Ex 7.1

Q1

We have,

$$\sin A = \frac{4}{5} \quad \text{and} \quad \cos B = \frac{5}{13}$$

$\therefore \quad \cos A = \sqrt{1 - \sin^2 A} \quad \text{and} \quad \sin B = \sqrt{1 - \cos^2 B}$

$\Rightarrow \quad \cos A = \sqrt{1 - \left(\frac{4}{5}\right)^2} \quad \text{and} \quad \sin B = \sqrt{1 - \left(\frac{5}{13}\right)^2}$

$\Rightarrow \quad \cos A = \sqrt{1 - \frac{16}{25}} \quad \text{and} \quad \sin B = \sqrt{1 - \frac{25}{169}}$

$\Rightarrow \quad \cos A = \sqrt{\frac{25-16}{25}} \quad \text{and} \quad \sin B = \sqrt{\frac{169-25}{169}}$

$\Rightarrow \quad \cos A = \sqrt{\frac{9}{25}} \quad \text{and} \quad \sin B = \sqrt{\frac{144}{169}}$

$\Rightarrow \quad \cos A = \frac{3}{5} \quad \text{and} \quad \sin B = \frac{12}{13}$

Now,

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$= \frac{4}{5} \times \frac{5}{13} + \frac{3}{5} \times \frac{12}{13}$$

$$= \frac{20}{65} + \frac{36}{65}$$

$$= \frac{20+36}{65}$$

$$= \frac{56}{65}$$

We have,

$$\sin A = \frac{4}{5} \text{ and } \cos B = \frac{5}{13}$$

∴ $\cos A = \sqrt{1 - \sin^2 A}$ and $\sin B = \sqrt{1 - \cos^2 B}$

⇒ $\cos A = \sqrt{1 - \left(\frac{4}{5}\right)^2}$ and $\sin B = \sqrt{1 - \left(\frac{5}{13}\right)^2}$

⇒ $\cos A = \sqrt{1 - \frac{16}{25}}$ and $\sin B = \sqrt{1 - \frac{25}{169}}$

⇒ $\cos A = \sqrt{\frac{25 - 16}{25}}$ and $\sin B = \sqrt{\frac{169 - 25}{169}}$

⇒ $\cos A = \sqrt{\frac{9}{25}}$ and $\sin B = \sqrt{\frac{144}{169}}$

⇒ $\cos A = \frac{3}{5}$ and $\sin B = \frac{12}{13}$

Now,

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$= \frac{3}{5} \times \frac{5}{13} - \frac{4}{5} \times \frac{12}{13}$$

$$= \frac{15}{65} - \frac{48}{65}$$

$$= \frac{15 - 48}{65}$$

$$= \frac{-33}{65}$$

We have,

$$\sin A = \frac{4}{5} \text{ and } \cos B = \frac{5}{13}$$

$\therefore$ $\cos A = \sqrt{1 - \sin^2 A}$ and $\sin B = \sqrt{1 - \cos^2 B}$

$\Rightarrow$ $\cos A = \sqrt{1 - \left(\frac{4}{5}\right)^2}$ and $\sin B = \sqrt{1 - \left(\frac{5}{13}\right)^2}$

$\Rightarrow$ $\cos A = \sqrt{1 - \frac{16}{25}}$ and $\sin B = \sqrt{1 - \frac{25}{169}}$

$\Rightarrow$ $\cos A = \sqrt{\frac{25 - 16}{25}}$ and $\sin B = \sqrt{\frac{169 - 25}{169}}$

$\Rightarrow$ $\cos A = \sqrt{\frac{9}{25}}$ and $\sin B = \sqrt{\frac{144}{169}}$

$\Rightarrow$ $\cos A = \frac{3}{5}$ and $\sin B = \frac{12}{13}$

Now,

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$= \frac{4}{5} \times \frac{5}{13} - \frac{3}{5} \times \frac{12}{13}$$

$$= \frac{20}{65} - \frac{36}{65}$$

$$= \frac{20 - 36}{65}$$

$$= -\frac{16}{65}$$

We have,

$$\sin A = \frac{4}{5} \text{ and } \cos B = \frac{5}{13}$$

$\therefore \quad \cos A = \sqrt{1 - \sin^2 A} \text{ and } \sin B = \sqrt{1 - \cos^2 B}$

$\Rightarrow \quad \cos A = \sqrt{1 - \left(\frac{4}{5}\right)^2} \text{ and } \sin B = \sqrt{1 - \left(\frac{5}{13}\right)^2}$

$\Rightarrow \quad \cos A = \sqrt{1 - \frac{16}{25}} \text{ and } \sin B = \sqrt{1 - \frac{25}{169}}$

$\Rightarrow \quad \cos A = \sqrt{\frac{25 - 16}{25}} \text{ and } \sin B = \sqrt{\frac{169 - 25}{169}}$

$\Rightarrow \quad \cos A = \sqrt{\frac{9}{25}} \text{ and } \sin B = \sqrt{\frac{144}{169}}$

$\Rightarrow \quad \cos A = \frac{3}{5} \text{ and } \sin B = \frac{12}{13}$

Now,

$$\begin{aligned}
\cos(A - B) &= \cos A \cos B + \sin A \sin B \\
&= \frac{3}{5} \times \frac{5}{13} + \frac{4}{5} \times \frac{12}{13} \\
&= \frac{15}{65} + \frac{48}{65} \\
&= \frac{15 + 48}{65} \\
&= \frac{63}{65}
\end{aligned}$$

Q2

We have,

$$\sin A = \frac{12}{13} \text{ and } \sin B = \frac{4}{5}$$

$$\therefore \quad \cos A = -\sqrt{1 - \sin^2 A} \text{ and } \cos B = \sqrt{1 - \sin^2 B}$$

$$[\because \text{ In the second quadrant } \cos\theta \text{ is negative}]$$

$$\Rightarrow \quad \cos A = -\sqrt{1 - \left(\frac{12}{13}\right)^2} \text{ and } \cos B = \sqrt{1 - \left(\frac{4}{5}\right)^2}$$

$$\Rightarrow \quad \cos A = -\sqrt{1 - \frac{144}{169}} \text{ and } \cos B = \sqrt{1 - \frac{16}{25}}$$

$$\Rightarrow \quad \cos A = -\sqrt{\frac{25}{169}} \text{ and } \cos B = \sqrt{\frac{9}{25}}$$

$$\Rightarrow \quad \cos A = \frac{-5}{13} \text{ and } \cos B = \frac{3}{5}$$

Now,

(i)

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$= \frac{12}{13} \times \frac{3}{5} - \frac{5}{13} \times \frac{4}{5}$$

$$= \frac{36}{65} - \frac{20}{65}$$

$$= \frac{16}{65}$$

(ii)

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$= \frac{-5}{13} \times \frac{3}{5} - \frac{12}{13} \times \frac{4}{5}$$

$$= \frac{-15}{65} - \frac{48}{65}$$

$$= \frac{-63}{65}$$

We have,

$$\sin A = \frac{3}{5} \text{ and } \cos B = \frac{-12}{13}$$

∴ $\cos A = -\sqrt{1 - \sin^2 A}$ and $\sin B = \sqrt{1 - \cos^2 B}$

[∵ In the second quadrant $\cos\theta$ is negative]

$$\Rightarrow \quad \cos A = -\sqrt{1 - \left(\frac{3}{5}\right)^2} \text{ and } \sin B = \sqrt{1 - \left(\frac{-12}{13}\right)^2}$$

$$\Rightarrow \quad \cos A = -\sqrt{1 - \frac{9}{25}} \text{ and } \sin B = \sqrt{1 - \frac{144}{169}}$$

$$\Rightarrow \quad \cos A = -\sqrt{\frac{16}{25}} \text{ and } \sin B = \sqrt{\frac{25}{169}}$$

$$\Rightarrow \quad \cos A = -\frac{4}{5} \text{ and } \sin B = \frac{5}{13}$$

Now,

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$= \frac{3}{5} \times \left(\frac{-12}{13}\right) - \frac{4}{5} \times \frac{5}{13}$$

$$= -\frac{36}{65} - \frac{20}{65}$$

$$= -\frac{56}{65}$$

∴ $\sin(A + B) = -\frac{56}{65}$

Q3

We have,

$$\cos A = -\frac{24}{25} \text{ and } \cos B = \frac{3}{5}$$

$$\therefore \quad \sin A = -\sqrt{1-\cos^2 A} \text{ and } \sin B = -\sqrt{1-\cos^2 B}$$

$$[\because \text{ In the 3rd and 4th quadrant } \sin\theta \text{ is negative}]$$

$$\Rightarrow \quad \sin A = -\sqrt{1-\left(-\frac{24}{25}\right)^2} \text{ and } \sin B = -\sqrt{1-\left(\frac{3}{5}\right)^2}$$

$$\Rightarrow \quad \sin A = -\sqrt{1-\frac{576}{625}} \text{ and } \sin B = -\sqrt{1-\frac{9}{25}}$$

$$\Rightarrow \quad \sin A = -\sqrt{\frac{49}{625}} \text{ and } \sin B = -\sqrt{\frac{16}{25}}$$

$$\Rightarrow \quad \sin A = -\frac{7}{25} \text{ and } \sin B = -\frac{4}{5}$$

Now,

(i)
$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$= -\frac{7}{25}\times\frac{3}{5} - \frac{24}{25}\times\left(-\frac{4}{5}\right)$$

$$= -\frac{21}{25} + \frac{96}{125}$$

$$= \frac{75}{125}$$

$$= \frac{3}{5}$$

(ii)
$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$= -\frac{24}{25}\times\frac{3}{5} - \left(-\frac{7}{25}\right)\times\left(-\frac{4}{5}\right)$$

$$= -\frac{72}{125} - \frac{28}{125}$$

$$= \frac{-72-28}{125}$$

$$= \frac{-100}{125} = \frac{-4}{5}$$

Q4

We have,

$$\tan A = \frac{3}{4}, \text{ and } \cos B = \frac{9}{41}$$

$$\therefore \quad \sin B = \sqrt{1 - \cos^2 B}$$

$$= \sqrt{1 - \left(\frac{9}{41}\right)^2}$$

$$= \sqrt{1 - \frac{81}{1681}}$$

$$= \sqrt{\frac{1600}{1681}}$$

$$= \frac{40}{41}$$

$$\therefore \quad \tan B = \frac{\sin B}{\cos B} = \frac{\frac{40}{41}}{\frac{9}{41}} = \frac{40}{9}$$

Now,

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{3}{4} + \frac{40}{9}}{1 - \frac{3}{4} \times \frac{40}{9}}$$

$$= \frac{\frac{27 + 160}{36}}{\frac{36 - 120}{36}}$$

$$= \frac{\frac{187}{36}}{\frac{-84}{36}}$$

$$= -\frac{187}{84}$$

Q5

We have,

$$\sin A = \frac{1}{2} \text{ and } \cos B = \frac{12}{13}$$

$\therefore$ $\cos A = -\sqrt{1 - \sin^2 A}$ and $\sin B = -\sqrt{1 - \cos^2 B}$

$$\left[\because \text{cosine is negative in second quadrant and} \atop \text{sine is negative in fourth quadrant} \right]$$

$\Rightarrow$ $\cos A = -\sqrt{1 - \left(\frac{1}{2}\right)^2}$ and $\sin B = -\sqrt{1 - \left(\frac{12}{13}\right)^2}$

$\Rightarrow$ $\cos A = -\sqrt{1 - \frac{1}{4}}$ and $\sin B = -\sqrt{1 - \frac{144}{169}}$

$\Rightarrow$ $\cos A = -\sqrt{\frac{3}{4}}$ and $\sin B = -\sqrt{\frac{25}{169}}$

$\Rightarrow$ $\cos A = -\frac{\sqrt{3}}{2}$ and $\sin B = -\frac{5}{13}$

$\therefore$ $\tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2}}{\frac{-\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$

and, $\tan B = \frac{\sin B}{\cos B} = \frac{-\frac{5}{13}}{\frac{12}{13}} = \frac{-5}{12}$

Now, $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$= \frac{\frac{-1}{\sqrt{3}} - \left(\frac{-5}{12}\right)}{1 + \left(\frac{-1}{\sqrt{3}}\right) \times \left(\frac{-5}{12}\right)}$$

$$= \frac{\frac{-1}{\sqrt{3}} + \frac{5}{12}}{1 + \frac{5}{12\sqrt{3}}}$$

$$= \frac{\frac{-12 + 5\sqrt{3}}{12\sqrt{3}}}{\frac{12\sqrt{3} + 5}{12\sqrt{3}}}$$

$$= \frac{5\sqrt{3} - 12}{5 + 12\sqrt{3}}$$

$\Rightarrow$ $\tan(A - B) = \frac{5\sqrt{3} - 12}{5 + 12\sqrt{3}}$

Q6

We have,

$$\sin A = \frac{1}{2} \text{ and } \cos B = \frac{\sqrt{3}}{2}$$

$$\therefore \quad \cos A = -\sqrt{1 - \sin^2 A} \text{ and } \sin B = \sqrt{1 - \cos^2 B}$$

$$[\because \text{cosine is negative in second quadrant}]$$

$$\Rightarrow \quad \cos A = -\sqrt{1 - \left(\frac{1}{2}\right)^2} \text{ and } \sin B = \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow \quad \cos A = -\sqrt{1 - \frac{1}{4}} \text{ and } \sin B = \sqrt{1 - \frac{3}{4}}$$

$$\Rightarrow \quad \cos A = -\sqrt{\frac{3}{4}} \text{ and } \sin B = \sqrt{\frac{1}{4}}$$

$$\Rightarrow \quad \cos A = -\frac{\sqrt{3}}{2} \text{ and } \sin B = \frac{1}{2}$$

$$\therefore \quad \tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2}}{\frac{-\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$$

$$\text{and,} \quad \tan B = \frac{\sin B}{\cos B} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

Now,

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{-1}{\sqrt{3}} + \frac{1}{\sqrt{3}}}{1 - \left(\frac{-1}{\sqrt{3}}\right) \times \left(\frac{1}{\sqrt{3}}\right)}$$

$$= 0$$

$$\therefore \quad \tan(A + B) = 0$$

We have,

$$\sin A = \frac{1}{2} \text{ and } \cos B = \frac{\sqrt{3}}{2}$$

$\therefore$ $\cos A = -\sqrt{1 - \sin^2 A}$ and $\sin B = \sqrt{1 - \cos^2 B}$ [$\because$ cosine is negative in second quadrant]

$\Rightarrow$ $\cos A = -\sqrt{1 - \left(\frac{1}{2}\right)^2}$ and $\sin B = \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2}$

$\Rightarrow$ $\cos A = -\sqrt{1 - \frac{1}{4}}$ and $\sin B = \sqrt{1 - \frac{3}{4}}$

$\Rightarrow$ $\cos A = -\sqrt{\frac{3}{4}}$ and $\sin B = \sqrt{\frac{1}{4}}$

$\Rightarrow$ $\cos A = -\frac{\sqrt{3}}{2}$ and $\sin B = \frac{1}{2}$

$\therefore$ $\tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2}}{\frac{-\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$

and, $\tan B = \frac{\sin B}{\cos B} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

Now, $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$= \frac{\frac{-1}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{-1}{\sqrt{3}}\right) \times \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{-2}{\sqrt{3}}}{1 - \frac{1}{3}}$$

$$= \frac{\frac{-2}{\sqrt{3}}}{\frac{3 - 1}{3}}$$

$$= \frac{\frac{-2}{\sqrt{3}}}{\frac{2}{3}}$$

$$= \frac{-3}{\sqrt{3}}$$

$$= \frac{-\sqrt{3} \times \sqrt{3}}{\sqrt{3}} = -\sqrt{3}$$

$\therefore$ $\tan(A - B) = -\sqrt{3}$

Q7

(i)

$\sin 78° \cos 18° - \cos 78° \sin 18°$ $\left[\sin(A-B) = \sin A \cos B - \cos A \sin B\right]$

$= \sin(78° - 18°)$

$= \sin 60°$

$= \dfrac{\sqrt{3}}{2}$

(ii)

$\cos 47° \cos 13° - \sin 47° \sin 13°$ $\left[\cos(A+B) = \cos A \cos B - \sin A \sin B\right]$

$= \cos(47° + 13°)$

$= \cos 60°$

$= \dfrac{1}{2}$

(iii)

$\sin 36° \cos 9° + \cos 36° \sin 9°$ $\left[\sin(A+B) = \sin A \cos B + \cos A \sin B\right]$

$= \sin(36° + 9°)$

$= \sin 45°$

$= \dfrac{1}{\sqrt{2}}$

(iv)

$\cos 80° \cos 20° + \sin 80° \sin 20°$ $\left[\cos(A-B) = \cos A \cos B + \sin A \sin B\right]$

$= \cos(80° - 20°)$

$= \cos 60°$

$= \dfrac{1}{2}$

Q8

We have,

$$\cos A = \frac{-12}{13} \text{ and } \cot B = \frac{24}{7}$$

$$\therefore \quad \sin A = \sqrt{1 - \cos^2 A} = \sqrt{1 - \left(\frac{-12}{13}\right)^2} = \sqrt{1 - \frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

and,

$$\text{cosec } B = -\sqrt{1 + \cot^2 B} \qquad [\because \text{cosec is negative in third quadrant}]$$

$$= -\sqrt{1 + \left(\frac{24}{7}\right)^2} = -\sqrt{1 + \frac{576}{49}} = -\sqrt{\frac{49 + 576}{49}} = -\sqrt{\frac{625}{49}} = -\frac{25}{7}$$

$$\Rightarrow \quad \sin B = \frac{-7}{25} \qquad \left[\because \text{cosec} B = \frac{1}{\sin B}\right]$$

Now,

$$\cos B = -\sqrt{1 - \sin^2 B} \qquad [\because \cos\theta \text{ is negative in third quadrant}]$$

$$= -\sqrt{1 - \left(\frac{-7}{25}\right)^2} = -\sqrt{1 - \frac{49}{625}} = -\sqrt{\frac{625 - 49}{625}} = -\sqrt{\frac{576}{625}} = \frac{-24}{25}$$

Now,

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$= \frac{5}{13} \times \left(\frac{-24}{25}\right) + \left(\frac{-12}{13}\right) \times \left(\frac{-7}{25}\right)$$

$$= \frac{-120}{325} + \frac{84}{325}$$

$$= \frac{-120 + 84}{325}$$

$$= \frac{-36}{325}$$

We have,

$$\cos A = \frac{-12}{13} \text{ and } \cot B = \frac{24}{7}$$

$$\therefore \quad \sin A = \sqrt{1 - \cos^2 A} = \sqrt{1 - \left(\frac{-12}{13}\right)^2} = \sqrt{1 - \frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

and,

$$\operatorname{cosec} B = -\sqrt{1 + \cot^2 B} \qquad [\because \operatorname{cosec} \text{ is negative in third quadrant}]$$

$$= -\sqrt{1 + \left(\frac{24}{7}\right)^2} = -\sqrt{1 + \frac{576}{49}} = -\sqrt{\frac{49 + 576}{49}} = -\sqrt{\frac{625}{49}} = -\frac{25}{7}$$

$$\Rightarrow \quad \sin B = \frac{-7}{25} \qquad \left[\because \operatorname{cosec} B = \frac{1}{\sin B}\right]$$

Now,

$$\cos B = -\sqrt{1 - \sin^2 B} \qquad [\because \cos\theta \text{ is negative in third quadrant}]$$

$$= -\sqrt{1 - \left(\frac{-7}{25}\right)^2} = -\sqrt{1 - \frac{49}{625}} = -\sqrt{\frac{625 - 49}{625}} = -\sqrt{\frac{576}{625}} = \frac{-24}{25}$$

Now,

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$= \left(\frac{-12}{13}\right) \times \left(\frac{-24}{25}\right) - \left(\frac{5}{13}\right) \times \left(\frac{-7}{25}\right)$$

$$= \frac{288}{325} + \frac{35}{325}$$

$$= \frac{323}{325}$$

We have,

$$\cos A = \frac{-12}{13} \text{ and } \cot B = \frac{24}{7}$$

$$\therefore \quad \sin A = \sqrt{1 - \cos^2 A} = \sqrt{1 - \left(\frac{-12}{13}\right)^2} = \sqrt{1 - \frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

and,

$$\operatorname{cosec} B = -\sqrt{1 + \cot^2 B} \qquad [\because \operatorname{cosec} \text{ is negative in third quadrant}]$$

$$= -\sqrt{1 + \left(\frac{24}{7}\right)^2} = -\sqrt{1 + \frac{576}{49}} = -\sqrt{\frac{49 + 576}{49}} = -\sqrt{\frac{625}{49}} = -\frac{25}{7}$$

$$\Rightarrow \quad \sin B = \frac{-7}{25} \qquad \left[\because \operatorname{cosec} B = \frac{1}{\sin B}\right]$$

Now,

$$\cos B = -\sqrt{1 - \sin^2 B} \qquad [\because \cos \theta \text{ is negative in third quadrant}]$$

$$= -\sqrt{1 - \left(\frac{-7}{25}\right)^2} = -\sqrt{1 - \frac{49}{625}} = -\sqrt{\frac{625 - 49}{625}} = -\sqrt{\frac{576}{625}} = \frac{-24}{25}$$

Now,

$$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{5}{13}}{\frac{-12}{13}} = \frac{-5}{12} \qquad \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta}\right]$$

and,

$$\tan B = \frac{\sin B}{\cos B} = \frac{\frac{-7}{25}}{\frac{-24}{25}} = \frac{7}{24} \qquad \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta}\right]$$

$$\therefore \quad \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{-5}{12} + \frac{7}{24}}{1 - \left(\frac{-5}{12}\right) \times \frac{7}{24}}$$

$$= \frac{\frac{-10 + 7}{24}}{1 + \frac{35}{288}}$$

$$= \frac{\frac{-3}{24}}{\frac{288 + 35}{288}}$$

Q9

LHS: $\cos 105° + \cos 15°$

       $= \cos(90° + 15°) + \cos(90° - 75°)$

       $= -\sin 15° + \sin 75°$

       $= \sin 75° - \sin 15°$

$\begin{bmatrix} \because \cos(90 + \theta) = -\sin\theta \\ \text{and } \cos(90 - \theta) = \sin\theta \end{bmatrix}$

∴ $\cos 105° + \cos 15° = \sin 75° - \sin 15°$

Hence proved.

Q10

LHS: $\dfrac{\tan A + \tan B}{\tan A - \tan B}$

$= \dfrac{\dfrac{\sin A}{\cos A} + \dfrac{\sin B}{\cos B}}{\dfrac{\sin A}{\cos A} - \dfrac{\sin B}{\cos B}}$

$\begin{bmatrix} \because \tan\theta = \dfrac{\sin\theta}{\cos\theta} \end{bmatrix}$

$= \dfrac{\dfrac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}}{\dfrac{\sin A \cos B - \cos A \sin B}{\cos A \cos B}}$

$= \dfrac{\sin A \cos B + \cos A \sin B}{\sin A \cos B - \cos A \sin B}$

$= \dfrac{\sin(A + B)}{\sin(A - B)}$

$\begin{bmatrix} \because \qquad \sin(A + B) = \sin A \cos B + \cos A \sin B \\ \text{and, } \sin(A - B) = \sin A \cos B - \cos A \sin B \end{bmatrix}$

∴ $\dfrac{\tan A + \tan B}{\tan A - \tan B} = \dfrac{\sin(A + B)}{\sin(A - B)}$

Hence proved.

Q11

LHS: $\dfrac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°}$

Dividing numerator and denominator by $\cos 11°$, we get

$$\dfrac{\dfrac{\cos 11°}{\cos 11°} + \dfrac{\sin 11°}{\cos 11°}}{\dfrac{\cos 11°}{\cos 11°} - \dfrac{\sin 11°}{\cos 11°}}$$

$$= \dfrac{1 + \tan 11°}{1 - \tan 11°} \qquad \left[\because \tan\theta = \dfrac{\sin\theta}{\cos\theta} \right]$$

$$= \dfrac{\tan 45° + \tan 11°}{1 - \tan 45° \times \tan 11°} \qquad \left[\tan 45° = 1 \right]$$

$$= \tan(45° + 11°) \qquad \left[\because \tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B} \right]$$

$$= \tan 56°$$

$\therefore \quad \dfrac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°} = \tan 56°$

Hence proved.

LHS: $\dfrac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°}$

$$\dfrac{\dfrac{\cos 9°}{\cos 9°} + \dfrac{\sin 9°}{\cos 9°}}{\dfrac{\cos 9°}{\cos 9°} - \dfrac{\sin 9°}{\cos 9°}}$$

$\left[\begin{array}{l}\text{Dividing numerator and}\\\text{denominator by } \cos 9°\end{array}\right]$

$= \dfrac{1 + \tan 9°}{1 - \tan 9°}$

$\left[\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right]$

$= \dfrac{\tan 45° + \tan 9°}{1 - \tan 45° \times \tan 9°}$

$[\tan 45° = 1]$

$= \tan(45° + 9°)$

$\left[\because \tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}\right]$

$= \tan 54°$

$= \text{RHS}$

∴   LHS = RHS

Hence proved.

LHS: $\dfrac{\cos 8° - \sin 8°}{\cos 8° + \sin 8°}$

$$\dfrac{\dfrac{\cos 8°}{\cos 8°} - \dfrac{\sin 8°}{\cos 8°}}{\dfrac{\cos 8°}{\cos 8°} + \dfrac{\sin 8°}{\cos 8°}}$$

$\left[\begin{array}{l}\text{Dividing numerator and}\\\text{denominator by } \cos 8°\end{array}\right]$

$= \dfrac{1 - \tan 8°}{1 + \tan 8°}$

$\left[\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right]$

$= \dfrac{\tan 45° - \tan 8°}{1 + \tan 45° \times \tan 8°}$

$[\tan 45° = 1]$

$= \tan(45° - 8°)$

$\left[\because \tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}\right]$

$= \tan 37°$

$= \text{RHS}$

∴   LHS = RHS

Hence proved.

Q12

LHS: $\sin(60° - \theta)\cos(30° + \theta) + \cos(60° - \theta) \times \sin(30° + \theta)$

$= \sin\left[(60° - \theta) + (30° + \theta)\right]$ $\left[\sin(A + B) = \sin A \cos B + \cos A \sin B\right]$

$= \sin\left[60° - \theta + 30° + \theta\right]$

$= \sin(90°)$

$= 1$

$= RHS$

∴   LHS = RHS

Hence proved.

Q13

LHS: $\dfrac{\tan 69° + \tan 66°}{1 - \tan 69° \tan 66°}$

$= \tan(69° + 66°)$ $\left[\because \tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}\right]$

$= \tan(135°)$

$= \tan(90° + 45°)$

$= -\cot 45°$ $\left[\because \tan\theta \text{ is negative in second quadrant}\right]$

$= -1$

$= RHS$

∴   LHS = RHS

Hence proved.

Q14

We have,

$$\tan A = \frac{5}{6} \quad \text{and} \quad \tan B = \frac{1}{11}$$

Now,

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{5}{6} + \frac{1}{11}}{1 - \frac{5}{6} \times \frac{1}{11}}$$

$$= \frac{\frac{55+6}{66}}{1 - \frac{5}{66}}$$

$$= \frac{\frac{61}{66}}{\frac{66-5}{66}}$$

$$= \frac{\frac{61}{66}}{\frac{61}{66}}$$

$$= \frac{61}{66} \times \frac{66}{61}$$

$$= 1$$

$$= \tan\frac{\pi}{4} \qquad \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$

$$\Rightarrow \quad \tan(A+B) = \tan\frac{\pi}{4}$$

$$\Rightarrow \quad A + B = \frac{\pi}{4}$$

Hence proved.

We have,

$$\tan A = \frac{m}{m-1} \quad \text{and} \quad \tan B = \frac{1}{2m-1}$$

Now,
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{\dfrac{m}{m-1} - \dfrac{1}{2m-1}}{1 + \dfrac{m}{m-1} \times \dfrac{1}{2m-1}}$$

$$= \frac{\dfrac{m(2m-1) - (m-1)}{(m-1)(2m-1)}}{1 + \dfrac{m}{(m-1)(2m-1)}}$$

$$= \frac{\dfrac{m(2m-1) - (m-1)}{(m-1)(2m-1)}}{\dfrac{(m-1)(2m-1) + (m)}{(m-1)(2m-1)}}$$

$$= \frac{m(2m-1) - (m-1)}{(m-1)(2m-1) + (m)}$$

$$= \frac{2m^2 - m - m + 1}{2m^2 - m - 2m + 1 + m}$$

$$= \frac{2m^2 - m - m + 1}{2m^2 - 2m + 1}$$

$$= \frac{2m^2 - 2m + 1}{2m^2 - 2m + 1}$$

$$= 1$$

$\therefore \quad \tan(A-B) = 1 = \tan\left(\frac{\pi}{4}\right) \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$

$\Rightarrow \quad \tan(A-B) = \tan\left(\frac{\pi}{4}\right)$

$\Rightarrow \quad A - B = \left(\frac{\pi}{4}\right)$

Q15

LHS: $\cos^2 45° - \sin^2 15°$

$= \left(\dfrac{1}{\sqrt{2}}\right)^2 - \sin^2 15°$ $\left[\because \cos 45 = \dfrac{1}{\sqrt{2}}\right]$

$= \dfrac{1}{2} - \left(\dfrac{1 - \cos 2 \times 15°}{2}\right)$ $\left[\because \cos 2\theta = 1 - \sin^2 \theta\right]$

$= \dfrac{1}{2} - \left(\dfrac{1 - \cos 30°}{2}\right)$

$= \dfrac{1 - 1 + \cos 30°}{2}$

$= \dfrac{\cos 30°}{2}$

$= \dfrac{\sqrt{3}}{2} \times \dfrac{1}{2}$

$= \dfrac{\sqrt{3}}{4}$

$= $ RHS

$\therefore$ LHS = RHS

Hence proved.

We have,

LHS $\sin^2 (n + 1) A - \sin^2 nA$

$= \sin[(n+1)A + nA] \sin[(n+1)A - nA]$

$\qquad\qquad\qquad \left[\because \sin^2 A - \sin^2 B = \sin(A + B)\sin(A - B)\right]$

$= \sin[nA + A + nA] \sin[nA + A - nA]$

$= \sin(2nA + A) \sin(A)$

$= \sin(2n + 1) A \sin A$

$= $ RHS

$\because$ LHS = RHS

Hence proved.

Q16

We have,

$$\text{LHS} = \frac{\sin(A+B) + \sin(A-B)}{\cos(A+B) + \cos(A-B)}$$

$$= \frac{2\sin A\cos B}{2\cos A\cos B} \qquad \left[\begin{array}{l} \because \quad 2\sin A\cos B = \sin(A+B) + \sin(A-B) \\ \text{and, } 2\cos A\cos B = \cos(A+B) + \cos(A-B) \end{array}\right]$$

$$= \frac{\sin A}{\cos A}$$

$$= \tan A$$

$$= \text{RHS}$$

$\therefore$ LHS = RHS

Hence proved.

$$\text{LHS} = \frac{\sin(A-B)}{\cos A\cos B} + \frac{\sin(B-C)}{\cos B\cos C} + \frac{\sin(C-A)}{\cos C\cos A}$$

$$= \frac{\sin A\cos B - \cos A\sin B}{\cos A\cos B} + \frac{\sin B\cos C - \cos B\sin C}{\cos B\cos C} + \frac{\sin C\cos A - \cos C\sin A}{\cos C\cos A}$$

$$= \frac{\sin A\cos B}{\cos A\cos B} - \frac{\cos A\sin B}{\cos A\cos B} + \frac{\sin B\cos C}{\cos B\cos C} - \frac{\cos B\sin C}{\cos B\cos C} + \frac{\sin C\cos A}{\cos C\cos A}$$

$$\qquad\qquad - \frac{\cos C\sin A}{\cos C\cos A}$$

$$= \tan A - \tan B + \tan B - \tan C + \tan C - \tan A$$

$$= 0$$

$$= \text{RHS}$$

$\therefore$ LHS = RHS

Hence proved.

We have,

LHS $= \dfrac{\sin(A - B)}{\sin A \sin B} + \dfrac{\sin(B - C)}{\sin B \sin C} + \dfrac{\sin(C - A)}{\sin C \sin A}$

$= \dfrac{\sin A \cos B - \cos A \sin B}{\sin A \sin B} + \dfrac{\sin B \cos C - \cos B \sin C}{\sin B \sin C} + \dfrac{\sin C \cos A - \cos C \sin A}{\sin C \sin A}$

$= \dfrac{\sin A \cos B}{\sin A \sin B} - \dfrac{\cos A \sin B}{\sin A \sin B} + \dfrac{\sin B \cos C}{\sin B \sin C} - \dfrac{\cos B \sin C}{\sin B \sin C} + \dfrac{\sin C \cos A}{\sin C \sin A}$

$\quad\quad - \dfrac{\cos C \sin A}{\sin C \sin A}$

$= \dfrac{\cos B}{\sin B} - \dfrac{\cos A}{\sin A} + \dfrac{\cos C}{\sin C} - \dfrac{\cos B}{\sin B} + \dfrac{\cos A}{\sin A} - \dfrac{\cos C}{\sin C}$

$= \cot B - \cot A + \cot C - \cot B + \cot A - \cot C \quad\quad \left[\because \cot \theta = \dfrac{\cos \theta}{\sin \theta} \right]$

$= 0$

$= \text{RHS}$

∴   LHS = RHS

Hence proved.

We have,

RHS $= \sin^2 A + \sin^2 (A - B) - 2 \sin A \cos B \sin(A - B)$

$= \sin^2 A + \sin(A - B) \left[\sin(A - B) - 2 \sin A \cos B \right]$

$= \sin^2 A + \sin(A - B) \left[\sin A \cos B - \cos A \sin B - 2 \sin A \cos B \right]$

$= \sin^2 A + \sin(A - B) \left[- \sin A \cos B - \cos A \sin B \right]$

$= \sin^2 A - \sin(A - B) \left(\sin A \cos B + \cos A \sin B \right)$

$= \sin^2 A - \sin(A - B) \left(\sin(A + B) \right)$

$= \sin^2 A - \sin(A - B) \sin(A + B)$

$= \sin^2 A - \left(\sin^2 A - \sin^2 B \right) \quad \left[\because \sin(A - B) \sin(A + B) = \sin^2 A - \sin^2 B \right]$

$= \sin^2 A - \sin^2 A + \sin^2 B$

$= \sin^2 B$

$= \text{LHS}$

∴   LHS = RHS

Hence proved.

RHS $= \cos^2 A + \cos^2 B - 2 \cos A \cos B \cos(A+B)$

$= \cos^2 A + \left(1 - \sin^2 B\right) - 2 \cos A \cos B \cos(A+B)$

$= \left[\cos^2 A - \sin^2 B\right] - 2 \cos A \cos B \cos(A+B) + 1$

$= \left[\cos(A+B)\cos(A-B)\right] - 2 \cos A \cos B \cos(A+B) + 1$

$= \cos(A+B)\left[\cos(A-B) - 2 \cos A \cos B\right] + 1$

$= \cos(A+B)\left[\cos A \cos B + \sin A \sin B - 2 \cos A \cos B\right] + 1$

$= \cos(A+B)\left[-\cos A \cos B + \sin A \sin B\right] + 1$

$= -\cos(A+B)\left[\cos A \cos B - \sin A \sin B\right] + 1$

$= -\cos(A+B)\left[\cos(A+B)\right] + 1$

$= -\cos^2(A+B) + 1$

$= 1 - \cos^2(A+B)$

$= \sin^2(A+B)$ $\qquad \left[\sin^2\theta = 1 - \cos^2\theta\right]$

$= $ RHS

∴ LHS = RHS

Hence proved.

We have,

$$\text{LHS} = \frac{\tan(A+B)}{\cot(A-B)}$$

$$= \frac{\tan(A+B)}{\frac{1}{\tan(A-B)}} \qquad \left[\because \cot\theta = \frac{1}{\tan\theta}\right]$$

$$= \tan(A+B)\tan(A-B)$$

$$= \left[\frac{\tan A + \tan B}{1 - \tan A \tan B}\right]\left[\frac{\tan A - \tan B}{1 + \tan A \tan B}\right]$$

$$= \frac{(\tan A + \tan B)(\tan A - \tan B)}{(1 - \tan A \tan B)(1 + \tan A \tan B)}$$

$$= \frac{\tan^2 A - \tan^2 B}{1 - (\tan A \tan B)^2} \qquad \left[\because (a-b)(a+b) = a^2 - b^2\right]$$

$$= \frac{\tan^2 A - \tan^2 B}{1 - \tan^2 A \tan^2 B}$$

$$= \text{RHS}$$

$\therefore \quad \text{LHS} = \text{RHS}$

Hence proved.

Q17

We have,

$$8\theta = 6\theta + 2\theta$$

$\Rightarrow \quad \tan 8\theta = \tan(6\theta + 2\theta)$

$\Rightarrow \quad \tan 8\theta = \dfrac{\tan 6\theta + \tan 2\theta}{1 - \tan 6\theta \tan 2\theta} \qquad \left[\because \tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}\right]$

$\Rightarrow \quad \tan 8\theta(1 - \tan 6\theta \tan 2\theta) = \tan 6\theta + \tan 2\theta$

$\Rightarrow \quad \tan 8\theta - \tan 8\theta \tan 6\theta \tan 2\theta = \tan 6\theta + \tan 2\theta$

$\Rightarrow \quad \tan 8\theta - \tan 6\theta - \tan 2\theta = \tan 8\theta \tan 6\theta \tan 2\theta$

Hence proved.

We have,

$$45° = 30° + 15°$$

$\Rightarrow$ $\tan 45° = \tan(30° + 15°)$

$\Rightarrow$ $1 = \dfrac{\tan 30° + \tan 15°}{1 - \tan 30° \tan 15°}$ $\left[\because \tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}\right]$

$\Rightarrow$ $1 - \tan 30° \tan 15° = \tan 15° + \tan 30°$

$\Rightarrow$ $1 = \tan 15° + \tan 30° + \tan 30° \tan 15°$

$\Rightarrow$ $\tan 15° + \tan 30° + \tan 15° \tan 30° = 1$

Hence proved.

We have,

$$45° = 9° + 36°$$

$\Rightarrow$ $\tan 45° = \tan(9° + 36°)$

$\Rightarrow$ $1 = \dfrac{\tan 9° + \tan 36°}{1 - \tan 9° \tan 36°}$ $\left[\because \tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}\right]$

$\Rightarrow$ $1 - \tan 9° \tan 36° = \tan 9° + \tan 36°$

$\Rightarrow$ $1 = \tan 9° + \tan 36° + \tan 9° \tan 36°$

$\Rightarrow$ $\tan 9° + \tan 36° + \tan 9° \tan 36° = 1$

Hence proved.

We have,

$$13\theta = 9\theta + 4\theta$$

$\Rightarrow$ $\tan 13\theta = \tan(9\theta + 4\theta)$

$\Rightarrow$ $\tan 13\theta = \dfrac{\tan 9\theta + \tan 4\theta}{1 - \tan 9\theta \tan 4\theta}$ $\left[\because \tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}\right]$

$\Rightarrow$ $\tan 13\theta\left(1 - \tan 9\theta \tan 4\theta\right) = \tan 9\theta + \tan 4\theta$

$\Rightarrow$ $\tan 13\theta - \tan 13\theta \tan 9\theta \tan 4\theta = \tan 9\theta + \tan 4\theta$

$\Rightarrow$ $\tan 13\theta - \tan 9\theta - \tan 4\theta = \tan 13\theta \tan 9\theta \tan 4\theta$

Hence proved.

Q18

We have,

$$\text{RHS} = \tan 3\theta \tan \theta$$

$$= \tan(2\theta + \theta) \times \tan(2\theta - \theta)$$

$$= \left[\frac{\tan 2\theta + \tan\theta}{1 - \tan 2\theta \tan\theta}\right] \times \left[\frac{\tan 2\theta - \tan\theta}{1 + \tan 2\theta \tan\theta}\right]$$

$$= \frac{(\tan 2\theta + \tan\theta)(\tan 2\theta - \tan\theta)}{(1 - \tan 2\theta \tan\theta)(1 + \tan 2\theta \tan\theta)}$$

$$= \frac{\tan^2 2\theta - \tan^2 \theta}{1 - \tan^2 2\theta \tan^2 \theta} \qquad \left[\because (a-b)(a+b) = a^2 - b^2\right]$$

$$= \text{LHS}$$

$$\therefore \quad \text{LHS} = \text{RHS}$$

Hence proved

Q19

$$\frac{\sin x.\cos y + \sin y.\cos x}{\sin x.\cos y - \sin y.\cos x} = \frac{a - b}{a - b}$$

$$\rightarrow \frac{\sin x.\cos y + \sin y.\cos x - \sin x.\cos y - \sin y.\cos x}{\sin x.\cos y + \sin y.\cos x - \sin x.\cos y + \sin y.\cos x} = \frac{a - b + a - b}{a - b - a + b} \quad [\text{Using Componendo and Dividendo}]$$

$$\rightarrow \frac{2\sin x.\cos y}{2\sin y.\cos x} = \frac{2a}{2b}$$

$$\Rightarrow \frac{\tan x}{\tan y} = \frac{a}{b}$$

Hence Proved

Q20

We have,

$$\tan A = x \tan B$$

$$\frac{\sin A}{\cos A} = x \frac{\sin B}{\cos B} \qquad\qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta} \right]$$

$$\Rightarrow \quad \sin A \cos B = x \cos A \sin B \qquad\qquad ---(i)$$

Now,
$$\frac{\sin(A-B)}{\sin(A+B)} = \frac{\sin A \cos B - \sin B \cos A}{\sin A \cos B + \cos A \sin B}$$

$$= \frac{x \cos A \sin B - \cos A \sin B}{x \cos A \sin B + \cos A \sin B} \qquad\qquad \left[\text{Using equation } (i) \right]$$

$$= \frac{\cos A \sin B \,(x-1)}{\cos A \sin B \,(x+1)}$$

$$= \frac{x-1}{x+1}$$

$$\therefore \quad \frac{\sin(A-B)}{\sin(A+B)} = \frac{x-1}{x+1}$$

Hence proved.

Q21

We have,

$$\tan(A+B) = x \text{ and } \tan(A-B) = y$$

Now, $\tan 2A = \tan\left[(A+B)+(A-B)\right]$

$$= \frac{\tan(A+B)+\tan(A-B)}{1-\tan(A+B)\times\tan(A-B)}$$

$$= \frac{x+y}{1-xy}$$

$$\therefore \quad \tan 2A = \frac{x+y}{1-xy}$$

Now, $\tan 2B = \tan\left[(A+B)-(A-B)\right]$

$$= \frac{\tan(A+B)-\tan(A-B)}{1+\tan(A+B)\times\tan(A-B)}$$

$$= \frac{x-y}{1+xy}$$

$$\therefore \quad \tan 2B = \frac{x-y}{1+xy}$$

Q22

We have,

$$\cos A + \sin B = m \quad \text{and} \quad \sin A + \cos B = n$$

Now, $m^2 + n^2 - 2$

$$= \left(\cos A + \sin B\right)^2 + \left(\sin A + \cos B\right)^2 - 2$$

$$= \cos^2 A + \sin^2 B + 2\cos A \sin B + \sin^2 A + \cos^2 B + 2\sin A \cos B - 2$$

$$= \left(\sin^2 A + \cos^2 A\right) + \left(\sin^2 B + \cos^2 B\right) + 2\cos A \sin B + 2\sin A \cos B - 2$$

$$= 1 + 1 + 2\cos A \sin B + 2\sin A \cos B - 2$$

$$= 2 + 2\left(\sin A \cos B + \cos A \sin B\right) - 2$$

$$= 2\left(\sin A \cos B + \cos A \sin B\right)$$

$$= 2\sin\left(A + B\right) \qquad \left[\because \sin\left(A + B\right) = \sin A \cos B + \cos A \sin B\right]$$

$$\therefore \quad 2\sin\left(A + B\right) = m^2 + n^2 - 2$$

Hence proved

Q23

We have,

$$\tan A + \tan B = a \quad \text{and} \quad \cot A + \cot B = b$$

Now, $\cot A + \cot B = b$

$$\Rightarrow \quad \frac{1}{\tan A} + \frac{1}{\tan B} = b \qquad \qquad \left[\because \cot\theta = \frac{1}{\tan\theta}\right]$$

$$\Rightarrow \quad \frac{\tan B + \tan A}{\tan A \tan B} = b$$

$$\Rightarrow \quad \frac{a}{\tan A \tan B} = b \qquad \qquad \left[\because \tan A + \tan B = a\right]$$

$$\Rightarrow \quad \frac{a}{b} = \tan A \tan B$$

$$\therefore \quad \cot(A+B) = \frac{1}{\tan(A+B)}$$

$$= \frac{1}{\dfrac{\tan A + \tan B}{1 - \tan A \tan B}}$$

$$= \frac{1 - \tan A \tan B}{\tan A + \tan B}$$

$$= \frac{1 - \dfrac{a}{b}}{a} \qquad \qquad \left[\because \tan A \tan B = \frac{a}{b}\right]$$

$$= \frac{b - a}{ab}$$

$$= \frac{b}{ab} - \frac{a}{ab}$$

$$= \frac{1}{a} - \frac{1}{b}$$

$$\therefore \quad \cot(A+B) = \frac{1}{a} - \frac{1}{b}$$

Hence proved.

Q24

We have,

$$\cos\theta = \frac{8}{17}$$

$$\therefore \quad \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{64}{289}}$$

$$= \sqrt{\frac{225}{289}}$$

$$= \frac{15}{17}$$

Now, $\cos\left(\dfrac{\pi}{6}+\theta\right)+\cos\left(\dfrac{\pi}{4}-\theta\right)+\cos\left(\dfrac{2\pi}{3}-\theta\right)$

$=\left[\cos\dfrac{\pi}{6}\cos\theta-\sin\dfrac{\pi}{6}\sin\theta\right]+\left[\cos\dfrac{\pi}{4}\cos\theta+\sin\dfrac{\pi}{4}\sin\theta\right]$

$\qquad+\left[\cos\dfrac{2\pi}{3}\cos\theta+\sin\dfrac{2\pi}{3}\sin\theta\right]$

$=\left[\cos\dfrac{\pi}{6}+\cos\dfrac{\pi}{4}+\cos\dfrac{2\pi}{3}\right]\cos\theta+\sin\theta\left[-\sin\dfrac{\pi}{6}+\sin\dfrac{\pi}{4}+\sin\dfrac{2\pi}{3}\right]$

$=\left[\dfrac{\sqrt{3}}{2}+\dfrac{1}{\sqrt{2}}+\cos\left(\dfrac{\pi}{2}+\dfrac{\pi}{6}\right)\right]\times\dfrac{8}{17}+\dfrac{15}{17}\times\left[-\dfrac{1}{2}+\dfrac{1}{\sqrt{2}}+\sin\left(\dfrac{\pi}{2}+\dfrac{\pi}{6}\right)\right]$

$=\left[\dfrac{\sqrt{3}}{2}+\dfrac{1}{\sqrt{2}}-\sin\dfrac{\pi}{6}\right]\times\dfrac{8}{17}+\dfrac{15}{17}\times\left[-\dfrac{1}{2}+\dfrac{1}{\sqrt{2}}+\cos\dfrac{\pi}{6}\right]$

$=\left[\dfrac{\sqrt{3}}{2}+\dfrac{1}{\sqrt{2}}-\dfrac{1}{2}\right]\times\dfrac{8}{17}+\dfrac{15}{17}\times\left[-\dfrac{1}{2}+\dfrac{1}{\sqrt{2}}+\dfrac{\sqrt{3}}{2}\right]$

$\qquad\qquad\qquad\left[\because \cos A \text{ is negative in second quadrant}\right]$

$=\left[\dfrac{\sqrt{3}-1}{2}+\dfrac{1}{\sqrt{2}}\right]\times\dfrac{8}{17}+\dfrac{15}{17}\times\left[\dfrac{\sqrt{3}-1}{2}+\dfrac{1}{\sqrt{2}}\right]$

$=\left(\dfrac{\sqrt{3}-1}{2}+\dfrac{1}{\sqrt{2}}\right)\left(\dfrac{8}{17}+\dfrac{15}{17}\right)$

$=\left(\dfrac{\sqrt{3}-1}{2}+\dfrac{1}{\sqrt{2}}\right)\left(\dfrac{8+15}{17}\right)$

$=\left(\dfrac{\sqrt{3}-1}{2}+\dfrac{1}{\sqrt{2}}\right)\times\dfrac{23}{17}$

$\therefore\quad \cos\left(\dfrac{\pi}{6}+\theta\right)+\cos\left(\dfrac{\pi}{4}-\theta\right)+\cos\left(\dfrac{2\pi}{3}-\theta\right)=\left(\dfrac{\sqrt{3}-1}{2}+\dfrac{1}{\sqrt{2}}\right)\times\dfrac{23}{17}$

Hence proved.

Q25

We have,

$$\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$$

$$\Rightarrow \quad \tan x + \left[\frac{\tan x + \tan\frac{\pi}{3}}{1 - \tan x \tan\frac{\pi}{3}}\right] + \left[\frac{\tan x + \tan\left(\frac{2\pi}{3}\right)}{1 - \tan x \tan\frac{2\pi}{3}}\right] = 3$$

$$\Rightarrow \quad \tan x + \left[\frac{\tan x + \sqrt{3}}{1 - \sqrt{3}\tan x}\right] + \left[\frac{\tan x + \tan\left(\frac{\pi}{2} + \frac{\pi}{3}\right)}{1 - \tan x \tan\left(\frac{\pi}{2} + \frac{\pi}{3}\right)}\right] = 3$$

$$\Rightarrow \quad \tan x + \frac{\tan x + \sqrt{3}}{1 - \sqrt{3}\tan x} + \frac{\tan x - \cot\frac{\pi}{3}}{1 + \tan x \cot\frac{\pi}{3}} = 3 \qquad \begin{bmatrix} \because \tan\theta \text{ is negative in} \\ \text{second quadrant} \end{bmatrix}$$

$$\Rightarrow \quad \tan x + \frac{\tan x + \sqrt{3}}{1 - \sqrt{3}\tan x} + \frac{\tan x - \sqrt{3}}{1 + \sqrt{3}\tan x} = 3$$

$$\Rightarrow \quad \tan x + \frac{\left(\tan x + \sqrt{3}\right)\left(1 + \sqrt{3}\tan x\right) + \left(\tan x - \sqrt{3}\right)\left(1 - \sqrt{3}\tan x\right)}{\left(1 - \sqrt{3}\tan x\right)\left(1 + \sqrt{3}\tan x\right)} = 3$$

$$\Rightarrow \quad \tan x + \frac{\tan x + \sqrt{3}\tan^2 x + \sqrt{3} + 3\tan x + \tan x - \sqrt{3}\tan^2 x - \sqrt{3} + 3\tan x}{1 - \left(\sqrt{3}\tan x\right)^2} = 3$$

$$\Rightarrow \quad \tan x + \frac{8\tan x}{1 - 3\tan^2 x} = 3$$

$$\Rightarrow \quad \frac{\tan x\left(1 - 3\tan^2 x\right) + 8\tan x}{1 - 3\tan^2 x} = 3$$

$$\Rightarrow \quad \frac{\tan x - 3\tan^3 x + 8\tan x}{1 - 3\tan^2 x} = 3$$

$$\Rightarrow \quad \frac{9\tan x - 3\tan^3 x}{1 - 3\tan^2 x} = 3$$

$$\Rightarrow \quad \frac{3\left(3\tan x - \tan^3 x\right)}{1 - 3\tan^2 x} = 3$$

$$\Rightarrow \quad \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x} = 1$$

Hence proved.

Q26

We have,

$$\sin(\alpha + \beta) = 1$$

$$\Rightarrow \quad \sin(\alpha + \beta) = \sin\frac{\pi}{2}$$

$$\Rightarrow \quad \alpha + \beta = \frac{\pi}{2} \qquad\qquad ---(i)$$

and, $\quad \sin(\alpha - \beta) = \frac{1}{2}$

$$\Rightarrow \quad \sin(\alpha - \beta) = \sin\frac{\pi}{6}$$

$$\Rightarrow \quad \alpha - \beta = \frac{\pi}{6} \qquad\qquad ---(ii)$$

Adding equations (i) and (ii), we get

$$2\alpha = \frac{\pi}{2} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

$$\Rightarrow \quad \alpha = \frac{\pi}{3}$$

Putting $\alpha = \frac{\pi}{3}$ in equation (i), we get

$$\frac{\pi}{3} + \beta = \frac{\pi}{2}$$

$$\Rightarrow \quad \beta = \frac{\pi}{2} - \frac{\pi}{3}$$

$$\Rightarrow \quad \beta = \frac{3\pi - 2\pi}{6}$$

$$= \frac{\pi}{6}$$

$$\Rightarrow \quad \beta = \frac{\pi}{6}$$

Now, $\tan(\alpha + 2\beta) = \tan\left(\dfrac{\pi}{3} + 2 \times \dfrac{\pi}{6}\right)$

$= \tan\left(\dfrac{\pi}{3} + \dfrac{\pi}{3}\right)$

$= \tan\dfrac{2\pi}{3}$

$= \tan\left(\dfrac{\pi}{2} + \dfrac{\pi}{6}\right)$

$= -\cot\dfrac{\pi}{6}$ $\qquad \left[\because \begin{array}{l} \tan\theta \text{ is negative in} \\ \text{second quadrant} \end{array}\right]$

$= -\sqrt{3}$

∴ $t(\alpha + 2\beta) = -\sqrt{3}$

and, $\tan(2\alpha + \beta) = \tan\left(2 \times \dfrac{\pi}{3} + \dfrac{\pi}{6}\right)$

$= \tan\left(\dfrac{2\pi}{3} + \dfrac{\pi}{6}\right)$

$= \tan\left(\dfrac{4\pi + \pi}{6}\right)$

$= \tan\left(\dfrac{5\pi}{6}\right)$

$= \tan\left(\dfrac{\pi}{2} + \dfrac{\pi}{3}\right)$

$= -\cot\dfrac{\pi}{3}$ $\qquad \left[\because \begin{array}{l} \tan\theta \text{ is negative in} \\ \text{second quadrant} \end{array}\right]$

$= -\dfrac{1}{\sqrt{3}}$

∴ $\tan(2\alpha + \beta) = \dfrac{-1}{\sqrt{3}}$

Q27

We have,

$$6\cos\theta + 8\sin\theta = 9 \qquad\qquad ---(i)$$

$\Rightarrow\qquad 8\sin\theta = 9 - 6\cos\theta$

$\Rightarrow\qquad (8\sin\theta)^2 = (9 - 6\cos\theta)^2 \qquad\qquad [\because \text{Squaring both sides}]$

$\Rightarrow\qquad 64\sin^2\theta = 81 + 36\cos^2\theta - 108\cos\theta$

$\Rightarrow\qquad 64\sin^2\theta = 81 + 36\cos^2\theta - 108\cos\theta$

$\Rightarrow\qquad 64\left(1 - \cos^2\theta\right) = 81 + 36\cos^2\theta - 108\cos\theta$

$\Rightarrow\qquad 64 - 64\cos^2\theta = 81 + 36\cos^2\theta - 108\cos\theta$

$\Rightarrow\qquad 36\cos^2\theta + 64\cos^2\theta - 108\cos\theta + 81 - 64 = 0$

$\Rightarrow\qquad 100\cos^2\theta - 108\cos\theta + 17 = 0 \qquad\qquad ---(ii)$

Since α, β are roots of equation (ii).

Therefore, $\cos\alpha$ and $\cos\beta$ are roots of equation (ii)

$$\therefore\qquad \cos\alpha + \cos\beta = \frac{17}{100} \qquad\qquad ---(iii)$$

Again, $6\cos\theta + 8\sin\theta = 9$

$\Rightarrow\qquad 6\cos\theta = 9 - 8\sin\theta$

$\Rightarrow\qquad (6\cos\theta)^2 = (9 - 8\sin\theta)^2 \qquad\qquad [\because \text{Squaring both sides}]$

$\Rightarrow\qquad 36\cos^2\theta = 81 + 64\sin^2\theta - 144\sin\theta$

$\Rightarrow\qquad 36\left(1 - \sin^2\theta\right) = 81 + 64\sin^2\theta - 144\cos\theta$

$\Rightarrow\qquad 36 - 36\sin^2\theta = 81 + 64\sin^2\theta - 144\sin\theta$

$\Rightarrow\qquad 64\sin^2\theta + 36\sin^2\theta - 144\sin\theta + 81 - 36 = 0$

$\Rightarrow\qquad 100\sin^2\theta - 144\sin\theta + 45 = 0 \qquad\qquad ---(iv)$

$\therefore$ $\sin \alpha \times \sin \beta = \dfrac{45}{100}$ $---(v)$

Now, $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$

$= \dfrac{17}{100} - \dfrac{45}{100}$ [Using equation (iii) and (v)]

$= -\dfrac{28}{100}$

$= -\dfrac{7}{25}$

Now, $\sin(\alpha + \beta) = \sqrt{1 - (\cos\theta)^2}$

$= \sqrt{1 - \left(-\dfrac{7}{25}\right)^2}$

$= \sqrt{1 - \dfrac{49}{625}}$

$= \sqrt{\dfrac{625 - 49}{625}}$

$= \sqrt{\dfrac{576}{625}}$

$= \dfrac{24}{25}$

$\therefore$ $\sin(\alpha + \beta) = \dfrac{24}{25}$

Q28

$$\sin(\alpha + \beta) = \sqrt{1 - \cos^2(\alpha + \beta)}$$

$$b^2 + a^2 = (\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2$$

$$\Rightarrow \quad b^2 + a^2 = (\cos^2\alpha + \sin^2\alpha) + (\cos^2\beta + \sin^2\beta) + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$$

$$\Rightarrow \quad b^2 + a^2 = 1 + 1 + 2\cos(\alpha - \beta) = 2 + 2\cos(\alpha - \beta)$$

and, $\quad b^2 - a^2 = (\cos\alpha + \cos\beta)^2 - (\sin\alpha + \sin\beta)^2$

$$b^2 - a^2 = \cos^2\alpha + \cos^2\beta - \sin^2\alpha - \sin^2\beta + 2(\cos\alpha\cos\beta - \sin\alpha\sin\beta)$$

$$\Rightarrow \quad b^2 - a^2 = (\cos^2\alpha - \sin^2\beta) + (\cos^2\beta - \sin^2\alpha) + 2\cos(\alpha + \beta)$$

$$\Rightarrow \quad b^2 - a^2 = \cos(\alpha + \beta)\cos(\alpha - \beta) + \cos(\beta + \alpha)\cos(\beta - \alpha) + 2\cos(\alpha + \beta)$$

$$\Rightarrow \quad b^2 - a^2 = 2\cos(\alpha + \beta)\cos(\alpha - \beta) + 2\cos(\alpha + \beta)$$

$$\left[\because \cos(\beta - \alpha) = \cos\{-(\alpha - \beta)\} = \cos(\alpha - \beta)\right]$$

$$\Rightarrow \quad b^2 - a^2 = \cos(\alpha + \beta)\{2\cos(\alpha - \beta) + 2\}$$

$$\Rightarrow \quad b^2 - a^2 = \cos(\alpha + \beta)(b^2 + a^2) \qquad \qquad [\text{Using (i)}]$$

Thus, $\quad b^2 - a^2 = (b^2 + a^2)\cos(\alpha + \beta)$

$$\Rightarrow \quad \cos(\alpha + \beta) = \frac{b^2 - a^2}{b^2 + a^2}$$

$$\Rightarrow \quad \sin(\alpha + \beta) = \sqrt{1 - \left(\frac{b^2 - a^2}{b^2 - a^2}\right)^2} = \sqrt{\frac{4a^2b^2}{(a^2 + b^2)^2}} = \frac{2ab}{a^2 + b^2}$$

$$b^2 + a^2 = (\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2$$

$$\Rightarrow \quad b^2 + a^2 = (\cos^2\alpha + \sin^2\alpha) + (\cos^2\beta + \sin^2\beta) + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$$

$$\Rightarrow \quad b^2 + a^2 = 1 + 1 + 2\cos(\alpha - \beta) = 2 + 2\cos(\alpha - \beta)$$

and, $\quad b^2 - a^2 = (\cos\alpha + \cos\beta)^2 - (\sin\alpha + \sin\beta)^2$

$$b^2 - a^2 = \cos^2\alpha + \cos^2\beta - \sin^2\alpha - \sin^2\beta + 2(\cos\alpha\cos\beta - \sin\alpha\sin\beta)$$

$$\Rightarrow \quad b^2 - a^2 = (\cos^2\alpha - \sin^2\beta) + (\cos^2\beta - \sin^2\alpha) + 2\cos(\alpha + \beta)$$

$$\Rightarrow \quad b^2 - a^2 = \cos(\alpha + \beta)\cos(\alpha - \beta) + \cos(\beta + \alpha)\cos(\beta - \alpha) + 2\cos(\alpha + \beta)$$

$$\Rightarrow \quad b^2 - a^2 = 2\cos(\alpha + \beta)\cos(\alpha - \beta) + 2\cos(\alpha + \beta)$$

$$\left[\because \cos(\beta - \alpha) = \cos\{-(\alpha - \beta)\} = \cos(\alpha - \beta)\right]$$

$$\Rightarrow \quad b^2 - a^2 = \cos(\alpha + \beta)\{2\cos(\alpha - \beta) + 2\}$$

$$\Rightarrow \quad b^2 - a^2 = \cos(\alpha + \beta)(b^2 + a^2) \qquad\qquad\qquad [\text{Using (i)}]$$

Thus, $\quad b^2 - a^2 = (b^2 + a^2)\cos(\alpha + \beta)$

$$\Rightarrow \quad \cos(\alpha + \beta) = \frac{b^2 - a^2}{b^2 + a^2}$$

Q29

LHS $\dfrac{1}{\sin(x-a)\sin(x-b)}$

$= \dfrac{1}{\sin(a-b)} \left[\dfrac{\sin(a-b)}{\sin(x-a)\sin(x-b)} \right]$

$= \dfrac{1}{\sin(a-b)} \left[\dfrac{\sin\{(x-b)-(x-a)\}}{\sin(x-a)\sin(x-b)} \right]$

$= \dfrac{1}{\sin(a-b)} \left[\dfrac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)} \right]$

$= \dfrac{1}{\sin(a-b)} \left[\dfrac{\sin(x-b)\cos(x-a)}{\sin(x-a)\sin(x-b)} - \dfrac{\cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)} \right]$

$= \dfrac{1}{\sin(a-b)} \left[\cot(x-a)-\cot(x-b) \right]$

$= \dfrac{\cot(x-a)-\cot(x-b)}{\sin(a-b)}$

$=$ RHS

$\therefore$ LHS = RHS

Hence proved

LHS $\dfrac{1}{\sin(x-a)\cos(x-b)}$

$= \dfrac{1}{\cos(a-b)} \left[\dfrac{\cos(a-b)}{\sin(x-a)\cos(x-b)} \right]$

$= \dfrac{1}{\cos(a-b)} \left[\dfrac{\cos\{(x-b)-(x-a)\}}{\sin(x-a)\cos(x-b)} \right]$

$= \dfrac{1}{\cos(a-b)} \left[\dfrac{\cos(x-b)\cos(x-a) + \sin(x-b)\sin(x-a)}{\sin(x-a)\cos(x-b)} \right]$

$= \dfrac{1}{\cos(a-b)} \left[\dfrac{\cos(x-b)\cos(x-a)}{\sin(x-a)\cos(x-b)} + \dfrac{\sin(x-b)\sin(x-a)}{\sin(x-a)\cos(x-b)} \right]$

$= \dfrac{1}{\cos(a-b)} \left[\dfrac{\cos(x-a)}{\sin(x-a)} + \dfrac{\sin(x-b)}{\cos(x-b)} \right]$

$= \dfrac{1}{\cos(a-b)} \left[\cot(x-a) + \tan(x-b) \right]$

$= \dfrac{\cot(x-a) + \tan(x-b)}{\cos(a-b)}$

= RHS

∴ LHS=RHS

Hence proved

LHS $\dfrac{1}{\cos(x-a)\cos(x-b)}$

$= \dfrac{1}{\sin(a-b)}\left[\dfrac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\right]$

$= \dfrac{1}{\sin(a-b)}\left[\dfrac{\sin\{(x-b)-(x-a)\}}{\cos(x-a)\cos(x-b)}\right]$

$= \dfrac{1}{\sin(a-b)}\left[\dfrac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)}\right]$

$= \dfrac{1}{\sin(a-b)}\left[\dfrac{\sin(x-b)\cos(x-a)}{\cos(x-a)\cos(x-b)}-\dfrac{\cos(x-b)\;\sin(x-a)}{\cos(x-a)\cos(x-b)}\right]$

$= \dfrac{1}{\sin(a-b)}\left[\dfrac{\sin(x-b)}{\cos(x-b)}-\dfrac{\sin(x-a)}{\cos(x-a)}\right]$

$= \dfrac{1}{\sin(a-b)}\left[\tan(x-b)-\tan(x-s)\right]$

$= \dfrac{\tan(x-b)-\tan(x-a)}{\sin(a-b)}$

= RHS

∴   LHS=RHS

Hence proved

Q30

We have,

$$\sin\alpha \sin\beta - \cos\alpha \cos\beta + 1 = 0$$

$\Rightarrow \quad -(\cos\alpha \cos\beta - \sin\alpha \sin\beta) = -1$

$\Rightarrow \quad \cos(\alpha + \beta) = 1 \qquad\qquad\qquad ---(i)$

$\therefore \quad \sin(\alpha + \beta) = \sqrt{1 - \cos^2(\alpha + \beta)}$

$\qquad\qquad = \sqrt{1 - 1^2}$

$\qquad\qquad = 0$

$\Rightarrow \quad \sin(\alpha + \beta) = 0 \qquad\qquad\qquad ---(ii)$

Now,

$$1 + \cot\alpha \tan\beta = 1 + \frac{\cos\alpha}{\sin\alpha} \times \frac{\sin\beta}{\cos\beta}$$

$$= \frac{\sin\alpha \times \cos\beta + \cos\alpha \times \sin\beta}{\sin\alpha \times \cos\beta}$$

$$= \frac{\sin(\alpha + \beta)}{\sin\alpha \times \cos\beta}$$

$$= \frac{0}{\sin\alpha \times \cos\beta} \qquad\qquad \left[\text{Using equation (ii)}\right]$$

$$= 0$$

$\therefore \quad 1 + \cot\alpha \tan\beta = 0$

Hence proved

Q31

We have,

$$\tan\alpha = x + 1 \text{ and } \tan\beta = x - 1$$

Now, $2\cot(\alpha - \beta)$

$$= \frac{2}{\tan(\alpha - \beta)}$$

$$= \frac{2}{\frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}}$$

$$= \frac{2(1 + \tan\alpha\tan\beta)}{\tan\alpha - \tan\beta}$$

$$= \frac{2[1 + (x+1)(x-1)]}{x + 1 - (x - 1)}$$

$$= \frac{2[1 + x^2 - 1]}{x + 1 - x + 1}$$

$$= \frac{2 \times x^2}{2} = x^2$$

∴ $\quad 2\cot(\alpha - \beta) = x^2$

Hence proved

Q32

Let the two parts of the angle be θ and $\theta - \emptyset$.

$\tan(\theta - \emptyset) = \lambda \tan \emptyset$ [According to question]

$\Rightarrow \dfrac{\tan(\theta - \emptyset)}{\tan \emptyset} = \dfrac{\lambda}{1}$

$\Rightarrow \dfrac{\tan(\theta - \emptyset)}{\tan \emptyset} = \dfrac{\lambda}{1}$

$\Rightarrow \dfrac{\tan(\theta - \emptyset) + \tan \emptyset}{\tan(\theta - \emptyset) - \tan \emptyset} = \dfrac{\lambda + 1}{\lambda - 1}$ [Using Componendo and Dividendo]

$\Rightarrow \dfrac{\dfrac{\tan \theta - \tan \emptyset}{1 + \tan \theta . \tan \emptyset} + \tan \emptyset}{\dfrac{\tan \theta - \tan \emptyset}{1 + \tan \theta . \tan \emptyset} - \tan \emptyset} = \dfrac{\lambda + 1}{\lambda - 1}$

$\Rightarrow \dfrac{\dfrac{\tan \theta - \tan \emptyset + \tan \emptyset(1 + \tan \theta . \tan \emptyset)}{1 + \tan \theta . \tan \emptyset}}{\dfrac{\tan \theta - \tan \emptyset - \tan \emptyset(1 + \tan \theta . \tan \emptyset)}{1 + \tan \theta . \tan \emptyset}} = \dfrac{\lambda + 1}{\lambda - 1}$

$\Rightarrow \dfrac{\tan \theta - \tan \emptyset + \tan \emptyset + \tan \theta . \tan^2 \emptyset}{\tan \theta - \tan \emptyset - \tan \emptyset - \tan \theta . \tan^2 \emptyset} = \dfrac{\lambda + 1}{\lambda - 1}$

$\Rightarrow \dfrac{\tan \theta + \tan \theta . \tan^2 \emptyset}{\tan \theta - 2\tan \emptyset - \tan \theta . \tan^2 \emptyset} = \dfrac{\lambda + 1}{\lambda - 1}$

$\Rightarrow \dfrac{\tan}{\underline{}}$

Q33

$$\tan \theta = \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}$$

$$\Rightarrow \tan \theta = \frac{\tan \alpha - 1}{\tan \alpha + 1} [\textit{Dividing both Numerator and Denominator by } \cos \alpha]$$

$$\Rightarrow \tan \theta = \frac{\tan \alpha - \tan \frac{\pi}{4}}{1 + \tan \frac{\pi}{4}.\tan \alpha}$$

$$\Rightarrow \tan \theta = \tan \left(\alpha - \frac{\pi}{4}\right)$$

$$\Rightarrow \theta = \alpha - \frac{\pi}{4} \quad [\textit{Removing } \tan \textit{ from both sides}]$$

$$\Rightarrow \cos \theta = \cos \left(\alpha - \frac{\pi}{4}\right) [\textit{Taking } \cos \textit{ on both sides}]$$

$$\Rightarrow \cos \theta = \cos \alpha.\cos \frac{\pi}{4} + \sin \alpha.\sin \frac{\pi}{4}$$

$$\Rightarrow \cos \theta = \cos \alpha.\frac{1}{\sqrt{2}} + \sin \alpha.\frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos \theta = \frac{\cos \alpha + \sin \alpha}{\sqrt{2}}$$

$$\Rightarrow \sqrt{2} \cos \theta = \sin \alpha + \cos \alpha$$

Hence Proved

Q34

RHS,

$$\frac{p \quad q}{1-pq}$$

$$= \frac{\tan(A+B)+\tan(A-B)}{1-\tan(A+B).\tan(A-B)}$$

$$= \frac{\dfrac{\tan A+\tan B}{1-\tan A.\tan B}+\dfrac{\tan A-\tan B}{1+\tan A.\tan B}}{1-\dfrac{\tan A+\tan B}{1-\tan A.\tan B}.\dfrac{\tan A-\tan B}{1+\tan A.\tan B}}$$

$$= \frac{\dfrac{(\tan A+\tan B)(1-\tan A.\tan B)+(\tan A-\tan B)(1-\tan A.\tan B)}{(1-\tan A.\tan B)(1+\tan A.\tan B)}}{\dfrac{(1-\tan A.\tan B)(1+\tan A.\tan B)-(\tan A+\tan B).(\tan A-\tan B)}{(1-\tan A.\tan B)(1+\tan A.\tan B)}}$$

$$= \frac{\tan A+\tan B+\tan^2 A.\tan B-\tan A.\tan^2 B+\tan A-\tan B-\tan^2 A.\tan B+\tan A.\tan^2 B}{1-\tan^2 A.\tan^2 B-\tan^2 A+\tan^2 B}$$

$$= \frac{2\tan A+2\tan A.\tan^2 B}{(1-\tan^2 A)(1+\tan^2 B)} = \frac{2\tan A(1+\tan^2 B)}{(1-\tan^2 A)(1-\tan^2 B)} = \frac{2\tan A}{1-\tan^2 A} = \tan 2A = LHS$$

Hence Proved

Ex 7.2

Q1

Let $f(\theta) = 12\sin\theta - 5\cos\theta$

We know that

$$-\sqrt{(12)^2 + (-5)^2} \le f(\theta) \le \sqrt{(12)^2 + (-5)^2}$$

$$\Rightarrow \quad -\sqrt{144 + 25} \le f(\theta) \le \sqrt{144 + 25}$$

$$\Rightarrow \quad -\sqrt{169} \le f(\theta) \le \sqrt{169}$$

$$\Rightarrow \quad -13 \le f(\theta) \le 13$$

Hence, minimum and maximum values of $12\sin\theta - 5\cos\theta$ are -13 and 13 respectively.

Let $f(\theta) = 12\cos\theta + 5\sin\theta + 4$

We know that

$$-\sqrt{(12)^2 + (5)^2} \le 12\cos\theta + 5\sin\theta \le \sqrt{(12)^2 + (-5)^2}$$

$$\Rightarrow \quad -\sqrt{144 + 25} \le 12\cos\theta + 5\sin\theta \le \sqrt{144 + 25}$$

$$\Rightarrow \quad -\sqrt{169} \le 12\cos\theta + 5\sin\theta \le \sqrt{169}$$

$$\Rightarrow \quad -13 \le 12\cos\theta + 5\sin\theta \le 13$$

$$\Rightarrow \quad -13 + 4 \le 12\cos\theta + 5\sin\theta + 4 \le 13 + 4$$

$$\Rightarrow \quad -9 \le 12\cos\theta + 5\sin\theta + 4 \le 17$$

$$\Rightarrow \quad -9 \le f(\theta) \le 17$$

Hnece, minimum and maximum values of $12\cos\theta + 5\sin\theta + 4$ are -9 and 17 respectively.

Let $f(\theta) = 5\cos\theta + 3\sin\left(\dfrac{\pi}{6} - \theta\right) + 4$

Then, $f(\theta) = 5\cos\theta + 3\left[\sin\dfrac{\pi}{6}\cos\theta - \cos\dfrac{\pi}{6}\sin\theta\right] + 4$

$\qquad = 5\cos\theta + 3\left[\dfrac{1}{2}\cos\theta - \dfrac{\sqrt{3}}{2}\sin\theta\right] + 4$

$\qquad = 5\cos\theta + \dfrac{3}{2}\cos\theta - \dfrac{3\sqrt{3}}{2}\sin\theta + 4$

$\qquad = \left(5 + \dfrac{3}{2}\right)\cos\theta - \dfrac{3\sqrt{3}}{2}\sin\theta + 4$

$\qquad = \dfrac{13}{2}\cos\theta - \dfrac{3\sqrt{3}}{2}\sin\theta + 4$

$\qquad = \dfrac{13}{2}\cos\theta - \left(\dfrac{-3\sqrt{3}}{2}\right)\sin\theta + 4$

We know that

$$-\sqrt{\left(\dfrac{13}{2}\right)^2 + \left(\dfrac{-3\sqrt{3}}{2}\right)^2} \leq \dfrac{13}{2}\cos\theta - \left(\dfrac{-3\sqrt{3}}{2}\right)\sin\theta \leq \sqrt{\left(\dfrac{13}{2}\right)^2 + \left(\dfrac{-3\sqrt{3}}{2}\right)^2}$$

$\Rightarrow \qquad -\sqrt{\dfrac{169}{4} + \dfrac{27}{4}} \leq \dfrac{13}{2}\cos\theta - \left(\dfrac{-3\sqrt{3}}{2}\right)\sin\theta \leq \sqrt{\dfrac{169}{4} + \dfrac{27}{4}}$

$\Rightarrow \qquad -\sqrt{\dfrac{196}{4}} \leq \dfrac{13}{2}\cos\theta - \left(\dfrac{-3\sqrt{3}}{2}\right)\sin\theta \leq \sqrt{\dfrac{196}{4}}$

$\Rightarrow \qquad -\dfrac{14}{2} \leq \dfrac{13}{2}\cos\theta - \dfrac{3\sqrt{3}}{2}\sin\theta \leq \dfrac{14}{2}$

$\Rightarrow \qquad -7 \leq \dfrac{13}{2}\cos\theta - \dfrac{3\sqrt{3}}{2}\sin\theta \leq 7$

$\Rightarrow \qquad -7 + 4 \leq \dfrac{13}{2}\cos\theta - \dfrac{3\sqrt{3}}{2}\sin\theta + 4 \leq 7 + 4$

$\Rightarrow \qquad -3 \leq \dfrac{13}{2}\cos\theta - \left(\dfrac{-3\sqrt{3}}{2}\right)\sin\theta + 4 \leq 11$

$\Rightarrow \qquad -3 \leq f(\theta) \leq 11$

Let $f(\theta) = \sin\theta - \cos\theta + 1$. Then,

$$f(\theta) = \sin\theta + (-1)\cos\theta + 1$$
$$= (-1)\cos\theta + \sin\theta + 1$$

We know that

$$-\sqrt{(-1)^2 + (1)^2} \leq -\cos\theta + \sin\theta \leq \sqrt{(-1)^2 + (1)^2}$$
$$\Rightarrow \quad -\sqrt{1+1} \leq -\cos\theta + \sin\theta \leq \sqrt{1+1}$$
$$\Rightarrow \quad -\sqrt{2} \leq -\cos\theta + \sin\theta \leq \sqrt{2}$$
$$\Rightarrow \quad -\sqrt{2} + 1 \leq -\cos\theta + \sin\theta + 1 \leq \sqrt{2} + 1$$
$$\Rightarrow \quad 1 - \sqrt{2} \leq f(\theta) \leq 1 + \sqrt{2}$$

Hence, minimum and maximum values of $\sin\theta - \cos\theta + 1$ are $1 - \sqrt{2}$ and $1 + \sqrt{2}$ respectively.

Q2

Let $f(\theta) = \sqrt{3}\sin\theta - \cos\theta$

Multiplying and dividing by $\sqrt{\left(\sqrt{3}\right)^2 + (-1)^2}$, we get

$$f(\theta) = \sqrt{\left(\sqrt{3}\right)^2 + (-1)^2}\left[\frac{\sqrt{3}\sin\theta}{\sqrt{\left(\sqrt{3}\right)^2 + (-1)^2}} - \frac{\cos\theta}{\sqrt{\left(\sqrt{3}\right)^2 + (-1)^2}}\right]$$

$$= \sqrt{3+1}\left[\frac{\sqrt{3}\sin\theta}{\sqrt{3+1}} - \frac{\cos\theta}{\sqrt{3+1}}\right]$$

$\Rightarrow \quad f(\theta) = 2\left[\frac{\sqrt{3}\sin\theta}{2} - \frac{\cos\theta}{2}\right] \qquad \qquad ---(i)$

$\Rightarrow \quad f(\theta) = 2\left[\frac{\sqrt{3}}{2}\times\sin\theta - \frac{1}{2}\times\cos\theta\right]$

$$= 2\left[\cos\frac{\pi}{6}\times\sin\theta - \sin\frac{\pi}{6}\times\cos\theta\right]$$

$$= 2\left[\sin\theta\times\cos\frac{\pi}{6} - \cos\theta\times\sin\frac{\pi}{6}\right]$$

$$= 2\sin\left(\theta - \frac{\pi}{6}\right) \qquad \qquad \left[\because \sin(A-B) = \sin A\cos B - \cos A\sin B\right]$$

$\Rightarrow \quad f(\theta) = 2\sin\left(\theta - \frac{\pi}{6}\right)$

Again,

$$f(\theta) = 2\left[\frac{\sqrt{3}}{2}\sin\theta - \frac{\cos\theta}{2}\right]$$

$$= -2\left[\frac{1}{2}\times\cos\theta - \frac{\sqrt{3}}{2}\times\sin\theta\right]$$

$$= -2\left[\cos\frac{\pi}{3}\times\cos\theta - \sin\frac{\pi}{3}\times\sin\theta\right]$$

$$= -2\cos\left(\frac{\pi}{3} + \theta\right)$$

Let $f(\theta) = \cos\theta - \sin\theta$

Multiplying and dividing by $\sqrt{1^2 + 1^2}$, we get

$$f(\theta) = \sqrt{1^2 + 1^2}\left[\frac{\cos\theta}{\sqrt{1^2 + 1^2}} - \frac{\sin\theta}{\sqrt{1^2 + 1^2}}\right]$$

$$\Rightarrow \quad f(\theta) = \sqrt{2}\left[\frac{\cos\theta}{\sqrt{2}} - \frac{\sin\theta}{\sqrt{2}}\right] \qquad \qquad ---(i)$$

Now, $f(\theta) = \sqrt{2}\left[\dfrac{1}{\sqrt{2}} \times \cos\theta - \dfrac{1}{\sqrt{2}} \times \sin\theta\right]$

$\qquad\qquad = \sqrt{2}\left[\sin\dfrac{\pi}{4} \times \cos\theta - \cos\dfrac{\pi}{4} \times \sin\theta\right]$

$\qquad\qquad = \sqrt{2}\sin\left(\dfrac{\pi}{4} - \theta\right) \qquad\qquad \left[\because \sin(A - B) = \sin A \cos B - \cos A \sin B\right]$

$\Rightarrow \quad f(\theta) = \sqrt{2}\sin\left(\dfrac{\pi}{4} - \theta\right)$

Again,

$f(\theta) = \sqrt{2}\left[\dfrac{1}{\sqrt{2}} \times \cos\theta - \dfrac{1}{\sqrt{2}} \times \sin\theta\right]$

$\qquad\qquad = \sqrt{2}\left[\cos\dfrac{\pi}{4} \times \cos\theta - \sin\dfrac{\pi}{4} \times \sin\theta\right]$

$\qquad\qquad = \sqrt{2}\cos\left(\dfrac{\pi}{4} + \theta\right) \qquad\qquad \left[\because \cos(A + B) = \cos A \cos B - \sin A \sin B\right]$

$\Rightarrow \quad f(\theta) = \sqrt{2}\cos\left(\dfrac{\pi}{4} + \theta\right)$

Let $f(\theta) = 24\cos\theta + 7\sin\theta$

Multiplying and dividing by $\sqrt{(24)^2 + (7)^2}$, we get

$$f(\theta) = \sqrt{(24)^2 + 7^2}\left[\frac{24\cos\theta}{\sqrt{24^2 + 7^2}} + \frac{7\sin\theta}{\sqrt{24^2 + 7^2}}\right]$$

$$= \sqrt{576 + 49}\left[\frac{24\cos\theta}{\sqrt{576 + 49}} + \frac{7\sin\theta}{\sqrt{576 + 49}}\right]$$

$$= \sqrt{625}\left[\frac{24\cos\theta}{\sqrt{625}} + \frac{7\sin\theta}{\sqrt{625}}\right]$$

$$= 25\left[\frac{24}{25} \times \cos\theta + \frac{7}{25} \times \sin\theta\right]$$

$\Rightarrow \qquad f(\theta) = 25\left[\frac{24}{25} \times \cos\theta + \frac{7}{25} \times \sin\theta\right]$ \qquad\qquad --- (i)

Now, $\quad f(\theta) = 25\left[\frac{24}{25} \times \cos\theta + \frac{7}{25} \times \sin\theta\right]$

$$= 25\left[\sin\alpha \times \cos\theta + \cos\alpha \times \sin\theta\right]$$

$$\text{where } \sin\alpha = \frac{24}{25} \text{ and } \cos\alpha = \frac{7}{25}$$

$\Rightarrow \qquad f(\theta) = 25\sin(\alpha + \theta)$, where $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{24}{7}$

Again,

$$f(\theta) = 25\left[\frac{24}{25} \times \cos\theta + \frac{7}{25} \times \sin\theta\right]$$

$$= 25\left[\cos\alpha \times \cos\theta + \sin\alpha \times \sin\theta\right], \text{ where } \cos\alpha = \frac{24}{25} \text{ and } \sin\alpha = \frac{7}{25}$$

$\Rightarrow \qquad f(\theta) = 25\cos(\alpha - \theta)$, where $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{7}{24}$

Q3

We have,

$$\sin 100° - \sin 10°$$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}} \times \sin 100° - \frac{1}{\sqrt{2}} \times \cos 100°\right) \qquad \begin{bmatrix} \text{Multiplying and dividing} \\ \text{by } \sqrt{1^2+1^2} \text{ ie., by } \sqrt{2} \end{bmatrix}$$

$$= \sqrt{2}\left(\cos 45° \times \sin 100° - \sin 45° \times \cos 100°\right)$$

$$= \sqrt{2}\left(\sin 100° \times \cos 45° - \cos 100° \times \sin 45°\right)$$

$$= \sqrt{2}\left(\sin(100° - 45°)\right)$$

$$= \sqrt{2} \sin 55°, \text{ which is positive real number.}$$

$$\begin{bmatrix} \because \sin\theta \text{ is positive in first quadrant} \end{bmatrix}$$

Q4

$(2\sqrt{3}+3)\sin\theta + 2\sqrt{3}\cos\theta$

assume $a = 2\sqrt{3}+3, b = 2\sqrt{3}$

$\sqrt{a^2+b^2} = \sqrt{12+9+12\sqrt{3}+12} = \sqrt{33+12\sqrt{3}}$

Dividing and multiplying the above equation with above value

we get, $\sqrt{33+12\sqrt{3}}\left(\dfrac{2\sqrt{3}+3}{\sqrt{33+12\sqrt{3}}}\sin\theta + \dfrac{2\sqrt{3}}{\sqrt{33+12\sqrt{3}}}\cos\theta\right)$

Assume $\tan\phi = \dfrac{a}{b}$, we have $\sin\phi = \dfrac{a}{\sqrt{a^2+b^2}}, \cos\phi = \dfrac{b}{\sqrt{a^2+b^2}}$

so above expressions changes to $\sqrt{33+12\sqrt{3}}(\sin\phi\sin\theta + \cos\phi\cos\theta)$

which is equal to $\sqrt{33+12\sqrt{3}}\cos(\theta-\phi)$

We know that maximum and minimum value of any cosine term is +1 and -1

$\sqrt{33+12\sqrt{3}} = \sqrt{15+12+6+12\sqrt{3}}$

we know that $12\sqrt{3}+6 < 12\sqrt{5}$ becasue value of $\sqrt{5}-\sqrt{3}$ is more than 0.5

so if we replace $12\sqrt{3}+6$ with $12\sqrt{5}$ the above inequality still holds

So range of above expression can be $\sqrt{15+12+12\sqrt{5}} = 2\sqrt{3}+\sqrt{15}$

$-(2\sqrt{3}+\sqrt{15}) < \sqrt{33+12\sqrt{3}}\cos(\theta-\phi) < 2\sqrt{3}+\sqrt{15}$