

Let p denote the probability of having defective item, so

$$p = 6\% = \frac{6}{100} = \frac{3}{50}$$

So, $q = 1 - p$

$$= 1 - \frac{3}{50} \quad [\text{Since } p + q = 1]$$

$$= \frac{47}{50}$$

Let X denote the number of defective items in a sample of 8 items. Then, the probability of getting r defective bulks is

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^8C_r \left(\frac{3}{50}\right)^r \left(\frac{47}{50}\right)^{8-r} \quad \dots (1)$$

Therefore, probability of getting not more than one defective item

$$= P(X = 0) + P(X = 1)$$

$$= {}^8C_0 \left(\frac{3}{50}\right)^0 \left(\frac{47}{50}\right)^{8-0} + {}^8C_1 \left(\frac{3}{50}\right)^1 \left(\frac{47}{50}\right)^{8-1} \quad [\text{Using equation (1)}]$$

$$= 1.1 \cdot \left(\frac{47}{50}\right)^8 + 8 \cdot \frac{3}{50} \cdot \left(\frac{47}{50}\right)^7$$

$$= \left(\frac{47}{50}\right)^7 \left(\frac{47}{50} + \frac{24}{50}\right)$$

$$= \left(\frac{71}{50}\right) \left(\frac{47}{50}\right)^7$$

$$= (1.42) \times (0.94)^7$$

The required probability is,

$$(1.42) \times (0.94)^7$$

Probability of getting head on one throw of coin = $\frac{1}{2}$

So, $p = \frac{1}{2}$

$$q = 1 - \frac{1}{2}$$

$$q = \frac{1}{2} \quad [\text{Since } p + q = 1]$$

The coin is tossed 5 times. Let X denote the number of getting head as 5 tosses of coins.

So probability of getting r heads in n tosses of coin is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r} \quad \dots (1)$$

Probability of getting at least 3 heads

$$= P(X = 3) + P(X = 4) + P(X = 5)$$

$$= {}^5C_3 \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^{5-3} + {}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{5-4} + {}^5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0 \quad [\text{Using (1)}]$$

$$= {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 + {}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right) + {}^5C_5 \left(\frac{1}{2}\right)^5 \cdot 1$$

$$= \frac{5 \cdot 4}{2} \cdot \left(\frac{1}{2}\right)^5 + 5 \left(\frac{1}{2}\right)^5 + 1 \cdot \left(\frac{1}{2}\right)^5$$

$$= \left(\frac{1}{2}\right)^5 [10 + 5 + 1]$$

$$= 16 \cdot \frac{1}{32}$$

$$= \frac{1}{2}$$

The required probability is = $\frac{1}{2}$

Binomial Distribution Ex 33.1 Q3

Let p be the probability getting tail on a toss of a fair coin, so

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2}$$

$$q = \frac{1}{2} \quad [\text{Since } p + q = 1]$$

Let X denote the number tail obtained on the toss of coin 5 times. So probability of getting r tails in n tosses of coin is given by

$$\begin{aligned} P(X = r) &= {}^nC_r p^r q^{n-r} \\ &= {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r} \quad \dots (1) \end{aligned}$$

Probability of getting tail an odd number of times

$$\begin{aligned} &= P(X = 1) + P(X = 3) + P(X = 5) \\ &= {}^5C_3 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{5-1} + {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3} + {}^5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0 \quad [\text{Using (1)}] \\ &= 5 \cdot \left(\frac{1}{2}\right)^5 + \frac{5 \cdot 4}{2} \cdot \left(\frac{1}{2}\right)^5 + 1 \cdot \left(\frac{1}{2}\right)^5 \\ &= \left(\frac{1}{2}\right)^5 [5 + 10 + 1] \\ &= 16 \left(\frac{1}{2}\right)^5 \\ &= 16 \cdot \frac{1}{32} \\ &= \frac{1}{2} \end{aligned}$$

The required probability is $= \frac{1}{2}$

Binomial Distribution Ex 33.1 Q4

Let p be the probability of getting a sum of 9 and it considered as success.

Sum of a 9 on a pair of dice

$$= \{(3, 6), (4, 5), (5, 4), (6, 3)\}$$

$$\text{So, } p = \frac{4}{36}$$

$$p = \frac{1}{9}$$

$$q = 1 - \frac{1}{9}$$

$$q = \frac{8}{9}$$

$$[\text{Since } p + q = 1]$$

Let X denote the number of success in throw of a pair of dice 6 times. So probability of getting r success out of n is given by

$$P(X = r) = {}^nC_r p^r q^{n-r} \quad \text{--- (1)}$$

Probability of getting at least 5 success

$$= P(X = 5) + P(X = 6)$$

$$= {}^6C_5 \left(\frac{1}{9}\right)^5 \left(\frac{8}{9}\right)^{6-5} + {}^6C_6 \left(\frac{1}{9}\right)^6 \left(\frac{8}{9}\right)^{6-6} \quad [\text{Using (1)}]$$

$$= 6 \left(\frac{1}{9}\right)^5 \left(\frac{8}{9}\right)^1 + 1 \cdot \left(\frac{1}{9}\right)^6 \left(\frac{8}{9}\right)^0$$

$$= \left(\frac{1}{9}\right)^5 \left[\frac{48}{9} + \frac{1}{9} \right]$$

$$= \frac{49}{9} \times \left(\frac{1}{9}\right)^5$$

$$= \frac{49}{9^6}$$

So,

$$\text{Required probability} = \frac{49}{9^6}$$

Let p be the probability of getting head in a throw of coin. So,

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2}$$

$$q = \frac{1}{2} \quad [\text{Since } p + q = 1]$$

Let X denote the number of heads on tossing the coin 6 times. Probability of getting r in tossing the coin n times is given by

$$P(X = r) = {}^nC_r p^r q^{n-r} \quad \text{--- (1)}$$

Probability of getting at least three heads

$$= P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6)$$

$$= 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$$

$$= 1 - \left[{}^6C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{6-0} + {}^6C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{6-1} + {}^6C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{6-2} \right] \quad [\text{Using (1)}]$$

$$= 1 - \left[1 \cdot \left(\frac{1}{2}\right)^6 + 6 \left(\frac{1}{2}\right)^6 + \frac{6 \cdot 5}{2} \cdot \left(\frac{1}{2}\right)^6 \right]$$

$$= 1 - \left[\left(\frac{1}{2}\right)^6 (1 + 6 + 15) \right]$$

$$= 1 - \left[\frac{22}{64} \right]$$

$$= \frac{64 - 22}{64}$$

$$= \frac{42}{64}$$

$$= \frac{21}{32}$$

$$\text{Required probability} = \frac{21}{32}$$

Binomial Distribution Ex 33.1 Q6

Let p denote the 4 turning up in a toss of a fair die, so

$$p = \frac{1}{6}$$

$$q = 1 - \frac{1}{6}$$

$$q = \frac{5}{6} \quad [\text{Since } p + q = 1]$$

Let X denote the variable showing the number of turning 4 up in 2 tosses of die.

Probability of getting 4, r times in n tosses of a die is given by

$$\begin{aligned} P(X = r) &= {}^nC_r p^r q^{n-r} \\ &= {}^2C_r \left(\frac{1}{6}\right)^r \left(\frac{5}{6}\right)^{2-r} \quad \text{--- (1)} \end{aligned}$$

Probability of getting 4 at least once in tow tosses of a fair die

$$\begin{aligned} &= P(X = 1) + P(X = 2) \\ &= 1 - P(X = 0) \\ &= 1 - \left[{}^2C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{2-0} \right] \quad [\text{Using (1)}] \\ &= 1 - \left[1 \cdot 1 \cdot \left(\frac{5}{6}\right)^2 \right] \\ &= 1 - \left[\frac{25}{36} \right] \\ &= \frac{36 - 25}{36} \\ &= \frac{11}{36} \end{aligned}$$

So,

$$\text{Required probability} = \frac{11}{36}$$

Binomial Distribution Ex 33.1 Q7

Let p denote the probability of getting head in a toss of fair coin. So

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2}$$

$$q = \frac{1}{2} \quad [\text{Since } p + q = 1]$$

Let X denote the variable representing number of heads on 5 tosses of a fair coin.

Probability of getting r an n tosses of a fair coin, so

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r} \quad \text{--- (1)}$$

Probability of getting head on an even number of tosses of coin

$$= P(X = 0) + P(X = 2) + P(X = 4)$$

$$= {}^5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{5-0} + {}^5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{5-2} + {}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{5-4} \quad [\text{Using (1)}]$$

$$= 1 \cdot 1 \cdot \left(\frac{1}{2}\right)^5 + \frac{5 \cdot 4}{2} \cdot \left(\frac{1}{2}\right)^5 + 5 \cdot \left(\frac{1}{2}\right)^5$$

$$= \left(\frac{1}{2}\right)^5 [1 + 10 + 5]$$

$$= 16 \times \frac{1}{32}$$

$$= \frac{1}{2}$$

$$\text{Required probability} = \frac{1}{2}$$

Binomial Distribution Ex 33.1 Q8

Let p be the probability of hitting the target, so

$$p = \frac{1}{4}$$

$$q = 1 - p \quad [\text{Since } p + q = 1]$$

$$= 1 - \frac{1}{4}$$

$$q = \frac{3}{4}$$

Let X denote the variable representing the number of times hitting the target out of 7 fires. Probability of hitting the target r times out of n fires is given by,

$$\begin{aligned} P(X = r) &= {}^nC_r p^r q^{n-r} \\ &= {}^7C_r \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{7-r} \quad \text{--- (1)} \end{aligned}$$

Probability of hitting the target at least twice

$$\begin{aligned} &= P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7) \\ &= 1 - [P(X = 0) + P(X = 1)] \\ &= 1 - \left[{}^7C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{7-0} + {}^7C_1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^{7-1} \right] \quad [\text{Using (1)}] \\ &= 1 - \left[1 \cdot 1 \cdot \left(\frac{3}{4}\right)^7 + 7 \cdot \frac{1}{4} \cdot \left(\frac{3}{4}\right)^6 \right] \\ &= 1 - \left(\frac{3}{4}\right)^6 \left(\frac{3}{4} + \frac{7}{4}\right) \\ &= 1 - \left(\frac{3}{4}\right)^6 \left(\frac{10}{4}\right) \\ &= 1 - \frac{7290}{16384} \\ &= \frac{9194}{16384} \\ &= \frac{4547}{8192} \end{aligned}$$

Binomial Distribution Ex 33.1 Q9

Let the probability of one telephone number out of 15 is busy between 2 PM and 3 PM be 'p'. then

$P = 1/15$; probability that number is not busy, $q = 1-p$

$Q = 14/15$. Binomial distribution is $\left(\frac{14}{15} + \frac{1}{15}\right)^6$

Since 6 numbers are called we find the probability for none of the numbers are busy is $P(0)$

One number is busy $P(1)$; Two numbers are busy is $P(2)$

Three numbers are busy is $P(3)$; Four numbers are busy is $P(4)$; Five numbers are busy is $P(5)$; Six numbers are busy is $P(6)$.

$$P(0) = {}^6C_0 \left(\frac{14}{15}\right)^6$$

$$P(1) = {}^6C_1 \left(\frac{14}{15}\right)^5 \left(\frac{1}{15}\right)^1$$

$$P(2) = {}^6C_2 \left(\frac{14}{15}\right)^4 \left(\frac{1}{15}\right)^2$$

$$P(3) = {}^6C_3 \left(\frac{14}{15}\right)^3 \left(\frac{1}{15}\right)^3$$

$$P(4) = {}^6C_4 \left(\frac{14}{15}\right)^2 \left(\frac{1}{15}\right)^4$$

$$P(5) = {}^6C_5 \left(\frac{14}{15}\right)^1 \left(\frac{1}{15}\right)^5$$

$$P(6) = {}^6C_6 \left(\frac{14}{15}\right)^0 \left(\frac{1}{15}\right)^6$$

Probability that at least 3 of the numbers will be busy

$$P(3) + P(4) + P(5) + P(6) = 0.05$$

Binomial Distribution Ex 33.1 Q10

p denote the probability of success

p = Probability of getting 5 or 6 in a throw of die.

$$= \frac{2}{6}$$

$$p = \frac{1}{3}$$

$$q = 1 - \frac{1}{3} \quad [\text{Since } p + q = 1]$$

$$q = \frac{2}{3}$$

Let X denote the number of success in six throws of a die. Probability of getting r success in six throws of an unbiased die is given by

$$\begin{aligned} P(X = r) &= {}^nC_r p^r q^{n-r} \\ &= {}^6C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{6-r} \quad \text{--- (1)} \end{aligned}$$

$$P(X \geq 4)$$

$$= P(X = 4) + P(X = 5) + P(X = 6)$$

$$= {}^6C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^{6-4} + {}^6C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^{6-5} + {}^6C_6 \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^{6-6}$$

$$= \frac{6 \cdot 5}{2} \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^2 + 6 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right) + 1 \cdot \left(\frac{1}{3}\right)^6 \cdot 1$$

$$= 15 \cdot \frac{1}{81} \cdot \frac{4}{9} + 6 \cdot \frac{1}{243} \cdot \frac{2}{3} + \frac{1}{729}$$

$$= \frac{60}{729} + \frac{12}{729} + \frac{1}{729}$$

$$= \frac{73}{729}$$

$$\text{Required probability} = \frac{73}{729}$$

Binomial Distribution Ex 33.1 Q11

Let p denote the probability of getting head on a throw of fair coin, so

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2} \quad \text{[Since } p + q = 1\text{]}$$

$$q = \frac{1}{2}$$

Let X denote the variable representing the number of getting heads on throw of 8 coins.

Probability of getting r heads in a throw of n coins is given by

$$\begin{aligned} P(X = r) &= {}^nC_r p^r q^{n-r} \\ &= {}^8C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{8-r} \quad \text{--- (1)} \end{aligned}$$

Probability of getting at least six heads

$$\begin{aligned} &= P(X = 6) + P(X = 7) + P(X = 8) \\ &= {}^8C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^{8-6} + {}^8C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^{8-7} + {}^8C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^{8-8} \quad \text{[Using (1)]} \\ &= \frac{8 \cdot 7}{2} \left(\frac{1}{2}\right)^8 + 8 \left(\frac{1}{2}\right)^8 + 1 \cdot \left(\frac{1}{2}\right)^8 \cdot 1 \\ &= \left(\frac{1}{2}\right)^8 [28 + 8 + 1] \\ &= \frac{1}{256} (37) \\ &= \frac{37}{256} \end{aligned}$$

$$\text{Required probability} = \frac{37}{256}$$

Binomial Distribution Ex 33.1 Q12

Let p denote the probability of getting one spade out of a deck of 52 cards, so

$$p = \frac{13}{52}$$

$$p = \frac{1}{4}$$

$$q = 1 - \frac{1}{4} \quad \text{[Since } p + q = 1\text{]}$$

$$q = \frac{3}{4}$$

Let X denote the random variable of number of spades out of 5 cards. Probability of getting r spades out of n cards is given by

$$\begin{aligned} P(X = r) &= {}^nC_r p^r q^{n-r} \\ &= {}^5C_r \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{5-r} \end{aligned} \quad \text{--- (1)}$$

(i)

Probability of getting all five spades

$$\begin{aligned} &= P(X = 5) \\ &= {}^5C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^{5-5} \\ &= \frac{1}{1024} \end{aligned}$$

$$\text{Probability of getting 5 spades} = \frac{1}{1024}$$

(ii)

Probability of getting only 3 spades

$$\begin{aligned} &= P(X = 3) \\ &= {}^5C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^{5-3} \\ &= \frac{5 \cdot 4}{2} \left(\frac{1}{64}\right) \left(\frac{9}{16}\right) \\ &= \frac{45}{512} \end{aligned}$$

$$\text{Probability of getting 3 spades} = \frac{45}{512}$$

(iii)

Probability that none is spade

$$\begin{aligned} &= P(X = 0) \\ &= {}^5C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{5-0} \\ &= \frac{243}{1024} \end{aligned}$$

$$\text{Probability of getting non spade} = \frac{243}{1024}$$

Binomial Distribution Ex 33.1 Q13

Let p be the probability of getting 1 white ball out of 7 red, 5 white and 8 black balls. So

$$p = \frac{5}{20}$$

$$p = \frac{1}{4}$$

$$q = 1 - \frac{1}{4}$$

$$[\text{Since } p + q = 1]$$

$$q = \frac{3}{4}$$

Let X denote the random variable of number of selecting white ball with replacement out of 4 balls. Probability of getting r white balls out of n balls is given by

$$\begin{aligned} P(X = r) &= {}^nC_r p^r q^{n-r} \\ &= {}^4C_r \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{4-r} \quad \text{--- (1)} \end{aligned}$$

(i)

Probability of getting none white ball

$$\begin{aligned} &= P(X = 0) \\ &= {}^4C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{4-0} \quad [\text{Using (1)}] \\ &= \left(\frac{3}{4}\right)^4 \\ &= \frac{81}{256} \end{aligned}$$

$$\text{Probability of getting none white ball} = \frac{81}{256}$$

(ii)

Probability of getting all white balls

$$\begin{aligned} &= P(X = 4) \\ &= {}^4C_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^{4-0} \\ &= \left(\frac{1}{4}\right)^4 \\ &= \frac{1}{256} \end{aligned}$$

$$\text{Probability of getting all white balls} = \frac{1}{256}$$

(iii)

Probability of getting any two are white

$$\begin{aligned} &= P(X = 2) \\ &= {}^4C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^{4-2} \\ &= \frac{4 \cdot 3}{2} \cdot \frac{1}{16} \cdot \frac{9}{16} \\ &= \frac{27}{128} \end{aligned}$$

$$\text{Probability of getting any two are white balls} = \frac{27}{128}$$

Binomial Distribution Ex 33.1 Q14

Let p denote the probability of getting a ticket bearing number divisible by 10, So

$$p = \frac{10}{100} \quad \left[\begin{array}{l} \text{Since there are 10, 20, 30, 40, 50, 60, 70, 80,} \\ \text{90, 100 which are divisible by 10} \end{array} \right]$$

$$p = \frac{1}{10}$$

$$q = 1 - \frac{1}{10} \quad [\text{Since } p + q = 1]$$

$$q = \frac{9}{10}$$

Let X denote the variable representing the number of tickets bearing a number divisible by 10 out of 5 tickets. Probability of getting r tickets bearing a number divisible by 10 out of n tickets is given by

$$\begin{aligned} P(X = r) &= {}^nC_r p^r q^{n-r} \\ &= {}^5C_r \left(\frac{1}{10}\right)^r \left(\frac{9}{10}\right)^{5-r} \quad \text{--- (1)} \end{aligned}$$

Probability of getting all the tickets bearing a number divisible by 10

$$\begin{aligned} &= {}^5C_5 \left(\frac{1}{10}\right)^5 \left(\frac{9}{10}\right)^{5-5} \quad [\text{Using (1)}] \\ &= 1 \cdot \left(\frac{1}{10}\right)^5 \left(\frac{9}{10}\right)^0 \\ &= \left(\frac{1}{10}\right)^5 \end{aligned}$$

$$\text{Required probability} = \left(\frac{1}{10}\right)^5$$

Binomial Distribution Ex 33.1 Q15

Let p denote the probability of getting a ball marked with 0. So

$$p = \frac{1}{10} \quad \text{[Since balls are marked with 0, 1, 2, 3, 4, 5, 6, 7, 8, 9]}$$

$$q = 1 - \frac{1}{10} \quad \text{[Since } p + q = 1\text{]}$$

$$q = \frac{9}{10}$$

Let X denote the variable presenting the number of balls marked with 0 out of four balls drawn. Probability of drawing r balls out of n balls that are marked 0 is given by

$$\begin{aligned} P(X = r) &= {}^nC_r p^r q^{n-r} \\ &= {}^4C_r \left(\frac{1}{10}\right)^r \left(\frac{9}{10}\right)^{4-r} \end{aligned} \quad \text{--- (1)}$$

Probability of getting none balls marked with 0

$$\begin{aligned} &= P(X = 0) \\ &= {}^4C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{4-0} \\ &= 1 \cdot 1 \cdot \left(\frac{9}{10}\right)^4 \\ &= \left(\frac{9}{10}\right)^4 \end{aligned}$$

Probability of getting none balls marked with 0 = $\left(\frac{9}{10}\right)^4$

Binomial Distribution Ex 33.1 Q16

Let p denote the probability of getting one defective item out of hundred. So

$$p = 5\% \quad [\text{Since } 5\% \text{ are defective items}]$$

$$= \frac{5}{100}$$

$$p = \frac{1}{20}$$

$$q = 1 - \frac{1}{20} \quad [\text{Since } p + q = 1]$$

$$q = \frac{19}{20}$$

Let X denote the random variable representing the number of defective items out of 10 items. Probability of getting r defective items out of n items selected is given by,

$$\begin{aligned} P(X = r) &= {}^nC_r p^r q^{n-r} \\ &= {}^{10}C_r \left(\frac{1}{20}\right)^r \left(\frac{19}{20}\right)^{10-r} \quad \dots (1) \end{aligned}$$

Probability of getting not more than one defective items

$$\begin{aligned} &= P(X = 0) + P(X = 1) \\ &= {}^{10}C_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^{10-0} + {}^{10}C_1 \left(\frac{1}{20}\right)^1 \left(\frac{19}{20}\right)^{10-1} \\ &= 1 \cdot 1 \cdot \left(\frac{19}{20}\right)^{10} + 10 \cdot \frac{1}{20} \left(\frac{19}{20}\right)^9 \\ &= \left(\frac{19}{20}\right)^9 \left[\frac{19}{20} + \frac{10}{20}\right] \\ &= \frac{29}{20} \left(\frac{19}{20}\right)^9 \end{aligned}$$

$$\text{The required probability} = \frac{29}{20} \left(\frac{19}{20}\right)^9$$

Binomial Distribution Ex 33.1 Q17

Let p denote the probability that one bulb produced will fuse after 150 days, so

$$p = 0.05$$

$$= \frac{5}{100}$$

[It is given]

$$q = 1 - \frac{1}{20} \quad [\text{Since } p + q = 1]$$

$$q = \frac{19}{20}$$

Let X denote the number of fuse bulb out of 5 bulbs. Probability that r bulbs out of n will fuse in 150 days is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$= {}^5C_r \left(\frac{1}{20}\right)^r \left(\frac{19}{20}\right)^{5-r} \quad \text{--- (1)}$$

(i)

Probability that none is fuse = $P(X = 0)$

$$= {}^5C_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^{5-0}$$

$$= \left(\frac{19}{20}\right)^5$$

Probability that none will fuse = $\left(\frac{19}{20}\right)^5$

(ii)

Probability that not more than 1 will fuse

$$= P(X = 0) + P(X = 1)$$

$$= \left(\frac{19}{20}\right)^5 + {}^5C_1 \left(\frac{1}{20}\right)^1 \left(\frac{19}{20}\right)^{5-1}$$

$$= \left(\frac{19}{20}\right)^4 \left[\frac{19}{20} + \frac{5}{20}\right]$$

$$= \left(\frac{24}{20}\right) \left(\frac{19}{20}\right)^4$$

Probability not more than one will fuse = $\left(\frac{6}{5}\right) \left(\frac{19}{20}\right)^4$

(iii)

Probability that more than one will fuse

$$\begin{aligned} &= P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) \\ &= 1 - [P(X = 0) + P(X = 1)] \\ &= 1 - \left[\frac{6}{5} \left(\frac{19}{20} \right)^4 \right] \end{aligned}$$

$$\text{Probability that more than one will fuse} = 1 - \left[\frac{6}{5} \left(\frac{19}{20} \right)^4 \right]$$

(iv)

Probability that at least one will fuse

$$\begin{aligned} &= P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) \\ &= 1 - P(X = 0) \\ &= 1 - \left[{}^5C_0 \left(\frac{1}{20} \right)^0 \left(\frac{19}{20} \right)^{5-0} \right] \\ &= 1 - \left[\left(\frac{19}{20} \right)^5 \right] \end{aligned}$$

$$\text{Probability that at least one will fuse} = 1 - \left(\frac{19}{20} \right)^5$$

Binomial Distribution Ex 33.1 Q18

A person can be either right-handed or left-handed.

It is given that 90% of the people are right-handed.

$$\therefore p = P(\text{right-handed}) = \frac{9}{10}$$

$$q = P(\text{left-handed}) = 1 - \frac{9}{10} = \frac{1}{10}$$

Using binomial distribution, the probability that more than 6 people are right-handed is given by,

$$\sum_{r=7}^{10} {}^{10}C_r p^r q^{10-r} = \sum_{r=7}^{10} {}^{10}C_r \left(\frac{9}{10}\right)^r \left(\frac{1}{10}\right)^{10-r}$$

Therefore, the probability that at most 6 people are right-handed

$$= 1 - P(\text{more than 6 are right-handed})$$

$$= 1 - \sum_{r=7}^{10} {}^{10}C_r (0.9)^r (0.1)^{10-r}$$

Binomial Distribution Ex 33.1 Q19

Let p denote the probability of getting 1 red ball out of 7 green, 4 white and 5 red balls, so

$$p = \frac{5}{16}$$

$$q = 1 - \frac{5}{16}$$

[Since $p + q = 1$]

$$q = \frac{11}{16}$$

Let X denote the number of red balls drawn out of four balls. Probability of getting r red balls out of n drawn balls is given by

$$\begin{aligned} P(X = r) &= {}^nC_r p^r q^{n-r} \\ &= {}^4C_r \left(\frac{5}{16}\right)^r \left(\frac{11}{16}\right)^{4-r} \end{aligned} \quad \text{--- (1)}$$

Probability of getting one red ball

$$\begin{aligned} &= P(X = 1) \\ &= {}^4C_1 \left(\frac{5}{16}\right)^1 \left(\frac{11}{16}\right)^{4-1} \\ &= 4 \cdot \left(\frac{5}{16}\right) \left(\frac{11}{16}\right)^3 \\ &= \left(\frac{5}{4}\right) \left(\frac{11}{16}\right)^3 \end{aligned}$$

$$\text{Required probability} = \left(\frac{5}{4}\right) \left(\frac{11}{16}\right)^3$$

Binomial Distribution Ex 33.1 Q20

| X | $P(X)$ |
|-----|---|
| 0 | $\frac{7}{9} \times \frac{6}{8} = \frac{21}{36}$ |
| 1 | $\frac{7}{9} \times \frac{2}{8} \times 2 = \frac{14}{36}$ |
| 2 | $\frac{2}{9} \times \frac{1}{8} = \frac{1}{36}$ |

Binomial Distribution Ex 33.1 Q21

| X | $P(X)$ |
|-----|---|
| 0 | ${}^3C_0 \left(\frac{3}{7}\right)^0 \left(\frac{4}{7}\right)^{3-0} = \left(\frac{4}{7}\right)^3 = \frac{64}{343}$ |
| 1 | ${}^3C_1 \left(\frac{3}{7}\right)^1 \left(\frac{4}{7}\right)^{3-1} = 3 \cdot \left(\frac{3}{7}\right) \left(\frac{4}{7}\right)^2 = \frac{144}{343}$ |
| 2 | ${}^3C_2 \left(\frac{3}{7}\right)^2 \left(\frac{4}{7}\right)^{3-2} = 3 \cdot \left(\frac{3}{7}\right)^2 \left(\frac{4}{7}\right) = \frac{108}{343}$ |
| 3 | ${}^3C_3 \left(\frac{3}{7}\right)^3 \left(\frac{4}{7}\right)^{3-3} = \left(\frac{3}{7}\right)^3 = \frac{27}{343}$ |

Binomial Distribution Ex 33.1 Q22

Let p be the probability of getting doublet is a throw of a pair of dice, so

$$p = \frac{6}{36} \quad [\text{Since } (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) \text{ are doublets}]$$

$$p = \frac{1}{6}$$

$$q = 1 - \frac{1}{6} \quad [\text{Since } p + q = 1]$$

$$= \frac{5}{6}$$

Let X denote the number of getting doublets out of 4 times. So probability distribution is given by

Binomial Distribution Ex 33.1 Q23

| X | $P(X)$ |
|-----|---|
| 0 | ${}^3C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{3-0} = \left(\frac{5}{6}\right)^3 = \frac{125}{216}$ |
| 1 | ${}^3C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^{3-1} = 3 \left(\frac{1}{6}\right) \left(\frac{5}{6}\right)^2 = \frac{25}{72}$ |
| 2 | ${}^3C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{3-2} = 3 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right) = \frac{5}{72}$ |
| 3 | ${}^3C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^{3-3} = \left(\frac{1}{6}\right)^3 = \frac{1}{216}$ |

Binomial Distribution Ex 33.1 Q24

We know that, probability of getting head in a toss of coin $p = \frac{1}{2}$

Probability of not getting head $q = 1 - \frac{1}{2}$

$$q = \frac{1}{2}$$

The coin is tossed 5 times. Let X denote the number of times head occur in 5 tosses.

$$\begin{aligned}
 P(X = r) &= {}^nC_r p^r q^{n-r} \\
 &= {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r}
 \end{aligned}$$

Probability distribution is given by

Binomial Distribution Ex 33.1 Q25

Let p be the probability of a getting a number greater than 4 in a toss of die, so

$$p = \frac{2}{6} \quad \left[\text{Since, numbers greater than 4 coin a die} = 5, 6 \right]$$

$$p = \frac{1}{3}$$

$$q = 1 - \frac{1}{3} \quad \left[\text{Since } p + q = 1 \right]$$

$$q = \frac{2}{3}$$

Let X denote the number of success in 2 throws of a die. Probability of getting r success in n thrown of a die is given by

$$\begin{aligned} P(X = r) &= {}^nC_r p^r q^{n-r} \\ &= {}^2C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{2-r} \end{aligned} \quad \text{--- (1)}$$

Probability distribution of number of success is given by

| X | $P(X)$ |
|-----|---|
| 0 | ${}^2C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{2-0} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$ |
| 1 | ${}^2C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^{2-1} = 2 \cdot \left(\frac{1}{3}\right) \left(\frac{2}{3}\right) = \frac{4}{9}$ |
| 2 | ${}^2C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{2-2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$ |

Binomial Distribution Ex 33.1 Q26

Let n denote the number of throws required to get a head and X denote the amount won/lost.

He may get head on first toss or lose first and 2nd toss or lose first and won second toss probability distribution for X

Number of throws (n): 1 2 2
 Amount won/lost (X): 1 0 -2

Probability $P(X)$: $\frac{1}{2}$ $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

So probability distribution is given by

| X | $P(X)$ |
|-----|---------------|
| 0 | $\frac{1}{4}$ |
| 1 | $\frac{1}{2}$ |
| -2 | $\frac{1}{4}$ |

Binomial Distribution Ex 33.1 Q27

Let p denote the probability of getting 3, 4 or 5 in a throw of die. So

p = probability of success

$$= \frac{3}{6}$$

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2} \quad [\text{Since } p + q = 1]$$

$$q = \frac{1}{2}$$

Let X denote the number of success in throw of 5 dice simultaneously. Probability of getting r success out of n throws of die is given by

$$\begin{aligned} P(X = r) &= {}^nC_r p^r q^{n-r} \\ &= {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r} \quad \dots (1) \end{aligned}$$

Probability getting at least 3 success

$$\begin{aligned} &= P(X = 3) + P(X = 4) + P(X = 5) \\ &= {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3} + {}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{5-4} + {}^5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^{5-5} \\ &= \frac{5 \cdot 4}{2} \left(\frac{1}{2}\right)^5 + 5 \cdot \left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^5 \\ &= \left(\frac{1}{2}\right)^5 [10 + 5 + 1] \\ &= \frac{16}{32} \\ &= \frac{1}{2} \end{aligned}$$

Required probability = $\frac{1}{2}$

Binomial Distribution Ex 33.1 Q28

Let p denote the probability of getting defective items out of 100 items, so

$$p = 10\% \\ = \frac{10}{100}$$

$$p = \frac{1}{10}$$

$$q = 1 - \frac{1}{10} \quad [\text{Since } p + q = 1]$$

$$q = \frac{9}{10}$$

Let X denote the number of defective items drawn out of 8 items. Probability of getting r defective items out of a sample of 8 items is given by

$$P(X = r) = {}^n C_r p^r q^{n-r} \\ = {}^8 C_r \left(\frac{1}{10}\right)^r \left(\frac{9}{10}\right)^{8-r} \quad \dots (1)$$

Probability of getting 2 defective items

$$= P(X = 2) \\ = {}^8 C_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^{8-2} \\ = \frac{8 \times 7}{2} \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^6 \\ = \frac{28 \times 9^6}{10^8}$$

$$\text{Required probability} = \frac{28 \times 9^6}{10^8}$$

Binomial Distribution Ex 33.1 Q29

Let p denote the probability of drawing a heart from a deck of 52 cards, so

$$p = \frac{13}{52} \quad [\because \text{There are 13 hearts in deck}]$$

$$p = \frac{1}{4}$$

$$q = 1 - \frac{1}{4} \quad [\text{Since } p + q = 1]$$

$$q = \frac{3}{4}$$

Let the card is drawn n times. So Binomial distribution is given by

$$P(X = r) = {}^n C_r p^r q^{n-r}$$

where X denote the number of spades drawn and $r = 0, 1, 2, 3, \dots, n$

(i)

We have to find the smallest value of n for which $P(X = 0)$ is less than $\frac{1}{4}$

$$P(X = 0) < \frac{1}{4}$$

$${}^n C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{n-0} < \frac{1}{4}$$

$$\left(\frac{3}{4}\right)^n < \frac{1}{4}$$

$$\text{Put } n = 1, \left(\frac{3}{4}\right) \not< \frac{1}{4}$$

$$n = 2, \left(\frac{3}{4}\right)^2 \not< \frac{1}{4}$$

$$n = 3, \left(\frac{3}{4}\right)^3 \not< \frac{1}{4}$$

So, smallest value of $n = 3$

\therefore We must draw cards at least 3 times

(ii)

Given, the probability of drawing a heart $> \frac{3}{4}$

$$1 - P(X = 0) > \frac{3}{4}$$

$$1 - {}^nC_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{n-0} > \frac{3}{4}$$

$$1 - \left(\frac{3}{4}\right)^n > \frac{3}{4}$$

$$1 - \frac{3}{4} > \left(\frac{3}{4}\right)^n$$

$$\frac{1}{4} > \left(\frac{3}{4}\right)^n$$

For $n = 1$, $\left(\frac{3}{4}\right)^1 \not< \frac{1}{4}$

$n = 2$, $\left(\frac{3}{4}\right)^2 \not< \frac{1}{4}$

$n = 3$, $\left(\frac{3}{4}\right)^3 \not< \frac{1}{4}$

$n = 4$, $\left(\frac{3}{4}\right)^4 \not< \frac{1}{4}$

$n = 5$, $\left(\frac{3}{4}\right)^5 < \frac{1}{4}$

So, card must be drawn 5 times.

Binomial Distribution Ex 33.1 Q30

Here $n = 8, p = \frac{1}{2}, q = \frac{1}{2}$

Let there be k desks and X be the number of students studying in office.

Then we want that

$$P(X \leq k) > .90$$

$$\Rightarrow P(X > k) < .10$$

$$\Rightarrow P(X = k+1, k+2, \dots, 8) < .10$$

Clearly $P(X > 6) = P(X = 7 \text{ or } X = 8)$

$$\begin{aligned} &= {}^8C_7 \left(\frac{1}{2}\right)^8 + {}^8C_8 \left(\frac{1}{2}\right)^8 \\ &= .04 \end{aligned}$$

$$\begin{aligned} \text{and } P(X > 5) &= P(X = 6, X = 7 \text{ or } X = 8) \\ &= .15 \end{aligned}$$

$$\therefore P(X > 6) < 0.10$$

\Rightarrow If there are 6 desks then there is at least 90% chance for every graduate assistant to get a desk.

Binomial Distribution Ex 33.1 Q31

Binomial Distribution formula is given by

$$P(x) = {}^nC_x p^x q^{n-x}, \text{ where } x = 0, 1, 2, \dots, n$$

Let x = No. of heads in a toss

We need probability of 6 or more heads

$$X = 6, 7, 8$$

Here $p = \frac{1}{2}$ and $q = \frac{1}{2}$

$$P(6) = \text{Prob of getting 6 heads, 2 tails} = {}^8C_6 \left(\frac{1}{2}\right)^6 \times \left(\frac{1}{2}\right)^2$$

$$P(7) = \text{Prob of getting 7 heads, 1 tails} = {}^8C_7 \left(\frac{1}{2}\right)^7 \times \left(\frac{1}{2}\right)^1$$

$$P(8) = \text{Prob of getting 8 heads, 0 tails} = {}^8C_8 \left(\frac{1}{2}\right)^8 \times \left(\frac{1}{2}\right)^0$$

The probability of getting at least 6 heads (not more than 2 tails) is then

$${}^8C_6 \left(\frac{1}{2}\right)^6 \times \left(\frac{1}{2}\right)^2 + {}^8C_7 \left(\frac{1}{2}\right)^7 \times \left(\frac{1}{2}\right)^1 + {}^8C_8 \left(\frac{1}{2}\right)^8 \times \left(\frac{1}{2}\right)^0$$

$$= \frac{1}{256} + 8 \frac{1}{256} + 28 \frac{1}{256} = \frac{37}{256}$$

Binomial Distribution Ex 33.1 Q32

Let p represents the probability of getting head in a toss of fair coin, so

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2} \quad [\text{Since } p + q = 1]$$

$$q = \frac{1}{2}$$

Let X denote the random variable representing the number heads in 6 tosses of coin. Probability of getting r sixes in n tosses of a fair coin is given by,

$$\begin{aligned} P(X = r) &= {}^nC_r p^r q^{n-r} \\ &= {}^6C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{6-r} \quad \text{--- (1)} \end{aligned}$$

(i)

Probability of getting 3 heads

$$\begin{aligned} &= P(X = 3) \\ &= {}^6C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{6-3} \\ &= \frac{6 \times 5 \times 4}{3 \times 2} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^3 \\ &= \frac{20}{64} \end{aligned}$$

$$\text{Probability of getting 3 heads} = \frac{20}{64} = \frac{5}{16}$$

(ii)

Probability of getting no heads

$$\begin{aligned} &= P(X = 0) \\ &= {}^6C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{6-0} \\ &= \left(\frac{1}{2}\right)^6 \\ &= \frac{1}{64} \end{aligned}$$

$$\text{Probability of getting no heads} = \frac{1}{64}$$

(iii)

Probability of getting at least one head

$$\begin{aligned} &= P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6) \\ &= 1 - P(X = 0) \\ &= 1 - \frac{1}{64} \\ &= \frac{63}{64} \end{aligned}$$

$$\text{Probability of getting at least one head} = \frac{63}{64}$$

Binomial Distribution Ex 33.1 Q33

Let p be the probability that a tube function for more than 500 hours. So

$$p = 0.2$$

$$p = \frac{1}{5}$$

$$q = 1 - \frac{1}{5} \quad \text{[Since } p + q = 1\text{]} \\ = \frac{4}{5}$$

Let X denote the random variable representing the number of tube that functions for more than 500 hours out of 4 tubes. Probability of functioning r tubes out n tubes selected for more than 500 hours is given by,

$$P(X = r) = {}^nC_r p^r q^{n-r} \\ = {}^4C_r \left(\frac{1}{5}\right)^r \left(\frac{4}{5}\right)^{4-r} \quad \text{--- (1)}$$

Probability that exactly 3 tube will function for more than 500 hours

$$= {}^4C_3 \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)^{4-3} \\ = 4 \cdot \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right) \\ = \frac{16}{625}$$

$$\text{Required probability} = \frac{16}{625}$$

Binomial Distribution Ex 33.1 Q34

Let p be the probability that component survive the shock test. So

$$p = \frac{3}{4}$$

$$q = 1 - \frac{3}{4} \quad [\text{Since } p + q = 1]$$

$$q = \frac{1}{4}$$

Let X denote the random variable representing the number of components that survive shock test out of 5 components. Probability of that r components that survive shock test out of n components is given by

$$\begin{aligned} P(X = r) &= {}^nC_r p^r q^{n-r} \\ &= {}^5C_r \left(\frac{3}{4}\right)^r \left(\frac{1}{4}\right)^{5-r} \quad \text{--- (1)} \end{aligned}$$

(i)

Probability that exactly 2 will survive the shock test

$$\begin{aligned} &= P(X = 2) \\ &= {}^5C_2 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^{5-2} \\ &= \frac{5 \cdot 4}{2} \left(\frac{9}{16}\right) \left(\frac{1}{64}\right) \\ &= \frac{45}{512} = 0.0879 \end{aligned}$$

Probability that exactly 2 survive = 0.0879

(ii)

Probability that at most 3 will survive

$$\begin{aligned} &= P(X = 0) + P(X = 1) + P(X = 3) + P(X = 4) \\ &= 1 - [P(X = 4) + P(X = 5)] \\ &= 1 - \left[{}^5C_4 \left(\frac{3}{4}\right)^4 \left(\frac{1}{4}\right)^{5-4} + {}^5C_5 \left(\frac{3}{4}\right)^5 \left(\frac{1}{4}\right)^{5-5} \right] \\ &= 1 - \left[5 \cdot \frac{81}{1024} + \frac{243}{1024} \right] \\ &= 1 - \left[\frac{405 + 243}{1024} \right] \\ &= \frac{1024 - 648}{1024} \\ &= \frac{376}{1024} = 0.3672 \end{aligned}$$

Binomial Distribution Ex 33.1 Q35

Probability that bomb strikes a target $p = 0.2$

Probability that a bomb misses the target $= 0.8$

$n = 6$

let x = number of bombs that strike the target

$P(x=2)$ = exactly 2 bombs strike the target

$$= {}^6C_2 \left(\frac{2}{10}\right)^2 \times \left(\frac{8}{10}\right)^4 = 15 \times \frac{16384}{10^6} = 0.24576$$

$P(x \geq 2)$ = at least 2 bombs strike the target

$$= 1 - P(x < 2)$$

$$= 1 - [P(x=0) + P(x=1)]$$

$$= 1 - \left[{}^6C_0 \left(\frac{2}{10}\right)^0 \times \left(\frac{8}{10}\right)^6 + {}^6C_1 \left(\frac{2}{10}\right)^1 \times \left(\frac{8}{10}\right)^5 \right]$$

$$\begin{aligned} &= 1 - [0.0262144 + 0.393216] = 1 - 0.65536 \\ &= 0.34464 \end{aligned}$$

Binomial Distribution Ex 33.1 Q36

Let p be the probability that a mouse get contract the disease. So

$$p = 40\%$$

$$= \frac{40}{100}$$

$$= \frac{2}{5}$$

$$q = 1 - \frac{2}{5} \quad [\text{Since } p + q = 1]$$

$$q = \frac{3}{5}$$

Let X denote the variable representing number of mice contract the disease out of 5 mice.

Probability the r mice get contract the disease out of n mice inoculated is given by

$$\begin{aligned} P(X = r) &= {}^nC_r p^r q^{n-r} \\ &= {}^5C_r \left(\frac{2}{5}\right)^r \left(\frac{3}{5}\right)^{5-r} \quad \text{--- (1)} \end{aligned}$$

(i)

Probability that none contract the disease = $P(X = 0)$

$$\begin{aligned} &= {}^5C_0 \left(\frac{2}{5}\right)^0 \left(\frac{3}{5}\right)^{5-0} \\ &= \left(\frac{3}{5}\right)^5 \end{aligned}$$

Probability that none contract the disease = $\left(\frac{3}{5}\right)^5$

(ii)

Probability that more than 3 contract disease

$$\begin{aligned} &= P(X = 4) + P(X = 5) \\ &= {}^5C_4 \left(\frac{2}{5}\right)^4 \left(\frac{3}{5}\right)^{5-4} + {}^5C_5 \left(\frac{2}{5}\right)^5 \left(\frac{3}{5}\right)^{5-5} \\ &= 5 \cdot \left(\frac{2}{5}\right)^4 \left(\frac{3}{5}\right) + \left(\frac{2}{5}\right)^5 \end{aligned}$$

$$= \binom{5}{5} \left[\frac{2}{5} \right]^5$$

$$= \frac{17}{5} \left(\frac{2}{5} \right)^4$$

Binomial Distribution Ex 33.1 Q37

SUPERCOP

Let p be the probability of success in experiments, q be the probability of failure,

Given, $p = 2q$

but $p + q = 1$

$$2q + q = 1$$

$$3q = 1$$

$$q = \frac{1}{3}$$

$$p = \frac{2}{3}$$

Let X denote the random variable representing the number of success out of 6 experiments.

Probability of getting r success out of n experiments is given by

$$\begin{aligned} P(X = r) &= {}^nC_r p^r q^{n-r} \\ &= {}^6C_r \left(\frac{2}{3}\right)^r \left(\frac{1}{3}\right)^{6-r} \end{aligned} \quad \text{--- (1)}$$

Probability of getting at least 4 success

$$\begin{aligned} &= P(X = 4) + P(X = 5) + P(X = 6) \\ &= {}^6C_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^{6-4} + {}^6C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^{6-5} + {}^6C_6 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^{6-6} \\ &= \frac{6 \times 5}{2} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 + 6 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^1 + \left(\frac{2}{3}\right)^6 \\ &= \left(\frac{2}{3}\right)^4 \left[\frac{15}{9} + \frac{4}{3} + \frac{4}{9} \right] \\ &= \left(\frac{2}{3}\right)^4 \left[\frac{15 + 12 + 4}{9} \right] \\ &= \left(\frac{31}{9}\right) \left(\frac{2}{3}\right)^4 \\ &= \frac{496}{729} \end{aligned}$$

$$\text{Required probability} = \frac{496}{729}$$

Let x = number of out of service machines
 p = probability that machine will be out of service on the same day
 $= 2/100$
 q = probability that machine will be in service on the same day
 $= 8/100$
 $P(x=3)$ = probability exactly 3 machines will be out of service on the same day

$$P(x=3) = {}^{20}C_3 \times \left(\frac{2}{100}\right)^3 \left(\frac{8}{100}\right)^0 = 1140 \times 0.000008 \\ = 0.00912$$

For low probability events Poisson's distribution is used instead of Binomial distribution. Then,
 $\lambda = np = 20 \times 0.02 = 0.4$

$$P(x=r) = \frac{e^{-\lambda} \times \lambda^r}{r!}$$

$$P(x=3) = \frac{e^{-0.4} \times 0.4^3}{3!} = 0.6703 \times 0.064 / 6 = 0.0071$$

Binomial Distribution Ex 33.1 Q39

Let p be the probability that a student entering a university will graduate, so

$$p = 0.4$$

$$q = 1 - 0.4$$

$$= 0.6$$

[Since $p + q = 1$]

Let X denote the random variable representing the number of students entering a university will graduate out of 3 students of university. Probability that r students will graduate out of n entering the university is given by

$$P(X=r) = {}^nC_r p^r q^{n-r} \\ = {}^3C_r (0.4)^r (0.6)^{3-r} \quad \text{--- (1)}$$

(i)

Probability that none will graduate

$$= P(X=0)$$

$$= {}^3C_0 (0.4)^0 (0.6)^{3-0}$$

$$= 0.216$$

Probability that none will graduate = 0.216

(ii)

Probability that one will graduate

$$= P(X = 1)$$

$$= {}^3C_1 (0.4)^1 (0.6)^{3-1}$$

$$= 3 \times (0.4) (0.36)$$

$$= 0.432$$

Probability that only one will graduate = 0.432

(iii)

Probability that all will graduate

$$= P(X = 3)$$

$$= {}^3C_3 (0.4)^3 (0.6)^{3-3}$$

$$= (0.4)^3$$

$$= 0.064$$

Probability that all will graduate = 0.064

Binomial Distribution Ex 33.1 Q40

Let X denote the number of defective eggs in the 10 eggs drawn.

Since the drawing is done with replacement, the trials are Bernoulli trials.

Clearly, X has the binomial distribution with $n=10$ and $p = \frac{10}{100} = \frac{1}{10}$

Therefore, $q = 1 - \frac{1}{10} = \frac{9}{10}$

Now, $P(\text{at least one defective egg}) = P(X \geq 1) = 1 - P(X = 0)$

$$= 1 - {}^{10}C_0 \left(\frac{9}{10}\right)^{10} = 1 - \frac{9^{10}}{10^{10}}$$

Binomial Distribution Ex 33.1 Q41

Let p be the probability of answering a true. So

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2} \quad [\text{Since } p + q = 1]$$

$$= \frac{1}{2}$$

Thus the probability that he answers at least 12 questions correctly among 20 questions is

$$P(X \geq 12) = P(X = 12) + P(X = 13) + P(X = 14) + P(X = 15) + P(X = 16) +$$

$$P(X = 17) + P(X = 18) + P(X = 19) + P(X = 20)$$

$$= \left(\frac{1}{2}\right)^{20} \{ {}^{20}C_{12} + {}^{20}C_{13} + {}^{20}C_{14} + {}^{20}C_{15} + {}^{20}C_{16} + {}^{20}C_{17} + {}^{20}C_{18} + {}^{20}C_{19} + {}^{20}C_{20} \}$$

$$= \frac{{}^{20}C_{12} + {}^{20}C_{13} + {}^{20}C_{14} + {}^{20}C_{15} + {}^{20}C_{16} + {}^{20}C_{17} + {}^{20}C_{18} + {}^{20}C_{19} + {}^{20}C_{20}}{2^{20}}$$

Therefore, the required answer is

$$\frac{{}^{20}C_{12} + {}^{20}C_{13} + {}^{20}C_{14} + {}^{20}C_{15} + {}^{20}C_{16} + {}^{20}C_{17} + {}^{20}C_{18} + {}^{20}C_{19} + {}^{20}C_{20}}{2^{20}}$$

Binomial Distribution Ex 33.1 Q42

X is the random variable whose binomial distribution is $B\left(6, \frac{1}{2}\right)$.

Therefore, $n = 6$ and $p = \frac{1}{2}$

$$\therefore q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\begin{aligned}\text{Then, } P(X = x) &= {}^nC_x q^{n-x} p^x \\ &= {}^6C_x \left(\frac{1}{2}\right)^{6-x} \cdot \left(\frac{1}{2}\right)^x \\ &= {}^6C_x \left(\frac{1}{2}\right)^6\end{aligned}$$

It can be seen that $P(X = x)$ will be maximum, if 6C_x will be maximum.

$$\text{Then, } {}^6C_0 = {}^6C_6 = \frac{6!}{0!6!} = 1$$

$${}^6C_1 = {}^6C_5 = \frac{6!}{1!5!} = 6$$

$${}^6C_2 = {}^6C_4 = \frac{6!}{2!4!} = 15$$

$${}^6C_3 = \frac{6!}{3!3!} = 20$$

The value of 6C_3 is maximum. Therefore, for $x = 3$, $P(X = x)$ is maximum.

Thus, $X = 3$ is the most likely outcome.

Binomial Distribution Ex 33.1 Q43

The repeated guessing of correct answers from multiple choice questions are Bernoulli trials. Let X represent the number of correct answers by guessing in the set of 5 multiple choice questions.

Probability of getting a correct answer is, $p = \frac{1}{3}$

$$\therefore q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$$

Clearly, X has a binomial distribution with $n = 5$ and $p = \frac{1}{3}$

$$\begin{aligned}\therefore P(X = x) &= {}^nC_x q^{n-x} p^x \\ &= {}^5C_x \left(\frac{2}{3}\right)^{5-x} \cdot \left(\frac{1}{3}\right)^x\end{aligned}$$

$P(\text{guessing more than 4 correct answers}) = P(X \geq 4)$

$$\begin{aligned}&= P(X = 4) + P(X = 5) \\ &= {}^5C_4 \left(\frac{2}{3}\right) \cdot \left(\frac{1}{3}\right)^4 + {}^5C_5 \left(\frac{1}{3}\right)^5 \\ &= 5 \cdot \frac{2}{3} \cdot \frac{1}{81} + 1 \cdot \frac{1}{243} \\ &= \frac{10}{243} + \frac{1}{243} \\ &= \frac{11}{243}\end{aligned}$$

Binomial Distribution Ex 33.1 Q44

$$(b) P(\text{winning exactly once}) = P(X = 1)$$

$$= {}^{50}C_1 \left(\frac{99}{100}\right)^{49} \cdot \left(\frac{1}{100}\right)^1$$

$$= 50 \left(\frac{1}{100}\right) \left(\frac{99}{100}\right)^{49}$$

$$= \frac{1}{2} \left(\frac{99}{100}\right)^{49}$$

$$(c) P(\text{at least twice}) = P(X \geq 2)$$

$$= 1 - P(X < 2)$$

$$= 1 - P(X \leq 1)$$

$$= 1 - [P(X = 0) + P(X = 1)]$$

$$= [1 - P(X = 0)] - P(X = 1)$$

$$= 1 - \left(\frac{99}{100}\right)^{50} - \frac{1}{2} \cdot \left(\frac{99}{100}\right)^{49}$$

$$= 1 - \left(\frac{99}{100}\right)^{49} \cdot \left[\frac{99}{100} + \frac{1}{2}\right]$$

$$= 1 - \left(\frac{99}{100}\right)^{49} \cdot \left(\frac{149}{100}\right)$$

$$= 1 - \left(\frac{149}{100}\right) \left(\frac{99}{100}\right)^{49}$$

Binomial Distribution Ex 33.1 Q45

Let the shooter fire n times.

n fires are Bernoulli trials.

In each trial, $p =$ probability of hitting the target $= \frac{3}{4}$

And $q =$ probability of not hitting the target $= 1 - \frac{3}{4} = \frac{1}{4}$

$$\text{Then, } P(X = x) = {}^nC_x q^{n-x} p^x = {}^nC_x \left(\frac{1}{4}\right)^{n-x} \left(\frac{3}{4}\right)^x = {}^nC_x \frac{3^x}{4^n}$$

Now, given that

$$P(\text{hitting the target atleast once}) > 0.99$$

$$\text{i.e. } P(x \geq 1) > 0.99$$

$$\Rightarrow 1 - P(x = 0) > 0.99$$

$$\Rightarrow 1 - {}^nC_0 \frac{1}{4^n} > 0.99$$

$$\Rightarrow {}^nC_0 \frac{1}{4^n} < 0.01$$

$$\Rightarrow \frac{1}{4^n} < 0.01$$

$$\Rightarrow 4^n > \frac{1}{0.01} = 100$$

The minimum value of n to satisfy this inequality is 4

Thus, the shooter must fire 4 times.

Binomial Distribution Ex 33.1 Q46

Let the man toss the coin n times. The n tosses are n Bernoulli trials.

Probability (p) of getting a head at the toss of a coin is $\frac{1}{2}$.

$$\Rightarrow p = \frac{1}{2} \Rightarrow q = \frac{1}{2}$$

$$\therefore P(X=x) = {}^nC_x p^{n-x} q^x = {}^nC_x \left(\frac{1}{2}\right)^{n-x} \left(\frac{1}{2}\right)^x = {}^nC_x \left(\frac{1}{2}\right)^n$$

It is given that,

$$P(\text{getting at least one head}) > \frac{90}{100}$$

$$P(X \geq 1) > 0.9$$

$$1 - P(X = 0) > 0.9$$

$$1 - {}^nC_0 \cdot \frac{1}{2^n} > 0.9$$

$${}^nC_0 \cdot \frac{1}{2^n} < 0.1$$

$$\frac{1}{2^n} < 0.1$$

$$2^n > \frac{1}{0.1}$$

$$2^n > 10 \quad \dots(1)$$

The minimum value of n that satisfies the given inequality is 4.

Thus, the man should toss the coin 4 or more than 4 times.

Binomial Distribution Ex 33.1 Q47

Let the man toss the coin n times.

Probability (p) of getting a head at the toss of a coin is $\frac{1}{2}$.

So,

$$p = \frac{1}{2}$$

$$q = 1 - \frac{1}{2} \quad [\text{Since } p + q = 1]$$

$$= \frac{1}{2}$$

$$\therefore P(X=x) = {}^nC_x p^{n-x} q^x$$

$$= {}^nC_x \left(\frac{1}{2}\right)^{n-x} \left(\frac{1}{2}\right)^x$$

$$= {}^nC_x \left(\frac{1}{2}\right)^n$$

It is given that

$$p(\text{getting at least one head}) > \frac{80}{100}$$

$$P(x \geq 1) > 0.8$$

$$1 - P(x=0) > 0.8$$

$$1 - {}^nC_0 \cdot \frac{1}{2^n} > 0.8$$

$${}^nC_0 \cdot \frac{1}{2^n} < 0.2$$

$$\frac{1}{2^n} < 0.2$$

$$2^n > \frac{1}{0.2}$$

$$2^n > 5$$

The minimum value of n that satisfies the given inequality is 3.

Thus, the man should toss the coin 3 or more than 3 times.

Binomial Distribution Ex 33.1 Q48

Let p be the probability of getting a doublet in a throw of a pair of dice, so

$$p = \frac{6}{36} \quad [\text{Since } (1,1), (2,2), (3,3), (4,4), (5,5), (6,6)]$$

$$= \frac{1}{6}$$

$$q = 1 - \frac{1}{6} \quad [\text{Since } p + q = 1]$$

$$= \frac{5}{6}$$

Let X denote the number of getting doublets i.e. success out of 4 times. So, probability distribution is given by

| X | $P(X)$ |
|-----|--|
| 0 | ${}^4C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{4-0} = \left(\frac{5}{6}\right)^4$ |
| 1 | ${}^4C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^{4-1} = 4 \left(\frac{1}{6}\right) \left(\frac{5}{6}\right)^3 = \frac{2}{3} \left(\frac{5}{6}\right)^3$ |
| 2 | ${}^4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{4-2} = \frac{4 \cdot 3}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 = \frac{25}{216}$ |
| 3 | ${}^4C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^{4-3} = \frac{4 \cdot 3}{2} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right) = \frac{5}{324}$ |
| 4 | ${}^4C_4 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^{4-4} = \left(\frac{1}{6}\right)^4 = \frac{1}{1296}$ |

Binomial Distribution Ex 33.1 Q49

Let p be the probability of defective bulbs, so

$$p = \frac{6}{30}$$

$$= \frac{1}{5}$$

$$q = 1 - \frac{1}{5} \quad [\text{Since } p + q = 1]$$

$$= \frac{4}{5}$$

Here, 4 bulbs are drawn at random with replacement. So, probability distribution is given by

| X | $P(X)$ |
|-----|---|
| 0 | ${}^4C_0 \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^{4-0} = \frac{256}{625}$ |
| 1 | ${}^4C_1 \left(\frac{1}{5}\right)^1 \left(\frac{4}{5}\right)^{4-1} = \frac{4}{5} \times \frac{4^3}{5^3} = \frac{256}{625}$ |
| 2 | ${}^4C_2 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^{4-2} = \frac{6}{5^2} \times \frac{4^2}{5^2} = \frac{96}{625}$ |
| 3 | ${}^4C_3 \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)^{4-3} = \frac{4}{5^3} \times \frac{4}{5} = \frac{16}{625}$ |
| 4 | ${}^4C_4 \left(\frac{1}{5}\right)^4 \left(\frac{4}{5}\right)^{4-4} = 1 \cdot \frac{1}{625} = \frac{1}{625}$ |

Binomial Distribution Ex 33.1 Q50

Here success is a score which is multiple of 3 i.e. 3 or 6.

$$\therefore p(3 \text{ or } 6) = \frac{2}{6} = \frac{1}{3}$$

The probability of r successes in 10 throws is given by

$$P(r) = {}^{10}C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{10-r}$$

$$\text{Now } P(\text{at least 8 successes}) = P(8) + P(9) + P(10)$$

$$= {}^{10}C_8 \left(\frac{1}{3}\right)^8 \left(\frac{2}{3}\right)^2 + {}^{10}C_9 \left(\frac{1}{3}\right)^9 \left(\frac{2}{3}\right)^1 + {}^{10}C_{10} \left(\frac{1}{3}\right)^{10} \left(\frac{2}{3}\right)^0$$

$$= \frac{1}{3^{10}} [45 \times 4 + 10 \times 2 + 1]$$

$$= \frac{201}{3^{10}}$$

Binomial Distribution Ex 33.1 Q51

Here success is an odd number i.e. 1, 3 or 5.

$$\therefore p(1, 3 \text{ or } 5) = \frac{3}{6} = \frac{1}{2}$$

The probability of r successes in 5 throws is given by

$$P(r) = {}^5C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r}$$

$$\text{Now } P(\text{exactly 3 times}) = P(3)$$

$$= {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$$

$$= \frac{10}{2^5}$$

$$= \frac{5}{16}$$

Probability of a man hitting a target is 0.25.

$$\therefore p = 0.25 = \frac{1}{4}, \quad q = 1 - p = \frac{3}{4}$$

The probability of r successes in 7 shoots is given by

$$P(r) = {}^7C_r (0.25)^r (0.75)^{7-r}$$

Now $P(\text{at least twice}) = 1 - P(\text{less than 2})$

$$= 1 - {}^7C_0 (0.25)^0 (0.75)^7 + {}^7C_1 (0.25)^1 (0.75)^6$$

$$= 1 - \frac{3^7}{4^7} + 7 \times \frac{3^6}{4^7}$$

$$= \frac{4547}{8192}$$

Binomial Distribution Ex 33.1 Q53

Probability of a bulb to be defective is $\frac{1}{50}$.

$$\therefore p = \frac{1}{50}, \quad q = 1 - p = \frac{49}{50}$$

The probability of r defective bulbs in 10 bulbs is given by

$$P(r) = {}^{10}C_r \left(\frac{1}{50}\right)^r \left(\frac{49}{50}\right)^{10-r}$$

$$(i) \quad P(\text{none of the bulb is defective}) = P(0)$$

$$= {}^{10}C_0 \left(\frac{1}{50}\right)^0 \left(\frac{49}{50}\right)^{10}$$

$$= \left(\frac{49}{50}\right)^{10}$$

$$(ii) \quad P(\text{exactly two bulbs are defective}) = P(2)$$

$$= {}^{10}C_2 \left(\frac{1}{50}\right)^2 \left(\frac{49}{50}\right)^8$$

$$= 45 \times \frac{(49)^8}{(50)^{10}}$$

(iii) $P(\text{more than 8 bulbs work properly})$

$= P(\text{at most two bulbs are defective})$

$$= {}^{10}C_0 \left(\frac{1}{50}\right)^0 \left(\frac{49}{50}\right)^{10} + {}^{10}C_1 \left(\frac{1}{50}\right)^1 \left(\frac{49}{50}\right)^9 + {}^{10}C_2 \left(\frac{1}{50}\right)^2 \left(\frac{49}{50}\right)^8$$

$$= \left(\frac{49}{50}\right)^{10} + 10 \times \frac{(49)^9}{(50)^{10}} + 45 \times \frac{(49)^8}{(50)^{10}}$$

$$= \frac{(49)^8}{(50)^{10}} [(49)^2 + 490 + 45]$$

$$= \frac{(49)^8 \times 2936}{(50)^{10}}$$

Ex 33.2

Binomial Distribution Ex 33.2 Q1

Let X be a binomial variate with parameters n and p .

$$\text{Mean} = np$$

$$\text{Variance} = npq$$

$$\begin{aligned}\text{Mean} - \text{Variance} &= np - npq \\ &= np(1 - q) \\ &= np.p \\ &= np^2\end{aligned}$$

$$\text{Mean} - \text{Variance} > 0$$

$$\text{Mean} > \text{Variance}$$

So, mean can never be less than variance.

Binomial Distribution Ex 33.2 Q2

Let X denote the variance with parameters n and p

$$p + q = 1$$

$$q = 1 - p$$

$$\text{Given, Mean} = np = 9$$

$$\text{--- (i)}$$

$$\text{Variance} = npq = \frac{9}{4}$$

$$\text{--- (ii)}$$

$$\frac{npq}{np} = \frac{\frac{9}{4}}{9}$$

[By dividing (i) by (ii)]

$$q = \frac{1}{4}$$

$$\text{So, } p = 1 - q$$

$$= 1 - \frac{1}{4}$$

$$p = \frac{3}{4}$$

Put p in equation (i),

$$n\left(\frac{3}{4}\right) = 9$$

$$\Rightarrow n = \frac{36}{3}$$

$$\text{So, } n = 12$$

The distribution is given by

$$= {}^nC_r p^r (q)^{n-r}$$

$$P(X = r) = {}^{12}C_r \left(\frac{3}{4}\right)^r \left(\frac{1}{4}\right)^{12-r}$$

$$\text{for } r = 0, 1, 2, \dots, 12$$

Binomial Distribution Ex 33.2 Q3

Let n and p be parameters of binomial distribution. Here

$$\text{Mean} = np = 9 \quad \text{--- (i)}$$

$$\text{Variance} = npq = 6 \quad \text{--- (ii)}$$

$$\frac{npq}{np} = \frac{6}{9}$$

$$q = \frac{2}{3}$$

$$\text{So, } p = 1 - \frac{2}{3} \quad [\text{Since } p + q = 1]$$

$$p = \frac{1}{3}$$

Using equation (i), $np = 9$

$$n \left(\frac{1}{3} \right) = 9$$
$$n = 27$$

Hence, binomial distribution is given by

$$P(X = r) = {}^{27}C_r \left(\frac{1}{3} \right)^r \left(\frac{2}{3} \right)^{27-r}$$
$$r = 0, 1, 2, \dots, 27$$

Binomial Distribution Ex 33.2 Q4

Given that,

$$n = 5$$

Also, Mean + Variance = 4.8

$$np + npq = 4.8$$

$$np(1+q) = 4.8$$

$$5p(1+q) = 4.8$$

$$5(1-q)(1+q) = 4.8 \quad [\text{Since } p + q = 1]$$

$$5(1-q^2) = 4.8$$

$$1-q^2 = \frac{4.8}{5}$$

$$q^2 = 1 - \frac{4.8}{5}$$

$$= \frac{0.2}{5}$$

$$q^2 = \frac{1}{25}$$

$$q = \frac{1}{5}$$

$$\Rightarrow p = 1 - q$$

$$= 1 - \frac{1}{5}$$

$$p = \frac{4}{5}$$

$$\text{So, } n = 5, p = \frac{4}{5}, q = \frac{1}{5}$$

Here binomial distribution is

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^5C_r \left(\frac{4}{5} \right)^r \left(\frac{1}{5} \right)^{5-r}$$

$$r = 0, 1, 2, 3, \dots, 5$$

Binomial Distribution Ex 33.2 Q5

Given that,

$$\text{Mean} = np = 20 \quad \text{--- (i)}$$

$$\text{Variance} = npq = 16 \quad \text{--- (ii)}$$

Let n and p be the parameters of distribution dividing equation (ii) by (i)

$$\frac{npq}{np} = \frac{16}{20}$$

$$q = \frac{4}{5}$$

$$\text{So, } p = 1 - q \quad [\text{Since } p + q = 1]$$

$$= 1 - \frac{4}{5}$$

$$p = \frac{1}{5}$$

Put p in equation (i),

$$np = 20$$

$$n\left(\frac{1}{5}\right) = 20$$

$$n = 20 \times 5$$

$$n = 100$$

So, binomial distribution is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^{100}C_r \left(\frac{1}{5}\right)^r \left(\frac{4}{5}\right)^{100-r}$$

$$r = 0, 1, 2, 3, \dots, 100$$

Binomial Distribution Ex 33.2 Q6

Let n and p be the parameters of distribution binomial distribution. So

$$q = 1 - p \quad \text{as } p + q = 1$$

$$\text{Mean} + \text{Variance} = \frac{25}{3}$$

$$np + npq = \frac{25}{3}$$

$$np(1+q) = \frac{25}{3}$$

$$np = \frac{25}{3(1+q)} \quad \text{--- (1)}$$

$$\text{Mean} \times \text{Variance} = \frac{50}{3}$$

$$np \times npq = \frac{50}{3}$$

$$n^2 p^2 q = \frac{50}{3}$$

$$\left[\frac{25}{3(1+q)} \right]^2 \cdot q = \frac{50}{3} \quad \text{[Using (1)]}$$

$$625q = \frac{50}{3} [9(1+q)^2]$$

$$625q = 150(1+q)^2$$

$$25q = 6(1+q)^2$$

$$6 + 6q^2 + 12q - 25q = 0$$

$$6q^2 - 13q + 6 = 0$$

$$6q^2 - 9q - 4q + 6 = 0$$

$$3q(2q - 3) - 2(2q - 3) = 0$$

$$(2q - 3)(3q - 2) = 0$$

$$\Rightarrow 2q - 3 = 0 \quad \text{or} \quad 3q - 2 = 0$$

$$\Rightarrow q = \frac{3}{2} \quad \text{or} \quad q = \frac{2}{3}$$

Since $q \leq 1$, so

$$q = \frac{2}{3}$$

$$p = 1 - q$$

$$= 1 - \frac{2}{3}$$

$$p = \frac{1}{3}$$

Binomial Distribution Ex 33.2 Q7

Let n and p be the parameters of binomial distribution.

Given that,

$$\text{Mean} = np = 20 \quad \text{--- (i)}$$

$$\text{Standard deviation} = \sqrt{npq} = 4$$

Squaring both the sides,

$$npq = 16 \quad \text{--- (ii)}$$

Dividing equation (ii) by (i),

$$\frac{npq}{np} = \frac{16}{20}$$

$$q = \frac{4}{5}$$

$$\text{So, } p = 1 - q \quad [\text{Since } p + q = 1]$$

$$= 1 - \frac{4}{5}$$

$$p = \frac{1}{5}$$

Put value of p in equation (i),

$$np = 20$$

$$\frac{n}{5} = 20$$

$$n = 100$$

$$p = \frac{1}{5}$$

Binomial Distribution Ex 33.2 Q8

Let p denotes the probability of selecting a defective bolt, so

$$p = 0.1$$

$$p = \frac{1}{10}$$

$$q = 1 - \frac{1}{10} \quad [\text{Since } p + q = 1]$$

$$q = \frac{9}{10}$$

Given, $n = 400$

(i)

$$\text{Mean} = np$$

$$= 400 \times \frac{1}{10}$$

$$\text{Mean} = 40$$

(ii)

$$\text{Standard deviation} = \sqrt{npq}$$

$$= \sqrt{400 \times \frac{1}{10} \times \frac{9}{10}}$$

$$= \sqrt{36}$$

$$\text{Standard deviation} = 6$$

Binomial Distribution Ex 33.2 Q9

Let n and p be the parameters of binomial distribution.

Given, Mean = $np = 5$ --- (i)

Variance = $npq = \frac{10}{3}$ --- (ii)

Dividing (ii) by (i)

$$\frac{npq}{np} = \frac{\frac{10}{3}}{5}$$

$$q = \frac{2}{3}$$

So, $p = 1 - q$ [Since $p + q = 1$]

$$= 1 - \frac{2}{3}$$

$$p = \frac{1}{3}$$

Put the value of p in equation (i),

$$np = 5$$

$$n = 5 \times 3$$

$$n = 15$$

Hence, the binomial distribution is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^{15}C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{15-r}$$
$$r = 0, 1, 2, \dots, 15$$

Binomial Distribution Ex 33.2 Q10

Let p be the probability of a ship returning safely to parts, so

$$p = \frac{9}{10}$$

$$q = 1 - \frac{9}{10} \quad [\text{Since } p + q = 1]$$

$$q = \frac{1}{10}$$

Given, $n = 500$

$$\text{Mean} = np$$

$$= 500 \times \frac{9}{10}$$

$$\text{Mean} = 450$$

$$\text{Standard deviation} = \sqrt{npq}$$

$$= \sqrt{500 \times \frac{9}{10} \times \frac{1}{10}}$$

$$= \sqrt{45}$$

$$= 6.71$$

$$\text{Mean} = 450$$

$$\text{Standard deviation} = 6.71$$

Binomial Distribution Ex 33.2 Q11

Given that, parameters for binomial distribution are n and p .

$$\text{Also, Mean} = np = 16 \quad \text{--- (i)}$$

$$\text{Variance} = npq = 8 \quad \text{--- (ii)}$$

Dividing (ii) by (i)

$$\frac{npq}{np} = \frac{8}{16}$$

$$q = \frac{1}{2}$$

$$\text{So, } p = 1 - \frac{1}{2} \quad [\text{as } p + q = 1]$$

$$p = \frac{1}{2}$$

Put the value of p in equation (i),

$$np = 16$$

$$n\left(\frac{1}{2}\right) = 16$$

$$n = 32$$

Hence, binomial distribution is given by,

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^{32}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{32-r} \quad \text{--- (iii)}$$

$$P(X = 0)$$

$$= {}^{32}C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{32-0}$$

[Using (iii)]

$$= \left(\frac{1}{2}\right)^{32}$$

$$P(X = 1)$$

$$= {}^{32}C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{32-1}$$

$$= 32 \cdot \frac{1}{2} \left(\frac{1}{2}\right)^{31}$$

$$= \left(\frac{1}{2}\right)^{27}$$

$$P(X \geq 2)$$

$$= 1 - [P(X = 0) + P(X = 1)]$$

$$= 1 - \left[\left(\frac{1}{2}\right)^{32} + \left(\frac{1}{2}\right)^{27} \right]$$

$$= 1 - \left(\frac{1}{2}\right)^{27} \left(\frac{1}{32} + 1\right)$$

$$= 1 - \left(\frac{1}{2}\right)^{27} \left(\frac{33}{32}\right)$$

$$= 1 - \frac{33}{2^{32}}$$

Hence

$$P(X = 0) = \left(\frac{1}{2}\right)^{32}, P(X = 1) = \left(\frac{1}{2}\right)^{27}, P(X \geq 2) = 1 - \frac{33}{2^{32}}$$

Binomial Distribution Ex 33.2 Q12

Let p be the probability of success in a single throw of die

$$p = \frac{2}{6} \quad [\text{Since success is occurrence of 5 or 6}]$$

$$p = \frac{1}{3}$$

$$q = 1 - \frac{1}{3} \quad [\text{Since } p + q = 1]$$

$$q = \frac{2}{3}$$

Given, $n = 8$

$$\begin{aligned} \text{Mean} &= np \\ &= \frac{8}{3} \\ &= 2.66 \end{aligned}$$

$$\begin{aligned} \text{Standard deviation} &= \sqrt{npq} \\ &= \sqrt{8 \times \frac{1}{3} \times \frac{2}{3}} \\ &= \frac{4}{3} \\ &= 1.33 \end{aligned}$$

Mean = 2.66, Standard deviation = 1.33

Binomial Distribution Ex 33.2 Q13

Let n and p be the parameters of binomial distribution.

Let p = probability of having a boy in the family

Given, $p = q$

Since, $p + q = 1$

$$p + p = 1$$

$$2p = 1$$

$$p = \frac{1}{2}$$

$$q = \frac{1}{2}$$

$$n = 8$$

The expected number of boys = np

$$\begin{aligned} &= 8 \times \frac{1}{2} \\ &= 4 \end{aligned}$$

The expected number of boys = 4

Binomial Distribution Ex 33.2 Q14

Let p denote the probability of a defective item produced in the factory, so

$$p = 0.02$$

$$= \frac{2}{100}$$

$$p = \frac{1}{50}$$

$$q = 1 - \frac{1}{50}$$

$$= \frac{49}{50}$$

[Since $p + q = 1$]

Given $n = 10,000$

Expected number of defective item = np

$$= 10000 \times \frac{1}{50}$$

$$= 200$$

Standard deviation = \sqrt{npq}

$$= \sqrt{10000 \times \frac{1}{50} \times \frac{49}{50}}$$

$$= 14$$

Expected No. of defective items = 200

Standard deviation = 14

Binomial Distribution Ex 33.2 Q15

Let p be the probability of success, so

$$p = \frac{2}{6}$$

[Since success in occurrence of 1 or 6 on the die]

$$p = \frac{1}{3}$$

Given, $n = 3$

$$q = 1 - p$$

[Since $p + q = 1$]

$$= 1 - \frac{1}{3}$$

$$q = \frac{2}{3}$$

Mean = np

$$= 3 \left(\frac{1}{3} \right)$$

$$= 1$$

Variance = npq

$$= 3 \times \left(\frac{1}{3} \right) \left(\frac{2}{3} \right)$$

$$= \frac{2}{3}$$

Mean = 1

Variance = $\frac{2}{3}$

Binomial Distribution Ex 33.2 Q16

Let n and p be the parameters of binomial distribution

Given,

$$\text{Mean} = np = 3 \quad \text{--- (i)}$$

$$\text{Variance} = npq = \frac{3}{2} \quad \text{--- (ii)}$$

Dividing equation (ii) by (i),

$$\frac{npq}{np} = \frac{\frac{3}{2}}{3}$$

$$q = \frac{1}{2}$$

$$p = 1 - \frac{1}{2} \quad [\text{as } p + q = 1]$$

$$p = \frac{1}{2}$$

Put the value of p in equation (i)

$$np = 3$$

$$n \left(\frac{1}{2} \right) = 3$$

$$n = 6$$

Hence, binomial distribution is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^6C_r \left(\frac{1}{2} \right)^r \left(\frac{1}{2} \right)^{6-r} \quad \text{--- (iii)}$$

$$P(X \leq 5)$$

$$= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$$

$$= 1 - P(X = 6)$$

$$= 1 - {}^6C_6 \left(\frac{1}{2} \right)^6 \left(\frac{1}{2} \right)^{6-6}, \quad [\text{Using (iii)}]$$

$$= 1 - \left(\frac{1}{2} \right)^6$$

$$= 1 - \frac{1}{64}$$

$$= \frac{63}{64}$$

$$P(X \leq 5) = \frac{63}{64}$$

Binomial Distribution Ex 33.2 Q17

Let n and p be the parameters of binomial distribution.

Given,

$$\text{Mean} = np = 4 \quad \text{--- (i)}$$

$$\text{Variance} = npq = 2 \quad \text{--- (ii)}$$

Dividing equation (ii) by (i),

$$\frac{npq}{np} = \frac{2}{4}$$

$$q = \frac{1}{2}$$

$$p = 1 - \frac{1}{2} \quad [\text{Since } p + q = 1]$$

$$p = \frac{1}{2}$$

Put the value of p in equation (i),

$$np = 4$$

$$n\left(\frac{1}{2}\right) = 4$$

$$n = 8$$

Hence, binomial distribution is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = {}^8C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{8-r} \quad \text{--- (iii)}$$

$$P(X \geq 5)$$

$$= P(X = 5) + P(X = 6) + P(X = 7) + P(X = 8)$$

$$= {}^8C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^3 + {}^8C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^2 + {}^8C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right) + {}^8C_8 \left(\frac{1}{2}\right)^8$$

[Using equation (iii)]

$$= \frac{8 \times 7 \times 6}{3 \times 2} \left(\frac{1}{2}\right)^8 + \frac{8 \times 7}{2} \left(\frac{1}{2}\right)^8 + 8 \left(\frac{1}{2}\right)^8 + \left(\frac{1}{2}\right)^8$$

$$= \left(\frac{1}{2}\right)^8 [56 + 28 + 8 + 1]$$

$$= \frac{93}{256}$$

$$P(X \geq 5) = \frac{93}{256}$$

Binomial Distribution Ex 33.2 Q18

$$= 1 - \left(\frac{2}{3}\right)^4$$

$$= 1 - \frac{16}{81}$$

$$= \frac{65}{81}$$

$$P(X \geq 4) = \frac{65}{81}$$

Binomial Distribution Ex 33.2 Q19

Let n and p be the parameters of binomial distribution,

Given, $n = 6$

$$\text{Mean} + \text{Variance} = \frac{10}{3}$$

$$np + npq = \frac{10}{3}$$

$$6p + 6pq = \frac{10}{3}$$

$$6p(1+q) = \frac{10}{3}$$

$$6(1-q)(1+q) = \frac{10}{3} \quad [\text{Since } p+q=1]$$

$$6(1-q^2) = \frac{10}{3}$$

$$1-q^2 = \frac{10}{18}$$

$$-q^2 = \frac{5}{9} - 1$$

$$-q^2 = -\frac{4}{9}$$

$$q^2 = \frac{4}{9}$$

$$q = \frac{2}{3}$$

$$p = 1 - q$$

$$= 1 - \frac{2}{3}$$

$$p = \frac{1}{3}$$

Hence, the binomial distribution is given by,

$$P(X=r) = {}^nC_r p^r q^{n-r}$$

$$P(X=r) = {}^6C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{6-r}$$

as $r = 0, 1, 2, \dots, 6$

Binomial Distribution Ex 33.2 Q20

Throwing a doublet i.e. $\{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

Total number of outcomes = 36

Let p be the probability of success therefore

$$p = 6/36 = 1/6$$

Let q be the probability of failure therefore $q = 1 - p = 1 - 1/6 = 5/6$

Since the dice is thrown 4 times, $n = 4$

Let X be the random variable for getting doublet, therefore X can take at max 4 values.

$$P(X=0) = {}^4C_0 p^0 q^4 = \left(\frac{5}{6}\right)^4 = \frac{625}{1296}$$

$$P(X=1) = {}^4C_1 p^1 q^3 = 4 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^3 = \frac{500}{1296}$$

$$P(X=2) = {}^4C_2 p^2 q^2 = \frac{4 \cdot 3}{2} \cdot \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^2 = \frac{150}{1296}$$

$$P(X=3) = {}^4C_3 p^3 q^1 = 4 \cdot \left(\frac{1}{6}\right)^3 \cdot \frac{5}{6} = \frac{20}{1296}$$

$$P(X=4) = {}^4C_4 p^4 q^0 = 1 \cdot \left(\frac{1}{6}\right)^4 \cdot \left(\frac{5}{6}\right)^0 = \frac{1}{1296}$$

Mean

$$\begin{aligned} \mu &= \sum_{i=1}^4 X_i P(X_i) = 0 \cdot \frac{625}{1296} + 1 \cdot \frac{500}{1296} + 2 \cdot \frac{150}{1296} + 3 \cdot \frac{20}{1296} + 4 \cdot \frac{1}{1296} \\ &= \frac{500 + 300 + 60 + 4}{1296} = \frac{864}{1296} = \frac{2}{3} \end{aligned}$$

Hence the mean is $= \frac{2}{3}$

Binomial Distribution Ex 33.2 Q21

Throwing a doublet i.e. $\{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

Total number of outcomes = 36

Let p be the probability of success therefore

$$p = 6/36 = 1/6$$

Let q be the probability of failure therefore $q = 1 - p = 1 - 1/6 = 5/6$

Since there is three rows of dice so $n = 3$

Let X be the random variable for getting doublet, therefore X can take at max 3 values.

$$P(X=0) = {}^3C_0 p^0 q^3 = \left(\frac{5}{6}\right)^3 = \frac{125}{216}$$

$$P(X=1) = {}^3C_1 p^1 q^2 = 3 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^2 = \frac{75}{216}$$

$$P(X=2) = {}^3C_2 p^2 q^1 = 3 \cdot \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right) = \frac{15}{216}$$

$$P(X=3) = {}^3C_3 p^3 q^0 = \left(\frac{1}{6}\right)^3 = \frac{1}{216}$$

Mean

$$\begin{aligned} \mu &= \sum_{i=1}^3 X_i P(X_i) = 0 \cdot \frac{125}{216} + 1 \cdot \frac{75}{216} + 2 \cdot \frac{15}{216} + 3 \cdot \frac{1}{216} \\ &= \frac{75 + 30 + 3}{216} = \frac{108}{216} = \frac{1}{2} \end{aligned}$$

Hence the mean is $= \frac{1}{2}$

Binomial Distribution Ex 33.2 Q22

Out of 15 bulbs 5 are defective.

Hence, the probability that the drawn bulb is defective is

$$P(\text{Defective}) = \frac{5}{15} = \frac{1}{3}$$

$$P(\text{Not defective}) = \frac{10}{15} = \frac{2}{3}$$

Let X denote the number of defective bulbs out of 4.

Then, X follows binomial distribution with

$n = 4$, $p = \frac{1}{3}$ and $q = \frac{2}{3}$ such that

$$P(X=r) = {}^4C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{4-r}; r=0,1,2,3,4$$

$$\begin{aligned} \text{Mean} &= \sum_{r=0}^4 rP(r) = 1 \times {}^4C_1 \left(\frac{1}{3}\right) \left(\frac{2}{3}\right)^3 + 2 \times {}^4C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^2 \\ &+ 3 \times {}^4C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right) + 4 \times {}^4C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^0 \\ &= \frac{32}{81} + \frac{48}{81} + \frac{24}{81} + \frac{4}{81} = \frac{108}{81} = \frac{4}{3} \end{aligned}$$

Binomial Distribution Ex 33.2 Q23

Let p be the probability of getting 2 when a dice is thrown.

Then $p = \frac{1}{6}$

Clearly, X follows binomial distribution with $n = 3$, $p = \frac{1}{6}$.

$$\therefore \text{Expectation} = E(X) = np = 3 \times \frac{1}{6} = \frac{1}{2}$$

Binomial Distribution Ex 33.2 Q24

Let p be the probability of getting an even number on the toss when a dice is thrown.

Let q be the probability of not getting an even number on the toss when a dice is thrown.

Then $p = \frac{3}{6} = \frac{1}{2}$ and $q = \frac{1}{2}$

Clearly, X follows binomial distribution with $n = 2$, $p = \frac{1}{2}$.

$$\therefore \text{Variance} = npq = 2 \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$$

Binomial Distribution Ex 33.2 Q25

Let p be the probability of getting a spade card.

Let q be the probability of getting a spade card.

$$\text{Then } p = \frac{13}{52} = \frac{1}{4} \text{ and } q = \frac{3}{4}$$

Clearly, X follows binomial distribution with $n = 3$, $p = \frac{1}{4}$ and $q = \frac{3}{4}$.

Probability distribution is given by,

$$P(X = r) = {}^3C_r \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{3-r}; r = 0, 1, 2$$

$$\therefore \text{Mean} = np = 3 \times \frac{1}{4} = \frac{3}{4}$$

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