

CHAPTER – 23

HEAT AND TEMPERATURE

EXERCISES

1. Ice point = 20° (L_0) $L_1 = 32^\circ$
 Steam point = 80° (L_{100})

$$T = \frac{L_1 - L_0}{L_{100} - L_0} \times 100 = \frac{32 - 20}{80 - 20} \times 100 = 20^\circ\text{C}$$
2. $P_{tr} = 1.500 \times 10^4$ Pa
 $P = 2.050 \times 10^4$ Pa
 We know, For constant volume gas Thermometer

$$T = \frac{P}{P_{tr}} \times 273.16 \text{ K} = \frac{2.050 \times 10^4}{1.500 \times 10^4} \times 273.16 = 373.31$$
3. Pressure Measured at M.P = $2.2 \times$ Pressure at Triple Point

$$T = \frac{P}{P_{tr}} \times 273.16 = \frac{2.2 \times P_{tr}}{P_{tr}} \times 273.16 = 600.952 \text{ K} \approx 601 \text{ K}$$
4. $P_{tr} = 40 \times 10^3$ Pa, $P = ?$
 $T = 100^\circ\text{C} = 373 \text{ K}, \quad T = \frac{P}{P_{tr}} \times 273.16 \text{ K}$

$$\Rightarrow P = \frac{T \times P_{tr}}{273.16} = \frac{373 \times 40 \times 10^3}{273.16} = 54620 \text{ Pa} = 5.42 \times 10^3 \text{ pa} \approx 55 \text{ K Pa}$$
5. $P_1 = 70 \text{ K Pa}, \quad P_2 = ?$
 $T_1 = 273 \text{ K}, \quad T_2 = 373 \text{ K}$

$$T = \frac{P_1}{P_{tr}} \times 273.16 \quad \Rightarrow 273 = \frac{70 \times 10^3}{P_{tr}} \times 273.16 \quad \Rightarrow P_{tr} = \frac{70 \times 273.16 \times 10^3}{273}$$

$$T_2 = \frac{P_2}{P_{tr}} \times 273.16 \quad \Rightarrow 373 = \frac{P_2 \times 273}{70 \times 273.16 \times 10^3} \quad \Rightarrow P_2 = \frac{373 \times 70 \times 10^3}{273} = 95.6 \text{ K Pa}$$
6. $P_{\text{ice point}} = P_0 = 80 \text{ cm of Hg}$
 $P_{\text{steam point}} = P_{100^\circ} = 90 \text{ cm of Hg}$
 $P_0 = 100 \text{ cm}$

$$t = \frac{P - P_0}{P_{100} - P_0} \times 100^\circ = \frac{80 - 100}{90 - 100} \times 100 = 200^\circ\text{C}$$
7. $T' = \frac{V}{V - V'} T_0 \quad T_0 = 273,$
 $V = 1800 \text{ CC}, \quad V' = 200 \text{ CC}$

$$T' = \frac{1800}{1600} \times 273 = 307.125 \approx 307$$
8. $R_t = 86\Omega; \quad R_0 = 80\Omega; \quad R_{100^\circ} = 90\Omega$

$$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100 = \frac{86 - 80}{90 - 80} \times 100 = 60^\circ\text{C}$$
9. R at ice point (R_0) = 20Ω
 R at steam point (R_{100}) = 27.5Ω
 R at Zinc point (R_{420}) = 50Ω
 $R_\theta = R_0 (1 + \alpha\theta + \beta\theta^2)$
 $\Rightarrow R_{100} = R_0 + R_0 \alpha\theta + R_0 \beta\theta^2$
 $\Rightarrow \frac{R_{100} - R_0}{R_0} = \alpha\theta + \beta\theta^2$

$$\Rightarrow \frac{27.5 - 20}{20} = \alpha \times 100 + \beta \times 10000$$

$$\Rightarrow \frac{7.5}{20} = 100 \alpha + 10000 \beta$$

$$R_{420} = R_0 (1 + \alpha\theta + \beta\theta^2) \Rightarrow \frac{50 - R_0}{R_0} = \alpha\theta + \beta\theta^2$$

$$\Rightarrow \frac{50 - 20}{20} = 420 \times \alpha + 176400 \times \beta \quad \Rightarrow \frac{3}{2} = 420 \alpha + 176400 \beta$$

$$\Rightarrow \frac{7.5}{20} = 100 \alpha + 10000 \beta \quad \Rightarrow \frac{3}{2} = 420 \alpha + 176400 \beta$$

10. $L_1 = ?$, $L_0 = 10 \text{ m}$, $\alpha = 1 \times 10^{-5}/^\circ\text{C}$, $t = 35$
 $L_1 = L_0 (1 + \alpha t) = 10(1 + 10^{-5} \times 35) = 10 + 35 \times 10^{-4} = 10.0035 \text{ m}$

11. $t_1 = 20^\circ\text{C}$, $t_2 = 10^\circ\text{C}$, $L_1 = 1 \text{ cm} = 0.01 \text{ m}$, $L_2 = ?$
 $\alpha_{\text{steel}} = 1.1 \times 10^{-5}/^\circ\text{C}$
 $L_2 = L_1 (1 + \alpha_{\text{steel}} \Delta T) = 0.01(1 + 101 \times 10^{-5} \times 10) = 0.01 + 0.01 \times 1.1 \times 10^{-4}$
 $= 10^{-4} \times 10^{-6} + 1.1 \times 10^{-6} = 10^{-6} (10000 + 1.1) = 10001.1$
 $= 1.00011 \times 10^{-2} \text{ m} = 1.00011 \text{ cm}$

12. $L_0 = 12 \text{ cm}$, $\alpha = 11 \times 10^{-5}/^\circ\text{C}$
 $t_w = 18^\circ\text{C}$ $t_s = 48^\circ\text{C}$
 $L_w = L_0 (1 + \alpha t_w) = 12 (1 + 11 \times 10^{-5} \times 18) = 12.002376 \text{ m}$
 $L_s = L_0 (1 + \alpha t_s) = 12 (1 + 11 \times 10^{-5} \times 48) = 12.006336 \text{ m}$
 $\Delta L = 12.006336 - 12.002376 = 0.00396 \text{ m} \approx 0.4 \text{ cm}$

13. $d_1 = 2 \text{ cm} = 2 \times 10^{-2}$
 $t_1 = 0^\circ\text{C}$, $t_2 = 100^\circ\text{C}$
 $\alpha_{\text{Al}} = 2.3 \times 10^{-5}/^\circ\text{C}$
 $d_2 = d_1 (1 + \alpha \Delta t) = 2 \times 10^{-2} (1 + 2.3 \times 10^{-5} \times 10^2)$
 $= 0.02 + 0.000046 = 0.020046 \text{ m} = 2.0046 \text{ cm}$

14. $L_{\text{st}} = L_{\text{Al}}$ at 20°C $\alpha_{\text{Al}} = 2.3 \times 10^{-5}/^\circ\text{C}$ $\alpha_{\text{st}} = 1.1 \times 10^{-5}/^\circ\text{C}$
 So, $L_{\text{st}} (1 - \alpha_{\text{st}} \times 20) = L_{\text{Al}} (1 - \alpha_{\text{Al}} \times 20)$
 (a) $\Rightarrow \frac{L_{\text{st}}}{L_{\text{Al}}} = \frac{(1 - \alpha_{\text{Al}} \times 20)}{(1 - \alpha_{\text{st}} \times 20)} = \frac{1 - 2.3 \times 10^{-5} \times 20}{1 - 1.1 \times 10^{-5} \times 20} = \frac{0.99954}{0.99978} = 0.999$
 (b) $\Rightarrow \frac{L_{\text{40st}}}{L_{\text{40Al}}} = \frac{(1 - \alpha_{\text{Al}} \times 40)}{(1 - \alpha_{\text{st}} \times 40)} = \frac{1 - 2.3 \times 10^{-5} \times 40}{1 - 1.1 \times 10^{-5} \times 40} = \frac{0.99954}{0.99978} = 0.999$
 $= \frac{L_{\text{Al}}}{L_{\text{st}}} \times \frac{1 + 2.3 \times 10^{-5} \times 10}{273} = \frac{0.99977 \times 1.00092}{1.00044} = 1.0002496 \approx 1.00025$
 $\frac{L_{\text{100Al}}}{L_{\text{100St}}} = \frac{(1 + \alpha_{\text{Al}} \times 100)}{(1 + \alpha_{\text{st}} \times 100)} = \frac{0.99977 \times 1.00092}{1.00011} = 1.00096$

15. (a) Length at $16^\circ\text{C} = L$
 $L = ?$ $T_1 = 16^\circ\text{C}$, $T_2 = 46^\circ\text{C}$
 $\alpha = 1.1 \times 10^{-5}/^\circ\text{C}$
 $\Delta L = L \alpha \Delta \theta = L \times 1.1 \times 10^{-5} \times 30$
 $\% \text{ of error} = \left(\frac{\Delta L}{L} \times 100 \right) \% = \left(\frac{L \alpha \Delta \theta}{L} \times 100 \right) \% = 1.1 \times 10^{-5} \times 30 \times 100 \% = 0.033 \%$

(b) $T_2 = 6^\circ\text{C}$
 $\% \text{ of error} = \left(\frac{\Delta L}{L} \times 100 \right) \% = \left(\frac{L \alpha \Delta \theta}{L} \times 100 \right) \% = -1.1 \times 10^{-5} \times 10 \times 100 = -0.011 \%$

16. $T_1 = 20^\circ\text{C}$, $\Delta L = 0.055\text{mm} = 0.55 \times 10^{-3}\text{m}$
 $t_2 = ?$ $\alpha_{\text{st}} = 11 \times 10^{-6}/^\circ\text{C}$

We know,

$$\Delta L = L_0 \alpha \Delta T$$

In our case,

$$0.055 \times 10^{-3} = 1 \times 1.1 \times 10^{-6} \times (T_1 + T_2)$$

$$0.055 = 11 \times 10^{-3} \times 20 \pm 11 \times 10^{-3} \times T_2$$

$$T_2 = 20 + 5 = 25^\circ\text{C} \quad \text{or } 20 - 5 = 15^\circ\text{C}$$

The expt. Can be performed from 15 to 25°C

17. $f_{0^\circ\text{C}} = 0.098\text{ g/m}^3$, $f_{4^\circ\text{C}} = 1\text{ g/m}^3$

$$f_{0^\circ\text{C}} = \frac{f_{4^\circ\text{C}}}{1 + \gamma \Delta T} \Rightarrow 0.998 = \frac{1}{1 + \gamma \times 4} \Rightarrow 1 + 4\gamma = \frac{1}{0.998}$$

$$\Rightarrow 4 + \gamma = \frac{1}{0.998} - 1 \Rightarrow \gamma = 0.0005 \approx 5 \times 10^{-4}$$

As density decreases $\gamma = -5 \times 10^{-4}$

18. Iron rod L_{Fe} $\alpha_{\text{Fe}} = 12 \times 10^{-8}/^\circ\text{C}$ Aluminium rod L_{Al} $\alpha_{\text{Al}} = 23 \times 10^{-8}/^\circ\text{C}$

Since the difference in length is independent of temp. Hence the difference always remains constant.

$$L'_{\text{Fe}} = L_{\text{Fe}}(1 + \alpha_{\text{Fe}} \times \Delta T) \quad \dots(1)$$

$$L'_{\text{Al}} = L_{\text{Al}}(1 + \alpha_{\text{Al}} \times \Delta T) \quad \dots(2)$$

$$L'_{\text{Fe}} - L'_{\text{Al}} = L_{\text{Fe}} - L_{\text{Al}} + L_{\text{Fe}} \times \alpha_{\text{Fe}} \times \Delta T - L_{\text{Al}} \times \alpha_{\text{Al}} \times \Delta T$$

$$\frac{L_{\text{Fe}}}{L_{\text{Al}}} = \frac{\alpha_{\text{Al}}}{\alpha_{\text{Fe}}} = \frac{23}{12} = 23 : 12$$

19. $g_1 = 9.8\text{ m/s}^2$, $g_2 = 9.788\text{ m/s}^2$
 $T_1 = 2\pi \frac{\sqrt{l_1}}{g_1}$ $T_2 = 2\pi \frac{\sqrt{l_2}}{g_2} = 2\pi \frac{\sqrt{l_1(1 + \Delta T)}}{g}$

$$\alpha_{\text{Steel}} = 12 \times 10^{-6}/^\circ\text{C}$$

$$T_1 = 20^\circ\text{C} \quad T_2 = ?$$

$$T_1 = T_2$$

$$\Rightarrow 2\pi \frac{\sqrt{l_1}}{g_1} = 2\pi \frac{\sqrt{l_1(1 + \Delta T)}}{g_2} \Rightarrow \frac{l_1}{g_1} = \frac{l_1(1 + \Delta T)}{g_2}$$

$$\Rightarrow \frac{1}{9.8} = \frac{1 + 12 \times 10^{-6} \times \Delta T}{9.788} \Rightarrow \frac{9.788}{9.8} = 1 + 12 \times 10^{-6} \times \Delta T$$

$$\Rightarrow \frac{9.788}{9.8} - 1 = 12 \times 10^{-6} \Delta T \Rightarrow \Delta T = \frac{-0.00122}{12 \times 10^{-6}}$$

$$\Rightarrow T_2 - 20 = -101.6 \Rightarrow T_2 = -101.6 + 20 = -81.6 \approx -82^\circ\text{C}$$

20. Given

$$d_{\text{St}} = 2.005\text{ cm}, \quad d_{\text{Al}} = 2.000\text{ cm}$$

$$\alpha_{\text{S}} = 11 \times 10^{-6}/^\circ\text{C} \quad \alpha_{\text{Al}} = 23 \times 10^{-6}/^\circ\text{C}$$

$$d's = 2.005 (1 + \alpha_{\text{S}} \Delta T) \text{ (where } \Delta T \text{ is change in temp.)}$$

$$\Rightarrow d's = 2.005 + 2.005 \times 11 \times 10^{-6} \Delta T$$

$$d'_{\text{Al}} = 2(1 + \alpha_{\text{Al}} \Delta T) = 2 + 2 \times 23 \times 10^{-6} \Delta T$$

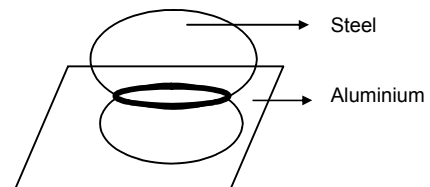
The two will slip i.e. the steel ball will fall when both the diameters become equal.

So,

$$\Rightarrow 2.005 + 2.005 \times 11 \times 10^{-6} \Delta T = 2 + 2 \times 23 \times 10^{-6} \Delta T$$

$$\Rightarrow (46 - 22.055)10^{-6} \times \Delta T = 0.005$$

$$\Rightarrow \Delta T = \frac{0.005 \times 10^6}{23.945} = 208.81$$



Now $\Delta T = T_2 - T_1 = T_2 - 10^\circ\text{C}$ [$\therefore T_1 = 10^\circ\text{C}$ given]

$$\Rightarrow T_2 = \Delta T + T_1 = 208.81 + 10 = 281.81$$

21. The final length of aluminium should be equal to final length of glass.

Let the initial length of aluminium = l

$$l(1 - \alpha_{Al}\Delta T) = 20(1 - \alpha_g\Delta\theta)$$

$$\Rightarrow l(1 - 24 \times 10^{-6} \times 40) = 20(1 - 9 \times 10^{-6} \times 40)$$

$$\Rightarrow l(1 - 0.00096) = 20(1 - 0.00036)$$

$$\Rightarrow l = \frac{20 \times 0.99964}{0.99904} = 20.012 \text{ cm}$$

Let initial breadth of aluminium = b

$$b(1 - \alpha_{Al}\Delta T) = 30(1 - \alpha_g\Delta\theta)$$

$$\Rightarrow b = \frac{30 \times (1 - 9 \times 10^{-6} \times 40)}{(1 - 24 \times 10^{-6} \times 40)} = \frac{30 \times 0.99964}{0.99904} = 30.018 \text{ cm}$$

22. $V_g = 1000 \text{ CC},$ $T_1 = 20^\circ\text{C}$
 $V_{Hg} = ?$ $\gamma_{Hg} = 1.8 \times 10^{-4} / ^\circ\text{C}$
 $\gamma_g = 9 \times 10^{-6} / ^\circ\text{C}$

ΔT remains constant

$$\text{Volume of remaining space} = V'_g - V'_{Hg}$$

Now

$$V'_g = V_g(1 + \gamma_g\Delta T) \quad \dots(1)$$

$$V'_{Hg} = V_{Hg}(1 + \gamma_{Hg}\Delta T) \quad \dots(2)$$

Subtracting (2) from (1)

$$V'_g - V'_{Hg} = V_g - V_{Hg} + V_g\gamma_g\Delta T - V_{Hg}\gamma_{Hg}\Delta T$$

$$\Rightarrow \frac{V_g}{V_{Hg}} = \frac{\gamma_{Hg}}{\gamma_g} \Rightarrow \frac{1000}{V_{Hg}} = \frac{1.8 \times 10^{-4}}{9 \times 10^{-6}}$$

$$\Rightarrow V_{HG} = \frac{9 \times 10^{-3}}{1.8 \times 10^{-4}} = 500 \text{ CC.}$$

23. Volume of water = 500 cm^3

$$\text{Area of cross section of can} = 125 \text{ m}^2$$

Final Volume of water

$$= 500(1 + \gamma\Delta\theta) = 500[1 + 3.2 \times 10^{-4} \times (80 - 10)] = 511.2 \text{ cm}^3$$

The aluminium vessel expands in its length only so area expansion of base can be neglected.

$$\text{Increase in volume of water} = 11.2 \text{ cm}^3$$

Considering a cylinder of volume = 11.2 cm^3

$$\text{Height of water increased} = \frac{11.2}{125} = 0.089 \text{ cm}$$

24. $V_0 = 10 \times 10 \times 10 = 1000 \text{ CC}$

$$\Delta T = 10^\circ\text{C}, \quad V'_{HG} - V'_g = 1.6 \text{ cm}^3$$

$$\alpha_g = 6.5 \times 10^{-6} / ^\circ\text{C}, \quad \gamma_{Hg} = ?, \quad \gamma_g = 3 \times 6.5 \times 10^{-6} / ^\circ\text{C}$$

$$V'_{Hg} = V_{HG}(1 + \gamma_{Hg}\Delta T) \quad \dots(1)$$

$$V'_g = V_g(1 + \gamma_g\Delta T) \quad \dots(2)$$

$$V'_{Hg} - V'_g = V_{HG} - V_g + V_{HG}\gamma_{Hg}\Delta T - V_g\gamma_g\Delta T$$

$$\Rightarrow 1.6 = 1000 \times \gamma_{Hg} \times 10 - 1000 \times 6.5 \times 3 \times 10^{-6} \times 10$$

$$\Rightarrow \gamma_{Hg} = \frac{1.6 + 6.3 \times 3 \times 10^{-2}}{10000} = 1.789 \times 10^{-4} \approx 1.8 \times 10^{-4} / ^\circ\text{C}$$

25. $f_w = 880 \text{ Kg/m}^3,$ $f_b = 900 \text{ Kg/m}^3$
 $T_1 = 0^\circ\text{C},$ $\gamma_w = 1.2 \times 10^{-3} / ^\circ\text{C},$
 $\gamma_b = 1.5 \times 10^{-3} / ^\circ\text{C}$

The sphere begins to sink when,

$$(mg)_{\text{sphere}} = \text{displaced water}$$

$$\Rightarrow V f'_{\omega} g = V f'_{\text{b}} g$$

$$\Rightarrow \frac{f_{\omega}}{1 + \gamma_{\omega} \Delta\theta} = \frac{f_{\text{b}}}{1 + \gamma_{\text{b}} \Delta\theta}$$

$$\Rightarrow \frac{880}{1 + 1.2 \times 10^{-3} \Delta\theta} = \frac{900}{1 + 1.5 \times 10^{-3} \Delta\theta}$$

$$\Rightarrow 880 + 880 \times 1.5 \times 10^{-3} (\Delta\theta) = 900 + 900 \times 1.2 \times 10^{-3} (\Delta\theta)$$

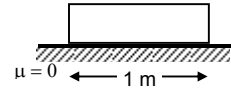
$$\Rightarrow (880 \times 1.5 \times 10^{-3} - 900 \times 1.2 \times 10^{-3}) (\Delta\theta) = 20$$

$$\Rightarrow (1320 - 1080) \times 10^{-3} (\Delta\theta) = 20$$

$$\Rightarrow \Delta\theta = 83.3^{\circ}\text{C} \approx 83^{\circ}\text{C}$$

26. $\Delta L = 100^{\circ}\text{C}$

A longitudinal strain develops if and only if, there is an opposition to the expansion. Since there is no opposition in this case, hence the longitudinal strain here = Zero.



27. $\theta_1 = 20^{\circ}\text{C}$, $\theta_2 = 50^{\circ}\text{C}$

$$\alpha_{\text{steel}} = 1.2 \times 10^{-5} /^{\circ}\text{C}$$

Longitudinal strain = ?

$$\text{Strain} = \frac{\Delta L}{L} = \frac{L \alpha \Delta\theta}{L} = \alpha \Delta\theta$$

$$= 1.2 \times 10^{-5} \times (50 - 20) = 3.6 \times 10^{-4}$$

28. $A = 0.5 \text{ mm}^2 = 0.5 \times 10^{-6} \text{ m}^2$

$$T_1 = 20^{\circ}\text{C}, T_2 = 0^{\circ}\text{C}$$

$$\alpha_s = 1.2 \times 10^{-5} /^{\circ}\text{C}, Y = 2 \times 10^{11} \text{ N/m}^2$$

$$\text{Decrease in length due to compression} = L \alpha \Delta\theta \quad \dots(1)$$

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F}{A} \times \frac{L}{\Delta L} \Rightarrow \Delta L = \frac{FL}{AY} \quad \dots(2)$$

Tension is developed due to (1) & (2)

Equating them,

$$L \alpha \Delta\theta = \frac{FL}{AY} \Rightarrow F = \alpha \Delta\theta AY$$

$$= 1.2 \times 10^{-5} \times (20 - 0) \times 0.5 \times 10^{-6} \times 2 \times 10^{11} = 24 \text{ N}$$

29. $\theta_1 = 20^{\circ}\text{C}$, $\theta_2 = 100^{\circ}\text{C}$

$$A = 2 \text{ mm}^2 = 2 \times 10^{-6} \text{ m}^2$$

$$\alpha_{\text{steel}} = 12 \times 10^{-6} /^{\circ}\text{C}, Y_{\text{steel}} = 2 \times 10^{11} \text{ N/m}^2$$

Force exerted on the clamps = ?

$$\frac{\left(\frac{F}{A}\right)}{\text{Strain}} = Y \Rightarrow F = \frac{Y \times \Delta L}{L} \times L = \frac{Y L \alpha \Delta\theta A}{L} = Y A \alpha \Delta\theta$$

$$= 2 \times 10^{11} \times 2 \times 10^{-6} \times 12 \times 10^{-6} \times 80 = 384 \text{ N}$$

30. Let the final length of the system at system of temp. $0^{\circ}\text{C} = \ell_0$

Initial length of the system = ℓ_0

When temp. changes by θ .

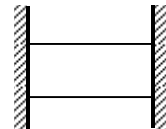
$$\text{Strain of the system} = \ell_1 - \frac{\ell_0}{\ell_0}$$

$$\text{But the total strain of the system} = \frac{\text{total stress of system}}{\text{total young's modulus of system}}$$

Now, total stress = Stress due to two steel rod + Stress due to Aluminium

$$= \gamma_s \alpha_s \theta + \gamma_s ds \theta + \gamma_{\text{al}} \text{ at } \theta = 2\% \alpha_s \theta + \gamma_2 A \ell \theta$$

$$\text{Now young'' modulus of system} = \gamma_s + \gamma_s + \gamma_{\text{al}} = 2\gamma_s + \gamma_{\text{al}}$$



Steel
Aluminium
Steel

$$\therefore \text{Strain of system} = \frac{2\gamma_s \alpha_s \theta + \gamma_s \alpha_{al} \theta}{2\gamma_s + \gamma_{al}}$$

$$\Rightarrow \frac{\ell_\theta - \ell_0}{\ell_0} = \frac{2\gamma_s \alpha_s \theta + \gamma_s \alpha_{al} \theta}{2\gamma_s + \gamma_{al}}$$

$$\Rightarrow \ell_\theta = \ell_0 \left[\frac{1 + \alpha_{al} \gamma_{al} + 2\alpha_s \gamma_s \theta}{\gamma_{al} + 2\gamma_s} \right]$$

31. The ball tries to expand its volume. But it is kept in the same volume. So it is kept at a constant volume. So the stress arises

$$\frac{P}{\left(\frac{\Delta V}{V}\right)} = B \Rightarrow P = B \frac{\Delta V}{V} = B \times \gamma \Delta \theta$$

$$= B \times 3\alpha \Delta \theta = 1.6 \times 10^{11} \times 10^{-6} \times 3 \times 12 \times 10^{-6} \times (120 - 20) = 57.6 \times 10^7 \approx 5.8 \times 10^8 \text{ pa.}$$

32. Given

I_0 = Moment of Inertia at 0°C

α = Coefficient of linear expansion

To prove, $I = I_0 (1 + 2\alpha\theta)$

Let the temp. change to θ from 0°C

$$\Delta T = \theta$$

Let 'R' be the radius of Gyration,

$$\text{Now, } R' = R(1 + \alpha\theta), \quad I_0 = MR^2 \quad \text{where M is the mass.}$$

$$\text{Now, } I' = MR'^2 = MR^2(1 + \alpha\theta)^2 \approx MR^2(1 + 2\alpha\theta)$$

[By binomial expansion or neglecting $\alpha^2 \theta^2$ which given a very small value.]

$$\text{So, } I = I_0(1 + 2\alpha\theta) \quad (\text{proved})$$

33. Let the initial m.I. at 0°C be I_0

$$T = 2\pi \sqrt{\frac{I}{K}}$$

$$I = I_0(1 + 2\alpha\Delta\theta) \quad (\text{from above question})$$

$$\text{At } 5^\circ\text{C}, \quad T_1 = 2\pi \sqrt{\frac{I_0(1 + 2\alpha\Delta\theta)}{K}} = 2\pi \sqrt{\frac{I_0(1 + 2\alpha 5)}{K}} = 2\pi \sqrt{\frac{I_0(1 + 10\alpha)}{K}}$$

$$\text{At } 45^\circ\text{C}, \quad T_2 = 2\pi \sqrt{\frac{I_0(1 + 2\alpha 45)}{K}} = 2\pi \sqrt{\frac{I_0(1 + 90\alpha)}{K}}$$

$$\frac{T_2}{T_1} = \sqrt{\frac{1 + 90\alpha}{1 + 10\alpha}} = \sqrt{\frac{1 + 90 \times 2.4 \times 10^{-5}}{1 + 10 \times 2.4 \times 10^{-5}}} = \sqrt{\frac{1.00216}{1.00024}}$$

$$\% \text{ change} = \left(\frac{T_2}{T_1} - 1 \right) \times 100 = 0.0959\% = 9.6 \times 10^{-2}\%$$

34. $T_1 = 20^\circ\text{C}, \quad T_2 = 50^\circ\text{C}, \quad \Delta T = 30^\circ\text{C}$

$$\alpha = 1.2 \times 10^{-5} / ^\circ\text{C}$$

ω remains constant

$$(I) \omega = \frac{V}{R} \quad (II) \omega = \frac{V'}{R'}$$

$$\text{Now, } R' = R(1 + \alpha\Delta\theta) = R + R \times 1.2 \times 10^{-5} \times 30 = 1.00036R$$

From (I) and (II)

$$\frac{V}{R} = \frac{V'}{R'} = \frac{V'}{1.00036R}$$

$$\Rightarrow V' = 1.00036 V$$

$$\% \text{ change} = \frac{(1.00036V - V)}{V} \times 100 = 0.00036 \times 100 = 3.6 \times 10^{-2}$$

