Exercise -1.1

1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Sol:

Yes, zero is a rotational number. It can be written in the form of $\frac{p}{q}$ where q to as such as

$\frac{0}{3}, \frac{0}{5}, \frac{0}{11}, etc.........$

2. Find five rational numbers between 1 and 2.

Sol:

Given to find five rotational numbers between 1 and 2

A rotational number lying between 1 and 2 is

$$(1+2)\div 2 = 3\div 2 = \frac{3}{2} \quad i.e., 1 < \frac{3}{2} < 2$$

Now, a rotational number lying between 1 and $\frac{3}{2}$ is

$$\left(1+\frac{3}{2}\right)\div 2 = \left(\frac{2+3}{2}\right)\div 2 = \frac{5}{2}\div 2 = \frac{5}{2}\times\frac{1}{2} = \frac{5}{4}$$

i.e., $1 < \frac{5}{4} < \frac{3}{2}$

Similarly, a rotational number lying between 1 and $\frac{5}{4}$ is

$$\left(1+\frac{5}{4}\right)\div 2 = \left(\frac{4+5}{2}\right)\div 2 = \frac{9}{4}\div 2 = \frac{9}{4}\times\frac{1}{2} = \frac{9}{8}$$

i.e., $1 < \frac{9}{8} < \frac{5}{4}$

Now, a rotational number lying between $\frac{3}{2}$ and 2 is

$$\left(1+\frac{5}{4}\right)\div 2 = \left(\frac{4+5}{4}\right)\div 2 = \frac{9}{4}\div 2 = \frac{9}{4}\times\frac{1}{2} = \frac{9}{8}$$

i.e., $1 < \frac{9}{8} < \frac{5}{4}$

Now, a rotational number lying between $\frac{3}{2}$ and 2 is

$$\left(\frac{3}{2}+2\right)\div 2 = \left(\frac{3+4}{2}\right)\div 2 = \frac{7}{2}\times\frac{1}{2} = \frac{7}{4}$$

i.e., $\frac{3}{2} < \frac{7}{4} < 2$

Similarly, a rotational number lying between $\frac{7}{4}$ and 2 is

$$\left(\frac{7}{4}+2\right) \div 2 = \left(\frac{7+8}{4}\right) \div 2 = \frac{15}{4} \times \frac{1}{2} = \frac{15}{8}$$

i.e., $\frac{7}{4} < \frac{15}{8} < 2$

$$\therefore 1 < \frac{9}{8} < \frac{5}{4} < \frac{3}{2} < \frac{7}{4} < \frac{15}{8} < 2$$

Recall that to find a rational number between r and s, you can add r and s and divide the sum by 2, that is $\frac{r+s}{2}$ lies between r and s So, $\frac{3}{2}$ is a number between 1 and 2. you can proceed in this manner to find four more rational numbers between 1 and 2, These four numbers are, $\frac{5}{4}, \frac{11}{8}, \frac{13}{8}$ *and* $\frac{7}{4}$

3. Find six rational numbers between 3 and 4.

Sol:

Given to find six rotational number between 3 and 4

We have,

$$3 \times \frac{7}{7} = \frac{21}{7} \text{ and } 4 \times \frac{7}{7} = \frac{28}{7}$$

We know that

$$21 < 22 < 23 < 24 < 25 < 26 < 27 < 28$$

$$\Rightarrow \frac{21}{7} < \frac{22}{7} < \frac{23}{7} < \frac{24}{7} < \frac{25}{7} < \frac{26}{7} < \frac{27}{7} < \frac{28}{7}$$

$$\Rightarrow 3 < \frac{22}{7} < \frac{23}{7} < \frac{24}{7} < \frac{25}{7} < \frac{26}{7} < \frac{27}{7} < 4$$

Hence, 6 rotational number between 3 and 4 are

$$\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$$

4. Find five rational numbers between $\frac{3}{4}$ *and* $\frac{4}{5}$

Sol:

Given to find 5 rotational numbers lying between $\frac{3}{5}$ *and* $\frac{4}{5}$.

We have,

$\frac{3}{5}\times\frac{6}{6}=\frac{18}{100}$ and $\frac{4}{5}\times\frac{6}{6}=\frac{24}{30}$

We know that

$18<19<20<21<22<23<24$

$\Rightarrow \frac{18}{30}<\frac{19}{30}<\frac{20}{30}<\frac{21}{30}<\frac{22}{30}<\frac{23}{30}<\frac{24}{30}$

$\Rightarrow \frac{3}{5}<\frac{19}{30}<\frac{20}{30}<\frac{21}{30}<\frac{22}{30},\frac{23}{30},\frac{4}{5}$

$\Rightarrow \frac{3}{5}<\frac{19}{30}<\frac{2}{3}<\frac{7}{10}<\frac{11}{15}<\frac{23}{30}<\frac{4}{5}$

Hence, 5 rotational number between $\frac{3}{5}$ and $\frac{4}{5}$ are

$\frac{19}{30},\frac{2}{3},\frac{7}{10},\frac{11}{15},\frac{23}{30}.$

5. Are the following statements true or false? Give reasons for your answer.

(i) Every whole number is a rational number.

(ii) Every integer is a rational number.

(iii) Every rational number is a integer.

(iv) Every natural number is a whole number.

(v) Every integer is a whole number.

(vi) Evert rational number is a whole number.

Sol:

(i) False. As whole numbers include zero, whereas natural number does not include zero

(ii) True. As integers are a part of rotational numbers.

(iii) False. As integers are a part of rotational numbers.

(iv) True. As whole numbers include all the natural numbers.

(v) False. As whole numbers are a part of integers

(vi) False. As rotational numbers includes all the whole numbers.

Exercise – 1.2

Express the following rational numbers as decimals:

1. (i) $\frac{42}{100}$ (ii) $\frac{327}{500}$ (iii) $\frac{15}{4}$

Sol:

(i) By long division, we have

$$100\overline{)42.00}\ 0\cdot42$$

$$\begin{array}{r} \underline{400} \\ 200 \\ \underline{200} \\ 0 \end{array}$$

$\therefore \boxed{\frac{42}{100} = 0\cdot42}$

(ii) By long division, we have

$$500\overline{)327\cdot000}\ 0.654$$

$$\begin{array}{r} \underline{3000} \\ 2700 \\ \underline{2500} \\ 2000 \\ \underline{2000} \\ 0 \end{array}$$

$\therefore \boxed{\frac{327}{500} = 0\cdot654}$

(iii) By long division, we have

$$4\overline{)15\cdot00}\quad 3\cdot75$$

$$\begin{array}{r} \underline{12} \\ 30 \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

$\therefore \boxed{\frac{15}{4} = 3\cdot75}$

2. (i) $\frac{2}{3}$ (ii) $-\frac{4}{9}$ (iii) $\frac{-2}{15}$ (iv) $-\frac{22}{13}$ (v) $\frac{437}{999}$

Sol:

(i) By long division, we have

$$
\begin{array}{r|l}
3 & 2\cdot0000\,(0.6666 \\
 & \underline{18} \\
 & 20 \\
 & \underline{18} \\
 & 20 \\
 & \underline{18} \\
 & 20 \\
 & \underline{18} \\
 & 2
\end{array}
$$

$\therefore \boxed{\frac{2}{3} = 0\cdot6666..... = 0.\overline{6}}$

(ii) By long division, we have

$$
\begin{array}{r|l}
9 & 4\cdot0000\,(0\cdot4444 \\
 & \underline{36} \\
 & 40 \\
 & \underline{36} \\
 & 40 \\
 & \underline{36} \\
 & 40 \\
 & \underline{36} \\
 & 4
\end{array}
$$

$\therefore \boxed{\frac{4}{9} = 0\cdot4444..... = 0.\overline{4}}$

Hence, $\boxed{-\frac{4}{9} = -0.\overline{4}}$

(iii) By long division, we have

$$
\begin{array}{r|l}
5 & 2\cdot0000\,(0.13333 \\
 & \underline{15} \\
 & 50 \\
 & \underline{45} \\
 & 50 \\
 & \underline{45}
\end{array}
$$

$$\begin{array}{r} 50 \\ \underline{45} \\ 5 \end{array}$$

$\therefore \frac{2}{15} = 0.1333 \ldots = 0.1\bar{3}$

Hence, $\boxed{\frac{-2}{15} = -0.1\bar{3}}$

(iv) By long division, we have

$$\begin{array}{l} 13\overline{)22.0000}\ (1.692307692307 \\ \underline{-13} \\ \quad 90 \\ \underline{\quad -78} \\ \quad\ 120 \\ \underline{\quad -117} \\ \qquad 30 \\ \underline{\quad -26} \\ \quad\ 40 \\ \underline{\quad -39} \\ \quad\ 100 \\ \underline{\quad -91} \\ \quad\ 90 \\ \underline{\quad -78} \\ \quad\ 120 \\ \underline{\quad -117} \\ \quad\ 30 \\ \quad\ 26 \end{array}$$

$\therefore \frac{22}{13} = 1\cdot 692307692307\ldots\ldots. = 1.\overline{692307} \Rightarrow \boxed{-\frac{22}{13} = 1.\overline{692307}}$

(v) By long division, we have

$$\begin{array}{l} 999\overline{)437.000000}\ (0.437437 \\ \underline{3996} \\ 3740 \\ \underline{2997} \\ 7430 \\ \underline{6993} \end{array}$$

4370
3996
3740
2997
7430
6993
4370

$$\therefore \boxed{\frac{437}{999} = 0.437437......... = 0.\overline{437}}$$

(vi) By long division, we have

$$26\overline{)33.0000000000000}(1.2692307692307$$

26
70
_52
180
_156
240
_234
60
_52
80
_78
200
_182
180
_156
240
_234
60
_52
80
_78
200
_182
18

$$\therefore \frac{33}{26} = 1.2692307698307........... = 1.2\overline{692307}$$

3. Look at several examples of rational numbers in the form $\frac{p}{q}$ $(q \neq 0)$, where p and q are integers with no common factors other than 1 and having terminating decimal representations. Can you guess what property q must satisfy?

Sol:

A rational number $\frac{p}{q}$ is a terminating decimal only, when prime factors of q are q and 5 only. Therefore, $\frac{p}{q}$ is a terminating decimal only, when prime factorization of q must have only powers of 2 or 5 or both.

Exercise -1.3

1. Express each of the following decimals in the form $\frac{p}{q}$:

(i) 0.39
(ii) 0.750
(iii) 2.15
(iv) 7.010
(v) 9.90
(vi) 1.0001

Sol:

(i) We have,

$$0\cdot 39 = \frac{39}{100}$$

$$\Rightarrow \boxed{0\cdot 39 = \frac{39}{100}}$$

(ii) We have,

$$0\cdot 750 = \frac{750}{1000} = \frac{750 \div 250}{1000 \div 250} = \frac{3}{4}$$

$$\therefore \boxed{0.750 = \frac{3}{4}}$$

(iii) We have

$$2\cdot 15 = \frac{215}{100} = \frac{215 \div 5}{100 \div 5} = \frac{43}{20}$$

$$\therefore \boxed{2\cdot 15 = \frac{43}{20}}$$

(iv) We have,

$$7\cdot 010=\frac{7010}{1000}=\frac{7010\div 10}{1000\div 10}=\frac{701}{100}$$

$$\therefore \boxed{7010=\frac{701}{100}}$$

(v) We have,

$$9\cdot 90=\frac{990}{100}=\frac{990\div 10}{100\div 10}=\frac{99}{10}$$

$$\therefore \boxed{9\cdot 90=\frac{99}{10}}$$

(vi) We have,

$$1\cdot 0001=\frac{10001}{10000}$$

$$\therefore \boxed{1\cdot 0001=\frac{10001}{10000}}$$

2. Express each of the following decimals in the form $\frac{p}{q}$:

(i) $0.\overline{4}$

(ii) $0.\overline{37}$

Sol:

(i) Let $x=0\cdot\overline{4}$

Now, $x=0\cdot\overline{4}=0.444....$ $\quad ---(1)$

Multiplying both sides of equation (1) by 10, we get,

$10x=4.444....$ $\quad ---(2)$

Subtracting equation (1) by (2)

$\therefore 10x-x=4.444...-0.444...$

$\Rightarrow 9x=4$

$\Rightarrow x=\frac{4}{9}$

Hence, $0\cdot\overline{4}=\frac{4}{9}$

(ii) Let $x=0\cdot\overline{37}$

Now, $x=0.3737...$ $\quad .....(1)$

Multiplying equation (1) by l0.

$\therefore 10x=3.737....$ $\quad ---(2)$

$100x = 37.3737...\qquad ---(3)$

Subtracting equation (1) by equation (3)

$\therefore 100x - x = 37$

$\Rightarrow 99x = 37$

$\Rightarrow x = \frac{37}{99}$

Hence, $0\cdot\overline{37} = \frac{37}{99}$

Exercise -1.4

1. Define an irrational number.

Sol:

A number which can neither be expressed as a terminating decimal nor as a repeating decimal is called an irrational number. For example,

1.01001000100001…

2. Explain, how irrational numbers differ from rational numbers?

Sol:

A number which can neither be expressed as a terminating decimal nor as a repeating decimal is called an irrational number, For example,

0.33033003300033...

On the other hand, every rational number is expressible either as a terminating decimal or as a repeating decimal. For examples, $3.2\overline{4}$ and 6.2876 are rational numbers

3. Examine, whether the following numbers are rational or irrational:

(i) $\sqrt{7}$

(ii) $\sqrt{4}$

(iii) $2+\sqrt{3}$

(iv) $\sqrt{3}+\sqrt{2}$

(v) $\sqrt{3}+\sqrt{5}$

(vi) $\left(\sqrt{2}-2\right)^2$

(vii) $\left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)$

(viii) $\left(\sqrt{2}+\sqrt{3}\right)^2$

(ix) $\sqrt{5}-2$

(x) $\sqrt{23}$

(xi) $\sqrt{225}$

(xii) 0.3796

(xiii) 7.478478……

(xiv) 1.101001000100001……

Sol:

$\sqrt{7}$ is not a perfect square root, so it is an irrational number.

We have,

$\sqrt{4} = 2 = \frac{2}{1}$

$\therefore$ $\sqrt{4}$ can be expressed in the form of $\frac{p}{q}$, so it is a rational number.

The decimal representation of $\sqrt{4}$ is 2.0.

2 is a rational number, whereas $\sqrt{3}$ is an irrational number.

Because, sum of a rational number and an irrational number is an irrational number, so $2+\sqrt{3}$ is an irrational number.

$\sqrt{2}$ is an irrational number. Also $\sqrt{3}$ is an irrational number.

The sum of two irrational numbers is irrational.

$\therefore \sqrt{3}+\sqrt{2}$ is an irrational number.

$\sqrt{5}$ is an irrational number. Also $\sqrt{3}$ is an irrational number.

The sum of two irrational numbers is irrational.

$\therefore \sqrt{3}+\sqrt{5}$ is an irrational number.

We have,

$\left(\sqrt{2}-2\right)^2 = \left(\sqrt{2}\right)^2 - 2\times\sqrt{2}\times 2 + \left(2\right)^2$

$= 2-4\sqrt{2}+4$

$= 6-4\sqrt{2}$

Now, 6 is a rational number, whereas $4\sqrt{2}$ is an irrational number.

The difference of a rational number and an irrational number is an irrational number.

So, $6-4\sqrt{2}$ is an irrational number.

$\therefore \left(\sqrt{2}-2\right)^2$ is an irrational number.

We have,

$\left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right) = \left(2\right)^2 - \left(\sqrt{2}\right)^2$ $\quad \left[\because (a-b)(a+b) = a^2-b^2\right]$

$= 4-2$

$= 2 = \frac{2}{1}$

Since, 2 is a rational number.

$\therefore \left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)$ is a rational number.

We have,

$\left(\sqrt{2}+\sqrt{3}\right)^2 = \left(\sqrt{2}\right)^2 + 2\times\sqrt{2}\times\sqrt{3} + \left(\sqrt{3}\right)^2$

$= 2 + 2\sqrt{6} + 3$

$= 5 + 2\sqrt{6}$

The sum of a rational number and an irrational number is an irrational number, so $5+2\sqrt{6}$ is an irrational number.

$\therefore \left(\sqrt{2}+\sqrt{3}\right)^2$ is an irrational number.

The difference of a rational number and an irrational number is an irrational number

$\therefore \sqrt{5}-2$ is an irrational number.

$\sqrt{23} = 4.79583152331........$

$\sqrt{225} = 15 = \frac{15}{1}$

Rational number as it can be represented in $\frac{p}{q}$ form.

0.3796

As decimal expansion of this number is terminating, so it is a rational number.

$7.478478............ = 7.\overline{478}$

As decimal expansion of this number is non-terminating recurring so it is a rational number.

4. Identify the following as rational numbers. Give the decimal representation of rational numbers:

(i) $\sqrt{4}$

(ii) $3\sqrt{18}$

(iii) $\sqrt{1.44}$

(iv) $\sqrt{\frac{9}{27}}$

(v) $-\sqrt{64}$

(vi) $\sqrt{100}$

Sol:

We have

$\sqrt{4} = 2 = \frac{2}{1}$

$\sqrt{4}$ can be written in the form of $\frac{p}{q}$, so it is a rational number.

Its decimal representation is 2.0.
We have,
$3\sqrt{18} = 3\sqrt{2\times3\times3}$
$= 3\times3\sqrt{2}$
$= 9\sqrt{2}$
Since, the product of a rations and an irrational is an irrational number.
$\therefore 9\sqrt{2}$ is an irrational
$\Rightarrow 3\sqrt{18}$ is an irrational number.
We have,
$\sqrt{1\cdot44} = \sqrt{\frac{144}{100}}$
$= \frac{12}{10}$
$= 1.2$
Every terminating decimal is a rational number, so 1.2 is a rational number.
Its decimal representation is 1.2.
We have,
$\sqrt{\frac{9}{27}} = \frac{3}{\sqrt{27}} = \frac{3}{\sqrt{3\times3\times3}}$
$= \frac{3}{3\sqrt{3}}$
$= \frac{1}{\sqrt{3}}$
Quotient of a rational and an irrational number is irrational numbers so $\frac{1}{\sqrt{3}}$ is an irrational number.
$\Rightarrow \sqrt{\frac{9}{27}}$ is an irrational number.
We have,
$-\sqrt{64} = -\sqrt{8\times8}$
$= -8$
$= -\frac{8}{1}$
$-\sqrt{64}$ can be expressed in the form of $\frac{p}{q}$, so $-\sqrt{64}$ is a rotational number.
Its decimal representation is -8.0.
We have,

$\sqrt{100} = 10$

$= \frac{10}{1}$

$\sqrt{100}$ can be expressed in the form of $\frac{p}{q}$. so $\sqrt{100}$ is a rational number.

The decimal representation of $\sqrt{100}$ is 10.0.

5. In the following equations, find which variables x, y, z etc. represent rational or irrational numbers:

(i) $x^2 = 5$

(ii) $y^2 = 9$

(iii) $z^2 = 0.04$

(iv) $u^2 = \frac{17}{4}$

(v) $v^2 = 3$

(vi) $w^2 = 27$

(vii) $t^2 = 0.4$

Sol:

(i) We have

$x^2 = 5$

Taking square root on both sides.

$\Rightarrow \sqrt{x^2} = \sqrt{5}$

$\Rightarrow x = \sqrt{5}$

$\sqrt{5}$ is not a perfect square root, so it is an irrational number.

(ii) We have

$y^2 = 9$

$\Rightarrow y = \sqrt{9}$

$= 3$

$= \frac{3}{1}$

$\sqrt{9}$ can be expressed in the form of $\frac{p}{q}$, so it a rational number.

(iii) We have

$z^2 = 0.04$

Taking square root on the both sides, we get,

$\sqrt{z^2} = \sqrt{0.04}$

$\Rightarrow z = \sqrt{0.04}$

$= 0.2$

$= \frac{2}{10}$

$= \frac{1}{5}$

z can be expressed in the form of $\frac{p}{q}$, so it is a rational number.

(iv) We have

$u^2 = \frac{17}{4}$

Taking square root on both sides, we get,

$\sqrt{u^2} = \sqrt{\frac{17}{4}}$

$\Rightarrow u = \sqrt{\frac{17}{2}}$

Quotient of an irrational and a rational number is irrational, so u is an irrational number.

(v) We have

$v^2 = 3$

Taking square root on both sides, we get,

$\sqrt{v^2} = \sqrt{13}$

$\Rightarrow v = \sqrt{3}$

$\sqrt{3}$ is not a perfect square root, so y is an irrational number.

(vi) We have

$w^2 = 27$

Taking square root on both des, we get,

$\sqrt{w^2} = \sqrt{27}$

$\Rightarrow w == \sqrt{3 \times 3 \times 3}$

$= 3\sqrt{3}$

Product of a rational and an irrational is irrational number, so w is an irrational number.

(vii) We have

$t^2 = 0.4$

Taking square root on both sides, we get

$\sqrt{t^2} = \sqrt{0.4}$

$\Rightarrow t = \sqrt{\frac{4}{10}}$

$= \frac{2}{\sqrt{10}}$

Since, quotient of a rational and an irrational number is irrational number, so t is an irrational number.

6. Give an example of each, of two irrational numbers whose:
(i) difference is a rational number.
(ii) difference is an irrational number.
(iii) sum is a rational number.
(iv) sum is an irrational number.
(v) product is a rational number.
(vi) product is an irrational number.
(vii) quotient is a rational number.
(viii) quotient is an irrational number.

Sol:

(i) $\sqrt{3}$ is an irrational number.
Now, $\left(\sqrt{3}\right)-\left(\sqrt{3}\right)=0$
0 is the rational number.

(ii) Let two irrational numbers are $5\sqrt{2}$ and $\sqrt{2}$
Now, $\left(5\sqrt{2}\right)-\left(\sqrt{2}\right)=4\sqrt{2}$
$4\sqrt{2}$ is the rational number.

(iii) Let two irrational numbers are $\sqrt{11}$ and $-\sqrt{11}$
Now, $\left(\sqrt{11}\right)+\left(-\sqrt{11}\right)=0$
0 is the rational number.

(iv) Let two irrational numbers are $4\sqrt{6}$ and $\sqrt{6}$
Now, $\left(4\sqrt{6}\right)+\left(\sqrt{6}\right)=5\sqrt{6}$
$5\sqrt{6}$ is the rational number.

(v) Let two irrational numbers are $2\sqrt{3}$ and $\sqrt{3}$
Now, $2\sqrt{3}\times\sqrt{3}=2\times 3$
$=6$
6 is the rational number.

(vi) Let two irrational numbers are $\sqrt{2}$ and $\sqrt{5}$
Now, $\sqrt{2}\times\sqrt{5}=\sqrt{10}$

$\sqrt{10}$ is the rational number.

(vii) Let two irrational numbers are $3\sqrt{6}$ and $\sqrt{6}$

Now, $\frac{3\sqrt{6}}{\sqrt{6}} = 3$

3 is the rational number.

(viii) Let two irrational numbers are $\sqrt{6}$ and $\sqrt{2}$

Now, $\frac{\sqrt{6}}{\sqrt{2}} = \frac{\sqrt{3+2}}{\sqrt{2}}$

$= \frac{\sqrt{3}\times\sqrt{2}}{\sqrt{2}}$

$= \sqrt{3}$

$\sqrt{3}$ is an irrational number.

7. Give two rational numbers lying between 0.232332333233332 . . . and 0.212112111211112.

Sol:

Let, a = 0.212112111211112

And, b = 0.232332333233332...

Clearly, $a < b$ because in the second decimal place a has digit 1 and b has digit 3

If we consider rational numbers in which the second decimal place has the digit 2, then they will lie between a and b.

Let,

x = 0.22

y = 0.22112211...

Then,

$a < x < y < b$

Hence, x, and y are required rational numbers.

8. Give two rational numbers lying between 0.515115111511115 ... and 0.5353353335 ...

Sol:

Let, a = 0.515115111511115...

And, b = 0.5353353335...

We observe that in the second decimal place a has digit 1 and b has digit 3, therefore,

$a < b.$ So if we consider rational numbers

$x = 0.52$

y = 0.52052052...

We find that,

$a < x < y < b$

Hence x, and y are required rational numbers.

9. Find one irrational number between 0.2101 and 0.2222 . . . = $0.\overline{2}$

Sol:

Let, a = 0.2101

And, b = 0.2222...

We observe that in the second decimal place a has digit 1 and b has digit 2, therefore $a < b.$ in the third decimal place a has digit 0. So, if we consider irrational numbers

$x = 0.211011001100011....$

We find that

$a < x < b$

Hence, x is required irrational number.

10. Find a rational number and also an irrational number lying between the numbers 0.3030030003 ... and 0.3010010001 ...

Sol:

Let, $a = 0.3010010001$

And, $b = 0.3030030003...$

We observe that in the third decimal place a has digit 1 and b has digit 3, therefore $a < b.$ in the third decimal place a has digit 1. so, if we consider rational and irrational numbers

$x = 0.302$

$y = 0.302002000200002.....$

We find that

$a < x < b$

And, $a < y < b$

Hence, x and y are required rational and irrational numbers respectively.

11. Find two irrational numbers between 0.5 and 0.55.

Sol:

Let $a = 0.5 = 0.50$

And, $b = 0.55$

We observe that in the second decimal place a has digit 0 and b has digit 5, therefore $a < b.$ so, if we consider irrational numbers

$x = 0.51051005100051...$

$y = 0.530535305353530...$

We find that

$a < x < y < b$

Hence, x and y are required irrational numbers.

12. Find two irrational numbers lying between 0.1 and 0.12.

Sol:

Let, $a = 0.1 = 0.10$

And, $b = 0.12$

We observe that in the second decimal place a has digit 0 and b has digit 2, Therefore $a < b$. So, if we consider irrational numbers

$x = 0.11011001100011...$

$y = 0.111011110111110...$

We find that,

$a < x < y < b$

Hence, x and y are required irrational numbers.

13. Prove that $\sqrt{3} + \sqrt{5}$ is an irrational number.

Sol:

If possible, let $\sqrt{3}+\sqrt{5}$ be a rational number equal to x. Then,

$x = \sqrt{3}+\sqrt{5}$

$\Rightarrow x^2 = \left(\sqrt{3}+\sqrt{5}\right)^2$

$\Rightarrow x^2 = \left(\sqrt{3}\right)^2 + \left(\sqrt{5}\right)^2 + 2\times\sqrt{3}\times\sqrt{5}$

$= 3+5+2\sqrt{15}$

$= 8+2\sqrt{15}$

$\Rightarrow x^2 - 8 = 2\sqrt{15}$

$\Rightarrow \dfrac{x^2-8}{2} = \sqrt{15}$

Now, x is rational

$\Rightarrow x^2$ is rational

$\Rightarrow \dfrac{x^2-8}{2}$ is rational

$\Rightarrow \sqrt{15}$ is rational

But, $\sqrt{15}$ is rational

Thus, we arrive at a contradiction. So, our supposition that $\sqrt{3}+\sqrt{5}$ is rational is wrong.

Hence, $\sqrt{3}+\sqrt{5}$ is an irrational number.

14. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$

Sol:

$\frac{5}{7} = 0.\overline{714285}$

$\frac{9}{11} = 0.\overline{81}$

3 irrational numbers are

0.73073007300073000073.......

0.75075007500075000075.......

0.79079007900079000079.......

Exercise -1.5

1. Complete the following sentences:

(i) Every point on the number line corresponds to a ______ number which many be either ______ or ______

(ii) The decimal form of an irrational number is neither ______ nor ______

(iii) The decimal representation of a rational number is either ______ or ______

(iv) Every real number is either ______ number or ______ number.

Sol:

(i) Every point on the number line corresponds to a **Real** number which may be either **rational** or **irrational**.

(ii) The decimal form of an irrational number is neither **terminating** nor **repeating**

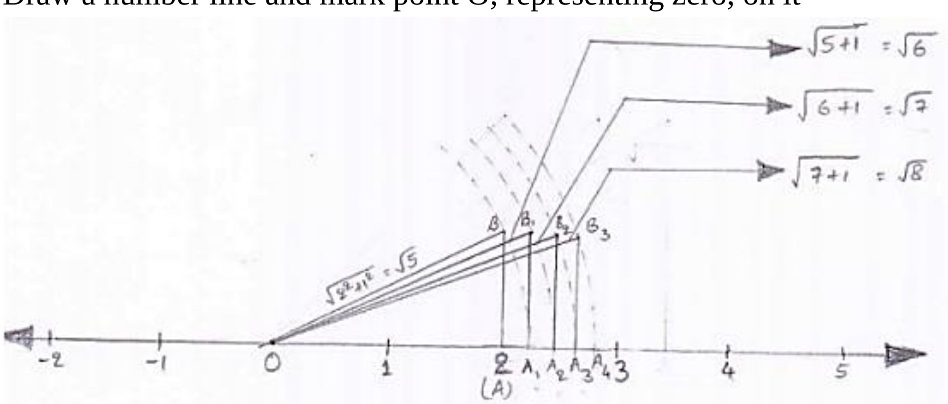
(iii) The decimal representation of a rational number is either **terminating, non-terminating** or **recurring**.

(iv) Every real number is either **a rational** number or **an irrational** number.

2. Represent $\sqrt{6}, \sqrt{7}, \sqrt{8}$ on the number line.

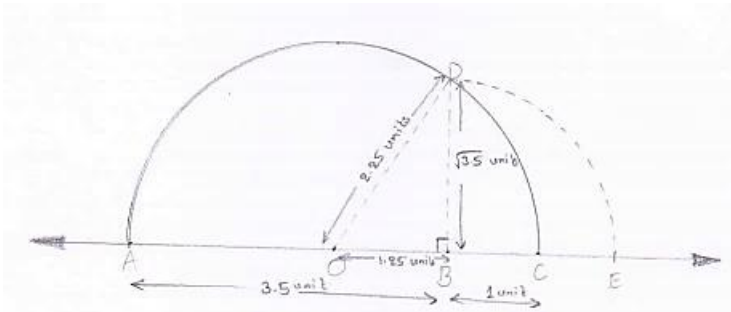
Sol:

Draw a number line and mark point O, representing zero, on it



Suppose point A represents 2 as shown in the figure
Then $OA = 2.$ Now, draw a right triangle OAB such that $AB = 1.$
By Pythagoras theorem, we have
$OB^2 = OA^2 + AB^2$
$\Rightarrow OB^2 = 2^2 + 1^2$
$\Rightarrow OB^2 = 4 + 1 = 5 \Rightarrow OB = \sqrt{5}$
Now, draw a circle with center O and radius OB.
We fine that the circle cuts the number line at A
Clearly, $OA_1 = OB =$ radius of circle $= \sqrt{5}$
Thus, A_1 represents $\sqrt{5}$ on the number line.
But, we have seen that $\sqrt{5}$ is not a rational number. Thus we find that there is a point on the number which is not a rational number.
Now, draw a right triangle $OA_1B_1,$ Such that $A_1B_1 = AB = 1$
Again, by Pythagoras theorem, we have
$(OB_1)^2 = (OA_1)^2 + (A_1B_1)^2$
$\Rightarrow (OB_1)^2 = (\sqrt{5})^2 + (1)^2$
$\Rightarrow (OB_1^2) = 5 + 1 = 6 \Rightarrow OB_1 = \sqrt{6}$
Draw a circle with center O and radius $OB_1 = \sqrt{6}.$ This circle cuts the number line at A_2 as shown in figure
Clearly $OA_2 = OB_1 = \sqrt{6}$
Thus, A_2 represents $\sqrt{6}$ on the number line.
Also, we know that $\sqrt{6}$ is not a rational number.
Thus, A_2 is a point on the number line not representing a rational number
Continuing in this manner, we can represent $\sqrt{7}$ and $\sqrt{8}$ also on the number lines as shown in the figure
Thus, $OA_3 = OB_2 = \sqrt{7}$ and $OA_4 = OB_3 = \sqrt{8}$

3. Represent $\sqrt{3.5}, \sqrt{9.4}, \sqrt{10.5}$ on the real number line.
Sol:
Given to represent $\sqrt{3\cdot 5}, \sqrt{9\cdot 4}, \sqrt{10\cdot 5}$ on the real number line
Representation of $\sqrt{3\cdot 5}$ on real number line:
Steps involved:
(i) Draw a line and mark A on it.



(ii) Mark a point B on the line drawn in step - (i) such that $AB = 3\cdot 5$ units

(iii) Mark a point C on AB produced such that BC = 1unit

(iv) Find mid-point of AC. Let the midpoint be O

$\Rightarrow AC = AB + BC = 3\cdot 5 + 1 = 4\cdot 5$

$\Rightarrow AO = OC = \dfrac{AC}{2} = \dfrac{4\cdot 5}{2} = 2\cdot 25$

(v) Taking O as the center and $OC = OA$ as radius drawn a semi-circle. Also draw a line passing through B perpendicular to OB. Suppose it cuts the semi-circle at D. Consider triangle OBD, it is right angled at B.

$BD^2 = OD^2 - OB^2$

$\Rightarrow BD^2 = OC^2 - (OC - BC)^2$ $\qquad [\because OC = OD = \text{radius}]$

$\Rightarrow BD^2 = 2OC \cdot BC - (BC)^2$

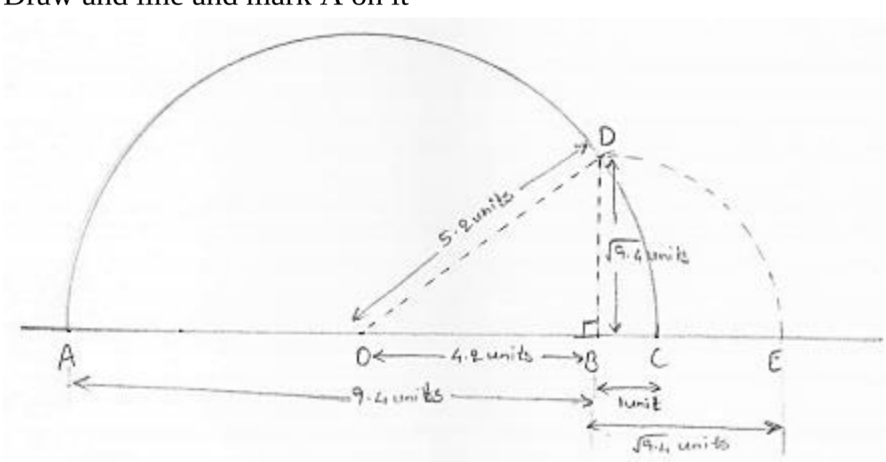
$\Rightarrow BD = \sqrt{2 \times 2\cdot 25 \times 1 - (1)^2} \Rightarrow BD = \sqrt{35}$

(vi) Taking B as the center and BD as radius draw an arc cutting OC produced at E. point E so obtained represents $\sqrt{3\cdot 5}$ as $BD = BE = \sqrt{3\cdot 5} =$ radius

Thus, E represents the required point on the real number line.

Representation of $\sqrt{9\cdot 4}$ on real number line steps involved:

(i) Draw and line and mark A on it



(ii) Mark a point B on the line drawn in step (i) such that $AB = 9\cdot 4$ units

(iii) Mark a point C on AB produced such that $BC = 1$ unit.

(iv) Find midpoint of AC. Let the midpoint be O.

$\Rightarrow AC = AB + BC = 9\cdot 4 + 1 = 10\cdot 4$ units

$\Rightarrow AD = OC = \dfrac{AC}{2} = \dfrac{10\cdot 4}{2} = 5\cdot 2$ units

(v) Taking O as the center and $OC = OA$ as radius draw a semi-circle. Also draw a line passing through B perpendicular to OB. Suppose it cuts the semi-circle at D.

Consider triangle OBD, it is right angled at B.

$\Rightarrow BD^2 = OD^2 - OB^2$

$\Rightarrow BD^2 = OC^2 - (OC - BC)^2$ $\quad [\because OC = OD = \text{radius}]$

$\Rightarrow BD^2 = OC^2 - \left(OC^2 - 2OC \cdot BC + (BC)^2\right)$

$\Rightarrow BD^2 = 2OC \cdot \left(BC - \left(BC^2\right)\right)$

$\Rightarrow BD^2 = \sqrt{2 \times (5\cdot 2) \times 1 - 1^2} \Rightarrow BD = \sqrt{9\cdot 4}$ units

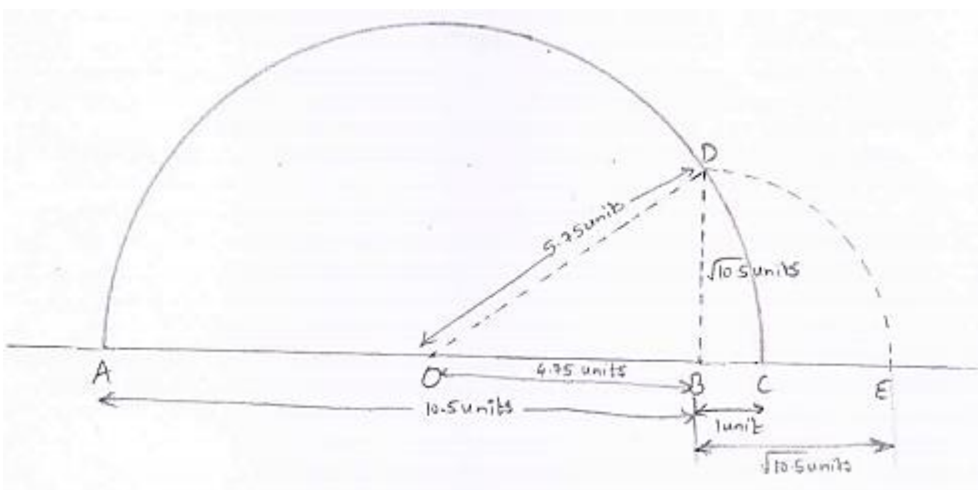
(vi) Taking B as center and BD as radius draw an arc cutting OC produced at E so obtained represents $\sqrt{9\cdot 4}$ as $BD = BE = \sqrt{9\cdot 4} = \text{radius}$

Thus, E represents the required point on the real number line.

Representation of $\sqrt{10\cdot 5}$ on the real number line:

Steps involved:

(i) Draw a line and mark A on it



(ii) Mark a point B on the line drawn in step (i) such that $AB = 10\cdot 5$ units

(iii) Mark a point C on AB produced such that $BC = 1$ unit

(iv) Find midpoint of AC. Let the midpoint be 0.

$\Rightarrow AC = AB + BC = 10\cdot 5 + 1 = 11\cdot 5$ units

$\Rightarrow AO = OC = \frac{AC}{2} = \frac{11\cdot5}{2} = 5\cdot75$ units

(v) Taking O as the center and $OC = OA$ as radius, draw a semi-circle. Also draw a line passing through B perpendicular to DB. Suppose it cuts the semi-circle at D. consider triangle OBD, it is right angled at B

$\Rightarrow BD^2 = OD^2 - OB^2$

$\Rightarrow BD^2 = OC^2 - (OC - BC)^2$ $[\because OC = OD = \text{radius}]$

$\Rightarrow BD^2 = OC^2 - \left[OC^2 - 2OC\cdot BC + (BC)^2\right]$

$\Rightarrow BD^2 = 2OC\cdot BC - BC^2$

$\Rightarrow BC^2 = OD^2 - OB^2$

$\Rightarrow BD^2 = OC^2 - (OC - BC)^2$ $[\because OC = OD = \text{radius}]$

$\Rightarrow BD^2 = OC^2 - \left[OC^2 - 2OC\cdot BC + (BC)^2\right]$

$\Rightarrow BD^2 = 2OC\cdot BC - BC^2$

$\Rightarrow BD = \sqrt{2\times575\times1-(1)^2} \Rightarrow BD = \sqrt{10\cdot5}$

(vi) Taking B as the center and BD as radius draw on arc cutting OC produced at E. point E so obtained represents $\sqrt{10\cdot5}$ as $BD = BE = \sqrt{10\cdot5} =$ radius arc

Thus, E represents the required point on the real number line

4. Find whether the following statements are true or false.

(i) Every real number is either rational or irrational.

(ii) it is an irrational number.

(iii) Irrational numbers cannot be represented by points on the number line.

Sol:

(i) True

As we know that rational and irrational numbers taken together from the set of real numbers.

(ii) True

As, π is ratio of the circumference of a circle to its diameter, it is an irrational number

$\Rightarrow \pi = \frac{2\pi r}{2r}$

(iii) False

Irrational numbers can be represented by points on the number line.

Exercise -1.6

Mark the correct alternative in each of the following:

1. Which one of the following is a correct statement?

(a) Decimal expansion of a rational number is terminating

(b) Decimal expansion of a rational number is non-terminating

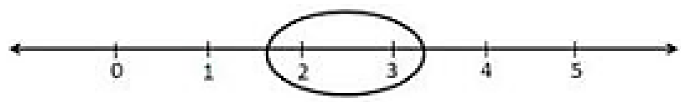
(c) Decimal expansion of an irrational number is terminating

(d) Decimal expansion of an irrational number is non-terminating and non-repeating

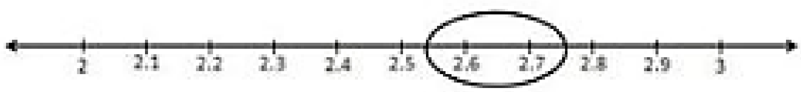
Sol:

The following steps for successive magnification to visualise 2.665 are:

(1) We observe that 2.665 is located somewhere between 2 and 3 on the number line. So, let us look at the portion of the number line between 2 and 3.

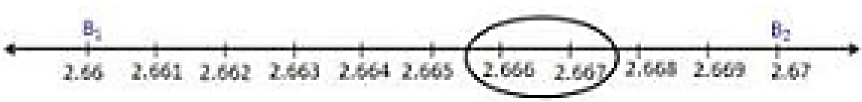


(2) We divide this portion into 10 equal parts and mark each point of division. The first mark to the right of 2 will represent 2.1, the next 2.2 and soon. Again we observe that 2.665 lies between 2.6 and 2.7.



(3) We mark these points A_1 and A_2 respectively. The first mark on the right side of A_1, will represent 2.61, the number 2.62, and soon. We observe 2.665 lies between 2.66 and 2.67.

(4) Let us mark 2.66 as B_1 and 2.67 as B_2. Again divide the B_1B_2 into ten equal parts. The first mark on the right side of B_1 will represent 2.661. Then next 2.662, and so on. Clearly, fifth point will represent 2.665.



2. Which one of the following statements is true?

(a) The sum of two irrational numbers is always an irrational number

(b) The sum of two irrational numbers is always a rational number

(c) The sum of two irrational numbers may be a rational number or an irrational number

(d) The sum of two irrational numbers is always an integer

Sol:

Once again we proceed by successive magnification, and successively decrease the lengths of the portions of the number line in which $5.3\overline{7}$ is located. First, we see that $5.3\overline{7}$ is located between 5 and 6. In the next step, we locate $5.3\overline{7}$ between 5.3 and 5.4. To get a more accurate visualization of the representation, we divide this portion of the number line into lo equal parts and use a magnifying glass to visualize that $5\cdot 3\overline{7}$ lies between 5.37 and 5.38. To visualize $5.3\overline{7}$ more accurately, we again divide the portion between 5.37 and 5.38 into ten equal parts and use a magnifying glass to visualize that S.S lies between 5.377 and 5.378. Now to visualize $5.3\overline{7}$ still more accurately, we divide the portion between 5.377 and 5.378 into 10 equal parts, and visualize the representation of $5.3\overline{7}$ as in fig.,(iv) . Notice that $5.3\overline{7}$ is located closer to 5.3778 than to 5.3777(iv)