Ex 13.1

Derivatives as a Rate Measurer Ex 13.1 Q1

Let total suface area of the cylinder be A

$$A = 2\pi r(h+r)$$

Differentiating it with respect to r as r varies

$$\frac{dA}{dr} = 2\pi r(0+1) + (h+r)2\pi$$

$$= 2\pi r + 2\pi h + 2\pi r$$

$$\frac{dA}{dr} = 4\pi r + 2\pi h$$

Derivatives as a Rate Measurer Ex 13.1 Q2

Let D be the diatmeter and r be the radius of sphere,

So, volume of sphere $= \frac{4}{3}\pi r^2$

$$v = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3$$

$$v = \frac{4}{24}\pi D^3$$

Differentiating it with respect to D.

$$\frac{dv}{dD} = \frac{12}{24}\pi D^2$$

$$\frac{dv}{dD} = \frac{\pi D^2}{2}$$

Derivatives as a Rate Measurer Ex 13.1 Q3

Given, radius of sphere $(r) = 2\text{cm}$.

We know that,

$$v = \frac{4}{3}\pi r^2$$

$$\frac{dv}{dr} = 4\pi r^2 \qquad \text{---(i)}$$

And $A = 4\pi r^2$

$$\frac{dA}{dr} = 8\pi r^2 \qquad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{dv}{dr}}{\frac{dA}{dr}} = \frac{4\pi r^2}{8\pi r}$$

$$\frac{dv}{dA} = \frac{r}{2}$$

$$\left(\frac{dv}{dA}\right)_{r=2} = 1$$

Derivatives as a Rate Measurer Ex 13.1 Q4

Let r be two radius of circular disc.

We know that,

Area $A = \pi r^2$

$$\frac{dA}{dr} = 2\pi r \qquad \text{---(i)}$$

Circumference $C = 2\pi r$

$$\frac{dc}{dr} = 2\pi \qquad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{dA}{dr}}{\frac{dc}{dr}} = \frac{2\pi r}{2\pi}$$

$$\frac{dA}{dc} = r$$

$$\left(\frac{dA}{dc}\right)_{r=3} = 3$$

Derivatives as a Rate Measurer Ex 13.1 Q5

Let r be the radius, v be the volume of cone and h be height

$$v = \frac{1}{3}\pi r^2 h$$

$$\frac{dv}{dr} = \frac{2}{3}\pi rh.$$

Derivatives as a Rate Measurer Ex 13.1 Q6

Let r be radius and A be area of circle, so

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

$$\left(\frac{dA}{dr}\right)_{r=5} = 2\pi(5)$$

$$\left(\frac{dA}{dr}\right)_{r=5} = 10\pi$$

Derivatives as a Rate Measurer Ex 13.1 Q7

Here, $r = 2$ cm

$$v = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\left(\frac{dV}{dr}\right)_{r=2} = 4\pi(2)^2$$

$$\left(\frac{dV}{dr}\right)_{r=2} = 16\pi$$

Derivatives as a Rate Measurer Ex 13.1 Q8

Marginal cost is the rate of change of total cost with respect to output.

$$\therefore \text{Marginal cost (MC)} = \frac{dC}{dx} = 0.007(3x^2) - 0.003(2x) + 15$$

$$= 0.021x^2 - 0.006x + 15$$

When $x = 17$, MC $= 0.021\ (17^2) - 0.006\ (17) + 15$

$= 0.021(289) - 0.006(17) + 15$

$= 6.069 - 0.102 + 15$

$= 20.967$

Hence, when 17 units are produced, the marginal cost is Rs. 20.967

Derivatives as a Rate Measurer Ex 13.1 Q9

Marginal revenue is the rate of change of total revenue with respect to the number of units sold.

$$\therefore \text{Marginal Revenue (MR)} = \frac{dR}{dx} = 13(2x) + 26 = 26x + 26$$

When $x = 7$,

MR $= 26(7) + 26 = 182 + 26 = 208$

Hence, the required marginal revenue is Rs 208.

Derivatives as a Rate Measurer Ex 13.1 Q10

$$R(x) = 3x^2 + 36x + 5$$

$$\frac{dR}{dx} = 6x + 36$$

$$\left.\frac{dR}{dx}\right|_{x=5} = 6 \times 5 + 36$$

$= 30 + 36$

$= 66$

This, as per the question, indicates the money to be spent
on the welfare of the employess, when the number of employees is 5.

Ex 13.2

Derivatives as a Rate Measurer Ex 13.2 Q1

Let x be the side of square.

Given, $\frac{dx}{dt} = 4$ cm/min, $x = 8$ cm

We know that

$$\text{Area } (A) = x^2$$

$$\frac{dA}{dt} = 2x\frac{dx}{dt}$$

$$\left(\frac{dA}{dt}\right)_{8\text{ cm}} = 2 \times (8)(4)$$

$$\frac{dA}{dt} = 64 \text{ cm}^2/\text{min}$$

Area increases at a rate of $64 \text{ cm}^2/\text{min}$.

Derivatives as a Rate Measurer Ex 13.2 Q2

Let edge of the cube is x cm.

$$\frac{dx}{dt} = 3 \text{ cm/sec}, x = 10 \text{ cm}$$

Let V be volume of cube,

$$V = x^3$$

$$\frac{dV}{dt} = 3x^2\frac{dx}{dt}$$

$$= 3(10)^2 \times (3)$$

$$= 900 \text{ cm}^3/\text{sec}$$

So,

Volume increases at a rate of $900 \text{ cm}^3/\text{sec}$.

Derivatives as a Rate Measurer Ex 13.2 Q3

Let x be the side of the square.

Here, $\frac{dx}{dt} = 0.2$ cm/sec.

$$P = 4x$$
$$\frac{dP}{dt} = 4\frac{dx}{dt}$$
$$= 4 \times (0.2)$$
$$\frac{dP}{dt} = 0.8 \text{ cm/sec}$$

So, perimeter increases at the rate of 0.8 cm /sec.

Derivatives as a Rate Measurer Ex 13.2 Q4

The circumference of a circle (C) with radius (r) is given by

$C = 2\pi r$.

Therefore, the rate of change of circumference (C) with respect to time (t) is given by,

$$\frac{dC}{dt} = \frac{dC}{dr} \cdot \frac{dr}{dt} \text{ (By chain rule)}$$

$$= \frac{d}{dr}(2\pi r)\frac{dr}{dt}$$

$$= 2\pi \cdot \frac{dr}{dt}$$

It is given that $\frac{dr}{dt} = 0.7$ cm/s.

Hence, the rate of increase of the circumference is $2\pi(0.7) = 1.4\pi$ cm/s.

Derivatives as a Rate Measurer Ex 13.2 Q5

Let r be the radius of the spherical soap bubble.

Here, $\frac{dr}{dt} = 0.2$ cm/sec, $r = 7$ cm

Surface Area $(A) = 4\pi r^2$

$$\frac{dA}{dt} = 4\pi (2r)\frac{dr}{dt}$$
$$\left(\frac{dA}{dt}\right)_{r=7} = 4\pi (2 \times 7) \times 0.$$
$$= 11.2\pi \text{ cm}^2\text{/sec.}$$

So, area of bubble increases at the rate of 11.2π cm^2/sec.

Derivatives as a Rate Measurer Ex 13.2 Q6

The volume of a sphere (V) with radius (r) is given by,

$$V = \frac{4}{3}\pi r^3$$

$\therefore$Rate of change of volume (V) with respect to time (t) is given by,

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt} \text{ [By chain rule]}$$

$$= \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right) \cdot \frac{dr}{dt}$$

$$= 4\pi r^2 \cdot \frac{dr}{dt}$$

It is given that $\frac{dV}{dt} = 900 \text{ cm}^3/\text{s}$.

$$\therefore 900 = 4\pi r^2 \cdot \frac{dr}{dt}$$
$$\Rightarrow \frac{dr}{dt} = \frac{900}{4\pi r^2} = \frac{225}{\pi r^2}$$

Therefore, when radius = 15 cm,

$$\frac{dr}{dt} = \frac{225}{\pi(15)^2} = \frac{1}{\pi}$$

Hence, the rate at which the radius of the balloon increases when the radius is 15 cm is $\frac{1}{\pi}$ cm/s.

Derivatives as a Rate Measurer Ex 13.2 Q7

Let r be the radius of the air bubble.

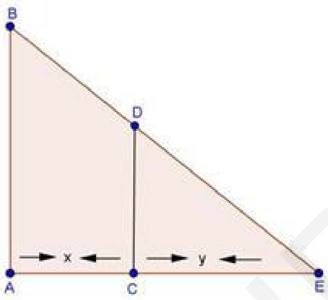
Here, $\frac{dr}{dt} = 0.5$ cm/sec, $r = 1$ cm

Volume $(V) = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = \frac{4}{3}\pi\left(3r^2\right)\frac{dr}{dt}$$
$$= 4\pi r^2 \frac{dr}{dt}$$
$$= 4\pi(1)^2 \times (0.5)$$
$$\frac{dV}{dt} = 2\pi \text{ cm}^3/\text{sec.}$$

So, volume of air bubble increases at the rate of 2π cm^3/sec.

Derivatives as a Rate Measurer Ex 13.2 Q8



Let AB be the lamp-post. Suppose at time t, the man CD is at a distance of x meters from the lamp-post and y meters be the length of his shadow CE.

Here, $\frac{dx}{dt} = 5$ km/hr

$CD = 2$ m, $AB = 6$ m

Here, ΔABE and ΔCDE are similar, so

$$\frac{AB}{CD} = \frac{AE}{CE}$$
$$\frac{6}{2} = \frac{x+y}{y}$$
$$3y = x + y$$
$$2y = x$$
$$2\frac{dy}{dt} = \frac{dx}{dt}$$
$$\frac{dy}{dt} = \frac{5}{2} \text{ km/hr}$$

So, the length of his shadow increases at the rate of $\frac{5}{2}$ km/hr.

Derivatives as a Rate Measurer Ex 13.2 Q9

The area of a circle (A) with radius (r) is given by $A = \pi r^2$.

Therefore, the rate of change of area (A) with respect to time (t) is given by,

$$\frac{dA}{dt} = \frac{d}{dt}\left(\pi r^2\right) = \frac{d}{dr}\left(\pi r^2\right)\frac{dr}{dt} = 2\pi r\frac{dr}{dt} \text{ [By chain rule]}$$

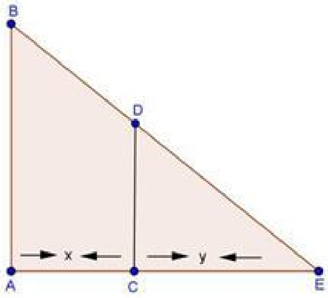
It is given that $\frac{dr}{dt} = 4$ cm/s.

Thus, when $r = 10$cm,

$$\frac{dA}{dt} = 2\pi(10)(4) = 80\pi$$

Hence, when the radius of the circular wave is 8 cm, the enclosed area is increasing at the rate of 80π cm^2/s

Derivatives as a Rate Measurer Ex 13.2 Q10



Let AB be the height of pole. Suppose at time t, the man CD is at a distance of x meters from the lamp-post and y meters be the length of his shadow CE, then

$$\frac{dx}{dt} = 1.1 \text{ m/sec}$$

ΔABE is similar to ΔCDE,

$$\frac{AB}{CD} = \frac{AE}{CE}$$

$$\frac{600}{160} = \frac{x+y}{y}$$

$$\frac{15}{4} = \frac{x+y}{y}$$

$$15y = 4x + 4y$$

$$11y = 4x$$

$$11\frac{dy}{dx} = 4\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{4}{11}(1.1)$$

$$\frac{dy}{dt} = 0.4 \text{ m/sec}$$

Rate of increasing of shadow = 0.4 m/sec.

Derivatives as a Rate Measurer Ex 13.2 Q11

Let AB be the height of source of light. Suppose at time t, the man CD is at a distance of x meters from the lamp-post and y meters be the length of his shadow CE, then

$$\frac{dx}{dt} = 2 \text{ m/sec}$$

ΔABE is similar to ΔCDE,

$$\frac{AB}{CD} = \frac{AE}{CE}$$

$$\frac{900}{180} = \frac{x+y}{y}$$

$$5y = x + y$$

$$4y = x$$

$$4\frac{dy}{dt} = \frac{dx}{dt}$$

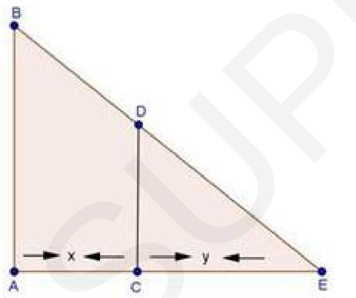
$$\frac{dy}{dt} = \frac{2}{4}$$

$$= \frac{1}{2}$$

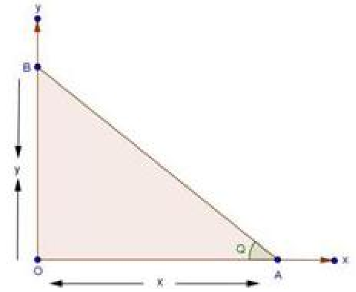
$$\frac{dy}{dt} = 0.5 \text{ m/sec}$$

So, rate of increase of shadow is 0.5 m/sec.

The diagram of the problem is shown below



Derivatives as a Rate Measurer Ex 13.2 Q12



Let AB be the position of the ladder, at time t, such that $OA = x$ and $OB = y$

Here,

$$OA^2 + OB^2 = AB^2$$
$$x^2 + y^2 = (13)^2$$
$$x^2 + y^2 = 169 \qquad \text{---(i)}$$

And $\dfrac{dx}{dt} = 1.5$ m/sec

From figure, $\tan\theta = \dfrac{y}{x}$

Differentiating equation (i) with respect to t,

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$
$$2(1.5)x + 2y\frac{dy}{dt} = 0$$
$$3x + 2y\frac{dy}{dt} = 0$$
$$\frac{dy}{dt} = -\frac{3x}{2y}$$

Differentiating equation (ii) with respect to t,

$$\sec^2\theta\frac{d\theta}{dt} = \frac{d\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}$$
$$= \frac{x \times \left(-\frac{3x}{2y}\right) - y(1.5)}{x^2}$$
$$= \frac{-1.5x^2 - 1.5y^2}{yx^2}$$
$$\frac{d\theta}{dt} = \frac{-1.5\left(x^2 + y^2\right)}{x^2 y \sec^2\theta}$$
$$= \frac{-1.5\left(x^2 + y^2\right)}{x^2 y\left(1 + \tan^2\theta\right)}$$
$$\frac{d\theta}{dt} = \frac{-1.5\left(x^2 + y^2\right)}{x^2 y\left(1 + \frac{y^2}{x^2}\right)}$$
$$= \frac{-1.5\left(x^2 + y^2\right) \times x^2}{x^2 y\left(x^2 + y^2\right)}$$
$$= \frac{-1.5}{y}$$
$$= \frac{-1.5}{\sqrt{169 - x^2}}$$
$$= \frac{-1.5}{\sqrt{169 - 144}}$$
$$= \frac{-1.5}{5}$$
$$= -0.3 \text{ radian/sec}$$

So, angle between ladder and ground is decreasing at the rate of 0.3 radian/sec.

Derivatives as a Rate Measurer Ex 13.2 Q13

Here, curve is

$$y = x^2 + 2x$$

And $\quad \frac{dy}{dt} = \frac{dx}{dt}$ ---(i)

$$y = x^2 + 2x$$

$$\Rightarrow \quad \frac{dy}{dt} = 2x\frac{dx}{dt} + 2\frac{dx}{dt}$$

$$\Rightarrow \quad \frac{dy}{dt} = \frac{dx}{dt}(2x + 2)$$

Using equation (i),

$$2x + 2 = 1$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

So, $\quad y = x^2 + 2x$

$$= \left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right)$$

$$= \frac{1}{4} - 1$$

$$y = -\frac{3}{4}$$

So, required points is $\left(-\frac{1}{2}, -\frac{3}{4}\right)$.

Derivatives as a Rate Measurer Ex 13.2 Q14

Here,

$$\frac{dx}{dt} = 4 \text{ units/sec, and } x = 2$$

And, $\quad y = 7x - x^3$

Slope of the curve $(S) = \frac{dy}{dx}$

$$S = 7 - 3x^2$$

$$\frac{ds}{dt} = -6x\frac{dx}{dt}$$

$$= -6(2)(4)$$

$$= -48 \text{ units/sec}$$

So, slope is decreasing at the rate of 48 units/sec.

Derivatives as a Rate Measurer Ex 13.2 Q15

Here,

$$\frac{dy}{dt} = 3\frac{dx}{dt} \quad \text{---(i)}$$

And, $\quad y = x^3$

$$\frac{dy}{dt} = 3x^2\frac{dx}{dt}$$

$$3\frac{dx}{dt} = 3x^2\frac{dx}{dt} \quad \text{[Using equation (i)]}$$

$$3x^2 = 3$$

$$x^2 = 1$$

$$x = \pm 1$$

Put $x = 1 \Rightarrow y = (1)^3 = 1$

Put $x = -1 \Rightarrow y = (-1)^3 = -13$

So, the required points are $(1,1)$ and $(-1,-1)$.

Derivatives as a Rate Measurer Ex 13.2 Q16(i)

Here,

$$2\frac{d(\sin\theta)}{dt} = \frac{d\theta}{dt}$$

$$2 \times \cos\theta \frac{d\theta}{dt} = \frac{d\theta}{dt}$$

$$2\cos\theta = 1$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}.$$

Derivatives as a Rate Measurer Ex 13.2 Q16(ii)

$$\frac{d\theta}{dt} = -2\frac{d}{dt}(\cos\theta)$$

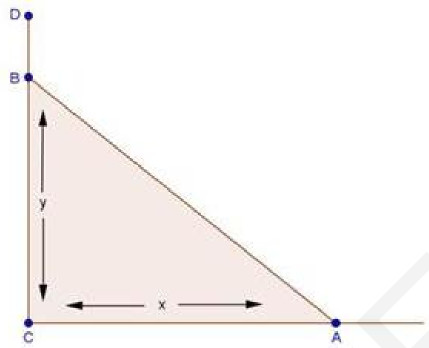
$$\frac{d\theta}{dt} = -2(-\sin\theta)\frac{d\theta}{dt}$$

$$1 = 2\sin\theta$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}$$

Derivatives as a Rate Measurer Ex 13.2 Q17



Let CD be the wall and AB is the ladder

Here, $AB = 6$ meter and $\left(\frac{dx}{dt}\right)_{x=4} = 0.5$ m/sec.

From figure,

$$AB^2 = x^2 + y^2$$

$$(6)^2 = x^2 + y^2$$

$$36 = x^2 + y^2$$

Differentiating it with respect to t,

$$0 = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt} \qquad ---(i)$$

$$\left(\frac{dy}{dt}\right)_{x=4} = \frac{4(0.5)}{\sqrt{36-x^2}}$$

$$= -\frac{2}{\sqrt{36-16}}$$

$$= -\frac{2}{2\sqrt{5}}$$

$$= -\frac{1}{\sqrt{5}} \text{ m/sec.}$$

So, ladder top is sliding at the rate of $\frac{1}{\sqrt{5}}$ m/sec.

Now, to find x when $\frac{dx}{dt} = -\frac{dy}{dt}$

From equation (i),

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$-\frac{dx}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$x = y$$

Now,

$$36 = x^2 + y^2$$

$$36 = x^2 + x^2$$

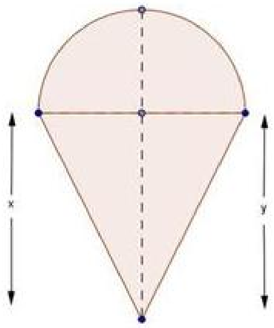
$$2x^2 = 36$$

$$x^2 = 18$$

$$x = 3\sqrt{2} \text{ m}$$

When foot and top are moving at the same rate, foot of wall is $3\sqrt{2}$ meters away from the wall

Derivatives as a Rate Measurer Ex 13.2 Q18



Let height of the cone is x cm. and radius of sphere is r cm.

Here given,

$$x = 2r \qquad \text{---(i)}$$

$$h = x + r$$

$$h = 2r + r$$

$$h = 3r \qquad \text{---(ii)}$$

$$v = \text{volume of cone} + \text{volume of hemisphere}$$

$$= \frac{1}{3}\pi r^2 x + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2 (2r) + \frac{2}{3}\pi r^3 \qquad \left[\text{Using equation (ii)}\right]$$

$$v = \frac{2}{3}\pi r^3 + \frac{2}{3}\pi r^3$$

$$= \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi\left(\frac{h}{3}\right)^3$$

$$v = \frac{4}{81}\pi h^3$$

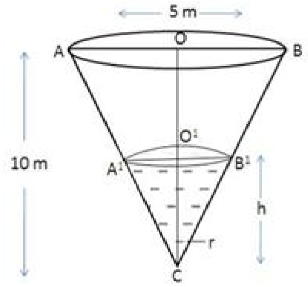
$$\frac{dv}{dh} = \frac{4}{81}\pi \times 3h^2$$

$$\left(\frac{dv}{dh}\right)_{h=9} = \frac{12}{81}\pi(9)^2$$

$$\left(\frac{dv}{dh}\right)_{h=9} = 12\pi \text{ cm}^2$$

Volume is changing at the rate 12π cm^2 with respect to total height.

Derivatives as a Rate Measurer Ex 13.2 Q19



Let α be the semi vertical angle of the cone CAB whose height CO is 10 m and radius $OB = 5$ m.

Now,

$$\tan\alpha = \frac{OB}{CO}$$
$$= \frac{5}{10}$$
$$\tan\alpha = \frac{1}{2}$$

Let V be the volume of the water in the cone, then

$$v = \frac{1}{3}\pi\left(O'B'\right)^2\left(CO'\right)$$
$$= \frac{1}{3}\pi\left(h\tan\alpha\right)^2(h)$$
$$v = \frac{1}{3}\pi h^3\tan^2\alpha$$
$$v = \frac{\pi}{12}h^2 \qquad \left[\because \tan\alpha = \frac{1}{2}\right]$$
$$\frac{dv}{dt} = \frac{\pi}{12}3h^2\frac{dh}{dt}$$
$$\pi = \frac{\pi}{4}h^2\frac{dh}{dt} \qquad \left[\because \frac{dV}{dt} = \text{m}^3/\text{min}\right]$$
$$\frac{dh}{dt} = \frac{4}{h^2}$$
$$\left(\frac{dh}{dt}\right)_{2.5} = \frac{4}{(2.5)^2} \qquad \left[\because h = 10 - 7.5 = 2.5 \text{ m}\right]$$
$$= \frac{4}{6.25}$$
$$= 0.64 \text{ m/min}$$

So, water level is rising at the rate of 0.64 m/min.

Derivatives as a Rate Measurer Ex 13.2 Q20

Let AB be the lamp-post. Suppose at time t, the man CD is at a distance x m. from the lamp-post and y m be the length of the shadow CE.

Here, $\frac{dx}{dt} = 6$ km/hr

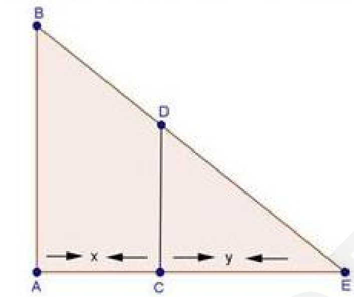
$CD = 2$ m, $AB = 6$ m

Here, ΔABE and ΔCDE are similar

So,
$$\frac{AB}{CD} = \frac{AE}{CE}$$
$$\frac{6}{2} = \frac{x+y}{y}$$
$$3y = x + y$$
$$2y = x$$
$$2\frac{dy}{dt} = \frac{dx}{dt}$$
$$2\frac{dy}{dt} = 6$$
$$\frac{dy}{dt} = 3 \text{ km/hr}$$

So, length of his shadow increases at the rate of 3 km/hr.

The diagram of the problem is shown below



Derivatives as a Rate Measurer Ex 13.2 Q21

Here, $\frac{dA}{dt} = 2 \text{ cm}^2\text{/sec}$

To find $\frac{dV}{dt}$ at $r = 6$ cm
$$A = 4\pi r^2$$
$$\frac{dA}{dt} = 8\pi r\frac{dr}{dt}$$
$$2 = 8\pi r\frac{dr}{dt}$$
$$\frac{dr}{dt} = \frac{1}{4\pi r} \text{ cm/sec}$$

Now,
$$V = \frac{4}{3}\pi r^3$$
$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$$
$$= 4\pi r^2\left(\frac{1}{4\pi r}\right)$$
$$= r$$
$$\frac{dV}{dt} = 6 \text{ cm}^3\text{/sec}$$

So, volume of bubble is increasing at the rate of 6 cm^3/sec.

Derivatives as a Rate Measurer Ex 13.2 Q22

Here, $\frac{dr}{dt} = 2$ cm/sec, $\frac{dh}{dt} = -3$ cm/sec

To find $\frac{dV}{dt}$ when $r = 3$ cm, $h = 5$ cm

Now, V = volume of cylinder

$$V = \pi r^2 h$$
$$\frac{dV}{dt} = \pi\left[2r\frac{dr}{dt}\times h + r^2\frac{dh}{dt}\right]$$
$$= \pi\left[2(3)(2)(5) + (3)^2(-3)^2\right]$$
$$= \pi[60 - 27]$$
$$\frac{dV}{dt} = 33\pi \text{ cm}^3/\text{sec}$$

So, volume of cylinder is increasing at the rate of 33π cm^3/sec.

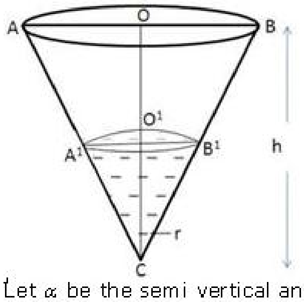
Derivatives as a Rate Measurer Ex 13.2 Q23

Let V be volume of sphere with miner radius r and onter radius R, then

$$V = \frac{4}{3}\pi\left(R^3 - r^3\right)$$
$$\frac{dV}{dt} = \frac{4}{3}\pi\left(3R^2\frac{dR}{dt} - 3r^2\frac{dr}{dt}\right)$$
$$0 = \frac{4\pi}{3}3\left(R^2\frac{dR}{dt} - r^2\frac{dr}{dt}\right) \qquad [\text{Since volume } V \text{ is constant}]$$
$$R^2\frac{dR}{dt} = r^2\frac{dr}{dt}$$
$$(8)^2\frac{dR}{dt} = (4)^2(1)$$
$$\frac{dR}{dt} = \frac{16}{64}$$
$$\frac{dR}{dt} = \frac{1}{4} \text{ cm/sec}$$

Rate of increasing of onter radius = $\frac{1}{4}$ cm/sec.

Derivatives as a Rate Measurer Ex 13.2 Q24



Let α be the semi vertical angle of the cone CAB whose height CO is half of radius OB.

Now,

$$\tan\alpha = \frac{OB}{CO}$$

$$= \frac{OB}{2OB} \qquad [\because CO = 2OB]$$

$$\tan\alpha = \frac{1}{2}$$

Let V be the volume of the sand in the cone

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$$

$$= \frac{\pi}{12}h^3$$

$$\frac{dV}{dt} = \frac{3\pi}{12}h^2\frac{dh}{dt}$$

$$50 = \frac{3\pi}{12}h^2\frac{dh}{dt} \qquad \left[\because \frac{dV}{dt} = 50\ \text{cm}^3/\text{min}\right]$$

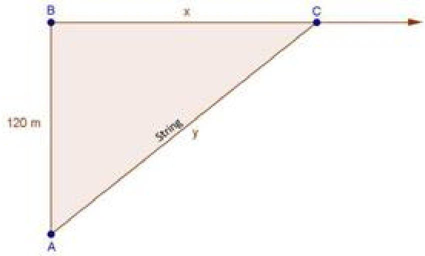
$$\frac{dh}{dt} = \frac{200}{\pi h^2}$$

$$= \frac{200}{\pi(5)^2}$$

$$\frac{dh}{dt} = \frac{8}{3.14}\ \text{cm/min}$$

Rate of increasing of height = $\frac{8}{\pi}$ cm/min

Derivatives as a Rate Measurer Ex 13.2 Q25



Let C be the position of kite and AC be the string.

Here, $y^2 = x^2 + (120)^2$ ---(i)

$$2y\frac{dy}{dt} = 2x\frac{dx}{dt}$$

$$y\frac{dy}{dt} = x\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{x}{y}(52) \quad \text{---(ii)} \quad \left[\because \frac{dx}{dt} = 52 \text{ m/sec}\right]$$

From equation (i),

$$y^2 = x^2 + (120)^2$$

$$(130)^2 = x^2 + (120)^2$$

$$x^2 = 16900 - 14400$$

$$x^2 = 2500$$

$$x = 50$$

Using equation (ii),

$$\frac{dy}{dt} = \frac{x}{y}(52)$$

$$= \frac{50}{130}(52)$$

$$= 20 \text{ m/sec}$$

So, string is being paid out at the rate of 20 m/sec.

Derivatives as a Rate Measurer Ex 13.2 Q26

Here,

$$\frac{dy}{dt} = 2\frac{dx}{dt} \quad \text{---(i)}$$

and $y = \left(\frac{2}{3}\right)x^3 + 1$

$$\frac{dy}{dt} = \frac{2}{3} \times 3x^2\frac{dx}{dt}$$

$$2\frac{dx}{dt} = 2x^2\frac{dx}{dt} \quad [\text{Using equation (i)}]$$

$$2 = 2x^2$$

$\Rightarrow$ $x = \pm 1$

$$y = \left(\frac{2}{3}\right)x^3 + 1$$

Put $x = 1$, $y = \frac{2}{3} + 1 = \frac{5}{3}$

Put $x = -1$, $y = \frac{2}{3}(-1) + 1 = \frac{1}{3}$

So, required point $\left(1, \frac{5}{3}\right)$ and $\left(-1, \frac{1}{3}\right)$.

Derivatives as a Rate Measurer Ex 13.2 Q27

Here,

$$\frac{dx}{dt} = \frac{dy}{dt} \qquad \text{---(i)}$$

and curve is

$$y^2 = 8x$$

$$2y\frac{dy}{dt} = 8\frac{dx}{dt}$$

$$2y = 8 \qquad \left[\text{using equation (i)}\right]$$

$$y = 4$$

$$\Rightarrow \quad (4)^2 = 8x$$

$$\Rightarrow \quad x = 2$$

So, required point $= (2, 4)$.

Derivatives as a Rate Measurer Ex 13.2 Q28

Let edge of cube be x cm

Here,

$$\frac{dV}{dt} = 9 \text{ cm}^3/\text{sec}$$

To find $\frac{dA}{dt}$ when $x = 10$ cm

We know that

$$V = x^3$$

$$\frac{dV}{dt} = 3x^2\left(\frac{dx}{dt}\right)$$

$$9 = 3(10)^2\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{3}{100} \text{ cm/sec}$$

Now, $A = 6x^2$

$$\frac{dA}{dt} = 12x\frac{dx}{dt}$$

$$= 12(10)\left(\frac{3}{100}\right)$$

$$\frac{dA}{dt} = 3.6 \text{ cm}^2/\text{sec.}$$

Derivatives as a Rate Measurer Ex 13.2 Q29

Given, $\frac{dV}{dt} = 25 \text{ cm}^3/\text{sec}$

To find $\frac{dA}{dt}$ when $r = 5$ cm

We know that,

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{3}\pi\left(3r^2\right)\frac{dr}{dt}$$

$$25 = 4\pi(5)^2\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4\pi} \text{ cm/sec}$$

Now, $A = 4\pi r^2$

$$\frac{dA}{dt} = 8\pi r\frac{dr}{dt}$$

$$= 8\pi(5)\left(\frac{1}{4\pi}\right)$$

$$\frac{dA}{dt} = 10 \text{ cm}^2/\text{sec.}$$

Derivatives as a Rate Measurer Ex 13.2 Q30

Given,

$$\frac{dx}{dt} = -5 \text{ cm/min}$$

$$\frac{dy}{dt} = 4 \text{ cm/min}$$

(i) To find $\frac{dP}{dt}$ when $x = 8$ cm, $y = 6$ cm

$$P = 2(x + y)$$

$$\frac{dP}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$$

$$= 2(-5 + 4)$$

$$\frac{dP}{dt} = -2 \text{ cm/min}$$

(ii) To find $\frac{dA}{dt}$ when $x = 8$ cm and $y = 6$ cm

$$A = xy$$

$$\frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$$

$$= (8)(4) + (6)(-5)$$

$$= 32 - 30$$

$$\frac{dA}{dt} = 2 \text{ cm}^2/\text{min}.$$

Derivatives as a Rate Measurer Ex 13.2 Q31

Let r be the radius of the given disc and A be its area.

Then, $A = \pi r^2$

$\therefore \quad \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ [by chain rule]

Now, the approximate increase of radius $= dr = \frac{dr}{dt}\Delta t = 0.05 \text{ cm/sec}$

$\therefore$ the approximate rate of increase in area is given by

$$dA = \frac{dA}{dt}(\Delta t) = 2\pi r\left(\frac{dr}{dt}\Delta t\right) = 2\pi(3.2)(0.05) = 0.320\pi \text{ cm}^3/\text{s}$$