SOLUTIONS TO CONCEPTS
CHAPTER 12

1. Given, r = 10cm.
   At t = 0, x = 5 cm.
   T = 6 sec.
   So, w = $\frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3}\sec^{-1}$
   At, t = 0, x = 5 cm.
   So, 5 = 10 sin (w × 0 + ϕ) = 10 sin ϕ [y = r sin wt]
   Sin ϕ = 1/2 $\Rightarrow \phi = \frac{\pi}{6}$
   $\therefore$ Equation of displacement x = (10cm) sin $\left(\frac{\pi}{3}\right)$
   (ii) At t = 4 second
   x = 10 sin $\left[\frac{\pi}{3}\times 4+\frac{\pi}{6}\right]$ = 10 sin $\left[\frac{8\pi+\pi}{6}\right]$
   = 10 sin $\left(\frac{3\pi}{2}\right)$ = 10 sin $\left(\pi+\frac{\pi}{2}\right)$ = - 10 sin $\left(\frac{\pi}{2}\right)$ = -10
   Acceleration a = – w^2x = $-\left(\frac{\pi^2}{9}\right)$ × (–10) = 10.9 $\approx$ 0.11 cm/sec.

2. Given that, at a particular instant,
   X = 2cm = 0.02m
   V = 1 m/sec
   A = 10 $msec^{-2}$
   We know that a = ω^2x
   $\Rightarrow \omega = \sqrt{\frac{a}{x}} = \sqrt{\frac{10}{0.02}} = \sqrt{500} = 10\sqrt{5}$
   $T = \frac{2\pi}{\omega} = \frac{2\pi}{10\sqrt{5}} = \frac{2\times 3.14}{10\times 2.236}$ = 0.28 seconds.
   Again, amplitude r is given by v = $\omega\left(\sqrt{r^2-x^2}\right)$
   $\Rightarrow v^2 = \omega^2(r^2 - x^2)$
   1 = 500 (r^2 – 0.0004)
   $\Rightarrow$ r = 0.0489 $\approx$ 0.049 m
   $\therefore$ r = 4.9 cm.

3. r = 10cm
   Because, K.E. = P.E.
   So (1/2) m ω^2 (r^2– y^2) = (1/2) m ω^2y^2
   $r^2 - y^2 = y^2 \Rightarrow 2y^2 = r^2 \Rightarrow y = \frac{r}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 5\sqrt{2}$ cm form the mean position.

4. v_{max} = 10 cm/sec.
   $\Rightarrow r\omega = 10$
   $\Rightarrow \omega^2 = \frac{100}{r^2}$ …(1)
   $A_{max} = \omega^2 r$ = 50 cm/sec
   $\Rightarrow \omega^2 = \frac{50}{y} = \frac{50}{r}$ …(2)

$\therefore \dfrac{100}{r^2} = \dfrac{50}{r} \Rightarrow r = 2$ cm.

$\therefore \omega = \sqrt{\dfrac{100}{r^2}} = 5\ \text{sec}^2$

Again, to find out the positions where the speed is 8m/sec,

$v^2 = \omega^2 (r^2 - y^2)$

$\Rightarrow 64 = 25\ (4 - y^2)$

$\Rightarrow 4 - y^2 = \dfrac{64}{25} \Rightarrow y^2 = 1.44 \Rightarrow y = \sqrt{1.44} \Rightarrow y = \pm 1.2$ cm from mean position.

5. $x = (2.0\text{cm})\sin[(100\text{s}^{-1})\,t + (\pi/6)]$

m = 10g.

a) Amplitude = 2cm.

$\omega = 100\ \text{sec}^{-1}$

$\therefore T = \dfrac{2\pi}{100} = \dfrac{\pi}{50}$ sec = 0.063 sec.

We know that $T = 2\pi\sqrt{\dfrac{m}{k}} \Rightarrow T^2 = 4\pi^2 \times \dfrac{m}{k} \Rightarrow k = \dfrac{4\pi^2}{T^2}m$

$= 10^5$ dyne/cm = 100 N/m. [because $\omega = \dfrac{2\pi}{T} = 100\ \text{sec}^{-1}$]

b) At t = 0

$x = 2\text{cm} \sin\left(\dfrac{\pi}{6}\right) = 2 \times (1/2) = 1$ cm. from the mean position.

We know that $x = A \sin(\omega t + \phi)$

$v = A \cos(\omega t + \phi)$

$= 2 \times 100 \cos(0 + \pi/6) = 200 \times \dfrac{\sqrt{3}}{2} = 100\sqrt{3}\ \text{sec}^{-1} = 1.73$m/s

c) $a = -\omega^2 x = 100^2 \times 1 = 100\ \text{m/s}^2$

6. $x = 5 \sin(20t + \pi/3)$

a) Max. displacement from the mean position = Amplitude of the particle.

At the extreme position, the velocity becomes '0'.

$\therefore$ x = 5 = Amplitude.

$\therefore 5 = 5 \sin(20t + \pi/3)$

$\sin(20t + \pi/3) = 1 = \sin(\pi/2)$

$\Rightarrow 20t + \pi/3 = \pi/2$

$\Rightarrow t = \pi/120$ sec., So at $\pi/120$ sec it first comes to rest.

b) $a = \omega^2 x = \omega^2\,[5 \sin(20t + \pi/3)]$

For a = 0, $5 \sin(20t + \pi/3) = 0 \Rightarrow \sin(20t + \pi/3) = \sin(\pi)$

$\Rightarrow 20\,t = \pi - \pi/3 = 2\pi/3$

$\Rightarrow t = \pi/30$ sec.

c) $v = A\,\omega \cos(\omega t + \pi/3) = 20 \times 5 \cos(20t + \pi/3)$

when, v is maximum i.e. $\cos(20t + \pi/3) = -1 = \cos\pi$

$\Rightarrow 20t = \pi - \pi/3 = 2\pi/3$

$\Rightarrow t = \pi/30$ sec.

7. a) $x = 2.0 \cos(50\pi t + \tan^{-1} 0.75) = 2.0 \cos(50\pi t + 0.643)$

$v = \dfrac{dx}{dt} = -100 \sin(50\pi t + 0.643)$

$\Rightarrow \sin(50\pi t + 0.643) = 0$

As the particle comes to rest for the 1st time

$\Rightarrow 50\pi t + 0.643 = \pi$

$\Rightarrow t = 1.6 \times 10^{-2}$ sec.

b) Acceleration $a = \frac{dv}{dt} = -100\pi \times 50\pi \cos(50\pi t + 0.643)$

For maximum acceleration $\cos(50\pi t + 0.643) = -1 \cos\pi$ (max) (so a is max)

$\Rightarrow t = 1.6 \times 10^{-2}$ sec.

c) When the particle comes to rest for second time,

$50\pi t + 0.643 = 2\pi$

$\Rightarrow t = 3.6 \times 10^{-2}$ s.

8. $y_1 = \frac{r}{2}$, $y_2 = r$ (for the two given position)

Now, $y_1 = r \sin \omega t_1$

$\Rightarrow \frac{r}{2} = r \sin \omega t_1 \Rightarrow \sin \omega t_1 = \frac{1}{2} \Rightarrow \omega t_1 = \frac{\pi}{2} \Rightarrow \frac{2\pi}{t} \times t_1 = \frac{\pi}{6} \Rightarrow t_1 = \frac{t}{12}$

Again, $y_2 = r \sin \omega t_2$

$\Rightarrow r = r \sin \omega t_2 \Rightarrow \sin \omega t_2 = 1 \Rightarrow \omega t_2 = \pi/2 \Rightarrow \left(\frac{2\pi}{t}\right) t_2 = \frac{\pi}{2} \Rightarrow t_2 = \frac{t}{4}$

So, $t_2 - t_1 = \frac{t}{4} - \frac{t}{12} = \frac{t}{6}$

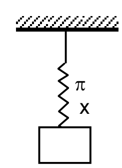
9. $k = 0.1$ N/m

$T = 2\pi\sqrt{\frac{m}{k}} = 2$ sec [Time period of pendulum of a clock = 2 sec]

So, $4\pi^{2+}\left(\frac{m}{k}\right) = 4$

$\therefore m = \frac{k}{\pi^2} = \frac{0.1}{10} = 0.01\text{kg} \approx 10$ gm.

10. Time period of simple pendulum $= 2\pi\sqrt{\frac{1}{g}}$



Time period of spring is $2\pi\sqrt{\frac{m}{k}}$

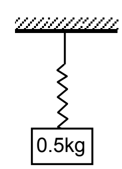
$T_p = T_s$ [Frequency is same]

$\Rightarrow \sqrt{\frac{1}{g}} = \sqrt{\frac{m}{k}} \qquad \Rightarrow \frac{1}{g} = \frac{m}{k}$

$\Rightarrow 1 = \frac{mg}{k} = \frac{F}{k} = x$. (Because, restoring force = weight = F =mg)

$\Rightarrow 1 = x$ (proved)

11. $x = r = 0.1$ m



$T = 0.314$ sec

$m = 0.5$ kg.

Total force exerted on the block = weight of the block + spring force.

$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow 0.314 = 2\pi\sqrt{\frac{0.5}{k}} \Rightarrow k = 200$ N/m

$\therefore$ Force exerted by the spring on the block is

$F = kx = 201.1 \times 0.1 = 20N$

$\therefore$ Maximum force = F + weight = 20 + 5 = 25N

12. $m = 2$kg.

$T = 4$ sec.

$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow 4 = 2\pi\sqrt{\frac{2}{K}} \Rightarrow 2 = \pi\sqrt{\frac{2}{K}}$

$\Rightarrow 4 = \pi^2 \left(\frac{2}{k}\right) \Rightarrow k = \frac{2\pi^2}{4} \Rightarrow k = \frac{\pi^2}{2} = 5$ N/m

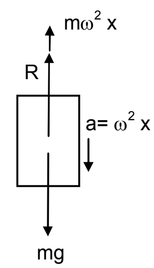
But, we know that F = mg = kx

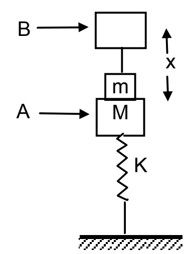
$\Rightarrow x = \frac{mg}{k} = \frac{2 \times 10}{5} = 4$

$\therefore$ Potential Energy = (1/2) k x^2 = (1/2) × 5 × 16 = 5 × 8 = 40J

13. x = 25cm = 0.25m
E = 5J
f = 5
So, T = 1/5sec.
Now P.E. = (1/2) kx^2
$\Rightarrow$(1/2) $kx^2 = 5 \Rightarrow$ (1/2) k $(0.25)^2 = 5 \Rightarrow$ k = 160 N/m.
Again, $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow \frac{1}{5} = 2\pi\sqrt{\frac{m}{160}} \Rightarrow$ m = 0.16 kg.

14. a) From the free body diagram,





$\therefore R + m\omega^2 x - mg = 0$ …(1)

Resultant force $m\omega^2 x = mg - R$

$\Rightarrow m\omega^2 x = m\left(\frac{k}{M+m}\right) \Rightarrow x = \frac{mkx}{M+m}$

[$\omega = \sqrt{k/(M+m)}$ for spring mass system]

b) $R = mg - m\omega^2 x = mg - m\frac{k}{M+m}x = mg - \frac{mkx}{M+m}$

For R to be smallest, $m\omega^2 x$ should be max. i.e. x is maximum.

The particle should be at the high point.

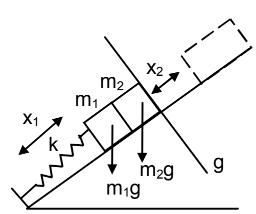
c) We have $R = mg - m\omega^2 x$

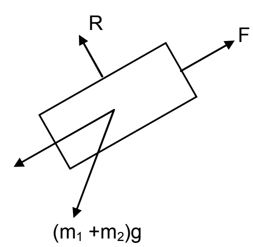
The tow blocks may oscillates together in such a way that R is greater than 0. At limiting condition, R = 0, $mg = m\omega^2 x$

$X = \frac{mg}{m\omega^2} = \frac{mg(M+m)}{mk}$

So, the maximum amplitude is = $\frac{g(M+m)}{k}$

15. a) At the equilibrium condition,





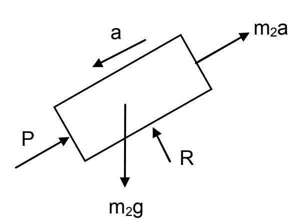
$kx = (m_1 + m_2) g \sin\theta$

$\Rightarrow x = \frac{(m_1+m_2)g\sin\theta}{k}$

b) $x_1 = \frac{2}{k}(m_1 + m_2) g \sin\theta$ (Given)

when the system is released, it will start to make SHM

where $\omega = \sqrt{\frac{k}{m_1+m_2}}$



When the blocks lose contact, P = 0

So $m_2 g \sin\theta = m_2 x_2 \omega^2 = m_2 x_2 \left(\frac{k}{m_1+m_2}\right)$

$\Rightarrow x_2 = \frac{(m_1+m_2)g\sin\theta}{}k$

So the blocks will lose contact with each other when the springs attain its natural length.

c) Let the common speed attained by both the blocks be v.

$1/2\,(m_1 + m_2)\,v^2 - 0 = 1/2\,k(x_1 + x_2)^2 - (m_1 + m_2)\,g \sin\theta\,(x + x_1)$

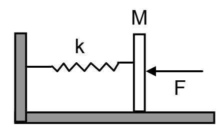
[$x + x_1$ = total compression]

$\Rightarrow (1/2)\,(m_1 + m_2)\,v^2 = [(1/2)\,k\,(3/k)\,(m_1 + m_2)\,g \sin\theta - (m_1 + m_2)\,g \sin\theta]\,(x + x_1)$

$\Rightarrow (1/2)\,(m_1 + m_2)\,v^2 = (1/2)\,(m_1 + m_2)\,g \sin\theta \times (3/k)\,(m_1 + m_2)\,g \sin\theta$

$\Rightarrow v = \sqrt{\dfrac{3}{k(m_1 + m_2)}}\,g \sin\theta.$

16. Given, k = 100 N/m, M = 1kg and F = 10 N



a) In the equilibrium position,

compression $\delta = F/k = 10/100 = 0.1$ m = 10 cm

b) The blow imparts a speed of 2m/s to the block towards left.

$\therefore$ P.E. + K.E. $= 1/2\,k\delta^2 + 1/2\,Mv^2$

$= (1/2) \times 100 \times (0.1)^2 + (1/2) \times 1 \times 4 = 0.5 + 2 = 2.5$ J

c) Time period $= 2\pi\sqrt{\dfrac{M}{k}} = 2\pi\sqrt{\dfrac{1}{100}} = \dfrac{\pi}{5}$ sec

d) Let the amplitude be 'x' which means the distance between the mean position and the extreme position.

So, in the extreme position, compression of the spring is $(x + \delta)$.

Since, in SHM, the total energy remains constant.

$(1/2)\,k\,(x + \delta)^2 = (1/2)\,k\delta^2 + (1/2)\,mv^2 + Fx = 2.5 + 10x$

[because $(1/2)\,k\delta^2 + (1/2)\,mv^2 = 2.5$]

So, $50(x + 0.1)^2 = 2.5 + 10x$

$\therefore\ 50\,x^2 + 0.5 + 10x = 2.5 + 10x$

$\therefore 50x^2 = 2 \Rightarrow x^2 = \dfrac{2}{50} = \dfrac{4}{100} \Rightarrow x = \dfrac{2}{10}$ m = 20cm.

e) Potential Energy at the left extreme is given by,

P.E. $= (1/2)\,k\,(x + \delta)^2 = (1/2) \times 100\,(0.1 + 0.2)^2 = 50 \times 0.09 = 4.5$J

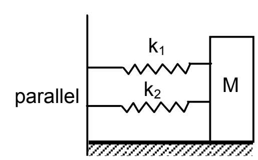
f) Potential Energy at the right extreme is given by,

P.E. $= (1/2)\,k\,(x + \delta)^2 - F(2x)$ [2x = distance between two extremes]

$= 4.5 - 10(0.4) = 0.5$J

The different values in (b) (e) and (f) do not violate law of conservation of energy as the work is done by the external force 10N.

17. a) Equivalent spring constant $k = k_1 + k_2$ (parallel)



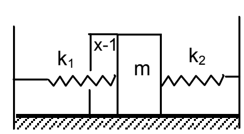
(a)

$T = 2\pi\sqrt{\dfrac{M}{k}} = 2\pi\sqrt{\dfrac{m}{k_1 + k_2}}$

b) Let us, displace the block m towards left through displacement 'x'

Resultant force $F = F_1 + F_2 = (k_1 + k_2)x$

Acceleration (F/m) $= \dfrac{(k_1 + k_2)x}{m}$

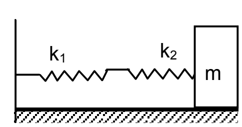


Time period $T = 2\pi\sqrt{\dfrac{\text{displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\dfrac{x}{\dfrac{m(k_1 + k_2)}{m}}} = 2\pi\sqrt{\dfrac{m}{k_1 + k_2}}$

The equivalent spring constant $k = k_1 + k_2$

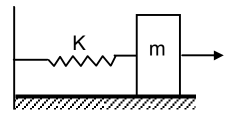
c) In series conn equivalent spring constant be k.

So, $\dfrac{1}{k} = \dfrac{1}{k_1} + \dfrac{1}{k_2} = \dfrac{k_2 + k_1}{k_1k_2} \Rightarrow k = \dfrac{k_1k_2}{k_1 + k_2}$



$T = 2\pi\sqrt{\dfrac{M}{k}} = 2\pi\sqrt{\dfrac{m(k_1 + k_2)}{k_1k_2}}$

18. a) We have F = kx ⇒ $x = \frac{F}{k}$



Acceleration = $\frac{F}{m}$

Time period $T = 2\pi\sqrt{\frac{\text{displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\frac{F/k}{F/m}} = 2\pi\sqrt{\frac{m}{k}}$

Amplitude = max displacement = F/k

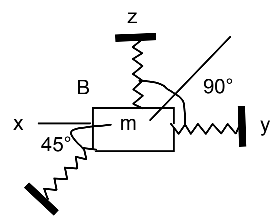
b) The energy stored in the spring when the block passes through the equilibrium position

$(1/2)\,kx^2 = (1/2)\,k\,(F/k)^2 = (1/2)\,k\,(F^2/k^2) = (1/2)\,(F^2/k)$

c) At the mean position, P.E. is 0. K.E. is $(1/2)\,kx^2 = (1/2)\,(F^2/x)$

19. Suppose the particle is pushed slightly against the spring 'C' through displacement 'x'.

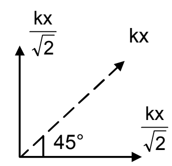
Total resultant force on the particle is kx due to spring C and $\frac{kx}{\sqrt{2}}$ due to spring A and B.



∴ Total Resultant force = $kx + \sqrt{\left(\frac{kx}{\sqrt{2}}\right)^2 + \left(\frac{kx}{\sqrt{2}}\right)^2} = kx + kx = 2kx$.

Acceleration = $\frac{2kx}{m}$

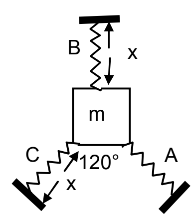
Time period $T = 2\pi\sqrt{\frac{\text{displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\frac{x}{\frac{2kx}{m}}} = 2\pi\sqrt{\frac{m}{2k}}$



[Cause:- When the body pushed against 'C' the spring C, tries to pull the block towards XL. At that moment the spring A and B tries to pull the block with force $\frac{kx}{\sqrt{2}}$ and $\frac{kx}{\sqrt{2}}$ respectively towards xy and xz respectively. So the total force on the block is due to the spring force 'C' as well as the component of two spring force A and B.]

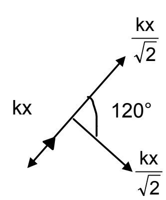
20. In this case, if the particle 'm' is pushed against 'C' a by distance 'x'.

Total resultant force acting on man 'm' is given by,



$F = kx + \frac{kx}{2} = \frac{3kx}{2}$

[Because net force A & B = $\sqrt{\left(\frac{kx}{2}\right)^2 + \left(\frac{kx}{2}\right)^2 + 2\left(\frac{kx}{2}\right)\left(\frac{kx}{2}\right)\cos 120^\circ} = \frac{kx}{2}$



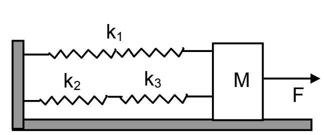
$\therefore a = \frac{F}{m} = \frac{3kx}{2m}$

$\Rightarrow \frac{a}{x} = \frac{3k}{2m} = \omega^2 \quad \Rightarrow \omega = \sqrt{\frac{3k}{2m}}$

∴Time period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2m}{3k}}$

21. K_2 and K_3 are in series.

Let equivalent spring constant be K_4



$\therefore \frac{1}{K_4} = \frac{1}{K_2} + \frac{1}{K_3} = \frac{K_2 + K_3}{K_2K_3} \Rightarrow K_4 = \frac{K_2K_3}{K_2 + K_3}$

Now K_4 and K_1 are in parallel.

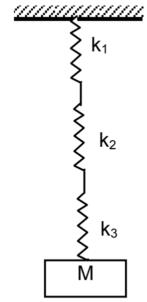
So equivalent spring constant $k = k_1 + k_4 = \frac{K_2K_3}{K_2 + K_3} + k_1 = \frac{k_2k_3 + k_1k_2 + k_1k_3}{k_2 + k_3}$

$\therefore T = 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{M(k_2 + k_3)}{k_2k_3 + k_1k_2 + k_1k_3}}$

b) frequency = $\frac{1}{T}$ = $\frac{1}{2\pi}\sqrt{\frac{k_2k_3 + k_1k_2 + k_1k_3}{M(k_2 + k_3)}}$

c) Amplitude x = $\frac{F}{k}$ = $\frac{F(k_2 + k_3)}{k_1k_2 + k_2k_3 + k_1k_3}$

22. k_1, k_2, k_3 are in series,



$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$ $\Rightarrow k = \frac{k_1k_2k_3}{k_1k_2 + k_2k_3 + k_1k_3}$

Time period T = $2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m(k_1k_2 + k_2k_3 + k_1k_3)}{k_1k_2k_3}} = 2\pi\sqrt{m\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right)}$

Now, Force = weight = mg.

$\therefore$ At k_1 spring, $x_1 = \frac{mg}{k_1}$

Similarly $x_2 = \frac{mg}{k_2}$ and $x_3 = \frac{mg}{k_3}$

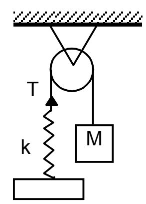
$\therefore PE_1 = (1/2)\, k_1\, x_1^2 = \frac{1}{2}k_1\left(\frac{Mg}{k_1}\right)^2 = \frac{1}{2}k_1\frac{m^2g^2}{k_1^2} = \frac{m^2g^2}{2k_1}$

Similarly $PE_2 = \frac{m^2g^2}{2k_2}$ and $PE_3 = \frac{m^2g^2}{2k_3}$

23. When only 'm' is hanging, let the extension in the spring be 'ℓ'

So $T_1 = k\ell = mg$.

When a force F is applied, let the further extension be 'x'



$\therefore T_2 = k(x + \ell)$

$\therefore$ Driving force = $T_2 - T_1 = k(x + \ell) - k\ell = kx$

$\therefore$ Acceleration = $\frac{K\ell}{m}$

$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\frac{x}{\frac{kx}{m}}} = 2\pi\sqrt{\frac{m}{k}}$

24. Let us solve the problem by 'energy method'.

Initial extension of the sprig in the mean position,

$\delta = \frac{mg}{k}$

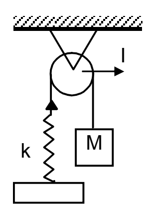
During oscillation, at any position 'x' below the equilibrium position, let the velocity of 'm' be v and angular velocity of the pulley be 'ω'. If r is the radius of the pulley, then v = rω.

At any instant, Total Energy = constant (for SHM)

$\therefore$ $(1/2)\, mv^2 + (1/2)\, I\, \omega^2 + (1/2)\, k[(x + \delta)^2 - \delta^2] - mgx$ = Cosntant

$\Rightarrow (1/2)\, mv^2 + (1/2)\, I\, \omega^2 + (1/2)\, kx^2 - kx\delta - mgx$ = Cosntant

$\Rightarrow (1/2)\, mv^2 + (1/2)\, I\, (v^2/r^2) + (1/2)\, kx^2$ = Constant $(\delta = mg/k)$



Taking derivative of both sides eith respect to 't',

$mv\frac{dv}{dt} + \frac{I}{r^2}v\frac{dv}{dt} + k \times \frac{dv}{dt} = 0$

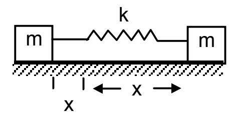
$\Rightarrow a\left(m + \frac{I}{r^2}\right) = kx$ $\left(\therefore x = \frac{dx}{dt} \text{ and } a = \frac{dx}{dt}\right)$

$\Rightarrow \frac{a}{x} = \frac{k}{m + \frac{I}{r^2}} = \omega^2 \Rightarrow T = 2\pi\sqrt{\frac{m + \frac{I}{r^2}}{k}}$

25. The centre of mass of the system should not change during the motion. So, if the block 'm' on the left moves towards right a distance 'x', the block on the right moves towards left a distance 'x'. So, total compression of the spring is 2x.

By energy method, $\frac{1}{2}k(2x)^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = C \Rightarrow mv^2 + 2kx^2 = C.$

Taking derivative of both sides with respect to 't'.



$m \times 2v\frac{dv}{dt} + 2k \times 2x\frac{dx}{dt} = 0$

$\therefore ma + 2kx = 0$ [because v = dx/dt and a = dv/dt]

$\Rightarrow \frac{a}{x} = -\frac{2k}{m} = \omega^2 \Rightarrow \omega = \sqrt{\frac{2k}{m}}$

$\Rightarrow$ Time period $T = 2\pi\sqrt{\frac{m}{2k}}$

26. Here we have to consider oscillation of centre of mass

Driving force $F = mg\sin\theta$

Acceleration $= a = \frac{F}{m} = g\sin\theta.$

For small angle θ, $\sin\theta = \theta$.

$\therefore a = g\theta = g\left(\frac{x}{L}\right)$ [where g and L are constant]

$\therefore a \propto x$,

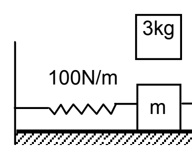
So the motion is simple Harmonic

Time period $T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\frac{x}{\left(\frac{gx}{L}\right)}} = 2\pi\sqrt{\frac{L}{g}}$

27. Amplitude = 0.1m

Total mass = 3 + 1 = 4kg (when both the blocks are moving together)

$\therefore T = 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{4}{100}} = \frac{2\pi}{5}$ sec.



$\therefore$ Frequency $= \frac{5}{2\pi}$ Hz.

Again at the mean position, let 1kg block has velocity v.

KE. $= (1/2)\,mv^2 = (1/2)\,mx^2$ where $x \rightarrow$ Amplitude = 0.1m.

$\therefore (1/2) \times (1 \times v^2) = (1/2) \times 100\,(0.1)^2$

$\Rightarrow v = 1$m/sec …(1)

After the 3kg block is gently placed on the 1kg, then let, 1kg +3kg = 4kg block and the spring be one system. For this mass spring system, there is so external force. (when oscillation takes place). The momentum should be conserved. Let, 4kg block has velocity v′.

$\therefore$ Initial momentum = Final momentum

$\therefore 1 \times v = 4 \times v' \Rightarrow v' = 1/4$ m/s (As v = 1m/s from equation (1))

Now the two blocks have velocity 1/4 m/s at its mean poison.

$KE_{mass} = (1/2)\,m'v'^2 = (1/2)\,4 \times (1/4)^2 = (1/2) \times (1/4).$

When the blocks are going to the extreme position, there will be only potential energy.

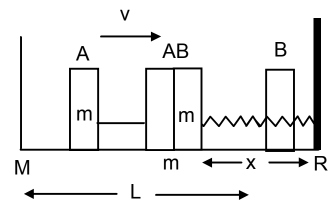
$\therefore PE = (1/2)\,k\delta^2 = (1/2) \times (1/4)$ where $\delta \rightarrow$ new amplitude.

$\therefore 1/4 = 100\,\delta^2 \Rightarrow \delta = \sqrt{\frac{1}{400}} = 0.05\text{m} = 5\text{cm}.$

So Amplitude = 5cm.

28. When the block A moves with velocity 'V' and collides with the block B, it transfers all energy to the block B. (Because it is a elastic collision). The block A will move a distance 'x' against the spring, again the block B will return to the original point and completes half of the oscillation.

So, the time period of B is $\dfrac{2\pi\sqrt{\dfrac{m}{k}}}{2} = \pi\sqrt{\dfrac{m}{k}}$



The block B collides with the block A and comes to rest at that point. The block A again moves a further distance 'L' to return to its original position.

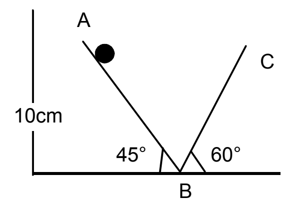
$\therefore$ Time taken by the block to move from M $\rightarrow$ N and N $\rightarrow$ M

is $\dfrac{L}{V} + \dfrac{L}{V} = 2\left(\dfrac{L}{V}\right)$

$\therefore$ So time period of the periodic motion is $2\left(\dfrac{L}{V}\right) + \pi\sqrt{\dfrac{m}{k}}$

29. Let the time taken to travel AB and BC be t_1 and t_2 respectively

Fro part AB, a_1 = g sin 45°. $s_1 = \dfrac{0.1}{\sin 45°} = 2m$



Let, v = velocity at B

$\therefore v^2 - u^2 = 2a_1 s_1$

$\Rightarrow v^2 = 2 \times g \sin 45° \times \dfrac{0.1}{\sin 45°} = 2$

$\Rightarrow v = \sqrt{2}$ m/s

$\therefore t_1 = \dfrac{v-u}{a_1} = \dfrac{\sqrt{2}-0}{\dfrac{g}{\sqrt{2}}} = \dfrac{2}{g} = \dfrac{2}{10} = 0.2$ sec

Again for part BC, $a_2 = -g \sin 60°$, $u = \sqrt{2}$, v = 0

$\therefore t_2 = \dfrac{0-\sqrt{2}}{-g\left(\dfrac{\sqrt{3}}{2}\right)} = \dfrac{2\sqrt{2}}{\sqrt{3}g} = \dfrac{2\times(1.414)}{(1.732)\times 10} = 0.165$sec.

So, time period = 2 ($t_1 + t_2$) = 2(0.2 + 0.155) = 0.71sec

30. Let the amplitude of oscillation of 'm' and 'M' be x_1 and x_2 respectively.

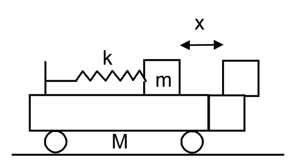
a) From law of conservation of momentum,

$mx_1 = Mx_2$ …(1) [because only internal forces are present]

Again, $(1/2)\, kx_0^2 = (1/2)\, k\, (x_1 + x_2)^2$

$\therefore x_0 = x_1 + x_2$ …(2)

[Block and mass oscillates in opposite direction. But x $\rightarrow$ stretched part]



From equation (1) and (2)

$\therefore x_0 = x_1 + \dfrac{m}{M} x_1 = \left(\dfrac{M+m}{M}\right) x_1$

$\therefore x_1 \dfrac{Mx_0}{M+m}$

So, $x_2 = x_0 - x_1 = x_0 \left[1 - \dfrac{M}{M+m}\right] = \dfrac{mx_0}{M+m}$ respectively.

b) At any position, let the velocities be v_1 and v_2 respectively.

Here, v_1 = velocity of 'm' with respect to M.

By energy method

Total Energy = Constant

$(1/2)\, Mv^2 + (1/2)\, m(v_1 - v_2)^2 + (1/2)\, k(x_1 + x_2)^2$ = Constant …(i)

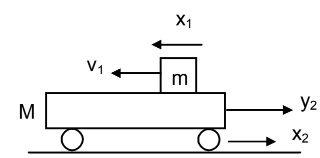
[$v_1 - v_2$ = Absolute velocity of mass 'm' as seen from the road.]

Again, from law of conservation of momentum,

$mx_2 = mx_1 \Rightarrow x_1 = \dfrac{M}{m} x_2$ ...(1)

$mv_2 = m(v_1 - v_2) \Rightarrow (v_1 - v_2) = \dfrac{M}{m} v_2$ …(2)

Putting the above values in equation (1), we get



$\dfrac{1}{2} M v_2^2 + \dfrac{1}{2} m \dfrac{M^2}{m^2} v_2^2 + \dfrac{1}{2} k x_2^2 \left(1 + \dfrac{M}{m}\right)^2 = \text{constant}$

$\therefore M\left(1 + \dfrac{M}{m}\right) v_2 + k\left(1 + \dfrac{M}{m}\right)^2 x_2^2 = \text{Constant.}$

$\Rightarrow m v_2^2 + k\left(1 + \dfrac{M}{m}\right) x_2^2 = \text{constant}$

Taking derivative of both sides,

$M \times 2v_2 \dfrac{dv_2}{dt} + k \dfrac{(M+m)}{m} - ex_2^2 \dfrac{dx_2}{dt} = 0$

$\Rightarrow ma_2 + k\left(\dfrac{M+m}{m}\right) x_2 = 0$ [because, $v_2 = \dfrac{dx_2}{dt}$]

$\Rightarrow \dfrac{a_2}{x_2} = -\dfrac{k(M+m)}{Mm} = \omega^2$

$\therefore \omega = \sqrt{\dfrac{k(M+m)}{Mm}}$

So, Time period, $T = 2\pi \sqrt{\dfrac{Mm}{k(M+m)}}$

31. Let 'x' be the displacement of the plank towards left. Now the centre of gravity is also displaced through 'x'
In displaced position
$R_1 + R_2 = mg.$
Taking moment about G, we get
$R_1(\ell/2 - x) = R_2(\ell/2 + x) = (mg - R_1)(\ell/2 + x)$ …(1)\
So, $R_1 (\ell/2 - x) = (mg - R_1)(\ell/2 + x)$

$\Rightarrow R_1 \dfrac{\ell}{2} - R_1 x = mg \dfrac{\ell}{2} - R_1 x + mgx - R_1 \dfrac{\ell}{2}$

$\Rightarrow R_1 \dfrac{\ell}{2} + R_1 \dfrac{\ell}{2} = mg\left(x + \dfrac{\ell}{2}\right)$

$\Rightarrow R_1 \left(\dfrac{\ell}{2} + \dfrac{\ell}{2}\right) = mg \left(\dfrac{2x + \ell}{2}\right)$

$\Rightarrow R_1 \ell = \dfrac{mg(2x + \ell)}{2}$

$\Rightarrow R_1 = \dfrac{mg(2x + \ell)}{2\ell}$ …(2)

Now $F_1 = \mu R_1 = \dfrac{\mu mg(\ell + 2x)}{2\ell}$

Similarly $F_2 = \mu R_2 = \dfrac{\mu mg(\ell - 2x)}{2\ell}$

Since, $F_1 > F_2$. $\Rightarrow F_1 - F_2 = ma = \dfrac{2\mu mg}{\ell} x$

$\Rightarrow \dfrac{a}{x} = \dfrac{2\mu g}{\ell} = \omega^2 \Rightarrow \omega = \sqrt{\dfrac{2\mu g}{\ell}}$

$\therefore$ Time period $= 2\pi \sqrt{\dfrac{\ell}{2rg}}$

32. T = 2sec.

$T = 2\pi\sqrt{\frac{\ell}{g}}$

$\Rightarrow 2 = 2\pi\sqrt{\frac{\ell}{10}} \Rightarrow \frac{\ell}{10} = \frac{1}{\pi^2} \Rightarrow \ell = 1\text{cm}$ $(\therefore \pi^2 \approx 10)$

33. From the equation,

$\theta = \pi \sin [\pi \text{ sec}^{-1} t]$

$\therefore \omega = \pi \text{ sec}^{-1}$ (comparing with the equation of SHM)

$\Rightarrow \frac{2\pi}{T} = \pi \Rightarrow T = 2$ sec.

We know that $T = 2\pi\sqrt{\frac{\ell}{g}}$ $\Rightarrow 2 = 2\sqrt{\frac{\ell}{g}}$ $\Rightarrow 1 = \sqrt{\frac{\ell}{g}}$ $\Rightarrow \ell = 1$m.

$\therefore$ Length of the pendulum is 1m.

34. The pendulum of the clock has time period 2.04sec.

Now, No. or oscillation in 1 day = $\frac{24 \times 3600}{2}$ = 43200

But, in each oscillation it is slower by (2.04 – 2.00) = 0.04sec.

So, in one day it is slower by,

= 43200 × (0.04) = 12 sec = 28.8 min

So, the clock runs 28.8 minutes slower in one day.

35. For the pendulum, $\frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}}$

Given that, T_1 = 2sec, g_1 = 9.8m/s^2

$T_2 = \frac{24 \times 3600}{\left(\frac{24 \times 3600 - 24}{2}\right)} = 2 \times \frac{3600}{3599}$

Now, $\frac{g^2}{g_1} = \left(\frac{T_1}{T_2}\right)^2$

$\therefore g_2 = (9.8)\left(\frac{3599}{3600}\right)^2 = 9.795\text{m/s}^2$

36. L = 5m.

a) $T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{0.5} = 2\pi(0.7)$

$\therefore$ In $2\pi(0.7)$sec, the body completes 1 oscillation,

In 1 second, the body will complete $\frac{1}{2\pi(0.7)}$ oscillation

$\therefore f = \frac{1}{2\pi(0.7)} = \frac{10}{14\pi} = \frac{0.70}{\pi}$ times

b) When it is taken to the moon

$T = 2\pi\sqrt{\frac{\ell}{g'}}$ where $g' \rightarrow$ Acceleration in the moon.

$= 2\pi\sqrt{\frac{5}{1.67}}$

$\therefore f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{1.67}{5}} = \frac{1}{2\pi}(0.577) = \frac{1}{2\pi\sqrt{3}}$ times.

37. The tension in the pendulum is maximum at the mean position and minimum on the extreme position.

Here $(1/2)\ mv^2 - 0 = mg\ \ell(1 - \cos\theta)$

$v^2 = 2g\ell(1 - \cos\theta)$

Now, $T_{max} = mg + 2\ mg\ (1 - \cos\theta)$ $\quad$ $[\ T = mg + (mv^2/\ell)]$

Again, $T_{min} = mg \cos\theta$.

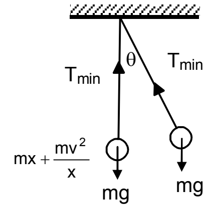
According to question, $T_{max} = 2T_{min}$

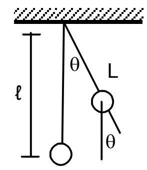
$\Rightarrow mg + 2mg - 2mg\cos\theta = 2mg\cos\theta$

$\Rightarrow 3mg = 4mg\cos\theta$

$\Rightarrow \cos\theta = 3/4$

$\Rightarrow \theta = \cos^{-1}(3/4)$





38. Given that, R = radius.

Let N = normal reaction.

Driving force F = mg sinθ.

Acceleration =a = g sin θ

As, sin θ is very small, $\sin\theta \to \theta$

$\therefore$Acceleration a = gθ

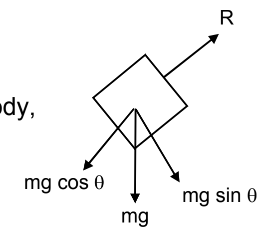
Let 'x' be the displacement from the mean position of the body,

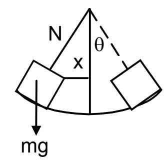
$\therefore\ \theta = x/R$

$\Rightarrow a = g\theta = g(x/R) \Rightarrow (a/x) = (g/R)$

So the body makes S.H.M.

$\therefore T = 2\pi\sqrt{\dfrac{\text{Displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\dfrac{x}{gx/R}} = 2\pi\sqrt{\dfrac{R}{g}}$





39. Let the angular velocity of the system about the point os suspension at any time be 'ω'

So, $v_c = (R - r)\omega$

Again $v_c = r\omega_1$ [where, ω_1 = rotational velocity of the sphere]

$\omega_1 = \dfrac{v_c}{r} = \left(\dfrac{R+-r}{r}\right)\omega$ $\quad$ …(1)

By Energy method, Total energy in SHM is constant.

So, $mg(R - r)(1 - \cos\theta) + (1/2)\ mv_c^2 + (1/2)\ I\omega_1^2$ = constant

$\therefore\ mg(R - r)\ (1 - \cos\theta) + (1/2)\ m(R - r)^2\ \omega^2 + (1/2)\ mr^2\left(\dfrac{R-r}{r}\right)^2\ \omega^2$ = constant

$\Rightarrow g(R - r)\ 1 - \cos\theta) + (R - r)^2\ \omega^2\left[\dfrac{1}{2}+\dfrac{1}{5}\right]$ = constant

Taking derivative, $g(R - r)\sin\theta\dfrac{d\theta}{dt} = \dfrac{7}{10}(R - r)^2 2\omega\dfrac{d\omega}{dt}$

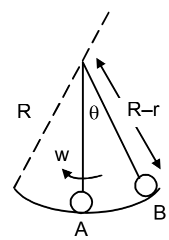
$\Rightarrow g\sin\theta = 2 \times \dfrac{7}{10}(R - r)\alpha$

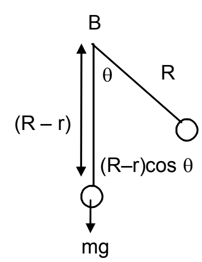
$\Rightarrow g\sin\theta = \dfrac{7}{5}\ (R - r)\alpha$

$\Rightarrow \alpha = \dfrac{5g\sin\theta}{7(R-r)} = \dfrac{5g\theta}{7(R-r)}$

$\therefore \dfrac{\alpha}{\theta} = \omega^2 = \dfrac{5g\theta}{7(R-r)}$ = constant

So the motion is S.H.M. Again $\omega = \omega\sqrt{\dfrac{5g}{7(R-r)}} \Rightarrow T = 2\pi\sqrt{\dfrac{7(R-r)}{5g}}$





40. Length of the pendulum = 40cm = 0.4m.

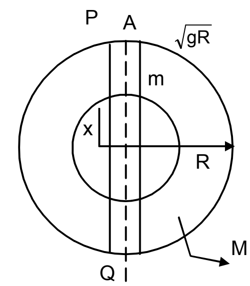
Let acceleration due to gravity be g at the depth of 1600km.

$\therefore gd = g(1\text{-}d/R) = 9.8\left(1-\dfrac{1600}{6400}\right) = 9.8\left(1-\dfrac{1}{4}\right) = 9.8 \times \dfrac{3}{4} = 7.35 m/s^2$

$\therefore$ Time period $T' = 2\pi \sqrt{\dfrac{\ell}{g\delta}}$

$= 2\pi\sqrt{\dfrac{0.4}{7.35}} = 2\pi \sqrt{0.054} = 2\pi \times 0.23 = 2 \times 3.14 \times 0.23 = 1.465 \approx 1.47$sec.

41. Let M be the total mass of the earth.
At any position x,



$\therefore \dfrac{M'}{M} = \dfrac{\rho \times \left(\frac{4}{3}\right)\pi \times x^3}{\rho \times \left(\frac{4}{3}\right)\pi \times R^3} = \dfrac{x^3}{R^3} \Rightarrow M' = \dfrac{Mx^3}{R^3}$

So force on the particle is given by,

$\therefore F_X = \dfrac{GM'm}{x^2} = \dfrac{GMm}{R^3}x$ …(1)

So, acceleration of the mass 'M' at that position is given by,

$a_x = \dfrac{GM}{R^2}x \Rightarrow \dfrac{a_x}{x} = w^2 = \dfrac{GM}{R^3} = \dfrac{g}{R}$ $\left(\because g = \dfrac{GM}{R^2}\right)$

So, $T = 2\pi\sqrt{\dfrac{R}{g}}$ = Time period of oscillation.

a) Now, using velocity – displacement equation.

$V = \omega \sqrt{(A^2 - R^2)}$ [Where, A = amplitude]

Given when, y = R, $v = \sqrt{gR}$, $\omega = \sqrt{\dfrac{g}{R}}$

$\Rightarrow \sqrt{gR} = \sqrt{\dfrac{g}{R}} \sqrt{(A^2 - R^2)}$ [because $\omega = \sqrt{\dfrac{g}{R}}$]

$\Rightarrow R^2 = A^2 - R^2 \Rightarrow A = \sqrt{2}\, R$

[Now, the phase of the particle at the point P is greater than $\pi/2$ but less than π and at Q is greater than π but less than $3\pi/2$. Let the times taken by the particle to reach the positions P and Q be t_1 & t_2 respectively, then using displacement time equation]

$y = r \sin \omega t$

We have, $R = \sqrt{2}\ R \sin \omega t_1$ $\Rightarrow \omega t_1 = 3\pi/4$

& $-R = \sqrt{2}\ R \sin \omega t_2$ $\Rightarrow \omega t_2 = 5\pi/4$

So, $\omega(t_2 - t_1) = \pi/2 \Rightarrow t_2 - t_1 = \dfrac{\pi}{2\omega} = \dfrac{\pi}{2\sqrt{(R/g)}}$

Time taken by the particle to travel from P to Q is $t_2 - t_1 = \dfrac{\pi}{2\sqrt{(R/g)}}$ sec.

b) When the body is dropped from a height R, then applying conservation of energy, change in P.E. = gain in K.E.

$\Rightarrow \dfrac{GMm}{R} - \dfrac{GMm}{2R} = \dfrac{1}{2}mv^2$ $\Rightarrow v = \sqrt{gR}$

Since, the velocity is same at P, as in part (a) the body will take same time to travel PQ.

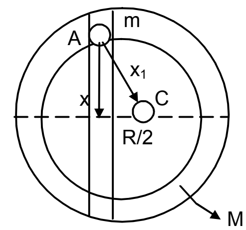
c) When the body is projected vertically upward from P with a velocity $\sqrt{gR}$, its velocity will be Zero at the highest point.

The velocity of the body, when reaches P, again will be $v = \sqrt{gR}$, hence, the body will take same time $\dfrac{\pi}{2\sqrt{(R/g)}}$ to travel PQ.

42. $M = 4/3\ \pi R^3 \rho$.

$M^1 = 4/3\ \pi x_1^3 \rho$

$M^1 = \left(\frac{M}{R^3}\right) x_1^3$

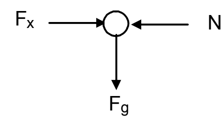


a) F = Gravitational force exerted by the earth on the particle of mass 'x' is,

$$F = \frac{GM^1 m}{x_1^2} = \frac{GMm}{R^3} \frac{x_1^3}{x_1^2} = \frac{GMm}{R^3} x_1 = \frac{GMm}{R^3} \sqrt{x^2 + \left(\frac{R^2}{4}\right)}$$

b) $F_y = F \cos\theta = \frac{GMmx_1}{R^3} \frac{x}{x_1} = \frac{GMmx}{R^3}$

$F_x = F \sin\theta = \frac{GMmx_1}{R^3} \frac{R}{2x_1} = \frac{GMm}{2R^2}$



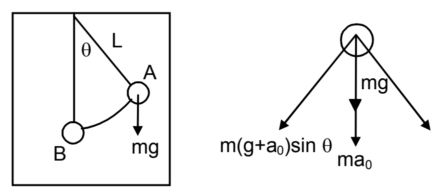
c) $F_x = \frac{GMm}{2R^2}$ [since Normal force exerted by the wall $N = F_x$]

d) Resultant force = $\frac{GMmx}{R^3}$

e) Acceleration = $\frac{\text{Driving force}}{\text{mass}} = \frac{GMmx}{R^3 m} = \frac{GMx}{R^3}$

So, $a \propto x$ (The body makes SHM)

$\therefore \frac{a}{x} = w^2 = \frac{GM}{R^3} \Rightarrow w = \sqrt{\frac{GM}{R^3}} \Rightarrow T = 2\pi \sqrt{\frac{R^3}{GM}}$



43. Here driving force $F = m(g + a_0) \sin\theta$ …(1)

Acceleration $a = \frac{F}{m} = (g + a_0) \sin\theta = \frac{(g + a_0)\, x}{\ell}$

(Because when θ is small $\sin\theta \to \theta = x/\ell$)

$\therefore a = \frac{(g + a_0)\, x}{\ell}$.

$\therefore$ acceleration is proportional to displacement.

So, the motion is SHM.

Now $\omega^2 = \frac{(g + a_0)}{\ell}$

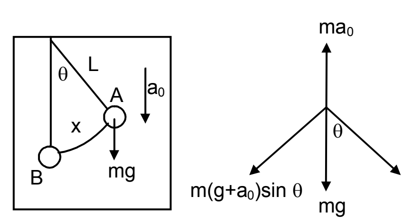
$\therefore T = 2\pi \sqrt{\frac{\ell}{g + a_0}}$

b) When the elevator is going downwards with acceleration a_0

Driving force = $F = m\,(g - a_0) \sin\theta$.

Acceleration = $(g - a_0) \sin\theta = \frac{(g - a_0)x}{\ell} = \omega^2 x$

$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{\ell}{g - a_0}}$



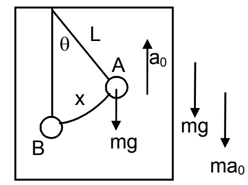
c) When moving with uniform velocity $a_0 = 0$.

For, the simple pendulum, driving force = $\frac{mgx}{\ell}$

$\Rightarrow a = \frac{gx}{\ell} \Rightarrow \frac{x}{a} = \frac{\ell}{g}$

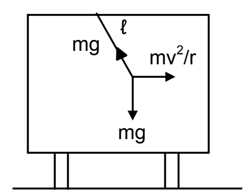
$T = 2\pi \sqrt{\frac{\text{displacement}}{\text{acceleration}}} = 2\pi \sqrt{\frac{\ell}{g}}$

44. Let the elevator be moving upward accelerating 'a_0'
Here driving force $F = m(g + a_0) \sin\theta$
Acceleration = $(g + a_0) \sin\theta$
$= (g + a_0)\theta$ $\quad$ $(\sin\theta \to \theta)$
$= \dfrac{(g+a_0)x}{\ell} = \omega^2 x$

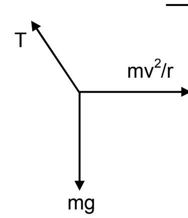


$T = 2\pi\sqrt{\dfrac{\ell}{g+a_0}}$
Given that, $T = \pi/3$ sec, ℓ = 1ft and g = 32 ft/sec^2
$\dfrac{\pi}{3} = 2\pi\sqrt{\dfrac{1}{32+a_0}}$
$\dfrac{1}{9} = 4\left(\dfrac{1}{32+a}\right)$
$\Rightarrow 32 + a = 36$ $\quad\Rightarrow a = 36 - 32 = 4$ ft/sec^2

45. When the car moving with uniform velocity
$T = 2\pi\sqrt{\dfrac{\ell}{g}} \Rightarrow 4 = 2\pi\sqrt{\dfrac{\ell}{g}}$ $\quad$ …(1)
When the car makes accelerated motion, let the acceleration be a_0
$T = 2\pi\sqrt{\dfrac{\ell}{\sqrt{g^2 + a_0^{\,2}}}}$
$\Rightarrow 3.99 = 2\pi\sqrt{\dfrac{\ell}{\sqrt{g^2 + a_0^{\,2}}}}$
Now $\dfrac{T}{T'} = \dfrac{4}{3.99} = \dfrac{(g^2 + a_0^{\,2})^{1/4}}{\sqrt{g}}$
Solving for 'a_0' we can get $a_0 = g/10$ ms^{-2}

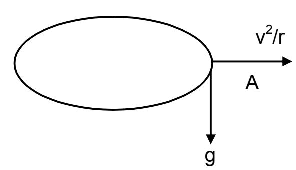


46. From the freebody diagram,
$T = \sqrt{(mg)^2 + \left(\dfrac{mv^2}{r^2}\right)}$
$= m\sqrt{g^2 + \dfrac{v^4}{r^2}} = ma$, where a = acceleration = $\left(g^2 + \dfrac{v^4}{r^2}\right)^{1/2}$



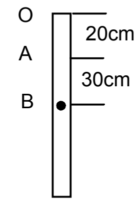
The time period of small accellations is given by,
$T = 2\pi\sqrt{\dfrac{\ell}{g}} = 2\pi\sqrt{\dfrac{\ell}{\left(g^2 + \dfrac{v^4}{r^2}\right)^{1/2}}}$

47. a) ℓ = 3cm = 0.03m.
$T = 2\pi\sqrt{\dfrac{\ell}{g}} = 2\pi\sqrt{\dfrac{0.03}{9.8}} = 0.34$ second.



b) When the lady sets on the Merry-go-round the ear rings also experience centrepetal acceleration
$a = \dfrac{v^2}{r} = \dfrac{4^2}{2} = 8$ m/s^2
Resultant Acceleration $A = \sqrt{g^2 + a^2} = \sqrt{100 + 64} = 12.8$ m/s^2
Time period $T = 2\pi\sqrt{\dfrac{\ell}{A}} = 2\pi\sqrt{\dfrac{0.03}{12.8}} = 0.30$ second.

48. a) M.I. about the pt A = I = $I_{C.G.}$ + Mh^2

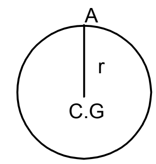


$= \frac{m\ell^2}{12} + MH^2 = \frac{m\ell^2}{12} + m\,(0.3)^2 = M\left(\frac{1}{12} + 0.09\right) = M\left(\frac{1+1.08}{12}\right) = M\left(\frac{2.08}{12}\right)$

$\therefore\ T = 2\pi\sqrt{\frac{I}{mg\ell'}} = 2\pi\sqrt{\frac{2.08m}{m \times 9.8 \times 0.3}}$ (ℓ' = dis. between C.G. and pt. of suspension)

$\approx$ 1.52 sec.

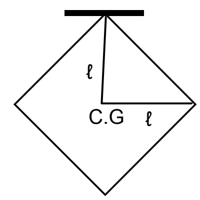
b) Moment of in isertia about A



$I = I_{C.G.} + mr^2 = mr^2 + mr^2 = 2\,mr^2$

$\therefore$ Time period $= 2\pi\sqrt{\frac{I}{mg\ell}} = 2\pi\sqrt{\frac{2mr^2}{mgr}} = 2\pi\sqrt{\frac{2r}{g}}$

c) I_{ZZ} (corner) $= m\left(\frac{a^2 + a^2}{3}\right) = \frac{2ma^2}{3}$



In the ΔABC, $\ell^2 + \ell^2 = a^2$

$\therefore\ \ell = \frac{a}{\sqrt{2}}$

$\therefore\ T = 2\pi\sqrt{\frac{I}{mg\ell}} = 2\pi\sqrt{\frac{2ma^2}{3mg\ell}} = 2\pi\sqrt{\frac{2a^2}{3ga\sqrt{2}}} = 2\pi\sqrt{\frac{\sqrt{8}a}{3g}}$

d) h = r/2, ℓ = r/2 = Dist. Between C.G and suspension point.

M.I. about A, $I = I_{C.G.} + Mh^2 = \frac{mc^2}{2} + n\left(\frac{r}{2}\right)^2 = mr^2\left(\frac{1}{2} + \frac{1}{4}\right) = \frac{3}{4}mr^2$

$\therefore\ T = 2\pi\sqrt{\frac{I}{mg\ell}} = 2\pi\sqrt{\frac{3mr^2}{4mg\ell}} = 2\pi\sqrt{\frac{3r^2}{4g\left(\frac{r}{2}\right)}} = 2\pi\sqrt{\frac{3r}{2g}}$

49. Let A $\rightarrow$ suspension of point.
B $\rightarrow$ Centre of Gravity.
$\ell' = \ell/2$, $h = \ell/2$
Moment of inertia about A is

$I = I_{C.G.} + mh^2 = \frac{m\ell^2}{12} + \frac{m\ell^2}{4} = \frac{m\ell^2}{3}$

$\Rightarrow T = 2\pi\sqrt{\frac{I}{mg\left(\frac{\ell}{2}\right)}} = 2\pi\sqrt{\frac{2m\ell^2}{3mgl}} = 2\pi\sqrt{\frac{2\ell}{3g}}$

Let, the time period 'T' is equal to the time period of simple pendulum of length 'x'.

$\therefore\ T = 2\pi\sqrt{\frac{x}{g}}$. So, $\frac{2\ell}{3g} = \frac{x}{g} \Rightarrow x = \frac{2\ell}{3}$

$\therefore$ Length of the simple pendulum $= \frac{2\ell}{3}$

50. Suppose that the point is 'x' distance from C.G.
Let m = mass of the disc., Radius = r
Here $\ell = x$
M.I. about A = $I_{C.G.} + mx^2 = mr^2/2 + mx^2 = m(r^2/2 + x^2)$

$T = 2\pi\sqrt{\frac{I}{mg\ell}} = 2\pi\sqrt{\frac{m\left(\frac{r^2}{2} + x^2\right)}{mgx}} = 2\pi\sqrt{\frac{m\left(r^2 + 2x^2\right)}{2mgx}} = 2\pi\sqrt{\frac{r^2 + 2x^2}{2gx}}$ …(1)

For T is minimum $\frac{dt^2}{dx}$ =0

$\therefore\ \frac{d}{dx}T^2 = \frac{d}{dx}\left(\frac{4\pi^2r^2}{2gx} + \frac{4\pi^2 2x^2}{2gx}\right)$

$\Rightarrow \frac{2\pi^2r^2}{g}\left(-\frac{1}{x^2}\right) + \frac{4\pi^2}{g} = 0$

$\Rightarrow -\frac{\pi^2r^2}{gx^2} + \frac{2\pi^2}{g} = 0$

$\Rightarrow \frac{\pi^2r^2}{gx^2} = \frac{2\pi^2}{g} \Rightarrow 2x^2 = r^2 \Rightarrow x = \frac{r}{\sqrt{2}}$

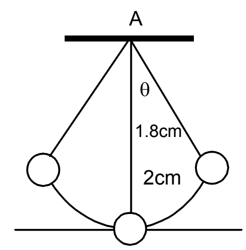
So putting the value of equation (1)

$T = 2\pi\sqrt{\frac{r^2 + 2\left(\frac{r^2}{2}\right)}{2gx}} = 2\pi\sqrt{\frac{2r^2}{2gx}} = 2\pi\sqrt{\frac{r^2}{g\left(\frac{r}{\sqrt{2}}\right)}} = 2\pi\sqrt{\frac{\sqrt{2}\,r^2}{gr}} = 2\pi\sqrt{\frac{\sqrt{2}r}{g}}$

51. According to Energy equation,
$mg\ell\,(1 - \cos\theta) + (1/2)\,I\omega^2 = \text{const.}$
$mg(0.2)\,(1 - \cos\theta) + (1/2)\,I\omega^2 = C.$ (I)
Again, $I = 2/3\ m(0.2)^2 + m(0.2)^2$

$= m\left[\frac{0.008}{3} + 0.04\right]$

$= m\left(\frac{0.1208}{3}\right)$m. Where I $\to$ Moment of Inertia about the pt of suspension A



From equation
Differenting and putting the value of I and 1 is

$\frac{d}{dt}\left[mg(0.2)(1-\cos\theta) + \frac{1}{2}\frac{0.1208}{3}m\omega^2\right] = \frac{d}{dt}(C)$

$\Rightarrow mg\ (0.2)\ \sin\theta\ \frac{d\theta}{dt} + \frac{1}{2}\left(\frac{0.1208}{3}\right)m2\omega\frac{d\omega}{dt} = 0$

$\Rightarrow 2\sin\theta = \frac{0.1208}{3}\alpha$ [because, $g = 10m/s^2$]

$\Rightarrow \frac{\alpha}{\theta} = \frac{6}{0.1208} = \omega^2 = 58.36$

$\Rightarrow \omega = 7.3$. So $T = \frac{2\pi}{\omega} = 0.89$sec.

For simple pendulum $T = 2\pi\sqrt{\frac{0.19}{10}} = 0.86$sec.

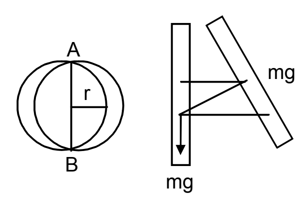
% more = $\frac{0.89 - 0.86}{0.89} = 0.3$.

$\therefore$ It is about 0.3% larger than the calculated value.

52. (For a compound pendulum)

a) $T = 2\pi\sqrt{\frac{I}{mg\ell}} = 2\pi\sqrt{\frac{I}{mgr}}$

The MI of the circular wire about the point of suspension is given by
$\therefore\ I = mr^2 + mr^2 = 2\ mr^2$ is Moment of inertia about A.



$\therefore 2 = 2\pi \sqrt{\dfrac{2mr^2 mgr}{}} = 2\pi \sqrt{\dfrac{2r}{g}}$

$\Rightarrow \dfrac{2r}{g} = \dfrac{1}{\pi^2} \Rightarrow r = \dfrac{g}{2\pi^2} = 0.5\pi = 50\text{cm. (Ans)}$

b) $(1/2)\ \omega^2 - 0 = mgr\ (1 - \cos\theta)$

$\Rightarrow (1/2)\ 2mr^2 - \omega^2 = mgr\ (1 - \cos 2°)$

$\Rightarrow \omega^2 = g/r\ (1 - \cos 2°)$

$\Rightarrow \omega = 0.11$ rad/sec [putting the values of g and r]

$\Rightarrow v = \omega \times 2r = 11$ cm/sec.

c) Acceleration at the end position will be centripetal.

$= a_n = \omega^2\ (2r) = (0.11)^2 \times 100 = 1.2\ \text{cm/s}^2$

The direction of 'a_n' is towards the point of suspension.

d) At the extreme position the centrepetal acceleration will be zero. But, the particle will still have acceleration due to the SHM.

Because, T = 2 sec.

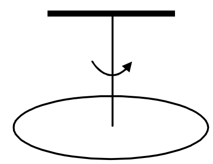
Angular frequency $\omega = \dfrac{2\pi}{T}$ (π= 3.14)

So, angular acceleration at the extreme position,

$\alpha = \omega^2\theta = \pi^2 \times \dfrac{2\pi}{180} = \dfrac{2\pi^3}{180}$ [$1° = \dfrac{\pi}{180}$ radious]

So, tangential acceleration = $\alpha\ (2r) = \dfrac{2\pi^3}{180} \times 100 = 34\ \text{cm/s}^2$.

53. M.I. of the centre of the disc. = $mr^2/2$



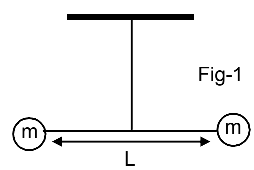
$T = 2\pi \sqrt{\dfrac{I}{k}} = 2\pi \sqrt{\dfrac{mr^2}{2K}}$ [where K = Torsional constant]

$T^2 = 4\pi^2 \dfrac{mr^2}{2K} = 2\pi^2 \dfrac{mr^2}{K}$

$\Rightarrow 2\pi^2\ mr^2 = KT^2 \quad \Rightarrow K = \dfrac{2mr^2\pi^2}{T^2}$

$\therefore$ Torsional constant $K = \dfrac{2mr^2\pi^2}{T^2}$

54. The M.I of the two ball system



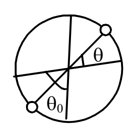
$I = 2m\ (L/2)^2 = m\ L^2/2$

At any position θ during the oscillation, [fig-2]

Torque = $k\theta$

So, work done during the displacement 0 to θ_0,

$W = \int_0^{\theta} k\theta d\theta = k\ \theta_0^2/2$



By work energy method,

$(1/2)\ I\omega^2 - 0 = \text{Work done} = k\ \theta_0^2/2$

$\therefore \omega^2 = \dfrac{k\theta_0^2}{2I} = \dfrac{k\theta_0^2}{mL^2}$

Now, from the freebody diagram of the rod,

$T_2 = \sqrt{(m\omega^2 L)^2 + (mg)^2}$

$= \sqrt{\left(m\dfrac{k\theta_0^2}{mL^2} \times L\right)^2 + m^2g^2} = \dfrac{k^2\theta_0^4}{L^2} + m^2g^2$

55. The particle is subjected to two SHMs of same time period in the same direction/
Given, r_1 = 3cm, r_2 = 4cm and ϕ = phase difference.

Resultant amplitude = R = $\sqrt{r_1^2 + r_2^2 + 2r_1r_2\cos\phi}$

a) When ϕ = 0°,

R = $\sqrt{(3^2 + 4^2 + 2\times 3\times 4\cos 0°}$ = 7 cm

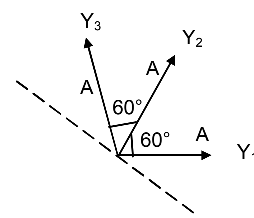
b) When ϕ = 60°

R = $\sqrt{(3^2 + 4^2 + 2\times 3\times 4\cos 60°}$ = 6.1 cm

c) When $\phi = 90°$

R = $\sqrt{(3^2 + 4^2 + 2\times 3\times 4\cos 90°}$ = 5 cm

56. Three SHMs of equal amplitudes 'A' and equal time periods in the same dirction combine.
The vectors representing the three SHMs are shown it the figure.



Using vector method,
Resultant amplitude = Vector sum of the three vectors
= A + A cos 60° + A cso 60° = A + A/2 + A/2 = 2A
So the amplitude of the resultant motion is 2A.

57. $x_1 = 2\sin 100\pi t$
$x_2 = w\sin(120\pi t + \pi/3)$
So, resultant displacement is given by,
$x = x_1 + x_2 = 2[\sin(100\pi t) + \sin(120\pi t + \pi/3)]$

a) At t = 0.0125s,
$x = 2[\sin(100\pi \times 0.0125) + \sin(120\pi \times 0.0125 + \pi/3)]$
$= 2[\sin 5\pi/4 + \sin(3\pi/2 + \pi/3)]$
= 2 [(–0.707) + (–0.5)] = – 2.41cm.

b) At t = 0.025s.
$x = 2[\sin(100\pi \times 0.025) + \sin(120\pi \times 0.025 + \pi/3)]$
$= 2[\sin 5\pi/2 + \sin(3\pi + \pi/3)]$
=2[1+(–0.8666)] = 0.27 cm.

58. The particle is subjected to two simple harmonic motions represented by,
$x = x_0 \sin wt$
$s = s_0 \sin wt$
and, angle between two motions = θ = 45°
$\therefore$Resultant motion will be given by,

R = $\sqrt{(x^2 + s^2 + 2xs\cos 45°)}$

= $\sqrt{\{x_0^2 \sin^2 wt + s_0^2 \sin^2 wt + 2x_0s_0 \sin^2 wt x(1/\sqrt{2})\}}$

= $[x_0^2 + s_0^2 = \sqrt{2}\, x_0s_0]^{1/2} \sin wt$

$\therefore$ Resultant amplitude = $[x_0^2 + s_0^2 = \sqrt{2}\, x_0s_0]^{1/2}$

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