# Ex 6.1

**Determinants Ex 6.1 Q1(i)**

Let $M_{ij}$ and $C_{ij}$ represents the minor and co-factor respectively of an element which is placed at the $i^{th}$ row and $j^{th}$ column.

Now,

$M_{11} = -1$

$M_{21} = 20$

$\begin{bmatrix} \text{In a } 2 \times 2 \text{ matrix, the minor is obtained for a particular element, by} \\ \text{deleting that row and column where the element is present.} \end{bmatrix}$

$C_{11} = (-1)^{1+1} \times M_{11}$      $\left[ \because C_{ij} = (-1)^{i+j} \times M_{ij} \right]$

$\quad = (+1)(-1)$

$\quad = -1$

$C_{21} = (-1)^{2+1} M_{21}$

$\quad = (-1)^3 \times 20$

$\quad = -20$

Also,

$|A| = 5(-1) - (0) \times (20)$      $\left[ \text{If } A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{then } |A| = a_{11}a_{22} - a_{21}a_{12} \right]$

$\quad = -5$

**Determinants Ex 6.1 Q1(ii)**

Let $M_{ij}$ and $C_{ij}$ represents the minor and co-factor respectively of an element which is present at the $i^{th}$ row and $j^{th}$ column.

Now,

$M_{11} = 3$ $\left[\begin{array}{l}\text{In a } 2 \times 2 \text{ matrix, the minor of an element is obtained by} \\ \text{deleting that row and that column in which it is present.}\end{array}\right]$

$M_{21} = 4$

$C_{11} = (-1)^{1+1} \times M_{11}$ $\left[C_{ij} = (-1)^{i+j} \times M_{ij}\right]$

$C_{21} = (-1)^{2+1} \times M_{21}$

$\quad = (-1)^3 \times 4$

$\quad = -4$

Also,

$|A| = (-1) \times (3) - (2) \times (4)$

$\quad = -3 - 8$

$\quad = -11$

## Determinants Ex 6.1 Q1(iii)

Let $M_{ij}$ and $C_{ij}$ represents the minor and co-factor respectively of an element which is placed at the $i^{th}$ row and $j^{th}$ column.

Now,

$M_{11} = \begin{bmatrix} -1 & 2 \\ 5 & 2 \end{bmatrix}$ $\left[\begin{array}{l}\text{In a } 3 \times 3 \text{ matrix, } M_{ij} \text{ equals to the determinant of the } 2 \times 2 \\ \text{sub-matrix obtained by leaving the } i^{th} \text{ row and } j^{th} \text{ column of } A.\end{array}\right]$

$\quad = (-1) \times (2) - (5) \times (2)$

$\quad = -2 - 10$

$\quad = -12$

$M_{21} = \begin{bmatrix} -3 & 2 \\ 5 & 2 \end{bmatrix} = (-3) \times (2) - (5) \times (2) = -6 - 10 = -16$

$M_{31} = \begin{bmatrix} -3 & 2 \\ -1 & 2 \end{bmatrix} = (-3)(2) - (-1)(2) = -6 + 2 = -4$

$C_{11} = (-1)^{1+1} M_{11}$ $\left(C_{ij} = (-1)^{i+j} \times M_{ij}\right)$

$\quad = (+)(-12) = -12$

$C_{21} = (-1)^{2+1} M_{21} = (-1)^3 (-16) = 16$

$C_{31} = (-1)^{3+1} M_{31} = (-1)^4 (-4) = -4$

Also, expanding the determinant along the first column.

$|A| = a_{11} \times \left((-1)^{1+1} \times M_{11}\right) + a_{21} \times \left((-1)^{2+1} \times M_{21}\right) + a_{31} \times \left((-1)^{3+1} \times M_{31}\right)$

$= a_{11} \times C_{11} + a_{21} \times C_{21} + a_{31} \times C_{31}$

$= 1 \times (-12) + 4 \times 16 + 3 \times (-4)$

$= -12 + 48 - 12 = 24$

## Determinants Ex 6.1 Q1(iv)

Let $M_{ij}$ and $C_{ij}$ are respectively the minor and co-factor of the element $a_{ij}$.

Now,

$$M_{11} = \begin{bmatrix} b & ca \\ c & ab \end{bmatrix}$$
$$= ab^2 - ac^2$$

$$M_{21} = \begin{bmatrix} a & bc \\ c & ab \end{bmatrix}$$
$$= a^2b - c^2b$$

$$M_{31} = \begin{bmatrix} a & bc \\ b & ca \end{bmatrix}$$
$$= a^2c - b^2c$$

$$C_{11} = (-1)^{1+1} \times M_{11} = +\left(ab^2 - ac^2\right)$$
$$C_{21} = (-1)^{2+1} \times M_{21} = -\left(a^2b - c^2b\right)$$
$$C_{31} = (-1)^{3+1} \times M_{31} = +\left(a^2c - b^2c\right)$$

Also, expanding the determinant, along the first column.

$$|A| = a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31}$$
$$= 1\left(ab^2 - ac^2\right) + 1\left(c^2b - a^2b\right) + 1 \times \left(a^2c - b^2c\right)$$
$$= ab^2 - ac^2 + c^2b - a^2b + a^2c - b^2c$$

## Determinants Ex 6.1 Q1(v)

Let $M_{ij}$ and $C_{ij}$ are respectively the minor and co-factor of the element $a_{ij}$.

Now,

$$M_{11} = \begin{bmatrix} 5 & 0 \\ 7 & 1 \end{bmatrix} = 5 - 0 = 5$$
$$M_{21} = \begin{bmatrix} 2 & 6 \\ 7 & 1 \end{bmatrix} = 2 - 42 = -40$$
$$M_{31} = \begin{bmatrix} 2 & 6 \\ 5 & 0 \end{bmatrix} = 0 - 30 = -30$$

$$C_{11} = (-1)^{1+1} \times M_{11} = +5$$
$$C_{21} = (-1)^{2+1} \times M_{21} = (-)(-40) = 40$$
$$C_{31} = (-1)^{3+1} \times M_{31} = +(-30) = -30$$

Now, expanding the determinant along the first column.

$$|A| = a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31}$$
$$= 0 \times 5 + 1 \times (40) + 3 \times (-30)$$
$$= 40 - 90$$
$$= -50$$

## Determinants Ex 6.1 Q1(vi)

Let $M_{ij}$ and $C_{ij}$ are respectively the minor and co-factor of the element $a_{ij}$.

Now,

$$M_{11} = \begin{bmatrix} b & f \\ f & c \end{bmatrix} = bc - f^2$$

$$M_{21} = \begin{bmatrix} h & g \\ f & c \end{bmatrix} = hc - gf$$

$$M_{31} = \begin{bmatrix} h & g \\ b & f \end{bmatrix} = hf - bg$$

Also $C_{11} = (-1)^{1+1} M_{11} = bc - f^2$

$C_{21} = (-1)^{2+1} M_{21} = -(hc - gf)$

$C_{31} = (-1)^{3+1} M_{31} = hf - bg$

Also, expanding along the first column.

$$|A| = a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31}$$
$$= a(bc - f^2) + h(-)(hc - gf) + g(hf - bg)$$
$$= abc - af^2 + hgf - h^2c + ghf - bg^2$$


## Determinants Ex 6.1 Q1(vii)

We have,

$$\begin{bmatrix} 2 & -1 & 0 & 1 \\ -3 & 0 & 1 & -2 \\ 1 & 1 & -1 & 1 \\ 2 & -1 & 5 & 0 \end{bmatrix}$$

Here, $M_{11} = \begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & 1 \\ -1 & 5 & 0 \end{bmatrix} = -1(0 + 10) - 1(1 - 2) = -9$

$$M_{21} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 1 \\ -1 & 5 & 0 \end{bmatrix} = 9$$

$$M_{31} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -2 \\ -1 & 5 & 0 \end{bmatrix} = -9$$

$$M_{41} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & - & 1 \end{bmatrix} = 0$$

$\therefore$    $C_{11} = (-1)^{1+1} M_{11} = -9$

$C_{21} = (-1)^3 M_{21} = -9$

$C_{31} = (-4)^4 M_{31} = -9$

$C_{41} = (-1)^5 M_{41} = 0$

Hence,

$$\begin{bmatrix} 2 & -1 & 0 & 1 \\ -3 & 0 & 1 & -2 \\ 1 & 1 & -1 & 1 \\ 2 & -1 & 5 & 0 \end{bmatrix} = 2 \times C_{11} + (-3)C_{21} + 1 \times C_{31} + 2 \times C_{41} = -9[2 - 3 + 1] = 0$$


## Determinants Ex 6.1 Q2(i)

Let $A = \begin{vmatrix} x & -7 \\ x & 5x + 1 \end{vmatrix}$

$|A| = x(5x + 1) + 7 \times x$
$= 5x^2 + x + 7x$
$= 5x^2 + 8x$

Hence $|A| = 5x^2 + 8x$

**Determinants Ex 6.1 Q2(ii)**

Let $A = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

$|A| = \cos\theta \times \cos\theta + \sin\theta \times \sin\theta$

$\quad = \cos^2\theta + \sin^2\theta$

$\quad = 1$

Hence $|A| = 1$

**Determinants Ex 6.1 Q2(iii)**

Let $A = \begin{vmatrix} \cos 15° & \sin 15° \\ \sin 75° & \cos 75° \end{vmatrix}$

$|A| = \cos 15° \cos 75° - \sin 15° \sin 75°$

$\quad = \cos(75 + 15)$ $\qquad\qquad (\because \cos A \cos B - \sin A \sin B = \cos(A + B))$

$\quad = \cos 90°$

$\quad = 0$

Hence $|A| = 0$

**Determinants Ex 6.1 Q2(iv)**

Let $A = \begin{vmatrix} a + ib & c + id \\ -c + id & a - ib \end{vmatrix}$

$|A| = (a + ib)(a - ib) - (c + id)(-c + id)$

$\quad = (a^2 + b^2) + (c + id)(c - id)$ $\qquad (\text{Taking }(-)\text{ sign common from } -c + id)$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad (\text{Also }(a + ib)(a - ib) = a^2 + b^2)$

$= a^2 + b^2 + c^2 + d^2$

Hence $|A| = a^2 + b^2 + c^2 + d^2$

**Determinants Ex 6.1 Q3**

Since $|AB| = |A| \times |B|$

Hence $|A|^2 = |A| \times |A|$ $\qquad\qquad - - - (1)$

Now let $A = \begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 15 & 20 & 12 \end{vmatrix}$

Expanding along the first column, we get

$|A| = 2\begin{vmatrix} 17 & 5 \\ 20 & 12 \end{vmatrix} - 3\begin{vmatrix} 13 & 5 \\ 15 & 12 \end{vmatrix} + 7\begin{vmatrix} 13 & 17 \\ 15 & 20 \end{vmatrix}$

$\quad = 2(204 - 100) - 3(156 - 75) + 7(260 - 255)$

$\quad = 2(104) - 3(81) + 7(5)$

$\quad = 208 - 243 + 35$

$\quad = 243 - 243$

$\quad = 0$

Hence from eq.(1)

$|A|^2 = |A| \times |A| = 0 \times 0 = 0$

**Determinants Ex 6.1 Q4**

Evaluating the given determinant

$\sin 10° \times \cos 80° + \cos 10° \sin 80°$

$= \sin\left(10° + 80°\right)$ $\qquad$ $\left[\because \sin A \cos B + \cos A \sin B = \sin\left(A + B\right)\right]$

$= \sin 90°$

$= 1$

Hence proved

### Determinants Ex 6.1 Q5

We will evaluate the given determinant
(i) Along the first row
(ii) Along the first column

(i) Along the first row

$$|A| = 2\begin{vmatrix} 1 & -2 \\ 4 & 1 \end{vmatrix} - 3\begin{vmatrix} 7 & -2 \\ -3 & 1 \end{vmatrix} - 5\begin{vmatrix} 7 & 1 \\ -3 & 4 \end{vmatrix}$$

$$= 2(1+8) - 3(7-6) - 5(28+3)$$

$$= 2(9) - 3(1) - 5(31)$$

$$= 18 - 3 - 155 = -140$$

(ii) Along the first column

$$|A| = 2\begin{vmatrix} 1 & -2 \\ 4 & 1 \end{vmatrix} - 7\begin{vmatrix} 3 & -5 \\ 4 & 1 \end{vmatrix} - 3\begin{vmatrix} 3 & -5 \\ 1 & -2 \end{vmatrix}$$

$$= 2(1+8) - 7(3+20) - 3(-6+5)$$

$$= 18 - 7(23) - 3(-1)$$

$$= 18 - 161 + 3$$

$$= 21 - 161$$

$$= -140$$

We can see, the answer is same with both the methods.

### Determinants Ex 6.1 Q6

$$\Delta = \begin{vmatrix} 0 & \sin\alpha & -\cos\alpha \\ -\sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{vmatrix}$$

$$= -\sin\alpha(-\sin\beta\cos\alpha) - \cos\alpha(\sin\alpha\sin\beta)$$

$$= 0$$

### Determinants Ex 6.1 Q7

$$\Delta = \begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$$

Expanding along $C_3$, we have:

$$\Delta = -\sin\alpha\left(-\sin\alpha\sin^2\beta - \cos^2\beta\sin\alpha\right) + \cos\alpha\left(\cos\alpha\cos^2\beta + \cos\alpha\sin^2\beta\right)$$

$$= \sin^2\alpha\left(\sin^2\beta + \cos^2\beta\right) + \cos^2\alpha\left(\cos^2\beta + \sin^2\beta\right)$$

$$= \sin^2\alpha(1) + \cos^2\alpha(1)$$

$$= 1$$

### Determinants Ex 6.1 Q8

Let $A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$

$\Rightarrow |A| = 2 - 10 = -8$

$B = \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$

$\Rightarrow |B| = 20 + 6 = 26$

Now $AB = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$

$= \begin{bmatrix} 2 \times 4 + 5 \times 2 & 2 \times (-3) + 5 \times 5 \\ 2 \times 4 + 1 \times 2 & 2 \times (-3) + 1 \times 5 \end{bmatrix}$

$= \begin{bmatrix} 8 + 10 & -6 + 25 \\ 8 + 2 & -6 + 5 \end{bmatrix}$

$= \begin{bmatrix} 18 & 19 \\ 10 & -1 \end{bmatrix}$

$\Rightarrow |AB| = 18 \times (-1) - (10)(19)$

$\qquad = -18 - 190 = -208$

Now $|AB| = |A| \times |B|$

$\qquad -208 = (-8) \times (26)$

$\qquad -208 = -208$

Hence verified.

## Determinants Ex 6.1 Q9

Let $A = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$

Evaluating the determinant along the first column

$|A| = 1\begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 \\ 0 & 4 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix}$

$\quad = 1 \times (4 - 0) - 0 + 0$

$\quad = 4$

Again $3A = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$ (every element of A will be multiplied by 3)

Now, evaluating this determinant

$|3A| = 3\begin{vmatrix} 3 & 6 \\ 0 & 12 \end{vmatrix} - 0\begin{vmatrix} 0 & 3 \\ 0 & 12 \end{vmatrix} + 0\begin{vmatrix} 0 & 3 \\ 3 & 6 \end{vmatrix}$

$\quad = 3(36 - 0) - 0 + 0$

$\quad = 108$

Now, according to the question

$|3A| = 27|A|$

$108 = 27(4)$ (Substituting values)

$108 = 108$

Hence proved

## Determinants Ex 6.1 Q10

(i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

$\Rightarrow 2 \times 1 - 5 \times 4 = 2x \times x - 6 \times 4$

$\Rightarrow 2 - 20 = 2x^2 - 24$

$\Rightarrow 2x^2 = 6$

$\Rightarrow x^2 = 3$

$\Rightarrow x = \pm\sqrt{3}$

(ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

$\Rightarrow 2 \times 5 - 3 \times 4 = x \times 5 - 3 \times 2x$

$\Rightarrow 10 - 12 = 5x - 6x$

$\Rightarrow -2 = -x$

$\Rightarrow x = 2$

(iii)

$\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$

$3 - x^2 = 3 - 8$

$x^2 = 8$

$x = \pm 2\sqrt{2}$

(iv)

$\begin{vmatrix} 3x & 7 \\ 2 & 4 \end{vmatrix} = 10$

$12x - 14 = 10$

$12x = 24$

$x = 2$

**Determinants Ex 6.1 Q11**

Let $A = \begin{vmatrix} x^2 & x & 1 \\ 0 & 2 & 1 \\ 3 & 1 & 4 \end{vmatrix}$

Expanding the given determinant along the first column

$|A| = x^2 \begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix} - 0 \begin{vmatrix} x & 1 \\ 1 & 4 \end{vmatrix} + 3 \begin{vmatrix} x & 1 \\ 2 & 1 \end{vmatrix}$

$28 = x^2(8 - 1) - 0(4x - 1) + 3(x - 2)$

$28 = 7x^2 + 3x - 6$

or

$7x^2 + 3x - 6 = 28$

$7x^2 + 3x - 34 = 0$

Solving using quadratic formula, we get $x = 2$.

**Determinants Ex 6.1 Q12(i)**

A matrix A is called singular if $|A| = 0$

Now expanding along the first row $|A|$

$$= (x-1)\begin{vmatrix} x-1 & 1 \\ 1 & x-1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & x-1 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ x-1 & 1 \end{vmatrix}$$

$$= (x-1)\left[(x-1)^2 - 1\right] - 1\left[x-1-1\right] + 1\left[1 - x + 1\right]$$

$$= (x-1)\left(x^2 + 1 - 2x - 1\right) - 1(x-2) + 1(2-x)$$

$$= (x-1)\left(x^2 - 2x\right) - x + 2 + 2 - x$$

$$= (x-1) \times x \times (x-2) + (4 - 2x)$$

$$= (x-1) \times x \times (x-2) + 2(2-x)$$

$$= (x-1) \times x \times (x-2) - 2(x-2)$$

$$= (x-2)\left[x(x-1) - 2\right] \qquad \qquad \left(\text{Taking } (x-2) \text{ common}\right)$$

Since A is a singular matrix, so $|A| = 0$

$i.e\ (x-2)\left(x^2 - x - 2\right) = 0$

either $(x-2) = 0$       or $x^2 - x - 2 = 0$

         $x = 2$         or $x^2 - 2x + x - 2 = 0$

                $x(x-2) + 1(x-2) = 0$

                   $(x-2)(x+1) = 0$

                     $x = 2, -1$

$x = 2$ or $-1$

### Determinants Ex 6.1 Q12(ii)

A matrix A is said to be singular if $|A| = 0$

Now

$$\begin{vmatrix} 1+x & 7 \\ 3-x & 8 \end{vmatrix} = 0$$

$$8 + 8x - 21 + 7x = 0$$

$$15x = 13$$

$$x = \frac{13}{15}$$

# Ex 6.2

**Chapter 6 Determinants Ex 6.2 Q1-i**

$$\begin{vmatrix} 1 & 3 & 5 \\ 2 & 6 & 10 \\ 31 & 11 & 38 \end{vmatrix} = 2 \begin{vmatrix} 1 & 3 & 5 \\ 1 & 3 & 5 \\ 31 & 11 & 38 \end{vmatrix} = 0$$

**Chapter 6 Determinants Ex 6.2 Q1-ii**

*Consider the determinant*

$$\Delta = \begin{vmatrix} 67 & 19 & 21 \\ 39 & 13 & 14 \\ 81 & 24 & 26 \end{vmatrix}$$

*Applying* $C_1 \rightarrow C_1 - 4C_3$, *we get,*

$$\Delta = \begin{vmatrix} 4 & 19 & 21 \\ -3 & 13 & 14 \\ -3 & 24 & 26 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 4 & 19 & 21 \\ -3 & 13 & 14 \\ -3 & 24 & 26 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 32 & 35 \\ -3 & 13 & 14 \\ 0 & 11 & 12 \end{vmatrix} \quad [Applying \ R_3 \rightarrow R_3 - R_2 \ and \ R_1 \rightarrow R_1 + R_2]$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 32 & 35 \\ 0 & 109 & 119 \\ 0 & 11 & 12 \end{vmatrix} \quad [Applying \ R_2 \rightarrow 3R_1 + R_2]$$

$$\Rightarrow \Delta = 1(109 \times 12 - 119 \times 11)$$

$$\Rightarrow \Delta = -1$$

## Chapter 6 Determinants Ex 6.2 Q1-iii

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

$= a(bc - f^2) - h(hc - fg) + g(hf - bg)$

$= abc - af^2 - h^2c + hfg + ghf - bg^2$

## Chapter 6 Determinants Ex 6.2 Q1-iv

$$\begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix} = 2\begin{vmatrix} 1 & -3 & 1 \\ 4 & -1 & 1 \\ 3 & 5 & 1 \end{vmatrix} = 2\begin{vmatrix} 1 & -3 & 1 \\ 3 & 2 & 0 \\ 2 & 8 & 0 \end{vmatrix} = 2(24 - 4) = 40$$

## Chapter 6 Determinants Ex 6.2 Q1-v

*Let $\Delta$ be the determinant.*

$$\Delta = \begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$$

*Applying $R_3 \rightarrow R_3 - R_2$, we get,*

$$\Delta = \begin{vmatrix} 1 & 4 & 9-4 \\ 4 & 9 & 16-9 \\ 9 & 16 & 25-16 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 4 & 5 \\ 4 & 9 & 7 \\ 9 & 16 & 9 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 5 & 5 \\ 4 & 13 & 7 \\ 9 & 25 & 9 \end{vmatrix} \qquad [Applying\ C_2 \rightarrow C_1 + C_2]$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 0 & 0 \\ 4 & -7 & -13 \\ 9 & -20 & -36 \end{vmatrix} \qquad [Applying\ C_2 \rightarrow 5C_1 - C_2\ and\ C_3 \rightarrow 5C_1 - C_3]$$

$$\Rightarrow \Delta = 1(7 \times 36 - 13 \times 20) = 252 - 260 = -8$$

**Chapter 6 Determinants Ex 6.2 Q1-vi**

$$\begin{vmatrix} 6 & -3 & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{vmatrix}$$

Apply: $R_1 \to R_1 + (-3)R_2$ and $R_3 \to R_3 + 5R_2$

$$= \begin{vmatrix} 0 & 0 & -4 \\ 2 & -1 & 2 \\ 0 & 0 & 12 \end{vmatrix} = 0$$

**Chapter 6 Determinants Ex 6.2 Q1-vii**

$$\begin{vmatrix} 1 & 3 & 9 & 27 \\ 3 & 9 & 27 & 1 \\ 9 & 27 & 1 & 3 \\ 27 & 1 & 3 & 9 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 3^2 & 3^3 \\ 3 & 3^2 & 3^3 & 1 \\ 3^2 & 3^3 & 1 & 3 \\ 3^3 & 1 & 3 & 3^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1+3+3^2+3^3 & 3 & 3^2 & 3^3 \\ 1+3+3^2+3^3 & 3^2 & 3^3 & 1 \\ 1+3+3^2+3^3 & 3^3 & 1 & 3 \\ 1+3+3^2+3^3 & 1 & 3 & 3^2 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right)\begin{vmatrix} 1 & 3 & 3^2 & 3^3 \\ 1 & 3^2 & 3^3 & 1 \\ 1 & 3^3 & 1 & 3 \\ 1 & 1 & 3 & 3^2 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right)\begin{vmatrix} 1 & 3 & 3^2 & 3^3 \\ 0 & 3^2-3 & 3^3-3^2 & 1-3^3 \\ 0 & 3^3-3 & 1-3^2 & 3-3^3 \\ 0 & 1-3 & 3-3^2 & 3^2-3^3 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right)\begin{vmatrix} 6 & 18 & -26 \\ 24 & -8 & -24 \\ -2 & -6 & -18 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right)2^3\begin{vmatrix} 3 & -9 & 13 \\ 12 & 4 & 12 \\ -1 & 3 & 9 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right)2^3\begin{vmatrix} 0 & 0 & 40 \\ 12 & 4 & 12 \\ -1 & 3 & 9 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right)2^3 \times 40(36+4) = 512000$$

**Chapter 6 Determinants Ex 6.2 Q1-viii**

Let $\Delta = \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$

Applying $R_3 \to 17R_2 - R_3$, we get,

$$\Delta = \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 0 & 48 & 62 \end{vmatrix}$$

Applying $R_2 \to 102R_2 - R_1$, we get,

$$\Delta = \begin{vmatrix} 102 & 18 & 36 \\ 0 & 288 & 372 \\ 0 & 48 & 62 \end{vmatrix}$$

Thus,

$\Delta = 102(288 \times 62 - 372 \times 48)$

$\Rightarrow \Delta = 0$

**Chapter 6 Determinants Ex 6.2 Q2(i)**

$$\begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 16 & 4 & 3 \end{vmatrix}$$

Apply: $R_3 \to R_3 - R_2$

$$= \begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 4 & 1 & -2 \end{vmatrix}$$

Apply: $R_2 \to R_2 - R_1$

$$= \begin{vmatrix} 8 & 2 & 7 \\ 4 & 1 & -2 \\ 4 & 1 & -2 \end{vmatrix}$$

Since, $R_3 = R_2$, the value of the determinant is zero.

## Chapter 6 Determinants Ex 6.2 Q2(ii)

$$\begin{vmatrix} 6 & -3 & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{vmatrix}$$

Taking $(-2)$ common from $C_1$, we get

$$= (-2)\begin{vmatrix} -3 & -3 & 2 \\ -1 & -1 & 2 \\ 5 & 5 & 2 \end{vmatrix}$$

$$= 0$$

$\because C_1$ and $C_2$ are identical.

## Chapter 6 Determinants Ex 6.2 Q2(iii)

$$\begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 15 & 20 & 12 \end{vmatrix}$$

Use: $R_3 \to R_3 - R_2$

$$= \begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 2 & 3 & 7 \end{vmatrix}$$

$$= 0$$

$\because R_3 = R_1$

## Chapter 6 Determinants Ex 6.2 Q2(iv)

$$\begin{vmatrix} \dfrac{1}{a} & a^2 & bc \\ \dfrac{1}{b} & b^2 & ca \\ \dfrac{1}{c} & c^2 & ab \end{vmatrix}$$

Multiply: $R_1, R_2$ and $R_3$ by $a, b$ and $c$ respectively, we get

$$= \frac{1}{abc}\begin{vmatrix} 1 & a^3 & abc \\ 1 & b^3 & bca \\ 1 & c^3 & cab \end{vmatrix}$$

Take $abc$ common from $C_3$, we get,

$$= \begin{vmatrix} 1 & a^3 & 1 \\ 1 & b^3 & 1 \\ 1 & c^3 & 1 \end{vmatrix}$$

$$= 0$$

$\because \qquad C_1 = C_3$

## Chapter 6 Determinants Ex 6.2 Q2(v)

$$\begin{vmatrix} a+b & 2a+b & 3a+b \\ 2a+b & 3a+b & 4a+b \\ 4a+b & 5a+b & 6a+b \end{vmatrix}$$

Apply: $C_3 \to C_3 - C_2$

$$= \begin{vmatrix} a+b & 2a+b & a \\ 2a+b & 3a+b & a \\ 4a+b & 5a+b & a \end{vmatrix}$$

Apply: $C_2 \to C_2 - C_1$

$$= \begin{vmatrix} a+b & a & a \\ 2a+b & a & a \\ 4a+b & a & a \end{vmatrix}$$

$= 0$

$\because \qquad C_3 = C_2$

## Chapter 6 Determinants Ex 6.2 Q2(vi)

$$\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ac \\ 1 & c & c^2 - ab \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ac \\ 1 & c & ab \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2 - a^2 \\ 0 & c-a & c^2 - a^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 0 & b-a & (a-b)c \\ 0 & c-a & (a-c)b \end{vmatrix}$$

$$= (b-a)(c-a)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix} - (b-a)(c-a)\begin{vmatrix} 1 & a & bc \\ 0 & 1 & -c \\ 0 & 1 & -b \end{vmatrix}$$

$= (b-a)(c-a)(c+a-b-a) - (b-a)(c-a)(-b+c)$

$= (b-a)(c-a)(c-b) - (b-a)(c-a)(-b+c)$

$= 0$

## Chapter 6 Determinants Ex 6.2 Q2(vii)

$$\begin{vmatrix} 49 & 1 & 6 \\ 39 & 7 & 4 \\ 26 & 2 & 3 \end{vmatrix}$$

Apply: $C_1 \to C_1 + (-8)C_3$

$$= \begin{vmatrix} 1 & 1 & 6 \\ 7 & 7 & 4 \\ 2 & 2 & 3 \end{vmatrix} = 0$$

$\because \qquad C_1 = C_2$

## Chapter 6 Determinants Ex 6.2 Q2(viii)

$$\begin{vmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{vmatrix}$$

Multiply $C_1$, $C_2$ and $C_3$ by z, y, and x respectively

$$= \frac{1}{xyz}\begin{vmatrix} 0 & xy & yx \\ -xz & 0 & zx \\ -yz & -zy & 0 \end{vmatrix}$$

Take y, x, and z common from $R_1$, $R_2$ and $R_3$ respectively

$$= \begin{vmatrix} 0 & x & x \\ -z & 0 & z \\ -y & -y & 0 \end{vmatrix}$$

Apply: $C_2 \to C_2 - C_3$

$$= \begin{vmatrix} 0 & 0 & x \\ -z & -z & z \\ -y & -y & 0 \end{vmatrix}$$

$= 0$

$\because C_1 = C_2$

## Chapter 6 Determinants Ex 6.2 Q2(ix)

$$\begin{vmatrix} 1 & 43 & 6 \\ 7 & 35 & 4 \\ 3 & 17 & 2 \end{vmatrix}$$

Apply: $C_2 \to C_2 + (-7)C_3$

$$= \begin{vmatrix} 1 & 1 & 6 \\ 7 & 7 & 4 \\ 3 & 3 & 2 \end{vmatrix}$$

$$= 0$$

$\because \quad C_1 = C_2$

## Chapter 6 Determinants Ex 6.2 Q2(x)

$$\begin{vmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{vmatrix}$$

Apply: $C3 \to C3 - C2, C4 \to C4 - C1$

$$= \begin{vmatrix} 1^2 & 2^2 & 3^2-2^2 & 4^2-1^2 \\ 2^2 & 3^2 & 4^2-3^2 & 5^2-2^2 \\ 3^2 & 4^2 & 5^2-4^2 & 6^2-3^2 \\ 4^2 & 5^2 & 6^2-5^2 & 7^2-4^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1^2 & 2^2 & 5 & 15 \\ 2^2 & 3^2 & 7 & 21 \\ 3^2 & 4^2 & 9 & 27 \\ 4^2 & 5^2 & 11 & 33 \end{vmatrix}$$

Take 3 common from $C_4$

$$= 3\begin{vmatrix} 1^2 & 2^2 & 5 & 5 \\ 2^2 & 3^2 & 7 & 7 \\ 3^2 & 4^2 & 9 & 9 \\ 4^2 & 5^2 & 11 & 11 \end{vmatrix}$$

$$= 0$$

$\because \quad C_3 = C_4$

## Chapter 6 Determinants Ex 6.2 Q2(xi)

$$\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ 2a+2x & 2b+2y & 2c+2z \\ x+a & y+b & z+c \end{vmatrix}$$

$$= 2\begin{vmatrix} a & b & c \\ a+x & b+y & c+z \\ x+a & y+b & z+c \end{vmatrix}$$

$$= 0$$

## Chapter 6 Determinants Ex 6.2 Q3

$$\begin{vmatrix} a & b+c & a^2 \\ b & c+a & b^2 \\ c & a+b & c^2 \end{vmatrix}$$

Apply: $C_2 \to C_2 + C_1$.

$$= \begin{vmatrix} a & b+c+a & a^2 \\ b & c+a+b & b^2 \\ c & a+b+c & c^2 \end{vmatrix}$$

Take $(a+b+c)$ common from $C_2$

$$= (b+c+a)\begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix}$$

Apply: $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$= (b+c+a)\begin{vmatrix} a & 1 & a^2 \\ b-a & 0 & b^2-a^2 \\ c-a & 0 & c^2-a^2 \end{vmatrix}$$

$$= (b+c+a)(b-a)(c-a)\begin{vmatrix} a & 1 & a^2 \\ 1 & 0 & b+a \\ 1 & 0 & c+a \end{vmatrix}$$

$$= (b+c+a)(b-a)(c-a)(b-c)$$

**Chapter 6 Determinants Ex 6.2 Q4**

Let $\Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$ we get,

$$\Delta = \begin{vmatrix} 1 & a & bc \\ 0 & b-a & ca-bc \\ 0 & c-a & ab-ba \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & a & bc \\ 0 & b-a & c(a-b) \\ 0 & c-a & b(a-c) \end{vmatrix}$$

Taking $(a-b)$ and $(a-c)$ common, we have

$$\Delta = (a-b)(a-c)\begin{vmatrix} 1 & a & bc \\ 0 & -1 & c \\ 0 & -1 & b \end{vmatrix}$$

$$\Rightarrow \Delta = (a-b)(c-a)(b-c)$$

**Chapter 6 Determinants Ex 6.2 Q5**

Let $\Delta = \begin{vmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$ we get,

$$\Delta = \begin{vmatrix} 3x+\lambda & x & x \\ 3x+\lambda & x+\lambda & x \\ 3x+\lambda & x & x+\lambda \end{vmatrix}$$

Taking $(3x+\lambda)$ common, we have

$$\Delta = (3x+\lambda)\begin{vmatrix} 1 & x & x \\ 1 & x+\lambda & x \\ 1 & x & x+\lambda \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$, we get,

$$\Delta = (3x+\lambda)\begin{vmatrix} 1 & x & x \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix}$$

$$\Rightarrow \Delta = \lambda^2(3x+\lambda)$$

**Chapter 6 Determinants Ex 6.2 Q6**

Let $\Delta = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$ we get,

$\Delta = \begin{vmatrix} a+b+c & b & c \\ a+b+c & a & b \\ a+b+c & c & a \end{vmatrix}$

Taking $(a+b+c)$ common, we have

$\Delta = (a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & a & b \\ 1 & c & a \end{vmatrix}$

Applying $R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - R_1$, we get,

$\Delta = (a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & a-b & b-c \\ 0 & c-b & a-c \end{vmatrix}$

$\Rightarrow \Delta = (a+b+c)[(a-b)(a-c) - (b-c)(c-b)]$

$\Rightarrow \Delta = (a+b+c)[a^2 - ac - ab + bc + b^2 + c^2 - 2bc]$

$\Rightarrow \Delta = (a+b+c)[a^2 + b^2 + c^2 - ac - ab - bc]$

**Chapter 6 Determinants Ex 6.2 Q7**

$\begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix} = \begin{vmatrix} 2+x & 1 & 1 \\ 2+x & x & 1 \\ 2+x & 1 & x \end{vmatrix} = (2+x)\begin{vmatrix} 1 & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$

$= (2+x)\begin{vmatrix} 1 & 1 & 1 \\ 0 & x-1 & 0 \\ 0 & 0 & x-1 \end{vmatrix}$

$= (2+x)(x-1)^2$

**Chapter 6 Determinants Ex 6.2 Q8**

$\begin{vmatrix} 0 & xy^2 & xz^2 \\ x^2y & 0 & yz^2 \\ x^2z & zy^2 & 0 \end{vmatrix}$

$= 0(0 - y^3z^3) - xy^2(0 - x^2yz^3) + xz^2(x^2y^3z - 0)$

$= 0 + x^3y^3z^3 + x^3y^3z^3$

$= 2x^3y^3z^3$

**Chapter 6 Determinants Ex 6.2 Q9**

Let $\Delta = \begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$

Applying $R_1 \rightarrow R_1 - R_2$ and $R_3 \rightarrow R_3 - R_2$

$\Delta = \begin{vmatrix} a & -a & 0 \\ x & a+y & z \\ 0 & -a & a \end{vmatrix}$

Applying $C_2 \rightarrow C_2 + C_1$

$\Delta = \begin{vmatrix} a & 0 & 0 \\ x & a+x+y & z \\ 0 & -a & a \end{vmatrix}$

$\Delta = a[a(a+x+y) + az] + 0 + 0$

$\Delta = a^2(a+x+y+z)$

**Chapter 6 Determinants Ex 6.2 Q10**

$$\Delta + \Delta_1 = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ yz & zx & xy \\ x & y & z \end{vmatrix}$$

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + \begin{vmatrix} 1 & yz & x \\ 1 & zx & y \\ 1 & xy & z \end{vmatrix} \quad \ldots\ldots\ldots \left[ \because |A| = |A^T| \right]$$

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} - \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$$

$$\ldots\ldots \left[ \begin{matrix} \text{If any two rows (columns) of the determinant are interchanged} \\ \text{then value of the determinant changes in sign.} \end{matrix} \right]$$

$$= \begin{vmatrix} 0 & 0 & x^2 - yz \\ 0 & 0 & y^2 - zx \\ 0 & 0 & z^2 - xy \end{vmatrix}$$

$$= 0 \ldots\ldots\ldots \left[ \because C_1 \text{ and } C_2 \text{ are identical} \right]$$

**Chapter 6 Determinants Ex 6.2 Q11**

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$$

$$\text{LHS} = \begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$$

Apply: $C_1 \to C_1 + C_2 + C_3$.

$$= \begin{vmatrix} a+b+c & b & c \\ 0 & b-c & c-a \\ 2(a+b+c) & c+a & a+b \end{vmatrix}$$

Take $(a+b+c)$ common from $C_1$

$$= (a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 2 & c+a & a+b \end{vmatrix}$$

Apply: $R_3 \to R_3 - 2R_1$

$$= (a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 0 & c+a-2b & a+b-2c \end{vmatrix}$$

$$= (a+b+c) \left[ (b-c)(a+b-2c) - (c-a)(c+a-2b) \right]$$

$$= a^3 + b^3 + c^3 - 3abc$$

$$= \text{RHS}$$

**Chapter 6 Determinants Ex 6.2 Q12**

$$\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$$

$$\text{LHS} = \begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$$

$$= \begin{vmatrix} b+c+a & -b & a \\ c+a+b & -c & b \\ a+b+c & -a & c \end{vmatrix}$$

$$= -(b+c+a) \begin{vmatrix} 1 & b & a \\ 1 & c & b \\ 1 & a & c \end{vmatrix}$$

$$= -(b+c+a) \begin{vmatrix} 1 & b & a \\ 0 & c-b & b-a \\ 0 & a-b & c-a \end{vmatrix}$$

$$= -(b+c+a) \left[ (c-b)(c-a) - (b-a)(a-b) \right]$$

$$= 3abc - a^3 - b^3 - c^3$$

$$= RHS$$

## Chapter 6 Determinants Ex 6.2 Q13

$$\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$\text{LHS} = \begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix}$$

Apply: $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2(a+b+c) & b+c & c+a \\ 2(a+b+c) & c+a & a+b \\ 2(a+b+c) & a+b & b+c \end{vmatrix}$$

$$= 2 \begin{vmatrix} a+b+c & b+c & c+a \\ a+b+c & c+a & a+b \\ a+b+c & a+b & b+c \end{vmatrix}$$

Apply: $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$$= 2 \begin{vmatrix} a+b+c & -a & -b \\ a+b+c & -b & -c \\ a+b+c & -c & -a \end{vmatrix}$$

$$= 2 \begin{vmatrix} a+b+c & a & b \\ a+b+c & b & c \\ a+b+c & c & a \end{vmatrix}$$

$$= 2 \left( \begin{vmatrix} c & a & b \\ a & b & c \\ b & c & a \end{vmatrix} + \begin{vmatrix} a & a & b \\ b & b & c \\ c & c & a \end{vmatrix} + \begin{vmatrix} b & a & b \\ c & b & c \\ a & c & a \end{vmatrix} \right)$$

$$= 2 \begin{vmatrix} c & a & b \\ a & b & c \\ b & c & a \end{vmatrix}$$

$$= 2 \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= RHS$$

## Chapter 6 Determinants Ex 6.2 Q14

We need to prove the following identity:

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$, we have,

$$L.H.S = \begin{vmatrix} 2a+2b+2c & a & b \\ 2a+2b+2c & b+c+2a & b \\ 2a+2b+2c & a & c+a+2b \end{vmatrix}$$

Taking the term $2a + 2b + 2$ as common, we have

$$L.H.S = (2a+2b+2c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

$$\Rightarrow L.H.S = 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$ we have

$$L.H.S = 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 0 & a+b+c & 0 \\ 0 & 0 & a+b+c \end{vmatrix}$$

Thus, we have,

$$L.H.S = 2(a+b+c)\left[1 \times (a+b+c)^2\right]$$
$$= 2(a+b+c)(a+b+c)^2$$
$$= 2(a+b+c)^3$$

**Chapter 6 Determinants Ex 6.2 Q15**

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

$$LHS = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Apply: $R_1 \rightarrow R_1 + R_2 + R_3$.

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Take $(a+b+c)$ common from $R_1$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Apply: $C_2 \rightarrow C_2 - C_1$, $C_3 \rightarrow C_3 - C_1$

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -b-c-a & 0 \\ 2c & 0 & -c-a-b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & b+c+a & 0 \\ 2c & 0 & b+c+a \end{vmatrix}$$

$$= (a+b+c)^3$$
$$= RHS$$

**Chapter 6 Determinants Ex 6.2 Q16**

$$LHS = \begin{vmatrix} 1 & b+c & b^2+c^2 \\ 1 & c+a & c^2+a^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & b+c & b^2+c^2 \\ 0 & a-b & a^2-b^2 \\ 0 & a-c & a^2-c^2 \end{vmatrix}$$

$$= (a-b)(a-c)\begin{vmatrix} 1 & b+c & b^2+c^2 \\ 0 & 1 & a+b \\ 0 & 1 & a+c \end{vmatrix}$$

$$= (a-b)(a-c)\begin{vmatrix} 1 & b+c & b^2+c^2 \\ 0 & 1 & a+b \\ 0 & 0 & c-b \end{vmatrix}$$

$$= (a-b)(b-c)(c-a)$$
$$= RHS$$

**Chapter 6 Determinants Ex 6.2 Q17**

$$LHS = \begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$$

$$= \begin{vmatrix} 3a+3b & 3a+3b & 3a+3b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$$

$$= (3a+3b)\begin{vmatrix} 1 & 1 & 1 \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$$

$$= 3(a+b)\begin{vmatrix} 0 & 1 & 0 \\ 2b & a & b \\ -b & a+2b & -2b \end{vmatrix}$$

$$= 3(a+b)b^2\begin{vmatrix} 0 & 1 & 0 \\ 2 & a & 1 \\ -1 & a+2b & -2 \end{vmatrix}$$

$$= 9(a+b)b^2$$
$$= RHS$$

**Chapter 6 Determinants Ex 6.2 Q18**

$$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

Apply $R_1 \to R_1 a$, $R_2 \to R_2 b$, $R_3 \to R_3 c$

$$= \frac{1}{abc}\begin{vmatrix} a & a^2 & abc \\ b & b^2 & cab \\ c & c^2 & abc \end{vmatrix}$$

$$= \frac{abc}{abc}\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

$$= -\begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

**Chapter 6 Determinants Ex 6.2 Q19**

$$\begin{vmatrix} z & x & y \\ z^2 & x^2 & y^2 \\ z^4 & x^4 & y^4 \end{vmatrix} = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^4 & y^4 & z^4 \end{vmatrix} = \begin{vmatrix} x^2 & y^2 & z^2 \\ x^4 & y^4 & z^4 \\ x & y & z \end{vmatrix} = xyz\,(x - y)(y - z)(z - x)(x + y + z)$$

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^4 & y^4 & z^4 \end{vmatrix}$$

$$= xyz \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix}$$

$$= xyz \begin{vmatrix} 0 & 1 & 0 \\ x - y & y & z - y \\ x^3 - y^3 & y^3 & z^3 - y^3 \end{vmatrix}$$

$$= xyz\,(x - y)(z - y) \begin{vmatrix} 0 & 1 & 0 \\ 1 & y & 1 \\ x^2 + y^2 + xy & y^3 & z^2 + y^2 + zy \end{vmatrix}$$

$$= -xyz\,(x - y)(z - y)\left[ z^2 + y^2 + zy - x^2 - y^2 - xy \right]$$

$$= -xyz\,(x - y)(z - y)\left[ (z - x)(z + x) + y\,(z - x) \right]$$

$$= -xyz\,(x - y)(z - y)(z - x)\left[ z + x + y \right]$$

$$= xyz\,(x - y)(y - z)(z - x)(x + y + z)$$

$$= RHS$$

**Chapter 6 Determinants Ex 6.2 Q20**

$$\begin{vmatrix} (b + c)^2 & a^2 & bc \\ (c + a)^2 & b^2 & ca \\ (a + b)^2 & c^2 & ab \end{vmatrix} = (a - b)(b - c)(c - a)(a + b + c)\left(a^2 + b^2 + c^2\right)$$

$$LHS = \begin{vmatrix} (b + c)^2 & a^2 & bc \\ (c + a)^2 & b^2 & ca \\ (a + b)^2 & c^2 & ab \end{vmatrix}$$

Apply: $C_1 \to C_1 + C_2 - 2C_3$

$$= \begin{vmatrix} (b + c)^2 + a^2 - 2bc & a^2 & bc \\ (c + a)^2 + b^2 - 2ca & b^2 & ca \\ (a + b)^2 + c^2 - 2ab & c^2 & ab \end{vmatrix}$$

$$= \begin{vmatrix} a^2 + b^2 + c^2 & a^2 & bc \\ a^2 + b^2 + c^2 & b^2 & ca \\ a^2 + b^2 + c^2 & c^2 & ab \end{vmatrix}$$

Take $\left(a^2 + b^2 + c^2\right)$ common from $C_1$

$$= \left(a^2 + b^2 + c^2\right) \begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix}$$

$$= \left(a^2 + b^2 + c^2\right) \begin{vmatrix} 1 & a^2 & bc \\ 0 & b^2 - a^2 & ca - bc \\ 0 & c^2 - a^2 & ab - bc \end{vmatrix}$$

$$= \left(a^2 + b^2 + c^2\right)(b - a)(c - a) \begin{vmatrix} 1 & a^2 & bc \\ 0 & b + a & -c \\ 0 & c + a & -b \end{vmatrix}$$

$$= \left(a^2 + b^2 + c^2\right)(b - a)(c - a)\left[ (b + a)(-b) - (-c)(c + a) \right]$$

$$= (a - b)(b - c)(c - a)(a + b + c)\left(a^2 + b^2 + c^2\right)$$

$$= RHS$$

**Chapter 6 Determinants Ex 6.2 Q21**

$$\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} = -2$$

$$\text{LHS} = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$$

Apply $R_3 \rightarrow R_3 - R_2$

$$= \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)2 & 1 & 0 \end{vmatrix}$$

Apply $R_2 \rightarrow R_2 - R_1$

$$= \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)2 & 1 & 0 \\ (a+3)2 & 1 & 0 \end{vmatrix}$$

$$= \left[ (2a+4)(1) - (1)(2a+6) \right]$$
$$= -2$$
$$= \text{RHS}$$

**Chapter 6 Determinants Ex 6.2 Q22**

$$\begin{vmatrix} a^2 & a^2 - (b-c)^2 & bc \\ b^2 & b^2 - (c-a)^2 & ca \\ c^2 & c^2 - (a-b)^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

$$\text{LHS} = \begin{vmatrix} a^2 & a^2 - (b-c)^2 & bc \\ b^2 & b^2 - (c-a)^2 & ca \\ c^2 & c^2 - (a-b)^2 & ab \end{vmatrix}$$

Apply: $C_2 \rightarrow C_2 - 2C_1 - 2C_3$

$$= \begin{vmatrix} a^2 & a^2 - (b-c)^2 - 2a^2 - 2bc & bc \\ b^2 & b^2 - (c-a)^2 - 2b^2 - 2ca & ca \\ c^2 & c^2 - (a-b)^2 - 2c^2 - 2ab & ab \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & -(b^2+c^2+a^2) & bc \\ b^2 & -(b^2+c^2+a^2) & ca \\ c^2 & -(b^2+c^2+a^2) & ab \end{vmatrix}$$

Take $-(a^2+b^2+c^2)$ common from $C_2$

$$= -(b^2+c^2+a^2)\begin{vmatrix} a^2 & 1 & bc \\ b^2 & 1 & ca \\ c^2 & 1 & ab \end{vmatrix}$$

$$= -(b^2+c^2+a^2)\begin{vmatrix} a^2 & 1 & bc \\ b^2 - a^2 & 0 & ca - bc \\ c^2 - a^2 & 0 & ab - bc \end{vmatrix}$$

$$= -(b^2+c^2+a^2)(a-b)(c-a)\begin{vmatrix} a^2 & 1 & bc \\ -(b+a) & 0 & c \\ c+a & 0 & -b \end{vmatrix}$$

$$= -(b^2+c^2+a^2)(a-b)(c-a)\left[ (-(b+a))(-b) - (c)(c+a) \right]$$

$$= (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

$$= \text{RHS}$$

**Chapter 6 Determinants Ex 6.2 Q23**

$$\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix} = -(a-b)(b-c)(c-a)(a^2+b^2+c^2)$$

$$\text{LHS} = \begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

Apply: $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & a^2+bc & a^3 \\ 0 & b^2+ca-a^2-bc & b^3-a^3 \\ 0 & c^2+ab\,b^2+ca-a^2-bc & c^3-a^3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a^2+bc & a^3 \\ 0 & (b^2-a^2)-c(b-a) & b^3-a^3 \\ 0 & (c^2-a^2)-b(c-a) & c^3-a^3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a^2+bc & a^3 \\ 0 & (b-a)(b+a-c) & b^3-a^3 \\ 0 & (c-a)(c+a-b) & c^3-a^3 \end{vmatrix}$$

$$= (b-a)(c-a)\begin{vmatrix} 1 & a^2+bc & a^3 \\ 0 & (b+a-c) & b^2+a^2+ab \\ 0 & (c+a-b) & c^2+a^2+ac \end{vmatrix}$$

$$= (b-a)(c-a)\Big[((b+a-c))(c^2+a^2+ac)-(b^2+a^2+ab)(c^2+a^2+ac)\Big]$$

$$= -(a-b)(b-c)(c-a)(a^2+b^2+c^2)$$

$$= \text{RHS}$$

**Chapter 6 Determinants Ex 6.2 Q24**

We need to prove the following identity:

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

Taking the term $a,b,c$ common from $C_1$, $C_2$ and $C_3$, respectively, we have,

$$\text{L.H.S} = abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, we have,

$$\text{L.H.S} = abc\begin{vmatrix} 2a+2c & c & a+c \\ 2a+2b & b & a \\ 2b+2c & b+c & c \end{vmatrix}$$

$$\Rightarrow \text{L.H.S} = 2abc\begin{vmatrix} a+c & c & a+c \\ a+b & b & a \\ b+c & b+c & c \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we have,

$$L.H.S = 2abc \begin{vmatrix} a+c & -a & 0 \\ a+b & -a & -b \\ b+c & 0 & -b \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$, we have,

$$\Rightarrow L.H.S = 2abc \begin{vmatrix} c & -a & 0 \\ 0 & -a & -b \\ c & 0 & -b \end{vmatrix}$$

Taking $c$, $a$, and $b$ from $C_1$, $C_2$ and $C_3$ respectively, we have,

$$L.H.S = 2a^2b^2c^2 \begin{vmatrix} 1 & -1 & 0 \\ 0 & -1 & -1 \\ 1 & 0 & -1 \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 - R_1$, we have

$$L.H.S = 2a^2b^2c^2 \begin{vmatrix} 1 & -1 & 0 \\ 0 & -1 & -1 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= 4a^2b^2c^2$$

**Chapter 6 Determinants Ex 6.2 Q25**

We need to prove the following identity:

$$\begin{vmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix} = 16(3x+4)$$

Let us consider the L.H.S of the above equation.

$$\Delta = \begin{vmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$, we get,

$$\Delta = \begin{vmatrix} 3x+4 & x & x \\ 3x+4 & x+4 & x \\ 3x+4 & x & x+4 \end{vmatrix}$$

Taking the common term $3x+4$, we get,

$$\Delta = (3x+4)\begin{vmatrix} 1 & x & x \\ 1 & x+4 & x \\ 1 & x & x+4 \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we get,

$$\Delta = (3x+4)\begin{vmatrix} 1 & x & x \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{vmatrix}$$

$$\Rightarrow \Delta = 16(3x+4)$$

**Chapter 6 Determinants Ex 6.2 Q26**

*We need to prove the following identity:*

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1$$

*Let us consider the L.H.S of the above equation.*

*Applying $C_2 \rightarrow C_2 - pC_1$ and $C_3 \rightarrow C_3 - qC_1$, we get*

$$\Delta = \begin{vmatrix} 1 & 1 & 1+p \\ 2 & 3 & 4+3p \\ 3 & 6 & 10+6p \end{vmatrix}$$

*Applying $C_3 \rightarrow C_3 - pC_2$, we get*

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 6 & 10 \end{vmatrix}$$

*Applying $C_2 \rightarrow C_2 - pC_1$ and $C_3 \rightarrow C_3 - qC_1$, we get*

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 2 \\ 3 & 3 & 7 \end{vmatrix}$$

$$\Rightarrow \Delta = 1[7-6] = 1$$

**Chapter 6 Determinants Ex 6.2 Q27**

$$\begin{vmatrix} a & b-c & c-b \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$$

$R_1 \rightarrow R_1 - R_2 - R_3$

$$= \begin{vmatrix} -a+c+b & -b-c+a & -c-b+a \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$$

$$= (b+c-a) \begin{vmatrix} 1 & -1 & -1 \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$$

$$= (b+c-a) \begin{vmatrix} 1 & 0 & 0 \\ a-c & b+a-c & 0 \\ a-b & 0 & c+a-b \end{vmatrix}$$

$$= (a+b-c)(b+c-a)(c+a-b)$$

$$= RHS$$

**Chapter 6 Determinants Ex 6.2 Q28**

$$LHS = \begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 + b^2 + 2ab & 2ab & b^2 \\ a^2 + b^2 + 2ab & a^2 & 2ab \\ a^2 + b^2 + 2ab & b^2 & a^2 \end{vmatrix}$$

$$= \left(a^2 + b^2 + 2ab\right) \begin{vmatrix} 1 & 2ab & b^2 \\ 1 & a^2 & 2ab \\ 1 & b^2 & a^2 \end{vmatrix}$$

$$= \left(a^2 + b^2 + 2ab\right) \begin{vmatrix} 1 & 2ab & b^2 \\ 0 & a^2 - 2ab & 2ab - b^2 \\ 0 & b^2 - 2ab & a^2 - b^2 \end{vmatrix}$$

$$= \left(a^2 + b^2 + 2ab\right) \begin{vmatrix} 1 & 2ab & b^2 \\ 0 & a^2 - b^2 & 2ab - a^2 \\ 0 & b^2 - 2ab & a^2 - b^2 \end{vmatrix}$$

$$= (a+b)^2 \left[ \left(a^2 - b^2\right)\left(a^2 - b^2\right) - \left(2ab - a^2\right)\left(b^2 - 2ab\right) \right]$$

$$= (a+b)^2 \left(a^2 + b^2 - ab\right)^2$$

$$= \left(a^3 + b^3\right)^2$$

$$= RHS$$

## Chapter 6 Determinants Ex 6.2 Q29

*We need to prove the following identity:*

$$\begin{vmatrix} a^2 + 1 & ab & ac \\ ab & b^2 + 1 & bc \\ ca & cb & c^2 + 1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

*Let us consider the L.H.S of the above equation.*

$$\Delta = \begin{vmatrix} a^2 + 1 & ab & ac \\ ab & b^2 + 1 & bc \\ ca & cb & c^2 + 1 \end{vmatrix}$$

*Applying* $R_1 \to R_1(a), R_2 \to R_2(b)$ *and* $R_3 \to R_3(c)$, *we get*

$$\Delta = \frac{1}{abc} \begin{vmatrix} a(a^2 + 1) & a^2 b & a^2 c \\ ab^2 & b(b^2 + 1) & b^2 c \\ c^2 a & c^2 b & c(c^2 + 1) \end{vmatrix}$$

*Taking a,b, and c common from* $C_1, C_2$ *and* $C_3$, *respectively, we get,*

$$\Delta = \frac{abc}{abc} \begin{vmatrix} (a^2 + 1) & a^2 & a^2 \\ b^2 & (b^2 + 1) & b^2 \\ c^2 & c^2 & (c^2 + 1) \end{vmatrix}$$

*Applying* $R_1 \to R_1 + R_2 + R_3$, *we get,*

$$\Delta = \frac{abc}{abc} \begin{vmatrix} (a^2 + b^2 + c^2 + 1) & (a^2 + b^2 + c^2 + 1) & (a^2 + b^2 + c^2 + 1) \\ b^2 & (b^2 + 1) & b^2 \\ c^2 & c^2 & (c^2 + 1) \end{vmatrix}$$

Taking the term, $\left(a^2 + b^2 + c^2 + 1\right)$ common from the above equation, we have,

$$\Delta = \left(a^2 + b^2 + c^2 + 1\right)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & \left(b^2 + 1\right) & b^2 \\ c^2 & c^2 & \left(c^2 + 1\right) \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$, $C_3 \rightarrow C_3 - C_1$, we get,

$$\Delta = \left(a^2 + b^2 + c^2 + 1\right)\begin{vmatrix} 1 & 0 & 1 \\ b^2 & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \left(a^2 + b^2 + c^2 + 1\right)$$

**Chapter 6 Determinants Ex 6.2 Q30**

Let us consider the L.H.S of the given equation.

$$\text{Let } \Delta = \begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$, we have,

$$\Delta = \begin{vmatrix} 1 + a + a^2 & a & a^2 \\ 1 + a + a^2 & 1 & a \\ 1 + a + a^2 & a^2 & 1 \end{vmatrix}$$

Taking the term $\left(1 + a + a^2\right)$ common, we have,

$$\Delta = \left(1 + a + a^2\right)\begin{vmatrix} 1 & a & a^2 \\ 1 & 1 & a \\ 1 & a^2 & 1 \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we have

$$\Delta = \left(1 + a + a^2\right)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 - a & a(1-a) \\ 0 & -a(1-a) & (1-a)(1+a) \end{vmatrix}$$

Taking the term $(1-a)$ common from $R_2$ and $R_3$, we have

$$\Rightarrow \Delta = \left(1 + a + a^2\right)(1-a)^2\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & a \\ 0 & -a & (1+a) \end{vmatrix}$$

$$\Rightarrow \Delta = \left(1 + a + a^2\right)(1-a)^2\left(1 + a + a^2\right)$$

$$\Rightarrow \Delta = \left(1 + a + a^2\right)^2(1-a)^2$$

$$\Rightarrow \Delta = \left[\left(1 + a + a^2\right)(1-a)\right]^2$$

$$\Rightarrow \Delta = \left[\left(a^3 - 1\right)\right]^2$$

**Chapter 6 Determinants Ex 6.2 Q31**

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$$

$$\text{LHS} = \begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$

Apply: $C_1 \to C_1 + C_3$ and $C_2 \to C_2 + C_3$

$$= \begin{vmatrix} a+c & -(c+b) & -b \\ -(c+a) & b+c & -a \\ a+c & b+c & a+b+c \end{vmatrix}$$

$$= (c+a)(c+b) \begin{vmatrix} 1 & -1 & -b \\ -1 & 1 & -a \\ 1 & 1 & a+b+c \end{vmatrix}$$

$$= (c+a)(c+b) \begin{vmatrix} 1 & -1 & -b \\ 0 & 0 & -a-b \\ 0 & 2 & a+c \end{vmatrix}$$

$$= 2(a+b)(b+c)(c+a)$$

$$= \text{RHS}$$

## Chapter 6 Determinants Ex 6.2 Q32

We need to prove the following identity:

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

Let us consider the L.H.S of the above equation.

$$\Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, we have,

$$\Delta = \begin{vmatrix} 2(b+c) & 2(a+c) & 2(a+b) \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Taking 2 common from the above equation, we have,

$$\Delta = 2 \begin{vmatrix} (b+c) & (a+c) & (a+b) \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we have,

$$\Delta = 2 \begin{vmatrix} (b+c) & (a+c) & (a+b) \\ -c & 0 & -a \\ -b & -a & 0 \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, we have,

$$\Delta = 2 \begin{vmatrix} 0 & c & b \\ -c & 0 & -a \\ -b & -a & 0 \end{vmatrix}$$

$$\Rightarrow \Delta = 2(0 + 2abc + abc)$$

$$\Rightarrow \Delta = 4abc$$

## Chapter 6 Determinants Ex 6.2 Q33

$$\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$$

$$LHS = \begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix}$$

Multiply $R_1, R_2$ and $R_3$ by $a, b$ and $c$ respectively.

$$= \frac{1}{abc}\begin{vmatrix} ab^2+ac^2 & a^2b & a^2c \\ b^2a & bc^2+ba^2 & b^2c \\ c^2a & c^2b & ca^2+cb^2 \end{vmatrix}$$

Take $a, b$ and $c$ common from $C_1, C_2$ and $C_3$ respectively.

$$= \frac{abc}{abc}\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

Now apply $R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 2(b^2+c^2) & 2(c^2+a^2) & 2(a^2+b^2) \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$$= 2\begin{vmatrix} (b^2+c^2) & (c^2+a^2) & (a^2+b^2) \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$$= 2\begin{vmatrix} c^2 & 0 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$$= 2\left[c^2\left\{(c^2+a^2)(a^2+b^2) - b^2c^2\right\} + a^2\left\{b^2c^2 - (c^2+a^2)c^2\right\}\right]$$

$$= 4a^2b^2c^2$$

$$= RHS$$

**Chapter 6 Determinants Ex 6.2 Q34**

$$\begin{vmatrix} 0 & b^2a & c^2a \\ a^2b & 0 & c^2b \\ a^2c & b^2c & 0 \end{vmatrix} = 2a^3b^3c^3$$

$$LHS = \begin{vmatrix} 0 & b^2a & c^2a \\ a^2b & 0 & c^2b \\ a^2c & b^2c & 0 \end{vmatrix}$$

$$= a^2b^2c^2\begin{vmatrix} 0 & a & a \\ b & 0 & b \\ c & c & 0 \end{vmatrix}$$

$$= a^3b^3c^3\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= a^3b^3c^3\begin{vmatrix} 0 & 0 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= 2a^3b^3c^3$$

$$= RHS$$

**Chapter 6 Determinants Ex 6.2 Q35**

$$\begin{vmatrix} \dfrac{a^2+b^2}{c} & c & c \\ a & \dfrac{b^2+c^2}{a} & a \\ b & b & \dfrac{c^2+a^2}{b} \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} a^2+b^2 & c^2 & c^2 \\ a^2 & c^2+b^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} 0 & -2b^2 & -2a^2 \\ a^2 & c^2+b^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

$$= \frac{-2}{abc}\begin{vmatrix} 0 & b^2 & a^2 \\ a^2 & c^2+b^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

$$= \frac{-2}{abc}\begin{vmatrix} 0 & b^2 & a^2 \\ a^2 & c^2+b^2 & a^2 \\ b^2 & 0 & c^2 \end{vmatrix}$$

$$= \frac{-2}{abc}\begin{vmatrix} 0 & b^2 & a^2 \\ a^2 & c^2 & 0 \\ b^2 & 0 & c^2 \end{vmatrix}$$

$$= \frac{-2}{abc}\left[\left(-a^2\right)\left(b^2c^2\right)+\left(b^2\right)\left(-a^2c^2\right)\right]$$

$$= \frac{-2}{abc}\left(-2a^2b^2c^2\right)$$

$$= 4abc$$

$$= RHS$$

**Chapter 6 Determinants Ex 6.2 Q36**

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$$

Multiply $R_1, R_2$ and $R_3$ by $a,b$ and $c$ respectively

$$= \frac{1}{abc}\begin{vmatrix} -abc & ab^2+abc & ac^2+abc \\ a^2b+abc & -abc & bc^2+abc \\ a^2c+abc & b^2c+abc & -abc \end{vmatrix}$$

Take $a,b$ and $c$ common from $C_1, C_2$ and $C_3$ respectively.

$$= \frac{abc}{abc}\begin{vmatrix} -bc & ab+ac & ac+ab \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

Apply: $R_1 \to R_1+R_2+R_3$

$$= \begin{vmatrix} ab+bc+ca & ab+bc+ca & ab+bc+ca \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$$= (ab+bc+ca)\begin{vmatrix} 1 & 1 & 1 \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$$= (ab+bc+ca)\begin{vmatrix} 0 & 1 & 0 \\ ab+bc+ac & -ac & bc+ab+ac \\ 0 & bc+ac & -ab-bc-ac \end{vmatrix}$$

$$= (ab+bc+ca)^3\begin{vmatrix} 0 & 1 & 0 \\ 1 & -ac & 1 \\ 0 & bc+ac & -1 \end{vmatrix}$$

$$= (ab+bc+ca)^3$$

$$= RHS$$

**Chapter 6 Determinants Ex 6.2 Q37**

L.H.S.,

$$\begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix}$$

$$= \begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} [C_1 \to C_1 - C_3, C_2 \to C_2 - C_3]$$

$$= \begin{vmatrix} \lambda-x & 0 & 2x \\ 0 & \lambda-x & 2x \\ x-\lambda & x-\lambda & x+\lambda \end{vmatrix}$$

$$= (\lambda-x)(\lambda-x)\begin{vmatrix} 1 & 0 & 2x \\ 0 & 1 & 2x \\ -1 & -1 & x+\lambda \end{vmatrix}$$

$$= (\lambda-x)^2\begin{vmatrix} 1 & 0 & 2x \\ 0 & 1 & 2x \\ -1 & -1 & x+\lambda \end{vmatrix}$$

$$= (\lambda-x)^2[1(x+\lambda) + 2x + 2x(0+1)]$$
$$= (\lambda-x)^2[x+\lambda + 2x + 2x]$$
$$= (\lambda-x)^2[5x+\lambda]$$
$$= R.H.S$$
Hence Proved

**Chapter 6 Determinants Ex 6.2 Q38**

$$LHS = \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

Apply $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 5x+4 & 2x & 2x \\ 5x+4 & x+4 & 2x \\ 5x+4 & 2x & x+4 \end{vmatrix}$$

$$= (5x+4)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x+4 & 2x \\ 1 & 2x & x+4 \end{vmatrix}$$

$$= (5x+4)\begin{vmatrix} 1 & 2x & 2x \\ 0 & -x+4 & 0 \\ 0 & 0 & -x+4 \end{vmatrix}$$

$$= (5x+4)(4-x)^2\begin{vmatrix} 1 & 2x & 2x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= (5x+4)(4-x)^2$$
$$= RHS$$

**Chapter 6 Determinants Ex 6.2 Q39**

Let $\Delta = \begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$

Applying $R_1 \rightarrow R_1 - R_2$

$\Delta = \begin{vmatrix} y & -x & y-x \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$

Applying $R_1 \rightarrow R_1 - R_3$

$\Delta = \begin{vmatrix} 0 & -2x & -2x \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$

$\Delta = 2x\left[z(x+y) - xy\right] - 2x\left[zx - y(z+x)\right]$

$\Delta = 2x\left[zx + zy - xy - zx + yz + yx\right]$

$\Delta = 4xyz$

**Chapter 6 Determinants Ex 6.2 Q40**

$\begin{vmatrix} -a(b^2+c^2-a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2) \end{vmatrix} = abc(a^2+b^2+c^2)^3$

LHS $= \begin{vmatrix} -a(b^2+c^2-a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2) \end{vmatrix}$

Take $a, b$ and $c$ common from $C_1, C_2$ and $C_3$ respectively.

$= abc\begin{vmatrix} -(b^2+c^2-a^2) & 2b^2 & 2c^2 \\ 2a^2 & -(c^2+a^2-b^2) & 2c^2 \\ 2a^2 & 2b^2 & -(a^2+b^2-c^2) \end{vmatrix}$

Apply: $R_1 \rightarrow R_1 - R_3, R_2 \rightarrow R_2 - R_3$

$= abc\begin{vmatrix} -(b^2+c^2-a^2)-2a^2 & 0 & 2c^2+(a^2+b^2-c^2) \\ 0 & -(c^2+a^2-b^2)-2b^2 & 2c^2+(a^2+b^2-c^2) \\ 2a^2 & 2b^2 & -(a^2+b^2-c^2) \end{vmatrix}$

$= abc\begin{vmatrix} -(b^2+c^2+a^2) & 0 & (a^2+b^2+c^2) \\ 0 & -(c^2+a^2+b^2) & (a^2+b^2+c^2) \\ 2a^2 & 2b^2 & -(a^2+b^2-c^2) \end{vmatrix}$

$= abc(b^2+c^2+a^2)^2\begin{vmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2a^2 & 2b^2 & -(a^2+b^2-c^2) \end{vmatrix}$

$= abc(b^2+c^2+a^2)^2\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 1 \\ 2a^2 & 2b^2 & -(a^2+b^2-c^2)+2a^2 \end{vmatrix}$

$= abc(b^2+c^2+a^2)^2\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 1 \\ 2a^2 & 2b^2 & -b^2+c^2+a^2 \end{vmatrix}$

$= -abc(b^2+c^2+a^2)^2\left[(-1)(-b^2+c^2+a^2) - (1)(2b^2)\right]$

$abc(a^2+b^2+c^2)^3$

$= RHS$

**Chapter 6 Determinants Ex 6.2 Q41**

$$LHS = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$$

$$= \begin{vmatrix} 3+a & 3+a & 3+a \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$$

$$= (3+a)\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$$

$$= (3+a)\begin{vmatrix} 1 & 1 & 1 \\ 0 & a & 0 \\ 0 & 1 & a \end{vmatrix}$$

$$= (3+a)a^2$$

$$= a^3 + 3a^2$$

$$= RHS$$

**Chapter 6 Determinants Ex 6.2 Q42**

L.H.S.,

$$\begin{vmatrix} 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix}$$

$$= \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix} [R_1 = R_1 + R_2 + R_3]$$

$$= (x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} 1 & 0 & 0 \\ 2z & 0 & -x-y-z \\ x-y-z & x+y+z & x+y+z \end{vmatrix} [C_2 = C_2 - C_1, C_3 = C_3 - C_1]$$

$$= (x+y+z)[1\{0 + (x+y+z)(x+y+z)\}]$$

$$= (x+y+z)^3$$

$$= R.H.S.$$

Hence Proved

**Chapter 6 Determinants Ex 6.2 Q43**

$$\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$$

$$= \begin{vmatrix} 2(y+z+x) & y+z+x & y+z+x \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} 2 & 1 & 1 \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} 0 & 1 & 1 \\ z+x-z-x & z & x \\ x+y-y-z & y & z \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} 0 & 1 & 1 \\ 0 & z & x \\ x-z & y & z \end{vmatrix}$$

$$= (x+y+z)(x-z)^2$$

$$= RHS$$

**Chapter 6 Determinants Ex 6.2 Q44**

L.H.S. =

$$\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$$

$$= \begin{vmatrix} a+x+y+z & y & z \\ a+x+y+z & a+y & z \\ a+x+y+z & x & a+z \end{vmatrix} \quad [C_1 = C_1 + C_2 + C_3]$$

$$= (a+x+y+z)\begin{vmatrix} 1 & y & z \\ 1 & a+y & z \\ 1 & x & a+z \end{vmatrix}$$

$$= (a+x+y+z)\begin{vmatrix} 1 & y & z \\ 0 & a & 0 \\ 0 & x-y & a \end{vmatrix} \quad [R_2 \to R_2 - R_1, R_3 \to R_3 - R_1]$$

$$= (a+x+y+z)[1(a^2 - 0)]$$

$$= a^2(a+x+y+z)$$

$$= \text{R.H.S.}$$

Hence Proved.

**Chapter 6 Determinants Ex 6.2 Q45**

Let $\Delta = \begin{vmatrix} a^3 & 2 & a \\ b^3 & 2 & b \\ c^3 & 2 & c \end{vmatrix}$

$\Delta = 2\begin{vmatrix} a^3 & 1 & a \\ b^3 & 1 & b \\ c^3 & 1 & c \end{vmatrix}$

$\Delta = 2\left\{ a^3(c-b) - 1(b^3c - bc^3) + a(b^3 - c^3) \right\}$

$\Delta = 2\left\{ a^3(c-b) - bc(b-c)(b+c) + a(b-c)(b^2 + bc + c^2) \right\}$

$\Delta = 2(b-c)\left\{ -a^3 - bc(b+c) + a(b^2 + bc + c^2) \right\}$

$\Delta = 2(a-b)(b-c)(c-a)(a+b+c)$

**Chapter 6 Determinants Ex 6.2 Q46**

$$\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = -\begin{vmatrix} x & y & z \\ a & b & c \\ p & q & r \end{vmatrix} = (-1)^2\begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix} = \begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix}$$

$$= (-1)\begin{vmatrix} y & x & z \\ q & p & r \\ b & a & c \end{vmatrix}$$

$$= (-1)^2\begin{vmatrix} y & x & z \\ b & a & c \\ q & p & r \end{vmatrix}$$

Taking transpose, we get

$$= \begin{vmatrix} y & b & p \\ x & a & q \\ z & c & r \end{vmatrix}$$

**Chapter 6 Determinants Ex 6.2 Q47**

Consider the determinant $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$, where $a, b, c$ are in A.P.

Let $\Delta = \begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$, we have,

$\Delta = \begin{vmatrix} 3x+1+2+a & x+2 & x+a \\ 3x+2+3+b & x+3 & x+b \\ 3x+3+4+c & x+4 & x+c \end{vmatrix}$

$\Rightarrow \Delta = \begin{vmatrix} 3x+3+a & x+2 & x+a \\ 3x+5+b & x+3 & x+b \\ 3x+7+c & x+4 & x+c \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_2$, we have,

$\Rightarrow \Delta = \begin{vmatrix} 3x+3+a & x+2 & x+a \\ 2+b-a & 1 & b-a \\ 2+c-b & 1 & c-b \end{vmatrix}$

Since $a, b$ and $c$ are in arithmetic progression, we have
$b-a = c-b = k(say)$.
Thus,

$\Delta = \begin{vmatrix} 3x+3+a & x+2 & x+a \\ 2+k & 1 & k \\ 2+k & 1 & k \end{vmatrix}$

Since the second row and the third row are identical, we have
$\Delta = 0$

**Chapter 6 Determinants Ex 6.2 Q48**

Since, $\alpha, \beta, \gamma$ are in A.P, $2\beta = \alpha + \gamma$

$LHS = \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix}$

$R_2 \to R_2 - \dfrac{R_1}{2} - \dfrac{R_3}{2}$

$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ (x-2)-\dfrac{x-3}{2}-\dfrac{x-1}{2} & (x-3)-\dfrac{x-4}{2}-\dfrac{x-2}{2} & (x-\beta)-\dfrac{x-\alpha}{2}-\dfrac{x-\gamma}{2} \\ x-1 & x-2 & x-\gamma \end{vmatrix}$

$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ 0 & 0 & 0 \\ x-1 & x-2 & x-\gamma \end{vmatrix} \qquad [\because 2\beta = \alpha + \gamma]$

$= 0$

**Chapter 6 Determinants Ex 6.2 Q49**

$$\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, we have:

$$\Delta = \begin{vmatrix} 2(a+b+c) & 2(a+b+c) & 2(a+b+c) \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

$$= 2(a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we have:

$$\Delta = 2(a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ c+a & b-c & b-a \\ a+b & c-a & c-b \end{vmatrix}$$

Expanding along $R_1$, we have:

$$\Delta = 2(a+b+c)(1) \left[ (b-c)(c-b) - (b-a)(c-a) \right]$$

$$= 2(a+b+c) \left[ -b^2 - c^2 + 2bc - bc + ba + ac - a^2 \right]$$

$$= 2(a+b+c) \left[ ab + bc + ca - a^2 - b^2 - c^2 \right]$$

It is given that $\Delta = 0$.

$$(a+b+c) \left[ ab + bc + ca - a^2 - b^2 - c^2 \right] = 0$$

$\Rightarrow$ Either $a+b+c = 0$, or $ab + bc + ca - a^2 - b^2 - c^2 = 0$.

Now,

$$ab + bc + ca - a^2 - b^2 - c^2 = 0$$

$$\Rightarrow -2ab - 2bc - 2ca + 2a^2 + 2b^2 + 2c^2 = 0$$

$$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 = 0$$

$$\Rightarrow (a-b)^2 = (b-c)^2 = (c-a)^2 = 0 \qquad \left[ (a-b)^2, (b-c)^2, (c-a)^2 \text{ are non-negative} \right]$$

$$\Rightarrow (a-b) = (b-c) = (c-a) = 0$$

$$\Rightarrow a = b = c$$

Hence, if $\Delta = 0$, then either $a + b + c = 0$ or $a = b = c$.

Chapter 6 Determinants Ex 6.2 Q50

$$\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} p-a & 0 & c-r \\ 0 & q-b & c-r \\ a & b & r \end{vmatrix} = 0 \, [R_1 = R_1 - R_3, R_2 = R_2 - R_3]$$

$$\Rightarrow (p-a)[r(q-b) - b(c-r)] + (c-r)[0 - a(q-b)] = 0$$

$$\Rightarrow (p-a)r(q-b) - (p-a)b(c-r) - (c-r)a(q-b) = 0$$

$$\Rightarrow \frac{(p-a)r(q-b)}{(p-a)(q-b)(r-c)} - \frac{(p-a)b(c-r)}{(p-a)(q-b)(r-c)} - \frac{(c-r)a(q-b)}{(p-a)(q-b)(r-c)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{b}{(q-b)} + \frac{a}{(p-a)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{b-q+q}{(q-b)} + \frac{a+p-p}{(p-a)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{q}{(q-b)} + \frac{(b-q)}{(q-b)} + \frac{(a-p)}{(p-a)} + \frac{p}{(p-a)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{q}{(q-b)} - 1 - 1 + \frac{p}{(p-a)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{q}{(q-b)} + \frac{p}{(p-a)} = 2$$

$$\therefore \frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$$

**Chapter 6 Determinants Ex 6.2 Q51**

Let us show that x = 2 is a root of the given equation:
Putting x = 2 in the LHS, we get

$$\begin{vmatrix} 2 & -6 & -1 \\ 2 & -6 & -1 \\ -3 & 4 & 4 \end{vmatrix} = 0$$

$\because R_1 = R_2$

Hence, x = 2 is a root of the given equation.

Now, we see if there are any other roots. For this we need to solve the equation:

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & -6 & -1 \\ x-1 & -3x & x-3 \\ x-1 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)\begin{vmatrix} 1 & -6 & -1 \\ 1 & -3x & x-3 \\ 1 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)\begin{vmatrix} 1 & -6 & -1 \\ 1 & -3x & x-3 \\ 1 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)\begin{vmatrix} 1 & -6 & -1 \\ 0 & -3x+6 & x-3+1 \\ 0 & 2x+6 & x+2+1 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)\begin{vmatrix} 1 & -6 & -1 \\ 0 & -3(x-2) & x-2 \\ 0 & 2(x+3) & x+3 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)(x-2)(x+3)\begin{vmatrix} 1 & -6 & -1 \\ 0 & -3 & 1 \\ 0 & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)(x-2)(x+3) = 0$$

$$\Rightarrow (x-1) = 0 \quad (x-2) = 0 \quad (x+3) = 0$$

$$\Rightarrow x = 1 \qquad x = 2 \qquad x = -3$$

**Chapter 6 Determinants Ex 6.2 Q52-i**

$$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$$

Apply $C_1 \rightarrow C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ x+a+b+c & b & x+c \end{vmatrix} = 0$$

$$\Rightarrow (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix} = 0$$

$$\Rightarrow (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix} = 0$$

$$\Rightarrow (x+a+b+c)x^2 = 0$$

$$\Rightarrow x = -(a+b+c) \quad or \quad x = 0$$

**Chapter 6 Determinants Ex 6.2 Q52-ii**

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$, we get:

$$\begin{vmatrix} 3x+a & 3x+a & 3x+a \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

$$\Rightarrow (3x+a)\begin{vmatrix} 1 & 1 & 1 \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we have:

$$(3x+a)\begin{vmatrix} 1 & 0 & 0 \\ x & a & 0 \\ x & 0 & a \end{vmatrix} = 0$$

Expanding along $R_1$, we have:

$$(3x+a)\left[ 1 \times a^2 \right] = 0$$

$$\Rightarrow a^2(3x+a) = 0$$

But $a \neq 0$.

Therefore, we have:

$$3x + a = 0$$

$$\Rightarrow x = -\frac{a}{3}$$

**Chapter 6 Determinants Ex 6.2 Q52-iii**

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

Apply $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \quad \begin{vmatrix} 3x-2 & 3 & 3 \\ 3x-2 & 3x-8 & 3 \\ 3x-2 & 3 & 3x-8 \end{vmatrix} = 0$$

$$\Rightarrow \quad (3x-2)\begin{vmatrix} 1 & 3 & 3 \\ 1 & 3x-8 & 3 \\ 1 & 3 & 3x-8 \end{vmatrix} = 0$$

$$\Rightarrow \quad (3x-2)\begin{vmatrix} 1 & 3 & 3 \\ 0 & 3x-11 & 0 \\ 0 & 0 & 3x-11 \end{vmatrix} = 0$$

$$\Rightarrow \quad (3x-2)(3x-11)^2 = 0$$

$$\Rightarrow \quad (3x-2) = 0 \quad or \quad (3x-11)^2 = 0$$

$$\Rightarrow \quad x = \frac{2}{3} \quad or \quad x = \pm\frac{11}{3}$$

**Chapter 6 Determinants Ex 6.2 Q52-iv**

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & a & a^2 \\ 1 & b & b^2 \end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix} 1 & x & x^2 \\ 0 & a-x & a^2-x^2 \\ 0 & b-x & b^2-x^2 \end{vmatrix} = 0$$

$$\Rightarrow \quad (a-x)(b-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & a+x \\ 0 & 1 & b+x \end{vmatrix} = 0$$

$$\Rightarrow \quad (a-x)(b-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & a+x \\ 0 & 0 & b-a \end{vmatrix} = 0$$

$$\Rightarrow \quad (a-x)(b-x)(b-a) = 0$$

$$\Rightarrow \quad (a-x) = 0 \quad or \quad (b-x) = 0$$

$$\Rightarrow \quad a = x \quad or \quad b = x$$

**Chapter 6 Determinants Ex 6.2 Q52-v**

$$\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$$

Apply $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \quad \begin{vmatrix} x+9 & 3 & 5 \\ x+9 & x+2 & 5 \\ x+9 & 3 & x+4 \end{vmatrix} = 0$$

$$\Rightarrow \quad (x+9)\begin{vmatrix} 1 & 3 & 5 \\ 1 & x+2 & 5 \\ 1 & 3 & x+4 \end{vmatrix} = 0$$

$$\Rightarrow \quad (x+9)\begin{vmatrix} 1 & 3 & 5 \\ 0 & x-1 & 0 \\ 0 & 0 & x-1 \end{vmatrix} = 0$$

$$\Rightarrow \quad (x+9)(x-1)^2 = 0$$

$$\Rightarrow \quad (x+9) = 0 \quad or \quad (x-1)^2 = 0$$

$$\Rightarrow \quad x = -9 \quad or \quad x = 1$$

**Chapter 6 Determinants Ex 6.2 Q52-vi**

$$\Rightarrow \quad \begin{vmatrix} 1 & x & x^3 \\ 0 & b-x & b^3-x^3 \\ 0 & c-x & c^3-x^3 \end{vmatrix} = 0$$

$$\Rightarrow \quad (b-x)(c-x)\begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & b^2+x^2+bx \\ 0 & 1 & c^2+x^2+cx \end{vmatrix} = 0$$

$$\Rightarrow \quad (b-x)(c-x)\begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & b^2+x^2+bx \\ 0 & 0 & c^2+x^2+cx-\left(b^2+x^2+bx\right) \end{vmatrix} = 0$$

$$\Rightarrow \quad (b-x)(c-x)\begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & b^2+x^2+bx \\ 0 & 0 & c^2-b^2+cx-bx \end{vmatrix} = 0$$

$$\Rightarrow \quad (b-x)(c-x)(c-b)\begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & b^2+x^2+bx \\ 0 & 0 & b+c+x \end{vmatrix} = 0$$

$$\Rightarrow \quad (b-x)(c-x)(c-b)(b+c+x) = 0$$

$$\Rightarrow \quad (b-x)=0 \quad (c-x)=0 \quad (b+c+x)=0$$

$$\Rightarrow \quad x=b \quad x=c \quad x=-(b+c)$$

**Chapter 6 Determinants Ex 6.2 Q52-vii**

$$\begin{vmatrix} 15-2x & 11-3x & 7-x \\ 11 & 17 & 14 \\ 10 & 16 & 13 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 15-2x & 11-3x & 7-x \\ 1 & 1 & 1 \\ 10 & 16 & 13 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 15-2x & -x-4 & 7-x \\ 1 & 0 & 1 \\ 10 & 6 & 13 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 8-x & -x-4 & 7-x \\ 0 & 0 & 1 \\ -3 & 6 & 13 \end{vmatrix} = 0$$

$$\Rightarrow -\left[(8-x)(6)-(-x-4)(-3)\right] = 0$$

$$\Rightarrow -\left[36-9x\right] = 0$$

$$\Rightarrow x = 4$$

**Chapter 6 Determinants Ex 6.2 Q52-viii**

$$\begin{vmatrix} 1 & 1 & x \\ p+1 & p+1 & p+x \\ 3 & x+1 & x+2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & x \\ p & p & p \\ 2 & x & 2 \end{vmatrix} = 0$$

$$\Rightarrow p\begin{vmatrix} 1 & 1 & x \\ 1 & 1 & 1 \\ 2 & x & 2 \end{vmatrix} = 0$$

$$\Rightarrow p\begin{vmatrix} 1 & 1 & x \\ 0 & 0 & 1-x \\ 2 & x & 2 \end{vmatrix} = 0$$

$$\Rightarrow p(x-1)(x-2) = 0$$

$$\Rightarrow (x-1)=0 \qquad (x-2)=0$$

$$\Rightarrow x=1 \qquad x=2$$

**Chapter 6 Determinants Ex 6.2 Q52-ix**

$$\begin{vmatrix} 3 & -2 & \sin 3\theta \\ -7 & 8 & \cos 2\theta \\ -11 & 14 & 2 \end{vmatrix} = 0$$

$\Rightarrow 3(16 - 14\cos 2\theta) + 2(-14 + 11\cos 2\theta) + \sin 3\theta(-98 + 88) = 0$

$\Rightarrow 20(1 - \cos 2\theta) + 10\sin 3\theta = 0$

$\Rightarrow 20(2\sin^2 \theta) + 10(3\sin \theta - 4\sin^3 \theta) = 0$

$\Rightarrow 4\sin^2 \theta + 3\sin \theta - 4\sin^3 \theta = 0$

$\Rightarrow 4\sin \theta + 3 - 4\sin^2 \theta = 0$

$\Rightarrow 4\sin^2 \theta - 4\sin \theta - 3 = 0$

$\Rightarrow (2\sin \theta + 1)(2\sin \theta - 3) = 0$

$\Rightarrow \sin \theta = -\dfrac{1}{2}$ or $\sin \theta = \dfrac{3}{2} = 1.5$

As $\sin \theta \in [-1, 1]$

$\therefore \sin \theta = -\dfrac{1}{2}$

$\Rightarrow \theta = n\pi + (-1)^n \dfrac{\pi}{6}, n \in Z$

# Ex 6.3

**Chapter Determinants Ex 6.3 Q1(i)**

If the vertices of a triangle are $(x_1, y_1), (x_2, y_2)$ and $(x_3, y_3)$ then the area of the triangle is given by :

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Substituting the values

$$\Delta = \frac{1}{2} \begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & -1 & 1 \end{vmatrix}$$

expanding the determinant along $R_1$

$$= \frac{1}{2} \left[ 3 \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} - 8 \begin{vmatrix} -4 & 1 \\ 5 & 1 \end{vmatrix} + 1 \begin{vmatrix} -4 & 2 \\ 5 & -1 \end{vmatrix} \right]$$

$$= \frac{1}{2} \left[ 3(3) - 8(-9) + 1(-6) \right]$$

$$= \frac{1}{2} \left[ 9 + 72 - 6 \right] = \frac{75}{2} \text{ sq. units}$$

The area of the $\Delta$ is $\frac{75}{2}$ sq. units

**Chapter Determinants Ex 6.3 Q1(ii)**

The area is given by:

$$\Delta = \frac{1}{2} \begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

expanding along $R_1$

$$= \frac{1}{2}\left[2(-7) - 7(-9) + 1(-2)\right]$$

$$= \frac{1}{2}\left[-14 + 63 - 2\right]$$

$$= \frac{47}{2} \text{ sq. units}$$

The area of the $\Delta$ is $\frac{47}{2}$ sq. units

## Chapter Determinants Ex 6.3 Q1(iii)

The area is given by:

$$\Delta = \frac{1}{2} \begin{vmatrix} -1 & -8 & 1 \\ -2 & -3 & 1 \\ 3 & 2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[-1(-5) + 8(-5) + 1(5)\right]$$

$$= \frac{1}{2}\left[5 - 40 + 5\right] = \frac{-30}{2} = 15 \text{ sq. units}$$

∵ Area can not be negative, so answer will be 15 sq. units.

The area of the $\Delta$ is 15 sq. units.

## Chapter Determinants Ex 6.3 Q1(iv)

The area is given by:

$$\Delta = \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

Expanding along $R_1$

$$= \frac{1}{2}\left[0 - 0 + 1(18)\right] = 9 \text{ sq. units}$$

The area is 9 sq. units

## Chapter Determinants Ex 6.3 Q2(i)

If 3 points are collinear, then the area of the triangle then form will be zero.

Hence

$$\frac{1}{2}\begin{vmatrix} 5 & 5 & 1 \\ -5 & 1 & 1 \\ 10 & 7 & 1 \end{vmatrix} = 0$$

Expanding along $R_1$

$$= \frac{1}{2}\left[5(-6) - 5(-15) + 1(-35 - 10)\right]$$

$$= \frac{1}{2}\left[-35 + 75 - 45\right]$$

$$= \frac{1}{2}[0]$$

$$= 0$$

Since the area of the triangle is zero, hence the points are collinear.

## Chapter Determinants Ex 6.3 Q2(ii)

If 3 points are collinear, then the area of the triangle then form will be zero.

Hence

$$\frac{1}{2}\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 4 & 5 & 1 \end{vmatrix} = 0$$

Expanding along $R_1$

$$= \frac{1}{2}\left[1(-4) + 1(-2) + 1(6)\right]$$

$$= 0$$

Since the area of the triangle is zero, hence the points are collinear.

## Chapter Determinants Ex 6.3 Q2(iii)

If the points are collinear, then the area of the triangle will be zero.

So

$$\frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ 8 & 8 & 1 \\ 5 & 2 & 1 \end{vmatrix} = 0$$

L.H.S

Expanding along $R_1$

$$= \frac{1}{2}\left[3(6) + 2(3) + 1(-24)\right]$$

$$= \frac{1}{2}\left[18 + 6 - 24\right]$$

$$= \frac{1}{2}[0]$$

$$= 0$$

Since the area of the triangle is zero, hence given points are collinear.

## Chapter Determinants Ex 6.3 Q2(iv)

If given points are collinear, then the area of the triangle must be zero.

Hence

$$= \frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ -1 & -2 & 1 \\ 5 & 8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[2(-10) - 3(-6) + 1(2)\right]$$

$$= \frac{1}{2}\left[-20 + 18 + 2\right]$$

$$= \frac{1}{2}[0]$$

$$= 0$$

Hence the given points are collinear.

## Chapter Determinants Ex 6.3 Q3

If the given points are collinear, the area of the triangle must be zero.

Hence

$$\frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

Expanding along $R_1$

$$= \frac{1}{2}\left[a(b-1) - 0(0-1) + 1(-b)\right] = 0$$

or $ab - a - 0 - b = 0$

or $ab = a + b$

Hence proved

## Chapter Determinants Ex 6.3 Q4

If the given points are collinear, then the area of the triangle must be zero.

Hence

$$\frac{1}{2}\begin{vmatrix} a & b & 1 \\ a' & b' & 1 \\ a-a' & b-b' & 1 \end{vmatrix} = 0$$

or

$$\frac{1}{2}\left[a\left(b'-b+b'\right)-b\left(a'-a+a'\right)+1\left(a'b-a'b'-ab'+a'b'\right)\right] = 0$$

or $\frac{1}{2}\left[ab'-ab+ab'-a'b+ab-a'b+a'b-ab'\right] = 0$

or $ab'-a'b = 0$

$\qquad ab' = a'b$

Hence proved

### Chapter Determinants Ex 6.3 Q5

If the points are collinear, then the area of the triangle must be zero.

Hence

$$\begin{vmatrix} 1 & -5 & 1 \\ -4 & 5 & 1 \\ \lambda & 7 & 1 \end{vmatrix} = 0$$

Expanding along $R_1$

$1(-2)+5(-4-\lambda)+1(-28-5\lambda) = 0$

$\qquad -2-20-5\lambda-28-5\lambda = 0$

$\qquad\qquad -50-10\lambda = 0$

$\qquad\qquad\qquad \lambda = -5$

Hence $\lambda = -5$

### Chapter Determinants Ex 6.3 Q6

$$\text{Area} = \left|\frac{1}{2}\begin{vmatrix} x & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}\right|$$

$$\pm 2 \times 35 = \begin{vmatrix} x & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}$$

$\pm 70 = x(-10)-4(-3)+1(38)$

$\pm 70 = -10x+12+38$

$\pm 70 = -10x+50 \qquad\qquad ---(1)$

Taking $(+)$ sign

$+70 = -10x+50$

$10x = -20$ or $x = -2$

Again taking $(-)$ sign

$-70 = -10x+50$

$10x = 120$ or $x = 12$

Hence $x = -2, 12$

### Chapter Determinants Ex 6.3 Q7

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 1 & 4 & 1 \\ 2 & 3 & 1 \\ -5 & -3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[1(6) - 4(7) + 1(-6 + 15)\right]$$

$$= \frac{1}{2}\left[6 - 28 + 9\right]$$

$$= \frac{1}{2}\left[-13\right]$$

$$= \frac{13}{2} \text{ sq. units} \qquad [\because \text{Area can not be negative}]$$

Also, since the area of the triangle is non-zero.

Hence these points are non-collinear.

### Chapter Determinants Ex 6.3 Q8

$$\text{Area} = \frac{1}{2}\begin{vmatrix} -3 & 5 & 1 \\ 3 & -6 & 1 \\ 7 & 2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[-3(-8) - 5(-4) + 1(48)\right]$$

$$= \frac{1}{2}\left[24 + 20 + 48\right]$$

$$= 46 \text{ sq. units}$$

Hence the area is 46 sq. units.

### Chapter Determinants Ex 6.3 Q9

If the given points are collinear, then the area of the triangle must be zero.

$$\text{so} \quad \frac{1}{2}\begin{vmatrix} k & 2-2k & 1 \\ -k+1 & 2k & 1 \\ -4-k & 6-2k & 1 \end{vmatrix} = 0$$

expanding along $R_1$

$$k(2k - 6 + 2k) - (2 - 2k)(-k + 1 + 4 + k) + 1(1 - k) \times (6 - 2k) - 2k(-4 - k) = 0$$

$$k(4k - 6) - (2 - 2k)(5) + 1[6 - 2k - 6k + 2k^2 + 8k + 2k^2] = 0$$

$$4k^2 - 6k - 10 + 10k + 6 + 4k^2 = 0$$

$$8k^2 + 4k - 4 = 0$$

$$8k^2 + 8k - 4k - 4 = 0 \qquad \text{(Middle term splitting)}$$

$$8k(k + 1) - 4(k + 1) = 0$$

$$(8k - 4)(k + 1) = 0$$

If $8k - 4 = 0$     or     if $k + 1 = 0$

$k = \frac{1}{2}$                        $k = -1$

Hence $k = -1, \frac{1}{2}$

### Chapter Determinants Ex 6.3 Q10

Since the points are collinear, hence the area of the triangle must be zero.

$$\text{so} \quad \frac{1}{2}\begin{vmatrix} x & -2 & 1 \\ 5 & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} = 0$$

or $x(-6) + 2(-3) + 1(24) = 0$

or $-6x - 6 + 24 = 0$

$-6x + 18 = 0$

$x = 3$

Hence $x = 3$

## Chapter Determinants Ex 6.3 Q11

Since the points are collinear, hence the area of the triangle must be zero.

$$\frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ x & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} = 0$$

$$3(-6) + 2(x - 8) + 1(8x - 16) = 0$$
$$-18 + 2x - 16 + 8x - 16 = 0$$
$$10x = 50$$
$$x = 5$$

Hence $x = 5$

## Chapter Determinants Ex 6.3 Q12(i)

Let $A(x, y)$, $B(1, 2)$ and $C(3, 6)$ are 3 points in a line.

Since these points are collinear, hence area of the triangle must be zero.

$$\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0$$

Expanding along $R_1$
$$x(-4) - y(-2) + 1(0) = 0$$
$$-4x + 2y = 0$$
$$\text{or } 2x - y = 0$$
$$\text{or } y = 2x$$

Hence the equation is $y = 2x$

## Chapter Determinants Ex 6.3 Q12(ii)

Let $A(x, y)$, $B(3, 1)$ and $C(9, 3)$ are 3 points in a line.

Since these points are collinear, hence the area of the triangle $ABC$ must be zero.

$$\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$$

Expanding along $R_1$
$$x(-2) - y(-6) + 1(0) = 0$$
$$-2x + 6y = 0$$
$$x - 3y = 0$$

Hence the equation of the line is $x - 3y = 0$

## Chapter Determinants Ex 6.3 Q13(i)

$$\text{Area} = \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$$

$$\pm 4 = \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$$

Expanding along $R_1$
$$\pm 8 = k(-2) - 0(4 - 0) + 1(8)$$
$$\pm 8 = -2k + 8$$

Taking positive $(+)$ sign
$$+8 = -2k + 8 \qquad \text{or } k = 0$$

Taking negative $(-)$ sign
$$-8 = -2k + 8 \qquad \text{or } k = 8$$

Hence $k = 0, 8$

**Chapter Determinants Ex 6.3 Q13(ii)**

$$4 = \frac{1}{2} \begin{Vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{Vmatrix}$$

$$\pm 8 = \begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}$$

Expanding along $R_1$

$\pm 8 = -2(4 - k) - 0(0 - 0) + 1(0)$

$\pm 8 = -8 + 2k$

Taking positive $(+)$ sign

$+8 = -8 + 2k$         or $k = 8$

Taking negative $(-)$ sign

$-8 = -8 + 2k$         or $k = 0$

Hence $k = 0, 8$

# Ex 6.4

**Chapter 6 Determinants Ex 6.4 Q1**

Let $D = \begin{vmatrix} 1 & -2 \\ -3 & 5 \end{vmatrix} = 5 - 6 = -1$

$D_1 = \begin{vmatrix} 4 & -2 \\ -7 & 5 \end{vmatrix} = 20 - 14 = 6$

$D_2 = \begin{vmatrix} 1 & 4 \\ -3 & -7 \end{vmatrix} = -7 + 12 = 5$

by definition $x = \dfrac{D_1}{D} = \dfrac{6}{-1} = -6$

$y = \dfrac{D_2}{D} = \dfrac{5}{-1} = -5$

Hence $x = -6$
$y = -5$

**Chapter 6 Determinants Ex 6.4 Q2**

Let $D = \begin{vmatrix} 2 & -1 \\ 7 & -2 \end{vmatrix} = -4 + 7 = 3$

$D_1 = \begin{vmatrix} 1 & -1 \\ -7 & -2 \end{vmatrix} = -9$

$D_2 = \begin{vmatrix} 2 & 1 \\ 7 & -7 \end{vmatrix} = -21$

Now, $x = \dfrac{D_1}{D} = \dfrac{-9}{3} = -3$

$y = \dfrac{+D_2}{D} = \dfrac{-21}{3} = -7$

Hence $x = -3$
$y = -7$

**Chapter 6 Determinants Ex 6.4 Q3**

$$\text{Let } D = \begin{vmatrix} 2 & -1 \\ 3 & 5 \end{vmatrix} = 13$$

$$D_1 = \begin{vmatrix} 17 & -1 \\ 6 & 5 \end{vmatrix} = 91$$

$$D_2 = \begin{vmatrix} 2 & 17 \\ 3 & 6 \end{vmatrix} = -39$$

$$x = \frac{D_1}{D} = \frac{91}{13} = 7$$

$$y = \frac{D_2}{D} = \frac{-39}{13} = -3$$

Hence $x = 7$

$y = -3$

**Chapter 6 Determinants Ex 6.4 Q4**

$$\text{Let } D = \begin{vmatrix} 3 & 1 \\ 3 & -1 \end{vmatrix} = -6$$

$$D_1 = \begin{vmatrix} 19 & 1 \\ 23 & -1 \end{vmatrix} = -42$$

$$D_2 = \begin{vmatrix} 3 & 19 \\ 3 & 23 \end{vmatrix} = 12$$

$$x = \frac{D_1}{D} = \frac{-42}{-6} = 7$$

$$y = \frac{D_2}{D} = \frac{12}{-6} = -2$$

Hence $x = 7$

$y = -2$

**Chapter 6 Determinants Ex 6.4 Q5**

$$\text{Let } D = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} = 11$$

$$D_1 = \begin{vmatrix} -2 & -1 \\ 3 & 4 \end{vmatrix} = -5$$

$$D_2 = \begin{vmatrix} 2 & -2 \\ 3 & 3 \end{vmatrix} = 12$$

$$x = \frac{D_1}{D} = \frac{-5}{11}$$

$$y = \frac{D_2}{D} = \frac{12}{11}$$

**Chapter 6 Determinants Ex 6.4 Q6**

$$\text{Let } D = \begin{vmatrix} 3 & a \\ 2 & a \end{vmatrix} = a$$

$$D_1 = \begin{vmatrix} 4 & a \\ 2 & a \end{vmatrix} = 2a$$

$$D_2 = \begin{vmatrix} 3 & 4 \\ 4 & 2 \end{vmatrix} = -2$$

$$x = \frac{D_1}{D} = \frac{2a}{a} = 2$$

$$y = \frac{D_2}{D} = \frac{-2}{a}$$

**Chapter 6 Determinants Ex 6.4 Q7**

Let $D = \begin{vmatrix} 2 & 3 \\ 1 & 6 \end{vmatrix} = 9$

$D_1 = \begin{vmatrix} 10 & 3 \\ 4 & 6 \end{vmatrix} = 48$

$D_2 = \begin{vmatrix} 2 & 10 \\ 1 & 4 \end{vmatrix} = -2$

$x = \dfrac{D_1}{D} = \dfrac{48}{9} = \dfrac{16}{3}$

$y = \dfrac{D_2}{D} = \dfrac{-2}{9}$

**Chapter 6 Determinants Ex 6.4 Q8**

Let $D = \begin{vmatrix} 5 & 7 \\ 4 & 6 \end{vmatrix} = 2$

$D_1 = \begin{vmatrix} -2 & 7 \\ -3 & 6 \end{vmatrix} = 9$

$D_2 = \begin{vmatrix} 5 & -2 \\ 4 & -3 \end{vmatrix} = -7$

$x = \dfrac{D_1}{D} = \dfrac{9}{2}$

$y = \dfrac{D_2}{D} = \dfrac{-7}{2}$

**Chapter 6 Determinants Ex 6.4 Q9**

Let $D = \begin{vmatrix} 9 & 5 \\ -2 & 3 \end{vmatrix} = 37$

$D_1 = \begin{vmatrix} 10 & 5 \\ 8 & 3 \end{vmatrix} = -10$

$D_2 = \begin{vmatrix} 9 & 10 \\ -2 & 8 \end{vmatrix} = 92$

$x = \dfrac{D_1}{D} = \dfrac{-10}{37}$

$y = \dfrac{D_2}{D} = \dfrac{92}{37}$

**Chapter 6 Determinants Ex 6.4 Q10**

Let $D = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -5$

$D_1 = \begin{vmatrix} 1 & 2 \\ 4 & 1 \end{vmatrix} = -7$

$D_2 = \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} = 1$

$x = \dfrac{D_1}{D} = \dfrac{7}{5}$

$y = \dfrac{D_2}{D} = \dfrac{-1}{5}$

**Chapter 6 Determinants Ex 6.4 Q11**

Let $D = \begin{vmatrix} 3 & 1 & 1 \\ 2 & -4 & 3 \\ 4 & 1 & -3 \end{vmatrix}$

Expanding along $R_1$

$= 3(9) + (-1)(-18) + 1(18)$

$= 27 + 18 + 18 = 63$

Again $D_1 = \begin{vmatrix} 2 & 1 & 1 \\ -1 & -4 & 3 \\ -11 & 1 & -3 \end{vmatrix}$

Expanding along $R_1$

$= 2(9) + (-1)(36) + 1(-45)$

$= 18 - 36 - 45 = -63$

Again $D_2 = \begin{vmatrix} 3 & 2 & 1 \\ 2 & -1 & 3 \\ 4 & -11 & -3 \end{vmatrix}$

Expanding along $R_1$ $\quad = 3(3+33) - 2(-18) + 1(-22+4)$

$\qquad\qquad = 108 + 36 - 18 = 126$

Also $D_3 = \begin{vmatrix} 3 & 1 & 2 \\ 2 & -4 & -1 \\ 4 & 1 & -11 \end{vmatrix}$

Expanding along $R_1$

$= 3(45) - 1(-18) + 2(18) = 135 + 18 + 36 = 189$

Now $\quad x = \dfrac{D_1}{D} = \dfrac{-63}{63} = -1$

$\qquad y = \dfrac{D_2}{D} = \dfrac{126}{63} = 2$

$\qquad z = \dfrac{D_3}{D} = \dfrac{189}{63} = 3$

**Chapter 6 Determinants Ex 6.4 Q12**

Let $D = \begin{vmatrix} 1 & -4 & -1 \\ 2 & -5 & 2 \\ -3 & 2 & 1 \end{vmatrix}$

Expanding along $R_1$

$\qquad = 1(-9) + 4(8) - 1(-11) = -9 + 32 + 11 = 34$

Again $D_1 = \begin{vmatrix} 11 & -4 & -1 \\ 39 & -5 & 2 \\ 1 & 2 & 1 \end{vmatrix}$

Expanding along $R_1$

$\qquad = 11(-9) + 4(37) - 1(83) \qquad = -99 + 148 - 83$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad = 148 - 182$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad = -34$

Also $D_2 = \begin{vmatrix} 1 & 11 & -1 \\ 2 & 39 & 2 \\ -3 & 1 & 1 \end{vmatrix}$

Expanding along $R_1$

$\qquad = 1(37) - 11(8) - 1(119)$

$\qquad = 37 - 88 - 119 = -170$

Also $D_3 = \begin{vmatrix} 1 & -4 & 11 \\ 2 & -5 & -39 \\ -3 & 2 & 1 \end{vmatrix}$

Expanding along $R_1$

$\qquad = 1(-5 - 78) + 4(2 + 117) + 11(4 - 15)$

$\qquad = -83 + 476 - 121 = 272$

Now $\quad x = \dfrac{D_1}{D} = \dfrac{-34}{34} = -1$

$\qquad\quad y = \dfrac{D_2}{D} = \dfrac{-170}{34} = -5$

$\qquad\quad z = \dfrac{D_3}{D} = \dfrac{272}{34} = 8$

Hence $x = -1, y = -5, z = 8$

**Chapter 6 Determinants Ex 6.4 Q13**

Let $D = \begin{vmatrix} 6 & 1 & -3 \\ 1 & 3 & -2 \\ 2 & 1 & 4 \end{vmatrix}$

Expanding along $R_1$

$= 6(14) - 1(8) - 3(-5)$

$= 84 - 8 + 15 = 91$

Also $D_1 = \begin{vmatrix} 5 & 1 & -3 \\ 5 & -3 & -2 \\ 8 & 1 & 4 \end{vmatrix}$

Expanding along $R_1$

$= 5(14) - 1(36) - 3(-19) = 70 - 36 + 57 = 91$

Again $D_2 = \begin{vmatrix} 6 & 5 & -3 \\ 1 & 5 & -2 \\ 2 & 8 & 4 \end{vmatrix}$

Expanding along $R_1$

$= 6(36) - 5(8) - 3(-2) = 216 - 40 + 6 = 182$

Also $D_3 = \begin{vmatrix} 6 & 1 & 5 \\ 1 & 3 & 5 \\ 2 & 1 & 8 \end{vmatrix}$

Expanding along $R_1$

$= 6(19) - 1(-2) + 5(-5) = 114 + 2 - 25 = 91$

Now $x = \dfrac{D_1}{D} = \dfrac{91}{91} = 1$

$y = \dfrac{D_2}{D} = \dfrac{182}{91} = 2$

Also $z = \dfrac{D_3}{D} = \dfrac{91}{91} = 1$

Hence $x = 1, y = 2, z = 1$

**Chapter 6 Determinants Ex 6.4 Q14**

Let $D = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$

Expanding along $R_1$

$\qquad = 1(1) - 1(-1) + 0(-1) = 1 + 1 + 0 = 2$

Also $D_1 = \begin{vmatrix} 5 & 1 & 0 \\ 3 & 1 & 1 \\ 4 & 0 & 1 \end{vmatrix}$

Expanding along $R_1$

$\qquad = 5(1) - 1(-1) + 0(-4) = 5 + 1 + 0 = 6$

Again $D_2 = \begin{vmatrix} 1 & 5 & 0 \\ 0 & 3 & 1 \\ 1 & 4 & 1 \end{vmatrix}$

Expanding along $R_1$

$\qquad = 1(-1) - 5(-1) + 0(-3) = -1 + 5 + 0 = 4$

Also $D_3 = \begin{vmatrix} 1 & 1 & 5 \\ 0 & 1 & 3 \\ 1 & 0 & 4 \end{vmatrix}$

$\qquad = 1(4) - 1(-3) + 5(-1) = 4 + 3 - 5 = 2$

Now $\quad x = \dfrac{D_1}{D} = \dfrac{6}{2} = 3$

$\qquad y = \dfrac{D_2}{D} = \dfrac{4}{2} = 2$

$\qquad z = \dfrac{D_3}{D} = \dfrac{2}{2} = 1$

Hence $x = 3, y = 2, z = 1$

**Chapter 6 Determinants Ex 6.4 Q15**

Let $D = \begin{vmatrix} 0 & 2 & -3 \\ 1 & 3 & 0 \\ 3 & 4 & 0 \end{vmatrix}$

Expanding along $R_1$

$= 0(0) - 2(0) - 3(-5) = 15$

Also $D_1 = \begin{vmatrix} 0 & 2 & -3 \\ -4 & 3 & 0 \\ 3 & 4 & 0 \end{vmatrix}$

Expanding along $R_1$

$= 0(0) - 2(0) - 3(-25) = 75$

Again $D_2 = \begin{vmatrix} 0 & 0 & -3 \\ 1 & -4 & 0 \\ 3 & 3 & 0 \end{vmatrix}$

Expanding along $R_1$

$= 0(0) - 0(0) - 3(15) = -45$

Also $D_3 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 3 & -4 \\ 3 & 4 & 3 \end{vmatrix}$

$= 0(25) - 2(15) + 0(1) = -30$

Now $x = \dfrac{D_1}{D} = \dfrac{75}{15} = 5$

$y = \dfrac{D_2}{D} = \dfrac{-45}{15} = -3$

$z = \dfrac{D_3}{D} = \dfrac{-30}{15} = -2$

Hence $x = 5, y = -3, z = -2$

## Chapter 6 Determinants Ex 6.4 Q16

Here $D = \begin{vmatrix} 5 & -7 & 1 \\ 6 & -8 & -1 \\ 3 & 2 & -6 \end{vmatrix}$ $= 5(48 + 2) + 7(-33) + 1(36)$

$= 250 - 231 + 36 = 55$

$D_1 = \begin{vmatrix} 11 & -7 & 1 \\ 15 & -8 & -1 \\ 7 & 2 & -6 \end{vmatrix}$ $= 11(50) + 7(-83) + 1(86)$

$= 550 - 581 + 86 = 55$

$D_2 = \begin{vmatrix} 5 & 11 & 1 \\ 6 & 15 & -1 \\ 3 & 7 & -6 \end{vmatrix}$ $= 5(-83) - 11(-33) + 1(-3)$

$= -415 + 363 - 3 = -55$

$D_3 = \begin{vmatrix} 5 & -7 & 11 \\ 6 & -8 & 15 \\ 3 & 2 & 7 \end{vmatrix}$ $= 5(-86) + 7(-3) + 11(36)$

$= -430 - 21 + 396$

$= -55$

Now $x = \dfrac{D_1}{D} = \dfrac{55}{55} = 1$

$y = \dfrac{D_2}{D} = \dfrac{-55}{55} = -1$

$z = \dfrac{D_3}{D} = \dfrac{-55}{55} = -1$

Hence $x = 1, y = -1, z = -1$

**Chapter 6 Determinants Ex 6.4 Q17**

$2x - 3y - 4z = 29$
$-2x + 5y - z = -15$
$3x - y + 5z = -11$

From the given system of equation we have

$$D = \begin{vmatrix} 2 & -3 & 4 \\ -2 & 5 & -1 \\ 3 & -1 & 5 \end{vmatrix} = 2(25-1) + 3(-10+3) + 4(2-15) = 48 - 21 - 52 = -25$$

$$D_1 = \begin{vmatrix} 29 & -3 & 4 \\ -15 & 5 & -1 \\ 11 & -1 & 5 \end{vmatrix} = 29(25-1) + 3(-75+11) + 4(15-55) = 696 - 192 - 160 = 344$$

$$D_2 = \begin{vmatrix} 2 & 29 & 4 \\ -2 & -15 & -1 \\ 3 & 11 & 5 \end{vmatrix} = 2(-75+11) - 29(-10+3) + 4(-22+45) = -128 + 203 + 92 = 167$$

$$D_3 = \begin{vmatrix} 2 & -3 & 29 \\ -2 & 5 & -15 \\ 3 & -1 & 11 \end{vmatrix} = 2(55-15) + 3(-22+45) + 29(2-15) = 80 + 69 - 377 = -228$$

*So*, by Cramer's Rule, we obtain

$$x = \frac{D_1}{D} = -\frac{344}{25}$$

$$y = \frac{D_2}{D} = -\frac{167}{25}$$

$$z = \frac{D_3}{D} = \frac{228}{25}$$

Note: Answer given in the book is incorrect.

**Chapter 6 Determinants Ex 6.4 Q18**

Here $\quad D = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & -2 \end{vmatrix} \qquad = 1(1) - 1(-3) = 1 + 3 = 4$

$$D_1 = \begin{vmatrix} 1 & 1 & 0 \\ -6 & 0 & 1 \\ 3 & -1 & -2 \end{vmatrix} \qquad = 1(1) - 1(9) \quad = -8$$

$$D_2 = \begin{vmatrix} 1 & 1 & 0 \\ 1 & -6 & 1 \\ 1 & 3 & -2 \end{vmatrix} \qquad = 1(9) - 1(-3) = 12$$

$$D_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & -6 \\ 1 & -1 & 3 \end{vmatrix} \qquad = 1(-6) - 1(9) + 1(-1) = -6 - 9 - 1 \quad = -16$$

Now $\quad x = \dfrac{D_1}{D} = \dfrac{-8}{4} = -2$

$\qquad y = \dfrac{D_2}{D} = \dfrac{12}{4} = 3$

$\qquad z = \dfrac{D_3}{D} = \dfrac{-16}{4} = -4$

Hence $x = -2, y = 3, z = -4$

**Chapter 6 Determinants Ex 6.4 Q19**

$$D = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$$c_2 \to c_2 - c_1, c_3 \to c_3 - c_1$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \end{vmatrix}$$

Now taking $(b-a)$ from $c_2$, and $(c-a)$ $c_3$ common

$$= (b-a)(c-a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b+a & c+a \end{vmatrix}$$

Expanding along $R_1$

$= (b-a)(c-a)[c+a-b-a]$

$= (b-a)(c-a)(c-b)$

$= (a-b)(b-c)(c-a)$

Again $D_1 = - \begin{vmatrix} 1 & 1 & 1 \\ d & b & c \\ d^2 & b^2 & c^2 \end{vmatrix}$

$$c_2 \to c_2 - c_1, c_3 \to c_3 - c_1$$

$$D_1 = - \begin{vmatrix} 1 & 0 & 0 \\ d & b-d & c-d \\ d^2 & b^2-d^2 & c^2-d^2 \end{vmatrix}$$

Taking $(b-d)$ common from $c_2$ and $(c-d)$ from $c_3$

$$- (b-d)(c-d) \begin{vmatrix} 1 & 0 & 0 \\ d & 1 & 1 \\ d^2 & b+d & c+d \end{vmatrix}$$

Expanding along $R_1$

$= -(b-d)(c-d)[1(c+d-b-d)]$

$= -(b-d)(c-d)(c-b)$

$= -(b-c)(c-d)(d-b)$

Again $D_2 = - \begin{vmatrix} 1 & 1 & 1 \\ a & d & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

$$c_2 \to c_2 - c_1, c_3 \to c_3 - c_1$$

$$= - \begin{vmatrix} 1 & 0 & 0 \\ a & d-a & c-a \\ a^2 & d^2-a^2 & c^2-a^2 \end{vmatrix}$$

Taking $(d-a)$ common from $c_2$ and $(c-a)$ from $c_3$

$$= - (d-a)(c-a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & d+a & c+a \end{vmatrix}$$

Expanding along $R_1$

$= - (d-a)(c-a) \times 1[c+a-d-a]$

$= - (d-a)(c-a)(c-d)$

$= - (a-d)(d-c)(c-a)$

Also $\quad D_3 = \; - \begin{vmatrix} 1 & 1 & 1 \\ a & b & d \\ a^2 & b^2 & d^2 \end{vmatrix}$

$c_2 \to c_2 - c_1, c_3 \to c_3 - c_1$

$$= - \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & d-a \\ a^2 & b^2-a^2 & d^2-a^2 \end{vmatrix}$$

Now, taking $(b-a)$ common from $c_2$ and $(d-a)$ from $c_3$

$$= -(b-a)(d-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b+a & d+a \end{vmatrix}$$

Expanding along $R_1$

$= -(b-a)(d-a) \times 1[d+a-b-a]$

$= -(b-a)(d-a)(d-b)$

$= -(a-b)(b-d)(d-a)$

Now $\quad x = \dfrac{D_1}{D} = - \dfrac{(b-c)(c-d)(d-b)}{(a-b)(b-c)(c-a)}$

$\qquad y = \dfrac{D_2}{D} = - \dfrac{(a-d)(d-c)(c-a)}{(a-b)(b-c)(c-a)}$

$\qquad z = \dfrac{D_3}{D} = - \dfrac{(a-b)(b-d)(d-a)}{(a-b)(b-c)(c-a)}$

**Chapter 6 Determinants Ex 6.4 Q20**

Here $D = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -2 & 2 & 2 \\ 2 & 1 & -2 & 2 \\ 3 & -1 & 3 & -3 \end{vmatrix}$

$\therefore D = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & -3 & 1 & 1 \\ 2 & -1 & -4 & 0 \\ 3 & -4 & 0 & -6 \end{vmatrix} = 1\begin{vmatrix} -3 & 1 & 1 \\ -1 & -4 & 0 \\ -4 & 0 & -6 \end{vmatrix} \qquad \begin{bmatrix} C2 \to C2 - C1 \\ C3 \to C3 - C1 \\ C4 \to C4 - C1 \end{bmatrix}$

$\therefore D = \begin{vmatrix} 0 & 0 & 1 \\ -1 & -4 & 0 \\ -22 & 6 & -6 \end{vmatrix} \qquad \begin{bmatrix} C1 \to C1 + 3C3 \\ C2 \to C2 - C3 \end{bmatrix}$

$= 1(-6 - 88) = -94$

$D_1 = \begin{vmatrix} 2 & 1 & 1 & 1 \\ -6 & -2 & 2 & 2 \\ -5 & 1 & -2 & 2 \\ -3 & -1 & 3 & -3 \end{vmatrix} = 188$

$D_2 = \begin{vmatrix} 1 & 2 & 1 & 1 \\ 1 & -6 & 2 & 2 \\ 2 & -5 & -2 & 2 \\ 3 & -3 & 3 & -3 \end{vmatrix} = -282$

$D_3 = \begin{vmatrix} 1 & 1 & 2 & 1 \\ 1 & -2 & -6 & 2 \\ 2 & 1 & -5 & 2 \\ 3 & -1 & -3 & -3 \end{vmatrix} = -141$

$D_4 = \begin{vmatrix} 1 & 1 & 1 & 2 \\ 1 & -2 & 2 & -6 \\ 2 & 1 & -2 & -5 \\ 3 & -1 & 3 & -3 \end{vmatrix} = 47$

Now $x = \dfrac{D_1}{D} = \dfrac{188}{-94} = -2$

$y = \dfrac{D_2}{D} = \dfrac{-282}{-94} = 3$

$z = \dfrac{D_3}{D} = \dfrac{-141}{-94} = \dfrac{3}{2}$

$w = \dfrac{D_4}{D} = \dfrac{47}{-94} = -\dfrac{1}{2}$

Hence $x = -2, y = 3, z = \dfrac{3}{2}, w = -\dfrac{1}{2}$

**Chapter 6 Determinants Ex 6.4 Q21**

Here $D = \begin{vmatrix} 2 & 0 & -3 & 1 \\ 1 & -1 & 0 & 2 \\ 0 & -3 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$

$\therefore D = \begin{vmatrix} 2 & -2 & -5 & 1 \\ 1 & -2 & -1 & 2 \\ 0 & -3 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{vmatrix} = -1 \begin{vmatrix} -2 & -5 & 1 \\ -2 & -1 & 2 \\ -3 & 1 & 1 \end{vmatrix}$ $\qquad \begin{bmatrix} C2 \to C2 - C1 \\ C3 \to C3 - C1 \end{bmatrix}$

$\therefore \quad D = -1 \begin{vmatrix} 1 & -6 & 1 \\ 4 & -3 & 2 \\ 0 & 0 & 1 \end{vmatrix}$ $\qquad \begin{bmatrix} C1 \to C1 + 3C3 \\ C2 \to C2 - C3 \end{bmatrix}$

$= -1(-3 + 24) = -21$

$D_1 = \begin{vmatrix} 1 & 0 & -3 & 1 \\ 1 & -1 & 0 & 2 \\ 1 & -3 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = -21$

$D_2 = \begin{vmatrix} 2 & 1 & -3 & 1 \\ 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = -6$

$D_3 = \begin{vmatrix} 2 & 0 & 1 & 1 \\ 1 & -1 & 1 & 2 \\ 0 & -3 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = -6$

$D_4 = \begin{vmatrix} 2 & 0 & -3 & 1 \\ 1 & -1 & 0 & 1 \\ 0 & -3 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 3$

Now $\quad x = \dfrac{D_1}{D} = \dfrac{-21}{-21} = 1$

$y = \dfrac{D_2}{D} = \dfrac{-6}{-21} = \dfrac{2}{7}$

$z = \dfrac{D_3}{D} = \dfrac{-6}{-21} = \dfrac{2}{7}$

$w = \dfrac{D_4}{D} = \dfrac{3}{-21} = -\dfrac{1}{7}$

## Chapter 6 Determinants Ex 6.4 Q22

Let $D = \begin{vmatrix} 2 & -1 \\ 4 & -2 \end{vmatrix}$

Expanding along $R_1$

$= -4 + 4 = 0$

Also $D_1 = \begin{vmatrix} 5 & -1 \\ 7 & -2 \end{vmatrix} = -3$

Also $D_2 = \begin{vmatrix} 2 & 5 \\ 4 & 7 \end{vmatrix} = -6$

And since $D = 0$ and $D_1$ and $D_2$ are non-zero, hence the given system of equations is inconsistent.

Hence proved.

## Chapter 6 Determinants Ex 6.4 Q23

$$D = \begin{vmatrix} 3 & 1 \\ -6 & -2 \end{vmatrix} = -6 + 6 = 0$$

$$D_1 = \begin{vmatrix} 5 & 1 \\ 9 & -2 \end{vmatrix} = -10 - 9 = -19 \neq 0$$

Since $D = 0$ but $D_1 \neq 0$

Hence the given system of equations is inconsistent.

### Chapter 6 Determinants Ex 6.4 Q24

Here $D = \begin{vmatrix} 3 & -1 & 2 \\ 2 & 1 & 3 \\ 1 & -2 & -1 \end{vmatrix}$

Expanding along $R_1$

$$= 3(5) + 1(-5) + 2(-5)$$
$$= 15 - 5 - 10 = 15 - 15 = 0$$

Also $D_1 = \begin{vmatrix} 3 & -1 & 2 \\ 5 & 1 & 3 \\ 1 & -2 & -1 \end{vmatrix}$

Expanding along $R_1$

$$= 3(5) + 1(-8) + 2(-11)$$
$$= 15 - 8 - 22$$
$$= -15 \neq 0$$

Since $D = 0$ and $D_1 \neq 0$

Hence the given system of equations is inconsistent.

### Chapter 6 Determinants Ex 6.4 Q25

Here $D = \begin{vmatrix} 3 & -1 & 2 \\ 2 & -1 & 1 \\ 3 & 6 & 5 \end{vmatrix} = 3(-11) + 1(7) + 2(15) = -33 + 7 + 30 = 4$

$$D_1 = \begin{vmatrix} 6 & -1 & 2 \\ 2 & -1 & 1 \\ 20 & 6 & 5 \end{vmatrix} = 12$$

$$D_2 = \begin{vmatrix} 3 & 6 & 2 \\ 2 & 2 & 1 \\ 3 & 20 & 5 \end{vmatrix} = -4$$

$$D_3 = \begin{vmatrix} 3 & -1 & 6 \\ 2 & -1 & 2 \\ 3 & 6 & 20 \end{vmatrix} = 28$$

Now $x = \dfrac{D_1}{D} = \dfrac{12}{4} = -3$

$y = \dfrac{D_2}{D} = \dfrac{-4}{4} = -1$

$z = \dfrac{D_3}{D} = \dfrac{28}{4} = 7$

### Chapter 6 Determinants Ex 6.4 Q26

We have,

$$D = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ -1 & -2 & 2 \end{vmatrix} = \begin{vmatrix} 0 & -1 & 0 \\ 3 & 1 & 0 \\ -3 & -2 & 0 \end{vmatrix} = 0$$

$$D_1 = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & -2 & 2 \end{vmatrix} = \begin{vmatrix} 3 & -1 & 0 \\ 2 & 1 & 0 \\ 1 & -2 & 0 \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 1 & 3 & 1 \\ 2 & 2 & -1 \\ -1 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & -4 & -3 \\ -1 & 4 & 3 \end{vmatrix} = 1(-12 + 12) = 0$$

$$D_3 = \begin{vmatrix} 1 & -1 & 3 \\ 2 & 1 & 2 \\ -1 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 3 & -4 \\ -1 & -3 & 4 \end{vmatrix} = 1(12 - 12) = 0$$

$\therefore$     $D = D_1 = D_2 = D_3 = 0$

So, either the system is consistent with infinitely many solutions or it is inconsistent.

Consider the first two equations, written as

$x - y = 3 - z$

$2x + y = 2 + z$

To solve these equations we use Cramer's rule.

Here,

$$D = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 1 + 2 = 3$$

$$D_1 = \begin{vmatrix} 3 - z & -1 \\ 2 + z & 1 \end{vmatrix} = (3 - z) + (2 + z) = 5$$

$$D_2 = \begin{vmatrix} 1 & 3 - z \\ 2 & 2 + z \end{vmatrix} = (2 + z) - (6 - 2z) = -4 + 3z$$

$\therefore$     $x = \dfrac{D_1}{D} = \dfrac{5}{3}$

$y = \dfrac{D_2}{D} = \dfrac{-4 + 3z}{3}$

Let $z = k$, then the equations have the solution.

$x = \dfrac{5}{3}$, $y = \dfrac{-4 + 3k}{3}$, $z = k$

## Chapter 6 Determinants Ex 6.4 Q27

Here,

$$D = \begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} = 6 - 6 = 0$$

$$D_1 = \begin{vmatrix} 5 & 2 \\ 15 & 6 \end{vmatrix} = 30 - 30 = 0$$

$$D_2 = \begin{vmatrix} 1 & 5 \\ 3 & 15 \end{vmatrix} = 15 - 15 = 0$$

So, $D = D_1 = D_2 = 0$

Let $y = k$, then we have,

$x + 2y = 5$

$\Rightarrow$     $x = 5 - 2y = 5 - 2k$

$\therefore$     $x = 5 - 2k$, $y = k$ are the infinite solutions of the given system.

## Chapter 6 Determinants Ex 6.4 Q28

Here,

$$D = \begin{vmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ 3 & 6 & -5 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & -3 & 2 \\ 3 & 3 & -2 \end{vmatrix} = 1(6-6) = 0$$

$$D_1 = \begin{vmatrix} 0 & 1 & -1 \\ 0 & -2 & 1 \\ 0 & 6 & -5 \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 1 & 0 & -1 \\ 1 & 0 & 1 \\ 3 & 0 & -5 \end{vmatrix} = 0$$

$$D_3 = \begin{vmatrix} 1 & 1 & 0 \\ 1 & -2 & 0 \\ 3 & 6 & 0 \end{vmatrix} = 0$$

So, $D = D_1 = D_2 = D_3 = 0$

The given system either has infinite solutions or it is inconsistent.

Consider the first two equations, written as

$$x + y = z$$
$$x - 2y = -z$$

To solve this we will use Cramer's rule

Here,

$$D = \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -2 - 1 = -3$$

$$D_1 = \begin{vmatrix} z & 1 \\ -z & -2 \end{vmatrix} = -2z - z = -z$$

$$D_2 = \begin{vmatrix} 1 & z \\ 1 & -z \end{vmatrix} = -z - z = -2z$$

$$\therefore \quad x = \frac{D_1}{D} = \frac{-z}{-3} = \frac{z}{3}$$

$$y = \frac{D_2}{D} = \frac{-2z}{-3} = \frac{2z}{3}$$

Let $z = k$, then the solutions of the given system are

$$x = \frac{k}{3}, \; y = \frac{2k}{3}, \; z = k$$

## Chapter 6 Determinants Ex 6.4 Q29

Here,

$$D = \begin{vmatrix} 2 & 1 & -2 \\ 1 & -2 & 1 \\ 5 & -5 & 1 \end{vmatrix} = \begin{vmatrix} 12 & 9 & -2 \\ -4 & -3 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 1(-36+36) = 0$$

$$D_1 = \begin{vmatrix} 4 & 1 & -2 \\ -2 & -2 & 1 \\ -2 & -5 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 1 & -2 \\ 0 & -2 & 1 \\ 0 & -5 & 1 \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 2 & 4 & -2 \\ 1 & -2 & 1 \\ 5 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 0 & -2 \\ 1 & 0 & 1 \\ 5 & 0 & 1 \end{vmatrix} = 0$$

$$D_3 = \begin{vmatrix} 2 & 1 & 4 \\ 1 & -2 & -2 \\ 5 & -5 & -2 \end{vmatrix} = \begin{vmatrix} 4 & -3 & 0 \\ 1 & -2 & -2 \\ 4 & -3 & 0 \end{vmatrix} = 2(-12+12) = 0$$

So, $D = D_1 = D_2 = D_3 = 0$

So, the given system is either inconsistent or has infinite solutions.

Consider the 2nd and 3rd equation, written as

$$x - 2y = -2 - z$$
$$5x - 5y = -2 - z$$

Then,

$$D = \begin{vmatrix} 1 & -2 \\ 5 & -5 \end{vmatrix} = -5 - (-10) = 5$$

$$D_1 = \begin{vmatrix} -2-z & -2 \\ -2-z & -5 \end{vmatrix} = (2+z)(5) - 2(2+z) = 3(2+z) = 6+3z$$

$$D_2 = \begin{vmatrix} 1 & -(2+z) \\ 5 & -(2+z) \end{vmatrix} = -(2+z) + 5(2+z) = 4(2+z) = 8+4z$$

$$\therefore \quad x = \frac{D_1}{D} = \frac{6+3z}{5}$$

$$y = \frac{D_2}{D} = \frac{8+4z}{5}$$

Let $z = k$, then

$$x = \frac{6+3k}{5}, \; y = \frac{8+4k}{5}, \; z = k \text{ are the infinite solution of the given system of equations.}$$

## Chapter 6 Determinants Ex 6.4 Q30

Here,

$$D = \begin{vmatrix} 1 & -1 & 3 \\ 1 & 3 & -3 \\ 5 & 3 & 3 \end{vmatrix} = \begin{vmatrix} 2 & 2 & 0 \\ 1 & 3 & -3 \\ 6 & 6 & 0 \end{vmatrix} = 3(12 - 12) = 0$$

$$D_1 = \begin{vmatrix} 6 & -1 & 3 \\ -4 & 3 & -3 \\ 10 & 3 & 3 \end{vmatrix} = \begin{vmatrix} 2 & 2 & 0 \\ -4 & 3 & -3 \\ 6 & 6 & 0 \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 1 & 6 & 3 \\ 1 & -4 & -3 \\ 5 & 10 & 3 \end{vmatrix} = \begin{vmatrix} 2 & 2 & 0 \\ 1 & -4 & -3 \\ 6 & 6 & 0 \end{vmatrix} = 0$$

$$D_3 = \begin{vmatrix} 1 & -1 & 6 \\ 1 & 3 & -4 \\ 5 & 3 & 10 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 6 \\ 0 & 4 & -10 \\ 0 & 8 & -20 \end{vmatrix} = 1(-80 + 80) = 0$$

So, $D = D_1 = D_2 = D_3 = 0$

So, the given system is either inconsistent or has infinite solutions.

Consider the first to equations, written as

$$x - y = 6 - 3z$$
$$x + 3y = -4 + 3z$$

Here,

$$D = \begin{vmatrix} 1 & -1 \\ 1 & 3 \end{vmatrix} = 3 + 1 = 4$$

$$D_1 = \begin{vmatrix} 6 - 3z & -1 \\ -4 + 3z & 3 \end{vmatrix} = 3(6 - 3z) + (-4 + 3z) = 14 - 6z$$

$$D_2 = \begin{vmatrix} 1 & 6 - 3z \\ 1 & -4 + 3z \end{vmatrix} = (-4 + 3z) - (6 - 3z) = -10 + 6z$$

$$\therefore \quad x = \frac{D_1}{D} = \frac{14 - 6z}{4} = \frac{7 - 3z}{2}$$

$$y = \frac{D_2}{D} = \frac{6z - 10}{4} = \frac{3z - 5}{2}$$

Let $z = k$, then

$$x = \frac{7 - 3k}{2}, \ y = \frac{3k - 5}{2}, \ z = k \text{ are the infinite solution of the given system of equations.}$$

## Chapter 6 Determinants Ex 6.4 Q31

Let the rates of commissions on items $A$, $B$ and $C$ be $x$, $y$ and $z$ respectively.

Then we can express the given model as a system of linear equations

$$90x + 100y + 20z = 800$$
$$130x + 50y + 40z = 900$$
$$60x + 100y + 30z = 850$$

We will solve this using the Cramer's rule.

Here,

$$D = \begin{vmatrix} 90 & 100 & 20 \\ 130 & 50 & 40 \\ 60 & 100 & 30 \end{vmatrix} = \begin{vmatrix} -170 & 0 & -60 \\ 130 & 50 & 40 \\ -200 & 0 & -50 \end{vmatrix} = 50(8500 - 12000) = -175000$$

$$D_1 = \begin{vmatrix} 800 & 100 & 20 \\ 900 & 50 & 40 \\ 850 & 100 & 30 \end{vmatrix} = \begin{vmatrix} -1000 & 0 & -60 \\ 900 & 50 & 40 \\ -950 & 0 & -50 \end{vmatrix} = 50(50000 - 57000) = -350000$$

$$D_2 = \begin{vmatrix} 90 & 800 & 20 \\ 130 & 900 & 40 \\ 60 & 850 & 30 \end{vmatrix} = \begin{vmatrix} 90 & 800 & 20 \\ -50 & -700 & 0 \\ -75 & -350 & 0 \end{vmatrix} = 20(17500 - 52500) = -700000$$

$$D_3 = \begin{vmatrix} 90 & 100 & 800 \\ 130 & 50 & 900 \\ 60 & 100 & 850 \end{vmatrix} = \begin{vmatrix} -170 & 0 & -1000 \\ 130 & 50 & 900 \\ -200 & 0 & -950 \end{vmatrix} = 50(161500 - 200000) = -1925000$$

$$\therefore \quad x = \frac{D_1}{D} = \frac{-350000}{-175000} = 2$$

$$y = \frac{D_2}{D} = \frac{-700000}{-175000} = 4$$

$$z = \frac{D_3}{D} = \frac{-1925000}{-175000} = 11$$

$\therefore$ The rates of commission of items $A$, $B$ and $C$ are 2%, 4% and 11% respectively.

## Chapter 6 Determinants Ex 6.4 Q32

Expressing the given information as a system of linear equations we get

$$2x + 3y + 4z = 29$$
$$x + y + 2z = 13$$
$$3x + 2y + z = 16$$

Where $x$, $y$, $z$ is the number of cars $C_1$, $C_2$ and $C_3$ produced.

We use Cramer's rule to solve this system.

Here,

$$D = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 1 & 2 \\ 3 & 2 & 1 \end{vmatrix} = \begin{vmatrix} -10 & -5 & 0 \\ -5 & -3 & 0 \\ 3 & 2 & 1 \end{vmatrix} = 1(30 - 25) = 5$$

$$D_1 = \begin{vmatrix} 29 & 3 & 4 \\ 13 & 1 & 2 \\ 16 & 2 & 1 \end{vmatrix} = \begin{vmatrix} -35 & -5 & 0 \\ -19 & -3 & 0 \\ 16 & 2 & 1 \end{vmatrix} = 1(105 - 95) = 10$$

$$D_2 = \begin{vmatrix} 0 & 29 & 4 \\ 1 & 13 & 2 \\ 3 & 16 & 1 \end{vmatrix} = \begin{vmatrix} -10 & -35 & 0 \\ -5 & -19 & 0 \\ 3 & 16 & 1 \end{vmatrix} = 1(190 - 175) = 15$$

$$D_3 = \begin{vmatrix} 2 & 3 & 29 \\ 1 & 1 & 13 \\ 3 & 2 & 16 \end{vmatrix} = \begin{vmatrix} -2 & 0 & 0 \\ 1 & 1 & 13 \\ 3 & 2 & 16 \end{vmatrix} = -2(16 - 26) = 20$$

$$\therefore \quad x = \frac{D_1}{D} = \frac{10}{5} = 2$$

$$y = \frac{D_2}{D} = \frac{15}{5} = 3$$

$$\text{and} \quad z = \frac{D_3}{D} = \frac{20}{5} = 4$$

Hence, the number of cars produced of type $C_1$, $C_2$ and $C_3$ are 2, 3 and 4 respectively.

# Ex 6.5

**Chapter 6 Determinants Ex 6.5 Q1**

Here $D = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$

$\qquad = 1(3) - 1(-3) - 2(3)$

$\qquad = 3 + 3 - 6$

$\qquad = 0$

Since $D = 0$, so the system has infinite solutions:

Now let $z = k$,

$\qquad x + y = 2k$

$\qquad 2x + y = 3k$

Solving there equations by cramer's Rule

$x = \dfrac{D_1}{D} = \dfrac{\begin{vmatrix} 2k & 1 \\ 3k & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}} = \dfrac{-k}{-1} = k$

$y = \dfrac{D_2}{D} = \dfrac{\begin{vmatrix} 1 & 2k \\ 2 & 3k \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}} = \dfrac{-k}{-1} = k$

thus, we have $x = k, y = k, z = k$

and there values satisfy eq.$(3)$

Hence $x = k, y = k, z = k$

**Chapter 6 Determinants Ex 6.5 Q2**

Here $D = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix}$

$= 2(4) - 3(1) + 4(-3)$

$= 8 - 3 - 7$

$= -2$

$\neq 0$

So, the given system of equations has only the trivial solutions i.e $x = 0 = y = z$:

Hence $x = 0$

$\quad\quad y = 0$

$\quad\quad z = 0$

### Chapter 6 Determinants Ex 6.5 Q3

Here $D = \begin{vmatrix} 3 & 1 & 1 \\ 1 & -4 & 3 \\ 2 & 5 & -2 \end{vmatrix}$

$= 3(8 - 15) - 1(-2 - 6) + 1(13)$

$= -21 + 8 + 13$

$= 0$

So, the system has infinite solutions:

Let $z = k$,

so, $\quad 3x + y = -k$

$\quad\quad x - 4y = -3k$

Now,

$x = \dfrac{D_1}{D} = \dfrac{\begin{vmatrix} -k & 1 \\ -3k & -4 \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 1 & -4 \end{vmatrix}} = \dfrac{7k}{-13}$

$y = \dfrac{D_2}{D} = \dfrac{\begin{vmatrix} 3 & -k \\ 1 & -3k \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 1 & -4 \end{vmatrix}} = \dfrac{-8k}{-13}$

$x = \dfrac{-7k}{13}, y = \dfrac{8k}{13}, z = k$

and there values satisfy eq.(3)

Hence $x = -7k, y = 8k, z = 13k$

### Chapter 6 Determinants Ex 6.5 Q4

$D = \begin{vmatrix} 2\lambda & -2 & 3 \\ 1 & \lambda & 2 \\ 2 & 0 & \lambda \end{vmatrix}$

$= 3\lambda^3 + 2\lambda - 8 - 6\lambda$

$= 2\lambda^3 - 4\lambda - 8$

which is satisfied by $\lambda = 2$ $\quad \left[ \because \quad\quad \text{for non-trivial solutions } \lambda = 2 \right]$

Now Let $z = k$,

$\quad\quad 4x - 2y = -3k$

$\quad\quad x + 2y = -3k$

$x = \dfrac{D_1}{D} = \dfrac{\begin{vmatrix} -3k & -2 \\ -2k & 2 \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ 1 & 2 \end{vmatrix}} = \dfrac{-10k}{10} = -k$

$y = \dfrac{D_2}{D} = \dfrac{\begin{vmatrix} 4 & -3k \\ 1 & -2k \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ 1 & 2 \end{vmatrix}} = \dfrac{-5k}{10} = \dfrac{-k}{2}$

Hence solution is $x = -k, y = \dfrac{-k}{2}, z = k$

### Chapter 6 Determinants Ex 6.5 Q5

$$D = \begin{vmatrix} (a-1) & -1 & -1 \\ -1 & (b-1) & -1 \\ -1 & -1 & (c-1) \end{vmatrix}$$

Now for non-trivial solution, $D = 0$

$$0 = (a-1)\big[(b-1)(c-1) - 1\big] + 1\big[-c + \cancel{1} - \cancel{1}\big] - \big[\cancel{1} + b - \cancel{1}\big]$$

$$0 = (a-1)\big[bc - b - c + \cancel{1} - \cancel{1}\big] - c - b$$

$$0 = abc - ab - ac + \cancel{b} + \cancel{c} - \cancel{c} - \cancel{b}$$

$$ab + bc + ac = abc$$

Hence proved