Ex 21.1

Q1

Let T_n be the nth term of this series then,

$$T_n = \left[1 + (n-1)2\right]^3$$
$$= (2n-1)^3$$
$$= (2n)^3 - 3(2n)^2.1 + 3.1^2.2n - 1^3 \qquad \left[\because (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b\right]$$
$$= 8n^3 - 12n^2 + 6n - 1$$

$\therefore$ $1^3 + 3^3 + 5^3 + \ldots$ to n terms

$$= \sum_{k=1}^{n} T_k$$

$$= \sum_{k=1}^{n} \left(8k^3 - 12k^2 + 6k - 1\right)$$

$$= 8\sum_{k=1}^{n} k^3 - 12\sum_{k=1}^{n} k^2 + 6\sum_{k=1}^{n} k - \sum_{k=1}^{n} 1$$

$$= 8\left[\frac{n(n+1)}{2}\right]^2 - 12\left[\frac{n(n+1)(2n+1)}{6}\right] + 6\left[\frac{n(n+1)}{2}\right] - n$$

$$= 8\frac{n^2(n+1)^2}{4} - 12\left[n(n+1)(2n+1)\right] + 3\left[n(n+1)\right] - n$$

$$= 2n^2(n+1)^2 - 2n(n+1)(2n+1) + 3(n+1) - n$$
$$= (n+1)\left[2n^2(n+1) - 2n(2n+1) + 3n\right] - n$$
$$= (n+1)\left[2n^3 + 2n^2 - 4n^2 - 2n + 3n\right] - n$$
$$= (n+1)\left[an^3 - 2n^2 + n\right] - n$$
$$= 2n^4 - 2n^3 + n^2 + 2n^3 - 2n^2 + n - n$$
$$= 2n^4 + n^2 - 2n^2$$
$$= 2n^4 - n^2$$
$$= n^2\left(2n^2 - 1\right)$$

$\therefore$ $1^3 + 3^3 + 5^3 + 7^3 \ldots\ldots$ to n terms $= n^2\left(2n^2 - 1\right)$.

Q2

Let T_n be the nth term of this series. Then,

$$T_n = (2n)^3$$
$$= 8n^3$$

Let S_n be the sum to n terms of the given series; Then,

$$S_n = \sum_{k=1}^{n} 8k^3$$

$$= 8\sum_{k=1}^{n} k^3$$

$$= 8\left[\frac{n(n+1)}{2}\right]^2$$

$$= 8 \times \frac{n^2(n+1)^2}{4}$$

$$= 2n^2(n+1)^2$$

Hence, $S_n = 2n^2(n+1)^2$

Q3

Let T_n be the nth term of the given series. Then,

$$T_n = (n\text{th term of } 1,2,3....) \times (n\text{th term of } 2,3,4...) \times (n\text{th term of } 5,6,7...)$$

$$= [1+(n-1)\times 1]\times[2+(n-1)\times 1]\times[5+(n-1)\times 1]$$

$$= [1+n-1]\times[2+n-1]\times[5+n-1]$$

$$= n\times(n+1)(n+4)$$

$$= (n^2+n)(n+4)$$

$$= n^3+4n^2+n^2+4n$$

$$= n^3+5n^2+4n$$

$$= T_n = n^3+5n^2+4n$$

Let S_n denote the sum to n terms of the give series. Then,

$$S_n = \sum_{n=1}^{n} T_n = \sum_{n=1}^{n}\left(n^3+5n^2+4n\right)$$

$$= \sum_{n=1}^{n} n^3 + \sum_{n=1}^{n} 5n^2 + \sum_{n=1}^{n} 4n$$

$$= \sum_{n=1}^{n} n^3 + 5\sum_{n=1}^{n} n^2 + 4\sum_{n=1}^{n} n$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + 5\left[\frac{n(n+1)(2n+1)}{6}\right] + 4\left[\frac{n(n+1)}{2}\right]$$

$$= \frac{n^2(n+1)^2}{4} + \frac{5n(n+1)(2n+1)}{6} + 2n(n+1)$$

$$= \frac{3n^2(n+1)^2 + 10n(n+1)(2n+1) + 24n(n+1)}{12}$$

$$= \frac{n(n+1)}{12}\left[3n(n+1)+10(2n+1)+24\right]$$

$$= \frac{n(n+1)}{12}\left[3n^2+3n+20n+10+24\right]$$

$$= \frac{n(n+1)}{12}\left[3n^2+23n+34\right]$$

$$= \frac{n(n+1)\left(3n^2+23n+34\right)}{12}$$

Hence, $S_n = \dfrac{n(n+1)\left(3n^2+23n+34\right)}{12}$

Q4

Let T_n be the nth term of the given series. Then,

$$T_n = (n\text{th term of } 1,2,3\ldots) \times (n\text{th term of } 2,3,4\ldots) \times (n\text{th term of } 4,7,10\ldots)$$

$$= [1+(n-1)\times 1].[2+(n-1)\times 1].[4+(n-1)\times 3]$$

$$= [1+n-1].[2+n-1].[4+3n-3]$$

$$= n(n+1)(3n+1)$$

$$= (n^2+n)(3n+1)$$

$$= 3n^3+n^2+3n^2+n$$

$$= 3n^3+4n^2+n$$

$$\therefore \quad = T_n = 3n^3+4n^2+n$$

Let S_n denote the sum to n terms of the given series. Then,

$$S_n = \sum_{n=1}^{n} T_n = \sum_{n=1}^{n}\left(3n^3+4n^2+n\right)$$

$$= \sum_{n=1}^{n} 3n^3 + \sum_{n=1}^{n} 4n^2 + \sum_{n=1}^{n} n$$

$$= 3\sum_{n=1}^{n} n^3 + 4\sum_{n=1}^{n} n^2 + \sum_{n=1}^{n} n$$

$$= 3\left[\frac{n(n+1)}{2}\right]^2 + 4\left[\frac{n(n+1)(2n+1)}{6}\right] + \left[\frac{n(n+1)}{2}\right]$$

$$= \frac{3}{4}[n(n+1)]^2 + \frac{2n(n+1)(2n+1)}{3} + \frac{n(n+1)}{2}$$

$$= \frac{9[n(n+1)]^2 + 8n(n+1)(2n+1) + 6n(n+1)}{12}$$

$$= \frac{n(n+1)}{12}[9n(n+1)+8(2n+1)+6]$$

$$= \frac{n}{12}(n+1)[9n^2+9n+16n+8+6]$$

$$= \frac{n}{12}(n+1)[9n^2+25n+14]$$

Hence, $S_n = \dfrac{n}{12}(n+1)(9n^2+25n+14)$

Q5

Let T_n be the nth term of the given series, Then,

$$
\begin{aligned}
T_n &= 1+2+3+\ldots.+n \\
&= \frac{n}{2}\left[2\times 1+(n-1)\times 1\right] \\
&= \frac{n}{12}\left[2+n-1\right] \\
&= \frac{n}{12}(n+1) \\
&= \frac{n^2}{2}+\frac{n}{2}
\end{aligned}
$$

Let S_n denote the sum to n terms of the given series. Then,

$$S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n}\left[\frac{k^2}{2}+\frac{k}{2}\right]$$

$$= \sum_{k=1}^{n}\frac{k^2}{2}+\sum_{k=1}^{n}\frac{k}{2}$$

$$\Rightarrow \quad S_n = \frac{1}{2}\sum_{k=1}^{n}k^2+\frac{1}{2}\sum_{k=1}^{n}k$$

$$
\begin{aligned}
&= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6}\right]+\frac{1}{2}\left[\frac{n(n+1)}{2}\right] \\
&= \frac{n(n+1)(2n+1)}{12}+\frac{n(n+1)}{4} \\
&= \frac{n(n+1)(2n+1)+3n(n+1)}{12} \\
&= \frac{n(n+1)}{12}\left[2n+1+3\right] \\
&= \frac{n(n+1)}{12}\left[2n+4\right] \\
&= \frac{n(n+1)}{12}\times 2(n+2) \\
&= \frac{n}{6}(n+1)(n+2)
\end{aligned}
$$

Hence, $S_n = \frac{n}{6}(n+1)(n+2)$

Q6

Let T_n be the nth term of the given series. Then,

$T_n = (n\text{th term of } 1,2,3\ldots) \times (n\text{th term of } 2,3,4\ldots)$

$$= [1 + (n+1)\times 1].[2 + (n+1)\times 1]$$

$$= [1 + n - 1].[2 + n - 1]$$

$$= n(n+1)$$

$$= n^2 + n$$

Let S_n denote the sum to n terms of the given series. Then,

$$S_n = \sum_{n=1}^{n} T_n = \sum_{n=1}^{n}(n^2+n) = \sum_{n=1}^{n} n^2 + \sum_{n=1}^{n} n$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{n)n+1}{2}$$

$$= \frac{n(n+1)(2n+1) + 3n(n+1)}{6}$$

$$= \frac{n(n+1)[2n+1+3]}{6}$$

$$= \frac{n(n+1)[2n+4]}{6}$$

$$= \frac{n(n+1)\times 2(n+2)}{6}$$

$$= \frac{n}{6}(n+1)(n+2)$$

Hence, $S_n = \frac{n}{3}(n+1)(n+2)$

Q7

Let T_n the nth term of the given series. Then,

$$T_n = (n\text{th term of } 3,5,7\ldots) \times (n\text{th term of } 1^2,2^2,3^2\ldots)$$
$$= \left[3+(n-1)2\right].\left[n^2\right]$$
$$= \left[3+2n-2\right].\left(n^2\right)$$
$$= \left[2n+1\right]\left[n^2\right]$$
$$= 2n^3+n^2$$

$$\therefore \quad T_n = 2n^3+n^2$$

Let S_n denote the sum of n terms of the given series. Then,

$$S_n = \sum_{n=1}^{n} T_n = \sum_{n=1}^{n}\left(2n^3+n^2\right)$$

$$= \sum_{n=1}^{n} 2n^3 + \sum_{n=1}^{n} n^2 = 2\sum_{n=1}^{n} n^3 + \sum_{n=1}^{n} n^2$$

$$= 2\left[\frac{n(n+1)}{2}\right]^2 + \left[\frac{n(n+1)(2n+1)}{6}\right]$$
$$= \frac{2}{4}\left[n(n+1)\right]^2 + \frac{\left[n(n+1)(2n+1)\right]}{6}$$
$$= \frac{\left[n(n+1)\right]^2}{2} + \frac{n(n+1)(2n+1)}{6}$$
$$= \frac{3\left[n(n+1)\right]^2 + n(n+1)(2n+1)}{6}$$
$$= \frac{n(n+1)}{6}\left[3n(n+1)+(2n+1)\right]$$
$$= \frac{n(n+1)}{6}\left[3n^2+3n+2n+1\right]$$
$$= \frac{n}{6}(n+1)\left(3n^2+5n+1\right)$$

Hence, $S_n = \dfrac{n}{6}(n+1)\left(3n^2+5n+1\right)$

Q8(i)

We have,

$$T_n = 2n^3 + 3n^2 - 1$$

Let S_n denote the sum of n terms of the series whose nth term is T_n. Then,

$$S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n} \left(2k^3 + 3k^2 - 1\right) = \sum_{k=1}^{n} 2k^3 + \sum_{k=1}^{n} 3k^2 - \sum_{k=1}^{n} 1$$

$$= 2\sum_{k=1}^{n} k^3 + 3\sum_{k=1}^{n} k^2 - \sum_{k=1}^{n} 1$$

$$= 2\left[\frac{n(n+1)}{2}\right]^2 + 3\left[\frac{n(n+1)(2n+1)}{6}\right] - n$$

$$= \frac{2}{4}\left[n(n+1)\right]^2 + \frac{n(n+1)(2n+1) - n}{2}$$

$$= \frac{\left[n(n+1)\right]^2 + n(n+1)(2n+1) - n}{2}$$

$$= \frac{\left[n(n+1)\right]^2 + (n+1)(2n+1) - 2n}{2}$$

$$= \frac{n}{2}\left[n(n+1)^2 + (n+1)(2n+1) - 2\right]$$

$$= \frac{n}{2}\left[n^3 + n + 2n^2 + 2n^2 + 3n - 1\right]$$

$$= \frac{n}{2}\left[n^3 + 4n^2 + 4n - 1\right]$$

Hence, $S_n = \dfrac{n}{2}\left(n^3 + 4n^2 + 4n - 1\right)$

Q8(ii)

We have,

$$T_n = n^3 - 3^n$$

Let S_n denote the sum of $n$terms of the series whose nth term is T_n. Then,

$$S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n} \left(k^3 - 3^k\right) = \sum_{k=1}^{n} k^3 - \sum_{k=1}^{n} 3^k$$

$$\Rightarrow \quad S_n = \sum_{k=1}^{n} k^3 - \sum_{k=1}^{n} 3^k$$

$$= \left[\frac{n(n+1)}{2}\right]^2 - \left(3^1 + 3^2 .... + 3^n\right)$$

$$= \frac{n^2(n+1)^2}{4} - 3\left(\frac{3^n - 1}{3 - 1}\right)$$

$$= \frac{n^2(n+1)^2}{4} - \frac{3}{2}\left(3^n - 1\right)$$

Hence, $S_n = \left[\frac{n(n+1)}{2}\right]^2 - \frac{3}{2}\left(3^n - 1\right)$

Q8(iii)

We have,

$$T_n = n(n+1)(n+4) = (n^2+n)(n+4) = n^3+5n^2+4n$$

Let S_n denote the sum of n terms of the series nth term is T_n. Then,

$$S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n}\left(k^3+5k^2+4k\right)$$

$$= \sum_{k=1}^{n} k^3 + \sum_{k=1}^{n} 5k^2 + \sum_{k=1}^{n} 4k$$

$$= \sum_{k=1}^{n} k^3 + 5\sum_{k=1}^{n} k^2 + 4\sum_{k=1}^{n} k$$

$$\Rightarrow \quad S_n = \sum_{k=1}^{n} k^2 + 5\sum_{k=1}^{n} k^2 + 4\sum_{k=1}^{n} k$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + 5\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{4n(n+1)}{2}$$

$$= \frac{1}{4}\left[n(n+1)\right]^2 + \frac{5n(n+1)(2n+1)}{6} + 2n(n+1)$$

$$= \frac{3\left[n(n+1)\right]^2 + 10n(n+1)(2n+1) + 24n(n+1)}{12}$$

$$= \frac{n(n+1)}{12}\left[3n(n+1)+10(2n+1)+24\right]$$

$$= \frac{n(n+1)}{12}\left[3n^2+3n+20n+10+24\right]$$

$$= \frac{n(n+1)}{12}\left[3n^2+23n+34\right]$$

Hence, $S_n = \frac{n}{12}(n+1)\left(3n^2+23n+34\right)$

Q8(iv)

We have,

$$T_n = (2n-1)^2$$

$$= (2n)^2 + 1 - 2 \times 2n \times 1$$

$$= 4n^2 + 1 - 4n$$

$$= 4n^2 - 4n + 1$$

$$\therefore \quad T_n = 4n^2 - 4n + 1$$

Let S_n denote the sum of n terms of th series whose nth term is T_n. Then,

$$S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n} 4k + \sum_{k=1}^{n} 1$$

$$S_n = \sum_{k=1}^{n} k^2 - 4\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1$$

$$\Rightarrow \quad S_n = 4\sum_{k=1}^{n} k^2 - 4\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1$$

$$= 4\left[\frac{n(n+1)(2n+1)}{6}\right] - 4\frac{n(n+1)}{2} + n$$

$$= \frac{2}{3}n(n+1)(2n+1) - 2n(n+1) + n$$

$$= \frac{2}{3}n(n+1)(2n+1) - 2n^2 - 2n + n$$

$$= \frac{2}{3}n(n+1)(2n+1) - 2n^2 - n$$

$$= \frac{2n}{3}(n+1)(2n+1) - n(2n+1)$$

$$= \frac{2n(n+1)(2n+1) - 3n(2n+1)}{3}$$

$$= \frac{n}{3}(2n+1)\left[2(n+1) - 3\right]$$

$$= \frac{n}{3}(2n+1)(2n+2-3)$$

$$= \frac{n}{3}(2n+1)(2n-1)$$

Hence, $S_n = \frac{n}{3}(2n+1)(2n-1)$

Q9

Here the nth term of the series is:

$$T_n = 2n(2n+2)$$

Thus the 20th term will be:

$$T_{20} = 2\times 20(2\times 20+2) = 1680$$

The infinite series can be written as:

$$2\times 4 + 4\times 6 + 6\times 8 + \cdots = \sum_{n=1}^{\infty} 2n(2n+2)$$

Therefore the sum up to 20th term will be:

$$\begin{aligned}\sum_{n=1}^{20} 2n(2n+2) &= \sum_{n=1}^{20} 4n^2 + \sum_{n=1}^{20} 4n \\ &= 4\sum_{n=1}^{20} n^2 + 4\sum_{n=1}^{20} n \\ &= 4\cdot\frac{20(20+1)(2\cdot 20+1)}{6} + 4\cdot\frac{20(20+1)}{2} \\ &= 12320\end{aligned}$$

Ex 21.2

Q1

we have,

$$3+5+9+15+23+\ldots+T_{n-1}+T_n$$

The difference between the successive terms are $5-3=2$, $9-5=4$, $15-9=6$...clearly, these difference are in A.P.

Let, S_n denote the sum to n terms of the given series.

Then,

$$S_n = 3+5+9+15+23+\ldots..T_{n-1}+T_n\ldots.(i)$$

Also, $S_n = 3-5-9-15\ldots..T_{n-1}+T_n\ldots(i)$

Subtrating (ii) from (i), we get

$$0-3+[2+4+6+8\ldots.(T_n-T_{n-1})]-T_n$$

$$T_n = 3+\frac{(n-1)}{2}[2\times2+(n-1-1)\times2]$$

$$T_n = 3+\frac{(n-1)}{2}\times2[2+n-2]$$

$$= 3+(n-1)(n)$$

$$= 3+n^2-n$$

$$= n^2-n+3$$

$$\therefore \quad S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n}\left(k^2-k+3\right)$$

$$= \sum_{k=1}^{n} k^2 - \sum_{k=1}^{n} k + \sum_{k=1}^{n} 3$$

$$- \sum_{k=1}^{n} k^2 - \sum_{k=1}^{n} k + \sum_{k=1}^{n} 3$$

Q2

We have,
$2+5+10+17+26+\ldots.$

The sequence of the differences between the successive terms of this series is 3, 5, 7, 9.....
Cleary, it is an A.P. with common difference 2.
Let T_n be the nth term and S_n denote the sum of n terms of the given series.

Then, $S_n = 2+5+10+17\ldots T_{n-1}+T_n\ldots(\text{i})$
Also, $S_n = 2+5+10+17\ldots.+T_{n-1}+T_n\ldots..(\text{ii})$

Subtracting (ii) from (i), we get.

$$0 = 2+\left[3+5+7\ldots\left(T_n - T_{n-1}\right)\right]-T_n$$

$$\Rightarrow \quad T_n = 2+\left(3+5+7+\ldots.T_n - T_{n-1}\right)$$

$$= 2+\frac{(n-1)}{2}\left[2\times 3+(n-1-1)\times 2\right]$$

$$= 2+\frac{(n-1)}{2}\times 2\left[3+n-2\right]$$

$$= 2+(n-1)(n+1)$$

$$= 2+n^2+n-n-1$$

$$= 2+n^2-1$$

$$= n^2+1$$

Q3

We have,
$1+3+7+13+21+\ldots\ldots.$

The sequence of the differences between the successive terms of this series is 2, 4, 6, 8....
clearly, it is an A.P. with common difference 2.
Let T_n be the nth term and S_n denote the sum of n terms of the given series.

Then, $S_n = 1+3+7+13+21+\ldots T_{n-1}+T_n \ldots\ldots(\mathrm{i})$
Also, $S_n = 1+3+7+13\ldots\ldots T_{n-1}+T_n \ldots.(\mathrm{ii})$

Subtracting (ii) from (i), we get

$$0 = 1+\left[2+4+6+8\ldots\ldots\left(T_n - T_{n-1}\right)\right]-T_n$$

$$\Rightarrow \quad T_n = 1+\left[2+4+6+8\ldots.\left(T_n - T_{n-1}\right)\right]$$

$$\Rightarrow \quad T_n = 1+\frac{(n-1)}{2}\left[2\times 2+(n-1-1)\times 2\right]$$

$$= 1+\frac{(n-1)}{2}\times 2\left[2+(n-2)\right]$$

$$= 1+(n-1)(n)$$

$$= 1+n^2-n$$

$$= n^2-n+1$$

$$\Rightarrow \quad S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n}\left(k^2-k+1\right) = \sum_{k=1}^{n} k^2 - \sum_{k=1}^{n} k + \sum_{k=1}^{n} 1$$

Q4

We have,

$3 + 7 + 14 + 24 + 37 + \ldots\ldots$

The sequence of the differences between the successive terms of this series is $4, 7, 10, 13 + \ldots$

clearly, it is an A.P. with common difference 3.

Let T_n be the nth term and S_n denote the sum of n terms of the given series.

Then, $S_n = 3 + 7 + 14 + 24 + 37 + \ldots T_{n-1} + T_n \ldots\ldots \text{(i)}$

Also, $S_n = 3 + 7 + 14 + 24 + \ldots\ldots + T_{n-1} + T_n \ldots \text{(ii)}$

Subtracting (ii) from (i), we get

$$0 = 3 + [4 + 7 + 10 \ldots + (T_n - T_{n-1})] - T_n$$

$$\Rightarrow \quad T_n = 3 + [4 + 7 + 10 \ldots + (T_n - T_{n-1})]$$

$$\Rightarrow \quad T_n = 3 + \frac{(n-1)}{2}[2 \times 4 + (n - 1 - 1) \times 3]$$

$$= 3 + \frac{(n-1)}{2}[8 + (n-2)3]$$

$$= 3 + \frac{(n-1)}{2}[8 + 3n - 6]$$

$$= 3 + \frac{(n-1)}{2}[2 + 3n]$$

$$= \frac{6 + (n-1)(2+3n)}{2}$$

$$= \frac{6 + 2n + 3n^2 - 2 - 3n}{2}$$

$$= \frac{6 + 3n^2 - n - 2}{2}$$

$$= \frac{3n^2 - n + 4}{2}$$

Q5

We have,

$$1+3+6+10+15+.......$$

The sequence of the differences between the successive terms of this series is 2, 3, 4, 5 + ...
clearly, it is an A.P. with common difference 1.
Let T_n be the nth term and S_n denote the sum of n terms of the given series.

Then, $S_n = 1+3+6+10+15+...T_{n-1}+T_n......(i)$
Also, $S_n = 1+3+6+10+.....+T_{n-1}+T_n....(ii)$

Subtracting (ii) from (i), we get

$$0 = 1+\left[2+3+4+5.....\left(T_n - T_{n-1}\right)\right]-T_n$$

$$\Rightarrow \quad T_n = 1+\left[2+3+4+5....\left(T_n - T_{n-1}\right)\right]$$

$$\Rightarrow \quad T_n = 1+\frac{(n-1)}{2}\left[2\times 2+(n-1-1)\times 1\right]$$

$$= 1+\frac{(n-1)}{2}\left[4+n-2\right]$$

$$= 1+\frac{(n-1)}{2}(n+2)$$

$$= 1+\frac{n^2+2n-n-2}{2}$$

$$= 1+\frac{n^2+n-2}{2}$$

$$= \frac{2+n^2+n-2}{2}$$

$$= \frac{n^2+n}{2}$$

$$\therefore \quad = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n}\left(\frac{k^2+k}{2}\right) = \frac{1}{2}\sum_{k=1}^{n} k^2 + \frac{1}{2}\sum_{k=1}^{n} k$$

Q6

We have,

$$1+4+13+40+121+\ldots\ldots.$$

The sequence of the differences between the successive terms of this series is 3, 9, 27, 81....
clearly, it is a G.P. with common difference 3.
Let T_n be the nth term and S_n denote the sum of n terms of the given series.

Then, $S_n = 1+4+13+40+\ldots\ldots+T_{n-1}+T_n\ldots\ldots(i)$

Also, $S_n = 1+4+13\ldots\ldots+T_{n-1}+T_n\ldots.(ii)$

Subtracting (ii) from (i), we get

$$0 = 1+\left[3+9+27+81\ldots.(T_n-T_{n-1})\right]-T_n$$

$$\Rightarrow \quad T_n = 1+\left[3+9+27+81\ldots(T_n-T_{n-1})\right]$$

$$\Rightarrow \quad T_n = 1+\frac{3\left(3^{n-1}-1\right)}{(3-1)}$$

$$\Rightarrow \quad T_n = 1+\frac{3}{2}\left(3^{n-1}-1\right)$$

$$= 1+\frac{3}{2}-3^{n-1}-\frac{3}{2}$$

$$= 1-\frac{3}{2}+\frac{3^n}{2}$$

$$= -\frac{1}{2}+\frac{3^n}{2}$$

$$= \frac{3^n}{2}-\frac{1}{2}$$

$$\therefore \quad S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n}\left(\frac{3^n}{n}-\frac{1}{2}\right)$$

Q7

We have,

$$4+6+9+13+18+\ldots\ldots.$$

The sequence of the differences between the successive terms of this series is 2, 3, 4, 5....
clearly, it is an A.P. with common difference 1.
Let T_n be the nth term and S_n denote the sum of n terms of the given series.

Then, $S_n = 4+6+13+18\ldots\ldots T_{n-1}+T_n\ldots\ldots(\text{i})$
Also, $S_n = 4+6+9+13\ldots\ldots T_{n-1}+T_n\ldots\ldots(\text{ii})$

Subtracting (ii) from (i), we get

$$0 = 4+\left[2+3+4+5\ldots\ldots\left(T_n-T_{n-1}\right)\right]-T_n$$

$$\Rightarrow \quad T_n = 4+\left[2+3+4+5\ldots\left(T_n-T_{n-1}\right)\right]$$

$$\Rightarrow \quad T_n = 4+\frac{(n-1)}{2}\left[2\times 2+(n-1-1)\times 1\right]$$

$$= 4+\frac{(n-1)}{2}\left[4+n-2\right]$$

$$= 4+\frac{(n-1)}{2}(n+2)$$

$$= \frac{8+n^2+2n-n-2}{2}$$

$$= \frac{n^2+n+6}{2}$$

$$\therefore \quad S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n}\left(\frac{k^2+k+6}{2}\right)$$

$$= \frac{1}{2}\sum_{k=1}^{n} k^2+\frac{1}{2}\sum_{k=1}^{n} k+\sum_{k=1}^{n} 3$$

$$\Rightarrow \quad S_n = \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6}\right]+\frac{n(n+1)}{2\times 2}+3n$$

Q8

We have,

$$2+4+7+11+16+\ldots\ldots.$$

The sequence of the differences between the successive terms of this series is 2, 3, 4, 5....
clearly, it is an A.P. with common difference 1.
Let T_n be the nth term and S_n denote the sum of n terms of the given series.

Then, $S_n = 2+4+7+11+16\ldots\ldots+T_{n-1}+T_n\ldots\ldots(i)$
Also, $S_n = 2+4+7+11+\ldots\ldots T_{n-1}+T_n\ldots\ldots(ii)$

Subtracting (ii) from (i), we get

$$0 = 2+\left[2+3+4+5\ldots\ldots\left(T_n - T_{n-1}\right)\right]-T_n$$

$$\Rightarrow \quad T_n = 2+\frac{(n-1)}{2}\left[2\times 2+(n-1-1)\times 1\right]$$

$$= 2+\frac{(n-1)}{2}\left[4+n-2\right]$$

$$= 2+\frac{(n-1)}{2}(n+2)$$

$$= \frac{4+n^2+2n-n-2}{2}$$

$$= \frac{n^2+n+2}{2}$$

$$= \frac{n^2}{2}+\frac{n}{2}+1$$

$$\Rightarrow \quad S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n}\left(\frac{k^2}{2}+\frac{k}{2}+1\right)$$

Q9

We have,

$$\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots..$$

Let T_r be the rth term of the given series. Then,

$$T_r=\frac{1}{(3r-2)(3r+1)}, r=1,2\ldots,n$$

$$\Rightarrow \quad T_r=\frac{1}{3}\left[\frac{1}{3r-2}-\frac{1}{3r+1}\right]$$

$$\therefore \quad \text{required sum}=\frac{1}{3}\sum_{r=1}^{n}T_r$$

$$=\frac{1}{3}\sum_{r=1}^{n}\left[\frac{1}{3n-2}-\frac{1}{3n+1}\right]$$

$$=\frac{1}{3}\left[\left(1-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{10}\right)\ldots.\left(\frac{1}{3n-2}-\frac{1}{3n+1}\right)\right]$$

$$=\frac{1}{3}\left[1-\frac{1}{3n+1}\right]$$

$$=\frac{1}{3}\left[\frac{3n+1-1}{3n+1}\right]$$

$$=\frac{1}{3}\times\frac{3n}{3n+1}$$

$$=\frac{n}{3n+1}$$

Hence, required sum $=\dfrac{n}{3n+1}$

Q10

We have,

$$\frac{1}{1.6}+\frac{1}{6.11}+\frac{1}{11.14}+\frac{1}{14.19}+\ldots\ldots\frac{1}{(5n-4)(5n+1)}$$

Let T_r be the rth term of the given series. Then,

$$T_r=\frac{1}{(5r-4)(5r+1)},r=1,2\ldots,n$$

$$\Rightarrow \qquad T_r=\frac{1}{5}\left[\frac{1}{5r-2}-\frac{1}{5r+1}\right]$$

$$\therefore \quad \text{required sum}=\frac{1}{5}\sum_{r=1}^{n}T_r$$

$$=\frac{1}{5}\sum_{r=1}^{n}\left[\frac{1}{5r-4}-\frac{1}{5r+1}\right]$$

$$=\frac{1}{5}\left[\left(1-\frac{1}{6}\right)+\left(\frac{1}{6}-\frac{1}{11}\right)+\left(\frac{1}{11}-\frac{1}{14}\right)\ldots\left(\frac{1}{5n-4}-\frac{1}{5n+1}\right)\right]$$

$$=\frac{1}{5}\left[1-\frac{1}{5n+1}\right]$$

$$=\frac{1}{5}\left[\frac{5n+1-1}{5n+1}\right]$$

$$=\frac{1}{5}\times\frac{5n}{5n+1}$$

$$=\frac{n}{5n+1}$$

Hence, required sum $=\dfrac{n}{5n+1}$