Heron's Formula
Exercise 12.1

1. An isosceles right triangle has area 8 cm². The length of its hypotenuse is

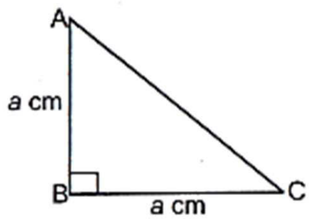
(a) $\sqrt{32}cm$

(b) $\sqrt{16}cm$

(c) $\sqrt{48}cm$

(d) $\sqrt{24}cm$

Sol. ABC is an isosceles right triangle. We have



AB = BC = a

Area of $\Delta = \frac{1}{2} base \times Height$

$\Rightarrow \quad 8 = \frac{1}{2} \times a \times a \qquad [\because AB = BC = a]$

$\Rightarrow \quad a^2 = 16, \therefore a = +\sqrt{16} = 4cm$

Now, hyp. $AC = \sqrt{4^2 + 4^2} = \sqrt{16+16} = \sqrt{32}cm$

Hence, (a) is the correct answer.

2. The perimeter of an equilateral triangle is 60 m, then its area is:

(a) $10\sqrt{3}m^2$

(b) $15\sqrt{3}m^2$

(c) $20\sqrt{3}m^2$

(d) $100\sqrt{3}m^2$

Sol. Perimeter of triangle = 3a

Now, $3a = 60 \Rightarrow a = 60 \div 3 = 20m$

Area of equilateral $\Delta = \frac{\sqrt{3}}{4}(side)^2 = \frac{\sqrt{3}}{4} \times (20)^2 = 100\sqrt{3}m^2$

Hence, (d) is the correct answer.

3. The sides of a triangle are 56 cm, 60 cm and 52 cm long. Then the area of the triangle is

(A) 1322 cm²

(B) 1311 cm²

(C) 1344 cm²

(D) 1392 cm²

Sol. Since, the three sides of triangle are a = 56 cm, ab = 60 cm and c = 52 cm.
Then the semi-perimeter of triangle,

$$s = \frac{a+b+c}{2} = \frac{56+60+52}{2} = \frac{168}{2} = 84cm$$

Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ [By heron's formula]

$$= \sqrt{84(84-56)(84-60)(84-52)}$$
$$= \sqrt{84 \times 28 \times 24 \times 32}$$
$$= \sqrt{4 \times 7 \times 3 \times 4 \times 7 \times 4 \times 2 \times 3 \times 4 \times 4 \times 2}$$
$$= \sqrt{(4)^6 \times (7)^2 \times (3)^2}$$
$$= (4)^2 \times 7 \times 3$$

= 1344 cm^2.

Hence, the area of triangle is 1344 cm^2.
Therefore, (c) is the correct answer.

4. The area of an equilateral triangle with side $2\sqrt{3}cm$ is

(a) 5.196 cm^2
(b) 0.886 cm^2
(c) 3.496 cm^2
(d) 1.732 cm^2

Sol. Area of equilateral $\Delta = \frac{\sqrt{3}}{4}$ (side)^2

$$= \frac{\sqrt{3}}{4}(2\sqrt{3})^2 = 3\sqrt{3} = 3 \times 1.732$$

= 5196 cm2

Hence, (a) is the correct answer.

5. The length of each side of an equilateral triangle having an area of $9\sqrt{3}cm^2$ is

(A) 8 cm
(B) 36 cm
(C) 4 cm
(D) 6 cm

Sol. Area of equilateral Δ i.e., $9\sqrt{3} = \frac{\sqrt{3}}{4}$ (side)^2

$$\Rightarrow \quad (side)^2 = \frac{9\sqrt{3} \times 4}{\sqrt{3}} = 36$$

$$\therefore \quad side = +\sqrt{36} = 6cm$$

Hence, (d) is the correct answer.

6. If the area of an equilateral triangle is $16\sqrt{3}cm^2$ then the perimeter of the triangle is

(A) 48 cm
(B) 24 cm
(C) 12 cm
(D) 36 cm

Sol. Area of equilateral $\Delta = \frac{\sqrt{3}}{4}$ (side)^2

$\Rightarrow \quad 16\sqrt{3} = \frac{\sqrt{3}}{4}$ (side)^2

$\Rightarrow \quad (side)^2 = \frac{16\sqrt{3}\times 4}{\sqrt{3}} = 64$

$\therefore \quad Side = +\sqrt{64} = 8cm$

So, perimeter of triangle = 8 + 8 + 8 = 24 cm

Hence, (b) is the correct answer.

7. The sides of a triangle are 35 cm, 54 cm and 61 cm, respectively. The length of its longest altitude

(a) $16\sqrt{5}cm$

(b) $10\sqrt{5}cm$

(c) $24\sqrt{5}cm$

(d) 28 cm

Sol. Sides of the triangle are 35 cm, 54 cm and 61 cm

$$s = \frac{35+54+61}{2} = 75cm$$

Area of $\Delta = \sqrt{75(75-35)(75-54)(75-61)}$

$$= \sqrt{75\times 40\times 21\times 14}$$

$$= \sqrt{5\times 5\times 3\times 2\times 2\times 2\times 5\times 3\times 7\times 7\times 2}$$

$$= 5\times 3\times 2\times 2\times 7\sqrt{5} = 420\sqrt{5}cm^2$$

Now, longest altitude will be the perpendicular on the smallest side of the triangle from the opposite vertex.

$\therefore$ Length of longest altitude $= \frac{2(Area\ of\ \Delta)}{35}$

$$= \frac{2\times 420\sqrt{5}}{35} = 24\sqrt{5}cm$$

Hence, (c) is the correct answer.

8. The area of an isosceles triangle having base 2 cm and the length of its equal sides 4 cm, is

(a) $\sqrt{15}cm^2$

(b) $\sqrt{\frac{15}{2}}cm^2$

(c) $2\sqrt{15}cm^2$

(d) $4\sqrt{15}cm^2$

Sol. Here, $s=\dfrac{4+4+2}{2}=5cm$

Area of $\Delta=\sqrt{5(5-2)(5-4)(5-4)}$

$=\sqrt{5\times3\times1\times1}=\sqrt{15}cm^2$

Hence, (a) is the correct answer.

9. The edges of a triangular board are 6 cm, 8 cm and 10 cm. The cost of painting it at the rate of 9 paise per cm^2 is

(A) Rs2.00

(B) Rs2.16

(C) Rs2.48

(D) Rs3.00

Sol. Here, $2s=6+8+10=24\Rightarrow s=24\div2=12cm$

Area of $\Delta s=\sqrt{12(12-6)(12-8)(12-10)}$

$=\sqrt{2\times6\times6\times4\times2}=2\times6\times2=24cm^2$

Cost of painting at the rate of 9 paise per $cm^2=Rs(24\times0.09)=Rs2.16$

Hence, (b) is the correct answer.

Heron's Formula
Exercise 12.2

Write whether the following statements are True or False and justify your answer:

1. The area of a triangle with base 4 cm and height 6 cm is 24 cm².

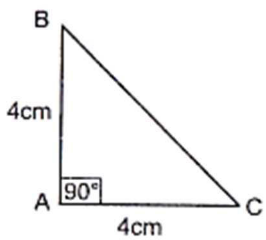
Sol. Area of $\Delta = \frac{1}{2} \times base \times height = \frac{1}{2} \times 4 \times 6 = 12 cm^2$

Hence, the statement is false.

2. The area of Δ ABC is 8 cm² in which AB = AC = 4 cm and ∠A= 90º.

Sol. Area of $\Delta = \frac{1}{2} \times base \times height$

$= \frac{1}{2} \times 4 \times 4 = 8 cm^2$



Hence, the given statement is true.

3. The area of the isosceles triangle is $\frac{5}{4}\sqrt{11} cm^2$ if the perimeter is 11 cm and the base is 5 cm.

Sol. Let the equal sides of the isosceles triangle be 'a' and base of the triangle be 'b'.

Perimeter of $\Delta s = 5 + a + a = 11$

$\Rightarrow \quad 2a = 11 - 5 = 6; a = 6 \div 2 = 3 cm$

Area of isosceles $\Delta = \frac{b}{4}\sqrt{4a^2 - b^2} = \frac{5}{4}\sqrt{4(3)^2 - 5^2}$

$= \frac{5}{4}\sqrt{11} cm^2$

Hence, the given statement is true.

4. The area of the equilateral triangle is $20\sqrt{3} cm^2$ whose each side is 8 cm.

Sol. Area of equilateral $\Delta = \frac{\sqrt{3}}{4}(side)^2$

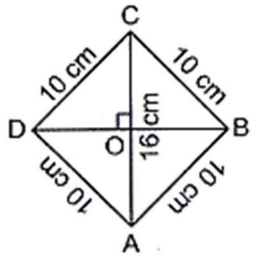
$= \frac{\sqrt{3}}{4}(8)^2 = \frac{\sqrt{3}}{4} \times 64 = 16\sqrt{3} cm^2$

Hence, the given statement is false.

5. If the side of a rhombus is 10 cm and one diagonal is 16 cm, the area of the rhombus is 96 cm².

Sol. Let ABCD be the rhombus whose one diagonal AC is 16cm. Each side of rhombus is 10cm.
We know that diagonal of a rhombus bisect each other at right angles, so

OA = OC = 8 cm and OB = OD.



In ΔAOB, we have $\angle AOB = 90^0$

$\therefore \quad AB^2 = OA^2 + OB^2$

$\Rightarrow \quad OB^2 = AB^2 - OA^2$

$= (10)^2 - 8^2 = 100 - 64 = 36$

$\therefore \quad OB = +\sqrt{36} = 6cm$

$\therefore \quad DB = 2(OB) = 2 \times 6 = 12cm$

Hence, area of rhombus $= \frac{1}{2} \times$ Product of diagonals

$= \frac{1}{2} \times 16 \times 12 = 96cm^2$

Hence, the given statement is true.

6. The base and the corresponding altitude of a parallelogram are 10 cm and 3.5 cm, respectively. The area of the parallelogram is 30 cm².

Sol. The base of the parallelogram is 10 cm and the corresponding altitude is 3.5 cm.

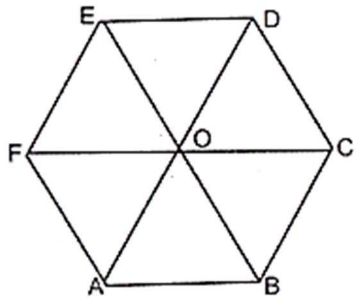
Area of ||gm = base × corresponding altitude.

= 10 cm × 3.5 cm = 35 cm².

Hence, the given statement is false.

7. The area of a regular hexagon of side 'a' is the sum of the areas of the five equilateral triangles with side a.

Sol. We see a regular hexagon is divided into six equilateral triangles.
So, the area of the regular hexagon is divided side 'a' is the sum of area of the six equilateral triangles with side a.



Hence, the given statement that the area of a regular hexagon of side 'a' is the sum of the areas of the five equilateral triangles with side a is false.

8. The cost of levelling the ground in the form of a triangle having the sides 51 m, 37 m and 20 m at the rate of Rs3 per m² is Rs918.

Sol. We have 2s = 51 m + 37 m + 20 m= 180 m

$\Rightarrow \quad 108 \div 2 = 54m$

Area of $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{54(54-51)(54-37)(54-20)}$

$= \sqrt{54 \times 3 \times 17 \times 34}$

$= \sqrt{3 \times 3 \times 3 \times 2 \times 3 \times 17 \times 17 \times 2}$

$= 3 \times 3 \times 17 \times 2 = 306m^2$

Cost of leveling the ground = Rs(306 × 3) Rs918

Hence, the given statement is true.

9. In a triangle, the sides are given as 11 cm, 12 cm and 13 cm. The length of the altitude is 10.25 cm corresponding to the side having length 12 cm.

Sol. We have the length of the altitude corresponding to the side having length 12 cm.

2s = 11 cm + 12 cm + 13 cm = 36 cm

$\Rightarrow \quad s = 36 \div 2 = 18cm$

Area of $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{18(18-11)(18-12)(18-13)}$

$= \sqrt{18 \times 7 \times 6 \times 5} = \sqrt{2 \times 3 \times 3 \times 7 \times 2 \times 3 \times 5}$

$= 2 \times 3\sqrt{105} = 6\sqrt{105}cm^2$

Length of altitude $= \dfrac{2 Area\, of\, \Delta}{Base} = \dfrac{2 \times 6\sqrt{105}}{12}$

$= \sqrt{105} = 10.25cm$

Hence, the given statement is true.

Heron's Formula
<u>Exercise 12.3</u>

1. Find the cost of laying grass in a triangular field of sides 50 m, 65 m and 65 m at the rate of Rs7 per m².

Sol. We have, 2s = 50 m + 65 m + 65 m = 180 m

S = 180 ÷ 2 = 90 m

$$\text{Area of } \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{90(90-50)(90-65)(90-65)}$$

$$= \sqrt{90\times40\times25\times25} = 60\times25$$

= 1500m².

Cost of laying grass at the rate of Rs7 per m² = Rs(1500 × 7) = Rs10,500

2. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 13 m, 14 m and 15 m. The advertisements yield an earning of Rs2000 per m² a year. A company hired one of its walls for 6 months. How much rent did it pay?

Sol. The sides of triangular side walls of flyover which have been used for advertisements are 13 m, 14 m, 15 m.

$$s = \frac{13+14+15}{2} = \frac{42}{2} = 21m$$

$$= \sqrt{21(21-13)(21-14)(21-15)}$$

$$= \sqrt{21\times8\times7\times6}$$

$$= \sqrt{7\times3\times2\times2\times2\times7\times3\times2}$$

$$= 7\times3\times2\times2 = 84m^2$$

It is given that the advertisement yield an earning of Rs2,000 per m² a year.

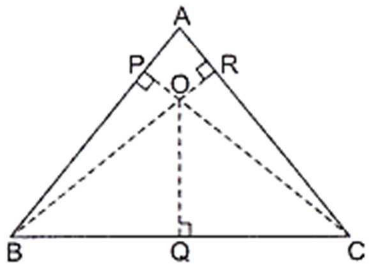
$\therefore$ Rent for 1 m² for 1 year = Rs2000

So, rent for 1 m² for 6 months or $\frac{1}{2}\, year = Rs(\frac{1}{2}\times2000) = Rs1,000.$

$\therefore$ Rent for 84 m² for 6 months = Rs(1000 × 84) = Rs84,000.

3. From a point in the interior of an equilateral triangle, perpendiculars are drawn on the three sides. The lengths of the perpendiculars are 14 cm, 10 cm and 6 cm. Find the area of the triangle.

Sol. Let ABC be an equilateral triangle, O be the interior point and OQ, OR and OC are the perpendicular drawn from points O. Let the sides of an equilateral triangle be a m.



Area of $\Delta AOB = \frac{1}{2} \times AB \times OP$

[$\because$ Area of a triangle $= \frac{1}{2} \times (base \times height)$]

$= \frac{1}{2} \times a \times 14 = 7a\,cm^2$ ...(1)

Area of $\Delta OBC = \frac{1}{2} \times BC \times OQ = \frac{1}{2} \times a \times 10$

= 5a cm^2 ...(2)

Area of $\Delta OAC = \frac{1}{2} \times AC \times OR = \frac{1}{2} \times a \times 6$

= 3a cm^2 ...(3)

$\therefore$ Area of an equilateral ΔABC

$= Area\,of\ (\Delta OAB + \Delta OBC + \Delta OAC)$

= (7a + 5a + 3a) cm^2

= 15a cm^2 ...(4)

We have, semi-perimeter $s = \dfrac{a+a+a}{2}$

$\Rightarrow \quad s = \frac{3a}{2} cm$

$\therefore$ Area of an equilateral $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

[By Heron's formula]

$$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2} - a\right)\left(\frac{3a}{2} - a\right)\left(\frac{3a}{2} - a\right)}$$

$$= \sqrt{\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}}$$

$$= \frac{\sqrt{3}}{4} a^2 \quad ...(5)$$

From equations (4) and (5), we get

$$\frac{\sqrt{3}}{4} a^2 = 15a$$

$\Rightarrow \qquad a = \frac{15 \times 4}{\sqrt{3}} = \frac{60}{\sqrt{3}}$

$\Rightarrow \qquad a = \frac{60}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}} = 20\sqrt{3}cm$

On putting $a = 20\sqrt{3}$ in equation (5), we get

Area of $\Delta ABC = \frac{\sqrt{3}}{4}(20\sqrt{3})^2 = \frac{\sqrt{3}}{4} \times 400 \times 3 = 300\sqrt{3}cm^2$

Hence, the area of an equilateral triangle is $300\sqrt{3}cm^2$.

4. The perimeter of an isosceles triangle is 32 cm. The ratio of the equal side to its base is 3: 2. Find the area of the triangle.

Sol. As the sides of the equal to the base of an isosceles triangle is 3: 2, so let the sides of an isosceles triangle be 3x, 3x and 2x.

Now, perimeter of triangle = 3x + 3x + 2x = 8x

Given Perimeter of triangle = 32 m

$\therefore 8x = 32; x = 32 \div 8 = 4$

So, the sides of the isosceles triangle are $(3 \times 4)cm, (3 \times 4)cm, (2 \times 4)cm$ i.e., 12 cm, 12 cm and 8cm

$\therefore \qquad s = \frac{12+12+8}{2} = \frac{32}{2} = 16cm$

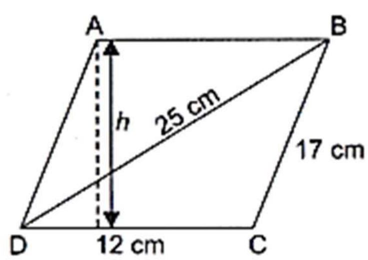
$= \sqrt{16(16-12)(16-12)(16-18)}$

$= \sqrt{16 \times 4 \times 4 \times 8} = \sqrt{4 \times 4 \times 4 \times 4 \times 4 \times 2}$

$= 4 \times 4 \times 2\sqrt{2} = 32\sqrt{2}cm^2$

5. Find the area of a parallelogram given in Fig. 12.2. Also find the length of the altitude from vertex A on the side DC.

Sol. Area of parallelogram ABCD = 2 (Area of ΔBCD) ...(1)



Now, the sides of ΔBCD are

a = 12 cm, b = 17 cm and c = 25 cm.

$\therefore$ Semi-perimeter of ΔBCD,

$$s = \frac{a+b+c}{2}$$

$$= \frac{12+17+25}{2}$$

$$= \frac{54}{2} = 27cm$$

$\therefore$ Area of $\Delta BCD = \sqrt{s(s-a)(s-b)(s-c)}$ [By Heron's formula]

$$= \sqrt{27(27-12)(27-17)(27-25)}$$

$$= \sqrt{25\times5\times10\times2}$$

$$= \sqrt{9\times3\times3\times5\times5\times2\times2} = 3\times3\times5\times2$$

$$= 90cm^2$$

From equation (1), we get

Area of parallelogram ABCD $= 2\times90 = 180cm^2$

Let the altitude of parallelogram be h.

Also, area of parallelogram ABCD = Base × Altitude

$\Rightarrow \quad 180 = DC\times h$

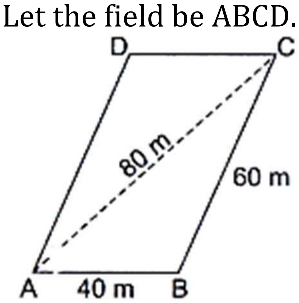
$\Rightarrow \quad 180 = 12\times h$

$\Rightarrow \quad h = \frac{180}{12} = 15cm$

Hence, the area of parallelogram is 180 cm^2 and the length of altitude is 15 cm.

6. A field in the form of a parallelogram has sides 60 m and 40 m and one of its diagonals is 80 m long. Find the area of the parallelogram.

Sol. Let the field be ABCD.



Area of the parallelogram ABCD = 2(area of ΔABC) ...(1)

Now, the sides of ΔABC are

a = 40 m, b = 60 m and c = 80 m

$\therefore$ Semi-perimeter of ΔABC,

$$s = \frac{a+b+c}{2}$$

$$= \frac{40+60+80}{2}$$

$$= \frac{180}{2} = 90m$$

$\therefore$ Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$ [By Heron's Formula]

$$= \sqrt{90(90-40)(90-60)(90-80)}$$
$$= \sqrt{90\times50\times30\times10}$$
$$= \sqrt{3\times30\times5\times10\times30\times10}$$
$$= 300\sqrt{15}cm^2 = 1161.895m^2$$

Formula equation (1), we get

Area of parallelogram ABCD = 2 × 1161.895 = 2323.79 m^2.

7. The perimeter of a triangular field is 420 m and its sides are in the ratio 6 : 7 : 8. Find the area of the triangular field.

Sol. Suppose that the sides in metres are 6x, 7x and 8x.

Now, 6x + 7x + 8x = perimeter = 420

$\Rightarrow \quad 21x = 420$

$\Rightarrow \quad x = \dfrac{420}{21}$

$\Rightarrow \quad x = 20$

$\therefore$ The sides of the triangular field are $6\times20m, 7\times20m, 8\times20m,$ i.e., 120 m, 140 m and 160 m.

Now, s = Half the perimeter of triangular field

$$= \frac{1}{2}\times420m = 210m$$

Using Heron's formula,

Area of triangular field $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{210(210-120)(210-140)(210-160)}$$
$$= \sqrt{210\times90\times70\times50}$$
$$= \sqrt{66150000} = 8133.265m^2$$

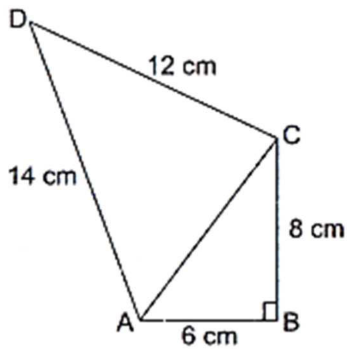
Hence, the area of the triangular field = 8133.265 m^2.

8. The sides of a quadrilateral ABCD are 6 cm, 8 cm, 12 cm and 14 cm (taken in order) respectively, and the angle between the first two sides is a right angle. Find its area.

Sol. We have to find that the area of quadrilateral ABCD. ABC is a right triangle.

$\therefore$ By Pythagoras theorem, we have

$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{(6)^2 + (8)^2}$$
$$= \sqrt{36+64}$$
$$= \sqrt{100} = 10cm$$



Area of $\Delta ABC = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 6 \times 8 cm^2 = 24 cm^2$

Let a = 10 cm, b = 12 cm and c = 14 m

$\therefore \quad s = \frac{a+b+c}{2}$

$= \frac{10+12+14}{2} = \frac{32}{2} = 18 cm$

Area of $\Delta ACD = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{18(18-10)(18-12)(18-14)}$

$= \sqrt{18 \times 8 \times 6 \times 4} = \sqrt{3456}$

$= 58.787 cm^2$ (approx.)

$\therefore$ Area of quadrilateral ABCD

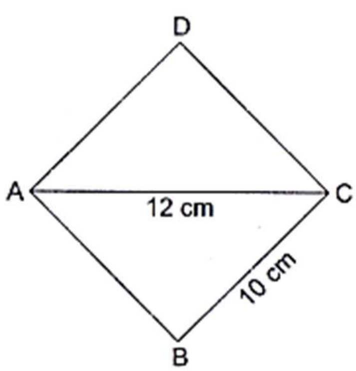
= Area of ΔABC + Area of ΔACD

$= 24 cm^2 + 58.787 cm^2$

$= 82.787 cm^2$ (approx.)

9. A rhombus shaped sheet with perimeter 40 cm and one diagonal 12 cm, is painted on both sides at the rate of Rs5 per m^2. Find the cost of painting.

Sol. Perimeter of rhombus = 40 cm



$\therefore$ 4 × side = 40

$\Rightarrow$ side $= \dfrac{40}{4} = 10cm$

One diagonal = 12 cm

As rhombus is also a parallelogram, so it diagonal divide it into two congruent triangles of equal area.

$\therefore$ Area of rhombus = 2(Area of triangle with sides 10cm, 10cm and 12cm)

So, let a = 10 cm, b = 10 cm and c = 12 cm

$\therefore s = \dfrac{a+b+c}{2} = \dfrac{10+10+10}{2} = \dfrac{32}{2} = 16cm$

$\therefore$ Area of $\Delta ABC = \sqrt{s(s-a)(s-b(s-c)}$

$= \sqrt{(16(16-10)(16-10)(16-12)}$

$= \sqrt{16\times 6\times 6\times 4} = \sqrt{2304} = 48cm^2$

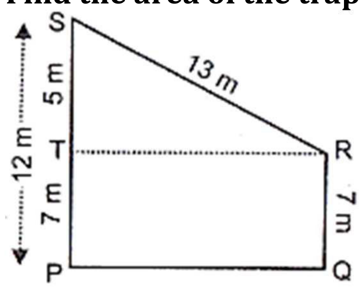
Now, area of rhombus ABCD = 2(area of ΔABC)

= 2 × 48 cm^2 = 86 cm^2.

Now, cost of painting both sides of rhombus shaped sheet ABCD

= Rs2 × 5 × 96 = Rs960

10. Find the area of the trapezium PQRS with height PQ given in Fig. 12.3



Sol. Draw RT $\perp$ PS from the figure, it is clear that

ST = PS – PT

= 12 m – 7 m

= 5 m

Now, from right triangle RTS, we have

$RS^2 = RT^2 + ST^2$

$\Rightarrow RT^2 = RS^2 - ST^2 = (13)^2 - 5^2$

$\therefore \quad RT^2 = 169 - 25 = 144 \Rightarrow RT = +\sqrt{144} = 12m$

Now area of trapezium PQRS

$= (PS + QR)\times RT = \dfrac{1}{2}(12m + 7m)\times 12m$

$= \dfrac{1}{2}\times 19m\times 12m = \dfrac{1}{2}\times 228m^2 = 14m^2$

Heron's Formula

Exercise 12.4

1. **How much paper of each shade is needed to make a kite given in Fig. 12.4, in which ABCD is a square with diagonal 44 cm.**

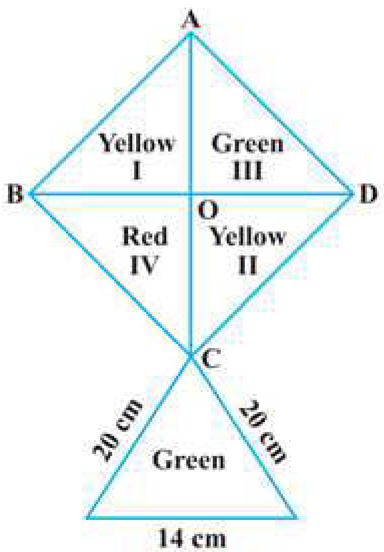


Fig. 12.4

Sol. Each diagonal of square = 44 cm and as diagonal of a square bisect each other at right angles.

$\therefore$ Area of square ABCD

= 2(area of ΔABC)

$= 2(\frac{1}{2}\times 44\times 22) = 2(44\times 11)$

= 968 cm^2.

$\therefore$ Paper of Red shade needed to make the kite

$= \frac{1}{4}(968 cm^2) = 242 cm^2$

Paper of yellow shade needed to make the kite = (242 + 242) = 484 cm^2.

Let us find the area of a triangle with sides 20 cm, 20 cm and 14 cm which is at the bottom of the square ABCD.

Now, semi-perimeter

$$s = \frac{20+20+14}{2} = \frac{54}{2} = 27 cm$$

Area of $\Delta = \sqrt{s(s-a)(s-b(s-c)}$

$= \sqrt{27(27-20)(27-20)(27-14)}$

$= \sqrt{27(27\times 7\times 7\times 13} = 21\sqrt{39}$

$= 21\times 6.245 = 131.15 cm^2$

Paper of Green shade needed to make the kite

= (242 + 131.15) cm^2 = 373.15 cm^2.

2. The perimeter of a triangle is 50 cm. One side of a triangle is 4 cm longer than the smaller side and the third side is 6 cm less than twice the smaller side. Find the area of the triangle.

Sol. Let the smaller side of the triangle be x cm. therefore, the second side will be (x + 4) cm, and third side is (2x – 6) cm.

Now, perimeter of triangle = x (x + 4) + (2x – 6)

= (4x – 2) cm

Also, perimeter of triangle = 50 cm.

4x = 52; x = 52 ÷ 4 = 13

Therefore, the three sides are 13 cm, 17 cm, 20 cm

$$s=\frac{13+17+20}{2}=\frac{50}{2}=25cm$$

∴ Area of $\Delta=\sqrt{25(25-13)(25-17)(25-20)}$

$$=\sqrt{25\times12\times8\times5}=\sqrt{5\times5\times4\times3\times4\times2\times5}$$

$$=5\times4\times\sqrt{3\times2\times5}=20\sqrt{30}cm^2$$

3. The area of a trapezium is 475 cm^2 and the height is 19 cm. Find the lengths of its two parallel sides if one side is 4 cm greater than the other.

Sol. Area of trapezium $=\frac{1}{2}\times$(Sum of the parallel side) $\times$height

$$\Rightarrow \quad 475=\frac{1}{2}\times(x+x+4)\times19cm$$

$$\Rightarrow \quad 2x+4=\frac{950}{19}=50$$

$$\Rightarrow \quad 2x=50-4=46; x=46\div2=23$$

Hence, the length of two parallel sides are 23 cm and (23 + 4) cm i.e., 23 cm and 27 cm.

4. A rectangular plot is given for constructing a house, having a measurement of 40 m long and 15 m in the front. According to the laws, a minimum of 3 m, wide space should be left in the front and back each and 2 m wide space on each of other sides. Find the largest area where house can be constructed.

Sol. The length of the rectangular plot = 40 m

And the breath of the plot = 15 m.

As a minimum of 3 m wide space should be left in the front and back 2 m wide space each of other side, so the largest area where the house can be constructed.

$$=[40-2(3)][15-2(2)]=34\times11=374m^2$$

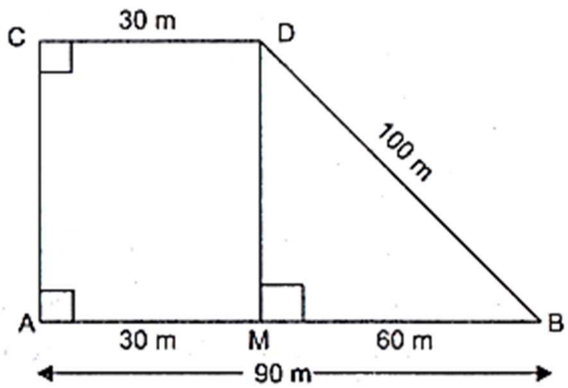
5. A field is in the shape of a trapezium having parallel sides 90 m and 30 m. These sides meet the third side at right angles. The length of the fourth side is 100 m. If it costs Rs4 to plough 1m^2 of the field, find the total cost of ploughing the field.

Sol. The two-parallel side are AB = 90 cm and CD = 30 m. DM $\perp$ AB

Now, MB = AB – AM = 90 m – 30 m = 60 m.

In right triangle DMB, we have

$$DM^2 = DB^2 - MB^2 = (100)^2 - (60)^2$$



$DM^2 = 10,000 - 3600 = 6400$

$\Rightarrow \quad DM = +\sqrt{6400} = 80m$

$\therefore$ The area of the field ABDC which is trapezium in shape

$$= \frac{1}{2} \times (\text{Sum of the parallel sides}) \times \text{height}$$

$$= \frac{1}{2} \times (90 + 30) \times 80 m^2$$

$$= \frac{1}{2} \times 120 \times 80 = 4800 m^2$$

Total cost of ploughing the field at the rate of Rs4 per m^2 = Rs(4800 × 4) = Rs19,200.

6. In Fig. 12.5, Δ ABC has sides AB = 7.5 cm, AC = 6.5 cm and BC = 7 cm. On base BC a parallelogram DBCE of same area as that of Δ ABC is constructed. Find the height DF of the parallelogram.

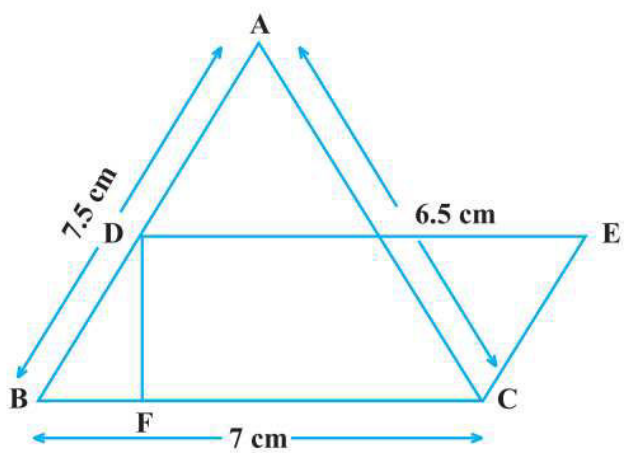


Fig. 12.5

Sol. Sides of triangle ABC are 7.5cm, 7cm and 6.5cm.

The semi-perimeter of ΔABC

$$s=\frac{7.5+7+6.5}{2}=\frac{21}{2}=10.5cm$$

Area of $\Delta ABC=\sqrt{s(s-a)(s-b(s-c)}$

$$=\sqrt{10.5(10.5-7.5)(10.5-7)(10.5-6.5)}$$

$$=\sqrt{10.5\times3\times3.5\times4}=\sqrt{31.5\times14}$$

$$=\sqrt{441}=21cm^2$$

Now, as on base BC a parallelogram DBCE of same area of as that of ΔABC is constructed.
Therefore, area of ||gm $\Delta BCE=BC\times DF$

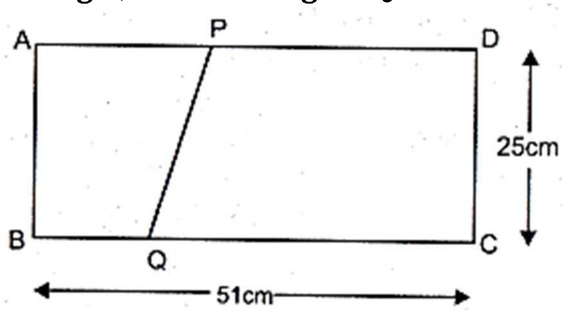
$\therefore \quad BC\times DF=21cm^2$

$\Rightarrow \quad 7\times DF=21cm^2$

$\Rightarrow \quad DF=21cm^2\div7cm=3cm$

Hence, the height DF of the parallelogram = 3cm.

7. The dimensions of a rectangle ABCD are 51 cm × 25 cm. A trapezium with its parallel sides QC and PD in the ratio 9 : 8, is cut off from the rectangle as shown in the Fig. 12. 6. If the area of the trapezium PQCD is $\frac{5}{6}th$ part of the area of the rectangle, find the lengths QC and PD.



Sol. ABCD is a rectangle in which AB = 51 cm and BC = 25 cm.
Since parallel sides QC and PD are in the ratio 9:8, so let QC = 9x and PD = 8x.

Now, are of trapezium PACD $=\frac{1}{2}\times(9x+8x)\times25cm^2$

$$=\frac{1}{2}\times17x\times25$$

Area of rectangle ABCD = BC × CD = 51 × 25

It is given that area of trapezium PQCD $=\frac{5}{6}\times$Area of rectangle ABCD.

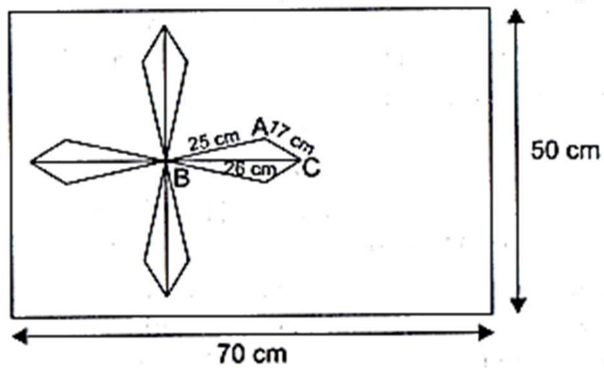
$\therefore \quad \frac{1}{2}\times17x\times25=\frac{5}{6}\times51\times25$

$\Rightarrow \quad x=\frac{5}{6}\times51\times25\times2\times\frac{1}{17\times25}=5$

Hence, the length QC = 9x = 9 × 5 = 45 cm.

And the length PD = 8x = 8 × 5 = 40 cm.

8. **A design is made on a rectangular tile of dimensions 50 cm × 70 cm as shown in Fig. 12.7. The design shows 8 triangles, each of sides 26 cm, 17 cm and 25 cm. Find the total area of the design and the remaining area of the tile.**



Sol. Given, the dimension of rectangular tile are 50 cm × 70 cm

$\therefore$ Area of rectangular tile = 50 × 70 = 3500 cm^2.

The sides of a design of one triangle be a = 25 cm, b = cm and c = 26 cm.

Now, semi-perimeter, $s = \frac{a+b+c}{2}$

$$= \frac{25+17+26}{2} = \frac{68}{2} = 34cm$$

$\therefore$ Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

[By heron's formula]

$$= \sqrt{34 \times 9 \times 17 \times 8}$$

$$= \sqrt{17 \times 2 \times 3 \times 3 \times 17 \times 2 \times 2 \times 2}$$

$$= 17 \times 3 \times 2 \times 2$$

= 204 cm^2.

$\therefore$ Total area of eight triangles = 240 × 8 = 1632 cm^2

Now, area of the design = Total area of eight triangles

= 1632 cm^2

Also, remaining area of the tile = Area of the rectangle – Area of the design

= 3500 cm^2 – 1632 cm^2

= 1868 cm^2

Hence, the total area of the design is 1632 cm^2 and the remaining area of the tile is 1868 cm^2.