Exercise 6.1

Prove the following trigonometric identities:

1. $\left(1-\cos^2 A\right)\cos ec^2 A=1$

Sol:

We know $\sin^2 A+\cos^2 A=1$

$\sin^2 A=1-\cos^2 A$

$\Rightarrow \sin^2 A\cdot \cos ec^2 A$

$\Rightarrow \sin^2 A\cdot \frac{1}{\sin^2 A}=1 \qquad \therefore L.H.S=R.H.S$

2. $\left(1+\cos^2 A\right)\sin^2 A=1$

Sol:

We know that $\cos ec^2 A-a^2-A=1$

$\cos ec^2 A=1+\cot^2 A$

$\Rightarrow \cos ec^2 A\cdot \sin^2 A=1$

$\frac{1}{\sin A}\cdot n^2 A\cdot 1$

$1=1 \qquad L.H.S=R.H.S$

3. $\tan^2\theta\cos^2\theta=1-\cos^2\theta$

Sol:

$L.H.S\Rightarrow \frac{\sin^2\theta}{\cos^2\theta}\cdot\cos^2\theta=\sin^2\theta$

$R.H.S\Rightarrow 1-\cos^2\theta \qquad \left[1=\sin\theta+\cos^2\theta\right]$

$\Rightarrow \sin^2\theta \qquad \left[\therefore \sin^2\theta=1-\cos^2\theta\right]$

$L.H.S=R.H.S$

4. $\cos ec\theta\sqrt{1-\cos^2\theta}=1$

Sol:

$LHS=\cos ec\theta\sqrt{\sin^2\theta} \qquad \left[\because 1-\cos^2\theta=\sin^2\theta\right]$

$=\cos ec\theta\cdot\sin\theta$

$=1$

$\therefore L.H.S=R.H.S$

5. $\left(\sec^2\theta-1\right)\left(\cos ec^2\theta-1\right)=1$

Sol:

We know that $\sec^2\theta-\tan^2\theta=1$

$\Rightarrow \sec^2\theta=1,\tan\theta$

$\cos ec^2\theta-\cos^2\theta=1$

$\cos ec^2\theta-\cot^2\theta$

$\tan^2\theta\cdot\cot^2\theta=\tan^2\theta\dfrac{1}{\tan^2\theta}$

6. $\tan\theta\dfrac{1}{\tan\theta}=\sec\theta\cos ec\theta$

Sol:

$LHS=\tan\theta+\dfrac{1}{\tan\theta}\Rightarrow\dfrac{\sin\theta}{\cos\theta}+\dfrac{1}{\dfrac{\sin\theta}{\cos\theta}}$

$\Rightarrow\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}$

$\Rightarrow\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}=\dfrac{1}{\sin\theta\cos\theta}$

$\Rightarrow\sec\theta\cos ec\theta$

Hence L.H.S = R.H.S

7. $\dfrac{\cos\theta}{1-\sin\theta}=\dfrac{1+\sin\theta}{\cos\theta}$

Sol:

$\cos\theta-\cos 2\cdot\dfrac{\theta}{2}=\cos^2\dfrac{\theta}{2}-\sin^2\dfrac{\theta}{2}$

$\sin\theta=\sin\dfrac{\theta}{2}\cdot 2\Rightarrow 2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}$

$\Rightarrow LHS=\dfrac{\cos^2\dfrac{\theta}{2}-\sin^2\dfrac{\theta}{2}}{\left[\cos^2\dfrac{\theta}{2}+\sin^2\dfrac{\theta}{2}\right]-2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}}\qquad\left[\therefore 1=\cos^2\dfrac{\theta}{2}+\sin^2\dfrac{\theta}{2}\right]$

$\Rightarrow\dfrac{\left[\cos\dfrac{\theta}{2}-\sin\dfrac{\theta}{2}\right]\left[\cos\dfrac{\theta}{2}+\sin\dfrac{\theta}{2}\right]}{\left[\cos\dfrac{\theta}{2}-\sin\dfrac{\theta}{2}\right]}\qquad\left[\because a^2-b^2=\left(a-b\right)\left(a+b\right)\left(a-b\right)^2=a^2+b^2-2ab\right]$

$$\Rightarrow \frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}.$$

$$R.H.S\,\frac{1+\sin\theta}{\cos\theta} \Rightarrow \frac{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}}$$

$$\Rightarrow \frac{\left[\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right]}{\left[\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right]}$$

$$\Rightarrow \frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}$$

$$\therefore L.H.S = R.H.S$$

8. $\dfrac{\cos\theta}{1+\sin\theta} = \dfrac{1-\sin\theta}{\cos\theta}$

Sol:

$$\cos\theta = \cos 2\frac{\theta}{2} = \cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}$$

$$\sin\theta = \sin 2\cdot\frac{\theta}{2} = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$1 = \cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}$$

$$LHS = \frac{\cos\theta}{1+\sin\theta} = \frac{\cos^2\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cot\frac{\theta}{2}}$$

$$\Rightarrow \frac{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}{\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)^2}$$

$$\Rightarrow \frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}$$

$$RHS=\frac{1-\sin\theta}{\cos\theta}=\frac{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}}$$

$$\Rightarrow\frac{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)^2}{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}$$

$$\Rightarrow\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}$$

$\therefore$ LHS = RHS

9. $\cos^2 A+\frac{1}{1+\cot^2 A}=1$

Sol:

$1+\cot^2 A=\cos ec^2 A$ $\left[\because \cos ec^2 A-\cot^2 A=1\right]$

$\cos ec^2 A=1+\cot^2 A.$

$\Rightarrow \cot^2 A+\frac{1}{\cos ec^2 A}$

$\Rightarrow \cos^2 A+\sin^2 A=1$ $\therefore LHS=RHS$

10. $\sin^2 A+\frac{1}{1+\tan^2 A}=1$

Sol:

$1+\tan^2 A=\sec^2$ $\left[\because \sec^2 A-\tan^2 A=1\right]$

$\Rightarrow \sin^2 A+\frac{1}{\sec^2}$ $\left[1+\tan^2 A-\sec^2 A\right]$

$\Rightarrow \sin^2 A+\cos^2 A=1$

$\therefore LHS=RHS$

11. $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\cos ec\theta-\cot\theta.$

Sol:

$\text{L.H.S}=\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$ Rationalize numerator with $\sqrt{1-\cos\theta}$

$$\Rightarrow \frac{\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}} \times \frac{\sqrt{1-\cos\theta}}{1-\cos\theta}$$

$$= \frac{\left(\sqrt{1-\cos\theta}\right)^2}{\sqrt{(1-\cos\theta)(1+\cos\theta)}}$$

$$= \frac{1-\cos\theta}{\sqrt{1-\cos^2\theta}} = \frac{1-\cos\theta}{\sqrt{\sin^2\theta}} = \frac{1-\cos\theta}{\sin\theta}$$

$$= \cos ec\theta - \cot\theta$$

12. $\dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta}$

Sol:

$$1 = \cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}$$

$$\cos\theta = \cos 2\cdot\frac{\theta}{2} = \cos^2\frac{\theta}{2} - 8\sin^2\frac{\theta}{2}$$

$$\sin\theta = \sin 2\cdot\frac{\theta}{2} = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$LHS = \frac{1-\cos\theta}{\sin\theta} = \frac{\cos^2\frac{\theta}{2} = \sin^2\frac{\theta}{2} - \left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right)}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$\frac{\cos\frac{\theta}{2} + \sin^2\frac{\theta}{2} - \cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$= \frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \tan\frac{\theta}{2}.$$

$$RHS = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}}$$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = \tan\frac{\theta}{2}$$

$$\therefore L.H.S = R.H.S$$

13. $\frac{\sin\theta}{1-\cos\theta}-\cos ec\theta+\cot\theta$

Sol:

$LHS=\frac{\sin\theta}{1-\cos\theta}$

Rationalizer both Nr and Or with $1+\cos\theta$

$\Rightarrow\frac{\sin\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}$

$\Rightarrow\frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$ $\quad\left[\because(a-b)(a+b)=a^2-b^2\right]$

$\Rightarrow\frac{\sin\theta+\sin\theta}{\sin^2\theta}$ $\quad\left[\because 1-\cos^2\theta-\sin^2\theta\right]$

$\Rightarrow\frac{\sin\theta}{\sin^2\theta}+\frac{\sin\theta\cos\theta}{\sin^2\theta}$

$\Rightarrow\frac{1}{\sin\theta}\frac{\cos\theta}{\sin\theta}\Rightarrow\cos ec\theta+\cot\theta$

$\therefore LHS=RHS$

14. $\frac{1-\sin\theta}{1+\sin\theta}-(\sec\theta-\tan\theta)^2$

Sol:

$LHS=\frac{1-\sin\theta}{1+\sin\theta}$

Rationalize both Nr and Or with $(1-\sin\theta)$ multiply

$\Rightarrow\quad\frac{1-\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}$

$\Rightarrow\quad\frac{(1-\sin\theta)^2}{\cos^2\theta}$ $\quad\left[\because(1-\sin\theta)(1+\sin\theta)=\cos^2\theta\right]$

$\Rightarrow\left[\frac{1-\sin\theta}{\cos\theta}\right]^2\Rightarrow\left[\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}\right]^2$

$\Rightarrow[\sec\theta-\tan\theta]^2$

$=LHS=RHS$ Hence proved

15. $(\cos ec\theta+\sin\theta)(\cos ec\theta-\sin\theta)=\cos^2\theta=\cos^2\theta$

Sol:

$LHS\Rightarrow\cos ec^2\theta-\sin^2\theta$ $\quad\left[(a+b)(a-b)=a^2-b^2\right]$

$\Rightarrow 1+\cot^2\theta-\left(1-\cos^2\theta\right) \qquad \left[\because \cos ec^2\theta=1+\cot^2\theta \sin^2\theta=1-\cos^2\theta\right]$

$\Rightarrow 1+\cot^2-1+\cos^2\theta$

$\Rightarrow \cot^2\theta+\cos^2\theta$

$=LHS=RHS$ Hence proved

16. $\dfrac{\left(1+\cot^2\theta\right)\tan\theta}{\sec^2\theta}=\cot\theta$

Sol:

$LHS=\dfrac{\left(1+\cot^2\theta\right)\tan\theta}{\sec^2\theta} \qquad \left[\because \cos ec^2\theta=1+\cot^2\theta\right]$

$\Rightarrow \dfrac{\cos ec^2\theta\cdot\tan\theta}{\sec^2}\Rightarrow\dfrac{1}{\sin^2\theta}\cdot\dfrac{\cos^2\theta}{1}\cdot\dfrac{\sin\theta}{\cos\theta}$

$\Rightarrow \dfrac{\cos\theta}{\sin\theta}=\cot\theta$

$=LHS=RHS$ Hence proved

17. $\left(\sec\theta+\cos\theta\right)\left(\sec\theta-\cos\theta\right)=\tan^2\theta+\sin^2\theta$

Sol:

$LHS=\sec^2\theta-\cos^2\theta \qquad \left[\because \left(\sec\theta+\cos\theta\right)\left(\sec\theta-\cos\theta\right)-\sec^2\theta-\cos^2\theta\right]$

$\Rightarrow 1+\tan^2\theta-\left(1-\sin^2\theta\right) \qquad \left[\because \sec^2\theta=1+\tan^2\theta\cos^2\theta=1-\tan^2\theta\right]$

$\Rightarrow 1+\tan^2\theta-l+\sin^2\theta$

$\tan^2\theta+\sin^2\theta$

$=LHS=RHS$ Hence proved

18. $\sec A\left(1-\sin A\right)\left(\sec A+\tan A\right)=1$

Sol:

$LHS=\dfrac{1}{\cos+1}=\left(1-\sin A\right)\times\left[\dfrac{1}{\cos A}+\dfrac{\sin A}{\cos A}\right] \qquad \left[\because \sec A=\dfrac{1}{\cos A}\ \textit{and}\ \tan A=\dfrac{\sin A}{\cos A}\right]$

$\Rightarrow \dfrac{1}{\cos A}\times\left(1-\sin A\right)\dfrac{\left(1+\sin A\right)}{\cos A}$

$=\dfrac{\cos^2 A}{\cos^2 A}=1 \qquad \left[\because \left(1-\sin A\right)\left(1+\sin A\right)\cdot\cos^2 A=1-\sin^2 A\right]$

$=LHS=RHS$ Hence proved

19. $\left(\cos ecA-\sin A\right)\left(\sec A-\cos A\right)\left(\tan A+\cot A\right)=1$

Sol:

$$LHS=\left[\frac{1}{\sin A}-\sin A\right]\left[\frac{1}{\cos A}-\cos A\right]\left[\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}\right]$$

$$\Rightarrow\frac{1-\sin^2 A}{\sin A}\times\frac{1-\cos^2 A}{\cos A}\times\frac{\sin^2 A+\cos^2 A}{\sin A\cos A}$$

$$\Rightarrow\frac{\cos^2 A\cdot\sin^2 A\cdot 1}{\sin^2 A\cos^2 A}\qquad\left[\begin{array}{c}\because \cos ec\ A=\dfrac{1}{\sin}A\\ \sec A=\dfrac{1}{\cos}A\\ \tan A=\dfrac{\sin A}{\cos A}\\ \cot A=\dfrac{\cos A}{\sin A}\end{array}\right]$$

$$=1\qquad\left[\begin{array}{c}\because 1-\sin^2 A=\cos^2 A\\ 1-\cos^2 A=\sin^2 A\\ \sin^2 A+\cos^2 A=1\end{array}\right]$$

$= LHS = RHS$ Hence proved

20. $\tan^2\theta-\sin^2\theta\ \ \tan^2\theta\sin^2\theta$

Sol:

$$LHS=\tan^2\theta-\sin^2\theta=\frac{\sin^2\theta}{\cos^2\theta}-\sin^2\theta\qquad\left[\because \tan^2\theta=\frac{\sin^2\theta}{\cos^2\theta}\right]$$

$$\Rightarrow\sin^2\theta\left[\frac{1}{\cos^2\theta}-1\right]$$

$$\sin^2\theta\left[\frac{1-\cos^2\theta}{\cos^2\theta}\right]$$

$$\Rightarrow\sin^2\theta\cdot\frac{\sin^2\theta}{\cos^2\theta}=\sin^2\theta\tan^2\theta$$

$= LHS = RHS$ Hence proved

21. $\left(1+\tan^2\theta\right)\left(1-\sin\theta\right)\cdot\left(1+\sin\theta\right)=1$

Sol:

$$LHS=\left(1+\tan^2\theta\right)\left(1-\sin^2\theta\right)\qquad\left[\because\left(a-b\right)\left(a+b\right)=a^2-b^2\right]$$

$$\Rightarrow\qquad\sec^2\theta\cdot\cos^2\theta\qquad\left[\because \sec^2\theta=1+\tan^2\theta\right]$$

$=1$
$= LHS = RHS$ Hence proved

22. $\sin^2 A \cot^2 A + \cos^2 A \tan^2 A = 1$
Sol:

$$LHS = \sin^2 A \cdot \frac{\cos^2 A}{\sin^2 A} + \cos^2 A \cdot \frac{\sin^2 A}{\cos^2 A}$$

$$= \cos^2 A + \sin^2 A \qquad \left[\because \cot^2 A = \cos^2 \frac{A}{\sin^2 A} \tan^2 A = \frac{\sin^2 A}{\cos^2 A}\right]$$

$= LHS = RHS$ Hence proved

23. (i) $\cos\theta - \tan\theta = \dfrac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$
Sol:

$$L.H.S = \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta} = \frac{\cos^2\theta}{\sin\theta\cos\theta} \qquad \left[\because \cos^2\theta - \sin^2\theta = \cos\theta\right]$$

$$\left[\because \cos^2 = 2\cos^2\theta - 1\right]$$

$$= \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$$

$= LHS = RHS$ Hence proved

(ii) $\tan\theta - \cot\theta = \dfrac{2\sin^2\theta - 1}{\sin\theta\cos\theta}$
Sol:

$$LHS = \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$$

$$\Rightarrow \frac{\sin^2\theta - \cos^2}{\cos\theta\sin\theta}$$

$$\Rightarrow \frac{\sin^2\theta - \left(1 - \sin^2\theta\right)}{\cos\theta\sin\theta} \qquad \left[\because \cos^2\theta = 1\sin^2\theta\right]$$

$$\Rightarrow \frac{2\sin^2\theta - 1}{\sin\theta\cos\theta}$$

$\therefore LHS = RHS$ Hence proved

24. $\dfrac{\cos^2\theta}{\sin\theta}-\cos ec\theta+\sin\theta=\theta$

Sol:

$LHS=\dfrac{\left(15^2\right)-\sin\theta\cos ec\theta+\sin^2\theta}{\sin\theta}$

$\Rightarrow\dfrac{\cos^2\theta+\sin^2\theta-1}{\sin\theta}$ $\left[\because\sin\theta\cos ec\theta=1\right]$

$=0$

$\therefore LHS=RHS$ Hence proved

25. $\dfrac{1}{1+\sin A}+\dfrac{1}{1-\sin A}=2\sec^2 A$

Sol:

$LHS=\dfrac{1-\sin A+1+\sin A}{(1+\sin A)(1-\sin A)}$

$\Rightarrow\dfrac{2}{1-\sin^2 A}$ $\left[\because(1+\sin A)(1-\sin A)=1-\sin^2 A\right]$

$\Rightarrow\dfrac{2}{\cos^2 A}\Rightarrow 2\sec^2 A$ $\left[\because 1-\sin A=\cos A\right]$

$\therefore LHS=RHS$ Hence proved

26. $\dfrac{1+\sin\theta}{\cos\theta}+\dfrac{\cos A}{1+\sin\theta}=2\sec\theta$

Sol:

$LHS=\dfrac{(1+\sin\theta)^2+\cos^2\theta}{\cos\theta(1+\sin\theta)}$

$=\dfrac{1+\sin^2\theta+2\sin\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}$

$\Rightarrow\dfrac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}=2\sec\theta$

$\therefore LHS=RHS$ Hence proved

27. $\dfrac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta}=\dfrac{1+\sin^2\theta}{1-\sin^2\theta}$

Sol:

$LHS=\dfrac{1+\sin^2\theta+2\sin\theta+1+\sin^2\theta-2\sin\theta}{2\cos\theta}$

$$\Rightarrow \frac{2\left(1+\sin^2\theta\right)}{2\cos^2\theta} \Rightarrow \frac{1+\sin^2\theta}{1-\sin^2\theta} \qquad \left[\because \cos^2\theta = 1-\sin^2\theta\right]$$

$\therefore LHS = RHS$ Hence proved

28. $\dfrac{1+\tan^2\theta}{1+\cot^2\theta} - \left[\dfrac{1-\tan\theta}{\cot\theta}\right]^2 - \tan^2\theta$

Sol:

$$LHS \Rightarrow \frac{1+\tan^2\theta}{1+\cot^2\theta} = \frac{\sec^2\theta}{\cos ec^2\theta} \qquad \left[\because \tan^2\theta + 1 = \sec^2\theta \;\; 1+\cot^2\theta = \cos ec^2\theta\right]$$

$$= \frac{1}{\cos^2\theta \cdot 1}\sin^2 = \tan^2\theta$$

$$\Rightarrow \left[\frac{1-\tan\theta}{1-\cot\theta}\right]^2 \Rightarrow \left[\frac{1-\tan\theta}{1-\dfrac{1}{\tan\theta}}\right]^2$$

$$\Rightarrow \left[\frac{1-\tan\theta}{\left(1-\tan\theta\right)} \cdot \tan\theta\right]^2 = \tan^2\theta$$

$\therefore LHS = RHS$ Hence proved

29. $\dfrac{1+\sec\theta}{\sec\theta} = \dfrac{\sin^2\theta}{1-\cos\theta}$

Sol:

$$LHS = \frac{1+\sec\theta}{\sec\theta} = \frac{1+\dfrac{1}{\cos\theta}}{\dfrac{1}{\cos\theta}}$$

$$= \frac{\cos\theta + 1}{\cos\theta} \cdot \cos\theta$$

$$= 1+\cos\theta$$

$$RHS = \frac{\sin^2\theta}{1-\cos\theta} \Rightarrow \frac{1-\cos^2\theta}{1-\cos\theta}$$

$$\Rightarrow \frac{\left(1-2\sqrt{5}b\right)+\left(\cos\theta\right)}{1-48} = 1+\cos\theta$$

$\therefore LHS = RHS$ Hence proved

30. $\frac{\tan\theta}{1-\cot\theta}=\frac{\cot\theta}{1-\tan\theta}=1+\tan\theta+\cot\theta.$

Sol:

$$LHS=\frac{\tan\theta}{1-\frac{1}{\tan\theta}}+\frac{\cot\theta}{1-\tan\theta}$$

$$\Rightarrow -\frac{\tan^2\theta}{(1-\tan\theta)}+\frac{\cot\theta}{1-\tan\theta}$$

$$\frac{1}{1-\tan\theta}\left[\frac{1}{\tan\theta}-\tan^2\theta\right]$$

$$\frac{1}{1-\tan\theta}\left[\frac{1-\tan^3\theta}{\tan\theta}\right]$$

$$\Rightarrow \frac{1}{1-\tan\theta}\frac{(1-\tan\theta)(1+\tan\theta+\tan^2\theta)}{\tan\theta} \qquad \left[\because a^3-b^3=(a-b)(a^2+ab+b^2)\right]$$

$$\Rightarrow \frac{1+\tan\theta+\tan^2\theta}{\tan\theta}$$

$$\Rightarrow \cot\theta+1+\tan\theta$$

$\therefore LHS=RHS$ Hence proved

31. $\sec^6\theta=\tan^6\theta+3\tan^2\theta\sec^2\theta+1$

Sol:

We know that $\sec^2\theta-\tan^2\theta:$

Cubing on both sides

$$\left(\sec\theta-\tan^2\theta\right)^3=1$$

$$\sec^6\theta\cdot\tan^6\theta-3\sec^2\theta\left(\sec^2\theta-\tan^2\theta\right)=1$$

$$\tan^2\theta \qquad \left[\because (a-b)^3=a^3-b^3-3ab(a-b)\right]$$

$$\Rightarrow \sec\theta-\tan^6\theta=3\sec^2\theta\tan^2\theta=1$$

$$\Rightarrow \sec^6\theta=\tan^6\theta+1+3\tan^2\theta\sec^2\theta$$

Hence proved

32. $\cos ec^6\theta=\cot^6\theta+3\cot^2\theta\cos ec^2\theta+1$

Sol:

We know that $\cos ec^2\theta-\cot^2\theta=1$

Cubing on both sides

$$\left(\cos ec^2\theta-\cot^2\theta\right)^3=(1)^3$$

$\Rightarrow \cos ec^6\theta - \cot^6\theta - 3\cos ec^2\theta\cot^2\theta\left(\cos ec^2\theta - \cot^2\theta\right) = 1$

$\left[\because \left(a-b\right)^3 - a^3 - b^3 - 3ab\left(a-b\right)\right]$

$\Rightarrow \cos ec^6 = 1 + 3\cos ec^2\theta\cot^2\theta + \cot^6\theta$

Hence proved

33. $\dfrac{\left(1+\tan^2\theta\right)\cot\theta}{\cos ec^2\theta} = \tan\theta$

Sol:

$\sec^2\theta = \tan^2\theta = 1$

$\therefore \sec^2\theta = 1 + \tan^2\theta$

$LHS = \dfrac{\sec^2\theta\cdot\cot\theta}{\cos ec^2\theta} \Rightarrow \dfrac{1\cdot\sin^6\theta}{\cos^2\theta}\cdot\dfrac{\cos\theta}{\sin\theta}$

$\left[\because \sec\theta = \dfrac{1}{\cos\theta}, \cos ec\theta = \dfrac{1}{\sin\theta}\cot\theta = \dfrac{\cos\theta}{\sin\theta}\right]$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} = \tan\theta$

$\therefore LHS = RHS$ Hence proved

34. $\dfrac{1+\cos A}{\sin^2 A} = \dfrac{1}{1-\cos A}$

Sol:

We know that $\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A$

$\Rightarrow \sin^2 A = \left(1-\cos A\right)\left(1+\cos A\right)$

$\Rightarrow LHS = \dfrac{\left(1+\cos A\right)}{\left(1-\cos A\right)\left(1+\cos A\right)} = \dfrac{1}{1-\cos A}$

$\therefore L.H.S = R.H.S$ Hence proved

35. $\dfrac{\sec A - \tan A}{\sec A + \tan A} = \dfrac{\cos^2 A}{\left(1+\sin A\right)^2}$

Sol:

$LHS = \dfrac{\sec\theta - \tan\theta}{\sec A + \tan A}$

Rationalizing the denominator y multiply and diving with $\sec A + \tan A$ we get

$$\frac{(\sec A-\tan A)}{(\sec A+\tan A)}\times\frac{(\sec A+\tan A)}{(\sec A+\tan A)}=\frac{\sec^2 A-\tan^2 A}{(\sec A+\tan A)^2}=\frac{1}{(\sec A+\tan A)^2}$$

$\left[\because \sec^2 A-\tan^2 A=1\right]$

$$=\frac{1}{\sec^2 A+\tan^2 A+2\sec A\tan A}=\frac{1}{\frac{1}{\cos^2 A}+\frac{\sin^2 A}{\cos^2 A}+\frac{2\sin A}{\cos^2 A}}$$

$$\Rightarrow \frac{\cos^2}{1+\sin^2 A+2\sin A}=\frac{\cos^2 A}{(1+\sin A)^2}$$

$\therefore L.H.S=R.H.S$ Hence proved

36. $\dfrac{1+\cos A}{\sin A}=\dfrac{\sin A}{1-\cos A}$

Sol:

$$LHS=\frac{1+\cos A}{\sin A} \qquad ...(1)$$

Multiply both Nr and Dr with $(1-\cos A)$ we get

$$\frac{(1+\cos A)(1-\cos A)}{\sin A(1-\cos A)}=\frac{1-\cos^2 A}{\sin A(1-\cos A)}$$

$$=\frac{\sin^2 A}{\sin A(1-\cos A)}=\frac{1-\cos^2 A}{\sin A(1-\cos A)}$$

$$=\frac{\sin^2}{\sin A(1-\cos A)} \qquad \left[\because \cos^2 A=\sin^2 A\right]$$

$$=\frac{\sin A}{1-\cos A}$$

$\therefore L.H.S=R.H.S$ Hence proved

37. $\sqrt{\dfrac{1+\sin A}{1-\sin A}}=\sin A+\tan A$

Sol:

$$LHS=\sqrt{\frac{1+\sin A}{1-\sin A}}.$$

Rationalize the Nr. By multiplying both Nr and Dr with $\sqrt{1+\sin A}.$

$$\Rightarrow \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1+\sin A)(1-\sin A)}}=\sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} \qquad \left[\because (1+\sin A)(1-\sin A)=\cos^2 A\right]$$

$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$

$\sec A + \tan A$

$\therefore L.H.S = R.H.S$ Hence proved

38. $\sqrt{\frac{1-\cos A}{1+\cos A}} = \cos ecA - \cot A$

Sol:

Rationalizing both Nr and Or by multiplying both with $\sqrt{1-\cos A}$ we get

$\Rightarrow \sqrt{\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}}.$ $\quad \left[\because (1+\cos A)(1-\cos A) = 1-\cos^2 A = \sin^2 A\right]$

$\sqrt{\frac{(1-\cos A)^2}{\sin^2 A}}$

$= \frac{1-\cos A}{\sin A}$

$= \cos ecA - \cot A.$

$\therefore L.H.S = R.H.S$ Hence proved

39. $(\sec A - \tan A)^2 = \frac{1-\sin A}{1+\sin A}$

Sol:

$LHS = (\sec A - \tan A)^2$

$\Rightarrow \left[\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right]^2 \Rightarrow \frac{(1-\sin A)^2}{\cos^2 A.}$

$\Rightarrow \frac{(1-\sin A)^2}{1-\sin^2 A}$ $\quad \left[\because 1-\sin^2 A = \cos^2 A\right]$

$\Rightarrow \frac{(1-\sin A)^2}{(1-\sin A)(1+\sin A)}$ $\quad \left[\because a^2 - b^2 = (a-b)(a+b)\right]$

$= \frac{1-\sin A}{1+\sin A}$

$\therefore L.H.S = R.H.S$ Hence proved

40. $\frac{1-\cos A}{1+\cos A}=(\cot A-\operatorname{cosec} A)^2$

Sol:

$LHS=\frac{1-\cos A}{1+\cos A}$

Rationalizing Nr by multiplying and dividing with $1-\cos A$.

$=\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}$

$\Rightarrow \frac{(1-\cos A)^2}{1-\cos^2 A}$

$\Rightarrow \frac{(1-\cos A)^2}{\sin^2 A}$ $\quad\left[\because (a+b)(a-b)=a^2-b^2 \quad 1-\cos^2 A=\sin^2 A\right]$

$=\left[\frac{1}{\sin A}-\frac{\cos A}{\sin A}\right]^2$ $\quad(\operatorname{cosec} A-\cot A)^2$

$=(\cot A-\operatorname{cosec} A)^2$

$\therefore L.H.S=R.H.S$ Hence proved

41. $\frac{1}{\sec A-1}=\frac{1}{\sec A+1}=2\operatorname{cosec} A\cot A$

Sol:

$LHS=\frac{\sec A+1+\sec A-1}{(\sec A+1)(\sec A-1)}=\frac{2\sec A}{(\sec^2 A-1)}$

$\left[\because (a+b)(a-b)=a^2-b^2 \sec^2 A-1=\tan^2 A\right]$

$\Rightarrow \frac{2\sec A}{\tan^2 A}=\frac{2\cdot 1\cos^2 A}{\cos A\cdot\sin^2 A}$ $\quad\left[\because \sec A=\frac{1}{\cos A}\tan^2 A=\frac{\sin^2 A}{\cos^2 A}\right]$

$\Rightarrow 2\operatorname{cosec} A\cot A$

$\therefore L.H.S=R.H.S$ Hence proved

42. $\frac{\cos A}{1-\tan A}+\frac{\sin A}{(1-\cot A)}=\sin A+\cos A$

Sol:

$LHS=\frac{\cos A}{1-\tan A}+\frac{\sin A}{\left(1-\frac{1}{\tan A}\right)}$

$=\frac{\cos A}{1-\tan A}-\frac{\sin A\cdot\tan A}{1-\tan A}$

$$\Rightarrow \frac{\cos A-\sin A\tan A}{(1-\tan A)}$$

$$\Rightarrow \frac{\cos A-\sin A\cdot\frac{\sin A}{\cos A}}{1-\frac{\sin A}{\cos A}}$$

$$\Rightarrow \frac{\cos^2 A-\sin^2 A\cos A}{(\cos A-\sin A)\cos A}=\frac{(\cos A-\sin A)(\cos A+\sin A)}{\cos A-\sin A}$$

$$\Rightarrow \cos A+\sin A.$$

$\therefore L.H.S=R.H.S$ Hence proved

43. $\frac{\cos ecA}{\cos ecA-1}+\frac{\cos ecA}{\cos ecA+1}=2\sec^2 A.$

Sol:

$$LHS\ \cos ecA\left[\frac{\cos ec+1+\cos ec-1}{\cos ec^2 A-1}\right] \qquad \left[\because \cos ec^2 A-1=\cot^2 A\right]$$

$$\Rightarrow \cos ecA\left[\frac{2\cos ecA}{\cot^2 A}\right]$$

$$\Rightarrow \frac{2}{\sin^2 A}\frac{\sin^2 A}{\cos^2 A}=2\sec^2 A.$$

$\therefore LHS=RHS$ Hence proved.

44. $\left(1+\tan^2 A\right)+\left(1+\frac{1}{\tan^2 A}\right)=\frac{1}{\sin^2 A-\sin^4 A}$

Sol:

$$LHS=\left[1+\frac{\sin^2 A}{\cos^2 A}\right]+\left[1+\frac{\cos^2 A}{\sin^2 A^2}\right]$$

$$\Rightarrow \frac{\cos^2 A+\sin^2 A}{\cos^2 A}+\frac{\sin^2 A+\cos^2 A}{\sin^2 A}$$

$$\Rightarrow \frac{1}{\cos^2 A}+\frac{1}{\sin^2 A} \qquad \left[\because \sin^2 A+\cos^2 A=1\right]$$

$$\Rightarrow \frac{\sin^2 A+\cos^2 A}{\sin^2 A\cos^2 A}=\frac{1}{\sin^2 A(1-\sin A)} \qquad \left[\cos^2 A=1-\sin^2 A\right]$$

$$\Rightarrow \frac{1}{\sin^2 A-\sin^2 A}$$

$\therefore LHS=RHS$ Hence proved.

45. $\dfrac{\tan^2 A}{1+\tan^2 A}+\dfrac{\cot^2 A}{1+\cot A}$

Sol:

We know that

$\sec^2 A = 1+\tan^2 A$

$\cos ec^2 A = 1+\cot^2 A.$

$\therefore LHS = \dfrac{\tan^2}{\sec^2}+\dfrac{\cot^2}{\cos ec^2 A}$

$\Rightarrow \dfrac{\sin^2 A}{\cos^2 A}\times\dfrac{\cos^2 A}{1}+\dfrac{\cos^2 A}{\sin^2 A}=\dfrac{\sin^2 A}{1}$

$\left[\because \tan A = \sin\dfrac{A}{\cos A} \sec A = \dfrac{1}{\cos A} \cot A = \dfrac{\cos A}{\sin A} \cos ec = \dfrac{1}{\sin A}\right]$

$\Rightarrow \sin^2 A+\cos^2 A$

$=1$

$\therefore LHS = RHS$ Hence proved.

46. $\dfrac{\cot A-\cos A}{\cos A+\cos A}=\dfrac{\cos ecA-1}{\cos ecA+1}$

Sol:

$$=\dfrac{\dfrac{\cos A}{\sin A}-\cos A}{\dfrac{\cos A}{\sin A}+\cos A} \qquad \left[\because \cot A = \dfrac{\cos A}{\sin A}\right]$$

$$=\dfrac{\cos A\left[\dfrac{1}{\sin A}-1\right]}{\cos A\left[\dfrac{1}{\sin A}+1\right]}$$

$$=\dfrac{\cos ecA-1}{\cos ecA+1}$$

47. (i) $\dfrac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}$

(ii) $\dfrac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}$

(iii) $\dfrac{\cos\theta-\sin\theta+1}{\cos\theta+\sin\theta-1}=\cos ec+\cot\theta$

Sol:

(i) $\Rightarrow \dfrac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}$

Dividing the equation with $\cos\theta$ we get or both Nr and Dr

$$\frac{\frac{1+\cos\theta+\sin\theta}{\cos\theta}}{\frac{1+\cos\theta-\sin\theta}{\cos\theta}}=\frac{\frac{1}{\cos\theta}+\frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}-\frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}$$

$$=\frac{\sec\theta+1+ta\theta}{\sec\theta+1-\tan\theta}$$

$$=\frac{\sec\theta+\tan\theta+\sec^2\theta-\tan^2\theta}{\sec^2\theta-\tan\theta+1} \qquad \left[\because \sec^2\theta-\tan^2\theta=1\right]$$

Or

$$\frac{\sec\theta+\tan\theta+1}{\sec\theta-\tan\theta+1}$$

$$\frac{\frac{1}{\sec\theta-\tan\theta}+1}{\sec\theta-\tan\theta+1} \qquad \left[\because \sec\theta+\tan\theta=\frac{1}{\sec\theta-\tan\theta}\right]$$

Or

$$\frac{\sec\theta+\tan\theta+1}{\sec\theta-\tan\theta+1}$$

$$\frac{\frac{1}{\sec\theta-\tan\theta}+1}{\sec\theta-\tan\theta+1} \qquad \left[\because \sec\theta+\tan\theta=\frac{1}{\sec\theta-\tan\theta}\right]$$

$$=\frac{1+\sec\theta-\tan\theta}{1+\sec\theta-\tan\theta}\times\frac{1}{\sec\theta-\tan\theta}$$

$$=\frac{1}{\sec\theta-\tan\theta}=\sec\theta+\tan\theta$$

$$=\sec\theta+\tan\theta$$

$$=\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}$$

$$=\frac{1+\cos\theta}{\cos\theta}$$

(ii) $\dfrac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}$

Divide Nr and Dr with $\cos\theta$, we get

$$\frac{\frac{\sin\theta-\cos\theta+1}{\cos\theta}}{\frac{\sin\theta+\cos\theta-1}{\cos\theta}}=\frac{\tan\theta-1+\sec\theta}{\tan\theta+1-\sec\theta}.$$

$$=\frac{\frac{1}{\sec\theta-\tan\theta}-1}{\tan\theta-\sec\theta+1}$$

$$=\frac{1-\sec\theta+\tan\theta}{1-\sec\theta+\tan\theta}\times\frac{1}{\sec\theta-\tan\theta}$$

$$=\frac{1}{\sec\theta-\tan\theta}$$

(iii) $\frac{\cos\theta-\sin\theta+1}{\cos\theta+\sin\theta-1}=\cos ec+\cot\theta$

Divide both Nr and Dr with $\sin\theta$

$$\frac{\frac{\cos\theta-\sin\theta+1}{\sin\theta}}{\frac{\cos\theta+\sin\theta-1}{\sin\theta}}$$

$$=\frac{\cot\theta-1+\cos ec}{\cot\theta+1-\cos ec}$$

$$=\frac{\cot\theta+\cos ec\theta-\left(\cos ec^2\theta-\cot^2\theta\right)}{\cot\theta-\cos ec\theta+1}$$

$$=\frac{\cot\theta+\cos ec\theta-\left(\cos ec^2\theta+\cot^2\theta\right)}{\cot\theta-\cos ec\theta+1}$$

$$=\frac{\cot\theta+\cos ec\left(1-\left(\cos ec-\cot\theta\right)\right)}{\cot\theta-\cos ec\theta+1}$$

$$=\cot\theta+\cos ec\theta$$

48. $\frac{1}{\sec A+\tan A}-\frac{1}{\cos A}=\frac{1}{\cos A}-\frac{1}{\sec A-\tan A}$

Sol:

$$LHS:\sec A-\tan A\left[\therefore\frac{1}{\sec A+\tan A}=\sec A-\tan A\right]$$

$$=-\tan A$$

$RHS\ \dfrac{1}{\cos A}-\dfrac{1}{\sec A-\tan A}$

$\sec A-(\sec A+\tan A)$

$\left[\because \dfrac{1}{\sec A-\tan A}=\sec A+\tan A\right]$

$=-\tan A$

$LHS = RHS$

49. $\tan^2 A+\cot^2 A=\sec^2 A\cos ec^2 A-2$

Sol:

$\tan^2 A+\cot^2 A=\dfrac{\sin^2 A}{\cos^2 A}+\dfrac{\cos^2 A}{\sin^2 A}$

$=\dfrac{\sin^4 A+\cos^4 A}{\cos^2 A\sin^2 A}$

$=\dfrac{1-2\sin^2 A\cos^2 A}{\sin^2 A\cos^2 A}$ $\left[\because \sin^4 A+\cos^4 A=1-2\sin^2 A\cos^2 A\right]$

$=\sec^2 A\cos ec^2 A-2$

$\sin^4 A+\cos^4 A$ is in the form of a^4+b^4

$a^4+b^4=\left(a^2+b^2\right)^2-2a^2b^2$

Here $a=\sin A, b=\cos A$

$=\left(\sin^2 A+\cos^2 A\right)^2-2\sin^2 A\cos^2 A$

$=1-2\sin^2 A\cos^2 14$

50. $\dfrac{1-\tan^2 A}{\cot^2 A-1}=\tan^2 A.$

Sol:

$\dfrac{1-\dfrac{\sin^2 A}{\cos^2 A}}{\dfrac{\cos^2}{\sin^2}-1}=\dfrac{\dfrac{\cos^2 A-\sin^2 A}{\cos^2 A}}{\dfrac{\cos^2 A-\sin^2 A}{\sin^2 A}}$

$=\dfrac{\sin^2 A}{\cos^2 A}$

$=\tan^2 A.$

51. $1+\frac{\cot^2\theta}{1+\cos ec\theta}=\cos ec\theta$

Sol:

$1+\frac{\cos ec^2\theta-1}{1+\cos ec\theta}$ $\left[\because \cos ec^2\theta-\cot^2\theta=1, \cot^2\theta=\cos ec^2\theta-1\right]$

$1+\frac{(\cos ec\theta-1)(\cos ec\theta+1)}{1+\cos ec\theta}$

$=1+\cos ec\theta-1$ $\left[\because (a+b)(a-b)=a^2-b^2 a=\cos ec\theta, b=1.\right]$

$=\cos ec\theta$

52. $\frac{\cos\theta}{\cos ec\theta+1}+\frac{\cos\theta}{\cos ec\theta-1}=2\tan\theta$

Sol:

$\frac{\cos\theta}{\frac{1}{\sin\theta}+1}+\frac{\cos\theta}{\frac{1}{\sin\theta}-1}$

$\frac{\frac{\cos\theta}{1+\sin\theta}}{\sin\theta}+\frac{\frac{\cos\theta}{1-\sin\theta}}{\sin\theta}$

$\frac{(\cos\theta)(\sin\theta)}{1+\sin\theta}+\frac{(\cos\theta)(\sin\theta)}{1-\sin\theta}$

$\frac{(1-\sin\theta)(\sin\theta\cos\theta)+(\sin\theta\cos\theta)}{(1+\sin\theta)(1-\sin\theta)}$

$\frac{\sin\theta\cos\theta-\sin\theta\cos\theta+\sin\theta\cos\theta+\sin^2\theta\cos^2}{1-\sin^2\theta}$

$=\frac{\sin\theta\cos\theta}{\cos^2\theta}$

$=\frac{2\sin\theta}{\cos\theta}$

$=2\tan\theta$

53. $\dfrac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}=\cot\theta$

Sol:

$\dfrac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}$

$=\dfrac{1-\sin^2\theta+\cos\theta}{\sin\theta(1+\cos\theta)}$

$=\dfrac{\cos^2\theta+\cos\theta}{\sin\theta(1+\cos\theta)}$

$=\dfrac{\cos\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

$=\cot\theta.$

54. $\dfrac{\tan^3\theta}{1+\tan^2\theta}+\dfrac{\cot^3\theta}{1+\cot^2\theta}=\sec\theta\cos ec\theta-2\sin\theta\cos\theta$

Sol:

$\dfrac{\tan^3\theta}{\sec^2\theta}+\dfrac{\cos^2\theta}{\cos ec^2\theta}$ $\left[\because \sec^2\theta-\tan^2\theta=1\cos ec^2\theta-\cot^2\theta=1\right]$

$\cos ec^2\theta=1+\cot^2\theta.$

$\tan\theta+\cos^2\theta+\cot^3\theta\times\sin^3\theta$ $\left[\because \dfrac{1}{\sec^2\theta}=\cos^2\theta,\dfrac{1}{\cos ec^2\theta}=1+\cot^2\theta\right]$

$\dfrac{\sin^3\theta}{\cos^3\theta}\times\cos^2\theta+\dfrac{\cos^3\theta}{\sin^3\theta}\times\sin^2\theta$

$\dfrac{\sin^3\theta}{\cos\theta}+\dfrac{\cos^3\theta}{\sin\theta}$

$=\dfrac{\sin^4\theta+\cos^4\theta}{\sin\theta\cos\theta}$

$\dfrac{1-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$

$\dfrac{1}{\sin\theta\cos\theta}-\dfrac{2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$

$\sec\theta\cos ec\theta-2\sin\theta\cos\theta.$

55. $If\ T_n = \sin^n\theta + \cos^n\theta,\ prove\, that\, \frac{T_3 - T_5}{T_1} = \frac{T_5 - T_7}{T_3}.$

Sol:

$$LHS = \frac{\left(\sin^3\theta + \cos^3\theta\right) - \left(\sin^3\theta + \cos^5\theta\right)}{\sin\theta + \cos\theta}$$

$$= \frac{\sin^3\theta\left(1 - \sin^2\theta\right) + \cos^3\theta + 1 - \cos^2\theta}{\sin\theta + \cos\theta}$$

$$= \frac{\sin^3\theta \times \cos^3\theta + \cos^3\theta \times \sin^2\theta}{\sin\theta + \cos\theta}$$

$$= \frac{\sin^2\theta + \cos\theta\left(\sin\theta + \cos\theta\right)}{\sin\theta + \cos\theta}$$

$$= \sin^3\theta\cos^2\theta$$

$$\frac{T_5 - T_9}{T_3} = \frac{\left(\sin^5\theta + \cos^5\theta\right) - \left(\sin^7\theta + \cos^7\theta\right)}{\sin^3\theta + \cos^3\theta}$$

$$= \frac{\sin^5\theta\left(1 - \sin^2\theta\right) + \cos^5\theta\left(\sin^2\theta\right)}{\sin^3\theta + \cos^3\theta}$$

$$= \frac{\sin^5\theta + \cos^2 + \cos^5\theta\left(\sin^2\theta\right)}{\sin^3\theta + \cos^3\theta}$$

$$= \frac{\sin^2\theta\cos^2\theta\left(\sin^3\theta + \cos^3\theta\right)}{\sin^3\theta + \cos^3\theta}$$

$$= \sin^2\theta\cos^2\theta$$

$L.H.S = R.H.S$ Hence Proval.

$$= \frac{\sin^2\theta\cos^2\theta\left(\sin\theta + \cos\theta\right)}{\sin^2\theta + \cos\theta}$$

$$= \sin^2\theta\cos^2\theta$$

$L.H.S = R.H.S$

56. $\left[\tan\theta + \frac{1}{\cos\theta}\right]^2 + \left[\tan\theta - \frac{1}{\cos\theta}\right]^2 = 2\left(\frac{1 + \sin^2\theta}{1 - \sin^2\theta}\right)$

Sol:

$$\Rightarrow \left(\tan\theta + \sec\theta\right)^2 + \left(\tan\theta - \sec\theta\right)^2$$

$$= \tan^2\theta + \sec^2\theta + 2\tan\theta\sec\theta + \tan^2\theta + \sec^2\theta + 2\tan\theta\sec\theta.$$

$$= 2\tan^2\theta + 2\sec^2\theta$$

$$= 2\left[\tan^2\theta + \sec^2\right]$$

$$= 2\left[\frac{\sin^2\theta}{\cos^2\theta} + \frac{1}{\cos^2\theta}\right]$$

$$= 2\left(\frac{\sin + \sin^2\theta}{\cos^2\theta}\right)$$

57. $\left[\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\cos ec^2\theta - \sin^2\theta}\right]\sin^2\theta\cos^2\theta = \frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}.$

Sol:

$$\Rightarrow \left[\frac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \frac{1}{\cos ec^2\theta - \sin^2\theta}\right]\sin^2\theta\cos^2\theta.$$

$$= \left[\frac{1}{\frac{1-\cos^4\theta}{\cos^2\theta}} + \frac{1}{\frac{1-\sin^4\theta}{\sin^2\theta}}\right]\sin^2\theta\cos^2\theta.$$

$$= \left[\frac{\cos^2\theta}{1-\cos^4\theta} + \frac{\sin^2\theta}{1-\sin^4\theta}\right]\sin^2\theta\cos^2\theta$$

$$= \left[\frac{\cos^2\theta}{\cos^2\theta + \sin^2\theta - \cos^4\theta} + \frac{\sin^2}{\cos^2\theta + \sin^2\theta - \sin^4\theta}\right]\sin^2\theta\cos^2\theta.$$

$$= \left[\frac{\cos^2\theta}{\cos^2\theta\left(1-\cos^2\theta\right) + \sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta\left(1-\sin^2\theta\right) + \cos^2\theta}\right]\sin^2\theta\cos^2\theta.$$

$$= \left[\frac{\cos^2\theta}{\cos^2\theta\sin^2\theta + \sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta\cos^2\theta + \cos^2\theta}\right]\sin^2\theta\cos^2\theta$$

$$= \left[\frac{\cos^2\theta}{\sin^2\theta\left(\cos^2\theta + 1\right)} + \frac{\sin^2\theta}{\cos^2\theta\left(\sin^2\theta + 1\right)}\right]\sin^2\theta\cos^2\theta.$$

$$= \left[\frac{\cos^4\theta\left(1+\sin^2\theta\right) + \sin^4\theta\left(1+\cos^2\theta\right)}{\sin^2\theta\cos^2\theta\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}\right]\sin^2\theta\cos^2\theta$$

$$= \frac{\cos^4\theta\left(1+\sin^2\theta\right) + \sin^4\theta\left(1+\cos^2\theta\right)}{\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}$$

$$= \frac{\cos^4\theta + \cos^4\theta\sin^2\theta + \sin^4\theta + \sin^4\theta\cos^2\theta}{1 + \sin^2\theta + \cos^2\theta + \cos^2\theta\sin^2\theta}.$$

$$= \frac{1 - 2\sin^2\theta\cos^2\theta + \sin^2\theta\cos^2\theta\left(\cos^2\theta + \sin^2\theta\right)}{1 + 1 + \cos^2\theta\sin^2\theta} \quad \left(\because \cos^2\theta + \sin^2\theta = 1\right)$$

$$= \frac{1 - \sin^2\theta\cos^2\theta}{2 + \sin^2\theta\cos^2\theta}.$$

58. $\left[\frac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta}\right]^2 = \frac{1 - \cos\theta}{1 + \cos\theta}$

Sol:

$$\Rightarrow \left(\frac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta} \times \frac{1 + \sin\theta - \cos\theta}{1 + \sin\theta - \cos\theta}\right)^2$$

$$\Rightarrow \left[\frac{\left(1 + \sin\theta - \cos\theta\right)^2}{\left(1 + \sin\theta\right)^2 - \cos^2\theta}\right]$$

$$= \left[\frac{\left(1\right)^2 + \sin^2\theta + \cos^2\theta + 2\times1\times\sin\theta + 2\times\sin\theta\left(-\cos\theta\right) - 2\cos\theta}{1 - \cos^2\theta + \sin^2\theta + 2\sin\theta}\right]$$

(Since, $\sin^2\theta + \cos^2\theta = 1$]

$$= \left[\frac{1 + 1 + 2\sin\theta - 2\sin\theta\cos\theta - 2\cos\theta}{\sin^2\theta - \sin^2\theta + 2\sin\theta}\right]^2$$

$$= \left[\frac{2\times2\sin\theta - 2\sin\theta\cos\theta - 2\cos\theta}{2\sin^2\theta + 2\sin\theta}\right]^2$$

$$= \left[\frac{2\left(1 + \sin\theta\right) - 2\cos\theta\left(\sin\theta + 1\right)}{2\sin\theta\left(\sin\theta + 1\right)}\right]^2$$

$$= \left[\frac{\left(1 + \sin\theta\right)\left(2 - 2\cos\theta\right)}{2\sin\theta\left(\sin\theta + 1\right)}\right]^2$$

$$= \left[\frac{2 - 2\cos\theta}{2\sin\theta}\right]^2$$

$$= \left[\frac{2}{2} - \left(\frac{1 - \cos\theta}{\sin\theta}\right)\right]^2$$

$$=\left[\frac{1-\cos\theta}{\sin\theta}\right]^2$$
$$=\frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$
$$=\frac{(1-\cos\theta)\times(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$
$$=\frac{1-\cos\theta}{1+\cos\theta}.$$

59. $(\sec A+\tan A-1)(\sec A-\tan A+1)=2\tan A$

Sol:

$$=\left(\sec A+\tan A-\{\sec^2 A-\tan^2 A\}\right)\left[\sec A-\tan A+\left(\sec^2 A-\tan^2 A\right)\right]$$
$$=(\sec A+\tan A-\sec A+\tan A)(\sec A-\tan A)\left(\sec A-\tan A+(\sec A+\tan A)(\sec A-\tan A)\right)$$
$$=(\sec A+\tan A)(1-(\sec A-\tan A))(\sec A-\tan A)(1+\sec A\tan A)$$
$$=(\sec A+\tan A)(1-\sec A+\tan A)(\sec A-\tan A)(1+\sec A\tan A)$$
$$=(\sec A+\tan A)(\sec A-\tan A)(1-\sec A+\tan A)(1+\sec A\tan A)$$
$$=\left(\sec^2 A-\tan^2 A\right)(1-\sec A+\tan A)(1-\sec A\tan A)$$
$$=\left[1-\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right]\left[1+\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right]$$
$$=\left(\frac{\cos A-1+\sin A}{\cos A}\right)\left(\frac{\cos A+1+\sin A}{\cos A}\right)$$
$$=\left(\frac{\cos A+\sin A^2-1}{\cos^2 A}\right)$$
$$=\frac{\cos^2 A+\sin^2 A+2\sin A\cos A-1}{\cos^2 A}$$
$$=\frac{1+2\sin A\cos A}{\cos^2 A}-1$$
$$=\frac{2\sin A\cos A}{\cos^2 A}\qquad\left[\because \sin^2 A+\cos^2 A=1\right]$$
$$=2\tan A$$

60. $(1+\cot A-cosec\ A)(1+\tan A+\sec A)=2$

Sol:

$LHS=(1+\cot A-\cos ecA)(1+\tan A+\sec A)$

$=\left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)$

$=\left(\frac{\sin A+\cos A-1}{\sin}\right)\left(\frac{\cos A+\sin A+1}{\cos A}\right)$

$=\frac{(\sin A+\cos A)^2-1}{\sin A\cos A}$

$=\frac{1+2\sin A\cos A-1}{\sin A\cos A}$ $\quad\left[\because \sin^2 A+\cos^2 A=1\right]$

$=2.$

61. $(\cos ec\theta-\sec\theta)(\cot\theta-\tan\theta)(\cos ec\theta+\sec\theta)(\sec\theta\cos ec\theta-2)$

Sol:

LHS

$(\cos ec\theta-\sec\theta)(\cot\theta-\tan\theta)$

$\left[\frac{1}{\sin\theta}-\frac{1}{\cos\theta}\right]\left[\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}\right]$

$\left[\frac{\cos\theta-\sin\theta}{\sin\theta\cos\theta}\right]\left[\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}\right]$

$\left[\frac{(\cos\theta-\sin\theta)^2(\cos\theta+\sin\theta)}{\cos^2\theta\sin^2\theta}\right]$

RHS

$(\cos ec\theta+\sec\theta)(\sec\theta\cos ec\theta-2)$

$=\left[\frac{1}{\sin\theta}+\frac{1}{\cos\theta}\right]\left[\frac{1}{\cos\theta}-\frac{1}{\sin\theta}-2\right]$

$=\left[\frac{\sin\theta-\cos\theta}{\sin\theta\cos\theta}\right]\left[\frac{1-2\sin\theta\cos\theta}{\sin\theta\cos\theta}\right]$

$=\left[\frac{\sin\theta+\cos\theta}{\sin\theta\cos\theta}\right]\left[\frac{\cos^2\theta+\sin^2\theta-2\sin\theta\cos\theta}{\sin\theta\cos\theta}\right]$

$=\frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)^2}{\sin^2\theta\cos^2\theta}$ $\quad\left[\because \cos^2\theta+\sin^2\theta=1\right]$

$L.H.S=R.H.S$ Hence proved

62. $(\sec A-\operatorname{cosec}A)(1+\tan A+\cot A)=\tan A\sec A-\cot A\operatorname{cosec}A$

Sol:

$LHS=(\sec A-\operatorname{cosec}A)(1+\tan A+\cot A)$

$=\left[\frac{1}{\cos A}-\frac{1}{\sin A}\right]\left[1+\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}\right]$

$=\left[\frac{\sin A-\cos A}{\sin A\cos A}\right]\left[\frac{\cos A\sin A+\sin^2 A+\cos^2 A}{\sin A\cos A}\right]$

$=\frac{(\sin A-\cos A)(\sin^2 A+\cos A\sin A+\cos^2 A)}{\sin^2 A\cos^2 A}$

$=\frac{(\sin^3 A-\cos^3 A)}{\sin^2 A\cos^2 A}$ $\left[\therefore (a-b)(a^2+ab)+b=(a^3-b^3)\right]$

$RHS=\tan A\sec A-\cot A\operatorname{cosec}A$

$=\frac{\sin A}{\cos A}\times\frac{1}{\cos A}-\frac{\cos A}{\sin A}\times\frac{1}{\cos A}$

$=\frac{\sin A}{\cos^2 A}-\frac{\cos A}{\sin A}$

$=\frac{\sin^3 A-\cos^3 A}{\sin^2 A\cos^2 A}$

$L.H.S=R.H.S$ Hence proved.

63. $\frac{\cos A\operatorname{cosec}A-\sin A\sec A}{\cos A+\sin A}=\operatorname{cosec}A-\sec A$

Sol:

$$LHS \frac{\cos A \operatorname{cosec} A - \sin A \sec A}{\cos A + \sin A}$$

$$= \frac{\cos A \times \frac{1}{\sin A} - \sin A \times \frac{1}{\cos A}}{\cos A + \sin A}$$

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\cos A}}{\cos A + \sin A}$$

$$= \frac{\frac{\cos^2 A - \sin^2 A}{\sin A \cos A}}{\cos A + \sin A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\sin A \cos A} \times \frac{1}{\cos A + \sin A}$$

$$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\sin A \cos A \times (\cos A + \sin A)}$$

$$= \frac{\cos A - \sin A}{\sin A \cos A}$$

$$= \frac{\cos A}{\sin A \cos A} - \frac{\sin A}{\sin A \cos A}$$

$$= \frac{1}{\sin A} - \frac{1}{\cos A}$$

$$= \operatorname{cosec} A - \sec A$$

$$= R.H.S$$

Hence proved.

64. $$\frac{\sin A}{\sec A + \tan A - 1} + \frac{\cot A}{\operatorname{cosec} A + \cot A - 1} = 1$$

Sol:

$$LHS = \frac{\sin A}{\frac{1}{\cos A} + \frac{\sin A}{\cos A} - 1} + \frac{\cos A}{\frac{1}{\sin A} + \frac{\cos A}{\sin A} - 1}$$

$$= \frac{\sin A}{\frac{1 + \sin A - \cos A}{\cos A}} + \frac{\cos A}{\frac{1 + \cos A - \sin A}{\sin A}}$$

$$= \frac{\sin A \cos A}{1 + \sin A - \cos A} + \frac{\sin A \cos A}{1 + \cos A - \sin A}$$

$$= \sin A \cos A \left[\frac{1}{1 + \sin A - \cos A} + \frac{1}{1 + \cos A} - \sin A \right]$$

$$=\sin A\cos A\left[\frac{1+\cos A-\sin A+\cot A\sin A-\cos A}{(1+\sin\theta-\cos\theta)(1+\cos A-\sin A)}\right]$$

$$=\sin A\cos A\left[\frac{2}{\cos A-\sin A+\sin A+\sin A\cos A-\sin^2 A-\cos A-\cos^2 A+\cos A\sin A}\right]$$

$$=\sin A\cos A\left[\frac{2}{1-\sin^2 A-\cos^2 A+2\sin A\cos A}\right]$$

$$=\sin A\cos A\left[\frac{2}{1-\left(\sin^2 A+\cos^2 A\right)+2\sin A\cos A}\right]$$

$$=\sin A\cos A\left[\frac{2}{1-1+2\sin A\cos A}\right]\left(\because \sin^2 A+\cos^2 A=1\right)$$

$$=\sin A\times\cos A\times\frac{2}{2\sin A\cos A}$$

$$=1$$

$L.H.S = R.H.S$

65. $$\frac{\tan A}{\left(1+\tan^2 A\right)}+\frac{\cos A}{\left(1+\cot^2 A\right)^2}=\sin A\cos A$$

Sol:

$$=\frac{\tan A}{\left(\sec^2 A\right)^2}+\frac{\cos A}{\left(\cos ec^2 A\right)^2} \qquad \left[\begin{array}{l}\because 1+\tan^2 A=\sec^2 A\\ 1+\cot^2 A=\cos ec^2 A\end{array}\right]$$

$$=\frac{\frac{\sin A}{\cos A}}{\sec^4 A}+\frac{\frac{\cot A}{\sin A}}{\cos ec^4 A}$$

$$=\frac{\frac{\sin A}{\cos A}}{\frac{1}{\cos^4 A}}+\frac{\frac{\cos A}{\sin A}}{\frac{1}{\sin^4 A}}$$

$$=\frac{\sin A}{\cos A}\times\frac{\cos^4 A}{1}+\frac{\cos A}{\sin A}\times\frac{\sin^4}{1}$$

$$=\sin A\times\cos^3 A+\cos A-\sin^3$$

$$=\sin A\cos A\left(\cos^2 A+\sin^2 A\right)$$

$$=\sin A\cos A$$

L.H.S = R.H.S

Hence proved.

66. $\sec^4 A\left(1-\sin^4 A\right)-2\tan^4 A=1$

Sol:

$LHS=\sec^4 A\left(1-\sin^4 A\right)-2\tan^4 A$

$=\sec^4 A-\sec^4 A\times\sin^4 A-2\tan^2 A$

$=\sec^4 A-\dfrac{1}{\cos^4 A}\times\sin^4-2\tan^2 A$

$=\sec^4 A-\tan^4 A-2\tan^2 A$

$=\left(\sec^2 A\right)^2=\tan^4 A-2\tan^2 A$

$=\left(1+\tan^2 A\right)^2-\tan^4 A-2\tan^2 A\qquad\left[\because \sec^2 A-\tan^2 A=1\right]$

$=1+\tan^4 A+2\tan^2 A-\tan^4 A-2\tan^2 A$

$=1=RHS$

Hence proved.

67. $\dfrac{\cot^2 A\left(\sec A-1\right)}{1+\sin A}=\sec^2\left[\dfrac{1-\sin A}{1+\sec A}\right]$

Sol:

$=\dfrac{\dfrac{\cos^2 A}{\sin^2 A}\left(\dfrac{1}{\cos A}-1\right)}{1+\sin A}$

$=\dfrac{\dfrac{\cos^2 A}{\sin^2 A}\left(\dfrac{1-\cos A}{\cos A}\right)}{1+\sin A}\qquad\left[\because \sin^2 A+\cos^2 A=1\right]$

$=\dfrac{\dfrac{\left(\cos A\times\cos A\right)}{\left(1-\cos^2 A\right)}\left[\dfrac{1-\cos A}{\cos A}\right]}{1+\sin A}$

$=\dfrac{\left(\cos A\right)\left(1-\cos A\right)}{\left(1+\cos A\right)\left(1-\cos A\right)}=\dfrac{1}{1+\sin A}$

$=\dfrac{\cos A}{\left(1+\cos A\right)\left(1+\sin A\right)}$

Solving

$RHS = \sec^2\left[\frac{1-\sin A}{1+\sec A}\right]$

$= \frac{1}{\cos^2 A}\left[\frac{1-\sec A}{1+\sec A}\right]$

$= \frac{1}{\cos^2 A}\left[\frac{1-\sec A}{\cos A+1}\right](\cos A)$

$= \frac{(1-\sin A)}{(\cos A)(\cos A+1)}$

By multiplying Nr and Dr with $(1+\sin A)$

$= \frac{(1-\sin A)}{(\cos A)(1+\cos A)} \times \frac{1+\sin A}{1+\sin A}$

$= \frac{(1)^2 - \sin^2 A}{\cos A(1+\cos A)(1+\sin A)}$

$= \frac{\cos^2 A}{\cos A(1+\cos A)(1+\sin A)}$

$= \frac{\cos^2 A}{(1+\cos A)(1+\sin A)}$

$L.H.S = R.H.S$ hence proved.

68. $(1+\cot A+\tan A)(\sin A-\cos A) = \frac{\sec A}{\cos ec^2 A} - \frac{\cos ecA}{\sec^2 A} == \sin A \tan A - \cos A \cot A$

Sol:

$(1+\cot A+\tan A)(\sin A-\cos A)$

$\sin A - \cos A + \cot A \sin A - \cot A \cos A + \sin A \tan A - \tan A \cos A$

$\sin A - \cos A + \frac{\cos A}{\sin A} \times \sin A - \cot A \cos A + \sin A \tan A - \frac{\sin A}{\cos A} \times \cos A$

$\sin A - \cos + \cos A - \cot A \cos A + \sin A \tan A - \sin A$

$= \sin A \cos A \ \cos A \cot A$

Solving:

$\frac{\sec A}{\cos ec^2 A} - \frac{\cos ecA}{\sec^2 A}$

$$\frac{\frac{1}{\cos A}-\frac{1}{\sin A}}{\frac{1}{\sin^2 A}\quad\frac{1}{\cos^2 A}}$$

$$\frac{\sin^2 A}{\cos A}-\frac{\cos^2 A}{\sin A}$$

$$\frac{\sin^3 A-\cos^3 A}{\sin A\cos A}$$

$$=\sin A\times\frac{\sin A}{\cos A}-\cos A\times\frac{\cos A}{\sin A}$$

$$=\sin A\tan A-\cos A\cot A$$

$$L.H.S=R.H.S$$

69. $\sin^2 A\cos^2 B-\cos^2 A\sin^2 B=\sin^2 A-\sin^2 B$

Sol:

$$LHS=\sin^2 A\cos^2 B-\cos^2 A\sin^2 B.$$

$$=\sin^2 A\left(1-\sin^2 B\right)-\left(1-\sin^2 A\right)\left(\sin^2 A\right)\qquad\left(\because\cos^2 A=1-\sin^2 A\right)$$

$$=\sin^2 A-\sin^2 A\sin^2 B-\sin^2 B+\sin^2 A\sin^2 B$$

$$=\sin^2 A-\sin^2 B$$

$R.H.S$ Hence Proved.

70. $\dfrac{\cot A+\tan B}{\cot B+\tan A}=\cot A\tan B$

Sol:

$$LHS=\frac{\cot A+\tan B}{\cot B+\tan A}$$

$$=\frac{\frac{\cos A}{\sin A}+\frac{\sin B}{\cos B}}{\frac{\cos B}{\sin B}+\frac{\cos ecA}{\cos A}}$$

$$=\frac{\frac{\cos A\cos B-\sin A\sin B}{\sin A\cos B}}{\frac{\cos A\cos B+\sin A\sin B}{\cos A\sin B}}$$

$$= \frac{\cos A\cos B+\sin A\sin B}{\sin A\cos B}\times\frac{\cos A\sin B}{\cos A\cos B+\sin A\sin B}$$
$$= \frac{\cos A\sin B}{\sin A\cos B}$$
$$= \cot A\tan B$$
$$= RHS$$
Hence proved

71. $\frac{\tan A+\tan B}{\cot A+\cot B}=\tan A\tan B$

Sol:

$$LHSS = \frac{\tan A+\tan B}{\cot A+\cot B}$$
$$= \frac{\frac{\sin A}{\cos A}+\frac{\sin B}{\cos B}}{\frac{\cos A}{\sin A}+\frac{\cos B}{\sin B}}$$
$$= \frac{\frac{\sin A\cos B+\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\sin B+\cos B\sin A}{\sin A\sin B}}$$
$$= \frac{\sin A\cos B+\cos A\sin B}{\cos A\cos B}\times\frac{\sin A\sin B}{\cos A\sin B+\cos B\sin A}$$
$$= \frac{\sin A\sin B}{\cos A\cos B}$$
$$= \tan A+\tan B = RHS$$
Hence proved

72. $\cot^2 A\cos ec^2 B-\cot^2 B\cos ec^2 A=\cot^2 A-\cot^2 B$

Sol:

$$LHS = \cot^2 A\cos ec^2 B-\cot^2 B\cos ec^2 A$$
$$= \cot^2 A\left(1+\cot^2 B\right)-\cot^2 B\left(1+\cot^2 B\right) \qquad \left[\because \cos ec^2\theta = 1+\cot^2\theta\right]$$
$$= \cot^2 A+\cot^2 A\cot^2 B-\cot^2 B-\cot^2 B\cot^2 A$$
$$= \cot^2 A-\cot^2 B.$$
Hence proved

73. $\tan^2 A\sec^2 B-\sec^2 A\tan^2 B=\tan^2 A-\tan^2 B$

Sol:

$$LHS = \tan^2 A \sec^2 B - \sec^2 A \tan^2 B$$
$$= \tan^2 A + \left(1 + \tan^2 B\right) - \sec^2 A\left(\tan^2 A\right)$$
$$= \tan^2 A + \tan^2 A \tan^2 B - \tan^2 B\left(1 + \tan^2 A\right) \qquad \left(\because \sec^2 A = 4\tan^2 A\right)$$
$$= \tan^2 + \tan^2 A \tan^2 B - \tan^2 B - \tan^2 B \tan^2 A$$
$$= \tan^2 A - \tan^2 B$$
$$= RHS$$

74. If x = a sec θ + b tan θ and y $=$ a tan $\theta +$ b sec θ, prove that $x^2 - y^2 = a^2 - b^2$

Sol:

$$L.H.S = x^2 - y^2$$
$$= \left(a\sec\theta + b\tan\theta\right)^2 - \left(a\tan\theta + b\sec\theta\right)^2$$
$$= a^2\sec^2\theta + b^2\tan^2\theta + 2ab\sec\theta\tan\theta - a^2\tan^2\theta - b^2\sec\theta - 2ab\sec\theta\tan\theta.$$
$$= a^2 - \sec^2\theta - b^2\sec^2\theta + b^2\tan^2\theta - a^2\tan^2\theta$$
$$= \sec^2\theta\left(a^2 - b^2\right) + \tan^2\theta\left(b^2 - a^2\right)$$
$$= \sec^2\theta\left(a^2 - b^2\right) - \tan^2\theta\left(a^2 - b^2\right)$$
$$= \left(a^2 - b^2\right)\left(\sec^2\theta - \tan^2\theta\right) \qquad \left[\because \sec^2\theta - \tan^2\theta = 1\right]$$
$$= a^2 - b^2$$

75. If $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ *and* $\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$, *prove that* $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

Sol:

$$\left[\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right]^2 + \left[\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right]^2 = (1)^2 + (1)^2$$
$$\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta \frac{2xy}{ab}\cos\theta\sin\theta + \frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta$$
$$-\frac{2xy}{ab}\sin\theta\cos\theta = 1 + 1$$
$$\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + \frac{x^2}{a^2}\sin^2\theta = 2$$
$$\cos^2\theta\left[\frac{x^2}{a^2} + \frac{y^2}{b^2}\right] + \sin^2\theta\left(\frac{x^2}{a^2} + \frac{y^2}{a^2}\right) = 2$$

$$\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)\left(\cos^2\theta+\sin^2\theta\right)=2$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}= \qquad \left(\therefore \cos^2\theta+\sin^2\theta=1\right)$$

76. If cosec θ – sin $\theta = a^3, \sec\theta - \cos\theta = b^3, prove\ that\ a^2b^2(a^2+b^2)=1$

Sol:

$$\cos ec\theta-\sin\theta=a^3$$

$$\frac{1}{\sin\theta}-\sin\theta=a^3$$

$$\frac{1-\sin^2}{\sin\theta}=a^3$$

$$\frac{\cos^2\theta}{\sin\theta}=a^3$$

$$a=\frac{\cos^{\frac{1}{3}}\theta}{\sin^{\frac{1}{3}}\theta}$$

$$\Rightarrow a^2=\frac{\cos\frac{4}{3}\theta}{\sin\frac{2}{3}\theta}$$

$$\sec\theta-\cos\theta=b^3$$

$$\frac{1}{\cos\theta}\cos\theta=b^3$$

$$\frac{1-\cos^2\theta}{\cos\theta}=b^3$$

$$\frac{\sin^2\theta}{\cos\theta}=b^3$$

$$b=\frac{\sin^{\frac{2}{3}}\theta}{\cos^{\frac{1}{3}}\theta}$$

Now, $a^2b^2\left(a^2+b^2\right)$

$$=\frac{\cos^{\frac{4}{3}}\theta}{\sin^{\frac{2}{3}}\theta}\times\frac{\sin^{\frac{4}{3}}\theta}{\cos^{\frac{2}{3}}\theta}\left(\frac{\cos^{\frac{4}{3}}\theta}{\sin^{\frac{2}{3}}\theta}=\frac{\sin^{\frac{4}{3}}\theta}{\cos^{\frac{2}{3}}\theta}\right)$$

$$= \cos^{\frac{4}{3}-\frac{2}{3}}\theta \times \sin^{\frac{4-2}{3}}\left(\frac{\cos^{\frac{4}{3}}\theta}{\sin^{\frac{2}{3}}\theta} + \frac{\sin^{\frac{4}{3}}\theta}{\cos^{\frac{2}{3}}\theta}\right)$$

$$= \cos^{\frac{2}{3}}\theta \sin^{\frac{2}{3}}\left(\frac{1}{\sin^{\frac{2}{3}}\theta \cos^{\frac{2}{3}}\theta}\right) \quad \left(\because \cos^2\theta + \sin^2\theta = 1\right)$$

$$= 1$$

$$L.H.S = R.H.S$$

77. If a $\cos^3\theta + 3a\cos\theta\sin^2\theta = m, a\sin^3\theta + 3\ a\cos^2\theta\sin\theta = n, prove\ that$

$$(m+n)^{\frac{2}{3}} + (m-n)^{\frac{2}{3}}$$

Sol:

$$= \left(a\cos^3\theta + 3a\cos\theta\sin^2\theta + a\sin^3\theta + 3a\cos^2\theta\sin\theta\right)^{\frac{2}{3}}$$

$$+\left(a\cos^3\theta + 3a\cos\theta\sin^2\theta - a\sin^3\theta - 3a\cos^2\theta\sin\theta\right)^{\frac{2}{3}}$$

$$= a^{\frac{1}{3}}\left(\cos^3\theta + 3\cos\theta\sin^2\theta + \sin^3\theta + 3\cos^2\theta\sin\theta\right)^{\frac{2}{3}}$$

$$+a^{\frac{2}{3}}\left(\cos^3\theta + 3\cos\theta\sin^2\theta + \sin^3\theta - 3\cos^2\theta\sin\theta\right)^{\frac{2}{3}}$$

$$= a^{\frac{1}{3}}\left[\left(\cos\theta + \sin\theta\right)^3\right]^{\frac{2}{3}} + a^{\frac{2}{3}}\left(\cos\theta - \sin\theta\right)^3\Big]^{\frac{2}{3}}$$

$$= a^{\frac{2}{3}}\left[\left(\cos\theta + \sin\theta\right)^2\right] + a^{\frac{2}{3}}\left(\cos\theta - \sin\theta\right)^2$$

$$= a^{\frac{2}{3}}\left[\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta\right]$$

$$= a^{\frac{2}{3}}\left[\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta\right] + a^{\frac{2}{3}}\left[\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta\right]$$

$$= a^{\frac{2}{3}}\left[1 + 2\sin\theta\cos\theta\right] + a^{\frac{2}{3}}\left[1 - 2\sin\theta\cos\theta\right]$$

$$= a^{\frac{2}{3}}\left[1 + 2\sin\theta\cos\theta + 1 - 2\sin\theta\cos\theta\right]$$

$$= a^{\frac{1}{3}}(1+1) = 2a^{\frac{2}{3}}$$

$$= R.H.S$$

Hence proved.

78. If x = a $\cos^3\theta, y = b\sin^3\theta, prove\ that\ \left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$

Sol:

$x = a\cos^3\theta : y = b\sin^3\theta$

$\frac{x}{a} = \cos^3\theta : \frac{y}{b} = \sin^3\theta$

$L.H.S = \left[\frac{x}{a}\right]^{\frac{2}{3}} + \left[\frac{y}{b}\right]^{\frac{2}{3}}$

$= \left(\cos^3\theta\right)^{\frac{2}{3}} + \left(\sin^3\theta\right)^{\frac{2}{3}}$

$= \cos^2\theta + \sin^2\theta$ $\quad\left(\therefore \cos^2\theta + \sin^2\theta = 1\right)$

$= 1$

Hence proved

79. If 3 sin θ + 5 cos θ = 5, prove that 5sin θ – 3 cos $\theta = \pm$ 3.

Sol:

Given $3\sin\theta + 5\cos\theta = 5$

$3\sin\theta = 5 - 5\cos\theta$

$3\sin\theta = 5(1-\cos\theta)$

$3\sin\theta = \frac{5(1-\cos\theta)(1-\cos\theta)}{1+\cos\theta}$

$3\sin\theta = \frac{5(1-\cos^2\theta)}{(1+\cos\theta)}$

$3\sin\theta = \frac{5\sin^2\theta}{1+\cos\theta}$

$3 + 3\cos\theta = 5\sin\theta$

$3 = 5\sin - 3\cos\theta$

$= RHS$

Hence proved.

80. If a cos θ + b sin θ = m and a sin θ – b cos θ = n, prove that $a^2 + b^2 = m^2 + n^2$

Sol:

$R.H.S = m^2\sin^2$

$= (a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2$

$= a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta$

$+ a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta$

$= a^2\cos^2\theta + b^2\cos^2\theta + b^2\sin^2\theta + a^2\sin^2\theta$

$= a^2\left(\sin^2\theta + \cos^2\theta\right) + b^2\left(\sin^2\theta + \cos^2\theta\right)$

$= a^2 + b^2 \qquad \left(\because \sin^2\theta + \cos^2\theta = 1\right)$

81. If $\cos\theta + \cot\theta = \text{m}$ and $\text{cosec}\,\theta - \cot\theta = \text{n}$, prove that $\text{m n} = 1$

Sol:

$LHS = mn$

$= \left(\cos ec\theta + \cot\theta\right)\left(\cos ec\theta - \cot\theta\right)$

$= \cos ec^2\theta - \cot^2\theta$

$= 1 \qquad \left[\because \left(a+b\right)\left(a-b\right) = a^2 - b^2 \cos ec^2\theta - \cot^2\theta = 1\right]$

$= R.H.S$

82. If cos A + cos^2 A = 1, prove that sin^2 A + sin^4 A = 1

Sol:

$\cos A + \cos^2 A = 1$

$\cos A = 1 - \cos^2 A$

$\cos A = \sin^2 A$

$LHS = \sin^2 A + \sin^4 A$

$= \sin^2 A + \left(\sin^2 A\right)$

$= \sin^2 A + \left(\cos A\right)^2$

$= \sin^2 A + \cos A$

$= 1$

83. Prove that:

$(i)\ \sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = 2cosec\ \theta$

$(ii)\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$

$(iii)\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = 2cosec\ \theta$

$(iv)\ \frac{\sec\theta - 1}{\sec\theta + 1} = \left(\frac{\sin\theta}{1+\cos\theta}\right)^2$

Sol:

$LHS = \sqrt{\frac{\frac{1}{\cos\theta} - 1}{\frac{1}{\cos\theta} + 1}} + \sqrt{\frac{\frac{1}{\cos\theta} + 1}{\frac{1}{\cos\theta} - 1}}$

$$= \sqrt{\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{1+\cos\theta}{\cos\theta}}} + \sqrt{\frac{\frac{1+\cos\theta}{\cos\theta}}{\frac{1-\cos\theta}{\cos\theta}}}$$

$$= \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} + \sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$$

$$= \sqrt{\frac{(1-\cos\theta)}{(1+\cos\theta)} \times \frac{(1-\cos\theta)}{1-\cos\theta}} + \sqrt{\frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}} + \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}}$$

$$= \frac{1-\cos\theta}{\sin\theta} + \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1-\cos\theta+1+\cos\theta}{\sin\theta}$$

$$= \frac{2}{\sin\theta}$$

$$= 2\cos ec$$

(2) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta} \times \frac{(1+\sin\theta)}{1+\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}}$$

$$\sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}} + \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$$

$$= \frac{1+\cos\theta}{\sin\theta} + \frac{1-\cos\theta}{\sin\theta}$$

$$= \frac{2}{\sin\theta} = 2\cos ec\theta$$

(3) Not given

(4) $\frac{\sec\theta-1}{\sec\theta+1}$

$=\frac{\frac{1}{\cos\theta}-1}{\frac{1}{\cos\theta}+1}$

$=\frac{1-\cos\theta}{1+\cos\theta}$

$=\frac{1-\cos\theta}{1+\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}$

$=\frac{1-\cos^2\theta}{(1+\cos\theta)^2}$

$=\frac{\sin^3}{(1+\cos\theta)^2}$

$=\left[\frac{\sin\theta}{1+\cos\theta}\right]^2$

$=RHS$

Hence proved.

84. If $\cos\theta + \cos^2\theta = 1$, prove that

$\sin^{12}\theta+3\sin^{10}\theta+3\sin^8\theta+\sin^6\theta+2\sin^4\theta+2\sin^2\theta-2=1$

Sol:

$\cos\theta+\cos^2\theta=1$

$\cos\theta=1-\cos^2\theta$

$\cos\theta=\sin^2\theta \qquad .....(1)$

Now, $\sin^{12}\theta+3\sin^{10}\theta+3\sin^8\theta+\sin^6\theta+2\sin^4\theta+2\sin^2\theta-2$

$=(\sin^4\theta)^3+3\sin^4\theta\cdot\sin^2\theta(\sin^4\theta+\sin^2\theta)$

$+(\sin^2\theta)^3+2(\sin^2\theta)^2+2\sin^2\theta 2$

Using $(a+b)^3=a^3+b^3+3ab(a+b)$and also from

(1) $\sin^2\theta\cos\theta$

$(\sin^4\theta+\sin^2\theta)^3+2\cos\theta^2+2\cos\theta-2.$

$\left((\sin^2\theta)^2+\sin^2\theta\right)+2\cos^2\theta+2\cos\theta-2$

$(\cos^2+\sin^2\theta)^3+2\cos^2\theta+2\cos\theta-2$

$\left(\cos^2 + \sin\right)^3 + 2\cos^2\theta + 2\sin^2\theta - 2 \quad \left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$1 + 2\left(\sin^2\theta + \cos^2\theta\right) - 2$

$1 + 2(1) - 2$

$= 1$

$L.H.S = R.H.S$

Hence proved.

85. Given that $(1+\cos\alpha)(1+\cos\beta)(1+\cos\gamma) = (1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma)$

Show that one of the values of each member of this equality is $\sin\alpha \sin\beta \sin\gamma$

Sol:

L.H.S

We know that $1+\cos\theta = 1+\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} = 2\cos^2\frac{\theta}{2}$

$\therefore \Rightarrow 2\cos^2\frac{\alpha}{2}\cdot 2\cos^2\frac{\beta}{2}\cdot 2\cos^2\frac{\gamma}{2} \qquad ....(1)$

Multiply (1) with $\sin\alpha\sin\beta\sin\gamma$ and divide it with same we get

$$\frac{8\cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2}\cos^2\frac{\gamma}{2}}{\sin\alpha\sin\beta\sin\gamma} \times \sin\alpha\sin\beta\sin\gamma$$

$$\Rightarrow \frac{2\cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2}\cos^2\frac{\gamma}{2} \times \sin\alpha\sin\beta\sin\gamma}{\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}}$$

$$\Rightarrow \sin\alpha\sin\beta\sin\gamma \times \cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$$

$RHS\ (1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma)$

We know that $1-\cos\theta = 1-\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} = 2\sin^2\frac{\theta}{2}$

$$\Rightarrow 2\sin^2\frac{\alpha}{2} 2\cdot\sin^2\frac{\beta}{2}\cdot 2\sin^2\frac{\gamma}{2}$$

Multiply and divide by $\sin\alpha\sin\beta\sin\gamma$ we get

$$\frac{2\sin^2\frac{\alpha}{2} 2\sin^2\frac{\beta}{2} 2\sin^2\frac{\gamma}{2}\cdot\sin\alpha\sin\beta\sin\gamma}{\sin\alpha\sin\beta\sin\gamma}$$

$$\Rightarrow \frac{2\sin^2\frac{\alpha}{2}\cdot 2\sin^2\frac{\beta}{2}\cdot 2\sin^2\frac{\gamma}{2}\cdot \sin\alpha\sin\beta\sin\gamma}{2\sin\frac{\alpha}{2}\cos\frac{\beta}{2}2\sin\frac{\beta}{2}\cos\frac{\beta}{2}2\sin\frac{\gamma}{2}\cos\frac{\gamma}{2}}$$

$$\Rightarrow \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}\sin\alpha\sin\beta\sin\gamma$$

Hence $\sin\alpha\sin\beta\sin\gamma$ is the member of equality.

86. $if\ \sin\theta+\cos\theta=x$ P.T $\sin^6\theta+\cos^6\theta=\frac{4-3\left(x^2-1\right)^2}{4}$

Sol:

$\sin\theta+\cos\theta=x$

Squaring on both sides

$\left(\sin\theta+\cos\theta\right)^2=x^2$

$\Rightarrow \sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=x^2$

$\therefore \sin\theta\cos\theta=\frac{x^2-1}{2}$ ......(1)

We know $\sin^2\theta+\cos^2\theta=1$

Cubing on both sides

$\left(\sin^2\theta+\cos^2\theta\right)^3=(1)^3$

$\sin^6\theta+\cos^6\theta+3\sin^2\theta\cos^2\theta\left(\sin^2\theta+\cos^2\theta\right)=1$

$\Rightarrow \sin^6\theta+\cos^6 7=1-3\sin^2\theta\cos^2\theta$

$=1-3\frac{\left(x^2-1\right)^2}{4}$ from (1)

$\therefore \sin^6\theta+\cos^6\theta=\frac{4-3\left(x^2-1\right)^2}{4}$

Hence proved

87. $if\ x=a\sec\theta\cos\phi y=b\sec\theta\sin\phi\, and\ z=c\tan\theta,\ S.T\ \frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

Sol:

$x^2=a^2\sec^2\theta\cos^2\theta$ .....(i)

$y^2=b^2\sec^2\theta\sin^2\theta$ ....(ii)

$z^2=c^2\tan^2\theta$ ....(iii)

Exercise 6.2

1. *If* $\cos\theta = \frac{4}{5}$, *find all other trigonometric ratios of angle* θ

Sol:

We have $\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{4}{5}\right)^2}$

$= \sqrt{1-\frac{16}{25}}$

$= \sqrt{25-\frac{16}{25}}$

$= \sqrt{\frac{9}{25}} = \frac{3}{5}$

$\therefore \sin\theta = \frac{3}{5}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3/5}{4/5} = \frac{3}{4} \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$

$\cos ec = \frac{1}{\sec\theta} = \frac{1}{\frac{3}{5}} = \frac{5}{3} \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{3}{4}} = \frac{4}{3}.$

2. *If* $\sin\theta = \frac{1}{\sqrt{2}}$, *find all other trigonometric ratios of angle* θ

Sol:

We have $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{1}{2}\right)^2}$

$= \sqrt{\frac{1-1}{2}} = \sqrt{\frac{1}{2}}$

$\therefore \cos\theta = \frac{1}{\sqrt{2}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$

$\cos ec\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{1} = 1$$

3. $If\ \tan\theta = \frac{1}{\sqrt{2}}, Find\ the\ value\ of\ \frac{\operatorname{cos}ec^2\theta - \sec^2\theta}{\operatorname{cos}ec^2\theta + \cot^2\theta}$

Sol:

We know that $\sec\theta = \sqrt{1+\tan^2\theta}$

$$= \sqrt{1+\left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= \sqrt{1+\frac{1}{2}} = \sqrt{\frac{3}{2}}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

$$\operatorname{cos}ec\theta = \sqrt{1+\cot^2\theta} \Rightarrow \sqrt{1+2} = \sqrt{3}$$

Substituting it in (1) we get

$$\Rightarrow \frac{\left(\sqrt{3}\right)^2 - \left(\sqrt{3}\right)^2}{\left(\sqrt{3}\right)^2 + \left(\sqrt{3}\right)^2} = \frac{3-\frac{3}{2}}{3+2} = \frac{\frac{3}{2}}{5}$$

$$= \frac{3}{10}$$

4. $If\ \tan\theta = \frac{3}{4},\ find\ the\ value\ of\ \frac{1-\cos\theta}{1+\cos\theta}$

Sol:

$$\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1+\left(\frac{3}{4}\right)^2} = \sqrt{1+\frac{9}{16}}$$

$$\Rightarrow \sqrt{\frac{16-9}{16}} = \frac{5}{4}$$

$$\therefore \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{5}{4}} = \frac{4}{5} = \cos\theta$$

$$\therefore \text{We get } \frac{1-\frac{4}{5}}{1+\frac{4}{5}} = \frac{\frac{1}{5}}{\frac{9}{5}} = \frac{1}{9}.$$

5. $If\ \tan\theta = \frac{12}{5}$, *find the value of* $\frac{1+\sin\theta}{1-\sin\theta}$

Sol:

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{12}{5}} = \frac{5}{12}$$

$$\cos ec = \sqrt{1+\cot^2\theta} = \sqrt{1+\left[\frac{5}{12}\right]^2} = \sqrt{\frac{144+25}{(12)^2}} = \sqrt{\frac{169}{144}} = \frac{13}{12}.$$

$$\sin\theta = \frac{1}{\cos ec\theta} = \frac{1}{\frac{13}{12}} = \frac{12}{13}.$$

$$\text{We get } \frac{1+\frac{12}{13}}{1-\frac{12}{13}} = \frac{\frac{13+12}{18}}{\frac{13-12}{18}} = \frac{25}{1} = 25$$

6. $If\ \cot\theta = \frac{1}{\sqrt{3}}$, *find the value of* $\frac{1-\cos^2\theta}{2-\sin^2\theta}$

Sol:

$$\cos ec\theta = \sqrt{1+\cot^2\theta} = \sqrt{1+\frac{1}{3}} = \sqrt{\frac{4}{3}}$$

$$\therefore \cos ec\theta = \frac{2}{\sqrt{3}}$$

$$\sin\theta = \frac{1}{\cos ec\theta} = \frac{1}{\frac{2}{\sqrt{3}}} = \frac{\sqrt{3}}{2}$$

$$and\ \frac{1}{\cot\theta} = \frac{\sin\theta}{\cos\theta} = \cos = \frac{\sin\theta}{\frac{1}{\cos\theta}} \Rightarrow \frac{\frac{\sqrt{3}}{2}}{\sqrt{3}} = \frac{1}{2}.$$

$\therefore$ on substituting we get

$$\frac{1-\frac{1}{4}}{2-\frac{3}{4}}=\frac{\frac{3}{4}}{\frac{5}{4}}=\frac{3}{5}.$$

7. $If\ \cos ec=\sqrt{2},\ find\ the\ value\ of\ \dfrac{2\sin^2 A+3\cot^2 A}{4\left(\tan^2 A-\cos^2 A\right)}$

Sol:

We know that $\cot A=\sqrt{\cos ec^2 A-1}$

$=\sqrt{(2)^2-1}=\sqrt{2-1}$

$-1.$

$$\tan A=\frac{1}{\cot A}=\frac{1}{1}=1$$

$$\sin A=\frac{1}{\cos ec}A=\frac{1}{\sqrt{2}}\ \therefore \sin A=\frac{1}{\sqrt{2}}$$

$$\cos A\sqrt{1-\sin^2 A}=\sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2}-\sqrt{\frac{1}{\sqrt{2}}}=\frac{1}{\sqrt{2}}.$$

On substituting we get

$$\frac{2\left[\frac{1}{\sqrt{2}}\right]^3+3[1]^2}{4\left[(1)-\left(\frac{1}{\sqrt{2}}\right)^2\right]}=\frac{2=\frac{1}{2}+3}{4\left[1-\frac{1}{2}\right]}$$

$$\Rightarrow \frac{1+3}{4\cdot\frac{1}{2}}=\frac{4}{2}=2.$$

8. $If\ \cot\theta\sqrt{3},\ find\ the\ value\ of\ \dfrac{\cos ec^2\theta+\cot^2\theta}{\cos ec^2\theta-\cot^2\theta}$

Sol:

$$\cos ec\theta=\sqrt{1+\cot^2\theta}=\sqrt{1+\left(\sqrt{3}\right)^2}=\sqrt{1+3}=2$$

$$\sin\theta=\frac{1}{\cos ec}\theta=\frac{1}{2}\cot\theta=\frac{\cos\theta}{\sin\theta}\qquad \therefore \cos\theta=\cot\theta\cdot\sin\theta$$

$$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{2}{\sqrt{3}}$$

On substituting we get

$$\frac{(2)^2 + \left(\sqrt{3}\right)^2}{(2)^2 - \left(\frac{2}{\sqrt{3}}\right)^2} = \frac{4+3}{\frac{12-4}{3}} = \frac{7}{\frac{8}{3}}$$

$$= \frac{21}{8}.$$

9. $If\ 3\cos\theta = 1,\ find\ the\ value\ of\ \ \frac{6\sin^2\theta + \tan^2\theta}{4\cos\theta}$

Sol:

$$\cos\theta = \frac{1}{3} \qquad \sin = \sqrt{1+\cos^2\theta}$$

$$= \sqrt{1-\frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2\sqrt{2}}{3\cdot\frac{1}{3}} = 2\sqrt{2}$$

On substituting in (1) we get

$$\frac{6\left[\frac{2\sqrt{2}}{3}\right]^2 + \left(2\sqrt{2}\right)^2}{4\cdot\frac{1}{3}} = \frac{6\cdot\frac{3}{5}}{\frac{4}{5}} = \frac{\frac{16+24}{3}}{\frac{4}{3}}$$

$$= \frac{40}{4} = 10$$

10. $If\ \sqrt{3}\tan\theta = \sin\theta,\ find\ the\ value\ of\ \sin^2\theta - \cos^2\theta$

Sol:

$$\sqrt{3}\cdot\frac{\sin\theta}{\cos\theta} = \sin\theta$$

$$\cos\theta = \frac{\sqrt{3}}{3} \Rightarrow \frac{1}{\sqrt{3}}$$

$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{1}{\sqrt{3}}\right)^2}$

$\therefore \sin^2\theta - \cos^2\theta = \left(\sqrt{\frac{2}{3}}\right)^2 - \left[\frac{1}{\sqrt{3}}\right]^2$

$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

11. $If\ \cos ec\theta = \frac{13}{12},\ find\ the\ value\ of\ \frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$

Sol:

$\sin\theta = \frac{1}{\cos ec\theta} = \frac{1}{\frac{13}{12}} = \frac{12}{13}$

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left[\frac{12}{13}\right]^2} = \sqrt{1-\frac{144}{169}}$

$= \sqrt{\frac{25}{169}} = \frac{5}{13}$

$\Rightarrow \frac{2\cdot\frac{12}{13} - 3\cdot\frac{5}{13}}{4\cdot\frac{12}{13} - 9\cdot\frac{5}{13}} = \frac{\frac{24-15}{13}}{\frac{48-15}{13}} = \frac{9}{3} = 3$

12. $If\ \sin\theta + \cos\theta = \sqrt{2}\cos\left(90° - \theta\right),\ find\ \cot\theta$ find $\cot\theta$

Sol:

$L.H.S \Rightarrow \sin\theta + \cos\theta = \sqrt{2}\sin\theta \qquad \left[\because \cos\left(90-\theta\right) = \sin\theta\right]$

$\Rightarrow \cos\theta - \sin\theta\left(\sqrt{2}\right) - \sin\theta$

$\cos\theta - \sin\theta\left(\sqrt{2}-1\right)$

Divide both sides with $\sin\theta$ we get

$\frac{\cos\theta}{\sin\theta} = \frac{\sin\theta}{\sin\theta}\left(\sqrt{2}-1\right)$

$= \cot\theta = \sqrt{2}-1$