

# Ex 24.1

## Scalar or Dot Product Ex 24.1 Q1

(i)

$$\begin{aligned}\vec{a} \cdot \vec{b} &= (\hat{i} - 2\hat{j} + \hat{k}) \cdot (4\hat{i} - 4\hat{j} + 7\hat{k}) \\ &= (1)(4) + (-2)(-4) + (1)(7) \\ &= 4 + 8 + 7 \\ &= 19\end{aligned}$$

$$\vec{a} \cdot \vec{b} = 19$$

(ii)

$$\begin{aligned}\vec{a} \cdot \vec{b} &= (\hat{j} + 2\hat{k}) \cdot (2\hat{i} + \hat{k}) \\ &= (0 \times \hat{i} + \hat{j} + 2\hat{k}) \cdot (2\hat{i} + 0 \times \hat{j} + \hat{k}) \\ &= (0)(2) + (1)(0) + (2)(1) \\ &= 0 + 0 + 2\end{aligned}$$

$$\vec{a} \cdot \vec{b} = 2$$

(iii)

$$\begin{aligned}\vec{a} \cdot \vec{b} &= (\hat{j} - \hat{k}) \cdot (2\hat{i} + 3\hat{j} - 2\hat{k}) \\ &= (0 \times \hat{i} + \hat{j} - \hat{k}) \cdot (2\hat{i} + 3\hat{j} - 2\hat{k}) \\ &= (0)(2) + (1)(3) + (-1)(-2) \\ &= 0 + 3 + 2\end{aligned}$$

$$\vec{a} \cdot \vec{b} = 5$$

## Scalar or Dot Product Ex 24.1 Q2

(i)

$\vec{a}$  and  $\vec{b}$  are perpendicular

$$\begin{aligned}\Rightarrow \vec{a} \cdot \vec{b} &= 0 \\ \Rightarrow (\lambda\hat{i} + 2\hat{j} + \hat{k}) \cdot (4\hat{i} - 9\hat{j} + 2\hat{k}) &= 0 \\ \Rightarrow (\lambda)(4) + (2)(-9) + (1)(2) &= 0 \\ \Rightarrow 4\lambda - 18 + 2 &= 0 \\ \Rightarrow 4\lambda - 16 &= 0 \\ \Rightarrow 4\lambda &= 16 \\ \Rightarrow \lambda &= \frac{16}{4} \\ \Rightarrow \lambda &= 4\end{aligned}$$

(ii)

$\vec{a}$  and  $\vec{b}$  are perpendicular

$$\begin{aligned}\Rightarrow \vec{a} \cdot \vec{b} &= 0 \\ \Rightarrow (\lambda\hat{i} + 2\hat{j} + \hat{k}) \cdot (5\hat{i} - 9\hat{j} + 2\hat{k}) &= 0 \\ \Rightarrow (\lambda)(5) + (2)(-9) + (1)(2) &= 0 \\ \Rightarrow 5\lambda - 18 + 2 &= 0 \\ \Rightarrow 5\lambda - 16 &= 0 \\ \Rightarrow 5\lambda &= 16 \\ \Rightarrow \lambda &= \frac{16}{5}\end{aligned}$$

(iii)

$\vec{a}$  and  $\vec{b}$  are perpendicular

$$\begin{aligned}\Rightarrow \vec{a} \cdot \vec{b} &= 0 \\ \Rightarrow (2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (3\hat{i} + 2\hat{j} - \lambda\hat{k}) &= 0 \\ \Rightarrow (2)(3) + (3)(2) + (4)(-\lambda) &= 0 \\ \Rightarrow 6 + 6 - 4\lambda &= 0 \\ \Rightarrow 12 - 4\lambda &= 0 \\ \Rightarrow -4\lambda &= -12 \\ \Rightarrow \lambda &= \frac{-12}{-4} \\ \Rightarrow \lambda &= 3\end{aligned}$$

(iv)

$\vec{a}$  and  $\vec{b}$  are perpendicular

$$\begin{aligned}\Rightarrow \vec{a} \cdot \vec{b} &= 0 \\ \Rightarrow (\lambda\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (\hat{i} - \hat{j} + 3\hat{k}) &= 0 \\ \Rightarrow (\lambda)(1) + (3)(-1) + (2)(3) &= 0 \\ \Rightarrow \lambda - 3 + 6 &= 0 \\ \Rightarrow \lambda + 3 &= 0 \\ \Rightarrow \lambda &= -3\end{aligned}$$

### Scalar or Dot Product Ex 24.1 Q3

We know that, if  $\theta$  is the angle between  $\vec{a}$  and  $\vec{b}$ , then

$$\begin{aligned}\cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \\&= \frac{6}{4 \times 3} \\&= \frac{6}{12} \\ \cos \theta &= \frac{1}{2} \\ \theta &= \cos^{-1} \left( \frac{1}{2} \right) \\ \theta &= \frac{\pi}{3}\end{aligned}$$

Angle between  $\vec{a}$  and  $\vec{b} = \frac{\pi}{3}$

### Scalar or Dot Product Ex 24.1 Q4

$$\begin{aligned}(\vec{a} - 2\vec{b}) &= (\hat{i} - \hat{j}) - 2(-\hat{j} + 2\hat{k}) \\&= (\hat{i} - \hat{j}) + 2\hat{j} - 4\hat{k} \\&= (\hat{i} + \hat{j} - 4\hat{k})\end{aligned}$$

$$\begin{aligned}(\vec{a} + \vec{b}) &= (\hat{i} - \hat{j}) + (-\hat{j} + 2\hat{k}) \\&= \hat{i} - \hat{j} - \hat{j} + 2\hat{k} \\&= (\hat{i} - 2\hat{j} + 2\hat{k})\end{aligned}$$

Now,

$$\begin{aligned}(\vec{a} - 2\vec{b}) \cdot (\vec{a} + \vec{b}) &= (\hat{i} + \hat{j} - 4\hat{k}) \cdot (\hat{i} - 2\hat{j} + 2\hat{k}) \\&= (1)(1) + (1)(-2) + (-4)(2) \\&= 1 - 2 - 8 \\&= -9\end{aligned}$$

$$(\vec{a} - 2\vec{b}) \cdot (\vec{a} + \vec{b}) = -9$$

### Scalar or Dot Product Ex 24.1 Q5(i)

Let  $\theta$  be the angle between  $\vec{a}$  and  $\vec{b}$ , then

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \quad \dots (i)$$

$$\begin{aligned}\vec{a} \cdot \vec{b} &= (\hat{i} - \hat{j}) (\hat{j} + \hat{k}) \\ &= (\hat{i} - \hat{j} + 0 \times \hat{k}) (0 \times \hat{i} + \hat{j} + \hat{k}) \\ &= (1)(0) + (-1)(1) + (0)(1) \\ &= 0 - 1 + 0 \\ \vec{a} \cdot \vec{b} &= -1\end{aligned}$$

$$\begin{aligned}|\vec{a}| &= |\hat{i} - \hat{j}| \\ &= |\hat{i} - \hat{j} + 0 \times \hat{k}| \\ &= \sqrt{(1)^2 + (-1)^2 + (0)^2} \\ &= \sqrt{1+1+0} \\ |\vec{a}| &= \sqrt{2}\end{aligned}$$

$$\begin{aligned}|\vec{b}| &= |\hat{j} + \hat{k}| \\ &= |0 \times \hat{i} + \hat{j} + \hat{k}| \\ &= \sqrt{(0)^2 + (1)^2 + (1)^2} \\ &= \sqrt{0+1+1} \\ |\vec{b}| &= \sqrt{2}\end{aligned}$$

Put  $\vec{a} \cdot \vec{b}$ ,  $|\vec{a}|$  and  $|\vec{b}|$  in equation (i)

$$\begin{aligned}\cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \\ &= \frac{-1}{\sqrt{2} \times \sqrt{2}}\end{aligned}$$

$$\cos \theta = \frac{-1}{2}$$

$$\theta = \cos^{-1} \left( -\frac{1}{2} \right)$$

$$\theta = \pi - \frac{\pi}{3}$$

$$\theta = \frac{2\pi}{3}$$

Angle between  $\vec{a}$  and  $\vec{b} = \frac{2\pi}{3}$

### Scalar or Dot Product Ex 24.1 Q5(ii)

Let  $\theta$  be the angle between two vector  $\vec{a} = 3\hat{i} - 2\hat{j} - 6\hat{k}$  and  $\vec{b} = 4\hat{i} - \hat{j} + 8\hat{k}$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \dots (1)$$

$$\begin{aligned}\vec{a} \cdot \vec{b} &= (3\hat{i} - 2\hat{j} - 6\hat{k}) (4\hat{i} - \hat{j} + 8\hat{k}) \\ &= 3 \times 4 + (-2)(-1) + (-6)8 \\ &= 12 + 2 - 48 \\ &= -34\end{aligned}$$

$$\begin{aligned}|\vec{a}| &= \sqrt{3^2 + (-2)^2 + (-6)^2} \\ &= \sqrt{49} \\ &= 7\end{aligned}$$

$$\begin{aligned}|\vec{b}| &= \sqrt{4^2 + (-1)^2 + 8^2} \\ &= \sqrt{81} \\ &= 9\end{aligned}$$

Putting value of  $|\vec{a}|$ ,  $|\vec{b}|$  and  $\vec{a} \cdot \vec{b}$  in equation (1)

$$\begin{aligned}\cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \\ &= \frac{-34}{7 \times 9} \\ &= \frac{-34}{63} \\ \theta &= \cos^{-1} \left( \frac{-34}{63} \right) \\ &= 122.66^\circ\end{aligned}$$

### Scalar or Dot Product Ex 24.1 Q5(iii)

Let the angle between  $\vec{a}$  and  $\vec{b}$  be  $\theta$ , then

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \quad \dots (i)$$

$$\begin{aligned} \vec{a} \cdot \vec{b} &= (2\hat{i} - \hat{j} + 2\hat{k}) (4\hat{i} + 4\hat{j} - 2\hat{k}) \\ &= (2)(4) + (-1)(4) + (2)(-2) \\ &= 8 - 4 - 4 \end{aligned}$$

$$\vec{a} \cdot \vec{b} = 0$$

$$\begin{aligned} |\vec{a}| &= |2\hat{i} - \hat{j} + 2\hat{k}| \\ &= \sqrt{(2)^2 + (-1)^2 + (2)^2} \\ &= \sqrt{4 + 1 + 4} \\ &= \sqrt{9} \\ |\vec{a}| &= 3 \end{aligned}$$

$$\begin{aligned} |\vec{b}| &= |4\hat{i} + 4\hat{j} - 2\hat{k}| \\ &= \sqrt{(4)^2 + (4)^2 + (-2)^2} \\ &= \sqrt{16 + 16 + 4} \\ &= \sqrt{36} \\ |\vec{b}| &= 6 \end{aligned}$$

Put  $\vec{a} \cdot \vec{b}$ ,  $|\vec{a}|$  and  $|\vec{b}|$  in equation (i)

$$\begin{aligned} \cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \\ &= \frac{0}{3 \times 6} \\ &= \frac{0}{18} \\ \cos \theta &= 0 \\ \theta &= \cos^{-1}(0) \end{aligned}$$

Angle between  $\vec{a}$  and  $\vec{b}$  is  $\frac{\pi}{2}$

### Scalar or Dot Product Ex 24.1 Q5(iv)

Let  $\theta$  be the angle between vector  $\vec{a}$  and  $\vec{b}$ , then

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \quad \dots (i)$$

$$\begin{aligned} \vec{a} \cdot \vec{b} &= (2\hat{i} - 3\hat{j} + \hat{k}) (\hat{i} + \hat{j} - 2\hat{k}) \\ &= (2)(1) + (-3)(1) + (1)(-2) \\ &= 2 - 3 - 2 \\ \vec{a} \cdot \vec{b} &= -3 \end{aligned}$$

$$\begin{aligned} |\vec{a}| &= |2\hat{i} - 3\hat{j} + \hat{k}| \\ &= \sqrt{(2)^2 + (-3)^2 + (1)^2} \\ &= \sqrt{4 + 9 + 1} \\ &= \sqrt{14} \end{aligned}$$

$$\begin{aligned} |\vec{b}| &= |\hat{i} + \hat{j} - 2\hat{k}| \\ &= \sqrt{(1)^2 + (1)^2 + (-2)^2} \\ &= \sqrt{1 + 1 + 4} \\ |\vec{b}| &= \sqrt{6} \end{aligned}$$

Put  $\vec{a} \cdot \vec{b}$ ,  $|\vec{a}|$  and  $|\vec{b}|$  in equation (i),

$$\begin{aligned} \cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \\ &= \frac{-3}{\sqrt{14} \times \sqrt{6}} \\ \cos \theta &= \frac{-3}{\sqrt{84}} \\ \theta &= \cos^{-1} \left( \frac{-3}{\sqrt{84}} \right) \end{aligned}$$

Angle between vector  $\vec{a}$  and  $\vec{b}$  =  $\cos^{-1} \left( \frac{-3}{\sqrt{84}} \right)$

### Scalar or Dot Product Ex 24.1 Q5(v)

Let  $\theta$  be the angle between vector  $\vec{a}$  and  $\vec{b}$ , then

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \quad \dots (i)$$

$$\begin{aligned}\vec{a} \cdot \vec{b} &= (\hat{i} + 2\hat{j} - \hat{k}) (\hat{i} - \hat{j} + \hat{k}) \\ &= (1)(1) + (2)(-1) + (-1)(1) \\ &= 1 - 2 - 1 \\ \vec{a} \cdot \vec{b} &= -2\end{aligned}$$

$$\begin{aligned}|\vec{a}| &= |\hat{i} + 2\hat{j} - \hat{k}| \\ &= \sqrt{(1)^2 + (2)^2 + (-1)^2} \\ &= \sqrt{1 + 4 + 1} \\ &= \sqrt{6}\end{aligned}$$

$$\begin{aligned}|\vec{b}| &= |\hat{i} - \hat{j} + \hat{k}| \\ &= \sqrt{(1)^2 + (-1)^2 + (1)^2} \\ &= \sqrt{1 + 1 + 1} \\ |\vec{b}| &= \sqrt{3}\end{aligned}$$

Put  $\vec{a} \cdot \vec{b}$ ,  $|\vec{a}|$ ,  $|\vec{b}|$  in equation (i),

$$\begin{aligned}\cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \\ &= \frac{-2}{\sqrt{6}\sqrt{3}} \\ &= \frac{-2}{\sqrt{18}} \\ &= \frac{-2 \times \sqrt{2}}{3\sqrt{2} \times \sqrt{2}} \\ &= \frac{-2\sqrt{2}}{3 \times 2} \\ \cos \theta &= \frac{-\sqrt{2}}{3} \\ \theta &= \cos^{-1}\left(\frac{-\sqrt{2}}{3}\right)\end{aligned}$$

$$\text{Angle between vector } \vec{a} \text{ and } \vec{b} = \cos^{-1}\left(\frac{-\sqrt{2}}{3}\right)$$

### Scalar or Dot Product Ex 24.1 Q6

Component along x-, y- and z-axis are  $\hat{i}$ ,  $\hat{j}$  and  $\hat{k}$  respectively.

Let  $\theta_1$  be the angle between  $\vec{a}$  and  $\hat{i}$ .

$$\begin{aligned}\cos \theta_1 &= \frac{\vec{a} \cdot \hat{i}}{|\vec{a}| |\hat{i}|} \\ &= \frac{(\hat{i} - \hat{j} + \sqrt{2}\hat{k}) (\hat{i} + 0\hat{j} + 0\hat{k})}{|\hat{i} - \hat{j} + \sqrt{2}\hat{k}| |\hat{i} + 0\hat{j} + 0\hat{k}|} \\ &= \frac{(1)(1) + (-1)(0) + (\sqrt{2})(0)}{\sqrt{(1)^2 + (-1)^2 + (\sqrt{2})^2} \cdot \sqrt{(1)^2 + (0)^2 + (0)^2}} \\ &= \frac{1 + 0 + 0}{\sqrt{4}\sqrt{1}} \\ \cos \theta_1 &= \frac{1}{2}\end{aligned}$$

$$\theta_1 = \frac{\pi}{3}$$

Let  $\theta_2$  be the angle between  $\vec{a}$  and  $\hat{j}$ .

$$\begin{aligned}\cos \theta_2 &= \frac{\vec{a} \cdot \hat{j}}{|\vec{a}| |\hat{j}|} \\ &= \frac{(\hat{i} - \hat{j} + \sqrt{2}\hat{k}) (0\hat{i} + \hat{j} + 0\hat{k})}{\sqrt{(1)^2 + (-1)^2 + (\sqrt{2})^2} \cdot \sqrt{(0)^2 + (1)^2 + (0)^2}} \\ &= \frac{(1)(0) + (-1)(1) + (\sqrt{2})(0)}{\sqrt{1+1+2} \sqrt{1}} \\ &= \frac{-1}{\sqrt{4}\sqrt{1}} \\ &= \frac{-1}{2} \\ \cos \theta_2 &= -\frac{1}{2} \\ \theta_2 &= \pi - \frac{\pi}{3} \\ \theta_2 &= \frac{2\pi}{3}\end{aligned}$$

Let  $\theta_3$  be the angle between  $\vec{a}$  and  $\hat{k}$ , then

$$\begin{aligned}\cos \theta_3 &= \frac{\vec{a} \cdot \hat{k}}{|\vec{a}| |\hat{k}|} \\ &= \frac{(\hat{i} - \hat{j} + \sqrt{2}\hat{k}) (0\hat{i} + 0\hat{j} + \hat{k})}{\sqrt{(1)^2 + (-1)^2 + (\sqrt{2})^2} \cdot \sqrt{(0)^2 + (0)^2 + (1)^2}} \\ &= \frac{(1)(0) + (-1)(0) + (\sqrt{2})(1)}{\sqrt{1+1+2} \sqrt{1}} \\ &= \frac{\sqrt{2}}{\sqrt{4}\sqrt{1}} \\ \cos \theta_3 &= \frac{1}{\sqrt{2}} \\ \theta_3 &= \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) \\ \theta_3 &= \frac{\pi}{4}\end{aligned}$$

So, the angle between vector  $\vec{a}$  and x-axis is  $\frac{\pi}{3}$ , vector  $\vec{a}$  and y-axis is  $\frac{2\pi}{3}$ ,  
vector  $\vec{a}$  and z-axis is  $\frac{\pi}{4}$ .



**Scalar or Dot Product Ex 24.1 Q7(i)**

Let the required vector be  $x\hat{i} + y\hat{j} + z\hat{k}$

According to question,

$$\begin{aligned}(x\hat{i} + y\hat{j} + z\hat{k})(\hat{i} + \hat{j} - 3\hat{k}) &= 0 \\ (x)(1) + (y)(1) + (z)(-3) &= 0 \\ x + y - 3z &= 0 \quad \text{--- (i)}\end{aligned}$$

And,

$$\begin{aligned}(x\hat{i} + y\hat{j} + z\hat{k})(\hat{i} + 3\hat{j} - 2\hat{k}) &= 5 \\ (x)(1) + (y)(3) + (z)(-2) &= 5 \\ x + 3y - 2z &= 5 \quad \text{--- (ii)}\end{aligned}$$

And,

$$\begin{aligned}(x\hat{i} + y\hat{j} + z\hat{k})(2\hat{i} + \hat{j} + 4\hat{k}) &= 8 \\ (x)(2) + (y)(1) + (z)(4) &= 8 \\ 2x + y + 4z &= 8 \quad \text{--- (iii)}\end{aligned}$$

Subtracting (i) from (ii),

$$\begin{array}{rcl}x + 3y - 2z &= & 5 \\ x + y - 3z &= & 0 \\ \hline (-)(-) & (+) & \\ 2y + z &= & 5 \quad \text{--- (iv)}\end{array}$$

Subtracting  $2 \times$  (ii) from (iii),

$$\begin{array}{rcl}2x + y + 4z &= & 8 \\ 2x + 6y - 4z &= & 10 \\ \hline (-) & (-) & (+) & (-) \\ -5y + 8z &= & -2 \quad \text{--- (v)}\end{array}$$

Subtracting  $8 \times$  (iv) from (v),

$$\begin{array}{rcl}-5y + 8z &= & -2 \\ 18y + 8z &= & 40 \\ \hline (-) & (-) & (-) \\ -21y &= & -42 \quad \text{--- (vi)} \\ y &= & \frac{-42}{-21} \\ y &= & 2\end{array}$$

Put  $y = 2$  in equation (iv),

$$\begin{aligned}2y + z &= 5 \\ 2(2) + z &= 5 \\ 4 + z &= 5 \\ z &= 5 - 4 \\ z &= 1\end{aligned}$$

Put  $y = 2$  and  $z = 1$  in equation (i),

$$\begin{aligned}x + y - 3z &= 0 \\ x + (2) - 3(1) &= 0 \\ x + 2 - 3 &= 0 \\ x - 1 &= 0 \\ x &= 1\end{aligned}$$

The required vector  $= x\hat{i} + y\hat{j} + z\hat{k}$

The required vector  $= \hat{i} + 2\hat{j} + \hat{k}$

### Scalar or Dot Product Ex 24.1 Q8(i)

Here,  $\hat{a}$  and  $\hat{b}$  are unit vectors, then

$$|\hat{a}| = |\hat{b}| = 1$$

$$\begin{aligned} |\hat{a} + \hat{b}|^2 &= (\hat{a} + \hat{b})^2 \\ &= (\hat{a})^2 + (\hat{b})^2 + 2\hat{a} \cdot \hat{b} \\ &= |\hat{a}|^2 + |\hat{b}|^2 + 2\hat{a} \cdot \hat{b} \\ &= (1)^2 + (1)^2 + 2\hat{a} \cdot \hat{b} \\ |\hat{a} + \hat{b}|^2 &= 2 + 2\hat{a} \cdot \hat{b} \end{aligned}$$

$$|\hat{a} + \hat{b}|^2 = 2 + 2 \times |\hat{a}| |\hat{b}| \cos \theta \quad \left[ \text{Since } \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta \right]$$

$$\begin{aligned} |\hat{a} + \hat{b}|^2 &= 2 + 2 \times 1 \times 1 \times \cos \theta \\ &= 2 + 2 \cos \theta \end{aligned}$$

$$|\hat{a} + \hat{b}|^2 = 2(1 + \cos \theta)$$

$$|\hat{a} + \hat{b}|^2 = 2 \left( 2 \cos^2 \frac{\theta}{2} \right) \quad \left[ \text{Since } 1 + \cos \theta = 2 \cos^2 \frac{\theta}{2} \right]$$

$$|\hat{a} + \hat{b}|^2 = 4 \cos^2 \frac{\theta}{2}$$

$$|\hat{a} + \hat{b}| = \sqrt{4 \cos^2 \frac{\theta}{2}}$$

$$|\hat{a} + \hat{b}| = 2 \cos \frac{\theta}{2}$$

$$\cos \frac{\theta}{2} = \frac{1}{2} |\hat{a} + \hat{b}|$$

### Scalar or Dot Product Ex 24.1 Q8(ii)

Here,  $\hat{a}$  and  $\hat{b}$  are unit vectors

$$|\hat{a}| = |\hat{b}| = 1$$

$$\begin{aligned} \frac{|\hat{a} - \hat{b}|^2}{|\hat{a} + \hat{b}|^2} &= \frac{(\hat{a} - \hat{b})^2}{(\hat{a} + \hat{b})^2} \\ &= \frac{(\hat{a})^2 + (\hat{b})^2 - 2\hat{a} \cdot \hat{b}}{(\hat{a})^2 + (\hat{b})^2 + 2\hat{a} \cdot \hat{b}} \\ &= \frac{|\hat{a}|^2 + |\hat{b}|^2 - 2\hat{a} \cdot \hat{b}}{|\hat{a}|^2 + |\hat{b}|^2 + 2\hat{a} \cdot \hat{b}} \end{aligned}$$

$$\frac{|\hat{a} - \hat{b}|^2}{|\hat{a} + \hat{b}|^2} = \frac{(1)^2 + (1)^2 - 2|\hat{a}| |\hat{b}| \cos \theta}{(1)^2 + (1)^2 + 2|\hat{a}| |\hat{b}| \cos \theta} \quad \left[ \text{Since } \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta \right]$$

$$\frac{|\hat{a} - \hat{b}|^2}{|\hat{a} + \hat{b}|^2} = \frac{1 + 1 - 2(1)(1) \cos \theta}{1 + 1 + 2(1)(1) \cos \theta}$$

$$\begin{aligned} &= \frac{2 - 2 \cos \theta}{2 + 2 \cos \theta} \\ &= \frac{2(1 - \cos \theta)}{2(1 + \cos \theta)} \end{aligned}$$

$$= \frac{2 \times \sin^2 \frac{\theta}{2}}{2 \times \cos^2 \frac{\theta}{2}} \quad \left[ \text{Since } 1 - \cos \theta = 2 \sin^2 \frac{\theta}{2}, 1 + \cos \theta = 2 \cos^2 \frac{\theta}{2} \right]$$

$$\frac{|\hat{a} - \hat{b}|^2}{|\hat{a} + \hat{b}|^2} = \tan^2 \frac{\theta}{2}$$

$$\tan \frac{\theta}{2} = \frac{|\hat{a} - \hat{b}|}{|\hat{a} + \hat{b}|}$$



### Scalar or Dot Product Ex 24.1 Q9

Let  $\hat{a}$  and  $\hat{b}$  are two unit vectors

$$\text{Then, } |\hat{a}| = |\hat{b}| = 1$$

And sum of  $\hat{a}$  and  $\hat{b}$  is a unit vector, then

$$|\hat{a} + \hat{b}| = 1$$

Taking square of both the sides,

$$|\hat{a} + \hat{b}|^2 = (1)^2$$

$$(\hat{a} + \hat{b})^2 = 1$$

$$(\hat{a})^2 + (\hat{b})^2 + 2\hat{a} \cdot \hat{b} = 1$$

$$|\hat{a}|^2 + |\hat{b}|^2 + 2\hat{a} \cdot \hat{b} = 1$$

$$(1)^2 + (1)^2 + 2\hat{a} \cdot \hat{b} = 1$$

$$2 + 2\hat{a} \cdot \hat{b} = 1$$

$$2\hat{a} \cdot \hat{b} = 1 - 2$$

$$2\hat{a} \cdot \hat{b} = -1$$

$$\hat{a} \cdot \hat{b} = \frac{-1}{2} \quad \dots (i)$$

$$|\hat{a} - \hat{b}|^2 = (\hat{a} - \hat{b})^2$$

$$= (\hat{a})^2 + (\hat{b})^2 - 2\hat{a} \cdot \hat{b}$$

$$= |\hat{a}|^2 + |\hat{b}|^2 - 2 \times \hat{a} \cdot \hat{b}$$

$$= (1)^2 + (1)^2 - 2 \times \left(\frac{-1}{2}\right)$$

$$\text{Using equation (i)}$$

$$= 1 + 1 + \frac{2}{2}$$

$$= 1 + 1 + 1$$

$$|\hat{a} - \hat{b}|^2 = 3$$

$$|\hat{a} - \hat{b}| = \sqrt{3}$$

### Scalar or Dot Product Ex 24.1 Q10

Given that  $\vec{a}, \vec{b}, \vec{c}$  are mutually perpendicular, so,

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$$

and  $\vec{a}, \vec{b}$  and  $\vec{c}$  are unit vectors, so

$$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

Now,

$$|\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c})^2$$

$$= (\vec{a})^2 + (\vec{b})^2 + (\vec{c})^2 + 2\vec{a}\vec{b} + 2\vec{b}\vec{c} + 2\vec{c}\vec{a}$$

$$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(0) + 2(0) + 2(0)$$

$$= (1)^2 + (1)^2 + (1)^2 + 0$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = 1 + 1 + 1$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 = 3$$

$$|\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}$$



### Scalar or Dot Product Ex 24.1 Q11

Here,  $|\vec{a} + \vec{b}| = 60$

Squaring both the sides,

$$|\vec{a} + \vec{b}|^2 = (60)^2$$

$$(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = (60)^2$$

$$(\vec{a})^2 + (\vec{b})^2 + 2\vec{a} \cdot \vec{b} = 3600$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = 3600 \quad \text{--- (i)}$$

Now,  $|\vec{a} - \vec{b}| = 40$

Squaring both the sides,

$$|\vec{a} - \vec{b}|^2 = (40)^2$$

$$|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b} = 1600 \quad \text{--- (ii)}$$

Adding (i) and (ii),

$$2|\vec{a}|^2 + 2|\vec{b}|^2 + 2\vec{a} \cdot \vec{b} - 2\vec{a} \cdot \vec{b} = 3600 - 1600$$

$$2|\vec{a}|^2 + 2(46)^2 = 5200$$

$$2|\vec{a}|^2 = 5200 - 4232$$

$$2|\vec{a}|^2 = 968$$

$$|\vec{a}|^2 = \frac{968}{2}$$

$$|\vec{a}|^2 = 484$$

$$|\vec{a}| = \sqrt{484}$$

$$|\vec{a}| = 22$$

### Scalar or Dot Product Ex 24.1 Q12

Let  $\theta$  be the angle between  $\hat{i} + \hat{j} + \hat{k}$  and  $\hat{i}$

Then,

$$\begin{aligned} \cos \theta &= \frac{(\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{i})}{|\hat{i} + \hat{j} + \hat{k}| |\hat{i}|} \\ &= \frac{1}{\sqrt{3}} \\ &= \frac{1}{\sqrt{3}} \end{aligned}$$

Similarly, if  $\alpha$  and  $\gamma$  are angles that  $\hat{i} + \hat{j} + \hat{k}$  make with  $\hat{j}$  and  $\hat{k}$

Then,

$$\cos \alpha = \frac{1}{\sqrt{3}}$$

$$\text{and } \cos \gamma = \frac{1}{\sqrt{3}}$$

Therefore,  $\hat{i} + \hat{j} + \hat{k}$  is equally inclined to the three axes.

### Scalar or Dot Product Ex 24.1 Q13

We have,

$$\vec{a} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\vec{b} = \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k})$$

$$\vec{c} = \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k})$$

Then,

$$\begin{aligned} \vec{a} \cdot \vec{b} &= \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}) \cdot \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k}) \\ &= \frac{1}{49}(6 - 18 + 12) = 0 \end{aligned}$$

Similarly,

$$\vec{b} \cdot \vec{c} = \vec{a} \cdot \vec{c} = 0$$

$\therefore \vec{a}, \vec{b}, \vec{c}$  are mutually perpendicular

**Scalar or Dot Product Ex 24.1 Q14**

$$\text{Let } (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$$

$$|\vec{a}|^2 - |\vec{b}|^2 = 0$$

$$|\vec{a}|^2 = |\vec{b}|^2$$

$$|\vec{a}| = |\vec{b}|$$

$$\text{Let } |\vec{a}| = |\vec{b}|$$

Squaring both the sides.

$$|\vec{a}|^2 = |\vec{b}|^2$$

$$|\vec{a}|^2 - |\vec{b}|^2 = 0$$

$$(\vec{a})^2 - (\vec{b})^2 = 0$$

$$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$$

Thus,

$$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0 \Leftrightarrow |\vec{a}| = |\vec{b}|$$

**Scalar or Dot Product Ex 24.1 Q15**

If  $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ ,  $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$  and  $\vec{c} = \hat{i} + 3\hat{j} - \hat{k}$ ,  
find  $\lambda$

Given that  $\vec{a}$  is perpendicular to  $\lambda\vec{b} + \vec{c}$

$$\therefore \vec{a} \cdot (\lambda\vec{b} + \vec{c}) = 0$$

$$\lambda\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = 0$$

$$\lambda(2\hat{i} - \hat{j} + \hat{k}) \cdot (\hat{i} + \hat{j} - 2\hat{k}) + (2\hat{i} - \hat{j} + \hat{k}) \cdot (\hat{i} + 3\hat{j} - \hat{k}) = 0$$

$$\lambda(2 - 1 - 2) + (2 - 3 - 1) = 0$$

$$-\lambda - 2 = 0$$

$$\lambda = -2$$

**Scalar or Dot Product Ex 24.1 Q16**

$$\vec{p} = 5\hat{i} + \lambda\hat{j} - 3\hat{k} \text{ and } \vec{q} = \hat{i} + 3\hat{j} - 5\hat{k}$$

$$\vec{p} + \vec{q}$$

$$= 5\hat{i} + \lambda\hat{j} - 3\hat{k} + \hat{i} + 3\hat{j} - 5\hat{k}$$

$$= 6\hat{i} + (\lambda + 3)\hat{j} - 8\hat{k}$$

$$\vec{p} - \vec{q}$$

$$= 5\hat{i} + \lambda\hat{j} - 3\hat{k} - \hat{i} - 3\hat{j} + 5\hat{k}$$

$$= 4\hat{i} + (\lambda - 3)\hat{j} + 2\hat{k}$$

$$(\vec{p} + \vec{q}) \cdot (\vec{p} - \vec{q}) = 0$$

$$\Rightarrow [6\hat{i} + (\lambda + 3)\hat{j} - 8\hat{k}] \cdot [4\hat{i} + (\lambda - 3)\hat{j} + 2\hat{k}] = 0$$

$$\Rightarrow 24 + (\lambda^2 - 9) - 16 = 0$$

$$\Rightarrow \lambda^2 - 9 + 8 = 0$$

$$\Rightarrow \lambda^2 - 1 = 0$$

$$\therefore \lambda = \pm 1$$

### Scalar or Dot Product Ex 24.1 Q17

According to question  $\vec{\beta}_1$  is parallel to  $\vec{\alpha}$ . So

$$\begin{aligned}\vec{\beta}_1 &= \gamma \vec{\alpha} \\ &= \gamma (3\hat{i} + 4\hat{j} + 5\hat{k})\end{aligned}$$

$$\begin{aligned}\vec{\beta} &= \vec{\beta}_1 + \vec{\beta}_2 \\ 2\hat{i} + \hat{j} - 4\hat{k} &= \gamma (3\hat{i} + 4\hat{j} + 5\hat{k}) + \vec{\beta}_2 \quad (\text{putting } \vec{\beta} \text{ and } \vec{\beta}_1) \\ \vec{\beta}_2 &= (2 - 3\gamma)\hat{i} + (1 - 4\gamma)\hat{j} - (4 + 5\gamma)\hat{k}\end{aligned}$$

Again  $\vec{\beta}_2$  is perpendicular to  $\vec{\alpha}$ . So

$$\begin{aligned}\vec{\beta}_2 \cdot \vec{\alpha} &= 0 \\ [(2 - 3\gamma)\hat{i} + (1 - 4\gamma)\hat{j} - (4 + 5\gamma)\hat{k}] \cdot (3\hat{i} + 4\hat{j} + 5\hat{k}) &= 0 \\ 6 - 9\gamma + 4 - 16\gamma - 20 - 25\gamma &= 0 \\ -50\gamma &= 10 \\ \gamma &= -\frac{1}{5}\end{aligned}$$

$$\begin{aligned}\vec{\beta}_1 &= -\frac{1}{5}(3\hat{i} + 4\hat{j} + 5\hat{k}) \\ \vec{\beta} &= \vec{\beta}_1 + \vec{\beta}_2 \\ 2\hat{i} + \hat{j} - 4\hat{k} &= -\frac{1}{5}(3\hat{i} + 4\hat{j} + 5\hat{k}) + \vec{\beta}_2 \quad (\text{putting } \vec{\beta} \text{ and } \vec{\beta}_1) \\ \vec{\beta}_2 &= \frac{1}{5}(13\hat{i} + 9\hat{j} - 15\hat{k}) \\ \vec{\beta} &= -\frac{1}{5}(3\hat{i} + 4\hat{j} + 5\hat{k}) + \frac{1}{5}(13\hat{i} + 9\hat{j} - 15\hat{k})\end{aligned}$$

### Scalar or Dot Product Ex 24.1 Q18

Consider  $\vec{a} = 2\hat{i} + 4\hat{j} + 3\hat{k}$  and  $\vec{b} = 3\hat{i} + 3\hat{j} - 6\hat{k}$ .

Then,

$$\vec{a} \cdot \vec{b} = 2 \cdot 3 + 4 \cdot 3 + 3(-6) = 6 + 12 - 18 = 0$$

We now observe that:

$$|\vec{a}| = \sqrt{2^2 + 4^2 + 3^2} = \sqrt{29}$$

$$\therefore \vec{a} \neq \vec{0}$$

$$|\vec{b}| = \sqrt{3^2 + 3^2 + (-6)^2} = \sqrt{54}$$

$$\therefore \vec{b} \neq \vec{0}$$

Hence, the converse of the given statement need not be true.

### Scalar or Dot Product Ex 24.1 Q19

Here,

$$\begin{aligned}\vec{b} + \vec{c} &= (\hat{i} - 3\hat{j} + 5\hat{k}) (2\hat{i} + \hat{j} - 4\hat{k}) \\ &= 3\hat{i} + 2\hat{j} + \hat{k} \\ \vec{b} + \vec{c} &= \vec{a}\end{aligned}$$

$\therefore \vec{a}, \vec{b}, \vec{c}$  are represents the sides of a triangle.

$$\begin{aligned}|\vec{a}| &= \sqrt{(3)^2 + (-2)^2 + (1)^2} \\ &= \sqrt{9 + 4 + 1} \\ &= \sqrt{14}\end{aligned}$$

$$\begin{aligned}|\vec{b}| &= \sqrt{(1)^2 + (-3)^2 + (5)^2} \\ &= \sqrt{1 + 9 + 25} \\ |\vec{b}| &= \sqrt{35}\end{aligned}$$

$$\begin{aligned}|\vec{c}| &= \sqrt{(2)^2 + (1)^2 + (-4)^2} \\ &= \sqrt{4 + 1 + 16} \\ &= \sqrt{21}\end{aligned}$$

$$\begin{aligned}(\sqrt{21})^2 + (\sqrt{14})^2 &= (\sqrt{35})^2 \\ 21 + 14 &= 35 \\ 35 &= 35\end{aligned}$$

$$|\vec{c}|^2 + |\vec{a}|^2 = |\vec{b}|^2$$

$\therefore$  By the pythagorous theorem,

Triangle formed by  $\vec{a}, \vec{b}, \vec{c}$  is a right angled triangled.

### Scalar or Dot Product Ex 24.1 Q20

The given vectors are  $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$ ,  $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ , and  $\vec{c} = 3\hat{i} + \hat{j}$ .

Now,

$$\vec{a} + \lambda \vec{b} = (2\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + 2\hat{j} + \hat{k}) = (2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}$$

If  $(\vec{a} + \lambda \vec{b})$  is perpendicular to  $\vec{c}$ , then

$$(\vec{a} + \lambda \vec{b}) \cdot \vec{c} = 0.$$

$$\Rightarrow [(2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}] \cdot (3\hat{i} + \hat{j}) = 0$$

$$\Rightarrow (2 - \lambda)3 + (2 + 2\lambda)1 + (3 + \lambda)0 = 0$$

$$\Rightarrow 6 - 3\lambda + 2 + 2\lambda = 0$$

$$\Rightarrow -\lambda + 8 = 0$$

$$\Rightarrow \lambda = 8$$

Hence, the required value of  $\lambda$  is 8.

# Scalar or Dot Product Ex 24.1 Q21

$$\vec{A} = 0\hat{i} - \hat{j} - 2\hat{k}$$

$$\vec{B} = 3\hat{i} + \hat{j} + 4\hat{k}$$

$$\vec{C} = 5\hat{i} + 7\hat{j} + \hat{k}$$

$$\overrightarrow{AB} = \vec{B} - \vec{A}$$

$$= (3\hat{i} + \hat{j} + 4\hat{k}) - (0\hat{i} - \hat{j} - 2\hat{k})$$

$$= 3\hat{i} + \hat{j} + 4\hat{k} - 0\hat{i} + \hat{j} + 2\hat{k}$$

$$\overrightarrow{AB} = 3\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\overrightarrow{BC} = \vec{C} - \vec{B}$$

$$= (5\hat{i} + 7\hat{j} + \hat{k}) - (3\hat{i} + \hat{j} + 4\hat{k})$$

$$= 5\hat{i} + 7\hat{j} + \hat{k} - 3\hat{i} - \hat{j} - 4\hat{k}$$

$$\overrightarrow{BC} = 2\hat{i} + 6\hat{j} - 3\hat{k}$$

$$\overrightarrow{AC} = \vec{C} - \vec{A}$$

$$= (5\hat{i} + 7\hat{j} + \hat{k}) - (-\hat{j} - 2\hat{k})$$

$$= 5\hat{i} + 7\hat{j} + \hat{k} + \hat{j} + 2\hat{k}$$

$$\overrightarrow{AC} = 5\hat{i} + 8\hat{j} + 3\hat{k}$$

Angle between  $\overrightarrow{AB}$  and  $\overrightarrow{AC}$ ,

$$\cos A = \frac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{|\overrightarrow{AB}| |\overrightarrow{AC}|}$$

$$= \frac{(3\hat{i} + 2\hat{j} + 6\hat{k}) \cdot (5\hat{i} + 8\hat{j} + 3\hat{k})}{\sqrt{(3)^2 + (2)^2 + (6)^2} \sqrt{(5)^2 + (8)^2 + (3)^2}}$$

$$= \frac{(3)(5) + (2)(8) + (6)(3)}{\sqrt{9 + 4 + 36} \sqrt{25 + 64 + 9}}$$

$$= \frac{15 + 16 + 18}{\sqrt{49} \sqrt{98}}$$

$$= \frac{49}{\sqrt{49} \sqrt{49 \times 2}}$$

$$\cos A = \frac{49}{49\sqrt{2}}$$

$$\cos A = \frac{1}{\sqrt{2}}$$

$$A = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{\pi}{4}$$

$$\angle A = \frac{\pi}{4}$$

Angle between  $\overrightarrow{BC}$  and  $\overrightarrow{BA}$

$$\cos B = \frac{\overrightarrow{BC} \cdot \overrightarrow{BA}}{|\overrightarrow{BC}| |\overrightarrow{BA}|}$$

$$= \frac{(2\hat{i} + 6\hat{j} - 3\hat{k}) \cdot (-3\hat{i} - 2\hat{j} - 6\hat{k})}{\sqrt{(2)^2 + (6)^2 + (-3)^2} \sqrt{(-3)^2 + (-2)^2 + (-6)^2}}$$

$$= \frac{(2)(-3) + (6)(-2) + (-3)(-6)}{\sqrt{4 + 36 + 9} \sqrt{9 + 4 + 36}}$$

$$= \frac{-6 - 12 + 18}{\sqrt{49} \sqrt{98}}$$

$$\cos B = \frac{-18 + 18}{49}$$

$$= \frac{0}{49}$$

$$\cos B = 0$$

$$B = \cos^{-1}(0)$$

$$\angle B = \frac{\pi}{2}$$

We know that,

$$\angle A + \angle B + \angle C = \pi$$

$$\frac{\pi}{4} + \frac{\pi}{2} + \angle C = \pi$$

$$\frac{3\pi}{4} + \angle C = \pi$$

$$\angle C = \frac{\pi}{4} - \frac{3\pi}{4}$$

$$\angle C = \frac{4\pi - 3\pi}{4}$$

$$\angle C = \frac{\pi}{4}$$

$$\angle A = \frac{\pi}{4}$$

$$\angle B = \frac{\pi}{2}$$

#### Scalar or Dot Product Ex 24.1 Q22

Let  $\theta$  be the angle between the vectors  $\vec{a}$  and  $\vec{b}$ .

It is given that  $|\vec{a}| = |\vec{b}|$ ,  $\vec{a} \cdot \vec{b} = \frac{1}{2}$ , and  $\theta = 60^\circ$ . ... (1)

We know that  $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$ .

$$\therefore \frac{1}{2} = |\vec{a}| |\vec{a}| \cos 60^\circ \quad [\text{Using (1)}]$$

$$\Rightarrow \frac{1}{2} = |\vec{a}|^2 \times \frac{1}{2}$$

$$\Rightarrow |\vec{a}|^2 = 1$$

$$\Rightarrow |\vec{a}| = |\vec{b}| = 1$$

#### Scalar or Dot Product Ex 24.1 Q23

Given

$$\vec{a} = 4\hat{i} - 3\hat{j} + \hat{k}$$

$$\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$$

$$\vec{c} = \hat{i} - \hat{j}$$

$\overrightarrow{AB}$  = Position vector of B - Position vector of A

$$= (2\hat{i} - 4\hat{j} + 5\hat{k}) - (4\hat{i} - 3\hat{j} + \hat{k})$$

$$= 2\hat{i} - 4\hat{j} + 5\hat{k} - 4\hat{i} + 3\hat{j} - \hat{k}$$

$$\overrightarrow{AB} = -2\hat{i} - \hat{j} + 4\hat{k}$$

$\overrightarrow{BC}$  = Position vector of C - Position vector of B

$$= (\hat{i} - \hat{j}) - (2\hat{i} - 4\hat{j} + 5\hat{k})$$

$$= \hat{i} - \hat{j} - 2\hat{i} + 4\hat{j} - 5\hat{k}$$

$$= -\hat{i} + 3\hat{j} - 5\hat{k}$$

$\overrightarrow{CA}$  = Position vector of A - Position vector of C

$$= (4\hat{i} - 3\hat{j} + \hat{k}) - (\hat{i} - \hat{j})$$

$$= 4\hat{i} - 3\hat{j} + \hat{k} - \hat{i} + \hat{j}$$

$$= 3\hat{i} - 2\hat{j} + \hat{k}$$

Now,  $\overrightarrow{AB} \cdot \overrightarrow{CA}$

$$= (-2\hat{i} - \hat{j} + 4\hat{k}) \cdot (3\hat{i} - 2\hat{j} + \hat{k})$$

$$= (-2)(3) + (-1)(-2) + (4)(1)$$

$$= -6 + 2 + 4$$

$$= -6 + 6$$

$$= 0$$

So,  $\overrightarrow{AB}$  is perpendicular to  $\overrightarrow{CA}$

$\angle A$  is right angle.

Hence,  $ABC$  is a right triangle



# Scalar or Dot Product Ex 24.1 Q24

Given,

$$A = (1, 2, 3)$$

$$B = (-1, 0, 0)$$

$$C = (0, 1, 2)$$

$$\text{Position vector of } A = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\text{Position vector of } B = -\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\text{Position vector of } C = 0\hat{i} + \hat{j} + 2\hat{k}$$

$$\overrightarrow{AB} = \text{Position vector of } B - \text{Position vector of } A$$

$$= (-\hat{i} + 0\hat{j} + 0\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= -2\hat{i} - 2\hat{j} - 3\hat{k}$$

$$\overrightarrow{BC} = \text{Position vector of } C - \text{Position vector of } B$$

$$= (0\hat{i} + \hat{j} + 2\hat{k}) - (-\hat{i} + 0\hat{j} + 0\hat{k})$$

$$= \hat{i} + \hat{j} + 2\hat{k}$$

$$\overrightarrow{AC} = \text{Position vector of } C - \text{Position vector of } A$$

$$= (0\hat{i} + \hat{j} + 2\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= -\hat{i} - \hat{j} - \hat{k}$$

$$\overrightarrow{AB} \cdot \overrightarrow{BC} = (-2\hat{i} - 2\hat{j} - 3\hat{k}) \cdot (\hat{i} + \hat{j} + 2\hat{k})$$

$$= -2 - 2 - 6$$

$$= -10$$

$$\angle ABC = \frac{\overrightarrow{AB} \cdot \overrightarrow{BC}}{|\overrightarrow{AB}| |\overrightarrow{BC}|}$$

$$= \frac{-10}{\sqrt{(-2)^2 + (-2)^2 + (-3)^2} \sqrt{1^2 + 1^2 + 2^2}}$$

$$= \frac{-10}{\sqrt{17} \sqrt{6}}$$

$$= \frac{-10}{\sqrt{102}}$$

$$\angle ABC = \cos^{-1} \left( \frac{-10}{\sqrt{102}} \right)$$

## Scalar or Dot Product Ex 24.1 Q25

Given

$$A = (0, 1, 1)$$

$$B = (3, 1, 5)$$

$$C = (0, 3, 3)$$

$$\text{Position vector of } A = 0\hat{i} + \hat{j} + \hat{k}$$

$$\text{Position vector of } B = 3\hat{i} + \hat{j} + 5\hat{k}$$

$$\text{Position vector of } C = 0\hat{i} + 3\hat{j} + 3\hat{k}$$

$$\overrightarrow{AB} = \text{Position vector of } B - \text{Position vector of } A$$

$$= (3\hat{i} + \hat{j} + 5\hat{k}) - (0\hat{i} + \hat{j} + \hat{k})$$

$$= 3\hat{i} + \hat{j} + 5\hat{k} - \hat{j} - \hat{k}$$

$$\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$$

$$\overrightarrow{BC} = \text{Position vector of } C - \text{Position vector of } B$$

$$= (0\hat{i} + 3\hat{j} + 3\hat{k}) - (3\hat{i} + \hat{j} + 5\hat{k})$$

$$\overrightarrow{BC} = 3\hat{j} + 3\hat{k} - 3\hat{i} - \hat{j} - 5\hat{k}$$

$$= -3\hat{i} + 2\hat{j} - 2\hat{k}$$

$$\overrightarrow{AC} = \text{Position vector of } C - \text{Position vector of } A$$

$$= (-3\hat{j} + 3\hat{k}) - (\hat{j} + \hat{k})$$

$$= 3\hat{j} + 3\hat{k} - \hat{j} - \hat{k}$$

$$= 2\hat{j} + 2\hat{k}$$

$$\begin{aligned}
 & \overrightarrow{BC} \cdot \overrightarrow{AC} \\
 &= (-3\hat{i} + 2\hat{j} - 2\hat{k}) \cdot (2\hat{j} + 2\hat{k}) \\
 &= (-3)(0) + (2)(2) + (-2)(2) \\
 &= 0 + 4 - 4 \\
 &= 0
 \end{aligned}$$

So,  $\overrightarrow{BC}$  and  $\overrightarrow{AC}$  is perpendicular

$\Rightarrow \angle C$  is right angle.

#### Scalar or Dot Product Ex 24.1 Q26

Projection of  $(\vec{b} + \vec{c})$  on  $\vec{a}$

$$\begin{aligned}
 &= \frac{(\vec{b} + \vec{c}) \cdot \vec{a}}{|\vec{a}|} \\
 &= \frac{\vec{b} \cdot \vec{a} + \vec{c} \cdot \vec{a}}{\sqrt{(2)^2 + (-2)^2 + (1)^2}} \\
 &= \frac{(\hat{i} + 2\hat{j} - 2\hat{k}) \cdot (2\hat{i} - 2\hat{j} + \hat{k}) + (2\hat{i} - \hat{j} + 4\hat{k}) \cdot (2\hat{i} - 2\hat{j} + \hat{k})}{\sqrt{4 + 4 + 1}} \\
 &= \frac{(1)(2) + (2)(-2) + (-2)(1) + (2)(2) + (-1)(-2) + (4)(1)}{\sqrt{9}} \\
 &= \frac{2 - 4 - 2 + 4 + 2 + 4}{3} \\
 &= \frac{12 - 6}{3} = \frac{6}{3} = 2
 \end{aligned}$$

Projection of  $(\vec{b} + \vec{c}) = 2$

#### Scalar or Dot Product Ex 24.1 Q27

$$\begin{aligned}
 \vec{a} + \vec{b} &= (5\hat{i} - \hat{j} - 3\hat{k}) + (\hat{i} + 3\hat{j} - 5\hat{k}) \\
 &= 5\hat{i} - \hat{j} - 3\hat{k} + \hat{i} + 3\hat{j} - 5\hat{k} \\
 \vec{a} + \vec{b} &= 6\hat{i} + 2\hat{j} - 8\hat{k} \quad \text{--- (i)}
 \end{aligned}$$

$$\begin{aligned}
 \vec{a} - \vec{b} &= (5\hat{i} - \hat{j} - 3\hat{k}) - (\hat{i} + 3\hat{j} - 5\hat{k}) \\
 &= 5\hat{i} - \hat{j} - 3\hat{k} - \hat{i} - 3\hat{j} + 5\hat{k} \\
 \vec{a} - \vec{b} &= 4\hat{i} - 4\hat{j} + 2\hat{k} \quad \text{--- (ii)}
 \end{aligned}$$

$$\begin{aligned}
 \text{Now, } (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) &= (6\hat{i} + 2\hat{j} - 8\hat{k}) \cdot (4\hat{i} - 4\hat{j} + 2\hat{k}) \\
 &= (6)(4) + (2)(-4) + (-8)(2) \\
 &= 24 - 8 - 16 \\
 &= 0
 \end{aligned}$$

So,  $(\vec{a} + \vec{b})$  and  $(\vec{a} - \vec{b})$  are perpendicular.

#### Scalar or Dot Product Ex 24.1 Q28

Let unit vector  $\vec{a}$  have  $(a_1, a_2, a_3)$  components.

$$\Rightarrow \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

Since  $\vec{a}$  is a unit vector,  $|\vec{a}| = 1$ .

Also, it is given that  $\vec{a}$  makes angles  $\frac{\pi}{4}$  with  $\hat{i}$ ,  $\frac{\pi}{3}$  with  $\hat{j}$ , and an acute angle  $\theta$  with  $\hat{k}$ .

Then, we have:

$$\cos \frac{\pi}{4} = \frac{a_1}{|\vec{a}|}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = a_1 \quad [|\vec{a}| = 1]$$

$$\cos \frac{\pi}{3} = \frac{a_2}{|\vec{a}|}$$

$$\Rightarrow \frac{1}{2} = a_2 \quad [|\vec{a}| = 1]$$

$$\text{Also, } \cos \theta = \frac{a_3}{|\vec{a}|}$$

$$\Rightarrow a_3 = \cos \theta$$

Now,

$$|a| = 1$$

$$\Rightarrow \sqrt{a_1^2 + a_2^2 + a_3^2} = 1$$

$$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2 \theta = 1$$

$$\Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2 \theta = 1$$

$$\Rightarrow \frac{3}{4} + \cos^2 \theta = 1$$

$$\Rightarrow \cos^2 \theta = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$\therefore a_3 = \cos \frac{\pi}{3} = \frac{1}{2}$$

Hence,  $\theta = \frac{\pi}{3}$  and the components of  $\vec{a}$  are  $\left(\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}\right)$ .

#### Scalar or Dot Product Ex 24.1 Q29

$$\begin{aligned} & (3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b}) \\ &= 3\vec{a} \cdot 2\vec{a} + 3\vec{a} \cdot 7\vec{b} - 5\vec{b} \cdot 2\vec{a} - 5\vec{b} \cdot 7\vec{b} \\ &= 6\vec{a} \cdot \vec{a} + 21\vec{a} \cdot \vec{b} - 10\vec{a} \cdot \vec{b} - 35\vec{b} \cdot \vec{b} \\ &= 6|\vec{a}|^2 + 11\vec{a} \cdot \vec{b} - 35|\vec{b}|^2 \\ &= \mathbf{6 \cdot 2^2 + 11 \cdot 1 - 35 \cdot 1^2} \\ &= \mathbf{35 - 35} \\ &= \mathbf{0} \end{aligned}$$

#### Scalar or Dot Product Ex 24.1 Q30(i)

We have,

$$\begin{aligned} & (\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 8 \\ & \Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 8 \\ & \Rightarrow |\vec{x}|^2 - 1^2 = 8 \quad \text{since } |\vec{a}| = 1 \\ & \Rightarrow |\vec{x}|^2 = 8 + 1 \\ & \Rightarrow |\vec{x}|^2 = 9 \\ & \Rightarrow |\vec{x}| = 3 \end{aligned}$$

#### Scalar or Dot Product Ex 24.1 Q30(ii)

We have,

$$\begin{aligned} & (\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12 \\ & \Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 12 \\ & \Rightarrow |\vec{x}|^2 - 1^2 = 12 \quad \text{since } |\vec{a}| = 1 \\ & \Rightarrow |\vec{x}|^2 = 12 + 1 \\ & \Rightarrow |\vec{x}|^2 = 13 \\ & \Rightarrow |\vec{x}| = \sqrt{13} \end{aligned}$$

### Scalar or Dot Product Ex 24.1 Q31(i)

$$\text{Here, } (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 12$$

$$|\vec{a}|^2 - |\vec{b}|^2 = 12$$

$$(2|\vec{b}|)^2 - |\vec{b}|^2 = 12$$

$$\left[ \text{Using } |\vec{a}| = 2|\vec{b}| \right]$$

$$4|\vec{b}|^2 - |\vec{b}|^2 = 12$$

$$3|\vec{b}|^2 = 12$$

$$|\vec{b}|^2 = \frac{12}{3}$$

$$|\vec{b}|^2 = 4$$

$$|\vec{b}| = 2$$

$$|\vec{a}| = 2|\vec{b}| = 2(2)$$

$$|\vec{a}| = 4$$

$$|\vec{b}| = 2$$

### Scalar or Dot Product Ex 24.1 Q31(ii)

$$(\vec{a} \cdot \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$$

$$\Rightarrow \vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} - \vec{b} \cdot \vec{b} = 8$$

$$\Rightarrow |\vec{a}|^2 - |\vec{b}|^2 = 8$$

$$\Rightarrow (8|\vec{b}|)^2 - |\vec{b}|^2 = 8$$

$$\left[ |\vec{a}| = 8|\vec{b}| \right]$$

$$\Rightarrow 64|\vec{b}|^2 - |\vec{b}|^2 = 8$$

$$\Rightarrow 63|\vec{b}|^2 = 8$$

$$\Rightarrow |\vec{b}|^2 = \frac{8}{63}$$

$$\Rightarrow |\vec{b}| = \sqrt{\frac{8}{63}}$$

[Magnitude of a vector is non-negative]

$$\Rightarrow |\vec{b}| = \frac{2\sqrt{2}}{3\sqrt{7}}$$

$$|\vec{a}| = 8|\vec{b}| = \frac{8 \times 2\sqrt{2}}{3\sqrt{7}} = \frac{16\sqrt{2}}{3\sqrt{7}}$$

### Scalar or Dot Product Ex 24.1 Q31(iii)

$$\text{Here, } (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 3$$

$$|\vec{a}|^2 - |\vec{b}|^2 = 3$$

$$(2|\vec{b}|)^2 - |\vec{b}|^2 = 3$$

$$\left[ \text{Using } |\vec{a}| = 2|\vec{b}| \right]$$

$$4|\vec{b}|^2 - |\vec{b}|^2 = 3$$

$$3|\vec{b}|^2 = 3$$

$$|\vec{b}|^2 = \frac{3}{3}$$

$$|\vec{b}|^2 = 1$$

$$|\vec{b}| = 1$$

$$|\vec{a}| = 2|\vec{b}|$$

$$= 2(1)$$

$$|\vec{a}| = 2$$

$$|\vec{b}| = 1$$

Scalar or Dot Product Ex 24.1 Q32(i)

$$\begin{aligned} |\vec{a} - \vec{b}|^2 &= |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b} \\ &= (2)^2 + (5)^2 - 2(8) \\ &= 4 + 25 - 16 \\ |\vec{a} - \vec{b}|^2 &= 13 \end{aligned}$$

$$|\vec{a} - \vec{b}| = \sqrt{13}$$

Scalar or Dot Product Ex 24.1 Q32(ii)

$$\begin{aligned} |\vec{a} - \vec{b}|^2 &= |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b} \\ &= (3)^2 + (4)^2 - 2(1) \\ &= 9 + 16 - 2 \\ |\vec{a} - \vec{b}|^2 &= 23 \end{aligned}$$

$$|\vec{a} - \vec{b}| = \sqrt{23}$$

Scalar or Dot Product Ex 24.1 Q32(iii)

We have

$$\begin{aligned} |\vec{a} - \vec{b}|^2 &= (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b}) \\ &= \vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} - \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} \\ &= |\vec{a}|^2 - 2(\vec{a} \cdot \vec{b}) + |\vec{b}|^2 = (2)^2 - 2(4) + (3)^2 = 5 \end{aligned}$$

$$\therefore |\vec{a} - \vec{b}| = \sqrt{5}$$

Scalar or Dot Product Ex 24.1 Q33(i)

We have,

$$|\vec{a}| = \sqrt{3}, |\vec{b}| = 2 \text{ and } \vec{a} \cdot \vec{b} = \sqrt{6}$$

Let  $\theta$  be the angle between  $\vec{a}$  and  $\vec{b}$ . Then

$$\begin{aligned} \cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \\ &= \frac{\sqrt{6}}{\sqrt{3} \times 2} \\ &= \frac{1}{\sqrt{2}} \\ \theta &= \cos^{-1} \left( \frac{1}{\sqrt{2}} \right) \\ &= \frac{\pi}{4} \end{aligned}$$

Scalar or Dot Product Ex 24.1 Q33(ii)

Let the angle between  $\vec{a}$  and  $\vec{b}$  is  $\theta$ , then

$$\begin{aligned} \cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \\ &= \frac{1}{3 \cdot 3} \\ \cos \theta &= \frac{1}{9} \\ \theta &= \cos^{-1} \left( \frac{1}{9} \right) \end{aligned}$$

### Scalar or Dot Product Ex 24.1 Q34

$$\text{Let } \vec{a} = \vec{u} + \vec{v}$$

$$5\hat{i} - 2\hat{j} + 5\hat{k} = \vec{u} + \vec{v} \quad \text{--- (i)}$$

Such that  $\vec{u}$  is parallel to  $\vec{b}$  and  $\vec{v}$  is perpendicular to  $\vec{b}$ .

Now,  $\vec{u}$  is parallel to  $\vec{b}$

$$\vec{u} = \lambda \vec{b}$$

$$= \lambda (3\hat{i} + \hat{k})$$

$$\vec{u} = 3\lambda\hat{i} + \lambda\hat{k} \quad \text{--- (ii)}$$

Put value of  $\vec{u}$  in equation (i),

$$5\hat{i} - 2\hat{j} + 5\hat{k} = (3\lambda\hat{i} + \lambda\hat{k}) + \vec{v}$$

$$\vec{v} = 5\hat{i} - 2\hat{j} + 5\hat{k} - 3\lambda\hat{i} - \lambda\hat{k}$$

$$\vec{v} = (5 - 3\lambda)\hat{i} + (-2)\hat{j} + (5 - \lambda)\hat{k}$$

$\vec{v}$  is perpendicular to  $\vec{b}$

$$\text{Then, } \vec{v} \cdot \vec{b} = 0$$

$$[(5 - 3\lambda)\hat{i} + (-2)\hat{j} + (5 - \lambda)\hat{k}] \cdot (3\hat{i} + 0\hat{j} + \hat{k}) = 0$$

$$(5 - 3\lambda)(3) + (-2)(0) + (5 - \lambda)(1) = 0$$

$$15 - 9\lambda + 0 + 5 - \lambda = 0$$

$$20 - 10\lambda = 0$$

$$-10\lambda = -20$$

$$\lambda = \frac{-20}{-10}$$

$$\lambda = 2$$

Put  $\lambda$  in equation (ii)

$$\vec{u} = 3\lambda\hat{i} + \lambda\hat{k}$$

$$= 3(2)\hat{i} + (2)\hat{k}$$

$$\vec{u} = 6\hat{i} + 2\hat{k}$$

Put the value of  $\vec{u}$  in equation (i)

$$5\hat{i} - 2\hat{j} + 5\hat{k} = \vec{u} + \vec{v}$$

$$5\hat{i} - 2\hat{j} + 5\hat{k} = (6\hat{i} + 2\hat{k}) + \vec{v}$$

$$\vec{v} = 5\hat{i} - 2\hat{j} + 5\hat{k} - 6\hat{i} - 2\hat{k}$$

$$\vec{v} = -\hat{i} - 2\hat{j} + 3\hat{k}$$

$$\vec{a} = (6\hat{i} + 2\hat{k}) + (-\hat{i} - 2\hat{j} + 3\hat{k})$$

### Scalar or Dot Product Ex 24.1 Q35

Vectors  $\vec{a}$  and  $\vec{b}$  have same magnitude, then

$$|\vec{a}| = |\vec{b}| = x \quad (\text{Say})$$

Let  $\theta$  be the angle between  $\vec{a}$  and  $\vec{b}$ , then

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$$\cos 30^\circ = \frac{3}{x \cdot x}$$

$$\frac{\sqrt{3}}{2} = \frac{3}{x^2}$$

$$\sqrt{3}x^2 = 6$$

$$x^2 = \frac{6}{\sqrt{3}}$$

Rationalizing the denominator,

$$x^2 = \frac{6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$x^2 = \frac{6\sqrt{3}}{3}$$

$$x^2 = 2\sqrt{3}$$

$$x = \sqrt{2\sqrt{3}}$$

$$|\vec{a}| = |\vec{b}| = \sqrt{2\sqrt{3}}$$



# Scalar or Dot Product Ex 24.1 Q36

$$\text{Let } (2\hat{i} - \hat{j} + 3\hat{k}) = \vec{a} + \vec{b} \quad \text{--- (i)}$$

Such that  $\vec{a}$  is a vector parallel to vector  $(2\hat{i} + 4\hat{j} - 2\hat{k})$  and  $\vec{b}$  is a vector perpendicular to the vector  $(2\hat{i} + 4\hat{j} - 2\hat{k})$ .

Since,  $\vec{a}$  is parallel to  $(2\hat{i} + 4\hat{j} - 2\hat{k})$

$$\begin{aligned} \vec{a} &= \lambda (2\hat{i} + 4\hat{j} - 2\hat{k}) \\ \vec{a} &= 2\lambda\hat{i} + 4\lambda\hat{j} - 2\lambda\hat{k} \quad \text{--- (ii)} \end{aligned}$$

Put value of  $\vec{a}$  in equation (i),

$$\begin{aligned} (2\hat{i} - \hat{j} + 3\hat{k}) &= (2\lambda\hat{i} + 4\lambda\hat{j} - 2\lambda\hat{k}) + \vec{b} \\ \vec{b} &= 2\hat{i} - \hat{j} + 3\hat{k} - 2\lambda\hat{i} - 4\lambda\hat{j} + 2\lambda\hat{k} \\ \vec{b} &= (2 - 2\lambda)\hat{i} + (-1 - 4\lambda)\hat{j} + (3 + 2\lambda)\hat{k} \end{aligned}$$

$\vec{b}$  is a vector perpendicular to the vector  $(2\hat{i} + 4\hat{j} - 2\hat{k})$ , then

$$\begin{aligned} \vec{b} \cdot (2\hat{i} + 4\hat{j} - 2\hat{k}) &= 0 \\ [(2 - 2\lambda)\hat{i} + (-1 - 4\lambda)\hat{j} + (3 + 2\lambda)\hat{k}] \cdot (2\hat{i} + 4\hat{j} - 2\hat{k}) &= 0 \\ (2 - 2\lambda)(2) + (-1 - 4\lambda)(4) + (3 + 2\lambda)(-2) &= 0 \\ 4 - 4\lambda - 4 - 16\lambda - 6 - 4\lambda &= 0 \\ -6 - 24\lambda &= 0 \\ -24\lambda &= 6 \\ \lambda &= -\frac{1}{4} \end{aligned}$$

Put  $\lambda$  in equation (ii),

$$\begin{aligned} \vec{a} &= 2\lambda\hat{i} + 4\lambda\hat{j} - 2\lambda\hat{k} \\ &= 2\left(-\frac{1}{4}\right)\hat{i} + 4\left(-\frac{1}{4}\right)\hat{j} - 2\left(-\frac{1}{4}\right)\hat{k} \\ \vec{a} &= -\frac{1}{2}\hat{i} - \hat{j} + \frac{1}{2}\hat{k} \end{aligned}$$

Put the value of  $\vec{a}$  in equation (i),

$$\begin{aligned} (2\hat{i} - \hat{j} + 3\hat{k}) &= \left(-\frac{1}{2}\hat{i} - \hat{j} + \frac{1}{2}\hat{k}\right) + \vec{b} \\ \vec{b} &= 2\hat{i} - \hat{j} + 3\hat{k} + \frac{1}{2}\hat{i} + \hat{j} - \frac{1}{2}\hat{k} \\ &= \frac{4\hat{i} - 2\hat{j} + 6\hat{k} + \hat{i} + 2\hat{j} - \hat{k}}{2} \\ &= \frac{5\hat{i} + 5\hat{k}}{2} \\ \vec{b} &= \frac{5}{2}(\hat{i} + \hat{k}) \\ (2\hat{i} - \hat{j} + 3\hat{k}) &= \left(-\frac{1}{2}\hat{i} - \hat{j} + \frac{1}{2}\hat{k}\right) + \frac{5}{2}(\hat{i} + \hat{k}) \end{aligned}$$



### Scalar or Dot Product Ex 24.1 Q37

$$\text{Let } (6\hat{i} - 3\hat{j} - 6\hat{k}) = \vec{a} + \vec{b} \quad \text{--- (i)}$$

Such that  $\vec{a}$  is parallel to  $(\hat{i} + \hat{j} + \hat{k})$  and  $\vec{b}$  is perpendicular to  $(\hat{i} + \hat{j} + \hat{k})$ .

Since,  $\vec{a}$  is parallel to  $(\hat{i} + \hat{j} + \hat{k})$

$$\vec{a} = \lambda (\hat{i} + \hat{j} + \hat{k}) \quad \text{--- (ii)}$$

Put  $\vec{a}$  in equation (i),

$$(6\hat{i} - 3\hat{j} - 6\hat{k}) = (\lambda\hat{i} + \lambda\hat{j} + \lambda\hat{k}) + \vec{b}$$

$$\vec{b} = 6\hat{i} - \lambda\hat{i} - 3\hat{j} - \lambda\hat{j} - 6\hat{k} - \lambda\hat{k}$$

$$\vec{b} = (6 - \lambda)\hat{i} + (-3 - \lambda)\hat{j} + (-6 - \lambda)\hat{k}$$

$\vec{b}$  is a vector perpendicular to the vector  $(\hat{i} + \hat{j} + \hat{k})$ , then

$$\vec{b} \cdot (\hat{i} + \hat{j} + \hat{k}) = 0$$

$$[(6 - \lambda)\hat{i} + (-3 - \lambda)\hat{j} + (-6 - \lambda)\hat{k}] \cdot (\hat{i} + \hat{j} + \hat{k}) = 0$$

$$(6 - \lambda)(1) + (-3 - \lambda)(1) + (-6 - \lambda)(1) = 0$$

$$6 - \lambda - 3 - \lambda - 6 - \lambda = 0$$

$$-3 - 3\lambda = 0$$

$$-3 = 3\lambda$$

$$\lambda = \frac{-3}{3}$$

$$\lambda = -1$$

Put value of  $\lambda$  in (ii),

$$\vec{a} = -1 \cdot (\hat{i} + \hat{j} + \hat{k})$$

$$\vec{a} = -\hat{i} - \hat{j} - \hat{k}$$

Using  $\vec{a}$  in equation (i),

$$(6\hat{i} - 3\hat{j} - 6\hat{k}) = (-\hat{i} - \hat{j} - \hat{k}) + \vec{b}$$

$$\vec{b} = 6\hat{i} + \hat{i} - 3\hat{j} + \hat{j} - 6\hat{k} + \hat{k}$$

$$\vec{b} = 7\hat{i} - 2\hat{j} - 5\hat{k}$$

Thus,

$$\text{Vector } \vec{a} = -\hat{i} - \hat{j} - \hat{k} \text{ and}$$

$$\vec{b} = 7\hat{i} - 2\hat{j} - 5\hat{k}$$

are required vectors.

### Scalar or Dot Product Ex 24.1 Q38

Here,  $(\vec{a} + \vec{b})$  is orthogonal to  $(\vec{a} - \vec{b})$

$$\text{Then, } (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$$

$$|\vec{a}|^2 - |\vec{b}|^2 = 0$$

$$\left\{ \sqrt{(5)^2 + (-1)^2 + (7)^2} \right\}^2 - \left\{ \sqrt{(1)^2 + (-1)^2 + (\lambda)^2} \right\}^2 = 0$$

$$(25 + 1 + 49) - (1 + 1 + \lambda^2) = 0$$

$$75 - (2 + \lambda^2) = 0$$

$$75 - 2 - \lambda^2 = 0$$

$$-\lambda^2 = -73$$

$$\lambda = \sqrt{73}$$

**Scalar or Dot Product Ex 24.1 Q39**

It is given that  $\vec{a} \cdot \vec{a} = 0$  and  $\vec{a} \cdot \vec{b} = 0$ .

Now,

$$\vec{a} \cdot \vec{a} = 0 \Rightarrow |\vec{a}|^2 = 0 \Rightarrow |\vec{a}| = 0$$

$\therefore \vec{a}$  is a zero vector.

Hence, vector  $\vec{b}$  satisfying  $\vec{a} \cdot \vec{b} = 0$  can be any vector

**Scalar or Dot Product Ex 24.1 Q40**

Given that  $\vec{c}$  is perpendicular to both  $\vec{a}$  and  $\vec{b}$ , so,

$$\vec{a} \cdot \vec{c} = 0 \text{ and } \vec{b} \cdot \vec{c} = 0$$

$$\begin{aligned} \text{Now, } \vec{c} \cdot (\vec{a} + \vec{b}) &= \vec{c} \cdot \vec{a} + \vec{c} \cdot \vec{b} \\ &= 0 + 0 \\ &= 0 \end{aligned}$$

$\therefore \vec{c}$  is perpendicular to  $(\vec{a} + \vec{b})$

$$\begin{aligned} \vec{c} \cdot (\vec{a} - \vec{b}) &= \vec{c} \cdot \vec{a} - \vec{c} \cdot \vec{b} \\ &= 0 - 0 \\ &= 0 \end{aligned}$$

$\therefore \vec{c}$  is perpendicular to  $(\vec{a} - \vec{b})$

**Scalar or Dot Product Ex 24.1 Q41**

Here  $|\vec{a}| = a$ ,  $|\vec{b}| = b$

$$\begin{aligned} \text{LHS} &= \left( \frac{\vec{a}}{a^2} - \frac{\vec{b}}{b^2} \right)^2 \\ &= \left( \frac{\vec{a}}{a^2} \right)^2 + \left( \frac{\vec{b}}{b^2} \right)^2 - 2 \frac{\vec{a}}{a^2} \cdot \frac{\vec{b}}{b^2} \\ &= \frac{|\vec{a}|^2}{a^4} + \frac{|\vec{b}|^2}{b^4} - \frac{2\vec{a}\vec{b}}{a^2b^2} \\ &= \frac{a^2}{a^4} + \frac{b^2}{b^4} - \frac{2\vec{a}\vec{b}}{a^2b^2} \quad \left[ \text{Since } |\vec{a}| = a, |\vec{b}| = b \right] \\ &= \frac{1}{a^2} + \frac{1}{b^2} - \frac{2\vec{a}\vec{b}}{a^2b^2} \\ &= \frac{b^2 + a^2 - 2\vec{a}\vec{b}}{a^2b^2} \\ &= \frac{|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\vec{b}}{a^2b^2} \\ &= \frac{(\vec{a} - \vec{b})^2}{a^2b^2} \\ &= \left( \frac{\vec{a} - \vec{b}}{ab} \right)^2 \\ &= \text{RHS} \end{aligned}$$

Hence proved

$$\therefore \left( \frac{\vec{a}}{a^2} - \frac{\vec{b}}{b^2} \right)^2 = \left( \frac{\vec{a} - \vec{b}}{ab} \right)^2$$

### Scalar or Dot Product Ex 24.1 Q42

Given that

$\vec{a}, \vec{b}, \vec{c}$  are three non-coplanar vectors such that  
 $\vec{d} \cdot \vec{a} = \vec{d} \cdot \vec{b} = \vec{d} \cdot \vec{c} = 0$

Given that

$$\begin{aligned} \vec{d} \cdot \vec{a} &= 0 \\ \Rightarrow \vec{d} &\text{ perpendicular to } \vec{a} \\ \text{or } \vec{d} &= 0 \quad \text{--- (i)} \end{aligned}$$

$$\begin{aligned} \vec{d} \cdot \vec{b} &= 0 \\ \Rightarrow \vec{d} &\text{ is perpendicular to } \vec{b} \text{ or } \vec{d} = 0 \quad \text{--- (ii)} \end{aligned}$$

$$\begin{aligned} \vec{d} \cdot \vec{c} &= 0 \\ \Rightarrow \vec{d} &\text{ is perpendicular to } \vec{c} \text{ or } \vec{d} = 0 \quad \text{--- (iii)} \end{aligned}$$

From (i), (ii), (iii), we get

$\vec{d}$  is perpendicular to  $\vec{a}, \vec{b}, \vec{c}$  or  $\vec{d} = 0$ , but  $\vec{d}$  can not be perpendicular to  $\vec{a}, \vec{b}$  and  $\vec{c}$  because  $\vec{a}, \vec{b}, \vec{c}$  are three non-coplanar vectors, so

$$\vec{d} = 0$$

### Scalar or Dot Product Ex 24.1 Q43

Given that

$\vec{a}$  is perpendicular to  $\vec{b}$  and  $\vec{c}$

It means,

$$\vec{a} \cdot \vec{b} = 0 \text{ and } \vec{a} \cdot \vec{c} = 0 \quad \text{--- (i)}$$

Let  $\vec{r}$  be some vector in the plane of  $\vec{b}$  and  $\vec{c}$

Then,  $\vec{r}, \vec{b}, \vec{c}$  are coplanar

We know that,

Three vectors are coplanar if one of them is expressible as linear combination of other two vectors.

$$\begin{aligned} \text{Let } \vec{r} &= x\vec{b} + y\vec{c} \\ &\text{where } x \text{ and } y \text{ are same scalar} \end{aligned}$$

$$\begin{aligned} \vec{r} \cdot \vec{a} &= (x\vec{b} + y\vec{c}) \cdot \vec{a} && \text{[Taking dot product with } \vec{a} \text{ on both the side]} \\ \vec{r} \cdot \vec{a} &= x\vec{b} \cdot \vec{a} + y\vec{c} \cdot \vec{a} \\ &= x \cdot 0 + y \cdot 0 && \text{[Using (i)]} \\ \vec{r} \cdot \vec{a} &= 0 + 0 \\ \vec{r} \cdot \vec{a} &= 0 \end{aligned}$$

So,  $\vec{r}$  is perpendicular to  $\vec{a}$

Thus,

$\vec{a}$  is perpendicular to every vector in the plane of  $\vec{b}$  and  $\vec{c}$

### Scalar or Dot Product Ex 24.1 Q44

We have,

$$\begin{aligned} \vec{a} + \vec{b} + \vec{c} &= \vec{0} \\ \vec{b} + \vec{c} &= -\vec{a} \end{aligned}$$

Squaring both the sides.

$$\begin{aligned} (\vec{b} + \vec{c})^2 &= (-\vec{a})^2 \\ |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b} \cdot \vec{c} &= |\vec{a}|^2 \\ 2\vec{b} \cdot \vec{c} &= |\vec{a}|^2 - |\vec{b}|^2 - |\vec{c}|^2 \\ 2|\vec{b}||\vec{c}|\cos\theta &= |\vec{a}|^2 - |\vec{b}|^2 - |\vec{c}|^2 && \text{[Since } \vec{b} \cdot \vec{c} = |\vec{b}||\vec{c}|\cos\theta] \\ \cos\theta &= \frac{|\vec{a}|^2 - |\vec{b}|^2 - |\vec{c}|^2}{2|\vec{b}||\vec{c}|} \end{aligned}$$

# Scalar or Dot Product Ex 24.1 Q45

Here,  $\vec{u} + \vec{v} + \vec{w} = 0$

Squaring both the sides,

$$(\vec{u} + \vec{v} + \vec{w})^2 = (0)^2$$

$$|\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 + 2\vec{u}\vec{v} + 2\vec{v}\vec{w} + 2\vec{w}\vec{u} = 0$$

$$(3)^2 + (4)^2 + (5)^2 + 2(\vec{u}\vec{v} + \vec{v}\vec{w} + \vec{w}\vec{u}) = 0$$

$$9 + 16 + 25 + 2(\vec{u}\vec{v} + \vec{v}\vec{w} + \vec{w}\vec{u}) = 0$$

$$2(\vec{u}\vec{v} + \vec{v}\vec{w} + \vec{w}\vec{u}) = -50$$

$$\vec{u}\vec{v} + \vec{v}\vec{w} + \vec{w}\vec{u} = \frac{-50}{2}$$

$$\vec{u}\vec{v} + \vec{v}\vec{w} + \vec{w}\vec{u} = -25$$

## Scalar or Dot Product Ex 24.1 Q46

Given

$$\vec{a} = x^2\hat{i} + 2\hat{j} - 2\hat{k}$$

$$\vec{b} = \hat{i} - \hat{j} + \hat{k}$$

$$\vec{c} = x^2\hat{i} + 5\hat{j} - 4\hat{k}$$

Let  $\theta$  be the angle between  $\vec{a}$  and  $\vec{b}$ , then

$$\vec{a}\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$\cos\theta = \frac{\vec{a}\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\begin{aligned} &= \frac{(x^2\hat{i} + 2\hat{j} - 2\hat{k})(\hat{i} - \hat{j} + \hat{k})}{\sqrt{(x^2)^2 + (2)^2 + (-2)^2} \sqrt{(1)^2 + (-1)^2 + (1)^2}} \\ &= \frac{(x^2)(1) + (2)(-1) + (-2)(1)}{\sqrt{x^4 + 4 + 4} \sqrt{1 + 1 + 1}} \\ &= \frac{x^2 - 2 - 2}{\sqrt{8 + x^4} \sqrt{3}} \\ \cos\theta &= \frac{x^2 - 4}{\sqrt{3}\sqrt{8 + x^4}} \end{aligned}$$

Since  $\theta$  is an acute angle, so

$$\cos\theta > 0$$

$$\frac{x^2 - 4}{\sqrt{3}\sqrt{8 + x^4}} > 0$$

$$x^2 - 4 > 0$$

$$x^2 > 4$$

$$\Rightarrow x < -2 \text{ or } x > 2 \quad \text{--- (i)}$$

Again, let  $\phi$  be the angle between  $\vec{b}$  and  $\vec{c}$ ,

$$\cos\phi = \frac{\vec{b}\vec{c}}{|\vec{b}||\vec{c}|}$$

$$= \frac{(\hat{i} - \hat{j} + \hat{k})(x^2\hat{i} + 5\hat{j} - 4\hat{k})}{\sqrt{(1)^2 + (-1)^2 + (1)^2} \sqrt{(x^2)^2 + (5)^2 + (-4)^2}}$$

$$\cos\phi = \frac{(1)(x^2) + (-1)(5) + (1)(-4)}{\sqrt{3}\sqrt{x^4 + 25 + 16}}$$

$$\cos\phi = \frac{x^2 - 5 - 4}{\sqrt{3}\sqrt{x^4 + 41}}$$

$$\cos\phi = \frac{x^2 - 9}{\sqrt{3}\sqrt{x^4 + 41}}$$

Since  $\phi$  is an obtuse angle, so

$$\cos\phi < 0$$

$$\frac{x^2 - 9}{\sqrt{3}\sqrt{x^4 + 41}} < 0$$

$$x^2 - 9 < 0$$

$$x^2 < 9$$

$$\Rightarrow x > -3 \text{ and } x < 3 \quad \text{--- (ii)}$$

From

$$-3 < x < -2 \text{ and } 2 < x < 3$$

$$x \in (-3, -2) \cup (2, 3)$$

### Scalar or Dot Product Ex 24.1 Q47

Here,  $\vec{a}$  and  $\vec{b}$  are mutually perpendicular, then

$$\vec{a} \cdot \vec{b} = 0$$

$$(3\hat{i} + x\hat{j} - \hat{k}) \cdot (2\hat{i} + \hat{j} + y\hat{k}) = 0$$

$$(3)(2) + (x)(1) + (-1)(y) = 0$$

$$6 + x - y = 0$$

$$x - y = -6 \quad \text{--- (i)}$$

Also,  $\vec{a}$  and  $\vec{b}$  have equal magnitude,

$$|\vec{a}| = |\vec{b}|$$

$$\sqrt{(3)^2 + (x)^2 + (-1)^2} = \sqrt{(2)^2 + (1)^2 + (y)^2}$$

$$9 + x^2 + 1 = 4 + 1 + y^2$$

$$x^2 + 10 = 5y^2$$

$$x^2 - y^2 = 5 - 10$$

$$x^2 - y^2 = -5$$

$$(x + y)(x - y) = -5$$

$$(x + y)(-6) = -5 \quad \text{[Using (i)]}$$

$$-6x - 6y = -5$$

$$-(6x + 6y) = -5$$

$$6x + 6y = 5 \quad \text{--- (ii)}$$

Solving (i) and (ii),

$$6x + 6y = 5$$

$$6x - 6y = -36 \quad \text{[(i) } \times 6]$$

$$12x = -31$$

$$x = \frac{-31}{12}$$

Put value of  $x$  in equation (i),

$$x - y = -6$$

$$\frac{-31}{12} - y = -6$$

$$-y = \frac{-6}{1} + \frac{31}{12}$$

$$-y = \frac{-72 + 31}{12}$$

$$y = \frac{41}{12}$$

### Scalar or Dot Product Ex 24.1 Q48

Given

$\vec{a}$  and  $\vec{b}$  are unit vectors

Then,  $|\vec{a}| = |\vec{b}| = 1$

$$|\vec{a} + \vec{b}| = \sqrt{3}$$

Squaring both the sides,

$$|\vec{a} + \vec{b}|^2 = (\sqrt{3})^2$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = 3$$

$$1 + 1 + 2\vec{a} \cdot \vec{b} = 3$$

$$2 + 2\vec{a} \cdot \vec{b} = 3$$

$$2\vec{a} \cdot \vec{b} = 3 - 2$$

$$2\vec{a} \cdot \vec{b} = 1$$

$$\vec{a} \cdot \vec{b} = \frac{1}{2}$$

Now,  $(2\vec{a} - 5\vec{b}) \cdot (3\vec{a} + \vec{b})$

$$= 2\vec{a} \cdot 3\vec{a} + 2\vec{a} \cdot \vec{b} - 5\vec{b} \cdot 3\vec{a} - 5\vec{b} \cdot \vec{b}$$

$$= 6(\vec{a})^2 + 2\vec{a} \cdot \vec{b} - 15\vec{a} \cdot \vec{b} - 5(\vec{b})^2$$

$$= 6|\vec{a}|^2 - 13\vec{a} \cdot \vec{b} - 5|\vec{b}|^2$$

$$= 6(1)^2 - 13\left(\frac{1}{2}\right) - 5(1)^2$$

$$= \frac{6}{1} - \frac{13}{2} - \frac{5}{1}$$

$$= \frac{12 - 13 - 10}{2}$$

$$= \frac{12 - 23}{2}$$

$$= -\frac{11}{2}$$

$$(2\vec{a} - 5\vec{b}) \cdot (3\vec{a} + \vec{b}) = -\frac{11}{2}$$

# Scalar or Dot Product Ex 24.1 Q49

$$|\vec{a} + \vec{b}|^2 = |\vec{b}|^2$$

$$\Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = \vec{b} \cdot \vec{b}$$

$$\Rightarrow \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} = \vec{b} \cdot \vec{b}$$

$$\Rightarrow \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b} = \vec{b} \cdot \vec{b}$$

$$\Rightarrow \vec{a} \cdot \vec{a} + 2\vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow \vec{a} \cdot (\vec{a} + 2\vec{b}) = 0$$

$\therefore \vec{a} + 2\vec{b}$  is perpendicular to  $\vec{a}$ .

$$\text{Let } (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$$

$$|\vec{a}|^2 - |\vec{b}|^2 = 0$$

$$|\vec{a}|^2 = |\vec{b}|^2$$

$$|\vec{a}| = |\vec{b}|$$

$$\text{Let } |\vec{a}| = |\vec{b}|$$

Squaring both the sides.

$$|\vec{a}|^2 = |\vec{b}|^2$$

$$|\vec{a}|^2 - |\vec{b}|^2 = 0$$

$$(\vec{a})^2 - (\vec{b})^2 = 0$$

$$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$$

Thus,

$$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0 \Leftrightarrow |\vec{a}| = |\vec{b}|$$

# Ex 24.2

## Scalar or Dot Product Ex 24.2 Q1

Let  $\vec{o}$ ,  $\vec{a}$  and  $\vec{b}$  be the position vector of the O, A and B.

P and Q are points of trisection of AB.

$$\text{Position vector of point P} = \frac{2\vec{a} + \vec{b}}{3}$$

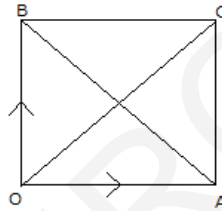
$$\text{Position vector of point Q} = \frac{\vec{a} + 2\vec{b}}{3}$$

$$\vec{OP} = \frac{2\vec{a} + \vec{b}}{3} - \vec{o} = \frac{2\vec{a} + \vec{b} - 3\vec{o}}{3} = \frac{2\vec{OA} + \vec{OB}}{3}$$

$$\vec{OQ} = \frac{\vec{a} + 2\vec{b}}{3} - \vec{o} = \frac{\vec{a} + 2\vec{b} - 3\vec{o}}{3} = \frac{\vec{OA} + 2\vec{OB}}{3}$$

$$\begin{aligned} OP^2 + OQ^2 &= \left( \frac{2\vec{OA} + \vec{OB}}{3} \right)^2 + \left( \frac{\vec{OA} + 2\vec{OB}}{3} \right)^2 \\ &= \frac{5(OA^2 + OB^2) + 8(OA)(OB)\cos 90^\circ}{9} \\ &= \frac{5}{9}AB^2 \dots\dots\dots [\because OA^2 + OB^2 = AB^2 \text{ and } \cos 90^\circ = 0] \end{aligned}$$

## Scalar or Dot Product Ex 24.2 Q2



Let OACB be a quadrilateral such that its diagonal bisect each other at right angles.

We know that if the diagonals of a quadrilateral bisect each other then its a parallelogram.

$\therefore$  OACB is a parallelogram.

$\Rightarrow OA = BC$  and  $OB = AC$ .

Taking O as origin let  $\vec{a}$  and  $\vec{b}$  be the position vector of the A and B.

AB and OC be the diagonals of quadrilateral which bisect each other at right angles.

$$\therefore \vec{OC} \cdot \vec{AB} = 0$$

$$\Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{b} - \vec{a}) = 0$$

$$\Rightarrow |\vec{b}|^2 = |\vec{a}|^2$$

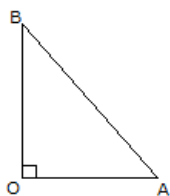
$$\Rightarrow OB = OA$$

Similarly we can show that

$$OA = OB = BC = CA$$

Hence OACB is a rhombus.

### Scalar or Dot Product Ex 24.2 Q3



Let OAC be a right triangle, right angled at O.

Taking O as origin let  $\vec{a}$  and  $\vec{b}$  be the position vector of the  $\vec{OA}$  and  $\vec{OB}$ .

$\vec{OA}$  is perpendicular to  $\vec{OB}$

$$\therefore \vec{OA} \cdot \vec{OB} = 0$$

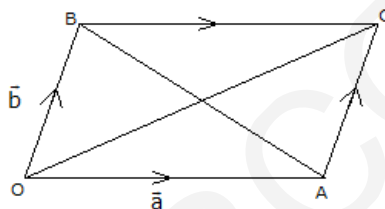
$$\vec{a} \cdot \vec{b} = 0$$

Now,

$$AB^2 = (\vec{b} - \vec{a})^2 = (\vec{a})^2 + (\vec{b})^2 - 2\vec{a} \cdot \vec{b} = (\vec{a})^2 + (\vec{b})^2 - 0 = (\vec{OA})^2 + (\vec{OB})^2$$

Hence proved.

### Scalar or Dot Product Ex 24.2 Q4



Let OAC be a right triangle, right angled at O.

Taking O as origin let  $\vec{a}$  and  $\vec{b}$  be the position vector of the  $\vec{OA}$  and  $\vec{OB}$ .

$\vec{OA}$  is perpendicular to  $\vec{OB}$

$$\therefore \vec{OA} \cdot \vec{OB} = 0$$

$$\vec{a} \cdot \vec{b} = 0$$

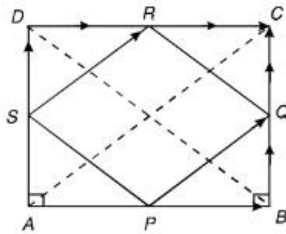
Now,

$$AB^2 = (\vec{b} - \vec{a})^2 = (\vec{a})^2 + (\vec{b})^2 - 2\vec{a} \cdot \vec{b} = (\vec{a})^2 + (\vec{b})^2 - 0 = (\vec{OA})^2 + (\vec{OB})^2$$

Hence proved.



### Scalar or Dot Product Ex 24.2 Q5



ABCD be a rectangle.

Let P, Q, R and S be the midpoints of the sides AB, BC, CD and DA respectively.

Now,

$$\overrightarrow{PQ} = \overrightarrow{PB} + \overrightarrow{BQ} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{BC}) = \frac{1}{2}\overrightarrow{AC} \dots \dots \dots (i)$$

$$\overrightarrow{SR} = \overrightarrow{SD} + \overrightarrow{DR} = \frac{1}{2}(\overrightarrow{AD} + \overrightarrow{DC}) = \frac{1}{2}\overrightarrow{AC} \dots \dots \dots (ii)$$

From (i) and (ii), we have

$\overrightarrow{PQ} = \overrightarrow{SR}$  i.e. sides PQ and SR are equal and parallel.

$\therefore$  PQRS is a parallelogram.

$$(\overrightarrow{PQ})^2 = \overrightarrow{PQ} \cdot \overrightarrow{PQ} = (\overrightarrow{PB} + \overrightarrow{BQ}) \cdot (\overrightarrow{PB} + \overrightarrow{BQ}) = |\overrightarrow{PB}|^2 + |\overrightarrow{BQ}|^2 \dots \dots \dots (iii)$$

$$(\overrightarrow{PS})^2 = \overrightarrow{PS} \cdot \overrightarrow{PS} = (\overrightarrow{PA} + \overrightarrow{AS}) \cdot (\overrightarrow{PA} + \overrightarrow{AS}) = |\overrightarrow{PA}|^2 + |\overrightarrow{AS}|^2 = |\overrightarrow{PB}|^2 + |\overrightarrow{BQ}|^2 \dots \dots \dots (iv)$$

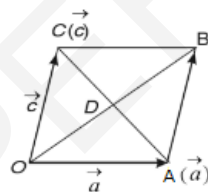
From (iii) and (iv) we get,

$$(\overrightarrow{PQ})^2 = (\overrightarrow{PS})^2 \text{ i. e. } PQ = PS$$

$\Rightarrow$  The adjacent sides of PQRS are equal.

$\therefore$  PQRS is a rhombus.

### Scalar or Dot Product Ex 24.2 Q6



Let OABC be a rhombus, whose diagonals OB and AC intersect at point D.

Let O be the origin.

Let the position vector of A and C be  $\vec{a}$  and  $\vec{c}$  respectively then,

$$\overrightarrow{OA} = \vec{a} \text{ and } \overrightarrow{OC} = \vec{c}$$

$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + \overrightarrow{OC} = \vec{a} + \vec{c} \dots \dots \dots [\because \overrightarrow{AB} = \overrightarrow{OC}]$$

$$\text{Position vector of mid-point of } \overrightarrow{OB} = \frac{1}{2}(\vec{a} + \vec{c})$$

$$\text{Position vector of mid-point of } \overrightarrow{AC} = \frac{1}{2}(\vec{a} + \vec{c})$$

$\therefore$  Midpoints of OB and AC coincide.

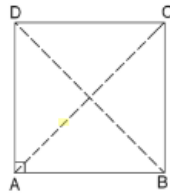
$\therefore$  Diagonal OB and AC bisect each other.

$$\overrightarrow{OB} \cdot \overrightarrow{AC} = (\vec{a} + \vec{c}) \cdot (\vec{c} - \vec{a}) = (\vec{c} + \vec{a}) \cdot (\vec{c} - \vec{a}) = |\vec{c}|^2 - |\vec{a}|^2 = \overrightarrow{OC} \cdot \overrightarrow{OA} = 0$$

[ $\because$  OC and OA are sides of the rhombus]

$$\Rightarrow \overrightarrow{OB} \perp \overrightarrow{AC}$$

### Scalar or Dot Product Ex 24.2 Q7



Let ABCD be a rectangle.

Take A as origin.

Let position vectors of point B, D be  $\vec{a}$  and  $\vec{b}$  respectively.

By parallelogram law,

$$\vec{AC} = \vec{a} + \vec{b} \text{ and } \vec{BD} = \vec{a} - \vec{b}$$

As ABCD is a rectangle,  $AB \perp AD$

$$\Rightarrow \vec{a} \cdot \vec{b} = 0 \dots \dots \dots (i)$$

Now, diagonals AC and BD are perpendicular iff  $\vec{AC} \cdot \vec{BD} = 0$

$$\Rightarrow (\vec{a} + \vec{b})(\vec{a} - \vec{b}) = 0$$

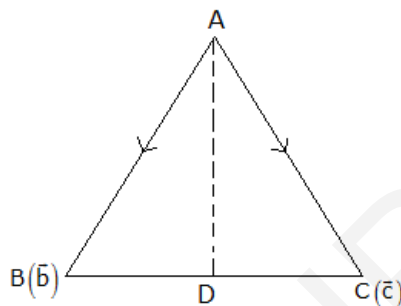
$$\Rightarrow (\vec{a})^2 - (\vec{b})^2 = 0$$

$$\Rightarrow |\vec{AB}|^2 = |\vec{AD}|^2$$

$$\Rightarrow |AB| = |AD|$$

Hence ABCD is a square.

### Scalar or Dot Product Ex 24.2 Q8



Take A as origin, let the position vectors of B and C are  $\vec{b}$  and  $\vec{c}$  respectively.

Position vector of D =  $\frac{\vec{b} + \vec{c}}{2}$ ,  $\vec{AB} = \vec{b}$  and  $\vec{AC} = \vec{c}$ .

$$\vec{AD} = \frac{\vec{b} + \vec{c}}{2} - \vec{0} = \frac{\vec{b} + \vec{c}}{2}$$

Consider,  $2(AD^2 + CD^2)$

$$= 2 \left[ \left( \frac{\vec{b} + \vec{c}}{2} \right)^2 + \left( \frac{\vec{b} + \vec{c}}{2} - \vec{c} \right)^2 \right]$$

$$= 2 \left[ \left( \frac{\vec{b} + \vec{c}}{2} \right)^2 + \left( \frac{\vec{b} - \vec{c}}{2} \right)^2 \right]$$

$$= \frac{1}{2} [(\vec{b} + \vec{c})^2 + (\vec{b} - \vec{c})^2]$$

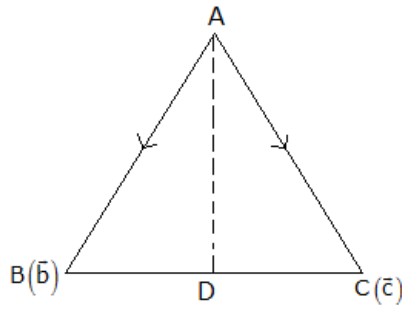
$$= (\vec{b})^2 + (\vec{c})^2$$

$$= (\vec{AB})^2 + (\vec{AC})^2$$

$$= AB^2 + AC^2$$

Hence proved.

### Scalar or Dot Product Ex 24.2 Q9



Take A as origin, let the position vectors of B and C are  $\vec{b}$  and  $\vec{c}$  respectively.

Position vector of D =  $\frac{\vec{b} + \vec{c}}{2}$ ,  $\overrightarrow{AB} = \vec{b}$  and  $\overrightarrow{AC} = \vec{c}$ .

$$\overrightarrow{AD} = \frac{\vec{b} + \vec{c}}{2} - \vec{0} = \frac{\vec{b} + \vec{c}}{2}$$

AD is perpendicular to BC

$$\Rightarrow \overrightarrow{AD} \cdot \overrightarrow{BC} = 0$$

$$\Rightarrow \left( \frac{\vec{b} + \vec{c}}{2} \right) \cdot (\vec{c} - \vec{b}) = 0$$

$$\Rightarrow (\vec{b} + \vec{c}) \cdot (\vec{c} - \vec{b}) = 0$$

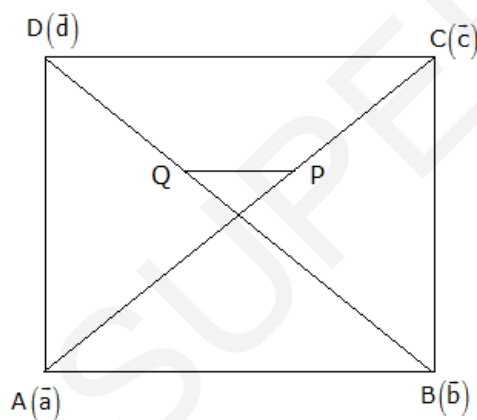
$$\Rightarrow |\vec{c}|^2 = |\vec{b}|^2$$

$$\Rightarrow |\vec{c}| = |\vec{b}|$$

$$\Rightarrow AC = AB$$

Hence  $\triangle ABC$  is an isosceles triangle.

### Scalar or Dot Product Ex 24.2 Q10



Take O as origin, let the position vectors of A, B, C and D are  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$  and  $\vec{d}$  respectively.

$$\text{Position vector of P} = \frac{\vec{a} + \vec{c}}{2}$$

$$\text{Position vector of Q} = \frac{\vec{a} + \vec{d}}{2}$$

$$\begin{aligned} \text{LHS} &= AB^2 + BC^2 + CD^2 + DA^2 \\ &= (\vec{b} - \vec{a})^2 + (\vec{c} - \vec{b})^2 + (\vec{d} - \vec{c})^2 + (\vec{d} - \vec{a})^2 \\ &= 2 \left[ (\vec{a})^2 + (\vec{b})^2 + (\vec{c})^2 + (\vec{d})^2 - \vec{a}\vec{b} \cos \theta_1 - \vec{b}\vec{c} \cos \theta_2 - \vec{c}\vec{d} \cos \theta_3 - \vec{c}\vec{a} \cos \theta_4 \right] \end{aligned}$$

$$\begin{aligned} \text{RHS} &= AC^2 + BD^2 + 4PQ^2 \\ &= (\vec{c} - \vec{a})^2 + (\vec{d} - \vec{b})^2 + 4 \left( \frac{\vec{a} + \vec{d}}{2} - \frac{\vec{a} + \vec{c}}{2} \right)^2 \\ &= 2 \left[ (\vec{a})^2 + (\vec{b})^2 + (\vec{c})^2 + (\vec{d})^2 - \vec{a}\vec{b} \cos \theta_1 - \vec{b}\vec{c} \cos \theta_2 - \vec{c}\vec{d} \cos \theta_3 - \vec{c}\vec{a} \cos \theta_4 \right] \\ &= \text{LHS} \end{aligned}$$

Hence proved.