Q1(i)

First arrange the given numbers in ascending order

write these numbers in ascending order

3011, 2780, 3020, 2354, 3541, 4150, 5000

we get 2354, 2780, 3011, 3020, 3541, 4150, 5000

Clearly, the middle number is median, 3020

Calculation of Mean Deviations

x_i	$\lvert d_i \rvert = \lvert x_i - 3020 \rvert$
3011	9
2780	240
3020	0
2354	666
3541	521
4150	1130
5000	1980
Total	$d_i = \sum \lvert x_i - 3020 \rvert = 4546$

$$\text{M.D} = \frac{\sum d_i}{n} = \frac{4546}{7} = 649.428$$

Q1(ii)

Clearly, the middle observations are 46 and 48. So, median = 47

34
38
42
44
46
48
54
55
63
70

We have,

$$\Sigma \lvert x_i - 47 \rvert = \Sigma d_i = 86$$

$$\therefore \quad \text{M.D} = \frac{1}{n}\Sigma \lvert d_i \rvert = \frac{1}{10}[86] = 8.6$$

Q1(iii)

30
34 38 40
42
44 50
51
60
66

Q1(iv)

22
24 25 27
28
29 30
31
41
42

Q1(v)

Arranging the observations in ascending order of magnitude, we have

34
38
42
44
47
48
53
55
63
70

Q2(i)

$$\text{Mean} = \frac{1}{n}\Sigma|x_i| = \frac{80}{8} = 10$$

Calculation of Mean Deviation

X-values	Deviation From Mean
4	6
7	3
8	2
9	1
10	0
12	2
13	3
17	7
Total	24

Q2(ii)

$$\text{Mean} = \frac{1}{n}\Sigma |x_i| = \frac{168}{12} = 14$$

Calculation of Mean Deviation

X-values	Deviation From Mean
13	1
17	3
16	2
14	0
11	3
13	1
10	4
16	2
11	3
18	4
12	2
17	3
Total	28

Q2(iii)

$$\text{Mean} = \frac{1}{n}\Sigma|x_i| = \frac{500}{10} = 50$$

Calculation of Mean Deviation

X-values	Deviation From Mean
38	12
70	20
48	2
40	10
42	8
55	5
63	13
46	4
54	4
44	6
Total	84

Q2(iv)

Calculation of Mean Deviation

X-values	Deviation From Mean
38	12
70	20
48	2
40	10
42	8
55	5
63	13
46	4
54	4
44	6
Total	84

Q2(v)

First arrange the given numbers in ascending order
write these numbers in ascending order
57, 64, 43, 67, 49, 59, 44, 47, 61, 59
we get 43, 44, 47, 49, 57, 59, 59, 61, 64, 67
Let X be the mean of given data, we get

$$X = \frac{43+44+47+49+57+59+59+61+64+67}{10} = 55$$

Calculation of Mean Deviations from mean

x_i	$\lvert d_i \rvert = \lvert x_i - 55 \rvert$
43	12
44	11
47	8
49	6
57	2
59	4
59	4
61	6
64	9
67	12
Total	74

$$M.D = \frac{\sum d_i}{n} = \frac{74}{10} = 7.4$$

Q3

Arrange the given data for income group I in assending order, middle observation is 4400.
So, median = 4400.
Mean deviation for group I

x_i	$\lvert d_i \rvert = \lvert x_i - 4400 \rvert$
4000	400
4200	200
4400	0
4600	200
4800	400
Total	$\sum \lvert d_i \rvert = 1000$

$$M.D. = \frac{1}{n}\sum \lvert d_i \rvert = \frac{1000}{5} = 200$$

Q4

First arrange the given numbers in ascending order

write these numbers in ascending order

40.0, 52.3, 55.2, 72.9, 52.8, 79.0, 32.5, 15.2, 27.9, 30.2

we get 15.2, 27.9, 30.2, 32.5, 40.0, 52.3, 52.8, 55.2, 72.9, 79.0

Clearly, Median $= \frac{40.0+52.3}{2} = 46.15$

Let $\overline{X}$ be the mean of given data, we get

$$\overline{X} = \frac{15.2+27.9+30.2+32.5+40.0+52.3+52.8+55.2+72.9+79.0}{10} = 45.8$$

Q5(i)

$$\text{Mean} = \frac{1}{n}\Sigma|x_i| = \frac{455}{10} = 45.5$$

X-values	Deviation From Mean
34	11.5
66	20.5
30	15.5
38	7.5
44	1.5
50	4.5
40	5.5
60	14.5
42	3.5
51	5.5
Total	90

Q5(ii)

$$\text{Mean} = \frac{1}{n}\Sigma|x_i| = \frac{299}{10} = 29.9$$

X-values	Deviation From Mean
22	7.9
24	5.9
30	0.1
27	2.9
29	0.9
31	1.1
25	4.9
28	1.9
41	11.1
42	12.1
Total	**48.8**

Q5(iii)

$$\text{Mean} = \frac{1}{n}\Sigma|x_i| = \frac{494}{10} = 49.4$$

38	11.4
70	20.6
48	1.4
34	15.4
63	13.6
42	7.4
55	5.6
44	5.4
53	3.6
47	2.4
	86.8

Q6

$$\sigma = \sqrt{\frac{1}{n}\Sigma\left(x_i - \bar{x}\right)^2}$$

$$\sigma' = \sqrt{\frac{1}{n}\Sigma x_i^2 - \bar{x}^2}$$

$$\left(x_i - \bar{x}\right)^2 = x_i^2 + \bar{x}^2 - 2x_i\bar{x}$$

$$\sum 2x_i\bar{x} = 2\bar{x}\sum x_i = 2n\bar{x}^2$$

$$\frac{1}{n}\Sigma\left(x_i - \bar{x}\right)^2 = \frac{\sum\left(x_i^2 + \bar{x}^2 - 2x_i\bar{x}\right)}{n}$$

$$= \frac{\sum x_i^2 + \sum \bar{x}^2 - \sum 2x_i\bar{x}}{n}$$

$$= \frac{1}{n}\sum x_i^2 + \frac{n\bar{x}^2 - 2n\bar{x}^2}{n}$$

$$= \frac{1}{n}\sum x_i^2 - \bar{x}^2$$

EX - 32.2

Q1

x_i	f_i	Cum. Freq	$\|d_i\| = \|x_i - 61\|$	$f_i\|d_i\|$
58	15	15	3	45
59	20	35	2	40
60	32	67	1	32
61	35	102	0	0
62	35	137	1	35
63	22	159	2	44
64	20	179	3	60
65	10	189	4	40
66	8	197	5	40
	$N = 197$			$Total = 336$

$N = 197, \frac{N}{2} = 98.5$

Corresponding value for median is 61

Mean Deviation $= \frac{336}{197} = 1.705$

Q2

We have to calculate mean deviation from the median. So, first we calculate the median.

x	f	cf	d = (x-med)	fd
0	14	14	4	56
1	21	35	3	63
2	25	60	2	50
3	43	103	1	43
4	51	154	0	0
5	40	194	1	40
6	39	233	2	78
7	12	245	3	36
	245			366

Q3

x_i	f_i	Cum. fre	$\lvert d_i \rvert = \lvert x_i - 13 \rvert$	$f_i \lvert d_i \rvert$
5	2	2	8	16
7	4	6	6	24
9	6	12	4	24
11	8	20	2	16
13	10	30	0	0
15	12	42	2	24
17	8	50	4	32
	$N = 50$			$Total = 136$

$\frac{N}{2} = 25$

Value corresponding to 25 is Median=13

M.D= $\frac{136}{50} = 2.72$

Q4(i)

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x_i	f_i	$f_i x_i$	$\lvert d_i \rvert = \lvert x_i - 9 \rvert$	$f_i \lvert d_i \rvert$
5	8	40	4	32
7	6	42	2	12
9	2	18	0	0
10	2	20	1	2
12	2	24	3	6
15	6	90	6	36
	26	$Total = 234$		$Total = 88$

$\frac{\sum f_i x_i}{26} = 9$

Mean=9

M.D= $\frac{88}{26} \approx 3.39$

Q4(ii)

x	f	xf	d=(x-mean)	fd
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
	25	350		158

Q4(iii)

x	f	xf	d=(x-mean)	fd
10	4	40	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
	80	4000		1280

Q4(iv)

x_i	f_i	$f_i x_i$	$\lvert d_i \rvert = \lvert x_i - 21.65 \rvert$	$f_i \lvert d_i \rvert$
20	6	120	1.65	9.9
21	4	84	0.65	2.6
22	5	110	0.35	1.75
23	1	23	1.35	1.35
24	4	96	2.35	9.40
	20	*Total* = 433		*Total* = 25

$$\frac{\sum f_i x_i}{20} = 21.65$$

Mean=21.65

$$\text{M.D} = \frac{25}{20} \approx 1.25$$

Q5

x_i	f_i	*Cum Freq*	$\lvert d_i \rvert = \lvert x_i - 30 \rvert$	$f_i \lvert d_i \rvert$
15	3	3	15	45
21	5	8	9	45
27	6	14	3	18
30	7	21	0	0
35	8	29	5	40
	29			*Total* $= 148$

$\frac{N}{2} = 14.5$

Median=30

M.D= $\frac{148}{29} \approx 5.10$

Q1

We have to calculate mean deviation from the median. So, first we calculate the median.

CI	x	f	cf	d = (x-med)	fd
0-10	5	5	5	20	100
10-20	15	10	15	10	100
20-30	25	20	35	0	0
30-40	35	5	91	10	50
40-50	45	10	101	20	200
		50			450

$$\text{M.D} = \frac{1}{n}\Sigma f_i |d_i| = \frac{1}{50}[450] = 9$$

Q2(i)

CI	x	f	xf	d=(x-mean)	fd
0-100	50	4	200	308	1232
100-200	150	8	1200	208	1664
200-300	250	9	2250	108	972
300-400	350	10	3500	8	80
400-500	450	7	3150	92	644
500-600	550	5	2750	192	960
600-700	650	4	2600	292	1168
700-800	750	3	2250	392	1176
		50	17900		7896

$$\text{Mean} = \frac{1}{n}\Sigma f_i x_i = \frac{17900}{50} = 358$$

$$\therefore \quad \text{M.D} = \frac{1}{n}\Sigma f_i |d_i| = \frac{1}{50}[7896] = 157.92$$

Q2(ii)

Classes	f_i	x_i	d_i	$f_i d_i$	$\lvert x_i - \overline{X} \rvert$	$f_i \lvert x_i - \overline{X} \rvert$
95−105	9	100	−3	−27	28.58	257.22
105−115	13	110	−2	−26	18.58	241.54
115−125	16	120	−1	−16	8.58	137.28
125−135	26	130	0	0	1.42	36.92
135−145	30	140	1	30	11.42	342.6
145−155	12	150	2	24	21.42	257.04
	$N = 106$			$Total = -15$		$Total = 1272.60$

$N = 106$

$a = 130$

$h = 10$

$$\overline{X} = a + h\left(\frac{\sum f_i d_i}{N}\right) = 128.58$$

$$M.D = \frac{\sum f_i \lvert x_i - \overline{X} \rvert}{N} = \frac{1272.60}{106} = 12.005$$

Q2(iii)

CI	x	f	xf	d=(x-mean)	fd
0-10	5	6	30	22	132
10-20	15	8	120	12	96
20-30	25	14	350	2	28
30-40	35	16	560	8	128
40-50	45	4	180	18	72
50-60	55	2	110	28	56
		50	1350		512
	Mean		27		
	Mean Deviation		10.24		

$$\text{Mean} = \frac{1}{n}\Sigma f_i x_i = \frac{1350}{50} = 27$$

$$\therefore \quad \text{M.D} = \frac{1}{n}\Sigma f_i |d_i| = \frac{1}{50}[512] = 10.24$$

Q3

Find the mean deviation from the mean for the data:

Classes	0-10	10-20	20-30	30-40	40-50	50-60
Frequencies	6	8	14	16	4	2

$$\text{Mean} = \frac{1}{n}\Sigma f_i x_i = \frac{5390}{110} = 49$$

$$\therefore \quad \text{M.D} = \frac{1}{n}\Sigma f_i |d_i| = \frac{1}{110}[1644] = 14.95$$

Q4

We have to calculate mean deviation from the median. So, first we calculate the median.

CI	x	f	cf	d = (x-med)	fd
17-19.5	18.25	5	5	20	100
20-25.5	22.75	16	21	15.5	248
26-35.5	30.75	12	33	7.5	90
36-40.5	38.25	26	59	0	0
41-50.5	45.75	14	73	7.5	105
51-55.5	53.25	12	85	15	180
56-60.5	58.25	6	91	20	120
61-70.5	65.75	5	96	27.5	137.5
		96			980.5

We have $N = 96 \Rightarrow N/2 = 48$

The cumulative frequency just greater than N/2 is 59 and the corresponding value of x is 38.25.

Hence, median = 38.25

$$\therefore \quad \text{M.D} = \frac{1}{n}\Sigma f_i |d_i| = \frac{1}{96}[980.5] = 10.21$$

Q5

M.D from Median

Marks	Students	x_i	Cum.Freq	$\lvert d_i\rvert = \left\lvert x_i - \frac{70}{3}\right\rvert$	$f_i d_i$
$0-10$	5	5	5	$\frac{55}{3}$	$\frac{275}{3}$
$10-20$	8	15	13	$\frac{25}{3}$	$\frac{200}{3}$
$20-30$	15	25	28	$\frac{5}{3}$	$\frac{75}{3}$
$30-40$	16	35	44	$\frac{35}{3}$	$\frac{560}{3}$
$40-50$	6	45	50	$\frac{65}{3}$	$\frac{390}{3}$
	$N = 50$				$Total = 500$

$$Median = l + \frac{\frac{N}{2} - F}{f} \times h = 20 + \frac{30-25}{15} \times 10 = 20 + \frac{10}{3} = \frac{70}{3}$$

Q6

Converting the given data into continuous frequency distribution by subtracing 0.5 from the lower limit and adding 0.5 to the upper limit of each class interval.

Age	x_i	f_i	Cumulativefrequency	$\lvert d_i \rvert = \lvert x_i - 38 \rvert$	$f_i \lvert d_i \rvert$
15.5 – 20.5	18	5	5	20	100
20.5 – 25.5	23	6	11	15	90
25.5 – 30.5	28	12	23	10	120
30.5 – 35.5	33	14	37	5	70
35.5 – 40.5	38	26	63	0	0
40.5 – 45.5	43	12	75	5	60
45.5 – 50.5	48	16	91	10	160
50.5 – 55.5	53	9	100	15	135
		$N = \sum f_i = 100$			$\sum f_i \lvert d_i \rvert = 735$

Clearly, $N = 100 \Rightarrow \frac{N}{2} = 50.$

Cumulative frequency is just greater than $\frac{N}{2}$ is 63 and the corresponding class is 35.5 - 40.5.

$l = 35.5,\ f = 26,\ h = 5,\ F = 37$

Therefore, median $= l + \dfrac{\frac{N}{2} - F}{f} \times h = 35.5 + \dfrac{50 - 37}{26} \times 5 = 38$

$$\text{M.D.} = \frac{1}{N}\sum f_i \lvert d_i \rvert = \frac{735}{100} = 7.35$$

Q7

Classes	f_i	x_i	$f_i x_i$	$\lvert x_i - 9.2 \rvert$	$f_i \lvert x_i - 9.2 \rvert$
0 – 4	4	2	8	7.2	28.8
4 – 8	6	6	36	3.2	19.2
8 – 12	8	10	80	0.8	6.4
12 – 16	5	14	70	4.8	24.0
16 – 20	2	18	36	8.8	17.6
	$N = 25$		$Total = 230$		$Total = 96.0$

$$Mean = \frac{230}{25} = 9.2$$

$$M.D = \frac{96}{25} = 3.84$$

Q8

$Classes$	f_i	x_i	$f_i x_i$	$\lvert x_i - 14.1\rvert$	$f_i \lvert x_i - 14.1\rvert$
$0-6$	4	3	12	11.1	44.4
$6-12$	5	9	45	5.1	25.5
$12-18$	3	15	45	0.9	2.7
$18-24$	6	21	126	6.9	41.4
$24-30$	2	27	54	12.9	25.8
	$N = 20$		$Total = 282$		$Total = 139.8$

$$Mean = \frac{282}{20} = 14.1$$

$$M.D = \frac{139.8}{20} = 6.99$$

Q1(i)

x	d=(x- Mean)	d^2
2	-5	25
4	-3	9
5	-2	4
6	-1	1
8	1	1
17	10	100
42		**140**

$$\overline{x} = \frac{1}{n}\Sigma x_i = \frac{1}{6}[42] = 7$$

$$\text{var}(x) = \frac{1}{n}\left\{\Sigma\left(x_i - \overline{x}\right)^2\right\} = \frac{1}{6}\{140\} = 23.33$$

$$S.D(x) = \sqrt{\text{var}(x)} = \sqrt{23.33} = 4.8$$

Q1(ii)

x	d=(x- Mean)	d^2
6	-3	9
7	-2	4
10	1	1
12	3	9
13	4	16
4	-5	25
8	-1	1
12	3	9
72		**74**

$$\overline{x} = \frac{1}{n}\Sigma x_i = \frac{1}{8}[72] = 9$$

$$\text{var}(x) = \frac{1}{n}\left\{\Sigma\left(x_i - \overline{x}\right)^2\right\} = \frac{1}{8}\{74\} = 9.25$$

$$S.D(x) = \sqrt{\text{var}(x)} = \sqrt{9.25} = 3.04$$

Q1(iii)

x_i	$d_i = x_i - 299$	d_i^2
227	-72	5184
235	-64	4096
255	-44	1936
269	-30	900
292	-7	49
299	0	0
312	13	169
321	22	484
333	34	1156
348	49	2401
	$Total = -99$	$Total = 16375$

$$\overline{X} = 299 + \frac{-99}{10} = 289.1$$

$$Var = \frac{16375}{10} - \left(\frac{-99}{10}\right)^2 = 1637.5 - 98.01 = 1539.49$$

$$S.D = \sqrt{1539.49} = 39.24$$

Q1(iv)

x_i	$d_i = x_i - 15$	d_i^2
15	0	0
22	7	49
27	12	144
11	-4	16
9	-6	36
21	6	36
14	-1	1
9	-6	36
	$Total = 8$	$Total = 318$

$$Mean = 15 + \frac{8}{8} = 16$$

$$Var = \frac{318}{8} - 1 = 38.75$$

$$SD = \sqrt{38.75} = 6.22$$

Q2

We have, $n = 20$, and $\sigma^2 = 5$

Now each observation is multiplied by 2.
Suppose $X = 2x$ be the new data.

$$\therefore \quad \overline{X} = \frac{1}{20}\Sigma\, 2x_i = \frac{1}{20} \times 2\Sigma x_i = 2\overline{x}$$

$$\Rightarrow \quad \Sigma X_i^{\,2} = 4\Sigma x_i^{\,2}$$

Since, $\sigma^2 = 5$

$$\Rightarrow \quad \frac{1}{n}\Sigma x_i^2 - \left(\overline{x}\right)^2 = 5$$

Now, for the new data:

$$\sigma^2 = \frac{1}{n}\Sigma X_i^2 - \left(\overline{X}\right)^2 = 4\Sigma x_i^{\,2} - \left(2\overline{x}\right)^2 = 4\left(\Sigma x_i^{\,2} - \left(\overline{x}\right)^2\right) = 4 \times 5 = 20$$

Q3

We have, $n = 15$, and $\sigma^2 = 4$

Now each observation is increased by 9.
Suppose $X = x + 9$ be the new data.

$$\therefore \quad \overline{X} = \frac{1}{15}\Sigma(x_i + 9) = \left(\frac{1}{15} \times \Sigma x_i\right) + 9 = \overline{x} + 9$$

$$\Rightarrow \quad \Sigma X_i^{\,2} = \Sigma(x_i + 9)^2 = \Sigma x_i^{\,2} + \Sigma 18x_i + \Sigma 9^2$$

Since, $\sigma^2 = 5$

$$\Rightarrow \quad \frac{1}{n}\Sigma x_i^{\,2} - \left(\overline{x}\right)^2 = 4$$

Now, for the new data:

$$\sigma^2 = \frac{1}{n}\Sigma X_i^{\,2} - \left(\overline{X}\right)^2 = \frac{1}{15}\left(\Sigma x_i^{\,2} + \Sigma 18x_i + \Sigma 9^2\right) - \left(\overline{x} + 9\right)^2 =$$

$$= \frac{1}{15}\Sigma x_i^{\,2} + \frac{1}{15}\Sigma 18x_i + \frac{1}{15}\Sigma 9^2 - (9)^2 - \left(18\overline{x}\right) - \left(\overline{x}\right)^2$$

$$= \left[\frac{1}{15}\Sigma x_i^{\,2} - \left(\overline{x}\right)^2\right] + \left[\frac{1}{15}\Sigma 18x_i - \left(18\overline{x}\right)\right] + \left[\frac{1}{15}\Sigma 9^2 - (9)^2\right]$$

$$= \left[\frac{1}{15}\Sigma x_i^{\,2} - \left(\overline{x}\right)^2\right] + \left[18 \times \frac{1}{15}\Sigma x_i - \left(18\overline{x}\right)\right] + \left[\frac{1}{15} \times 15 \times (9)^2 - (9)^2\right]$$

$$= \frac{1}{15}\Sigma x_i^{\,2} - \left(\overline{x}\right)^2$$

$$= 4$$

Q4

Let the other two be x and y

1 + 2 + 6 + x + y = 5 * 4.4 because of the mean
x + y = 13

Variance = [(1 – 4.4)^2 + (2 – 4.4)^2 +(6 – 4.4)^2 + (x – 4.4)^2 + (y – 4.4)^2]/5

Hence
11.56 + 5.76 + 2.56 + (x – 4.4)^2 +(y – 4.4)^2 = 41.2
(x – 4.4)^2 + (y – 4.4)^2 = 21.32

Solve simultaneously
(x – 4.4)^2 + (13 - x - 4.4)^2 = 21.32
(x – 4.4)^2 + (8.6 - x)^2 = 21.32
x^2 – 8.8x + 19.36 + 73.96 – 17.2x + x^2 = 21.32
2x^2 - 26x + 72 = 0
x^2 - 13x + 36 = 0
(x – 4)(x – 9) = 0
x = 4 or x = 9
If x = 4, y = 9 and

The other two observations are 4 and 9.

Q5

If mean and SD of observations are $\overline{X}$ and σ respectively,
then mean and SD of observations multiplied by a constant 'k' are
Mean=$k\overline{X}$
SD=$|k|\sigma$

In this question, it is given that k=3
So New mean = $8\times3=24$
New SD= $4\times3=12$

Q6

Let x and y be the remaining two observations. Then,

$$\text{Mean} = 9$$

$$\Rightarrow \quad \frac{6+7+10+12+12+13+x+y}{8} = 9$$

$$\Rightarrow \quad 60 + x + y = 72$$

$$\Rightarrow \quad x + y = 12 \quad \text{-------(i)}$$

$$\text{Variance} = 9.25$$

$$\Rightarrow \quad \frac{1}{8}\left(6^2+7^2+10^2+12^2+12^2+13^2+x^2+y^2\right) - (\text{Mean})^2 = 9.25$$

$$\Rightarrow \quad \frac{1}{8}\left(36+49+100+144+144+169+x^2+y^2\right) - 81 = 9.25$$

$$\Rightarrow \quad 642 + x^2 + y^2 = 722$$

$$\Rightarrow \quad x^2 + y^2 = 80 \quad \text{------(ii)}$$

Now, $(x+y)^2 + (x-y)^2 = 2\left(x^2+y^2\right)$

$$\Rightarrow \quad 144 + (x-y)^2 = 2 \times 80$$

$$\Rightarrow \quad (x-y)^2 = 16$$

$$\Rightarrow \quad x - y = \pm 4$$

if $x - y = 4$, then $x + y = 12$ and $x - y = 4 \Rightarrow x = 8,\ y = 4$

if $x - y = -4$, then $x + y = 12$ and $x - y = -4 \Rightarrow \quad x = 4,\ y = 8$

Hence, the remaining two observations are 4 and 8.

Q7

We have,

$n = 200, \overline{X} = 40, \sigma = 15.$

$\therefore \quad \overline{X} = \frac{1}{n}\Sigma x_i = \overline{X} = 200 \times 40 = 8000.$

Corrected Σx_i = Incorrect Σx_i – (sum of incorrect values) + (sum of correct values)

$= 8000 - 34 - 53 + 43 + 35 = 7991$

$\therefore \quad$ Corrected mean$= \frac{\text{corrected } \Sigma x_i}{n} = \frac{7991}{200} = 39.955$

Now, $\quad \sigma = 15$

$\Rightarrow \quad 15^2 = \frac{1}{200}\left(\Sigma x_i^2\right) - \left(\frac{1}{200}\Sigma x_i\right)^2$

$\Rightarrow \quad 255 = \frac{1}{200}\left(\Sigma x_i^2\right) - \left(\frac{8000}{200}\right)^2$

$\Rightarrow \quad 255 = \frac{1}{200}\left(\Sigma x_i^2\right) - 1600$

$\Rightarrow \quad \Sigma x_i^2 = 200 \times 1825 = 365000$

$\Rightarrow \quad$ Incorrect $\Sigma x_i^2 = 365000.$

corrected$\Sigma x_i^2 = \left(\text{incorrect}\Sigma x_i^2\right)$ – (sum of squares of incorrect values)

+ (sum of squares of correct values)

$= 365000 - (34)^2 - 53^2 + (43)^2 + 35^2 = 364109$

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so, $\quad$ Corrected $\sigma = \sqrt{\frac{1}{n}\Sigma x_i^2 - \left(\frac{1}{n}\Sigma x_i\right)^2} = \sqrt{\frac{364109}{200} - \left(\frac{7991}{200}\right)^2}$

$= \sqrt{1820.545 - 1596.402} = 14.97$

Q8

We have,

$n = 100, \overline{X} = 40, \sigma = 5.1$

$\therefore \quad \overline{X} = \frac{1}{n}\Sigma x_i = \overline{X} = 100 \times 40 = 4000.$

Corrected Σx_i = Incorrect Σx_i – (sum of incorrect values) + (sum of correct values)

$= 4000 - 50 + 40 = 3990$

$\therefore \quad$ Corrected mean $= \frac{\text{corrected } \Sigma x_i}{n} = \frac{3990}{100} = 39.9$

Now, $\sigma = 5.1$

$\Rightarrow \quad 5.1^2 = \frac{1}{100}\left(\Sigma x_i^2\right) - \left(\frac{1}{100}\Sigma x_i\right)^2$

$\Rightarrow \quad 26.01 = \frac{1}{100}\left(\Sigma x_i^2\right) - \left(\frac{4000}{100}\right)^2$

$\Rightarrow \quad 26.01 = \frac{1}{100}\left(\Sigma x_i^2\right) - 1600$

$\Rightarrow \quad \Sigma x_i^2 = 100 \times 1626.01 = 162601$

$\Rightarrow \quad$ Incorrect $\Sigma x_i^2 = 162601.$

corrected Σx_i^2 = (incorrect Σx_i^2) – (sum of squares of incorrect values)

+ (sum of squares of correct values)

$= 162601 - (50)^2 + (40)^2 = 161701$

so, Corrected $\sigma = \sqrt{\frac{1}{n}\Sigma x_i^2 - \left(\frac{1}{n}\Sigma x_i\right)^2} = \sqrt{\frac{161701}{100} - \left(\frac{3990}{100}\right)^2}$

$= \sqrt{1617.01 - 1592.01} = 5$

Q9

We have, $n = 20$, $\overline{x} = 10$ and $\sigma = 2$

$\therefore \quad \overline{x} = \frac{1}{n}\Sigma x_i$

$\Rightarrow \quad \Sigma x_i = n\overline{x} = 20 \times 10 = 200$

$\Rightarrow \quad$ Incorrected $\Sigma x_i = 200$

and,

$\sigma = 2$

$\Rightarrow \quad \sigma^2 = 4$

$\Rightarrow \quad \frac{1}{n}\Sigma x_i^2 - (\text{Mean})^2 = 4$

$\Rightarrow \quad \frac{1}{20}\Sigma x_i^2 - 100 = 4$

$\Rightarrow \quad \Sigma x_i^2 = 104 \times 20$

$\Rightarrow \quad$ Incorrected $\Sigma x_i^2 = 2080.$

Q10

We have, $n = 100$, $\overline{x} = 20$ and $\sigma = 3$

Since $\overline{x} = \frac{1}{n}\Sigma x_i$

$\Rightarrow \quad \Sigma x_i = n\overline{x} = 20 \times 100 = 2000$

$\Rightarrow \quad$ Incorrect $\Sigma x_i = 2000$

and,

$\sigma = 3$

$\Rightarrow \quad \sigma^2 = 9$

$\Rightarrow \quad \frac{1}{n}\Sigma x_i^2 - (\text{Mean})^2 = 9$

$\Rightarrow \quad \frac{1}{100}\Sigma x_i^2 - 400 = 9$

$\Rightarrow \quad \Sigma x_i^2 = 409 \times 100$

$\Rightarrow \quad$ Incorrect $\Sigma x_i^2 = 40900.$

Q11

We have $\sum(x_i - \overline{X})^2 = \sum\left(x_i^2 - 2x_i\overline{X} + \overline{X}^2\right)$

$= \sum\left(x_i^2\right) + \sum\left(-2x_i\overline{X}\right) + \sum\left(\overline{X}\right)^2$

$= \sum\left(x_i^2\right) - 2\overline{X}\sum\left(x_i\right) + \left(\overline{X}\right)^2\sum 1$

$= \sum\left(x_i^2\right) - 2\overline{X}\left(n\overline{X}\right) + n\left(\overline{X}\right)^2$

$= \sum\left(x_i^2\right) - n\left(\overline{X}\right)^2$

Dividing both the sides by n we get,

$\frac{1}{n}\sum(x_i - \overline{X})^2 = \frac{1}{n}\sum\left(x_i^2\right) - n\left(\overline{X}\right)^2$

Taking square root on both the sides

$\sqrt{\frac{1}{n}\sum(x_i - \overline{X})^2} = \sqrt{\frac{1}{n}\sum\left(x_i^2\right) - n\left(\overline{X}\right)^2}$

Q1

x	f	fx	x-mean	(x-mean)²	f(x-mean)²
4.5	1	4.5	-33.14	1098.45	1098.45
14.5	5	72.5	-23.14	535.59	2677.96
24.5	12	294	-13.14	172.73	2072.82
34.5	22	759	-3.14	9.88	217.31
44.5	17	756.5	6.86	47.02	799.35
54.5	9	490.5	16.86	284.16	2557.47
64.5	4	258	26.86	721.31	2885.22
	N=70	**2635**			**12308.57**

Here, $N = 70$, $\Sigma f_i x_i = 2635$

$\therefore$ $\bar{x} = \frac{1}{N}(\Sigma f_i x_i) = \frac{2635}{70} = 37.64$

We have, $\Sigma f_i (x_i - \bar{x})^2 = 12308.57$

$\therefore$ $\text{var}(x) = \frac{1}{N}\left[\Sigma f_i (x_i - \bar{x})^2\right] = \frac{12308.57}{70} = 175.84$

$S.D. = \sqrt{\text{var}(x)} = \sqrt{175.84} = 13.26$

Q2

X	F	Fx	x-mean	F(x-mean)	$(x-mean)^2$	$F(x-mean)^2$
0	51	0	-6	-306	36	1836
1	203	203	-5	-1015	25	5075
2	383	766	-4	-1532	16	6128
3	525	1575	-3	-1575	9	4725
4	532	2128	-2	-1064	4	2128
5	408	2040	-1	-408	1	408
6	273	1638	0	0	0	0
7	139	973	1	139	1	139
8	43	344	2	86	4	172
9	27	243	3	81	9	243
10	10	100	4	40	16	160
11	4	44	5	20	25	100
12	2	24	6	12	36	72
	2600	10078		-5522		21186

Here, $N = 2600$, $\Sigma f_i x_i = 10078$

$$\therefore \quad \bar{x} = \frac{1}{N}(\Sigma f_i x_i) = \frac{10078}{2600} = 3.88$$

Since ,

$$\text{var}(x) = h^2\left(\frac{1}{N}\sum f_i(x-mean)_i^2 - \left\{\frac{1}{N}\sum f_i(x-mean)_i\right\}^2\right)$$

$$\sigma^2 = 1\left(\frac{21186}{2600} - \left(\frac{-5522}{2600}\right)^2\right)$$

$$\sigma^2 = 8.14846 - 4.51072$$

$$\sigma^2 = \boxed{3.64}$$

Q3

i)

x_i	Cum Freq	f_i	$f_i x_i$	$f_i x_i^2$
10	15	15	150	1500
20	32	17	340	6800
30	51	19	570	17100
40	78	27	1080	43200
50	97	19	950	47500
60	109	12	720	43200
		$N = 109$	Total = 3810	Total = 159300

$$Mean = \frac{3810}{109} = 34.95$$

$$Var = \frac{159300}{109} - (34.95)^2 = 239.96$$

$$SD = \sqrt{239.96} = 15.49$$

Q4

(i)

x	f	fx	x-mean	(x-mean)²	f(x-mean)²
3	7	21	-9.79	95.88	671.13
8	10	80	-4.79	22.96	229.60
13	15	195	0.21	0.04	0.65
18	10	180	5.21	27.13	271.26
23	6	138	10.21	104.21	625.26
	48	614			1797.92

Here, $N = 48, and\ \Sigma f_i x_i = 614$

$$\therefore \quad \bar{x} = \frac{1}{N}(\Sigma f_i x_i) = \frac{614}{48} = 12.79$$

$$\Sigma f_i (x_i - \bar{x})^2 = 1797.92$$

$$\therefore \quad var(x) = \frac{1}{N}\left[\Sigma f_i (x_i - \bar{x})^2\right] = \frac{1797.92}{48} = 37.46$$

$$S.D. = \sqrt{var(x)} = \sqrt{37.46} = 6.12$$

Q1

CI	f	x	u=(x-A)/h	fu	u²	fu²
0-10	14	5	-2	-28	4	56
10-20	13	15	-1	-13	1	13
20-30	27	25	0	0	0	0
30-40	21	35	1	21	1	21
40-50	15	45	2	30	4	60
	90			**10**		**150**

Here, $N = 90$, $A = 25$, $\Sigma f_i u_i = 10$, $\Sigma f_i u_i^2 = 150$ and $h = 10$

$\therefore$ Mean $= \bar{x} = A + h\left(\frac{1}{N}\Sigma f_i u_i\right)$

$\Rightarrow$ $\bar{x} = 25 + 10\left(\frac{10}{90}\right) = 26.11$

$$\text{var}(x) = h^2\left[\frac{1}{N}\Sigma f_i u_i^2 - \left(\frac{1}{N}\Sigma f_i u_i\right)^2\right] = 100\left[\frac{150}{90} - \left(\frac{10}{90}\right)^2\right] = 165.4$$

$\therefore$ $S.D. = \sqrt{\text{var}(x)} = \sqrt{165.4} = 12.86$

Q2

CI	f	x	u=(x-A)/h	f*u	u^2	fu^2
0-30	9	15	-3	-27	9	81
30-60	17	45	-2	-34	4	68
60-90	43	75	-1	-43	1	43
90-120	82	105	0	0	0	0
120-150	81	135	1	81	1	81
150-180	44	165	2	88	4	176
180-210	24	195	3	72	9	216
	300			137		665

Here, $N = 300$, $A = 105$, $\Sigma f_i u_i = 137$, $\Sigma f_i u_i^2 = 665$ and $h = 30$

$\therefore$ Mean $= \bar{x} = A + h\left(\frac{1}{N}\Sigma f_i u_i\right)$

$\Rightarrow$ $\bar{x} = 105 + 30\left(\frac{137}{300}\right) = 118.7$

$$\text{var}(x) = h^2\left[\frac{1}{N}\Sigma f_i u_i^2 - \left(\frac{1}{N}\Sigma f_i u_i\right)^2\right] = 900\left[\frac{665}{300} - \left(\frac{137}{300}\right)^2\right] = 1807.31$$

$\therefore$ $S.D. = \sqrt{\text{var}(x)} = \sqrt{1807.31} = 42.51$

Q3

CI	f	x	u=(x-A)/h	f*u	u²	fu²
0-10	18	5	-3	-54	9	162
10-20	16	15	-2	-32	4	64
20-30	15	25	-1	-15	1	15
30-40	12	35	0	0	0	0
40-50	10	45	1	10	1	10
50-60	5	55	2	10	4	20
60-70	2	65	3	6	9	18
70-80	1	75	4	4	16	16
	79			**-71**		**305**

Here, $N = 79, A = 35, \Sigma f_i u_i = -71, \Sigma f_i u_i^2 = 305$ and $h = 10$

$\therefore$ Mean $= \bar{x} = A + h\left(\frac{1}{N}\Sigma f_i u_i\right)$

$\Rightarrow$ $\bar{x} = 35 + 10\left(\frac{-71}{79}\right) = 26.01$

$$\text{var}(x) = h^2\left[\frac{1}{N}\Sigma f_i u_i^2 - \left(\frac{1}{N}\Sigma f_i u_i\right)^2\right] = 100\left[\frac{305}{79} - \left(\frac{-71}{79}\right)^2\right] = 305.30$$

$\therefore$ $S.D. = \sqrt{\text{var}(x)} = \sqrt{305.30} = 17.47$

Q4

We have, $n = 100$, $\bar{x} = 40$ and $\sigma = 5.1$

$\therefore$ $\bar{x} = \frac{1}{n}\Sigma x_i$

$\Rightarrow$ $\Sigma x_i = n\bar{x} = 100 \times 40 = 4000$

$\therefore$ Incorrect $\Sigma x_i = 4000$

and,

$\sigma = 5.1$

$\Rightarrow$ $\sigma^2 = 26.01$

$\Rightarrow$ $\frac{1}{n}\Sigma x_i^2 - (\text{Mean})^2 = 26.01$

$\Rightarrow$ $\frac{1}{100}\Sigma x_i^2 - 1600 = 26.01$

$\Rightarrow$ $\Sigma x_i^2 = 1626.01 \times 100$

$\therefore$ Incorrect $\Sigma x_i^2 = 162601$

Q5

CI	Freq	MidValue	u_i	f_iu_i	$f_iu_i^2$
31 − 35	2	33	−4	−8	32
36 − 40	3	38	−3	−9	27
41 − 45	8	43	−2	−16	32
46 − 50	12	48	−1	−12	12
51 − 55	16	53	0	0	0
56 − 60	5	58	1	5	5
61 − 65	2	63	2	4	8
66 − 70	2	68	3	6	18
	$N = 50$			$Total = -30$	$Total = 134$

$$Mean = 53 + 5 \times \frac{-30}{50} = 50$$

$$Var = 25 \times \left(\frac{134}{50} - \frac{9}{25}\right) = 58$$

$$SD = \sqrt{58} = 7.62$$

Q6

Converting the given data into continuous frequency distribution by subtracing 0.5 from the lower limit and adding 0.5 to the upper limit of each class interval.

Class interval	f_i	Mid-value x_i	$u_i = \frac{x_i - 5.5}{1}$	f_iu_i	u_i^2	$f_iu_i^2$
1 − 2	6	1.5	−4	−24	16	96
3 − 4	4	3.5	−2	−8	4	16
5 − 6	5	5.5	0	0	0	0
7 − 8	1	7.5	2	2	4	4
	$N = \sum f_i = 16$			$\sum f_iu_i = -30$		$\sum f_iu_i^2 = 116$

$N = 16$, $\sum f_iu_i = -30$, $\sum f_iu_i^2 = 116$, $A = 5.5$ and $h = 1$

$$\text{Mean} = A + h\left(\frac{1}{N}\sum f_iu_i\right) = 5.5 + 1\left(\frac{1}{16} \times (-30)\right) = 3.625$$

$$Var(X) = h^2\left\{\left(\frac{1}{N}\sum f_iu_i^2\right) - \left(\frac{1}{N}\sum f_iu_i\right)^2\right\} = 1\left\{\left(\frac{1}{16} \times 116\right) - \left(\frac{1}{16} \times (-30)\right)^2\right\} = \{7.25 - 3.51\} = 3.74$$

Note: Answer given in the book is incorrect.

Q7

CI	x_i	f_i	u_i	f_iu_i	$f_iu_i^2$
$200-201$	200.5	13	-1.5	-19.5	29.25
$201-202$	201.5	27	-1	-27	27
$202-203$	202.5	18	-0.5	-9	4.5
$203-204$	203.5	10	0	0	0
$204-205$	204.5	1	0.5	0.5	0.25
$205-206$	205.5	1	1	1	1
		$N=70$		$Total=-54$	$Total=62$

$$Mean = 203.5 + 2\left(\frac{-54}{70}\right) = 201.9$$

$$Var = 4\left(\frac{62}{70} - \left(\frac{-54}{70}\right)^2\right) = 0.98$$

$$SD = \sqrt{0.98} = 0.99$$

Q8

$$Mean = 40$$
$$SD = 10$$
$$n = 100$$
$$\sum x_i = 40 \times 100 = 4000$$
$$Corrected\ Sum = 4000 - 30 - 70 + 3 + 27 = 3930$$
$$Corrected\ \ Mean = \frac{3930}{100} = 39.3$$
$$Variance = 100$$
$$100 = \frac{\sum x_i^2}{100} - (40)^2$$
$$Incorrect \sum x_i^2 = 170000$$
$$Corrected \sum x_i^2 =$$
$$Incorrect \sum x_i^2 - (\text{Sum of squares of incorrect values}) +$$
$$(\text{Sum of squares of corrected values})$$
$$Corrected \sum x_i^2 = 170000 - (900 + 4900) + (9 + 729)$$
$$Corrected \sum x_i^2 = 164938$$
$$Corrected\ \sigma = \sqrt{\frac{Corrected \sum x_i^2}{n} - (Corrected\ \ Mean)^2}$$
$$Corrected\ \sigma = \sqrt{\frac{164938}{100} - (39.3)^2} = 10.24$$

Q9

$Mean = 45$

$Variance = 16$

$n = 10$

$\sum x_i = 450$

$Corrected\ Sum = 450 - 52 + 25 = 423$

$Corrected\ Mean = 42.3$

$Variance = 16$

$16 = \frac{\sum x_i^2}{10} - (45)^2$

$Incorrect \sum x_i^2 = 20410$

$Corrected \sum x_i^2 =$

$Incorrect \sum x_i^2 - (\text{Sum of squares of incorrect values}) +$

$(\text{Sum of squares of corrected values})$

$Corrected \sum x_i^2 = 20410 - 2704 + 625 = 18331$

$Corrected\ \sigma = \sqrt{\frac{Corrected \sum x_i^2}{n} - (Corrected\ \ Mean)^2}$

$Corrected\ \sigma = \sqrt{\frac{18331}{10} - (42.3)^2} = 6.62$

$Corrected\ Variance = 6.62 * 6.62 = 43.82$

Q10

Class interval	f_i	Mid-value x_i	$u_i = \frac{x_i - 35}{10}$	$f_i u_i$	u_i^2	$f_i u_i^2$
0 – 10	11	5	−3	−33	9	99
10 – 20	29	15	−2	−58	4	116
20 – 30	18	25	−1	−18	1	18
30 – 40	4	35	0	0	0	0
40 – 50	5	45	1	5	1	5
50 – 60	3	55	2	6	4	12
	$N = \sum f_i = 70$			$\sum f_i u_i = -98$		$\sum f_i u_i^2 = 250$

$N = 70, \sum f_i u_i = -98, \sum f_i u_i^2 = 250, A = 35$ and $h = 10$

$$\text{Mean} = A + h\left(\frac{1}{N}\sum f_i u_i\right) = 35 + 10\left(\frac{-98}{70}\right) = -21$$

$$Var(X) = h^2\left\{\left(\frac{1}{N}\sum f_i u_i^2\right) - \left(\frac{1}{N}\sum f_i u_i\right)^2\right\} = 100\left\{\left(\frac{1}{70}\times 250\right) - \left(\frac{1}{70}\times(-98)\right)^2\right\} = 100\left(3.57 - 1.96\right) = 161$$

$$SD = \sqrt{Var(X)} = \sqrt{161} = 12.7$$

EX 32.7

Q1

We observe that the average monthly wages in both firms is same i.e. Rs. 2500. Therefore the plant with greater variance will have greater variability. Thus plant B has greater variability in individual wages.

Q2

We observe that the average weights and heights for the 50 students is same i.e. 63.2.
Therefore, the parameter with greater variance will have more variability.
Thus, *height* has greater variability than weights.

Q3

Coefficient of variation = $\frac{\sigma}{\bar{x}} \times 100$

So, we have:

$$60\% = \frac{21}{\bar{x}} \times 100 \Rightarrow \bar{x} = \frac{21}{.60} \times 100 = 35$$

$$70\% = \frac{16}{\bar{x}} \times 100 \Rightarrow \bar{x} = \frac{16}{.70} \times 100 = 22.85$$

Q4

CI	f	x	u=(x-A)/h	fu	u^2	fu^2
1000-1700	12	1350	-2	-24	4	48
1700-2400	18	2050	-1	-18	1	18
2400-3100	20	2750	0	0	0	0
3100-3800	25	3450	1	25	1	25
3800-4500	35	4150	2	70	4	140
4500-5200	10	4850	3	30	9	90
	120			**83**		**321**

Here, $N = 120, A = 2750, \Sigma f_i u_i = 83, \Sigma f_i u_i^2 = 321$ and $h = 700$

$\therefore$ $$\text{Mean} = \bar{x} = A + h\left(\frac{1}{N}\Sigma f_i u_i\right)$$

$\Rightarrow$ $$\bar{x} = 2750 + 700\left(\frac{83}{120}\right) = 3234.17$$

$$\text{var}(x) = h^2\left[\frac{1}{N}\Sigma f_i u_i^2 - \left(\frac{1}{N}\Sigma f_i u_i\right)^2\right] = 490000\left[\frac{321}{120} - \left(\frac{83}{120}\right)^2\right] = 1076332.64$$

$\therefore$ $$S.D. = \sqrt{\text{var}(x)} = \sqrt{1076332.64} = 1037.46$$

Coefficient of variation $= \frac{S.D}{\bar{X}_1} \times 100 = \frac{1037.46}{3234.17} \times 100 = 32.08$

Q5

(*i*)

Total wages paid by firm A = (Average wages) $\times$(Number of employees)

$= 52.5 \times 587 =$ Rs 30817.50

Total wages paid by firm B = (Average wages) $\times$(Number of employees)

$= 47.5 \times 648 =$ Rs 30780

So, firm A pays higher total wages.

(ii)

In order to compare the variability of wages among the two firms, we have to calculate their coefficients of variation.

Let σ_1 and σ_2 denote the standard deviations of Firm A and Firm B respectively. Further, let $\overline{X_1}$ and $\overline{X_2}$ be the mean wages in firms A and B respectively.

We have,

$$\overline{X_1} = 52.5, \qquad \overline{X_2} = 47.5$$

$$\sigma_1^2 = 100 \qquad \text{and} \qquad \sigma_2^2 = 121$$

$$\Rightarrow \qquad \sigma_1 = \sqrt{100} = 10 \qquad \text{and} \qquad \sigma_2 = \sqrt{121} = 11$$

Now,

$$\text{Coefficient of variation in wages in firm A} = \frac{\sigma_1}{\overline{X_1}} \times 100$$

$$= \frac{10}{52.5} \times 100 = 19.05$$

and,

$$\text{Coefficient of variation in wages in firm B} = \frac{\sigma_2}{\overline{X_2}} \times 100$$

$$= \frac{11}{47.5} \times 100 = 23.16$$

Clearly, coefficient of variation in wages is greater for firm B than for firm A.

So, firm B shows more variability in wages.

Q6

In order to compare the variability of weight in boys and girls, we have to calculate their coefficients of variation.

Let σ_1 and σ_2 denote the standard deviations of weight in boys and girls respectively. Further, let $\overline{X_1}$ and $\overline{X_2}$ be the mean weight of boys and girls respectively.

We have,

$$\overline{X_1} = 60, \qquad \overline{X_2} = 45$$

$$\sigma_1^2 = 9 \text{ and } \sigma_2^2 = 4$$

$$\Rightarrow \quad \sigma_1 = \sqrt{9} = 3 \quad \text{and} \quad \sigma_2 = \sqrt{4} = 2$$

Now,

$$\text{Coefficient of variation in weights in boys} = \frac{\sigma_1}{\overline{X_1}} \times 100$$

$$= \frac{3}{60} \times 100 = 5$$

and,

$$\text{Coefficient of variation in weights in girls} = \frac{\sigma_2}{\overline{X_2}} \times 100$$

$$= \frac{2}{45} \times 100 = 4.44$$

Clearly, coefficient of variation in weights is greater in boys than in girls.

So, weights shows more variability in boys.

Q7

In order to compare the variability of marks in Math, Physics, and Chemistry, we have to calculate their coefficients of variation.

Let σ_1, σ_2 and σ_3 denote the standard deviations of marks in Math, Physics and Chemistry respectively. Further, let $\overline{X_1}$, $\overline{X_2}$ and $\overline{X_3}$ be the mean scores in Math, Physics and Chemistry respectively.

We have,

$$\overline{X_1} = 42, \qquad \overline{X_2} = 32 \qquad \overline{X_3} = 40.9$$

$$\Rightarrow \quad \sigma_1 = 12 \qquad \sigma_2 = 15 \qquad \sigma_3 = 20$$

Now,

$$\text{Coefficient of variation in Maths} = \frac{\sigma_1}{\overline{X_1}} \times 100 = \frac{12}{42} \times 100 = 28.57$$

$$\text{Coefficient of variation in Physics} = \frac{\sigma_2}{\overline{X_2}} \times 100 = \frac{15}{32} \times 100 = 46.88$$

$$\text{Coefficient of variation in Chemistry} = \frac{\sigma_2}{\overline{X_2}} \times 100 = \frac{20}{40.9} \times 100 = 48.90$$

Clearly, coefficient of variation in marks is greatest in Chemistry and lowest in Math.

So, marks in chemistry show highest variability and marks in maths show lowest variability.

Q8

Let's first find the coefficient of variable for Group G_1

CI	f	x	u=(x-A)/h	fu	u^2	fu^2
10-20	9	15	-3	-27	9	81
20-30	17	25	-2	-34	4	68
30-40	32	35	-1	-32	1	32
40-50	33	45	0	0	0	0
50-60	40	55	1	40	1	40
60-70	10	65	2	20	4	40
70-80	9	75	3	27	9	81
	150			-6		342

Here, $N = 150, A = 45, \Sigma f_i u_i = -6, \Sigma f_i u_i^2 = 342$ and $h = 10$

$\therefore$ Mean $= \overline{x} = A + h\left(\frac{1}{N}\Sigma f_i u_i\right)$

$\Rightarrow$ $\overline{x} = 45 + 10\left(\frac{-6}{150}\right) = 44.6$

$$\text{var}(x) = h^2\left[\frac{1}{N}\Sigma f_i u_i^2 - \left(\frac{1}{N}\Sigma f_i u_i\right)^2\right] = 100\left[\frac{342}{150} - \left(\frac{-6}{150}\right)^2\right] = 227.84$$

$\therefore$ $S.D. = \sqrt{\text{var}(x)} = \sqrt{227.84} = 15.09$

Coefficient of variation $= \frac{S.D}{\overline{X_1}} \times 100 = \frac{15.09}{44.6} \times 100 = 33.83$

Q9

CI	f	x	u=(x-A)/h	fu	u^2	fu^2
10-15	2	12.5	-2	-4	4	8
15-20	8	17.5	-1	-8	1	8
20-25	20	22.5	0	0	0	0
25-30	35	27.5	1	35	1	35
30-35	20	32.5	2	40	4	80
35-40	15	37.5	3	45	9	135
	100			108		266

Here, $N = 100, A = 22.5, \sum f_i u_i = 108, \sum f_i u_i^2 = 266$ and $h = 5$

$\therefore$ Mean $= \bar{x} = A + h\left(\frac{1}{N}\sum f_i u_i\right)$

$\Rightarrow$ $\bar{x} = 22.5 + 5\left(\frac{108}{100}\right) = 27.90$

$$\text{var}(x) = h^2\left[\frac{1}{N}\sum f_i u_i^2 - \left(\frac{1}{N}\sum f_i u_i\right)^2\right] = 25\left[\frac{266}{100} - \left(\frac{108}{100}\right)^2\right] = 37.34$$

$\therefore$ $S.D. = \sqrt{\text{var}(x)} = \sqrt{37.34} = 6.11$

Coefficient of variation $= \frac{S.D}{\bar{X}_1} \times 100 = \frac{6.11}{27.90} \times 100 = 21.9$

Q10

x	d=(x- Mean)	d^2
35	-13	169
24	-24	576
52	4	16
53	5	25
56	8	64
58	10	100
52	4	16
50	2	4
51	3	9
49	1	1
480		**980**

$$\bar{x} = \frac{1}{n}\Sigma x_i = \frac{1}{10}[480] = 48$$

$$\text{var}(x) = \frac{1}{n}\left\{\Sigma\left(x_i - \bar{x}\right)^2\right\} = \frac{1}{10}\{980\} = 98$$

$$S.D(x) = \sqrt{\text{var}(x)} = \sqrt{98} = 9.9$$

$$\text{Coefficient of variation} = \frac{S.D}{\overline{\overline{X_1}}}\times 100 = \frac{9.9}{48}\times 100 = 20.6$$

Q11

Factory A

Length of life	Mid value x_i	f_i	$u_i = \dfrac{x_i - 800}{100}$	f_iu_i	$f_iu_i^2$
550 – 650	600	10	–2	–20	40
650 – 750	700	22	–1	–22	22
750 – 850	800	52	0	0	0
850 – 950	900	20	1	20	20
950 – 1050	1000	16	2	32	64
		$N = \sum f_i = 120$		$\sum f_iu_i = 10$	$\sum f_iu_i^2 = 146$

$N = 120, \sum f_iu_i = 10, \sum f_iu_i^2 = 146, A = 800$ and $h = 100$

$$\overline{X_A} = A + h\left(\frac{1}{N}\sum f_iu_i\right) = 800 + 100\left(\frac{10}{120}\right) = 808.33$$

$$\sigma_A^2 = h^2\left\{\left(\frac{1}{N}\sum f_iu_i^2\right) - \left(\frac{1}{N}\sum f_iu_i\right)^2\right\} = 10000\left\{\left(\frac{1}{120}\times 146\right) - \left(\frac{1}{120}\times(10)\right)^2\right\} = 10000\{1.2166 - 0.0069\} = 12097$$

$$\Rightarrow \sigma_A = \sqrt{12097} = 109.98 \approx 110$$

Q12

Ravi

x_i	f_i	f_ix_i	$f_ix_i^2$
1	25	25	25
2	50	100	200
3	45	135	405
4	30	120	480
5	70	350	1750
6	42	252	1512
7	36	252	1764
8	45	360	2880
9	35	315	2835
10	60	600	6000
	$N = \sum f_i = 438$	$\sum f_ix_i = 2509$	$\sum f_ix_i^2 = 17851$

$$\text{Variance} = \left(\frac{1}{N}\sum f_ix_i^2\right) - \left(\frac{1}{N}\sum f_ix_i\right)^2 = \left(\frac{1}{438}\times 17851\right) - \left(\frac{1}{438}\times(2509)\right)^2 = \{40.75 - 32.81\} = 7.94$$

$$\Rightarrow SD = \sqrt{7.94} = 2.82$$