CHAPTER – 27
SPECIFIC HEAT CAPACITIES OF GASES

1. N = 1 mole, W = 20 g/mol, V = 50 m/s

   K.E. of the vessel = Internal energy of the gas

   $= (1/2)\ mv^2 = (1/2) \times 20 \times 10^{-3} \times 50 \times 50 = 25$ J

   $25 = n\frac{3}{2}r(\Delta T) \Rightarrow 25 = 1 \times \frac{3}{2} \times 8.31 \times \Delta T \Rightarrow \Delta T = \frac{50}{3 \times 8.3} \approx 2$ k.

2. m = 5 g, $\Delta t = 25 - 15 = 10°C$

   $C_V = 0.172$ cal/g-°CJ = 4.2 J/Cal.

   dQ = du + dw

   Now, V = 0 (for a rigid body)

   So, dw = 0.

   So dQ = du.

   Q = msdt = 5 × 0.172 × 10 = 8.6 cal = 8.6 × 4.2 = 36.12 Joule.

3. $\gamma = 1.4$, w or piston = 50 kg., A of piston = 100 cm^2

   Po = 100 kpa, g = 10 m/s^2, x = 20 cm.

   $dw = pdv = \left(\frac{mg}{A} + Po\right)Adx = \left(\frac{50 \times 10}{100 \times 10^{-4}} + 10^5\right)100 \times 10^{-4} \times 20 \times 10^{-2} = 1.5 \times 10^5 \times 20 \times 10^{-4} = 300$ J.

   $nRdt = 300 \Rightarrow dT = \frac{300}{nR}$

   $dQ = nCpdT = nCp \times \frac{300}{nR} = \frac{n\gamma R300}{(\gamma - 1)nR} = \frac{300 \times 1.4}{0.4} = 1050$ J.

4. $C_VH_2 = 2.4$ Cal/g°C, $C_PH^2 = 3.4$ Cal/g°C

   M = 2 g/ Mol, $R = 8.3 \times 10^7$ erg/mol-°C

   We know, $C_P - C_V = 1$ Cal/g°C

   So, difference of molar specific heats

   $= C_P \times M - C_V \times M = 1 \times 2 = 2$ Cal/g°C

   Now, $2 \times J = R \Rightarrow 2 \times J = 8.3 \times 10^7$ erg/mol-°C $\Rightarrow J = 4.15 \times 10^7$ erg/cal.

5. $\frac{C_P}{C_V} = 7.6$, n = 1 mole, $\Delta T = 50K$

   (a) Keeping the pressure constant, dQ = du + dw,

   $\Delta T = 50$ K, $\gamma = 7/6$, m = 1 mole,

   $dQ = du + dw \Rightarrow nC_VdT = du + RdT \Rightarrow du = nCpdT - RdT$

   $= 1 \times \frac{R\gamma}{\gamma - 1} \times dT - RdT = \frac{R \times \frac{7}{6}}{\frac{7}{6} - 1}dT - RdT$

   = DT – RdT = 7RdT – RdT = 6 RdT = 6 × 8.3 × 50 = 2490 J.

   (b) Kipping Volume constant, $dv = nC_VdT$

   $= 1 \times \frac{R}{\gamma - 1} \times dt = \frac{1 \times 8.3}{\frac{7}{6} - 1} \times 50$

   = 8.3 × 50 × 6 = 2490 J

   (c) Adiabetically dQ = 0, du = – dw

   $= \left[\frac{n \times R}{\gamma - 1}(T_1 - T_2)\right] = \frac{1 \times 8.3}{\frac{7}{6} - 1}(T_2 - T_1) = 8.3 \times 50 \times 6 = 2490$ J

6. m = 1.18 g, $\quad V = 1 \times 10^3\ cm^3 = 1\ L \quad T = 300\ k, \quad P = 10^5\ Pa$

$PV = nRT$ or $\quad n = \frac{PV}{RT} = 10^5 = atm.$

$N = \frac{PV}{RT} = \frac{1}{8.2 \times 10^{-2} \times 3 \times 10^2} = \frac{1}{8.2 \times 3} = \frac{1}{24.6}$

Now, $C_v = \frac{1}{n} \times \frac{Q}{dt} = 24.6 \times 2 = 49.2$

$C_p = R + C_v = 1.987 + 49.2 = 51.187$

$Q = nC_p dT = \frac{1}{24.6} \times 51.187 \times 1 = 2.08$ Cal.

7. $V_1 = 100\ cm^3, \quad V_2 = 200\ cm^3 \quad P = 2 \times 10^5\ Pa, \ \Delta Q = 50J$

(a) $\Delta Q = du + dw \Rightarrow 50 = du + 20\times 10^5(200 - 100 \times 10^{-6}) \Rightarrow 50 = du + 20 \Rightarrow du = 30\ J$

(b) $30 = n \times \frac{3}{2} \times 8.3 \times 300 \quad [\ U = \frac{3}{2} nRT$ for monoatomic$]$

$\Rightarrow n = \frac{2}{3 \times 83} = \frac{2}{249} = 0.008$

(c) $du = nC_v dT \Rightarrow C_v = \frac{dndTu}{} = \frac{30}{0.008 \times 300} = 12.5$

$C_p = C_v + R = 12.5 + 8.3 = 20.3$

(d) $C_v = 12.5$ (Proved above)

8. Q = Amt of heat given

Work done = $\frac{Q}{2}, \quad \Delta Q = W + \Delta U$

for monoatomic gas $\Rightarrow \Delta U = Q - \frac{Q}{2} = \frac{Q}{2}$

$V = n\frac{3}{2}RT = \frac{Q}{2} = nT \times \frac{3}{2}R = 3R \times nT$

Again Q = n CpdT Where C_P > Molar heat capacity at const. pressure.

$3RnT = ndTC_P \Rightarrow C_P = 3R$

9. $P = KV \Rightarrow \frac{nRT}{V} = KV \Rightarrow RT = KV^2 \Rightarrow R\ \Delta T = 2KV\ \Delta U \Rightarrow \frac{R\Delta T}{2KV} = dv$

$dQ = du + dw \Rightarrow mcdT = C_V dT + pdv \Rightarrow msdT = C_V\ dT + \frac{PRdF}{2KV}$

$\Rightarrow ms = C_V + \frac{RKV}{2KV} \Rightarrow C_P + \frac{R}{2}$

10. $\frac{C_P}{C_V} = \gamma, \quad C_P - C_V = R, \quad C_V = \frac{r}{\gamma - 1}, \quad C_P = \frac{\gamma R}{\gamma - 1}$

$Pdv = \frac{1}{b+1}(Rdt)$

$\Rightarrow 0 = C_V dT + \frac{1}{b+1}(Rdt) \Rightarrow \frac{1}{b+1} = \frac{-C_V}{R}$

$\Rightarrow b + 1 = \frac{-R}{C_V} = \frac{-(C_P - C_V)}{C_V} = -\gamma + 1 \Rightarrow b = -\gamma$

11. Considering two gases, in Gas(1) we have,

γ, Cp_1 (Sp. Heat at const. 'P'), Cv_1 (Sp. Heat at const. 'V'), n_1 (No. of moles)

$\frac{Cp_1}{Cv_1} = \gamma$ & $Cp_1 - Cv_1 = R$

$\Rightarrow \gamma Cv_1 - Cv_1 = R \Rightarrow Cv_1 (\gamma - 1) = R$

$\Rightarrow Cv_1 = \dfrac{R}{\gamma - 1}$ & $Cp_1 = \dfrac{\gamma R}{\gamma - 1}$

In Gas(2) we have, γ, Cp_2 (Sp. Heat at const. 'P'), Cv_2 (Sp. Heat at const. 'V'), n_2 (No. of moles)

$\dfrac{Cp_2}{Cv_2} = \gamma$ & $Cp_2 - Cv_2 = R \Rightarrow \gamma Cv_2 - Cv_2 = R \Rightarrow Cv_2 (\gamma - 1) = R \Rightarrow Cv_2 = \dfrac{R}{\gamma - 1}$ & $Cp_2 = \dfrac{\gamma R}{\gamma - 1}$

Given $n_1 : n_2 = 1 : 2$

$dU_1 = nCv_1\, dT$ & $dU_2 = 2nCv_2\, dT = 3nCvdT$

$$\Rightarrow C_V = \frac{Cv_1 + 2Cv_2}{3} = \frac{\frac{R}{\gamma - 1} + \frac{2R}{\gamma - 1}}{3} = \frac{3R}{3(\gamma - 1)} = \frac{R}{\gamma - 1} \qquad \ldots(1)$$

$$\&Cp = \gamma Cv = \frac{\gamma r}{\gamma - 1} \quad \ldots(2)$$

So, $\dfrac{Cp}{Cv} = \gamma$ [from (1) & (2)]

12. $Cp' = 2.5\ RCp'' = 3.5\ R$

$Cv' = 1.5\ R \qquad Cv'' = 2.5\ R$

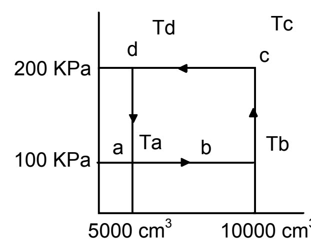
$n_1 = n_2 = 1$ mol $\qquad (n_1 + n_2)C_V dT = n_1\, C_v dT + n_2\, C_v dT$

$$\Rightarrow C_V = \frac{n_1 Cv' + n_2 Cv''}{n_1 + n_2} = \frac{1.5R + 2.5R}{2}\ 2R$$

$C_P = C_V + R = 2R + R = 3R$

$$\gamma = \frac{C_p}{C_V} = \frac{3R}{2R} = 1.5$$

13. $n = \dfrac{1}{2}$ mole, $\qquad R = \dfrac{25}{3}$ J/mol-k, $\qquad \gamma = \dfrac{5}{3}$



(a) Temp at A = T_a, $P_a V_a = nRT_a$

$$\Rightarrow T_a = \frac{P_a V_a}{nR} = \frac{5000 \times 10^{-6} \times 100 \times 10^3}{\frac{1}{2} \times \frac{25}{3}} = 120\ \text{k}.$$

Similarly temperatures at point b = 240 k at C it is 480 k and at D it is 240 k.

(b) For ab process,

$dQ = nCpdT \qquad$ [since ab is isobaric]

$$= \frac{1}{2} \times \frac{R\gamma}{\gamma - 1}(T_b - T_a) = \frac{1}{2} \times \frac{\frac{35}{3} \times \frac{5}{3}}{\frac{5}{3} - 1} \times (240 - 120) = \frac{1}{2} \times \frac{125}{9} \times \frac{3}{2} \times 120 = 1250\ \text{J}$$

For bc, $\qquad dQ = du + dw \qquad$ [dq = 0, Isochorie process]

$$\Rightarrow dQ = du = nC_v dT = \frac{nR}{\gamma - 1}(T_c - T_a) = \frac{1}{2} \times \frac{\frac{25}{3}}{\left(\frac{5}{3} - 1\right)}(240) = \frac{1}{2} \times \frac{25}{3} \times \frac{3}{2} \times 240 = 1500\ \text{J}$$

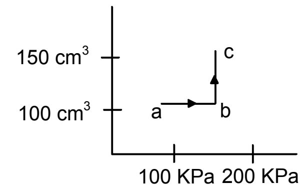
(c) Heat liberated in cd = $-\, nC_p dT$

$$= \frac{-1}{2} \times \frac{nR}{\gamma - 1}(T_d - T_c) = \frac{-1}{2} \times \frac{125}{3} \times \frac{3}{2} \times 240 = 2500\ \text{J}$$

Heat liberated in da = $-\, nC_v dT$

$$= \frac{-1}{2} \times \frac{R}{\gamma - 1}(T_a - T_d) = \frac{-1}{2} \times \frac{25}{2} \times (120 - 240) = 750\ \text{J}$$

14. (a) For a, b 'V' is constant



So, $\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow \frac{100}{300} = \frac{200}{T_2} \Rightarrow T_2 = \frac{200 \times 300}{100} = 600$ k

For b,c 'P' is constant

So, $\frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow \frac{100}{600} = \frac{150}{T_2} \Rightarrow T_2 = \frac{600 \times 150}{100} = 900$ k

(b) Work done = Area enclosed under the graph 50 cc × 200 kpa = $50 \times 10^{-6} \times 200 \times 10^3$ J = 10 J

(c) 'Q' Supplied = $nC_v dT$

Now, $n = \frac{PV}{RT}$ considering at pt. 'b'

$C_v = \frac{R}{\gamma - 1} dT = 300$ a, b.

$Q_{bc} = \frac{PV}{RT} \times \frac{R}{\gamma - 1} dT = \frac{200 \times 10^3 \times 100 \times 10^{-6}}{600 \times 0.67} \times 300 = 14.925$ $(\therefore \gamma = 1.67)$

Q supplied to be $nC_p dT$ $[\therefore C_p = \frac{\gamma R}{\gamma - 1}]$

$= \frac{PV}{RT} \times \frac{\gamma R}{\gamma - 1} dT = \frac{200 \times 10^3 \times 150 \times 10^{-6}}{8.3 \times 900} \times \frac{1.67 \times 8.3}{0.67} \times 300 = 24.925$

(d) $Q = \Delta U + w$

Now, $\Delta U = Q - w$ = Heat supplied – Work done = (24.925 + 14.925) – 1 = 29.850

15. In Joly's differential steam calorimeter

$C_v = \frac{m_2 L}{m_1(\theta_2 - \theta_1)}$

m_2 = Mass of steam condensed = 0.095 g, L = 540 Cal/g = 540 × 4.2 J/g

m_1 = Mass of gas present = 3 g, $\theta_1 = 20°C$, $\theta_2 = 100°C$

$\Rightarrow C_v = \frac{0.095 \times 540 \times 4.2}{3(100 - 20)} = 0.89 \approx 0.9$ J/g-K

16. $\gamma = 1.5$

Since it is an adiabatic process, So PV^γ = const.

(a) $P_1V_1^\gamma = P_2V_2^\gamma$ Given $V_1 = 4$ L, $V_2 = 3$ L, $\frac{P_2}{P_1} = ?$

$\Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma = \left(\frac{4}{3}\right)^{1.5} = 1.5396 \approx 1.54$

(b) $TV^{\gamma-1}$ = Const.

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \Rightarrow \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{4}{3}\right)^{0.5} = 1.154$

17. $P_1 = 2.5 \times 10^5$ Pa, $V_1 = 100$ cc, $T_1 = 300$ k

(a) $P_1V_1^\gamma = P_2V_2^\gamma$

$\Rightarrow 2.5 \times 10^5 \times V^{1.5} = \left(\frac{V}{2}\right)^{1.5} \times P_2$

$\Rightarrow P_2 = 2^{1.5} \times 2.5 \times 10^5 = 7.07 \times 10^5 \approx 7.1 \times 10^5$

(b) $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \Rightarrow 300 \times (100)^{1.5-1} = T_2 \times (50)^{1.5-1}$

$\Rightarrow T_2 = \frac{3000}{7.07} = 424.32$ k ≈ 424 k

(c) Work done by the gas in the process

$W = \frac{mR}{\gamma - 1}\left[T_2 - T_1\right] = \frac{P_1V_1}{T(\gamma-1)}\left[T_2 - T_1\right]$

$= \frac{2.5 \times 10^5 \times 100 \times 10^{-6}}{300(1.5-1)}[424 - 300] = \frac{2.5 \times 10}{300 \times 0.5} \times 124 = 20.72 \approx 21$ J

18. $\gamma = 1.4$, $T_1 = 20°C = 293$ k, $P_1 = 2$ atm, $p_2 = 1$ atm

We know for adiabatic process,

$P_1^{1-\gamma} \times T_1^{\gamma} = P_2^{1-\gamma} \times T_2^{\gamma}$ or $(2)^{1-1.4} \times (293)^{1.4} = (1)^{1-1.4} \times T_2^{1.4}$

$\Rightarrow (2)^{0.4} \times (293)^{1.4} = T_2^{1.4} \Rightarrow 2153.78 = T_2^{1.4} \Rightarrow T_2 = (2153.78)^{1/1.4} = 240.3$ K

19. $P_1 = 100$ KPa $= 10^5$ Pa, $V_1 = 400\ cm^3 = 400 \times 10^{-6}\ m^3$, $T_1 = 300$ k,

$\gamma = \frac{C_P}{C_V} = 1.5$

(a) Suddenly compressed to $V_2 = 100\ cm^3$

$P_1V_1^{\gamma} = P_2V_2^{\gamma} \Rightarrow 10^5\ (400)^{1.5} = P_2 \times (100)^{1.5}$

$\Rightarrow P_2 = 10^5 \times (4)^{1.5} = 800$ KPa

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \Rightarrow 300 \times (400)^{1.5-1} = T_2 \times (100)^{1.5-1} \Rightarrow T_2 = \frac{300 \times 20}{10} = 600$ K

(b) Even if the container is slowly compressed the walls are adiabatic so heat transferred is 0.

Thus the values remain, $P_2 = 800$ KPa, $T_2 = 600$ K.

20. Given $\frac{C_P}{C_V} = \gamma$ P_0 (Initial Pressure), V_0 (Initial Volume)

(a) (i) Isothermal compression, $P_1V_1 = P_2V_2$ or, $P_0V_0 = \frac{P_2V_0}{2} \Rightarrow P_2 = 2P_0$

(ii) Adiabatic Compression $P_1V_1^{\gamma} = P_2V_2^{\gamma}$ or $2P_0\left(\frac{V_0}{2}\right)^{\gamma} = P1\left(\frac{V_0}{4}\right)^{\gamma}$

$\Rightarrow P' = \frac{V_0^{\gamma}}{2^{\gamma}} \times 2P_0 \times \frac{4^{\gamma}}{V_0^{\gamma}} = 2^{\gamma} \times 2\ P_0 \Rightarrow P_0 2^{\gamma+1}$

(b) (i) Adiabatic compression $P_1V_1^{\gamma} = P_2V_2^{\gamma}$ or $P_0V_0^{\gamma} = P'\left(\frac{V_0}{2}\right)^{\gamma} \Rightarrow P' = P_0 2^{\gamma}$

(ii) Isothermal compression $P_1V_1 = P_2V_2$ or $2^{\gamma}\ P_0 \times \frac{V_0}{2} = P_2 \times \frac{V_0}{4} \Rightarrow P_2 = P_0 2^{\gamma+1}$

21. Initial pressure = P_0

Initial Volume = V_0

$\gamma = \frac{C_P}{C_V}$

(a) Isothermally to pressure $\frac{P_0}{2}$

$P_0V_0 = \frac{P_0}{2} V_1 \Rightarrow V_1 = 2\ V_0$

Adiabetically to pressure = $\frac{P_0}{4}$

$\frac{P_0}{2}(V_1)^{\gamma} = \frac{P_0}{4}(V_2)^{\gamma} \Rightarrow \frac{P_0}{2}(2V_0)^{\gamma} = \frac{P_0}{4}(V_2)^{\gamma}$

$\Rightarrow 2^{\gamma+1}\ V_0^{\gamma} = V_2^{\gamma} \Rightarrow V_2 = 2^{(\gamma+1)/\gamma}\ V_0$

$\therefore$ Final Volume = $2^{(\gamma+1)/\gamma}\ V_0$

(b) Adiabetically to pressure $\frac{P_0}{2}$ to P_0

$P_0 \times (2^{\gamma+1} V_0^{\gamma}) = \frac{P_0}{2} \times (V')^{\gamma}$

Isothermal to pressure $\frac{P_0}{4}$

$\frac{P_0}{2} \times 2^{1/\gamma} V_0 = \frac{P_0}{4} \times V'' \Rightarrow V'' = 2^{(\gamma+1)/\gamma} V_0$

$\therefore$ Final Volume = $2^{(\gamma+1)/\gamma} V_0$

22. PV = nRT

Given P = 150 KPa = 150×10^3 Pa, V = $150\ cm^3 = 150 \times 10^{-6}\ m^3$, T = 300 k

(a) $n = \frac{PV}{RT} = \frac{150 \times 10^3 \times 150 \times 10^{-6}}{8.3 \times 300} = 9.036 \times 10^{-3} = 0.009$ moles.

(b) $\frac{C_P}{C_V} = \gamma \Rightarrow \frac{\gamma R}{(\gamma - 1)C_V} = \gamma$ $\left[\therefore C_P = \frac{\gamma R}{\gamma - 1}\right]$

$\Rightarrow C_V = \frac{R}{\gamma - 1} = \frac{8.3}{1.5 - 1} = \frac{8.3}{0.5} = 2R = 16.6$ J/mole

(c) Given P_1 = 150 KPa = 150×10^3 Pa, P_2 =?

$V_1 = 150\ cm^3 = 150 \times 10^{-6}\ m^3$, $\gamma = 1.5$

$V_2 = 50\ cm^3 = 50 \times 10^{-6}\ m^3$, T_1 = 300 k, T_2 = ?

Since the process is adiabatic Hence – $P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$\Rightarrow 150 \times 10^3 (150 \times 10^{-6})^{\gamma} = P_2 \times (50 \times 10^{-6})^{\gamma}$

$\Rightarrow P_2 = 150 \times 10^3 \times \left(\frac{150 \times 10^{-6}}{50 \times 10^{-6}}\right)^{1.5} = 150000 \times 3^{1.5} = 779.422 \times 10^3\ Pa \approx 780$ KPa

(d) $\Delta Q = W + \Delta U$ or $W = -\Delta U$ [$\therefore \Delta U = 0$, in adiabatic]

$= -nC_VdT = -0.009 \times 16.6 \times (520 - 300) = -0.009 \times 16.6 \times 220 = -32.8\ J \approx -33$ J

(e) $\Delta U = nC_VdT = 0.009 \times 16.6 \times 220 \approx 33$ J

23. $V_A = V_B = V_C$

For A, the process is isothermal

$P_AV_A = P_A'V_A' \Rightarrow P_A' = P_A \frac{V_A}{V_A'} = P_A \times \frac{1}{2}$

For B, the process is adiabatic,

$P_A(V_B)^{\gamma} = P_A'(V_B)^{\gamma} = P_B' = P_B\left(\frac{V_B}{V_B'}\right)^{\gamma} = P_B \times \left(\frac{1}{2}\right)^{1.5} = \frac{P_B}{2^{1.5}}$

For, C, the process is isobaric

$\frac{V_C}{T_C} = \frac{V_C'}{T_C'} \Rightarrow \frac{V_C}{T_C} = \frac{2V_C'}{T_C'} \Rightarrow T_C' = \frac{2}{T_C}$

Final pressures are equal.

$= \frac{p_A}{2} = \frac{P_B}{2^{1.5}} = P_C \Rightarrow P_A : P_B : P_C = 2 : 2^{1.5} : 1 = 2 : 2\sqrt{2} : 1$

24. P_1 = Initial Pressure V_1 = Initial Volume P_2 = Final Pressure V_2 = Final Volume

Given, $V_2 = 2V_1$, Isothermal workdone = $nRT_1 \ln\left(\frac{V_2}{V_1}\right)$

Adiabatic workdone = $\frac{P_1V_1 - P_2V_2}{\gamma - 1}$

Given that workdone in both cases is same.

Hence $nRT_1 \text{ Ln}\left(\frac{V_2}{V_1}\right) = \frac{P_1V_1 - P_2V_2}{\gamma - 1} \Rightarrow (\gamma - 1) \ln\left(\frac{V_2}{V_1}\right) = \frac{P_1V_1 - P_2V_2}{nRT_1}$

$\Rightarrow (\gamma - 1)\ln\left(\frac{V_2}{V_1}\right) = \frac{nRT_1 - nRT_2}{nRT_1} \Rightarrow (\gamma - 1) \ln 2 = \frac{T_1 - T_1}{T_1}$ …(i) $[\because V_2 = 2V_1]$

We know $TV^{\gamma-1}$ = const. in adiabatic Process.

$T_1V_1^{\gamma-1} = T_2\, V_2^{\gamma-1}$, or $T_1\, (V_2)^{\gamma-1} = T_2 \times (2)^{\gamma-1} \times (V_1)^{\gamma-1}$

Or, $T_1 = 2^{\gamma-1} \times T_2$ or $T_2 = T_1^{1-\gamma}$ …(ii)

From (i) & (ii)

$(\gamma - 1) \ln 2 = \frac{T_1 - T_1 \times 2^{1-\gamma}}{T_1} \Rightarrow (\gamma - 1) \ln 2 = 1 - 2^{1-\gamma}$

25. $\gamma = 1.5$, $T = 300$ k, $V = 1Lv = \frac{1}{2}l$

(a) The process is adiabatic as it is sudden,

$P_1\, V_1^{\gamma} = P_2\, V_2^{\gamma} \Rightarrow P_1\, (V_0)^{\gamma} = P_2\left(\frac{V_0}{2}\right)^{\gamma} \Rightarrow P_2 = P_1\left(\frac{1}{1/2}\right)^{1.5} = P_1\,(2)^{1.5} \Rightarrow \frac{P_2}{P_1} = 2^{1.5} = 2\sqrt{2}$

(b) $P_1 = 100$ KPa $= 10^5$ Pa $W = \frac{nR}{\gamma - 1}[T_1 - T_2]$

$T_1\, V_1^{\gamma-1} = P_2\, V_2^{\gamma-1} \Rightarrow 300 \times (1)^{1.5-1} = T_2\,(0.5)^{1.5-1} \Rightarrow 300 \times 1 = T_2\ \sqrt{0.5}$

$T_2 = 300 \times \sqrt{\frac{1}{0.5}} = 300\sqrt{2}$ K

$P_1\, V_1 = nRT_1 \Rightarrow n = \frac{P_1V_1}{RT_1} = \frac{10^5 \times 10^{-3}}{R \times 300} = \frac{1}{3R}$ (V in m^3)

$w = \frac{nR}{\gamma - 1}[T_1 - T_2] = \frac{1R}{3R(1.5-1)}\left[300 - 300\sqrt{2}\right] = \frac{300}{3 \times 0.5}\left(1 - \sqrt{2}\right) = -82.8 \text{ J} \approx -82 \text{ J.}$

(c) Internal Energy,

$dQ = 0$, $\Rightarrow du = -dw = -(-82.8)J = 82.8 \text{ J} \approx 82 \text{ J.}$

(d) Final Temp $= 300\sqrt{2} = 300 \times 1.414 \times 100 = 424.2$ k ≈ 424 k.

(e) The pressure is kept constant. $\therefore$ The process is isobaric.

Work done = nRdT $= \frac{1}{3R} \times R \times (300 - 300\sqrt{2})$ Final Temp = 300 K

$= -\frac{1}{3} \times 300\,(0.414) = -41.4$ J. Initial Temp $= 300\sqrt{2}$

(f) Initial volume $\Rightarrow \frac{V_1}{T_1} = \frac{V_1'}{T_1'} = V_1' = \frac{V_1}{T_1} \times T_1' = \frac{1}{2 \times 300 \times \sqrt{2}} \times 300 = \frac{1}{2\sqrt{2}}$ L.

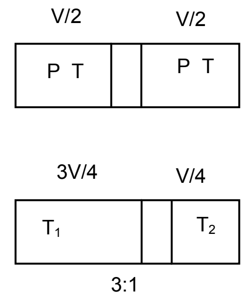
Final volume = 1L

Work done in isothermal $= nRT\ln\frac{V_2}{V_1}$

$= \frac{1}{3R} \times R \times 300 \ln\left(\frac{1}{1/2\sqrt{2}}\right) = 100 \times \ln\left(2\sqrt{2}\right) = 100 \times 1.039 \approx 103$

(g) Net work done $= W_A + W_B + W_C = -82 - 41.4 + 103 = -20.4$ J.

26. Given $\gamma = 1.5$



We know fro adiabatic process $TV^{\gamma-1}$ = Const.

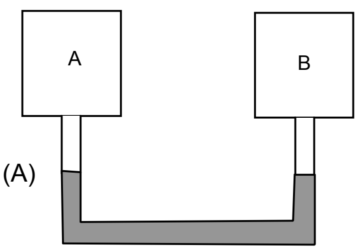
So, $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ …(eq)

As, it is an adiabatic process and all the other conditions are same. Hence the above equation can be applied.

So, $T_1 \times \left(\frac{3V}{4}\right)^{1.5-1} = T_2 \times \left(\frac{V}{4}\right)^{1.5-1} \Rightarrow T_1 \times \left(\frac{3V}{4}\right)^{0.5} = T_2 \times \left(\frac{V}{4}\right)^{0.5}$

$\Rightarrow \frac{T_1}{T_2} = \left(\frac{V}{4}\right)^{0.5} \times \left(\frac{4}{3V}\right)^{0.5} = \frac{1}{\sqrt{3}}$ So, $T_1 : T_2 = 1 : \sqrt{3}$

27. $V = 200\ cm^3$, $C = 12.5$ J/mol-k, $T = 300$ k, $P = 75$ cm



(a) No. of moles of gas in each vessel,

$\frac{PV}{RT} = \frac{75 \times 13.6 \times 980 \times 200}{8.3 \times 10^7 \times 300} = 0.008$

(b) Heat is supplied to the gas but dv = 0

$dQ = du \Rightarrow 5 = nC_V dT \Rightarrow 5 = 0.008 \times 12.5 \times dT \Rightarrow dT = \frac{5}{0.008 \times 12.5}$ for (A)

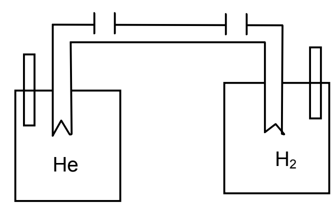
For (B) $dT = \frac{10}{0.008 \times 12.5}$ $\because \frac{P}{T} = \frac{P_A}{T_A}$ [For container A]

$\Rightarrow \frac{75}{300} = \frac{P_A \times 0.008 \times 12.5}{5} \Rightarrow P_A = \frac{75 \times 5}{300 \times 0.008 \times 12.5} = 12.5$ cm of Hg.

$\because \frac{P}{T} = \frac{P_B}{T_B}$ [For Container B] $\Rightarrow \frac{75}{300} = \frac{P_B \times 0.008 \times 12.5}{10} \Rightarrow P_B = 2\ P_A = 25$ cm of Hg.

Mercury moves by a distance $P_B - P_A = 25 - 12.5 = 12.5$ Cm.

28. mHe = 0.1 g, $\gamma = 1.67$, $\mu = 4$ g/mol, mH_2 =?



$\mu = 28$/mol $\gamma_2 = 1.4$

Since it is an adiabatic surrounding

He $dQ = nC_V dT = \frac{0.1}{4} \times \frac{R}{\gamma - 1} \times dT = \frac{0.1}{4} \times \frac{R}{(1.67 - 1)} \times dT$ …(i)

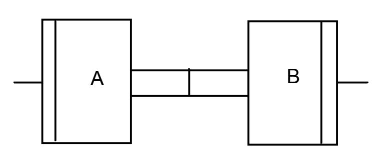
$H_2 = nC_V dT = \frac{m}{2} \times \frac{R}{\gamma - 1} \times dT = \frac{m}{2} \times \frac{R}{1.4 - 1} \times dT$ [Where m is the rqd. Mass of H_2]

Since equal amount of heat is given to both and ΔT is same in both.

Equating (i) & (ii) we get

$\frac{0.1}{4} \times \frac{R}{0.67} \times dT = \frac{m}{2} \times \frac{R}{0.4} \times dT \Rightarrow m = \frac{0.1}{2} \times \frac{0.4}{0.67} = 0.0298 \approx 0.03$ g

29. Initial pressure = P_0, Initial Temperature = T_0



Initial Volume = V_0

$\frac{C_P}{C_V} = \gamma$

(a) For the diathermic vessel the temperature inside remains constant

$P_1 V_1 - P_2 V_2 \Rightarrow P_0 V_0 = P_2 \times 2V_0 \Rightarrow P_2 = \frac{P_0}{2}$, Temperature = T_o

For adiabatic vessel the temperature does not remains constant. The process is adiabatic

$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow T_0 V_0^{\gamma-1} = T_2 \times (2V_0)^{\gamma-1} \Rightarrow T_2 = T_0 \left(\frac{V_0}{2V_0}\right)^{\gamma-1} = T_0 \times \left(\frac{1}{2}\right)^{\gamma-1} = \frac{T_0}{2^{\gamma-1}}$

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \Rightarrow P_0 V_0^{\gamma} = p_1 (2V_0)^{\gamma} \Rightarrow\ P_1 = P_0\left(\frac{V_0}{2V_0}\right)^{\gamma} = \frac{P_0}{2^{\gamma}}$

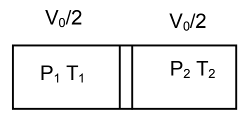
(b) When the values are opened, the temperature remains T_0 through out

$P_1 = \frac{n_1RT_0}{4V_0}$, $P_2 = \frac{n_2RT_0}{4V_0}$ [Total value after the expt = $2V_0 + 2V_0 = 4V_0$]

$P = P_1 + P_2 = \frac{(n_1+n_2)RT_0}{4V_0} = \frac{2nRT_0}{4V_0} = \frac{nRT_0}{2V} = \frac{P_0}{2}$

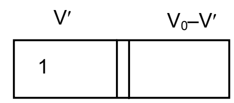
30. For an adiabatic process, Pv^{γ} = Const.

There will be a common pressure 'P' when the equilibrium is reached



Hence $P_1\left(\frac{V_0}{2}\right)^{\gamma} = P(V')^{\gamma}$

For left $P = P_1\left(\frac{V_0}{2}\right)^{\gamma}(V')^{\gamma}$ …(1)



For Right $P = P_2\left(\frac{V_0}{2}\right)^{\gamma}(V_0 - V')^{\gamma}$ …(2)

Equating 'P' for both left & right

$= \frac{P_1}{(V')^{\gamma}} = \frac{P_2}{(V_0 - V')^{\gamma}}$ or $\frac{V_0 - V'}{V'} = \left(\frac{P_2}{P_1}\right)^{1/\gamma}$

$\Rightarrow \frac{V_0}{V'} - 1 = \frac{P_2^{1/\gamma}}{P_1^{1/\gamma}} \Rightarrow \frac{V_0}{V'} = \frac{P_2^{1/\gamma} + P_1^{1/\gamma}}{P_1^{1/\gamma}} \Rightarrow V' = \frac{V_0P_1^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}}$ For left …….(3)

Similarly $V_0 - V' = \frac{V_0P_2^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}}$ For right ……(4)

(b) Since the whole process takes place in adiabatic surroundings. The separator is adiabatic. Hence heat given to the gas in the left part = Zero.

(c) From (1) Final pressure $P = \frac{P_1\left(\frac{V_0}{2}\right)^{y}}{(V')^{\gamma}}$

Again from (3) $V' = \frac{V_0P_1^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}}$ or $P = \frac{P_1\frac{(V_0)^{\gamma}}{2^{\gamma}}}{\left(\frac{V_0P_1^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}}\right)^{\gamma}} = \frac{P_1(V_0)^{\gamma}}{2^{\gamma}} \times \frac{\left(P_1^{1/\gamma} + P_2^{1/\gamma}\right)^{\gamma}}{(V_0)^{\gamma}P_1} = \left(\frac{P_1^{1/\gamma} + P_2^{1/\gamma}}{2}\right)^{\gamma}$

31. $A = 1\ cm^2 = 1 \times 10^{-4}\ m^2$, $M = 0.03\ g = 0.03 \times 10^{-3}$ kg,

$P = 1$ atm $= 10^5$ pascal, $L = 40$ cm $= 0.4$ m.

$L_1 = 80$ cm $= 0.8$ m, $P = 0.355$ atm

The process is adiabatic

$P(V)^{\gamma} = P(V')^{\gamma} = \Rightarrow 1 \times (AL)^{\gamma} = 0.355 \times (A2L)^{\gamma} \Rightarrow 1\ \ \mathit{1 = 0.355}\ \ 2^{\gamma} \Rightarrow \frac{1}{0.355} = 2^{\gamma}$

$= \gamma \log 2 = \log\left(\frac{1}{0.355}\right) = 1.4941$

$V = \sqrt{\frac{\gamma P}{f}} = \sqrt{\frac{1.4941 \times 10^5}{m/v}} = \sqrt{\frac{1.4941 \times 10^5}{\left(\frac{0.03 \times 10^{-3}}{10^{-4} \times 1 \times 0.4}\right)}} = \sqrt{\frac{1.441 \times 10^5 \times 4 \times 10^{-5}}{3 \times 10^{-5}}} = 446.33 \approx 447$ m/s

32. V = 1280 m/s, T = 0°C, $f oH_2$ = 0.089 kg/m^3, rR = 8.3 J/mol-k,
At STP, P = 10^5 Pa, We know

$$V_{sound} = \sqrt{\frac{\gamma P}{f o}} \Rightarrow 1280 = \sqrt{\frac{\gamma \times 10^5}{0.089}} \Rightarrow (1280)^2 = \frac{\gamma \times 10^5}{0.089} \Rightarrow \gamma = \frac{0.089 \times (1280)^2}{10^5} \approx 1.458$$

Again,

$$C_V = \frac{R}{\gamma - 1} = \frac{8.3}{1.458 - 1} = 18.1 \text{ J/mol-k}$$

Again, $\frac{C_P}{C_V} = \gamma$ or $C_P = \gamma C_V = 1.458 \times 18.1 = 26.3$ J/mol-k

33. $\mu = 4g = 4 \times 10^{-3}$ kg, V = 22400 cm^3 = 22400 × 10^{-6} m^3
C_P = 5 cal/mol-ki = 5 × 4.2 J/mol-k = 21 J/mol-k

$$C_P = \frac{\gamma R}{\gamma - 1} = \frac{\gamma \times 8.3}{\gamma - 1}$$

$$\Rightarrow 21(\gamma - 1) = \gamma\,(8.3) \Rightarrow 21\,\gamma - 21 = 8.3\,\gamma \Rightarrow \gamma = \frac{21}{12.7}$$

Since the condition is STP, P = 1 atm = 10^5 pa

$$V = \sqrt{\frac{\gamma f}{f}} = \sqrt{\frac{\frac{21}{12.7} \times 10^5}{\frac{4 \times 10^{-3}}{22400 \times 10^{-6}}}} = \sqrt{\frac{21 \times 10^5 \times 22400 \times 10^{-6}}{12.7 \times 4 \times 10^{-3}}} = 962.28 \text{ m/s}$$

34. Given $f o = 1.7 \times 10^{-3}$ g/cm^3 = 1.7 kg/m^3, P = 1.5 × 10^5 Pa, R = 8.3 J/mol-k,
f = 3.0 KHz.

Node separation in a Kundt′′ tube = $\frac{\lambda}{2}$ = 6 cm, $\Rightarrow \lambda$ = 12 cm = 12 × 10^{-3} m

So, V = $f\lambda$ = 3 × 10^3 × 12 × 10^{-2} = 360 m/s

We know, Speed of sound = $\sqrt{\frac{\gamma P}{f o}} \Rightarrow (360)^2 = \frac{\gamma \times 1.5 \times 10^5}{1.7} \Rightarrow \gamma = \frac{(360)^2 \times 1.7}{1.5 \times 10^5} = 1.4688$

But $C_V = \frac{R}{\gamma - 1} = \frac{8.3}{1.488 - 1} = 17.72$ J/mol-k

Again $\frac{C_P}{C_V} = \gamma$ So, $C_P = \gamma C_V$ = 17.72 × 1.468 = 26.01 ≈ 26 J/mol-k

35. f = 5 × 10^3 Hz, T = 300 Hz, $\frac{\lambda}{2}$ = 3.3 cm $\Rightarrow \lambda$ = 6.6 × 10^{-2} m

V = $f\lambda$ = 5 × 10^3 × 6.6 × 10^{-2} = (66 × 5) m/s

$$V = \frac{\lambda P}{f} \quad \left[Pv = nRT \Rightarrow P = \frac{m}{mV} \times Rt \Rightarrow PM = f oRT \Rightarrow \frac{P}{f o} = \frac{RT}{m}\right]$$

$$= \sqrt{\frac{\gamma RT}{m}} (66 \times 5) = \sqrt{\frac{\gamma \times 8.3 \times 300}{32 \times 10^{-3}}} \Rightarrow (66 \times 5)^2 = \frac{\gamma \times 8.3 \times 300}{32 \times 10^{-3}} \Rightarrow \gamma = \frac{(66 \times 5)^2 \times 32 \times 10^{-3}}{8.3 \times 300} = 1.3995$$

$$C_v = \frac{R}{\gamma - 1} = \frac{8.3}{0.3995} = 20.7 \text{ J/mol-k,}$$

$C_P = C_V + R$ = 20.77 + 8.3 = 29.07 J/mol-k.

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