

Ex 27.1

Q1

Let $S(-1, 1)$ be the focus and $P(x, y)$ be a point on the hyperbola. Draw PM perpendicular from P on the directrix. Then, by definition.

$$\begin{aligned}
 SP &= ePM \\
 \Rightarrow SP^2 &= e^2 PM^2 \\
 \Rightarrow (x+1)^2 + (y-1)^2 &= (3)^2 \left[\frac{x-y+3}{\sqrt{1^2 + (-1)^2}} \right]^2 && [\because e = 3] \\
 \Rightarrow x^2 + 1 + 2x + y^2 + 1 - 2y &= \frac{9[x-y+3]^2}{2} \\
 \Rightarrow 2[x^2 + y^2 + 2x - 2y + 2] &= 9[x-y+3]^2 \\
 \Rightarrow 2x^2 + 2y^2 + 4x - 4y + 4 &= 9[x^2 - y^2 + 3^2 + 2 \times x \times (-y) + 2 \times (-y) \times 3 + 2 \times 3 \times x] \\
 \Rightarrow 2x^2 + 2y^2 + 4x - 4y - 4 &= 9[x^2 + y^2 + 9 - 2xy - 6y + 6x] \\
 \Rightarrow 2x^2 + 2y^2 + 4x - 4y + 4 &= 9x^2 + 9y^2 + 81 - 18xy - 54y + 4y + 81 - 4 = 0 \\
 \Rightarrow 7x^2 + 7y^2 - 18xy + 50x - 50y + 77 &= 0
 \end{aligned}$$

This is the required equation of the hyperbola.

Q2(i)

Let $S(0, 3)$ be the focus and $P(x, y)$ be a point on the hyperbola. Draw PM perpendicular from P on the directrix. Then, by definition

$$\begin{aligned}
 SP &= ePM \\
 \Rightarrow SP^2 &= e^2 PM^2 \\
 \Rightarrow (x-0)^2 + (y-3)^2 &= 2^2 \left[\frac{x+y-1}{\sqrt{1^2 + 1^2}} \right]^2 && [\because e = 2] \\
 \Rightarrow x^2 + y^2 + 9 - 6y &= \frac{4[x+y-1]^2}{2} \\
 \Rightarrow x^2 + y^2 - 6y + 9 &= 2(x+y-1)^2 \\
 \Rightarrow x^2 + y^2 - 6y + 9 &= 2[x^2 + y^2 + (-1)^2 + 2xy + 2 \times y \times (-1) + 2 \times (-1) \times x] \\
 \Rightarrow x^2 + y^2 - 6y + 9 &= 2[x^2 + y^2 + 1 + 2xy - 2y - 2x] \\
 \Rightarrow x^2 + y^2 - 6y + 9 &= 2x^2 + 2y^2 + 2 + 4xy - 4y - 4x \\
 \Rightarrow 2x^2 - x^2 + 2y^2 - y^2 + 4xy - 4x - 4y + 6y + 2 - 9 &= 0 \\
 \Rightarrow x^2 + y^2 + 4xy - 4x + 2y - 7 &= 0
 \end{aligned}$$

This is the required equation of the hyperbola.

Q2(ii)

Let $S(1, 1)$ be the focus and $P(x, y)$ be a point on the hyperbola.

Draw PM perpendicular from P on the directrix. Then, by definition

$$\begin{aligned} & SP = ePM \\ \Rightarrow & SP^2 = e^2 PM^2 \\ \Rightarrow & (x-1)^2 + (y-1)^2 = 2^2 \left[\frac{3x+4y+8}{\sqrt{3^2+4^2}} \right]^2 \quad [\because e=2] \\ \Rightarrow & x^2+1-2x+y^2+1-2y = 4 \left[\frac{3x+4y+8}{\sqrt{25}} \right]^2 \\ \Rightarrow & x^2+y^2-2x-2y+2 = \frac{4(3x+4y+8)^2}{25} \\ \Rightarrow & 25x^2+25y^2-50x-50y+50 = 4(3x+4y+8)^2 \\ \Rightarrow & 25x^2+25y^2-50x-50y+50 = 4[9x^2+16y^2+6y+24xy+64y+48x] \\ \Rightarrow & 25x^2+25y^2-50x-50y+50 = 36x^2+64y^2+256+96xy+256y+192x \\ \Rightarrow & 36x^2-25x^2+64y^2-25y^2+96xy+192x+50x+256y+50y+256-50=0 \\ \Rightarrow & 11x^2+39y^2+96xy+242x+306y+206=0 \end{aligned}$$

This is the required equation of the hyperbola.

Q2(iii)

Let $S(1, 1)$ be the focus and $P(x, y)$ be a point on the hyperbola.

Draw PM perpendicular from P on the directrix. Then, by definition

$$\begin{aligned} & SP = ePM \\ \Rightarrow & SP^2 = e^2 PM^2 \\ \Rightarrow & (x-1)^2 + (y-1)^2 = (\sqrt{3})^2 \left[\frac{2x+y-1}{\sqrt{2^2+1^2}} \right]^2 \quad [\because e=2] \\ \Rightarrow & x^2+1-2x+y^2+1-2y = \frac{3[2x+y-1]^2}{5} \\ \Rightarrow & 5[x^2+y^2-2x-2y+2] = 3(2x+y-1)^2 \\ \Rightarrow & 5x^2+5y^2-10x-10y+10 = 3[(2x)^2+y^2+(-1)^2+2 \times 2x \times y+2 \times y \times (-1)+2 \times (-1) \times 2x] \\ \Rightarrow & 5x^2+5y^2-10x-10y+10 = 3[4x^2+y^2+1+4xy-2y-4x] \\ \Rightarrow & 5x^2+5y^2-10x-10y+10 = 12x^2+3y^2+3+12xy-6y-12x \\ \Rightarrow & 12x^2-5x^2+3y^2-5y^2+12xy-12x+10x-6y+10y+3-10=0 \\ \Rightarrow & 7x^2-2y^2+12xy-2x+4y-7=0 \end{aligned}$$

This is the required equation of the hyperbola.

Q2(iv)

Let $S(2, -1)$ be the focus and $P(x, y)$ be a point on the hyperbola.

Draw PM perpendicular from P on the directrix. Then, by definition

$$\begin{aligned} SP &= ePM \\ \Rightarrow SP^2 &= e^2 PM^2 \\ \Rightarrow (x-2)^2 + (y+1)^2 &= 2^2 \left[\frac{2x+3y-1}{\sqrt{2^2+3^2}} \right]^2 && [\because e=2] \\ \Rightarrow x^2 + 4 - 4x + y^2 + 1 + 2y &= \frac{4[2x+3y-1]^2}{13} \\ \Rightarrow 13[x^2 + y^2 - 4x + 2y + 5] &= 4(2x+3y-1)^2 \\ \Rightarrow 13x^2 + 13y^2 - 52x + 26y + 65 &= 4[2x+3y-1]^2 \\ \Rightarrow 13x^2 + 13y^2 - 52x + 26y + 65 &= 4[(2x)^2 + (3y)^2 + (-1)^2 + 2 \times 2x \times 3y + 2 \times 3y \times (-1) + 2 \times (-1) \times 2x] \\ \Rightarrow 13x^2 + 13y^2 - 52x + 26y + 65 &= 4[4x^2 + 9y^2 + 1 + 12xy - 6y - 4x] \\ \Rightarrow 13x^2 + 13y^2 - 52x + 26y + 65 &= 16x^2 + 36y^2 + 4 + 48xy - 24y - 16x \\ \Rightarrow 16x^2 - 13x^2 + 36y^2 - 13y^2 + 48xy - 16x + 52x - 24y - 26y + 4 - 65 &= 0 \\ \Rightarrow 3x^2 + 23y^2 + 48xy + 36x - 50y - 61 &= 0 \end{aligned}$$

This is the required equation of the hyperbola.

Q2(v)

Let $S(a, 0)$ be the focus and $P(x, y)$ be a point on the hyperbola.

Draw PM perpendicular from P on the directrix. Then, by definition

$$\begin{aligned} SP &= ePM \\ \Rightarrow SP^2 &= e^2 PM^2 \\ \Rightarrow (x-a)^2 + (y-0)^2 &= \left(\frac{4}{3}\right)^2 \left[\frac{2x-y+a}{\sqrt{2^2+(-1)^2}} \right]^2 && \left[\because e = \frac{4}{3} \right] \\ \Rightarrow x^2 + a^2 - 2ax + y^2 &= \frac{16}{9} \times \frac{[2x-y+a]^2}{5} \\ \Rightarrow 45[x^2 + y^2 - 2ax + a^2] &= 16[2x-y+a]^2 \\ \Rightarrow 45x^2 + 45y^2 - 90ax + 45a^2 &= 16[(2x)^2 + (-y)^2 + a^2 + 2 \times 2x(-y) + 2 \times (-y) \times a + 2 \times a \times 2x] \\ \Rightarrow 45x^2 + 45y^2 - 90ax + 45a^2 &= 16[4x^2 + y^2 + a^2 - 4xy - 2ay + 4ax] \\ \Rightarrow 45x^2 + 45y^2 - 90ax + 45a^2 &= 64x^2 + 16y^2 + 16a^2 - 64xy - 32ay + 64ax \\ \Rightarrow 64x^2 - 45x^2 + 16y^2 - 45y^2 - 64xy + 64ax + 90ax - 32ay + 16a^2 - 45a^2 &= 0 \\ \Rightarrow 19x^2 - 29y^2 - 64xy + 154ax - 32ay - 29a^2 &= 0 \end{aligned}$$

This is the required equation of the hyperbola.

Q2(vi)

Let $S(2, 2)$ be the focus and $P(x, y)$ be a point on the hyperbola.

Draw PM perpendicular from P on the directrix. Then, by definition

$$\begin{aligned} SP &= ePM \\ \Rightarrow SP^2 &= e^2 PM^2 \\ \Rightarrow (x-2)^2 + (y-2)^2 &= 2^2 \left[\frac{x+y-9}{\sqrt{1^2+1^2}} \right]^2 && \left[\because e = \frac{4}{3} \right] \\ \Rightarrow x^2 + 4 - 4x + y^2 + 4 - 4y &= \frac{4[x+y-9]^2}{2} \\ \Rightarrow x^2 + y^2 - 4x - 4y + 8 &= 2[x+y-9]^2 \\ \Rightarrow x^2 + y^2 - 4x - 4y + 8 &= 2[x^2 + y^2 + (-9)^2 + 2 \times x \times y + 2 \times y \times (-9) + 2 \times (-9) \times x] \\ \Rightarrow x^2 + y^2 - 4x - 4y + 8 &= 2[x^2 + y^2 + 81 + 2xy - 18y + 18x] \\ \Rightarrow x^2 + y^2 - 4x - 4y + 8 &= [2x^2 + 2y^2 + 162 + 4xy - 36y - 36x] \\ \Rightarrow 2x^2 - x^2 + 2y^2 - y^2 + 4xy - 36x + 4x - 36y + 4y + 162 - 8 &= 0 \\ \Rightarrow x^2 + y^2 + 4xy - 32x - 32y + 154 &= 0 \end{aligned}$$

This is the required equation of the hyperbola.

Q3(i)

We have,

$$9x^2 - 16y^2 = 144$$

$$\Rightarrow \frac{9x^2}{144} - \frac{16y^2}{144} = 1$$

$$\Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a^2 = 16$ and $b^2 = 9$

Eccentricity: The eccentricity e is given by

$$\begin{aligned} e &= \sqrt{1 + \frac{b^2}{a^2}} \\ &= \sqrt{1 + \frac{9}{16}} \\ &= \sqrt{\frac{25}{16}} \\ &= \frac{5}{4} \end{aligned}$$

Foci: The coordinates of the foci are $(\pm ae, 0)$ i.e., $(\pm 5, 0)$

Equations of the directrices: The equations of the directrices are

$$\begin{aligned} x &= \pm \frac{a}{e} \text{ i.e., } x = \pm \frac{16}{5} \\ \therefore 5x &= \pm 16 \\ \Rightarrow 5x \mp 16 &= 0 \end{aligned}$$

Length of latus-rectum: The length of the latus-rectum

$$= \frac{2b^2}{a} = \frac{2 \times 9}{4} = \frac{9}{2}$$

Q3(ii)

We have,

$$\begin{aligned} 16x^2 - 9y^2 &= -144 \\ \Rightarrow \frac{16x^2}{144} - \frac{9y^2}{144} &= -1 \\ \Rightarrow \frac{x^2}{9} - \frac{y^2}{16} &= -1 \end{aligned}$$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$, where $a^2 = 9$ and $b^2 = 16$

$$\therefore a = 3 \text{ and } b = 4$$

Eccentricity: The eccentricity e is given by

$$\begin{aligned} e &= \sqrt{1 + \frac{a^2}{b^2}} \\ &= \sqrt{1 + \frac{9}{16}} \\ &= \sqrt{\frac{25}{16}} \\ &= \frac{5}{4} \end{aligned}$$

Foci: The coordinates of the foci are $(0, \pm be)$.

$$\begin{aligned} \therefore (0, \pm be) &= \left(0, \pm 4 \times \frac{5}{4}\right) \\ &= (0, \pm 5) \end{aligned}$$

\therefore the coordinates of the foci are $(0, \pm 5)$

Equations of the directrices: The equations of the directrices are

$$\begin{aligned} y &= \frac{\pm b}{e} \\ \Rightarrow y &= \pm \frac{4}{\frac{5}{4}} = \pm \frac{16}{5} \\ \Rightarrow 5y \mp 16 &= 0 \end{aligned}$$

Latus-rectum: The length of the latus-rectum

$$\begin{aligned} &= \frac{2a^2}{b} \\ &= \frac{2 \times 9}{4} = \frac{9}{2} \end{aligned}$$

Q3(iii)

We have,

$$4x^2 - 3y^2 = 36$$

$$\Rightarrow \frac{4x^2}{36} - \frac{3y^2}{36} = 1$$

$$\Rightarrow \frac{x^2}{9} - \frac{y^2}{12} = 1$$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$, where $a^2 = 9$ and $b^2 = 12$

$$\therefore a = 3 \text{ and } b = \sqrt{12} = 2\sqrt{3}$$

Eccentricity: The eccentricity e is given by

$$\begin{aligned} e &= \sqrt{1 + \frac{b^2}{a^2}} \\ &= \sqrt{1 + \frac{12}{9}} \\ &= \sqrt{1 + \frac{4}{3}} \\ &= \sqrt{\frac{7}{3}} \end{aligned}$$

Q3(iv)

We have,

$$\begin{aligned}3x^2 - y^2 &= 4 \\ \Rightarrow \frac{3x^2}{4} - \frac{y^2}{4} &= 1 \\ \Rightarrow \frac{x^2}{\frac{4}{3}} - \frac{y^2}{4} &= 1 \\ \Rightarrow \frac{x^2}{\left(\frac{2}{\sqrt{3}}\right)^2} - \frac{y^2}{2^2} &= 1\end{aligned}$$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a = \frac{2}{\sqrt{3}}$ and $b = 2$

Eccentricity: The eccentricity e is given by

$$\begin{aligned}e &= \sqrt{1 + \frac{b^2}{a^2}} \\ &= \sqrt{1 + \frac{4}{\frac{4}{3}}} \\ &= \sqrt{1 + 3} \\ &= \sqrt{4} \\ &= 2\end{aligned}$$

Q3(v)

Foci: The coordinates of the foci are $(\pm ae, 0)$

$$\therefore \quad \pm ae = \pm \frac{2}{\sqrt{3}} \times 2 = \pm \frac{4}{\sqrt{3}}$$

The coordinates of the foci are $\left(\pm \frac{4}{\sqrt{3}}, 0\right)$

Equations of the directrices: The equations of the directrices are

$$\begin{aligned}x &= \pm \frac{a}{e} \\&= \pm \frac{\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}} \\&= \pm \frac{1}{\sqrt{3}} \\\Rightarrow \quad \sqrt{3}x \mp 1 &= 0\end{aligned}$$

Latus-rectum: The length of the latus-rectum = $\frac{2b^2}{a}$.

$$\begin{aligned}\therefore \quad \frac{2b^2}{a} &= 2 \times \frac{4}{\frac{2}{\sqrt{3}}} \\&= 4\sqrt{3}\end{aligned}$$

Q4

We have,

$$25x^2 - 36y^2 = 225$$

$$\Rightarrow \frac{25x^2}{225} - \frac{36y^2}{225} = 1$$

$$\Rightarrow \frac{x^2}{9} - \frac{4y^2}{25} = 1$$

$$\Rightarrow \frac{x^2}{9} - \frac{y^2}{\frac{25}{4}} = 1$$

$$\Rightarrow \frac{x^2}{(3)^2} - \frac{y^2}{\left(\frac{5}{2}\right)^2} = 1$$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a = 3$ and $b = \frac{5}{2}$

Length of the transverse axis: The length of the transverse axis

$$= 2a$$

$$= 2 \times 3 = 6$$

Q5(i)

We have,

$$\begin{aligned}
 & 16x^2 - 9y^2 + 32x + 36y - 164 = 0 \\
 \Rightarrow & 16x^2 + 32x - 9y^2 + 36y - 164 = 0 \\
 \Rightarrow & 16(x^2 + 2x) - 9(y^2 + 4y) - 164 = 0 \\
 \Rightarrow & 16[x^2 + 2x + 1 - 1] - 9[y^2 + 4y + 4 - 4] - 164 = 0 \\
 \Rightarrow & 16[(x+1)^2 - 1] - 9[(y+2)^2 - 4] - 164 = 0 \\
 \Rightarrow & 16(x+1)^2 - 16 - 9(y+2)^2 + 36 - 164 = 0 \\
 \Rightarrow & 16(x+1)^2 - 9(y+2)^2 + 20 - 164 = 0 \\
 \Rightarrow & 16(x+1)^2 - 9(y+2)^2 - 144 = 0 \\
 \Rightarrow & 16(x+1)^2 - 9(y+2)^2 = 144 \\
 \Rightarrow & \frac{16(x+1)^2}{144} - \frac{9(y+2)^2}{144} = 1 \\
 \Rightarrow & \frac{(x+1)^2}{9} - \frac{(y+2)^2}{16} = 1 \quad \text{---(i)}
 \end{aligned}$$

Shifting the origin at $(-1, 2)$ without rotating the coordinate axes and denoting the new coordinates with respect to the new axes by X and y ,

We have,

$$x = X - 1 \text{ and } y = Y + 2 \quad \text{---(ii)}$$

Q5(ii)

We have,

$$\begin{aligned}
 & x^2 - y^2 + 4x = 0 \\
 \Rightarrow & x^2 + 4x - y^2 = 0 \\
 \Rightarrow & x^2 + 4x + 4 - 4 - y^2 = 0 \\
 \Rightarrow & (x + 2)^2 - y^2 = 4 \\
 \Rightarrow & \frac{(x + 2)^2}{4} - \frac{y^2}{4} = 1 \quad \text{---(i)}
 \end{aligned}$$

Shifting the origin at $(-2, 0)$ without rotating the axes and denoting the new coordinates w.r.t these axes by X and Y ,

We have,

$$x = X - 2 \text{ and } y = Y \quad \text{---(ii)}$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{4} - \frac{Y^2}{4} = 1 \quad \text{---(ii)}$$

This is of the form $\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$, where $a^2 = 4$ and $b^2 = 4$. so,

We have,

Centre: The coordinates of the centre w.r.t the new axes are $(X = 0, Y = 0)$

Putting $X = 0$ and $Y = 0$ in equation (ii), we get

$$x = -2 \text{ and } y = 0.$$

So, the coordinates of the centre w.r.t the old axes are $(-2, 0)$.

Q5(iii)

We have,

$$\begin{aligned} & x^2 - 3y^2 - 2x = 8 \\ \Rightarrow & x^2 - 2x - 3y^2 = 8 \\ \Rightarrow & x^2 - 2x + 1 - 1 - 3y^2 = 8 \\ \Rightarrow & (x-1)^2 - 1 - 3y^2 = 8 \\ \Rightarrow & (x-1)^2 - 3y^2 = 9 \\ \Rightarrow & \frac{(x-1)^2}{9} - \frac{3y^2}{9} = 1 \\ \Rightarrow & \frac{(x-1)^2}{9} - \frac{y^2}{3} = 1 \end{aligned} \quad \text{---(i)}$$

Shifting the origin at $(1,0)$ without rotating the axes and denoting the new coordinates w.r.t these axes by X and y , We have,

$$x = X + 1 \text{ and } y = Y \quad \text{---(ii)}$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{9} - \frac{Y^2}{3} = 1 \quad \text{---(ii)}$$

This is of the form $\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$, where $a^2 = 9$ and $b^2 = 3$. so,

We have,

Centre: The coordinates of the centre w.r.t the new axes are $(X = 0, Y = 0)$

Putting $X = 0$ and $Y = 0$ in equation (ii), we get

$$x = 1 \text{ and } y = 0.$$

So, the coordinates of the centre w.r.t the old axes are $(1,0)$.

Q6(i)

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{---(i)}$$

Then,

Distance between the foci = 16

$$\Rightarrow 2ae = 16 \quad [\because \text{Distance between foci} = 2ae]$$

$$\Rightarrow ae = 8$$

$$\Rightarrow a \times \sqrt{2} = 8 \quad [\because e = \sqrt{2}]$$

$$\Rightarrow a = \frac{8}{\sqrt{2}}$$

$$\Rightarrow a^2 = \frac{64}{2} = 32$$

Now,

$$\begin{aligned} b^2 &= a^2(e^2 - 1) \\ &= 32\left((\sqrt{2})^2 - 1\right) \\ &= 32 \times (2 - 1) \\ &= 32 \end{aligned}$$

Putting $a^2 = 32$ and $b^2 = 32$ in equation (i), we get

$$\begin{aligned} \frac{x^2}{32} - \frac{y^2}{32} &= 1 \\ \Rightarrow x^2 - y^2 &= 32 \end{aligned}$$

Hence, the equation of the required hyperbola is $x^2 - y^2 = 32$.

Q6(ii)

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{--- (i)}$$

Then,

The length of the conjugate axis = $2b$

$$\therefore 2b = 5$$

[\because Conjugate axis = 5]

$$\Rightarrow b = \frac{5}{2}$$

$$\Rightarrow b^2 = \frac{25}{4}$$

And, the distance between foci = $2ae$

$$\therefore 2ae = 13$$

[\because The distance between foci is 13]

$$\Rightarrow a^2e^2 = \frac{169}{4}$$

Q6(iii)

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{---(i)}$$

Then,

The length of the conjugate axis = $2b$

$$\therefore 2b = 7 \quad [\because \text{Conjugate axis is } = 5]$$

$$\Rightarrow b = \frac{7}{2}$$

$$\Rightarrow b^2 = \frac{49}{4} \quad \text{---(ii)}$$

The required hyperbola passes through the point $(3, -2)$.

$$\therefore \frac{(3)^2}{a^2} - \frac{(-2)^2}{b^2} = 1$$

$$\Rightarrow \frac{9}{a^2} - \frac{4}{\frac{49}{4}} = 1$$

$$\Rightarrow \frac{9}{a^2} - \frac{16}{49} = 1$$

$$\Rightarrow \frac{9}{a^2} = 1 + \frac{16}{49}$$

$$\Rightarrow \frac{9}{a^2} = \frac{65}{49}$$

$$\Rightarrow a^2 = \frac{49 \times 9}{65}$$

$$\Rightarrow a^2 = \frac{441}{65}$$

Putting $a^2 = \frac{441}{65}$ and $b^2 = \frac{49}{4}$ in equation (i), we get

$$\frac{x^2}{\frac{441}{65}} - \frac{y^2}{\frac{49}{4}} = 1$$

$$\Rightarrow \frac{65x^2}{441} - \frac{4y^2}{49} = 1$$

$$\Rightarrow \frac{65x^2 - 36y^2}{441} = 1$$

$$\Rightarrow 65x^2 - 36y^2 = 441$$

Hence, the equation of the required hyperbola is $65x^2 - 36y^2 = 441$.

Q7(i)

The centre of the hyperbola is the mid-point of the line joining the two foci.

So, the coordinates of the centre are $\left(\frac{6+4}{2}, \frac{4+4}{2}\right)$ i.e., $(1, 4)$.

Let $2a$ and $2b$ be the length of transverse and conjugate axes and let e be the eccentricity.

Then, the equation of the hyperbola is

$$\frac{(x-1)^2}{a^2} - \frac{(y-4)^2}{b^2} = 1 \quad \text{---(i)}$$

Now, distance between two foci = $2ae$

$$\Rightarrow \sqrt{(6+4)^2 + (4-4)^2} = 2ae \quad [\because \text{Foci} = (6, 4) \text{ and } (-4, 4)]$$

$$\Rightarrow \sqrt{(10)^2} = 2ae$$

$$\Rightarrow 10 = 2ae$$

$$\Rightarrow 2ae = 10$$

$$\Rightarrow 2a \times 2 = 10 \quad [\because e = 2]$$

$$\Rightarrow a = \frac{10}{4}$$

$$\Rightarrow a = \frac{5}{2}$$

$$\Rightarrow a^2 = \frac{25}{4}$$

Q7(ii)

The centre of the hyperbola is the mid-point of the line joining the two vertices.

So, the coordinates of the centre are $\left(\frac{16-8}{2}, \frac{-1-1}{2}\right)$ i.e., $(4, -1)$.

Let $2a$ and $2b$ be the length of transverse and conjugate axes and let e be the eccentricity. Then, the equation of the hyperbola is

$$\frac{(x-4)^2}{a^2} - \frac{(y+1)^2}{b^2} = 1 \quad \text{---(i)}$$

Now,

The distance between two vertices = $2a$

$$\therefore \sqrt{(16-8)^2 + (-1-1)^2} = 2ae \quad [\because \text{vertices} = (-8, -1) \text{ and } (16, -1)]$$

$$\Rightarrow 24 = 2a$$

$$\Rightarrow a = 12$$

$$\Rightarrow a^2 = 144$$

and, the distance between the focus and vertex is = $ae - a$

Q7(iii)

The centre of the hyperbola is the mid-point of the line joining the two foci.

So, the coordinates of the centre are $\left(\frac{4+8}{2}, \frac{2+2}{2}\right)$ i.e., $(6, 2)$.

Let $2a$ and $2b$ be the length of transverse and conjugate axes and let e be the eccentricity.

Then, the equation of the hyperbola is

$$\frac{(x-6)^2}{a^2} - \frac{(y-2)^2}{b^2} = 1 \quad \text{---(i)}$$

Now, distance between two foci = $2ae$

$$\Rightarrow \sqrt{(8-4)^2 + (2-2)^2} = 2ae \quad [\because \text{Foci} = (4, 2) \text{ and } (8, 2)]$$

$$\Rightarrow \sqrt{(4)^2} = 2ae$$

$$\Rightarrow 2ae = 4$$

$$\Rightarrow 2 \times a \times 2 = 4 \quad [\because e = 2]$$

$$\Rightarrow a = \frac{4}{4} = 1$$

$$\Rightarrow a^2 = 1$$

Now,

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow b^2 = 1(2^2 - 1) \quad [\because e = 2]$$

$$\Rightarrow b^2 = 4 - 1$$

$$\Rightarrow b^2 = 3$$

Q7(iv)

Since, the vertices are on y -axis, so let the equation of the required hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \quad \text{--- (i)}$$

The coordinates of its vertices and foci are $(0, \pm b)$ and $(0, \pm be)$ respectively.

$$\begin{aligned} \therefore b &= 7 & [\because \text{vertices} &= (0, \pm 7)] \\ \Rightarrow b^2 &= 49 \end{aligned}$$

and,

$$\begin{aligned} be &= \frac{28}{3} & [\because \text{Foci} &= (0, \pm \frac{28}{3})] \\ \Rightarrow 7 \times e &= \frac{28}{3} \\ \Rightarrow e &= \frac{4}{3} \\ \Rightarrow e^2 &= \frac{16}{9} \end{aligned}$$

Now,

$$\begin{aligned} a^2 &= b^2(e^2 - 1) \\ \Rightarrow a^2 &= 49\left(\frac{16}{9} - 1\right) \\ \Rightarrow a^2 &= 49 \times \frac{7}{9} \\ \Rightarrow a^2 &= \frac{343}{9} \end{aligned}$$

Putting $a^2 = \frac{343}{9}$ and $b^2 = 49$ in equation (i), we get

$$\frac{x^2}{\frac{343}{9}} - \frac{y^2}{49} = -1$$

This is the equation of the required hyperbola.

Q8

Let $2a$ and $2b$ be the transverse and conjugate axes and e be the eccentricity. Then,

The length of conjugate axis = $\frac{3}{4}$ [length of transverse axis]

$$\Rightarrow 2b = \frac{3}{4} \times (2a)$$

$$\Rightarrow \frac{b}{a} = \frac{3}{4}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{9}{16}$$

Now,

$$\begin{aligned} e &= \sqrt{1 + \frac{b^2}{a^2}} \\ &= \sqrt{1 + \frac{9}{16}} \\ &= \sqrt{\frac{25}{16}} \\ &= \frac{5}{4} \end{aligned}$$

Hence, $e = \frac{5}{4}$

Q9(i)

Let (x_2, y_2) be the coordinates of the second vertex.

We know that, the centre of the hyperbola is the mid-point of the line-joining the two vertices.

$$\therefore \frac{x_1 + 4}{2} = 3 \text{ and } \frac{y_1 + 2}{2} = 2 \quad [\because \text{Centre} = (3, 2) \text{ and vertex} = (4, 2)]$$

$$\Rightarrow x_1 = 2 \text{ and } y_2 = 2$$

\therefore The coordinates of the second vertex is $(2, 2)$

Let $2a$ and $2b$ be the length of transverse and conjugate axes and let e be eccentricity. Then, the equation of hyperbola is

$$\frac{(x-3)^2}{a^2} - \frac{(y-2)^2}{b^2} = 1 \quad \text{---(i)}$$

Now, distance between the two vertices $= 2a$

$$\Rightarrow \sqrt{(4-2)^2 + (2-2)^2} = 2a \quad [\because \text{Vertices} = (4, 2) \text{ and } (2, 2)]$$

$$\Rightarrow \sqrt{2^2} = 2a$$

$$\Rightarrow 2a = 2$$

$$\Rightarrow a = 1 \quad \text{---(ii)}$$

Q9(ii)

Let (x_1, y_1) be the coordinates of the second focus of the required hyperbola.

We know that, the centre of the hyperbola is the mid-point of the line-joining the two foci.

$$\begin{aligned} \therefore \quad \frac{x_1 + 4}{2} &= 6 \text{ and } \frac{y_1 + 2}{2} = 2 & [\because \text{Centre} = (6, 2) \text{ and focus} = (4, 2)] \\ \Rightarrow \quad x_1 &= 8 \text{ and } y_1 = 2 \end{aligned}$$

\therefore The coordinates of the second focus is $(8, 2)$

Let $2a$ and $2b$ be the length of transverse and conjugate axes and let e be the eccentricity. Then, the equation of hyperbola is

$$\frac{(x - 6)^2}{a^2} - \frac{(y - 2)^2}{b^2} = 1 \quad \text{---(i)}$$

Now, distance between the two vertices $= 2ae$

$$\begin{aligned} \Rightarrow \quad \sqrt{(8 - 4)^2 + (2 - 2)^2} &= 2ae & [\because \text{foci} = (4, 2) \text{ and } (8, 2)] \\ \Rightarrow \quad \sqrt{2^2} &= 2ae \\ \Rightarrow \quad 2a &= 2 \\ \Rightarrow \quad a &= 1 & \text{---(ii)} \end{aligned}$$

Now, the distance between the vertex and focus is $= ae - a$

$$\begin{aligned} \Rightarrow \quad \sqrt{(8 - 4)^2 + (2 - 2)^2} &= ae - a & [\because \text{Focus} = (8, 2) \text{ and vertex} = (4, 2)] \\ \Rightarrow \quad \sqrt{4^2} &= 2ae \\ \Rightarrow \quad 2ae &= 4 \\ \Rightarrow \quad 2 \times a \times e &= 4 & [\because e = 2] \\ \Rightarrow \quad a &= 1 \\ \Rightarrow \quad a^2 &= 1 \end{aligned}$$

Q10

For a hyperbole if the length of semi transverse and semi conjugate axes are equal.

Then $a = b$

Equation of the given hyperbole is

$$x^2 - y^2 = a^2 \dots\dots(1)$$

$$\text{Then } e = \sqrt{2}, C = (0, 0), S = (\sqrt{2}a, 0), S' = (-\sqrt{2}a, 0)$$

Let coordinates of any point P on hyperbole be (α, β) . Since P lies on (1)

$$\alpha^2 - \beta^2 = a^2 \dots\dots(2)$$

$$\text{Now } SP^2 = (\sqrt{2}a - \alpha)^2 + \beta^2 = 2a^2 + \alpha^2 + \beta^2 - 2\sqrt{2}a\alpha$$

$$\text{and } S'P^2 = (-\sqrt{2}a - \alpha)^2 + \beta^2 = 2a^2 + \alpha^2 + \beta^2 + 2\sqrt{2}a\alpha$$

$$\begin{aligned} \text{Now } SP \cdot S'P^2 &= (2a^2 + \alpha^2 + \beta^2)^2 - 8a^2\alpha^2 \\ &= 4a^4 + 4a^2(\alpha^2 + \beta^2) + (\alpha^2 + \beta^2)^2 - 8a^2\alpha^2 \\ &= 4a^2(\alpha^2 - 2a^2) + 4a^2(\alpha^2 + \beta^2) + (\alpha^2 + \beta^2)^2 \\ &= 4a^2(\alpha^2 - \beta^2 - 2a^2) + 4a^2(\alpha^2 + \beta^2) + (\alpha^2 + \beta^2)^2 \\ &= (\alpha^2 + \beta^2)^2 = CP^2 \end{aligned}$$

$$SP \cdot S'P = CP^2$$

Q11(i)

Let the equation of hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \dots\dots(i)$$

The coordinates of its vertices and foci are $(\pm a, 0)$ and $(\pm ae, 0)$ respectively.

$$\therefore a = 2 \quad [\because \text{vertices} = (\pm 2, 0)]$$

$$\Rightarrow a^2 = 4$$

and,

$$ae = 3 \quad [\because \text{Foci} = (\pm 3, 0)]$$

$$\Rightarrow 2 \times e = 3 \quad [\because a = 2]$$

$$\Rightarrow e = \frac{3}{2}$$

Now,

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow b^2 = 2^2 \left[\left(\frac{3}{2} \right)^2 - 1 \right]$$

$$\Rightarrow b^2 = 4 \left[\frac{9}{4} - 1 \right]$$

$$\Rightarrow b^2 = 4 \left[\frac{9 - 4}{4} \right]$$

$$= 4 \times \frac{5}{4}$$

$$= 5$$

Putting $a^2 = 4$ and $b^2 = 5$ in equation (1), we get

$$\frac{x^2}{4} - \frac{y^2}{5} = 1$$

Hence, the equation of the required hyperbola is $\frac{x^2}{4} - \frac{y^2}{5} = 1$.

Q11(ii)

Since, the vertices lie on y-axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \quad \text{---(i)}$$

The coordinates of its vertices and foci are $(0, \pm b)$ and $(0, \pm be)$ respectively.

$$\therefore b = 5 \quad [\because \text{vertices} = (0, \pm 5)]$$

$$\Rightarrow b^2 = 25$$

$$\text{and, } be = 8 \quad [\because \text{Foci} = (0, \pm 8)]$$

$$\Rightarrow 5 \times e = 8 \quad [\because b = 5]$$

$$\Rightarrow e = \frac{8}{5}$$

$$\Rightarrow e^2 = \frac{64}{25}$$

Now,

$$a^2 = b^2(e^2 - 1)$$

$$\Rightarrow a^2 = 25 \left(\frac{64}{25} - 1 \right) \quad \left[\because b^2 = 25 \text{ and } e^2 = \frac{64}{25} \right]$$

$$\Rightarrow a^2 = 25 \times \frac{39}{25}$$

$$\Rightarrow a^2 = 39$$

Putting $a^2 = 39$ and $b^2 = 25$ in equation (i), we get

$$\frac{x^2}{39} - \frac{y^2}{25} = -1$$

Hence, the equation of the required hyperbola is

$$\frac{x^2}{39} - \frac{y^2}{25} = -1.$$

Q11(iii)

Since, the vertices lie on y -axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \quad \text{---(i)}$$

The coordinates of its vertices and foci are $(0, \pm b)$ and $(0, \pm be)$ respectively.

$$\therefore b = 3 \quad [\because \text{vertices} = (0, \pm 3)]$$

$$\Rightarrow b^2 = 9$$

$$\text{and, } be = 5 \quad [\because \text{Foci} = (0, \pm 5)]$$

$$\Rightarrow e \times 3 = 5$$

$$\Rightarrow e = \frac{5}{3}$$

$$\Rightarrow e^2 = \frac{25}{9}$$

Now,

$$\begin{aligned} a^2 &= b^2(e^2 - 1) \\ \Rightarrow a^2 &= 9\left(\frac{25}{9} - 1\right) \\ &= 9 \times \left(\frac{25 - 9}{9}\right) \\ &= 9 \times \frac{16}{9} \\ &= 16 \end{aligned}$$

Putting $a^2 = 16$ and $b^2 = 9$ in equation (i), we get

$$\frac{x^2}{16} - \frac{y^2}{9} = -1$$

Hence, the equation of the required hyperbola is

$$\frac{x^2}{16} - \frac{y^2}{9} = -1.$$

Q11(iv)

Since, the vertices lie on x-axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{---(i)}$$

The length of transverse axis = 8

$$\therefore 2a = 8 \quad [\because \text{transverse axis is } 2a]$$

$$\Rightarrow a = 4$$

$$\Rightarrow a^2 = 16$$

This coordinates of foci of the required hyperbola is $(\pm ae, 0)$

$$\therefore ae = 5 \quad [\because \text{foci} = (\pm 5, 0)]$$

$$\Rightarrow 4 \times e = 5 \quad [\because a = 4]$$

$$\Rightarrow e = \frac{5}{4}$$

$$\rightarrow e^2 = \frac{25}{16}$$

Now,

$$\begin{aligned} b^2 &= a^2(e^2 - 1) \\ &= 16\left(\frac{25}{16} - 1\right) \\ &= 16 \times \frac{9}{16} \\ &= 9 \end{aligned}$$

Putting $a^2 = 16$ and $b^2 = 9$ in equation (i), we get

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Hence, the equation of the required hyperbola is

$$\frac{x^2}{16} - \frac{y^2}{9} = 1.$$

Q11(v)

Since, the vertices lie on x-axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \quad \text{---(i)}$$

The length of conjugate axis of the required hyperbola is 24.

$$\therefore 2a = 24 \quad [\because \text{conjugate axis is } 2a]$$

$$\Rightarrow a = \frac{24}{2} = 12$$

$$\Rightarrow a^2 = 144$$

This coordinates of foci of the required hyperbola is $(0, \pm be)$

$$\therefore be = 13 \\ b^2e^2 = 169$$

Now,

$$a^2 = b^2(e^2 - 1)$$

$$\Rightarrow 144 = b^2e^2 - b^2$$

$$\Rightarrow 144 = 169 - b^2$$

$$\Rightarrow b^2 = 169 - 144 = 25$$

Putting $a^2 = 144$ and $b^2 = 25$ in equation (i), we get

$$\frac{x^2}{144} - \frac{y^2}{25} = -1$$

Hence, the equation of the required hyperbola is

$$\frac{x^2}{144} - \frac{y^2}{25} = -1$$

Q11(vi)

Since, the vertices line on x-axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{---(i)}$$

The length of conjugate axis of the required hyperbola is 8.

$$\begin{aligned} \therefore \quad \frac{2b^2}{a} &= 8 \\ \Rightarrow \quad b^2 &= \frac{8}{2} \times a \\ \Rightarrow \quad b^2 &= 4a \end{aligned}$$

---(ii)

Now,

This coordinates of foci of the required hyperbola is $(\pm ae, 0)$

$$\begin{aligned} \therefore \quad ae &= 3\sqrt{5} & [\because \text{Foci} = (\pm 3\sqrt{5}, 0)] \\ \Rightarrow \quad e &= \frac{3\sqrt{5}}{a} \\ \Rightarrow \quad e^2 &= \frac{45}{a^2} \end{aligned}$$

---(iii)

Q11(vii)

Since, the vertices line on x-axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{---(i)}$$

The length of the latus-rectum of the required hyperbola is 12

$$\begin{aligned} \therefore \quad \frac{2b^2}{a} &= 12 \\ \Rightarrow \quad b^2 &= 6a \end{aligned}$$

---(ii)

Now,

The coordinates of foci of the required hyperbola is $(\pm ae, 0)$

$$\begin{aligned} \therefore \quad ae &= 4 \\ \Rightarrow \quad e &= \frac{4}{a} \\ \Rightarrow \quad e^2 &= \frac{16}{a^2} \end{aligned}$$

---(iii)

Q11(viii)

Since, the vertices lie on x -axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{---(i)}$$

The length of the vertices of the required hyperbola are $(\pm a, 0)$.

$$\begin{aligned} \therefore a &= 7 & [\because \text{vertices} &= (\pm 7, 0)] \\ \Rightarrow a^2 &= 49 & \text{---(ii)} \end{aligned}$$

Now,

$$\begin{aligned} b^2 &= a^2 (e^2 - 1) \\ \Rightarrow b^2 &= 49 \left[\left(\frac{4}{3} \right)^2 - 1 \right] & [\because e = \frac{4}{3}] \\ \Rightarrow b^2 &= 49 \left[\frac{16}{9} - 1 \right] \\ \Rightarrow b^2 &= 49 \left[\frac{7}{9} \right] \\ \Rightarrow b^2 &= \frac{343}{9} \end{aligned}$$

Putting $a^2 = 49$ and $b^2 = \frac{343}{9}$ in equation (i), we get

$$\begin{aligned} \frac{x^2}{49} - \frac{y^2}{\frac{343}{9}} &= 1 \\ \Rightarrow \frac{x^2}{49} - \frac{9y^2}{343} &= 1 \end{aligned}$$

Hence, the equation of the required hyperbola is

$$\frac{x^2}{49} - \frac{9y^2}{343} = 1.$$

Q11(ix)

Since, the vertices lie on y-axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \quad \text{---(i)}$$

It passes through (2,3)

$$\therefore \frac{(2)^2}{a^2} - \frac{(3)^2}{b^2} = -1$$

$$\Rightarrow \frac{4}{a^2} - \frac{9}{b^2} = -1$$

$$\Rightarrow \frac{4}{a^2} - \frac{9}{a^2(e^2 - 1)} = -1 \quad \left[\because b^2 = a^2(e^2 - 1) \right]$$

$$\Rightarrow \frac{4}{a^2} - \frac{9}{a^2 e^2 - a^2} = -1 \quad \text{---(ii)}$$

The coordinates of foci of the required hyperbola are $(0, \pm ae)$.

$$\therefore ae = \sqrt{10}$$

$$\Rightarrow a^2 e^2 = 10 \quad \text{---(iii)}$$

Q11(x)

Since, the vertices lie on x- axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \quad \text{----- (i)}$$

The length of the latus-rectum of the required hyperbola is 36.

$$\frac{2a^2}{b} = 36$$

$$a^2 = 18b \quad \text{-----(ii)}$$

Now,

The coordinates of foci of the required hyperbola is $(0, \pm be)$.

$$be = 12$$

$$e = \frac{12}{b}$$

$$e^2 = \frac{144}{b^2}$$

Now,

$$a^2 = b^2(e^2 - 1)$$

$$18b = b^2 \left(\frac{144}{b^2} - 1 \right)$$

$$18b = 144 - b^2$$

$$b^2 + 18b - 144 = 0$$

$$(b - 6)(b + 24) = 0$$

$$b_1 = 6, -24$$

Consider the positive value of $b = 6$.

On putting $b^2 = 36$, $a^2 = 18(6) = 108$ in equation (i), we get

$$\frac{x^2}{108} - \frac{y^2}{36} = -1$$

$$\frac{x^2 - 3y^2}{108} = -1$$

$$x^2 - 3y^2 = -108$$

$$3y^2 - x^2 = 108$$

Therefore, the equation of the hyperbola is $3y^2 - x^2 = 108$.

Q12

Eccentricity $= e = \sqrt{2}$

Distance between foci is

$$2ae = 16$$

$$2a\sqrt{2} = 16$$

$$a = \frac{16}{2\sqrt{2}} = 4\sqrt{2}$$

$$e = \frac{\sqrt{a^2 + b^2}}{a}$$

$$\sqrt{2} = \frac{\sqrt{32 + b^2}}{4\sqrt{2}}$$

$$8 = \sqrt{32 + b^2}$$

$$64 = 32 + b^2$$

$$b^2 = 32$$

Equation of hyperbola is $\frac{x^2}{32} - \frac{y^2}{32} = 1$

Rewriting we get, $x^2 - y^2 = 32$

Q13

Let P (x,y) be a point of the set.

$$\text{Distance of P(x,y) from (4,0)} = \sqrt{(x-4)^2 + y^2}$$

$$\text{Distance of P(x,y) from (-4,0)} = \sqrt{(x+4)^2 + y^2}$$

Difference between distance = 2

$$\sqrt{(x-4)^2 + y^2} - \sqrt{(x+4)^2 + y^2} = 2$$

$$\sqrt{(x-4)^2 + y^2} = 2 + \sqrt{(x+4)^2 + y^2}$$

Squaring both sides, we get,

$$(x-4)^2 + y^2 = 4 + 4\sqrt{(x+4)^2 + y^2} + (x+4)^2 + y^2$$

$$(x-4)^2 + y^2 - (x+4)^2 - y^2 = 4 + 4\sqrt{(x+4)^2 + y^2}$$

$$(x-4-x-4)(x-4+x+4) = 4 + 4\sqrt{(x+4)^2 + y^2}$$

$$-16x - 4 = 4\sqrt{(x+4)^2 + y^2}$$

$$-4x - 1 = \sqrt{(x+4)^2 + y^2}$$

Squaring both sides, we get,

$$16x^2 + 8x + 1 = x^2 + 8x + 16 + y^2$$

$$15x^2 - y^2 = 15$$

This is a hyperbola.