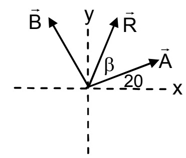
SOLUTIONS TO CONCEPTS
CHAPTER – 2

1. As shown in the figure,

   The angle between $\vec{A}$ and $\vec{B}$ = 110° – 20° = 90°

   $|\vec{A}|$ = 3 and $|\vec{B}|$ = 4m

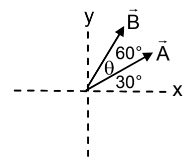
   Resultant R = $\sqrt{A^2 + B^2 + 2AB\cos\theta}$ = 5 m

   Let β be the angle between $\vec{R}$ and $\vec{A}$

   $\beta = \tan^{-1}\left(\dfrac{4\sin 90°}{3 + 4\cos 90°}\right) = \tan^{-1}(4/3) = 53°$

   ∴ Resultant vector makes angle (53° + 20°) = 73° with x-axis.

2. Angle between $\vec{A}$ and $\vec{B}$ is θ = 60° – 30° =30°

   $|\vec{A}|$ and $|\vec{B}|$ = 10 unit

   R = $\sqrt{10^2 + 10^2 + 2.10.10.\cos 30°}$ = 19.3

   β be the angle between $\vec{R}$ and $\vec{A}$

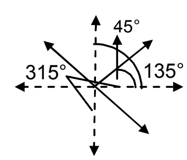
   $\beta = \tan^{-1}\left(\dfrac{10\sin 30°}{10 + 10\cos 30°}\right) = \tan^{-1}\left(\dfrac{1}{2+\sqrt{3}}\right) = \tan^{-1}(0.26795) = 15°$

   ∴ Resultant makes 15° + 30° = 45° angle with x-axis.

3. x component of $\vec{A}$ = 100 cos 45° = $100/\sqrt{2}$ unit

   x component of $\vec{B}$ = 100 cos 135° = $100/\sqrt{2}$

   x component of $\vec{C}$ = 100 cos 315° = $100/\sqrt{2}$

   Resultant x component = $100/\sqrt{2}$ – $100/\sqrt{2}$ + $100/\sqrt{2}$ = $100/\sqrt{2}$

   y component of $\vec{A}$ = 100 sin 45° = $100/\sqrt{2}$ unit

   y component of $\vec{B}$ = 100 sin 135° = $100/\sqrt{2}$

   y component of $\vec{C}$ = 100 sin 315° = $-100/\sqrt{2}$

   Resultant y component = $100/\sqrt{2}$ + $100/\sqrt{2}$ – $100/\sqrt{2}$ = $100/\sqrt{2}$

   Resultant = 100

   Tan α = $\dfrac{\text{y component}}{\text{x component}}$ = 1

   ⇒ α = $\tan^{-1}$ (1) = 45°

   The resultant is 100 unit at 45° with x-axis.

4. $\vec{a} = 4\vec{i} + 3\vec{j}$, $\vec{b} = 3\vec{i} + 4\vec{j}$

   a) $|\vec{a}| = \sqrt{4^2 + 3^2} = 5$

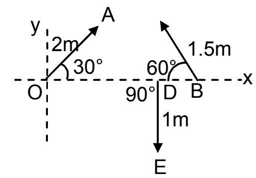
   b) $|\vec{b}| = \sqrt{9 + 16} = 5$

   c) $|\vec{a} + \vec{b}| = |7\vec{i} + 7\vec{j}| = 7\sqrt{2}$

   d) $\vec{a} - \vec{b} = (-3+4)\hat{i} + (-4+3)\hat{j} = \hat{i} - \hat{j}$

   $|\vec{a} - \vec{b}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$.

5. x component of $\overrightarrow{OA}$ = 2cos30° = $\sqrt{3}$



x component of $\overrightarrow{BC}$ = 1.5 cos 120° = –0.75

x component of $\overrightarrow{DE}$ = 1 cos 270° = 0

y component of $\overrightarrow{OA}$ = 2 sin 30° = 1

y component of $\overrightarrow{BC}$ = 1.5 sin 120° = 1.3

y component of $\overrightarrow{DE}$ = 1 sin 270° = –1

R_x = x component of resultant = $\sqrt{3} - 0.75 + 0$ = 0.98 m

R_y = resultant y component = 1 + 1.3 – 1 = 1.3 m

So, R = Resultant = 1.6 m

If it makes and angle α with positive x-axis

Tan α = $\dfrac{\text{y component}}{\text{x component}}$ = 1.32

$\Rightarrow \alpha = \tan^{-1} 1.32$

6. $|\vec{a}|$ = 3m $|\vec{b}|$ = 4

a) If R = 1 unit $\Rightarrow \sqrt{3^2 + 4^2 + 2.3.4.\cos\theta}$ = 1

θ = 180°

b) $\sqrt{3^2 + 4^2 + 2.3.4.\cos\theta}$ = 5

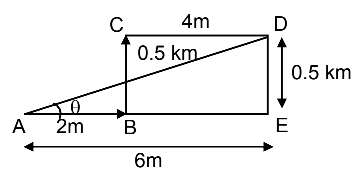
θ = 90°

c) $\sqrt{3^2 + 4^2 + 2.3.4.\cos\theta}$ = 7

θ = 0°

Angle between them is 0°.

7. $\overrightarrow{AD} = 2\hat{i} + 0.5\hat{J} + 4\hat{K}$ = $6\hat{i} + 0.5\hat{j}$



AD = $\sqrt{AE^2 + DE^2}$ = 6.02 KM

Tan θ = DE / AE = 1/12

$\theta = \tan^{-1}$ (1/12)

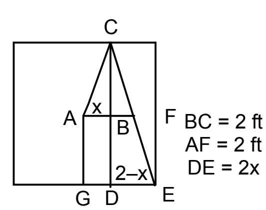
The displacement of the car is 6.02 km along the distance $\tan^{-1}$ (1/12) with positive x-axis.

8. In ΔABC, tanθ = x/2 and in ΔDCE, tanθ = (2 – x)/4 tan θ = (x/2) = (2 – x)/4 = 4x

$\Rightarrow$ 4 – 2x = 4x

$\Rightarrow$ 6x = 4 $\Rightarrow$ x = 2/3 ft

a) In ΔABC, AC = $\sqrt{AB^2 + BC^2}$ = $\dfrac{2}{3}\sqrt{10}$ ft

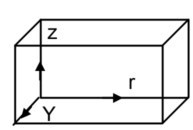


b) In ΔCDE, DE = 1 – (2/3) = 4/3 ft

CD = 4 ft. So, CE = $\sqrt{CD^2 + DE^2}$ = $\dfrac{4}{3}\sqrt{10}$ ft

c) In ΔAGE, AE = $\sqrt{AG^2 + GE^2}$ = $2\sqrt{2}$ ft.

9. Here the displacement vector $\vec{r} = 7\hat{i} + 4\hat{j} + 3\hat{k}$



a) magnitude of displacement = $\sqrt{74}$ ft

b) the components of the displacement vector are 7 ft, 4 ft and 3 ft.

10. $\vec{a}$ is a vector of magnitude 4.5 unit due north.

   a) $3|\vec{a}| = 3 \times 4.5 = 13.5$

   $3\vec{a}$ is along north having magnitude 13.5 units.

   b) $-4|\vec{a}| = -4 \times 1.5 = -6$ unit

   $-4\vec{a}$ is a vector of magnitude 6 unit due south.

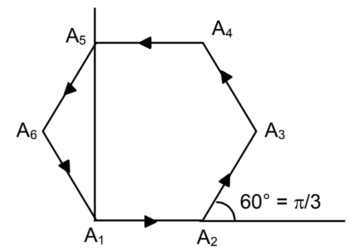
11. $|\vec{a}| = 2$ m, $|\vec{b}| = 3$ m

   angle between them $\theta = 60°$

   a) $\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cos 60° = 2 \times 3 \times 1/2 = 3\ m^2$

   b) $|\vec{a}\times\vec{b}| = |\vec{a}|\cdot|\vec{b}|\sin 60° = 2 \times 3 \times \sqrt{3/2} = 3\sqrt{3}\ m^2$.

12. We know that according to polygon law of vector addition, the resultant of these six vectors is zero.



   Here A = B = C = D = E = F (magnitude)

   So, $Rx = A\cos\theta + A\cos\pi/3 + A\cos 2\pi/3 + A\cos 3\pi/3 + A\cos 4\pi/4 + A\cos 5\pi/5 = 0$

   [As resultant is zero. X component of resultant $R_x = 0$]

   $= \cos\theta + \cos\pi/3 + \cos 2\pi/3 + \cos 3\pi/3 + \cos 4\pi/3 + \cos 5\pi/3 = 0$

   Note : Similarly it can be proved that,

   $\sin\theta + \sin\pi/3 + \sin 2\pi/3 + \sin 3\pi/3 + \sin 4\pi/3 + \sin 5\pi/3 = 0$

13. $\vec{a} = 2\vec{i} + 3\vec{j} + 4\vec{k};\ \vec{b} = 3\vec{i} + 4\vec{j} + 5\vec{k}$

   $\vec{a}\cdot\vec{b} = ab\cos\theta \Rightarrow \theta = \cos^{-1}\dfrac{\vec{a}\cdot\vec{b}}{ab}$

   $\Rightarrow \cos^{-1}\dfrac{2\times3 + 3\times4 + 4\times5}{\sqrt{2^2+3^2+4^2}\sqrt{3^2+4^2+5^2}} = \cos^{-1}\left(\dfrac{38}{\sqrt{1450}}\right)$

14. $\vec{A}\cdot(\vec{A}\times\vec{B}) = 0$ (claim)

   As, $\vec{A}\times\vec{B} = AB\sin\theta\hat{n}$

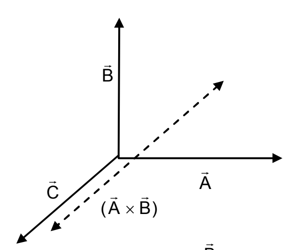
   $AB\sin\theta\ \hat{n}$ is a vector which is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$, this implies that it is also perpendicular to $\vec{A}$. As dot product of two perpendicular vector is zero.

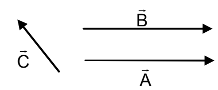
   Thus $\vec{A}\cdot(\vec{A}\times\vec{B}) = 0$.

15. $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{B} = 4\hat{i} + 3\hat{j} + 2\hat{k}$

   $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 4 & 3 & 2 \end{vmatrix} \Rightarrow \hat{i}(6-12) - \hat{j}(4-16) + \hat{k}(6-12) = -6\hat{i} + 12\hat{j} - 6\hat{k}$.

16. Given that $\vec{A}$, $\vec{B}$ and $\vec{C}$ are mutually perpendicular





   $\vec{A} \times \vec{B}$ is a vector which direction is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

   Also $\vec{C}$ is perpendicular to $\vec{A}$ and $\vec{B}$

   $\therefore$ Angle between $\vec{C}$ and $\vec{A} \times \vec{B}$ is 0° or 180° (fig.1)

   So, $\vec{C} \times (\vec{A} \times \vec{B}) = 0$

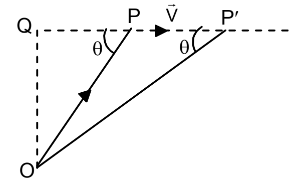
   The converse is not true.

   For example, if two of the vector are parallel, (fig.2), then also

   $\vec{C} \times (\vec{A} \times \vec{B}) = 0$

   So, they need not be mutually perpendicular.

17. The particle moves on the straight line PP' at speed v.



From the figure,

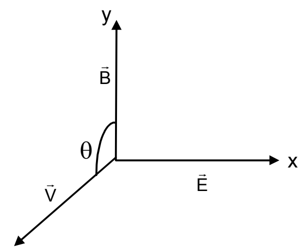
$\overrightarrow{OP} \times v = (OP)v\sin\theta\hat{n}$ = v(OP) sin θ $\hat{n}$ = v(OQ)$\hat{n}$

It can be seen from the figure, OQ = OP sin θ = OP' sin θ'

So, whatever may be the position of the particle, the magnitude and direction of $\overrightarrow{OP} \times \vec{v}$ remain constant.

$\therefore$ $\overrightarrow{OP} \times \vec{v}$ is independent of the position P.

18. Give $\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B}) = 0$



$\Rightarrow \vec{E} = -(\vec{v} \times \vec{B})$

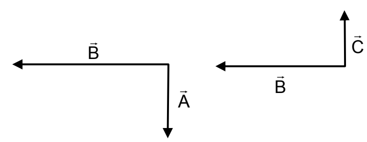
So, the direction of $\vec{v} \times \vec{B}$ should be opposite to the direction of $\vec{E}$. Hence, $\vec{v}$ should be in the positive yz-plane.

Again, E = vB sin θ ⇒ v = $\dfrac{E}{B\sin\theta}$

For v to be minimum, θ = 90° and so v_{min} = F/B

So, the particle must be projected at a minimum speed of E/B along +ve z-axis (θ = 90°) as shown in the figure, so that the force is zero.

19. For example, as shown in the figure,



$\vec{A} \perp \vec{B}$ $\vec{B}$ along west

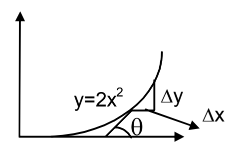
$\vec{B} \perp \vec{C}$ $\vec{A}$ along south

$\vec{C}$ along north

$\vec{A} \cdot \vec{B} = 0$ $\therefore$ $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{C}$

$\vec{B} \cdot \vec{C} = 0$ But $\vec{B} \neq \vec{C}$

20. The graph y = $2x^2$ should be drawn by the student on a graph paper for exact results.



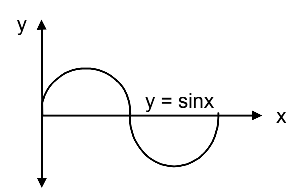
To find slope at any point, draw a tangent at the point and extend the line to meet x-axis. Then find tan θ as shown in the figure.

It can be checked that,

Slope = tan θ = $\dfrac{dy}{dx} = \dfrac{d}{dx}(2x^2)$ = 4x

Where x = the x-coordinate of the point where the slope is to be measured.

21. y = sinx



So, y + Δy = sin (x + Δx)

Δy = sin (x + Δx) – sin x

= $\left(\dfrac{\pi}{3} + \dfrac{\pi}{100}\right) - \sin\dfrac{\pi}{3}$ = 0.0157.

22. Given that, i = $i_0 e^{-t/RC}$

$\therefore$ Rate of change of current = $\dfrac{di}{dt} = \dfrac{d}{dt} i_0 e^{-i/RC} = i_0 \dfrac{d}{dt} e^{-t/RC}$ = $\dfrac{-i_0}{RC} \times e^{-t/RC}$

When   a) t = 0, $\dfrac{di}{dt} = \dfrac{-i}{RC}$

b) when t = RC, $\dfrac{di}{dt} = \dfrac{-i}{RCe}$

c) when t = 10 RC, $\dfrac{di}{dt} = \dfrac{-i_0}{RCe^{10}}$

23. Equation i = $i_0 e^{-t/RC}$

i_0 = 2A, R = 6 × 10^{-5} Ω, C = 0.0500 × 10^{-6} F = 5 × 10^{-7} F

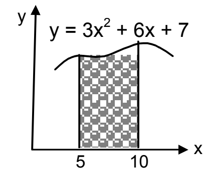
a) i = $2 \times e^{\left(\frac{-0.3}{6\times0^3\times5\times10^{-7}}\right)} = 2 \times e^{\left(\frac{-0.3}{0.3}\right)} = \frac{2}{e}\text{amp}$.

b) $\frac{di}{dt} = \frac{-i_0}{RC}e^{-t/RC}$ when t = 0.3 sec ⇒ $\frac{di}{dt} = -\frac{2}{0.30}e^{(-0.3/0.3)} = \frac{-20}{3e}\text{Amp/sec}$

c) At t = 0.31 sec, i = $2e^{(-0.3/0.3)} = \frac{5.8}{3e}\text{Amp}$.

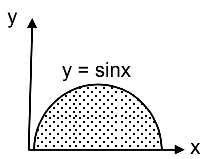
24. y = $3x^2$ + 6x + 7

∴ Area bounded by the curve, x axis with coordinates with x = 5 and x = 10 is given by,



Area = $\int_0^y dy$ = $\int_5^{10} (3x^2 + 6x + 7)dx$ = $3\left.\frac{x^3}{3}\right]_5^{10} + 5\left.\frac{x^2}{3}\right]_5^{10} + 7x]_5^{10}$ = 1135 sq.units.

25. Area = $\int_0^y dy$ = $\int_0^{\pi} \sin x dx = -[\cos x]_0^{\pi}$ = 2

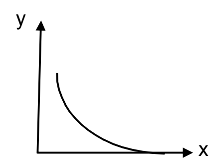


26. The given function is y = e^{-x}

When x = 0, y = e^{-0} = 1

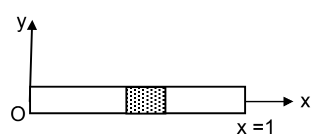
x increases, y value deceases and only at x = ∞, y = 0.

So, the required area can be found out by integrating the function from 0 to ∞.



So, Area = $\int_0^{\infty} e^{-x}dx = -[e^{-x}]_0^{\infty} = 1$.

27. $\rho = \frac{\text{mass}}{\text{length}} = a + bx$



a) S.I. unit of ‘a’ = kg/m and SI unit of ‘b’ = kg/m^2 (from principle of homogeneity of dimensions)

b) Let us consider a small element of length ‘dx’ at a distance x from the origin as shown in the figure.

∴ dm = mass of the element = ρ dx = (a + bx) dx

So, mass of the rod = m = $\int dm = \int_0^L (a + bx)dx$ = $\left[ax + \frac{bx^2}{2}\right]_0^L = aL + \frac{bL^2}{2}$

28. $\frac{dp}{dt}$ = (10 N) + (2 N/S)t

momentum is zero at t = 0

∴ momentum at t = 10 sec will be

dp = [(10 N) + 2Ns t]dt

$\int_0^p dp = \int_0^{10} 10dt + \int_0^{10} (2tdt)$ = $10t]_0^{10} + 2\left.\frac{t^2}{2}\right]_0^{10}$ = 200 kg m/s.

29. The change in a function of y and the independent variable x are related as $\frac{dy}{dx} = x^2$.

$\Rightarrow dy = x^2\, dx$

Taking integration of both sides,

$\int dy = \int x^2 dx \Rightarrow y = \frac{x^3}{3} + c$

$\therefore$ y as a function of x is represented by $y = \frac{x^3}{3} + c$.

30. The number significant digits

a) 1001	No.of significant digits = 4
b) 100.1	No.of significant digits = 4
c) 100.10	No.of significant digits = 5
d) 0.001001	No.of significant digits = 4

31. The metre scale is graduated at every millimeter.

1 m = 100 mm

The minimum no.of significant digit may be 1 (e.g. for measurements like 5 mm, 7 mm etc) and the maximum no.of significant digits may be 4 (e.g.1000 mm)

So, the no.of significant digits may be 1, 2, 3 or 4.

32. a) In the value 3472, after the digit 4, 7 is present. Its value is greater than 5.
So, the next two digits are neglected and the value of 4 is increased by 1.
$\therefore$ value becomes 3500

b) value = 84

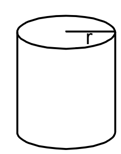
c) 2.6

d) value is 28.

33. Given that, for the cylinder

Length = l = 4.54 cm, radius = r = 1.75 cm

Volume = $\pi r^2 l = \pi \times (4.54) \times (1.75)^2$



Since, the minimum no.of significant digits on a particular term is 3, the result should have 3 significant digits and others rounded off.

So, volume $V = \pi r^2 l = (3.14) \times (1.75) \times (1.75) \times (4.54) = 43.6577\ cm^3$

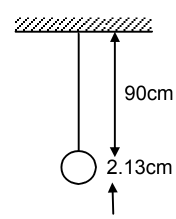
Since, it is to be rounded off to 3 significant digits, $V = 43.7\ cm^3$.

34. We know that,

Average thickness = $\frac{2.17 + 2.17 + 2.18}{3}$ = 2.1733 mm

Rounding off to 3 significant digits, average thickness = 2.17 mm.

35. As shown in the figure,



Actual effective length = (90.0 + 2.13) cm

But, in the measurement 90.0 cm, the no. of significant digits is only 2.

So, the addition must be done by considering only 2 significant digits of each measurement.

So, effective length = 90.0 + 2.1 = 92.1 cm.

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