

Exercise -10.1

1. In Fig. 10.22, the sides BA and CA have been produced such that: $BA = AD$ and $CA = AE$. Prove that segment $DE \parallel BC$.

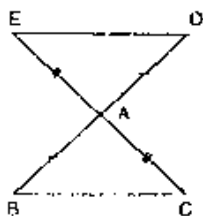


Fig. 10.22

Sol:

Given that, the sides BA and CA have been produced such that $BA = AD$ and $CA = AE$ and given to prove $DE \parallel BC$

Consider triangle BAC and DAE ,

We have

$$BA = AD \text{ and } CA = AE$$

[\because given in the data]

$$\text{And also } \angle BAC = \angle DAE$$

[\because vertically opposite angles]

angles]

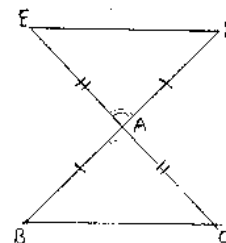
So, by SAS congruence criterion, we have $\triangle BAC \cong \triangle DAE$

$$\Rightarrow BC = DE \text{ and } \angle DEA = \angle BCA, \angle EDA = \angle CBA$$

[Corresponding parts of congruent triangles are equal]

Now, DE and BC are two lines intersected by a transversal DB such that $\angle DEA = \angle BCA$, i.e., alternate angles are equal

Therefore, $DE \parallel BC$



2. In a $\triangle PQR$, if $PQ = QR$ and L, M and N are the mid-points of the sides PQ, QR and RP respectively. Prove that: $LN = MN$.

Sol:

Given that, in $\triangle PQR$, $PQ = QR$ and L, M, N are midpoints of the sides PQ, QR and RP respectively and given to prove that $LN = MN$

Here we can observe that PQR is an isosceles triangle

$$\Rightarrow PQ = QR \text{ and } \angle QPR = \angle QRP \quad \dots\dots\dots(1)$$

And also, L and M are midpoints of PQ and QR respectively

$$\Rightarrow PL = LQ = \frac{PQ}{2}, QM = MR = \frac{QR}{2}$$

And also, $PQ = QR$

$$\Rightarrow PL = LQ = QM = MR = \frac{PQ}{2} = \frac{QR}{2} \quad \dots\dots\dots(2)$$

Now, consider $\triangle LPN$ and $\triangle MRN$,

$$LP = MR \quad [\text{From } - (2)]$$

$$\angle LPN = \angle MRN \quad [\text{From } - (1)]$$

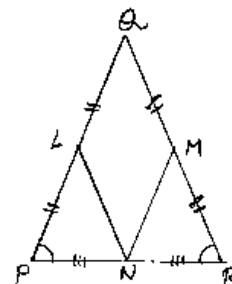
$\therefore \angle QPN$ and $\angle LPN$ and $\angle QRM$ and $\angle MRN$ are same

$$PN = NR \quad [\because N \text{ is midpoint of } PR]$$

So, by SAS congruence criterion, we have $\triangle LPN \cong \triangle MRN$

$$\Rightarrow LN = MN$$

$[\because \text{Corresponding parts of congruent triangles are equal}]$



3. In Fig. 10.23, PQRS is a square and SRT is an equilateral triangle. Prove that

(i) $PT = QT$ (ii) $\angle TQR = 15^\circ$

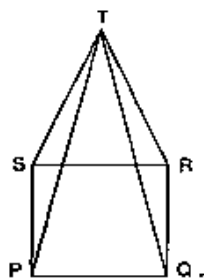


Fig. 10.23

Sol:

Given that PQRS is a square and SRT is an equilateral triangle. And given to prove that

(i) $PT = QT$ and (ii) $\angle TQR = 15^\circ$

Now, PQRS is a square

$$\Rightarrow PQ = QR = RS = SP \quad \dots\dots\dots(1)$$

$$\text{And } \angle SPQ = \angle PQR = \angle QRS = \angle RSP = 90^\circ = \text{right angle}$$

And also, SRT is an equilateral triangle.

$$\Rightarrow SR = RT = TS \quad \dots\dots\dots(2)$$

$$\text{And } \angle TSR = \angle SRT = \angle RTS = 60^\circ$$

From (1) and (2)

$$PQ = QR = SP = SR = RT = TS \quad \dots\dots\dots(3)$$

And also,

$$\angle TSR = \angle TSR + \angle RSP = 60^\circ + 90^\circ = 150^\circ$$

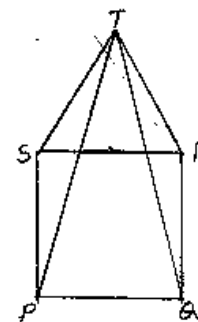
$$\angle TRQ = \angle TRS + \angle SRQ = 60^\circ + 90^\circ = 150^\circ$$

$$\Rightarrow \angle TSR = \angle TRQ = 150^\circ \quad \dots\dots\dots(4)$$

Now, in $\triangle TSR$ and $\triangle TRQ$

$$TS = TR \quad [\text{From } (3)]$$

$$\angle TSP = \angle TRQ \quad [\text{From } (4)]$$



$$SP = RQ \quad [\text{From (3)}]$$

So, by SAS congruence criterion we have

$$\Delta TSP \cong \Delta TRQ$$

$$\Rightarrow \boxed{PT = QT} \quad [\text{Corresponding parts of congruent triangles are equal}]$$

Consider ΔTQR ,

$$QR = TR \quad [\text{From (3)}]$$

$\Rightarrow \Delta TQR$ is an isosceles triangle

$$\Rightarrow \angle QTR = \angle TQR \quad [\text{angles opposite to equal sides}]$$

Now,

Sum of angles in a triangle is equal to 180°

$$\Rightarrow \angle QTR + \angle TQR + \angle TRQ = 180^\circ$$

$$\Rightarrow 2\angle TQR + 150^\circ = 180^\circ \quad [\text{From (4)}]$$

$$\Rightarrow 2\angle TQR = 180^\circ - 150^\circ$$

$$\Rightarrow 2\angle TQR = 30^\circ \Rightarrow \boxed{\angle TQR = 15^\circ}$$

\therefore Hence proved

4. Prove that the medians of an equilateral triangle are equal.

Sol:

Given to prove that the medians of an equilateral triangle are equal

Median: The line joining the vertex and midpoint of opposite side.

Now, consider an equilateral triangle ABC

Let D, E, F are midpoints of BC, CA and AB.

Then, AD, BE and CF are medians of ΔABC .

Now,

$$D \text{ is midpoint of } BC \Rightarrow BD = DC = \frac{BC}{2}$$

$$\text{Similarly, } CE = EA = \frac{AC}{2}$$

$$AF = FB = \frac{AB}{2}$$

$$\text{Since } \Delta ABC \text{ is an equilateral triangle } \Rightarrow AB = BC = CA \quad \dots\dots(1)$$

$$\Rightarrow BD = DC = CE = EA = AF = FB = \frac{BC}{2} = \frac{AC}{2} = \frac{AB}{2} \quad \dots\dots(2)$$

$$\text{And also, } \angle ABC = \angle BCA = \angle CAB = 60^\circ \quad \dots\dots(3)$$

Now, consider ΔABD and ΔBCE

$$AB = BC \quad [\text{From (1)}]$$

$$BD = CE \quad [\text{From (2)}]$$

$\angle ABD = \angle BCE$ [From (3)] [$\angle ABD$ and $\angle ABC$ and $\angle BCE$ and $\angle BCA$ are same]

So, from SAS congruence criterion, we have

$$\boxed{\triangle ABD \cong \triangle BCE}$$

$$\Rightarrow \boxed{AD = BE} \quad \dots\dots\dots(4)$$

[Corresponding parts of congruent triangles are equal]

Now, consider $\triangle BCE$ and $\triangle CAF$,

$$BC = CA \quad \text{[From (1)]}$$

$$\angle BCE = \angle CAF \quad \text{[From (3)]}$$

[$\angle BCE$ and $\angle BCA$ and $\angle CAF$ and $\angle CAB$ are same]

$$CE = AF \quad \text{[From (2)]}$$

So, from SAS congruence criterion, we have $\boxed{\triangle BCE \cong \triangle CAF}$

$$\Rightarrow \boxed{BE = CF} \quad \dots\dots\dots(5)$$

[Corresponding parts of congruent triangles are equal]

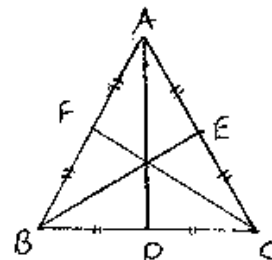
From (4) and (5), we have

$$AD = BE = CF$$

$$\Rightarrow \text{Median } AD = \text{Median } BE = \text{Median } CF$$

\therefore The medians of an equilateral triangle are equal

\therefore Hence proved



5. In a $\triangle ABC$, if $\angle A = 120^\circ$ and $AB = AC$. Find $\angle B$ and $\angle C$.

Sol:

Consider a $\triangle ABC$

Given that $\angle A = 120^\circ$ and $AB = AC$ and given to find $\angle B$ and $\angle C$

We can observe that $\triangle ABC$ is an isosceles triangle since $AB = AC$

$$\Rightarrow \boxed{\angle B = \angle C} \quad \dots\dots\dots(1) \quad \text{[Angles opposite to equal sides are equal]}$$

We know that sum of angles in a triangle is equal to 180°

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

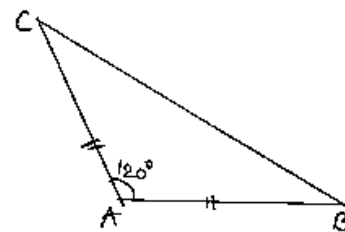
$$\Rightarrow \angle A + \angle B + \angle B = 180^\circ \quad \text{[From (1)]}$$

$$\Rightarrow 120^\circ + 2\angle B = 180^\circ$$

$$\Rightarrow 2\angle B = 180^\circ - 120^\circ$$

$$\Rightarrow 2\angle B = 60^\circ \Rightarrow \boxed{\angle B = 30^\circ}$$

$$\Rightarrow \angle C = \angle B = 30^\circ$$



6. In a $\triangle ABC$, if $AB = AC$ and $\angle B = 70^\circ$, find $\angle A$.

Sol:

Consider $\triangle ABC$, we have $\angle B = 70^\circ$ and $AB = AC$

Since, $AB = AC$ $\triangle ABC$ is an isosceles triangle

$$\Rightarrow \angle B = \angle C \quad [\text{Angles opposite to equal sides are equal}]$$

$$\Rightarrow \angle B = \angle C = 70^\circ$$

And also,

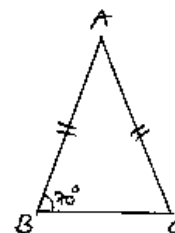
Sum of angles in a triangle = 180°

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A + 70^\circ + 70^\circ = 180^\circ$$

$$\Rightarrow \angle A + 140^\circ = 180^\circ$$

$$\Rightarrow \angle A = 180^\circ - 140^\circ \Rightarrow \boxed{\angle A = 40^\circ}$$



7. The vertical angle of an isosceles triangle is 100° . Find its base angles.

Sol:

Consider an isosceles $\triangle ABC$ such that $AB = AC$

Given that vertical angle A is 100° . Given to find the base angles

Since $\triangle ABC$ is isosceles

$$\angle B = \angle C \quad [\text{Angles opposite to equal sides are equal}]$$

And also,

Sum of the interior angles of a triangle = 180°

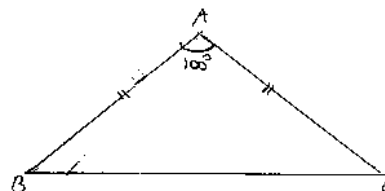
$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 100^\circ + \angle B + \angle B = 180^\circ$$

$$\Rightarrow 2\angle B = 180^\circ - 100^\circ \Rightarrow 2\angle B = 80^\circ$$

$$\Rightarrow \boxed{\angle B = 40^\circ}$$

$$\therefore \angle B = \angle C = 40^\circ$$



8. In Fig. 10.24, $AB = AC$ and $\angle ACD = 105^\circ$, find $\angle BAC$.

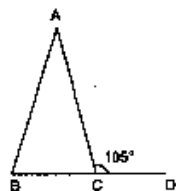


Fig. 10.24

Sol:

Consider the given figure

We have,

$$AB = AC \text{ and } \angle ACD = 105^\circ$$

Since,

$$\angle BCD = 180^\circ = \text{Straight angle}$$

$$\Rightarrow \angle BCA + \angle ACD = 180^\circ$$

$$\Rightarrow \angle BCA + 105^\circ = 180^\circ$$

$$\Rightarrow \angle BCA = 180^\circ - 105^\circ \Rightarrow \boxed{\angle BCA = 75^\circ} \quad \dots\dots(1)$$

And also,

$$\triangle ABC \text{ is an isosceles triangle} \quad [\because AB = AC]$$

$$\Rightarrow \angle ABC = \angle ACB \quad [\text{Angles opposite to equal sides are equal}]$$

From (1), we have

$$\angle ACB = 75^\circ \Rightarrow \angle ABC = \angle ACB = 75^\circ$$

And also,

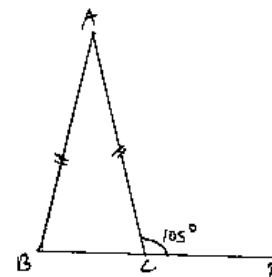
$$\text{Sum of interior angles of a triangle} = 180^\circ$$

$$\Rightarrow \angle ABC + \angle BCA + \angle CAB = 180^\circ$$

$$\Rightarrow 75^\circ + 75^\circ + \angle CAB = 180^\circ$$

$$\Rightarrow 150^\circ + \angle BAC = 180^\circ \Rightarrow \angle BAC = 180^\circ - 150^\circ = 30^\circ$$

$$\therefore \boxed{\angle BAC = 30^\circ}$$



9. Find the measure of each exterior angle of an equilateral triangle.

Sol:

Given to find the measure of each exterior angle of an equilateral triangle consider an equilateral triangle ABC.

We know that for an equilateral triangle

$$AB = BC = CA \text{ and } \angle ABC = \angle BCA = \angle CAB = \frac{180^\circ}{3} = 60^\circ \quad \dots\dots(1)$$

Now,

Extend side BC to D, CA to E and AB to F.

Here

BCD is a straight line segment

$$\Rightarrow \angle BCD = \text{Straight angle} = 180^\circ$$

$$\angle BCA + \angle ACD = 180^\circ$$

$$\Rightarrow 60^\circ + \angle ACD = 180^\circ \quad [\text{From (1)}]$$

$$\Rightarrow \boxed{\angle ACD = 120^\circ}$$

Similarly, we can find $\angle FAB$ and $\angle FBC$ also as 120° because ABC is an equilateral triangle

$$\therefore \angle ACD = \angle EAB = \angle FBC = 120^\circ$$

Hence, the measure of each exterior angle of an equilateral triangle is 120°

10. If the base of an isosceles triangle is produced on both sides, prove that the exterior angles so formed are equal to each other.

Sol:

ED is a straight line segment and B and C are points on it.

$$\Rightarrow \angle EBC = \angle BCD = \text{straight angle} = 180^\circ$$

$$\Rightarrow \angle EBA + \angle ABC = \angle ACB + \angle ACD$$

$$\Rightarrow \angle EBA = \angle ACD + \angle ACB - \angle ABC$$

$$\Rightarrow \angle EBA = \angle ACD \quad \left[\text{From (1) } \angle ABC = \angle ACB \right]$$

$$\Rightarrow \boxed{\angle ABE = \angle ACD}$$

\therefore Hence proved

11. In Fig. 10.25, $AB = AC$ and $DB = DC$, find the ratio $\angle ABD : \angle ACD$.

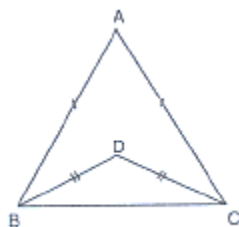


Fig. 10.25

Sol:

Consider the figure

Given

$AB = AC$, $DB = DC$ and given to find the ratio

$$\angle ABD = \angle ACD$$

Now, $\triangle ABC$ and $\triangle DBC$ are isosceles triangles since $AB = AC$ and $DB = DC$ respectively

$$\Rightarrow \angle ABC = \angle ACB \text{ and } \angle DBC = \angle DCB \quad [\because \text{angles opposite to equal sides are equal}]$$

Now consider,

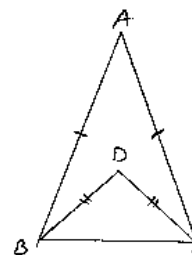
$$\angle ABD : \angle ACD$$

$$\Rightarrow (\angle ABC - \angle DBC) : (\angle ACB - \angle DCB)$$

$$\Rightarrow (\angle ABC - \angle DBC) : (\angle ABC - \angle DBC) \quad [\because \angle ABC = \angle ACB \text{ and } \angle DBC = \angle DCB]$$

$$\Rightarrow 1:1$$

$$\therefore \angle ABD : \angle ACD = 1:1$$



12. Determine the measure of each of the equal angles of a right-angled isosceles triangle.

OR

ABC is a right-angled triangle in which $\angle A = 90^\circ$ and $AB = AC$. Find $\angle B$ and $\angle C$.

Sol:

Given to determine the measure of each of the equal angles of right – angled isosceles triangle

Consider on a right – angled isosceles triangle ABC such that

$$\angle A = 90^\circ \text{ and } AB = AC$$

$$\text{Since, } AB = AC \Rightarrow \angle C = \angle B \quad \dots\dots\dots(1)$$

[Angles opposite to equal sides are equal]

Now,

$$\text{Sum of angles in a triangle} = 180^\circ$$

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

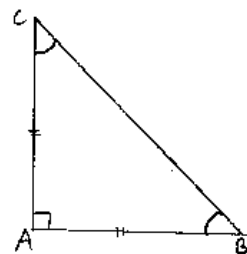
$$\Rightarrow 90^\circ + \angle B + \angle B = 180^\circ \quad [\because \angle A = 90^\circ \text{ and } \angle B = \angle C]$$

$$\Rightarrow 2\angle B = 90^\circ$$

$$\Rightarrow \boxed{\angle B = 45^\circ} \Rightarrow \angle C = 45^\circ$$

$$\therefore \angle B = \angle C = 45^\circ$$

Hence, the measure of each of the equal angles of a right-angled isosceles triangle is 45° .



13. AB is a line segment. P and Q are points on opposite sides of AB such that each of them is equidistant from the points A and B (See Fig. 10.26). Show that the line PQ is perpendicular bisector of AB.

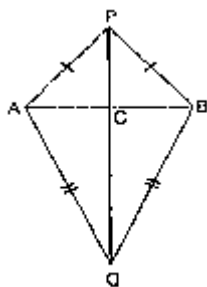


Fig. 10.26

Sol:

Consider the figure,

We have

AB is a line segment and P, Q are points on opposite sides of AB such that

$$AP = BP \quad \dots\dots\dots(1)$$

$$AQ = BQ \quad \dots\dots\dots(2)$$

We have to prove that PQ is perpendicular bisector of AB.

Now consider $\triangle PAQ$ and $\triangle PBQ$,

$$\text{We have } AP = BP \quad [\because \text{From (1)}]$$

$$AQ = BQ \quad [\because \text{From (2)}]$$

$$\text{And } PQ = PQ \quad [\text{Common side}]$$

$$\Rightarrow \boxed{\triangle PAQ \cong \triangle PBQ} \quad \dots\dots\dots(3) \quad [\text{From SSS congruence}]$$

Now, we can observe that $\triangle APB$ and $\triangle ABQ$ are isosceles triangles. (From 1 and 2)

$$\Rightarrow \angle PAB = \angle PBA \text{ and } \angle QAB = \angle QBA$$

Now consider $\triangle PAC$ and $\triangle PBC$,

C is the point of intersection of AB and PQ.

$$PA = PB \quad [\text{From (1)}]$$

$$\angle APC = \angle BPC \quad [\text{From (3)}]$$

$$PC = PC \quad [\text{Common side}]$$

So, from SAS congruency of triangle $\triangle PAC \cong \triangle PBC$

$$\Rightarrow AC = CB \text{ and } \angle PCA = \angle PCB \quad \dots\dots(4)$$

[\therefore Corresponding parts of congruent triangles are equal]

And also, ACB is line segment

$$\Rightarrow \angle ACP + \angle BCP = 180^\circ$$

But $\angle ACP = \angle PCB$

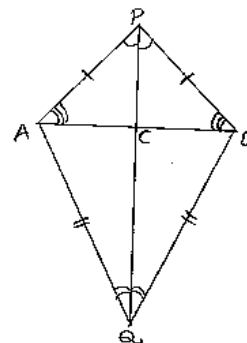
$$\Rightarrow \boxed{\angle ACP = \angle PCB = 90^\circ} \quad \dots\dots(5)$$

We have $AC = CB \Rightarrow C$ is the midpoint of AB

From (4) and (5)

We can conclude that PC is the perpendicular bisector of AB

Since C is a point on the line PQ, we can say that PQ is the perpendicular bisector of AB.



Exercise -10.2

1. In Fig. 10.40, it is given that $RT = TS$, $\angle 1 = 2\angle 2$ and $\angle 4 = 2\angle 3$. Prove that $\triangle RBT \cong \triangle SAT$.

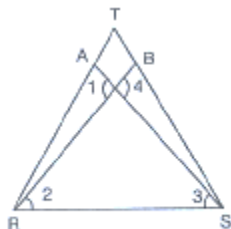


Fig. 10.40

Sol:

In the figure given that

$$RT = TS \quad \dots\dots(1)$$

$$\angle 1 = 2\angle 2 \quad \dots\dots(2)$$

$$\text{And } \angle 4 = 2\angle 3 \quad \dots\dots(3)$$

And given to prove $\triangle RBT \cong \triangle SAT$

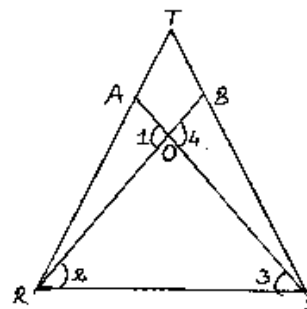
Let the point of intersection of RB and SA be denoted by O

Since RB and SA intersect at O.

$$\therefore \angle AOR = \angle BOS \quad [\text{Vertically opposite angles}]$$

$$\Rightarrow \angle 1 = \angle 4$$

$$\Rightarrow 2\angle 2 = 2\angle 3 \quad [\text{From (2) and (3)}]$$



$$\Rightarrow \boxed{\angle 2 = \angle 3} \quad \dots\dots(4)$$

Now we have $RT = TS$ in $\triangle TRS$

$\Rightarrow \triangle TRS$ is an isosceles triangle

$$\therefore \boxed{\angle TRS = \angle TSR} \quad \dots\dots(5) \quad [\text{Angles opposite to equal sides are equal}]$$

But we have

$$\angle TRS = \angle TRB + \angle 2 \quad \dots\dots(6)$$

$$\text{And } \angle TSR = \angle TSA + \angle 3 \quad \dots\dots(7)$$

Putting (6) and (7) in (5) we get

$$\angle TRB + \angle 2 = \angle TSA + \angle 3$$

$$\Rightarrow \boxed{\angle TRB = \angle TSA} \quad [\because \text{From (4)}]$$

Now consider $\triangle RBT$ and $\triangle SAT$

$$RT = ST \quad [\text{From (1)}]$$

$$\angle TRB = \angle TSA \quad [\text{From (4)}]$$

$$\angle RTB = \angle STA \quad [\text{Common angle}]$$

From ASA criterion of congruence, we have $\boxed{\triangle RBT \cong \triangle SAT}$

2. Two lines AB and CD intersect at O such that BC is equal and parallel to AD. Prove that the lines AB and CD bisect at O.

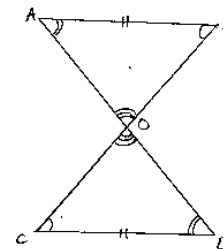
Sol:

Given that lines AB and CD intersect at O

Such that $BC \parallel AD$ and $BC = AD \quad \dots\dots(1)$

We have to prove that AB and CD bisect at O.

To prove this first we have to prove that $\triangle AOD \cong \triangle BOC$



3. BD and CE are bisectors of $\angle B$ and $\angle C$ of an isosceles $\triangle ABC$ with $AB = AC$. Prove that $BD = CE$.

Sol:

Given that $\triangle ABC$ is isosceles with $AB = AC$ and BD and CE are bisectors of $\angle B$ and $\angle C$

We have to prove $BD = CE$

$$\text{Since } AB = AC \Rightarrow \angle ABC = \angle ACB \quad \dots\dots(1)$$

$[\because \text{Angles opposite to equal sides are equal}]$

Since BD and CE are bisectors of $\angle B$ and $\angle C$

$$\Rightarrow \angle ABD = \angle DBC = \angle BCE = \angle ECA = \frac{\angle B}{2} = \frac{\angle C}{2} \quad \dots\dots(2)$$

Now,

Consider $\triangle EBC$ and $\triangle DCB$

$$\angle EBC = \angle DCB \quad [\because \angle B = \angle C] \text{ from (1)}$$

$$BC = BC \quad [\text{Common side}]$$

$$\angle BCE = \angle CBD \quad [\because \text{From (2)}]$$

So, by ASA congruence criterion, we have $\triangle EBC \cong \triangle DCB$

Now,

$$CE = BD \quad [\because \text{Corresponding parts of congruent triangles are equal}]$$

or $\boxed{BD = CE}$

\therefore Hence proved

Since $AD \parallel BC$ and transversal AB cuts at A and B respectively

$$\therefore \boxed{\angle DAO = \angle OBC} \quad \dots\dots\dots(2) \text{ [alternate angle]}$$

And similarly $AD \parallel BC$ and transversal DC cuts at D and C respectively

$$\therefore \boxed{\angle ADO = \angle OCB} \quad \dots\dots\dots(3) \text{ [alternate angle]}$$

Since AB and CD intersect at O.

$$\therefore \angle AOD = \angle BOC \quad [\text{Vertically opposite angles}]$$

Now consider $\triangle AOD$ and $\triangle BOC$

$$\angle DAO = \angle OBC \quad [\because \text{From (2)}]$$

$$AD = BC \quad [\because \text{From (1)}]$$

$$\text{And } \angle ADO = \angle OCB \quad [\text{From (3)}]$$

So, by ASA congruence criterion, we have

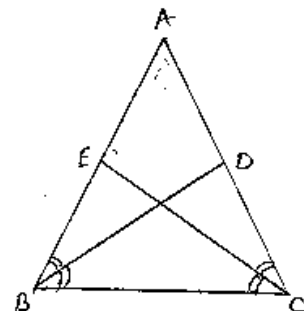
$$\boxed{\triangle AOD \cong \triangle BOC}$$

Now,

$$AO = OB \text{ and } DO = OC \quad [\because \text{Corresponding parts of congruent triangles are equal}]$$

\Rightarrow Lines AB and CD bisect at O.

\therefore Hence proved



Exercise -10.3

1. In two right triangles one side and an acute angle of one are equal to the corresponding side and angle of the other. Prove that the triangles are congruent.

Sol:

Given that, in two right triangles one side and acute angle of one are equal to the corresponding side and angles of the other.

We have to prove that the triangles are congruent.

Let us consider two right triangles such that

$$\angle B = \angle E = 90^\circ \quad \dots\dots(1)$$

$$AB = DE \quad \dots\dots(2)$$

$$\angle C = \angle F \quad \dots\dots(3)$$

Now observe the two triangles ABC and DEF

$$\angle C = \angle F \quad [\text{From (3)}]$$

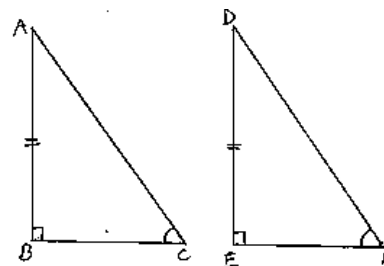
$$\angle B = \angle E \quad [\text{From (1)}]$$

$$\text{and } AB = DE \quad [\text{From (2)}]$$

So, by AAS congruence criterion, we have $\triangle ABC \cong \triangle DEF$

\therefore The two triangles are congruent

Hence proved



2. If the bisector of the exterior vertical angle of a triangle be parallel to the base. Show that the triangle is isosceles.

Sol:

Given that the bisector of the exterior vertical angle of a triangle is parallel to the base and we have to prove that the triangle is isosceles

Let ABC be a triangle such that AD is the angular bisector of exterior vertical angle EAC and $AD \parallel BC$

$$\text{Let } \angle EAD = (1), \angle DAC = (2), \angle ABC = (3) \text{ and } \angle ACB = (4)$$

We have,

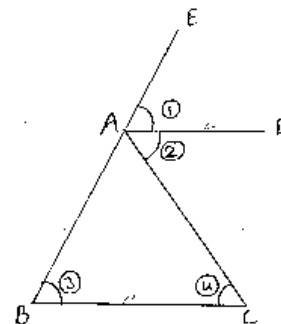
$$(1) = (2) \quad [\because AD \text{ is bisector of } \angle EAC]$$

$$(1) = (3) \quad [\text{Corresponding angles}]$$

$$\text{and } (2) = (4) \quad [\text{alternative angle}]$$

$$\Rightarrow (3) = (4) \Rightarrow \boxed{AB = AC}$$

Since, in $\triangle ABC$, two sides AB and AC are equal we can say that $\triangle ABC$ is isosceles



3. In an isosceles triangle, if the vertex angle is twice the sum of the base angles, calculate the angles of the triangle.

Sol:

Let $\triangle ABC$ be isosceles such that $AB = AC$.

$$\Rightarrow \angle B = \angle C$$

Given that vertex angle A is twice the sum of the base angles B and C.

$$\text{i.e., } \angle A = 2(\angle B + \angle C)$$

$$\Rightarrow \angle A = 2(\angle B + \angle B) \quad [\because \angle B = \angle C]$$

$$\Rightarrow \angle A = 2(2\angle B)$$

$$\Rightarrow \boxed{\angle A = 4\angle B}$$

Now,

We know that sum of angles in a triangle $= 180^\circ$

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 4\angle B + \angle B + \angle B = 180^\circ \quad [\because \angle A = 4\angle B \text{ and } \angle B = \angle C]$$

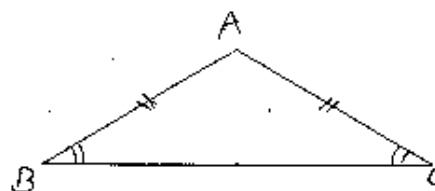
$$\Rightarrow 6\angle B = 180^\circ$$

$$\Rightarrow \angle B = \frac{180^\circ}{6} = 30^\circ \quad \therefore \boxed{\angle B = 30^\circ}$$

Since, $\angle B = \angle C \Rightarrow \angle B = \angle C = 30^\circ$

And $\angle A = 4\angle B \Rightarrow \angle A = 4 \times 30^\circ = 120^\circ$

\therefore Angles of the given triangle are $120^\circ, 30^\circ, 30^\circ$.



4. PQR is a triangle in which $PQ = PR$ and S is any point on the side PQ. Through S, a line is drawn parallel to QR and intersecting PR at T. Prove that $PS = PT$.

Sol:

Given that $\triangle PQR$ is a triangle such that $PQ = PR$ and S is any point on the side PQ and $ST \parallel QR$.

We have to prove $PS = PT$

Since, $PQ = PR \Rightarrow \triangle PQR$ is isosceles

$$\Rightarrow \angle Q = \angle R \text{ (or) } \angle PQR = \angle PRQ$$

Now,

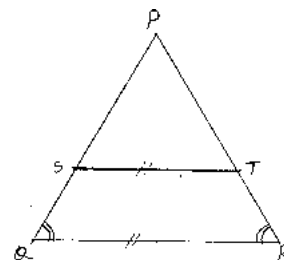
$$\angle PST = \angle PQR \text{ and } \angle PTS = \angle PRQ \quad [\text{Corresponding angles as } ST \parallel QR]$$

$$\text{Since, } \angle PQR = \angle PRQ \Rightarrow \boxed{\angle PST = \angle PTS}$$

Now, In $\triangle PST$, $\angle PST = \angle PTS$

$\Rightarrow \triangle PST$ is an isosceles triangle

$$\Rightarrow \boxed{PS = PT}$$



5. In a $\triangle ABC$, it is given that $AB = AC$ and the bisectors of $\angle B$ and $\angle C$ intersect at O. If M is a point on BO produced, prove that $\angle MOC = \angle ABC$.

Sol:

Given that in $\triangle ABC$,

$AB = AC$ and the bisector of $\angle B$ and $\angle C$ intersect at O and M is a point on BO produced

We have to prove $\angle MOC = \angle ABC$

Since,

$AB = AC \Rightarrow \triangle ABC$ is isosceles $\Rightarrow \angle B = \angle C$ (or) $\angle ABC = \angle ACB$

Now,

BO and CO are bisectors of $\angle ABC$ and $\angle ACB$ respectively

$$\Rightarrow \angle ABO = \angle OBC = \angle ACO = \angle OCB = \frac{1}{2} \angle ABC = \frac{1}{2} \angle ACB \quad \dots\dots\dots(1)$$

We have, in $\triangle OBC$

$$\angle OBC + \angle OCB + \angle BOC = 180^\circ \quad \dots\dots\dots(2)$$

And also

$$\angle BOC + \angle COM = 180^\circ \quad \dots\dots\dots(3) \text{ [Straight angle]}$$

Equating (2) and (3)

$$\Rightarrow \angle OBC + \angle OCB + \cancel{\angle BOC} = \cancel{\angle BOC} + \angle MOC$$

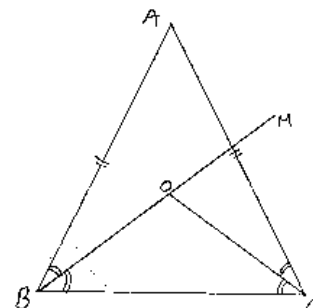
$$\Rightarrow \angle OBC + \angle OCB = \angle MOC \quad [\because \text{From (1)}]$$

$$\Rightarrow 2\angle OBC = \angle MOC$$

$$\Rightarrow 2\left(\frac{1}{2} \angle ABC\right) = \angle MOC \quad [\because \text{From (1)}]$$

$$\Rightarrow \angle ABC = \angle MOC$$

$$\therefore \boxed{\angle MOC = \angle ABC}$$



6. P is a point on the bisector of an angle $\angle ABC$. If the line through P parallel to AB meets BC at Q, prove that triangle BPQ is isosceles.

Sol:

Given that P is a point on the bisector of an angle $\angle ABC$, and $PQ \parallel AB$.

We have to prove that $\triangle BPQ$ is isosceles

Since,

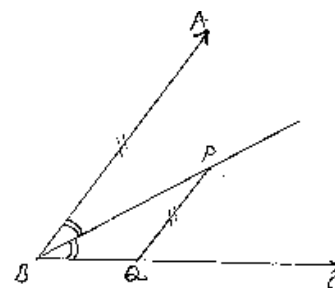
$$BP \text{ is bisector of } \angle ABC \Rightarrow \angle ABP = \angle PBC \quad \dots\dots\dots(1)$$

Now,

$$PQ \parallel AB$$

$$\Rightarrow \angle BPQ = \angle ABP \quad \dots\dots\dots(2)$$

[alternative angles]



From (1) and (2), we get

$$\angle BPQ = \angle PBC \text{ (or) } \angle BPQ = \angle PBQ$$

Now,

In $\triangle BPQ$,

$$\angle BPQ = \angle PBQ$$

$\Rightarrow \triangle BPQ$ is an isosceles triangle.

\therefore Hence proved

7. Prove that each angle of an equilateral triangle is 60° .

Sol:

Given to prove that each angle of an equilateral triangle is 60°

Let us consider an equilateral triangle ABC

Such that $AB = BC = CA$

Now,

$$AB = BC \Rightarrow \angle A = \angle C \quad \dots\dots\dots(1) \text{ [Opposite angles to equal sides are equal]}$$

$$\text{and } BC = AC \Rightarrow \angle B = \angle A \quad \dots\dots\dots(2)$$

From (1) and (2), we get

$$\angle A = \angle B = \angle C \quad \dots\dots\dots(3)$$

We know that

Sum of angles in a triangle = 180°

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

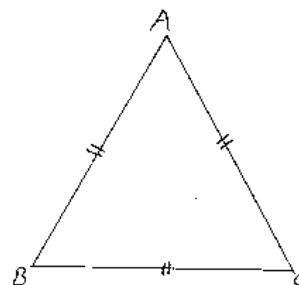
$$\Rightarrow \angle A + \angle A + \angle A = 180^\circ \quad [\because \text{From (3)}]$$

$$\Rightarrow 3\angle A = 180^\circ$$

$$\Rightarrow \angle A = \frac{180^\circ}{3} = 60^\circ$$

$$\therefore \angle A = \angle B = \angle C = 60^\circ$$

Hence, each angle of an equilateral triangle is 60° .



8. Angles A, B, C of a triangle ABC are equal to each other. Prove that $\triangle ABC$ is equilateral.

Sol:

Given that angles A, B, C of a triangle ABC equal to each other.

We have to prove that $\triangle ABC$ is equilateral

We have, $\angle A = \angle B = \angle C$

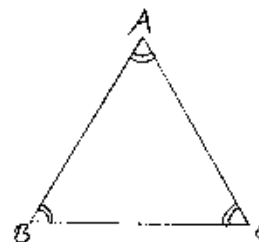
Now,

$$\angle A = \angle B \Rightarrow BC = AC$$

[Opposite sides to equal angles are equal]

$$\text{and } \angle B = \angle C \Rightarrow AC = AB$$

From the above we get



$$\boxed{AB = BC = AC}$$

$\Rightarrow \triangle ABC$ is equilateral

9. ABC is a triangle in which $\angle B = 2\angle C$. D is a point on BC such that AD bisects $\angle BAC$ and $AB = CD$. Prove that $\angle BAC = 72^\circ$.

Sol:

Given that in $\triangle ABC$, $\angle B = 2\angle C$ and D is a point on BC such that AD bisects $\angle BAC$ and $AB = CD$.

We have to prove that $\angle BAC = 72^\circ$

Now, draw the angular bisector of $\angle ABC$, which meets AC in P . join PD

Let $C = \angle ACB = y \Rightarrow \angle B = \angle ABC = 2\angle C = 2y$ and also

Let $\angle BAD = \angle DAC \Rightarrow \angle BAC = 2x$ [$\because AD$ is the bisector of $\angle BAC$]

Now, in $\triangle BPC$,

$$\angle CBP = y \quad [\because BP \text{ is the bisector of } \angle ABC]$$

$$\angle PCB = y$$

$$\Rightarrow \angle CBP = \angle PCB = y \quad \therefore \boxed{PC = BP}$$

Consider, $\triangle ABP$ and $\triangle DCP$, we have

$$\angle ABP = \angle DCP = y$$

$$AB = DC \quad [\text{Given}]$$

$$\text{And } PC = BP \quad [\text{From above}]$$

So, by SAS congruence criterion, we have $\boxed{\triangle ABP \cong \triangle DCP}$

Now,

$$\angle BAP = \angle CDP \text{ and } AP = DP \quad [\text{Corresponding parts of congruent triangles are equal}]$$

$$\Rightarrow \angle BAP = \angle CDP = 2x$$

Consider, $\triangle APD$,

$$\text{We have } AP = DP \Rightarrow \angle ADP = \angle DAP$$

$$\text{But } \angle DAP = x \Rightarrow \angle ADP = \angle DAP = x$$

Now

In $\triangle ABD$,

$$\angle ABD + \angle BAD + \angle ADB = 180^\circ$$

$$\text{And also } \angle ADB + \angle ADC = 180^\circ \quad [\text{Straight angle}]$$

From the above two equations, we get



$$\angle ABD + \angle BAD + \angle ADB = \angle ADB + \angle ADC$$

$$\Rightarrow 2y + x = \angle ADP + \angle PDC$$

$$\Rightarrow 2y + x = x + 2x$$

$$\Rightarrow 2y = 2x$$

$$\Rightarrow y = x \text{ (or) } \boxed{x = y}$$

We know,

Sum of angles in a triangle = 180°

So, in $\triangle ABC$,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 2x + 2y + y = 180^\circ \quad [\because \angle A = 2x, \angle B = 2y, \angle C = y]$$

$$\Rightarrow 2(y) + 3y = 180^\circ \quad [\because x = y]$$

$$\Rightarrow 5y = 180^\circ$$

$$\Rightarrow y = \frac{180^\circ}{5} = 36^\circ \quad \therefore \boxed{x = y = 36^\circ}$$

Now,

$$\angle A = \angle BAC = 2x = 2 \times 36^\circ = 72^\circ$$

$$\therefore \boxed{\angle BAC = 72^\circ}$$

10. ABC is a right angled triangle in which $\angle A = 90^\circ$ and $AB = AC$. Find $\angle B$ and $\angle C$.

Sol:

Given that ABC is a right angled triangle such that $\angle A = 90^\circ$ and $AB = AC$

Since,

$AB = AC \Rightarrow \triangle ABC$ is also isosceles

\therefore We can say that $\triangle ABC$ is right angled isosceles triangle

$$\Rightarrow \angle C = \angle B \text{ and } \angle A = 90^\circ \quad \dots\dots\dots(1)$$

Now, we have

Sum of angles in a triangle = 180°

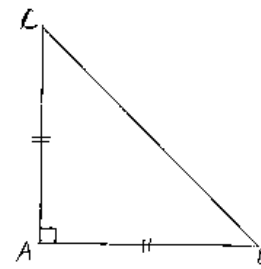
$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 90^\circ + \angle B + \angle B = 180^\circ \quad [\because \text{From (1)}]$$

$$\Rightarrow 2\angle B = 180^\circ - 90^\circ$$

$$\Rightarrow \angle B = \frac{90^\circ}{2} = 45^\circ$$

$$\therefore \boxed{\angle B = \angle C = 45^\circ}$$



Exercise -10.4

1. In Fig. 10.92, it is given that $AB = CD$ and $AD = BC$. Prove that $\triangle ADC \cong \triangle CBA$.

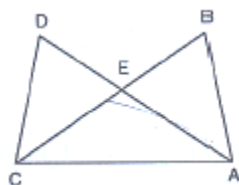


Fig. 10.92

Sol:

Given that in the figure $AB = CD$ and $AD = BC$.

We have to prove

$$\triangle ADC \cong \triangle CBA$$

Now,

Consider $\triangle ADC$ and $\triangle CBA$,

We have

$$AB = CD \quad [\text{Given}]$$

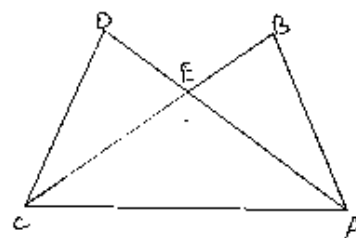
$$BC = AD \quad [\text{Given}]$$

$$\text{And } AC = AC \quad [\text{Common side}]$$

So, by SSS congruence criterion, we have

$$\boxed{\triangle ADC \cong \triangle CBA}$$

\therefore Hence proved



2. In a $\triangle PQR$, if $PQ = QR$ and L , M and N are the mid-points of the sides PQ , QR and RP respectively. Prove that $LN = MN$.

Sol:

Given that in $\triangle PQR$, $PQ = QR$ and L , M and N are mid-points of PQ , QR and RP respectively

We have to prove $LN = MN$.

Join L and M , M and N , N and L

We have

$$PL = LQ, QM = MR \text{ and } RN = NP$$

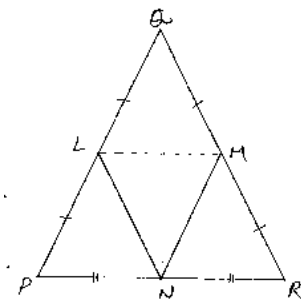
[$\because L, M$ and N are mid-points of PQ, QR and RP respectively]

And also

$$PQ = QR \Rightarrow PL = LQ = QM = MR = \frac{PQ}{2} = \frac{QR}{2}$$

theorem, we have

$$MN \parallel PQ \text{ and } MN = \frac{1}{2}PQ \Rightarrow \boxed{MN = PL = LQ}$$



.....(1) Using mid-point

.....(2)

Similarly, we have

$$LN \parallel QR \text{ and } LN = \frac{1}{2}QR \Rightarrow \boxed{LN = QM = MR} \quad \dots\dots\dots(3)$$

From equation (1), (2) and (3), we have

$$PL = LQ = QM = MR = MN = LN$$

$$\therefore \boxed{LN = MN}$$

Exercise -10.5

1. ABC is a triangle and D is the mid-point of BC. The perpendiculars from D to AB and AC are equal. Prove that the triangle is isosceles.

Sol:

Given that, in two right triangles one side and acute angle of one are equal to the corresponding side and angle of the other

We have to prove that the triangles are congruent

Let us consider two right triangles such that

$$\angle B = \angle E = 90^\circ \quad \dots\dots\dots(1)$$

$$AB = DE \quad \dots\dots\dots(2)$$

$$\angle C = \angle F \quad \dots\dots\dots(3)$$

Now observe the two triangles ABC and DEF

$$\angle C = \angle F \quad \left[\text{From (3)} \right]$$

$$\angle B = \angle E \quad \left[\text{From (1)} \right]$$

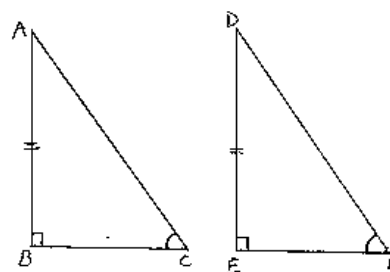
$$\text{and } AB = DE \quad \left[\text{From (2)} \right]$$

So, by AAS congruence criterion, we have

$$\boxed{\triangle ABC \cong \triangle DEF}$$

\therefore The two triangles are congruent

Hence proved



2. ABC is a triangle in which BE and CF are, respectively, the perpendiculars to the sides AC and AB. If BE = CF, prove that $\triangle ABC$ is isosceles.

Sol:

Given that ABC is a triangle in which BE and CF are perpendicular to the sides AC and AB respectively such that $BE = CF$.

We have to prove that $\triangle ABC$ is isosceles

Now, consider $\triangle BCF$ and $\triangle CBE$,

We have

$$\angle BFC = \angle CEB = 90^\circ \quad [\text{Given}]$$

$$BC = CB \quad [\text{Common side}]$$

$$\text{And } CF = BE \quad [\text{Given}]$$

So, by RHS congruence criterion, we have $\triangle BFC \cong \triangle CEB$

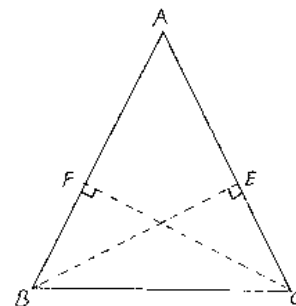
Now,

$$\angle FBC = \angle ECB \quad [\because \text{Incongruent triangles corresponding parts are equal}]$$

$$\Rightarrow \angle ABC = \angle ACB$$

$$\Rightarrow AC = AB \quad [\because \text{Opposite sides to equal angles are equal in a triangle}]$$

$\therefore \triangle ABC$ is isosceles



3. If perpendiculars from any point within an angle on its arms are congruent, prove that it lies on the bisector of that angle.

Sol:

Given that, if perpendicular from any point within, an angle on its arms is congruent, prove that it lies on the bisector of that angle

Now,

Let us consider an angle ABC and let BP be one of the arm within the angle

Draw perpendicular PN and PM on the arms BC and BA such that they meet BC and BA in N and M respectively.

Now, in $\triangle BPM$ and $\triangle BPN$

$$\text{We have } \angle BMP = \angle BNP = 90^\circ \quad [\text{given}]$$

$$BP = BP \quad [\text{Common side}]$$

$$\text{And } MP = NP \quad [\text{given}]$$

So, by RHS congruence criterion, we have

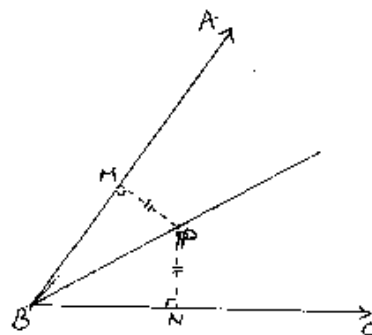
$$\triangle BPM \cong \triangle BPN$$

Now,

$$\angle MBP = \angle NBP \quad [\because \text{Corresponding parts of congruent triangles are equal}]$$

$$\Rightarrow BP \text{ is the angular bisector of } \angle ABC.$$

\therefore Hence proved



4. In Fig. 10.99, $AD \perp CD$ and $CB \perp CD$. If $AQ = BP$ and $DP = CQ$, prove that $\angle DAQ = \angle CBP$.

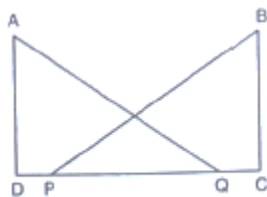


Fig. 10.99

Sol:

Given that, in the figure $AD \perp CD$ and $CB \perp CD$ and $AQ = BP, DP = CQ$

We have to prove that $\angle DAQ = \angle CBP$

Given that $DP = QC$

Add PQ on both sides

Given that $DP = QC$

Add PQ on both sides

$$\Rightarrow DP + PQ = PQ + QC$$

$$\Rightarrow \boxed{DQ = PC} \quad \dots\dots\dots(1)$$

Now, consider triangle DAQ and CBP ,

We have

$$\angle ADQ = \angle BCP = 90^\circ \quad [\text{given}]$$

$$AQ = BP \quad [\text{given}]$$

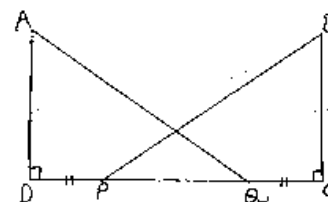
$$\text{And } DQ = PC \quad [\text{From (1)}]$$

So, by RHS congruence criterion, we have $\boxed{\triangle DAQ \cong \triangle CBP}$

Now,

$$\boxed{\angle DAQ = \angle CBP} \quad [\because \text{Corresponding parts of congruent triangles are equal}]$$

\therefore Hence proved



5. ABCD is a square, X and Y are points on sides AD and BC respectively such that $AY = BX$. Prove that $BY = AX$ and $\angle BAY = \angle ABX$.

Sol:

Given that ABCD is a square, X and Y are points on sides AD and BC respectively such that $AY = BX$.

We have to prove $BY = AX$ and $\angle BAY = \angle ABX$

Join B and X, A and Y.

Since, ABCD is a square $\Rightarrow \angle DAB = \angle CBA = 90^\circ$

$$\Rightarrow \boxed{\angle XAB = \angle YBA = 90^\circ} \quad \dots\dots(1)$$

Now, consider triangle XAB and YBA

We have

$$\angle XAB = \angle YBA = 90^\circ \quad [\text{From (1)}]$$

$$BX = AY \quad [\text{given}]$$

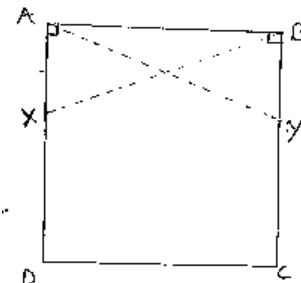
$$\text{And } AB = BA \quad [\text{Common side}]$$

So, by RHS congruence criterion, we have $\boxed{\triangle XAB \cong \triangle YBA}$

Now, we know that corresponding parts of congruent triangles are equal.

$$\therefore \boxed{BY = AX \text{ and } \angle BAY = \angle ABX}$$

\therefore Hence proved



6. Which of the following statements are true (T) and which are false (F):

- (i) Sides opposite to equal angles of a triangle may be unequal.
- (ii) Angles opposite to equal sides of a triangle are equal
- (iii) The measure of each angle of an equilateral triangle is 60°
- (iv) If the altitude from one vertex of a triangle bisects the opposite side, then the triangle may be isosceles.
- (v) The bisectors of two equal angles of a triangle are equal.
- (vi) If the bisector of the vertical angle of a triangle bisects the base, then the triangle may be isosceles.
- (vii) The two altitudes corresponding to two equal sides of a triangle need not be equal.
- (viii) If any two sides of a right triangle are respectively equal to two sides of other right triangle, then the two triangles are congruent.
- (ix) Two right triangles are congruent if hypotenuse and a side of one triangle are respectively equal to the hypotenuse and a side of the other triangle.

Sol:

- (i) False (F)
Reason: Sides opposite to equal angles of a triangle are equal
- (ii) True (T)
Reason: Since the sides are equal, the corresponding opposite angles must be equal
- (iii) True (T)
Reason: Since all the three angles of an equilateral triangles are equal and sum of the three angles is 180° , each angle will be equal to $\frac{180^\circ}{3} \Rightarrow 60^\circ$
- (iv) False (F)
Reason: Here the altitude from the vertex is also the perpendicular bisector of the opposite side.
 \Rightarrow The triangle must be isosceles and may be an equilateral triangle.
- (v) True (T)

Reason: Since it is an isosceles triangle, the lengths of bisectors of the two equal angles are equal

(vi) False (F)

Reason: The angular bisector of the vertex angle is also a median

\Rightarrow The triangle must be an isosceles and also may be an equilateral triangle.

(vii) False (F)

Reason: Since two sides are equal, the triangle is an isosceles triangle.

\Rightarrow The two altitudes corresponding to two equal sides must be equal.

(viii) False (F)

Reason: The two right triangles may or may not be congruent

(ix) True (T)

Reason: According to RHS congruence criterion the given statement is true.

7. Fill the blanks in the following so that each of the following statements is true.

(i) Sides opposite to equal angles of a triangle are

(ii) Angle opposite to equal sides of a triangle are

(iii) In an equilateral triangle all angles are

(iv) In a $\triangle ABC$ if $\angle A = \angle C$, then $AB = \dots$

(v) If altitudes CE and BF of a triangle ABC are equal, then $AB = \dots$

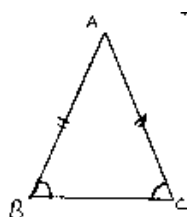
(vi) In an isosceles triangle ABC with $AB = AC$, if BD and CE are its altitudes, then BD is CE .

(vii) In right triangles ABC and DEF , if hypotenuse $AB = EF$ and side $AC = DE$, then $\triangle ABC \cong \triangle \dots$

Sol:

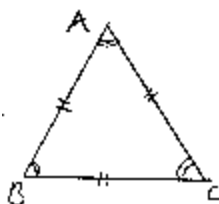
(i) Sides opposite to equal angles of a triangle are equal

(ii) Angles opposite to equal sides of a triangle are equal



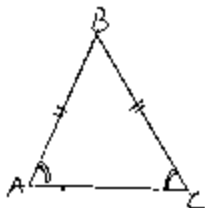
(iii) In an equilateral triangle all angles are equal

Reason: Since all sides are equal in an equilateral triangle, the angles opposite to equal sides will be equal



- (iv) In a $\triangle ABC$ if $\angle A = \angle C$, then $AB = BC$

Reason: Since, the sides opposite to equal angles are equal, the side opposite to $\angle A$ i.e., BC and $\angle C$ i.e., AB are equal



- (v) If altitudes CE and BF of a triangle ABC are equal, then $AB = AC$

Reason: From RHS congruence criterion $\triangle BEC \cong \triangle CFB$
 $\Rightarrow \angle EBC = \angle FCB \Rightarrow \angle ABC = \angle ACB \Rightarrow AC = AB$
 [\because Sides opposite to equal angles are equal]



- (vi) In an isosceles triangle ABC with $AB = AC$, if BD and CE are its altitudes, then BD is equal to CE

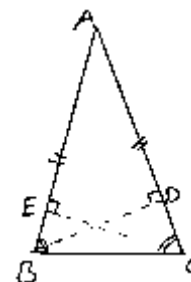
Reason: Since angles opposite to equal sides are equal, so
 $\angle ABC = \angle ACB$

$$\Rightarrow \angle EBC = \angle DCB$$

So, by ASA congruence criterion

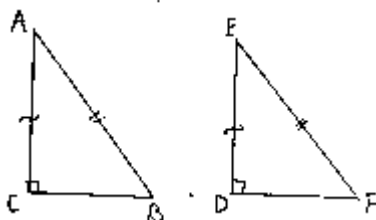
$$\triangle EBC \cong \triangle DCB$$

$\Rightarrow CE = BD$ [Corresponding parts of congruent triangles are equal]



- (vii) In right triangles ABC and DEF , if hypotenuse $AB = EF$ and side $AC = DE$, then.
 $\triangle ABC \cong \triangle EFD$

Reason: From RHS congruence criterion we have $\triangle ABC \cong \triangle EFD$



Exercise -10.6

1. In $\triangle ABC$, if $\angle A = 40^\circ$ and $\angle B = 60^\circ$. Determine the longest and shortest sides of the triangle.

Sol:

Given that in $\triangle ABC$, $\angle A = 40^\circ$ and $\angle B = 60^\circ$

We have to find longest and shortest side

We know that,

Sum of angles in a triangle 180°

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 40^\circ + 60^\circ + \angle C = 180^\circ$$

$$\Rightarrow \angle C = 180^\circ - (40^\circ + 60^\circ)$$

$$= 180^\circ - 100^\circ = 80^\circ$$

$$\therefore \boxed{\angle C = 80^\circ}$$

Now,

$$\Rightarrow 40^\circ < 60^\circ < 80^\circ \Rightarrow \angle A < \angle B < \angle C$$

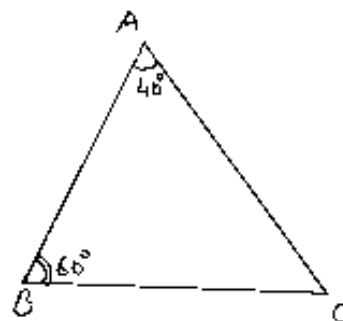
$\Rightarrow \angle C$ is greater angle and $\angle A$ is smaller angle

Now, $\angle A < \angle B < \angle C$

$$\Rightarrow BC < AC < AB$$

[\because Side opposite to greater angle is larger and side opposite to smaller angle is smaller]

$\therefore AB$ is longest and BC is smallest or shortest side.



2. In a $\triangle ABC$, if $\angle B = \angle C = 45^\circ$, which is the longest side?

Sol:

Given that in $\triangle ABC$,

$$\angle B = \angle C = 45^\circ$$

We have to find longest side

We know that,

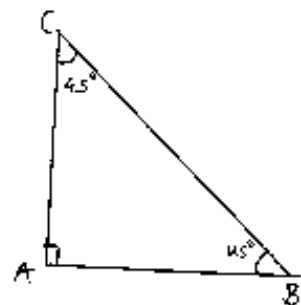
Sum of angles in a triangle $= 180^\circ$

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A + 45^\circ + 45^\circ = 180^\circ$$

$$\Rightarrow \angle A = 180^\circ - (45^\circ + 45^\circ) = 180^\circ - 90^\circ = 90^\circ$$

$$\therefore \boxed{\angle A = 90^\circ}$$



3. In $\triangle ABC$, side AB is produced to D so that $BD = BC$. If $\angle B = 60^\circ$ and $\angle A = 70^\circ$, prove that: (i) $AD > CD$ (ii) $AD > AC$

Sol:

Given that in $\triangle ABC$, side AB is produced to D So that $BD = BC$ and $\angle B = 60^\circ$, $\angle A = 70^\circ$

We have to prove that

(i) $AD > CD$ (ii) $AD > AC$

First join C and D

Now, in $\triangle ABC$

$$\angle A + \angle B + \angle C = 180^\circ \quad [\because \text{Sum of angles in a triangle} = 180^\circ]$$

$$\Rightarrow \angle C = 180^\circ - 70^\circ - 60^\circ$$

$$= 180^\circ - 130^\circ = 50^\circ$$

$$\therefore \boxed{\angle C = 50^\circ} \Rightarrow \angle ACB = 50^\circ \quad \dots\dots(1)$$

And also in $\triangle BDC$,

$$\angle DBC = 180^\circ - \angle ABC \quad [\because ABD \text{ is a straight angle}]$$

$$= 180^\circ - 60^\circ = 120^\circ$$

and also $BD = BC$ [given]

$$\Rightarrow \angle BCD = \angle BDC \quad [\because \text{Angles opposite to equal sides are equal}]$$

Now,

$$\angle DBC + \angle BCD + \angle BDC = 180^\circ \quad [\because \text{Sum of angles in a triangle} = 180^\circ]$$

$$\Rightarrow 120^\circ + \angle BCD + \angle BCD = 180^\circ$$

$$\Rightarrow 2\angle BCD = 180^\circ - 120^\circ \Rightarrow \angle BCD = \frac{60^\circ}{2} = 30^\circ$$

$$\therefore \boxed{\angle BCD = \angle BDC = 30^\circ} \quad \dots\dots(2)$$

Now, consider $\triangle ADC$,

$$\angle BAC \Rightarrow \angle DAC = 70^\circ \quad [\text{given}]$$

$$\angle BDC \Rightarrow \angle ADC = 30^\circ \quad [\because \text{From (2)}]$$

$$\angle ACD = \angle ACB + \angle BCD$$

$$= 50^\circ + 30^\circ$$

$$[\because \text{From (1) and (2)}]$$

$$= 80^\circ$$

Now, $\angle ADC < \angle DAC < \angle ACD$

$$\Rightarrow \boxed{AC < DC < AD}$$

$[\because \text{Side opposite to greater angle is longer and smaller angle is smaller}]$

$$\Rightarrow AD > CD \text{ and } AD > AC$$

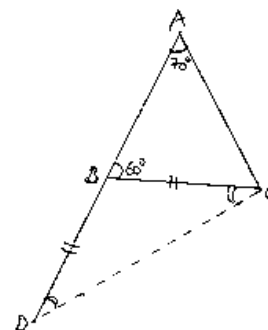
\therefore Hence proved

Or

We have, $\angle ACD > \angle DAC$ and $\angle ACD > \angle ADC$

$$\Rightarrow \boxed{AD > DC} \text{ and } \boxed{AD > AC}$$

$[\because \text{Side opposite to greater angle is longer and smaller angle is smaller}]$



4. Is it possible to draw a triangle with sides of length 2 cm, 3 cm and 7 cm?

Sol:

Given lengths of sides are 2cm, 3cm and 7cm we have to check whether it is possible to draw a triangle with the given lengths of sides

We know that,

A triangle can be drawn only when the sum of any two sides is greater than the third side.

So, let's check the rule.

$$2 + 3 \nless 7 \text{ or } 2 + 3 < 7$$

$$2 + 7 > 3$$

$$\text{and } 3 + 7 > 2$$

Here, $2 + 3 \nless 7$ So, the triangle does not exist.

5. O is any point in the interior of $\triangle ABC$. Prove that

(i) $AB + AC > OB + OC$

(ii) $AB + BC + CA > OA + OB + OC$

(iii) $OA + OB + OC > \frac{1}{2}(AB + BC + CA)$

Sol:

Given that O is any point in the interior of $\triangle ABC$

We have to prove

(i) $AB + AC > OB + OC$

(ii) $AB + BC + CA > OA + OB + OC$

(iii) $OA + OB + OC > \frac{1}{2}(AB + BC + CA)$

We know that, in a triangle the sum of any two sides is greater than the third side

So, we have

In $\triangle ABC$

$$AB + BC > AC$$

$$BC + AC > AB$$

$$AC + AB > BC$$

In $\triangle OBC$

$$OB + OC > BC \quad \dots\dots(1)$$

In $\triangle OAC$

$$OA + OC > AC \quad \dots\dots(2)$$

In $\triangle OAB$

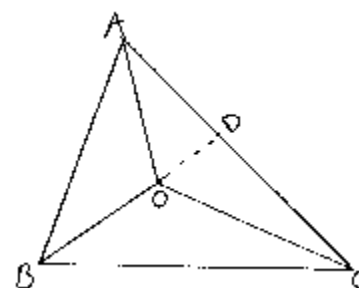
$$OA + OB > AB \quad \dots\dots(3)$$

Now, extend (or) produce BO to meet AC in D .

Now, in $\triangle ABD$, we have

$$AB + AD > BD$$

$$\Rightarrow AB + AD > BO + OD \quad \dots\dots(4) [\because BD = BO + OD]$$



Similarly in $\triangle ODC$, we have

$$OD + DC > OC \quad \dots\dots\dots(5)$$

(i) Adding (4) and (5), we get

$$AB + AD + OD + DC > BO + OD + OC$$

$$\Rightarrow AB + (AD + DC) > OB + OC$$

$$\Rightarrow \boxed{AB + AC > OB + OC} \quad \dots\dots\dots(6)$$

Similarly, we have

$$BC + BA > OA + OC \quad \dots\dots\dots(7)$$

$$\text{and } CA + CB > OA + OB \quad \dots\dots\dots(8)$$

(ii) Adding equation (6), (7) and (8), we get

$$AB + AC + BC + BA + CA + CB > OB + OC + OA + OC + OA + OB$$

$$\Rightarrow 2AB + 2BC + 2CA > 2OA + 2OB + 2OC$$

$$\Rightarrow 2(AB + BC + CA) > 2(OA + OB + OC)$$

$$\Rightarrow \boxed{AB + BC + CA > OA + OB + OC}$$

(iii) Adding equations (1), (2) and (3)

$$OB + OC + OA + OC + OA + OB > BC + AC + AB$$

$$\Rightarrow 2OA + 2OB + 2OC > AB + BC + CA$$

$$\text{We get } \Rightarrow 2(OA + OB + OC) > AB + BC + CA$$

$$\therefore \boxed{(OA + OB + OC) > \frac{1}{2}(AB + BC + CA)}$$

6. Prove that the perimeter of a triangle is greater than the sum of its altitudes.

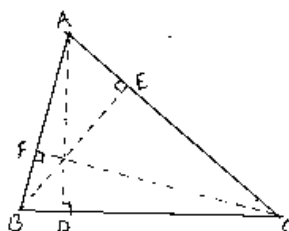
Sol:

Given: A $\triangle ABC$ in which $AD \perp BC$, $BE \perp AC$ and $CF \perp AB$.

To prove:

$$AD + BE + CF < AB + BC + AC$$

Figure:



Proof:

We know that of all the segments that can be drawn to a given line, from a point not lying on it, the perpendicular distance i.e., the perpendicular line segment is the shortest.

Therefore,

$$AD \perp BC$$

$$\Rightarrow AB > AD \text{ and } AC > AD$$

$$\Rightarrow \boxed{AB + AC > 2AD} \quad \dots\dots(1)$$

Similarly $BE \perp AC$

$$\Rightarrow BA > BE \text{ and } BC > BE$$

$$\Rightarrow \boxed{BA + BC > 2BE} \quad \dots\dots(2)$$

And also $CF \perp AB$

$$\Rightarrow CA > CF \text{ and } CB > CF$$

$$\Rightarrow \boxed{CA + CB > 2CF} \quad \dots\dots(3)$$

Adding (1), (2) and (3), we get

$$AB + AC + BA + BC + CA + CB > 2AD + 2BE + 2CF$$

$$\Rightarrow 2AB + 2BC + 2CA > 2(AD + BE + CF)$$

$$\Rightarrow 2(AB + BC + CA) > 2(AD + BE + CF)$$

$$\Rightarrow \boxed{AB + BC + CA > AD + BE + CF}$$

\Rightarrow The perimeter of the triangle is greater than the sum of its altitudes

\therefore Hence proved

7. In Fig. 10.131, prove that: (i) $CD + DA + AB + BC > 2AC$ (ii) $CD + DA + AB > BC$

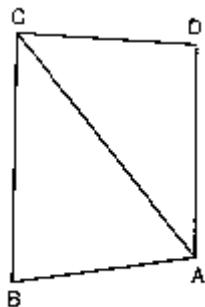


Fig. 10.131

Sol:

Given to prove

(i) $CD + DA + AB + BC > 2AC$

(ii) $CD + DA + AB > BC$

From the given figure,

We know that, in a triangle sum of any two sides is greater than the third side

(i) So,

In $\triangle ABC$, we have

$$AB + BC > AC \quad \dots\dots(1)$$

In $\triangle ADC$, we have

$$CD + DA > AC \quad \dots\dots\dots(2)$$

Adding (1) and (2) we get

$$AB + BC + CD + DA > AC + AC$$

$$\Rightarrow \boxed{CD + DA + AB + BC > 2AC}$$

(ii) Now, in $\triangle ABC$, we have

$$\boxed{AB + AC > BC} \quad \dots\dots\dots(3)$$

and in $\triangle ADC$, we have

$$CD + DA > AC$$

Add AB on both sides

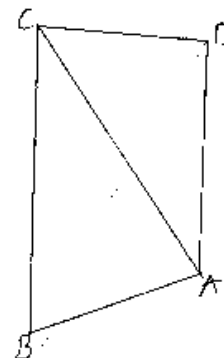
$$\Rightarrow \boxed{CD + DA + AB > AC + AB} \quad \dots\dots\dots(4)$$

From equation (3) and (4), we get

$$CD + DA + AB > AC + AB > BC$$

$$\Rightarrow \boxed{CD + DA + AB > BC}$$

\therefore Hence proved



8. Which of the following statements are true (T) and which are false (F)?

- (i) Sum of the three sides of a triangle is less than the sum of its three altitudes.
- (ii) Sum of any two sides of a triangle is greater than twice the median drawn to the third side.
- (iii) Sum of any two sides of a triangle is greater than the third side.
- (iv) Difference of any two sides of a triangle is equal to the third side.
- (v) If two angles of a triangle are unequal, then the greater angle has the larger side opposite to it.
- (vi) Of all the line segments that can be drawn from a point to a line not containing it, the perpendicular line segment is the shortest one.

Sol:

- (i) False (F)

Reason: Sum of these sides of a triangle is greater than sum of its three altitudes

- (ii) True (F)

- (iii) True (T)

- (iv) False (F)

Reason: The difference of any two sides of a triangle is less than third side.

- (v) True (T)

Reason: The side opposite to greater angle is longer and smaller angle is shorter in a triangle

- (vi) True (T)

Reason: The perpendicular distance is the shortest distance from a point to a line not containing it.

9. Fill in the blanks to make the following statements true.
- (i) In a right triangle the hypotenuse is the side.
 - (ii) The sum of three altitudes of a triangle is than its perimeter.
 - (iii) The sum of any two sides of a triangle is than the third side.
 - (iv) If two angles of a triangle are unequal, then the smaller angle has the side opposite to it.
 - (v) Difference of any two sides of a triangle is than the third side.
 - (vi) If two sides of a triangle are unequal, then the larger side has angle opposite to it.

Sol:

- (i) In a right triangle the hypotenuse is the largest side
Reason: Since a triangle can have only one right angle, other two angles must be less than 90°
 \Rightarrow The right angle is the largest angle
 \Rightarrow Hypotenuse is the largest side.
 - (ii) The sum of three altitudes of a triangle is less than its perimeter
 - (iii) The sum of any two sides of a triangle is greater than the third side.
 - (iv) If two angles of a triangle are unequal, then the smaller angle has the smaller side opposite to it.
 - (v) Difference of any two sides of a triangle is less than the third side.
 - (vi) If two sides of a triangle are unequal, then the larger side has greater angle opposite to it.
-