Exercise – 5.1

Factorize:

1. $x^3 + x - 3x^2 - 3$

Sol:

$x^3 + x - 3x^2 - 3$

Taking x common in $\left(x^3 + x\right)$

$= x\left(x^2 + 1\right) - 3x^2 - 3$

Taking –3 common in $\left(-3x^2 - 3\right)$

$= x\left(x^2 + 1\right) - 3\left(x^2 + 1\right)$

Now, we take $\left(x^2 + 1\right)$ common

$= \left(x^2 + 1\right)\left(x - 3\right)$

$\therefore x^3 + x - 3x^2 - 3 = \left(x^2 + 1\right)\left(x - 3\right)$

2. $a\left(a+b\right)^3 - 3a^2b\left(a+b\right)$

Sol:

Taking $\left(a+b\right)$ common in two terms

$= \left(a+b\right)\left\{a\left(a+b\right)^2 - 3a^2b\right\}$

Now, using $\left(a+b\right)^2 = a^2 + b^2 + 2ab$

$= \left(a+b\right)\left\{a\left(a^2 + b^2 + 2ab\right) - 3a^2b\right\}$

$= \left(a+b\right)\left\{a^3 + ab^2 + 2a^2b - 3a^2b\right\}$

$= \left(a+b\right)\left\{a^3 + ab^2 - a^2b\right\}$

$= \left(a+b\right)a\left\{a^2 + b^2 - ab\right\}$

$= a\left(a+b\right)\left(a^2 + b^2 - ab\right)$

$\therefore a\left(a+b\right)^3 - 3a^2b\left(a+b\right) = a\left(a+b\right)\left(a^2 + b^2 - ab\right)$

3. $x\left(x^3-y^3\right)+3xy\left(x-y\right)$

Sol:

Elaborating x^3-y^3 using identity $a^3-b^3=\left(a-b\right)\left(a^2+ab+b^2\right)$

$=x\left(x-y\right)\left(x^2+xy+y^2\right)+3xy\left(x-y\right)$

Taking common $x\left(x-y\right)$ in both the terms

$=x\left(x-y\right)\left\{x^2+xy+y^2+3y\right\}$

$\therefore x\left(x^3-y^3\right)+3xy\left(x-y\right)=x\left(x-y\right)\left(x^2+xy+y^2+3y\right)$

4. $a^2x^2+\left(ax^2+1\right)x+a$

Sol:

We multiply $x\left(ax^2+1\right)=ax^3+x$

$=a^2x^2+ax^3+x+a$

Taking common ax^2 in $\left(a^2x^2+ax^3\right)$ and 1 in $\left(x+a\right)$

$=ax^2\left(a+x\right)+1\left(x+a\right)$

$=ax^2\left(a+x\right)+1\left(a+x\right)$

Taking $\left(a+x\right)$ common in both the terms

$=\left(a+x\right)\left(ax^2+1\right)$

$\therefore a^2x^2+\left(ax^2+1\right)x+a=\left(a+x\right)\left(ax^2+1\right)$

5. $x^2+y-xy-x$

Sol:

On rearranging

$x^2-xy-x+y$

Taking x common in the $\left(x^2+y\right)$ and -1 in $\left(-x+y\right)$

$=x\left(x-y\right)-1\left(x-y\right)$

Taking $\left(x-y\right)$ common in both the terms

$=\left(x-y\right)\left(x-1\right)$

$\therefore x^2+y-xy-x=\left(x-y\right)\left(x-1\right)$

6. $x^3-2x^2y+3xy^2-6y^3$

Sol:

Taking x^2 common in $\left(x^3-2x^2y\right)$ and $+3y^2$ common in $\left(3xy^2-6y^3\right)$

$=x^2\left(x-2y\right)+3y^2\left(x-2y\right)$

$=\left(x-2y\right)\left(x^2+3y^2\right)$

$\therefore x^3-2x^2y+3xy^2-6y^3=\left(x-2y\right)\left(x^2+3y^2\right)$

7. $6ab-b^2+12ac-2bc$

Sol:

Taking common b in $\left(6ab-b^2\right)$ and 2c in $\left(12ac-2bc\right)$

$=b\left(6a-b\right)+2c\left(6a-b\right)$

Taking $\left(6a-b\right)$ common in both terms

$=\left(6a-b\right)\left(b+2c\right)$

$\therefore 6ab-b^2+12ac-2bc=\left(6a-b\right)\left(b+2c\right)$

8. $\left[x^2+\frac{1}{x^2}\right]-4\left[x+\frac{1}{x}\right]+6$

Sol:

$=x^2+\frac{1}{x^2}-4x-\frac{4}{x}+4+2$

$=x^2+\frac{1}{x^2}+4+2-\frac{4}{x}-4x$

$=\left(x^2\right)+\left(\frac{1}{x}\right)^2+\left(-2\right)^2+2\times x\times x\frac{1}{x}+2x\frac{1}{x}\times\left(-2\right)+2\left(-2\right)x$

Using identity

$a^2+b^2+c^2+2ab+2bc+2ca=\left(a+b+c\right)^2$

We get,

$=\left[x+\frac{1}{x}+\left(-2\right)\right]^2$

$=\left[x+\frac{1}{x}-2\right]^2$

$=\left[x+\frac{1}{x}-2\right]\left[x+\frac{1}{x}-2\right]$

$\therefore\left[x^2+\frac{1}{x^2}\right]-4\left[x+\frac{1}{x}\right]+6=\left[x+\frac{1}{x}-2\right]\left[x+\frac{1}{x}-2\right]$

9. $x(x-2)(x-4)+4x-8$

Sol:

$=x(x-2)(x-4)+4(x-2)$

Taking $(x-2)$ common in both terms

$=(x-2)\{x(x-4)+4\}$

$=(x-2)\{x^2-4x+4\}$

Now splitting middle term of x^2-4x+4

$=(x-2)\{x^2-2x-2x+4\}$

$=(x-2)\{x(x-2)-2(x-2)\}$

$=(x-2)\{(x-2)(x-2)\}$

$=(x-2)(x-2)(x-2)$

$=(x-2)^3$

$\therefore x(x-2)(x-4)+4x-8=(x-2)^3$

10. $(x+2)(x^2+25)-10x^2-20x$

Sol:

$=(x+2)(x^2+25)-10x(x+2)$

Taking $(x+2)$ common in both terms

$=(x+2)(x^2+25-10x)$

$=(x+2)(x^2-10x+25)$

Splitting middle term of $x^2-10x+25$

$=(x+2)\{x^2-5x-5x+25\}$

$=(x+2)\{x(x-5)-5(x-5)\}$

$=(x+2)(x-5)(x-5)$

$\therefore (x+2)(x^2+25)-10x^2-20x=(x+2)(x-5)(x-5)$

11. $2a^2+2\sqrt{6}ab+3b^2$

Sol:

$=\left(2\sqrt{a}\right)^2+2\times\sqrt{2}a\times\sqrt{3}b+\left(\sqrt{3}b\right)^2$

Using identity $a^2+2ab+b^2=(a+b)^2$

$=\left(\sqrt{2}a+\sqrt{3}b\right)^2$

$=\left(\sqrt{2}a+\sqrt{3}b\right)\left(\sqrt{2}a+\sqrt{3}b\right)$

$\therefore 2a^2+2\sqrt{6}ab+3b^2=\left(\sqrt{2}a+\sqrt{3}b\right)\left(\sqrt{2}a+\sqrt{3}b\right)$

12. $(a-b+c)^2+(b-c+a)^2+2(a-b+c)(b-c+a)$

Sol:

Let $(a-b+c)=x$ and $(b-c+a)=y$

$=x^2+y^2+2xy$

Using identity $a^2+b^2+2ab=(a+b)^2$

$=(x+y)^2$

Now, substituting x and y

$=(a-b+c+b-c+a)^2$

Cancelling $-b,+b$ and $+c,-c$

$=(2a)^2$

$=4a^2$

$\therefore(a-b+c)^2+(b-c+a)^2+2(a-b+c)(b-c+a)=4a^2$

13. $a^2+b^2+2(ab+bc+ca)$

Sol:

$=a^2+b^2+2ab+2bc+2ca$

Using identity $a^2+b^2+2ab=(a+b)^2$

We get,

$=(a+b)^2+2bc+2ca$

$=(a+b)^2+2c(b+a)$

or $(a+b)^2+2c(a+b)$

Taking $(a+b)$ common

$=(a+b)(a+b+2c)$

$\therefore a^2+b^2+2(ab+bc+ca)=(a+b)(a+b+2c)$

14. $4(x-y)^2-12(x-y)(x+y)+9(x+y)^2$

Sol:

Let $(x-y)=a,(x+y)=b$

$=4a^2-12ab+9b^2$

Splitting middle term $-12=-6-6$ also $4\times 9=-6\times -6$

$=4a^2-6ab-6ab+9b^2$

$=2a(2a-3b)-3b(2a-3b)$

$=(2a-3b)(2a-3b)$

$=(2a-3b)^2$

Substituting $a=x-y$ and $b=x+y$

$=\left[2(x-y)-3(x+y)\right]^2$

$=\left[2x-2y-3x-3y\right]^2$

$=\left[2x-3x-2y-3y\right]^2$

$=\left[-x-5y\right]^2$

$=\left[(-1)(x+5y)\right]^2$

$=(x+5y)^2$ $\left[\because (-1)^2=1\right]$

$\therefore 4(x-y)^2-12(x-y)(x+y)+9(x+y)^2=(x+5y)^2$

15. $a^2-b^2+2bc-c^2$

Sol:

$=a^2-\left(b^2-2bc+c^2\right)$

Using identity $a^2-2ab+b^2=(a-b)^2$

$=a^2-(b-c)^2$

Using identity $a^2-b^2=(a+b)(a-b)$

$=(a+b-c)(a-(b-c))$

$=(a+b-c)(a-b+c)$

$\therefore a^2-b^2+2bc-c^2=(a+b-c)(a-b+c)$

16. $a^2+2ab+b^2-c^2$

Sol:

Using identity $a^2+2ab+b^2=(a+b)^2$

$=(a+b)^2-c^2$

Using identity $a^2-b^2=(a+b)(a-b)$

$=(a+b+c)(a+b-c)$

$\therefore a^2+2ab+b^2-c^2=(a+b+c)(a+b-c)$

17. $a^2+4b^2-4ab-4c^2$

Sol:

On rearranging

$=a^2-4ab+4b^2-4c^2$

$=(a)^2-2\times a\times 2b+(2b)^2-4c^2$

Using identity $a^2-2ab+b^2=(a-b)^2$

$=(a-2b)^2-4c^2$

$=(a-2b)^2-(2c)^2$

Using identity $a^2-b^2=(a+b)(a-b)$

$=(a-2b+2c)(a-2b-2c)$

$\therefore a^2+4b^2-4ab-4c^2=(a-2b+2c)(a-2b-2c)$

18. xy^9-yx^9

Sol:

xy^9-yx^9

$=xy\left(y^8-x^8\right)$

$=xy\left(\left(y^4\right)^2-\left(x^4\right)^2\right)$

Using identity $a^2-b^2=(a+b)(a-b)$

$=xy\left(y^4+x^4\right)\left(y^4-x^4\right)$

$=xy\left(y^4+x^4\right)\left(\left(y^2\right)^2-\left(x^2\right)^2\right)$

Using identity $a^2-b^2=(a+b)(a-b)$

$= xy\left(y^4+x^4\right)\left(y^2+x^2\right)\left(y^2-x^2\right)$

$= xy\left(y^4+x^4\right)\left(y^2+x^2\right)\left(y+x\right)\left(y-x\right)$

$= xy\left(x^4+y^4\right)\left(x^2+y^2\right)\left(x+y\right)\left(-1\right)\left(x-y\right)$

$\because \left(b-a\right)=-1\left(a-b\right)$

$= -xy\left(x^4+y^4\right)\left(x^2+y^2\right)\left(x+y\right)\left(x-y\right)$

$\therefore xy^9-yx^9=-xy\left(x^4+y^4\right)\left(x^2+y^2\right)\left(x+y\right)\left(x-y\right)$

19. $x^4+x^2y^2+y^4$

Sol:

Adding x^2y^2 and subtracting x^2y^2 to the given equation

$= x^4+x^2y^2+y^4+x^2y^2-x^2y^2$

$= x^4+2x^2y^2+y^4-x^2y^2$

$= \left(x^2\right)^2+2\times x^2\times y^2+\left(y^2\right)^2-\left(xy\right)^2$

Using identity $a^2+2ab+b^2=\left(a+b\right)^2$

$= \left(x^2+y^2\right)^2-\left(xy\right)^2$

Using identity $a^2-b^2=\left(a+b\right)\left(a-b\right)$

$= \left(x^2+y^2+xy\right)\left(x^2+y^2-xy\right)$

$\therefore x^4+x^2y^2+y^4=\left(x^2+y^2+xy\right)\left(x^2+y^2-xy\right)$

20. $x^2-y^2-4xz+4z^2$

Sol:

On rearranging the terms

$= x^2-4xz+4z^2-y^2$

$= \left(x\right)^2-2\times x\times 2z+\left(2z\right)^2-y^2$

Using identity $a^2-2ab+b^2=\left(a-b\right)^2$

$= \left(x-2z\right)^2-y^2$

Using identity $a^2-b^2=\left(a+b\right)\left(a-b\right)$

$= \left(x-2z+y\right)\left(x-2z-y\right)$

$\therefore x^2-y^2-4xz+4z^2=\left(x-2z+y\right)\left(x-2z-y\right)$

21. $x^2+6\sqrt{2}x+10$

Sol:

Splitting middle term,

$=x^2+5\sqrt{2}x+\sqrt{2}x+10 \qquad \left[\because 6\sqrt{2}=5\sqrt{2}+\sqrt{2} \text{ and } 5\sqrt{2}\times\sqrt{2}=10\right]$

$=x\left(x+5\sqrt{2}\right)+\sqrt{2}\left(x+5\sqrt{2}\right)$

$=\left(x+5\sqrt{2}\right)\left(x+\sqrt{2}\right)$

$\therefore x^2+6\sqrt{2}x+10=\left(x+5\sqrt{2}\right)\left(x+\sqrt{2}\right)$

22. $x^2-2\sqrt{2}x-30$

Sol:

Splitting the middle term,

$=x^2-5\sqrt{2}x+3\sqrt{2}x-30 \qquad \left[\because -2\sqrt{2}=-5\sqrt{2}+3\sqrt{2} \text{ also} -5\sqrt{2}\times3\sqrt{2}=-30\right]$

$x\left(x-5\sqrt{2}\right)+3\sqrt{2}\left(x-5\sqrt{2}\right)$

$=\left(x-5\sqrt{2}\right)\left(x+3\sqrt{2}\right)$

$\therefore x^2-2\sqrt{2}x-30=\left(x-5\sqrt{2}\right)\left(x+3\sqrt{2}\right)$

23. $x^2-\sqrt{3}x-6$

Sol:

Splitting the middle term,

$=x^2-2\sqrt{3}x+\sqrt{3}x-6 \qquad \left[\because -\sqrt{3}=-2\sqrt{3}+\sqrt{3} \text{ also } -2\sqrt{3}\times\sqrt{3}=-6\right]$

$=x\left(x-2\sqrt{3}\right)+\sqrt{3}\left(x-2\sqrt{3}\right)$

$=\left(x-2\sqrt{3}\right)\left(x+\sqrt{3}\right)$

$\therefore x^2-\sqrt{3}x-6=\left(x-2\sqrt{3}\right)\left(x+\sqrt{3}\right)$

24. $x^2+5\sqrt{5}x+30$

Sol:

Splitting the middle term,

$=x^2+2\sqrt{5}x+3\sqrt{5}x+30 \qquad \left[\because 5\sqrt{5}=2\sqrt{5}+3\sqrt{5} \text{ also } 2\sqrt{5}\times\sqrt{3}=30\right]$

$=x\left(x+2\sqrt{5}\right)+3\sqrt{5}\left(x+2\sqrt{5}\right)$

$=\left(x+2\sqrt{5}\right)\left(x+3\sqrt{5}\right)$

$\therefore x^2+5\sqrt{5}x+30=\left(x+2\sqrt{5}\right)\left(x+3\sqrt{5}\right)$

25. $x^2+2\sqrt{3}x-24$

Sol:

Splitting the middle term,

$=x^2+4\sqrt{3}x-2\sqrt{3}x-24 \qquad \left[\because 2\sqrt{3}=4\sqrt{3}-2\sqrt{3} \text{ also } 4\sqrt{3}\left(-2\sqrt{3}\right)=-24\right]$

$=x\left(x+4\sqrt{3}\right)-2\sqrt{3}\left(x+4\sqrt{3}\right)$

$=\left(x+4\sqrt{3}\right)\left(x-2\sqrt{3}\right)$

$\therefore x^2+2\sqrt{3}x-24=\left(x+4\sqrt{3}\right)\left(x-2\sqrt{3}\right)$

26. $2x^2-\frac{5}{6}x+\frac{1}{12}$

Sol:

Splitting the middle term,

$=2x^2-\frac{x}{2}-\frac{x}{3}+\frac{1}{12} \qquad \left[\because -\frac{5}{6}=-\frac{1}{2}-\frac{1}{3} \text{ also } -\frac{1}{2}\times-\frac{1}{3}=2\times\frac{1}{12}\right]$

$=x\left(2x-\frac{1}{2}\right)-\frac{1}{6}\left(2x-\frac{1}{2}\right)$

$=\left(2x-\frac{1}{2}\right)\left(x-\frac{1}{6}\right)$

$\therefore 2x^2-\frac{5}{6}x+\frac{1}{12}=\left(2x-\frac{1}{2}\right)\left(x-\frac{1}{6}\right)$

27. $x^2+\frac{12}{35}x+\frac{1}{35}$

Sol:

Splitting the middle term,

$=x^2+\frac{5}{35}x+\frac{7}{35}x+\frac{1}{35} \qquad \left[\because \frac{12}{35}=\frac{5}{35}+\frac{7}{35} \text{ and } \frac{5}{35}\times\frac{7}{35}=\frac{1}{35}\right]$

$=x^2+\frac{x}{7}+\frac{x}{5}+\frac{1}{35}$

$=x\left(x+\frac{1}{7}\right)+\frac{1}{5}\left(x+\frac{1}{7}\right)$

$=\left(x+\frac{1}{7}\right)\left(x+\frac{1}{5}\right)$

$\therefore x^2+\frac{12}{35}x+\frac{1}{35}=\left(x+\frac{1}{7}\right)\left(x+\frac{1}{5}\right)$

28. $21x^2 - 2x + \frac{1}{21}$

Sol:

$= \left(\sqrt{21}x\right)^2 - 2 \times \sqrt{21}x \times \frac{1}{\sqrt{21}} + \left(\frac{1}{\sqrt{21}}\right)^2$

Using identity $a^2 - 2ab + b^2 = (a-b)^2$

$= \left(\sqrt{21}x - \frac{1}{\sqrt{21}}\right)^2$

$\therefore 21x^2 - 2x + \frac{1}{21} = \left(\sqrt{21}x - \frac{1}{\sqrt{21}}\right)^2$

29. $5\sqrt{5}x^2 + 20x + 3\sqrt{5}$

Sol:

Splitting the middle term,

$= 5\sqrt{5}x^2 + 15x + 5x + 3\sqrt{5}$ $\left[\because 20 = 15 + 5 \text{ and } 15 \times 5 = 5 = 5\sqrt{5} \times 3\sqrt{5}\right]$

$= 5x\left(\sqrt{5}x + 3\right) + \sqrt{5}\left(\sqrt{5}x + 3\right)$

$= \left(\sqrt{5}x + 3\right)\left(5x + \sqrt{5}\right)$

$\therefore 5\sqrt{5}x^2 + 20x + 3\sqrt{5} = \left(\sqrt{5}x + 3\right)\left(5x + \sqrt{5}\right)$

30. $2x^2 + 3\sqrt{5}x + 5$

Sol:

Splitting the middle term,

$= 2x^2 + 2\sqrt{5}x + \sqrt{5}x + 5$ $\left[\because 3\sqrt{5} = 2\sqrt{5} + \sqrt{5} \text{ also } 2\sqrt{5} \times \sqrt{5} = 2 \times 5\right]$

$= 2x\left(x + \sqrt{5}\right) + \sqrt{5}\left(x + \sqrt{5}\right)$

$= \left(x + \sqrt{5}\right)\left(2x + \sqrt{5}\right)$

$\therefore 2x^2 + 3\sqrt{5}x + 5 = \left(x + \sqrt{5}\right)\left(2x + \sqrt{5}\right)$

31. $9(2a-b)^2 - 4(2a-b) - 13$

Sol:

Let $2a - b = x$

$= 9x^2 - 4x - 13$

Splitting the middle term,

$=9x^2-13x+9x-13$

$=x(9x-13)+1(9x-13)$

$=(9x-13)(x+1)$

Substituting $x=2a-b$

$=[9(2a-b)-13](2a-b+1)$

$=(18a-9b-13)(2a-b+1)$

$\therefore 9(2a-b)^2-4(2a-b)-13=(18a-9b-13)(2a-b+1)$

32. $7(x-2y)^2-25(x-2y)+12$

Sol:

Let $x-2y=P$

$=7P^2-25P+12$

Splitting the middle term,

$=7P^2-21P-4P+12$

$=7P(P-3)-4(P-3)$

$=(P-3)(7P-4)$

Substituting $P=x-2y$

$=(x-2y-3)(7(x-2y)-4)$

$=(x-2y-3)(7x-14y-4)$

$\therefore 7(x-2y)^2-25(x-2y)+12=(x-2y-3)(7x-14y-4)$

33. $2(x+y)^2-9(x+y)-5$

Sol:

Let $x+y=z$

$=2z^2-9z-5$

Splitting the middle term,

$=2z^2-10z+z-5$

$=2z(z-5)+1(z-5)$

$=(z-5)(2z+1)$

Substituting $z=x+y$

$=(x+y-5)(2(x+y)+1)$

$=(x+y-5)(2x+2y+1)$

$\therefore 2(x+y)^2-9(x+y)-5=(x+y-5)(2x+2y+1)$

34. Given possible expressions for the length and breadth of the rectangle having $35y^2 + 13y - 12$ as its area.

Sol:

Area $= 35y^2 + 13y - 12$

Splitting the middle term,

Area $= 35y^2 + 28y - 15y - 12$

$= 7y(5y+4) - 3(5y+4)$

Area $= (5y+4)(7y-3)$

Also area of rectangle = Length $\times$ Breadth

$\therefore$ Possible length $= (5y+4)$ and breadth $= (7y-3)$

Or Possible length $= (7y-3)$ and breadth $= (5y+4)$

35. What are the possible expressions for the dimensions of the cuboid whose volume is $3x^2 - 12x$.

Sol:

Here volume $= 3x^2 - 12x$

$= 3x(x-4)$

$= 3 \times x(x-4)$

Also volume = Length $\times$ Breadth $\times$ Height

$\therefore$ Possible expressions for dimensions of the cuboid are $= 3, x, (x-4)$

Exercise – 5.2

Factorize each of the following expressions :

1. p^3+27

Sol:

p^3+27

$=p^3+3^3$

$=(p+3)(p^2-3p+3^2)$ $\quad\left[\because a^3+b^3=(a+b)(a^2-ab+b^2)\right]$

$=(p+3)(p^2-3p+9)$

$\therefore p^3+27=(p+3)(p^2-3p+9)$

2. y^3+125

Sol:

$=y^3+5^3$

$=(y+5)(y^2-5y+5^2)$ $\quad\left[\because a^3+b^3=(a+b)(a^2-ab+b^2)\right]$

$=(y+5)(y^2-5y+25)$

$\therefore y^3+125=(y+5)(y^2-5y+25)$

3. $1-27a^3$

Sol:

$=(1)^3-(3a)^3$

$=(1-3a)\left(1^2+1\times 3a+(3a)^2\right)$ $\quad\left[\because a^3-b^3=(a-b)(a^2+ab+b^2)\right]$

$=(1-3a)(1+3a+9a^2)$

$\therefore 1-27a^3=(1-3a)(1+3a+9a^2)$

4. $8x^3y^3+27a^3$

Sol:

$=(2xy)^3+(3a)^3$

$=(2xy+3a)\left((2xy)^2-2xy\times 3a+(3a)^2\right)$ $\quad\left[\because a^3+b^3=(a+b)(a^2-ab+b^2)\right]$

$=(2xy+3a)(4x^2y^2-6xya+9a^2)$

$\therefore 8x^3y^3+27a^3=(2xy+3a)(4x^2y^2-6xya+9a^2)$

5. $64a^3 - b^3$

Sol:

$= (4a)^3 - b^3$

$= (4a - b)\left((4a)^2 + 4a \times b + b^2\right)$ $\left[\because a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)\right]$

$= (4a - b)\left(16a^2 + 4ab + b^2\right)$

$\therefore 64a^3 - b^3 = (4a - b)\left(16a^2 + 4ab + b^2\right)$

6. $\frac{x^3}{216} - 8y^3$

Sol:

$= \left(\frac{x}{6}\right)^3 - (2y)^3$

$= \left(\frac{x}{6} - 2y\right)\left(\left(\frac{x}{6}\right)^2 + \frac{x}{6} \times 2y + (2y)^2\right)$ $\left[\because a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)\right]$

$= \left(\frac{x}{6} - 2y\right)\left(\frac{x^2}{36} + \frac{xy}{3} + 4y^2\right)$

$\therefore \frac{x^3}{216} - 8y^3 = \left(\frac{x}{6} - 2y\right)\left(\frac{x^2}{36} + \frac{xy}{3} + 4y^2\right)$

7. $10x^4y - 10xy^4$

Sol:

$10x^4y - 10xy^4$

$= 10xy\left(x^3 - y^3\right)$

$= 10xy(x - y)\left(x^2 + xy + y^2\right)$ $\left[\because a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)\right]$

$\therefore 10x^4y - 10xy^4 = 10xy(x - y)\left(x^2 + xy + y^2\right)$

8. $54x^6y + 2x^3y^4$

Sol:

$54x^6y+2x^3y^4$

$=2x^3y\left(27x^3+y^3\right)$

$=2x^3y\left((3x)^3+y^3\right)$

$=2x^3y(3x+y)\left((3x)^2-3\times xy+y^2\right)$ $\left[\because a^3+b^3=(a+b)\left(a^2-ab+b^2\right)\right]$

$=2x^3y(3x+y)\left(9x^2-3xy+y^2\right)$

$\therefore 54x^6y+2x^3y^4=2x^3y(3x+y)\left(9x^2-3xy+y^2\right)$

9. $32a^3 + 108b^3$

Sol:

$32a^3+108b^3$

$=4\left(8a^3+27b^3\right)$

$=4\left((2a)^3+(3b)^3\right)$ [Using $a^3+b^3=(a+b)\left(a^2-ab+b^2\right)$]

$=4\left[(2a+3b)\left((2a)^2-2a\times 3b+(3b)^2\right)\right]$

$=4(2a+3b)\left(4a^2-6ab+9b^2\right)$

$\therefore 32a^3+108b^3=4(2a+3b)\left(4a^2-6ab+9b^2\right)$

10. $(a-2b)^3-512b^3$

Sol:

$(a-2b)^3-512b^3$

$=(a-2b)^3-(8b)^3$

$=(a-2b-8b)\left((a-2b)^2+(a-2b)8b+(8b)^2\right)$ $\left[\because a^3-b^3=(a-b)\left(a^2+ab+b^2\right)\right]$

$=(a-10b)\left(a^2+4b^2-4ab+8b(a-2b)+(8b)^2\right)$ $\left[\because (a-b)^2=a^2+b^2-2ab\right]$

$=(a-10b)\left(a^2+4b^2-4ab+8ab-16b^2+64b^2\right)$

$=(a=10b)\left(a^2+68b^2-16b^2-4ab+8ab\right)$

$=(a-10b)\left(a^2+52b^2+4ab\right)$

$\therefore (a-2b)^3-512b^3=(a-10b)\left(a^2+4ab+52b^2\right)$

11. $(a + b)^3 - 8(a - b)^3$

Sol:

$(a+b)^3 - 8(a-b)^3$

$= (a+b)^3 - [2(a-b)]^3$

$= (a+b)^3 - (2a-2b)^3$ [Using $a^3 - b^3 = (a-b)(a^2+ab+b^2)$]

$= (a+b-(2a-2b))((a+b)^2 + (a+b)(2a-2b) + (2a-2b)^2)$

$= (a+b-2a+2b)(a^2+b^2+2ab+(a+b)(2a-2b)+(2a-2b)^2)$ $\left[\because (a+b)^2 = a^2+b^2+2ab\right]$

$= (3b-a)(a^2+b^2+2ab+2a^2-2ab+2ab-2b^2+(2a-2b)^2)$

$= (3b-a)(3a^2+2ab-b^2+(2a-2b)^2)$

$= (3b-a)(3a^2+2ab-b^2+4a^2+4b^2-8ab)$ $\left[\because (a-b)^2 = a^2+b^2-2ab\right]$

$= (3b-a)(3a^2+4a^2-b^2+4b^2+2ab-8ab)$

$= (3b-a)(7a^2+3b^2-6ab)$

$\therefore (a+b)^3 - 8(a-b)^3 = (-a+3b)(7a^2-6ab+3b^2)$

12. $(x+2)^3 + (x-2)^3$

Sol:

$= (x+2+x-2)((x+2)^2 - (x+2)(x-2) + (x-2)^2)$ $\left[\because a^3+b^3 - (a+b)(a^2-ab+b^2)\right]$

$= 2x(x^2+4x+4-(x+2)(x-2)+x^2-4x+4)$ $\left[\because (a+b)^2 = a^2+2ab+b^2, (a-b)^2 = a^2-2ab+b^2\right]$

$= 2x(2x^2+8-(x^2-2^2))$ $\left[\because (a+b)(a-b) = a^2-b^2\right]$

$= 2x(2x^2+8-x^2+4)$

$= 2x(x^2+12)$

$\therefore (x+2)^3 + (x-2)^3 = 2x(x^2+12)$

13. $8x^2y^3 - x^5$

Sol:

$= x^2(8y^3 - x^3)$

$= x^2((2y)^3 - x^3)$

$= x^2(2y-x)\left((2y)^2+2y(x)+x^2\right)$ $\left[\because a^3-b^3=(a-b)\left(a^2+ab+b^2\right)\right]$

$= x^2(2y-x)\left(4y^2+2xy+x^2\right)$

$\therefore 8x^2y^3-x^5=x^2(2y-x)\left(4y^2+2xy+x^2\right)$

14. $1029 - 3x^3$

Sol:

$= 3\left(343-x^3\right)$

$= 3\left(7^3-x^3\right)$

$= 3(7-x)\left(7^2+7\times x+x^2\right)$ $\left[\because a^3-b^3=(a-b)\left(a^2+ab+b^2\right)\right]$

$= 3(7-x)\left(49+7x+x^2\right)$

$\therefore 1029-3x^3=3(7-x)\left(49+7x+x^2\right)$

15. $x^6 + y^6$

Sol:

x^6+y^6

$=\left(x^2\right)^3+\left(y^2\right)^3$

$=\left(x^2+y^2\right)\left(\left(x^2\right)^2-x^2y^2+\left(y^2\right)^2\right)$ $\left[\because a^3+b^3=(a+b)\left(a^2-ab+b^2\right)\right]$

$=\left(x^2+y^2\right)\left(x^4-x^2y^2+y^4\right)$

$\therefore x^6+y^6=\left(x^2+y^2\right)\left(x^4-x^2y^2+y^4\right)$

16. $x^3y^3 + 1$

Sol:

$=(xy)^3+1^3$

$=(xy+1)\left((xy)^2-xy\times1+1^2\right)$ $\left[\because a^3+b^3=(a+b)\left(a^2-ab+b^2\right)\right]$

$=(xy+1)\left(x^2y^2-xy+1\right)$

$\therefore x^3y^3+1=(xy+1)\left(x^2y^2-xy+1\right)$

17. $x^4y^4 - xy$

Sol:

$= xy\left(x^3y^3 - 1\right)$

$= xy\left(\left(xy\right)^3 - 1^3\right)$

$= xy\left(xy-1\right)\left(\left(xy\right)^2 + \left(xy\right)1 + 1^2\right)$ $\left[\because a^3 - b^3 = \left(a-b\right)\left(a^2+ab+b^2\right)\right]$

$= xy\left(xy-1\right)\left(x^2y^2 + xy + 1\right)$

$\therefore x^4y^4 - xy = xy\left(xy-1\right)\left(x^2y^2+xy+1\right)$

18. $a^{12} + b^{12}$

Sol:

$= \left(a^4\right)^3 + \left(b^4\right)^3$

$= \left(a^4\right)^3 + \left(b^4\right)^3$

$= \left(a^4+b^4\right)\left(\left(a^4\right)^2 - a^4 \times b^4 + \left(b^4\right)^2\right)$ $\left[\because a^3 + b^3 = \left(a+b\right)\left(a^2-ab+b^2\right)\right]$

$= \left(a^4+b^4\right)\left(a^8 - a^4b^4 + b^8\right)$

$\therefore a^{12} + b^{12} = \left(a^4+b^4\right)\left(a^8-a^4b^4+b^8\right)$

19. $x^3 + 6x^2 + 12x + 16$

Sol:

$= x^3 + 6x^2 + 12x + 8 + 8$

$= x^3 + 3\times x^2 \times 2 + 3 \times x \times 2^2 + 2^3 + 8$

$= \left(x = 2\right)^3 + 8$ $\left[\because a^3 + 3a^2b + 3ab^2 + b^3 = \left(a+b\right)^3\right]$

$= \left(x+2\right)^3 + 2^3$

$= \left(x+2+2\right)\left(\left(x+2\right)^2 - 2\left(x+2\right) + 2^2\right)$ $\left[\because a^3 + b^3 = \left(a+b\right)\left(a^2-ab+b^2\right)\right]$

$= \left(x+4\right)\left(x^2+4x+4-2x-4+4\right)$ $\left[\because \left(a+b\right)^2 = a^2 + 2ab + b^2\right]$

$= \left(x+4\right)\left(x^2+2x+4\right)$

$\therefore x^3 + 6x^2 + 12x + 16 = \left(x+4\right)\left(x^2+2x+4\right)$

20. a^3+b^3+a+b

Sol:

$=\left(a^3+b^3\right)+1(a+b)$

$=(a+b)\left(a^2-ab+b^2\right)+1(a+b)$

$=(a+b)\left(a^2-ab+b^2+1\right)$

$\therefore a^3+b^3+a+b=(a+b)\left(a^2-ab+b^2+1\right)$

21. $a^3-\frac{1}{a^3}-2a+\frac{2}{a}$

Sol:

$=\left(a^3-\frac{1}{a^3}\right)-2\left(a-\frac{1}{a}\right)$

$=\left(a^3-\left(\frac{1}{a}\right)^3\right)-2\left(a-\frac{1}{a}\right)$

$=\left(a-\frac{1}{a}\right)\left(a^2+a\times\frac{1}{a}+\left(\frac{1}{a}\right)^2\right)-2\left(a-\frac{1}{a}\right)$ $\left[\because a^3-b^3=(a-b)\left(a^2+ab+b^2\right)\right]$

$=\left(a-\frac{1}{a}\right)\left(a^2+1+\frac{1}{a^2}\right)-2\left(a-\frac{1}{a}\right)$

$=\left(a-\frac{1}{a}\right)\left(a^2+1+\frac{1}{a^2}-2\right)$

$=\left(a-\frac{1}{a}\right)\left(a^2+\frac{1}{a^2}-1\right)$

$\therefore a^3-\frac{1}{a^3}-2a+\frac{2}{a}=\left(a-\frac{1}{a}\right)\left(a^2+\frac{1}{a^2}-1\right)$

22. $a^3+3a^2b+3ab^2+b^3-8$

Sol:

$=(a+b)^3-8$ $\left[\because a^3+3a^2b+3ab^2+b^3=(a+b)^3\right]$

$=(a+b)^3-2^3$

$=(a+b-2)\left((a+b)^2+(a+b)2+2^2\right)$

$=(a+b-2)\left(a^2+b^2+2ab+2a+2b+4\right)$

$\therefore a^3+3a^2b+3ab^2+b^3-8=(a+b-2)\left(a^2+b^2+2ab+2a+2b+4\right)$

23. $8a^3 - b^3 - 4ax + 2bx$

Sol:

$= 8a^3 - b^3 - 2x(2a - b)$

$= (2a)^3 - b^3 - 2x(2a - b)$

$= (2a - b)\left((2a)^2 + 2a \times b + b^2\right) - 2x(2a - b)$ $\left[\because a^3 - b^3 = (a - b)\left(a^2 + ab + b^2\right)\right]$

$= (2a - b)\left(4a^2 + 2ab + b^2\right) - 2x(2a - b)$

$= (2a - b)\left(4a^2 + 2ab + b^2 - 2x\right)$

$\therefore 8a^3 - b^3 - 4ax + 2bx = (2a - b)\left(4a^2 + 2ab + b^2 - 2x\right)$

24. Simplify:

Sol:

(i) $\dfrac{173 \times 173 \times 173 + 127 \times 127 \times 127}{173 \times 173 - 173 \times 127 + 127 \times 127}$

$= \dfrac{173^2 + 127^3}{173^2 - 173 \times 127 + 127^2}$

$= \dfrac{(173 + 127)\left(173^2 - 173 \times 127 + 127^2\right)}{\left(173^2 - 173 \times 127 + 127^2\right)}$ $\left[\because a^3 + b^3 = (a + b)\left(a^2 - ab + b^2\right)\right]$

$= (173 + 127) = 300$

(ii) $\dfrac{155 \times 155 \times 155 - 55 \times 55 \times 55}{155 \times 155 + 155 \times 55 + 55 \times 55}$

$= \dfrac{155^3 - 55^3}{155^2 + 155 \times 55 + 55^2}$

$= \dfrac{(155 - 55)\left(155^2 + 155 \times 55 + 55^2\right)}{\left(155^2 + 155 \times 55 + 55^2\right)}$ $\left[\because a^3 - b^3 = (a - b)\left(a^2 + ab + b^2\right)\right]$

$= (155 - 55) = 100$

Exercise – 5.3

Factorize:

1. $64a^3+125b^3+240a^2b+300ab^2$

Sol:

$64a^3+125b^3+240a^2b+300ab^2$

$=(4a)^3+(5b)^3+3\times(4a)^2\times 5b+3(4a)(5b)^2$

$=(4a+5b)^3$ $\left[\because a^3+b^3+3a^2b+3ab^2=(a+b)^3\right]$

$=(4a+5b)(4a+5b)(4a+5b)$

$\therefore 64a^3+125b^3+240a^2b+300ab^2=(4a+5b)(4a+5b)(4a+5b)$

2. $125x^3-27y^3-225x^2y+135xy^2$

Sol:

$125x^3-27y^3-225x^2y+135xy^2$

$=(5x)^3-(3y)^3-3\times(5x)^2\times 3y+3\times(5x)(3y)^2$

$=(5x-3y)^3$ $\left[\because a^3-b^3-3a^2b+3ab^2=(a-b)^3\right]$

$=(5x-3y)(5x-3y)(5x-3y)$

$\therefore 125x^3-27y^3-225x^2y+135xy^2=(5x-3y)(5x-3y)(5x-3y)$

3. $\frac{8}{27}x^3+1+\frac{4}{3}x^2+2x$

Sol:

$=\left(\frac{2}{3}x\right)^3+(1)^3+3\times\left(\frac{2}{3}x\right)^2\times 1+3(1)^2\times\left(\frac{2}{3}x\right)$

$=\left(\frac{2}{3}x+1\right)^3$ $\left[\because a^3+b^3+3a^2b+3ab^2=(a+b)^3\right]$

$=\left(\frac{2}{3}x+1\right)\left(\frac{2}{3}x+1\right)\left(\frac{2}{3}x+1\right)$

$\therefore \frac{8}{27}x^3+1+\frac{4}{3}x^2+2x=\left(\frac{2}{3}x+1\right)\left(\frac{2}{3}x+1\right)\left(\frac{2}{3}x+1\right)$

4. $8x^3+27y^3+36x^2y+54xy^2$

Sol:

$8x^3+27y^3+36x^2y+54xy^2$

$=(2x)^3+(3y)^3+3\times(2x)^2\times 3y+3\times(2x)(3y)^2$

$=(2x+3y)^3$ $\left[\because a^3+b^3+3a^2b+3ab^2=(a+b)^3\right]$

$=(2x+3y)(2x+3y)(2x+3y)$

$\therefore\ \ 8x^3+27y^3+36x^2y+54xy^2=(2x+3y)(2x+3y)(2x+3y)$

5. $a^3 - 3a^2b + 3ab^2 - b^3 + 8$

Sol:

$a^3-3a^2b+3ab^2-b^3+8$

$=(a-b)^3+8$ $\left[\because a^3-b^3-3a^2b+3ab^2=(a-b)^3\right]$

$=(a-b)^3+2^3$

$=(a-b+2)\left((a-b)^2-(a-b)2+2^2\right)$ $\left[\because a^3+b^3=(a+b)\left(a^2-ab+b^2\right)\right]$

$=(a-b+2)\left(a^2+b^2-2ab-2(a-b)+4\right)$

$=(a-b+2)\left(a^2+b^2-2ab-2a+2b+4\right)$

$\therefore a^3-3a^2b+3ab^2-b^3+8=(a-b+2)\left(a^2+b^2-2ab-2a+2b+4\right)$

6. $x^3+8y^3+6x^2y+12xy^2$

Sol:

$=(x)^3+(2y)^3+3\times x^2\times 2y+3\times x\times(2y)^2$

$=(x+2y)^3$ $\left[\because a^3+b^3+3a^2b+3ab^2=(a+b)^3\right]$

$=(x+2y)(x+2y)(x+2y)$

$\therefore x^3+8y^3+6x^2y+12xy^2=(x+2y)(x+2y)(x+2y)$

7. $8x^2+y^3+12x^2y+6xy^2$

Sol:

$8x^2+y^3+12x^2y+6xy^2$

$=(2x)^3+y^3+3\times(2x)^2\times y+3(2x)\times y^2$

$=(2x+y)^3$ $\left[\because a^3+b^3+3a^2b+3ab^2=(a+b)^3\right]$

$=(2x+y)(2x+y)(2x+y)$

$\therefore 8x^3+y^3+12x^2y+6xy^2=(2x+y)(2x+y)(2x+y)$

8. $8a^3 + 27b^3 + 36a^2b + 54ab^2$

Sol:

$= (2a)^3 + (3b)^3 + 3\times(2a)^2 \times 3b + 3\times(2a)(3b)^2$

$= (2a+3b)^3$ $\left[\because a^3 + b^3 + 3a^2b + 3ab^2 = (a+b)^3\right]$

$= (2a+3b)(2a+3b)(2a+3b)$

$\therefore 8a^3 + 27b^3 + 36a^2b + 54ab^2 = (2a+3b)(2a+3b)(2a+3b)$

9. $8a^3 - 27b^3 - 36a^2b + 54ab^2$

Sol:

$8a^3 - 27b^3 - 36a^2b + 54ab^2$

$= (2a)^3 - (3b)^3 - 3\times(2a)^2 3b + 3(2a)(3b)^2$

$= (2a-3b)^3$ $\left[\because a^3 - b^3 - 3a^2b + 3ab^2 = (a-b)^3\right]$

$= (2a-3b)(2a-3b)(2a-3b)$

$\therefore 8a^3 - 27b^3 - 36a^2b + 54ab^2 = (2a-3b)(2a-3b)(2a-3b)$

10. $x^3 - 12x(x-4) - 64$

Sol:

$x^3 - 12x(x-4) - 64$

$= x^3 - 12x^2 + 48x - 64$

$= (x)^3 - 3\times x^2 \times 4 + 3\times 4^2 \times x - 4^3$

$= (x-4)^3$ $\left[\because a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3\right]$

$= (x-4)(x-4)(x-4)$

$\therefore x^3 - 12x(x-4) - 64 = (x-4)(x-4)(x-4)$

11. $a^3x^3 - 3a^2bx^2 + 3ab^2x - b^3$

Sol:

$= (ax)^3 - 3(ax)^2 \times b + 3(ax)b^2 - b^3$

$= (ax-b)^3$ $\left[\because a^2 - 3a^2b + 3ab^2 - b^3 = (a-b)^3\right]$

$= (ax-b)(ax-b)(ax-b)$

$\therefore a^3x^3 - 3a^2bx^2 + 3ab^2x - b^3 = (ax-b)(ax-b)(ax-b)$

Exercise – 5.4

1. $a^3+8b^3+64c^3-24abc$

Sol:

$$a^3+8b^3+64c^3-24abc$$
$$=(a)^3+(2b)^3+(4c)^3-3\times a\times 2b\times 4c$$
$$=(a+2b+4c)\left(a^2+(2b)^2+(4c)^2-a\times 2b-2b\times 4c-4c\times a\right)$$
$$\left[\because a^3+b^3+c^3-3abc=(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)\right]$$
$$=(a+2b+4c)\left(a^2+4b^2+16c^2-2ab-8bc-4ac\right)$$
$$\therefore a^3+8b^3+64c^3-24abc=(a+2b+4c)\left(a^2+4b^2+16c^2-2ab-8bc-4ac\right)$$

2. $x^3-8y^3+27z^3+18xyz$

Sol:

$$=x^3+(-2y)^3+(3z)^3-3\times x\times(-2y)(3z)$$
$$=(x+(-2y)+3z)\left(x^2+(-2y)^2+(3z)^2-x(-2y)-(-2y)(3z)-3z(x)\right)$$
$$\left[\because a^3+b^3+c^3-3abc=(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)\right]$$
$$=(x-2y+3z)\left(x^2+4y^2+9z^2+2xy+6yz-3zx\right)$$
$$\therefore x^3-8y^3+27z^3+18xyz=(x-2y+3z)\left(x^2+4y^2+9z^2+2xy+6yz-3zx\right)$$

3. $\frac{1}{27}x^3-y^3+125z^3+5xyz$

Sol:

$$\frac{1}{27}x^3-y^3+125z^3+5xyz$$
$$=\left(\frac{x}{3}\right)^3+(-y)^3+(5z)^3-3\times\frac{x}{3}(-y)(5z)$$
$$=\left(\frac{x}{3}+(-y)+5z\right)\left(\left(\frac{x}{3}\right)^2+(-y)^2+(5z)^2-\frac{x}{3}(-y)-(-y)5z-5z\left(\frac{x}{3}\right)\right)$$
$$=\left(\frac{x}{3}-y+5z\right)\left(\frac{x^2}{9}+y^2+25z^2+\frac{xy}{3}+5xyz-\frac{5}{3}zx\right)$$
$$\therefore\frac{1}{27}x^3-y^3+125z^3+5xyz=\left(\frac{x}{3}-y+5z\right)\left(\frac{x^2}{9}+y^2+25z^2+\frac{xy}{3}+5yz-\frac{5}{3}zx\right)$$

4. $8x^3+27y^3-216z^3+108xyz$

Sol:

$=(2x)^3+(3y)^3+(-6z)^3-3(2x)(3y)(-6z)$

$=(2x+3y+(-6z))\left((2x)^2+(3y)^2+(-6z)^2-2x\times 3y-3y(-6z)-(-6z)2x\right)$

$=(2x+3y-6z)\left(4x^2+9y^2+36z^2-6xy+18yz+12zx\right)$

$\therefore 8x^3+27y^3-216z^3+108xyz=(2x+3y-6z)\left(4x^2+9y^2+36z^2-6xy+18yz+12zx\right)$

5. $125+8x^3-27y^3+90xy$

Sol:

$=5^3+(2x)^3+(-3y)^3-3\times 5\times 2x\times(-3y)$

$=(5+2x+(-3y))\left(5^2+(2x)^2+(-3y)^2-5(2x)-2x(-3y)-(-3y)5\right)$

$=(5+2x-3y)\left(25+4x^2+9y^2-10x+6xy+15y\right)$

$\therefore 125+8x^3-27y^3+90xy=(5+2x-3y)\left(25+4x^2+9y^2-10x+6xy+15y\right)$

6. $(3x-2y)^3+(2y-4z)^3+(4z-3x)^3$

Sol:

Let $(3x-2y)=a,(2y-4z)=b,(4z-3x)=c$

$\therefore a+b+c=3x-2y+2y-4z+4z-3x=0$

$\because a+b+c=0 \therefore a^3+b^3+c^3=3abc$

$\therefore (3x-2y)^3+(2y-4z)^3+(4z-3x)^3=3(3x-2y)(2y-4z)(4z-3x)$

7. $(2x-3y)^3+(4z-2x)^3+(3y-4z)^3$

Sol:

Let $2x-3y=a,4z-2x=b,3y-4z=c$

$\therefore a+b+c=2x-3y+4z-2x+3y-4z=c$

$\because a+b+c=0 \qquad \therefore a^3+b^3+c^3=3abc$

$\therefore (2x-3y)^3+(4z-2x)^3+(3y-4z)^3=3(2x-3y)(4z-2x)(3y-4z)$

8. $\left[\frac{x}{2}+y+\frac{z}{3}\right]^3+\left[\frac{x}{2}+\frac{2y}{3}+z\right]^3+\left[-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right]^3$

Sol:

Let $\left(\frac{x}{2}+y+\frac{z}{3}\right)=a,\left(\frac{x}{3}-\frac{2y}{3}+z\right)=b,\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right)=c$

$a+b+c=\frac{x}{2}+y+\frac{z}{3}+\frac{x}{3}-\frac{2y}{3}+z-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}$

$a+b+c=\left(\frac{x}{2}+\frac{x}{3}-\frac{5x}{6}\right)+\left(y-\frac{2y}{3}-\frac{y}{3}\right)+\left(\frac{z}{3}+z-\frac{4z}{3}\right)$

$a+b+c=\frac{3x}{6}+\frac{2x}{6}-\frac{5x}{6}+\frac{3y}{3}-\frac{2y}{3}-\frac{y}{3}+\frac{z}{3}+\frac{3z}{3}-\frac{4z}{3}$

$a+b+c=\frac{5x-5x}{6}+\frac{3y-3y}{3}+\frac{4z-4z}{3}$

$a+b+c=0$

$\because a+b+c=0 \qquad \therefore a^3+b^3+c^3=3abc$

$\therefore\left[\frac{x}{2}+y+\frac{z}{3}\right]^3+\left[\frac{x}{3}-\frac{2y}{3}+z\right]^3+\left[-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right]^3=3\left(\frac{x}{2}+y+\frac{z}{3}\right)\left(\frac{x}{3}-\frac{2y}{3}+z\right)\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right)$

9. $(a – 3b)^3 + (3b – c)^3 + (c – a)^3$

Sol:

Let $(a-3b)=x,(3b-c)=y,(c-a)=z$

$x+y+z=a-3b+3b-c+c-a=0$

$\because x+y+z=0 \qquad \therefore x^3+y^3+z^3=3xyz$

$\therefore(a-3b)^3+(3b-c)^3+(c-a)^3=3(a-3b)(3b-c)(c-a)$

10. $2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6}abc$

Sol:

$=\left(\sqrt{2}a\right)^3+\left(\sqrt{3}b\right)^3+c^3-3\times\sqrt{2}a\times\sqrt{3}b\times c$

$=\left(\sqrt{2}a+\sqrt{3}b+c\right)\left(\left(\sqrt{2}a\right)^2+\left(\sqrt{3}b\right)^2+c^2-\left(\sqrt{2}a\right)\left(\sqrt{3}b\right)-\left(\sqrt{3}b\right)c-\left(\sqrt{2}a\right)c\right)$

$=\left(\sqrt{2}a+\sqrt{3}b+c\right)\left(2a^2+3b^2+c^2-\sqrt{6}ab-\sqrt{3}bc-\sqrt{2}ac\right)$

$\therefore 2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6}abc=\left(\sqrt{2}a+\sqrt{3}b+c\right)\left(2a^2+3b^2+c^2-\sqrt{6}ab-\sqrt{3}bc-\sqrt{2}ac\right)$

11. $3\sqrt{3}a^3 - b^3 - 5\sqrt{5}c^3 - 3\sqrt{15}abc$

Sol:

$= \left(\sqrt{3}a\right)^3 + (-b)^3 + \left(-\sqrt{5}c\right)^3 - 3\times\left(\sqrt{3}a\right)(-b)\left(-\sqrt{5}c\right)$

$= \left(\sqrt{3}a + (-b) + \left(-\sqrt{5}c\right)\right)\left(\left(\sqrt{3}a\right)^2 + (-b)^2 + \left(-\sqrt{5}c\right)^2 - \sqrt{3}a(-b) - (-b)\left(-\sqrt{5}c\right) - \left(-\sqrt{5}c\right)\sqrt{3}a\right)$

$= \left(\sqrt{3}a - b - \sqrt{5}c\right)\left(3a^2 + b^2 + 5c^2 + \sqrt{3}ab - \sqrt{5}bc + \sqrt{15}ac\right)$

$\therefore 3\sqrt{3}a^3 - b^3 - 5\sqrt{5}c^3 - 3\sqrt{15}abc = \left(\sqrt{3}a - b - \sqrt{5}c\right)\left(3a^2 + b^2 + 5c^2 + \sqrt{3}ab - \sqrt{5}bc + \sqrt{15}ac\right)$

12. $8x^3 - 125y^3 + 180xy + 216$

Sol:

$8x^3 - 125y^3 + 180xy + 216$

or, $8x^3 - 125y^3 + 216 + 180xy$

$= (2x)^3 + (-5y)^3 + 6^3 - 3\times(2x)(-5y)(6)$

$= (2x + (-5y) + 6)\left((2x)^2 + (-5y)^2 + 6^2 - 2x(-5y) - (-5y)6 - 6(2x)\right)$

$= (2x - 5y + 6)\left(4x^2 + 25y^2 + 36 + 10xy + 30y - 12x\right)$

$\therefore 8x^3 - 125y^3 + 180xy + 216 = (2x - 5y + 6)\left(4x^2 + 25y^2 + 36 + 10xy + 30y - 12x\right)$

13. $2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc$

Sol:

$= \left(\sqrt{2}a\right)^3 + \left(2\sqrt{2}b\right)^3 + c^3 - 3\times\sqrt{2}a\times 2\sqrt{2}b\times c$

$= \left(\sqrt{2}a + 2\sqrt{2}b + c\right)\left(\left(\sqrt{2}a\right)^2 + \left(2\sqrt{2}b\right)^2 + c^2 - \left(\sqrt{2}a\right)\left(2\sqrt{2}b\right) - \left(2\sqrt{2}b\right)c - \left(\sqrt{2}a\right)c\right)$

$= \left(\sqrt{2}a + 2\sqrt{2}b + c\right)\left(2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ac\right)$

$2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc = \left(\sqrt{2}a + 2\sqrt{2}b + c\right)\left(2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ac\right)$