
Exercise – 2.1

1. Assuming that x, y, z are positive real numbers, simplify each of the following:

(i) $\left(\sqrt{x^{-3}}\right)^5$

(ii) $\sqrt{x^3 y^{-2}}$

(iii) $\left(x^{-\frac{2}{3}} y^{-\frac{1}{2}}\right)^2$

(iv) $\left(\sqrt{x}\right)^{\frac{2}{3}} \sqrt{y^4} \div \sqrt{xy^{-\frac{1}{2}}}$

(v) $\sqrt[5]{243x^{10}y^5z^{10}}$

(vi) $\left(\frac{x^{-4}}{y^{-10}}\right)^{\frac{5}{4}}$

Sol:

We have,

$$\left(\sqrt{x^{-3}}\right)^5 = \left(\sqrt{\frac{1}{x^3}}\right)^5$$

$$= \left(\frac{1}{x^{\frac{3}{2}}}\right)^5$$

$$= \frac{1}{x^{\frac{3}{2} \times 5}}$$

$$= \frac{1}{x^{\frac{15}{2}}}$$

$$\Rightarrow \left(\sqrt{x^{-3}}\right)^5 = \frac{1}{x^{\frac{15}{2}}}$$

We have,

$$\sqrt{x^3 y^{-2}} = \sqrt{\frac{x^3}{y^2}}$$

$$= \left(\frac{x^3}{y^2}\right)^{\frac{1}{2}}$$

$$= \frac{x^{3 \times \frac{1}{2}}}{y^{2 \times \frac{1}{2}}}$$

$$= \frac{x^{\frac{3}{2}}}{y}$$

$$\Rightarrow \sqrt{x^3 y^{-2}} = \frac{x^{\frac{3}{2}}}{y}$$

We have,

$$\left(x^{-\frac{2}{3}} y^{-\frac{1}{2}} \right)^2 = \left(\frac{1}{x^{\frac{2}{3}} y^{\frac{1}{2}}} \right)^2$$

$$= \left(\frac{1}{x^{\frac{2}{3} \times 2} y^{2 \times \frac{1}{2}}} \right)^2$$

$$= \frac{1}{x^{\frac{4}{3}} y^1}$$

$$= \frac{1}{x^{\frac{4}{3}} y}$$

$$\Rightarrow \left(x^{-\frac{2}{3}} y^{-\frac{1}{2}} \right)^2 = \frac{1}{x^{\frac{4}{3}} y}$$

We have,

$$\left(\sqrt{x} \right)^{-\frac{2}{3}} \sqrt{y^4} \div \sqrt{xy^{-\frac{1}{2}}}$$

$$= \left(x^{\frac{1}{2}} \right)^{-\frac{2}{3}} (y^2) \div \sqrt{xy^{-\frac{1}{2}}}$$

$$= \frac{x^{\frac{1}{2} \times \frac{2}{3}} y^2}{\left(xy^{-\frac{1}{2}} \right)^{\frac{1}{2}}}$$

$$= \frac{x^{\frac{1}{3}} y^2}{x^{\frac{1}{2}} y^{-\frac{1}{2} \times \frac{1}{2}}}$$

$$= \left(x^{-\frac{1}{3}} \times x^{-\frac{1}{2}} \right) \times \left(y^2 \times y^{\frac{1}{4}} \right)$$

$$= \left(x^{-\frac{1}{3} - \frac{1}{2}} \right) \left(y^{2 + \frac{1}{4}} \right)$$

$$= \left(x^{-\frac{2+3}{6}} \right) \left(y^{\frac{8+1}{4}} \right)$$

$$= \left(x^{-\frac{5}{6}} \right) \left(y^{\frac{9}{4}} \right)$$

$$= \frac{y^{\frac{9}{4}}}{x^{\frac{5}{6}}}$$

$$\Rightarrow \left(\sqrt{x} \right)^{-\frac{2}{3}} \sqrt{y^4} \div \sqrt{xy^{-\frac{1}{2}}} = \frac{y^{\frac{9}{4}}}{x^{\frac{5}{6}}}$$

We have,

$$\sqrt[5]{243x^{10}y^5z^{10}} = \left(243x^{10}y^5z^{10} \right)^{\frac{1}{5}}$$

$$= \left(243 \right)^{\frac{1}{5}} x^{\frac{10}{5}} y^{\frac{5}{5}} z^{\frac{10}{5}}$$

$$= \left(3^5 \right)^{\frac{1}{5}} x^2 y^1 z^2$$

$$= 3^{5 \times \frac{1}{5}} x^2 yz^2$$

$$= 3x^2 yz^2$$

$$\Rightarrow \sqrt[5]{243x^{10}y^5z^{10}} = 3x^2 yz^2$$

We have,

$$\left(\frac{x^{-4}}{y^{-10}} \right)^{\frac{5}{4}} = \left(\frac{y^{10}}{x^4} \right)^{\frac{5}{4}}$$

$$= \frac{y^{10 \times \frac{5}{4}}}{x^{4 \times \frac{5}{4}}}$$

$$= \frac{y^{\frac{25}{2}}}{x^5}$$

$$\Rightarrow \left(\frac{x^{-4}}{y^{-10}} \right)^{\frac{5}{4}} = \frac{y^{\frac{25}{2}}}{x^5}$$

2. Simplify:

(i) $\left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}}$

(ii) $\sqrt[3]{(342)^{-2}}$

(iii) $(0.001)^{\frac{1}{3}}$

(iv) $\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}}$

(v) $\left(\frac{\sqrt{2}}{5}\right)^8 \div \left(\frac{\sqrt{2}}{5}\right)^{13}$

(vi) $\left[\frac{5^{-1} \times 7^2}{5^2 \times 7^{-4}}\right]^{\frac{7}{2}} \times \left[\frac{5^{-2} \times 7^3}{5^3 \times 7^{-5}}\right]^{-\frac{5}{2}}$

Sol:

(i) We have

$$\left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}} = (16)^{-\frac{1}{5} \times \frac{5}{2}} = (16)^{-\frac{1}{2}} = (4^2)^{-\frac{1}{2}} = 4^{2 \times -\frac{1}{2}} = 4^{-1} = \frac{1}{4}$$

Hence, $\boxed{\left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}} = \frac{1}{4}}$

(ii) We have,

$$\begin{aligned}\sqrt[3]{(342)^{-2}} &= \left[(342)^{-2}\right]^{\frac{1}{3}} = (342)^{-2 \times \frac{1}{3}} \\ &= (7^3)^{-\frac{2}{3}} \\ &= 7^{3 \times -\frac{2}{3}} \\ &= 7^{-2} = \frac{1}{7^2} = \frac{1}{49}\end{aligned}$$

Hence, $\boxed{\sqrt[3]{(342)^{-2}} = \frac{1}{49}}$

(iii) We have,

$$(0.001)^{\frac{1}{3}} = \left(\frac{1}{1000}\right)^{\frac{1}{3}} = \left(\frac{1}{10^3}\right)^{\frac{1}{3}}$$

$$= \frac{1^{\frac{1}{3}}}{(10^3)^{\frac{1}{3}}} = \frac{1}{10^{3 \times \frac{1}{3}}} = \frac{1}{10} = 0.01$$

Hence, $\boxed{(0.001)^{\frac{1}{3}} = 0.1}$

(iv) We have,

$$\begin{aligned} \frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}} &= \frac{(5^2)^{\frac{3}{2}} \times (3^5)^{\frac{3}{5}}}{(2^4)^{\frac{5}{4}} \times (2^3)^{\frac{4}{3}}} \\ &= \frac{5^{2 \times \frac{3}{2}} \times 3^{5 \times \frac{3}{5}}}{2^{4 \times \frac{5}{4}} \times 2^{3 \times \frac{4}{3}}} \\ &= \frac{5^3 \times 3^3}{2^5 \times 2^4} = \frac{125 \times 27}{32 \times 16} = \frac{3375}{512} \end{aligned}$$

Hence, $\boxed{\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}} = \frac{3375}{512}}$

(v) We have,

$$\begin{aligned} \left(\frac{\sqrt{2}}{5}\right)^8 \div \left(\frac{\sqrt{2}}{5}\right)^{13} &= \frac{\left(\frac{\sqrt{2}}{5}\right)^8}{\left(\frac{\sqrt{2}}{5}\right)^{13}} \\ \Rightarrow \left(\frac{\sqrt{2}}{5}\right)^{8-13} &= \left(\frac{\sqrt{2}}{5}\right)^{-5} = \frac{\left(2^{\frac{1}{2}}\right)^{-5}}{(5)^{-5}} = \frac{2^{\frac{1}{2} \times -5}}{5^{-5}} = \frac{2^{-\frac{5}{2}}}{5^{-5}} \\ \Rightarrow \frac{1}{2^{\frac{5}{2}}} \times \frac{5^5}{1} &= \frac{5^5}{2^{\frac{5}{2}}} = \frac{3125}{4\sqrt{2}} \end{aligned}$$

Hence $\boxed{\left(\frac{\sqrt{2}}{5}\right)^8 \div \left(\frac{\sqrt{2}}{5}\right)^{13} = \frac{3125}{4\sqrt{2}}}$

(vi) We have,

$$\left[\frac{5^{-1} \times 7^2}{5^2 \times 7^{-4}}\right]^{\frac{7}{2}} \times \left[\frac{5^{-2} \times 7^3}{5^3 \times 7^{-5}}\right]^{-\frac{5}{2}}$$

$$\Rightarrow \frac{(5^1 \times 7^2)^{\frac{7}{2}}}{(5^2 \times 7^{-4})^{\frac{7}{2}}} \times \frac{(5^{-2} \times 7^3)^{-\frac{5}{2}}}{(5^3 \times 7^{-5})^{-\frac{5}{2}}}$$

$$\Rightarrow \frac{(5^{-1})^{\frac{7}{2}} \times (7^2)^{\frac{7}{2}}}{(5^2)^{\frac{7}{2}} \times (7^{-4})^{\frac{7}{2}}} \times \frac{(5^{-2})^{-\frac{5}{2}} \times (7^3)^{-\frac{5}{2}}}{(5^3)^{-\frac{5}{2}} \times (7^{-5})^{-\frac{5}{2}}}$$

$$\Rightarrow \frac{5^{-1 \times \frac{7}{2}} \times 7^{2 \times \frac{7}{2}}}{5^{2 \times \frac{7}{2}} \times 7^{-4 \times \frac{7}{2}}} \times \frac{5^{-2 \times -\frac{5}{2}} \times 7^{3 \times -\frac{5}{2}}}{5^{3 \times -\frac{5}{2}} \times 7^{-5 \times -\frac{5}{2}}}$$

$$\Rightarrow \frac{5^{\frac{-7}{2}} \times 7^7}{5^7 \times 7^{-14}} \times \frac{5^5 \times 7^{\frac{-15}{2}}}{5^{\frac{-15}{2}} \times 7^{\frac{25}{2}}}$$

$$\Rightarrow \frac{7^{7+14}}{5^{\frac{7}{2}+7}} \times \frac{5^{5+\frac{15}{2}}}{7^{\frac{25}{2}+\frac{15}{2}}}$$

$$\Rightarrow \frac{5^{\frac{-7}{2}} \times 7^7}{5^7 \times 7^{-14}} \times \frac{5^5 \times 7^{\frac{-15}{2}}}{5^{\frac{-15}{2}} \times 7^{\frac{25}{2}}}$$

$$\Rightarrow \frac{7^{7+14}}{5^{\frac{7}{2}+7}} \times \frac{5^{5+\frac{15}{2}}}{7^{\frac{25}{2}+\frac{15}{2}}}$$

$$\Rightarrow \frac{7^{21}}{5^{\frac{21}{2}}} \times \frac{5^{\frac{25}{2}}}{7^{\frac{40}{2}}}$$

$$\Rightarrow \frac{7^{21}}{7^{\frac{20}{2}}} \times \frac{5^{\frac{25}{2}}}{5^{\frac{21}{2}}}$$

$$\Rightarrow 7^{21-20} \times 5^{\frac{25}{2}-\frac{21}{2}}$$

$$\Rightarrow 7^1 \times 5^{\frac{4}{2}}$$

$$\Rightarrow 7^1 \times 5^2 \Rightarrow 7 \times 25 \Rightarrow 175$$

$$\text{Hence, } \left[\frac{5^{-1} \times 7^2}{5^2 \times 7^{-4}} \right]^{\frac{7}{2}} \times \left[\frac{5^{-2} \times 7^3}{5^3 \times 7^{-5}} \right]^{-\frac{5}{2}} = 175$$

3. Prove that:

$$(i) \quad 9^{\frac{3}{2}} - 3 \times 5^0 - \left(\frac{1}{81}\right)^{-\frac{1}{2}} = 15$$

$$(ii) \quad \left(\frac{1}{4}\right)^{-2} - 3 \times 8^{\frac{2}{3}} \times 4^0 + \left(\frac{9}{16}\right)^{-\frac{1}{2}} = \frac{16}{3}$$

$$(iii) \quad \frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times 4^{\frac{1}{4}}}{10^{-\frac{1}{5}} \times 5^{\frac{3}{5}}} \div \frac{3^{\frac{4}{3}} \times 5^{\frac{-7}{5}}}{4^{\frac{-3}{5}} \times 6} = 10$$

$$(iv) \quad \frac{(0.6)^0 - (0.1)^{-1}}{\left(\frac{3}{8}\right)^{-1} \left(\frac{3}{2}\right)^3 + \left(-\frac{1}{3}\right)^{-1}} = -\frac{3}{2}$$

$$(v) \quad \sqrt{\frac{1}{4}} + (0.01)^{-\frac{1}{2}} - (27)^{\frac{2}{3}} = \frac{3}{2}$$

$$(vi) \quad \frac{2^n + 2^{n-1}}{2^{n+1} - 2^n} = \frac{2^n + 2^n \times 2^{-1}}{2^n \times 2^1 - 2^n}$$

$$(vii) \quad \left(\frac{64}{125}\right)^{-\frac{2}{3}} + \frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)$$

$$(viii) \quad \frac{3^{-3} \times 6^2 \times \sqrt{98}}{5^2 \times \sqrt[3]{\frac{1}{25}} \times (15)^{-\frac{4}{3}} \times 3^{\frac{1}{3}}}$$

Sol:

(i) We have,

$$\begin{aligned} & 9^{\frac{3}{2}} - 3 \times 5^0 - \left(\frac{1}{81}\right)^{-\frac{1}{2}} \\ &= (3^2)^{\frac{3}{2}} - 3 \times 1 - \left(\frac{1}{9^2}\right)^{-\frac{1}{2}} \\ &= 3^{2 \times \frac{3}{2}} - 3 - (9^{-2})^{-\frac{1}{2}} \\ &= 3^3 - 3 - 9^{-2 \times \left(-\frac{1}{2}\right)} \\ &= 3^3 - 3 - 9 \\ &= 27 - 3 - 9 \\ &= 27 - 12 \\ &= 15 \end{aligned}$$

$$\Rightarrow 9^{\frac{3}{2}} - 3 \times 5^0 - \left(\frac{1}{81}\right)^{-\frac{1}{2}} = 15$$

(ii) We have,

$$\begin{aligned} & \left(\frac{1}{4}\right)^{-2} - 3 \times 8^{\frac{2}{3}} \times 4^0 + \left(\frac{9}{16}\right)^{-\frac{1}{2}} \\ &= \left(\frac{1}{2^2}\right)^{-2} - 3 \times 8^{\frac{2}{3}} \times 1 + \left(\frac{3^2}{4^2}\right)^{-\frac{1}{2}} \\ &= \left(2^{-2}\right)^{-2} - 3 \times 8^{\frac{2}{3}} \times 1 + \left(\frac{3^{2 \times \frac{-1}{2}}}{4^{2 \times \frac{-1}{2}}}\right) \\ &= 2^{(-2) \times (-2)} - 3 \times 8^{\frac{2}{3}} + \left(\frac{3^{-1}}{4^{-1}}\right) \\ &= 2^4 - 3 \times 2^{3 \times \frac{2}{3}} + \frac{4}{3} \\ &= 2^4 - 3 \times 2^2 + \frac{4}{3} \\ &= 2^4 - 3 \times 4 + \frac{4}{3} \\ &= 16 - 12 + \frac{4}{3} \\ &= 4 + \frac{4}{3} = \frac{12+4}{3} \\ &= \frac{16}{3} \\ &\Rightarrow \left(\frac{1}{4}\right)^{-2} - 3 \times 8^{\frac{2}{3}} \times 4^0 + \left(\frac{9}{16}\right)^{-\frac{1}{2}} = \frac{16}{3} \end{aligned}$$

(iii) We have,

$$\begin{aligned} & \frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times 4^{\frac{1}{4}}}{10^{\frac{-1}{5}} \times 5^{\frac{3}{5}}} \div \frac{3^{\frac{4}{3}} \times 5^{\frac{-7}{5}}}{4^{\frac{-3}{5}} \times 6} \\ &= \frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times 4^{\frac{1}{4}}}{10^{\frac{-1}{5}} \times 5^{\frac{3}{5}}} \times \frac{4^{\frac{-3}{5}} \times 6}{3^{\frac{4}{3}} \times 5^{\frac{-7}{5}}} \end{aligned}$$

$$\begin{aligned}
&= \frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times (2^2)^{\frac{1}{4}} \times (2^2)^{-\frac{3}{5}} \times (2 \times 3)}{(2 \times 5)^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}} \\
&= \frac{\left(2^{\frac{1}{2}} \times 2^{\frac{1}{2}} \times 2^{-\frac{6}{5}} \times 2^1\right) \times \left(3^{\frac{1}{3}} \times 3^1\right)}{2^{-\frac{1}{5}} \times 5^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}} \\
&= \frac{\left(2 \times 2^{-\frac{6}{5}} \times 2\right) \times \left(3^{\frac{1}{3}} \times 3^1 \times 3^{-\frac{4}{3}}\right)}{2^{-\frac{1}{5}} \times \left(5^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 5^{-\frac{7}{5}}\right)} \\
&= \frac{\left(2 \times 2^{-\frac{6}{5}} \times 2 \times 2^{\frac{1}{5}}\right) \times \left(3^{\frac{1}{3}} \times 3^1 \times 3^{-\frac{4}{3}}\right)}{\left(5^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 5^{-\frac{7}{5}}\right)} \\
&= \frac{(2)^{1-\frac{6}{5}+1+\frac{1}{5}} \times (3)^{\frac{1}{3}+1-\frac{4}{3}}}{(5)^{-\frac{1}{5}+\frac{3}{5}-\frac{7}{5}}} \\
&= \frac{(2)^{2-\frac{6}{5}+\frac{1}{5}} \times (3)^{\frac{1+3-4}{3}}}{(5)^{\frac{-1+3-7}{5}}} \\
&= \frac{(2)^{2-\frac{5}{5}} \times (3)^{\frac{0}{3}}}{(5)^{-\frac{5}{5}}} \\
&= \frac{(2)^{2-1} \times (3)^0}{(5)^{-1}} \\
&= 2^1 \times 1 \times 5^1 \\
&= 10 \\
&\Rightarrow \frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times 4^{\frac{1}{4}}}{10^{-\frac{1}{5}} \times 5^{\frac{3}{5}}} \div \frac{3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}}{4^{-\frac{3}{5}} \times 6} = 10
\end{aligned}$$

(iv) We have,

$$\frac{(0.6)^0 - (0.1)^{-1}}{\left(\frac{3}{8}\right)^{-1} \left(\frac{3}{2}\right)^3 + \left(-\frac{1}{3}\right)^{-1}}$$

$$\begin{aligned}
 &= \frac{1 - \frac{1}{0.1}}{\left(\frac{8}{3}\right)\left(\frac{3}{2}\right)^3 + (-3)^1} \\
 &= \frac{1 - 10}{\frac{8}{3} \times \frac{3^3}{2^3} - 3} \\
 &= \frac{-9}{3^2 - 3} \\
 &= \frac{-9}{9 - 3} = -\frac{9}{6} = -\frac{3}{2}
 \end{aligned}$$

(v) We have,

$$\begin{aligned}
 &\sqrt{\frac{1}{4}} + (0.01)^{-\frac{1}{2}} - (27)^{\frac{2}{3}} \\
 &= \frac{1}{2} + \frac{1}{(0.01)^{\frac{1}{2}}} - (3^3)^{\frac{2}{3}} \\
 &= \frac{1}{2} + \frac{1}{(0.1)^{2 \times \frac{1}{2}}} - 3^{3 \times \frac{2}{3}} \\
 &= \frac{1}{2} + \frac{1}{0.1} - 3^2 \\
 &= \frac{1}{2} + 10 - 9 \\
 &= \frac{1}{2} + 1 = \frac{3}{2} \\
 &\Rightarrow \sqrt{\frac{1}{4}} (0.01)^{-\frac{1}{2}} - (27)^{\frac{2}{3}} = \frac{3}{2}
 \end{aligned}$$

(vi) We have,

$$\begin{aligned}
 \frac{2^n + 2^{n-1}}{2^{n+1} - 2^n} &= \frac{2^n + 2^n \times 2^{-1}}{2^n \times 2^1 - 2^n} \\
 &= \frac{2^n [1 + 2^{-1}]}{2^n [2 - 1]} \\
 &= \frac{1 + \frac{1}{2}}{1} \\
 &= 1 + \frac{1}{2}
 \end{aligned}$$

$$= \frac{3}{2}$$

$$\Rightarrow \frac{2^n + 2^{n-1}}{2^{n+1} - 2^n} = \frac{3}{2}$$

(vii) We have,

$$\left(\frac{64}{125}\right)^{\frac{2}{3}} + \frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)$$

$$= \left(\frac{125}{64}\right)^{\frac{2}{3}} + \frac{1}{\left(\frac{4^4}{5^4}\right)^{\frac{1}{4}}} + \left(\frac{5}{(64)^{\frac{1}{3}}}\right)$$

$$= \left(\frac{5^3}{4^3}\right)^{\frac{2}{3}} + \frac{1}{\frac{4}{5}} + \left(\frac{5}{4^{\frac{3 \times 1}{3}}}\right)$$

$$= \frac{5^2}{4^2} + \frac{5}{4} + \frac{5}{4}$$

$$= \frac{25}{16} + \frac{10}{4}$$

$$= \frac{25 + 40}{16} = \frac{65}{16}$$

(viii) We have,

$$\frac{3^{-3} \times 6^2 \times \sqrt{98}}{5^2 \times 3 \sqrt[3]{\frac{1}{25}} \times (15)^{-\frac{4}{3}} \times 3^{\frac{1}{3}}}$$

$$= \frac{3^{-3} \times 36 \times \sqrt{7 \times 7 \times 2}}{5^2 \times \left(\frac{1}{25}\right)^{\frac{1}{3}} \times (15)^{-\frac{4}{3}} \times 3^{\frac{1}{3}}}$$

$$= \frac{3^{-3} \times 36 \times 7\sqrt{2}}{5^2 \times \left(\frac{1}{5^{\frac{2 \times 1}{3}}}\right) \times \frac{1}{(15)^{\frac{4}{3}}} \times 3^{\frac{1}{3}}}$$

$$= \frac{3^{-3} \times 36 \times 7\sqrt{2}}{5^2 \times 5^{-\frac{2}{3}} \times \frac{1}{(5 \times 3)^{\frac{4}{3}}} \times 3^{\frac{1}{3}}}$$

$$\begin{aligned}
&= \frac{3^{-3} \times 36 \times 7\sqrt{2}}{\left(5^2 \times 5^{\frac{2}{3}} \times 5^{\frac{4}{3}}\right) \times 3^{\frac{4}{3}} \times 3^{\frac{1}{3}}} \\
&= \frac{3^{-3} \times 36 \times 7\sqrt{2} \times 3^{\frac{4}{3}} \times 3^{\frac{1}{3}}}{(5)^{2-\frac{2}{3}-\frac{4}{3}}} \\
&= \frac{3^{-3} \times 36 \times 7\sqrt{2} \times 3^{\frac{4}{3}} \times 3^{\frac{1}{3}}}{(5)^{\frac{6-2-4}{3}}} \\
&= \frac{3^{-3+\frac{4}{3}+\frac{1}{3}} \times 36 \times 7\sqrt{2}}{5^0} \\
&= 3^{-3+\left(\frac{4+1}{3}\right)} \times 36 \times 7\sqrt{2} \\
&= 3^{-3+\frac{5}{3}} \times 36 \times 7\sqrt{2} \\
&= 3^{-3+1} \times 36 \times 7\sqrt{2} \\
&= 3^{-2} \times 36 \times 7\sqrt{2} \\
&= \frac{1}{9} \times 36 \times 7\sqrt{2} \\
&= 4 \times 7\sqrt{2} \\
&= 28\sqrt{2} \\
&\Rightarrow \frac{3^{-3} \times 6^2 \times \sqrt{98}}{5^2 \times \sqrt[3]{\frac{1}{25}} \times (15)^{-\frac{4}{3}} \times 3^{\frac{1}{3}}} = 28\sqrt{2}
\end{aligned}$$

4. If $27^x = \frac{9}{3^x}$, find x

Sol:

We have,

$$\begin{aligned}
(27^x) &= \frac{9}{3^x} \\
\Rightarrow (3^3)^x &= \frac{9}{3^x} \\
\Rightarrow 3^{3 \times x} &= \frac{3^2}{3^x} \\
\Rightarrow 3^{3x} &= 3^{2-x} \\
\Rightarrow 3x &= 2-x && \text{[On equating exponents]}
\end{aligned}$$

$$\Rightarrow 3x + x = 2$$

$$\Rightarrow 4x = 2 \Rightarrow x = \frac{2}{4} \Rightarrow \boxed{x = \frac{1}{2}}$$

Hence, value of x is $\frac{1}{2}$

5. Find the values of x in each of the following:

(i) $2^{5x} \div 2^x = \sqrt[5]{2^{20}}$

(ii) $(2^3)^4 = (2^2)^x$

(iii) $\left(\frac{3}{5}\right)^x \left(\frac{5}{x}\right)^{2x} = \frac{125}{27}$

(iv) $5^{x-2} \times 3^{2x-3} = 135$

(v) $2^{x-5} \times 5^{x-4} = 5$

(vi) $2^{x-7} \times 5^{x-4} = 1250$

Sol:

(i) We have

$$2^{5x} \div 2^x = \sqrt[5]{2^{20}}$$

$$\frac{2^{5x}}{2^x} = (2^{20})^{\frac{1}{5}}$$

$$2^{5x-x} = 2^{20 \times \frac{1}{5}}$$

$$2^{4x} = 2^4$$

$$\Rightarrow 4x = 4$$

[On equating exponents]

$$\Rightarrow \boxed{x = 1}$$

Hence value of x is 1

(ii) We have,

$$(2^3)^4 = (2^2)^x$$

$$\Rightarrow 2^{3 \times 4} = 2^{2 \times x}$$

$$\Rightarrow 12 = 2x$$

[On equating exponents]

$$\Rightarrow 2x = 12$$

$$\Rightarrow \boxed{x = 6}$$

Hence, value of x is 6.

(iii) We have,

$$\left(\frac{3}{5}\right)^x \left(\frac{5}{x}\right)^{2x} = \frac{125}{27}$$

$$\Rightarrow \frac{(3)^x (5)^{2x}}{(5)^x (3)^{2x}} = \frac{5^3}{3^3}$$

$$\Rightarrow \frac{5^{2x-x}}{3^{2x-x}} = \left(\frac{5}{3}\right)^3$$

$$\Rightarrow \frac{5^x}{3^x} = \left(\frac{5}{3}\right)^3$$

$$\Rightarrow \left(\frac{5}{3}\right)^x = \left(\frac{5}{3}\right)^3$$

[On equating exponents]

$$\Rightarrow \boxed{x=3}$$

Hence, value of x is 3

(iv) We have

$$5^{x-2} \times 3^{2x-3} = 135$$

$$\Rightarrow 5^{x-2} \times 3^{2x-3} = 5 \times 27$$

$$\Rightarrow 5^{x-2} \times 3^{2x-3} = 5^1 \times 3^3$$

$$\Rightarrow x-2=1, \quad 2x-3=3$$

[On equating exponents]

$$\Rightarrow x=2+1, \quad 2x=3+3$$

$$\Rightarrow x=3, \quad 2x=6 \Rightarrow x=3$$

Hence, the value of x is 3

(v) We have,

$$2^{x-5} \times 5^{x-4} = 5$$

$$\Rightarrow 2^{x-5} \times 5^{x-4} = 5^1 \times 2^0$$

$$\Rightarrow x-5=0, \quad x-4=1$$

$$\Rightarrow x=5, \quad x=4+1$$

$$\Rightarrow \boxed{x=5}$$

Hence, the value of x is 5

(vi) We have,

$$2^{x-7} \times 5^{x-4} = 1250$$

$$\Rightarrow 2^{x-7} \times 5^{x-4} = 2 \times 625$$

$$\Rightarrow 2^{x-7} \times 5^{x-4} = 2^1 \times 5^4$$

$$\Rightarrow x-7=1$$

$$\Rightarrow x=8, \quad x-4=4$$

$$\Rightarrow \boxed{x=8}$$

Hence, the value of x is 8