Ex 17.1

Increasing and Decreasing Functions Ex 17.1 Q1

Let $x_1,\ x_2 \in (0,\infty)$

We have,

$$x_1 < x_2$$
$$\Rightarrow \quad \log_e x_1 < \log_e x_2$$
$$\Rightarrow \quad f(x_1) < f(x_2)$$

So, $f(x)$ is increasing in $(0,\infty)$.

Increasing and Decreasing Functions Ex 17.1 Q2

Case I

When $a > 1$

Let $x_1, x_2 \in (0,\infty)$

We have

$$x_1 < x_2$$
$$\Rightarrow \quad \log_a x_1 < \log_a x_2$$
$$\Rightarrow \quad f(x_1) < f(x_2)$$

Thus, $f(x)$ is increasing on $(0,\infty)$

Case II

When $0 < a < 1$

$$f(x) = \log_a x = \frac{\log x}{\log a}$$

When $a < 1 \Rightarrow \log a < 0$

Let $x_1 < x_2$

$$\Rightarrow \quad \log x_1 < \log x_2$$
$$\Rightarrow \quad \frac{\log x_1}{\log a} > \frac{\log x_2}{\log a} \qquad [\because \log a < 0]$$
$$\Rightarrow \quad f(x_1) > f(x_2)$$

So, $f(x)$ is decreasing on $(0,\infty)$.

Increasing and Decreasing Functions Ex 17.1 Q3

We have,

$$f(x) = ax + b,\ a > 0$$

Let $x_1, x_2 \in R$ and $x_1 > x_2$

$\Rightarrow \quad ax_1 > ax_2$ for some $a > 0$

$\Rightarrow \quad ax_1 + b > ax_2 + b$ for some b

$\Rightarrow \quad f(x_1) > f(x_2)$

$\therefore\ f(x)$ is increasing function of R.

Increasing and Decreasing Functions Ex 17.1 Q4

We have,

$$f(x) = ax + b,\ a < 0$$

Let $x_1, x_2 \in R$ and $x_1 > x_2$

$\Rightarrow \quad ax_1 < ax_2$ for some $a < 0$

$\Rightarrow \quad ax_1 + b < ax_2 + b$ for some b

$\Rightarrow \quad f(x_1) < f(x_2)$

Hence, $x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$

$\therefore\ f(x)$ is decreasing function of R.

Increasing and Decreasing Functions Ex 17.1 Q5

We have,

$$f(x) = \frac{1}{x}$$

Let $x_1, x_2 \in (0, \infty)$ and $x_1 > x_2$

$\Rightarrow \quad \frac{1}{x_1} < \frac{1}{x_2}$

$\Rightarrow \quad f(x_1) < f(x_2)$

Thus, $x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$

So, $f(x)$ is decreasing function.

Increasing and Decreasing Functions Ex 17.1 Q6

We have,

$$f(x) = \frac{1}{1+x^2}$$

Case I

When $x \in [0, \infty)$

Let $x_1, x_2 \in (0, \infty]$ and $x_1 > x_2$

$\Rightarrow \quad x_1^2 > x_2^2$

$\Rightarrow \quad 1 + x_1^2 > 1 + x_2^2$

$\Rightarrow \quad \frac{1}{1+x_1^2} < \frac{1}{1+x_2^2}$

$\Rightarrow \quad f(x_1) < f(x_2)$

So, $f(x)$ is decreasing on $[0, \infty)$

Case II

When $x \in (-\infty, 0]$

Let $x_1 > x_2$

$\Rightarrow \quad x_1^2 < x_2^2 \qquad [\because -2 > -3 \Rightarrow 4 < 9]$

$\Rightarrow \quad 1 + x_1^2 < 1 + x_2^2$

$\Rightarrow \quad \frac{1}{1+x_1^2} > \frac{1}{1+x_2^2}$

$\Rightarrow \quad f(x_1) > f(x_2)$

So, $f(x)$ is increasing on $(-\infty, 0]$

Increasing and Decreasing Functions Ex 17.1 Q7

We have,

$$f(x) = \frac{1}{1+x^2}$$

Case I

When $x \in [0,\infty)$

Let $x_1 > x_2$

$\Rightarrow \quad x_1^2 > x_2^2$

$\Rightarrow \quad 1+x_1^2 > 1+x_2^2$

$\Rightarrow \quad \frac{1}{1+x_1^2} < \frac{1}{1+x_2^2}$

$\Rightarrow \quad f(x_1) < f(x_2)$

$\therefore$ $f(x)$ is decreasing on $[0,\infty)$.

Case II

When $x \in (-\infty,0]$

Let $x_1 > x_2$

$\Rightarrow \quad x_1^2 < x_2^2$

$\Rightarrow \quad 1+x_1^2 < 1+x_2^2$

$\Rightarrow \quad \frac{1}{1+x_1^2} > \frac{1}{1+x_2^2}$

$\Rightarrow \quad f(x_1) > f(x_2)$

So, $f(x)$ is increasing on $(-\infty,0]$

Thus, $f(x)$ is neither increasing nor decreasing on R.

Increasing and Decreasing Functions Ex 17.1 Q8

We have,

$$f(x) = |x| = \begin{cases} x, & x > 0 \\ -x, & x < 0 \end{cases}$$

(a)

Let $x_1,\ x_2 \in (0,\infty)$ and $x_1 > x_2$

$\Rightarrow \quad f(x_1) > f(x_2)$

So, $f(x)$ is increasing in $(0,\infty)$

(b)

Let $x_1,\ x_2 \in (-\infty,0)$ and $x_1 > x_2$

$\Rightarrow \quad -x_1 < -x_2$

$\Rightarrow \quad f(x_1) < f(x_2)$

$\therefore$ $f(x)$ is strictly decreasing on $(-\infty,0)$.

Increasing and Decreasing Functions Ex 17.1 Q9

$$f(x) = 7x - 3$$

Let $x_1,\ x_2 \in R$ and $x_1 > x_2$

$\Rightarrow \quad 7x_1 > 7x_2$

$\Rightarrow \quad 7x_1 - 3 > 7x_2 - 3$

$\Rightarrow \quad f(x_1) > f(x_2)$

$\therefore$ $f(x)$ is strictly increasing on R.

Ex 17.2

Increasing and Decreasing Functions Ex 17.2 Q1(i)

We have,

$f(x) = 10 - 6x - 2x^2$

$\therefore f'(x) = -6 - 4x$

Now,

$f'(x) = 0 \Rightarrow x = -\frac{3}{2}$

The point $x = -\frac{3}{2}$ divides the real line into two disjoint intervals i.e., $\left(-\infty, -\frac{3}{2}\right)$ and $\left(-\frac{3}{2}, \infty\right)$.

In interval $\left(-\infty, -\frac{3}{2}\right)$ i.e., when $x < -\frac{3}{2}$, $f'(x) = -6 - 4x < 0$.

$\therefore f$ is strictly increasing for $x < -\frac{3}{2}$.

In interval $\left(-\frac{3}{2}, \infty\right)$ i.e., when $x > -\frac{3}{2}$, $f'(x) = -6 - 4x < 0$.

$\therefore f$ is strictly decreasing for $x > -\frac{3}{2}$.

Increasing and Decreasing Functions Ex 17.2 Q1(ii)

We have,

$f(x) = x^2 + 2x - 5$

$\therefore f'(x) = 2x + 2$

Now,

$f'(x) = 0 \Rightarrow x = -1$

Point $x = -1$ divides the real line into two disjoint intervals i.e., $(-\infty, -1)$ and $(-1, \infty)$.

In interval $(-\infty, -1)$, $f'(x) = 2x + 2 < 0$.

$\therefore f$ is strictly decreasing in interval $(-\infty, -1)$.

Thus, f is strictly decreasing for $x < -1$.

In interval $(-1, \infty)$, $f'(x) = 2x + 2 > 0$.

$\therefore f$ is strictly increasing in interval $(-1, \infty)$.

Thus, f is strictly increasing for $x > -1$.

Increasing and Decreasing Functions Ex 17.2 Q1(iii)

We have,

$f(x) = 6 - 9x - x^2$

$\therefore f'(x) = -9 - 2x$

Now,.

$f'(x) = 0$ gives $x = -\frac{9}{2}$

The point $x = -\frac{9}{2}$ divides the real line into two disjoint intervals i.e., $\left(-\infty, -\frac{9}{2}\right)$ and $\left(-\frac{9}{2}, \infty\right)$.

In interval $\left(-\infty, -\frac{9}{2}\right)$ i.e., for $x < -\frac{9}{2}$, $f'(x) = -9 - 2x > 0$.

$\therefore f$ is strictly increasing for $x < -\frac{9}{2}$.

In interval $\left(-\frac{9}{2}, \infty\right)$ i.e., for $x > -\frac{9}{2}$, $f'(x) = -9 - 2x < 0$.

$\therefore f$ is strictly decreasing for $x > -\frac{9}{2}$.

Increasing and Decreasing Functions Ex 17.2 Q1(iv)

$f(x) = 2x^3 - 12x^2 + 18x + 15$

$\therefore \quad f'(x) = 6x^2 - 24x + 18$

$\quad\quad = 6(x^2 - 4x + 3)$

$\quad\quad = 6(x-3)(x-1)$

Critical point

$f'(x) = 0$

$\Rightarrow \quad 6(x-3)(x-1) = 0$

$\Rightarrow \quad x = 3,\ 1$

Clearly, $f(x) > 0$ if $x < 1$ and $x > 3$

and $f(x) < 0$ if $1 < x < 3$

Thus, $f(x)$ increases on $(-\infty, 1) \cup (3, \infty)$, decreases on $(1, 3)$.

Increasing and Decreasing Functions Ex 17.2 Q1(v)

We have,

$f(x) = 5 + 36x + 3x^2 - 2x^3$

$\therefore \quad f'(x) = 36 + 6x - 6x^2$

Critical point

$f'(x) = 0$

$\Rightarrow \quad 36 + 6x - 6x^2 = 0$

$\Rightarrow \quad -6(x^2 - x - 6) = 0$

$\Rightarrow \quad (x-3)(x+2) = 0$

$\therefore \quad x = 3,\ -2$

Clearly, $f'(x) > 0$ if $-2 < x < 3$

Also $f'(x) < 0$ if $x < -2$ and $x > 3$

Thus, increases if $x \in (-2, 3)$, decreases if $x \in (-\infty, -2) \cup (3, \infty)$

Increasing and Decreasing Functions Ex 17.2 Q1(vi)

We have,

$f(x) = 8 + 36x + 3x^2 - 2x^3$

$\therefore \quad f'(x) = 36 + 6x - 6x^2$

Critical points

$f'(x) = 0$

$\Rightarrow \quad 6(6 + x - x^2) = 0$

$\Rightarrow \quad (3 - x)(2 + x) = 0$

$\Rightarrow \quad x = 3,\ -2$

Clearly, $f'(x) > 0$ if $-2 < x < 3$

and $f'(x) < 0$ if $-\infty < x < -2$ and $3 < x < \infty$

Thus, increases in $(-2, 3)$, decreases in $(-\infty, -2) \cup (3, \infty)$

Increasing and Decreasing Functions Ex 17.2 Q1(vii)

We have,

$f(x) = 5x^3 - 15x^2 - 120x + 3$

$\therefore \quad f'(x) = 15x^2 - 30x - 120$

Critical points

$f'(x) = 0$

$\Rightarrow \quad 15(x^2 - 2x - 8) = 0$

$\Rightarrow \quad (x-4)(x+2) = 0$

$\Rightarrow \quad x = 4,\ -2$

Clearly, $f'(x) > 0$ if $x < -2$ and $x > 4$

and $f'(x) < 0$ if $-2 < x < 4$

Thus, increases in $(-\infty, -2) \cup (4, \infty)$, decreases in $(-2, 4)$

Increasing and Decreasing Functions Ex 17.2 Q1(viii)

$f(x) = x^3 - 6x^2 - 36x + 2$

$\therefore \quad f'(x) = 3x^2 - 12x - 36$

Critical point

$f'(x) = 0$

$\Rightarrow \quad 3(x^2 - 4x - 12) = 0$

$\Rightarrow \quad (x-6)(x+2) = 0$

$\Rightarrow \quad x = 6, \; -2$

Clearly, $f'(x) > 0$ if $x < -2$ and $x > 6$

$f'(x) < 0$ if $-2x < x < 6$

Thus, increases in $(-\infty, -2) \cup (6, \infty)$, decreases in $(-2, 6)$.

Increasing and Decreasing Functions Ex 17.2 Q1(ix)

We have,

$f(x) = 2x^3 - 15x^2 + 36x + 1$

$\therefore \quad f'(x) = 6x^2 - 30x + 36$

Critical points

$\Rightarrow \quad 6(x^2 - 5x + 6) = 0$

$\Rightarrow \quad (x-3)(x-2) = 0$

$\Rightarrow \quad x = 3, \; 2$

Clearly, $f'(x) > 0$ if $x < 2$ and $x > 3$

$f'(x) < 0$ if $2 < x < 3$

Thus, $f(x)$ increases in $(-\infty, 2) \cup (3, \infty)$, decreases in $(2, 3)$.

Increasing and Decreasing Functions Ex 17.2 Q1(x)

We have,

$f(x) = 2x^3 + 9x^2 + 12x - 1$

$\therefore \quad f'(x) = 6x^2 + 18x + 12$

Critical ponts

$f'(x) = 0$

$\Rightarrow \quad 6(x^2 + 3x + 2) = 0$

$\Rightarrow \quad (x+2)(x+1) = 0$

$\Rightarrow \quad x = -2, \; -1$

Increasing and Decreasing Functions Ex 17.2 Q1(xi)

We have,

$f(x) = 2x^3 - 9x^2 + 12x - 5$

$\therefore \quad f'(x) = 6x^2 - 18x + 12$

Critical points

$f'(x) = 0$

$\Rightarrow \quad 6(x^2 - 3x + 2) = 0$

$\Rightarrow \quad (x-2)(x-1) = 0$

$\Rightarrow \quad x = 2, \; 1$

Clearly, $f'(x) > 0$ if $x < 1$ and $x > 2$

$f'(x) < 0$ if $1 < x < 2$

Thus, $f(x)$ increases in $(-\infty, 1) \cup (2, \infty)$, decreases in $(1, 2)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xii)

We have,

$$f(x) = 6 + 12x + 3x^2 - 2x^3$$

$\therefore \quad f'(x) = 12 + 6x - 6x^2$

Critical points

$$f'(x) = 0$$

$\Rightarrow \quad 6\left(2 + x - x^2\right) = 0$

$\Rightarrow \quad (2 - x)(1 + x) = 0$

$\Rightarrow \quad x = 2, \ -1$

Clearly, $f'(x) > 0$ if $-1 < x < 2$

$f'(x) < 0$ if $x < -1$ and $x > 2$.

Thus, $f(x)$ increases in $(-1, 2)$, decreases in $(-\infty, -1) \cup (2, \infty)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xiii)

We have,

$$f(x) = 2x^3 - 24x + 107$$

$\therefore \quad f'(x) = 6x^2 - 24$

Critical points

$$f'(x) = 0$$

$\Rightarrow \quad 6\left(x^2 - 4\right) = 0$

$\Rightarrow \quad (x - 2)(x + 2) = 0$

$\Rightarrow \quad x = 2, -2$

Clearly, $f'(x) > 0$ if $x < -2$ and $x > 2$

$f'(x) < 0$ if $-2 < x < 2$

Thus, $f(x)$ increases in $(-\infty, -2) \cup (2, \infty)$, decreases in $(-2, 2)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xiv)

We have

$$f(x) = -2x^3 - 9x^2 - 12x + 1$$

$$f'(x) = -6x^2 - 18x - 12$$

Critical points

$$f'(x) = 0$$

$$-6x^2 - 18x - 12 = 0$$

$$x^2 + 3x + 2 = 0$$

$$(x+2)(x+1) = 0$$

$$x = -2, -1$$

Clearly, $f'(x) > 0$ if $x < -1$ and $x < -2$

$f'(x) < 0$ if $-2 < x < -1$

Thus, $f(x)$ is increasing in $(-2, -1)$, decreasing in $(-\infty, -2) \cup (-1, \infty)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xv)

We have,

$$f(x) = (x - 1)(x - 2)^2$$

$\therefore \quad f'(x) = (x - 2)^2 + 2(x - 1)(x - 2)$

$f'(x) = (x - 2)(x - 2 + 2x - 2)$

$\Rightarrow \quad f'(x) = (x - 2)(3x - 4)$

Critical points

$$f'(x) = 0$$

$\Rightarrow \quad (x - 2)(3x - 4) = 0$

$\Rightarrow \quad x = 2, \ \frac{4}{3}$

Clearly, $f'(x) > 0$ if $x < \frac{4}{3}$ and $x > 2$

$f'(x) < 0$ if $\frac{4}{3} < x < 2$

Thus, $f(x)$ increases in $\left(-\infty, \frac{4}{3}\right) \cup (2, \infty)$, decreases in $\left(\frac{4}{3}, 2\right)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xvi)

We have,

$$f(x) = x^3 - 12x^2 + 36x + 17$$

$$\therefore \quad f'(x) = 3x^2 - 24x + 36$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \quad 3(x^2 - 8x + 12) = 0$$

$$\Rightarrow \quad (x-6)(x-2) = 0$$

$$\Rightarrow \quad x = 6, 2$$

Clearly, $f'(x) > 0$ if $x < 2$ and $x > 6$

$f'(x) < 0$ if $2 < x < 6$

Thus, $f(x)$ increases in $(-\infty, 2) \cup (6, \infty)$, decreases in $(2, 6)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xvii)

We have

$$f(x) = 2x^3 - 24x + 7$$

$$f'(x) = 6x^2 - 24$$

Critical points

$$f'(x) = 0$$

$$6x^2 - 24 = 0$$

$$6x^2 = 24$$

$$x^2 = 4$$

$$x = 2, -2$$

Clearly, $f'(x) > 0$ if $x > 2$ and $x < -2$

$f'(x) < 0$ if $-2 \le x \le 2$

Thus, $f(x)$ is increasing in $(-\infty, -2) \cup (2, \infty)$, decreasing in $(-2, 2)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xviii)

We have $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$

$$\therefore \quad f'(x) = \frac{3}{10}(4x^3) - \frac{4}{5}(3x^2) - 3(2x) + \frac{36}{5}$$

$$= \frac{6}{5}(x-1)(x+2)(x-3)$$

Now $f'(x) = 0$

$\Rightarrow \frac{6}{5}(x-1)(x+2)(x-3) = 0$

$\Rightarrow x = 1, -2$ or 3

The points $x = 1, -2$ and 3 divide the number line into four disjoint intervals namely, $(-\infty, -2)$, $(-2, 1)$, $(1, 3)$ and $(3, \infty)$.

Consider the interval $(-\infty, -2)$, i.e $-\infty < x < -2$

In this case, we have $x - 1 < 0$, $x + 2 < 0$ and $x - 3 < 0$

$\therefore \quad f'(x) < 0$ when $-\infty < x < -2$

Thus, the function f is strictly decreasing in $(-\infty, -2)$

Consider the interval $(-2, 1)$, i.e $-2 < x < 1$

In this case, we have $x - 1 < 0$, $x + 2 > 0$ and $x - 3 < 0$

$\therefore \quad f'(x) > 0$ when $-2 < x < 1$

Thus, the function f is strictly increasing in $(-2, 1)$

Now, consider the interval $(1, 3)$, i.e $1 < x < 3$

In this case, we have $x - 1 > 0$, $x + 2 > 0$ and $x - 3 < 0$

$\therefore \quad f'(x) < 0$ when $1 < x < 3$

Thus, the function f is strictly decreasing in $(1, 3)$

Finally consider the interval $(3, \infty)$, i.e $3 < x < \infty$

In this case, we have $x - 1 > 0$, $x + 2 > 0$ and $x - 3 > 0$

$\therefore \quad f'(x) > 0$ when $x > 3$

Thus, the function f is strictly increasing in $(3, \infty)$

Increasing and Decreasing Functions Ex 17.2 Q1(xix)

We have,

$$f(x) = x^4 - 4x$$

$\therefore$ $f'(x) = 4x^3 - 4$

Critical points,

$f'(x) = 0$

$\Rightarrow$ $4(x^3 - 1) = 0$

$\Rightarrow$ $x = 1$

Clearly, $f'(x) > 0$ if $x > 1$

$f'(x) < 0$ if $x < 1$

Thus, $f(x)$ increases in $(1,\infty)$, decreases in $(-\infty, 1)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xx)

$f(x) = \frac{x^4}{4} + \frac{2}{3}x^3 - \frac{5}{2}x^2 - 6x + 7$

$\therefore$ $f'(x) = x^3 + 2x^2 - 5x - 6$

Critical points

$f'(x) = 0$

$\Rightarrow$ $x^3 + 2x^2 - 5x - 6 = 0$

$\Rightarrow$ $(x+1)(x+3)(x-2) = 0$

$\Rightarrow$ $x = -1, \ -3, \ 2$

Clearly, $f'(x) > 0$ if $-3 < x < -1$ and $x > 2$

$f'(x) < 0$ if $x < -3$ and $-1 < x < 2$

Thus, $f(x)$ increases in $(-3,-1) \cup (2,\infty)$, decreases in $(-\infty,-3) \cup (-1,2)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxi)

$f(x) = x^4 - 4x^3 + 4x^2 + 15$

$\therefore$ $f'(x) = 4x^3 - 12x^2 + 8x$

Critical points

$f'(x) = 0$

$\Rightarrow$ $4x(x^2 - 3x + 2) = 0$

$\Rightarrow$ $4x(x-2)(x-1) = 0$

$\Rightarrow$ $x = 0, \ 2, \ 1$

Clealry, $f'(x) > 0$ if $0 < x < 1$ and $x > 2$

$f'(x) < 0$ if $x < 0$ and $1 < x < 2$

Thus, $f(x)$ increases in $(0,1) \cup (2,\infty)$, decreases in $(-\infty,0) \cup (1,2)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxii)

We have,

$f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}; \ x > 0$

$\therefore$ $f'(x) = \frac{15}{2}x^{\frac{1}{2}} - \frac{15}{2}x^{\frac{3}{2}}$

Critical points

$f'(x) = 0$

$\Rightarrow$ $\frac{15}{2}x^{\frac{1}{2}} - \frac{15}{2}x^{\frac{3}{2}} = 0$

$\Rightarrow$ $\frac{15}{2}x^{\frac{1}{2}}(1 - x) = 0$

$\Rightarrow$ $x = 0, \ 1$

Clearly, $f'(x) > 0$ if $0 < x < 1$

and $f'(x) < 0$ if $x > 1$

Thus, $f(x)$ increases in $(0,1)$, decreases in $(1,\infty)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxiii)

We have,

$$f(x) = x^8 + 6x^2$$

$\therefore \quad f'(x) = 8x^7 + 12x$

Critical points

$$f'(x) = 0$$

$\Rightarrow \quad 8x^7 + 12x = 0$

$\Rightarrow \quad 4x(2x^6 + 3) = 0$

$\Rightarrow \quad x = 0$

Clearly, $f'(x) > 0$ if $x > 0$

$f'(x) < 0$ if $x < 0$

Thus, $f(x)$ increases in $(0, \infty)$, decreases in $(-\infty, 0)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxiv)

We have,

$$f(x) = x^3 - 6x^2 + 9x + 15$$

$\therefore \quad f'(x) = 3x^2 - 12x + 9$

Critical points

$$f'(x) = 0$$

$\Rightarrow \quad 3(x^2 - 4x + 3) = 0$

$\Rightarrow \quad (x-3)(x-1) = 0$

$\Rightarrow \quad x = 3,\ 1$

Clearly, $f'(x) > 0$ if $x < 1$ and $x > 3$

$f'(x) < 0$ if $1 < x < 3$

Thus, $f(x)$ increases in $(-\infty, 1) \cup (3, \infty)$, decreases in $(1, 3)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxv)

We have,

$$y = [x(x-2)]^2 = [x^2 - 2x]^2$$

$$\therefore \frac{dy}{dx} = y' = 2(x^2 - 2x)(2x - 2) = 4x(x-2)(x-1)$$

$$\therefore \frac{dy}{dx} = 0 \Rightarrow x = 0, x = 2, x = 1.$$

The points $x = 0$, $x = 1$, and $x = 2$ divide the real line into four disjoint intervals i.e., $(-\infty, 0)$, $(0, 1)$ $(1, 2)$, and $(2, \infty)$.

In intervals $(-\infty, 0)$ and $(1, 2)$, $\frac{dy}{dx} < 0$.

$\therefore y$ is strictly decreasing in intervals $(-\infty, 0)$ and $(1, 2)$.

However, in intervals (0, 1) and $(2, \infty)$, $\frac{dy}{dx} > 0$.

$\therefore y$ is strictly increasing in intervals (0, 1) and $(2, \infty)$.

$\therefore y$ is strictly increasing for $0 < x < 1$ and $x > 2$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxvi)

Consider the given function

$f(x)=3x^4-4x^3-12x^2+5$

$\Rightarrow f'(x)=12x^3-12x^2-24x$

$\Rightarrow f'(x)=12x\left(x^2-x-2\right)$

$\Rightarrow f'(x)=12x(x+1)(x-2)$

For f(x) to be increasing, we must have,

$f'(x)>0$

$\Rightarrow 12x(x+1)(x-2)>0$

$\Rightarrow x(x+1)(x-2)>0$

$\Rightarrow -1<x<0$ or $2<x<\infty$

$\Rightarrow x\in(-1,0)\cup(2,\infty)$

So, f(x) *i*s increa sing in $(-1,0)\cup(2,\infty)$

For f(x) to be decreasing, we must have,

$f'(x)<0$

$\Rightarrow 12x(x+1)(x-2)<0$

$\Rightarrow x(x+1)(x-2)<0$

$\Rightarrow -\infty<x<-1$ or $0<x<2$

$\Rightarrow x\in(-\infty,-1)\cup(0,2)$

So, f(x) *i*s *decrea*sing in $(-\infty,-1)\cup(0,2)$

Increasing and Decreasing Functions Ex 17.2 Q1(xxvii)

Consider the given function

$f(x)=\frac{3}{2}x^4-4x^3-45x^2+51$

$\Rightarrow f'(x)=4\times\frac{3}{2}x^3-12x^2-90x$

$\Rightarrow f'(x)=6x^3-12x^2-90x$

$\Rightarrow f'(x)=6x\left(x^2-2x-15\right)$

$\Rightarrow f'(x)=6x(x+3)(x-5)$

For f(x) to be increasing, we must have,

$f'(x)>0$

$\Rightarrow 6x(x+3)(x-5)>0$

$\Rightarrow x(x+3)(x-5)>0$

$\Rightarrow -3<x<0$ or $5<x<\infty$

$\Rightarrow x\in(-3,0)\cup(5,\infty)$

So, f(x) *i*s increasing in $(-3,0)\cup(5,\infty)$

For f(x) to be decreasing, we must have,

$f'(x)<0$

$\Rightarrow 6x(x+3)(x-5)<0$

$\Rightarrow x(x+3)(x-5)<0$

$\Rightarrow -\infty<x<-3$ or $0<x<5$

$\Rightarrow x\in(-\infty,-3)\cup(0,5)$

So, f(x) *i*s *decrea*sing in $(-\infty,-3)\cup(0,5)$

Increasing and Decreasing Functions Ex 17.2 Q1(xxviii)

Consider the given function

$$f(x) = \log(2+x) - \frac{2x}{2+x}, x \in R$$

$$\Rightarrow f'(x) = \frac{1}{2+x} - \frac{(2+x)2 - 2x \times 1}{(2+x)^2}$$

$$\Rightarrow f'(x) = \frac{1}{2+x} - \frac{4+2x-2x}{(2+x)^2}$$

$$\Rightarrow f'(x) = \frac{1}{2+x} - \frac{4}{(2+x)^2}$$

$$\Rightarrow f'(x) = \frac{2+x-4}{(2+x)^2}$$

$$\Rightarrow f'(x) = \frac{x-2}{(2+x)^2}$$

For $f(x)$ to be increasing, we must have,

$f'(x) > 0$

$\Rightarrow x - 2 > 0$

$\Rightarrow 2 < x < \infty$

$\Rightarrow x \in (2, \infty)$

So, $f(x)$ *i* s*increa*sing in $(2, \infty)$

For $f(x)$ to be decreasing, we must have,

$f'(x) < 0$

$\Rightarrow x - 2 < 0$

$\Rightarrow -\infty < x < 2$

$\Rightarrow x \in (-\infty, 2)$

So, $f(x)$ is decreasing in $(-\infty, 2)$

Increasing and Decreasing Functions Ex 17.2 Q2

We have,

$$f(x) = x^2 - 6x + 9$$

$\therefore \quad f'(x) = 2x - 6$

Critical points

$$f'(x) = 0$$

$\Rightarrow \quad 2(x-3) = 0$

$\Rightarrow \quad x = 3$

Clearly, $f'(x) > 0$ if $x > 3$

$$f'(x) < 0 \text{ if } x < 3$$

Thus, $f(x)$ increases in $(3, \infty)$, decreases in $(-\infty, 3)$

IInd part

The given equation of curves

$$y = x^2 - 6x + 9 \qquad \text{---(i)}$$

$$y = x + 5 \qquad \text{---(ii)}$$

Slope of (i)

$$m_1 = \frac{dy}{dx} = 2x - 6$$

Slope of (ii)

$$m_2 = 1$$

Given that slope of normal to (i) is parallelt to (ii)

$\therefore \quad \frac{-1}{2x-6} = 1$

$\Rightarrow \quad 2x - 6 = -1$

$\Rightarrow \quad x = \frac{5}{2}$

From (i)

$$y = \frac{25}{4} - 15 + 9$$

$$= \frac{25}{4} - 6$$

$$= \frac{1}{4}$$

Thus, the required point is $\left(\frac{5}{2}, \frac{1}{4}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q3

We have,

$$f(x) = \sin x - \cos x, \quad 0 < x < 2\pi$$

$\therefore \quad f'(x) = \cos x + \sin x$

Critical points

$$f'(x) = 0$$

$\Rightarrow \quad \cos x + \sin x = 0$

$\Rightarrow \quad \tan x = -1$

$\Rightarrow \quad x = \frac{3\pi}{4}, \frac{7\pi}{4}$

Clearly, $f'(x) > 0$ if $0 < x < \frac{3\pi}{4}$ and $\frac{7\pi}{4} < x < 2\pi$

$$f'(x) < 0 \text{ if } \frac{3\pi}{4} < x < \frac{7\pi}{4}$$

Thus, $f(x)$ increases in $\left(0, \frac{3\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$, decreases in $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q4

We have,

$$f(x) = e^{2x}$$

$\therefore \quad f'(x) = 2e^{2x}$

We know that

$f(x)$ is increasing if $f'(x) > 0$

$\Rightarrow \quad 2e^{2x} > 0$

$\Rightarrow \quad e^{2x} > 0$

Since, the value of e lies between 2 and 3

So, any power of e will be greater than zero.

Thus, $f(x)$ is increasing on R.

Increasing and Decreasing Functions Ex 17.2 Q5

We have,

$$f(x) = e^{\frac{1}{x}}, \quad x \neq 0$$

$$f'(x) = e^{\frac{1}{x}} \times \left(\frac{-1}{x^2}\right)$$

$\therefore \quad f'(x) = -\frac{e^{\frac{1}{x}}}{x^2}$

Now,

$x \in R,\ x \neq 0$

$\Rightarrow \quad \frac{1}{x^2} > 0$ and $e^{\frac{1}{x}} > 0$

$\Rightarrow \quad \frac{e^{\frac{1}{x}}}{x^2} > 0$

$\Rightarrow \quad -\frac{e^{\frac{1}{x}}}{x^2} < 0$

$\Rightarrow \quad f'(x) < 0$

Hence, $f(x)$ is a decreasing function for all $x \neq 0$.

Increasing and Decreasing Functions Ex 17.2 Q6

We have,

$$f(x) = \log_a x, \ \ 0 < a < 1$$

$\Rightarrow \quad f'(x) = \frac{1}{x \log a}$

$\therefore \quad 0 < a < 1$

$\Rightarrow \quad \log a < 0$

Now,

$x > 0$

$\Rightarrow \quad \frac{1}{x} > 0$

$\Rightarrow \quad \frac{1}{x \log a} < 0$

$\Rightarrow \quad f'(x) < 0$

Thus, $f(x)$ is a decreasing function for $x > 0$.

Increasing and Decreasing Functions Ex 17.2 Q7

The given function is $f(x) = \sin x$.

$\therefore f'(x) = \cos x$

(a) Since for each $x \in \left(0, \frac{\pi}{2}\right)$, $\cos x > 0$, we have $f'(x) > 0$.

Hence, f is strictly increasing in $\left(0, \frac{\pi}{2}\right)$.

(b) Since for each $x \in \left(\frac{\pi}{2}, \pi\right)$, $\cos x < 0$, we have $f'(x) < 0$.

Hence, f is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

(c) From the results obtained in (a) and (b), it is clear that f is neither increasing nor decreasing in $(0, \pi)$.

Increasing and Decreasing Functions Ex 17.2 Q8

We have,

$f(x) = \log \sin x$

$\therefore f'(x) = \frac{1}{\sin x} \cos x = \cot x$

In interval $\left(0, \frac{\pi}{2}\right)$, $f'(x) = \cot x > 0$.

$\therefore f$ is strictly increasing in $\left(0, \frac{\pi}{2}\right)$.

In interval $\left(\frac{\pi}{2}, \pi\right)$, $f'(x) = \cot x < 0$.

$\therefore f$ is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

Increasing and Decreasing Functions Ex 17.2 Q9

We have,

$f(x) = x - \sin x$

$\therefore \quad f'(x) = 1 - \cos x$

Now,

$x \in R$

$\Rightarrow \quad -1 < \cos x < 1$

$\Rightarrow \quad -1 > \cos x > 0$

$\Rightarrow \quad f'(x) > 0$

Hence, $f(x)$ is increasing for all $x \in R$.

Increasing and Decreasing Functions Ex 17.2 Q10

We have,

$$f(x) = x^3 - 15x^2 + 75x - 50$$
$$\therefore \quad f'(x) = 3x^2 - 30x + 75$$
$$\Rightarrow \quad f'(x) = 3(x^2 - 10x + 25)$$
$$= 3(x-5)^2$$

Now,

$$x \in R$$
$$\Rightarrow \quad (x-5)^2 > 0$$
$$\Rightarrow \quad 3(x-5)^2 > 0$$
$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function for all $x \in R$.

Increasing and Decreasing Functions Ex 17.2 Q11

We have,

$$f(x) = \cos^2 x$$
$$\therefore \quad f'(x) = 2\cos x(-\sin x)$$
$$\Rightarrow \quad f'(x) = -2\sin x \cos x$$
$$\Rightarrow \quad f'(x) = -\sin 2x$$

Now,

$$x \in \left(0, \frac{\pi}{2}\right)$$
$$\Rightarrow \quad 2x \in (0, \pi)$$
$$\Rightarrow \quad \sin 2x > 0 \text{ when } 2x \in (0, \pi)$$
$$\Rightarrow \quad -\sin 2x < 0$$
$$\Rightarrow \quad f'(x) < 0$$

Hence, $f(x)$ is a decreasing function on $\left(0, \frac{\pi}{2}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q12

We have

$$f(x) = \sin x$$
$$f'(x) = \cos x$$

Now,

$$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$
$$\Rightarrow \cos x > 0$$
$$\Rightarrow f'(x) > 0$$

Therefore, $f(x) = \sin x$ is an increasing function on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q13

We have,

$$f(x) = \cos x$$
$$\therefore \quad f'(x) = -\sin x$$

Now,

If $x \in (0, \pi)$
$\Rightarrow \quad \sin x > 0$
$\Rightarrow \quad -\sin x < 0$

Hence, $f(x)$ is decreasing function on $(0, \pi)$

If $x \in (-\pi, 0)$
$\Rightarrow \quad \sin x < 0 \qquad [\because \sin(-\theta) = -\sin\theta]$
$\Rightarrow \quad -\sin x > 0$

Hence, $f(x)$ is increasing function on $(-\pi, 0)$

If $x \in (-\pi, \pi)$
Thus, $\sin x > 0$ for $x \in (0, \pi)$
and $\quad \sin x < 0$ for $x \in (-\pi, 0)$
$\Rightarrow \quad -\sin x < 0$ for $x \in (0, \pi)$
and $\quad -\sin x > 0$ for $x \in (-\pi, 0)$

Hence, $f(x)$ is neither increasing nor decreasing on $(-\pi, \pi)$.

Increasing and Decreasing Functions Ex 17.2 Q14

We have,

$$f(x) = \tan x$$
$$\therefore \quad f'(x) = \sec^2 x$$

Now,

$$x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$$
$\Rightarrow \quad \sec^2 x > 0$
$\Rightarrow \quad f'(x) > 0$

Hence, $f(x)$ is increasing function on $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q15

We have,

$$f(x) = \tan^{-1}(\sin x + \cos x)$$
$$\therefore \quad f'(x) = \frac{1}{1 + (\sin x + \cos x)^2} \times (\cos x - \sin x)$$
$$= \frac{\cos x - \sin x}{1 + \sin^2 x + \cos^2 x + 2 \sin x \cos x}$$
$$= \frac{\cos x - \sin x}{2(1 + \sin x \cos x)}$$

Now,

$$x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$
$\Rightarrow \quad \cos x - \sin x < 0$
$\Rightarrow \quad \frac{\cos x - \sin x}{2(1 + \sin x \cos x)} < 0 \qquad [\because 2(1 + \sin x \cos x) > 0]$
$\Rightarrow \quad f'(x) < 0$

Hence, $f(x)$ is decreasing function on $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q16

We have,

$$f(x) = \sin\left(2x + \frac{\pi}{4}\right)$$

$$\therefore \quad f'(x) = \cos\left(2x + \frac{\pi}{4}\right) \times 2$$

$$\therefore \quad f'(x) = 2\cos\left(2x + \frac{\pi}{4}\right)$$

Now,

$$x \in \left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$$

$$\Rightarrow \quad \frac{3\pi}{8} < x < \frac{5\pi}{8}$$

$$\Rightarrow \quad \frac{3\pi}{4} < 2x < \frac{5\pi}{4}$$

$$\Rightarrow \quad \pi < 2x < \frac{\pi}{4} < \frac{3\pi}{2}$$

$\Rightarrow \quad 2x + \frac{\pi}{4}$ lies in IIIrd quadrant

$$\Rightarrow \quad \cos\left(2x + \frac{\pi}{4}\right) < 0$$

$$\Rightarrow \quad 2\cos\left(2x + \frac{\pi}{4}\right) < 0$$

$$\Rightarrow \quad f'(x) < 0$$

Hence, $f(x)$ is decreasing on $\left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q17

We have,

$$f(x) = \tan x - 4x$$

$$\therefore \quad f'(x) = \sec^2 x - 4$$

$$= \frac{1 - 4\cos^2 x}{\cos^2 x}$$

$$= \frac{(1 + 2\cos x)(1 - 2\cos x)}{\cos^2 x}$$

$$= 4\sec^2 x\left(\frac{1}{2} + \cos x\right)\left(\frac{1}{2} - \cos x\right)$$

Now,

$$x \in \left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$$

$$\Rightarrow \quad -\frac{\pi}{3} < x < \frac{\pi}{3}$$

$$\Rightarrow \quad \cos x > \frac{1}{2}$$

$$\Rightarrow \quad \left(\frac{1}{2} - \cos x\right) < 0$$

$$\Rightarrow \quad 4\sec^2 x\left(\frac{1}{2} + \cos x\right)\left(\frac{1}{2} - \cos x\right) < 0$$

$$\Rightarrow \quad f'(x) < 0$$

Hence, $f(x)$ is decreasing function on $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q18

We have,

$$f(x) = (x - 1)e^x + 1$$

$$\therefore \quad f'(x) = e^x + (x - 1)e^x$$

$$\Rightarrow \quad f'(x) = e^x(1 + x - 1) = xe^x$$

Now,

$$x > 0$$

$$\Rightarrow \quad e^x > 0$$

$$\Rightarrow \quad xe^x > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function for all $x > 0$.

Increasing and Decreasing Functions Ex 17.2 Q19

We have,

$$f(x) = x^2 - x + 1$$

$$\therefore \quad f'x = 2x - 1$$

Now,

$$x \in (0,1)$$

$$\Rightarrow \quad 2x - 1 > 0 \text{ if } x > \frac{1}{2}$$

$$\text{and} \quad 2x - 1 < 0 \text{ if } x < \frac{1}{2}$$

$$\Rightarrow \quad f'(x) > 0 \text{ if } x > \frac{1}{2}$$

$$\text{and} \quad f'(x) < 0 \text{ if } x < \frac{1}{2}$$

Thus, $f(x)$ is neither increasing nor decreasing on $(0,1)$.

Increasing and Decreasing Functions Ex 17.2 Q20

We have,

$$f(x) = x^9 + 4x^7 + 11$$

$$f'(x) = 9x^8 + 28x^6$$

$$= x^6\left(9x^2 + 28\right)$$

Now,

$$x \in R$$

$$\Rightarrow \quad x^6 > 0 \text{ and } 9x^2 + 28 > 0$$

$$\Rightarrow \quad x^6\left(9x^2 + 28\right) > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Thus, $f(x)$ is an increasing function for $x \in R$.

Increasing and Decreasing Functions Ex 17.2 Q21

We have,

$$f(x) = x^3 - 6x^2 + 12x - 18$$

$$\therefore \quad f'(x) = 3x^2 - 12x + 12$$

$$= 3\left(x^2 - 4x + 4\right)$$

$$= 3(x-2)^2$$

Now,

$$x \in R$$

$$\Rightarrow \quad (x-2)^2 > 0$$

$$\Rightarrow \quad 3(x-2)^2 > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Thus, $f(x)$ is on increasing function for $x \in R$.

Increasing and Decreasing Functions Ex 17.2 Q22

A function $f(x)$ is said to be increasing on $[a,b]$ if $f(x) > 0$

Now, we have,

$$f(x) = x^2 - 6x + 3$$

$$\therefore \quad f'(x) = 2x - 6$$

$$= 2(x-3)$$

Again,

$$x \in [4,6]$$

$$\Rightarrow \quad 4 \le x \le 6$$

$$\Rightarrow \quad 1 \le x - 3 \le 3$$

$$\Rightarrow \quad (x-3) > 0$$

$$\Rightarrow \quad 2(x-3) > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function for $x \in [4,6]$.

Increasing and Decreasing Functions Ex 17.2 Q23

We have,

$$f(x) = \sin x - \cos x$$

$$\therefore \quad f'(x) = \cos x + \sin x$$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right)$$

$$= \sqrt{2}\left(\frac{\sin \pi}{4}\cos x + \frac{\cos \pi}{4}\sin x\right)$$

$$= \sqrt{2}\ \sin\left(\frac{\pi}{4} + x\right)$$

Now,

$$x \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$$

$$\Rightarrow \quad -\frac{\pi}{4} < x < \frac{\pi}{4}$$

$$\Rightarrow \quad 0 < \frac{\pi}{4} + x < \frac{\pi}{2}$$

$$\Rightarrow \quad \sin 0^\circ < \sin\left(\frac{\pi}{4} + x\right) < \sin\frac{\pi}{2}$$

$$\Rightarrow \quad 0 < \sin\left(\frac{\pi}{4} + x\right) < 1$$

$$\Rightarrow \quad \sqrt{2}\sin\left(\frac{\pi}{4} + x\right) > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function on $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q24

We have,

$$f(x) = \tan^{-1} x - x$$

$$\therefore \quad f'(x) = \frac{1}{1+x^2} - 1$$

$$= \frac{-x^2}{1+x^2}$$

Now,

$$x \in R$$

$$\Rightarrow \quad x^2 > 0 \text{ and } 1 + x^2 > 0$$

$$\Rightarrow \quad \frac{x^2}{1+x^2} > 0$$

$$\Rightarrow \quad \frac{-x^2}{1+x^2} < 0$$

$$\Rightarrow \quad f'(x) < 0$$

Hence, $f(x)$ is a decreasing function for $x \in R$.

Increasing and Decreasing Functions Ex 17.2 Q25

We have,

$$f(x) = -\frac{x}{2} + \sin x$$

$$\therefore \quad f'(x) = -\frac{1}{2} + \cos x$$

Now,

$$x \in \left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$$

$$\Rightarrow \quad -\frac{\pi}{3} < x < \frac{\pi}{3}$$

$$\Rightarrow \quad \cos\left(-\frac{\pi}{3}\right) < \cos x < \cos\frac{\pi}{3}$$

$$\Rightarrow \quad \cos\frac{\pi}{3} < \cos x < \cos\frac{\pi}{3}$$

$$\Rightarrow \quad \frac{1}{2} < \cos x < \frac{1}{2}$$

$$\Rightarrow \quad -\frac{1}{2} + \cos x + 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function on $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q26

We have,

$$f(x) = \log(1+x) - \frac{x}{1+x}$$

$$\therefore \quad f'(x) = \frac{1}{1+x} - \left(\frac{(1+x)-x}{(1+x)^2}\right)$$

$$= \frac{1}{1+x} - \frac{1}{(1+x)^2}$$

$$= \frac{x}{(1+x)^2}$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{x}{(1+x)^2} = 0$$

$$\Rightarrow \quad x = 0, \ -1$$

Clearly, $f'(x) > 0$ if $x > 0$

and $\quad f'(x) < 0$ if $-1 < x < 0$ or $x < -1$

Hence, $f(x)$ increases in $(0, \infty)$, decreases in $(-\infty, -1) \cup (-1, 0)$.

Increasing and Decreasing Functions Ex 17.2 Q27

We have,

$$f(x) = (x+2)e^{-x}$$

$$\therefore \quad f'(x) = e^{-x} - e^{-x}(x+2)$$

$$= e^{-x}(1 - x - 2)$$

$$= -e^{-x}(x+1)$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \quad -e^{-x}(x+1) = 0$$

$$\Rightarrow \quad x = -1$$

Clearly, $f'(x) > 0$ if $x < -1$

$f'(x) < 0$ if $x > -1$

Hence, $f(x)$ increases in $(-\infty, -1)$, decreases in $(-1, \infty)$

Increasing and Decreasing Functions Ex 17.2 Q28

We have,

$$f(x) = 10^x$$

$$\therefore \quad f'(x) = 10^x \times \log 10$$

Now,

$$x \in R$$

$$\Rightarrow \quad 10^x > 0$$

$$\Rightarrow \quad 10^x \log 10 > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ in an increasing function for all x.

Increasing and Decreasing Functions Ex 17.2 Q29

We have,

$$f(x) = x - [x]$$

$$\therefore \quad f'(x) = 1 > 0$$

$\therefore$ $f(x)$ in an increasing function on $(0, 1)$.

Increasing and Decreasing Functions Ex 17.2 Q30

We have,

$$f(x) = 3x^5 + 40x^3 + 240x$$

$$\therefore \quad f'(x) = 15x^4 + 120x^2 + 240$$

$$= 15\left(x^4 + 8x^2 + 16\right)$$

$$= 15\left(x^2 + 4\right)^2$$

Now,

$$x \in R$$

$$\Rightarrow \quad \left(x^2 + 4\right)^2 > 0$$

$$\Rightarrow \quad 15\left(x^2 + 4\right)^2 > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function for all x.

Increasing and Decreasing Functions Ex 17.2 Q31

We have,

$$f(x) = \log \cos x$$

$$\therefore f'(x) = \frac{1}{\cos x}(-\sin x) = -\tan x$$

In interval $\left(0, \frac{\pi}{2}\right)$, $\tan x > 0 \Rightarrow -\tan x < 0$.

$$\therefore f'(x) < 0 \text{ on } \left(0, \frac{\pi}{2}\right)$$

$\therefore f$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

In interval $\left(\frac{\pi}{2}, \pi\right)$, $\tan x < 0 \Rightarrow -\tan x > 0$.

$$\therefore f'(x) > 0 \text{ on } \left(\frac{\pi}{2}, \pi\right)$$

Increasing and Decreasing Functions Ex 17.2 Q32

Given $f(x) = x^3 - 3x^2 + 4x$

$\therefore \quad f'(x) = 3x^2 - 6x + 4$

$= 3(x^2 - 2x + 1) + 1$

$= 3(x-1)^2 + 1 > 0, \text{ for all } x \in \mathbf{R}$

Hence, f is strictly increasing on **R**.

Increasing and Decreasing Functions Ex 17.2 Q33

Given $f(x) = \cos x$

$\therefore \quad f'(x) = -\sin x$

(i) Since for each $x \in (0, \pi)$, $\sin x > 0$

$\Rightarrow \quad f'(x) < 0$

So f is strictly decreasing in $(0, \pi)$

(ii) Since for each $x \in (\pi, 2\pi)$, $\sin x < 0$

$\Rightarrow \quad f'(x) > 0$

So f is strictly increasing in $(\pi, 2\pi)$

(iii) Clearly from (i) & (ii) above, f is neither increasing nor decreasing in $(0, 2\pi)$

Increasing and Decreasing Functions Ex 17.2 Q34

We have,

$f(x) = x^2 - x\sin x$

$\therefore \quad f'(x) = 2x - \sin x - x\cos x$

Now,

$x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow \quad 0 \le \sin x \le 1, \quad 0 \le \cos x \le 1$

$\Rightarrow \quad 2x - \sin x - x\cos x > 0$

$\Rightarrow \quad f'(x) \ge 0$

Hence, $f(x)$ is an increasing function on $\left(0, \frac{\pi}{2}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q35

We have,

$f(x) = x^3 - ax$

$\therefore \quad f'(x) = 3x^2 - a$

Given that $f(x)$ is on increasing function

$\therefore \quad f'(x) > 0 \quad \text{for all } x \in R$

$\Rightarrow \quad 3x^2 - a > 0 \quad \text{for all } x \in R$

$\Rightarrow \quad a < 3x^2 \quad \text{for all } x \in R$

But the last value of $3x^2 = 0$ for $x = 0$

$\therefore \quad a \le 0$

Increasing and Decreasing Functions Ex 17.2 Q36

We have,

$f(x) = \sin x - bx + c$

$\therefore \quad f'(x) = \cos x - b$

Given that $f(x)$ is a decreasing function on R

$\therefore \quad f'(x) < 0 \quad \text{for all } x \in R$

$\Rightarrow \quad \cos x - b < 0 \quad \text{for all } x \in R$

$\Rightarrow \quad b > \cos x \quad \text{for all } x \in R$

But man value of $\cos x$ in 1

$\therefore \quad b \ge 1$

Increasing and Decreasing Functions Ex 17.2 Q37

We have,

$$f(x) = x + \cos x - a$$

$$\therefore \quad f'(x) = 1 - \sin x = \frac{2\cos^2 x}{2}$$

Now,

$$x \in R$$

$$\Rightarrow \quad \frac{\cos^2 x}{2} > 0$$

$$\Rightarrow \quad \frac{2\cos^2 x}{2} > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function for $x \in R$.

Increasing and Decreasing Functions Ex 17.2 Q38

As $f(0) = f(1)$ and f is differentiable, hence by Rolles theorem:

$f'(c) = 0$ for some $c \in [0, 1]$

Let us now apply LMVT (as function is twice differentiable) for point c and $x \in [0, 1]$, hence

$$\frac{|f'(x) - f(c)|}{x - c} = f''(d)$$

$$\Rightarrow \frac{|f'(x) - 0|}{x - c} = f''(d)$$

$$\Rightarrow \frac{|f'(x)|}{x - c} = f''(d)$$

As given that $|f''(d)| \leq 1$ for $x \in [0, 1]$

$$\Rightarrow \frac{|f'(x)|}{x - c} \leq 1$$

$$\Rightarrow |f'(x)| \leq x - c$$

Now as both x and c lie in $[0, 1]$, hence $x - c \in (0, 1)$

$\Rightarrow |f'(x)| < 1$ for all $x \in [0, 1]$

Increasing and Decreasing Functions Ex 17.2 Q39(i)

Consider the given function,

$f(x) = x|x|, \; x \in R$

$$\Rightarrow f(x) = \begin{cases} -x^2, & x<0 \\ x^2, & x>0 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} -2x, & x<0 \\ 2x, & x>0 \end{cases}$$

$\Rightarrow f'(x) > 0$, for values of x

Therefore, $f(x)$ is an increasing function for all real values.

Increasing and Decreasing Functions Ex 17.2 Q39(ii)

Consider the function

$f(x) = \sin x + |\sin x|,\ 0 < x \le 2\pi$

$\Rightarrow f(x) = \begin{cases} 2\sin x, & 0 < x < \pi \\ 0, & \pi < x < 2\pi \end{cases}$

$\Rightarrow f'(x) = \begin{cases} 2\cos x, & 0 < x < \pi \\ 0, & \pi < x < 2\pi \end{cases}$

The function 2cosx will be positive between $\left(0, \frac{\pi}{2}\right)$.

Hence the function $f(x)$ is increasing in the interval $\left(0, \frac{\pi}{2}\right)$.

The function 2cosx will be negative between $\left(\frac{\pi}{2}, \pi\right)$.

Hence the function $f(x)$ is decreasing in the interval $\left(\frac{\pi}{2}, \pi\right)$.

The value of $f'(x) = 0$, when $\pi \le x < 2\pi$.

Therefore, the function $f(x)$ is neither increasing nor decreasing in the interval $(\pi, 2\pi)$

Increasing and Decreasing Functions Ex 17.2 Q39(iii)

Consider the function,

$f(x) = \sin x(1 + \cos x),\ 0 < x < \frac{\pi}{2}$

$\Rightarrow f'(x) = \cos x + \sin x(-\sin x) + \cos x(\cos x)$

$\Rightarrow f'(x) = \cos x - \sin^2 x + \cos^2 x$

$\Rightarrow f'(x) = \cos x + (\cos^2 x - 1) + \cos^2 x$

$\Rightarrow f'(x) = \cos x + 2\cos^2 x - 1$

$\Rightarrow f'(x) = 2\cos^2 x + \cos x - 1$

$\Rightarrow f'(x) = 2\cos^2 x + 2\cos x - \cos x - 1$

$\Rightarrow f'(x) = 2\cos x(\cos x + 1) - 1(\cos x + 1)$

$\Rightarrow f'(x) = (2\cos x - 1)(\cos x + 1)$

For $f(x)$ to be increasing, we must have,

$f'(x) > 0$

$\Rightarrow f'(x) = (2\cos x - 1)(\cos x + 1) > 0$

$\Rightarrow 0 < x < \frac{\pi}{3}$

$\Rightarrow x \in \left(0, \frac{\pi}{3}\right)$

So, $f(x)$ is *increasing* in $\left(0, \frac{\pi}{3}\right)$

For $f(x)$ to be decreasing, we must have,

$f'(x) < 0$

$\Rightarrow f'(x) = (2\cos x - 1)(\cos x + 1) < 0$

$\Rightarrow \frac{\pi}{3} < x < \frac{\pi}{2}$

$\Rightarrow x \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

So, $f(x)$ is decreasing in $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$