Exercise – 9.1

1. In a ΔABC, if ∠A = 55°, ∠B = 40°, find ∠C.

Sol:

Given $\angle A = 55°, \angle B = 40°$ then $\angle C = ?$

We know that

In $a\Delta ABC$ sum of all angles of triangle is $180°$

$i.e., \angle A + \angle B + \angle C = 180°$

$\Rightarrow 55° + 40° \angle C = 180°$

$\Rightarrow 95° + \angle C = 180°$

$\Rightarrow \angle C = 180° - 95°$

$\Rightarrow \angle C = 85°$

2. If the angles of a triangle are in the ratio 1: 2 : 3, determine three angles.

Sol:

Given that the angles of a triangle are in the ratio $1:2:3$

Let the angles be $a, 2a, 3a$

$\therefore$ We know that

Sum of all angles of triangles is $180°$

$a + 2a + 3a = 180°$

$\Rightarrow 6a = 180°$

$\Rightarrow a = \frac{180°}{6}$

$\Rightarrow a = 30°$

Since $a = 30°$

$2a = 2(30)° = 60°$

$3a = 3(30)° = 90°$

$\therefore$ angles are $a = 30°, 2a = 60°, 3a = 90°$

$\therefore$ Hence angles are $30°, 60°$ and $90°$

3. The angles of a triangle are (x − 40)°, (x − 20)° and ($\frac{1}{2}x - 10$)°. Find the value of x.

Sol:

Given that

The angles of a triangle are

$(x - 40°), (x - 20)°$ and $\left(\frac{x}{2} - 10\right)^{\circ}$

We know that

Sum of all angles of triangle is $180°$

$\therefore x-40°+x-20°+\frac{x}{2}-10°=180°$

$2x+\frac{x}{2}-70°=180°$

$\frac{5x}{2}=180+70°$

$5x=250°(2)$

$x=50°(2)$

$x=100°$

$\therefore x=100°$

4. The angles of a triangle are arranged in ascending order of magnitude. If the difference between two consecutive angles is 10°, find the three angles.

Sol:

Given that,

The difference between two consecutive angles is $10°$

Let $x, x+10, x+20$ be the consecutive angles differ by $10°$

$W \cdot K \cdot T$ sum of all angles of triangle is $180°$

$x+x+10+x+20=180°$

$3x+30=180°$

$\Rightarrow 3x=180-30° \Rightarrow 3x=150°$

$\Rightarrow x=50°$

$\therefore x=50°$

$\therefore$ The required angles are

$x, x+10$ and $x+20$

$x=50$

$x+10=50+10=60$

$x+20=50+10+10=70$

The difference between two consecutive angles is $10°$ then three angles are $50°, 60°$ and $70°$.

5. Two angles of a triangle are equal and the third angle is greater than each of those angles by 30°. Determine all the angles of the triangle.

Sol:

Given that,

Two angles are equal and the third angle is greater than each of those angles by $30°$.

Let $x, x, x+30$ be the angles of a triangle

We know that

Sum of all angles of a triangle is $180°$

$x+x+x+30=180°$

$3x+30=180°$

$\Rightarrow 3x=180°-30°$

$\Rightarrow 3x=150°$

$\Rightarrow x=\frac{150°}{3}$

$\Rightarrow x=50°$

$\therefore$ The angles are $x, x, x+30$

$x=50°$

$x+30=80°$

$\therefore$ The required angles are $50°, 50°, 80°$

6. If one angle of a triangle is equal to the sum of the other two, show that the triangle is a right triangle.

Sol:

If one angle of a triangle is equal to the sum of other two

i.e., $\angle B=\angle A+\angle C$

Now, in ΔABC

(Sum of all angles of triangle $180°$)

$\angle A+\angle B+\angle C=180°$

$\angle B+\angle B=180°$ $\quad\left[\because \angle B=\angle A+\angle C\right]$

$2\angle B=180°$

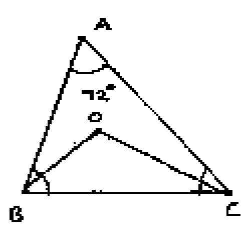
$\angle B=\frac{180°}{2}$

$\angle B=90°$

$\therefore ABC$ is a right angled a triangle.

7. ABC is a triangle in which ∠A — 72°, the internal bisectors of angles B and C meet in O. Find the magnitude of ∠ROC.

Sol:



Given,

ABC is a triangle

$\angle A = 72°$ and internal bisector of angles B and C meeting O

In $\Delta ABC = \angle A + \angle B + \angle C = 180°$

$\Rightarrow 72° + \angle B + \angle C = 180°$

$\Rightarrow \angle B + \angle C = 180° - 72°$ divide both sides by '2'

$\Rightarrow \frac{\angle B}{2} + \frac{\angle C}{2} = \frac{108°}{2}$ ....(1)

$\Rightarrow \angle OBC + \angle OCB = 54°$ ....(1)

Now in $\Delta BOC \Rightarrow \angle OBC + \angle OCB + \angle BOC = 180°$

$\Rightarrow 54° + \angle BOC = 180°$

$\Rightarrow \angle BOC = 180° - 54° = 126°$

$\therefore \angle BOC = 126°$

8. The bisectors of base angles of a triangle cannot enclose a right angle in any case.

Sol:

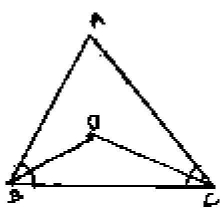
In a ΔABC

Sum of all angles of triangles is $180°$

i.e., $\angle A + \angle B + \angle C = 180°$ divide both sides by '2'

$\Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C = 180°$

$\Rightarrow \frac{1}{2}\angle A + \angle OBC + \angle OBC = 90°$ [$\because OB, OC$ insects $\angle B$ and $\angle C$]



$\Rightarrow \angle OBC + \angle OCB = 90° - \frac{1}{2}A$

Now in ΔBOC

$\therefore \angle BOC + \angle OBC + \angle OCB = 180°$

$\Rightarrow \angle BOC + 90° - \frac{1}{2}\angle A = 180°$

$\Rightarrow \angle BOC = 90° - \frac{1}{2}\angle A$

Hence, bisectors of a base angle cannot enclose right angle.

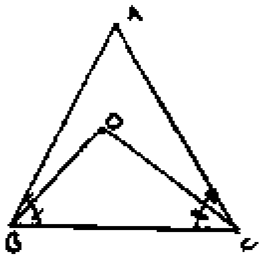
9. If the bisectors of the base angles of a triangle enclose an angle of 135°, prove that the triangle is a right triangle.

Sol:

Given the bisectors the base angles of an triangle enclose an angle of $135°$

i.e., $\angle BOC = 135°$

But, W.K.T



$\angle BOC = 90° + \frac{1}{2}\angle B$

$\Rightarrow 135° = 90° + \frac{1}{2}\angle A$

$\Rightarrow \frac{1}{2}\angle A = 135° - 90°$

$\Rightarrow \angle A = 45°(2)$

$\Rightarrow \angle A = 90°$

$\therefore \Delta ABC$ is right angled triangle right angled at A.

10. In a ΔABC, ∠ABC = ∠ACB and the bisectors of ∠ABC and ∠ACB intersect at O such that ∠BOC = 120°. Show that ∠A = ∠B = ∠C = 60°.

Sol:

Given,

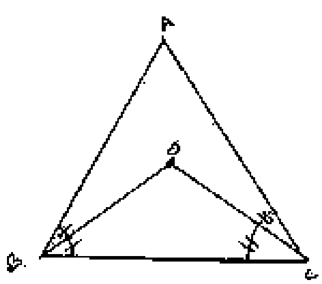
In ΔABC

$\angle ABC = \angle ACB$

Divide both sides by '2'

$\frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$

$\Rightarrow \angle OBC = \angle OCB$ [$\because OB, OC$ bisects $\angle B$ and $\angle C$]



Now

$\angle BOC = 90° + \frac{1}{2}\angle A$

$\Rightarrow 120° - 90° = \frac{1}{2}\angle A$

$\Rightarrow 30° \times (2) = \angle A$

$\Rightarrow \angle A = 60°$

Now in ΔABC

$\angle A + \angle ABC + \angle ACB = 180°$ (Sum of all angles of a triangle)

$\Rightarrow 60° + 2\angle ABC = 180°$ $[\because \angle ABC = \angle ACB]$

$\Rightarrow 2\angle ABC = 180° - 60°$

$\Rightarrow \angle ABC = \frac{120°}{2} = 90°$

$\Rightarrow \angle ABC = \angle ACB$

$\therefore \angle ACB = 60°$

Hence proved.

11. Can a triangle have:

(i) Two right angles?

(ii) Two obtuse angles?

(iii) Two acute angles?

(iv) All angles more than 60°?

(v) All angles less than 60°?

(vi) All angles equal to 60°?

Justify your answer in each case.

Sol:

(i) No,

Two right angles would up to $180°$, So the third angle becomes zero. This is not possible, so a triangle cannot have two right angles.

[Since sum of angles in a triangle is $180°$]

(ii) No,

A triangle can't have 2 obtuse angles. Obtuse angle means more than $90°$ So that the sum of the two sides will exceed $180°$ which is not possible. As the sum of all three angles of a triangle is $180°$.

(iii) Yes

A triangle can have 2 acute angle. Acute angle means less the $90°$ angle

(iv) No,

Having angles-more than $60°$ make that sum more than $18°$. Which is not possible

[$\because$ The sum of all the internal angles of a triangle is $180°$]

(v) No,
Having all angles less than $60°$ will make that sum less than $180°$ which is not possible.
[$\because$ The sum of all the internal angles of a triangle is $180°$]

(vi) Yes
A triangle can have three angles are equal to $60°$. Then the sum of three angles equal to the $180°$. Which is possible such triangles are called as equilateral triangle.
[$\because$ The sum of all the internal angles of a triangle is $180°$]

12. If each angle of a triangle is less than the sum of the other two, show that the triangle is acute angled.

Sol:

Given each angle of a triangle less than the sum of the other two

$\therefore \angle A + \angle B + \angle C$

$\Rightarrow \angle A + \angle A < \angle A + \angle B + \angle C$

$\Rightarrow 2\angle A < 180°$ [Sum of all angles of a triangle]

$\Rightarrow \angle A < 90°$

Similarly $\angle B < 90°$ and $\angle C < 90°$

Hence, the triangles acute angled.