Ex 12.1

Higher Order Derivatives Ex 12.1 Q1(i)

We have $f(x) = x^3 + \tan x$

$\Rightarrow \quad f'(x) = 3x^2 + \sec^2 x$

$\Rightarrow \quad f''(x) = 6x + 2\sec x \times \sec x \tan x$

$\Rightarrow \quad f''(x) = 6x + 2\sec^2 x \tan x.$

Higher Order Derivatives Ex 12.1 Q1(ii)

Let $y = \sin(\log x)$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin(\log x)\right] = \cos(\log x)\cdot\frac{d}{dx}(\log x) = \frac{\cos(\log x)}{x}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{\cos(\log x)}{x}\right]$$

$$= \frac{x\cdot\frac{d}{dx}\left[\cos(\log x)\right] - \cos(\log x)\cdot\frac{d}{dx}(x)}{x^2}$$

$$= \frac{x\cdot\left[-\sin(\log x)\cdot\frac{d}{dx}(\log x)\right] - \cos(\log x).1}{x^2}$$

$$= \frac{-x\sin(\log x).\frac{1}{x} - \cos(\log x)}{x^2}$$

$$= \frac{-\left[\sin(\log x) + \cos(\log x)\right]}{x^2}$$

Higher Order Derivatives Ex 12.1 Q1(iii)

Let $y = \log(\sin x)$

Differentiating with repect to x, we get,

$$\frac{dy}{dx} = \frac{\cos x}{\sin x}$$

Again differentiating with respect to x, we get,

$$\frac{d^2y}{dx^2} = \frac{-\sin x \times \sin x - \cos x \times \cos x}{\sin^2 x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-1}{\sin^2 x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\operatorname{cosec}^2 x$$

Higher Order Derivatives Ex 12.1 Q1(iv)

Let $y = e^x \sin 5x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x \sin 5x\right) = \sin 5x \cdot \frac{d}{dx}\left(e^x\right) + e^x \frac{d}{dx}(\sin 5x)$$

$$= \sin 5x \cdot e^x + e^x \cdot \cos 5x \cdot \frac{d}{dx}(5x) = e^x \sin 5x + e^x \cos 5x \cdot 5$$

$$= e^x\left(\sin 5x + 5\cos 5x\right)$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left[e^x\left(\sin 5x + 5\cos 5x\right)\right]$$

$$= \left(\sin 5x + 5\cos 5x\right) \cdot \frac{d}{dx}\left(e^x\right) + e^x \cdot \frac{d}{dx}\left(\sin 5x + 5\cos 5x\right)$$

$$= \left(\sin 5x + 5\cos 5x\right)e^x + e^x\left[\cos 5x \cdot \frac{d}{dx}(5x) + 5(-\sin 5x) \cdot \frac{d}{dx}(5x)\right]$$

$$= e^x\left(\sin 5x + 5\cos 5x\right) + e^x\left(5\cos 5x - 25\sin 5x\right)$$

$$= e^x\left(10\cos 5x - 24\sin 5x\right) = 2e^x\left(5\cos 5x - 12\sin 5x\right)$$

Higher Order Derivatives Ex 12.1 Q1(v)

Let $y = e^{6x}\cos 3x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{6x} \cdot \cos 3x\right) = \cos 3x \cdot \frac{d}{dx}\left(e^{6x}\right) + e^{6x} \cdot \frac{d}{dx}(\cos 3x)$$

$$= \cos 3x \cdot e^{6x} \cdot \frac{d}{dx}(6x) + e^{6x} \cdot (-\sin 3x) \cdot \frac{d}{dx}(3x)$$

$$= 6e^{6x}\cos 3x - 3e^{6x}\sin 3x \qquad \ldots(1)$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left(6e^{6x}\cos 3x - 3e^{6x}\sin 3x\right) = 6 \cdot \frac{d}{dx}\left(e^{6x}\cos 3x\right) - 3 \cdot \frac{d}{dx}\left(e^{6x}\sin 3x\right)$$

$$= 6 \cdot \left[6e^{6x}\cos 3x - 3e^{6x}\sin 3x\right] - 3 \cdot \left[\sin 3x \cdot \frac{d}{dx}\left(e^{6x}\right) + e^{6x} \cdot \frac{d}{dx}(\sin 3x)\right] \qquad [\text{Using } (1)]$$

$$= 36e^{6x}\cos 3x - 18e^{6x}\sin 3x - 3\left[\sin 3x \cdot e^{6x} \cdot 6 + e^{6x} \cdot \cos 3x \cdot 3\right]$$

$$= 36e^{6x}\cos 3x - 18e^{6x}\sin 3x - 18e^{6x}\sin 3x - 9e^{6x}\cos 3x$$

$$= 27e^{6x}\cos 3x - 36e^{6x}\sin 3x$$

$$= 9e^{6x}\left(3\cos 3x - 4\sin 3x\right)$$

Higher Order Derivatives Ex 12.1 Q1(vi)

Let $y = x^3 \log x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left[x^3 \log x\right] = \log x \cdot \frac{d}{dx}\left(x^3\right) + x^3 \cdot \frac{d}{dx}(\log x)$$
$$= \log x \cdot 3x^2 + x^3 \cdot \frac{1}{x} = \log x \cdot 3x^2 + x^2$$
$$= x^2(1 + 3\log x)$$
$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left[x^2(1+3\log x)\right]$$
$$= (1+3\log x)\cdot\frac{d}{dx}\left(x^2\right) + x^2\frac{d}{dx}(1+3\log x)$$
$$= (1+3\log x)\cdot 2x + x^2 \cdot \frac{3}{x}$$
$$= 2x + 6x\log x + 3x$$
$$= 5x + 6x\log x$$
$$= x(5 + 6\log x)$$

Higher Order Derivatives Ex 12.1 Q1(vii)

Let $y = \tan^{-1} x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1}x\right) = \frac{1}{1+x^2}$$
$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{1+x^2}\right) = \frac{d}{dx}\left(1+x^2\right)^{-1} = (-1)\cdot\left(1+x^2\right)^{-2}\cdot\frac{d}{dx}\left(1+x^2\right)$$
$$= \frac{-1}{\left(1+x^2\right)^2}\times 2x = \frac{-2x}{\left(1+x^2\right)^2}$$

Higher Order Derivatives Ex 12.1 Q1(viii)

Let $y = x \cdot \cos x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}(x\cdot\cos x) = \cos x\cdot\frac{d}{dx}(x) + x\frac{d}{dx}(\cos x) = \cos x\cdot 1 + x(-\sin x) = \cos x - x\sin x$$
$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left[\cos x - x\sin x\right] = \frac{d}{dx}(\cos x) - \frac{d}{dx}(x\sin x)$$
$$= -\sin x - \left[\sin x\cdot\frac{d}{dx}(x) + x\cdot\frac{d}{dx}(\sin x)\right]$$
$$= -\sin x - (\sin x + x\cos x)$$
$$= -(x\cos x + 2\sin x)$$

Higher Order Derivatives Ex 12.1 Q1(ix)

Let $y = \log(\log x)$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log(\log x)\right] = \frac{1}{\log x}\cdot\frac{d}{dx}(\log x) = \frac{1}{x\log x} = (x\log x)^{-1}$$
$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left[(x\log x)^{-1}\right] = (-1)\cdot(x\log x)^{-2}\cdot\frac{d}{dx}(x\log x)$$
$$= \frac{-1}{(x\log x)^2}\cdot\left[\log x\cdot\frac{d}{dx}(x) + x\cdot\frac{d}{dx}(\log x)\right]$$
$$= \frac{-1}{(x\log x)^2}\cdot\left[\log x\cdot 1 + x\cdot\frac{1}{x}\right] = \frac{-(1+\log x)}{(x\log x)^2}$$

Higher Order Derivatives Ex 12.1 Q2

$y = e^{-x}\cos x$

differentiating both sides w.r.t x

$$\Rightarrow \quad \frac{dy}{dx} = e^{-x}(-\sin x) + (\cos x)\left(-e^{-x}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = -e^{-x}\sin x - e^{-x}\cos x = -e^{-x}(\sin x + \cos x)$$

again differentiating both sides w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -e^{-x}(\cos x - \sin x) + e^{-x}(\sin x + \cos x)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 2e^{-x}\sin x$$

Higher Order Derivatives Ex 12.1 Q3

$y = x + \tan x$

differentiating both sides w.r.t x

$$\Rightarrow \quad \frac{dy}{dx} = 1 + \sec^2 x$$

differentiating w.r.t x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 0 + 2\sec^2 x \tan x$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{2\sin x}{\cos^3 x}$$

$$\Rightarrow \quad \cos^2 x \frac{d^2y}{dx^2} = 2\tan x + 2x - 2x$$

$$\Rightarrow \quad \cos^2 x \frac{d^2y}{dx^2} = 2(x + \tan x) - 2x$$

$$\Rightarrow \quad \cos^2 x \frac{d^2y}{dx^2} = 2y - 2x$$

$$\Rightarrow \quad \cos^2 x \frac{d^2y}{dx^2} - 2y + 2x = 0$$

Higher Order Derivatives Ex 12.1 Q4

$y = x^3 \log x$

differentiating w.r.t x

$$\Rightarrow \quad \frac{dy}{dx} = 3x^2 \log x + \frac{x^3}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} = 3x^2 \log x + x^2$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = (\log x)(3 \times 2x) + \frac{3x^2}{x} + 2x$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 6x \log x + 5x$$

differentiating w.r.t x

$$\Rightarrow \quad \frac{d^3y}{dx^3} = \frac{6x}{x} + 6\log x + 5$$

$$\Rightarrow \quad \frac{d^3y}{dx^3} = 6\log x + 11$$

differentiating w.r.t x

$$\Rightarrow \quad \frac{d^4y}{dx^4} = \frac{6}{x} + 0$$

$$\Rightarrow \quad \frac{d^4y}{dx^4} = \frac{6}{x}$$

Higher Order Derivatives Ex 12.1 Q5

$y = \log(\sin x)$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = \frac{d(\log(\sin x))}{d(\sin x)} \times \frac{d(\sin x)}{dx} \text{(chain rule)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{\sin x} \times \cos x = \cot x$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\operatorname{cosec}^2 x$$

differentiating w.r.t x

$$\Rightarrow \quad \frac{d^3y}{dx^3} = (-2\operatorname{cosec} x) \times (-\cot x \operatorname{cosec} x)$$

$$\Rightarrow \quad \frac{d^3y}{dx^3} = \frac{2\operatorname{cosec}^2 \cos x}{\sin x}$$

$$\Rightarrow \quad \frac{d^3y}{dx^3} = 2\operatorname{cosec}^3 x \cos x$$

Higher Order Derivatives Ex 12.1 Q6

$y = 2\sin x + 3\cos x$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = 2\cos x + 3(-\sin x) = 2\cos x - 3\sin x$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 2(-\sin x) - 3\cos x = -(2\sin x + 3\cos x) = -y$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + y = 0$$

Higher Order Derivatives Ex 12.1 Q7

$y = \frac{\log x}{x}$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x\left(\frac{1}{x}\right) - (\log x)(i)}{x^2}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1 - \log x}{x^2}$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{x^2\left(-\frac{1}{x}\right) - (1 - \log x)(2x)}{x^4}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-x - 2x + 2x\log x}{x^4} = \frac{x(2\log x - 3)}{x^4}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{2\log x - 3}{x^3}$$

Higher Order Derivatives Ex 12.1 Q8

$x = a\sec\theta \qquad y = b\tan\theta$
differentiating both w.r.t.θ

$$\Rightarrow \quad \frac{dx}{d\theta} = a\sec\theta\tan\theta \qquad \text{........}(1)$$

$$\Rightarrow \quad \frac{dy}{d\theta} = b\sec^2\theta \qquad \text{.....}(2)$$

Dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{b\sec^2\theta}{a\sec\theta\tan\theta} \qquad \text{....}(3)$$

Differentiating (3) w.r.t.θ

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{b}{a}\left[\frac{\tan\theta\left(\sec\theta\tan\theta\right) - \sec\theta\left(\sec^2\theta\right)}{\tan^2\theta}\right]$$

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{b}{a}\left[\frac{\sec\theta\left(\tan^2\theta\right) - \sec^2\theta}{\tan^2\theta}\right] \text{.....}(4)$$

Dividing (4) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{b\sec\theta\left(\tan^2\theta - \sec^2\theta\right)}{a \times a\sec\theta\tan\theta \times \tan^2\theta}$$

Multiplying & dividing RHS by b^3

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-b^4}{a^2 \times b^3\tan^3\theta}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

Higher Order Derivatives Ex 12.1 Q9

It is given that, $x = a\left(\cos t + t\sin t\right)$ and $y = a\left(\sin t - t\cos t\right)$

$$\therefore \frac{dx}{dt} = a\cdot\frac{d}{dt}(\cos t + t\sin t)$$
$$= a\left[-\sin t + \sin t\cdot\frac{d}{dt}(t) + t\cdot\frac{d}{dt}(\sin t)\right]$$
$$= a\left[-\sin t + \sin t + t\cos t\right] = at\cos t$$

$$\frac{dy}{dt} = a\cdot\frac{d}{dt}(\sin t - t\cos t)$$
$$= a\left[\cos t - \left\{\cos t\cdot\frac{d}{dt}(t) + t\cdot\frac{d}{dt}(\cos t)\right\}\right]$$
$$= a\left[\cos t - \left\{\cos t - t\sin t\right\}\right] = at\sin t$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{at\sin t}{at\cos t} = \tan t$$

Then, $$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(\tan t) = \sec^2 t\cdot\frac{dt}{dx}$$

$$= \sec^2 t\cdot\frac{1}{at\cos t} \qquad \left[\frac{dx}{dt} = at\cos t \Rightarrow \frac{dt}{dx} = \frac{1}{at\cos t}\right]$$

$$= \frac{\sec^3 t}{at}, 0 < t < \frac{\pi}{2}$$

Higher Order Derivatives Ex 12.1 Q10

$y = e^x \cos x$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = e^x(-\sin x) + e^x \cos x = e^x(\cos x - \sin x)$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = e^x(-\cos x - \sin x) + e^x(\cos x - \sin x)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -2e^x \sin x$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 2e^x \cos\left(x + \frac{\pi}{2}\right)$$

Higher Order Derivatives Ex 12.1 Q11

$x = a\cos\theta$

differentiating w.r.t. θ

$$\Rightarrow \quad \frac{dy}{d\theta} = -a\sin\theta \quad .....(1)$$

$$\Rightarrow \quad \frac{dy}{d\theta} = b\cos\theta \quad .....(2)$$

Dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-b\cos\theta}{a\sin\theta} \quad ......(3)$$

differentiating (3) w.r.t. θ

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{-b}{a}\left\{\frac{\sin\theta(-\sin\theta) - \cos\theta(\cos\theta)}{\sin^2\theta}\right\} = \frac{b}{a}\frac{\left(\sin^2\theta + \cos^2\theta\right)}{\sin^2\theta} = \frac{b}{a\sin^2\theta} \quad ......(4)$$

Dividing (4) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-b}{a^2\sin^3\theta} \times \frac{b^3}{b^3}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

Higher Order Derivatives Ex 12.1 Q12

$x = a\left(1 - \cos^3\theta\right); \quad y = a\sin^3\theta$

differentiating both w.r.t.θ

$$\Rightarrow \quad \frac{dx}{d\theta} = a\left(0 - 3\cos^2\theta(-\sin\theta)\right); \quad \frac{dy}{d\theta} = a\left(3\sin^2\theta \times \cos\theta\right) ......(2)$$

$$\Rightarrow \quad \frac{dy}{d\theta} = 3a\sin\theta\cos^2\theta; \quad \frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{3a\sin^2\theta\cos\theta}{3a\sin\theta\cos^2\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

Differentiating w.r.t.θ

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \sec^2\theta \quad ....(3)$$

Dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{\sec^2\theta}{3a\sin\theta\cos^2\theta}$$

Putting $\theta = \pi/6$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{\frac{2}{\sqrt{3}} \times \frac{2}{\sqrt{3}}}{3a \times \frac{1}{2} \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}} = \frac{2^5}{3a \times \left(\sqrt{3}\right)^4} = \frac{32}{27a}$$

Higher Order Derivatives Ex 12.1 Q13

$x = a(\theta + \sin\theta)$; $y = a(1 + \cos\theta)$

differentiating both w.r.t.θ

$$\Rightarrow \quad \frac{dx}{d\theta} = a(1 + \cos\theta); \quad (1)$$

$$\Rightarrow \quad \frac{dy}{d\theta} = a(0 - \sin\theta) \quad (2)$$

Dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{-a\sin\theta}{a(1 + \cos\theta)}$$

Differentiating w.r.t.θ

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = -\left\{\frac{(1 + \cos\theta)(\cos\theta) - (\sin\theta)(0 - \sin\theta)}{(1 + \cos\theta)^2}\right\} = -\left\{\frac{\cos\theta + \cos^2\theta + \sin^2\theta}{(1 + \cos\theta)^2}\right\}$$

$$= -\left\{\frac{\cos\theta + 1}{(\cos + 1)^2}\right\}$$

$$= \frac{-1}{1 + \cos\theta} \quad \ldots\ldots(3)$$

dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-1 \times a}{a(1 + \cos\theta)^2 \times a} = \frac{-a}{y^2}$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q14

$x = a(\theta - \sin\theta);\ y = a(1 + \cos\theta)$

Diiferentiating the above functions with respect to θ, we get,

$$\frac{dx}{d\theta} = a(1 - \cos\theta) \quad ...(1)$$

$$\frac{dy}{d\theta} = a(-\sin\theta) \qquad ...(2)$$

Dividing equation (2) by (1), we have,

$$\frac{dy}{dx} = \frac{a(-\sin\theta)}{a(1-\cos\theta)} = \frac{-\sin\theta}{1-\cos\theta}$$

Differentiating with respect to θ, we have,

$$\frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{(1-\cos\theta)(-\cos\theta) + \sin\theta(\sin\theta)}{(1-\cos\theta)^2}$$

$$= \frac{-\cos\theta + \cos^2\theta + \sin^2\theta}{(1-\cos\theta)^2}$$

$$= \frac{1-\cos\theta}{(1-\cos\theta)^2}$$

$$\frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{1}{1-\cos\theta} ...(3)$$

Dividing equation (3) by (1), we have,

$$\frac{d^2y}{dx^2} = \frac{1}{1-\cos\theta} \times \frac{1}{a(1-\cos\theta)}$$

$$= \frac{1}{a(1-\cos\theta)^2}$$

$$= \frac{1}{a\left(2\sin^2\frac{\theta}{2}\right)^2}$$

$$= \frac{1}{4a\sin^4\left(\frac{\theta}{2}\right)}$$

$$= \frac{1}{4a}\operatorname{cosec}^4\left(\frac{\theta}{2}\right)$$

Higher Order Derivatives Ex 12.1 Q15

$x = a(1 - \cos\theta)$; $\quad y = a(\theta + \sin\theta)$

Differentiating both w.r.t.θ

$$\Rightarrow \quad \frac{dx}{d\theta} = a(0 + \sin\theta); \quad \frac{dy}{d\theta} = a(1 + \cos\theta)$$

Dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{a(1 + \cos\theta)}{a\sin\theta}$$

Differentiating w.r.t.θ

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{\sin\theta(0 - \sin\theta) - (1 + \cos\theta)\cos\theta}{\sin^2\theta} = -\frac{\sin^2\theta - \cos\theta - \cos^2\theta}{\sin^2\theta}$$

$$= -\frac{(1 + \cos\theta)}{\sin^2\theta} \qquad \ldots\ldots(3)$$

dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\frac{(1 + \cos\theta)}{\sin^2\theta \times a\sin\theta}$$

Putting $\theta = \pi/2$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\frac{1}{a}$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q17

$x = \cos\theta;\ y = \sin^3\theta$
Differentiating both w.r.t.θ

$$\Rightarrow \quad \frac{dx}{d\theta} = -\sin\theta; \qquad (1)$$
$$\frac{dy}{d\theta} = 3\sin^2\theta\cos\theta \qquad (2)$$

Dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = -\frac{3\sin^2\theta\cos\theta}{\sin\theta} = -3\sin\theta\cos\theta$$

Differentiating w.r.t.θ

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = -3\{\sin\theta(-\sin\theta) + \cos\theta(\cos\theta)\} = -3\left(\cos^2\theta - \sin^2\theta\right)\ldots\ldots\ldots(3)$$

Dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{+3\left(\cos^2\theta - \sin^2\theta\right)}{\sin\theta} \times \frac{\sin^2\theta}{\sin^2\theta}$$

$$\Rightarrow \quad \sin^3\theta\frac{d^2y}{dx^2} = 3\sin^2\theta\left(\cos^2\theta - \sin^2\theta\right)$$

$$\Rightarrow \quad y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 3\sin^2\theta\left(\cos^2\theta - \sin^2\theta\right) + \left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow \quad y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 3\sin^2\theta\cos^2\theta - 3\sin^4\theta + 9\sin^2\theta\cos^2\theta$$

adding and subtracting $3\sin^2\theta\cos^2\theta$ on RHS

$$\Rightarrow \quad y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 12\sin^2\theta\cos^2\theta - 3\sin^4\theta + 3\sin^2\theta\cos^2\theta - 3\sin^2\theta\cos^2\theta$$

$$\Rightarrow \quad y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 15\sin^2\theta\cos^2\theta - 3\sin^2\theta\left(\sin^2\theta + \cos^2\theta\right)$$
$$= 15\sin^2\theta\cos^2\theta - 3\sin^2\theta$$

$$\Rightarrow \quad y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 3\sin^2\theta\{5\cos^2\theta - 1\}$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q18

$y = \sin(\sin x)$
differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = \frac{d(\sin(\sin x))}{d(\sin x)} \times \frac{d(\sin x)}{dx} = \cos(\sin x) \times \cos x$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = (\cos(\sin x))(-\sin x) + (\cos x)(-\sin(\sin x))(\cos x)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\sin x \cos(\sin x) \times \frac{\cos x}{\cos x} - y\cos^2 x$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\tan x \frac{dy}{dx} - y\cos^2 x$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y\cos^2 x = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q19

$x = \sin t; \quad y = \sin pt$
differentiating both w.r.t. t

$$\Rightarrow \quad \frac{dy}{dt} = \cos t \;\ldots\ldots(1); \qquad \frac{dy}{dt} = p\cos pt \;\ldots\ldots(2)$$

dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = p\frac{\cos pt}{\cos t}$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{dt} = p\left\{\frac{p\cos t(-\sin pt) - (\cos pt)(-\sin t)}{\cos^2 t}\right\}$$

$$= p\left\{\frac{\sin t\cos pt - p\cos t\sin pt(-\sin t)}{\cos^2 t}\right\} \ldots\ldots(3)$$

$\Rightarrow$ dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = p\left\{\frac{\sin t\cos pt - p\cos t\sin pt}{\cos^3 t}\right\} = \left\{\frac{\tan t\cos t - p\sin pt}{\cos^2 t}\right\}$$

$\because \quad \sin^2 t + \cos^2 t = 1$

$\Rightarrow \quad 1 - \sin^2 t = \cos^2 t$

$\Rightarrow \quad 1 - x^2 = \cos^2 t$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = p\left\{\frac{\tan t\cos pt - p\sin pt}{1 - x^2}\right\}$$

$$\Rightarrow \quad (1 - x^2)\frac{d^2y}{dx^2} = p\frac{\sin t\cos pt}{\cos t} - p^2\sin pt = \frac{dy}{dx} - p^2y$$

$$\Rightarrow \quad (1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q20

$y = e^{\tan^{-1} x}$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = e^{\tan^{-1} x}\left(\frac{1}{1+x^2}\right)$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{\left(1+x^2\right)\left(e^{\tan^{-1} x}\right) \times \frac{1}{1+x^2} - e^{\tan^{-1} x}(2x)}{\left(1+x^2\right)^2}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} = \frac{e^{\tan^{-1} x} - 2xe^{\tan^{-1} x}}{1+x^2}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} = \frac{e^{\tan^{-1} x}}{1+x^2}(1-2x) = \frac{dy}{dx}(1-2x)$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q21

$y = e^{\tan^{-1} x}$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = e^{\tan^{-1} x}\left(\frac{1}{1+x^2}\right)$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{\left(1+x^2\right)\left(e^{\tan^{-1} x}\right) \times \frac{1}{1+x^2} - e^{\tan^{-1} x}(2x)}{\left(1+x^2\right)^2}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} = \frac{e^{\tan^{-1} x} - 2xe^{\tan^{-1} x}}{1+x^2}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} = \frac{e^{\tan^{-1} x}}{1+x^2}(1-2x) = \frac{dy}{dx}(1-2x)$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q22

It is given that, $y = 3\cos(\log x) + 4\sin(\log x)$

Then,

$$y_1 = 3 \cdot \frac{d}{dx}\left[\cos(\log x)\right] + 4 \cdot \frac{d}{dx}\left[\sin(\log x)\right]$$
$$= 3 \cdot \left[-\sin(\log x) \cdot \frac{d}{dx}(\log x)\right] + 4 \cdot \left[\cos(\log x) \cdot \frac{d}{dx}(\log x)\right]$$
$$\therefore y_1 = \frac{-3\sin(\log x)}{x} + \frac{4\cos(\log x)}{x} = \frac{4\cos(\log x) - 3\sin(\log x)}{x}$$
$$\therefore y_2 = \frac{d}{dx}\left(\frac{4\cos(\log x) - 3\sin(\log x)}{x}\right)$$
$$= \frac{x\{4\cos(\log x) - 3\sin(\log x)\}' - \{4\cos(\log x) - 3\sin(\log x)\}(x)'}{x^2}$$
$$= \frac{x\left[4\{\cos(\log x)\}' - 3\{\sin(\log x)\}'\right] - \{4\cos(\log x) - 3\sin(\log x)\}.1}{x^2}$$
$$= \frac{x\left[-4\sin(\log x).(\log x)' - 3\cos(\log x).(\log x)'\right] - 4\cos(\log x) + 3\sin(\log x)}{x^2}$$
$$= \frac{x\left[-4\sin(\log x).\frac{1}{x} - 3\cos(\log x).\frac{1}{x}\right] - 4\cos(\log x) + 3\sin(\log x)}{x^2}$$
$$= \frac{-4\sin(\log x) - 3\cos(\log x) - 4\cos(\log x) + 3\sin(\log x)}{x^2}$$
$$= \frac{-\sin(\log x) - 7\cos(\log x)}{x^2}$$
$$\therefore x^2 y_2 + x y_1 + y$$
$$= x^2\left(\frac{-\sin(\log x) - 7\cos(\log x)}{x^2}\right) + x\left(\frac{4\cos(\log x) - 3\sin(\log x)}{x}\right) + 3\cos(\log x) + 4\sin(\log x)$$
$$= -\sin(\log x) - 7\cos(\log x) + 4\cos(\log x) - 3\sin(\log x) + 3\cos(\log x) + 4\sin(\log x)$$
$$= 0$$

Hence, proved.

Higher Order Derivatives Ex 12.1 Q23

$y = e^{2x}(ax + b)$

differentiating w.r.t. x

$\Rightarrow \quad \frac{dy}{dx} = e^{2x}(a) + 2(ax + b)(e^{2x})$

$\Rightarrow \quad \frac{dy}{dx} = ae^{2x} + 2y$

differentiating w.r.t. x

$\Rightarrow \quad \frac{d^2y}{dx^2} = 2ae^{2x} + 2\frac{dy}{dx}$

$\Rightarrow \quad \frac{d^2y}{dx^2} = 2\frac{dy}{dx} + 2ae^{2x} + 4y - 4y = 2\frac{dy}{dx} + 2\frac{dy}{dx} - 4y$

$\Rightarrow \quad \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

$\Rightarrow \quad y_2 - 4y_1 + 4y = 0$

Hence proved!

Higher Order Derivatives Ex 12.1 Q24

$$x = \sin\left(\frac{1}{a}\log y\right)$$

$$\Rightarrow \quad \sin^{-1} x = \frac{1}{a}\log y$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{1}{\sqrt{1-x^2}} = \frac{1}{ay}\frac{dy}{dx}$$

$$\Rightarrow \quad y_1 = \frac{dy}{dx} = \frac{ay}{\sqrt{1-x^2}}$$

differentiating w.r.t.x

$$\Rightarrow \quad y_2 = \frac{d^2y}{dx^2} = a\left[\frac{\sqrt{1-x^2}\frac{dy}{dx} + \frac{y \times 2x}{2\sqrt{1-x^2}}}{1-x^2}\right]$$

$$\Rightarrow \quad \left(1-x^2\right)y_2 = a\sqrt{1-x^2}\frac{dy}{dx} + \frac{ayx}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad \left(1-x^2\right)y_2 = x\frac{dy}{dx} + a\sqrt{1-x^2} \times \frac{ay}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad \left(1-x^2\right)y_2 - xy_1 - a^2y = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q25

$$\log y = \tan^{-1} x$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \frac{1}{1+x^2}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{dy}{dx} = y$$

differentiating w.r.t.x

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = \frac{dy}{dx}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} + \left(2x-1\right)\frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q26

$$y = \tan^{-1} x$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{1+x^2}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{dy}{dx} = 1$$

differentiating w.r.t.x

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q27

$$y = \left[\log\left(x+\sqrt{1+x^2}\right)\right]^2$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = 2\log\left(x+\sqrt{1+x^2}\right) \times \frac{1}{x+\sqrt{1+x^2}} \times \left(1+\frac{1\times 2x}{2\sqrt{1+x^2}}\right)$$

$$\Rightarrow \quad y_1 = \frac{2\log\left(x+\sqrt{1+x^2}\right)}{x+\sqrt{1+x^2}} \times \frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}} = \frac{2\log\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}$$

squaring both sides

$$\Rightarrow \quad (y_1)^2 = \frac{4}{1+x^2}\left[\log\left(x+\sqrt{1+x^2}\right)\right]^2 = \frac{4y}{1+x^2}$$

$$\Rightarrow \quad \left(1+x^2\right)(y_1)^2 = 4y$$

differentiating w.r.t. x

$$\Rightarrow \quad \left(1+x^2\right)2y_1y_2 + 2x\,(y_1)^2 = 4y_1$$

$$\Rightarrow \quad \left(1+x^2\right)y_2 + xy_1 = 2$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q28

The given relationship is $y = \left(\tan^{-1} x\right)^2$

Then,

$$y_1 = 2\tan^{-1} x \frac{d}{dx}\left(\tan^{-1} x\right)$$

$$\Rightarrow y_1 = 2\tan^{-1} x.\frac{1}{1+x^2}$$

$$\Rightarrow \left(1+x^2\right)y_1 = 2\tan^{-1} x$$

Again differentiating with respect to x on both the sides, we obtain

$$\left(1+x^2\right)y_2 + 2xy_1 = 2\left(\frac{1}{1+x^2}\right)$$

$$\Rightarrow \left(1+x^2\right)^2 y_2 + 2x\left(1+x^2\right)y_1 = 2$$

Hence, proved.

Higher Order Derivatives Ex 12.1 Q29

$$y = \cot x$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = -\operatorname{cosec}^2 x$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\left[2\operatorname{cosec} x\left(-\operatorname{cosec} x \cot x\right)\right] = 2\operatorname{cosec}^2 x \cot x = -2\frac{dy}{dx}.y$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + 2y\frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q30

$$y = \log\left(\frac{x^2}{e^2}\right)$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{x^2/e^2} \times \frac{1}{e^2} \times 2x = \frac{2}{x}$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 2\left(\frac{-1}{x^2}\right) = \frac{-2}{x^2}$$

Higher Order Derivatives Ex 12.1 Q31

$y = ae^{2x} + be^{-x}$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = 2ae^{2x} + be^{-x}(-1) = 2ae^{2x} - be^{-x}$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 2ae^{2x}(2) - be^{-x}(-1) = 4ae^{2x} + be^{-x}$$

Adding and subtracting be^{-x} on RHS

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 4ae^{2x} + 2be^{-x} - be^{-x} = 2\left(ae^{2x} + be^{-x}\right) + 2ae^{2x} - be^{-x} = 2y + \frac{dy}{dx}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$$

Higher Order Derivatives Ex 12.1 Q32

$$y = e^x(\sin x + \cos x)$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = e^x(\cos x - \sin x) + (\sin x + \cos x)e^x$$

$$\Rightarrow \quad \frac{dy}{dx} = y + e^x(\cos x - \sin x)$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{dy}{dx} + e^x(-\sin x - \cos x) + (\cos x - \sin x)e^x$$

$$= \frac{dy}{dx} - y + (\cos x - \sin x)e^x$$

Adding and subtracting y on RHS

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{dy}{dx} - y + (\cos x - \sin x)e^x + y - y = 2\frac{dy}{dx} - 2y$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q33

It is given that, $y = \cos^{-1} x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}} = -\left(1-x^2\right)^{\frac{-1}{2}}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[-\left(1-x^2\right)^{\frac{-1}{2}}\right]$$

$$= -\left(-\frac{1}{2}\right)\cdot\left(1-x^2\right)^{\frac{-3}{2}}\cdot\frac{d}{dx}\left(1-x^2\right)$$

$$= \frac{1}{2\sqrt{\left(1-x^2\right)^3}} \times (-2x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-x}{\sqrt{\left(1-x^2\right)^3}} \qquad \ldots\text{(i)}$$

$y = \cos^{-1} x \Rightarrow x = \cos y$

Putting $x=\cos y$ in equation (i), we obtain

$$\frac{d^2y}{dx^2}=\frac{-\cos y}{\sqrt{\left(1-\cos^2 y\right)^3}}$$

$$\Rightarrow \frac{d^2y}{dx^2}=\frac{-\cos y}{\sqrt{\left(\sin^2 y\right)^3}}$$

$$=\frac{-\cos y}{\sin^3 y}$$

$$=\frac{-\cos y}{\sin y}\times\frac{1}{\sin^2 y}$$

$$\Rightarrow \frac{d^2y}{dx^2}=-\cot y\cdot\operatorname{cosec}^2 y$$

Higher Order Derivatives Ex 12.1 Q34

It is given that, $y=e^{a\cos^{-1}x}$

Taking logarithm on both the sides, we obtain

$\log\ y=a\cos^{-1}x\log e$

$\log\ y=a\cos^{-1}x$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{y}\frac{dy}{dx}=a\times\frac{-1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx}=\frac{-ay}{\sqrt{1-x^2}}$$

By squaring both the sides, we obtain

$$\left(\frac{dy}{dx}\right)^2=\frac{a^2y^2}{1-x^2}$$

$$\Rightarrow \left(1-x^2\right)\left(\frac{dy}{dx}\right)^2=a^2y^2$$

$$\left(1-x^2\right)\left(\frac{dy}{dx}\right)^2=a^2y^2$$

Again differentiating both sides with respect to x, we obtain

$$\left(\frac{dy}{dx}\right)^2\frac{d}{dx}\left(1-x^2\right)+\left(1-x^2\right)\times\frac{d}{dx}\left[\left(\frac{dy}{dx}\right)^2\right]=a^2\frac{d}{dx}\left(y^2\right)$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2(-2x)+\left(1-x^2\right)\times 2\frac{dy}{dx}.\frac{d^2y}{dx^2}=a^2.2y.\frac{dy}{dx}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2(-2x)+\left(1-x^2\right)\times 2\frac{dy}{dx}.\frac{d^2y}{dx^2}=a^2.2y.\frac{dy}{dx}$$

$$\Rightarrow -x\frac{dy}{dx}+\left(1-x^2\right)\frac{d^2y}{dx^2}=a^2.y \qquad \left[\frac{dy}{dx}\neq 0\right]$$

$$\Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-a^2y=0$$

Hence, proved.

Higher Order Derivatives Ex 12.1 Q35

It is given that, $y = 500e^{7x} + 600e^{-7x}$

Then,

$$\frac{dy}{dx} = 500.\frac{d}{dx}\left(e^{7x}\right) + 600.\frac{d}{dx}\left(e^{-7x}\right)$$
$$= 500 \cdot e^{7x} \cdot \frac{d}{dx}(7x) + 600 \cdot e^{-7x} \cdot \frac{d}{dx}(-7x)$$
$$= 3500e^{7x} - 4200e^{-7x}$$
$$\therefore \frac{d^2y}{dx^2} = 3500 \cdot \frac{d}{dx}\left(e^{7x}\right) - 4200.\frac{d}{dx}\left(e^{-7x}\right)$$
$$= 3500 \cdot e^{7x}.\frac{d}{dx}(7x) - 4200 \cdot e^{-7x} \cdot \frac{d}{dx}(-7x)$$
$$= 7 \times 3500 \cdot e^{7x} + 7 \times 4200 \cdot e^{-7x}$$
$$= 49 \times 500e^{7x} + 49 \times 600e^{-7x}$$
$$= 49\left(500e^{7x} + 600e^{-7x}\right)$$
$$= 49y$$

Hence, proved

Higher Order Derivatives Ex 12.1 Q36

$$y = 2\cos t - \cos 2t; \quad y = 2\sin t - \sin 2t$$

differentiating w.r.t. t

$$\Rightarrow \quad \frac{dy}{dt} = 2(-\sin t) - 2(-\sin 2t); \ \frac{dy}{dt} = 2\cos t - 2\cos 2t$$

dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2(\cos t - \cos 2t)}{2(\sin 2t - \sin t)}$$

differentiating w.r.t. t

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{dt} = \frac{(\sin 2t - \sin t)(-\sin t + 2\sin 2t) - (\cos t - \cos 2t)(2\cos 2t - \cos t)}{(\sin 2t - \sin t)^2} \quad \ldots\ldots\ldots(3)$$

dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{(\sin 2t - \sin t)(2\sin 2t - \sin t) - (\cos t - \cos 2t)(2\cos 2t - \cos t)}{2(\sin 2t - \sin t)^3}$$

Putting $t = \pi/2$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{(0-1)(0-1) - (0-(-1))(2(-1)-0)}{2(0-1)^3} = \frac{1+2}{-2} = \frac{-3}{2}$$

Higher Order Derivatives Ex 12.1 Q37

$x = 4z^2 + 5 \qquad y = 6z^2 + 72 + 3$

differentiating both w.r.t. z

$$\Rightarrow \quad \frac{dx}{dz} = 8z + 0 \qquad \frac{dy}{dz} = 12z + 7$$

$$\Rightarrow \quad \frac{dx}{dz} = \frac{12z+7}{8z} = \frac{12z}{8z} + \frac{7}{8z}$$

differentiating w.r.t. z

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{dz} = 0 + \frac{7}{8}\left(\frac{-1}{z^2}\right) \qquad \ldots\ldots(3)$$

dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-7}{8z^2 \times 8z} = \frac{-7}{64z^3}$$

Higher Order Derivatives Ex 12.1 Q38

$$y = \log(1+\cos x)$$

differentiating w.r.t.x

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{1+\cos x} \times -\sin x = \frac{-\sin x}{1+\cos x}$$

differentiating w.r.t.x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\left[\frac{(1+\cos x)\cos x - \sin x(-\sin x)}{(1+\cos x)^2}\right]$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\left[\frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}\right] = -\left[\frac{1+\cos x}{(1+\cos x)^2}\right] = \frac{-1}{1+\cos x}$$

differentiating w.r.t.x

$$\Rightarrow \quad \frac{d^3y}{dx^3} = -\left(\frac{+1}{(1+\cos x)^2} \times +\sin x\right) = -\left(\frac{-\sin x}{1+\cos x}\right) \times \left(\frac{-1}{1+\cos x}\right) = -\frac{dy}{dx} \cdot \frac{d^2y}{dx^2}$$

$$\Rightarrow \quad \frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} \cdot \frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q39

$$y = \sin(\log x)$$

$$\Rightarrow \quad \frac{dy}{dx} = \cos(\log x) \times \frac{1}{x}$$

$$\Rightarrow \quad x\frac{dy}{dx} = \cos(\log x)$$

$$\Rightarrow \quad x\frac{d^2y}{dx^2} + \frac{dy}{dx} = -\sin(\log x) \times \frac{1}{x}$$

$$\Rightarrow \quad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y$$

$$\Rightarrow \quad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q40

Given $y = 3e^{2x} + 2e^{3x}$

Then, $\frac{dy}{dx} = 6e^{2x} + 6e^{3x} = 6\left(e^{2x} + e^{3x}\right)$

$$\therefore \quad \frac{d^2y}{dx^2} = 12e^{2x} + 18e^{3x} = 6\left(2e^{2x} + 3e^{3x}\right)$$

Hence,

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 6\left(2e^{2x} + 3e^{3x}\right) - 30\left(e^{2x} + e^{3x}\right) + 6\left(3e^{2x} + 2e^{3x}\right)$$
$$= 0$$

Higher Order Derivatives Ex 12.1 Q41

$$y = \left(\cot^{-1} x\right)^2$$

differentiating w.r.t.x

$$\Rightarrow \quad \frac{dy}{dx} = y_1 = 2\cot^{-1} x \, \frac{-1}{1+x^2}$$

$$= \frac{-2\cot^{-1} x}{1+x^2} \text{(chain rule)}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{dy}{dx} = -2\cot^{-1} x$$

differentiating w.r.t.x

$$\Rightarrow \quad \left(1+x^2\right)y_2 + 2xy_1 = +2\left(\frac{+1}{1+x^2}\right)$$

(multiplication rule on LHS)

$$\Rightarrow \quad \left(1+x^2\right)^2 y_2 + 2x\left(1+x^2\right)y_1 = 2$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q42

We know that, $\frac{d}{dx}\left(\text{cosec}^{-1}x\right) = \frac{-1}{|x|\sqrt{x^2-1}}$

Let $y = \text{cosec}^{-1}x$

$$\frac{dy}{dx} = \frac{-1}{|x|\sqrt{x^2-1}}$$

Since $x > 1$, $|x| = x$

Thus,

$$\frac{dy}{dx} = \frac{-1}{x\sqrt{x^2-1}} \quad ...(1)$$

Differentiating the above function with respect to x*, we have,*

$$\frac{d^2y}{dx^2} = \frac{x\frac{2x}{2\sqrt{x^2-1}} + \sqrt{x^2-1}}{x^2\left(x^2-1\right)}$$

$$= \frac{\frac{x^2}{\sqrt{x^2-1}} + \sqrt{x^2-1}}{x^2\left(x^2-1\right)}$$

$$= \frac{x^2 + x^2 - 1}{x^2\left(x^2-1\right)^{\frac{3}{2}}}$$

$$= \frac{2x^2 - 1}{x^2\left(x^2-1\right)^{\frac{3}{2}}}$$

Thus, $x\left(x^2-1\right)\frac{d^2y}{dx^2} = \frac{2x^2-1}{x\sqrt{x^2-1}} \quad ...(2)$

Similarly, from (1), *we have*

$$\left(2x^2-1\right)\frac{dy}{dx} = \frac{-2x^2+1}{x\sqrt{x^2-1}} \quad ...(3)$$

Thus, from (2) *and* (3), *we have,*

$$x\left(x^2-1\right)\frac{d^2y}{dx^2} + \left(2x^2-1\right)\frac{dy}{dx} = \frac{2x^2-1}{x\sqrt{x^2-1}} + \left(\frac{-2x^2+1}{x\sqrt{x^2-1}}\right) = 0$$

Hence proved.

Higher Order Derivatives Ex 12.1 Q43

Given that, $x = \cos t + \log\tan\frac{t}{2}$, $y = \sin t$

Differentiating with respect to t, we have,

$$\frac{dx}{dt} = -\sin t + \frac{1}{\tan\frac{t}{2}} \times sec^2\frac{t}{2} \times \frac{1}{2}$$

$$= -\sin t + \frac{1}{\frac{\sin\frac{t}{2}}{\cos\frac{t}{2}}} \times \frac{1}{\cos^2\frac{t}{2}} \times \frac{1}{2}$$

$$= -\sin t + \frac{1}{\frac{\sin\frac{t}{2}}{\cos\frac{t}{2}}} \times \frac{1}{\cos^2\frac{t}{2}} \times \frac{1}{2}$$

$$= -\sin t + \frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}$$

$$= -\sin t + \frac{1}{\sin t}$$

$$= \frac{1-\sin^2 t}{\sin t}$$

$$= \frac{\cos^2 t}{\sin t}$$

$$= \cos t \times \cot t$$

Now find the value of $\frac{dy}{dt}$:

$$\frac{dy}{dt} = \cos t$$

Thus, $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \cos t \times \frac{1}{\cos t \times \cot t}$

$$\Rightarrow \frac{dy}{dx} = \tan t$$

Since $\frac{dy}{dt} = \cos t$, *we have* $\frac{d^2y}{dt^2} = -\sin t$

At $t = \frac{\pi}{4}$, $\left(\frac{d^2y}{dt^2}\right)_{t=\frac{\pi}{4}} = -\sin\left(\frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}}$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

$$= \frac{\frac{d}{dt}(\tan t)}{\cos t \times \cot t}$$

$$= \frac{sec^2 t}{\cos t \times \cot t}$$

$$= \frac{sec^2 t}{\cos t \times \frac{\cos t}{\sin t}}$$

$$= \frac{sec^2 t}{\cos^2 t} \times \sin t$$

$$= sec^4 t \times \sin t$$

Thus, $\left(\frac{d^2y}{dx^2}\right)_{t=\frac{\pi}{4}} = sec^4\left(\frac{\pi}{4}\right) \times \sin\frac{\pi}{4} = 2$

Higher Order Derivatives Ex 12.1 Q44

$x = a\sin t$ and $y = a\left(\cos t + \log\tan\frac{t}{2}\right)$

$$\frac{dx}{dt} = a\cos t$$

$$\frac{d^2x}{dt^2} = -a\sin t$$

$$\frac{dy}{dt} = -a\sin t + a\frac{1}{\tan\frac{t}{2}} \times \sec^2\frac{t}{2} \times \frac{1}{2}$$

$$= -a\sin t + a\frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}$$

$$= -a\sin t + a\operatorname{cosec} t$$

$$\frac{d^2y}{dt^2} = -a\cos t - a\operatorname{cosec} t\cot t$$

$$\frac{d^2y}{dx^2} = \frac{\frac{dx}{dt}\frac{d^2y}{dt^2} - \frac{dy}{dt}\frac{d^2x}{dt^2}}{\left(\frac{dx}{dt}\right)^3}$$

$$= \frac{a\cos t(-a\cos t - a\operatorname{cosec} t\cot t) - (-a\sin t + a\operatorname{cosec} t)(-a\sin t)}{(a\cos t)^3}$$

$$= \frac{-a^2\cos^2 t - a^2\cot^2 t - a^2\sin^2 t + a^2}{a^3\cos^3 t}$$

$$= \frac{-a^2\cos^2 t - a^2\sin^2 t - a^2\cot^2 t + a^2}{a^3\cos^3 t}$$

$$= \frac{-a^2(\cos^2 t + \sin^2 t) - a^2\cot^2 t + a^2}{a^3\cos^3 t}$$

$$= -\frac{1}{a\sin^2 t\cos t}$$

Higher Order Derivatives Ex 12.1 Q45

$x = a(\cos t + t\sin t)$

$$\frac{dx}{dt} = -a\sin t + at\cos t + a\sin t$$

$$= at\cos t$$

$$\frac{d^2x}{dt^2} = -at\sin t + a\cos t$$

$y = a(\sin t - t\cos t)$

$$\frac{dy}{dt} = a\cos t - a\cos t + at\sin t$$

$$= at\sin t$$

$$\frac{d^2y}{dt^2} = at\cos t + a\sin t$$

$$\frac{d^2y}{dx^2} = \frac{\frac{dx}{dt}\frac{d^2y}{dt^2} - \frac{dy}{dt}\frac{d^2x}{dt^2}}{\left(\frac{dx}{dt}\right)^3}$$

$$= \frac{at\cos t(at\cos t + a\sin t) - at\sin t(-at\sin t + a\cos t)}{(at\cos t)^3}$$

$$= \frac{a^2t^2\cos^2 t + a^2t\cos t\sin t + a^2t^2\sin^2 t - a^2t\sin t\cos t}{(at\cos t)^3}$$

$$= \frac{a^2t^2}{a^3t^3\cos^3 t} = \frac{1}{at\cos^3 t}$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{4}} = \frac{1}{a \times \frac{\pi}{4}\cos^3\frac{\pi}{4}} = \frac{8\sqrt{2}}{\pi a}$$

Higher Order Derivatives Ex 12.1 Q46

$x=a\left(\cos t + \log \tan\frac{t}{2}\right)$ and $y=a\sin t$

$$\frac{dx}{dt} = -a\sin t + a\frac{1}{\tan\frac{t}{2}} \times \sec^2\frac{t}{2} \times \frac{1}{2}$$
$$= -a\sin t + a\frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}$$
$$= -a\sin t + a\,\mathrm{cosec}\,t$$
$$\frac{d^2x}{dt^2} = -a\cos t - a\,\mathrm{cosec}\,t\cot t$$

$$\frac{dy}{dt} = a\cos t$$
$$\frac{d^2y}{dt^2} = -a\sin t$$
$$\frac{d^2y}{dx^2} = \frac{\frac{dx}{dt}\frac{d^2y}{dt^2} - \frac{dy}{dt}\frac{d^2x}{dt^2}}{\left(\frac{dx}{dt}\right)^3}$$
$$= \frac{(-a\sin t + a\,\mathrm{cosec}\,t)(-a\sin t) - (a\cos t)(-a\cos t - a\,\mathrm{cosec}\,t\cot t)}{(-a\sin t + a\,\mathrm{cosec}\,t)^3}$$
$$= \frac{a^2\sin^2 t + a^2\cos^2 t - a^2 + a^2\cot^2 t}{\left(-a\sin t + \frac{a}{\sin t}\right)^3}$$
$$= \frac{a^2\cot^2 t}{a^3\cos^6 t} \times \sin^3 t = \frac{1}{a} \times \frac{\sin t}{\cos^4 t}$$
$$\left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{3}} = \frac{1}{a} \times \frac{\sin\frac{\pi}{3}}{\cos^4\frac{\pi}{3}} = \frac{8\sqrt{3}}{a}$$

Higher Order Derivatives Ex 12.1 Q47

$$x = a\,(\cos 2t + 2t\sin 2t)$$
$$\frac{dx}{dt} = -2a\sin 2t + 2a\sin 2t + 4at\cos 2t = 4at\cos 2t$$

$$y = a(\sin 2t - 2t\cos 2t)$$
$$\frac{dy}{dt} = 2a\cos 2t - 2a\cos 2t + 4at\sin 2t = 4at\sin 2t$$

$$\frac{dy}{dx} = \tan 2t$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(\tan 2t)$$
$$\frac{d^2y}{dx^2} = \sec^2 2t\frac{d}{dx}(2t)$$
$$\frac{d^2y}{dx^2} = 2\sec^2 2t\frac{d}{dx}(t)$$
$$\frac{d^2y}{dx^2} = 2\sec^2 2t \times \frac{1}{4at\cos 2t}$$
$$\frac{d^2y}{dx^2} = \frac{1}{2a}\sec^3 2t$$

Higher Order Derivatives Ex 12.1 Q48

$x = a\sin t - b\cos t; \quad y = a\cos t + b\sin t$

Differentiating both w.r.t.t

$$\Rightarrow \quad \frac{dx}{dt} = a\cos t + b\sin t; \quad \frac{dy}{dt} = -a\sin t + b\cos t$$

$$\Rightarrow \quad \frac{dx}{dt} = y \;\ldots\ldots(1) \quad ; \quad \frac{dy}{dt} = -x \ldots\ldots(2)$$

Dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} == -\frac{x}{y}$$

Differentiating w.r.t.t

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{dt} = -\left\{\frac{y\frac{dx}{dt} - x\frac{dy}{dt}}{y^2}\right\}$$

Putting values from (1) and (2)

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{dt} = -\left\{\frac{y^2 + x^2}{y^2}\right\} \ldots\ldots\ldots(3)$$

Dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\left\{\frac{y^2 + x^2}{y^2 \times y}\right\} = -\left\{\frac{x^2 + y^2}{y^3}\right\}$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q49

$y = A \sin 3x + B \cos 3x$
differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = 3A \cos 3x + 3B\left(-\sin 3x\right)$$

again differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 3A\left(-\sin 3x\right) \times 3 - 3B\left(\cos 3x\right) \times 3$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -9\left(A \sin 3x + B \cos 3x\right) = -9y$$

Now adding $\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 3y$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \frac{4dy}{dx} + 3y = -9y + 4\left(3A \cos 3x - 3B \sin 3x\right) + 3y$$

$$= 12\left(A \cos 3x - B \sin 3x\right) - 6\left(A \sin 3x + B \cos 3x\right)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \frac{4dy}{dx} + 3y = \left(12A - 6B\right) \cos 3x - \left(12B + 6A\right) \sin 3x$$

But given,

$$\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 3y = 10 \cos 3x$$

Thus, $12A - 6B = 10$ ....(1)
and $-\left(12B + 6A\right) = 0$ ....(2)

solving (2)
$12B + 6A = 0 \Rightarrow 6A = -12B \Rightarrow A = -2B$

Putting value of A in (1)

$\Rightarrow \quad 12\left(-2B\right) - 63 = 10$

$\Rightarrow \quad -24B - 6B = 10$

$\Rightarrow \quad -30B = 10$

$\Rightarrow \quad B = \frac{-1}{3}$

$\Rightarrow \quad \therefore \quad A = -2 \times \frac{-1}{3} = \frac{2}{3}$

and $A = \frac{2}{3}; \quad B = \frac{-1}{3}$

Higher Order Derivatives Ex 12.1 Q50

$y = Ae^{-kt}\cos(pt+c)$

differentiating w.r.t. t

$\Rightarrow \quad \frac{dy}{dt} = A\left\{e^{-kt}\left(-\sin(pt+c)\times p\right) + \left(\cos(pt+c)\right)\left(-re^{-kt}\right)\right\}$

$\Rightarrow \quad -Ape^{-kt}\sin(pt+c) - kAe^{-kt}\cos(pt+c)$

$\Rightarrow \quad \frac{dy}{dt} = -Ape^{-kt}\sin(pt+c) - ky$

differentiating w.r.t. t

$\Rightarrow \quad \frac{d^2y}{dt^2} = -Ap\left\{e^{-kt}\left(\cos(pt+c)\times p\right) + \left(\sin(pt+c)\right)\left(e^{-kt}\times -R\right) - ky^1\right\}$

$= -p^2y + Apke^{-kt}\sin(pt+c) - ky^1$

Adding & subtracting ky^1 on RHS

$\Rightarrow \quad \frac{d^2y}{dt^2} = +Apke^{-kt}\sin(pt+c) - p^2y - 2ky^1 + ky^1$

$\frac{d^2y}{dt^2} = Apke^{-kt}\sin(pt+c) - p^2y - 2ky^1 - kApe^{-kt}\sin(pt+c) - k^2y$

$\Rightarrow \quad \frac{d^2y}{dt^2} = -\left(p^2+k^2\right)y - 2k\frac{dy}{dx}$

$\Rightarrow \quad \frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + n^2y = 0$

Hence proved!

Higher Order Derivatives Ex 12.1 Q51

$y = x^n\left\{a\cos(\log x) + b\sin(\log x)\right\}$

$y = ax^n\cos(\log x) + bx^n\sin(\log x)$

$\frac{dy}{dx} = anx^{n-1}\cos(\log x) - ax^{n-1}\sin(\log x) + bnx^{n-1}\sin(\log x) + bx^{n-1}\cos(\log x)$

$\frac{dy}{dx} = x^{n-1}\cos(\log x)(na+b) + x^{n-1}\sin(\log x)(bn-a)$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(x^{n-1}\cos(\log x)(na+b) + x^{n-1}\sin(\log x)(bn-a)\right)$

$\frac{d^2y}{dx^2} = (na+b)\left[(n-1)x^{n-2}\cos(\log x) - x^{n-2}\sin(\log x)\right] + (bn-a)\left[(n-1)x^{n-2}\sin(\log x) + x^{n-2}\cos(\log x)\right]$

$\frac{d^2y}{dx^2} = (na+b)x^{n-2}\left[(n-1)\cos(\log x) - \sin(\log x)\right] + (bn-a)x^{n-2}\left[(n-1)\sin(\log x) + \cos(\log x)\right]$

$x^2\frac{d^2y}{dx^2} + (1-2n)\frac{dy}{dx} + \left(1+n^2\right)y$

$= (na+b)x^n\left[(n-1)\cos(\log x) - \sin(\log x)\right] + (bn-a)x^n\left[(n-1)\sin(\log x) + \cos(\log x)\right]$

$+ (1-2n)x^{n-1}\cos(\log x)(na+b) + (1-2n)x^{n-1}\sin(\log x)(bn-a)$

$+ a\left(1+n^2\right)x^n\cos(\log x) + b\left(1+n^2\right)x^n\sin(\log x)$

$= 0$

Higher Order Derivatives Ex 12.1 Q52

$$y = a\left\{x+\sqrt{x^2+1}\right\}^n + b\left\{x-\sqrt{x^2+1}\right\}^{-n},$$

$$\frac{dy}{dx} = na\left\{x+\sqrt{x^2+1}\right\}^{n-1}\left[1+x\left(x^2+1\right)^{-\frac{1}{2}}\right] - nb\left\{x-\sqrt{x^2+1}\right\}^{-n-1}\left[1-x\left(x^2+1\right)^{-\frac{1}{2}}\right]$$

$$\frac{dy}{dx} = \frac{na}{\sqrt{x^2+1}}\left\{x+\sqrt{x^2+1}\right\}^n + \frac{nb}{\sqrt{x^2+1}}\left\{x-\sqrt{x^2+1}\right\}^{-n}$$

$$\frac{dy}{dx} = \frac{n}{\sqrt{x^2+1}}\left[a\left\{x+\sqrt{x^2+1}\right\}^n + b\left\{x-\sqrt{x^2+1}\right\}^{-n}\right]$$

$$x\frac{dy}{dx} = \frac{nx}{\sqrt{x^2+1}}y$$

$$\frac{d^2y}{dx^2} = \frac{nx}{\sqrt{x^2+1}}\frac{dy}{dx} + y\left[\frac{\sqrt{x^2+1} - x^2\left(x^2+1\right)^{-\frac{1}{2}}}{x^2+1}\right]$$

$$\frac{d^2y}{dx^2} = \frac{n^2x^2}{x^2+1} + y\left[\frac{1}{\left(x^2+1\right)\sqrt{x^2+1}}\right]$$

$$\frac{d^2y}{dx^2} = \frac{n^2x^2\left(\sqrt{x^2+1}\right)+y}{\left(x^2+1\right)\sqrt{x^2+1}}$$

$$\left(x^2-1\right)\frac{d^2y}{dx^2} = \frac{n^2x^4\left(\sqrt{x^2+1}\right)+x^2y}{\left(x^2+1\right)\sqrt{x^2+1}} - \frac{n^2x^2\left(\sqrt{x^2+1}\right)+y}{\left(x^2+1\right)\sqrt{x^2+1}}$$

Now

$$\left(x^2-1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - ny$$

$$= \frac{n^2x^4\left(\sqrt{x^2+1}\right)+x^2y}{\left(x^2+1\right)\sqrt{x^2+1}} - \frac{n^2x^2\left(\sqrt{x^2+1}\right)+y}{\left(x^2+1\right)\sqrt{x^2+1}} + \frac{nx}{\sqrt{x^2+1}}y - ny$$

$$= 0$$