Ex 25.1

Vector or Cross Product Ex 25.1 Q1

If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and
$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$, then

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ -1 & 0 & 3 \end{vmatrix}$$

$$= \hat{i}(9-0) - \hat{j}(3-2) + \hat{k}(0+3)$$

$$\vec{a}\times\vec{b} = 9\hat{i} - \hat{j} + 3\hat{k}$$

$$\left|\vec{a}\times\vec{b}\right| = \sqrt{(9)^2 + (-1)^2 + (3)^2}$$
$$= \sqrt{81+1+9}$$

$$\left|\vec{a}\times\vec{b}\right| = \sqrt{91}$$

Vector or Cross Product Ex 25.1 Q2(i)

If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and
$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$, then

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(4-0) - \hat{j}(3-0) + \hat{k}(3-4)$$
$$= 4\hat{i} - 3\hat{j} - \hat{k}$$

$$\left|\vec{a}\times\vec{b}\right| = \sqrt{(4)^2 + (-3)^2 + (-1)^2}$$
$$= \sqrt{16+9+1}$$

$$\left|\vec{a}\times\vec{b}\right| = \sqrt{26}$$

Vector or Cross Product Ex 25.1 Q2(ii)

If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and
$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$, then

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(0-1) - \hat{j}(2-1) + \hat{k}(2-0)$$
$$= -\hat{i} - \hat{j} + 2\hat{k}$$

$$\left|\vec{a}\times\vec{b}\right| = \sqrt{(-1)^2 + (-1)^2 + (2)^2}$$
$$= \sqrt{1+1+4}$$

$$\left|\vec{a}\times\vec{b}\right| = \sqrt{6}$$

Magnitude of $\vec{a}\times\vec{b} = \sqrt{6}$.

Vector or Cross Product Ex 25.1 Q3(i)

A vector perpendicular to both $\vec{a}$ and $\vec{b} = \vec{a} \times \vec{b}$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$\vec{c}\,(\text{say}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ -2 & 1 & -2 \end{vmatrix}$$

$$\vec{c} = \hat{i}\,(2-3) - \hat{j}\,(-8+6) + \hat{k}\,(4-2)$$
$$\vec{c} = -\hat{i} + 2\hat{j} + 2\hat{k}$$

$\vec{c}$ is a vector perpendicular to both $\vec{a}$ and $\vec{b}$.

$$\hat{c} = \frac{\vec{c}}{|\vec{c}|}$$

$$= \frac{-\hat{i} + 2\hat{j} + 2\hat{k}}{\sqrt{(-1)^2 + (2)^2 + (2)^2}}$$
$$= \frac{-\hat{i} + 2\hat{j} + 2\hat{k}}{\sqrt{1+4+4}}$$

$$= \frac{1}{3}\left(-\hat{i} + 2\hat{j} + 2\hat{k}\right)$$

So, unit vector perpendicular to both $\vec{a}$ and $\vec{b} = \frac{1}{3}\left(-\hat{i} + 2\hat{j} + 2\hat{k}\right)$.

Vector or Cross Product Ex 25.1 Q3(ii)

A vector perpendicular to the plane containing the vector $\vec{a}$ and $\vec{b}$ is given by
$\vec{a} \times \vec{b} = \pm\vec{c}$ (Say)

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$\vec{c} = \hat{i}\,(1-2) - \hat{j}\,(2-1) + \hat{k}\,(4-1)$$
$$\vec{c} = -\hat{i} - \hat{j} + 3\hat{k}$$

$$\hat{c} = \frac{\vec{c}}{|\vec{c}|}$$

$$= \frac{-\hat{i} - \hat{j} + 3\hat{k}}{\sqrt{(-1)^2 + (-1)^2 + (3)^2}}$$
$$= \frac{-\hat{i} - \hat{j} + 3\hat{k}}{\sqrt{1+1+9}}$$

$$= \frac{1}{\sqrt{11}}\left(-\hat{i} - \hat{j} + 3\hat{k}\right)$$

Unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b} = \pm\frac{1}{\sqrt{11}}\left(-\hat{i} - \hat{j} + 3\hat{k}\right)$.

Vector or Cross Product Ex 25.1 Q4

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4 & 3 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= \hat{i}\,(-4-3) - \hat{j}\,(0-3) + \hat{k}\,(0-4)$$
$$= -7\hat{i} + 3\hat{j} - 4\hat{k}$$

$$\left|\vec{a} \times \vec{b}\right| = \sqrt{(-7)^2 + (3)^2 + (-4)^2}$$
$$= \sqrt{49 + 9 + 16}$$

$$\left|\vec{a} \times \vec{b}\right| = \sqrt{74}$$

Vector or Cross Product Ex 25.1 Q5

$$\vec{b} = \hat{i} - 2\hat{k}$$
$$\hat{b} = \frac{\vec{b}}{\left|\vec{b}\right|}$$

$$= \frac{\hat{i} - 2\hat{k}}{\sqrt{(1)^2 + (-2)^2}}$$
$$= \frac{\hat{i} - 2\hat{k}}{\sqrt{1+4}}$$

$$= \frac{\hat{i} - 2\hat{k}}{\sqrt{5}}$$

$$2\hat{b} = \frac{2}{\sqrt{5}}\hat{i} - \frac{4}{\sqrt{5}}\hat{k}$$

And, $\vec{a} = 4\hat{i} + 3\hat{j} + \hat{k}$

If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and
$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$2\hat{b} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{2}{\sqrt{5}} & 0 & -\frac{4}{\sqrt{5}} \\ 4 & 3 & 1 \end{vmatrix}$$

$$= \hat{i}\left(0 + \frac{12}{\sqrt{5}}\right) - \hat{j}\left(\frac{2}{\sqrt{5}} + \frac{16}{\sqrt{5}}\right) + \hat{k}\left(\frac{6}{\sqrt{5}} - 0\right)$$

$$2\hat{b} \times \vec{a} = \frac{12}{\sqrt{5}}\hat{i} - \frac{18}{\sqrt{5}}\hat{j} + \frac{6}{\sqrt{5}}\hat{k}$$

$$\left|2\hat{b} \times \vec{a}\right| = \sqrt{\left(\frac{12}{\sqrt{5}}\right)^2 + \left(-\frac{18}{\sqrt{5}}\right)^2 + \left(\frac{6}{\sqrt{5}}\right)^2}$$
$$= \sqrt{\frac{144}{5} + \frac{324}{5} + \frac{36}{5}}$$

$$\left|2\hat{b} \times \vec{a}\right| = \sqrt{\frac{504}{5}}$$

Vector or Cross Product Ex 25.1 Q6

$$\vec{a}+2\vec{b} = \left(3\hat{i}-\hat{j}-2\hat{k}\right)+2\left(2\hat{i}+3\hat{j}+\hat{k}\right)$$
$$= 3\hat{i}-\hat{j}-2\hat{k}+4\hat{i}+6\hat{j}+2\hat{k}$$

$$\vec{a}+2\vec{b} = 7\hat{i}+5\hat{j}$$

$$2\vec{a}-\vec{b} = 2\left(3\hat{i}-\hat{j}-2\hat{k}\right)-\left(2\hat{i}+3\hat{j}+\hat{k}\right)$$
$$= 6\hat{i}-2\hat{j}-4\hat{k}-2\hat{i}-3\hat{j}-\hat{k}$$

$$= 4\hat{i}-5\hat{j}-5\hat{k}$$

We know that if $\vec{a} = a_1\hat{i}+b_1\hat{j}+c_1\hat{k}$ and $\vec{b} = a_2\hat{i}+b_2\hat{j}+c_2\hat{k}$ then,

$$\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}$$

Therefore,

$$\left(\vec{a}+2\vec{b}\right)\times\left(2\vec{a}-\vec{b}\right) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 7 & 5 & 0\\ 4 & -5 & -5\end{vmatrix}$$

$$= \hat{i}\left(-25-0\right)-\hat{j}\left(-35-0\right)+\hat{k}\left(-35-20\right)$$
$$= -25\hat{i}+35\hat{j}-55\hat{k}$$

$$\left(\vec{a}+2\vec{b}\right)\times\left(2\vec{a}-\vec{b}\right) = -25\hat{i}+35\hat{j}-55\hat{k}$$

Vector or Cross Product Ex 25.1 Q7(i)

Let, $\vec{a} = 2\hat{i}+3\hat{j}+6\hat{k}$, $\vec{b} = 3\hat{i}-6\hat{j}+2\hat{k}$

If $\vec{a} = a_1\hat{i}+b_1\hat{j}+c_1\hat{k}$ and
$\vec{b} = a_2\hat{i}+b_2\hat{j}+c_2\hat{k}$, then,

$$\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}$$

$$= \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 6\\ 3 & -6 & 2\end{vmatrix}$$

$$= \hat{i}\left(6+36\right)-\hat{j}\left(4-18\right)+\hat{k}\left(-12-9\right)$$
$$= 42\hat{i}+14\hat{j}-21\hat{k}$$

$$= 7\left(6\hat{i}+2\hat{j}-3\hat{k}\right)$$

$$\left|\vec{a}\times\vec{b}\right| = 7\sqrt{(6)^2+(2)^2+(-3)^2}$$
$$= 7\sqrt{36+4+9}$$

$$\left|\vec{a}\times\vec{b}\right| = 7\sqrt{49}$$
$$\left|\vec{a}\times\vec{b}\right| = 7\times 7$$

$$\left|\vec{a}\times\vec{b}\right| = 49$$

Vector or Cross Product Ex 25.1 Q7(ii)

Vector perpendicular to $\vec{a}$ and $\vec{b}$

with magnitude 1 = $\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

$$= \frac{1}{49}\left(7\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)\right)$$

$$= \frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$$

vector of magnitude 49, which is perpendicular to $\vec{a}$ and $\vec{b}$

$$= 49\left(\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}\right)$$

$$= 49\left[\frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)\right]$$

$$= 42\hat{i} + 14\hat{j} - 21\hat{k}$$

The required vector = $42\hat{i} + 14\hat{j} - 21\hat{k}$

Vector or Cross Product Ex 25.1 Q7(iii)

If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and

$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$, then,

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -4 \\ 6 & 5 & -2 \end{vmatrix}$$

$$= \hat{i}\left(-2 + 20\right) - \hat{j}\left(-6 + 24\right) + \hat{k}\left(15 - 6\right)$$

$$= 18\hat{i} - 18\hat{j} + 9\hat{k}$$

$$= 9\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$$

$$|\vec{a}\times\vec{b}| = 9\sqrt{2^2 + (-2)^2 + (1)^2}$$

$$= 9\sqrt{4 + 4 + 1}$$

$$|\vec{a}\times\vec{b}| = 9\sqrt{9}$$

$$|\vec{a}\times\vec{b}| = 9\times 3$$

$$|\vec{a}\times\vec{b}| = 27$$

Vector or Cross Product Ex 25.1 Q7(iv)

Unit vector perpendicular to the vector

$\vec{a}$ and $\vec{b} = \dfrac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|}$

$$= \frac{1}{27}\left(9\left(2\hat{i}-2\hat{j}+\hat{k}\right)\right)$$

$$= \frac{1}{3}\left(2\hat{i}-2\hat{j}+\hat{k}\right)$$

vector with length 3 and which is perpendicular to both $\vec{a}$ and $\vec{b}$

$$= 3\left(\frac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|}\right)$$

$$= 3\left[\frac{1}{3}\left(2\hat{i}-2\hat{j}+\hat{k}\right)\right]$$

$$= 2\hat{i}-2\hat{j}+\hat{k}$$

Required vector $= 2\hat{i}-2\hat{j}+\hat{k}$

Vector or Cross Product Ex 25.1 Q8(i)

Here, $\vec{a} = 2\hat{i}+0.\hat{j}+0.\hat{k}$

$\vec{b} = 0.\hat{i}+3\hat{j}+0.\hat{k}$,

$$\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}$$

$$= \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 0 & 0\\ 0 & 3 & 0\end{vmatrix}$$

$$= \hat{i}(0-0)-\hat{j}(0-0)+\hat{k}(6-0)$$
$$= 6\hat{k}$$

Area of parallelogram $= \left|\vec{a}\times\vec{b}\right|$

$$= \left|0\hat{i}+0.\hat{j}+6\hat{k}\right|$$

$$= \sqrt{(0)^2+(0)^2+(6)^2}$$

Area of parallelogram = 6 sq.unit

Vector or Cross Product Ex 25.1 Q8(ii)

Let, $\vec{a} = 2\hat{i}+\hat{j}+3\hat{k}$

$\vec{b} = \hat{i}-\hat{j}$

$$\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}$$

$$= \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & 3\\ 1 & -1 & 0\end{vmatrix}$$

$$= \hat{i}(0+3)-\hat{j}(0-3)+\hat{k}(-2-1)$$
$$= 3\hat{i}+3\hat{j}-3\hat{k}$$
$$= 3\left(\hat{i}+\hat{j}-\hat{k}\right)$$

Area of parallelogram $= \left|\vec{a}\times\vec{b}\right|$

$$= 3\sqrt{(1)^2+(1)^2+(-1)^2}$$

$$= 3\sqrt{3}$$

Area of parallelogram $= 3\sqrt{3}$ sq.unit

Vector or Cross Product Ex 25.1 Q8(iii)

Let, $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$

$\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$$

$$= \hat{i}\,(4 - 6) - \hat{j}\,(12 + 2) + \hat{k}\,(-9 - 1)$$
$$= -2\,\hat{i} - 14\hat{j} - 10\hat{k}$$
$$= -2\left(\hat{i} + 7\hat{j} + 5\hat{k}\right)$$

Area of parallelogram $= \left|\vec{a} \times \vec{b}\right|$

$$= 2\sqrt{(1)^2 + (7)^2 + (5)^2}$$

$$= 2\sqrt{1 + 49 + 25}$$
$$= 2\sqrt{75}$$
$$= 10\sqrt{3}$$

Area of parallelogram $= 10\sqrt{3}$ sq.unit

Vector or Cross Product Ex 25.1 Q8(iv)

Let, $\vec{a} = \hat{i} - 3\hat{j} + \hat{k}$

$\vec{b} = \hat{i} + \hat{j} + \hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}\,(-3 - 1) - \hat{j}\,(1 - 1) + \hat{k}\,(1 + 3)$$
$$= -4\hat{i} - 0.\hat{j} + 4\hat{k}$$

Area of parallelogram $= \left|\vec{a} \times \vec{b}\right|$

$$= \sqrt{(-4)^2 + (0)^2 + (4)^2}$$

$$= \sqrt{16 + 0 + 16}$$
$$= \sqrt{32}$$

$$= 4\sqrt{2}$$

Area of parallelogram $= 4\sqrt{2}$ sq.unit

Vector or Cross Product Ex 25.1 Q9(i)

Area of parallelogram = $\frac{1}{2}\left|\vec{d_1} \times \vec{d_2}\right|$

Here, $d_1 = 4\hat{i} - \hat{j} - 3\hat{k}$

$d_2 = -2\hat{i} + \hat{j} - 2\hat{k}$

$$\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & -3 \\ -2 & 1 & -2 \end{vmatrix}$$

$$= \hat{i}(2+3) - \hat{j}(-8-6) + \hat{k}(4-2)$$
$$= 5\hat{i} + 14\hat{j} + 2\hat{k}$$

$$\left|\vec{d_1} \times \vec{d_2}\right| = \sqrt{(5)^2 + (14)^2 + (2)^2}$$

$$= \sqrt{25 + 196 + 4}$$
$$= \sqrt{225}$$

$$= 15$$

Area of parallelogram = $\frac{1}{2}\left|\vec{d_1} \times \vec{d_2}\right|$

Area of parallelogram = $\frac{15}{2}$ sq.unit

Vector or Cross Product Ex 25.1 Q9(ii)

Given, $d_1 = 2\hat{i} + \hat{k}$

$d_2 = \hat{i} + \hat{j} + \hat{k}$

$$\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(0-1) - \hat{j}(2-1) + \hat{k}(2-0)$$
$$= -\hat{i} - \hat{j} + 2\hat{k}$$

$$\left|\vec{d_1} \times \vec{d_2}\right| = \sqrt{(-1)^2 + (-1)^2 + (2)^2}$$

$$= \sqrt{1+1+4}$$

$$= \sqrt{6}$$

Area of parallelogram = $\frac{1}{2}\left|\vec{d_1} \times \vec{d_2}\right|$

Area of parallelogram = $\frac{1}{2}\sqrt{6}$ sq.unit

Vector or Cross Product Ex 25.1 Q9(iii)

Given, $d_1 = 3\hat{i} + 4\hat{j}$

$d_2 = \hat{i} + \hat{j} + \hat{k}$

$$\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(4-0) - \hat{j}(3-0) + \hat{k}(3-4)$$

$$= 4\hat{i} - 3\hat{j} - \hat{k}$$

$$\left|\vec{d_1} \times \vec{d_2}\right| = \sqrt{(4)^2 + (-3)^2 + (-1)^2}$$

$$= \sqrt{16+9+1}$$

$$= \sqrt{26}$$

Area of parallelogram $= \frac{1}{2}\left|\vec{d_1} \times \vec{d_2}\right|$

Area of parallelogram $= \frac{\sqrt{26}}{2}$ sq.unit

Vector or Cross Product Ex 25.1 Q9(iv)

Here, $d_1 = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$d_2 = 3\hat{i} - 6\hat{j} + 2\hat{k}$

$$\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 3 & -6 & 2 \end{vmatrix}$$

$$= \hat{i}(6+36) - \hat{j}(4-18) + \hat{k}(-12-9)$$

$$= 42\hat{i} + 14\hat{j} - 21\hat{k}$$

$$= 7\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$$

$$\left|\vec{d_1} \times \vec{d_2}\right| = 7\sqrt{(6)^2 + (2)^2 + (-3)^2}$$

$$= 7\sqrt{36+4+9}$$

$$= 7\sqrt{49}$$

$$= 7 \times 7$$

$$= 49$$

Area of parallelogram $= \frac{1}{2}\left|\vec{d_1} \times \vec{d_2}\right|$

Area of parallelogram $= \frac{49}{2}$ sq.unit

Vector or Cross Product Ex 25.1 Q10

Given, $\vec{a} = 2\hat{i} + 5\hat{j} - 7\hat{k}$,
$\vec{b} = -3\hat{i} + 4\hat{j} + \hat{k}$,
$\vec{c} = \hat{i} - 2\hat{j} - 3\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 5 & -7 \\ -3 & 4 & 1 \end{vmatrix}$$

$$= \hat{i}(5+28) - \hat{j}(2-21) + \hat{k}(8+15)$$
$$= 33\hat{i} + 19\hat{j} + 23\hat{k}$$

$$\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 33 & 19 & 23 \\ 1 & -2 & -3 \end{vmatrix}$$

$$= \hat{i}(-57+46) - \hat{j}(-99-23) + \hat{k}(-66-19)$$
$$\left(\vec{a} \times \vec{b}\right) \times \vec{c} = -11\hat{i} + 122\hat{j} - 85\hat{k} \qquad ---(i)$$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 4 & 1 \\ 1 & -2 & -3 \end{vmatrix}$$

$$= \hat{i}(-12+2) - \hat{j}(9-1) + \hat{k}(6-4)$$
$$= -10\hat{i} - 8\hat{j} + 2\hat{k}$$

$$\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 5 & -7 \\ -10 & -8 & 2 \end{vmatrix}$$

$$= \hat{i}(10+56) - \hat{j}(4-70) + \hat{k}(-16+50)$$

$$\vec{a} \times \left(\vec{b} \times \vec{c}\right) = 66\hat{i} + 66\hat{j} + 36\hat{k} \qquad ---(ii)$$

From equation (i) and (ii)

$$\vec{a} \times \left(\vec{b} \times \vec{c}\right) \neq \left(\vec{a} \times \vec{b}\right) \times \vec{c}$$

Vector or Cross Product Ex 25.1 Q11

We know that, if θ be the angle between $\vec{a}$ and $\vec{b}$, then,

$$\vec{a} \times \vec{b} = \left|\vec{a}\right| . \left|\vec{b}\right| . \sin\theta . \hat{n}$$
$$\left|\vec{a} \times \vec{b}\right| = \left|\vec{a}\right| \left|\vec{b}\right| . \left|\sin\theta\right| . \left|\hat{n}\right|$$
$$8 = 2.5.\sin\theta.1 \qquad \left[\text{As } \hat{n} \text{ is a unit vector}\right]$$

$$\sin\theta = \frac{8}{10}$$
$$\sin\theta = \frac{4}{5}$$
$$\cos^2\theta = 1 - \sin^2\theta$$
$$= 1 - \left(\frac{4}{5}\right)^2$$
$$= 1 - \frac{16}{25}$$

$$= \frac{25-16}{25}$$
$$= \frac{9}{25}$$

$$\cos\ \theta = \frac{3}{5}$$

$$\vec{a}.\vec{b} = \left|\vec{a}\right| . \left|\vec{b}\right| . \cos\theta$$
$$= 2.5.\frac{3}{5}$$

$$\vec{a}.\vec{b} = 6$$

Given, $\vec{a} = \frac{1}{7}\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$

$\vec{b} = \frac{1}{7}\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right)$

$\vec{c} = \frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$,

$$\vec{a} \times \vec{b} = \frac{1}{7} \times \frac{1}{7}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 3 & -6 & 2 \end{vmatrix}$$

$$= \frac{1}{49}\left[\hat{i}\left(6 + 36\right) - \hat{j}\left(4 - 18\right) + \hat{k}\left(-12 - 9\right)\right]$$

$$= \frac{1}{49}\left[42\hat{i} + 14\hat{j} - 21\hat{k}\right]$$

$$= \frac{7\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)}{49}$$

$$= \frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$$

$$\vec{a} \times \vec{b} = \vec{c} \qquad ---(i)$$

$$\vec{b} \times \vec{c} = \frac{1}{7} \times \frac{1}{7}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & 2 \\ 6 & 2 & -3 \end{vmatrix}$$

$$= \frac{1}{49}\left[\hat{i}\left(18 - 4\right) - \hat{j}\left(-9 - 12\right) + \hat{k}\left(6 + 36\right)\right]$$

$$= \frac{1}{49}\left[14\hat{i} + 21\hat{j} + 42\hat{k}\right]$$

$$= \frac{7\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)}{49}$$

$$\vec{c} \times \vec{a} = \frac{1}{7} \times \frac{1}{7}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 2 & -3 \\ 2 & 3 & 6 \end{vmatrix}$$

$$\vec{c} \times \vec{a} = \frac{1}{49}\left[\hat{i}\left(12 + 9\right) - \hat{j}\left(36 + 6\right) + \hat{k}\left(18 - 4\right)\right]$$

$$= \frac{1}{49}\left[21\hat{i} - 42\hat{j} + 14\hat{k}\right]$$

$$= \frac{7\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right)}{49}$$

$$= \frac{1}{7}\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right)$$

$$\vec{c} \times \vec{a} = \vec{b} \qquad ---(iii)$$

From (i), (ii), and (iii),

$$\left.\begin{aligned} \vec{a} \times \vec{b} = \vec{c} \\ \vec{b} \times \vec{c} = \vec{a} \\ \vec{c} \times \vec{a} = \vec{b} \end{aligned}\right\} - (A)$$

$$|\vec{a}| = \frac{1}{7}\sqrt{(2)^2 + (3)^2 + (6)^2}$$

$$= \frac{1}{7}\sqrt{4 + 9 + 36}$$

$$= \frac{1}{7}\sqrt{49}$$

$$= \frac{1}{7} \times 7$$

$$|\vec{a}| = 1 \qquad ---(iv)$$

$$|\vec{b}| = \frac{1}{7}\sqrt{(3)^2 + (-6)^2 + (2)^2}$$

$$= \frac{1}{7}\sqrt{9 + 36 + 4}$$

$$= \frac{1}{7}\sqrt{49}$$

$$= \frac{7}{7}$$

$$|\vec{b}| = 1 \qquad ---(v)$$

$$|\vec{c}| = \frac{1}{7}\sqrt{(6)^2 + (2)^2 + (-3)^2}$$

$$= \frac{1}{7} 36 + 4 + 9$$

$$= \frac{1}{7}\sqrt{49}$$

$$= \frac{7}{7}$$

$$|\vec{c}| = 1 \qquad ---(vi)$$

From equation (iv), (v), (vi),

$$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1 \qquad ---(B)$$

From (A) and (B), We can say that

$\vec{a}, \vec{b}, \vec{c}$ is a right handed orthogonal system of unit vectors

Vector or Cross Product Ex 25.1 Q13

We know that, if θ is angle between $\vec{a}$ and $\vec{b}$,

$$\vec{a}\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$
$$60 = 13.5.\cos\theta$$
$$\cos\theta = \frac{60}{65}$$
$$\cos\theta = \frac{12}{13}$$
$$\sin^2\theta = 1 - \cos^2\theta$$
$$= 1 - \left(\frac{12}{13}\right)^2$$
$$= 1 - \frac{144}{169}$$
$$= \frac{169 - 144}{169}$$
$$= \frac{25}{169}$$
$$\sin\ \theta = \pm\sqrt{\frac{25}{169}}$$
$$= \pm\frac{5}{13}$$

$$|\sin\theta| = \frac{5}{13}$$

We know that,

$$\vec{a}\times\vec{b} = |\vec{a}|.|\vec{b}|.\sin\theta.\hat{n}$$
$$|\vec{a}\times\vec{b}| = |\vec{a}|.|\vec{b}|.|\sin\theta|.|\hat{n}|$$
$$= 13.5.\frac{5}{13}.1 \qquad \left[\text{Since, } \hat{n} \text{ is a unit vector}\right]$$

$$|\vec{a}\times\vec{b}| = 25$$

Vector or Cross Product Ex 25.1 Q14

We know that, if θ be the angle between $\vec{a}$ and $\vec{b}$, then

$$\vec{a}.\vec{b} = |\vec{a}|\ |\vec{b}|\cos\theta \qquad ---(i)$$
$$\text{And, } \vec{a}\times\vec{b} = |\vec{a}|\ |\vec{b}|.\sin\theta.\hat{n}$$
$$|\vec{a}\times\vec{b}| = |\vec{a}|.|\vec{b}|.|\sin\theta|.|\hat{n}|$$
$$= |\vec{a}|\ |\vec{b}||\sin\theta|.1 \qquad \left[\text{Since, } \hat{n} \text{ is a unit vector}\right]$$

$$|\vec{a}\times\vec{b}| = |\vec{a}|\ |\vec{b}|\sin\theta \qquad ---(ii)$$

Given that, $|\vec{a}\times\vec{b}| = \vec{a}.\vec{b}$

$$|\vec{a}|\ |\vec{b}|\sin\theta = |\vec{a}|.|\vec{b}|\cos\theta$$
$$\sin\theta = \cos\theta$$

$$\theta = \frac{\pi}{4}$$

Angle between $\vec{a}$ and $\vec{b}$ = $\frac{\pi}{4}$

Vector or Cross Product Ex 25.1 Q15

We have,

$\vec{a}\times\vec{b}=\vec{b}\times\vec{c}$

$\left(\vec{a}\times\vec{b}\right)-\left(\vec{b}\times\vec{c}\right)=\vec{0}$

$\left(\vec{a}\times\vec{b}\right)+\left(\vec{c}\times\vec{b}\right)=\vec{0}$ $\quad\left[\text{Since, }\left(\vec{b}\times\vec{c}\right)=-\left(\vec{c}\times\vec{b}\right)\right]$

$\left(\vec{a}+\vec{c}\right)\times\vec{b}=\vec{0}$ $\quad$ [Using distributive property]

We know that, if $\vec{a}\times\vec{b}=\vec{0}$, then vector $\vec{a}$ is parallel to vector $\vec{b}$.

Thus, $\left(\vec{a}+\vec{c}\right)$ is parallel to $\vec{b}$

$\left(\vec{a}+\vec{c}\right)=\overline{mb}$

Where m is any scalar

Vector or Cross Product Ex 25.1 Q16

We know that,

$\vec{a}\times\vec{b}=|\vec{a}|\,|\vec{b}|\sin\theta\,\hat{n}$

$|\vec{a}\times\vec{b}|=|\vec{a}|\,|\vec{b}|\,|\sin\theta|\,|\hat{n}|$

$=|\vec{a}|\,|\vec{b}|\,|\sin\theta|.1$ $\quad$ [as $\hat{n}$ is a unit vector]

$|\vec{a}\times\vec{b}|=|\vec{a}|\,|\vec{b}|\,|\sin\theta|$

$\sqrt{(3)^2+(2)^2+(6)^2}=2.7.|\sin\theta|$

$\sqrt{9+4+36}=14.|\sin\theta|$

$\sqrt{49}=14|\sin\theta|$

$\sin\theta=\frac{7}{14}$

$\sin\theta=\frac{1}{2}$

$\theta=\sin^{-1}\left(\frac{1}{2}\right)$

$\theta=\frac{\pi}{6}$

Angle between $\vec{a}$ and $\vec{b}=\frac{\pi}{6}$

Vector or Cross Product Ex 25.1 Q17

Given that $\vec{a}\times\vec{b}=\vec{0}$

This gives us four conclusions about $\vec{a}$ and $\vec{b}$

(i) $\vec{a}=\vec{0}$ or

(ii) $\vec{b}=\vec{0}$ or

(iii) $\vec{a}=\vec{b}=\vec{0}$ or

(iv) $\vec{a}$ is parallel to $\vec{b}$.

Also, it is given that $\vec{a}.\vec{b}=0$

This also gives us four canclusions about $\vec{a}$ and $\vec{b}$.

(i) $\vec{a}=\vec{0}$ or

(ii) $\vec{b}=\vec{0}$ or

(iii) $\vec{a}=\vec{b}=\vec{0}$ or

(iv) $\vec{a}$ is perpendicular to $\vec{b}$.

Now,

$\vec{a}$ parallel $\vec{b}$ and $\vec{a}$ is perpendicular to $\vec{b}$ are not possible simultaneously.

So,

$\vec{a}=0$ or $\vec{b}=0$ or $\vec{a}=\vec{b}=\vec{0}$

Vector or Cross Product Ex 25.1 Q18

Given that $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that

$\vec{a} \times \vec{b} = \vec{c},\ \vec{b} \times \vec{c} = \vec{a},\ \vec{c} \times \vec{a} = \vec{b},$

$\vec{a} \times \vec{b} = \vec{c}$

$\Rightarrow$ $\vec{c}$ is a vector perpendicular to both $\vec{a}$ and $\vec{b}$ ---(i)

$\vec{b} \times \vec{c} = \vec{a}$

$\Rightarrow$ $\vec{a}$ is a vector perpendicular to $\vec{b}$ and $\vec{c}$ ---(ii)

$\vec{c} \times \vec{a} = \vec{b}$

$\Rightarrow$ $\vec{b}$ is a vector perpendicular to $\vec{a}$ and $\vec{c}$ ---(iii)

Using (i), (ii) and (iii), we can see that $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular unit vectors.

Since,
$$\vec{a} \times \vec{b} = \vec{c}$$
$$\vec{b} \times \vec{c} = \vec{a}$$
$$\vec{c} \times \vec{a} = \vec{b}$$

Therefore,
$\vec{a}, \vec{b}, \vec{c}$ form an orthonormal right handed traid of unit vectors.

Vector or Cross Product Ex 25.1 Q19

Here, Position vector of $A = \left(3\hat{i} - \hat{j} + 2\hat{k}\right)$

Position vector of $B = \left(\hat{i} - \hat{j} - 3\hat{k}\right)$

Position vector of $C = \left(4\hat{i} - 3\hat{j} + \hat{k}\right)$

$$\overrightarrow{AB} = \vec{B} - \vec{A}$$
$$= \left(\hat{i} - \hat{j} - 3\hat{k}\right) - \left(3\hat{i} - \hat{j} + 2\hat{k}\right)$$
$$= \hat{i} - \hat{j} - 3\hat{k} - 3\hat{i} + \hat{j} - 2\hat{k}$$
$$= -2\hat{i} - 5\hat{k}$$

$$\overrightarrow{AC} = \vec{C} - \vec{A}$$
$$= \left(4\hat{i} - 3\hat{j} + \hat{k}\right) - \left(3\hat{i} - \hat{j} + 2\hat{k}\right)$$
$$= 4\hat{i} - 3\hat{j} + \hat{k} - 3\hat{i} + \hat{j} - 2\hat{k}$$
$$= \hat{i} - 2\hat{j} - \hat{k}$$

Vector perpendicular to the plane ABC

$$= \overrightarrow{AC} \times \overrightarrow{AB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -1 \\ -2 & 0 & -5 \end{vmatrix}$$

$$\overrightarrow{AC} \times \overrightarrow{AB} = \hat{i}(10 - 0) - \hat{j}(-5 - 2) + \hat{k}(0 - 4)$$
$$= 10\hat{i} + 7\hat{j} - 4\hat{k}$$

$$\left|\overrightarrow{AC} \times \overrightarrow{AB}\right| = \sqrt{(10)^2 + (7)^2 + (-4)^2}$$
$$= \sqrt{100 + 49 + 16}$$
$$= \sqrt{165}$$

Therefore, unit vector perpendicular to the plane $ABC = \dfrac{\overrightarrow{AC} \times \overrightarrow{AB}}{\left|\overrightarrow{AC} \times \overrightarrow{AB}\right|}$

$$= \frac{1}{\sqrt{165}}\left(10\hat{i} + 7\hat{j} - 4\hat{k}\right)$$

Unit vector perpendicular to the plane $ABC = \dfrac{1}{\sqrt{165}}\left(10\hat{i} + 7\hat{j} - 4\hat{k}\right)$

Vector or Cross Product Ex 25.1 Q20

Here, It is given that

In $\triangle ABC$

$\overrightarrow{BC}+\overrightarrow{CA}+\overrightarrow{AB}$
$=\overrightarrow{BA}+\overrightarrow{AB}$
$=\overrightarrow{BA}-\overrightarrow{BA}$ $\quad\left[\text{Since, } \overrightarrow{BA}=-\overrightarrow{AB}\right]$

$=\vec{0}$

Given that, $\left|\overrightarrow{BC}\right|=a$
$\left|\overrightarrow{CA}\right|=b$
$\left|\overrightarrow{AB}\right|=c$

Let, $\overrightarrow{BC}=\vec{a}$, $\overrightarrow{CA}=\vec{b}$ and $\overrightarrow{AB}=\vec{c}$

We have,

$$\overrightarrow{BC}+\overrightarrow{CA}+\overrightarrow{AB}=\vec{0}$$

$$\Rightarrow \overrightarrow{BC}+\overrightarrow{CA}=-\overrightarrow{AB}$$
$$\Rightarrow \overrightarrow{BC}+\overrightarrow{CA}=\overrightarrow{BA}$$
$$\Rightarrow \vec{a}+\vec{b}=-\vec{c}$$
$$\Rightarrow \vec{a}+\vec{b}+\vec{c}=\vec{0}$$
$$\Rightarrow \vec{a}\times\left(\vec{a}+\vec{b}+\vec{c}\right)=\vec{a}\times\vec{0}$$
$$\Rightarrow \vec{a}\times\vec{a}+\vec{a}\times\vec{b}+\vec{a}\times\vec{c}=\vec{0}$$
$$\Rightarrow \vec{0}+\vec{a}\times\vec{b}+\vec{a}\times\vec{c}=\vec{0} \qquad \left[\text{Since, } \vec{a}\times\vec{a}=\vec{0}\right]$$
$$\Rightarrow \vec{a}\times\vec{b}=-\left(\vec{a}\times\vec{c}\right)$$

$$\Rightarrow \vec{a}\times\vec{b}=\vec{c}\times\vec{a} \qquad ---\text{(i)}$$

Again, $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$$\Rightarrow \vec{b}\times\left(\vec{a}+\vec{b}+\vec{c}\right)=\vec{b}\times\vec{0}$$
$$\Rightarrow \vec{b}\times\vec{a}+\vec{b}\times\vec{b}+\vec{b}\times\vec{c}=\vec{0}$$
$$\Rightarrow \vec{b}\times\vec{a}+\vec{0}+\vec{b}\times\vec{c}=\vec{0} \qquad \left[\text{Since, } \vec{b}\times\vec{b}=\vec{0}\right]$$
$$\Rightarrow \vec{b}\times\vec{a}+\vec{b}\times\vec{c}=\vec{0}$$
$$\Rightarrow \vec{b}\times\vec{c}=-\left(\vec{b}\times\vec{a}\right)$$

$$\Rightarrow \vec{b}\times\vec{c}=\vec{a}\times\vec{b} \qquad ---\text{(ii)}$$

From equation (i) and (ii), we get

$$\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$$

$$\Rightarrow \left|\vec{a}\times\vec{b}\right|=\left|\vec{b}\times\vec{c}\right|=\left|\vec{c}\times\vec{a}\right|$$
$$\Rightarrow \left|\vec{a}\right|\left|\vec{b}\right|\sin\left(\pi-C\right)=\left|\vec{b}\right|\left|\vec{c}\right|\sin\left(\pi-A\right)=\left|\vec{c}\right|\left|\vec{a}\right|\sin\left(\pi-B\right)$$
$$\Rightarrow ab\sin C=bc\sin A=ca\sin B$$

Dividing by abc

$$\Rightarrow \frac{ab\sin C}{abc}=\frac{bc\sin A}{abc}=\frac{ca\sin B}{abc}$$
$$\Rightarrow \frac{\sin C}{c}=\frac{\sin A}{a}=\frac{\sin B}{b}$$

$$\Rightarrow \frac{c}{\sin C}=\frac{a}{\sin A}=\frac{b}{\sin B}$$

Vector or Cross Product Ex 25.1 Q21

Here, $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - 5\hat{k}$,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 3 & -5 \end{vmatrix}$$

$$= \hat{i}\left(10 - 9\right) - \hat{j}\left(-5 - 6\right) + \hat{k}\left(3 + 4\right)$$

$$= \hat{i} + 11\hat{j} + 7\hat{k}$$

Now, $\vec{a}.\left(\vec{a} \times \vec{b}\right) = \left(\hat{i} - 2\hat{j} + 3\hat{k}\right).\left(\hat{i} + 11\hat{j} + 7\hat{k}\right)$

$$\vec{a}.\left(\vec{a} \times \vec{b}\right) = (1)(1) + (-2)(11) + (3)(7)$$

$$= 1 - 22 + 21$$

$$= 22 - 22$$

$$\vec{a} \times \left(\vec{a} \times \vec{b}\right) = 0$$

Dot product of $\vec{a}$ and $\vec{a} \times \vec{b}$ is zero, then,

$\vec{a}$ is perpendicular to $\left(\vec{a} \times \vec{b}\right)$

Vector or Cross Product Ex 25.1 Q22

Given $\vec{p}$ and $\vec{q}$ be unit vector with angle $30°$ between then

$$\left|\vec{p}\right| = \left|\vec{q}\right| = 1$$

$$\vec{p} \times \vec{q} = \left|\vec{p}\right| \left|\vec{q}\right| \sin 30° \hat{n}$$

$$= 1.1.\left(\frac{1}{2}\right)\hat{n}$$

$$\left|\vec{p} \times \vec{q}\right| = \left|\frac{\hat{n}}{2}\right|$$

$$\left|\vec{p} \times \vec{q}\right| = \frac{1}{2} \qquad ---(i) \qquad \left[\text{Since, } \hat{n} \text{ is a unit vector}\right]$$

Area of parallelogram $= \frac{1}{2}\left|\vec{a} \times \vec{b}\right|$

$$= \frac{1}{2}\left|\left(\vec{p} + 2\vec{q}\right) \times \left(2\vec{p} + \vec{q}\right)\right|$$

$$= \frac{1}{2}\left|\vec{p} \times 2\vec{p} + \vec{p} \times \vec{q} + 2\vec{q} \times 2\vec{p} + 2\vec{q} \times \vec{q}\right|$$

$$= \frac{1}{2}\left|\vec{0} + \vec{p} \times \vec{q} + 2\vec{q} \times 2\vec{p} + \vec{0}\right| \qquad \left[\text{Since, } \vec{p} \times 2\vec{q} = \vec{0} \text{ and } 2\vec{q} \times \vec{q} = \vec{0}\right]$$

$$= \frac{1}{2}\left|\vec{p} \times \vec{q} + 4\left(\vec{q} \times \vec{p}\right)\right|$$

$$= \frac{1}{2}\left|\left(\vec{p} \times \vec{q}\right) - 4\left(\vec{p} \times \vec{q}\right)\right| \qquad \left[\text{Since, } \vec{q} \times \vec{p} = -\vec{p} \times \vec{q}\right]$$

$$= \frac{1}{2}\left|-3\left(\vec{p} \times \vec{q}\right)\right|$$

$$= \frac{3}{2}\left|\vec{p} \times \vec{q}\right|$$

$$= \frac{3}{2} \times \frac{1}{2} \qquad \left[\text{Using (i)}\right]$$

$$= \frac{3}{4}$$

Area of parallelogram $= \frac{3}{4}$ sq. unit

Vector or Cross Product Ex 25.1 Q23

We know that

$$\vec{a}\times\vec{b} = |\vec{a}|.|\vec{b}|.\sin\theta.\hat{n}$$

$$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta.|\hat{n}|$$

$$= |\vec{a}||\vec{b}|\sin\theta.1 \qquad \left[\text{Since, } \hat{n} \text{ is unit vector}\right]$$

$$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$$

Squaring both the sides,

$$|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2\sin^2\theta$$

$$= |\vec{a}|^2|\vec{b}|^2\left(1-\cos^2\theta\right)$$

$$= |\vec{a}|^2|\vec{b}|^2 - |\vec{a}|^2|\vec{b}|^2\cos^2\theta$$

$$= |\vec{a}|^2|\vec{b}|^2 - \left(|\vec{a}||\vec{b}|\cos\theta\right)^2$$

$$= \left(\vec{a}\right)^2\left(\vec{b}\right)^2 - \left(\vec{a}.\vec{b}\right)^2 \qquad \left[\text{Since, } |\vec{a}||\vec{b}|\cos\theta = \vec{a}.\vec{b}\right]$$

$$|\vec{a}\times\vec{b}|^2 = \left(\vec{a}.\vec{a}\right)\left(\vec{b}.\vec{b}\right) - \left(\vec{a}.\vec{b}\right)\left(\vec{b}.\vec{a}\right) \qquad \left[\text{Since, } \left(\vec{a}.\vec{b}\right) = \left(\vec{b}.\vec{a}\right)\right]$$

$$|\vec{a}\times\vec{b}|^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} \end{vmatrix}$$

Vector or Cross Product Ex 25.1 Q24

Define of $\vec{a}\times\vec{b}$: – Let $\vec{a}$, $\vec{b}$ be two non-zero, non-parallel vectors. Then $\vec{a}\times\vec{b}$, in that order, is defined as a vector whose magnitude is $|\vec{a}|\,|\vec{b}|\sin\theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$ and whose direction is perpendicular to the plane of $\vec{a}$ and $\vec{b}$ and this constitute a right handed system.

$$\vec{a}\times\vec{b} = |\vec{a}|\,|\vec{b}|\sin\theta\hat{n}$$

Where $\hat{n}$ is a unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b}$ such that $\vec{a},\vec{b},\hat{n}$ form a right handed system.

Now,

$$\vec{a}\times\vec{b} = |\vec{a}|\,|\vec{b}|\sin\theta\ \hat{n}$$

$$|\vec{a}\times\vec{b}| = |\vec{a}|\,|\vec{b}||\sin\theta|\,|\hat{n}|$$

$$= |\vec{a}|\,|\vec{b}||\sin\theta|.1$$

$$|\vec{a}\times\vec{b}| = |\vec{a}|\,|\vec{b}||\sin\theta|$$

$$= \frac{\vec{a}.\vec{b}}{\cos\theta}.\sin\theta \qquad \left[\text{Since, } \cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\right]$$

$$|\vec{a}\times\vec{b}| = \vec{a}.\vec{b}.\tan\theta$$

Vector or Cross Product Ex 25.1 Q25

$\vec{a} \times \vec{b} = |\vec{a}|\ |\vec{b}| \sin\theta . \hat{n}$

$|\vec{a} \times \vec{b}| = |\vec{a}|\ |\vec{b}|\ |\sin\theta|\ |\hat{n}|$

$35 = \sqrt{26}.7\ |\sin\theta|.1$

$\sin\theta = \dfrac{35}{\sqrt{26}.7}$

$\sin\theta = \dfrac{5}{\sqrt{26}}$

$\cos^2\theta = 1 - \sin^2\theta$

$= 1 - \left(\dfrac{5}{\sqrt{26}}\right)^2$

$= \dfrac{1}{1} - \dfrac{25}{26}$

$= \dfrac{26 - 25}{26}$

$= \dfrac{1}{26}$

$\cos\theta = \dfrac{1}{\sqrt{26}}$

$\vec{a}.\vec{b} = |\vec{a}|\ |\vec{b}| \cos\theta$

$= \sqrt{26}.7.\dfrac{1}{\sqrt{26}}$

$\vec{a}.\vec{b} = 7$

Vector or Cross Product Ex 25.1 Q26

Area of triangle $= \dfrac{1}{2}\left|\overrightarrow{OA} \times \overrightarrow{OB}\right|$

$$\left|\overrightarrow{OA} \times \overrightarrow{OB}\right| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -3 & -2 & 1 \end{vmatrix}$$

$= \hat{i}(2+6) - \hat{j}(1+9) + \hat{k}(-2+6)$

$= 8\hat{i} - 10\hat{j} + 4\hat{k}$

$= 2\left(4\hat{i} - 5\hat{j} + 2\hat{k}\right)$

Area of triangle $= \dfrac{1}{2}\left|\overrightarrow{OA} \times \overrightarrow{OB}\right|$

$= \dfrac{1}{2}\left[2\sqrt{(4)^2 + (-5)^2 + (2)^2}\right]$

$= \dfrac{1}{2}\left[2\sqrt{16 + 25 + 4}\right]$

$= \sqrt{45}$

$= 3\sqrt{5}$

Area of triangle $= 3\sqrt{5}$ Sq.unit

Vector or Cross Product Ex 25.1 Q27

Let $\vec{d} = d_1\hat{i} + d_2\hat{j} + d_3\hat{k}$.

Since $\vec{d}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, we have:

$$\vec{d}\cdot\vec{a} = 0$$
$$\Rightarrow d_1 + 4d_2 + 2d_3 = 0 \qquad \text{...(i)}$$
And,
$$\vec{d}\cdot\vec{b} = 0$$
$$\Rightarrow 3d_1 - 2d_2 + 7d_3 = 0 \qquad \text{...(ii)}$$

Also, it is given that:

$$\vec{c}\cdot\vec{d} = 15$$
$$\Rightarrow 2d_1 - d_2 + 4d_3 = 15 \qquad \text{...(iii)}$$

On solving (i), (ii), and (iii), we get:

$$d_1 = \frac{160}{3}, d_2 = -\frac{5}{3} \text{ and } d_3 = -\frac{70}{3}$$
$$\therefore \vec{d} = \frac{160}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{70}{3}\hat{k} = \frac{1}{3}\left(160\hat{i} - 5\hat{j} - 70\hat{k}\right)$$

Hence, the required vector is $\frac{1}{3}\left(160\hat{i} - 5\hat{j} - 70\hat{k}\right)$.

Vector or Cross Product Ex 25.1 Q28

Given, $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$
$\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

Let, $\vec{d} = \vec{a} + \vec{b}$
$= \left(3\hat{i} + 2\hat{j} + 2\hat{k}\right) + \left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$
$\vec{d} = 4\hat{i} + 4\hat{j} - 0\hat{k}$

And, $\vec{e} = \vec{a} - \vec{b}$
$= \left(3\hat{i} + 2\hat{j} + 2\hat{k}\right) - \left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$
$\vec{e} = 2\hat{i} + 4\hat{k}$

Let, $\vec{f}$ be any vector perpendicular to both $\vec{d}$ and $\vec{e}$

$$\vec{f} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix}$$

$= \hat{i}(16 - 0) - \hat{j}(16 - 0) + \hat{k}(0 - 8)$
$\vec{f} = 16\hat{i} - 16\hat{j} - 8\hat{k}$

$= 8\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$

Let $\vec{g}$ be the required vector, then

$\vec{g} = \lambda\vec{f}$ and $\left|\vec{g}\right| = 1$
$$\vec{g} = 8\lambda\left(2\hat{i} - 2\hat{j} - \hat{k}\right) \qquad ---(i)$$

$\left|\vec{g}\right| = 1$

$8\lambda\sqrt{(2)^2 + (-2)^2 + (-1)^2} = 1$
$8\lambda\sqrt{4 + 4 + 1} = 1$
$8\lambda\sqrt{9} = 1$
$24\lambda = 1$

$\lambda = \frac{1}{24}$

Put λ in (i)

$$\vec{g} = 8\left(\frac{1}{24}\right)\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$$

$$\vec{g} = \frac{1}{3}\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$$

Thus,

Unit vector perpendicular to $\left(\vec{a} + \vec{b}\right)$ and $\left(\vec{a} - \vec{b}\right) = \frac{1}{3}\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$

Vector or Cross Product Ex 25.1 Q29

Given, $A = (2, 3, 5)$

$B = (3, 5, 8)$

$C = (2, 7, 8)$

Position vector of $A = 2\hat{i} + 3\hat{j} + 5\hat{k}$

Position vector of $B = 3\hat{i} + 5\hat{j} + 8\hat{k}$

Position vector of $C = 2\hat{i} + 7\hat{j} + 8\hat{k}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= \left(3\hat{i} + 5\hat{j} + 8\hat{k}\right) - \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right)$

$= 3\hat{i} + 5\hat{j} + 8\hat{k} - 2\hat{i} - 3\hat{j} - 5\hat{k}$

$\overrightarrow{AB} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{AC}$ = Position vector of C – Position vector of A

$= \left(2\hat{i} + 7\hat{j} + 8\hat{k}\right) - \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right)$

$= 2\hat{i} + 7\hat{j} + 8\hat{k} - 2\hat{i} - 3\hat{j} - 5\hat{k}$

$\overrightarrow{AC} = 4\hat{j} + 3\hat{k}$

Area of triangle $= \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix}$$

$= \hat{i}\,(6 - 12) - \hat{j}\,(3 - 0) + \hat{k}\,(4 - 0)$

$\overrightarrow{AB} \times \overrightarrow{AC} = -6\hat{i} - 3\hat{j} + 4\hat{k}$

$\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \sqrt{(-6)^2 + (-3)^2 + (4)^2}$

$= \sqrt{36 + 9 + 16}$

$= \sqrt{61}$

Area of triangle $= \frac{1}{2}\sqrt{61}$ Sq. unit

Vector or Cross Product Ex 25.1 Q30

Let $\vec{d} = d_1\hat{i} + d_2\hat{j} + d_3\hat{k}$.

Since $\vec{d}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, we have:

$$\vec{d} \cdot \vec{a} = 0$$
$$\Rightarrow d_1 + 4d_2 + 2d_3 = 0 \qquad \text{...(i)}$$

And,

$$\vec{d} \cdot \vec{b} = 0$$
$$\Rightarrow 3d_1 - 2d_2 + 7d_3 = 0 \qquad \text{...(ii)}$$

Also, it is given that:

$$\vec{c} \cdot \vec{d} = 15$$
$$\Rightarrow 2d_1 - d_2 + 4d_3 = 15 \qquad \text{...(iii)}$$

On solving (i), (ii), and (iii), we get:

$$d_1 = \frac{160}{3}, d_2 = -\frac{5}{3} \text{ and } d_3 = -\frac{70}{3}$$
$$\therefore \vec{d} = \frac{160}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{70}{3}\hat{k} = \frac{1}{3}\left(160\hat{i} - 5\hat{j} - 70\hat{k}\right)$$

Hence, the required vector is $\frac{1}{3}\left(160\hat{i} - 5\hat{j} - 70\hat{k}\right)$.

Vector or Cross Product Ex 25.1 Q31

$$\vec{a} \cdot \vec{b} = 0$$

Then,

(i) Either $|\vec{a}| = 0$ or $|\vec{b}| = 0$, or $\vec{a} \perp \vec{b}$ $\left(\text{in case } \vec{a} \text{ and } \vec{b} \text{ are non-zero}\right)$

$$\vec{a} \times \vec{b} = 0$$

(ii) Either $|\vec{a}| = 0$ or $|\vec{b}| = 0$, or $\vec{a} \parallel \vec{b}$ $\left(\text{in case } \vec{a} \text{ and } \vec{b} \text{ are non-zero}\right)$

But, $\vec{a}$ and $\vec{b}$ cannot be perpendicular and parallel simultaneously.

Hence, $|\vec{a}| = 0$ or $|\vec{b}| = 0$.

Vector or Cross Product Ex 25.1 Q32

Take any parallel non-zero vectors so that $\vec{a} \times \vec{b} = \vec{0}$.

Let $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = 4\hat{i} + 6\hat{j} + 8\hat{k}$.

Then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 4 & 6 & 8 \end{vmatrix} = \hat{i}(24-24) - \hat{j}(16-16) + \hat{k}(12-12) = 0\hat{i} + 0\hat{j} + 0\hat{k} = \vec{0}$$

It can now be observed that:

$$|\vec{a}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$$
$$\therefore \vec{a} \neq \vec{0}$$
$$|\vec{b}| = \sqrt{4^2 + 6^2 + 8^2} = \sqrt{116}$$
$$\therefore \vec{b} \neq \vec{0}$$

Hence, the converse of the given statement need not be true.

Vector or Cross Product Ex 25.1 Q33

We have,

$$\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k},\ \vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k},\ \vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$$

$$\left(\vec{b}+\vec{c}\right)=\left(b_1+c_1\right)\hat{i}+\left(b_2+c_2\right)\hat{j}+\left(b_3+c_3\right)\hat{k}$$

$$\text{Now, } \vec{a}\times\left(\vec{b}+\vec{c}\right)\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1+c_1 & b_2+c_2 & b_3+c_3\end{vmatrix}$$

$$=\hat{i}\left[a_2\left(b_3+c_3\right)-a_3\left(b_2+c_2\right)\right]-\hat{j}\left[a_1\left(b_3+c_3\right)-a_3\left(b_1+c_1\right)\right]+\hat{k}\left[a_1\left(b_2+c_2\right)-a_2\left(b_1+c_1\right)\right]$$
$$=\hat{i}\left[a_2b_3+a_2c_3-a_3b_2-a_3c_2\right]+\hat{j}\left[-a_1b_3-a_1c_3+a_3b_1+a_3c_1\right]+\hat{k}\left[a_1b_2+a_1c_2-a_2b_1-a_2c_1\right]\quad ...(1)$$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$
$$=\hat{i}\left[a_2b_3-a_3b_2\right]+\hat{j}\left[b_1a_3-a_1b_3\right]+\hat{k}\left[a_1b_2-a_2b_1\right]\qquad(2)$$
$$\vec{a}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ c_1 & c_2 & c_3\end{vmatrix}$$
$$=\hat{i}\left[a_2c_3-a_3c_2\right]+\hat{j}\left[a_3c_1-a_1c_3\right]+\hat{k}\left[a_1c_2-a_2c_1\right]\qquad(3)$$

Vector or Cross Product Ex 25.1 Q34

Given that

$A=(1,1,2)$

$B=(2,3,5)$

$C=(1,5,5)$

Position vector of $A=\hat{i}+\hat{j}+2\hat{k}$

Position vector of $B=2\hat{i}+3\hat{j}+5\hat{k}$

Position vector of $C=\hat{i}+5\hat{j}+5\hat{k}$

$\overrightarrow{AB}$ = Position vector of B − Position vector of A

$=2\hat{i}+3\hat{j}+5\hat{k}-\left(\hat{i}+\hat{j}+2\hat{k}\right)$

$=\hat{i}+2\hat{j}+3\hat{k}$

$\overrightarrow{AC}$ − Position vector of C − Position vector of A

$=\hat{i}+5\hat{j}+5\hat{k}-\left(\hat{i}+\hat{j}+2\hat{k}\right)$

$=4\hat{j}+3\hat{k}$

Area of triangle $=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ 0 & 4 & 3\end{vmatrix}$$

$=\hat{i}(6-12)-\hat{j}(3-0)+\hat{k}(4-0)$

$=-6\hat{i}-3\hat{j}+4\hat{k}$

$\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|=\sqrt{(-6)^2+(-3)^2+4^2}$

$=\sqrt{36+9+16}$

$=\sqrt{61}$

Area of the triangle $=\frac{1}{2}\sqrt{61}$ Sq.unit

Vector or Cross Product Ex 25.1 Q35

Let

$$\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$$
$$\vec{b} = \hat{i} - 2\hat{j} - 3\hat{k}$$

The unit vector parallel to one of its diagonals is $\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$.

Now

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 5 \\ 1 & -2 & -3 \end{vmatrix}$$
$$= \hat{i}(12+10) - \hat{j}(-6-5) + \hat{k}(-4+4)$$
$$= 22\hat{i} + 11\hat{j}$$
$$= 11(2\hat{i} + \hat{j})$$

$$|\vec{a}\times\vec{b}| = 11\sqrt{2^2+1^2}$$
$$= 11\sqrt{5}$$

Therefore

$$\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{11(2\hat{i}+\hat{j})}{11\sqrt{5}}$$
$$= \frac{1}{\sqrt{5}}(2\hat{i}+\hat{j})$$

The unit vector parallel to one of its diagonals is $\dfrac{1}{\sqrt{5}}(2\hat{i}+\hat{j})$.

Again, the area of the parallelogram is $|\vec{a}\times\vec{b}| = 11\sqrt{5}$ Sq. unit