Exercise – 10A

1. Which of the following are quadratic equation in x?

(i) $x^2 - x + 3 = 0$

(ii) $2x^2 + \frac{5}{2}x - \sqrt{3} = 0$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2}$

(iv) $\frac{1}{3}x^2 + \frac{1}{5}x - 2 = 0$

(v) $x^2 - 3x - \sqrt{x} + 4 = 0$

(vi) $x - \frac{6}{x} = 3$

(vii) $x^2 - \frac{2}{x} = x^2$

(viii) $x^2 - \frac{1}{x^2} = 5$

(ix) $(x+2)^3 = x^3 - 8$

(x) $(2x+3)(3x+2) = 6(x-1)(x-2)$

(xi) $\left(x + \frac{1}{x}\right)^2 = 2\left(x + \frac{1}{x}\right) + 3$

Sol:

(i) $\left(x^2 - x + 3\right)$ is a quadratic polynomial

$\therefore x^2 - x + 3 = 0$ is a quadratic equation.

(ii) Clearly, $\left(2x^2 + \frac{5}{2}x - \sqrt{3}\right)$ is a quadratic polynomial.

$\therefore 2x^2 + \frac{5}{2}x - \sqrt{3} = 0$ is a quadratic equation.

(iii) Clearly, $\left(\sqrt{2}x^2 + 7x + 5\sqrt{2}\right)$ is a quadratic polynomial.

$\therefore \sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ is a quadratic equation.

(iv) Clearly, $\left(\frac{1}{3}x^2 + \frac{1}{5}x - 2\right)$ is a quadratic polynomial.

$\therefore \frac{1}{3}x^2 + \frac{1}{5}x - 2 = 0$ is a quadratic equation.

(v) $\left(x^2 - 3x - \sqrt{x} + 4\right)$ contains a term with $\sqrt{x}$, i.e., $x^{\frac{1}{2}}$, where $\frac{1}{2}$ is not a ineger.

Therefore, it is not a quadratic polynomial.

$\therefore x^2 - 3x - \sqrt{x} + 4 = 0$ is not a quadratic equation.

(vi) $x - \frac{6}{x} = 3$

$\Rightarrow x^2 - 6 = 3x$

$\Rightarrow x^2 - 3x - 6 = 0$

$\left(x^3 - 3x - 6\right)$ is not quadratic polynomial; therefore, the given equation is quadratic.

(vii) $x^2 - \frac{2}{x} = x^2$

$\Rightarrow x^2 + 2 = x^3$

$\Rightarrow x^3 - x^2 - 2 = 0$

$\left(x^3 - x^2 - 2\right)$ is not a quadratic polynomial.

$\therefore x^3 - x^2 - 2 = 0$ is not a quadratic equation.

(viii) $x^2 - \frac{1}{x^2} = 5$

$\Rightarrow x^4 - 1 = 5x^2$

$\Rightarrow x^4 - 5x^2 - 1 = 0$

$\left(x^4 - 5x^2 - 1\right)$ is a polynomial with degree 4.

$\therefore x^4 - 5x^2 - 1 = 0$ is not a quadratic equation.

(ix) $(x+2)^3 = x^3 - 8$

$\Rightarrow x^3 + 6x^2 + 12x + 8 = x^3 - 8$

$\Rightarrow 6x^2 + 12x + 16 = 0$

This is of the form $ax^2 + bx + c = 0$

Hence, the given equation is a quadratic equation.

(x) $(2x+3)(3x+2) = 6(x-1)(x-2)$

$\Rightarrow 6x^2 + 4x + 9x + 6 = 6\left(x^2 - 3x + 2\right)$

$\Rightarrow 6x^2 + 13x + 6 = 6x^2 - 18x + 12$

$\Rightarrow 31x - 6 = 0$

This is of the form $ax^2 + bx + c = 0$

Hence, the given equation is not a quadratic equation.

(xi) $\left(x + \frac{1}{x}\right)^2 = 2\left(x + \frac{1}{x}\right) + 3$

$$\Rightarrow \left(\frac{x^2+1}{x}\right)^2 = 2\left(\frac{x^2+1}{x}\right)+3$$

$$\Rightarrow \left(x^2+1\right)^2 = 2x\left(x^2+1\right)+3x^2$$

$$\Rightarrow x^4+2x^2+1 = 2x^3+2x+3x^2$$

$$\Rightarrow x^4-2x^3-x^2-2x+1=0$$

This is not of the form $ax^2+bx+c=0$

Hence, the given equation is not a quadratic equation.

2. Which of the following are the roots of $3x^2+2x-1=0$?

(i) -1 (ii) $\frac{1}{3}$ (iii) $-\frac{1}{2}$

Sol:

The given equation is $\left(3x^2+2x-1=0\right)$.

(i) $x=\left(-1\right)$

L.H.S. $= x^2+2x-1$

$=3\times\left(-1\right)^2+2\times\left(-1\right)-1$

$=3-2-1$

$=0$

$=R.H.S.$

Thus, $\left(-1\right)$ is a root of $\left(3x^2+2x-1=0\right)$.

(ii) On subtracting $x=\frac{1}{3}$ in the given equation, we get:

L.H.S. $=3x^2+2x-1$

$=3\times\left(\frac{1}{3}\right)^2+2\times\frac{1}{3}-1$

$=3\times\frac{1}{9}+\frac{2}{3}-1$

$=\frac{1+2-3}{3}$

$=\frac{0}{3}$

$=0$

$=R.H.S.$

Thus, $\left(\frac{1}{3}\right)$ is a root of $\left(3x^2+2x-1=0\right)$

(iii) On subtracting $x = \left(-\frac{1}{2}\right)$ in the given equation, we get

L.H.S. $= 3x^2 + 2x - 1$

$= 3 \times \left(-\frac{1}{2}\right)^2 + 2 \times \left(-\frac{1}{2}\right) - 1$

$= 3 \times \frac{1}{4} - 1 - 1$

$= \frac{3}{4} - 2$

$- \frac{3-8}{4}$

$= \frac{-5}{4} \neq 0$

Thus, $L.H.S = R.H.S.$

Hence, $\left(-\frac{1}{2}\right)$ is a solution of $\left(3x^2 + 2x - 1 = 0\right)$.

3. Find the value of *k* for which $x = 1$ is a root of the equation $x^2 + kx + 3 = 0$.

Sol:

It is given that $(x = 1)$ is a root of $\left(x^2 + kx + 3 = 0\right)$.

Therefore, $(x = 1)$ must satisfy the equation.

$\Rightarrow (1)^2 + k \times 1 + 3 = 0$

$\Rightarrow k + 4 = 0$

$\Rightarrow k = -4$

Hence, the required value of *k* is –4.

4. Find the value of *a* and *b* for which $x = \frac{3}{4}$ and $x = -2$ are the roots of the equation

$ax^2 + bx - 6 = 0$

Sol:

It is given that $\frac{3}{4}$ is a root of $ax^2 + bx - 6 = 0$; therefore, we have:

$a \times \left(\frac{3}{4}\right)^2 + b \times \frac{3}{4} - 6 = 0$

$\Rightarrow \frac{9a}{16}+\frac{3b}{4}=6$

$\Rightarrow \frac{9a+12b}{16}=6$

$\Rightarrow 9a+12b-96=0$

$\Rightarrow 3a+4b=32 \qquad ....(i)$

Again, (-2) is a root of $ax^2+bx-6=0;$ therefore, we have:

$a\times(-2)^2+b\times(-2)-6=0$

$\Rightarrow 4a-2b=6$

$\Rightarrow 2a-b=3 \qquad ....(ii)$

On multiplying (ii) by 4 and adding the result with (i), we get:

$\Rightarrow 3a+4b+8a-4b=32+12$

$\Rightarrow 11a=44$

$\Rightarrow a=4$

Putting the value of a in (ii), we get:

$2\times4-b=3$

$\Rightarrow 8-b=3$

$\Rightarrow b=5$

Hence, the required values of *a* and *b* are 4 and 5, respectively.

5. $(2x-3)(3x+1)=0$

Sol:

$(2x-3)(3x+1)=0$

$\Rightarrow 2x-3=0 \; or \; 3x+1=0$

$\Rightarrow 2x=3 \; or \; 3x=-1$

$\Rightarrow x=\frac{3}{2} \; or \; x=-\frac{1}{3}$

Hence the roots of the given equation are $\frac{3}{2} \; and \; -\frac{1}{3}.$

6. $4x^2+5x=0$

Sol:

$4x^2+5x=0$

$\Rightarrow x(4x+5)=0$

$\Rightarrow x=0 \; or \; 4x+5=0$

$\Rightarrow x = 0 \ or \ x = -\frac{5}{4}$

Hence, the roots of the given equation are 0 and $-\frac{5}{4}$.

7. $3x^2 - 243 = 0.$

Sol:

Given:

$3x^2 - 243 = 0$

$\Rightarrow 3\left(x^2 - 81\right) = 0$

$\Rightarrow (x)^2 - (9)^2 = 0$

$\Rightarrow (x+9)(x-9) = 0$

$\Rightarrow x + 9 = 0 \ or \ x - 9 = 0$

$\Rightarrow x = -9 \ or \ x = 9$

Hence, -9 and 9 are the roots of the equation $3x^2 - 243 = 0.$

8. $2x^2 + x - 6 = 0$

Sol:

We write, $x = 4x - 3x$ as $2x^2 \times (-6) = -12x^2 = 4x \times (-3x)$

$\therefore 2x^2 + x - 6 = 0$

$\Rightarrow 2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow 2x(x+2) - 3(x+2) = 0$

$\Rightarrow (x+2)(2x-3) = 0$

$\Rightarrow x + 2 = 0 \ or \ 2x - 3 = 0$

$\Rightarrow x = -2 \ or \ x = \frac{3}{2}$

Hence, the roots of the given equation are -2 and $\frac{3}{2}$.

9. $x^2 + 6x + 5 = 0$

Sol:

We write, $6x = x + 5x$ as $x^2 \times 5 = 5x^2 = x \times 5x$

$\therefore x^2 + 6x + 5 = 0$

$\Rightarrow x^2 + x - 5x + 5 = 0$

$\Rightarrow x(x+1)+5(x+1)=0$

$\Rightarrow (x+1)(x+5)=0$

$\Rightarrow x+1=0$ *or* $x+5=0$

$\Rightarrow x=-1$ *or* $x=-5$

Hence, the roots of the given equation are -1 and -5.

10. $9x^2-3x-2=0$

Sol:

We write, $-3x=3x-6x$ as $9x^2\times(-2)=-18x^2=3x\times(-6x)$

$\therefore 9x^2-3x-2=0$

$\Rightarrow 9x^2+3x-6x-2=0$

$\Rightarrow 3x(3x+1)-2(3x+1)=0$

$\Rightarrow (3x+1)(3x-2)=0$

$\Rightarrow 3x+1=0$ *or* $3x-2=0$

$\Rightarrow x=-\frac{1}{3}$ *or* $x=\frac{2}{3}$

Hence, the roots of the given equation are $-\frac{1}{3}$ and $\frac{2}{3}$.

11. $x^2+12x+35=0$

Sol:

Given:

$x^2+12x+35=0$

$\Rightarrow x^2+7x+5x+35=0$

$\Rightarrow x(x+7)+5(x+7)=0$

$\Rightarrow (x+5)(x+7)=0$

$\Rightarrow x+5=0$ *or* $x+7=0$

$\Rightarrow x=-5$ *or* $x=-7$

Hence, -5 and -7 are the roots of the equation $x^2+12x+35=0$.

12. $x^2=18x-77$

Sol:

Given

$x^2=18x-77$

$\Rightarrow x^2-18x+77=0$

$\Rightarrow x^2 - (11x + 7x) + 77 = 0$

$\Rightarrow x^2 - 11x - 7x + 77 = 0$

$\Rightarrow x(x-11) - 7(x-11) = 0$

$\Rightarrow (x-7)(x-11) = 0$

$\Rightarrow x - 7 = 0 \ or \ x - 11 = 0$

$\Rightarrow x = 7 \ or \ x = 11$

Hence, 7 and 11 are the roots of the equation $x^2 = 18x - 77$.

13. $6x^2 + 11x + 3 = 0.$

Sol:

Given:

$6x^2 + 11x + 3 = 0$

$\Rightarrow 6x^2 + 9x + 2x + 3 = 0$

$\Rightarrow 3x(2x+3) + 1(2x+3) = 0$

$\Rightarrow (3x+1)(2x+3) = 0$

$\Rightarrow 3x + 1 = 0 \ or \ 2x + 3 = 0$

$\Rightarrow x = \frac{-1}{3} \ or \ x = \frac{-3}{2}$

Hence, $\frac{-1}{3}$ and $\frac{-3}{2}$ are the roots of the equation $6x^2 + 11x + 3 = 0.$

14. $6x^2 + x - 12 = 0.$

Sol:

Given:

$6x^2 + x - 12 = 0$

$\Rightarrow 6x^2 + 9x - 8x - 12 = 0$

$\Rightarrow 3x(2x+3) - 4(2x+3) = 0$

$\Rightarrow (3x-4)(2x+3) = 0$

$\Rightarrow 3x - 4 = 0 \ or \ 2x + 3 = 0$

$\Rightarrow x = \frac{4}{3} \ or \ x = \frac{-3}{2}$

Hence, $\frac{4}{3}$ and $\frac{-3}{2}$ are the roots of the equation $6x^2 + x - 12 = 0.$

15. $3x^2-2x-1=0$

Sol:

We write, $-2x=-3x+x$ as $3x^2\times(-1)=-3x^2=(-3x)\times x$

$\therefore 3x^2-2x-1=0$

$\Rightarrow 3x^2-3x+x-1=0$

$\Rightarrow 3x(x-1)+1(x-1)=0$

$\Rightarrow (x-1)(3x+1)=0$

$\Rightarrow x-1=0 \; or \; 3x+1=0$

$\Rightarrow x=1 \text{ or } x=-\frac{1}{3}$

Hence, the roots of the given equation are 1 and $-\frac{1}{3}$.

16. $4x^2-9x=100$

Sol:

Given:

$4x^2-9x=100$

$\Rightarrow 4x^2-9x-100=0$

$\Rightarrow 4x^2-(25x-16x)-100=0$

$\Rightarrow 4x^2-25x+16x-100=0$

$\Rightarrow x(4x-25)+4(4x-25)=0$

$\Rightarrow (4x-25)(x+4)=0$

$\Rightarrow 4x-25=0 \; or \; x+4=0$

$\Rightarrow x=\frac{25}{4} \; or \; x=-4$

Hence, the roots of the equation are $\frac{25}{4}$ *and* -4.

17. $15x^2-28=x$

Sol:

Given:

$15x^2-28=x$

$\Rightarrow 15x^2-x-28=0$

$\Rightarrow 15x^2-(21x-20x)-28=0$

$\Rightarrow 15x^2-21x+20x-28=0$

$\Rightarrow 3x(5x-7)+4(5x-7)=0$

$\Rightarrow (3x+4)(5x-7)=0$

$\Rightarrow 3x+4=0$ or $5x-7=0$

$\Rightarrow x=\frac{-4}{3}$ *or* $x=\frac{7}{5}$

Hence, the roots of the equation are $\frac{-4}{3}$ and $\frac{7}{5}$.

18. $4-11x=3x^2$

Sol:

Given:

$4-11x=3x^2$

$\Rightarrow 3x^2+11x-4=0$

$\Rightarrow 3x^2+12x-x-4=0$

$\Rightarrow 3x(x+4)-1(x+4)=0$

$\Rightarrow (x+4)(3x-1)=0$

$\Rightarrow x+4=0$ *or* $3x-1=0$

$\Rightarrow x=-4$ *or* $x=\frac{1}{3}$

Hence, the roots of the equation are -4 and $\frac{1}{3}$.

19. $48x^2-13x-1=0$

Sol:

Given:

$48x^2-13x-1=0$

$\Rightarrow 48x^2-(16x-3x)-1=0$

$\Rightarrow 48x^2-16x+3x-1=0$

$\Rightarrow 16x(3x-1)+1(3x-1)=0$

$\Rightarrow (16x+1)(3x-1)=0$

$\Rightarrow 16x+1=0$ *or* $3x-1=0$

$\Rightarrow x=\frac{-1}{16}$ *or* $x=\frac{1}{3}$

Hence, the roots of the equation are $\frac{-1}{16}$ and $\frac{1}{3}$.

20. $x^2+2\sqrt{2}x-6=0$

Sol:

We write:

$2\sqrt{2}x=3\sqrt{2}x-\sqrt{2}x$ as $x^2\times(-6)=-6x^2=3\sqrt{2}x\times\left(-\sqrt{2}x\right)$

$\therefore x^2+2\sqrt{2}x-6=0$

$\Rightarrow x^2+2\sqrt{2}x-\sqrt{2}x-6=0$

$\Rightarrow x\left(x+3\sqrt{2}\right)-\sqrt{2}\left(x+3\sqrt{2}\right)=0$

$\Rightarrow \left(x+3\sqrt{2}\right)\left(x-\sqrt{2}\right)=0$

$\Rightarrow x+3\sqrt{2}=0$ *or* $x-\sqrt{2}=0$

$\Rightarrow x=-3\sqrt{2}$ or $x=\sqrt{2}$

Hence, the roots of the given equation are $-3\sqrt{2}$ and $\sqrt{2}$

21. $\sqrt{3}x^2+10x+7\sqrt{3}=0$

Sol:

We write: $10x=3x+7x$ as $\sqrt{3}x^2\times 7\sqrt{3}=21x^2=3x\times 7x$

$\therefore \sqrt{3}x^2+10x+7\sqrt{3}=0$

$\Rightarrow \sqrt{3}x^2+3x+7x+7\sqrt{3}=0$

$\Rightarrow \sqrt{3}x\left(x+\sqrt{3}\right)+7\left(x+\sqrt{3}\right)=0$

$\Rightarrow \left(x+\sqrt{3}\right)\left(\sqrt{3}x+7\right)=0$

$\Rightarrow x+\sqrt{3}=0$ *or* $\sqrt{3}x+7=0$

$\Rightarrow x=-\sqrt{3}$ or $x=-\frac{7}{\sqrt{3}}=-\frac{7\sqrt{3}}{3}$

Hence, the roots of the given equation are $-\sqrt{3}$ and $-\frac{7\sqrt{3}}{3}$.

22. $\sqrt{3}x^2+11x+6\sqrt{3}=0$

Sol:

Given:

$\sqrt{3}x^2+11x+6\sqrt{3}=0$

$\Rightarrow \sqrt{3}x^2+9x+2x+6\sqrt{3}=0$

$\Rightarrow \sqrt{3}x\left(x+3\sqrt{3}\right)+2\left(x+3\sqrt{3}\right)=0$

$\Rightarrow \left(x+3\sqrt{3}\right)\left(\sqrt{3}x+2\right)=0$

$\Rightarrow x+3\sqrt{3}=0 \ or \ \sqrt{3}x+2=0$

$\Rightarrow x=-3\sqrt{3} \ or \ x=\dfrac{-2}{\sqrt{3}}=\dfrac{-2\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}=\dfrac{-2\sqrt{3}}{3}$

Hence, the roots of the equation are $-3\sqrt{3}$ and $\dfrac{-2\sqrt{3}}{3}$.

23. $3\sqrt{7}x^2+4x-\sqrt{7}=0$

Sol:

Given:

$3\sqrt{7}x^2+4x-\sqrt{7}=0$

$\Rightarrow 3\sqrt{7}x^2+7x-3x-\sqrt{7}=0$

$\Rightarrow \sqrt{7}x\left(3x+\sqrt{7}\right)-1\left(3x+\sqrt{7}\right)=0$

$\Rightarrow \left(3x+\sqrt{7}\right)\left(\sqrt{7}-1\right)=0$

$\Rightarrow 3x+\sqrt{7}=0 \ or \ \sqrt{7}x-1=0$

$\Rightarrow x=\dfrac{-\sqrt{7}}{3} or \ x=\dfrac{1}{\sqrt{7}}=\dfrac{1\times\sqrt{7}}{\sqrt{7}\times\sqrt{7}}=\dfrac{\sqrt{7}}{7}$

Hence, the roots of the equation are $\dfrac{-\sqrt{7}}{3}$ and $\dfrac{\sqrt{7}}{7}$.

24. $\sqrt{7}x^2-6x-13\sqrt{7}=0$

Sol:

We write, $-6x=7x-13x$ as $\sqrt{7}x^2\times\left(-13\sqrt{7}\right)=-91x^2=7x\times\left(-13x\right)$

$\therefore \sqrt{7}x^2-6x-13\sqrt{7}=0$

$\Rightarrow \sqrt{7}x^2+7x-13x-13\sqrt{7}=0$

$\Rightarrow \sqrt{7}x\left(x+\sqrt{7}\right)-13\left(x+\sqrt{7}\right)=0$

$\Rightarrow \left(x+\sqrt{7}\right)\left(\sqrt{7}x-13\right)=0$

$\Rightarrow x+\sqrt{7}=0 \ or \ \sqrt{7}x-13=0$

$\Rightarrow x=-\sqrt{7}$ or $x=\dfrac{13}{\sqrt{7}}=\dfrac{13\sqrt{7}}{7}$

Hence, the roots of the given equation are $-\sqrt{7}$ and $\dfrac{13\sqrt{7}}{7}$.

25. $4\sqrt{6}x^2-13x-2\sqrt{6}=0$

Sol:

Given:

$4\sqrt{6}x^2-13x-2\sqrt{6}=0$

$\Rightarrow 4\sqrt{6}x^2-16x+3x-2\sqrt{6}=0$

$\Rightarrow 4\sqrt{2}x\left(\sqrt{3}x-2\sqrt{2}\right)+\sqrt{3}\left(\sqrt{3}x-2\sqrt{2}\right)=0$

$\Rightarrow \left(4\sqrt{2}x+\sqrt{3}\right)\left(\sqrt{3}x-2\sqrt{2}\right)=0$

$\Rightarrow 4\sqrt{2}x+\sqrt{3}=0$ *or* $\sqrt{3}x-2\sqrt{2}=0$

$\Rightarrow x=\frac{-\sqrt{3}}{4\sqrt{2}}=\frac{-\sqrt{3}\times\sqrt{2}}{4\sqrt{2}\times\sqrt{2}}=\frac{-\sqrt{8}}{8}$ or $x=\frac{2\sqrt{2}}{\sqrt{3}}=\frac{2\sqrt{2}\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}=\frac{2\sqrt{6}}{3}$

Hence, the roots of the equation are $\frac{-\sqrt{6}}{8}$ and $\frac{2\sqrt{6}}{3}$.

26. $3x^2-2\sqrt{6}x+2=0$

Sol:

We write, $-2\sqrt{6}x=-\sqrt{6}x$ ad $3x^2\times 2=6x^2=\left(-\sqrt{6}x\right)\times\left(-\sqrt{6}x\right)$

$\therefore 3x^2-2\sqrt{6}x+2=0$

$\Rightarrow 3x^2-\sqrt{6}x-\sqrt{6}x+2=0$

$\Rightarrow \sqrt{3}x\left(\sqrt{3}x-\sqrt{2}\right)-\sqrt{2}\left(\sqrt{3}x-\sqrt{2}\right)=0$

$\Rightarrow \left(\sqrt{3}x-\sqrt{2}\right)\left(\sqrt{3}x-\sqrt{2}\right)=0$

$\Rightarrow \left(\sqrt{3}x-\sqrt{2}\right)^2=0$

$\Rightarrow \sqrt{3}x-\sqrt{2}=0$

$\Rightarrow x=\frac{\sqrt{2}}{\sqrt{3}}=\frac{\sqrt{6}}{3}$

Hence, $\frac{\sqrt{6}}{3}$ is the repeated root of the given equation.

27. $\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0$

Sol:

We write, $-2\sqrt{2}x=-3\sqrt{2}x+\sqrt{2}x$ as $\sqrt{3}x^2\times\left(-2\sqrt{3}\right)=-6x^2=\left(-3\sqrt{2}x\right)\times\left(\sqrt{2}x\right)$

$\therefore \sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0$

$\Rightarrow \sqrt{3}x^2 - 3\sqrt{2}x + \sqrt{2}x - 2\sqrt{3} = 0$

$\Rightarrow \sqrt{3}x\left(x - \sqrt{6}\right) + \sqrt{2}\left(x - \sqrt{6}\right) = 0$

$\Rightarrow \left(x - \sqrt{6}\right)\left(\sqrt{3}x + \sqrt{2}\right) = 0$

$\Rightarrow x - \sqrt{6} = 0 \ or \ \sqrt{3}x + \sqrt{2} = 0$

$\Rightarrow x - \sqrt{6} = 0 \ or \ x = -\frac{\sqrt{2}}{\sqrt{3}} = -\frac{\sqrt{6}}{3}$

Hence, the roots of the given equation are $\sqrt{6}$ and $-\frac{\sqrt{6}}{3}$.

28. $x^2 - 3\sqrt{5}x + 10 = 0$

Sol:

We write, $-3\sqrt{5}x = -2\sqrt{5}x - \sqrt{5}x$ as $x^2 \times 10 = 10x^2 = \left(-2\sqrt{5}x\right) \times \left(-\sqrt{5}x\right)$

$\therefore x^2 - 3\sqrt{5}x + 10 = 0$

$\Rightarrow x^2 - 2\sqrt{5}x - \sqrt{5}x + 10 = 0$

$\Rightarrow x\left(x - 2\sqrt{5}\right) - \sqrt{5}\left(x - 2\sqrt{5}\right) = 0$

$\Rightarrow \left(x - 2\sqrt{5}\right)\left(x - \sqrt{5}\right) = 0$

$\Rightarrow x\left(x - 2\sqrt{5}\right) - \sqrt{5}\left(x - 2\sqrt{5}\right) = 0$

Hence, the roots of the given equation are $\sqrt{5}$ and $2\sqrt{5}$.

29. $x^2 - \left(\sqrt{3} + 1\right)x + \sqrt{3} = 0$

Sol:

$x^2 - \left(\sqrt{3} + 1\right)x + \sqrt{3} = 0$

$\Rightarrow x^2 - \sqrt{3}x - x + \sqrt{3} = 0$

$\Rightarrow x\left(x - \sqrt{3}\right) - 1\left(x - \sqrt{3}\right) = 0$

$\Rightarrow \left(x - \sqrt{3}\right)\left(x - 1\right) = 0$

$\Rightarrow x - \sqrt{3} = 0 \ or \ x - 1 = 0$

$\Rightarrow x = \sqrt{3} \ or \ x = 1$

Hence, 1 and $\sqrt{3}$ are the roots of the given equation.

30. $x^2+3\sqrt{3}x-30=0$

Sol:

We write, $3\sqrt{3}x=5\sqrt{3}x-2\sqrt{3}x$ as $x^2\times(-30)=-30x^2=5\sqrt{3}x\times\left(-2\sqrt{3}x\right)$

$\therefore x^2+3\sqrt{3}x-30=0$

$\Rightarrow x^2+5\sqrt{3}x-2\sqrt{3}x-30=0$

$\Rightarrow x\left(x+5\sqrt{3}\right)-2\sqrt{3}\left(x+5\sqrt{3}\right)=0$

$\Rightarrow \left(x+5\sqrt{3}\right)\left(x-2\sqrt{3}\right)=0$

$\Rightarrow x+5\sqrt{3}=0 \text{ or } x-2\sqrt{3}=0$

$\Rightarrow x=-5\sqrt{3} \text{ or } x=2\sqrt{3}$

Hence, the roots of the given equation are $-5\sqrt{3}$ and $2\sqrt{3}$

31. $\sqrt{2}x^2+7x+5\sqrt{2}=0$

Sol:

We write, $7x=5x+2x$ as $\sqrt{2}x^2\times5\sqrt{2}=10x^2=5x\times2x$

$\therefore \sqrt{2}x^2+7x+5\sqrt{2}=0$

$\Rightarrow \sqrt{2}x^2+5x+2x+5\sqrt{2}=0$

$\Rightarrow x\left(\sqrt{2}x+5\right)+\sqrt{2}\left(\sqrt{2}x+5\right)=0$

$\Rightarrow \left(\sqrt{2}x+5\right)\left(x+\sqrt{2}\right)=0$

$\Rightarrow x+\sqrt{2}=0 \text{ or } \sqrt{2}x+5=0$

$\Rightarrow x=-\sqrt{2} \text{ or } x=-\dfrac{5}{\sqrt{2}}=-\dfrac{5\sqrt{2}}{2}$

Hence, the roots of the given equation are $-\sqrt{2}$ and $-\dfrac{5\sqrt{2}}{2}$.

32. $5x^2+13x+8=0$

Sol:

We write, $13x=5x+8x$ as $5x^2\times8=40x^2=5x\times8x$

$\therefore 5x^2+13x+8=0$

$\Rightarrow 5x^2+5x+8x+8=0$

$\Rightarrow 5x(x+1)+8(x+1)=0$

$\Rightarrow (x+1)(5x+8)=0$

$\Rightarrow x+1=0 \text{ or } 5x+8=0$

$x = -1 \text{ or } x = -\frac{8}{5}$

Hence, -1 and $-\frac{8}{5}$ are the roots of the given equation.

33. $x^2 - \left(1+\sqrt{2}\right)x + \sqrt{2} = 0$

Sol:

Given:

$x^2 - \left(1+\sqrt{2}\right)x + \sqrt{2} = 0$

$\Rightarrow x^2 - x - \sqrt{2}x + \sqrt{2} = 0$

$\Rightarrow x(x-1) - \sqrt{2}(x-1) = 0$

$\Rightarrow \left(x-\sqrt{2}\right)(x-1) = 0$

$\Rightarrow x - \sqrt{2} = 0 \text{ or } x - 1 = 0$

$\Rightarrow x = \sqrt{2} \text{ or } x = 1$

Hence, the roots of the equation are $\sqrt{2}$ and 1.

34. $9x^2 + 6x + 1 = 0$

Sol:

Given:

$9x^2 + 6x + 1 = 0$

$\Rightarrow 9x^2 + 3x + 3x + 1 = 0$

$\Rightarrow 3x(3x+1) + 1(3x+1) = 0$

$\Rightarrow (3x+1)(3x+1) = 0$

$\Rightarrow 3x + 1 = 0 \text{ or } 3x + 1 = 0$

$\Rightarrow x = \frac{-1}{3} \text{ or } x = \frac{-1}{3}$

Hence, $\frac{-1}{3}$ is the root of the equation $9x^2 + 6x + 1 = 0$.

35. $100x^2 - 20x + 1 = 0$

Sol:

We write, $-20x = -10x - 10x$ as $100x^2 \times 1 = 100x^2 = (-10x) \times (-10x)$

$\therefore 100x^2 - 20x + 1 = 0$

$\Rightarrow 100x^2 - 10x - 10x + 1 = 0$

$\Rightarrow 10x(10x-1)-1(10x-1)=0$

$\Rightarrow (10x-1)(10x-1)=0$

$\Rightarrow (10x-1)^2=0$

$\Rightarrow 10x-1=0$

$\Rightarrow x=\frac{1}{10}$

Hence, $\frac{1}{10}$ is the repeated root of the given equation.

36. $2x^2-x+\frac{1}{8}=0$

Sol:

We write, $-x=-\frac{x}{2}-\frac{x}{2}$ as $2x^2\times\frac{1}{8}=\frac{x^2}{4}=\left(-\frac{x}{2}\right)\times\left(-\frac{x}{2}\right)$

$\therefore 2x^2-x+\frac{1}{8}=0$

$\Rightarrow 2x^2-\frac{x}{2}-\frac{x}{2}+\frac{1}{8}=0$

$\Rightarrow 2x\left(x-\frac{1}{4}\right)-\frac{1}{2}\left(x-\frac{1}{4}\right)=0$

$\Rightarrow \left(x-\frac{1}{4}\right)\left(2x-\frac{1}{2}\right)=0$

$\Rightarrow x-\frac{1}{4}=0 \text{ or } 2x-\frac{1}{2}=0$

$\Rightarrow x=\frac{1}{4} \text{ or } x=\frac{1}{4}$

Hence, $\frac{1}{4}$ is the repeated root of the given equation.

37. $10x-\frac{1}{x}=3$

Sol:

Given:

$10x-\frac{1}{x}=3$

$\Rightarrow 10x^2-1=3x$ [Multiplying both sides by x]

$\Rightarrow 10x^2 - 3x - 1 = 0$
$\Rightarrow 10x^2 - (5x - 2x) - 1 = 0$
$\Rightarrow 10x^2 - 5x + 2x - 1 = 0$
$\Rightarrow 5x(2x - 1) + 1(2x - 1) = 0$
$\Rightarrow (2x - 1)(5x + 1) = 0$
$\Rightarrow 2x - 1 = 0$ or $5x + 1 = 0$
$\Rightarrow x = \frac{1}{2}$ or $x = \frac{-1}{5}$

Hence, the roots of the equation are $\frac{1}{2}$ and $\frac{-1}{5}$.

38. $\frac{2}{x^2} - \frac{5}{x} + 2 = 0$

Sol:

Given:

$\frac{2}{x^2} - \frac{5}{x} + 2 = 0$
$\Rightarrow 2 - 5x + 2x^2 = 0$ [Multiplying both sides by x^2]
$\Rightarrow 2x^2 - 5x + 2 = 0$
$\Rightarrow 2x^2 - (4x + x) + 2 = 0$
$\Rightarrow 2x^2 - 4x - x + 2 = 0$
$\Rightarrow 2x(x - 2) - 1(x - 2) = 0$
$\Rightarrow (2x - 1)(x - 2) = 0$
$\Rightarrow 2x - 1 = 0 \ or \ x - 2 = 0$
$\Rightarrow x = \frac{1}{2} \ or \ x = 2$

Hence, the roots of the equation are $\frac{1}{2}$ and 2.

39. $2x^2 + ax - a^2 = 0$

Sol:

We write, $ax = 2ax - ax$ as $2x^2 \times (-a^2) = -2a^2x^2 = 2ax \times (-ax)$

$\therefore 2x^2 + ax - a^2 = 0$
$\Rightarrow 2x^2 + 2ax - ax - a^2 = 0$
$\Rightarrow 2x(x + a) - a(x + a) = 0$
$\Rightarrow (x + a)(2x - a) = 0$

$\Rightarrow x+a=0 \; or \; 2x-a=0$

$\Rightarrow x=-a \; or \; x=\frac{a}{2}$

Hence, $-a$ and $\frac{a}{2}$ are the roots of the given equation.

40. $4x^2+4bx-\left(a^2-b^2\right)=0$

Sol:

We write, $4bx=2(a+b)x-2(a-b)x$ as

$4x^2\times\left[-\left(a^2-b^2\right)\right]=-4\left(a^2-b^2\right)x^2=2(a+b)x\times\left[-2(a-b)x\right]$

$\therefore 4x^2+4bx-\left(a^2-b^2\right)=0$

$\Rightarrow 4x^2+2(a+b)x-2(a-b)x-(a-b)(a+b)=0$

$\Rightarrow 2x\left[2x+(a+b)\right]-(a-b)\left[2x+(a+b)\right]=0$

$\Rightarrow \left[2x+(a+b)\right]\left[2x-(a-b)\right]=0$

$\Rightarrow 2x+(a+b)=0 \; or \; 2x-(a-b)=0$

$\Rightarrow x=-\frac{a+b}{2} \; or \; x=\frac{a-b}{2}$

Hence, $-\frac{a+b}{2}$ and $\frac{a-b}{2}$ are the roots of the given equation.

41. $4x^2-4a^2x+\left(a^4-b^4\right)=0$

Sol:

We write, $-4a^2x=-2\left(a^2+b^2\right)x-2\left(a^2-b^2\right)x$ as

$4x^2\times\left(a^4-b^4\right)=4\left(a^4-b^4\right)x^2=\left[-2\left(a^2+b^2\right)\right]x\times\left[-2\left(a^2-b^2\right)\right]x$

$\therefore 4x^2-4a^2x+\left(a^4-b^4\right)=0$

$\Rightarrow 4x^2-2\left(a^2+b^2\right)x-2\left(a^2-b^2\right)x+\left(a^2-b^2\right)\left(a^2+b^2\right)=0$

$\Rightarrow 2x\left[2x-\left(a^2+b^2\right)\right]-\left(a^2-b^2\right)\left[2x-\left(a^2+b^2\right)\right]=0$

$\Rightarrow \left[2x-\left(a^2+b^2\right)\right]\left[2x-\left(a^2-b^2\right)\right]=0$

$\Rightarrow 2x-\left(a^2+b^2\right)=0 \; or \; 2x-\left(a^2-b^2\right)=0$

$\Rightarrow x=\frac{a^2+b^2}{2} \; or \; x=\frac{a^2-b^2}{2}$

Hence, $\frac{a^2+b^2}{2}$ and $\frac{a^2-b^2}{2}$ are the roots of the given equation.

42. $x^2+5x-\left(a^2+a-6\right)=0$

Sol:

We write, $5x=(a+3)x-(a-2)x$ as

$x^2\times\left[-\left(a^2+a-6\right)\right]=-\left(a^2+a-6\right)x^2=(a+3)x\times\left[-(a-2)x\right]$

$\therefore x^2+5x-\left(a^2+a-6\right)=0$

$\Rightarrow x^2+(a+3)x-(a-2)x-(a+3)(a-2)=0$

$\Rightarrow x\left[x+(a+3)\right]-(a-2)\left[x+(a+3)\right]=0$

$\Rightarrow\left[x+(a+3)\right]\left[x-(a-2)\right]=0$

$\Rightarrow x+(a+3)=0\ x-(a-2)=0$

$\Rightarrow x=-(a+3)\ or\ x=a-2$

Hence, $-(a+3)$ and $(a-2)$ are the roots of the given equation.

43. $x^2-2ax-\left(4b^2-a^2\right)=0$

Sol:

We have, $-2ax=(2b-a)x-(2b+a)x$ as

$x^2\times\left[-\left(4b^2-a^2\right)\right]=-\left(4b^2-a^2\right)x^2=(2b-a)x\times\left[-(2b+a)x\right]$

$\therefore x^2-2ax-\left(4b^2-a^2\right)=0$

$\Rightarrow x^2+(2b-a)x-(2b+a)x-(2b-a)(2b+a)=0$

$\Rightarrow x\left[x+(2b-a)\right]-(2b+a)\left[x+(2b-a)\right]=0$

$\Rightarrow\left[x+(2b-a)\right]\left[x-(2b+a)\right]=0$

$\Rightarrow x+(2b-a)=0\ or\ x-(2b+a)=0$

$x=-(2b-a)$ or $x=2b+a$

$\Rightarrow x=a-2b\ or\ x=a+2b$

Hence, $a-2b$ and $a+2b$ are the roots of the given equation.

44. $x^2-(2b-1)x+\left(b^2-b-20\right)=0$

Sol:

We write, $-(2b-1)x=-(b-5)x-(b+4)x$ as

$x^2\times(b^2-b-20)=(b^2-b-20)x^2=[-(b-5)x]\times[-(b+4)x]$

$\therefore x^2-(2b-1)x+(b^2-b-20)=0$

$\Rightarrow x^2-(b-5)x-(b+4)x+(b-5)(b+4)=0$

$\Rightarrow x[x-(b-5)]-(b+4)[x-(b-5)]=0$

$\Rightarrow [x-b-5][x-(b+4)]=0$

$\Rightarrow x-(b-5)=0$ or $x-(b+4)=0$

$\Rightarrow x=b-5$ or $x=b+4$

Hence, $b-5$ and $b+4$ are the roots of the given equation.

45. $x^2+6x-(a^2+2a-8)=0$

Sol:

We write, $6x=(a+4)x-(a-2)x$ as

$x^2\times[-(a^2+2a-8)]=-(a^2+2a-8)x^2=(a+4)x\times[-(a-2)x]$

$\therefore x^2+6x-(a^2+2a-8)=0$

$\Rightarrow x^2+(a+4)x-(a-2)x-(a+4)(a-2)=0$

$\Rightarrow x[x+(a+4)]-(a-2)[x+(a+4)]=0$

$\Rightarrow [x+(a+4)][x-(a-2)]=0$

$\Rightarrow x+(a+4)=0$ *or* $x-(a-2)=0$

$\Rightarrow x=-(a+4)$ or $x=a-2$

Hence, $-(a+4)$ and $(a-2)$ are the roots of the given equation.

46. $abx^2+(b^2-ac)x-bc=0$

Sol:

$abx^2+(b^2-ac)x-bc=0$

$\Rightarrow abx^2+b^2x-acx-bc=0$

$\Rightarrow bx(ax+b)-c(ax+b)=0$

$\Rightarrow (bx-c)(ax+b)=0$

$\Rightarrow bx-c=0$ or $ax+b=0$

$\Rightarrow x=\frac{c}{b}$ *or* $x=\frac{-b}{a}$

Hence, the roots of the equation are $\frac{c}{b}$ and $\frac{-b}{a}$.

47. $x^2 - 4ax - b^2 + 4a^2 = 0$

Sol:

We write, $-4ax = -(b+2a)x + (b-2a)x$ as

$x^2 \times (-b^2 + 4a^2) = (-b^2 + 4a^2)x^2 = -(b+2a)x \times (b-2a)x$

$\therefore x^2 - 4ax - b^2 + 4a^2 = 0$

$\Rightarrow x^2 - (b+2a)x(b-2a)x - (b-2a)(b+2a) = 0$

$\Rightarrow x[x-(b+2a)] + (b-2a)[x-(b+2a)] = 0$

$\Rightarrow [x-(b+2a)][x+(b-2a)] = 0$

$\Rightarrow x-(b+2a) = 0 \; or \; x+(b-2a) = 0$

$\Rightarrow x = 2a+b \; or \; x = -(b-2a)$

$\Rightarrow x = 2a+b \; or \; x = 2a-b$

Hence, $(2a+b)$ and $(2a-b)$ are the roots of the given equation.

48. $4x^2 - 2(a^2+b^2)x + a^2b^2 = 0$

Sol:

Given:

$4x^2 - 2(a^2+b^2)x + a^2b^2 = 0$

$\Rightarrow 4x^2 - 2a^2x - 2b^2x + a^2b^2 = 0$

$\Rightarrow 2x(2x-a^2) - b^2(2x-a^2) = 0$

$\Rightarrow (2x-b^2)(2x-a^2) = 0$

$\Rightarrow 2x - b^2 = 0 \; or \; 2x - a^2 = 0$

$\Rightarrow x = \frac{b^2}{2} \; or \; x = \frac{a^2}{2}$

Hence, the roots of the equation are $\frac{b^2}{2}$ and $\frac{a^2}{2}$.

49. $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

Sol:

Given:

$12abx^2 - \left(9a^2 - 8b^2\right)x - 6ab = 0$

$\Rightarrow 12abx^2 - 9a^2x + 8b^2x - 6ab = 0$

$\Rightarrow 3ax\left(4bx - 3a\right) + 2b\left(4bx - 3a\right) = 0$

$\Rightarrow \left(3ax + 2b\right)\left(4bx - 3a\right) = 0$

$\Rightarrow 3ax + 2b = 0 \text{ } or \text{ } 4bx - 3a = 0$

$\Rightarrow x = \dfrac{-2b}{3a} \text{ } or \text{ } x = \dfrac{3a}{4b}$

Hence, the roots of the equation are $\dfrac{-2b}{3a}$ and $\dfrac{3a}{4b}$.

50. $a^2b^2x^2 + b^2x - a^2x - 1 = 0$

Sol:

Given:

$a^2b^2x^2 + b^2x - a^2x - 1 = 0$

$\Rightarrow b^2x\left(a^2x + 1\right) - 1\left(a^2x + 1\right) = 0$

$\Rightarrow \left(b^2x - 1\right)\left(a^2x + 1\right) = 0$

$\Rightarrow \left(b^2x - 1\right) = 0 \text{ } or \text{ } \left(a^2x + 1\right) = 0$

$\Rightarrow x = \dfrac{1}{b^2} \text{ } or \text{ } x = \dfrac{-1}{a^2}$

Hence, $\dfrac{1}{b^2}$ and $\dfrac{-1}{a^2}$ are the roots of the given equation.

51. $9x^2 - 9\left(a + b\right)x + \left(2a^2 + 5ab + 2b^2\right) = 0$

Sol:

We write, $-9\left(a+b\right)x = -3\left(2a+b\right)x - 3\left(a+2b\right)x$ as

$9x^2 \times \left(2a^2 + 5ab + 2b^2\right) = 9\left(2a^2 + 5ab + 2b^2\right)x^2 = \left[-3\left(2a+b\right)x\right] \times \left[-3\left(a+2b\right)x\right]$

$\therefore 9x^2 - 9\left(a+b\right)x + \left(2a^2 + 5ab + 2b^2\right) = 0$

$\Rightarrow 9x^2 - 3\left(2a+b\right)x - 3\left(a+2b\right)x + \left(2a+b\right)\left(a+2b\right) = 0$

$\Rightarrow 3x\left[3x - \left(2a+b\right)\right] - \left(a+2b\right)\left[3x - \left(2a+b\right)\right] = 0$

$\Rightarrow \left[3x - \left(2a+b\right)\right]\left[3x - \left(a+2b\right)\right] = 0$

$\Rightarrow 3x - \left(2a+b\right) = 0 \text{ } or \text{ } 3x - \left(a+2b\right) = 0$

$\Rightarrow x = \frac{2a+b}{3} \; or \; x = \frac{a+2b}{3}$

Hence, $\frac{2a+b}{3}$ and $\frac{a+2b}{3}$ are the roots of the given equation.

52. $\frac{16}{x} - 1 = \frac{15}{x+1}, x \neq 0, -1$

Sol:

$\frac{16}{x} - 1 = \frac{15}{x+1}, x \neq 0, -1$

$\Rightarrow \frac{16}{x} - \frac{15}{x+1} = 1$

$\Rightarrow \frac{16x+16-15x}{x(x+1)} = 1$

$\Rightarrow \frac{x+16}{x^2+x} = 1$

$\Rightarrow x^2 + x = x + 16$ (Cross multiplication)

$\Rightarrow x^2 - 16 = 0$

$\Rightarrow (x+4)(x-4) = 0$

$\Rightarrow x+4 = 0 \; or \; x-4 = 0$

$\Rightarrow x = -4 \; or \; x = 4$

Hence, -4 and 4 are the roots of the given equation.

53. $\frac{4}{x} - 3 = \frac{5}{2x+3}, x \neq 0, -\frac{3}{2}$

Sol:

$\frac{4}{x} - 3 = \frac{5}{2x+3}, x \neq 0, -\frac{3}{2}$

$\Rightarrow \frac{4}{x} - \frac{5}{2x+3} = 3$

$\Rightarrow \frac{8x+12-5x}{x(2x+3)} = 3$

$\Rightarrow \frac{3x+12}{2x^2+3x} = 3$

$\Rightarrow \frac{x+4}{2x^2+3x} = 1$

$\Rightarrow 2x^2+3x=x+4$ (Cross multiplication)

$\Rightarrow 2x^2+2x-4=0$

$\Rightarrow x^2+x-2=0$

$\Rightarrow x^2+2x-x-2=0$

$\Rightarrow x(x+2)-1(x+2)=0$

$\Rightarrow (x+2)(x-1)=0$

$\Rightarrow x+2=0$ or $x-1=0$

$\Rightarrow x=-2$ or $x=1$

Hence, -2 and 1 are the roots of the given equation.

54. $\frac{3}{x+1}-\frac{1}{2}=\frac{2}{3x-1}, x\neq -1, \frac{1}{3}$

Sol:

$\frac{3}{x+1}-\frac{1}{2}=\frac{2}{3x-1}, x\neq -1, \frac{1}{3}$

$\Rightarrow \frac{3}{x+1}-\frac{2}{3x-1}=\frac{1}{2}$

$\Rightarrow \frac{9x-3-2x-2}{(x+1)(3x-1)}=\frac{1}{2}$

$\Rightarrow \frac{7x-5}{3x^2+2x-1}=\frac{1}{2}$

$\Rightarrow 3x^2+2x-1=14x-10$ (Cross multiplication)

$\Rightarrow 3x^2-12x+9=0$

$\Rightarrow x^2-4x+3=0$

$\Rightarrow x^2-3x-x+3=0$

$\Rightarrow x(x-3)-1(x-3)=0$

$\Rightarrow (x-3)(x-1)=0$

$\Rightarrow x-3=0$ *or* $x-1=0$

$\Rightarrow x=3$ *or* $x=1$

Hence, 1 and 3 are the roots of the given equation.

55. $\frac{1}{x-1}-\frac{1}{x+5}=\frac{6}{7}, x\neq 1,-5$

Sol:

$\frac{1}{x-1}-\frac{1}{x+5}=\frac{6}{7}, x\neq 1,-5$

$\Rightarrow \frac{x+5-x+1}{(x-1)(x+5)}=\frac{6}{7}$

$\Rightarrow \frac{6}{x^2+4x-5}=\frac{6}{7}$

$\Rightarrow x^2+4x-5=7$

$\Rightarrow x^2+4x-12=0$

$\Rightarrow x^2+6x-2x-12=0$

$\Rightarrow x(x+6)-2(x+6)=0$

$\Rightarrow (x+6)(x-2)=0$

$\Rightarrow x+6=0 \text{ or } x-2=0$

$\Rightarrow x=-6 \text{ or } x=2$

Hence, -6 and 2 are the roots of the given equation.

56. $\frac{1}{2a+b+2x}=\frac{1}{2a}+\frac{1}{b}+\frac{1}{2x}$

Sol:

$\frac{1}{2a+b+2x}=\frac{1}{2a}+\frac{1}{b}+\frac{1}{2x}$

$\Rightarrow \frac{1}{2a+b+2x}-\frac{1}{2x}=\frac{1}{2a}+\frac{1}{b}$

$\Rightarrow \frac{2x-2a-b-2x}{2x(2a+b+2x)}=\frac{2a+b}{2ab}$

$\Rightarrow \frac{-(2a+b)}{4x^2+4ax+2bx}=\frac{2a+b}{2ab}$

$\Rightarrow 4x^2+4ax+2bx=-2ab$

$\Rightarrow 4x^2+4ax+2bx+2ab=0$

$\Rightarrow 4x(x+a)+2b(x+a)=0$

$\Rightarrow (x+a)(4x+2b)=0$

$\Rightarrow x+a=0 \text{ or } 4x+2b=0$

$\Rightarrow x=-a \text{ or } x=-\frac{b}{2}$

Hence, $-a$ and $-\frac{b}{2}$ are the roots of the give equation.

57. $\frac{x+3}{x-2}-\frac{1-x}{x}=4\frac{1}{4}, x\neq 2,0$

Sol:

Given:

$$\frac{(x+3)}{(x-2)}-\frac{(1-x)}{x}=\frac{17}{4}$$

$$\Rightarrow \frac{x(x+3)-(1-x)(x-2)}{(x-2)x}=\frac{17}{4}$$

$$\Rightarrow \frac{x^2+3x-(x-2-x^2+2x)}{x^2-2x}=\frac{17}{4}$$

$$\Rightarrow \frac{x^2+3x+x^2-3x+2}{x^2-2x}=\frac{17}{4}$$

$$\Rightarrow \frac{2x^2+2}{x^2-2x}=\frac{17}{4}$$

$\Rightarrow 8x^2+8=17x^2-34x$ [On cross multiplying]

$$\Rightarrow -9x^2+34x+8=0$$

$$\Rightarrow 9x^2-34x-8=0$$

$$\Rightarrow 9x^2-36x+2x-8=0$$

$$\Rightarrow 9x(x-4)+2(x-4)=0$$

$$\Rightarrow (x-4)(9x+2)=0$$

$$\Rightarrow x-4=0 \ or \ 9x+2=0$$

$$\Rightarrow x=4 \ or \ x=\frac{-2}{9}$$

Hence, the roots of the equation are 4 and $\frac{-2}{9}$.

58. $\frac{3x-4}{7}+\frac{7}{3x-4}=\frac{5}{2}, x\neq\frac{4}{3}$

Sol:

$$\frac{3x-4}{7}+\frac{7}{3x-4}=\frac{5}{2}, x\neq\frac{4}{3}$$

$$\Rightarrow \frac{(3x-4)^2+49}{7(3x-4)}=\frac{5}{2}$$

$$\Rightarrow \frac{9x^2 - 24x + 16 + 49}{21x - 28} = \frac{5}{2}$$

$$\Rightarrow \frac{9x^2 - 24x + 65}{21x - 28} = \frac{5}{2}$$

$$\Rightarrow 18x^2 - 48x + 130 = 105x - 140$$

$$\Rightarrow 18x^2 - 153x + 270 = 0$$

$$\Rightarrow 2x^2 - 17x + 30 = 0$$

$$\Rightarrow 2x^2 - 12x - 5x + 30 = 0$$

$$\Rightarrow 2x(x-6) - 5(x-6) = 0$$

$$\Rightarrow (x-6)(2x-5) = 0$$

$$\Rightarrow x - 6 = 0 \ or \ 2x - 5 = 0$$

$$\Rightarrow x = 6 \ or \ x = \frac{5}{2}$$

Hence, 6 and $\frac{5}{2}$ are the roots of the given equation.

59. $\frac{x}{x-1} + \frac{x-1}{x} = 4\frac{1}{4}, x \neq 0,1$

Sol:

$$\frac{x}{x-1} + \frac{x-1}{x} = 4\frac{1}{4}, x \neq 0,1$$

$$\Rightarrow \frac{x^2 + (x-1)^2}{x(x-1)} = \frac{17}{4}$$

$$\Rightarrow \frac{x^2 + x^2 - 2x + 1}{x^3 - x} = \frac{17}{4}$$

$$\Rightarrow \frac{2x^2 - 2x + 1}{x^2 - 1} = \frac{17}{4}$$

$$\Rightarrow 8x^2 - 8x + 4 = 17x^2 - 17x$$

$$\Rightarrow 9x^2 - 9x - 4 = 0$$

$$\Rightarrow 9x^2 - 12x + 3x - 4 = 0$$

$$\Rightarrow 3x(3x-4) + 1(3x-4) = 0$$

$$\Rightarrow (3x-4)(3x+1) = 0$$

$$\Rightarrow 3x - 4 = 0 \ or \ 3x + 1 = 0$$

$$\Rightarrow x = \frac{4}{3} \ or \ x = -\frac{1}{3}$$

Hence, $\frac{4}{3}$ and $-\frac{1}{3}$ are the roots of the given equation.

60. $\frac{x}{x+1}+\frac{x+1}{x}=2\frac{4}{15}, x\neq 0,-1$

Sol:

$\frac{x}{x+1}+\frac{x+1}{x}=2\frac{4}{15}, x\neq 0,-1$

$\Rightarrow \frac{x^2+(x+1)^2}{x(x+1)}=\frac{34}{15}$

$\Rightarrow \frac{x^2+x^2+2x+1}{x^2+x}=\frac{34}{15}$

$\Rightarrow \frac{2x^2+2x+1}{x^2+x}=\frac{34}{15}$

$\Rightarrow 30x^2+30x+15=34x^2+34x$

$\Rightarrow 4x^2+4x-15=0$

$\Rightarrow 4x^2+10x-6x-15x=0$

$\Rightarrow 2x(2x+5)-3(2x+5)=0$

$\Rightarrow (2x+5)(2x-3)=0$

$\Rightarrow 2x+5=0 \text{ or } 2x-3=0$

$\Rightarrow x=-\frac{5}{2} \text{ or } 2x-3=0$

Hence, $-\frac{5}{2}$ and $\frac{3}{2}$ are the roots of the given equation.

61. $\frac{x-4}{x-5}+\frac{x-6}{x-7}=3\frac{1}{3}, x\neq 5,7$

Sol:

$\frac{x-4}{x-5}+\frac{x-6}{x-7}=3\frac{1}{3}, x\neq 5,7$

$\Rightarrow \frac{(x-4)(x-7)+(x-5)(x-6)}{(x-5)(x-7)}=\frac{10}{3}$

$\Rightarrow \frac{x^2-11x+28+x^2-11x+30}{x^2-12x+35}=\frac{10}{3}$

$\Rightarrow \frac{2x^2-22x+58}{x^2-12x+35}=\frac{10}{3}$

$\Rightarrow \frac{x^2-11x+29}{x^2-12x+35}=\frac{5}{3}$

$\Rightarrow 3x^2-33x+87=5x^2-60x+175$

$\Rightarrow 2x^2-27x+88=0$

$\Rightarrow 2x^2-16x-11x+88=0$

$\Rightarrow 2x(x-8)-11(x-8)=0$

$\Rightarrow (x-8)(2x-11)=0$

$\Rightarrow x-8=0$ or $2x-11=0$

$\Rightarrow x=8$ *or* $x=\frac{11}{2}$

Hence, 8 and $\frac{11}{2}$ are the roots of the given equation.

62. $\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3}, x\neq 2,4$

Sol:

$\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3}, x\neq 2,4$

$\Rightarrow \frac{(x-1)(x-4)+(x-2)(x-3)}{(x-2)(x-4)}=\frac{10}{3}$

$\Rightarrow \frac{x^2-5x+4+x^2-5x+6}{x^2-6x+8}=\frac{10}{3}$

$\Rightarrow \frac{2x^2-10x+10}{x^2-6x+8}=\frac{10}{3}$

$\Rightarrow \frac{x^2-5x+5}{x^2-6x+8}=\frac{5}{3}$

$\Rightarrow 3x^2-15x+15=5x^2-30x+40$

$\Rightarrow 2x^2-15x+25=0$

$\Rightarrow 2x^2-10x-5x+25=0$

$\Rightarrow 2x(x-5)-5(x-5)=0$

$\Rightarrow (x-5)(2x-5)=0$

$\Rightarrow x-5=0$ *or* $2x-5=0$

$\Rightarrow x=5$ *or* $x=\frac{5}{2}$

Hence, 5 and $\frac{5}{2}$ are the roots of the given equation.

63. $\frac{1}{(x-2)}+\frac{2}{(x-1)}=\frac{6}{x}, x\neq 0,1,2$

Sol:

$\frac{1}{(x-2)}+\frac{2}{(x-1)}=\frac{6}{x}$

$\Rightarrow \frac{(x-1)+2(x-2)}{(x-1)(x-2)}=\frac{6}{x}$

$\Rightarrow \frac{3x-5}{x^2-3x+2}=\frac{6}{x}$

$\Rightarrow 3x^2-5x=6x^2-18x+12$ [On cross multiplying]

$\Rightarrow 3x^2-13x+12=0$

$\Rightarrow 3x^2-(9+4)x+12=0$

$\Rightarrow 3x^2-9x-4x+12=0$

$\Rightarrow 3x(x-3)-4(x-3)=0$

$\Rightarrow (3x-4)(x-3)=0$

$\Rightarrow 3x-4=0 \ or \ x-3=0$

$\Rightarrow x=\frac{4}{3} \ or \ x=3$

64. $\frac{1}{x+1}+\frac{2}{x+2}=\frac{5}{x+4}, x\neq -1,-2,-4$

Sol:

$\frac{1}{x+1}+\frac{2}{x+2}=\frac{5}{x+4}, x\neq -1,-2,-4$

$\Rightarrow \frac{x+2+2x+2}{(x+1)(x+2)}=\frac{5}{x+4}$

$\Rightarrow \frac{3x+4}{x^2+3x+2}=\frac{5}{x+4}$

$\Rightarrow (3x+4)(x+4)=5(x^2+3x+2)$

$\Rightarrow 3x^2+16x+16=5x^2+15x+10$

$\Rightarrow 2x^2-x-6=0$

$\Rightarrow 2x^2-4x+3x-6=0$

$\Rightarrow 2x(x-2)+3(x-2)=0$

$\Rightarrow (x-2)(2x+3)=0$

$\Rightarrow 3x^2+16x+16=5x^2+15x+10$

$\Rightarrow 2x^2-x-6=0$

$\Rightarrow 2x^2-4x+3x-6=0$

$\Rightarrow 2x(x-2)+3(x-2)=0$

$\Rightarrow (x-2)(2x+3)=0$

$\Rightarrow x-2=0 \ or \ 2x+3=0$

$\Rightarrow x=2 \ or \ x=-\frac{3}{2}$

Hence, 2 and $-\frac{3}{2}$ are the roots of the given equation.

65. $3\left(\frac{3x-1}{2x+3}\right)-2\left(\frac{2x+3}{3x-1}\right)=5, x\neq\frac{1}{3},-\frac{3}{2}$

Sol:

$3\left(\frac{3x-1}{2x+3}\right)-2\left(\frac{2x+3}{3x-1}\right)=5, x\neq\frac{1}{3},-\frac{3}{2}$

$\Rightarrow \frac{3(3x-1)^2-2(2x+3)^2}{(2x+3)(3x-1)}=5$

$\Rightarrow \frac{3(9x^2-6x+1)-2(4x^2+12x+9)}{6x^2+7x-3}=5$

$\Rightarrow \frac{27x^2-18x+3-8x^2-24x-18}{6x^2+7x-3}=5$

$\Rightarrow \frac{19x^2-42x-15}{6x^2+7x-3}=5$

$\Rightarrow 19x^2-42x-15=30x^2+35x-15$

$\Rightarrow 11x^2+77x=0$

$\Rightarrow 11x(x+7)=0$

$\Rightarrow x=0 \ or \ x+7=0$

$\Rightarrow x=0 \ or \ x=-7$

Hence, 0 and -7 are the roots of the given equation.

66. $3\left(\frac{7x+1}{5x-3}\right)-4\left(\frac{5x-3}{7x+1}\right)=11, x\neq\frac{3}{5},-\frac{1}{7}$

Sol:

$$3\left(\frac{7x+1}{5x-3}\right)-4\left(\frac{5x-3}{7x+1}\right)=11, x\neq\frac{3}{5},-\frac{1}{7}$$

$$\Rightarrow\frac{3(7x+1)^2-4(5x-3)^2}{(5x-3)(7x+1)}=11$$

$$\Rightarrow\frac{3\left(49x^2+14x+1\right)-4\left(25x^3-30x+9\right)}{35x^3-16x-3}=11$$

$$\Rightarrow\frac{147x^2+42x+3-100x^2+120x-36}{35x^2-16x-3}=11$$

$$\Rightarrow\frac{47x^2+162x-33}{35x^2-16x-3}=11$$

$$\Rightarrow 47x^2+162x-33=385x^2-176x-33$$

$$\Rightarrow 338x^2-338x=0$$

$$\Rightarrow 338x(x-1)=0$$

$$\Rightarrow x=0 \; or \; x-1=0$$

$$\Rightarrow x=0 \; or \; x=1$$

Hence, 0 and 1 are the roots of the given equation.

67. $\left(\frac{4x-3}{2x+1}\right)-10\left(\frac{2x+1}{4x-3}\right)=3, x\neq-\frac{1}{2},\frac{3}{4}$

Sol:

Given:

$$\left(\frac{4x-3}{2x+1}\right)-10\left(\frac{2x+1}{4x-3}\right)=3$$

Putting $\frac{4x-3}{2x+1}=y,$ we get:

$$y-\frac{10}{y}=3$$

$$\Rightarrow\frac{y^2-10}{y}=3$$

$$\Rightarrow y^2-10=3y \qquad \text{[On cross multiplying]}$$

$$\Rightarrow y^2-3y-10=0$$

$$\Rightarrow y^2-(5-2)y-10=0$$

$$\Rightarrow y^2-5y+2y-10=0$$

$$\Rightarrow y(y-5)+2(y-5)=0$$

$$\Rightarrow (y-5)(y+2)=0$$

$\Rightarrow y-5=0$ *or* $y+2=0$

$\Rightarrow y=5$ *or* $y=-2$

Case I:

If $y=5,$ we get:

$\frac{4x-3}{2x+1}=5$

$\Rightarrow 4x-3=5(2x+1)$ [On cross multiplying]

$\Rightarrow 4x-3=10x+5$

$\Rightarrow -6x=8$

$\Rightarrow -6x=8$

$\Rightarrow x=\frac{8}{6}$

$\Rightarrow x=-\frac{4}{3}$

Case II:

If $y=-2,$ we get:

$\frac{4x-3}{2x+1}=-2$

$\Rightarrow 4x-3=-2(2x+1)$

$\Rightarrow 4x-3=-4x-2$

$\Rightarrow 8x=1$

$\Rightarrow x=\frac{1}{8}$

Hence, the roots of the equation are $-\frac{4}{3}$ and $\frac{1}{8}$.

68. $\left(\frac{x}{x+1}\right)^2-5\left(\frac{x}{x+1}\right)+6=0, x\neq b, a$

Sol:

$\left(\frac{x}{x+1}\right)^2-5\left(\frac{x}{x+1}\right)+6=0$

Putting $\frac{x}{x+1}=y,$ we get:

$y^2-5y+6=0$

$\Rightarrow y^2-5y+6=0$

$\Rightarrow y^2-(3+2)y+6=0$

$\Rightarrow y^2-3y-2y+6=0$

$\Rightarrow y(y-3)-2(y-3)=0$

$\Rightarrow (y-3)(y-2)=0$

$\Rightarrow y-3=0 \ or \ y-2=0$

$\Rightarrow y=3 \ or \ y=2$

Case I:

If $y=3,$ we get

$\frac{x}{x+1}=3$

$\Rightarrow x=3(x+1)$ [On cross multiplying]

$\Rightarrow x=3x+3$

$\Rightarrow x=\frac{-3}{2}$

Case II:

If $y=2,$ we get:

$\frac{x}{x+1}=2$

$\Rightarrow x=2(x+1)$

$\Rightarrow x=2x+2$

$\Rightarrow -x=2$

$\Rightarrow x=-2$

Hence, the roots of the equation are $\frac{-3}{2}$ and $-2.$

69. $\frac{a}{(x-b)}+\frac{b}{(x-a)}=2, x\neq b, a$

Sol:

$\frac{a}{(x-b)}+\frac{b}{(x-a)}=2$

$\Rightarrow \left[\frac{a}{(x-b)}-1\right]+\left[\frac{b}{(x-b)}-1\right]=0$

$\Rightarrow \frac{a-(x-b)}{x-b}+\frac{b-(x-b)}{x-b}=0$

$$\Rightarrow (a-x+b)\left[\frac{1}{(x-b)}+\frac{1}{(x-a)}\right]=0$$

$$\Rightarrow (a-x+b)\left[\frac{(x-a)+(x-b)}{(x-b)(x-a)}\right]=0$$

$$\Rightarrow (a-x+b)\left[\frac{2x-(a+b)}{(x-b)(x-a)}\right]=0$$

$$\Rightarrow (a-x+b)\left[2x-(a+b)\right]=0$$

$$\Rightarrow a-x+b=0 \text{ or } 2x-(a+b)=0$$

$$\Rightarrow x=a+b \text{ or } x=\frac{a+b}{2}$$

Hence, the roots of the equation are $(a+b)$ and $\left(\frac{a+b}{2}\right)$.

70. $\frac{a}{(ax-1)}+\frac{b}{(bx-1)}=(a+b), x\neq\frac{1}{a},\frac{1}{b}$

Sol:

$$\frac{a}{(ax-1)}+\frac{b}{(bx-1)}=(a+b)$$

$$\Rightarrow \left[\frac{a}{(ax-1)}-b\right]+\left[\frac{b}{(bx-1)}-a\right]=0$$

$$\Rightarrow \frac{a-b(ax-1)}{ax-1}+\frac{b-a(bx-1)}{bx-1}=0$$

$$\Rightarrow \frac{a-abx+b}{ax-1}+\frac{a-abx+b}{bx-1}=0$$

$$\Rightarrow (a-abx+b)\left[\frac{1}{ax-1}+\frac{1}{(bx-1)}\right]=0$$

$$\Rightarrow (a-abx+b)\left[\frac{(bx-1)+(ax-1)}{(ax-1)(bx-1)}\right]=0$$

$$\Rightarrow (a-abx+b)\left[\frac{(a+b)x-2}{(ax-1)(bx-1)}\right]=0$$

$$\Rightarrow (a-abx+b)\left[(a+b)x-2\right]=0$$

$$\Rightarrow a-abx+b=0 \text{ or } (a+b)x-2=0$$

$\Rightarrow x=\frac{(a+b)}{ab}$ or $x=\frac{2}{(a+b)}$

Hence, the roots of the equation are $\frac{(a+b)}{ab}$ and $\frac{2}{(a+b)}$.

71. $3^{(x+2)}+3^{-x}=10$

Sol:

$3^{(x+2)}+3^{-x}=10$

$3^{x}.9+\frac{1}{3^{x}}=10$

Let 3^{x} be equal to y.

$\therefore 9y+\frac{1}{y}=10$

$\Rightarrow 9y^{2}+1=10y$

$\Rightarrow 9y^{2}-10y+1=0$

$\Rightarrow (y-1)(9y-1)=0$

$\Rightarrow y-1=0$ or $9y-1=0$

$\Rightarrow y=1$ or $y=\frac{1}{9}$

$\Rightarrow 3x^{x}=1$ or $3^{x}=\frac{1}{9}$

$\Rightarrow 3^{x}=3^{0}$ or $3^{x}=3^{-2}$

$\Rightarrow x=0$ or $x=-2$

Hence, 0 and -2 are the roots of the given equation.

72. $4^{(x+1)}+4^{(1-x)}=10$

Sol:

Given:

$4^{(x+1)}+4^{(1-x)}=10$

$\Rightarrow 4^{x}.4+4^{1}.\frac{1}{4^{4}}=10$

Let 4^{x} be y.

$\therefore 4y+\frac{4}{y}=10$

$\Rightarrow 4y^{2}-10y+4=0$

$\Rightarrow 4y^{2}-8y-2y+4=0$

$\Rightarrow 4y(y-2)-2(y-2)=0$

$\Rightarrow y=2 \text{ or } y=\frac{2}{4}=\frac{1}{2}$

$\Rightarrow 4^x=2 \text{ or } \frac{1}{2}$

$\Rightarrow 4^x=2^{2x}=2^1 \text{ or } 2^{2x}=2^{-1}$

$\Rightarrow x=\frac{1}{2} \text{ or } x=\frac{-1}{2}$

Hence, $\frac{1}{2}$ and $\frac{-1}{2}$ are roots of the given equation.

73. $2^{2x}-3.2^{(x+2)}+32=0$

Sol:

$2^{2x}-3.2^{(x+2)}+32=0$

$\Rightarrow (2^x)^2-3.2^x.2^2+32=0$

Let 2^x be y.

$\therefore y^2-12y+32=0$

$\Rightarrow y^2-8y-4y+32=0$

$\Rightarrow y(y-8)-4(y-8)=0$

$\Rightarrow (y-8)=0 \text{ or } (y-4)=0$

$\Rightarrow y=8 \text{ or } y=4$

$\therefore 2^x=8 \text{ or } 2^x=4$

$\Rightarrow 2^x=2^3 \text{ or } 2^x=2^2$

$\Rightarrow x-2 \text{ or } 3$

Hence, 2 and 3 are the roots of the given equation.

Exercise - 10B

1. $x^2-6x+3=0$

Sol:

$x^2-6x+3=0$

$\Rightarrow x^2-6x=-3$

$\Rightarrow x^2-2\times x\times 3+3^2=-3+3^2$ (Adding 3^2 on both sides)

$\Rightarrow (x-3)^2=-3+9=6$

$\Rightarrow x-3=\pm\sqrt{6}$ (Taking square root on the both sides)

$\Rightarrow x-3=\sqrt{6}$ *or* $x-3=-\sqrt{6}$

$\Rightarrow x=3+\sqrt{6}$ *or* $x=3-\sqrt{6}$

Hence, $3+\sqrt{6}$ and $3-\sqrt{6}$ are the roots of the given equation.

2. $x^2-4x+1=0$

Sol:

$x^2-4x+1=0$

$\Rightarrow x^2-4x=-1$

$\Rightarrow x^2-2\times x\times 2+2^2=-1+2^2$ (Adding 2^2 on both sides)

$\Rightarrow (x-2)^2=-1+4=3$

$\Rightarrow x-2=\pm\sqrt{3}$ (Taking square root on the both sides)

$\Rightarrow x-2=\sqrt{3}$ *or* $x-2=-\sqrt{3}$

$\Rightarrow x=2+\sqrt{3}$ *or* $x=2-\sqrt{3}$

Hence, $2+\sqrt{3}$ and $2-\sqrt{3}$ are the roots of the given equation.

3. $x^2+8x-2=0$

Sol:

$x^2+8x-2=0$

$\Rightarrow x^2+8x=2$

$\Rightarrow x^2+2\times x\times 4+4^2=2+4^2$ (Adding 4^2 on both sides)

$\Rightarrow (x+4)^2=2+16=18$

$\Rightarrow x+4=\pm\sqrt{18}=\pm 3\sqrt{2}$ (Taking square root on the both sides)

$\Rightarrow x+4=3\sqrt{2}$ *or* $x+4=-3\sqrt{2}$

$\Rightarrow x=-4+3\sqrt{2}$ *or* $x=-4-3\sqrt{2}$

Hence, $\left(-4+3\sqrt{2}\right)$ and $\left(-4-3\sqrt{2}\right)$ are the roots of the given equation.

4. $4x^2+4\sqrt{3}x+3=0$

Sol:

$4x^2+4\sqrt{3}x+3=0$

$\Rightarrow 4x^2+4\sqrt{3}x=-3$

$\Rightarrow (2x)^2+2\times 2x\times\sqrt{3}+\left(\sqrt{3}\right)^2=-3+\left(\sqrt{3}\right)^2$ [Adding $\left(\sqrt{3}\right)^2$ on both sides]

$\Rightarrow \left(2x+\sqrt{3}\right)^2 = -3+3=0$

$\Rightarrow 2x+\sqrt{3}=0$

$\Rightarrow x=-\dfrac{\sqrt{3}}{2}$

Hence, $-\dfrac{\sqrt{3}}{2}$ is the repeated root of the given equation.

5. $2x^2+5x-3=0$

Sol:

$2x^2+5x-3=0$

$\Rightarrow 4x^2+10x-6=0$ (Multiplying both sides by 2)

$\Rightarrow 4x^2+10x=6$

$\Rightarrow (2x)^2+2\times 2x\times\dfrac{5}{2}+\left(\dfrac{5}{2}\right)^2=6+\left(\dfrac{5}{2}\right)^2$ [Adding $\left(\dfrac{5}{2}\right)^2$ on both sides]

$\Rightarrow \left(2x+\dfrac{5}{2}\right)^2=6+\dfrac{25}{4}=\dfrac{24+25}{4}=\dfrac{49}{4}=\left(\dfrac{7}{2}\right)^2$

$\Rightarrow 2x+\dfrac{5}{2}=\pm\dfrac{7}{2}$ (Taking square root on both sides)

$\Rightarrow 2x+\dfrac{5}{2}=\dfrac{7}{2}$ *or* $2x+\dfrac{5}{2}=-\dfrac{7}{2}$

$\Rightarrow 2x=\dfrac{7}{2}-\dfrac{5}{2}=\dfrac{2}{3}=1$ or $2x=-\dfrac{7}{2}-\dfrac{5}{2}=-\dfrac{12}{2}=-6$

$x=\dfrac{1}{2}$ or $x=-3$

Hence, $\dfrac{1}{2}$ and -3 are the roots of the given equation.

6. $3x^2-x-2=0$

Sol:

$3x^2-x-2=0$

$\Rightarrow 9x^2-3x-6=0$ (Multiplying both sides by 3)

$\Rightarrow 9x^2-3x=6$

$\Rightarrow (3x)^2-2\times 3x\times\dfrac{1}{2}+\left(\dfrac{1}{2}\right)^2=6+\left(\dfrac{1}{2}\right)^2$ [Adding $\left(\dfrac{1}{2}\right)^2$ on both sides]

$\Rightarrow \left(3x-\frac{1}{2}\right)^2 = 6+\frac{1}{4}=\frac{25}{4}=\left(\frac{5}{2}\right)^2$

$\Rightarrow 3x-\frac{1}{2}=\pm\frac{5}{2}$ (Taking square root on both sides)

$\Rightarrow 3x-\frac{1}{2}=\frac{5}{2}$ *or* $3x-\frac{1}{2}=-\frac{5}{2}$

$\Rightarrow 3x=\frac{5}{2}+\frac{1}{2}=\frac{6}{2}=3$ or $3x=-\frac{5}{2}+\frac{1}{2}=-\frac{4}{2}=-2$

$\Rightarrow x=1$ *or* $x=-\frac{2}{3}$

Hence, 1 and $-\frac{2}{3}$ are the roots of the given equation.

7. $8x^2-14x-15=0$

Sol:

$8x^2-14x-15=0$

$\Rightarrow 16x^2-28x-30=0$ (Multiplying both sides by 2)

$\Rightarrow 16x^2-28x=30$

$\Rightarrow (4x)^2-2\times 4x\times\frac{7}{2}+\left(\frac{7}{2}\right)^2=30+\left(\frac{7}{2}\right)^2$ [Adding $\left(\frac{7}{2}\right)^2$ on both sides]

$\Rightarrow \left(4x-\frac{7}{2}\right)^2=30+\frac{49}{4}=\frac{169}{4}=\left(\frac{13}{2}\right)^2$

$\Rightarrow 4x-\frac{7}{2}=\pm\frac{13}{2}$ (Taking square root on both sides)

$\Rightarrow 4x-\frac{7}{2}=\frac{13}{2}$ *or* $4x-\frac{7}{2}=\frac{13}{2}$

$\Rightarrow 4x=\frac{13}{2}+\frac{7}{2}=\frac{20}{2}=10$ or $4x=-\frac{13}{2}+\frac{7}{2}=-\frac{6}{2}=-3$

$\Rightarrow x=\frac{5}{2}$ *or* $x=-\frac{3}{4}$

Hence, $\frac{5}{2}$ and $-\frac{3}{4}$ are the roots of the given equation.

8. $7x^2+3x-4=0$

Sol:

$7x^2+3x-4=0$

$\Rightarrow 49x^2+21x-28=0$ (Multiplying both sides by 7)

$\Rightarrow 49x^2 + 21x = 28$

$\Rightarrow (7x)^2 + 2 \times 7x \times \frac{3}{2} + \left(\frac{3}{2}\right)^2 = 28 + \left(\frac{3}{2}\right)^2$ [Adding $\left(\frac{3}{2}\right)^2$ on both sides]

$\Rightarrow \left(7x + \frac{3}{2}\right)^2 = 28 + \frac{9}{4} = \frac{121}{4} = \left(\frac{11}{2}\right)^2$

$\Rightarrow 7x + \frac{3}{2} = \pm\frac{11}{2}$ (Taking square root on both sides)

$\Rightarrow 7x + \frac{3}{2} = \frac{11}{2}$ *or* $7x + \frac{3}{2} = -\frac{11}{2}$

$\Rightarrow 7x = \frac{11}{2} - \frac{3}{2} = \frac{8}{2} = 4$ or $7x = -\frac{11}{2} - \frac{3}{2} = -\frac{14}{2} = -7$

$\Rightarrow x = \frac{4}{7}$ *or* $x = -1$

Hence, $\frac{4}{7}$ and -1 are the roots of the given equation.

9. $3x^2 - 2x - 1 = 0$

Sol:

$3x^2 - 2x - 1 = 0$

$\Rightarrow 9x^2 - 6x - 3 = 0$ (Multiplying both sides by 3)

$\Rightarrow 9x^2 - 6x = 3$

$\Rightarrow (3x)^2 - 2 \times 3x \times 1 + 1^2 = 3 + 1^2$ [Adding 1^2 on both sides]

$\Rightarrow (3x - 1)^2 = 3 + 1 = 4 = (2)^2$

$\Rightarrow 3x - 1 = \pm 2$ (Taking square root on both sides)

$\Rightarrow 3x - 1 = 2$ or $3x - 1 = -2$

$\Rightarrow 3x = 3$ or $3x = -1$

$\Rightarrow x = 1$ or $x = -\frac{1}{3}$

Hence, 1 and $-\frac{1}{3}$ are the roots of the given equation.

10. $5x^2 - 6x - 2 = 0$

Sol:

$5x^2 - 6x - 2 = 0$

$\Rightarrow 25x^2 - 30x - 10 = 0$ (Multiplying both sides by 5)

$\Rightarrow 25x^2 - 30x = 10$

$\Rightarrow (5x)^2 - 2\times 5x\times 3 + 3^2 = 10 + 3^2$ (Adding 3^2 on both sides)

$\Rightarrow (5x-3)^2 = 10+9-19$

$\Rightarrow 5x-3 = \pm\sqrt{19}$ (Taking square root on both)

$\Rightarrow 5x-3=\sqrt{19}$ *or* $5x-3=-\sqrt{19}$

$\Rightarrow 5x = 3+\sqrt{19}$ *or* $5x = 3-\sqrt{19}$

$\Rightarrow x = \frac{3+\sqrt{19}}{5}$ *or* $x = \frac{3-\sqrt{19}}{5}$

Hence, $\frac{3+\sqrt{19}}{5}$ and $\frac{3-\sqrt{19}}{5}$ are $\frac{3-\sqrt{19}}{5}$ are the roots of the given equation.

11. $\frac{2}{x^2}-\frac{5}{x}+2=0$

Sol:

$\frac{2}{x^2}-\frac{5}{x}+2=0$

$\Rightarrow \frac{2-5x+2x^2}{x^2}=0$

$\Rightarrow 2x^2-5x+2=0$

$\Rightarrow 4x^2-10x+4=0$ (Multiplying both sides by 2)

$\Rightarrow 4x^2-10x=-4$

$\Rightarrow (2x)^2 - 2\times 2x\times\frac{5}{2}+\left(\frac{5}{2}\right)^2 = -4+\left(\frac{5}{2}\right)^2$ [Adding $\left(\frac{5}{2}\right)^2$ on both sides]

$\Rightarrow \left(2x-\frac{5}{2}\right)^2 = -4+\frac{25}{4}=\frac{9}{4}=\left(\frac{3}{2}\right)^2$

$\Rightarrow 2x-\frac{5}{2}=\pm\frac{3}{2}$ (Taking square root on both sides)

$\Rightarrow 2x-\frac{5}{2}=\frac{3}{2}$ or $2x-\frac{5}{2}=-\frac{3}{2}$

$\Rightarrow 2x=\frac{3}{2}+\frac{5}{2}=\frac{8}{2}=4$ or $2x=-\frac{3}{2}+\frac{5}{2}=\frac{2}{2}=1$

$\Rightarrow x=2$ or $x=\frac{1}{2}$

Hence, 2 and $\frac{1}{2}$ are the roots of the given equation.

12. $4x^2+4bx-\left(a^2-b^2\right)=0$

Sol:

$4x^2+4bx-\left(a^2-b^2\right)=0$

$\Rightarrow 4x^2+4bx=a^2-b^2$

$\Rightarrow (2x)^2+2\times 2x\times b+b^2=a^2-b^2+b^2$ (Adding b^2 on both sides)

$\Rightarrow (2x+b)^2=a^2$

$\Rightarrow 2x+b=\pm a$ (Taking square root on both sides)

$\Rightarrow 2x+b=a \text{ or } 2x+b=-a$

$\Rightarrow 2x=a-b \text{ or } 2x=-a-b$

$\Rightarrow x=\dfrac{a-b}{2} \text{ or } x=-\dfrac{a+b}{2}$

Hence, $\dfrac{a-b}{2}$ and $-\dfrac{a+b}{2}$ are the roots of the given equation.

13. $x^2-\left(\sqrt{2}+1\right)x+\sqrt{2}=0$

Sol:

$x^2-\left(\sqrt{2}+1\right)x+\sqrt{2}=0$

$\Rightarrow x^2-\left(\sqrt{2}+1\right)x=-\sqrt{2}$

$\Rightarrow x^2-2\times x\times\left(\dfrac{\sqrt{2}+1}{2}\right)+\left(\dfrac{\sqrt{2}+1}{2}\right)^2=-\sqrt{2}+\left(\dfrac{\sqrt{2}+1}{2}\right)^2$

[Adding $\left(\dfrac{\sqrt{2}+1}{2}\right)^2$ on both sides]

$\Rightarrow \left[x-\left(\dfrac{\sqrt{2}+1}{2}\right)\right]^2=\dfrac{-4\sqrt{2}+2+1+2\sqrt{2}}{4}=\dfrac{2-2\sqrt{2}+1}{4}=\left(\dfrac{\sqrt{2}-1}{2}\right)^2$

$\Rightarrow x-\left(\dfrac{\sqrt{2}+1}{2}\right)=\pm\left(\dfrac{\sqrt{2}-1}{2}\right)$ (Taking square root on both sides)

$\Rightarrow x-\left(\dfrac{\sqrt{2}+1}{2}\right)=\left(\dfrac{\sqrt{2}-1}{2}\right) \text{ or } x-\left(\dfrac{\sqrt{2}+1}{2}\right)=-\left(\dfrac{\sqrt{2}-1}{2}\right)$

$\Rightarrow x=\dfrac{\sqrt{2}+1}{2}+\dfrac{\sqrt{2}-1}{2}$ or $x=\dfrac{\sqrt{2}+1}{2}-\dfrac{\sqrt{2}-1}{2}$

$\Rightarrow x = \frac{2\sqrt{2}}{2} = \sqrt{2}$ or $x = \frac{2}{2} = 1$

Hence, $\sqrt{2}$ and 1 are the roots of the given equation.

14. $\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$

Sol:

$\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$

$\Rightarrow 2x^2 - 3\sqrt{2}x - 4 = 0$ (Multiplying both sides by $\sqrt{2}$)

$\Rightarrow 2x^2 - 3\sqrt{2}x = 4$

$\Rightarrow \left(\sqrt{2}x\right)^2 - 2 \times \sqrt{2}x \times \frac{3}{2} + \left(\frac{3}{2}\right)^2 = 4 + \left(\frac{3}{2}\right)^2$ [Adding $\left(\frac{3}{2}\right)^2$ on both sides]

$\Rightarrow \left(\sqrt{2}x - \frac{3}{2}\right)^2 = 4 + \frac{9}{4} = \frac{25}{4} = \left(\frac{5}{2}\right)^2$

$\Rightarrow \sqrt{2}x - \frac{3}{2} = \pm\frac{5}{2}$ (Taking square root on both sides)

$\Rightarrow \sqrt{2}x - \frac{3}{2} = \frac{5}{2}$ or $\sqrt{2}x - \frac{3}{2} = -\frac{5}{2}$

$\Rightarrow \sqrt{2}x = \frac{5}{2} + \frac{3}{2} = \frac{8}{2} = 4$ or $\sqrt{2}x = -\frac{5}{2} + \frac{3}{2} = -\frac{2}{2} = -1$

$\Rightarrow x = \frac{4}{\sqrt{2}} = 2\sqrt{2}$ or $x = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

Hence, $2\sqrt{2}$ and $-\frac{\sqrt{2}}{2}$ are the roots of the given equation.

15. $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

Sol:

$\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

$\Rightarrow 3x^2 + 10\sqrt{3}x + 21 = 0$ (Multiplying both sides by $\sqrt{3}$)

$\Rightarrow 3x^2 + 10\sqrt{3}x = -21$

$\Rightarrow \left(\sqrt{3}x\right)^2 + 2 \times \sqrt{3}x \times 5 + 5^2 = -21 + 5^2$ (Adding 5^2 on both sides)

$\Rightarrow \left(\sqrt{3}x + 5\right)^2 = 21 + 25 = 4 = 2^2$

$\Rightarrow \sqrt{3}x + 5 = \pm 2$ (Taking square root on both sides)

$\Rightarrow \sqrt{3}x + 5 = 2$ or $\sqrt{3}x + 5 = -2$

$\Rightarrow \sqrt{3}x = -3$ or $\sqrt{3}x = -7$

$\Rightarrow x = -\frac{3}{\sqrt{3}} = -\sqrt{3}$ or $x = -\frac{7}{\sqrt{3}} = -\frac{7\sqrt{3}}{3}$

Hence, $-\sqrt{3}$ and $-\frac{7\sqrt{3}}{3}$ are the roots of the given equation.

16. By using the method of completing the square, show that the equation $2x^2 + x + 4 = 0$ has no real roots.

Sol:

$2x^2 + x + 4 = 0$

$\Rightarrow 4x^2 + 2x + 8 = 0$ (Multiplying both sides by 2)

$\Rightarrow 4x^2 + 2x = -8$

$\Rightarrow (2x)^2 + 2 \times 2x \times \frac{1}{2} + \left(\frac{1}{2}\right)^2 = -8 + \left(\frac{1}{2}\right)^2$ [Adding $\left(\frac{1}{2}\right)^2$ on both sides]

$\Rightarrow \left(2x + \frac{1}{2}\right)^2 = -8 + \frac{1}{4} = -\frac{31}{4} < 0$

But, $\left(2x + \frac{1}{2}\right)^2$ cannot be negative for any real value of *x*.

So, there is no real value of x satisfying the given equation.

Hence, the given equation has no real roots.

Exercise -10C

1. $2x^2 - 7x + 6 = 0$

Sol:

(i) $2x^2 - 7x + 6 = 0$

Here,

$a = 2,$

$b = -7,$

$c = 6$

Discriminant D is diven by:

$D = b^2 - 4ac$

$= (-7)^2 - 4 \times 2 \times 6$

$= 49 - 48$

$= 1$

(ii) $3x^2 - 2x + 8 = 0$

Here,

$a = 3,$

$b = -2,$

$c = 8$

Discriminant D is given by:

$D = b^2 - 4ac$

$= (-2)^2 - 4 \times 3 \times 8$

$= 4 - 96$

$= -92$

(iii) $2x^2 - 5\sqrt{2x} + 4 = 0$

Here,

$a = 2,$

$b = -5\sqrt{2},$

$c = 4$

Discriminant D is given by:

$D = b^2 - 4ac$

$= \left(-5\sqrt{2}\right)^2 - 4 \times 2 \times 4$

$= (25 \times 2) - 32$

$= 50 - 32$

$= 18$

(iv) $\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$

Here,

$a = \sqrt{3}$

$b = 2\sqrt{2},$

$c = -2\sqrt{3}$

Discriminant D is given by:

$D = b^2 - 4ac$

$= \left(2\sqrt{2}\right)^2 - 4 \times \sqrt{3} \times \left(-2\sqrt{3}\right)$

$= (4 \times 2) + (8 \times 3)$

$= 8 + 24$

$= 32$

(v) $(x-1)(2x-1) = 0$

$\Rightarrow 2x^2 - 3x + 1 = 0$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a=2, b=-3$ *and* $c=1$

$\therefore$ Discriminant, $D=b^2-4ac=(-3)^2-4\times2\times1=9-8=1$

(vi) $1-x=2x^2$

$\Rightarrow 2x^2+x-1=0$

Here,

$a=2,$

$b=1,$

$c=-1$

Discriminant D is given by:

$D=b^2-4ac$

$=1^2-4\times2(-1)$

$=1+8$

$=9$

Find the roots of the each of the following equations, if they exist, by applying the quadratic formula:

2. $x^2-4x-1=0$

Sol:

Given:

$x^2-4x-1=0$

On comparing it with $ax^2+bx+c=0,$ we get:

$a=1, b=-4$ *and* $c=-1$

Discriminant D is given by:

$D=(b^2-4ac)$

$=(-4)^2-4\times1\times(-1)$

$=16+4$

$=20$

$=20>0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$$\alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-(-4)+\sqrt{20}}{2\times1}=\frac{4+2\sqrt{5}}{2}=\frac{2(2+\sqrt{5})}{2}=(2+\sqrt{5})$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-(-4)-\sqrt{20}}{2}=\frac{4-2\sqrt{5}}{2}=\frac{2(2-\sqrt{5})}{2}=(2-\sqrt{5})$$

Thus, the roots of the equation are $\left(2+\sqrt{5}\right)$ and $\left(2-\sqrt{5}\right)$.

3. $x^2-6x+4=0$

Sol:

Given:

$x^2-6x+4=0$

On comparing it with $ax^2+bx+c=0,$ we get:

$a=1, b=-6$ *and* $c=4$

Discriminant D is given by:

$D=\left(b^2-4ac\right)$

$=\left(-6\right)^2-4\times1\times4$

$=36-16$

$=20>0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$$\alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-\left(-6\right)+\sqrt{20}}{2\times1}=\frac{6+2\sqrt{5}}{2}=\frac{2\left(3+\sqrt{3}\right)}{2}=\left(3+\sqrt{5}\right)$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-\left(-6\right)-\sqrt{20}}{2}=\frac{6-2\sqrt{5}}{2}=\frac{2\left(3-\sqrt{5}\right)}{2}=\left(3-\sqrt{5}\right)$$

Thus, the roots of the equation are $\left(3+2\sqrt{5}\right)$ and $\left(3-2\sqrt{5}\right)$.

4. $2x^2+x-4=0.$

Sol:

The given equation is $2x^2+x-4=0.$

Comparing it with $ax^2+bx+c=0,$ we get

$a=2, b=1$ and $c=-4$

$\therefore$ Discriminant, $D=b^2-4ac=\left(1\right)^2-4\times2\times\left(-4\right)=1+32=33>0$

So, the given equation has real roots.

Now, $\sqrt{D}=\sqrt{33}$

$$\therefore\alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-1+\sqrt{33}}{2\times2}=\frac{-1+\sqrt{33}}{4}$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-1-\sqrt{33}}{2\times2}=\frac{-1-\sqrt{33}}{4}$$

Hence, $\frac{-1+\sqrt{33}}{4}$ and $\frac{-1-\sqrt{33}}{4}$ are the roots of the given equation.

5. $25x^2+30x+7=0$

Sol:

Given:

$25x^2+30x+7=0$

On comparing it with $ax^2+bx+x=0$, we get;

$a=25, b=30 \; and \; c=7$

Discriminant D is given by:

$D=\left(b^2-4ac\right)$

$=30^2-4\times 25\times 7$

$=900-700$

$=200$

$=200>0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$$\alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-30+\sqrt{200}}{2\times 25}=\frac{-30+10\sqrt{2}}{50}=\frac{10\left(-3+\sqrt{2}\right)}{50}=\frac{\left(-3+\sqrt{2}\right)}{5}$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-30-\sqrt{200}}{2\times 25}=\frac{-30-10\sqrt{2}}{50}=\frac{10\left(-3-\sqrt{2}\right)}{50}=\frac{\left(-3-\sqrt{2}\right)}{5}$$

Thus, the roots of the equation are $\frac{\left(-3+\sqrt{2}\right)}{5} \; and \; \frac{\left(-3-\sqrt{2}\right)}{5}$.

6. $16x^2+24x+1$

Sol:

Given:

$16x^2+24x+1$

$\Rightarrow 16x^2-24x-1=0$

On comparing it with $ax^2+bx+x=0$, we get;

$a=16, b=-24 \; and \; c=-1$

Discriminant D is given by:

$D=\left(b^2-4ac\right)$

$=\left(-24\right)^2-4\times 16\times\left(-1\right)$

$= 576 + (64)$

$= 640 > 0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-(-24)+\sqrt{640}}{2\times 16} = \frac{24+8\sqrt{10}}{32} = \frac{8\left(3+\sqrt{10}\right)}{32} = \frac{\left(3+\sqrt{10}\right)}{4}$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-(-24)-\sqrt{640}}{2\times 16} = \frac{24-8\sqrt{10}}{32} = \frac{8\left(3-\sqrt{10}\right)}{32} = \frac{\left(3-\sqrt{10}\right)}{4}$$

Thus, the roots of the equation are $\frac{\left(3+\sqrt{10}\right)}{4}$ *and* $\frac{\left(3-\sqrt{10}\right)}{4}$.

7. $15x^2 - 28 = x$

Sol:

Given:

$15x^2 - 28 = x$

$\Rightarrow 15x^2 - x - 28 = 0$

On comparing it with $ax^2 + bx + c = 0,$ we get;

$a = 25, b = -1$ *and* $c = -28$

Discriminant D is given by:

$D = \left(b^2 - 4ac\right)$

$= (-1)^2 - 4\times 15\times(-28)$

$= 1 - (-1680)$

$= 1 + 1680$

$= 1681$

$= 1681 > 0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-(-1)+\sqrt{1681}}{2\times 25} = \frac{1+41}{30} = \frac{42}{30} = \frac{7}{5}$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-(-1)-\sqrt{1681}}{2\times 25} = \frac{1-41}{30} = \frac{-40}{30} = \frac{-4}{3}$$

Thus, the roots of the equation are $\frac{7}{5}$ and $\frac{-4}{3}$.

8. $2x^2 - 2\sqrt{2}x + 1 = 0$

Sol:

The given equation is $2x^2 - 2\sqrt{2}x + 1 = 0$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = 2, b = -2\sqrt{2}$ *and* $c = 1$

$\therefore$ Discriminant, $D = b^2 - 4ac = \left(-2\sqrt{2}\right)^2 - 4\times2\times1 = 8 - 8 = 0$

So, the given equation has real roots.

Now, $\sqrt{D} = 0$

$$\therefore \alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-\left(-2\sqrt{2}\right)}{2\times2} = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-\left(-2\sqrt{2}\right)-\sqrt{0}}{2\times2} = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$$

Hence, $\frac{\sqrt{2}}{2}$ is the repeated root of the given equation.

9. $\sqrt{2}x^2 + 7 + 5\sqrt{2} = 0.$

Sol:

The given equation is $\sqrt{2}x^2 + 7 + 5\sqrt{2} = 0.$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = \sqrt{2}, b = 7$ *and* $c = 5\sqrt{2}$

$\therefore$ Discriminant, $D = b^2 - 4ac = (7)^2 - 4\times\sqrt{2}\times5\sqrt{2} = 49 - 40 = 9 > 0$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{9} = 3$

$$\therefore \alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-7+3}{2\times\sqrt{2}} = \frac{-4}{2\sqrt{2}} = -\sqrt{2}$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-7-3}{2\times\sqrt{2}} = \frac{-10}{2\sqrt{2}} = -\frac{5\sqrt{2}}{2}$$

Hence, $-\sqrt{2}$ and $-\frac{5\sqrt{2}}{2}$ are the root of the given equation.

10. $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

Sol:

Given:

$\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

On comparing it with $ax^2 + bx + x = 0,$ we get;

$a = \sqrt{3}, b = 10 \ and \ c = -8\sqrt{3}$

Discriminant D is given by:

$D = \left(b^2 - 4ac\right)$

$= (10)^2 - 4\times\sqrt{3}\times\left(-8\sqrt{3}\right)$

$= 100 + 96$

$= 196 > 0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-10+\sqrt{196}}{2\sqrt{3}} = \frac{-10+14}{2\sqrt{3}} = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-(10)-\sqrt{196}}{2\sqrt{3}} = \frac{-10-14}{2\sqrt{3}} = \frac{-24}{2\sqrt{3}} = \frac{-12}{\sqrt{3}} = \frac{-12}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{-12\sqrt{3}}{3} = -4\sqrt{3}$$

Thus, the roots of the equation are $\frac{2\sqrt{3}}{3}$ and $-4\sqrt{3}.$

11. $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0.$

Sol:

The given equation is $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0.$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = \sqrt{3}, b = -2\sqrt{2} \ and \ c = -2\sqrt{3}$

$\therefore$ Discriminant, $D = b^2 - 4ac = \left(-2\sqrt{2}\right)^2 - 4\times\sqrt{3}\times\left(-2\sqrt{3}\right) = 8 + 24 = 32 > 0$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{32} = 4\sqrt{2}$

$$\therefore \alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-\left(-2\sqrt{2}\right)+4\sqrt{2}}{2\times\sqrt{3}} = \frac{6\sqrt{2}}{2\sqrt{3}} = \sqrt{6}$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-\left(-2\sqrt{2}\right)-4\sqrt{2}}{2\times\sqrt{3}} = \frac{-2\sqrt{2}}{2\sqrt{3}} = -\frac{\sqrt{6}}{3}$$

Hence, $\sqrt{6}$ and $-\frac{\sqrt{6}}{3}$ are the root of the given equation.

12. $2x^2+6\sqrt{3}x-60=0.$

Sol:

The given equation is $2x^2+6\sqrt{3}x-60=0.$

Comparing it with $ax^2+bx+c=0,$ we get

$a=2, b=6\sqrt{3}$ *and* $c=-60$

$\therefore$ Discriminant, $D=b^2-4ac=\left(6\sqrt{3}\right)^2-4\times2\times(-60)=180+480=588>0$

So, the given equation has real roots.

Now, $\sqrt{D}=\sqrt{588}=14\sqrt{3}$

$$\therefore \alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-6\sqrt{3}+14\sqrt{3}}{2\times2}=\frac{8\sqrt{3}}{4}=2\sqrt{3}$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-6\sqrt{3}-14\sqrt{3}}{2\times2}=\frac{-20\sqrt{3}}{4}=-5\sqrt{3}$$

Hence, $2\sqrt{3}$ and $-5\sqrt{3}$ are the root of the given equation.

13. $4\sqrt{3}x^2+5x-2\sqrt{3}=0$

Sol:

The given equation is $4\sqrt{3}x^2+5x-2\sqrt{3}=0$

Comparing it with $ax^2+bx+c=0,$ we get

$a=4\sqrt{3}, b=5$ *and* $c=-2\sqrt{3}$

$\therefore$ Discriminant, $D=b^2-4ac=5^2-4\times4\sqrt{3}\times\left(-2\sqrt{3}\right)=25+96=121>0$

So, the given equation has real roots.

Now, $\sqrt{D}=\sqrt{121}=11$

$$\therefore \alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-5-11}{2\times4\sqrt{3}}=\frac{6}{8\sqrt{3}}=\frac{\sqrt{3}}{4}$$

$$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-5-11}{2\times4\sqrt{3}}=\frac{-16}{8\sqrt{3}}=-\frac{2\sqrt{3}}{3}$$

Hence, $\frac{\sqrt{3}}{4}$ and $-\frac{2\sqrt{3}}{3}$ are the root of the given equation.

14. $3x^2-2\sqrt{6}x+2=0$

Sol:

The given equation is $3x^2-2\sqrt{6}x+2=0$

Comparing it with $ax^2+bx+c=0,$ we get

$a = 3, b = -2\sqrt{6}$ *and* $c = 2$

$\therefore$ Discriminant, $D = b^2 - 4ac = \left(-2\sqrt{6}\right)^2 - 4\times3\times2 = 24 - 24 = 0$

So, the given equation has real roots.

Now, $\sqrt{D} = 0$

$\therefore \alpha = \dfrac{-b+\sqrt{D}}{2a} = \dfrac{-\left(-2\sqrt{6}\right)+0}{2\times3} = \dfrac{2\sqrt{6}}{6} = \dfrac{\sqrt{6}}{3}$

$\beta = \dfrac{-b-\sqrt{D}}{2a} = \dfrac{-\left(-2\sqrt{6}\right)}{2\times3} = \dfrac{2\sqrt{6}}{6} = \dfrac{\sqrt{6}}{3}$

Hence, $\dfrac{\sqrt{6}}{3}$ are the repeated of the given equation.

15. $2\sqrt{3}x^2 - 5x + \sqrt{3} = 0$

Sol:

The given equation is $2\sqrt{3}x^2 - 5x + \sqrt{3} = 0$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = 2\sqrt{3}, b = -5$ *and* $c = \sqrt{3}$

$\therefore$ Discriminant, $D = b^2 - 4ac = (-5)^2 - 4\times2\sqrt{3}\times\sqrt{3} = 25 - 25 = 1 > 0$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{1} = 1$

$\therefore \alpha = \dfrac{-b+\sqrt{D}}{2a} = \dfrac{-(-5+1)+1}{2\times2\sqrt{3}} = \dfrac{6}{4\sqrt{3}} = \dfrac{\sqrt{3}}{2}$

$\beta = \dfrac{-b-\sqrt{D}}{2a} = \dfrac{-(-5)-1}{2\times2\sqrt{3}} = \dfrac{4}{4\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

Hence, $\dfrac{\sqrt{3}}{2}$ and $\dfrac{\sqrt{3}}{3}$ are the roots of the given equation.

16. $x^2 + x + 2 = 0.$

Sol:

The given equation is $x^2 + x + 2 = 0.$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = 1, b = 1$ and $c = 2$

$\therefore$ Discriminant $D = b^2 - 4ac = 1^2 - 4\times1\times2 = 1 - 8 = -7 < 0$

Hence, the given equation has no real roots (or real roots does not exist).

17. $2x^2 + ax - a^2 = 0.$

Sol:

The given equation is $2x^2 + ax - a^2 = 0.$

Comparing it with $Ax^2 + Bx + C = 0,$ we get

$A = 2, B = a \text{ and } C = -a^2$

$\therefore$ Discriminant, $D = B^2 - 4AC = a^2 - 4\times 2\times -a^2 = a^2 + 8a^2 = 9a^2 \geq 0$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{9a^2} = 3a$

$$\therefore \alpha = \frac{-B+\sqrt{D}}{2A} = \frac{-a+3a}{2\times 2} = \frac{2a}{4} = \frac{a}{2}$$

$$\beta = \frac{-B-\sqrt{D}}{2A} = \frac{-a-3a}{2\times 2} = \frac{-4a}{4} = -a$$

Hence, $\frac{a}{2}$ and $-a$ are the roots of the given equation.

18. $x^2 - \left(\sqrt{3}+1\right)x + \sqrt{3} = 0.$

Sol:

The given equation is $x^2 - \left(\sqrt{3}+1\right)x + \sqrt{3} = 0.$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = 1, b = -\left(\sqrt{3}+1\right)$ and $c = \sqrt{3}$

$\therefore$ Discriminant,

$$D = b^2 - 4ac = \left[-\left(\sqrt{3}+1\right)\right]^2 - 4\times 1\times\sqrt{3} = 3+1+2\sqrt{3}-4\sqrt{3} = 3-2\sqrt{3}+1 = \left(\sqrt{3}-1\right)^2 > 0$$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{\left(\sqrt{3}-1\right)^2} = \sqrt{3}-1$

$$\therefore \alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-\left[-\left(\sqrt{3}+1\right)\right]+\left(\sqrt{3}-1\right)}{2\times 1} = \frac{\sqrt{3}+1+\sqrt{3}-1}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-\left[-\left(\sqrt{3}+1\right)\right]-\left(\sqrt{3}-1\right)}{2\times 1} = \frac{\sqrt{3}+1-\sqrt{3}+1}{2} = \frac{2}{2} = 1$$

Hence, $\sqrt{3}$ and 1 are the roots of the given equation.

19. $2x^2 + 5\sqrt{3}x + 6 = 0.$

Sol:

The given equation is $2x^2 + 5\sqrt{3}x + 6 = 0.$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = 2, b = 5\sqrt{3}$ *and* $c = 6$

$\therefore$ Discriminant, $D = b^2 - 4ac = \left(5\sqrt{3}\right)^2 - 4\times 2\times 6 = 75 - 48 = 27 > 0$

So, the given equation has real roots.

Now, $\sqrt{D} = \sqrt{27} = 3\sqrt{3}$

$$\therefore \alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-5\sqrt{3}+3\sqrt{3}}{2\times 2} = \frac{-2\sqrt{3}}{4} = -\frac{\sqrt{3}}{2}$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-5\sqrt{3}-3\sqrt{3}}{2\times 2} = \frac{-8\sqrt{3}}{4} = -2\sqrt{3}$$

Hence, $-\frac{\sqrt{3}}{2}$ and $-2\sqrt{3}$ are the roots of the given equation.

20. $3x^2 - 2x + 2 = 0.$ b

Sol:

The given equation is $3x^2 - 2x + 2 = 0.$

Comparing it with $ax^2 + bx + c = 0,$ we get

$a = 3, b = -2$ and $c = 2$

$\therefore$ Discriminant $D = b^2 - 4ac = (-2)^2 - 4\times 3\times 2 = 4 - 24 = -20 < 0$

Hence, the given equation has no real roots (or real roots does not exist).

21. $x + \frac{1}{x} = 3, x \neq 0$

Sol:

The given equation is

$$x + \frac{1}{x} = 3, x \neq 0$$

$$\Rightarrow \frac{x^2+1}{x} = 3$$

$$\Rightarrow x^2 + 1 = 3x$$

$$\Rightarrow x^2 - 3x + 1 = 0$$

This equation is of the form $ax^2 + bx + c = 0,$ where, $a = 1, b = -3$ and $c = 1.$

$\therefore$ Discriminant, $D = b^2 - 4ac = (-3)^2 - 4\times 1\times 1 = 9 - 4 = 5 > 0$

So, the given equation has real roots.

Now, $\sqrt{D}=\sqrt{5}$

$\therefore a=\dfrac{-b+\sqrt{D}}{2a}=\dfrac{-(-3)+\sqrt{5}}{2\times 1}=\dfrac{3+\sqrt{5}}{2}$

$\beta=\dfrac{-b-\sqrt{D}}{2a}=\dfrac{-(-3)-\sqrt{5}}{2\times 1}=\dfrac{3-\sqrt{5}}{2}$

Hence, $\dfrac{3+\sqrt{5}}{2}$ and $\dfrac{3-\sqrt{5}}{2}$ are the roots of the given equation.

22. $\dfrac{1}{x}-\dfrac{1}{x-2}=3, x\neq 0,2$

Sol:

The given equation is

$\dfrac{1}{x}-\dfrac{1}{x-2}=3, x\neq 0,2$

$\Rightarrow \dfrac{x-2-x}{x(x-2)}=3$

$\Rightarrow \dfrac{-2}{x^2-2x}=3$

$\Rightarrow -2=3x^2-6x$

$\Rightarrow 3x^2-6x+2=0$

This equation is of the form $ax^2+bx+c=0,$ where $a=3, b=-6$ and $c=2.$

$\therefore$ Discriminant, $D=b^2-4ac=(-6)^2-4\times 3\times 2=36-24=12>0$

So, the given equation has real roots.

Now, $\sqrt{D}=\sqrt{12}=2\sqrt{3}$

$\therefore a=\dfrac{-b+\sqrt{D}}{2a}=\dfrac{-(-6)+2\sqrt{3}}{2\times 3}=\dfrac{6+2\sqrt{3}}{6}=\dfrac{3+\sqrt{3}}{3}$

$\beta=\dfrac{-b-\sqrt{D}}{2a}=\dfrac{-(-6)-2\sqrt{3}}{2\times 3}=\dfrac{6-2\sqrt{3}}{6}=\dfrac{3-\sqrt{3}}{3}$

Hence, $\dfrac{3+\sqrt{3}}{3}$ and $\dfrac{3-\sqrt{3}}{3}$ are the roots of the given equation.

23. $x-\dfrac{1}{x}=3, x\neq 0$

Sol:

The given equation is

$x-\frac{1}{x}=3, x\neq 0$

$\Rightarrow \frac{x^2-1}{x}=3$

$\Rightarrow x^2-1=3x$

$\Rightarrow x^2-3x-1=0$

This equation is of the form $ax^2+bx+c=0,$ where $a=1, b=-3$ and $c=-1.$

$\therefore$ Discriminant, $D=b^2-4ac=(-3)^2-4\times1\times(-1)=9+4=13>0$

So, the given equation has real roots.

Now, $\sqrt{D}=\sqrt{13}$

$\therefore a=\frac{-b+\sqrt{D}}{2a}=\frac{-(-3)+\sqrt{13}}{2\times1}=\frac{3+\sqrt{13}}{2}$

$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{-(-3)-\sqrt{13}}{2\times1}=\frac{3-\sqrt{13}}{2\times1}=\frac{3-\sqrt{13}}{2}$

Hence, $\frac{3+\sqrt{13}}{3}$ and $\frac{3-\sqrt{13}}{3}$ are the roots of the given equation.

24. $\frac{m}{n}x^2\frac{n}{m}=1-2x$

Sol:

The given equation is

$\frac{m}{n}x^2\frac{n}{m}=1-2x$

$\Rightarrow \frac{m^2x^2+n^2}{mn}=1-2x$

$\Rightarrow m^2x^2+n^2=mn-2mnx$

$\Rightarrow m^2x^2+2mnx+n^2-mn=0$

This equation is of the form $ax^2+bx+c=0,$ where $a=m^2, b=2mn$ and $c=n^2-mn$

$\therefore$ Discriminant,

$D=b^2-4ac=(2mn)^2-4\times m^2\times\left(n^2-mn\right)=4m^2n^2-4m^2n^2+4m^3n^2=4m^3n>0$

So, the given equation has real roots.

Now, $\sqrt{D}=\sqrt{4m^3n}=2m\sqrt{mn}$

$\therefore a=\frac{-b+\sqrt{D}}{2a}=\frac{-2mn+2m\sqrt{mn}}{2\times m^2}=\frac{2mn\left(-n+\sqrt{mn}\right)}{2m^2}=\frac{-n+\sqrt{mn}}{m}$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-2mn-2m\sqrt{mn}}{2\times m^2} = \frac{-2m\left(n+\sqrt{mn}\right)}{2m^2} = \frac{-n+\sqrt{mn}}{m}$$

Hence, $\frac{-n+\sqrt{mn}}{m}$ and $\frac{-n-\sqrt{mn}}{m}$ are the roots of the given equation.

25. $36x^2 - 12ax + \left(a^2 - b^2\right) = 0$

Sol:

The given equation is $36x^2 - 12ax + \left(a^2 - b^2\right) = 0$

Comparing it with $Ax^2 + Bx + C = 0,$ we get

$A = 36, B = -12a \text{ and } C = a^2 - b^2$

$\therefore$ Discriminant,

$D = B^2 - 4AC = \left(-12a\right)^2 - 4\times 36\times\left(a^2 - b^2\right) = 144a^2 - 144a^2 + 144b^2 = 144b^2 > 0$

So, the given equation has real roots

Now, $\sqrt{D} = \sqrt{144b^2} = 12b$

$$\therefore \alpha = \frac{-B+\sqrt{D}}{2A} = \frac{-\left(-12a\right)+12b}{2\times 36} = \frac{12\left(a+b\right)}{72} = \frac{a+b}{0}$$

$$\beta = \frac{-B-\sqrt{D}}{2A} = \frac{-\left(-12a\right)-12b}{2\times 36} = \frac{12\left(a-b\right)}{72} = \frac{a-b}{6}$$

Hence, $\frac{a+b}{6}$ and $\frac{a-b}{6}$ are the roots of the given equation.

26. $x^2 - 2ax + \left(a^2 - b^2\right) = 0$

Sol:

Given:

$x^2 - 2ax + \left(a^2 - b^2\right) = 0$

On comparing it with $Ax^2 + Bx + C = 0,$ we get:

$A = 1, B = -2a$ and $C = \left(a^2 - b^2\right)$

Discriminant D is given by:

$D = B^2 - 4AC$

$= \left(-2a\right)^2 - 4\times 1\times\left(a^2 - b^2\right)$

$= 4a^2 - 4a^2 + 4b^2$

$= 4b^2 > 0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-(-2a)+\sqrt{4b^2}}{2\times 1} = \frac{2a+2b}{2} = \frac{2(a+b}{2} = (a+b)$$

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-(-2a)-\sqrt{4b^2}}{2\times 1} = \frac{2a-2b}{2} = \frac{2(a-b)}{2} = (a-b)$$

Hence, the roots of the equation are $(a+b)$ and $(a-b)$.

27. $x^2 - 2ax - (4b^2 - a^2) = 0$

Sol:

The given equation is $x^2 - 2ax - (4b^2 - a^2) = 0$

Comparing it with $Ax^2 + Bx + C = 0,$ we get

$A = 1, B = -2a \text{ and } C = -(4b^2 - a^2)$

$\therefore$ Discriminant,

$B^2 - 4AC = (-2a)^2 - 4\times 1\times\left[-(4b^2 - a^2)\right] = 4a^2 + 16b^2 - 4a^2 = 16b^2 > 0$

So, the given equation has real roots

Now, $\sqrt{D} = \sqrt{16b^2} = 4b$

$$\therefore \alpha = \frac{-B+\sqrt{D}}{2A} = \frac{-(-2a)+4b}{2\times 1} = \frac{2(a+2b)}{2} = a+2b$$

$$\beta = \frac{-B-\sqrt{D}}{2A} = \frac{-(-2a)-4b}{2\times 1} = \frac{2(a-2b)}{2} = a-2b$$

Hence, $a+2b$ and $a-2b$ are the roots of the given equation.

28. $x^2 + 6x - (a^2 + 2a - 8) = 0.$

Sol:

The given equation is $x^2 + 6x - (a^2 + 2a - 8) = 0.$

Comparing it with $Ax^2 + Bx + C = 0,$ we get

$A = 1, B = 6 \text{ and } C = -(a^2 + 2a - 8)$

$\therefore$ Discriminant, $D =$

$B^2 - 4AC = 6^2 - 4\times 1\times\left[-(a^2 + 2a - 8)\right] = 36 + 4a^2 + 8a - 32 = 4a^2 + 8a - 32 = 4a^2 + 8a + 4$

$= 4(a^2 + 2a + 1) = 4(a+1)^2 > 0$

So, the given equation has real roots

Now, $\sqrt{D} = \sqrt{4(a+1)^2} = 2(a+1)$

$\therefore \alpha = \dfrac{-B+\sqrt{D}}{2A} = \dfrac{-6+2(a+1)}{2\times 1} = \dfrac{2a-4}{2} = a-2$

$\beta = \dfrac{-B-\sqrt{D}}{2A} = \dfrac{-6-2(a+1)}{2\times 1} = \dfrac{-2a-8}{2} = -a-4 = -(a+4)$

Hence, $(a-2)$ and $-(a+4)$ are the roots of the given equation.

29. $x^2+5x-(a^2+a-6)=0.$

Sol:

The given equation is $x^2+5x-(a^2+a-6)=0.$

Comparing it with $Ax^2+Bx+C=0,$ we get

$A=1, B=5$ *and* $C=-(a^2+a-8)$

$\therefore$ Discriminant, $D =$

$B^2-4AC = 5^2-4\times 1\times\left[-(a^2+a-6)\right] = 25+4a^2+4a-24 = 4a^2+4a^2+4a+1$

$=(2a+1)^2>0$

So, the given equation has real roots

Now, $\sqrt{D}=\sqrt{(2a+1)^2}=2a+1$

$\therefore \alpha = \dfrac{-B+\sqrt{D}}{2A} = \dfrac{-5+2a+1}{2\times 1} = \dfrac{2a-4}{2} = a-2$

$\beta = \dfrac{-B-\sqrt{D}}{2A} = \dfrac{-5-(2a+1)}{2\times 1} = \dfrac{-2a-6}{2} = -a-3 = -(a+3)$

Hence, $(a-2)$ and $-(a+3)$ are the roots of the given equation.

30. $x^2-4ax-b^2+4a^2=0.$

Sol:

The given equation is $x^2-4ax-b^2+4a^2=0.$

Comparing it with $Ax^2+Bx+C=0,$ we get

$A=1, B=-4a$ *and* $C=-b^2+4a^2$

$\therefore$ Discriminant, $D=B^2-4AC=(-4a)^2-4\times 1\times(-b^2+4a^2)=16a^2+4b^2-16a^2=4b^2>0$

So, the given equation has real roots

Now, $\sqrt{D}=\sqrt{4b^2}=2b$

$\therefore \alpha = \dfrac{-B+\sqrt{D}}{2A} = \dfrac{-(-4a)+2b}{2\times 1} = \dfrac{4a+2b}{2} = 2a+b$

$$\beta = \frac{-B-\sqrt{D}}{2A} = \frac{-(-4a)-2b}{2\times 1} = \frac{4a-2b}{2} = 2a-b$$

Hence, $(2a+b)$ and $(2a-b)$ are the roots of the given equation.

31. $4x^2 - 4a^2x + (a^4 - b^4) = 0.$

Sol:

The given equation is $4x^2 - 4a^2x + (a^4 - b^4) = 0.$

Comparing it with $Ax^2 + Bx + C = 0,$ we get

$A = 4, B = -4a^2$ *and* $C = a^4 - b^4$

$\therefore$ Discriminant, $B^2 - 4AC = (-4a^2)^2 - 4\times 4\times(a^2 - b^4) = 16a^4 - 16a^4 + 16b^4 = 16b^4 > 0$

So, the given equation has real roots

Now, $\sqrt{D} = \sqrt{16b^4} = 4b^2$

$$\therefore \alpha = \frac{-B+\sqrt{D}}{2A} = \frac{-(-4a^2)+4b^2}{2\times 4} = \frac{4(a^2+b^2)}{8} = \frac{a^2+b^2}{2}$$

$$\beta = \frac{-B-\sqrt{D}}{2A} = \frac{-(-4a^2)-4b^2}{2\times 4} = \frac{4(a^2-b^2)}{8} = \frac{a^2-b^2}{2}$$

Hence, $\frac{1}{2}(a^2+b^2)$ and $\frac{1}{2}(a^2-b^2)$ are the roots of the given equation.

32. $4x^2 - 4bx - (a^2 - b^2) = 0.$

Sol:

The given equation is $4x^2 - 4bx - (a^2 - b^2) = 0.$

Comparing it with $Ax^2 + Bx + C = 0,$ we get

$A = 4, B = 4b$ *and* $C = -(a^2 - b^2)$

$\therefore$ Discriminant,

$D = B^2 - 4AC = (4b)^2 - 4\times 4\times\left[-(a^2-b^2)\right] = 16b^2 + 16a^2 - 16b^2 = 16a^2 > 0$

So, the given equation has real roots

Now, $\sqrt{D} = \sqrt{16a^2} = 4a$

$$\therefore \alpha = \frac{-B+\sqrt{D}}{2A} = \frac{-4b+4a}{2\times 4} = \frac{4(a-b)}{8} = \frac{a-b}{2}$$

$$\beta = \frac{-B-\sqrt{D}}{2A} = \frac{-4b-4a}{2\times 4} = \frac{-4(a+b)}{8} = -\frac{a+b}{2}$$

Hence, $\frac{1}{2}(a-b)$ and $-\frac{1}{2}(a+b)$ are the roots of the given equation.

33. $x^2-(2b-1)x+(b^2-b-20)=0.$

Sol:

The given equation is $x^2-(2b-1)x+(b^2-b-20)=0.$

Comparing it with $Ax^2+Bx+C=0,$ we get

$A=1, B=-(2b-1)$ *and* $C=b^2-b-20$

$\therefore$ Discriminant,

$D=B^2-4AC=\left[-(2b-1)\right]^2-4\times1\times(b^2-b-20)=4b^2-4b+1-4b^2+4b+80=81>0$

So, the given equation has real roots

Now, $\sqrt{D}=\sqrt{18}=9$

$\therefore \alpha=\dfrac{-B+\sqrt{D}}{2A}=\dfrac{-\left[-(2b-1)\right]+9}{2\times1}=\dfrac{2b+8}{2}=b+4$

$\beta=\dfrac{-B-\sqrt{D}}{2A}=\dfrac{-\left[-(2b-1)\right]-9}{2\times1}=\dfrac{2b-10}{2}=b-5$

Hence, $(b+4)$ and $(b-5)$ are the roots of the given equation.

34. $3a^2x^2+8abx+4b^2=0$

Sol:

Given:

$3a^2x^2+8abx+4b^2=0$

On comparing it with $Ax^2+Bx+C=0,$ we get:

$A=3a^2, B=8ab$ *and* $C=4b^2$

Discriminant D is given by:

$D=(B^2-4AC)$

$=(8ab)^2-4\times3a^2\times4b^2$

$=16a^2b^2>0$

Hence, the roots of the equation are real.

Roots α and β are given by:

$\alpha=\dfrac{-b+\sqrt{D}}{2a}=\dfrac{-8ab+\sqrt{16a^2b^2}}{2\times3a^2}=\dfrac{-8ab+4ab}{6a^2}=\dfrac{-4ab}{6a^2}=\dfrac{-2b}{3a}$

$\beta=\dfrac{-b-\sqrt{D}}{2a}=\dfrac{-8ab-\sqrt{16a^2b^2}}{2\times3a^2}=\dfrac{-8ab-4ab}{6a^2}=\dfrac{-12ab}{6a^2}=\dfrac{-2b}{a}$

Thus, the roots of the equation are $\dfrac{-2b}{3a}$ *and* $\dfrac{-2b}{a}$.

35. $a^2b^2x^2 - \left(4b^4 - 3a^4\right)x - 12a^2b^2 = 0, a \neq 0\, and\, b \neq 0$

Sol:

The given equation is $a^2b^2x^2 - \left(4b^4 - 3a^4\right)x - 12a^2b^2 = 0.$

Comparing it with $Ax^2 + Bx + C = 0,$ we get

$A = a^2b^2, B = -\left(4b^4 - 3a^4\right)$ *and* $c = -12a^2b^2$

$\therefore$ Discriminant,

$B^2 - 4AC = \left[-\left(4b^4 - 3a^4\right)\right]^2 - 4 \times a^2b^2 \times \left(-12a^2b^2\right) = 16b^8 - 24a^4b^4 + 9a^8 + 48a^4b^4$

$= 16b^8 + 24a^4b^4 + 9a^8 = \left(4b^4 + 3a^4\right)^2 > 0$

So, the given equation has real roots

Now, $\sqrt{D} = \sqrt{\left(4b^4 + 3a^4\right)^2} = 4b^4 + 3a^4$

$$\therefore \alpha = \frac{-B + \sqrt{D}}{2A} = \frac{-\left[-\left(4b^4 - 3a^4\right)\right] + \left(4b^4 + 3a^4\right)}{2 \times a^2b^2} = \frac{8b^4}{2a^2b^2} = \frac{4b^2}{a^2}$$

$$\beta = \frac{-B - \sqrt{D}}{2A} = \frac{-\left[-\left(4b^4 - 3a^4\right)\right] - \left(4b^4 + 3a^4\right)}{2 \times a^2b^2} = \frac{-6a^4}{2a^2b^2} = -\frac{3a^2}{b^2}$$

Hence, $\frac{4b^2}{a^2}$ and $-\frac{3a^2}{b^2}$ are the roots of the given equation.

36. $12abx^2 - \left(9a^2 - 8b^2\right)x - 6ab = 0, where\, a \neq 0\, and\, b \neq 0$

Sol:

Given:

$12abx^2 - \left(9a^2 - 8b^2\right)x - 6ab = 0$

On comparing it with $Ax^2 + Bx + C = 0,$ we get:

$A = 12ab, B = -\left(9a^2 - 8b^2\right)$ and $C = -6ab$

Discriminant D is given by:

$D = B^2 - 4AC$

$= \left[-\left(9a^2 - 8b^2\right)\right]^2 - 4 \times 12ab \times \left(-6ab\right)$

$= 81a^4 - 144a^2b^2 + 64b^4 + 288a^2b^2$

$= 81a^4 + 144a^2b^2 + 64b^4$

$= \left(9a^2 + 8b^2\right)^2 > 0$

Hence, the roots of the equation are equal.

Roots α and β are given by:

$$\alpha=\frac{-B+\sqrt{D}}{2A}=\frac{-\left[-\left(9a^2-8b^2\right)\right]+\sqrt{\left(9a^2+8b^2\right)^2}}{2\times 12ab}=\frac{9a^2-8b^2+9a^2+8b^2}{24ab}=\frac{18a^2}{24ab}=\frac{3a}{4b}$$

$$\beta=\frac{-B-\sqrt{D}}{2A}=\frac{-\left[-\left(9a^2-8b^2\right)\right]-\sqrt{\left(9a^2+8b^2\right)^2}}{2\times 12ab}=\frac{9a^2-8b^2-9a^2-8b^2}{24ab}=\frac{-16a^2}{24ab}=\frac{-2b}{3a}$$

Thus, the roots of the equation are $\frac{3a}{4b}$ and $\frac{-2b}{3a}$.

Exercise - 10D

1. Find the nature of roots of the following quadratic equations:

(i) $2x^2-8x+5=0.$

(ii) $3x^2-2\sqrt{6}x+2=0.$

(iii) $5x^2-4x+1=0.$

(iv) $5x\left(x-2\right)+6=0$

(v) $12x^2-4\sqrt{15}x+5=0$

(vi) $x^2-x+2=0.$

Sol:

(i) The given equation is $2x^2-8x+5=0.$

This is of the form $ax^2+bx+c=0,$ where $a=2, b=-8$ *and* $c=5.$

$\therefore$ Discriminant, $D=b^2-4ac=\left(-8\right)^2-4\times 2\times 5=64-40=24>0$

Hence, the given equation has real and unequal roots.

(ii) The given equation is $3x^2-2\sqrt{6}x+2=0.$

This is of the form $ax^2+bx+c=0,$ where $a=3, b=-2\sqrt{6}$ *and* $c=2.$

$\therefore$ Discriminant, $D=b^2-4ac=\left(-2\sqrt{6}\right)^2-4\times 3\times 2=24-24=0$

Hence, the given equation has real and equal roots.

(iii) The given equation is $5x^2-4x+1=0.$

This is of the form $ax^2+bx+c=0,$ where $a=5, b=-4$ *and* $c=1.$

$\therefore$ Discriminant, $D=b^2-4ac=\left(-4\right)^2-4\times 5\times 1=16-20=-4<0$

Hence, the given equation has no real roots.

(iv) The given equation is

$5x(x-2)+6=0$

$\Rightarrow 5x^2-10x+6=0$

This is of the form $ax^2+bx+c=0$, where $a=5, b=-10$ and $c=6$.

$\therefore$ Discriminant, $D=b^2-4ac=(-10)^2-4\times5\times6=100-120=-20<0$

Hence, the given equation has no real roots.

(v) The given equation is $12x^2-4\sqrt{15}x+5=0$

This is of the form $ax^2+bx+c=0$, where $a=12, b=-4\sqrt{15}$ and $c=5$.

$\therefore$ Discriminant, $D=b^2-4ac=\left(-4\sqrt{15}\right)^2-4\times12\times5=240-240=0$

Hence, the given equation has real and equal roots.

(vi) The given equation is $x^2-x+2=0$.

This is of the form $ax^2+bx+c=0$, where $a=1, b=-1$ and $c=2$.

$\therefore$ Discriminant, $D=b^2-4ac=(-1)^2-4\times1\times2=1-8=-7<0$

Hence, the given equation has no real roots.

2. If a and b are distinct real numbers, show that the quadratic equations $2\left(a^2+b^2\right)x^2+2(a+b)x+1=0.$ has no real roots.

Sol:

The given equation is $2\left(a^2+b^2\right)x^2+2(a+b)x+1=0.$

$\therefore D=\left[2(a+b)\right]^2-4\times2\left(a^2+b^2\right)\times1$

$=4\left(a^2+2ab+b^2\right)-8\left(a^2+b^2\right)$

$=4a^2+8ab+4b^2-8a^2-8b^2$

$=-4a^2+8ab-4b^2$

$=-4\left(a^2-2ab+b^2\right)$

$=-4(a-b)^2<0$

Hence, the given equation has no real roots.

3. Show that the roots of the equation $x^2+px-q^2=0$ are real for all real values of p and q.

Sol:

Given:

$x^2+px-q^2=0$

Here,

$a=1, b=p$ and $c=-q^2$

Discriminant D is given by:

$D = \left(b^2 - 4ac\right)$

$= p^2 - 4 \times 1 \times \left(-q^2\right)$

$= \left(p^2 + 4q^2\right) > 0$

$D > 0$ for all real values of p and q.

Thus, the roots of the equation are real.

4. For what values of k are the roots of the quadratic equation $3x^2 + 2kx + 27 = 0$ real and equal?

Sol:

Given:

$3x^2 + 2kx + 27 = 0$

Here,

$a = 3, b = 2k \text{ and } c = 27$

It is given that the roots of the equation are real and equal; therefore, we have:

$D = 0$

$\Rightarrow (2k)^2 - 4 \times 3 \times 27 = 0$

$\Rightarrow 4k^2 - 324 = 0$

$\Rightarrow 4k^2 = 324$

$\Rightarrow k^2 = 81$

$\Rightarrow k = \pm 9$

$\therefore k = 9 \text{ or } k = -9$

5. For what value of k are the roots of the quadratic equation $kx\left(x - 2\sqrt{5}\right) + 10 = 0$ real and equal.

Sol:

The given equation is

$kx\left(x - 2\sqrt{5}\right) + 10 = 0$

$\Rightarrow kx^2 - 2\sqrt{5}kx + 10 = 0$

This is of the form $ax^2 + bx + c = 0,$ where $a = k, b = -2\sqrt{5}k \text{ and } c = 10.$

$\therefore D = b^2 - 4ac = \left(-2\sqrt{5}k\right)^2 - 4 \times k \times 10 = 20k^2 - 40k$

The given equation will have real and equal roots if $D = 0$.

$\therefore 20k^2 - 40k = 0$

$\Rightarrow 20k(k-2) = 0$

$\Rightarrow k = 0 \ or \ k - 2 = 0$

$\Rightarrow k = 0 \ or \ k = 2$

But, for $k = 0,$ we get $10 = 0,$ which is not true

Hence, 2 is the required value of *k*.

6. For what values of p are the roots of the equation $4x^2 + px + 3 = 0.$ real and equal?

Sol:

The given equation is $4x^2 + px + 3 = 0.$

This is of the form $ax^2 + bx + c = 0,$ where $a = 4, b = p \ and \ c = 3.$

$\therefore D = b^2 - 4ac = p^2 - 4 \times 4 \times 3 = p^2 - 48$

The given equation will have real and equal roots if $D = 0.$

$\therefore p^2 - 48 = 0$

$\Rightarrow p^2 = 48$

$\Rightarrow p = \pm\sqrt{48} = \pm 4\sqrt{3}$

Hence, $4\sqrt{3}$ and $-4\sqrt{3}$ are the required values of *p*.

7. Find the nonzero value of k for which the roots of the quadratic equation $9x^2 - 3kx + k = 0.$ are real and equal.

Sol:

The given equation is $9x^2 - 3kx + k = 0.$

This is of the form $ax^2 + bx + c = 0,$ where $a = 9, b = -3k \ and \ c = k.$

$\therefore D = b^2 - 4ac = (-3k)^2 - 4 \times 9 \times k = 9k^2 - 36k$

The given equation will have real and equal roots if $D = 0.$

$\therefore 9k^2 - 36k = 0$

$\Rightarrow 9k(k-4) = 0$

$\Rightarrow k = 0 \ or \ k - 4 = 0$

$\Rightarrow k = 0 \ or \ k = 4$

But, $k \neq 0$ (Given)

Hence, the required values of *k* is 4.

8. Find the values of k for which the quadratic equation $(3k+1)x^2 + 2(k+1)x + 1 = 0.$ has real and equal roots.

Sol:

The given equation is $(3k+1)x^2+2(k+1)x+1=0.$

This is of the form $ax^2+bx+c=0,$ where $a=3k+1, b=2(k+1)$ *and* $c=1.$

$\therefore D=b^2-4ac$

$=[2(k+1)]^2-4\times(3k+1)\times1$

$=4(k^2+2k+1)-4(3k+1)$

$=4k^2+8k+4-12k-4$

$=4k^2-4k$

The given equation will have real and equal roots if $D=0.$

$\therefore 4k^2-4k=0$

$\Rightarrow 4k(k-1)=0$

$\Rightarrow k=0$ *or* $k-1=0$

$\Rightarrow k=0$ *or* $k=1$

Hence, 0 and 1are the required values of k.

9. Find the value of p for which the quadratic equation $(2p+1)x^2-(7p+2)x+(7p-3)=0.$ has real and equal roots.

Sol:

The given equation is $(2p+1)x^2-(7p+2)x+(7p-3)=0.$

This is of the form $ax^2+bx+c=0,$ where $a=2p+1, b=-(7p+2)$ *and* $c=7p-3.$

$\therefore D=b^2-4ac$

$=-[-(7p+2)]^2-4\times(2p+1)\times(7p-3)$

$=(49p^2+28p+4)-4(14p^2+p-3)$

$=49p^2+28p+4-56p^2-4p+12$

$=-7p^2+24p+16$

The given equation will have real and equal roots if $D=0.$

$\therefore -7p^2+24p+16=0$

$\Rightarrow 7p^2-24p-16=0$

$\Rightarrow 7p^2-28p+4p-16=0$

$\Rightarrow 7p(p-4)+4(p-4)=0$

$\Rightarrow (p-4)(7p+4)=0$

$\Rightarrow p-4=0$ *or* $7p+4=0$

$\Rightarrow p = 4 \ or \ p = -\frac{4}{7}$

Hence, 4 and $-\frac{4}{7}$ are the required values of *p*.

10. Find the values of p for which the quadratic equation $(p+1)x^2 - 6(p+1)x + 3(p+9) = 0.$, $p \neq -1$ has equal roots. Hence find the roots of the equation.

Sol:

The given equation is $(p+1)x^2 - 6(p+1)x + 3(p+9) = 0.$

This is of the form $ax^2 + bx + c = 0,$ where $a = p+1, b = -6(p+1) \ and \ c = 3(p+9).$

$\therefore D = b^2 - 4ac$

$= [-6(p+1)]^2 - 4 \times (p+1) \times 3(p+9)$

$= 12(p+1)[3(p+1) - (p+9)]$

$= 12(p+1)(2p-6)$

The given equation will have real and equal roots if $D = 0.$

$\therefore 12(p+1)(2p-6) = 0$

$\Rightarrow p+1 = 0 \ or \ 2p-6 = 0$

$\Rightarrow p = -1 \ or \ p = 3$

But, $p \neq -1$ (Given)

Thus, the value of *p* is 3

Putting $p = 3,$ the given equation becomes $4x^2 - 24x + 36 = 0$

$4x^2 - 24x + 36 = 0$

$\Rightarrow 4(x^2 - 6x + 9) = 0$

$\Rightarrow (x-3)^2 = 0$

$\Rightarrow x - 3 = 0$

$\Rightarrow x = 3$

Hence, 3 is the repeated root of this equation.

11. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0.$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.

Sol:

It is given that -5 is a root of the quadratic equation $2x^2 + px - 15 = 0.$

$\therefore 2(-5)^2 + p \times (-5) - 15 = 0$

$\Rightarrow -5p+35=0$

$\Rightarrow p=7$

The roots of the equation $px^2+px+k=0=0$ are equal.

$\therefore D=0$

$\Rightarrow p^2-4pk=0$

$\Rightarrow (7)^2-4\times7\times k=0$

$\Rightarrow 49-28k=0$

$\Rightarrow k=\dfrac{49}{28}=\dfrac{7}{4}$

Thus, the value of k is $\dfrac{7}{4}$.

12. If 3 is a root of the quadratic equation $x^2-x+k=0.$, find the value of p so that the roots of the equation $x^2+2kx+\left(k^2+2k+p\right)=0$ are equal.

Sol:

It is given that 3 is a root of the quadratic equation $x^2-x+k=0.$

$\therefore (3)^2-3+k=0$

$\Rightarrow k+6=0$

$\Rightarrow k=-6$

The roots of the equation $x^2+2kx+\left(k^2+2k+p\right)=0$ are equal.

$\therefore D=0$

$\Rightarrow (2k)^2-4\times1\times\left(k^2+2k+p\right)=0$

$\Rightarrow 4k^2-4k^2-8k-4p=0$

$\Rightarrow -8k-4p=0$

$\Rightarrow p=\dfrac{8k}{-4}=-2k$

$\Rightarrow p=-2\times(-6)=12$

Hence, the value of p is 12.

13. If -4 is a root of the equation $x^2+2x+4p=0.$ find the value of k for the which the quadratic equation $x^2+px(1+3k)+7(3+2k)=0$ has equal roots.

Sol:

It is given that -4 is a root of the quadratic equation $x^2+2x+4p=0.$

$\therefore (-4)^2 + 2\times(-4) + 4p = 0$
$\Rightarrow 16 - 8 + 4p = 0$
$\Rightarrow 4p + 8 = 0$
$\Rightarrow p = -2$

The equation $x^2 + px(1+3k) + 7(3+2k) = 0$ has real roots.

$\therefore D = 0$
$\Rightarrow \left[p(1+3k)\right]^2 - 4\times1\times7(3+2k) = 0$
$\Rightarrow \left[-2(1+3k)\right]^2 - 28(3+2k) = 0$
$\Rightarrow 4(1+6k+9k^2) - 28(3+2k) = 0$
$\Rightarrow 4(1+6k+9k^2-21-14k) = 0$
$\Rightarrow 9k^2 - 8k - 20 = 0$
$\Rightarrow 9k^2 - 18k + 10k - 20 = 0$
$\Rightarrow 9k(k-2) + 10(k-2) = 0$
$\Rightarrow (k-2)(9k+10) = 0$
$\Rightarrow k - 2 = 0 \text{ or } 9k + 10 = 0$
$\Rightarrow k = 2 \text{ or } k = -\frac{10}{9}$

Hence, the required value of k is *2* or $-\frac{10}{9}$.

14. If the quadratic equation $(1+m^2)x^2 + 2mcx + (c^2 - a^2) = 0$ has equal roots, prove that $c^2 = a^2(1+m^2)$.

Sol:
Given:
$(1+m^2)x^2 + 2mcx + (c^2 - a^2) = 0$
Here,
$a = (1+m^2), b = 2mc \text{ and } c = (c^2 - a^2)$
It is given that the roots of the equation are equal; therefore, we have:
$D = 0$
$\Rightarrow (b^2 - 4ac) = 0$
$\Rightarrow (2mc)^2 - 4\times(1+m^2)\times(c^2 - a^2) = 0$

$\Rightarrow 4m^2c^2 - 4\left(c^2 - a^2 + m^2c^2 - m^2a^2\right) = 0$

$\Rightarrow 4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2 = 0$

$\Rightarrow -4c^2 + 4a^2 + 4m^2a^2 = 0$

$\Rightarrow a^2 + m^2a^2 = c^2$

$\Rightarrow a^2\left(1 + m^2\right) = c^2$

$\Rightarrow c^2 = a^2\left(1 + m^2\right)$

Hence proved.

15. If the roots of the quadratic equation $\left(c^2 - ab\right)x^2 - 2\left(a^2 - bc\right)x + \left(b^2 - ac\right) = 0$ are real and equal, show that either a = 0 or $\left(a^3 + b^3 + c^3 = 3abc\right)$

Sol:

Given:

$\left(c^2 - ab\right)x^2 - 2\left(a^2 - bc\right)x + \left(b^2 - ac\right) = 0$

Here,

$a = \left(c^2 - ab\right), b = -2\left(a^2 - bc\right), c = \left(b^2 - ac\right)$

It is given that the roots of the equation are real and equal; therefore, we have:

$D = 0$

$\Rightarrow \left(b^2 - 4ac\right) = 0$

$\Rightarrow \left\{-2\left(a^2 - bc\right)\right\}^2 - 4 \times \left(c^2 - ab\right) \times \left(b^2 - ac\right) = 0$

$\Rightarrow 4\left(a^4 - 2a^2bc + b^2c^2\right) - 4\left(b^2c^2 - ac^3 - ab^3 + a^2bc\right) = 0$

$\Rightarrow a^4 - 2a^2bc + b^2c^2 - b^2c^2 + ac^3 + ab^3 - a^2bc = 0$

$\Rightarrow a^4 - 3a^2bc + ac^3 + ab^3 = 0$

$\Rightarrow a\left(a^3 - 3abc + c^3 + b^3\right) = 0$

Now,

$a = 0 \ or \ a^3 - 3abc + c^3 + b^3 = 0$

$a = 0 \ or \ a^3 + b^3 + c^3 = 3abc$

16. Find the value of p for which the quadratic equation $2x^2 + px + 8 = 0$ has real roots.

Sol:

Given:

$2x^2 + px + 8 = 0$

Here,

$a=2, b=p \ and \ c=8$

Discriminant D is given by:

$D=\left(b^2-4ac\right)$

$=p^2-4\times2\times8$

$=\left(p^2-64\right)$

If $D\geq0$, the roots of the equation will be real

$\Rightarrow\left(p^2-64\right)\geq0$

$\Rightarrow(p+8)(p-8)\geq0$

$\Rightarrow p\geq8 \ and \ p\leq-8$

Thus, the roots of the equation are real for $p\geq8$ and $p\leq-8$.

17. Find the value of a for which the equation $\left(\alpha-12\right)x^2+2\left(\alpha-12\right)x+2=0$ has equal roots.

Sol:

Given:

$\left(\alpha-12\right)x^2+2\left(\alpha-12\right)x+2=0$

Here,

$a=\left(\alpha=12\right), b=2\left(\alpha-12\right) \ and \ c=2$

It is given that the roots of the equation are equal; therefore, we have

$D=0$

$\Rightarrow\left(b^2-4ac\right)=0$

$\Rightarrow\left\{2\left(\alpha-12\right)\right\}^2-4\times\left(\alpha-12\right)\times2=0$

$\Rightarrow4\left(\alpha^2-24\alpha+144\right)-8\alpha+96=0$

$\Rightarrow4\alpha^2-96\alpha+576-8\alpha+96=0$

$\Rightarrow4\alpha^2-104\alpha+672=0$

$\Rightarrow\alpha^2-26\alpha+168=0$

$\Rightarrow\alpha^2-14\alpha-12\alpha+168=0$

$\Rightarrow\alpha\left(\alpha-14\right)-12\left(\alpha-14\right)=0$

$\Rightarrow\left(\alpha-14\right)\left(\alpha-12\right)=0$

$\therefore\alpha=14 \ or \ \alpha=12$

If the value of α is 12, the given equation becomes non-quadratic.

Therefore, the value of α will be 14 for the equation to have equal roots.

18. Find the value of k for which the roots of $9x^2+8kx+16=0$ are real and equal

Sol:

Given:

$9x^2+8kx+16=0$

Here,

$a=9, b=8k \text{ and } c=16$

It is given that the roots of the equation are real and equal; therefore, we have:

$D=0$

$\Rightarrow \left(b^2-4ac\right)=0$

$\Rightarrow (8k)^2-4\times 9\times 16=0$

$\Rightarrow 64k^2-576=0$

$\Rightarrow 64k^2=576$

$\Rightarrow k^2=9$

$\Rightarrow k=\pm 3$

$\therefore k=3 \text{ or } k=-3$

19. Find the values of k for which the given quadratic equation has real and distinct roots:

(i) $kx^2+6x+1=0.$ (ii) $x^2-kx+9=0.$

(iii) $9x^2+3kx+4=0.$ (iv) $5x^2-kx+1=0.$

Sol:

(i) The given equation is $kx^2+6x+1=0.$

$\therefore D=6^2-4\times k\times 1=36-4k$

The given equation has real and distinct roots if $D>0.$

$\therefore 36-4k>0$

$\Rightarrow 4k<36$

$\Rightarrow k<9$

(ii) The given equation is $x^2-kx+9=0.$

$\therefore D=(-k)^2-4\times 1\times 9=k^2-36$

The given equation has real and distinct roots if $D>0.$

$\therefore k^2-36>0$

$\Rightarrow (k-6)(k+6)>0$

$\Rightarrow k<-6 \text{ or } k>6$

(iii) The given equation is $9x^2+3kx+4=0.$

$\therefore D=(3k)^2-4\times 9\times 4=9k^2-144$

The given equation has real and distinct roots if $D>0.$

$\therefore 9k^2 - 144 > 0$

$\Rightarrow 9(k^2 - 16) > 0$

$\Rightarrow (k-4)(k+4) > 0$

$\Rightarrow k < -4 \ or \ k > 4$

(iv) The given equation is $5x^2 - kx + 1 = 0$.

$\therefore D = (-k)^2 - 4 \times 5 \times 1 = k^2 - 20$

The given equation has real and distinct roots if $D > 0$.

$\therefore k^2 - 20 > 0$

$\Rightarrow k^2 - (2\sqrt{5})^2 > 0$

$\Rightarrow (k - 2\sqrt{5})(k + 2\sqrt{5}) > 0$

$\Rightarrow k < -2\sqrt{5} \ or \ k > 2\sqrt{5}$

20. If a and b are real and $a \neq b$ then show that the roots of the equation $(a-b)x^2 + 5(a+b)x - 2(a-b) = 0.$ are equal and unequal.

Sol:

The given equation is $(a-b)x^2 + 5(a+b)x - 2(a-b) = 0.$

$\therefore D = [5(a+b)]^2 - 4 \times (a-b) \times [-2(a-b)]$

$= 25(a+b)^2 + 8(a-b)^2$

Since a and b are real and $a \neq b$, so $(a-b)^2 > 0 \ and \ (a+b)^2 > 0.$

$\therefore 8(a-b)^2 > 0$ ....(1) (Product of two positive numbers is always positive)

Also, $25(a+b)^2 > 0$ ....(2) (Product of two positive numbers is always positive)

Adding (1) and (2), we get

$25(a+b)^2 + 8(a-b)^2 > 0$ (Sum of two positive numbers is always positive)

$\Rightarrow D > 0$

Hence, the roots of the given equation are real and unequal.

21. If the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$

Sol:

It is given that the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal.

$\therefore D = 0$

$\Rightarrow \left[-2(ac+bd)\right]^2 - 4\left(a^2+b^2\right)\left(c^2+d^2\right) = 0$

$\Rightarrow 4\left(a^2c^2+b^2d^2+2abcd\right) - 4\left(a^2c^2+a^2d^2+b^2c^2+b^2d^2\right) = 0$

$\Rightarrow 4\left(a^2c^2+b^2d^2+2abcd-a^2c^2-a^2d^2-b^2c^2-b^2d^2\right) = 0$

$\Rightarrow \left(-a^2d^2+2abcd-b^2c^2\right) = 0$

$\Rightarrow -\left(a^2d^2-2abcd+b^2c^2\right) = 0$

$\Rightarrow (ad-bc)^2 = 0$

$\Rightarrow ad-bc = 0$

$\Rightarrow ad = bc$

$\Rightarrow \frac{a}{b} = \frac{c}{d}$

Hence proved.

22. If the roots of the equations $ax^2+2bx+c=0$ and $bx^2-2\sqrt{acx}+b=0$ are simultaneously real then prove that $b^2 = ac$

Sol:

It is given that the roots of the equation $ax^2+2bx+c=0$ are real.

$\therefore D_1 = (2b)^2 - 4\times a\times c \geq 0$

$\Rightarrow 4\left(b^2-ac\right) \geq 0$

$\Rightarrow b^2 - ac \geq 0 \qquad ....(1)$

Also, the roots of the equation $bx^2-2\sqrt{ac}x+b=0$ are real.

$\therefore D_2 = \left(-2\sqrt{ac}\right)^2 - 4\times b\times b \geq 0$

$\Rightarrow 4\left(ac-b^2\right) \geq 0$

$\Rightarrow -4\left(b^2-ac\right) \geq 0$

$\Rightarrow b^2 - ac \geq 0 \qquad ....(2)$

The roots of the given equations are simultaneously real if (1) and (2) holds true together. This is possible if

$b^2 - ac = 0$

$\Rightarrow b^2 = ac$

Exercise 10E

1. The sum of a natural number and its square is 156. Find the number.

Sol:

Let the required natural number be x.

According to the given condition,

$x+x^2=156$

$\Rightarrow x^2+x-156=0$

$\Rightarrow x^2+13x-12x-156=0$

$\Rightarrow x(x+13)-12(x+13)=0$

$\Rightarrow (x+13)(x-12)=0$

$\Rightarrow x+13=0$ or $x-12=0$

$\Rightarrow x=-13$ or $x=12$

$\therefore x=12$ (x cannot be negative)

Hence, the required natural number is 12.

2. The sum of natural number and its positive square root is 132. Find the number.

Sol:

Let the required natural number be x.

According to the given condition,

$x+\sqrt{x}=132$

Putting $\sqrt{x}=y$ or $x=y^2$, we get

$y^2+y=132$

$\Rightarrow y^2+y-132=0$

$\Rightarrow y^2+12y-11y-132=0$

$\Rightarrow y(y+12)-11(y+12)=0$

$\Rightarrow (y+12)(y-11)=0$

$\Rightarrow y+12=0$ or $y-11=0$

$\Rightarrow y=-12$ or $y=11$

$\therefore y=11$ (y cannot be negative)

Now,

$\sqrt{x}=11$

$\Rightarrow x=(11)^2=121$

Hence, the required natural number is 121.

3. The sum of two natural number is 28 and their product is 192. Find the numbers.

Sol:

Let the required number be x and $(28-x)$.

According to the given condition,

$x(28-x)=192$

$\Rightarrow 28x-x^2=192$

$\Rightarrow x^2-28x+192=0$

$\Rightarrow x^2-16x-12x+192=0$

$\Rightarrow x(x-16)-12(x-16)=0$

$\Rightarrow (x-12)(x-16)=0$

$\Rightarrow x-12=0 \text{ or } x-16=0$

$\Rightarrow x=12 \text{ or } x=16$

When $x=12$,

$28-x=28-12=16$

When $x=16$,

$28-x=28-16=12$

Hence, the required numbers are 12 and 16.

4. The sum of the squares of two consecutive positive integers is 365. Find the integers.

Sol:

Let the required two consecutive positive integers be x and $(x+1)$.

According to the given condition,

$x^2+(x+1)^2=365$

$\Rightarrow x^2+x^2+2x+1=365$

$\Rightarrow 2x^2+2x-364=0$

$\Rightarrow x^2+x-182=0$

$\Rightarrow x^2+14x-13x-182=0$

$\Rightarrow x(x+14)-13(x+14)=0$

$\Rightarrow (x+14)(x-13)=0$

$\Rightarrow x+14=0 \text{ or } x-13=0$

$\Rightarrow x=-14 \text{ or } x=13$

$\therefore x=13$ (x is a positive integers)

When $x=13$,

$x+1=13+1=14$

Hence, the required positive integers are 13 and 14.

5. The sum of the squares to two consecutive positive odd numbers is 514. Find the numbers.

Sol:

Let the two consecutive positive odd numbers be x and $(x+2)$.

According to the given condition,

$x^2+(x+2)^2=514$

$\Rightarrow x^2+x^2+4x+4=514$

$\Rightarrow 2x^2+4x-510=0$

$\Rightarrow x^2+2x-255=0$

$\Rightarrow x^2+17x-15x-255=0$

$\Rightarrow x(x+17)-15(x+17)=0$

$\Rightarrow (x+17)(x-15)=0$

$\Rightarrow x+17=0$ *or* $x-15=0$

$\Rightarrow x=-17$ *or* $x=15$

$\therefore x=15$ (x is a positive odd number)

When $x=15$,

$x+2=15+2=17$

Hence, the required positive integers are 15 and 17.

6. The sum of the squares of two consecutive positive even numbers is 452. Find the numbers.

Sol:

Let the two consecutive positive even numbers be x and $(x+2)$.

According to the given condition,

$x^2+(x+2)^2=452$

$\Rightarrow x^2+x^2+4x+4=452$

$\Rightarrow 2x^2+4x-448=0$

$\Rightarrow x^2+2x-224=0$

$\Rightarrow x^2+16x-14x-224=0$

$\Rightarrow x(x+16)-14(x+16)=0$

$\Rightarrow (x+16)(x-14)=0$

$\Rightarrow x+16=0$ *or* $x-14=0$

$\Rightarrow x=-16$ *or* $x=14$

$\therefore x=14$ (x is a positive even number)

When $x=14$,

$x+2=14+2=16$

Hence, the required numbers are 14 and 16.

7. The product of two consecutive positive integers is 306. Find the integers.

Sol:

Let the two consecutive positive integers be x and $(x+1)$.

According to the given condition,

$x(x+1)=306$

$\Rightarrow x^2+x-306=0$

$\Rightarrow x^2+18x-17x-306=0$

$\Rightarrow x(x+18)-17(x+18)=0$

$\Rightarrow (x+18)(x-17)=0$

$\Rightarrow x+18=0$ *or* $x-17=0$

$\Rightarrow x=-18$ *or* $x=17$

$\therefore x=17$ (x is a positive integers)

When $x=17$,

$x+1=17+1=18$

Hence, the required integers are 17 and 18.

8. Two natural number differ by 3 and their product is 504. Find the numbers.

Sol:

Let the required numbers be x and $(x+3)$.

According to the question:

$x(x+3)=504$

$\Rightarrow x^2+3x=504$

$\Rightarrow x^2+3x-504=0$

$\Rightarrow x^2+(24-21)x-504=0$

$\Rightarrow x^2+24x-21x-504=0$

$\Rightarrow x(x+24)-21(x+24)=0$

$\Rightarrow (x+24)(x-21)=0$

$\Rightarrow x+24=0$ *or* $x-21=0$

$\Rightarrow x=-24$ *or* $x=21$

If $x=-24$, the numbers are -24 and $\{(-24+3)=-21\}$.

If $x=21$, the numbers are 21 and $\{(21+3)=24\}$.

Hence, the numbers are $(-24,-21)$ and $(21,24)$.

9. Find two consecutive multiples of 3 whose product is 648.

Sol:

Le the required consecutive multiples of 3 be $3x$ and $3(x+1)$.

According to the given condition,

$3x \times 3(x+1) = 648$

$\Rightarrow 9(x^2 + x) = 648$

$\Rightarrow x^2 + x = 72$

$\Rightarrow x^2 + x - 72 = 0$

$\Rightarrow x^2 + 9x - 8x - 72 = 0$

$\Rightarrow x(x+9) - 8(x+9) = 0$

$\Rightarrow (x+9)(x-8) = 0$

$\Rightarrow x+9 = 0 \ or \ x-8 = 0$

$\Rightarrow x = -9 \ or \ x = 8$

$\therefore x = 8$ (Neglecting the negative value)

When $x = 8$,

$3x = 3 \times 8 = 24$

$3(x+1) = 3 \times (8+1) = 3 \times 9 = 27$

Hence, the required multiples are 24 and 27.

10. Find the tow consecutive positive odd integer whose product s 483.

Sol:

Let the two consecutive positive odd integers be x and $(x+2)$.

According to the given condition,

$x(x+2) = 483$

$\Rightarrow x^2 + 2x - 483 = 0$

$\Rightarrow x^2 + 23x - 21x - 483 = 0$

$\Rightarrow x(x+23) - 21(x+23) = 0$

$\Rightarrow (x+23)(x+21) = 0$

$\Rightarrow x+23 = 0 \ or \ x-21 = 0$

$\Rightarrow x = -23 \ or \ x = 21$

$\therefore x = 21$ (x is a positive odd integer)

When $x = 21$,

$x + 2 = 21 + 2 = 23$

Hence, the required integers are 21 and 23.

11. Find the two consecutive positive even integers whose product is 288.

Sol:

Let the two consecutive positive even integers be x and $(x+2)$.

According to the given condition,

$x(x+2)=288$

$\Rightarrow x^2+2x-288=0$

$\Rightarrow x^2+18x-16x-288=0$

$\Rightarrow x(x+18)-16(x+18)=0$

$\Rightarrow (x+18)(x-16)=0$

$\Rightarrow x+18=0 \; or \; x-16=0$

$\Rightarrow x=-18 \; or \; x=16$

$\therefore x=16$ (x is a positive even integer)

When $x=16$,

$x+2=16+2=18$

Hence, the required integers are 16 and 18.

12. The sum of two natural numbers is 9 and the sum of their reciprocals is $\frac{1}{2}$. Find the numbers.

Sol:

Let the required natural numbers be x and $(9,-x)$.

According to the given condition,

$\frac{1}{x}+\frac{1}{9-x}=\frac{1}{2}$

$\Rightarrow \frac{9-x+x}{x(9-x)}=\frac{1}{2}$

$\Rightarrow \frac{9}{9x-x^2}=\frac{1}{2}$

$\Rightarrow 9x-x^2=18$

$\Rightarrow x^2-9x+18=0$

$\Rightarrow x^2-6x-3x+18=0$

$\Rightarrow x(x-6)-3(x-6)=0$

$\Rightarrow x-3=0 \; or \; x-6=0$

$\Rightarrow x=3 \; or \; x=6$

When $x=3$,

$9-x=9-3=6$

When $x=6$,

$9-x=9-6=3$

Hence, the required natural numbers are 3 and 6.

13. The sum of two natural numbers is 15 and the sum of their reciprocals is $\frac{3}{10}$. Find the numbers.

Sol:

Let the required natural numbers be x and $(15-x)$.

According to the given condition,

$\frac{1}{x}+\frac{1}{15-x}=\frac{3}{10}$

$\Rightarrow \frac{15-x+x}{x(15-x)}=\frac{3}{10}$

$\Rightarrow \frac{15}{15x-x^2}=\frac{3}{10}$

$\Rightarrow 15x-x^2=50$

$\Rightarrow x^2-15x+50=0$

$\Rightarrow x^2-10x-5x+50=0$

$\Rightarrow x(x-10)-5(x-10)=0$

$\Rightarrow (x-5)(x-10)=0$

$\Rightarrow x-5=0$ *or* $x-10=0$

$\Rightarrow x=5$ *or* $x=10$

When $x=5$,

$15-x=15-5=10$

When $x=10$,

$15-x=15-10=5$

Hence, the required natural numbers are 5 and 10.

14. The difference of two natural number is 3 and the difference of their reciprocals is $\frac{3}{28}$. Find the numbers.

Sol:

Let the required natural numbers be x and $(x+3)$.

Now, $x<x+3$

$\therefore \frac{1}{x} > \frac{1}{x+3}$

According to the given condition,

$\frac{1}{x} - \frac{1}{x+3} = \frac{3}{28}$

$\Rightarrow \frac{x+3-x}{x(x+3)} = \frac{3}{28}$

$\Rightarrow \frac{3}{x^2+3x} = \frac{3}{28}$

$\Rightarrow x^2+3x = 28$

$\Rightarrow x^2+3x-28 = 0$

$\Rightarrow x^2+7x-4x-28 = 0$

$\Rightarrow x(x+7)-4(x+7) = 0$

$\Rightarrow (x+7)(x-4) = 0$

$\Rightarrow x+7 = 0 \ or \ x-4 = 0$

$\Rightarrow x = -7 \ or \ x = 4$

$\therefore x = 4$ (-7 is not a natural number)

When $x = 4$,

$x+3 = 4+3 = 7$

Hence, the required natural numbers are 4 and 7.

15. The difference of two natural numbers is 5 and the difference of heir reciprocals is $\frac{5}{14}$. Find the numbers.

Sol:

Let the required natural numbers be x and $(x+5)$.

Now, $x < x+5$

$\therefore \frac{1}{x} > \frac{1}{x+5}$

According to the given condition,

$\frac{1}{x} - \frac{1}{x+5} = \frac{5}{14}$

$\Rightarrow \frac{x+5-x}{x(x+5)} = \frac{5}{14}$

$\Rightarrow \frac{5}{x^2+5x} = \frac{5}{14}$

$\Rightarrow x^2+5x = 14$

$\Rightarrow x^2+5x-14=0$

$\Rightarrow x^2+7x-2x-14=0$

$\Rightarrow x(x+7)-2(x+7)=0$

$\Rightarrow (x+7)(x-2)=0$

$\Rightarrow x+7=0 \ or \ x-2=0$

$\Rightarrow x=-7 \ or \ x=2$

$\therefore x=2$ (-7 is not a natural number)

When $x=2$,

$x+5=2+5=7$

Hence, the required natural numbers are 2 and 7.

16. The sum of the squares two consecutive multiples of 7 is 1225. Find the multiples.

Sol:

Let the required consecutive multiplies of 7 be $7x$ and $7(x+1)$.

According to the given condition,

$(7x)^2+[7(x+1)]^2=1225$

$\Rightarrow 49x^2+49(x^2+2x+1)=1225$

$\Rightarrow 49x^2+49x^2+98x+49=1225$

$\Rightarrow 98x^2+98x-1176=0$

$\Rightarrow x^2+x-12=0$

$\Rightarrow x^2+4x-3x-12=0$

$\Rightarrow x(x+4)-3(x+4)=0$

$\Rightarrow (x+4)(x-3)=0$

$\Rightarrow x+4=0 \ or \ x-3=0$

$\Rightarrow x=-4 \ or \ x=3$

$\therefore x=3$ (Neglecting the negative value)

When $x=3$,

$7x=7\times 3=21$

$7(x+1)=7(3+1)=7\times 4=28$

Hence, the required multiples are 21 and 28.

17. The sum of natural number and its reciprocal is $\frac{65}{8}$. Find the number.

Sol:

Let the natural number be x.
According to the given condition,

$$x+\frac{1}{x}=\frac{65}{8}$$
$$\Rightarrow \frac{x^2+1}{x}=\frac{65}{8}$$
$$\Rightarrow 8x^2+8=65x$$
$$\Rightarrow 8x^2-65x+8=0$$
$$\Rightarrow 8x^2-64x-x+8=0$$
$$\Rightarrow 8x(x-8)-1(x-8)=0$$
$$\Rightarrow (x-8)(8x-1)=0$$
$$\Rightarrow x-8=0 \text{ or } 8x-1=0$$
$$\Rightarrow x=8 \text{ or } x=\frac{1}{8}$$

$\therefore x=8$ (x is a natural number)
Hence, the required number is 8.

18. Divide 57 into two parts whose product is 680.
Sol:
Let the two parts be x and $(57-x)$.
According to the given condition,

$$x(57-x)=680$$
$$\Rightarrow 57x-x^2=680$$
$$\Rightarrow x^2-57x+680=0$$
$$\Rightarrow x^2-40x-17x+680=0$$
$$\Rightarrow x(x-40)-17(x-40)=0$$
$$\Rightarrow (x-40)(x-17)=0$$
$$\Rightarrow x-40=0 \text{ or } x-17=0$$
$$\Rightarrow x=40 \text{ or } x=17$$

When $x=40$,
$57-x=57-40=17$
When $x=17$,
$57-x=57-17=40$
Hence, the required parts are 17 and 40.

19. Divide 27 into two parts such that the sum of their reciprocal is $\frac{3}{20}$.

Sol:

Let the two parts be x and $(27-x)$.

According to the given condition,

$$\frac{1}{x}+\frac{1}{27-x}=\frac{3}{20}$$

$$\Rightarrow \frac{27-x+x}{x(27-x)}=\frac{3}{20}$$

$$\Rightarrow \frac{27}{27x-x^2}=\frac{3}{20}$$

$$\Rightarrow 27x-x^2=180$$

$$\Rightarrow x^2-27x+180=0$$

$$\Rightarrow x^2-15x-12x+180=0$$

$$\Rightarrow x(x-15)-12(x-15)=0$$

$$\Rightarrow (x-12)(x-15)=0$$

$$\Rightarrow x-12=0 \text{ or } x-15=0$$

$$\Rightarrow x=12 \text{ or } x=15$$

When $x=12$,

$27-x=27-12=15$

When $x=15$,

$27-x=27-15=12$

Hence, the required parts are 12 and 15.

20. Divide 16 into two parts such that twice the square of the larger part exceeds the square of the smaller part by 164.

Sol:

Let the larger and smaller parts be x and y, respectively.

According to the question:

$x+y=16$ ....(i)

$2x^2=y^2+164$ ....(ii)

From (i), we get:

$x=16-y$ ......(iii)

From (ii) and (iii), we get:

$2(16-y)^2 = y^2 + 164$

$\Rightarrow 2(256-32y+y^2) = y^2 + 164$

$\Rightarrow 512-64y+2y^2 = y^2 + 164$

$\Rightarrow y^2 - 64y + 348 = 0$

$\Rightarrow y^2 - (58+6)y + 348 = 0$

$\Rightarrow y^2 - 58y - 6y + 348 = 0$

$\Rightarrow y(y-58) - 6(y-58) = 0$

$\Rightarrow (y-58)(y-6) = 0$

$\Rightarrow y-58 = 0 \ or \ y-6 = 0$

$\Rightarrow y = 6 (\because y < 16)$

Putting the value of y in equation (iii), we get

$x = 16-6 = 10$

Hence, the two natural numbers are 6 and 10.

21. Divide two natural numbers, the sum of whose squares is 25 times their sum and also equal to 50 times their difference.

Sol:

Let the two natural numbers be *x* and *y*.

According to the question:

$x^2 + y^2 = 25(x+y)$ ......(i)

$x^2 + y^2 = 50(x-y)$ .....(ii)

From (i) and (ii), we get:

$25(x+y) = 50(x-y)$

$\Rightarrow x+y = 2(x-y)$

$\Rightarrow x+y = 2x-2y$

$\Rightarrow y+2y = 2x-x$

$\Rightarrow 3y = x$ ....(iii)

From (ii) and (iii), we get:

$(3y)^2 + y^2 = 50(3y-y)$

$\Rightarrow 9y^2 + y^2 = 100y$

$\Rightarrow 10y^2 = 100y$

$\Rightarrow y = 10$

From (iii), we have:

$3 \times 10 = x$

$\Rightarrow 30 = x$

Hence, the two natural numbers are 30 and 10.

22. The difference of the squares of two natural numbers is 45. The square of the smaller number is four times the larger number. Find the numbers.

Sol:

Let the greater number be x and the smaller number be y.

According to the question:

$x^2 - y^2 = 45 \qquad ....(i)$

$y^2 = 4x \qquad ....(ii)$

From (i) and (ii), we get:

$x^2 - 4x = 45$

$\Rightarrow x^2 - 4x - 45 = 0$

$\Rightarrow x^2 - (9-5)x - 45 = 0$

$\Rightarrow x^2 - 9x + 5x - 45 = 0$

$\Rightarrow x(x-9) + 5(x-9) = 0$

$\Rightarrow (x-9)(x+5) = 0$

$\Rightarrow x - 9 = 0 \text{ or } x + 5 = 0$

$\Rightarrow x = 9 \text{ or } x = -5$

$\Rightarrow x = 9 \qquad (\because x \text{ is a natural number})$

Putting the value of x in equation (ii), we get:

$y^2 = 4 \times 9$

$\Rightarrow y^2 = 36$

$\Rightarrow y = 6$

Hence, the two numbers are 9 and 6.

23. Three consecutive positive integers are such that the sum of the square of the first and product of the other two is 46. Find the integers.

Sol:

Let the three consecutive positive integers be $x, x+1$ and $x+2$.

According to the given condition,

$x^2 + (x+1)(x+2) = 46$

$\Rightarrow x^2 + x^2 + 3x + 2 = 46$

$\Rightarrow 2x^2 + 3x - 44 = 0$

$\Rightarrow 2x^2 + 11x - 8x - 44 = 0$

$\Rightarrow x(2x+11)-4(2x+11)=0$
$\Rightarrow (2x+11)(x-4)=0$
$\Rightarrow 2x+11=0 \ or \ x-4=0$
$\Rightarrow x=-\frac{11}{2} \ or \ x=4$
$\therefore x=4$ (x is a positive integer)
When $x=4,$
$x+1=4+1=5$
$x+2=4+2=6$
Hence, the required integers are 4, 5 and 6.

24. A two-digit number is 4 times the sum of its digits and twice the product of digits. Find the number.
Sol:
Let the digits at units and tens places be *x* and *y*, respectively.
Original number $=10y+x$
According to the question:
$10y+x=4(x+y)$
$\Rightarrow 10y+x=4x+4y$
$\Rightarrow 3x-6y=0$
$\Rightarrow 3x=6y$
$\Rightarrow x=2y$ ......(i)
Also,
$10y+x=2xy$
$\Rightarrow 10y+2y=2.2y.y$ $[From(i)]$
$\Rightarrow 12y=4y^2$
$\Rightarrow y=3$
From (i), we get:
$x=2\times 3=6$
$\therefore$ Original number $=10\times 3+6=36$

25. A two-digit number is such that the product of its digits is 14. If 45 is added to the number, the digit interchange their places. Find the number.
Sol:
Let the digits at units and tens places be *x* and *y*, respectively.
$\therefore xy=14$

$\Rightarrow y = \dfrac{14}{x}$ ....(i)

According to the question:

$(10y + x) + 45 = 10x + y$

$\Rightarrow 9y - 9x = -45$

$\Rightarrow y - x = -5$ ....(ii)

From (i) and (ii), we get:

$\dfrac{14}{x} - x = -5$

$\Rightarrow \dfrac{14 - x^2}{x} = -5$

$\Rightarrow 14 - x^2 = -5x$

$\Rightarrow x^2 - 5x - 14 = 0$

$\Rightarrow x^2 - (7 - 2)x - 14 = 0$

$\Rightarrow x^2 - 7x + 2x - 14 = 0$

$\Rightarrow x(x - 7) + 2(x - 7) = 0$

$\Rightarrow (x - 7)(x + 2) = 0$

$\Rightarrow x - 7 = 0 \ or \ x + 2 = 0$

$\Rightarrow x = 7 \ or \ x = -2$

$\Rightarrow x = 7$ ($\because$ the digit cannot be negative)

Putting $x = 7$ in equation (i), we get:

$y = 2$

$\therefore$ Required number $= 10 \times 2 + 7 = 27$

26. The denominator of a fraction is 3 more than its numerator. The sum of the fraction and its reciprocal is $2\dfrac{9}{10}$. Find the fraction.

Sol:

Let the numerator be *x*.

$\therefore$ Denominator $= x + 3$

$\therefore$ Original number $= \dfrac{x}{x+3}$

According to the question:

$\dfrac{x}{x+3} + \dfrac{1}{\left(\dfrac{x}{x+3}\right)} = 2\dfrac{9}{10}$

$\Rightarrow \frac{x}{x+3}+\frac{x+3}{x}=\frac{29}{10}$

$\Rightarrow \frac{x^2+(x+3)^2}{x(x+3)}=\frac{29}{10}$

$\Rightarrow \frac{x^2+x^2+6x+9}{x^2+3x}=\frac{29}{10}$

$\Rightarrow \frac{2x^2+6x+9}{x^2+3x}=\frac{29}{10}$

$\Rightarrow 29x^2+87x=20x^2+60x+90$

$\Rightarrow 9x^2+27x-90=0$

$\Rightarrow 9(x^2+3x-10)=0$

$\Rightarrow x^2+3x-10=0$

$\Rightarrow x^2+5x-2x-10=0$

$\Rightarrow x(x+5)-2(x+5)=0$

$\Rightarrow (x-2)(x+5)=0$

$\Rightarrow x-2=0 \text{ or } x+5=0$

$\Rightarrow x=2 \text{ or } x=-5$ (rejected)

So, number $= x = 2$

denominator $= x+3=2+3=5$

So, required fraction $=\frac{2}{5}$

27. The numerator of a fraction is 3 less than its denominator. If 1 is added to the denominator, the fraction is decreased by $\frac{1}{15}$. Find the fraction.

Sol:

Let the denominator of the required fraction be x.

Numerator of the required fraction $= x-3$

$\therefore$ Original fraction $=\frac{x-3}{x}$

If 1 is added to the denominator, then the new fraction obtained is $\frac{x-3}{x+1}$

According to the given condition,

$\frac{x-3}{x+1}=\frac{x-3}{x}-\frac{1}{15}$

$\Rightarrow \frac{x-3}{x}-\frac{x-3}{x+1}=\frac{1}{15}$

$\Rightarrow \frac{(x-3)(x+1)-x(x-3)}{x(x+1)}=\frac{1}{15}$

$\Rightarrow \frac{x^2-2x-3-x^2+3x}{x^2+x}=\frac{1}{15}$

$\Rightarrow \frac{x-3}{x^2+x}=\frac{1}{15}$

$\Rightarrow x^2+x=15x-45$

$\Rightarrow x^2-14x+45=0$

$\Rightarrow x^2-9x-5x+45=0$

$\Rightarrow x(x-9)-5(x-9)=0$

$\Rightarrow (x-5)(x-9)=0$

$\Rightarrow x-5=0 \text{ or } x-9=0$

$\Rightarrow x=5 \text{ or } x=9$

When $x=5$,

$\frac{x-3}{x}=\frac{5-3}{5}=\frac{2}{5}$

When $x=9$,

$\frac{x-3}{x}=\frac{9-3}{9}=\frac{6}{9}=\frac{2}{3}$ (This fraction is neglected because this does not satisfies the given condition.)

Hence, the required fraction is $\frac{2}{5}$.

28. The sum of a number and its reciprocal is $2\frac{1}{30}$. Find the number.

Sol:

Let the required number be x.

According to the given condition,

$x+\frac{1}{x}=2\frac{1}{30}$

$\Rightarrow \frac{x^2+1}{x}=\frac{61}{30}$

$\Rightarrow 30x^2+30=61x$

$\Rightarrow 30x^2-61x+30=0$

$\Rightarrow 30x^2 - 36x - 25x + 30 = 0$

$\Rightarrow 6x(5x-6) - 5(5x-6) = 0$

$\Rightarrow (5x-6)(6x-5) = 0$

$\Rightarrow 5x - 6 = 0 \text{ or } 6x - 5 = 0$

$\Rightarrow x = \frac{6}{5} \text{ or } x = \frac{5}{6}$

Hence, the required number is $\frac{5}{6}$ or $\frac{6}{5}$.

29. A teacher on attempting to arrange the students for mass drill in the form of solid square found that 24 students were left. When he increased the size of the square by one student, he found that he was short of 25 students. Find the number of students.

Sol:

Let there be x rows.

Then, the number of students in each row will also be x.

$\therefore$ Total number of students $= (x^2 + 24)$

According to the question:

$(x+1)^2 - 25 = x^2 + 24$

$\Rightarrow x^2 + 2x + 1 - 25 - x^2 - 24 = 0$

$\Rightarrow 2x - 48 = 0$

$\Rightarrow 2x = 48$

$\Rightarrow x = 24$

$\therefore$ Total number of students $= 24^2 + 24 = 576 + 24 = 600$

30. 300 apples are distributed equally among a certain number of students. Had there been 10 more students, each would have received one apple less. Find the number of students.

Sol:

Let the total number of students be x.

According to the question:

$\frac{300}{x} - \frac{300}{x+10} = 1$

$\Rightarrow \frac{300(x+10) - 300x}{x(x+10)} = 1$

$\Rightarrow \frac{300x + 3000 - 300x}{x^2 + 10x} = 1$

$\Rightarrow 3000 = x^2 + 10x$

$\Rightarrow x^2 + 10x - 3000 = 0$

$\Rightarrow x^2 + (60-50)x = 3000 = 0$

$\Rightarrow x^2 + 60x - 50x - 3000 = 0$

$\Rightarrow x(x+60) - 50(x+60) = 0$

$\Rightarrow (x+60)(x-50) = 0$

$\Rightarrow x = 50 \text{ or } x = -60$

x cannot be negative; therefore, the total number of students is 50.

31. In a class test, the sum of Kamal's marks in mathematics and English is 40. Had he got 3 marks more in mathematics and 4 marks less in English, the product of the marks would have been 360. Find his marks in two subjects separately.

Sol:

Let the marks of Kamal in mathematics and English be x and y, respectively.

According to the question:

$x + y = 40 \qquad \text{......}(i)$

Also,

$(x+3)(y-4) = 360$

$\Rightarrow (x+3)(40-x-4) = 360$ [From (i)]

$\Rightarrow (x+3)(36-x) = 360$

$\Rightarrow 36x - x^2 + 108 - 3x = 360$

$\Rightarrow 33x - x^2 - 252 = 0$

$\Rightarrow -x^2 + 33x - 252 = 0$

$\Rightarrow x^2 - 33x - 252 = 0$

$\Rightarrow x^2 - (21+12)x + 252 = 0$

$\Rightarrow x^2 - 21x - 12x + 252 = 0$

$\Rightarrow x(x-21) - 12(x-21) = 0$

$\Rightarrow (x-21)(x-12) = 0$

$\Rightarrow x = 21 \text{ or } x = 12$

If $x = 21$,

$y = 40 - 21 = 119$

Thus, Kamal scored 21 and 19 marks in mathematics and English, respectively.

If $x = 12$,

$y = 40 - 12 = 28$

Thus, Kamal scored 12 and 28 marks in mathematics and English, respectively.

32. Some students planned a picnic. The total budget for food was ₹ 2000. But, 5 students failed to attend the picnic and thus the cost for food for each member increased by ₹ 20. How many students attended the picnic and how much did each student pay for the food?

Sol:

Let x be the number of students who planned a picnic.

$\therefore$ Original cost of food for each member = ₹ $\frac{2000}{x}$

Five students failed to attend the picnic. So, $(x-5)$ students attended the picnic.

$\therefore$ New cost of food for each member = ₹ $\frac{2000}{(x-5)}$

According of the given condition,

$₹\frac{2000}{x-5} - ₹\frac{2000}{x} = ₹20$

$\Rightarrow \frac{2000x-2000x+10000}{x(x-5)} = 20$

$\Rightarrow \frac{10000}{x^2-5x} = 20$

$\Rightarrow x^2-5x = 500$

$\Rightarrow x^2-5x-500 = 0$

$\Rightarrow x^2-25x+20x-500 = 0$

$\Rightarrow x(x-25)+20(x-25) = 0$

$\Rightarrow (x-25)(x+20) = 0$

$\Rightarrow x-25 = 0 \text{ or } x+20 = 0$

$\Rightarrow x = 25 \text{ or } x = -20$

$\therefore x = 25$ (Number of students cannot be negative)

Number of students who attended the picnic $= x-5 = 23-5 = 20$

Amount paid by each student for the food = ₹ $\frac{2000}{(25-5)}$ = ₹ $\frac{2000}{20}$ =₹ 100

33. If the price of a book is reduced by ₹ 5, a person can buy 4 more books for ₹ 600. Find the original price of the book.

Sol:

Let the original price of the book be ₹ x.

$\therefore$ Number of books bought at original price for ₹ $600 = \frac{600}{x}$

If the price of a book is reduced by ₹ 5, then the new price of the book is ₹$(x-5)$.

$\therefore$ Number of books bought at reduced price for ₹ $600 = \frac{600}{x-5}$

According to the given condition,

$$\frac{600}{x-5} - \frac{600}{x} = 4$$

$$\Rightarrow \frac{600x - 600x + 3000}{x(x-5)} = 4$$

$$\Rightarrow \frac{3000}{x^2 - 5x} = 4$$

$$\Rightarrow x^2 - 5x = 750$$

$$\Rightarrow x^2 - 5x - 750 = 0$$

$$\Rightarrow x^2 - 30x + 25x - 750 = 0$$

$$\Rightarrow x(x-30) + 25(x-30) = 0$$

$$\Rightarrow x - 30 = 0 \text{ or } x + 25 = 0$$

$$\Rightarrow x = 30 \text{ or } x = -25$$

$\therefore x = 30$ (Price cannot be negative)

Hence, the original price of the book is ₹30.

34. A person on tour has ₹ 10800 for his expenses. If he extends his tour by 4 days, he has to cut down his daily expenses by ₹ 90. Find the original duration of the tour.

Sol:

Let the original duration of the tour be x days.

$\therefore$ Original daily expenses = ₹ $\frac{10,800}{x}$

If he extends his tour by 4 days, then his new daily expenses =₹ $\frac{10,800}{x+4}$

According to the given condition,

$$₹\,\frac{10,800}{x} - ₹\,\frac{10,800}{x+4} = ₹\ 90$$

$$\Rightarrow \frac{10800x + 43200 - 10800x}{x(x+4)} = 90$$

$$\Rightarrow \frac{43200}{x^2 + 4x} = 90$$

$\Rightarrow x^2 + 4x = 480$
$\Rightarrow x^2 + 4x - 480 = 0$
$\Rightarrow x^2 + 24x - 20x - 480 = 0$
$\Rightarrow x(x+24) - 20x - 480 = 0$
$\Rightarrow x(x+24) - 20(x+24) = 0$
$\Rightarrow (x+24)(x-20) = 0$
$\Rightarrow x + 24 = 0 \text{ or } x - 20 = 0$
$\Rightarrow x = -24 \text{ or } x = 20$

$\therefore x = 20$ (Number of days cannot be negative)

Hence, the original duration of the tour is 20 days.

35. In a class test, the sum of the marks obtained by P in mathematics and science is 28. Had he got 3 more marks in mathematics and 4 marks less in science, the product of marks obtained in the two subjects would have been 180. Find the marks obtained by him in the two subjects separately.

Sol:

Let the marks obtained by P in mathematics and science be x and $(28-x)$, respectively.

According to the given condition,

$(x+3)(28-x-4) = 180$
$\Rightarrow (x+3)(24-x) = 180$
$\Rightarrow -x^2 + 21x + 72 = 180$
$\Rightarrow x^2 - 21x + 108 = 0$
$\Rightarrow x^2 - 12x - 9x + 108 = 0$
$\Rightarrow x(x-12) - 9(x-12) = 0$
$\Rightarrow (x-12)(x-9) = 0$
$\Rightarrow x - 12 = 0 \text{ or } x - 9 = 0$
$\Rightarrow x = 12 \text{ or } x = 9$

When $x = 12$,

$28 - x = 28 - 12 = 16$

When $x = 9$,

$28 - x = 28 - 9 = 19$

Hence, he obtained 12 marks in mathematics and 16 marks in science or 9 marks in mathematics and 19 marks in science.

36. A man buys a number of pens for ₹ 180. If he had bought 3 more pens for the same amount, each pen would have cost him ₹ 3 less. How many pens did he buy?

Sol:

Let the total number of pens be x.

According to the question:

$$\frac{80}{x}-\frac{80}{x+4}=1$$

$$\Rightarrow \frac{80(x+4)-80x}{x(x+4)}=1$$

$$\Rightarrow \frac{80+320-80x}{x^2+4x}=1$$

$$\Rightarrow 320=x^2+4x$$

$$\Rightarrow x^2+4x-320=0$$

$$\Rightarrow x^2+(20-16)x-320=0$$

$$\Rightarrow x^2+20x-16x-320=0$$

$$\Rightarrow x(x+20)-16(x+20)=0$$

$$\Rightarrow (x+20)(x-16)=0$$

$$\Rightarrow x=-20 \ or \ x=16$$

The total number of pens cannot be negative; therefore, the total number of pens is 16.

37. A dealer sells an article for ₹ 75 and gains as much per cent as the cost priced of the article. Find the cost price of the article.

Sol:

Let the cost price of the article be x

$\therefore$ Gain percent $=x\%$

According to the given condition,

$$₹\, x+₹\left(\frac{x}{100}\times x\right)= ₹75 \qquad \text{(Cost price + Gain = Selling price)}$$

$$\Rightarrow \frac{100x+x^2}{100}=75$$

$$\Rightarrow x^2+100x=7500$$

$$\Rightarrow x^2+100x-7500=0$$

$$\Rightarrow x^2+150x-50x-7500=0$$

$$\Rightarrow x(x+150)-50(x+150)=0$$

$$\Rightarrow (x-50)(x+150)=0$$

$$\Rightarrow x-50=0 \ or \ x+150=0$$

$\Rightarrow x = 50 \ or \ x = -150$

$\therefore x = 50$ (Cost price cannot be negative)

Hence, the cost price of the article is ₹50.

38. One year ago, man was 8 times as old as his son. Now, his age is equal to the square of his son's age. Find their present ages.

Sol:

Let the present age of the son be x years.

$\therefore$ Present age of the man $= x^2$ years

One year ago,

Age of the son $= (x-1)$ years

Age of the man $= (x^2-1)$ years

According to the given condition,

Age of the man $= 8 \times$ Age of the son

$\therefore x^2 - 1 = 8(x-1)$

$\Rightarrow x^2 - 1 = 8x - 8$

$\Rightarrow x^2 - 8x + 7 = 0$

$\Rightarrow x^2 - 7x - x + 7 = 0$

$\Rightarrow x(x-7) - 1(x-7) = 0$

$\Rightarrow (x-1)(x-7) = 0$

$\Rightarrow x - 1 = 0 \ or \ x - 7 = 0$

$\Rightarrow x = 1 \ or \ x = 7$

$\therefore x = 7$ (Man's age cannot be 1 year)

Present age of the son $= 7$ years

Present age of the man $= 7^2$ years = 49 years.

39. The sum of reciprocals of Meena's ages (in years) 3 years ago and 5 years hence $\frac{1}{3}$. Find her present ages.

Sol:

Let the present age of Meena be x years

Meena's age 3 years ago $= (x-3)$ years

Meena's age 5 years hence $= (x+5)$ years

According to the given condition,

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$\Rightarrow \frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$

$\Rightarrow \frac{2x+2}{x^2+2x-15} = \frac{1}{3}$

$\Rightarrow x^2+2x-15 = 6x+6$

$\Rightarrow x^2-4x-21 = 0$

$\Rightarrow x^2-7x+3x-21 = 0$

$\Rightarrow x(x-7)+3(x-7) = 0$

$\Rightarrow (x-7)(x+3) = 0$

$\Rightarrow x-7 = 0 \text{ or } x+3 = 0$

$\Rightarrow x = 7 \text{ or } x = -3$

$\therefore x = 7$ (Age cannot be negative)

Hence, the present age of Meena is 7 years.

40. The sum of the ages of a boy and his brother is 25 years, and the product of their ages in years is 126. Find their ages.

Sol:

Let the present ages of the boy and his brother be x years and $(25-x)$ years.

According to the question:

$x(25-x) = 126$

$\Rightarrow 25x - x^2 = 126$

$\Rightarrow x^2-(18-7)x+126 = 0$

$\Rightarrow x^2-18x-7x+126 = 0$

$\Rightarrow x(x-18)-7(x-18) = 0$

$\Rightarrow (x-18)(x-7) = 0$

$\Rightarrow x-18 = 0 \text{ or } x-7 = 0$

$\Rightarrow x = 18 \text{ or } x = 7$

$\Rightarrow x = 18$ ($\because$ Present age of the boy cannot be less than his brother)

If $x = 18$, we have

Present ages of the boy = 18 years

Present age of his brother $= (25-18)$ years = 7 years

Thus, the present ages of the boy and his brother are 18 years and 7 years, respectively.

41. The product of Tanvy's age (in years) 5 years ago and her age is 8 years later is 30. Find her present age.

Sol:

Let the present age of Meena be x years.

According to the question:

$(x-5)(x+8)=30$

$\Rightarrow x^2+3x-40=30$

$\Rightarrow x^2+3x-70=0$

$\Rightarrow x^2+(10-7)x-70=0$

$\Rightarrow x^2+10x-7x-70=0$

$\Rightarrow x(x+10)-7(x+10)=0$

$\Rightarrow (x+10)(x-7)=0$

$\Rightarrow x+10=0 \ or \ x-7=0$

$\Rightarrow x=-10 \ or \ x=7$

$\Rightarrow x=7$ ($\because$ Age cannot be negative)

Thus, the present age of Meena is 7 years.

42. Two years ago, man's age was three times the square of his son's age. In three years' time, his age will be four time his son's age. Find their present ages.

Sol:

Let son's age 2 years ago be x years. Then,

Man's age 2 years ago $=3x^2$ years

$\therefore$ Sons present age $=(x+2)$ years

Man's present age $=(3x^2+2)$ years

In three years time,

Son's age $=(x+2+3)$ years $=(x+5)$ years

Man's age $=(3x^2+2+3)$ years $=(3x^2+5)$ years

According to the given condition,

Man's age = $4\times$Son's age

$\therefore 3x^2+5=4(x+5)$

$\Rightarrow 3x^2+5=4x+20$

$\Rightarrow 3x^2-4x-15=0$

$\Rightarrow 3x^2-9x+5x-15=0$

$\Rightarrow 3x(x-3)+5(x-3)=0$

$\Rightarrow (x-3)(3x+5)=0$

$\Rightarrow x-3=0 \ or \ 3x+5=0$

$\Rightarrow x=3 \ or \ x=-\dfrac{5}{3}$

$\therefore x=3$ (Age cannot be negative)

Son's present age $=(x+2)$ years $=(3+2)$ years $=5$ years

Man's present age $=(3x^2+2)$ years $=(3\times 9+2)$ years $=29$ years

43. A truck covers a distance of 150 km at a certain average speed and then covers another 200 km at an average speed which is 20 km per hour more than the first speed. If the truck covers the total distance in 5 hours, find the first speed of the truck.

Sol:

Let the first speed of the truck be $x \ km/h$.

$\therefore$ Time taken to cover 150 $km=\dfrac{150}{x}h$ $\left(Time=\dfrac{Distance}{Speed}\right)$

New speed of the truck $=(x+20)km/h$

$\therefore$ Time taken to cover 200 km $=\dfrac{200}{x+20}h$

According to the given condition,

Time taken to cover 150 km + Time taken to cover 200 km = 5 h

$\therefore \dfrac{150}{x}+\dfrac{200}{x+20}=5$

$\Rightarrow \dfrac{150x+3000+200x}{x(x+20)}=5$

$\Rightarrow 350x+3000=5(x^2+20x)$

$\Rightarrow 350x+3000=5x^2+100x$

$\Rightarrow 5x^2-250x-3000=0$

$\Rightarrow x^2-50x-600=0$

$\Rightarrow x^2-60x+10x-600=0$

$\Rightarrow x(x-60)+10(x-60)=0$

$\Rightarrow (x-60)(x+10)=0$

$\Rightarrow x-60=0 \ or \ x+10=0$

$\Rightarrow x=60 \ or \ x=-10$

$\therefore x=60$ (Speed cannot be negative)

Hence, the first speed of the truck is 60 km/h.

44. While boarding an aeroplane, a passengers got hurt. The pilot showing promptness and concern, made arrangements to hospitalize the injured and so the plane started late by 30 minutes. To reach the destination, 1500 km away, in time, the pilot increased the speed by 100 km/hour. Find the original speed of the plane.
Do you appreciate the values shown by the pilot, namely promptness in providing help to the injured and his efforts to reach in time?

Sol:

Let the original speed of the plane be x km/h.

$\therefore$ Actual speed of the plane $= (x+100) km/h$

Distance of the journey $= 1500\ km$

Time taken to reach the destination at original speed $= \frac{1500}{x}h$ $\left(Time = \frac{Distance}{Speed}\right)$

Time taken to reach the destination at actual speed $= \frac{1500}{x+100}h$

According to the given condition,

Time taken to reach the destination at original speed = Time taken to reach the destination at actual speed + 30 min

$\therefore \frac{1500}{x} = \frac{1500}{x+100} + \frac{1}{2}$ $\left(30 \text{ min} = \frac{30}{60}h = \frac{1}{2}h\right)$

$\Rightarrow \frac{1500}{x} - \frac{1500}{x+100} = \frac{1}{2}$

$\Rightarrow \frac{1500x + 150000 - 1500x}{x(x+100)} = \frac{1}{2}$

$\Rightarrow \frac{150000}{x^2+100x} = \frac{1}{2}$

$\Rightarrow x^2 + 100x = 300000$

$\Rightarrow x^2 + 100x = 300000 = 0$

$\Rightarrow x^2 + 600x - 500x - 300000 = 0$

$\Rightarrow x(x+600) - 500(x+600) = 0$

$\Rightarrow (x+600)(x-500) = 0$

$\Rightarrow x + 600 = 0 \text{ or } x - 500 = 0$

$\Rightarrow x = -600 \text{ or } x = 500$

$\therefore x = 500$ (Speed cannot be negative)

Hence, the original speed of the plane is 500 km/h.

Yes, we appreciate the values shown by the pilot, namely promptness in providing help to the injured and his efforts to reach in time. This reflects the caring nature of the pilot and his dedication to the work.

45. A train covers a distance of 480 km at a uniform speed. If the speed had been 8 km/hr less then it would have taken 3 hours more to cover the same distance. Find the usual speed of the train.

Sol:

Let the usual speed of the train be x km/h.

$\therefore$ Reduced speed of the train $=(x-8)km/h$

Total distance to be covered = 480 km

Time taken by the train to cover the distance at usual speed $=\frac{480}{x}h$ $\left(Time=\frac{Distance}{Speed}\right)$

Time taken by the train to cover the distance at reduced speed $=\frac{480}{x-8}h$

According to the given condition,

Time taken by the train to cover the distance at reduced speed = Time taken by the train to cover the distance at usual speed + 3 h

$\therefore \frac{480}{x-8}=\frac{480}{x}+3$

$\Rightarrow \frac{480}{x-8}-\frac{480}{x}=3$

$\Rightarrow \frac{480x-480x+3840}{x(x-8)}=3$

$\Rightarrow \frac{3840}{x^2-8x}=3$

$\Rightarrow x^2-8x=1280$

$\Rightarrow x^2-8x-1280=0$

$\Rightarrow x(x-40)+32(x-40)=0$

$\Rightarrow (x-40)(x+32)=0$

$\Rightarrow x-40=0$ or $x+32=0$

$\Rightarrow x=40$ or $x=-32$

$\therefore x=40$ (Speed cannot be negative)

Hence, the usual speed of the train is 40 km/h.

46. A train travels at a certain average speed for a distanced of 54 km and then travels a distance of 63 km at an average speed of 6 km/hr more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed?

Sol:

Let the first speed of the train be x km/h.

Time taken to cover $54\ km = \frac{54}{x} h$ $\left(Time = \frac{Distance}{Speed} \right)$

New speed of the train $= (x+6)\ km/h$

$\therefore$ Time taken to cover $63 km = \frac{63}{x+6} h$

According to the given condition,

Time taken to cover 54 km + Time taken to cover 63 km = 3 h

$\therefore \frac{54}{x} + \frac{63}{x+6} = 3$

$\Rightarrow \frac{54x + 324 + 63x}{x(x+6)} = 3$

$\Rightarrow 117x + 324 = 3(x^2 + 6x)$

$\Rightarrow 117x + 324 = 3x^2 + 18x$

$\Rightarrow 3x^2 - 99x - 324 = 0$

$\Rightarrow x^2 - 33x - 324 = 0$

$\Rightarrow x^2 - 33x - 108 = 0$

$\Rightarrow x^2 - 36x + 3x - 108 = 0$

$\Rightarrow x(x-36) + 3(x-36) = 0$

$\Rightarrow (x-36)(x+3) = 0$

$\Rightarrow x - 36 = 0\ or\ x + 3 = 0$

$\Rightarrow x = 36\ or\ x = -3$

$\therefore x = 36$ (Speed cannot be negative)

Hence, the first speed of the train is 36 km/h.

47. A train travels 180 km at a uniform speed. If the speed had been 9 km/hr more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Sol: 36km/hr

48. A train covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hr more, it would have taken 30 minutes less for the journey. Find the original speed of the train.

Sol:

Let the original speed of the train be *x* km/hr.

According to the question:

$\frac{90}{x} - \frac{90}{(x+15)} = \frac{1}{2}$

$\Rightarrow \dfrac{90(x+15)-90x}{x(x+15)}=\dfrac{1}{2}$

$\Rightarrow \dfrac{90x+1350-90x}{x^2+15x}=\dfrac{1}{2}$

$\Rightarrow \dfrac{1350}{x^2+15x}=\dfrac{1}{2}$

$\Rightarrow 2700=x^2+15x$

$\Rightarrow x^2+(60-45)x-2700=0$

$\Rightarrow x^2+60x-45x-2700=0$

$\Rightarrow x(x+60)-45x(x+60)=0$

$\Rightarrow (x+60)(x-45)=0$

$\Rightarrow x=-60 \text{ or } x=45$

x cannot be negative; therefore, the original speed of train is 45 km/hr.

49. A passenger train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hr from its usual speed. Find its usual speed.

Sol:

Let the usual speed x km/hr.

According to the question:

$\dfrac{300}{x}-\dfrac{300}{(x+5)}=2$

$\Rightarrow \dfrac{300(x+5)-300x}{x(x+5)}=2$

$\Rightarrow \dfrac{300x+1500-300x}{x^2+5x}=2$

$\Rightarrow 1500=2(x^2+5x)$

$\Rightarrow 1500=2x^2+10x$

$\Rightarrow x^2+5x-750=0$

$\Rightarrow x^2+(30-25)x-750=0$

$\Rightarrow x^2+30x-25x-750=0$

$\Rightarrow x(x+30)-25(x+30)=0$

$\Rightarrow (x+30)(x-25)=0$

$\Rightarrow x=-30 \text{ or } x=25$

The usual speed cannot be negative; therefore, the speed is 25 km/hr.

50. The distance between Mumbai and Pune is 192 km. Travelling by the Deccan Queen, it takes 48 minutes less than another train. Calculate the speed of the Deccan Queen if the speeds of the two train differ by 20km/hr.

Sol:

Let the speed of the Deccan Queen be x km/hr.

According to the question:

Speed of another train $=(x-20)km/hr$

$$\therefore \frac{192}{x-20}-\frac{192}{x}=\frac{48}{60}$$

$$\Rightarrow \frac{4}{x-20}-\frac{4}{x}=\frac{1}{60}$$

$$\Rightarrow \frac{4x-4(x-20)}{(x-20)x}=\frac{1}{60}$$

$$\Rightarrow \frac{4x-4x+80}{x^2-20x}=\frac{1}{60}$$

$$\Rightarrow \frac{80}{x^2-20x}=\frac{1}{60}$$

$$\Rightarrow x^2-20x=4800$$

$$\Rightarrow x^2-20x-4800=0$$

$$\Rightarrow x^2-(80-60)x-4800=0$$

$$\Rightarrow x^2-80x+60x-4800=0$$

$$\Rightarrow x(x-80)+60(x-80)=0$$

$$\Rightarrow (x-80)(x+60)=0$$

$$\Rightarrow x=80 \ or \ x=-60$$

The value of

x cannot be negative; therefore, the original speed of Deccan Queen 180 km/hr.

51. A motor boat whose speed in still water is 178 km/hr, takes 1 hour more to go 24 km upstream than to return to the same spot. Find the speed of the stream

Sol:

Let the speed of the stream be $x \ km/hr$.

Given:

Speed of the boat $=18 \ km/hr$

$\therefore$ Speed downstream $=(18+x) \ km/hr$

Speed upstream $=(18-x) \ km/hr$

$\therefore \frac{24}{(18-x)} - \frac{24}{(18-x)} = 1$

$\Rightarrow \frac{1}{(18-x)} - \frac{1}{(18+x)} = \frac{1}{24}$

$\Rightarrow \frac{18+x-18+x}{(18-x)(18+x)} = \frac{1}{24}$

$\Rightarrow \frac{2x}{18^2 - x^2} = \frac{1}{24}$

$\Rightarrow 324 - x^2 = 48x$

$\Rightarrow 324 - x^2 - 48x = 0$

$\Rightarrow x^2 + 48x - 324 = 0$

$\Rightarrow x^2 + (54-6)x - 324 = 0$

$\Rightarrow x^2 + 54x - 6x - 324 = 0$

$\Rightarrow x(x+54) - 6(x+54) = 0$

$\Rightarrow (x+54)(x-6) = 0$

$\Rightarrow x = -54 \ or \ x = 6$

The value of x cannot be negative; therefore, the speed of the stream is 6 km/hr.

52. The speed of a boat in still water is 8 km/hr. It can go 15 km upstream and 22 km downstream is 5 hours. Fid the speed of the stream

Sol:

Speed of the boat in still water $= 8\ km/hr.$

Let the speed of the stream be x km/hr.

$\therefore$ Speed upstream $= (8-x) km/hr.$

Speed downstream $= (8+x) km/hr.$

Time taken to go 22 km downstream $= \frac{22}{(8+x)} hr$

Time taken to go 15 km upstream $= \frac{15}{(8-x)} hr$

According to the question:

$\Rightarrow \frac{22}{(8+x)} + \frac{15}{(8-x)} = 5$

$\Rightarrow \frac{22}{(8+x)} + \frac{15}{(8-x)} - 5 = 0$

$\Rightarrow \dfrac{22(8-x)+15(8+x)-5(8-x)(8+x)}{(8-x)(8+x)}=0$

$\Rightarrow 176-22x+120+15x-320+5x^2=0$

$\Rightarrow 5x^2-7x-24=0$

$\Rightarrow 5x^2-(15-8)x-24=0$

$\Rightarrow 5x^2-15x+8x-24=0$

$\Rightarrow 5x(x-3)-8(x-3)=0$

$\Rightarrow (x-3)(5x-8)=0$

$\Rightarrow x-3=0 \ or \ 5x-8=0$

$\Rightarrow x=3 \ or \ x=\dfrac{8}{5}$

$\Rightarrow x=3$ ($\because$ Speed cannot be a fraction)

$\therefore$ Speed of the stream $=3\ km/hr$

53. A motorboat whose speed is 9 km/hr in still water, goes 15 km downstream and comes back in a total time of 3 hours 45 minutes. Find the speed of the stream.

Sol:

Let the speed of the stream be x km/hr.

$\therefore$ Downstream speed $=(9+x)km/hr$.

Upstream speed $=(9-x)km/hr$

Distance covered downstream = Distance covered upstream = 15 km

Total time taken = 3hours 45 minutes $=\left(3+\dfrac{45}{60}\right)$ minutes $=\dfrac{225}{60}$ minutes $=\dfrac{15}{4}$ minutes

$\therefore \dfrac{15}{(9+x)}+\dfrac{15}{(9-x)}=\dfrac{15}{4}$

$\Rightarrow \dfrac{1}{(9+x)}+\dfrac{1}{(9-x)}=\dfrac{1}{4}$

$\Rightarrow \dfrac{9-x+9+x}{(9+x)(9-x)}=\dfrac{1}{4}$

$\Rightarrow \dfrac{18}{9^2-x^2}=\dfrac{1}{4}$

$\Rightarrow \dfrac{18}{81-x^2}=\dfrac{1}{4}$

$\Rightarrow 81-x^2=72$

$\Rightarrow 81-x^2-72=0$

$\Rightarrow -x^2+9=0$

$\Rightarrow x^2=9$

$\Rightarrow x=3 \text{ or } x=-3$

The value of x cannot be negative; therefore, the speed of the stream is 3 km/hr.

54. A takes 10 days less than the time taken by B to finish a piece of work. If both A and B together can finish the work in 12 days, find the time taken by B to finish the work.

Sol:

Let B takes x days to complete the work.

Therefore, A will take $(x-10)$ days.

$\therefore \frac{1}{x}+\frac{1}{(x-10)}=\frac{1}{12}$

$\Rightarrow \frac{(x-10)+x}{x(x-10)}=\frac{1}{12}$

$\Rightarrow \frac{2x-10}{x^2-10x}=\frac{1}{12}$

$\Rightarrow x^2-10x=12(2x-10)$

$\Rightarrow x^2-10x=24x-120$

$\Rightarrow x^2-34x+120=0$

$\Rightarrow x^2-(30+4)x+120=0$

$\Rightarrow x^2-30x-4x+120=0$

$\Rightarrow x(x-30)-4(x-30)=0$

$\Rightarrow (x-30)(x-4)=0$

$\Rightarrow x=30 \text{ or } x=4$

Number of days to complete the work by

B cannot be less than that by A; therefore, we get: x = 30

Thus, B completes the work in 30 days.

55. Two pipes running together can fill a cistern in $3\frac{1}{13}$ minutes. If one pipe takes 3 minutes more than the other to fill it, find the time in which each pipe would fill the cistern.

Sol:

Let one pipe fills the cistern in x mins.

Therefore, the other pipe will fill the cistern in $(x+3)$ mins.

Time taken by both, running together, to fill the cistern $=3\frac{1}{13}\text{ mins}=\frac{40}{13}\text{ mins}$

Part filled by one pipe in 1 min $= \frac{1}{x}$

Part filled by the other pipe in 1 min $= \frac{1}{x+3}$

Part filled by both pipes, running together, in 1 min $= \frac{1}{x} + \frac{1}{x+3}$

$\therefore \frac{1}{x} + \frac{1}{x+3} = \frac{1}{\frac{40}{13}}$

$\Rightarrow \frac{(x+3)+x}{x(x+3)} = \frac{13}{40}$

$\Rightarrow \frac{2x+3}{x^2+3a} = \frac{13}{40}$

$\Rightarrow 13x^2 + 39x = 80x + 120$

$\Rightarrow 13x^2 - 41x - 120 = 0$

$\Rightarrow 13x^2 - (65-24)x - 120 = 0$

$\Rightarrow 13x^2 - 65x + 24x - 120 = 0$

$\Rightarrow 13x(x-5) + 24(x-5) = 0$

$\Rightarrow (x-5)(13x+24) = 0$

$\Rightarrow x - 5 = 0 \text{ or } 13x + 24 = 0$

$\Rightarrow x = 5 \text{ or } x = \frac{-24}{13}$

$\Rightarrow x = 5$ ($\because$ Speed cannot be a negative fraction)

Thus, one pipe will take 5 mins and other will take $\{(5+3) = 8\}$ mins to fill the cistern.

56. Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If on pipe takes 5 minutes more than the other to fill the tank separately, find the time in which each pipe would fill the tank separately.

Sol:

Let the time taken by one pipe to fill the tank be x minutes.

$\therefore$ Time taken by the other pipe to fill the tank $= (x+5)$ min

Suppose the volume of the tank be V.

Volume of the tank filled by one pipe in x minutes = V

$\therefore$ Volume of the tank filled by one pipe in 1 minute $= \frac{V}{x}$

$\Rightarrow$ Volume of the tank filled by one pipe in $11\frac{1}{9}$ minutes $= \frac{V}{x} \times 11\frac{1}{9} = \frac{V}{x} \times \frac{100}{9}$

Similarly,

Volume of the tank filled by the other pipe in $11\frac{1}{9}$ minutes $= \frac{V}{(x+5)} \times 11\frac{1}{9} = \frac{V}{(x+5)} \times \frac{100}{9}$

Now,

Volume of the tank filled by one pipe in $11\frac{1}{9}$ minutes + Volume of the tank filled by the other pipe in $11\frac{1}{9}$ minutes $= V$

$\therefore V\left(\frac{1}{x} + \frac{1}{x+5}\right) \times 100 = V$

$\Rightarrow \frac{1}{x} + \frac{1}{x+5} = \frac{9}{100}$

$\Rightarrow \frac{x+5+x}{x(x+5)} = \frac{9}{100}$

$\Rightarrow \frac{2x+5}{x^2+5x} = \frac{9}{100}$

$\Rightarrow 200x + 500 = 9x^2 + 45x$

$\Rightarrow 9x^2 - 155x - 500 = 0$

$\Rightarrow 9x^2 - 180x + 25x - 500 = 0$

$\Rightarrow 9x(x-20) + 25(x-20) = 0$

$\Rightarrow (x-20)(9x+25) = 0$

$\Rightarrow x - 20 = 0 \text{ or } 9x + 25 = 0$

$\Rightarrow x = 20 \text{ or } x = -\frac{25}{9}$

$\therefore x = 20$ (Time cannot be negative)

Time taken by one pipe to fill the tank = 20 min

Time taken by other pipe to fill the tank = (20 + 5) 25 min

57. Two water taps together can fill a tank in 6 hours. The tap of larger diameter takes 9 hours less than the smaller one to fill the tank separately. Find the time which each tap can separately fill the tank.

Sol:

Let the tap of smaller diameter fill the tank in x hours.

$\therefore$ Time taken by the tap of larger diameter to fill the tank $= (x-9)h$

Suppose the volume of the tank be V.

Volume of the tank filled by the tap of smaller diameter in x hours = V

$\therefore$ Volume of the tank filled by the tap of smaller diameter in 1 hour $= \dfrac{V}{x}$

$\Rightarrow$ Volume of the tank filled by the tap of smaller diameter in 6 hours $= \dfrac{V}{x} \times 6$

Similarly,

Volume of the tank filled by the tap of larger diameter in 6 hours $= \dfrac{V}{(x-9)} \times 6$

Now,
Volume of the tank filled by the tap of smaller diameter in 6 hours + Volume of the tank filled by the tap of larger diameter in 6 hours = V

$$\therefore V\left(\frac{1}{x}+\frac{1}{x-9}\right)\times 6 = V$$
$$\Rightarrow \frac{1}{x}+\frac{1}{x-9}=\frac{1}{6}$$
$$\Rightarrow \frac{x-9+x}{x(x-9)}=\frac{1}{6}$$
$$\Rightarrow \frac{2x-9}{x^2-9x}=\frac{1}{6}$$
$$\Rightarrow 12x-54 = x^2-9x$$
$$\Rightarrow x^2-21x+54=0$$
$$\Rightarrow x^2-81x-3x+54=0$$
$$\Rightarrow x(x-18)-3(x-18)=0$$
$$\Rightarrow (x-18)(x-3)=0$$
$$\Rightarrow x-18=0 \; or \; x-3=0$$
$$\Rightarrow x=18 \; or \; x=3$$

For $x=3$, time taken by the tap of larger diameter to fill the tank is negative which is not possible.
$\therefore x=18$
Time taken by the tap of smaller diameter to fill the tank = 18 h
Time taken by the tap of larger diameter to fill the tank $=(18-9)=9h$

Hence, the time taken by the taps of smaller and larger diameter to fill the tank is 18 hours and 9 hours, respectively.

58. The length of rectangle is twice its breadth and its areas is 288 cm $288cm^2$. Find the dimension of the rectangle.

Sol:
Let the length and breadth of the rectangle be $2x$ m and x m, respectively.

According to the question:

$2x \times x = 288$

$\Rightarrow 2x^2 = 288$

$\Rightarrow x^2 = 144$

$\Rightarrow x = 12 \ or \ x = -12$

$\Rightarrow x = 12$ ($\because x$ cannot be negative)

$\therefore \text{Length} = 2 \times 12 = 24m$

Breath $= 12m$

59. The length of a rectangular field is three times its breadth. If the area of the field be 147 sq meters, find the length of the field.

Sol:

Let the length and breadth of the rectangle be $3x$ m and x m, respectively.

According to the question:

$3x \times x = 147$

$\Rightarrow 3x^2 = 147$

$\Rightarrow x^2 = 49$

$\Rightarrow x = 7 \ or \ x = -7$

$\Rightarrow x = 7$ ($\because x$ cannot be negative)

$\therefore \text{Length} = 3 \times 7 = 21m$

Breath $= 7m$

60. The length of a hall is 3 meter more than its breadth. If the area of the hall is 238 sq meters, calculate its length and breadth.

Sol:

Let the breath of the rectangular hall be x meter.

Therefore, the length of the rectangular hall will be $(x+3)$ meter.

According to the question:

$x(x+3) = 238$

$\Rightarrow x^2 + 3x = 238$

$\Rightarrow x^2 + 3x - 238 = 0$

$\Rightarrow x^2 + (17-14)x - 238 = 0$

$\Rightarrow x^2 + 17x - 14x - 238 = 0$

$\Rightarrow x(x+17) - 14(x+17) = 0$

$\Rightarrow (x+17)(x-14) = 0$

$\Rightarrow x = -17 \ or \ x = 14$

But the value x cannot be negative.
Therefore, the breath of the hall is 14 meter and the length is 17 meter.

61. The perimeter of a rectangular plot is 62 m and its area is 288 sq meters. Find the dimension of the plot

Sol:

Let the length and breadth of the rectangular plot be x and y meter, respectively.
Therefore, we have:

Perimeter $=2(x+y)=62$ .....(i) and

Area $=xy=228$

$\Rightarrow y=\dfrac{228}{x}$

Putting the value of y in (i), we get

$\Rightarrow 2\left(x+\dfrac{228}{x}\right)=62$

$\Rightarrow x+\dfrac{228}{x}=31$

$\Rightarrow \dfrac{x^2+228}{x}=31$

$\Rightarrow x^2+228=31x$

$\Rightarrow x^2-31x+228=0$

$\Rightarrow x^2-(19+12)x+228=0$

$\Rightarrow x^2-19x-12x+228=0$

$\Rightarrow x(x-19)-12(x-19)=0$

$\Rightarrow (x-19)(x-12)=0$

$\Rightarrow x=19 \text{ or } x=12$

If $x=19\,m, y=\dfrac{228}{19}=12\,m$

Therefore, the length and breadth of the plot are 19 m and 12 m, respectively.

62. A rectangular filed in 16 m long and 10 m wide. There is a path of uniform width all around it, having an area of $120m^2$. Find the width of the path

Sol:

Let the width of the path be x m.

$\therefore$ Length of the field including the path $=16+x+x=16+2x$

Breadth of the field including the path $=10+x+x=10+2x$

Now,

(Area of the field including path) - (Area of the field excluding path) =
Area of the path

$\Rightarrow (16+2x)(10+2x)-(16\times10)=120$

$\Rightarrow 160+32x+20x+4x^2-160=120$

$\Rightarrow 4x^2+52x-120=0$

$\Rightarrow x^2+13x-30=0$

$\Rightarrow x^2+(15-2)x+30=0$

$\Rightarrow x^2+15x-2x+30=0$

$\Rightarrow x(x+15)-2(x+15)=0$

$\Rightarrow (x-2)(x+15)=0$

$\Rightarrow x-2=0$ *or* $x+15=0$

$\Rightarrow x=2$ *or* $x=-15$

$\Rightarrow x=2$ ($\because$ Width cannot be negative)

Thus, the width of the path is 2 m.

63. The sum of the areas of two squares is $640\,m^2$. If the difference in their perimeter be 64m, find the sides of the two square

Sol:

Let the length of the side of the first and the second square be *x* and *y*. respectively.

According to the question:

$x^2+y^2=640 \qquad ....(i)$

Also,

$4x-4y=64$

$\Rightarrow x-y=16$

$\Rightarrow x=16+y$

Putting the value of *x* in (i), we get:

$x^2+y^2=640$

$\Rightarrow (16+y)^2+y^2=640$

$\Rightarrow 256+32y+y^2+y^2=640$

$\Rightarrow 2y^2+32y-384=0$

$\Rightarrow y^2+16y-192=0$

$\Rightarrow y^2+(24-8)y-192=0$

$\Rightarrow y^2+24y-8y-192=0$

$\Rightarrow y(y+24)-8(y+24)=0$

$\Rightarrow (y+24)(y-8)=0$

$\Rightarrow y=-24 \ or \ y=8$

$\therefore y=8$ (∵ Side cannot be negative)

$\therefore x=16+y=16+8=24 \ m$

Thus, the sides of the squares are 8 m and 24 m.

64. The length of a rectangle is thrice as long as the side of a square. The side of the square is 4 cm more than the width of the rectangle. Their areas being equal, find the dimensions.

Sol:

Let the breadth of rectangle be x cm.

According to the question:

Side of the square $=(x+4)cm$

Length of the rectangle = {3(x + 4)} cm

It is given that the areas of the rectangle and square are same.

$\therefore 3(x+4)\times x=(x+4)^2$

$\Rightarrow 3x^2+12x=(x+4)^2$

$\Rightarrow 3x^2+12x=x^2+8x+16$

$\Rightarrow 2x^2+4x-16=0$

$\Rightarrow x^2+2x-8=0$

$\Rightarrow x^2+(4-2)x-8=0$

$\Rightarrow x^2+4x-2x-8=0$

$\Rightarrow x(x+4)-2(x+4)=0$

$\Rightarrow (x+4)(x-2)=0$

$\Rightarrow x=-4 \ or \ x=2$

$\therefore x=2$ (∵ The value of x cannot be negative)

Thus, the breadth of the rectangle is 2 cm and length is {3(2 + 4) =18} cm.

Also, the side of the square is 6 cm.

65. A farmer prepares rectangular vegetable garden of area 180 sq meters. With 39 meters of barbed wire, he can fence the three sides of the garden, leaving one of the longer sides unfenced. Find the dimensions of the garden.

Sol:

Let the length and breadth of the rectangular garden be x and y meter, respectively.

Given:

$xy=180 \ sq \ m$ ...(i) and

$2y + x = 39$
$\Rightarrow x = 39 - 2y$
Putting the value of x in (i), we get:
$(39 - 2y)\,y = 180$
$\Rightarrow 39 - 2y^2 = 180$
$\Rightarrow 39y - 2y^2 - 180 = 0$
$\Rightarrow 2y^2 - 39y + 180 = 0$
$\Rightarrow 2y^2 - (24 + 15)\,y + 180 = 0$
$\Rightarrow 2y^2 - 24y - 15y + 180 = 0$
$\Rightarrow 2y(y - 12) - 15(y - 12) = 0$
$\Rightarrow (y - 12)(2y - 15) = 0$
$\Rightarrow y = 12 \ or \ y = \frac{15}{2} = 7.5$
If $y = 12, x = 39 - 24 = 15$
If $y = 7.5, x = 39 - 15 = 24$
Thus, the
length and breadth of the garden are (15 m and 12 m) or (24 m and 7.5 m), respectively.

66. The area of a right triangle is $600\,cm^2$. If the base of the triangle exceeds the altitude by 10 cm, find the dimensions of the triangle.
Sol:
Let the altitude of the triangle be x cm
Therefore, the base of the triangle will be $(x + 10)\,cm$
Area of triangle $= \frac{1}{2}x(x + 10) = 600$
$\Rightarrow (x + 10) = 1200$
$\Rightarrow x^2 + 10x - 1200 = 0$
$\Rightarrow x^2 + (40 - 30)\,x - 1200 = 0$
$\Rightarrow x^2 + 40x - 30x - 1200 = 0$
$\Rightarrow x(x + 40) - 30(x + 40) = 0$
$\Rightarrow (x + 40)(x - 30) = 0$
$\Rightarrow x = -40 \ or \ x = 30$
$\Rightarrow x = 30$ [$\because$ Altitude cannot be negative]
Thus, the altitude and base of the triangle are 30 cm and (30 + 10 = 40) cm, respectively.

$(Hypotenuse)^2 = (Altitude)^2 + (Base)^2$

$\Rightarrow (Hypotenuse)^2 = (30)^2 (40)^2$

$\Rightarrow (Hypotenuse)^2 = 900 + 1600 = 2500$

$\Rightarrow (Hypotenuse)^2 = (50)^2$

$\Rightarrow (Hypotenuse) = 50$

Thus, the dimensions of the triangle are:
Hypotenuse = 50 cm
Altitude = 30 cm
Base = 40 cm

67. The area of right-angled triangle is 96 sq meters. If the base is three time the altitude, find the base.

Sol:

Let the altitude of the triangle be x m.
Therefore, the base will be $3x$ m.

Area of a triangle $= \frac{1}{2} \times$ Base $\times$ Altitude

$\therefore \frac{1}{2} \times 3x \times x = 96 (\because Area = 96\, sq\, m)$

$\Rightarrow \frac{x^2}{2} = 32$

$\Rightarrow x^2 = 64$

$\Rightarrow x = \pm 8$

The value of x cannot be negative

Therefore, the altitude and base of the triangle are 8 m and $(3 \times 8 = 24m)$, respectively.

68. The area of right -angled triangle is 165 sq meters. Determine its base and altitude if the latter exceeds the former by 7 meters.

Sol:

Let the base be x m.
Therefore, the altitude will be $(x+7)m$.

Area of a triangle $= \frac{1}{2} \times$ Base $\times$ Altitude

$\therefore \frac{1}{2} \times x \times (x+7) = 165$

$\Rightarrow x^2 + 7x = 330$

$\Rightarrow x^2 + 7x - 330 = 0$

$\Rightarrow x^2+(22-15)x-330=0$

$\Rightarrow x^2+22x-15x-330=0$

$\Rightarrow x(x+22)-15(x+22)=0$

$\Rightarrow (x+22)(x-15)=0$

$\Rightarrow x=-22 \ or \ x=15$

The value of x cannot be negative

Therefore, the base is 15 m and the altitude is {(15 + 7) = 22 m}.

69. The hypotenuse of a right=-angled triangle is 20 meters. If the difference between the lengths of the other sides be 4 meters, find the other sides

Sol:

Let one side of the right-angled triangle be x m and the other side be $(x+4)m$.

On applying Pythagoras theorem, we have:

$20^2=(x+4)^2+x^2$

$\Rightarrow 400=x^2+8x+16+x^2$

$\Rightarrow 2x^2+8x-384=0$

$\Rightarrow x^2+4x-192=0$

$\Rightarrow x^2+(16-12)x-192=0$

$\Rightarrow x^2+16x-12x-192=0$

$\Rightarrow x^2(x+16)-12(x+16)=0$

$\Rightarrow (x+16)(x-12)=0$

$\Rightarrow x=-16 \ or \ x=12$

The value of x cannot be negative.

Therefore, the base is 12 m and the other side is {(12 + 4) = 16 m).

70. The length of the hypotenuse of a right-angled triangle exceeds the length of the base by 2 cm and exceeds twice the length of the altitude by 1 cm. Find the length of each side of the triangle.

Sol:

Let the base and altitude of the right-angled triangle be x and y cm, respectively

Therefore, the hypotenuse will be (x + 2) cm.

$\therefore (x+2)^2=y^2+x^2 \qquad ....(i)$

Again, the hypotenuse exceeds twice the length of the altitude by 1 cm.

$\therefore h=(2y+1)$

$\Rightarrow x+2=2y+1$

$\Rightarrow x=2y-1$

Putting the value of x in (i), we get:

$(2y-1+2)^2 = y^2 + (2y-1)^2$

$\Rightarrow (2y+1)^2 = y^2 + 4y^2 - 4y + 1$

$\Rightarrow 4y^2 + 4y + 1 = 5y^2 - 4y + 1$

$\Rightarrow -y^2 + 8y = 0$

$\Rightarrow y^2 - 8y = 0$

$\Rightarrow y(y-8) = 0$

$\Rightarrow y = 8\ cm$

$\therefore x = 16 - 1 = 15\ cm$

$\therefore h = 16 + 1 = 17\ cm$

Thus, the base, altitude and hypotenuse of the triangle are 15 cm, 8 cm and 17 cm, respectively.

71. The hypotenuse of a right-angled triangle is 1 meter less than twice the shortest side. If the third side 1 meter more than the shortest side, find the side, find the sides of the triangle.

Sol:

Let the shortest side be *x* m.

Therefore, according to the question:

Hypotenuse $= (2x-1)m$

Third side $= (x+1)m$

On applying Pythagoras theorem, we get:

$(2x-1)^2 = (x+1)^2 + x^2$

$\Rightarrow 4x^2 - 4x + 1 = x^2 + 2x + 1 + x^2$

$\Rightarrow 2x^2 - 6x = 0$

$\Rightarrow 2x(x-3) = 0$

$\Rightarrow x = 0\ or\ x = 3$

The length of the side cannot be 0; therefore, the shortest side is 3 m.

Therefore,

Hypotenuse $= (2 \times 3 - 1) = 5m$

Third side $= (3+1) = 4m$

Exercise - 10F

1. Which of the following is a quadratic equation?

(a) $x^3 - 3\sqrt{x} + 2 = 0$ (b) $x + \frac{1}{x} = x^2$

(c) $x^2 + \frac{1}{x^2} = 5$ (d) $2x^2 - 5x = (x-1)^2$

Answer: (d) $2x^2 - 5x = (x-1)^2$

Sol:

A quadratic equation is the equation with degree 2.

$\because 2x^2 - 5x = (x-1)^2$

$\Rightarrow 2x^2 - 5x = x^2 - 2x + 1$

$\Rightarrow 2x^2 - 5x - x^2 + 2x - 1 = 0$

$\Rightarrow x^2 - 3x - 1 = 0$, which is a quadratic equation

2. Which of the following is a quadratic equation?

(a) $(x^2 + 1) = (2 - x)^2 + 3$ (b) $x^3 - x^2 = (x-1)^3$

(c) $2x^2 + 3 = (5 + x)(2x - 3)$ (d) None of these

Answer: (b) $x^3 - x^2 = (x-1)^3$

Sol:

$\because x^3 - x^2 = (x-1)^3$

$\Rightarrow x^3 - x^2 = x^3 - 3x^2 + 3x - 1$

$\Rightarrow 2x^2 - 3x + 1 = 0$, which is a quadratic equation

3. Which of the following is not a quadratic equation?

(a) $3x - x^2 = x^2 + 5$ (b) $(x+2)^2 = 2(x^2 - 5)$

(c) $(\sqrt{2}x + 3)^2 = 2x^2 + 6$ (d) $(x-1)^2 = 3x^2 + x - 2$

Answer: (c) $(\sqrt{2}x + 3)^2 = 2x^2 + 6$

Sol:

$\because (\sqrt{2}x + 3)^2 = 2x^2 + 6$

$\Rightarrow 2x^2 + 9 + 6\sqrt{2}x = 2x^2 + 6$

$\Rightarrow 6\sqrt{2}x + 3 = 0$, which is not a quadratic equation

4. If $x = 3$ is a solution of the equation $3x^2 + (k-1)x + 9 = 0$, then $k = ?$

(a) 11 (b) –11 (c) 13 (d) –13

Answer: (b) –11

Sol:

It is given that $x = 3$ is a solution of $3x^2 + (k-1)x + 9 = 0$; therefore, we have:

$3(3)^2 + (k-1)\times 3 + 9 = 0$

$\Rightarrow 27 + 3(k-1) + 9 = 0$

$\Rightarrow 3(k-1) = -36$

$\Rightarrow (k-1) = -12$

$\Rightarrow k = -11$

5. If one root of the equation $2x^2 + ax + 6 = 0$ is 2 then a=?

(a) 7 (b) -7 (c) $\frac{7}{2}$ (d) $\frac{-7}{2}$

Answer: (b) -7

Sol:

It is given that one root of the equation $2x^2 + ax + 6 = 0$ is 2.

$\therefore 2\times 2^2 + a\times 2 + 6 = 0$

$\Rightarrow 2a + 14 = 0$

$\Rightarrow a = -7$

6. The sum of the roots of the equation $x^2 - 6x + 2 = 0$ is

(a) 2 (b) -2 (c) 6 (d) -6

Answer: (b) -2

Sol:

Sum of the roots of the equation $x^2 - 6x + 2 = 0$ is

$\alpha + \beta = \frac{-b}{a} = \frac{-(-6)}{1} = 6$, where α and β are the roots of the equation.

7. If the product of the roots of the equation $x^2 - 3x + k = 10$ is -2 then the value of k is

(a) -2 (b) -8 (c) 8 (d) 12

Answer: (c) 8

Sol:

It is given that the product of the roots of the equation $x^2 - 3x + k = 10$ is -2.

The equation can be rewritten as:

$x^2 - 3x + (k-10) = 0$

Product of the roots of a quadratic equation $= \frac{c}{a}$

$\Rightarrow \frac{c}{a} = -2$

$\Rightarrow \frac{(k-10)}{1} = -2$

$\Rightarrow k = 8$

8. The ratio of the sum and product of the roots of the equation $7x^2 - 12x + 18 = 0$ is

(a) $7:12$ (b) $7:18$ (c) $3:2$ (d) $2:3$

Answer : (d) $2:3$

Sol:

Given:

$7x^2 - 12x + 18 = 0$

$\therefore \alpha + \beta = \frac{12}{7}$ *and* $\beta = \frac{18}{7}$, where α and β are the roots of the equation

$\therefore$ Ratio of the sum and product of the roots $= \frac{12}{7} : \frac{18}{7}$

$= 12:18$

$= 2:3$

9. If one root of the equation $3x^2 - 10x + 3 = 0$ is $\frac{1}{3}$ then the other root is

(a) $\frac{-1}{3}$ (b) $\frac{1}{3}$ (c) -3 (d)3

Answer: (d)3

Sol:

Given:

$3x^2 - 10x + 3 = 0$

One root of the equation is $\frac{1}{3}$.

Let the other root be α.

Product of the roots $= \frac{c}{a}$

$\Rightarrow \frac{1}{3} \times \alpha = \frac{3}{3}$

$\Rightarrow \alpha = 3$

10. If one root of $5x^2 + 13x + k = 0$ be the reciprocal of the other root then the value of k is

(a) 0 (b) 1 (c)2(d)5
Answer: (d)5
Sol:
Let the roots of the equation $\frac{-2}{3}$be α and $\frac{1}{\alpha}$.

$\therefore$ Product of the roots $=\frac{c}{a}$

$\Rightarrow \alpha \times \frac{1}{\alpha}=\frac{k}{5}$

$\Rightarrow a=\frac{k}{5}$

$\Rightarrow k=5$

11. If the sum of the roots of the equation $kx^2+2x+3k=0$ is equal to their product then the value of k
(a) $\frac{1}{3}$ (b) $\frac{-1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$

Answer: (d) $\frac{-2}{3}$

Sol:
Given:
$kx^2+2x+3k=0$
Sum of the roots = Product of the roots

$\Rightarrow \frac{-2}{k}=\frac{3k}{k}$

$\Rightarrow 3k=-2$

$\Rightarrow k=\frac{-2}{3}$

12. The roots of a quadratic equation are 5 and -2. Then, the equation is
(a) $x^2-3x+10=0$(b) $x^2-3x-10=0$(c) $x^2+3x-10=0$(d) $x^2+3x+10=0$
Answer: (b) $x^2-3x-10=0$
Sol:
It is given that the roots of the quadratic equation are 5 and -2.
Then, the equation is:
$x^2-(5-2)x+5\times(-2)=0$
$\Rightarrow x^2-3x-10=0$

13. If the sum of the roots of a quadratic equation is 6 and their product is 6, the equation is

(a) $x^2-6x+6=0$ (b) $x^2+6x+6=0$ (c) $x^2-6x-6=0$ (d) $x^2+6x+6=0$

Answer: (a) $x^2-6x+6=0$

Sol:

Given:

Sum of roots = 6

Product of roots = 6

Thus, the equation is:

$x^2-6x+6=0$

14. If α *and* β are the roots of the equation $3x^2+8x+2=0$ then $\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)=?$

(a) $\frac{-3}{8}$ (b) $\frac{2}{3}$ (c) -4 (d) 4

Answer: (c) -4

Sol:

It is given that α and β are the roots of the equation $3x^2+8x+2=0$

$\therefore \alpha+\beta=-\frac{8}{3}$ *and* $\alpha\beta=\frac{2}{3}$

$$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{-\frac{8}{3}}{\frac{2}{3}}=-4$$

15. The roots of the equation $ax^2+bx+c=0$ will be reciprocal each other if

(a) a = b (b) b = c (c) c = a (d) none of these

Answer: (c) c = a

Sol:

Let the roots of the equation $\left(ax^2+bx+c=0\right)$ be α and $\frac{1}{\alpha}$.

$\therefore$ Product of the roots $=\alpha\times\frac{1}{\alpha}=1$

$\Rightarrow \frac{c}{a}=1$

$\Rightarrow c=a$

16. If the roots of the equation $ax^2+bx+c=0$ are equal then c=?

(a) $\frac{-b}{2a}$ (b) $\frac{b}{2a}$ (c) $\frac{-b^2}{4a}$ (d) $\frac{b^2}{4a}$

Answer: (d) $\frac{b^2}{4a}$

Sol:

It is given that the roots of the equation $\left(ax^2+bx+c=0\right)$ are equal.

$\therefore\left(b^2-4ac\right)=0$

$\Rightarrow b^2=4ac$

$\Rightarrow c=\frac{b^2}{4a}$

17. If the equation $9x^2+6kx+4=0$ has equal roots then k=?

(a) 0 or 0 (b) -2 or 0 (c) 2 or -2 (d) 0 only

Answer: (c) 2 or -2

Sol:

It is given that the roots of the equation $\left(9x^2+6kx+4=0\right)$ are equal.

$\therefore\left(b^2-4ac\right)=0$

$\Rightarrow\left(6k\right)^2-4\times9\times4=0$

$\Rightarrow 36k^2=144$

$\Rightarrow k^2=4$

$\Rightarrow k=\pm2$

18. If the equation $x^2+2\left(k+2\right)x+9k=0$ has equal roots then k = ?

(a) 1 or 4 (b)-1 or 4 (c) 1 or -4 (d) -1 or –4

Answer: (a) 1 or 4

Sol:

It is given that the roots of the equation $\left(x^2+2\left(k+2\right)x+9k=0\right)$ are equal.

$\therefore\left(b^2-4ac\right)=0$

$\Rightarrow\left\{2\left(k+2\right)\right\}^2-4\times1\times9k=0$

$\Rightarrow 4\left(k^2+4k+4\right)-36k=0$

$\Rightarrow 4k^2+16k+16-36k=0$

$\Rightarrow 4k^2-20+16=0$

$\Rightarrow k^2 - 5k + 4 = 0$

$\Rightarrow k^2 - 4k - k + 4 = 0$

$\Rightarrow k(k-4) - (k-4) = 0$

$\Rightarrow (k-4)(k-1) = 0$

$\Rightarrow k = 4 \text{ or } k = 1$

19. If the equation $4x^2 - 3kx + 1 = 0$ has equal roots then value of k=?

(a) $\pm\frac{2}{3}$ (b) $\pm\frac{1}{3}$ (c) $\pm\frac{3}{4}$ (d) $\pm\frac{4}{3}$

Answer: (d) $\pm\frac{4}{3}$

Sol:

It is given that the roots of the equation $(4x^2 - 3kx + 1 = 0)$ are equal.

$\therefore (b^2 - 4ac) = 0$

$\Rightarrow (3k)^2 - 4 \times 4 \times 1 = 0$

$\Rightarrow 9k^2 = 16$

$\Rightarrow k^2 = \frac{16}{9}$

$\Rightarrow k = \pm\frac{4}{3}$

20. The roots of $ax^2 + bx + c = 0, a \neq 0$ are real and unequal, if $(b^2 - 4ac)$ is

(a) > 0 (b) = 0 (c) < 0 (d) none of these

Answer: (a) > 0

Sol:

The roots of the equation are real and unequal when $(b^2 - 4ac) > 0$.

21. In the equation $ax^2 + bx + c = 0$, it is given that $D = (b^2 - 4ac) > 0$. Then, the roots of the equation are

(a) real and equal (b) real and unequal (c) imaginary (d) none of these

Answer: (b) real and unequal

Sol:

We know that when discriminant, $D > 0$, the roots of the given quadratic equation are real and unequal.

22. The roots of the equation $2x^2 - 6x + 7 = 0$ are

(a) real, unequal and rational (b) real, unequal and irrational (c) real and equal (d) imaginary

Answer: (d) imaginary

Sol:

$\because D = (b^2 - 4ac)$

$= (-6)^2 - 4 \times 2 \times 7$

$= 36 - 56$

$= -20 < 0$

Thus, the roots of the equation are imaginary

23. The roots of the equation $2x^2 - 6x + 3 = 0$ are

(a) real, unequal and rational (b) real, unequal and irrational (c) real and equal (d) imaginary

Answer: (b) real, unequal and irrational

Sol:

$\because D = (b^2 - 4ac)$

$= (-6)^2 - 4 \times 2 \times 3$

$= 36 - 24$

$= 12$

12 is greater than

0 and it is not a perfect square; therefore, the roots of the equation are real, unequal and irrational.

24. If the roots of $5x^2 - k + 1 = 0$ are real and distinct then

(a) $-2\sqrt{5} < k < 2\sqrt{5}$ (b) $k > 2\sqrt{5}$ only (c) $k < -2\sqrt{5}$ (d) either $k > 2\sqrt{5}$ or $k < -2\sqrt{5}$

Answer: (d) either $k > 2\sqrt{5}$ *or* $k < -2\sqrt{5}$

Sol:

It is given that the roots of the equation $(5x^2 - k + 1 = 0)$ are real and distinct.

$\therefore (b^2 - 4ac) > 0$

$\Rightarrow (-k)^2 - 4 \times 5 \times 1 > 0$

$\Rightarrow k^2 - 20 > 0$

$\Rightarrow k^2 > 20$

$\Rightarrow k > \sqrt{20} \text{ or } k < -\sqrt{20}$

$\Rightarrow k > 2\sqrt{5} \text{ or } k < -2\sqrt{5}$

25. If the equation $x^2 + 5kx + 16 = 0$ has no real roots then

(a) $k > \frac{8}{5}$ (b) $k < \frac{-8}{5}$ (c) $\frac{-8}{5} < k < \frac{8}{5}$ (d) None of these

Answer: (c) $\frac{-8}{5} < k < \frac{8}{5}$

Sol:

It is given that the equation $\left(x^2 + 5kx + 16 = 0\right)$ has no real roots.

$\therefore \left(b^2 - 4ac\right) < 0$

$\Rightarrow (5k)^2 - 4 \times 1 \times 16 < 0$

$\Rightarrow 25k^2 - 64 < 0$

$\Rightarrow k^2 < \frac{64}{25}$

$\Rightarrow \frac{-8}{5} < k < \frac{8}{5}$

26. If the equation $x^2 - kx + 1 = 0$ has no real roots then

(a) $k < -2$ (b) k > 2 (c) -2 < k < 2 (d) None of these

Answer: c) -2 < k < 2

Sol:

It is given that the equation $x^2 - kx + 1 = 0$ has no real roots.

$\therefore \left(b^2 - 4ac\right) < 0$

$\Rightarrow (-k)^2 - 4 \times 1 \times 1 < 0$

$\Rightarrow k^2 < 4$

$\Rightarrow -2 < k < 2$

27. For what value of k, the equation $kx^2 - 6x - 2 = 0$ has real roots?

(a) $k \le \frac{-9}{2}$ (b) $k \ge \frac{-9}{2}$ (c) $k \le -2$ (d)None of these

Answer: (b) $k \ge \frac{-9}{2}$

Sol:

It is given that the roots of the equation $\left(kx^2 - 6x - 2 = 0\right)$ are real.

$\therefore D \ge 0$

$\Rightarrow \left(b^2 - 4ac\right) \ge 0$

$\Rightarrow (-6)^2 - 4 \times k \times (-2) \ge 0$

$\Rightarrow 36+8k \geq 0$

$\Rightarrow k \geq \frac{-36}{8}$

$\Rightarrow k \geq \frac{-9}{2}$

28. The sum of a number and its reciprocal is $2\frac{1}{20}$. The number is

(a) $\frac{5}{4}$ *or* $\frac{4}{5}$ (b) $\frac{4}{3}$ *or* $\frac{3}{4}$ (c) $\frac{5}{6}$ *or* $\frac{6}{5}$ (d) $\frac{1}{6}$ *or* 6

Answer: (a) $\frac{5}{4}$ *or* $\frac{4}{5}$

Sol:

Let the required number be *x*.

According to the question:

$x+\frac{1}{x}=\frac{41}{20}$

$\Rightarrow \frac{x^2+1}{x}=\frac{41}{20}$

$\Rightarrow 20x^2-41x+20=0$

$\Rightarrow 20x^2-25x-16x+20=0$

$\Rightarrow 5x(4x-5)-4(4x-5)=0$

$\Rightarrow (4x-5)(5x-4)=0$

$\Rightarrow x=\frac{5}{4}$ *or* $x=\frac{4}{5}$

29. The perimeter of a rectangle is 82m and its area is $400m^2$. The breadth of the rectangle is

(a) 25 m (b) 20 m (c) 16 m (d) 9m

Answer: (c) 16 m

Sol:

Let the length and breadth of the rectangle be *l* and *b*.

Perimeter of the rectangle $=82m$

$\Rightarrow 2\times(l+b)=82$

$\Rightarrow l+b=41$

$\Rightarrow l=(41-b)$ ....(i)

Area of the rectangle $=400\ m^2$

$\Rightarrow l\times b=400m^2$

$\Rightarrow (41-b)b = 400$ (using (i))

$\Rightarrow 41b - b^2 = 400$

$\Rightarrow b^2 - 41b + 400 = 0$

$\Rightarrow b^2 - 25b - 16b + 400 = 0$

$\Rightarrow b(b-25) - 16(b-25) = 0$

$\Rightarrow (b-25)(b-16) = 0$

$\Rightarrow b = 25 \text{ or } b = 16$

If $b = 25$, we have:

$l = 41 - 25 = 16$

Since, l cannot be less than b,

$\therefore b = 16m$

30. The length of a rectangular field exceeds its breadth by 8 m and the area of the field is $240m^2$. The breadth of the field is

(a) 20 m (b) 30 m (c) 12 m (d) 16 m

Sol:

Let the breadth of the rectangular field be x m.

$\therefore$ Length of the rectangular field $= (x+8)m$

Area of the rectangular field $= 240\,m^2$ (Given)

$\therefore (x+8) \times x = 240$ (Area=Length $\times$ Breadth)

$\Rightarrow x^2 + 8x - 240 = 0$

$\Rightarrow x^2 + 20x - 12x - 240 = 0$

$\Rightarrow x(x+20) - 12(x+20) = 0$

$\Rightarrow (x+20)(x-12) = 0$

$\Rightarrow x + 20 = 0 \text{ or } x - 12 = 0$

$\Rightarrow x = -20 \text{ or } x = 12$

$\therefore x = 12$ (Breadth cannot be negative)

Thus, the breadth of the field is 12 m

Hence, the correct answer is option C.

31. The roots of the quadratic equation $2x^2 - x - 6 = 0$. are

(a) $-2, \frac{3}{2}$ (b) $2, \frac{-3}{2}$ (c) $-2, \frac{-3}{2}$ (d) $2, \frac{3}{2}$

Answer: (b) $2, \frac{-3}{2}$

Sol:

The given quadratic equation is $2x^2 - x - 6 = 0$.

$2x^2 - x - 6 = 0$

$\Rightarrow 2x^2 - 4x + 3x - 6 = 0$

$\Rightarrow 2x(x-2) + 3(x-2) = 0$

$\Rightarrow (x-2)(2x+3) = 0$

$\Rightarrow x - 2 = 0 \ or \ 2x + 3 = 0$

$\Rightarrow x = 2 \ or \ x = \frac{-3}{2}$

Thus, the roots of the given equation are 2 and $\frac{-3}{2}$

Hence, the correct answer is option B.

32. The sum of two natural numbers is 8 and their product is 15., Find the numbers.

Sol:

Let the required natural numbers be x and $(8-x)$.

It is given that the product of the two numbers is 15.

$\therefore x(8-x) = 15$

$\Rightarrow 8x - x^2 = 15$

$\Rightarrow x^2 - 8x + 15 = 0$

$\Rightarrow x^2 - 5x - 3x + 15 = 0$

$\Rightarrow x(x-5) - 3(x-5) = 0$

$\Rightarrow (x-5)(x-3) = 0$

$\Rightarrow x - 5 = 0 \ or \ x - 3 = 0$

$\Rightarrow x = 5 \ or \ x = 3$

Hence, the required numbers are 3 and 5.

33. Show the x= -3 is a solution of $x^2 + 6x + 9 = 0$

Sol:

The given equation is $x^2 + 6x + 9 = 0$

Putting $x = -3$ in the given equation, we get

$LHS = (-3)^2 + 6 \times (-3) + 9 = 9 - 18 + 9 = 0 = RHS$

$\therefore x = -3$ is a solution of the given equation.

34. Show that x= -2 is a solution of $3x^2+13x+14=0.$

Sol:

The given equation is $3x^2+13x+14=0.$

Putting $x=-2$ in the given equation, we get

$LHS-3\times(-2)^2+13\times(-2)+14=12-26+14=0=RHS$

$\therefore\ x=-2$ is a solution of the given equation.

35. If $x=\frac{-1}{2}$ is a solution of the quadratic equation $3x^2+2kx-3=0.$ Find the value of k.

Sol:

It is given that $x=\frac{-1}{2}$ is a solution of the quadratic equation $3x^2+2kx-3=0.$

$\therefore 3\times\left(\frac{-1}{2}\right)^2+2k\times\left(\frac{-1}{2}\right)-3=0$

$\Rightarrow \frac{3}{4}-k-3=0$

$\Rightarrow k=\frac{3-12}{4}=-\frac{9}{4}$

Hence, the value of k is $-\frac{9}{4}$.

36. Find the roots of the quadratic equation $2x^2-x-6=0.$

Sol:

The given quadratic equation is $2x^2-x-6=0.$

$2x^2-x-6=0$

$\Rightarrow 2x^2-4x+3x-6=0$

$\Rightarrow 2x(x-2)+3(x-2)=0$

$\Rightarrow (x-2)(2x+3)=0$

$\Rightarrow x-2=0\ or\ 2x+3=0$

$\Rightarrow x=2\ or\ x=-\frac{3}{2}$

Hence, the roots of the given equation are 2 and $-\frac{3}{2}$.

37. Find the solution of the quadratic equation $3\sqrt{3}x^2+10x+\sqrt{3}=0$.

Sol:

The given quadratic equation is $3\sqrt{3}x^2+10x+\sqrt{3}=0$

$3\sqrt{3}x^2+10x+\sqrt{3}=0$

$\Rightarrow 3\sqrt{3}x^2+9x+x+\sqrt{3}=0$

$\Rightarrow 3\sqrt{3}x\left(x+\sqrt{3}\right)+1\left(x+\sqrt{3}\right)=0$

$\Rightarrow \left(x+\sqrt{3}\right)\left(3\sqrt{3}x+1\right)=0$

$\Rightarrow x+\sqrt{3}=0 \ or \ 3\sqrt{3}x+1=0$

$\Rightarrow x=-\sqrt{3} \ or \ x=-\frac{1}{3\sqrt{3}}=-\frac{\sqrt{3}}{9}$

Hence, $-\sqrt{3}$ and $-\frac{\sqrt{3}}{9}$ are the solutions of the given equation.

38. If the roots of the quadratic equation $2x^2+8x+k=0$ are equal then find the value of k.

Sol:

It is given that the roots of the quadratic equation $2x^2+8x+k=0$ are equal.

$\therefore D=0$

$\Rightarrow 8^2-4\times 2\times k=0$

$\Rightarrow 64-8k=0$

$\Rightarrow k=8$

Hence, the value of k is 8.

39. If the quadratic equation $px^2-2\sqrt{5}px+15=0$ has two equal roots then find the value of p.

Sol:

It is given that the quadratic equation $px^2-2\sqrt{5}px+15=0$ has two equal roots.

$\therefore D=0$

$\Rightarrow \left(-2\sqrt{5}p\right)^2-4\times p\times 15=0$

$\Rightarrow 20p^2-60p=0$

$\Rightarrow 20p\left(p-3\right)=0$

$\Rightarrow p=0 \ or \ p-3=0$

$\Rightarrow p=0 \ or \ p=3$

For $p=0$, we get $15=0$, which is not true.

$\therefore p\neq 0$

Hence, the value of p is 3.

40. If 1 is a root of the equation $ay^2+ay+3=0.$ and $y^2+y+b=0.$ then find the value of ab.

Sol:

It is given that $y=1$ is a root of the equation $ay^2+ay+3=0.$

$\therefore a\times(1)^2+a\times1+3=0$

$\Rightarrow a+a+3=0$

$\Rightarrow 2a+3=0$

$\Rightarrow a=-\frac{3}{2}$

Also, y = 1 is a root of the equation $y^2+y+b=0.$

$\therefore (1)^2+1+b=0$

$\Rightarrow 1+1+b=0$

$\Rightarrow b+2=0$

$\Rightarrow b=-2$

$\therefore ab=\left(-\frac{3}{2}\right)\times(-2)=3$

Hence, the value of ab is 3.

41. If one zero of the polynomial x^2-4x+1 is $\left(2+\sqrt{3}\right)$, write the other zero.

Sol:

Let the other zero of the given polynomial be α.

Now,

Sum of the zeroes of the given polynomial $=\frac{-(-4)}{1}=4$

$\therefore \alpha+\left(2+\sqrt{3}\right)=4$

$\Rightarrow \alpha=4-2-\sqrt{3}=2-\sqrt{3}$

Hence, the other zero of the given polynomial is $\left(2-\sqrt{3}\right)$.

42. If one root of the quadratic equation $3x^2-10x+k=0.$ is reciprocal of the other , find the value of k.

Sol:

Let α and β be the roots of the equation $3x^2-10x+k=0.$

$\therefore \alpha = \frac{1}{\beta}$ (Given)

$\Rightarrow \alpha\beta = 1$

$\Rightarrow \frac{k}{3} = 1$ (Product of the roots $= \frac{c}{a}$)

$\Rightarrow k = 3$

Hence, the value of k is 3.

43. If the roots of the quadratic equation $px(x-2)+=0$ are equal, find the value of p.

Sol:

It is given that the roots of the quadratic equation $px^2 - 2px + 6 = 0$ are equal.

$\therefore D = 0$

$\Rightarrow (-2p)^2 - 4 \times p \times 6 = 0$

$\Rightarrow 4p^2 - 24p = 0$

$\Rightarrow 4p(p-6) = 0$

$\Rightarrow p = 0 \ or \ p - 6 = 0$

$\Rightarrow p = 0 \ or \ p = 6$

For $p = 0,$ we get $6 = 0,$ which is not true.

$\therefore p \neq 0$

Hence, the value of p is 6.

44. Find the value of k so that the quadratic equation $x^2 - 4kx + k = 0$ has equal roots.

Sol:

It is given that the quadratic equation $x^2 - 4kx + k = 0$ has equal roots.

$\therefore D = 0$

$\Rightarrow (-4k)^2 - 4 \times 1 \times k = 0$

$\Rightarrow 16k^2 - 4k = 0$

$\Rightarrow 4k(4k-1) = 0$

$\Rightarrow k = 0 \ or \ 4k - 1 = 0$

$\Rightarrow k = 0 \ or \ k = \frac{1}{4}$

Hence, 0 and $\frac{1}{4}$ are the required values of k.

45. Find the value of k for which the quadratic equation $9x^2 - 3kx + k = 0$ has equal roots.

Sol:

It is given that the quadratic equation $9x^2 - 3kx + k = 0$ has equal roots.

$\therefore D = 0$

$\Rightarrow (-3k)^2 - 4 \times 9 \times k = 0$

$\Rightarrow 9k^2 - 36k = 0$

$\Rightarrow 9k(k-4) = 0$

$\Rightarrow k = 0 \ or \ k - 4 = 0$

$\Rightarrow k = 0 \ or \ k = 4$

Hence, 0 and 4 are the required values of k.

46. Solve $x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$

Sol:

$x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$

$\Rightarrow x^2 - \sqrt{3}x - x + \sqrt{3} = 0$

$\Rightarrow x(x - \sqrt{3}) - 1(x - \sqrt{3}) = 0$

$\Rightarrow (x - \sqrt{3})(x - 1) = 0$

$\Rightarrow x - \sqrt{3} = 0 \ or \ x - 1 = 0$

$\Rightarrow x = \sqrt{3} \ or \ x = 1$

Hence, 1 and $\sqrt{3}$ are the roots of the given equation.

47. Solve $2x^2 + ax - a^2 = 0$

Sol:

$2x^2 + ax - a^2 = 0$

$\Rightarrow 2x^2 + 2ax - ax - a^2 = 0$

$\Rightarrow 2x(x+a) - a(x+a) = 0$

$\Rightarrow (x+a)(2x-a) = 0$

$\Rightarrow x + a = 0 \ or \ 2x - a = 0$

$\Rightarrow x = -a \ or \ x = \frac{a}{2}$

Hence, $-a$ and $\frac{a}{2}$ are the roots of the given equation.

48. Solve $3x^2+5\sqrt{5}x-10=0$

Sol:

$3x^2+5\sqrt{5}x-10=0$

$\Rightarrow 3x^2+6\sqrt{5}x-\sqrt{5}x-10=0$

$\Rightarrow 3x\left(x+2\sqrt{5}\right)-\sqrt{5}\left(x+2\sqrt{5}\right)=0$

$\Rightarrow \left(x+2\sqrt{5}\right)\left(3x-\sqrt{5}\right)=0$

$\Rightarrow x+2\sqrt{5}=0 \ or \ 3x-\sqrt{5}=0$

$\Rightarrow x=-2\sqrt{5} \ or \ x=\dfrac{\sqrt{5}}{3}$

Hence, $-2\sqrt{5}$ and $\dfrac{\sqrt{5}}{3}$ are the roots of the given equation.

49. Solve $\sqrt{3}x^2+10x-8\sqrt{3}=0$

Sol:

$\sqrt{3}x^2+10x-8\sqrt{3}=0$

$\Rightarrow \sqrt{3}x^2+12x-2x-8\sqrt{3}=0$

$\Rightarrow \sqrt{3}x\left(x+4\sqrt{3}\right)-2\left(x+4\sqrt{3}\right)=0$

$\Rightarrow \left(x+4\sqrt{3}\right)\left(\sqrt{3}x-2\right)=0$

$\Rightarrow x+4\sqrt{3}=0 \ or \ \sqrt{3}x-2=0$

$\Rightarrow x=-4\sqrt{3} \ or \ x=\dfrac{2}{\sqrt{3}}=\dfrac{2\sqrt{3}}{3}$

Hence, $-4\sqrt{3}$ and $\dfrac{2\sqrt{3}}{3}$ are the roots of the given equation.

50. Solve $\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0$

Sol:

$\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0$

$\Rightarrow \sqrt{3}x^2-3\sqrt{2}x+\sqrt{2}x-2\sqrt{3}=0$

$\Rightarrow \sqrt{3}x\left(x-\sqrt{6}\right)+\sqrt{2}\left(x-\sqrt{6}\right)=0$

$\Rightarrow \left(x-\sqrt{6}\right)\left(\sqrt{3}+\sqrt{2}\right)=0$

$\Rightarrow x-\sqrt{6}=0 \ or \ \sqrt{3}x+\sqrt{2}=0$

$\Rightarrow x = \sqrt{6} \ or \ x = -\frac{\sqrt{2}}{\sqrt{3}} = -\frac{\sqrt{6}}{3}$

Hence, $\sqrt{6}$ and $-\frac{\sqrt{6}}{3}$ are the roots of the given equation.

51. Solve $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

Sol:

$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

$\Rightarrow 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$

$\Rightarrow 4x\left(\sqrt{3}x + 2\right) - \sqrt{3}\left(\sqrt{3}x + 2\right) = 0$

$\Rightarrow \left(\sqrt{3}x + 2\right)\left(4x - \sqrt{3}\right) = 0$

$\Rightarrow \sqrt{3}x + 2 = 0 \ or \ 4x - \sqrt{3} = 0$

$\Rightarrow x = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3} \ or \ x = \frac{\sqrt{3}}{4}$

Hence, $-\frac{2\sqrt{3}}{3}$ and $\frac{\sqrt{3}}{4}$ are the roots of the given equation.

52. Solve $4x^2 + 4bx - \left(a^2 - b^2\right) = 0$

Sol:

$4x^2 + 4bx - \left(a^2 - b^2\right) = 0$

$\Rightarrow 4x^2 + 4bx - (a-b)(a+b) = 0$

$\Rightarrow 4x^2 + 2\left[(a+b) - (a-b)\right]x - (a-b)(a+b) = 0$

$\Rightarrow 4x^2 + 2(a+b)x - 2(a-b)x - (a-b)(a+b) = 0$

$\Rightarrow 2x\left[2x + (a+b)\right] - (a-b)\left[2x + (a+b)\right] = 0$

$\Rightarrow \left[2x + (a+b)\right]\left[2x - (a-b)\right] = 0$

$\Rightarrow 2x + (a+b) = 0 \ or \ 2x - (a-b) = 0$

$\Rightarrow x = -\frac{a+b}{2} \ or \ x = \frac{a-b}{2}$

Hence, $-\frac{a+b}{2}$ and $\frac{a-b}{2}$ are the roots of the given equation.

53. Solve $x^2+5x-\left(a^2+a-6\right)=0$

Sol:

$x^2+5x-\left(a^2+a-6\right)=0$

$\Rightarrow x^2+5x-(a+3)(a-2)=0$

$\Rightarrow x^2+\left[(a+3)-(a-2)\right]x-(a+3)(a-2)=0$

$\Rightarrow x^2+(a+3)x-(a-2)x-(a+3)(a-2)=0$

$\Rightarrow x\left[x+(a+3)\right]-(a-2)\left[x+(a+3)\right]=0$

$\Rightarrow \left[x+(a+3)\right]\left[x-(a-2)\right]=0$

$\Rightarrow x+(a+3)=0 \text{ or } x-(a-2)=0$

$\Rightarrow x=-(a+3) \text{ or } x=(a-2)$

Hence, $-(a+3)$ and $(a-2)$ are the roots of the given equation.

54. Solve $x^2+6x-\left(a^2+2a-8\right)=0$

Sol:

$x^2+6x-\left(a^2+2a-8\right)=0$

$\Rightarrow x^2+6x-(a+4)(a-2)=0$

$\Rightarrow x^2+\left[(a+4)-(a-2)\right]x-(a+4)(a-2)=0$

$\Rightarrow x^2+(a+4)x-(a-2)x-(a+4)(a-2)=0$

$\Rightarrow x\left[x+(a+4)\right]-(a-2)\left[x+(a+4)\right]=0$

$\Rightarrow \left[x+(a+4)\right]\left[x-(a-2)\right]=0$

$\Rightarrow x+(a+4)=0 \text{ or } x-(a-2)=0$

$\Rightarrow x=-(a+4) \text{ or } x=(a-2)$

Hence, $-(a+4)$ and $(a-2)$ are the roots of the given equation.

55. Solve $x^2-4ax+4a^2-b^2=0$

Sol:

$x^2-4ax+4a^2-b^2=0$

$\Rightarrow x^2-4ax+(2a+b)(2a-b)=0$

$\Rightarrow x^2-\left[(2a+b)+(2a-b)\right]x+(2a+b)(2a-b)=0$

$\Rightarrow x^2-(2a+b)x-(2a-b)x+(2a+b)(2a-b)=0$

$\Rightarrow x\left[x-(2a+b)\right]-(2a-b)\left[x-(2a+b)\right]=0$

$\Rightarrow \left[x-(2a+b)\right]\left[x-(2a-b)\right]=0$

$\Rightarrow x-(2a+b)=0 \ or \ x-(2a-b)=0$

$\Rightarrow x=(2a+b) \ or \ x=(2a-b)$

Hence, $(2a+b)$ and $(2a-b)$ are the roots of the given equation.