Linear Programming

<u>Short Answer Type Questions</u>

1. Determine the maximum value of $Z = 4x + 3y$ if the feasible region for an LPP is shown in Fig. 12.1.

Sol. The feasible region is bounded. Therefore, maximum of Z must occur at the corner point of the feasible region (Fig. 12.1).

Corner Point	Value of Z
O, (0, 0)	4 (0) + 3 (0) = 0
A (25, 0)	4 (25) + 3 (0) = 100
B (16, 16)	4 (16) + 3 (16) = **112** (maximum)
C (0, 24)	4 (0) + 3 (24) = 72

Hence, the maximum value of Z is 112.

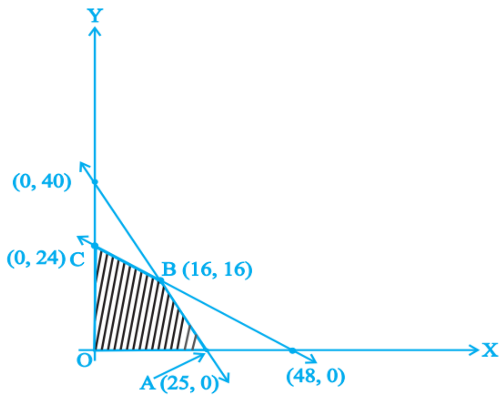


Fig. 12.1

2. Determine the minimum value of $Z = 3x + 2y$ (if any), if the feasible region for an LPP is shown in Fig.12.2.

Sol. The feasible region (R) is unbounded. Therefore, minimum of Z may or may not exist. If it exists, it will be at the corner point (Fig.12.2).

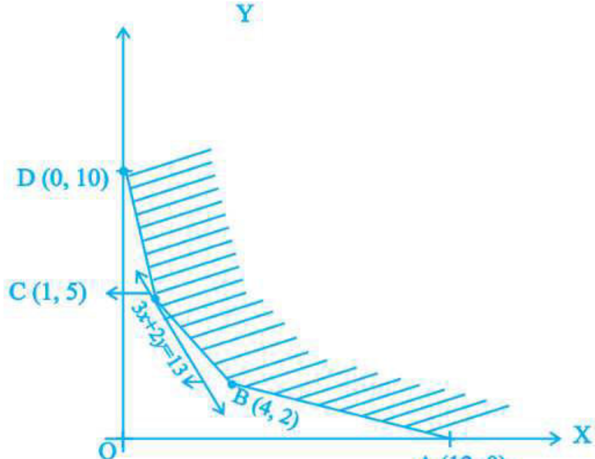


Fig. 12.2

Corner Point	Value of Z
A (12, 0)	3 (12) + 2 (0) = 36
B (4, 2)	3 (4) + 2 (2) = 16
C (1, 5)	3 (1) + 2 (5) = **13** (smallest)
D (0, 10)	3 (0) + 2 (10) = 20

Let us graph $3x+2y<13$. We see that the open half plane determined by $3x+2y<13$ and R do not have a common point. So, the smallest value 13 is the minimum value of Z.

3. Solve the following LPP graphically:

Maximise $Z=2x+3y$**,**

subject to $x+y\leq 4,\ x\geq 0,\ y\geq 0$

Sol. The shaded region (OAB) in the Fig. 12.3 is the feasible region determined by the system of constraints $x\geq 0,\ y\geq 0$ *and* $x+y\leq 4$.

The feasible region OAB is bounded, so, maximum value will occur at a corner point of the feasible region.

Corner Points are $O(0,\ 0),\ A(4,\ 0)$ *and* $B(0,\ 4)$.

Evaluate Z at each of these corner point.

Corner Point	Value of Z
O, (0, 0)	2 (0) + 3 (0) = 0
A (4, 0)	2 (4) + 3 (0) = 8
B (0, 4)	2 (0) + 3 (4) = **12** (Maximum)

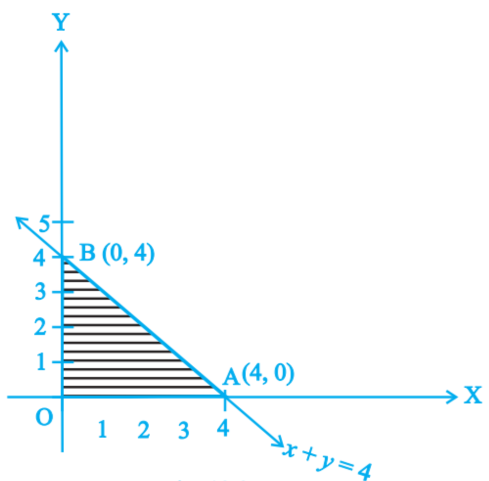


Fig. 12.3

Hence, the maximum value of Z is 12 at the point $(0,\ 4)$.

4. A manufacturing company makes two types of television sets; one is black and white and the other is colour. The company has resources to make at most 300 sets a week. It takes Rs 1800 to make a black and white set and Rs 2700 to make a coloured set. The company can spend not more than Rs

648000 a week to make television sets. If it makes a profit of Rs 510 per black and white set and Rs 675 per coloured set, how many sets of each type should be produced so that the company has maximum profit? Formulate this problem as a LPP given that the objective is to maximise the profit.

Sol. Let x and y denote, respectively, the number of black and white sets and coloured sets made each week. Thus

$x \geq 0,\ y \geq 0$

Since the company can make at most 300 sets a week, therefore,

$x + y \leq 300$

Weekly cost (in Rs) of manufacturing the set is

$1800x + 2700y$

and the company can spend upto Rs. 648000. Therefore,

$1800x + 2700y \leq 648000,\ i.e.,\ or\ 2x + 3y \leq 720$

The total profit on x black and white sets and y colour sets is $Rs\ (510x + 675y)$.

Let $Z = 510x + 675y$. This is the **objective function**.

Thus, the mathematical formulation of the problem is

Maximise $Z = 510x + 675y$

Subject the constraints:

$$\left.\begin{array}{l} x + y \leq 300 \\ 2x + 3y \leq 720 \\ x \geq 0,\ y \geq 0 \end{array}\right\}$$

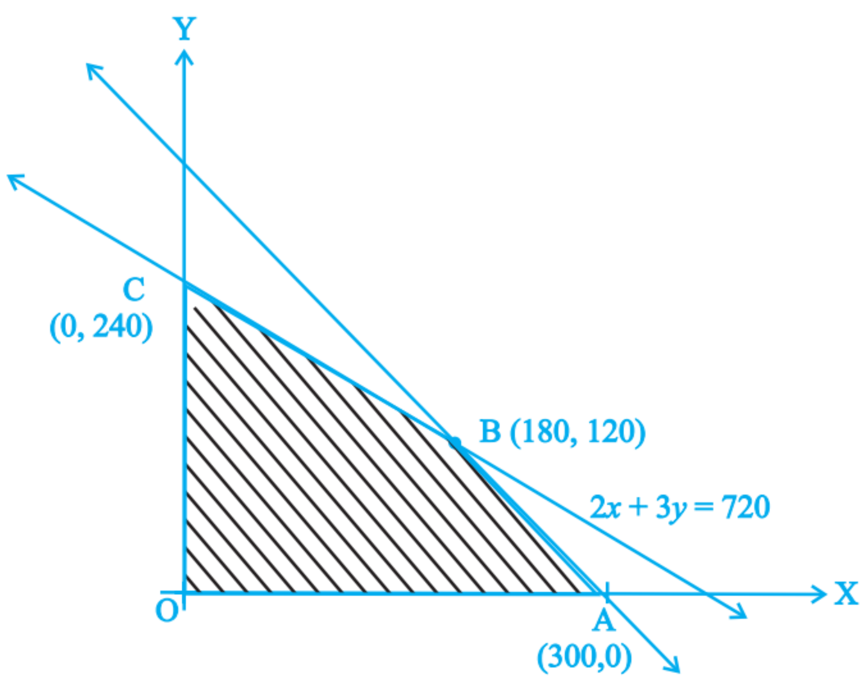


Fig. 12.4

Long Answer Type Questions

5. Refer to Example 4. Solve the LPP.

Sol. The problem is:

Maximise $Z = 510x + 675y$

Subjects to the constraints:

$$\left.\begin{array}{l} x + y \leq 300 \\ 2x + 3y \leq 720 \\ x \geq 0,\ y \geq 0 \end{array}\right\}$$

The feasible region OABC is shown in the Fig. 12.4.

Since the feasible region is bounded, therefore maximum of Z must occur at the corner point of OBC.

Corner Point	Value of Z
O (0, 0)	510 (0) + 675 (0) = 0
A (300, 0)	510 (300) + 675 (0) = 153000
B (180, 120)	510 (180) + 675 (120) = 172800 (maximum)
C (0, 240)	510 (0) + 675 (240) = 162000

Thus, maximum Z is 172800 at the point $(180,\ 120)$, i.e., the company should produce 180 black and white television sets and 120 coloured television sets to get maximum profit.

6. Minimise $Z = 3x + 5y$ subject to the constraints:

$x + 2y \geq 10$

$x + y \geq 6$

$3x + y \geq 8$

$x, y \geq 0$

Sol. We first draw the graphs of $x + 2y = 10,\ x + y = 6,\ 3x + y = 8$. The shaded region ABCD is the feasible region (R) determined by the above constraints. The feasible region is unbounded. Therefore, minimum of Z may or may not occur. If it occurs, it will be on the corner point.

Corner Point	Value of Z
A (0, 8)	40
B (1, 5)	28
C (2, 4)	26 (smallest)
D (10, 0)	30

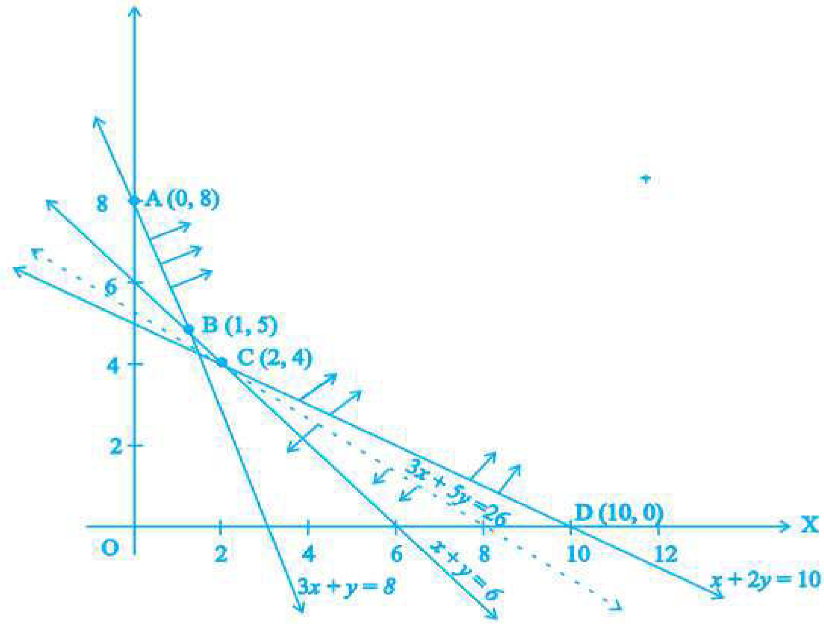


Fig. 12.5

Let us draw the graph of $3x+5y<26$ as shown in Fig. 12.5 by dotted line.

We see that the open half plane determined by $3x+5y<26$ and R do not have a point in common. Thus, 26 is the minimum value of Z.

Objective Type Questions

Choose the correct answer from the given four options in each of the Examples 7 to 8.

7. The corner points of the feasible region determined by the system of linear constraints are $(0,10),\ (5,5),\ (15,15),\ (0,20)$. Let $Z = px+qy,\ where\ p,\ q>0$. Condition on p and q so that the maximum of Z occurs at both the points $(15,15)\ and\ (0,20)$ is

(A) $p=q$

(B) $p=2q$

(C) $q=2p$

(D) $q=3p$

Sol. The correct answer is (D). Since Z occurs maximum at $(15,15)\ and\ (0,20)$, therefore, $15p+15q=0.\ p+20q \ \Rightarrow q=3p$.

8. Feasible region (shaded) for a LPP is shown in the Fig. 14.6. Minimum of $Z=4x+3y$ occurs at the point

(A) (0, 8)

(B) (2, 5)

(C) (4, 3)

(D) (9, 0)

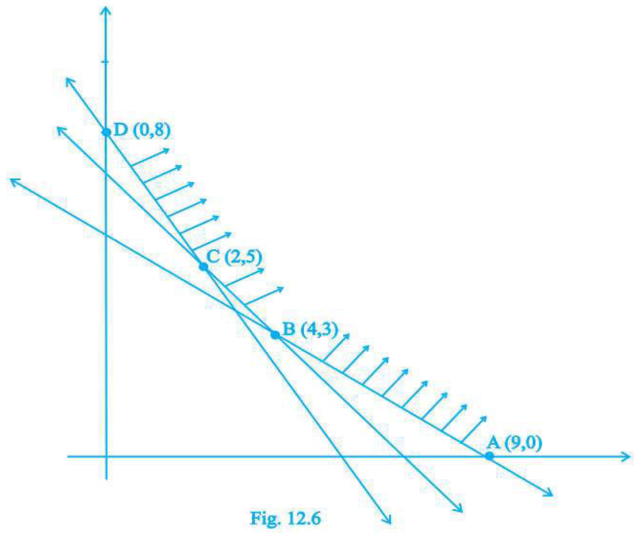


Fig. 12.6

Sol. The correct answer is (B).

Fill in the blanks in each of the Examples 9 and 10:

9. In a LPP, the linear function which has to be maximised or minimised is called a linear _________ function.

Sol. Objective.

10. The common region determined by all the linear constraints of a LPP is called the ______ region.

Sol. Feasible.

State whether the statements in Examples 11 and 12 are True or False.

11. If the feasible region for a linear programming problem is bounded, then the objective function $Z = ax + by$ has both a maximum and a minimum value on R.

Sol. True

12. The minimum value of the objective function $Z = ax + by$ in a linear programming problem always occurs at only one corner point of the feasible region.

Sol. False

The minimum value can also occur at more than one corner points of the feasible region.

Linear Programming

Objective Type Questions

Choose the correct answer from the given four options in each of the Exercises 26 to 34.

26. **The corner points of the feasible region determined by the system of linear constraints are** $(0,0)$, $(0,40)$, $(20,40)$, $(60,20)$, $(60,0)$. **The objective function is** $Z = 4x + 3y$.

Compare the quantity in Column A and Column B

Column A	**Column B**
Maximum of Z	325

(A) The quantity in column A is greater
(B) The quantity in column B is greater
(C) The two quantities are equal
(D) The relationship cannot be determined on the basis of the information Supplied

Sol. (B)

Corner points	Corresponding value of Z = 4x + 3y
(0, 0)	0
(0, 40)	120
(20, 40)	200
(60, 20)	300 **(Maximum)**
(60, 0)	240

Hence, maximum value of $Z = 300 < 325$.

So, the quantity in column B is greater.

27. **The feasible solution for a LPP is shown in Fig. 12.12. Let** $Z = 3x - 4y$ **be the**

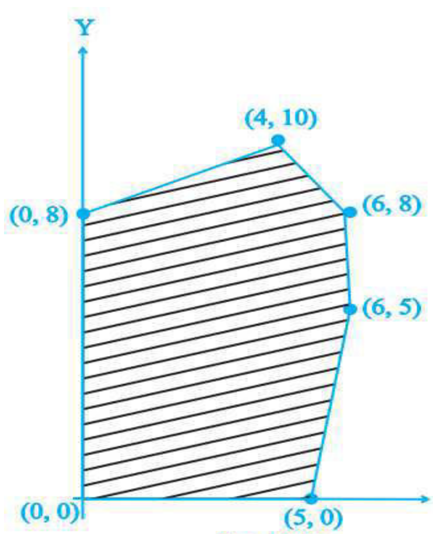


Fig. 12.12

objective function. Minimum of Z occurs at

(A) (0, 0)

(B) (0, 8)

(C) (5, 0)

(D) (4, 10)

Sol. (B)

Corner Points	Corresponding value of $Z = 3x - 4y$
(0, 0)	0
(5, 0)	15 **(maximum)**
(6, 5)	-2
(6, 8)	-14
(4, 10)	-28
(0, 8)	-32 **(minimum)**

Hence, the minimum of Z occurs at (0, 8) and its minimum value is (-32).

28. Refer to Exercise 27. Maximum of Z occurs at

(A) (5, 0)

(B) (6, 5)

(C) (6, 8)

(D) (4, 10)

Sol. (A) Refer to solution 27, maximum of Z occurs at (5, 0).

29. Refer to Exercise 27. (Maximum value of Z + Minimum value of Z) is equal to

(A) 13

(B) 1

(C) –13

(D) –17

Sol. (D) Refer to solution 27, maximum value of Z + minimum value of Z.

$= 15 - 32 = -17$

30. The feasible region for an LPP is shown in the Fig. 12.13. Let F = 3x– 4y be the objective function. Maximum value of F is.

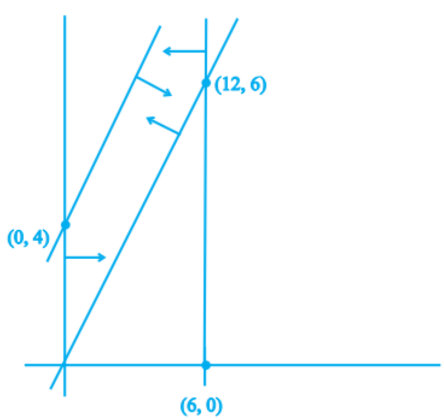


Fig. 12.13

(A) 0
(B) 8
(C) 12
(D) –18

Sol. (C) The feasible region as shown in the figure, has objective function F = 3x – 4y

Corner Points	Corresponding value of F = 3x – 4y
(0, 0)	0
(12, 6)	12 **(maximum)**
(0, 4)	-16 **(minimum)**

Hence, the maximum value of F is 12.

31. Refer to Exercise 30. Minimum value of F is
(A) 0
(B) –16
(C) 12
(D) does not exist

Sol. (B) Referring to solution 30, minimum value of F is –16 at (0, 4).

32. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) **and** $(0,5)$.
Let $F = 4x + 6y$ **be the objective function.**
The Minimum value of F occurs at
(A) (0, 2) only
(B) (3, 0) only
(C) the midpoint of the line segment joining the points (0, 2) and (3, 0) only
(D) any point on the line segment joining the points (0, 2) and (3, 0).

Sol. (D)

Corner Points	Corresponding value of F = 4x + 6y
(0, 2)	12 **(minimum)**
(3, 0)	12 **(minimum)**
(6, 0)	24
(6, 8)	72 **(maximum)**
(0, 5)	30

Hence, minimum value of F occurs at any points on the line segment joining the points (0, 2) and (3, 0).

33. Refer to Exercise 32, Maximum of F – Minimum of F =
(A) 60
(B) 48
(C) 42
(D) 18

Sol. (A) Referring to the solution 32, maximum of F - minimum of $F = 72 - 12 = 60$.

34. Corner points of the feasible region determined by the system of linear constraints are $(0,3)$, $(1,1)$ *and* $(3,0)$. Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the minimum of Z occurs at $(3,0)$ *and* $(1,1)$ is

(A) p = 2q

(B) $p = \frac{q}{2}$

(C) p = 3q

(D) p = q

Sol. (B)

Corner Points	Corresponding value of Z = px + qy; q > 0
(0, 3)	3q
(1, 1)	p + q
(3, 0)	3p

So, condition of p and q, so that the minimum of Z occurs at $(3,0)$ *and* $(1,1)$ is

$p + q = 3p \Rightarrow 2p = q$

$\therefore\ p = \frac{q}{2}$

Fill in the blanks in each of the Exercises 35 to 41.

35. In a LPP, the linear inequalities or restrictions on the variables are called ________.

Sol. In a LPP, the linear inequalities or restrictions on the variables are called linear constraints.

36. In a LPP, the objective function is always ________

Sol. In a LPP, objective function is always linear.

37. If the feasible region for a LPP is ________, then the optimal value of the objective function $Z = ax + by$ may or may not exist.

Sol. If the feasible region for a LPP is unbounded, then the optimal value of objective function $Z = ax + by$ may or may not exits.

38. In a LPP if the objective function $Z = ax + by$ has the same maximum value on two corner points of the feasible region, then every point on the line segment joining these two points give the same ________ value.

Sol. In a LPP, if the objective function $Z = ax + by$ has the same maximum value on two corner points of the feasible region, then every point on the line segment joining these two points give the same maximum value.

39. A feasible region of a system of linear inequalities is said to be ________ if it can be enclosed within a circle.

Sol. A feasible region of a system of linear inequality is said to be bounded, if it can be enclosed within a circle.

40. A corner point of a feasible region is a point in the region which is the ________ of two boundary lines.

Sol. A corner point of feasible region is a point in the region which is the intersection of two boundary lines.

41. The feasible region for an LPP is always a ________ polygon.

Sol. The feasible region for an LPP is always a **convex** polygon.

State whether the statements in Exercises 42 to 45 are True or False.

42. If the feasible region for a LPP is unbounded, maximum or minimum of the objective function $Z = ax + by$ may or may not exist.

Sol. Ture

43. Maximum value of the objective function $Z = ax + by$ in a LPP always occurs at only one corner point of the feasible region.

Sol. False

44. In a LPP, the minimum value of the objective function $Z = ax + by$ is always 0 if origin is one of the corner point of the feasible region.

Sol. False

45. In a LPP, the maximum value of the objective function $Z = ax + by$ is always finite.

Sol. True

Linear Programming

Short Answer Type Questions

1. Determine the maximum value of $Z = 11x + 7y$ subject to the constraints:

$2x + y \leq 6,\ x \leq 2,\ x \geq 0,\ y \geq 0$.

Sol. We have, maximise $Z = 11x + 7y$ ...(i)

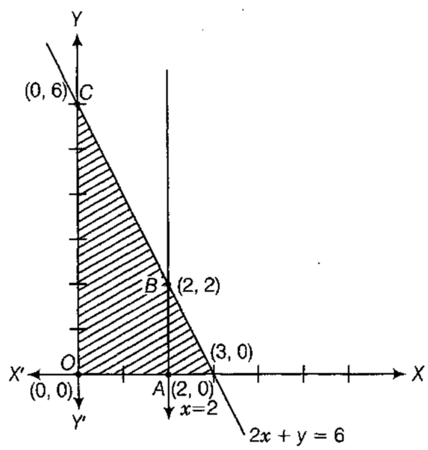
Subject to the constraints

$2x + y \leq 6$ ...(ii)

$x \leq 2$ ...(iii)

$x \geq 0, y \geq 0$ ...(iv)

We see that, the feasible region as shaded determined by the system of constraint (ii) to (iv) is OABC and is bounded. So, now we shall use corner point method to determine the maximum value of Z.



Corner Points	Corresponding value of Z
(0,0)	0
(2, 0)	22
(2, 2)	36
(0, 6)	42 **(Maximum)**

Hence, the maximum value of Z is 42 at (0, 6).

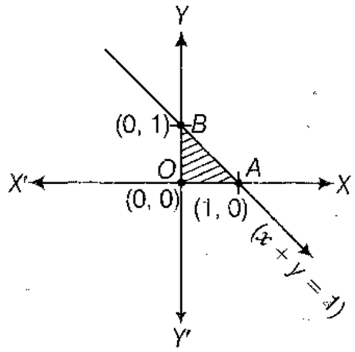
2. Maximise $Z = 3x + 4y$, subject to the constraints: $x + y \leq 1,\ x \geq 0,\ y \geq 0$.

Sol. Maximise $Z = 3x + 4y$. Subject to the constraints

$x + y \leq 1,\ x \geq 0,\ y \geq 0$

The Shaded region shown in the figure as OAB is bounded and the coordinates of corner points O, A and B are $(0,0)$, $(1,0)$ *and* $(0,1)$, respectively.

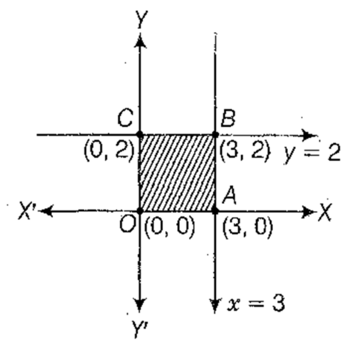
Corner Points	Corresponding value of Z
(0,0)	0
(1, 0)	3
(0, 1)	4**(Maximum)**



Hence, the maximum value of Z is 4 at $(0,\ 1)$.

3. **Maximise the function** $Z=11x+7y$, **subject to the constraints:** $x\le 3,\ y\le 2,\ x\ge 0,\ y\ge 0$

Sol. Maximise $Z=11x+7y$, subject to the constraints $x\le 3,\ y\le 2,\ x\ge 0,\ y\ge 0$.



The shaded region as shown in the figure as OABC is bounded and the coordinates of corner points are $(0,0)$, $(3,0)$, $(3,2)$, *and* $(0,2)$, respectively.

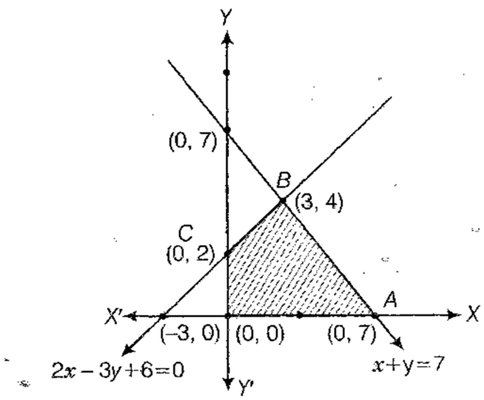
Corner Points	Corresponding value of Z
(0,0)	0
(3, 0)	33
(3, 2)	47 **(Maximum)**
(0, 2)	14

Hence, Z is maximise at $(3,\ 2)$ and its maximum value is 47.

4. **Minimise** $Z=13x-15y$ **subject to the constraints:** $x+y\le 7,\ 2x-3y+6\ge 0,$ $x\ge 0\ y\ge 0$**.**

Sol. Minimise $Z=13x-15y$ subject to the constraints: $x+y\le 7,\ 2x-3y+6\ge 0,$ $x\ge 0\ y\ge 0$.

Shaded region shown as OABC is bounded and coordinates of its corner points are $(0,0)$, $(7,0)$, $(3,4)$ and $(0,\ 2)$, respectively.



Corner Points	Corresponding value of Z
(0, 0)	0
(7, 0)	91
(3, 4)	-21
(0, 2)	-30 **(Minimum)**

Hence, the minimum value of Z is (-30) *at* $(0,2)$.

5. Determine the maximum value of $Z=3x+4y$ if the feasible region (shaded) for a LPP is shown in Fig.12.7.

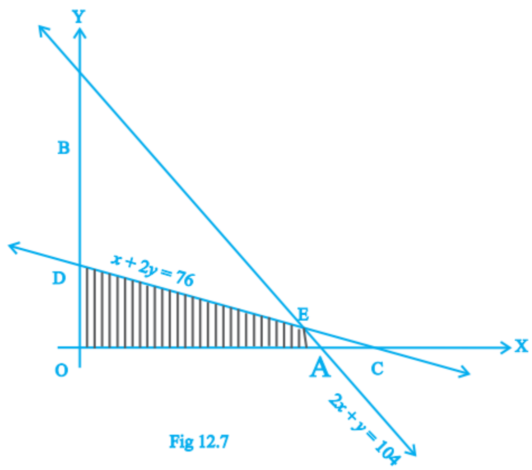


Fig 12.7

Sol. As clear from the graph, corner points are O, A, E *and* D with coordinate $(0,0)$, $(52,0)$, $(144,16)$ and $(0,\ 38)$, respectively. Also given region is bounded.

Here, $Z=3x+4y$

$\because\ 2x+y=104$ *and* $2x+4y=152$

$\Rightarrow -3y=-48$

$\Rightarrow\ y=16$ *and* $x=44$

Corner Points	Corresponding value of Z
(0, 0)	0
(52, 0)	156
(44, 16)	196 **(maximum)**
(0, 38)	152

Hence, Z is at $(44, \ 16)$ is maximum and its maximum value is 196.

6. Feasible region (shaded) for a LPP is shown in Fig. 12.8. Maximise $Z = 5x + 7y$.

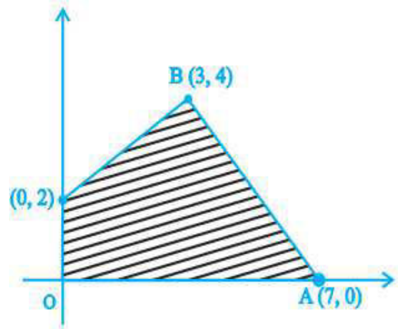


Fig. 12.8

Sol. The Shaded region is bounded and has coordinate of corner points as $(0,0)$, $(7,0)$, $(3,4)$ and $(0,2)$, Also, $Z = 5x + 7y$.

Corner Points	Corresponding value of Z
(0, 0)	0
(7, 0)	35
(3, 4)	43 **(Maximum)**
(0, 2)	14

Hence, the maximum value of Z is 43 at $(3,4)$.

7. The feasible region for a LPP is shown in Fig. 12.9. Find the minimum value of $Z = 11x + 7y$.

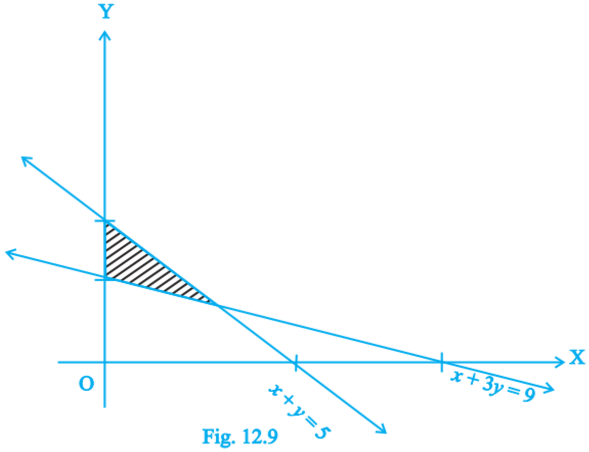


Fig. 12.9

Sol. From the figure, it is clear that feasible region is bounded with coordinate of corner points as $(0,3)$, $(3,2)$ *and* $(0,5)$. Here $Z = 11x + 7y$.

$\because\ x + 3y = 9$ *and* $x + y = 5$

$\Rightarrow 2y = 4$

$\therefore\ y = 2$ *and* $x = 3$

So, intersection points of $x + y = 5$ and $x + 3y = 9$ *is* $(3,2)$.

Corner Points	Corresponding value of Z
(0, 3)	21 **(minimum)**
(3, 2)	47
(0, 5)	35

Hence, the minimum value of Z is 21 at $(0,3)$.

8. Refer to Exercise 7 above. Find the maximum value of Z.

Sol. From Question 7, above, it is clear that Z is maximum at $(3,2)$ and its maximum value is 47.

9. The feasible region for a LPP is shown in Fig. 12.10. Evaluate $Z = 4x + y$ at each of the corner points of this region. Find the minimum value of Z, if it exists.

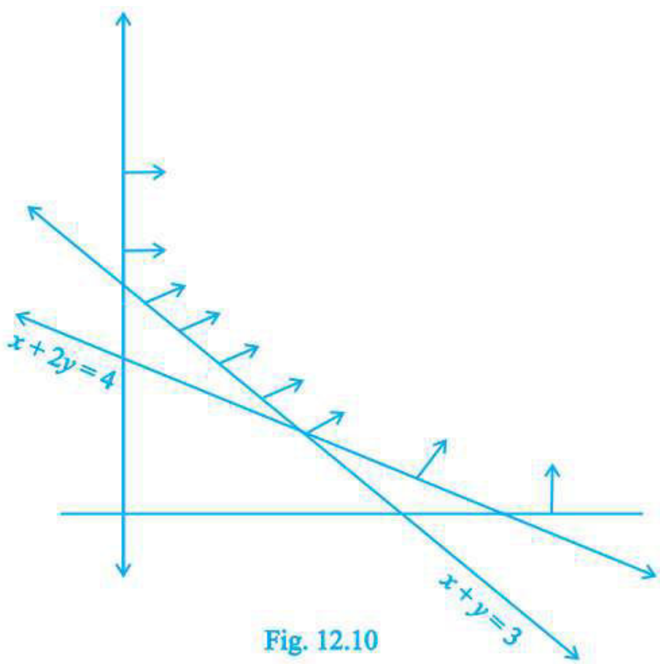
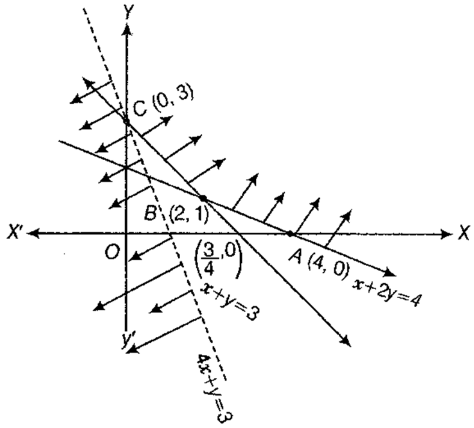


Fig. 12.10

Sol. From the shaded region, it is clear that feasible region is unbounded with the corner points $A(4,0)$, $B(2,1)$ *and* $C(0,3)$.

Also, we have $Z = 4x + y$.

$[since,\ x + 2y = 4 \text{ and } x + y = 3 \Rightarrow y = 1 \text{ and } x = 2]$



Corner Points	Corresponding value of Z
(4, 0)	16
(2, 1)	9
(0, 3)	3 **(minimum)**

Now, we see that 3 is the smallest value of Z at the corner point (0, 3). Note that here we see that the region is unbounded, therefore 3 may or may not be the minimum value of Z.

To decide this issue, we graph the inequality $4x+y<3$ and check whether the resulting open half plan has no point in common with feasible region otherwise, Z has no minimum value.

From the shown graph above, it is clear that there is no point in common with feasible region and hence Z has minimum value of 3 at $(0,\ 3)$.

10. In Fig. 12.11, the feasible region (shaded) for a LPP is shown. Determine the maximum and minimum value of $Z = x + 2y$.

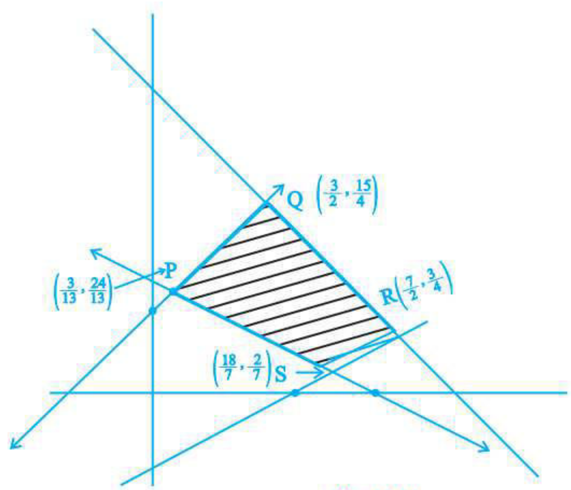


Fig. 12.11

Sol. From the shaded bounded region, it is clear that the coordinates of corner points are $\left(\frac{3}{13},\frac{24}{13}\right),\left(\frac{18}{7},\frac{2}{7}\right),\left(\frac{7}{2},\frac{3}{4}\right)$ and $\left(\frac{3}{2},\frac{15}{4}\right)$

Also, we have to determine maximum and minimum value of $Z = x + 2y$.

Corner Points	Corresponding value of Z
$\left(\frac{3}{13}, \frac{24}{13}\right)$	$\frac{3}{13} + \frac{48}{13} = \frac{51}{13} = 3\frac{12}{13}$
$\left(\frac{18}{7}, \frac{2}{7}\right)$	$\frac{18}{7} + \frac{4}{7} = \frac{22}{7} = 3\frac{1}{7}$ **(Minimum)**
$\left(\frac{7}{2}, \frac{3}{4}\right)$	$\frac{7}{2} + \frac{6}{4} = \frac{20}{4} = 5$
$\left(\frac{3}{2}, \frac{15}{4}\right)$	$\frac{3}{2} + \frac{30}{4} = \frac{36}{4} = 9$ **(Maximum)**

Hence, the maximum and minimum value of are 9 *and* $3\frac{1}{7}$, respectively.

11. A manufacturer of electronic circuits has a stock of 200 resistors, 120 transistors and 150 capacitors and is required to produce two types of circuits A and B. Type A requires 20 resistors, 10 transistors and 10 capacitors. Type B requires 10 resistors, 20 transistors and 30 capacitors. If the profit on type A circuit is Rs 50 and that on type B circuit is Rs 60, formulate this problem as a LPP so that the manufacturer can maximise his profit.

Sol. Let the Manufacture produce x units of type A circuit and y units of type B circuits.

Form the given information, we have following corresponding constraint table.

	Type A(x)	Type B(y)	Maximum stock
Resistors	20	10	200
Transistors	10	20	120
Capacitors	10	30	150
Profit	Rs 50	Rs 60	

Thus, we see that total profit $Z = 50x + 60y$ (in Rs).

Now, we have the following mathematical model for the given problem.

Maximise $Z = 50x + 60y$ ...(i)

Subject to the constraints.

$20x + 10y \le 200$ [resistor constraint]

$\Rightarrow 2x + y \le 20$ ...(ii)

And $10x + 20y \le 120$ [transistor constraint]

$\Rightarrow x + 2y \le 12$ ...(iii)

And $10x + 30y \le 150$ [capacitor constraint]

$\Rightarrow x + 3y \le 15$ ...(iv)

And $x \le 0, y \le 0$ ...(v) [non-negative constraint]

So, Maximise $Z = 50x + 60y$, subject to $2x + y \le 20, x + 2y \le 12,\ x + 3y \le 15,$
$x \le 0, y \le 0$.

12. A firm has to transport 1200 packages using large vans which can carry 200 packages each and small vans which can take 80 packages each. The cost for engaging each large van is Rs 400 and each small van is Rs 200. Not more than Rs 3000 is to be spent on the job and the number of large vans cannot exceed the number of small vans. Formulate this problem as a LPP given that the objective is to minimise cost.

Sol. Let the firm has x number of large vans and y number of small vans. From the given information, we have following corresponding constraint table.

	Large vans (x)	Small vans (y)	Maximum / Minimum
Packages	200	80	1200
Cost	400	200	3000

Thus, we see that objective function for minimum cost is $Z = 400x + 200y$.

Subject to constraints

$200x + 80y \le 1200$ [package constraint]

$\Rightarrow 5x + 2y \le 30$ ...(i)

And $400x + 200y \le 3000$ [cost constraint]

$\Rightarrow 2x + y \le 15$ ...(ii)

And $x \le y$ [van constraint] ...(iii)

And $x \ge 0, y \ge 0$ [non-negative constraints] ...(iv)

Thus, required LPP to minimise cot is minimise $Z = 400x + 200y$, subject to

$5x + 2y \ge 30$.

$2x + y \le 15$

$x \le y$

$x \ge 0, y \ge 0$

13. A company manufactures two types of screws A and B. All the screws have to pass through a threading machine and a slotting machine. A box of Type A screws requires 2 minutes on the threading machine and 3 minutes on the slotting machine. A box of type B screws requires 8 minutes of threading on the threading machine and 2 minutes on the slotting machine. In a week, each machine is available for 60 hours.

On selling these screws, the company gets a profit of Rs 100 per box on type A screws and Rs 170 per box on type B screws.

Formulate this problem as a LPP given that the objective is to maximise profit.

Sol. Let the company manufacture x boxes of type A screws and y boxes of type B screws. From the given information, we have following corresponding constraint table

	Type A(x)	Type B(x)	Maximum time available on each machine in a week
Time required for screws on threading machine	2	8	60×60 (min)
Time required for screws on slotting machine	3	2	60×60 (min)
Profit	Rs 100	Rs 170	

Thus, we see that objective function for maximum profit is $Z=100x+170y$.

Subject to constraints.

$2x+8y\le 60\times 60$ [time constraint for threading machine]

$\Rightarrow x+4y\le 1800 \ \ldots(i)$

And $3x+2y\le 60\times 60$ [time constraint for slotting machine]

$\Rightarrow 3x+2y\le 3600 \ \ldots(ii)$

Also, $x\le 0, y\le 0$ [non-negative constraints] ...(iii)

$\therefore$ Required LPP is,

Maximise $Z=100x+170y$

Subject to constraints $x+4y\le 1800, 3x+2y\le 3600, x\le 0, y\le 0$.

14. A company manufactures two types of sweaters: type A and type B. It costs Rs 360 to make a type A sweater and Rs 120 to make a type B sweater. The company can make at most 300 sweaters and spend at most Rs 72000 a day. The number of sweaters of type B cannot exceed the number of sweaters of type A by more than 100. The company makes a profit of Rs 200 for each sweater of type A and Rs 120 for every sweater of type B. Formulate this problem as a LPP to maximise the profit to the company.

Sol. Let the company manufactures x number of type A sweaters and y number of type B sweaters.

From the given information we see that cost to make a type A sweater is Rs 360 and cost to make a type B sweater is Rs 120.

Also, the company spent at most Rs 72000 a day

$\therefore 360x+120y\le 72000$

$\Rightarrow 3x+y\le 600 \ \ldots(i)$

Also, company can make at most 300 sweaters.

$\therefore x+y\le 300 \ \ldots(ii)$

Further, the number of sweaters of type B cannot exceed the number of sweaters of type A by more than 100 i.e.,

$x+100\ge y$

$\Rightarrow x-y\ge -100 \ \ldots(iii)$

Also, we have non-negative constraints for x and y i.e., $x\ge 0, y\ge 0 \ \ldots(iv)$

Hence, the company makes a profit of Rs 200 for each sweater of type A and Rs 120 for each sweater of type B i.e.,

$Profit(Z)=200x+120y$

Thus, the required LPP to maximise the profit is Maximise $Z = 200x + 120y$ is subject to constraints.

$3x + y \leq 600$

$x = y \leq 300$

$x - y \geq -100$

$x \geq 0, y \geq 0$

15. A man rides his motorcycle at the speed of 50 km/hour. He has to spend Rs 2 per km on petrol. If he rides it at a faster speed of 80 km/hour, the petrol cost increases to Rs 3 per km. He has at most Rs 120 to spend on petrol and one hour's time. He wishes to find the maximum distance that he can travel. Express this problem as a linear programming problem.

Sol. Let the man rides to his motorcycle to a distance x km at the speed of 50 km/h and to a distance y km at the speed of 80 km/h.

Therefore, cost on petrol is 2x + 3y.

Since, he has to spend Rs 120 at most on petrol.

$\therefore\ 2x + 3y \leq 120 \ \ ...(i)$

Also, he has at most one hour's time.

$\therefore\ \frac{x}{50} + \frac{y}{80} \leq 1$

$\Rightarrow 8x + 5y \leq 400 \ \ ...(ii)$

Also, we have $x \geq 0, y \geq 0$ [non-negative constraints]

Thus, required LPP to travel maximum distance by him is

Maximise $Z = x + y,$ subject to $2x + 3y \leq 120, 8x + 5y \leq 400, x \geq 0, y \geq 0$

Long Answer type Questions

16. Refer to Exercise 11. How many of circuits of Type A and of Type B, should be produced by the manufacturer so as to maximise his profit? Determine the maximum profit.

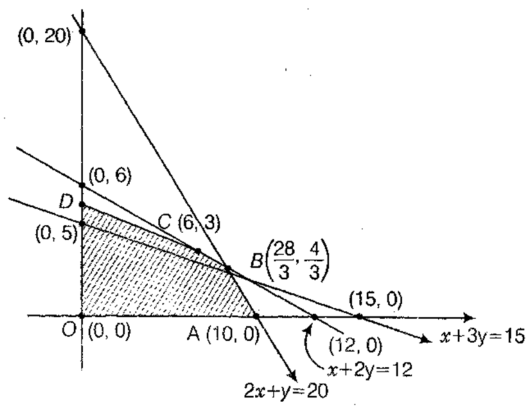
Sol. Referring to solution 11, we have

Maximise $Z = 50x + 60y$, subject to

$2x + y \le 20, x + 2y \le 12, x + 3y \le 15, x \ge 0, y \ge 0$

From the shaded region it is clear that the feasible region determined by the system of constraint is OABCD and is bounded and the coordinates of corner points are $(0,0)$, $(10,0)$, $\left(\frac{28}{3}, \frac{4}{3}\right)$, $(6,3)$ *and* $(0,5)$, respectively.

[Since, $x + 2y = 12$ *and* $2x + y = 20 \Rightarrow x = \frac{28}{3}, y = \frac{4}{3}$ and $x + 3y = 15$ *and* $x + 2y = 12 \Rightarrow y = 3$ *and* $x = 16$]



Corner Points	Corresponding value of Z = 50x + 60y
(0, 0)	0
(10, 0)	500
$\left(\frac{28}{3}, \frac{4}{3}\right)$	$\frac{1400}{3} + \frac{240}{3} = \frac{1640}{3} = 546.66$ **(maximum)**
(6, 3)	480
(0, 5)	300

Since, the manufacturer is required to produce two types of circuits A and B and it is clear that parts of resistor, transistor and capacitor cannot be in fraction, So the required maximum profit is 480 where circuits of type A is 6 and circuits of type B is 3.

17. Refer to Exercise 12. What will be the minimum cost?

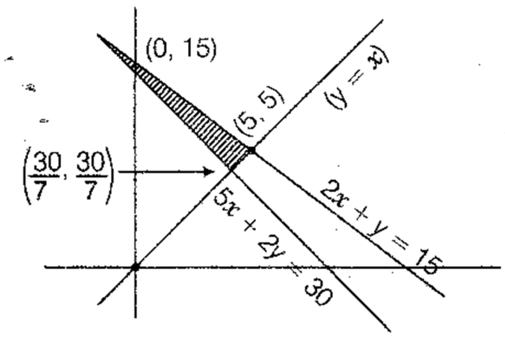
Sol. Referring to solution 12, we have minimise $Z = 400x + 200y$, subject to

$5x + 2y \ge 30.$

$2x+y\ge 15, x\le y, x\ge 0, y\ge 0.$

On solving $x-y=0$ *and* $5x+2y=30,$ we get

$y=\frac{30}{7}, x=\frac{30}{7}$



On solving $x-y=0$ *and* $2x+y=15,$ we get $x=5,\ y=5$

So, from the shaded feasible region it is clear that coordinates of corner points are $(0,15)$, $(5,5)$ and $\left(\frac{30}{7}, \frac{30}{7}\right)$.

Corner Points	Corresponding value of X = 400x + 200y
(0, 15)	3000
(5, 5)	3000
$\left(\frac{30}{7}, \frac{30}{7}\right)$	$400\times\frac{30}{7}+200\times\frac{30}{7}=\frac{18000}{7}$
	= 2571.43 **(minimum)**

Hence, the minimum cost is Rs 2571.43.

18. Refer to Exercise 13. Solve the linear programming problem and determine the maximum profit to the manufacturer.

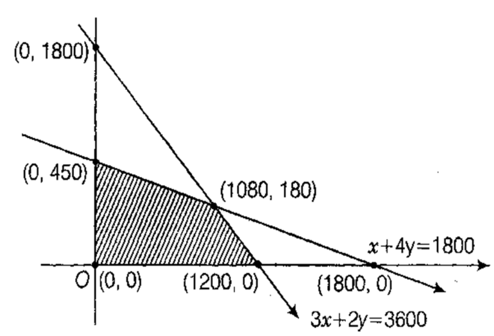
Sol. Referring to solution 13, we have

Maximise $Z=100x+170y$ subject to

$3x+2y\le 3600, x+4y\le 1800, x\ge 0, y\ge 0$

From the shaded feasible region it is clear that the coordinates of corner points are $(0,0)$, $(1200,0)$, $(1080,180)$ and $(0,450)$.

On solving $x+4y=1800$ *and* $3x+2y=3600,$ we get $x=1080$ *and* $y=180$.



Corner Points	Corresponding value of Z = 100x + 170y
(0, 0)	0
(1200, 0)	$1200\times100=12000$
(1080, 180)	$100\times1080+170\times180=138600$ **(maximum)**
(0, 450)	$0+170\times450=76500$

Hence, the maximum profit to the manufacture is 138600.

19. Refer to Exercise 14. How many sweaters of each type should the company make in a day to get a maximum profit? What is the maximum profit.

Sol. Referring to solution 14, we have maximise $Z=200x+120y$

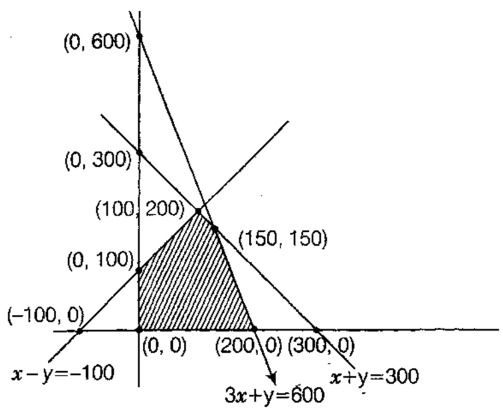
Subject to $x+y\le300, 3x+y\le600, x-y\ge-100, x\ge0, y\ge0.$

On solving $x+y=300$ and $3x+y=600,$ we get

$x=150,\ y=150$

On solving $x-y=-100$ *and* $x+y=300,$ we get

$x=100,\ y=200$



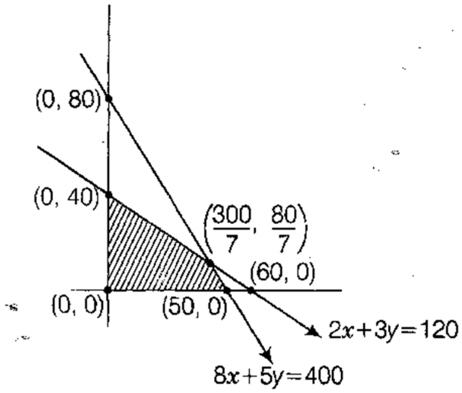
From the shaded feasible region it is clear that coordinate of corner points are $(0,0)$, $(200,0)$, $(150,150)$, $(100,200)$ and $(0,100)$.

Corner Points	Corresponding value of Z = 200x + 120y
(0, 0)	0
(200, 0)	40000
(150, 150)	$150\times200+120\times150=48000$ **(maximum)**
(100, 200)	$100\times200+120\times200=44000$
(0, 100)	$120\times100=12000$

Hence, 150 sweaters of each type made by company and maximum profit = Rs 48000.

20. Refer to Exercise 15. Determine the maximum distance that the man can travel.

Sol. Referring of solution 15, we have



Maximise $Z = x + y$, subject to

$2x + 3y \leq 120, 8x + 5y \leq 400, x \geq 0, y \geq 0$

On solving, we get

$8x + 5y = 400$ *and* $2x + 3y = 120$, we get

$x = \frac{300}{7}, y = \frac{80}{7}$

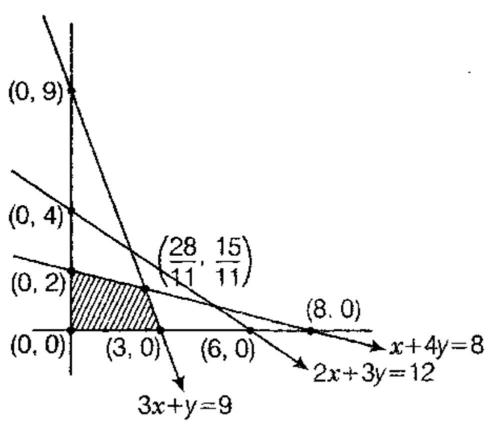
From the shaded feasible region, it is clear that coordinates of corner points are (0, 0), (50, 0), $\left(\frac{30}{7}, \frac{80}{7}\right)$ and (0, 40).

Corner Points	Corresponding value of Z = x + y
(0, 0)	0
(50, 0)	50
$\frac{300}{7}, \frac{80}{7}$	$\frac{380}{7} = 54\frac{2}{7}$km **(maximum)**
(0, 40)	40

Hence, the maximum distance that the man can travel is $54\frac{2}{7}$km.

21. Maximise $Z = x + y$ **subject to** $x + 4y \leq 8,\ 2x + 3y \leq 12,\ 3x + y \leq 9,\ x \geq 0,\ y \geq 0$**.**

Sol. Here, the given LPP is,



Maximise $Z = x + y$ subject to,

$x + 4y \le 8, 2x + 3y \le 12, 3x + y \le 9, x \ge 0, y \ge 0$

On solving $x + 4y = 8$ *and* $3x + y = 9$, we get

$x = \frac{28}{11}, y = \frac{15}{11}$

From the feasible region, it is clear that coordinates of corner points are

$(0,0)$, $(3,0)$, $\left(\frac{28}{11}, \frac{15}{11}\right)$ and $(0,2)$.

Corner Points	value of Z = x + y
(0, 0)	0
(3, 0)	3
$\left(\frac{28}{11}, \frac{15}{11}\right)$	$\frac{43}{11} = 3\frac{10}{11}$ **(maximum)**
(0, 2)	2

Hence, the maximum value of $3\frac{10}{11}$.

22. A manufacturer produces two Models of bikes - Model X and Model Y. Model X takes a 6 man-hours to make per unit, while Model Y takes 10 man-hours per unit. There is a total of 450 man-hour available per week. Handling and Marketing costs are Rs 2000 and Rs 1000 per unit for Models X and Y respectively. The total funds available for these purposes are Rs 80,000 per week. Profits per unit for Models X and Y are Rs 1000 and Rs 500, respectively. How many bikes of each model should the manufacturer produce so as to yield a maximum profit? Find the maximum profit.

Sol. Let the manufacturer produces x number of Models X and y number of model Y bikes. Model X takes 6 man-hours to make per unit and model Y takes 10 man-hours to make per unit.

There is total of 450 man-hours available per week.

$\therefore 6x + 10y \le 450$

$\Rightarrow 3x + 5y \le 225 \;\; ...(i)$

For model X and Y, handling and marketing costs are Rs 2000 and Rs 1000, respectively, total funds available for these purposes are Rs 80000 per week.

$\therefore 2000x + 1000y \le 80000$

$\Rightarrow 2x + y \le 80 \;\; ...(ii)$

Also, $x \ge 0, y \ge 0$

Hence, the profits per unit for models X and Y are Rs 1000 and Rs 500, respectively,

$\therefore$ Required LPP is

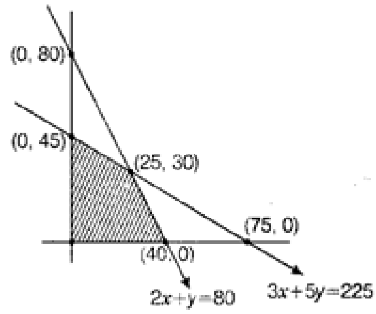
Maximise $Z = 1000x + 500y$

Subject to, $3x + 5y \le 225, 2x + y \le 80, x \ge 0, y \ge 0$

From the shaded feasible region, it is clear that coordinates of corner points are $(0,0)$, $(40,0)$, $(25,30)$ and $(0,45)$.

On solving $3x+5y=225$ *and* $2x+y=80$, we get

$x=25,\ y=30$



Corner Points	Value of Z = 1000x + 500y
(0, 0)	0
(40, 0)	40000 **(Maximum)**
(25, 30)	25000 + 15000 = 40000 **(Maximum)**
(0, 45)	22500

So, the manufacture should produce 25 bikes of model X and 30 bikes of model Y to get a maximum profit of Rs 40000.

Since, the question it is asked that each model bikes should be produced.

23. In order to supplement daily diet, a person wishes to take some X and some wishes Y tablets. The contents of iron, calcium and vitamins in X and Y (in milligrams per tablet) are given as below:

Tablets	**Iron**	**Calcium**	**Vitamin**
X	**6**	**3**	**2**
Y	**2**	**3**	**4**

The person needs at least 18 milligrams of iron, 21 milligrams of calcium and 16 milligram of vitamins. The price of each tablet of X and Y is Rs 2 and Re 1 respectively. How many tablets of each should the person take in order to satisfy the above requirement at the minimum cost?

Sol. Let the person takes x units of table X and y unit of table Y.

So, from the given information, we have

$6x+2y\geq 18 \Rightarrow 3x+y\geq 9 \ \ldots(i)$

$3x+3y\geq 21 \Rightarrow x+y\geq 7 \ \ldots(ii)$

And $2x+4y\geq 16 \Rightarrow x+2y\geq 8 \ \ldots(iii)$

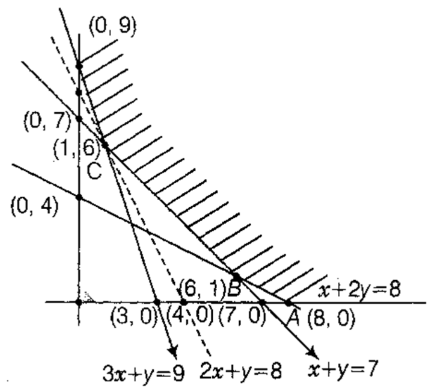
Also, we know that here, $x\geq 0, y\geq 0 \ \ldots(iv)$

The price of each tablet of X and Y is Rs 2 and Rs 1, respectively.

So, the corresponding LPP is minimise $Z = 2x + y$, subject to $3x + y \geq 9$, $x + y \geq 7$, $x + 2y \geq 8, x \geq 0, y \geq 0$

From the shaded graph, we see that for the shown unbounded region, we have coordinates of corner points A, B, C and D as $(8,0)$, $(6,1)$, $(1,6)$ *and* $(0,9)$, respectively.

[On solving $x + 2y = 8$ *and* $x + y = 7$, we get $x = 6$, $y = 1$ and on solving $3x + y = 9$ and $x + y = 7$, we get $x = 1$, $y = 6$]



Corner Points	Value of Z = 2x + y
(8, 0)	16
(6, 1)	13
(1, 6)	8 **(minimum)**
(0, 9)	9

Thus, we see that 8 is the minimum value of Z at the corner point (1, 6). Here we see that the feasible region is unbounded. Therefore, 8 may or may not be the minimum value of Z. To decide this issue, we graph the inequality

$2x + y < 8 \ \ldots(v)$

And check whether the resulting open half has points in common with feasible region or not. If it has common point, then 8 will not be the minimum value of Z, otherwise 8 will be the minimum value of Z.

Thus, from the graph it is clear that, it has no common point.

Therefore, $Z = 2x + y$ has 8 as minimum value subject to the given constraint.

Hence, the person should take 1 unit of X tablet and 6 unit of Y tablets to satisfy the given requirements and at the minimum cost of Rs 8.

24. A company makes 3 model of calculators: A, B and C at factory I and factory II. The company has orders for at least 6400 calculators of model A, 4000 calculator of model B and 4800 calculator of model C. At factory I, 50 calculators of model A, 50 of model B and 30 of model C are made every day; at factory II, 40 calculators of model A, 20 of model B and 40 of model C are made everyday. It costs Rs 12000 and Rs 15000 each day to operate factory I and II, respectively. Find the number of days each factory should operate to minimise the operating costs and still meet the demand.

Sol. Let the factory I operate for x days and the factory II operate for y days.

At factory I, 50 calculators of model A and at the factory II, 40 calculators of model A are made everyday. Also, company has ordered for at least 6400 calculators of model A.

$\therefore 50x+40y \geq 6400 \Rightarrow 5x+4y \geq 640 \ \ldots(i)$

Also, at factory I, 50 calculators of model B and at factory II, 20 calculators of model B are made everyday.

Since, the company has ordered at least 4000 calculators of model B.

$\therefore 50x+20y \geq 4000 \Rightarrow 5x+2y \geq 400 \ \ldots(ii)$

Similarly, for model C, $30x+40y \geq 4800$

$\Rightarrow 3x+4y \geq 480 \ \ldots(iii)$

Also, $x \geq 0, y \geq 0 \ \ldots(iv)$

[since, x and y are non-negative]

It costs Rs 12000 and Rs 15000 each day to operate factories factories I and II, respectively.

$\therefore$ Corresponding LPP is

Minimise $Z = 12000x + 15000y$, subject to

$5x+4y \geq 640$

$5x+2y \geq 400$

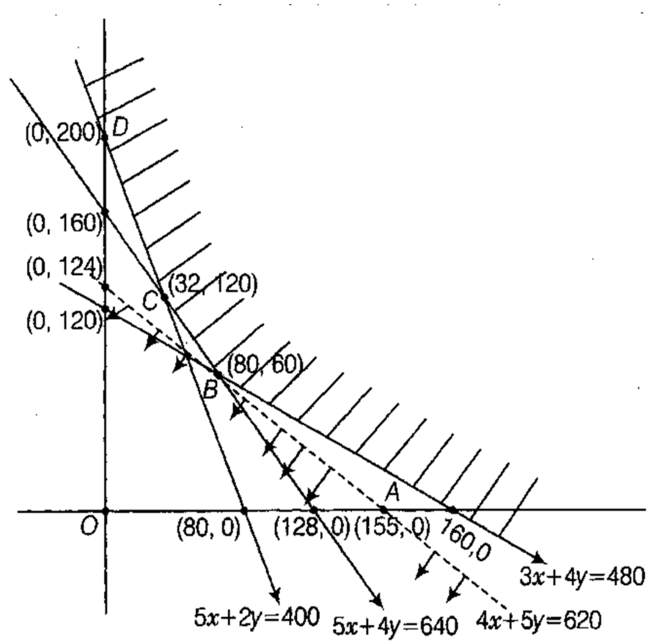
$3x+4y \geq 480$

$x \geq 0, y \geq 0$

On solving $3x+4y=480$ *and* $5x+4y=640$, we get $x=80$ *and*, $y=60$.

On solving $5x+4y=640$ and $5x+2y=400$, we get $x=32$, $y=120$

Thus, from the graph, it is clear that feasible region is unbounded and the coordinates of corner points A, B, C and D are $(160,0)$, $(80,60)$, $(32,120)$ and $(0,200)$, respectively.



Corner Points	Value of Z = 12000x + 15000y
(160, 0)	$160\times12000=1920000$
(80, 60)	$(80\times12+60\times15)\times1000=1860000$ **(minimum)**
(32, 120)	$(32\times12+120\times15)\times1000=2184000$
(0, 200)	$0+200\times15000=3000000$

From the above table, it is clear that for given unbounded region the minimum value of Z may or may not be 1860000.

Now, for deciding this, we graph the inequality

$12000x+15000y<1860000$

$\Rightarrow 4x+5y<620$

And check whether the resulting open half plane has point in common with feasible region or not.

Thus, as shown in the figure, it has no common points so, $Z=12000x+15000y$ has minimum value of 1860000.

So, number of days factor I should be operated is 80 and number of days factory II should be operate is 60 for the minimum cost and satisfying the given constraints.

25. Maximise and Minimise Z = 3x– 4y. subject to

$x-2y\le 0$

$-3x+y\le 4$

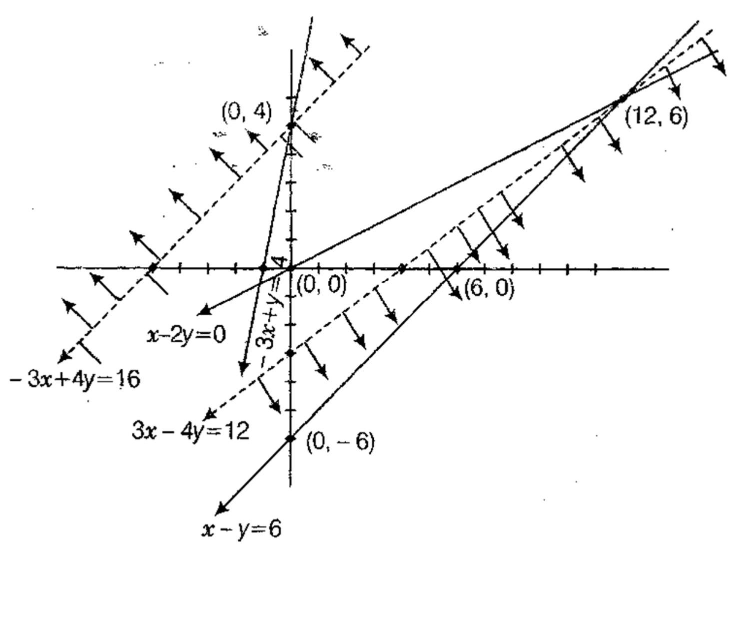
$x-y\le 6$

$x, y\ge 0$

Sol. Given LPP is

Maximise and minimise $Z=3x-4y$ subject to $x-2y\le 0, -3x+y\le 4,\ x-y\le 6,$ $x, y\ge 0.$

[On solving $x-y=6$ *and* $x-2y=0,$ we get $x=12,\ y=6$]



From the shown graph, for the feasible region, we see that it is unbounded and coordinates of corner points are $(0,0)$, $(12,6)$ *and* $(0,4)$.

Corner Points	Corresponding value of Z = 3x – 4y
(0, 0)	0
(0, 4)	-16 **(minimum)**
(12, 6)	12 **(maximum)**

For given unbounded region the minimum value of Z may or may not be –16. So, for deciding this, we graph the inequality.

$3x-4y<-16$

And check whether the resulting open half plane has common points with feasible region or not.

Thus, from the figures it shows it has common points with feasible region, So, it does not have any minimise value.

Also, similarly for maximum value, we graph the inequality $3x-4y>12$

And see that resulting open half plane has no common points with the feasible region and hence maximum value of 12 exits for $Z=3x-4y$.