

Ex 28.1

Q1(i)

All are positive, so octant is XOYZ

Q1(ii)

X is negative and rest are positive, so octant is X'OYZ

Q1(iii)

Y is negative and rest are positive, so octant is XOY'Z

Q1(iv)

Z is negative and rest are positive, so octant is XOYZ'

Q1(v)

X and Y are negative and Z is positive, so octant is X'OY'Z

Q1(vi)

All are negative, so octant is X'OY'Z'

Q1(vii)

X and Z are negative, so octant is X'OYZ'

Q2(i)

YZ plane is x-axis, so sign of x will be changed. So answer is (2, 3, 4)

Q2(ii)

XZ plane is y-axis, so sign of y will be changed. So answer is (-5, -4, -3)

Q2(iii)

XY-plane is z-axis, so sign of Z will change. So answer is (5, 2, 7)

Q2(iv)

XZ plane is y-axis, so sign of Y will change, So answer is (-5, 0, 3)

Q2(v)

XY plane is Z-axis, so sign of Z will change So answer is (-4, 0, 0)

Q3

Vertices of cube are

(1, 0, -1) (1, 0, 4) (1, -5, -1)

(1, -5, 4) (-4, 0, -1) (-4, -5, -4)

(-4, -5, -1) (4, 0, 4) (1, 0, 4)

Q4

$3 - (-2) = 5$, $|0 - 5| = 5$, $|-1 - 4| = 5$

5, 5, 5 are lengths of edges

Q5

$5 - 3 = 2$, $0 - (-2) = 2$, $5 - 2 = 3$

2, 2, 3 are lengths of edges

Q6

(-4, 3, 5)

x-axis: $\sqrt{9+25} = \sqrt{34}$

y-axis: $\sqrt{16+25} = \sqrt{41}$

z-axis: $\sqrt{9+16} = 5$

Q7

$(-3, -2, -5)$ $(-3, -2, 5)$ $(3, -2, -5)$ $(-3, 2, -5)$ $(3, 2, 5)$

$(3, 2, -5)$ $(-3, 2, 5)$

SUPERCOP

Ex 28.2

Q1

(i) Distance between points P and Q

$$\begin{aligned}PQ &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \\&= \sqrt{(1 - 2)^2 + (-1 - 1)^2 + (0 - 2)^2} \\&= \sqrt{(-1)^2 + (-2)^2 + (-2)^2} \\&= \sqrt{1 + 4 + 4}\end{aligned}$$

$$PQ = 3 \text{ units}$$

(ii) Distance between points A and B

$$\begin{aligned}AB &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \\&= \sqrt{(3 + 1)^2 + (2 + 1)^2 + (-1 + 1)^2} \\&= \sqrt{(4)^2 + (3)^2 + (0)^2} \\&= \sqrt{16 + 9 + 0} \\&= \sqrt{25}\end{aligned}$$

$$AB = 5 \text{ units}$$

Q2

Distance between points P and Q

$$\begin{aligned}PQ &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \\&= \sqrt{(-2 - 2)^2 + (3 - 1)^2 + (1 - 2)^2} \\&= \sqrt{(-4)^2 + (2)^2 + (-1)^2} \\&= \sqrt{16 + 4 + 1}\end{aligned}$$

$$PQ = \sqrt{21} \text{ units}$$

Q3(i)

$A(4, -3, -1)$, $B(5, -7, 6)$ and $C(3, 1, -8)$

$$\begin{aligned}AB &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \\&= \sqrt{(4 - 5)^2 + (-3 - 7)^2 + (-1 - 6)^2} \\&= \sqrt{(-1)^2 + (-4)^2 + (-7)^2} \\&= \sqrt{1 + 16 + 49} \\&= \sqrt{66} \text{ units}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(5 - 3)^2 + (-7 - 1)^2 + (6 + 8)^2} \\&= \sqrt{(2)^2 + (-8)^2 + (14)^2} \\&= \sqrt{4 + 64 + 196} \\&= \sqrt{264} \\&= 2\sqrt{66} \text{ units}\end{aligned}$$

$$\begin{aligned}AC &= \sqrt{(4 - 3)^2 + (-3 - 1)^2 + (-1 + 8)^2} \\&= \sqrt{(1)^2 + (-4)^2 + (7)^2} \\&= \sqrt{1 + 16 + 49} \\&= \sqrt{66} \text{ units}\end{aligned}$$

Since $AC + AB = BC$
so, A, B, C are collinear.

Q3(ii)

$$P(0, 7, -7), Q(1, 4, -5), R(-1, 10, -9)$$

$$PQ = \sqrt{(0-1)^2 + (7-4)^2 + (-7+5)^2}$$

$$= \sqrt{1^2 + 3^2 + (-2)^2}$$

$$= \sqrt{1+9+4}$$

$$= \sqrt{14} \text{ units}$$

$$QR = \sqrt{(1+1)^2 + (4-10)^2 + (-5+9)^2}$$

$$= \sqrt{2^2 + (-6)^2 + 4^2}$$

$$= \sqrt{4+36+16}$$

$$= 2\sqrt{14} \text{ units}$$

$$PR = \sqrt{(0+1)^2 + (7-10)^2 + (-7+9)^2}$$

$$= \sqrt{1^2 + (-3)^2 + 2^2}$$

$$= \sqrt{1+9+4}$$

$$= \sqrt{14} \text{ units}$$

Since $PQ + PR = QR$

so, P, Q, R are collinear

Q3(iii)

$A(3, -5, 1)$, $B(-1, 0, 8)$, and $C(7, -10, -6)$

$$AB = \sqrt{(3+1)^2 + (-5-0)^2 + (1-8)^2}$$

$$= \sqrt{4^2 + (-5)^2 + (-7)^2}$$

$$= \sqrt{16+25+49}$$

$$= \sqrt{90}$$

$$= 3\sqrt{10} \text{ units}$$

$$BC = \sqrt{(-1-7)^2 + (0+10)^2 + (8+6)^2}$$

$$= \sqrt{(-8)^2 + (10)^2 + (14)^2}$$

$$= \sqrt{64+100+196}$$

$$= \sqrt{360}$$

$$= 6\sqrt{10} \text{ units}$$

$$CA = \sqrt{(3-7)^2 + (-5+10)^2 + (1+6)^2}$$

$$= \sqrt{(-4)^2 + (5)^2 + (7)^2}$$

$$= \sqrt{16+25+49}$$

$$= \sqrt{90}$$

$$= 3\sqrt{10} \text{ units}$$

Since $AB + AC = BC$

so, A , B , and C are collinear

Q4(i)

Let the point on xy - plane be $P(x, y, 0)$.

Now P is equidistance from $A(1, -1, 0)$, $B(2, 1, 2)$ and $C(3, 2, -1)$.

So, $AP = BP = CP$

Now,

$$(AP)^2 = (x-1)^2 + (y+1)^2 + (0-0)^2$$

$$(BP)^2 = (x-2)^2 + (y-1)^2 + (0-2)^2$$

$$(CP)^2 = (x-3)^2 + (y-2)^2 + (0+1)^2$$

$$(AP)^2 = (BP)^2 \Rightarrow (x-1)^2 + (y+1)^2 = (x-2)^2 + (y-1)^2 + 4$$

$$\Rightarrow x^2 + 1 - 2x + y^2 + 1 + 2y + 2 = x^2 + 4 - 4x + y^2 + 1 - 2y + 4$$

$$\Rightarrow 2x + 4y = 7 \dots (1)$$

$$(BP)^2 = (CP)^2 \Rightarrow (x-2)^2 + (y-1)^2 + 4 = (x-3)^2 + (y-2)^2 + 1$$

$$\Rightarrow x^2 + 4 - 4x + y^2 + 1 - 2y + 2 + 4 = x^2 + 9 - 6x + y^2 + 4 - 4y + 1$$

$$\Rightarrow 2x + 2y = 5 \dots (2)$$

$$(AP)^2 = (CP)^2 \Rightarrow (x-1)^2 + (y+1)^2 = (x-3)^2 + (y-2)^2 + 1$$

$$\Rightarrow x^2 + 1 - 2x + y^2 + 1 + 2y = x^2 + 9 - 6x + y^2 + 4 - 4y + 1$$

$$\Rightarrow 4x + 6y = 12 \dots (3)$$

Solving equation (1) and (2) we get

$$y = 1, \quad x = 3/2$$

Put x and y in equation (3)

$$4(3/2) + 6(1) = 12$$

$$12 = 12$$

So, the required point is $(3/2, 1, 0)$

Q4(ii)

Let $Q(0, y, z)$ be the required point.

So

$$\begin{aligned} (AQ)^2 &= (BQ)^2 \Rightarrow (0-1)^2 + (y+1)^2 + (z-0)^2 = (0-2)^2 + (y-1)^2 + (z-2)^2 \\ \Rightarrow 1 + y^2 + 1 + 2y + z^2 &= 4 + y^2 + 1 - 2y + z^2 + 4 - 4z \\ \Rightarrow 4y + 4z &= 7 \dots (1) \end{aligned}$$

$$\begin{aligned} (BQ)^2 &= (CQ)^2 \Rightarrow (0-1)^2 + (y+1)^2 + (z-0)^2 = (0-3)^2 + (y-2)^2 + (z+1)^2 \\ \Rightarrow 4 + y^2 + 1 - 2y + z^2 + 4 - 4z &= 9 + y^2 + 4 - 4y + z^2 + 1 + 2z \\ \Rightarrow 2y - 6z &= 5 \dots (2) \end{aligned}$$

$$\begin{aligned} (AQ)^2 &= (CQ)^2 \Rightarrow (0-1)^2 + (y+1)^2 + (z-0)^2 = (0-3)^2 + (y-2)^2 + (z+1)^2 \\ \Rightarrow 1 + y^2 + 2y + 1 + z^2 &= 9 + y^2 - 4y + 4 + z^2 + 1 + 2z \\ \Rightarrow 6y - 2z &= 12 \dots (3) \end{aligned}$$

Solving equation (1) and (2), we get

$$z = \frac{-3}{16} \text{ and } y = \frac{31}{16}$$

Put the value of y and z in equation (3)

$$6y - 2z = 12 = 12$$

$$6\left(\frac{31}{16}\right) - 2\left(\frac{-3}{16}\right) = 12$$

$$\frac{186}{16} + \frac{6}{16} = 12$$

$$\frac{192}{16} = 12$$

$$12 = 12$$

LHS = RHS.

so,

$$y = \frac{31}{16}, z = \frac{-3}{16}$$

$$\text{Required point} = \left(0, \frac{31}{16}, \frac{-3}{16}\right)$$

Q4(iii)

Let $R(x, 0, z)$ be the required point.

So

$$\begin{aligned} (AR)^2 &= (BR)^2 \Rightarrow (1-x)^2 + (-1-0)^2 + (0-z)^2 = (2-x)^2 + (1-0)^2 + (2-z)^2 \\ \Rightarrow 1+x^2-2x+1+z^2 &= 4+x^2-4x+1+4+z^2-4z \\ \Rightarrow 2x+4z &= 7 \dots (1) \end{aligned}$$

$$\begin{aligned} (BR)^2 &= (OR)^2 \Rightarrow (z-z)^2 + (1-0)^2 + (2-z)^2 = (3-x)^2 + (2-0)^2 + (-1-z)^2 \\ \Rightarrow 4+x^2-4x+4+z^2-4z &= 9+x^2-6x+4+1+z^2+2z \\ \Rightarrow 2x-6z &= 5 \dots (2) \end{aligned}$$

$$\begin{aligned} (AR)^2 &= (OR)^2 \Rightarrow (1-x)^2 + (1-0)^2 + (0-z)^2 = (3-x)^2 + (2-0)^2 + (-1-z)^2 \\ \Rightarrow 1+x^2-2x+1+z^2 &= 9+6x+4+1+z^2+2z \\ \Rightarrow 4x-2z &= 12 \dots (3) \end{aligned}$$

Solving equation (1) and (2), we get

$$z = \frac{1}{5}, \quad x = \frac{31}{10}$$

Put the value of x and z in equation (3)

$$4x - 2z = 12$$

$$4\left(\frac{31}{10}\right) - 2\left(\frac{1}{5}\right) = 12$$

$$\frac{124}{10} - \frac{2}{10} = 12$$

$$\frac{120}{10} = 12$$

$$12 = 12$$

LHS = RHS.

so,

$$x = \frac{31}{10}, \quad z = \frac{1}{5}$$

$$\text{Required point} = \left(\frac{31}{10}, 0, \frac{1}{5}\right)$$

Q5

Let $P(0, 0, z)$ be the point equidistant from $Q(1, 5, 7)$ and $R(5, 1, -4)$.

So,

$$(PQ)^2 = (PR)^2 \Rightarrow (0-1)^2 + (0-5)^2 + (z-7)^2 = (0-5)^2 + (0-1)^2 + (z+4)^2$$

$$\Rightarrow 1 + 25 + (z-7)^2 = 25 + 1 + (z+4)^2$$

$$\Rightarrow 26 + z^2 + 49 - 14z = 26 + z^2 + 8z + 16$$

$$\Rightarrow -14z - 8z = 16 - 49$$

$$\Rightarrow -22z = -33$$

$$\Rightarrow z = \frac{-33}{-22}$$

$$\Rightarrow z = \frac{3}{2}$$

Required point = $(0, 0, 3/2)$

Q6

Let $P(0, y, 0)$ be a point on y-axis which is equidistant from $Q(3, 1, 2)$ and $R(5, 5, 2)$.

So,

$$(PR)^2 = (PQ)^2 \Rightarrow (0-5)^2 + (y-5)^2 + (0-2)^2 = (0-3)^2 + (y-1)^2 + (0-2)^2$$

$$\Rightarrow 25 + y^2 + 25 - 10y + 4 = 9 + y^2 + 1 - 2y + 4$$

$$\Rightarrow -10y + 2y = 14 - 54$$

$$\Rightarrow -14z - 8z = 16 - 49$$

$$\Rightarrow -8y = -40$$

$$\Rightarrow y = 5$$

So, the required point is $(0, 5, 0)$

Q7

Let $P(0, 0, z)$ be at a distance of $\sqrt{21}$ from $Q(1, 2, 3)$.

So

$$PQ = \sqrt{(0-1)^2 + (0-2)^2 + (z-3)^2}$$

$$\sqrt{21} = \sqrt{(1)^2 + (2)^2 + (z-3)^2}$$

$$21 - 5 = (z-3)^2$$

$$16 = (z-3)^2$$

$$z-3 = \pm 4$$

$$z = 7 \text{ and } z = -1$$

So, the required points are $(0, 0, 7)$ and $(0, 0, -1)$

Q8

Let the triangle formed be $\triangle ABC$

$$\begin{aligned} AB &= \sqrt{(1-2)^2 + (2-3)^2 + (3-1)^2} \\ &= \sqrt{(-1)^2 + (-1)^2 + (2)^2} \\ &= \sqrt{6} \text{ units} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(2-3)^2 + (3-1)^2 + (1-2)^2} \\ &= \sqrt{(-1)^2 + (2)^2 + (-1)^2} \\ &= \sqrt{6} \text{ units} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(1-3)^2 + (2-1)^2 + (3-2)^2} \\ &= \sqrt{(-2)^2 + (1)^2 + (1)^2} \\ &= \sqrt{6} \text{ units} \end{aligned}$$

since, $AB = BC = CA$

So, $\triangle ABC$ is an equilateral \triangle

Q9

Let $A = (0, 7, 10)$, $B = (-1, 6, 6)$ and $C = (-4, 9, 6)$

$$\begin{aligned} AB &= \sqrt{(0+1)^2 + (7-6)^2 + (10-6)^2} \\ &= \sqrt{(1)^2 + (1)^2 + (4)^2} \\ &= \sqrt{18} \\ &= 3\sqrt{2} \text{ units} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(-1+4)^2 + (6-9)^2 + (6-6)^2} \\ &= \sqrt{(3)^2 + (3)^2 + 0} \\ &= \sqrt{18} \\ &= 3\sqrt{2} \text{ units} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(0+4)^2 + (7-9)^2 + (10-6)^2} \\ &= \sqrt{(4)^2 + (-2)^2 + (4)^2} \\ &= \sqrt{36} \\ &= 6 \text{ units} \end{aligned}$$

$$\begin{aligned} (AB)^2 + (BC)^2 &= (3\sqrt{2})^2 + (3\sqrt{2})^2 \\ &= 18 + 18 \\ &= 36 \\ &= (AC)^2 \end{aligned}$$

Also $l(AB) = l(BC)$

Hence $(0, 7, 10)$, $(-1, 6, 6)$ and $(-4, 9, 6)$ are the vertices of an isosceles right-angled triangle.

Q10

Here points are $A(3, 3, 3)$, $B(0, 6, 3)$, $C(1, 7, 7)$ and $D(4, 4, 7)$.

$$\begin{aligned}AB &= \sqrt{(3-0)^2 + (3-6)^2 + (3-3)^2} \\&= \sqrt{9+9} \\&= 3\sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(0-1)^2 + (6-7)^2 + (3-7)^2} \\&= \sqrt{1+1+16} \\&= 3\sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}AC &= \sqrt{(3-1)^2 + (3-7)^2 + (3-7)^2} \\&= \sqrt{4+16+16} \\&= 6 \text{ units}\end{aligned}$$

$$\begin{aligned}BD &= \sqrt{(0-4)^2 + (6-4)^2 + (3-7)^2} \\&= \sqrt{16+4+16} \\&= 6 \text{ units}\end{aligned}$$

$$\begin{aligned}CD &= \sqrt{(1-4)^2 + (7-4)^2 + (7-7)^2} \\&= \sqrt{9+9} \\&= 3\sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}AD &= \sqrt{(3-4)^2 + (3-4)^2 + (3-7)^2} \\&= \sqrt{1+1+16} \\&= 3\sqrt{2} \text{ units}\end{aligned}$$

Since,

$$AB = BC = CD = DA$$

And $AC = BD$

So,

A, B, C, D are vertices of a square.

Q11

Here,

$$\begin{aligned}AB &= \sqrt{(1+5)^2 + (3-5)^2 + (0-2)^2} \\&= \sqrt{36 + 4 + 4} \\&= \sqrt{44} \\&= 2\sqrt{11} \text{ units}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(-5+9)^2 + (5+1)^2 + (2-2)^2} \\&= \sqrt{16 + 36} \\&= \sqrt{52} \\&= 2\sqrt{13} \text{ units}\end{aligned}$$

$$\begin{aligned}CD &= \sqrt{(-9+3)^2 + (-1+3)^2 + (2-0)^2} \\&= \sqrt{36 + 4 + 4} \\&= 2\sqrt{11} \text{ units}\end{aligned}$$

$$\begin{aligned}DA &= \sqrt{(-3-4)^2 + (-3-3)^2 + 0} \\&= \sqrt{16 + 36} \\&= \sqrt{52} \\&= 2\sqrt{13} \text{ units}\end{aligned}$$

$$\begin{aligned}AC &= \sqrt{(1+9)^2 + (3+1)^2 + (0-2)^2} \\&= \sqrt{150 + 16 + 4} \\&= \sqrt{120} \\&= 4\sqrt{3} \text{ units}\end{aligned}$$

$$\begin{aligned}BD &= \sqrt{(-3+5)^2 + (-3-5)^2 + (0-2)^2} \\&= \sqrt{4 + 64 + 4} \\&= \sqrt{72} \\&= 6\sqrt{2} \text{ units}\end{aligned}$$

Since,

$$AB = CD \text{ and } BC = DA$$

\Rightarrow $ABCD$ is a parallelogram $\therefore BD$

but, $AC \neq BD$

\Rightarrow $ABCD$ is not a rectangle.

Q12

Here,

$$\begin{aligned} AB &= \sqrt{(1+1)^2 + (3-6)^2 + (4-10)^2} \\ &= \sqrt{4+9+36} \\ &= 7 \text{ units} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(-1+7)^2 + (6-4)^2 + (0-7)^2} \\ &= \sqrt{36+4+9} \\ &= 7 \text{ units} \end{aligned}$$

$$\begin{aligned} CD &= \sqrt{(-7+5)^2 + (4-1)^2 + (7-1)^2} \\ &= \sqrt{4+9+36} \\ &= 7 \text{ units} \end{aligned}$$

$$\begin{aligned} DA &= \sqrt{(-5-1)^2 + (1-3)^2 + (1-4)^2} \\ &= \sqrt{36+4+9} \\ &= \sqrt{52} \\ &= 7 \text{ units} \end{aligned}$$

Since, $AB = BC = CD = DA$

So, $ABCD$ is a rhombus.

Q13

Here,

$$\begin{aligned}AB &= \sqrt{(0-1)^2 + (1-0)^2 + (1-1)^2} \\&= \sqrt{1+1} \\&= \sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(1-1)^2 + (0-1)^2 + (1-0)^2} \\&= \sqrt{1+1} \\&= \sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}CA &= \sqrt{(1-0)^2 + (1-1)^2 + (0-1)^2} \\&= \sqrt{1+0+1} \\&= \sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}DA &= \sqrt{(0-0)^2 + (0-1)^2 + (0-1)^2} \\&= \sqrt{1+1} \\&= \sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}OB &= \sqrt{(0-1)^2 + (0-0)^2 + (0-1)^2} \\&= \sqrt{1+1} \\&= \sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}OC &= \sqrt{(0-1)^2 + (0-1)^2 + (0-0)^2} \\&= \sqrt{1+1} \\&= \sqrt{2} \text{ units}\end{aligned}$$

Since, $OA = OB = OC = AB = BC = CA$

So, O, A, B, C represent a regular tetrahedron

Q14

Here,

$$\begin{aligned}OA &= \sqrt{(1-3)^2 + (3-2)^2 + (4-2)^2} \\&= \sqrt{4+1+4} \\&= 3 \text{ units}\end{aligned}$$

$$\begin{aligned}OB &= \sqrt{(1+1)^2 + (3-1)^2 + (4-3)^2} \\&= \sqrt{4+4+1} \\&= 3 \text{ units}\end{aligned}$$

$$\begin{aligned}OC &= \sqrt{(1-0)^2 + (3-5)^2 + (4-6)^2} \\&= \sqrt{1+4+4} \\&= 3 \text{ units}\end{aligned}$$

$$\begin{aligned}OD &= \sqrt{(1-2)^2 + (3-1)^2 + (4-2)^2} \\&= \sqrt{1+4+4} \\&= 3 \text{ units}\end{aligned}$$

Since, $OA = OC = OD = OB$, points A, B, C, D lie on a sphere with centre O.

Radius = 3 units

Q15

Let the required point be $P(x_1, y_1, z_1)$

Here, $O(0, 0, 0)$, $A(2, 0, 0)$, $B(0, 3, 0)$, $C(0, 0, 8)$

Since, $(OP)^2 = (PA)^2$

$$(x-0)^2 + (y-0)^2 + (z-0)^2 = (x-2)^2 + (y-0)^2 + (z-0)^2$$

$$x^2 + y^2 + z^2 = x^2 - 4x + 4 + y^2 + z^2$$

$$4x = 4$$

$$x = 1$$

$$(OP)^2 = (PB)^2$$

$$(x-0)^2 + (y-0)^2 + (z-0)^2 = (x-0)^2 + (y-3)^2 + (z-0)^2$$

$$x^2 + y^2 + z^2 = x^2 + y^2 - 6y + 9 + z^2$$

$$6y = 9$$

$$y = \frac{3}{2}$$

$$(OP)^2 = (PC)^2$$

$$(x-0)^2 + (y-0)^2 + (z-0)^2 = (x-0)^2 + (y-0)^2 + (z-8)^2$$

$$x^2 + y^2 + z^2 = x^2 + y^2 + z^2 - 16z + 64$$

$$16z = 64$$

$$z = 4$$

The required point = $\left(1, \frac{3}{2}, 4\right)$

Q16

Let P be (x, y, z) , here, $A(-2, 2, 3)$ and $B(13, -3, 13)$
and $3PA = 2PB$

$$\Rightarrow 3\sqrt{(x+2)^2 + (y-2)^2 + (z-3)^2} = 2\sqrt{(x-13)^2 + (y+3)^2 + (z-13)^2}$$

squaring both the sides,

$$\begin{aligned} \Rightarrow & 9[x^2 + 4x + 4 + y^2 + 4 - 4y + z^2 + 9 - 6z] \\ & = 4[x^2 + 169 - 26x + y^2 + 9 + 6y + z^2 + 169 - 26z] \\ \Rightarrow & 9x^2 - 4x^2 + 36x + 104x + 36 - 676 + 9y^2 - 4y^2 \\ & + 36 - 36 - 36y - 24y + 9z^2 - 4z^2 + 81 - 676 - 54z + 6yz = 0 \\ \Rightarrow & 5x^2 + 5y^2 + 5z^2 + 140x - 60y + 50z - 1235 = 0 \\ \Rightarrow & 5(x^2 + y^2 + z^2) + 140x - 60y + 50z - 1235 = 0 \end{aligned}$$

Q17

Let $P(x, y, z)$, here, $A(3, 4, 5)$, $B(-1, 3, -7)$
 $PA^2 + PB^2 = 2k^2$

$$\begin{aligned} \Rightarrow & (x-3)^2 + (y-4)^2 + (z-5)^2 + (x+1)^2 + (y-3)^2 + (z+7)^2 = 2k^2 \\ \Rightarrow & x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z + x^2 + 1 + 2x \\ & + y^2 + 9 - 6y + z^2 + 49 + 14z = 2k^2 \\ \Rightarrow & 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = 2k^2 \\ \Rightarrow & 2(x^2 + y^2 + z^2) - 4x - 14y + 4z + 109 - 2k^2 = 0 \end{aligned}$$

Q18

Here, $A(a, b, c)$, $B(b, c, a)$, $C(c, a, b)$

$$\begin{aligned}AB &= \sqrt{(a-b)^2 + (b-c)^2 + (c-a)^2} \\&= \sqrt{a^2 + b^2 - 2ab + b^2 + c^2 - 2bc + c^2 + a^2 - 2ac}\end{aligned}$$

$$AB = \sqrt{2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac}$$

$$\begin{aligned}BC &= \sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2} \\&= \sqrt{b^2 + c^2 - 2bc + c^2 + a^2 - 2ca + a^2 + b^2 - 2ab}\end{aligned}$$

$$BC = \sqrt{2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca}$$

$$\begin{aligned}CA &= \sqrt{(a-c)^2 + (b-a)^2 + (c-b)^2} \\&= \sqrt{a^2 + c^2 - 2ac + b^2 + a^2 - 2ab + b^2 + c^2 - 2bc}\end{aligned}$$

$$CA = \sqrt{2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca}$$

Since, $AB = BC = CA$, so

$\triangle ABC$ is an isosceles \triangle

Q19

Here, $A(3, 6, 9)$, $B(10, 20, 30)$, $C(25, 41, 5)$

$$\begin{aligned}(AB)^2 &= (3-10)^2 + (6-20)^2 + (9-30)^2 \\&= (-7)^2 + (-14)^2 + (-21)^2 \\&= 49 + 196 + 441 \\&= 586\end{aligned}$$

$$\begin{aligned}(BC)^2 &= (10-25)^2 + (20-41)^2 + (30-5)^2 \\&= (-15)^2 + (-21)^2 + (25)^2 \\&= 225 + 441 + 625 \\&= 1291\end{aligned}$$

$$\begin{aligned}(CA)^2 &= (3-25)^2 + (6-41)^2 + (9-5)^2 \\&= (-22)^2 + (-35)^2 + (4)^2 \\&= 484 + 1225 + 16 \\&= 1725\end{aligned}$$

Since, $AB^2 + BC^2 \neq AC^2$

$$AB^2 + AC^2 \neq BC^2$$

$$BC^2 + AC^2 \neq AB^2$$

So, $\triangle ABC$ is not a right triangle.

Q20(i)

Here, $A(0, 7, -10)$, $B(1, 6, -6)$, $C(4, 9, -6)$

$$\begin{aligned}AB &= \sqrt{(0-1)^2 + (7-6)^2 + (-10+6)^2} \\&= \sqrt{1+1+16} \\&= 3\sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(1-4)^2 + (6-9)^2 + (-6+6)^2} \\&= \sqrt{9+9} \\&= 3\sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}AC &= \sqrt{(0-4)^2 + (7-9)^2 + (-10+6)^2} \\&= \sqrt{16+4+16} \\&= 6 \text{ units}\end{aligned}$$

Since, $AB = BC$

So, $\triangle ABC$ is an isosceles \triangle

Q20(ii)

Here, $A(0, 7, 10)$, $B(-1, 6, 6)$, $C(-4, 9, 6)$

$$\begin{aligned}AB &= \sqrt{(0+1)^2 + (7-6)^2 + (10-6)^2} \\&= \sqrt{1+1+16} \\&= 3\sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(-1+4)^2 + (6-9)^2 + (6-6)^2} \\&= \sqrt{9+9+0} \\&= 3\sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}CA &= \sqrt{(-4-0)^2 + (9-7)^2 + (6+10)^2} \\&= \sqrt{16+4+16} \\&= \sqrt{36} \text{ units}\end{aligned}$$

Since, $(AB)^2 + (BC)^2 = (AC)^2$

So, $\triangle ABC$ is a right triangle.

Q20(iii)

Here, $A(-1, 2, 1)$, $B(1, -2, 5)$, $C(4, -7, 8)$, $D(2, -3, 4)$

$$\begin{aligned}AB &= \sqrt{(-1-1)^2 + (2+2)^2 + (1-5)^2} \\&= \sqrt{4+16+16} \\&= 6 \text{ units}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(1-4)^2 + (-2+7)^2 + (5-8)^2} \\&= \sqrt{9+25+9} \\&= \sqrt{43} \text{ units}\end{aligned}$$

$$\begin{aligned}CD &= \sqrt{(4-2)^2 + (-7+3)^2 + (8-4)^2} \\&= \sqrt{4+16+16} \\&= 6 \text{ units}\end{aligned}$$

$$\begin{aligned}DA &= \sqrt{(2+1)^2 + (-3-2)^2 + (4-1)^2} \\&= \sqrt{9+25+9} \\&= \sqrt{43} \text{ units}\end{aligned}$$

Since, $AB = CD$ and $BC = DA$

So, $\triangle ABC$ is a parallelogram

Q20(iv)

Let $A(5, -1, 1)$, $B(7, -4, 7)$, $C(1, -6, 10)$ and $D(-1, -3, 4)$ be the given points.

$$AB = \sqrt{(7-5)^2 + (-4+1)^2 + (7-1)^2} = \sqrt{4+9+36} = 7$$

$$BC = \sqrt{(1-7)^2 + (-6+4)^2 + (10-7)^2} = \sqrt{36+4+9} = 7$$

$$CD = \sqrt{(-1-1)^2 + (-3+6)^2 + (4-10)^2} = \sqrt{4+9+36} = 7$$

$$AD = \sqrt{(-1-5)^2 + (-3+1)^2 + (4-1)^2} = \sqrt{36+4+9} = 7$$

So $AB = BC = CD = AD$

Hence ABCD is a rhombus.

Q21

Let the point $P(x_1, y_1, z)$ which is equidistance from $A(1, 2, 3)$ and $B(3, 2, -1)$, so

$$AP = BP$$

$$(AP)^2 = (BP)^2$$

$$(x-1)^2 + (y-2)^2 + (z-3)^2 = (x-3)^2 + (y-2)^2 + (z+1)^2$$

$$x^2 + 1 - 2x + y^2 + 4 - 4y + z^2 + 9 - 6z = x^2 + 9 - 6x + y^2 + 4 - 4y + z^2 + 1 + 2z$$

$$4x - 8z = 14 - 14$$

$$4x - 8z = 0$$

$$x - 2z = 0$$

Q22

Let locus of $P(x_1, y_1, z)$ is the required locus, so

$$PA + P = 10$$

$$\Rightarrow \sqrt{(x-4)^2 + (y-0)^2 + (z-0)^2} + \sqrt{(x+4)^2 + (y-0)^2 + (z-0)^2} = 10$$

$$\Rightarrow \sqrt{x^2 + 16 - 8x + y^2 + z^2} = 10 - \sqrt{x^2 + 8x + 16 + y^2 + z^2}$$

$$\Rightarrow x^2 + y^2 + z^2 - 8x + 16 = (10)^2 + (x^2 + y^2 + z^2 + 8x + 16) - 20\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$

$$\Rightarrow -8x + 16 - 100 - 8x - 16 = -20\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$

$$\Rightarrow -16x - 100 = -20\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$

$$\Rightarrow -4(4x + 25) = -20\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$

$$\Rightarrow (4x + 25) = 5\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$

squaring both the sides,

$$(4x + 25)^2 = 25(x^2 + y^2 + z^2 + 8x + 16)$$

$$\Rightarrow 16x^2 + 625 + 200x = 25(x^2 + y^2 + z^2 + 8x + 16)$$

$$\Rightarrow 16x^2 + 625 + 200x = 25x^2 + 25y^2 + 25z^2 + 200x + 400$$

$$\Rightarrow 9x^2 + 25y^2 + 25z^2 - 225 = 0$$

Q23

$$\begin{aligned}
 AB &= \sqrt{(1+1)^2 + (2+2)^2 + (3+1)^2} \\
 &= \sqrt{4+16+16} \\
 &= 6 \text{ units}
 \end{aligned}$$

$$\begin{aligned}
 BC &= \sqrt{(-1-2)^2 + (-2-3)^2 + (-1-2)^2} \\
 &= \sqrt{9+25+9} \\
 &= \sqrt{43} \text{ units}
 \end{aligned}$$

$$\begin{aligned}
 CD &= \sqrt{(2-4)^2 + (3-7)^2 + (2-6)^2} \\
 &= \sqrt{4+16+16} \\
 &= 6 \text{ units}
 \end{aligned}$$

$$\begin{aligned}
 DA &= \sqrt{(4-1)^2 + (7-2)^2 + (6-3)^2} \\
 &= \sqrt{9+25+9} \\
 &= \sqrt{43} \text{ units}
 \end{aligned}$$

Q24

Let the point be P (x, y, z)

Given

$$A=(3, 4, -5)$$

$$B=(-2, 1, 4)$$

$$PA=PB \Rightarrow PA^2=PB^2$$

$$PA^2 = (x-3)^2 + (y-4)^2 + (z+5)^2$$

$$PB^2 = (x+2)^2 + (y-1)^2 + (z-4)^2$$

$$PA^2=PB^2 \Rightarrow$$

$$(x-3)^2 + (y-4)^2 + (z+5)^2 = (x+2)^2 + (y-1)^2 + (z-4)^2$$

All square terms will be cancelled on both sides, we get

$$-6x+9-8y+16+10z+25=4x+4-2y+1-8z+16$$

$$10x+6y-18z-29=0 \text{ is the required equation}$$

Ex 28.3

Q1

We know that angle bisector divides opposite side in ratio of other two sides

\Rightarrow D divides BC in ratio of AB : AC

A(5, 4, 6), B(1, -1, 3) and C(4, 3, 2)

$$AB = \sqrt{16+25+9} = \sqrt{50} = 5\sqrt{2}$$

$$AC = \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$$

$$AB:AC=5:3=m:n$$

$$D(x,y,z) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n} \right)$$

Substitute values for m:n=5:3,

$$(x_1, y_1, z_1) = (1, -1, 3)$$

$$(x_2, y_2, z_2) = (4, 3, 2)$$

$$D = \left(\frac{23}{8}, \frac{3}{2}, \frac{19}{8} \right)$$

Q2

z-coordinate 8

A(2, -3, 4) and B(8, 0, 10)

DR's of AB = (6, 3, 6)

DR's of BC = (x-8, y-0, 8-10)

Given A, B, C lie on same line

So values of DR's should be proportional

$$\frac{x-8}{6} = \frac{y}{3} = \frac{8-10}{6}$$

So $x=6, y=-1$

point is (6, -1, 8)

Q3

If points are collinear then all points lie on same line
and DR's should be proportional

A(2, 3, 4), B(-1, 2, -3) and C(-4, 1, -10)

DR's of AB=(3, 1, 7)

DR's of BC=(3, 1, 7)

So A, B, C are collinear

Length of AC= $\sqrt{36+4+196} = \sqrt{236}$

Length of AB= $\sqrt{9+1+49} = \sqrt{59}$

Ratio is AC:AB=2:1

So C divides AB in ratio 2:1 externally

Q4

yz plane means $x=0$

Given (2, 4, 5) and (3, 5, 4)

assume ratio to be m:n

lets equate x-term

$$0 = \frac{3m+2n}{m+n}$$

$$3m = -2n$$

$$m:n = -2:3$$

which means yz plane divides the line in 2:3 ratio externally

Q5

(2, -1, 3) and (-1, 2, 1)

$$x+y+z=5$$

Assume plane divides line in ratio $\lambda:1$

so point P which is dividing line in $\lambda:1$ ratio is

$$P = \left(\frac{-\lambda+2}{\lambda+1}, \frac{2\lambda-1}{\lambda+1}, \frac{\lambda+3}{\lambda+1} \right)$$

P lies on plane $x+y+z=5$

$$-\lambda+2+2\lambda-1+\lambda+3=5\lambda+5$$

$$3\lambda = -1 \Rightarrow \lambda = -1:3$$

So plane dividing line in 1:3 ratio externally

Q6

$A(3, 2, -4), B(9, 8, -10)$ and $C(5, 4, -6)$

$$AC = \sqrt{4+4+4} = 2\sqrt{3}$$

$$AB = \sqrt{36+36+36} = 6\sqrt{3}$$

$$BC = \sqrt{16+16+16} = 4\sqrt{3}$$

$$AC : BC = 1 : 2$$

Q7

Given midpoints $D(-2, 3, 5), E(4, -1, 7)$ and $F(6, 5, 3)$

Assume D is midpoint of AB , E is midpoint of BC

F is midpoint of CA

$A(x_1, y_1, z_1) B(x_2, y_2, z_2) C(x_3, y_3, z_3)$

From midpoint formula, we get following equations

$$x_1 + x_2 = 4, x_2 + x_3 = 8, x_3 + x_1 = 12$$

$$y_1 + y_2 = 6, y_2 + y_3 = -2, y_3 + y_1 = 10$$

$$z_1 + z_2 = 10, z_2 + z_3 = 14, z_3 + z_1 = 6$$

Solving above set of equations we get

$$A = (0, 9, 1)$$

$$B = (-4, -3, 9)$$

$$C = (12, 1, 5)$$

Q8

$A(1, 2, 3), B(0, 4, 1), C(-1, -1, -3)$

Angle bisector at A divides BC in ratio of $AB : AC$

$$AB = \sqrt{1+4+4} = 3$$

$$AC = \sqrt{4+9+36} = 7$$

Assume D divides BC

$$m : n = 3 : 7$$

$$\text{so } D = \left(\frac{-3}{10}, \frac{25}{10}, \frac{-2}{10} \right)$$

Q9

(12, -4, 8) and (27, -9, 18)

Assume point P is dividing line in $\lambda:1$ ratio, we get

$$P = \left(\frac{27\lambda + 12}{\lambda + 1}, \frac{-9\lambda - 4}{\lambda + 1}, \frac{18\lambda + 8}{\lambda + 1} \right)$$

P lies on Sphere, so substitute in Sphere equation

$$x^2 + y^2 + z^2 = 504$$

$$9(9\lambda + 4)^2 + (9\lambda + 4)^2 + 4(9\lambda + 4)^2 = 504(\lambda + 1)^2$$

$$729\lambda^2 + 81\lambda^2 + 324\lambda^2 + 648\lambda + 72\lambda + 288\lambda + 144 + 16 + 64 = 504\lambda^2 + 1008\lambda + 504$$

$$(1134 - 504)\lambda^2 + (1008 - 1008)\lambda + 224 - 504 = 0$$

$$630\lambda^2 = 280$$

$$\lambda^2 = \frac{4}{9}$$

$$\lambda = 2:3$$

Q10

Assume ratio is $\lambda:1$

Plane is $ax + by + cz + d = 0$

points (x_1, y_1, z_1) and (x_2, y_2, z_2)

Assume point of intersection of line and plane is D

$$D = \left(\frac{\lambda x_2 + x_1}{\lambda + 1}, \frac{\lambda y_2 + y_1}{\lambda + 1}, \frac{\lambda z_2 + z_1}{\lambda + 1} \right)$$

As D lies on plane, substitute D in plane equation, we get

$$\lambda(ax_2 + by_2 + cz_2 + d) + ax_1 + by_1 + cz_1 + d = 0$$

$$\Rightarrow \lambda = -\frac{ax_1 + by_1 + cz_1 + d}{ax_2 + by_2 + cz_2 + d}$$

Q11

(1, 2, -3), (3, 0, 1) and (-1, 1, -4)

Centroid of Triangle is given by

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3} \right)$$

We know that

$$x_1+x_2=2$$

$$x_2+x_3=6$$

$$x_1+x_3=-2$$

$$\text{Adding all gives } \Rightarrow 2(x_1+x_2+x_3)=6$$

$$\text{so } x_1+x_2+x_3=3$$

$$\text{similarly, } y_1+y_2+y_3=3; z_1+z_2+z_3=-6$$

$$\text{Centroid} = (1, 1, -2)$$

Q12

Given Centroid (1, 1, 1)

A(3, -5, 7) and B(-1, 7, -6)

Equating terms, we get

$$1 = \frac{3-1+x_3}{3}$$

$$1 = \frac{-5+7+y_3}{3}$$

$$1 = \frac{7-6+z_3}{3}$$

$$(x_3, y_3, z_3) = (1, 1, 2)$$

Q13

Trisection points are those which divide line in ratio 1:2 or 2:1

P(4, 2, -6) and Q(10, -16, 6)

Consider 1:2 case, we get

$$\left(\frac{10+8}{3}, \frac{-16+4}{3}, \frac{6-12}{3} \right) = (6, -4, -2)$$

Consider 2:1 case, we get

$$\left(\frac{20+4}{3}, \frac{-32+2}{3}, \frac{12-6}{3} \right) = (8, -10, 2)$$

(6, -4, -2) and (8, -10, 2) are trisection points

Q14

A(2, -3, 4), B(-1, 2, 1) and C(0, 1/3, 2)

DR's of AB are (3, -5, 3)

DR's of BC are $(-1, \frac{5}{3}, -1)$

DR's of AC are $(2, \frac{-10}{3}, 2)$

Its clear that all DR's are proportional

Q15

P(3, 2, -4), Q(5, 4, -6) and R(9, 8, -10)

$$PQ = \sqrt{4+4+4} = 2\sqrt{3}$$

$$QR = \sqrt{16+16+16} = 4\sqrt{3}$$

$$PQ : QR = 1 : 2$$

Q16

(4, 8, 10) and (6, 10, -8) is divided by the yz-plane.

Equation of yz-plane is $x=0$

assume ratio is m:n

Equating x-term, we get

$$0 = \frac{6m+4n}{m+n}$$

$$m : n = -2 : 3$$

So YZ plane divides the line segment in ratio 2:3 externally