

# NCERT Solutions for Class 10 Maths Unit 8

## Introduction to Trigonometry Class 10

Unit 8 Introduction to Trigonometry Exercise 8.1, 8.2, 8.3, 8.4 Solutions

**Exercise 8.1 : Solutions of Questions on Page Number : 181**

**Q1 :**

In  $\triangle ABC$  right angled at B, AB = 24 cm, BC = 7 m. Determine

(i)  $\sin A$ ,  $\cos A$

(ii)  $\sin C$ ,  $\cos C$

**Answer :**

Applying Pythagoras theorem for  $\triangle ABC$ , we obtain

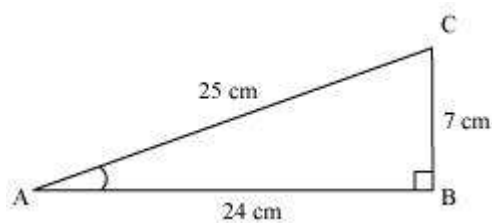
$$AC^2 = AB^2 + BC^2$$

$$= (24 \text{ cm})^2 + (7 \text{ cm})^2$$

$$= (576 + 49) \text{ cm}^2$$

$$= 625 \text{ cm}^2$$

$$\therefore AC = \sqrt{625} \text{ cm} = 25 \text{ cm}$$

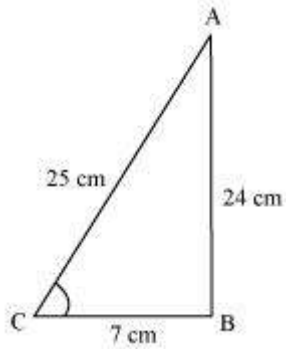


$$(i) \sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{7}{25}$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$$

(ii)



$$\sin C = \frac{\text{Side opposite to } \angle C}{\text{Hypotenuse}} = \frac{AB}{AC}$$

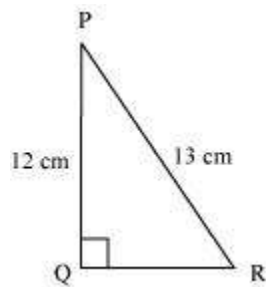
$$= \frac{24}{25}$$

$$\cos C = \frac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{7}{25}$$

**Q2 :**

In the given figure find  $\tan P - \cot R$



**Answer :**

Applying Pythagoras theorem for  $\triangle PQR$ , we obtain

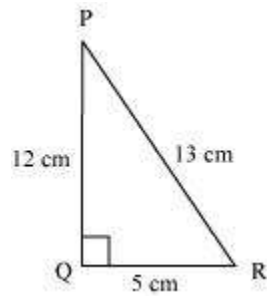
$$PR^2 = PQ^2 + QR^2$$

$$(13 \text{ cm})^2 = (12 \text{ cm})^2 + QR^2$$

$$169 \text{ cm}^2 = 144 \text{ cm}^2 + QR^2$$

$$25 \text{ cm}^2 = QR^2$$

$$QR = 5 \text{ cm}$$



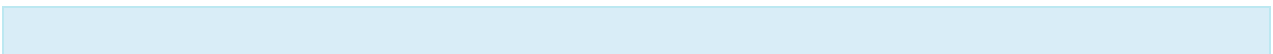
$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

$$\cot R = \frac{\text{Side adjacent to } \angle R}{\text{Side opposite to } \angle R} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

$$\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$$

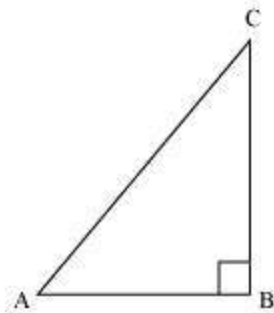


**Q3 :**

If  $\sin A = \frac{3}{4}$ , calculate  $\cos A$  and  $\tan A$ .

**Answer :**

Let  $\triangle ABC$  be a right-angled triangle, right-angled at point B.



Given that,

$$\sin A = \frac{3}{4}$$

$$\frac{BC}{AC} = \frac{3}{4}$$

Let BC be  $3k$ . Therefore, AC will be  $4k$ , where  $k$  is a positive integer.

Applying Pythagoras theorem in  $\triangle ABC$ , we obtain

$$AC^2 = AB^2 + BC^2$$

$$(4k)^2 = AB^2 + (3k)^2$$

$$16k^2 - 9k^2 = AB^2$$

$$7k^2 = AB^2$$

$$AB = \sqrt{7}k$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A}$$

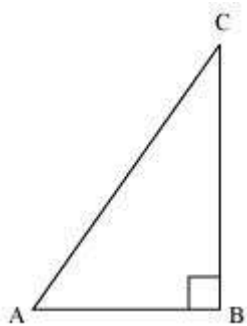
$$= \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$$

**Q4 :**

**Given  $15 \cot A = 8$ . Find  $\sin A$  and  $\sec A$**

**Answer :**

Consider a right-angled triangle, right-angled at B.



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$= \frac{AB}{BC}$$

It is given that,

$$\cot A = \frac{8}{15}$$

$$\frac{AB}{BC} = \frac{8}{15}$$

Let AB be  $8k$ . Therefore, BC will be  $15k$ , where  $k$  is a positive integer.

Applying Pythagoras theorem in  $\triangle ABC$ , we obtain

$$AC^2 = AB^2 + BC^2$$

$$= (8k)^2 + (15k)^2$$

$$= 64k^2 + 225k^2$$

$$= 289k^2$$

$$AC = 17k$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{15k}{17k} = \frac{15}{17}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle A}$$

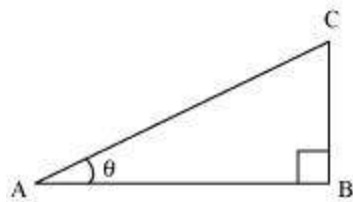
$$= \frac{AC}{AB} = \frac{17}{8}$$

**Q5 :**

Given  $\sec \theta = \frac{13}{12}$ , calculate all other trigonometric ratios.

**Answer :**

Consider a right-angle triangle  $\triangle ABC$ , right-angled at point B.



$$\sec \theta = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle \theta}$$

$$\frac{13}{12} = \frac{AC}{AB}$$

If AC is  $13k$ , AB will be  $12k$ , where  $k$  is a positive integer.

Applying Pythagoras theorem in  $\triangle ABC$ , we obtain

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(13k)^2 = (12k)^2 + (BC)^2$$

$$169k^2 = 144k^2 + BC^2$$

$$25k^2 = BC^2$$

$$BC = 5k$$

$$\sin \theta = \frac{\text{Side opposite to } \angle \theta}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\cos \theta = \frac{\text{Side adjacent to } \angle \theta}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$$

$$\tan \theta = \frac{\text{Side opposite to } \angle \theta}{\text{Side adjacent to } \angle \theta} = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$$

$$\cot \theta = \frac{\text{Side adjacent to } \angle \theta}{\text{Side opposite to } \angle \theta} = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$$

$$\operatorname{cosec} \theta = \frac{\text{Hypotenuse}}{\text{Side opposite to } \angle \theta} = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5}$$

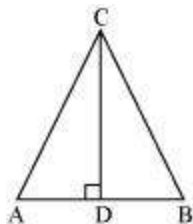
**Q6 :**

If  $\angle A$  and  $\angle B$  are acute angles such that  $\cos A = \cos B$ , then show that

$\angle A = \angle B$ .

**Answer :**

Let us consider a triangle ABC in which  $CD \perp AB$ .

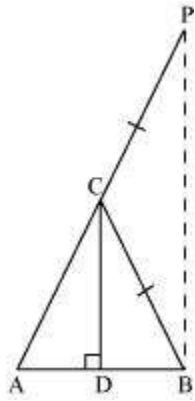


It is given that

$$\cos A = \cos B$$

$$\Rightarrow \frac{AD}{AC} = \frac{BD}{BC} \dots (1)$$

We have to prove  $\angle A = \angle B$ . To prove this, let us extend AC to P such that  $BC = CP$ .



From equation (1), we obtain

$$\begin{aligned} \frac{AD}{BD} &= \frac{AC}{BC} \\ \Rightarrow \frac{AD}{BD} &= \frac{AC}{CP} \quad \text{(By construction, we have } BC = CP) \quad \dots (2) \end{aligned}$$

By using the converse of B.P.T,

$CD \parallel BP$

$\Rightarrow \angle ACD = \angle CPB$  (Corresponding angles) ... (3)

And,  $\angle BCD = \angle CBP$  (Alternate interior angles) ... (4)

By construction, we have  $BC = CP$ .

$\therefore \angle CBP = \angle CPB$  (Angle opposite to equal sides of a triangle) ... (5)

From equations (3), (4), and (5), we obtain

$\angle ACD = \angle BCD$  ... (6)

In  $\triangle CAD$  and  $\triangle CBD$ ,

$\angle ACD = \angle BCD$  [Using equation (6)]

$\angle CDA = \angle CDB$  [Both  $90^\circ$ ]

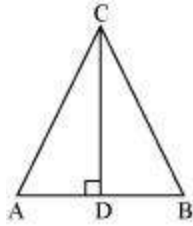
Therefore, the remaining angles should be equal.

$\therefore \angle CAD = \angle CBD$

$\Rightarrow \angle A = \angle B$

**Alternatively,**

Let us consider a triangle ABC in which  $CD \perp AB$ .



It is given that,

$$\cos A = \cos B$$

$$\Rightarrow \frac{AD}{AC} = \frac{BD}{BC}$$

$$\Rightarrow \frac{AD}{BD} = \frac{AC}{BC}$$

$$\text{Let } \frac{AD}{BD} = \frac{AC}{BC} = k$$

$$\Rightarrow AD = k BD \dots (1)$$

$$\text{And, } AC = k BC \dots (2)$$

Using Pythagoras theorem for triangles CAD and CBD, we obtain

$$CD^2 = AC^2 - AD^2 \dots (3)$$

$$\text{And, } CD^2 = BC^2 - BD^2 \dots (4)$$

From equations (3) and (4), we obtain

$$AC^2 - AD^2 = BC^2 - BD^2$$

$$\Rightarrow (k BC)^2 - (k BD)^2 = BC^2 - BD^2$$

$$\Rightarrow k^2 (BC^2 - BD^2) = BC^2 - BD^2$$

$$\Rightarrow k^2 = 1$$

$$\Rightarrow k = 1$$

Putting this value in equation (2), we obtain

$$AC = BC$$

$$\Rightarrow \angle A = \angle B (\text{Angles opposite to equal sides of a triangle})$$

**Q7 :**

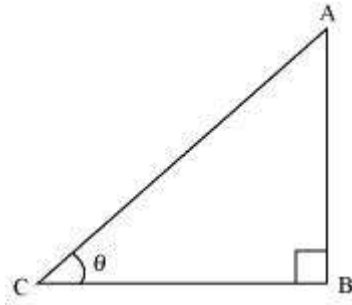
If  $\cot \theta = \frac{7}{8}$ , evaluate

$$(i) \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} \quad (ii) \cot^2 \theta$$



**Answer :**

Let us consider a right triangle ABC, right-angled at point B.



$$\begin{aligned}\cot \theta &= \frac{\text{Side adjacent to } \angle \theta}{\text{Side opposite to } \angle \theta} = \frac{BC}{AB} \\ &= \frac{7}{8}\end{aligned}$$

If BC is  $7k$ , then AB will be  $8k$ , where  $k$  is a positive integer.

Applying Pythagoras theorem in  $\triangle ABC$ , we obtain

$$AC^2 = AB^2 + BC^2$$

$$= (8k)^2 + (7k)^2$$

$$= 64k^2 + 49k^2$$

$$= 113k^2$$

$$AC = \sqrt{113}k$$

$$\begin{aligned}\sin \theta &= \frac{\text{Side opposite to } \angle \theta}{\text{Hypotenuse}} = \frac{AB}{AC} \\ &= \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}\end{aligned}$$

$$\begin{aligned}\cos \theta &= \frac{\text{Side adjacent to } \angle \theta}{\text{Hypotenuse}} = \frac{BC}{AC} \\ &= \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}\end{aligned}$$

$$(i) \quad \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} = \frac{(1 - \sin^2 \theta)}{(1 - \cos^2 \theta)}$$

$$= \frac{1 - \left(\frac{8}{\sqrt{113}}\right)^2}{1 - \left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1 - \frac{64}{113}}{1 - \frac{49}{113}}$$

$$= \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$$

$$(ii) \cot^2 \theta = (\cot \theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

**Q8 :**

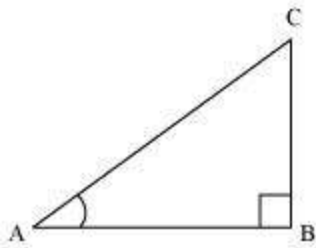
If  $3 \cot A = 4$ , Check whether  $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$  or not.

**Answer :**

It is given that  $3 \cot A = 4$

$$\text{Or, } \cot A = \frac{4}{3}$$

Consider a right triangle ABC, right-angled at point B.



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$\frac{AB}{BC} = \frac{4}{3}$$

If AB is  $4k$ , then BC will be  $3k$ , where  $k$  is a positive integer.

In  $\triangle ABC$ ,

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$= (4k)^2 + (3k)^2$$

$$= 16k^2 + 9k^2$$

$$= 25k^2$$

$$AC = 5k$$

$$\begin{aligned}\cos A &= \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} \\ &= \frac{4k}{5k} = \frac{4}{5}\end{aligned}$$

$$\begin{aligned}\sin A &= \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC} \\ &= \frac{3k}{5k} = \frac{3}{5}\end{aligned}$$

$$\begin{aligned}\tan A &= \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AB} \\ &= \frac{3k}{4k} = \frac{3}{4}\end{aligned}$$

$$\begin{aligned}\frac{1 - \tan^2 A}{1 + \tan^2 A} &= \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}} \\ &= \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}\end{aligned}$$

$$\begin{aligned}\cos^2 A - \sin^2 A &= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 \\ &= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}\end{aligned}$$

$$\therefore \frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$$

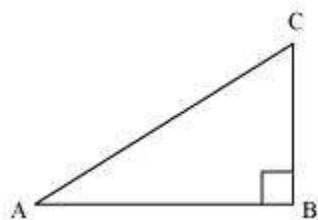
**Q9 :**

In  $\triangle ABC$ , right angled at B. If  $\tan A = \frac{1}{\sqrt{3}}$ , find the value of

(i)  $\sin A \cos C + \cos A \sin C$

(ii)  $\cos A \cos C - \sin A \sin C$

Answer :



$$\tan A = \frac{1}{\sqrt{3}}$$

$$\frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

If BC is  $k$ , then AB will be  $\sqrt{3}k$ , where  $k$  is a positive integer.

In  $\triangle ABC$ ,

$$AC^2 = AB^2 + BC^2$$

$$= (\sqrt{3}k)^2 + (k)^2$$

$$= 3k^2 + k^2 = 4k^2$$

$$\therefore AC = 2k$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{\text{Side opposite to } \angle C}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

(i)  $\sin A \cos C + \cos A \sin C$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{3}{4}$$

$$= \frac{4}{4} = 1$$

(ii)  $\cos A \cos C - \sin A \sin C$

$$= \left( \frac{\sqrt{3}}{2} \right) \left( \frac{1}{2} \right) - \left( \frac{1}{2} \right) \left( \frac{\sqrt{3}}{2} \right) = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

**Q10 :**

In  $\Delta PQR$ , right angled at Q,  $PR + QR = 25$  cm and  $PQ = 5$  cm. Determine the values of  $\sin P$ ,  $\cos P$  and  $\tan P$ .

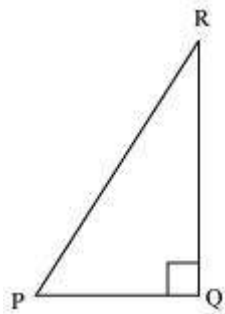
**Answer :**

Given that,  $PR + QR = 25$

$PQ = 5$

Let  $PR$  be  $x$ .

Therefore,  $QR = 25 - x$



Applying Pythagoras theorem in  $\Delta PQR$ , we obtain

$$PR^2 = PQ^2 + QR^2$$

$$x^2 = (5)^2 + (25 - x)^2$$

$$x^2 = 25 + 625 + x^2 - 50x$$

$$50x = 650$$

$$x = 13$$

Therefore,  $PR = 13$  cm

$$QR = (25 - 13) \text{ cm} = 12 \text{ cm}$$

$$\sin P = \frac{\text{Side opposite to } \angle P}{\text{Hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$$

$$\cos P = \frac{\text{Side adjacent to } \angle P}{\text{Hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$$

$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{12}{5}$$

Q11 :

State whether the following are true or false. Justify your answer.

(i) The value of  $\tan A$  is always less than 1.

(ii)  $\sec A = \frac{12}{5}$  for some value of angle  $A$ .

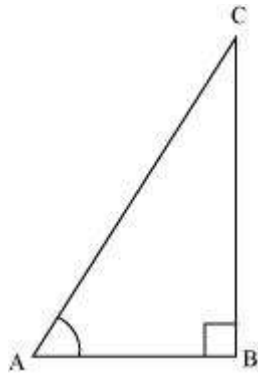
(iii)  $\cos A$  is the abbreviation used for the cosecant of angle  $A$ .

(iv)  $\cot A$  is the product of  $\cot$  and  $A$

(v)  $\sin \theta = \frac{4}{3}$ , for some angle  $\theta$

Answer :

(i) Consider a  $\triangle ABC$ , right-angled at  $B$ .



$$\begin{aligned}\tan A &= \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A} \\ &= \frac{12}{5}\end{aligned}$$

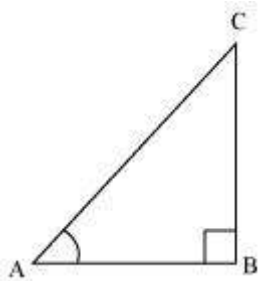
But  $\frac{12}{5} > 1$

$\therefore \tan A > 1$

So,  $\tan A < 1$  is not always true.

Hence, the given statement is false.

(ii)  $\sec A = \frac{12}{5}$



$$\frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle A} = \frac{12}{5}$$

$$\frac{AC}{AB} = \frac{12}{5}$$

Let AC be  $12k$ , AB will be  $5k$ , where  $k$  is a positive integer.

Applying Pythagoras theorem in  $\triangle ABC$ , we obtain

$$AC^2 = AB^2 + BC^2$$

$$(12k)^2 = (5k)^2 + BC^2$$

$$144k^2 = 25k^2 + BC^2$$

$$BC^2 = 119k^2$$

$$BC = 10.9k$$

It can be observed that for given two sides  $AC = 12k$  and  $AB = 5k$ ,

BC should be such that,

$$AC - AB < BC < AC + AB$$

$$12k - 5k < BC < 12k + 5k$$

$$7k < BC < 17k$$

However,  $BC = 10.9k$ . Clearly, such a triangle is possible and hence, such value of  $\sec A$  is possible.

Hence, the given statement is true.

(iii) Abbreviation used for cosecant of angle A is cosec A. And cos A is the abbreviation used for cosine of angle A.

Hence, the given statement is false.

(iv)  $\cot A$  is not the product of cot and A. It is the cotangent of  $\angle A$ .

Hence, the given statement is false.

$$(v) \sin \theta = \frac{4}{3}$$

We know that in a right-angled triangle,

$$\sin \theta = \frac{\text{Side opposite to } \angle \theta}{\text{Hypotenuse}}$$

In a right-angled triangle, hypotenuse is always greater than the remaining two sides. Therefore, such value of  $\sin \theta$  is not possible.

Hence, the given statement is false

**Exercise 8.2 : Solutions of Questions on Page Number : 187**

**Q1 :**

**Evaluate the following**

(i)  $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

(ii)  $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

(iii)  $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

(iv)  $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

(v)  $\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

**Answer :**

(i)  $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$$\begin{aligned} &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \\ &= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1 \end{aligned}$$

(ii)  $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$$\begin{aligned} &= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 \\ &= 2 + \frac{3}{4} - \frac{3}{4} = 2 \end{aligned}$$

(iii)  $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$



$$\begin{aligned}
&= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}} \\
&= \frac{\sqrt{3}}{\sqrt{2}(2+2\sqrt{3})} = \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}} \\
&= \frac{\sqrt{3}(2\sqrt{6}-2\sqrt{2})}{(2\sqrt{6}+2\sqrt{2})(2\sqrt{6}-2\sqrt{2})} \\
&= \frac{2\sqrt{3}(\sqrt{6}-\sqrt{2})}{(2\sqrt{6})^2 - (2\sqrt{2})^2} = \frac{2\sqrt{3}(\sqrt{6}-\sqrt{2})}{24-8} = \frac{2\sqrt{3}(\sqrt{6}-\sqrt{2})}{16} \\
&= \frac{\sqrt{18}-\sqrt{6}}{8} = \frac{3\sqrt{2}-\sqrt{6}}{8}
\end{aligned}$$

$$(iv) \quad \frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

$$\begin{aligned}
&= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{3}{2} + \frac{2}{\sqrt{3}}}
\end{aligned}$$

$$\begin{aligned}
&= \frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{3\sqrt{3}+4}{2\sqrt{3}}} = \frac{(3\sqrt{3}-4)}{(3\sqrt{3}+4)}
\end{aligned}$$

$$= \frac{(3\sqrt{3}-4)(3\sqrt{3}-4)}{(3\sqrt{3}+4)(3\sqrt{3}-4)} = \frac{(3\sqrt{3}-4)^2}{(3\sqrt{3})^2 - (4)^2}$$

$$= \frac{27+16-24\sqrt{3}}{27-16} = \frac{43-24\sqrt{3}}{11}$$

$$(v) \quad \frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

$$\begin{aligned}
&= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \\
&= \frac{5\left(\frac{1}{4}\right) + \left(\frac{16}{3}\right) - 1}{\frac{1}{4} + \frac{3}{4}} \\
&= \frac{\frac{15 + 64 - 12}{12}}{\frac{4}{4}} = \frac{67}{12}
\end{aligned}$$

**Q2 :**

Choose the correct option and justify your choice.

(i)  $\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} =$

- (A).  $\sin 60^\circ$
- (B).  $\cos 60^\circ$
- (C).  $\tan 60^\circ$
- (D).  $\sin 30^\circ$

(ii)  $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} =$

- (A).  $\tan 90^\circ$
- (B). 1
- (C).  $\sin 45^\circ$
- (D). 0

(iii)  $\sin 2A = 2 \sin A$  is true when  $A =$

- (A).  $0^\circ$
- (B).  $30^\circ$
- (C).  $45^\circ$
- (D).  $60^\circ$

(iv)  $\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} =$

(A).  $\cos 60^\circ$

(B).  $\sin 60^\circ$

(C).  $\tan 60^\circ$

(D).  $\sin 30^\circ$

Answer :

$$\begin{aligned} & \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} \\ \text{(i)} \quad &= \frac{2 \left( \frac{1}{\sqrt{3}} \right)}{1 + \left( \frac{1}{\sqrt{3}} \right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} \\ &= \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2} \end{aligned}$$

Out of the given alternatives, only  $\sin 60^\circ = \frac{\sqrt{3}}{2}$

Hence, (A) is correct.

$$\begin{aligned} & \frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} \\ \text{(ii)} \quad &= \frac{1 - (1)^2}{1 + (1)^2} = \frac{1 - 1}{1 + 1} = \frac{0}{2} = 0 \end{aligned}$$

Hence, (D) is correct.

(iii) Out of the given alternatives, only  $A = 0^\circ$  is correct.

As  $\sin 2A = \sin 0^\circ = 0$

$2 \sin A = 2 \sin 0^\circ = 2(0) = 0$

Hence, (A) is correct.

$$\begin{aligned} & \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} \\ \text{(iv)} \quad &= \frac{2 \left( \frac{1}{\sqrt{3}} \right)}{1 - \left( \frac{1}{\sqrt{3}} \right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} \\ &= \sqrt{3} \end{aligned}$$

Out of the given alternatives, only  $\tan 60^\circ = \sqrt{3}$

Hence, (C) is correct.

**Q3 :**

If  $\tan(A + B) = \sqrt{3}$  and  $\tan(A - B) = \frac{1}{\sqrt{3}}$ ;

$0^\circ < A + B \leq 90^\circ$ ,  $A > B$  find A and B.

**Answer :**

$$\tan(A + B) = \sqrt{3}$$

$$\Rightarrow \tan(A + B) = \tan 60$$

$$\Rightarrow A + B = 60 \dots (1)$$

$$\tan(A - B) = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan(A - B) = \tan 30$$

$$\Rightarrow A - B = 30 \dots (2)$$

On adding both equations, we obtain

$$2A = 90$$

$$\Rightarrow A = 45$$

From equation (1), we obtain

$$45 + B = 60$$

$$B = 15$$

Therefore,  $\angle A = 45^\circ$  and  $\angle B = 15^\circ$

**Q4 :**

State whether the following are true or false. Justify your answer.

(i)  $\sin(A + B) = \sin A + \sin B$

(ii) The value of  $\sin \theta$  increases as  $\theta$  increases

(iii) The value of  $\cos \theta$  increases as  $\theta$  increases

(iv)  $\sin \theta = \cos \theta$  for all values of  $\theta$

(v)  $\cot A$  is not defined for  $A = 0^\circ$

**Answer :**

(i)  $\sin(A + B) = \sin A + \sin B$

Let  $A = 30^\circ$  and  $B = 60^\circ$

$$\sin(A + B) = \sin(30^\circ + 60^\circ)$$

$$= \sin 90^\circ$$

$$= 1$$

$$\sin A + \sin B = \sin 30^\circ + \sin 60^\circ$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1 + \sqrt{3}}{2}$$

Clearly,  $\sin(A + B) \neq \sin A + \sin B$

Hence, the given statement is false.

(ii) The value of  $\sin \theta$  increases as  $\theta$  increases in the interval of  $0^\circ < \theta < 90^\circ$  as

$$\sin 0^\circ = 0$$

$$\sin 30^\circ = \frac{1}{2} = 0.5$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}} = 0.707$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} = 0.866$$

$$\sin 90^\circ = 1$$

Hence, the given statement is true.

(iii)  $\cos 0^\circ = 1$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} = 0.866$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = 0.707$$

$$\cos 60^\circ = \frac{1}{2} = 0.5$$

$$\cos 90^\circ = 0$$

It can be observed that the value of  $\cos \theta$  does not increase in the interval of  $0^\circ < \theta < 90^\circ$ .

Hence, the given statement is false.

(iv)  $\sin \theta = \cos \theta$  for all values of  $\theta$ .

This is true when  $\theta = 45^\circ$

$$\text{As } \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

It is not true for all other values of  $\theta$ .

$$\text{As } \sin 30^\circ = \frac{1}{2} \text{ and } \cos 30^\circ = \frac{\sqrt{3}}{2},$$

Hence, the given statement is false.

(v)  $\cot A$  is not defined for  $A = 0^\circ$

$$\text{As } \cot A = \frac{\cos A}{\sin A},$$

$$\cot 0^\circ = \frac{\cos 0^\circ}{\sin 0^\circ} = \frac{1}{0} = \text{undefined}$$

Hence, the given statement is true.

### Exercise 8.3 : Solutions of Questions on Page Number : 189

**Q1 :**

**Evaluate**

$$(I) \frac{\sin 18^\circ}{\cos 72^\circ}$$

$$(II) \frac{\tan 26^\circ}{\cot 64^\circ}$$

$$(III) \cos 48^\circ - \sin 42^\circ$$

$$(IV) \operatorname{cosec} 31^\circ - \sec 59^\circ$$

**Answer :**

$$(I) \frac{\sin 18^\circ}{\cos 72^\circ} = \frac{\sin(90^\circ - 72^\circ)}{\cos 72^\circ}$$

$$= \frac{\cos 72^\circ}{\cos 72^\circ} = 1$$

$$(II) \frac{\tan 26^\circ}{\cot 64^\circ} = \frac{\tan(90^\circ - 64^\circ)}{\cot 64^\circ}$$

$$= \frac{\cot 64^\circ}{\cot 64^\circ} = 1$$

$$(III) \cos 48^\circ - \sin 42^\circ = \cos (90^\circ - 42^\circ) - \sin 42^\circ$$

$$= \sin 42^\circ - \sin 42^\circ$$

$$= 0$$

$$(IV) \operatorname{cosec} 31^\circ - \sec 59^\circ = \operatorname{cosec} (90^\circ - 59^\circ) - \sec 59^\circ$$

$$= \sec 59^\circ - \sec 59^\circ$$

$$= 0$$

**Q2 :**

**Show that**

$$(I) \tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$$

$$(II) \cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$$

**Answer :**

$$(I) \tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$$

$$= \tan (90^\circ - 42^\circ) \tan (90^\circ - 67^\circ) \tan 42^\circ \tan 67^\circ$$

$$= \cot 42^\circ \cot 67^\circ \tan 42^\circ \tan 67^\circ$$

$$= (\cot 42^\circ \tan 42^\circ) (\cot 67^\circ \tan 67^\circ)$$

$$= (1) (1)$$

$$= 1$$

$$(II) \cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ$$

$$= \cos (90^\circ - 52^\circ) \cos (90^\circ - 38^\circ) - \sin 38^\circ \sin 52^\circ$$

$$= \sin 52^\circ \sin 38^\circ - \sin 38^\circ \sin 52^\circ$$

$$= 0$$

**Q3 :**

**If  $\tan 2A = \cot (A - 18^\circ)$ , where  $2A$  is an acute angle, find the value of  $A$ .**

**Answer :**

Given that,

$$\tan 2A = \cot (A - 18^\circ)$$

$$\cot (90^\circ - 2A) = \cot (A - 18^\circ)$$

$$90^\circ - 2A = A - 18^\circ$$

$$108^\circ = 3A$$

$$A = 36^\circ$$

**Q4 :**

**If  $\tan A = \cot B$ , prove that  $A + B = 90^\circ$**

**Answer :**

Given that,

$$\tan A = \cot B$$

$$\tan A = \tan (90^\circ - B)$$

$$A = 90^\circ - B$$

$$A + B = 90^\circ$$

**Q5 :**

**If  $\sec 4A = \operatorname{cosec} (A - 20^\circ)$ , where  $4A$  is an acute angle, find the value of  $A$ .**

**Answer :**

Given that,

$$\sec 4A = \operatorname{cosec} (A - 20^\circ)$$

$$\operatorname{cosec} (90^\circ - 4A) = \operatorname{cosec} (A - 20^\circ)$$

$$90^\circ - 4A = A - 20^\circ$$

$$110^\circ = 5A$$

$$A = 22^\circ$$

**Q6 :**

**If  $A$ ,  $B$  and  $C$  are interior angles of a triangle  $ABC$  then show that**

$$\sin \left( \frac{B+C}{2} \right) = \cos \frac{A}{2}$$

**Answer :**

We know that for a triangle  $ABC$ ,

$$\angle A + \angle B + \angle C = 180^\circ$$



- $\angle B + \angle C = 180^\circ - \angle A$

$$\begin{aligned}\frac{\angle B + \angle C}{2} &= 90^\circ - \frac{\angle A}{2} \\ \sin\left(\frac{B+C}{2}\right) &= \sin\left(90^\circ - \frac{A}{2}\right) \\ &= \cos\left(\frac{A}{2}\right)\end{aligned}$$

**Q7 :**

Express  $\sin 67^\circ + \cos 75^\circ$  in terms of trigonometric ratios of angles between  $0^\circ$  and  $45^\circ$ .

**Answer :**

$$\begin{aligned}\sin 67^\circ + \cos 75^\circ \\ &= \sin (90^\circ - 23^\circ) + \cos (90^\circ - 15^\circ) \\ &= \cos 23^\circ + \sin 15^\circ\end{aligned}$$

**Exercise 8.4 :** Solutions of Questions on Page Number : 193

**Q1 :**

Express the trigonometric ratios  $\sin A$ ,  $\sec A$  and  $\tan A$  in terms of  $\cot A$ .

**Answer :**

We know that,

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\frac{1}{\operatorname{cosec}^2 A} = \frac{1}{1 + \cot^2 A}$$

$$\sin^2 A = \frac{1}{1 + \cot^2 A}$$

$$\sin A = \pm \frac{1}{\sqrt{1 + \cot^2 A}}$$

$\sqrt{1 + \cot^2 A}$  will always be positive as we are adding two positive quantities.

$$\sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$$

Therefore,

$$\tan A = \frac{\sin A}{\cos A}$$

We know that,

However,  $\cot A = \frac{\cos A}{\sin A}$

Therefore,  $\tan A = \frac{1}{\cot A}$

Also,  $\sec^2 A = 1 + \tan^2 A$

$$= 1 + \frac{1}{\cot^2 A}$$

$$= \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

**Q2 :**

Write all the other trigonometric ratios of  $\angle A$  in terms of  $\sec A$ .

**Answer :**

We know that,

$$\cos A = \frac{1}{\sec A}$$

Also,  $\sin^2 A + \cos^2 A = 1$

$$\sin^2 A = 1 - \cos^2 A$$

$$\begin{aligned} \sin A &= \sqrt{1 - \left(\frac{1}{\sec A}\right)^2} \\ &= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A} \end{aligned}$$

$$\tan^2 A + 1 = \sec^2 A$$

$$\tan^2 A = \sec^2 A - 1$$

$$\tan A = \sqrt{\sec^2 A - 1}$$

$$\cot A = \frac{\cos A}{\sin A} = \frac{\frac{1}{\sec A}}{\frac{1}{\sqrt{\sec^2 A - 1}}}$$

$$= \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

**Q3 :**

**Evaluate**

$$(i) \frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$$

$$(ii) \sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$$

**Answer :**

$$(i) \frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$$

$$= \frac{[\sin(90^\circ - 27^\circ)]^2 + \sin^2 27^\circ}{[\cos(90^\circ - 73^\circ)]^2 + \cos^2 73^\circ}$$

$$= \frac{[\cos 27^\circ]^2 + \sin^2 27^\circ}{[\sin 73^\circ]^2 + \cos^2 73^\circ}$$

$$= \frac{\cos^2 27^\circ + \sin^2 27^\circ}{\sin^2 73^\circ + \cos^2 73^\circ}$$

$$= \frac{1}{1} \quad (\text{As } \sin^2 A + \cos^2 A = 1)$$

$$= 1$$

$$(ii) \sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$$

$$= (\sin 25^\circ) \{ \cos(90^\circ - 25^\circ) \} + \cos 25^\circ \{ \sin(90^\circ - 25^\circ) \}$$

$$= (\sin 25^\circ)(\sin 25^\circ) + (\cos 25^\circ)(\cos 25^\circ)$$

$$= \sin^2 25^\circ + \cos^2 25^\circ$$

$$= 1 \text{ (As } \sin^2 A + \cos^2 A = 1)$$

**Q4 :**

**Choose the correct option. Justify your choice.**

(i)  $9 \sec^2 A - 9 \tan^2 A =$

(A) 1

(B) 9

(C) 8

(D) 0

(ii)  $(1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta)$

(A) 0

(B) 1

(C) 2

(D) - 1

(iii)  $(\sec A + \tan A) (1 - \sin A) =$

(A)  $\sec A$

(B)  $\sin A$

(C)  $\operatorname{cosec} A$

(D)  $\cos A$

(iv)  $\frac{1 + \tan^2 A}{1 + \cot^2 A}$

(A)  $\sec^2 A$

(B) - 1

(C)  $\cot^2 A$

(D)  $\tan^2 A$

**Answer :**

(i)  $9 \sec^2 A - 9 \tan^2 A$

$$= 9 (\sec^2 A - \tan^2 A)$$

$$= 9 (1) \text{ [As } \sec^2 A - \tan^2 A = 1]$$

$$= 9$$

Hence, alternative (B) is correct.

(ii)

$$(1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta)$$

$$\begin{aligned}
 &= \left(1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}\right) \left(1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}\right) \\
 &= \left(\frac{\cos \theta + \sin \theta + 1}{\cos \theta}\right) \left(\frac{\sin \theta + \cos \theta - 1}{\sin \theta}\right) \\
 &= \frac{(\sin \theta + \cos \theta)^2 - (1)^2}{\sin \theta \cos \theta} \\
 &= \frac{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta} \\
 &= \frac{1 + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta} \\
 &= \frac{2 \sin \theta \cos \theta}{\sin \theta \cos \theta} = 2
 \end{aligned}$$

Hence, alternative (C) is correct.

$$(iii) (\sec A + \tan A) (1 - \sin A)$$

$$\begin{aligned}
 &= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right) (1 - \sin A) \\
 &= \left(\frac{1 + \sin A}{\cos A}\right) (1 - \sin A) \\
 &= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A}
 \end{aligned}$$

$$= \cos A$$

Hence, alternative (D) is correct.

$$\begin{aligned}
 (iv) \quad \frac{1 + \tan^2 A}{1 + \cot^2 A} &= \frac{1 + \frac{\sin^2 A}{\cos^2 A}}{1 + \frac{\cos^2 A}{\sin^2 A}} \\
 &= \frac{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\frac{\sin^2 A + \cos^2 A}{\sin^2 A}} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} \\
 &= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A
 \end{aligned}$$

Hence, alternative (D) is correct.

Q5 :

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

Answer :

$$(i) \quad (\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$$

$$\begin{aligned} \text{L.H.S.} &= (\operatorname{cosec} \theta - \cot \theta)^2 \\ &= \left( \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)^2 \\ &= \frac{(1 - \cos \theta)^2}{(\sin \theta)^2} = \frac{(1 - \cos \theta)^2}{\sin^2 \theta} \\ &= \frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta} = \frac{(1 - \cos \theta)^2}{(1 - \cos \theta)(1 + \cos \theta)} = \frac{1 - \cos \theta}{1 + \cos \theta} \\ &= \text{R.H.S.} \end{aligned}$$

$$(ii) \quad \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$$

$$\begin{aligned} \text{L.H.S.} &= \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} \\ &= \frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A)(\cos A)} \\ &= \frac{\cos^2 A + 1 + \sin^2 A + 2 \sin A}{(1 + \sin A)(\cos A)} \\ &= \frac{\sin^2 A + \cos^2 A + 1 + 2 \sin A}{(1 + \sin A)(\cos A)} \\ &= \frac{1 + 1 + 2 \sin A}{(1 + \sin A)(\cos A)} = \frac{2 + 2 \sin A}{(1 + \sin A)(\cos A)} \\ &= \frac{2(1 + \sin A)}{(1 + \sin A)(\cos A)} = \frac{2}{\cos A} = 2 \sec A \\ &= \text{R.H.S.} \end{aligned}$$

$$(iii) \quad \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$$

$$\begin{aligned}
 \text{L.H.S.} &= \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} \\
 &= \frac{\frac{\sin \theta}{\cos \theta}}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{1 - \frac{\sin \theta}{\cos \theta}} \\
 &= \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta - \cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{\frac{\cos \theta - \sin \theta}{\cos \theta}} \\
 &= \frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)} - \frac{\cos^2 \theta}{\sin \theta (\sin \theta - \cos \theta)}
 \end{aligned}$$