL + \angle ALC = 180^0 - \angle BAL$ ...(5) $\left[\angle ALC = \angle ALB = 90^0\right]$

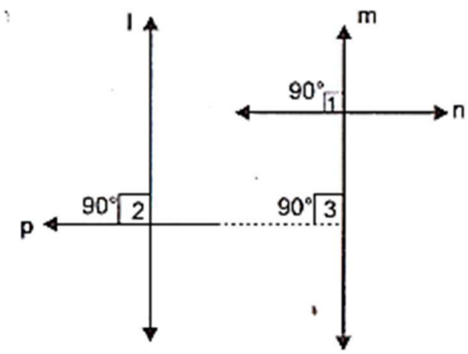
Substituting the value from equation (4) and (5) in equation (3), we get

$180^0 - \angle ACB = 180^0 - \angle BAL \Rightarrow \angle ACB = \angle BAL$

Hence, proved.

10. Two lines are respectively perpendicular to two parallel lines. Show that they are parallel to each other.

Sol.



Two lines p and n are respectively perpendicular to two parallel line l and m, i.e., $p \perp l$ and $n \perp m.$

We have to show that p is parallel to n.

As $n \perp m,$ so $\angle 1 = 90^0$ ...(1)

Again, $p \perp l,$ So $\angle 2 = 90^0.$

But, l is parallel to m, so

$\angle 2 = \angle 3$ [corres. $\angle s$]

$\therefore$ $\angle 2 = \angle 90^0$ ...(2) [$\because \angle 2 = 90^0$]

From (1) and (2), we get

$\Rightarrow$ $\angle 1 = \angle 3$ [Each = 90°]

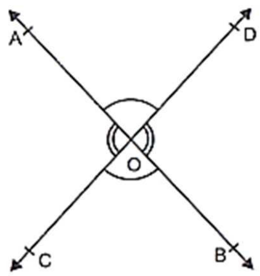
But, these are corresponding angles.

Hence, p||n.

Lines and Angles
<u>Exercise 6.4</u>

1. If two lines intersect, prove that the vertically opposite angles are equal.

Sol.



Given: Two lines AB and CD intersect at point O.

To prove: (i) $\angle AOC = \angle BOD$

(ii) $\angle AOD = \angle BOC$

Proof: (i) Since, ray OA stands on line CD.

$\therefore \quad \angle AOC = \angle AOD = 180^0 \quad ...(1)$

[Linear pair axiom]

Similarly, ray OD stands on line AB.

$\therefore \quad \angle AOD = \angle BOD = 180^0 \quad ...(2)$

From equations (1) and (2), we get

$\angle AOC = \angle AOD = \angle AOD + \angle BOD$

$\Rightarrow \quad \angle AOC = \angle BOD$

Hence, proved.

(ii) Since, ray OD stands on line AB.

$\therefore \quad \angle AOD + \angle BOD = 180^0 \quad ...(3)$ [Linear pair axiom]

Similarly, ray OB stands on line CD.

$\therefore \quad \angle DOB + \angle BOC = 180^0 \quad ...(4)$

From equations (3) and (4), we get

$\angle AOD + \angle BOD = \angle DOB + \angle BOC \Rightarrow \angle AOD = \angle BOC$

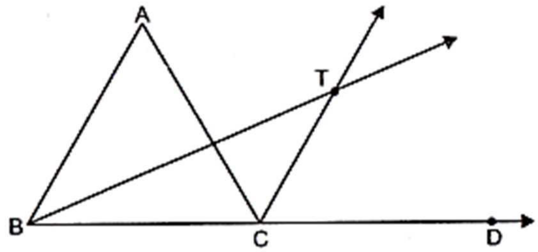
Hence, proved.

2. Bisectors of interior ∠B and exterior ∠ACD of a ΔABC intersect at the point T. Prove that

$\angle BTC = \frac{1}{2}\angle BAC$

Sol. Given: ΔABC, produce BC to D and the bisectors of $\angle ABC$ and $\angle ACD$ meet at point T.

To prove: $\angle BTC = \frac{1}{2}\angle BAC$



Proof: In $\Delta ABC, \angle ACD$ is an exterior angle.

$\therefore \quad \angle ACD = \angle ABC + \angle CAB$

[Exterior angle of a triangle is equal to the sum of two opposite angles]

$\Rightarrow \quad \frac{1}{2}\angle ACD = \frac{1}{2}\angle CAB + \frac{1}{2}\angle ABC$ [Dividing both sides by 2]

$\Rightarrow \quad \angle TCD = \frac{1}{2}\angle CAB + \frac{1}{2}\angle ABC$ ...(1)

[$\because$ CT is a bisector of $\angle ACD \Rightarrow \frac{1}{2}\angle ACD = \angle TCD$]

In $\Delta BTC, \angle TCD = \angle BTC + \angle CBT$

[Exterior angle of the triangle is equal to the sum of two opposite angles]

$\Rightarrow \quad \angle TCD = \angle BTC + \frac{1}{2}\angle ABC$ ...(2)

[$\because$ BT is bisector of $\Delta ABC \Rightarrow \angle CBT = \frac{1}{2}\angle ABC$]

From equation (1) and (2), we get

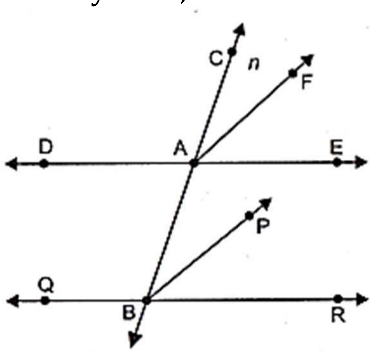
$$\frac{1}{2}\angle CAB + \frac{1}{2}\angle ABC = \angle BTC + \frac{1}{2}\angle ABC$$

$\Rightarrow \quad \frac{1}{2}\angle CAB = \angle BTC$ or $\frac{1}{2}\angle BAC = \angle BTC$

Hence, proved.

3. A transversal intersects two parallel lines. Prove that the bisectors of any pair of corresponding angles so formed are parallel.

Sol. Given: Two lines DE and QR are parallel and are intersected by transversal at A and B respectively. Also, BP and AF are the bisector of angles $\angle ABR$ and $\angle CAE$ respectively.



To prove: EP||FQ

Proof: Given, $DE \parallel QR \Rightarrow \angle CAE = \angle ABR$ [Corresponding angles]

$\Rightarrow \quad \frac{1}{2}\angle CAE = \frac{1}{2}\angle ABR$ [Dividing both sides by 2]

$\Rightarrow \quad \angle CAE = \angle ABP$

[$\because$ BP and AF are the bisector of angles $\angle ABR$ and $\angle CAE$ respectively.

As these are the corresponding angles on the transversal line n and are equal.

Here, EP||FQ.

4. Prove that through a given point, we can draw only one perpendicular to a given line.

[Hint: Use proof by contradiction].

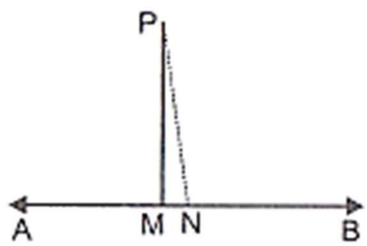
Sol. From the point P, a perpendicular PM is drawn to the given line AB.

$\therefore \quad \angle PMB = 90^0$

Let if possible, we can draw another perpendicular PN to the line AB. Then,

$\angle PMB = 90^0$

$\therefore \quad \angle PMB = \angle PNB,$ which is possible only when PM and PN coincide with each other.



Hence, through a given point, we can draw only one perpendicular to a given line.

5. Prove that two lines that are respectively perpendicular to two intersecting lines intersect each other.

[Hint: Use proof by contradiction].

Sol. Given: Let lines l and m are two intersecting lines. Again, let n and p be another two lines which are perpendicular to the intersecting lines meet at point D.

To prove: Two lines n and p intersecting at a point.

Proof: Let us consider lines n and p are not intersecting, then it means they are parallel to each other i.e., n||P. ...(1)

Since, lines n and p are perpendicular to m and l respectively.

But from equation (1), n||p, it implies that l and m. It is a contradiction

Thus, our assumption is wrong. Hence, lines n and p intersect at a point.

6. Prove that a triangle must have at least two acute angles.

Sol. If the triangle is an acute angled triangle, then all its three angles are acute angle. Each of these angles is less than 90^o, so they can make three angles sum equal to 180^o.

If a triangle is a right triangle, then one angle which is right angle will be equal to 90^o and the other two acute angles can make the three angles sum equal to 180^o.

Hence, we can say that a triangle must have a least two acute angles.

7. In Fig. 6.17, ∠Q > ∠R, PA is the bisector of ∠QPR and PM ⊥ QR. Prove that $\angle APM = \frac{1}{2}(\angle Q - \angle R)$

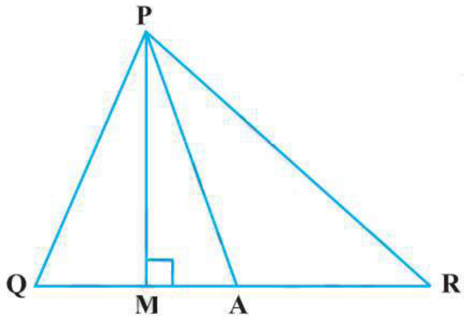


Fig. 6.17

Sol. Given: $\Delta PQR, \angle Q < \angle R,$ PA is the bisector of $\angle QPR$ and $PM \perp QR.$

To prove: $\angle APM = \frac{1}{2}(\angle Q - \angle R)$

Proof: Since, PA is the bisector of $\angle QPR$

$\angle QPA = \angle APR$

In $\angle PQM, \angle Q + \angle PMQ + \angle QPM = 180^0$ ...(1)

[Angle sum property of a triangle]

$\Rightarrow \quad \angle Q + 90^0 + \angle QPM = 180^0$ $[\therefore \angle PMR = 90^0]$

$\Rightarrow \quad \angle Q = 90^0 - \angle QPM$ ...(2)

In $\Delta PMR, \angle PMR + \angle R + \angle RPM = 180^0$

[Angle sum property of a triangle]

$\Rightarrow 90^0 + \angle R + \angle RPM = 180^0$ $[\because \angle PMR = 90^0]$

$\Rightarrow \quad \angle R = 180^0 - 90^0 - \angle RPM$

$\Rightarrow \quad \angle Q = 90^0 - \angle QPM$

$\Rightarrow \quad \angle PRM = 90^0 - \angle RPM$ ...(3)

Subtracting equation (3) from equation (2), we get

$\angle Q - \angle R = (90^0 - \angle QPM) - (90^0 - \angle RPM)$

$\Rightarrow \quad \angle Q - \angle R = \angle RPM - \angle QPM$

$\Rightarrow \quad \angle Q - \angle R = (\angle RPM + \angle APM) - (\angle QPA - \angle APM)$ ...(4)

$\Rightarrow \quad \angle Q - \angle R = \angle QPA + \angle APM - \angle QPA + \angle APM$ [Using equation (1)]

$\Rightarrow \quad \angle Q - \angle R = 2\angle APM$

$\Rightarrow \quad \angle APM = \frac{1}{2}(\angle Q - \angle R)$

Hence, proved.