

# Ex 1.1

## Q1

Well-defined collections are sets.

Example:

The collection of good teachers in a school is not a set, It is a collection.

Thus, we can say that every set is a collection, but every collection is not necessarily a set.

The collection of vowels in English alphabets is a set.

## Q2

**Answer :**

(i) The collection of all natural numbers less than 50 is a set because it is well defined.

(ii) The collection of good hockey players is not a set because the goodness of a hockey player is not defined here. So, it is not a set.

(iii) The collection of all girls in a class is a set, as it is well defined that all girls of the class are being talked about.

(iv) The collection of the most talented writers of India is a set because it is well defined.

(v) The collection of difficult topics in mathematics is not a set because a topic can be easy for one student while difficult for the other student.

(vi) The collection of all months of a year beginning with the letter J is a set given by {January, June, July}

(vii) A collection of novels written by Munshi Prem Chand is a set because one can determine whether the novel is written by Munshi Prem Chand or not.

(Viii) The collection of all question in this chapter is a set because one can easily check whether it is a question of the chapter or not.

(ix) A collection of most dangerous animals of the world is not a set because we cannot decide whether the animal is dangerous or not.

(x) The collection of prime integers is set given by {2, 3, 5.....}

## Q3

(i)  $4 \in$

(ii)  $-4 \notin$

(iii)  $12 \notin$

(iv)  $9 \in$

(v)  $0 \in$

(vi)  $-2 \notin$

## Ex 1.2

### Q1(i)

In Roster form, we describe a set by listing its elements, separated by commas and the elements are written within braces  $\{ \}$ . If a set has infinitely many elements, then comma is followed by  $\dots$ , where the dots stand for 'and so on'.

The above set in Roster form can be written as  $\{a, b, c, d, e\}$ . Since the letters  $a, b, c$ , and  $d$  precedes  $e$  in the english alphabet.

### Q1(ii)

In Roster form, we describe a set by listing its elements, separated by commas and the elements are written within braces  $\{ \}$ . If a set has infinitely many elements, then comma is followed by  $\dots$ , where the dots stand for 'and so on'.

$$1 \in N \because 1^2 = 1 < 25$$

$$2 \in N \because 2^2 = 4 < 25$$

$$3 \in N \because 3^2 = 9 < 25$$

$$4 \in N \because 4^2 = 16 < 25$$

Hence, the above set can be written as  $\{1, 2, 3, 4\}$

### Q1(iii)

In Roster form, we describe a set by listing its elements, separated by commas and the elements are written within braces  $\{ \}$ . If a set has infinitely many elements, then comma is followed by  $\dots$ , where the dots stand for 'and so on'.

We note that  $a < x < b$  means that  $x$  is more than  $a$  but less than  $b$ .

The prime numbers which are more than 10 but less than 20 are 11, 13, 17 and 19.

Hence the above set can be written as  $\{11, 13, 17, 19\}$

### Q1(iv)

In Roster form, we describe a set by listing its elements, separated by commas and the elements are written within braces  $\{ \}$ . If a set has infinitely many elements, then comma is followed by  $\dots$ , where the dots stand for 'and so on'.

The above set can be written as  $\{2, 4, 6, 8, \dots\}$  since all those natural numbers, which can be written as a multiple of 2 are the even natural numbers.

### Q1(v)

In Roster form, we describe a set by listing its elements, separated by commas and the elements are written within braces  $\{ \}$ . If a set has infinitely many elements, then comma is followed by  $\dots$ , where the dots stand for 'and so on'.

We know that given any  $x \in R$ ,  $x$  is always less than or equal to itself, i.e  $x \leq x$   
Hence the above set is empty, i.e  $\emptyset$ .

### Q1(vi)

In Roster form, we describe a set by listing its elements, separated by commas and the elements are written within braces  $\{ \}$ . If a set has infinitely many elements, then comma is followed by  $\dots$ , where the dots stand for 'and so on'.

The Prime divisors of 60 are 2,3,5.  
Hence the above set can be written as  $\{2, 3, 5\}$

### Q1(vii)

In Roster form, we describe a set by listing its elements, separated by commas and the elements are written within braces  $\{ \}$ . If a set has infinitely many elements, then comma is followed by  $\dots$ , where the dots stand for 'and so on'.

The above set can be written as  
 $\{17, 26, 35, 44, 53, 62, 71, 80\}$

### Q1(viii)

In Roster form, we describe a set by listing its elements, separated by commas and the elements are written within braces  $\{ \}$ . If a set has infinitely many elements, then comma is followed by  $\dots$ , where the dots stand for 'and so on'.

As repetition is not allowed in a set, the distinct letters are T,R,I,G,O,N,M,E,Y.  
Hence the above set can be written as

$$\{T, R, I, G, O, N, M, E, Y\}$$

### Q1(ix)

In Roster form, we describe a set by listing its elements, separated by commas and the elements are written within braces  $\{ \}$ . If a set has infinitely many elements, then comma is followed by  $\dots$ , where the dots stand for 'and so on'.

The distinct letters are B, E, T, R.

Hence the set can be written as

$$\{B, E, T, R\}$$

### Q2(i)

In set Builder form, a set is described by some characterizing property  $P(x)$  of its elements  $x$ .

In this case a set can be described as  $\{x : P(x) \text{ hold}\}$  or  $\{x | P(x) \text{ holds}\}$  which is read as 'the set of all  $x$  such that  $P(x)$  holds'.

The symbols ':' or '|' is read as 'such that'.

So, the above set  $A$  in Set-Builder form may be written as

$$A = \{x \in N : x < 7\}$$

i.e  $A$  is the set of natural numbers  $x$  such that  $x$  is less than 7.

or

$$A = \{x \in N | 1 \leq x \leq 6\},$$

i.e  $A$  is the set of natural numbers  $x$  such that  $x$  is greater than or equal 1 and less than or equal to 6.

## Q2(ii)

In set Builder form, a set is described by some characterizing property  $P(x)$  of its elements  $x$ .

In this case a set can be described as  $\{x : P(x) \text{ hold}\}$  or  $\{x | P(x) \text{ holds}\}$  which is read as 'the set of all  $x$  such that  $P(x)$  holds'.

The symbols ':' or '|' is read as 'such that'.

$$B = \left\{ x : x = \frac{1}{n}, n \in \mathbb{N} \right\}$$

i.e  $B$  is the set of all those  $x$  such that  $x = \frac{1}{n}$ , where  $n \in \mathbb{N}$

## Q2(iii)

In set Builder form, a set is described by some characterizing property  $P(x)$  of its elements  $x$ .

In this case a set can be described as  $\{x : P(x) \text{ hold}\}$  or  $\{x | P(x) \text{ holds}\}$  which is read as 'the set of all  $x$  such that  $P(x)$  holds'.

The symbols ':' or '|' is read as 'such that'.

$$C = \{x : x = 3k, k \in \mathbb{Z}^+, \text{ the set of positive integers}\},$$

i.e  $C$  is the set of multiples of 3 including 0

### Q2(iv)

In set Builder form, a set is described by some characterizing property  $P(x)$  of its elements  $x$ .

In this case a set can be described as  $\{x : P(x) \text{ hold}\}$  or  $\{x | P(x) \text{ holds}\}$  which is read as 'the set of all  $x$  such that  $P(x)$  holds'.

The symbols ':' or '|' is read as 'such that'.

$$D = \{x \in N : 9 < x < 16\},$$

i.e  $D$  is the set of natural numbers which are more than 9 but less than 16.

### Q2(v)

In set Builder form, a set is described by some characterizing property  $P(x)$  of its elements  $x$ .

In this case a set can be described as  $\{x : P(x) \text{ hold}\}$  or  $\{x | P(x) \text{ holds}\}$  which is read as 'the set of all  $x$  such that  $P(x)$  holds'.

The symbols ':' or '|' is read as 'such that'.

$$E = \{x \in Z : -1 < x < 1\}$$

or

$$E = \{x \in Z : x = 0\}$$

## Q2(vi)

In set Builder form, a set is described by some characterizing property  $P(x)$  of its elements  $x$ .

In this case a set can be described as  $\{x : P(x) \text{ hold}\}$  or  $\{x | P(x) \text{ holds}\}$  which is read as 'the set of all  $x$  such that  $P(x)$  holds'.

The symbols ':' or '|' is read as 'such that'.

$$\begin{aligned} \text{As } 1^2 &= 1 \\ 2^2 &= 4 \\ 3^2 &= 9 \\ &\vdots \\ &\vdots \\ 10^2 &= 100 \end{aligned}$$

$\therefore$  The above set may be described as

$$\{x^2 : x \in \mathbb{N} \text{ \& } 1 \leq x \leq 10\}$$

## Q2(vii)

In set Builder form, a set is described by some characterizing property  $P(x)$  of its elements  $x$ .

In this case a set can be described as  $\{x : P(x) \text{ hold}\}$  or  $\{x | P(x) \text{ holds}\}$  which is read as 'the set of all  $x$  such that  $P(x)$  holds'.

The symbols ':' or '|' is read as 'such that'.

The given set can be described as

$$\{x : x = 2n, n \in \mathbb{N}\} (\because 2, 4, 6, \dots \text{ are multiples of } 2)$$

### Q2(viii)

In set Builder form, a set is described by some characterizing property  $P(x)$  of its elements  $x$ .

In this case a set can be described as  $\{x : P(x) \text{ hold}\}$  or  $\{x | P(x) \text{ holds}\}$  which is read as 'the set of all  $x$  such that  $P(x)$  holds'.

The symbols ':' or '|' is read as 'such that'.

$$\therefore 5^1 = 5$$

$$5^2 = 25$$

$$5^3 = 125$$

$$5^4 = 625$$

$\therefore$  The above set can be described as

$$\{x : x = 5^n, 1 \leq n \leq 4\}$$

### Q3(i)

The integers whose squares are less than or equal to 10 are:

$$(-3)^2 = 9 < 10$$

$$(-2)^2 = 4 < 10$$

$$(-1)^2 = 1 < 10$$

$$0^2 = 0 < 10$$

$$1^2 = 1 < 10$$

$$2^2 = 4 < 10$$

$$3^2 = 9 < 10$$

The square of other integers are more than 10

$$\text{Hence } A = \{0, \pm 1, \pm 2, \pm 3\}$$

or

$$A = \{0, -1, -2, -3, 1, 2, 3\}$$



**Q3(ii)**

Let's find the values of  $x = \frac{1}{2n-1}$ , for  $1 \leq n \leq 5$

$$\text{for } n = 1, x = \frac{1}{1} = 1$$

$$\text{for } n = 2, x = \frac{1}{2 \times 2 - 1} = \frac{1}{4 - 1} = \frac{1}{3}$$

$$\text{for } n = 3, x = \frac{1}{2 \times 3 - 1} = \frac{1}{6 - 1} = \frac{1}{5}$$

$$\text{for } n = 4, x = \frac{1}{2 \times 4 - 1} = \frac{1}{8 - 1} = \frac{1}{7}$$

$$\text{for } n = 5, x = \frac{1}{2 \times 5 - 1} = \frac{1}{10 - 1} = \frac{1}{9}$$

$$\text{Hence, } B = \left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}\right\}$$

**Q3(iii)**

The integers which lie between  $\frac{-1}{2}$  and  $\frac{9}{2}$  are 0, 1, 2, 3, 4

$$\text{Hence } C = \{0, 1, 3, 4\}$$

**Q3(iv)**

The vowels in the word EQUATION are E, U, A, I, O.

Since the order in which the elements of a set are written is immaterial,  $D = \{A, E, I, O, U\}$

**Q3(v)**

A month has either 28, 29, 30 or 31 days.

Out of the 12 months in a year, the months that have 31 days are:

January, March, May, July, August, October, December.

$$\therefore E = \{\text{February, April, June, September, November}\}$$

**Q3(vi)**

The distinct letters of the word 'MISSISSIPPI' are M, I, S, P

$$\text{Hence } F = \{M, I, S, P\}$$

#### Q4

- (i)  $\{A, P, L, E\} \leftrightarrow \{x : x \text{ is a letter of the word "APPLE"}\}$
- (ii) The solution set of  $x^2 - 25 = 0$  is  $x = \pm 5$   
Hence,  $\{-5, 5\} \leftrightarrow \{x : x^2 - 25 = 0\}$
- (iii) The solution set of  $x + 5 = 5$  is  $x = 0$   
Hence,  $\{0\} \leftrightarrow \{x : x + 5 = 5, x \in \mathbb{Z}\}$
- (iv) The natural numbers which are divisor of 10 are 1, 2, 5, 10  
Hence,  $\{1, 2, 5, 10\} \leftrightarrow \{x : x \text{ is a natural number and divisor of } 10\}$
- (v) The distinct letters of the word "RAJASTHAN" are A, H, J, R, S, T, N  
Hence,  $\{A, H, J, R, S, T, N\} \leftrightarrow \{x : x \text{ is a letter of the word "RAJASTHAN"}\}$
- (vi) The prime natural numbers which are divisor of 10 are 2, 5  
Hence,  $\{2, 5\} \leftrightarrow \{x : x \text{ is a prime natural number and a divisor of } 10\}$

#### Q5

The vowels which precede  $q$ , that is, come before  $q$  are  $a, e, i, o$

Hence the set of vowels in the English alphabet which precede  $q$  are

$$\{a, e, i, o\}$$

#### Q6

As the cube of an odd integer is odd, and an odd positive integer has the form  $2n + 1$  for some  $n \geq 0$ ,

Hence the set of all positive integers whose cube is odd may be written in set builder form as  $\{x \in \mathbb{Z}, x = 2n + 1, n \geq 0\}$

#### Q7

$$\begin{aligned}\text{As } 2 &= 1^2 + 1 \\ 5 &= 2^2 + 1 \\ 10 &= 3^2 + 1 \\ &\vdots \\ &\vdots \\ 50 &= 7^2 + 1\end{aligned}$$

So, the above set in set builder form can be written as

$$\left\{ \frac{n}{n^2 + 1} : n \in \mathbb{N}, 1 \leq n \leq 7 \right\}$$

## Ex 1.3

### Q1

- (i) This set is non-empty as 10 is an even natural number divisible by 5.
- (ii) As 2 belongs to this set, so it is non-empty.
- (iii)  $x^2 - 2 = 0 \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2} \notin \mathbb{Q}$ , the set of rational numbers  
So, this set is empty.
- (iv) This set is empty as there is no natural number  $x$  such that  $x < 8$  and simultaneously  $x > 12$ .
- (v) This set is empty as any two parallel lines never intersect each other.

### Q2

- (i) Infinite, since with a common centre infinitely many circles can be drawn in a plane.
- (ii) Finite, as there are only 26 letters of English Alphabet.
- (iii) Infinite,  $\because \{x \in \mathbb{N} : x > 5\} = \{6, 7, 8, \dots\}$  Which is infinite.
- (iv) Finite,  $\because \{x \in \mathbb{N} : x, 200\} = \{1, 2, 3, \dots, 199\}$  Which is finite.
- (v) Infinite,  $\because \{x \in \mathbb{Z} : x < 5\} = \{\dots, -3, -2, -1, 0, 1, 2, 3, 4\}$  Which is infinite.
- (vi)  $\{x \in \mathbb{R} : 0 < x < 1\}$  is an infinite set  $\because$  an interval is an infinite set.

### Q3

$$A = \{1, 2, 3\}$$

$$\begin{aligned} B &= \{x \in R : (x - 1)^2 = 0\} \\ &= \{x \in R : x = 1, 1\} \\ &= \{1\} \end{aligned}$$

$$C = \{1, 2, 3\} \text{ (}\therefore \text{ repetition is not allowed in a set)}$$

$$\begin{aligned} D &= \{x \in R : x^3 - 6x^2 + 11x - 6 = 0\} \\ &= \{x \in R : (x - 1)(x^2 - 5x + 6) = 0\} && [\therefore x = 1 \text{ satisfies the above equation}] \\ &= \{x \in R : (x - 1)(x - 2)(x - 3) = 0\} \\ &= \{x \in R : x = 1, 2, 3\} \\ &= \{1, 2, 3\} \end{aligned}$$

Hence the set  $A, C$  and  $D$  are equal.

### Q4

$$A = \{a, e, p, r\}$$

$$B = \{a, e, p, r\} \text{ (repetition of 'p' is not allowed)}$$

$$C = \{e, o, p, r\}$$

as  $A = B \neq C$ ,  $\therefore$  the sets are not equal

### Q5

Two finite sets are said to be equivalent if they have the same number of elements. As  $A$  and  $C$  have same number of elements, and  $B$  and  $D$  also have same number of elements.

$\therefore A$  is equivalent to  $C$  &  $B$  is equivalent to  $D$ .

## Q6

(i)

Two sets  $A$  and  $B$  are said to be equal if every elements of  $A$  is an elements of  $B$  and vice-versa.

We have,  $A = \{2, 3\}$

$$\begin{aligned}\text{and } B &= \{x : x \text{ is a solution of } x^2 + 5x + 6 = 0\} \\ &= \{x : x^2 + 3x + 2x + 6 = 0\} \\ &= \{x : x(x + 3) + 2(x + 3) = 0\} \\ &= \{x : (x + 3)(x + 2) = 0\} \\ &= \{x : x = -2, -3\} \\ &= \{-2, -3\}\end{aligned}$$

Hence  $A \neq B$ .

(ii)

$$A = \{W, O, L, F\}$$

$$B = \{F, O, L, W\}$$

$$= \{W, O, L, F\}$$

Hence  $A = B$

[ $\therefore$  repetition is not allowed]

[The order in which the elements are written does not matter.]

### Q7

$$A = \{0, a\}$$

$$B = \{1, 2, 3, 4\}$$

$$C = \{4, 8, 12\}$$

$$D = \{3, 1, 2, 4\}$$

$$= \{1, 2, 3, 4\}$$

$$E = \{1, 0\}$$

$$F = \{8, 4, 12\}$$

$$= \{4, 8, 12\}$$

$$G = \{1, 5, 7, 11\}$$

$$H = \{a, b\}$$

The sets  $B$  and  $D$  are equal.

The sets  $C$  and  $F$  are equal.

As  $A, E$  and  $H$  have same number of elements so they are equivalent.

As  $B, D$  and  $G$  have same number of elements, so they are equivalent

Also  $C$  and  $F$  have same number of elements, so they are equivalent.

### Q8

$$A = \{1, 2\}$$

$$B = \{1, 2\}$$

$$C = \{3, 1\}$$

$$D = \{1, 3\} \quad [\because \text{the odd natural numbers less than 5 are 1 and 3}]$$

$$E = \{1, 2\} \quad [\because \text{repetition is not allowed}]$$

$$F = \{1, 3\} \quad [\because \text{repetition is not allowed}]$$

$\therefore$   $A, B$  and  $E$  are equal

Also,  $C, D$  and  $F$  are equal

### Q9

The set formed by distinct letters of the word "CATARACT" are  $\{C, A, T, R\}$ .

The set formed by distinct letters of the word "TRACT" are  $\{T, R, A, C\}$

Hence the two set are equal.

## Ex 1.4

### Q1

- (i) False,  $\because$  the two sets  $A$  and  $B$  need not be comparable.
- (ii) False,  $\because \{1\}$  is a finite subset of the infinite set  $N$  of natural numbers.
- (iii) True,  $\because$  the order (or cardinal number) of any subset of a set is less than or equal to the order of the set.  
(order (or cardinal number) of a set is the number of elements in the set).
- (iv) False,  $\because$  the empty set  $\emptyset$  has no proper subset.
- (v) False,  $\because \{a, b, a, b, \dots\} = \{a, b\}$  (repetition is not allowed)  
 $\therefore \{a, b, a, b, \dots\}$  is a finite set.
- (vi) True,  $\because$  equivalent sets have the same cardinal number.
- (vii) False,  
One knows that if the cardinal number of a set  $A$  is  $n$ , then the power set of  $A$  denoted by  $P(A)$  which is the set of all subsets of  $A$ , has the cardinal number  $2^n$ .  
If the cardinal number of  $A$  is infinite, then the cardinal number of  $P(A)$  is also infinite.  
Hence, the above statement is true provided the set is infinite.

### Q2

- (i) True,  $\because 1$  is an element of the set  $\{1, 2, 3\}$ .
- (ii) False,  $\because a$  is an element and not a subset of the set  $\{b, c, a\}$ .
- (iii) False,  $\because \{a\}$  is a subset of the set  $\{a, b, c\}$  and not an element.
- (iv) True,  $\because$  repetition is not allowed in a set.
- (v) False,  $\because$  the set  $\{x : x + 8 = 8\}$  is the single ton set  $\{0\}$  which is not the null set  $\emptyset$ .

### Q3

We have,

$$\begin{aligned}A &= \{x : x \text{ satisfies } x^2 - 8x + 12 = 0\} \\&= \{x : x^2 - 6x - 2x + 12 = 0\} \\&= \{x : x(x - 6) - 2(x - 6) = 0\} \\&= \{x : (x - 6)(x - 2) = 0\} \\&= \{x : x = 6, 2\} \\&= \{6, 2\}\end{aligned}$$

$$B = \{2, 4, 6\}$$

$$C = \{2, 4, 6, 8, \dots\}$$

$$D = \{6\}$$

We know that if  $E$  and  $F$  are two sets, then  $E$  is a subset of  $F$ , i.e.,  $E \subseteq F$  if  $x \in E \Rightarrow x \in F$ .  $E$  is called a proper subset of  $F$  if  $E$  is strictly contained in  $F$  and is denoted by  $E \subset F$ .

Clearly,

$$D \subset A \{\because 6 \in D \text{ and } 6 \in A\}$$

$$A \subset B \{\because 2, 6 \in A \text{ and they also belong to } B\}$$

Similarly,  $B \subset C$

Hence,  $D \subset A \subset B \subset C$ .

### Q4(i)

The given statement is 'True'.

If  $m \in \mathbb{Z}$ , then  $m$  can be written as  $\frac{m}{1}$ , which is of the form  $\frac{p}{q}$ ,

where  $p$  and  $q$  are relatively prime integers and  $q \neq 0$ .

This implies that  $m \in \mathbb{Q}$ , the set of rational numbers.

Thus,  $m \in \mathbb{Z} \Rightarrow m \in \mathbb{Q}$

Hence  $\mathbb{Z} \subseteq \mathbb{Q}$



#### Q4(ii)

The given statement is 'True'.

$\therefore$  Crows are also Birds.

#### Q4(iii)

The given statement is 'False'.

$\therefore$  A rectangle need not be a square.

#### Q4(iv)

The given statement is 'True'.

If  $z$  is a complex number, then it can be written as  $z = x + iy$ , where  $x$  and  $y$  are real numbers and are called the real and imaginary parts of the complex number  $z$ .

If  $x$  is a real number, then

$$x = x + i \cdot 0 \in C,$$

where  $C$  is the set of complex numbers.

$$\text{Thus } x \in R \Rightarrow x \in C$$

Hence, the set of all real numbers is contained in the set of all complex numbers.

#### Q4(v)

False,  $\therefore a \in P$  but  $a \notin B$

Note that  $\{a\}$  is an element of  $B$  which is different from the element ' $a$ '.

#### Q4(vi)

$$A = \{L, I, T, E\} \quad [\therefore \text{repetition is not allowed}]$$

$$B = \{T, I, L, E\} \quad [\therefore \text{repetition is not allowed}]$$

$$= \{L, I, T, E\} \quad \left[ \begin{array}{l} \therefore \text{the manner in which the elements are} \\ \text{listed does not matter} \end{array} \right]$$

$\therefore$  Each element of  $A$  is an element of  $B$  and vice-versa

$$\therefore A = B$$

Hence, the given statement is true.

## Q5

(i) False,

The correct statement is  $a \in \{a, b, c\}$ .

(ii) False,  $\because \{a\}$  is a subset and not an element of  $\{a, b, c\}$ .

The correct form is  $\{a\} \subset \{a, b, c\}$ .

(iii) False,  $\because a$  is not an element of  $\{\{a\}, b\}$ .

The correct form is  $\{a\} \in \{\{a\}, b\}$ .

(iv) False,  $\because \{a\}$  is not a subset of  $\{\{a\}, b\}$  hence it cannot be contained in it.

The correct form is  $\{a\} \in \{\{a\}, b\}$ . Another correct form could be  $\{\{a\}\} \subset \{\{a\}, b\}$ .

(v) False,  $\because \{b, c\}$  is an element and not a subset of  $\{a, \{b, c\}\}$ .

The correct form is  $\{b, c\} \in \{a, \{b, c\}\}$ .

(vi) False,  $\because \{a, b\}$  is not a subset of  $\{a, \{b, c\}\}$ .

The correct form is  $\{a, b\} \not\subset \{a, \{b, c\}\}$ .

(vii) False,  $\because \emptyset$  is not an element of  $\{a, b\}$ .

The correct form is  $\emptyset \subset \{a, b\}$ .

(viii) True,  $\because$  empty set  $\emptyset$  is a subset of every set.

(ix) False,  $\because \{x : x + 3 = 3\} = \{x : x = 0\} = \{0\}$ .

The correct form is  $\{x : x + 3 = 3\} \neq \emptyset$ .

## Q6

- (i) False,  $\{c, d\}$  is an element of  $A$  and not a subset of  $A$ .
- (ii) True,  $\because \{c, d\}$  is indeed an element of  $A$ .
- (iii) True,  $\because \{c, d\}$  is a subset of  $A$ .
- (iv) True,
- (v) False,  $\because a$  belongs to  $A$  and not a subset of  $A$ . An element of a set belongs to it whereas a subset of it is contained in it.
- (vi) True,  $\because \{a, b, e\}$  is a subset of  $A$ .
- (vii) False,  $\because \{a, b, e\}$  is a subset of  $A$ , so it does not belong to  $A$ .
- (viii) False,  $\because \{a, b, c\}$  is not a subset of  $A$ .
- (ix) False,  $\because \emptyset$  is a subset and not an element of  $A$ .
- (x) False,  $\because \emptyset$  and not  $\{\emptyset\}$  is a subset of  $A$ .

## Q7

- (i) False,  $\because 1$  is not an element of  $A$ .
- (ii) False,  $\because \{1, 2, 3\}$  is not a subset of  $A$ , it is an element of  $A$ .
- (iii) True,  $\because \{6, 7, 8\}$  is indeed an element of  $A$ .
- (iv) True,  $\because \{\{4, 5\}\}$  is indeed a subset of  $A$ .
- (v) False,  $\because \emptyset$  is a subset and not an element of  $A$ .
- (vi) True,  $\because \emptyset$  is a subset of every set, and hence a subset of  $A$ .

### Q8

- (i) True,  $\because \emptyset$  indeed belongs to  $A$ .
- (ii) True,  $\because \{\emptyset\}$  is an element of  $A$ .
- (iii) False,  $\because \{1\}$  is not an element of  $A$ .
- (iv) True,  $\because \{2, \emptyset\}$  is a subset of  $A$ .
- (v) False,  $\because 2$  is not a subset of  $A$ , it is an element of  $A$ .
- (vi) True,  $\because \{2, \{1\}\}$  is not a subset of  $A$ .
- (vii) True,  $\because \{\{2\}, \{1\}\}$  is not a subset of  $A$ .
- (viii) True,  $\because \{\emptyset, \{\emptyset\}, \{1, \emptyset\}\}$  is a subset of  $A$ .
- (ix) True,  $\because \{\{\emptyset\}\}$  is a subset of  $A$ .

### Q9

- (i) We know that, if a set has  $n$  elements, then its power set has  $2^n$  elements.

Here,  $n = 1$ , so there  $2^1 = 2$  subsets of the given set.

The possible subsets are  $\emptyset, \{a\}$ .

- (ii) The set has two elements, so power set has  $2^2 = 4$  elements, namely  $\emptyset, \{0\}, \{1\}, \{0, 1\}$ .

- (iii) The set has 3 elements, so power set has  $2^3 = 8$  elements,  
namely  $\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}$ .

- (iv) The set has 2 elements, so power set has  $2^2 = 4$  elements, namely  $\emptyset, \{1\}, \{\{1\}\}, \{1, \{1\}\}$ .

- (v) The set has 1 element, so power set has  $2^1 = 2$  elements, namely  $\emptyset, \{\emptyset\}$ .

### Q10

(i) We know that if  $A$  is a set and  $B$  a subset of  $A$ , then  $B$  is called a proper subset of  $A$  if  $B \subseteq A$  and  $B \neq A$ ,  $\emptyset$  and is written as  $B \subset A$  or  $B \subseteq A$ .

Hence, the proper subsets are given by  $\{1\}, \{2\}$ .

(ii) The proper subsets are given by  $\{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}$ .

(iii) The only subsets of the given set are  $\emptyset$  &  $\{1\}$ .

Hence, there are no proper subsets.

### Q11

We know that, if  $A$  is a set having  $n$  elements then power set of  $A$ , namely  $P(A)$  has  $2^n$  elements. Out of this  $A$  is not proper subset.

Hence, the total number of proper subsets of a set consisting of  $n$  elements is  $2^n - 1$ .

### Q12

The symbol ' $\Leftrightarrow$ ' stands for if and only if (in short if).

In order to show that two sets  $A$  and  $B$  are equal we show that  $A \subseteq B$  and  $B \subseteq A$ .

We have  $A \subseteq \emptyset$ .  $\therefore \emptyset$  is a subset of every set

$$\therefore \emptyset \subseteq A$$

Hence  $A = \emptyset$

To show the backward implication, suppose that  $A = \emptyset$

$\therefore$  every set is a subset of itself

$$\therefore \emptyset = A \subseteq \emptyset$$

Hence, proved.

**Q13**

We have  $A \subseteq B$ ,  $B \subseteq C$  and  $C \subseteq A$ , so  $A \subseteq B \subseteq C \subseteq A$   
Now,  $A$  is a subset of  $B$  and  $B$  is a subset of  $C$ , so

$A$  is a subset of  $C$ , i.e.,  $A \subseteq C$

Also,  $C \subseteq A$

Hence,  $A = C$

**Q14**

$\therefore$  an empty set has zero element.

$\therefore$  power set of  $\emptyset$  has  $2^0 = 1$  element.

**Q15**

(i)

The set of right triangles is a subset of the set of all triangles in the plane. So, the set of all triangles in the plane forms a universal set for the set of right triangles.

(ii)

The set of isosceles triangles forms a subset of the set of all triangles in the plane.

Hence the set of all triangles in the plane forms a universal set for the set of isosceles triangles.

**Q16**

$$X = \{8^n - 7n - 1 : n \in \mathbb{N}\}$$

$$Y = \{4n(n-1) : n \in \mathbb{N}\}$$

In order to show that  $X \subseteq Y$  we show that every element of  $X$  is an element of  $Y$ .

So let  $x \in X \Rightarrow x = 8^m - 7m - 1$  for some  $m \in \mathbb{N}$

$$\begin{aligned} \Rightarrow x &= (1+7)^m - 7m - 1 \\ &= \left( {}^mC_0 1^m + {}^mC_1 1^{m-1} 7 + \dots + {}^mC_{m-1} 1^1 7^{m-1} + {}^mC_m 7^m \right) - 7m - 1 \\ &\quad \text{[using binomial expansion]} \\ &= 1 + 7m + {}^mC_2 7^2 + {}^mC_3 7^3 + \dots + {}^mC_m 7^m - 7m - 1 \\ &= {}^mC_2 7^2 + {}^mC_3 7^3 + \dots + {}^mC_m 7^m \\ &= 49 \left( {}^mC_2 + {}^mC_3 7 + \dots + {}^mC_m 7^{m-2} \right), \quad m \geq 2 \\ &= 49t_m, \quad m \geq 2, \quad \text{where } t_m = {}^mC_2 + {}^mC_3 7 + \dots + {}^mC_m 7^{m-2} \end{aligned}$$

Is some positive integer depending on  $m \geq 2$

For  $m = 1$

$$\begin{aligned} x &= 8^1 - 7 \times 1 - 1 \\ &= 8 - 8 \\ &= 0 \end{aligned}$$

Hence,  $X$  contains all positive integral multiples of 49.

Also,  $Y$  consists of all positive integral multiples of 49, including 0, for  $n = 1$ .

Thus, we conclude that  $X \subseteq Y$ .

## Ex 1.5

### Q1

(i)

$A \cap B$  denotes intersection of the two sets  $A$  and  $B$ , which consists of elements which are common to both  $A$  and  $B$ .

Since  $A \subset B$ , every element of  $A$  is already an element of  $B$ .

$$\therefore A \cap B = A$$

(ii)

$A \cup B$  denotes the union of the sets  $A$  and  $B$  which consists of elements which are either in  $A$  or  $B$  or in both  $A$  and  $B$ .

Since  $A \subset B$ , every element of  $A$  is already an element of  $B$ .

$$\therefore A \cup B = B$$

### Q2(i)

$$A = \{1, 2, 3, 4, 5\}$$

$$B = \{4, 5, 6, 7, 8\}$$

$$\begin{aligned} \text{So, } A \cup B &= \{x : x \in A \text{ or } x \in B\} \\ &= \{1, 2, 3, 4, 5, 6, 7, 8\} \end{aligned}$$

### Q2(ii)

$$\begin{aligned} A \cup C &= \{x : x \in A \text{ or } x \in C\} \\ &= \{1, 2, 3, 4, 5, 7, 8, 9, 10, 11\} \end{aligned}$$

### Q2(iii)

$$\begin{aligned} B \cup C &= \{x : x \in B \text{ or } x \in C\} \\ &= \{4, 5, 6, 7, 8, 9, 10, 11\} \end{aligned}$$

### Q2(iv)

$$\begin{aligned} B \cup D &= \{x : x \in B \text{ or } x \in D\} \\ &= \{4, 5, 6, 7, 8, 10, 11, 12, 13, 14\} \end{aligned}$$



**Q2(v)**

$$\begin{aligned}A \cup B \cup C &= \{x | x \in A \text{ or } x \in B \text{ or } x \in C\} \\&= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}\end{aligned}$$

**Q2(vi)**

$$\begin{aligned}A \cup B \cup D &= \{x : x \in A \text{ or } x \in B \text{ or } x \in D\} \\&= \{1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14\}\end{aligned}$$

**Q2(vii)**

$$\begin{aligned}B \cup C \cup D &= \{x | x \in B \text{ or } x \in C \text{ or } x \in D\} \\&= \{4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}\end{aligned}$$

**Q2(viii)**

$$\begin{aligned}A \cap (B \cup C) &= \text{all those elements which are common} \\&\quad \text{to } A \text{ and } B \cup C \\&= \{x | x \in A \text{ and } x \in B \cup C\}\end{aligned}$$

$$\text{Now, } B \cup C = \{4, 5, 6, 7, 8, 9, 10, 11\}$$

$$\begin{aligned}\therefore A \cap (B \cup C) &= \{1, 2, 3, 4, 5\} \cap \{4, 5, 6, 7, 8, 9, 10, 11\} \\&= \{4, 5\}\end{aligned}$$

**Q2(ix)**

$$(A \cap B) \cap (B \cap C) = \{x \mid x \in (A \cap B) \text{ and } x \in (B \cap C)\}$$

Now,

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

i.e., elements which are common to  $A$  &  $B$

$$\begin{aligned}\therefore A \cap B &= \{1, 2, 3, 4, 5\} \cap \{4, 5, 6, 7, 8\} \\ &= \{4, 5\}\end{aligned}$$

Also,

$$\begin{aligned}B \cap C &= \{4, 5, 6, 7, 8\} \cap \{7, 8, 9, 10, 11\} \\ &= \{7, 8\}\end{aligned}$$

$$\text{Hence, } (A \cap B) \cap (B \cap C) = \{4, 5\} \cap \{7, 8\}$$

$$= \emptyset$$

$\left[ \because \text{there is no element common in } \{4, 5\} \text{ and } \{7, 8\} \right]$

**Q2(x)**

$$(A \cup D) \cap (B \cup C) = \{x \mid x \in (A \cup D) \text{ or } x \in (B \cup C)\}$$

Now,

$$A \cup D = \{1, 2, 3, 4, 5, 10, 11, 12, 13, 14\}$$

$$\text{and } B \cup C = \{4, 5, 6, 7, 8, 9, 10, 11\}$$

$$\therefore (A \cup D) \cap (B \cup C) = \{4, 5, 10, 11\}$$

**Q3(i)**

We have,

$$\begin{aligned} A &= \{x : x \in N\} \\ &= \{1, 2, 3, \dots\}, \text{ the set of natural numbers} \end{aligned}$$

$$\begin{aligned} B &= \{x : x = 2n, x \in N\} \\ &= \{2, 4, 6, 8, \dots\}, \text{ the set of even natural numbers} \end{aligned}$$

$$\begin{aligned} \therefore A \cap B &= \{x : x \in A \text{ and } x \in B\} \\ &= \{2, 4, 6, \dots\} \\ &= B \end{aligned} \quad [\because B \subset A]$$

**Q3(ii)**

We have,

$$\begin{aligned} A &= \{x : x \in N\} \\ &= \{1, 2, 3, \dots\}, \text{ the set of natural numbers} \end{aligned}$$

$$\begin{aligned} C &= \{x : x = 2n - 1, x \in N\} \\ &= \{1, 3, 5, \dots\}, \text{ the set of odd natural numbers} \end{aligned}$$

$$\begin{aligned} A \cap C &= \{x : x \in A \text{ and } x \in C\} \\ &= C \end{aligned} \quad [\because C \subset A]$$

### Q3(iii)

We have,

$$\begin{aligned} A &= \{x : x \in N\} \\ &= \{1, 2, 3, \dots\}, \text{ the set of natural numbers} \end{aligned}$$

$$\begin{aligned} \text{and } D &= \{x : x \text{ is a prime natural number}\} \\ &= \{2, 3, 5, 7, \dots\} \end{aligned}$$

$$\begin{aligned} A \cap D &= \{x : x \in A \text{ and } x \in D\} \\ &= D \end{aligned} \quad [\because D \subset A]$$

### Q3(iv)

We have,

$$\begin{aligned} B &= \{x : x = 2n, x \in N\} \\ &= \{2, 4, 6, 8, \dots\}, \text{ the set of even natural numbers} \end{aligned}$$

and

$$\begin{aligned} C &= \{x : x = 2n-1, x \in N\} \\ &= \{1, 3, 5, \dots\}, \text{ the set of odd natural numbers} \end{aligned}$$

$$\begin{aligned} B \cap C &= \{x : x \in B \text{ and } x \in C\} \\ &= \emptyset \end{aligned} \quad \left[ \because B \text{ and } C \text{ are disjoint sets, i.e., } \right. \\ \left. \text{have no elements in common} \right]$$

### Q3(v)

Here,

$$\begin{aligned} B &= \{x : x = 2n, x \in N\} \\ &= \{2, 4, 6, 8, \dots\}, \text{ the set of even natural numbers} \end{aligned}$$

$$\begin{aligned} \text{and } D &= \{x : x \text{ is a prime natural number}\} \\ &= \{2, 3, 5, 7, \dots\} \end{aligned}$$

$$\begin{aligned} B \cap D &= \{x : x \in B \text{ and } x \in D\} \\ &= \{2\} \end{aligned}$$

### Q3(vi)

Here,

$$\begin{aligned}C &= \{x : x = 2n - 1, x \in \mathbb{N}\} \\&= \{1, 3, 5, \dots\}, \text{ the set of odd natural numbers}\end{aligned}$$

$$\begin{aligned}\text{and } D &= \{x : x \text{ is a prime natural number}\} \\&= \{2, 3, 5, 7, \dots\}\end{aligned}$$

$$C \cap D = \{x : x \in C \text{ and } x \in D\}$$

We observe that except, the element 2, every other element in  $D$  is an odd natural number.

$$\begin{aligned}\text{Hence, } C \cap D &= D - \{2\} \\&= \{x \in D : x \neq 2\}\end{aligned}$$

### Q4

We have,

$$\begin{aligned}A &= \{3, 6, 12, 15, 18, 21\} \\B &= \{4, 8, 12, 16, 20\} \\C &= \{2, 4, 6, 8, 10, 12, 14, 16\} \\D &= \{5, 10, 15, 20\}\end{aligned}$$

If  $A$  and  $B$  are two sets, then the set  $A - B$  is defined as

$$A - B = \{x \in A : x \notin B\}.$$

- (i)  $A - B = \{x \in A : x \notin B\} = \{3, 6, 15, 18, 21\}$
- (ii)  $A - C = \{x \in A : x \notin C\} = \{3, 15, 18, 21\}$
- (iii)  $A - D = \{x \in A : x \notin D\} = \{3, 6, 12, 18, 21\}$
- (iv)  $B - A = \{x \in B : x \notin A\} = \{4, 8, 16, 20\}$
- (v)  $C - A = \{x \in C : x \notin A\} = \{2, 4, 8, 10, 14, 16\}$
- (vi)  $D - A = \{x \in D : x \notin A\} = \{5, 10, 20\}$
- (vii)  $B - C = \{x \in B : x \notin C\} = \{20\}$
- (viii)  $B - D = \{x \in B : x \notin D\} = \{4, 8, 12, 16\}$

### Q5

(i)

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}, A = \{1, 2, 3, 4\}, B = \{2, 4, 6, 8\}, C = \{3, 4, 5, 6\}$$

By the complement of a set  $A$ , which respect to the universal set  $U$ , denoted by  $A'$  or  $A^c$  or  $U - A$ , we mean  $\{x \in U : x \notin A\}$ .

$$\text{Hence, } A' = \{x \in U : x \notin A\} = \{5, 6, 7, 8, 9\}$$

(ii)  $B' = \{x \in U : x \notin B\} = \{1, 3, 5, 7, 9\}$

(iii)  $(A \cap C)' = \{x \in U : x \notin A \cap C\}$

Now,

$$A \cap C = \{x : x \in A \text{ and } x \in C\} = \{3, 4\}$$

$$\therefore (A \cap C)' = \{1, 2, 5, 6, 7, 8, 9\}$$

## Q6

(i)

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$A = \{2, 4, 6, 8\}$$

$$B = \{2, 3, 5, 7\}$$

We have,

$$\begin{aligned} A \cup B &= \{x : x \in A \text{ or } x \in B\} \\ &= \{2, 3, 4, 5, 6, 7, 8\} \end{aligned}$$

$$\begin{aligned} \therefore (A \cup B)' &= \{x \in U : x \notin A \cup B\} \\ &= \{1, 9\} \end{aligned}$$

$$\begin{aligned} A' &= \{x \in U : x \notin A\} \\ &= \{1, 3, 5, 7, 9\} \end{aligned}$$

$$\begin{aligned} B' &= \{x \in U : x \notin B\} \\ &= \{1, 4, 6, 8, 9\} \end{aligned}$$

$$\text{Hence, } A' \cap B' = \{1, 9\}$$

$$\text{Hence, } (A \cup B)' = A' \cap B' = \{1, 9\}$$

(ii)

$$\begin{aligned} A \cap B &= \{x : x \in A \text{ and } x \in B\} \\ &= \{2\} \end{aligned}$$

$$\begin{aligned} \therefore (A \cap B)' &= \{x \in U : x \notin A \cap B\} \\ &= \{1, 3, 4, 5, 6, 7, 8, 9\} \end{aligned}$$

Also,

$$\begin{aligned} A' \cup B' &= \{x : x \in A' \text{ or } x \in B'\} \\ &= \{1, 3, 4, 5, 6, 7, 8, 9\} \end{aligned}$$

$$\text{Hence, } (A \cap B)' = A' \cup B' = \{1, 3, 4, 5, 6, 7, 8, 9\}$$

## Ex 1.6

### Q1

The smallest set  $A$  such that

$$A \cup \{1, 2\} = \{1, 2, 3, 5, 9\} \text{ is } \{3, 5, 9\}$$

$$\therefore \{3, 5, 9\} \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$$

Any other set  $B$  such that  $B \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$  will

contain  $A$ . For example we can take  $B$  to be  $\{1, 3, 5, 9\}$  or  $\{1, 2, 3, 5, 9\}$ .

Clearly  $B$  contains  $A = \{3, 5, 9\}$ .

### Q2(i)

$$\text{i. } A = \{1, 2, 4, 5\}, B = \{2, 3, 5, 6\}, C = \{4, 5, 6, 7\}$$

$$B \cap C = \{5, 6\}$$

$$A \cup (B \cap C) = \{1, 2, 4, 5, 6\} \dots\dots\dots (1)$$

$$(A \cup B) = \{1, 2, 3, 4, 5, 6\}$$

$$(A \cup C) = \{1, 2, 4, 5, 6, 7\}$$

$$(A \cup B) \cap (A \cup C) = \{1, 2, 4, 5, 6\} \dots\dots\dots (2)$$

From eq<sup>n</sup> (1) and eq<sup>n</sup> (2), we get

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

### Q2(ii)

$$\text{ii. } A = \{1, 2, 4, 5\}, B = \{2, 3, 5, 6\}, C = \{4, 5, 6, 7\}$$

$$B \cup C = \{2, 3, 4, 5, 6, 7\}$$

$$A \cap (B \cup C) = \{2, 4, 5\} \dots\dots\dots (1)$$

$$(A \cap B) = \{2, 5\}$$

$$(A \cap C) = \{4, 5\}$$

$$(A \cap B) \cup (A \cap C) = \{2, 4, 5\} \dots\dots\dots (2)$$

From eq<sup>n</sup> (1) and eq<sup>n</sup> (2), we get

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$



**Q2(iii)**

$$\text{iii. } A = \{1, 2, 4, 5\}, B = \{2, 3, 5, 6\}, C = \{4, 5, 6, 7\}$$

$$B - C = \{2, 3\}$$

$$A \cap (B - C) = \{2\} \dots \dots \dots (1)$$

$$(A \cap B) = \{2, 5\}$$

$$(A \cap C) = \{4, 5\}$$

$$(A \cap B) - (A \cap C) = \{2\} \dots \dots \dots (2)$$

From eq<sup>n</sup> (1) and eq<sup>n</sup> (2), we get

$$A \cap (B - C) = (A \cap B) - (A \cap C)$$

**Q2(iv)**

$$\text{iv. } A = \{1, 2, 4, 5\}, B = \{2, 3, 5, 6\}, C = \{4, 5, 6, 7\}$$

$$B \cup C = \{2, 3, 4, 5, 6, 7\}$$

$$A - (B \cup C) = \{1\} \dots \dots \dots (1)$$

$$(A - B) = \{1, 4\}$$

$$(A - C) = \{1, 2\}$$

$$(A - B) \cap (A - C) = \{1\} \dots \dots \dots (2)$$

From eq<sup>n</sup> (1) and eq<sup>n</sup> (2), we get

$$A - (B \cup C) = (A - B) \cap (A - C)$$

**Q2(v)**

$$v. A = \{1, 2, 4, 5\}, B = \{2, 3, 5, 6\}, C = \{4, 5, 6, 7\}$$

$$B \cap C = \{5, 6\}$$

$$A - (B \cap C) = \{1, 2, 4\} \dots \dots \dots (1)$$

$$(A - B) = \{1, 4\}$$

$$(A - C) = \{1, 2\}$$

$$(A - B) \cup (A - C) = \{1, 2, 4\} \dots \dots \dots (2)$$

From eq<sup>n</sup> (1) and eq<sup>n</sup> (2), we get

$$A - (B \cap C) = (A - B) \cup (A - C)$$

**Q2(vi)**

$$vi. A = \{1, 2, 4, 5\}, B = \{2, 3, 5, 6\}, C = \{4, 5, 6, 7\}$$

$$B \Delta C = (B - C) \cup (C - B) = \{2, 3\} \cup \{4, 7\} = \{2, 3, 4, 7\}$$

$$A \cap (B \Delta C) = \{2, 4\} \dots \dots \dots (1)$$

$$(A \cap B) = \{2, 5\}$$

$$(A \cap C) = \{4, 5\}$$

$$(A \cap B) \Delta (A \cap C) = [(A \cap B) - (A \cap C)] \cup [(A \cap C) - (A \cap B)]$$

$$(A \cap B) \Delta (A \cap C) = \{2\} \cup \{4\} = \{2, 4\} \dots \dots \dots (2)$$

From eq<sup>n</sup> (1) and eq<sup>n</sup> (2), we get

$$A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$$

**Q3(i)**

$U = \{2, 3, 5, 7, 9\}$  is the universal set

$$A = \{3, 7\}, B = \{2, 5, 7, 9\}$$

$$\begin{aligned} A \cup B &= \{x : x \in A \text{ or } x \in B\} \\ &= \{2, 3, 5, 7, 9\} \end{aligned}$$

$$\begin{aligned} \text{LHS} &= (A \cup B)' \\ &= \{2, 3, 5, 7, 9\}' \\ &= U - A \cup B \\ &= \emptyset \end{aligned}$$

$$\begin{aligned} \text{RHS} &= A' \cap B' \\ A' &= \{x \in U : x \notin A\} \\ &= \{2, 5, 9\} \\ B' &= \{x \in U : x \notin B\} \\ &= \{3\} \end{aligned}$$

$$\begin{aligned} \therefore A' \cap B' &= \{2, 5, 9\} \cap \{3\} \\ &= \emptyset \end{aligned}$$

[ $\because$  the two sets are disjoint]

$$\therefore \text{LHS} = \text{RHS} \quad \text{Proved}$$

**Q3(ii)**

$$\text{LHS} = (A \cap B)'$$

Now,

$$\begin{aligned} A \cap B &= \{x \mid x \in A \text{ and } x \in B\} \\ &= \{7\} \end{aligned}$$

$$\begin{aligned} \therefore (A \cap B)' &= \{7\}' \\ &= \{x \in U : x \notin 7\} \\ &= \{2, 3, 5, 9\} \end{aligned}$$

$$\text{RHS} = A' \cup B'$$

$$\text{Now, } A' = \{2, 5, 9\}$$

[from (i)]

$$\text{and } B' = \{3\}$$

[from (i)]

$$\therefore A' \cup B' = \{2, 3, 5, 9\}$$

$$\text{Hence, LHS} = \text{RHS} \quad \text{Proved}$$

**Q4(i)**

i. Let  $x \in B$ . Then

$$\Rightarrow x \in B \cup A$$

$$\Rightarrow x \in A \cup B$$

$$\therefore B \subset (A \cup B)$$

**Q4(ii)**

ii. Let  $x \in A \cap B$ . Then

$$\Rightarrow x \in A \text{ and } x \in B$$

$$\Rightarrow x \in B$$

$$\therefore (A \cap B) \subset B$$

**Q4(iii)**

iii. Let  $x \in A \subset B$ . Then

$$\Rightarrow x \in B$$

Let and  $x \in A \cap B$

$$\Leftrightarrow x \in A \text{ and } x \in B$$

$$\Leftrightarrow x \in A \text{ and } x \in A \quad (\because A \subset B)$$

$$\therefore (A \cap B) = A$$

## Q5

(i)

In order to show that the following four statements are equivalent, we need to show that  $(1) \Rightarrow (2)$ ,  $(2) \Rightarrow (3)$ ,  $(3) \Rightarrow (4)$  and  $(4) \Rightarrow (1)$

We first show that  $(1) \Rightarrow (2)$

We assume that  $A \subset B$ , and use this to show that  $A - B = \emptyset$

Now  $A - B = \{x \in A : x \notin B\}$ . As  $A \subset B$ ,

$\therefore$  Each element of  $A$  is an element of  $B$ ,

$\therefore A - B = \emptyset$

Hence, we have proved that  $(1) \Rightarrow (2)$ .

(ii)

We now show that  $(2) \Rightarrow (3)$

So assume that  $A - B = \emptyset$

To show:  $A \cup B = B$

$\because A - B = \emptyset$

$\therefore$  Every element of  $A$  is an element of  $B$

[ $\because A - B = \emptyset$  only when there is some element in  $A$  which is not in  $B$ ]

So  $A \subset B$  and therefore  $A \cup B = B$

So  $(2) \Rightarrow (3)$  is true.

(iii)

We now show that  $(3) \Rightarrow (4)$

Assume that  $A \cup B = B$

To show:  $A \cap B = A$

$\because A \cup B = B$

$\therefore A \subset B$  and so  $A \cap B = A$

So  $(3) \Rightarrow (4)$  is true.

(iv)

Finally we show that  $(4) \Rightarrow (1)$ , which will prove the equivalence of the four statements.

So, assume that  $A \cap B = A$

To show:  $A \subset B$

$\because A \cap B = A$ , therefore  $A \subset B$ , and so  $(4) \Rightarrow (1)$  is true.

Hence,  $(1) \Leftrightarrow (2) \Leftrightarrow (3) \Leftrightarrow (4)$ .

### Q6(i)

Let  $A = \{1, 2, 3\}$ ,  $B = \{2, 4, 6\}$  and  $C = \{2, 5, 7\}$

Then,

$$A \cap B = \{2\}$$

$$\text{and } A \cap C = \{2\}$$

Hence,  $A \cap B = A \cap C$ , but clearly  $B \neq C$ .

### Q6(ii)

Given  $A \subset B$

To show:  $C - B \subset C - A$

Let  $x \in C - B$

$\Rightarrow x \in C$  and  $x \notin B$  [by definition of  $C - B$ ]

$\Rightarrow x \in C$  and  $x \notin A$  [ $\because A \subset B$ ]

This can be seen by the venn diagram above

$\Rightarrow x \in C - A$  [by definition of  $C - A$ ]

Thus  $x \in C - B \Rightarrow x \in C - A$ . This is true for all  $x \in C - B$

$\therefore C - B \subset C - A$

### Q7

(i)

$$\begin{aligned} A \cup (A \cap B) &= (A \cup A) \cap (A \cup B) && [\because \text{union } \cup \text{ is distributive over intersection } \cap] \\ &= A \cap (A \cup B) && [\because A \cup A = A] \\ &= A && [\because A \subset (A \cup B), \text{ as union of two sets is bigger than each of the individual sets}] \end{aligned}$$

Hence,  $A \cup (A \cap B) = A$  Proved.

(ii)

$$\begin{aligned} A \cap (A \cup B) &= (A \cap A) \cup (A \cap B) && [\because \cap \text{ distributes over } \cup] \\ &= A \cup (A \cap B) && [\because A \cap A = A] \\ &= A && [\text{using (i)}] \end{aligned}$$

### Q8

To find sets  $A, B$  and  $C$  such that  $A \cap B \neq \emptyset$ ,  $A \cap C = \emptyset$   
and  $B \cap C = \emptyset$  and  $A \cap B \cap C = \emptyset$

Take  $A = \{1, 2, 3\}$

$$B = \{2, 4, 6\}$$

and  $C = \{3, 4, 7\}$

Then,

$$A \cap B = \{2\}$$

$$\therefore A \cap B \neq \emptyset$$

$$A \cap C = \{3\}$$

$$\therefore A \cap C \neq \emptyset$$

$$B \cap C = \{4\}$$

$$\therefore B \cap C \neq \emptyset$$

However  $A, B$  and  $C$  have no elements in common,

$$\therefore A \cap B \cap C = \emptyset$$

### Q9

Given  $A \cap B = \emptyset$ , i.e.,  $A$  and  $B$  are disjoint sets this can  
be represented by venn diagram as follows

To show:  $A \subseteq B^c$

This is clear from the venn diagram itself

$\because A$  is lying in the complement of  $B$ , but we give a proof of it.

So let  $x \in A$

$$\because A \cap B = \emptyset,$$

$$\therefore x \notin B$$

$$\text{and so } x \in B^c \quad [\because x \notin B \Rightarrow x \in B^c]$$

Thus  $x \in A \Rightarrow x \in B^c$ . This is true for all  $x \in A$

Hence,  $A \subseteq B^c$



### Q10

We need to show that  $(A - B) \cap (A \cap B) = \emptyset$ ,  $(A \cap B) \cap (B - A) = \emptyset$   
and  $(A - B) \cap (B - A) = \emptyset$

The 3 sets  $A - B$ ,  $A \cap B$  and  $B - A$  may be represented by a venn diagram as follows

It is clear from the diagram that the 3 sets are pairwise disjoint, but we shall give a proff of it.

We first show that  $(A - B) \cap (A \cap B) = \emptyset$

Let  $x \in (A - B)$

$\Rightarrow x \in A$  and  $x \notin B$  [by definition of  $A - B$ ]

$\Rightarrow x \notin A \cap B$ . This is true for all  $x \in (A - B)$

Hence  $(A - B) \cap (A \cap B) = \emptyset$

On a similar lines, it can be seen that  $(A \cap B) \cap (B - A) = \emptyset$

Finally, we show that  $(A - B) \cap (B - A) = \emptyset$

We have,

$$A - B = \{x \in A : x \notin B\}$$

$$\text{and } B - A = \{x \in B : x \notin A\}$$

Hence,  $(A - B) \cap (B - A) = \emptyset$ .

### Q11

We need to show  $(A \cup B) \cap (A \cap B') = A$

Now,

$$(A \cup B) \cap (A \cap B') = ((A \cup B) \cap A) \cap B'$$

$$= ((A \cap A) \cup (B \cap A)) \cap B'$$

$$= A \cap B'$$

$$= A$$

[Using associative property]

[ $\because A \cap A = A$  and  $B \cap A = A \cap B$ ,]  
[by commutative law]

[ $\because A \cup (A \cap B) = A$ ]

**Q12(i)**

We have  $A \cup B = U$ , the universal set

To show:  $A \subset B$

Let,  $x \in A$

$$\Rightarrow x \notin A' \quad [\because A \cap A' = \emptyset]$$

$$\therefore x \in A \text{ and } A \subset U$$

$$\Rightarrow x \in U$$

$$\Rightarrow x \in (A \cup B) \quad [\because U = A \cup B]$$

$$\Rightarrow x \in A' \text{ or } x \in B$$

But,  $x \notin A'$ ,

$$\therefore x \in B$$

Thus,  $x \in A \Rightarrow x \in B$

This is true for all  $x \in A$

$$\therefore A \subset B$$

**Q12(ii)**

We have  $B' \subset A'$

To show:  $A \subset B$

Let,  $x \in A$

$$\Rightarrow x \notin A' \quad [\because A \cap A' = \emptyset]$$

$$\Rightarrow x \notin B' \quad [\because B' \subset A']$$

$$\Rightarrow x \in B \quad [\because B \cap B' = \emptyset]$$

Thus,  $x \in A \Rightarrow x \in B$

This is true for all  $x \in A$

$$\therefore A \subset B$$

**Q13**

This is a false statement

Let,  $A = \{1\}$  and  $B = \{2\}$

Then,

$$P(A) = \{\emptyset, \{1\}\}$$

and  $P(B) = \{\emptyset, \{2\}\}$

$$\therefore P(A) \cup P(B) = \{\emptyset, \{1\}, \{2\}\}$$

Now,

$$A \cup B = \{1, 2\}$$

and  $P(A \cup B) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$

$$\text{Hence, } P(A) \cup P(B) \neq P(A \cup B)$$

**Q14(i)**

i. We know that  $(A \cap B) \subset A$  and  $(A - B) \subset A$

$$\Rightarrow (A \cap B) \cup (A - B) \subset A \dots \dots \dots (1)$$

$$\text{Let and } x \in (A \cap B) \cap (A - B)$$

$$\Rightarrow x \in (A \cap B) \text{ and } x \in (A - B)$$

$$\Rightarrow x \in A \text{ and } x \in B \text{ and } x \in A \text{ and } x \notin B$$

$$\Rightarrow x \in A \text{ and } x \in A \text{ [} \because x \in B \text{ and } x \notin B \text{ are not possible simultaneously]}$$

$$\Rightarrow x \in A$$

$$\therefore (A \cap B) \cap (A - B) \subset A \dots \dots \dots (2)$$

From (1) and (2), we get

$$A = (A \cap B) \cup (A - B)$$

**Q14(ii)**

ii. Let  $x \in A \cup (B - A)$

$$\Rightarrow x \in A \text{ or } x \in (B - A)$$

$$\Rightarrow x \in A \text{ or } x \in B \text{ and } x \notin A$$

$$\Rightarrow x \in A \text{ or } x \in B$$

$$\Rightarrow x \in (A \cup B)$$

$$\therefore A \cup (B - A) \subset (A \cup B) \dots \dots \dots (1)$$

Let and  $x \in (A \cup B)$

$$\Rightarrow x \in A \text{ or } x \in B$$

$$\Rightarrow x \in A \text{ or } x \in B \text{ and } x \notin A$$

$$\Rightarrow x \in A \text{ or } x \in (B - A)$$

$$\Rightarrow x \in A \cup (B - A)$$

$$\therefore (A \cup B) \subset A \cup (B - A) \dots \dots \dots (2)$$

From (1) and (2), we get

$$A \cup (B - A) = A \cup B$$

**Q15**

Since each  $X_r$  has 5 elements and each element of  $S$  belongs to exactly 10 of  $X_r$ 's.

$$\therefore S = \bigcup_{r=1}^{20} X_r \Rightarrow \frac{1}{10} \sum_{r=1}^{20} n(X_r) = \frac{1}{10} (5 \times 20) = 10 \dots \dots \dots (i)$$

Since each  $Y_r$  has 2 elements and each element of  $S$  belongs to exactly 4 of  $X_r$ 's.

$$\therefore S = \bigcup_{r=1}^n X_r \Rightarrow \frac{1}{4} \sum_{r=1}^n n(Y_r) = \frac{1}{4} (2n) = \frac{n}{2} \dots \dots (ii)$$

From (i) and (ii), we get

$$10 = \frac{n}{2} \Rightarrow n = 20$$

## Ex 1.7

### Q1

To show  $A' - B' = B - A$

We show that  $A' - B' \subseteq B - A$  and vice versa

Let,  $x \in A' - B'$

$$\Rightarrow x \in A' \text{ and } x \notin B'$$

$$\Rightarrow x \notin A \text{ and } x \in B$$

$$\Rightarrow x \in B \text{ and } x \notin A$$

$$\Rightarrow x \in B - A$$

$$[\because A \cap A' = \emptyset \text{ and } B \cap B' = \emptyset]$$

This is true for all  $x \in A' - B'$

Hence  $A' - B' \subseteq B - A$

Conversely,

Let,  $x \in B - A$

$$\Rightarrow x \in B \text{ and } x \notin A$$

$$\Rightarrow x \notin B' \text{ and } x \in A'$$

$$\Rightarrow x \in A' \text{ and } x \notin B'$$

$$\Rightarrow x \in A' - B'$$

$$[\because B \cap B' = \emptyset \text{ and } A \cap A' = \emptyset]$$

This is true for all  $x \in B - A$

Hence  $B - A \subseteq A' - B'$

$\therefore A' - B' = B - A$  Proved.

### Q2(i)

$$\begin{aligned} \text{LHS} &= A \cap (A' \cup B) \\ &= (A \cap A') \cup (A \cap B) \\ &= \emptyset \cup (A \cap B) \\ &= A \cap B \\ &= \text{RHS} \end{aligned}$$

$$[\because \cap \text{ distributes over } (i)]$$

$$[\because A \cap A' = \emptyset]$$

$$[\because \emptyset \cup x = x \text{ for any set } x]$$

$\therefore \text{LHS} = \text{RHS}$  Proved.

## Q2(ii)

For any sets  $A$  and  $B$  we have by De-morgan's laws

$$(A \cup B)' = A' \cap B', \quad (A \cap B)' = A' \cup B'$$

Also,

$$\begin{aligned} \text{LHS} &= A - (A - B) \\ &= A \cap (A - B)' \\ &= A \cap (A \cap B')' \\ &= A \cap (A' \cup (B')') && [\text{By De-morgan's law}] \\ &= A \cap (A' \cup B) && [\because (B')' = B] \\ &= (A \cap A') \cup (A \cap B) \\ &= \emptyset \cup (A \cap B) && [\because A \cap A' = \emptyset] \\ &= A \cap B && [\because \emptyset \cup x = x, \text{ for any set } x] \\ &= \text{RHS} \end{aligned}$$

$\therefore$  LHS = RHS Proved.

## Q2(iii)

$$\begin{aligned} \text{LHS} &= A \cap (A \cup B') \\ &= A \cap (A' \cap B') && [\text{By De-morgan's law}] \\ &= (A \cap A') \cap B' && [\text{By associative law}] \\ &= \emptyset \cap B' && [\because A \cap A' = \emptyset] \\ &= \emptyset \\ &= \text{RHS} \end{aligned}$$

$\therefore$  LHS = RHS Proved.

### Q2(iv)

$$\begin{aligned}\text{RHS} &= A \Delta (A \cap B) \\&= (A - (A \cap B)) \cup (A \cap B - A) \\&= (A \cap (A \cap B)') \cup (A \cap B \cap A') \\&= (A \cap (A' \cup B')) \cup (A \cap A' \cap B) \\&= (A \cap A') \cup (A \cap B') \cup (\emptyset \cap B) \\&= \emptyset \cup (A \cap B') \cup \emptyset \\&= A \cap B' \\&= A - B \\&= \text{LHS}\end{aligned}$$

$$\begin{aligned}[\because E \Delta F &= (E - F) \cup (F - E)] \\&[\because E - F = E \cap F'] \\&[\text{By De-morgan's law \& associative law}] \\&[\because \cap \text{ distributes over } \cup \text{ and}] \\&[A \cap A' = \emptyset] \\&[\because \emptyset \cap B = \emptyset] \\&[\because \emptyset \cup x = x \text{ for any set } x] \\&[\because A \cap B' = A - B]\end{aligned}$$

$\therefore$  LHS = RHS Proved.

### Q3

We have,  $A \subset B$

To show:  $C - B \subset C - A$

Let,  $x \in C - B$

$$\begin{aligned}\Rightarrow x &\in C \text{ and } x \notin B \\ \Rightarrow x &\in C \text{ and } x \notin A & [\because A \subset B] \\ \Rightarrow x &\in C - A\end{aligned}$$

Thus,  $x \in C - B \Rightarrow x \in C - A$

This is true for all  $x \in C - B$

$\therefore C - B \subset C - A$

### Q4(i)

$$\begin{aligned}\text{i. } (A \cup B) - B &= (A - B) \cup (B - B) \\&= (A - B) \cup \emptyset \\&= A - B\end{aligned}$$

**Q4(ii)**

$$\begin{aligned}
 \text{ii. } A - (A \cap B) &= (A - A) \cap (A - B) \\
 &= \phi \cap (A - B) \\
 &= A - B
 \end{aligned}$$

**Q4(iii)**

$$\begin{aligned}
 \text{iii. Let } x \in A - (A - B) &\Leftrightarrow x \in A \text{ and } x \notin (A - B) \\
 &\Leftrightarrow x \in A \text{ and } x \in (A \cap B) \\
 &\Leftrightarrow x \in A \cap (A \cap B) \\
 &\Leftrightarrow x \in (A \cap B)
 \end{aligned}$$

$$\therefore A - (A - B) = (A \cap B)$$

**Q4(iv)**

$$\begin{aligned}
 \text{iv. Let } x \in A \cup (B - A) &\Rightarrow x \in A \text{ or } x \in (B - A) \\
 &\Rightarrow x \in A \text{ or } x \in B \text{ and } x \notin A \\
 &\Rightarrow x \in B \\
 &\Rightarrow x \in (A \cup B) \quad [\because B \subset (A \cup B)]
 \end{aligned}$$

This is true for all  $x \in A \cup (B - A)$

$$\therefore A \cup (B - A) \subset (A \cup B) \dots \dots \dots (1)$$

Conversely,

$$\begin{aligned}
 \text{Let, } x \in (A \cup B) \\
 &\Rightarrow x \in A \text{ or } x \in B \\
 &\Rightarrow x \in A \text{ or } x \in (B - A) \quad [\because B \subset (B - A)] \\
 &\Rightarrow x \in A \cup (B - A)
 \end{aligned}$$

$$\therefore (A \cup B) \subset A \cup (B - A) \dots \dots \dots (2)$$

From (1) and (2), we get

$$A \cup (B - A) = (A \cup B)$$



#### Q4(v)

v. Let  $x \in A$ .

Then either  $x \in (A - B)$  or  $x \in (A \cap B)$

$$\Rightarrow x \in (A - B) \cup (A \cap B)$$

$$\therefore A \subset (A - B) \cup (A \cap B) \dots \dots \dots (1)$$

Conversely,

Let  $x \in (A - B) \cup (A \cap B)$

$$\Rightarrow x \in (A - B) \text{ or } x \in (A \cap B)$$

$$\Rightarrow x \in A \text{ and } x \notin B \text{ or } x \in A \text{ and } x \in B$$

$$\Rightarrow x \in A$$

$$\therefore (A - B) \cup (A \cap B) \subset A \dots \dots \dots (2)$$

$\therefore$  From (1) and (2), we get

$$(A - B) \cup (A \cap B) = A$$

## Ex 1.8

### Q1

$n(A \cup B) = 50$ ,  $n(A) = 28$ ,  $n(B) = 32$ , where  $n(x)$  denotes the cardinal number of the set  $x$ .

We know that  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$\Rightarrow 50 = 28 + 32 - n(A \cap B)$$

$$\Rightarrow 50 = 60 - n(A \cap B)$$

$$\Rightarrow n(A \cap B) = 60 - 50$$

$$= 10$$

$$\therefore n(A \cap B) = 10$$

### Q2

We have,

$n(P) = 40$ ,  $n(P \cup Q) = 60$ ,  $n(P \cap Q) = 10$ , to find  $n(Q)$ .

We know  $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$

$$\Rightarrow 60 = 40 + n(Q) - 10$$

$$\Rightarrow 60 = 30 + n(Q)$$

$$\Rightarrow n(Q) = 60 - 30$$

$$= 30$$

Hence,  $Q$  has 30 elements.

### Q3

Let  $n(P)$  denote the number of teachers who teach Physics and  
 $n(Q)$  denote the number of teachers who teach Mathematics.

We have,

$$n(P \text{ or } M) = 20$$

$$\text{i.e. } n(P \cup M) = 20$$

$$n(M) = 12$$

$$\text{and } n(P \cap M) = 4$$

To find:  $n(P)$

We know  $n(P \cup M) = n(P) + n(M) - n(P \cap M)$

$$\Rightarrow 20 = n(P) + 12 - 4$$

$$\Rightarrow 20 = n(P) + 8$$

$$\begin{aligned}\Rightarrow n(P) &= 20 - 8 \\ &= 12\end{aligned}$$

$\therefore$  There are 12 Physics teachers.

#### Q4

Let,

$n(P)$  denote the total number of people

$n(C)$  denote the number of people who like coffee and

$n(T)$  denote the number of people who like tea.

$$\text{Then, } n(P) = 70$$

$$n(C) = 37$$

$$n(T) = 52$$

We are given that each person likes at least one of the two drinks, i.e.,  $P = C \cup T$

To find:  $n(C \cap T)$

$$\text{We know } n(P) = n(C) + n(T) - n(C \cap T)$$

$$\Rightarrow 70 = 37 + 52 - n(C \cap T)$$

$$\Rightarrow 70 = 89 - n(C \cap T)$$

$$\begin{aligned}\Rightarrow n(C \cap T) &= 89 - 70 \\ &= 19\end{aligned}$$

Hence, 19 people like both coffee and tea.

#### Q5(i)

$$n(A) = 20, n(A \cup B) = 42 \text{ and } n(A \cap B) = 4, \text{ to find: } n(B)$$

$$\text{We know } n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\Rightarrow 42 = 20 + n(B) - 4$$

$$\Rightarrow 42 = 16 + n(B)$$

$$\begin{aligned}\Rightarrow n(B) &= 42 - 16 \\ &= 26\end{aligned}$$

$$\therefore n(B) = 26$$

### Q5(ii)

To find:  $n(A - B)$

We know that if  $A$  and  $B$  are disjoint sets, then

$$A \cap B = \emptyset$$

$$\begin{aligned}\therefore n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= n(A) + n(B) - n(\emptyset)\end{aligned}$$

$$\Rightarrow n(A \cup B) = n(A) + n(B) \quad [\because n(\emptyset) = 0]$$

Now,

$$A = (A - B) \cup (A \cap B)$$

i.e  $A$  is the disjoint union of  $A - B$  and  $A \cap B$

$$\begin{aligned}\therefore n(A) &= n(A - B) \cup (A \cap B) \\ &= n(A - B) + n(A \cap B) \quad [\because A - B \text{ and } A \cap B \text{ are disjoint}]\end{aligned}$$

$$\Rightarrow 20 = n(A - B) + 4$$

$$\begin{aligned}\Rightarrow n(A - B) &= 20 - 4 \\ &= 16\end{aligned}$$

$$\therefore n(A - B) = 16$$

### Q5(iii)

To find:  $B - A$

On a similar lines we have  $B$  is the disjoint union of  $B - A$  and  $A \cap B$

i.e  $B = (B - A) \cup (A \cap B)$

$$\therefore n(B) = n(B - A) + n(A \cap B)$$

$$\Rightarrow 26 = n(B - A) + 4 \quad [\text{using (i)}]$$

$$\begin{aligned}\Rightarrow n(B - A) &= 26 - 4 \\ &= 22\end{aligned}$$

$$\therefore n(B - A) = 22$$

## Q6

Let  $n(P)$  denote the total percentage of Indians  $n(O)$  denotes the percentage of Indians who like oranges, and  $n(B)$  denotes the percentage of Indians who like bananas.

Then,  $n(P) = 100$ ,  $n(O) = 76$  and  $n(B) = 62$

To find:  $n(O \cap B)$

Now,

$$n(P) = n(O) + n(B) - n(O \cap B)$$

$$\Rightarrow 100 = 76 + 62 - n(O \cap B)$$

$$\Rightarrow 100 = 138 - n(O \cap B)$$

$$\Rightarrow n(O \cap B) = 138 - 100 \\ = 38$$

$\therefore$  38% of Indians like both oranges and bananas.

## Q7

(i)

Let,

$n(P)$  denote the total number of persons,

$n(H)$  denote the number of persons who speak Hindi and

$n(E)$  denote the number of persons who speak English.

Then,

$$n(P) = 950, n(H) = 750, n(E) = 460$$

To find:  $n(H \cap E)$

$$n(P) = n(H) + n(E) - n(H \cap E)$$

$$\Rightarrow 950 = 750 + 460 - n(H \cap E)$$

$$\Rightarrow 950 = 2110 - n(H \cap E)$$

$$\Rightarrow n(H \cap E) = 2110 - 950 \\ = 260$$

Hence, 260 persons can speak both Hindi and English.

(ii)

Clearly  $H$  is the disjoint union of  $H - E$  &  $H \cap E$

$$\text{i.e } H = (H - E) \cup (H \cap E)$$

$$\therefore n(H) = n(H - E) + n(H \cap E) \quad \left[ \because \text{if } A \text{ \& } B \text{ are disjoint then } \right. \\ \left. n(A \cup B) = n(A) + n(B) \right]$$

$$\Rightarrow 750 = n(H - E) + 260$$

$$\Rightarrow n(H - E) = 750 - 260 \\ = 490$$

Hence, 490 persons can speak Hindi only.

(iii)

On a similar lines we have

$$E = (E - H) \cup (H \cap E)$$

i.e  $E$  is the disjoint union of  $E - H$  &  $H \cap E$

$$\therefore n(E) = n(E - H) + n(H \cap E)$$

$$\Rightarrow 460 = n(E - H) + 260$$

$$\Rightarrow n(E - H) = 460 - 260 \\ = 200$$

Hence, 200 persons can speak English only.

## Q8

(i)

Let,

$n(P)$  denote the total number of persons,

$n(T)$  denote number of persons who drink tea and

$n(C)$  denote number of persons who drink coffee.

Then,  $n(P) = 50$ ,  $n(T - C) = 14$ ,  $n(T) = 30$

To find:  $n(T \cap C)$

Clearly  $T$  is the disjoint union of  $T - C$  and  $T \cap C$

$$\therefore T = (T - C) \cup (T \cap C)$$

$$\therefore n(T) = n(T - C) + n(T \cap C)$$

$$\Rightarrow 30 = 14 + n(T \cap C)$$

$$\Rightarrow n(T \cap C) = 30 - 14 \\ = 16$$

Hence, 16 persons drink tea and coffee both.

(ii)

To find:  $C - T$

We know  $n(P) = n(C) + n(T) - n(T \cap C)$

$$\Rightarrow 50 = n(C) + 30 - 16$$

$$\Rightarrow 50 = n(C) + 14$$

$$\Rightarrow n(C) = 50 - 14 \\ = 36$$

New  $C$  is the disjoint union of  $C - T$  and  $T \cap C$

$$\therefore C = (C - T) \cup (C \cap T)$$

$$\Rightarrow n(C) = n(C - T) + n(C \cap T)$$

$$\Rightarrow 36 = n(C - T) + 16$$

$$\Rightarrow n(C - T) = 36 - 16 \\ = 20$$

$$[\because n(T \cap C) = n(C \cap T) = 16]$$

Hence, 20 persons drink coffee but not tea.



## Q9

(i)

Let  $n(P)$  denote total number of people  $n(H)$  denote number of people who read newspaper  $H$   $n(T)$  denote number of people who read newspaper  $T$  and  $n(I)$  denote number of people who read newspaper  $I$

Then,  $n(P) = 60$ ,  $n(H) = 25$ ,  $n(T) = 26$ ,  $n(I) = 26$

$$n(H \cap I) = 9, n(H \cap T) = 11, n(T \cap I) = 8, n(H \cap T \cap I) = 3$$

We need to find the number of people who read at least one of the newspaper, i.e.,  $n(H \text{ or } T \text{ or } I)$ , i.e.,  $n(H \cup T \cup I)$  we know that if  $A, B, C$  are 3 sets, then,

$$\begin{aligned} n(A \cup B \cup C) &= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C) \\ \therefore n(H \cup T \cup I) &= n(H) + n(T) + n(I) - n(H \cap T) - n(T \cap I) - n(H \cap I) + n(H \cap T \cap I) \\ &= 25 + 26 + 26 - 9 - 11 - 8 + 3 \\ &= 25 + 52 - 28 + 3 \\ &= 25 + 52 - 25 \\ &= 52 \end{aligned}$$

Hence, 52 people read at least one of the newspaper.

(ii)

The venn diagram representing people reading newspapers  $H, T$  and  $I$  is shown above.

The shaded region shows the number of people who read newspaper  $H$  only, newspaper  $T$  only and newspaper  $I$  only respectively.

The number of people who read newspaper  $H$  only equals

$$\begin{aligned} &25 - (8 + 3 + 6) \\ &= 25 - 17 \\ &= 8 \end{aligned}$$

The number of people who read newspaper  $T$  only

$$\begin{aligned} &= 26 - (8 + 3 + 5) \\ &= 26 - 16 \\ &= 10 \end{aligned}$$

And, the number of people who read newspaper  $I$  only

$$\begin{aligned} &= 26 - (6 + 3 + 5) \\ &= 26 - 14 \\ &= 12 \end{aligned}$$

Hence, the number of people, who read exactly one newspaper =  $8 + 10 + 12 = 30$ .

### Q10

Let,

$n(P)$  denote total number of members,

$n(B)$  denote number of members in the basket ball team

$n(H)$  denote number of members in the hockey team and

$n(F)$  denote number of members in the football team.

Then,  $n(B) = 21$ ,  $n(H) = 26$ , and  $n(F) = 29$

Also,  $n(H \cap B) = 14$ ,  $n(H \cap F) = 15$ ,  $n(F \cap B) = 12$ ,  $n(H \cap B \cap F) = 8$

Now,

$$P = B \cup H \cup F$$

$$\therefore n(P) = n(B \cup H \cup F)$$

$$= n(B) + n(H) + n(F) - n(B \cap H) - n(H \cap F) - n(B \cap F) + n(B \cap H \cap F)$$

$$\Rightarrow n(P) = 21 + 26 + 29 - 14 - 15 - 12 + 8$$

$$= 76 - 41 + 8$$

$$= 43$$

Hence, there are 43 members in all.

### Q11

Let,

$n(P)$  denote the total number of people,

$n(H)$  the number of people who speak Hindi and

$n(B)$  the number of people who speak Bengali.

Then,  $n(P) = 1000$ ,  $n(H) = 750$ ,  $n(B) = 400$

We have  $P = (H \cup B)$

$$\begin{aligned}\therefore n(P) &= n(H \cup B) \\ &= n(H) + n(B) - n(H \cap B) \\ \Rightarrow 1000 &= 750 + 400 - n(H \cap B) \\ \Rightarrow 1000 &= 1150 - n(H \cap B) \\ \Rightarrow n(H \cap B) &= 1150 - 1000 \\ &= 150\end{aligned}$$

Hence, 150 people can speak both Hindi and Bengali now  $H = (H - B) \cup (H \cap B)$ , the union being disjoint

$$\begin{aligned}\therefore n(H) &= n(H - B) + n(H \cap B) \\ \Rightarrow 750 &= n(H - B) + 150 \\ \Rightarrow n(H - B) &= 750 - 150 \\ &= 600\end{aligned}$$

Hence, 600 people can speak Hindi only

On a similar lines we have  $B = (B - H) \cup (H \cap B)$

$$\begin{aligned}\Rightarrow n(B) &= n(B - H) + n(H \cap B) \\ \Rightarrow 400 &= n(B - H) + 150 \\ \Rightarrow n(B - H) &= 400 - 150 \\ &= 250\end{aligned}$$

Hence, 250 people can speak Bengali only.

## Q12

Let,

$n(P)$  denote the total number of television viewers,

$n(F)$  be the number of people who watch football,

$n(H)$  be the number of people who watch hockey and

$n(B)$  be the number of people who watch basket ball.

Then,  $n(P) = 500$ ,  $n(F) = 285$ ,  $n(H) = 195$ ,  $n(B) = 115$ ,  $n(F \cap B) = 45$ ,  $n(F \cap H) = 70$ ,  
 $n(H \cap B) = 50$  and  $n(F \cup H \cup B) = 50$

Now,

$$\begin{aligned}n((F \cup H \cup B)') &= n(P) - n(F \cup H \cup B) \\ \Rightarrow 50 &= 500 - (n(F) + n(H) + n(B) - n(F \cap H) - n(H \cap B) - n(F \cap B) + n(F \cap H \cap B)) \\ \Rightarrow 50 &= 500 - (285 + 195 + 115 - 70 - 50 - 45 + n(F \cap H \cap B)) \\ \Rightarrow 50 &= 500 - 430 - n(F \cap H \cap B) \\ \Rightarrow 50 &= 70 - n(F \cap H \cap B) \\ \Rightarrow n(F \cap H \cap B) &= 70 - 50 \\ &= 20\end{aligned}$$

Hence, 20 people watch all the 3 games

Number of people who watch only football

$$\begin{aligned}&= 285 - (50 + 20 + 25) \\ &= 285 - 95 \\ &= 190\end{aligned}$$

Number of people who watch only hockey

$$\begin{aligned}&= 195 - (50 + 20 + 30) \\ &= 195 - 100 \\ &= 95\end{aligned}$$

And, number of people who watch only basket ball

$$\begin{aligned}&= 115 - (25 + 20 + 30) \\ &= 115 - 75 \\ &= 40\end{aligned}$$

Number of people who watch exactly one of the three games

= number of people who watch either football only or hockey only or  
basket ball only

$$= 190 + 95 + 40 \quad [\because \text{they are pairwise disjoint}]$$

$$= 325$$

Hence, 325 people watch exactly one of the three games.

### Q13

(i)

Let  $n(P)$  denote total number of persons

$n(A)$  denote number of people who read magazine  $A$

$n(B)$  denote number of people who read magazine  $B$

and  $n(C)$  denote number of people who read magazine  $C$

$$\text{Then, } n(P) = 100, n(A) = 28, n(B) = 30, n(C) = 42, n(A \cap B) = 8,$$

$$n(A \cap C) = 10, n(B \cap C) = 5, n(A \cap B \cap C) = 3$$

Now,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$= 28 + 30 + 42 - 8 - 10 - 5 + 3$$

$$= 100 - 23 + 3$$

$$= 100 - 20$$

$$= 80$$

$\therefore$  Number of people who read none of the three magazines

$$= n(A \cup B \cup C)'$$

$$= n(P) - n(A \cup B \cup C)$$

$$= 100 - 80$$

$$= 20$$

Hence, 20 people read none of the three magazines.

(ii)

$$n(C \text{ only}) = 42 - (7 + 3 + 2)$$

$$= 42 - 12$$

$$= 30$$

### Q14

(i)

Let  $n(P)$  denote total number of students

$n(E)$  denote number of students studying English language

$n(H)$  denote number of students studying Hindi language and

$n(S)$  denote number of students studying Sanskrit language

Then,  $n(P) = 100$ ,  $n(E - H) = 23$ ,  $n(E \cap S) = 8$ ,  $n(E) = 26$ ,  $n(S) = 48$ ,

$$n(S \cap H) = 8, n((E \cup H \cup S)') = 24$$

Number of students studying English only = 18

We have,

$$n((E \cup H \cup S)') = 24$$

$$\Rightarrow n(P) - n(E \cup H \cup S) = 24$$

$$\Rightarrow 100 - 24 = n(E \cup H \cup S)$$

$$\Rightarrow n(E \cup H \cup S) = 76$$

$$\text{We have } n(E \cup H \cup S) = n(E) + n(H) + n(S) - n(E \cap H) - n(H \cap S) - n(E \cap S) + n(E \cap H \cap S)$$

$$\Rightarrow 76 = 26 + n(H) + 48 - 3 - 8 - 8 + 3$$

$$\Rightarrow 76 = 26 + n(H) + 48 - 16$$

$$\Rightarrow 76 = 26 + 32 + n(H)$$

$$\Rightarrow n(H) = 76 - 58$$

$$= 18$$

$\therefore$  18 students were studying Hindi.

(ii)

From (i) we have  $n(E \cap H) = 3$

$\therefore$  3 students were studying both English and Hindi.

### Q15

Let  $n(p_1)$  be the number of persons liking product  $p_1$   
 $n(p_2)$  be the number of persons liking product  $p_2$   
and  $n(p_3)$  be the number of persons liking product  $p_3$

Then,  $n(p_1) = 21$ ,  $n(p_2) = 26$ ,  $n(p_3) = 29$ ,  $n(p_1 \cap p_2) = 14$ ,  
 $n(p_1 \cap p_3) = 12$ ,  $n(p_2 \cap p_3) = 14$ ,  $n(p_1 \cap p_2 \cap p_3) = 8$

$\therefore$  Number of people liking product  $p_3$  only  
=  $29 - (4 + 8 + 6)$   
=  $29 - 18$   
=  $11$

Hence, 11 persons liked product  $p_3$  only.