Quadrilaterals

Exercise – 8.1

Write the correct answer in each of the following:

1. Three angles of a quadrilateral are 75º, 90º and 75º. The fourth angle is

(A) 90º
(B) 95º
(C) 105º
(D) 120º

Sol. Fourth angel of the quadrilateral

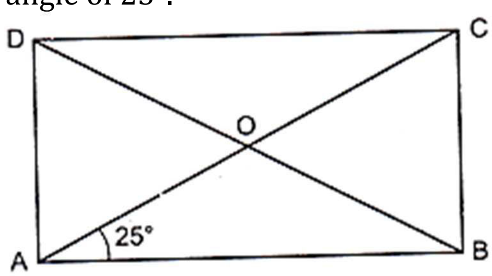
$= 360^o - (75^o + 90^o + 75^o)$
$= 360^o - 240^o$
$= 120^o$

Hence, (d) is the correct answer.

2. A diagonal of a rectangle is inclined to one side of the rectangle at 25º. The acute angle between the diagonals is

(A) 55º
(B) 50º
(C) 40º
(D) 25º

Sol. ABCD is a rectangle in which diagonal AC is inclined to one side AB of the rectangle at an angle of 25°.



Now, AC = BD

[∵ Diagonal of a rectangle are equal]

$\therefore \quad \frac{1}{2}AC = \frac{1}{2}BD$

$\Rightarrow \quad$ OA = OB

In ΔAOB, we have OA = OB

$\therefore \quad \angle OBA = \angle OAB = 25^0$

$\therefore \quad \angle AOB = 180^0 - (25^0 + 25^0) = 130^0$

$\angle AOB$ and $\angle AOD$ from angle of a linear pair.

$\therefore \angle AOB + \angle AOD = 180^o$

$\Rightarrow \angle AOD = 180^o - 130^o = 50^o$

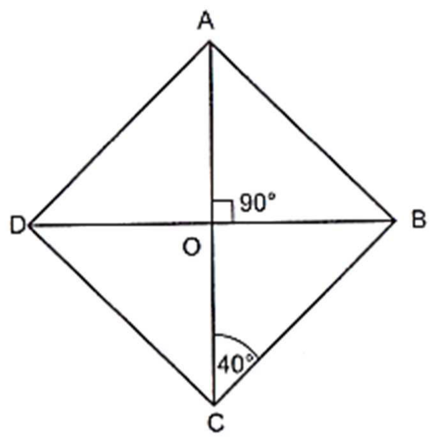
Hence, the acute angel between the diagonal is 50°.

Therefore, (b) is the correct answer.

3. ABCD is a rhombus such that ∠ACB = 40º. Then ∠ADB is

(A) 40º
(B) 45º
(C) 50º
(D) 60º

Sol. ABCD is a rhombus such that ∠ACB = 40º.
We know that diagonals of rhombus bisect each other at right angles.
In right ΔBOC, we have



$$\angle OBC = 180^0 - (\angle BOC + \angle BCO)$$
$$= 180^0 - (90^0 + 40^0) = 50^0$$
$$\therefore \angle DBC = \angle OBC = 50^0$$

Now,

$\angle ADB = \angle DBC$ [Alt. int. $\angle s$]

$\therefore \angle ADB = 50^0$ $\left[\therefore \angle DBC = 50^0\right]$

Hence, (c) is the correct answer.

4. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral PQRS, taken in order, is a rectangle, if

(A) PQRS is a rectangle
(B) PQRS is a parallelogram
(C) diagonals of PQRS are perpendicular
(D) diagonals of PQRS are equal.

Sol. If diagonals of PQRS are perpendicular.
Hence, (c) is the correct answer.

5. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral PQRS, taken in order, is a rhombus, if

(A) PQRS is a rhombus
(B) PQRS is a parallelogram
(C) diagonals of PQRS are perpendicular
(D) diagonals of PQRS are equal.

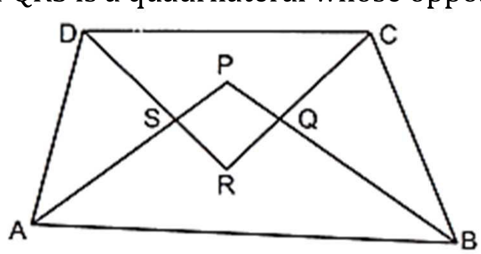
Sol. If diagonals of PQRS are equal.
Hence, (d) is the correct answer.

6. If angles A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio 3:7:6:4, then ABCD is a
(A) rhombus
(B) parallelogram
(C) trapezium
(D) kite

Sol. As angle A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio 3 : 7 : 6 : 4, so let the angles A, B, C and D be 3x, 7x, 6x and 4x.
Now, sum of the angle of a quadrilateral is 360°.
$3x+7x+6x+4x=360^0$
$\Rightarrow 20x=360^0 \Rightarrow x=360^0 \div 20=18^0$
So, the angles A, B, C and D of quadrilateral ABCD are $3\times18^0, 7\times18^0, 6\times18^0$ and 4×18^0
i.e., 54°, 126°, 180° and 72°
Now, AD and BC are two lines which are cut by a transversal CD such that the sum of angles $\angle C$ and $\angle D$ on the same side of transversal is $\angle C+\angle D=180^0+72^0=180^0$
$\therefore AD \parallel BC$
So, ABCD is a quadrilateral in which one pair of opposite sides are parallel. Hence, ABCD is a trapezium.
Hence, (c) is the correct answer.

7. If bisectors of ∠A and ∠B of a quadrilateral ABCD intersect each other at P, of ∠B and ∠C at Q, of ∠C and ∠D at R and of ∠D and ∠A at S, then PQRS is a
(A) rectangle
(B) rhombus
(C) parallelogram
(D) quadrilateral whose opposite angles are supplementary

Sol. PQRS is a quadrilateral whose opposite angles are supplementary.

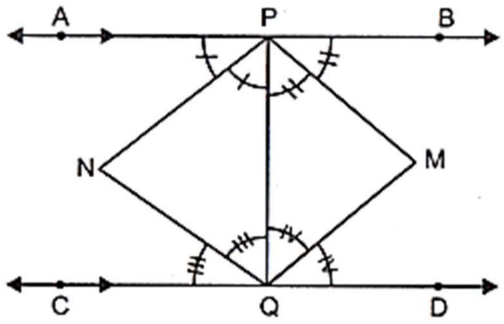


Hence, (d) is the correct answer.

8. If APB and CQD are two parallel lines, then the bisectors of the angles APQ, BPQ, CQP and PQD form
(A) a square
(B) a rhombus
(C) a rectangle

(D) any other parallelogram

Sol. PNQM is a rectangle.



Hence, (C) is the correct answer.

9. The figure obtained by joining the mid-points of the sides of a rhombus, taken in order, is

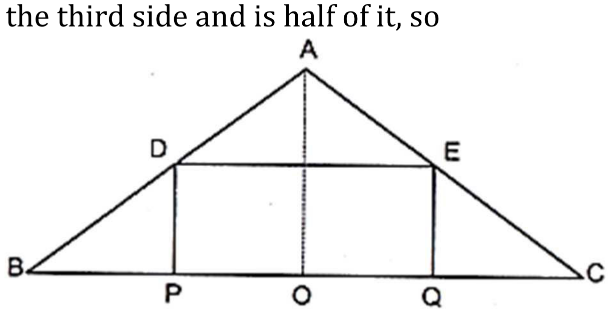
(A) a rhombus
(B) a rectangle
(C) a square
(D) any parallelogram

Sol. The figure will be a rectangle.
Hence, (b) is the correct answer.

10. D and E are the mid-points of the sides AB and AC of ΔABC and O is any point on side BC. O is joined to A. If P and Q are the mid-points of OB and OC respectively, then DEQP is

(A) a square
(B) a rectangle
(C) a rhombus
(D) a parallelogram

Sol. Since the line segment joining the mid-points of any two sides of a triangle is parallel to the third side and is half of it, so



$\therefore \quad DE = \frac{1}{2} BC$ and DE||BC

Similarly, $DP = \frac{1}{2} AO$ and DP||AO

And $EQ = \frac{1}{2} AO$ and EQ||AO

$\therefore$ $DP = EQ$ [$\because$ Each $= \frac{1}{2} AO$]

And DP||EQ [$\because$ DP||AO and EQ||AO]

Now, DEOQ is quadrilateral in which one pair of its opposite sides is equal and parallel.
Therefore, quadrilateral DEQP is a parallelogram.
Hence, (d) is the correct answer.

11. The figure formed by joining the mid-points of the sides of a quadrilateral ABCD, taken in order, is a square only if,

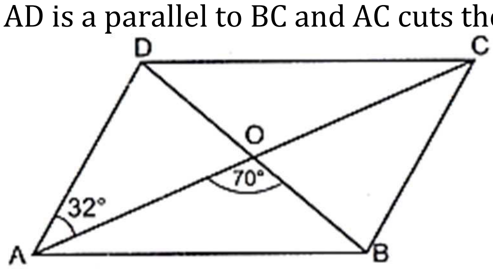
(A) ABCD is a rhombus
(B) diagonals of ABCD are equal
(C) diagonals of ABCD are equal and perpendicular
(D) diagonals of ABCD are perpendicular

Sol. If diagonal of ABCD are equal and perpendicular.
Hence, (c) is the correct answer.

12. The diagonals AC and BD of a parallelogram ABCD intersect each other at the point O. If ∠DAC = 32º and ∠AOB = 70º, then ∠DBC is equal to

(A) 24º
(B) 86º
(C) 38º
(D) 32º

Sol. AD is a parallel to BC and AC cuts them,



$\angle DAC = \angle ACB$ [Alt. int. $\angle s$]

$\angle DAC = 32^0$

$\therefore$ $\angle ACB = 32^0$

Now, in ΔBOC, CO is produced to A

$\therefore$ Ext. $\angle BOA = \angle OCB + \angle OBC$ [By exterior angle theorem]

$\Rightarrow$ $70^0 = 32^0 + \angle OBC$

$\therefore$ $\angle OBC = 70^0 - 32^0 = 38^0$

Hence, $\angle DBC = 38^0$.

Therefore, (C) is the correct answer.

13. Which of the following is not true for a parallelogram?

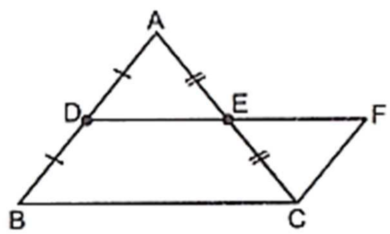
(A) opposite sides are equal
(B) opposite angles are equal
(C) opposite angles are bisected by the diagonals
(D) diagonals bisect each other.

Sol. Opposite angles are bisected by the diagonals. This is not true for a parallelogram.
Hence, (c) is the correct answer.

14. D and E are the mid-points of the sides AB and AC respectively of Δ ABC. DE is produced to F. To prove that CF is equal and parallel to DA, we need an additional information which is

(A) ∠DAE = ∠EFC
(B) AE = EF
(C) DE = EF
(D) ∠ADE = ∠ECF.

Sol. We need DE = EF.



Hence, (c) is the correct answer.

Quadrilaterals

Exercise 8.2

1. Diagonals AC and BD of a parallelogram ABCD intersect each other at O. If OA = 3 cm and OD = 2 cm, determine the lengths of AC and BD.

Sol. We know that diagonals of a parallelogram bisect each other.

$\therefore$ $AC = 2 \times OA = 2 \times 3 \text{ cm} = 6 \text{ cm}$

And $BD = 2OD = 2 \times 2 \text{ cm} = 4 \text{ cm}$

Hence, lengths of AC and BD are 6 cm and 4 cm respectively.

2. Diagonals of a parallelogram are perpendicular to each other. Is this statement true? Give reason for your answer.

Sol. This statement not true. Diagonals of a parallelogram bisect each other.

3. Can the angles 110º, 80º, 70º and 95º be the angles of a quadrilateral? Why or why not?

Sol. Sum of these angles $110^\circ + 80^\circ + 70^\circ + 95^\circ = 355^\circ$

But, sum of the angles of a quadrilateral is always 360°. Hence, 110º, 80º, 70º and 95º cannot be the angles of a quadrilateral.

4. In quadrilateral ABCD, $\angle A + \angle D = 180^\circ$. What special name can be given to this quadrilateral?

Sol. In quadrilateral ABCD, $\angle A + \angle D = 180^\circ$ i.e., the sum of two consecutive angles is 180°. So, pair of opposite side AB and CD are parallel.

Therefore, quadrilateral ABCD is trapezium. Hence, special name which can be given to this quadrilateral ABCD is trapezium.

5. All the angles of a quadrilateral are equal. What special name is given to this quadrilateral?

Sol. All the angle of a quadrilateral are equal. Also, the sum of angles of a quadrilateral is 360°. Therefore, each angle of quadrilateral is 90°. So, the given quadrilateral is a rectangle. Hence, special name given quadrilateral is rectangle.

6. Diagonals of a rectangle are equal and perpendicular. Is this statement true? Give reason for your answer.

Sol. The given statement is not true. Diagonals of a rectangle need not to be perpendicular.

7. Can all the four angles of a quadrilateral be obtuse angles? Give reason for your answer.

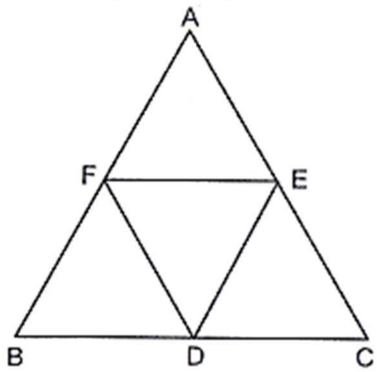
Sol. No, because then the sum of four angles of the quadrilateral will be more than 360° whereas sum of four angles of a quadrilateral is always equal to 360°.

8. In Δ ABC, AB = 5 cm, BC = 8 cm and CA= 7 cm. If D and E are respectively the mid-points of AB and BC, determine the length of DE.

Sol. In Δ ABC, AB = 5 cm, BC = 8 cm and CA= 7 cm. D and E are respectively the mid-points of AB and BC.

$\therefore \quad DE = \frac{1}{2}AC = \frac{1}{2} \times 7cm = 3.5cm$ [Using the mid-point theorem]

9. In Fig.8.1, it is given that BDEF and FDCE are parallelograms. Can you say that BD = CD? Why or why not?



Sol. BDEF is a parallelogram.

$\therefore$ BD = EF ...(1)

[Opposite side of a parallelogram]

FDCE is a parallelogram

$\therefore$ CD = EF ...(2)

From (1) and (2), we get

BD = CD

10. In Fig.8.2, ABCD and AEFG are two parallelograms. If ∠C = 55º, determine ∠F.

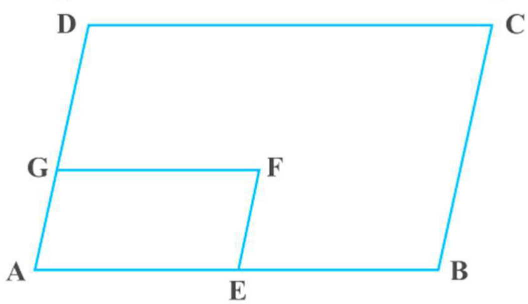


Fig. 8.2

Sol. We know that opposite angle of parallelogram are equal.

In parallelogram ABCD, we have

$\angle A = \angle C$

But, $\angle C = 55^0$ [Given]

$\therefore \quad \angle A = 55^0$

Now, in parallelogram AEFG, we have

$\angle F = \angle A = 55^0$

Hence, $\angle F = 55^0$.

11. **Can all the angles of a quadrilateral be acute angles? Give reason for your answer.**

Sol. We know that an acute angle is less than 90º. All the angles of a quadrilateral cannot be acute angles because than equal sum of a quadrilateral will be than a 360º, whereas angle sum of quadrilateral is 360º.

12. **Can all the angles of a quadrilateral be right angles? Give reason for your answer.**

Sol. Yes, all the angles of a quadrilateral can be right angles. Angles sum of a quadrilateral will be 360º, which is true.

13. **Diagonals of a quadrilateral ABCD bisect each other. If ∠A= 35º, determine ∠B.**

Sol. As the diagonals of a quadrilateral ABCD bisect each other, so ABCD is a parallelogram.
Now, ABCD is a parallelogram

$\therefore \quad \angle A + \angle B = 180^0$

[∵ Adjacent angles of a parallelogram are supplementary]

$\therefore \quad 35^0 + \angle B = 180^0$

$\Rightarrow \quad \angle B = 180^0 - 35^0 = 145^0$

14. **Opposite angles of a quadrilateral ABCD are equal. If AB = 4 cm, determine CD.**

Sol. Since opposite angles of a quadrilateral ACBD are equal. If AB = cm, determine CD.

[∵ Opposite sides of a parallelogram are equal]

But, AB = 4cm, therefore CD = 4cm.
Hence, CD = 4cm.

Quadrilaterals
Exercise 8.3

1. One angle of a quadrilateral is of 108º and the remaining three angles are equal. Find each of the three equal angles.

Sol. One angle of a quadrilateral is of 180º and let each of the three remaining equal angles be x^0.

As the sum of the angles of a quadrilateral is 360º.

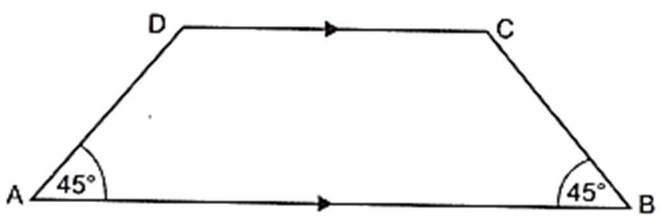
$\therefore 180^0 + x + x + x = 360^0 \Rightarrow 3x = 360^0 - 180^0 = 252^0$

$\therefore \quad x = 252^o \div 3 = 84^o$

Hence, each of the three angles be 84º.

2. ABCD is a trapezium in which AB || DC and ∠A = ∠B = 45º. Find angles C and D of the trapezium.

Sol. ABCD is a trapezium in which AB||DC.



Now, AB||DC and AD is transversal.

$\therefore \quad \angle A + \angle D = 180^0$

[∵ Sum of interior angles on the side of the transversal is 180º]

$\Rightarrow \quad 45^0 + \angle D = 180^0$

$\Rightarrow \quad \angle D = 180^0 - 45^0 = 135^0$

Similarly, $\quad \angle B + \angle C = 180^0$

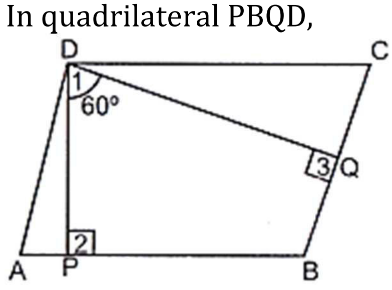
$\Rightarrow \quad 45^0 + \angle C = 180^0$

$\Rightarrow \quad \angle C = 180^0 - 45^0 = 135^0$

Hence, $\angle A = \angle B = 45^0$ and $\angle C = \angle D = 135^0$.

3. The angle between two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 60º. Find the angles of the parallelogram.

Sol. In quadrilateral PBQD,



$\angle 1 + \angle 2 + \angle B + \angle 3 = 360^0$

$\Rightarrow 60^0 + 90^0 + \angle B + 90^0 = 360^0$

$\Rightarrow \quad \angle B + 240^0 = 360^0$

$\Rightarrow \quad \angle B = 360^0 - 240^0$

$\Rightarrow \quad \angle B = 120^0$

Now, $\angle ADC = \angle B = 120^0$

[$\because$ Opposite angles of a parallelogram are equal]

$\angle A + \angle B = 180^0$

[$\because$ Sum of consecutive interior angles is 180°]

$\Rightarrow \quad \angle A + 120^0 = 180^0$

$\Rightarrow \quad \angle A = 180^0 - 120^0$

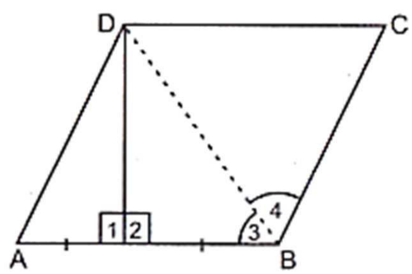
$\Rightarrow \quad \angle A = 60^0$

But, $\angle C = \angle A = 60^0$

[$\because$ Opposite angles of a parallelogram are equal]

4. ABCD is a rhombus in which altitude from D to side AB bisects AB. Find the angles of the rhombus.

Sol. In ΔAPD and ΔBPD, we have



AP = BP [Given]

$\angle 1 = \angle 2$ [$\because$ Each equal to 90°]

PD = PD [Common side]

So, by SAS Criterion of congruence, we have

$\Delta APD \cong \Delta BPD$

$\therefore \quad \angle A = \angle 3$ [CPCT]

But, $\angle 3 = \angle 4$ [$\because$ Diagonals bisect opposite angles of a rhombus]

$\Rightarrow \quad \angle A = \angle 3 = \angle 4$ ...(1)

Now, AD||BC

SO, $\angle A + \angle ABC = 180^0$ [$\because$ Sum of consecutive interior angles is 180°]

$\Rightarrow \quad \angle A + \angle 3 + \angle 4 = 180^0$

$\Rightarrow \quad \angle A + \angle A + \angle A = 180^0$ [Using (1)]

$\Rightarrow \quad 3\angle A = 180^0$

$\Rightarrow \quad \angle A = \dfrac{180^0}{3} = 60^0$

Now, $\angle ABC = \angle 3 + \angle 4$

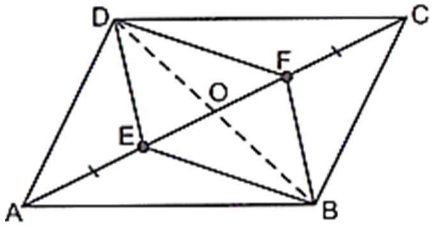
$= 60^0 + 60^0$

$= 120^0$ [$\because$ Opposite angles of a rhombus are equal]

$\therefore$ $\angle ADC = \angle ABC = 120^0$ [Same reason as above]

5. E and F are points on diagonal AC of a parallelogram ABCD such that AE = CF. Show that BFDE is a parallelogram.

Sol. Given: A parallelogram ABCD: E and F are points of diagonal AC of parallelogram ABCD such that AE = CF.



To prove: BFDE is parallelogram.

Proof: ABCD is s parallelogram.

$\therefore$ OD = OB ...(1) [$\because$ Diagonals of parallelogram bisect each other]

OA = OC ...(2) [Same reason as above]

AE = CF ...(3) [Given]

Subtracting (3) from (2), we get

OA – AE = OC – CF

$\Rightarrow$ OE = OF ...(4)

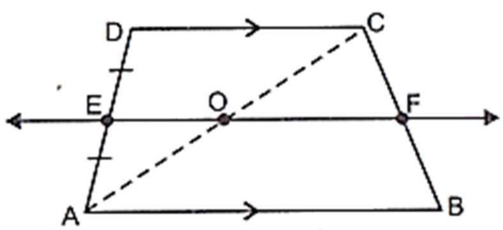
$\therefore$ BFDE is parallelogram. [$\because$ OD = OB and OE = OF]

Hence, proved.

6. E is the mid-point of the side AD of the trapezium ABCD with AB||DC. A line through E drawn parallel to AB intersect BC at F . Show that F is the mid-point of BC. [Hint: Join AC]

Sol. Given: A trapezium ABCD in which AB||CD and E is mid-point of the side AB. Also, EF||AB.

To prove: F is the mid-point of BC.



Construction: Join AC which intersect EF at O.

Proof: In ΔADC, E is the mid-point of AD and EF||DC.

[$\because$ EF||AB and DC||AB $\Rightarrow$ AB||EF||DC]

$\therefore$ O is the mid-point of AC. [Converse of mid – point theorem]

Now, in ΔCAB, O is the mid-point of AC and OF||AB.

$\Rightarrow$ OF bisects BC.

Or F is the mid-point of BC.

Hence, proved.

7. Through A, B and C, lines RQ, PR and QP have been drawn, respectively parallel to sides BC, CA and AB of a Δ ABC as shown in Fig.8.5. Show that $BC = \frac{1}{2}QR$.

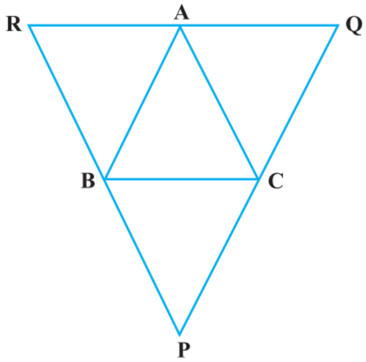


Fig. 8.5

Sol. Given: Triangle ABC and PQR in which AB||PQ, BC||RQ and CA||PR.

To prove: $BC = \frac{1}{2}QR$

Proof: Quadrilateral RBCA is a parallelogram.

[∵ RA||BC and BR||CA]

∴ RA = BC ...(1) [∵ Opposite side of parallelogram]

Now, quadrilateral BCQA is a parallelogram.

∴ AQ = BC ...(2) [∵ Opposite side of parallelogram]

Adding (1) and (2), we get

RA + AQ = 2BC

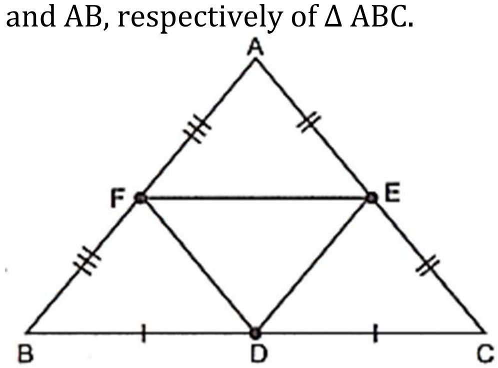
⇒ QR = 2BC

⇒ $BC = \frac{1}{2}QR$

Hence, proved.

8. D, E and F are the mid-points of the sides BC, CA and AB, respectively of an equilateral triangle ABC. Show that Δ DEF is also an equilateral triangle.

Sol. Given: Δ ABC is an equilateral triangle. D, E and F are the mid-points of the sides BC, CA and AB, respectively of Δ ABC.



To prove: Δ DEF is an equilateral triangle.

Proof: EF joints mid-points of sides of AB and AC respectively.

$\therefore \quad EF = \frac{1}{2}BC$ ...(1) [Mid-point theorem]

Similarly, $DE = \frac{1}{2}BC$ ...(2) [Mid-point theorem]

$DF = \frac{1}{2}AC$ ...(3) [Mid-point theorem]

But, AB = BC = CA ...(4) [Sides of an equilateral Δ ABC]

From (1), (2), (3) and (4), we have

DE = EF = FD

$\therefore \Delta DEF$ is an equilateral triangle.

Hence, proved.

9. Points P and Q have been taken on opposite sides AB and CD, respectively of a parallelogram ABCD such that AP= CQ (Fig. 8.6). Show that AC and PQ bisect each other.

Sol. Points P and Q have been taken on opposite sides AB and CD respectively of a parallelogram ABCD such that AP = CQ.

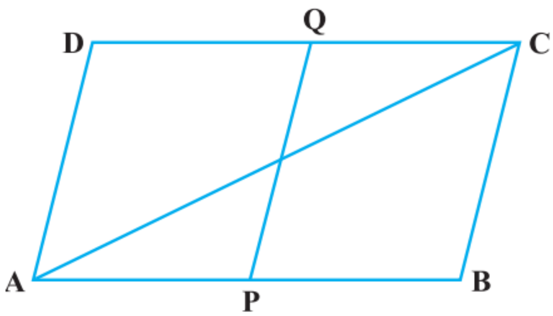


Fig. 8.6

In ΔAOP and ΔCOQ, we have

AP = CQ [Given]

$\angle 1 = \angle 2$ [Alt. int. $\angle s$ are equal]

$\angle 3 = \angle 4$ [Vertically opp. $\angle s$]

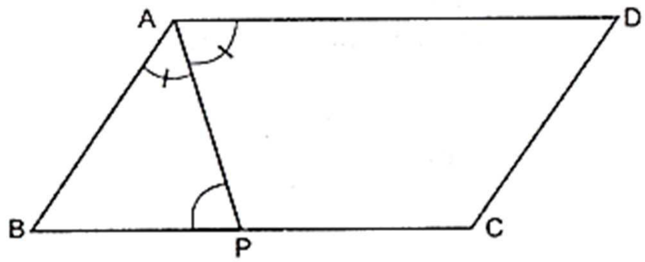
$\therefore \quad \Delta AOP \cong \Delta COQ$ [By SAS congruence rule]

$\therefore$ OA = OC and OP = OQ [CPCT]

Hence, AC and PQ bisect each other.

10. In Fig. 8.7, P is the mid-point of side BC of a parallelogram ABCD such that ∠BAP = ∠DAP. Prove that AD = 2CD.

Sol. $\angle BAP = \angle DAP = \frac{1}{2}\angle A$ ...(1)



Since ABCD is a parallelogram, we have

$$\angle A + \angle B = 180^0 \qquad ...(2)$$

[$\because$ Sum of interior angles on the same sides of transversal is 180°]

In ΔABP, we have

$$\angle BAP + \angle B + \angle APB = 180^0$$

$$\Rightarrow \frac{1}{2}\theta\angle A = 180^0 - \angle A + \angle APB = 180^0 \qquad \text{[Using (1) and (2)]}$$

$$\Rightarrow \angle APB = \frac{1}{2}\angle A \qquad ...(3)$$

From (1) and (3), we get

$$\angle BAP = \angle APB$$

BP = AB ...(4)

[$\because$ Side of opposite to equal angles are equal]

Since opposite sides of a parallelogram are equal, we have

$$AD = BC \Rightarrow \frac{1}{2}AD = \frac{1}{2}BC$$

$$\Rightarrow \quad \frac{1}{2}AD = BP \qquad \text{[}\because\text{ P is the mid-point of BC]}$$

$$\Rightarrow \quad \frac{1}{2}AD = AB \qquad \text{[}\because\text{ From (4), BP = AB]}$$

Since, opposite sides of a parallelogram are equal, we have

$$\frac{1}{2}AD = CD \Rightarrow AD = 2CD$$

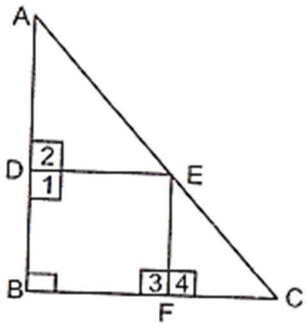
Hence, proved.

Quadrilaterals
Exercise 8.4

1. A square is inscribed in an isosceles right triangle so that the square and the triangle have one angle common. Show that the vertex of the square opposite the vertex of the common angle bisects the hypotenuse.

Sol. ABC is an isosceles right triangle with AB = BC. A square BFED is inscribed in triangle ABC so that $\angle B$ = common = 90°.

In ΔADE and ΔEFC, we have



DE = EF ...(1) [∵ Sides of a square are equal]

$\angle 1 + \angle 2 = 180^0$ [Linear pair axiom]

$\Rightarrow 90^0 + \angle 2 = 180^0$ [∵ Each angle of a square = 90°]

$\Rightarrow \angle 2 = 90^0$

Similarly, $\angle 4 = 90^0$

$\therefore \angle 2 = \angle 4$ ...(2) [∵ Each = 90°]

Now, AB = BC [Given]

$\therefore \angle C = \angle A$ ...(3) [∵ Angles opp. To equal sides are equal]

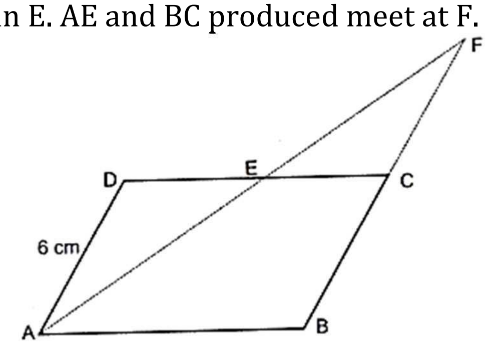
From (1), (2) and (3), we get

$\Delta ADE \cong \Delta EFC$ [By AAS Congruence rule]

Hence, AE = EC [CPCT]

2. In a parallelogram ABCD, AB = 10 cm and AD = 6 cm. The bisector of ∠A meets DC in E. AE and BC produced meet at F. Find the length of CF

Sol. ABCD is a parallelogram, in which AB = 10cm and AD = 6cm. The bisector of ∠A meets DC in E. AE and BC produced meet at F.



$\angle BAE = \angle EAD$ ...(1) [∵ bisect of ∠A]

$\angle EAD = \angle EFB$ ...(2) [Alt. $\angle s$]

$\Rightarrow$ $\angle BAE = \angle EFB$ [From (1) and (2)]

$\therefore$ BF = AB [$\because$ Sides opposite to equal $\angle s$ are equal]

$\Rightarrow$ BF = 10 cm [$\because$ AB = 10 cm]

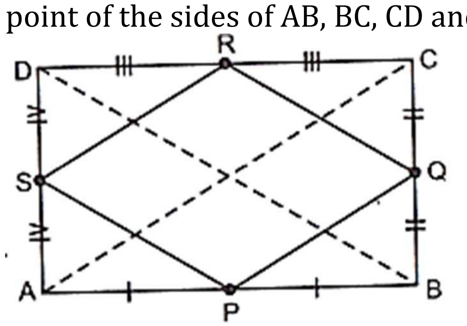
$\Rightarrow$ BC + CF = 10 cm $\Rightarrow$ 6 cm + CF = 10 cm

[$\because$ BC = AD = 6 cm, opposite sides of a ||gm]

$\Rightarrow$ CF = 10 – 6 cm = 4 cm

3. P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD in which AC = BD. Prove that PQRS is a rhombus.

Sol. Given: A quadrilateral ABCD in which AC = BD and P, Q, R and S are respectively the mid-point of the sides of AB, BC, CD and DA of quadrilateral ABCD.



To prove: PQRS is a rhombus.

Proof: In ΔABC, P and Q are the mid-points of AB and BC respectively.

That is, PQ joins mid-points of sides AB and BC.

$\therefore$ $PQ \parallel AC$ ...(1)

And $PQ = \frac{1}{2} AC$ ...(2) [Mid-point theorem]

In ΔADC, R and S are the mid-points of CD and AD respectively.

$\therefore$ $SR \parallel AC$ ...(3)

And $SR = \frac{1}{2} AC$ ...(4) [Mid-point theorem]

From (1) and (3), we get

PQ||SR

From (2) and (4), we get

PQ = RS

$\Rightarrow$ PQRS is a parallelogram.

In ΔDAB, SP joins mid-points of sides of DA and AB respectively.

$\therefore$ $SP = \frac{1}{2} BD$ ...(5) [Mid-point theorem]

AC = BD ...(6) [Given]

From equations (2), (5) and (6), we get

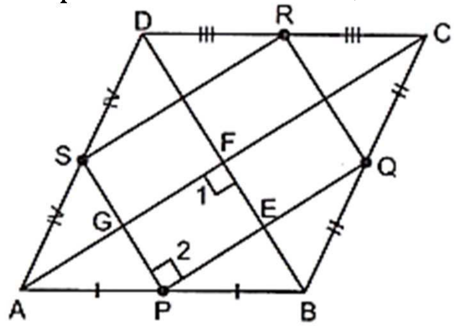
SP = PQ

$\therefore$ Parallelogram PQRS is a rhombus.

Hence, proved.

4. P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD such that AC ⊥BD. Prove that PQRS is a rectangle.

Sol. Given: A quadrilateral ABCD in which $AC \perp BD$ and P, Q, R and S are respectively the mid-points of the sides AB, BC CD and DA of quadrilateral ABCD.



To Prove: In ΔABC, P and Q are the mid-point od sides AB and BC respectively.

That is, PQ joins mid-points of sides AB and BC.

$\therefore \quad PQ \parallel AC$ ...(1)

And $PQ = \frac{1}{2}AC$ ...(2) [Mid-point theorem]

In ΔADC, R and S are the mid-points of CD and AD respectively.

$\therefore \quad SR \parallel AC$ ...(3)

And $SR = \frac{1}{2}AC$ ...(4) [Mid-point theorem]

From (1) and (3), we get

PQ||SR

From (2) and (4), we get

PQ = RS

$\Rightarrow$ PQRS is a parallelogram.

PQ||AC [Proved above]

$\Rightarrow$ PG||GF

In ΔABD, PS joins mid-points of sides AB and AD respectively.

$\therefore$ PS||MD [Mid-point theorem]

$\Rightarrow$ PG||EF

$\Rightarrow$ PEFG is a parallelogram [$\because$ PE||GF and PG||EF]

$\Rightarrow \quad \angle 1 = \angle 2$ [$\because$ Opposite angles of a parallelogram are equal]

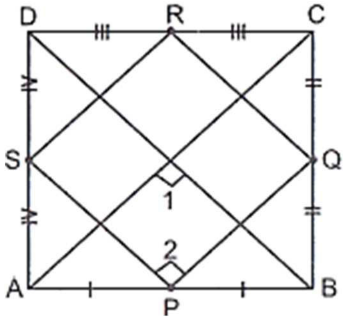
But, $\angle 1 = 90^0$ $[\because AC \perp BD]$

$\therefore \quad \angle 2 = 90^0$

$\Rightarrow$ Parallelogram PQRS is a rectangle.

5. P, Q, R and S are respectively the mid-points of sides AB, BC, CD and DA of quadrilateral ABCD in which AC = BD and AC⊥BD. Prove that PQRS is a square.

Sol. Given: A quadrilateral ABCD is which AC = BD and AC⊥BD. P, Q, R and S respectively the mid-points of sides AB, BC, CD and DA of quadrilateral ABCD.



To prove: PQRS is a square.
Proof: Parallelogram PQRS is a rectangle.

[Same as in Q4]

$PQ = \frac{1}{2} AC$ ...(1) [Proved as in Q4]

PS joins mid-points of sides AB and AD respectively.

$PS = \frac{1}{2} BD$ ...(2) [Mid-point theorem]

AC = BD ...(3) [Given]

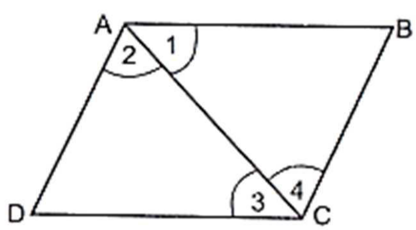
From (1), (2) and (3), we get

PS = PQ

$\Rightarrow$ Rectangle PQRS is a square.

6. A diagonal of a parallelogram bisects one of its angles. Show that it is a rhombus.

Sol. ABCD is a parallelogram and diagonal AC bisect $\angle A.$ We have to show that ABCD is a rhombus.



$\angle 1 = \angle 2$ ...(1) [$\because$ AC bisect $\angle A$]

$\angle 2 = \angle 4$ ...(2) [Alt. interior angles]

From (1) and (2), we get

$\angle 1 = \angle 4$

Now, in $\Delta ABC,$ we have

$\angle 1 = \angle 4$ [Proved above]

$\therefore$ BC = AB [$\because$ Side. Opp. To equal $\angle s$ are equal]

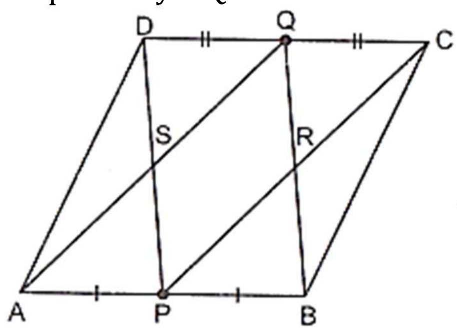
Also, AB = DC and AD = BC [$\because$ Opposite sides of a parallelogram are equal]

So, ABCD is a parallelogram in which its sides AB = BC = CD = AD.

Hence, ABCD is a rhombus.

7. P and Q are the mid-points of the opposite sides AB and CD of a parallelogram ABCD. AQ intersects DP at S and BQ intersects CP at R. Show that PRQS is a parallelogram.

Sol. Given: A quadrilateral ABCD in which P and Q are the mid-points of the sides AB and CD respectively. AQ intersect DP at S and BQ intersect CP at R.



To prove: PRQS is a parallelogram.

Proof: DC||AB [∵ Opposite sides of a parallelogram are parallel]

⇒ AP||QC

DC||AB [∵ Opposite sides of a parallelogram are equal]

⇒ $\frac{1}{2}DC = \frac{1}{2}AB$

⇒ $QC = AP$ [∵ P is the mid-point of AB and Q is mid-points of CD]

⇒ APCQ is a parallelogram. [∵ AP||QC and QC = AP]

∴ AQ||PC [∵ Opposite sides of a ||gm are parallelogram]

⇒ SQ||PR

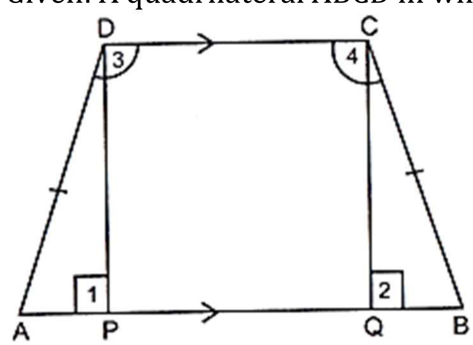
Similarly, SP||QR

∴ Quadrilateral PRQS is a parallelogram.

Hence, proved.

8. ABCD is a quadrilateral in which AB || DC and AD = BC. Prove that ∠A = ∠B and ∠C = ∠D.

Sol. Given: A quadrilateral ABCD in which AB||CD and AD = BC.



To prove: ∠A = ∠B and ∠C = ∠D.

Construction: Draw $DP \perp AB$ and $CQ \perp AB$.

Proof: In ΔAPD and ΔBQC, we have

$\angle 1 = \angle 2$ [∵ Each equal to 90°]

AB = BC [Given]

[Distance between parallel line]

So, By RHS criterion of congruence, we have

$\Delta APD \cong \Delta BQC$ [CPCT]

$\therefore$ $\angle A = \angle B$

Now, DC||AB

$\angle A + \angle 3 = 180^0$ ...(1) [$\because$ Sum of consecutive interior angles is 180°]

$\angle B + \angle 4 = 180^0$ ...(2) [Same reason]

From (1) and (2), we get

$\angle A + \angle 3 = \angle B + \angle 4$

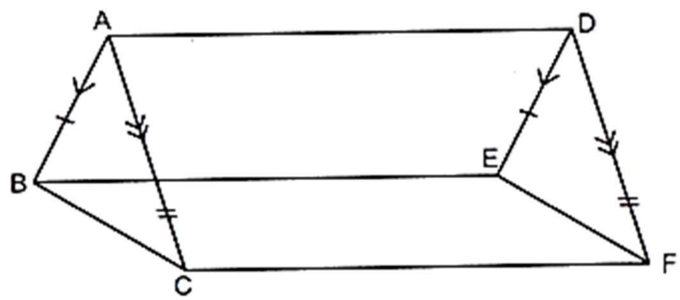
$\Rightarrow$ $\angle 3 = \angle 4$ $[\because \angle A = \angle B]$

$\Rightarrow$ $\angle C = \angle D$

Hence, proved.

9. In Fig. 8.11, AB || DE, AB = DE, AC||DF and AC = DF. Prove that BC||EF and BC = EF.

Sol.



Given: AB||DE, AB = DE, AC||DF and AC = DF

To prove: BC||EF and BC = EF

Proof: AC||DF [Given]

And AC = DF [Given]

$\therefore$ ACFD is a parallelogram.

$\Rightarrow$ AD||CF ...(1) [$\because$ Opposite side of a ||gm are parallel]

And AD||CF ...(2) [$\because$ Opposite sides of a ||gm are equal]

Now, AB||DE [Given]

And AB = DE [Given]

$\therefore$ ABED is a parallelogram.

$\Rightarrow$ AD||BE ...(3) [$\because$ Opposite sides of a are||gm are parallel]

And AD = BE ...(4) [$\because$ Opposite sides of a ||gm are equal]

From (1) and (3), we get

CF||BE

And, from (2) and (4) we get

CF||BE

$\therefore$ BDFE is a parallelogram.

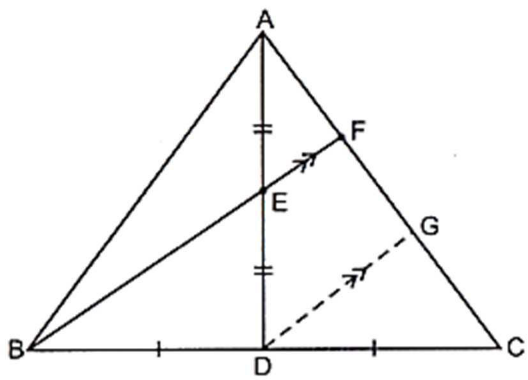
$\Rightarrow$ BC||EF [$\because$ Opposite sides of a ||gm are parallel]

And BC = EF [$\because$ Opposite sides of a ||gm are equal]

Hence, proved.

10. E is the mid-point of a median AD of ΔABC and BE is produced to meet AC at F. Show that $AF = \frac{1}{2}AC.$

Sol. Given: A ΔABC in which E is the mid-point of median AB and BE is produced to meet AC at F.



To prove: $AF = \frac{1}{2}AC$

Construction: Draw DG||BF intersecting AC at G.

Proof: In ΔADG, E is the mid-point of AD and EF||DG.

∴ AF = FG ...(1) [Converse of mid-point theorem]

In ΔFBC, D is the mid-point of BC and DG||BF.

∴ FG = GC ...(2) [Converse of mid-point theorem]

From equation (1) and (2), we get

AF = FG = GC ...(3)

But, AC = AF + FG + GC

⇒ AC = AF + AF + AF [Using (3)]

⇒ AC = 3AF

⇒ $AF = \frac{1}{2}AC$

Hence, proved.

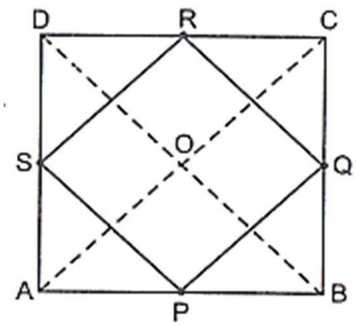
11. Show that the quadrilateral formed by joining the mid-points of the consecutive sides of a square is also a square.

Sol. Given: A square ABCD in which P, Q, R, S are the mid-points of sides AB, BC, CD, DA respectively. PQ, QR, RS and SP are joined.

To prove: PQRS is a square.

Construction: Join AC and BD.

Proof: In ΔABC, P and Q are the mid-points of sides AB and BC respectively.



$\therefore \quad PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ ...(1)

In ΔADC, R and S are the mid-points of CD and AD respectively.

$\therefore \quad RS \parallel AC$ and $RS = \frac{1}{2} AC$ ...(2)

From eqs. (1) and (2), we get

PQ||RS and PQ = RS ...(3)

Thus, in quadrilateral PQRS one pair of opposite sides are equal and parallel.

Hence, PQRS is a parallelogram.

Since ABCD is a square.

$\therefore$ AB = BC = CD = DA

$\Rightarrow \quad \frac{1}{2} AB = \frac{1}{2} CD$ and $\frac{1}{2} AB = \frac{1}{2} BC$

$\Rightarrow$ PB = RC and BQ = CQ

Thus, in Δs PBQ and RCQ, we have

PB = RC

BQ = CQ [$\Rightarrow$ PB = CR and BQ = CQ]

And $\angle PBQ = \angle RCQ$ [$\because$ Each equal to 90^o]

So, By SAS criterion of congruence, we have

$\Delta PBQ \cong \Delta RCQ$

$\Rightarrow$ PQ = QR [CPCT]...(4)

From (3) and (4), we have

PQ = QR = RS

But, PQRS is a parallelogram

$\therefore$ PQ = PS

So, PQ = QR = RS = PS ...(5)

Now, PQ||AC [From (1)]

$\Rightarrow$ PM||NO ...(6)

Since, P and S are the mid-points of AB and AD respectively.

PS||BD

$\Rightarrow$ PN||MO ...(7)

Thus, in quadrilateral PMON, we have

PM||NO [From (6)]

And PN||MO [From (7)]

So, PMON is a parallelogram.

$\Rightarrow \quad \angle MPN = \angle MON$

$\Rightarrow \quad \angle MPN = \angle BOA \qquad [\because \angle MPN = \angle BOA]$

$\Rightarrow \quad \angle MPN = 90^0$

[$\because$ Diagonals of square are $\perp \therefore AC \perp BD \Rightarrow \angle BOA = 90^0$]

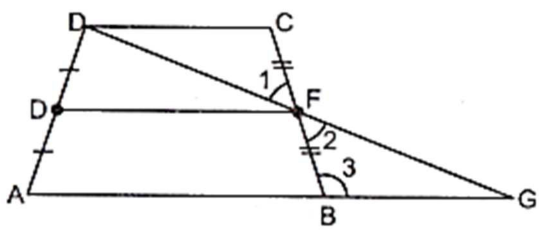
$\Rightarrow \quad \angle QPS = 90^0$

Thus, PQRS is a quadrilateral such that PQ = QR = RS = SP and $\angle QPS = 90^0$.

12. E and F are respectively the mid-points of the non-parallel sides AD and BC of a trapezium ABCD. Prove that EF || AB and $EF = \frac{1}{2}(AB + CD)$.

[Hint: Join BE and produce it to meet CD produced at G.]

Sol.



Given: A trapezium ABCD in which E and F are respectively the mid-points of the non-parallel sides AD and BC.

To prove: EF||AB and $EF = \frac{1}{2}(AB + CD)$

Construction: Join DF and produce it to intersect AB produced at G.

Proof: In ΔCFD and ΔBFG, we have

DC||AB

$\therefore \quad \angle C = \angle 3 \qquad$ [Alternate interior angles]

CF = BF

$\angle 1 = \angle 2 \qquad$ [Vertically opposite angles]

So, By ASA criterion of congruence, we have

$\Delta CFD \cong \Delta BFG$

$\therefore$ CD = BG [CPCT]

EF joins mid-points of sides AD and GD respectively

$\therefore$ EF||AG [$\because$ Mid-point theorem]

$\Rightarrow$ EF||AB

So, $EF = \frac{1}{2} AG \qquad$ [Mid-point theorem]

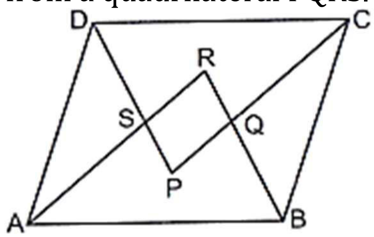
$\Rightarrow \quad EF = \frac{1}{2}(AB + AG)$

$\Rightarrow \quad EF = \frac{1}{2}(AB + CD) \qquad$ [$\because$ CD = BG]

Hence, proved.

13. Prove that the quadrilateral formed by the bisectors of the angles of a parallelogram is a rectangle.

Sol. Given: A quadrilateral ABCD in which bisectors of angles A, B, C, D intersect at P, Q, R, S to from a quadrilateral PQRS.



To Prove: PQRS is a rectangle.

Proof: Since ABCD is a parallelogram.

Therefore, AB||DC

Now, AB||DC and transversal AD intersect them at D and A respectively.

Therefore,

$\angle A + \angle D = 180^0$ [$\because$ Sum of consecutive interior angles is 180°]

$\Rightarrow \quad \frac{1}{2}\angle A = \frac{1}{2}\angle D = 90^0$

$\Rightarrow \quad \angle DAS + \angle ADS = 90^0$ ...(1)

[$\because$ DS and AS are bisectors of $\angle A$ and $\angle D$ respectively]

But, in ΔDAS, we have

$\angle DAS + \angle ASD + \angle ADS = 180^0$

[$\because$ Sum of the angles of a triangle is 180°]

$\Rightarrow \angle 90^0 + \angle ASD = 180^0$ [Using (1)]

$\Rightarrow \quad \angle ASD = 90^0$

$\Rightarrow \quad \angle PSR = 90^0$ [$\because \angle ASD$ and $\angle PSR$ are vertically opposite angles

$\therefore \angle PSR = \angle ASD$]

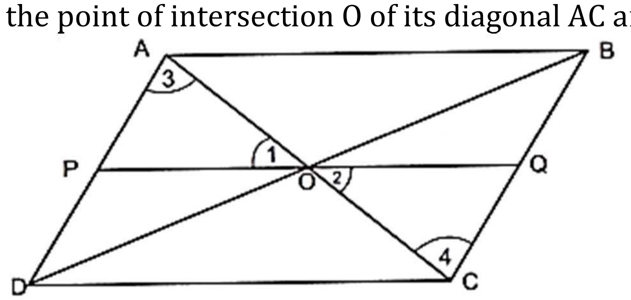
Similarly, we can prove that

$\angle SRQ = 90^0, \angle RQP = 90^0$ and $\angle SPQ = 90^0$

Hence, PQRS is a rectangle.

14. P and Q are points on opposite sides AD and BC of a parallelogram ABCD such that PQ passes through the point of intersection O of its diagonals AC and BD. Show that PQ is bisected at O.

Sol. ABCD is a parallelogram. Its diagonal AC and BD bisect each other at O. PQ passes through the point of intersection O of its diagonal AC and BD.



In ΔAOP and ΔCOQ, we have

$\angle 3 = \angle 4$ [Alternate int. $\angle s$]

OA = OC [Diagonals of a ||gm bisects each other]

$\angle 1 = \angle 2$ [Vertically opposite angles]

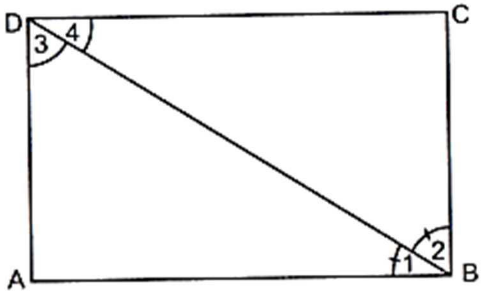
$\therefore$ $\Delta AOP \cong \Delta COQ$ [By ASA Congruence rule]

So, OP = OQ [CPCT]

Hence, PQ is bisected at O.

15. ABCD is a rectangle in which diagonal BD bisects ∠B. Show that ABCD is a square.

Sol. Given: A rectangle ABCD in which diagonal BD bisects $\angle B$.



To prove: ABCD is a square.

Proof: DC||AB

[∵ Opposite sides of a rectangle are parallel]

$\Rightarrow$ $\angle 4 = \angle 1$ ..(1) [Alternate interior angles]

Similarly, $\angle 3 = \angle 2$ ...(2) [Alternate interior angles]

And $\angle 1 = \angle 2$ ...(3) [Given]

From equation (1), (2) and (3), we get

$\angle 3 = \angle 4$

In ΔBDA and ΔBDC, we have

$\angle 1 = \angle 2$ [Given]

BD = BD [Common side]

$\angle 3 = \angle 4$ [proved above]

So, By ASA criterion of congruence, we have

$\Delta BDA \cong \Delta BDC$

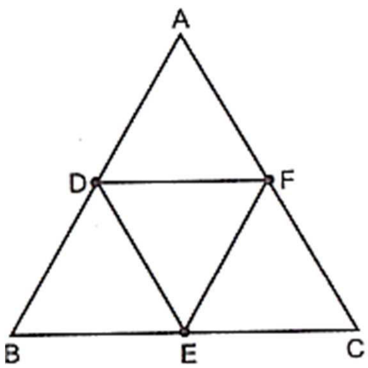
$\therefore$ AB = BC [CPCT]

So, ABCD is a square.

Hence, proved.

16. D, E and F are respectively the mid-points of the sides AB, BC and CA of a triangle ABC. Prove that by joining these mid-points D, E and F, the triangles ABC is divided into four congruent triangles.

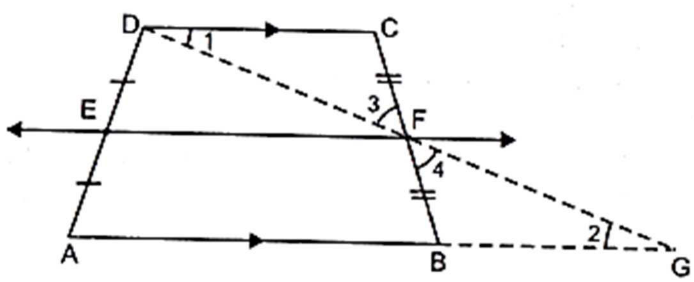
Sol. Given: A ΔABC and ΔDEF which is formed by joining the mid-point D, E and F of the sides AB, BC and CA of ΔABC.



To prove: $\Delta DEF \cong \Delta EDB \cong \Delta CFE \cong \Delta FAD$
Proof: DF joins mid-points of sides AB and AC respectively of ΔABC
$\therefore$ DF||BC [Mid-point theorem]
$\Rightarrow$ DF||BE
Similarly, EF||BD
So, quadrilateral BEFD is a parallelogram.
$\Rightarrow$ $\Delta DEF \cong \Delta EDB$ ...(1)
[$\because$ Diagonal of parallelogram divides it into two congruent triangles]
Similarly, $\Delta DEF \cong \Delta CFE$ ...(2)
And $\Delta DEF \cong \Delta FAD$ ...(3)
From equations (1), (2) and (3), we get
$\Delta DEF \cong \Delta EDB \cong \Delta DEF \cong \Delta FAD$
Hence, proved.

17. Prove that the line joining the mid-points of the diagonals of a trapezium is parallel to the parallel sides of the trapezium.

Sol.



Given: A trapezium ABCD in which E and F are the mid-points of sides AD and BC respectively.
To prove: EF||AB||DC
Construction: Join DF are produce it to intersect AB produced at G.
Proof: In ΔDCF and ΔGBF, we have
$\angle 1 = \angle 2$ [Alternate interior angles because DC||BG]
$\angle 3 = \angle 4$ [Vertically opposite angles]
CF = BF [$\because$ F is the mid-point of BC]
So, By AAS criterion of congruence, we have

$\Delta DCF \cong \Delta GBF$

$\therefore$ DF = GF [CPCT]

In $\Delta DAG,$ EF joins mid-points of sides DA and DG respectively.

$\therefore$ EF||AG [Mid-point theorem]

$\Rightarrow$ EF||AB

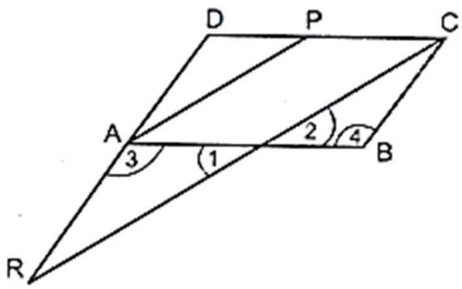
But, AB||DC [Given]

$\therefore$ EF||BC||DC

Hence, proved.

18. P is the mid-point of the side CD of a parallelogram ABCD. A line through C parallel to PA intersects AB at Q and DA produced at R. Prove that DA = AR and CQ = QR.

Sol. ABCD is parallelogram. P is the mid-point of CD. CR which intersects AB at Q is parallel to AP.



In $\Delta DCR,$ P is the mid-point of CD and AP||CR,

$\therefore$ A is the mid-points of DR, i.e., AD = AR.

[$\because$ The line drawn through the mid-point of one side of a triangle parallel to another side intersects the third side at its mid-point.]

In ΔARQ and ΔBCQ we have

AR = BC [$\because$ AD = AR (Proved above) and AD = BC]

$\angle 1 = \angle 2$ [Vertically opposite angles]

$\angle 3 = \angle 4$ [Alt. $\angle s$]

$\therefore$ $\Delta ARQ \cong \Delta BCQ$ [By ASS Congruence rule]

CQ = QR [CPCT]

Hence, DA = AR and CQ = QR is proved.