Areas of Parallelogram and Triangles

Exercise 9.1

Write the correct answer in each of the following:

1. The median of a triangle divides it into two

(A) triangles of equal area
(B) congruent triangles
(C) right triangles
(D) isosceles triangles

Sol. The median of a triangle divides it into triangle of equal area.
Hence, (a) is the correct answer.

2. In which of the following figures (Fig. 9.3), you find two polygons on the same base and between the same parallels?

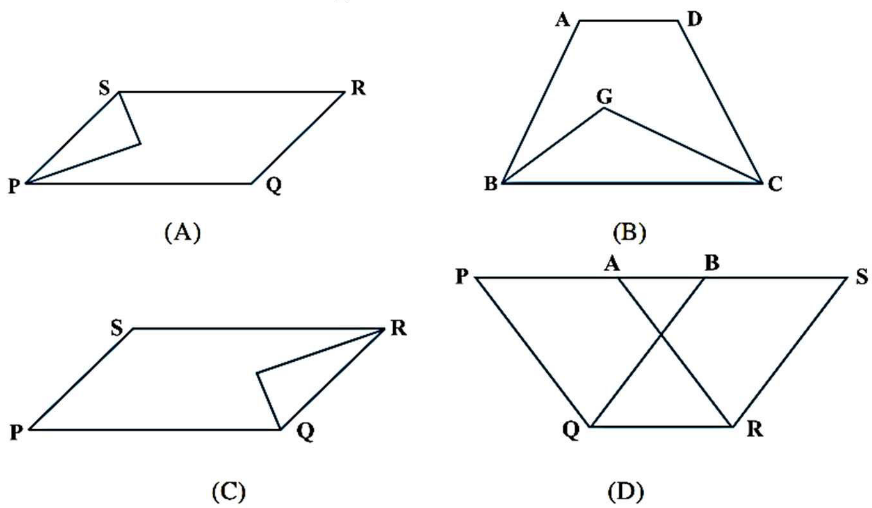
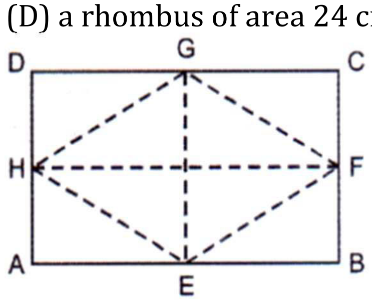


Fig. 9.3

Sol. In figure (d), we find two polygons (parallelogram) on the same base and between the same parallels.

3. The figure obtained by joining the mid-points of the adjacent sides of a rectangle of sides 8 cm and 6 cm is:

(A) a rectangle of area 24 cm^2
(B) a square of area 25 cm^2
(C) a trapezium of area 24 cm^2
(D) a rhombus of area 24 cm^2



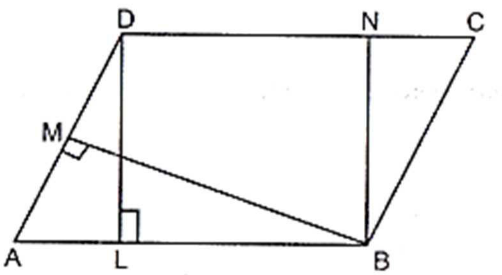
Sol. ABCD is a rectangle and E, F, G and H are the mid-points of the sides AB, BC, CD and DA respectively. The figure obtained is rhombus whose area

$$=\frac{1}{2}\times EG\times FH=\frac{1}{2}\times 6cm\times 8cm=24cm^2$$

Hence, (d) is the correct answer.

4. In Fig. 9.4, the area of parallelogram ABCD is:

(A) AB × BM
(B) BC × BN
(C) DC × DL
(D) AD × DL



Sol. Area of parallelogram = Base × Corresponding altitude
= AB × DL = DC × DL
[∵ AB = DC (opposite side of a ||gm]

Hence, (c) is the correct answer.

5. In Fig. 9.5, if parallelogram ABCD and rectangle ABEM are of equal area, then:

(A) Perimeter of ABCD = Perimeter of ABEM
(B) Perimeter of ABCD < Perimeter of ABEM
(C) Perimeter of ABCD > Perimeter of ABEM
(D) Perimeter of ABCD $=\frac{1}{2}$(Perimeter of ABEM)

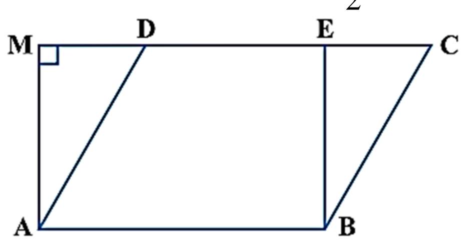


Fig. 9.5

Sol. If parallelogram ABCD and rectangle ABEM are of equal area, then perimeter of ABCD > Perimeter of ABEM because of all the line segments to a given line from a point outside it, the perpendicular is the least. Hence, (c) is the correct answer.

6. The mid-point of the sides of a triangle along with any of the vertices as the fourth point make a parallelogram of area equal to

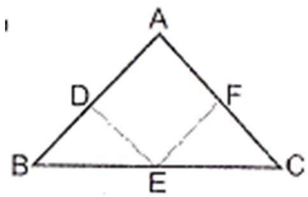
(a) $\frac{1}{2}ar(\Delta ABC)$

(b) $\frac{1}{3}ar(\Delta ABC)$

(c) $\frac{1}{4}ar(\Delta ABC)$

(d) $ar(\Delta ABC)$

Sol. Since medina of a triangle divides it into two triangles of equal area



$\therefore \quad ar(\Delta ADE) = ar(\Delta BDE) \quad \quad ...(1)$

$ar(\Delta AEF) = ar(\Delta EFC) \quad \quad ...(2)$

Since AE is the diagonal of a parallelogram ADEF. It divides it into two triangles of equal area.

$\therefore \quad ar(\Delta ADE) = ar(\Delta AFE) \quad \quad ...(3)$

From (1), (2) and (3), we get

$\therefore ar(\Delta ADE) = ar(\Delta BDE) = ar(\Delta AEE) = ar(\Delta EFC)$

Hence, $ar(\Delta ADEF) = \frac{1}{2}ar(\Delta ABC)$

So, (a) is the correct answer.

7. Two parallelograms are on equal bases and between the same parallels. The ratio of their areas is

(A) 1 : 2

(B) 1 : 1

(C) 2 : 1

(D) 3 : 1

Sol. We know that parallelogram on the same or equal bases and between the same parallels are equal in area.

So, the ratio of their area is 1 : 1.

Hence, (b) is the correct answer.

8. ABCD is a quadrilateral whose diagonal AC divides it in two parts, equal in area, then ABCD

(A) is a rectangle

(B) is always a rhombus

(C) is a parallelogram

(D) need not be any of (A), (B) or (C)

Sol. Since diagonal of a parallelogram divides it into two triangles of equal area and rectangle and a rhombus are also parallelograms. Then ABCD need not be any of (a), (b) or (c). Hence, (d) is the correct answer.

9. If a triangle and a parallelogram are on the same base and between same parallels, then the ratio of the area of the triangle to the area of parallelogram is

(A) 1: 3
(B) 1: 2
(C) 3: 1
(D) 1: 4

Sol. We know that a triangle and a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to half the area of the parallelogram. Hence the ratio of the area of the triangle to the area of parallelogram is 1 : 2.

Hence, (b) is the correct answer.

10. ABCD is a trapezium with parallel sides AB = a cm and DC = b cm (Fig. 9.6). E and F are the mid-points of the non-parallel sides. The ratio of ar (ABFE) and ar (EFCD) is

(A) a : b
(B) (3 a+ b) : (a + 3b)
(C) (a + 3b) : (3a + b)
(D) (2a + b) : (3a + b)

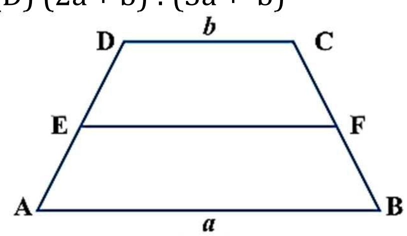


Fig. 9.6

Sol. ABCD is a trapezium in which AB||DC. E and F are the mid-points of AD and BC, so

$$EF = \frac{1}{2}(a+b)$$

ABEF and EFCD are also trapeziums.

$$\text{ar (ABEF)} = \frac{1}{2}\left[\frac{1}{2}(a+b)+a\right]\times h = \frac{h}{4}(3a+b)$$

$$\text{ar (EFCD)} = \frac{1}{2}\left[b+\frac{1}{2}(a+b)\right]\times h = \frac{h}{4}(a+3b)$$

$$\therefore \quad \frac{ar(ABEF)}{ar(EFCD)} = \frac{\frac{h}{4}(3a+b)}{\frac{h}{4}(a+3b)} = \frac{(3a+b)}{(a+3b)}$$

So, the required ratio is (3a + b) : (a + 3b).

Hence, (b) is the correct answer.

Areas of Parallelogram and Triangles

Exercise 9.2

Write True or False and justify your answer:

1. ABCD is a parallelogram and X is the mid-point of AB. If ar (AXCD) = 24 cm², then ar (Δ ABC) = 24cm².

Sol. We have ABCD is a parallelogram and X is the mid – point of AB.

Now, ar (ABCD) = ar (AXCD) + ar (Δ XBC) ...(1)

$\because$ Diagonal AC of a parallelogram divides it into two triangles of equal area.

$\therefore$ ar (ABCD) = 2ar (Δ ABC) ...(2)

Again, X is the mid-point of AB, So

$$ar(\Delta CXB) = \frac{1}{2} ar(\Delta ABC) \quad ...(3)$$

[$\because$ Median divides the triangle in two triangles of equal area]

$$\therefore \quad 2ar(\Delta ABC) = 24 + \frac{1}{2} ar(\Delta ABC) \quad \text{[Using (1), (2) and (3)]}$$

$$\therefore 2ar(\Delta ABC) - \frac{1}{2} ar(\Delta ABC) = 24$$

$$\Rightarrow \quad \frac{3}{2} ar(\Delta ABC) = 24$$

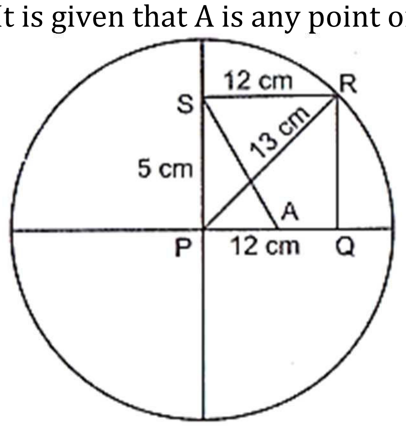
$$\Rightarrow \quad ar(\Delta ABC) = \frac{2 \times 24}{3} = 16 cm^2$$

Hence, the given statement is false.

2. PQRS is a rectangle inscribed in a quadrant of a circle of radius 13 cm. A is any point on PQ. If PS = 5 cm, then ar (PAS) = 30 cm².

Sol. It is given that A is any point on PQ, therefore, PA < PQ.

It is given that A is any point on PQ, therefore PA < PQ.



Now, $ar(\Delta PQR) = \frac{1}{2} \times base \times height$

Now, $ar(\Delta PQR) = \frac{1}{2} \times PQ \times QR = \frac{1}{2} \times 12 \times 5 = 30cm^2$

[$\because$ PQRS is a rectangle $\therefore$ RQ = SP = 5 cm]

As PA < PQ (= 12 cm)

So $ar(\Delta PAS) < ar(\Delta PQR)$

Or $ar(\Delta PAS) < 30cm^2$ $\left[ar(\Delta PQR) = 30cm^2\right]$

Hence, the given statement is false.

3. PQRS is a parallelogram whose area is 180cm² and A is any point on the diagonal QS. The area of Δ ASR = 90cm².

Sol. PQRS is a parallelogram.

We know that diagonal (QS) of a parallelogram divides parallelogram into two triangles of equal area, so

$$\therefore \quad ar(\Delta QRS) = \frac{1}{2} ar(\| gm PQRS)$$

$$= \frac{1}{2} \times 180 = 90cm^2$$

$\because$ A is any point on SQ

$$\therefore \quad ar(\Delta ASR) < ar(\Delta QRS)$$

Hence, $ar(\Delta ASR) < 90cm^2$

Hence, the given statement is false.

4. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Then $ar(\Delta BDE) = \frac{1}{4} ar(\Delta ABC)$.

Sol. ΔABC and ΔBDE are two equilateral triangles.

Let each sides of triangle ABC be x.

Again, D is the mid-point of BC, so each side of triangle BDE is $\frac{x}{2}$.

$$\text{Now,} \quad \frac{ar(\Delta BDE)}{ar(\Delta ABC)} = \frac{\frac{\sqrt{3}}{4} \times \left(\frac{x}{2}\right)^2}{\frac{\sqrt{3}}{4} \times x^2} = \frac{x^2}{4x^4} = \frac{1}{4}$$

$$\text{Hence,} \quad ar(\Delta BDE) = \frac{1}{4} ar(\Delta ABC)$$

$\therefore$ The given statement is true.

5. In the given figure, ABCD and EFGD are two parallelogram and G is the mid-point of CD. Then $ar(\Delta DPC) = \frac{1}{2} ar(\| gm EFGD)$.

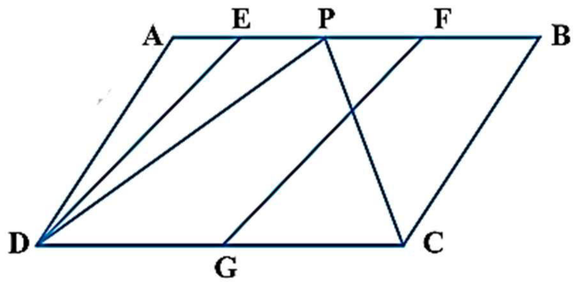


Fig. 9.8

Sol. As ΔDPC and ||gm ABCD are on the same base DC and between the same parallels AB and DC, So

$$ar(\Delta DPC) = \frac{1}{2} ar(\| \, gm.ABCD)$$

= ar(||gm EFGD) [∵ G is the point of DC]

Hence, the given statement is false.

Areas of Parallelogram and Triangles

Exercise 9.3

1. In Fig.9.11, PSDA is a parallelogram. Points Q and R are taken on PS such that PQ = QR = RS and PA||QB||RC. Prove that ar (PQE) = ar (CFD).

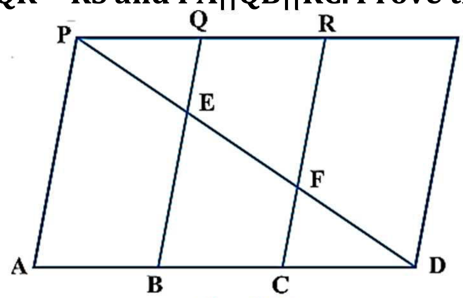


Fig. 9.11

Sol. PSDA is a parallelogram. Points Q and R are taken on Ps such that PQ = RS = RS and PA||QB||RC.

We have to prove that $ar(\Delta PQE) = ar(\Delta CFD)$.

Now, PS = AD [Opp. Sides of a ||gm]

$\therefore \quad \frac{1}{3}PS = \frac{1}{2}AD \Rightarrow PQ = CD$ ...(1)

Again, PS||AD and QB cut them,

$\therefore \quad \angle PQE = \angle CBE$ [Alt. $\angle s$] ...(2)

Now, QB||RS and AD cut them

$\therefore \quad \angle QBD = \angle RCD$ [Corres. $\angle s$] ...(3)

So, $\angle PQE = \angle FCD$ ...(4)

[From (2) and (3), $\angle CBE$ and $\angle QBD$ are same and $\angle RCD$ and $\angle FCD$ are same]

Now, in ΔPQE and ΔCFD

$\angle PQE = \angle CDF$ [Alt. $\angle s$]

PQ = CD [From (1)]

And $\angle PQE = \angle FCD$ [From (4)]

$\therefore \quad \Delta PQE \cong \Delta CFD$ [By ASA congruence rule]

Hence, $ar(\Delta PQE) = ar(\Delta CFD)$ [Congruence Δs are equal in area]

2. X and Y are points on the side LN of the triangle LMN such that LX = XY= YN. Through X, a line is drawn parallel to LM to meet MN at Z (See Fig. 9.12). Prove that ar (LZY) = ar (MZYX).

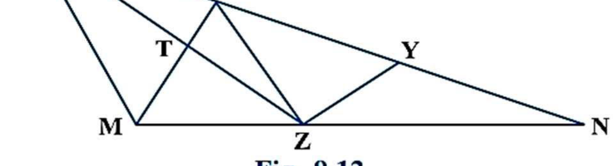


Fig. 9.12

Sol. We have to prove that $ar(\Delta LZY) = ar(MZYX)$

Since ΔLZY and ΔXMZ are on the same base and between the same parallels LM and XZ, we have

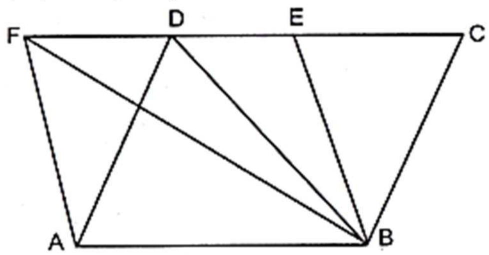
$$ar(\Delta LXZ) = ar(\Delta XMZ) \qquad ..(1)$$

Adding $ar(\Delta XYZ)$ to both sides of (1), we get

$$ar(\Delta LXZ) + ar(\Delta XYZ) = ar(\Delta XMZ) + ar(\Delta XYZ)$$

$\Rightarrow \quad ar(\Delta LZY) = ar(MZYX)$

3. The area of the parallelogram ABCD is 90 cm² (see fig). Find
(i) ar (ABEF)
(ii) ar (ABD)
(iii) ar (BEF)



Sol. (i) Since parallelograms on the same base and between the same parallels are equal in area, so we have

ar (||gm ABEF) = ar (||gm ABCD)

Hence, ar (||gm ABEF) = ar (||gm ABCD) = 90 cm²

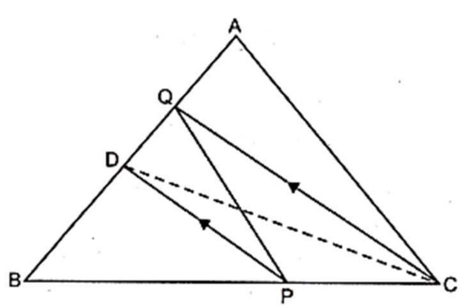
(ii) $ar(\Delta ABD) = \frac{1}{2} ar(\| gm ABCD)$

[$\because$ A diagonal of a parallelogram divides the parallelogram in two triangles of equal area]

$$= \frac{1}{2} \times 90 cm^2 = 45 cm^2$$

(iii) $ar(\Delta BEF) = \frac{1}{2} ar(\| gm ABEF) = \frac{1}{2} \times 90 cm^2 = 45 cm^2$

4. In Δ ABC, D is the mid-point of AB and P is any point on BC. If CQ || PD meets AB in Q (Fig. 9.14), then prove that $ar(\Delta BPQ) = \frac{1}{2} ar(\Delta ABC)$.



Sol. D is the mid-point of AB and P is any point on BC of Δ ABC. CQ||PD meets AB in Q, we have to prove that $ar(\Delta BPQ) = \frac{1}{2} ar(\Delta ABC)$.

Join CD. Since median of a triangle divides it into two triangles of equal area, so we have

$$ar(\Delta BCD) = \frac{1}{2} ar(\Delta ABC) \qquad ...(1)$$

Since, triangles on the same base and between the same parallels are equal in area, so we have

$$ar(\Delta DPQ) = ar(\Delta DPC) \qquad ...(2)$$

[∵ Triangle DPQ and DPC are on the same base DP and between the same parallels DP and CQ]

$$ar(\Delta DPQ) + ar(\Delta DPB) = ar(\Delta DPC) + ar(\Delta DPB)$$

Hence, $ar(\Delta BPQ) = ar(\Delta BCD) = \frac{1}{2} ar(\Delta ABC)$ [Using (1)]

5. ABCD is a square. E and F are respectively the midpoints of BC and CD. If R is the mid-point of EF (Fig. 9.15), prove that ar (Δ AER) = ar (Δ AFR).

Sol. ABCD is a square. E and F are respectively the mid-points of BC and CD. If R is the mid-point of EF, we have to prove that $ar(\Delta AER) = ar(\Delta AFR)$.

In ΔABE and ΔADF, we have

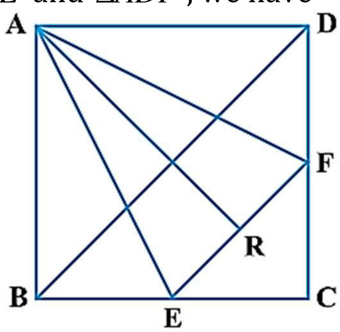


Fig. 9.15

AB = AD [Sides of a square are equal]

$\angle ABE = \angle ADF$ [Each 90^o]

BE = DF [∵ E is the mid-point of BC and F is the mid-point of CD. Also $\frac{1}{2} BC = \frac{1}{2} CD$]

$ar(\Delta ABE) = ar(\Delta ADF)$ [By SAS Congruence rule]

∴ AE = AF [CPCT] ...(1)

Now, in ΔAER and ΔAFR, we have

AE = AF [From (1)]

ER = RF [∵ R is mid-point of EF]

And AR = AR [Common side]

∴ $ar(\Delta AER) = ar(\Delta AFR)$ [By SSS rule of congruence]

Hence, as $ar(\Delta AER) = ar(\Delta AFR)$ [∵ Congruent triangles are equal in area]

6. **O is any point on the diagonal PR of a parallelogram PQRS (Fig. 9.16). Prove that ar (PSO) = ar (PQO).**

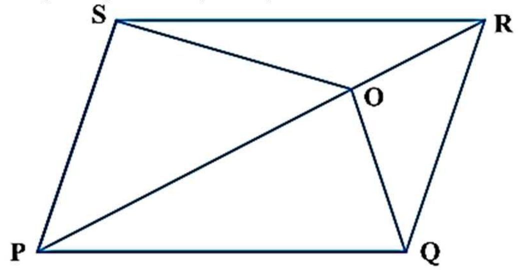


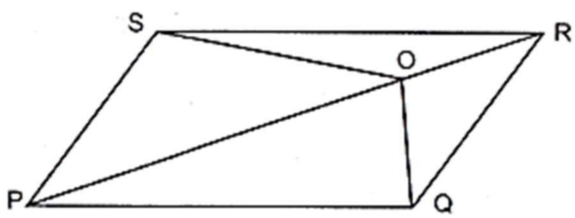
Fig. 9.16

Sol. Join SQ, bisect the diagonal PM at M. Since diagonals of a parallelogram bisect each other, so SM = MQ.

Therefore, PM is a median of ΔPQS

$$ar(\Delta PSM) = ar(\Delta PQM) \quad ..(1)$$

[∵ Median divides a triangle into two triangles of equal area]



Again, as Om is the median of triangle ΔOSQ, so

$$ar(\Delta OSM) = ar(\Delta OQM) \quad ...(2)$$

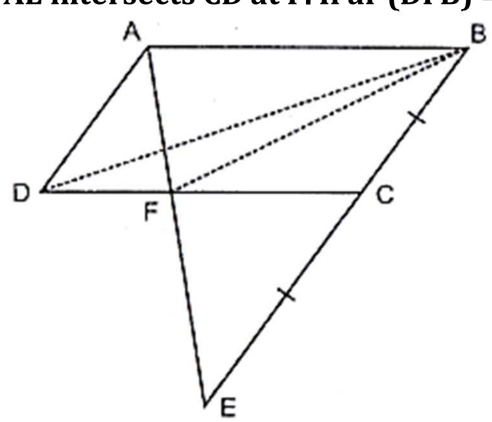
Adding (1) and (2), we get

$$ar(\Delta PSM) + ar(\Delta OSM) = ar(\Delta PQM) + ar(\Delta OQM)$$

$\Rightarrow \quad ar(\Delta PSO) = ar(\Delta PQO)$

Hence, proved.

7. **ABCD is a parallelogram in which BC is produced to E such that CE = BC (Fig. 9.17). AE intersects CD at F. If ar (DFB) = 3 cm², find the area of the parallelogram ABCD.**



Sol. In ΔADF and ΔEFC, we have

$\angle DAF = \angle CEF$ [Alt. interior $\angle s$]

AD = CE [$\because$ AD = BC = CE [Given]]

$\angle ADF = \angle FCE$ [Alt interior $\angle s$]

$\therefore$ $\Delta ADF \cong \Delta ECF$ [By SAS rule of congruence]

$\therefore$ DF = CF [CPCT]

As BF is the median of ΔBCD,

$\therefore$ $ar(\Delta BDF) = \frac{1}{2} ar(\Delta BCD)$ ...(1)

[$\because$ Median divides a triangle into two triangles of equal area]

Now, if a triangle and parallelogram are on the same base and between the same parallels, then the area of the triangles is equal to half the area of the parallelogram.

$\therefore$ $ar(\Delta BCD) = \frac{1}{2} ar(\| gm ABCD)$ ...(2)

$\therefore$ By (1), we have $ar(\Delta BDF) = \frac{1}{2}\left\{\frac{1}{2} ar(\| gm ABCD)\right\}$

$\Rightarrow$ $3cm^2 = \frac{1}{4} ar(\| gm ABCD)$

$\Rightarrow$ $ar(\| gm ABCD) = 12 cm^2$

Hence, the area of the parallelogram ABCD is 12 cm^2.

8. In trapezium ABCD, AB || DC and L is the mid-point of BC.
Through L, a line PQ||AD has been drawn which meets AB in P and DC produced in Q (Fig. 9.18). Prove that ar (ABCD) = ar (APQD)

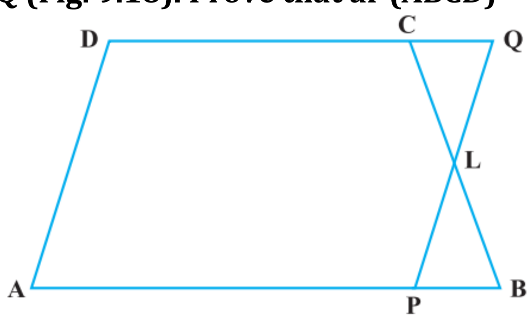


Fig. 9.18

Sol. AS AB||DC, so AB||DQ

In ΔCLQ and ΔBLP, we have

$\therefore$ $\angle QCL = \angle LBP$ [Alt. $\angle s$]

CL = LB [$\because$ L is the mid-point of BC]

$\angle CLQ = \angle BLP$ [Vertically opposite $\angle s$]

$\therefore$ $\Delta QCL \cong \angle BLP$ [By ASA congruence rule]

$\Rightarrow$ $ar(\Delta CLQ) = ar(\Delta BLP)$ ...(1)[Congruence Δs are equal in area]

Adding ar (APLCD) to both sides of (1), we get

$ar(\Delta CLQ) + ar(APLCD) = ar(\Delta BLP) + ar(APLCD)$

$\Rightarrow$ ar(APQD) = ar(ABCD)

Hence, ar(ABCD) = ar(APQD)

9. If the mid-points of the sides of a quadrilateral are joined in order, prove that the area of the parallelogram so formed will be half of the area of the given quadrilateral (Fig. 9.19). [Hint: Join BD and draw perpendicular from A on BD.]

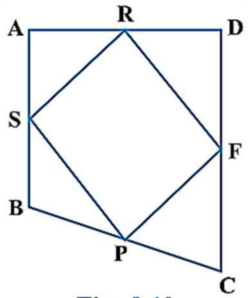
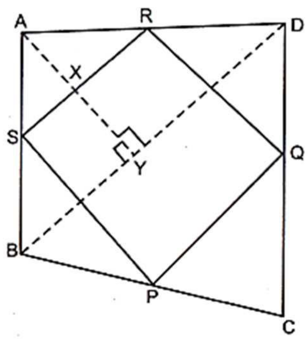


Fig. 9.19

Sol.



Given: A quadrilateral ABCD in which the mid-point of the sides of it are joined in order of form parallelogram PQRS.

To prove: $ar(\| gmPQRS) = \frac{1}{2} ar(\square ABCD)$

Construction: Join BD and draw perpendicular from A and BD which intersect SR and BD at X and Y respectively.

Proof: In ΔABD, S and R are the mid-points of sides AB and AD respectively.

$\therefore$ SR||BD

$\Rightarrow$ SX||BY

$\Rightarrow$ X is the mid-point of AY [Converse of mid-point theorem]

$\Rightarrow$ AX = XY ...(1) [$\because$ S is the mid-point of AB and SX||BY]

And, $SR = \frac{1}{2} BD$ ...(2) [$\because$ Mid-point theorem]

Now, $ar(\Delta ABD) = \frac{1}{2} \times BD \times AY$

And $ar(\Delta ASR) = \frac{1}{2} \times SR \times AX$

$\Rightarrow$ $ar(\Delta ASR) = \frac{1}{2} \times \left(\frac{1}{2} BD\right) \times \left(\frac{1}{2} AX\right)$ [Using (1) and (2)]

$$\Rightarrow \quad ar(\Delta ASR) = \frac{1}{4} \times \left(\frac{1}{2} \times BD \times AX \right)$$

$$\Rightarrow \quad ar(\Delta ASR) = \frac{1}{4} \times (\Delta ABD) \qquad ...(3)$$

Similarly,

$$ar(\Delta CPQ) = \frac{1}{4} ar(\Delta CBD) \qquad ...(4)$$

$$ar(\Delta BPS) = \frac{1}{4} ar(\Delta BAC) \qquad ...(5)$$

$$ar(\Delta DRQ) = \frac{1}{4} ar(\Delta DAC) \qquad ...(6)$$

Adding (3), (4), (5) and (6), we get

$$ar(\Delta ASR) + ar(\Delta CPQ) + ar(\Delta BPS) + ar(\Delta DRQ)$$

$$= \frac{1}{4} ar(\Delta ABD) + \frac{1}{4} ar(\Delta CBD) = \frac{1}{4} ar(\Delta ABD) + \frac{1}{4} ar(\Delta CBD)$$

$$= \frac{1}{4} [ar(\Delta ABD) + ar(\Delta CBD) + ar(\Delta BAC) + ar(\Delta DAC)]$$

$$= \frac{1}{4} [ar(\square ABCD) + ar(\square ABCD)]$$

$$= \frac{1}{4} \times 2ar(\square ABCD)$$

$$= \frac{1}{2} \times ar(\square ABCD)$$

$$\therefore ar(\Delta ASR) + ar(\Delta CPQ) + ar(\Delta BPS) + ar(\Delta DRQ)$$

$$= \frac{1}{2} ar(\square ABCD)$$

$$\Rightarrow ar(\square ABCD) - ar(\| gmPQRS) = \frac{1}{2} ar(\square ABCD)$$

$$\Rightarrow ar(\| gmPQRS) = ar(\square ABCD) - \frac{1}{2} ar(\square ABCD)$$

$$\Rightarrow ar(\| gmPQRS) = \frac{1}{2} ar(\square ABCD)$$

Hence, proved.

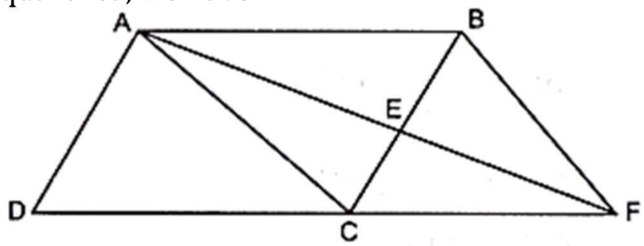
Areas of Parallelogram and Triangles

Exercise 9.4

1. A point E is taken on the side BC of a parallelogram ABCD. AE and DC are produced to meet at F. Prove that ar (ΔADF) = ar (ΔABFC)

Sol. Given: ABCD is a parallelogram. A point E is taken on the Side BC. AE and DC are produced to meet at F.

Proof: Since ABCD is a parallelogram and diagonal AC divides it into two triangles of equal area, we have



$$ar(\Delta ADC) = ar(\Delta ABC) \quad ...(1)$$

As DC||AB, So CF||AB

Since triangles on the same base and between the same parallels are equal in area, so we have

$$ar(\Delta ACF) = ar(\Delta BCF) \quad ...(2)$$

Adding (1) and (2), we get

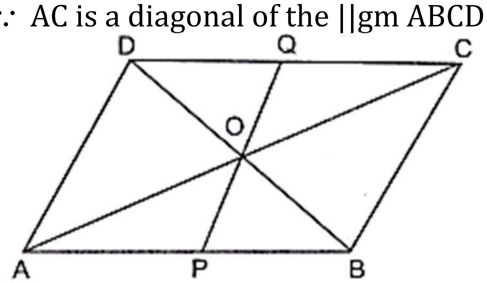
$$ar(\Delta ADC) + ar(\Delta ACF) = ar(\Delta ABC) + ar(\Delta BCF)$$

$$\Rightarrow \quad ar(\Delta ADF) = ar(ABFC)$$

Hence, proved.

2. The diagonals of a parallelogram ABCD intersect at a point O. Through O, a line is drawn to intersect AD at P and BC at Q. Show that PQ divides the parallelogram into two parts of equal area.

Sol. $\because$ AC is a diagonal of the ||gm ABCD



$$\therefore \quad ar(\Delta ACD) = \frac{1}{2} ar(\| gm ABCD) \quad ..(1)$$

Now, in ΔAOP and ΔCOQ

AO = CO [$\because$ Diagonals of a ||gm bisects each other]

$\angle AOP = \angle COQ$ [Vert. opp. $\angle s$]

$\angle OAP = \angle OCQ$ [Alt. $\angle s$; AB||CD]

$\therefore \quad \Delta AOP \cong \Delta COQ$ [By ASA cong. Rule]

Hence, $ar(\Delta AOP) = ar(\Delta COQ)$ [Cong. Area axiom] ...(2)

Adding ar(quad. AOQD) to both sides of (2), we get

$ar(quad.AOQD) + ar(\Delta AOP) = ar(quad.AOQD) + ar(\Delta COQ)$

$\Rightarrow$ $ar(quad.APQD) = ar(\Delta ACD)$

But, $ar(\Delta ACD) = \frac{1}{2} ar(\| gmABCD)$ [From (1)]

Hence, $ar(quad.APQD) = \frac{1}{2} ar(\| gmABCD)$.

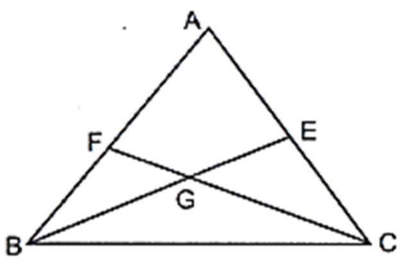
3. The medians BE and CF of a triangle ABC intersect at G. Prove that the area of ΔGBC = area of the quadrilateral AFGE.

Sol. BE and CF are medians of a triangle ABC intersect at G. We have to prove that the ar(ΔGBC) = area of the quadrilateral AFGF.

Since, median (CF) divides a triangle into two triangles of equal area, so we have

$ar(\Delta BCF) = ar(\Delta ACF)$

$\Rightarrow$ $ar(\Delta GBF) + ar(\Delta GBC) = ar(AFGE) + ar(\Delta GCE)$ ..(1)



Since, median (BE) divides a triangle into two triangle of equal area, so we have

$\Rightarrow$ $ar(\Delta GBF) + ar(AFGE) = ar(\Delta GCE) + ar(\Delta GBE)$ ...(2)

Subtracting (2) and (1), we get

$ar(\Delta GBC) - ar(AFGE) = ar(AFGE) - ar(\Delta GBC)$

$\Rightarrow$ $ar(\Delta GBC) + ar(\Delta GBC) = ar(AFGE) + ar(AFGE)$

$\Rightarrow$ $2ar(\Delta GBC) = 2ar(AFGE)$

Hence, $ar(\Delta GBC) = ar(AFGE)$

4. In Fig. 9.24, CD||AE and CY||BA. Prove that ar (ΔCBX) = ar (ΔAXY)

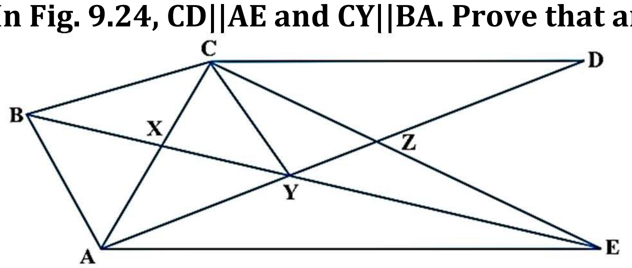


Fig. 9.24

Sol. CD||AE and CY||BA. We have to prove that $ar(\Delta CBX) = ar(\Delta AXY)$.

Since triangle on the same base and between the same parallels are equal in area, so we have

$$ar(\Delta ABC) = ar(\Delta ABY)$$

$\Rightarrow \quad ar(\Delta CBX) + ar(\Delta ABX) = ar(\Delta ABX) + ar(\Delta AXY)$

Hence, $\quad ar(\Delta CBX) = ar(\Delta AXY) \quad$ [Cancelling $ar(\Delta ABX)$ from both sides]

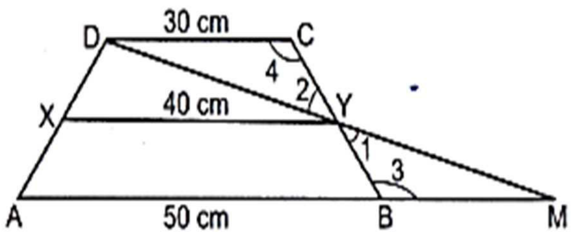
5. ABCD is a trapezium in which AB||DC, DC = 30 cm and AB = 50 cm. If X and Y are, respectively the mid-points of AD and BC, prove that

$ar(DCYX) = \frac{7}{9} ar(XYBA).$

Sol. In ΔMBY and ΔDCY, we have

$\angle 1 = \angle 2 \quad$ [Vertically opposite $\angle s$]

$\angle 3 = \angle 4 \quad$ [$\because$ AB||DC and alt. $\angle s$ are equal]



BY = CY $\quad$ [$\because$ Y is the mid-point of BC]

$\therefore \quad \Delta MBY \cong \Delta DCY \quad$ [By ASA Cong. Rule]

So, MB = DC = 30 cm $\quad$ [CPCT]

Now, AM = AB + BM = 50 cm + 30 cm = 80 cm

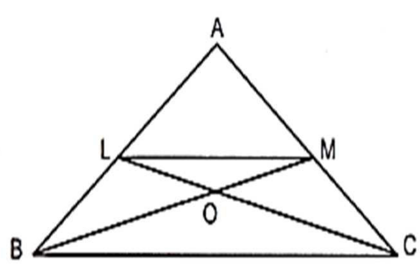
In ΔADM, we have $XY = \frac{1}{2} AM = \frac{1}{2} \times 80cm = 40cm$

As AB||XY||DC and X and Y are the mid-points of AD and BC, so height of trapezium DCXY and XYBA are equal. Let the equal height be h cm.

$$\frac{ar(DCXY)}{ar(XYBA)} = \frac{\frac{1}{2}(30+40)\times h}{\frac{1}{2}\times(40+50)\times h} = \frac{70}{90} = \frac{7}{9}$$

Hence, $ar(DCXY) = \frac{7}{9} ar(XYBA)$

6. In Δ ABC, if L and M are the points on AB and AC, respectively such that LM || BC. Prove that ar (ΔLOB) = ar (ΔMOC).



Sol. Since triangles on the same base and between the same parallels are equal in area, So we have

$\therefore \quad ar(\Delta LBM) = ar(\Delta LCM)$

[ΔLBM and ΔLCM are on the same base LM and between the same parallels LM and BC]

$\therefore \quad ar(\Delta LBM) = ar(\Delta LCM)$

$\Rightarrow \quad ar(\Delta LOM) + ar(\Delta LOB) = ar(\Delta LOM) + ar(\Delta MOC)$

Hence, $ar(\Delta LOB) = ar(\Delta MOC)$ [Cancelling (ΔLOM) from both sides]

7. In Fig. 9.25, ABCDE is any pentagon. BP drawn parallel to AC meets DC produced at P and EQ drawn parallel to AD meets CD produced at Q. Prove that ar (ABCDE) = ar (ΔAPQ)

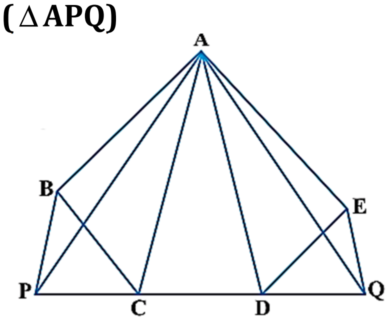


Fig. 9.25

Sol. BP||AC and AD||EQ.

Since, triangles on the same base and between the same parallels are equal in area

$ar(\Delta ABC) = ar(\Delta APC)$ ...(1)

And $ar(\Delta ADE) = ar(\Delta ADQ)$ ...(2)

Adding (1) and (2), we get

$ar(\Delta ABC) + ar(\Delta ADE) = ar(\Delta APC) + ar(\Delta ADQ)$

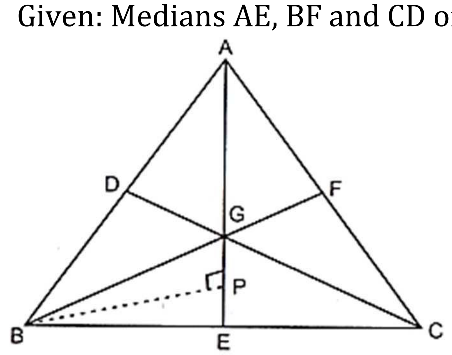
Adding $ar(\Delta ACD)$ to both sides, we get

$ar(\Delta ABC) + ar(\Delta ADE) + ar(\Delta ACD) = ar(\Delta APC) + ar(\Delta ADQ) + ar(\Delta ACD)$

Hence, $ar(ABCDE) = ar(\Delta APQ)$

8. If the medians of a Δ ABC intersect at G, show that ar (AGB) = ar (AGC) = ar (BGC) $= \frac{1}{2} ar(ABC).$

Sol. Given: Medians AE, BF and CD of Δ ABC intersect at G.



To prove: $ar(\Delta AGB) = ar(\Delta AGC) = ar(\Delta BGC)$

$$= \frac{1}{3} ar(\Delta ABC)$$

Construction: Draw $BP \perp EG$.

Proof: $AG = \frac{2}{3} AE$ [$\because$ Centroid divides the median in the ration 2:1]

Now, $ar(\Delta AGB) = \frac{1}{2} \times AG \times BP$

$$= \frac{1}{2} \times \frac{2}{3} \times AE \times BP$$

$$= \frac{2}{3} \times \frac{1}{2} \times AE \times BP$$

$$= \frac{2}{3} ar(\Delta ABE)$$

$$= \frac{2}{3} \times \frac{1}{2} ar(\Delta ABC)$$ [$\because$ Median divides a triangle into two triangles equal in area]

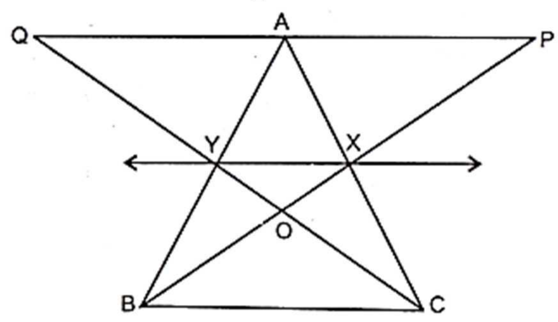
$$= \frac{1}{3} ar(\Delta ABC)$$

Similarly, $ar(\Delta ABC) = ar(\Delta BGC) = \frac{1}{3} ar(\Delta ABC)$

$\therefore ar(\Delta AGB) = ar(\Delta AGC) = ar(\Delta BGC) = \frac{1}{3} ar(\Delta ABC)$

Hence, proved.

9. In Fig. 9.26, X and Y are the mid-points of AC and AB respectively, QP||BC and CYQ and BXP are straight lines. Prove that ar (ΔABP) = ar (ΔACQ).



Sol. In triangle ABC, X and Y are the mid-points of AB and AC.

$\therefore$ XY||BC [BY mid-point theorem]

Since triangles on the same base (BC) and between the same parallels (XY||BC) are equal in area

$\therefore$ $ar(\Delta BYC) = ar(\Delta BXC)$ ...(1)

Subtracting $ar(\Delta BOC)$ from both sides, we get

$$ar(\Delta BYC) - ar(\Delta BOC) = ar(\Delta BXC) - ar(\Delta BOC)$$

$\Rightarrow \quad ar(\Delta BOY) = ar(\Delta COX) \quad ...(2)$

Adding $ar(\Delta XOY)$ to both sides of (2), we get

$ar(\Delta BOY) + ar(\Delta XOY) = ar(\Delta COX) + ar(\Delta XOY) \quad ...(3)$

Now, quad. XYAP and XYQA are on the same base XY and between the same parallels XY and PQ.

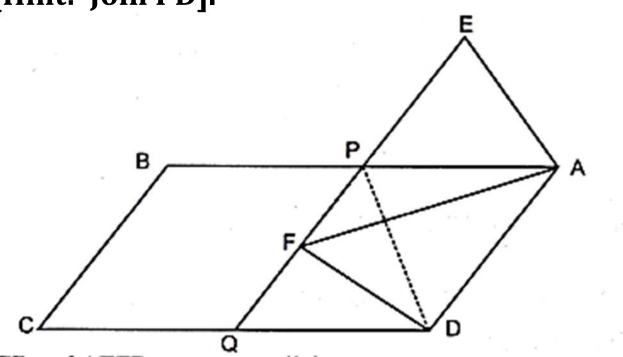
$\therefore \quad ar(XYAP) = ar(XYQA) \quad ...(4)$

Adding (3) and (4), we get

$$ar(\Delta BXY) + ar(XYAP) = ar(\Delta CXY) + ar(XYAQ)$$

Hence, $ar(\Delta ABP) = ar(\Delta ACQ)$

10. In Fig. 9.27, ABCD and AEFD are two parallelograms. Prove that ar (ΔPEA) = ar (Δ QFD)
[Hint: Join PD].



Sol. ABCD and AEFD are two parallelograms.

We have to prove that $ar(\Delta PEA) = ar(\Delta QFD)$. Join PD.

In ΔPEA and ΔQFD, we have

$\angle APE = \angle DQF$ [$\because$ Corresp. $\angle s$ are equal as AB||CD]

$\angle AEP = \angle DFQ$ [$\because$ Corresp. $\angle s$ are equal as AE||DF]

AE = DF [$\because$ opposite sides of a ||gm are equal]

$\therefore \quad \Delta PEA \cong \Delta QFD$ [By AAS cong. Rule]

Hence, ar(ΔPEA) = ar(ΔQED)