
Exercise – 2A

1. Find the zeros of the polynomial $f(x) = x^2 + 7x + 12$ and verify the relation between its zeroes and coefficients.

Sol:

$$x^2 + 7x + 12 = 0$$

$$\Rightarrow x^2 + 4x + 3x + 12 = 0$$

$$\Rightarrow x(x+4) + 3(x+4) = 0$$

$$\Rightarrow (x+4)(x+3) = 0$$

$$\Rightarrow (x+4) = 0 \text{ or } (x+3) = 0$$

$$\Rightarrow x = -4 \text{ or } x = -3$$

$$\text{Sum of zeroes} = -4 + (-3) = \frac{-7}{1} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = (-4)(-3) = \frac{12}{1} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

2. Find the zeroes of the polynomial $f(x) = x^2 - 2x - 8$ and verify the relation between its zeroes and coefficients.

Sol:

$$x^2 - 2x - 8 = 0$$

$$\Rightarrow x^2 - 4x + 2x - 8 = 0$$

$$\Rightarrow x(x-4) + 2(x-4) = 0$$

$$\Rightarrow (x-4)(x+2) = 0$$

$$\Rightarrow (x-4) = 0 \text{ or } (x+2) = 0$$

$$\Rightarrow x = 4 \text{ or } x = -2$$

$$\text{Sum of zeroes} = 4 + (-2) = 2 = \frac{2}{1} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = (4)(-2) = \frac{-8}{1} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

3. Find the zeroes of the quadratic polynomial $f(x) = x^2 + 3x - 10$ and verify the relation between its zeroes and coefficients.

Sol:

We have:

$$f(x) = x^2 + 3x - 10$$

$$= x^2 + 5x - 2x - 10$$

$$= x(x+5) - 2(x+5)$$

$$= (x-2)(x+5)$$

$$\therefore f(x) = 0 \Rightarrow (x-2)(x+5) = 0$$

$$\Rightarrow x-2 = 0 \text{ or } x+5 = 0$$

$$\Rightarrow x = 2 \text{ or } x = -5.$$

So, the zeroes of $f(x)$ are 2 and -5.

$$\text{Sum of zeroes} = 2 + (-5) = -3 = \frac{-3}{1} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = 2 \times (-5) = -10 = \frac{-10}{1} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

4. Find the zeroes of the quadratic polynomial $f(x) = 4x^2 - 4x - 3$ and verify the relation between its zeroes and coefficients.

Sol:

We have:

$$\begin{aligned} f(x) &= 4x^2 - 4x - 3 \\ &= 4x^2 - (6x - 2x) - 3 \\ &= 4x^2 - 6x + 2x - 3 \\ &= 2x(2x - 3) + 1(2x - 3) \\ &= (2x + 1)(2x - 3) \\ \therefore f(x) = 0 &\Rightarrow (2x + 1)(2x - 3) = 0 \\ &\Rightarrow 2x + 1 = 0 \text{ or } 2x - 3 = 0 \\ &\Rightarrow x = \frac{-1}{2} \text{ or } x = \frac{3}{2} \end{aligned}$$

So, the zeroes of $f(x)$ are $\frac{-1}{2}$ and $\frac{3}{2}$.

$$\text{Sum of zeroes} = \left(\frac{-1}{2}\right) + \left(\frac{3}{2}\right) = \frac{-1+3}{2} = \frac{2}{2} = 1 = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = \left(\frac{-1}{2}\right) \times \left(\frac{3}{2}\right) = \frac{-3}{4} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

5. Find the zeroes of the quadratic polynomial $f(x) = 5x^2 - 4 - 8x$ and verify the relationship between the zeroes and coefficients of the given polynomial.

Sol:

We have:

$$\begin{aligned} f(x) &= 5x^2 - 4 - 8x \\ &= 5x^2 - 8x - 4 \\ &= 5x^2 - (10x - 2x) - 4 \\ &= 5x^2 - 10x + 2x - 4 \\ &= 5x(x - 2) + 2(x - 2) \\ &= (5x + 2)(x - 2) \\ \therefore f(x) = 0 &\Rightarrow (5x + 2)(x - 2) = 0 \\ &\Rightarrow 5x + 2 = 0 \text{ or } x - 2 = 0 \\ &\Rightarrow x = \frac{-2}{5} \text{ or } x = 2 \end{aligned}$$

So, the zeroes of $f(x)$ are $\frac{-2}{5}$ and 2.

$$\text{Sum of zeroes} = \left(\frac{-2}{5}\right) + 2 = \frac{-2+10}{5} = \frac{8}{5} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = \left(\frac{-2}{5}\right) \times 2 = \frac{-4}{5} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

6. Find the zeroes of the polynomial $f(x) = 2\sqrt{3}x^2 - 5x + \sqrt{3}$ and verify the relation between its zeroes and coefficients.

Sol:

$$2\sqrt{3}x^2 - 5x + \sqrt{3}$$

$$\Rightarrow 2\sqrt{3}x^2 - 2x - 3x + \sqrt{3}$$

$$\Rightarrow 2x(\sqrt{3}x - 1) - \sqrt{3}(\sqrt{3}x - 1) = 0$$

$$\Rightarrow (\sqrt{3}x - 1) \text{ or } (2x - \sqrt{3}) = 0$$

$$\Rightarrow (\sqrt{3}x - 1) = 0 \text{ or } (2x - \sqrt{3}) = 0$$

$$\Rightarrow x = \frac{1}{\sqrt{3}} \text{ or } x = \frac{\sqrt{3}}{2}$$

$$\Rightarrow x = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} \text{ or } x = \frac{\sqrt{3}}{2}$$

$$\text{Sum of zeroes} = \frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{6} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = \frac{\sqrt{3}}{3} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{6} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

7. Find the zeroes of the quadratic polynomial $2x^2 - 11x + 15$ and verify the relation between the zeroes and the coefficients.

Sol:

$$f(x) = 2x^2 - 11x + 15$$

$$= 2x^2 - (6x + 5x) + 15$$

$$= 2x^2 - 6x - 5x + 15$$

$$= 2x(x - 3) - 5(x - 3)$$

$$= (2x - 5)(x - 3)$$

$$\therefore f(x) = 0 \Rightarrow (2x - 5)(x - 3) = 0$$

$$\Rightarrow 2x - 5 = 0 \text{ or } x - 3 = 0$$

$$\Rightarrow x = \frac{5}{2} \text{ or } x = 3$$

So, the zeroes of $f(x)$ are $\frac{5}{2}$ and 3.

$$\text{Sum of zeroes} = \frac{5}{2} + 3 = \frac{5+6}{2} = \frac{11}{2} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = \frac{5}{2} \times 3 = \frac{-15}{2} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

8. Find the zeroes of the quadratic polynomial $4x^2 - 4x + 1$ and verify the relation between the zeroes and the coefficients.

Sol:

$$4x^2 - 4x + 1 = 0$$

$$\Rightarrow (2x)^2 - 2(2x)(1) + (1)^2 = 0$$

$$\Rightarrow (2x - 1)^2 = 0 \quad [\because a^2 - 2ab + b^2 = (a-b)^2]$$

$$\Rightarrow (2x - 1)^2 = 0$$

$$\Rightarrow x = \frac{1}{2} \text{ or } x = \frac{1}{2}$$

$$\text{Sum of zeroes} = \frac{1}{2} + \frac{1}{2} = 1 = \frac{1}{1} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

9. Find the zeroes of the quadratic polynomial $(x^2 - 5)$ and verify the relation between the zeroes and the coefficients.

Sol:

We have:

$$f(x) = x^2 - 5$$

It can be written as $x^2 + 0x - 5$.

$$= (x^2 - (\sqrt{5})^2)$$

$$= (x + \sqrt{5})(x - \sqrt{5})$$

$$\therefore f(x) = 0 \Rightarrow (x + \sqrt{5})(x - \sqrt{5}) = 0$$

$$\Rightarrow x + \sqrt{5} = 0 \text{ or } x - \sqrt{5} = 0$$

$$\Rightarrow x = -\sqrt{5} \text{ or } x = \sqrt{5}$$

So, the zeroes of $f(x)$ are $-\sqrt{5}$ and $\sqrt{5}$.

Here, the coefficient of x is 0 and the coefficient of x^2 is 1.

$$\text{Sum of zeroes} = -\sqrt{5} + \sqrt{5} = \frac{0}{1} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = -\sqrt{5} \times \sqrt{5} = \frac{-5}{1} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

10. Find the zeroes of the quadratic polynomial $(8x^2 - 4)$ and verify the relation between the zeroes and the coefficients.

Sol:

We have:

$$f(x) = 8x^2 - 4$$

It can be written as $8x^2 + 0x - 4$

$$= 4 \{ (\sqrt{2}x)^2 - (1)^2 \}$$

$$= 4 (\sqrt{2}x + 1)(\sqrt{2}x - 1)$$

$$\therefore f(x) = 0 \Rightarrow (\sqrt{2}x + 1)(\sqrt{2}x - 1) = 0$$

$$\Rightarrow (\sqrt{2}x + 1) = 0 \text{ or } \sqrt{2}x - 1 = 0$$

$$\Rightarrow x = \frac{-1}{\sqrt{2}} \text{ or } x = \frac{1}{\sqrt{2}}$$

So, the zeroes of $f(x)$ are $\frac{-1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$

Here the coefficient of x is 0 and the coefficient of x^2 is $\sqrt{2}$

$$\text{Sum of zeroes} = \frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{-1+1}{\sqrt{2}} = \frac{0}{\sqrt{2}} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = \frac{-1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{-1 \times 1}{2 \times 1} = \frac{-1}{2} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

- 11.** Find the zeroes of the quadratic polynomial $(5y^2 + 10y)$ and verify the relation between the zeroes and the coefficients.

Sol:

We have,

$$f(u) = 5u^2 + 10u$$

It can be written as $5u(u+2)$

$$\therefore f(u) = 0 \Rightarrow 5u = 0 \text{ or } u + 2 = 0$$

$$\Rightarrow u = 0 \text{ or } u = -2$$

So, the zeroes of $f(u)$ are -2 and 0 .

$$\text{Sum of the zeroes} = -2 + 0 = -2 = \frac{-2 \times 5}{1 \times 5} = \frac{-10}{5} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } u^2)}$$

$$\text{Product of zeroes} = -2 \times 0 = 0 = \frac{0 \times 5}{1 \times 5} = \frac{-0}{5} = \frac{\text{constant term}}{(\text{coefficient of } u^2)}$$

- 12.** Find the zeroes of the quadratic polynomial $(3x^2 - x - 4)$ and verify the relation between the zeroes and the coefficients.

Sol:

$$3x^2 - x - 4 = 0$$

$$\Rightarrow 3x^2 - 4x + 3x - 4 = 0$$

$$\Rightarrow x(3x - 4) + 1(3x - 4) = 0$$

$$\Rightarrow (3x - 4)(x + 1) = 0$$

$$\Rightarrow (3x - 4) \text{ or } (x + 1) = 0$$

$$\Rightarrow x = \frac{4}{3} \text{ or } x = -1$$

$$\text{Sum of zeroes} = \frac{4}{3} + (-1) = \frac{1}{3} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = \frac{4}{3} \times (-1) = \frac{-4}{3} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

- 13.** Find the quadratic polynomial whose zeroes are 2 and -6. Verify the relation between the coefficients and the zeroes of the polynomial.

Sol:

Let $\alpha = 2$ and $\beta = -6$

Sum of the zeroes, $(\alpha + \beta) = 2 + (-6) = -4$

Product of the zeroes, $\alpha\beta = 2 \times (-6) = -12$

$$\therefore \text{Required polynomial} = x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - (-4)x - 12 \\ = x^2 + 4x - 12$$

$$\text{Sum of the zeroes} = -4 = \frac{-4}{1} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = -12 = \frac{-12}{1} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

- 14.** Find the quadratic polynomial whose zeroes are $\frac{2}{3}$ and $\frac{-1}{4}$. Verify the relation between the coefficients and the zeroes of the polynomial.

Sol:

$$\text{Let } \alpha = \frac{2}{3} \text{ and } \beta = \frac{-1}{4}.$$

$$\text{Sum of the zeroes} = (\alpha + \beta) = \frac{2}{3} + \left(\frac{-1}{4}\right) = \frac{8-3}{12} = \frac{5}{12}$$

$$\text{Product of the zeroes, } \alpha\beta = \frac{2}{3} \times \left(\frac{-1}{4}\right) = \frac{-2}{12} = \frac{-1}{6}$$

$$\therefore \text{Required polynomial} = x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - \frac{5}{12}x + \left(\frac{-1}{6}\right) \\ = x^2 - \frac{5}{12}x - \frac{1}{6}$$

$$\text{Sum of the zeroes} = \frac{5}{12} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{Product of zeroes} = \frac{-1}{6} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

- 15.** Find the quadratic polynomial, sum of whose zeroes is 8 and their product is 12. Hence, find the zeroes of the polynomial.

Sol:

Let α and β be the zeroes of the required polynomial $f(x)$.

$$\text{Then } (\alpha + \beta) = 8 \text{ and } \alpha\beta = 12$$

$$\therefore f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

$$\Rightarrow f(x) = x^2 - 8x + 12$$

$$\text{Hence, required polynomial } f(x) = x^2 - 8x + 12$$

$$\therefore f(x) = 0 \Rightarrow x^2 - 8x + 12 = 0$$

$$\Rightarrow x^2 - (6x + 2x) + 12 = 0$$

$$\Rightarrow x^2 - 6x - 2x + 12 = 0$$

$$\Rightarrow x(x - 6) - 2(x - 6) = 0$$

$$\Rightarrow (x - 2)(x - 6) = 0$$

$$\Rightarrow (x - 2) = 0 \text{ or } (x - 6) = 0$$

$$\Rightarrow x = 2 \text{ or } x = 6$$

So, the zeroes of $f(x)$ are 2 and 6.

- 16.** Find the quadratic polynomial, sum of whose zeroes is 0 and their product is -1. Hence, find the zeroes of the polynomial.

Sol:

Let α and β be the zeroes of the required polynomial $f(x)$.

Then $(\alpha + \beta) = 0$ and $\alpha\beta = -1$

$$\therefore f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

$$\Rightarrow f(x) = x^2 - 0x + (-1)$$

$$\Rightarrow f(x) = x^2 - 1$$

Hence, required polynomial $f(x) = x^2 - 1$.

$$\therefore f(x) = 0 \Rightarrow x^2 - 1 = 0$$

$$\Rightarrow (x + 1)(x - 1) = 0$$

$$\Rightarrow (x + 1) = 0 \text{ or } (x - 1) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 1$$

So, the zeroes of $f(x)$ are -1 and 1.

- 17.** Find the quadratic polynomial, sum of whose zeroes is $(\frac{5}{2})$ and their product is 1. Hence, find the zeroes of the polynomial.

Sol:

Let α and β be the zeroes of the required polynomial $f(x)$.

Then $(\alpha + \beta) = \frac{5}{2}$ and $\alpha\beta = 1$

$$\therefore f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

$$\Rightarrow f(x) = x^2 - \frac{5}{2}x + 1$$

$$\Rightarrow f(x) = 2x^2 - 5x + 2$$

Hence, the required polynomial is $f(x) = 2x^2 - 5x + 2$

$$\therefore f(x) = 0 \Rightarrow 2x^2 - 5x + 2 = 0$$

$$\Rightarrow 2x^2 - (4x + x) + 2 = 0$$

$$\Rightarrow 2x^2 - 4x - x + 2 = 0$$

$$\Rightarrow 2x(x - 2) - 1(x - 2) = 0$$

$$\Rightarrow (2x - 1)(x - 2) = 0$$

$$\Rightarrow (2x - 1) = 0 \text{ or } (x - 2) = 0$$

$$\Rightarrow x = \frac{1}{2} \text{ or } x = 2$$

So, the zeros of $f(x)$ are $\frac{1}{2}$ and 2.

18. Find the quadratic polynomial, sum of whose zeroes is $\sqrt{2}$ and their product is $(\frac{1}{3})$.

Sol:

We can find the quadratic equation if we know the sum of the roots and product of the roots by using the formula

$$x^2 - (\text{Sum of the roots})x + \text{Product of roots} = 0$$

$$\Rightarrow x^2 - \sqrt{2}x + \frac{1}{3} = 0$$

$$\Rightarrow 3x^2 - 3\sqrt{2}x + 1 = 0$$

19. If $x = \frac{2}{3}$ and $x = -3$ are the roots of the quadratic equation $ax^2 + 2ax + 5x + 10$ then find the value of a and b .

Sol:

$$\text{Given: } ax^2 + 7x + b = 0$$

Since, $x = \frac{2}{3}$ is the root of the above quadratic equation

Hence, it will satisfy the above equation.

Therefore, we will get

$$a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = 0$$

$$\Rightarrow \frac{4}{9}a + \frac{14}{3} + b = 0$$

$$\Rightarrow 4a + 42 + 9b = 0$$

$$\Rightarrow 4a + 9b = -42 \quad \dots(1)$$

Since, $x = -3$ is the root of the above quadratic equation

Hence, It will satisfy the above equation.

Therefore, we will get

$$a(-3)^2 + 7(-3) + b = 0$$

$$\Rightarrow 9a - 21 + b = 0$$

$$\Rightarrow 9a + b = 21 \quad \dots(2)$$

From (1) and (2), we get

$$a = 3, b = -6$$

20. If $(x + a)$ is a factor of the polynomial $2x^2 + 2ax + 5x + 10$, find the value of a .

Sol:

$$\text{Given: } (x + a) \text{ is a factor of } 2x^2 + 2ax + 5x + 10$$

So, we have

$$x + a = 0$$

$$\Rightarrow x = -a$$

Now, it will satisfy the above polynomial.

Therefore, we will get

$$2(-a)^2 + 2a(-a) + 5(-a) + 10 = 0$$

$$\Rightarrow 2a^2 - 2a^2 - 5a + 10 = 0$$

$$\Rightarrow -5a = -10$$

$$\Rightarrow a = 2$$

21. One zero of the polynomial $3x^3 + 16x^2 + 15x - 18$ is $\frac{2}{3}$. Find the other zeros of the polynomial.

Sol:

Given: $x = \frac{2}{3}$ is one of the zero of $3x^3 + 16x^2 + 15x - 18$

Now, we have

$$x = \frac{2}{3}$$

$$\Rightarrow x - \frac{2}{3} = 0$$

Now, we divide $3x^3 + 16x^2 + 15x - 18$ by $x - \frac{2}{3}$ to find the quotient

$$\begin{array}{r}
 3x^2 + 18x + 27 \\
 x - \frac{2}{3} \overline{) 3x^3 + 16x^2 + 15x - 18} \\
 \underline{3x^3 - 2x^2} \\
 18x^2 + 15x \\
 \underline{18x^2 - 12x} \\
 27x - 18 \\
 \underline{27x - 18} \\
 0
 \end{array}$$

So, the quotient is $3x^2 + 18x + 27$

Now,

$$3x^2 + 18x + 27 = 0$$

$$\Rightarrow 3x^2 + 9x + 9x + 27 = 0$$

$$\Rightarrow 3x(x + 3) + 9(x + 3) = 0$$

$$\begin{aligned}\Rightarrow (x + 3)(3x + 9) &= 0 \\ \Rightarrow (x + 3) &= 0 \text{ or } (3x + 9) = 0 \\ \Rightarrow x &= -3 \text{ or } x = -3\end{aligned}$$

Exercise – 2B

1. Verify that 3, -2, 1 are the zeros of the cubic polynomial $p(x) = (x^3 - 2x^2 - 5x + 6)$ and verify the relation between its zeros and coefficients.

Sol:

The given polynomial is $p(x) = (x^3 - 2x^2 - 5x + 6)$

$$\therefore p(3) = (3^3 - 2 \times 3^2 - 5 \times 3 + 6) = (27 - 18 - 15 + 6) = 0$$

$$p(-2) = [(-2)^3 - 2 \times (-2)^2 - 5 \times (-2) + 6] = (-8 - 8 + 10 + 6) = 0$$

$$p(1) = (1^3 - 2 \times 1^2 - 5 \times 1 + 6) = (1 - 2 - 5 + 6) = 0$$

\therefore 3, -2 and 1 are the zeroes of $p(x)$,

Let $\alpha = 3$, $\beta = -2$ and $\gamma = 1$. Then we have:

$$(\alpha + \beta + \gamma) = (3 - 2 + 1) = 2 = \frac{-(\text{coefficient of } x^2)}{(\text{coefficient of } x^3)}$$

$$(\alpha\beta + \beta\gamma + \gamma\alpha) = (-6 - 2 + 3) = \frac{-5}{1} = \frac{\text{coefficient of } x}{\text{coefficient of } x^3}$$

$$\alpha\beta\gamma = \{3 \times (-2) \times 1\} = \frac{-6}{1} = \frac{-(\text{constant term})}{(\text{coefficient of } x^3)}$$

2. Verify that 5, -2 and $\frac{1}{3}$ are the zeroes of the cubic polynomial $p(x) = (3x^3 - 10x^2 - 27x + 10)$ and verify the relation between its zeroes and coefficients.

Sol:

$$p(x) = (3x^3 - 10x^2 - 27x + 10)$$

$$p(5) = (3 \times 5^3 - 10 \times 5^2 - 27 \times 5 + 10) = (375 - 250 - 135 + 10) = 0$$

$$p(-2) = [3 \times (-2)^3 - 10 \times (-2)^2 - 27 \times (-2) + 10] = (-24 - 40 + 54 + 10) = 0$$

$$\begin{aligned}p\left(\frac{1}{3}\right) &= \left\{3 \times \left(\frac{1}{3}\right)^3 - 10 \times \left(\frac{1}{3}\right)^2 - 27 \times \frac{1}{3} + 10\right\} = \left(3 \times \frac{1}{27} - 10 \times \frac{1}{9} - 9 + 10\right) \\ &= \left(\frac{1}{9} - \frac{10}{9} + 1\right) = \left(\frac{1-10+9}{9}\right) = \left(\frac{0}{9}\right) = 0\end{aligned}$$

\therefore 5, -2 and $\frac{1}{3}$ are the zeroes of $p(x)$.

Let $\alpha = 5$, $\beta = -2$ and $\gamma = \frac{1}{3}$. Then we have:

$$(\alpha + \beta + \gamma) = \left(5 - 2 + \frac{1}{3}\right) = \frac{10}{3} = \frac{-(\text{coefficient of } x^2)}{(\text{coefficient of } x^3)}$$

$$(\alpha\beta + \beta\gamma + \gamma\alpha) = \left(-10 - \frac{2}{3} + \frac{5}{3}\right) = \frac{-27}{3} = \frac{\text{coefficient of } x}{\text{coefficient of } x^3}$$

$$\alpha\beta\gamma = \left\{5 \times (-2) \times \frac{1}{3}\right\} = \frac{-10}{3} = \frac{-(\text{constant term})}{(\text{coefficient of } x^3)}$$

Remainder $r(x) = 7x - 9$

7. If $f(x) = x^4 - 3x^2 + 4x + 5$ is divided by $g(x) = x^2 - x + 1$

Sol:

$$\begin{array}{r}
 \quad \quad \quad x^2 + x - 3 \\
 x^2 - x + 1 \overline{) \begin{array}{l} x^4 + 0x^3 - 3x^2 + 4x + 5 \\ x^4 - x^3 + x^2 \end{array} } \\
 \hline
 \quad \quad \quad - \quad + \quad - \\
 \quad \quad \quad x^3 - 4x^2 + 4x + 5 \\
 \quad \quad \quad x^3 - x^2 + x \\
 \hline
 \quad \quad \quad - \quad + \quad - \\
 \quad \quad \quad - 3x^2 + 3x + 5 \\
 \quad \quad \quad - 3x^2 + 3x - 3 \\
 \hline
 \quad \quad \quad + \quad - \quad + \\
 \quad \quad \quad 8 \\
 \hline
 \hline
 \end{array}$$

Quotient $q(x) = x^2 + x - 3$

Remainder $r(x) = 8$

8. If $f(x) = x^4 - 5x + 6$ is divided by $g(x) = 2 - x^2$.

Sol:

We can write

$f(x)$ as $x^4 + 0x^3 + 0x^2 - 5x + 6$ and $g(x)$ as $-x^2 + 2$

$$\begin{array}{r}
 \quad \quad \quad -x^2 - 2 \\
 -x^2 + 2 \overline{) \begin{array}{l} x^4 + 0x^3 + 0x^2 - 5x + 6 \\ x^4 - 2x^2 \end{array} } \\
 \hline
 \quad \quad \quad - \quad + \\
 \quad \quad \quad 2x^2 - 5x + 6 \\
 \quad \quad \quad 2x^2 - 4 \\
 \hline
 \quad \quad \quad - \quad + \\
 \quad \quad \quad -5x + 10 \\
 \hline
 \hline
 \end{array}$$

Quotient $q(x) = -x^2 - 2$

Remainder $r(x) = -5x + 10$

9. By actual division, show that $x^2 - 3$ is a factor of $2x^4 + 3x^3 - 2x^2 - 9x - 12$.

Sol:

Let $f(x) = 2x^4 + 3x^3 - 2x^2 - 9x - 12$ and $g(x)$ as $x^2 - 3$

$$\begin{array}{r}
 \\
 2x^2 + 3x + 4 \\
 \hline
 x^2 - 3 \quad \overline{) 2x^4 + 3x^3 - 2x^2 - 9x - 12} \\
 \underline{2x^4 - 6x^2} \\
 - + \\
 3x^3 + 4x^2 - 9x - 12 \\
 \underline{3x^3 - 9x} \\
 - + \\
 4x^2 - 12 \\
 \underline{4x^2 - 12} \\
 - + \\
 x
 \end{array}$$

Quotient $q(x) = 2x^2 + 3x + 4$

Remainder $r(x) = 0$

Since, the remainder is 0.

Hence, $x^2 - 3$ is a factor of $2x^4 + 3x^3 - 2x^2 - 9x - 12$

10. On dividing $3x^3 + x^2 + 2x + 5$ is divided by a polynomial $g(x)$, the quotient and remainder are $(3x - 5)$ and $(9x + 10)$ respectively. Find $g(x)$.

Sol:

By using division rule, we have

Dividend = Quotient \times Divisor + Remainder

$$\therefore 3x^3 + x^2 + 2x + 5 = (3x - 5)g(x) + 9x + 10$$

$$\Rightarrow 3x^3 + x^2 + 2x + 5 - 9x - 10 = (3x - 5)g(x)$$

$$\Rightarrow 3x^3 + x^2 - 7x - 5 = (3x - 5)g(x)$$

$$\Rightarrow g(x) = \frac{3x^3 + x^2 - 7x - 5}{3x - 5}$$

$$\begin{array}{r}
 \\
 x^2 + 2x + 1 \\
 \hline
 3x - 5 \quad \overline{) 3x^3 + x^2 - 7x - 5} \\
 \underline{3x^3 - 5x^2} \\
 - + \\
 6x^2 - 7x - 5 \\
 \underline{6x^2 - 10x} \\
 - + \\
 3x - 5 \\
 \underline{3x - 5} \\
 - + \\
 X
 \end{array}$$

$$\therefore g(x) = x^2 + 2x + 1$$

$$\begin{aligned}
 f(x) &= x^3 + 2x^2 - 11x - 12 \\
 &= (x + 1)(x^2 + x - 12) \\
 &= (x + 1)\{x^2 + 4x - 3x - 12\} \\
 &= (x + 1)\{x(x + 4) - 3(x + 4)\} \\
 &= (x + 1)(x - 3)(x + 4)
 \end{aligned}$$

$$\therefore f(x) = 0 \Rightarrow (x + 1)(x - 3)(x + 4) = 0$$

$$\Rightarrow (x + 1) = 0 \text{ or } (x - 3) = 0 \text{ or } (x + 4) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 3 \text{ or } x = -4$$

Thus, all the zeroes are -1 , 3 and -4 .

- 13.** If 1 and -2 are two zeroes of the polynomial $(x^3 - 4x^2 - 7x + 10)$, find its third zero.

Sol:

$$\text{Let } f(x) = x^3 - 4x^2 - 7x + 10$$

Since 1 and -2 are the zeroes of $f(x)$, it follows that each one of $(x-1)$ and $(x+2)$ is a factor of $f(x)$.

Consequently, $(x-1)(x+2) = (x^2 + x - 2)$ is a factor of $f(x)$.

On dividing $f(x)$ by $(x^2 + x - 2)$, we get:

$$\begin{array}{r}
 x^2 + x - 2 \overline{) \begin{array}{l} x^3 - 4x^2 - 7x + 10 \\ x^3 + x^2 - 2x \end{array} } \quad \begin{array}{l} (x - 5) \\ \\ \end{array} \\
 \underline{\begin{array}{r} - \quad - \quad + \\ -5x^2 - 5x + 10 \\ -5x^2 - 5x + 10 \\ + \quad + \quad - \\ \hline \text{X} \end{array}}
 \end{array}$$

$$f(x) = 0 \Rightarrow (x^2 + x - 2)(x - 5) = 0$$

$$\Rightarrow (x - 1)(x + 2)(x - 5) = 0$$

$$\Rightarrow x = 1 \text{ or } x = -2 \text{ or } x = 5$$

Hence, the third zero is 5 .

- 14.** If 3 and -3 are two zeroes of the polynomial $(x^4 + x^3 - 11x^2 - 9x + 18)$, find all the zeroes of the given polynomial.

Sol:

$$\text{Let } x^4 + x^3 - 11x^2 - 9x + 18$$

Since 3 and -3 are the zeroes of $f(x)$, it follows that each one of $(x + 3)$ and $(x - 3)$ is a factor of $f(x)$.

Consequently, $(x - 3)(x + 3) = (x^2 - 9)$ is a factor of $f(x)$.

On dividing $f(x)$ by $(x^2 - 9)$, we get:

$$\begin{array}{r}
 x^2 - 9 \overline{) x^4 + x^3 - 11x^2 - 9x + 18} \left(x^2 + x - 2 \right. \\
 \underline{x^4 - 9x^2} \\
 - + \\
 \underline{x^3 - 2x^2 - 9x + 18} \\
 x^3 - 9x \\
 - + \\
 \underline{-2x^2 + 18} \\
 -2x^2 + 18 \\
 + - \\
 \underline{x}
 \end{array}$$

$$\begin{aligned}
 f(x) = 0 &\Rightarrow (x^2 + x - 2)(x^2 - 9) = 0 \\
 &\Rightarrow (x^2 + 2x - x - 2)(x - 3)(x + 3) \\
 &\Rightarrow (x - 1)(x + 2)(x - 3)(x + 3) = 0 \\
 &\Rightarrow x = 1 \text{ or } x = -2 \text{ or } x = 3 \text{ or } x = -3
 \end{aligned}$$

Hence, all the zeroes are 1, -2, 3 and -3.

- 15.** If 2 and -2 are two zeroes of the polynomial $(x^4 + x^3 - 34x^2 - 4x + 120)$, find all the zeroes of the given polynomial.

Sol:

$$\text{Let } f(x) = x^4 + x^3 - 34x^2 - 4x + 120$$

Since 2 and -2 are the zeroes of $f(x)$, it follows that each one of $(x - 2)$ and $(x + 2)$ is a factor of $f(x)$.

Consequently, $(x - 2)(x + 2) = (x^2 - 4)$ is a factor of $f(x)$.

On dividing $f(x)$ by $(x^2 - 4)$, we get:

$$\begin{array}{r}
 x^2 - 4 \overline{) x^4 + x^3 - 34x^2 - 4x + 120} \left(x^2 + x - 2 \right. \\
 \underline{x^4 - 4x^2} \\
 - + \\
 \underline{x^3 - 30x^2 - 4x + 120} \\
 x^3 - 4x \\
 - + \\
 \underline{-30x^2 + 120} \\
 -30x^2 + 120 \\
 + - \\
 \underline{x}
 \end{array}$$

$$\begin{aligned}
 f(x) &= 0 \\
 &\Rightarrow (x^2 + x - 30)(x^2 - 4) = 0
 \end{aligned}$$

$$\Rightarrow (x^2 + 6x - 5x - 30) (x - 2) (x + 2)$$

$$\Rightarrow [x(x + 6) - 5(x + 6)] (x - 2) (x + 2)$$

$$\Rightarrow (x - 5) (x + 6) (x - 2) (x + 2) = 0$$

$$\Rightarrow x = 5 \text{ or } x = -6 \text{ or } x = 2 \text{ or } x = -2$$

Hence, all the zeroes are 2, -2, 5 and -6.

- 16.** Find all the zeroes of $(x^4 + x^3 - 23x^2 - 3x + 60)$, if it is given that two of its zeroes are $\sqrt{3}$ and $-\sqrt{3}$.

Sol:

$$\text{Let } f(x) = x^4 + x^3 - 23x^2 - 3x + 60$$

Since $\sqrt{3}$ and $-\sqrt{3}$ are the zeroes of $f(x)$, it follows that each one of $(x - \sqrt{3})$ and $(x + \sqrt{3})$ is a factor of $f(x)$.

Consequently, $(x - \sqrt{3})(x + \sqrt{3}) = (x^2 - 3)$ is a factor of $f(x)$.

On dividing $f(x)$ by $(x^2 - 3)$, we get:

$$\begin{array}{r}
 x^2 - 3 \overline{) \begin{array}{r} x^4 + x^3 - 23x^2 - 3x + 60 \\ x^4 - 3x^2 \\ \hline x^3 - 20x^2 - 3x + 60 \\ x^3 - 3x \\ \hline - 20x^2 + 60 \\ -20x^2 + 60 \\ \hline 0 \\ x \\ \hline 0 \end{array} } \\
 \overline{) \begin{array}{r} x^2 + x - 20 \\ x^2 - 3 \\ \hline x - 23 \\ x^2 - 3 \\ \hline - 20 \\ -20 \\ \hline 0 \end{array} }
 \end{array}$$

$$f(x) = 0$$

$$\Rightarrow (x^2 + x - 20) (x^2 - 3) = 0$$

$$\Rightarrow (x^2 + 5x - 4x - 20) (x^2 - 3)$$

$$\Rightarrow [x(x + 5) - 4(x + 5)] (x^2 - 3)$$

$$\Rightarrow (x - 4) (x + 5) (x - \sqrt{3}) (x + \sqrt{3}) = 0$$

$$\Rightarrow x = 4 \text{ or } x = -5 \text{ or } x = \sqrt{3} \text{ or } x = -\sqrt{3}$$

Hence, all the zeroes are $\sqrt{3}$, $-\sqrt{3}$, 4 and -5.

17. Find all the zeroes of $(2x^4 - 3x^3 - 5x^2 + 9x - 3)$, it is being given that two of its zeroes are $\sqrt{3}$ and $-\sqrt{3}$.

Sol:

The given polynomial is $f(x) = 2x^4 - 3x^3 - 5x^2 + 9x - 3$

Since $\sqrt{3}$ and $-\sqrt{3}$ are the zeroes of $f(x)$, it follows that each one of $(x - \sqrt{3})$ and $(x + \sqrt{3})$ is a factor of $f(x)$.

Consequently, $(x - \sqrt{3})(x + \sqrt{3}) = (x^2 - 3)$ is a factor of $f(x)$.

On dividing $f(x)$ by $(x^2 - 3)$, we get:

$$\begin{array}{r}
 x^2 - 3 \overline{) 2x^4 - 3x^3 - 5x^2 + 9x - 3} \quad \left(2x^2 - 3x + 1 \right. \\
 \underline{2x^4 - 6x^2} \\
 -3x^3 + x^2 + 9x - 3 \\
 \underline{-3x^3 + 9x} \\
 + x^2 - 3 \\
 x^2 - 3 \\
 \underline{- +} \\
 x
 \end{array}$$

$$f(x) = 0$$

$$\Rightarrow 2x^4 - 3x^3 - 5x^2 + 9x - 3 = 0$$

$$\Rightarrow (x^2 - 3)(2x^2 - 3x + 1) = 0$$

$$\Rightarrow (x^2 - 3)(2x^2 - 2x - x + 1) = 0$$

$$\Rightarrow (x - \sqrt{3})(x + \sqrt{3})(2x - 1)(x - 1) = 0$$

$$\Rightarrow x = \sqrt{3} \text{ or } x = -\sqrt{3} \text{ or } x = \frac{1}{2} \text{ or } x = 1$$

Hence, all the zeroes are $\sqrt{3}$, $-\sqrt{3}$, $\frac{1}{2}$ and 1.

18. Obtain all other zeroes of $(x^4 + 4x^3 - 2x^2 - 20x - 15)$ if two of its zeroes are $\sqrt{5}$ and $-\sqrt{5}$.

Sol:

The given polynomial is $f(x) = x^4 + 4x^3 - 2x^2 - 20x - 15$.

Since $(x - \sqrt{5})$ and $(x + \sqrt{5})$ are the zeroes of $f(x)$ it follows that each one of $(x - \sqrt{5})$ and $(x + \sqrt{5})$ is a factor of $f(x)$.

Consequently, $(x - \sqrt{5})(x + \sqrt{5}) = (x^2 - 5)$ is a factor of $f(x)$.

On dividing $f(x)$ by $(x^2 - 5)$, we get:

$$\begin{array}{c} + \quad - \quad + \\ \hline x \\ \hline \end{array}$$

$$f(x) = 0$$

$$\Rightarrow 2x^4 - 11x^3 + 7x^2 + 13x - 7 = 0$$

$$\Rightarrow (x^2 - 6x + 7)(2x^2 + x - 7) = 0$$

$$\Rightarrow (x + 3 + \sqrt{2})(x + 3 - \sqrt{2})(2x - 1)(x + 1) = 0$$

$$\Rightarrow x = -3 - \sqrt{2} \text{ or } x = -3 + \sqrt{2} \text{ or } x = \frac{1}{2} \text{ or } x = -1$$

Hence, all the zeroes are $(-3 - \sqrt{2})$, $(-3 + \sqrt{2})$, $\frac{1}{2}$ and -1 .

Exercise – 2C

1. If one zero of the polynomial $x^2 - 4x + 1$ is $(2 + \sqrt{3})$, write the other zero.

Sol:

Let the other zeroes of $x^2 - 4x + 1$ be a .

By using the relationship between the zeroes of the quadratic polynomial.

We have, sum of zeroes = $\frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$

$$\therefore 2 + \sqrt{3} + a = \frac{-(-4)}{1}$$

$$\Rightarrow a = 2 - \sqrt{3}$$

Hence, the other zeroes of $x^2 - 4x + 1$ is $2 - \sqrt{3}$.

2. Find the zeroes of the polynomial $x^2 + x - p(p + 1)$

Sol:

$$f(x) = x^2 + x - p(p + 1)$$

By adding and subtracting px , we get

$$f(x) = x^2 + px + x - px - p(p + 1)$$

$$= x^2 + (p + 1)x - px - p(p + 1)$$

$$= x[x + (p + 1)] - p[x + (p + 1)]$$

$$= [x + (p + 1)](x - p)$$

$$f(x) = 0$$

$$\Rightarrow [x + (p + 1)](x - p) = 0$$

$$\Rightarrow [x + (p + 1)] = 0 \text{ or } (x - p) = 0$$

$$\Rightarrow x = -(p + 1) \text{ or } x = p$$

So, the zeroes of $f(x)$ are $-(p + 1)$ and p .

3. Find the zeroes of the polynomial $x^2 - 3x - m(m + 3)$

Sol:

$$f(x) = x^2 - 3x - m(m + 3)$$

By adding and subtracting mx , we get

$$f(x) = x^2 - mx - 3x + mx - m(m + 3)$$

$$= x[x - (m + 3)] + m[x - (m + 3)]$$

$$= [x - (m + 3)](x + m)$$

$$f(x) = 0 \Rightarrow [x - (m + 3)](x + m) = 0$$

$$\Rightarrow [x - (m + 3)] = 0 \text{ or } (x + m) = 0$$

$$\Rightarrow x = m + 3 \text{ or } x = -m$$

So, the zeroes of $f(x)$ are $-m$ and $+3$.

4. Find α , β are the zeros of polynomial $\alpha + \beta = 6$ and $\alpha\beta = 4$ then write the polynomial.

Sol:

If the zeroes of the quadratic polynomial are α and β then the quadratic polynomial can be found as $x^2 - (\alpha + \beta)x + \alpha\beta$ (1)

Substituting the values in (1), we get

$$x^2 - 6x + 4$$

5. If one zero of the quadratic polynomial $kx^2 + 3x + k$ is 2, then find the value of k .

Sol:

Given: $x = 2$ is one zero of the quadratic polynomial $kx^2 + 3x + k$

Therefore, it will satisfy the above polynomial.

Now, we have

$$k(2)^2 + 3(2) + k = 0$$

$$\Rightarrow 4k + 6 + k = 0$$

$$\Rightarrow 5k + 6 = 0$$

$$\Rightarrow k = -\frac{6}{5}$$

6. If 3 is a zero of the polynomial $2x^2 + x + k$, find the value of k .

Sol:

Given: $x = 3$ is one zero of the polynomial $2x^2 + x + k$

Therefore, it will satisfy the above polynomial.

Now, we have

$$2(3)^2 + 3 + k = 0$$

$$\Rightarrow 21 + k = 0$$

$$\Rightarrow k = -21$$

7. If -4 is a zero of the polynomial $x^2 - x - (2k + 2)$ is -4, then find the value of k.

Sol:

Given: $x = -4$ is one zero of the polynomial $x^2 - x - (2k + 2)$

Therefore, it will satisfy the above polynomial.

Now, we have

$$(-4)^2 - (-4) - (2k + 2) = 0$$

$$\Rightarrow 16 + 4 - 2k - 2 = 0$$

$$\Rightarrow 2k = -18$$

$$\Rightarrow k = 9$$

8. If 1 is a zero of the quadratic polynomial $ax^2 - 3(a - 1)x - 1$ is 1, then find the value of a.

Sol:

Given: $x = 1$ is one zero of the polynomial $ax^2 - 3(a - 1)x - 1$

Therefore, it will satisfy the above polynomial.

Now, we have

$$a(1)^2 - (a - 1)1 - 1 = 0$$

$$\Rightarrow a - 3a + 3 - 1 = 0$$

$$\Rightarrow -2a = -2$$

$$\Rightarrow a = 1$$

9. If -2 is a zero of the polynomial $3x^2 + 4x + 2k$ then find the value of k.

Sol:

Given: $x = -2$ is one zero of the polynomial $3x^2 + 4x + 2k$

Therefore, it will satisfy the above polynomial.

Now, we have

$$3(-2)^2 + 4(-2)1 + 2k = 0$$

$$\Rightarrow 12 - 8 + 2k = 0$$

$$\Rightarrow k = -2$$

10. Write the zeros of the polynomial $f(x) = x^2 - x - 6$.

Sol:

$$f(x) = x^2 - x - 6$$

$$= x^2 - 3x + 2x - 6$$

$$= x(x - 3) + 2(x - 3)$$

$$= (x - 3)(x + 2)$$

$$f(x) = 0 \Rightarrow (x - 3)(x + 2) = 0$$

$$\Rightarrow (x - 3) = 0 \text{ or } (x + 2) = 0$$

$$\Rightarrow x = 3 \text{ or } x = -2$$

So, the zeroes of $f(x)$ are 3 and -2 .

11. If the sum of the zeros of the quadratic polynomial $kx^2 - 3x + 5$ is 1 write the value of k .

Sol:

By using the relationship between the zeroes of the quadratic polynomial.

We have

$$\text{Sum of zeroes} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$$

$$\Rightarrow 1 = \frac{-(-3)}{k}$$

$$\Rightarrow k = 3$$

12. If the product of the zero of the polynomial $(x^2 - 4x + k)$ is 3. Find the value of k .

Sol:

By using the relationship between the zeroes of the quadratic polynomial.

We have

$$\text{Product of zeroes} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

$$\Rightarrow 3 = \frac{k}{1}$$

$$\Rightarrow k = 3$$

13. If $(x + a)$ is a factor of $(2x^2 + 2ax + 5x + 10)$, then find the value of a .

Sol:

Given: $(x + a)$ is a factor of $2x^2 + 2ax + 5x + 10$

We have

$$x + a = 0$$

$$\Rightarrow x = -a$$

Since, $(x + a)$ is a factor of $2x^2 + 2ax + 5x + 10$

Hence, It will satisfy the above polynomial

$$\therefore 2(-a)^2 + 2a(-a) + 5(-a) + 10 = 0$$

$$\Rightarrow -5a + 10 = 0$$

$$\Rightarrow a = 2$$

14. If $(a-b)$, a and $(a+b)$ are zeros of the polynomial $2x^3 - 6x^2 + 5x - 7$ write the value of a .

Sol:

By using the relationship between the zeroes of the quadratic polynomial.

We have

$$\text{Sum of zeroes} = \frac{-(\text{coefficient of } x^2)}{\text{coefficient of } x^3}$$

$$\Rightarrow a - b + a + a + b = \frac{-(-6)}{2}$$

$$\Rightarrow 3a = 3$$

$$\Rightarrow a = 1$$

15. If $x^3 + x^2 - ax + b$ is divisible by $(x^2 - x)$, write the value of a and b .

Sol:

Equating $x^2 - x$ to 0 to find the zeroes, we will get

$$x(x - 1) = 0$$

$$\Rightarrow x = 0 \text{ or } x - 1 = 0$$

$$\Rightarrow x = 0 \text{ or } x = 1$$

Since, $x^3 + x^2 - ax + b$ is divisible by $x^2 - x$.

Hence, the zeroes of $x^2 - x$ will satisfy $x^3 + x^2 - ax + b$

$$\therefore (0)^3 + 0^2 - a(0) + b = 0$$

$$\Rightarrow b = 0$$

And

$$(1)^3 + 1^2 - a(1) + 0 = 0 \quad [\because b = 0]$$

$$\Rightarrow a = 2$$

16. If α and β be the zeroes of the polynomial $2x^2 - 7x + k$ write the value of $(\alpha + \beta + \alpha\beta)$.

Sol:

By using the relationship between the zeroes of the quadratic polynomial.

We have

$$\text{Sum of zeroes} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2} \text{ and Product of zeroes} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

$$\therefore \alpha + \beta = \frac{-7}{2} \text{ and } \alpha\beta = \frac{5}{2}$$

$$\text{Now, } \alpha + \beta + \alpha\beta = \frac{-7}{2} + \frac{5}{2} = -1$$

17. State Division Algorithm for Polynomials.

Sol:

“If $f(x)$ and $g(x)$ are two polynomials such that degree of $f(x)$ is greater than degree of $g(x)$ where $g(x) \neq 0$, there exists unique polynomials $q(x)$ and $r(x)$ such that

$$f(x) = g(x) \times q(x) + r(x),$$

where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$.

18. Find the sum of the zeros and the product of zeros of a quadratic polynomial, are $-\frac{1}{2}$ and -3 respectively. Write the polynomial.

Sol:

We can find the quadratic polynomial if we know the sum of the roots and product of the roots by using the formula

$$x^2 - (\text{sum of the zeroes})x + \text{product of zeroes}$$

$$\Rightarrow x^2 - \left(-\frac{1}{2}\right)x + (-3)$$

$$\Rightarrow x^2 + \frac{1}{2}x - 3$$

Hence, the required polynomial is $x^2 + \frac{1}{2}x - 3$.

19. Find the zeroes of the quadratic polynomial $f(x) = 6x^2 - 3$.

Sol:

To find the zeroes of the quadratic polynomial we will equate $f(x)$ to 0

$$\therefore f(x) = 0$$

$$\Rightarrow 6x^2 - 3 = 0$$

$$\Rightarrow 3(2x^2 - 1) = 0$$

$$\Rightarrow 2x^2 - 1 = 0$$

$$\Rightarrow 2x^2 = 1$$

$$\Rightarrow x^2 = \frac{1}{2}$$

$$\Rightarrow x = \pm \frac{1}{\sqrt{2}}$$

Hence, the zeroes of the quadratic polynomial $f(x) = 6x^2 - 3$ are $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$.

20. Find the zeroes of the quadratic polynomial $f(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$.

Sol:

To find the zeroes of the quadratic polynomial we will equate $f(x)$ to 0

$$\therefore f(x) = 0$$

$$\Rightarrow 4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$$

$$\Rightarrow 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$$

$$\Rightarrow 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$$

$$\Rightarrow (\sqrt{3}x + 2) = 0 \text{ or } (4x - \sqrt{3}) = 0$$

$$\Rightarrow x = -\frac{2}{\sqrt{3}} \text{ or } x = \frac{\sqrt{3}}{4}$$

Hence, the zeroes of the quadratic polynomial $f(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ are $-\frac{2}{\sqrt{3}}$ or $\frac{\sqrt{3}}{4}$

21. If α, β are the zeroes of the polynomial $f(x) = x^2 - 5x + k$ such that $\alpha - \beta = 1$, find the value of $k = ?$

Sol:

By using the relationship between the zeroes of the quadratic polynomial.

We have

$$\text{Sum of zeroes} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2} \text{ and Product of zeroes} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

$$\therefore \alpha + \beta = \frac{-(-5)}{1} \text{ and } \alpha\beta = \frac{k}{1}$$

$$\Rightarrow \alpha + \beta = 5 \text{ and } \alpha\beta = \frac{k}{1}$$

Solving $\alpha - \beta = 1$ and $\alpha + \beta = 5$, we will get

$$\alpha = 3 \text{ and } \beta = 2$$

Substituting these values in $\alpha\beta = \frac{k}{1}$, we will get

$$k = 6$$

22. If α and β are the zeros of the polynomial $f(x) = 6x^2 + x - 2$ find the value of $\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$

Sol:

By using the relationship between the zeroes of the quadratic polynomial.

We have

$$\text{Sum of zeroes} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2} \text{ and Product of zeroes} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

$$\therefore \alpha + \beta = \frac{-1}{6} \text{ and } \alpha\beta = -\frac{1}{3}$$

$$\begin{aligned} \text{Now, } \frac{\alpha}{\beta} + \frac{\beta}{\alpha} &= \frac{\alpha^2 + \beta^2}{\alpha\beta} \\ &= \frac{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta}{\alpha\beta} \\ &= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} \\ &= \frac{\left(\frac{-1}{6}\right)^2 - 2\left(-\frac{1}{3}\right)}{-\frac{1}{3}} \\ &= \frac{\frac{1}{36} + \frac{2}{3}}{-\frac{1}{3}} \\ &= -\frac{25}{12} \end{aligned}$$

23. If α, β are the zeroes of the polynomial $f(x) = 5x^2 - 7x + 1$, then $\frac{1}{\alpha} + \frac{1}{\beta} = ?$

Sol:

By using the relationship between the zeroes of the quadratic polynomial.

We have

Sum of zeroes = $\frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$ and Product of zeroes = $\frac{\text{constant term}}{\text{coefficient of } x^2}$

$$\therefore \alpha + \beta = \frac{-(-7)}{5} \text{ and } \alpha\beta = \frac{1}{5}$$

$$\Rightarrow \alpha + \beta = \frac{7}{5} \text{ and } \alpha\beta = \frac{1}{5}$$

$$\begin{aligned} \text{Now, } \frac{1}{\alpha} + \frac{1}{\beta} &= \frac{\alpha + \beta}{\alpha\beta} \\ &= \frac{\frac{7}{5}}{\frac{1}{5}} \\ &= 7 \end{aligned}$$

24. If α, β are the zeroes of the polynomial $f(x) = x^2 + x - 2$, then $\left(\frac{\alpha}{\beta} - \frac{\alpha}{\beta}\right)$.

Sol:

By using the relationship between the zeroes of the quadratic polynomial.

We have

Sum of zeroes = $\frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$ and Product of zeroes = $\frac{\text{constant term}}{\text{coefficient of } x^2}$

$$\therefore \alpha + \beta = \frac{-1}{1} \text{ and } \alpha\beta = \frac{-2}{1}$$

$$\Rightarrow \alpha + \beta = -1 \text{ and } \alpha\beta = -2$$

$$\text{Now, } \left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^2 = \left(\frac{\beta - \alpha}{\alpha\beta}\right)^2$$

$$= \frac{(\alpha + \beta)^2 - 4\alpha\beta}{(\alpha\beta)^2} \quad [\because (\beta - \alpha)^2 = (\alpha + \beta)^2 - 4\alpha\beta]$$

$$= \frac{(-1)^2 - 4(-2)}{(-2)^2} \quad [\because \alpha + \beta = -1 \text{ and } \alpha\beta = -2]$$

$$= \frac{(-1)^2 - 4(-2)}{4}$$

$$= \frac{9}{4}$$

$$\therefore \left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^2 = \frac{9}{4}$$

$$\Rightarrow \frac{1}{\alpha} - \frac{1}{\beta} = \pm \frac{3}{2}$$

25. If the zeroes of the polynomial $f(x) = x^3 - 3x^2 + x + 1$ are $(a - b)$, a and $(a + b)$, find the values of a and b .

Sol:

By using the relationship between the zeroes of the quadratic polynomial.

We have, Sum of zeroes = $\frac{-(\text{coefficient of } x^2)}{\text{coefficient of } x^3}$

$$\therefore a - b + a + a + b = \frac{-(-3)}{1}$$

$$\Rightarrow 3a = 3$$

$$\Rightarrow a = 1$$

$$\text{Now, Product of zeroes} = \frac{-(\text{constant term})}{\text{coefficient of } x^3}$$

$$\therefore (a - b)(a)(a + b) = \frac{-1}{1}$$

$$\Rightarrow (1 - b)(1)(1 + b) = -1 \quad [\because a = 1]$$

$$\Rightarrow 1 - b^2 = -1$$

$$\Rightarrow b^2 = 2$$

$$\Rightarrow b = \pm\sqrt{2}$$

Exercise – MCQ

1. Which of the following is a polynomial?

(a) $x^2 - 5x + 6\sqrt{x} + 3$

(b) $x^{3/2} - x + x^{1/2} + 1$

(c) $\sqrt{x} + \frac{1}{\sqrt{x}}$

(d) None of these

Sol:

(d) none of these

A polynomial in x of degree n is an expression of the form $p(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$, where $a_n \neq 0$.

2. Which of the following is not a polynomial?

(a) $\sqrt{3}x^2 - 2\sqrt{3}x + 5$

(b) $9x^2 - 4x + \sqrt{2}$

(c) $\frac{3}{2}x^3 + 6x^2 - \frac{1}{\sqrt{2}}x - 8$

(d) $x + \frac{3}{x}$

Sol:

(d) $x + \frac{3}{x}$ is not a polynomial.

It is because in the second term, the degree of x is -1 and an expression with a negative degree is not a polynomial.

3. The Zeroes of the polynomial $x^2 - 2x - 3$ are

(a) $-3, 1$

(b) $-3, -1$

(c) $3, -1$

(d) $3, 1$

Sol:

(c) $3, -1$

$$\text{Let } f(x) = x^2 - 2x - 3 = 0$$

$$= x^2 - 3x + x - 3 = 0$$

$$= x(x - 3) + 1(x - 3) = 0$$

$$= (x - 3)(x + 1) = 0$$

$$\Rightarrow x = 3 \text{ or } x = -1$$

4. The zeroes of the polynomial $x^2 - \sqrt{2}x - 12$ are
(a) $\sqrt{2}, -\sqrt{2}$ (b) $3\sqrt{2}, -2\sqrt{2}$ (c) $-3\sqrt{2}, 2\sqrt{2}$ (d) $3\sqrt{2}, 2\sqrt{2}$

Sol:

(b) $3\sqrt{2}, -2\sqrt{2}$

Let $f(x) = x^2 - \sqrt{2}x - 12 = 0$

$$\Rightarrow x^2 - 3\sqrt{2}x + 2\sqrt{2}x - 12 = 0$$

$$\Rightarrow x(x - 3\sqrt{2}) + 2\sqrt{2}(x - 3\sqrt{2}) = 0$$

$$\Rightarrow (x - 3\sqrt{2})(x + 2\sqrt{2}) = 0$$

$$\Rightarrow x = 3\sqrt{2} \text{ or } x = -2\sqrt{2}$$

5. The zeroes of the polynomial $4x^2 + 5\sqrt{2}x - 3$ are
(a) $-3\sqrt{2}, \sqrt{2}$ (b) $-3\sqrt{2}, \frac{\sqrt{2}}{2}$ (c) $\frac{-3}{\sqrt{2}}, \frac{\sqrt{2}}{4}$ (d) none of these

Sol:

(c) $-\frac{3}{\sqrt{2}}, \frac{\sqrt{2}}{4}$

Let $f(x) = 4x^2 + 5\sqrt{2}x - 3 = 0$

$$\Rightarrow 4x^2 + 6\sqrt{2}x - \sqrt{2}x - 3 = 0$$

$$\Rightarrow 2\sqrt{2}x(\sqrt{2}x + 3) - 1(\sqrt{2}x + 3) = 0$$

$$\Rightarrow (\sqrt{2}x + 3)(2\sqrt{2}x - 1) = 0$$

$$\Rightarrow x = -\frac{3}{\sqrt{2}} \text{ or } x = \frac{1}{2\sqrt{2}}$$

$$\Rightarrow x = -\frac{3}{\sqrt{2}} \text{ or } x = \frac{1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$$

6. The zeros of the polynomial $x^2 + \frac{1}{6}x - 2$ are
(a) $-3, 4$ (b) $\frac{-3}{2}, \frac{4}{3}$ (c) $\frac{-4}{3}, \frac{3}{2}$ (d) none of these

Sol:

(b) $\frac{-3}{2}, \frac{4}{3}$

Let $f(x) = x^2 + \frac{1}{6}x - 2 = 0$

$$\Rightarrow 6x^2 + x - 12 = 0$$

$$\Rightarrow 6x^2 + 9x - 8x - 12 = 0$$

$$\Rightarrow 3x(2x + 3) - 4(2x + 3) = 0$$

$$\Rightarrow (2x + 3)(3x - 4) = 0$$

$$\therefore x = \frac{-3}{2} \text{ or } x = \frac{4}{3}$$

7. The zeros of the polynomial $7x^2 - \frac{11}{3}x - \frac{2}{3}$ are
 (a) $\frac{2}{3}, \frac{-1}{7}$ (a) $\frac{2}{7}, \frac{-1}{3}$ (c) $\frac{-2}{3}, \frac{1}{7}$ (d) none of these

Sol:

(a) $\frac{2}{3}, \frac{-1}{7}$

Let $f(x) = 7x^2 - \frac{11}{3}x - \frac{2}{3} = 0$

$\Rightarrow 21x^2 - 11x - 2 = 0$

$\Rightarrow 21x^2 - 14x + 3x - 2 = 0$

$\Rightarrow 7x(3x - 2) + 1(3x - 2) = 0$

$\Rightarrow (3x - 2)(7x + 1) = 0$

$\Rightarrow x = \frac{2}{3} \text{ or } x = \frac{-1}{7}$

8. The sum and product of the zeroes of a quadratic polynomial are 3 and -10 respectively.

The quadratic polynomial is

(a) $x^2 - 3x + 10$ (b) $x^2 + 3x - 10$ (c) $x^2 - 3x - 10$ (d) $x^2 + 3x + 10$

Sol:

(c) $x^2 - 3x - 10$

Given: Sum of zeroes, $\alpha + \beta = 3$

Also, product of zeroes, $\alpha\beta = -10$

\therefore Required polynomial $= x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - 3x - 10$

9. A quadratic polynomial whose zeroes are 5 and -3, is

(a) $x^2 + 2x - 15$ (b) $x^2 - 2x + 15$ (c) $x^2 - 2x - 15$ (d) none of these

Sol:

(c) $x^2 - 2x - 15$

Here, the zeroes are 5 and -3.

Let $\alpha = 5$ and $\beta = -3$

So, sum of the zeroes, $\alpha + \beta = 5 + (-3) = 2$

Also, product of the zeroes, $\alpha\beta = 5 \times (-3) = -15$

The polynomial will be $x^2 - (\alpha + \beta)x + \alpha\beta$

\therefore The required polynomial is $x^2 - 2x - 15$.

10. A quadratic polynomial whose zeroes are $\frac{3}{5}$ and $\frac{-1}{2}$, is

(a) $10x^2 + x + 3$ (b) $10x^2 + x - 3$ (c) $10x^2 - x + 3$ (d) $x^2 - \frac{1}{10}x - \frac{3}{10}$

Sol:

$$(d) x^2 - \frac{1}{10}x - \frac{3}{10}$$

Here, the zeroes are $\frac{3}{5}$ and $\frac{-1}{2}$

Let $\alpha = \frac{3}{5}$ and $\beta = \frac{-1}{2}$

So, sum of the zeroes, $\alpha + \beta = \frac{3}{5} + \left(\frac{-1}{2}\right) = \frac{1}{10}$

Also, product of the zeroes, $\alpha\beta = \frac{3}{5} \times \left(\frac{-1}{2}\right) = \frac{-3}{10}$

The polynomial will be $x^2 - (\alpha + \beta)x + \alpha\beta$.

\therefore The required polynomial is $x^2 - \frac{1}{10}x - \frac{3}{10}$.

11. The zeroes of the quadratic polynomial $x^2 + 88x + 125$ are

- (a) both positive (b) both negative
(c) one positive and one negative (d) both equal

Sol:

(b) both negative

Let α and β be the zeroes of $x^2 + 88x + 125$.

Then $\alpha + \beta = -88$ and $\alpha \times \beta = 125$

This can only happen when both the zeroes are negative.

12. If α and β are the zeros of $x^2 + 5x + 8$, then the value of $(\alpha + \beta)$ is

- (a) 5 (b) -5 (c) 8 (d) -8

Sol:

(b) -5

Given: α and β be the zeroes of $x^2 + 5x + 8$.

If $\alpha + \beta$ is the sum of the roots and $\alpha\beta$ is the product, then the required polynomial will be $x^2 - (\alpha + \beta)x + \alpha\beta$.

$\therefore \alpha + \beta = -5$

13. If α and β are the zeroes of $2x^2 + 5x - 9$, then the value of $\alpha\beta$ is

- (a) $\frac{-5}{2}$ (b) $\frac{5}{2}$ (c) $\frac{-9}{2}$ (d) $\frac{9}{2}$

Sol:

(c) $\frac{-9}{2}$

Given: α and β be the zeroes of $2x^2 + 5x - 9$.

If $\alpha + \beta$ are the zeroes, then $x^2 - (\alpha + \beta)x + \alpha\beta$ is the required polynomial.

The polynomial will be $x^2 - \frac{5}{2}x - \frac{9}{2}$.

$\therefore \alpha\beta = \frac{-9}{2}$

14. If one zero of the quadratic polynomial $kx^2 + 3x + k$ is 2, then the value of k is

(a) $\frac{5}{6}$ (b) $\frac{-5}{6}$ (c) $\frac{6}{5}$ (d) $\frac{-6}{5}$

Sol:

(d) $\frac{-6}{5}$

Since 2 is a zero of $kx^2 + 3x + k$, we have:

$$k \times (2)^2 + 3(2) + k = 0$$

$$\Rightarrow 4k + k + 6 = 0$$

$$\Rightarrow 5k = -6$$

$$\Rightarrow k = \frac{-6}{5}$$

15. If one zero of the quadratic polynomial $(k - 1)x^2 - kx + 1$ is -4, then the value of k is

(a) $\frac{-5}{4}$ (b) $\frac{5}{4}$ (c) $\frac{-4}{3}$ (d) $\frac{4}{3}$

Sol:

(b) $\frac{5}{4}$

Since -4 is a zero of $(k - 1)x^2 - kx + 1$, we have:

$$(k - 1) \times (-4)^2 - k \times (-4) + 1 = 0$$

$$\Rightarrow 16k - 16 - 4k + 1 = 0$$

$$\Rightarrow 12k - 15 = 0$$

$$\Rightarrow k = \frac{15}{12}$$

$$\Rightarrow k = \frac{5}{4}$$

16. If -2 and 3 are the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$, then

(a) $a = -2$, $b = 6$ (b) $a = 2$, $b = -6$
(c) $a = -2$, $b = -6$ (d) $a = 2$, $b = 6$

Sol:

(c) $a = -2$, $b = -6$

Given: -2 and 3 are the zeroes of $x^2 + (a + 1)x + b$.

$$\text{Now, } (-2)^2 + (a + 1) \times (-2) + b = 0 \Rightarrow 4 - 2a - 2 + b = 0$$

$$\Rightarrow b - 2a = -2 \quad \dots(1)$$

$$\text{Also, } 3^2 + (a + 1) \times 3 + b = 0 \Rightarrow 9 + 3a + 3 + b = 0$$

$$\Rightarrow b + 3a = -12 \quad \dots(2)$$

On subtracting (1) from (2), we get $a = -2$

$$\therefore b = -2 - 4 = -6 \quad [\text{From (1)}]$$

17. If one zero of $3x^2 - 8x + k$ be the reciprocal of the other, then $k = ?$

(a) 3 (b) -3 (c) $\frac{1}{3}$ (d) $\frac{-1}{3}$

Sol:

(a) $k = 3$

Let α and $\frac{1}{\alpha}$ be the zeroes of $3x^2 - 8x + k$.

Then the product of zeroes $= \frac{k}{3}$

$$\Rightarrow \alpha \times \frac{1}{\alpha} = \frac{k}{3}$$

$$\Rightarrow 1 = \frac{k}{3}$$

$$\Rightarrow k = 3$$

18. If the sum of the zeroes of the quadratic polynomial $kx^2 + 2x + 3k$ is equal to the product of its zeroes, then $k = ?$

(a) $\frac{1}{3}$ (b) $\frac{-1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$

Sol:

(d) $\frac{-2}{3}$

Let α and β be the zeroes of $kx^2 + 2x + 3k$.

Then $\alpha + \beta = \frac{-2}{k}$ and $\alpha\beta = 3$

$$\Rightarrow \alpha + \beta = \alpha\beta$$

$$\Rightarrow \frac{-2}{k} = 3$$

$$\Rightarrow k = \frac{-2}{3}$$

19. If α, β are the zeroes of the polynomial $x^2 + 6x + 2$, then $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = ?$

(a) 3 (b) -3 (c) 12 (d) -12

Sol:

(b) -3

Since α and β be the zeroes of $x^2 + 6x + 2$, we have:

$$\alpha + \beta = -6 \text{ and } \alpha\beta = 2$$

$$\therefore \left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = \left(\frac{\alpha + \beta}{\alpha\beta}\right) = \frac{-6}{2} = -3$$

20. If α, β, γ be the zeroes of the polynomial $x^3 - 6x^2 - x + 30$, then $(\alpha\beta + \beta\gamma + \gamma\alpha) = ?$

(a) -1 (b) 1 (c) -5 (d) 30

Sol:

(a) -1

It is given that α , β and γ are the zeroes of $x^3 - 6x^2 - x + 30$.

$$\therefore (\alpha\beta + \beta\gamma + \gamma\alpha) = \frac{\text{co-efficient of } x}{\text{co-efficient of } x^3} = \frac{-1}{1} = -1$$

21. If α , β , γ be the zeroes of the polynomial $2x^3 + x^2 - 13x + 6$, then $\alpha\beta\gamma = ?$

(a) -3 (b) 3 (c) $\frac{-1}{2}$ (d) $\frac{-13}{2}$

Sol:

(a) -3

Since, α , β and γ are the zeroes of $2x^3 + x^2 - 13x + 6$, we have:

$$\alpha\beta\gamma = \frac{-(\text{constant term})}{\text{co-efficient of } x^3} = \frac{-6}{2} = -3$$

22. If α , β , γ be the zeroes of the polynomial $p(x)$ such that $(\alpha + \beta + \gamma) = 3$, $(\alpha\beta + \beta\gamma + \gamma\alpha) = -10$ and $\alpha\beta\gamma = -24$, then $p(x) = ?$

(a) $x^3 + 3x^2 - 10x + 24$ (b) $x^3 + 3x^2 + 10x - 24$
 (c) $x^3 - 3x^2 - 10x + 24$ (d) none of these

Sol:(c) $x^3 - 3x^2 - 10x + 24$ Given: α , β and γ are the zeroes of polynomial $p(x)$.Also, $(\alpha + \beta + \gamma) = 3$, $(\alpha\beta + \beta\gamma + \gamma\alpha) = -10$ and $\alpha\beta\gamma = -24$

$$\begin{aligned} \therefore p(x) &= x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma \\ &= x^3 - 3x^2 - 10x + 24 \end{aligned}$$

23. If two of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ are 0, then the third zero is

(a) $\frac{-b}{a}$ (b) $\frac{b}{a}$ (c) $\frac{c}{a}$ (d) $\frac{-d}{a}$

Sol:(a) $\frac{-b}{a}$ Let α , 0 and 0 be the zeroes of $ax^3 + bx^2 + cx + d = 0$ Then the sum of zeroes = $\frac{-b}{a}$

$$\Rightarrow \alpha + 0 + 0 = \frac{-b}{a}$$

$$\Rightarrow \alpha = \frac{-b}{a}$$

Hence, the third zero is $\frac{-b}{a}$.

24. If one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is 0, then the product of the other two zeroes is

(a) $\frac{-c}{a}$ (b) $\frac{c}{a}$ (b) 0 (b) $\frac{-b}{a}$

Sol:

(b) $\frac{c}{a}$

Let α , β and 0 be the zeroes of $ax^3 + bx^2 + cx + d$.

Then, sum of the products of zeroes taking two at a time is given by

$$(\alpha\beta + \beta \times 0 + \alpha \times 0) = \frac{c}{a}$$

$$\Rightarrow \alpha\beta = \frac{c}{a}$$

\therefore The product of the other two zeroes is $\frac{c}{a}$.

25. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is -1, then the product of the other two zeroes is

(a) $a - b - 1$ (b) $b - a - 1$ (c) $1 - a + b$ (d) $1 + a - b$

Sol:

(c) $1 - a + b$

Since -1 is a zero of $x^3 + ax^2 + bx + c$, we have:

$$(-1)^3 + a \times (-1)^2 + b \times (-1) + c = 0$$

$$\Rightarrow a - b + c + 1 = 0$$

$$\Rightarrow c = 1 - a + b$$

Also, product of all zeroes is given by

$$\alpha\beta \times (-1) = -c$$

$$\Rightarrow \alpha\beta = c$$

$$\Rightarrow \alpha\beta = 1 - a + b$$

26. If α , β be the zeroes of the polynomial $2x^2 + 5x + k$ such that $(\alpha + \beta)^2 - \alpha\beta = \frac{21}{4}$, then $k = ?$

(a) 3 (b) -3 (c) -2 (d) 2

Sol:

(d) 2

Since α and β are the zeroes of $2x^2 + 5x + k$, we have:

$$\alpha + \beta = \frac{-5}{2} \text{ and } \alpha\beta = \frac{k}{2}$$

$$\text{Also, it is given that } \alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}.$$

$$\Rightarrow (\alpha + \beta)^2 - \alpha\beta = \frac{21}{4}$$

$$\Rightarrow \left(\frac{-5}{2}\right)^2 - \frac{k}{2} = \frac{21}{4}$$

$$\Rightarrow \frac{25}{4} - \frac{k}{2} = \frac{21}{4}$$

$$\Rightarrow \frac{k}{2} = \frac{25}{4} - \frac{21}{4} = \frac{4}{4} = 1$$

$$\Rightarrow k = 2$$

27. On dividing a polynomial $p(x)$ by a non-zero polynomial $q(x)$, let $g(x)$ be the quotient and $r(x)$ be the remainder, then $p(x) = q(x) \cdot g(x) + r(x)$, where

- (a) $r(x) = 0$ always
- (b) $\deg r(x) < \deg g(x)$ always
- (c) either $r(x) = 0$ or $\deg r(x) < \deg g(x)$
- (d) $r(x) = g(x)$

Sol:

- (c) either $r(x) = 0$ or $\deg r(x) < \deg g(x)$

By division algorithm on polynomials, either $r(x) = 0$ or $\deg r(x) < \deg g(x)$.

28. Which of the following is a true statement?

- (a) $x^2 + 5x - 3$ is a linear polynomial.
- (b) $x^2 + 4x - 1$ is a binomial
- (c) $x + 1$ is a monomial
- (d) $5x^2$ is a monomial

Sol:

- (d) $5x^2$ is a monomial.

$5x^2$ consists of one term only. So, it is a monomial.

Exercise – Formative Assessment

1. The zeroes of the polynomial $P(x) = x^2 - 2x - 3$ are

- (a) -3, 1
- (b) -3, -1
- (c) 3, -1
- (d) 3, 1

Sol:

- (c) 3, -1

Here, $p(x) = x^2 - 2x - 3$

Let $x^2 - 2x - 3 = 0$

$$\Rightarrow x^2 - (3 - 1)x - 3 = 0$$

$$\Rightarrow x^2 - 3x + x - 3 = 0$$

$$\Rightarrow x(x - 3) + 1(x - 3) = 0$$

$$\Rightarrow (x - 3)(x + 1) = 0$$

$$\Rightarrow x = 3, -1$$

2. If α, β, γ be the zeroes of the polynomial $x^3 - 6x^2 - x + 3$, then the values of $(\alpha\beta + \beta\gamma + \gamma\alpha) = ?$

(a) -1 (b) 1 (c) -5 (d) 3

Sol:

(a) -1

Here, $p(x) = x^3 - 6x^2 - x + 3$

Comparing the given polynomial with $x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$, we get: $(\alpha\beta + \beta\gamma + \gamma\alpha) = -1$

3. If α, β are the zeros of $kx^2 - 2x + 3k$ is equal $\alpha + \beta = \alpha\beta$ then $k = ?$

(a) $\frac{1}{3}$ (b) $\frac{-1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$

Sol:

(c) $\frac{2}{3}$

Here, $p(x) = x^2 - 2x + 3k$

Comparing the given polynomial with $ax^2 + bx + c$, we get:

$a = 1, b = -2$ and $c = 3k$

It is given that α and β are the roots of the polynomial.

$$\therefore \alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow \alpha + \beta = -\left(\frac{-2}{1}\right)$$

$$\Rightarrow \alpha + \beta = 2 \quad \dots(i)$$

$$\text{Also, } \alpha\beta = \frac{c}{a}$$

$$\Rightarrow \alpha\beta = \frac{3k}{1}$$

$$\Rightarrow \alpha\beta = 3k \quad \dots(ii)$$

Now, $\alpha + \beta = \alpha\beta$

$$\Rightarrow 2 = 3k \quad [\text{Using (i) and (ii)}]$$

$$\Rightarrow k = \frac{2}{3}$$

4. It is given that the difference between the zeroes of $4x^2 - 8kx + 9$ is 4 and $k > 0$. Then, $k = ?$

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{7}{2}$

Sol:

(c) $\frac{5}{2}$

Let the zeroes of the polynomial be α and $\alpha + 4$

Here, $p(x) = 4x^2 - 8kx + 9$

Comparing the given polynomial with $ax^2 + bx + c$, we get:

$a = 4, b = -8k$ and $c = 9$

Now, sum of the roots = $\frac{-b}{a}$

$$\Rightarrow \alpha + \alpha + 4 = \frac{-(-8)}{4}$$

$$\Rightarrow 2\alpha + 4 = 2k$$

$$\Rightarrow \alpha + 2 = k$$

$$\Rightarrow \alpha = (k - 2) \quad \dots(i)$$

Also, product of the roots, $\alpha\beta = \frac{c}{a}$

$$\Rightarrow \alpha (\alpha + 4) = \frac{9}{4}$$

$$\Rightarrow (k - 2) (k - 2 + 4) = \frac{9}{4}$$

$$\Rightarrow (k - 2) (k + 2) = \frac{9}{4}$$

$$\Rightarrow k^2 - 4 = \frac{9}{4}$$

$$\Rightarrow 4k^2 - 16 = 9$$

$$\Rightarrow 4k^2 = 25$$

$$\Rightarrow k^2 = \frac{25}{4}$$

$$\Rightarrow k = \frac{5}{2} \quad (\because k > 0)$$

5. Find the zeroes of the polynomial $x^2 + 2x - 195$.

Sol:

Here, $p(x) = x^2 + 2x - 195$

Let $p(x) = 0$

$$\Rightarrow x^2 + (15 - 13)x - 195 = 0$$

$$\Rightarrow x^2 + 15x - 13x - 195 = 0$$

$$\Rightarrow x(x + 15) - 13(x + 15) = 0$$

$$\Rightarrow (x + 15)(x - 13) = 0$$

$$\Rightarrow x = -15, 13$$

Hence, the zeroes are -15 and 13 .

6. If one zero of the polynomial $(a^2 + 9)x^2 - 13x + 6a$ is the reciprocal of the other, find the value of a .

Sol:

$$(a^2 + 9)x^2 - 13x + 6a = 0$$

Here, $A = (a^2 + 9)$, $B = 13$ and $C = 6a$

Let α and $\frac{1}{\alpha}$ be the two zeroes.

Then, product of the zeroes = $\frac{C}{A}$

$$\Rightarrow \alpha \cdot \frac{1}{\alpha} = \frac{6a}{a^2 + 9}$$

$$\Rightarrow 1 = \frac{6a}{a^2 + 9}$$

$$\begin{aligned}\Rightarrow a^2 + 9 &= 6a \\ \Rightarrow a^2 - 6a + 9 &= 0 \\ \Rightarrow a^2 - 2 \times a \times 3 + 3^2 &= 0 \\ \Rightarrow (a - 3)^2 &= 0 \\ \Rightarrow a - 3 &= 0 \\ \Rightarrow a &= 3\end{aligned}$$

7. Find a quadratic polynomial whose zeroes are 2 and -5.

Sol:

It is given that the two roots of the polynomial are 2 and -5.

Let $\alpha = 2$ and $\beta = -5$

Now, the sum of the zeroes, $\alpha + \beta = 2 + (-5) = -3$

Product of the zeroes, $\alpha\beta = 2 \times (-5) = -10$

\therefore Required polynomial $= x^2 - (\alpha + \beta)x + \alpha\beta$

$$= x^2 - (-3)x + (-10)$$

$$= x^2 + 3x - 10$$

8. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $(a - b)$, a and $(a + b)$, find the values of a and b .

Sol:

The given polynomial $= x^3 - 3x^2 + x + 1$ and its roots are $(a - b)$, a and $(a + b)$.

Comparing the given polynomial with $Ax^3 + Bx^2 + Cx + D$, we have:

$$A = 1, B = -3, C = 1 \text{ and } D = 1$$

$$\text{Now, } (a - b) + a + (a + b) = \frac{-B}{A}$$

$$\Rightarrow 3a = -\frac{-3}{1}$$

$$\Rightarrow a = 1$$

$$\text{Also, } (a - b) \times a \times (a + b) = \frac{-D}{A}$$

$$\Rightarrow a(a^2 - b^2) = \frac{-1}{1}$$

$$\Rightarrow 1(1^2 - b^2) = -1$$

$$\Rightarrow 1 - b^2 = -1$$

$$\Rightarrow b^2 = 2$$

$$\Rightarrow b = \pm\sqrt{2}$$

$$\therefore a = 1 \text{ and } b = \pm\sqrt{2}$$

9. Verify that 2 is a zero of the polynomial $x^3 + 4x^2 - 3x - 18$.

Sol:

$$\text{Let } p(x) = x^3 + 4x^2 - 3x - 18$$

$$\text{Now, } p(2) = 2^3 + 4 \times 2^2 - 3 \times 2 - 18 = 0$$

$\therefore 2$ is a zero of $p(x)$.

10. Find the quadratic polynomial, the sum of whose zeroes is -5 and their product is 6 .

Sol:

Given:

Sum of the zeroes $= -5$

Product of the zeroes $= 6$

\therefore Required polynomial $= x^2 - (\text{sum of the zeroes})x + \text{product of the zeroes}$

$$= x^2 - (-5)x + 6$$

$$= x^2 + 5x + 6$$

11. Find a cubic polynomial whose zeroes are 3 , 5 and -2 .

Sol:

Let α , β and γ are the zeroes of the required polynomial.

Then we have:

$$\alpha + \beta + \gamma = 3 + 5 + (-2) = 6$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 3 \times 5 + 5 \times (-2) + (-2) \times 3 = -1$$

$$\text{and } \alpha\beta\gamma = 3 \times 5 \times (-2) = -30$$

$$\text{Now, } p(x) = x^3 - x^2(\alpha + \beta + \gamma) + x(\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma$$

$$= x^3 - x^2 \times 6 + x \times (-1) - (-30)$$

$$= x^3 - 6x^2 - x + 30$$

So, the required polynomial is $p(x) = x^3 - 6x^2 - x + 30$.

12. Using remainder theorem, find the remainder when $p(x) = x^3 + 3x^2 - 5x + 4$ is divided by $(x - 2)$.

Sol:

$$\text{Given: } p(x) = x^3 + 3x^2 - 5x + 4$$

$$\text{Now, } p(2) = 2^3 + 3(2^2) - 5(2) + 4$$

$$= 8 + 12 - 10 + 4$$

$$= 14$$

13. Show that $(x + 2)$ is a factor of $f(x) = x^3 + 4x^2 + x - 6$.

Sol:

$$\text{Given: } f(x) = x^3 + 4x^2 + x - 6$$

$$\text{Now, } f(-2) = (-2)^3 + 4(-2)^2 + (-2) - 6$$

$$= -8 + 16 - 2 - 6$$

$$= 0$$

$\therefore (x + 2)$ is a factor of $f(x) = x^3 + 4x^2 + x - 6$.

14. If α, β, γ are the zeroes of the polynomial $p(x) = 6x^3 + 3x^2 - 5x + 1$, find the value of $\left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right)$.

Sol:

$$\begin{aligned}\text{Given: } p(x) &= 6x^3 + 3x^2 - 5x + 1 \\ &= 6x^3 - (-3)x^2 + (-5)x - 1\end{aligned}$$

Comparing the polynomial with $x^3 - x^2(\alpha + \beta + \gamma) + x(\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma$, we get:

$$\alpha\beta + \beta\gamma + \gamma\alpha = -5$$

$$\text{and } \alpha\beta\gamma = -1$$

$$\therefore \left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right)$$

$$= \left(\frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}\right)$$

$$= \left(\frac{-5}{-1}\right)$$

$$= 5$$

15. If α, β are the zeroes of the polynomial $f(x) = x^2 - 5x + k$ such that $\alpha - \beta = 1$, find the value of k .

Sol:

$$\text{Given: } x^2 - 5x + k$$

The co-efficients are $a = 1$, $b = -5$ and $c = k$.

$$\therefore \alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow \alpha + \beta = \frac{-(-5)}{1}$$

$$\Rightarrow \alpha + \beta = 5 \quad \dots(1)$$

$$\text{Also, } \alpha - \beta = 1 \quad \dots(2)$$

From (1) and (2), we get:

$$2\alpha = 6$$

$$\Rightarrow \alpha = 3$$

Putting the value of α in (1), we get $\beta = 2$.

$$\text{Now, } \alpha\beta = \frac{c}{a}$$

$$\Rightarrow 3 \times 2 = \frac{k}{1}$$

$$\therefore k = 6$$

16. Show that the polynomial $f(x) = x^2 + 4x + 6$ has no zero.

Sol:

$$\text{Let } t = x^2$$

$$\text{So, } f(t) = t^2 + 4t + 6$$

Now, to find the zeroes, we will equate $f(t) = 0$

$$\Rightarrow t^2 + 4t + 6 = 0$$

$$\begin{aligned}\text{Now, } t &= \frac{-4 \pm \sqrt{16-24}}{2} \\ &= \frac{-4 \pm \sqrt{-8}}{2} \\ &= -2 \pm \sqrt{-2}\end{aligned}$$

$$\text{i.e., } x^2 = -2 \pm \sqrt{-2}$$

$$\Rightarrow x = \sqrt{-2 \pm \sqrt{-2}}, \text{ which is not a real number.}$$

The zeroes of a polynomial should be real numbers.

\therefore The given $f(x)$ has no zeroes.

- 17.** If one zero of the polynomial $p(x) = x^3 - 6x^2 + 11x - 6$ is 3, find the other two zeroes.

Sol:

$$p(x) = x^3 - 6x^2 + 11x - 6 \text{ and its factor, } x + 3$$

Let us divide $p(x)$ by $(x - 3)$.

$$\begin{aligned}\text{Here, } x^3 - 6x^2 + 11x - 6 &= (x - 3)(x^2 - 3x + 2) \\ &= (x - 3)[(x^2 - (2 + 1)x + 2)] \\ &= (x - 3)(x^2 - 2x - x + 2) \\ &= (x - 3)[x(x - 2) - 1(x - 2)] \\ &= (x - 3)(x - 1)(x - 2)\end{aligned}$$

\therefore The other two zeroes are 1 and 2.

- 18.** If two zeroes of the polynomial $p(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2$ are $\sqrt{2}$ and $-\sqrt{2}$, find its other two zeroes.

Sol:

$$\text{Given: } p(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2 \text{ and the two zeroes, } \sqrt{2} \text{ and } -\sqrt{2}$$

$$\text{So, the polynomial is } (x + \sqrt{2})(x - \sqrt{2}) = x^2 - 2.$$

Let us divide $p(x)$ by $(x^2 - 2)$

$$\begin{aligned}\text{Here, } 2x^4 - 3x^3 - 3x^2 + 6x - 2 &= (x^2 - 2)(2x^2 - 3x + 1) \\ &= (x^2 - 2)[(2x^2 - (2 + 1)x + 1)] \\ &= (x^2 - 2)(2x^2 - 2x - x + 1) \\ &= (x^2 - 2)[(2x(x - 1) - 1(x - 1))] \\ &= (x^2 - 2)(2x - 1)(x - 1)\end{aligned}$$

The other two zeroes are $\frac{1}{2}$ and 1.

- 19.** Find the quotient when $p(x) = 3x^4 + 5x^3 - 7x^2 + 2x + 2$ is divided by $(x^2 + 3x + 1)$.

Sol:

Given: $p(x) = 3x^4 + 5x^3 - 7x^2 + 2x + 2$

Dividing $p(x)$ by $(x^2 + 3x + 1)$, we have:

[illegible]

∴ The quotient is $3x^2 - 4x + 2$

20. Use remainder theorem to find the value of k, it being given that when $x^3 + 2x^2 + kx + 3$ is divided by $(x - 3)$, then the remainder is 21.

Sol:

Let $p(x) = x^3 + 2x^2 + kx + 3$

$$\begin{aligned}\text{Now, } p(3) &= (3)^3 + 2(3)^2 + 3k + 3 \\ &= 27 + 18 + 3k + 3 \\ &= 48 + 3k\end{aligned}$$

It is given that the remainder is 21

$$\therefore 3k + 48 = 21$$

$$\Rightarrow 3k = -27$$

$$\Rightarrow k = -9$$