Ex 5.1

Q1

$$
\begin{aligned}
\text{LHS} &= \sec^4\theta - \sec^2\theta \\
&= \sec^2\theta\left(\sec^2\theta - 1\right) \\
&= \left(1 + \tan^2\theta\right)\tan^2\theta && \left[\because \sec^2\theta = 1 + \tan^2\theta\right] \\
&= \tan^2\theta + \tan^4\theta \\
&= \tan^4\theta + \tan^2\theta \\
&= \text{RHS}
\end{aligned}
$$

LHS = RHS

Proved

Q2

$$
\begin{aligned}
\text{LHS} &= \sin^6\theta + \cos^6\theta \\
&= \left(\sin^2\theta\right)^3 + \left(\cos^2\theta\right)^3 \\
&= \left(\sin^2\theta + \cos^2\theta\right)\left[\left(\sin^2\theta\right)^2 - \sin^2\theta\cos^2\theta + \left(\cos^2\theta\right)^2\right] && \left(\because a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)\right) \\
&= \left(\sin^2\theta\right)^2 + \left(\cos^2\theta\right)^2 + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta)
\end{aligned}
$$

$\left[\begin{array}{l}\text{adding and subtracting } 2\sin^2\theta\cos^2\theta \text{ and} \\ \text{using identity } \sin^2\theta + \cos^2\theta = 1\end{array}\right]$

$$
\begin{aligned}
&= \left(\sin^2\theta + \cos^2\theta\right)^2 - 3\sin^2\theta\cos^2\theta \\
&= \left(\sin^2\theta + \cos^2\theta\right)^2 - 3\sin^2\theta\cos^2\theta \\
&= 1^2 - 3\sin^2\theta\cos^2\theta && \left(\because \sin^2\theta + \cos^2\theta = 1\right) \\
&= 1 - 3\sin^2\theta\cos^2\theta \\
&= \text{RHS}
\end{aligned}
$$

$\therefore$ LHS = RHS

Proved

Q3

$$LHS = (\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right) \quad \left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}, \sec\theta = \frac{1}{\cos\theta}, \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta - \cos^2\theta}{\cos\theta\sin\theta}\right)$$

$$= \frac{\cos^2\theta.\sin^2\theta\ 1}{\sin^2\theta.\cos^2\theta} \left(\begin{array}{l}\because \sin^2\theta + \cos^2\theta = 1 \\ \Rightarrow 1 - \sin^2\theta = \cos^2\theta, \text{ and} \\ \qquad 1 - \cos^2\theta = \sin^2\theta\end{array}\right)$$

$= 1$

$= RHS$

Proved

Q4

$$LHS = \operatorname{cosec}\theta(\sec\theta - 1) - \cot\theta(1 - \cos\theta)$$

$$= \frac{1}{\sin\theta}\left(\frac{1}{\cos\theta} - 1\right) - \frac{\cos\theta}{\sin\theta}(1 - \cos\theta) \quad \left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}, \sec\theta = \frac{1}{\cos\theta} \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$= \frac{(1-\cos\theta)}{\sin\theta\cos\theta} \quad \frac{\cos\theta(1-\cos\theta)}{\sin\theta}$$

$$= \frac{(1-\cos\theta) - \cos^2\theta(1-\cos\theta)}{\sin\theta\cos\theta}$$

$$= \frac{(1-\cos\theta)(1-\cos^2\theta)}{\sin\theta\cos\theta}$$

$$= \frac{(1-\cos\theta)\sin^2\theta}{\sin\theta\cos\theta} \quad (\because 1 - \cos^2\theta = \sin^2\theta)$$

$$= (1-\cos\theta)\frac{\sin\theta}{\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} - \sin\theta$$

$$= \tan\theta - \sin\theta \quad (\because \tan\theta = \sin\theta - \cos\theta)$$

$= RHS$

Proved

Q5

$$LHS = \frac{1-\sin A\cos A}{\cos A\left(\sec A-\operatorname{cosec}A\right)}\cdot\frac{\sin^2 A-\cos^2 A}{\sin^3 A+\cos^3 A}$$

$$= \frac{1-\sin A\cos A}{\cos A\left(\frac{1}{\cos A}-\frac{1}{\sin A}\right)}\cdot\frac{(\sin A-\cos A)(\sin A+\cos A)}{(\sin A+\cos A)\left(\sin^2 A+\cos^2 A-\sin A\cos A\right)}$$

$$\left[\begin{array}{l}\text{Using } a^2-b^2=(a-b)(a+b)\\ \text{and } a^3+b^3=(a+b)\left(a^2+b^2-ab\right)\end{array}\right.$$

$$= \frac{(1-\sin A\cos A)}{\cos A\left(\frac{\sin A-\cos A}{\cos A\sin A}\right)}\cdot\frac{(\sin A-\cos A)}{(1-\sin A\cos A)}\quad\left(\because \sin^2 A+\cos^2 A=1\right)$$

$$= \frac{\cos A\sin A}{\cos A}$$

$= \sin A$

$= RHS$

Proved

Q6

$$LHS = \frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$$

$$= \frac{(\sin A/\cos A)}{\left(1-\frac{\cos A}{\sin A}\right)}+\frac{(\cos A/\sin A)}{1-\frac{\sin A}{\cos A}}$$

$$= \frac{\sin A}{\cos A\frac{(\sin A-\cos A)}{\sin A}}+\frac{\cos A}{\sin A\frac{(\cos A-\sin A)}{\cos A}}$$

$$= \frac{\sin^2 A}{\cos A(\sin A-\cos A)}+\frac{\cos^2 A}{\sin A(\cos A-\sin A)}$$

$$= \frac{\sin^3 A-\cos^3 A}{\cos A\sin A(\sin A-\cos A)}$$

$$= \frac{(\sin A-\cos A)\left(\sin^2 A+\cos^2 A+\sin A\cos A\right)}{\cos A\sin A(\sin A-\cos A)}\qquad\left[\text{Using } a^3-b^3=(a-b)\left(a^2+b^2+ab\right)\right]$$

$$= \frac{1+\sin A\cos A}{\sin A\cos A}\qquad\left[\because \sin^2 A+\cos^2 A=1\right]$$

$$= \frac{1}{\sin A\cos A}+\frac{\sin A\cos A}{\sin A\cos A}$$

$$= \sec A\operatorname{cosec}A+1\qquad\left[\because \frac{1}{\cos A}=\sec A,\ \frac{1}{\sin A}=\operatorname{cosec}A\right]$$

$= RHS$

Proved

Q7

$$\text{LHS} = \frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A}$$

$$= \frac{(\sin A + \cos A)(\sin^2 A + \cos^2 A - \sin A \cos A)}{(\sin A + \cos A)} + \frac{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A \cos A)}{\sin A - \cos A}$$

$$\left[\text{Using } a^3 + b^3 = (a+b)(a^2 + b^2 - ab) \text{ and } a^3 - b^3 = (a-b)(a^2 + b^2 + ab)\right]$$

$$= (1 - \sin A \cos A) + (1 + \sin A \cos A) \left(\because \sin^2 A + \cos^2 A = 1\right)$$

$$= 2$$

$$= \text{RHS}$$

Q8

$$\text{LHS} = (\sec A \sec B + \tan A \tan B)^2 - (\sec A \tan B + \tan A \sec B)^2$$

$$= (\sec A \sec B)^2 + (\tan A \tan B)^2 + 2 \sec A \sec B \tan A \tan B$$

$$-\left((\sec A \tan B)^2 + (\tan A \sec B)^2 + 2 \sec A \tan B \tan A \sec B\right) \quad \left[\text{Using} (a+b)^2 = a^2 + b^2 + 2ab\right]$$

$$= \sec^2 A \sec^2 B + \tan^2 A \tan^2 B + 2 \sec A \sec B \tan A \tan B$$

$$- \sec^2 A \tan^2 B - \tan^2 A \sec^2 B - 2 \sec A \sec B \tan A \tan B \left[\text{Using } (ab)^2 = a^2 b^2\right]$$

$$= \sec^2 A \sec^2 B - \sec^2 A \tan^2 B + \tan^2 A \tan^2 B - \tan^2 A \sec^2 B$$

$$= \sec^2 A \left(\sec^2 B - \tan^2 B\right) + \tan^2 A \left(\tan^2 B - \sec^2 B\right)$$

$$= \sec^2 A . 1 - \tan^2 A . 1 \qquad \begin{bmatrix} \because \sec^2 \theta = 1 + \tan^2 \theta \\ \Rightarrow \sec^2 \theta - \tan^2 \theta = 1 \end{bmatrix}$$

$$= 1 + \tan^2 A - \tan^2 A$$

$$= 1$$

$$= \text{RHS}$$

Proved

Q9

$$\text{RHS} = \frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}$$

$$= \frac{\left((1+\cos\theta)+\sin\theta\right)}{(1+\cos\theta)-\sin\theta} \times \frac{\left((1+\cos\theta)+\sin\theta\right)}{(1+\cos\theta+\sin\theta)}$$

$$= \frac{\left((1+\cos\theta)+\sin\theta\right)^2}{(1+\cos\theta)^2-\sin^2\theta} \qquad \left(\begin{array}{l}\text{Using } (a+b)(a+b)=(a+b)^2 \\ \quad \& \ (a+b)(a-b)=a^2b^2\end{array}\right)$$

$$= \frac{(1+\cos\theta)^2+\sin^2\theta+2\sin\theta(1+\cos\theta)}{1+\cos^2\theta+2\cos\theta-\sin^2\theta} \qquad \left(\text{Using } (a+b)^2=a^2+b^2+2ab\right)$$

$$= \frac{1+\cos^2\theta+2.1\cos\theta+\sin^2\theta+2\sin\theta(1+\cos\theta)}{1+\cos^2+2\cos\theta-(1-\cos^2\theta)} \qquad \left(\text{Using } \sin^2\theta=1-\cos^2\theta\right)$$

$$= \frac{1+1+2\cos\theta+2\sin\theta(1+\cos\theta)}{1-1+\cos^2\theta+\cos^2\theta+2\cos\theta} \qquad \left(\text{Using } \sin^2\theta+\cos^2\theta=1\right)$$

$$= \frac{2+2\cos\theta+2\sin\theta(1+\cos\theta)}{2\cos^2\theta+2\cos\theta}$$

$$= \frac{2(1+\cos\theta)+2\sin\theta(1+\cos\theta)}{2\cos\theta(\cos\theta+1)}$$

$$= \frac{(1+\cos\theta)(2+2\sin\theta)}{2\cos\theta(1+\cos\theta)}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$$

$$= \frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$$

$$= \frac{\cos\theta}{1-\sin\theta}$$

Q10

$$\text{LHS} = \frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1-\cot^2\theta}$$

$$= \frac{\sin^3\theta}{\cos^3\theta\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)} + \frac{\cos^3\theta}{\sin^3\theta\left(1-\frac{\cos^2\theta}{\sin^2\theta}\right)} \qquad \left(\because \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}\right)$$

$$= \frac{\sin^3\theta\cos^2\theta}{\cos^3\theta\left(\cos^2\theta+\sin^2\theta\right)} + \frac{\cos^3\theta\sin^2\theta}{\sin^3\theta\left(\sin^2\theta+\cos^2\theta\right)}$$

$$= \frac{\sin^3\theta}{\cos\theta} + \frac{\cos^3\theta}{\sin\theta} \qquad \left(\because \cos^2\theta+\sin^2\theta = 1\right)$$

$$= \frac{\sin^4\theta+\cos^4\theta}{\sin\theta\cos\theta}$$

$$= \frac{\left(\sin^2\theta\right)^2+\left(\cos^2\theta\right)^2+2\sin^2\theta\cos^2\theta-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta} \quad \left\{\text{adding and subtracting } 2\sin^2\theta\cos^2\theta\right\}$$

$$= \frac{\left(\sin^2\theta+\cos^2\theta\right)^2-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1^2-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta} \qquad \left(\because \sin^2\theta+\cos^2\theta = 1\right)$$

$$= \frac{1-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

= RHS

Proved

Q11

$$\text{LHS} = 1 - \frac{\sin^2\theta}{1+\cot\theta} - \frac{\cos^2\theta}{1+\tan\theta}$$

$$= 1 - \frac{\sin^2\theta}{1+\frac{\cos\theta}{\sin\theta}} - \frac{\cos^2\theta}{1+\frac{\sin\theta}{\cos\theta}} \left(\because\ \cot\theta = \frac{\cos\theta}{\sin\theta}, \tan\theta = \frac{\sin\theta}{\cos\theta}\right)$$

$$= 1 - \frac{\sin^2\theta}{\frac{\sin\theta+\cos\theta}{\sin\theta}} - \frac{\cos^2\theta}{\frac{\cos\theta+\sin\theta}{\cos\theta}}$$

$$= 1 - \frac{\sin^3\theta}{\sin\theta+\cos\theta} - \frac{\cos^3\theta}{\cos\theta+\sin\theta}$$

$$= \frac{\sin\theta+\cos\theta-\left(\sin^3+\cos^3\theta\right)}{\sin\theta+\cos\theta}$$

$$= \frac{\sin\theta+\cos\theta-\left(\sin\theta+\cos\theta\right)\left(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta\right)}{\sin\theta+\cos\theta}$$

$$\left(\text{Using } a^3+b^3 = (a+b)\left(a^2+b^2-ab\right)\right)$$

$$\frac{\left(\sin\theta+\cos\theta\right)\left(1-\left(1-\sin\theta\cos\theta\right)\right)}{\sin\theta+\cos\theta} \qquad \left(\text{Using} \sin^2\theta+\cos^2\theta = 1\right)$$

$= \sin\theta\cos\theta$

$=$ RHS

Proved

Q12

$$LHS = \left(\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\operatorname{cosec}^2\theta - \sin^2\theta}\right)\sin^2\theta\cos^2\theta$$

$$= \left(\frac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \frac{1}{\left(\frac{1}{\sin^2\theta} - \sin^2\theta\right)}\right)\sin^2\theta\cos^2\theta$$

$$= \left(\frac{1}{\frac{1-\cos^4\theta}{\cos^2\theta}} + \frac{1}{\frac{1-\sin^4\theta}{\sin^2\theta}}\right)\sin^2\theta\cos^2\theta$$

$$= \left(\frac{\cos^2\theta}{\left(1-\cos^2\theta\right)\left(1+\cos^2\theta\right)} + \frac{\sin^2\theta}{\left(1-\sin^2\theta\right)\left(1+\sin^2\theta\right)}\right)\sin^2\theta\cos^2\theta$$

$$\left(\begin{aligned}\text{Using } 1-a^4 &= 1-\left(a^2\right)^2 \\ &= \left(1-a^2\right)\left(1+a^2\right)\end{aligned}\right)$$

$$= \left(\frac{\cos^2\theta}{\sin^2\theta\left(1+\cos^2\theta\right)} + \frac{\sin^2\theta}{\cos^2\theta\left(1+\sin^2\right)}\right)\sin^2\theta\cos^2\theta \qquad \left(\begin{aligned}&\text{Using } 1-\cos^2\theta = \sin^2\theta \\ &\& \quad 1-\sin^2\theta = \cos^2\theta\end{aligned}\right)$$

$$= \left(\frac{\cos^4\theta\left(1+\sin^2\theta\right) + \sin^4\theta\left(1-\cos^2\theta\right)}{\sin^2\theta\cos^2\theta\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}\right).\sin^2\theta\cos^2\theta$$

$$= \frac{\cos^4\theta + \sin^2\theta\cos^4\theta + \sin^4\theta + \cos^2\theta\sin^4\theta}{\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}$$

$$= \frac{\left(\cos^2\theta\right)^2 + \left(\sin^2\theta\right)^2 + 2\cos^2\theta\sin^2\theta - 2\cos^2\theta\sin^2\theta + \sin^2\theta\cos^4\theta + \cos^2\theta\sin^4\theta}{\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}$$

$$\left(\text{adding and subtracting } 2\cos^2\theta\sin^2\theta\right)$$

$$= \frac{\left(\cos^2\theta + \sin^2\theta\right)^2 - 2\cos^2\theta\sin^2\theta + \sin^2\theta\cos^2\theta\left(\cos^2\theta + \sin^2\theta\right)}{1 + \sin^2\theta + \cos^2\theta + \sin^2\theta\cos^2\theta}$$

$$= \frac{1^2 - 2\cos^2\theta\sin^2\theta + \sin^2\theta\cos^2\theta.1}{1 + 1 + \sin^2\theta\cos^2\theta}$$

$$= \frac{1 - \sin^2\theta\cos^2\theta}{2 + \sin^2\theta\cos^2\theta}$$

= RHS

Proved

Q13

$$\text{LHS} = \left(1 + \tan\alpha\tan\beta\right)^2 + \left(\tan\alpha - \tan\beta\right)^2$$

$$= 1 + \left(\tan\alpha + \tan\beta\right)^2 + 2.1\tan\alpha\tan\beta + \left(\tan\alpha\right)^2 + \left(\tan\beta\right)^2 - 2\tan\alpha.\tan\beta$$

$$\left(\text{Using } (a+b)^2 = a^2 + b^2 + 2ab \text{ and } (a-b)^2 = a^2 + b^2 - 2ab\right)$$

$$= 1 + \tan^2\alpha + \tan^2\beta + 2\tan\alpha\tan\beta + \tan^2\alpha + \tan^2\beta - 2\tan\alpha\tan\beta$$

$$= 1 + \tan^2\alpha + \tan^2\alpha + \tan^2\beta + \tan^2\beta$$

$$= \sec^2\alpha + \tan^2\beta\left(1 + \tan^2\alpha\right) \qquad \left(\because 1 + \tan^2\alpha = \sec^2\alpha\right)$$

$$= \sec^2\alpha + \tan^2\beta.\sec^2\alpha$$

$$= \sec^2\alpha\left(1 + \tan^2\beta\right)$$

$$= \sec^2\alpha.\sec^2\beta \qquad \left(\because 1 + \tan^2\beta = \sec^2\beta\right)$$

= RHS

Proved

Q14

$$\text{LHS} = \frac{(1+cct\,\theta+\tan\theta)(\sin\theta-\cos\theta)}{\sec^3\theta-\cos ec^3\theta}$$

$$= \frac{\left(1+\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}\right)}{\left(\frac{1}{\cos^3\theta}-\frac{1}{\sin^3\theta}\right)} \qquad \left(\begin{array}{l}\because \cot\theta = \frac{\cos\theta}{\sin\theta}, \tan\theta = \frac{\sin\theta}{\cos\theta} \\ \sec\theta = \frac{1}{\cos\theta}, \cos ec\theta = \frac{1}{\sin\theta}\end{array}\right)$$

$$= \left(\frac{1+\frac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}}{\frac{\sin^3\theta-\cos^3\theta}{\cos^3\theta\sin^3\theta}}\right)(\sin\theta-\cos\theta)$$

$$= \frac{(\sin\theta\cos\theta+1)\sin^3\theta\cos^3 6}{\sin\theta\cos\theta.(\sin^3\theta-\cos^3\theta)}.(\sin\theta-\cos\theta) \qquad (\because \sin^2\theta-\cos^2\theta=1)$$

$$= \frac{(1+\sin\theta\cos\theta)\sin^2\theta\cos^2 6.(\sin\theta-\cos\theta)}{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)} \qquad (\because a^3-b^3=(a-b)(a^2+b^2+ab))$$

$$= \frac{(1+\sin\theta\cos\theta).\sin^2\theta\cos^2 9}{(1+\sin\theta\cos\theta)}$$

$= \sin^2\theta\cos^2 9$

= RHS

Proved

Q15

$$\text{LHS} = \frac{2\sin\theta\cos\theta - \cos\theta}{1 - \sin\theta + \sin^2\theta - \cos^2\theta}$$

$$= \frac{\cos\theta\left(2\sin\theta - 1\right)}{1 - \cos^2\theta + \sin^2\theta - \sin\theta}$$

$$= \frac{\cos\theta\left(2\sin\theta - 1\right)}{\sin^2\theta + \sin^2\theta - \sin\theta} \qquad \left(\because 1 - \cos^2\theta = \sin^2\theta\right)$$

$$= \frac{\cos\theta\left(2\sin\theta - 1\right)}{2\sin^2\theta - \sin\theta}$$

$$= \frac{\cos\theta\left(2\sin\theta - 1\right)}{\sin\theta\left(2\sin\theta - 1\right)}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$= \cot\theta$

$=$ RHS

Proved

Q16

$$\text{LHS} = \cos\theta\,(\tan\theta + 2)(2\tan\theta + 1)$$

$$= \cos\theta\left(\frac{\sin\theta}{\cos\theta} + 2\right)\left(\frac{2\sin\theta}{\cos\theta} + 1\right)\left(\because\ \tan\theta = \frac{\sin\theta}{\cos\theta}\right)$$

$$= \cos\frac{(\sin\theta + 2\cos\theta)(2\sin\theta + \cos\theta)}{\cos\theta.\cos\theta}$$

$$= \frac{(2\sin^2\theta + \sin\theta\cos\theta + 4\sin\theta\cos\theta + 2\cos^2\theta)}{\cos\theta}$$

$$= \frac{2(\sin^2\theta + \cos^2\theta) + 5\sin\theta\cos\theta}{\cos\theta}$$

$$= \frac{2 + 5\sin\theta\cos\theta}{\cos\theta}\ (\because\ \sin^2\theta + \cos^2\theta) = 1$$

$$= \frac{2}{\cos\theta} + \frac{5\sin\theta\cos\theta}{\cos\theta}$$

$$= 2\sec\theta + 5\sin\theta$$

$= \text{RHS}$

Proved

Q17

$$\frac{2\sin\theta}{1+\cos\theta+\sin\theta} = x$$

$$\Rightarrow \frac{2\sin\theta(1-\cos\theta+\sin\theta)}{(1+\cos\theta-\sin\theta)(1-\cos\theta+\sin\theta)} = x \quad [\textit{Rationalizing the denominator}]$$

$$\Rightarrow \frac{2\sin\theta(1-\cos\theta+\sin\theta)}{(1-\sin\theta)^2-\cos^2\theta} = x$$

$$\Rightarrow \frac{2\sin\theta-2\sin\theta\cos\theta+2\sin^2\theta}{1+\sin^2\theta+2\sin\theta-\cos^2\theta} = x$$

$$\Rightarrow \frac{2\sin\theta(1+\cos\theta-\sin\theta)}{2\sin^2\theta+2\sin\theta} = x$$

$$\Rightarrow \frac{2\sin\theta(1+\cos\theta-\sin\theta)}{2\sin\theta(1+\sin\theta)} = x$$

$$\Rightarrow \frac{1+\cos\theta-\sin\theta}{1+\sin\theta} = x \; [\textit{Cancelling the 2 sin } \theta \textit{ in both Numerator and Denominator}]$$

Hence Proved

Q18

Now, $\cos\theta = \sqrt{1 - \sin^2\theta}$

$$= \sqrt{\frac{1 - \left(a^2 - b^2\right)^2}{\left(a^2 + b^2\right)^2}} \qquad \left[\because \sin\theta = \frac{a^2 - b^2}{a^2 + b^2}\right]$$

$$= \sqrt{\frac{\left(a^2 + b^2\right)^2 - \left(a^2 - b^2\right)^2}{\left(a^2 + b^2\right)^2}}$$

$$= \sqrt{\frac{\left(a^2 + b^2 + a^2 - b^2\right)\left(a^2 + b^2 - a^2 + b^2\right)}{a^2 + b^2}} \quad \left(\text{Using } x^2 - y^2 = (x - y)(x + y)\right)$$

$$= \sqrt{\frac{2a^2 \times 2b^2}{a^2 + b^2}}$$

$$= \frac{2ab}{a^2 + b^2} \text{ ....(ii)}$$

Now $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$$= \frac{\dfrac{a^2 - b^2}{a^2 + b^2}}{\dfrac{2ab}{a^2 + b^2}}$$

$$= \frac{a^2 - b^2}{2ab}$$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{a^2 + b^2}{2ab}$ (from (ii))

and $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{a^2 + b^2}{a^2 - b^2}$ (from (i))

Q19

$$\sqrt{\frac{a+b}{a-b}}+\sqrt{\frac{a-b}{a+b}}$$

$$=\sqrt{\frac{\frac{a}{b}+1}{\frac{a}{b}-1}}+\sqrt{\frac{\frac{a}{b}-1}{\frac{a}{b}+1}}\ [\textit{Dividing both Numerator and denominator by } b]$$

$$=\sqrt{\frac{\tan\theta+1}{\tan\theta-1}}+\sqrt{\frac{\tan\theta-1}{\tan\theta+1}}$$

$$=\sqrt{\frac{\frac{\sin\theta}{\cos\theta}+1}{\frac{\sin\theta}{\cos\theta}-1}}+\sqrt{\frac{\frac{\sin\theta}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}+1}}$$

$$=\sqrt{\frac{\frac{\sin\theta+\cos\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\cos\theta}}}+\sqrt{\frac{\frac{\sin\theta-\cos\theta}{\cos\theta}}{\frac{\sin\theta+\cos\theta}{\cos\theta}}}$$

$$=\sqrt{\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}}+\sqrt{\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}}$$

$$=\frac{\sin\theta+\cos\theta+\sin\theta-\cos\theta}{\sqrt{\sin^2\theta-\cos^2\theta}}$$

$$=\frac{2\sin\theta}{\sqrt{\sin^2\theta-\cos^2\theta}}$$

Q20

Given = $\tan\theta = \frac{a}{b}$

To show: $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2 - b^2}{a^2 + b^2}$

Since, $\tan\theta = \frac{a}{b}$

$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = \frac{a}{b}$

$\Rightarrow \quad b\sin\theta = a\cos\theta = \lambda\ (\text{say})$

$\Rightarrow \quad \sin\theta = \frac{\lambda}{b}$ and $\cos\theta = \frac{\lambda}{a}$

how $\quad \frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{\frac{a.\lambda}{b} - \frac{b.\lambda}{a}}{\frac{a.\lambda}{b} + \frac{b.\lambda}{a}}$

$$= \frac{\lambda\left(\frac{a}{b} - \frac{b}{a}\right)}{\lambda\left(\frac{a}{b} + \frac{b}{a}\right)}$$

$$= \frac{\frac{a}{b} - \frac{b}{a}}{\frac{a}{b} + \frac{b}{a}}$$

$$= \frac{\frac{a^2 - b^2}{ab}}{\frac{a^2 + b^2}{ab}}$$

$$= \frac{a^2 - b^2}{a^2 + b^2}$$

Proved

Q21

Given, $\operatorname{cosec}\theta - \sin\theta = a^3$, $\sec\theta - \cos\theta = b^3$

To show: $a^2b^2\left(a^2+b^2\right) = 1$

Since, $\operatorname{cosec}\theta - \sin\theta = a^3$

$$\Rightarrow \quad \frac{1}{\sin\theta} - \sin\theta = a^3 \qquad \left(\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right)$$

$$\Rightarrow \quad \frac{1-\sin^2\theta}{\sin\theta} = a^3$$

$$\Rightarrow \quad \frac{\cos^2\theta}{\sin\theta} = a^3 \qquad \left(\because 1-\sin^2\theta = \cos^2\theta\right)$$

$$\Rightarrow \quad a = \frac{\cos^{2/3}\theta}{\sin^{1/3}\theta}$$

Since, $\frac{1}{\cos\theta} - \cos\theta = b^3 \qquad \left(\because \sec\theta = \frac{1}{\cos\theta}\right)$

$$\Rightarrow \quad \frac{1-\cos^2\theta}{\cos\theta} = b^3$$

$$\Rightarrow \quad \frac{\sin^2\theta}{\cos\theta} = b^3 \qquad \left(\because 1-\cos^2\theta = \sin^2\theta\right)$$

$$\Rightarrow \quad b = \frac{\sin^{2/3}\theta}{\cos^{1/3}\theta}$$

Now, $a^2b^2\left(a^2+b^2\right) = \frac{\cos^{4/3}\theta}{\sin^{2/3}\theta} \times \frac{\sin^{4/3}\theta}{\cos^{2/3}\theta}\left(\frac{\cos^{4/3}\theta}{\sin^{2/3}\theta} + \frac{\sin^{4/3}\theta}{\cos^{2/3}\theta}\right)$

$$= \cos^{2/3}\theta \times \sin^{2/3}\theta \, \frac{\left(\cos^{6/3}\theta + \sin^{6/3}\theta\right)}{\sin^{2/3}\theta . \cos^{2/3}\theta}$$

$$= \cos^2\theta + \sin^2\theta$$

$$= 1$$

Proved

Q22

Let,

$$\cot\theta(1+\sin\theta) = 4m \qquad \text{---(i)}$$

and, $$\cot\theta(1-\sin\theta) = 4n \qquad \text{---(ii)}$$

To show: $\left(m^2-n^2\right)^2 = mn$

From (i) and (ii), we get

$$m = \frac{\cot\theta(1+\sin\theta)}{4} \quad \& \quad n = \frac{\cot\theta(1-\sin\theta)}{4}$$

$$
\begin{aligned}
\text{LHS} &= \left(m^2-n^2\right)^2 \\
&= \left((m+n)(m-n)\right)^2 \\
&= (m+n)^2(m-n)^2 \\
&= \left(\frac{\cot\theta(1+\sin\theta)+\cot\theta(1-\sin\theta)}{4}\right)^2\left(\frac{\cot\theta(1+\sin\theta)-\cot\theta(1-\sin\theta)}{4}\right)^2 \\
&= \left(\frac{\cot\theta(1+\sin\theta+1-\sin\theta)}{4}\right)^2 \times \left(\frac{\cot\theta(1+\sin\theta-1+\sin\theta)}{4}\right)^2 \\
&= \frac{\cot^2\theta}{16}\times 4\times\frac{\cot^2\theta}{16}\times 4\sin^2\theta \\
&= \frac{\cot^2\theta}{16}\times\frac{\cos^2\theta}{\sin^2\theta}\sin^2\theta && \left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right] \\
&= \frac{\cot\theta}{4}\times\frac{\cot\theta}{4}\times\left(1-\sin^2\theta\right) && \left[\because \cos^2\theta = 1-\sin^2\theta\right] \\
&= \frac{\cot\theta(1+\sin\theta)}{4}\times\frac{\cot\theta(1-\sin\theta)}{4} \\
&= mn
\end{aligned}
$$

Q23

To show: $sin^6\theta + cos^6\theta = \dfrac{4-3\left(m^2-1\right)^2}{4}$, where $m^2 \le 2$

Since, $sin\,\theta + cos\,\theta = m$ ...(i)

$\Rightarrow \quad \left(sin\,\theta + cos\,\theta\right)^2 = m^2$

$\Rightarrow \quad sin^2\theta + cos^2\theta + 2\,sin\,\theta\,cos\,\theta = m^2$

$\Rightarrow \quad 1 + 2\,sin\,\theta\,cos\,\theta = m^2 \quad \left(\because sin^2\theta + cos^2\theta = 1\right)$

$\Rightarrow \quad 2\,sin\,\theta\,cos\,\theta = m^2 - 1$

$\Rightarrow \quad sin\,\theta\,cos\,\theta = \dfrac{m^2-1}{2}$ ...(ii)

$\therefore$ LHS $= sin^6\theta + cos^2\theta$

$= \left(sin^2\theta\right)^3 + \left(cos^2\theta\right)^3$

$= \left(sin^2\theta + cos^2\theta\right)\left(\left(sin^2\theta\right)^2 + \left(cos^2\theta\right)^2 - sin^2\theta\,cos^2\theta\right)$

$= 1.\left(\left(sin^2\theta\right)^2 + \left(cos^2\theta\right)^2 + 2\,sin^2\theta\,cos^2\theta - 2\,sin^2\theta\,cos^2\theta - sin^2\theta\,cos^2\theta\right)$

(adding and subtracting $2\,sin^2\theta\,cos^2\theta$)

$= \left(sin^2\theta + cos^2\theta\right)^2 - 3\,sin^2\theta\,cos^2\theta$

$= 1 - 3\,sin^2\theta\,cos^2\theta$

$= 1 - 3\left(sin\,\theta\,cos\,\theta\right)^2$

$= 1 - 3\dfrac{\left(m^2-1\right)^2}{4}$ (from (ii))

$= \dfrac{4-3\left(m^2-1\right)^2}{4}$, where $m^2 \le 2$

= RHS

Proved

Q24

$LHS = ab - a - b + 1$

$= (\sec\theta - \tan\theta)(\operatorname{cosec}\theta + \cot\theta) + \sec\theta - \tan\theta - \operatorname{cosec}\theta - \cot\theta + 1$

$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right) + \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} + 1$

$= \frac{1}{\sin\theta\cos\theta} + \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} \times \frac{1}{\sin\theta} - \tan\theta \times \cot\theta + \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} + 1$

$= \frac{1}{\sin\theta\cos\theta} - \frac{1}{\sin\theta} - \frac{1}{\cos\theta} - 1 + \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} + 1$

$= \frac{1}{\sin\theta\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$

$= \frac{1}{\sin\theta\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$

$= \frac{1 - \sin^2\theta - \cos^2\theta}{\sin\theta.\cos\theta}$

$= \frac{1 - (\cos^2\theta - \sin^2\theta)}{\sin\theta.\cos\theta}$

$= \frac{1-1}{\sin\theta.\cos\theta} = 0 = RHS.$ *Hence Proved*

Q25

$$\text{LHS} = \left|\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} + \sqrt{\frac{1+\sin\theta}{1-\sin\theta}}\right|$$

$$= \left|\frac{\left(\sqrt{1-\sin\theta}\right)^2 + \left(\sqrt{1+\sin\theta}\right)^2}{\sqrt{(1+\sin\theta)(1-\sin\theta)}}\right|$$

$$= \left|\frac{1-\sin\theta+1+\sin\theta}{\sqrt{1-\sin^2\theta}}\right|$$

$$= \left|\frac{2}{\cos\theta}\right| \qquad \left(\because\ 1-\sin^2\theta = \cos^2\theta \Rightarrow \sqrt{1-\sin^2\theta} = \cos\theta\right)$$

$$= \frac{-2}{\cos\theta} \qquad \left(\because \frac{\pi}{2} < \theta < \pi \Rightarrow \cos\theta < 0\right)$$

$= \text{RHS}$

Q26

We have,

$$T_n = \sin^n\theta + \cos^n\theta \qquad \text{(i)}$$

To show: $\dfrac{T_3 - T_5}{T_1} = \dfrac{T_5 - T_7}{T_3}$

$$\text{LHS} = \frac{T_3 - T_5}{T_1}$$

$$= \frac{\left(\sin^3\theta + \cos^3\theta\right) - \left(\sin^5\theta + \cos^5\theta\right)}{\sin\theta + \cos\theta} \qquad \left[\begin{array}{l}\text{Substituting the values of} \\ T_3,\ T_5 \text{ and } T_1 \text{ from (i)}\end{array}\right]$$

$$= \frac{\sin^3\theta - \sin^5\theta + \cos^3\theta - \cos^5\theta}{\sin\theta + \cos\theta}$$

$$= \frac{\sin^3\theta\left(1 - \sin^2\theta\right) + \cos^3\theta\left(1 - \cos^2\theta\right)}{\sin\theta + \cos\theta}$$

$$= \frac{\sin^3\theta\cos^2\theta + \cos^3\theta\sin^2\theta}{\sin\theta + \cos\theta} \qquad \left[\begin{array}{l}\because 1 - \sin^2\theta = \cos^2\theta \\ \text{and } 1 - \cos^2\theta = \sin^2\theta\end{array}\right]$$

$$= \frac{\sin^2\theta\cos^2\theta + \left(\sin\theta + \cos\theta\right)}{\sin\theta + \cos\theta}$$

$$= \sin^2\theta\cos^2\theta$$

$$\text{RHS} = \frac{\sin^5\theta + \cos^5\theta - \left(\sin^7\theta + \cos^7\theta\right)}{\sin^3\theta + \cos^3\theta}$$

$$= \frac{\sin^5\theta - \sin^7\theta + \cos^5\theta - \cos^7\theta}{\sin^3\theta + \cos^3\theta}$$

$$= \frac{\sin^5\theta\left(1 - \sin^2\theta\right) + \cos^5\theta\left(1 - \cos^2\theta\right)}{\sin^3\theta + \cos^3\theta}$$

$$= \frac{\sin^5\theta\cos^2\theta + \cos^5\theta\sin^2\theta}{\sin^3\theta + \cos^3\theta}$$

$$= \frac{\sin^2\theta\cos^2\theta\left(\sin^3\theta + \cos^3\theta\right)}{\sin^3\theta + \cos^3\theta}$$

$$= \sin^2\theta\cos^2\theta$$

$\therefore$ LHS = RHS Proved.

$$
\begin{aligned}
\text{LHS} &= 2T_6 - 3T_4 + 1 \\
&= 2\left(\sin^6\theta + \cos^6\theta\right) - 3\left(\sin^4\theta + \cos^4\theta\right) + 1 \\
&= 2\left(\left(\sin^2\theta\right)^3 + \left(\cos^2\theta\right)^3 - 3\left(\sin^2\theta\right)^2 + \left(\cos^2\theta\right)^2\right) + 1 \\
&= 2\left(\left(\sin^2\theta + \cos^2\theta\right)\left(\sin^2\theta\right)^2 - \left(\cos^2\theta\right)^2 - \left(\sin^2\theta\cos^2\theta\right)\right) - \\
&\qquad 3\left(\left(\sin^2\theta\right)^2 + \left(\cos^2\theta\right)^2 + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta\right) + 1
\end{aligned}
$$

$$\left[\begin{array}{l}\text{Using } a^3 + b^3 = (a+b)\left(a^2 - b^2 - ab\right) \text{ and adding and subtracting} \\ 2\sin^2\theta\cos^2\theta\end{array}\right]$$

$$
\begin{aligned}
&= 2\left(\left(\sin^2\theta + \cos^2\theta\right)^2 - 3\sin^2\theta\cos^2\theta\right) - 3\left(1 - 2\sin^2\theta\cos^2\theta\right) + 1 \\
&= 2\left(1 - 3\sin^2\theta\cos^2\theta\right) - 3 + 6\sin^2\theta\cos^2\theta + 1 \\
&= 2 - 6\sin^2\theta\cos^2\theta - 2 + 6\sin^2\theta\cos^2\theta \\
&= 0 \\
&= \text{RHS Proved.}
\end{aligned}
$$

$$\text{LHS} = 6T_{10} - 15T_8 + 10T_6 - 1$$

$$= 6\left(\sin^{10}\theta + \cos^{10}\theta\right) - 15\left(\sin^8\theta + \cos^8\theta\right) + 10\left(\sin^6\theta + \cos^6\theta\right) - 1$$

$$= 6\sin^{10}\theta - 15\sin^8\theta + 10\sin^6\theta + 6\cos^{10}\theta - 15\cos^8\theta + 10\cos^6\theta - 1$$

$$= \sin^6\theta\left(6\sin^4\theta - 15\sin^2\theta + 10\right) + \cos^6\theta\left(6\cos^4\theta - 15\cos^2\theta + 10\right) - \left(\sin^2\theta + \cos^2\theta\right)^3$$

$$\left[\because 1 = \sin^2\theta + \cos^2\theta\right]$$

$$= \sin^6\theta\left(6\sin^4\theta - 15\sin^2\theta + 10\right) + \cos^6\theta\left(6\cos^4\theta - 15\cos^2\theta + 10\right) - \left(\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta\left(\sin^2\theta + \cos^2\theta\right)\right)$$

$$\left[\text{Using } (a+b)^3 = a^3 + b^3 + 3ab(a+b)\right]$$

$$= \sin^6\theta\left(6\sin^4\theta - 15\sin^2\theta + 10 - 1\right) + \cos^6\theta\left(6\cos^4\theta - 15\cos^2\theta + 10 - 1\right) - 3\sin^2\theta\cos^2\theta \times 1$$

$$\left[\because \cos^2\theta + \sin^2\theta = 1\right]$$

$$= \sin^6\theta\left(6\sin^4\theta - 9\sin^2\theta - 6\sin^2\theta + 9\right) + \cos^6\theta\left(6\cos^4\theta - 9\cos^2\theta - 6\cos^2\theta + 9\right) - 3\sin^2\theta\cos^2\theta$$

$$\left[\text{On splitting the middle term}\right]$$

$$= \sin^6\theta\left(3\sin^2\theta\left(2\sin^2\theta - 3\right) - 3\left(2\sin^2\theta - 3\right)\right) + \cos^6\theta\left(3\cos^2\theta\left(2\cos^2\theta - 3\right) - 3\left(2\cos^2\theta - 3\right)\right) - 3\sin^2\theta\cos^2\theta$$

$$= \sin^6\theta\left(2\sin^2\theta - 3\right)\left(3\sin^2\theta - 3\right) + \cos^6\theta\left(2\cos^2\theta - 3\right)\left(3\cos^2\theta - 3\right) - 3\sin^2\theta\cos^2\theta$$

$$= \sin^6\theta \times (-3)\left(2\sin^2\theta - 3\right)\left(1 - \sin^2\theta\right) + \cos^6\theta \times (-3)\left(2\cos^2\theta - 3\right)\left(1 - \cos^2\theta\right) - 3\sin^2\theta\cos^2\theta$$

$$= -3\sin^6\theta\left(2\sin^2\theta - 3\right)\cos^2\theta - 3\cos^6\theta\left(2\cos^2\theta - 3\right)\sin^2\theta - 3\sin^2\theta\cos^2\theta$$

$$= -6\sin^8\theta\cos^2\theta + 9\sin^6\theta\cos^2\theta - 6\cos^8\theta\sin^2\theta + 9\cos^6\theta\sin^2\theta - 3\sin^2\theta\cos^2\theta$$

$$= -6\sin^2\theta\cos^2\theta\left(\sin^6\theta + \cos^6\theta\right) + 9\sin^2\theta\cos^2\theta\left(\sin^4\theta + \cos^4\theta\right) - 3\sin^2\theta\cos^2\theta$$

$$= -6\sin^2\theta\cos^2\theta\left(\left(\sin^2\theta\right)^3 + \left(\cos^2\theta\right)^3\right) + 9\sin^2\theta\cos^2\theta\left(\left(\sin^2\theta\right)^2 + \left(\cos^2\theta\right)^2\right) - 3\sin^2\theta\cos^2\theta$$

$$= -6\sin^2\theta\cos^2\theta\left(\sin^2\theta + \cos^2\theta\right)\left(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta\right)$$
$$+ 9\sin^2\theta\cos^2\theta\left(\sin^4\theta + \cos^4\theta\right) - 3\sin^2\theta\cos^2\theta$$

$$\left(\text{Using } a^3 + b^3 = (a+b)\left(a^2 + b^2 - ab\right)\right)$$

$$= -6\sin^2\theta\cos^2\theta\left(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta\right) + 9\sin^2\theta\cos^2\theta\left(\sin^4\theta + \cos^4\theta\right)$$
$$- 3\sin^2\theta\cos^2\theta \qquad \left(\because \cos^2\theta + \sin^2\theta = 1\right)$$

$= -6\sin^2\theta\cos^2\theta\left(\sin^4\theta+\cos^4\theta\right) - 6\sin^4\theta\cos^4\theta + 9\sin^2\theta\cos^2\theta\left(\sin^4\theta+\cos^4\theta\right) - 3\sin^2\theta\cos^2\theta$

$= 3\sin^2\theta\cos^2\theta\left(\sin^4\theta+\cos^4\theta\right) + 6\sin^4\theta\cos^4\theta - 3\sin^2\theta\cos^2\theta$

$= 3\sin^2\theta\cos^2\theta\left(\left(\sin^2\theta\right)^2+\left(\cos^2\theta\right)^2 + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta\right)$

$+ 6\sin^4\theta\cos^4\theta - 3\sin^2\theta\cos^2\theta$ (adding and subtracting $2\sin^2\theta\cos^2\theta$)

$= 3\sin^2\theta\cos^2\theta\left(\left(\sin^2\theta+\cos^2\theta\right)^2 - 2\sin^2\theta\cos^2\theta\right) + 6\sin 4\theta\cos^4\theta - 3\sin^2\theta\cos^2\theta$

$= 3\sin^2\theta\cos^2\theta\left(1-2\sin^2\theta\cos^2\theta\right) + 6\sin^4\theta\cos^4\theta - 3\sin^2\theta\cos^2\theta$

$= 3\sin^2\theta\cos^2\theta - 6\sin^4\theta\cos^4\theta + 6\sin^4\theta\cos^4\theta - 3\sin^2\theta\cos^2\theta$

$= 0$

$=$ RHS

Proved

Ex 5.3

Q1

We have,

$$\operatorname{cosec}^2\theta - \cot^2\theta = 1$$

$$\Rightarrow \quad \operatorname{cosec}^2\theta = 1 + \cot^2\theta$$

$$\Rightarrow \quad \operatorname{cosec}\theta = \pm\sqrt{1+\cot^2\theta}$$

In the third quadrant $\operatorname{cosec}\theta$ is negative

$$\therefore \quad \operatorname{cosec}\theta = -\sqrt{1+\cot^2\theta}$$

$$= -\sqrt{1+\left(\frac{12}{5}\right)^2} \qquad \left[\because \quad \cot\theta = \frac{12}{5}\right]$$

$$= -\sqrt{1+\frac{144}{25}}$$

$$= -\sqrt{\frac{169}{25}}$$

$$= -\frac{13}{5}$$

$$\therefore \quad \operatorname{cosec}\theta = -\frac{13}{5}$$

Now, $\tan\theta = \dfrac{1}{\cot\theta}$

$$= \frac{1}{\frac{12}{5}}$$

$$= \frac{5}{12}$$

We have,

$$\sin^2\theta + \cos^2\theta = 1$$

$\Rightarrow \quad \sin^2\theta = 1 - \cos^2\theta$

$\Rightarrow \quad \sin\theta = \pm\sqrt{1-\cos^2\theta}$

In the 2^{nd} quadrant $\sin\theta$ is positive and $\tan\theta$ is negative

$\therefore \quad \sin\theta = \sqrt{1-\cos^2\theta}$

$$= \sqrt{1-\left(-\frac{1}{2}\right)^2} \qquad \left[\because \quad \cos\theta = -\frac{1}{2}\right]$$

$$= \sqrt{1-\frac{1}{4}}$$

$$= \sqrt{\frac{3}{4}}$$

$$= \frac{\sqrt{3}}{2}$$

and, $\quad \tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$

Now, $\quad \operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\frac{\sqrt{3}}{2}} = \dfrac{2}{\sqrt{3}}$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-\frac{1}{2}} = -2$$

and $\quad \cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{-\sqrt{3}} = \dfrac{-1}{\sqrt{3}}$

Hence, $\sin\theta = \dfrac{\sqrt{3}}{2}$, $\quad \tan\theta = -\sqrt{3}$,

$\operatorname{cosec}\theta = \dfrac{2}{\sqrt{3}}$, $\sec\theta = -2$ and $\cot\theta = \dfrac{-1}{\sqrt{3}}$

In the third quadrant cosecθ is negative

$$\therefore \quad \operatorname{cosec}\theta = -\sqrt{1+\cot^2\theta}$$

$$= -\sqrt{1+\left(\frac{4}{3}\right)^2}$$

$$= -\sqrt{1+\frac{16}{9}}$$

$$= -\sqrt{\frac{25}{9}}$$

$$= -\frac{5}{3}$$

Now, $\sin\theta = \dfrac{1}{\operatorname{cosec}\theta} = \dfrac{1}{\frac{-5}{3}} = \dfrac{-3}{5}$

and, $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{\frac{-5}{4}} = \dfrac{-4}{5}$

Hence, $\sin\theta = \dfrac{-3}{5}$, $\cos\theta = \dfrac{-4}{5}$,

$\operatorname{cosec}\theta = -\dfrac{5}{3}$, $\sec\theta = \dfrac{-5}{4}$ and $\cot\theta = \dfrac{4}{3}$

We have,

$$\sin^2\theta + \cos^2\theta = 1$$

$\Rightarrow \quad \cos^2\theta = 1 - \sin^2\theta$

$\Rightarrow \quad \cos\theta = \pm\sqrt{1-\sin^2\theta}$

In the 1st quadrant $\cos\theta$ is positive and $\tan\theta$ is also positive

$\therefore \quad \cos\theta = \sqrt{1-\sin^2\theta}$

$$= \sqrt{1-\left(\frac{3}{5}\right)^2} \qquad \left[\because \quad \sin\theta = \frac{3}{5}\right]$$

$$= \sqrt{1-\frac{9}{25}}$$

$$= \sqrt{\frac{16}{25}}$$

$$= \frac{4}{5}$$

and, $\quad \tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{3}{5}}{\frac{4}{5}} = \dfrac{3}{4}$

Now, $\quad \operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\frac{3}{5}} = \dfrac{5}{3}$

$\quad \sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\frac{4}{5}} = \dfrac{5}{4}$

and, $\quad \cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{\frac{3}{4}} = \dfrac{4}{3}$

Hence, $\cos\theta = \dfrac{4}{5}$, $\quad \operatorname{cosec}\theta = \dfrac{5}{3}$ $\quad$, $\tan\theta = \dfrac{3}{4}$,

$\sec\theta = \dfrac{5}{4}$, $\quad$ and $\cot\theta = \dfrac{4}{3}$

Q2

We have,

$$\sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \quad \cos^2\theta = 1 - \sin^2\theta$$

$$\Rightarrow \quad \cos\theta = \pm\sqrt{1-\sin^2\theta}$$

In the 2^{nd} quadrant $\cos\theta$ is negative and $\tan\theta$ is also negative

$$\therefore \quad \cos\theta = -\sqrt{1-\sin^2\theta}$$

$$= -\sqrt{1-\left(\frac{12}{13}\right)^2} \qquad \left[\because \quad \sin\theta = \frac{12}{13}\right]$$

$$= -\sqrt{1-\frac{144}{169}}$$

$$= -\sqrt{\frac{25}{169}}$$

$$= -\frac{5}{13}$$

and, $$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{12}{13}}{\frac{-5}{13}} = -\frac{12}{5}$$

Now, $$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{-5}{13}} = -\frac{13}{5}$$

$$\therefore \quad \sec\theta + \tan\theta = -\frac{13}{5} - \frac{12}{5}$$

$$= \frac{-13-12}{5}$$

$$= -\frac{25}{5}$$

$$= -5$$

$$\Rightarrow \quad \sec\theta + \tan\theta = -5$$

Q3

We have,

$$\sin\theta = \frac{3}{5}, \quad \tan\phi = \frac{1}{2} \quad \text{and } \frac{\pi}{2} < \theta < \pi < \frac{3\pi}{2}$$

$\Rightarrow$ θ lies in the second quadrant and ϕ lies in the third quadrant.

Now, $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow$ $\cos^2\theta = 1 - \sin^2\theta$

$\Rightarrow$ $\cos\theta = \pm\sqrt{1 - \sin^2\theta}$

In the 2^{nd} quadrant $\cos\theta$ is negative and $\tan\theta$ is also negative

$\therefore$ $\cos\theta = -\sqrt{1 - \sin^2\theta}$

$$= -\sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$= -\sqrt{1 - \frac{9}{25}}$$

$$= -\sqrt{\frac{16}{25}}$$

$$= -\frac{4}{5}$$

$\Rightarrow$ $\cos\theta = -\frac{4}{5}$

and, $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{3}{5}}{\frac{-4}{5}} = -\dfrac{3}{4}$ - - - - - (i)

Now, $\sec^2\phi - \tan^2\phi = 1$

$\Rightarrow$ $\sec^2\phi = 1 + \tan^2\phi$

$\Rightarrow$ $\sec\phi = \pm\sqrt{1 + \tan^2\phi}$

In the third quadrant $\sec\phi$ is negative

$$\therefore \quad \sec\phi = -\sqrt{1+\left(\frac{1}{2}\right)^2}$$

$$= -\sqrt{1+\frac{1}{4}}$$

$$= -\sqrt{\frac{5}{4}}$$

$$\Rightarrow \quad \sec\phi = -\frac{\sqrt{5}}{2} \text{ - - - - - - - (ii)}$$

$$\therefore \quad 8\tan\theta - \sqrt{5}\sec\phi$$

$$= 8\times\left(\frac{-3}{4}\right) - \sqrt{5}\times\left(-\frac{\sqrt{5}}{2}\right) \qquad \text{[by equations (i) and (ii)]}$$

$$= -2\times 3 + \frac{5}{2}$$

$$= -6 + \frac{5}{2}$$

$$= \frac{-12+5}{2}$$

$$= \frac{-7}{2}$$

$$\therefore \quad 8\tan\theta - \sqrt{5}\sec\phi = -\frac{7}{2}$$

Q4

We have,

$$\sin\theta + \cos\theta = 0$$

$\Rightarrow \quad \sin\theta = -\cos\theta \text{ ------- (i)}$

$\Rightarrow \quad \dfrac{\sin\theta}{\cos\theta} = -1$

$\Rightarrow \quad \tan\theta = -1$

We know that,

$$\sec^2\theta - \tan^2\theta = 1$$

$\Rightarrow \quad \sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow \quad \sec\theta = \pm\sqrt{1 + \tan^2\theta}$

In the 4[th] quadrant $\sec\theta$ is positive.

$$\therefore \quad \sec\theta = \sqrt{1 + \tan^2\theta}$$

$$= \sqrt{1 + (-1)^2}$$

$$= \sqrt{1 + 1}$$

$$= \sqrt{2}$$

$$\therefore \quad \cos\theta = \frac{1}{\sec\theta} = \frac{1}{\sqrt{2}}$$

putting $\cos\theta = \dfrac{1}{\sqrt{2}}$ in equation (i), we get,

$$\sin\theta = -\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}}$$

Hence, $\sin\theta = -\dfrac{1}{\sqrt{2}}$ and $\cos\theta = \dfrac{1}{\sqrt{2}}$.

Q5

We have,

$$\cos\theta = -\frac{3}{5}, \quad \text{and } \pi < \theta < \frac{3\pi}{2}$$

$\Rightarrow$ θ lies in the 3rd quadrant

We know that,

$\Rightarrow$ $\sin\theta = \pm\sqrt{1-\cos^2\theta}$

In the 3rd quadrant $\sin\theta$ is negative and $\tan\theta$ is positive.

$\therefore$ $\sin\theta = -\sqrt{1-\cos^2\theta}$

$$= -\sqrt{1-\left(-\frac{3}{5}\right)^2} \qquad \left[\because \quad \cos\theta = -\frac{3}{5}\right]$$

$$= -\sqrt{1-\frac{9}{25}}$$

$$= -\sqrt{\frac{16}{25}}$$

$$= -\frac{4}{5}$$

$\Rightarrow$ $\sin\theta = -\frac{4}{5}$

and, $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{-4}{5}}{\frac{-3}{5}} = \dfrac{4}{3}$

Now, $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\frac{-4}{5}} = \dfrac{-5}{4}$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\frac{-3}{5}} = \dfrac{-5}{3}$

and, $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{\frac{4}{3}} = \dfrac{3}{4}$

$$\therefore \qquad \frac{\operatorname{cosec}\theta + \cot\theta}{\sec\theta - \tan\theta} = \frac{\frac{-5}{4} + \frac{3}{4}}{\frac{-5}{3} - \frac{4}{3}}$$

$$= \frac{\frac{-5+3}{4}}{\frac{-5-4}{3}}$$

$$= \frac{-\frac{2}{4}}{-\frac{9}{3}}$$

$$= \frac{2}{4} \times \frac{3}{9}$$

$$= \frac{1}{6}$$

$$\therefore \qquad \frac{\operatorname{cosec}\theta + \cot\theta}{\sec\theta - \tan\theta} = \frac{1}{6}$$

Ex 5.3

Q1(i)

$$\sin\frac{5\pi}{3} = \sin\left(2\pi - \frac{\pi}{3}\right)$$
$$= -\sin\frac{\pi}{3} \qquad \left(\because \sin(2\pi - \theta) = -\sin\theta\right)$$
$$= \frac{-\sqrt{3}}{2}$$

Q1(ii)

$$3060^\circ = 17\pi \qquad \left(\because \pi = 180^\circ\right)$$
$$\therefore \sin 3060^\circ = \sin 17\pi$$
$$= 0 \qquad \left(\because \sin n\pi = 0 \text{ for all } n \in z\right)$$

Q1(iii)

$$\tan\frac{11\pi}{6} = \tan\left(2\pi - \frac{\pi}{6}\right)$$
$$= -\tan\frac{\pi}{6} \qquad \left(\because \tan(2\pi - \theta) = -\tan\theta\right)$$
$$= \frac{-1}{\sqrt{3}}$$

Q1(iv)

$$1125^\circ = 6\pi + \frac{\pi}{4} \left(\pi = 180^\circ\right)$$
$$\cos\left(-1125^\circ\right) = \cos\left(-\left(6\pi + \frac{\pi}{4}\right)\right)$$
$$= \cos\left(6\pi + \frac{\pi}{4}\right) \qquad \left(\because \cos(-\theta) = \cos\theta\right)$$
$$= \cos\left(2 \times 3\pi + \frac{\pi}{4}\right)$$
$$= \cos\frac{\pi}{4} \qquad \left(\because \cos(2k\pi + \theta) = \cos\theta, k \in n\right)$$
$$= \frac{1}{\sqrt{2}}$$

Q1(v)

$$\tan 315^\circ = \tan\left(2\pi - \frac{\pi}{4}\right)$$

$$= -\tan\frac{\pi}{4} \qquad \left(\because \tan(2\pi - \theta) = -\tan\theta\right)$$

$$= -1$$

Q1(iv)

$$\sin 510^\circ = \sin\left(3\pi - \frac{\pi}{6}\right)$$

$$= \sin\frac{(\pi)}{6} \qquad \left(\because 3\pi - \frac{\pi}{6} \text{ lies in second quadrant}\right)$$

$$= \frac{1}{2}$$

Alternative solution

$$\sin 510^\circ = \sin\left(3\pi - \frac{\pi}{6}\right)$$

$$= \sin\left(2\pi + \left(\pi - \frac{\pi}{6}\right)\right)$$

$$= \sin\left(\pi - \frac{\pi}{6}\right) \qquad \left(\because \sin(2\pi + \theta) = \sin\theta, \text{ as sine is periodic with period } 2\pi\right)$$

$$= \sin\frac{\pi}{6} \qquad \left(\because \sin(\pi - \theta) = \sin\theta\right)$$

$$= \frac{1}{2}$$

Q1(vii)

$$\cos 570^\circ = \cos\left(3\pi + \frac{\pi}{6}\right)$$

$$= \cos\left(2\pi + \left(\pi + \frac{\pi}{6}\right)\right)$$

$$= \cos\left(\pi + \frac{\pi}{6}\right) \qquad \left(\begin{array}{l}\because \cos(2\pi + \theta) = \cos\theta, \text{ as cosine} \\ \text{is periodoc with period } 2\pi\end{array}\right)$$

$$= -\cos\frac{\pi}{6} \qquad \left(\because \cos(\pi + \theta) = -\cos\theta\right)$$

$$= \frac{-\sqrt{3}}{2}$$

Q1(viii)

$$\sin\left(-330^\circ\right) = \sin\left(-\left(2\pi - \frac{\pi}{6}\right)\right)$$

$$= \sin\left(2\pi - \frac{\pi}{6}\right) \qquad \left(\because \sin(-\theta) = -\sin\theta\right)$$

$$= -\left(-\sin\frac{\pi}{6}\right) \qquad \left(\because \sin(2\pi - \theta) = -\sin\theta\right)$$

$$= \sin\frac{\pi}{6}$$

$$= \frac{1}{2}$$

Q1(ix)

$$\operatorname{cosec}\left(-1200^\circ\right) = \operatorname{cosec}\left(-\left(7\pi - \frac{\pi}{3}\right)\right)$$

$$= \operatorname{cosec}\left(7\pi - \frac{\pi}{3}\right) \qquad \left(\because \operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta\right)$$

$$= -\operatorname{cosec}\left(2\times 3\pi + \left(\pi - \frac{\pi}{3}\right)\right)$$

$$= -\operatorname{cosec}\left(\pi - \frac{\pi}{3}\right) \qquad \left(\begin{array}{l}\because \operatorname{cosec} \text{ is periodic of period } 2\pi,\\ \therefore \operatorname{cosec}(2\pi + \theta) = \operatorname{cosec}(2n\pi + \theta)\\ \qquad = \operatorname{cosec}\theta \text{ for all } n \in N\end{array}\right)$$

$$= -\operatorname{cosec}\frac{\pi}{3} \qquad \left(\therefore \operatorname{cosec}(\pi - \theta) = \operatorname{cosec}\theta\right)$$

$$= \frac{-2}{\sqrt{3}}$$

Q1(x)

$$\tan\left(-585^\circ\right) = -\tan(585) \qquad \left(\because \tan(-\theta) = -\tan\theta\right)$$

$$= -\tan\left(3\pi + \frac{\pi}{4}\right)$$

$$= -\tan\left(2\pi + \left(\pi + \frac{\pi}{4}\right)\right) \qquad \left(\because \tan(2\pi + \theta) = \tan\theta\right)$$

$$= -\tan\frac{\pi}{4} \qquad \left(\because \tan(\pi + \theta) = \tan\theta\right)$$

$$= -1$$

Q1(xi)

$$\begin{aligned}
\cos\left(855^\circ\right) &= \cos\left(5\pi - \frac{\pi}{4}\right) \\
&= \cos\left(2\times 2\pi + \left(\pi - \frac{\pi}{4}\right)\right) \\
&= \cos\left(\pi - \frac{\pi}{4}\right) && \left(\because\ \cos\left(2k\pi + \theta\right) = \cos\theta \text{ for all } k \in N\right) \\
&= -\cos\frac{\pi}{4} && \left(\because \cos\left(\pi - \theta\right) = -\cos\theta\right) \\
&= \frac{-1}{\sqrt{2}}
\end{aligned}$$

Q1(xii)

$$\begin{aligned}
\sin 1845^\circ &= \sin\left(10\pi + \frac{\pi}{4}\right) \\
&= \left(2\times 5\pi + \frac{\pi}{4}\right) \\
&= \sin\pi && \left(\because\ \sin\left(2k\pi + \theta\right) = \sin\theta,\ \text{ for all } k \in N\right) \\
&= \frac{1}{\sqrt{2}}
\end{aligned}$$

Q1(xiii)

$$\begin{aligned}
\cos 1755^\circ &= \cos\left(10\pi - \frac{\pi}{4}\right) \\
&= \cos\left(2\times 5\pi - \frac{\pi}{4}\right) \\
&= \cos\frac{\pi}{4} && \left(\because \cos\left(2k\pi - \theta\right) = \cos\theta, k \in N\right) \\
&= \frac{1}{\sqrt{2}}
\end{aligned}$$

Q1(xiv)

$$4530^\circ = \left(25\pi + \frac{\pi}{6}\right)$$

$$\therefore \quad \sin 4530 = \sin\left(25\pi + \frac{\pi}{6}\right)$$

$$= \sin\left(2 \times 12\pi + \left(\pi + \frac{\pi}{6}\right)\right)$$

$$= \sin\left(\pi \frac{\pi}{6}\right) \qquad \left(\because \sin\left(2k\pi + \theta\right) = \sin\theta, k \in N\right)$$

$$= -\sin\frac{\pi}{6} \qquad \left(\because \sin\left(\pi + \theta\right) = -\sin\theta\right)$$

$$= \frac{-1}{2}$$

Q2(i)

$$\text{LHS} = \tan 225^\circ \ \cot 405^\circ + \tan 765^\circ \cot 675^\circ$$

$$= \tan\left(\pi + \frac{\pi}{4}\right)\cot\left(2\pi + \frac{\pi}{4}\right) + \tan\left(4\pi + \frac{\pi}{4}\right)\cot\left(4\pi - \frac{\pi}{4}\right)$$

$$= \tan\frac{\pi}{4}.\ \cot\frac{\pi}{4} + \tan\frac{\pi}{4} \times \left(-\cot\frac{\pi}{4}\right) \qquad \left(\because \cot\left(4\pi - \frac{\pi}{4}\right) = -\cot\frac{\pi}{4}\right)$$

$$= 1.1 + 1.\left(-1\right)$$

$$= 0$$

$$= \text{RHS}$$

Proved

Q2(ii)

$$\text{LHS} = \sin\frac{8\pi}{3}\cos\frac{23\pi}{6} + \cos\frac{13\pi}{3}\sin\frac{35\pi}{6}$$

$$= \sin\left(3\pi - \frac{\pi}{3}\right)\cos\left(4\pi - \frac{\pi}{6}\right) + \cos\left(4\pi + \frac{\pi}{3}\right)\sin\left(6\pi - \frac{\pi}{6}\right)$$

$$= \sin\frac{\pi}{3}\cos\frac{\pi}{6} + \cos\frac{\pi}{3}\left(-\sin\frac{\pi}{6}\right) \qquad \left(\because\ \sin(6\pi - \theta) = -\sin\theta\right)$$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{22} + \frac{1}{2} \times \left(\frac{-1}{2}\right)$$

$$= \frac{3}{4} - \frac{1}{4}$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

= RHS

Proved

Q2(iii)

$$\text{LHS} = \cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ$$

$$= \cos 24^\circ + \cos 204^\circ + \cos 55^\circ + \cos 125^\circ + \cos 300^\circ$$

$$= \cos 24^\circ + \cos\left(\pi + 24^\circ\right) + \cos 55^\circ + \cos\left(\pi - 55^\circ\right) + \cos\left(2\pi - \frac{\pi}{3}\right)$$

$$= \cos 24^\circ - \cos 24^\circ + \cos 55^\circ - \cos 55^\circ + \cos\frac{\pi}{3}$$

$$= \cos\frac{\pi}{3}$$

$$= \frac{1}{2}$$

= RHS

Proved

Q2(iv)

$$\text{LHS} = \tan\left(-225^\circ\right)\cot\left(-405^\circ\right) - \tan\left(-765^\circ\right)\cot\left(675^\circ\right)$$

$$= -\tan 225^\circ\left(-\cot 405^\circ\right) + \tan 765^\circ \cot 765^\circ \qquad \left(\begin{array}{l}\because \tan(-\theta) = -\tan\theta \\ \& \ \cot(-\theta) = -\cot\theta\end{array}\right)$$

$$= \tan\left(\pi + \frac{\pi}{4}\right)\cot\left(2\pi \frac{\pi}{4}\right) + \tan\left(4\pi + \frac{\pi}{4}\right)\cot\left(4\pi - \frac{\pi}{4}\right)$$

$$= \tan\frac{\pi}{4}\cot\frac{\pi}{4} + \tan\frac{\pi}{4} \times \left(-\cot\frac{\pi}{4}\right) \qquad \left(\because \cot(4\pi - \theta) = -\cot\theta\right)$$

$$= 1.1 + 1(-1)$$

$$= 1 - 1$$

$$= 0$$

$= \text{RHS}$

Proved

Q2(v)

$$\text{LHS} = \cos 570^\circ \sin 510^\circ + \sin\left(-330^\circ\right)\cos\left(-390^\circ\right)$$

$$= \cos\left(3\pi + \frac{\pi}{6}\right)\sin\left(3\pi - \frac{\pi}{6}\right) - \sin 330^\circ \cos 390^\circ \qquad \left(\begin{array}{l}\because \sin(-\theta) = -\sin\theta \text{ and} \\ \cos(-\theta) = \cos\theta\end{array}\right)$$

$$= -\cos\frac{\pi}{6}\sin\frac{\pi}{6} - \sin\left(2\pi - \frac{\pi}{6}\right)\cos\left(2\pi + \frac{\pi}{6}\right)$$

$$= -\sin\frac{\pi}{6}\cos\frac{\pi}{6} + \sin\frac{\pi}{6}.\cos\frac{\pi}{6} \qquad \left(\because \sin(2\pi - \theta) = -\sin\theta\right)$$

$$= 0$$

$= \text{RHS}$

Proved

Q2(vi)

$$\text{LHS} = \tan\frac{11\pi}{3} - 2\sin\frac{4\pi}{6} - \frac{3}{4}\operatorname{cosec}^2\frac{\pi}{4} + 4\cos^2\frac{17\pi}{6}$$

$$= \tan\left(4\pi - \frac{\pi}{3}\right) - 2\sin\frac{2\pi}{3} - \frac{3}{4}\times\left(\sqrt{2}\right)^2 + 4\cos^2\left(3\pi - \frac{\pi}{6}\right)$$

$$= -\tan\frac{\pi}{3} - 2\sin\left(\pi - \frac{\pi}{3}\right) - \frac{3}{4}\times 2 + 4\cos^2\frac{\pi}{6}$$

$$\left(\because \tan\left(4\pi - \frac{\pi}{3}\right) = -\tan\frac{\pi}{3}, \cos\left(3\pi - \frac{\pi}{6}\right) = -\cos\frac{\pi}{6}\right)$$

$$= -\sqrt{3} - 2\sin\frac{\pi}{3} - \frac{3}{2} + 4\times\left(\frac{\sqrt{3}}{2}\right)^2$$

$$= -\sqrt{3} - 2\times\frac{\sqrt{3}}{2} - \frac{3}{2} + 4\times\frac{3}{4}$$

$$= -\sqrt{3} - \sqrt{3} - \frac{3}{2} + 3$$

$$= -2\sqrt{3}\,\frac{-3+6}{2}$$

$$= -2\sqrt{3} + \frac{3}{2}$$

$$= \frac{3 - 4\sqrt{3}}{2}$$

= RHS

Proved

Q2(vii)

$$\text{LHS} = 3\sin\frac{\pi}{6}\sec\frac{\pi}{3} - 4\sin\frac{5\pi}{6}\cot\frac{\pi}{4}$$

$$= 3\times\frac{1}{2}\times 2 - 4\sin\left(\pi - \frac{\pi}{6}\right)\times 1$$

$$= 3 - 4\sin\frac{\pi}{6} \qquad \left(\because \sin(\pi - \theta) = \sin\theta\right)$$

$$= 3 - 4\times\frac{1}{2}$$

$$= 3 - 2$$

$$= 1$$

= RHS

Proved

Q3(i)

$$\text{LHS} = \frac{\cos(2\pi+\theta)\operatorname{cos\,ec}(2\pi+\theta)\tan\left(\frac{\pi}{2}+\theta\right)}{\sec\left(\frac{\pi}{2}+\theta\right)\cos\theta\cot(\pi+\theta)}$$

$$= \frac{\cos\theta \times \operatorname{cos\,ec}\theta\,(-\cot\theta)}{-\operatorname{cos\,ec}\theta.\cos\theta\cot\theta} \qquad \left(\begin{array}{l}\because \tan\left(\frac{\pi}{2}+\theta\right) = -\cot\theta \\ \ \&\ \sec\left(\frac{\pi}{2}+\theta\right) = -\operatorname{cos\,ec}\theta\end{array}\right)$$

$= 1$

$= \text{RHS}$

Proved

Q3(ii)

$$\text{LHS} = \frac{\operatorname{cos\,ec}(90^\circ+\theta)+\cot(450^\circ+\theta)}{\operatorname{cos\,ec}(90^\circ-\theta)+\tan(180^\circ-\theta)} + \frac{\tan(180^\circ+\theta)+\sec(180^\circ-\theta)}{\tan(360^\circ+\theta)-\sec(-\theta)}$$

$$= \frac{\sec\theta+\cot\left(2\pi+\frac{\pi}{2}+\theta\right)}{\sec\theta-\tan\theta} + \frac{\tan\theta-\sec\theta}{\tan\theta-\sec\theta}$$

$$\left(\because \operatorname{cos\,ec}(90^\circ+\theta) = \sec\theta, \operatorname{cos\,ec}(90^\circ+\theta) = \sec\theta, \tan(180^\circ-\theta) = -\tan\theta\,\sec(-\theta) = \sec\theta\right)$$

$$= \frac{\sec\theta+\cot\left(\frac{\pi}{2}+\theta\right)}{\sec\theta-\tan\theta} + 1 \qquad \left(\because \cot(2\pi+\theta) = \cot\theta\right)$$

$$= \frac{\sec\theta-\tan\theta}{\sec\theta-\tan\theta} + 1 \qquad \left(\because \cot\left(\frac{\pi}{2}+\theta\right) = -\tan\theta\right)$$

$= 1 + 1$

$= 2$

$= \text{RHS}$

Proved

Q3(iii)

$$\text{LHS} = \frac{\sin(180^\circ+\theta)\cos(90^\circ+\theta)\tan(270^\circ-\theta)\cot(360^\circ-\theta)}{\sin(360^\circ-\theta)\cos(360^\circ+\theta)\operatorname{cosec}(-\theta)\sin(270^\circ+\theta)}$$

$$= \frac{\sin\theta(-\sin\theta)\cot\theta(-\cot\theta)}{-\sin\theta\cos\theta(-\operatorname{cosec}\theta)(-\cos\theta)} \quad \left(\begin{array}{l}\because \tan(270^\circ-\theta)=\cot\theta \\ \& \sin(270^\circ+\theta)=-\cos\theta\end{array}\right)$$

$$= \frac{-\sin\theta\times\sin\theta\times\cos\theta\times\cos\theta\times\sin\theta}{-\sin\theta\times\cos\theta\times\sin\theta\times\sin\theta\times\cos\theta} \quad \left(\begin{array}{l}\because \cot\theta=\dfrac{\cos\theta}{\sin\theta} \\ \& \operatorname{cosec}\theta=\dfrac{1}{\sin\theta}\end{array}\right)$$

$= 1$

$=$ RHS

Proved

Q3(iv)

$$\text{LHS} = \left\{1+\cot\theta-\sec\left(\frac{\pi}{2}+\theta\right)\right\}\left\{1+\cot\theta+\sec\left(\frac{\pi}{2}+\theta\right)\right\}$$

$$= \{1+\cot\theta-(-\operatorname{cosec}\theta)\}\{1+\cot\theta-\operatorname{cosec}\theta\}$$

$$\left(\because \sec\left(\frac{\pi}{2}+\theta\right)=-\operatorname{cosec}\theta\right)$$

$$= \{(1+\cot\theta)+\operatorname{cosec}\theta\}\{(1+\cot\theta)-\operatorname{cosec}\theta\}$$

$$= (1+\cot\theta)^2-\operatorname{cosec}^2\theta$$

$$= 1+\cot^2\theta+2\cot\theta-\operatorname{cosec}^2\theta$$

$$= \operatorname{cosec}^2+2\cot\theta-\operatorname{cosec}^2 \quad \{\because 1+\cot^2\theta=\operatorname{cosec}^2\theta\}$$

$$= 2\cot\theta$$

$=$ RHS

Proved

Q3(v)

$$\text{LHS} = \frac{\tan(90^\circ - \theta)\sec(180^\circ - \theta)\sin(-\theta)}{\sin(180^\circ + \theta)\cot(360^\circ - \theta)\operatorname{cosec}(90^\circ - \theta)}$$

$$= \frac{\cot\theta \times (-\sec\theta) \times (-\sin\theta)}{-\sin\theta \times (-\cot\theta) \times \sec\theta}$$

$= 1$

$= \text{RHS}$

Proved

Q4

$$\text{LHS} = \sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18} + \sin^2\frac{4\pi}{9}$$

$$= \sin^2\frac{\pi}{18} + \sin^2\frac{4\pi}{9} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18}$$

$$= \sin^2\left(\frac{\pi}{2} - \frac{4\pi}{9}\right) + \sin^2\frac{4\pi}{9} + \sin^2\frac{\pi}{9} + \sin^2\left(\frac{\pi}{2} - \frac{\pi}{9}\right) \qquad \left(\because \frac{\pi}{18} = \frac{\pi}{2} - \frac{4\pi}{9} \text{ and } \frac{7\pi}{18} = \frac{\pi}{2} - \frac{\pi}{9}\right)$$

$$= \cos^2\frac{4\pi}{9} + \sin^2\frac{4\pi}{9} + \sin^2\frac{\pi}{9} + \cos^2\frac{\pi}{9} \qquad \left(\because \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta\right)$$

$= 1 + 1 \quad (\because \sin^2\theta + \cos^2\theta = 1)$

$= 2$

$= \text{RHS}$

Proved

Q5

$$\text{LHS} = \sec\left(\frac{3\pi}{2} - \theta\right)\sec\left(\theta - \frac{5\pi}{2}\right) + \tan\left(\frac{5\pi}{2} + \theta\right)\tan\left(\theta - \frac{3\pi}{2}\right)$$

$$= \sec\left(\frac{3\pi}{2} - \theta\right)\sec\left(-\left(\frac{5\pi}{2} - \theta\right)\right) + \tan\left(\frac{5\pi}{2} + \theta\right)\tan\left(-\left(\frac{3\pi}{2} - \theta\right)\right)$$

$$= -\operatorname{cosec}\theta . \sec\left(\frac{5\pi}{2} - \theta\right) - \cot\theta \times (-)\tan\left(\frac{3\pi}{2} - \theta\right)$$

$$\left[\because \left(\sec\left(\frac{3\pi}{2} - \theta\right)\right) = -\operatorname{cosec}\theta,\ \sec(-\theta) = \sec\theta, \tan\left(\frac{5\pi}{2} + \theta\right) = -\cot\theta \ \& \tan(-\theta) = -\tan\theta)\right]$$

$$= -\operatorname{cosec}\theta \times \operatorname{cosec}\theta - \cot\theta \times (-1) \times \cot\theta \qquad \left[\left(\because \sec\left(\frac{5\pi}{2} - \theta\right)\right) = \operatorname{cosec}\theta \ \& \tan\left(\frac{3\pi}{2} - \theta\right) = \cot\theta\right]$$

$$= -\operatorname{cosec}^2\theta + \cot^2\theta$$

$$= -\operatorname{cosec}^2\theta + \operatorname{cosec}^2\theta - 1 \qquad \left(\because \operatorname{cosec}^2\theta = 1 + \cot^2\theta\right)$$

$$= -1$$

= RHS

Proved

Q6(i)

We have $A + B + C = \pi$ $\qquad \left(\because \text{sum of 3 angles of a triangle is } \pi = 180^\circ\right)$

$\Rightarrow \quad A + B = \pi - C$

$\Rightarrow \quad \cos(A + B) = \cos(\pi - C)$

$\Rightarrow \quad\quad = -\cos C$ $\qquad \left(\because \cos(\pi - \theta) = -\cos\theta\right)$

$\Rightarrow \quad \cos(A + B) + \cos C = 0$

Proved

Q6(ii)

We have $A+B+C=\pi$ $\left(\because \text{ sum of 3 angles of a triangle is } \pi = 180^\circ\right)$

$\Rightarrow \quad A+B=\pi-C$

$\Rightarrow \quad \dfrac{A+B}{2}=\dfrac{\pi-C}{2}$

$\Rightarrow \quad \dfrac{A+B}{2}=\dfrac{\pi}{2}-\dfrac{C}{2}$

$\Rightarrow \quad \cos=\left(\dfrac{A+B}{2}\right)=\cos\left(\dfrac{\pi}{2}-\dfrac{C}{2}\right)$

$\Rightarrow \quad =\sin\dfrac{C}{2}$ $\left(\because \cos\left(\dfrac{\pi}{2}-\theta\right)=\sin\theta\right)$

Hence $\cos\left(\dfrac{A+B}{2}\right)=\sin\dfrac{C}{2}$

Proved

Q6(iii)

We have $A+B+C=\pi$ $\left(\because \text{ sum of 3 angles of a triangle is } \pi = 180^\circ\right)$

$\Rightarrow \quad A+B=\pi-C$

$\Rightarrow \quad \dfrac{A+B}{2}=\dfrac{\pi-C}{2}$

$\Rightarrow \quad \dfrac{A+B}{2}=\dfrac{\pi}{2}-\dfrac{C}{2}$

$\Rightarrow \quad \tan\left(\dfrac{A+B}{2}\right)=\tan\left(\dfrac{\pi}{2}-\dfrac{C}{2}\right)$

$=\cot\dfrac{C}{2}$ $\left(\because \tan\left(\dfrac{\pi}{2}-\theta\right)=\cot\theta\right)$

Hence $\tan\left(\dfrac{A+B}{2}\right)=\cot\dfrac{C}{2}$

Proved

Q7

$\because$ A, B, C, D are the angles of a cyclic quadrilateral in order,

$\therefore$ $A + C = \pi$ & $B + D = \pi$

$\Rightarrow$ $\pi - A = C$ & $\pi - D = B$

$\Rightarrow$ $\cos(\pi - A) = \cos C$ ....(i)

& $\cos(\pi - D) = \cos B$ ....(ii)

Now, $\cos(180^\circ - A) + \cos(180^\circ + B) + (180^\circ + C) - \sin(90^\circ + D)$

$= \cos C + (-\cos B) - \cos C - \cos D$

$(\because \cos(180^\circ + B) = -\cos B, \cos(180^\circ + C) = -\cos C$ & using (i))

$= -\cos B - \cos D$

$= -\cos B - (-\cos B)$ (using (ii))

$= -\cos B + \cos B$

$= 0$

Proved

Q8(i)

$\operatorname{cosec}(90^\circ + \theta) + x\cos\theta\cot(90^\circ + \theta) = \sin(90^\circ + \theta)$

$\Rightarrow$ $\sec\theta + x\cos\theta \times (-\tan\theta) = \cos\theta$

$\Rightarrow$ $\dfrac{1}{\cos\theta} + x\cos\theta \times \dfrac{(-\sin\theta)}{\cos\theta} = \cos\theta$

$\Rightarrow$ $\dfrac{1}{\cos\theta} - x\sin\theta = \cos\theta$

$\Rightarrow$ $\dfrac{1 - x\sin\theta\cos\theta}{\cos\theta} = \cos\theta$

$\Rightarrow$ $1 - x\sin\theta\cos\theta = \cos^2\theta$

$\Rightarrow$ $1 - \cos^2\theta = x\sin\theta\cos\theta$

$\Rightarrow$ $\sin^2\theta = x\sin\theta\cos\theta$

$\Rightarrow$ $\sin\theta = x\cos\theta$

$\Rightarrow$ $x = \dfrac{\sin\theta}{\cos\theta}$

$= \tan\theta$

Hence $x = \tan\theta$

Q8(ii)

We have $x \cot\left(90^\circ + \theta\right) + \tan\left(90^\circ + \theta\right) \sin\theta + \operatorname{cosec}\left(90^\circ + \theta\right) = 0$

$\Rightarrow \quad x\left(-\tan\theta\right) - \cot\theta \times \sin\theta + \sec\theta = 0$

$\Rightarrow \quad -x\tan\theta - \dfrac{\cos\theta}{\sin\theta} \times \sin\theta + \dfrac{1}{\cos\theta} = 0$

$\Rightarrow \quad -x\dfrac{\sin\theta}{\cos\theta} - \cos\theta + \dfrac{1}{\cos\theta} = 0$

$\Rightarrow \quad \dfrac{-x\sin\theta - \cos^2\theta + 1}{\cos\theta} = 0$

$\Rightarrow \quad -x\sin\theta + 1 - \cos^2\theta = 0$

$\Rightarrow \quad -x\sin\theta + \sin^2\theta = 0$

$\Rightarrow \quad x\sin\theta = \sin^2\theta$

$\Rightarrow \quad x = \dfrac{\sin^2\theta}{\sin\theta}$

$\Rightarrow \quad x = \sin\theta$

Q9(i)

LHS $= \tan 720^\circ - \cos 270^\circ - \sin 150^\circ \cos 120^\circ$

$= \tan 4\pi - \cos\left(\dfrac{3\pi}{2}\right) - \sin\left(\pi - \dfrac{\pi}{6}\right)\cos\left(\dfrac{\pi}{2} + \dfrac{\pi}{6}\right) \left(\because \pi = 180^\circ\right)$

$= 0 - 0 - \sin\dfrac{\pi}{6}\left(-\sin\dfrac{\pi}{6}\right) \qquad \left(\because \tan n\pi = 0 \text{ for all } n \in Z \ \& \ \cos\dfrac{3\pi}{2} = 0\right)$

$= \sin^2\dfrac{\pi}{6}$

$= \left(\dfrac{1}{2}\right)^2$

$= \dfrac{1}{4}$

= RHS

Proved

Q9(ii)

LHS = $\sin 780° \sin 480° + \cos 120° \sin 150°$

$= \sin\left(4\pi + \frac{\pi}{3}\right)\sin\left(3\pi - \frac{\pi}{3}\right) + \cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right)\sin\left(\pi - \frac{\pi}{6}\right)$ $\left(\because \pi = 180°\right)$

$= \sin\frac{\pi}{3} \times \sin\frac{\pi}{3} + \left(-\sin\frac{\pi}{6}\right)\sin\frac{\pi}{6}$ $\left(\begin{array}{l}\because \sin\left(4\pi + \frac{\pi}{3}\right) = \sin\frac{\pi}{3} \\ \ \& \sin\left(3\pi - \frac{\pi}{3}\right) = \sin\frac{\pi}{3}\end{array}\right)$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{4} - \frac{1}{4}$

$= \frac{2}{4}$

$= \frac{1}{2}$

= RHS

Proved

Q9(iii)

LHS = $\sin 780° \sin 120° + \cos 240° \sin 390°$

$= \sin\left(4\pi + \frac{\pi}{3}\right)\sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right) + \cos\left(\pi + \frac{\pi}{6}\right)\sin\left(2\pi + \frac{\pi}{6}\right)$

$= \sin\frac{\pi}{3} \times \cos\frac{\pi}{6} - \cos\frac{\pi}{3} \times \left(+\sin\frac{\pi}{6}\right)$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{4} - \frac{1}{4}$

$= \frac{1}{2}$

$= \frac{1}{2}$

= RHS

Proved

Q9(iv)

$$\text{LHS} = \sin 600^\circ \cos 390^\circ + \cos 480^\circ \sin 150^\circ$$

$$= \sin\left(3\pi + \frac{\pi}{3}\right)\cos\left(2\pi + \frac{\pi}{6}\right) + \cos\left(3\pi - \frac{\pi}{3}\right)\sin\left(\pi - \frac{\pi}{6}\right)$$

$$= -\sin\frac{\pi}{3}\cos\frac{\pi}{6} - \cos\frac{\pi}{3} - \sin\frac{\pi}{6} \qquad \left(\because \sin\left(3\pi + \frac{\pi}{3}\right) = -\sin\frac{\pi}{3} \;\&\; \cos\left(3\pi - \frac{\pi}{3}\right) = -\cos\frac{\pi}{3}\right)$$

$$= \frac{-\sqrt{3}}{2} \times \frac{-\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{-3}{4} - \frac{1}{4}$$

$$= \frac{-4}{4}$$

$$= -1$$

$= \text{RHS}$

Proved

Q9(v)

$$\text{LHS} = \tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ$$

$$= \tan\left(\pi + \frac{\pi}{4}\right)\cot\left(2\pi + \frac{\pi}{4}\right) + \tan\left(4\pi + \frac{\pi}{4}\right)\cot\left(4\pi - \frac{\pi}{4}\right)$$

$$= \tan\frac{\pi}{4}\cot\frac{\pi}{4} + \tan\frac{\pi}{4}\left(-\cot\frac{\pi}{4}\right)$$

$$= 1.1 + 1.(-1)$$

$$= 1 - 1$$

$$= 0$$

$= \text{RHS}$

Proved