Ex 29.1

Q1

$$\lim_{x \to 0} \frac{x}{|x|}$$

We know that $|x| = \begin{cases} x, \text{if } x \geq 0 \\ -x, \text{if } x < 0 \end{cases}$

$\therefore \quad \lim_{x \to 0^+} \frac{x}{|x|} = \lim_{x \to 0^+} \frac{x}{x} = \lim_{x \to 0^+} 1 = 1$

Also, $\lim_{x \to 0^-} \frac{x}{|x|} = \lim_{x \to 0^-} \frac{x}{-x} = \lim_{x \to 0^-} -1 = -1$

$\Rightarrow \quad$ LHL of $f(x) \neq$ RHL of $f(x)$

$\Rightarrow \quad \lim_{x \to 0} \frac{x}{|x|}$ does not exist

Q2

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (2x + 3)$$
$$= 2(2) + 3$$
$$= 7$$

$\therefore \quad \lim_{x \to 2^-} f(x) = 7$

Also,

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x + k)$$
$$= (2 + k)$$

Since, $\lim_{x \to 2} f(x)$ exists (given)

$\therefore \quad \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

$\Rightarrow \quad 7 = 2 + k$

$\Rightarrow \quad k = 5$

Q3

Let $f(x) = \frac{1}{x}$, this function is defined for every value of x except at $x = 0$

As $x \to 0^+, \frac{1}{x} \to \infty.$

As $x \to 0^-, \frac{1}{x} \to -\infty$

$\therefore \quad \lim_{x\to 0} \frac{1}{x}$ does not exist.

Q4

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{3x}{-x+2x} = \lim_{x\to 0^-} \frac{3x}{x} = 3 \qquad \left[\because \text{ as } x \to 0^-, |x| = -x\right]$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{3x}{x+2x} = 1 \qquad \left[\because \text{ as } x \to 0^+, |x| = x\right]$

thus, $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

$\therefore \quad \lim_{x\to 0} f(x)$ does not exist.

Q5

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x + 1$$
$$= \lim_{h\to 0} (0+h) + 1 = 1$$

Also, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x - 1$

$\lim_{h\to 0} (0-h) - 1 = -1$

$\Rightarrow \quad \lim_{x\to 0^+} f(x) \neq \lim_{x\to 0^-} f(x)$

Hence, limit does not exist.

Q6

$$\begin{aligned}
\text{LHL} &= \lim_{x\to 0^-} f(x) \\
&= \lim_{h\to 0} f(0-h) \\
&= \lim_{h\to 0} -h-4 \\
&= 0-4 \\
&= -4
\end{aligned}$$

$$\begin{aligned}
\text{RHL} &= \lim_{x\to 0^+} f(x) \\
&= \lim_{h\to 0} f(0+h) \\
&= \lim_{h\to 0} h+5 \\
&= 0+5 \\
&= 5
\end{aligned}$$

$\therefore \lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

Hence $\lim_{x\to 0} f(x)$ does not exist.

Q7

$\lim_{x\to 3^+} f(x) = 4$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (x+1) = \lim_{h\to 0} (3-h+1) = 3+1 = 4$

Since, $\lim_{x\to 3^+} f(x) = 4 = \lim_{x\to 3^-} f(x)$

$\therefore \quad \lim_{x\to 3} f(x)$ is 4

Q8

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} 3(x+1) = 3$$

and

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} 2(x) + 3 = 3$$

$$\therefore \qquad \lim_{x \to 0} f(x) = 3$$

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} 3(x+1) = 3 + 3 = 6$$

$$\therefore \qquad \lim_{x \to 1} f(x) = 6$$

Q9

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2 - 1 = \lim_{h \to 0} (-1-h)^2 - 1 = 1 - 1 = 0$$
$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} -x^2 - 1 = \lim_{h \to 0} -(1+h)^2 - 1 = -2$$

Since, $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$

$\therefore$ $\lim_{x \to 1} f(x)$ does not exist.

Q10

$$\text{LHL} = \lim_{x \to 0^-} f(x)$$

$$\Rightarrow \quad \lim_{x \to 0^-} \frac{|x|}{x}$$

$$\Rightarrow \quad \lim_{h \to 0} \frac{|0-h|}{0-h}$$

$$\Rightarrow \quad \lim_{h \to 0} \frac{+h}{-h} = -1 \qquad \text{---(i)}$$

and,

$$\text{RHL} = \lim_{x \to 0^+} f(x)$$

$$\Rightarrow \quad \lim_{x \to 0^+} \frac{|x|}{x}$$

$$\Rightarrow \quad \lim_{h \to 0} \frac{|0+h|}{0+h} = \lim_{h \to 0} \frac{h}{h} = 1 \qquad \text{---(ii)}$$

So, LHL $\neq$ RHL

$\therefore \quad \lim_{x \to 0} f(x)$ does not exist.

Q11

$$\lim_{x \to a_1} f(x)$$

$$\Rightarrow \quad \lim_{x \to a_1} (x-a_1)(x-a_2)\ldots(x-a_n) \qquad [\text{Putting limit } x \to a_1]$$

$$\Rightarrow \quad (a_1-a_1)(a_1-a_2)\ldots(a_1-a_n)$$

$$\Rightarrow \quad 0$$

And, $\lim_{x \to a} f(x)$

$$\Rightarrow \quad \lim_{x \to a} (x-a_1)(x-a_2)\ldots(x-a_n) \qquad [\text{Putting limit } x \to a]$$

$$\Rightarrow \quad (a-a_1)(a-a_2)\ldots(a-a_n).$$

Q12

$$\lim_{x \to 1^+} \frac{1}{(x-1)} = \lim_{h \to 0} \frac{1}{(1+h-1)} = \lim_{h \to 0} \frac{1}{h} = \infty$$

Q13(i)

$$\lim_{x\to 2^+}\frac{x-3}{x^2-4}$$

$$=\lim_{h\to 0}\frac{(2+h)-3}{(2+h)^2-2^2} \qquad \left[\because \lim_{x\to 2^+} f(x)=\lim_{h\to 0} f(2+h)\right]$$

$$=\lim_{h\to 0}\frac{(2-3+h)}{(2+h-2)(2+h+2)}$$

$$=\lim_{h\to 0}\frac{(h-1)}{(h)(4+h)}$$

$$=\lim_{h\to 0}\frac{1-\frac{1}{h}}{4+h}$$

$$=\frac{1-\frac{1}{0}}{4}=-\infty$$

Q13(ii)

$$\lim_{x\to 2^-}\frac{x-3}{\left(x^2-4\right)}=\lim_{h\to 0}\frac{(2-h)-3}{(2-h)^2-4}$$

$$=\lim_{h\to 0}\frac{(2-h-3)}{(2-h+2)(2-h-2)} \qquad \left[\because \lim_{x\to 2^-} f(x)=\lim_{h\to 0} f(2-h)\right]$$

$$=\lim_{h\to 0}\frac{-1-h}{(4-h)(-h)}$$

$$=\lim_{h\to 0}\frac{\frac{1}{h}+1}{(4-h)}$$

$$=\frac{\frac{1}{0}+1}{4}=\infty$$

Q13(iii)

$$\lim_{x\to 0^+}\frac{1}{3x}$$

$$=\lim_{h\to 0}\frac{1}{3(0+h)}$$

$$=\lim_{h\to 0}\frac{1}{0+3h}$$

$$=\frac{1}{0}=\infty$$

Q13(iv)

$$\lim_{x\to -8^+}\frac{2x}{x+8}$$

$$= \lim_{h\to 0}\frac{2(-8+h)}{(-8+h)+8}$$

$$= \lim_{h\to 0}\frac{-16+2h}{h}$$

$$= \lim_{h\to 0}\frac{-16}{h}+2$$

$$\Rightarrow \quad \frac{-16}{0}+2=\infty$$

Q13(v)

$$\lim_{x\to 0^+}\frac{2}{x^{\frac{1}{5}}}$$

$$= \lim_{h\to 0}\frac{2}{(0+h)^{\frac{1}{5}}}$$

$$\Rightarrow \frac{2}{0}=\infty$$

Q13(vi)

$$\lim_{x\to \frac{\pi}{2}^-}\tan x$$

$$= \lim_{h\to 0}\tan\left(\frac{\pi}{2}-h\right)$$

$$= \tan\left(\frac{\pi}{2}-0\right)$$

$$\Rightarrow \quad \tan\frac{\pi}{2}=\infty$$

Q13(vii)

$$\lim_{x \to -\frac{\pi}{2}^+} \sec x$$

$$= \lim_{h \to 0} \sec\left(-\frac{\pi}{2} + h\right)$$

$$= \sec\left(-\frac{\pi}{2} + 0\right)$$

$$= \sec\left(-\frac{\pi}{2}\right)$$

$$= \frac{1}{\cos\left(-\frac{\pi}{2}\right)}$$

$$= \frac{-1}{\left(\cos\frac{\pi}{2}\right)}$$

$$= \frac{-1}{0} = -\infty$$

Q13(viii)

$$\lim_{x \to 0^-} \frac{x^2 - 3x + 2}{x^3 - 2x^2}$$

$$= \lim_{x \to 0^-} \frac{x^2 - x - 2x + 2}{x^2(x-2)}$$

$$= \lim_{x \to 0^-} \frac{x(x-1) - 2(x-1)}{x^2(x-2)}$$

$$= \lim_{x \to 0^-} \frac{(x-1)(x-2)}{x^2(x-2)}$$

$$= \lim_{x \to 0^-} \frac{(x-1)}{x^2}$$

$$= \lim_{h \to 0} \frac{(0-h-1)}{(0-h)^2}$$

$$= \frac{-h}{h^2} = \frac{-1}{h} = \frac{-1}{0} = -\infty$$

Q13(ix)

$$\lim_{x \to -2^+} \frac{x^2 - 1}{2x + 4}$$

$$= \lim_{x \to 2^+} \frac{(x-1)(x+1)}{2(x+2)}$$

$$= \lim_{h \to 0} \frac{(-2+h-1)(-2+h+1)}{2(-2+h+2)}$$

$$= \lim_{h \to 0} \frac{(-3+h)(h-1)}{2h}$$

$$\Rightarrow \quad \frac{-3 \times -1}{2 \times 0} = \frac{1}{0} = \infty$$

Q13(x)

$$\lim_{x \to 0^-} 2 - \cot x$$

$$= \lim_{h \to 0} \; 2 - \cot(0-h)$$

$$= \lim_{h \to 0} \; 2 - (-1)\cot h$$

$$= \lim_{h \to 0} \; 2 + \cot h$$

$$= \lim_{h \to 0} \; 2 + \frac{1}{\tan h}$$

$$\Rightarrow \quad 2 + \frac{1}{0} \Leftarrow \infty$$

Q13(xi)

$$\lim_{x \to 0^-} \; 1 + \operatorname{cosec} x$$

$$= \lim_{x \to 0^-} \; 1 + \operatorname{cosec}(0-h)$$

$$= \lim_{h \to 0} \; 1 - \operatorname{cosec} h$$

$$= \lim_{h \to 0} \; 1 - \frac{1}{\sin h}$$

$$\Rightarrow \quad 1 - \frac{1}{0} = -\infty$$

Q14

$$\lim_{x \to 0} e^{\frac{-1}{x}}$$

$$\lim_{x \to 0^+} e^{\frac{-1}{x}} = \lim_{x \to 0^+} \frac{1}{e^{\frac{1}{x}}} = \lim_{h \to 0} \frac{1}{e^{\frac{1}{0+h}}} = \frac{1}{e^{\frac{1}{0}}} = \frac{1}{e^{\infty}} \quad = \frac{1}{\infty} = 0$$

$$And, \lim_{x \to 0^-} e^{\frac{-1}{x}} = \lim_{x \to 0^-} \frac{1}{e^{\frac{1}{x}}} = \lim_{h \to 0} \frac{1}{e^{\frac{1}{0-h}}} = \lim_{h \to 0} \frac{1}{e^{-\frac{1}{h}}} = \frac{1}{e^{-\frac{1}{0}}} = \frac{1}{e^{-\infty}} = e^{\infty} = \infty$$

$$\Rightarrow \quad \lim_{x \to 0^+} e^{\frac{-1}{x}} \neq \lim_{x \to 0^-} e^{\frac{-1}{x}}$$

$$\therefore \quad \lim_{x \to 0} e^{\frac{-1}{x}} \text{ does not exist .}$$

Q15

(i) $\lim_{x\to 2}[x]$

$$\lim_{x\to 2^-}[x]=1$$
$$\lim_{x\to 2^+}[x]=2$$

Thus, $\lim_{x\to 2}[x]$ does not exist.

(ii) $\lim_{x\to \frac{5}{2}}[x]$

$$\lim_{x\to \frac{5}{2}^+}[x]=2$$
$$\lim_{x\to \frac{5}{2}^-}[x]=2$$

$\Rightarrow \quad \lim_{x\to \frac{5}{2}}[x]=2$

(iii) $\lim_{x\to 1}[x]$

$$\lim_{x\to 1^-}[x]=0$$
$$\lim_{x\to 1^+}[x]=1$$

$\Rightarrow \quad \lim_{x\to 1^-}[x]\neq \lim_{x\to 1^+}[x]$

Thus, $\lim_{x\to 1}[x]$ does not exist

Q16

$$\lim_{x\to a^+}[x]$$

$\Rightarrow \quad \lim_{h\to 0^-}[a+h]=[a]$

$\Rightarrow \quad \lim_{h\to 0^-}[x]=[a]\forall a\in R$

Also, $\lim_{x\to 1^-}[x]$

$=\lim_{h\to 0}[1-h]$

$=0$

$\Rightarrow \quad \lim_{x\to 1^-}[x]=0$

Q17

$$\lim_{x\to 2^-}\frac{x}{[x]} = \lim_{x\to 2^-}\frac{x}{1} = \frac{2}{1} = 2 \qquad \left[\because \lim_{x\to k^-}[x] = k-1\right]$$

Also, $\lim_{x\to 2^+}\frac{x}{[x]} = \lim_{x\to 2^+}\frac{x}{3} = \frac{2}{3}$ $\qquad \left[\lim_{x\to k^+}[x] = k+1\right]$

$$\Rightarrow \quad \lim_{x\to 2^-}\frac{x}{[x]} \neq \lim_{x\to 2^+}\frac{x}{[x]}$$

Q18

$$\lim_{x\to 3^+}\frac{x}{[x]} = \lim_{x\to 3^+}\frac{x}{3} = \frac{3}{3} = 1$$

$$\lim_{x\to 3^-}\frac{x}{[x]} = \lim_{x\to 3^-}\frac{x}{2} = \frac{3}{2} = 1.5$$

Therefore, $\lim_{x\to 3^+}\frac{x}{[x]} \neq \lim_{x\to 3^-}\frac{x}{[x]}$

Q19

$$\lim_{x\to \frac{5}{2}}[x]$$

$$\lim_{x\to \frac{5}{2}}[x] = \left[\frac{5}{2}\right],$$

$$= [2.5] = 2 \qquad \text{[By definition of greatest integer function]}$$

$$\Rightarrow \quad \lim_{x\to \frac{5}{2}}[x] = 2$$

Q20

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(x-[x])$$
$$= \lim_{x\to 2^-} x - \lim_{x\to 2^-}[x]$$
$$= 2-1=1 \qquad \left[\because \lim_{x\to k^-}[x] = k-1\right]$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(3x-5)$$
$$= 3(2)-5 \qquad [\because x>2]$$
$$= 6-5$$
$$= 1$$

Thus, $\lim_{x\to 2^-} f(x) = 1 = \lim_{x\to 2^+} f(x)$

$\Rightarrow \quad \lim_{x\to 2} f(x) = 1$

Q21

$$\lim_{x\to 0^-} \sin\frac{1}{x} = \lim_{h\to 0}\sin\frac{1}{0-h} = -\lim_{h\to 0}\sin\frac{1}{h}$$
= −(An oscillating number which oscillates between −1 and 1).

So, $\lim_{x\to 0^-} \sin\frac{1}{x}$ does not exist.

Similarly, $\lim_{x\to 0^+} \sin\frac{1}{x}$ does not exist.

$\lim_{x\to 0} \sin\frac{1}{x}$ does not exist.

Q22

Let $f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x}, & \text{where } x \neq \dfrac{\pi}{2}\\ 3, & \text{where } x = \dfrac{\pi}{2}\end{cases}$ and if $\lim\limits_{x\to\pi/2} f(x) = f\left(\dfrac{\pi}{2}\right)$, find the value of k.

$$\text{LHL} = \lim_{x\to\frac{\pi}{2}^-} f(x)$$

$$= \lim_{h\to\frac{\pi}{2}} f\left(\frac{\pi}{2}-h\right)$$

$$= \lim_{h\to\frac{\pi}{2}} \frac{k\cos\left(\frac{\pi}{2}-h\right)}{\pi-2\left(\frac{\pi}{2}-h\right)}$$

$$= \lim_{h\to\frac{\pi}{2}} \frac{k\cos\left(\frac{\pi}{2}-h\right)}{2h}$$

$$= \frac{k\cos\left(\frac{\pi}{2}-\frac{\pi}{2}\right)}{\pi}$$

$$= \frac{k}{\pi}$$

Ex 29.2

Q1

$$\lim_{x\to 1}\frac{x^2+1}{x+1}=\frac{(1)^2+1}{1+1}=\frac{1+1}{2}=\frac{2}{2}=1$$

Q2

$$\lim_{x\to 0}\frac{2x^2+3x+4}{x^2+3x+2}=\lim_{x\to 0}\frac{2x^2+3x+4}{(x+2)(x+1)}=\frac{2(0)+3(0)+4}{(0+2)(0+1)}=\frac{4}{2}=2$$

Q3

$$\lim_{x\to 3}\frac{\sqrt{2x+3}}{x+3}=\frac{\sqrt{9}}{6}=\frac{1}{2}$$

Q4

$$\lim_{x\to 1}\frac{\sqrt{x+8}}{\sqrt{x}}=\frac{\sqrt{(1+8)}}{\sqrt{1}}=\sqrt{9}=3$$

Q5

$$\lim_{x\to a}\frac{\sqrt{x}+\sqrt{a}}{x+a}=\frac{\sqrt{a}+\sqrt{a}}{a+a}=\frac{2\sqrt{a}}{2a}=\frac{1}{\sqrt{a}}$$

Q6

$$\lim_{x\to 1}\frac{1+(x-1)^2}{1+x^2}=\frac{1+0^2}{1+1}=\frac{1}{2}$$

Q7

$$\lim_{x\to 0}\frac{x^{\frac{2}{3}}-9}{x-27}=\frac{-9}{-27}=\frac{1}{3}$$

Q8

$$\lim_{x \to 0} 9 = 9$$

Q9

$$\lim_{x \to 2} (3 - x) = (3 - 2) = 1$$

Q10

$$\lim_{x \to -1} \left(4x^2 + 2\right) = \left(4(-1)^2 + 2\right) = 6$$

Q11

$$\lim_{x \to -1} \frac{x^3 - 3x + 1}{x - 1} = \frac{(-1)^3 - 3(-1) + 1}{(-1 - 1)} = \frac{-1 + 3 + 1}{-2} = \frac{3}{-2} = \frac{-3}{2}$$

Q12

$$\lim_{x \to 0} \frac{3x + 1}{x + 3} = \frac{3(0) + 1}{(0 + 3)} = \frac{1}{3}$$

Q13

$$\lim_{x \to 3} \frac{x^2 - 9}{x + 2} = \frac{3^2 - 9}{3 + 2} = 0$$

Q14

$$\lim_{x \to 0} \frac{ax + b}{(x + d)} = \frac{a \times 0 + b}{(0 + d)} = \frac{b}{d}$$

Ex 29.3

Q1

$$\lim_{x\to -5}\frac{2x^2+9x-5}{x+5}=\lim_{x\to -5}\frac{(x+5)(2x-1)}{(x+5)}\quad=\lim_{x\to -5}(2x-1)=2(-5)-1=-10-1=-11$$

Q2

$$\lim_{x\to 3}\frac{x^2-4x+3}{x^2-2x-3}$$
$$=\lim_{x\to 3}\frac{x^2-3x-x+3}{x^2+x-3x-3}$$
$$=\lim_{x\to 3}\frac{x(x-1)-3(x-1)}{x(x+1)-3(x+1)}$$
$$=\lim_{x\to 3}\frac{(x-1)(x-3)}{(x+1)(x-3)}$$
$$=\lim_{x\to 3}\frac{x-1}{x+1}$$
$$=\frac{3-1}{3+1}$$
$$=\frac{2}{4}=\frac{1}{2}$$

Q3

$$\lim_{x\to 3}\frac{x^4-81}{x^2-9}=\lim_{x\to 3}\frac{(x^2-9)(x^2+9)}{(x^2-9)}=\lim_{x\to 3}x^2+9\quad=(3)^2+9=9+9=18$$

Q4

$$\lim_{x\to 2}\frac{x^3-8}{x^2-4}=\lim_{x\to 2}\frac{(x-2)(x^2+4+2x)}{(x-2)(x+2)}=\frac{(2)^2+4+2(2)}{2+2}=\frac{4+4+4}{4}=\frac{12}{4}=3$$

Q5

$$\lim_{x\to-\frac{1}{2}}\frac{8x^3+1}{2x+1} = \lim_{x\to-\frac{1}{2}}\frac{8\left(x^3+\frac{1}{8}\right)}{2\left(x+\frac{1}{2}\right)}$$

$$= \frac{8}{2}\lim_{x\to-\frac{1}{2}}\frac{\left(x^3+\left(\frac{1}{2}\right)^3\right)}{x+\frac{1}{2}}$$

$$= 4\lim_{x\to-\frac{1}{2}}\frac{\left(x+\frac{1}{2}\right)\left(x^2+\frac{1}{4}-\frac{1}{2}x\right)}{\left(x+\frac{1}{2}\right)}$$

$$= 4\left(\left(\frac{-1}{2}\right)^2+\frac{1}{4}-\frac{1}{2}\left(\frac{-1}{2}\right)\right)$$

$$= 4\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right)$$

$$= 3$$

Q6

$$\lim_{x\to 4}\frac{x^2-7x+12}{x^2-3x-4}$$

$$= \lim_{x\to 4}\frac{x^2-3x-4x+12}{x^2+x-4x-4}$$

$$= \lim_{x\to 4}\frac{x(x-3)-4(x-3)}{x(x+1)-1(x+1)}$$

$$= \lim_{x\to 4}\frac{(x-3)(x-4)}{(x-4)(x+1)}$$

$$= \lim_{x\to 4}\frac{x-3}{x+1}$$

$$= \frac{4-3}{4+1}$$

$$= \frac{1}{5}$$

Q7

$$\lim_{x\to 2}\frac{x^4-16}{x-2}$$

$$=\lim_{x\to 2}\frac{\left(x^2-4\right)\left(x^2+4\right)}{(x-2)}$$

$$=\lim_{x\to 2}\frac{(x-2)(x+2)\left(x^2+4\right)}{(x-2)}$$

$$=\lim_{x\to 2}(x+2)\left(x^2+4\right)$$

$$=(2+2)(4+4)$$

$$=4(8)$$

$$=32$$

Q8

$$\lim_{x\to 5}\frac{x^2-9x+20}{x^2-6x+5}$$

$$=\lim_{x\to 5}\frac{x^2-4x-5x+20}{x^2-x-5x+5}$$

$$=\lim_{x\to 5}\frac{x(x-4)-5(x-4)}{x(x-1)-5(x-1)}$$

$$=\lim_{x\to 5}\frac{(x-5)(x-4)}{(x-5)(x-1)}$$

$$=\lim_{x\to 5}\frac{x-4}{x-1}$$

$$=\frac{5-4}{5-1}$$

$$=\frac{1}{4}$$

Q9

$$\lim_{x \to -1} \frac{x^3+1}{x+1}$$

$$= \lim_{x \to -1} \frac{(x+1)\left(x^2-x+1\right)}{(x+1)} \qquad \left[a^3+b^3=(a+b)\left(a^2+b^2-ab\right)\right]$$

$$= \lim_{x \to -1} \left(x^2-x+1\right)$$

$$= (-1)^2-(-1)+1$$

$$= 1+1+1$$

$$= 3$$

Q10

$$\lim_{x \to 5} \frac{x^3-125}{x^2-7x+10} = \lim_{x \to 5} \frac{(x-5)\left(x^2+25+5x\right)}{(x-2)(x-5)} = \frac{(5)^2+25+5(5)}{(5-2)} = \frac{25+25+25}{3} = \frac{75}{3} = 25$$

Q11

$$\lim_{x \to \sqrt{2}} \frac{x^2-2}{x^2+\sqrt{2}x-4}$$

$$= \lim_{x \to \sqrt{2}} \frac{\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)}{x^2+2\sqrt{2}x-\sqrt{2}x-4}$$

$$= \lim_{x \to \sqrt{2}} \frac{\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)}{x\left(x+2\sqrt{2}\right)-\sqrt{2\left(x+2\sqrt{2}\right)}}$$

$$= \lim_{x \to \sqrt{2}} \frac{\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)}{\left(x+2\sqrt{2}\right)\left(x-\sqrt{2}\right)}$$

$$= \frac{\sqrt{2}+\sqrt{2}}{\sqrt{2}+2\sqrt{2}} = \frac{2\sqrt{2}}{3\sqrt{2}}$$

$$= \frac{2}{3}$$

Q12

$$\lim_{x\to\sqrt{3}} \frac{x^2 - 3}{x^2 + 3\sqrt{3}x - 12}$$

$$= \lim_{x\to\sqrt{3}} \frac{\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)}{x^2 + 4\sqrt{3x} - \sqrt{3x} - 12}$$

$$= \lim_{x\to\sqrt{3}} \frac{\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)}{x\left(x + 4\sqrt{3}\right) - \sqrt{3}\left(x + 4\sqrt{3}\right)}$$

$$= \lim_{x\to\sqrt{3}} \frac{\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)}{\left(x - \sqrt{3}\right)\left(x + 4\sqrt{3}\right)}$$

$$= \frac{\sqrt{3} + \sqrt{3}}{\sqrt{3} + 4\sqrt{3}} = \frac{2\sqrt{3}}{5\sqrt{3}}$$

$$= \frac{2}{5}$$

Q13

$$\lim_{x\to\sqrt{3}} \frac{x^4 - 9}{x^2 + 4\sqrt{3}x - 15} = \lim_{x\to\sqrt{3}} \frac{\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)\left(x^2 + 3\right)}{\left(x - \sqrt{3}\right)\left(x + 5\sqrt{3}\right)} = \lim_{x\to\sqrt{3}} \frac{\left(x + \sqrt{3}\right)\left(x^2 + 3\right)}{\left(x + 5\sqrt{3}\right)}$$

$$= \frac{\left(\sqrt{3} + \sqrt{3}\right)(3 + 3)}{\left(\sqrt{3} + 5\sqrt{3}\right)} = \frac{\left(2\sqrt{3}\right)(6)}{6\sqrt{3}} = 2$$

Q14

$$\lim_{x\to 2}\left(\frac{x}{x-2}-\frac{4}{x^2-2x}\right)$$
$$=\lim_{x\to 2}\left(\frac{x}{x-2}-\frac{4}{x(x-2)}\right)$$
$$=\lim_{x\to 2}\left(\frac{x(x)-4}{x(x-2)}\right)$$
$$=\lim_{x\to 2}\left(\frac{x^2-4}{x(x-2)}\right)$$
$$=\lim_{x\to 2}\frac{(x-2)(x+2)}{x(x-2)}$$
$$=\lim_{x\to 2}\frac{(x+2)}{x}$$
$$=\frac{2+2}{2}$$
$$=\frac{4}{2}$$
$$=2$$

Q15

$$\lim_{x\to 1}\left(\frac{1}{x^2+x-2}-\frac{x}{x^3-1}\right)$$
$$=\lim_{x\to 1}\left(\frac{1}{x^2+x-2}-\frac{x}{(x-1)(x^2+x+1)}\right)$$
$$=\lim_{x\to 1}\left(\frac{x^3-1-x^3-x^2+2x}{(x^3-1)(x^2+x-2)}\right)$$
$$=\lim_{x\to 1}\left(-\frac{(x^2-2x+1)}{(x^3-1)(x^2+x-2)}\right)$$
$$=\lim_{x\to 1}\left(-\frac{(x-1)(x-1)}{(x-1)(x^2+1+x)(x^2+x-2)}\right)$$
$$=-\lim_{x\to 1}\left(\frac{x-1}{(x^2+1+x)(x+2)(x-1)}\right)$$
$$=-\frac{1}{(1+1+1)(1+2)}$$
$$=-\frac{1}{9}$$

Q16

$$\lim_{x\to 3}\left(\frac{1}{x-3}-\frac{2}{x^2-4x+3}\right)$$
$$=\lim_{x\to 3}\left(\frac{1}{x-3}-\frac{2}{(x-3)(x-1)}\right)$$
$$=\lim_{x\to 3}\left(\frac{x-1-2}{(x-1)(x-3)}\right)$$
$$=\lim_{x\to 3}\left(\frac{x-3}{(x-1)(x-3)}\right)$$
$$=\lim_{x\to 3}\frac{1}{x-1}$$
$$=\frac{1}{3-1}$$
$$=\frac{1}{2}$$

Q17

$$\lim_{x\to 2}\left(\frac{1}{x-2}-\frac{2}{x^2-2x}\right)$$
$$=\lim_{x\to 2}\left(\frac{1}{x-2}-\frac{2}{x(x-2)}\right)$$
$$=\lim_{x\to 2}\left(\frac{x-2}{(x-2)(x)}\right)$$
$$=\lim_{x\to 2}\left(\frac{\cancel{(x-2)}}{\cancel{(x-2)}(x)}\right)$$
$$=\lim_{x\to 2}\frac{1}{x}$$
$$=\frac{1}{2}$$

Q18

$$\lim_{x\to\frac{1}{4}}\frac{4x-1}{2\sqrt{x}-1}=\lim_{x\to\frac{1}{4}}\frac{4\left(x-\frac{1}{4}\right)}{2\left(\sqrt{x}-\frac{1}{2}\right)}=4\lim_{x\to\frac{1}{4}}\frac{\left(\sqrt{x}-\frac{1}{2}\right)\left(\sqrt{x}+\frac{1}{2}\right)}{2\left(\sqrt{x}-\frac{1}{2}\right)}=4\lim_{x\to\frac{1}{4}}\frac{\left(\sqrt{x}+\frac{1}{2}\right)}{2}=\frac{4\left(\frac{1}{2}+\frac{1}{2}\right)}{2}=\frac{4(1)}{2}=2$$

Q19

$$\lim_{x\to 4}\frac{x^2-16}{\sqrt{x}-2}$$
$$=\lim_{x\to 4}\frac{(x-4)(x+4)}{\left(\sqrt{x}-2\right)}$$
$$=\lim_{x\to 4}\frac{\left(\sqrt{x}-2\right)\left(\sqrt{x}+2\right)(x+4)}{\left(\sqrt{x}-2\right)}$$
$$=\lim_{x\to 4}\left(\sqrt{x}+2\right)(x+4)$$
$$=(2+2)(4+4)$$
$$=4(8)$$
$$=32$$

Q20

$$\lim_{x\to 0}\frac{(a+x)^2-a^2}{x}$$
$$=\lim_{x\to 0}\frac{(a+x-a)(a+x+a)}{x}$$
$$=\lim_{x\to 0}\frac{(x)(2a+x)}{x}$$
$$=\lim_{x\to 0}(2a+x)$$
$$=2a$$

Q21

$$\lim_{x\to 2}\left(\frac{1}{x-2}-\frac{4}{x^3-2x^2}\right)$$
$$=\lim_{x\to 2}\left(\frac{1}{x-2}-\frac{4}{x^2(x-2)}\right)$$
$$=\lim_{x\to 2}\left(\frac{x^2-4}{x^2(x-2)}\right)$$
$$=\lim_{x\to 2}\frac{(x-2)(x+2)}{x^2(x-2)}$$
$$=\lim_{x\to 2}\frac{(x+2)}{x^2}$$
$$=\frac{(2+2)}{2^2}=\frac{4}{4}$$
$$=1$$

Q22

$$\lim_{x\to 3}\left(\frac{1}{x-3}-\frac{3}{x^2-3x}\right)$$

$$=\lim_{x\to 3}\left(\frac{1}{x-3}-\frac{3}{x(x-3)}\right)$$

$$=\lim_{x\to 3}\left(\frac{x-3}{x(x-3)}\right)$$

$$=\lim_{x\to 3}\left(\frac{1}{x}\right)$$

$$=\frac{1}{3}$$

Q23

$$\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{2}{x^2-1}\right)$$

$$=\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{2}{(x-1)(x+1)}\right)$$

$$=\lim_{x\to 1}\left(\frac{x+1-2}{(x-1)(x+1)}\right)$$

$$=\lim_{x\to 1}\left(\frac{x-1}{(x-1)(x+1)}\right)$$

$$=\lim_{x\to 1}\left(\frac{1}{x+1}\right)$$

$$=\frac{1}{1+1}=\frac{1}{2}$$

Q24

$$\lim_{x\to 3}\left(x^2-9\right)\left[\frac{1}{x+3}+\frac{1}{x-3}\right]$$

$$=\lim_{x\to 3}\left(x^2-9\right)\left[\frac{x-3+x+3}{(x+3)(x-3)}\right]$$

$$=\lim_{x\to 3}\left(x^2-9\right)\left[\frac{2x}{(x+3)(x-3)}\right]$$

$$=\lim_{x\to 3}\frac{(x-3)(x+3)(2x)}{(x+3)(x-3)}$$

$$=\lim_{x\to 3}2(x)=2(3)=6$$

Q25

$$\lim_{x\to 1}\frac{x^4-3x^3+2}{x^3-5x^2+3x+1}$$

Dividing x^4-3x^3+2 by x^3-5x^2+3x+1

$$\begin{array}{r|l}
 & \ \ x+2 \\
x^3-5x^3+3x+1 & x^4-3x^3+2 \\
 & \underline{\pm x^4 \mp 5x^3 \pm 3x^2 \pm x} \\
 & 2x^3-3x^2-x+2 \\
 & \underline{\pm 2x^3 \mp 10x^2 \pm 6x \pm 2} \\
 & \qquad 7x^2-7x
\end{array}$$

$$\Rightarrow \lim_{x\to 1}\frac{x^4-3x^3+2}{x^3-5x^2+3x+1}=\lim_{x\to 1} x+2+\lim_{x\to 1}\frac{7x^2-7x}{x^3-5x^3+3x+1}$$

$$=\lim_{x\to 1} x+2+\lim_{x\to 1}\frac{7x(x-1)}{x^3-5x^3+3x+1}$$

Dividing x^3-5x^2+3x+1 by $x-1$

$$\begin{array}{r|l}
 & \quad x^2-4x-1 \\
x-1 & x^3-5x^2+3x+1 \\
 & \underline{\pm x^3 \mp x^2} \\
 & -4x^2+3x+1 \\
 & \underline{\underset{+}{-4x^2}+\underset{-}{4x}} \\
 & \quad -x+1 \\
 & \quad \underline{-x+1} \\
 & \qquad \underline{\times}
\end{array}$$

Q26

$$\lim_{x\to 2}\frac{x^3+3x^2-9x-2}{x^3-x-6}$$

Divide x^3+3x^2-9x-2 by x^3-x-6

$$\begin{array}{r|l} & \quad 1 \\ x^3-x-6 & x^3+3x^2-9x-2 \\ & \underline{\underset{-}{+x^3} \quad \underset{+}{-x} \quad \underset{+}{-6}} \\ & \quad \underline{\underline{3x^2-8x+4}} \end{array}$$

$$\Rightarrow \quad \lim_{x\to 2}\frac{x^3+3x^2-9x-2}{x^3-x-6} = \lim_{x\to 2}1+\lim_{x\to 2}\frac{3x^2-8x+4}{x^3-x-6}$$

$$= 1+\lim_{x\to 2}\frac{3x^2-2x-6x+4}{x^3-x-6}$$

$$= 1+\lim_{x\to 2}\frac{3x^2-2x-6x+4}{x^3-x-6}$$

$$\Rightarrow \quad \lim_{x\to 2}\frac{x^3+3x^2-9x-2}{x^3-x-6} = 1+\lim_{x\to 2}\frac{(3x-2)(x-2)}{x^3-x-6}$$

Divide x^3-x-6 by $x-2$

$$\begin{array}{r|l} & x^2+2x+3 \\ x-2 & x^3-x-6 \\ & \underline{\pm x^3 \underset{+}{-2x^2}} \\ & 2x^2-x-6 \\ & \underline{2x^2 \mp 4x} \\ & \quad 3x-6 \\ & \quad \underline{3x-6} \\ & \quad \times \end{array}$$

$$\Rightarrow \quad \lim_{x\to 2}\frac{x^3+3x^2-9x-2}{x^3-x-6} = 1+\lim_{x\to 2}\frac{(3x-2)(x-2)}{(x-2)\left(x^2+2x+3\right)}$$

$$= 1+\lim_{x\to 2}\frac{(3x-2)}{\left(x^2+2x+3\right)}$$

$$= 1+\frac{3\times 2-2}{2^2+2\times 2+3}$$

$$= 1+\frac{4}{11}$$

$$= \frac{15}{11}$$

Q27

$$\lim_{x\to 1}\frac{1-x^{\frac{-1}{3}}}{1-x^{\frac{-2}{3}}}$$

$$=\lim_{x\to 1}\frac{1-\frac{1}{x^{\frac{1}{3}}}}{1-\frac{1}{x^{\frac{2}{3}}}}$$

$$=\lim_{x\to 1}\frac{\left(x^{\frac{1}{3}}-1\right)}{\left(x^{\frac{1}{3}}-1\right)\left(x^{\frac{1}{3}}+1\right)}\times x^{\frac{1}{3}}$$

$$=\lim_{x\to 1}\frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}+1}$$

$$=\frac{1}{1+1}$$

$$=\frac{1}{2}$$

Q28

$$\lim_{x \to 3} \frac{x^2 - x - 6}{x^3 - 3x^2 + x - 3}$$

Now $\quad x^2 - x - 6$

$= x^2 - 3x + 2x - 6$

$= x(x-3) + 2(x-3)$

$= (x+2)(x-3) \qquad$ ---(i)

Dividing $x^3 - 3x^2 + x - 3$ by $(x-3)$, we get

$$\begin{array}{r|l} & x^2 + 1 \\ \hline x-3 & x^3 - 3x^2 + x - 3 \\ & \underline{\pm x^3 \mp 3x^2} \\ & \qquad\quad x - 3 \\ & \qquad\quad \underline{x - 3} \\ & \qquad\qquad \times \end{array}$$

Thus $(x-3)$ is a factor of $x^3 - 3x^2 + x - 3 \qquad$ ---(ii)

Substituting (i) and (ii) in the given expression

$= \lim_{x \to 3} \frac{(x+2)(x-3)}{(x^2+1)(x-3)}$

$= \frac{x+2}{x^2+1} = \frac{3+2}{9+1} = \frac{5}{10}$

$= \frac{1}{2}$

Q29

$$\lim_{x \to -2} \frac{x^3 + x^2 + 4x + 12}{x^3 - 3x + 2} = \lim_{x \to -2} \frac{(x+2)(x^2 - x + 6)}{(x+2)(x^2 - 2x + 1)}$$

$$\lim_{x \to -2} \frac{x^2 - x + 6}{x^2 - 2x + 1}$$

$$= \frac{(-2)^2 - (-2) + 6}{(-2)^2 - 2(-2) + 1}$$

$$= \frac{4 + 2 + 6}{4 + 4 + 1}$$

$$= \frac{12}{9}$$

$$= \frac{4}{3}$$

Q30

$$\lim_{x \to 1} \frac{x^3 + 3x^2 - 6x + 2}{x^3 + 3x^2 - 3x - 1} = \lim_{x \to 1} \frac{(x-1)(x^2 + 4x - 2)}{(x-1)(x^2 + 4x + 1)}$$

$$= \frac{(1)^2 + 4(1) - 2}{(1)^2 + 4(1) + 1} = \frac{1 + 4 - 2}{1 + 4 + 1}$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

Q31

$$\lim_{x\to 2}\left[\frac{1}{x-2}-\frac{2(2x-3)}{x^3-3x^2+2x}\right]$$

$$=\lim_{x\to 2}\left[\frac{1}{x-2}-\frac{2(2x-3)}{x(x^2-2x-x+2)}\right]$$

$$=\lim_{x\to 2}\left[\frac{1}{x-2}-\frac{2(2x-3)}{x(x-2)(x-1)}\right]$$

$$=\lim_{x\to 2}\left[\frac{x(x-1)-2(2x-3)}{x(x-1)(x-2)}\right]$$

$$=\lim_{x\to 2}\left[\frac{x^2-x-4x+6}{x(x-1)(x-2)}\right]$$

$$=\lim_{x\to 2}\left[\frac{x^2-2x-3x+6}{x(x-1)(x-2)}\right]$$

$$=\lim_{x\to 2}\left[\frac{x(x-2)-3(x-2)}{x(x-1)(x-2)}\right]$$

$$=\lim_{x\to 2}\left[\frac{(x-2)(x-3)}{x(x-1)(x-2)}\right]$$

$$=\lim_{x\to 2}\left[\frac{1}{x-2}-\frac{2(2x-3)}{x(x-2)(x-1)}\right]$$

$$=\frac{-1}{2}$$

Q32

$$\lim_{x \to 1} \frac{\sqrt{x^2 - 1} + \sqrt{x - 1}}{\sqrt{x^2 - 1}}$$

$$= \lim_{x \to 1} \frac{\left(\sqrt{x^2 - 1} + \sqrt{x - 1}\right)}{\sqrt{x^2 - 1}} \times \frac{\left(\sqrt{x^2 - 1} - \sqrt{x - 1}\right)}{\left(\sqrt{x^2 - 1} - \sqrt{x - 1}\right)} \times \frac{\sqrt{x^2 - 1}}{\sqrt{x^2 - 1}}$$

$$= \lim_{x \to 1} \frac{\left[\left(x^2 - 1\right) - (x - 1)\right] \times \sqrt{x^2 - 1}}{\left(x^2 - 1\right)\left(\sqrt{x^2 - 1} - \sqrt{x - 1}\right)}$$

$$= \lim_{x \to 1} \frac{\left(x^2 - x\right)\sqrt{x^2 - 1}}{\left(x^2 - 1\right)\left(\sqrt{x^2 - 1} - \sqrt{x - 1}\right)}$$

$$= \lim_{x \to 1} \frac{x\,(x - 1)\sqrt{x^2 - 1}}{(x - 1)(x + 1)\left(\sqrt{x^2 - 1} - \sqrt{x - 1}\right)}$$

$$\lim_{x \to 1} \frac{x\,\cancel{\left(\sqrt{x - 1}\right)}\left(\sqrt{x + 1}\right)}{(x + 1)\,\cancel{\left(\sqrt{x - 1}\right)}\left(\sqrt{x + 1} - 1\right)}$$

$$= \frac{\sqrt{2}}{2\left(\sqrt{2} - 1\right)}$$

$$= \frac{\sqrt{2}}{2 \times \left(\sqrt{2} - 1\right)} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$$

$$= \frac{\sqrt{2} + 1}{\sqrt{2}}$$

Q33

$$\lim_{x\to 1}\left\{\frac{x-2}{x^2-x}-\frac{1}{x^3-3x^2+2x}\right\}$$

$$=\lim_{x\to 1}\left\{\frac{x-2}{x(x-1)}-\frac{1}{x\left(x^2-3x+2\right)}\right\}$$

$$=\lim_{x\to 1}\left\{\frac{x-2}{x(x-1)}-\frac{1}{x\left(x^2-3x+2\right)}\right\}$$

$$=\lim_{x\to 1}\left\{\frac{x-2}{x(x-1)}-\frac{1}{x\left(x^2-1x-2x+2\right)}\right\}$$

$$=\lim_{x\to 1}\left\{\frac{x-2}{x(x-1)}-\frac{1}{x(x-1)(x-2)}\right\}$$

$$=\lim_{x\to 1}\left\{\frac{(x-2)^2-1}{x(x-1)(x-2)}\right\}$$

$$=\lim_{x\to 1}\left\{\frac{x^2+4-4x-1}{x(x-1)(x-2)}\right\}$$

$$=\lim_{x\to 1}\left\{\frac{x^2-4x+3}{x(x-1)(x-2)}\right\}$$

$$=\lim_{x\to 1}\left[\frac{x^2-x-3x+3}{x(x-1)(x-2)}\right]$$

$$=\lim_{x\to 1}\left[\frac{x(x-1)-3(x-1)}{x(x-1)(x-2)}\right]$$

$$=\lim_{x\to 1}\left[\frac{(x-3)(x-1)}{x(x-1)(x-2)}\right]$$

$$=\frac{(1-3)}{1(1-2)}$$

$$=\frac{-2}{-1}$$

$$=2$$

Q34

$$\lim_{x\to 1}\frac{x^7-2x^5+1}{x^3-3x^2+2}=\lim_{x\to 1}\frac{(x-1)(x^6+x^5-x^4-x^3-x^2-x-1)}{(x-1)(x^2-2x-2)}$$

$$=\lim_{x\to 1}\frac{(x^6+x^5-x^4-x^3-x^2-x-1)}{(x^2-2x-2)}$$

$$=\frac{(1+1-1-1-1-1-1)}{(1-2-2)}$$

$$=\frac{-3}{-3}$$

$$=1$$

Ex 29.4

Q1

$$\lim_{x\to 0}\frac{\sqrt{1+x+x^2}-1}{x}$$

$$= \lim_{x\to 0}\frac{\left(\sqrt{1+x+x^2}-1\right)}{x}\frac{\left(\sqrt{1+x+x^2}+1\right)}{\left(\sqrt{1+x+x^2}+1\right)}$$

$$= \lim_{x\to 0}\frac{\left(\left(1+x+x^2\right)-1\right)}{x\left(\sqrt{1+x+x^2}+1\right)}$$

$$= \lim_{x\to 0}\frac{x\left(1+x\right)}{x\left(\sqrt{1+x+x^2}+1\right)}$$

$$= \lim_{x\to 0}\frac{1+x}{\sqrt{1+x+x^2}+1}$$

$$= \frac{1+0}{\sqrt{1+0+0}+1}$$

$$= \frac{1}{1+1}$$

$$= \frac{1}{2}$$

Q2

$$\lim_{x\to 0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}}$$

$$= \lim_{x\to 0}\frac{2x}{\left(\sqrt{a+x}-\sqrt{a-x}\right)}\times\frac{\sqrt{a+x}+\sqrt{a-x}}{\left(\sqrt{a+x}+\sqrt{a-x}\right)}$$

$$= \lim_{x\to 0}\frac{2x\left(\sqrt{a+x}+\sqrt{a-x}\right)}{\left(\left(a+x\right)-\left(a-x\right)\right)}$$

$$= \lim_{x\to 0}\frac{2x\left(\sqrt{a+x}+\sqrt{a-x}\right)}{2x}$$

$$= \lim_{x\to 0}\left(\sqrt{a+x}+\sqrt{a-x}\right)$$

$$= \sqrt{a}+\sqrt{a}$$

$$= 2\sqrt{a}$$

Q3

$$\lim_{x\to 0}\frac{\sqrt{a^2+x^2}-a}{x^2}$$

$$= \lim_{x\to 0}\frac{\left(\sqrt{a^2+x^2}-a\right)}{x^2}\times\frac{\left(\sqrt{a^2+x^2}+a\right)}{\left(\sqrt{a^2+x^2}+a\right)}$$

$$= \lim_{x\to 0}\frac{\left(a^2+x^2-a^2\right)}{x^2\sqrt{a^2+x^2}+a}$$

$$= \lim_{x\to 0}\frac{x^2}{x^2\left(\sqrt{a^2+x^2}+a\right)}$$

$$= \lim_{x\to 0}\frac{1}{\sqrt{a^2+x^2}+a}$$

$$= \frac{1}{a+a}$$

$$= \frac{1}{2a}$$

Q4

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{2x}$$

$$= \lim_{x\to 0}\frac{\left(\sqrt{1+X}-\sqrt{1-X}\right)}{2X}\times\frac{\left(\sqrt{1+x}+\sqrt{1-x}\right)}{\left(\sqrt{1+x}+\sqrt{1-x}\right)}$$

$$= \lim_{x\to 0}\frac{(1+x)-(1-x)}{2x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$$

$$= \lim_{x\to 0}\frac{2x}{2x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$$

$$= \lim_{x\to 0}\frac{1}{\left(\sqrt{1+x}+\sqrt{1-x}\right)}$$

$$= \frac{1}{\sqrt{1}+\sqrt{1}}$$

$$= \frac{1}{2}$$

Q5

$$\lim_{x \to 2} \frac{\sqrt{3-x}-1}{2-x}$$

$$= \lim_{x \to 2} \frac{\left(\sqrt{3-x}-1\right)}{2-x} \times \frac{\left(\sqrt{3-x}+1\right)}{\left(\sqrt{3-x}+1\right)}$$

$$= \lim_{x \to 2} \frac{(3-x)-1}{(2-x)\left(\sqrt{3-x}+1\right)}$$

$$= \lim_{x \to 2} \frac{(2-x)}{(2-x)\left(\sqrt{3-x}+1\right)}$$

$$= \lim_{x \to 2} \frac{1}{\left(\sqrt{3-x}+1\right)}$$

$$= \frac{1}{\sqrt{3-2}+1} = \frac{1}{1+1}$$

$$= \frac{1}{2}$$

Q6

$$\lim_{x \to 3} \frac{x-3}{\sqrt{x-2}-\sqrt{4-x}}$$

$$= \lim_{x \to 3} \frac{x-3}{\left(\sqrt{x-2}-\sqrt{4-x}\right)} \times \frac{\left(\sqrt{x-2}+\sqrt{4-x}\right)}{\left(\sqrt{x-2}+\sqrt{4-x}\right)}$$

$$= \lim_{x \to 3} \frac{(x-3)\left(\sqrt{x-2}+\sqrt{4-x}\right)}{(x-2)-(4-x)}$$

$$= \lim_{x \to 3} \frac{(x-3)\left(\sqrt{x-2}+\sqrt{4-x}\right)}{x-2-4+x}$$

$$= \lim_{x \to 3} \frac{(x-3)\left(\sqrt{x-2}+\sqrt{4-x}\right)}{2(x-3)}$$

$$= \frac{1}{2} \lim_{x \to 3} \left(\sqrt{x-2}+\sqrt{4-x}\right)$$

$$= \frac{1}{2}\left(\sqrt{3-2}+\sqrt{4-3}\right)$$

$$= \frac{1}{2}\left(\sqrt{1}+\sqrt{1}\right)$$

$$= \frac{1}{2}(1+1) = \frac{2}{2}$$

$$= 1$$

Q7

$$\lim_{x\to 0}\frac{x}{\left(\sqrt{1+x}-\sqrt{1-x}\right)}$$

$$=\lim_{x\to 0}\frac{x}{\left(\sqrt{1+x}-\sqrt{1-x}\right)}\times\frac{\left(\sqrt{1+x}+\sqrt{1-x}\right)}{\left(\sqrt{1+x}+\sqrt{1-x}\right)}$$

$$=\lim_{x\to 0}\frac{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}{\left(\sqrt{1+x}\right)^2-\left(\sqrt{1-x}\right)^2}$$

$$=\lim_{x\to 0}\frac{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}{1+x-1+x}$$

$$=\lim_{x\to 0}\frac{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}{2x}$$

$$=\frac{1}{2}\lim_{x\to 0}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{x}\right)x$$

$$=\frac{1}{2}\lim_{x\to 0}\left(\sqrt{1+x}+\sqrt{1-x}\right)$$

$$=\frac{1}{2}\left(\sqrt{1}+\sqrt{1}\right)$$

$$=\frac{2}{2}=1$$

Q8

$$\lim_{x\to 1}\frac{\left(\sqrt{5x-4}-\sqrt{x}\right)}{x-1}$$

$$=\lim_{x\to 1}\frac{\left(\sqrt{5x-4}-\sqrt{x}\right)}{x-1}\times\frac{\left(\sqrt{5x-4}+\sqrt{x}\right)}{\left(\sqrt{5x-4}+\sqrt{x}\right)}$$

$$=\lim_{x\to 1}\frac{\left((5x-4)-x\right)}{(x-1)\left(\sqrt{5x-4}+\sqrt{x}\right)}$$

$$=4\lim_{x\to 1}\frac{(x-1)}{(x-1)\left(\sqrt{5x-4}+\sqrt{x}\right)}$$

$$=4\lim_{x\to 1}\frac{1}{\sqrt{5x-4}+\sqrt{x}}$$

$$=4\times\frac{1}{\sqrt{5-4}+\sqrt{1}}$$

$$=4\times\frac{1}{\sqrt{1}+\sqrt{1}}$$

$$=\frac{4}{2}=2$$

Q9

$$\lim_{x\to 1}\frac{(x-1)}{\left(\sqrt{x^2+3}-2\right)}$$

$$=\lim_{x\to 1}\frac{(x-1)\times\left(\sqrt{x^2+3}+2\right)}{\left(\sqrt{x^2+3}-2\right)\left(\sqrt{x^2+3}+2\right)}$$

$$=\lim_{x\to 1}\frac{(x-1)\left(\sqrt{x^2+3}+2\right)}{\left(x^2+3-4\right)}$$

$$=\lim_{x\to 1}\frac{(x-1)\left(\sqrt{x^2+3}+2\right)}{\left(x^2-1\right)}$$

$$=\lim_{x\to 1}\frac{\sqrt{x^2+3}+2}{x+1}$$

Putting the value $x=1$

$$\Rightarrow \quad \frac{\sqrt{1+3}+2}{1+1}$$

$$=\frac{2+2}{2}$$

$$=\frac{4}{2}=2$$

Q10

$$\lim_{x\to 3}\frac{\left(\sqrt{x+3}-\sqrt{6}\right)}{\left(x^2-9\right)}$$

$$=\lim_{x\to 3}\frac{\left(\sqrt{x+3}-\sqrt{6}\right)}{(x-3)(x+3)}\frac{\left(\sqrt{x+3}-\sqrt{6}\right)}{\left(\sqrt{x+3}+\sqrt{6}\right)}$$

$$=\lim_{x\to 3}\frac{\left((x+3)-6\right)}{(x-3)(x+3)\left(\sqrt{x+3}+\sqrt{6}\right)}$$

$$=\lim_{x\to 3}\frac{(x-3)}{(x-3)(x+3)\left(\sqrt{x+3}+\sqrt{6}\right)}$$

$$=\lim_{x\to 3}\frac{1}{(x+3)\left(\sqrt{x+3}+\sqrt{6}\right)}$$

$$=\frac{1}{(3+3)\sqrt{3+3}+\sqrt{6}}$$

$$=\frac{1}{6\left(\sqrt{6}+\sqrt{6}\right)}=\frac{1}{6\times 2\sqrt{6}}$$

$$=\frac{1}{12\sqrt{6}}$$

Q11

$$\lim_{x\to 1}\frac{\left(\sqrt{5x-4}-\sqrt{x}\right)}{\left(x^2-1\right)}$$

$$=\lim_{x\to 1}\frac{\left(\sqrt{5x-4}-\sqrt{x}\right)}{(x-1)(x+1)}\times\frac{\left(\sqrt{5x-4}+\sqrt{x}\right)}{\left(\sqrt{5x-4}+\sqrt{x}\right)}$$

$$=\lim_{x\to 1}\frac{\left((5x-4)-x\right)}{(x-1)(x+1)\left(\sqrt{5x-4}+\sqrt{x}\right)}$$

$$=\lim_{x\to 1}\frac{4(x-1)}{(x-1)(x+1)\left(\sqrt{5x-4}+\sqrt{x}\right)}$$

$$=\lim_{x\to 1}\frac{4}{(x+1)\left(\sqrt{5x-4}+\sqrt{x}\right)}$$

$$=\frac{4}{(1+1)\left(\sqrt{5-4}+\sqrt{1}\right)}$$

$$=\frac{4}{2(1+1)}$$

$$=\frac{4}{4}=1$$

Q12

$$\lim_{x\to 0}\frac{\left(\sqrt{1+x}-1\right)}{x}$$

$$=\lim_{x\to 0}\frac{\left(\sqrt{1+x}-1\right)}{x}\times\frac{\left(\sqrt{1+x}+1\right)}{\left(\sqrt{1+x}+1\right)}$$

$$=\lim_{x\to 0}\frac{(1+x-1)}{x\left(\sqrt{1+x}+1\right)}$$

$$=\lim_{x\to 0}\frac{x}{x\left(\sqrt{1+x}+1\right)}$$

$$=\lim_{x\to 0}\frac{1}{\left(\sqrt{1+x}+1\right)}$$

$$=\frac{1}{\sqrt{1}+1}=\frac{1}{2}$$

Q13

$$\lim_{x\to 2}\frac{\sqrt{x^2+1}-\sqrt{5}}{(x-2)}$$

$$=\lim_{x\to 2}\frac{\sqrt{x^2+1}-\sqrt{5}}{(x-2)}\times\frac{\left(\sqrt{x^2+1}+\sqrt{5}\right)}{\left(\sqrt{x^2+1}+\sqrt{5}\right)}$$

$$=\lim_{x\to 2}\frac{\left(x^2+1-5\right)}{(x-2)\left(\sqrt{x^2+1}+\sqrt{5}\right)}$$

$$=\lim_{x\to 2}\frac{(x+2)(x-2)}{(x-2)\left(\sqrt{x^2+1}+\sqrt{5}\right)}$$

$$=\lim_{x\to 2}\frac{(x+2)}{\left(\sqrt{x^2+1}+\sqrt{5}\right)}$$

$$=\frac{(2+2)}{\sqrt{4+1}+\sqrt{5}}$$

$$=\frac{4}{2\sqrt{5}}=\frac{2}{\sqrt{5}}$$

Q14

$$\lim_{x\to 2}\frac{x-2}{\sqrt{x}-\sqrt{2}}$$

$$=\lim_{x\to 2}\frac{(x-2)\left(\sqrt{x}+\sqrt{2}\right)}{\left(\sqrt{x}-\sqrt{2}\right)\left(\sqrt{x}+\sqrt{2}\right)}$$

$$=\lim_{x\to 2}\frac{(x-2)\left(\sqrt{x}+\sqrt{2}\right)}{(x-2)}$$

$$=\sqrt{2}+\sqrt{2}$$

$$=2\sqrt{2}$$

Q15

$$\lim_{x\to 7}\frac{4-\sqrt{9+x}}{1-\sqrt{8-x}}$$

$$=\lim_{x\to 7}\frac{\left(4-\sqrt{9+x}\right)}{1-\sqrt{8-x}}\times\frac{\left(4+\sqrt{9+x}\right)}{\left(4+\sqrt{9+x}\right)}\times\frac{\left(1+\sqrt{8-x}\right)}{\left(\sqrt{1+\sqrt{8-x}}\right)}$$

$$=\lim_{x\to 7}\frac{\left((4)^2-\left(\sqrt{9+x}\right)^2\right)}{\left((1)^2-\left(\sqrt{8-x}\right)^2\right)}\times\frac{1+\sqrt{8-x}}{4+\sqrt{9+x}}$$

$$=\lim_{x\to 7}\frac{(16-9-x)\times\left(1+\sqrt{8-x}\right)}{(1-8+x)\times\left(4+\sqrt{9+x}\right)}$$

$$=\lim_{x\to 7}\frac{7-x}{(-1)(7-x)}\frac{\left(1+\sqrt{8-x}\right)}{\left(4+\sqrt{9+x}\right)}$$

$$=\frac{1}{(-1)}\times\frac{(1+1)}{(4+4)}=\frac{-1}{4}$$

Q16

$$\lim_{x\to 0}\frac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a^2+ax}}$$

$$=\lim_{x\to 0}\frac{\left(\sqrt{a+x}-\sqrt{a}\right)}{x\sqrt{a^2+ax}}\times\frac{\left(\sqrt{a+x}+\sqrt{a}\right)}{\left(\sqrt{a+x}+\sqrt{a}\right)}$$

$$=\lim_{x\to 0}\frac{(a+x)-a}{x\sqrt{a^2+ax}\left(\sqrt{a+x}+\sqrt{a}\right)}$$

$$=\lim_{x\to 0}\frac{x}{x\sqrt{a^2+ax}\left(\sqrt{a+x}+\sqrt{a}\right)}$$

$$=\lim_{x\to 0}\frac{1}{\left(\sqrt{a^2+ax}\right)\left(\sqrt{a+x}+\sqrt{a}\right)}$$

$$=\frac{1}{a\left(2\sqrt{a}\right)}$$

$$=\frac{1}{2a\sqrt{a}}$$

Q17

$$\lim_{x\to 5}\frac{x-5}{\sqrt{6x-5}-\sqrt{4x+5}}$$

$$= \lim_{x\to 5}\frac{x-5}{\left(\sqrt{6x-5}-\sqrt{4x+5}\right)}\times\frac{\left(\sqrt{6x-5}+\sqrt{4x+5}\right)}{\left(\sqrt{6x-5}+\sqrt{4x+5}\right)}$$

$$= \lim_{x\to 5}\frac{(x-5)\left(\sqrt{6x-5}+\sqrt{4x+5}\right)}{(6x-5)-(4x+5)}$$

$$= \lim_{x\to 5}\frac{(x-5)\left(\sqrt{6x-5}+\sqrt{4x+5}\right)}{2x-10}$$

$$= \lim_{x\to 5}\frac{(x-5)\left(\sqrt{6x-5}+\sqrt{4x+5}\right)}{2(x-5)}$$

$$= \frac{\sqrt{6(5)-5}+\sqrt{4(5)+5}}{2}$$

$$= \frac{\sqrt{25}+\sqrt{25}}{2}$$

$$= \frac{5+5}{2} = 5$$

Q18

$$\lim_{x\to1}\frac{\sqrt{5x-4}-\sqrt{x}}{x^3-1}$$

$$=\lim_{x\to1}\frac{\left(\sqrt{5x-4}-\sqrt{x}\right)}{(x-1)\left(x^2+1+1\right)}\times\frac{\left(\sqrt{5x-4}+\sqrt{x}\right)}{\left(\sqrt{5x-4}+\sqrt{x}\right)}$$

$$=\lim_{x\to1}\frac{(5x-4-x)}{(x-1)\left(x^2+1+x\right)\left(\sqrt{5x-4}+\sqrt{x}\right)}$$

$$=\lim_{x\to1}\frac{4(x-1)}{(x-1)\left(x^2+x+1\right)\left(\sqrt{5x-4}+\sqrt{x}\right)}$$

$$=\frac{4}{(1+1+1)\left(\sqrt{5-4}+\sqrt{1}\right)}$$

$$=\frac{4}{(3)(1+1)}$$

$$=\frac{4}{3\times2}=\frac{2}{3}$$

Q19

$$\lim_{x\to2}\frac{\sqrt{1+4x}-\sqrt{5+2x}}{x-2}$$

$$=\lim_{x\to2}\frac{\left(\sqrt{1+4x}-\sqrt{5+2x}\right)}{(x-2)}\times\frac{\left(\sqrt{1+4x}+\sqrt{5+2x}\right)}{\left(\sqrt{1+4x}+\sqrt{5+2x}\right)}$$

$$=\lim_{x\to2}\frac{(1+4x)-(5+2x)}{(x-2)\left(\sqrt{1+4x}+\sqrt{5+2x}\right)}$$

$$=\lim_{x\to2}\frac{-4+2x}{(x-2)\left(\sqrt{1+4x}+\sqrt{5+2x}\right)}$$

$$=\lim_{x\to2}\frac{2(x-2)}{(x-2)\left(\sqrt{1+4x}+\sqrt{5+2x}\right)}$$

$$=\frac{2}{\sqrt{1+8}+\sqrt{5+4}}=\frac{2}{\sqrt{9}+\sqrt{9}}$$

$$=\frac{2}{6}=\frac{1}{3}$$

Q20

$$\lim_{x \to 1} \frac{\sqrt{3+x} - \sqrt{5-x}}{x^2 - 1}$$

$$= \lim_{x \to 1} \frac{\left(\sqrt{3+x} - \sqrt{5-x}\right)}{\left(x^2 - 1\right)} \times \frac{\left(\sqrt{3+x} + \sqrt{5-x}\right)}{\left(\sqrt{3+x} + \sqrt{5-x}\right)}$$

$$= \lim_{x \to 1} \frac{(3+x) - (5-x)}{(x-1)(x+1)\left(\sqrt{3+x} + \sqrt{5-x}\right)}$$

$$= \lim_{x \to 1} \frac{-2+2x}{(x-1)(x+1)\left(\sqrt{3+x} + \sqrt{5-x}\right)}$$

$$= \lim_{x \to 1} \frac{2(x-1)}{(x-1)(x+1)\left(\sqrt{3+x} + \sqrt{5-x}\right)}$$

$$= \lim_{x \to 1} \frac{2}{(x+1)\left(\sqrt{3+x} + \sqrt{5-x}\right)}$$

$$= \frac{2}{(1+1)\left(\sqrt{3+1} + \sqrt{5-1}\right)} = \frac{2}{(2)(2+2)}$$

$$= \frac{1}{4}$$

Q21

$$\lim_{x\to 0}\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{x}$$

$$= \lim_{x\to 0}\frac{\left(\sqrt{1+x^2}-\sqrt{1-x^2}\right)}{x}\times\frac{\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)}{\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)}$$

$$= \lim_{x\to 0}\frac{\left(1+x^2\right)-\left(1-x^2\right)}{x\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)}$$

$$= \lim_{x\to 0}\frac{2x^2}{x\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)}$$

$$= \frac{2\times 0}{\left(\sqrt{1}+\sqrt{1}\right)}$$

$$= \frac{2}{2}\times 0$$

$$= 0$$

Q22

$$\lim_{x\to 0}\frac{\sqrt{1+x+x^2}-\sqrt{x+1}}{2x^2}$$

$$= \lim_{x\to 0}\frac{\left(\sqrt{1+x+x^2}-\sqrt{x+1}\right)\times\left(\sqrt{1+x+x^2}+\sqrt{x+1}\right)}{2x^2\left(\sqrt{1+x+x^2}+\sqrt{x+1}\right)}$$

$$= \lim_{x\to 0}\frac{\left(1+x+x^2\right)-(x+1)}{2x^2\left(\sqrt{1+x+x^2}+\sqrt{x+1}\right)}$$

$$= \lim_{x\to 0}\frac{x^2}{2x^2\left(\sqrt{1+x+x^2}+\sqrt{x+1}\right)}$$

$$= \frac{1}{2\left(\sqrt{1}+\sqrt{1}\right)}$$

$$= \frac{1}{2\times 2}$$

$$= \frac{1}{4}$$

Q23

$$\lim_{x \to 4} \frac{2-\sqrt{x}}{4-x}$$

$$= \lim_{x \to 4} \frac{\left(2-\sqrt{x}\right)\left(2+\sqrt{x}\right)}{(4-x)\left(2+\sqrt{x}\right)}$$

$$= \lim_{x \to 4} \frac{(2)^2-\left(\sqrt{x}\right)^2}{(4-x)\left(2+\sqrt{x}\right)}$$

$$= \lim_{x \to 4} \frac{(4-x)}{(4-x)\left(2+\sqrt{x}\right)}$$

$$= \frac{1}{2+\sqrt{4}}$$

$$= \frac{1}{2+2}$$

$$= \frac{1}{4}$$

Q24

$$\lim_{x \to a} \frac{x-a}{\sqrt{x}-\sqrt{a}}$$

$$= \lim_{x \to a} \frac{(x-a)\left(\sqrt{x}+\sqrt{a}\right)}{\left(\sqrt{x}-\sqrt{a}\right)\left(\sqrt{x}+\sqrt{a}\right)}$$

$$= \lim_{x \to a} \frac{(x-a)\left(\sqrt{x}+\sqrt{a}\right)}{(x-a)}$$

$$= \lim_{x \to a} \left(\sqrt{x}+\sqrt{a}\right)$$

$$= 2\sqrt{a}$$

Q25

$$\lim_{x \to 0} \frac{\sqrt{1+3x}-\sqrt{1-3x}}{x}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{1+3x}-\sqrt{1-3x}\right)}{x} \times \frac{\left(\sqrt{1+3x}+\sqrt{1-3x}\right)}{\left(\sqrt{1+3x}+\sqrt{1-3x}\right)}$$

$$= \lim_{x \to 0} \frac{(1+3x)-(1-3x)}{x\left(\sqrt{1+3x}+\sqrt{1-3x}\right)}$$

$$= \lim_{x \to 0} \frac{6x}{x\left(\sqrt{1+3x}+\sqrt{1-3x}\right)}$$

$$= \lim_{x \to 0} \frac{6}{\left(\sqrt{1+3x}+\sqrt{1-3x}\right)}$$

$$= \frac{6}{\sqrt{1}+\sqrt{1}}$$

$$= \frac{6}{2}$$

$$= 3$$

Q26

$$\lim_{x \to 0} \frac{\sqrt{2-x} - \sqrt{2+x}}{x}$$

$$= \lim_{x \to 0} \frac{\left(\sqrt{2-x} + \sqrt{2+x}\right) \times \left(\sqrt{2-x} + \sqrt{2+x}\right)}{x \times \left(\sqrt{2-x} + \sqrt{2+x}\right)}$$

$$= \lim_{x \to 0} \frac{(2-x) - (2+x)}{x\left(\sqrt{2-x} + \sqrt{2+x}\right)}$$

$$= \lim_{x \to 0} \frac{-2x}{x\left(\sqrt{2-x} + \sqrt{2+x}\right)}$$

$$= \frac{-2}{\sqrt{2} + \sqrt{2}}$$

$$= \frac{-2}{2\sqrt{2}}$$

$$= \frac{-1}{\sqrt{2}}$$

Q27

$$\lim_{x \to 1} \frac{\sqrt{3+x} - \sqrt{5-x}}{x^2 - 1}$$

$$= \lim_{x \to 1} \frac{\left(\sqrt{3+x} - \sqrt{5-x}\right)}{(x-1)(x+1)} \times \frac{\left(\sqrt{3+x} + \sqrt{5-x}\right)}{\left(\sqrt{3+x} + \sqrt{5-x}\right)}$$

$$= \lim_{x \to 1} \frac{\left((3+x) - (5-x)\right)}{(x-1)(x+1)\left(\sqrt{3+x} + \sqrt{5-x}\right)}$$

$$= \lim_{x \to 1} \frac{-2 + 2x}{(x-1)(x+1)\left(\sqrt{3+x} + \sqrt{5-x}\right)}$$

$$= \lim_{x \to 1} \frac{-2(x-1)}{(x-1)(x+1)\left(\sqrt{3+x} + \sqrt{5-x}\right)}$$

$$= \lim_{x \to 1} \frac{-2}{(x+1)\left(\sqrt{3+x} + \sqrt{5-x}\right)}$$

$$= \frac{2}{(1+1)\left(\sqrt{3+1} + \sqrt{5-1}\right)}$$

$$= \frac{2}{(2)(2+2)}$$

$$= \frac{1}{4}$$

Q28

$$\lim_{x\to 1}\frac{(2x-3)\left(\sqrt{x}-1\right)}{3x^2+3x-6}$$

$$=\lim_{x\to 1}\frac{(2x-3)\left(\sqrt{x}-1\right)}{3\left[x^2+x-2\right]}$$

$$=\lim_{x\to 1}\frac{(2x-3)\left(\sqrt{x}-1\right)}{3\left[x^2+2x-x-2\right]}$$

$$=\lim_{x\to 1}\frac{(2x-3)\left(\sqrt{x}-1\right)}{3(x+2)(x-1)}$$

$$=\lim_{x\to 1}\frac{(2x-3)\left(\sqrt{x}-1\right)}{3(x+2)\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)}$$

$$=\lim_{x\to 1}\frac{(2x-3)}{3(x+2)\left(\sqrt{x}+1\right)}$$

$$=\frac{(2-3)}{3(1+2)(1+1)}$$

$$=\frac{-1}{3\times 3\times 2}$$

$$=-\frac{1}{18}$$

Q29

$$\lim_{x\to 0}\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}$$

$$=\lim_{x\to 0}\frac{\left(\sqrt{1+x^2}-\sqrt{1+x}\right)}{\left(\sqrt{1+x^3}-\sqrt{1+x}\right)}\times\frac{\left(\sqrt{1+x^2}+\sqrt{1+x}\right)}{\left(\sqrt{1+x^2}+\sqrt{1+x}\right)}$$

$$=\lim_{x\to 0}\frac{\left(1+x^2\right)-(1+x)\times\left(\sqrt{1+x^3}+\sqrt{1+x}\right)}{\left(\sqrt{1+x^3}-\sqrt{1+x}\right)\left(\sqrt{1+x^2}+\sqrt{1+x}\right)\left(\sqrt{1+x^3}+\sqrt{1+x}\right)}$$

$$=\lim_{x\to 0}\frac{\left(x^2-x\right)\left(\sqrt{1+x^3}+\sqrt{1+x}\right)}{\left(\sqrt{1+x^2}+\sqrt{1+x}\right)\left(1+x^3-1-x\right)}$$

$$=\lim_{x\to 0}\frac{x(x-1)\left(\sqrt{1+x^3}+\sqrt{1+x}\right)}{\left(\sqrt{1+x^2}+\sqrt{1+x}\right)x\left(x^2-1\right)}$$

$$=\lim_{x\to 0}\frac{x(x-1)\left(\sqrt{1+x^3}+\sqrt{1+x}\right)}{\left(\sqrt{1+x^2}+\sqrt{1+x}\right)(x)(x-1)(x+1)}$$

$$=\lim_{x\to 0}\frac{\left(\sqrt{1+x^3}+\sqrt{1+x}\right)}{\left(\left(\sqrt{1+x^2}+\sqrt{1+x}\right)(x+1)\right)}$$

$$=\frac{2}{2}=1$$

Q30

$$\lim_{x \to 1} \frac{x^2 - \sqrt{x}}{\sqrt{x} - 1}$$

$$= \lim_{x \to 1} \frac{\left(x^2 - \sqrt{x}\right)\left(x^2 + \sqrt{x}\right)}{\left(\sqrt{x} - 1\right)\left(x^2 + \sqrt{x}\right)}$$

$$= \lim_{x \to 1} \frac{x^4 - x}{\left(\sqrt{x} - 1\right)\left(x^2 + \sqrt{x}\right)}$$

$$= \lim_{x \to 1} \frac{x\left(x^3 - 1\right)}{\left(\sqrt{x} - 1\right)\left(x^2 + \sqrt{x}\right)}$$

$$= \lim_{x \to 1} \frac{x\left(x - 1\right)\left(x^2 + 1 + x\right)}{\left(\sqrt{x} - 1\right)\left(x^2 + \sqrt{x}\right)}$$

$$= \lim_{x \to 1} \frac{x\left(\sqrt{x} - 1\right)\left(\sqrt{x} + 1\right)\left(x^2 + 1 + x\right)}{\left(\sqrt{x} - 1\right)\left(x^2 + \sqrt{x}\right)}$$

$$= \lim_{x \to 1} \frac{x\left(\sqrt{x} + 1\right)\left(x^2 + 1 + x\right)}{\left(x^2 + \sqrt{x}\right)}$$

$$= \frac{1(1+1)(1+1+1)}{1+1}$$

$$= \frac{6}{2}$$

$$= 3$$

Q31

$$\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{\left(\sqrt{x+h} - \sqrt{x}\right)}{h} \times \frac{\left(\sqrt{x+h} + \sqrt{x}\right)}{\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \lim_{h \to 0} \frac{(x + h - x)}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \frac{1}{\sqrt{x} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

Q32

$$\lim_{x\to\sqrt{10}}\frac{\sqrt{7+2x}-\left(\sqrt{5}+\sqrt{2}\right)}{x^2-10}$$

$$=\lim_{x\to\sqrt{10}}\frac{\sqrt{7+2x}-\left(\sqrt{\left(\sqrt{5}+\sqrt{2}\right)^2}\right)}{x^2-10}$$

$$=\lim_{x\to\sqrt{10}}\frac{\sqrt{7+2x}-\left(\sqrt{7+2\sqrt{10}}\right)}{x^2-10}$$

$$=\lim_{x\to\sqrt{10}}\frac{\sqrt{7+2x}-\left(\sqrt{7+2\sqrt{10}}\right)}{x^2-10}\times\frac{\sqrt{7+2x}+\left(\sqrt{7+2\sqrt{10}}\right)}{\sqrt{7+2x}+\left(\sqrt{7+2\sqrt{10}}\right)}$$

$$=\lim_{x\to\sqrt{10}}\frac{7+2x-7-2\sqrt{10}}{\left(x^2-10\right)\left(\sqrt{7+2x}+\left(\sqrt{7+2\sqrt{10}}\right)\right)}$$

$$=\lim_{x\to\sqrt{10}}\frac{2\left(x-\sqrt{10}\right)}{\left(x^2-10\right)\left(\sqrt{7+2x}+\left(\sqrt{7+2\sqrt{10}}\right)\right)}$$

$$=\lim_{x\to\sqrt{10}}\frac{2}{\left(x+\sqrt{10}\right)\left(\sqrt{7+2x}+\left(\sqrt{7+2\sqrt{10}}\right)\right)}$$

$$=\frac{2}{\left(\sqrt{10}+\sqrt{10}\right)\left(\sqrt{7+2\sqrt{10}}+\left(\sqrt{7+2\sqrt{10}}\right)\right)}$$

$$=\frac{2}{\left(2\sqrt{10}\right)\left(2\sqrt{7+2\sqrt{10}}\right)}$$

$$=\frac{1}{\left(2\sqrt{10}\right)\left(\sqrt{7+2\sqrt{10}}\right)}$$

$$=\frac{1}{\left(2\sqrt{10}\right)\left(\sqrt{5}+\sqrt{2}\right)}$$

$$=\frac{\left(\sqrt{5}-\sqrt{2}\right)}{\left(6\sqrt{10}\right)}$$

Q33

$$\lim_{x\to\sqrt{6}}\frac{\sqrt{5+2x}-\left(\sqrt{3}+\sqrt{2}\right)}{x^2-6}$$

$$=\lim_{x\to\sqrt{6}}\frac{\sqrt{5+2x}-\left(\sqrt{\left(\sqrt{3}+\sqrt{2}\right)^2}\right)}{x^2-6}$$

$$=\lim_{x\to\sqrt{6}}\frac{\sqrt{5+2x}-\left(\sqrt{5+2\sqrt{6}}\right)}{x^2-6}$$

$$=\lim_{x\to\sqrt{6}}\frac{\sqrt{5+2x}-\left(\sqrt{5+2\sqrt{6}}\right)}{x^2-6}\times\frac{\sqrt{5+2x}+\left(\sqrt{5+2\sqrt{6}}\right)}{\sqrt{5+2x}+\left(\sqrt{5+2\sqrt{6}}\right)}$$

$$=\lim_{x\to\sqrt{6}}\frac{5+2x-5-2\sqrt{6}}{\left(x^2-6\right)\left(\sqrt{5+2x}+\left(\sqrt{5+2\sqrt{6}}\right)\right)}$$

$$=\lim_{x\to\sqrt{6}}\frac{2\left(x-\sqrt{6}\right)}{\left(x^2-6\right)\left(\sqrt{5+2x}+\left(\sqrt{5+2\sqrt{6}}\right)\right)}$$

$$=\lim_{x\to\sqrt{6}}\frac{2}{\left(x+\sqrt{6}\right)\left(\sqrt{5+2x}+\left(\sqrt{5+2\sqrt{6}}\right)\right)}$$

$$=\frac{2}{\left(\sqrt{6}+\sqrt{6}\right)\left(\sqrt{5+2\sqrt{6}}+\left(\sqrt{5+2\sqrt{6}}\right)\right)}$$

$$=\frac{2}{\left(2\sqrt{6}\right)\left(2\sqrt{5+2\sqrt{6}}\right)}$$

$$=\frac{1}{\left(2\sqrt{6}\right)\left(\sqrt{5+2\sqrt{6}}\right)}$$

$$=\frac{1}{\left(2\sqrt{6}\right)\left(\sqrt{3}+\sqrt{2}\right)}$$

$$=\frac{\left(\sqrt{3}-\sqrt{2}\right)}{\left(2\sqrt{6}\right)}$$

Q34

$$\lim_{x\to\sqrt{2}}\frac{\sqrt{3+2x}-\left(\sqrt{2}+1\right)}{x^2-2}$$

$$=\lim_{x\to\sqrt{2}}\frac{\sqrt{3+2x}-\left(\sqrt{\left(\sqrt{2}+1\right)^2}\right)}{x^2-2}$$

$$=\lim_{x\to\sqrt{2}}\frac{\sqrt{3+2x}-\left(\sqrt{3+2\sqrt{2}}\right)}{x^2-2}$$

$$=\lim_{x\to\sqrt{2}}\frac{\sqrt{3+2x}-\left(\sqrt{3+2\sqrt{2}}\right)}{x^2-2}\times\frac{\sqrt{3+2x}+\left(\sqrt{3+2\sqrt{2}}\right)}{\sqrt{3+2x}+\left(\sqrt{3+2\sqrt{2}}\right)}$$

$$=\lim_{x\to\sqrt{2}}\frac{3+2x-3-2\sqrt{2}}{\left(x^2-2\right)\left(\sqrt{3+2x}+\left(\sqrt{3+2\sqrt{2}}\right)\right)}$$

$$=\lim_{x\to\sqrt{2}}\frac{2\left(x-\sqrt{2}\right)}{\left(x^2-2\right)\left(\sqrt{3+2x}+\left(\sqrt{3+2\sqrt{2}}\right)\right)}$$

$$=\lim_{x\to\sqrt{2}}\frac{2}{\left(x+\sqrt{2}\right)\left(\sqrt{3+2x}+\left(\sqrt{3+2\sqrt{2}}\right)\right)}$$

$$=\frac{2}{\left(\sqrt{2}+\sqrt{2}\right)\left(\sqrt{3+2\sqrt{2}}+\left(\sqrt{3+2\sqrt{2}}\right)\right)}$$

$$=\frac{2}{\left(2\sqrt{2}\right)\left(2\sqrt{3+2\sqrt{2}}\right)}$$

$$=\frac{1}{\left(2\sqrt{2}\right)\left(\sqrt{3+2\sqrt{2}}\right)}$$

$$=\frac{1}{\left(2\sqrt{2}\right)\left(\sqrt{2}+1\right)}$$

$$=\frac{\left(\sqrt{2}-1\right)}{\left(2\sqrt{2}\right)}$$

Ex 29.5

Q1

$$\lim_{x \to a} \frac{(x+2)^{\frac{5}{2}} - (a+2)^{\frac{5}{2}}}{x-a}$$

$$= \lim_{x \to a} \frac{(x+2)^{\frac{5}{2}} - (a+2)^{\frac{5}{2}}}{(x+2)-(a+2)}$$

$$= \lim_{y \to b} \frac{y^{\frac{5}{2}} - b^{\frac{5}{2}}}{y-b}, \text{ where } x+2 = y \text{ and } a+2 = b$$

$$= \frac{5}{2} b^{\frac{5}{2}-1} \qquad \left[\text{Using formula } \lim_{x \to a} \frac{x^n - a^n}{x-a} = na^{n-1}\right]$$

$$= \frac{5}{2}(a+2)^{\frac{5}{2}-1}$$

$$= \frac{5}{2}(a+2)^{\frac{3}{2}}$$

Q2

$$\lim_{x \to a} \frac{(x+2)^{\frac{3}{2}} - (a+2)^{\frac{3}{2}}}{x-a}$$

$$= \lim_{x \to a} \frac{(x+2)^{\frac{3}{2}} - (a+2)^{\frac{3}{2}}}{(x+2)-(a+2)}$$

Let $x+2 = y,\ a+2 = b$

$$\Rightarrow \lim_{(x+2) \to (a+2)} \frac{(y)^{\frac{3}{2}} - (b)^{\frac{3}{2}}}{(y)-(b)} \qquad \left[\text{Using formula } \lim_{x \to a} \frac{x^n - a^n}{x-a} = na^{n-1}\right]$$

$$= \frac{3}{2}(b)^{\frac{3}{2}-1}$$

$$= \frac{3}{2}(a+2)^{\frac{3}{2}-1}$$

$$= \frac{3}{2}(a+2)^{\frac{1}{2}}$$

Q3

$$\lim_{x\to 0}\frac{(1+x)^6-1}{(1+x)^2-1}$$

$$= \lim_{x\to 0}\frac{\dfrac{(1+x)^6-1^6}{1+x-1}}{\dfrac{(1+x)^2-1^2}{1+x-1}}$$

$\Rightarrow$ Let $1+x=y$, as $x\to 0$, $y\to 1$

$$= \frac{\lim\limits_{y\to 1}\dfrac{y^6-1^6}{y-1}}{\lim\limits_{y\to 1}\dfrac{y^2-1}{y-1}}$$

$$= \frac{6(1)^{6-1}}{2(1)^{2-1}} \qquad \left[\text{Using formula } \lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$

$$= \frac{6}{2}$$

$$= 3$$

Q4

$$\lim_{x\to a}\frac{x^{\frac{2}{7}}-a^{\frac{2}{7}}}{x-a}$$

Applying formula $\lim\limits_{x\to a}\dfrac{x^n-a^n}{x-a}=na^{n-1}$, here, $n=\dfrac{2}{7}$

$$\Rightarrow \lim_{x\to a}\frac{x^{\frac{2}{7}}-a^{\frac{2}{7}}}{x-a}=\frac{2}{7}(a)^{\frac{2}{7}-1}$$

$$= \frac{2}{7}a^{\frac{-5}{7}}$$

Q5

$$\lim_{x \to a} \frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x^{\frac{2}{7}} - a^{\frac{2}{7}}}$$

$$= \lim_{x \to a} \frac{\frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x - a}}{\frac{x^{\frac{2}{7}} - a^{\frac{2}{7}}}{x - a}} \qquad \left[\text{Dividing numerator and denominator by } x - a\right]$$

$$= \frac{\lim_{x \to a} \frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x - a}}{\lim_{x \to a} \frac{x^{\frac{2}{7}} - a^{\frac{2}{7}}}{x - a}}$$

Applying formula $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

Here, $n = \frac{5}{7}$ is numerator and applying $\lim_{x \to a} \frac{x^m - a^m}{x - a} = ma^{m-1}$ in denominator, where $m = \frac{2}{7}$

$$\Rightarrow \quad \frac{\lim_{x \to a} \frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x - a}}{\lim_{x \to a} \frac{x^{\frac{2}{7}} - a^{\frac{2}{7}}}{x - a}} = \frac{\frac{5}{7}(a)^{\frac{5}{7}-1}}{\frac{2}{7}(a)^{\frac{2}{7}-1}}$$

$$= \frac{\frac{5}{7}a^{\frac{-2}{7}}}{\frac{2}{7}a^{\frac{-5}{7}}}$$

$$= \frac{5}{2}a^{\frac{-2}{7}+\frac{5}{7}}$$

$$= \frac{5}{2}a^{\frac{3}{7}}$$

Q6

$$\lim_{x \to -\frac{1}{2}} \frac{8x^3 + 1}{2x + 1}$$

$$= \frac{8}{2} \lim_{x \to -\frac{1}{2}} \frac{x^3 + \left(\frac{1}{2}\right)^3}{x + \frac{1}{2}}$$

$$= 4 \lim_{x \to -\frac{1}{2}} \frac{x^3 - \left(-\frac{1}{2}\right)^3}{x - \left(-\frac{1}{2}\right)}$$

Applying formula $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

Here, $n = 3$, $a = \frac{-1}{2}$

$$\Rightarrow \quad 4 \lim_{x \to -\frac{1}{2}} \frac{x^3 - \left(-\frac{1}{2}\right)^3}{x - \left(-\frac{1}{2}\right)} = 4 \times 3 \left(-\frac{1}{2}\right)^{3-1}$$

$$= 4 \times 3 \times \frac{1}{4}$$

$$= 3$$

Q7

$$\lim_{x \to 27} \frac{\left(x^{\frac{1}{3}} + 3\right)\left(x^{\frac{1}{3}} - 3\right)}{x - 27}$$

$$= \lim_{x \to 27} \frac{\left(x^{\frac{2}{3}} - 9\right)}{x - 27}$$

$$= \lim_{x \to 27} \frac{x^{\frac{2}{3}} - 27^{\frac{2}{3}}}{x - 27}$$

Applying formula $\lim_{x \to 27} \frac{x^n - a^n}{x - a} = na^{n-1}$

$$= \frac{2}{3}(27)^{\frac{2}{3}-1}$$

$$= \frac{2}{3}(27)^{\frac{-1}{3}}$$

$$= \frac{2}{3} \times \frac{1}{(27)^{\frac{1}{3}}}$$

$$= \frac{2}{3} \times \frac{1}{3}$$

$$= \frac{2}{9}$$

Q8

$$\lim_{x \to 4} \frac{x^3 - 64}{x^2 - 16}$$

$$= \lim_{x \to 4} \frac{x^3 - 4^3}{x^2 - 4^2}$$

$$= \lim_{x \to 4} \frac{\frac{x^3 - 4^3}{x - 4}}{\frac{x^2 - 4^2}{x - 4}}$$ [Dividing numerator and denominator by $x - 4$]

$$= \frac{\lim_{x \to 4} \frac{x^3 - 4^3}{x - 4}}{\lim_{x \to 4} \frac{x^2 - 4}{x - 4}}$$

Applying $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$ in numerator and $\lim_{x \to a} \frac{x^m - a^m}{x - a} = ma^{m-1}$ in denominator

$\Rightarrow \quad n = 3, \ m = 2$

$$\Rightarrow \quad \frac{\lim_{x \to 4} \frac{x^3 - 4^3}{x - 4}}{\lim_{x \to 4} \frac{x^2 - 4}{x - 4}} = \frac{3(4)^{3-1}}{2(4)^{2-1}} = \frac{3(4)^2}{2(4)}$$

$= 6$

Q9

$$\lim_{x\to 1}\frac{x^{15}-1}{x^{10}-1}$$

$$=\lim_{x\to 1}\frac{\frac{x^{15}-1^{15}}{x-1}}{\frac{x^{10}-1^{10}}{x-1}}$$ [Dividing numerator and denomintoar by $(x-1)$]

$$=\frac{\lim_{x\to 1}\frac{x^{15}-1^{15}}{x-1}}{\lim_{x\to 1}\frac{x^{10}-1^{10}}{x-1}}$$

Applying formula $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$ in numerator and $\lim_{x\to a}\frac{x^m-a^m}{x-a}=ma^{m-1}$ in denominator

Here, $n=15$, $m=10$

$$\Rightarrow \quad \frac{\lim_{x\to 1}\frac{x^{15}-1^{15}}{x-1}}{\lim_{x\to 1}\frac{x^{10}-1^{10}}{x-1}}=\frac{15\,(1)^{15-1}}{10\,(1)^{10-1}}$$

$$=\frac{15}{10}$$

$$=\frac{3}{2}$$

Q10

$$\lim_{x\to -1}\frac{x^3+1}{x+1}$$

$$=\lim_{x\to -1}\frac{x^3-(-1)^3}{x-(-1)}$$

Applying formula $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$

Here, $n=3$, $a=-1$

$$\Rightarrow \quad \lim_{x\to -1}\frac{x^3-(-1)^3}{x-(-1)}=na^{n-1}$$

$$=3(-1)^{3-1}$$

$$=3(-1)^2$$

$$=3$$

Q11

$$\lim_{x \to a} \frac{x^{\frac{2}{3}} - a^{\frac{2}{3}}}{x^{\frac{3}{4}} - a^{\frac{3}{4}}}$$

$$= \lim_{x \to a} \frac{\dfrac{x^{\frac{2}{3}} - a^{\frac{2}{3}}}{x - a}}{\dfrac{x^{\frac{3}{4}} - a^{\frac{3}{4}}}{x - a}}$$ [Dividing numerator and denominator by $x - a$]

$$= \frac{\lim\limits_{x \to a} \dfrac{x^{\frac{2}{3}} - a^{\frac{2}{3}}}{x - a}}{\lim\limits_{x \to a} \dfrac{x^{\frac{3}{4}} - a^{\frac{3}{4}}}{x - a}}$$

Applying the formula $\lim\limits_{x \to a} \dfrac{x^n - a^n}{x - a} = na^{n-1}$ in numerator and $\lim\limits_{x \to a} \dfrac{x^m - a^m}{x - a} = ma^{m-1}$ in denominator respectively

Here, $n = \dfrac{2}{3}$, $m = \dfrac{3}{4}$

$$\Rightarrow \quad \frac{\lim\limits_{x \to a} \dfrac{x^{\frac{2}{3}} - a^{\frac{2}{3}}}{x - a}}{\lim\limits_{x \to a} \dfrac{x^{\frac{3}{4}} - a^{\frac{3}{4}}}{x - a}} = \frac{\frac{2}{3}(a)^{\frac{2}{3}-1}}{\frac{3}{4}(a)^{\frac{3}{4}-1}}$$

$$= \frac{8}{9} a^{\frac{-1}{3}+\frac{1}{4}}$$

$$= \frac{8}{9} a^{\frac{-1}{12}}$$

Q12

$$\lim_{x\to 3}\frac{x^n-3^n}{x-3}=108$$

LHS $=\lim_{x\to 3}\frac{x^n-3^n}{x-3}$

Applying the formula $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$

Here, $n=n,\ a=3$

$\Rightarrow\quad \lim_{x\to 3}\frac{x^n-3^n}{x-3}=n(3)^{n-1}$

It is given that $n(3)^{n-1}=108$

$$\begin{aligned}\Rightarrow\quad n(3)^{n-1}&=2\times2\times3\times3\times3\\&=(2)^2\times(3)^3\\&=4(3)^{4-1}\end{aligned}$$

$\Rightarrow\quad n=4$

Q13

If $\lim_{x\to a}\frac{x^9-a^9}{x-a}=9$ ---(i)

LHS $=\lim_{x\to a}\frac{x^9-a^9}{x-a}$

$=9(a)^{9-1}$

$=9a^8$

It is given that $9a^8=9$ [From (i)]

$\Rightarrow\quad a^8=\frac{9}{9}=1$

$\Rightarrow\quad a^4=1$

$a^2=1$

$a=\pm1$

$\Rightarrow\quad a=1$ and $a=-1$

Q14

If $\lim_{x \to a} \frac{x^5 - a^5}{x - a} = 405$ ---(i)

LHS = $\lim_{x \to a} \frac{x^5 - a^5}{x - a}$

$= 5(a)^{5-1}$

$= 5a^4$

It is given that $5a^4 = 405$

$\Rightarrow$ $5a^4 = 405$

$a^4 = \frac{405}{5} = 81$

$a^4 = (3)^4$, $a^2 = 9$

$a = \pm 3$

$\Rightarrow$ $a = 3$ and $a = -3$

Q15

If $\lim_{x \to a} \frac{x^9 - a^9}{x - a} = \lim_{x \to 5} (4 + x)$ ---(i)

LHS = $\lim_{x \to a} \frac{x^9 - a^9}{x - a}$

$= 9(a)^{9-1} = 9a^8$ ---(ii)

RHS = $\lim_{x \to 5} (4 + x)$

$= 4 + 5 = 9$ ---(iii)

Substituting (ii) and (iii) in (i),

$9a^8 = 9$

$a^8 = 1$

$\Rightarrow$ $a^2 = 1$

$\Rightarrow$ $a = 1$, $a = -1$

Q16

If $\lim\limits_{x \to a} \dfrac{x^3 - a^3}{x - a} = \lim\limits_{x \to 1} \dfrac{x^4 - 1}{x - 1}$ ---(i)

LHS = If $\lim\limits_{x \to a} \dfrac{x^3 - a^3}{x - a}$

$= 3(a)^{3-1}$

$= 3a^2$ ---(ii)

RHS = $\lim\limits_{x \to a} \dfrac{x^4 - 1}{x - 1}$

$= \lim\limits_{x \to a} \dfrac{x^4 - 1}{x - 1}$

$= 4(1)^{4-1}$

$= 4$ ---(iii)

Substituting (ii) and (iii) in (i),

$3a^2 = 4$

$a^2 = \dfrac{4}{3}$

$a = \pm \dfrac{2}{\sqrt{3}}$

Ex 29.6

Q1

$$\lim_{x\to\infty} \frac{(3x-1)(4x-2)}{(x+8)(x-1)} \qquad \left[\text{Expression is } \frac{\infty}{\infty}\right]$$

$$= \lim_{x\to\infty} \frac{(12x^2-10x+2)}{(x^2+9x-8)}$$

$$= \lim_{x\to\infty} \left(\frac{12-\frac{10}{x}+\frac{2}{x^2}}{1+\frac{9}{x}-\frac{8}{x^2}}\right)$$

$$= \frac{12-0+0}{1+0-1}$$

$$= 12$$

Q2

$$\lim_{x\to\infty} \frac{3x^3-4x^2+6x-1}{2x^3+x^2-5x+7}$$

$$= \lim_{x\to\infty} \frac{3-\frac{4}{x}+\frac{6}{x^2}-\frac{1}{x^3}}{2+\frac{1}{x}-\frac{5}{x^2}+\frac{7}{x^3}}$$

$$= \frac{3-0+0-0}{2+0-0+0}$$

$$= \frac{3}{2}$$

Q3

$$\lim_{x\to\infty} \frac{5x^3-6}{\sqrt{9+4x^6}}$$

$$= \lim_{x\to\infty} \frac{5-\frac{6}{x^3}}{\sqrt{\frac{9}{x^6}+\frac{4x^6}{x^6}}}$$

$$= \lim_{x\to\infty} \frac{\left(5-\frac{6}{x^3}\right)}{\sqrt{\frac{9}{x^6}+4}}$$

$$= \frac{5}{\sqrt{4}} = \frac{5}{2}$$

Q4

$$\lim_{x\to\infty} \sqrt{x^2+cx}-x$$

$$= \lim_{x\to\infty}\left(\left(\sqrt{x^2+cx}-x\right)\frac{\left(\sqrt{x^2+cx}+x\right)}{\sqrt{x^2+cx}+x}\right)$$

$$= \lim_{x\to\infty}\frac{\left(x^2+cx-x^2\right)}{\sqrt{x^2+cx}+x}$$

$$= \lim_{x\to\infty}\frac{cx}{\sqrt{x^2+cx}+x} \qquad \left[\frac{\infty}{\infty}\text{ form}\right]$$

$$= \lim_{x\to\infty}\frac{c}{\sqrt{1+\frac{c}{x}}+1}$$

$$= \frac{c}{1+1} = \frac{c}{2}$$

Q5

$$\lim_{x\to\infty}\sqrt{x+1}-\sqrt{x}$$

$$= \lim_{x\to\infty}\frac{\left(\sqrt{x+1}-\sqrt{x}\right)\left(\sqrt{x+1}+\sqrt{x}\right)}{\left(\sqrt{x+1}+\sqrt{x}\right)}$$

$$= \lim_{x\to\infty}\frac{(x+1-x)}{\sqrt{x+1}+\sqrt{x}}$$

$$= \lim_{x\to\infty}\left(\frac{1}{\sqrt{x+1}+\sqrt{x}}\right)$$

$$= \frac{1}{\infty}$$

$$= 0$$

Q6

$$\lim_{x\to\infty} \sqrt{x^2+7x}-x$$

$$= \lim_{x\to\infty} \left(\frac{\left(\sqrt{x^2+7x}-x\right)\left(\sqrt{x^2+7x}+x\right)}{\sqrt{x^2+7x}+x} \right)$$

$$= \lim_{x\to\infty} \left(\frac{\left(x^2+7x\right)-x^2}{\sqrt{x^2+7x}+x} \right)$$

$$= \lim_{x\to\infty} \frac{7x}{\sqrt{x^2+7x}+x}$$

$$= \lim_{x\to\infty} \frac{7}{\sqrt{\frac{x^2}{x^2}+\frac{7x}{x^2}}+1}$$

$$= \lim_{x\to\infty} \frac{7}{\sqrt{1+\frac{7}{x}}+1}$$

$$= \frac{7}{2}$$

Q7

$$\lim_{x\to\infty} \frac{x}{\sqrt{4x^2+1}-1}$$

$$= \lim_{x\to\infty} \frac{1}{\sqrt{4+\frac{1}{x^2}}-\frac{1}{x}}$$

$$= \frac{1}{\sqrt{4}-0}$$

$$= \frac{1}{2}$$

Q8

$$\lim_{n\to\infty} \frac{n^2}{1+2+3+\ldots+n}$$

$$= \lim_{n\to\infty} \frac{n^2}{\frac{1}{2}n(n+1)} \qquad \left[\because 1+2+3+\ldots+n = \frac{n(n+1)}{2}\right]$$

$$= \lim_{n\to\infty} \frac{2n^2}{n^2+n}$$

$$= 2\lim_{n\to\infty} \frac{n^2}{n^2+n}$$

$$= 2\lim_{n\to\infty} \frac{n^2}{n^2\left(1+\frac{1}{n}\right)}$$

$$= 2\times\frac{1}{1+0}$$

$$= 2$$

Q9

$$\lim_{n\to\infty} \frac{\frac{3}{x}+\frac{4}{x^2}}{\frac{5}{x}+\frac{6}{x^2}} \qquad \left[\frac{0}{0}\ \text{form}\right]$$

$$= \lim_{x\to\infty} \frac{\frac{1}{x}\left(3+\frac{4}{x}\right)}{\frac{1}{x}\left(5+\frac{6}{x}\right)}$$

$$= \lim_{x\to\infty} \frac{(3+0)}{(5+0)} = \frac{3}{5}$$

Q10

$$\lim_{x\to\infty}\frac{\sqrt{x^2+a^2}-\sqrt{x^2+b^2}}{\sqrt{x^2+c^2}-\sqrt{x^2+d^2}} \qquad \left[\frac{\infty}{\infty}\text{ form}\right]$$

$$\lim_{x\to\infty}\frac{\left(\sqrt{x^2+a^2}-\sqrt{x^2+b^2}\right)}{\left(\sqrt{x^2+c^2}-\sqrt{x^2+d^2}\right)}\times\frac{\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}{\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(\left(x^2+a^2\right)-\left(x^2-b^2\right)\right)}{\left(\sqrt{x^2+c^2}-\sqrt{x^2+d^2}\right)\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(a^2-b^2\right)}{\left(\sqrt{x^2+c^2}-\sqrt{x^2+d^2}\right)\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(a^2-b^2\right)\left(\sqrt{x^2+c^2}+\sqrt{x^2+d^2}\right)}{\left(\sqrt{x^2+c^2}-\sqrt{x^2+d^2}\right)\left(\sqrt{x^2+c^2}+\sqrt{x^2+d^2}\right)\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(a^2-b^2\right)\left(\sqrt{x^2+c^2}+\sqrt{x^2+d^2}\right)}{\left(x^2+c^2-x^2-d^2\right)\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(a^2-b^2\right)\left(\sqrt{x^2+c^2}+\sqrt{x^2+d^2}\right)}{\left(c^2-d^2\right)\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(a^2-b^2\right)\left(x\sqrt{1+\frac{c^2}{x^2}}+x\sqrt{1+\frac{d^2}{x^2}}\right)}{\left(c^2-d^2\right)\left(x\sqrt{1+\frac{a^2}{x^2}}+x\sqrt{1+\frac{b^2}{x^2}}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(a^2-b^2\right)\left(\sqrt{1+\frac{c^2}{x^2}}+\sqrt{1+\frac{d^2}{x^2}}\right)}{\left(c^2-d^2\right)\left(\sqrt{1+\frac{a^2}{x^2}}+\sqrt{1+\frac{b^2}{x^2}}\right)}$$

$$=\frac{\left(a^2-b^2\right)\left(\sqrt{1+0}+\sqrt{1+0}\right)}{\left(c^2-d^2\right)\left(\sqrt{1+0}+\sqrt{1+0}\right)}$$

$$=\frac{\left(a^2-b^2\right)(1+1)}{\left(c^2-d^2\right)(1+1)}$$

$$=\frac{\left(a^2-b^2\right)(2)}{\left(c^2-d^2\right)(2)}=\frac{a^2-b^2}{c^2-d^2}$$

Q11

$$\lim_{n\to\infty} \frac{(n+2)!+(n+1)!}{(n+2)!-(n+1)!}$$

We know that $(n+2) = (n+2)(n+1)!$

$$\Rightarrow \quad \lim_{n\to\infty} \frac{(n+2)(n+1)!+(n+1)!}{(n+2)(n+1)!-(n+1)!}$$

$$= \lim_{n\to\infty} \frac{(n+1)!\left[(n+2)+1\right]}{(n+1)\left[(n+2)-1\right]}$$

$$= \lim_{n\to\infty} \frac{n+3}{n+1} \qquad \left[\frac{\infty}{\infty} \text{ form}\right]$$

$$= \lim_{n\to\infty} \frac{1+\frac{3}{n}}{1+\frac{1}{n}}$$

$$= \frac{1+0}{1+0} = 1$$

$$= 1$$

Q12

$$\lim_{x\to\infty} x\left[\sqrt{x^2+1}-\sqrt{x^2-1}\right]$$

$$= \lim_{x\to\infty} x\left[\sqrt{x^2+1}-\sqrt{x^2-1}\right] \times \frac{\left(\sqrt{x^2+1}+\sqrt{x^2-1}\right)}{\left(\sqrt{x^2+1}+\sqrt{x^2+1}\right)}$$

$$= \lim_{x\to\infty} x\,\frac{x\left(x^2+1-x^2+1\right)}{\sqrt{x^2+1}+\sqrt{x^2-1}}$$

$$= \lim_{x\to\infty} \frac{x(2)}{x\left(\sqrt{1+\frac{1}{x^2}}+\sqrt{1-\frac{1}{x^2}}\right)}$$

$$= \lim_{x\to\infty} \frac{2}{\sqrt{1+\frac{1}{x^2}}+\sqrt{1-\frac{1}{x^2}}}$$

$$= \frac{2}{2} = 1$$

Q13

$$\lim_{x\to\infty}\left[\sqrt{x+1}-\sqrt{x}\right]\sqrt{x+2}$$

$$=\lim_{x\to\infty}\left[\sqrt{x+1}-\sqrt{x}\right]\frac{\left[\sqrt{x+1}+\sqrt{x}\right]}{\left[\sqrt{x+1}+\sqrt{x}\right]}\times\frac{\sqrt{x+2}\times\sqrt{x+2}}{\sqrt{x+2}}$$

$$=\lim_{x\to\infty}\frac{(x+1-x)}{\sqrt{x+1}+\sqrt{x}}\times\frac{(x+2)}{\sqrt{x+2}}$$

$$=\lim_{x\to\infty}\frac{1(x+2)}{\left(\sqrt{x+1}+\sqrt{x}\right)\left(\sqrt{x+2}\right)}$$

$$=\lim_{x\to\infty}\frac{x\left(1+\frac{2}{x}\right)}{\sqrt{x}\left(\sqrt{1+\frac{1}{x}}+\sqrt{1}\right)\left(\sqrt{1+\frac{2}{x}}\right)\sqrt{x}}$$

$$=\lim_{x\to\infty}\frac{\left(1+\frac{2}{x}\right)}{\left(\sqrt{1+\frac{1}{x}}+\sqrt{1}\right)\left(\sqrt{1+\frac{2}{x}}\right)}$$

$$=\lim_{x\to\infty}\frac{(1+0)}{(1+1)\times 1}=\frac{1}{2}$$

Q14

$$\lim_{n\to\infty}\frac{1^2+2^2+3^2+\ldots+n^2}{n^3}$$

$$=\lim_{n\to\infty}\frac{\frac{1}{6}n(n+1)(2n+1)}{n^3}\qquad\left|1^2+2^2+3^2+\ldots+n^2=\frac{1}{6}n(n+1)(2n+1)\right|$$

$$=\frac{1}{6}\lim_{n\to\infty}\frac{(n^2+n)(2n+1)}{n^3}$$

$$=\frac{1}{6}\lim_{n\to\infty}\frac{(2n^3+n^2+2n^2+n)}{n^3}$$

$$=\frac{1}{6}\lim_{n\to\infty}\frac{(2n^3+3n^2+n)}{n^3}\qquad\left[\frac{\infty}{\infty}\text{ form}\right]$$

$$=\frac{1}{6}\lim_{n\to\infty}\frac{\left(2+\frac{3}{n}+\frac{1}{n^2}\right)}{1}$$

$$=\frac{1}{6}\frac{(2+0+0)}{1}=\frac{1}{3}$$

Q15

$$\lim_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+\ldots+\frac{n-1}{n^2}\right)$$

$$\lim_{n\to\infty}\left(\frac{1+2+3+\ldots+(n-1)}{n^2}\right)$$

$$=\lim_{n\to\infty}\frac{(n-1)(n)}{2\times n^2} \qquad \left[1+2+3+\ldots+(n-1)=\frac{(n-1)(n)}{2}\right]$$

$$=\lim_{n\to\infty}\frac{n^2-n}{2n^2} \qquad \left[\frac{\infty}{\infty}\text{ form}\right]$$

$$=\lim_{n\to\infty}\frac{1\quad\frac{1}{n}}{2}$$

$$=\frac{1-0}{2}=\frac{1}{2}$$

$$=\frac{1}{2}$$

Q16

$$\lim_{n\to\infty}\frac{1^3+2^3+3^3-\ldots+n^3}{n^4}$$

$$=\lim_{n\to\infty}\frac{\left[\frac{1}{2}n(n+1)\right]^2}{n^4} \qquad \left[1^3+2^3+3^3+\ldots+n^3=\left(\frac{1}{2}n(n+1)\right)^2\right]$$

$$=\lim_{n\to\infty}\frac{\frac{1}{4}n^2(n+1)^2}{n^4}$$

$$=\lim_{n\to\infty}\frac{1}{4}\frac{\left(n^2\left(n^2+1+2n\right)\right)}{n^4} \qquad [\text{Multiplying the term}]\left[\frac{\infty}{\infty}\text{ form}\right]$$

$$=\frac{1}{4}\lim_{n\to\infty}\frac{\left(1+\frac{1}{n^2}+\frac{2}{n}\right)}{1}$$

$$=\frac{1}{4}$$

Q17

$$\lim_{n \to \infty} \frac{1^3 + 2^3 + \ldots + n^3}{(n-1)^4}$$

$$= \lim_{n \to \infty} \frac{\left[\frac{1}{2}(n)(n+1)\right]^2}{(n-1)^4} \qquad \left[1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{1}{2}n(n+1)\right]^2\right]$$

$$= \lim_{n \to \infty} \left(\frac{\frac{1}{4}n^2(n^2+1+2n)}{(n-1)^4}\right)$$

$$= \frac{1}{4} \lim_{n \to \infty} \left(\frac{n^4 + n^2 + 2n^3}{(n-1)^2(n-1)^2}\right)$$

$$= \frac{1}{4} \lim_{n \to \infty} \left(\frac{n^4 + n^2 + 2n^3}{n^4 + n^2 - 2n^3 + n^2 + 1 - 2n - 2n^3 - 2n + 4n^2}\right) \qquad \left[\frac{\infty}{\infty} \text{ form}\right]$$

$$= \frac{1}{4} \lim_{n \to \infty} \frac{\left(1 + \frac{1}{n^2} + \frac{2}{n}\right)}{\left(1 + \frac{1}{n^2} - \frac{2}{n} + \frac{1}{n^2} + \frac{1}{n^4} - \frac{2}{n^3} - \frac{2}{n} - \frac{2}{n^3} + \frac{4}{n^2}\right)}$$

$$= \frac{1}{4}\left(\frac{1}{1}\right)$$

$$= \frac{1}{4}$$

Q18

$$\lim_{n\to\infty} \sqrt{x}\left(\sqrt{x+1}-\sqrt{x}\right)$$

$$= \lim_{n\to\infty}\left(\sqrt{x^2+x}-x\right)$$

$$= \lim_{n\to\infty}\left(\left(\sqrt{x^2+x}-x\right)\times\frac{\left(\sqrt{x^2+x}+x\right)}{\sqrt{x^2+x}+x}\right)$$

$$= \lim_{n\to\infty}\left(\frac{\left(x^2+x\right)-x^2}{\sqrt{x^2+x}+x}\right)$$

$$= \lim_{n\to\infty}\left(\frac{x}{\sqrt{x^2+x}+x}\right) \qquad \left[\frac{\infty}{\infty}\ \text{form}\right]$$

$$= \lim_{n\to\infty}\left(\frac{1}{\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}}+1}\right)$$

$$= \lim_{n\to\infty}\left(\frac{1}{\sqrt{1+\frac{1}{x}}+1}\right)$$

$$= \frac{1}{1+1}$$

$$= \frac{1}{2}$$

Q19

$$\lim_{n\to\infty}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots+\frac{1}{3^n}\right) \qquad \text{---(i)}$$

This is G.P of common ratio $\frac{1}{3}$

$\therefore$ Sum of n terms of G.P with $a=\frac{1}{3}, r=\frac{-1}{3}$

$$Sn = a\left(\frac{1-r^n}{1-r}\right)$$

$$Sn = \frac{1}{3}\left(\frac{1-\left(\frac{1}{3}\right)^n}{1-\frac{1}{3}}\right)$$

$$= \frac{1}{3}\left(\frac{1-\frac{1}{3^n}}{\frac{2}{3}}\right)$$

$$= \frac{1}{3}\times\frac{3}{2}\left(1-\frac{1}{3^n}\right)$$

$$Sn = \frac{1}{2}\left(1-\frac{1}{3^n}\right)$$

Substituting value of Sn in (i), *we get*

$$\lim_{n\to\infty} Sn = \lim_{n\to\infty}\frac{1}{2}\left(1-\frac{1}{3^n}\right)$$

$$= \frac{1}{2}\lim_{n\to\infty}\left(1-\frac{1}{3^n}\right)$$

$$= \frac{1}{2}(1-0)$$

$$= \frac{1}{2}$$

Q20

$$\lim_{x \to \infty} \frac{x^4 + 7x^3 + 46x + a}{x^4 + 6} \qquad \left[\frac{\infty}{\infty} \text{ form}\right]$$

$$= \lim_{x \to \infty} \frac{1 + \frac{7}{x} + \frac{46}{x^3} + \frac{a}{x^4}}{1 + \frac{6}{x^4}}$$

$$= 1$$

Q21

$f(x) = \frac{ax^2 + b}{x^2 + 1}$

Also $\lim_{x \to 0} f(x) = 1$ ---(i) [given]

$\Rightarrow \quad \lim_{x \to 0} \frac{ax^2 + b}{x^2 + 1} = 1$

$\Rightarrow \quad \frac{\lim_{x \to 0} ax^2 + b}{\lim_{x \to 0} x^2 + 1} = 1$

$\Rightarrow \quad b = 1$ [from (i)]

Also, it is given that $\lim_{x \to \infty} f(x) = 1$

$\therefore \quad \lim_{x \to \infty} \frac{ax^2 + b}{x^2 + 1} = 1$ ---(ii)

$\Rightarrow \quad \lim_{x \to \infty} \frac{ax^2 + 1}{x^2 + 1} = 1$

$\Rightarrow \quad \lim_{x \to \infty} \frac{a + \frac{1}{x^2}}{1 + \frac{1}{x^2}} = 1$ $\left[\frac{\infty}{\infty} \text{ form}\right]$

$\Rightarrow \quad a = 1$

Thus, $f(x) = \frac{ax^2 + b}{x^2 + 1} = \frac{x^2 + 1}{x^2 + 1} = 1$ [from (ii)]

$f(-2) = 1$

$f(2) = 1$

$f(-2) = 1 = f(2)$

Hence, proved.

Q22

RHS $= \lim_{x\to\infty}\left(\sqrt{x^2+1}-x\right)$

$= \lim_{x\to\infty}\frac{\left(\sqrt{x^2+1}-x\right)\left(\sqrt{x^2+1}+x\right)}{\left(\sqrt{x^2+1}+x\right)}$

$= \lim_{x\to\infty}\frac{x^2+1-x^2}{x\sqrt{1+\frac{1}{x^2}}+x}$

$= \lim_{x\to\infty}\frac{1}{x\sqrt{1+\frac{1}{x^2}}+x}$

$= \lim_{x\to\infty}\frac{1}{x}\left(\frac{1}{\sqrt{1+\frac{1}{x^2}}+1}\right)$

$= 0$

Also,

LHS $= \lim_{x\to\infty}\left(\sqrt{x^2+x+1}-x\right)$

$= \lim_{x\to\infty}\frac{\left(\sqrt{x^2+x+1}-x\right)\left(\sqrt{x^2+x+1}+x\right)}{\left(\sqrt{x^2+x+1}+x\right)}$

$= \lim_{x\to\infty}\frac{\left(x^2+x+1-x^2\right)}{\sqrt{x^2+x+1}+x}$

$= \lim_{x\to\infty}\frac{x\left(1+\frac{1}{x}\right)}{x\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1\right)}$

$= \lim_{x\to\infty}\frac{1+\frac{1}{x}}{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1}$

$= \frac{1}{1+1} = \frac{1}{2}$

LHS $\neq$ RHS

$\therefore$ $\lim_{x\to\infty}\left(\sqrt{x^2+x+1}-x\right)$ is not equal to $\lim_{x\to\infty}\left(\sqrt{x^2+1}-x\right)$.

Q23

$$\lim_{x \to -\infty}\left(\sqrt{4x^2 - 7x} + 2x\right)$$

Substitute $y = -x$

$$= \lim_{y \to \infty}\left(\sqrt{4y^2 + 7y} - 2y\right)$$

$$= \lim_{y \to \infty}\frac{\left(\sqrt{4y^2 + 7y} - 2y\right)\left(\sqrt{4y^2 + 7y} + 2y\right)}{\sqrt{4y^2 + 7y} + 2y}$$

$$= \lim_{y \to \infty}\frac{\left(4y^2 + 7y - 4y^2\right)}{\sqrt{4y^2 + 7y} + 2y}$$

$$= \lim_{y \to \infty}\frac{(7y)}{\sqrt{4y^2 + 7y} + 2y}$$

$$= \lim_{y \to \infty}\frac{7}{\sqrt{4 + \frac{7}{y}} + 2}$$

$$= \frac{7}{2+2} = \frac{7}{4}$$

Q24

$$\lim_{x \to -\infty} \left(\sqrt{x^2 - 8x} + x\right)$$

$$= \lim_{y \to \infty} \left(\sqrt{y^2 + 8y} - y\right), \text{ where } y = -x \text{ on rationalising}$$

$$= \lim_{y \to \infty} \frac{\left\{\sqrt{y^2 + 8y} - y\right\}\left\{\sqrt{y^2 + 8y} + y\right\}}{\left\{\sqrt{y^2 + 8y} + y\right\}}$$

$$= \lim_{y \to \infty} \frac{y^2 + 8y - y^2}{\sqrt{y^2 + 8y} + y}$$

$$= \lim_{y \to \infty} \frac{8y}{y\sqrt{1 + \frac{8}{y}} + y} \qquad \left[\frac{\infty}{\infty} \text{ form}\right]$$

$$= \lim_{y \to \infty} \frac{8}{\sqrt{1 + \frac{8}{y}} + 1}$$

$$= \frac{8}{1+1} = \frac{8}{2}$$

$$= 4$$

Q25

$$\lim_{n \to \infty} \frac{1^4 + 2^4 + 3^4 + \ldots + n^4}{n^5} - \lim_{n \to \infty} \frac{1^3 + 2^3 + \ldots + n^3}{n^5}$$

$$= \lim_{n \to \infty} \frac{\frac{n(1+n)(1+2n)(-1+3n+3n^2)}{30}}{n^5} - \lim_{n \to \infty} \frac{\left(\frac{n(n+1)}{2}\right)^2}{n^5}$$

$$= \lim_{n \to \infty} \frac{\left(\frac{1}{n}+1\right)\left(\frac{1}{n}+2\right)\left(-\frac{1}{n^2}+\frac{3}{n}+3\right)}{30} - \lim_{n \to \infty} \frac{1}{n^5}\left(\frac{n^2(n^2+2n+1)}{4}\right)$$

$$= \lim_{n \to \infty} \frac{\left(\frac{1}{n}+1\right)\left(\frac{1}{n}+2\right)\left(-\frac{1}{n^2}+\frac{3}{n}+3\right)}{30} - \lim_{n \to \infty} \frac{\left(\frac{1}{n}+\frac{2}{n^2}+\frac{1}{n^3}\right)}{4}$$

$$= \frac{1 \times 2 \times 3}{30} - 0$$

$$= \frac{1}{5}$$

Q26

$$\lim_{n\to\infty}\frac{1.2+2.3+3.4+\ldots+n(n+1)}{n^3}=\lim_{n\to\infty}\frac{\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}}{n^3}$$

$$=\lim_{n\to\infty}\frac{n(n+1)\left[\frac{(2n+1)+3}{6}\right]}{n^3}$$

$$=\lim_{n\to\infty}\frac{\frac{n(n+1)(2n+4)}{6}}{n^3}$$

$$=\lim_{n\to\infty}\frac{\left(1+\frac{1}{n}\right)\left(2+\frac{4}{n}\right)}{6}$$

$$=\frac{1\times 2}{6}$$

$$=\frac{1}{3}$$

Ex 29.7

Q1

$$\lim_{x\to 0}\frac{\sin 3x}{5x}$$
$$=\frac{1}{5}\lim_{x\to 0}\frac{\sin 3x}{x}$$
$$=\frac{1}{5}\lim_{x\to 0}\frac{\sin 3x}{3x}\times 3$$
$$=\frac{3}{5}\lim_{3x\to 0}\frac{\sin 3x}{3x}$$
$$=\frac{3}{5}\times 1 \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$
$$=\frac{3}{5}$$

Q2

$$\lim_{x\to 0}\frac{\sin x^\circ}{x}$$
$$=\lim_{x\to 0}\frac{\sin\frac{x\times\pi}{180}}{x} \qquad \left[\because 1^\circ=\frac{\pi}{180}\text{ radians}\right]$$
$$=\lim_{x\to 0}\frac{\sin\frac{\pi x}{180}}{x\times\frac{\pi}{180}}\times\frac{\pi}{180}$$
$$=\frac{\pi}{180}\lim_{x\to 0}\frac{\frac{\sin\pi x}{180}}{\frac{\pi x}{180}}$$
$$=\frac{\pi}{180}\lim_{x\to 0}\frac{\frac{\sin\pi x}{180}}{\frac{\pi x}{180}}$$
$$=\frac{\pi}{180}\times 1=\frac{\pi}{180} \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$
$$=\frac{\pi}{180}$$

Q3

$$\lim_{x\to 0}\frac{x^2}{\sin x^2}$$

$$= \lim_{x\to 0}\frac{1}{\frac{\sin x^2}{x^2}}$$

$$= \frac{1}{\lim\limits_{x\to 0}\frac{\sin x^2}{x^2}}$$

$$= \frac{1}{1} \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x} = 1\right]$$

$$= 1$$

Q4

$$\lim_{x\to 0}\frac{\sin x \cos x}{3x}$$

$$= \frac{1}{3}\lim_{x\to 0}\frac{\sin x \cos x}{x}$$

$$= \frac{1}{3}\lim_{x\to 0}\frac{\sin x}{x} \times \lim_{x\to 0}\cos x$$

$$= \frac{1}{3}\times 1\times 1 \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x} = 1,\ \lim_{x\to 0}\cos x = \cos 0^\circ = 1\right]$$

$$= \frac{1}{3}$$

Q5

$$\lim_{x\to 0}\frac{3\sin x - 4\sin^3 x}{x}$$

$$= \lim_{x\to 0}\frac{\sin 3x}{x} \qquad \left[\because \sin 3x = 3\sin x - 4\sin^3 x\right]$$

$$= \lim_{x\to 0}\frac{\sin 3x}{3x}\times 3$$

$$= 3\times\lim_{x\to 0}\frac{\sin 3x}{3x}$$

$$= 3\times\lim_{3x\to 0}\frac{\sin 3x}{3x} \qquad \left[\because x\to 0,\ 3x\to 0\right]$$

$$= 3\times 1 \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x} = 1\right]$$

$$= 3$$

Q6

$$\lim_{x\to 0}\frac{\tan 8x}{\sin 2x}$$

$$=\frac{\lim\limits_{x\to 0}\tan 8x}{\lim\limits_{x\to 0}\sin 2x}$$

$$=\frac{\lim\limits_{x\to 0}\frac{\tan 8x}{8x}\times 8x}{\lim\limits_{x\to 0}\frac{\sin 2x}{2x}\times 2x}$$

$$=\frac{\lim\limits_{8x\to 0}\frac{\tan 8x}{8x}\times\frac{8x}{2x}}{\lim\limits_{2x\to 0}\frac{\sin 2x}{2x}} \qquad \left[\begin{array}{l}\because x\to 0\\ 8x\to 0\\ 2x\to 0\end{array}\right]$$

$$=\frac{1\times\frac{8}{2}}{1} \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1 \text{ and } \lim_{x\to 0}\frac{\tan x}{x}=1\right]$$

$$=\frac{8}{2}$$

$$=4$$

Q7

$$\lim_{x\to 0}\frac{\tan mx}{\tan nx}$$

$$=\frac{\lim\limits_{x\to 0}\tan mx}{\lim\limits_{x\to 0}\tan nx}$$

$$=\frac{\lim\limits_{mx\to 0}\frac{\tan mx}{mx}\times mx}{\lim\limits_{nx\to 0}\frac{\tan nx}{nx}\times nx} \qquad \left[\because \text{If } x\to 0 \text{ then } mx\to 0 \text{ also } nx\to 0\right]$$

$$=\frac{1\times m}{1\times n} \qquad \left[\because \lim_{x\to 0}\frac{\tan x}{x}=1\right]$$

$$=\frac{m}{n}$$

Q8

$$\lim_{x\to 0}\frac{\sin 5x}{\tan 3x}$$

$$=\frac{\lim\limits_{x\to 0}\sin 5x}{\lim\limits_{3x\to 0}\tan 3x}$$

$$=\frac{\lim\limits_{5x\to 0}\frac{\sin 5x}{5x}\times 5}{\lim\limits_{3x\to 0}\frac{\tan 3x}{3x}\times 3} \qquad \left[\because \text{If } x\to 0 \text{ then } 3x\to 0, 5x\to 0\right]$$

$$=\frac{5}{3}\times 1 \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1 \text{ also } \lim_{x\to 0}\frac{\tan x}{x}=1\right]$$

$$=\frac{5}{3}$$

Q9

$$\lim_{x\to 0}\frac{\sin x^\circ}{x^\circ}$$

$$=\lim_{x\to 0}\frac{\sin\frac{x\times\pi}{180}}{x\times\frac{\pi}{180}} \qquad \left[\because 1^\circ=\frac{\pi}{180}\text{ radians}\right]$$

$$=\lim_{\frac{\pi x}{180}\to 0}\frac{\sin\frac{\pi x}{180}}{\frac{\pi x}{180}} \qquad \left[\because \text{If } x\to 0 \text{ then } \frac{\pi x}{180}\to 0\right]$$

$$=1 \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

Q10

$$\lim_{x\to 0}\frac{7x\cos x - 3\sin x}{4x + \tan x}$$

$$= \lim_{x\to 0}\frac{7\cos x - \frac{3\sin x}{x}}{4 + \frac{\tan x}{x}}$$

$$= \frac{\lim_{x\to 0} 7\cos x - \lim_{x\to 0}\frac{3\sin x}{x}}{\lim_{x\to 0} 4 + \lim_{x\to 0}\frac{\tan x}{x}}$$

$$= \frac{7\times\lim_{x\to 0}\cos x - 3\lim_{x\to 0}\frac{\sin x}{x}}{4 + \lim_{x\to 0}\frac{\tan x}{x}}$$

$$= \frac{7\times 1 - 3\times 1}{4+1} \qquad \left[\begin{array}{l}\because \lim_{x\to 0}\frac{\sin x}{x} = 1\\ \lim_{x\to 0}\frac{\tan x}{x} = 1\end{array}\right]$$

$$= \frac{4}{5}$$

Q11

$$\lim_{x\to0}\frac{\cos ax-\cos bx}{\cos cx-\cos dx}=\lim_{x\to0}\frac{\left(-2\sin\left(\frac{a+b}{2}\right)x\sin\left(\frac{a-b}{2}\right)x\right)}{-2\sin\left(\frac{c+d}{2}\right)x\sin\left(\frac{c-d}{2}\right)x}$$

$$=\frac{\lim\limits_{x\to0}\sin\left(\frac{a+b}{2}\right)x\ \lim\limits_{x\to0}\sin\left(\frac{a+b}{2}\right)x}{\lim\limits_{x\to0}\sin\left(\frac{c+d}{2}\right)x\ \lim\limits_{x\to0}\sin\left(\frac{c-d}{2}\right)x}$$

$$=\frac{\left(\lim\limits_{x\to0}\frac{\sin\left(\frac{a+b}{2}\right)x}{\left(\frac{a+b}{2}\right)x}\times\left(\frac{a+b}{2}\right)x\right)\left(\lim\limits_{x\to0}\frac{\sin\left(\frac{a-b}{2}\right)x}{\left(\frac{a-b}{2}\right)x}\times\left(\frac{a-b}{2}\right)x\right)}{\left(\lim\limits_{x\to0}\frac{\sin\left(\frac{c+d}{2}\right)x}{\left(\frac{c+d}{2}\right)x}\times\left(\frac{c+d}{2}\right)x\right)\left(\lim\limits_{x\to0}\frac{\sin\left(\frac{c-d}{2}\right)x}{\left(\frac{c-d}{2}\right)x}\times\left(\frac{c-d}{2}\right)x\right)}$$

$$=\frac{(a+b)(a-b)}{(c+d)(c-d)}\qquad\left[\because\lim_{\theta\to0}\frac{\sin\theta}{\theta}=1\right]$$

$$=\frac{a^2-b^2}{c^2-d^2}$$

Q12

$$\lim_{x\to0}\frac{\tan^2 3x}{x^2}$$

$$=\left(\lim_{x\to0}\frac{\tan 3x}{x}\right)^2$$

$$=\left(\lim_{x\to0}\frac{\tan 3x}{3x}\right)^2\times9$$

$$=1\times9\qquad\left[\because\lim_{x\to0}\frac{\tan x}{x}=1\right]$$

$$=9$$

Q13

$$\lim_{x\to 0}\frac{1-\cos mx}{x^2}$$

$$=\lim_{x\to 0}\frac{2\sin^2\frac{mx}{2}}{x^2}$$

$$=2\times\left(\lim_{x\to 0}\frac{\sin\frac{mx}{2}}{x}\right)^2$$

$$=2\times\left(\lim_{x\to 0}\frac{\sin\frac{mx}{2}}{\frac{mx}{2}}\right)^2\times\left(\frac{m}{2}\right)^2$$

$$=2\times\frac{m^2}{4}=\frac{m^2}{2} \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

$$=\frac{m^2}{2}$$

Q14

$$\lim_{x\to 0}\frac{3\sin 2x+2x}{3x+2\tan 3x}$$

$$=\lim_{x\to 0}\frac{3\frac{\sin 2x}{x}+2}{3+2\tan\frac{3x}{x}}$$

$$=\frac{\lim_{x\to 0}\frac{3\sin 2x}{x}+\lim_{x\to 0}2}{\lim_{x\to 0}3+\lim_{x\to 0}\frac{2\tan 3x}{x}}$$

$$=\frac{\left(3\lim_{2x\to 0}\frac{\sin 2x}{2x}\times 2\right)+2}{3+\left(2\times\lim_{3x\to 0}\frac{\tan 3x}{3x}\times 3\right)}$$

$$=\frac{(3\times 2)+2}{3+(2\times 3)} \qquad \left[\because \lim_{x\to a}\frac{\sin x}{x}=1 \text{ also } \lim_{x\to 0}\frac{\tan x}{x}=1\right]$$

$$=\frac{8}{9}$$

Q15

$$\lim_{x\to 0}\frac{\cos 3x-\cos 7x}{x^2}$$

$$=\lim_{x\to 0}\left(\frac{-2\sin\left(\frac{3x+7x}{2}\right)\sin\left(\frac{3x-7x}{2}\right)}{x^2}\right)$$

$$=\lim_{x\to 0}\left(\frac{-2\sin 5x\sin\left(\frac{-4x}{2}\right)}{x^2}\right)$$

$$=\left(\lim_{x\to 0}\frac{-2\sin 5x}{x}\right)\times\left(\lim_{x\to 0}\frac{\sin(-2x)}{x}\right)$$

$$=\left(-2\lim_{x\to 0}\frac{\sin 5x}{5x}\times 5\right)\times\left(-1\lim_{x\to 0}\frac{\sin 2x}{2x}\times 2\right)$$

$$=(-2\times 5)(-1\times 2)$$

$$=20$$

Q16

$$\frac{\lim\limits_{\theta\to 0}\sin 3\theta}{\lim\limits_{\theta\to 0}\tan 2\theta}$$

$$=\frac{\lim\limits_{3\theta\to 0}\frac{\sin 3\theta}{3\theta}\times 3\theta}{\lim\limits_{2\theta\to 0}\frac{\tan 2\theta}{2\theta}\times 2\theta}$$

$$=\frac{\left(\lim\limits_{3\theta\to 0}\frac{\sin 3\theta}{3\theta}\right)}{\left(\lim\limits_{2\theta\to 0}\frac{\sin 2\theta}{2\theta}\right)}\times\frac{3\theta}{2\theta}$$

$$=\frac{1}{1}\times\frac{3}{2}\qquad\left[\because \lim_{x\to 0}\frac{\sin x}{x}=1 \text{ and } \lim_{x\to 0}\frac{\tan x}{x}=1\right]$$

$$=\frac{3}{2}$$

Q17

$$\lim_{x\to 0}\frac{\sin x^2\left(1-\cos x^2\right)}{x^6}$$

$$=\lim_{x\to 0}\frac{\sin x^2\times 2\sin^2\frac{x^2}{2}}{x^6}$$

$$=\lim_{x\to 0}\frac{\sin x^2}{x^2}\times\lim_{x\to 0}\frac{2\sin^2\frac{x^2}{2}}{x^4}$$

$$=\left(\lim_{x\to 0}\frac{\sin x}{x}\right)^2\times 2\times\left(\lim_{x\to 0}\frac{\sin\frac{x^2}{2}}{\frac{x^2}{2}}\right)^2\times\frac{1}{4}$$

$$=(1)^2\times 2\times 1\times\frac{1}{4}\qquad\left[\because\lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

$$=\frac{1}{2}$$

Q18

$$\lim_{x\to 0}\frac{\sin^2 4x^2}{x^4}$$

$$=\lim_{x\to 0}\frac{\left(\sin 4x^2\right)^2}{x^4}$$

$$=\lim_{x\to 0}\frac{\left(\sin 4x^2\right)^2}{\left(x^2\right)^2}$$

$$=\left(\lim_{x\to 0}\frac{\sin 4x^2}{x^2}\right)^2$$

$$=\left(\lim_{4x^2\to 0}\frac{\sin 4x^2}{4x^2}\right)\times 16$$

$$=1\times 16=16\qquad\left[\because\lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

$$=16$$

Q19

$$\lim_{x\to 0}\frac{x\cos x+2\sin x}{x^2+\tan x}$$

Dividing each term by x

$$\lim_{x\to 0}\frac{\cos x+\frac{2\sin x}{x}}{x+\frac{\tan x}{x}}$$

$$=\frac{\lim\limits_{x\to 0}\cos x+\lim\limits_{x\to 0}\frac{2\sin x}{x}}{\lim\limits_{x\to 0}x+\lim\limits_{x\to 0}\frac{\tan x}{x}}$$

$$=\frac{\cos 0+2\lim\limits_{x\to 0}\frac{\sin x}{x}}{0+\lim\limits_{x\to 0}\frac{\tan x}{x}}$$

$$=\frac{1+2}{0+1}=3 \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1,\ \lim_{x\to 0}\frac{\tan x}{x}=1\right]$$

$$=3$$

Q20

$$\lim_{x\to 0}\frac{2x-\sin x}{\tan x+x}$$

Dividing each term by x

$$=\lim_{x\to 0}\frac{2-\frac{\sin x}{x}}{\frac{\tan x}{x}+1}$$

$$=\frac{\lim\limits_{x\to 0}2-\lim\limits_{x\to 0}\frac{\sin x}{x}}{\lim\limits_{x\to 0}\frac{\tan x}{x}+\lim\limits_{x\to 0}1}$$

$$=\frac{2-1}{1+1} \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1,\lim_{x\to 0}\frac{\tan x}{x}=1\right]$$

$$=\frac{1}{2}$$

Q21

$$\lim_{x\to 0}\frac{5x\cos x+3\sin x}{3x^2+\tan x}$$

$$\lim_{x\to 0}\frac{5\cos x+3\dfrac{\sin x}{x}}{3x+\dfrac{\tan x}{x}}$$

$$=\frac{\lim\limits_{x\to 0}5\cos x+\lim\limits_{x\to 0}\dfrac{3\sin x}{x}}{\lim\limits_{x\to 0}3x+\lim\limits_{x\to 0}\dfrac{\tan x}{x}}$$

$$=\frac{5\lim\limits_{x\to 0}\cos x+3\lim\limits_{x\to 0}\dfrac{\sin x}{x}}{3\lim\limits_{x\to 0}x+\lim\limits_{x\to 0}\dfrac{\tan x}{x}}$$

$$=\frac{5\times\cos 0+3\times 1}{3\times 0+1}\qquad\left[\because \lim_{x\to 0}\frac{\sin x}{x}=1 \text{ and } \lim_{x\to 0}\frac{\tan x}{x}=1\right]$$

$$=\frac{5+3}{1}$$

$$=8$$

Q22

$$\lim_{x\to 0}\frac{\sin 3x-\sin x}{\sin x}$$

$$=\lim_{x\to 0}\left(\frac{2\cos\left(\dfrac{3x+x}{2}\right)\sin\left(\dfrac{3x-x}{2}\right)}{\sin x}\right)$$

$$=\lim_{x\to 0}\left(\frac{2\cos 2x\sin x}{\sin x}\right)$$

$$=\lim_{x\to 0}(2\cos x)$$

$$=2\lim_{x\to 0}\cos 2x$$

$$=2\times\cos 0$$

$$=2\times 1=2$$

$$=2$$

Q23

$$\lim_{x \to 0} \frac{\sin 5x - \sin 3x}{\sin x}$$

$$= \lim_{x \to 0} \frac{2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)}{\sin x}$$

$$= \lim_{x \to 0} \frac{2\cos 4x \sin x}{\sin x}$$

$$= 2\lim_{x \to 0} \cos 4x$$

$$= 2 \times \cos 0$$

$$= 2$$

Q24

$$\lim_{x \to 0} \frac{\cos 3x - \cos 5x}{x^2}$$

$$= \lim_{x \to 0} \frac{\left(-2\sin\left(\frac{3x+5x}{2}\right)\sin\left(\frac{3x-5x}{2}\right)\right)}{x^2}$$

$$= \lim_{x \to 0} \left(\frac{-2\sin 4x \sin(-x)}{x^2}\right)$$

$$= \lim_{x \to 0} \frac{2\sin 4x \sin x}{x^2}$$

$$= \lim_{x \to 0} \frac{2\sin 4x}{x} \times \lim_{x \to 0} \frac{\sin x}{x}$$

$$= \left(2\lim_{x \to 0} \frac{\sin 4x}{4x} \times 4\right) \times \left(\lim_{x \to 0} \frac{\sin x}{x}\right) \qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

$$= 8$$

Q25

$$\lim_{x \to 0} \frac{\frac{\tan 3x}{x} - \frac{\tan 2x}{x}}{\frac{3x}{x} - \frac{\sin^2 x}{x}}$$

$$= \frac{\left(\lim_{x \to 0} \frac{\tan 3x}{3x} \times 3\right) - \left(\lim_{x \to 0} \frac{\tan 2x}{2x} \times 2\right)}{\left(\lim_{x \to 0} \frac{3x}{2}\right) - \lim_{x \to 0} \frac{(\sin x)^2}{x}}$$

$$= \frac{3 - 2}{3 - \left(\frac{\sin x}{x}\right)^2 \times x} \qquad \left[\because \lim_{x \to 0} \frac{\tan x}{x} = 1\right]$$

$$= \frac{3 - 2}{3 - 0} = \frac{1}{3} \qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

$$= \frac{1}{3}$$

Q26

$$\lim_{x \to 0} \frac{\sin(2 + x) - \sin(2 - x)}{x}$$

$$= \lim_{x \to 0} 2\cos \frac{\left(\frac{2 + x + 2 - x}{2}\right) \times \sin\left(\frac{2 + x - 2 + x}{2}\right)}{x}$$

$$= 2 \lim_{x \to 0} \frac{\cos(2) \times \sin x}{x}$$

$$= 2 \lim_{x \to 0} \cos 2 \times \lim_{x \to 0} \frac{\sin x}{x} \qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

$$= 2\cos 2 \times 1$$

$$= 2\cos 2$$

Q27

$$\lim_{h\to 0}\frac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$$

$$=\lim_{h\to 0}\frac{\left(a^2+h^2+2ah\right)\sin(a+h)-a^2\sin a}{h}$$

$$=\lim_{h\to 0}\frac{a^2\sin(a+h)+h^2\sin(a+h)+2ah\sin(a+h)-a^2\sin a}{h}$$

$$=\lim_{h\to 0}\left[\frac{a^2\left(\sin(a+h)-\sin a\right)}{h}+\frac{h^2\sin(a+h)}{h(a+h)}\times(a+h)+\frac{2ah}{h}\left(\sin(a+h)\right)\right]$$

$$=\left[a^2\lim_{h\to 0}\frac{2\cos\left(\frac{a+h+a}{2}\right)\sin\left(\frac{a+h-a}{2}\right)}{h}\right]+[0]+2a\lim_{h\to 0}\sin(a+h)$$

$$=\left(2a^2\lim_{h\to 0}\cos\left(a+\frac{h}{2}\right)\frac{\sin\frac{h}{2}}{\frac{h}{2}}\times\frac{1}{2}\right)+(2a\times\sin a)$$

$$=\left(2a^2\cos a\times\frac{1}{2}\right)+(2a\sin a)$$

$$=a^2\cos a+2a\sin a$$

Q28

$$\lim_{x \to 0} \frac{\tan x - \sin x}{\sin 3x - 3\sin x}$$

$$\lim_{x \to 0} \frac{\frac{\sin x}{\cos x} - \sin x}{3\sin x - 4\sin^3 x - 3\sin x}$$

$$= \lim_{x \to 0} \frac{\sin x\left(\frac{1}{\cos x} - 1\right)}{-4\sin^3 x}$$

$$= \frac{-1}{4} \lim_{x \to 0} \frac{\frac{1}{\cos x} - 1}{\sin^2 x}$$

$$= \frac{-1}{4} \lim_{x \to 0} \frac{\frac{1 - \cos x}{\cos x}}{1 - \cos^2 x}$$

$$= \frac{-1}{4} \lim_{x \to 0} \frac{1 - \cos x}{(\cos x)(1 - \cos x)(1 + \cos x)}$$

$$= \frac{-1}{4} \lim_{x \to 0} \frac{1}{(\cos x)(1 + \cos x)}$$

$$= \frac{-1}{4} \times \frac{1}{1(1+1)}$$

$$= \frac{-1}{4} \times \frac{1}{2}$$

$$= -\frac{1}{8}$$

Q29

$$\lim_{x\to 0}\frac{\sec 5x-\sec 3x}{\sec 3x-\sec x}=\lim_{x\to 0}\left(\frac{\frac{\cos 3x-\cos 5x}{\cos 3x\cos 5x}}{\frac{\cos x-\cos 3x}{\cos x\cos 3x}}\right)$$

$$=\lim_{x\to 0}\left(\frac{\cos 3x-\cos 5x}{\cos x-\cos 3x}\times\frac{\cos x\cos 3x}{\cos 3x\cos 5x}\right)$$

$$=\lim_{x\to 0}\left(\frac{-2\sin 4x\sin(-x)}{-2\sin(2x)\sin(-x)}\times\frac{\cos x}{\cos 5x}\right)$$

$$=\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\times\frac{\cos x}{\cos 5x}\right)$$

$$=\frac{\lim\limits_{x\to 0}\sin 4x\times\lim\limits_{x\to 0}\cos x}{\lim\limits_{x\to 0}\sin 2x\times\lim\limits_{x\to 0}\cos 5x}$$

$$=\frac{\left(\lim\limits_{4x\to 0}\frac{\sin 4x}{4x}\times 4x\right)\times\left(\lim\limits_{x\to 0}\cos x\right)}{\left(\lim\limits_{2x\to 0}\frac{\sin 2x}{2x}\times 2x\right)\times\left(\lim\limits_{x\to 0}\cos 5x\right)}$$

$$=\frac{(1\times 4x)\times 1}{1\times 2x\times 1}\qquad\left[\because\ \lim_{x\to 0}\frac{\sin x}{x}=1\text{ and }\lim_{x\to 0}\cos x=\cos 0=1\right]$$

$$=\frac{4x}{2x}$$

$$=2$$

Q30

$$\lim_{x\to 0}\frac{1-\cos 2x}{\cos 2x-\cos 8x}$$

$$=\lim_{x\to 0}\frac{2\sin^2 x}{-2\sin\left(\frac{2x+8x}{2}\right)\sin\left(\frac{2x-8x}{2}\right)}$$

$$=\lim_{x\to 0}\frac{\sin^2 x}{\sin 5x\times\sin(-3x)}$$

$$=\frac{\lim_{x\to 0}\sin^2 x}{-\left(\lim_{x\to 0}\sin 5x\right)\left(-\lim_{x\to 0}\sin 3x\right)}$$

$$=\frac{\left(\lim_{x\to 0}\frac{\sin x}{x}\right)^2\times x^2}{\left(\lim_{x\to 0}\frac{\sin 5x}{5x}\right)\times 5x\left(\lim_{x\to 0}\frac{\sin 3x}{3x}\right)\times 3x}$$

$$=\frac{1\times x^2}{1\times 5x\times 1\times 3x}\qquad\left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

$$=\frac{x^2}{15x^2}$$

$$=\frac{1}{15}$$

Q31

$$\lim_{x\to 0}\frac{1-\cos 2x+\tan^2 x}{x\sin x}$$

$$=\lim_{x\to 0}\frac{2\sin^2 x+\tan^2 x}{x\sin x}$$

$$=\frac{2\lim_{x\to 0}\sin^2 x+\lim_{x\to 0}\tan^2 x}{\lim_{x\to 0}x\sin x}$$

$$=\frac{\left(2\left(\lim_{x\to 0}\frac{\sin x}{x}\right)^2\times x^2\right)+\left(\lim_{x\to 0}\frac{\tan x}{x}\right)^2\times x^2}{\left(\lim_{x\to 0}\frac{\sin x}{x}\right)\times x^2}$$

$$=\frac{\left(2\times 1\times x^2\right)+\left(1\times x^2\right)}{\left(1\times x^2\right)}\qquad\left[\because \lim_{x\to 0}\frac{\sin x}{x}=1 \text{ and } \lim_{x\to 0}\frac{\tan x}{x}=1\right]$$

$$=\frac{3x^2}{x^2}$$

$$=3$$

Q32

$$\lim_{x\to 0}\frac{\sin(a+x)+\sin(a-x)-2\sin a}{x\sin x}$$

$$=\lim_{x\to 0}\frac{2\sin\left(\frac{a+x+a-x}{2}\right)\cos\left(\frac{a+x-a+x}{2}\right)-2\sin a}{x\sin x}$$

$$=\lim_{x\to 0}\frac{2\sin a(\cos x-1)}{x\sin x}$$

$$=\lim_{x\to 0}\frac{-2\sin a(1-\cos x)}{x\sin x}$$

$$=-2\sin a\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)x}$$

$$=-2\sin a\lim_{x\to 0}\frac{2\sin\frac{x}{2}}{\left(\cos\frac{x}{2}\right)x}$$

$$=-2\sin a\lim_{x\to 0}\frac{\tan\frac{x}{2}}{\frac{x}{2}}\times\frac{1}{2}$$

$$=-2\sin a\times 1\times\frac{1}{2}\qquad\left[\because\lim_{x\to 0}\frac{\tan x}{x}=1\right]$$

$$=-\sin a$$

$$=-\sin a$$

Q33

$$\lim_{x\to 0}\frac{x^2-\tan 2x}{\tan x}$$

$$=\lim_{x\to 0}\frac{\left(\frac{x^2}{2x}-\frac{\tan 2x}{2x}\right)\times 2x}{\frac{\tan x}{x}\times x}$$

$$=\lim_{x\to 0}\frac{2\left(\frac{x}{2}-\frac{\tan 2x}{2x}\right)}{\frac{\tan x}{x}}$$

$$=2\left(\frac{0-1}{1}\right)$$

$$=-2$$

Q34

$$\lim_{x\to 0}\frac{\sqrt{2}+\sqrt{1+\cos x}}{\sin^2 x}=\lim_{x\to 0}\frac{\sqrt{2}+\sqrt{1+\cos x}}{\sin^2 x}\times\frac{\sqrt{2}-\sqrt{1+\cos x}}{\sqrt{2}-\sqrt{1+\cos x}}$$

$$=\lim_{x\to 0}\frac{2-1-\cos x}{\sin^2 x}$$

$$=\lim_{x\to 0}\frac{1-\cos x}{\sin^2 x}$$

$$=\lim_{x\to 0}\frac{1-\cos x}{1-\cos^2 x}$$

$$=\lim_{x\to 0}\frac{1}{1+\cos x}$$

$$=\frac{1}{1+1}$$

$$=\frac{1}{2}$$

Q35

$$\lim_{x\to 0}\frac{x\tan x}{1-\cos x}$$

$$=\lim_{x\to 0}\frac{x\sin x}{\cos x\,(1-\cos x)}$$

$$=\lim_{x\to 0}\frac{x\sin x}{\cos x\left(2\sin^2\frac{x}{2}\right)}$$

$$=\lim_{x\to 0}\frac{x\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)}{\cos x\left(2\sin^2\frac{x}{2}\right)}$$

$$=\lim_{x\to 0}\frac{x\cos\frac{x}{2}}{\cos x\,\frac{\sin x}{2}}$$

$$=\lim_{x\to 0}\frac{1}{\cos x\times\frac{\frac{\tan x}{2}}{x}}$$

$$=\lim_{x\to 0}\frac{1}{\cos x}\times\frac{1}{\lim_{x\to 0}\frac{\frac{\tan x}{2}}{\frac{x}{2}}\times\frac{1}{2}}$$

$$=1\times 1\times 2$$

$$=2$$

Q36

$$\lim_{x\to 0}\frac{x^2+1-\cos x}{x\sin x}$$

$$=\lim_{x\to 0}\frac{x^2+2\sin^2\frac{x}{2}}{x\sin x}$$

$$=\lim_{x\to 0}\frac{x^2\left[1+2\left(\frac{\sin\frac{x}{2}}{x}\right)^2\right]}{x\sin x}$$

$$=\lim_{x\to 0}\frac{1+2\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2\times\frac{1}{4}}{\frac{\sin x}{x}}$$

$$=\lim_{x\to 0}\frac{1+\lim_{x\to 0}2\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2\times\frac{1}{4}}{\lim_{x\to 0}\frac{\sin x}{x}}$$

$$=\frac{1+2\times 1\times\frac{1}{4}}{1}=\frac{1+\frac{1}{2}}{1}$$

$$=\frac{3}{2}$$

Q37

$$\lim_{x\to 0}\frac{\sin 2x\left(\cos 3x-\cos x\right)}{x^3}$$

$$=\lim_{x\to 0}\frac{\sin 2x\left(-2\sin\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)\right)}{x^3}$$

$$=\lim_{x\to 0}\frac{\sin 2x\left(-2\sin 2x\sin x\right)}{x^3}$$

$$=\frac{-2\lim\limits_{x\to 0}\sin 2x\times\lim\limits_{x\to 0}\sin 2x\times\lim\limits_{x\to 0}\sin x}{x^3}$$

$$=-2\left(\lim_{x\to 0}\frac{\sin 2x}{2x}\times 2\right)\times\left(2\lim_{x\to 0}\frac{\sin 2x}{2x}\right)\times\left(\lim_{x\to 0}\frac{\sin x}{x}\right)$$

$$=-2\left(1\times 2\right)\times\left(2\right)\times\left(1\right)$$

$$=-8$$

Q38

$$\lim_{x\to 0}\frac{2\sin x^\circ-\sin 2x^\circ}{x^3}$$

$$=\lim_{x\to 0}\frac{2\sin\frac{\pi x}{180}-\sin\frac{2\pi x}{180}}{x^3}$$

$$=\lim_{x\to 0}\frac{2\sin\frac{\pi x}{180}-2\sin\frac{\pi x}{180}\cos\frac{\pi x}{180}}{x^3}$$

$$=\lim_{x\to 0}\frac{2\sin\frac{\pi x}{180}\left(2\sin^2\frac{\pi x}{360}\right)}{x^3}$$

$$=4\left(\lim_{x\to 0}\frac{\sin\frac{\pi x}{180}}{x}\right)\times\left(\lim_{x\to 0}\frac{\sin\frac{\pi x}{360}}{x}\right)\times\left(\lim_{x\to 0}\frac{\sin\frac{\pi x}{360}}{x}\right)$$

$$=4\left(\lim_{x\to 0}\frac{\sin\frac{\pi x}{180}}{\frac{\pi x}{180}}\times\frac{\pi}{180}\right)\times\left(\lim_{x\to 0}\frac{\sin\frac{\pi x}{360}}{\frac{\pi x}{360}}\times\frac{\pi}{360}\right)\times\left(\lim_{x\to 0}\frac{\sin\frac{\pi x}{360}}{\frac{\pi x}{360}}\times\frac{\pi}{360}\right)$$

$$=4\times\frac{\pi}{180}\times\frac{\pi}{360}\times\frac{\pi}{360}$$

$$=\left(\frac{\pi}{180}\right)^3$$

Q39

$$\lim_{x\to 0}\frac{x^3\cot x}{1-\cos x}$$

$$= \lim_{x\to 0}\frac{x^3}{\tan x\,(1-\cos x)}$$

$$= \lim_{x\to 0}\frac{x^3}{\tan x . 2\sin^2\frac{x}{2}}$$

$$= \lim_{x\to 0}\frac{1}{\frac{\tan x}{x}\times\frac{2\sin^2\frac{x}{2}}{x^2}}$$

$$= \frac{1}{\left(\lim_{x\to 0}\frac{\tan x}{x}\right)\times 2\left(\lim_{x\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2\times\frac{1}{4}}$$

$$= \frac{1}{1\times 2\times 1\times\frac{1}{4}} \qquad \left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1,\ \lim_{\theta\to 0}\frac{\tan\theta}{\theta}=1\right]$$

$$= 2$$

Q40

$$\lim_{x\to 0}\frac{x\tan x}{1-\cos 2x}$$

$$= \lim_{x\to 0}\frac{x\tan x}{2\sin^2 x}$$

$$= \lim_{x\to 0}\frac{\frac{\tan x}{x}}{\frac{2\sin^2 x}{x^2}}$$

$$= \frac{\lim_{x\to 0}\frac{\tan x}{x}}{2\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2} \qquad \left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1,\ \lim_{\theta\to 0}\frac{\tan\theta}{\theta}=1\right]$$

$$= \frac{1}{2\times 1}$$

$$= \frac{1}{2}$$

Q41

$$\lim_{x \to 0} \frac{\sin(3+x) - \sin(3-x)}{x}$$

$$= \lim_{x \to 0} \frac{2\cos\left(\frac{3+x+3-x}{2}\right)\sin\left(\frac{3+x-3+x}{2}\right)}{x}$$

$$= 2\lim_{x \to 0} \frac{\cos 3 . \sin x}{x}$$

$$= 2\cos 3 \lim_{x \to 0} \frac{\sin x}{x}$$

$$= 2\cos 3 \times 1$$

$$= 2\cos 3$$

Q42

$$\lim_{x\to 0} kx \operatorname{cosec} x = \lim_{x\to 0} x \operatorname{cosec} kx$$

$$\text{LHS} = \lim_{x\to 0} kx \operatorname{cosec} x = k \lim_{x\to 0} x \operatorname{cosec} x$$

$$= k \lim_{x\to 0} \frac{x}{\sin x}$$

$$= k \lim_{x\to 0} \frac{1}{\frac{\sin x}{x}}$$

$$= k \times 1 \qquad \left[\because \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1\right]$$

$$\text{Also, RHS} = \lim_{x\to 0} x \operatorname{cosec} kx$$

$$= \lim_{x\to 0} \frac{x}{\sin kx}$$

$$= \lim_{x\to 0} \frac{x}{\frac{\sin kx}{kx} \times kx}$$

$$= \frac{1}{k} \qquad \left[\because \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1\right]$$

As, LHS = RHS

$$\Rightarrow \quad k = \frac{1}{k}$$

$$k^2 = 1$$

$$k = \pm 1$$

Q43

$$\lim_{x\to 0}\frac{3\sin^2 x-2\sin x^2}{3x^2}$$

$$=\lim_{x\to 0}\frac{3\sin^2 x}{3x^2}-\lim_{x\to 0}\frac{2\sin x^2}{3x^2}$$

$$=\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2-\frac{2}{3}\lim_{x\to 0}\frac{\sin x^2}{x^2}$$

$$=1-\frac{2}{3}\times 1 \qquad \left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]$$

$$=1-\frac{2}{3}$$

$$=\frac{3-2}{3}$$

$$=\frac{1}{3}$$

Q44

$$\lim_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$$

$$=\lim_{x\to 0}\frac{\left(\sqrt{1+\sin x}-\sqrt{1-\sin x}\right)}{x}\frac{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$$

$$=\lim_{x\to 0}\frac{\left((1+\sin x)-(1-\sin x)\right)}{x\sqrt{1+\sin x}+\sqrt{1-\sin x}}$$

$$=\lim_{x\to 0}\frac{2\sin x}{x\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$$

$$=2\lim_{x\to 0}\frac{\sin x}{x}\times\frac{1}{\lim_{x\to 0}\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$$

$$=2\times 1\times\frac{1}{2}$$

$$=1$$

Q45

$$\lim_{x \to 0} \frac{1-\cos 4x}{x^2}$$

$$= \lim_{x \to 0} \frac{2\sin^2 2x}{x^2}$$

$$= 2\lim_{x \to 0}\left(\frac{\sin 2x}{x}\right)^2$$

$$= 2\lim_{x \to 0}\left(\frac{\sin 2x}{2x}\right)^2 \times (2)^2$$

$$= 2 \times 1 \times 4$$

$$= 8$$

Q46

$$\lim_{x \to 0} \frac{x\cos x + \sin x}{x^2 + \tan x}$$

$$= \lim_{x \to 0} \frac{\cos x + \frac{\sin x}{x}}{x + \frac{\tan x}{x}}$$

$$= \frac{\lim_{x \to 0} \cos x + \lim_{x \to 0} \frac{\sin x}{x}}{\lim_{x \to 0} x + \lim_{x \to 0} \frac{\tan x}{x}}$$

$$= \frac{1+1}{0+1} = \frac{2}{1}$$

$$= 2$$

Q47

$$\lim_{x \to 0} \frac{1-\cos 2x}{3\tan^2 x}$$

$$= \lim_{x \to 0} \frac{2\sin^2 x}{3\tan^2 x}$$

$$= \frac{2}{3} \lim_{x \to 0} \frac{\sin^2 x}{\left(\frac{\sin^2 x}{\cos^2 x}\right)}$$

$$= \frac{2}{3} \lim_{x \to 0} \cos^2 x$$

$$= \frac{2}{3}$$

Q48

$$\lim_{\theta \to 0} \frac{1-\cos 4\theta}{1-\cos 6\theta}$$

$$= \lim_{\theta \to 0} \frac{2\sin^2 2\theta}{2\sin^2 3\theta}$$

$$= \lim_{\theta \to 0} \frac{(\sin 2\theta)^2}{(\sin 3\theta)^2}$$

$$= \frac{\lim_{\theta \to 0} \left(\frac{\sin 2\theta}{2\theta}\right)^2 \times 4\theta^2}{\lim_{\theta \to 0} \left(\frac{\sin 3\theta}{3\theta}\right)^2 \times 9\theta^2}$$

$$= \frac{1 \times 4\theta^2}{1 \times 9\theta^2}$$

$$= \frac{4}{9}$$

Q49

$$\lim_{x\to 0} \frac{ax + x\cos x}{b\sin x}$$

$$= \lim_{x\to 0} \frac{a + \cos x}{\frac{b\sin x}{x}}$$

$$= \frac{\lim_{x\to 0} a + \lim_{x\to 0} \cos x}{\lim_{x\to 0} \frac{b\sin x}{x}}$$

$$= \frac{a+1}{b} \qquad \left[\because \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1\right]$$

$$= \frac{a+1}{b}$$

Q50

$$\lim_{\theta\to 0} \frac{\sin 4\theta}{\tan 3\theta}$$

$$= \frac{\lim_{\theta\to 0} \sin 4\theta}{\lim_{\theta\to 0} \tan 3\theta}$$

$$= \frac{\left(\lim_{\theta\to 0} \frac{\sin 4\theta}{4\theta}\right) \times 4\theta}{\left(\lim_{\theta\to 0} \frac{\tan 3\theta}{3\theta}\right) \times 3\theta}$$

$$= \frac{1 \times 4\theta}{1 \times 3\theta}$$

$$= \frac{4}{3}$$

Q51

$$\lim_{x\to0}\frac{2\sin x-\sin 2x}{x^3}=\lim_{x\to0}\frac{2\sin x-2\sin x\cos x}{x^3}$$

$$=\lim_{x\to0}\frac{2\sin x\left(1-\cos x\right)}{x^3}$$

$$=\lim_{x\to0}\frac{2\sin x\left(1-\cos x\right)}{x^3}\times\frac{1+\cos x}{1+\cos x}$$

$$=\lim_{x\to0}\frac{2\sin x\left(1-\cos^2 x\right)}{x^3\left(1+\cos x\right)}$$

$$=\lim_{x\to0}\frac{2\sin x\left(\sin^2 x\right)}{x^3\left(1+\cos x\right)}$$

$$=\lim_{x\to0}\frac{2\sin^3 x}{x^3\left(1+\cos x\right)}$$

$$=2\lim_{x\to0}\left(\frac{\sin x}{x}\right)^3\times\lim_{x\to0}\frac{1}{\left(1+\cos x\right)}$$

$$=2\times1\times\frac{1}{\left(1+1\right)}$$

$$=1$$

Q52

$$\lim_{x\to0}\frac{1-\cos 5x}{1-\cos 6x}$$

$$=\frac{\lim\limits_{x\to0}2\sin^2\frac{5x}{2}}{\lim\limits_{x\to0}2\sin^2 3x}$$

$$=\frac{2\left(\lim\limits_{x\to0}\frac{\sin\frac{5x}{2}}{\frac{5x}{2}}\right)^2\times\frac{25}{4}x^2}{2\left(\lim\limits_{x\to0}\frac{\sin 3x}{3x}\right)^2\times 9x^2}$$

$$=\frac{2\times1\times\frac{25}{4}x^2}{2\times1\times9x^2}$$

$$=\frac{25}{4\times9}$$

$$=\frac{25}{36}$$

Q53

$$\lim_{x\to 0}\frac{\operatorname{cosec}x-\cot x}{x}$$

$$=\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{\cos x}{\sin x}\right)\times\frac{1}{x}$$

$$=\lim_{x\to 0}\left(\frac{1}{\sin x}\left(\frac{1-\cos x}{x}\right)\right)$$

$$=\lim_{x\to 0}\left(\frac{1}{\sin x}\left(\frac{2\sin^2\frac{x}{2}}{x}\right)\right)$$

$$=2\lim_{x\to 0}\left(\frac{1}{\frac{\sin x}{x}}\times x\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2\times\frac{x}{4}\right)$$

$$=2\left(\lim_{x\to 0}\frac{1}{\frac{\sin x}{x}}\right)\times\frac{1}{x}\times\left(\lim_{x\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2\times\frac{x}{4}$$

$$=2\times\frac{1}{x}\times\frac{x}{4}$$

$$=\frac{1}{2}$$

Q54

$$\lim_{x\to 0}\frac{\sin 3x+7x}{4x+\sin 2x}$$

$$=\lim_{x\to 0}\frac{\left(\frac{\sin 3x}{x}+\frac{7x}{x}\right)}{\left(\frac{4x}{x}+\frac{\sin 2x}{x}\right)}$$

$$=\frac{\left(\lim_{x\to 0}\frac{\sin 3x}{3x}\times 3\right)+7}{4+\left(\lim_{x\to 0}\frac{\sin 2x}{2x}\right)\times 2}$$

$$=\frac{3+7}{4+2}$$

$$=\frac{10}{6}$$

$$=\frac{5}{3}$$

Q55

$$\lim_{x\to 0}\frac{5x+4\sin 3x}{4\sin 2x+7x}$$

$$=\lim_{x\to 0}\frac{5+\frac{4\sin 3x}{x}}{\frac{4\sin 2x}{x}+7}$$

$$=\frac{\lim_{x\to 0}5+4\lim_{x\to 0}\frac{\sin 3x}{3x}\times 3}{4\lim_{x\to 0}\frac{\sin 2x}{2x}\times 2+7}$$

$$=\frac{5+4\times 1\times 3}{4\times 2+7}$$

$$=\frac{5+12}{8+7}$$

$$=\frac{17}{15}$$

Q56

$$\lim_{x\to 0}\frac{3\sin x-\sin 3x}{x^3}$$

$$=\lim_{x\to 0}\frac{3\sin x-\left(3\sin x-4\sin^3 x\right)}{x^3}$$

$$=\lim_{x\to 0}\frac{4\sin^3 x}{x^3}$$

$$=4\left(\lim_{x\to 0}\frac{\sin x}{x}\right)^3$$

$$=4\times 1$$

$$=4$$

Q57

$$\lim_{x\to 0}\frac{\tan 2x - \sin 2x}{x^3}$$

$$= \lim_{x\to 0}\frac{\frac{\sin 2x}{\cos 2x} - \sin 2x}{x^3}$$

$$= \lim_{x\to 0}\frac{\sin 2x\left(\frac{1}{\cos 2x} - 1\right)}{x^3}$$

$$= \lim_{x\to 0}\frac{\sin 2x\,(1 - \cos 2x)}{x^3\cos 2x}$$

$$= \lim_{x\to 0}\frac{\sin 2x\,(2\sin^2 x)}{x^3\cos 2x}$$

$$= \frac{\left(\lim_{x\to 0}\frac{\sin 2x}{x}\right)\left(\lim_{x\to 0}\frac{2\sin^2 x}{x^2}\right)}{\left(\lim_{x\to 0}\cos 2x\right)}$$

$$= \frac{\left(\lim_{x\to 0}\frac{\sin 2x}{2x}\times 2\right)\left(\lim_{x\to 0} 2\left(\frac{\sin x}{x}\right)^2\right)}{\lim_{x\to 0}\cos 2x}$$

$$= \frac{(2\times 1)(2\times 1)}{1}$$

$$= 4$$

Q58

$$\lim_{x\to 0}\frac{\sin ax + bx}{ax + \sin bx}$$

$$= \lim_{x\to 0}\frac{\frac{\sin ax}{x} + b}{a + \frac{\sin bx}{x}}$$

$$= \frac{\lim_{x\to 0}\frac{\sin ax}{ax}\times a + b}{a + \lim_{x\to 0}\frac{\sin bx}{bx}\times b}$$

$$= \frac{a+b}{a+b}$$

$$= 1$$

Q59

$$\lim_{x\to 0}(\operatorname{cosec}x - \cot x)$$
$$= \lim_{x\to 0}\left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)$$
$$= \lim_{x\to 0}\left(\frac{1-\cos x}{\sin x}\right)$$
$$= \lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$$
$$= \lim_{x\to 0}\frac{\tan x}{2}$$
$$= \left(\lim_{x\to 0}\frac{\frac{\tan x}{2}}{\frac{x}{2}}\right)\times\frac{x}{2}$$
$$= \lim_{x\to 0}1\times\frac{x}{2}$$
$$= 0$$

Q60

$$\lim_{x\to 0}\frac{\{\sin(\alpha+\beta)x + \sin(\alpha-\beta)x + \sin 2\alpha x\}}{\cos^2\beta x - \cos^2\alpha x}$$
$$= \lim_{x\to 0}\frac{\left\{2\sin\frac{(\alpha+\beta+\alpha-\beta)}{2}x\cos\frac{(\alpha+\beta-\alpha+\beta)}{2}x + 2\sin\alpha\cos\alpha x\right\}}{(\cos\beta x - \cos\alpha x)(\cos\beta x + \cos\alpha x)}$$
$$= \lim_{x\to 0}\frac{\{2\sin\alpha x\cos\beta x + 2\sin\alpha x\cos\alpha x\}}{(\cos\beta x - \cos\alpha x)(\cos\beta x + \cos\alpha x)}$$
$$= \lim_{x\to 0}\frac{2\sin\alpha x(\cos\beta x + \cos\alpha x)}{(\cos\beta x - \cos\alpha x)(\cos\beta x + \cos\alpha x)}$$
$$= \lim_{x\to 0}\frac{2\sin\alpha x}{(\cos\beta x - \cos\alpha x)}$$
$$= \lim_{x\to 0}\frac{2\sin\alpha x}{\left(1-2\sin^2\left(\frac{\beta x}{2}\right) - 1 + 2\sin^2\left(\frac{\alpha x}{2}\right)\right)}$$
$$= \lim_{x\to 0}\frac{2\sin\alpha x}{2\sin^2\left(\frac{\alpha x}{2}\right) - 2\sin^2\left(\frac{\beta x}{2}\right)}$$
$$= \frac{2\alpha}{\alpha^2-\beta^2}$$

Q61

$$\lim_{x\to 0}\frac{\cos ax-\cos bx}{\cos cx-1}=\lim_{x\to 0}\frac{1-2\sin^2\left(\frac{ax}{2}\right)-1+2\sin^2\left(\frac{bx}{2}\right)}{1-2\sin^2\left(\frac{cx}{2}\right)-1}$$

$$=\lim_{x\to 0}\frac{-2\sin^2\left(\frac{ax}{2}\right)+2\sin^2\left(\frac{bx}{2}\right)}{-2\sin^2\left(\frac{cx}{2}\right)}$$

$$=\lim_{x\to 0}\frac{-\sin^2\left(\frac{ax}{2ax}\right)4a^2x^2+\sin^2\left(\frac{bx}{2}\right)4b^2x^2}{-\sin^2\left(\frac{cx}{2}\right)4c^2x^2}$$

$$=\frac{-a^2+b^2}{-c^2}$$

$$=\frac{a^2-b^2}{c^2}$$

Q62

$$\lim_{h\to 0}\frac{(a+h)^2\sin(a+h)-a^2\sin a}{h}=\lim_{h\to 0}\frac{(a+h)^2(\sin a\cos h+\cos a\sin h)-a^2\sin a}{h}$$

$$=\lim_{h\to 0}\frac{(a+h)^2(\sin a\cos h)-a^2\sin a+(a+h)^2\cos a\sin h}{h}$$

$$=\lim_{h\to 0}\frac{(a^2+2ah+h^2)(\sin a\cos h)-a^2\sin a-(a+h)^2\cos a\sin h}{h}$$

$$=\lim_{h\to 0}\frac{a^2\sin a(\cos h-1)+2ah\sin a\cos h+h^2\sin a\cos h+(a+h)^2\cos a\sin h}{h}$$

$$=\lim_{h\to 0}\frac{a^2\sin a(\cos h-1)}{h}+\lim_{h\to 0}\frac{2ah\sin a\cos h}{h}-\lim_{h\to 0}\frac{h^2\sin a\cos h}{h}+\lim_{h\to 0}\frac{(a+h)^2\cos a\sin h}{h}$$

$$=\lim_{h\to 0}\frac{-a^2\sin a\sin^2\left(\frac{h}{2}\right)}{\frac{h}{2}}+2a\sin a+0+a^2\cos a$$

$$=0+2a\sin a+a^2\cos a$$

$$=2a\sin a+a^2\cos a$$

Q63

$$\lim_{x\to 0} kx \operatorname{cosec} x = \lim_{x\to 0} x \operatorname{cosec} kx$$

$$\lim_{x\to 0} kx \frac{1}{\sin x} = \lim_{x\to 0} x \frac{1}{\sin kx}$$

$$k \lim_{x\to 0}\left(\frac{x}{\sin x}\right) = \frac{1}{k} \lim_{x\to 0}\left(\frac{kx}{\sin kx}\right)$$

$$k = \frac{1}{k}$$

$$k^2 = 1$$

$$k = \pm 1$$

Ex 29.8

Q1

$$\lim_{x \to \pi/2}\left(\frac{\pi}{2} - x\right)\tan x$$

$$Let\ y = \frac{\pi}{2} - x$$

$$as\ x \to \pi/2, \quad y \to 0$$

$$\lim_{x \to \pi/2}\left(\frac{\pi}{2} - x\right)\tan x$$

$$= \lim_{y \to 0} y \tan\left(\frac{\pi}{2} - y\right)$$

$$= \lim_{y \to 0} y \frac{\sin\left(\frac{\pi}{2} - y\right)}{\cos\left(\frac{\pi}{2} - y\right)}$$

$$= \lim_{y \to 0} y \frac{\cos y}{\sin y}$$

$$= \lim_{y \to 0} \cos y = \lim_{y \to 0} \frac{y}{\sin y}$$

$$= 1$$

Q2

$$\lim_{x \to \frac{\pi}{2}} \frac{\sin 2x}{\cos x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{2 \sin x \cos x}{\cos x}$$

$$= 2 \lim_{x \to \frac{\pi}{2}} \sin x$$

$$= 2 \times \sin \frac{\pi}{2}$$

$$= 2 \times 1$$

$$= 2$$

Q3

$$\lim_{x\to\frac{\pi}{2}} \frac{\cos^2 x}{1-\sin x}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{1-\sin^2 x}{1-\sin x}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{(1-\sin x)(1+\sin x)}{(1-\sin x)}$$

$$= \lim_{x\to\frac{\pi}{2}} (1+\sin x)$$

$$= 1+\sin\frac{\pi}{2}$$

$$= 1+1$$

$$= 2$$

Q4

$$\lim_{x\to\frac{\pi}{2}} \frac{1-\sin x}{\cos^2 x}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{1-\sin x}{1-\sin^2 x}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{(1-\sin x)}{(1-\sin x)(1+\sin x)}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{1}{1+\sin x}$$

$$= \frac{1}{1+\sin\frac{\pi}{2}}$$

$$= \frac{1}{1+1}$$

$$= \frac{1}{2}$$

Q5

$$\lim_{x \to a} \frac{\cos x - \cos a}{x - a}$$

$$= \lim_{x \to a} \frac{\left(-2\sin\left(\frac{x+a}{2}\right)\sin\left(\frac{x-a}{2}\right)\right)}{x-a}$$

$$= -2\lim_{x \to a} \sin\left(\frac{x+a}{2}\right)\lim_{x \to a} \frac{\sin\left(\frac{x-a}{2}\right)}{x-a}$$

$$= -2 \times \sin\left(\frac{a+a}{2}\right) \times \left(\lim_{x-a \to 0} \frac{\sin\left(\frac{x-a}{2}\right)}{\frac{x-a}{2}}\right) \times \frac{1}{2}$$

$$= -2\sin a \times 1 \times \frac{1}{2} \qquad \left[\because \lim_{x \to a} \frac{\sin x}{x} = 1\right]$$

$$= -\sin a$$

Q6

$$\lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{x - \frac{\pi}{4}}$$

If $x \to \frac{\pi}{4}$, then $x - \frac{\pi}{4} \to 0$

Let $x - \frac{\pi}{4} = y \Rightarrow y \to 0$

$$= \lim_{y \to 0} \frac{1 - \tan\left(y + \frac{\pi}{4}\right)}{y}$$

$$= \lim_{y \to 0} \frac{1 - \left(\frac{\tan y + \tan\frac{\pi}{4}}{1 - \tan y \tan\frac{\pi}{4}}\right)}{y}$$

$$= \lim_{y \to 0} \frac{(1 - \tan y - \tan y - 1)}{y(1 - \tan y)} \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$

$$= \lim_{y \to 0} \frac{(-2\tan y)}{y(1 - \tan y)}$$

$$= -2\lim_{y \to 0} \frac{\tan y}{y} \times \frac{1}{\lim_{y \to 0}(1 - \tan y)}$$

$$= -2 \times 1 \frac{1}{(1-0)} \qquad \left[\because \lim_{\theta \to 0} = \frac{\tan\theta}{\theta} = 1\right]$$

$$= -2$$

Q7

$$\lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\left(\frac{\pi}{2} - x\right)^2}$$

If $x \to \frac{\pi}{2}, \frac{\pi}{2} - x \to 0$

Let $\frac{\pi}{2} - x = y$ they $y \to 0$

$$= \lim_{y \to 0} \frac{1 - \sin\left(\frac{\pi}{2} - y\right)}{y^2}$$

$$= \lim_{y \to 0} \frac{1 - \cos y}{y^2}$$

$$= \lim_{y \to 0} \frac{2 \sin^2 \frac{y}{2}}{y^2}$$

$$= 2\left(\lim_{y \to 0} \frac{\sin \frac{y}{2}}{\frac{y}{2}}\right)^2 \times \frac{1}{4} \qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

$$= 2 \times 1 \times \frac{1}{4}$$

$$= \frac{1}{2}$$

Q8

$$\lim_{x \to \frac{\pi}{3}} \frac{\sqrt{3} - \tan x}{\pi - 3x}$$

If $x \to \frac{\pi}{3}, \frac{\pi}{3} - x \to 0, \pi - 3x \to 0$

Let $\frac{\pi}{3} - x = y$ then $y \to 0$

$$= \lim_{y \to 0} \frac{\sqrt{3} - \tan\left(\frac{\pi}{3} - y\right)}{3\left(\frac{\pi}{3} - x\right)}$$

$$= \lim_{y \to 0} \left(\frac{\left(\sqrt{3} - \dfrac{\tan\frac{\pi}{3} - \tan y}{1 + \tan\frac{\pi}{3} . \tan y} \right)}{3y} \right)$$

$$= \lim_{y \to 0} \left(\frac{\left(\sqrt{3} - \dfrac{\sqrt{3} - \tan y}{1 + \sqrt{3}\tan y} \right)}{3y} \right)$$

$$= \lim_{y \to 0} \frac{\left(\sqrt{3} + 3\tan y - \sqrt{3} + \tan y\right)}{3\left(1 + \sqrt{3}\tan y\right) y}$$

$$= \lim_{y \to 0} \frac{4\tan y}{3\left(1 + \sqrt{3}\tan y\right) y}$$

$$= \frac{4}{3} \times \lim_{y \to 0} \frac{\tan y}{y} \times \frac{1}{\lim_{y \to 0}\left(1 + \sqrt{3}\,\frac{\tan y}{y} \times y\right)}$$

$$= \frac{4 \times 1}{3} \times \frac{1}{1 + 0}$$

$$= \frac{4}{3}$$

Q9

$$\lim_{x\to a}\frac{a\sin x - x\sin a}{ax^2 - xa^2} = \lim_{x\to a}\frac{(a\sin x - x\sin a)}{ax(x-a)}$$

Let $\quad t = x - a$

Then, as $x \to a$, $t \to 0$

$$\therefore \lim_{x\to a}\frac{(a\sin x - x\sin a)}{ax(x-a)} \quad = \lim_{t\to 0}\frac{(a\sin(t+a) - (t+a)\sin a)}{a(t+a)t}$$

$$= \lim_{t\to 0}\frac{a\sin t\cos a + a\sin a\cos t - t\sin a - a\sin a}{a(t+a)t}$$

$$= \lim_{t\to 0}\frac{a\sin t\cos a + a\sin a(\cos t - 1) - t\sin a}{a(t+a)t}$$

$$= \lim_{t\to 0}\frac{a\sin t\cos a + a\sin a\left(2\sin^2(t/2)\right) - t\sin a}{a(t+a)t}$$

$$= \lim_{t\to 0}\frac{a\sin t\cos a}{a(t+a)t} + \lim_{t\to 0}\frac{a\sin a\left(2\sin^2(t/2)\right)}{a(t+a)t} - \lim_{t\to 0}\frac{t\sin a}{a(t+a)t}$$

$$= \frac{a\cos a}{a^2} + 0 - \frac{\sin a}{a^2}$$

$$= \frac{a\cos a - \sin a}{a^2}$$

Q10

$$\lim_{x \to \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\cos^2 x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\cos^2 x} \frac{\sqrt{2} + \sqrt{1 + \sin x}}{\sqrt{2} + \sqrt{1 + \sin x}}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{2 - 1 - \sin x}{\cos^2 x \left(\sqrt{2} + \sqrt{1 + \sin x}\right)}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\left(1 - \sin^2 x\right)\left(\sqrt{2} + \sqrt{1 + \sin x}\right)}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1}{\left(1 + \sin x\right)\left(\sqrt{2} + \sqrt{1 + \sin x}\right)}$$

$$= \frac{1}{(1+1)\left(\sqrt{2} + \sqrt{2}\right)}$$

$$= \frac{1}{\left(4\sqrt{2}\right)}$$

Q11

$$\lim_{x\to\frac{\pi}{2}} \frac{\sqrt{2-\sin x}-1}{\left(\frac{\pi}{2}-x\right)^2}$$

$\Rightarrow \quad x \to \frac{\pi}{2}$, then $\frac{\pi}{2}-x \to 0$, let $\frac{\pi}{2}-x = y$

$$\lim_{x-\frac{\pi}{2}\to 0} = \lim_{y\to 0} \frac{\sqrt{2-\sin\left(\frac{\pi}{2}-y\right)}-1}{y^2}$$

$$= \lim_{y\to 0} \frac{\sqrt{2-\cos y}-1}{y^2}$$

$$= \lim_{y\to 0} \frac{\left(\sqrt{2-\cos y}-1\right)}{y^2} \times \frac{\left(\sqrt{2-\cos y}+1\right)}{\left(\sqrt{2-\cos y}+1\right)}$$

$$= \lim_{y\to 0} \frac{(2-\cos y-1)}{y^2\left(\sqrt{2-\cos y}+1\right)}$$

$$= \lim_{y\to 0} \frac{(1-\cos y)}{\left(\sqrt{2-\cos y}+1\right)y^2}$$

$$= \lim_{y\to 0} \frac{2\sin^2\frac{y}{2}}{y^2\left(\sqrt{2-\cos y}+1\right)}$$

$$= 2 \times \left(\lim_{y\to 0} \frac{\sin\frac{y}{2}}{\frac{y}{2}}\right)^2 \times \frac{1}{4} \times \frac{1}{\lim\limits_{y\to 0}\sqrt{2-\cos y}+1}$$

$$= 2 \times 1 \times \frac{1}{4} \times \frac{1}{1+1} = \frac{1}{4}$$

Q12

$$\lim_{x\to\frac{\pi}{4}} \frac{\sqrt{2}-\cos x-\sin x}{\left(\frac{\pi}{4}-x\right)^2}$$

As $x \to \frac{\pi}{4}, \frac{\pi}{4}-x \to 0$, let $\frac{\pi}{4}-x = y$

$$= \lim_{y\to 0} \frac{\sqrt{2}-\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-y\right)}{y^2}$$

$$= \lim_{y\to 0} \frac{\sqrt{2}-\left[\cos\frac{\pi}{4}\cos y+\sin\frac{\pi}{4}\sin y+\sin\frac{\pi}{4}\cos y-\cos\frac{\pi}{4}\sin y\right]}{y^2}$$

$$= \lim_{y\to 0} \frac{\sqrt{2}-\left(\frac{\cos y}{\sqrt{2}}+\frac{\sin y}{\sqrt{2}}+\frac{\cos y}{\sqrt{2}}-\frac{\sin y}{\sqrt{2}}\right)}{y^2}$$

$$= \lim_{y\to 0} \frac{\sqrt{2}-\frac{2\cos y}{\sqrt{2}}}{y^2} = \lim_{y\to 0} \frac{\sqrt{2}-\sqrt{2}\cos y}{y^2}$$

$$= \sqrt{2}\lim_{y\to 0} \frac{(1-\cos y)}{y^2}$$

$$= \sqrt{2}\lim_{y\to 0} \frac{2\sin^2\frac{y}{2}}{\frac{y^2}{4}} \times \frac{1}{4}$$

$$= \sqrt{2}\times\ 2\times\frac{1}{4}\left(\lim_{y\to 0}\frac{\sin\frac{y}{2}}{\frac{y}{2}}\right)^2$$

$$= \sqrt{2}\times 2\times\frac{1}{4}\times 1 \qquad \left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]$$

$$= \frac{1}{\sqrt{2}}$$

Q13

$$\lim_{x \to \frac{\pi}{8}} \frac{\cot 4x - \cos 4x}{8^3\left(\frac{\pi}{8} - x\right)^3}$$

When $x \to \frac{\pi}{8}, \frac{\pi}{8} - x \to 0$, let $\frac{\pi}{8} - x = y$

$$= \lim_{y \to 0} \frac{\cot 4\left(\frac{\pi}{8} - y\right) - \cos 4\left(\frac{\pi}{8} - y\right)}{(8)^3 y^3}$$

$$= \lim_{y \to 0} \frac{\cot\left(\frac{\pi}{2} - 4y\right) - \cos\left(\frac{\pi}{2} - 4y\right)}{(8)^3 y^3}$$

$$= \lim_{y \to 0} \frac{\tan 4y - \sin 4y}{(8)^3 y^3}$$

$$= \lim_{y \to 0} \frac{\frac{\sin 4y}{\cos 4y} - \sin 4y}{8^3 y^3}$$

$$= \lim_{y \to 0} \frac{\sin 4y - \sin 4y \cos 4y}{\cos 4y \times y^3 \times 8^3}$$

$$= \lim_{y \to 0} \frac{\sin 4y\,(1 - \cos 4y)}{\cos 4y \times y^3 \times 8^3}$$

$$= \lim_{y \to 0} \frac{\sin 4y\left(2\sin^2 2y\right)}{\cos 4y \times y^3 \times 8^3}$$

$$= \frac{2}{8^3} \lim_{y \to 0} \frac{\sin 4y}{y} \times \frac{\sin^2 2y}{y^2} \times \frac{1}{\cos 4y}$$

$$= \frac{2}{8^3}\left(\lim_{y \to 0} \frac{\sin 4y}{4y} \times 4\right) \times \left(\lim_{y \to 0} \frac{\sin 2y}{2y}\right)^2 \times 4 \times \frac{1}{\lim_{y \to 0} \cos 4y}$$

$$= \frac{2}{8^3}(1 \times 4) \times (1) \times 4 \times \frac{1}{1} \qquad \left[\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1, \lim_{\theta \to 0} \cos\theta = 1\right]$$

$$= \frac{2 \times 4 \times 4}{8 \times 8 \times 8}$$

$$= \frac{1}{16}$$

Q14

$$\lim_{x \to a} \frac{\cos x - \cos a}{\sqrt{x} - \sqrt{a}}$$

$$= \lim_{x \to a} \frac{\left(-2\sin\left(\frac{x+a}{2}\right)\sin\left(\frac{x-a}{2}\right)\right) \times \left(\sqrt{x} + \sqrt{a}\right)}{\left(\sqrt{x} - \sqrt{a}\right)\left(\sqrt{x} + \sqrt{a}\right)}$$

$$= -2 \lim_{x \to a} \frac{\sin\left(\frac{x+a}{2}\right)\sin\left(\frac{x-a}{2}\right) \times \left(\sqrt{x} + \sqrt{a}\right)}{(x - a)}$$

$$= -2 \lim_{x \to a} \sin\left(\frac{x+a}{2}\right) \times \lim_{x \to a} \frac{\sin\left(\frac{x-a}{2}\right) \times \frac{1}{2}}{\left(\frac{x-a}{2}\right)} \lim_{x \to a} \left(\sqrt{x} + \sqrt{a}\right)$$

$$= -2 \times \sin(a) \times 1 \times \frac{1}{2} \times 2\sqrt{a} \qquad \left[\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right]$$

$$= -2\sqrt{a}\sin a$$

Q15

$$\lim_{x\to\pi}\frac{\sqrt{5+\cos x}-2}{(\pi-x)^2}$$

$\Rightarrow$ $x\to\pi$, then $\pi-x\to 0$, let $\pi-x=y$

$$=\lim_{y\to 0}\frac{\sqrt{5+\cos(\pi-y)}-2}{y^2}$$

$$=\lim_{y\to 0}\frac{\sqrt{5-\cos y}-2}{y^2}$$

$$=\lim_{y\to 0}\frac{\left(\sqrt{5-\cos y}-2\right)}{y^2}\times\frac{\left(\sqrt{5-\cos y}+2\right)}{\left(\sqrt{5-\cos y}+2\right)}$$

$$=\lim_{y\to 0}\frac{(5-\cos y-4)}{y^2\left(\sqrt{5-\cos y}+2\right)}$$

$$=\lim_{y\to 0}\frac{2\sin^2\frac{y}{2}}{y^2\left(\sqrt{5-\cos y}+2\right)}$$

$$=2\times\left(\lim_{y\to 0}\frac{\sin\frac{y}{2}}{\frac{y}{2}}\right)^2\times\frac{1}{4}\frac{1}{\lim_{y\to 0}\left(\sqrt{5-\cos y}+2\right)}$$

$$=2\times\frac{1}{4}\times\frac{1}{\sqrt{4}+2}=2\times\frac{1}{4}\times\frac{1}{4}$$

$$=\frac{1}{8}$$

Q16

$$\lim_{x\to a}\frac{\cos\sqrt{x}-\cos\sqrt{a}}{x-a}$$

$$=\lim_{x\to a}\frac{-2\sin\left(\frac{\sqrt{x}+\sqrt{a}}{2}\right)\sin\left(\frac{\sqrt{x}-\sqrt{a}}{2}\right)}{\left(\sqrt{x}-\sqrt{a}\right)\left(\sqrt{x}+\sqrt{a}\right)}$$

$$=-2\lim_{x\to a}\frac{\sin\left(\frac{\sqrt{x}+\sqrt{a}}{2}\right)\times\lim_{x\to a}\sin\left(\frac{\sqrt{x}-\sqrt{a}}{2}\right)}{\lim_{x\to a}\left(\sqrt{x}+\sqrt{a}\right)\times\left(\frac{\sqrt{x}-\sqrt{a}}{2}\right)}\times\frac{1}{2}$$

$$=-2\sin\sqrt{a}\times 1\times\frac{1}{2\sqrt{a}}\times\frac{1}{2}$$

$$=-\frac{1}{2\sqrt{a}}\sin\sqrt{a}$$

Q17

$$\lim_{x \to a} \frac{\sin\sqrt{x} - \sin\sqrt{a}}{x - a}$$

$$= \lim_{x \to a} \frac{\sin\sqrt{x} - \sin\sqrt{a}}{\left(\sqrt{x} - \sqrt{a}\right)\left(\sqrt{x} + \sqrt{a}\right)}$$

$$= \lim_{x \to a} \frac{2\sin\left(\frac{\sqrt{x} - \sqrt{a}}{2}\right)\cos\left(\frac{\sqrt{x} + \sqrt{a}}{2}\right)}{\left(\sqrt{x} - \sqrt{a}\right)\left(\sqrt{x} + \sqrt{a}\right)}$$

$$= 2\left(\lim_{x \to a} \frac{\sin\frac{\sqrt{x} - \sqrt{a}}{2}}{\left(\frac{\sqrt{x} - \sqrt{a}}{2}\right)}\right) \times \frac{1}{2} \frac{\lim_{x \to a} \cos\left(\frac{\sqrt{x} + \sqrt{a}}{2}\right)}{\lim_{x \to a}\left(\sqrt{x} + \sqrt{a}\right)}$$

$$= 2 \times 1 \times \frac{1}{2} \times \cos\sqrt{a} \times \frac{1}{2\sqrt{a}}$$

$$= \frac{\cos\sqrt{a}}{2\sqrt{a}}$$

Q18

$$\lim_{x \to 1} \frac{1 - x^2}{\sin 2\pi x}$$

$\Rightarrow \quad x \to 1$, then $x - 1 \to 0$, let $x - 1 = y$

$$= \lim_{(x-1) \to 0} \frac{(1 - x)(1 + x)}{\sin 2\pi x}$$

$$= \lim_{y \to 0} \frac{-y\,(1 + y + 1)}{\sin 2\pi\,(y + 1)}$$

$$= -\lim_{y \to 0} \frac{y\,(y + 2)}{\sin(2\pi y + 2\pi)}$$

$$= -\lim_{y \to 0} \frac{y\,(y + 2)}{\sin 2\pi y}$$

$$= -\lim_{y \to 0}(y + 2) \times \frac{y}{\left(\lim_{y \to 0} \sin\frac{2\pi y}{y \times 2\pi}\right) \times 2\pi y}$$

$$= -2 \times \frac{1}{1 \times 2\pi}$$

$$= -\frac{1}{\pi}$$

Q19

$$\lim_{x\to\frac{\pi}{4}}\left(\frac{\sin 2x-\sin 2\left(\frac{\pi}{4}\right)}{x-\frac{\pi}{4}}\right) \qquad \left[\because \text{ given } f(x)=\sin 2x\right]$$

$$=\lim_{x\to\frac{\pi}{4}}\left(\frac{\sin 2x-\sin\frac{\pi}{2}}{x-\frac{\pi}{4}}\right)$$

$\Rightarrow \quad x\to\frac{\pi}{4}\Rightarrow x-\frac{\pi}{4}\to 0,\ \text{let } x-\frac{\pi}{4}=y$

$$=\lim_{y\to 0}\left(\frac{\sin 2\left(y+\frac{\pi}{4}\right)-1}{y}\right)$$

$$=\lim_{y\to 0}\frac{\sin\left(\frac{\pi}{2}+2y\right)-1}{y}$$

$$=\lim_{y\to 0}\frac{\cos 2y-1}{y}$$

$$=-\lim_{y\to 0}\frac{1-\cos 2y}{y}$$

$$=-\lim_{y\to 0}\frac{2\sin^2 y}{y}$$

$$=-2\left(\lim_{y\to 0}\frac{\sin y}{y}\right)^2\times y \qquad \left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]$$

$$=-2\times 0$$

$$=0$$

Q20

$$\lim_{x\to 1}\frac{1+\cos\pi x}{(1-x)^2}$$

$\Rightarrow \quad x\to 1, x-1\to 0,\ \text{let } x-1=y$

$$=\lim_{y\to 0}\frac{1+\cos\pi(y+1)}{(-y)^2}$$

$$=\lim_{y\to 0}\frac{1+\cos(\pi+\pi y)}{y^2}$$

$$=\lim_{y\to 0}\frac{1-\cos(\pi y)}{y^2}$$

$$=\lim_{y\to 0}\frac{2\sin^2\frac{\pi y}{2}}{y^2}$$

$$=2\left(\lim_{y\to 0}\frac{\sin\frac{\pi y}{2}}{\frac{\pi y}{2}}\right)^2\times\frac{\pi^2}{4}$$

$$=2\times 1\times\frac{\pi^2}{4}$$

$$=\frac{\pi^2}{2}$$

Q21

$$\lim_{x \to 1} \frac{1 - x^2}{\sin \pi x}$$

$\Rightarrow \quad x \to 1 \Rightarrow x - 1 \to 0$, let $x - 1 = y \Rightarrow y \to 0$

$$\Rightarrow \quad \lim_{x \to 1} \frac{1 - x^2}{\sin \pi x} = \lim_{x-1 \to 0} \frac{(1 - x)(1 + x)}{\sin \pi x}$$

$$= \lim_{y \to 0} \frac{(-y)(1 + y + 1)}{\sin \pi (y + 1)}$$

$$= -\lim_{y \to 0} \frac{y(y + 2)}{\sin(\pi y + \pi)}$$

$$= -\lim_{y \to 0} \frac{y(y + 2)}{-\sin \pi y}$$

$$= \lim_{y \to 0} \frac{y(y + 2)}{\frac{\sin \pi y}{2}}$$

$$= \frac{\lim_{y \to 0} y(y + 2)}{\lim_{y \to 0} \frac{\sin \pi y}{\pi y} \times \pi y}$$

$$= \frac{2}{\pi}$$

Q22

$$\lim_{x\to\frac{\pi}{4}}\frac{1-\sin 2x}{1+\cos 4x}$$

$x\to\frac{\pi}{4}, x-\frac{\pi}{4}\to 0,\ \text{let } x-\frac{\pi}{4}=y$

$$\Rightarrow \quad \lim_{x\to\frac{\pi}{4}}\frac{1-\sin 2x}{1+\cos 4x}=\lim_{x-\frac{\pi}{4}\to 0}\frac{\left(1-\sin 2\left(y+\frac{\pi}{4}\right)\right)}{1+\cos 4\left(y+\frac{\pi}{4}\right)}$$

$$=\lim_{x-\frac{\pi}{4}\to 0}\left(\frac{1-\sin\left(\frac{\pi}{2}+2y\right)}{1+\cos(\pi+4y)}\right)$$

$$=\lim_{x-\frac{\pi}{4}\to 0}\frac{1-\cos 2y}{1-\cos 4y}$$

$$=\lim_{y\to 0}\frac{2\sin^2 y}{2\sin^2 2y}$$

$$=\frac{\lim_{y\to 0}\sin^2 y}{\lim_{y\to 0}\sin^2 2y}$$

$$=\frac{\left(\lim_{y\to 0}\frac{\sin y}{y}\right)^2\times y^2}{\left(\lim_{y\to 0}\frac{\sin 2y}{2y}\right)^2\times 4y^2}$$

$$=\frac{1\times y^2}{1\times 4y^2} \qquad \left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]$$

$$=\frac{1}{4}$$

Q23

$$\lim_{x\to\frac{\pi}{6}}\frac{\cot^2 x-3}{\operatorname{cosec}x-2}$$

$$=\lim_{x\to\frac{\pi}{6}}\frac{\left(\operatorname{cosec}^2 x-1\right)-3}{\operatorname{cosec}x-2}$$

$$=\lim_{x\to\frac{\pi}{6}}\frac{\left(\operatorname{cosec}^2 x-4\right)}{\operatorname{cosec}x-2}$$

$$=\lim_{x\to\frac{\pi}{6}}\frac{(\operatorname{cosec}x-2)(\operatorname{cosec}x+2)}{(\operatorname{cosec}x-2)}$$

$$=\lim_{x\to\frac{\pi}{6}}\operatorname{cosec}x+2$$

$$=\operatorname{cosec}\frac{\pi}{6}+2$$

$$=2+2$$

$$=4$$

Q24

$$\lim_{n \to \infty} n \sin\left(\frac{\pi}{4n}\right)\cos\left(\frac{\pi}{4n}\right)$$

$$= \lim_{n \to \infty} 2\left(n \sin\frac{\pi}{4n}\cos\frac{\pi}{4n}\right) \times \frac{1}{2}$$

$$= \lim_{n \to \infty} n \times \sin\frac{\pi}{2n} \times \frac{1}{2}$$

$n \to \infty$, then $\frac{1}{n} \to 0$, let $\frac{1}{n} = y$

$$= \frac{1}{2}\lim_{\frac{1}{n} \to 0} \frac{1}{y} \sin\left(\frac{\pi}{2}\right)\left(\frac{1}{n}\right)$$

$$= \frac{1}{2}\lim_{y \to 0} \frac{\sin\left(\frac{\pi}{2}\right)y}{y}$$

$$= \frac{1}{2}\left(\lim_{y \to 0} \frac{\sin\left(\frac{\pi y}{2}\right)}{\frac{\pi y}{2}}\right) \times \frac{\pi}{2}$$

$$= \frac{1}{2} \times 1 \times \frac{\pi}{2} \qquad \left[\because \lim_{\theta \to 0}\frac{\sin\theta}{\theta} = 1\right]$$

$$= \frac{\pi}{4}$$

Q25

$$\lim_{n \to \infty} 2^{n-1} \sin\left(\frac{a}{2^n}\right)$$

$$= \lim_{n \to \infty} \frac{2^n}{2} \sin\left(\frac{a}{2^n}\right)$$

$$= \lim_{n \to \infty} \frac{2^n}{2} \sin\frac{a}{2^n}$$

$$n \to \infty, \frac{1}{n} = 0, let\, h = \frac{1}{n}$$

$$= \lim_{h \to 0} \frac{2^{\frac{1}{h}}}{2} \sin\frac{a}{2^{\frac{1}{h}}}$$

$$= \lim_{h \to 0} \frac{2^{\frac{1}{h}}}{2} \frac{\sin\frac{a}{2^{\frac{1}{h}}}}{\frac{a}{2^{\frac{1}{h}}}} \times \frac{a}{2^{\frac{1}{h}}} \qquad \left[\because \lim_{\theta \to 0}\frac{\sin\theta}{\theta} = 1\right]$$

$$= \frac{a}{2}$$

Q26

$$= \frac{\lim_{n\to\infty} \sin\left(\frac{a}{2^n}\right)}{\lim_{n\to\infty} \sin\left(\frac{b}{2^n}\right)}$$

$n \to \infty, \frac{1}{n} = h \to 0$

$$= \frac{\lim_{h\to 0} \sin\left(\frac{a}{2^{\frac{1}{h}}}\right)}{\lim_{h\to 0} \sin\left(\frac{b}{2^{\frac{1}{h}}}\right)}$$

$$= \frac{\left(\lim_{h\to 0} \frac{\sin\frac{a}{2^{\frac{1}{h}}}}{\frac{a}{2^{\frac{1}{h}}}} \times \frac{a}{2^{\frac{1}{h}}}\right)}{\left(\lim_{h\to 0} \frac{\sin\frac{b}{2^{\frac{1}{h}}}}{\frac{b}{2^{\frac{1}{h}}}} \times \frac{b}{2^{\frac{1}{h}}}\right)}$$

$$= \frac{1 \times \frac{a}{2^{\frac{1}{h}}}}{1 \times \frac{b}{2^{\frac{1}{h}}}} = \frac{a}{b}$$

Q27

$$\lim_{x\to -1} \frac{x^2 - x - 2}{\left(x^2 + x\right) + \sin(x+1)}$$

$$= \lim_{x\to -1} \frac{(x-2)(x+1)}{x(x+1) + \sin(x+1)}$$

$$= \lim_{x\to -1} \frac{1}{\frac{x(x+1)}{(x-2)(x+1)} + \frac{\sin(x+1)}{(x-2)(x+1)}}$$

$$= \lim_{x\to -1} \frac{1}{\frac{x}{x-2} + \frac{\sin(x+1)}{(x-2)(x+1)}}$$

$$= \lim_{x\to -1} \frac{1}{(x-2)} \left(\frac{1}{x + \frac{\sin(x+1)}{x+1}}\right)$$

$$= \lim_{x\to -1} \frac{1}{x-2} \times \frac{1}{\lim_{x\to -1}(x) + \lim_{x+1\to 0} \sin\frac{x+1}{x+1}}$$

$$= \left(\frac{1}{-1-2}\right) \times \frac{1}{(-1)+1} \qquad \left[\because \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1\right]$$

$$= \frac{1}{0} \qquad \left[\because \frac{1}{0} = \infty\right]$$

$$= \infty$$

Q28

$$\lim_{x\to 2}\frac{x^2-x-2}{x^2-2x+\sin(x-2)}$$

$$=\lim_{x\to 2}\frac{(x-2)(x+1)}{x^2-2x+\sin(x-2)}$$

$$=\lim_{x\to 2}\frac{1}{\frac{x}{x+1}+\frac{\sin(x-2)}{(x-2)(x+1)}}$$

$$=\lim_{x\to 2}(x+1)\left(\frac{1}{x+\frac{\sin(x-2)}{x-2}}\right)$$

$$=\lim_{x\to 2}(x+1)\times\frac{1}{\lim_{x\to 2}(x)+\lim_{x\to 2}\sin\frac{(x-2)}{x-2}}$$

$$=(2+1)\times\frac{1}{(2)+\lim_{x-2\to 0}\frac{\sin(x-2)}{(x-2)}} \qquad \left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]$$

$$=3\times\frac{1}{2+1}$$

$$=1$$

Q29

$$\lim_{x\to 1}(1-x)\tan\frac{\pi x}{2}$$

When $x\to 1, x-1\to 0$, let $x-1=y$, then $y\to 0$

$$=\lim_{(x-1)\to 0}-(x-1)\tan\frac{\pi x}{2}$$

$$=-\lim_{y\to 0}y\tan\frac{\pi}{2}(y+1)$$

$$=-\lim_{y\to 0}y\times\tan\left(\frac{\pi}{2}+\frac{\pi}{2}y\right)$$

$$=\lim_{y\to 0}y\times\cot\frac{\pi}{2}y$$

$$=\lim_{y\to 0}\frac{y}{\tan\frac{\pi y}{2}}$$

$$=\lim_{y\to 0}\frac{\frac{\pi y}{2}\times\frac{2}{\pi}}{\tan\frac{\pi y}{2}}$$

$$=\frac{2}{\pi}$$

Q30

$$\lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{1 - \sqrt{2}\sin x}$$

$$x \to \frac{\pi}{4},\ \text{then } x - \frac{\pi}{4} \to 0,\ \text{let } x - \frac{\pi}{4} = y$$

$$\Rightarrow \quad \lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{1 - \sqrt{2}\sin x} = \lim_{x - \frac{\pi}{4} \to 0} \frac{1 - \tan x}{1 - \sqrt{2}\sin x}$$

$$= \lim_{y \to 0} \frac{1 - \tan\left(y + \frac{\pi}{4}\right)}{1 - \sqrt{2}\sin\left(y + \frac{\pi}{4}\right)}$$

$$= \lim_{y \to 0} \frac{1 - \left(\dfrac{\tan\frac{\pi}{4} + \tan y}{1 + \tan\frac{\pi}{4}\tan y}\right)}{1 - \sqrt{2}\left(\sin y \cos\frac{\pi}{4} + \cos y \sin\frac{\pi}{4}\right)}$$

$$= \lim_{y \to 0} \frac{\left(1 - \left(\dfrac{1 + \tan y}{1 - \tan y}\right)\right)}{1 - \sqrt{2}\left(\dfrac{\sin y}{\sqrt{2}} + \dfrac{\cos y}{\sqrt{2}}\right)}$$

$$= \lim_{y \to 0} \frac{(1 - \tan y - 1 - \tan y)}{(1 - \tan y)(1 - \sin y + \cos y)}$$

$$= \lim_{y \to 0} \left(\frac{-2\tan y}{(1 - \tan y)(1 - \sin y - \cos y)}\right)$$

$$= -2 \lim_{y \to 0} \frac{\tan y \times 1}{\lim_{y \to 0}(1 - \tan y) \times \lim_{y \to 0}(1 - \sin y - \cos y)}$$

$$= \frac{-2\left(\lim_{y \to 0} \dfrac{\tan y}{y}\right) \times y}{\left(\lim_{y \to 0}(1) - \lim_{y \to 0} \tan y\right) \times \left(1 - \lim_{y \to 0} \dfrac{\sin y}{y} \times y - \cos 0\right)}$$

$$= \frac{-2}{(1 - y)(1 - y - 1)} = \frac{-2y}{(1 - y)(-y)} = \frac{2}{1 - y}$$

$$= \lim_{y \to 0} \frac{2}{1 - y} = 2$$

$$= 2$$

Q31

$$\lim_{x\to\pi}\frac{\left(\sqrt{2+\cos x}-1\right)}{(\pi-x)^2}$$

$\Rightarrow \quad x\to\pi$ then $x-\pi\to 0$ or let $x-\pi=y$

$$\Rightarrow \quad \lim_{x\to\pi}\frac{\left(\sqrt{2+\cos x}-1\right)}{(\pi-x)^2}=\lim_{x-\pi\to 0}\frac{\sqrt{2+\cos(x)}-1}{(-1)^2(x-\pi)^2}$$

$$=\lim_{y\to 0}\frac{\sqrt{2+\cos(\pi+y)}-1}{y^2}$$

$$=\lim_{y\to 0}\frac{\sqrt{2-\cos y}-1}{y^2}$$

$$=\lim_{y\to 0}\frac{\left(\sqrt{2-\cos y}-1\right)\left(\sqrt{2-\cos y}+1\right)}{y^2\left(\sqrt{2-\cos y}+1\right)}$$

$$=\lim_{y\to 0}\frac{(2-\cos y-1)}{\left(\sqrt{2-\cos y}+1\right)y^2}$$

$$=\lim_{y\to 0}\frac{(1-\cos y)}{\left(\sqrt{2-\cos y}+1\right)y^2}$$

$$=\lim_{y\to 0}\frac{2\sin^2\frac{y}{2}}{y^2\left(\sqrt{2-\cos y}+1\right)}$$

$$=2\lim_{y\to 0}\left(\frac{\sin\frac{y}{2}}{\frac{y}{2}}\right)^2\times\frac{1}{4}\times\frac{1}{\lim_{y\to 0}\sqrt{2-\cos 0}+1}$$

$$=2\times\frac{1}{4}\times\frac{1}{\sqrt{2-1}+1}\qquad\left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]$$

$$=\frac{1}{4}$$

Q32

$$\lim_{x\to\frac{\pi}{4}} \frac{\sqrt{\cos x}-\sqrt{\sin x}}{x-\frac{\pi}{4}}$$

$$= \lim_{x-\frac{\pi}{4}\to 0} \frac{\left(\sqrt{\cos x}-\sqrt{\sin x}\right)}{\left(x-\frac{\pi}{4}\right)} \times \frac{\left(\sqrt{\cos x}+\sqrt{\sin x}\right)}{\left(\sqrt{\cos x}+\sqrt{\sin x}\right)}$$

$$= \lim_{x-\frac{\pi}{4}\to 0} \frac{(\cos x - \sin x)}{\left(x-\frac{\pi}{4}\right)\left(\sqrt{\cos x}+\sqrt{\sin x}\right)}$$

As $x \to \frac{\pi}{4} \Rightarrow x - \frac{\pi}{4} \to 0 \Rightarrow$ let $x - \frac{\pi}{4} = y$

$$= \lim_{y\to 0} \frac{\left(\cos\left(\frac{\pi}{4}+y\right) - \sin\left(\frac{\pi}{4}+y\right)\right)}{y\left(\sqrt{\cos\left(\frac{\pi}{4}+y\right)}+\sqrt{\sin\left(\frac{\pi}{4}+y\right)}\right)}$$

$$= \lim_{y\to 0} \frac{\left(\cos\frac{\pi}{4}\cos y - \sin\frac{\pi}{4}\sin y\right) - \left(\sin\frac{\pi}{4}\cos y + \cos\frac{\pi}{4}\sin y\right)}{y\left(\sqrt{\cos\left(\frac{\pi}{4}+y\right)}+\sqrt{\sin\left(\frac{\pi}{4}+y\right)}\right)}$$

$$= \lim_{y\to 0} \frac{\left(\frac{\cos y}{\sqrt{2}} - \frac{\sin y}{\sqrt{2}} - \frac{\cos y}{\sqrt{2}} - \frac{\sin y}{\sqrt{2}}\right)}{y\left(\sqrt{\cos\left(\frac{\pi}{4}+y\right)}+\sqrt{\sin\left(\frac{\pi}{4}+y\right)}\right)}$$

$$= \lim_{y\to 0} \frac{\left(-2\frac{\sin y}{\sqrt{2}}\right)}{y\left(\sqrt{\cos\left(y+\frac{\pi}{4}\right)}+\sqrt{\sin\left(y+\frac{\pi}{4}\right)}\right)}$$

$$= -\sqrt{2}\left(\lim_{y\to 0}\frac{\sin y}{y}\right) \times \frac{1}{\lim_{y\to 0}\sqrt{\cos\left(y+\frac{\pi}{4}\right)} + \lim_{y\to 0}\sqrt{\sin\left(y+\frac{\pi}{4}\right)}}$$

$$= -\sqrt{2} \times 1 \times \frac{1}{\sqrt{\cos\frac{\pi}{4}} + \sqrt{\sin\frac{\pi}{4}}}$$

$$= -\sqrt{2} \times \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^{\frac{1}{2}} + \left(\frac{1}{\sqrt{2}}\right)^{\frac{1}{2}}} \qquad \left[\because \cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}\right]$$

$$= \frac{-\sqrt{2}}{\left(\frac{1}{\sqrt{2}}\right)^{\frac{1}{2}}(1+1)^{\frac{1}{2}}}$$

$$= \frac{-\sqrt{2}}{\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)^{\frac{1}{2}}}$$

$$= -\frac{1}{2^{\frac{1}{4}}}$$

Q33

$$\lim_{x\to 1}\frac{1-\frac{1}{x}}{\sin\pi\left(x-1\right)}$$

As $x\to 1$, then $x-1\to 0$ let $x-1=y$

$$=\lim_{x-1\to 0}\frac{\left(x-1\right)}{x\times\sin\pi\left(x-1\right)}$$

$$=\lim_{y\to 0}\frac{y}{\left(y+1\right)\sin\left(\pi y\right)}$$

$$=\lim_{y\to 0}\frac{y}{\left(y+1\right)\left(\sin\pi y\right)}$$

$$=\lim_{y\to 0}\frac{1}{\frac{\left(y+1\right)\sin\pi y}{y}}$$

$$=\frac{1}{\left(\lim\limits_{y\to 0}\left(y+1\right)\right)\times\left(\lim\limits_{y\to 0}\frac{\sin\pi y}{y\times\pi}\times\pi\right)}$$

$$=\frac{1}{\left(1\right)\left(1\times\pi\right)}\qquad\left[\because\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]$$

$$=\frac{1}{\pi}$$

Q34

$$\lim_{x\to\frac{\pi}{4}} \frac{\sqrt{2}-\cos x-\sin x}{(4x-\pi)^2}$$

$x \to \frac{\pi}{4}$ then $x-\frac{\pi}{4} \to 0$, also $4x-\pi \to 0$ let $x-\frac{\pi}{4} \to y$

$$\lim_{x-\frac{\pi}{4}\to 0} \frac{\sqrt{2}-\cos x-\sin x}{(4)^2\left(x-\frac{\pi}{4}\right)^2}$$

$$=\lim_{y\to 0} \frac{\sqrt{2}-\cos\left(y+\frac{\pi}{4}\right)-\sin\left(y+\frac{\pi}{4}\right)}{16\times y^2}$$

$$=\frac{1}{16}\lim_{y\to 0} \frac{\sqrt{2}-\left(\cos y\cos\frac{\pi}{4}-\sin y\sin\frac{\pi}{4}\right)-\left(\sin y\cos\frac{\pi}{4}+\cos\sin\frac{\pi}{4}\right)}{y^2}$$

$$=\frac{1}{16}\lim_{y\to 0} \frac{\sqrt{2}-\left(\cos y\times\frac{1}{\sqrt{2}}-\sin y\times\frac{1}{\sqrt{2}}\right)-\left(\frac{\sin y}{\sqrt{2}}+\frac{\cos y}{\sqrt{2}}\right)}{y^2}$$

$$=\frac{1}{16}\lim_{y\to 0} \frac{\sqrt{2}-\frac{1}{\sqrt{2}}(\cos y-\sin y)-\frac{1}{\sqrt{2}}(\sin y+\cos y)}{y^2}$$

$$=\frac{1}{16}\lim_{y\to 0} \frac{\sqrt{2}-\frac{1}{\sqrt{2}}\left[(\cos y-\sin y)-(\sin y+\cos y)\right]}{y^2}$$

$$=\frac{1}{16}\lim_{y\to 0} \frac{\left(\sqrt{2}-\frac{1}{\sqrt{2}}\cos y+\frac{1}{\sqrt{2}}\sin y-\frac{1}{\sqrt{2}}\sin y-\frac{1}{\sqrt{2}}\cos y\right)}{y^2}$$

$$=\frac{1}{16}\lim_{y\to 0} \frac{\sqrt{2}+\frac{2}{\sqrt{2}}\cos y}{y^2}$$

$$=\frac{1}{16}\lim_{y\to 0} \frac{\sqrt{2}(1-\cos y)}{y^2}$$

$$=\frac{\sqrt{2}}{16}\lim_{y\to 0} \frac{2\sin^2\frac{y}{2}}{y^2}$$

$$=\frac{\sqrt{2}}{8}\lim_{y\to 0} \left(\frac{\sin\frac{y}{2}}{\frac{y}{2}}\right)^2\times\frac{1}{4}$$

$$=\frac{1}{16\sqrt{2}}$$

Q35

$$\lim_{x\to\frac{\pi}{2}} \frac{\left(\frac{\pi}{2}-x\right)\sin x-2\cos x}{\left(\frac{\pi}{2}-x\right)+\cot x}$$

If $x\to\frac{\pi}{2}$, then $\frac{\pi}{2}-x\to 0$

Let $\frac{\pi}{2}-x=y$

$$=\lim_{y\to 0}\frac{\left(y\sin\left(\frac{\pi}{2}-y\right)-2\cos\left(\frac{\pi}{2}-y\right)\right)}{y+\cot\left(\frac{\pi}{2}-y\right)}$$

$$=\lim_{y\to 0}\left(\frac{y\cos y-2\sin y}{1+\tan y}\right)$$

$$=\lim_{y\to 0}\left(\frac{\cos y-2\frac{\sin y}{y}}{1+\frac{\tan y}{y}}\right)$$

$$=\frac{\lim_{y\to 0}\cos y-2\lim_{y\to 0}\frac{\sin y}{y}}{1+\lim_{y\to 0}\frac{\tan y}{y}} \qquad \left[\because \lim_{y\to 0}\frac{\sin y}{y}=1, \lim_{y\to 0}\frac{\tan y}{y}=1\right]$$

$$=\frac{1-2}{1+1}=\frac{-1}{2}$$

$$=-\frac{1}{2}$$

Q36

$$\lim_{x \to \frac{\pi}{4}} \frac{\cos x - \sin x}{\left(\frac{\pi}{4} - x\right)(\cos x + \sin x)}$$

$\Rightarrow \quad x \to \frac{\pi}{4}$ then $x - \frac{\pi}{4} \to 0$, let $x - \frac{\pi}{4} = y$

$$= \lim_{y \to 0} \frac{\cos\left(\frac{\pi}{4} + y\right) - \sin\left(\frac{\pi}{4} + y\right)}{-y\left(\cos\left(\frac{\pi}{4} + y\right) + \sin\left(\frac{\pi}{4} + y\right)\right)}$$

$$= \lim_{y \to 0} \frac{\left[\left(\cos\frac{\pi}{4}\cos y - \sin\frac{\pi}{4}\sin y\right) - \left(\sin\frac{\pi}{4}\cos y + \cos\frac{\pi}{4}\sin y\right)\right]}{-y\left(\cos\left(\frac{\pi}{4} + y\right) + \sin\left(\frac{\pi}{4} + y\right)\right)}$$

$$= \lim_{y \to 0} \frac{\left[\frac{\cos y}{\sqrt{2}} - \frac{\sin y}{\sqrt{2}} - \frac{\cos y}{\sqrt{2}} - \frac{\sin y}{\sqrt{2}}\right]}{-y\left(\cos\left(\frac{\pi}{4} + y\right) + \sin\left(\frac{\pi}{4} + y\right)\right)}$$

$$= \lim_{y \to 0} \frac{\frac{-2\sin y}{\sqrt{2}}}{-y\left(\cos\left(\frac{\pi}{4} + y\right) + \sin\left(\frac{\pi}{4} + y\right)\right)}$$

$$= \sqrt{2} \lim_{y \to 0}\left(\frac{\sin y}{y}\right) \times \frac{1}{\lim_{y \to 0}\left(\cos\left(\frac{\pi}{4} + y\right) + \sin\left(\frac{\pi}{4} + y\right)\right)}$$

$$= \sqrt{2} \times 1 \times \frac{1}{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}} = \sqrt{2} \times \frac{1}{\frac{2}{\sqrt{2}}}$$

$$= \frac{\sqrt{2} \times \sqrt{2}}{2} = 1$$

Q37

$$\lim_{x\to\pi}\frac{1-\sin\frac{x}{2}}{\cos\frac{x}{2}\left(\cos\frac{x}{4}-\sin\frac{x}{4}\right)}=\lim_{h\to 0}\frac{1-\sin\left(\frac{\pi+h}{2}\right)}{\cos\left(\frac{\pi+h}{2}\right)\left(\cos\left(\frac{\pi+h}{4}\right)-\sin\left(\frac{\pi+h}{4}\right)\right)}$$

$$=\lim_{h\to 0}\frac{1-\cos\left(\frac{h}{2}\right)}{-\sin\left(\frac{h}{2}\right)\left(\frac{1}{\sqrt{2}}\cos\left(\frac{h}{4}\right)-\frac{1}{\sqrt{2}}\sin\left(\frac{h}{4}\right)-\frac{1}{\sqrt{2}}\sin\left(\frac{h}{4}\right)-\frac{1}{\sqrt{2}}\cos\left(\frac{h}{4}\right)\right)}$$

$$=\lim_{h\to 0}\frac{1-\cos\left(\frac{h}{2}\right)}{\sqrt{2}\sin\left(\frac{h}{2}\right)\sin\left(\frac{h}{4}\right)}$$

$$=\lim_{h\to 0}\frac{2\sin^2\left(\frac{h}{4}\right)}{\sqrt{2}\sin\left(\frac{h}{2}\right)\sin\left(\frac{h}{4}\right)}$$

$$=\sqrt{2}\lim_{h\to 0}\frac{\sin\left(\frac{h}{4}\right)}{\sin\left(\frac{h}{2}\right)}$$

Ex 29.9

Q1

$$= \lim_{x \to \pi} \frac{1+\cos x}{\tan^2 x}$$

As $x \to \pi, x - \pi \to 0$, let $x - \pi = y$

$$= \lim_{y \to 0} \frac{1+\cos(\pi+y)}{\tan^2(\pi+y)}$$

$$= \lim_{y \to 0} \frac{1-\cos y}{\tan^2 y}$$

$$= \lim_{y \to 0} \frac{2\sin^2 \frac{y}{2}}{\tan^2 y}$$

$$= \frac{\lim_{y \to 0} 2\sin^2 \frac{y}{2}}{\lim_{y \to 0} \tan^2 y}$$

$$= \frac{2\left(\lim_{y \to 0} \frac{\sin \frac{y}{2}}{\frac{y}{2}}\right)^2 \times \frac{y^2}{4}}{\left(\lim_{y \to 0} \frac{\tan y}{y}\right) \times y^2}$$

$$= \frac{2 \times 1 \times \frac{y^2}{4}}{1 \times y^2} \qquad \left[\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1 \atop \lim_{\theta \to 0} \frac{\tan\theta}{\theta} = 1\right]$$

$$= 2 \times 1 \times \frac{1}{4}$$

$$= \frac{1}{2}$$

Q2

$$\lim_{x \to \frac{\pi}{4}} \frac{\operatorname{cosec}^2 x - 2}{\cot x - 1}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\cot^2 x + 1 - 2}{\cot x - 1}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\cot^2 x - 1}{\cot x - 1}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{(\cot x - 1)(\cot x + 1)}{(\cot x - 1)}$$

$$= \lim_{x \to \frac{\pi}{4}} (\cot x + 1)$$

$$= \cot \frac{\pi}{4} + 1$$

$$= 1 + 1$$

$$= 2$$

Q3

$$\lim_{x \to \frac{\pi}{6}} \frac{\operatorname{cosec}^2 x - 1 - 3}{\operatorname{cosec} x - 2}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{\operatorname{cosec}^2 x - 4}{\operatorname{cosec} x - 2}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{(\operatorname{cosec} x - 2)(\operatorname{cosec} x + 2)}{(\operatorname{cosec} x - 2)}$$

$$= \lim_{x \to \frac{\pi}{6}} (\operatorname{cosec} x + 2)$$

$$= \operatorname{cosec} \frac{\pi}{6} + 2$$

$$= 2 + 2$$

$$= 4$$

Q4

$$\lim_{x \to \frac{\pi}{4}} \frac{2 - \operatorname{cosec}^2 x}{1 - \cot x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{2 - \left(1 + \cot^2 x\right)}{1 - \cot x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{2 - 1 - \cot^2 x}{1 - \cot x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{1 - \cot^2 x}{1 - \cot x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\left(1 - \cot x\right)\left(1 + \cot x\right)}{\left(1 - \cot x\right)}$$

$$= \lim_{x \to \frac{\pi}{4}} \left(1 + \cot x\right)$$

$$= 1 + \cot \frac{\pi}{4}$$

$$= 1 + 1$$

$$= 2$$

Q5

$$\lim_{x\to\pi}\frac{\sqrt{2+\cos x}-1}{(\pi-x)^2}$$

$$=\lim_{x\to\pi}\frac{\sqrt{2+\cos x}-1}{(\pi-x)^2}\times\frac{\sqrt{2+\cos x}+1}{\sqrt{2+\cos x}+1}$$

$$=\lim_{x\to\pi}\frac{(2+\cos x)-1}{(\pi-x)^2\left(\sqrt{2+\cos x}+1\right)}$$

$$=\lim_{x\to\pi}\frac{1+\cos x}{(\pi-x)^2\left(\sqrt{2+\cos x}+1\right)}$$

Let $\pi-x=y, x\to\pi, y\to 0$

$$\Rightarrow\quad \lim_{x\to\pi}\frac{1+\cos x}{(\pi-x)^2\left(\sqrt{2+\cos x}+1\right)}=\lim_{y\to 0}\frac{1+\cos(\pi-y)}{y^2\left(\sqrt{2+\cos(\pi-y)}+1\right)}$$

$$=\lim_{y\to 0}\frac{1-\cos y}{y^2\sqrt{2-\cos y}+1}$$

$$=\lim_{y\to 0}\frac{\frac{2\sin^2 y}{2}}{y^2\sqrt{2-\cos y}+1}$$

$$=2\lim_{y\to 0}\left(\frac{\frac{\sin y}{2}}{\frac{y}{2}}\right)^2\times\frac{1}{4}\frac{1}{\sqrt{2-\cos y}+1}$$

$$=2\times\left(\lim_{y\to 0}\frac{\sin y}{2}\right)^2\times\frac{1}{4}\frac{1}{\lim_{y\to 0}\sqrt{2-\cos y}+1}$$

$$=2\times 1\times\frac{1}{4}\times\frac{1}{\sqrt{2-\cos 0}+1}$$

$$=2\times 1\times\frac{1}{4}\times\frac{1}{\sqrt{2-1}+1}$$

$$=2\times 1\times\frac{1}{4}\times\frac{1}{1+1}$$

$$=2\times 1\times\frac{1}{4}\times\frac{1}{2}$$

$$=\frac{1}{4}$$

Q6

$$\lim_{x\to\frac{3\pi}{2}} \frac{1+\operatorname{cosec}^3 x}{\cot^2 x}$$

$$= \lim_{x\to\frac{3\pi}{2}} \frac{(1+\operatorname{cosec}x)\{1+\operatorname{cosec}^2 x-\operatorname{cosec}x\}}{\{\operatorname{cosec}^2 x-1\}}$$

$$= \lim_{x\to\frac{3\pi}{2}} \frac{(\operatorname{cosec}x+1)\{1+\operatorname{cosec}^2 x-\operatorname{cosec}x\}}{(\operatorname{cosec}x-1)(\operatorname{cosec}x+1)}$$

$$= \lim_{x\to\frac{3\pi}{2}} \frac{\{1+\operatorname{cosec}^2 x-\operatorname{cosec}x\}}{\operatorname{cosec}x-1}$$

$$= \frac{1+\operatorname{cosec}^2\frac{3\pi}{2}-\operatorname{cosec}\frac{3\pi}{2}}{\operatorname{cosec}\frac{3\pi}{2}-1}$$

$$= \frac{1+(-1)^2-(-1)}{(-1)-1} \qquad \left[\because \operatorname{cosec}\frac{3\pi}{2}=-1\right]$$

$$= \frac{1+1+1}{-2}$$

$$= \frac{-3}{2}$$

Ex 29.10

Q1

$$\lim_{x \to 0} \frac{5^x - 1}{\sqrt{4+x} - 2}$$

$$= \lim_{x \to 0} \frac{(5^x - 1)(\sqrt{4+x} + 2)}{(\sqrt{4+x} - 2)(\sqrt{4+x} + 2)}$$

$$= \lim_{x \to 0} \frac{(5^x - 1)(\sqrt{4+x} + 2)}{x}$$

$$= 4 \log 5$$

Q2

$$\lim_{x \to 0} \frac{\log(1+x)}{3^x - 1}$$

$$= \lim_{x \to 0} \frac{\log(1+x)}{x} \times \frac{1}{\lim_{x \to 0} \frac{3^x - 1}{x}}$$

$$= \frac{1}{\log 3}$$

Q3

$$\lim_{x \to 0} \frac{a^x + a^{-x} - 2}{x^2}$$

$$= \lim_{x \to 0} \frac{a^{2x} - 2a^x + 1}{a^x \cdot x^2}$$

$$= \lim_{x \to 0} \left(\frac{a^x - 1}{x}\right)^2 \times \frac{1}{a^x}$$

$$= (\log_e a)^2 \times \frac{1}{a^0}$$

$$= (\log_e a)^2$$

Q4

$$\lim_{x\to 0}\frac{a^{mx}-1}{b^{nx}-1},\ n\neq 0$$

$$=\lim_{x\to 0}\frac{a^{mx}-1}{mx}\times\frac{1}{\lim_{x\to 0}\frac{b^{nx}-1}{nx}}\times\frac{m}{n}$$

$$=\frac{m\log a}{n\log b},\ n\neq 0$$

Q5

$$\lim_{x\to 0}\frac{a^{x}+b^{x}-2}{x}$$

$$=\lim_{x\to 0}\frac{a^{x}-1}{x}+\lim_{x\to 0}\frac{b^{x}-1}{x}$$

$$=\log a+\log b$$

$$=\log(ab)$$

Q6

$$\lim_{x\to 0}\frac{9^{x}-2.6^{x}+4^{x}}{x^{2}}$$

$$=\lim_{x\to 0}\frac{(3^{x})^{2}-2.3^{x}2^{x}+(2^{x})^{2}}{x^{2}}$$

$$=\lim_{x\to 0}\left(\frac{3^{x}-2^{x}}{x}\right)^{2}$$

$$=\left(\lim_{x\to 0}\frac{3^{x}-1}{x}-\lim_{x\to 0}\left(\frac{2^{x}-1}{x}\right)\right)^{2}$$

$$=\left(\log\frac{3}{2}\right)^{2}$$

Q7

$$\lim_{x\to 0}\frac{8^x-4^x-2^x+1}{x^2}$$

$$=\lim_{x\to 0}\frac{(2^x-1)^2(2^x+1)}{x^2}$$

$$=\lim_{x\to 0}\left(\frac{(2^x-1)}{x}\right)^2\lim_{x\to 0}(2^x+1)$$

$$=2\,(\log 2)^2$$

Q8

$$\lim_{x\to 0}\frac{a^{mx}-b^{nx}}{x}$$

$$=\lim_{x\to 0}m\frac{a^{mx}-1}{mx}-\lim_{x\to 0}n\frac{b^{nx}-1}{nx}$$

$$=m\log a-n\log b$$

$$=\log\left(\frac{a^m}{b^n}\right)$$

Q9

$$\lim_{x\to 0}\frac{a^x+b^x+c^x-3}{x}$$

$$=\lim_{x\to 0}\frac{a^x-1}{x}+\lim_{x\to 0}\frac{b^x-1}{x}+\lim_{x\to 0}\frac{c^x-1}{x}$$

$$=\log a+\log b+\log c$$

$$=\log(abc)$$

Q10

$$Let\ x-2=h$$

$$\lim_{h\to 0}\frac{h}{\log_a(h+1)}$$

$$=\lim_{h\to 0}\frac{\log a}{\frac{\log(h+1)}{h}}$$

$$=\log a$$

Q11

$$\lim_{x \to 0} \frac{5^x + 3^x + 2^x - 3}{x}$$

$$= \lim_{x \to 0} \frac{5^x - 1}{x} + \lim_{x \to 0} \frac{3^x - 1}{x} + \lim_{x \to 0} \frac{2^x - 1}{x}$$

$$= \log 5 + \log 3 + \log 2$$

$$= \log 30$$

Q12

$$Let\ \frac{1}{x} = h$$

$$\lim_{h \to 0} \frac{(a^h - 1)}{h}$$

$$= \log a$$

Q13

$$\lim_{x \to 0} \frac{a^{mx} - b^{nx}}{\sin kx}$$

$$= \lim_{x \to 0} \frac{a^{mx} - b^{nx}}{kx \dfrac{\sin kx}{kx}}$$

$$= \frac{1}{k} \lim_{x \to 0} \frac{\dfrac{(a^{mx} - b^{nx})}{x}}{\dfrac{\sin kx}{kx}}$$

$$= \frac{1}{k} \log \frac{a^m}{b^n}$$

Q14

$$\lim_{x \to 0} \frac{a^x + b^x - c^x - d^x}{x}$$

$$= \lim_{x \to 0} \frac{a^x - 1}{x} + \lim_{x \to 0} \frac{a^x - 1}{x} - \lim_{x \to 0} \frac{c^x - 1}{x} - \lim_{x \to 0} \frac{d^x - 1}{x}$$

$$= \log a + \log b - \log c - \log d$$

$$= \log\left(\frac{ab}{cd}\right)$$

Q15

$$\lim_{x \to 0} \frac{e^x - 1 + \sin x}{x}$$

$$= \lim_{x \to 0} \frac{e^x - 1}{x} + \lim_{x \to 0} \frac{\sin x}{x}$$

$$= \log e + 1$$

$$= 2$$

Q16

$$\lim_{x \to 0} \frac{\sin 2x}{e^x - 1}$$

$$= \lim_{x \to 0} \frac{\sin 2x}{2x} \times 2 \lim_{x \to 0} \frac{x}{e^x - 1}$$

$$= 1 \times 2 \times \log e$$

$$= 2$$

Q17

$$\lim_{x \to 0} \frac{e^{\sin x} - 1}{x}$$

$$= \lim_{x \to 0} \frac{e^{\sin x} - 1}{\sin x} \times \lim_{x \to 0} \frac{\sin x}{x}$$

$$= \log e \times 1$$

$$= 1$$

Q18

$$\lim_{x\to 0}\frac{e^{2x}-e^{x}}{\sin 2x}$$
$$=\lim_{x\to 0}\frac{e^{2x}-1}{\sin 2x}-\lim_{x\to 0}\frac{e^{x}-1}{\sin 2x}$$
$$=\left(\lim_{x\to 0}\frac{e^{2x}-1}{2x}\times\lim_{x\to 0}\frac{2x}{\sin 2x}\right)-\frac{1}{2}\left(\lim_{x\to 0}\frac{e^{x}-1}{x}\times\lim_{x\to 0}\frac{2x}{\sin 2x}\right)$$
$$=1-\frac{1}{2}$$
$$=\frac{1}{2}$$

Q19

$$\lim_{x\to a}\frac{\log x-\log a}{x-a}$$
$$=\lim_{x\to a}\frac{\log\frac{x}{a}}{a\left(\frac{x}{a}-1\right)}$$
$$let\ h=\frac{x}{a}-1$$
$$=\frac{1}{a}\lim_{x\to a}\frac{\log(h+1)}{h}$$
$$=\frac{1}{a}$$

Q20

$$\lim_{x\to 0}\frac{\log(a+x)-\log(a-x)}{x}$$
$$=\lim_{x\to 0}\frac{\log\left(\frac{a+x}{a-x}\right)}{x}$$
$$=\lim_{x\to 0}\frac{\log\left(1+\frac{2x}{a-x}\right)}{\frac{2x}{a-x}}\times\lim_{x\to 0}\frac{2}{a-x}$$
$$=\frac{2}{a}$$

Q21

$$\lim_{x \to 0} \frac{\log(2+x)+\log(0.5)}{x}$$

$$= \lim_{x \to 0} \frac{\log\left(1+\frac{x}{2}\right)}{2\left(\frac{x}{2}\right)}$$

$$= \frac{1}{2}$$

Q22

$$\lim_{x \to 0} \frac{\log(a+x)-\log(a)}{x}$$

$$= \lim_{x \to 0} \frac{\log\left(1+\frac{x}{a}\right)}{a\left(\frac{x}{a}\right)}$$

$$= \frac{1}{a}$$

Q23

$$\lim_{x \to 0} \frac{\log(3+x)-\log(3-x)}{x}$$

$$= \lim_{x \to 0} \frac{\log\left(\frac{3+x}{3-x}\right)}{x}$$

$$= \lim_{x \to 0} \frac{\log\left(1+\frac{2x}{3-x}\right)}{x}$$

$$= \lim_{x \to 0} \frac{\log\left(1+\frac{2x}{3-x}\right)}{\frac{2x}{3-x}} \times \lim_{x \to 0} \frac{2}{3-x}$$

$$= \frac{2}{3}$$

Q24

$$\lim_{x \to 0} \frac{8^x - 2^x}{x}$$
$$= \lim_{x \to 0} \frac{8^x - 1}{x} - \lim_{x \to 0} \frac{2^x - 1}{x}$$
$$= \log 8 - \log 2$$
$$= \log 4$$

Q25

$$\lim_{x \to 0} \frac{x(2^x - 1)}{1 - \cos x}$$
$$= \lim_{x \to 0} \frac{x(2^x - 1)}{2\sin^2\left(\frac{x}{2}\right)}$$
$$= \lim_{x \to 0} \frac{(2^x - 1)}{x} \times \lim_{x \to 0} \frac{x^2}{\left(\frac{\sin\left(\frac{x}{2}\right)}{\frac{x}{2}}\right)^2 \times \frac{x^2}{2}}$$
$$= 2\log 2$$
$$= \log 4$$

Q26

$$\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{\log(1+x)}$$
$$= \lim_{x \to 0} \frac{(\sqrt{1+x} - 1)(\sqrt{1+x} + 1)}{\log(1+x)(\sqrt{1+x} + 1)}$$
$$= \lim_{x \to 0} \frac{x}{\log(1+x)(\sqrt{1+x} + 1)}$$
$$= \lim_{x \to 0} \frac{1}{\frac{\log(1+x)}{x}} \times \lim_{x \to 0} \frac{1}{(\sqrt{1+x} + 1)}$$
$$= 1 \times \frac{1}{2}$$
$$= \frac{1}{2}$$

Q27

$$\lim_{x\to 0}\frac{\log\left|1+x^3\right|}{\sin^3 x}$$

$$=\lim_{x\to 0}\frac{\log\left|1+x^3\right|}{\sin^3 x}\times\frac{1}{\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)^3}$$

$$=1\times 1$$

$$=1$$

Q28

$$\lim_{x\to\frac{\pi}{2}}\frac{a^{\cot x}-a^{\cos x}}{\cot x-\cos x}$$

$$=\lim_{x\to\frac{\pi}{2}}a^{\cos x}\left[\frac{a^{\cot x-\cos x}-1}{\cot x-\cos x}\right]$$

$$=1\times\log a$$

$$=\log a$$

Q29

$$\lim_{x\to 0}\frac{e^x-1}{\sqrt{1-\cos x}}$$

$$=\lim_{x\to 0}\frac{(e^x-1)\left(\sqrt{1+\cos x}\right)}{\left(\sqrt{1-\cos x}\right)\left(\sqrt{1+\cos x}\right)}$$

$$=\lim_{x\to 0}\frac{(e^x-1)\left(\sqrt{1+\cos x}\right)}{\sin x}$$

Both numerator and denominator are both zeros for $x=0$

hence limit can not exist

Q30

$$\lim_{x \to 0} \frac{e^{5+h} - e^5}{h}$$
$$= e^5 \lim_{x \to 0} \frac{e^h - 1}{h}$$
$$= e^5 \times 1$$
$$= e^5$$

Q31

$$\lim_{x \to 0} \frac{e^{x+2} - e^2}{x}$$
$$= e^2 \lim_{x \to 0} \frac{e^x - 1}{x}$$
$$= e^2$$

Q32

$$Let\ x - \frac{\pi}{2} = h$$
$$\lim_{h \to 0} \frac{e^{-\sin h} - 1}{-\sin h}$$
$$= \lim_{\sin h \to 0} \frac{e^{-\sin h} - 1}{-\sin h}$$
$$= 1$$

Q33

$$\lim_{x \to 0} \frac{e^{3+x} - \sin x - e^3}{x}$$
$$= e^3 \lim_{x \to 0} \frac{e^x - 1}{x} - \lim_{x \to 0} \frac{\sin x}{x}$$
$$= e^3 \log e - 1$$
$$= e^3 - 1$$

Q34

$$\lim_{x \to 0} \frac{e^x - x - 1}{x}$$
$$= \lim_{x \to 0} \frac{e^x - 1}{x} - 1$$
$$= 1 - 1$$
$$= 0$$

Q35

$$\lim_{x \to 0} \frac{e^{3x} - e^{2x}}{x}$$
$$= 3 \lim_{x \to 0} \frac{e^{3x} - 1}{3x} - \lim_{x \to 0} \frac{e^{2x} - 1}{2x}$$
$$= 3 - 2$$
$$= 1$$

Q36

$$\lim_{x \to 0} \frac{e^{\tan x} - 1}{\tan x}$$
$$= \lim_{\tan x \to 0} \frac{e^{\tan x} - 1}{\tan x}$$
$$= 1$$

Q37

$$\lim_{x \to 0} \frac{e^{bx} - e^{ax}}{x}$$
$$= b \lim_{x \to 0} \frac{e^{bx} - 1}{bx} - a \lim_{x \to 0} \frac{e^{ax} - 1}{ax}$$
$$= b - a$$

Q38

$$\lim_{x \to 0} \frac{e^{\tan x} - 1}{x}$$

$$= \lim_{x \to 0} \frac{e^{\tan x} - 1}{\tan x} \times \lim_{x \to 0} \frac{\tan x}{x}$$

$$= \log e \times 1$$

$$= 1$$

Q39

$$\lim_{x \to 0} \frac{e^{x} - e^{\sin x}}{x - \sin x}$$

$$= \lim_{x \to 0} e^{\sin x}\left[\frac{e^{x - \sin x} - 1}{x - \sin x}\right]$$

$$= 1 \times \log e$$

$$= 1$$

Q40

$$\lim_{x \to 0} \frac{3^{2+x} - 9}{x} = \lim_{x \to 0} \frac{3^2 \cdot 3^x - 9}{x}$$

$$= 9 \lim_{x \to 0} \frac{3^x - 1}{x}$$

$$= 9 \log_e 3$$

Q41

$$\lim_{x \to 0} \frac{a^x - a^{-x}}{x} = \lim_{x \to 0} \frac{a^{2x} - 1}{x a^x}$$

$$= \lim_{x \to 0} \frac{2\left(a^{2x} - 1\right)}{2x} \lim_{x \to 0} \frac{1}{a^x}$$

$$= 2 \log_e 2$$

Q42

$$\lim_{x\to 0}\frac{x(e^x-1)}{1-\cos x}$$

$$=\lim_{x\to 0}\frac{x(e^x-1)}{2\sin^2\left(\frac{x}{2}\right)}$$

$$=\lim_{x\to 0}\frac{(e^x-1)}{2x}\times\lim_{x\to 0}\frac{4}{\left(\frac{\sin\left(\frac{x}{2}\right)}{\frac{x}{2}}\right)^2}$$

$$=\frac{1}{2}\times 4$$

$$=2$$

Q43

$$\lim_{x\to\frac{\pi}{2}}\frac{2^{-\cos x}-1}{x\left(x-\frac{\pi}{2}\right)}=\lim_{x\to\frac{\pi}{2}}\frac{2^{\sin\left(x-\frac{\pi}{2}\right)}-1}{\left(x-\frac{\pi}{2}\right)}\times\frac{1}{x}$$

$$=\frac{2}{\pi}\log_e 2$$

Ex 29.11

Q1

$$\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^{\lim\limits_{n\to\infty}\left(\frac{x}{n}\right)n}$$

$$= e^x$$

Q2

$$\lim_{x\to0^+}\left\{1+\tan^2\sqrt{x}\right\}^{1/2x}$$

$$= e^{\lim\limits_{x\to0^+}\left\{\frac{\tan^2\sqrt{x}}{2x}\right\}}$$

$$= e^{\lim\limits_{x\to0^+}\left\{\frac{\tan^2\sqrt{x}}{2x}\right\}}$$

$$= e^{\lim\limits_{x\to0^+}\left\{\frac{\sin^2\sqrt{x}}{2x\cos^2\sqrt{x}}\right\}}$$

$$= e^{\frac{1}{2}\lim\limits_{x\to0^+}\left\{\left(\frac{\sin\sqrt{x}}{\sqrt{x}}\right)^2\right\}\lim\limits_{x\to0^+}\left\{\frac{1}{\cos^2\sqrt{x}}\right\}}$$

$$= e^{\frac{1}{2}}$$

$$= \sqrt{e}$$

Q3

$$\lim_{x\to 0}(\cos x)^{1/\sin x} = \lim_{x\to 0}(1+\cos x-1)^{1/\sin x}$$

$$= \lim_{x\to 0}(1-(1-\cos x))^{1/\sin x}$$

$$= \lim_{x\to 0}\left(1-2\sin^2\left(\frac{x}{2}\right)\right)^{1/\sin x}$$

$$= e^{\lim_{x\to 0}\left(-2\sin^2\left(\frac{x}{2}\right)\right)\times(1/\sin x)}$$

$$= e^{\lim_{x\to 0}\left(\frac{-2\sin^2\left(\frac{x}{2}\right)}{\sin x}\right)}$$

$$= e^{\lim_{x\to 0}\left(\frac{-2\sin^2\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}\right)}$$

$$= e^{\lim_{x\to 0} -\tan x}$$

$$= e^0$$

$$= 1$$

Q4

$$\lim_{x\to 0}(\cos x+\sin x)^{1/x}$$

$$= \lim_{x\to 0}(1+(\cos x+\sin x-1))^{1/x}$$

$$= e^{\lim_{x\to 0}\frac{(\cos x+\sin x-1)}{x}}$$

$$= e^{\lim_{x\to 0}\frac{(\sin x-(1-\cos x))}{x}}$$

$$= e^{\lim_{x\to 0}\frac{(\sin x-2\sin^2(x/2))}{x}}$$

$$= e^{\lim_{x\to 0}\frac{\sin x}{x}-\lim_{x\to 0}\frac{2\sin(x/2)\sin(x/2)}{2\left(\frac{x}{2}\right)}}$$

$$= e^{\lim_{x\to 0}\frac{\sin x}{x}-\lim_{x\to 0}\frac{\sin(x/2)\sin(x/2)}{\left(\frac{x}{2}\right)}}$$

$$= e^{1-0}$$

$$= e$$

Q5

$$\lim_{x\to 0}\left(\cos x + a\sin bx\right)^{1/x}$$
$$= \lim_{x\to 0}\left(1 + (\cos x + a\sin bx - 1)\right)^{1/x}$$
$$= e^{\lim_{x\to 0}\frac{(\cos x + a\sin bx - 1)}{x}}$$
$$= e^{\lim_{x\to 0}\frac{(a\sin bx - (1-\cos x))}{x}}$$
$$= e^{\lim_{x\to 0}\frac{\left(a\sin bx - 2\sin^2(x/2)\right)}{x}}$$
$$= e^{\lim_{x\to 0}\frac{ab\sin bx}{bx} - \lim_{x\to 0}\frac{2\sin(x/2)\sin(x/2)}{2\left(\frac{x}{2}\right)}}$$
$$= e^{\lim_{x\to 0}\frac{ab\sin bx}{bx} - \lim_{x\to 0}\frac{\sin(x/2)\sin(x/2)}{\left(\frac{x}{2}\right)}}$$
$$= e^{ab-0}$$
$$= e^{ab}$$

Q6

$$\lim_{x\to\infty}\left\{\frac{x^2+2x+3}{2x^2+x+5}\right\}^{\frac{3x-2}{3x+2}}$$
$$= e^{\lim_{x\to\infty}\left\{\left(\frac{3x-2}{3x+2}\right)\ln\left(\frac{x^2+2x+3}{2x^2+x+5}\right)\right\}}$$
$$= e^{\lim_{x\to\infty}\left\{\left(\frac{3-\frac{2}{x}}{3+\frac{2}{x}}\right)\left(\ln\left(\frac{x^2+2x+3}{2x^2+x+5}\right)\right)\right\}}$$
$$= e^{\lim_{x\to\infty}\left\{\left(\frac{3-\frac{2}{x}}{3+\frac{2}{x}}\right)\left(\ln\left(\frac{1+\frac{2}{x}+\frac{3}{x^2}}{2+\frac{1}{x}+\frac{5}{x^2}}\right)\right)\right\}}$$
$$= e^{1.\ln\left(\frac{1}{2}\right)}$$
$$= \frac{1}{2}$$

Q7

$$\lim_{x\to 1}\left\{\frac{x^3+2x^2+x+1}{x^2+2x+3}\right\}^{\frac{1-\cos(x-1)}{(x-1)^2}}$$

$$= e^{\lim_{x\to 1}\left\{\frac{1-\cos(x-1)}{(x-1)^2}\ln\left(\frac{x^3+2x^2+x+1}{x^2+2x+3}\right)\right\}}$$

$$= e^{\lim_{x\to 1}\left\{\frac{2\sin^2(x-1)}{4\left(\frac{x-1}{2}\right)^2}\ln\left(\frac{x^3+2x^2+x+1}{x^2+2x+3}\right)\right\}}$$

$$= e^{\frac{2}{4}\ln\left(\frac{5}{6}\right)}$$

$$= e^{\ln\left(\frac{5}{6}\right)^{1/2}}$$

$$= \left(\frac{5}{6}\right)^{1/2} = \sqrt{\frac{5}{6}}$$

Q8

$$\lim_{x\to 0}\left\{\frac{e^x+e^{-x}-2}{x^2}\right\}^{1/x^2}$$

$$= e^{\lim_{x\to 0}\frac{\left\{\frac{e^x+e^{-x}-2}{x^2}\right\}}{x^2}}$$

Applying L'Hospital's Rule

$$= e^{\lim_{x\to 0}\left\{\frac{2+e^x(-2+x)+x}{2(-1+e^x)x^2}\right\}}$$

Applying L'Hospital's Rule again

$$= e^{\frac{1}{2}\lim_{x\to 0}\left\{\frac{1+e^x(-1+x)}{x(-2+e^x(2+x))}\right\}}$$

Applying L'Hospital's Rule again

$$= e^{\frac{1}{2}\lim_{x\to 0}\left\{\frac{e^x x}{-2+e^x(2+4x+x^2))}\right\}}$$

Applying L'Hospital's Rule again

$$= e^{\frac{1}{2}\lim_{x\to 0}\left\{\frac{1+x}{6+6x+x^2}\right\}}$$

$$= e^{\frac{1}{2}\left\{\frac{\lim_{x\to 0}(1+x)}{\lim_{x\to 0}(6+6x+x^2)}\right\}}$$

$$= e^{\frac{1}{12}}$$

$$= \sqrt[12]{e}$$

Q9

$$\lim_{x\to a}\left\{\frac{\sin x}{\sin a}\right\}^{\frac{1}{x-a}}$$

$$=\lim_{x\to a}\left\{1+\left(\frac{\sin x}{\sin a}-1\right)\right\}^{\frac{1}{x-a}}$$

$$=e^{\lim_{x\to a}\left\{\frac{\left(\frac{\sin x}{\sin a}-1\right)}{x-a}\right\}}$$

$$=e^{\lim_{x\to a}\left\{\frac{\left(\frac{\sin x-\sin a}{\sin a}\right)}{x-a}\right\}}$$

$$=e^{\lim_{x\to a}\left\{\left(\frac{\sin x-\sin a}{\sin a(x-a)}\right)\right\}}$$

$$=e^{\lim_{x\to a}\left\{\left(\frac{2\cos\left(\frac{x+a}{2}\right)\sin\left(\frac{x-a}{2}\right)}{\sin a(x-a)}\right)\right\}}$$

$$=e^{\lim_{x\to a}\left\{\left(\frac{2\cos\left(\frac{x+a}{2}\right)}{\sin a}\right)\right\}\lim_{x\to a}\left\{\left(\frac{\sin\left(\frac{x-a}{2}\right)}{2\left(\frac{x-a}{2}\right)}\right)\right\}}$$

$$=e^{\frac{2\cos a}{2\sin a}}$$

$$=e^{\cot a}$$

Q10

$$\lim_{x\to\infty}\left\{\frac{3x^2+1}{4x^2-1}\right\}^{\frac{x^3}{1+x}}$$

$$=\lim_{x\to\infty}\left\{1+\frac{-x^2+2}{4x^2-1}\right\}^{\frac{x^3}{1+x}}$$

$$=e^{\lim_{x\to\infty}\left\{\left(\frac{-x^2+2}{4x^2-1}\right)\left(\frac{x^3}{1+x}\right)\right\}}$$

$$=e^{\lim_{x\to\infty}\left\{\left(\frac{-x^5+2x^3}{4x^2-1+4x^3-x}\right)\right\}}$$

$$=e^{-\infty}$$

$$=0$$