Ex 22.1

Q1

It is given that O is the origin.
Then,

$$OQ^2 = x_2^2 + y_2^2,$$

$$OP^2 = x_1^2 + y_1^2$$

and, $PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Using cosine fromula in $\triangle OPQ$, we have

$$PQ^2 = OP^2 + OQ^2 - 2.(OP)(OQ)\cos\alpha$$

$$\Rightarrow \quad (x_2 - x_1)^2 + (y_2 - y_1)^2 = x_2^2 + y_2^2 + x_1^2 + y_1^2 - 2(OP).(OQ)\cos\alpha$$

$$\Rightarrow \quad x_2^2 + x_1^2 - 2x_2x_1 + y_2^2 + y_1^2 - 2y_2y_1 = x_2^2 + y_2^2 + x_1^2 + y_1^2 - 2\,OP.OQ\cos\alpha$$

$$\Rightarrow \quad -2x_1x_2 - 2y_1y_2 = -2OP.OQ\cos\alpha$$

$$\Rightarrow \quad x_1x_2 + y_1y_2 = OP.\ OQ\cos\alpha$$

$$\Rightarrow \quad OP.OQ\cos\alpha = x_1x_2 + y_1y_2$$

Hence, proved.

Q2

We know that

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac},$$

where $a = BC$, $b = CA$ and $C = AB$ are the sides of the triangle ABC.

we have,

$$a = BC = \sqrt{(9-2)^2 + (2+1)^2} = \sqrt{49+9} = \sqrt{58}$$

$$b = CA = \sqrt{(0-9)^2 + (0+2)^2} = \sqrt{81+4} = \sqrt{85}$$

and, $c = AB = \sqrt{(2-0)^2 + (-1-0)^2} = \sqrt{4+1} = \sqrt{5}$

$$\therefore \quad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{58 + 5 - 85}{2 \times \sqrt{58} \times \sqrt{5}}$$

$$= \frac{63 - 85}{2\sqrt{290}}$$

$$= \frac{-22}{2\sqrt{290}} = \frac{-11}{\sqrt{290}}$$

Hence, $\cos B = \frac{-11}{\sqrt{290}}$.

Q3

$A(6,3), B(-3,5), C(4,-2), D(x,3x)$

$$\text{or } (\square DBC) = \frac{1}{2}\left[x_1(y_2 - y_1) + x_2(y_3 - y_2) + x_3(y_1 - y_2)\right]$$

$$= \frac{1}{2}\left[-3(-2 - 3x) + 4(3x - 5) + x(5 + 2)\right]$$

$$= \frac{1}{2}\left[6 + 9x + 12x - 20 + 5x + 2x\right]$$

$$= \frac{1}{2}\left[28x - 14\right]$$

$$= 7\left[2x - 1\right]$$

$$\text{or } (\square ABC) = \frac{1}{2}\left[6(5 + 2) - 3(-2 - 3) + 4(3 - 5)\right]$$

$$= \frac{1}{2}\left[42 + 15 - 8\right]$$

$$= \frac{49}{2}$$

$$\frac{\text{or } (\square DBC)}{\text{or } (\square ABC)} = \frac{1}{2}$$

$$\frac{7(2x - 1)}{\frac{49}{2}} = \frac{1}{2}$$

$$\frac{14(2x - 1)}{49} = \frac{1}{2}$$

$$\frac{28x - 14}{49} = \frac{1}{2}$$

$$56x - 28 = 49$$

$$56x = 28 + 49$$

$$56x = 77$$

$$x = \frac{11}{8}$$

Q4

It is given that $A(2, 0)$, $B(9, 1)$, $C(11, 6)$ and $D(4, 4)$ are the vertices of a quadrilateral.

Now,

Coordinates of the mid-point of AC are $\left(\frac{2+11}{2}, \frac{0+6}{2}\right) = \left(\frac{13}{2}, 3\right)$

Coordinates of the mid-point of BD are $\left(\frac{9+4}{2}, \frac{1+4}{2}\right) = \left(\frac{13}{2}, \frac{5}{2}\right)$

Thus, AC and BD do not have the same mid-point. Hence $ABCD$ is not a parallelogram.

$\therefore$ $ABCD$ is not a rhombus.

Q5

Let $A(-36, 7)$, $B(20, 7)$ and $C(0, -8)$ be the vertices of the triangle ABC.

Now,

$$a = BC = \sqrt{(0-20)^2 + (-8-7)^2}$$

$$= \sqrt{400+225}$$

$$= \sqrt{625}$$

$$= 25,$$

$$b = AC = \sqrt{(0+36)^2 + (-8-7)^2}$$

$$= \sqrt{1296+225}$$

$$= \sqrt{1521}$$

$$= 39$$

and, $$c = AB = \sqrt{(20+36)^2 + (7-7)^2}$$

$$= \sqrt{(56)^2}$$

$$= 56$$

The coordinates of the centre of the circle are

$$\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$$

or, $$\left[\frac{25\times(-36)+39\times 20+56\times 0}{25+39+56}, \frac{25\times 7+39\times 7+56\times(-8)}{25+39+56}\right]$$

or, $$\left[\frac{-900+780}{120}, \frac{175+273-448}{120}\right]$$

or, $$\left[\frac{-120}{120}, \frac{0}{120}\right]$$

or, $(-1, 0)$.

Hence, the coordinates of the centre of the circle are $(-1, 0)$.

Q6

It is given that ABC is an equilatral triangle.

$\therefore \quad AB = BC = AC = 2a$

Area of equilatral triangle

$$= \frac{\sqrt{3}}{4}(\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times (2a)^2$$

$$= \frac{\sqrt{3}}{4} \times 4 \times a^2$$

$$= \sqrt{3}\,a^2$$

But, area of triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$.

$\Rightarrow \quad \frac{1}{2} \times \text{Base} \times \text{Height} = \sqrt{3}\,a^2$

$\Rightarrow \quad \frac{1}{2} \times BC \times OA = \sqrt{3}\,a^2$

$\Rightarrow \quad \frac{1}{2} \times 2a \times OA = \sqrt{3}\,a^2$

$\Rightarrow \quad OA = \sqrt{3}\,a$

$\therefore$ Coordinates of A are $\left(\sqrt{3}\,a, 0\right)$ or $OA\left(-\sqrt{3}\,a, 0\right)$

Clearly, the coordinates of B and C are $(0, -a)$ and $(0, a)$ respectively.

Hence, the vertices of the triangle are $(0, a), (0, -a)$ and $\left(-\sqrt{3}\,a, 0\right)$ or $(0, a), (0, -a)$ and $\left(\sqrt{3}\,a, 0\right)$.

Q7

It is given that $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points

(i) PQ is parallel to the y-axis.

$\therefore \quad x_1 = x_2 \ \ldots\ldots\ldots\ldots (1)$

$$\therefore \quad PQ = \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right|$$

$$= \left|\sqrt{(x_2 - x_x)^2 + (y_2 - y_1)^2}\right| \quad \text{[Using equation 1]}$$

$$= \left|\sqrt{(y_2 - y_1)^2}\right|$$

$$= |y_2 - y_1|$$

(ii) PQ is parallel to the x-axis.

$\therefore \quad y_1 = y_2 \ \ldots\ldots\ldots (2)$

$$\therefore \quad PQ = \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right|$$

$$= \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right| \quad \text{[Using equation 2]}$$

$$= \left|\sqrt{(x_2 - x_1)^2}\right|$$

$$= \left|\sqrt{x_2 - x_1}\right|$$

$$\therefore \quad PQ = |x_2 - x_1|$$

Q8

It is given that C lie on the x-axis. *Let* coordinates of C be $(x, 0)$.

Now, C *is* equidistant from the points $A(7, 6)$ and $B(3, 4)$.

$\therefore \quad AC = BC \quad$ [given]

$\Rightarrow \quad AC^2 = BC^2$

$$\Rightarrow \quad \left[\sqrt{(x-7)^2 + (0-6)^2}\right]^2 = \left[\sqrt{(x-3)^2 + (0-4)^2}\right]^2$$

$$\Rightarrow \quad (x-7)^2 + (-6)^2 = (x-3)^2 + (-4)^2$$

$$\Rightarrow \quad x^2 + 49 - 14x + 36 = x^2 + 9 - 6x + 16$$

$$\Rightarrow \quad 49 + 36 - 36 - 16 - 9 = x^2 - x^2 - 6x + 14x$$

$\Rightarrow \quad 85 - 25 = 8x$

$\Rightarrow \quad 60 = 8x$

$\Rightarrow \quad 8x = 60$

$$\Rightarrow \quad x = \frac{60}{8} = \frac{15}{2}$$

Hence, coordinates of c are $\left(\frac{15}{2}, 0\right)$.

Ex 22.2

Q1

Let $P(h,k)$ be any point on the locus and let $A(2,4)$ and $B(0,k)$. Then,

$$PA = PB$$

$$\Rightarrow \quad PA^2 = PB^2$$

$$\Rightarrow \quad \left[\sqrt{(2-h)^2 + (4-k)^2}\right]^2 = \left[\sqrt{(0-h)^2 + (k-k)^2}\right]^2$$

$$\Rightarrow \quad (2-h)^2 + (4-k)^2 = (0-h)^2 + (0)^2$$

$$\Rightarrow \quad 4 + h^2 - 4h + 16 + k^2 - 8k = h^2$$

$$\Rightarrow \quad k^2 - 8k - 4h + 20 = 0$$

Hence, locus of (h,k) is $y^2 - 8y - 4x + 20 = 0$

Let $P(h,k)$ be any point on the locus and let $AC(2,4)$ and $B(0,k)$ be the given points.

Q2

Let $P(h,k)$ be any point on the locus and let $A(2,0)$ and $B(1,3)$. Then,

$$\frac{PA}{BP}=\frac{5}{4}$$

$$\Rightarrow \quad \frac{PA^2}{BP^2}=\frac{25}{16}$$

$$\Rightarrow \quad \frac{\left[\sqrt{(h-2)^2+(k-0)^2}\right]^2}{\left[\sqrt{(h-1)^2+(k-3)^2}\right]^2}=\frac{25}{16}$$

$$\Rightarrow \quad \frac{(h-2)^2+k^2}{(h-1)^2+(k^2-3)^2}=\frac{25}{16}$$

$$\Rightarrow \quad \frac{h^2+4-4h+k^2}{h^2+1-2h+k^2+9-6k}=\frac{25}{16}$$

$$\Rightarrow \quad \frac{(h^2-4h+k^2+4)}{h^2+k^2-2h-6k+10}=\frac{25}{16}$$

$$\Rightarrow \quad 16(h^2-4h+k^2+4)=25(h^2+k^2-2h-6k+10)$$

$$\Rightarrow \quad 16h^2-64h+16k^2+64=25h^2+25k^2-50h-150k+250$$

$$\Rightarrow \quad 25h^2-16h^2+25k^2-16k^2-50h+64h-150k+250-64=0$$

$$\Rightarrow \quad 9h^2+9k^2+14h-150k+186=0$$

Hence, locus of (h,k) is $9x^2+9y^2+14x-150y+186=0$

Q3

Let $P(h,k)$ be any point on the locus and let $A(ae,0)$ and $B(-ae,0)$ be the given points.
By the given condition

$$PA - PB = 2a$$

$$\Rightarrow \quad PA = 2a + PB$$

$$\Rightarrow \quad \sqrt{(ae-h)^2 + (0-k)^2} = 2a + \sqrt{(-ae-h)^2 + (0-k)^2}$$

$$\Rightarrow \quad (ae-h)^2 + k^2 = \left(2a + \sqrt{(ae+h)^2 + k^2}\right)^2 \qquad \text{[Taking square on both sides]}$$

$$\Rightarrow \quad (ae)^2 + h^2 - 2aeh + k^2 = 4a^2 + (ae+h)^2 + k^2 + 2 \times 2a \times \sqrt{(ae+h)^2 + k^2}$$

$$\Rightarrow \quad h^2 + k^2 + (ae)^2 - 2aeh = 4a^2 + (ae)^2 + h^2 + 2hae + k^2 + 4a\sqrt{(ae+h)^2 + k^2}$$

$$\Rightarrow \quad -4a^2 - 2aeh - 2aeh = 4a\sqrt{(ae+h)^2 + k^2}$$

$$\Rightarrow \quad -4a^2 - 4aeh = 4a\sqrt{(ae+h)^2 + k^2}$$

$$\Rightarrow \quad -4\left[a^2 + aeh\right] = 4a\sqrt{(ae+h)^2 + k^2}$$

$$\Rightarrow \quad -\left[a^2 + aeh\right] = a\sqrt{(ae+h)^2 + k^2}$$

$$\Rightarrow \quad -a\left[a + eh\right] = a\sqrt{(ae+h)^2 + k^2}$$

$$\Rightarrow \quad -\left[a + eh\right] = \sqrt{(ae+h)^2 + k^2}$$

Q4

Let $P(h,k)$ be any point on the locus and let $A(0,2)$ and $B(0,-2)$ be the given points.
By the given condition $PA + PB = 6$

$$\Rightarrow \quad \sqrt{(h-0)^2 + (k-2)^2} + \sqrt{(h-0)^2 + (k+2)^2} = 6$$

$$\Rightarrow \quad \sqrt{h^2 (k-2) h^2} = 6 - \sqrt{h^2 + (k+2)^2}$$

$$\Rightarrow \quad h^2 + (k-2) h^2 = 36 - 12\sqrt{h^2 + (k+2)^2} + h^2 + (k+2)^2$$

$$\Rightarrow \quad -8k - 36 = -12\sqrt{h^2 + (k+2)^2}$$

$$\Rightarrow \quad (2k+9) = 3\sqrt{h^2 + (k+2)^2}$$

$$\Rightarrow \quad (2k+9)^2 = 9\left(h^2 + (k+2)^2\right)$$

$$\Rightarrow \quad 4k^2 + 36k + 81 = 9h^2 + 9k^2 + 36k + 36$$

$$\Rightarrow \quad 9h^2 + 5k^2 = 45$$

Hence, locus of (h,k) is $9x^2 + 5y^2 = 45$.

Q5

Let $P(h,k)$ be any point on the locus and let $A(1,3)$ and $B(h,0)$. Then,

$PA = PB$

$$\Rightarrow \quad PA^2 = PB^2$$

$$\Rightarrow \quad (1-h)^2 + (3-k)^2 = (h-h)^2 + (0-k)^2$$

$$\Rightarrow \quad 1 + h^2 - 2h + 9 + k^2 - 6k = 0 + k^2$$

$$\Rightarrow \quad h^2 - 2h - 6k + 10 = 0$$

Hence, locus of (h,k) is $x^2 - 2x - 6y + 10 = 0$

Q6

Let $P(h,k)$ be any point on the locus and let $O(0,0)$ be the origin.
By the given condition

$$OP = 3k \qquad [\because\ k \text{ is the diffance of point from x-axis}]$$

$$\Rightarrow \quad OP^2 = 9k^2$$

$$\Rightarrow \quad \left(\sqrt{(0-h)^2+(0-k)^2}\right)^2 = 9k^2$$

$$\Rightarrow \quad h^2+k^2 = 9k^2$$

$$\Rightarrow \quad h^2 = 9k^2 - k^2$$

$$\Rightarrow \quad h^2 = 8k^2$$

Hence, locus of (h,k) is $x^2 = 8y^2$

Q7

Let $P(h,k)$ be any point on the locus. Then,
Area $(PAB) = 9$ sq units

$$\Rightarrow \quad \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x^3(y_1-y_2)\right| = 9$$

$$\Rightarrow \quad \left|5(-2-k)+3(k-3)+h(3+2)\right| = 18$$

$$\Rightarrow \quad \left|-10-5k+3k-9+5h\right| = 18$$

$$\Rightarrow \quad \left|5h-2k-19\right| = 18$$

$$\Rightarrow \quad 5h-2k-19 = \pm 18$$

$$\Rightarrow \quad 5h-2k-19 \mp 18 = 0$$

$$\Rightarrow \quad 5h-2k-37 = 0 \quad \text{or,} \quad 5h-2k-1 = 0$$

Hence, the locus of (h,k) is

$$5x-2y-37=0 \quad \text{or,} \quad 5x-2y-1=0.$$

Q8

Let $P(h,k)$ be the variable point and let $A(2,0)$ and $B(-2,0)$ be the given points.

Then $\angle APB = \pi/2$

$\Rightarrow \quad AB^2 = PA^2 + PB^2$

$\Rightarrow \quad (2+2)^2 + 0 = (2-h)^2 + (0-k^2) + (-2-h)^2 + (0-k)^2$

$\Rightarrow \quad 16 = 4 + h^2 - 4h + k^2 + 4 + h^2 + 4h + k^2$

$\Rightarrow \quad 16 = 2h^2 + 2k^2 + 8$

$\Rightarrow \quad 2h^2 + 2k^2 + 8 - 16 = 0$

$\Rightarrow \quad 2h^2 + 2k^2 - 8 = 0$

$\Rightarrow \quad h^2 + k^2 - 4 = 0$

Hence, the locus of (h,k) is $x^2 + y^2 = 4$.

Q9

Let $P(h,k)$ be any point on the locus. Then,

Area $(PAB) = 8$ sq units

$\Rightarrow \quad \frac{1}{2}\left|x_1(y_2-y_3) + (y_3-y_1) + x_3(y_1-y_2)\right| = 8$

$\Rightarrow \quad \frac{1}{2}\left|1(3-k) + 2(k-1) + h(1-3)\right| = 8$

$\Rightarrow \quad \frac{1}{2}\left|-3 + k + 2k - 2 - 2h\right| = 8$

$\Rightarrow \quad \frac{1}{2}\left|-2h + 3k - 5\right| = 8$

$\Rightarrow \quad \left|-2h + 3k - 5\right| = 16$

$\Rightarrow \quad -2h + 3k - 5 = \pm 16$

$\Rightarrow \quad 2h - 3k + 5 \pm 16 = 0$

$\Rightarrow \quad 2h - 3k + 21 = 0 \quad$ or, $\quad 2h - 3k - 11 = 0$

Hence, the locus of (h,k) is

$2x - 3y + 21 = 0 \quad$ or, $\quad 2x - 3y - 11 = 0$

Q10

Let the two perpendicular lines be the coordinate axes. Let AB be a rod length l. Let the coordinates of A and B be $(a,0)$ and $(0,b)$ respectively. As the rod slides the value of a and b change. so, a and b are two variables.
Let $P(h,k)$ be the point on the locus. Then,

$$h = \frac{2\times a + 1\times 0}{2+1}$$

$$\Rightarrow \quad h = \frac{2a}{3}$$

$$\Rightarrow \quad a = \frac{3h}{2}$$

and $k = \dfrac{2\times 0 + b\times 1}{2+1}$

$$\Rightarrow \quad k = \frac{b}{3}$$

$$\Rightarrow \quad b = 3k$$

from $\triangle AOB$, we have

$$AB^2 = OA^2 + OB^2$$

$$\Rightarrow \quad l^2 = \left[(a-0)^2 + (0,0)\right]^2 + \left[(0-0)^2 + (b-0)^2\right]$$

$$\Rightarrow \quad l^2 = a^2 + b^2$$

$$\Rightarrow \quad a^2 + b^2 = l^2$$

$$\Rightarrow \quad \left(\frac{3h}{2}\right)^2 + (3k)^2 = l^2$$

$$\Rightarrow \quad \frac{9h^2}{4} + 9k^2 = l^2$$

$$\Rightarrow \quad \frac{h^2}{4} + k^2 = \frac{l^2}{9}$$

Hence, the locus of (h,k) is $\dfrac{x^2}{4} + y^2 = \dfrac{l^2}{9}$

Q11

Gicen, line is

$$x\cos\alpha + y\sin\alpha = p$$

$$\frac{x}{\frac{p}{\cos\alpha}} + \frac{y}{\frac{p}{\sin\alpha}} = 1$$

Intercepts on x axis is $\frac{p}{\cos\alpha}$ and y - axis is $\frac{p}{\sin\alpha}$

Let $P(x, y)$ be the mid point of AB.

$$(x, y) = \left(\frac{\frac{p}{\cos\alpha}+0}{2}, \frac{\frac{p}{\sin\alpha}+0}{2}\right) = \left(\frac{p}{2\cos\alpha}, \frac{p}{2\sin\alpha}\right)$$

$$\therefore x = \frac{p}{2\cos\alpha},\ y = \frac{p}{2\sin\alpha}$$

$$2\cos\alpha = \frac{p}{x},\quad 2\sin\alpha = \frac{p}{y}$$

Square both sides,

$$4\cos^2\alpha = \frac{p^2}{x^2} \text{----} (1)$$

and

$$4\sin^2\alpha = \frac{p^2}{y^2} \text{-----} (2)$$

$[(1)+(2)]$

$$4\cos^2\alpha + 4\sin^2\alpha = \frac{p^2}{x^2} + \frac{p^2}{y^2}$$

$$4 = \frac{p^2\left(x^2+y^2\right)}{x^2y^2}$$

$$4x^2y^2 = p^2\left(x^2+y^2\right)$$

Q12

Let $P(h,k)$ be the point on the locus and let the coordinates of a are (a,b). Then,

$h = \frac{a+0}{2}$ and $\frac{b+0}{2} = k$ $\quad$ $[\because P$ is the mid-point of Q and the origino$]$

$h = \frac{a}{2}$ and $b = 2k$

$\Rightarrow$ $a = 2h$ and $b = 2k$

point Q lies on the $y^2 = x$. Then,

$b^2 = a$ $\quad$ $[\because Q : (a,b)]$

$\Rightarrow$ $(2k)^2 = 2h$ $\quad$ $[\because a = 2h$ and $b = 2k]$

$\Rightarrow$ $4k^2 = 2h$

$\Rightarrow$ $2k^2 = h$

Hence, the locus of (h,k) is $2y^2 = x$.

Ex 22.3

Q1

We have,

$$(x-a)^2+(y-b)^2=r^2 \quad \text{........}(i)$$

Substituting $x=X+(a-c)$, $y=Y+b$ in the equation (i), we get

$$[X+a-c-a]^2+[Y+b-b]^2=r^2$$

$$\Rightarrow \quad [X-c]^2+[Y]^2=r^2$$

$$\Rightarrow \quad X^2+c^2-2Xc+Y^2=r^2$$

$$\Rightarrow \quad X^2+y^2-2cX=r^2-c^2$$

Hence, the required equation is $X^2+y^2-2cX=r^2-c^2$

Q2

We have,

$$(a-b)\left(x^2+y^2\right)-2abx=0$$

Substituting $x=X+\frac{ab}{a-b}$, $y=Y$
in the given equation, we get

$$(a-b)\left[\left(X+\frac{ab}{a-b}\right)^2+Y^2\right]-2ab\left[X+\frac{ab}{a-b}\right]=0$$

$$\Rightarrow \quad (a-b)\left[X^2+\left(\frac{ab}{a-b}\right)^2+2\frac{Xab}{a-b}+Y^2\right]-2abX-2\frac{(ab)^2}{a-b}=0$$

$$\Rightarrow \quad (a-b)\left[\frac{X^2(a-b)^2+(ab)^2+2Xab(a-b)+Y^2(a-b)^2}{(a-b)^2}\right]-\frac{2abX(a-b)-2(ab)^2}{a-b}=0$$

$$\Rightarrow \quad \frac{X^2(a-b)^2+(ab)^2+2ab(a-b)+Y^2(a-b)^2}{a-b}=\frac{2ab(a-b)+2(ab)^2}{a-b}$$

$$\Rightarrow \quad X^2(a-b)^2+Y^2(a-b)^2+(ab)^2+2ab(a-b)=2ab(a-b)+2(ab)^2$$

$$\Rightarrow \quad (a-b)^2\left(X^2+Y^2\right)=(ab)^2$$

$$\Rightarrow \quad (a-b)^2\left(X^2+Y^2\right)=a^2b^2$$

Q3(i)

We have,

$$x^2 + xy - 3x - y + 2 = 0$$

Substituting $x = X + 1$, $Y + 1$ in the equation, we get

$$(X+1)^2 + (X+1)(Y+1) - 3(X+1) - (Y+1) + 2 = 0$$

$$\Rightarrow \quad X^2 + 1 + 2X + XY + X + Y + 1 - 3X - 3 - Y - 1 + 2 = 0$$

$$\Rightarrow \quad X^2 + XY = 0$$

Q3(ii)

We have,

$$x^2 - y^2 - 2x + 2y = 0$$

Substituting $x = X + 1$, $y = Y + 1$ in the equation, we get

$$(X+1)^2 - (Y-1)^2 - 2(X+1) + 2(Y+1) = 0$$

$$\Rightarrow \quad X^2 + 1 + 2X - (Y^2 + 1 + 2Y) - 2X - 2 + 2Y + 2 = 0$$

$$\Rightarrow \quad X^2 + 1 - Y^2 - 1 - 2Y + 2Y = 0$$

$$\Rightarrow \quad X^2 - Y^2 = 0$$

Q3(iii)

We have,

$$xy - x - y + 1 = 0$$

Substituting $x = X + 1$, $y = Y + 1$ in the equation, we get

$$(X+1)(Y+1) - (X+1) - (Y+1) + 1 = 0$$

$$\Rightarrow \quad XY + X + Y + 1 - X - 1 - Y - 1 + 1 = 0$$

$$\Rightarrow \quad XY = 0$$

Q3(iv)

We have,

$$xy - y^2 - x + y = 0$$

Substituting $x = X + 1, \quad y = Y + 1$ in the equation, we get

$$(X+1)(Y+1) - (Y+1)^2 - (X+1) + (Y+1) = 0$$

$$\Rightarrow \quad XY + Y + X + 1 - (Y^2 + 1 + 2Y) - X - 1 + Y + 1 = 0$$

$$\Rightarrow \quad XY + 2Y - Y^2 - 1 - 2Y + 1 = 0$$

$$\Rightarrow \quad XY - Y^2 = 0$$

Q4

We have,

$$x^2 + xy - 3x - y + 2 = 0 \ldots\ldots (i)$$

Let the origin be shifted to (h, k). Then $x = X + h$ and $y = Y + k$.

Substituting $x = X + h, \quad y = Y + k$
in the equation (i), we get

$$(X+h)^2 + (X+h)(Y+k) - 3(X+h) - (Y+k) + 2 = 0$$

$$\Rightarrow \quad X^2 + h^2 + 2Xh + XY + Xk + Yh + hk - 3X - 3h - Y - k + 2 = 0$$

$$\Rightarrow \quad X^2 + XY + 2Xh + Xk + Yh - Y - 3X + h^2 + hk - 3h - k + 2 = 0$$

$$\Rightarrow \quad X^2 + (2Xh + Xk - 3X) + XY + (Yh - Y) + (h^2 + hk - 3h - k + 2) = 0$$

$$\Rightarrow \quad X^2 + (2h + k - 3)X + XY + (h-1)Y + (h^2 + hk - 3h - k + 2) = 0$$

For this equation to be free from first degree and the constant term, we must have,

$$2h + k - 3 = 0 \ldots\ldots\ldots (i)$$
$$h - 1 = 0$$
$$\Rightarrow \quad h = 1 \ldots\ldots\ldots (ii)$$

and

$$h^2 + hk - 3h - k + 2 = 0 \ldots\ldots\ldots (iv)$$

Putting $h = 1$ in equation (i) we get

$$2 + k - 3 = 0$$
$$\Rightarrow \quad k = 1$$

Putting $h = 1$ and $k = 1$ in equation (iv), we get

$$(1)^2 + 1 - 3 - 1 + 2 = 0$$

Hence, the value of h and k satisfies the equation (iv)

$\therefore$ The origin is shifted at the point $(1, 1)$.

Q5

Let the vertices of a triangle be $A(2,3)$, $B(5,7)$ and $C(-3,-1)$.

Then, area of $\triangle ABC$ is given by

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|2(7+1) + 5(-1-3) - 3(-3-7)\right|$$
$$= \frac{1}{2}\left|2 \times 8 + 5 \times (-4) - 3 \times (-4)\right|$$
$$= \frac{1}{2}\left|16 - 20 + 12\right|$$
$$= \frac{8}{2}$$
$$= 4$$

$\Rightarrow \quad \Delta = 4 \text{sq unit}$

It is given that the origin is shifted at $(-1,3)$. Then new coordinates of the vertices are

$$A_1 = (2-3, 3+3) = (-1, 6)$$
$$B_1 = (5-1, 7+3) = (4, 10)$$

and $\quad C_1 = (-3-1, -1+3) = (-4, 2)$

Therefore, the area of the triangle in the new coordinate system is given by

$$\Delta_1 = \frac{1}{2}\left|\left[-1(10-2) + 4(2-6) - 4(6-10)\right]\right|$$
$$= \frac{1}{2}\left|\left[-1 \times 8 + 4 \times (-4) - 4 \times (-4)\right]\right|$$
$$= \frac{1}{2}\left|\left[-8 - 16 + 16\right]\right|$$
$$= \frac{1}{2}\left|-8\right|$$
$$= \frac{8}{2}$$

$\Rightarrow \quad \Delta_1 = 4 \ldots\ldots\ldots\ldots\ldots (2)$

From (i) and (ii), we get

$$\Delta = \Delta_1$$

Hence, the area of a triangle is invariant under the translation of the axes.

Q6(i)

We have,

$$x^2 + xy - 3y^2 - y + 2 = 0 \ldots\ldots\ldots (i)$$

Substituting $x = X + 1$, $y = Y + 1$
in equation (i), we get

$$(X+1)^2 + (X+1)(Y+1) - 3(Y+1)^2 - (Y+1) + 2 = 0$$

$\Rightarrow \quad X^2 + 1 + 2X + XY + X + Y + 1 - 3(Y^2 + 1 + 2Y) - Y - 1 + 2 = 0$

$\Rightarrow \quad X^2 + XY + 3X + 3 - 3Y^2 - 3 - 5Y = 0$

$\Rightarrow \quad X^2 - 3Y^2 - XY + 3X - 6Y = 0$

Q6(ii)

We have,

$$xy - y^2 - x + y = 0 \ldots\ldots (i)$$

Substituting $x = X + 1$, $y = Y + 1$
in equation (i), we get

$$(X+1)(Y+1) - (Y+1)^2 - (X+1) + (Y+1) = 0$$

$$\Rightarrow \quad XY + X + Y + 1 - \left(y^2 + 1 + 2Y\right) - X - 1 + Y + 1 = 0$$
$$\Rightarrow \quad XY + 2Y + 1 - Y^2 - 1 - 2Y = 0$$
$$\Rightarrow \quad XY - Y^2 = 0$$

Q6(iii)

We have,

$$xy - x - y + 1 = 0 \ldots\ldots (i)$$

Substituting $x = X + 1$, $y = Y + 1$
in equation (i), we get

$$(X+1)(Y+1) - (X+1) - (Y+1) + 1 = 0$$

$$\Rightarrow \quad XY + X + Y + 1 - X - 1 - Y - 1 + 1 = 0$$
$$\Rightarrow \quad XY = 0$$

Q6(iv)

We have,

$$x^2 - y^2 - 2x - 2y = 0 \ldots\ldots (i)$$

Substituting $x = X + 1$, $y = Y + 1$
in equation (i), we get

$$(X+1)^2 - (Y+1)^2 - 2(X+1) + 2(Y+1) = 0$$

$$\Rightarrow \quad X^2 + 1 + 2X - \left(Y^2 + 1 + 2Y\right) - 2X - 2 + 2Y + 2 = 0$$
$$\Rightarrow \quad X^2 + 1 - Y^2 - 1 - 2Y + 2X = 0$$

$$\Rightarrow \quad X^2 + 2X + 1 - \left(Y^2 + 2Y + 1\right)$$
$$\Rightarrow \quad (X+1)^2 - (Y+1)^2$$
$$\Rightarrow \quad x^2 - y^2 = 0$$

Q7(i)

Let the origin be shifted to (h,k). Then, $x = X + h$ and $y = Y + k$.

Substituting $x = X + h$, $\quad y = Y + k$
in the equation $y^2 + x^2 - 4x - 8y + 3 = 0$, we get

$$(Y+k)^2 + (X+h)^2 - 4(X-h) - 8(Y+k) + 3 = 0$$

$\Rightarrow \quad Y^2 + k^2 + 2Yk + X^2 + h^2 + 2Xh - 4X - 4h - 8Y - 8k + 3 = 0$

$\Rightarrow \quad Y^2 + X^2 + 2Yk - 8Y + 2Xh - 4X + k^2 + h^2 - 4h - 8k + 3 = 0$

$\Rightarrow \quad Y^2 + X^2 + (2k-8)Y + (2h-4)X + (k^2 + h^2 - 4h - 8k + 3) = 0$

For this equation to be free from the term of first degree, we must have

$2k - 8 = 0$ and $2h - 4 = 0$

$\Rightarrow \quad k = 4$ and $h = 2$

Hence, the origin is shifted at the point $(2,4)$.

Q7(ii)

Let the origin be shifted to (h,k). Then, $x = X + h$ and $y = Y + k$
Substituting $x = X + h$, $\quad y = Y + k$
in the equation $x^2 + y^2 - 5x + 2y - 5 = 0$, we get

$$(X+h)^2 + (Y+k)^2 - 5(X+h) + 2(Y+k) - 5 = 0$$

$\Rightarrow \quad X^2 + h^2 + 2xh + Y^2 + k^2 + 2Yk - 5X - 5h + 2Y + 2k - 5 = 0$

$\Rightarrow \quad X^2 + Y^2 + 2Yk + 2Y + 2Xh - 5x + h^2 + k^2 - 5h + 2k - 5 = 0$

$\Rightarrow \quad X^2 + Y^2 + (2k+2)Y + (2h-5)X + h^2 + k^2 - 5h + 2k - 5 = 0$

For this equation to be free from the term of first degree, we must have

$2k + 2 = 0$ and $2h - 5 = 0$

$\Rightarrow \quad k = -1$ and $h = \frac{5}{2}$

Hence, the origin is shifted at the point $\left(\frac{5}{2}, -1\right)$.

Q7(iii)

Let the origin be shifted to (h,k). Then, $x = X + h$ and $y = Y + k$
Substituting $x = X + h$, $\quad y = Y + k$
in the equation $x^2 - 12x + 4 = 0$, we get

$$(X+h)^2 - 12(X-h)^2 + 4 = 0$$

$\Rightarrow \quad X^2 + h^2 + 2Xh - 12X - 12h + 4 = 0$

$\Rightarrow \quad X^2 + (2h-12)X + h^2 - 12h + 4 = 0$

For this equation to be free from term of first degree, we must have

$2h - 12 = 0$

$\Rightarrow \quad h = \frac{12}{2}$

$\Rightarrow \quad h = 6$

Hence, the origin is shifted at the point $(6,k)\ k \in R$.

Q8

Let the co-ordinate of the vertex be A(4,6) B(7,10) and C(1,-2)

Now area of the ΔABC is given by

$$\Delta = \frac{1}{2}\left|\left(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right)\right|$$
$$= \frac{1}{2}\left|\left(4(10+2) + 7(-2-6) + 1(6-10)\right)\right|$$
$$= \frac{1}{2}\left|(48 - 56 - 4)\right|$$
$$= 6$$

After transforming the origin to (-2,1), the co-ordinate of the vertex will be

A(2,7),B(5,11) and C(-1,-1). Now the area will be

$$\Delta_1 = \frac{1}{2}\left|\left(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right)\right|$$
$$= \frac{1}{2}\left|\left(2(11+1) + 5(-1-7) - 1(7-11)\right)\right|$$
$$= \frac{1}{2}\left|(24 - 40 + 4)\right|$$
$$= 6$$

Here $\Delta = \Delta_1$

Hence proved.