Exercise – 14.1

1. Three angles of a quadrilateral are respectively equal to 110°, 50° and 40°. Find its fourth angles.

Sol:

Given

Three angles are $110°, 50°$ and $40°$

Let fourth angle be x

We have,

Sum of all angles of a quadrilaterals $= 360°$

$110° + 50° + 40° + x° = 360°$

$\Rightarrow x = 360° - 200°$

$\Rightarrow x = 160°$

Required fourth angle $= 160°$.

2. In a quadrilateral ABCD, the angles A, B, C and D are in the ratio 1 : 2 : 4 : 5. Find the measure of each angles of the quadrilateral.

Sol:

Let the angles of the quadrilateral be

$A = x, B = 2x, C = 4x$ and $D = 5x$ then,

$A + B + C + D = 360°$

$\Rightarrow x + 2x + 4x + 5x = 360°$

$\Rightarrow 12x = 360°$

$$\Rightarrow x = \frac{360°}{12}$$

$\Rightarrow x = 30°$

$\therefore A = x = 30°$

$B = 2x = 60°$

$C = 4x = 30°(4) = 120°$

$D = 5x = 5(30°) = 150°$

3. In a quadrilateral ABCD, CO and DO are the bisectors of ∠C and ∠D respectively. Prove that ∠COD = $\frac{1}{2}(\angle A + \angle B)$.

Sol:

In ΔDOC

$\angle 1 + \angle COD + \angle 2 = 180°$ [Angle sum property of a triangle]

$\Rightarrow \angle COD = 180 - \angle 1 - \angle 2$

$\Rightarrow \angle COD = 180 - \angle 1 + \angle 2$

$\Rightarrow \angle COD = 180 - \left[\frac{1}{2}\angle C + \frac{1}{2}\angle D\right]$

[$\because$ OC and OD are bisectors of $\angle C$ and $\angle D$ represents]

$\Rightarrow \angle COD = 180 - \frac{1}{2}\left(\angle C + \angle D\right)\big]$ .....(1)

In quadrilateral $ABCD$

$\angle A + \angle B + \angle C + \angle D = 360°$

$\angle C + \angle D = 360 - \angle A + \angle B$ ......(2) [Angle sum property of quadrilateral]

Substituting (ii) in (i)

$\Rightarrow \angle COD = 180 - \frac{1}{2}\left(360 - \left(\angle A + \angle B\right)\right)$

$\Rightarrow \angle COD = 180 - 180 + \frac{1}{2}\left(\angle A + \angle B\right)$

$\Rightarrow \angle COD = \frac{1}{2}\left(\angle A + \angle B\right)$

4. The angles of a quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

Sol:

Let the common ratio between the angle is 'x' so the angles will be $3x, 5x, 9x$ and $13x$ respectively

Since the sum of all interior angles of a quadrilateral is $360°$

$\therefore 3x + 5x + 9x + 13x = 360°$

$\Rightarrow 30x = 360°$

$\Rightarrow x = 12°$

Hence, the angles are

$3x = 3 \times 12 = 36°$

$5x = 5 \times 12 = 60°$

$9x = 9 \times 12 = 108°$

$13x = 13 \times 12 = 156°$

Exercise – 14.2

1. Two opposite angles of a parallelogram are (3x – 2)° and (50 – x)°. Find the measure of each angle of the parallelogram.

Sol:

We know that

Opposite sides of a parallelogram are equal

$\therefore 3x-2=50-x$

$\Rightarrow 3x+x=50+2$

$\Rightarrow 4x=52$

$\Rightarrow x=13°$

$\therefore (3x-2)°=(3\times 13-2)=37°$

$(50-x)°=(50-13°)=37°$

Adjacent angles of a parallelogram are supplementary

$\therefore x+37=180°$

$\therefore x=180°-37°=143°$

Hence, four angles are : $37°, 143°, 37°, 143°$

2. If an angle of a parallelogram is two-third of its adjacent angle, find the angles of the parallelogram.

Sol:

Let the measure of the angle be x

$\therefore$ The measure of the angle adjacent is $\frac{2x}{3}$

We know that the adjacent angle of a parallelogram is supplementary

Hence $x+\frac{2x}{3}=180°$

$2x+3x=540°$

$\Rightarrow 5x=540°$

$\Rightarrow x=108°$

Adjacent angles are supplementary

$\Rightarrow x+108°=180°$

$\Rightarrow x=180°-108°=72°$

$\Rightarrow x=72°$

Hence, four angles are : $180°, 72°, 108°, 72°$

3. Find the measure of all the angles of a parallelogram, if one angle is 24° less than twice the smallest angle.

Sol:

Let the smallest angle be x

Then, the other angle is $(3x-24)$

Now, $x+2x-24=180°$

$3x-24=180°$

$\Rightarrow 3x=180+24$

$\Rightarrow 3x=204°$

$\Rightarrow x=\frac{204}{3}=68°$

$\Rightarrow x=68°$

$\Rightarrow 2x-24°=2\times 68°-24°=136°-24°=112°$

Hence four angles are $68°, 112°, 68°, 112°$.

4. The perimeter of a parallelogram is 22 cm. If the longer side measures 6.5 cm what is the measure of the shorter side?

Sol:

Let the shorter side be x

$\therefore$ Perimeter $=x+6.5+6\cdot 5+x$ [sum of all sides]

$22=2(x+6\cdot 5)$

$11=x+6\cdot 5$

$\Rightarrow x=11-6\cdot 5=4\cdot 5\,\text{cm}$

$\therefore$ Shorter side $=4\cdot 5\text{cm}$

5. In a parallelogram ABCD, ∠D = 135°, determine the measures of ∠A and ∠B.

Sol:

In a parallelogram $ABCD$

Adjacent angles are supplementary

So, $\angle D+\angle C=180°$

$135°+\angle C=180° \Rightarrow \angle C=180°-135°$

$\angle C=45°$

In a parallelogram opposite sides are equal

$\angle A=\angle C=45°$

$\angle B=\angle D=135°$

6. ABCD is a parallelogram in which ∠A = 70°. Compute ∠B, ∠C and ∠D.

Sol:

In a parallelogram ABCD.

$\angle A = 70°$

$\angle A = \angle B = 180°$ [$\because$ Adjacent angles supplementary]

$70° + \angle B = 180°$ $[\because \angle A = 70°]$

$\angle B = 180° - 70°$

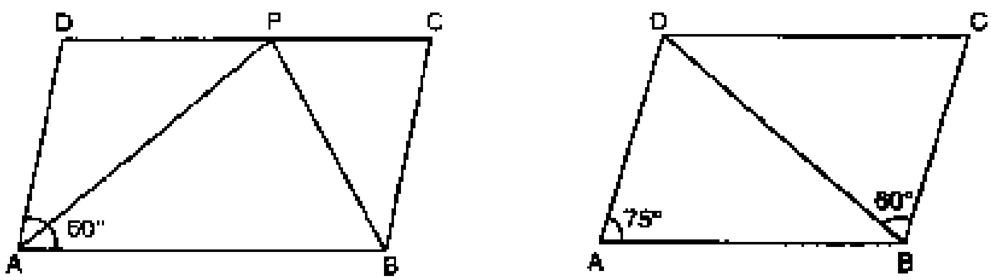
$= 110°$

In a parallelogram opposite sides are equal

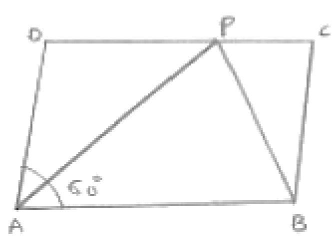
$\angle A = \angle C = 70°$

$\angle B = \angle D = 110°$

7. In Fig., below, ABCD is a parallelogram in which ∠A = 60°. If the bisectors of ∠A and ∠B meet at P, prove that AD = DP, PC = BC and DC = 2AD.



Sol:



AP bisects $\angle A$

Then, $\angle AP = \angle PAB = 30°$

Adjacent angles are supplementary

Then, $\angle A + \angle B = 180°$

$\angle B + 60° = 180°$ $\angle A = 60°$

$\angle B = 180° - 60°$

$\angle B = 120°$

BP bisects $\angle B$

Then, $\angle PBA \quad \angle PBC = 30°$

$\angle PAB = \angle APD = 30°$ [Alternative interior angles]

$\therefore AD = DP$ [$\because$ Sides opposite to equal angles are in equal length]

Similarly

$\angle PBA = \angle BPC = 60°$ [Alternative interior angle]

$\therefore PC = BC$

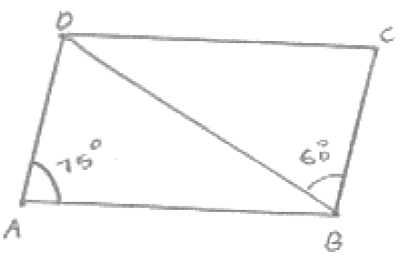
$DC = DP + PC$

$DC = AD + BC$ $[\because DP = AD, PC = BC]$

$DC = 2AD$ [$\because AD = BC$ Opposite sides of a parallelogram are equal].

8. In Fig. below, ABCD is a parallelogram in which ∠DAB = 75° and ∠DBC = 60°. Compute ∠CDB and ∠ADB.

Sol:



To find $\angle CDB$ and $\angle ADB$

$\angle CBD = \angle ABD = 60°$ [Alternative interior angle $AD \parallel BC$ and BD is the transversal]

In a parallelogram $ABCD$

$\angle A = \angle C = 75°$ [$\because$ Opposite side angles of a parallelogram are equal]

In $\angle BDC$

$\angle CBD + \angle C + \angle CDB = 180°$ [Angle sum property]

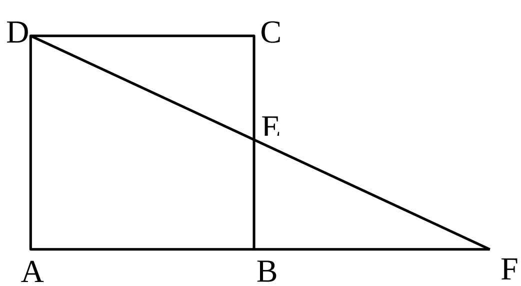
$\Rightarrow 60° + 75° + \angle CDB = 180°$

$\Rightarrow \angle CDB = 180° - (60° + 75°)$

$\Rightarrow \angle CDB = 45°$

Hence $\angle CDB = 45°, \angle ADB = 60°$

9. In below fig. ABCD is a parallelogram and E is the mid-point of side BC. If DE and AB when produced meet at F, prove that AF = 2AB.

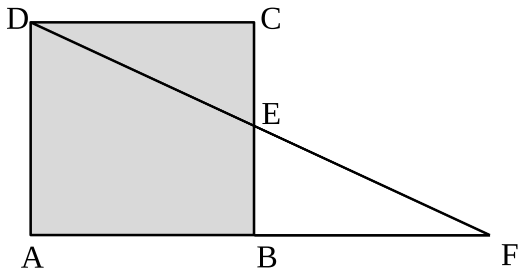


Sol:

In ΔBEF and ΔCED

$\angle BEF = \angle CED$ [Verified opposite angle]

$BE = CE$ [$\because$ E is the mid-point of BC]



$\angle EBF = \angle ECD$ [$\therefore$ Alternate interior angles are equal]

$\because \nabla BEF \cong \Delta CED$ [Angle side angle congruence]

$\because BF = CD$ [Corresponding Parts of Congruent Triangles]

$AF = AB + AF$

$AF = AB + AB$

$AF = 2AB$

10. Which of the following statements are true (T) and which are false (F)?

(i) In a parallelogram, the diagonals are equal.
(ii) In a parallelogram, the diagonals bisect each other.
(iii) In a parallelogram, the diagonals intersect each other at right angles.
(iv) In any quadrilateral, if a pair of opposite sides is equal, it is a parallelogram.
(v) If all the angles of a quadrilateral are equal, it is a parallelogram.
(vi) If three sides of a quadrilateral are equal, it is a parallelogram.
(vii) If three angles of a quadrilateral are equal, it is a parallelogram.
(viii) If all the sides of a quadrilateral are equal it is a parallelogram.

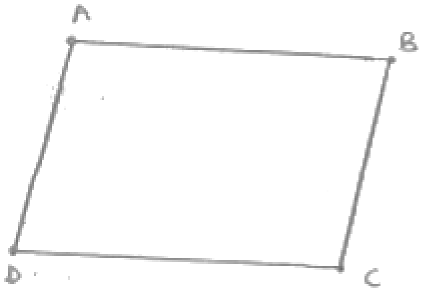
Sol:

(i) False
(ii) True
(iii) False
(iv) False
(v) True
(vi) False
(vii) False
(viii) True

Exercise – 14.3

1. In a parallelogram ABCD, determine the sum of angles ∠C and ∠D.

Sol:



$\angle C$ and $\angle D$ are consecutive interior angles on the same side of the transversal CD

$\therefore \angle C + \angle D = 180°$

2. In a parallelogram ABCD, if ∠B = 135°, determine the measures of its other angles.

Sol:

Given $\angle B = 135°$

$ABCD$ is a parallelogram

$\therefore \angle A = \angle C, \angle B = \angle D$ and $\angle A + \angle B = 180°$

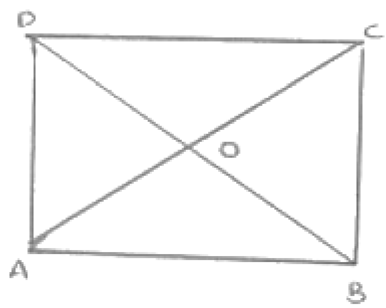
$\angle A + \angle B = 180°$

$\angle A = 45°$

$\Rightarrow \angle A = \angle C = 45°$ and $\angle B = \angle C = 135°$

3. ABCD is a square. AC and BD intersect at O. State the measure of ∠AOB.

Sol:

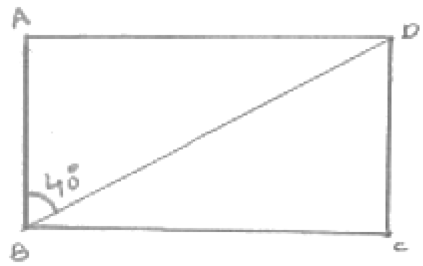


Since, diagonals of square bisect each other at right angle

$\therefore \angle ADB = 90°$

4. ABCD is a rectangle with ∠ABD = 40°. Determine ∠DBC.

Sol:



We have,

$\angle ABC = 90°$

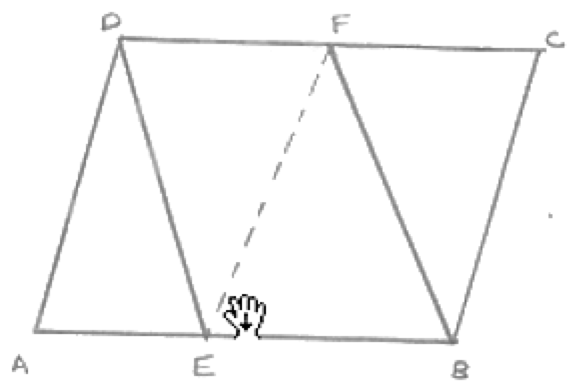
$\Rightarrow \angle ABD + \angle DBC = 90°$ $[\because \angle ABD = 40°]$

$\Rightarrow 40° + \angle DBC = 90°$

$\therefore \angle DBC = 50°$

5. The sides AB and CD of a parallelogram ABCD are bisected at E and F. Prove that EBFD is a parallelogram.

Sol:



Since ABCD is a parallelogram

$\therefore AB \parallel DC$ and $AB = DC$

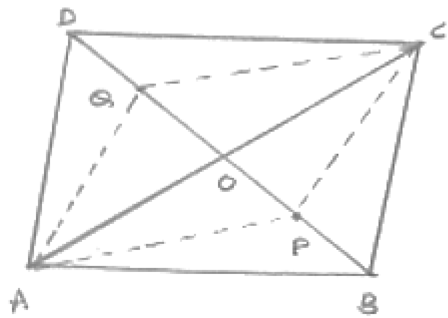
$\Rightarrow EB \parallel DF$ and $\frac{1}{2}AB = \frac{1}{2}DC$

$\Rightarrow EB \parallel DF$ and $EB = DF$

$EBFD$ is a parallelogram

6. P and Q are the points of trisection of the diagonal BD of a parallelogram ABCD. Prove that CQ is parallel to AP. Prove also that AC bisects PQ.

Sol:



We know that, diagonals of a parallelogram bisect each other

$\therefore OA = OC$ and $OB = OD$

Since P and Q are point of intersection of BD

$\therefore BP = PQ = QD$

Now, $OB = OD$ and $BP = QD$

$\Rightarrow OB - BP = OD - QD$

$\Rightarrow OP = OQ$

Thus in quadrilateral APCQ, we have

$OA = OC$ and $OP = OQ$

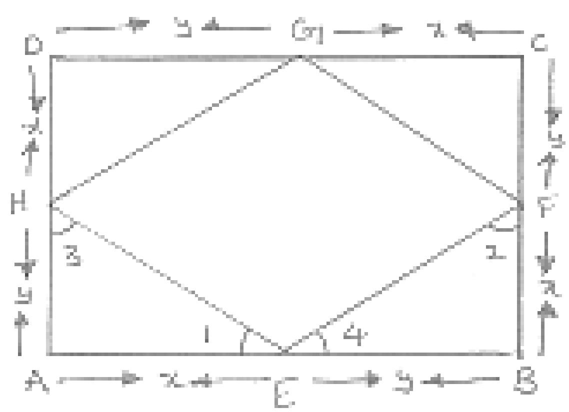
$\Rightarrow$ diagonals of quadrilateral APCQ bisect each other

$\therefore APCQ$ is a parallelogram

Hence $AP \| CQ$

7. ABCD is a square E, F, G and H are points on AB, BC, CD and DA respectively, such that AE = BF = CG = DH. Prove that EFGH is a square.

Sol:



We have

$AE = BF = CG = DH = x(\text{say})$

$\therefore BE = CF = DG = AH = y(\text{say})$

In $\Delta's\ AEH$ and BEF, we have

$AE = BF$

$\angle A = \angle B$

And $AH = BE$

So, by SAS configuration criterion, we have

$\Delta AEH \cong \Delta BFE$

$\Rightarrow \angle 1 = \angle 2$ and $\angle 3 = \angle 4$

But $\angle 1 + \angle 3 = 90°$ and $\angle 2 + \angle 4 = 90°$

$\Rightarrow \angle 1 + \angle 3 + \angle 2 + \angle 4 = 90° + 90°$

$\Rightarrow \angle 1 + \angle 4 + \angle 1 + \angle 4 = 180°$

$\Rightarrow 2(\angle 1 + \angle 4) = 180°$

$\Rightarrow \angle 1 + \angle 4 = 90°$

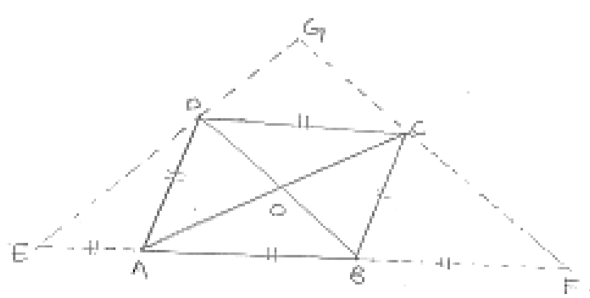
$HEF = 90°$

Similarly we have $\angle F = \angle G = \angle H = 90°$

Hence, $EFGH$ is a square

8. ABCD is a rhombus, EABF is a straight line such that EA = AB = BF. Prove that ED and FC when produced meet at right angles.

Sol:



We know that the diagonals of a rhombus are perpendicular bisector of each other

$\therefore OA = OC, OB = OD, \angle AOD = \angle COD = 90°$

And $\angle AOB = \angle COB = 90°$

In $\Delta BDE, A$ and O are mid points of BE and BD respectively

$OA \parallel DE$

$OC \parallel DG$

In $\Delta CFA, B$ and O are mid points of AF and AC respectively

$\therefore OB \parallel CF$

$OD \parallel GC$

Thus, in quadrilateral $DOCG$, we have

$OC \parallel DG$ and $OD \parallel GC$

$\Rightarrow DOCG$ is a parallelogram

$\angle DGC = \angle DOC$

$\angle DGC = 90°$

9. ABCD is a parallelogram, AD is produced to E so that DE = DC and EC produced meets AB produced in F. Prove that BF = BC.

Sol:

Draw a parallelogram $ABCD$ with AC and BD intersecting at O

Produce AD to E such that $DE = DC$

Join EC and produce it to meet AB produced at F.

In ΔDCE,

$\therefore \angle DCE = \angle DEC$ ....CD [In a triangle, equal sides have equal angles opposite]

$AB \parallel CD$ (Opposite sides of the parallelogram are parallel)

$\therefore AE \parallel CD$ $(AB$ Lies on $AF)$

$AF \parallel CD$ and EF is the transversal.

$\therefore \angle DCE = \angle BFC$ .....(2) [Pair of corresponding angles]

From (1) and (2), we get

$\angle DEC = \angle BFC$

In ΔAFE,

$\angle AFE = \angle AEF$ $(\angle DEC = \angle BFC)$

$\therefore AE = AF$ (In a triangle, equal angles have equal sides opposite to them)

$\Rightarrow AD + DE = AB + BF$

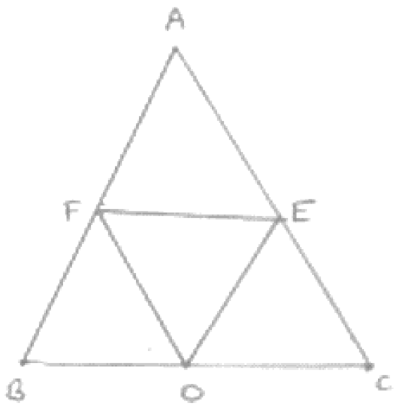
$\Rightarrow BC + AB = AB + BF$ $[\because AD = BC, DE = CD$ and $CD = AB, AB = DE]$

$\Rightarrow BC = BF$.

Exercise – 14.4

1. In a ΔABC, D, E and F are, respectively, the mid-points of BC, CA and AB. If the lengths of side AB, BC and CA are 7 cm, 8 cm and 9 cm, respectively, find the perimeter of ΔDEF.

Sol:



Given that

$AB = 7cm, BC = 8cm, AC = 9cm$.

In ΔABC

$\therefore F$ and E are the midpoint of AB and AC

$\therefore EF = \frac{1}{2} BC$ [Mid-points theorem]

Similarly

$DF = \frac{1}{2} AC, DE = \frac{1}{2} AB$

Perimeter of $\Delta DEF = DE + EF + DF$

$= \frac{1}{2} AB + \frac{1}{2} BC \frac{1}{2} AC$

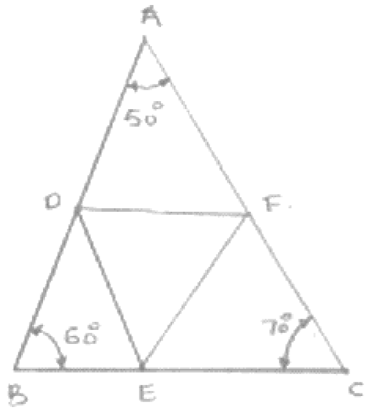
$= \frac{1}{2} \times 7 + \frac{1}{2} \times 8 + \frac{1}{2} \times 9$

$= 3 \cdot 5 + 4 + 4 \cdot 5 = 12cm$

$\therefore$ Perimeter of $\Delta DEF = 12cm$

2. In a triangle ∠ABC, ∠A = 50°, ∠B = 60° and ∠C = 70°. Find the measures of the angles of the triangle formed by joining the mid-points of the sides of this triangle.

Sol:



In ΔABC

D and E are midpoints of AB and BC

By midpoint theorem

$\therefore DE \parallel AC, DE = \frac{1}{2} AC.$

F is the midpoint of AC

Then, $DE = \frac{1}{2} AC = CF$

In a quadrilateral DECF

$DE \parallel AC, DE = CF$

Hence $DECF$ is a parallelogram

$\therefore \angle C = \angle D = 70°$ [Opposite sides of parallelogram]

Similarly

$BEFD$ is a parallelogram, $\angle B = \angle F = 60°$

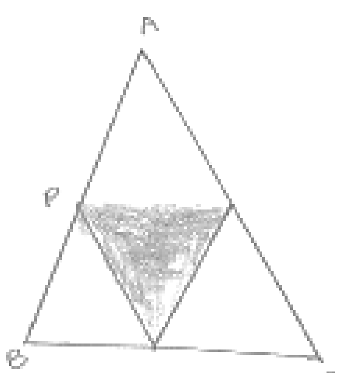
$ADEF$ is a parallelogram, $\angle A = \angle E = 50°$

$\therefore$ Angles of ΔDEF

$\angle D = 70°, \angle E = 50°, \angle F = 60°$

3. In a triangle, P, Q and R are the mid-points of sides BC, CA and AB respectively. If AC = 21 cm, BC = 29 cm and AB = 30 cm, find the perimeter of the quadrilateral ARPQ.

Sol:



In ΔABC

R and P are the midpoint of AB and BC

$\therefore RP \parallel AC, RP = \frac{1}{2}AC$ [By midpoint theorem]

In a quadrilateral

[A pair of side is parallel and equal]

$RP \parallel AQ, RP = AQ$

$\therefore RPQA$ is a parallelogram

$AR = \frac{1}{2}AB = \frac{1}{2} \times 30 = 15cm$

$AR = QP = 15$ [$\because$ Opposite sides are equal]

$\Rightarrow RP = \frac{1}{2}AC = \frac{1}{2} \times 21 = 10 \cdot 5cm$ [$\because$ Opposite sides are equal]

Now,

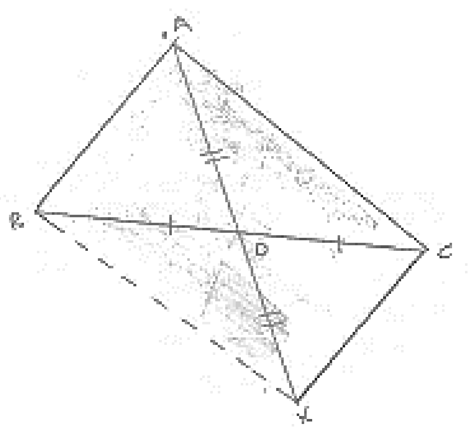
Perimeter of $ARPQ = AR + QP + RP + AQ$

$= 15 + 15 + 10 \cdot 5 + 10 \cdot 5$

$= 51cm$

4. In a ΔABC median AD is produced to X such that AD = DX. Prove that ABXC is a parallelogram.

Sol:



In a quadrilateral $ABXC$, we have

$AD = DX$ $[Given]$

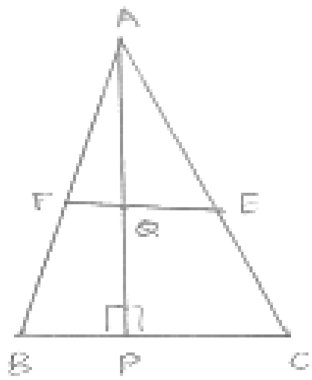
$BD = DC$ $[Given]$

So, diagonals AX and BC bisect each other

$\therefore ABXC$ is a parallelogram

5. In a ΔABC, E and F are the mid-points of AC and AB respectively. The altitude AP to BC intersects FE at Q. Prove that AQ = QP.

Sol:



In ΔABC

E and F are midpoints of AB and AC

$\therefore EF \parallel FE, \frac{1}{2}BC = FE$ [$\because$ By mid-point theorem]

In ΔABP

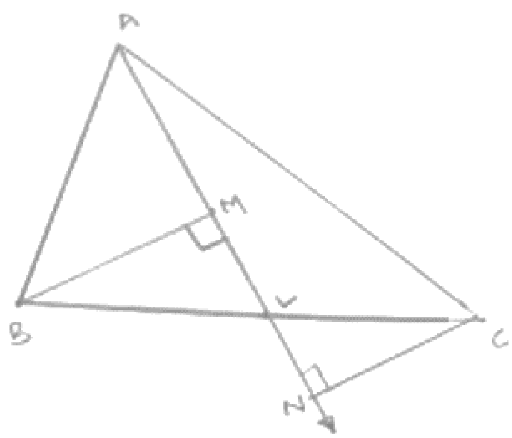
F is the midpoint of AB and $FQ \parallel BP$ $[\because EF \parallel BC]$

$\therefore Q$ is the midpoint of AP [By converse of midpoint theorem]

Hence, $AQ = QP$

6. In a ΔABC, BM and CN are perpendiculars from B and C respectively on any line passing through A. If L is the mid-point of BC, prove that ML = NL.

Sol:



In B

Given that

In ΔBLM and ΔCLN

$\angle BML = \angle CNL = 90°$

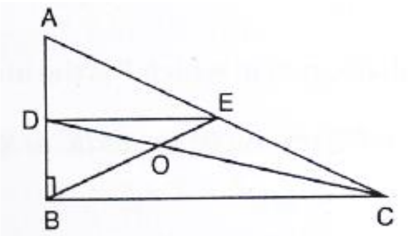
$BL = CL$ [L is the midpoint of BC]

$\angle MLB = \angle NLC$ [vertically opposite angle]

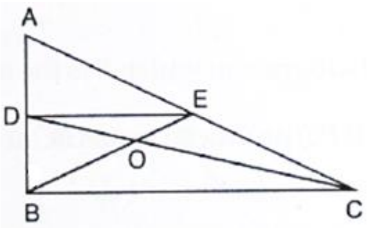
$\therefore \Delta BLM = \Delta CLN$ $(A \cdot L \cdot A \cdot S)$

$\therefore LM = LN$ [Corresponding plats parts of congruent triangles]

7. In Fig. below, triangle ABC is right-angled at B. Given that AB = 9 cm, AC = 15 cm and D, E are the mid-points of the sides AB and AC respectively, calculate
(i) The length of BC (ii) The area of ∆ADE.



Sol:



In right $\Delta ABC, \angle B = 90°$

By using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad 15^2 = 9^2 + BC^2$$

$$\Rightarrow \quad BC = \sqrt{15^2 - 9^2}$$

$$\Rightarrow \quad BC = \sqrt{225 - 81}$$

$$\Rightarrow \quad BC = \sqrt{144}$$

$$= 12cm$$

In ΔABC

D and E are midpoints of AB and AC

$\therefore DE \parallel BC, DE = \frac{1}{2}BC$ [By midpoint theorem]

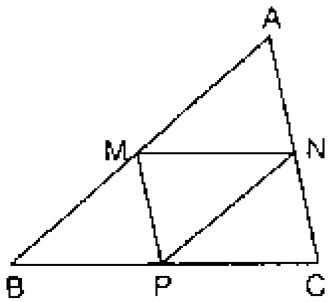
$AD = OB = \frac{AB}{2} = \frac{9}{2} = 4 \cdot 5cm$ [$\because D$ is the midpoint of AB]

$DE = \frac{BC}{2} = \frac{12}{2} = 6cm$

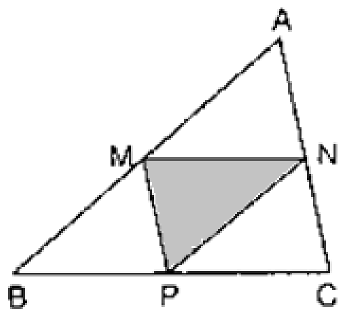
Area of $\Delta ADE = \frac{1}{2} \times AD \times DE$

$= \frac{1}{2} \times 4 \cdot 5 \times 6 = 13 \cdot 5 cm^2$

8. In Fig. below, M, N and P are the mid-points of AB, AC and BC respectively. If MN = 3 cm, NP = 3.5 cm and MP = 2.5 cm, calculate BC, AB and AC.



Sol:



Given $MN = 3cm, NP = 3 \cdot 5cm$ *and* $MP = 2 \cdot 5cm$

To find BC, AB and AC

In ΔABC

M and N are midpoints of AB and AC

$\therefore MN = \frac{1}{2} BC, MN \parallel BC$ [By midpoint theorem]

$\Rightarrow 3 = \frac{1}{2} BC$

$\Rightarrow 3 \times 2 = BC$

$\Rightarrow BC = 6cm$

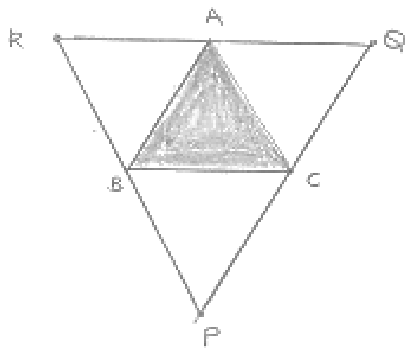
Similarly

$AC = 2MP = 2(2 \cdot 5) = 5cm$

$AB = 2NP = 2(3 \cdot 5) = 7cm$

9. ABC is a triangle and through A, B, C lines are drawn parallel to BC, CA and AB respectively intersecting at P, Q and R. Prove that the perimeter of ΔPQR is double the perimeter of ΔABC.

Sol:



Clearly ABCQ and ARBC are parallelograms.

$\therefore BC = AQ \text{ and } BC = AR$

$\Rightarrow AQ = AR$

$\Rightarrow A$ is the midpoint of QR.

Similarly B and C are the midpoints of PR and PQ respectively

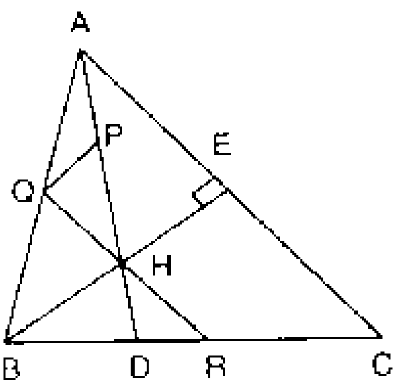
$\therefore AB = \frac{1}{2} PQ, BC = \frac{1}{2} QR, CA = \frac{1}{2} PR$

$\Rightarrow PQ = 2AB, QR = 2BC$ and $PR = 2CA$

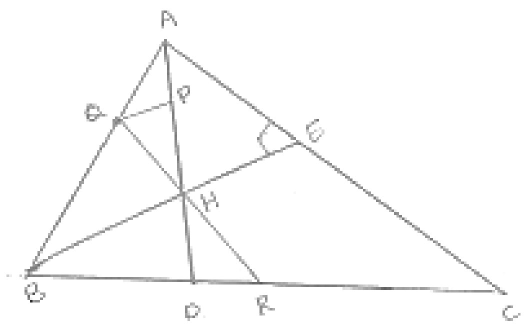
$\Rightarrow PQ + QR + RP = 2(AB + BC + CA)$

$\Rightarrow$ Perimeter of $\Delta PQR = 2$ [Perimeter of ΔABC]

10. In Fig. below, BE ⊥ AC. AD is any line from A to BC intersecting BE in H. P, Q and R are respectively the mid-points of AH, AB and BC. Prove that ∠PQR = 90°.



Sol:



Given

$BE \perp AC$ and P, Q and R are respectively midpoint of AH, AB and BC

To prove:

$\angle PQRD = 90°$

Proof: In $\Delta ABC, Q$ and R are midpoints of AB and BC respectively

$\therefore QR \parallel AC$ ......(i)

In $\Delta ABH, Q$ and P are the midpoints of AB and AH respectively

$\therefore QP \parallel BH$

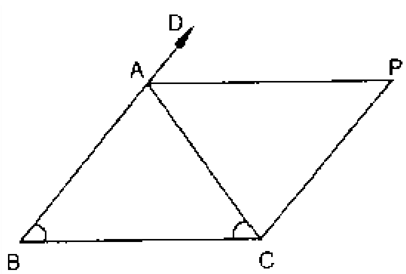
$\Rightarrow QP \parallel BE$ ......(ii)

But, $AC \perp BE \therefore$ from equation (i) and equation (ii) we have

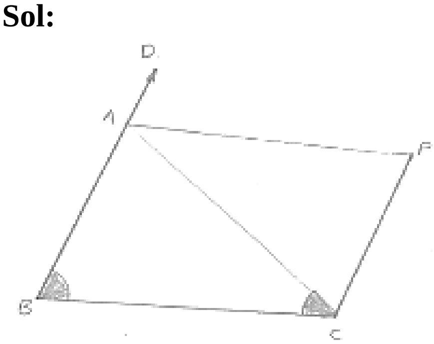
$QP \perp QR$

$\Rightarrow \angle PQR = 90°,$ hence proved.

11. In Fig. below, AB = AC and CP || BA and AP is the bisector of exterior ∠CAD of ΔABC. Prove that (i) ∠PAC = ∠BCA (ii) ABCP is a parallelogram.



Sol:



Given

$AB = AC$ and $CD \parallel BA$ and AP is the bisector of exterior

$\angle CAD$ of ΔABC

To prove:

(i) $\angle PAC = \angle BCA$

(ii) $ABCD$ is a parallelogram

Proof:

(i) We have,

AB = AC

$\Rightarrow \angle ACB = \angle ABC$ [Opposite angles of equal sides of triangle are equal]

Now, $\angle CAD = \angle ABC + \angle ACB$

$\Rightarrow \angle PAC + \angle PAD = 2\angle ACB$ $(\because \angle PAC = \angle PAD)$

$\Rightarrow 2\angle PAC = 2\angle ACB$

$\Rightarrow \angle PAC = \angle ACB$

(ii) Now,

$\angle PAC = \angle BCA$

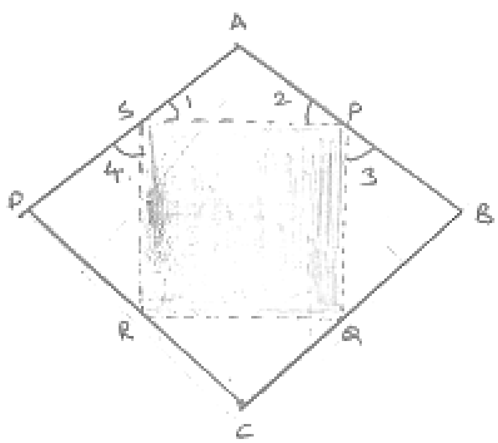
$\Rightarrow AP \parallel BC$

And, $CP \parallel BA$ [Given]

$\therefore ABCD$ is a parallelogram

12. ABCD is a kite having AB = AD and BC = CD. Prove that the figure formed by joining the mid-points of the sides, in order, is a rectangle.

Sol:



Given,

A kite $ABCD$ having $AB = AD$ and $BC = CD \cdot P, Q, R, S$ are the midpoint of sides AB, BC, CD and DA respectively PQ, QR, RS and spare joined

To prove:

$PQRS$ is a rectangle

Proof:

In $\Delta ABC, P$ and Q are the midpoints of AB and BC respectively.

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ ....(i)

In $\Delta ADC, R\ and\ S$ are the midpoint of CD and AD respectively.

$\therefore RS \parallel AC$ and $RS = \frac{1}{2}AC$ .....(ii)

From (i) and (ii), we have

$PQ \parallel RS$ and $PQ = RS$

Thus, in quadrilateral PQRS, a pair of opposite sides are equal and parallel. So PQRS is a parallelogram. Now, we shall prove that one angle of parallelogram PQRS it is a right angle

Since $AB = AD$

$\Rightarrow \frac{1}{2}AB = AD\left(\frac{1}{2}\right)$

$\Rightarrow AP = AS$ ...(iii) [$\because P$ and S are the midpoints of B and AD respectively]

$\Rightarrow \angle 1 = \angle 2$ ....(iv)

Now, in ΔPBQ and $\Delta SDR,$ we have

$PB = SD$ [$\because AD = AB \Rightarrow \frac{1}{2}AD = \frac{1}{2}AB$]

$BQ = DR$ $\therefore PB = SD$

And $PQ = SR$ [$\because PQRS$ is a parallelogram]

So by SSS criterion of congruence, we have

$\Delta PBQ \cong \Delta SOR$

$\Rightarrow \angle 3 = \angle 4$ $[CPCT]$

Now, $\angle 3 + \angle SPQ + \angle 2 = 180°$

And $\angle 1 + \angle PSR + \angle 4 = 180°$

$\therefore \angle 3 + \angle SPQ + \angle 2 = \angle 1 + \angle PSR + \angle 4$

$\Rightarrow \angle SPQ = \angle PSR$ $(\angle 1 = \angle 2$ and $\angle 3 = \angle 4)$

Now, transversal PS cuts parallel lines SR and PQ at S and P respectively. $\therefore \angle SPQ + \angle PSR = 180°$

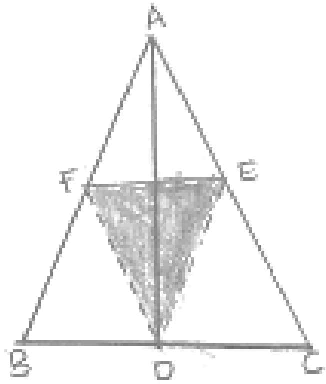
$\Rightarrow 2\angle SPQ = 180° = \angle SPQ = 90°$ $[\because \angle PSR = \angle SPQ]$

Thus, PQRS is a parallelogram such that $\angle SPQ = 90°$

Hence, PQRS is a parallelogram.

13. Let ABC be an isosceles triangle in which AB = AC. If D, E, F be the mid-points of the sides BC, CA and A B respectively, show that the segment AD and EF bisect each other at right angles.

Sol:



Since D, E and F are the midpoints of sides

BC, CA and AB respectively

$\therefore AB \parallel DF$ and $AC \parallel FD$

$AB \parallel DF$ and $AC \parallel FD$

$ABDF$ is a parallelogram

$AF = DE$ and $AE = DF$

$\frac{1}{2}AB = DE$ and $\frac{1}{2}AC = DF$

$DE = DF \qquad (\because AB = AC)$

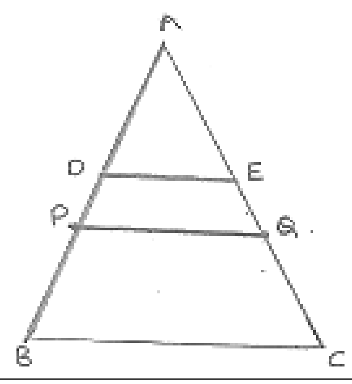
$AE = AF = DE = DF$

$ABDF$ is a rhombus

$\Rightarrow AD$ and FE bisect each other at right angle.

14. ABC is a triangle. D is a point on AB such that AD = $\frac{1}{4}$ AB and E is a point on AC such that AE = $\frac{1}{4}$ AC. Prove that DE = $\frac{1}{4}$ BC.

Sol:



Let P and Q be the midpoints of AB and AC respectively.
Then $PQ \parallel BC$ such that

$PQ = \frac{1}{2}BC \qquad ......(i)$

In ΔAPQ, D and E are the midpoint of AP and AQ are respectively

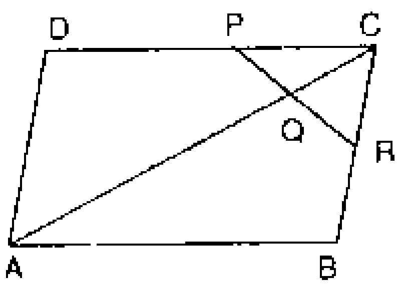
$\therefore DE \parallel PQ$ and $DE = \frac{1}{2}PQ \qquad ....(ii)$

From (1) and (2) $DE = \frac{1}{2}PQ = \frac{1}{2}PQ = \frac{1}{2}\left(\frac{1}{2}BC\right) \qquad ....$

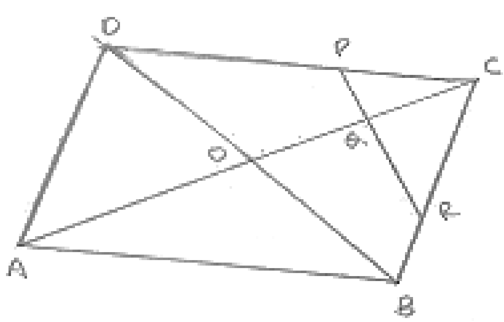
$DE = \frac{1}{4}BC$

Hence proved.

15. In below Fig, ABCD is a parallelogram in which P is the mid-point of DC and Q is a point on AC such that CQ = $\frac{1}{4}$ AC. If PQ produced meets BC at R, prove that R is a mid-point of BC.



Sol:



Join B and D, suppose AC and BD out at O.

Then $OC = \frac{1}{2}AC$

Now,

$CQ = \frac{1}{4}AC$

$$\Rightarrow CQ = \frac{1}{2}\left[\frac{1}{2}AC\right]$$

$$= \frac{1}{2} \times OC$$

In $\Delta DCO, P$ and Q are midpoints of DC and OC respectively

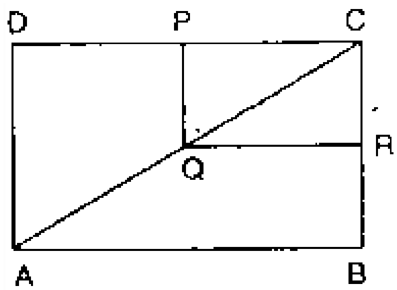
$\therefore PQ \parallel PO$

Also in $\Delta COB, Q$ is the midpoint of OC and $QR \parallel OB$

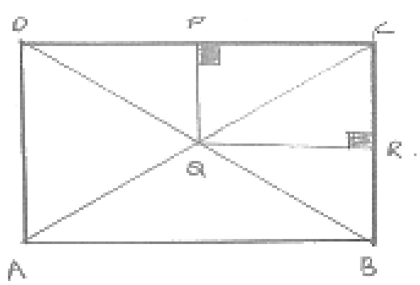
$\therefore R$ is the midpoint of BC

16. In the below Fig, ABCD and PQRC are rectangles and Q is the mid-point of AC. Prove that

(i) DP = PC (ii) PR = $\frac{1}{2}$ AC



Sol:



(i) In $\Delta ADC, Q$ is the midpoint of AC such that

$PQ \parallel AD$

$\therefore P$ is the midpoint of DC

$\Rightarrow DP = DC$ [Using converse of midpoint theorem]

(ii) Similarly, R is the midpoint of BC

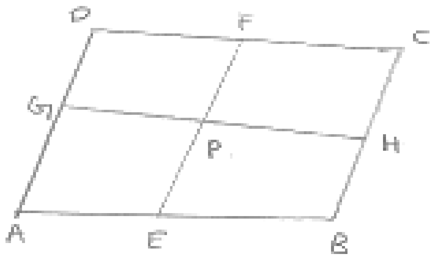
$$\therefore PR = \frac{1}{2}BD$$

[Diagonal of rectangle are equal $\therefore BD = AC$]

$$PR = \frac{1}{2}AC$$

17. ABCD is a parallelogram, E and F are the mid-points of AB and CD respectively. GH is any line intersecting AD, EF and BC at G, P and H respectively. Prove that GP = PH.

Sol:



Since E and F are midpoints of AB and CD respectively

$\therefore AE = BE = \frac{1}{2}AB$

And $CF = DF = \frac{1}{2}CD$

But, $AB = CD$

$\therefore \frac{1}{2}AB = \frac{1}{2}CD$

$\Rightarrow BE = CF$

Also, $BE \parallel CF$ $\quad [\because AB \parallel CD]$

$\therefore BEFC$ is a parallelogram

$\Rightarrow BC \parallel EF$ and $BF = PH$ $\quad ....(i)$

Now, $BC \parallel EF$

$\Rightarrow AD \parallel EF$ $\quad [\because BC \parallel AD$ as $ABCD$ is a parallel]

$\Rightarrow AEFD$ is parallelogram

$\Rightarrow AE = GP$

But is the midpoint of AB

$\therefore AE = BE$

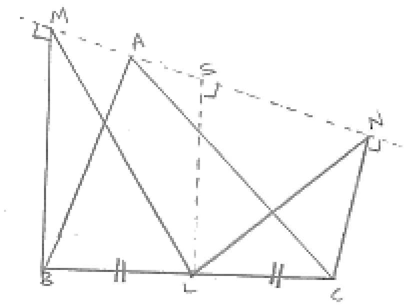
$\Rightarrow GP = PH$

18. BM and CN are perpendiculars to a line passing through the vertex A of a triangle ABC. If L is the mid-point of BC, prove that LM = LN.

Sol:

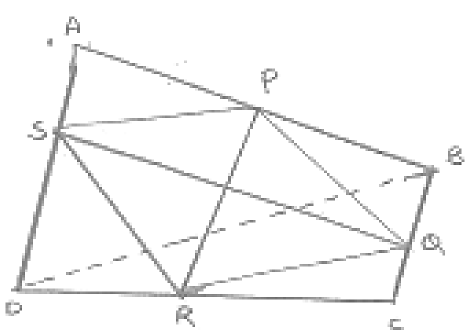
To prove $LM = LN$

Draw LS perpendicular to line MN



$\therefore$ The lines BM, LS and CN being the same perpendiculars, on line MN are parallel to each other.
According to intercept theorem,
If there are three or more parallel lines and the intercepts made by them on a transversal or equal. Then the corresponding intercepts on any other transversal are also equal.
In the drawn figure, MB and LS and NC are three parallel lines and the two transversal line are MN and BC
We have, $BL = LC$ (As L is the given midpoint of BC)
$\therefore$ using intercept theorem, we get
$MS = SN \quad ....(i)$
Now in ΔMLS *and* LSN
$MS = SN$ using ….(i)
$\angle LSM = \angle LSN = 90° LS \perp MN$ and $SL = LS$ common
$\therefore \Delta MLS \cong \Delta LSN$ (SAS congruency theorem)
$\therefore LM = LN \quad (CPCT)$

19. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.
Sol:



Let ABCD is a quadrilateral in which P, Q, R and S are midpoints of sides AB, BC, CD and DA respectively join PQ, QR, RS, SP and BD
In ΔABD, S and P are the midpoints of AD and AB respectively.
So, by using midpoint theorem we can say that

$SP \parallel BD \text{ and } SP = \frac{1}{2}BD \qquad ......(1)$

Similarly in ΔBCD

$QR \parallel BD$ and $QR = \frac{1}{2}BD \qquad .....(2)$

From equation (1) and (2) we have

$SP \parallel QR$ and $SP = QR$

As in quadrilateral SPQR one pair of opposite side are equal and parallel to each other.

So, SPQR is parallelogram

Since, diagonals of a parallelogram bisect each other.

Hence PR and QS bisect each other.

20. Fill in the blanks to make the following statements correct:

(i) The triangle formed by joining the mid-points of the sides of an isosceles triangle is ____

(ii) The triangle formed by joining the mid-points of the sides of a right triangle is _______

(iii) The figure formed by joining the mid-points of consecutive sides of a quadrilateral is ______

Sol:

(i) Isosceles

(ii) Right triangle

(iii) Parallelogram