

Ex 19.1

Q1

$a_n = n^2 - n + 1$ --- (i) is the given sequence

Then, first 5 terms are a_1, a_2, a_3, a_4 and a_5

$$a_1 = (1)^2 - 1 + 1 = 1$$

$$a_2 = (2)^2 - 2 + 1 = 3$$

$$a_3 = (3)^2 - 3 + 1 = 7$$

$$a_4 = (4)^2 - 4 + 1 = 13$$

$$a_5 = (5)^2 - 5 + 1 = 21$$

First 5 terms 1, 3, 7, 13 and 21.

Q2

$$a_n = n^3 - 6n^2 + 11n - 6 \quad n \in N.$$

The first three terms are a_1, a_2 and a_3

$$a_1 = (1)^3 - 6(1)^2 + 11(1) - 6 = 0$$

$$a_2 = (2)^3 - 6(2)^2 + 11(2) - 6 = 0$$

$$a_3 = (3)^3 - 6(3)^2 + 11(3) - 6 = 0$$

\therefore The 1st 3 terms are zero.

and

$$\begin{aligned} a_n &= n^3 - 6n^2 + 11n - 6 \\ &= (n-2)^3 - (n-2) \text{ is positive as } n \geq 4 \end{aligned}$$

$\therefore a_n$ is always positive.

Q3

$$a_n = 3a_{n-1} + 2 \text{ for } n > 1$$

$$\therefore a_2 = 3a_{2-1} + 2 = 3a_1 + 2$$

$$= 3(3) + 2 = 11$$

$$[\therefore a_1 = 3]$$

$$a_3 = 3a_{3-1} + 2 = 3a_2 + 2$$

$$= (11) + 2 = 35$$

$$[\therefore a_2 = 11]$$

$$a_4 = 3a_{4-1} + 2 = 3a_3 + 2$$

$$= 3(35) + 2 = 107$$

$$[\therefore a_3 = 35]$$

\therefore The first four terms of A.P are 3, 11, 35, 107.

Q4

$$(i) \quad a_1 = 1, \quad a_n = a_{n-1} + 2, \quad n \geq 2$$

$$a_2 = a_{2-1} + 2 = a_{1+2} = 3$$

$$[\because a_1 = 1]$$

$$a_3 = a_{3-1} + 2 = a_2 + 2 = 5$$

$$[\because a_2 = 3]$$

$$a_4 = a_{4-1} + 2 = a_3 + 2 = 7$$

$$[\because a_3 = 5]$$

$$a_5 = a_{5-1} + 2 = a_4 + 2 = 9$$

$$[\because a_4 = 7]$$

\therefore The first 5 terms of series are 1, 3, 5, 7, 11.

$$(ii) \quad a_1 = a_2 = 1$$

$$a_n = a_{n-1} + a_{n-2} \quad n > 2$$

$$\Rightarrow a_3 = a_{3-1} + a_{3-2}$$

$$= a_2 + a_1 = 1 + 1 = 2$$

$$\Rightarrow a_4 = a_{4-1} + a_{4-2}$$

$$= a_3 + a_2 = 2 + 1 = 3$$

$$\Rightarrow a_5 = a_{5-1} + a_{5-2}$$

$$= a_4 + a_3 = 5$$

\therefore The given sequence is 1, 1, 3, 5.

$$(iii) \quad a_1 = a_2 = 2$$

$$a_n = a_{n-1} - 1 \quad n > 2$$

$$\Rightarrow a_3 = a_{3-1} - 1$$

$$= a_2 - 1$$

$$= 2 - 1 = 1$$

$$\Rightarrow a_4 = a_{4-1} - 1$$

$$= a_3 - 1 = 1 - 1 = 0$$

$$\Rightarrow a_5 = a_{5-1} - 1$$

$$= 0 - 1 = -1$$

\therefore The first 5 terms of the sequence are 2, 2, 1, 0, -1.

Q5

$$\begin{aligned}
 a_n &= a_{n-1} + a_{n-2} && \text{for } n > 2 \\
 \Rightarrow a_3 &= a_{3-1} + a_{3-2} = a_2 + a_1 = 1 + 1 = 2 \\
 \Rightarrow a_4 &= a_{4-1} + a_{4-2} = a_3 + a_2 = 2 + 1 = 3 \\
 \Rightarrow a_5 &= a_{5-1} + a_{5-2} = a_4 + a_3 = 3 + 2 = 5 \\
 \Rightarrow a_6 &= a_{6-1} + a_{6-2} = a_5 + a_4 = 5 + 3 = 8
 \end{aligned}$$

\therefore For $n = 1$

$$\frac{a_{n+1}}{a_n} = \frac{a_2}{a_1} = \frac{1}{1} = 1$$

For $n = 2$

$$\frac{a^3}{a_2} = \frac{2}{1} = 2$$

For $n = 3$

$$\frac{a_4}{a_3} = \frac{3}{2} = 1.5$$

For $n = 4$ and $n = 5$

$$\frac{a_5}{a_4} = \frac{5}{3} \quad \text{and} \quad \frac{a_6}{a_5} = \frac{8}{5}$$

\therefore The required series is $1, 2, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \dots$

Q6(i)

$3, -1, -5, -9, \dots$

$$a_1 = 3, a_2 = -1, a_3 = -5, a_4 = -9$$

$$a_2 - a_1 = -1 - 3 = -4$$

$$a_3 - a_2 = -5 - (-1) = -4$$

$$a_4 - a_3 = -9 - (-5) = -4$$

\therefore Common difference is $d = -4$

$$a_4 - a_3 = a_3 - a_2 = d$$

\therefore The given sequence is an A.P

$$\therefore a_5 = 3 + (5 - 1)(-4) = -13$$

$$a_6 = 3 + (6 - 1)(-4) = -17$$

$$a_7 = 3 + (7 - 1)(-4) = -21$$

Q6(ii)

$$-1, \frac{1}{4}, \frac{3}{2}, \frac{11}{4} \dots$$

$$a_1 = -1, a_2 = \frac{1}{4}, a_3 = \frac{3}{2}, a_4 = \frac{11}{4}$$

$$a_4 - a_3 = a_3 - a_2 = a_2 - a_1 = \frac{5}{4}$$

∴ Common difference is $d = \frac{5}{4}$

∴ The given sequence is A.P

$$a_5 = -1 + (5 - 1) \frac{5}{4} = 4$$

$$a_6 = -1 + (6 - 1) \frac{5}{4} = \frac{21}{4}$$

$$a_7 = -1 + (7 - 1) \frac{5}{4} = \frac{26}{4} = \frac{13}{2}$$

Q6(iii)

$$(iii) \sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2} \dots$$

$$a_1 = \sqrt{2}, a_2 = 3\sqrt{2}, a_3 = 5\sqrt{2}, a_4 = 7\sqrt{2}$$

$$a_4 - a_3 = a_3 - a_2 = a_2 - a_1 = 2\sqrt{2}$$

∴ The common difference is $2\sqrt{2}$

and the given sequence is A.P

$$a_5 = \sqrt{2} + 2\sqrt{2}(5 - 1) = 9\sqrt{2}$$

$$a_6 = \sqrt{2} + 2\sqrt{2}(6 - 1) = 11\sqrt{2}$$

$$a_7 = \sqrt{2} + 2\sqrt{2}(7 - 1) = 13\sqrt{2}$$

Q6(iv)

$$9, 7, 5, 3, \dots$$

$$a_4 - a_3 = a_3 - a_2 = a_2 - a_1 = -2$$

∴ The common difference is -2

and the given sequence is A.P

$$a_5 = 9 + (-2)(5 - 1) = 1$$

$$a_6 = 9 + (-2)(6 - 1) = -1$$

$$a_7 = 9 + (-2)(7 - 1) = -3$$

Q7

$$a_n = 2n + 7$$

$$a_1 = 2(1) + 7 = 9$$

$$a_2 = 2(2) + 7 = 11$$

$$a_3 = 2(3) + 7 = 13$$

$$\text{Here, } a_3 - a_2 = a_2 - a_1 = 2$$

∴ The given sequence is A.P

$$a_7 = 2(7) + 7 = 21$$

7th term is 21.

Q8

$$a_n = 2n^2 + n + 1$$

$$a_1 = 2(1)^2 + (1) + 1 = 4$$

$$a_2 = 2(2)^2 + (2) + 1 = 11$$

$$a_3 = 2(3)^2 + (3) + 1 = 21$$

$$a_3 - a_2 \neq a_2 - a_1$$

∴ The given sequence is not as A.P as consecutive terms do not have a common difference.

Ex 19.2

Q1

(i) 10th term of A.P 1, 4, 7, 10, ...

Here, 1st term $= a_1 = 1$

and common difference $d = 4 - 1 = 3$

We know $a_n = a_1 + (n - 1)d$

$$\begin{aligned}\therefore a_{10} &= a_1 + (10 - 1)d \\ &= 1 + (10 - 1)3 \Rightarrow 28\end{aligned}$$

(ii) To find 18th term of A.P $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, \dots$

Here, 1st term $a_1 = \sqrt{2}$

and $d = \text{common difference} = 2\sqrt{2}$

$$\begin{aligned}\therefore a_n &= a_1 + (n - 1)d \\ a_{18} &= \sqrt{2} + 2\sqrt{2}(17) = 35\sqrt{2}\end{aligned}$$

(iii) Find n th term of A.P 13, 8, 3, -2

Here, $a_1 = 13$

$$d = -5$$

$$\begin{aligned}\therefore a_n &= a + (n - 1)d \\ &= 13 + (n - 1)(-5) \\ &= -5n + 18\end{aligned}$$

Q2

It is given that the sequence $\langle a_n \rangle$ is an A.P

$$\therefore a_n = a + (n - 1)d \quad \text{---(i)}$$

Similarly from (i)

$$a_{m+n} = a + (m + n - 1)d \quad \text{---(ii)}$$

$$a_{m-n} = a + (m - n - 1)d \quad \text{---(iii)}$$

Adding (ii) and (iii)

$$\begin{aligned}a_{m+n} + a_{m-n} &= \{a + (m + n - 1)d\} + \{a + (m - n - 1)d\} \\ &= 2a + (m + n - 1 + m - n - 1)d \\ &= 2a + 2d(m - 1) \\ &= 2\{a + (m - 1)d\} \\ &= 2a_m \text{ Hence proved.}\end{aligned}$$

Q3

(i) Let n th term of A.P = 248

$$\therefore a_n = 248 = a + (n - 1)d$$

$$\Rightarrow 248 = 3 + (n - 1)5$$

$$\therefore n = 50$$

\therefore 50th term of the given A.P is 248.

(ii) Which term of A.P 84, 80, 76 is 0?

Let n th term of A.P be 0

$$\text{Then, } a_n = 0 = a + (n - 1)d$$

$$0 = 84 + (n - 1)(-4)$$

$$\therefore n = 22$$

\therefore 22nd term of the given A.P is 0.

(iii) Which term of A.P is 4, 9, 14, ... is 254?

Let n th term of A.P be 254

$$a_n = a + (n - 1)d$$

$$254 = 4 + (n - 1)5$$

$$\therefore n = 51$$

\therefore 51st term of the given A.P is 254.

Q4

(i) Is 68 a term of A.P 7, 10, 13, ...?

Here, $a = 7$

and $x = 10 - 7 = 3$

$$\begin{aligned}\therefore a_n \text{ term is } &= a + (n - 1)d \\ &= 7 + (n - 1)3\end{aligned}$$

Let 68 be n th term of A.P

Then,

$$\begin{aligned}68 &= 7 + 3(n - 1) \\ \Rightarrow 68 &= 7 + 3n - 3 \\ \Rightarrow 68 - 4 &= 3n \\ \Rightarrow 64 &= 3n \\ \Rightarrow n &= \frac{64}{3}\end{aligned}$$

Which is not a natural number.

\therefore 68 is not a term of given A.P.

(ii) Is 302 a term of A.P 3, 8, 13

Let 302 be n th term of the given A.P

Here, $302 = 3 + (n - 1)5$

$$\begin{aligned}\frac{299}{5} &= (n - 1) \\ n &= \frac{304}{5}\end{aligned}$$

Which is not a natural number.

\therefore 302 is not a term of given A.P.

Q5

(i) The given sequence is $24, 23\frac{1}{4}, 22\frac{1}{2}, 21\frac{3}{4}, \dots$

Here, $a = 24$

$$d = 23\frac{1}{4} - 24 = \frac{93 - 96}{4} = \frac{-3}{4}$$

Let n th term be the 1st negative term.

$$a_n < 0$$

$$a + (n - 1)d < 0$$

$$24 - \frac{3}{4}(n - 1) < 0$$

$$96 - 3n + 3 < 0$$

$$99 < 3n$$

$$33 < n \quad \text{or} \quad n > 33$$

\therefore 34th term is 1st negative term.

(ii) The given sequence is $12 + 8i, 11 + 6i, 10 + 4i, \dots$

Here, $a = 12 + 8i$

$$d = -1 - 2i$$

Then, $a_n = a + (n - 1)d$

$$= 12 + 8i + (n - 1)(-1 - 2i)$$

$$= (13 - n) + i(10 - 2n)$$

Let n th term be purely real the $(10 - 2n) = 0$ or $n = 5$

So, 5th term is purely real.

Let n th term be purely imaginary. Then, $13 - n = 0$

$$\therefore n = 13$$

So, 13th term is purely imaginary.

Q6

(i) The given A.P is 7, 10, 13, ... 43.

Let there be n terms,

then, n term = 43

$$\text{or } 43 = a_n = a + (n - 1)d$$

$$\Rightarrow 43 = 7 + (n - 1)3$$

$$\Rightarrow n = 13$$

Thus, there are 13 terms in the given sequence.

(ii) The given A.P is $-1, \frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2}, \dots, \frac{10}{3}$?

Let there be n terms

$$\text{then, } n\text{th term} = \frac{10}{3}$$

$$\text{or } \frac{10}{3} = a_n = a + (n - 1)d$$

$$\Rightarrow \frac{10}{3} = -1 + (n - 1)\left(\frac{-5}{6} + 1\right)$$

$$\Rightarrow n = 27$$

Thus, there are 27 terms in the given sequence.

Q7

Given: $a = 5$

$$d = 3$$

$$a_n = \text{last term} = 80$$

Let there be n terms

$$\therefore a_n = 80 = a + (n - 1)d$$

$$80 = 5 + (n - 1)3$$

$$\Rightarrow n = 26$$

\therefore Thus, there are 26 terms in the given sequence.

Q8

Given that:

$$a_6 = 19 = a + (6 - 1)d \quad \text{--- (i)}$$

$$a_{17} = 41 = a + (17 - 1)d \quad \text{--- (ii)}$$

Solving (i) and (ii), we get

$$a = 9 \text{ and } d = 2$$

$$\begin{aligned} \therefore a_{40} &= a + (40 - 1)d \\ &= 9 + (40 - 1)2 \\ &= 9 + 39(2) \\ &= 87 \end{aligned}$$

40th term of the given sequence is 87.

Q9

Given:

$$a_9 = 0$$

$$\therefore a + 8d = 0$$

$$a = -8d \quad \text{--- (i)}$$

$$a_{19} = a + (19 - 1)d$$

$$= a + 18d$$

$$= -8d + 18d$$

$$= 10d$$

$$[\because a = -8d \text{ from (i)}]$$

$$\text{--- (ii)}$$

$$a_{29} = a + (29 - 1)d$$

$$= -8d + 28d$$

$$= 20d$$

$$[\because a = -8d \text{ from (i)}]$$

$$\text{--- (iii)}$$

From (ii) and (iii)

$$a_{29} = 2a_{19} \quad \text{Hence proved.}$$

Q10

Given:

$$\begin{aligned}10a_{10} &= 15a_{15} \\ \Rightarrow 10(a + (10 - 1)d) &= 15(a + (15 - 1)d) \\ \Rightarrow 10a + 90d &= 15a + 210d \\ \Rightarrow 5a + 120d &= 0 \\ \Rightarrow a + 24d &= 0 \quad \text{---(i)}\end{aligned}$$

$$\begin{aligned}a_{25} &= a + (25 - 1)d \\ &= a + 24d \\ &= 0 \quad \quad \quad [\because \text{from (i) } a + 24d = 0]\end{aligned}$$

Hence proved.

Q11

Given:

$$\begin{aligned}a_{10} &= 41 = a + 9d \quad \text{---(i)} \\ a_{18} &= 73 = a + 17d \quad \text{---(ii)}\end{aligned}$$

Solving (i) and (ii)

$$\begin{aligned}a + 9d &= 41 \\ a + 17d &= 73\end{aligned}$$

We get $a = 5$ and $d = 4$

$$\begin{aligned}\therefore a_{26} &= a + (26 - 1)d \\ &= 5 + 25(4) \\ &= 105\end{aligned}$$

26th term of the given A.P is 105.

Q12

Given:

$$\begin{aligned}a_{24} &= 2a_{10} \\ \Rightarrow a + 23d &= 2(a + 9d) \\ \Rightarrow a &= 5d \quad \text{---(i)}\end{aligned}$$

$$\begin{aligned}a_{72} &= a + (72 - 1)d \\ &= a + 71d \\ \Rightarrow &= 76d \quad \left[\because a = 5d \text{ from (i)} \right] \\ &\text{---(ii)}\end{aligned}$$

$$\begin{aligned}a_{34} &= a + (34 - 1)d \\ &= 5d + 33d \quad \left[\because a = 5d \text{ from (i)} \right] \\ &= 38d \quad \text{---(iii)}\end{aligned}$$

From (ii) and (iii)

$$a_{72} = 2a_{34} \quad \text{Hence proved.}$$

Q13

Given:

$$\begin{aligned}a_{m+1} &= 2a_{n+1} \\ \Rightarrow a + (m+1-1)d &= 2(a + (n+1-1)d) \\ \Rightarrow a + md &= 2a + 2nd \\ \Rightarrow a &= (m - 2n)d \quad \text{---(i)}\end{aligned}$$

Then,

$$\begin{aligned}a_{3m+1} &= a + (3m+1-1)d \\ &= a + 3md \\ &= 3d - 2nd + 3md \\ &= 2(2m - n)d \quad \text{---(ii)}\end{aligned}$$

$$\begin{aligned}a_{m+n+1} &= a + (m+n+1-1)d \\ &= md - 2nd + md + nd \\ &= (2m - n)d \quad \text{---(iii)}\end{aligned}$$

From (ii) and (iii)

$$a_{2m+1} = 2a_{m+n+1} \quad \text{Hence proved.}$$

Q14

The given A.P is 9, 7, 5, ... and 15, 12, 9

Here,

$$a = 9 \quad A = 15$$

$$d = -2 \quad D = 3$$

Let $a_n = A_n$ for same n .

$$\Rightarrow a + (n-1)d = A + (n-1)D$$

$$\Rightarrow 9 + (n-1)(-2) = 15 + (n-1)3$$

$$\Rightarrow n = 7$$

\therefore 7th term of both the A.P is same.

Q15

(i) A.P is 3, 5, 7, 9, ..., 201.

Here, $a = 3$

$$d = 2$$

n th term from the end is $l - (n-1)d$

i.e. $201 - (n-1)2$ or $203 - 2n$

---(i)

12th term from end is

$$203 - 2(12) = 179$$

(ii) A.P is 3, 8, 13, ..., 253.

Then, 12th term from end is $l - (n-1)d$ i.e.,

$$= 253 - (12-1)5$$

$$= 253 - 55$$

$$= 198$$

(iii) A.P is 1, 4, 7, 10, ..., 88

Then, 12th term from end is $l - (n-1)d$

$$= 88 - (12-1)3$$

$$= 88 - 33$$

$$= 55$$

Q16

Given,

$$a = 3a_1 \quad \text{---(i)}$$

$$a_7 = 2a_3 + 1 \quad \text{---(ii)}$$

Expanding (i) and (ii)

$$a + 3d = 2a$$

$$\therefore 2a = 3d \text{ or } a = \frac{3d}{2} \quad \text{---(iii)}$$

$$a + 6d = 2a + 4d + 1$$

$$a + 1 = 2d \quad \text{---(iv)}$$

From (iii) and (iv)

$$a = 3 \text{ and } d = 2$$

\therefore 1st term of the given A.P is 3, and common difference is 2.

Q17

$$a_6 = a + 5d = 12 \quad \text{---(i)}$$

$$a_8 = a + 7d = 22 \quad \text{---(ii)}$$

Solving (i) and (ii)

$$a = -13 \text{ and } d = 5$$

Then,

$$\begin{aligned} a_n &= a + (n - 1)d \\ &= -13 + (n - 1)5 \\ &= 5n - 18 \end{aligned}$$

and

$$\begin{aligned} a_2 &= a + (2 - 1)d \\ &= -13 + 5 \\ &= -8 \end{aligned}$$

Q18

The first two digit number divisible by 3 is 12.
and last two digit number divisible by 3 is 99.

So, the required series is 12, 15, 18, ... 99.

Let there be n terms then n th term = 99

$$\Rightarrow 99 = a + (n - 1)d$$

$$\Rightarrow 99 = 12 + (n - 1)3$$

$$\Rightarrow n = 30$$

30 two digit numbers are divisible by 3.

Q19

Given,

$$n = 60$$

$$a = 7$$

$$l = 125$$

$$\therefore a + (n - 1)d = 125$$

$$7 + (59)d = 125$$

$$d = 2$$

$$\therefore a_{32} = a + (32 - 1)d$$

$$= 7 + (31)2$$

$$= 69$$

32nd term is 69.

Q20

$$a_4 + a_8 = 24 \quad \text{[Given]}$$

$$\Rightarrow (a + 3d) + (a + 7d) = 24$$

$$\Rightarrow a + 5d = 12 \quad \text{---(i)}$$

$$a_6 + a_{10} = 34$$

$$\Rightarrow (a + 5d) + (a + 9d) = 34$$

$$\Rightarrow a + 7d = 17 \quad \text{---(ii)}$$

From (i) and (ii)

$$a = \frac{-1}{2} \text{ and } d = \frac{5}{2}$$

\therefore 1st term is $\frac{-1}{2}$ and common difference is $\frac{5}{2}$.

Q21

The n th term from starting

$$= a_n = a + (n - 1)d \quad \text{---(i)}$$

The n th term from end

$$= l - (n - 1)d \quad \text{---(ii)}$$

Adding (i) and (ii), we get

Sum of n th term from beginning and n th term from the end

$$= a + (n - 1)d + l - (n - 1)d$$

$$= a + l \text{ Hence proved.}$$

Q22

$$\frac{a_4}{a_7} = \frac{2}{3}$$

[Given]

$$\Rightarrow \frac{a+3d}{a+6d} = \frac{2}{3}$$

=

$$\Rightarrow 3a+9d = 2a+12d$$

$$\Rightarrow a = 3d$$

---(i)

$$\frac{a_6}{a_8} = \frac{a+5d}{a+7d}$$

$$\Rightarrow = \frac{3d+5d}{3d+7d}$$

[\because $3d$ from (i)]

$$\Rightarrow = \frac{8d}{10d}$$

$$\Rightarrow = \frac{4}{5}$$

$$\frac{a_6}{a_8} = \frac{4}{5}$$

Q23

$$\sec \theta_1 \sec \theta_2 + \sec \theta_2 \sec \theta_3 + \dots + \sec \theta_{n-1} \sec \theta_n = \frac{\tan \theta_n - \tan \theta_1}{\sin d}$$

$$\theta_2 - \theta_1 = \theta_3 - \theta_2 = \dots = d$$

$$\sec \theta_1 \sec \theta_2 = \frac{1}{\cos \theta_1 \cos \theta_2} = \frac{\sin d}{\sin d (\cos \theta_1 \cos \theta_2)}$$

$$= \frac{\sin (\theta_2 - \theta_1)}{\sin d (\cos \theta_1 \cos \theta_2)}$$

$$= \frac{\sin \theta_2 \cos \theta_1 - \cos \theta_2 \sin \theta_1}{\sin d (\cos \theta_1 \cos \theta_2)}$$

$$= \frac{1}{\sin d} \left[\frac{\sin \theta_2 \cos \theta_1}{(\cos \theta_1 \cos \theta_2)} - \frac{\cos \theta_2 \sin \theta_1}{(\cos \theta_1 \cos \theta_2)} \right]$$

$$= \frac{1}{\sin d} [\tan \theta_2 - \tan \theta_1]$$

$$\text{Similarly, } \sec \theta_2 \sec \theta_3 = \frac{1}{\sin d} [\tan \theta_3 - \tan \theta_2]$$

If we add up all terms, we get

$$= \frac{1}{\sin d} [\tan \theta_2 - \tan \theta_1 + \tan \theta_3 - \tan \theta_2 + \dots + \tan \theta_n - \tan \theta_{n-1}]$$

$$= \frac{1}{\sin d} [\tan \theta_n - \tan \theta_1]$$

Hence Proved

Ex 19.3

Q1

Let the 3rd term of A.P be

$$a - d, a, a + d$$

Then,

$$a - d + a + a + d = 21$$

$$3a = 21$$

$$\therefore a = 7$$

and

$$(a - d)(a + d) = a + 6$$

$$a^2 - d^2 = a + 6$$

$$7^2 - d^2 = 7 + 6 \quad [\because a = 7]$$

$$d^2 = 36$$

$$d = \pm 6$$

Since d can't be negative, therefore

\therefore The A.P is 1, 7, 13.

Q2

Let the 3 numbers in A.P are

$$a - d, a, a + d$$

Then,

$$a - d + a + a + d = 27$$

$$3a = 27$$

$$\therefore a = 9 \quad \text{---(i)}$$

and

$$(a - d)(a)(a + d) = 648$$

$$(9 - d)9(9 + d) = 648 \quad [\because a = 9]$$

$$9^2 - d^2 = 72$$

$$\therefore d = 3 \quad \text{---(ii)}$$

\therefore The given sequence is 6, 9, 12.

Q3

Let the four numbers in A.P be

$$a - 3d, a - d, a + d, a + 3d$$

$$(a - 3d) + (a - d) + (a + d) + (a + 3d) = 50$$

$$4a = 50$$

$$a = \frac{25}{2} \quad \text{--- (i)}$$

and

$$(a + 3d) = 4(a - 3d)$$

$$\frac{25 + 6d}{2} = 50 - 12d$$

$$30d = 75$$

$$d = \frac{25}{10} = \frac{5}{2} \quad \text{--- (ii)}$$

\therefore The required sequence is 5, 10, 15, 20.

Q4

Let three numbers be $a - d, a, a + d$

Then,

$$a - d + a + a + d = 12$$

$$3a = 12$$

$$a = 4$$

and

$$(a - d)^3 + a^3 + (a + d)^3 = \pm 288$$

$$a^3 + d^3 + 3ad(a + d) + a^3 + a^3 - a^3 - 3ad(a - d) = 288$$

$$\Rightarrow 2a^3 + 3a^2d + 3ad^2 - 3a^2d + 3ad^2 = 288$$

$$\Rightarrow 2a^3 + 3a^2d^2 = 288$$

$$\Rightarrow 128 + 48d^2 = 288$$

$$\therefore d = \pm 2$$

\therefore The required sequence is 2, 4, 6 or 6, 4, 2.

Q5

Let 3 numbers in A.P be

$$a - d, a \text{ and } a + d$$

$$\Rightarrow (a - d) + (a) + (a + d) = 24$$

$$3a = 24$$

$$a = 8$$

and

$$(a - d)(a)(a + d) = 440$$

$$8^2 - d^2 = 55$$

$$d = 3$$

\therefore The required sequence is 5, 8, 11.

Q6

Let the four angle be

$$a - 3d, a - d, a + d, a + 3d$$

Then,

$$\text{sum of all angles} = 360^\circ$$

$$a - 3d + a - d + a + d + a + 3d = 360^\circ$$

$$4a = 360^\circ$$

$$a = 90^\circ \quad \text{---(i)}$$

and

$$(a - d) - (a - 3d) = 10$$

$$2d = 10$$

$$d = 5$$

\therefore The angle of the given quadrilateral are $75^\circ, 85^\circ, 95^\circ$ and 105° .

Ex 19.4

Q1

(i) 50, 46, 42, ..., 10 terms

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\begin{aligned} S_{10} &= \frac{10}{2} [2 \times 50 + (10-1)(-4)] \\ &= 320 \end{aligned}$$

(ii) 13, 5, ..., 12 terms

$$\begin{aligned} S_{12} &= \frac{12}{2} [2 \times 13 + (12-1)(-8)] \\ &= 6 \times 24 = 144 \end{aligned}$$

(iii) $3, \frac{9}{2}, 6, \frac{15}{2}, \dots, 25$ terms

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\begin{aligned} S_{25} &= \frac{25}{2} \left(2 \times 3 + 24 \times \frac{3}{2} \right) \\ &= 525 \end{aligned}$$

(iv) 41, 36, 31, ..., 12 terms

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\begin{aligned} S_{12} &= \frac{12}{2} [2 \times 41 + (11)(-5)] \\ &= 162 \end{aligned}$$

(v) $a+b, a-b, a-3b, \dots$ to 22 terms

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\begin{aligned} S_{22} &= \frac{22}{2} [2a + 2b + 21(-2b)] \\ &= 22a - 440b \end{aligned}$$

(vi) $(x-y)^2, (x^2+y^2), (x+y)^2, \dots, x$ terms

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{n}{2} [2(x^2+y^2-2xy) + (n-1)(-2xy)]$$

$$= n[(x-y)^2 + (n-1)xy]$$

$$\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}, \dots \text{to } n \text{ terms}$$

$$n\text{th term in above sequence is } \frac{(2n-1)x - ny}{x+y}$$

Sum of n terms is given by

$$\frac{1}{x+y} [x + 3x + 5x + \dots + (2n-1)x - (y + 2y + 3y \dots + ny)]$$

$$= \frac{1}{x+y} \left[\frac{n}{2} (2x + (n-1)2x) - \frac{n(n+1)y}{2} \right]$$

$$= \frac{1}{2(x+y)} [2n^2x - 2n^2y - ny]$$

Q2

(i) $2 + 5 + 8 + \dots + 182.$

a_n term of given A.P is 182

$$a_n = a + (n - 1)d = 182$$

$$\Rightarrow 182 = 2 + (n - 1)3$$

or $n = 61$

Then,

$$\begin{aligned} S_n &= \frac{n}{2}[a + l] \\ &= \frac{61}{2}[2 + 182] \\ &= 61 \times 92 \\ &= 5612 \end{aligned}$$

(ii) $101 + 99 + 97 + \dots + 47$

a_n term of A.P of n terms is 47.

$$\therefore 47 = a + (n - 1)d$$

$$47 = 101 + (n - 1)(-2)$$

or $n = 28$

Then,

$$\begin{aligned} S_n &= \frac{n}{2}[a + l] \\ &= \frac{28}{2}[101 + 47] \\ &= 14 \times 148 \\ &= 2072 \end{aligned}$$

(iii) $(a - b)^2 + (a^2 + b^2) + (a + b)^2 + \dots + [(a + b)^2 + 6ab]$

Let number of terms be n

Then,

$$a_n = (a + b)^2 + 6ab$$

$$\Rightarrow (a - b)^2 + (n - 1)(2ab) = (a + b)^2 + 6ab$$

$$\Rightarrow a^2 + b^2 - 2ab + 2abn - 2ab = a^2 + b^2 + 2ab + 6ab$$

$$\Rightarrow n = 6$$

Then,

$$\begin{aligned} S_n &= \frac{n}{2}[a + l] \\ S_6 &= \frac{6}{2}[a^2 + b^2 - 2ab + a^2 + b^2 + 2ab + 6ab] \\ &= 6[a^2 + b^2 + 3ab] \end{aligned}$$

Q3

A.P formed is $1, 2, 3, 4, \dots, n$.

Here,

$$a = 1$$

$$d = 1$$

$$l = n$$

$$\text{So sum of } n \text{ terms } S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{n}{2} [2 + (n-1)1]$$

$$= \frac{n(n+1)}{2} \text{ is the sum of first } n \text{ natural numbers.}$$

Q4

The natural numbers which are divisible by 2 or 5 are:

$$2 + 4 + 6 + 8 + 10 + \dots + 100 = (2 + 4 + 6 + \dots + 100) + (5 + 15 + 25 + \dots + 95) \text{ Now}$$

$(2 + 4 + 6 + \dots + 100)$ and $(5 + 15 + 25 + \dots + 95)$ are AP with common difference 2 and 10 respectively.

Therefore

$$\begin{aligned} 2 + 4 + 6 + \dots + 100 &= 2 \frac{50}{2} (1 + 50) \\ &= 2550 \end{aligned}$$

Again

$$\begin{aligned} 5 + 15 + 25 + \dots + 95 &= 5(1 + 3 + 5 + \dots + 19) \\ &= 5 \left(\frac{10}{2} \right) (1 + 19) \\ &= 500 \end{aligned}$$

Therefore the sum of the numbers divisible by 2 or 5 is:

$$\begin{aligned} 2 + 4 + 6 + 8 + 10 + \dots + 100 &= 2550 + 500 \\ &= 3050 \end{aligned}$$

Q5

The series of n odd natural numbers are $1, 3, 5, \dots, n$

Where n is odd natural number

Then, sum of n terms is

$$\begin{aligned} S_n &= \frac{n}{2} [2a + (n-1)d] \\ &= \frac{n}{2} [2(1) + (n-1)(2)] \\ &= n^2 \end{aligned}$$

The sum of n odd natural numbers is n^2 .

Q6

The series so formed is 101, 103, 105, ..., 199

Let number of terms be n

Then,

$$a_n = a + (n - 1)d = 199$$

$$\Rightarrow 199 = 101 + (n - 1)2$$

$$\Rightarrow n = 50$$

The sum of n terms $= S_n = \frac{n}{2}[a + l]$

$$\begin{aligned} S_{50} &= \frac{50}{2}[101 + 199] \\ &= 7500 \end{aligned}$$

The sum of odd numbers between 100 and 200 is 7500.

Q7

The odd numbers between 1 and 100 divisible by 3 are 3, 9, 15, ..., 999

Let the number of terms be n then, n th term is 999.

$$a_n = a + (n - 1)d$$

$$999 = 3 + (n - 1)6$$

$$\Rightarrow n = 167$$

The sum of n terms

$$S_n = \frac{n}{2}[a + l]$$

$$\Rightarrow S_{167} = \frac{167}{2}[3 + 999]$$

$$= 83667 \quad \text{Hence proved.}$$

Q8

The required series is 85, 90, 95, ..., 715

Let there be n terms in the A.P

Then,

$$n\text{th term} = 715$$

$$715 = 85 + (n - 1)5$$

$$n = 127$$

Then,

$$S_n = \frac{n}{2}[a + l]$$

$$\begin{aligned} S_{127} &= \frac{127}{2}[85 + 715] \\ &= 50800 \end{aligned}$$

Q9

The series of integers divisible by 7 between 50 and 500 are

56, 63, 70, ..., 497

Let the number of terms be n then, n th term = 497

$$a_n = a + (n - 1)d$$

$$\Rightarrow 497 = 56 + (n - 1)7$$

$$\Rightarrow n = 64$$

$$\text{The sum } S_n = \frac{n}{2}[a + l]$$

$$\begin{aligned} \Rightarrow S_{64} &= \frac{64}{2}[56 + 497] \\ &= 32 \times 553 \\ &= 17696 \end{aligned}$$

Q10

All even integers will have common difference = 2

∴ A.P is 102, 104, 106, ..., 998

$$t_n = a + (n - 1)d$$

$$t_n = 998, a = 102, d = 2$$

$$998 = 102 + (n - 1)(2)$$

$$998 = 102 + 2n - 2$$

$$998 - 100 = 2n$$

$$2n = 898$$

$$n = 449$$

S_{449} can be calculated by

$$\begin{aligned} S_n &= \frac{n}{2}[a + l] \\ &= \frac{449}{2}[102 + 998] \\ &= \frac{449}{2} \times 1100 \\ &= 449 \times 550 \\ &= 246950 \end{aligned}$$

Q11

The series formed by all the integers between 100 and 550 which are divisible by 9 is
108, 117, 123, ..., 549

Let there be n terms in the A.P then, the n th term is 549

$$549 = a + (n - 1)d$$

$$549 = 108 + (n - 1)9$$

$$\Rightarrow n = 50$$

Then,

$$\begin{aligned} S_n &= \frac{n}{2}[a + l] \\ S_{50} &= \frac{50}{2}[108 + 549] \\ &= 16425 \end{aligned}$$

Q12

In the given series $3 + 5 + 7 + 9 + \dots$ to $3n$

Here,

$$a = 3$$

$$d = 2$$

$$\text{Number of terms} = 3n$$

The sum of n term is

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\begin{aligned}\Rightarrow S_{3n} &= \frac{3n}{2} [6 + (3n-1)2] \\ &= 3n(2n+3)\end{aligned}$$

Q13

The first number between 100 and 800 which on division by 16 leaves the remainder 7 is 112 and last number is 791.

Thus, the series so formed is $103, 119, \dots, 791$

Let number of terms be n , then

$$n\text{th term} = 791$$

Then,

$$a_n = a + (n-1)d$$

$$\Rightarrow 791 = 103 + (n-1)16$$

$$\Rightarrow n = 44$$

Then, sum of all terms of the given series is

$$\begin{aligned}S_{43} &= \frac{44}{2} [103 + 791] \\ &= \frac{44 \times 894}{2} \\ &= 19668\end{aligned}$$

Q14

$$(i) \quad 25 + 22 + 19 + 16 + \dots + x = 115$$

Here, sum of the given series of say n terms is 115

So, the n th term = x

Here, $a = 25$ and $d = 22 - 25 = -3$

$$\therefore a_n = a + (n - 1)d$$

$$\Rightarrow x = 25 - 3(n - 1)$$

$$\Rightarrow x = 28 - 3n \quad \text{--- (i)}$$

The sum of n terms

$$S_n = \frac{n}{2}[a + l]$$

$$\Rightarrow 115 = \frac{n}{2}[25 + 28 - 3n]$$

$$\Rightarrow 230 = 53n - 3n^2$$

$$\Rightarrow 3n^2 - 53n - 230 = 0$$

$$\Rightarrow 3n^2 - 30n - 23n - 230 = 0$$

$$\Rightarrow n = 10 \text{ or } \frac{23}{3}$$

But n can't be fraction

$$\therefore n = 10 \quad \text{--- (ii)}$$

From (i) and (ii)

$$x = 28 - 3n$$

$$= 28 - 3(10)$$

$$= -2$$

$$x = -2$$

Q15

Sum first n terms of the given AP is

$$S_n = 3n^2 + 2n$$

$$S_{n-1} = 3(n-1)^2 + 2(n-1)$$

$$a_n = S_n - S_{n-1}$$

$$a_n = 3n^2 + 2n - 3(n-1)^2 - 2(n-1)$$

$$a_n = 6n - 1$$

$$a_r = 6r - 1$$

r^{th} term is $6r - 1$.

Q16

Given,

$$a_1 = -14 = a + 0d \quad \text{--- (i)}$$

$$a_5 = 2 = a + 4d \quad \text{--- (ii)}$$

Solving (i) and (ii)

$$a_1 = a = -14 \text{ and } d = 4$$

Let there be n terms then sum of these n terms = 40

$$\therefore S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\Rightarrow 40 = \frac{n}{2} [-28 + (n-1)4]$$

$$\Rightarrow 4n^2 - 32n - 80 = 0$$

$$\text{or } n = 10 \text{ or } -2$$

But n can't be negative

$$\therefore n = 10$$

The given A.P has 10 terms.

Q17

Given,

$$a_7 = 10$$

$$S_{14} - S_7 = 17 \quad \text{---(i)}$$

$$\therefore S_{14} = 17 + S_7 = 17 + 10 = 27 \quad \text{---(ii)}$$

From (i) and (ii)

$$S_7 = \frac{7}{2}[2a + (7-1)d] \quad \left[\text{Using } S_n = \frac{n}{2}[2a + (n-1)d] \right]$$

$$\Rightarrow 10 = 7a + 21d \quad \text{---(iii)}$$

and

$$S_{14} = \frac{14}{2}[2a + 13d]$$

$$\Rightarrow 27 = 28a + 182d \quad \text{---(iv)}$$

Solving (iii) and (iv)

$$a = 1 \text{ and } d = \frac{1}{7}$$

\therefore The required A.P is

$$1, 1 + \frac{1}{7}, 1 + \frac{2}{7}, 1 + \frac{3}{7}, \dots, +\infty$$

$$\text{or } 1, \frac{8}{7}, \frac{9}{7}, \frac{10}{7}, \frac{11}{7}, \dots, \infty$$

Q18

Given,

$$a_3 = 7 = a + 2d \quad \text{---(i)}$$

$$a_7 = 3a_3 + 2$$

$$\therefore a_7 = 3(7) + 2 \quad [\because a_3 = 7]$$

$$= 23 = a + 6d \quad \text{---(ii)}$$

solving (i) and (ii)

$$a = -1, d = 4$$

Then, sum of 20 terms of this A.P

$$\begin{aligned} \Rightarrow S_{20} &= \frac{20}{2}[2 + (20-1)4] && \left[\text{Using } S_n = \frac{n}{2}[2a + (n-1)d] \right] \\ &= 10 \times 74 \\ &= 740 \end{aligned}$$

First term is -1 common difference $= 4$, sum of 20 terms $= 740$.

Q19

Given,

$$a = 2$$

$$l = 50$$

$$\therefore l = a + (n - 1)d$$

$$50 = 2 + (n - 1)d$$

$$(n - 1)d = 48 \quad \text{--- (i)}$$

S_n of all n terms is given 442

$$\therefore S_n = \frac{n}{2}[a + l]$$

$$442 = \frac{n}{2}[2 + 50]$$

$$\text{or } n = 17 \quad \text{--- (ii)}$$

From (i) and (ii)

$$d = \frac{48}{n - 1} = \frac{48}{16} = 3$$

The common difference is 3.

Q20

Let no. of terms be $2n$

$$\text{Odd terms sum} = 24 = T_1 + T_3 + \dots + T_{2n-1}$$

$$\text{Even terms sum} = 30 = T_2 + T_4 + \dots + T_{2n}$$

Subtract above two equations

$$nd = 6$$

$$T_{2n} = T_1 + \frac{21}{2}$$

$$T_{2n} - a = \frac{21}{2}$$

$$(2n-1)d = \frac{21}{2}$$

$$12 - \frac{21}{2} = d = \frac{3}{2}$$

$$\Rightarrow n = 6 \times \frac{2}{3} = 4$$

$$\text{Total terms} = 2n = 8$$

Substitute above values in equation of sum of even terms or odd terms, we get

$$a = \frac{3}{2}$$

$$\text{So series is } \frac{3}{2}, 3, \frac{9}{2}, \dots$$

Q21

Let a be the first term of the AP and d is the common difference. Then

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$n^2 p = \frac{n}{2}(2a + (n-1)d)$$

$$np = \frac{1}{2}[2a + (n-1)d]$$

$$2np = 2a + (n-1)d \quad \text{.....(1)}$$

Again

$$S_m = \frac{m}{2}(2a + (m-1)d)$$

$$m^2 p = \frac{m}{2}(2a + (m-1)d)$$

$$mp = \frac{1}{2}[2a + (m-1)d]$$

$$2mp = 2a + (m-1)d \quad \text{.....(2)}$$

Now subtract (1) from (2)

$$2p(m-n) = (m-n)d$$

$$d = 2p$$

Therefore

$$2mp = 2a + (m-1) \cdot 2p$$

$$2a = 2p$$

$$a = p$$

The sum up to p terms will be:

$$S_p = \frac{p}{2}(2a + (p-1)d)$$

$$= \frac{p}{2}(2p + (p-1) \cdot 2p)$$

$$= \frac{p}{2}(2p + 2p^2 - 2p)$$

$$= p^3$$

Hence it is shown.

Q22

$$a_{12} = a + 11d = -13$$

$$\text{---(i) [Given]}$$

$$s_4 = \frac{4}{2}(2a + 3d) = 24$$

$$\text{---(ii) [Given]}$$

From (i) and (ii)

$$d = -2 \text{ and } a = 9$$

Then,

Sum of first 10 terms is

$$\begin{aligned} s_{10} &= \frac{10}{2}[2 \times 9 + (9)(-2)] \\ &= 0 \end{aligned}$$

$$\left[\text{Using } s_n = \frac{n}{2}[2a + (n-1)d] \right]$$

Sum of first 10 terms is zero.

Q23

$$a_5 = a + 4d = 30$$

$$\text{---(i) [Given]}$$

$$a_{12} = a + 11d = 65$$

$$\text{---(ii) [Given]}$$

From (i) and (ii)

$$d = 5 \text{ and } a = 10$$

Then,

Sum of first 20 terms is

$$s_n = \frac{n}{2}[2a + (n-1)d]$$

$$\begin{aligned} \Rightarrow s_{20} &= \frac{20}{2}[2 \times 10 + (20-1)5] \\ &= 1150 \end{aligned}$$

Sum of first 20 terms is 1150.

Q24

Here,

$$a_k = 5k + 1$$

$$a_1 = 5 + 1 = 6$$

$$a_2 = 5(2) + 1 = 11$$

$$a_3 = 5(3) + 1 = 16$$

$$d = 11 - 6 = 16 - 11 = 5$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{n}{2} [2(6) + (n-1)(5)]$$

$$= \frac{n}{2} [12 + 5n - 5]$$

$$S_n = \frac{n}{2} (5n + 7)$$

Q25

sum of all two digit numbers which when divided by 4,
yields 1 as remainder, \Rightarrow all $4n+1$ terms with $n \geq 3$

13, 17, 21, 97

$$n = 22, a = 13, d = 4$$

$$\text{sum of terms} = \frac{22}{2} [26 + 21 \times 4] = 11 \times 110 = 1210$$

Q26

Sum of terms 25, 22, 19,....., is 116

$$\frac{n}{2}[50 + (n-1)(-3)] = 116$$

$$\frac{n}{2}[53 - 3n] = 116$$

$$53n - 3n^2 = 232$$

$$3n^2 - 53n + 232 = 0$$

$$3n^2 - 29n - 24n + 232 = 0$$

$$n(3n - 29) - 8(3n - 29) = 0$$

$$(3n - 29)(n - 8) = 0$$

$$\Rightarrow n = 8 \text{ or } \frac{29}{3}$$

n cannot be in fraction, so n=8

$$\text{last term} = 25 - 7 \times 3 = 4$$

Q27

Let the number of terms is n .

Now the sum of the series is:

$$1 + 3 + 5 + \dots + 2001$$

Here $l = 2001$ and $d = 2$.

Therefore

$$l = a + (n-1)d$$

$$2001 = 1 + (n-1) \cdot 2$$

$$2(n-1) = 2000$$

$$n-1 = 1000$$

$$n = 1001$$

Therefore the sum of the series is:

$$S = \frac{1001}{2} [2 + (1001-1)2]$$

$$= 1001^2$$

$$= 1002001$$

Q28

Let the number of terms to be added to the series is n .

Now $a = -6$ and $d = 0.5$.

Therefore

$$-25 = \frac{n}{2} [2(-6) + (n-1)(0.5)]$$

$$-50 = n[-12 + 0.5n - 0.5]$$

$$-12.5n + 0.5n^2 + 50 = 0$$

$$n^2 - 25n + 100 = 0$$

$$n = 20, 5$$

Therefore the value of n will be either 20 or 5.

Q29

Here the first term $a = 2$. Let the common difference is d .

Now

$$\frac{5}{2} [2a + (5-1)d] = \frac{1}{4} \left[\frac{5}{2} [2(a+5d) + (5-1)d] \right]$$

$$\frac{5}{2} [2 \cdot 2 + 4d] = \frac{5}{8} [2 \cdot 2 + 14d]$$

$$10 + 10d = \frac{5}{2} + \frac{35}{4}d$$

$$\frac{5}{4}d = -7.5$$

$$d = -6$$

The 20th term will be:

$$\begin{aligned} a + (n-1)d &= 2 + (20-1)(-6) \\ &= -112 \end{aligned}$$

Hence it is shown.

Q30

$$S_{(2n+1)} = S_1 = \frac{(2n+1)}{2} [2a + (2n+1-1)d]$$

$$S_1 = \frac{(2n+1)}{2} [2a + 2nd]$$

$$= (2n+1)(a + nd) \quad \text{--- (i)}$$

Sum of odd terms $= S_2$

$$S_2 = \frac{(n+1)}{2} [2a + (n+1-1)(2d)]$$

$$= \frac{(n+1)}{2} [2a + 2nd]$$

$$S_2 = (n+1)(a + nd) \quad \text{--- (ii)}$$

From equation (i) and (ii),

$$S_1 : S_2 = (2n+1)(a + nd) : (n+1)(a + nd)$$

$$S_1 : S_2 = (2n+1) : (n+1)$$

Q31

Here,

$$S_n = 3n^2 \quad \text{--- (i)} \quad \text{[Given]}$$

Where n is number of term

$$\therefore S_n = \frac{n}{2} [2a + (n-1)d] \quad \text{--- (ii)}$$

From (i) and (ii)

$$3n^2 = \frac{n}{2} [2a + (n-1)d]$$

$$6n = 2a + nd - d$$

Equating both sides

$$6n = nd$$

$$\therefore d = 6 \quad \text{--- (iii)}$$

and

$$0 = 2a - d$$

$$\text{or } d = 2a \quad \text{--- (iv)}$$

From (iii) and (iv)

$$a = 3 \text{ and } d = 6$$

\therefore The required A.P is 3, 9, 15, 21, ..., ∞

Q32

$$S_n = np + \frac{1}{2}n(n-1)Q \quad [\text{Given}]$$

$$S_n = \frac{n}{2}[2p + (n-1)Q] \quad \text{---(i)}$$

We know

$$S_n = \frac{n}{2}[2a + (n-1)d] \quad \text{---(ii)}$$

Where a = first term and d = common difference comparing (i) and (ii)

$$d = Q$$

\therefore The common difference is Q .

Q33

Let sum of n terms of two A.P be S_n and S'_n .

Then, $S_n = 5n + 4$ and $S'_n = 9n + 16$ respectively.

Then, if ratio of sum of n terms of 2A.P is given, then the ratio of there n th term is obtained by replacing n by $(2n-1)$.

$$\frac{a_n}{a'_n} = \frac{5(2n-1) + 4}{9(2n-1) + 16}$$

\therefore Ratio of there 18th term is

$$\begin{aligned} \frac{a_{18}}{a'_{18}} &= \frac{5(2 \times 18 - 1) + 4}{9(2 \times 18 - 1) + 16} \\ &= \frac{5 \times 35 + 4}{9 \times 35 + 16} \\ &= \frac{179}{321} \end{aligned}$$

Q34

Let sum of n term of 1 A.P series be S_n are other S_n

The, $S_n = 7n + 2$ ---(i).

$S_n = n + 4$ ---(ii)

If the ratio of sum of n terms of 2 A.P is given, then the ratio of there n th term is obtained by replacing n by $(2n - 1)$.

$$\frac{a_n}{a_{n'}} = \frac{7(2n - 1) + 2}{(2n - 1) + 4}$$

Putting $n = 5$ to get the ratio of 5th term, we get

$$\frac{a_5}{a'5} = \frac{7(2 \times 5 - 1) + 2}{(2 \times 5 - 1) + 4} = \frac{65}{13} = \frac{5}{1}$$

The ratio is 5 : 1.

Ex 19.5

Q1(i)

$\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ will be in A.P if $\frac{c+a}{b} - \frac{b+c}{a} = \frac{a+b}{c} - \frac{c+a}{b}$

if $\frac{ca+a^2-b^2-cb}{ab} = \frac{ab+b^2-c^2-ac}{bc}$

$$\begin{aligned} \text{LHS} &\Rightarrow \frac{ca+a^2-b^2-cb}{ab} \\ &\Rightarrow \frac{c^2a+a^2c-b^2c-c^2b}{abc} \\ &\Rightarrow \frac{c(a-b)[a+b+c]}{abc} \end{aligned} \quad \text{---(i)}$$

$$\begin{aligned} \text{RHS} &\Rightarrow \frac{ab+b^2-c^2-ac}{bc} \\ &\Rightarrow \frac{a^2b+ab^2-ac^2-a^2c}{abc} \\ &\Rightarrow \frac{a(b-c)[a+b+c]}{abc} \end{aligned} \quad \text{---(ii)}$$

and since $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$$\begin{aligned} \frac{1}{b} - \frac{1}{a} &= \frac{1}{c} - \frac{1}{b} \\ c(b-a) &= a(b-c) \end{aligned} \quad \text{---(iii)}$$

\therefore LHS = RHS and the given terms are in A.P.

Q1(ii)

$a(b+c), b(c+a), c(a+b)$ are in A.P if $b(c+a) - a(b+c) = c(a+b) - b(c+a)$

$$\begin{aligned} \text{LHS} &= b(c+a) - a(b+c) \\ &= bc+ab-ab-ac \\ &= c(b-a) \end{aligned} \quad \text{---(i)}$$

$$\begin{aligned} \text{RHS} &= c(a+b) - b(c+a) \\ &= ca+cb-bc-ba \\ &= a(c-b) \end{aligned} \quad \text{---(ii)}$$

and $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$$\begin{aligned} \therefore \frac{1}{a} - \frac{1}{b} &= \frac{1}{b} - \frac{1}{c} \\ \text{or } c(b-a) &= a(c-b) \end{aligned} \quad \text{---(iii)}$$

From (i), (ii) and (iii)

$a(b+c), b(c+a), c(a+b)$ are in A.P

Q2

$\frac{a}{b+c}, \frac{b}{a+c}, \frac{c}{a+b}$ are in A.P if $\frac{b}{a+c} - \frac{a}{b+c} = \frac{c}{a+b} - \frac{b}{a+c}$

$$\begin{aligned}\text{LHS} &= \frac{b}{a+c} - \frac{a}{b+c} \\ \Rightarrow & \frac{b^2 + bc - a^2 - ac}{(a+c)(b+c)} \\ \Rightarrow & \frac{(b-a)(a+b+c)}{(a+c)(b+c)} \quad \text{---(i)}\end{aligned}$$

$$\begin{aligned}\text{RHS} &= \frac{a}{a+b} - \frac{b}{a+c} \\ \Rightarrow & \frac{ca + c^2 - b^2 - ab}{(a+b)(b+c)} \\ \Rightarrow & \frac{(c-b)(a+b+c)}{(a+b)(b+c)} \quad \text{---(ii)}\end{aligned}$$

and a^2, b^2, c^2 are in A.P
 $\therefore b^2 - a^2 = c^2 - b^2 \quad \text{---(iii)}$

Substituting $b^2 - a^2$ with $c^2 - b^2$
(i) = (ii)

$$\therefore \frac{a}{b+c}, \frac{b}{a+c}, \frac{c}{a+b} \text{ are in A.P}$$

Q3(i)

$a^2(b+c), b^2(c+a), c^2(a+b)$ are in A.P.

$$\begin{aligned}\text{If } b^2(c+a) - a^2(b+c) &= c^2(a+b) - b^2(a+c) \\ \Rightarrow b^2c + b^2a - a^2b - a^2c &= c^2a + c^2b - b^2a - b^2c\end{aligned}$$

$$\begin{aligned}\text{Given, } b - a &= c - b & [a, b, c \text{ are in A.P}] \\ c(b^2 - a^2) + ab(b - a) &= a(c^2 - b^2) + bc(c - b) \\ (b - a)(ab + bc + ca) &= (c - b)(ab + bc + ca)\end{aligned}$$

Cancelling $ab + bc + ca$ from both sides
 $b - a = c - b$
 $2b = c + a$ which is true

Hence, $a^2(b+c), (c+a)b^2$ and $c^2(a+b)$ are also in A.P.

Q3(ii)

(ii) T.P $b+c-a, c+a-b, a+b-c$ are in A.P.

$b+c-a, c+a-b, a+b-c$ are in A.P only if $(c+a-b) - (b+c-a) = (a+b-c) - (c+a-b)$

$$\begin{aligned}\text{LHS} &\Rightarrow (c+a-b) - (b+c-a) \\ \Rightarrow 2a - 2b &\quad \text{---(i)}\end{aligned}$$

$$\begin{aligned}\text{RHS} &\Rightarrow (a+b-c) - (c+a-b) \\ \Rightarrow 2b - 2c &\quad \text{---(ii)}\end{aligned}$$

Since,

$$\begin{aligned}&a, b, c \text{ are in A.P} \\ \therefore b - a &= c - b \\ \text{or } a - b &= b - c &\quad \text{---(iii)}\end{aligned}$$

From (i), (ii) and (iii)

$$\text{LHS} = \text{RHS}$$

Thus, given numbers

$$b+c-a, c+a-b, a+b-c \text{ are in A.P}$$

Q3(iii)

To prove $bc - a^2, ca - b^2, ab - c^2$ are in A.P

$$(ca - b^2) - (bc - a^2) = (ab - c^2) - (ca - b^2)$$

$$\begin{aligned}\text{LHS} &= (a - b^2 - bc + a^2) \\ &= (a - b)[a + b + c] &\quad \text{---(i)}\end{aligned}$$

$$\begin{aligned}\text{RHS} &= ab - c^2 - ca + b^2 \\ &= (b - c)[a + b + c] &\quad \text{---(ii)}\end{aligned}$$

and since a, b, c are in A.P

$$\begin{aligned}&b - c = a - b \\ \therefore \text{LHS} &= \text{RHS}\end{aligned}$$

and

Thus, $bc - a^2, ca - b^2, ab - c^2$ are in A.P

Q4

(i) If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$$\begin{aligned} \frac{1}{b} - \frac{1}{a} &= \frac{1}{c} - \frac{1}{b} \\ \text{LHS} &= \frac{1}{b} - \frac{1}{a} \\ &= \frac{a-b}{ab} = \frac{c(a-b)}{abc} \end{aligned} \quad \text{---(i)}$$

$$\begin{aligned} \text{RHS} &= \frac{1}{c} - \frac{1}{b} \\ &= \frac{a(b-c)}{abc} \end{aligned} \quad \text{---(ii)}$$

Since, $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P

$$\begin{aligned} \frac{b+c}{a} - \frac{c+a}{b} &= \frac{c+a}{b} - \frac{a+b}{c} \\ \frac{b^2+cb-ac-a^2}{ab} &= \frac{c^2+ac-ab-b^2}{bc} \\ \Rightarrow \frac{(b-a)(a+b+c)}{ab} &= \frac{(c-b)(a+b+c)}{bc} \\ \text{or } \frac{a(b-c)}{abc} &= \frac{c(a-b)}{abc} \end{aligned} \quad \text{---(iii)}$$

From (i), (ii) and (iii)

$$\text{LHS} = \text{RHS}$$

Hence, $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

(ii) If bc, ca, ab are in A.P

Then,

$$\begin{aligned} ca - bc &= ab - ca \\ c(a-b) &= a(b-c) \end{aligned} \quad \text{---(i)}$$

If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$$\begin{aligned} \frac{1}{b} - \frac{1}{a} &= \frac{1}{c} - \frac{1}{b} \\ \Rightarrow c(a-b) &= a(b-c) \end{aligned} \quad \text{---(ii)}$$

Thus, the condition necessary to prove bc, ca, ab in A.P is fulfilled.

Thus, bc, ca, ab , are in A.P.

Q5

(i) If $(a - c)^2 = 4(a - b)(b - c)$

Then,

$$a^2 + c^2 - 2ac = 4(ab - b^2 - ac + bc)$$

$$\Rightarrow a^2 + c^2 - 4b^2 + 2ac - 4ab + 4bc = 0$$

$$\Rightarrow (a + c - 2b)^2 = 0$$

$$\left[\text{Using } (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc \right]$$

$$\therefore a + c - 2b = 0$$

$$\text{or } a + c = 2b$$

and since,

$$a, b, c \text{ are in A.P}$$

[Given]

$$a + c = 2b$$

Hence proved.

$$(a - c)^2 = 4(a - b)(b - c)$$

(ii) If $a^2 + c^2 + 4ac = 2(ab + bc + ca)$

Then,

$$a^2 + c^2 + 2ac - 2ab - 2bc = 0$$

$$\text{or } (a + c - b)^2 - b^2 = 0$$

$$\left[\because (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc \right]$$

$$\text{or } b = a + c - b$$

$$\text{or } 2b = a + c$$

$$b = \frac{a + c}{2}$$

and since,

$$a, b, c \text{ are in A.P}$$

$$b = \frac{a + c}{2}$$

Thus, $a^2 + c^2 + 4ac = 2(ab + bc + ca)$ Hence proved.

(iii) If $a^3 + c^3 + 6abc = 8b^3$

$$\text{or } a^3 + c^3 - (2b)^3 + 6abc = 0$$

$$\text{or } a^3 + (-2b)^3 + c^3 + 3 \times a \times (-2b) \times c = 0$$

$$\therefore (a - 2b + c) = 0$$

$$\left[\because x^3 + y^3 + z^3 + 3xyz = 0 \right]$$

$$\text{or } a + c = 2b$$

$$a - b = c - b$$

and since, a, b, c are in A.P

Thus, $a - b = c - b$

Hence proved. $a^3 + c^3 + 6abc = 8b^3$

Q6

Here,

$$a\left(\frac{1}{b} + \frac{1}{c}\right), b\left(\frac{1}{c} + \frac{1}{a}\right), c\left(\frac{1}{a} + \frac{1}{b}\right) \text{ are in A.P.}$$

$$\Rightarrow a\left(\frac{1}{b} + \frac{1}{c}\right) + 1, b\left(\frac{1}{c} + \frac{1}{a}\right) + 1, c\left(\frac{1}{a} + \frac{1}{b}\right) + 1 \text{ are in A.P.}$$

$$\Rightarrow \left(\frac{ac + ab + bc}{bc}\right), \left(\frac{ab + bc + ac}{ac}\right), \left(\frac{cb + ac + ab}{ab}\right) \text{ are in A.P.}$$

$$\Rightarrow \frac{1}{bc}, \frac{1}{ac}, \frac{1}{ab} \text{ are in A.P.}$$

$$\Rightarrow \frac{abc}{bc}, \frac{abc}{ac}, \frac{abc}{ab} \text{ are in A.P.}$$

$$\Rightarrow a, b, c \text{ are in A.P.}$$

Q7

x, y and z are in AP.

Let d be the common difference then,

$$y = x + d \text{ and } z = x + 2d$$

To show $x^2 + xy + y^2, z^2 + zx + x^2$ and $y^2 + yz + z^2$ are consecutive terms of an A.P., it is enough to show that,

$$(z^2 + zx + x^2) - (x^2 + xy + y^2) = (y^2 + yz + z^2) - (z^2 + zx + x^2)$$

$$\begin{aligned} \text{LHS} &= (z^2 + zx + x^2) - (x^2 + xy + y^2) \\ &= z^2 + zx - xy - y^2 \\ &= (x + 2d)^2 + (x + 2d)x - x(x + d) - (x + d)^2 \\ &= x^2 + 4xd + 4d^2 + x^2 + 2xd - x^2 - xd - x^2 - 2xd - d^2 \\ &= 3xd + 3d^2 \end{aligned}$$

$$\begin{aligned} \text{RHS} &= (y^2 + yz + z^2) - (z^2 + zx + x^2) \\ &= y^2 + yz - zx - x^2 \\ &= (x + d)^2 + (x + d)(x + 2d) - (x + 2d)x - x^2 \\ &= x^2 + 2dx + d^2 + x^2 + 2dx + xd + 2d^2 - x^2 - 2dx - x^2 \\ &= 3xd + 3d^2 \end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

$\therefore x^2 + xy + y^2, z^2 + zx + x^2$ and $y^2 + yz + z^2$ are consecutive terms of an A.P.

Ex 19.6

Q1

(i) 7 and 13

Let A be the arithmetic mean of 7 and 13.

Then,

7, A , 13 are in A.P

$$\Rightarrow A - 7 = 13 - A$$

$$\Rightarrow A = \frac{13 + 7}{2} = 10$$

\therefore A.M is 10.

(ii) 12 and -8

Let A be the arithmetic mean of 12 and -8

Then,

12, A , -8 are in A.P

$$\Rightarrow A - 12 = -8 - A$$

$$\Rightarrow A = \frac{12 + (-8)}{2} = 2$$

\therefore A.M is 2.

(iii) $(x - y)$ and $(x + y)$

Let A be the arithmetic mean of $(x - y)$ and $(x + y)$

Then,

$(x - y)$, A , $(x + y)$ are in A.P

$$\Rightarrow A - (x - y) = (x + y) - A$$

$$\Rightarrow A = \frac{(x - y) + (x + y)}{2} = \frac{2x}{2} = x$$

\therefore A.M is x .

Q2

Let A_1, A_2, A_3, A_4 be the 4 A.M.s between 4 and 19

Then,

4, $A_1, A_2, A_3, A_4, 19$ are in A.P of 6 terms

$$A_n = a + (n - 1)d$$

$$a_6 = 19 = 4 + (6 - 1)d$$

$$\text{or } d = 3 \quad \text{---(i)}$$

Now,

$$A_1 = a + d = 4 + 3 = 7$$

$$A_2 = A_1 + d = 7 + 3 = 10$$

$$A_3 = A_2 + d = 10 + 3 = 13$$

$$A_4 = A_3 + d = 13 + 3 = 16$$

The 4 A.M.s between 4 and 19 are 7, 10, 13, 16.

Q3

$$2, a_1, a_2, a_3, a_4, a_5, a_6, a_7, 17$$

$$17 = a + 8d$$

$$a = 2 \Rightarrow d = \frac{15}{8}$$

$$a_1 = 2 + \frac{15}{8} = \frac{31}{8}$$

$$a_2 = \frac{31}{8} + \frac{15}{8} = \frac{46}{8}$$

so we get our final series as

$$2, \frac{31}{8}, \frac{46}{8}, \frac{61}{8}, \frac{76}{8}, \frac{91}{8}, \frac{106}{8}, \frac{121}{8}, \frac{136}{8} = 17$$

Q4

Let $A_1, A_2, A_3, A_4, A_5, A_6$ be the 6 AM's between 15 and -13

Then,

15, $A_1, A_2, A_3, A_4, A_5, A_6, -13$ are in A.P of 8 terms

Here, $-13 = a_8 = a + 7d$

$$\Rightarrow -13 = 15 + 7d$$

$$\text{or } d = -4 \quad \text{---(i)}$$

$$\therefore A_1 = a + d = 15 - 4 = 11$$

$$A_2 = a + 2d = 15 - 2(4) = 7$$

$$A_3 = a + 3d = 15 - 4(3) = 3$$

$$A_4 = a + 4d = 15 - 4(4) = -1$$

$$A_5 = a + 5d = 15 - 4(5) = -5$$

$$A_6 = a + 6d = 15 - 4(6) = -9$$

The 6 A.M.s between 15 and -13 are 11, 7, 3, -1, -5 and -9.

Q5

Let the n A.M's between 3 and 17 be $A_1, A_2, A_3, \dots, A_n$

Then,

ATQ

$$\frac{A_n}{A_1} = \frac{3}{1} \quad \text{---(i)}$$

We know that

3, $A_1, A_2, A_3, \dots, A_n, 17$ are in A.P of $n+2$ terms

So, 17 is the $(n+2)$ th terms.

$$\text{i.e. } 17 = 3 + (n+2-1)d \quad \left[\text{Using } a_n = a + (n-1)d \right]$$

$$\text{or } d = \frac{14}{(n+1)} \quad \text{---(ii)}$$

$$\begin{aligned} \therefore A_n &= 3 + (n+1-1)d \\ &= 3 + \frac{14n}{n+1} = \frac{17n+3}{n+1} \end{aligned} \quad \text{---(iii)}$$

$$A_1 = 3 + d = \frac{3n+17}{n+1} \quad \text{---(iv)}$$

From (i), (iii) and iv

$$\frac{A_n}{A_1} = \frac{17n+3}{3n+17} = \frac{3}{1}$$

$$\therefore n = 6$$

There are 6 A'M between 3 and 17.

Q6

Let there be n A.M between 7 and 71 and let the A.M's be $A_1, A_2, A_3, \dots, A_n$.

So,

$7, A_1, A_2, A_3, \dots, A_n, 71$ are in A.P of $(n+2)$ terms

$$A_5 = a_6 = a + 5d = 27 \quad [\text{Given}]$$

$$\Rightarrow a + 5d = 27$$

$$\Rightarrow d = 4 \quad [\because a = 7] \quad \text{---(i)}$$

The $(n+2)$ th term of A.P is 71

$$\therefore a_{n+2} = 71 = a + (n+2-1)d$$

$$\text{or } n = 15$$

There are 15 AM's between 7 and 71.

Q7

Let $A_1, A_2, A_3, A_4, \dots, A_n$ be the n AMs inserted between two number a and b .

Then,

$a, A_1, A_2, A_3, A_4, \dots, A_n, b$ are in A.P

So, the mean of a and b

$$\text{A.M} = \frac{a+b}{2}$$

The mean of A_1 and A_n

$$\text{A.M} = \frac{a+d+b-d}{2} = \frac{a+b}{2}$$

Similarly mean of A_2 and A_{n-1}

$$\text{A.M} = \frac{a+2d+b-2d}{2} = \frac{a+b}{2}$$

Similarly we observe the means is equidistant from beginning and the end

is constant $\frac{a+b}{2}$.

The AM is $\frac{a+b}{2}$.

Q8

Here,

A_1 is the A.M of x and y ,

and A_2 is the A.M of y and z .

Then,

$$A_1 = \frac{x+y}{2} \quad \text{---(i)} \quad \left[\because \text{AM} = \frac{a+b}{2} \right]$$

$$A_2 = \frac{y+z}{2} \quad \text{---(ii)}$$

Let A.M be the arithmetic mean of A_1 and A_2

Then,

$$\begin{aligned} \text{A.M} &= \frac{A_1 + A_2}{4} \\ &= \frac{x+y+y+z}{4} \\ &= \frac{x+2y+z}{4} \quad \text{---(iii)} \end{aligned}$$

Since, x, y, z are in A.P

[Given]

$$y = \frac{x+a}{2} \quad \text{---(iv)}$$

From (iii) and (iv)

$$\text{A.M} = \frac{\left(\frac{x+a}{2}\right) + \left(\frac{2y}{2}\right)}{2} = \frac{y+y}{2} = y$$

Hence, proved A.M between A_1 and A_2 is y .

Q9

$$8, a_1, a_2, a_3, a_4, a_5, 26$$

$$a = 8$$

$$a + 6d = 26$$

$$\Rightarrow d = \frac{18}{6} = 3$$

So series is 8, 11, 14, 17, 20, 23, 26

Ex 19.7

Q1

Let the amount saved by the man in first year be x .

Then,

ATQ

$$x + (x + 100) + (x + 200) + \dots + (x + 900) = 16500$$

As his saving increased by Rs 100 every year.

$$\therefore 10x + 100 + 200 + \dots + 900 = 16500 \quad \text{--- (i)}$$

Here,

100 + 200 + 300 + ... + 900 form a series of

$$a = 100, d = 100 \text{ and } n = 9$$

So,

$$S_n = \frac{n}{2}[a + l]$$

$$S_9 = \frac{9}{2}[100 + 900] = 4500 \quad \text{--- (ii)}$$

From (i) and (ii)

$$10x + (4500) = 16500$$

$$10x = 12000$$

$$\text{or } x = 1200$$

The man saved Rs 1200 in the first year.

Q2

Let the man save Rs 200 in n numbers of years.

Then,

ATQ

$$32 + 36 + 40 + \dots = 200$$

It forms a series of n terms, with $a = 32$ and $d = 4$

$$\therefore S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow 200 = \frac{n}{2}[2(32) + (n - 1)4]$$

$$\Rightarrow 400 = 60n + 4n^2$$

$$\Rightarrow n^2 + 15n - 100 = 0$$

$$\Rightarrow n = 5 \text{ or } -20$$

$$\text{But, } n \neq -20$$

[It can't be negative]

$$\therefore n = 5$$

The man will save Rs 200 in 5 years.

Q3

Let the 40 annual instalments form an arithmetic series of common difference d and first instalment a . Then, series so formed is

$$a + (a + d) + (a + 2d) + \dots = 3600$$

$$\text{or } s_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\text{or } 3600 = 20 [2a + 39d]$$

$$2a + 39d = 180 \quad \text{---(i)}$$

and sum of first 30 terms is $\frac{2}{3}$ of 3600

$$= 2400$$

$$\Rightarrow 2400 = \frac{30}{2} [2a + (29)d]$$

$$\text{or } 2a + 29d = 160 \quad \text{---(ii)}$$

From (i) and (ii)

$$a = 51$$

The first installment paid by this man is Rs 51.

Q4

Let the number of Radio manufactured increase by x each year and number of radio manufacture in first year be a . So, A.P formed ATQ is

$$a, a + x, a + 2x, \dots$$

Here,

$$a_3 = a + 2x = 600 \quad \text{---(i)}$$

$$a_7 = a + 6x = 700 \quad \text{---(ii)}$$

From (i) and (ii)

$$a = 550, x = 25$$

(i) 550 Radio's were manufactured in the first year.

(ii) The total produce in 7 years is sum of produce in the first 7 years.

$$S_7 = \frac{7}{2} [550 + 700] \quad \left[\because S_n = \frac{n}{2} [a + l] \right]$$
$$= 4375$$

4375 Radio's were manufactured in first 7 years.

(iii) The product in 10th year

$$a_{10} = a + 9d$$
$$= 550 + 9(25) = 775$$

775 Radio's were manufactured in the 10th year.

Q5

There are 25 trees at equal distance of 5 m in a line with a well(w), and the distance of the well from the nearest tree = 10 m.

Thus,

The total distance travelled by gardener to tree 1 and back is $2 \times 10 \text{ m} = 20 \text{ m}$
Similarly for all the 25 trees.

The distance covered by gardener is

$$= 2[10 + (10 + 5) + (10 + 2 \times 5) + (10 + 3 \times 5) + \dots + (10 + 23 \times 5)] \quad \text{--- (i)}$$

This forms a series of 1st term $a = 10$, common difference $d = 5$ and $n = 25$

$$\therefore 10 + (10 + 5) + (10 + 2 \times 5) + \dots + (10 + 24 \times 5)$$

$$\Rightarrow S_{25} = \frac{25}{2} [2 \times 10 + (24) 5] = 25[10 + 60] = 1750 \text{ m} \quad \text{--- (ii)}$$

From (i) and (ii)

$$\text{Total distance} = 2 \times 1750 \text{ m} = 3500 \text{ m}.$$

Q6

The man counts at the rate of Rs 180 per minute for half an hour. After this he counts at the rate of Rs 3 less every minute than preceding minute.

Then, the amount counted in first 30 minute

$$= \text{Rs } 180 \times 30 = \text{Rs } 5400 \quad \text{--- (i)}$$

The amount left to be counted after 30 minute

$$= \text{Rs } 10710 - 5400 = \text{Rs } 5310 \quad \text{--- (ii)}$$

ATQ

$$\text{A.P formed is } (180 - 3) + (180 - 2 \times 3) + \dots = 5310$$

Let time taken to count 5310 be t

Then,

$$S_t = \frac{t}{2} [(180 - 3) + (t - 1)(-3)]$$

$$5310 = \frac{t}{2} [200 - 3t]$$

$$\text{or } t = 59 \text{ minute}$$

Thus, the total time taken by the man to count Rs 10710 is $(59 + 30) = 89$ minutes.

Q7

The piece of equipment depreciates 15% in first year i.e., $\frac{15}{100} \times 600,000 = \text{Rs } 90,000$

$$\therefore \text{Value after 1st year} = 600,000 - 90,000 \\ = \text{Rs } 510,000 \quad \text{---(i)}$$

The equipment depreciates at the rate 135% in 2nd year i.e., $\frac{135}{1000} \times 600,000 = 81000$

$$\therefore \text{Value after 2nd year} = 81000$$

The value after 3rd year = $\frac{12}{100} \times 600000 = 72000$

The total depreciation in 10 years

$$\Rightarrow S_{10} = \frac{10}{2} [2 \times 81000 + (9)(-9000)] \\ = 5[81000] \quad \left[\text{Using } S_n = \frac{n}{2} [2a + (n-1)d] \right] \\ = 405000$$

$$\therefore \text{The cost of machine after 10 years} = \text{Rs } 600000 - 405000 \\ = 105000.$$

Q8

Total cost of tractor

$$= 6000 + [(500 + 12\% \text{ of } 6000 \text{ for 1 year}) + (500 + 12\% \text{ of } 5500 \text{ 1 year}) + \dots + 12 \text{ times}] \\ = 6000 + 6000 + \frac{12}{100} (6000 + 5500 + \dots + 12 \text{ times}) \\ = 12000 + \frac{12}{100} \left[\frac{12}{2} (6000 + 5000) \right] \\ = 12000 + \frac{12}{100} \times \frac{12}{2} \times 6500 \\ = 12000 + (72 \times 65) \\ = 12000 + 4680 \\ = 16680$$

Total cost of tractor = Rs. 16680

Q9

Total cost of Scooter

$$\begin{aligned} &= \text{Rs } 4000 + \left[\begin{array}{l} \{ \text{Rs } 1000 + \text{S.I. on Rs Rs } 18000 \text{ for 1 year} \} \\ + \{ \text{Rs } 1000 + \text{S.I. on Rs Rs } 17000 \text{ for 1 year} \} \\ + \dots + 18 \text{ times} \end{array} \right] \\ &= (4000 + 18000) + \text{S.I. for 1 year on } (18000 + 17000 + \dots \text{ to 18 times}) \\ &= 22000 + \text{S.I. for 1 year on } \left\{ \frac{18}{2} (18000 + 1000) \right\} \\ &= 22000 + 9 (19000) \times \frac{10}{100} \\ &= 22000 + 17100 \\ &= \text{Rs } 39100 \end{aligned}$$

Total cost of Scooter = Rs. 39100

Q10

First year the person income is: 300,000

Second year his income will be: $300,000 + 10,000 = 310,000$

...

...

...

This way he receives the amount after 20 years will be:

$$300,000 + 310,000 + \dots + 490,000$$

This is an AP with first term $a = 300000$ and common difference $d = 10,000$.

Therefore

$$\begin{aligned} S &= \frac{20}{2} [2 \cdot 300000 + (20-1)10000] \\ &= 10 [600000 + 190000] \\ &= 7900000 \end{aligned}$$

Q11

In 1st installment the man paid 100 rupees.

In 2nd installment the man paid $(100 + 5) = 105$ rupees.

...

...

Likewise he pays up to the 30th installment as follows:

$$100 + 105 + \dots + (100 + 5 \times 29)$$

This is an AP with $a = 100$ and common difference $d = 5$.

Therefore at the 30th installment the amount he will pay

$$\begin{aligned} T_{30} &= 100 + (30-1)(5) \\ &= 100 + 145 \\ &= 245 \end{aligned}$$

Q12

Suppose carpenter took n days to finish his job.

First day carpenter made five frames

$$a_1 = 5$$

Each day after first day he made two more frames

$$d=2$$

\therefore On n^{th} day frames made by carpenter are,

$$a_n = a_1 + (n-1)d$$

$$\Rightarrow a_n = 5 + (n-1)2$$

Sum of all the frames till n^{th} day is

$$S = \frac{n}{2}[a_1 + a_n]$$

$$192 = \frac{n}{2}[5 + 5 + (n-1)2]$$

$$192 = 5n + n^2 - n$$

$$n^2 + 4n - 192 = 0$$

$$(n+16)(n-12) = 0$$

$$n = -16 \text{ or } n = 12$$

But number of days cannot be negative hence $n = 12$.

The carpenter took 12 days to finish his job.

Q13

We know that sum of interior angles of a polygon with n sides is given by,

$$a_n = 180^\circ(n - 2)$$

Sum of interior angles of a polygon with 3 sides is given by,

$$a_3 = 180^\circ(3 - 2) = 180^\circ \dots\dots\dots(i)$$

Sum of interior angles of a polygon with 4 sides is given by,

$$a_4 = 180^\circ(4 - 2) = 360^\circ \dots\dots\dots(ii)$$

Sum of interior angles of a polygon with 5 sides is given by,

$$a_5 = 180^\circ(5 - 2) = 540^\circ \dots\dots\dots(iii)$$

From eqⁿ (i), eqⁿ (ii) and eqⁿ (iii) we get,

$$a_4 = 360^\circ = 180^\circ + 180^\circ = a_3 + 180^\circ = a_3 + d$$

$$a_5 = 540^\circ = 180^\circ + 360^\circ = a_4 + 180^\circ = a_4 + d$$

Hence the sums of the interior angles of polygons with 3, 4, 5, 6,... sides form an arithmetic progression.

Sum of interior angles of 21 sided polygon

$$= 180^\circ(21 - 2)$$

$$= 3420^\circ$$

Q14

20 potatoes are placed in a line at intervals of 4 meters.

$$\therefore n = 20 \text{ and } d = 4$$

The first potato 24 meters from the starting point.

$$a_1 = 24$$

$$a_2 = a_1 + d = 24 + 4 = 28$$

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.

$$a_n = a_1 + (n-1)d$$

$$a_{20} = 24 + 19 \times 4 = 24 + 76 = 100$$

$$S = \frac{20}{2}[a_1 + a_{20}] = 10[24 + 100] = 1240$$

As contestant is required to bring the potatoes back to the starting point.

The distanced contestant would run

$$= 1240 + 1240$$

$$= 2480 \text{ m.}$$

Q15(i)

A man accepts a position with an initial salary of Rs.5200 per month.

$$a_1 = 5200$$

Man will receive an automatic increase of Rs.320.

$$d = 320$$

We need to find his salary for the n^{th} month is given by,

$$a_n = a_1 + (n-1)d$$

$$a_{10} = 5200 + 9 \times 320 = 8080$$

The salary of that man for tenth month is Rs.8080.

Q15(ii)

A man accepts a position with an initial salary of Rs.5200 per month.

$$a_1 = 5200$$

Man will receive an automatic increase of Rs.320.

$$d = 320$$

Man's salary for the n^{th} month is given by,

$$a_n = a_1 + (n-1)d$$

Total earnig of the man for the first year

$$= \frac{12}{2} [a_1 + a_{12}]$$

$$= 6 [5200 + 5200 + (12-1)320]$$

$$= 83520$$

Total earnig of the man for the first year is Rs. 83,520.

Q16

Suppose the man saved Rs. x in the first year

$$a_1 = x$$

In each succeeding year after the first year man saved Rs 200 more than what he saved in the previous year.

$$d = 200$$

Man saved Rs. 66000 in 20 years.

$$S = 66000$$

$$\frac{20}{2}[a_1 + a_1 + (20 - 1)200] = 66000$$

$$a_1 + 1900 = 3300$$

$$a_1 = 1400$$

Man saved Rs 1400 in the first year.

Q17

Suppose the award increases by Rs. x .

$$d = x$$

In cricket team tournament 16 teams participated.

$$n = 16$$

The last place team is awarded Rs. 275 in prize money

$$a_1 = 275$$

Sum of Rs. 8000 is to be awarded as prize money

$$S = 8000$$

$$\frac{16}{2}[a_1 + a_1 + (16 - 1)x] = 8000$$

$$2a_1 + 15x = 1000$$

$$550 + 15x = 1000$$

$$15x = 450$$

$$x = 30$$

The amount received by first place team

$$= a_{16}$$

$$= a_1 + (16 - 1)d$$

$$= 275 + 15 \times 30$$

$$= 275 + 450$$

$$= 725$$

The amount received by first place team is Rs. 725.