Ex 5.1

Algebra of Matrices Ex 5.1 Q1

We know that if a matrix is of the order $m \times n$, it has mn elements. Thus, to find all the possible orders of a matrix having 8 elements, we have to find all the ordered pairs of natural numbers whose products is 8.

The ordered pairs are: $(1 \times 8), (8 \times 1), (2 \times 4), (4 \times 2)$

$(1,5)$ *and* $(5,1)$ are the ordered pairs of natural numbers whose product is 5.

Hence, the possible orders of a matrix having 5 elements are 1×5 *and* 5×1

Algebra of Matrices Ex 5.1 Q2

If $A = [a_{ij}] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$ and $B = [b_{ij}] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$

(i) $a_{22} + b_{21} = 4 + (-3) = 1$

Hence, $a_{22} + b_{21} = 1$

(ii) $a_{11}\, b_{11} + a_{22} b_{22} = (2)(2) + (4)(4) = 4 + 16 = 20$

Hence,

$$a_{11}\, b_{11} + a_{22} b_{22} = 20$$

Algebra of Matrices Ex 5.1 Q3

Here, $A=\left[a_{ij}\right]_{3\times 4}$

R_1 = first row of $A = \left[a_{11}a_{12}a_{13}a_{14}\right]_{1\times 4}$

So, order of $R_1 = 1\times 4$

C_2 = Second column of A

$$= \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix}_{3\times 1}$$

Order of $C_2 = 3\times 1$

Algebra of Matrices Ex 5.1 Q4

Let $A = \left(a_{ij}\right)_{2\times 3}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \qquad ---(i)$$

(i) $\quad a_{ij} = i.j$

$a_{11} = 1.1 = 1,\ a_{12} = 1.2 = 2,\ a_{13} = 1.3 = 3$

$a_{21} = 2.1 = 2,\ a_{22} = 2.2 = 4,\ a_{23} = 2.3 = 6$

So, using equation (i)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$$

(ii) $a_{ij} = 2i - j$

$a_{11} = 2(1) - 1 = 1,\ a_{12} = 2(1) - 2 = 0,\ a_{13} = 2(1) - 3 = -1$

$a_{21} = 2(2) - 1 = 3,\ a_{22} = 2(2) - 2 = 2,\ a_{23} = 2(2) - 3 = 1$

Using equation (i)

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \end{bmatrix}$$

(iii) $a_{ij} = i + j$

$a_{11} = 1+1 = 2,\ a_{12} = 1+2 = 3,\ a_{13} = 1+3 = 4$

$a_{21} = 2+1 = 3,\ a_{22} = 2+2 = 4,\ a_{23} = 2+3 = 5$

Using equation (i)

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

(iv) $a_{ij} = \frac{(i+j)^2}{2}$

$a_{11} = \frac{(1+1)^2}{2} = 2,\ a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2},\ a_{13} = \frac{(1+3)^2}{2} = 8$

$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2},\ a_{22} = \frac{(2+2)^2}{2} = 8,\ a_{23} = \frac{(2+3)^2}{2} = \frac{25}{2}$

Using equation (i),

$$A = \begin{bmatrix} 2 & \frac{9}{2} & 8 \\ \frac{9}{2} & 8 & \frac{25}{2} \end{bmatrix}$$

Algebra of Matrices Ex 5.1 Q5

Here,

$$A = \left[a_{ij}\right]_{2\times 2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \text{---(i)}$$

(i) $\qquad a_{ij} = \dfrac{(i+j)^2}{2}$

$$a_{11} = \frac{(1+1)^2}{2} = 2,\; a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2},$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2},\; a_{22} = \frac{(2+2)^2}{2} = 8,$$

Using equation (i)

$$A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$$

(ii) $\qquad a_{ij} = \dfrac{(i-j)^2}{2}$

$$a_{11} = \frac{(1-1)^2}{2} = 0,\; a_{12} = \frac{(1+2)^2}{2} = \frac{1}{2},$$

$$a_{21} = \frac{(2-1)^2}{2} = \frac{1}{2},\; a_{22} = \frac{(2-2)^2}{2} = 0,$$

Using equation (i)

$$A = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$$

(iii) $\quad a_{ij} = \dfrac{(i - 2j)^2}{2}$

$$a_{11} = \frac{(1 - 2(1))^2}{2} = \frac{1}{2},\ a_{12} = \frac{(1 - 2(2))^2}{2} = \frac{9}{2},$$

$$a_{21} = \frac{(2 - 2(1))^2}{2} = 0,\ a_{22} = \frac{(2 - 2(2))^2}{2} = 2,$$

Using equation (i)

$$A = \begin{bmatrix} \frac{1}{2} & \frac{9}{2} \\ 0 & 2 \end{bmatrix}$$

(iv) $\quad a_{ij} = \dfrac{(2i + j)^2}{2}$

$$a_{11} = \frac{(2(1) + 1)^2}{2} = \frac{9}{2},\ a_{12} = \frac{(1(1) + 2)^2}{2} = 8,$$

$$a_{21} = \frac{(2(2) + 2)^2}{2} = \frac{25}{2},\ a_{22} = \frac{(2(2) + 2)^2}{2} = 18$$

Using equation (i)

$$A = \begin{bmatrix} \frac{9}{2} & 8 \\ \frac{25}{2} & 18 \end{bmatrix}$$

(v) $$a_{ij} = \frac{\left(\left|2i - 3j\right|\right)^2}{2}$$

$$a_{11} = \frac{\left|2(1) - 3(1)\right|}{2} = \frac{1}{2}, \quad a_{12} = \frac{\left|2(1) - 3(2)\right|}{2} = 2$$

$$a_{21} = \frac{\left|2(2) - 3(1)\right|}{2} = \frac{1}{2}, \quad a_{22} = \frac{\left|2(2) - 3(2)\right|}{2} = 1$$

Using equation (i)

$$A = \begin{bmatrix} \frac{1}{2} & 2 \\ \frac{1}{2} & 1 \end{bmatrix}$$

Algebra of Matrices Ex 5.1 Q6

Here, $A = \left(a_{ij}\right)_{3\times 4} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \quad ---(i)$

(i) $a_{ij} = i + j$

$a_{11} = 1 + 1 = 2, \ a_{12} = 1 + 2 = 3, \ a_{13} = 1 + 3 = 4, \ a_{14} = 1 + 4 = 5$

$a_{21} = 2 + 1 = 3, \ a_{22} = 2 + 2 = 4, \ a_{23} = 2 + 3 = 5, \ a_{24} = 2 + 4 = 6$

$a_{31} = 3 + 1 = 4, \ a_{32} = 3 + 2 = 5, \ a_{33} = 3 + 3 = 6, \ a_{34} = 3 + 4 = 7$

Using equation (i)

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$$

(ii) $a_{ij} = i - j$

$a_{11} = 1 - 1 = 0, \ a_{12} = 1 - 2 = -1, \ a_{13} = 1 - 3 = -2, \ a_{14} = 1 - 4 = -3$

$a_{21} = 2 - 1 = 1, \ a_{22} = 2 - 2 = 0, \ a_{23} = 2 - 3 = -1, \ a_{24} = 2 - 4 = -2$

$a_{31} = 3 - 1 = 2, \ a_{32} = 3 - 2 = 1, \ a_{33} = 3 - 3 = 0, \ a_{34} = 3 - 4 = -1$

Using equation (i)

$$A = \begin{bmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -1 & -2 \\ 2 & 1 & 0 & -1 \end{bmatrix}$$

(iii) $a_{ij} = 2i$

$a_{11} = 2(1) = 2,\ a_{12} = 2(1) = 2,\ a_{13} = 2(1) = 2,\ a_{14} = 2(1) = 2$

$a_{21} = 2(2) = 4,\ a_{22} = 2(2) = 4,\ a_{23} = 2(2) = 4,\ a_{24} = 2(2) = 4$

$a_{31} = 2(3) = 6,\ a_{32} = 2(3) = 6,\ a_{33} = 2(3) = 6,\ a_{34} = 2(3) = 6$

Using Equation (i),

$$A = \begin{bmatrix} 2 & 2 & 2 & 2 \\ 4 & 4 & 4 & 4 \\ 6 & 6 & 6 & 6 \end{bmatrix}$$

(iv) $a_{ij} = j$

$a_{11} = 1,\ a_{12} = 2,\ a_{13} = 3,\ a_{14} = 4$

$a_{21} = 1,\ a_{22} = 2,\ a_{13} = 3,\ a_{14} = 4$

$a_{31} = 1,\ a_{32} = 2,\ a_{33} = 3,\ a_{34} = 4$

Using Equation (i),

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

Algebra of Matrices Ex 5.1 Q7

Here,

$$A = \left[a_{ij}\right]_{4\times3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}$$

(a) $\quad a_{ij} = 2i + \frac{i}{j}$

$a_{11} = 2(1) + \frac{1}{1} = 3,\; a_{12} = 2(1) + \frac{1}{2} = \frac{5}{2},\; a_{13} = 2(1) + \frac{1}{3} = \frac{7}{3}$

$a_{21} = 2(2) + \frac{2}{1} = 6,\; a_{22} = 2(2) + \frac{2}{2} = 5,\; a_{23} = 2(2) + \frac{2}{3} = \frac{14}{3}$

$a_{31} = 2(3) + \frac{3}{1} = 9,\; a_{32} = 2(3) + \frac{3}{2} = \frac{15}{2},\; a_{33} = 2(3) + \frac{3}{3} = 7$

$a_{41} = 2(4) + \frac{4}{1} = 12,\; a_{42} = 2(4) + \frac{4}{2} = 10,\; a_{43} = 2(4) + \frac{4}{3} = \frac{28}{3}$

Using equation (i),

$$A = \begin{bmatrix} 3 & \frac{5}{2} & \frac{7}{3} \\ 6 & 5 & \frac{14}{3} \\ 9 & \frac{15}{2} & 7 \\ 12 & 10 & \frac{28}{3} \end{bmatrix}$$

(c) $a_{ij} = i$

$a_{11} = 1,\ a_{12} = 1,\ a_{13} = 1,$

$a_{21} = 2,\ a_{22} = 2,\ a_{23} = 2$

$a_{31} = 3,\ a_{32} = 3,\ a_{33} = 3$

$a_{41} = 4,\ a_{42} = 4,\ a_{43} = 4$

Using equation (i)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{bmatrix}$$

Algebra of Matrices Ex 5.1 Q8

Given,

$$\begin{bmatrix} 3x+4y & 2 & x-2y \\ a+b & 2a-b & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 4 \\ 5 & -5 & -1 \end{bmatrix}$$

Since corresponding entries of equal matrix are equal.
So,

$$3x+4y = 2 \qquad \text{---(i)}$$
$$x-2y = 4 \qquad \text{---(ii)}$$
$$a+b = 5 \qquad \text{---(iii)}$$
$$2a-b = -5 \qquad \text{---(iv)}$$

Solving equation (i) and (iii)

$$\begin{array}{l} 3x-4y = 2 \\ 3x-6y = 12 \\ \underline{(-)\quad(+)\quad(-)} \\ \qquad 10y = -10 \end{array}$$

$$y = \frac{-10}{10} = -1$$

Put $y = 1$ in equation (ii)

$$x-2y = 4$$
$$x-2(-1) = 4$$
$$x = 4-2$$
$$x = 2$$

Now, solving equation (iii) *and* (iv),

$$\begin{array}{l} 2a+2b = 10 \\ 2a-\ b = -5 \\ \underline{(-)\quad(+)\quad(+)} \\ \qquad 3b = 15 \end{array}$$

$$b = \frac{15}{3}$$
$$b = 5$$

Put the value of b in equation of (iii)

$$a+b = 5$$
$$a+5 = 5$$
$$a = 5-5$$
$$a = 0$$

Hence,

$$x = 2,\ y = -1,\ a = 0,\ b = 5$$

Algebra of Matrices Ex 5.1 Q9

Given,

$$\begin{bmatrix} 2x - 3y & a - b & 3 \\ 1 & x + 4y & 3a + 4b \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 6 & 29 \end{bmatrix}$$

Since corresponding entries of equal matrix are equal.

So,

$$2x - 3y = 1 \qquad \text{---(i)}$$

$$x - b = -2 \qquad \text{---(ii)}$$

$$x - 4y = 6 \qquad \text{---(iii)}$$

$$3a + 4b = 29 \qquad \text{---(iv)}$$

Solving equation (i) and (iii)

$$2x - 3y = 1$$

$$\underset{(-)}{2x} \underset{(-)}{- 8y} = \underset{(-)}{12}$$

$$-11y = -11$$

$$y = \frac{-11}{-11}$$

$$y = 1$$

Put the value of y in equation (i),

$$2x - 3y = 1$$

$$2x - 3(i) = 1$$

$$2x - 3 = 1$$

$$2x = 1 + 3$$

$$2x = 4$$

$$x = 2$$

Solving equation (ii) and (iv)

$$4a - 4b = -8$$

$$3a - 4b = 29$$

$$7a = 21$$

$$a = \frac{21}{7}$$

$$a = 3$$

Put $a = 3$ in equation (ii),

$$3 - b = -2$$
$$b = 3 + 2$$
$$b = 5$$

Hence,

$$x = 2,\ y = 1,\ a = 3,\ b = 5$$

Algebra of Matrices Ex 5.1 Q10

As the given matrices are equal, therefore their corresponding elements must be equal.

Comparing the corresponding elements, we get

$2a + b = 4$ ----(i)

$a - 2b = -3$ ----(ii)

$5c - d = 11$ ----(iii)

$4c + 3d = 24$ ----(iv)

Multiplying (i) by 2 and adding to (ii)

$5a = 5 \Rightarrow a = 1$

(i) $\Rightarrow b = 4 - 2.1 = 2$

Multiplying (iii)by 3 and adding to (iv)

$19c = 57 \Rightarrow c = 3$

(iii) $\Rightarrow d = 5.3 - 11 = 4$

Hence, $a = 1$, $b = 2$, $c = 3$, $d = 4$

Algebra of Matrices Ex 5.1 Q11

Given,

$$A = B$$

$$\begin{bmatrix} x-2 & 3 & 2z \\ 18z & y+2 & 6z \end{bmatrix} = \begin{bmatrix} y & z & 6 \\ 6y & x & 2y \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, So

$x - 2 = y$ ---(i)

$3 = z$ ---(ii)

$2z = 6$ ---(iii)

$18z = 6$ ---(iv)

$y + 2 = x$ ---(v)

$6z = 2y$ ---(vi)

Equation (ii) gives, $z = 3$

Put the value of z in equal (iv),

$$18z = 6y$$
$$18(3) = 6y$$
$$54 = 6y$$
$$y = \frac{54}{6}$$
$$y = 9$$

Put $y = 9$ in equation (v)

$$y + 2 = x$$
$$9 + 2 = x$$
$$11 = x$$

Hence,

$$x = 11, y = 9, z = 3$$

Algebra of Matrices Ex 5.1 Q12

Given,

$$\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-w \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, So

$x = 3$ ---(i)

$3x - y = 2$ ---(ii)

$2x + z = 4$ ---(iii)

$3y - w = 7$ ---(iv)

Put the value of $x = 3$ from equation on (i) in eqation (ii),

$3x - y = 2$

$3(3) - y = 2$

$9 - y = 2$

$y = 9 - 2$

$y = 7$

Put the value of $y = 7$ in equation (iv),

$3y - w = 7$

$3(7) - w = 7$

$w = 21 - 7$

$w = 14$

Put the value of $x = 3$ in equation (iii),

$2x + z = 4$

$2(3) + z = 4$

$6 + z = 4$

$z = 4 - 6$

$z = -2$

Hence,

$x = 3, y = 7, z = -2, w = 14$

Algebra of Matrices Ex 5.1 Q13

Given,

$$\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, So

$x - y = -1$ ---(i)

$z = 4$ ---(ii)

$2x - y = 0$ ---(iii)

$w = 5$ ---(iv)

Solving equation (i) and (iii)

$$\begin{array}{l} x - y = -1 \\ \underline{\underset{(-)}{2x} - \underset{(+)}{y} = \underset{(-)}{0}} \\ -x \quad = -1 \\ \qquad x = 1 \end{array}$$

Put $x = 1$ in equation (i),

$x - y = -1$

$1 - y = -1$

$-y = -1 - 1$

$-y = -2$

$y = 2$

equation (ii) and (iv) give the values of z and w respectively, so

$z = 4, \quad w = 5$

Hence,

$x = 1, y = 2, z = 4, w = 5$

Algebra of Matrices Ex 5.1 Q14

By the definition of equality of matrices we know that if two matrices

$A=\left[a_{ij}\right]_{m\times n}$ and $B=\left[b_{ij}\right]_{m\times n}$

are equal then $a_{ij}=b_{ij}$ for $i = 1,2,3\ldots\ldots m$ and $j = 1,2,3\ldots\ldots n$.

Given that $\begin{bmatrix} x+3 & z+4 & 2y-7 \\ 4x+6 & a-1 & 0 \\ b-3 & 3b & z+2c \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ 2x & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$

$\therefore$ Equating the entries gives:

$x + 3 = 0,\quad z + 4 = 6$ and $2y - 7 = 3y - 2$

$\Rightarrow\quad x = -3,\ z = 2$ and $2y - 3y = -2 + 7$

$\Rightarrow\quad x = -3,\ z = 2$ and $-y = 5$

$\Rightarrow\quad x = -3,\ z = 2$ and $y = -5$

Similarly, $a-1=-3$ and $2c + 2 = 0$

$\Rightarrow\quad a = -3 + 1$ and $2c = -2$

$\Rightarrow\quad a = -2$ and $c = -1$

Lastly, $b - 3 = 2b + 4$

$\Rightarrow\quad b - 2b = 4 + 3$

$\Rightarrow\quad -b = 7$

$\Rightarrow\quad b = -7$

The values of x, y, z, a, b, c are - 3, - 5 , 2 , -2 , - 7 , - 1 respectively.

Algebra of Matrices Ex 5.1 Q15

Given that $\begin{bmatrix} 2x+1 & 5x \\ 0 & y^2+1 \end{bmatrix} = \begin{bmatrix} x+3 & 10 \\ 0 & 26 \end{bmatrix}$

The corresponding entries of the equal matrices are equal.

$\Rightarrow\quad 2x + 1 = x + 3,\ y^2 + 1 = 26,$

$\Rightarrow\quad 2x - x = 2,\ y^2 = 25$

$\Rightarrow\quad x = 2,\ y = \pm 5$

$\Rightarrow\quad x = 2, y = 5$ or $x = 2,\ y = -5$

$\therefore\quad x + y = 7$ or -3

Algebra of Matrices Ex 5.1 Q16

$\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$

The corresponding entries of the two equal matrices are equal.

$\Rightarrow$ $xy = 8$ ......(1),

$w = 4$ ....(2),

$z + 6 = 0$ ....(3),

and $x + y = 6$ ....(4)

from equation (2) and equation(3) we get $z = -6$ and $w = 4$.

from equation(4) we have,

$x + y = 6,$

$\Rightarrow\ x = 6 - y,$

subsituting value of x in equation (1) we get,

$\Rightarrow\quad (6 - y)y = 8,$

$\Rightarrow\quad y^2 - 6y + 8 = 0,$

$\Rightarrow\quad (y - 2)(y - 4) = 0,$

$\Rightarrow\quad y = 2, 4$

subsituting the value of y in equation(1) we get ,

$\Rightarrow\ x = 4, 2$

Therefore, value of x , y , z ,w are 2 , 4 , - 6, 4 or 4, 2 , -6 , 4 .

Algebra of Matrices Ex 5.1 Q17

(i) We know that,

Order of a row matrix $= 1 \times n$

order of a column matrix $= m \times 1$

So, order of a row as well as column matrix $= 1 \times 1$

Therefore,

Required matrix $= [a]_{1\times1}$

(ii) A diagonal matrix has only a_{11}, a_{22}, a_{33} for a 3×3 matrix such that a_{11}, a_{22}, a_{33} are equal or different and all other entries zero while scalor matrix has $a_{11} = a_{22} = a_{33} = m$ (say) So, A diagonal matrix which is not scalar must have, $a_{11} \neq a_{22} \neq a_{33}$ and $aij = 0$ for $i \neq j$, So

$$\text{Required Matrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(iii) A triangular matrix is a square matrix $A = [aij]$ such that $aij = 0$ for all $i > j$, so

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 0 & 4 & 3 \\ 0 & 0 & -6 \end{bmatrix}$$

Algebra of Matrices Ex 5.1 Q18

Given data is,

For January 2013:

Dealer A	Deluxe	Premium	Standard Cars
	5	3	4
Dealer B	7	2	3

For January-February :

Dealer A	Deluxe	Premium	Standard Cars
	8	7	6
Dealer B	10	5	7

Hence,

$$A = \begin{array}{c} \\ \text{Dealer A} \\ \text{Dealer B} \end{array}\begin{array}{c} \begin{array}{ccc} \text{Deluxe} & \text{Premum} & \text{Standard} \end{array} \\ \begin{bmatrix} 5 & 3 & 4 \\ 7 & 2 & 3 \end{bmatrix} \end{array}$$

$$B = \begin{array}{c} \\ \text{Dealer A} \\ \text{Dealer B} \end{array}\begin{array}{c} \begin{array}{ccc} \textit{Deluxe} & \text{Premum} & \text{Standard} \end{array} \\ \begin{bmatrix} 8 & 7 & 6 \\ 10 & 5 & 7 \end{bmatrix} \end{array}$$

Algebra of Matrices Ex 5.1 Q19

Given,

$$A = B$$

$$\begin{bmatrix} 2x+1 & 2y \\ 0 & y^2-5y \end{bmatrix} = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$$

Since equal matrics has all corresponing entries equal,
So,

$$2x + 1 = x + 3 \qquad ---(i)$$

$$2y = y^2 + 2 \qquad ---(ii)$$

$$y^2 - 5y = -6 \qquad ---(iii)$$

Solving equation (i)

$$2x + 1 = x + 3$$

$$2x - x = 3 - 1$$

$$x = 2$$

Solving equation (ii)

$$2y = y^2 + 2$$

$$y^2 - 2y + 2 = 0$$

$$D = b^2 - 4ac$$

$$= (-2)^2 - 4(i)(ii)$$

$$= 4 - 8$$

$$= -2$$

So, There is no real value of y from equation (ii).

Solving equation (iii)

$$y^2 - 5y = -6$$

$$y^2 - 5y + 6 = 0$$

$$y^2 - 3y - 2y + 6 = 0$$

$$y(y-3) - 2(y-3) = 0$$

$$(y-3)(y-2) = 0$$

$$y = 3 \qquad or \qquad y = 2$$

From solution of equation (i), (ii) and (iii), We can say that
A and B can not be equal for any value of y.

Algebra of Matrices Ex 5.1 Q20

Given,

$$\begin{bmatrix} x+10 & y^2+2y \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 3x+4 & 3 \\ 0 & y^2-5y \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, So

$$x + 10 = 3x + 4 \qquad ---(i)$$

$$y^2 + 2y = 3 \qquad ---(ii)$$

$$-4 = y^2 - 5y \qquad ---(iii)$$

Solving equation (i),

$$x + 10 = 3x + 4$$
$$x - 3x = 4 - 10$$
$$-2x = -6$$
$$x = \frac{6}{2}$$

Solving equation (ii),

$$y^2 + 2y = 3$$
$$y^2 + 2y - 3 = 0$$
$$y^2 + 3y - y - 3 = 0$$
$$y(y+3)(y-1) = 0$$

$\Rightarrow \quad y = -3$ and $y = 1$

Solving equation (iii)

$$-4 = y^2 - 5y$$
$$y^2 - 5y + 4 = 0$$
$$y^2 - 4y - y + (y-4) = 0$$
$$y(y-4) - 1(y-4) = 0$$
$$(y-4)(y-1) = 0$$

$\Rightarrow \quad y = 4$ and $y = 1$

From equation (ii) *and* (iii),

The common value of $y = 1$

So, $\quad x = 3, y = 1$

Algebra of Matrices Ex 5.1 Q21

$$A = \begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix}, \; B = \begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-5b \end{bmatrix}$$

Given that A = B

Corresponding element of two equal matrices are equal

$\Rightarrow \quad a + 4 = 2a + 2$, $3b = b^2 + 2$ and $-6 = b^2 - 5b$

$\Rightarrow \quad a - 2a = 2 - 4$, $b^2 - 3b + 2 = 0$ and $b^2 - 5b + 6 = 0$

$\Rightarrow \quad -a = -2$, $(b-1)(b-2) = 0$ and $(b-2)(b-3) = 0$

$\Rightarrow \quad a = 2$, $b = 1,2$ and $b = 2,3$

So value of a = 2, b=2 respectively.

Ex 5.2

Algebra of Matrices Ex 5.2 Q1

(i) $$\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3-2 & -2+4 \\ 1+1 & 4+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix}$$

Hence,

$$\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix}$$

(ii) $$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 5 \\ -1 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & 1 \\ 0 & -3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+1 & 1-2 & 3+3 \\ 0+2 & 3+6 & 5+1 \\ -1+0 & 2-3 & 5+1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 & 6 \\ 2 & 9 & 6 \\ -1 & -1 & 6 \end{bmatrix}$$

Hence,

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 5 \\ -1 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & 1 \\ 0 & -3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 6 \\ 2 & 9 & 6 \\ -1 & -1 & 6 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q2

Given, $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$2A - 3B$

$$= 2\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - 3\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 8 \\ 6 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 9 \\ -6 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & 8-9 \\ 6+6 & 4-15 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ 12 & -11 \end{bmatrix}$$

Hence,

$$2A - 3B = \begin{bmatrix} 1 & -1 \\ 12 & -11 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q2(ii)

Given, $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$B - 4c$

$$= \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} - 4\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} - \begin{bmatrix} -8 & 20 \\ 12 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} 1+8 & 3-20 \\ -2-12 & 5-16 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -17 \\ -14 & -11 \end{bmatrix}$$

Hence,

$$B - 4c = \begin{bmatrix} 9 & -17 \\ -14 & -11 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q2(iii)

Given, $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$3A - C$

$$= 3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6+2 & 12-5 \\ 9-3 & 6-4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

Hence,

$$3A - C = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q2(iv)

Given, $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$3A - 2B + 3C$

$$= 3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} + 3\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ -4 & 10 \end{bmatrix} + \begin{bmatrix} -6 & 15 \\ 9 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 6-2-6 & 12-6+15 \\ 9+4+9 & 6-10+12 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 21 \\ 22 & 8 \end{bmatrix}$$

Hence,

$$3A - 2B + 3C = \begin{bmatrix} -2 & 21 \\ 22 & 8 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q3

Given, $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 4 & 1 \end{bmatrix}, C = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix} \sqsupset$

(i) $A + B$

$A + B$ is not possible as order of A is 2×2 and order of B is 2×3.
And we know that sum of matrix is possible only when their order is same.

Hence,

$A + B$ is not possible

$B + C$

$$= \begin{bmatrix} -1 & 0 & 2 \\ 3 & 4 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1-1 & 0+2 & 2+3 \\ 3+2 & 4+1 & 1+0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 2 & 5 \\ 5 & 5 & 1 \end{bmatrix}$$

So,

$$B + C = \begin{bmatrix} -2 & 2 & 5 \\ 5 & 5 & 1 \end{bmatrix}$$

We need to find $2B+3A$ *and* $3C-4B$

Thuss, $2B+3A$ *does not exist as the order of* A *and* B *are different.*

Let us find $3C-4B = 3\begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix} - 4\begin{bmatrix} -1 & 0 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$$= \begin{bmatrix} -3 & 6 & 9 \\ 6 & 3 & 0 \end{bmatrix} - \begin{bmatrix} -4 & 0 & 8 \\ 12 & 16 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 6 & 9 \\ 6 & 3 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & -8 \\ -12 & -16 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 6 & 1 \\ -6 & -13 & -4 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q4

Given, $A = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & -2 & 5 \\ 1 & -3 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & -5 & 2 \\ 6 & 0 & -4 \end{bmatrix}$

$2A - 3B + 4C$

$$= 2\begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix} - 3\begin{bmatrix} 0 & -2 & 5 \\ 1 & -3 & 1 \end{bmatrix} + 4\begin{bmatrix} 1 & -5 & 2 \\ 6 & 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 & 4 \\ 6 & 2 & 8 \end{bmatrix} - \begin{bmatrix} 0 & -6 & 15 \\ 3 & -9 & 3 \end{bmatrix} + \begin{bmatrix} 4 & -20 & 8 \\ 24 & 0 & -16 \end{bmatrix}$$

$$= \begin{bmatrix} -2-0+4 & 0+6-20 & 4-15+8 \\ 6-3+24 & 2+9+0 & 8-3-16 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -14 & -3 \\ 27 & 11 & -11 \end{bmatrix}$$

Hence,

$$2A - 3B + 4C = \begin{bmatrix} 2 & -14 & -3 \\ 27 & 11 & -11 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q5

Given, $A = diag\begin{pmatrix}2 & -5 & 9\end{pmatrix}, B = diag\begin{pmatrix}1 & 1 & -4\end{pmatrix}$

and $C = diag\begin{pmatrix}-b & 3 & 4\end{pmatrix}$

(i) $A - 2B$

$= diag\begin{pmatrix}2 & -5 & 9\end{pmatrix} - 2diag\begin{pmatrix}1 & 1 & 9-4\end{pmatrix}$

$= diag\begin{pmatrix}2 & -5 & 9\end{pmatrix} - diag\begin{pmatrix}2 & 2 & -8\end{pmatrix}$

$= diag\begin{pmatrix}2-2 & -5-2 & 9+8\end{pmatrix}$

$= diag\begin{pmatrix}0 & -7 & -17\end{pmatrix}$

So, $A - 2B = diag\begin{pmatrix}0 & -7 & 17\end{pmatrix}$

(ii) $B + C - 2A$

$= diag\begin{pmatrix}1 & 1 & -4\end{pmatrix} + diag\begin{pmatrix}-6 & 3 & 4\end{pmatrix} - 2diag\begin{pmatrix}2 & -5 & 9\end{pmatrix}$

$= diag\begin{pmatrix}1 & 1 & -4\end{pmatrix} + diag\begin{pmatrix}-6 & 3 & 4\end{pmatrix} - diag\begin{pmatrix}4 & -10 & 18\end{pmatrix}$

$= diag\begin{pmatrix}1-6-4 & 1+3+10 & -4+4-18\end{pmatrix}$

$= diag\begin{pmatrix}-9 & 14 & -18\end{pmatrix}$

So, $B + C - 2A = diag\begin{pmatrix}-9 & 14 & -18\end{pmatrix}$

(iii) $2A + 3B - 5C$

$= 2\ diag\begin{pmatrix}2 & -5 & 9\end{pmatrix} + 3\ diag\begin{pmatrix}1 & 1 & -4\end{pmatrix} - 5\ diag\begin{pmatrix}-6 & 3 & 4\end{pmatrix}$

$= diag\begin{pmatrix}4 & -10 & 18\end{pmatrix} + diag\begin{pmatrix}3 & 3 & -12\end{pmatrix} - diag\begin{pmatrix}-30 & 15 & 20\end{pmatrix}$

$= diag\begin{pmatrix}4+3+30 & -10+3-15 & 18-12-20\end{pmatrix}$

$= diag\begin{pmatrix}37 & -22 & -14\end{pmatrix}$

So,

$2A + 3B - 5C = diag\begin{pmatrix}37 & -22 & -14\end{pmatrix}$

Algebra of Matrices Ex 5.2 Q6

Given,

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 9 & 7 & -1 \\ 3 & 5 & 4 \\ 2 & 1 & 6 \end{bmatrix}, C = \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

$$\text{LHS} = (A+B)+C$$

$$= \left\{\begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 9 & 7 & -1 \\ 3 & 5 & 4 \\ 2 & 1 & 6 \end{bmatrix}\right\} + \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2+9 & 1+7 & 1-1 \\ 3+2 & -1+5 & 0+4 \\ 0+2 & 2+1 & 4+6 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 8 & 0 \\ 6 & 4 & 4 \\ 2 & 3 & 10 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 11+2 & 8-4 & 0+3 \\ 6+1 & 4-1 & 4+0 \\ 2+9 & 3+4 & 10+5 \end{bmatrix}$$

$$\text{LHS} = \begin{bmatrix} 13 & 4 & 3 \\ 7 & 3 & 4 \\ 11 & 7 & 15 \end{bmatrix} \qquad ---(i)$$

$$\text{RHS} = A+(B+C)$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \left\{\begin{bmatrix} 9 & 7 & -1 \\ 3 & 5 & 4 \\ 2 & 1 & 6 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}\right\}$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 9+2 & 7-4 & -1+3 \\ 3+1 & 5-1 & 4+0 \\ 2+9 & 1+4 & 6+5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 11 & 3 & 2 \\ 4 & 4 & 4 \\ 11 & 5 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 2+11 & 1+3 & 1+2 \\ 3+4 & -1+4 & 0+4 \\ 0+11 & 2+5 & 4+11 \end{bmatrix}$$

$$\text{RHS} = \begin{bmatrix} 13 & 4 & 3 \\ 7 & 3 & 4 \\ 11 & 7 & 15 \end{bmatrix} \qquad \text{---(ii)}$$

From equation (i) and , we get

$$(A+B)+C = A+(B+C)$$

Algebra of Matrices Ex 5.2 Q7

We have

$$(X+Y)+(X-Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow \quad 2X = \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$$

$$\Rightarrow \quad X = \frac{1}{2}\begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$$

$$\text{Also, } (X+Y)-(X-Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow \quad 2Y = \begin{bmatrix} 5-3 & 2-6 \\ 0-0 & 9+1 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix}$$

$$\Rightarrow \quad Y = \frac{1}{2}\begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q8

$$2X+Y=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$$

$$\Rightarrow 2X+\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$$

$$\Rightarrow 2X=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}-\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}=\begin{bmatrix}1-3 & 0-2\\ -3-1 & 2-4\end{bmatrix}$$

$$\Rightarrow 2X=\begin{bmatrix}-2 & -2\\ -4 & -2\end{bmatrix}$$

$$\therefore X=\frac{1}{2}\begin{bmatrix}-2 & -2\\ -4 & -2\end{bmatrix}=\begin{bmatrix}-1 & -1\\ -2 & -1\end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q9

Given,

$$2x-y=\begin{bmatrix}6 & -6 & 0\\ -4 & 2 & 1\end{bmatrix} \qquad \text{---(i)}$$

$$x+2y=\begin{bmatrix}3 & 2 & 5\\ -2 & 1 & -7\end{bmatrix} \qquad \text{---(ii)}$$

Now find

$$2(2x-y)+(x+2y)=2\begin{bmatrix}6 & -6 & 0\\ -4 & 2 & 1\end{bmatrix}+\begin{bmatrix}3 & 2 & 5\\ -2 & 1 & -7\end{bmatrix} \qquad \{\text{using equation (i) and (ii)}\}$$

$$\Rightarrow \quad 4x-2y+x+2y=\begin{bmatrix}12 & -12 & 0\\ -8 & 4 & 2\end{bmatrix}+\begin{bmatrix}3 & 2 & 5\\ -2 & 1 & -7\end{bmatrix}$$

$$\Rightarrow \quad 5x=\begin{bmatrix}12+3 & -12+2 & 0+5\\ -8-2 & 4+1 & 2-7\end{bmatrix}$$

$$\Rightarrow \quad 5x=\begin{bmatrix}15 & -10 & 5\\ -10 & 5 & -5\end{bmatrix}$$

$$\Rightarrow \quad 5x=5\begin{bmatrix}3 & -2 & 1\\ -2 & 1 & -1\end{bmatrix}$$

$$\Rightarrow \quad x=\begin{bmatrix}3 & -2 & 1\\ -2 & 1 & -1\end{bmatrix}$$

Now find,

$$(2x-y)-2(x+2y)=\begin{bmatrix}6 & -6 & 0\\ -4 & 2 & 1\end{bmatrix}-2\begin{bmatrix}3 & 2 & 5\\ -2 & 1 & -7\end{bmatrix} \qquad \{\text{using equation (i) and (ii)}\}$$

$$\Rightarrow \quad 2x=y-2x-4y=\begin{bmatrix}6 & -6 & 0\\ -4 & 2 & 1\end{bmatrix}-\begin{bmatrix}6 & 4 & 10\\ -4 & 2 & -14\end{bmatrix}$$

$$\Rightarrow \quad -y-4y=\begin{bmatrix}6-6 & -6-4 & 0-10\\ -4+4 & 2-2 & 1+14\end{bmatrix}$$

$$\Rightarrow \quad -5y=\begin{bmatrix}0 & -10 & -10\\ 0 & 0 & 15\end{bmatrix}$$

$$\Rightarrow \quad -5y=-5\begin{bmatrix}0 & 2 & 2\\ 0 & 0 & -3\end{bmatrix}$$

$$\Rightarrow \quad y=\begin{bmatrix}0 & 2 & 2\\ 0 & 0 & -3\end{bmatrix}$$

Hence,

$$x=\begin{bmatrix}3 & -2 & 1\\ -2 & 1 & -1\end{bmatrix}, y=\begin{bmatrix}0 & 2 & 2\\ 0 & 0 & -3\end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q10

Given,

$$x - y = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \quad x + y = \begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & 4 \\ 11 & 8 & 0 \end{bmatrix}$$

Now find,

$$(x - y) + (x + y) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & 4 \\ 11 & 8 & 0 \end{bmatrix}$$

$$\Rightarrow \quad 2x = \begin{bmatrix} 1+3 & 1+5 & 1+1 \\ 1-1 & 1+1 & 0+4 \\ 1+11 & 0+8 & 0+0 \end{bmatrix}$$

$$\Rightarrow \quad 2x = \begin{bmatrix} 4 & 6 & 2 \\ 0 & 2 & 4 \\ 12 & 8 & 0 \end{bmatrix}$$

$$\Rightarrow \quad 2x = 2\begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 6 & 4 & 0 \end{bmatrix}$$

$$\Rightarrow \quad x = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 6 & 4 & 0 \end{bmatrix}$$

Now find,

$$(x - y) - (x + y) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & 4 \\ 11 & 8 & 0 \end{bmatrix}$$

$$\Rightarrow \quad x - y - x - y = \begin{bmatrix} 1-3 & 1-5 & 1-1 \\ 1+1 & 1-1 & 0-4 \\ 1-11 & 0-8 & 0-0 \end{bmatrix}$$

$$\Rightarrow \quad -2y = \begin{bmatrix} -2 & -4 & 0 \\ 2 & 0 & -4 \end{bmatrix}$$

$$\begin{bmatrix} -10 & -8 & 0 \end{bmatrix}$$

$$\Rightarrow \quad -2y = -2\begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 2 \\ 5 & 4 & 0 \end{bmatrix}$$

$$\Rightarrow \quad y = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 2 \\ 5 & 4 & 0 \end{bmatrix}$$

Hence,

$$x = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 6 & 4 & 0 \end{bmatrix}, y = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 2 \\ 5 & 4 & 0 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q11

Given,

$$\begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & 9 \end{bmatrix} + A = \begin{bmatrix} 9 & -1 & 1 \\ 4 & -2 & 3 \end{bmatrix}$$

$$\Rightarrow \quad A = \begin{bmatrix} 9 & -1 & 1 \\ 4 & -2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & 9 \end{bmatrix}$$

$$\Rightarrow \quad A = \begin{bmatrix} 9-1 & -1-2 & 1-4 \\ 4+1 & -2-0 & 3-9 \end{bmatrix}$$

$$\Rightarrow \quad A = \begin{bmatrix} 8 & -3 & -3 \\ 5 & -2 & -6 \end{bmatrix}$$

Hence,

$$A = \begin{bmatrix} 8 & -3 & -3 \\ 5 & -2 & -6 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q12

Given, $A = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$

Let, $C = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$

Since, $5A + 3B + 2C$ is a null matrix, so

$5A + 3B + 2C = 0$

$$\Rightarrow \quad 5\begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix} + 3\begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix} + 2\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 45 & 5 \\ 35 & 40 \end{bmatrix} + \begin{bmatrix} 3 & 15 \\ 21 & 36 \end{bmatrix} + \begin{bmatrix} 2x & 2y \\ 2z & 2w \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 45 + 3 + 2x & 5 + 15 + 2y \\ 35 + 21 + 2z & 40 + 36 + 2w \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 48 + 2x & 20 + 2y \\ 56 + 2z & 76 + 2w \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Since, corresponding entries of equalmatrices are equal.

$48 + 2x = 0$

$x = -\dfrac{48}{2}$

$x = -24$

$20 + 2y = 0$

$y = -\dfrac{20}{2}$

$y = -10$

$56 + 2z = 0$

$z = -\dfrac{56}{2}$

$z = -28$

$76 + 2w = 0$

$w = -\dfrac{76}{2}$

$w = -38$

Hence, $C = \begin{bmatrix} -24 & -10 \\ -28 & -38 \end{bmatrix}$

Algebra of Matrices Ex 5.2 Q13

Given,

$$A = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}, B = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$$

And

$$2A + 3x = 5B$$

$$\Rightarrow \quad 3x = 5B - 2A$$

$$\Rightarrow \quad 3x = 5\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix} - 2\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$$

$$\Rightarrow \quad 3x = \begin{bmatrix} 40 & 0 \\ 20 & -10 \\ 15 & 30 \end{bmatrix} - \begin{bmatrix} 4 & -4 \\ 8 & 4 \\ -10 & 2 \end{bmatrix}$$

$$\Rightarrow \quad 3x = \begin{bmatrix} 40-4 & 0+4 \\ 20-8 & -10-4 \\ 15+10 & 30-2 \end{bmatrix}$$

$$\Rightarrow \quad 3x = \begin{bmatrix} 36 & 4 \\ 12 & -14 \\ 25 & 28 \end{bmatrix}$$

$$\Rightarrow \quad x = \begin{bmatrix} \frac{36}{3} & \frac{4}{3} \\ \frac{12}{3} & \frac{-14}{3} \\ \frac{25}{3} & \frac{28}{3} \end{bmatrix}$$

$$\Rightarrow \quad x = \begin{bmatrix} 12 & \frac{4}{3} \\ 4 & \frac{-14}{3} \\ \frac{25}{3} & \frac{28}{3} \end{bmatrix}$$

Hence,

$$\begin{bmatrix} 12 & \frac{4}{3} \\ & -14 \end{bmatrix}$$

$$x = \begin{bmatrix} 4 & \frac{[illegible]}{3} \\ \frac{25}{3} & \frac{28}{3} \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q14

Given,

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

And

$$A + B + C = 0$$

$$\Rightarrow \quad C = -A - B + 0$$

$$\Rightarrow \quad C = -A - B$$

$$\Rightarrow \quad C = -\begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow \quad C = \begin{bmatrix} -1 & 3 & -2 \\ -2 & 0 & -2 \end{bmatrix} - \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow \quad C = \begin{bmatrix} -1-2 & 3+1 & -2+1 \\ -2-1 & 0-0 & -2+1 \end{bmatrix}$$

$$\Rightarrow \quad C = \begin{bmatrix} -3 & 4 & -1 \\ -3 & 0 & -1 \end{bmatrix}$$

Hence,

$$C = \begin{bmatrix} -3 & 4 & -1 \\ -3 & 0 & -1 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q15(i)

$$\begin{bmatrix} x-y & 2 & -2 \\ 4 & x & 6 \end{bmatrix} + \begin{bmatrix} 3 & -2 & 2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 5 & 2x+y & 5 \end{bmatrix}$$

$$\begin{bmatrix} x-y+3 & 2-2 & -2+2 \\ 4+1 & x+0 & 6-1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 5 & 2x+y & 5 \end{bmatrix}$$

$$\begin{bmatrix} x-y+3 & 0 & 0 \\ 5 & x & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 5 & 2x+y & 5 \end{bmatrix}$$

Use know that, corresponding entries of equal matrices are equal. So,

$x - y + 3 = 6$

$\Rightarrow \quad x - y = 3 \qquad$ ---(i)

and $\quad x = 2x + y$

$\Rightarrow \quad 2x - x + y = 0$

$\Rightarrow \quad x + y = 0 \qquad$ ---(ii)

Adding equation (i), (ii),

$x - y + x + y = 3 + 0$

$\Rightarrow \quad 2x = 3$

$\Rightarrow \quad x = \frac{3}{2}$

Put in equation (i),

$x - y = 3$

$\Rightarrow \quad \frac{3}{2} - y = 3$

$\Rightarrow \quad -y = \frac{3-3}{2}$

$\Rightarrow \quad y = \frac{-3}{2}$

Hence,

$x = \frac{3}{2}, y = \frac{-3}{2}$

Algebra of Matrices Ex 5.2 Q15(ii)

$$[x \quad y+2 \quad z-3]+[y \quad 4 \quad 5]=[4 \quad 9 \quad 12]$$
$$\Rightarrow \quad [x+y \quad y+2+4 \quad z-3+5]=[4 \quad 9 \quad 12]$$
$$\Rightarrow \quad [x+y \quad y+6 \quad z+2]=[4 \quad 9 \quad 12]$$

We know that, corresponding entries, of equal matrices are equal, So

$$x+y=4 \qquad ---(i)$$
$$y+6=9 \qquad ---(ii)$$
$$z+2=12 \qquad ---(iii)$$

From equation (ii), We get

$$y=9-6$$
$$y=3$$

Put the value of y in equation (i),

$$x+y=4$$
$$\Rightarrow \quad x+3=4$$
$$\Rightarrow \quad x=4-3$$
$$\Rightarrow \quad x=1$$

From equation (iii)

$$z=2=12$$
$$z=12-2$$
$$z=10$$

Hence,

$$x=1, y=3, z=10$$

Algebra of Matrices Ex 5.2 Q16

Given,

$$2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix}+\begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$
$$\Rightarrow \quad \begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix}+\begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$
$$\Rightarrow \quad \begin{bmatrix} 6+1 & 8+y \\ 10+0 & 2x+1 \end{bmatrix}=\begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$
$$\Rightarrow \quad \begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix}=\begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, So

$$8+y=0$$
$$y=-8$$

And

$$2x+1=5$$
$$2x=5-1$$
$$x=\frac{4}{2}$$
$$x=2$$

Hence,

$$x=2, y=-8$$

Algebra of Matrices Ex 5.2 Q17

Given,

$$\lambda\begin{bmatrix}1 & 0 & 2\\ 3 & 4 & 5\end{bmatrix}+2\begin{bmatrix}1 & 2 & 3\\ -1 & -3 & 2\end{bmatrix}=\begin{bmatrix}4 & 4 & 10\\ 4 & 2 & 14\end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix}\lambda & 0 & 2\lambda\\ 3\lambda & 4\lambda & 5\lambda\end{bmatrix}+\begin{bmatrix}2 & 4 & 6\\ -2 & -6 & 4\end{bmatrix}=\begin{bmatrix}4 & 4 & 10\\ 4 & 2 & 14\end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix}\lambda+2 & 4 & 2\lambda+6\\ 3\lambda-2 & 4\lambda-6 & 5v+4\end{bmatrix}=\begin{bmatrix}4 & 4 & 10\\ 4 & 2 & 14\end{bmatrix}$$

Since corresponding entries of equal matrices are equal, So

$$\lambda = 2 = 4$$

$$\Rightarrow \quad \lambda = 2$$

and

$$3\lambda - 2 = 4$$

$$3\lambda = 6$$

$$\Rightarrow \quad \lambda = 2$$

Hence,

$$\lambda = 2$$

Algebra of Matrices Ex 5.2 Q18(i)

Given,

$$A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$$

and

$$2A + B + X = 0$$

$$\Rightarrow \quad X = -2A - B$$

$$\Rightarrow \quad X = -2\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$$

$$\Rightarrow \quad X = \begin{bmatrix} 2 & -4 \\ -6 & -8 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$$

$$\Rightarrow \quad X = \begin{bmatrix} 2-3 & -4+2 \\ -6-1 & -8-5 \end{bmatrix}$$

$$\Rightarrow \quad X = \begin{bmatrix} -1 & -2 \\ -7 & -13 \end{bmatrix}$$

Hence,

$$X = \begin{bmatrix} -1 & -2 \\ -7 & -13 \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q18(ii)

$$Given, A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix} and \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$$

Also we have $2A + 3X = 5B$

Thus, we have, $3X = 5B - 2A$

$$\Rightarrow 3x = 5\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix} - 2\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$$

$$\Rightarrow 3x = \begin{bmatrix} 10 & -10 \\ 20 & 10 \\ -25 & 5 \end{bmatrix} - \begin{bmatrix} 16 & 0 \\ 8 & -4 \\ 6 & 12 \end{bmatrix}$$

$$\Rightarrow 3x = \begin{bmatrix} 10-16 & -10-0 \\ 20-8 & 10-(-4) \\ -25-6 & 5-12 \end{bmatrix}$$

$$\Rightarrow 3x = \begin{bmatrix} -6 & -10 \\ 12 & 14 \\ -31 & -7 \end{bmatrix}$$

$$\Rightarrow x = \begin{bmatrix} -2 & \frac{-10}{3} \\ 4 & \frac{14}{3} \\ \frac{-31}{3} & \frac{-7}{3} \end{bmatrix}$$

Algebra of Matrices Ex 5.2 Q19(i)

$$3\begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2t \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+t & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3t \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+z+t & 2t+3 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

$$3x = x+4$$
$$\Rightarrow 2x = 4$$
$$\Rightarrow x = 2$$

$$3y = 6+x+y$$
$$\Rightarrow 2y = 6+x = 6+2 = 8$$
$$\Rightarrow y = 4$$

$$3t = 2t+3$$
$$\Rightarrow t = 3$$

$$3z = -1+z+t$$
$$\Rightarrow 2z = -1+t = -1+3 = 2$$
$$\Rightarrow z = 1$$

$$\therefore x = 2,\ y = 4,\ z = 1, \text{ and } t = 3$$

Algebra of Matrices Ex 5.2 Q19(ii)

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & 10 \\ 14 & 2y-6 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+3 & 14 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$$

Comparing the corresponding elements from both sides,

$2x+3 = 7 \Rightarrow 2x = 4 \Rightarrow x = 2$

$2y-4 = 14 \Rightarrow 2y = 18 \Rightarrow y = 9$

Hence, $x = 2$, $y = 9$

Algebra of Matrices Ex 5.2 Q20

Let us solve this problem using simultaneous linear equation and algebra of matrices.

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \quad \ldots\ldots\ldots (1)$$

$$3X + 2Y = \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix} \ldots\ldots\ldots\ldots (2)$$

multiplying the first equation by 3 and second equation by 2 we get,

$$6X + 9Y = 3\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \ldots\ldots\ldots (3),$$

$$6X + 4Y = 2\begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix} \ldots\ldots\ldots\ldots (4)$$

Subtracting equation (4) from equation (3) we have,

$$5Y = 3\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - 2\begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}$$

$$\Rightarrow 5Y = \begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix} - \begin{bmatrix} -4 & 4 \\ 2 & -10 \end{bmatrix}$$

$$\Rightarrow 5Y = \begin{bmatrix} 10 & 5 \\ 10 & 10 \end{bmatrix}$$

$$\Rightarrow Y = \frac{1}{5}\begin{bmatrix} 10 & 5 \\ 10 & 10 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$$

Similarly, multiplying the equation (1) by 2 and equation (2) by 3 we get,

$$4X + 6Y = 2\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \ldots\ldots\ldots (5),$$

$$9X + 6Y = 3\begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix} \ldots\ldots\ldots\ldots (6)$$

Subtracting equation (6) from equation (5) we have,

$$-5X = 2\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - 3\begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}$$

$$\Rightarrow -5X = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} - \begin{bmatrix} -6 & 6 \\ 3 & -15 \end{bmatrix}$$

$$\Rightarrow -5X = \begin{bmatrix} 10 & 0 \\ 5 & 15 \end{bmatrix}$$

$$\Rightarrow X = -\frac{1}{5}\begin{bmatrix} 10 & 0 \\ 5 & 15 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$$

Hence the value of $X = \begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$ and $Y = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$.

Algebra of Matrices Ex 5.2 Q21

Let A represent the post allocation matrix for a college, So

$$A = \begin{bmatrix} 15 \\ 6 \\ 1 \\ 1 \end{bmatrix} \begin{array}{l} \text{Peons} \\ \text{Clerks} \\ \text{Typist} \\ \text{Section officer} \end{array}$$

The total number of posts of each kind in 30 colleges in given by:

$$= 30A$$

$$= 30 \begin{bmatrix} 15 \\ 6 \\ 1 \\ 1 \end{bmatrix}$$

$$30A = \begin{bmatrix} 450 \\ 90 \\ 30 \\ 30 \end{bmatrix} \begin{array}{l} \text{Peons} \\ \text{Clerks} \\ \text{Typists} \\ \text{Section Officers} \end{array}$$

Ex 5.3

Algebra of Matrices Ex 5.3 Q1

(i) $$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$= \begin{bmatrix} (a)(a)+(b)(b) & (a)(-b)+(b)(a) \\ (-b)(a)+(a)(b) & (-b)(-b)+(a)(a) \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2 & -ab+ab \\ -ab+ab & b^2+a^2 \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$$

Hence,

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$$

(ii) $$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(1)+(-2)(-3) & (1)(2)+(-2)(2) & (1)(3)+(-2)(-1) \\ (2)(1)+(3)(-3) & (2)(2)+(3)(2) & (2)(3)+(3)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 1+6 & 2-4 & 3+2 \\ 2-9 & 4+6 & 6-3 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -2 & 5 \\ -7 & 10 & 3 \end{bmatrix}$$

Hence,

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 7 & -2 & 5 \\ -7 & 10 & 3 \end{bmatrix}$$

(iii) $$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} (2)(1)+(3)(0)+(4)(3) & (2)(-3)+(3)(2)+(4)(0) & (2)(5)+(3)(4)+(4)(5) \\ (3)(1)+(4)(0)+(5)(3) & (3)(-3)+(4)(2)+(5)(0) & (3)(5)+(4)(4)+(5)(5) \\ (4)(1)+(5)(0)+(6)(3) & (4)(-3)+(5)(2)+(6)(0) & (4)(5)+(5)(4)+(6)(5) \end{bmatrix}$$

$$= \begin{bmatrix} 2+0+12 & -6+6+0 & 10+12+20 \\ 3+0+15 & -9+8+0 & 15+16+25 \\ 4+0+18 & -12+10+0 & 20+20+30 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q2(i)

Given, $A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

$$AB = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix}$$

$$AB = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix} \quad ---(i)$$

$$BA = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix}$$

$$BA = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix} \quad ---(ii)$$

From equation (i) and (ii), we get

$$AB \neq BA$$

Algebra of Matrices Ex 5.3 Q2(ii)

$$\text{Given, } A = \begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1+0+0 & -2+1+0 & -3+0+0 \\ +0+01 & 0-1+1 & 0+0+0 \\ 2+0+4 & 4+3+4 & 6+0+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & -1 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \qquad ---(\text{i})$$

$$BA = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1+0+6 & 1-2+9 & 0+2+12 \\ 0+0+0 & 0-1+0 & 0+1+0 \\ -1+0+0 & 1-1+0 & 0+1+0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \qquad ---(\text{ii})$$

From (i) and (ii), $\quad AB \neq BC$

Algebra of Matrices Ex 5.3 Q2(iii)

Given, $A = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$

$$BA = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+3+0 & 1+0+0 & 0+0+0 \\ 0+1+0 & 1+0+0 & 0+0+0 \\ 0+1+0 & 4+0+0 & 0+0+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 4 & 0 \end{bmatrix} \quad ---(i)$$

$$BA = \begin{bmatrix} 0 & 10 & 1 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+0 & 0+1+0 & 0+0+0 \\ 1+0+0 & 3+0+0 & 0+0+0 \\ 0+5+4 & 0+5+1 & 0+0+0 \end{bmatrix} \quad ---(ii)$$

$$BA = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 0 \\ 9 & 6 & 0 \end{bmatrix} \quad ---(ii)$$

From equation (i) and (ii), we get

$$AB \neq BA$$

Algebra of Matrices Ex 5.3 Q3(i)

$A = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

Since order of A is 2×2 and order of B is 2×3,
So AB is possible but BA is not possible order of AB is 2×3.

$$AB = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(1)+(-2)(2) & (1)(2)+(-2)(3) & (1)(3)+(-2)(1) \\ (2)(1)+(3)(2) & (2)(2)+(3)(3) & (2)(3)+(3)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 1-4 & 2-6 & 3-2 \\ 2+6 & 4+9 & 6+3 \end{bmatrix}$$

$$AB = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

Hence,

$$AB = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

BA does not exits

Algebra of Matrices Ex 5.3 Q3(ii)

Here, $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$

Order of $A = 3 \times 2$ and order of $B = 2 \times 3$ So,

AB and BA Both exits and order of $AB = 3 \times 3$ and order of $BA = 2 \times 2$

$$AB = \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (3)(4)+(2)(0) & (3)(5)+(2)(1) & (3)(6)+(2)(2) \\ (-1)(4)+(0)(0) & (-1)(5)+(0)(0) & (-1)(6)+(0)(2) \\ (-1)(4)+(1)(0) & (-1)(5)+(1)(1) & (-1)(6)+(1)(2) \end{bmatrix}$$

$$= \begin{bmatrix} 12+0 & 15+2 & 18+4 \\ -4+0 & -5+0 & -6+0 \\ -4+0 & -5+0 & -6+2 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 17 & 22 \\ -4 & -5 & -6 \\ -4 & -4 & -4 \end{bmatrix}$$

$$BA = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (4)(3)+(5)(-1)+(6)(-1) & (4)(2)+(5)(0)+(6)(1) \\ (0)(3)+(1)(-1)+(2)(-1) & (0)(2)+(1)(0)+(2)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 12-5-6 & 8+0+6 \\ 0-1-2 & 0+0+2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 14 \\ -3 & 2 \end{bmatrix}$$

Hence,

$$AB = \begin{bmatrix} 12 & 17 & 22 \\ -4 & -5 & -6 \\ -4 & -4 & -4 \end{bmatrix}, BA = \begin{bmatrix} 1 & 14 \\ -3 & 2 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q3(iii)

Here,

$$A=\begin{bmatrix}1 & -1 & 2 & 3\end{bmatrix}, B=\begin{bmatrix}0\\1\\3\\2\end{bmatrix}$$

Order of A = 1×4 and order of B=4×1 So,
AB and BA both exist and order of AB = 1×1 and order of BA=4×4, So

$$AB=\begin{bmatrix}1 & -1 & 2 & 3\end{bmatrix}\begin{bmatrix}0\\1\\3\\2\end{bmatrix}$$

$$=\begin{bmatrix}(1)(0)+(-1)(1)+(2)(3)+(3)(2)\end{bmatrix}$$

$$=\begin{bmatrix}0-1+6+6\end{bmatrix}$$

$$AB=\begin{bmatrix}11\end{bmatrix}$$

$$BA=\begin{bmatrix}0\\1\\3\\2\end{bmatrix}\begin{bmatrix}1 & -1 & 2 & 3\end{bmatrix}$$

$$BA=\begin{bmatrix}(0)(1) & (0)(-1) & (0)(2) & (0)(3)\\(1)(1) & (1)(-1) & (1)(2) & (1)(3)\\(3)(1) & (3)(-1) & (3)(2) & (3)(3)\\(2)(1) & (2)(-1) & (3)(2) & (2)(3)\end{bmatrix}$$

$$BA=\begin{bmatrix}0 & 0 & 0 & 0\\1 & -1 & 2 & 3\\3 & -3 & 6 & 9\\2 & -2 & 4 & 6\end{bmatrix}$$

Hence,

$$AB=\begin{bmatrix}11\end{bmatrix}$$

$$BA=\begin{bmatrix}0 & 0 & 0 & 0\\1 & -1 & 2 & 3\\3 & -3 & 6 & 9\\2 & -2 & 4 & 6\end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q3(iv)

$$\begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} c \\ d \end{bmatrix}+\begin{bmatrix} a & b & c & d \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$= \left[ac + bd\right] + \left[a^2 + b^2 + c^2 + d^2\right]$$

$$= \left[ac + bd = a^2 + b^2 + c^2 + d^2\right]$$

Hence,

$$\begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} c \\ d \end{bmatrix}+\begin{bmatrix} a & b & c & d \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$= \left[ac + bd + a^2 + b^2 + c^2 + d^2\right]$$

Algebra of Matrices Ex 5.3 Q4(i)

$$A=\begin{bmatrix}1 & 3 & -1\\ 2 & -1 & -1\\ 3 & 0 & -1\end{bmatrix}, B=\begin{bmatrix}-2 & 3 & -1\\ -1 & 2 & -1\\ -6 & 2 & -4\end{bmatrix}$$

$$AB=\begin{bmatrix}1 & 3 & -1\\ 2 & -1 & -1\\ 3 & 0 & -1\end{bmatrix}\begin{bmatrix}-2 & 3 & -1\\ -1 & 2 & -1\\ -6 & 9 & -4\end{bmatrix}$$

$$=\begin{bmatrix}-2-3+6 & 3+6-9 & -1-3+4\\ -4+1+6 & 6-2-9 & -2+1+4\\ -6+0+6 & 9+0-9 & -3+0+4\end{bmatrix}$$

$$AB=\begin{bmatrix}1 & 0 & 0\\ 3 & -5 & 3\\ 0 & 0 & 1\end{bmatrix} \qquad ---(i)$$

$$BA=\begin{bmatrix}-2 & 3 & -1\\ -1 & 2 & -1\\ -6 & 9 & -4\end{bmatrix}\begin{bmatrix}1 & 2 & -1\\ 2 & -1 & -1\\ 3 & 0 & -1\end{bmatrix}$$

$$=\begin{bmatrix}-2+6-3 & -6-3+0 & 2-3+1\\ -1+4-3 & -3-2+0 & 1-2+1\\ -6+18-12 & -18-9+0 & 6-9+4\end{bmatrix}$$

$$BA=\begin{bmatrix}1 & -9 & 0\\ 0 & -5 & 0\\ 0 & -27 & 1\end{bmatrix} \qquad ---(ii)$$

From equation (i) and (ii),

$$AB \neq BA$$

Algebra of Matrices Ex 5.3 Q4(ii)

$$A=\begin{bmatrix}10&-4&-1\\-11&5&0\\9&-5&1\end{bmatrix}, B=\begin{bmatrix}1&2&1\\3&4&2\\1&3&2\end{bmatrix}$$

$$AB=\begin{bmatrix}10&-4&-1\\-11&5&0\\9&-5&1\end{bmatrix}\begin{bmatrix}1&2&1\\3&4&2\\1&3&2\end{bmatrix}$$

$$=\begin{bmatrix}10-12-1&20-16-3&10-8-2\\-11+15+0&-22+20+0&-11+10+0\\9-15+1&18-20+3&9-10+2\end{bmatrix}$$

$$AB=\begin{bmatrix}-3&1&0\\4&-2&-1\\-5&1&1\end{bmatrix} \qquad ---(i)$$

$$BA=\begin{bmatrix}1&2&1\\3&4&2\\1&3&2\end{bmatrix}\begin{bmatrix}10&-4&-1\\-11&5&0\\9&-5&1\end{bmatrix}$$

$$=\begin{bmatrix}10-22+9&-4+10-5&-9+0+1\\30-44+10&-12+20-10&-3+0+2\\10-33+18&-4+15-10&-1+0+2\end{bmatrix}$$

$$BA=\begin{bmatrix}-3&1&0\\4&-2&-1\\-5&1&1\end{bmatrix} \qquad ---(ii)$$

From equation (i) and (ii)

$$AB \neq BA$$

Algebra of Matrices Ex 5.3 Q5(i)

$$\left(\begin{bmatrix}1&3\\-1&-4\end{bmatrix}+\begin{bmatrix}3&-2\\-1&1\end{bmatrix}\right)\begin{bmatrix}1&3&5\\2&4&6\end{bmatrix}$$

$$=\left(\begin{bmatrix}1+3&3-2\\-1-1&-4+1\end{bmatrix}\right)\begin{bmatrix}1&3&5\\2&4&6\end{bmatrix}$$

$$=\begin{bmatrix}4&1\\-2&-3\end{bmatrix}\begin{bmatrix}1&3&5\\2&4&6\end{bmatrix}$$

$$=\begin{bmatrix}4+2&12+4&20+6\\-2-6&-6-12&-10-18\end{bmatrix}$$

$$=\begin{bmatrix}6&16&26\\-8&-18&-28\end{bmatrix}$$

Hence,

$$\left(\begin{bmatrix}1&3\\-1&-4\end{bmatrix}+\begin{bmatrix}3&-2\\-1&1\end{bmatrix}\right)\begin{bmatrix}1&3&5\\2&4&6\end{bmatrix}=\begin{bmatrix}6&16&26\\-8&-18&-28\end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q5(ii)

$$\begin{bmatrix}1 & 2 & 3\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\2 & 0 & 1\\0 & 1 & 2\end{bmatrix}\begin{bmatrix}2\\4\\6\end{bmatrix}$$

$$=\begin{bmatrix}1+4+0 & 0+0+3 & 2+0+6\end{bmatrix}\begin{bmatrix}2\\4\\6\end{bmatrix}$$

$$=\begin{bmatrix}5 & 3 & 10\end{bmatrix}\begin{bmatrix}2\\4\\6\end{bmatrix}$$

$$=\begin{bmatrix}10+12+60\end{bmatrix}$$

$$=\begin{bmatrix}82\end{bmatrix}$$

Hence,

$$\begin{bmatrix}1 & 2 & 3\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\2 & 0 & 1\\0 & 1 & 2\end{bmatrix}\begin{bmatrix}2\\4\\6\end{bmatrix}=\begin{bmatrix}82\end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q5(iii)

$$\begin{bmatrix}1 & -1\\0 & 2\\2 & 3\end{bmatrix}\left(\begin{bmatrix}1 & 0 & 2\\2 & 0 & 1\end{bmatrix}-\begin{bmatrix}0 & 1 & 2\\1 & 0 & 2\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & -1\\0 & 2\\2 & 3\end{bmatrix}\left(\begin{bmatrix}1-0 & 0-1 & 2-2\\2-1 & 0-0 & 1-2\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & -1\\0 & 2\\2 & 3\end{bmatrix}\begin{bmatrix}1 & -1 & 0\\1 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1-1 & -1+0 & 0+1\\0+2 & 0+0 & 0-2\\2+3 & -2+0 & 0-3\end{bmatrix}$$

$$=\begin{bmatrix}0 & -1 & 1\\2 & 0 & -2\\5 & -2 & -3\end{bmatrix}$$

Hence,

$$\begin{bmatrix}1 & -1\\0 & 2\\2 & 3\end{bmatrix}\left(\begin{bmatrix}1 & 0 & 2\\2 & 0 & 1\end{bmatrix}-\begin{bmatrix}0 & 1 & 2\\1 & 0 & 2\end{bmatrix}\right)=\begin{bmatrix}0 & -1 & 1\\2 & 0 & -2\\5 & -2 & -3\end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q6

Given, $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, C \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = I_2 \qquad \text{---(i)}$$

$$B^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B^2 = I_2 \qquad \text{---(ii)}$$

$$C^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix}$$

$$C^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C^2 = I_2 \qquad \text{---(iii)}$$

Hence,

From equation (i), (ii) and (iii),

$$A^2 = B^2 = C^2 = I_2$$

Algebra of Matrices Ex 5.3 Q7

Given, $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$

$$3A^2 - 2B + I$$

$$= 3\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} - 2\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 3\begin{bmatrix} 4-3 & -2-2 \\ 6+6 & -3+4 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 3\begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-0+1 & -12+8+0 \\ 36+2+0 & 3-14+1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$$

Hence,

$$3A^2 - 2B + I = \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q8

Given, $A=\begin{bmatrix}4 & 2\\ -1 & -1\end{bmatrix}$

$$(A-2I)(A-3I)$$

$$=\left(\begin{bmatrix}4 & 2\\ -1 & -1\end{bmatrix}-2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right)\left(\begin{bmatrix}4 & 2\\ -1 & -1\end{bmatrix}-3\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right)$$

$$=\left(\begin{bmatrix}4 & 2\\ -1 & -1\end{bmatrix}-\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}\right)\left(\begin{bmatrix}4 & 2\\ -1 & -1\end{bmatrix}-\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}\right)$$

$$=\left(\begin{bmatrix}4-2 & 2-0\\ -1-0 & 1-2\end{bmatrix}\right)\left(\begin{bmatrix}4-3 & 2-0\\ -1-0 & 1-3\end{bmatrix}\right)$$

$$=\begin{bmatrix}2 & 2\\ -1 & -1\end{bmatrix}\begin{bmatrix}1 & 2\\ -1 & -2\end{bmatrix}$$

$$=\begin{bmatrix}2-2 & 4-4\\ -1+1 & -2+2\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$=0$$

Hence,

$$(A-2I)(A-3I)=0$$

Algebra of Matrices Ex 5.3 Q9

Given, $A=\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$

$$A^2=\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1+0 & 1+1\\0+0 & 0+1\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$$

$$A^3=A^2\cdot A$$

$$=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1+0 & 1+2\\0+0 & 0+1\end{bmatrix}$$

$$A^3=\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}$$

Hence,

$$A^2=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}, A^3=\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q10

Given, $A=\begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$

$$A^2=\begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}\begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$$

$$=\begin{bmatrix} a^2b^2-a^2b^2 & ab^3-ab^3 \\ -a^3b+a^3b & -a^2b^2+a^2b^2 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$=0$$

Hence,

$$A^2=0$$

Algebra of Matrices Ex 5.3 Q11

Given, $A=\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$$A^2=A.A$$

$$=\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$=\begin{bmatrix} \cos^2 2\theta-\sin^2 2\theta & \cos 2\theta\sin^2+\cos 2\theta\sin^2\theta \\ -\cos 2\theta\sin^2\theta-\sin^2\theta\cos^2\theta & -\sin^2 2\theta+\cos^2 2\theta \end{bmatrix}$$

$$=\begin{bmatrix} \cos 4\theta & 2\sin^2\theta\cos^2\theta \\ -2\sin^2\cos 2\theta & \cos 4\theta \end{bmatrix}$$

$$\left\{\text{since } \cos^2\theta-\sin^2\theta=\cos 2\theta\right\}$$

$$=\begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$$

$$\left\{\text{since } \sin^2\theta=2\sin\theta\cos\theta\right\}$$

Hence,

$$A^2=\begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q12

Given, $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}, B = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -2-3+5 & 6+9-15 & 10+15+25 \\ 1+4-5 & -3-12+15 & -5-20+25 \\ -1-3+4 & 3+9-12 & 5+15-20 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$AB = O_{3\times 3} \qquad ---(i)$$

$$BA = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -2-3+5 & 3+12-15 & 5+15-20 \\ 2+3-5 & -3-12+15 & -5-15+20 \\ -2-3+5 & 3+12-15 & 5+15-20 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$BA = O_{3\times 3} \qquad ---(ii)$$

From equation (i) and (ii),

$$AB = BA = O_{3\times 3}$$

Algebra of Matrices Ex 5.3 Q13

$$\text{Given, } A = \begin{bmatrix} 0 & c & -b \\ -c & o & a \\ b & -a & 0 \end{bmatrix}, B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix} \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 + abc - abc & 0 + b^2c - b^2c & 0 + bc^2 - bc^2 \\ -a^2c + 0 + a^2c & -abc + 0 + abc & -ac^2 + 0 + ac^2 \\ a^2b - a^2b + 0 & ab^2 - ab^2 + 0 & abc - abc + 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$AB = O_{3\times 3} \qquad \text{---(ii)}$$

From equation (i) and (ii),

$$AB = BA = O_{3\times 3}$$

Algebra of Matrices Ex 5.3 Q14

Given, $A=\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}, B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3-5 & -4-9+10 & -8-12+15 \\ -2-4+5 & 2+12-10 & 4+16-15 \\ 2+3-4 & -2-9+18 & -4-12+12 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$AB = A$$

$$BA = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2-4 & -6-8+12 & -10-10+16 \\ -2-3+4 & 3+12-12 & 5+15-16 \\ 2+2-3 & -3-8+9 & -5-10+12 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$BA = B$$

Algebra of Matrices Ex 5.3 Q15

Given, $A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$

$$A^2 = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3-5 & -1-3-5 & 1+3-5 \\ -3-9+15 & 3+9+15 & -3-9+15 \\ -5+15+25 & 5-15+25 & -5+15+25 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & -9 & -1 \\ 3 & 27 & 3 \\ 35 & 15 & 35 \end{bmatrix} \qquad \text{---(i)}$$

$$B^2 = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix} \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0+4-3 & 0-12+12 & 0-12+12 \\ 0-3+3 & 4+9-12 & 3+9-12 \\ 0+4-4 & -4-12+16 & -3-12+16 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{---(ii)}$$

Subtracting equation (ii) from equation (i),

$$A^2 - B^2 = \begin{bmatrix} -1 & -9 & -1 \\ 3 & 27 & 3 \\ 35 & 15 & 35 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-1 & -9-0 & -1-0 \\ 3-0 & 27-1 & 3-0 \\ 35-0 & 15-0 & 35-1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -9 & -1 \\ 3 & 26 & 3 \\ 35 & 15 & 34 \end{bmatrix}$$

Hence,

$$A^2 - B^2 = \begin{bmatrix} -2 & -9 & -1 \\ 3 & 26 & 3 \\ 35 & 15 & 34 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q16(i)

$$\text{Given, } A=\begin{bmatrix}1 & 2 & 0\\ -1 & 0 & 1\end{bmatrix}, B=\begin{bmatrix}1 & 0\\ -1 & 2\\ 0 & 3\end{bmatrix} \text{ and}$$

$$C=\begin{bmatrix}1\\ -1\end{bmatrix}$$

$$(AB)C=\left(\begin{bmatrix}1 & 2 & 0\\ -1 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ -1 & 2\\ 0 & 3\end{bmatrix}\right)\begin{bmatrix}1\\ -1\end{bmatrix}$$

$$=\begin{bmatrix}1-2+0 & 0+4+0\\ -1+0+0 & +0+0+3\end{bmatrix}\begin{bmatrix}1\\ -1\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 4\\ -1 & 3\end{bmatrix}\begin{bmatrix}1\\ -1\end{bmatrix}$$

$$=\begin{bmatrix}-1 & -4\\ -1 & -3\end{bmatrix}$$

$$(AB)C=\begin{bmatrix}-5\\ -4\end{bmatrix} \qquad ---(i)$$

$$A(BC)=\begin{bmatrix}1 & 2 & 0\\ -1 & 0 & 1\end{bmatrix}\left(\begin{bmatrix}1 & 0\\ -1 & 2\\ 0 & 3\end{bmatrix}\begin{bmatrix}1\\ -1\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & 2 & 0\\ -1 & 0 & 1\end{bmatrix}\begin{bmatrix}1+0\\ -1-2\\ 0-3\end{bmatrix}$$

$$=\begin{bmatrix}1 & 2 & 0\\ -1 & 0 & 1\end{bmatrix}\begin{bmatrix}1\\ -3\\ -3\end{bmatrix}$$

$$=\begin{bmatrix}1-6+0\\ -1+0-3\end{bmatrix}$$

$$A(BC)=\begin{bmatrix}-5 \end{bmatrix} \qquad ---(ii)$$

$[-4]$

From equation (i) and (ii) we get,

$$(AB)C = A(BC)$$

(ii) Given,

$$A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(AB)C = \left(\begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}\right)\begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+6 & -4+2-3 & 4+4+3 \\ 1+0+4 & -1+1-2 & 1+2+2 \\ 3+0+2 & -3+0-1 & 3+0+1 \end{bmatrix}\begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -5 & 11 \\ 5 & -2 & 5 \\ 5 & -4 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 8 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10-15+0 & 20+0+0 & -10+5+11 \\ 5-6+0 & 10+0+0 & -5-2+5 \\ 5-12+0 & 10+0+0 & -5-4+4 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -5 & 20 & -4 \\ -1 & 10 & -2 \\ -7 & 10 & -5 \end{bmatrix} \qquad \text{---(i)}$$

$$A(BC) = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix}\left(\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1-3+0 & 2+0+0 & -1-1+1 \\ 0+3+0 & 0+0+0 & 0+1+2 \\ 2-3+0 & 4+0+0 & -2-1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix}\begin{bmatrix} -2 & 2 & -1 \\ 3 & 0 & 3 \\ -1 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -8+6-3 & 8+0+12 & -4+6-6 \\ -2+3-2 & 2+0+8 & -1+3-4 \\ -6+0-1 & 6+0+4 & -3+0-2 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -5 & 20 & -4 \\ -1 & 10 & -2 \\ -7 & 10 & -5 \end{bmatrix} \qquad \text{--(ii)}$$

From equation (i) and (ii),

$$(AB)C = A(BC)$$

Algebra of Matrices Ex 5.3 Q17(i)

Given, $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$

$$A(B+C) = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\left(\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -1+0 & 0+1 \\ 2+1 & 1-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1-3 & 1+0 \\ 0+6 & 0+0 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} -4 & 1 \\ 6 & 0 \end{bmatrix} \qquad ---(i)$$

$$AB = AC = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-2 & 0-1 \\ 0+4 & 0+2 \end{bmatrix} + \begin{bmatrix} 0+-1 & 1+1 \\ 0+2 & 0-2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -3-1 & -1+2 \\ 4+2 & 2-2 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} -4 & 1 \\ 6 & 0 \end{bmatrix} \qquad ---(ii)$$

Using equation (i) and (ii),

$$A(B+C) = AB + AC$$

Algebra of Matrices Ex 5.3 Q17(ii)

Given, $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$$A\left(B + C\right) = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}\left(\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 0+1 & 1-1 \\ 1+0 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 0+2 \\ 1+1 & 0+2 \\ -1+2 & 0+4 \end{bmatrix}$$

$$A\left(B + C\right) = \begin{bmatrix} 1 & -2 \\ 2 & 2 \\ 1 & 4 \end{bmatrix} \qquad \text{---(i)}$$

$$AB + AC = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1 & 2-1 \\ 0+1 & 1+1 \\ 0+2 & -1+2 \end{bmatrix} + \begin{bmatrix} 2+0 & -2-1 \\ 1+0 & -1+1 \\ -1+0 & 1+2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ 1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1+2 & 1-3 \\ 1+1 & 2+0 \\ 2-1 & 1+3 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 1 & -2 \\ 2 & 2 \\ 1 & 4 \end{bmatrix} \qquad \text{---(ii)}$$

From equation (i) and (ii),

$$A\left(B + C\right) = AB + AC$$

Algebra of Matrices Ex 5.3 Q18

Given,

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A(B-C) = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}\left[\begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\right]$$

$$= \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0-1 & 5-5 & -4-2 \\ -2+1 & 1-1 & 3-0 \\ -1-0 & 0+1 & 2-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 & -6 \\ -1 & 0 & 3 \\ -1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1+0+2 & 0+0-2 & -6+0-2 \\ -3+1+0 & 0+0+0 & -18-3+0 \\ 2-1-1 & 0+0+1 & 12+3+1 \end{bmatrix}$$

$$A(B-C) = \begin{bmatrix} 1 & -2 & -8 \\ -2 & 0 & -21 \\ 0 & 1 & 16 \end{bmatrix} \qquad ---(i)$$

$$AB - AC = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+2 & 5+0+0 & -4+0-4 \\ 0+2+0 & 15-1+0 & -12-3+0 \\ 0-2-1 & -10+1+0 & 8+3+2 \end{bmatrix} - \begin{bmatrix} 1+0+0 & 5+0+2 & 2+0-2 \\ 3+1+0 & 15-1+0 & 6+0+0 \\ 0-2-1 & -10+1+1 & -4+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 5 & -8 \\ 2 & 14 & -15 \\ -3 & -9 & 13 \end{bmatrix} - \begin{bmatrix} 1 & 7 & 0 \\ 4 & 14 & 6 \\ -3 & -10 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 5-7 & -8-0 \\ 2-4 & 14-14 & -14-6 \\ -3+3 & -9+10 & 13+3 \end{bmatrix}$$

$$AB - AC = \begin{bmatrix} 1 & -2 & -8 \\ -2 & 0 & -21 \\ 0 & 1 & 16 \end{bmatrix} \qquad ---(ii)$$

From equation (i) and (ii),

$$A(B-C) = AB - AC$$

Algebra of Matrices Ex 5.3 Q19

Given,

$$A=\begin{bmatrix}0&1&0\\2&0&2_{\,0}\\0&3&2\\4&0&4\end{bmatrix}\begin{bmatrix}2&-1\\-3&2\\4&3\end{bmatrix}\begin{bmatrix}0&1&-1&2&-2\\3&-3&4&-4&0\end{bmatrix}$$

$$A=\begin{bmatrix}0-3+0&0+2+0\\4+0+8&-2+0+6\\0-9+8&0+6+6\\8+0+16&-4+0+12\end{bmatrix}\begin{bmatrix}0&1&-1&2&-2\\3&-3&4&-4&0\end{bmatrix}$$

$$A=\begin{bmatrix}-3&2\\12&4\\-1&12\\24&8\end{bmatrix}\begin{bmatrix}0&1&-1&2&-2\\3&-3&4&-4&0\end{bmatrix}$$

$$A=\begin{bmatrix}0+6&-3-6&3+8&-6-8&6+0\\0+12&12-12&-12+16&24-16&-24+0\\0+36&-1-36&1+48&-2-48&2+0\\0+24&24-24&-24+34&48-32&-48+0\end{bmatrix}$$

$$A=\begin{bmatrix}6&-9&11&-14&6\\12&0&4&8&-24\\36&-37&49&-50&2\\24&0&8&16&-48\end{bmatrix}$$

Here, $a_{43}=8, a_{22}=0$

Algebra of Matrices Ex 5.3 Q20

Given,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$\begin{aligned}
A^2 &= A \times A \\
&= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix} \\
&= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+1+0 \\ 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
A^3 &= A^2 \times A \\
&= \begin{bmatrix} 0 & 0 & 1 \\ p & q & r \\ pr & p+qr & q+r^2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix} \\
&= \begin{bmatrix} 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \\ 0+0+pq+pr^2 & pr+0+q^2+qr^2 & 0+p+qr+qr+r^2 \end{bmatrix}
\end{aligned}$$

$$A^3 = \begin{bmatrix} p & q & r \\ pr & p+qr & q+r^2 \\ pq+pr^2 & pr+q^2+qr^2 & p+2qr+r^2 \end{bmatrix} \quad \text{---(i)}$$

$$\begin{aligned}
&pI + qA + rA^2 \\
&= p\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + q\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix} + r\begin{bmatrix} 0 & 0 & 1 \\ p & q & r \\ pr & p+qr & q+r^2 \end{bmatrix} \\
&= \begin{bmatrix} p+0+0 & 0+q+0 & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \\ 0+pq+pr^2 & 0+q^2+pr+qr^2 & p+qr+qr+r^2 \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
&pI + qA + rA^2 \\
&= \begin{bmatrix} p & q & r \\ pr & p+qr & q+r^2 \\ pq+pr^2 & pr+q^2+qr^2 & p+2qr+r^2 \end{bmatrix}
\end{aligned}$$

Algebra of Matrices Ex 5.3 Q21

Given, $\quad W$ is a complex cube root of unity,

$$\left(\begin{bmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{bmatrix} + \begin{bmatrix} w & w^2 & 1 \\ w^2 & 1 & w \\ w & w^2 & 1 \end{bmatrix}\right)\begin{bmatrix} 1 \\ w \\ w^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+w & w+w^2 & w^2+1 \\ w+w^2 & w^2+1 & 1+w \\ w^2+w & 1+w^2 & w+1 \end{bmatrix}\begin{bmatrix} 1 \\ w \\ w^2 \end{bmatrix}$$

$$= \begin{bmatrix} -w^2 & -1 & -w \\ -1 & -w & -w^2 \\ -1 & -w & -w^2 \end{bmatrix}\begin{bmatrix} 1 \\ w \\ w^2 \end{bmatrix} \qquad \begin{Bmatrix} \text{Since } 1+w+w^2 = 0 \\ \text{and } w^3 = 1 \end{Bmatrix} \qquad \text{---(i)}$$

$$= \begin{bmatrix} -w^2 - w - w^3 \\ -1 - w^2 - w^4 \\ -1 - w^2 - w^2 \end{bmatrix}$$

$$= \begin{bmatrix} -w\left(1+w+w^2\right) \\ -1 - w^2 - w^3 w \\ -1 - w^2 - w^3 w \end{bmatrix}$$

$$= \begin{bmatrix} -w.0 \\ -1 - w^2 - w \\ -1 - w^2 - w \end{bmatrix} \qquad \{\text{using reason (i)}\}$$

$$= \begin{bmatrix} 0 \\ -\left(1+w+w^2\right) \\ -\left(1+w+w^2\right) \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -(0) \\ -(0) \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q22

Given, $\quad A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$

$$A^2 = A, A$$

$$= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3-5 & -6-12+15 & -10-15+20 \\ -2-4+5 & 3+16-15 & 5+20-20 \\ 2+3-4 & -3-12+12 & -5-15+16 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$= A$$

Hence,

$$A^2 = A$$

Algebra of Matrices Ex 5.3 Q23

Given, $A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$

$$A^2 = A.A$$

$$= \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}\begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 16-3-12 & -4+0+4 & -16+4+12 \\ 12+0-12 & -3+0+4 & -12+0+12 \\ 12-3-9 & -3+0+3 & -12+4+9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= I_3$$

Hence,

$$A^2 = I_3$$

Algebra of Matrices Ex 5.3 Q24(i)

Given,

$$\begin{bmatrix}1 & 1 & x\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 1 & 0\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix}1+0+2x & 0+2+x & 2+1+0\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix}2x+1 & 2+x & 3\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix}2x+1+2+x+3\end{bmatrix} = 0$$

$$\Rightarrow \quad 3x+6 = 0$$

$$\Rightarrow \quad x = -\frac{6}{3}$$

$$\Rightarrow \quad x = -2$$

Algebra of Matrices Ex 5.3 Q24(ii)

Given that $\begin{bmatrix}2 & 3\\5 & 7\end{bmatrix}\begin{bmatrix}1 & -3\\-2 & 4\end{bmatrix} = \begin{bmatrix}-4 & 6\\-9 & x\end{bmatrix}$

By multiplication of matrices, we have,

$$\begin{bmatrix}2\times1+3\times(-2) & 2\times(-3)+3\times4\\5\times1+7\times(-2) & 5\times(-3)+7\times4\end{bmatrix} = \begin{bmatrix}-4 & 6\\-9 & x\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}-4 & 6\\-9 & 13\end{bmatrix} = \begin{bmatrix}-4 & 6\\-9 & x\end{bmatrix}$$

$$\Rightarrow x = 13$$

Algebra of Matrices Ex 5.3 Q25

Given,

$$\begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix} 2x+4+0 & x+0+2 & 2x+8-4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix} 2x+4 & x+2 & 2x+4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow \quad \left[(2x+4)x + 4(x+2) - 1(2x+4)\right] = 0$$

$$\Rightarrow \quad 2x^2 + 4x + 4x + 8 - 2x - 4 = 0$$

$$\Rightarrow \quad 2x + 6x + 4 = 0$$

$$\Rightarrow \quad 2x^2 + 2x + 4x + 4 = 0$$

$$\Rightarrow \quad 2x(x+1) + 4(x+1) = 0$$

$$\Rightarrow \quad (x+1)(2x+4) = 0$$

$$\Rightarrow \quad x+1 = 0 \; or \; 2x+4 = 0$$

$$\Rightarrow \quad x = -1 \text{ or } x = -2$$

Hence, $x = -1$ or -2

Algebra of Matrices Ex 5.3 Q26

Given,

$$\begin{bmatrix}1 & -1 & x\end{bmatrix}\begin{bmatrix}0 & 1 & -1\\2 & 1 & 3\\1 & 1 & 1\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix}0-2+x & 1-1+x & -1-3+x\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix}x-2 & x & x-4\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix} = 0$$

$$\Rightarrow \quad \left[0(x-2)+x.1+1.(x-4)\right] = 0$$

$$\Rightarrow \quad 0+x+x-4 = 0$$

$$\Rightarrow \quad 2x-4 = 0$$

$$\Rightarrow \quad x = 2$$

Hence,

$$x = 2$$

Algebra of Matrices Ex 5.3 Q27

Given, $A = \begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}, I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$$A^2 - A + 2I$$

$$= \begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}\begin{bmatrix}3 & -2\\4 & -2\end{bmatrix} - \begin{bmatrix}3 & -2\\4 & -2\end{bmatrix} + 2\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}9-8 & -6+4\\12-8 & -8+4\end{bmatrix} - \begin{bmatrix}3 & -2\\4 & -2\end{bmatrix} + \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}$$

$$= \begin{bmatrix}1 & -2\\4 & -4\end{bmatrix} - \begin{bmatrix}3 & -2\\4 & -2\end{bmatrix} + \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}$$

$$= \begin{bmatrix}1-3+2 & -2+2+0\\4-4+0 & -4+2+2\end{bmatrix}$$

$$= \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$

$$= 0$$

Hence,

$$A^2 - A + 2I = 0$$

Algebra of Matrices Ex 5.3 Q28

Given, $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

And

$A^2 = 5A + \lambda I$

$$\Rightarrow \quad \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 9-1 & 3+2 \\ -3+2 & -1+4 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 15+\lambda & 5 \\ -5 & 10+\lambda \end{bmatrix}$$

Since, Corresponding entries of equal matrices are equal, So

$8 = 15 + \lambda$

$\lambda = 8 - 15$

$\lambda = -7$

Algebra of Matrices Ex 5.3 Q29

Given, $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$A^2 - 5A + 7I_2$

$$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$= 0$

Hence, $A^2 - 5A + 7I_2 = 0$

Algebra of Matrices Ex 5.3 Q30

Given, $A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$

$$A^2 - 2A + 3I_2$$
$$= \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} - 2\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 4-3 & 6+0 \\ -2+0 & -3+0 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 1-4+3 & 6-6+0 \\ -2+2+0 & -3+0+3 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$= 0$$

Hence,
$$A^2 - 2A + 3I_2 = 0$$

Algebra of Matrices Ex 5.3 Q31

Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

$$A^3 = A^2.A$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 14+12 & 21+24 \\ 8+7 & 12+14 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix}$$

Hence, $A^3 - 4A^2 + A$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - 4\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 26-28+2 & 45-48+3 \\ 15-16+1 & 26-28+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

So, $A^3 - 4A^2 + A = 0$

Algebra of Matrices Ex 5.3 Q32

Given, $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$

$$A^2 - 12A - I$$

$$= \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix} - 12\begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 25+36 & 15+21 \\ 60+84 & 36+49 \end{bmatrix} - \begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 61 & 36 \\ 144 & 85 \end{bmatrix} - \begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 61-60-1 & 36-36-0 \\ 144-144-0 & 85-84-1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

Since $A^2 - 12A - I = 0$

So,

A is a root of the equation $A^2 - 12A - I = 0$

Algebra of Matrices Ex 5.3 Q33

Given, $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$$A^2 = 5A - 14I$$

$$= \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 5\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9+20 & -15-10 \\ -12-8 & 20+4 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 29-15-14 & -25+25-0 \\ -20+20-0 & 24-10-14 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

So,

$$A^2 - 5A - 14I = 0$$

Algebra of Matrices Ex 5.3 Q34

Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

$$A^3 = A^2.A$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 14+12 & 21+24 \\ 8+7 & 12+14 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix}$$

Hence, $A^3 - 4A^2 + A$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - 4\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 26-28+2 & 45-48+3 \\ 15-16+1 & 26-28+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

So, $\quad A^3 - 4A^2 + A = 0$

It is given that $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$

$$\therefore A^2 = A\cdot A = \begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$$

$$= \begin{bmatrix}3(3)+1(-1) & 3(1)+1(2)\\ -1(3)+2(-1) & -1(1)+2(2)\end{bmatrix}$$

$$= \begin{bmatrix}9-1 & 3+2\\ -3-2 & -1+4\end{bmatrix} = \begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}$$

$$\therefore \text{L.H.S.} = A^2 - 5A + 7I$$

$$= \begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix} - 5\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix} + 7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix} - \begin{bmatrix}15 & 5\\ -5 & 10\end{bmatrix} + \begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$$

$$= \begin{bmatrix}-7 & 0\\ 0 & -7\end{bmatrix} + \begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$$

$$= \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$= O = \text{R.H.S.}$$

$$\therefore A^2 - 5A + 7I = O$$

$Since\ A^2 - 5A + 7I = O,\ we\ have$

$A^2 = 5A - 7I$

$Therefore,\ A^4 = A^2 \times A^2 = (5A - 7I)(5A - 7I)$

$\Rightarrow A^4 = 25A^2 - 35AI - 35IA + 49I$

$\Rightarrow A^4 = 25A^2 - 70A + 49I$

$\Rightarrow A^4 = 25(5A - 7I) - 70A + 49I$

$\Rightarrow A^4 = 125A - 175I - 70A + 49I$

$\Rightarrow A^4 = 55A - 126I$

$$\Rightarrow A^4 = 55\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 126\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 165 & 55 \\ -55 & 110 \end{bmatrix} - \begin{bmatrix} 126 & 0 \\ 0 & 126 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 165 - 126 & 55 - 0 \\ -55 - 0 & 110 - 126 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q35

$$A^2 = A \cdot A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 3(3)+(-2)(4) & 3(-2)+(-2)(-2) \\ 4(3)+(-2)(4) & 4(-2)+(-2)(-2) \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

Now $A^2 = kA - 2I$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

Comparing the corresponding elements, we have:

$3k - 2 = 1$

$\Rightarrow 3k = 3$

$\Rightarrow k = 1$

Thus, the value of k is 1.

Algebra of Matrices Ex 5.3 Q36

Here,

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

And

$$A^2 - 8A + kI = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} - 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} + k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 1+0 & 0+0 \\ -1-7 & 0+49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1-8+k & 0+0+0 \\ -8+8+0 & 49-56+k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -7+k & 0 \\ 0 & -7+k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Since,

corresponding entries of equal matrices are equal, so

$-7 + k = 0$

$k = 7$

Algebra of Matrices Ex 5.3 Q37

Given,

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \text{ and } f(x) = x^2 - 2x - 3$$

$$f(A) = A^2 - 2A - 3I$$

$$= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4 & 2+2 \\ 2+2 & 4+1 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5-2-3 & 4-4-0 \\ 4-4-0 & 5-2-3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

So,

$$f(A) = 0$$

Algebra of Matrices Ex 5.3 Q38

Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Given,

$$A^2 = \lambda A + \mu I$$

$$\Rightarrow \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \lambda \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \mu \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} = \begin{bmatrix} 2\lambda & 3\lambda \\ \lambda & 2\lambda \end{bmatrix} + \begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix}$$

$$\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 2\lambda + \mu & 3\lambda \\ \lambda & 2\lambda + \mu \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, so

$$2\lambda + \mu = 7 \qquad \text{---(i)}$$

$$\lambda = 4 \qquad \text{---(ii)}$$

Put λ from equation (ii) in equation (i),

$2(4) + \mu = 7$

$\mu = 7 - 8$

$\mu = -1$

Hence. $\lambda = 4, \mu = -1$

Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^3 - 4A^2 + A$

$$= \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 14+12 & 21+24 \\ 8+7 & 12+14 \end{bmatrix} - \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 26-7+2 & 45-12+3 \\ 15-4+1 & 26-7+2 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 36 \\ 12 & 21 \end{bmatrix}$$

Hence, $A^3 - 4A^2 + A = \begin{bmatrix} 21 & 36 \\ 12 & 21 \end{bmatrix}$

Algebra of Matrices Ex 5.3 Q39

Given,

$$\begin{bmatrix}2 & 0 & 7\\0 & 1 & 0\\1 & -2 & 1\end{bmatrix}\begin{bmatrix}-x & 14x & 7x\\0 & 1 & 0\\x & -4x & -2x\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix}-2x+0+7x & 28x+0-28x & 14x+0-14x\\0+0+0 & 0+1+0 & 0+0+0\\-x+0+x & 14x-2-4x & 7x+0-2x\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix}5x & 0 & 0\\0 & 1 & 0\\0 & 10x-2 & 5x\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$$5x = 1 \qquad \text{and } 10x - 2 = 0$$

$$\Rightarrow \quad x = \frac{1}{5} \qquad \text{and } x = \frac{1}{5}$$

Hence, $x = \frac{1}{5}$

Algebra of Matrices Ex 5.3 Q40(i)

Here,

$$\begin{bmatrix}x & 1\end{bmatrix}\begin{bmatrix}1 & 0\\-2 & -3\end{bmatrix}\begin{bmatrix}x\\5\end{bmatrix}=0$$

$$\Rightarrow \quad \begin{bmatrix}x-2 & 0-3\end{bmatrix}\begin{bmatrix}x\\5\end{bmatrix}=0$$

$$\Rightarrow \quad \begin{bmatrix}(x-2)x-15\end{bmatrix}=0$$

$$\Rightarrow \quad x^2-2x-15=0$$

$$\Rightarrow \quad x^2-5x+3x-15=0$$

$$\Rightarrow \quad x(x-5)+3(x-5)=0$$

$$\Rightarrow \quad (x-5)(x+3)=0$$

$$\Rightarrow \quad x-5=0 \qquad \text{or} \qquad x+3=0$$

$$\Rightarrow \quad x=5 \qquad \text{or} \qquad x=-3$$

So,

$$x = 5 \text{ or } -3$$

Algebra of Matrices Ex 5.3 Q40(ii)

We have:

$$\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} x+0-2 & 0-10+0 & 2x-5-3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow \left[x(x-2)-40+2x-8\right] = O$$

$$\Rightarrow \left[x^2-2x-40+2x-8\right] = [0]$$

$$\Rightarrow \left[x^2-48\right] = [0]$$

$$\therefore x^2-48=0$$

$$\Rightarrow x^2=48$$

$$\Rightarrow x=\pm 4\sqrt{3}$$

Algebra of Matrices Ex 5.3 Q41

Given, $$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix}$$

$$A^2 - 4A + 3I_3$$
$$= \begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1+6+0 & 2-8+0 & 0+10+0 \\ 3-12+0 & 6+16-5 & 0-20+15 \\ 0-3+0 & 0+4-3 & 0-5+9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 0 \\ 12 & -16 & 20 \\ 0 & -4 & 12 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 7 & -6 & 10 \\ -9 & 17 & -5 \\ -3 & 1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 0 \\ 12 & -16 & 20 \\ 0 & -4 & 12 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 7-4+3 & -6-8+0 & 10-0+0 \\ -9-12+0 & 17+16+3 & -5-20+0 \\ -3-0+0 & 1+4+0 & 4-12+3 \end{bmatrix}$$
$$= \begin{bmatrix} 6 & -14 & 10 \\ -21 & 36 & -25 \\ -3 & 5 & -5 \end{bmatrix}$$

Hence,
$$A^2 - 4A + 3I_3 = \begin{bmatrix} 6 & -14 & 10 \\ -21 & 36 & -25 \\ -3 & 5 & -5 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q42

Given, $$A = \begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix}$$

And $f(x) = x^2 - 2x$

$$\Rightarrow \quad f(A) = A^2 - 2A$$
$$\Rightarrow \quad f(A) = \begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix} - 2\begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix}$$
$$\Rightarrow \quad f(A) = \begin{bmatrix} 0+4+0 & 0+5+4 & 0+0+6 \\ 0+20+0 & 4+25+0 & 8+0+0 \\ 0+8+0 & 0+10+6 & 0+0+9 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 4 \\ 8 & 10 & 0 \\ 0 & 4 & 6 \end{bmatrix}$$
$$\Rightarrow \quad f(A) = \begin{bmatrix} 4 & 9 & 6 \\ 20 & 29 & 8 \\ 8 & 16 & 9 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 4 \\ 8 & 10 & 0 \\ 0 & 4 & 6 \end{bmatrix}$$
$$\Rightarrow \quad f(A) = \begin{bmatrix} 4-0 & 9-2 & 6-4 \\ 20-8 & 29-10 & 8-0 \\ 8-0 & 16-4 & 9-6 \end{bmatrix}$$
$$\Rightarrow \quad f(A) = \begin{bmatrix} 4 & 7 & 2 \\ 12 & 19 & 8 \\ 8 & 12 & 3 \end{bmatrix}$$

Given,

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

And $\quad f(x) = x^3 + 4x^2 - x$

$\Rightarrow \quad f(x) = A^3 + 4A^2 - A \qquad$ ---(i)

$$A^2 = A \times A$$

$$= \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+2+2 & 0-3-2 & 0+0+0 \\ 0-6+0 & 2+9+0 & 4+0+0 \\ 0-2+0 & 1+3+0 & 0+0+0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4 & -5 & 0 \\ -6 & 11 & 4 \\ -2 & 4 & 2 \end{bmatrix}$$

$$A^3 = A^2 \times A$$

$$= \begin{bmatrix} 4 & -5 & 0 \\ -6 & 11 & 4 \\ -2 & 4 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0-10+0 & 4+15+0 & 8+0+0 \\ 0+22+4 & -6-33-4 & -12+0+0 \\ 0+8+2 & -2-12-2 & -4+0+0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} -10 & 19 & 8 \\ 26 & -43 & -12 \\ 10 & -16 & -4 \end{bmatrix}$$

Put the value of A, A^2, A^3 in equation (i)

$$f(A) = A^3 + 4A^2 - A$$

$$= \begin{bmatrix} -10 & 19 & 8 \\ 26 & -43 & -12 \\ 10 & -16 & -4 \end{bmatrix} + 4\begin{bmatrix} 4 & -15 & 0 \\ -6 & 11 & 4 \\ -2 & 4 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -10+16-0 & 19-20-1 & 8+0-2 \\ 26-24-2 & -43+44+3 & -12+16+0 \\ 10-8-1 & -16+16+1 & -4+8-0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -2 & 6 \\ 0 & 4 & 4 \\ 1 & 1 & 4 \end{bmatrix}$$

Hence,

$$f(A) = \begin{bmatrix} 6 & -2 & 6 \\ 0 & 4 & 4 \\ 1 & 1 & 4 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q44

$$Given\ that,\ A=\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix} and\ f(x)=x^3-6x^2+7x+2$$

$$Therefore,\ f(A)=A^3-6A^2+7A+2I_3$$

$$First\ find\ A^2:$$

$$A^2=A\times A=\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\times\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}=\begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}$$

$$Now,\ Let\ us\ find\ A^3:$$

$$A^3=A^2\times A=\begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}\times\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}=\begin{bmatrix}21&0&34\\12&8&23\\34&0&55\end{bmatrix}$$

$$Thus,$$

$$f(A)=A^3-6A^2+7A+2I_3$$

$$=\begin{bmatrix}21&0&34\\12&8&23\\34&0&55\end{bmatrix}-6\begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}+7\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}+2\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}21&0&34\\12&8&23\\34&0&55\end{bmatrix}-\begin{bmatrix}30&0&48\\12&24&30\\48&0&78\end{bmatrix}+\begin{bmatrix}7&0&14\\0&14&7\\14&0&21\end{bmatrix}+\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}$$

$$=\begin{bmatrix}21-30+7+2&0&34-48+14+0\\12-12+0&8-24+14+2&23-30+7+0\\34-48+14+0&0&55-78+21+2\end{bmatrix}$$

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}=O$$

$$Thus,\ A\ is\ a\ root\ of\ the\ polynomial.$$

Algebra of Matrices Ex 5.3 Q45

Given,

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$A^2 - 4A - 5I$

$$= \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9-4-5 & 8-8-0 & 8-8-0 \\ 8-8-0 & 9-4-5 & 8-8-0 \\ 8-8-0 & 8-8-0 & 9-4-5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= 0$$

Hence,

$$A^2 - 4A - 5I = 0$$

Algebra of Matrices Ex 5.3 Q46

Given,

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$A^2 - 7A + 10I_3$

$$= \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} - 7\begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} + 10\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9+2+0 & 6+8+0 & 0+0+0 \\ 3+4+0 & 2+16+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+25 \end{bmatrix} - \begin{bmatrix} 21 & 14 & 0 \\ 7 & 28 & 0 \\ 0 & 0 & 35 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix} - \begin{bmatrix} 21 & 14 & 0 \\ 7 & 28 & 0 \\ 0 & 0 & 35 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 11-21+10 & 14-14+0 & 0-0+0 \\ 7-7+0 & 18-28+10 & 0-0+0 \\ 0-0+0 & 0-0+0 & 25-35+10 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$= 0$

Hence,

$$A^2 - 7A + 10I_3 = 0$$

Algebra of Matrices Ex 5.3 Q47

Given,

$$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} x & y \\ z & u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 5x - 7z & 5y - 7u \\ -2x + 3z & -2y + 3u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$$5x - 7z = -16 \quad \text{---(i)}$$

$$-2x + 3z = 7 \quad \text{---(ii)}$$

$$5y - 7u = -6 \quad \text{---(iii)}$$

$$-2y + 3u = 2 \quad \text{---(iv)}$$

Solving equation (i) and (ii)

$$10x - 14z = -32$$

$$\underline{-10x + 15z = 35}$$

$$z = 3$$

Put the value of z in equation (i)

$$5x - 7(3) = -16$$

$$\Rightarrow \quad 5x = 16 + 21$$

$$\Rightarrow \quad 5x = 5$$

$$\Rightarrow \quad x = 1$$

Solving equation (iii) and (iv)

$$10y - 14u = -12$$

$$\underline{-10y + 15u = 10}$$

$$u = -2$$

Put the value of u in equation (iii)

$$5y - 7u = -6$$

$$\Rightarrow \quad 5y - 7(-2) = -6$$

$$\Rightarrow \quad 5y + 14 = -6$$

$$\Rightarrow \quad 5y = -20$$

$$\Rightarrow \quad y = -4$$

Algebra of Matrices Ex 5.3 Q48

Given,

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

Since, $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}_{2\times 2}$ $\quad A = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}_{2\times 3}$

$\Rightarrow \quad A$ is a matrix of order 2×3

So,

$$\text{Let } A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} a+d & b+e & c+c \\ 0+d & 0+e & 0+f \end{bmatrix} = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$d = 1,\ e = 0,\ f = 1$

And $\quad a + d = 3$

$a + 1 = 3$

$a = 3 - 1$

$a = 2$

$b + e = 3$

$b + 0 = 3$

$b = 3$

And $\quad c + f = 5$

$c + 1 = 5$

$c = 4$

Hence,

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 1 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q48(ii)

It is given that:

$$A\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

The matrix given on the R.H.S. of the equation is a 2×3 matrix and the one given on the L.H.S. of the equation is a 2×3 matrix. Therefore, X has to be a 2×2 matrix.

Now, let $X=\begin{bmatrix}a & c\\ b & d\end{bmatrix}$

Therefore, we have:

$$\begin{bmatrix}a & c\\ b & d\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}a+4c & 2a+5c & 3a+6c\\ b+4d & 2b+5d & 3b+6d\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

Equating the corresponding elements of the two matrices, we have:

$a+4c=-7,\quad 2a+5c=-8,\quad 3a+6c=-9$

$b+4d=2,\quad 2b+5d=4,\quad 3b+6d=6$

Now, $a+4c=-7 \Rightarrow a=-7-4c$

$\therefore 2a+5c=-8 \Rightarrow -14-8c+5c=-8$

$\Rightarrow -3c=6$

$\Rightarrow c=-2$

$\therefore a=-7-4(-2)=-7+8=1$

Now, $b+4d=2 \Rightarrow b=2-4d$

$\therefore 2b+5d=4 \Rightarrow 4-8d+5d=4$

$\Rightarrow -3d=0$

$\Rightarrow d=0$

$\therefore b=2-4(0)=2$

Thus, $a=1$, $b=2$, $c=-2$, $d=0$

Hence, the required matrix X is $\begin{bmatrix}1 & -2\\ 2 & 0\end{bmatrix}$.

Algebra of Matrices Ex 5.3 Q48(iii)

We know that two matrices B and C are eligible for the product BC only when number of columns of B is equal to number of rows in C. So, from the given definition we can conclude that the order of matrix A is 1×3 i.e. we can assume $A = \begin{bmatrix}x_1 & x_2 & x_3\end{bmatrix}$.

Therefore,

$$\begin{bmatrix}4\\1\\3\end{bmatrix}_{3\times1}\begin{bmatrix}x_1 & x_2 & x_3\end{bmatrix}_{1\times3}=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}_{3\times3},$$

$$\Rightarrow\quad\begin{bmatrix}4\times(x_1) & 4\times(x_2) & 4\times(x_3)\\ 1\times(x_1) & 1\times(x_2) & 1\times(x_3)\\ 3\times(x_1) & 3\times(x_2) & 3\times(x_3)\end{bmatrix}_{3\times3}=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}_{3\times3}$$

$$\Rightarrow\quad\begin{bmatrix}4x_1 & 4x_2 & 4x_3\\ x_1 & x_2 & x_3\\ 3x_1 & 3x_2 & 3x_3\end{bmatrix}_{3\times3}=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}_{3\times3}$$

$\Rightarrow \quad 4x_1 = -4, \quad 4x_2 = 8, \quad 4x_3 = 4$

Solving $\quad x_1 = -1$, $x_2 = 2$, $x_3 = 1$

So, matrix $A = \begin{bmatrix}-1 & 2 & 1\end{bmatrix}$.

Algebra of Matrices Ex 5.3 Q48(iv)

Using matrix multiplication,

Let, $A_1 = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}$, $A_2 = \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and $A_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

Now, $A_1.A_2 = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} =$

$$= \begin{bmatrix} (2\times -1)+(1\times -1)+(3\times 0) & (2\times 0)+(1\times 1)+(3\times 1) & (2\times -1)+(1\times 0)+(3\times 1) \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 4 & 1 \end{bmatrix}$$

and $(A_1.A_2)A_3 = \begin{bmatrix} -3 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$$= \begin{bmatrix} (-3\times 1)+(4\times 0)+(1\times -1) \end{bmatrix}$$

$$(A_1.A_2)A_3 = \begin{bmatrix} -4 \end{bmatrix} = A$$

Therefore matrix $A = \begin{bmatrix} -4 \end{bmatrix}$

Note : The problem can also be solved by calculating (A_2A_3) first then pre multiplying it with A_1 as matrix multiplication is associative but one must not change the order of multiplication.

Algebra of Matrices Ex 5.3 Q49

Let, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given,

$$A\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+b & -2a+4b \\ c+d & -2c+4d \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$$a+b = 6 \quad ---(i)$$

$$-2a+4b = 0 \quad ---(ii)$$

$$c+d = 0 \quad ---(iii)$$

$$-2c+4d = 6 \quad ---(iv)$$

Solving equation (i) and (ii)

$$4a+4b = 24$$

$$\begin{array}{l} -2a+4b = 0 \\ \ (+) \quad\ (-) \\ \hline 6a \qquad = 24 \end{array}$$

$$\Rightarrow a = \frac{24}{6}$$

$$a = 4$$

Put $a = 4$ in equation (i)

$$a + b = 6$$
$$4 + b = 6$$
$$b = 6 - 4$$
$$b = 2$$

Solving equation (iii) and (iv)

$$\begin{aligned} 2c + 2d &= 0 \\ -2c + 4d &= 6 \\ \hline 6d &= 6 \end{aligned}$$

$$d = \frac{6}{6}$$
$$d = 1$$

Put $d = 1$ in equation (iii)

$$c + d = 0$$
$$c = -1$$

Hence,

$$A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q50

Given,

$$A = \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$$

$$A^2 = A \times A$$

$$= \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

$$A^4 = A^2 \times A^2$$

$$= 0 \times 0$$

$$= 0$$

$$A^{16} = A^4 \times A^4$$

$$= 0 \times 0$$

$$= 0$$

So,

A^{16} is a nill matrix

Algebra of Matrices Ex 5.3 Q51

Solving the LHS of the given equation we have ,

$$\Rightarrow \quad A + B = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}$$

$$(A + B)^2 = \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}$$

$$(A + B)^2 = \begin{bmatrix} (0\times 0)+((-x+1)\times(x+1)) & (0\times(-x+1))+((-x+1)\times 0) \\ ((x+1)\times 0)+(0\times(x+1)) & ((x+1)\times(-x+1))+(0\times 0) \end{bmatrix}$$

$$(A + B)^2 = \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix}.$$

Solving the RHS we get,

$$\Rightarrow \quad A^2 + B^2 = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}\begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} (0\times 0)+((-x)\times(x)) & (0\times(-x))+((-x)\times 0) \\ ((x)\times 0)+(0\times(x)) & ((x)\times(-x))+(0\times 0) \end{bmatrix} + \begin{bmatrix} (0\times 0)+(1\times 1) & (0\times 1)+(1\times 0) \\ (1\times 0)+(0\times 1) & (1\times 1)+(0\times 0) \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} -x^2 & 0 \\ 0 & -x^2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix}$$

Subsituting the value of $x^2 = -1$ in the LHS and RHS above,

$$\Rightarrow \quad (A + B)^2 = \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix} = \begin{bmatrix} 1+1 & 0 \\ 0 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \text{ and}$$

$$A^2 + B^2 = \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix} = \begin{bmatrix} 1+1 & 0 \\ 0 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \quad (A + B)^2 = A^2 + B^2.$$

Algebra of Matrices Ex 5.3 Q52

Solving the LHS i.e.

$$A^2 + A = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}^2 + \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & -6 \\ 4 & 4 & 0 \\ 2 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -9 \\ 6 & 5 & 3 \\ 2 & 3 & 5 \end{bmatrix}$$

Solving the RHS i.e.

$$A(A + I) = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}\left(\begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 & -3 \\ 2 & 2 & 3 \\ 2 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -9 \\ 6 & 5 & 3 \\ 2 & 3 & 5 \end{bmatrix}$$

So, LHS = RHS verified.

Algebra of Matrices Ex 5.3 Q53

We have,

$$A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$\text{Now, } A^2 = AA = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} (3\times 3)+(-5\times -4) & (3\times -5)+(-5\times 2) \\ (-4\times 3)+(2\times -4) & (-4\times -5)+(2\times 2) \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix},$$

$$-5A = \begin{bmatrix} -15 & 25 \\ 20 & -10 \end{bmatrix} \text{ and } -14I = \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix}$$

$$\therefore \quad A^2-5A-14I = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + \begin{bmatrix} -15 & 25 \\ 20 & -10 \end{bmatrix} + \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix}$$

$$= \begin{bmatrix} 29-15-14 & -25+25+0 \\ -20+20+0 & 24-10+-14 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Now,

$$A^2-5A-14I = 0$$

$$\Rightarrow \quad A^2 = 5A + 14I$$

$$\Rightarrow \quad A^3 = A^2.A = (5A + 14I)A$$

$$\Rightarrow \quad A^3 = A^2.A = 5A^2 + 14A \qquad \begin{bmatrix} \text{By using dist. of matrices over} \\ \text{matrix addition} \end{bmatrix}$$

$$\Rightarrow \quad A^3 = 5\begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + 14\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$\Rightarrow \quad A^3 = \begin{bmatrix} 145 & -125 \\ -100 & 120 \end{bmatrix} + \begin{bmatrix} 42 & -70 \\ -56 & 28 \end{bmatrix}$$

$$\Rightarrow \quad A^3 = \begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q54

We have,

$$P(x).\,P(y) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}\begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$$

$$\Rightarrow P(x).\,P(y) = \begin{bmatrix} \cos x\cos y - \sin x\sin y & \sin y\cos x + \sin x\cos y \\ -\sin x\cos y - \cos x\sin y & -\sin x\sin y + \cos x\cos y \end{bmatrix}$$

$$\Rightarrow P(x).\,P(y) = \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} = P(x+y)$$

Now,

$$P(y).\,P(x) = \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$$\Rightarrow \quad P(y).\,P(x) = \begin{bmatrix} \cos y\cos x - \sin y\sin x & \sin x\cos y + \sin y\cos x \\ -\sin y\cos x - \cos y\sin x & -\sin y\sin x + \cos y\cos x \end{bmatrix}$$

$$\Rightarrow \quad P(y).\,P(x) = \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} = P(x+y)$$

$$\therefore \quad P(x).\,P(y) = P(x+y) = P(y).\,P(x)$$

We have,

$$P = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}, \ Q = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$\text{So,} \qquad PQ = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$= \begin{bmatrix} x \times a & 0 & 0 \\ 0 & y \times b & 0 \\ 0 & 0 & z \times c \end{bmatrix}$$

$$= \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix}$$

$$\text{and} \quad QP = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$$

$$= \begin{bmatrix} a \times x & 0 & 0 \\ 0 & b \times y & 0 \\ 0 & 0 & c \times z \end{bmatrix}$$

$$= \begin{bmatrix} ax & 0 & 0 \\ 0 & by & 0 \\ 0 & 0 & zc \end{bmatrix}$$

as, $xa = ax$, $yb = by$, $zc = cz$

$$\therefore \ PQ = \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix} = QP.$$

Algebra of Matrices Ex 5.3 Q55

We have,

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

Then ,

$$A^2 = A.A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 2\times2+0\times2+1\times1 & 2\times0+0\times1+1\times-1 & 2\times1+0\times3+1\times0 \\ 2\times2+1\times2+3\times1 & 2\times0+0\times1+1\times-1 & 2\times1+1\times3+3\times0 \\ 1\times2+-1\times2+0\times1 & 1\times0+-1\times1+0\times-1 & 1\times1+-1\times3+0\times0 \end{bmatrix}.$$

$$-5A = \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix}, \quad 4I = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\text{Hence,} \quad A^2\text{-}5A+4I = \begin{bmatrix} 5 & -1 & 5 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$A^2\text{-}5A+4I = \begin{bmatrix} 5\text{-}10\text{+}4 & \text{-}1\text{+}0\text{+}0 & 5\text{-}5\text{+}0 \\ 9\text{-}10\text{+}0 & \text{-}2\text{-}5\text{+}4 & 5\text{-}15\text{+}0 \\ 0\text{-}5\text{-}0 & \text{-}1\text{+}5\text{+}0 & \text{-}2\text{+}0\text{+}4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$$

Now, given is $A^2\text{-}5A+4I+X=O$

$\Rightarrow X = -(A^2\text{-}5A+4I)$

$$X = -\begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & 4 & 2 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q56

Given,

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

To prove $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ we will use the principle of mathematical induction.

Step 1: Put $n = 1$

$$A^1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

So,

A^n is true for $n = 1$

Step 2: Let, A^n be true for $n = k$, then

$$A^k = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \quad ---(i)$$

Step 3: We have to show that $A^{k+1} = \begin{bmatrix} 1 & k+1 \\ 0 & 1 \end{bmatrix}$

So,

$$A^{k+1} = A^k \times A$$

$$= \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad \{\text{using equation (i) and given}\}$$

$$= \begin{bmatrix} 1+0 & 1+k \\ 0+0 & 0+1 \end{bmatrix}$$

$$A^{k+1} = \begin{bmatrix} 1 & 1+k \\ 0 & 1 \end{bmatrix}$$

This shows that A^n is true for $n = k+1$ whenever it is true for $n = k$

Hence, by the principle of mathematical induction A^n is true for all positive integer.

Algebra of Matrices Ex 5.3 Q52

Given,

$$A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$$

To prove $A^n = \begin{bmatrix} a^n & \dfrac{b\left(a^n - 1\right)}{a - 1} \\ 0 & 1 \end{bmatrix}$ we will use the principle of mathematical induction.

Step 1: Put $n = 1$

$$A^1 = \begin{bmatrix} a^1 & \dfrac{b\left(a^1 - 1\right)}{a - 1} \\ 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$$

So,

A^n is true for $n = 1$

Step 2: Let, A^n is true for $n = k$, so,

$$A^k = \begin{bmatrix} a^k & \dfrac{b\,(ak - 1)}{a - 1} \\ 0 & 1 \end{bmatrix} \qquad \text{---(i)}$$

Step 3: We have to show that

$$A^{k+1} = \begin{bmatrix} a^{k+1} & \dfrac{b\left(a^k - 1\right)}{a - 1} \\ 0 & 1 \end{bmatrix}$$

Now,

$$A^{k+1} = A^k \times A$$

$$= \begin{bmatrix} a^k & \dfrac{b\left(a^k - 1\right)}{a - 1} \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix} \qquad \{\text{using equation (i) and given}\}$$

$$= \begin{bmatrix} a^{k+1} + 0 & a^k b + \dfrac{b\left(a^k - 1\right)}{a - 1} \\ 0 + 0 & 0 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} a^{k+1} & \dfrac{a^{k+1}b - a^k b + a^k b - b}{a - 1} \\ 0 & 1 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q57 pending

$$A^{k+1} = \begin{bmatrix} a^{k+1} & \dfrac{b\left(a^{k+1} - 1\right)}{a - 1} \\ 0 & 1 \end{bmatrix}$$

So,

A^n is true for $n = k + 1$ whenever it is true $n = k$.

Hence, by principle of mathematical induction A^n is true for all positive integer n.

Algebra of Matrices Ex 5.3 Q58

Given,

$$A=\begin{bmatrix}\cos\theta & i\sin\theta\\ i\sin\theta & \cos\theta\end{bmatrix}$$

To show that,

$$A^n=\begin{bmatrix}\cos n\theta & i\sin n\theta\\ i\sin n\theta & \cos n\theta\end{bmatrix}\text{ for all } n\in N.$$

Put $n=1$

$$A^1=\begin{bmatrix}\cos\theta & i\sin\theta\\ i\sin\theta & \cos\theta\end{bmatrix}$$

So,

A^n is true for $n=1$

Let, A^n is true for $n=k$, so

$$A^k=\begin{bmatrix}\cos k\theta & i\sin k\theta\\ i\sin k\theta & \cos k\theta\end{bmatrix}\qquad ---(i)$$

Now, we have to show that,

$$A^{k+1}=\begin{bmatrix}\cos(k+1)\theta & i\sin(k+1)\theta\\ i\sin(k+1)\theta & \cos(k+1)\theta\end{bmatrix}$$

Now, $A^{k+1}=A^k\times A$

$$=\begin{bmatrix}\cos k\theta & i\sin k\theta\\ i\sin k\theta & \cos k\theta\end{bmatrix}\begin{bmatrix}\cos\theta & i\sin\theta\\ i\sin\theta & \cos\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos k\theta\cos\theta+i^2\sin k\theta\sin\theta & i^2\cos k\theta\sin\theta+i\sin k\theta\cos\theta\\ i\sin k\theta\cos\theta+i\cos k\theta\sin\theta & i^2\sin k\theta\sin\theta+\cos\theta\cos k\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos k\theta\cos\theta-\sin k\theta\sin\theta & i\left(\cos k\theta\sin\theta+\sin k\theta\cos\theta\right)\\ i\left(\sin k\theta\cos k\theta\sin\theta\right) & \cos k\theta\cos\theta-\sin k\theta\sin\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos(k+1)\theta & i\sin(k+1)\theta\\ i\sin(k+1)\theta & \cos(k+1)\theta\end{bmatrix}$$

So, A^n is true for $n=k+1$ whenever it is true for $n=k$.

Hence, By principle of mathematical induction A^n is true for all positive integer.

Algebra of Matrices Ex 5.3 Q59

Given,

$$A = \begin{bmatrix} \cos\alpha + \sin\alpha & \sqrt{2}\sin\alpha \\ -\sqrt{2}\sin\alpha & \cos\alpha - \sin\alpha \end{bmatrix}$$

To prove P(n): $A^n = \begin{bmatrix} \cos n\alpha + \sin n\alpha & \sqrt{2}\sin n\alpha \\ -\sqrt{2}\sin n\alpha & \cos n\alpha - \sin n\alpha \end{bmatrix}$ we use mathematical induction.

Step 1: To show P(1) is true.

A^n is true for $n = 1$

Step 2: Let, $P(k)$ be true, so

$$A^k = \begin{bmatrix} \cos k\alpha + \sin k\alpha & \sqrt{2}\sin k\alpha \\ -\sqrt{2}\sin k\alpha & \cos k\alpha - \sin k\alpha \end{bmatrix} \quad \text{---(i)}$$

Step 3: Let, $P(k)$ is true.

Now, we have to show that

$$A^{k+1} = \begin{bmatrix} \cos(k+1)\alpha + \sin(k+1)\alpha & \sqrt{2}\sin(k+1)\alpha \\ -\sqrt{2}\sin(k1)\alpha & \cos(k+1)\alpha - \sin(k+1)\alpha \end{bmatrix}$$

Now,

$$A^{k+1} = A^k \times A$$

$$= \begin{bmatrix} \cos k\alpha + \sin k\alpha & \sqrt{2}\sin k\alpha \\ -\sqrt{2}\sin k\alpha & \cos k\alpha - \sin k\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha + \sin\alpha & \sqrt{2}\sin\alpha \\ -\sqrt{2}\sin\alpha & \cos\alpha - \sin\alpha \end{bmatrix}$$

$$= \begin{bmatrix} (\cos k\alpha + \sin k\alpha)(\cos\alpha + \sin\alpha) - 2\sin\alpha\sin k\alpha & \begin{matrix}(\cos k\alpha + \sin k\alpha)\sqrt{2}\sin\alpha \\ +\sqrt{2}\sin k\alpha(\cos\alpha - \sin\alpha)\end{matrix} \\ (\cos\alpha + \sin\alpha)(-\sqrt{2}\sin k\alpha) - \sqrt{2}\sin\alpha(\cos k\alpha - \sin k\alpha) & \begin{matrix}-2\sin k\alpha\sin\alpha + (\cos k\alpha - \sin k\alpha) \\ (\cos\alpha - \sin\alpha)\end{matrix} \end{bmatrix}$$

$$= \begin{bmatrix} \begin{matrix}\cos k\alpha\cos\alpha + \sin k\alpha\cos\alpha + \cos k\alpha\sin\alpha \\ +\sin\alpha\sin k\alpha - 2\sin\alpha\sin k\alpha\end{matrix} & \begin{matrix}\sqrt{2}\cos k\alpha\sin\alpha + \sqrt{2}\sin\alpha\sin k\alpha + \\ \sqrt{2}\sin k\alpha\cos\alpha - \sqrt{2}\sin k\alpha\sin\alpha\end{matrix} \\ \begin{matrix}-\sqrt{2}\cos\alpha\sin\alpha - \sqrt{2}\sin\alpha\sin k\alpha - \sqrt{2}\sin\alpha \\ \cos k\alpha + \sqrt{2}\sin\alpha\sin k\alpha\end{matrix} & \begin{matrix}-2\sin k\alpha\sin\alpha + \cos k\alpha\cos\alpha - \cos\alpha \\ \sin k\alpha - \sin\alpha\cos k\alpha\sin\alpha\sin k\alpha\end{matrix} \end{bmatrix}$$

$$= \begin{bmatrix} \begin{matrix}\cos\alpha\cos k\alpha + \sin\alpha\sin k\alpha \\ \sin\alpha\cos k\alpha + \sin k\alpha\cos\alpha\end{matrix} & \sqrt{2}(\sin k\alpha\cos\alpha + \cos k\alpha\sin\alpha) \\ -\sqrt{2}(\sin k\alpha\cos\alpha + \cos k\alpha\sin\alpha) & \begin{matrix}\cos k\alpha\cos\alpha - \sin k\alpha\sin\alpha - \\ (\sin k\alpha\cos\alpha + \sin\alpha\cos k\alpha)\end{matrix} \end{bmatrix}$$

$$= \begin{bmatrix} \cos(k+1)\alpha + \sin(k+1)\alpha & \sqrt{2}\sin(k+1)\alpha \\ -\sqrt{2}\sin(k+1)\alpha & \cos(k+1)\alpha - \sin(k+1)\alpha \end{bmatrix}$$

So, $P(k+1)$ is true whenever $P(k)$ is true.

Hence, by principle of mathematical induction $P(n)$ is true for all positive integer.

Algebra of Matrices Ex 5.3 Q60

Given,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

To prove, $A^n = \begin{bmatrix} 1 & n & \frac{n(n+1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$, we will use the principle of mathematical induction.

Step 1: Put $n = 1$

$$A^1 = \begin{bmatrix} 1 & 1 & \frac{1(1+1)}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

So, A^n is true for $n = 1$

Step 2: Let, A^n be true for $n = k$, so,

$$A^k = \begin{bmatrix} 1 & k & \frac{k(k+1)}{2} \\ 0 & 1 & (k+1) \\ 0 & 0 & 1 \end{bmatrix}$$

Step 3: We will prove that A^n be true for $n = k + 1$

Now,

$$A^{k+1} = A^k \times A$$

$$= \begin{bmatrix} 1 & k & \frac{k(k+1)}{2} \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \{\text{using equation (i) and given}\}$$

$$= \begin{bmatrix} 1+0+0 & 1+k+0 & 1+k+\frac{k(k+1)}{2} \\ 0+0+0 & 0+1+0 & 0+1+k \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & (k+1) & \frac{(k+1)(k+2)}{2} \\ 0 & 1 & (k+1) \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, A^n is true for $n = k + 1$ whenever it is true for $n = k$.

So, by principle of mathematical induction A^n is true for all positive integer n.

Algebra of Matrices Ex 5.3 Q61

We will prove P(n): $A^{n+1} = B^n\left[B + (n+1)C\right]$ is true for all natural numbers using mathematical induction.

Given,

$$A = B + C,\ BC = CB,\ C^2 = 0$$
$$A = B + C$$

Squaring both the sides, so

$$A^2 = (B + C)^2$$
$$\Rightarrow \quad A^2 = (B + C)(B + C)$$
$$\Rightarrow \quad A^2 = B \times B + BC + CB + C \times C \qquad \{\text{using distributive property}\}$$
$$\Rightarrow \quad A^2 = B^2 + BC + BC + C^2 \qquad \{\text{using } BC = CB \text{ given}\}$$
$$\Rightarrow \quad A^2 = B^2 + 2BC + 0 \qquad \left\{\text{since, given } C^2 = 0\right\}$$
$$\Rightarrow \quad A^2 = B^2 + 2BC \qquad \text{---(1)}$$
$$A^2 = B(B + 2C)$$

Now, consider

$$P(n): \ A^{n+1} = B^n\left[B + (n+1)C\right]$$

Step 1: Tp prove P(1) is true, put $n = 1$

$$A^{1+1} = B^1\left[B + (1+1)C\right]$$
$$A^2 = B\left[B + 2C\right]$$
$$A^2 = B^2 + 2BC$$

From equation (i), P(1) is true.

Step 2: Suppose P(k) is true.

$$\therefore A^{k+1} = B^k\left[B + (k+1)C\right] \qquad \text{---(2)}$$

Step 3: Now, we have to show that P(k+1) is true.

That is we need to prove that,

$$A^{k+2} = B^{k+1}\left[B + (k+2)C\right]$$

Now,

$$A^{k+2} = A^k \times A^2$$
$$= B^{(k-1)}\left[B + kC\right] \times \left[B(B + 2C)\right]$$
$$= B^k\left[B + kC\right] \times \left[B + 2C\right]$$
$$= B^k\left[B \times B + B \times 2C + kC \times B + 2kC^2\right]$$
$$= B^k\left[B^2 + 2BC + kBC + 2k \times 0\right] \qquad \left\{\text{since } BC = CB,\ C^2 = 0\right\}$$
$$= B^k\left[B^2 + BC(2 + k)\right]$$
$$= B^k \times B\left[B + (k+2)C\right]$$
$$= B^{k+1}\left[B + (k+2)C\right]$$

So, $P(n)$ is true for $n = k+1$ whenever $P(n)$ is true for $n = k$

Therefore by principle of mathematical induction $P(n)$ is true for all natural number.

Algebra of Matrices Ex 5.3 Q62

Given,

$$A = \text{diag}(a, b, c)$$

Show that,

$$A^n = \text{diag}\left(a^n, b^n, c^n\right)$$

Step 1: Put $n = 1$

$$A^1 = \text{diag}\left(a^1, b^1, c^1\right)$$

$$A = \text{diag}(a, b, c)$$

So,

A^n is true for $n = 1$

Step 2: Let, A^n be true for $n = k$, so,

$$A^k = \text{diag}\left(a^k, b^k, c^k\right) \quad \text{---(i)}$$

Step 3: Now, we have to show that,

$$A^{k+1} = \text{diag}\left(a^{k+1}, b^{k+1}, c^{k+1}\right)$$

Now,

$$A^{k+1} = A^k \times k3$$

$$= \text{diag}\left(a^k, b^k, c^k\right) \times \text{diag}(a, b, c) \quad \{\text{using equation (i) and given}\}$$

$$A^{k+1} = \begin{bmatrix} a^k & 0 & 0 \\ 0 & b^k & 0 \\ 0 & 0 & c^k \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$= \begin{bmatrix} a^k \times a + 0 + 0 & 0 + 0 + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + b^k \times b + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + 0 + 0 & 0 + 0 + c^k \times c \end{bmatrix}$$

$$= \begin{bmatrix} a^{k+1} & 0 & 0 \\ 0 & b^{k+1} & 0 \\ 0 & 0 & c^{k+1} \end{bmatrix}$$

$$A^{k+1} = \text{diag}\left(a^{k+1}, b^{k+1}, c^{k+1}\right)$$

So, $P(n)$ is true for $n = k + 1$ whenever $P(n)$ is true for $n = k$.

Hence, by principle of mathematical induction A^n is true for all positive integer.

Algebra of Matrices Ex 5.3 Q64

Given,

order of matrix $X = (a+b)\times(a+2)$

order of matrix $Y = (b+1)\times(a+3)$

Given, $X_{(a+b)\times(a+2)} \cdot Y_{(b+1)\times(a+3)}$ exist.

$\Rightarrow \quad a+2 = b+1$

$\Rightarrow \quad a-b = -1 \qquad ---(i)$

And

$Y_{(b+1)\times(a+3)} \cdot X_{(a+b)\times(a+2)}$ exists.

$\Rightarrow \quad a+3 = a+b$

$\Rightarrow \quad b = 3$

Put $b = 3$ in equation (i),

$a-b = -1$

$a-3 = -1$

$a = 3-1$

$a = 2$

So, $a = 2, b = 3$

So,

Order of $X = (a+b)\times(a+2)$

$= (2+3)\times(2+2)$

$= 5\times 4$

Order of $Y = (b+1)\times(a+3)$

$= (3+1)\times(2+3)$

$= 4\times 5$

Order of $X_{5\times 4} \cdot Y_{4\times 5} = 5\times 5$

Order of $X_{4\times 5} \cdot Y_{5\times 4} = 4\times 4$

So, *order* of XY and YX are not same and they are not equal but both are square matrices.

Algebra of Matrices Ex 5.3 Q65(i)

Let, $A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$, $B \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & ab+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & ab \\ 0 & 0 \end{bmatrix} \quad ---(i)$$

$$BA = \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

From equation (i) and (ii)

$$AB \neq BA$$

when $A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$

Algebra of Matrices Ex 5.3 Q65(ii)

Let, $$A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \neq 0$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix} \neq 0$$

$$AB = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence,

$$AB = 0$$

When,

$$A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \neq 0$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix} \neq 0$$

Let, $A = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}\begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$AB = 0$$

$$BA = \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & ab+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & ab \\ 0 & 0 \end{bmatrix}$$

$$BA \neq 0$$

Hence,

for $AB = 0$ and $BA \neq 0$ we have,

$$A = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}, \ B = \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q65(iv)

Let, $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Here,

$$A \neq 0,\ B \neq C$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

LHS = RHS

So,

for $A \neq 0,\ BC \neq 0$ but $AB = AC$

We have,

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix},\ C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q66

Given,

A and B are square matices of same order

$$\begin{aligned}(A+B)^2 &= (A+B)(A+B) \\ &= A(A+B) + B(A+B) \quad \{\text{using distributive property}\} \\ &= A \times A + AB + BA + B^2 \\ &= A^2 + AB + BA + B^2\end{aligned}$$

But,

$(A+B)^2 = A^2 + 2AB + B^2$ is possible only when $AB = BA$

Here, we can not say that $AB = BA$

So,

$(A+B)^2 = A^2 + 2AB + B^2$ does not hold.

Algebra of Matrices Ex 5.3 Q67

Given, A and B are square matrices of same order.

(i) $(A+B)^2 = (A+B)(A+B)$

$= A(A+B)+B(A+B)$ {using distributive property}

$= A\times A + AB + BA + B\times B$

$= A^2 + AB + BA + B^2$

$\neq A^2 + 2AB + B^2$

Since, in general matix multiplication is not commutative $(AB \neq BA)$

So, $(A+B)^2 \neq A^2 + 2AB + B^2$

(ii) $(A-B)^2 = (A-B)(A-B)$

$= A(A-B)-B(A-B)$ {using distributive property}.

$= A\times A - AB - BA + B\times B$

$= A^2 - AB - BA + B^2$

$\neq A^2 - 2AB + B^2$

Since, in general matrix multiplication is not commutative $(AB \neq BA)$, so

So, $(A-B)^2 \neq A^2 - 2AB + B^2$

(iii) $(A+B)(A-B) = A(A-B)+B(A-B)$ {using distributive property}

$= A\times A - AB + BA - B\times B$

$= A^2 - AB + BA - B^2$

$= A^2 - B^2$

Since, in general matix multiplication is not commutative $(AB \neq BA)$,

So, $(A+B)(A-B) \neq A^2 - B^2$

Algebra of Matrices Ex 5.3 Q68

The given equality is true only when we choose A and B to be a square matrix in such a way that AB = BA else the result is not true in general.

Example: Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Here $AB = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1\times 0+0\times 2+0\times 0 & 1\times 1+0\times 1+0\times 0 & 1\times 0+0\times 0+0\times 1 \\ 1\times 0+1\times 2+0\times 0 & 1\times 1+1\times 1+0\times 0 & 1\times 0+1\times 0+0\times 1 \\ 0\times 0+0\times 2+1\times 0 & 0\times 1+0\times 1+1\times 0 & 0\times 0+0\times 0+1\times 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

and $BA = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0\times 1+1\times 1+0\times 0 & 0\times 0+1\times 1+0\times 0 & 0\times 0+1\times 0+0\times 1 \\ 2\times 1+1\times 1+0\times 0 & 2\times 0+1\times 1+0\times 0 & 2\times 0+1\times 0+0\times 1 \\ 0\times 1+0\times 1+1\times 0 & 0\times 0+0\times 1+1\times 0 & 0\times 0+0\times 0+1\times 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$AB \neq BA$

Now, $(AB)^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 0\times0+1\times1+0\times0 & 0\times1+1\times2+0\times0 & 0\times0+1\times0+0\times1 \\ 1\times0+2\times1+0\times0 & 1\times1+1\times2+0\times0 & 1\times0+2\times0+0\times1 \\ 0\times0+0\times1+1\times0 & 0\times1+0\times2+1\times0 & 0\times0+0\times0+1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+0\times1+0\times0 & 1\times0+0\times1+0\times0 & 1\times0+0\times0+0\times1 \\ 1\times1+1\times1+0\times0 & 1\times0+1\times1+0\times0 & 1\times0+1\times0+0\times1 \\ 0\times1+0\times1+1\times0 & 0\times0+0\times1+1\times0 & 0\times0+0\times0+1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times0+1\times2+0\times0 & 0\times1+1\times1+0\times0 & 0\times0+1\times0+0\times1 \\ 2\times0+1\times2+0\times0 & 2\times1+1\times1+0\times0 & 2\times0+1\times0+0\times1 \\ 0\times0+0\times2+1\times0 & 0\times1+0\times1+1\times0 & 0\times0+0\times0+1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^2B^2 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+0\times1+0\times0 & 1\times1+0\times2+0\times0 & 1\times0+0\times0+0\times1 \\ 2\times1+1\times1+0\times0 & 2\times1+1\times2+0\times0 & 2\times0+1\times0+0\times1 \\ 0\times1+0\times1+1\times0 & 0\times1+0\times2+1\times0 & 0\times0+0\times0+1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We can see that if we have A and B two square matrices with $AB \neq BA$ then $(AB)^2 \neq A^2B^2$

Algebra of Matrices Ex 5.3 Q69

Given,

A and B two square matrices of same order such that $AB = BA$.

To prove : $(A+B)^2 = A^2 + 2AB + B^2$

Now, solving LHS gives,

$$(A+B)^2 = (A+B)(A+B)$$

$$= A(A+B) + B(A+B) \qquad \text{[by dist. of matrix multiplication over addition]}$$

$$= A^2 + AB + BA + B^2 \qquad \text{[by dist. of matrix multiplication over addition]}$$

$$= A^2 + 2AB + B^2 \qquad [\text{As, } AB = BA]$$

$$= \text{RHS}$$

Hence proved.

Algebra of Matrices Ex 5.3 Q70

Given, $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \\ 5 & 2 \\ -2 & 4 \end{bmatrix}, C = \begin{bmatrix} 4 & 2 \\ -3 & 5 \\ 5 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 5 & 2 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3+5-2 & 1+2+4 \\ 9+15-6 & 3+6+12 \end{bmatrix}$$

$$AB = \begin{bmatrix} 6 & 7 \\ 18 & 21 \end{bmatrix} \quad \text{---(i)}$$

$$AC = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ -3 & 5 \\ 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3+5 & 2+5+0 \\ 12-9+15 & 6+15+0 \end{bmatrix}$$

$$AC = \begin{bmatrix} 6 & 7 \\ 18 & 21 \end{bmatrix} \quad \text{---(ii)}$$

From equation (i) and (ii)

$$AB = AC$$

Algebra of Matrices Ex 5.3 Q71

The number of items purchased by A, B and C are represented in matrix form as,

$$X = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} \text{Notebook} & \text{Pens} & \text{Pencils} \end{array} \\ \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix} \end{array}$$

Now, matrix formed by the cost of each items is given by,

$$Y = \begin{bmatrix} 0.40 \\ 1.25 \\ 0.35 \end{bmatrix} \begin{array}{c} \text{Note book} \\ \text{Pen} \\ \text{Pencil} \end{array}$$

Individual bill can be calculated by

$$XY = \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix} \begin{bmatrix} 0.40 \\ 1.25 \\ 0.35 \end{bmatrix}$$

$$XY = \begin{bmatrix} 57.60 + 75.00 + 25.20 \\ 48.00 + 90.00 + 29.40 \\ 52.80 + 195.00 + 33.60 \end{bmatrix}$$

$$XY = \begin{bmatrix} 157.80 \\ 167.40 \\ 281.40 \end{bmatrix}$$

So,

Bill of A = Rs 157.80

Bill of B = Rs 167.40

Bill of C = Rs 281.40

Algebra of Matrices Ex 5.3 Q72

Matrix representation of stock of various types of book in the store is given by,

$$X = \begin{array}{c} \begin{array}{ccc} \text{Physics} & \text{Chemistry} & \text{Mathematics} \end{array} \\ \begin{bmatrix} 120 & 96 & 60 \end{bmatrix} \end{array}$$

Matrix representation of sellin price (Rs.) of each book is given by

$$Y = \begin{bmatrix} 8.30 \\ 3.45 \\ 4.50 \end{bmatrix} \begin{array}{c} \text{Physics} \\ \text{Chemistry} \\ \text{Mathematics} \end{array}$$

So, totaol amount recieved by the store from sellin all the items is given by,

$$XY = \begin{bmatrix} 120 & 96 & 60 \end{bmatrix} \begin{bmatrix} 8.30 \\ 3.45 \\ 4.50 \end{bmatrix}$$

$$= \left[(120)(8.30) + (96)(3.45) + (60)(4.50)\right]$$

$$= \left[996 + 331.20 + 270\right]$$

$$= \left[1597.20\right]$$

Required amount = Rs 1597.20

Algebra of Matrices Ex 5.3 Q73

Given,

The cost per contact (in paise) is given by

$$A = \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix} \begin{bmatrix} \text{Telephone} \\ \text{Housecall} \\ \text{Letter} \end{bmatrix}$$

The number of contact of each type made in two cities X and y is given by.

$$\begin{array}{cccc} & \text{Telephone} & \text{Housecall} & \text{Letter} \\ B = & \left[\begin{matrix} 1000 \\ 3000 \end{matrix}\right. & \begin{matrix} 500 \\ 1000 \end{matrix} & \left.\begin{matrix} 5000 \\ 10000 \end{matrix}\right] \end{array}$$

Total amount spent by the group in the two cities X and y can be given by

$$BA = \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix}$$

$$= \begin{bmatrix} 40000 + 50000 + 250000 \\ 120000 + 100000 + 500000 \end{bmatrix}$$

$$= \begin{bmatrix} 340000 \\ 720000 \end{bmatrix} \begin{matrix} X \\ Y \end{matrix}$$

Hence,

Amount spend on X = Rs 3400

Amount spend on Y = Rs 7200

Algebra of Matrices Ex 5.3 Q74

(a) Let Rs x be invested in the first bond. Then, the sum of money invested in the second bond will be Rs $(30000 - x)$.

It is given that the first bond pays 5% interest per year and the second bond pays 7% interest per year.

Therefore, in order to obtain an annual total interest of Rs 1800, we have:

$$\begin{bmatrix} x & (30000 - x) \end{bmatrix} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 1800 \qquad \left[\text{S.I. for 1 year} = \frac{\text{Principal} \times \text{Rate}}{100} \right]$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000 - x)}{100} = 1800$$

$$\Rightarrow 5x + 210000 - 7x = 180000$$

$$\Rightarrow 210000 - 2x = 180000$$

$$\Rightarrow 2x = 210000 - 180000$$

$$\Rightarrow 2x = 30000$$

$$\Rightarrow x = 15000$$

Thus, in order to obtain an annual total interest of Rs 1800, the trust fund should invest Rs 15000 in the first bond and the remaining Rs 15000 in the second bond.

(b) Let Rs x be invested in the first bond. Then, the sum of money invested in the second bond will be Rs $(30000-x)$.
Therefore, in order to obtain an annual total interest of Rs 2000, we have:

$$\begin{bmatrix} x & (30000-x) \end{bmatrix} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 2000$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000-x)}{100} = 2000$$
$$\Rightarrow 5x + 210000 - 7x = 200000$$
$$\Rightarrow 210000 - 2x = 200000$$
$$\Rightarrow 2x = 210000 - 200000$$
$$\Rightarrow 2x = 10000$$
$$\Rightarrow x = 5000$$

Thus, in order to obtain an annual total interest of Rs 2000, the trust fund should invest Rs 5000 in the first bond and the remaining Rs 25000 in the second bond

Algebra of Matrices Ex 5.3 Q75

The cost for each mode per attempt is represented by 3×1 matrix:

$$A = \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix}$$

The number of attempts made in the three villages X, Y, and Z are represented by a 3×3 matrix:

$$B = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}$$

The total cost incurred by the prganization for the three villages seperately is given by matrix multiplication

$$BA = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix}$$

$$BA = \begin{bmatrix} 400 \times 50 + 300 \times 20 + & 100 \times 40 \\ 300 \times 50 + 250 \times 20 + & 75 \times 40 \\ 500 \times 50 + 400 \times 20 + & 150 \times 40 \end{bmatrix}$$

$$= \begin{bmatrix} 30{,}000 \\ 23{,}000 \\ 39{,}000 \end{bmatrix}$$

Note: The answer given in the book is incorrect.

Algebra of Matrices Ex 5.3 Q76

Let F be the family matrix and R be the requirement matrix. Then,

$$F = \begin{array}{c} \\ \text{Family A} \\ \text{Family B} \end{array}\begin{array}{c} \begin{array}{ccc} \text{Men} & \text{Women} & \text{Children} \end{array} \\ \begin{bmatrix} 4 & 6 & 2 \\ 2 & 2 & 4 \end{bmatrix} \end{array}$$

$$R = \begin{array}{c} \\ \text{Men} \\ \text{Women} \\ \text{Children} \end{array}\begin{array}{c} \begin{array}{cc} \text{Calories} & \text{Portein} \end{array} \\ \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix} \end{array}$$

The requirement of calories and protein of each of the two families is given by the product matrix FR, as matrix F has number of columns equal to number of rows of R thus ,

$$FR = \begin{bmatrix} 4 & 6 & 2 \\ 2 & 2 & 4 \end{bmatrix}\begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix}$$

$$FR = \begin{bmatrix} 4 \times 2400 + 6 \times 1900 + 2 \times 1800 & 4 \times 45 + 6 \times 55 + 2 \times 33 \\ 2 \times 2400 + 2 \times 1900 + 4 \times 1800 & 2 \times 45 + 2 \times 55 + 4 \times 33 \end{bmatrix}$$

$$FR = \begin{array}{c} \\ \text{Family A} \\ \text{Family B} \end{array}\begin{array}{c} \begin{array}{cc} \text{Calories} & \text{Protein} \end{array} \\ \begin{bmatrix} 24600 & 576 \\ 15800 & 332 \end{bmatrix} \end{array}$$

we can say that balanced diet having the required amount of calories and protein must be taken by each of the family.

Algebra of Matrices Ex 5.3 Q77

The cost per contact (in paisa) is given in matrix A as

$$A = \begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix}\begin{array}{l} \text{Telephone} \\ \text{House calls} \\ \text{Letters} \end{array}$$

The number of contacts of each type made in two cities X and Y is given in the matrix B as

$$B = \begin{array}{c} \\ \text{City X} \\ \text{City Y} \end{array}\begin{array}{c} \begin{array}{ccc} \text{Telephone} & \text{House calls} & \text{Letters} \end{array} \\ \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \end{array}$$

The total amout of money spent by party in each of the city for the election is given by the matrix multiplication :

$$BA = \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix}\begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix}$$

$$= \begin{bmatrix} 1000 \times 140 + 500 \times 200 + 5000 \times 150 \\ 3000 \times 140 + 1000 \times 200 + 10000 \times 150 \end{bmatrix}$$

$$= \begin{array}{c} \text{City X} \\ \text{City Y} \end{array}\begin{bmatrix} 990000 \\ 2120000 \end{bmatrix}$$

The total amout of money spent by party in each of the city for the election in rupees is given by

$$= \left(\frac{1}{100}\right)\begin{array}{c} \text{City X} \\ \text{City Y} \end{array}\begin{bmatrix} 990000 \\ 2120000 \end{bmatrix}$$

$$= \begin{array}{c} \text{City X} \\ \text{City Y} \end{array}\begin{bmatrix} 9900 \\ 21200 \end{bmatrix}$$

One sould consider social activities before casting his/her vote to the party.

Ex 5.4

Algebra of Matrices Ex 5.4 Q1(i)

Given,

$$A = \begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$$(2A)^T = 2 \times A^T$$

$$\Rightarrow \quad \left(2\begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix}\right)^T = 2\begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} 4 & -6 \\ -14 & 10 \end{bmatrix}^T = 2\begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 4 & -14 \\ -6 & 10 \end{bmatrix} = \begin{bmatrix} 4 & -14 \\ -6 & 10 \end{bmatrix}$$

LHS = RHS

So,

$$(2A)^T = 2A^T$$

Algebra of Matrices Ex 5.4 Q1(ii)

Given,

$$A = \begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$$(A+B)^T = A^T + B^T$$

$$\left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}\right)^T = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T + \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} 2+1 & -3+0 \\ -7+2 & 5-4 \end{bmatrix}^T = \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 3 & -3 \\ -5 & 1 \end{bmatrix}^T = \begin{bmatrix} 2+1 & -7+2 \\ -3+0 & 5-4 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

So,

$$(A+B)^T = A^T + B^T$$

Algebra of Matrices Ex 5.4 Q1(iii)

Given,

$$A = \begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$(A - B)^T = A^T - B^T$

$$\Rightarrow \quad \left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix} \right)^T = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T - \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} 2-1 & -3-0 \\ -7-2 & 5+4 \end{bmatrix}^T = \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & -3 \\ -9 & 9 \end{bmatrix}^T = \begin{bmatrix} 2-1 & -7-2 \\ -3-0 & 5+4 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix} = \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix}$$

$\Rightarrow \quad$ LHS = RHS

So,

$$(A - B)^T = A^T - B^T$$

Algebra of Matrices Ex 5.4 Q1(iv)

Given,

$$A = \begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \quad \left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} 2-6 & 0+12 \\ -7+10 & 0-20 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} -4 & 12 \\ 3 & -20 \end{bmatrix}^T = \begin{bmatrix} 2-6 & -7+10 \\ 0+12 & 0-20 \end{bmatrix}.$$

$$\Rightarrow \quad \begin{bmatrix} -4 & 3 \\ 12 & -20 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 12 & -20 \end{bmatrix}$$

$$\Rightarrow \quad \text{HS} = \text{RHS}$$

So,

$$(AB)^T = B^T A^T$$

Algebra of Matrices Ex 5.4 Q2

Given,

$$A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 0 & 4 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \quad \left(\begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 4 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 0 & 4 \end{bmatrix}^T \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} 3 & 0 & 12 \\ 5 & 0 & 20 \\ 2 & 0 & 8 \end{bmatrix}^{.T} = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix} \begin{bmatrix} 3 & 5 & 2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix}$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

So,

$$(AB)^T = B^T A^T$$

Algebra of Matrices Ex 5.4 Q3(i)

Given,

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \quad A^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}, \; B^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$(A+B)^T = A^T + B^T$$

$$\Rightarrow \quad \left(\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}^T + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} 1+1 & -1+2 & 0+3 \\ 2+2 & 1+1 & 3+3 \\ 1+0 & 2+1 & 1+1 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 6 \\ 1 & 3 & 2 \end{bmatrix}^T = \begin{bmatrix} 1+1 & 2+2 & 1+0 \\ -1+2 & 1+1 & 2+1 \\ 0+3 & 3+3 & 1+1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 3 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 3 & 6 & 2 \end{bmatrix}$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

So,

$$(A+B)^T = A^T + B^T$$

Algebra of Matrices Ex 5.4 Q3(ii)

Given,

$$A=\begin{bmatrix}1 & -1 & 0\\ 2 & 1 & 3\\ 1 & 2 & 1\end{bmatrix},\ B=\begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 3\\ 0 & 1 & 1\end{bmatrix}$$

$$\Rightarrow \qquad A^T=\begin{bmatrix}1 & 2 & 1\\ -1 & 1 & 2\\ 0 & 3 & 1\end{bmatrix},\ B^T=\begin{bmatrix}1 & 2 & 0\\ 2 & 1 & 1\\ 3 & 3 & 1\end{bmatrix}$$

$$(AB)^T=B^TA^T$$

$$\Rightarrow \qquad \left(\begin{bmatrix}1 & -1 & 0\\ 2 & 1 & 3\\ 1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 3\\ 0 & 1 & 1\end{bmatrix}\right)^T=\begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 3\\ 0 & 1 & 1\end{bmatrix}^T\begin{bmatrix}1 & -1 & 0\\ 2 & 1 & 3\\ 1 & 2 & 1\end{bmatrix}^T$$

$$\Rightarrow \qquad \begin{bmatrix}1-2+0 & 2-1+0 & 3-3+0\\ 2+2+0 & 4+1+3 & 6+3+3\\ 1+4+0 & 2+2+1 & 3+6+1\end{bmatrix}^T=\begin{bmatrix}1 & 2 & 0\\ 2 & 1 & 1\\ 3 & 3 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 1\\ -1 & 1 & 2\\ 0 & 3 & 1\end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix}-1 & 1 & 0\\ 4 & 8 & 12\\ 5 & 5 & 10\end{bmatrix}^T=\begin{bmatrix}1-2+0 & 2+2+0 & 1+4+0\\ 2-1+0 & 4+1+3 & 2+2+1\\ 3-3+0 & 6+3+3 & 3+6+1\end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix}-1 & 4 & 5\\ 1 & 8 & 5\\ 0 & 12 & 10\end{bmatrix}=\begin{bmatrix}-1 & 4 & 5\\ 1 & 8 & 5\\ 0 & 12 & 10\end{bmatrix}$$

$\Rightarrow \qquad$ LHS = RHS

So,

$$(AB)^T=B^TA^T$$

Algebra of Matrices Ex 5.4 Q3(iii)

Given,

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \quad A^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}, \quad B^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$(2A)^T = 2A^T$$

$$\Rightarrow \quad \left(2\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}\right)^T = 2\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} 2 & -2 & 0 \\ 4 & 2 & 6 \\ 2 & 4 & 2 \end{bmatrix}^T = 2\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2 & 4 & 2 \\ -2 & 2 & 4 \\ 0 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 2 \\ -2 & 2 & 4 \\ 0 & 6 & 2 \end{bmatrix}$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

So,

$$(2A)^T = 2 \times A^T$$

Algebra of Matrices Ex 5.4 Q4

Given,

$$A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}, \ B = \begin{bmatrix} 1 & 3 & -6 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \quad \left(\begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix} \begin{bmatrix} 1 & 3 & -6 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 3 & -6 \end{bmatrix}^T \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} -2 & -6 & 12 \\ 4 & 12 & -24 \\ 5 & 15 & -30 \end{bmatrix}^T = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix} \begin{bmatrix} -2 & 4 & 5 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix} = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix}$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

So,

$$(2A)^T = 2 \times AT$$

Algebra of Matrices Ex 5.4 Q5

Given,

$$A = \begin{bmatrix} 2 & 4 & -1 \\ -1 & 0 & 2 \end{bmatrix}, \; B = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 2 & 1 \end{bmatrix}$$

$(AB)^T$

$$= \left(\begin{bmatrix} 2 & 4 & -1 \\ -1 & 0 & 2 \end{bmatrix}\right)\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 6-4-2 & 8+8-1 \\ -3+0+4 & -4+0+2 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 15 \\ 1 & -2 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 1 \\ 15 & -2 \end{bmatrix}$$

So,

$$(AB)^T = \begin{bmatrix} 0 & 1 \\ 15 & -2 \end{bmatrix}$$

Algebra of Matrices Ex 5.4 Q6

Given,

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \quad \left(\begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}^T \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} 2+0+15 & -2+20 \\ 4+0+0 & -4+2+0 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & 5 \\ -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 1 & 1 \\ 3 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 17 & 0 \\ 4 & -2 \end{bmatrix}^T = \begin{bmatrix} 2+0+15 & 4+0+0 \\ -2+2+0 & -4+2+0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 17 & 4 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 17 & 4 \\ 0 & -2 \end{bmatrix}$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

So,

$$(AB)^T = B^T A^T$$

Algebra of Matrices Ex 5.4 Q6(ii)

Given,

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix},\ B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \quad \left(\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}\right)^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}^T \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} 1+6 & 4+15 \\ 2+8 & 8+20 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 7 & 19 \\ 10 & 28 \end{bmatrix}^T = \begin{bmatrix} 1+6 & 2+8 \\ 4+15 & 8+20 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix}$$

$$\Rightarrow \quad \text{LHS} = \text{RHS}$$

So,

$$(AB)^T = B^T A^T$$

Algebra of Matrices Ex 5.4 Q7

$$\text{Given that } A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}.$$

$$\text{We need to find } A^T - B^T.$$

$$\text{Given that , } B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$\Rightarrow B^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$\text{Let us find } A^T - B^T:$$

$$A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$\Rightarrow A^T - B^T = \begin{bmatrix} 3+1 & 4-1 \\ -1-2 & 2-2 \\ 0-1 & 1-3 \end{bmatrix}$$

$$\Rightarrow A^T - B^T = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

Algebra of Matrices Ex 5.4 Q8

(i)

$$A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

$$\therefore A'=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$$A'A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$
$$=\begin{bmatrix}(\cos\alpha)(\cos\alpha)+(-\sin\alpha)(-\sin\alpha) & (\cos\alpha)(\sin\alpha)+(-\sin\alpha)(\cos\alpha)\\ (\sin\alpha)(\cos\alpha)+(\cos\alpha)(-\sin\alpha) & (\sin\alpha)(\sin\alpha)+(\cos\alpha)(\cos\alpha)\end{bmatrix}$$
$$=\begin{bmatrix}\cos^2\alpha+\sin^2\alpha & \sin\alpha\cos\alpha-\sin\alpha\cos\alpha\\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha & \sin^2\alpha+\cos^2\alpha\end{bmatrix}$$
$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

Hence, we have verified that $A'A=I$.

Algebra of Matrices Ex 5.4 Q9

(ii)

$$A=\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$$

$$\therefore A'=\begin{bmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}$$

$$A'A=\begin{bmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$$

$$\begin{bmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$$
$$=\begin{bmatrix}(\sin\alpha)(\sin\alpha)+(-\cos\alpha)(-\cos\alpha) & (\sin\alpha)(\cos\alpha)+(-\cos\alpha)(\sin\alpha)\\ (\cos\alpha)(\sin\alpha)+(\sin\alpha)(-\cos\alpha) & (\cos\alpha)(\cos\alpha)+(\sin\alpha)(\sin\alpha)\end{bmatrix}$$
$$=\begin{bmatrix}\sin^2\alpha+\cos^2\alpha & \sin\alpha\cos\alpha-\sin\alpha\cos\alpha\\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha & \cos^2\alpha+\sin^2\alpha\end{bmatrix}$$
$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

Hence, we have verified that $A'A=I$.

Algebra of Matrices Ex 5.4 Q10

Given,

l_i, m_i, n_i are direction cosines of three mutually perpendicular vectors

$$\Rightarrow \left.\begin{array}{l} l_1l_2 + m_1m_2 + n_1n_2 = 0 \\ l_2l_3 + m_2m_3 + n_2n_3 = 0 \\ l_1l_3 + m_1m_3 + n_1n_3 = 0 \end{array}\right\} \qquad ---(A)$$

And,

$$\left.\begin{array}{l} l_1^2 + m_1^2 + n_1^2 = 1 \\ l_2^2 + m_2^2 + n_2^2 = 1 \\ l_3^2 + m_3^2 + n_3^2 = 1 \end{array}\right\} \qquad ---(B)$$

Given,

$$A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}\begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}^T$$

$$= \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}\begin{bmatrix} l_1 & l_1 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix}$$

$$= \begin{bmatrix} l_1^2 + m_1^2 + n_1^2 & l_1l_2 + m_1m_2 + n_1n_2 & l_1l_3 + m_1m_3 + n_1n_3 \\ l_1l_2 + m_1m_2 + n_1n_2 & l_2^2 + m_2^2 + n_2^2 & l_2l_3 + m_2m_3 + n_2n_3` \\ l_1l_3 + m_1m_3 + n_1n_3 & l_3l_2 + m_3m_2 + n_3n_2 & l_3^2 + m_3^2 + n_3^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \{\text{Using (A) and (B)}\}$$

$$= I$$

Hence,

$$AA^T = I$$

Ex 5.5

Algebra of Matrices Ex 5.5 Q1

Given,

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$$\left(A - A^T\right) = \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^T\right)$$

$$= \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2-2 & 3-4 \\ 4-3 & 5-5 \end{bmatrix}$$

$$\left(A - A^T\right) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad \text{---(i)}$$

$$-\left(A - A^T\right)^T = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^T$$

$$= -\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$-\left(A - A^T\right)^T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad \text{---(ii)}$$

From (i) and (ii),

$$\left(A - A^T\right) = -\left(A - A^T\right)^T$$

We know that, x is a skew symmetric matrix if $x = -x^T$

So, $\left(A - A^T\right)$ is skew symmetric.

Algebra of Matrices Ex 5.5 Q2

Given,

$$A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$$

$$A-A^T=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}-\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}^T$$

$$=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}-\begin{bmatrix}3 & 1\\ -4 & -1\end{bmatrix}$$

$$=\begin{bmatrix}3-3 & -4-1\\ 1+4 & -1+1\end{bmatrix}$$

$$A-A^T=\begin{bmatrix}0 & -5\\ 5 & 0\end{bmatrix} \qquad \text{---(i)}$$

$$-\left(A-A^T\right)^T=-\begin{bmatrix}0 & -5\\ 5 & 0\end{bmatrix}^T$$

$$=-\begin{bmatrix}0 & 5\\ -5 & 0\end{bmatrix}$$

$$-\left(A-A^T\right)^T=\begin{bmatrix}0 & -5\\ 5 & 0\end{bmatrix} \qquad \text{---(ii)}$$

From equation (i) and (ii),

$$\left(A-A^T\right)=-\left(A-A^T\right)^T$$

We know that, x is skewsymmetric matrx if $x=-x^T$

So, $\left(A-A^T\right)$ is skewsymmetric matrix.

Algebra of Matrices Ex 5.5 Q3

Given,

$$A = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix} \text{ is a symmetric matrix.}$$

We know that $A = [aij]_{m \times n}$ is a symmetric matrix if $aij = aji$

So, $x = a_{13} = a_{31} = 4$

$y = a_{21} = a_{12} = 2$

$z = a_{22} = a_{22} = z$

$t = a_{32} = a_{32} = -3$

Hence,

$x = 4, y = 2, t = -3$ and z can have any value.

Algebra of Matrices Ex 5.5 Q4

Given,

$$A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} \qquad \Rightarrow \qquad A^T = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix}$$

$$\therefore X = \frac{1}{2}\left(A + A^T\right)$$

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 3+3 & 2+1 & 7-2 \\ 1+2 & 4+4 & 3+5 \\ -2+7 & 5+3 & 8+8 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 6 & 3 & 5 \\ 3 & 8 & 8 \\ 5 & 8 & 16 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix}$$

Now, $Y = \frac{1}{2}\left(A - A^T\right)$

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 3-3 & 2-1 & 7+2 \\ 1-2 & 4-4 & 3-5 \\ -2-7 & 5-3 & 8-8 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 1 & 9 \\ -1 & 0 & -2 \\ -9 & 2 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}$$

Now,

$$X^T = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix}^T = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix}^T = X$$

$\Rightarrow$ x is a symmetric matrix

Now,

$$-Y^T = -\begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-9}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{9}{2} & -1 & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & \frac{1}{2} & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \qquad -Y' = Y$$

$\therefore$ Y is skew symmetric matrix.

$$X + Y = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3+0 & \frac{3}{2}+\frac{1}{2} & \frac{5}{2}+\frac{9}{2} \\ \frac{3}{2}-\frac{1}{2} & 4+0 & 4-1 \\ \frac{5}{2}-\frac{9}{2} & 4+1 & 8-0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix}$$

$$= A$$

Hence,

$$X = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{4} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix}, \quad Y = \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}$$

Given,

$$A = \begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} \quad \Rightarrow \quad A^T = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{bmatrix}$$

Let $X = \frac{1}{2}\left(A + A^T\right)$

$$= \frac{1}{2}\left(\begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 4+4 & 2+3 & -1+1 \\ 3+2 & 5+5 & 7-2 \\ 1-1 & -2+7 & 1+1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 8 & 5 & 0 \\ 5 & 10 & 5 \\ 0 & 5 & 2 \end{bmatrix} = \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix}$$

$$X^T = \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix}^T = \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix}^T = X$$

$\therefore$ X is symmetric matrix

Now,

$$Y = \frac{1}{2}\left(A - A^T\right)$$

$$= \frac{1}{2}\left(\begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 4-4 & 2-3 & -1-1 \\ 3-2 & 5-5 & 7+2 \\ 1+1 & -2-7 & 1-1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 9 \\ 2 & -9 & 0 \end{bmatrix}$$

$$\therefore Y = \begin{bmatrix} 0 & \frac{-1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & \frac{-9}{2} & 0 \end{bmatrix}$$

$$\Rightarrow -Y^T = -\begin{bmatrix} 0 & \frac{-1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & \frac{-9}{2} & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & \frac{-1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & \frac{-9}{2} & 0 \end{bmatrix} = Y$$

$\Rightarrow$ Y is a skew symmetric maatrix.

Now,

$$X+Y=\begin{bmatrix}4 & \frac{5}{2} & 0\\ \frac{5}{2} & 5 & \frac{5}{2}\\ 0 & \frac{5}{2} & 1\end{bmatrix}+\begin{bmatrix}0 & \frac{-1}{2} & -1\\ \frac{1}{2} & 0 & \frac{9}{2}\\ 1 & \frac{-9}{2} & 0\end{bmatrix}$$

$$=\begin{bmatrix}4+0 & \frac{5}{2}-\frac{1}{2} & 0-1\\ \frac{5}{2}+\frac{1}{2} & 5+0 & \frac{5}{2}+\frac{9}{2}\\ 0+1 & \frac{5}{2}-\frac{9}{2} & 1+0\end{bmatrix}$$

$$=\begin{bmatrix}4 & 2 & -1\\ 3 & 5 & 7\\ 1 & -2 & 1\end{bmatrix}=A$$

Algebra of Matrices Ex 5.5 Q6

A square matrix A is called a symmetric matrix, if $A^T = A$

Here,

$$A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 4+5 \\ 5+4 & 6+6 \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix} \qquad \text{---(i)}$$

$$\left(A + A^T\right)^T = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix}^T$$

$$\left(A + A^T\right)^T = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix} \qquad \text{---(ii)}$$

From equation (i) and (ii),

$$\left(A + A^T\right)^T = \left(A + A^T\right)$$

So,

$\left(A + A^T\right)$ is a symmetric matrix.

Algebra of Matrices Ex 5.5 Q7

Here,

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \quad \Rightarrow A^T = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$$

$$Let, \quad X = \frac{1}{2}\left(A + A^T\right) = \frac{1}{2}\left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 3+3 & -4+1 \\ 1-4 & -1-1 \end{bmatrix} = \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix}$$

$$Now, \quad X^T = \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix}^T = \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix} = X$$

$\Rightarrow \quad X$ is symmetric matrix

$$Let \quad Y = \frac{1}{2}\left(A - A^T\right) = \frac{1}{2}\left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 3-3 & -4-1 \\ 1+4 & -1+1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$$

$$Now, \quad -Y^T = -\begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix} = Y$$

$\Rightarrow \quad Y$ is skew symmetric

$$Now, \quad X + Y = \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix} = \begin{bmatrix} 3+0 & \frac{-3}{2} - \frac{5}{2} \\ \frac{-3}{2} + \frac{5}{2} & -1+0 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = A$$

Algebra of Matrices Ex 5.5 Q8

Let,

$$A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} \quad \Rightarrow \quad A^T = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

Let, $X = \frac{1}{2}\left(A + A^T\right) = \frac{1}{2}\left(\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 3+3 & -2+3 & -4-1 \\ 3-2 & -2-2 & -5+1 \\ -1-4 & 1-5 & 2+2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix}$$

Now, $X^T = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix}^T = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix} = X$

$\Rightarrow$ X is a symmetric matrix

Let, $Y = \frac{1}{2}\left(A - A^T\right) = \frac{1}{2}\left(\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 3-3 & -2-3 & -4+1 \\ 3+2 & -2+2 & -5-1 \\ -1+4 & 1+5 & 2-2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix}$$

$-Y^T = -\begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix} = Y$

$\Rightarrow$ Y is a skew symmetric matrix

$X + Y = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix} = \begin{bmatrix} 3+0 & \frac{1}{2}-\frac{5}{2} & \frac{-5}{2}-\frac{3}{2} \\ \frac{1}{2}+\frac{5}{2} & -2+0 & -2-3 \\ \frac{-5}{2}+\frac{3}{2} & -2+3 & 2+0 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = A$

Hence, Symmetric matrix $X = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix}$