Exercise 2.1

1. Find the zeroes of each of the following quadratic polynomials and verify the relationship between the zeroes and their co efficient:

(i) f(x) = $x^2 - 2x - 8$

(ii) g(s) = $4s^2 - 4x + 1$

(iii) h(t) = $t^2 - 15$

(iv) p(x) = $x^2 + 2\sqrt{2}x + 6$

(v) q(x) = $\sqrt{3}x^2 + 10x + 7\sqrt{3}$

(vi) f(x) = $x^2 - (\sqrt{3} + 1)x + \sqrt{3}$

(vii) g(x) = $a(x^2 + 1) - x(a^2 + 1)$

(viii) $6x^2 - 3 - 7x$

Sol:

(i) f(x) = $x^2 - 2x - 8$

$f(x) = x^2 - 2x - 8 = x^2 - 4x + 2x - 8$

= $x(x - 4) + 2(x - 4)$

= $(x + 2)(x - 4)$

Zeroes of the polynomials are -2 and 4

Sum of the zeroes = $\frac{-\, co\ efficient\ of\ x}{co\ efficient\ of\ x}$

-2 + 4 = $\frac{-(-2)}{1}$

2 = 2

Product of the zeroes = $\frac{constant\ term}{co\ efficient\ of\ x^2}$

= 24 = $\frac{-8}{1}$

$-8 = -8$

∴ Hence the relationship verified

(ii) $9(5) = 45 - 45 + 1 = 45^2 - 25 - 25 + 1 = 25(25 - 1) - 1(25 - 1)$

= $(25 - 1)(25 - 1)$

Zeroes of the polynomials are $\frac{1}{2}\ and\ \frac{1}{2}$

Sum of zeroes = $\frac{-\, co\ efficient\ of\ s}{co\ efficient\ of\ s^2}$

$\frac{1}{2} + \frac{1}{2} = \frac{-(-4)}{4}$

1 = 1

Product of the zeroes = $\frac{constant\ term}{co\ efficient\ os\ s^2}$

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \Rightarrow \frac{1}{4} = \frac{1}{4}$

∴ Hence the relationship verified.

(iii) h(t) = $t^2 - 15 = (t^2) - (\sqrt{15})^2 = (t + \sqrt{15})(t - \sqrt{15})$

zeroes of the polynomials are $-\sqrt{15}\ and\ \sqrt{15}$

sum of zeroes = 0

$-\sqrt{15} + \sqrt{15} = 0$

$0 = 0$

Product of zeroes = $\frac{-15}{1}$

$-\sqrt{15} \times \sqrt{15} = -15$

-15 = -15

∴ Hence the relationship verified.

(iv) p(x) = $x^2 + 2\sqrt{2}x - 6 = x^2 + 3\sqrt{2}x + \sqrt{2} \times 3\sqrt{2}$

= $x(x + 3\sqrt{2}) - \sqrt{2}(2 + 3\sqrt{2}) = (x - \sqrt{2})(x + 3\sqrt{2})$

Zeroes of the polynomial are $3\sqrt{2}$ and -$3\sqrt{2}$

Sum of the zeroes = $\frac{-3\sqrt{2}}{1}$

$\sqrt{2} - 3\sqrt{2} = -2\sqrt{2}$

$-2\sqrt{2} = -2\sqrt{2}$

$Product\ of\ zeroes\ \Rightarrow\ \sqrt{2} \times -3\sqrt{2} = -\frac{6}{1}$

-6 = -6

$Hence\ the\ relatioship\ varified$

(v) 2(x) = $\sqrt{3}x^2 + 10x + 7\sqrt{3} = \sqrt{3}x^2 + 7x + 3x + 7\sqrt{3}$

= $\sqrt{3}x(x + \sqrt{3}) + 7(x + \sqrt{3})$

= $(\sqrt{3}x + 7)(x + \sqrt{3})$

Zeroes of the polynomials are $-\sqrt{3}, \frac{-7}{\sqrt{3}}$

Sum of zeroes = $\frac{-10}{\sqrt{3}}$

$\Rightarrow -\sqrt{3} - \frac{7}{\sqrt{3}} = \frac{-10}{\sqrt{3}} \Rightarrow \frac{-10}{\sqrt{3}} = \frac{-10}{\sqrt{3}}$

Product of zeroes = $\frac{7\sqrt{3}}{3} \Rightarrow \frac{\sqrt{3}x-7}{\sqrt{30}} = 7$

$\Rightarrow$ 7 = 7

Hence, relationship verified.

(vi) f(x) = $x^2 - (\sqrt{3} + 1)x + \sqrt{3} = x^2 - \sqrt{3}x - x + \sqrt{3}$

= x (x $-$ $\sqrt{3}$) – 1 (x - $\sqrt{3}$)

= (x – 1) (x $-$ $\sqrt{3}$)

Zeroes of the polynomials are 1 and $\sqrt{3}$

Sum of zeroes = $\frac{-\{coefficient\ of\ x\}}{co\ efficient\ of\ r^2} = \frac{-[-\sqrt{3}-1]}{1}$

1 + $\sqrt{3} = \sqrt{3} + 1$

Product of zeroes = $\frac{constant\ term}{co\ efficient\ of\ x^2} = \frac{\sqrt{3}}{1}$

$1 \times \sqrt{3} = \sqrt{3} = \sqrt{3} = \sqrt{3}$

∴ Hence, relationship verified

(vii) g(x) = $a[(x^2 + 1) - x(a^2 + 1)]^2 = ax^2 + a - a^2x - x$

= $ax^2 - [(a^2 + 1) - x] + 0 = ax^2 - a^2x - x + a$

$= ax(x-a)-1(x-a)=(x-a)(ax-1)$

Zeroes of the polynomials = $\frac{1}{a} and\ a$

Sum of the zeroes = $\frac{-[-a^2-1]}{a}$

$\Rightarrow \frac{1}{a}+a=\frac{a^2+1}{a} \Rightarrow \frac{a^2+1}{a}=\frac{a^2+1}{a}$

Product of zeroes = $\frac{a}{a}$

$\Rightarrow \frac{1}{a}\times a=\frac{a}{a} \Rightarrow \frac{a^2+1}{a}=\frac{a^2+1}{a}$

Product of zeroes = $\frac{a}{a} \Rightarrow 1=1$

Hence relationship verified

(viii) $6x^2-3-7x=6x^2-7x-3=(3x+11)(2x-3)$

Zeroes of polynomials are $+\frac{3}{2} and \frac{-1}{3}$

Sum of zeroes = $\frac{-1}{3}+\frac{3}{2}=\frac{7}{6}=\frac{-(-7)}{6}=\frac{-(co\ efficient\ of\ x)}{co\ efficient\ of\ x^2}$

Product of zeroes = $\frac{-1}{3}\times\frac{3}{2}=\frac{-1}{2}=\frac{-3}{6}=\frac{constant\ term}{co\ efficient\ of\ x^2}$

∴ Hence, relationship verified.

2. If α and β are the zeros of the quadratic polynomial f(x) = ax^2 + bx + c, then evaluate:

(i) $\alpha-\beta$

(ii) $\frac{1}{\alpha}-\frac{1}{\beta}$

(iii) $\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta$

(iv) $\alpha^2\beta+\alpha\beta^2$

(v) $\alpha^4+\beta^4$

(vi) $\frac{1}{a\alpha+b}+\frac{1}{a\beta+b}$

(vii) $\frac{\beta}{a\alpha+b}+\frac{\alpha}{a\beta+b}$

(viii) $a\left[\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right]+b\left[\frac{\alpha}{a}+\frac{\beta}{\alpha}\right]$

Sol:

f(x) = ax^2+bx+c

$\alpha+\beta=\frac{-b}{\boldsymbol{a}}$

$\alpha\beta=\frac{c}{a}$

$since\ \alpha+\beta\ are\ the\ roots\ (or) zeroes\ of\ the\ given\ polynomials$

(i) $\alpha-\beta$

The two zeroes of the polynomials are

$\frac{-b+\sqrt{b^2-4ac}}{2a}-\left(b\frac{-\sqrt{b^2-4ac}}{2a}\right)=-b+\frac{\sqrt{b^2-4ac}+b+\sqrt{b^2-4ac}}{2a}=\frac{2\sqrt{b^2-4ac}}{2a}=\frac{\sqrt{b^2-4ac}}{2a}$

(ii) $\frac{1}{\alpha}-\frac{1}{\beta}=\frac{\beta-\alpha}{\alpha\beta}=\frac{-(\alpha-\beta)}{\alpha\beta}$ ... (i)

From (i) we know that $\alpha-\beta=\frac{\sqrt{b^2-4ac}}{2a}$ $[from\ (i)]\alpha\beta=\frac{c}{a}$

Putting the values in the (a) = $-\left(\frac{\sqrt{b^2-4ac\times a}}{a\times c}\right)=\frac{-\sqrt{b^2-4ac}}{c}$

(iii) $\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta$

$\Rightarrow \left[\frac{\alpha+\beta}{\alpha\beta}\right] - 2\alpha\beta$

$\Rightarrow \frac{-b}{a} \times \frac{a}{c} - 2\frac{c}{a} = -2\frac{c}{a} - \frac{b}{c} = \frac{-ab-2c^2}{ac} - \left[\frac{b}{c} + \frac{2c}{a}\right]$

(iv) $\alpha^2\beta + \alpha\beta^2$

$\alpha\beta(\alpha+\beta)$

$= \frac{c}{a}\left(\frac{-b}{a}\right)$

$= \frac{-bc}{a^2}$

(v) $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2 + \beta^2$

$= ((\alpha+\beta)^2 - 2\alpha\beta)^2 - 2(\alpha\beta)^2$

$= \left[\left(-\frac{b}{a}\right)^2 - 2\frac{c}{a}\right]^2 - \left[2\left(\frac{c}{a}\right)^2\right]$

$= \left[\frac{b^2-2ac}{a^2}\right]^2 - \frac{2c^2}{a^2}$

$= \frac{\left(b^2 2ac\right)^2 - 2a^2c^2}{a^4}$

(vi) $\frac{1}{a\alpha+b} + \frac{1}{\alpha\beta+b}$

$\Rightarrow \frac{a\beta+b+a\alpha+b}{(3\alpha+b)(\alpha\beta+b)}$

$= \frac{a(\alpha+\beta)+2b}{a^2\alpha\beta+ab\,\alpha+ab\beta+b^2}$

$= \frac{a(\alpha+\beta)+b}{a^2\alpha\beta+\alpha\beta(\alpha^2\beta)+b^2}$

$= \frac{a\times\frac{a+2b}{a}}{a\times\frac{c}{a}+\frac{abc(-b)+b^2}{a}} = \frac{b}{ac-b^2+b^2} = \frac{b}{ac}$

(vii) $\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b}$

$= \frac{\beta(a\beta+b)+\alpha(a\alpha+b)}{(a\alpha+b)(\alpha\beta+b)}$

$= \frac{a\beta^2+b\beta+a\alpha^2+b\alpha}{a^2\alpha\beta+ab\alpha+ab\beta+b^2}$

$= \frac{a\alpha^2+a\beta^2+b\beta^2+b\alpha}{a\times\frac{c}{a}+ab(\alpha+\beta)+b^2}$

$= \frac{a\left[(\alpha^2+\beta^2)+b(\alpha+\beta)\right]}{ac+ab+x\left(\frac{-b}{a}\right)+b^2}$

$= \frac{a\left[(\alpha+\beta)^2-2\alpha\beta\right]+bx-\frac{b}{a}}{ac}$

$= \frac{a\left[\frac{b^2}{a}-\frac{2c}{a}\right]-\frac{b^2}{a}}{ac} = \frac{a\times\left[\frac{b^2-2c}{a}\right]-b^2}{ac} = \frac{-2}{a}$

(viii) $a\left[\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right] + b\left[\frac{\alpha}{a} + \frac{\beta}{\alpha}\right]$

$= a\left[\frac{\alpha^3+\beta^3}{\alpha\beta}\right] + b\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right)$

$= \frac{\alpha\left[(\alpha+\beta)^3-3\alpha\beta\,(\alpha+\beta)\right]}{\alpha\,\beta} + b(\alpha+\beta)^2 - 2\alpha\,\beta$

$= \frac{\alpha\left[\left(\frac{-b^3}{a^3}\right)+\frac{3b}{a}.\frac{c}{a}+b\left(\frac{b^2}{a^2}-\frac{2c}{a}\right)\right]}{\frac{c}{a}}$

$= \frac{a^2}{c}\left[\frac{-b^3}{a^3} + \frac{3bc}{a^2} + \frac{b^3}{a^2} - \frac{2bc}{a}\right]$

$= \frac{-a^2b^3}{ca^3} + \frac{3a^2bc}{ca^2} + \frac{b^3a^2}{a^2c} - \frac{2bca^2}{ac}$

$= \frac{-b^3}{ac} + 3b + \frac{b^3}{ac} - 2b$

= b

3. If α and β are the zeros of the quadratic polynomial f(x) = 6x^2 + x $-$ 2, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

Sol:

f(x) = $6x^2 - x - 2$

Since α and β are the zeroes of the given polynomial

∴ Sum of zeroes $[\alpha + \ \beta]$ = $\frac{-1}{6}$

Product of zeroes $(\alpha\beta)$ = $\frac{-1}{3}$

$= \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

$= \frac{\left(\frac{1}{6}\right)^2-2\times\left(\frac{-1}{3}\right)}{-\frac{1}{3}} = \frac{\frac{1}{6}-\frac{2}{3}}{\frac{-1}{3}} = \frac{\frac{1+24}{36}}{\frac{-1}{3}}$

$= \frac{\frac{2}{36}}{\frac{1}{3}} = \frac{-25}{12}$

4. If a and are the zeros of the quadratic polynomial f(x) = $x^2 - x - 4$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$

Sol:

Since $\alpha + \beta$ are the zeroes of the polynomial: $x^2 - x - 4$

Sum of the roots $(\alpha + \beta)$ = 1

Product of the roots $(\alpha\beta)$ = -4

$\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta = \frac{\alpha+\beta}{\alpha\beta} - \alpha\beta$

$= \frac{1}{-4} + 4 = \frac{-1}{4} + 4 = \frac{-1+16}{4} = \frac{15}{4}$

5. If α and β are the zeros of the quadratic polynomial p(x) = 4x^2 $-$ 5x $-$1, find the value of $\alpha^2\beta + \alpha\beta^2$.

Sol:

Since $\alpha \ and \ \beta$ are the roots of the polynomial: $4x^2 - 5x - 1$

$\therefore$ Sum of the roots $\alpha + \beta = \frac{5}{4}$

Product of the roots $\alpha\beta = \frac{-1}{4}$

Hence $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta) = \frac{5}{4}\left(\frac{-1}{4}\right) = \frac{-5}{16}$

6. If a and 3 are the zeros of the quadratic polynomial f(x) = x^2 + x − 2, find the value of $\frac{1}{\alpha} - \frac{1}{\beta}$.

Sol:

Since α and β are the roots of the polynomial x + x – 2

$\therefore$ Sum of roots α + β = 1

Product of roots $\alpha\beta$ $2 \Rightarrow -\frac{1}{\beta}$

= $\frac{\beta-\alpha}{\alpha\beta} \cdot \frac{(\alpha-\beta)}{\alpha\beta}$

= $\frac{\sqrt{(\alpha+\beta)^2-4\alpha\beta}}{\alpha\beta}$

= $\frac{\sqrt{1+8}}{+2} = \frac{3}{2}$

7. If α and β are the zeros of the quadratic polynomial f(x) = x^2 − 5x + 4, find the value of $\frac{1}{\alpha} - \frac{1}{\beta} - 2\alpha\beta$

Sol:

Since α *and* β are the roots of the quadratic polynomial

f(x) = $x^2 - 5x + 4$

Sum of roots = $\alpha + \beta = 5$

Product of roots = $\alpha\beta = 4$

$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{\beta+\alpha}{\alpha\beta} - 2\alpha\beta = \frac{5}{4} - 2 \times 4 = \frac{5}{4} - 8 = \frac{-27}{4}$

8. If α and β are the zeros of the quadratic polynomial f(t) = t^2 − 4t + 3, find the value of $\alpha^4\beta^3 + \alpha^3\beta^4$

Sol:

Since α *and* β are the zeroes of the polynomial f(t) = $t^2 - 4t + 3$

Since α + β = 4

Product of zeroes $\alpha\beta = 3$

Hence $\alpha^4\beta^3 + \alpha^3\beta^4 = \alpha^3\beta^3(\alpha + \beta) = [3]^3[4] = 108$

9. If α and β are the zeros of the quadratic polynomial p(y) = $5y^2$ − 7y + 1, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$

Sol:

Since α *and* β are the zeroes of the polynomials
p(y) = 54^2 = $5y^2$ – 7y + 1
Sum of the zeroes $\alpha\ \beta = \frac{1}{6}$
Product of zeroes = $\alpha\ \beta = \frac{1}{6}$
$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{7\times5}{5\times1} = 7$

10. If α and β are the zeros of the quadratic polynomial p(s) = $3s^2 - 6s + 4$, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta$

Sol:

Since α *and* β are the zeroes of the polynomials
Sum of the zeroes $\alpha + \beta = \frac{6}{3}$
Product of the zeroes $\alpha\beta = \frac{4}{3}$
$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta$
$\Rightarrow \frac{\alpha^2+\beta^2}{\alpha\beta} + 2\left[\frac{\alpha+\beta}{\alpha\beta}\right] + 3\alpha\beta$
$\Rightarrow \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta} + 2\left[\frac{\alpha+\beta}{\alpha\beta}\right] + 3\alpha\beta$
$= \frac{[2]^2-2\times\frac{4}{3}+2\left[\frac{2\times3}{4}\right]+3\left[\frac{4}{3}\right]}{\frac{4}{3}}$
$= \frac{4-\frac{8}{3}}{\frac{4}{3}} + 7 \Rightarrow \frac{4}{3}\times\frac{3}{4}(1+7) \Rightarrow 8$

11. If α and β are the zeros of the quadratic polynomial f(x) = $x^2 - px + q$, prove that $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2$

Sol:

Since α and β are the roots of the polynomials
f(x) = $x^2 - px + 2$
sum of zeroes = p = $\alpha + \beta$
Product of zeroes = q = $\alpha\beta$
LHS $= \frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2}$
$= \frac{\alpha^2+\beta^2}{\alpha\beta^2} = \frac{(\alpha^2+\beta^2)^2-2(\alpha\beta)^2}{(\alpha\beta)^2}$
$= \frac{[(\alpha+\beta)^2-2\alpha\beta]^2-2(\alpha\beta)^2}{(\alpha\beta)^2}$
$= \frac{[(p)^2-2q]^2-2q^2}{q}$

$= \frac{p^4+4q^2-2p^2.2q-2q^2}{q^2}$

$= \frac{p^4+2q^2-4p^2q}{q^2} = \frac{p^4}{q^2} + 2 - \frac{4p^2}{q}$

$= \frac{p^4}{q^2} - \frac{4p^2}{q^2} = \frac{p^4}{q^2} + 2 - \frac{4p^2}{q}$

$= \frac{p^4}{q^2} - \frac{4p^2}{q} + 2$

12. If the squared difference of the zeros of the quadratic polynomial f(x) = x^2 + px + 45 is equal to 144, find the value of p.

Sol:

Let the two zeroes of the polynomial be α and β

f(x) = $x^2 + px + 45$

sum of the zeroes = $-p$

Product of zeroes = 45

$\Rightarrow (\alpha - \beta)^2 - 4\alpha\beta = 144$

$\Rightarrow p^2 - 4 \times 45 = 144$

$\Rightarrow p^2 = 144 + 180$

$\Rightarrow p^2 = 324$

$p = \pm 1$

13. If the sum of the zeros of the quadratic polynomial f(t) = kt^2 + 2t + 3k is equal to their product, find the value of k.

Sol:

Let the two zeroes of the f(t) = $kt^2 + 2t + 3k\ be$ α and β

Sum of the zeroes $(\alpha + \beta)$

Product of the zeroes $\alpha\beta$

$\frac{-2}{\mathrm{k}} = \frac{3k}{\mathrm{k}}$

$-2k = 3k^2$

$2k + 3k^2 = 0$

k(3k + 2) = 0

k = 0

k = $\frac{-2}{3}$

14. If one zero of the quadratic polynomial f(x) = $4x^2 - 8kx - 9$ is negative of the other, find the value of k.

Sol:

Let the two zeroes of one polynomial

$f(x) = 4x^2 - 5k - 9\ be\ \alpha, -\alpha$

$\alpha \times \alpha = \frac{-9}{4}$
$t\alpha^2 = \frac{+9}{4}$
$\alpha = \frac{+3}{2}$
Sum of zeroes = $\frac{8k}{4} = 0$
Hence 8k = 0
Or k = 0

15. If α and β are the zeros of the quadratic polynomial f(x) = $x^2 - 1$, find a quadratic polynomial whose zeroes are $\frac{2\alpha}{\beta}$ *and* $\frac{2\beta}{\alpha}$

Sol:
f(x) = $x^2 - 1$
sum of zeroes $\alpha + \beta = 0$
Product of zeroes $\alpha\beta = -1$
Sum of zeroes = $\frac{2\alpha}{\beta} + \frac{2\beta}{\alpha} = \frac{2\alpha^2+2\beta^2}{\alpha\beta}$
= $\frac{2\left((\alpha+\beta)^2-2\alpha\beta\right)}{\alpha\beta}$
= $\frac{2[(0)^2-2\times -1]}{-1}$
= $\frac{2(2)1}{-1}$
= -4
Product of zeroes = $\frac{2\alpha\times 2\beta}{\alpha\beta} = \frac{4\alpha\beta}{\alpha\beta}$
Hence the quadratic equation is $x^2 - (sum\ of\ zeroes)x + product\ of\ zeroes$
= $k(x^2 + 4x + 14)$

16. If α and β are the zeros of the quadratic polynomial f(x) = $x^2 - 3x - 2$, find a quadratic polynomial whose zeroes are $\frac{1}{2\alpha+\beta} + \frac{1}{2\beta+\alpha}$.

Sol:
$f(x) = x^2 - 3x - 2$
Sum of zeroes $[\alpha + \beta] = 3$
Product of zeroes $[\alpha\beta] = -2$
Sum of zeroes = $\frac{1}{2\alpha+\beta} + \frac{1}{2\beta+\alpha}$
= $\frac{2\beta+\alpha+2\alpha+A}{(2\alpha+\beta)(2\beta+\alpha)}$
= $\frac{3\alpha+3\beta}{2(\alpha^2+\beta^2)+5\alpha\beta}$
= $\frac{3\times 3}{2[2\,(\alpha+\beta)^2-2\alpha\beta\,+5\times(-2)]}$

$= \frac{9}{2[9]-10} = \frac{9}{16}$

Product of zeroes = $\frac{1}{\alpha+\beta} \times \frac{1}{2\beta+\alpha} = \frac{1}{4\alpha\beta+\alpha\beta+2\alpha^2+2\beta^2}$

$= \frac{1}{5\times-2+2[(\alpha+\beta)^2-2\alpha\beta]}$

$= \frac{1}{-10+2[9+4]}$

$= \frac{1}{10+26}$

$= \frac{1}{16}$

Quadratic equation = $x^2 - [sum\ of\ zeroes]x + product\ of\ zeroes$

$= x^2 - \frac{9x}{16} + \frac{1}{16}$

$= k\left[x^2 - \frac{9x}{16} + \frac{1}{16}\right]$

17. If α and β are the zeros of a quadratic polynomial such that a + 13 = 24 and a $-\beta$ = 8, find a quadratic polynomial having α and β as its zeros.

Sol:

$\alpha + \beta = 24$

$\alpha\,\beta = 8$

…………

$2\,\alpha = 32$

$\alpha = 16$

$\beta = 8$

$\alpha\,\beta = 16 \times 8 = 128$

Quadratic equation

$\Rightarrow x^2 - (sum\ of\ zeroes) + product\ of\ zeroes$

$\Rightarrow k[x^2 - 24x + 128]$

18. If α and β are the zeros of the quadratic polynomial f(x) = $x^2 -$ p (x + 1) — c, show that (α + 1)(β +1) = 1$-$ c.

Sol:

$f(x) = x^2 - p(x+1)c = x - px = -p - c$

Sum of zeroes = $\alpha + \beta$ = p

Product of zeroes = $-$ p – c = $\alpha\,\beta$

(α + 1+ β+) = $\alpha\,\beta$+ α + β + 1 = $-$p –c + p + 1

= 1 – c = R.H.S

∴ Hence proved

19. If If α and β are the zeros of the quadratic polynomial f(x) = x^2 – 2x + 3, find a polynomial whose roots are (i) α + 2, β + 2 (ii) $\frac{\alpha-1}{\alpha+1}, \frac{\beta-1}{\beta+1}$

Sol:

$f(x) = x^2 - 2x + 3$

Sum of zeroes = 2 = $(\alpha + \beta)$

Product of zeroes = 3 = $(\alpha\ \beta)$

$(i)\ sum\ of\ zeroes = (\alpha + 2) + (\beta + 2) = \alpha + \beta + 4 = 2 + 4 = 6$

$Product\ of\ zeroes = (\alpha + 2)(\beta + 2)$

$= \alpha\beta + 2\alpha + 2\beta + 4 = 3 + 2(2) + 4 = 11$

$Quadratic\ equation = x^2 - 6x + 11 = k[x^2 - 6x + 11]$

(ii) sum of zeroes = $\frac{\alpha-1}{\alpha+1} + \frac{\beta-1}{\beta+1}$

= $\frac{(\alpha-1)(\beta+1)+(\beta-1)(\alpha+1)}{(\alpha+1)(\beta+1)}$

= $\frac{\alpha\beta+\alpha-\beta-1+\alpha\beta+\beta+\beta-\alpha-1}{3+2+1}$

= $\frac{3-1+3-1}{3+2+1} = 4 = \frac{2}{3}$

Product of zeroes = $\frac{\alpha-1}{\beta\alpha+1} \times \frac{\beta-1}{\alpha+1} - \frac{\alpha\ 1-\alpha-\alpha\beta+1}{\alpha\beta+\alpha+\beta+1}$

= $\frac{3-(\alpha+\beta)+1}{3+2+1} = \frac{2}{6} = \frac{1}{3}$

Quadratic equation on $x^2 - \frac{2}{3} \times \frac{+1}{3} = 1\left[\frac{x^2-2x}{3} + \frac{1}{3}\right]$

20. If α and β are the zeroes of the polynomial f(x) = $x^2 + px + q$, form a polynomial whose zeroes are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$.

Sol:

$f(x) = x^2 + p + q$

Sum of zeroes = p = $\alpha + \beta$

Product of zeroes = q = $\alpha\ \beta$

Sum of the new polynomial = $(\alpha + \beta)^2 + (\alpha - \beta)^2$

$= (-p)^2 + \alpha^2 + \beta^2 - 2\alpha\beta$

$= p^2 + (\alpha + \beta)^2 - 2\alpha\beta - 2\alpha\beta$

$= p^2 + p^2 - 4q$

$= 2p^2 - 4q$

Product of zeroes = $(\alpha + \beta)^2 \times (\alpha - \beta)^2 = [-p]^2 \times (p^2 - 4q) = (p^2 - 4q)p^2$

Quadratic equation = $x^2 - [2p^2 - 4q] + p^2[-4q + p]$

$$f(x) = k\{x^2 - 2(p^2 - 28)x + p^2(q^2 - 4q)\}$$

Exercise 2.2

1. Verify that the numbers given alongside of the cubic polynomials below are their zeros. Also, verify the relationship between the zeros and coefficients in each case:

$(i)\ f(x) = 2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, -2$

(ii) $g(x) = x^3 - 4x^2 + 5x - 2; 2, 1, 1$

Sol:

$(i)\ f(x) = 2x^3 + x^2 - 5x + 2$

$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$

$= \frac{2}{8} + \frac{1}{4} - \frac{5}{2} + 2 = \frac{-4}{2} + 2 = 0$

$f(1) = 2(1)^3 + (1)^2 - 5(1) + 2 = 2 + 1 - 5 + 2 = 0$

$f(-2) = q(-2)^3 + (-2)^2 - 5(-2) + 2$

$= -16 + 4 + 10 + 2$

$= -16 + 16 = 0$

$= \propto + \beta + \gamma = \frac{-b}{a}$

$\frac{1}{2} + 1 - 2 = \frac{-1}{2}$

$\frac{1}{2} - 1 = \frac{-1}{2}$

$\frac{1}{2} = \frac{-1}{2}$

$\alpha\beta . \beta\gamma + r\alpha = \frac{c}{a}$

$\frac{1}{2} \times 1 + 1 \times -2 + -2 \times \frac{1}{2} = \frac{-5}{2}$

$\frac{1}{2} - 2 - 1 = \frac{-5}{2}$

$\frac{-5}{2} = \frac{-5}{2}$

$(ii)\ g(x) = x^3 - 4x^2 + 5 \times -2$

$g(2) = (2)^3 - 4(2)^2 + 5(2) - 2 = 8 - 16 + 10 - 2 = 18 - 18 = 0$

$g(1) = [1]^3 - 4[1]^2 + 5[1] - 2 = 1 - 4 + 5 - 2 = 6 - 6 = 0$

$\alpha + \beta + \gamma = \frac{-b}{a}(2) + 1 + 1 = -(-4) = 4 = 4$

$\alpha\,\beta + \beta\,\gamma + \gamma\alpha = \frac{c}{a}$

$2 \times 1 + 1\,\alpha\,1 + 1 \times 2 = 5$

$2 + 1 + 2 = 5$

$5 = 5$

$\alpha\beta\gamma = -(-2)$

$2 \times 1 \times 1 = 2$

$2 = 2$

2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and product of its zeros as 3, -1 and -3 respectively.

Sol:

Any cubic polynomial is of the form $ax^3 + bx^2 + cx + d = x^3 - sum\ of\ zeroes\ (x^2)$[product of zeroes] + sum of the products of its zeroes × - product of zeroes

= $x^3 - 2x^2 + (3 - x) + 3$

= k $[x^3 - 3x^2 - x - 3]$

k is any non-zero real numbers

3. If the zeros of the polynomial f(x) = $2x^3 - 15x^2 + 37x - 30$ are in A.P., find them.

Sol:

Let α = a – d , β = a and γ = a + d be the zeroes of polynomial.

$f(x) = 2x^3 - 15x^2 + 37x - 30$

$\alpha + \beta + \gamma = -\left(\frac{-15}{2}\right) = \frac{15}{2}$

$\alpha\beta\gamma = -\left(\frac{-30}{2}\right) = 15$

a – d + a + a + d = $\frac{15}{2}$ and $a(a-d)(a+a) = 15$

3a = $\frac{15}{2}$, a = $\frac{5}{2}$

$a(a^2 - d^2) = 15$

$a^2 - a^2 = \frac{15\times2}{5} \Rightarrow \left(\frac{5}{2}\right)^2 - d^2 = 6 \Rightarrow\Rightarrow \frac{25-6}{4} = d^2$

$d^2 = \frac{1}{4} \Rightarrow d = \frac{1}{2}$

$\therefore \alpha = \frac{5}{2} - \frac{1}{2} = \frac{4}{2} = 2$

$\beta = \frac{5}{2} = \frac{5}{2}$

$\gamma = \frac{5}{2} + \frac{1}{2} = 3$

4. Find the condition that the zeros of the polynomial f(x) = $x^3 + 3px^2 + 3qx + r$ may be in A.P.

Sol:

$f(x) = x^3 + 3px^2 + 3qx + q$

$Let\ a - d, a, a + d$ be the zeroes of the polynomial

$The\ sum\ of\ zeroes = \frac{-b}{a}$

$a + a - d + a + d = \frac{b}{a}$

$3a = -3p$

a = $-p$

Since a is the zero of the polynomial f(x) therefore f(a) = 0 ⇒ $[a]^2 + 3pa^2 + 3qa + r = 0$

$\therefore f(a) = 0 \Rightarrow [a]^2 + 3pa^2 + 3qa + r = 0$
$\Rightarrow p^3 + 3p(-p)^2 + 3q(-p) + r = 0$
$\Rightarrow -p^3 + 3p^2 - pq + r = 0$
$\Rightarrow 2p^3 - pq + r = 0$

5. If the zeroes of the polynomial $f(x) = ax^3 + 3bx^2 + 3cx + d$ are in A.P., prove that $2b^3 - 3abc + a^2d = 0$

Sol:
Let a – d, a, a + d be the zeroes of the polynomial f(x)
The sum of zeroes ⇒ a – d + a + a + d = $\frac{-3b}{a}$
⇒ +3a $= -\frac{3b}{a} \Rightarrow a = \frac{-3b}{a\times 3} a = \frac{-b}{a}$
f(a) = 0 ⇒ a$(a)^2$ + 3b$(a)^2$ + 3 c(a) + d = 0
= a $\left(\frac{-b}{a}\right)^3 + \frac{3b^2}{a^2} - \frac{3bc}{a} + d = 0$
$\Rightarrow \frac{2b^3}{a^2} - \frac{3bc}{a} + d = 0$
$\Rightarrow \frac{2b^3 - 3abc + a^2d}{a^2} = 0$
$\Rightarrow 2b^3 - 3abc + a^2d = 0$

6. If the zeroes of the polynomial $f(x) = x^3 - 12x^2 + 39x + k$ are in A.P., find the value of k.

Sol:
$f(x) = x^3 - 12x^2 + 39x - k$
Let $a - d, a, a + d$ be the zeroes of the polynomial f(x)
The sum of the zeroes = 12
3a = 12
a = 4
$f(a), -a(x)^3 - l^2(4)^2 + 39(4) + k = 0$
64 – 192 + 156 + k = 0
= $-28 = k$
k = -28

Exercise 2.3

1. Apply division algorithm to find the quotient q(x) and remainder r(x) on dividing f(x) by g(x) in each of the following:
(i) $f(x) = x^3 - 6x^2 + 11x - 6$, g(x) = $x^2 + x + 1$
(ii) $f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 105(x) = 2x^2 + 7x + 1$
(iii) $f(x) = 4x^3 + 8x^2 + 8x + 7: 9(x) = 2x^2 - x + 1$
(iv) $f(x) = 15x^3 - 20x^2 + 13x - 12; g(x) = x^2 - 2x + 2$

Sol:

(i) $f(x) = x^3 - 6x^2 + 11x - 6$
g(x) = $x^2 + x + 1$

$$\begin{array}{r|l}
 & x - 7 \\ \hline
x^2 + x + 1 & x^3 - 6x^2 + 11x - 6 \\
 & x^3 + x^2 + x \\ \cline{2-2}
 & \quad -7x^2 - 7x - 7 \\
 & \quad -7x^2 - 7x - 7 \\ \cline{2-2}
 & \qquad\qquad 17x - 1
\end{array}$$

(ii) $f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 105(x) = 2x^2 + 7x + 1$

$$\begin{array}{r|l}
 & 5x^2 - 9x - 2 \\ \hline
2x^2 + 7x + 1 & 10x^4 + 17x^3 - 62x^2 + 30x - 3 \\
 & 10x^4 + 35x^3 + 5x^2 \\ \cline{2-2}
 & \qquad -18x^3 - 67x^2 + 30x \\
 & \qquad -18x^3 \pm 63x^2 + 9x \\ \cline{2-2}
 & \qquad\qquad -4x^2 + 39x - 3 \\
 & \qquad\qquad \pm 4x^2 \pm 14x \pm 2 \\ \cline{2-2}
 & \qquad\qquad\qquad 53x - 1
\end{array}$$

(iii) $f(x) = 4x^3 + 8x^2 + 8x + 7: 9(x) = 2x^2 - x + 1$

$$\begin{array}{r|l}
 & \qquad 2x - 5 \\ \hline
2x^2 - 2 + 1 & 4x^3 + 8x^2 + 8^2 + 7 \\
 & 4x^3 \mp 2x^2 \pm 2x \\ \cline{2-2}
 & \qquad 10x^2 + 6x + 7 \\
 & \qquad 10x^2 \pm 5x \pm 5 \\ \cline{2-2}
 & \qquad\qquad 11x - 2
\end{array}$$

(iv) $f(x) = 15x^3 - 20x^2 + 13x - 12; g(x) = x^2 - 2x + 2$

$$\begin{array}{r|l}
 & \qquad 15x + 10 \\ \hline
x^2 - 2x + 2 & 15x^3 - 20x^2 + 13x - 12 \\
 & 15x^3 \mp 30x^2 \pm 30x \\ \cline{2-2}
 & \qquad 10x^2 - 17x - 12 \\
 & \qquad 10x^2 \pm 20x + 20 \\ \cline{2-2}
 & \qquad\qquad 3x - 32
\end{array}$$

2. Check whether the first polynomial is a factor of the second polynomial by applying the division algorithm:
(i) $g(t) = t^2 - 3; f(t) = 2t^4 + 3t^3 - 2t^2 - 9t$
(ii) $g(x) = x^2 - 3x + 1, f(x) = x^5 - 4x^3 + x^2 + 3x + 1$
(iii) $g(x) = 2x^2 - x + 3, f(x) = 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$

Sol:

(i) $g(t) = t^2 - 3; f(t) = 2t^4 + 3t^3 - 2t^2 - 9t$

$$
\begin{array}{r|l}
 & 2t^2 + 3t + 4 \\ \hline
t^2 - 3 & 2t^4 + 3t^3 - 2t^2 - 9t \\
 & 2t^2 - 6t^2 \\ \hline
 & 3t^3 + 4t - 9t \\
 & 3t^3 + 4t - 9t \\ \hline
 & 4t^2 - 12 \\
 & 4t^2 \mp 12
\end{array}
$$

(ii) $g(x) = x^2 - 3x + 1, f(x) = x^5 - 4x^3 + x^2 + 3x + 1$

$$
\begin{array}{r|l}
 & x^2 - 1 \\ \hline
x^3 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\
 & x^5 - 3x^3 + x^2 \\ \hline
 & -x^3 + 3x + 1 \\
 & -x^3 + 3x - 1 \\ \hline
 & 2
\end{array}
$$

(iii) $g(x) = 2x^2 - x + 3, f(x) = 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$

$$
\begin{array}{r|l}
 & 3x^3 + x^2 - 2x - 5 \\ \hline
2x^2 - x + 3 & 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15 \\
 & 6x^5 - 3x^4 + 9x^3 \\ \hline
 & 2x^4 - 5x^3 - 5x^2 \\
 & 2x^4 \mp x^3 \pm 3x^2 \\ \hline
 & -4x^3 - 8x^2 - x \\
 & \mp 4x^3 \pm 2x^2 - 6x \\ \hline
 & -10x^2 - 5x - 15 \\
 & \mp 10x \pm 15x \mp 15 \\ \hline
 & 0
\end{array}
$$

3. Obtain all zeros of the polynomial $f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$, if two of its zeros are -2 and -1.

Sol:

$f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$

If the two zeroes of the polynomial are -2 and -1, then its factors are (x + 2) and (x + 1)

$(x + 2)(x + 1) = x^2 + x + 2x = x^2 + 3x + 2$

$$
\begin{array}{r|l}
 & 2x^2 - 5x - 3 \\
\hline
x^2 + 3x + 2 & 2x^4 + x^3 - 14x^2 - 19x - 6 \\
 & 2x^4 + 6x^3 + 4x^2 \\
\hline
 & \quad -5x^3 - 18x^2 - 19x \\
 & \quad -5x^3 \mp 15x^2 \mp 10x \\
\hline
 & \qquad -3x^2 - 9x - 6 \\
 & \qquad -3x^2 - 9x - 6
\end{array}
$$

$\therefore 2x^4 + x^3 - 14x^2 - 19x - 6$

$= (2x^2 - 5x - 3)[x^2 + 3x + 2] = [2x + 1][x - 3][x + 2][x + 1]$

$\therefore$ zero all x = $\frac{-1}{2}, 3, -2, -1$

4. Obtain all zeros of f(x) = $x^3 + 13x^2 + 32x + 20$, if one of its zeros is -2.

Sol:

$f(x) = x^3 + 13x^2 + 32x + 20$

$$
\begin{array}{r|l}
 & x^2 + 11x + 10 \\
\hline
x + 2 & x^3 + 13x^2 + 32x + 20 \\
 & x^3 \pm 2x^2 \\
\hline
 & \quad 11x^2 + 32x + 20 \\
 & \quad 11x^2 \pm 22x \\
\hline
 & \qquad 10x + 20 \\
 & \qquad 10x + 20 \\
\hline
 & \qquad \quad 0
\end{array}
$$

$(x^2 + 11x + 10) = x^2 + 10x + x + 20(x + 10) + 1(x + 10) = (x + 1)(x + 10)$

$\therefore$ The zeroes of the polynomial are -1, -10, -2.

5. Obtain all zeros of the polynomial $f(x) = x^4 - 3x^2 = x^2 + 9x - 6$ if two of its zeros are $-\sqrt{3}, and\ \sqrt{3}$.

Sol:

$f(x) = (x^2 - 3x + 2) = (x + \sqrt{3}) \& (x - \sqrt{3}) = x^2 - 3$

$$
\begin{array}{r|l}
 & x^2 - 3x + 2 \\
\hline
x^2 - 3 & x^4 - 3x^2 = x^2 + 9x - 6 \\
 & x^4 - 3x^2 \\
\hline
 & \quad -3x^2 + 2x^2 + 9x \\
 & \quad -3x^2 \qquad \pm 9x \\
\hline
 & \qquad 2x^2 - 6 \\
 & \qquad 2x^2 - 6
\end{array}
$$

$(x^2 - 3)(x^2 - 3x + 2) = (x + \sqrt{3})(x - \sqrt{3})(x^2 - 2x - x + 2)$

$= (x + \sqrt{3})(x - \sqrt{3})\ (x - 2)(x - 2)$

Zeroes are $-\sqrt{3}, \sqrt{3}, 1, 2$

6. Find all zeros of the polynomial $f(x) = 2x^4 - 2x^3 - 7x^2 + 3x + 6$, if its two zeroes are $-\sqrt{\frac{3}{2}}\ and\ \sqrt{\frac{3}{2}}$

Sol:

If the zeroes of the polynomial are $-\sqrt{\frac{3}{2}}\ and\ \sqrt{\frac{3}{2}}$

Its factors are $\left(x + \frac{\sqrt{3}}{2}\right)\left(x - \sqrt{\frac{3}{2}}\right) = \frac{x^2-3}{2}$

x = $-1, 2, \sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}}$

= $[2x^2 - 2x - 4]\left(x^2 - \frac{3}{2}\right)$

$= (2x^2 - 4x + 2x - 4)\left(x + \sqrt{\frac{3}{2}}\right)$

$= \left[2[x(x + 2) + 2(x - 2)]\right]$

$= \left[x + \frac{\sqrt{3}}{2}\right]\left[x - \sqrt{\frac{3}{2}}\right]$

$= (x + 2)(x - 2)\left[x + \sqrt{\frac{3}{2}}\right]\left[x - \sqrt{\frac{3}{2}}\right]$

$x = -1, 2, \sqrt{\frac{3}{2}} - \sqrt{\frac{3}{2}}$

7. What must be added to the polynomial $f(x) = x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$?

Sol:

	$x^2 - 1$
$x^2 + 2x - 3$	$x^4 + 2x^3 - 2x^2 + x - 1$
	$x^4 + 2x^3 - 3x^2$
	$x^2 + x - 1$
	$x^2 + 2x - 3$
	$-x + 2$

we must add x -2 in order to get the resulting polynomial exactly divisible by $x^2 + 2x - 3$

8. What must be subtracted from the polynomial $x^4 + 2x^3 - 13x^2 - 12x + 21$, so that the resulting polynomial is exactly divisible by $x^2 - 4x + 3$?

Sol:

$$
\begin{array}{r|l}
 & x^2 + 6x + 8 \\
\hline
x^2 - 4x + 3 & x^4 + 2x^3 - 13x^2 - 12x + 21 \\
 & x^4 - 4x^3 + 3x^2 \\
\hline
 & \quad 6x^3 - 16x^2 - 12x \\
 & \quad 6x^3 - 24x^2 - 18x \\
\hline
 & \qquad 8x^2 - 30x + 21 \\
 & \qquad 8x^2 - 32x + 21 \\
\hline
 & \qquad\qquad 2x - 2
\end{array}
$$

We must subtract $[2x - 2] + 10m$ the given polynomial so as to get the resulting polynomial exactly divisible by $x^2 - x + 3$

9. Find all the zeroes of the polynomial $x^4 + x^3 - 34x^2 - 4x + 120$, if two of its zeroes are 2 and -2.

Sol:

$\Rightarrow f(x) = x^4 + x^3 - 34x^2 - 4x + 120$

$\Rightarrow$ x = -2 is a solution

x = -2 is a factor

x = -2 is a solution

x = +2 is a factor

here,

$(x - 2)(x + 2) is\ a\ factor\ of\ f(x)$

$x^2 - 4\ is\ a\ factor$

$$
\begin{array}{r|l}
 & x^2 + x - 30 \\
\hline
x^2 - 4 & x^4 + x^3 - 34x^2 - 4x + 120 \\
 & -x^4 \qquad - 4x^2 \\
\hline
 & \quad x^3 - 30x^2 - 4x + 120 \\
 & \quad x^3 \qquad\quad - 4x \\
\hline
 & \qquad -30x^2 \quad + 120 \\
 & \qquad -30x^2 \quad + 120 \\
\hline
 & \qquad\qquad 0
\end{array}
$$

Hence, $x^4 + x^3 - 34x^2 - 4x + 120 = (x^2 - 4)(x^2 + x - 30)$

$x^4 + x^3 - 34x^2 - 4x + 120 = (x^2 - 4)(x^2 + 6x - 5x - 30)$

$x^4 + x^3 - 34x^2 - 4x + 120 = (x^2 - 4)\left[\left(x(x + 6) - 5(x + 6)\right)\right]$

$x^4 + x^3 - 34x^2 - 4x + 120 = (x^2 - 4)(x + 6)(x - 5)$

Other zeroes are

x + 6 = 0 $\Rightarrow$ x – 5 = 0

x = $-$ 6 x = 5

Set of zeroes for f(x) $[2, -2, -6, 5]$

10. Find all zeros of the polynomial $2x^4 + 7x^3 - 19x^2 - 14x + 30$, if two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$.

Sol:

$f(x) = 2x^4 + 7x^3 - 19x^2 - 14x + 30$

$x = \sqrt{2}\ is\ a\ solution$

$x - \sqrt{2}\ is\ a\ solution$

$x - \sqrt{2}\ is\ a\ solution$

$x + \sqrt{2}\ is\ a\ factor$

Here, $(x + \sqrt{2})(x - \sqrt{2}) is\ a\ factor\ of\ f(x)$

$x^2 - 2\ is\ a\ factor\ of\ f(x)$

$$
\begin{array}{r|l}
 & 2x^2 + 7x - 15 \\
\hline
x^2 - 2 & 2x^4 + 7x^3 - 19^2 - 14x + 30 \\
 & 2x^4 \qquad\quad - 4x^2 \\
\hline
 & \qquad\qquad 7x^3 - 15x^2 - 14x \\
 & \qquad\qquad 7x^3 - \qquad -14x \\
\hline
 & \qquad\qquad\qquad -15x^2 \qquad + 30 \\
 & \qquad\qquad\qquad -15x^2 \qquad + 30 \\
\hline
 & \qquad\qquad\qquad\qquad\qquad\quad 0
\end{array}
$$

Hence, $2x^4 + 7x^3 - 19x^2 - 14x + 30 = (x^2 - 2)(2x^2 + 7x - 15)$

$= (x^2 - 2)\ (2x^2 + 10x - 3x - 15)$

$= (x^2 - 2)\big(2x(x + 5) - 3(x + 5)\big)$

$= (x^2 - 2)(x + 5)(x - 3)$

Other zeroes are:

$x + 5 = 0$	$2x - 3 = 0$
$x = -5$	$2x = 3$
	$x = \frac{3}{2}$

Hence the set of zeroes for $f(x)$ $\left\{-5, \frac{3}{2}, \sqrt{2}, -\sqrt{2}\right\}$

11. Find all the zeros of the polynomial $2x^3 + x^2 - 6x - 3$, if two of its zeros are $-\sqrt{3}$ and $\sqrt{3}$.

Sol:

$f(x) = 2x^3 + x^2 - 6x - 3$

$x = -\sqrt{3}$ is a solution

$x + \sqrt{3}\ is\ a\ factor$

$x = \sqrt{3}\ is\ a\ solution$

$x - \sqrt{3}\ is\ a\ factor$

Here, $(x + \sqrt{3})(x - \sqrt{3}) is\ a\ factor\ of\ f(x)$

$x^2 - 3\ is\ a\ factor\ of\ f(x)$

$$\begin{array}{r|l} & 2x+1 \\ \hline x^2-3 & 2x^3+x^2-6x-3 \\ & 2x^3 \quad\quad -6x \\ \hline & \quad\quad x^2 \quad\quad -3 \\ & \quad\quad x^2 \quad\quad -3 \\ \hline & \quad\quad\quad\quad 0 \end{array}$$

Hence, $2x^3 + x^2 - 6x - 3 = (x^2 - 3)(2x + 1)$

Other zeroes of f(x) is $2 \times +1 = 0$

$x = -\frac{1}{2}$

$Set\ of\ zeroes\ \left\{\sqrt{3}, -\sqrt{3}, \frac{-1}{2}\right\}$

12. Find all the zeros of the polynomial $x^3 + 3x^2 - 2x - 6$, if two of its zeros are $-\sqrt{2}$ and $\sqrt{2}$.

Sol:

Since $-\sqrt{2}\ and\ \sqrt{2}$ are zeroes of polynomial $f(x) = x^3 + 3x^2 - 2x - 6$

$\left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right) = x^2 - 2\ is\ a\ factor\ of\ f(x)$

Now we divide $f(x) = x^3 + 3x^2 - 2x - 6$ by

$g(x) = x^2 - 2\ to\ b\ find\ the\ other\ zeroes\ of\ f(x)$

$$\begin{array}{r|l} & x+3 \\ \hline x^2-2 & x^3+3x^2-2x-6 \\ & x^3 \quad\quad -2x \\ \hline & \quad\quad 3x^2 \quad\quad -6 \\ & \quad\quad 3x^2 \quad\quad -6 \\ \hline & \quad\quad\quad\quad 0 \end{array}$$

By division algorithm, we have

$\Rightarrow x^3 + 3x^2 - 2 - 6 = (x^2 - 2)(x + 3)$

$\Rightarrow x^3 + 3x^2 - 2x - 6 = \left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right)(x + 3)$

Here the zeroes of the given polynomials are $-\sqrt{2}, \sqrt{2}\ and - 3$