

Ex 11.1

Differentiation Ex 11.1 Q1

$$\text{Let } f(x) = e^{-x}$$

$$\Rightarrow f(x+h) = e^{-(x+h)}$$

$$\begin{aligned}\frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{-(x+h)} - e^{-x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{-x} \times e^{-h} - e^{-x}}{h} \\ &= \lim_{h \rightarrow 0} e^{-x} \left\{ \frac{e^{-h} - 1}{-h} \right\} \times (-1) \\ &= -e^{-x}\end{aligned}$$

$$\left[\text{Since, } \lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1 \right]$$

So,

$$\frac{d}{dx}(e^{-x}) = -e^{-x}$$

Differentiation Ex 11.1 Q2

$$\text{Let } f(x) = e^{3x}$$

$$\Rightarrow f(x+h) = e^{3(x+h)}$$

$$\begin{aligned}\frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{3(x+h)} - e^{3x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{3x}e^{3h} - e^{3x}}{h} \\ &= \lim_{h \rightarrow 0} e^{3x} \left\{ \frac{e^{3h} - 1}{3h} \right\} \times 3 \\ &= 3e^{3x}\end{aligned}$$

$$\left[\text{Since, } \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1 \right]$$

Hence,

$$\frac{d}{dx}(e^{3x}) = 3e^{3x}$$

Differentiation Ex 11.1 Q3

Let $f(x) = e^{ax+b}$

$\Rightarrow f(x+h) = e^{a(x+h)+b}$

$$\begin{aligned} \therefore \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{a(x+h)+b} - e^{ax+b}}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{ax+b} e^{ah} - e^{ax+b}}{h} \\ &= \lim_{h \rightarrow 0} e^{ax+b} \left\{ \frac{e^{ah} - 1}{ah} \right\} \times a \\ &= ae^{ax+b} \quad \left[\text{Since, } \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1 \right] \end{aligned}$$

So,

$$\frac{d}{dx}(e^{ax+b}) = ae^{ax+b}$$

Differentiation Ex 11.1 Q4

Let $f(x) = e^{\cos x}$

$\Rightarrow f(x+h) = e^{\cos(x+h)}$

$$\begin{aligned} \therefore \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{\cos(x+h)} - e^{\cos x}}{h} \\ &= \lim_{h \rightarrow 0} e^{\cos x} \left[\frac{e^{\cos(x+h) - \cos x} - 1}{h} \right] \\ &= \lim_{h \rightarrow 0} e^{\cos x} \left[\frac{e^{\cos(x+h) - \cos x} - 1}{\cos(x+h) - \cos x} \right] \times \frac{\cos(x+h) - \cos x}{h} \\ &= \lim_{h \rightarrow 0} e^{\cos x} \times \left(\frac{\cos(x+h) - \cos x}{h} \right) \quad \left[\text{Since, } \lim_{h \rightarrow 0} \frac{e^x - 1}{x} = 1 \right] \\ &= \lim_{h \rightarrow 0} e^{\cos x} \times \left(\frac{-2 \sin \frac{x+h+x}{2} \times \sin \frac{x+h-x}{2}}{h} \right) \quad \left[\text{Since, } \cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2} \right] \\ &= e^{\cos x} \lim_{h \rightarrow 0} \frac{-2 \sin \left(\frac{2x+h}{2} \right) \times \sin \left(\frac{h}{2} \right)}{h} \\ &= e^{\cos x} \lim_{h \rightarrow 0} -2 \sin \left(\frac{2x+h}{2} \right) \times \frac{1}{2} \quad \left[\text{Since, } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \\ &= e^{\cos x} (-\sin x) \\ &= -\sin x e^{\cos x} \end{aligned}$$

Hence,

$$\frac{d}{dx}(e^{\cos x}) = -\sin x e^{\cos x}$$

Differentiation Ex 11.1 Q5

$$\text{Let } f(x) = e^{\sqrt{2x}}$$

$$\Rightarrow f(x+h) = e^{\sqrt{2(x+h)}}$$

$$\begin{aligned} \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{\sqrt{2(x+h)}} - e^{\sqrt{2x}}}{h} \\ &= \lim_{h \rightarrow 0} e^{\sqrt{2x}} \frac{e^{\sqrt{2(x+h)} - \sqrt{2x}} - 1}{h} \\ &= e^{\sqrt{2x}} \lim_{h \rightarrow 0} \left(\frac{e^{\sqrt{2(x+h)} - \sqrt{2x}} - 1}{\sqrt{2(x+h)} - \sqrt{2x}} \right) \left(\frac{\sqrt{2(x+h)} - \sqrt{2x}}{h} \right) \\ &= e^{\sqrt{2x}} \lim_{h \rightarrow 0} \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h} \\ &= e^{\sqrt{2x}} \lim_{h \rightarrow 0} \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h} \times \frac{\sqrt{2(x+h)} + \sqrt{2x}}{\sqrt{2(x+h)} + \sqrt{2x}} \\ &= e^{\sqrt{2x}} \lim_{h \rightarrow 0} \frac{2(x+h) - 2x}{h(\sqrt{2(x+h)} + \sqrt{2x})} \\ &= e^{\sqrt{2x}} \lim_{h \rightarrow 0} \frac{2x + 2h - 2x}{h(\sqrt{2(x+h)} + \sqrt{2x})} \\ &= e^{\sqrt{2x}} \lim_{h \rightarrow 0} \frac{2h}{h(\sqrt{2(x+h)} + \sqrt{2x})} \\ &= e^{\sqrt{2x}} \lim_{h \rightarrow 0} \frac{2}{\sqrt{2(x+h)} + \sqrt{2x}} \\ &= \frac{e^{\sqrt{2x}}}{\sqrt{2x}} \end{aligned}$$

$$\left[\text{Since, } \lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1 \right]$$

[Rationalizing the numerator]

So,

$$\frac{d}{dx}(e^{\sqrt{2x}}) = \frac{e^{\sqrt{2x}}}{\sqrt{2x}}$$

Differentiation Ex 11.1 Q6

$$\begin{aligned}\text{Let } f(x) &= \log \cos x \\ \Rightarrow f(x+h) &= \log \cos(x+h)\end{aligned}$$

$$\begin{aligned}\therefore \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\log \cos(x+h) - \log \cos x}{h} \\ &= \lim_{h \rightarrow 0} \frac{\log \frac{\cos(x+h)}{\cos x}}{h} && \left[\text{Since, } \log A - \log B = \log \frac{A}{B} \right] \\ &= \lim_{h \rightarrow 0} \frac{\log \left\{ 1 + \frac{\cos(x+h) - \cos x}{\cos x} \right\}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\log \left\{ 1 + \frac{\cos(x+h)}{\cos x} \right\}}{\left(\frac{\cos(x+h) - \cos x}{\cos x} \right) h \times \left(\frac{\cos x}{\cos(x+h) - \cos x} \right)} \\ &= \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{\cos x \times h} && \left[\text{Since, } \lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1 \right] \\ &= \lim_{h \rightarrow 0} \frac{-2 \sin \left(\frac{x+h+x}{2} \right) \sin \left(\frac{x+h-x}{2} \right)}{\cos x \times h} \\ &= -2 \lim_{h \rightarrow 0} \frac{\sin \left(\frac{2x+h}{2} \right) \times \left(\sin \frac{h}{2} \right)}{2 \cos x \left(\frac{h}{2} \right)} \\ &= \frac{-2 \sin x}{2 \cos x} && \left[\text{Since, } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \\ &= -\tan x\end{aligned}$$

So,

$$\frac{d}{dx}(\log \cos x) = -\tan x$$

Differentiation Ex 11.1 Q7

$$\begin{aligned}\text{Let } f(x) &= e^{\sqrt{\cot x}} \\ \Rightarrow f(x+h) &= e^{\sqrt{\cot(x+h)}}\end{aligned}$$

$$\begin{aligned}\frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{\sqrt{\cot(x+h)}} - e^{\sqrt{\cot x}}}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^{\sqrt{\cot x}} (e^{\sqrt{\cot(x+h)} - \sqrt{\cot x}} - 1)}{h} \\ &= e^{\sqrt{\cot x}} \lim_{h \rightarrow 0} \left(\frac{e^{\sqrt{\cot(x+h)} - \sqrt{\cot x}} - 1}{\sqrt{\cot(x+h)} - \sqrt{\cot x}} \right) \times \left(\frac{\sqrt{\cot(x+h)} - \sqrt{\cot x}}{h} \right) \\ &= e^{\sqrt{\cot x}} \lim_{h \rightarrow 0} \frac{(\sqrt{\cot(x+h)} - \sqrt{\cot x})}{h} \times \frac{\sqrt{\cot(x+h)} + \sqrt{\cot x}}{\sqrt{\cot(x+h)} + \sqrt{\cot x}}\end{aligned}$$

$$\left[\text{Since, } \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1 \text{ and rationalizing numerator} \right]$$

$$\begin{aligned}&= e^{\sqrt{\cot x}} \lim_{h \rightarrow 0} \frac{\cot(x+h) - \cot x}{h (\sqrt{\cot(x+h)} + \sqrt{\cot x})} \\ &= e^{\sqrt{\cot x}} \lim_{h \rightarrow 0} \frac{\cot(x+h) \cot x + 1}{h (\sqrt{\cot(x+h)} + \sqrt{\cot x})} \quad \left[\text{Since, } \cot(A-B) = \frac{\cot A \cot B + 1}{\cot A - \cot B} \right] \\ &= e^{\sqrt{\cot x}} \lim_{h \rightarrow 0} \frac{\cot(x+h) \cot x + 1}{\coth h (\sqrt{\cot(x+h)} + \sqrt{\cot x})} \\ &= e^{\sqrt{\cot x}} \lim_{h \rightarrow 0} \frac{(\cot(x+h) \cot x + 1)}{\left(\frac{h}{\tanh} \right) (\sqrt{\cot(x+h)} + \sqrt{\cot x})} \\ &= \frac{e^{\sqrt{\cot x}} \times (\cot^2 x + 1)}{2\sqrt{\cot x}} \quad \left[\text{Since, } \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \right] \\ &= \frac{e^{\sqrt{\cot x}} \times \operatorname{cosec}^2 x}{2\sqrt{\cot x}} \quad \left[\text{Since, } (1 + \cot^2 x) = \operatorname{cosec}^2 x \right]\end{aligned}$$

So,

$$\frac{d}{dx}(e^{\sqrt{\cot x}}) = \frac{e^{\sqrt{\cot x}} \times \operatorname{cosec}^2 x}{2\sqrt{\cot x}}$$

Differentiation Ex 11.1 Q8

$$\begin{aligned}\text{Let } f(x) &= x^2 e^x \\ \Rightarrow f(x+h) &= (x+h)^2 e^{(x+h)}\end{aligned}$$

$$\begin{aligned}\therefore \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^2 e^{(x+h)} - x^2 e^x}{h} \\ &= \lim_{h \rightarrow 0} \left(\frac{x^2 e^{(x+h)} - x^2 e^x}{h} + \frac{2x h e^{(x+h)}}{h} + \frac{h^2 e^{(x+h)}}{h} \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{x^2 e^x (e^{(x+h)-x} - 1)}{h} + 2x e^{(x+h)} + h e^{(x+h)} \right) \\ &= \lim_{h \rightarrow 0} \left[x^2 e^x \frac{(e^h - 1)}{h} + 2x e^{(x+h)} + h e^{(x+h)} \right] \\ &= x^2 e^x + 2x e^x + 0 \times e^x\end{aligned}$$

$$\left[\text{Since, } \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1 \right]$$

So,

$$\frac{d}{dx}(x^2 e^x) = e^x (x^2 + 2x)$$

Differentiation Ex 11.1 Q9

Let $f(x) = \log \operatorname{cosec} x$

$\Rightarrow f(x+h) = \log \operatorname{cosec}(x+h)$

$$\begin{aligned} \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\log \operatorname{cosec}(x+h) - \log \operatorname{cosec} x}{h} \\ &= \lim_{h \rightarrow 0} \frac{\log \left(\frac{\operatorname{cosec}(x+h)}{\operatorname{cosec} x} \right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\log \left(1 + \left(\frac{\sin x}{\sin(x+h)} - 1 \right) \right)}{h} \\ &= \lim_{h \rightarrow 0} \left\{ \frac{\log \left(1 + \left(\frac{\sin x - \sin(x+h)}{\sin(x+h)} \right) \right)}{\left(\frac{\sin x - \sin(x+h)}{\sin(x+h)} \right)} \right\} \left(\frac{\sin x - \sin(x+h)}{h} \right) \\ &= \lim_{h \rightarrow 0} \frac{2 \cos \left(\frac{x+x+h}{2} \right) \sin \left(\frac{x-x-h}{2} \right)}{\sin(x+h) h} \end{aligned}$$

$$\left[\text{Since, } \lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1 \text{ and } \sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right) \right]$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{2 \cos \left(\frac{2x+h}{2} \right) \left\{ \sin \left(-\frac{h}{2} \right) \right\}}{\sin(x+h) (-2) \left\{ -\frac{h}{2} \right\}} \quad \left[\text{Since, } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \\ &= -\cot x \end{aligned}$$

So,

$$\frac{d}{dx}(\log \operatorname{cosec} x) = -\cot x.$$

Differentiation Ex 11.1 Q10

$$\begin{aligned}
 \text{Let } f(x) &= \sin^{-1}(2x+3) \\
 \Rightarrow f(x+h) &= \sin^{-1}(2(x+h)+3) \\
 \Rightarrow f(x+h) &= \sin^{-1}(2x+2h+3)
 \end{aligned}$$

$$\begin{aligned}
 \therefore \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sin^{-1}(2x+2h+3) - \sin^{-1}(2x+3)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sin^{-1}\left[(2x+2h+3)\sqrt{1-(2x+3)^2} - (2x+3)\sqrt{1-(2x+2h+3)^2}\right]}{h} \\
 &\quad \left[\text{Since, } \sin^{-1}x - \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right] \right] \\
 &= \lim_{h \rightarrow 0} \frac{\sin^{-1}z}{z} \times \frac{z}{h}
 \end{aligned}$$

$$\text{Where, } z = (2x+2h+3)\sqrt{1-(2x+3)^2} - (2x+3)\sqrt{1-(2x+2h+3)^2} \text{ and } \lim_{h \rightarrow 0} \frac{\sin^{-1}h}{h} = 1$$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{z}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(2x+2h+3)\sqrt{1-(2x+3)^2} - (2x+3)\sqrt{1-(2x+2h+3)^2}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(2x+2h+3)^2 - (2x+3)^2 - (2x+3)^2\{1-(2x+2h+3)^2\}}{h\{(2x+2h+3)\sqrt{1-(2x+3)^2} + (2x+3)\sqrt{1-(2x+2h+3)^2}\}}
 \end{aligned}$$

[Since, rationalizing numerator]

$$\begin{aligned}
 &\quad \frac{\left[(2x+3)^2 + 4h^2 + 4h(2x+3)\right]\{1-(2x+3)^2\} - (2x+3)^2}{\left[1-(2x+3)^2 - 4h^2 - 4h(2x+3)\right]} \\
 &= \lim_{h \rightarrow 0} \frac{\left[1-(2x+3)^2 - 4h^2 - 4h(2x+3)\right]}{h\{(2x+2h+3)\sqrt{1-(2x+3)^2} + (2x+3)\sqrt{1-(2x+2h+3)^2}\}} \\
 &= \lim_{h \rightarrow 0} \frac{\left[(2x+3)^2 + 4h^2 + 4h(2x+3) - (2x+3)^4 - 4h^2(2x+3)^2 - 4h(2x+3)^3 - (2x+3)^2\right]}{h\{(2x+2h+3)\sqrt{1-(2x+3)^2} + (2x+3)\sqrt{1-(2x+2h+3)^2}\}} \\
 &= \lim_{h \rightarrow 0} \frac{4h[h + (2x+3)]}{h\{(2x+2h+3)\sqrt{1-(2x+3)^2} + (2x+3)\sqrt{1-(2x+2h+3)^2}\}} \\
 &= \frac{4(2x+3)}{(2x+3)\sqrt{1-(2x+3)^2} + (2x+3)\sqrt{1-(2x+3)^2}} \\
 &= \frac{4(2x+3)}{2(2x+3)\sqrt{1-(2x+3)^2}} \\
 &= \frac{2}{\sqrt{1-(2x+3)^2}}
 \end{aligned}$$

So,

$$\frac{d}{dx}(\sin^{-1}(2x+3)) = \frac{2}{\sqrt{1-(2x+3)^2}}$$

Ex 11.2

Differentiation Ex 11.2 Q1

Let,

$$y = \sin(3x + 5)$$

Differentiate y with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(\sin(3x + 5)) \\ &= \cos(3x + 5) \frac{d}{dx}(3x + 5) && \text{[using chain rule]} \\ &= \cos(3x + 5) \times [3(1) + 0] \\ &= 3 \cos(3x + 5)\end{aligned}$$

So,

$$\frac{d}{dx}(\sin(3x + 5)) = 3 \cos(3x + 5).$$

Differentiation Ex 11.2 Q2

Let,

$$y = \tan^2 x$$

Differentiate it with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= 2 \tan x \frac{d}{dx}(\tan x) && \text{[using chain rule]} \\ &= 2 \tan x \times \sec^2 x\end{aligned}$$

So,

$$\frac{d}{dx}(\tan^2 x) = 2 \tan x \sec^2 x.$$

Differentiation Ex 11.2 Q3

Let,

$$y = \tan(x^\circ + 45^\circ)$$
$$y = \tan\left\{(x^\circ + 45^\circ) \frac{\pi}{180^\circ}\right\}$$

Differentiate it with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \tan\left\{(x^\circ + 45^\circ) \frac{\pi}{180^\circ}\right\} \\ &= \sec^2\left\{(x^\circ + 45^\circ) \frac{\pi}{180^\circ}\right\} \times \frac{d}{dx} (x^\circ + 45^\circ) \frac{\pi}{180^\circ} \quad [\text{Using chain rule}] \\ &= \frac{\pi}{180^\circ} \sec^2(x^\circ + 45^\circ)\end{aligned}$$

So,

$$\frac{d}{dx} (\tan(x^\circ + 45^\circ)) = \frac{\pi}{180^\circ} \sec^2(x^\circ + 45^\circ).$$

Differentiation Ex 11.2 Q4

Let,

$$y = \sin(\log x)$$

Differentiate it with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \sin(\log x) \\ &= \cos(\log x) \frac{d}{dx} (\log x) \quad [\text{Using chain rule}] \\ &= \frac{1}{x} \cos(\log x)\end{aligned}$$

So,

$$\frac{d}{dx} (\sin(\log x)) = \frac{1}{x} \cos(\log x).$$

Differentiation Ex 11.2 Q5

Let,

$$y = e^{\sin \sqrt{x}}$$

Differentiate it with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} (e^{\sin \sqrt{x}}) \\ &= e^{\sin \sqrt{x}} \frac{d}{dx} (\sin \sqrt{x}) \quad [\text{Using chain rule}] \\ &= e^{\sin \sqrt{x}} \times \cos \sqrt{x} \frac{d}{dx} \sqrt{x} \quad [\text{Using chain rule}] \\ &= e^{\sin \sqrt{x}} \times \cos \sqrt{x} \times \frac{1}{2\sqrt{x}} \\ &= \frac{1}{2\sqrt{x}} \cos \sqrt{x} \times e^{\sin \sqrt{x}}\end{aligned}$$

So,

$$\frac{d}{dx} (e^{\sin \sqrt{x}}) = \frac{1}{2\sqrt{x}} \cos \sqrt{x} \times e^{\sin \sqrt{x}}.$$

Differentiation Ex 11.2 Q6

Let,

$$y = e^{\tan x}$$

Differentiate it with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} (e^{\tan x}) \\ &= e^{\tan x} \frac{d}{dx} (\tan x) \quad [\text{Using chain rule}] \\ &= e^{\tan x} \times \sec^2 x\end{aligned}$$

So,

$$\frac{d}{dx} (e^{\tan x}) = \sec^2 x \times e^{\tan x}.$$

Differentiation Ex 11.2 Q7

Let,

$$y = \sin^2(2x + 1)$$

Differentiate it with respect to x ,

$$\frac{dy}{dx} = \frac{d}{dx} [\sin^2(2x + 1)]$$

$$= 2 \sin(2x + 1) \frac{d}{dx} \sin(2x + 1)$$

[Using chain rule]

$$= 2 \sin(2x + 1) \cos(2x + 1) \frac{d}{dx} (2x + 1)$$

[Using chain rule]

$$= 4 \sin(2x + 1) \cos(2x + 1)$$

$$= 2 \sin 2(2x + 1)$$

[Since, $\sin^2 A = 2 \sin A \cos A$]

$$= 2 \sin(4x + 2)$$

So,

$$\frac{d}{dx} (\sin^2(2x + 1)) = 2 \sin(4x + 2).$$

Differentiation Ex 11.2 Q8

Let,

$$y = \log_7(2x - 3)$$

$$\Rightarrow y = \frac{\log(2x - 3)}{\log 7}$$

[Since, $\log_a b = \frac{\log b}{\log a}$]Differentiate it with respect to x ,

$$\frac{dy}{dx} = \frac{1}{\log 7} \frac{d}{dx} (\log(2x - 3))$$

$$= \frac{1}{\log 7} \times \frac{1}{(2x - 3)} \frac{d}{dx} (2x - 3)$$

[Using chain rule]

$$= \frac{2}{(2x - 3) \log 7}$$

Hence,

$$\frac{d}{dx} (\log_7(2x - 3)) = \frac{2}{(2x - 3) \log 7}.$$

Differentiation Ex 11.2 Q9

Let,

$$y = \tan 5x^\circ$$

$$\Rightarrow y = \tan \left(5x^\circ \times \frac{\pi}{180^\circ} \right)$$

Differentiate with respect to x ,

$$\frac{dy}{dx} = \frac{d}{dx} \tan \left(5x^\circ \times \frac{\pi}{180^\circ} \right)$$

$$= \sec^2 x \left(5x^\circ \times \frac{\pi}{180^\circ} \right) \frac{d}{dx} \left(5x^\circ \times \frac{\pi}{180^\circ} \right)$$

[Using chain rule]

$$= \left(\frac{5\pi}{180^\circ} \right) \sec^2 \left(5x^\circ \times \frac{\pi}{180^\circ} \right)$$

$$= \frac{5\pi}{180^\circ} \sec^2(5x^\circ)$$

Hence,

$$\frac{d}{dx} (\tan(5x^\circ)) = \frac{5\pi}{180^\circ} \sec^2(5x^\circ).$$

Differentiation Ex 11.2 Q10

Let,

$$y = 2^{x^3}$$

Differentiate with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} (2^{x^3}) \\ &= 2^{x^3} \times \log_2 \frac{d}{dx} (x^3) \quad [\text{Using chain rule}] \\ &= 3x^2 \times 2^{x^3} \times \log_2\end{aligned}$$

So,

$$\frac{d}{dx} (2^{x^3}) = 3x^2 \times 2^{x^3} \log_2.$$

Differentiation Ex 11.2 Q11

Let, $y = 3^{e^x}$

Differentiate it with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} (3^{e^x}) \\ &= 3^{e^x} \log 3 \frac{d}{dx} (e^x) \quad [\text{Using chain rule}] \\ &= e^x \times 3^{e^x} \log 3\end{aligned}$$

So,

$$\frac{d}{dx} (3^{e^x}) = e^x \times 3^{e^x} \log 3.$$

Differentiation Ex 11.2 Q12

Let $y = \log_x 3$

$$\Rightarrow y = \frac{\log 3}{\log x} \quad \left[\text{Since, } \log_a b = \frac{\log b}{\log a} \right]$$

Differentiate with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\frac{\log 3}{\log x} \right) \\ &= \log 3 \frac{d}{dx} (\log x)^{-1} \\ &= \log 3 \times \left[-1 (\log x)^{-2} \right] \frac{d}{dx} (\log x) \quad [\text{Using chain rule}] \\ &= -\frac{\log 3}{(\log x)^2} \times \frac{1}{x} \\ &= -\left(\frac{\log 3}{\log x} \right)^2 \times \frac{1}{x} \times \frac{1}{\log 3} \\ &= -\frac{1}{x \log 3 (\log_3 x)^2} \quad \left[\text{Since, } \frac{\log b}{\log a} = \log_a b \right]\end{aligned}$$

So,

$$\frac{d}{dx} (\log_x 3) = -\frac{1}{x \log 3 (\log_3 x)^2}.$$

Differentiation Ex 11.2 Q13

Let $y = 3^{x^2+2x}$

Differentiate with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(3^{x^2+2x} \right) \\ &= 3^{x^2+2x} \times \log 3 \times \frac{d}{dx} (x^2 + 2x) \quad \text{[Using chain rule]} \\ &= (2x + 2) \log 3 \times 3^{x^2+2x}\end{aligned}$$

So,

$$\frac{d}{dx} \left(3^{x^2+2x} \right) = (2x + 2) \log 3 \times 3^{x^2+2x}.$$

Differentiation Ex 11.2 Q14

Let $y = \sqrt{\frac{a^2 - x^2}{a^2 + x^2}}$

$$\Rightarrow y = \left(\frac{a^2 - x^2}{a^2 + x^2} \right)^{\frac{1}{2}}$$

Differentiate with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\frac{a^2 - x^2}{a^2 + x^2} \right)^{\frac{1}{2}} \\ &= \frac{1}{2} \left(\frac{a^2 - x^2}{a^2 + x^2} \right)^{\frac{1}{2}-1} \times \frac{d}{dx} \left(\frac{a^2 - x^2}{a^2 + x^2} \right) \quad \text{[Using chain rule]} \\ &= \frac{1}{2} \left(\frac{a^2 - x^2}{a^2 + x^2} \right)^{-\frac{1}{2}} \times \left\{ \frac{(a^2 + x^2) \frac{d}{dx} (a^2 - x^2) - (a^2 - x^2) \frac{d}{dx} (a^2 + x^2)}{(a^2 + x^2)^2} \right\} \quad \text{[Using chain rule]} \\ &= \frac{1}{2} \left(\frac{a^2 - x^2}{a^2 + x^2} \right)^{\frac{1}{2}} \left\{ \frac{-2x(a^2 + x^2) - 2x(a^2 - x^2)}{(a^2 + x^2)^2} \right\} \\ &= \frac{1}{2} \left(\frac{a^2 - x^2}{a^2 + x^2} \right)^{\frac{1}{2}} \left\{ \frac{-2xa^2 - 2x^3 - 2xa^2 + 2x^3}{(a^2 + x^2)^2} \right\} \\ &= \frac{1}{2} \left(\frac{a^2 - x^2}{a^2 + x^2} \right)^{\frac{1}{2}} \left(\frac{-4xa^2}{(a^2 + x^2)^2} \right) \\ &= \frac{-2xa^2}{\sqrt{a^2 - x^2} (a^2 + x^2)^{\frac{3}{2}}}\end{aligned}$$

So,

$$\frac{d}{dx} \left(\sqrt{\frac{a^2 - x^2}{a^2 + x^2}} \right) = \frac{-2a^2x}{\sqrt{a^2 - x^2} (a^2 + x^2)^{\frac{3}{2}}}.$$

Differentiation Ex 11.2 Q15

Let $y = 3^{x \log x}$

Differentiate with respect to x ,

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} (3^{x \log x}) \\
 &= 3^{x \log x} \times \log 3 \frac{d}{dx} (x \log x) && \text{[Using chain rule]} \\
 &= 3^{x \log x} \times \log 3 \left[x \frac{d}{dx} (\log x) + \log x \frac{d}{dx} (x) \right] && \text{[Using chain rule]} \\
 &= 3^{x \log x} \times \log 3 \left[\frac{x}{x} + \log x \right] \\
 &= 3^{x \log x} (1 + \log x) \times \log 3
 \end{aligned}$$

So,

$$\frac{d}{dx} (3^{x \log x}) = \log 3 \times 3^{x \log x} (1 + \log x).$$

Differentiation Ex 11.2 Q16

Let $y = \sqrt{\frac{1 + \sin x}{1 - \sin x}}$

Differentiate it with respect to x ,

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} \left(\frac{1 + \sin x}{1 - \sin x} \right)^{\frac{1}{2}} \\
 &= \frac{1}{2} \left(\frac{1 + \sin x}{1 - \sin x} \right)^{\frac{1}{2}-1} \frac{d}{dx} \left(\frac{1 + \sin x}{1 - \sin x} \right) \\
 &= \frac{1}{2} \left(\frac{1 + \sin x}{1 - \sin x} \right)^{-\frac{1}{2}} \left[\frac{(1 - \sin x)(\cos x) - (1 + \sin x)(-\cos x)}{(1 - \sin x)^2} \right] \\
 &= \frac{1}{2} \left(\frac{1 + \sin x}{1 - \sin x} \right)^{-\frac{1}{2}} \left[\frac{\cos x - \cos x \sin x + \cos x + \sin x \cos x}{(1 - \sin x)^2} \right] \\
 &= \frac{1}{2} \times \frac{2 \cos x}{\sqrt{1 + \sin x} (1 - \sin x)^{\frac{3}{2}}} \\
 &= \frac{\cos x}{\sqrt{1 + \sin x} (1 - \sin x)^{\frac{3}{2}}} \\
 &= \frac{\cos x}{\sqrt{1 + \sin x} \sqrt{1 - \sin x} (1 - \sin x)} \\
 &= \frac{\cos x}{\sqrt{1 - \sin^2 x} (1 - \sin x)} \\
 &= \frac{\cos x}{\cos x (1 - \sin x)} && \text{[Using } 1 - \sin^2 x = \cos^2 x \text{]} \\
 &= \frac{1}{(1 - \sin x)} \times \frac{(1 + \sin x)}{(1 + \sin x)} \\
 &= \frac{(1 + \sin x)}{(1 - \sin^2 x)} \\
 &= \frac{1 + \sin x}{\cos^2 x}
 \end{aligned}$$

$$\text{Thus, } \frac{dy}{dx} = \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}$$

$$\Rightarrow \frac{dy}{dx} = \sec^2 x + \tan x \sec x$$

$$\Rightarrow \frac{dy}{dx} = \sec x [\tan x + \sec x]$$

Differentiation Ex 11.2 Q17

Let $y = \sqrt{\frac{1-x^2}{1+x^2}}$

$$y = \left(\frac{1-x^2}{1+x^2} \right)^{\frac{1}{2}}$$

Differentiate it with respect to x ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \left(\frac{1-x^2}{1+x^2} \right)^{\frac{1}{2}} \\ &= \frac{1}{2} \left(\frac{1-x^2}{1+x^2} \right)^{\frac{1}{2}-1} \frac{d}{dx} \left(\frac{1-x^2}{1+x^2} \right) && \text{[Using chain rule]} \\ &= \frac{1}{2} \left(\frac{1-x^2}{1+x^2} \right)^{-\frac{1}{2}} \left[\frac{(1+x^2) \frac{d}{dx}(1-x^2) - (1-x^2) \frac{d}{dx}(1+x^2)}{(1+x^2)^2} \right] && \text{[Using quotient rule]} \\ &= \frac{1}{2} \left(\frac{1+x^2}{1-x^2} \right)^{\frac{1}{2}} \left[\frac{(1+x^2)(-2x) - (1-x^2)(2x)}{(1+x^2)^2} \right] \\ &= \frac{1}{2} \left(\frac{1+x^2}{1-x^2} \right)^{\frac{1}{2}} \left[\frac{-2x - 2x^3 - 2x + 2x^3}{(1+x^2)^2} \right] \\ &= \frac{1}{2} \frac{-4x}{\sqrt{1-x^2} (1+x^2)^{\frac{3}{2}}} \end{aligned}$$

So,

$$\frac{d}{dx} \left(\sqrt{\frac{1-x^2}{1+x^2}} \right) = \frac{-2x}{\sqrt{1-x^2} (1+x^2)^{\frac{3}{2}}}.$$

Differentiation Ex 11.2 Q18

Let $y = (\log \sin x)^2$

Differentiate with respect to x ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} (\log \sin x)^2 \\ &= 2 (\log \sin x) \frac{d}{dx} (\log \sin x) && \text{[Using chain rule]} \\ &= 2 (\log \sin x) \times \frac{1}{\sin x} \frac{d}{dx} (\log x) \\ &= 2 (\log \sin x) \times \frac{1}{\sin x} \times \frac{1}{x} \\ &= \frac{2 \log \sin x}{x \sin x} \end{aligned}$$

So,

$$\frac{d}{dx} (\log \sin x)^2 = \frac{2 \log \sin x}{x \sin x}$$

Differentiation Ex 11.2 Q19

$$\text{Let } y = \sqrt{\frac{1+x}{1-x}}$$

$$\Rightarrow y = \left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$$

Differentiate it with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\frac{1+x}{1-x}\right)^{\frac{1}{2}} \\ &= \frac{1}{2} \left(\frac{1+x}{1-x}\right)^{\frac{1}{2}-1} \frac{d}{dx} \left(\frac{1+x}{1-x}\right) && \text{[Using chain rule]} \\ &= \frac{1}{2} \left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}} \left[\frac{(1-x) \frac{d}{dx}(1+x) - (1+x) \frac{d}{dx}(1-x)}{(1-x)^2} \right] && \text{[Using chain rule]} \\ &= \frac{1}{2} \left(\frac{1-x}{1+x}\right)^{\frac{1}{2}} \left[\frac{(1-x)(1) - (1+x)(-1)}{(1-x)^2} \right] \\ &= \frac{1}{2} \left(\frac{1-x}{1+x}\right)^{\frac{1}{2}} \left[\frac{1-x+1+x}{(1-x)^2} \right] \\ &= \frac{1}{2} \frac{(1-x)^{\frac{1}{2}}}{(1+x)^{\frac{1}{2}}} \times \frac{2}{(1-x)^2} \\ &= \frac{1}{\sqrt{1+x} (1-x)^{3/2}}\end{aligned}$$

So,

$$\frac{d}{dx} \left(\sqrt{\frac{1+x}{1-x}} \right) = \frac{1}{\sqrt{1+x} (1-x)^{3/2}}$$

Differentiation Ex 11.2 Q20

$$\text{Let } y = \sin \left(\frac{1+x^2}{1-x^2} \right)$$

Differentiate it with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\sin \left(\frac{1+x^2}{1-x^2} \right) \right) \\ &= \cos \left(\frac{1+x^2}{1-x^2} \right) \frac{d}{dx} \left(\frac{1+x^2}{1-x^2} \right) && \text{[Using chain rule]} \\ &= \cos \left(\frac{1+x^2}{1-x^2} \right) \left[\frac{(1-x^2) \frac{d}{dx}(1+x^2) - (1+x^2) \frac{d}{dx}(1-x^2)}{(1-x^2)^2} \right] && \text{[Using quotient rule]} \\ &= \cos \left(\frac{1+x^2}{1-x^2} \right) \left[\frac{(1-x^2)(2x) - (1+x^2)(-2x)}{(1-x^2)^2} \right] \\ &= \cos \left(\frac{1+x^2}{1-x^2} \right) \left[\frac{2x - 2x^3 + 2x + 2x^3}{(1-x^2)^2} \right] \\ &= \frac{4x}{(1-x^2)^2} \cos \left(\frac{1+x^2}{1-x^2} \right)\end{aligned}$$

So,

$$\frac{d}{dx} \left(\sin \left(\frac{1+x^2}{1-x^2} \right) \right) = \frac{4x}{(1-x^2)^2} \cos \left(\frac{1+x^2}{1-x^2} \right).$$

Differentiation Ex 11.2 Q21

Let $y = e^{3x} \cos 2x$

Differentiate it with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} (e^{3x} \cos 2x) \\ &= e^{3x} \times \frac{d}{dx} (\cos 2x) + \cos 2x \frac{d}{dx} (e^{3x}) && \text{[Using product rule]} \\ &= e^{3x} \times (-\sin 2x) \frac{d}{dx} (2x) + \cos 2x e^{3x} \frac{d}{dx} (3x) && \text{[Using chain rule]} \\ &= -2e^{3x} \sin 2x + 3e^{3x} \cos 2x \\ &= e^{3x} (3 \cos 2x - 2 \sin 2x)\end{aligned}$$

so,

$$\frac{d}{dx} (e^{3x} \cos 2x) = e^{3x} (3 \cos 2x - 2 \sin 2x).$$

Differentiation Ex 11.2 Q22

Let $y = \sin(\log \sin x)$

Differentiate it with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \sin(\log \sin x) \\ &= \cos(\log \sin x) \frac{d}{dx} (\log \sin x) && \text{[Using chain rule]} \\ &= \cos(\log \sin x) \times \frac{1}{\sin x} \frac{d}{dx} \sin x \\ &= \cos(\log \sin x) \frac{\cos x}{\sin x} \\ &= \cos(\log \sin x) \times \cot x\end{aligned}$$

Hence,

$$\frac{d}{dx} (\sin(\log \sin x)) = \cos(\log \sin x) \times \cot x.$$

Differentiation Ex 11.2 Q23

Let $y = e^{\tan 3x}$

Differentiate it with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} (e^{\tan 3x}) \\ &= e^{\tan 3x} \frac{d}{dx} (\tan 3x) && \text{[Using chain rule]} \\ &= e^{\tan 3x} \times \sec^2 3x \times \frac{d}{dx} (3x) \\ &= 3e^{\tan 3x} \sec^2 3x\end{aligned}$$

So,

$$\frac{d}{dx} (e^{\tan 3x}) = 3e^{\tan 3x} \times \sec^2 3x$$

Differentiation Ex 11.2 Q24

$$\text{Let } y = e^{\sqrt{\cot x}}$$

$$\Rightarrow y = e^{(\cot x)^{\frac{1}{2}}}$$

Differentiate it with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(e^{(\cot x)^{\frac{1}{2}}} \right) \\ &= e^{(\cot x)^{\frac{1}{2}}} \times \frac{d}{dx} (\cot x)^{\frac{1}{2}} \quad [\text{Using chain rule}] \\ &= e^{\sqrt{\cot x}} \times \frac{1}{2} (\cot x)^{\frac{1}{2}-1} \frac{d}{dx} (\cot x) \\ &= -\frac{e^{\sqrt{\cot x}} \times \operatorname{cosec}^2 x}{2\sqrt{\cot x}}\end{aligned}$$

So,

$$\frac{d}{dx} \left(e^{\sqrt{\cot x}} \right) = -\frac{e^{\sqrt{\cot x}} \times \operatorname{cosec}^2 x}{2\sqrt{\cot x}}$$

Differentiation Ex 11.2 Q25

$$\text{Let } y = \log \left(\frac{\sin x}{1 + \cos x} \right)$$

Differentiating with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \log \left(\frac{\sin x}{1 + \cos x} \right) \\ &= \left(\frac{\sin x}{1 + \cos x} \right) \times \frac{d}{dx} \left(\frac{\sin x}{1 + \cos x} \right) \quad [\text{Using chain rule}] \\ &= \left(\frac{\sin x}{1 + \cos x} \right) \left[\frac{(1 + \cos x) \frac{d}{dx} (\sin x) - \sin x \frac{d}{dx} (1 + \cos x)}{(1 + \cos x)^2} \right] \quad [\text{Using quotient rule}] \\ &= \frac{\sin x}{1 + \cos x} \left[\frac{(1 + \cos x) (\cos x) - \sin x (-\sin x)}{(1 + \cos x)^2} \right] \\ &= \frac{\sin x}{1 + \cos x} \left[\frac{\cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)^2} \right] \\ &= \frac{\sin x}{1 + \cos x} \left[\frac{(1 + \cos x)}{(1 + \cos x)^2} \right] \\ &= \frac{1}{\sin x} \\ &= \operatorname{cosec} x\end{aligned}$$

So,

$$\frac{d}{dx} \left(\log \left(\frac{\sin x}{1 + \cos x} \right) \right) = \operatorname{cosec} x.$$

Differentiation Ex 11.2 Q26

$$\text{Let } y = \log \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

$$\Rightarrow y = \log \left(\frac{1 - \cos x}{1 + \cos x} \right)^{\frac{1}{2}}$$

$$\Rightarrow y = \frac{1}{2} \log \left(\frac{1 - \cos x}{1 + \cos x} \right) \quad [\text{Using } \log a^b = b \log a]$$

Differentiate it with respect to x ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \left\{ \frac{1}{2} \log \left(\frac{1 - \cos x}{1 + \cos x} \right) \right\} \\ &= \frac{1}{2} \times \frac{1}{\left(\frac{1 - \cos x}{1 + \cos x} \right)} \times \frac{d}{dx} \left(\frac{1 - \cos x}{1 + \cos x} \right) \quad [\text{Using chain rule}] \\ &= \frac{1}{2} \left(\frac{1 + \cos x}{1 - \cos x} \right) \left[\frac{(1 + \cos x) \frac{d}{dx} (1 - \cos x) - (1 - \cos x) \frac{d}{dx} (1 + \cos x)}{(1 + \cos x)^2} \right] \quad \left[\begin{array}{l} \text{Using} \\ \text{quotient} \\ \text{rule} \end{array} \right] \\ &= \frac{1}{2} \left(\frac{1 + \cos x}{1 - \cos x} \right) \left[\frac{(1 + \cos x) (\sin x) - (1 - \cos x) (-\sin x)}{(1 + \cos x)^2} \right] \\ &= \frac{1}{2} \left(\frac{1 + \cos x}{1 - \cos x} \right) \left[\frac{\sin x + \sin x \cos x + \sin x - \sin x \cos x}{(1 + \cos x)^2} \right] \\ &= \frac{1}{2} \left(\frac{1 + \cos x}{1 - \cos x} \right) \left[\frac{2 \sin x}{(1 + \cos x)^2} \right] \\ &= \frac{\sin x}{(1 - \cos x)(1 + \cos x)} \\ &= \frac{\sin x}{1 - \cos^2 x} \\ &= \frac{\sin x}{\sin^2 x} \quad [\text{Since } 1 - \cos^2 x = \sin^2 x] \\ &= \frac{1}{\sin x} \\ &= \operatorname{cosec} x \end{aligned}$$

So,

$$\frac{d}{dx} \left(\log \sqrt{\frac{1 - \cos x}{1 + \cos x}} \right) = \operatorname{cosec} x.$$

Differentiation Ex 11.2 Q27

$$\text{Let } y = \tan(e^{\sin x})$$

Differentiate it with respect to x ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} [\tan e^{\sin x}] \\ &= \sec^2(e^{\sin x}) \frac{d}{dx} (e^{\sin x}) \quad [\text{Using chain rule}] \\ &= \sec^2(e^{\sin x}) \times e^{\sin x} \times \frac{d}{dx} (\sin x) \\ &= \cos x \sec^2(e^{\sin x}) \times e^{\sin x} \end{aligned}$$

So,

$$\frac{d}{dx} (\tan e^{\sin x}) = \sec^2(e^{\sin x}) \times e^{\sin x} \times \cos x.$$

Differentiation Ex 11.2 Q28

Let $y = \log\left(x + \sqrt{x^2 + 1}\right)$

Differentiate with respect to x ,

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} \log\left(x + \sqrt{x^2 + 1}\right) \\
 &= \frac{1}{x + \sqrt{x^2 + 1}} \frac{d}{dx} \left(x + (x^2 + 1)^{\frac{1}{2}}\right) && \text{[Using chain rule]} \\
 &= \frac{1}{x + \sqrt{x^2 + 1}} \left[1 + \frac{1}{2}(x^2 + 1)^{\frac{1}{2}-1} \frac{d}{dx} (x^2 + 1)\right] \\
 &= \frac{1}{x + \sqrt{x^2 + 1}} \left[1 + \frac{1}{2\sqrt{x^2 + 1}} \times 2x\right] \\
 &= \frac{1}{x + \sqrt{x^2 + 1}} \left[\frac{\sqrt{x^2 + 1} + x}{\sqrt{x^2 + 1}}\right] \\
 &= \frac{1}{\sqrt{x^2 + 1}}
 \end{aligned}$$

So,

$$\frac{d}{dx} \left(\log\left(x + \sqrt{x^2 + 1}\right) \right) = \frac{1}{\sqrt{x^2 + 1}}.$$

Differentiation Ex 11.2 Q29

Let $y = \frac{e^x \log x}{x^2}$

Differentiate with respect to x ,

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{x^2 \frac{d}{dx} (e^x \log x) - (e^x \log x) \frac{d}{dx} (x^2)}{(x^2)^2} && \text{[Using quotient rule]} \\
 &= \frac{x^2 \left[e^x \frac{d}{dx} (\log x) + \log x \frac{d}{dx} (e^x) - e^x \log x \times 2x \right]}{x^4} && \text{[Using product rule]} \\
 &= \frac{x^2 \left[\frac{e^x}{x} + e^x \log x \right] - 2xe^x \log x}{x^4} \\
 &= \frac{\frac{x^2 e^x (1 + x \log x)}{x} - 2xe^x \log x}{x^4} \\
 &= \frac{xe^x [1 + x \log x - 2 \log x]}{x^4} \\
 &= \frac{xe^x}{x^3} \left[\frac{1}{x} + \frac{x \log x}{x} - \frac{2 \log x}{x} \right] \\
 &= e^x x^{-2} \left[\frac{1}{x} + \log x - \frac{2}{x} \log x \right]
 \end{aligned}$$

So,

$$\frac{d}{dx} \left[\frac{e^x \log x}{x^2} \right] = e^x x^{-2} \left[\frac{1}{x} + \log x - \frac{2}{x} \log x \right].$$

Differentiation Ex 11.2 Q30

Let $y = \log(\operatorname{cosec} x - \cot x)$

Differentiating with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \log(\operatorname{cosec} x - \cot x) \\ &= \frac{1}{(\operatorname{cosec} x - \cot x)} \frac{d}{dx} (\operatorname{cosec} x - \cot x) \quad \text{[Using chain rule]} \\ &= \frac{1}{(\operatorname{cosec} x - \cot x)} \times \{-\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x\} \\ &= \frac{\operatorname{cosec} x (\operatorname{cosec} x - \cot x)}{(\operatorname{cosec} x - \cot x)} \\ &= \operatorname{cosec} x\end{aligned}$$

So,

$$\frac{d}{dx} (\log(\operatorname{cosec} x - \cot x)) = \operatorname{cosec} x.$$

Differentiation Ex 11.2 Q31

Let $y = \frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$

Differentiating with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left[\frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}} \right] \\ &= \left[\frac{(e^{2x} - e^{-2x}) \frac{d}{dx} (e^{2x} + e^{-2x}) - (e^{2x} + e^{-2x}) \frac{d}{dx} (e^{2x} - e^{-2x})}{(e^{2x} - e^{-2x})^2} \right] \quad \text{[Using quotient rule and chain rule]} \\ &= \frac{(e^{2x} - e^{-2x}) \left[e^{2x} \frac{d}{dx} (2x) + e^{-2x} \frac{d}{dx} (-2x) \right] - (e^{2x} + e^{-2x}) \left[e^{2x} \frac{d}{dx} (2x) - e^{-2x} \frac{d}{dx} (-2x) \right]}{(e^{2x} - e^{-2x})^2} \\ &= \frac{(e^{2x} - e^{-2x}) (2e^{2x} - 2e^{-2x}) - (e^{2x} + e^{-2x}) (2e^{2x} + 2e^{-2x})}{(e^{2x} - e^{-2x})^2} \\ &= \frac{2(e^{2x} - e^{-2x})^2 - 2(e^{2x} + e^{-2x})^2}{(e^{2x} - e^{-2x})^2} \\ &= \frac{2[e^{4x} + e^{-4x} - 2e^{2x}e^{-2x} - e^{4x} - e^{-4x} - 2e^{2x}e^{-2x}]}{(e^{2x} - e^{-2x})^2} \\ &= \frac{-8}{(e^{2x} - e^{-2x})^2}\end{aligned}$$

So,

$$\frac{d}{dx} \left(\frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}} \right) = \frac{-8}{(e^{2x} - e^{-2x})^2}.$$

Differentiation Ex 11.2 Q32

Let $y = \log \left(\frac{x^2 + x + 1}{x^2 - x + 1} \right)$

Differentiating with respect to x ,

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} \left[\log \left(\frac{x^2 + x + 1}{x^2 - x + 1} \right) \right] \\
 &= \frac{1}{\left(\frac{x^2 + x + 1}{x^2 - x + 1} \right)} \frac{d}{dx} \left(\frac{x^2 + x + 1}{x^2 - x + 1} \right) \quad [\text{Using chain rule and quotient rule}] \\
 &= \left(\frac{x^2 - x + 1}{x^2 + x + 1} \right) \left[\frac{(x^2 - x + 1) \frac{d}{dx} (x^2 + x + 1) - (x^2 + x + 1) \frac{d}{dx} (x^2 - x + 1)}{(x^2 - x + 1)^2} \right] \\
 &= \left(\frac{x^2 - x + 1}{x^2 + x + 1} \right) \left[\frac{(x^2 - x + 1)(2x + 1) - (x^2 + x + 1)(2x - 1)}{(x^2 - x + 1)^2} \right] \\
 &= \left(\frac{x^2 - x + 1}{x^2 + x + 1} \right) \left[\frac{2x^3 - 2x^2 + 2x + x^2 - x + 1 - 2x^3 - 2x^2 - 2x + x^2 + x + 1}{(x^2 - x + 1)^2} \right] \\
 &= \frac{-4x^2 + 2x^2 + 2}{(x^2 + x + 1)(x^2 - x + 1)} \\
 &= \frac{-2(x^2 - 1)}{x^4 + 1 + 2x^2 - x^2} \\
 &= \frac{-2(x^2 - 1)}{x^4 + x^2 + 1}
 \end{aligned}$$

So,

$$\frac{d}{dx} \left(\log \frac{x^2 + x + 1}{x^2 - x + 1} \right) = \frac{-2(x^2 - 1)}{x^4 + x^2 + 1}$$

Differentiation Ex 11.2 Q33

Let $y = \tan^{-1}(e^x)$

Differentiate it with respect to x ,

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} (\tan^{-1} e^x) \\
 &= \frac{1}{1 + (e^x)^2} \frac{d}{dx} (e^x) \quad [\text{Using chain rule}] \\
 &= \frac{1}{1 + e^{2x}} \times e^x \\
 &= \frac{e^x}{1 + e^{2x}}
 \end{aligned}$$

So,

$$\frac{d}{dx} (\tan^{-1} e^x) = \frac{e^x}{1 + e^{2x}}.$$

Differentiation Ex 11.2 Q34

Let $y = e^{\sin^{-1} 2x}$

Differentiate it with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(e^{\sin^{-1} 2x} \right) \\ &= e^{\sin^{-1} 2x} \times \frac{d}{dx} \left(\sin^{-1} 2x \right) \quad \text{[Using chain rule]} \\ &= e^{\sin^{-1} 2x} \times \frac{1}{\sqrt{1 - (2x)^2}} \frac{d}{dx} (2x) \\ &= \frac{2e^{\sin^{-1} 2x}}{\sqrt{1 - 4x^2}}\end{aligned}$$

So,

$$\frac{d}{dx} \left(e^{\sin^{-1} 2x} \right) = \frac{2e^{\sin^{-1} 2x}}{\sqrt{1 - 4x^2}}.$$

Differentiation Ex 11.2 Q35

Let $y = \sin(2 \sin^{-1} x)$

Differentiate it with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\sin(2 \sin^{-1} x) \right) \\ &= \cos(2 \sin^{-1} x) \frac{d}{dx} (2 \sin^{-1} x) \quad \text{[Using chain rule]} \\ &= \cos(2 \sin^{-1} x) \times 2 \frac{1}{\sqrt{1 - x^2}} \\ &= \frac{2 \cos(2 \sin^{-1} x)}{\sqrt{1 - x^2}}\end{aligned}$$

So,

$$\frac{d}{dx} \left(\sin(2 \sin^{-1} x) \right) = \frac{2 \cos(2 \sin^{-1} x)}{\sqrt{1 - x^2}}.$$

Differentiation Ex 11.2 Q36

Let $y = e^{\tan^{-1} \sqrt{x}}$

Differentiate it with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(e^{\tan^{-1} \sqrt{x}} \right) \\ &= e^{\tan^{-1} \sqrt{x}} \frac{d}{dx} \left(\tan^{-1} \sqrt{x} \right) \quad \text{[Using chain rule]} \\ &= e^{\tan^{-1} \sqrt{x}} \times \frac{1}{1 + (\sqrt{x})^2} \frac{d}{dx} (\sqrt{x}) \\ &= \frac{e^{\tan^{-1} \sqrt{x}}}{1 + x} \times \frac{1}{2\sqrt{x}} \\ &= \frac{e^{\tan^{-1} \sqrt{x}}}{2\sqrt{x} (1 + x)}\end{aligned}$$

So,

$$\frac{d}{dx} \left(e^{\tan^{-1} \sqrt{x}} \right) = \frac{e^{\tan^{-1} \sqrt{x}}}{2\sqrt{x} (1 + x)}.$$

Differentiation Ex 11.2 Q37

$$\text{Let } y = \sqrt{\tan^{-1}\left(\frac{x}{2}\right)}$$

$$\Rightarrow y = \left(\tan^{-1}\left(\frac{x}{2}\right)\right)^{\frac{1}{2}}$$

Differentiate with respect to x ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \left(\tan^{-1}\left(\frac{x}{2}\right) \right)^{\frac{1}{2}} \\ &= \frac{1}{2} \left(\tan^{-1}\left(\frac{x}{2}\right) \right)^{\frac{1}{2}-1} \frac{d}{dx} \left(\tan^{-1}\left(\frac{x}{2}\right) \right) && [\text{Using chain rule}] \\ &= \frac{1}{2} \left(\tan^{-1}\left(\frac{x}{2}\right) \right)^{-\frac{1}{2}} \times \frac{1}{1 + \left(\frac{x}{2}\right)^2} \times \frac{d}{dx} \left(\frac{x}{2} \right) \\ &= \frac{4}{4 \sqrt{\tan^{-1}\left(\frac{x}{2}\right)} (4 + x^2)} \\ &= \frac{1}{(4 + x^2) \sqrt{\tan^{-1}\left(\frac{x}{2}\right)}} \end{aligned}$$

So,

$$\frac{d}{dx} \left(\sqrt{\tan^{-1}\left(\frac{x}{2}\right)} \right) = \frac{1}{(4 + x^2) \sqrt{\tan^{-1}\left(\frac{x}{2}\right)}}.$$

Differentiation Ex 11.2 Q38

$$\text{Let } y = \log(\tan^{-1} x)$$

Differentiate with respect to x ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \log(\tan^{-1} x) \\ &= \frac{1}{\tan^{-1} x} \times \frac{d}{dx} (\tan^{-1} x) && [\text{Using chain rule}] \\ &= \frac{1}{(1 + x^2) \tan^{-1} x} \end{aligned}$$

So,

$$\frac{d}{dx} (\log \tan^{-1} x) = \frac{1}{(1 + x^2) \tan^{-1} x}.$$

Differentiation Ex 11.2 Q39

Let $y = \frac{2^x \cos x}{(x^2 + 3)^2}$

Differentiating with respect to x ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \left[\frac{2^x \cos x}{(x^2 + 3)^2} \right] \\ &= \left[\frac{(x^2 + 3)^2 \frac{d}{dx}(2^x \cos x) - (2^x \cos x) \frac{d}{dx}(x^2 + 3)^2}{[(x^2 + 3)^2]^2} \right] \\ &\quad \text{[Using quotient rule, product rule and chain rule]} \\ &= \left[\frac{(x^2 + 3)^2 \left[2^x \frac{d}{dx} \cos x + \cos x \frac{d}{dx} 2^x \right] - (2^x \cos x) 2(x^2 + 3) \frac{d}{dx}(x^2 + 3)}{(x^2 + 3)^4} \right] \\ &= \left[\frac{(x^2 + 3)^2 [-2^x \sin x + \cos x 2^x \log 2] - 2(2^x \cos x)(x^2 + 3)(2x)}{(x^2 + 3)^4} \right] \\ &= \left[\frac{2^x (x^2 + 3) [(x^2 + 3)(\cos x \log 2 - \sin x)] - 4x \cos x}{(x^2 + 3)^4} \right] \\ &= \frac{2^x}{(x^2 + 3)^2} \left[\cos x \log 2 - \sin x - \frac{4x \cos x}{(x^2 + 3)} \right] \end{aligned}$$

So,

$$\frac{d}{dx} \left(\frac{2^x \cos x}{(x^2 + 3)^2} \right) = \frac{2^x}{(x^2 + 3)^2} \left[\cos x \log 2 - \sin x - \frac{4x \cos x}{(x^2 + 3)} \right].$$

Differentiation Ex 11.2 Q40

Let $y = x \sin 2x + 5^x + k^k + (\tan^2 x)^3$

Differentiate it with respect to x ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} [x \sin 2x + 5^x + k^k + (\tan^2 x)^3] \\ &= \frac{d}{dx} (x \sin 2x) + \frac{d}{dx} (5^x) + \frac{d}{dx} (k^k) + \frac{d}{dx} (\tan^6 x) \\ &= \left[x \frac{d}{dx} (\sin 2x) + \sin 2x \frac{d}{dx} (x) \right] + 5^x \log 5 + 0 + 6 \tan^5 x \frac{d}{dx} (\tan x) \\ &\quad \text{[Using product rule and chain rule]} \\ &= \left[x \cos 2x \frac{d}{dx} (2x) + \sin 2x \right] + 5^x \log 5 + 6 \tan^5 x \sec^2 x \\ &= 2x \cos 2x + \sin 2x + 5^x \log 5 + 6 \tan^5 x \sec^2 x \end{aligned}$$

so,

$$\frac{d}{dx} (x \sin 2x + 5^x + k^k + (\tan^2 x)^3) = 2x \cos 2x + \sin 2x + 5^x \log 5 + 6 \tan^5 x \sec^2 x.$$

Differentiation Ex 11.2 Q41

Let $y = \log(3x + 2) - x^2 \log(2x - 1)$

Differentiate with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} [\log(3x + 2) - x^2 \log(2x - 1)] \\ &= \frac{d}{dx} \log(3x + 2) - \frac{d}{dx} (x^2 \log(2x - 1)) \\ &= \frac{1}{(3x + 2)} \frac{d}{dx} (3x + 2) - \left[x^2 \frac{d}{dx} \log(2x - 1) + \log(2x - 1) \frac{d}{dx} (x^2) \right] \\ &\quad \text{[Using product rule and chain rule]} \\ &= \frac{3}{3x + 2} - \left[x^2 \times \frac{1}{(2x - 1)} \frac{d}{dx} (2x - 1) + \log(2x - 1) \times 2x \right] \\ &= \frac{3}{3x + 2} - \frac{2x^2}{(2x - 1)} - 2x \log(2x - 1)\end{aligned}$$

So,

$$\frac{d}{dx} (\log(3x + 2) - x^2 \log(2x - 1)) = \frac{3}{3x + 2} - \frac{2x^2}{(2x - 1)} - 2x \log(2x - 1).$$

Differentiation Ex 11.2 Q42

Let $y = \frac{3x^2 \sin x}{\sqrt{7 - x^2}}$

Differentiate it with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\frac{3x^2 \sin x}{(7 - x^2)^{\frac{1}{2}}} \right) \\ &= \frac{(7 - x^2)^{\frac{1}{2}} \times \frac{d}{dx} (3x^2 \sin x) - 3x^2 \sin x \frac{d}{dx} (7 - x^2)^{\frac{1}{2}}}{\left[(7 - x^2)^{\frac{1}{2}} \right]^2} \\ &\quad \text{[Using quotient rule, chain and product rule]} \\ &= \left[\frac{(7 - x^2)^{\frac{1}{2}} \times 3 \times \left[x^2 \frac{d}{dx} \sin x + \sin x \frac{d}{dx} x^2 \right] - 3x^2 \sin x \times \frac{1}{2} (7 - x^2)^{-\frac{1}{2}} \frac{d}{dx} (7 - x^2)}{(7 - x^2)} \right] \\ &= \left[\frac{(7 - x^2)^{\frac{1}{2}} 3 (x^2 \cos x + 2x \sin x) - 3x^2 \sin x \times \frac{1}{2} (7 - x^2)^{-\frac{1}{2}} (-2x)}{(7 - x^2)} \right] \\ &= \left[\frac{(7 - x^2)^{\frac{1}{2}} \times 3 (x^2 \cos x + 2x \sin x)}{(7 - x^2)} + \frac{3x^3 \sin x (7 - x^2)^{-\frac{1}{2}}}{(7 - x^2)} \right] \\ &= \left[\frac{6x \sin x + 3x^2 \cos x}{\sqrt{7 - x^2}} + \frac{3x^3 \sin x}{(7 - x^2)^{\frac{3}{2}}} \right]\end{aligned}$$

So,

$$\frac{d}{dx} \left(\frac{3x^2 \sin x}{\sqrt{7 - x^2}} \right) = \left[\frac{6x \sin x + 3x^2 \cos x}{\sqrt{7 - x^2}} + \frac{3x^3 \sin x}{(7 - x^2)^{\frac{3}{2}}} \right].$$

Differentiation Ex 11.2 Q43

Let $y = \sin^2 [\log(2x + 3)]$

Differentiate with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} [\sin^2 (\log(2x + 3))] \\ &= 2 \sin (\log(2x + 3)) \frac{d}{dx} \sin (\log(2x + 3)) \quad \text{Using chain rule} \\ &= 2 \sin (\log(2x + 3)) \cos (\log(2x + 3)) \frac{d}{dx} \log(2x + 3) \\ &= \sin (2 \log(2x + 3)) \times \frac{1}{(2x + 3)} \frac{d}{dx} (2x + 3) \\ &\quad \left[\text{Since, } 2 \sin A \cos A = \sin^2 A \right] \\ &= \sin (2 \log(2x + 3)) \times \frac{2}{(2x + 3)}\end{aligned}$$

So,

$$\frac{d}{dx} (\sin^2 \log(2x + 3)) = \sin (2 \log(2x + 3)) \times \frac{2}{(2x + 3)}.$$

Differentiation Ex 11.2 Q44

Let $y = e^x \log \sin 2x$

Differentiate with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} [e^x \log \sin 2x] \\ &= e^x \frac{d}{dx} \log \sin 2x + \log \sin 2x \frac{d}{dx} (e^x) \\ &\quad \left[\text{Using product rule and chain rule} \right] \\ &= e^x \frac{1}{\sin 2x} \frac{d}{dx} (\sin 2x) + \log \sin 2x (e^x) \\ &= \frac{e^x}{\sin 2x} \cos 2x \frac{d}{dx} (2x) + e^x \log \sin 2x \\ &= \frac{2 \cos 2x e^x}{\sin 2x} + e^x \log \sin 2x \\ &= e^x (2 \cot 2x + \log \sin 2x)\end{aligned}$$

so,

$$\frac{d}{dx} (e^x \log \sin 2x) = e^x (2 \cot 2x + \log \sin 2x).$$

Differentiation Ex 11.2 Q45

$$\text{Let } y = \frac{\sqrt{x^2+1} + \sqrt{x^2-1}}{\sqrt{x^2+1} - \sqrt{x^2-1}}$$

$$\Rightarrow y = \frac{(x^2+1)^{\frac{1}{2}} + (x^2-1)^{\frac{1}{2}}}{(x^2+1)^{\frac{1}{2}} - (x^2-1)^{\frac{1}{2}}}$$

Differentiate it with respect to x ,

$$\frac{dy}{dx} = \frac{d}{dx} \left[\frac{(x^2+1)^{\frac{1}{2}} + (x^2-1)^{\frac{1}{2}}}{(x^2+1)^{\frac{1}{2}} - (x^2-1)^{\frac{1}{2}}} \right]$$

$$= \left[\frac{\left\{ (x^2+1)^{\frac{1}{2}} - (x^2-1)^{\frac{1}{2}} \right\} \frac{d}{dx} \left\{ (x^2+1)^{\frac{1}{2}} + (x^2-1)^{\frac{1}{2}} \right\} - \left\{ (x^2+1)^{\frac{1}{2}} + (x^2-1)^{\frac{1}{2}} \right\} \frac{d}{dx} \left\{ (x^2+1)^{\frac{1}{2}} - (x^2-1)^{\frac{1}{2}} \right\}}{\left\{ (x^2+1)^{\frac{1}{2}} - (x^2-1)^{\frac{1}{2}} \right\}^2} \right]$$

[Using quotient rule and chain rule]

$$= \left[\frac{\left\{ (x^2+1)^{\frac{1}{2}} - (x^2-1)^{\frac{1}{2}} \right\} \left[\frac{1}{2} (x^2+1)^{-\frac{1}{2}} \frac{d}{dx} (x^2+1) + \frac{1}{2} (x^2-1)^{-\frac{1}{2}} \frac{d}{dx} (x^2-1) \right]}{\left[(x^2+1) + (x^2-1) - 2\sqrt{x^4-1} \right]} \right]$$

$$- \left[\frac{\left\{ (x^2+1)^{\frac{1}{2}} + (x^2-1)^{\frac{1}{2}} \right\} \left[\frac{1}{2} (x^2+1)^{-\frac{1}{2}} \frac{d}{dx} (x^2+1) - \frac{1}{2} (x^2-1)^{-\frac{1}{2}} \frac{d}{dx} (x^2-1) \right]}{\left[(x^2+1)(x^2-1) - 2\sqrt{x^4-1} \right]} \right]$$

$$\begin{aligned}
&= \left[\frac{\left(\sqrt{x^2+1} - \sqrt{x^2-1} \right) \left(\frac{2x}{2\sqrt{x^2+1}} + \frac{2x}{2\sqrt{x^2-1}} \right)}{\left[2x^2 - 2\sqrt{x^4-1} \right]} \right] - \\
&\quad \left[\frac{\left(\sqrt{x^2+1} + \sqrt{x^2-1} \right) \left(\frac{2x}{2\sqrt{x^2+1}} - \frac{2x}{2\sqrt{x^2-1}} \right)}{\left[2x^2 - 2\sqrt{x^4-1} \right]} \right] \\
&= \left[\frac{x \left(\sqrt{x^2+1} - \sqrt{x^2-1} \right) \left(\sqrt{x^2-1} + \sqrt{x^2+1} \right)}{2 \left[x^2 - \sqrt{x^4-1} \right] \left(\sqrt{x^2+1} \sqrt{x^2-1} \right)} \right] - \\
&\quad \left[\frac{x \left(\sqrt{x^2+1} + \sqrt{x^2-1} \right) \left(\sqrt{x^2-1} - \sqrt{x^2+1} \right)}{2 \left[x^2 - \sqrt{x^4-1} \right] \left(\sqrt{x^2+1} \sqrt{x^2-1} \right)} \right] \\
&= \left[\frac{x \left(x^2+1 - x^2+1 \right) - x \left(x^2-1 - x^2-1 \right)}{2 \left[x^2 - \sqrt{x^4-1} \right] \sqrt{x^4-1}} \right] \\
&= \left[\frac{4x}{2 \left(x^2 - \sqrt{x^4-1} \right) \sqrt{x^4-1}} \right] \\
&= 2x \left[\frac{1 \times \left(x^2 + \sqrt{x^4-1} \right)}{\left(x^2 - \sqrt{x^4-1} \right) \sqrt{x^4-1} \times \left(x^2 + \sqrt{x^4-1} \right)} \right]
\end{aligned}$$

Multiplying and divide by $\left\{ x^2 + \sqrt{x^4-1} \right\}$,

$$\begin{aligned}
&= 2x \left[\frac{x^2 + \sqrt{x^4-1}}{\left(x^4 - x^4 + 1 \right) \sqrt{x^4-1}} \right] \\
&= 2x \left[\frac{x^2 + \sqrt{x^4-1}}{\sqrt{x^4-1}} \right] \\
&= \frac{2x^3}{\sqrt{x^4-1}} + 2x
\end{aligned}$$

So,

$$\frac{d}{dx} \left[\frac{\sqrt{x^2+1} + \sqrt{x^2-1}}{\sqrt{x^2+1} - \sqrt{x^2-1}} \right] = \frac{2x^3}{\sqrt{x^4-1}} + 2x.$$

Differentiation Ex 11.2 Q46

Let $y = \log \left[x + 2 + \sqrt{x^2 + 4x + 1} \right]$

Differentiate with respect to x ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \log \left[x + 2 + \sqrt{x^2 + 4x + 1} \right] \\ &= \frac{1}{x + 2 + \sqrt{x^2 + 4x + 1}} \frac{d}{dx} \left[x + 2 + (x^2 + 4x + 1)^{\frac{1}{2}} \right] \\ &\quad \text{[using chain rule]} \\ &= \frac{1}{x + 2 + \sqrt{x^2 + 4x + 1}} \times \left[1 + 0 + \frac{1}{2} (x^2 + 4x + 1)^{-\frac{1}{2}} \frac{d}{dx} (x^2 + 4x + 1) \right] \\ &= \frac{1 + \frac{(2x + 4)}{2(\sqrt{x^2 + 4x + 1})}}{x + 2 + \sqrt{x^2 + 4x + 1}} \\ &= \frac{\sqrt{x^2 + 4x + 1} + x + 2}{[x + 2 + \sqrt{x^2 + 4x + 1}] \times \sqrt{x^2 + 4x + 1}} \\ &= \frac{1}{\sqrt{x^2 + 4x + 1}} \end{aligned}$$

So,

$$\frac{d}{dx} \left[\log \left(x + 2 + \sqrt{x^2 + 4x + 1} \right) \right] = \frac{1}{\sqrt{x^2 + 4x + 1}}.$$

Differentiation Ex 11.2 Q47

Let $y = (\sin^{-1} x^4)^4$

Differentiate with respect to x ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} (\sin^{-1} x^4)^4 \\ &= 4 (\sin^{-1} x^4)^3 \frac{d}{dx} (\sin^{-1} x^4) && \text{[Using chain rule]} \\ &= 4 (\sin^{-1} x^4)^3 \frac{1}{\sqrt{1 - (x^4)^2}} \frac{d}{dx} (x^4) \\ &= 4 (\sin^{-1} x^4)^3 \frac{4x^3}{\sqrt{1 - x^8}} \\ &= \frac{16x^3 (\sin^{-1} x^4)^3}{\sqrt{1 - x^8}} \end{aligned}$$

So,

$$\frac{d}{dx} (\sin^{-1} x^4) = \frac{16x^3 (\sin^{-1} x^4)^3}{\sqrt{1 - x^8}}.$$

Differentiation Ex 11.2 Q48

Let $y = \sin^{-1}\left(\frac{x}{\sqrt{x^2 + a^2}}\right)$

Differentiating with respect to x ,

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} \sin^{-1}\left(\frac{x}{\sqrt{x^2 + a^2}}\right) \\
 &= \frac{1}{\sqrt{1 - \left(\frac{x}{\sqrt{x^2 + a^2}}\right)^2}} \times \frac{d}{dx} \left(\frac{x}{\sqrt{x^2 + a^2}}\right) \quad [\text{Using chain rule and quotient rule}] \\
 &= \frac{1}{\sqrt{1 - \left(\frac{x}{\sqrt{x^2 + a^2}}\right)^2}} \times \left[\frac{(x^2 + a^2)^{\frac{1}{2}} \frac{d}{dx}(x) - \frac{d}{dx}(x^2 + a^2)^{\frac{1}{2}}}{\left[(x^2 + a^2)^{\frac{1}{2}}\right]^2} \right] \\
 &= \frac{\sqrt{x^2 + a^2}}{\sqrt{x^2 + a^2 - x^2}} \left[\frac{\sqrt{x^2 + a^2} - x \times \frac{1}{2\sqrt{x^2 + a^2}} \frac{d}{dx}(x^2 + a^2)}{(x^2 + a^2)} \right] \\
 &= \frac{\sqrt{x^2 + a^2}}{a(x^2 + a^2)} \left[\sqrt{x^2 + a^2} - \frac{x}{2\sqrt{x^2 + a^2}} \times 2x \right] \\
 &= \frac{\sqrt{x^2 + a^2}}{a(x^2 + a^2)} \left[\frac{x^2 + a^2 - x^2}{\sqrt{x^2 + a^2}} \right] \\
 &= \frac{a^2}{a(x^2 + a^2)} \\
 &= \frac{a}{(a^2 + x^2)}
 \end{aligned}$$

So,

$$\frac{d}{dx} \left(\sin^{-1} \frac{x}{\sqrt{x^2 + a^2}} \right) = \frac{a}{a^2 + x^2}$$

Differentiation Ex 11.2 Q49

Consider

$$y = \frac{e^x \sin x}{(x^2 + 2)^3}$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{(x^2 + 2)^3 \frac{d}{dx}(e^x \sin x) - e^x \sin x \frac{d}{dx}(x^2 + 2)^3}{\left[(x^2 + 2)^3\right]^2} \\
 &= \frac{(x^2 + 2)^3 [e^x \cos x + \sin x e^x] - e^x \sin x (x^2 + 2)^2 (2x)}{(x^2 + 2)^6} \\
 &= \frac{(x^2 + 2)^3 [e^x \cos x + \sin x e^x] - 6x e^x \sin x (x^2 + 2)^2}{(x^2 + 2)^6} \\
 &= \frac{(x^2 + 2)^2 [(x^2 + 2)(e^x \cos x + \sin x e^x) - 6x e^x \sin x]}{(x^2 + 2)^6} \\
 &= \frac{x^2 e^x \cos x + x^2 \sin x e^x + 2e^x \cos x + 2 \sin x e^x - 6x e^x \sin x}{(x^2 + 2)^4} \\
 &= \frac{e^x \sin x}{(x^2 + 2)^3} + \frac{e^x \cos x}{(x^2 + 2)^3} - \frac{6x e^x \sin x}{(x^2 + 2)^4}
 \end{aligned}$$

Therefore,

$$\frac{dy}{dx} = \frac{e^x \sin x}{(x^2 + 2)^3} + \frac{e^x \cos x}{(x^2 + 2)^3} - \frac{6x e^x \sin x}{(x^2 + 2)^4}$$

Differentiation Ex 11.2 Q50

Consider

$$y = 3e^{-2x} \log(1+x)$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\begin{aligned}\frac{dy}{dx} &= 3 \frac{d}{dx} [e^{-2x} \log(1+x)] \\ \frac{dy}{dx} &= 3 \left(e^{-2x} \frac{1}{1+x} + \log(1+x) (-3e^{-2x}) \right) \\ &= 3 \left(\frac{e^{-2x}}{1+x} - 3e^{-2x} \log(1+x) \right) \\ &= 3e^{-2x} \left(\frac{1}{1+x} - 3 \log(1+x) \right)\end{aligned}$$

Differentiation Ex 11.2 Q51

Consider

$$y = \frac{x^2 + 2}{\sqrt{\cos x}}$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\begin{aligned}\frac{dy}{dx} &= \frac{\sqrt{\cos x} \frac{d}{dx} (x^2 + 2) - (x^2 + 2) \frac{d}{dx} \sqrt{\cos x}}{(\sqrt{\cos x})^2} \\ &= \frac{2x\sqrt{\cos x} - (x^2 + 2) \left(-\frac{1}{2} \frac{\sin x}{\sqrt{\cos x}} \right)}{\cos x} \\ &= \frac{2x\sqrt{\cos x} + \frac{(x^2 + 2) \sin x}{2\sqrt{\cos x}}}{\cos x} \\ &= \frac{4x \cos x + (x^2 + 2) \sin x}{2(\cos x)^{\frac{3}{2}}} \\ &= \frac{2x}{\sqrt{\cos x}} + \frac{1}{2} \frac{(x^2 + 2) \sin x}{(\cos x)^{\frac{3}{2}}}\end{aligned}$$

Therefore,

$$\frac{dy}{dx} = \frac{2x}{\sqrt{\cos x}} + \frac{1}{2} \frac{(x^2 + 2) \sin x}{(\cos x)^{\frac{3}{2}}}$$

Differentiation Ex 11.2 Q52

Consider

$$y = \frac{x^2(1-x^2)^3}{\cos 2x}$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\begin{aligned}\frac{dy}{dx} &= \frac{\cos 2x \frac{d}{dx} x^2(1-x^2)^3 - x^2(1-x^2)^3 \frac{d}{dx} \cos 2x}{\cos^2 2x} \\ &= \frac{\cos 2x \left[x^2 \frac{d}{dx} (1-x^2)^3 + (1-x^2)^3 \frac{d}{dx} x^2 - x^2(1-x^2)^3 (-2 \sin 2x) \right]}{\cos^2 2x} \\ &= \frac{\cos 2x [-6x^3(1-x^2)^2 + (1-x^2)^3 2x + 2x^2(1-x^2)^3 \sin 2x]}{\cos^2 2x} \\ &= \frac{2x(1-x^2)^3}{\cos 2x} - \frac{6x^3(1-x^2)^2}{\cos 2x} + \frac{2x^2(1-x^2)^3 \sin 2x}{\cos^2 2x} \\ &= 2x(1-x^2)^3 \sec 2x \{1 - 4x^2 + x(1-x^2) \tan 2x\}\end{aligned}$$

Therefore,

$$\frac{dy}{dx} = 2x(1-x^2)^3 \sec 2x \{1 - 4x^2 + x(1-x^2) \tan 2x\}$$

Differentiation Ex 11.2 Q53

Consider

$$y = \log(3x+2) - x^2 \log(2x-1)$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} [\log(3x+2) - x^2 \log(2x-1)] \\ \frac{dy}{dx} &= \frac{3}{3x+2} - \left[x^2 \frac{d}{dx} \log(2x-1) + \log(2x-1) \frac{d}{dx} x^2 \right] \\ \frac{dy}{dx} &= \frac{3}{3x+2} - \left(\frac{2x^2}{2x-1} + 2x \log(2x-1) \right) \\ \frac{dy}{dx} &= \frac{3}{3x+2} - \frac{2x^2}{2x-1} - 2x \log(2x-1)\end{aligned}$$

Therefore,

$$\frac{dy}{dx} = \frac{3}{3x+2} - \frac{2x^2}{2x-1} - 2x \log(2x-1)$$

Differentiation Ex 11.2 Q54

Consider

$$y = e^{ax} \sec x \tan 2x$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} (e^{ax} \sec x \tan 2x) \\ &= e^{ax} \frac{d}{dx} \sec x \tan 2x + \sec x \tan 2x \frac{d}{dx} e^{ax} \\ &= e^{ax} [\sec x \tan x \tan 2x + (2 + 2 \tan^2 2x) \sec x] + a e^{ax} \sec x \tan 2x \\ &= e^{ax} [\sec x \tan x \tan 2x + 2 \sec x + 2 \tan^2 2x \sec x] + a e^{ax} \sec x \tan 2x \\ &= a e^{ax} \sec x \tan 2x + e^{ax} \sec x \tan x \tan 2x + e^{ax} \sec x (2 + 2 \tan^2 2x) \\ \frac{dy}{dx} &= e^{ax} \sec x \{a \tan 2x + \tan x \tan 2x + 2 \sec^2 2x\} \end{aligned}$$

Therefore,

$$\frac{dy}{dx} = e^{ax} \sec x \{a \tan 2x + \tan x \tan 2x + 2 \sec^2 2x\}$$

Differentiation Ex 11.2 Q55

Consider

$$y = \log(\cos x^2)$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \log(\cos x^2) \\ &= \frac{-2x \sin x^2}{\cos x^2} \\ \frac{dy}{dx} &= -2x \tan x^2 \end{aligned}$$

Therefore,

$$\frac{dy}{dx} = -2x \tan x^2$$

Differentiation Ex 11.2 Q56

Consider

$$y = \cos(\log x)^2$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \cos(\log x)^2 \\ &= -\sin(\log x)^2 \frac{d}{dx} (\log x)^2 \\ &= -\sin(\log x)^2 \frac{2 \log x}{x} \\ \frac{dy}{dx} &= \frac{-2 \log x \sin(\log x)^2}{x} \end{aligned}$$

Therefore,

$$\frac{dy}{dx} = \frac{-2 \log x \sin(\log x)^2}{x}$$

Differentiation Ex 11.2 Q57

Consider

$$y = \log \sqrt{\frac{x-1}{x+1}}$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\begin{aligned} y &= \log \left(\frac{x-1}{x+1} \right)^{\frac{1}{2}} \\ y &= \frac{1}{2} \log \left(\frac{x-1}{x+1} \right) \\ y &= \frac{1}{2} [\log(x-1) - \log(x+1)] \\ \frac{dy}{dx} &= \frac{1}{2} \left[\frac{d}{dx} \log(x-1) - \frac{d}{dx} \log(x+1) \right] \\ &= \frac{1}{2} \left(\frac{1}{x-1} - \frac{1}{x+1} \right) \\ &= \frac{1}{2} \left(\frac{2}{x^2-1} \right) \\ \frac{dy}{dx} &= \frac{1}{x^2-1} \end{aligned}$$

Therefore,

$$\frac{dy}{dx} = \frac{1}{x^2-1}$$

Differentiation Ex 11.2 Q58

Here $y = \log \{\sqrt{x-1} - \sqrt{x+1}\}$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \log \{\sqrt{x-1} - \sqrt{x+1}\} \\ \frac{dy}{dx} &= \frac{1}{\{\sqrt{x-1} - \sqrt{x+1}\}} \frac{d}{dx} (\sqrt{x-1} - \sqrt{x+1}) \\ &= \frac{1}{\{\sqrt{x-1} - \sqrt{x+1}\}} \left[\frac{d}{dx} \sqrt{x-1} - \frac{d}{dx} \sqrt{x+1} \right] \\ &= \frac{1}{\{\sqrt{x-1} - \sqrt{x+1}\}} \left[\frac{1}{2}(x-1)^{-\frac{1}{2}} - \frac{1}{2}(x+1)^{-\frac{1}{2}} \right] \\ &= \frac{1}{2} \frac{1}{\{\sqrt{x-1} - \sqrt{x+1}\}} \left(\frac{1}{\sqrt{x-1}} - \frac{1}{\sqrt{x+1}} \right) \\ &= \frac{1}{2} \frac{1}{\{\sqrt{x-1} - \sqrt{x+1}\}} \left(\frac{-\{\sqrt{x-1} - \sqrt{x+1}\}}{(\sqrt{x-1})(\sqrt{x+1})} \right) \\ &= \frac{-1}{2} \left(\frac{1}{(\sqrt{x-1})(\sqrt{x+1})} \right)\end{aligned}$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{x^2-1}}$$

Therefore,

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{x^2-1}}$$

Differentiation Ex 11.2 Q59

Here $y = \sqrt{x+1} + \sqrt{x-1}$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \sqrt{x+1} + \frac{d}{dx} \sqrt{x-1} \\ &= \frac{1}{2}(x+1)^{-\frac{1}{2}} + \frac{1}{2}(x-1)^{-\frac{1}{2}} \\ &= \frac{1}{2} \left(\frac{1}{\sqrt{x+1}} + \frac{1}{\sqrt{x-1}} \right) \\ &= \frac{1}{2} \left(\frac{\sqrt{x-1} + \sqrt{x+1}}{(\sqrt{x+1})(\sqrt{x-1})} \right) \\ \frac{dy}{dx} &= \frac{1}{2} \left(\frac{y}{\sqrt{x^2-1}} \right) \\ \sqrt{x^2-1} \frac{dy}{dx} &= \frac{1}{2} y\end{aligned}$$

Differentiation Ex 11.2 Q60

Here $y = \frac{x}{x+2}$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\frac{x}{x+2} \right) \\ &= \frac{(x+2) \frac{dx}{dx} - x \frac{d}{dx} (x+2)}{(x+2)^2} \\ &= \frac{x+2-x}{(x+2)^2} \\ &= \frac{x+2}{(x+2)^2} - \frac{x}{(x+2)^2} \\ &= \frac{1}{x+2} - \frac{xy^2}{x^2} \quad \left[\text{Since } x+2 = \frac{x}{y} \right] \\ &= \frac{y}{x} - \frac{y^2}{x} \\ \frac{dy}{dx} &= \frac{1}{x} y(1-y) \\ x \frac{dy}{dx} &= (1-y)y\end{aligned}$$

Differentiation Ex 11.2 Q61

Here $y = \log \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \log \left(x^{\frac{1}{2}} + x^{-\frac{1}{2}} \right) \\ &= \frac{1}{x^{\frac{1}{2}} + x^{-\frac{1}{2}}} \frac{d}{dx} \left(x^{\frac{1}{2}} + x^{-\frac{1}{2}} \right) \\ &= \frac{1}{\sqrt{x} + \frac{1}{\sqrt{x}}} \left(\frac{1}{2} x^{-\frac{1}{2}} - \frac{1}{2} x^{-\frac{3}{2}} \right) \\ &= \frac{1}{2x+1} \left(\frac{1}{\sqrt{x}} - \frac{1}{x\sqrt{x}} \right) \\ &= \frac{1}{2x+1} \left(\frac{x-1}{x\sqrt{x}} \right) \\ \frac{dy}{dx} &= \frac{x-1}{2x(x+1)} \end{aligned}$$

Differentiation Ex 11.2 Q62

Given, $y = \sqrt{\frac{1+e^x}{1-e^x}}$

Differentiate with respect to x ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \left(\sqrt{\frac{1+e^x}{1-e^x}} \right) \\ &= \frac{1}{2\sqrt{\frac{1+e^x}{1-e^x}}} \times \frac{d}{dx} \left(\frac{1+e^x}{1-e^x} \right) \quad \text{[Using chain rule, quotient rule]} \\ &= \frac{1}{2} \times \sqrt{\frac{1-e^x}{1+e^x}} \left[\frac{(1-e^x) \frac{d}{dx}(1+e^x) - (1+e^x) \frac{d}{dx}(1-e^x)}{(1-e^x)^2} \right] \\ &= \frac{1}{2} \sqrt{\frac{1-e^x}{1+e^x}} \left[\frac{(1-e^x)e^x + (1+e^x)e^x}{(1-e^x)^2} \right] \\ &= \frac{1}{2} \sqrt{\frac{1-e^x}{1+e^x}} \times \left[\frac{2e^x}{(1-e^x)^2} \right] \\ &= \frac{e^x}{\sqrt{(1+e^x)}\sqrt{(1-e^x)}} \times \frac{1}{(1-e^x)} \\ \frac{dy}{dx} &= \frac{e^x}{(1-e^x)\sqrt{1-e^{2x}}} \end{aligned}$$

Differentiation Ex 11.2 Q63

Given, $y = \sqrt{x} + \frac{1}{\sqrt{x}}$

Differentiate with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) \\ &= \frac{d}{dx} (\sqrt{x}) + \frac{d}{dx} \left(x^{-\frac{1}{2}} \right) \\ &= \frac{1}{2\sqrt{x}} + \left(-\frac{1}{2} \times x^{-\frac{1}{2}-1} \right) \\ &= \frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x}} \\ \frac{dy}{dx} &= \frac{x-1}{2x\sqrt{x}} \\ 2x \frac{dy}{dx} &= \frac{x-1}{\sqrt{x}} \\ \Rightarrow 2x \frac{dy}{dx} &= \frac{x}{\sqrt{x}} - \frac{1}{\sqrt{x}} \\ \Rightarrow 2x \frac{dy}{dx} &= \sqrt{x} - \frac{1}{\sqrt{x}}.\end{aligned}$$

Differentiation Ex 11.2 Q64

Given, $y = \frac{x \sin^{-1} x}{\sqrt{1-x^2}}$

Differentiate with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\frac{x \sin^{-1} x}{\sqrt{1-x^2}} \right) \\ &= \left[\frac{\sqrt{1-x^2} \frac{d}{dx} (x \sin^{-1} x) - (x \sin^{-1} x) \frac{d}{dx} (\sqrt{1-x^2})}{(\sqrt{1-x^2})^2} \right] \\ &\quad \text{[Using quotient rule, product rule, chain rule]} \\ &= \left[\frac{\sqrt{1-x^2} \left\{ x \frac{d}{dx} \sin^{-1} x + \sin^{-1} x \frac{d}{dx} (x) \right\} - (x \sin^{-1} x) \frac{1}{2\sqrt{1-x^2}} \frac{d}{dx} (1-x^2)}{(1-x^2)} \right] \\ &= \left[\frac{\sqrt{1-x^2} \left\{ \frac{x}{\sqrt{1-x^2}} + \sin^{-1} x \right\} - \frac{x \sin^{-1} x (-2x)}{2\sqrt{1-x^2}}}{(1-x^2)} \right] \\ &= \left[\frac{x + \sqrt{1-x^2} \sin^{-1} x + \frac{x^2 \sin^{-1} x}{\sqrt{1-x^2}}}{(1-x^2)} \right] \\ (1-x^2) \frac{dy}{dx} &= x + \frac{\sqrt{1-x^2} \sin^{-1} x}{1} + \frac{x^2 \sin^{-1} x}{\sqrt{1-x^2}} \\ \Rightarrow (1-x^2) \frac{dy}{dx} &= x + \left(\frac{(1-x^2) \sin^{-1} x + x^2 \sin^{-1} x}{\sqrt{1-x^2}} \right) \\ \Rightarrow (1-x^2) \frac{dy}{dx} &= x + \left(\frac{\sin^{-1} x - x^2 \sin^{-1} x + x^2 \sin^{-1} x}{\sqrt{1-x^2}} \right) \\ \Rightarrow (1-x^2) \frac{dy}{dx} &= x + \left(\frac{\sin^{-1} x}{\sqrt{1-x^2}} \right) \\ (1-x^2) \frac{dy}{dx} &= x + \frac{y}{x} \quad \left\{ \text{Since, given } y = \frac{x \sin^{-1} x}{\sqrt{1-x^2}} \right\}.\end{aligned}$$

Differentiation Ex 11.2 Q65

Given, $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

Differentiating with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\frac{e^x - e^{-x}}{e^x + e^{-x}} \right) \\ &= \left[\frac{(e^x + e^{-x}) \frac{d}{dx}(e^x - e^{-x}) - (e^x - e^{-x}) \frac{d}{dx}(e^x + e^{-x})}{(e^x + e^{-x})^2} \right] \\ &\quad \text{[Using quotient rule and chain rule]} \\ &= \left[\frac{(e^x + e^{-x}) \left[e^x - e^{-x} \frac{d}{dx}(-x) - (e^x - e^{-x}) \left(e^x + e^{-x} \frac{d}{dx}(-x) \right) \right]}{(e^x + e^{-x})^2} \right] \\ &= \left[\frac{(e^x + e^{-x}) (e^x + e^{-x}) - (e^x - e^{-x}) (e^x - e^{-x})}{(e^x + e^{-x})^2} \right] \\ &= \left[\frac{e^{2x} + e^{-2x} + 2e^x \times e^{-x} - e^{2x} - e^{-2x} + 2e^x e^{-x}}{(e^x + e^{-x})^2} \right] \\ \frac{dy}{dx} &= \left[\frac{4}{(e^x + e^{-x})^2} \right] \quad \text{---(i)}\end{aligned}$$

Now,

$$\begin{aligned}1 - y^2 &= 1 - \left(\frac{e^x - e^{-x}}{e^x + e^{-x}} \right)^2 \\ &= 1 - \frac{(e^x - e^{-x})^2}{(e^x + e^{-x})^2} \\ &= \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{(e^x + e^{-x})^2} \\ &= \frac{4}{(e^x + e^{-x})^2}\end{aligned}$$

Differentiation Ex 11.2 Q66

Given, $y = (x-1)\log(x-1) - (x+1)\log(x+1)$

Differentiating with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} [(x-1)\log(x-1) - (x+1)\log(x+1)] \\ &= \left[(x-1) \frac{d}{dx} \log(x-1) + \log(x-1) \frac{d}{dx}(x-1) \right] - \\ &\quad \left[(x+1) \frac{d}{dx} \log(x+1) + \log(x+1) \frac{d}{dx}(x+1) \right] \\ &\quad \text{[Using product rule, chain rule]} \\ &= \left[(x-1) \times \frac{1}{(x-1)} \frac{d}{dx}(x-1) + \log(x-1) \times (1) \right] - \\ &\quad \left[(x+1) \frac{1}{(x+1)} \times \frac{d}{dx}(x+1) + \log(x+1)(1) \right] \\ &= [(1) + \log(x-1)] - [1 + \log(x+1)] \\ &= \log(x-1) - \log(x+1)\end{aligned}$$

$$\frac{dy}{dx} = \log \left(\frac{x-1}{x+1} \right)$$

$$\left[\text{Since, } \log \left(\frac{a}{b} \right) = \log a - \log b \right].$$

Differentiation Ex 11.2 Q67

Given, $y = e^x \cos x$

Differentiating with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} (e^x \cos x) \\&= e^x \frac{d}{dx} \cos x + \cos x \frac{d}{dx} e^x && \text{[Using product rule]} \\&= e^x (-\sin x) + e^x \cos x \\&= e^x (\cos x - \sin x) \\&= \sqrt{2}e^x \left(\frac{\cos x}{\sqrt{2}} - \frac{\sin x}{\sqrt{2}} \right) && \text{[Multiplying and dividing by } \sqrt{2}] \\&= \sqrt{2}e^x \left(\cos \frac{\pi}{4} \cos x - \sin \frac{\pi}{4} \sin x \right) \\&= \sqrt{2}e^x \cos \left(x + \frac{\pi}{4} \right).\end{aligned}$$

Differentiation Ex 11.2 Q68

$$\begin{aligned}\text{Given, } y &= \frac{1}{2} \log \left(\frac{1 - \cos 2x}{1 + \cos 2x} \right) \\ \Rightarrow y &= \frac{1}{2} \log \left(\frac{2 \sin^2 x}{2 \cos^2 x} \right) && \left[\begin{array}{l} \text{Since, } 1 - \cos 2x = 2 \sin^2 x, \\ 1 + \cos 2x = 2 \cos^2 x \end{array} \right] \\ \Rightarrow y &= \frac{1}{2} \log (\tan^2 x) \\ \Rightarrow y &= \frac{2}{2} \log \tan x && \text{[Since, } \log a^b = b \log a] \\ \Rightarrow y &= \log \tan x\end{aligned}$$

Differentiate it with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= (\log \tan x) \\&= \frac{1}{\tan x} \times \frac{d}{dx} (\tan x) && \text{[Using chain rule]} \\&= \frac{\sec^2 x}{\tan x} \\&= \frac{1}{\cos^2 x \times \frac{\sin x}{\cos x}} \\&= \frac{1}{\sin x \cos x} \\&= \frac{2}{2 \sin x \cos x} \\&= \frac{2}{\sin 2x} && \left[\text{Since, } \frac{1}{\sin x} = \operatorname{cosec} x \right]\end{aligned}$$

So,

$$\frac{dy}{dx} = 2 \operatorname{cosec} 2x.$$

Differentiation Ex 11.2 Q69

Here, $y = x \sin^{-1} x + \sqrt{1-x^2}$

Differentiating with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left[x \sin^{-1} x + \sqrt{1-x^2} \right] \\ &= \frac{d}{dx} (x \sin^{-1} x) + \frac{d}{dx} (\sqrt{1-x^2}) \\ &= \left[x \frac{d}{dx} \sin^{-1} x + \sin^{-1} x \frac{d}{dx} (x) \right] + \frac{1}{2\sqrt{1-x^2}} \frac{d}{dx} (1-x^2)\end{aligned}$$

[Using product rule and chain rule]

$$\begin{aligned}&= \left[\frac{x}{\sqrt{1-x^2}} + \sin^{-1} x \right] - \frac{2x}{2\sqrt{1-x^2}} \\ &= \frac{x}{\sqrt{1-x^2}} + \sin^{-1} x - \frac{x}{\sqrt{1-x^2}} \\ &= \sin^{-1} x\end{aligned}$$

So,

$$\frac{dy}{dx} = \sin^{-1} x.$$

Differentiation Ex 11.2 Q70

Here, $y = \sqrt{x^2 + a^2}$

Differentiating with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} (\sqrt{x^2 + a^2}) \\ &= \frac{1}{2\sqrt{x^2 + a^2}} \frac{d}{dx} (x^2 + a^2) && \text{[Using chain rule]} \\ &= \frac{1}{2\sqrt{x^2 + a^2}} \times (2x) \\ &= \frac{x}{\sqrt{x^2 + a^2}}\end{aligned}$$

$$\frac{dy}{dx} = \frac{x}{y} \quad \left[\text{Since } \sqrt{x^2 + a^2} = y \right]$$

$$\Rightarrow y \frac{dy}{dx} = x$$

$$\Rightarrow y \frac{dy}{dx} - x = 0.$$

Differentiation Ex 11.2 Q71

Here, $y = e^x + e^{-x}$

Differentiating with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} (e^x + e^{-x}) \\ &= \frac{d}{dx} e^x + \frac{d}{dx} e^{-x} \\ &= e^x + e^{-x} \frac{d}{dx} (-x) && \text{[Using chain rule]} \\ &= e^x + e^{-x} (-1) \\ &= (e^x - e^{-x}) \\ &= \sqrt{(e^x + e^{-x})^2 - 4e^x \times e^{-x}} && \left[\text{Since } (a-b)^2 = (a+b)^2 - 4ab \right] \\ &= \sqrt{y^2 - 4} && \left[\text{Since } e^x + e^{-x} = y \right]\end{aligned}$$

So,

$$\frac{dy}{dx} = \sqrt{y^2 - 4}.$$

Differentiation Ex 11.2 Q72

Given, $y = \sqrt{a^2 - x^2}$

Differentiating with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\sqrt{a^2 - x^2} \right) \\ &= \frac{1}{2\sqrt{a^2 - x^2}} \frac{d}{dx} (a^2 - x^2) && \text{[Using chain rule]} \\ &= \frac{1}{2\sqrt{a^2 - x^2}} (-2x) \\ &= \frac{-x}{\sqrt{a^2 - x^2}} \\ \Rightarrow \frac{dy}{dx} &= \frac{-x}{y} && \text{[Since } \sqrt{a^2 - x^2} = y \text{]} \\ \Rightarrow y \frac{dy}{dx} &= -x \\ y \frac{dy}{dx} + x &= 0\end{aligned}$$

Differentiation Ex 11.2 Q73

Here, $xy = 4$

$$\Rightarrow y = \frac{4}{x}$$

Differentiate with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\frac{4}{x} \right) \\ &= 4 \frac{d}{dx} (x^{-1}) \\ &= 4 \left(-1 \times x^{-1-1} \right) \\ &= 4 \left(-\frac{1}{x^2} \right) \\ &= \frac{-4}{x^2} \\ &= -\frac{4}{\left(\frac{x}{y} \right)^2} && \text{[Since } x = \frac{4}{y} \text{]} \\ &= -\frac{4y^2}{16} \\ \frac{dy}{dx} &= -\frac{y^2}{4} \\ \Rightarrow 4 \frac{dy}{dx} &= -y^2 \\ \Rightarrow 4 \frac{dy}{dx} &= 3y^2 - 4y^2 \\ \Rightarrow 4 \frac{dy}{dx} + 4y^2 &= 3y^2 \\ \Rightarrow 4 \left(\frac{dy}{dx} + y^2 \right) &= 3y^2\end{aligned}$$

Dividing both the sides by x ,

$$\begin{aligned}\Rightarrow \frac{4}{x} \left(\frac{dy}{dx} + y^2 \right) &= \frac{3y^2}{x} \\ \Rightarrow y \left(\frac{dy}{dx} + y^2 \right) &= \frac{3y^2}{x} && \text{[Since } \frac{4}{x} = y \text{]} \\ \Rightarrow x \left(\frac{dy}{dx} + y^2 \right) &= \frac{3y^2}{y} \\ \Rightarrow x \left(\frac{dy}{dx} + y^2 \right) &= 3y.\end{aligned}$$

Differentiation Ex 11.2 Q74

$$\frac{d}{dx} \left\{ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right\} = \sqrt{a^2 - x^2}$$

$$\begin{aligned} \text{LHS} &= \frac{d}{dx} \left\{ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right\} \\ &= \frac{d}{dx} \left(\frac{x}{2} \sqrt{a^2 - x^2} \right) + \frac{d}{dx} \left(\frac{a^2}{2} \sin^{-1} \frac{x}{a} \right) \\ &= \frac{1}{2} \left[x \frac{d}{dx} \sqrt{a^2 - x^2} + \sqrt{a^2 - x^2} \frac{d}{dx} (x) \right] + \frac{a^2}{2} \times \frac{1}{\sqrt{1 - \left(\frac{x}{a} \right)^2}} \times \frac{d}{dx} \left(\frac{x}{a} \right) \end{aligned}$$

[Using product rule, chain rule]

$$\begin{aligned} &= \frac{1}{2} \left[x \times \frac{1}{2\sqrt{a^2 - x^2}} \frac{d}{dx} (a^2 - x^2) + \sqrt{a^2 - x^2} \right] + \left(\frac{a^2}{2} \right) \times \frac{1}{\sqrt{\frac{a^2 - x^2}{a^2}}} \times \left(\frac{1}{a} \right) \\ &= \frac{1}{2} \left[\frac{x(-2x)}{2\sqrt{a^2 - x^2}} + \sqrt{a^2 - x^2} \right] + \left(\frac{a^2}{2} \right) \frac{a}{\sqrt{a^2 - x^2}} \times \left(\frac{1}{a} \right) \\ &= \frac{1}{2} \left[\frac{-2x^2 + 2(a^2 - x^2)}{2\sqrt{a^2 - x^2}} \right] + \frac{a^2}{2\sqrt{a^2 - x^2}} \\ &= \frac{1}{2} \left[\frac{2(a^2 - 2x^2)}{2\sqrt{a^2 - x^2}} \right] + \frac{a^2}{2\sqrt{a^2 - x^2}} \\ &= \frac{a^2 - 2x^2}{2\sqrt{a^2 - x^2}} + \frac{a^2}{2\sqrt{a^2 - x^2}} \\ &= \frac{a^2 - 2x^2 + a^2}{2\sqrt{a^2 - x^2}} \\ &= \frac{2a^2 - 2x^2}{2\sqrt{a^2 - x^2}} \\ &= \frac{2(a^2 - x^2)}{2\sqrt{a^2 - x^2}} \\ &= \frac{(a^2 - x^2)}{\sqrt{a^2 - x^2}} \\ &= \sqrt{a^2 - x^2} \\ &= \text{RHS} \end{aligned}$$

Differentiation Ex 11.3 Q1

$$\text{Let } y = \cos^{-1} \left\{ 2x \sqrt{1-x^2} \right\}$$

$$\text{Put } x = \cos \theta$$

$$y = \cos^{-1} \left\{ 2 \cos \theta \sqrt{1 - \cos^2 \theta} \right\}$$

$$= \cos^{-1} \{ 2 \cos \theta \sin \theta \}$$

$$y = \cos^{-1} \{ \sin 2\theta \}$$

$$[\text{Since } \sin 2\theta = 2 \sin \theta \cos \theta, \sin^2 \theta + \cos^2 \theta = 1]$$

$$y = \cos^{-1} \left[\cos \left(\frac{\pi}{2} - \theta \right) \right]$$

$$\text{---(i)}$$

Now,

$$\frac{1}{\sqrt{2}} < x < 1$$

$$\Rightarrow \frac{1}{\sqrt{2}} < \cos \theta < 1$$

$$\Rightarrow 0 < \theta < \frac{\pi}{4}$$

$$\Rightarrow 0 < 2\theta < \frac{\pi}{2}$$

$$\Rightarrow 0 > -2\theta > -\frac{\pi}{2}$$

$$\Rightarrow \frac{\pi}{2} > \left(\frac{\pi}{2} - 2\theta \right) > 0$$

Hence, from equation (i),

$$y = \frac{\pi}{2} - 2\theta$$

$$[\text{Since } \cos^{-1}(\cos \theta) = \theta, \text{ if } \theta \in [0, \pi]]$$

$$y = \frac{\pi}{2} - 2 \cos^{-1} x$$

$$[\text{Since } x = \cos \theta]$$

Differentiating it with respect to x ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \left(\frac{\pi}{2} \right) - 2 \frac{d}{dx} (\cos^{-1} x) \\ &= 0 - 2 \left(\frac{-1}{\sqrt{1-x^2}} \right) \end{aligned}$$

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q2

$$\text{Let } y = \cos^{-1} \left\{ \sqrt{\frac{1+x}{2}} \right\}$$

$$\text{Put } x = \cos 2\theta$$

$$y = \cos^{-1} \left\{ \sqrt{\frac{1 + \cos 2\theta}{2}} \right\}$$

$$= \cos^{-1} \left\{ \sqrt{\frac{2 \cos^2 \theta}{2}} \right\}$$

$$y = \cos^{-1} \{ \cos \theta \}$$

$$\text{---(i)}$$

Here, $-1 < x < 1$

$$\rightarrow -1 < \cos 2\theta < 1$$

$$\Rightarrow 0 < 2\theta < \pi$$

$$\Rightarrow 0 < \theta < \frac{\pi}{2}$$

So, from equation (i),

$$y = \theta$$

$$[\text{Since } \cos^{-1}(\cos \theta) = \theta \text{ if } \theta \in [0, \pi]]$$

$$y = \frac{1}{2} \cos^{-1} x$$

$$[\text{Since } x = \cos 2\theta]$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} = - \frac{1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q3

$$\text{Let } y = \sin^{-1} \left\{ \sqrt{\frac{1-x}{2}} \right\}$$

$$\text{Let } x = \cos 2\theta$$

$$y = \sin^{-1} \left\{ \sqrt{\frac{1 - \cos 2\theta}{2}} \right\}$$

$$= \sin^{-1} \left\{ \sqrt{\frac{2 \sin^2 \theta}{2}} \right\}$$

$$y = \sin^{-1}(\sin \theta) \quad \text{---(i)}$$

$$\text{Here, } 0 < x < 1$$

$$\Rightarrow 0 < \cos 2\theta < 1$$

$$\Rightarrow 0 < 2\theta < \frac{\pi}{2}$$

$$\Rightarrow 0 < \theta < \frac{\pi}{4}$$

so, from equation (i),

$$y = \theta$$

$$y = \frac{1}{2} \cos^{-1} x$$

$$\left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right]$$

$$[\text{Since, } x = \cos 2\theta]$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} = -\frac{1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q4

$$\text{Let } y = \sin^{-1} \left\{ \sqrt{1-x^2} \right\}$$

$$\text{Let } x = \cos \theta$$

$$y = \sin^{-1} \left\{ \sqrt{1 - \cos^2 \theta} \right\}$$

$$y = \sin^{-1}(\sin \theta) \quad \text{---(i)}$$

$$\text{Here, } 0 < x < 1$$

$$\Rightarrow 0 < \cos \theta < 1$$

$$\Rightarrow 0 < \theta < \frac{\pi}{2}$$

From equation (i),

$$y = \theta$$

$$y = \cos^{-1} x$$

$$\left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right]$$

$$[\text{Since } x = \cos \theta]$$

Differentiating with respect to x ,

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q5

$$\text{Let } y = \tan^{-1} \left\{ \frac{x}{\sqrt{a^2 - x^2}} \right\}$$

$$\text{Let } x = a \sin \theta$$

$$y = \tan^{-1} \left\{ \frac{a \sin \theta}{\sqrt{a^2 - a^2 \sin^2 \theta}} \right\}$$

$$y = \tan^{-1} \left\{ \frac{a \sin \theta}{\sqrt{a^2 (1 - \sin^2 \theta)}} \right\}$$

$$y = \tan^{-1} \left\{ \frac{a \sin \theta}{a \cos \theta} \right\}$$

$$y = \tan^{-1} (\tan \theta) \quad \text{--- (i)}$$

$$\text{Here, } -a < x < a$$

$$\Rightarrow -1 < \frac{x}{a} < 1$$

$$\Rightarrow -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

From equatoin(i),

$$y = \theta$$

$$\left[\text{Since, } \tan^{-1} (\tan \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right]$$

$$y = \sin^{-1} \left(\frac{x}{a} \right)$$

$$[\text{Since } x = a \sin \theta]$$

Differentiating it with respect to x ,

Using chain rule,

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{\sqrt{1 - \left(\frac{x}{a}\right)^2}} \frac{d}{dx} \left(\frac{x}{a} \right) \\ &= \frac{a}{\sqrt{a^2 - x^2}} \times \left(\frac{1}{a} \right) \end{aligned}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{a^2 - x^2}}.$$

Differentiation Ex 11.3 Q6

$$\text{Let } y = \sin^{-1} \left\{ \frac{x}{\sqrt{x^2 + a^2}} \right\}$$

$$\text{Put } x = a \tan \theta$$

$$y = \sin^{-1} \left\{ \frac{a \tan \theta}{\sqrt{a^2 + \tan^2 \theta + a^2}} \right\}$$

$$= \sin^{-1} \left\{ \frac{a \tan \theta}{\sqrt{a^2 (\tan^2 \theta + 1)}} \right\}$$

$$= \sin^{-1} \left\{ \frac{a \tan \theta}{a \sec \theta} \right\}$$

$$= \sin^{-1} (\sin \theta)$$

$$= \theta$$

$$y = \tan^{-1} \left(\frac{x}{a} \right)$$

$$[x = a \tan \theta]$$

Differentiating it with respect to x using chain rule,

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{1 + \left(\frac{x}{a}\right)^2} \frac{d}{dx} \left(\frac{x}{a} \right) \\ &= \frac{a^2}{a^2 + x^2} \times \left(\frac{1}{a} \right) \end{aligned}$$

$$\frac{dy}{dx} = \frac{a}{a^2 + x^2}.$$

Differentiation Ex 11.3 Q7

$$\begin{aligned}
 \text{Let } y &= \sin^{-1}\{2x^2 - 1\} \\
 \text{Let } x &= \cos \theta \\
 y &= \sin^{-1}\{2 \cos^2 \theta - 1\} \\
 &= \sin^{-1}(\cos 2\theta) \\
 y &= \sin^{-1}\left\{\sin\left(\frac{\pi}{2} - 2\theta\right)\right\} \quad \text{---(i)}
 \end{aligned}$$

$$\begin{aligned}
 \text{Here, } 0 < x < 1 \\
 \Rightarrow 0 < \cos \theta < 1 \\
 \Rightarrow 0 < \theta < \frac{\pi}{2} \\
 \Rightarrow 0 < 2\theta < \pi \\
 \Rightarrow 0 > -2\theta > -\pi \\
 \Rightarrow \frac{\pi}{2} > \left(\frac{\pi}{2} - 2\theta\right) > -\frac{\pi}{2}
 \end{aligned}$$

So, from equatoin (i),

$$\begin{aligned}
 y &= \frac{\pi}{2} - 2\theta & \left[\text{Since, } \sin^{-1}(\cos \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \right] \\
 y &= \frac{\pi}{2} - 2 \cos^{-1} x & [\text{Since } x = \cos \theta]
 \end{aligned}$$

$$\begin{aligned}
 \frac{dy}{dx} &= 0 - 2 \frac{d}{dx}(\cos^{-1} x) \\
 &= -2 \left(-\frac{1}{\sqrt{1-x^2}} \right)
 \end{aligned}$$

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q8

$$\begin{aligned}
 \text{Let } y &= \sin^{-1}\{1 - 2x^2\} \\
 \text{Let } x &= \sin \theta, \text{ So,} \\
 y &= \sin^{-1}\{1 - 2 \sin^2 \theta\} \\
 &= \sin^{-1}(\cos 2\theta) \\
 y &= \sin^{-1}\left\{\sin\left(\frac{\pi}{2} - 2\theta\right)\right\} \quad \text{---(i)}
 \end{aligned}$$

$$\begin{aligned}
 \text{Here, } 0 < x < 1 \\
 \Rightarrow 0 < \sin \theta < 1 \\
 \Rightarrow 0 < \theta < \frac{\pi}{2} \\
 \Rightarrow 0 < 2\theta < \pi \\
 \Rightarrow 0 > -2\theta > -\pi \\
 \Rightarrow \frac{\pi}{2} > \left(\frac{\pi}{2} - 2\theta\right) > \frac{\pi}{2} - \pi \\
 \Rightarrow \frac{\pi}{2} > \left(\frac{\pi}{2} - 2\theta\right) > \left(-\frac{\pi}{2}\right)
 \end{aligned}$$

So, from equatoin (i),

$$\begin{aligned}
 y &= \frac{\pi}{2} - 2\theta & \left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \right] \\
 y &= \frac{\pi}{2} - 2 \sin^{-1} x & [\text{Since } x = \sin \theta]
 \end{aligned}$$

Differentiating with respect to x ,

$$\frac{dy}{dx} = 0 - 2 \left(\frac{1}{\sqrt{1-x^2}} \right)$$

$$\frac{dy}{dx} = -\frac{2}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q9

Let $y = \cos^{-1} \left\{ \frac{x}{\sqrt{x^2 + a^2}} \right\}$

Put $x = a \cot \theta,$

$$y = \cos^{-1} \left\{ \frac{a \cot \theta}{\sqrt{a^2 \cot^2 \theta + a^2}} \right\}$$

$$= \cos^{-1} \left\{ \frac{a \cot \theta}{a \operatorname{cosec} \theta} \right\}$$

$$= \cos^{-1} \left\{ \frac{\cos \theta}{\frac{1}{\sin \theta}} \right\}$$

$$= \cos^{-1} (\cos \theta)$$

$$= \theta$$

$$y = \cot^{-1} \left(\frac{x}{a} \right) \quad \text{[Since, } a \cot \theta = x \text{]}$$

Differentiating it with respect to x using chain rule,

$$\begin{aligned} \frac{dy}{dx} &= \frac{-1}{1 + \left(\frac{x}{a} \right)^2} \frac{d}{dx} \left(\frac{x}{a} \right) \\ &= \frac{-a^2}{a^2 + x^2} \times \left(\frac{1}{a} \right) \end{aligned}$$

$$\frac{dy}{dx} = \frac{-a}{a^2 + x^2}.$$

Differentiation Ex 11.3 Q10

Let $y = \sin^{-1} \left\{ \frac{\sin x + \cos x}{\sqrt{2}} \right\}$

$$= \sin^{-1} \left\{ \sin x \left(\frac{1}{\sqrt{2}} \right) + \cos x \times \left(\frac{1}{\sqrt{2}} \right) \right\}$$

$$= \sin^{-1} \left\{ \sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4} \right\}$$

$$y = \sin^{-1} \left\{ \sin \left(x + \frac{\pi}{4} \right) \right\}$$

Here, $\frac{-3\pi}{4} < x < \frac{\pi}{4}$

$$\Rightarrow \left(\frac{-3\pi}{4} + \frac{\pi}{4} \right) \quad \left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2} \right] \right]$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} = 1 + 0$$

$$\frac{dy}{dx} = 1$$

Differentiation Ex 11.3 Q11

$$\begin{aligned}
 \text{Let } y &= \cos^{-1} \left\{ \frac{\cos x + \sin x}{\sqrt{2}} \right\} \\
 y &= \cos^{-1} \left\{ \cos x \left(\frac{1}{\sqrt{2}} \right) + \sin x \left(\frac{1}{\sqrt{2}} \right) \right\} \\
 &= \cos^{-1} \left\{ \cos x \cos \left(\frac{\pi}{4} \right) + \sin x \sin \left(\frac{\pi}{4} \right) \right\} \\
 y &= \cos^{-1} \left[\cos \left(x - \frac{\pi}{4} \right) \right] \quad \text{---(i)}
 \end{aligned}$$

$$\begin{aligned}
 \text{Here, } -\frac{\pi}{4} < x < \frac{\pi}{4} \\
 \Rightarrow \left(-\frac{\pi}{4} - \frac{\pi}{4} \right) < \left(x - \frac{\pi}{4} \right) < \left(\frac{\pi}{4} - \frac{\pi}{4} \right) \\
 \Rightarrow -\frac{\pi}{2} < \left(x - \frac{\pi}{4} \right) < 0
 \end{aligned}$$

So, from equation (i),

$$\begin{aligned}
 y &= -\left(x - \frac{\pi}{4} \right) & \left[\text{Since, } \cos^{-1}(\cos \theta) = -\theta, \text{ if } \theta \in [-\pi, 0] \right] \\
 y &= -x + \frac{\pi}{4}
 \end{aligned}$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} = -1.$$

Differentiation Ex 11.3 Q12

$$\begin{aligned}
 \text{Let } y &= \tan^{-1} \left\{ \frac{x}{1 + \sqrt{1 - x^2}} \right\} \\
 \text{Put } x &= \sin \theta, \text{ so} \\
 y &= \tan^{-1} \left\{ \frac{\sin \theta}{1 + \sqrt{1 - \sin^2 \theta}} \right\} \\
 &= \tan^{-1} \left\{ \frac{\sin \theta}{1 + \cos \theta} \right\} \\
 &= \tan^{-1} \left\{ \frac{\frac{2 \sin \theta \cos \theta}{2}}{\frac{2 \cos^2 \theta}{2}} \right\} \\
 y &= \tan^{-1} \left\{ \frac{\tan \theta}{2} \right\} \quad \text{---(i)}
 \end{aligned}$$

$$\begin{aligned}
 \text{Here, } -1 < x < 1 \\
 \Rightarrow -1 < \sin \theta < 1 \\
 \Rightarrow -\frac{\pi}{2} < \theta < \frac{\pi}{2} \\
 \Rightarrow -\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{4}
 \end{aligned}$$

So, from equation (i),

$$\begin{aligned}
 y &= \frac{\theta}{2} & \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right] \\
 y &= \frac{1}{2} \sin^{-1} x & \left[\text{Since, } x = \sin \theta \right]
 \end{aligned}$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q13

Let $y = \tan^{-1} \left\{ \frac{x}{a + \sqrt{a^2 - x^2}} \right\}$

Put $x = a \sin \theta$, so

$$y = \tan^{-1} \left\{ \frac{a \sin \theta}{a + \sqrt{a^2 - a^2 \sin^2 \theta}} \right\}$$

$$= \tan^{-1} \left\{ \frac{a \sin \theta}{a + \sqrt{a^2 (1 - \sin^2 \theta)}} \right\}$$

$$= \tan^{-1} \left\{ \frac{a \sin \theta}{a + a \cos \theta} \right\}$$

$$= \tan^{-1} \left\{ \frac{a \sin \theta}{a (1 + \cos \theta)} \right\}$$

$$= \tan^{-1} \left(\frac{\sin \theta}{1 + \cos \theta} \right)$$

$$= \tan^{-1} \left(\frac{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}} \right)$$

$$y = \tan^{-1} \left(\tan \frac{\theta}{2} \right) \quad \text{--- (i)}$$

Here, $-a < x < a$

$$\Rightarrow -1 < \frac{x}{a} < 1$$

$$\Rightarrow -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$\Rightarrow -\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{4}$$

So, from equation (i),

$$y = \frac{\theta}{2}$$

$$\left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right]$$

$$y = \frac{1}{2} + \sin^{-1} \left(\frac{x}{a} \right)$$

$$\left[\text{Since, } x = a \sin \theta \right]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{\sqrt{1 - \left(\frac{x}{a} \right)^2}} \frac{d}{dx} \left(\frac{x}{a} \right)$$

$$= \frac{a}{2\sqrt{a^2 - x^2}} \times \left(\frac{1}{a} \right)$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{a^2 - x^2}}.$$

Differentiation Ex 11.3 Q14

Let $y = \sin^{-1} \left\{ \frac{x + \sqrt{1-x^2}}{\sqrt{2}} \right\}$

Put $x = \sin \theta$, so

$$\begin{aligned} &= \sin^{-1} \left\{ \frac{\sin \theta + \sqrt{1 - \sin^2 \theta}}{\sqrt{2}} \right\} \\ &= \sin^{-1} \left\{ \frac{\sin \theta + \cos \theta}{\sqrt{2}} \right\} \\ &= \sin^{-1} \left\{ \sin \theta \left(\frac{1}{\sqrt{2}} \right) + \cos \theta \left(\frac{1}{\sqrt{2}} \right) \right\} \\ &= \sin^{-1} \left\{ \sin \theta \cos \frac{\pi}{4} + \cos \theta \sin \frac{\pi}{4} \right\} \\ y &= \sin^{-1} \left\{ \sin \left(\theta + \frac{\pi}{4} \right) \right\} \end{aligned}$$

---(i)

Here, $-1 < x < 1$

$\Rightarrow -1 < \sin \theta < 1$

$\Rightarrow -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\Rightarrow \left(-\frac{\pi}{2} + \frac{\pi}{4} \right) < \left(\frac{\pi}{4} + \theta \right) < \frac{3\pi}{4}$

So, from equation (i),

$$y = \theta + \frac{\pi}{4}$$

$$y = \sin^{-1} x + \frac{\pi}{4}$$

$$\left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ as } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right]$$

$$[\text{Since, } \sin \theta = x]$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + 0$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q15

Let $y = \cos^{-1} \left\{ \frac{x + \sqrt{1-x^2}}{\sqrt{2}} \right\}$

Put $x = \sin \theta$, so

$$\begin{aligned} y &= \cos^{-1} \left\{ \frac{\sin \theta + \sqrt{1 - \sin^2 \theta}}{\sqrt{2}} \right\} \\ &= \cos^{-1} \left\{ \frac{\sin \theta + \cos \theta}{\sqrt{2}} \right\} \\ &= \cos^{-1} \left\{ \sin \theta \left(\frac{1}{\sqrt{2}} \right) + \cos \theta \left(\frac{1}{\sqrt{2}} \right) \right\} \\ &= \cos^{-1} \left\{ \sin \theta \times \sin \frac{\pi}{4} + \cos \theta \times \cos \frac{\pi}{4} \right\} \\ y &= \cos^{-1} \left\{ \cos \left(\theta - \frac{\pi}{4} \right) \right\} \end{aligned}$$

---(i)

Here, $-1 < x < 1$

$\Rightarrow -1 < \sin \theta < 1$

$\Rightarrow -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{2} + \frac{\pi}{4} < \left(\theta - \frac{\pi}{4} \right) < \frac{\pi}{2} - \frac{\pi}{4}$

$\Rightarrow \left(-\frac{3\pi}{4} \right) < \left(\theta - \frac{\pi}{4} \right) < \left(\frac{\pi}{4} \right)$

So, from equation (i),

$$y = - \left(\theta - \frac{\pi}{4} \right)$$

[Since, $\cos^{-1}(\cos \theta) = -\theta$, if $\theta \in [-\pi, 0]$]

$$y = -\theta + \frac{\pi}{4}$$

$$y = -\sin^{-1} x + \frac{\pi}{4}$$

[Since, $x = \sin \theta$]

Differentiating it with respect to x ,

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}} + 0$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q16

$$\text{Let } y = \tan^{-1} \left\{ \frac{4x}{1-4x^2} \right\}$$

$$\text{Put } 2x = \tan \theta, \text{ so}$$

$$y = \tan^{-1} \left\{ \frac{2 \tan \theta}{1 - \tan^2 \theta} \right\}$$

$$y = \tan^{-1} \{ \tan 2\theta \} \quad \text{---(i)}$$

$$\text{Here, } -\frac{1}{2} < x < \frac{1}{2}$$

$$\Rightarrow -1 < 2x < 1$$

$$\Rightarrow -1 < \tan \theta < 1$$

$$\Rightarrow -\frac{\pi}{4} < \theta < \frac{\pi}{4}$$

$$\Rightarrow -\frac{\pi}{2} < (2\theta) < \frac{\pi}{2}$$

So, from equation (i),

$$y = 2\theta \quad \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right]$$

$$y = 2 \tan^{-1}(2x) \quad [\text{Since, } 2x = \tan \theta]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = 2 \left(\frac{1}{1+(2x)^2} \right) \frac{d}{dx}(2x)$$

$$\frac{dy}{dx} = \frac{4}{1+4x^2}.$$

Differentiation Ex 11.3 Q17

$$\text{Let } y = \tan^{-1} \left\{ \frac{2^{x+1}}{1-4^x} \right\}$$

$$\text{Put } 2^x = \tan \theta, \text{ so,}$$

$$= \tan^{-1} \left\{ \frac{2^x \times 2}{1 - (2^x)^2} \right\}$$

$$= \tan^{-1} \left\{ \frac{2 \tan \theta}{1 - \tan^2 \theta} \right\}$$

$$y = \tan^{-1} \{ \tan(2\theta) \} \quad \text{---(i)}$$

$$\text{Here, } -\infty < x < \infty$$

$$\Rightarrow 2^{-\infty} < 2^x < 2^{\infty}$$

$$\Rightarrow 0 < 2^x < \infty$$

$$\Rightarrow 0 < \theta < \frac{\pi}{4}$$

$$\Rightarrow 0 < (2\theta) < \frac{\pi}{2}$$

From equation (i),

$$y = 2\theta \quad \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \right]$$

$$y = 2 \tan^{-1}(2^x)$$

Differentiate it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{2}{1+(2^x)^2} \frac{d}{dx}(2^x)$$

$$= \frac{2 \times 2^x \log 2}{1+4^x}$$

$$\frac{dy}{dx} = \frac{2^{x+1} \log 2}{1+4^x}.$$

Differentiation Ex 11.3 Q18

Let $y = \tan^{-1} \left\{ \frac{2a^x}{1-a^{2x}} \right\}$

Put $a^x = \tan \theta$,
 $y = \tan^{-1} \left\{ \frac{2 \tan \theta}{1 - \tan^2 \theta} \right\}$

$$y = \tan^{-1} \{ \tan(2\theta) \} \quad \text{--- (i)}$$

Here, $-\infty < x < 0$

$$\Rightarrow a^{-\infty} < a^x < 2^0$$

$$\Rightarrow 0 < \tan \theta < 1$$

$$\Rightarrow 0 < \theta < \frac{\pi}{4}$$

$$\Rightarrow 0 < (2\theta) < \frac{\pi}{2}$$

From equatoin (i),

$$y = 2\theta$$

$$\left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \right]$$

$$y = 2 \tan^{-1} (a^x)$$

Differentiate it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{2}{1+(a^x)^2} \frac{d}{dx} (a^x)$$

$$\frac{dy}{dx} = \frac{2a^x \log a}{1+a^{2x}}.$$

Differentiation Ex 11.3 Q19

Let $y = \sin^{-1} \left\{ \frac{\sqrt{1+x} + \sqrt{1-x}}{2} \right\}$

Put $x = \cos 2\theta$, so,

$$= \sin^{-1} \left\{ \frac{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}{2} \right\}$$

$$= \sin^{-1} \left\{ \frac{\sqrt{2\cos^2 \theta} + \sqrt{2\sin^2 \theta}}{2} \right\}$$

$$= \sin^{-1} \left\{ \frac{\sqrt{2}\cos \theta + \sqrt{2}\sin \theta}{2} \right\}$$

$$= \sin^{-1} \left\{ \cos \theta \left(\frac{1}{\sqrt{2}} \right) + \left(\frac{1}{\sqrt{2}} \right) \sin \theta \right\}$$

$$= \sin^{-1} \left\{ \cos \theta \sin \left(\frac{\pi}{4} \right) + \cos \frac{\pi}{4} \sin \theta \right\}$$

$$y = \sin^{-1} \left\{ \sin \left(\theta + \frac{\pi}{4} \right) \right\} \quad \text{---(i)}$$

Here, $0 < x < 1$

$$\Rightarrow 0 < \cos 2\theta < 1$$

$$\Rightarrow 0 < 2\theta < \frac{\pi}{2}$$

$$\Rightarrow 0 < \theta < \frac{\pi}{4}$$

$$\Rightarrow \frac{\pi}{4} < \left(\theta + \frac{\pi}{4} \right) < \frac{\pi}{2}$$

So, from equatoin (i),

$$y = \theta + \frac{\pi}{4}$$

$$\left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \right]$$

$$y = \frac{1}{2} \cos^{-1} x + \frac{\pi}{4}$$

Differentiate it with respect to x ,

$$\frac{dy}{dx} = \frac{1}{2} \left(\frac{-1}{\sqrt{1-x^2}} \right) + 0$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q20

$$\text{Let } y = \tan^{-1} \left(\frac{\sqrt{1+a^2x^2}-1}{ax} \right)$$

$$\text{Put } ax = \tan \theta$$

$$y = \tan^{-1} \left(\frac{\sqrt{1+a^2x^2}-1}{ax} \right)$$

$$= \tan^{-1} \left(\frac{\sec \theta - 1}{\tan \theta} \right)$$

$$= \tan^{-1} \left(\frac{1 - \cos \theta}{\sin \theta} \right)$$

$$= \tan^{-1} \left(\frac{\frac{2 \sin^2 \theta}{2}}{\frac{2 \sin \theta \cos \theta}{2}} \right)$$

$$y = \tan^{-1} \left(\frac{\tan \theta}{2} \right)$$

$$= \frac{\theta}{2}$$

$$y = \frac{1}{2} \tan^{-1}(ax)$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{2} \times \left(\frac{1}{1+(ax)^2} \right) \frac{d}{dx}(ax)$$

$$\frac{dy}{dx} = \frac{1}{2(1+a^2x^2)}(a)$$

$$\frac{dy}{dx} = \frac{a}{2(1+a^2x^2)}.$$

Differentiation Ex 11.3 Q21

$$\text{Let } f(x) = \tan^{-1} \left(\frac{\sin x}{1 + \cos x} \right)$$

This function is defined for all real numbers where $\cos x \neq -1$
i.e. at all odd multiples of π

$$f(x) = \tan^{-1} \left(\frac{\sin x}{1 + \cos x} \right)$$

$$= \tan^{-1} \left[\frac{2 \sin \left(\frac{x}{2} \right) \cos \left(\frac{x}{2} \right)}{2 \cos^2 \left(\frac{x}{2} \right)} \right]$$

$$= \tan^{-1} \left[\tan \left(\frac{x}{2} \right) \right] = \frac{x}{2}$$

$$\text{Thus, } f'(x) = \frac{d}{dx} \left(\frac{x}{2} \right) = \frac{1}{2}$$

Differentiation Ex 11.3 Q22

$$\text{Let } y = \sin^{-1} \left(\frac{1}{\sqrt{1+x^2}} \right)$$

$$\text{Put } x = \cot \theta$$

$$y = \sin^{-1} \left(\frac{1}{\sqrt{1+\cot^2 \theta}} \right)$$

$$= \sin^{-1} \left(\frac{1}{\sqrt{\operatorname{cosec}^2 \theta}} \right)$$

$$= \sin^{-1}(\sin \theta)$$

$$= \theta$$

$$y = \cot^{-1} x$$

[Since, $\cot \theta = x$]

Differentiating it with respect to x ,

$$\frac{dy}{dx} = -\frac{1}{(1+x^2)}.$$

Differentiation Ex 11.3 Q23

$$\text{Let } y = \cos^{-1} \left(\frac{1 - x^{2n}}{1 + x^{2n}} \right)$$

$$\text{Put } x^n = \tan \theta, \text{ so,}$$

$$y = \cos^{-1} \left(\frac{1 - (x^n)^2}{1 + (x^n)^2} \right)$$

$$= \cos^{-1} \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right)$$

$$y = \cos^{-1} (\cos 2\theta) \quad \text{---(i)}$$

$$\text{Here, } 0 < x < \infty$$

$$\Rightarrow 0 < x^n < \infty$$

$$\Rightarrow 0 < \theta < \frac{\pi}{2}$$

$$\Rightarrow 0 < (2\theta) < \pi$$

So, from equation (i),

$$y = 2\theta \quad \left[\text{Since, } \cos^{-1} (\cos \theta) = \theta, \text{ if } \theta \in [0, \pi] \right]$$

$$y = 2 \tan^{-1} (x^n)$$

Differentiating it with respect to x using chain rule,

$$\begin{aligned} \frac{dy}{dx} &= 2 \left(\frac{1}{1 + (x^n)^2} \right) \frac{d}{dx} (x^n) \\ &= \frac{2}{1 + x^{2n}} \times (nx^{n-1}) \end{aligned}$$

$$\frac{dy}{dx} = \frac{2nx^{n-1}}{1 + x^{2n}}.$$

Differentiation Ex 11.3 Q24

$$\text{Let } y = \sin^{-1} \left(\frac{1 - x^2}{1 + x^2} \right) + \sec^{-1} \left(\frac{1 + x^2}{1 - x^2} \right)$$

$$= \sin^{-1} \left(\frac{1 - x^2}{1 + x^2} \right) + \cos^{-1} \left(\frac{1 - x^2}{1 + x^2} \right)$$

$$y = \frac{\pi}{2}$$

$$\left[\text{Since, } \sec^{-1} x = \cos^{-1} \left(\frac{1}{x} \right) \right]$$

$$\left[\text{Since, } \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \right]$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} = 0.$$

Differentiation Ex 11.3 Q25

$$\text{Let } y = \tan^{-1} \left(\frac{a+x}{1-ax} \right)$$

$$y = \tan^{-1} a + \tan^{-1} x$$

$$\left[\text{Since, } \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right) \right]$$

Differentiating it with respect to x ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} (\tan^{-1} a) + \frac{d}{dx} (\tan^{-1} x) \\ &= 0 + \frac{1}{1+x^2} \end{aligned}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}.$$

Differentiation Ex 11.3 Q26

$$\text{Let } y = \tan^{-1} \left(\frac{\sqrt{x} + \sqrt{a}}{1 - \sqrt{xa}} \right)$$

$$y = \tan^{-1} \sqrt{x} + \tan^{-1} \sqrt{a}$$

$$\left[\text{Since, } \tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy} \right]$$

Differentiating it with respect to x using chain rule,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} (\tan^{-1} \sqrt{x}) + \frac{d}{dx} (\tan^{-1} \sqrt{a}) \\ &= \frac{1}{1+(\sqrt{x})^2} \frac{d}{dx} (\sqrt{x}) + 0 \\ &= \left(\frac{1}{1+x} \right) \left(\frac{1}{2\sqrt{x}} \right) \end{aligned}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}(1+x)}.$$

Differentiation Ex 11.3 Q27

$$\begin{aligned} \text{Let } y &= \tan^{-1} \left[\frac{a+b \tan x}{b-a \tan x} \right] \\ &= \tan^{-1} \left[\frac{\frac{a+b \tan x}{b}}{\frac{b-a \tan x}{b}} \right] \\ &= \tan^{-1} \left[\frac{\frac{a}{b} + \tan x}{1 + \frac{a}{b} \tan x} \right] \\ &= \tan^{-1} \left[\frac{\tan \left(\tan^{-1} \frac{a}{b} \right) + \tan x}{1 - \tan \left(\tan^{-1} \frac{a}{b} \right) \tan x} \right] \\ &= \tan^{-1} \left[\tan \left(\tan^{-1} \frac{a}{b} + x \right) \right] \\ y &= \tan^{-1} \left(\frac{a}{b} \right) + x \end{aligned}$$

Differentiate it with respect to x ,

$$\frac{dy}{dx} = 0 + 1$$

$$\frac{dy}{dx} = 1.$$

Differentiation Ex 11.3 Q28

$$\begin{aligned} \text{Let } y &= \tan^{-1} \left(\frac{a+bx}{b-ax} \right) \\ &= \tan^{-1} \left(\frac{\frac{a+bx}{b}}{\frac{b-ax}{b}} \right) \\ &= \tan^{-1} \left(\frac{\frac{a}{b} + \frac{bx}{b}}{\frac{b}{b} - \frac{ax}{b}} \right) \\ &= \tan^{-1} \left(\frac{\frac{a}{b} + x}{1 - \left(\frac{a}{b} \right) x} \right) \\ y &= \tan^{-1} \left(\frac{a}{b} \right) + \tan^{-1} x \end{aligned}$$

$$\left[\text{Since, } \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right) \right]$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} = 0 + \frac{1}{1+x^2}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}.$$

Differentiation Ex 11.3 Q29

$$\begin{aligned}
 \text{Let } y &= \tan^{-1} \left(\frac{x-a}{x+a} \right) \\
 &= \tan^{-1} \left(\frac{\frac{x-a}{x}}{\frac{x+a}{x}} \right) \\
 &= \tan^{-1} \left(\frac{\frac{x}{x} - \frac{a}{x}}{\frac{x}{x} + \frac{a}{x}} \right) \\
 &= \tan^{-1} \left(\frac{1 - \frac{a}{x}}{1 + 1 \times \frac{a}{x}} \right) \\
 y &= \tan^{-1}(1) - \tan^{-1} \left(\frac{a}{x} \right)
 \end{aligned}$$

Differentiating it with respect to x using chain rule,

$$\begin{aligned}
 \frac{dy}{dx} &= 0 - \frac{1}{1 + \left(\frac{a}{x} \right)^2} \frac{d}{dx} \left(\frac{a}{x} \right) \\
 &= - \frac{x^2}{x^2 + a^2} \left(\frac{-a}{x^2} \right)
 \end{aligned}$$

$$\frac{dy}{dx} = \frac{a}{a^2 + x^2}.$$

Differentiation Ex 11.3 Q30

$$\begin{aligned}
 \text{Let } y &= \tan^{-1} \left(\frac{x}{1+6x^2} \right) \\
 &= \tan^{-1} \left(\frac{3x-2x}{1+(3x)(2x)} \right) \\
 y &= \tan^{-1} 3x - \tan^{-1} 2x \quad \left[\text{Since, } \tan^{-1} x - \tan^{-1} y = \tan^{-1} \left(\frac{x-y}{1+xy} \right) \right]
 \end{aligned}$$

Differentiating it with respect to x using chain rule,

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{1}{1+(3x)^2} \frac{d}{dx} (3x) - \frac{1}{1+(2x)^2} \frac{d}{dx} (2x) \\
 &= \frac{1}{1+9x^2} (3) - \frac{1}{1+4x^2} (2) \\
 \frac{dy}{dx} &= \frac{3}{1+9x^2} - \frac{2}{1+4x^2}.
 \end{aligned}$$

Differentiation Ex 11.3 Q31

$$\begin{aligned}
 \text{Let } y &= \tan^{-1} \left(\frac{5x}{1-6x^2} \right) \\
 &= \tan^{-1} \left(\frac{3x+2x}{1-(3x)(2x)} \right) \\
 y &= \tan^{-1} (3x) + \tan^{-1} (2x) \quad \left[\text{Since, } \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right) \right]
 \end{aligned}$$

Differentiating it with respect to x using chain rule,

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{1}{1+(3x)^2} \frac{d}{dx} (3x) + \frac{1}{1+(2x)^2} \frac{d}{dx} (2x) \\
 &= \frac{1}{1+9x^2} (3) + \frac{1}{1+4x^2} (2) \\
 \frac{dy}{dx} &= \frac{3}{1+9x^2} + \frac{2}{1+4x^2}.
 \end{aligned}$$

Differentiation Ex 11.3 Q32

$$\begin{aligned}
 \text{Let } y &= \tan^{-1} \left[\frac{\cos x + \sin x}{\cos x - \sin x} \right] \\
 &= \tan^{-1} \left[\frac{\frac{\cos x + \sin x}{\cos x}}{\frac{\cos x - \sin x}{\cos x}} \right] \\
 &= \tan^{-1} \left[\frac{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}} \right] \\
 &= \tan^{-1} \left[\frac{1 + \tan x}{1 - \tan x} \right] \\
 &= \tan^{-1} \left[\frac{\frac{\tan \frac{\pi}{4}}{1} + \tan x}{1 - \frac{\tan \frac{\pi}{4}}{1} \tan x} \right] \\
 &= \tan^{-1} \left[\tan \left(\frac{\pi}{4} + x \right) \right] \\
 y &= \frac{\pi}{4} + x
 \end{aligned}$$

Differentiating it with respect to x ,

$$\begin{aligned}
 \frac{dy}{dx} &= 0 + 1 \\
 \frac{dy}{dx} &= 1.
 \end{aligned}$$

Differentiation Ex 11.3 Q33

$$\begin{aligned}
 \text{Let } y &= \tan^{-1} \left[\frac{x^{\frac{1}{3}} + a^{\frac{1}{3}}}{1 - (ax)^{\frac{1}{3}}} \right] \\
 y &= \tan^{-1} \left(x^{\frac{1}{3}} \right) + \tan^{-1} \left(a^{\frac{1}{3}} \right) \quad \left[\text{Since, } \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x + y}{1 - xy} \right) \right]
 \end{aligned}$$

Differentiating it with respect to x using chain rule,

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{1}{1 + \left(x^{\frac{1}{3}} \right)^2} \times \frac{d}{dx} \left(x^{\frac{1}{3}} \right) + 0 \\
 &= \frac{\left(\frac{1}{3} \times x^{\frac{1}{3}-1} \right)}{1 + x^{\frac{2}{3}}} \\
 \frac{dy}{dx} &= \frac{1}{3x^{\frac{2}{3}} \left(1 + x^{\frac{2}{3}} \right)}.
 \end{aligned}$$

Differentiation Ex 11.3 Q34

$$\text{Let } f(x) = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$$

To find the domain, we need to find all x such that

$$-1 \leq \frac{2^{x+1}}{1+4^x} \leq 1$$

Since the quantity in the middle is always positive, we need

$$\text{to find all } x \text{ such that } \frac{2^{x+1}}{1+4^x} \leq 1$$

$$\text{i.e. all } x \text{ such that } 2^{x+1} \leq 1+4^x$$

We may rewrite as $2 \leq \frac{1}{2^x} + 2^x$, which is true for all x

Hence the function is defined at all real numbers.

Putting $2^x = \tan \theta$

$$\begin{aligned} f(x) &= \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right) = \sin^{-1}\left(\frac{2^x \cdot 2}{1+(2^x)^2}\right) \\ &= \sin^{-1}\left[\frac{2 \tan \theta}{1+\tan^2 \theta}\right] = \sin^{-1}(\sin 2\theta) = 2\theta = 2 \tan^{-1}(2^x) \end{aligned}$$

$$\begin{aligned} \text{Thus, } f'(x) &= 2 \cdot \frac{1}{(2^x)^2} \frac{d}{dx}(2^x) \\ &= \frac{2}{1+4^x} \cdot (2^x) \log 2 = \frac{2^{x+1} \log 2}{1+4^x} \end{aligned}$$

Differentiation Ex 11.3 Q35

$$\begin{aligned} \text{Let } y &= \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right) \\ y &= \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) \end{aligned}$$

Put, $x = \tan \theta$

$$\begin{aligned} y &= \sin^{-1}\left(\frac{2 \tan \theta}{1+\tan^2 \theta}\right) + \cos^{-1}\left(\frac{1-\tan^2 \theta}{1+\tan^2 \theta}\right) \\ y &= \sin^{-1}(\sin 2\theta) + \cos^{-1}(\cos 2\theta) \quad \text{--- (i)} \end{aligned}$$

Here, $0 < x < 1$

$$\Rightarrow 0 < \tan \theta < 1$$

$$\Rightarrow 0 < \theta < \frac{\pi}{4}$$

$$\Rightarrow 0 < (2\theta) < \frac{\pi}{2}$$

So, from equation (i),

$$\begin{aligned} y &= 2\theta + 2\theta \\ y &= 4\theta \\ y &= 4 \tan^{-1} x \end{aligned} \quad \left[\begin{array}{l} \text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ \cos^{-1}(\cos \theta) = \theta, \text{ if } \theta \in [0, \pi] \end{array} \right]$$

[Since, $x = \tan \theta$]

Differentiating it with respect to x ,

$$\frac{dy}{dx} = \frac{4}{1+x^2}$$

Differentiation Ex 11.3 Q36

$$\text{Here, } y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) + \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$$

$$\text{Put } x = \tan \theta$$

$$y = \sin^{-1}\left(\frac{\tan \theta}{\sqrt{1+\tan^2 \theta}}\right) + \cos^{-1}\left(\frac{1}{\sqrt{1+\tan^2 \theta}}\right)$$

$$= \sin^{-1}\left(\frac{\sin \theta}{\sec \theta}\right) + \cos^{-1}\left(\frac{1}{\sec \theta}\right)$$

$$= \sin^{-1}\left(\frac{\sin \theta}{\frac{1}{\cos \theta}}\right) + \cos^{-1}(\cos \theta)$$

$$y = \sin^{-1}(\sin \theta) + \cos^{-1}(\cos \theta) \quad \text{---(i)}$$

$$\text{Here, } 0 < x < \infty$$

$$\Rightarrow 0 < \tan \theta < \infty$$

$$\Rightarrow 0 < \theta < \frac{\pi}{2}$$

So, from equation (i),

$$y = \theta + \theta$$

$$= 2\theta$$

$$y = 2 \tan^{-1} x$$

$$\left[\begin{array}{l} \text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ \text{and } \cos^{-1}(\cos \theta) = \theta, \text{ if } \theta \in [0, \pi] \end{array} \right]$$

$$[\text{Since, } x = \tan \theta]$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} = \frac{2}{1+x^2}.$$

Differentiation Ex 11.3 Q37

$$\text{Let } f(x) = \cos^{-1}(\sin x)$$

We observe that this function is defined for all real numbers.

$$f(x) = \cos^{-1}(\sin x)$$

$$= \cos^{-1}\left[\cos\left(\frac{\pi}{2} - x\right)\right] = \frac{\pi}{2} - x$$

$$\text{Thus, } f'(x) = \frac{d}{dx}\left(\frac{\pi}{2} - x\right) = -1$$

$$\text{Let } y = \cot^{-1}\left(\frac{1-x}{1+x}\right)$$

$$\text{Put } x = \tan \theta, \text{ so,}$$

$$y = \cot^{-1}\left(\frac{1-\tan \theta}{1+\tan \theta}\right)$$

$$= \cot^{-1}\left(\frac{\tan \frac{\pi}{4} - \tan \theta}{1 + \tan \frac{\pi}{4} \tan \theta}\right)$$

$$= \cot^{-1}\left[\tan\left(\frac{\pi}{4} - \theta\right)\right]$$

$$= \cot^{-1}\left[\cot\left(\frac{\pi}{2} - \frac{\pi}{4} + \theta\right)\right]$$

$$= \frac{\pi}{4} + \theta$$

$$y = \frac{\pi}{4} + \tan^{-1} x$$

$$[\text{Since } x = \tan \theta]$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} = 0 + \frac{1}{1+x^2}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}.$$

Differentiation Ex 11.3 Q38

$$\text{Let } y = \cot^{-1} \left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \right] \quad \dots(1)$$

$$\begin{aligned} \text{Then, } & \frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \\ &= \frac{(\sqrt{1+\sin x} + \sqrt{1-\sin x})^2}{(\sqrt{1+\sin x} - \sqrt{1-\sin x})(\sqrt{1+\sin x} + \sqrt{1-\sin x})} \\ &= \frac{(1+\sin x) + (1-\sin x) + 2\sqrt{(1-\sin x)(1+\sin x)}}{(1+\sin x) - (1-\sin x)} \\ &= \frac{2 + 2\sqrt{1-\sin^2 x}}{2\sin x} \\ &= \frac{1+\cos x}{\sin x} \\ &= \frac{2\cos^2 \frac{x}{2}}{2\sin \frac{x}{2} \cos \frac{x}{2}} \\ &= \cot \frac{x}{2} \end{aligned}$$

Therefore, equation (1) becomes

$$\begin{aligned} y &= \cot^{-1} \left(\cot \frac{x}{2} \right) \\ \Rightarrow y &= \frac{x}{2} \\ \therefore \frac{dy}{dx} &= \frac{1}{2} \frac{d}{dx} (x) \\ \Rightarrow \frac{dy}{dx} &= \frac{1}{2} \end{aligned}$$

Differentiation Ex 11.3 Q39

$$\text{Here, } y = \tan^{-1} \left(\frac{2x}{1-x^2} \right) + \sec^{-1} \left(\frac{1+x^2}{1-x^2} \right)$$

$$y = \tan^{-1} \left(\frac{2x}{1-x^2} \right) + \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$$

$$\text{Put } x = \tan \theta,$$

$$y = \tan^{-1} \left(\frac{2 \tan \theta}{1 - \tan^2 \theta} \right) + \cos^{-1} \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right)$$

$$y = \tan^{-1} (\tan 2\theta) + \cos^{-1} (\cos 2\theta) \quad \dots(i)$$

$$\text{Here, } -\infty < x < \infty$$

$$\Rightarrow 0 < \tan \theta < \infty$$

$$\Rightarrow 0 < \theta < \frac{\pi}{2}$$

$$\Rightarrow 0 < 2\theta < \pi$$

So, from equation (i),

$$y = 2\theta + 2\theta$$

$$y = 4\theta$$

$$y = 4 \tan^{-1} x$$

$$\left[\begin{array}{l} \text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \\ \text{and } \cos^{-1}(\cos \theta) = \theta, \text{ if } \theta \in [0, \pi] \end{array} \right]$$

[Using $x = \tan \theta$]

Differentiating it with respect to x ,

$$\frac{dy}{dx} = \frac{4}{1+x^2}$$

Differentiation Ex 11.3 Q40

Here, $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$

$$y = \cos^{-1}\left(\frac{x-1}{x+1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$$

$$y = \frac{\pi}{2}$$

$$\left[\text{Since, } \sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right) \right]$$

$$\left[\text{Since, } \cos^{-1}x + \sin^{-1}x = \frac{\pi}{2} \right]$$

Differentiating with respect to x ,

$$\frac{dy}{dx} = 0$$

Differentiation Ex 11.3 Q41

Here, $y = \sin\left[2 \tan^{-1}\left[\sqrt{\frac{1-x}{1+x}}\right]\right]$

Put $x = \cos 2\theta$, so,

$$y = \sin\left[2 \tan^{-1}\left[\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\right]\right]$$

$$= \sin\left[2 \tan^{-1}\left[\sqrt{\frac{2 \sin^2 \theta}{2 \cos^2 \theta}}\right]\right]$$

$$= \sin\left[2 \tan^{-1}\sqrt{\tan^2 \theta}\right]$$

$$= \sin\left[2 \tan^{-1}(\tan \theta)\right]$$

$$= \sin(2\theta)$$

$$= \sin\left[2 \times \frac{1}{2} \cos^{-1}x\right]$$

$$[\text{Since, } x = \cos 2\theta]$$

$$= \sin\left[\sin^{-1}\sqrt{1-x^2}\right]$$

$$y = \sqrt{1-x^2}$$

Differentiating with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}} \frac{d}{dx}(1-x^2).$$

Differentiation Ex 11.3 Q42

Here, $y = \cos^{-1}(2x) + 2 \cos^{-1} \sqrt{1-4x^2}$

Put $2x = \cos \theta$, so

$$\begin{aligned} y &= \cos^{-1}(\cos \theta) + 2 \cos^{-1} \sqrt{1 - \cos^2 \theta} \\ &= \cos^{-1}(\cos \theta) + 2 \cos^{-1}(\sin \theta) \\ &= \cos^{-1}(\cos \theta) + 2 \cos^{-1}\left(\cos\left(\frac{\pi}{2} - \theta\right)\right) \end{aligned} \quad \text{---(i)}$$

Here, $0 < x < \frac{1}{2}$

$$\Rightarrow 0 < 2x < 1$$

$$\Rightarrow 0 < \cos \theta < 1$$

$$\Rightarrow 0 < \theta < \frac{\pi}{2}$$

and

$$\Rightarrow 0 > -\theta > -\frac{\pi}{2}$$

$$\Rightarrow \frac{\pi}{2} > \left(\frac{\pi}{2} - \theta\right) > 0$$

So, from equation (i),

$$y = \theta + 2\left(\frac{\pi}{2} - \theta\right) \quad \left[\text{Since, } \cos^{-1}(\cos(\theta)) = \theta, \text{ if } \theta \in [0, \pi]\right]$$

$$= \theta + \pi - 2\theta$$

$$y = \pi - \theta$$

$$y = \pi - \cos^{-1}(2x) \quad \left[\text{Since, } 2x = \cos \theta\right]$$

Differentiating it with respect to x using chain rule,

$$\begin{aligned} \frac{dy}{dx} &= 0 - \left[\frac{-1}{\sqrt{1-(2x)^2}} \right] \frac{d}{dx}(2x) \\ &= \frac{1}{\sqrt{1-4x^2}}(2) \end{aligned}$$

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}.$$

Differentiation Ex 11.3 Q43

Here, $\frac{d}{dx} [\tan^{-1}(a+bx)] = 1$ at $x = 0$

So, using chain rule,

$$\left[\left\{ \frac{1}{1+(a+bx)^2} \right\} \frac{d}{dx}(a+bx) \right]_{x=0} = 1$$

$$\left[\frac{1}{1+(a+bx)^2} \times (b) \right]_{x=0} = 1$$

$$\Rightarrow \frac{b}{1+(a+0)^2} = 1$$

$$\Rightarrow b = 1+a^2.$$

Differentiation Ex 11.3 Q44

Here, $y = \cos^{-1}(2x) + 2 \cos^{-1} \sqrt{1-4x^2}$

Put $2x = \cos \theta$, so,

$$y = \cos^{-1}(\cos \theta) + 2 \cos^{-1} \sqrt{1 - \cos^2 \theta}$$

$$= \cos^{-1}(\cos \theta) + 2 \cos^{-1}(\sin \theta)$$

$$y = \cos^{-1}(\cos \theta) + 2 \cos^{-1}\left(\cos\left(\frac{\pi}{2} - \theta\right)\right) \quad \text{--- (i)}$$

Now, $-\frac{1}{2} < x < 0$

$$\Rightarrow -1 < 2x < 0$$

$$\Rightarrow -1 < \cos \theta < 0$$

$$\Rightarrow \frac{\pi}{2} < \theta < \pi$$

And

$$\Rightarrow -\frac{\pi}{2} > -\theta > -\pi$$

$$\Rightarrow \left(\frac{\pi}{2} - \frac{\pi}{2}\right) > \left(\frac{\pi}{2} - \theta\right) > \left(\frac{\pi}{2} - \pi\right)$$

$$\Rightarrow 0 > \left(\frac{\pi}{2} - \theta\right) > -\frac{\pi}{2}$$

So, from equation (i),

$$y = \theta + 2 \left[-\left(\frac{\pi}{2} - \theta\right) \right]$$

$$\left[\begin{array}{l} \text{Since, } \cos^{-1} \cos(\theta) = \theta, \text{ if } \theta \in [0, \pi] \\ \cos^{-1} \cos(\theta) = -\theta, \text{ if } \theta \in [-\pi, 0] \end{array} \right]$$

$$y = \theta - 2 \times \frac{\pi}{2} + 2\theta$$

$$y = -\pi + 3\theta$$

$$y = -\pi + 3 \cos^{-1}(2x) \quad [\text{Since, } 2x = \cos \theta]$$

Differentiating it with respect to x using chain rule,

$$\begin{aligned} \frac{dy}{dx} &= 0 + 3 \left(\frac{-1}{\sqrt{1-(2x)^2}} \right) \frac{d}{dx}(2x) \\ &= \frac{-3}{\sqrt{1-4x^2}} (2) \end{aligned}$$

$$\frac{dy}{dx} = -\frac{6}{\sqrt{1-4x^2}}$$

Differentiation Ex 11.3 Q45

Here, $y = \tan^{-1} \left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right)$

Put $x = \cos 2\theta$, so

$$\begin{aligned} y &= \tan^{-1} \left(\frac{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}} \right) \\ &= \tan^{-1} \left(\frac{\sqrt{2\cos^2 \theta} - \sqrt{2\sin^2 \theta}}{\sqrt{2\cos^2 \theta} + \sqrt{2\sin^2 \theta}} \right) \\ &= \tan^{-1} \left(\frac{\sqrt{2}(\cos \theta - \sin \theta)}{\sqrt{2}(\cos \theta + \sin \theta)} \right) \\ &= \tan^{(1)} \left(\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} \right) \end{aligned}$$

[Dividing numerator and denominator by $\cos \theta$]

$$\begin{aligned} &= \tan^{-1} \left(\frac{\frac{\cos \theta - \sin \theta}{\cos \theta}}{\frac{\cos \theta + \sin \theta}{\cos \theta}} \right) \\ &= \tan^{-1} \left(\frac{1 - \tan \theta}{1 + \tan \theta} \right) \\ &= \tan^{-1} \left(\frac{\frac{\tan \frac{\pi}{4} - \tan \theta}{1 + \frac{\tan \frac{\pi}{4} \tan \theta}}}{1 + \frac{\tan \frac{\pi}{4} \tan \theta}} \right) \\ &= \tan^{-1} \left[\tan \left(\frac{\pi}{4} - \theta \right) \right] \\ &= \frac{\pi}{4} - \theta \end{aligned}$$

$$y = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x \quad \text{[Using } x = \cos 2\theta \text{]}$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} = 0 - \frac{1}{2} \left(\frac{-1}{\sqrt{1-x^2}} \right)$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q46

$$\text{Here, } y = \cos^{-1} \left\{ \frac{2x - 3\sqrt{1-x^2}}{\sqrt{13}} \right\}$$

$$\text{Let } x = \cos \theta, \text{ so,}$$

$$\begin{aligned} y &= \cos^{-1} \left\{ \frac{2 \cos \theta - 3\sqrt{1-\cos^2 \theta}}{\sqrt{13}} \right\} \\ &= \cos^{-1} \left\{ \frac{2}{\sqrt{13}} \cos \theta - \frac{3}{\sqrt{13}} \sin \theta \right\} \end{aligned}$$

$$\text{Let } \cos \phi = \frac{2}{\sqrt{13}}$$

$$\begin{aligned} \Rightarrow \sin \phi &= \sqrt{1 - \cos^2 \phi} \\ &= \sqrt{1 - \left(\frac{2}{\sqrt{13}} \right)^2} \\ &= \sqrt{\frac{13-4}{13}} \\ &= \sqrt{\frac{9}{13}} \\ \sin \phi &= \frac{3}{\sqrt{13}} \end{aligned}$$

So,

$$\begin{aligned} y &= \cos^{-1} \{ \cos \phi \cos \theta - \sin \phi \sin \theta \} \\ &= \cos^{-1} [\cos(\theta + \phi)] \\ y &= \phi + \theta \end{aligned}$$

$$y = \cos^{-1} \left(\frac{2}{\sqrt{13}} \right) + \cos^{-1} x \quad \left[\text{Since, } x = \cos \theta, \cos \phi = \frac{2}{\sqrt{13}} \right]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 0 + \left(-\frac{1}{\sqrt{1-x^2}} \right)$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q47

Consider the given expression:

$$y = \sin^{-1} \left\{ \frac{2^{x+1} \times 3^x}{1 + (36)^x} \right\}$$

$$= \sin^{-1} \left\{ \frac{2 \times 2^x \times 3^x}{1 + (6^2)^x} \right\}$$

$$y = \sin^{-1} \left\{ \frac{2 \times 6^x}{1 + 6^{2x}} \right\} \dots (1)$$

Substituting $6^x = \tan \theta$ in the above equation, we get,

$$y = \sin^{-1} \left\{ \frac{2 \times 6^x}{1 + 6^{2x}} \right\}$$

$$= \sin^{-1} \left\{ \frac{2 \times \tan \theta}{1 + \tan^2 \theta} \right\}$$

$$= \sin^{-1} (\sin 2\theta)$$

$$= 2\theta$$

$$= 2 \tan^{-1} (6^x)$$

Differentiating the above function with respect to x, we have,

$$\frac{d}{dx} \left[\sin^{-1} \left\{ \frac{2^{x+1} \times 3^x}{1 + (36)^x} \right\} \right] = \frac{d}{dx} [2 \tan^{-1} (6^x)]$$

$$= 2 \times \frac{1}{1 + (6^x)^2} \times 6^x \log 6$$

$$= \frac{2 \times 6^x \log 6}{1 + 6^{2x}}$$

Ex 11.4

Differentiation Ex 11.4 Q1

Given,

$$xy = c^2$$

Differentiate with respect to x ,

$$\begin{aligned}\frac{d}{dx}(xy) &= \frac{d}{dx}(c^2) \\ \Rightarrow x \frac{dy}{dx} + y \frac{d}{dx}(x) &= 0 && \text{[Using product rule]} \\ \Rightarrow x \frac{dy}{dx} + y &= 0 \\ \Rightarrow x \frac{dy}{dx} &= -y \\ \Rightarrow \frac{dy}{dx} &= -\frac{y}{x}\end{aligned}$$

Differentiation Ex 11.4 Q2

Here, $y^3 - 3xy^2 = x^3 + 3x^2y$

Differentiating with respect to x ,

$$\begin{aligned}\Rightarrow \frac{d}{dx}(y^3) - \frac{d}{dx}(3xy^2) &= \frac{d}{dx}(x^3) + \frac{d}{dx}(3x^2y) \\ \Rightarrow 3y^2 \frac{dy}{dx} - 3 \left[x \frac{d}{dx} y^2 \frac{d}{dx}(x) \right] &= 3x^2 + 3 \left[x^2 \frac{d}{dx}(y) + y \frac{d}{dx}(x^2) \right] && \text{[Using product rule]} \\ \Rightarrow 3y^2 \frac{dy}{dx} - 3 \left[x(2y) \frac{dy}{dx} + y^2 \right] &= 3x^2 + 3 \left[x^2 \frac{dy}{dx} + y(2x) \right] \\ \Rightarrow 3y^2 \frac{dy}{dx} - 6xy \frac{dy}{dx} - 3y^2 &= 3x^2 + 3x^2 \frac{dy}{dx} + 6xy \\ \Rightarrow 3y^2 \frac{dy}{dx} - 6xy \frac{dy}{dx} - 3x^2 \frac{dy}{dx} &= 3x^2 + 6xy + 3y^2 \\ \Rightarrow 3 \frac{dy}{dx} (y^2 - 2xy - x^2) &= 3(x^2 + 2xy + y^2) \\ \Rightarrow \frac{dy}{dx} &= \frac{3(x+y)^2}{3(y^2 - 2xy - x^2)} \\ \Rightarrow \frac{dy}{dx} &= \frac{(x+y)^2}{y^2 - 2xy - x^2}\end{aligned}$$

Differentiation Ex 11.4 Q3

Here, $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

Differentiate it with respect to x ,

$$\begin{aligned} & \frac{d}{dx} \left(x^{\frac{2}{3}} \right) + \frac{d}{dx} \left(y^{\frac{2}{3}} \right) = \frac{d}{dx} \left(a^{\frac{2}{3}} \right) \\ \Rightarrow & \frac{2}{3} x^{\left(\frac{2}{3}-1\right)} + \frac{2}{3} y^{\left(\frac{2}{3}-1\right)} \frac{dy}{dx} = 0 \\ \Rightarrow & \frac{2}{3} x^{-\frac{1}{3}} + \frac{2}{3} y^{-\frac{1}{3}} \frac{dy}{dx} = 0 \\ \Rightarrow & \frac{2}{3} y^{-\frac{1}{3}} \frac{dy}{dx} = -\frac{2}{3} x^{-\frac{1}{3}} \\ \Rightarrow & \frac{dy}{dx} = -\frac{2}{3} x^{-\frac{1}{3}} \times \frac{3}{2y^{-\frac{1}{3}}} \\ \Rightarrow & \frac{dy}{dx} = -\frac{x^{-\frac{1}{3}}}{y^{\frac{1}{3}}} \\ \Rightarrow & \frac{dy}{dx} = -\frac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}} \\ \Rightarrow & \frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}} \end{aligned}$$

Differentiation Ex 11.4 Q4

Given, $4x + 3y = \log(4x - 3y)$

Differentiating with respect to x ,

$$\begin{aligned}\frac{d}{dx}(4x) + \frac{d}{dx}(3y) &= \frac{d}{dx}(\log(4x - 3y)) \\ \Rightarrow 4 + 3 \frac{dy}{dx} &= \frac{1}{(4x - 3y)} \frac{d}{dx}(4x - 3y) && \text{[Using chain rule]} \\ \Rightarrow 4 + 3 \frac{dy}{dx} &= \frac{1}{(4x - 3y)} \left(4 - 3 \frac{dy}{dx} \right) \\ \Rightarrow 4 + 3 \frac{dy}{dx} &= \frac{4}{(4x - 3y)} - \frac{3}{(4x - 3y)} \frac{dy}{dx} \\ \Rightarrow 3 \frac{dy}{dx} + \frac{3}{(4x - 3y)} \frac{dy}{dx} &= \frac{4}{(4x - 3y)} - 4 \\ \Rightarrow 3 \frac{dy}{dx} \left(1 + \frac{1}{(4x - 3y)} \right) &= 4 \left(\frac{1}{(4x - 3y)} - 1 \right) \\ \Rightarrow 3 \frac{dy}{dx} \left[\frac{4x - 3y + 1}{(4x - 3y)} \right] &= 4 \left[\frac{1 - 4x + 3y}{(4x - 3y)} \right] \\ \Rightarrow \frac{dy}{dx} &= \frac{4}{3} \left[\frac{1 - 4x + 3y}{(4x - 3y)} \right] \left[\frac{4x - 3y}{4x - 3y + 1} \right] \\ \Rightarrow \frac{dy}{dx} &= \frac{4}{3} \left(\frac{1 - 4x + 3y}{4x - 3y + 1} \right)\end{aligned}$$

Differentiation Ex 11.4 Q5

Given,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Differentiating with respect to x ,

$$\begin{aligned}\frac{d}{dx} \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} \right) &= \frac{d}{dx}(1) \\ \Rightarrow \frac{d}{dx} \left(\frac{x^2}{a^2} \right) + \frac{d}{dx} \left(\frac{y^2}{b^2} \right) &= 0 \\ \Rightarrow \frac{1}{a^2}(2x) + \frac{1}{b^2}(2y) \frac{dy}{dx} &= 0 \\ \Rightarrow \frac{2y}{b^2} \frac{dy}{dx} &= -\frac{2x}{a^2} \\ \Rightarrow \frac{dy}{dx} &= - \left(\frac{2x}{a^2} \right) \left(\frac{b^2}{2y} \right) \\ \Rightarrow \frac{dy}{dx} &= -\frac{b^2x}{a^2y}\end{aligned}$$

Differentiation Ex 11.4 Q6

Given,

$$x^5 + y^5 = 5xy$$

Differentiating with respect to x ,

$$\begin{aligned}\frac{d}{dx}(x^5) + \frac{d}{dx}(y^5) &= \frac{d}{dx}(5xy) \\ \Rightarrow 5x^4 + 5y^4 \frac{dy}{dx} &= 5 \left[x \frac{dy}{dx} + y \frac{dy}{dx}(x) \right] && \text{[Using product rule]} \\ \Rightarrow 5x^4 + 5y^4 \frac{dy}{dx} &= 5 \left[x \frac{dy}{dx} + y(1) \right] \\ \Rightarrow 5x^4 + 5y^4 \frac{dy}{dx} &= 5x \frac{dy}{dx} + 5y \\ \Rightarrow 5y^4 \frac{dy}{dx} - 5x \frac{dy}{dx} &= 5y - 5x^4 \\ \Rightarrow 5 \frac{dy}{dx} (y^4 - x) &= 5(y - x^4) \\ \Rightarrow \frac{dy}{dx} &= \frac{5(y - x^4)}{5(y^4 - x)} \\ \Rightarrow \frac{dy}{dx} &= \frac{y - x^4}{y^4 - x}\end{aligned}$$

Differentiation Ex 11.4 Q7

Given,

$$(x + y)^2 = 2axy$$

Differentiating with respect to x ,

$$\begin{aligned} \Rightarrow \frac{d}{dx}(x + y)^2 &= \frac{d}{dx}(2axy) \\ \Rightarrow 2(x + y) \frac{d}{dx}(x + y) &= 2a \left[x \frac{dy}{dx} + y \frac{d}{dx}(x) \right] && \text{[Using chain rule and product rule]} \\ \Rightarrow 2(x + y) \left[1 + \frac{dy}{dx} \right] &= 2a \left[x \frac{dy}{dx} + y(1) \right] \\ \Rightarrow 2(x + y) + 2(x + y) \frac{dy}{dx} &= 2ax \frac{dy}{dx} + 2ay \\ \Rightarrow \frac{dy}{dx} [2(x + y) - 2ax] &= 2ay - 2(x + y) \\ \Rightarrow \frac{dy}{dx} &= \frac{2[ay - x - y]}{2[x + y - ax]} \\ \Rightarrow \frac{dy}{dx} &= \left(\frac{ay - x - y}{x + y - ax} \right) \end{aligned}$$

Differentiation Ex 11.4 Q8

Given,

$$(x^2 + y^2)^2 = xy$$

Differentiating with respect to x ,

$$\begin{aligned} \frac{d}{dx} \left((x^2 + y^2)^2 \right) &= \frac{d}{dx}(xy) \\ \Rightarrow 2(x^2 + y^2) \frac{d}{dx}(x^2 + y^2) &= x \frac{dy}{dx} + y \frac{d}{dx}(x) && \text{[Using chain rule and product rule]} \\ \Rightarrow 2(x^2 + y^2) \left(2x + 2y \frac{dy}{dx} \right) &= x \frac{dy}{dx} + y(1) \\ \Rightarrow 4x(x^2 + y^2) + 4y(x^2 + y^2) \frac{dy}{dx} &= x \frac{dy}{dx} + y \\ \Rightarrow 4y(x^2 + y^2) \frac{dy}{dx} - x \frac{dy}{dx} &= y - 4x(x^2 + y^2) \\ \Rightarrow \frac{dy}{dx} [4yx^2 + 4y^3 - x] &= y - 4x^3 - 4xy^2 \\ \Rightarrow \frac{dy}{dx} &= \left(\frac{y - 4x^3 - 4xy^2}{4yx^2 + 4y^3 - x} \right) \end{aligned}$$

Differentiation Ex 11.4 Q9

Here,

$$\tan^{-1}(x^2 + y^2) = a$$

Differentiating with respect to x ,

$$\begin{aligned} \frac{d}{dx} \left(\tan^{-1}(x^2 + y^2) \right) &= \frac{d}{dx}(a) \\ \Rightarrow \frac{1}{1 + (x^2 + y^2)^2} \times \frac{d}{dx}(x^2 + y^2) &= 0 && \text{[Using chain rule]} \\ \Rightarrow \left[\frac{1}{1 + (x^2 + y^2)^2} \right] \left(2x + 2y \frac{dy}{dx} \right) &= 0 \\ \Rightarrow \left\{ \frac{2x}{1 + (x^2 + y^2)^2} \right\} + \left\{ \frac{2y}{1 + (x^2 + y^2)^2} \right\} \frac{dy}{dx} &= 0 \\ \Rightarrow \frac{2y}{1 + (x^2 + y^2)^2} \frac{dy}{dx} &= - \frac{2x}{1 + (x^2 + y^2)^2} \\ \Rightarrow \frac{dy}{dx} &= - \left(\frac{2x}{1 + (x^2 + y^2)^2} \right) \left(\frac{1 + (x^2 + y^2)^2}{2y} \right) \\ \Rightarrow \frac{dy}{dx} &= - \left(\frac{x}{y} \right) \end{aligned}$$

Differentiation Ex 11.4 Q10

Given,

$$e^{x-y} = \log\left(\frac{x}{y}\right)$$

Differentiating with respect to x ,

$$\begin{aligned} \frac{d}{dx}(e^{x-y}) &= \frac{d}{dx} \log\left(\frac{x}{y}\right) \\ \Rightarrow e^{(x-y)} \frac{d}{dx}(x-y) &= \frac{1}{\left(\frac{x}{y}\right)} \times \frac{d}{dx}\left(\frac{x}{y}\right) && \text{[Using chain rule and quotient rule]} \\ \Rightarrow e^{(x-y)} \left(1 - \frac{dy}{dx}\right) &= \frac{y}{x} \left[\frac{y \frac{d}{dx}(x) - x \frac{dy}{dx}}{y^2} \right] \\ \Rightarrow e^{(x-y)} - e^{(x-y)} \frac{dy}{dx} &= \frac{1}{xy} \left[y(1) - x \frac{dy}{dx} \right] \\ \Rightarrow e^{(x-y)} - e^{(x-y)} \frac{dy}{dx} &= \frac{y}{xy} - \frac{x}{xy} \frac{dy}{dx} \\ \Rightarrow e^{(x-y)} - e^{(x-y)} \frac{dy}{dx} &= \frac{1}{x} - \frac{1}{y} \frac{dy}{dx} \\ \Rightarrow \frac{1}{y} \frac{dy}{dx} - e^{(x-y)} \frac{dy}{dx} &= \frac{1}{x} - e^{(x-y)} \\ \Rightarrow \frac{dy}{dx} \left[\frac{1}{y} - \frac{e^{(x-y)}}{1} \right] &= \frac{1}{x} - \frac{e^{(x-y)}}{1} \\ \Rightarrow \frac{dy}{dx} \left[\frac{1 - ye^{(x-y)}}{y} \right] &= \frac{(1 - xe^{(x-y)})}{x} \\ \Rightarrow \frac{dy}{dx} &= \frac{y}{x} \left[\frac{1 - xe^{(x-y)}}{1 - ye^{(x-y)}} \right] \\ &= \frac{-y}{-x} \left[\frac{xe^{(x-y)} - 1}{ye^{(x-y)} - 1} \right] \\ \frac{dy}{dx} &= \frac{y}{x} \left[\frac{xe^{(x-y)} - 1}{ye^{(x-y)} - 1} \right] \end{aligned}$$

Differentiation Ex 11.4 Q11

Given,

$$\sin xy + \cos(x+y) = 1$$

Differentiating with respect to x ,

$$\begin{aligned} \frac{d}{dx} \sin xy + \frac{d}{dx} \cos(x+y) &= \frac{d}{dx}(1) \\ \Rightarrow \cos xy \frac{d}{dx}(xy) - \sin(x+y) \frac{d}{dx}(x+y) &= 0 && \text{[Using chain rule and product rule]} \\ \Rightarrow \cos(xy) \left[x \frac{dy}{dx} + y \frac{d}{dx}(x) \right] - \sin(x+y) \left[1 + \frac{dy}{dx} \right] &= 0 \\ \Rightarrow \cos(xy) \left[x \frac{dy}{dx} + y(1) \right] - \sin(x+y) + \sin(x+y) \frac{dy}{dx} &= 0 \\ \Rightarrow x \cos(xy) \frac{dy}{dx} + y \cos(xy) - \sin(x+y) + \sin(x+y) \frac{dy}{dx} &= 0 \\ \Rightarrow \left[x \cos(xy) + \sin(x+y) \right] \frac{dy}{dx} &= [\sin(x+y) - y \cos xy] \\ \Rightarrow \frac{dy}{dx} &= \left[\frac{\sin(x+y) - y \cos xy}{x \cos xy + \sin(x+y)} \right] \end{aligned}$$

Differentiation Ex 11.4 Q12

Here,

$$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$$

Let $x = \sin A, y = \sin B$, so

$$\Rightarrow \sqrt{1-\sin^2 A} + \sqrt{1-\sin^2 B} = a(\sin A - \sin B)$$

$$\Rightarrow \cos A + \cos B = a(\sin A - \sin B)$$

$$[\text{Since } (1 - \sin^2 \theta) = \cos^2 \theta]$$

$$\Rightarrow a = \frac{\cos A + \cos B}{\sin A - \sin B}$$

$$\Rightarrow a = \frac{2 \cos \frac{A+B}{2} \times \cos \frac{A-B}{2}}{2 \cos \frac{A+B}{2} \times \sin \frac{A-B}{2}}$$

$$\left[\begin{array}{l} \text{Since, } \sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right) \\ \cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right) \end{array} \right]$$

$$\Rightarrow a = \cot \left(\frac{A-B}{2} \right)$$

$$\Rightarrow \cot^{-1} a = \frac{A-B}{2}$$

$$\Rightarrow 2 \cot^{-1} a = A - B$$

$$\Rightarrow 2 \cot^{-1} a = \sin^{-1} x - \sin^{-1} y$$

$$[\text{Since } x = \sin A, y = \sin B]$$

Differentiating with respect to x ,

$$\frac{d}{dx} (2 \cot^{-1} a) = \frac{d}{dx} (\sin^{-1} x) - \frac{d}{dx} (\sin^{-1} y)$$

$$\Rightarrow 0 = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx}$$

$$\Rightarrow \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

Differentiation Ex 11.4 Q13

Here,

$$y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$$

Let $x = \sin A, y = \sin B$

$$\Rightarrow \sin B \sqrt{1-\sin^2 A} + \sin A \sqrt{1-\sin^2 B} = 1$$

$$\Rightarrow \sin B \cos A + \sin A \cos B = 1$$

$$\left[\begin{array}{l} \text{since } 1 - \sin^2 \theta = \cos^2 \theta \text{ and} \\ \sin(x+y) = \sin x \cos y + \cos x \sin y \end{array} \right]$$

$$\Rightarrow \sin(A+B) = 1$$

$$\Rightarrow A+B = \sin^{-1}(1)$$

$$\Rightarrow \sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$$

$$[\text{Since } x = \sin A, y = \sin B]$$

Differentiating with respect to x ,

$$\Rightarrow \frac{d}{dx} (\sin^{-1} x) + \frac{d}{dx} (\sin^{-1} y) = \frac{d}{dx} \left(\frac{\pi}{2} \right)$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$$

Differentiation Ex 11.4 Q14

Here,

$$xy = 1 \quad \text{---(i)}$$

Differentiating with respect to x ,

$$\begin{aligned} \frac{d}{dx}(xy) &= \frac{d}{dx}(1) \\ \Rightarrow x \frac{dy}{dx} + y \frac{d}{dx}(x) &= 0 && \text{[Using product rule]} \\ \Rightarrow x \frac{dy}{dx} + y(1) &= 0 \\ \Rightarrow \frac{dy}{dx} &= -\frac{y}{x} && \left[\text{Put } x = \frac{1}{y} \text{ from equation (i)} \right] \\ \Rightarrow \frac{dy}{dx} &= -\frac{y}{\frac{1}{y}} \\ \Rightarrow \frac{dy}{dx} &= -y^2 \\ \Rightarrow \frac{dy}{dx} + y^2 &= 0 \end{aligned}$$

Differentiation Ex 11.4 Q15

Here,

$$xy^2 = 1$$

Differentiating with respect to x ,

$$\begin{aligned} \frac{d}{dx}(xy^2) &= \frac{d}{dx}(1) \\ \Rightarrow x \frac{d}{dx}(y^2) + y^2 \frac{d}{dx}(x) &= 0 && \text{[Using product rule]} \\ \Rightarrow x(2y) \frac{dy}{dx} + y^2(1) &= 0 \\ \Rightarrow 2xy \frac{dy}{dx} &= -y^2 \\ \Rightarrow \frac{dy}{dx} &= \frac{-y^2}{2xy} \\ \Rightarrow \frac{dy}{dx} &= \frac{-y}{2x} \end{aligned}$$

Put $x = \frac{1}{y^2}$ from equation (i)

$$\begin{aligned} \Rightarrow \frac{dy}{dx} &= \frac{-y}{2\left(\frac{1}{y^2}\right)} \\ \Rightarrow 2 \frac{dy}{dx} &= -y^3 \\ 2 \frac{dy}{dx} + y^3 &= 0 \end{aligned}$$

Differentiation Ex 11.4 Q16

Given,

$$\begin{aligned} x\sqrt{1+y} + y\sqrt{1+x} &= 0 \\ \Rightarrow x\sqrt{1+y} &= -y\sqrt{1+x} \end{aligned}$$

Squaring both the sides,

$$\begin{aligned} \Rightarrow (x\sqrt{1+y})^2 &= (-y\sqrt{1+x})^2 \\ \Rightarrow x^2(1+y) &= y^2(1+x) \\ \Rightarrow x^2 + x^2y &= y^2 + y^2x \\ \Rightarrow x^2 - y^2 &= y^2x - x^2y \\ \Rightarrow (x-y)(x+y) &= xy(y-x) \\ \Rightarrow (x+y) &= -xy \\ \Rightarrow y + xy &= -x \\ \Rightarrow y(1+x) &= -x \\ \Rightarrow y &= \frac{-x}{(1+x)} \end{aligned}$$

Differentiating with respect to x using quotient rule,

$$\begin{aligned} \Rightarrow \frac{dy}{dx} &= \left[\frac{-(1+x) \frac{d}{dx}(x) + (-x) \frac{d}{dx}(x+1)}{(1+x)^2} \right] \\ \Rightarrow \frac{dy}{dx} &= \left[\frac{-(1+x)(1) + (-x)(1)}{(1+x)^2} \right] \\ \Rightarrow \frac{dy}{dx} &= \left[\frac{-1-x+x}{(1+x)^2} \right] \\ \Rightarrow \frac{dy}{dx} &= \frac{-1}{(1+x)^2} \\ \Rightarrow (1+x)^2 \frac{dy}{dx} &= -1 \\ \Rightarrow (1+x)^2 \frac{dy}{dx} + 1 &= 0 \end{aligned}$$

Differentiation Ex 11.4 Q17

Here,

$$\begin{aligned} \log \sqrt{x^2 + y^2} &= \tan^{-1} \left(\frac{x}{y} \right) \\ \Rightarrow \log (x^2 + y^2)^{\frac{1}{2}} &= \tan^{-1} \left(\frac{y}{x} \right) \\ \Rightarrow \frac{1}{2} \log (x^2 + y^2) &= \tan^{-1} \left(\frac{y}{x} \right) \end{aligned}$$

Differentiating with respect to x ,

$$\begin{aligned} \Rightarrow \frac{1}{2} \frac{d}{dx} \log (x^2 + y^2) &= \frac{d}{dx} \tan^{-1} \left(\frac{y}{x} \right) \\ \Rightarrow \frac{1}{2} \times \left(\frac{1}{x^2 + y^2} \right) \frac{d}{dx} (x^2 + y^2) &= \frac{1}{1 + \left(\frac{y}{x} \right)^2} \frac{d}{dx} \left(\frac{y}{x} \right) \quad \text{[Using chain rule, quotient rule]} \\ \Rightarrow \frac{1}{2} \left(\frac{1}{x^2 + y^2} \right) \left[2x + 2y \frac{dy}{dx} \right] &= \frac{x^2}{(x^2 + y^2)} \left[\frac{x \frac{dy}{dx} - y \frac{d}{dx}(x)}{x^2} \right] \\ \Rightarrow \frac{1}{2} \left(\frac{1}{x^2 + y^2} \right) \times 2 \left(x + y \frac{dy}{dx} \right) &= \frac{x^2}{(x^2 + y^2)} \left[\frac{x \frac{dy}{dx} - y(1)}{x^2} \right] \\ \Rightarrow x + y \frac{dy}{dx} &= x \frac{dy}{dx} - y \\ \Rightarrow y \frac{dy}{dx} - x \frac{dy}{dx} &= -y - x \\ \Rightarrow \frac{dy}{dx} (y - x) &= -(y + x) \\ \Rightarrow \frac{dy}{dx} &= \frac{-(y + x)}{y - x} \\ \Rightarrow \frac{dy}{dx} &= \frac{x + y}{x - y} \end{aligned}$$

Differentiation Ex 11.4 Q18

Here,

$$\sec\left(\frac{x+y}{x-y}\right) = a$$

$$\Rightarrow \frac{x+y}{x-y} = \sec^{-1}(a)$$

Differentiating with respect to x ,

$$\Rightarrow \left[\frac{(x-y) \frac{d}{dx}(x+y) - (x+y) \frac{d}{dx}(x-y)}{(x-y)^2} \right] = 0 \quad \text{[Using quotient rule]}$$

$$\Rightarrow (x-y) \left(1 + \frac{dy}{dx}\right) - (x+y) \left(1 - \frac{dy}{dx}\right) = 0$$

$$\Rightarrow (x-y) + (x-y) \frac{dy}{dx} - (x+y) + (x+y) \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} [x-y+x+y] = x+y-x+y$$

$$\Rightarrow \frac{dy}{dx} (2x) = 2y$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

Differentiation Ex 11.4 Q19

Here,

$$\tan^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = a$$

$$\Rightarrow \frac{x^2-y^2}{x^2+y^2} = \tan a$$

$$\Rightarrow x^2 - y^2 = \tan a (x^2 + y^2)$$

Differentiating with respect to x ,

$$\Rightarrow \frac{d}{dx}(x^2 - y^2) = \tan a \frac{d}{dx}(x^2 + y^2)$$

$$\Rightarrow \left(2x - 2y \frac{dy}{dx}\right) = \tan a \left(2x + 2y \frac{dy}{dx}\right)$$

$$\Rightarrow 2x - 2y \frac{dy}{dx} = 2x \tan a + 2y \tan a \frac{dy}{dx}$$

$$\Rightarrow 2y \tan a \frac{dy}{dx} + 2y \frac{dy}{dx} = 2x - 2x \tan a$$

$$\Rightarrow 2y \frac{dy}{dx} (1 + \tan a) = 2x (1 - \tan a)$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{y} \left(\frac{1 - \tan a}{1 + \tan a} \right)$$

Differentiation Ex 11.4 Q20

Here,

$$xy \log(x+y) = 1$$

Differentiating it with respect to x ,

$$\Rightarrow \frac{d}{dx} [xy \log(x+y)] = \frac{d}{dx} (1)$$

$$\Rightarrow xy \frac{d}{dx} \log(x+y) + x \log(x+y) \frac{dy}{dx} + y \log(x+y) \frac{d}{dx} (x) = 0$$

[Using chain rule and product rule]

$$\Rightarrow xy \times \left(\frac{1}{x+y} \right) \frac{d}{dx} (x+y) + x \log(x+y) \frac{dy}{dx} + y \log(x+y) (1) = 0$$

$$\Rightarrow \left(\frac{xy}{x+y} \right) \left(1 + \frac{dy}{dx} \right) + x \log(x+y) \frac{dy}{dx} + y \log(x+y) = 0$$

$$\Rightarrow \left(\frac{xy}{x+y} \right) \frac{dy}{dx} + \left(\frac{xy}{x+y} \right) + x \left(\frac{1}{xy} \right) \frac{dy}{dx} + y \left(\frac{1}{xy} \right) = 0$$

$$\left[\text{Since from equation (i) } \log(x+y) = \frac{1}{xy} \right]$$

$$\Rightarrow \frac{dy}{dx} \left[\frac{xy}{x+y} + \frac{1}{y} \right] = - \left[\frac{1}{x} + \frac{xy}{x+y} \right]$$

$$\frac{dy}{dx} \left[\frac{xy^2 + x + y}{(x+y)y} \right] = - \left[\frac{x + y + x^2y}{x(x+y)} \right]$$

$$\frac{dy}{dx} = - \left(\frac{x + y + x^2y}{x(x+y)} \right) \left(\frac{y(x+y)}{xy^2 + x + y} \right)$$

$$= - \frac{y}{x} \left(\frac{x + y + x^2y}{x + y + xy^2} \right)$$

So,

$$\frac{dy}{dx} = - \frac{y}{x} \left(\frac{x^2y + x + y}{xy^2 + x + y} \right)$$

Differentiation Ex 11.4 Q21

Here,

$$y = x \sin(a+y) \quad \text{---(i)}$$

Differentiating with respect to y ,

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} [x \sin(a+y)]$$

$$\Rightarrow \frac{dy}{dx} = x \frac{d}{dx} \sin(a+y) + \sin(a+y) \frac{d}{dx} (x)$$

[Using product rule, chain rule]

$$\Rightarrow \frac{dy}{dx} = x \cos(a+y) \frac{d}{dx} (a+y) + \sin(a+y) (1)$$

$$= x \cos(a+y) \left(0 + \frac{dy}{dx} \right) + \sin(a+y)$$

$$\Rightarrow \frac{dy}{dx} (1 - x \cos(a+y)) = \sin(a+y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin(a+y)}{1 - x \cos(a+y)}$$

Put x from equation (i), $x = \frac{y}{\sin(a+y)}$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin(a+y)}{1 - \frac{y}{\sin(a+y)} \cos(a+y)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin(a+y) - y \cos(a+y)}$$

Differentiation Ex 11.4 Q22

Here,

$$x \sin(a+y) + \sin a \cos(a+y) = 0 \quad \text{---(i)}$$

Differentiating with respect to x ,

$$\begin{aligned} \Rightarrow \quad & \frac{d}{dx} [x \sin(a+y)] + \frac{d}{dx} [\sin a \cos(a+y)] = 0 \\ \Rightarrow \quad & \left[x \frac{d}{dx} \sin(a+y) + \sin(a+y) \frac{d}{dx} (x) \right] + \sin a \frac{d}{dx} \cos(a+y) = 0 \end{aligned}$$

[Using product rule and chain rule]

$$\begin{aligned} \Rightarrow \quad & \left[x \cos(a+y) \frac{d}{dx} (a+y) + \sin(a+y)(1) \right] + \sin a \left[-\sin(a+y) \frac{d}{dx} (a+y) \right] = 0 \\ \Rightarrow \quad & \left[x \cos(a+y) \left(0 + \frac{dy}{dx} \right) + \sin(a+y) \right] - \sin a \sin(a+y) \left(0 + \frac{dy}{dx} \right) = 0 \\ \Rightarrow \quad & x \cos(a+y) \frac{dy}{dx} + \sin(a+y) - \sin a \sin(a+y) \frac{dy}{dx} = 0 \\ \Rightarrow \quad & \frac{dy}{dx} [x \cos(a+y) - \sin a \sin(a+y)] = -\sin(a+y) \end{aligned}$$

Put $x = -\sin a \frac{\cos(a+y)}{\sin(a+y)}$ from equation (i),

$$\begin{aligned} \Rightarrow \quad & \frac{dy}{dx} \left[-\sin a \frac{\cos^2(a+y)}{\sin(a+y)} - \sin a \sin(a+y) \right] = -\sin(a+y) \\ \Rightarrow \quad & -\frac{dy}{dx} \left[\frac{\sin a \cos^2(a+y) + \sin a \sin^2(a+y)}{\sin(a+y)} \right] = -\sin(a+y) \\ \Rightarrow \quad & \frac{dy}{dx} = \sin(a+y)^2 \left[\frac{\sin(a+y)}{\sin a (\cos^2(a+y) + \sin^2(a+y))} \right] \\ \Rightarrow \quad & \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a} \quad \left[\text{Since } \sin^2 \theta + \cos^2 \theta = 1 \right] \end{aligned}$$

Differentiation Ex 11.4 Q23

Here,

$$y = x \sin y$$

Differentiating with respect to x ,

$$\begin{aligned} \Rightarrow \quad & \frac{dy}{dx} = \frac{d}{dx} (x \sin y) \\ \Rightarrow \quad & \frac{dy}{dx} = x \frac{d}{dx} (\sin y) + \sin y \frac{d}{dx} (x) \quad \left[\text{Using product rule} \right] \\ \Rightarrow \quad & \frac{dy}{dx} = x \cos y \frac{dy}{dx} + \sin y (1) \\ \Rightarrow \quad & \frac{dy}{dx} (1 - x \cos y) = \sin y \\ \Rightarrow \quad & \frac{dy}{dx} = \frac{\sin y}{1 - x \cos y} \end{aligned}$$

Differentiation Ex 11.4 Q24

Here,

$$y\sqrt{x^2+1} = \log(\sqrt{x^2+1} - x)$$

Differentiating with respect to x ,

$$\Rightarrow \frac{d}{dx}(y\sqrt{x^2+1}) = \frac{d}{dx} \log(\sqrt{x^2+1} - x) \quad [\text{Using product rule and chain rule}]$$

$$\Rightarrow y \frac{d}{dx}(\sqrt{x^2+1}) + \sqrt{x^2+1} \frac{dy}{dx} = \frac{1}{(\sqrt{x^2+1} - x)} \times \frac{d}{dx}(\sqrt{x^2+1} - x)$$

$$\Rightarrow y \frac{1}{2\sqrt{x^2+1}} \times \frac{d}{dx}(x^2+1) + \sqrt{x^2+1} \frac{dy}{dx} = \frac{1}{(\sqrt{x^2+1} - x)} \times \left[\frac{1}{2\sqrt{x^2+1}} \frac{d}{dx}(x^2+1) - 1 \right]$$

$$\Rightarrow \frac{2xy}{2\sqrt{x^2+1}} + \sqrt{x^2+1} \frac{dy}{dx} = \frac{1}{(\sqrt{x^2+1} - x)} \left[\frac{2x}{2\sqrt{x^2+1}} - 1 \right]$$

$$\Rightarrow \sqrt{x^2+1} \frac{dy}{dx} = \left[\frac{1}{\sqrt{x^2+1} - x} \right] \left[\frac{x - \sqrt{x^2+1}}{\sqrt{x^2+1}} \right] - \frac{xy}{\sqrt{x^2+1}}$$

$$\Rightarrow \sqrt{x^2+1} \frac{dy}{dx} = \frac{-1}{\sqrt{x^2+1}} - \frac{xy}{\sqrt{x^2+1}}$$

$$\Rightarrow \sqrt{x^2+1} \frac{dy}{dx} = \frac{-(1+xy)}{\sqrt{x^2+1}}$$

$$\Rightarrow (x^2+1) \frac{dy}{dx} = -(1+xy)$$

$$\Rightarrow (x^2+1) \frac{dy}{dx} + 1 + xy = 0$$

Differentiation Ex 11.4 Q25

Here,

$$y = [\log_{\cos x} \sin x][\log_{\sin x} \cos x]^1 + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$y = [\log_{\cos x} \sin x][\log_{\cos x} \sin x] + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$[\text{Since, } \log_a a = (\log_a a)^{-1}]$$

$$y = \left[\frac{\log \sin x}{\log \cos x} \right]^2 + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$[\text{Since, } \log_a b = \frac{\log b}{\log a}]$$

Differentiating with respect to x ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \left[\frac{\log \sin x}{\log \cos x} \right]^2 + \frac{d}{dx} \left(\sin^{-1} \left(\frac{2x}{1+x^2} \right) \right) \\ &= 2 \left[\frac{\log \sin x}{\log \cos x} \right] \frac{d}{dx} \left(\frac{\log \sin x}{\log \cos x} \right) + \frac{1}{\sqrt{1 - \left(\frac{2x}{1+x^2} \right)^2}} \times \frac{d}{dx} \left[\frac{2x}{1+x^2} \right] \end{aligned}$$

$$\frac{dy}{dx} = 2 \left[\frac{\log \sin x}{\log \cos x} \right] \left[\frac{(\log \cos x) \frac{d}{dx}(\log \sin x) - \log \sin x \frac{d}{dx}(\log \cos x)}{(\log \cos x)^2} \right] +$$

$$[\text{Using chain rule, quotient rule}] \left(\frac{1+x^2}{\sqrt{1+x^4-2x^2}} \right) \left(\frac{(1+x^2)(2) - (2x)(2x)}{(1+x^2)^2} \right)$$

$$= 2 \left(\frac{\log \sin x}{\log \cos x} \right) \left(\frac{\log \cos x \times \frac{1}{\sin x} \frac{d}{dx}(\sin x) - \log \sin x \times \frac{1}{\cos x} \frac{d}{dx}(\cos x)}{(\log \cos x)^2} \right) +$$

$$\left(\frac{1+x^2}{\sqrt{1+x^4-2x^2}} \right) \left(\frac{(1+x^2)(2) - (2x)(2x)}{(1+x^2)^2} \right)$$

$$= 2 \left(\frac{\log \sin x}{\log \cos x} \right) \left(\frac{\log \cos x \left(\frac{\cos x}{\sin x} \right) + \log \sin x \left(\frac{\sin x}{\cos x} \right)}{(\log \cos x)^2} \right) +$$

$$\left(\frac{1+x^2}{\sqrt{(1-x^2)^2}} \right) \left(\frac{2+2x^2-4x^2}{(1+x^2)^2} \right)$$

$$\frac{dy}{dx} = 2 \frac{\log \sin x}{(\log \cos x)^3} (\cot x \log \cos x + \tan x \log \sin x) + \frac{2}{1+x^2}$$

Put $x = \frac{\pi}{4}$

$$\frac{dy}{dx} = 2 \left(\frac{\log \sin \frac{\pi}{4}}{(\log \cos \frac{\pi}{4})^3} \right) \left(\cot \frac{\pi}{4} \log \cos \frac{\pi}{4} + \tan \frac{\pi}{4} \log \sin \frac{\pi}{4} \right) + 2 \left(\frac{1}{1 + \left(\frac{\pi}{4} \right)^2} \right)$$

$$= 2 \left(\frac{1}{\left(\log \frac{1}{\sqrt{2}} \right)^2} \right) \left(1 \times \log \frac{1}{\sqrt{2}} + 1 \times \log \frac{1}{\sqrt{2}} \right) + 2 \left(\frac{16}{16 + \pi} \right)$$

$$= 2 \times \frac{2 \log \left(\frac{1}{\sqrt{2}} \right)}{\left(\log \left(\frac{1}{\sqrt{2}} \right) \right)} + \frac{32}{16 + \pi^2}$$

$$= 4 \frac{1}{\log \left(\frac{1}{\sqrt{2}} \right)} + \frac{32}{16 + \pi^2}$$

$$= 4 \frac{1}{-\frac{1}{2} \log 2} + \frac{32}{16 + \pi^2}$$

$$= -\frac{8}{\log 2} + \frac{32}{16 + \pi^2}$$

$$\left(\frac{dy}{dx} \right)_{x=\frac{\pi}{4}} = 8 \left[\frac{4}{16 + \pi^2} - \frac{1}{\log 2} \right]$$

Differentiation Ex 11.4 Q26

Here,

$$\sin(xy) + \frac{y}{x} = x^2 - y^2$$

Differentiating with respect to x ,

$$\Rightarrow \frac{d}{dx} (\sin xy) + \frac{d}{dx} \left(\frac{y}{x} \right) = \frac{d}{dx} (x^2) - \frac{d}{dx} (y^2)$$

$$\Rightarrow \cos(xy) \frac{d}{dx} (xy) + \left[\frac{x \frac{dy}{dx} - y \frac{d}{dx} (x)}{x^2} \right] = 2x - 2y \frac{dy}{dx} \quad \left[\begin{array}{l} \text{Using chain rule, quotient rule,} \\ \text{product rule} \end{array} \right]$$

$$\Rightarrow \cos(xy) \left[x \frac{dy}{dx} + y \frac{d}{dx} (x) \right] + \left[\frac{x \frac{dy}{dx} - y(1)}{x^2} \right] = 2x - 2y \frac{dy}{dx}$$

$$\Rightarrow \cos(xy) \left[x \frac{dy}{dx} + y(1) \right] + \frac{1}{x^2} \left(x \frac{dy}{dx} - y \right) = 2x - 2y \frac{dy}{dx}$$

$$\Rightarrow x \cos(xy) \frac{dy}{dx} + y \cos(xy) + \frac{1}{x} \frac{dy}{dx} - \frac{y}{x^2} = 2x - 2y \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} \left[x \cos(xy) + \frac{1}{x} + 2y \right] = \frac{y}{x^2} - y \cos(xy) + 2x$$

$$\Rightarrow \frac{dy}{dx} \left[\frac{x^2 \cos(xy) + 1 + 2xy}{x} \right] = \frac{1}{x^2} (y - x^2 y \cos xy + 2x^3)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x^3 + y - x^2 y \cos(xy)}{x \{ x^2 \cos xy + 1 + 2xy \}}$$

Differentiation Ex 11.4 Q27

Here,

$$\sqrt{y+x} + \sqrt{y-x} = c$$

Differentiating with respect to x ,

$$\begin{aligned} \Rightarrow \frac{d}{dx}(\sqrt{y+x}) + \frac{d}{dx}\sqrt{y-x} &= \frac{d}{dx}(c) \\ \Rightarrow \frac{1}{2\sqrt{y+x}} \frac{d}{dx}(y+x) + \frac{1}{2\sqrt{y-x}} \frac{d}{dx}(y-x) &= 0 \end{aligned}$$

Using chain rule

$$\begin{aligned} \Rightarrow \frac{1}{2\sqrt{y+x}} \left[\frac{dy}{dx} + 1 \right] + \frac{1}{2\sqrt{y-x}} \left[\frac{dy}{dx} - 1 \right] &= 0 \\ \Rightarrow \frac{dy}{dx} \left(\frac{1}{2\sqrt{y+x}} \right) + \frac{dy}{dx} \left(\frac{1}{2\sqrt{y-x}} \right) &= \frac{1}{2\sqrt{y-x}} - \frac{1}{2\sqrt{y+x}} \\ \Rightarrow \frac{dy}{dx} \times \frac{1}{2} \left[\frac{1}{\sqrt{y+x}} + \frac{1}{\sqrt{y-x}} \right] &= \frac{1}{2} \left[\frac{\sqrt{y-x} - \sqrt{y+x}}{\sqrt{y-x}\sqrt{y+x}} \right] \\ \Rightarrow \frac{dy}{dx} \left[\frac{\sqrt{y-x} + \sqrt{y+x}}{\sqrt{y+x}\sqrt{y-x}} \right] &= \left[\frac{\sqrt{y-x} - \sqrt{y+x}}{\sqrt{y-x}\sqrt{y+x}} \right] \\ \Rightarrow \frac{dy}{dx} = \frac{\sqrt{y-x} - \sqrt{y+x}}{\sqrt{y-x} + \sqrt{y+x}} \times \frac{(\sqrt{y-x} - \sqrt{y+x})}{(\sqrt{y-x} - \sqrt{y+x})} \end{aligned}$$

[rationalizing the denominator]

$$\begin{aligned} \Rightarrow \frac{dy}{dx} &= \frac{(y+x) + (y-x) - 2\sqrt{y+x}\sqrt{y-x}}{y+x - y+x} \\ \Rightarrow \frac{dy}{dx} &= \frac{2y - 2\sqrt{y^2 - x^2}}{2x} \\ \Rightarrow \frac{dy}{dx} &= \frac{2y}{2x} - \frac{2\sqrt{y^2 - x^2}}{2x} \\ \Rightarrow \frac{dy}{dx} &= \frac{y}{x} - \sqrt{\frac{y^2 - x^2}{x^2}} \\ \Rightarrow \frac{dy}{dx} &= \frac{y}{x} - \sqrt{\frac{y^2}{x^2} - 1} \end{aligned}$$

Differentiation Ex 11.4 Q28

Here,

$$\tan(x+y) + \tan(x-y) = 1$$

Differentiating with respect to x ,

$$\begin{aligned} \Rightarrow \frac{d}{dx} \tan(x+y) + \frac{d}{dx} \tan(x-y) &= \frac{d}{dx}(1) \\ \Rightarrow \sec^2(x+y) \frac{d}{dx}(x+y) + \sec^2(x-y) \frac{d}{dx}(x-y) &= 0 \\ \Rightarrow \sec^2(x+y) \left[1 + \frac{dy}{dx} \right] + \sec^2(x-y) \left[1 - \frac{dy}{dx} \right] &= 0 \\ \Rightarrow \sec^2(x+y) \frac{dy}{dx} - \sec^2(x-y) \frac{dy}{dx} &= -[\sec^2(x+y) + \sec^2(x-y)] \\ \Rightarrow \frac{dy}{dx} [\sec^2(x+y) - \sec^2(x-y)] &= -[\sec^2(x+y) + \sec^2(x-y)] \\ \Rightarrow \frac{dy}{dx} &= \frac{\sec^2(x+y) + \sec^2(x-y)}{\sec^2(x-y) - \sec^2(x+y)} \end{aligned}$$

[Using chain rule]

Differentiation Ex 11.4 Q29

Here,

$$e^x + e^y = e^{x+y}$$

Differentiating with respect to x using chain rule,

$$\Rightarrow \frac{d}{dx}(e^x) + \frac{d}{dx}e^y = \frac{d}{dx}(e^{x+y})$$

$$\Rightarrow e^x + e^y \frac{dy}{dx} = e^{x+y} \frac{d}{dx}(x+y)$$

$$\Rightarrow e^x + e^y \frac{dy}{dx} = e^{x+y} \left[1 + \frac{dy}{dx}\right]$$

$$\Rightarrow e^y \frac{dy}{dx} - e^{x+y} \frac{dy}{dx} = e^{x+y} - e^x$$

$$\Rightarrow \frac{dy}{dx}(e^y - e^{x+y}) = e^{x+y} - e^x$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{e^x \times e^y - e^x}{e^y - e^x \times e^y} \right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^x (e^y - 1)}{e^y (1 - e^x)}$$

$$\Rightarrow \frac{dy}{dx} = - \frac{e^x (e^y - 1)}{e^y (e^x - 1)}$$

Differentiation Ex 11.4 Q30

It is given that, $\cos y = x \cos(a+y)$

$$\therefore \frac{d}{dx}[\cos y] = \frac{d}{dx}[x \cos(a+y)]$$

$$\Rightarrow -\sin y \frac{dy}{dx} = \cos(a+y) \cdot \frac{d}{dx}(x) + x \cdot \frac{d}{dx}[\cos(a+y)]$$

$$\Rightarrow -\sin y \frac{dy}{dx} = \cos(a+y) + x \cdot [-\sin(a+y)] \frac{dy}{dx}$$

$$\Rightarrow [x \sin(a+y) - \sin y] \frac{dy}{dx} = \cos(a+y) \quad \dots(1)$$

$$\text{Since } \cos y = x \cos(a+y), x = \frac{\cos y}{\cos(a+y)}$$

Then, equation (1) reduces to

$$\left[\frac{\cos y}{\cos(a+y)} \cdot \sin(a+y) - \sin y \right] \frac{dy}{dx} = \cos(a+y)$$

$$\Rightarrow [\cos y \cdot \sin(a+y) - \sin y \cdot \cos(a+y)] \cdot \frac{dy}{dx} = \cos^2(a+y)$$

$$\Rightarrow \sin(a+y-y) \frac{dy}{dx} = \cos^2(a+b)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+b)}{\sin a}$$

Hence, proved.

Ex 11.5

Differentiation Ex 11.5 Q1

Let $y = x^{\frac{1}{x}}$ --- (i)

Taking log on both the sides,

$$\Rightarrow \log y = \log x^{\frac{1}{x}}$$

$$\Rightarrow \log y = \frac{1}{x} \log x \quad \left[\text{Since, } \log a^b = b \log a \right]$$

Differentiating with respect to x ,

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \frac{1}{x} \frac{d}{dx} (\log x) + \log x \frac{d}{dx} (x^{-1}) \quad \left[\text{Using product rule} \right]$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \frac{1}{x} \times \frac{1}{x} + (\log x) \times \left(-\frac{1}{x^2} \right)$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \frac{1}{x^2} - \frac{\log x}{x^2}$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \frac{(1 - \log x)}{x^2}$$

$$\Rightarrow \frac{dy}{dx} = y \left[\frac{1 - \log x}{x^2} \right]$$

Put the value of y from equation (i),

$$\frac{dy}{dx} = x^{\frac{1}{x}} \left[\frac{1 - \log x}{x^2} \right]$$

Differentiation Ex 11.5 Q2

Let $y = x^{\sin x}$ --- (i)

Taking log on both the sides,

$$\log y = \log x^{\sin x}$$

$$\log y = \sin x \log x \quad \left[\text{Since, } \log a^b = b \log a \right]$$

Differentiating with respect to x ,

$$\frac{1}{y} \frac{dy}{dx} = \sin x \frac{d}{dx} \log x + \log x \frac{d}{dx} \sin x \quad \left[\text{Using product rule} \right]$$

$$\frac{1}{y} \frac{dy}{dx} = \sin x \left(\frac{1}{x} \right) + \log x (\cos x)$$

$$\frac{dy}{dx} = y \left[\frac{\sin x}{x} + (\log x) (\cos x) \right]$$

Put the value of y ,

$$\frac{dy}{dx} = x^{\sin x} \left[\frac{\sin x}{x} + (\log x) (\cos x) \right]$$

Differentiation Ex 11.5 Q3

Let $y = (1 + \cos x)^x$ --- (i)

Taking log on both the sides,

$$\begin{aligned}\log y &= \log(1 + \cos x)^x \\ \log y &= x \log(1 + \cos x)\end{aligned}$$

Differentiating with respect to x ,

$$\begin{aligned}\frac{1}{y} \frac{dy}{dx} &= x \frac{d}{dx} \log(1 + \cos x) + \log(1 + \cos x) \frac{d}{dx} (x) && \text{[Using product rule and chain rule]} \\ \frac{1}{y} \frac{dy}{dx} &= x \frac{1}{(1 + \cos x)} \frac{d}{dx} (1 + \cos x) + \log(1 + \cos x) (1) \\ \frac{1}{y} \frac{dy}{dx} &= \frac{x}{(1 + \cos x)} (0 - \sin x) + \log(1 + \cos x) \\ \frac{1}{y} \frac{dy}{dx} &= \log(1 + \cos x) - \frac{x \sin x}{(1 + \cos x)} \\ \frac{dy}{dx} &= y \left[\log(1 + \cos x) - \frac{x \sin x}{1 + \cos x} \right] \\ \frac{dy}{dx} &= (1 + \cos x)^x \left[\log(1 + \cos x) - \frac{x \sin x}{(1 + \cos x)} \right] && \text{[Using equation (i)]}\end{aligned}$$

Differentiation Ex 11.5 Q4

Let $y = x^{\cos^{-1} x}$ --- (i)

Taking log on both the sides,

$$\begin{aligned}\log y &= \log x^{\cos^{-1} x} \\ \log y &= \cos^{-1} x \log x && \text{[Since, } \log a^b = b \log a \text{]}\end{aligned}$$

Differentiating it with respect to x using product rule,

$$\begin{aligned}\frac{1}{y} \frac{dy}{dx} &= \cos^{-1} x \frac{d}{dx} (\log x) + \log x \frac{d}{dx} (\cos^{-1} x) \\ &= \cos^{-1} x \left(\frac{1}{x} \right) + \log x \left(\frac{-1}{\sqrt{1-x^2}} \right) \\ \frac{1}{y} \frac{dy}{dx} &= \frac{\cos^{-1} x}{x} - \frac{\log x}{\sqrt{1-x^2}} \\ \frac{dy}{dx} &= y \left[\frac{\cos^{-1} x}{x} - \frac{\log x}{\sqrt{1-x^2}} \right] \\ \frac{dy}{dx} &= x^{\cos^{-1} x} \left[\frac{\cos^{-1} x}{x} - \frac{\log x}{\sqrt{1-x^2}} \right] && \text{[Using equation (i)]}\end{aligned}$$

Differentiation Ex 11.5 Q5

Let $y = (\log x)^x$ --- (i)

Taking log on both the sides,

$$\begin{aligned}\log y &= \log (\log x)^x \\ \log y &= x \log (\log x) && \text{[Since, } \log a^b = b \log a \text{]}\end{aligned}$$

Differentiating with respect to x , using product rule, chain rule,

$$\begin{aligned}\frac{1}{y} \frac{dy}{dx} &= x \frac{d}{dx} \log (\log x) + \log \log x \frac{d}{dx} (x) \\ &= x \frac{1}{\log x} \frac{d}{dx} (\log x) + \log \log x (1) \\ &= \frac{x}{\log x} \left(\frac{1}{x} \right) + \log \log x \\ \frac{1}{y} \frac{dy}{dx} &= \frac{1}{\log x} + \log \log x \\ \frac{dy}{dx} &= y \left[\frac{1}{\log x} + \log \log x \right] \\ \frac{dy}{dx} &= (\log x)^x \left[\frac{1}{\log x} + \log \log x \right] && \text{[Using equation (i)]}\end{aligned}$$

Differentiation Ex 11.5 Q6

Let $y = (\log x)^{\cos x}$ --- (i)

Taking log on both the sides,

$$\log y = \log(\log x)^{\cos x}$$

$$\log y = \cos x \log(\log x) \quad \left[\text{Since, } \log a^b = b \log a \right]$$

Differentiating with respect to x , using product rule, chain rule,

$$\frac{1}{y} \frac{dy}{dx} = \cos x \frac{d}{dx} \log(\log x) + \log \log x \frac{d}{dx} (\cos x)$$

$$= \frac{\cos x}{\log x} \frac{d}{dx} (\log x) + \log \log x \times (-\sin x)$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{\cos x}{\log x} \times \left(\frac{1}{x} \right) - \sin x \log \log x$$

$$\frac{dy}{dx} = y \left[\frac{\cos x}{x \log x} - \sin x \log \log x \right]$$

$$\frac{dy}{dx} = (\log x)^{\cos x} \left[\frac{\cos x}{x \log x} - \sin x \log \log x \right] \quad \left[\text{Using equation (i)} \right]$$

Differentiation Ex 11.5 Q7

Let $y = (\sin x)^{\cos x}$ --- (i)

Taking log on both the sides,

$$\log y = \log(\sin x)^{\cos x}$$

$$\log y = \cos x \log \sin x \quad \left[\text{Since, } \log a^b = b \log a \right]$$

Differentiating with respect to x , using product rule, chain rule,

$$\frac{1}{y} \frac{dy}{dx} = \cos x \frac{d}{dx} \log \sin x + \log \sin x \frac{d}{dx} \cos x$$

$$= \cos x \frac{1}{\sin x} \frac{d}{dx} (\sin x) + \log \sin x (-\sin x)$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{\cos x}{\sin x} (\cos x) - \sin x \log \sin x$$

$$\frac{dy}{dx} = y [\cos x \cot x - \sin x \log \sin x]$$

$$\frac{dy}{dx} = (\sin x)^{\cos x} [\cos x \cot x - \sin x \log \sin x]$$

Differentiation Ex 11.5 Q8

Let $y = e^{x \log x}$

$$\Rightarrow y = e^{\log x^x} \quad \left[\text{Since, } \log a^b = b \log a \right]$$

$$\Rightarrow y = x^x \quad \text{--- (i)} \quad \left[\text{Since, } e^{\log a} = a \right]$$

Taking log both the sides,

$$\log y = \log x^x$$

$$\log y = x \log x$$

Differentiating with respect to x , using product rule,

$$\frac{1}{y} \frac{dy}{dx} = x \frac{d}{dx} (\log x) + \log x \frac{d}{dx} (x)$$

$$= x \left(\frac{1}{x} \right) + \log x (1)$$

$$\frac{1}{y} \frac{dy}{dx} = 1 + \log x$$

$$\frac{dy}{dx} = y [1 + \log x]$$

$$\frac{dy}{dx} = x^x (1 + \log x) \quad \left[\text{Using equation (i)} \right]$$

Differentiation Ex 11.5 Q9

Let $y = (\sin x)^{\log x}$ ---(i)

Taking log on both the sides,

$$\begin{aligned}\log y &= \log(\sin x)^{\log x} \\ \log y &= \log x \log(\sin x) \quad \quad \quad \left[\text{Using } \log a^b = b \log a \right]\end{aligned}$$

Differentiating with respect to x , using product rule and chain rule,

$$\begin{aligned}\frac{1}{y} \frac{dy}{dx} &= \log x \frac{d}{dx}(\log \sin x) + \log \sin x \frac{d}{dx}(\log x) \\ &= \log x \left(\frac{1}{\sin x} \right) \frac{d}{dx}(\sin x) + \log \sin x \left(\frac{1}{x} \right) \\ &= \frac{\log x}{\sin x} \times \cos x + \frac{\log \sin x}{x} \\ \frac{1}{y} \frac{dy}{dx} &= \log x \cot x + \frac{\log \sin x}{x} \\ \frac{dy}{dx} &= y \left[\log x \cot x + \frac{\log \sin x}{x} \right] \\ \frac{dy}{dx} &= (\sin x)^{\log x} \left[\log x \cot x + \frac{\log \sin x}{x} \right] \quad \quad \quad \left[\text{Using equation (i)} \right]\end{aligned}$$

Differentiation Ex 11.5 Q10

Let $y = 10^{\log \sin x}$ ---(i)

Taking log on both the sides,

$$\begin{aligned}\log y &= \log 10^{\log \sin x} \\ \log y &= \log \sin x \log 10 \quad \quad \quad \left[\text{Since, } \log a^b = b \log a \right]\end{aligned}$$

Differentiating with respect to x , using chain rule,

$$\begin{aligned}\frac{1}{y} \frac{dy}{dx} &= \log 10 \frac{d}{dx}(\log \sin x) \\ &= \log 10 \left(\frac{1}{\sin x} \right) \frac{d}{dx}(\sin x) \\ \frac{1}{y} \frac{dy}{dx} &= \log 10 \left(\frac{1}{\sin x} \right) (\cos x) \\ \frac{dy}{dx} &= y [\log 10 \cot x] \\ \frac{dy}{dx} &= 10^{\log \sin x} [\log 10 \times \cot x] \quad \quad \quad \left[\text{Using equation (i)} \right]\end{aligned}$$

Differentiation Ex 11.5 Q11

Let $y = (\log x)^{\log x}$

Taking logarithm on both the sides, we obtain

$$\log y = \log x \cdot \log(\log x)$$

Differentiating both sides with respect to x , we obtain

$$\begin{aligned}\frac{1}{y} \frac{dy}{dx} &= \frac{d}{dx} [\log x \cdot \log(\log x)] \\ \Rightarrow \frac{1}{y} \frac{dy}{dx} &= \log(\log x) \cdot \frac{d}{dx}(\log x) + \log x \cdot \frac{d}{dx} [\log(\log x)] \\ \Rightarrow \frac{dy}{dx} &= y \left[\log(\log x) \cdot \frac{1}{x} + \log x \cdot \frac{1}{\log x} \cdot \frac{d}{dx}(\log x) \right] \\ \Rightarrow \frac{dy}{dx} &= y \left[\frac{1}{x} \log(\log x) + \frac{1}{x} \right] \\ \therefore \frac{dy}{dx} &= (\log x)^{\log x} \left[\frac{1}{x} + \frac{\log(\log x)}{x} \right]\end{aligned}$$

Differentiation Ex 11.5 Q12

Let $y = 10^{(10^x)}$ ---(i)

Taking log on both the sides,

$$\begin{aligned}\log y &= \log 10^{(10^x)} \\ \log y &= 10^x \log 10\end{aligned}$$

Differentiating it with respect to x ,

$$\begin{aligned}\frac{1}{y} \frac{dy}{dx} &= \log 10 \times 10^x \log 10 \\ \frac{1}{y} \frac{dy}{dx} &= 10^x \times (\log 10)^2 \\ \frac{dy}{dx} &= 10^{(10^x)} \times 10^x (\log 10)^2\end{aligned}\quad \text{[Using equation (i)]}$$

Differentiation Ex 11.5 Q13

Let $y = \sin x^x$
 $\Rightarrow \sin^{-1} y = x^x$

Taking log on both the sides,

$$\begin{aligned}\log(\sin^{-1} y) &= \log x^x \\ \log(\sin^{-1} y) &= x \log x\end{aligned}\quad \text{[Since, } \log a^b = b \log a]$$

Differentiating it with respect to x using chain rule and product rule,

$$\begin{aligned}\frac{1}{\sin^{-1} y} \frac{dy}{dx} &= (\sin^{-1} y) = x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x) \\ \frac{1}{\sin^{-1} y} \times \left(\frac{1}{\sqrt{1-y^2}} \right) \frac{dy}{dx} &= x \left(\frac{1}{x} \right) + \log x (1) \\ \frac{dy}{dx} &= \sin^{-1} y \sqrt{1-y^2} (1 + \log x) \\ &= \sin^{-1} (\sin x^x) \sqrt{1 - (\sin x^x)^2} (1 + \log x) \\ &= x^x \sqrt{\cos^2 x^x} (1 + \log x)\end{aligned}\quad \text{[Using equation (i)]}$$

$$\frac{dy}{dx} = x^x \cos x^x (1 + \log x)$$

Differentiation Ex 11.5 Q14

Let $y = (\sin^{-1} x)^x$

Taking log on both the sides,

$$\begin{aligned}\log y &= \log (\sin^{-1} x)^x \\ \log y &= x \log (\sin^{-1} x)\end{aligned}\quad \text{[Since, } \log a^b = b \log a]$$

Differentiating it with respect to x using product rule and chain rule,

$$\begin{aligned}\frac{1}{y} \frac{dy}{dx} &= x \frac{d}{dx} (\log \sin^{-1} x) + \log \sin^{-1} x \frac{d}{dx}(x) \\ &= x \times \frac{1}{\sin^{-1} x} \frac{d}{dx} (\sin^{-1} x) + \log \sin^{-1} x (1) \\ \frac{1}{y} \frac{dy}{dx} &= \frac{x}{\sin^{-1} x} \left(\frac{1}{\sqrt{1-x^2}} \right) + \log \sin^{-1} x \\ \frac{dy}{dx} &= y \left[\log \sin^{-1} x + \frac{x}{\sin^{-1} x \sqrt{1-x^2}} \right] \\ \frac{dy}{dx} &= (\sin^{-1} x)^2 \left[\log \sin^{-1} x + \frac{x}{\sin^{-1} x \sqrt{1-x^2}} \right]\end{aligned}\quad \text{[Using equation (i)]}$$

Differentiation Ex 11.5 Q15

Let $y = x^{\sin^{-1} x}$ ---(i)

Taking log on both the sides,

$$\log y = \log x^{\sin^{-1} x}$$

$$\log y = \sin^{-1} x \log x$$

$$[\text{Since, } \log a^b = b \log a]$$

Differentiating it with respect to x using product rule,

$$\frac{1}{y} \frac{dy}{dx} = \sin^{-1} x \frac{d}{dx} (\log x) + (\log x) \frac{d}{dx} (\sin^{-1} x)$$

$$\frac{1}{y} \frac{dy}{dx} = \sin^{-1} x \left(\frac{1}{x} \right) + (\log x) \left(\frac{1}{\sqrt{1-x^2}} \right)$$

$$\frac{dy}{dx} = y \left[\frac{\sin^{-1} x}{x} + \frac{\log x}{\sqrt{1-x^2}} \right]$$

$$\frac{dy}{dx} = x^{\sin^{-1} x} \left[\frac{\sin^{-1} x}{x} + \frac{\log x}{\sqrt{1-x^2}} \right]$$

$$[\text{Using equation (i)}]$$

Differentiation Ex 11.5 Q16

Let $y = (\tan x)^{\frac{1}{x}}$ ---(i)

Taking log on both the sides,

$$\log y = \log (\tan x)^{\frac{1}{x}}$$

$$\log y = \frac{1}{x} \log (\tan x)$$

$$[\text{Since, } \log a^b = b \log a]$$

Differentiating it with respect to x using product rule and chain rule,

$$\begin{aligned} \frac{1}{y} \frac{dy}{dx} &= \frac{1}{x} \frac{d}{dx} \log (\tan x) + \log (\tan x) \frac{d}{dx} \left(\frac{1}{x} \right) \\ &= \frac{1}{x} \times \frac{1}{\tan x} \frac{d}{dx} (\tan x) + \log (\tan x) \left(-\frac{1}{x^2} \right) \end{aligned}$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{x \tan x} (\sec^2 x) - \frac{\log (\tan x)}{x^2}$$

$$\frac{dy}{dx} = y \left[\frac{\sec^2 x}{x \tan x} - \frac{\log (\tan x)}{x^2} \right]$$

$$\frac{dy}{dx} = (\tan x)^{\frac{1}{x}} \left[\frac{\sec^2 x}{x \tan x} - \frac{\log \tan x}{x^2} \right]$$

$$[\text{Using equation (i)}]$$

Differentiation Ex 11.5 Q17

Let $y = x^{\tan^{-1} x}$ ---(i)

Taking log on both the sides,

$$\log y = \log x^{\tan^{-1} x}$$

$$\log y = \tan^{-1} x \log x$$

$$[\text{Since, } \log a^b = b \log a]$$

Differentiating it with respect to x using product rule,

$$\frac{1}{y} \frac{dy}{dx} = \tan^{-1} x \frac{d}{dx} (\log x) + \log x \frac{d}{dx} (\tan^{-1} x)$$

$$\frac{1}{y} \frac{dy}{dx} = \tan^{-1} x \left(\frac{1}{x} \right) + \log x \left(\frac{1}{1+x^2} \right)$$

$$\frac{dy}{dx} = y \left[\frac{\tan^{-1} x}{x} + \frac{\log x}{1+x^2} \right]$$

$$\frac{dy}{dx} = x^{\tan^{-1} x} \left[\frac{\tan^{-1} x}{x} + \frac{\log x}{1+x^2} \right]$$

$$[\text{Using equation (i)}]$$

Differentiation Ex 11.5 Q18(i)

Let $y = x^x \sqrt{x}$ ---(i)

Taking log on both the sides,

$$\log y = \log (x^x \sqrt{x})$$

$$= \log x^x + \log x^{\frac{1}{2}}$$

$$[\text{Since, } \log (a^b) = \log a + \log b]$$

$$\log y = x \log x + \frac{1}{2} \log x$$

$$[\text{Since, } \log a^b = b \log a]$$

Differentiating it with respect to x using product rule,

$$\frac{1}{y} \frac{dy}{dx} = x \frac{d}{dx} (\log x) + \log x \frac{d}{dx} (x) + \frac{1}{2} \frac{d}{dx} (\log x)$$

$$= x \left(\frac{1}{x} \right) + \log x (1) + \frac{1}{2} \left(\frac{1}{x} \right)$$

$$\frac{1}{y} \frac{dy}{dx} = 1 + \log x + \frac{1}{2x}$$

$$\frac{dy}{dx} = y \left(1 + \log x + \frac{1}{2x} \right)$$

$$\frac{dy}{dx} = x^x \sqrt{x} \left(1 + \log x + \frac{1}{2x} \right)$$

$$[\text{Using equation (i)}]$$

Differentiation Ex 11.5 Q18(ii)

$$\begin{aligned}\text{Let } y &= x^{(\sin x - \cos x)} + \left(\frac{x^2 - 1}{x^2 + 1} \right) \\ y &= e^{(\sin x - \cos x) \log x} + \left(\frac{x^2 - 1}{x^2 + 1} \right) \\ y &= e^{(\sin x - \cos x) \log x} + \left(\frac{x^2 - 1}{x^2 + 1} \right) \quad \left[\text{Since, } e^{\log a} = a, \log a^b = b \log a \right]\end{aligned}$$

Differentiating it with respect to x using chain rule and quotient rule,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left[e^{(\sin x - \cos x) \log x} \right] + \frac{d}{dx} \left[\frac{x^2 - 1}{x^2 + 1} \right] \\ &= e^{(\sin x - \cos x) \log x} \cdot \frac{d}{dx} \{ (\sin x - \cos x) \log x \} + \left[\frac{(x^2 + 1) \frac{d}{dx} (x^2 - 1) - (x^2 - 1) \frac{d}{dx} (x^2 + 1)}{(x^2 + 1)^2} \right] \\ &= e^{\log x^{(\sin x - \cos x)}} \left[(\sin x - \cos x) \frac{d}{dx} (\log x) + (\log x) \frac{d}{dx} (\sin x - \cos x) \right] + \left[\frac{(x^2 + 1)(2x) - (x^2 - 1)(2x)}{(x^2 + 1)^2} \right] \\ &= x^{(\sin x - \cos x)} \left[(\sin x - \cos x) \left(\frac{1}{x} \right) + \log x (\sin x + \cos x) \right] + \left[\frac{2x^3 + 2x - 2x^3 - 2x}{(x^2 + 1)^2} \right] \\ \frac{dy}{dx} &= x^{(\sin x - \cos x)} \left[\frac{(\sin x - \cos x)}{x} + \log x (\sin x + \cos x) \right] + \frac{4x}{(x^2 + 1)^2}\end{aligned}$$

Differentiation Ex 11.5 Q18(iii)

$$\text{Let } y = x^{x \cos x} + \frac{x^2 + 1}{x^2 - 1}$$

$$\text{Also, let } u = x^{x \cos x} \text{ and } v = \frac{x^2 + 1}{x^2 - 1}$$

$$\therefore y = u + v$$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \dots(1)$$

$$u = x^{x \cos x}$$

$$\Rightarrow \log u = \log (x^{x \cos x})$$

$$\Rightarrow \log u = x \cos x \log x$$

Differentiating both sides with respect to x , we obtain

$$\begin{aligned}\frac{1}{u} \frac{du}{dx} &= \frac{d}{dx} (x) \cdot \cos x \cdot \log x + x \cdot \frac{d}{dx} (\cos x) \cdot \log x + x \cos x \cdot \frac{d}{dx} (\log x) \\ \Rightarrow \frac{du}{dx} &= u \left[1 \cdot \cos x \cdot \log x + x \cdot (-\sin x) \log x + x \cos x \cdot \frac{1}{x} \right] \\ \Rightarrow \frac{du}{dx} &= x^{x \cos x} (\cos x \log x - x \sin x \log x + \cos x) \\ \Rightarrow \frac{du}{dx} &= x^{x \cos x} [\cos x (1 + \log x) - x \sin x \log x] \quad \dots(2)\end{aligned}$$

$$v = \frac{x^2 + 1}{x^2 - 1}$$

$$\Rightarrow \log v = \log (x^2 + 1) - \log (x^2 - 1)$$

Differentiating both sides with respect to x , we obtain

$$\begin{aligned}\frac{1}{v} \frac{dv}{dx} &= \frac{2x}{x^2+1} - \frac{2x}{x^2-1} \\ \Rightarrow \frac{dv}{dx} &= v \left[\frac{2x(x^2-1) - 2x(x^2+1)}{(x^2+1)(x^2-1)} \right] \\ \Rightarrow \frac{dv}{dx} &= \frac{x^2+1}{x^2-1} \times \left[\frac{-4x}{(x^2+1)(x^2-1)} \right] \\ \Rightarrow \frac{dv}{dx} &= \frac{-4x}{(x^2-1)^2} \quad \dots(3)\end{aligned}$$

From (1), (2), and (3), we obtain

$$\frac{dy}{dx} = x^{\cos x} \left[\cos x (1 + \log x) - x \sin x \log x \right] - \frac{4x}{(x^2-1)^2}$$

Differentiation Ex 11.5 Q18(iv)

$$\text{Let } y = (x \cos x)^x + (x \sin x)^{\frac{1}{x}}$$

$$\text{Also, let } u = (x \cos x)^x \text{ and } v = (x \sin x)^{\frac{1}{x}}$$

$$\therefore y = u + v$$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \dots(1)$$

$$u = (x \cos x)^x$$

$$\Rightarrow \log u = \log (x \cos x)^x$$

$$\Rightarrow \log u = x \log (x \cos x)$$

$$\Rightarrow \log u = x [\log x + \log \cos x]$$

$$\Rightarrow \log u = x \log x + x \log \cos x$$

Differentiating both sides with respect to x , we obtain

$$\begin{aligned}\frac{1}{u} \frac{du}{dx} &= \frac{d}{dx} (x \log x) + \frac{d}{dx} (x \log \cos x) \\ \Rightarrow \frac{du}{dx} &= u \left[\left\{ \log x \cdot \frac{d}{dx} (x) + x \cdot \frac{d}{dx} (\log x) \right\} + \left\{ \log \cos x \cdot \frac{d}{dx} (x) + x \cdot \frac{d}{dx} (\log \cos x) \right\} \right] \\ \Rightarrow \frac{du}{dx} &= (x \cos x)^x \left[\left(\log x \cdot 1 + x \cdot \frac{1}{x} \right) + \left\{ \log \cos x \cdot 1 + x \cdot \frac{1}{\cos x} \cdot \frac{d}{dx} (\cos x) \right\} \right] \\ \Rightarrow \frac{du}{dx} &= (x \cos x)^x \left[(\log x + 1) + \left\{ \log \cos x + \frac{x}{\cos x} \cdot (-\sin x) \right\} \right] \\ \Rightarrow \frac{du}{dx} &= (x \cos x)^x [(1 + \log x) + (\log \cos x - x \tan x)] \\ \Rightarrow \frac{du}{dx} &= (x \cos x)^x [1 - x \tan x + (\log x + \log \cos x)] \\ \Rightarrow \frac{du}{dx} &= (x \cos x)^x [1 - x \tan x + \log (x \cos x)] \quad \dots(2)\end{aligned}$$

$$\begin{aligned}
v &= (x \sin x)^{\frac{1}{x}} \\
\Rightarrow \log v &= \log (x \sin x)^{\frac{1}{x}} \\
\Rightarrow \log v &= \frac{1}{x} \log (x \sin x) \\
\Rightarrow \log v &= \frac{1}{x} (\log x + \log \sin x) \\
\Rightarrow \log v &= \frac{1}{x} \log x + \frac{1}{x} \log \sin x
\end{aligned}$$

Differentiating both sides with respect to x , we obtain

$$\begin{aligned}
\frac{1}{v} \frac{dv}{dx} &= \frac{d}{dx} \left(\frac{1}{x} \log x \right) + \frac{d}{dx} \left[\frac{1}{x} \log (\sin x) \right] \\
\Rightarrow \frac{1}{v} \frac{dv}{dx} &= \left[\log x \cdot \frac{d}{dx} \left(\frac{1}{x} \right) + \frac{1}{x} \cdot \frac{d}{dx} (\log x) \right] + \left[\log (\sin x) \cdot \frac{d}{dx} \left(\frac{1}{x} \right) + \frac{1}{x} \cdot \frac{d}{dx} \{ \log (\sin x) \} \right] \\
\Rightarrow \frac{1}{v} \frac{dv}{dx} &= \left[\log x \cdot \left(-\frac{1}{x^2} \right) + \frac{1}{x} \cdot \frac{1}{x} \right] + \left[\log (\sin x) \cdot \left(-\frac{1}{x^2} \right) + \frac{1}{x} \cdot \frac{1}{\sin x} \cdot \frac{d}{dx} (\sin x) \right] \\
\Rightarrow \frac{1}{v} \frac{dv}{dx} &= \frac{1}{x^2} (1 - \log x) + \left[-\frac{\log (\sin x)}{x^2} + \frac{1}{x \sin x} \cdot \cos x \right] \\
\Rightarrow \frac{dv}{dx} &= (x \sin x)^{\frac{1}{x}} \left[\frac{1 - \log x}{x^2} + \frac{-\log (\sin x) + x \cot x}{x^2} \right] \\
\Rightarrow \frac{dv}{dx} &= (x \sin x)^{\frac{1}{x}} \left[\frac{1 - \log x - \log (\sin x) + x \cot x}{x^2} \right] \\
\Rightarrow \frac{dv}{dx} &= (x \sin x)^{\frac{1}{x}} \left[\frac{1 - \log (x \sin x) + x \cot x}{x^2} \right] \quad \dots (3)
\end{aligned}$$

From (1), (2), and (3), we obtain

$$\frac{dy}{dx} = (x \cos x)^x [1 - x \tan x + \log (x \cos x)] + (x \sin x)^{\frac{1}{x}} \left[\frac{x \cot x + 1 - \log (x \sin x)}{x^2} \right]$$

Differentiation Ex 11.5 Q18(v)

$$\text{Let } y = \left(x + \frac{1}{x}\right)^x + x^{\left(1 + \frac{1}{x}\right)}$$

$$\text{Also, let } u = \left(x + \frac{1}{x}\right)^x \text{ and } v = x^{\left(1 + \frac{1}{x}\right)}$$

$$\therefore y = u + v$$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \dots(1)$$

$$\text{Then, } u = \left(x + \frac{1}{x}\right)^x$$

$$\Rightarrow \log u = \log \left(x + \frac{1}{x}\right)^x$$

$$\Rightarrow \log u = x \log \left(x + \frac{1}{x}\right)$$

Differentiating both sides with respect to x , we obtain

$$\frac{1}{u} \cdot \frac{du}{dx} = \frac{d}{dx}(x) \times \log \left(x + \frac{1}{x}\right) + x \times \frac{d}{dx} \left[\log \left(x + \frac{1}{x}\right) \right]$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = 1 \times \log \left(x + \frac{1}{x}\right) + x \times \left(\frac{1}{x + \frac{1}{x}}\right) \cdot \frac{d}{dx} \left(x + \frac{1}{x}\right)$$

$$\Rightarrow \frac{du}{dx} = u \left[\log \left(x + \frac{1}{x}\right) + \frac{x}{\left(x + \frac{1}{x}\right)} \times \left(1 - \frac{1}{x^2}\right) \right]$$

$$\Rightarrow \frac{du}{dx} = \left(x + \frac{1}{x}\right)^x \left[\log \left(x + \frac{1}{x}\right) + \frac{\left(x - \frac{1}{x}\right)}{\left(x + \frac{1}{x}\right)} \right]$$

$$\Rightarrow \frac{du}{dx} = \left(x + \frac{1}{x}\right)^x \left[\log \left(x + \frac{1}{x}\right) + \frac{x^2 - 1}{x^2 + 1} \right]$$

$$\Rightarrow \frac{du}{dx} = \left(x + \frac{1}{x}\right)^x \left[\frac{x^2 - 1}{x^2 + 1} + \log \left(x + \frac{1}{x}\right) \right]$$

$$\begin{aligned}
 v &= x^{\left(1+\frac{1}{x}\right)} \\
 \Rightarrow \log v &= \log \left[x^{\left(1+\frac{1}{x}\right)} \right] \\
 \Rightarrow \log v &= \left(1+\frac{1}{x}\right) \log x
 \end{aligned}$$

Differentiating both sides with respect to x , we obtain

$$\begin{aligned}
 \frac{1}{v} \cdot \frac{dv}{dx} &= \left[\frac{d}{dx} \left(1+\frac{1}{x}\right) \right] \times \log x + \left(1+\frac{1}{x}\right) \cdot \frac{d}{dx} \log x \\
 \Rightarrow \frac{1}{v} \frac{dv}{dx} &= \left(-\frac{1}{x^2} \right) \log x + \left(1+\frac{1}{x}\right) \cdot \frac{1}{x} \\
 \Rightarrow \frac{1}{v} \frac{dv}{dx} &= -\frac{\log x}{x^2} + \frac{1}{x} + \frac{1}{x^2} \\
 \Rightarrow \frac{dv}{dx} &= v \left[\frac{-\log x + x + 1}{x^2} \right] \\
 \Rightarrow \frac{dv}{dx} &= x^{\left(1+\frac{1}{x}\right)} \left(\frac{x+1-\log x}{x^2} \right) \quad \dots(3)
 \end{aligned}$$

Therefore, from (1), (2), and (3), we obtain

$$\frac{dy}{dx} = \left(x + \frac{1}{x}\right)^x \left[\frac{x^2-1}{x^2+1} + \log \left(x + \frac{1}{x}\right) \right] + x^{\left(1+\frac{1}{x}\right)} \left(\frac{x+1-\log x}{x^2} \right)$$

Differentiation Ex 11.5 Q18(vi)

$$\begin{aligned}
 \text{Let } y &= e^{\sin x} + (\tan x)^x \\
 y &= e^{\sin x} + e^{\log(\tan x)^x} \\
 y &= e^{\sin x} + e^{x \log(\tan x)} \quad \left[\text{Since, } \log a^b = b \log a, e^{\log a} = a \right]
 \end{aligned}$$

Differentiating it with respect to x using chain rule and product rule,

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} (e^{\sin x}) + \frac{d}{dx} (e^{x \log(\tan x)}) \\
 &= e^{\sin x} \frac{d}{dx} (\sin x) + e^{x \log(\tan x)} \times \frac{d}{dx} (x \log \tan x) \\
 &= e^{\sin x} (\cos x) + e^{\log(\tan x)^x} \left[x \frac{d}{dx} \log \tan x + \log \tan x \frac{d}{dx} (x) \right] \\
 &= e^{\sin x} (\cos x) + (\tan x)^x \left[\frac{x}{\tan x} \frac{d}{dx} (\tan x) + \log \tan x (1) \right] \\
 \frac{dy}{dx} &= \cos x e^{\sin x} + (\tan x)^x \left[\frac{x}{\tan x} (\sec^2 x) + \log \tan x \right]
 \end{aligned}$$

Differentiation Ex 11.5 Q18(vii)

$$\begin{aligned}
 \text{Let } y &= (\cos x)^{\frac{1}{x}} + (\sin x)^{\frac{1}{x}} \\
 y &= e^{\frac{\log(\cos x)}{x}} + e^{\frac{\log(\sin x)}{x}} \quad \left[\text{Since, } \log a^b = b \log a, e^{\log a} = a \right] \\
 y &= e^{\frac{1}{x} \log(\cos x)} + e^{\frac{1}{x} \log(\sin x)}
 \end{aligned}$$

Differentiating it with respect to x using chain rule and product rule,

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} e^{\frac{1}{x} \log \cos x} + \frac{d}{dx} e^{\frac{1}{x} \log \sin x} \\
 &= e^{\frac{1}{x} \log \cos x} \times \frac{d}{dx} \left(\frac{1}{x} \log \cos x \right) + e^{\frac{1}{x} \log \sin x} \times \frac{d}{dx} \left(\frac{1}{x} \log \sin x \right) \\
 &= e^{\frac{\log(\cos x)}{x}} \times \left[x \frac{d}{dx} \log \cos x + \log \cos x \times \frac{d}{dx} \left(\frac{1}{x} \right) \right] + e^{\frac{\log(\sin x)}{x}} \times \left[x \frac{d}{dx} \log \sin x + \log \sin x \times \frac{d}{dx} \left(\frac{1}{x} \right) \right] \\
 &= (\cos x)^{\frac{1}{x}} \left[x \times \left(\frac{1}{\cos x} \right) \frac{d}{dx} \cos x + \log \cos x (1) \right] + (\sin x)^{\frac{1}{x}} \left[x \times \frac{1}{\sin x} \times \frac{d}{dx} (\sin x) + \log \sin x \left(-\frac{1}{x^2} \right) \right] \\
 &= (\cos x)^{\frac{1}{x}} \left[x \left(\frac{1}{\cos x} \right) (-\sin x) + \log \cos x \right] + (\sin x)^{\frac{1}{x}} \left[\frac{1}{x} \times \frac{1}{\sin x} (\cos x) - \frac{1}{x^2} \log \sin x \right] \\
 \frac{dy}{dx} &= (\cos x)^{\frac{1}{x}} [\log \cos x - x \tan x] + (\sin x)^{\frac{1}{x}} \left[\frac{\cot x}{x} - \frac{1}{x^2} \log \sin x \right]
 \end{aligned}$$

Differentiation Ex 11.5 Q18(viii)

$$\text{Let } y = x^{x^2-3} + (x-3)^{x^2}$$

$$\text{Also, let } u = x^{x^2-3} \text{ and } v = (x-3)^{x^2}$$

$$\therefore y = u + v$$

Differentiating both sides with respect to x , we obtain

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \dots(1)$$

$$u = x^{x^2-3}$$

$$\therefore \log u = \log(x^{x^2-3})$$

$$\log u = (x^2 - 3) \log x$$

Differentiating with respect to x , we obtain

$$\begin{aligned}
 \frac{1}{u} \cdot \frac{du}{dx} &= \log x \cdot \frac{d}{dx} (x^2 - 3) + (x^2 - 3) \cdot \frac{d}{dx} (\log x) \\
 \Rightarrow \frac{1}{u} \frac{du}{dx} &= \log x \cdot 2x + (x^2 - 3) \cdot \frac{1}{x} \\
 \Rightarrow \frac{du}{dx} &= x^{x^2-3} \cdot \left[\frac{x^2 - 3}{x} + 2x \log x \right]
 \end{aligned}$$

Also,

$$v = (x-3)^{x^2}$$

$$\therefore \log v = \log(x-3)^{x^2}$$

$$\Rightarrow \log v = x^2 \log(x-3)$$

Differentiating both sides with respect to x , we obtain

$$\begin{aligned}\frac{1}{v} \cdot \frac{dv}{dx} &= \log(x-3) \cdot \frac{d}{dx}(x^2) + x^2 \cdot \frac{d}{dx}[\log(x-3)] \\ \Rightarrow \frac{1}{v} \frac{dv}{dx} &= \log(x-3) \cdot 2x + x^2 \cdot \frac{1}{x-3} \cdot \frac{d}{dx}(x-3) \\ \Rightarrow \frac{dv}{dx} &= v \left[2x \log(x-3) + \frac{x^2}{x-3} \cdot 1 \right] \\ \Rightarrow \frac{dv}{dx} &= (x-3)^{x^2} \left[\frac{x^2}{x-3} + 2x \log(x-3) \right]\end{aligned}$$

Substituting the expressions of $\frac{du}{dx}$ and $\frac{dv}{dx}$ in equation (1), we obtain

$$\frac{dy}{dx} = x^{x^2-3} \left[\frac{x^2-3}{x} + 2x \log x \right] + (x-3)^{x^2} \left[\frac{x^2}{x-3} + 2x \log(x-3) \right]$$

Differentiation Ex 11.5 Q19

Here,

$$\begin{aligned}y &= e^x + 10^x + x^x \\ &= e^x + 10^x + e^{\log x^x} \quad \left[\text{Since, } e^{\log_a a^b} = a, \log a^b = b \log a \right] \\ y &= e^x + 10^x + e^{x \log x}\end{aligned}$$

Differentiating it with respect to x using product rule, chain rule,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(e^x) + \frac{d}{dx}(10^x) + \frac{d}{dx}(e^{x \log x}) \\ &= e^x + 10^x \log 10 + e^{x \log x} \frac{d}{dx}(x \log x) \\ &= e^x + 10^x \log 10 + e^{x \log x} \left[x \times \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x) \right] \\ &= e^x + 10^x \log 10 + e^{\log x^x} \left[x \left(\frac{1}{x} \right) + \log x (1) \right] \\ &= e^x + 10^x \log 10 + x^x [1 + \log x] \\ &= e^x + 10^x \log 10 + x^x [\log e + \log x] \quad \left[\text{Since, } \log_e e = 1 \right] \\ \frac{dy}{dx} &= e^x + 10^x \log 10 + x^x (\log ex) \quad \left[\text{Since } \log A + \log B = \log AB \right]\end{aligned}$$

Differentiation Ex 11.5 Q20

Here,

$$\begin{aligned}y &= x^n + n^x + x^x + n^n \\ y &= x^n + n^n + e^{\log x^x} + n^n \quad \left[\text{Since, } e^{\log_a a^b} = a \text{ and } \log a^b = b \log a \right] \\ y &= x^n + n^x + e^{x \log x} + n^n\end{aligned}$$

Differentiating it with respect to x using chain rule and product rule,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(x^n) + \frac{d}{dx}(n^x) + \frac{d}{dx}(e^{x \log x}) + \frac{d}{dx}(n^n) \\ &= nx^{n-1} + n^x \log n + e^{\log x^x} \left[x \frac{d}{dx} \log x + \log x \frac{d}{dx}(x) \right] \\ &= nx^{n-1} + n^x \log n + x^x \left[x \left(\frac{1}{x} \right) + \log x \right] \\ &= nx^{n-1} + n^x \log n + x^x [1 + \log x] \\ &= nx^{n-1} + n^x \log n + x^x [\log e + \log x] \quad \left[\text{Since, } \log_e e = 1 \text{ and } \log A + \log B = \log(AB) \right] \\ \frac{dy}{dx} &= nx^{n-1} + n^x \log n + x^x \log(ex)\end{aligned}$$

Differentiation Ex 11.5 Q21

Here,

$$y = \frac{(x^2 - 1)^3 (2x - 1)}{\sqrt{(x - 3)(4x - 1)}} \quad \text{---(i)}$$

$$y = \frac{(x^2 - 1)^3 (2x - 1)}{(x - 3)^{\frac{1}{2}} (4x - 1)^{\frac{1}{2}}}$$

Taking log on both the sides,

$$\begin{aligned} \log y &= \log \left[\frac{(x^2 - 1)^3 (2x - 1)}{(x - 3)^{\frac{1}{2}} (4x - 1)^{\frac{1}{2}}} \right] \\ &= \log(x^2 - 1)^3 + \log(2x - 1) - \log(x - 3)^{\frac{1}{2}} - \log(4x - 1)^{\frac{1}{2}} \\ &\quad \left[\text{Since, } \log(AB) = \log A + \log B, \log\left(\frac{A}{B}\right) = \log A - \log B \right] \\ &= 3\log(x^2 - 1) + \log(2x - 1) - \frac{1}{2}\log(x - 3) - \frac{1}{2}\log(4x - 1) \end{aligned}$$

Differentiating it with respect to x using chain rule,

$$\begin{aligned} \frac{1}{y} \frac{dy}{dx} &= 3 \frac{d}{dx} \log(x^2 - 1) + \frac{d}{dx} \log(2x - 1) - \frac{1}{2} \frac{d}{dx} \log(x - 3) - \frac{1}{2} \frac{d}{dx} \log(4x - 1) \\ &= 3 \left(\frac{1}{x^2 - 1} \right) \frac{d}{dx} (x^2 - 1) + \frac{1}{(2x - 1)} \frac{d}{dx} (2x - 1) - \frac{1}{2} \left(\frac{1}{x - 3} \right) \frac{d}{dx} (x - 3) - \frac{1}{2} \left(\frac{1}{4x - 1} \right) \frac{d}{dx} (4x - 1) \\ &= 3 \left(\frac{1}{x^2 - 1} \right) (2x) + \frac{1}{(2x - 1)} (2) - \frac{1}{2} \left(\frac{1}{x - 3} \right) (1) - \frac{1}{2} \left(\frac{1}{4x - 1} \right) (4) \\ \frac{1}{y} \frac{dy}{dx} &= \left[\frac{6x}{x^2 - 1} + \frac{2}{2x - 1} - \frac{1}{2(x - 3)} - \frac{2}{4x - 1} \right] \\ \frac{dy}{dx} &= y \left[\frac{6x}{x^2 - 1} + \frac{2}{2x - 1} - \frac{1}{2(x - 3)} - \frac{2}{4x - 1} \right] \\ \frac{dy}{dx} &= \frac{(x^2 - 1)^3 (2x - 1)}{\sqrt{(x - 3)(4x - 1)}} \left[\frac{6x}{x^2 - 1} + \frac{2}{2x - 1} - \frac{1}{2(x - 3)} - \frac{2}{4x - 1} \right] \quad \text{[Using equation (i)]} \end{aligned}$$

Differentiation Ex 11.5 Q22

Here,

$$y = \frac{e^{2x} \sec x \log x}{\sqrt{1 - 2x}} \quad \text{---(i)}$$

$$\Rightarrow y = \frac{e^{2x} \times \sec x \times \log x}{(1 - 2x)^{\frac{1}{2}}}$$

Taking log on both the sides,

$$\begin{aligned} \log y &= \log e^{2x} + \log \sec x + \log \log x - \frac{1}{2} \log(1 - 2x) \quad \left[\text{Since, } \log\left(\frac{A}{B}\right) = \log A - \log B, \right. \\ &\quad \left. \log(AB) = \log A + \log B \right] \\ \log y &= 2x + \log \sec x + \log \log x - \frac{1}{2} \log(1 - 2x) \quad \left[\text{Since, } \log a^b = b \log a \text{ and } \log_e e = 1 \right] \end{aligned}$$

Differentiating it with respect to x using chain rule,

$$\begin{aligned} \frac{1}{y} \frac{dy}{dx} &= \frac{d}{dx} (2x) + \frac{d}{dx} (\log \sec x) + \frac{d}{dx} (\log \log x) - \frac{1}{2} \frac{d}{dx} \log(1 - 2x) \\ \frac{1}{y} \frac{dy}{dx} &= 2 + \frac{1}{\sec x} \frac{d}{dx} (\sec x) + \frac{1}{\log x} \frac{d}{dx} (\log x) - \frac{1}{2} \left(\frac{1}{1 - 2x} \right) \frac{d}{dx} (1 - 2x) \\ \frac{1}{y} \frac{dy}{dx} &= 2 + \frac{\sec x \tan x}{\sec x} + \frac{1}{(\log x)} \left(\frac{1}{x} \right) - \frac{1}{2} \left(\frac{1}{1 - 2x} \right) (-2) \\ \frac{dy}{dx} &= y \left[2 + \tan x + \frac{1}{x \log x} + \frac{1}{1 - 2x} \right] \\ \frac{dy}{dx} &= \frac{e^{2x} \sec x \log x}{\sqrt{1 - 2x}} \left[2 + \tan x + \frac{1}{x \log x} + \frac{1}{1 - 2x} \right] \quad \text{[Using equation (i)]} \end{aligned}$$

Differentiation Ex 11.5 Q23

Here,

$$y = e^{3x} \times \sin 4x \times 2^x \quad \text{--- (i)}$$

Taking log on both the sides,

$$\log y = \log e^{3x} + \log \sin 4x + \log 2^x$$

$$[\text{Since, } \log(AB) = \log A + \log B]$$

$$\log y = 3x \log e + \log \sin 4x + x \log 2$$

$$[\text{Since, } \log_e e = 1, \log a^b = b \log a]$$

$$\log y = 3x + \log \sin 4x + x \log 2$$

Differentiating it with respect to x ,

$$\frac{1}{y} \frac{dy}{dx} = \frac{d}{dx}(3x) + \frac{d}{dx}(\log \sin 4x) + \frac{d}{dx}(x \log 2)$$

$$= 3 + \frac{1}{\sin 4x} \frac{d}{dx}(\sin 4x) + \log 2(1)$$

$$= 3 + \frac{1}{\sin 4x} (\cos 4x) \frac{d}{dx}(4x) + \log 2$$

$$= 3 + \cot x (4) + \log 2$$

$$\frac{1}{y} \frac{dy}{dx} = 3 + 4 \cot 4x + \log 2$$

$$\frac{dy}{dx} = y [3 + 4 \cot 4x + \log 2]$$

$$\frac{dy}{dx} = e^{3x} \times \sin 4x \times 2^x [3 + 4 \cot 4x + \log 2]$$

Differentiation Ex 11.5 Q24

Here,

$$y = \sin x \sin 2x \sin 3x \sin 4x \quad \text{--- (i)}$$

Taking log on both the sides,

$$\log y = \log(\sin x \sin 2x \sin 3x \sin 4x)$$

$$\log y = \log \sin x + \log \sin 2x + \log \sin 3x + \log \sin 4x$$

Differentiating it with respect to x using chain rule,

$$\frac{1}{y} \frac{dy}{dx} = \frac{d}{dx} \log \sin x + \frac{d}{dx} \log \sin 2x + \frac{d}{dx} \log \sin 3x + \frac{d}{dx} \log \sin 4x$$

$$= \frac{1}{\sin x} \frac{d}{dx}(\sin x) + \frac{1}{\sin 2x} \frac{d}{dx}(\sin 2x) + \frac{1}{\sin 3x} \frac{d}{dx}(\sin 3x) + \frac{1}{\sin 4x} \frac{d}{dx}(\sin 4x)$$

$$= \frac{1}{\sin x} (\cos x) + \frac{1}{\sin 2x} (\cos 2x) \frac{d}{dx}(2x) + \frac{1}{\sin 3x} (\cos 3x) \frac{d}{dx}(3x) + \frac{1}{\sin 4x} (\cos 4x) \frac{d}{dx}(4x)$$

$$\frac{1}{y} \frac{dy}{dx} = [\cot x + \cot 2x (2) + \cot 3x (3) + \cot 4x (4)]$$

$$\frac{dy}{dx} = y [\cot x + 2 \cot 2x + 3 \cot 3x + 4 \cot 4x]$$

$$\frac{dy}{dx} = (\sin x \sin 2x \sin 3x \sin 4x) [\cot x + 2 \cot 2x + 3 \cot 3x + 4 \cot 4x] \quad [\text{Using equation (i)}]$$

Differentiation Ex 11.5 Q25

$$\text{Let } y = x^{\sin x} + (\sin x)^x$$

$$\text{Also, let } u = x^{\sin x} \text{ and } v = (\sin x)^x$$

$$\therefore y = u + v$$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \dots(1)$$

$$u = x^{\sin x}$$

$$\Rightarrow \log u = \log(x^{\sin x})$$

$$\Rightarrow \log u = \sin x \log x$$

Differentiating both sides with respect to x , we obtain

$$\begin{aligned} \frac{1}{u} \frac{du}{dx} &= \frac{d}{dx}(\sin x) \cdot \log x + \sin x \cdot \frac{d}{dx}(\log x) \\ \Rightarrow \frac{du}{dx} &= u \left[\cos x \log x + \sin x \cdot \frac{1}{x} \right] \\ \Rightarrow \frac{du}{dx} &= x^{\sin x} \left[\cos x \log x + \frac{\sin x}{x} \right] \quad \dots(2) \end{aligned}$$

$$v = (\sin x)^x$$

$$\Rightarrow \log v = \log(\sin x)^x$$

$$\Rightarrow \log v = x \log(\sin x)$$

Differentiating both sides with respect to x , we obtain

$$\begin{aligned} \frac{1}{v} \frac{dv}{dx} &= \frac{d}{dx}(x) \times \log(\sin x) + x \times \frac{d}{dx}[\log(\sin x)] \\ \Rightarrow \frac{dv}{dx} &= v \left[\log(\sin x) + x \cdot \frac{1}{\sin x} \cdot \frac{d}{dx}(\sin x) \right] \\ \Rightarrow \frac{dv}{dx} &= (\sin x)^x \left[\log \sin x + \frac{x}{\sin x} \cos x \right] \\ \Rightarrow \frac{dv}{dx} &= (\sin x)^x \left[\log \sin x + x \cot x \right] \\ \Rightarrow \frac{dv}{dx} &= (\sin x)^x \left[\log \sin x + x \cot x \right] \quad \dots(3) \end{aligned}$$

From (1), (2), and (3), we obtain

$$\frac{dy}{dx} = x^{\sin x} \left(\cos x \log x + \frac{\sin x}{x} \right) + (\sin x)^x \left[\log \sin x + x \cot x \right]$$

Here,

$$\begin{aligned}
 y &= (\sin x)^{\cos x} + (\cos x)^{\sin x} \\
 y &= e^{\log(\sin x)^{\cos x}} + e^{\log(\cos x)^{\sin x}} \\
 y &= e^{\cos x \log \sin x} + e^{\sin x \log \cos x} \quad \left[\text{Since, } \log_e e = 1 \text{ and } \log a^b = b \log a \right]
 \end{aligned}$$

Differentiating it with respect to x using chain rule and product rule,

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} (e^{\cos x \log \sin x}) + \frac{d}{dx} (e^{\sin x \log \cos x}) \\
 &= e^{\cos x \log \sin x} \frac{d}{dx} (\cos x \log \sin x) + e^{\sin x \log \cos x} \frac{d}{dx} (\sin x \log \cos x) \\
 &= e^{\log(\sin x)^{\cos x}} \left[\cos x \frac{d}{dx} \log \sin x + \log \sin x \frac{d}{dx} (\cos x) \right] + e^{\log(\cos x)^{\sin x}} \left[\sin x \frac{d}{dx} \log \cos x + \log \cos x \frac{d}{dx} (\sin x) \right] \\
 &= (\sin x)^{\cos x} \left[\cos x \left(\frac{1}{\sin x} \right) \frac{d}{dx} (\sin x) + \log \sin x \times (-\sin x) \right] + (\cos x)^{\sin x} \left[\sin x \left(\frac{1}{\cos x} \right) \frac{d}{dx} (\cos x) + \log \cos x (\cos x) \right] \\
 &= (\sin x)^{\cos x} [\cot x \times \cos x - \sin x \log \sin x] + (\cos x)^{\sin x} [\tan x (-\sin x) + \cos x \log \cos x] \\
 \frac{dy}{dx} &= (\sin x)^{\cos x} [\cot x \times \cos x - \sin x \log \sin x] + (\cos x)^{\sin x} [\cos x \log \cos x - \sin x \tan x]
 \end{aligned}$$

Differentiation Ex 11.5 Q27

Here,

$$\begin{aligned}
 y &= (\tan x)^{\cot x} + (\cot x)^{\tan x} \\
 y &= e^{\log(\tan x)^{\cot x}} + e^{\log(\cot x)^{\tan x}} \quad \left[\text{Since, } \log_e e = 1, \log a^b = b \log a \right] \\
 y &= e^{\cot x \log \tan x} + e^{\tan x \log \cot x}
 \end{aligned}$$

Differentiating it with respect to x using chain rule and product rule,

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} (e^{\cot x \log \tan x}) + \frac{d}{dx} (e^{\tan x \log \cot x}) \\
 &= e^{\cot x \log \tan x} \frac{d}{dx} (\cot x \log \tan x) + e^{\tan x \log \cot x} \frac{d}{dx} (\tan x \log \cot x) \\
 &= e^{\log(\tan x)^{\cot x}} \left[\cot x \frac{d}{dx} \log \tan x + \log \tan x \frac{d}{dx} \cot x \right] + e^{\log(\cot x)^{\tan x}} \left[\tan x \frac{d}{dx} \log \cot x + \log \cot x \frac{d}{dx} (\tan x) \right] \\
 &= (\tan x)^{\cot x} \left[\cot x \times \left(\frac{1}{\tan x} \right) \frac{d}{dx} (\tan x) + \log \tan x \{-\operatorname{cosec}^2 x\} \right] + (\cot x)^{\tan x} \left[\tan x \left(\frac{1}{\cot x} \right) \frac{d}{dx} (\cot x) + \log \cot x \{\sec^2 x\} \right] \\
 &= \tan x^{\cot x} [(1) \{\sec^2 x\} - \operatorname{cosec}^2 x \log \tan x] + (\cot x)^{\tan x} [(1) \{-\operatorname{cosec}^2 x\} + \sec^2 x \log \cot x] \\
 \frac{dy}{dx} &= (\tan x)^{\cot x} [\sec^2 x - \operatorname{cosec}^2 x \log \tan x] + (\cot x)^{\tan x} [\sec^2 x \log \cot x - \operatorname{cosec}^2 x]
 \end{aligned}$$

Differentiation Ex 11.5 Q28

Here,

$$\begin{aligned}
 y &= (\sin x)^x + \sin^{-1} \sqrt{x} \\
 &= e^{\log(\sin x)^x} + \sin^{-1} \sqrt{x} \\
 y &= e^{x \log \sin x} + \sin^{-1} \sqrt{x} \quad \left[\text{Since, } \log_e e = 1, \log a^b = b \log a \right]
 \end{aligned}$$

Differentiating it with respect to x using chain rule and product rule,

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} (e^{x \log \sin x}) + \frac{d}{dx} \sin^{-1} (\sqrt{x}) \\
 &= e^{x \log \sin x} \frac{d}{dx} (x \log \sin x) + \frac{1}{\sqrt{1 - (\sqrt{x})^2}} \frac{d}{dx} (\sqrt{x}) \\
 &= e^{\log(\sin x)^x} \left[x \frac{d}{dx} \log \sin x + \log \sin x \frac{d}{dx} (x) + \frac{1}{\sqrt{1 - x}} \times \frac{1}{2\sqrt{x}} \right] \\
 &= (\sin x)^x \left[x \times \frac{1}{\sin x} \frac{d}{dx} (\sin x) + \log \sin x (1) \right] + \frac{1}{2\sqrt{x - x^2}} \\
 &= (\sin x)^x \left[\frac{x}{\sin x} (\cos x) + \log \sin x \right] + \frac{1}{2\sqrt{x - x^2}} \\
 \frac{dy}{dx} &= (\sin x)^x [x \cot x + \log \sin x] + \frac{1}{2\sqrt{x - x^2}}
 \end{aligned}$$

Differentiation Ex 11.5 Q29

Here,

$$\begin{aligned}
 y &= x^{\cos x} + (\sin x)^{\tan x} \\
 y &= e^{\log x^{\cos x}} + e^{\log(\sin x)^{\tan x}} \quad \left[\text{Since, } e^{\log a^b} = a \text{ and } \log a^b = b \log a \right] \\
 y &= e^{\cos x \log x} + e^{\tan x \log \sin x}
 \end{aligned}$$

Differentiating it with respect to x using chain rule and product rule,

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} (e^{\cos x \log x}) + \frac{d}{dx} (e^{\tan x \log \sin x}) \\
 &= e^{\cos x \log x} \frac{d}{dx} (\cos x \log x) + e^{\tan x \log \sin x} \times \frac{d}{dx} (\tan x \log \sin x) \\
 &= e^{\log x^{\cos x}} \left[\cos x \frac{d}{dx} (\log x) + \log x \frac{d}{dx} (\cos x) \right] + e^{\log(\sin x)^{\tan x}} \left[\tan x \frac{d}{dx} \log \sin x + \log \sin x \frac{d}{dx} (\tan x) \right] \\
 &= x^{\cos x} \left[\cos x \left(\frac{1}{x} \right) + \log x (-\sin x) \right] + (\sin x)^{\tan x} \left[\tan x \left(\frac{1}{\sin x} \right) \frac{d}{dx} (\sin x) + \log \sin x (\sec^2 x) \right] \\
 &= x^{\cos x} \left[\frac{\cos x}{x} - \sin x \log x \right] + (\sin x)^{\tan x} \left[\tan x \left(\frac{1}{\sin x} \right) (\cos x) + \sec^2 x \log \sin x \right] \\
 \frac{dy}{dx} &= x^{\cos x} \left[\frac{\cos x}{x} - \sin x \log x \right] + (\sin x)^{\tan x} [1 + \sec^2 x \log \sin x]
 \end{aligned}$$

Here,

$$\begin{aligned}
 y &= x^x + (\sin x)^x \\
 &= e^{\log x^x} + e^{\log(\sin x)^x} \\
 y &= e^{x \log x} + e^{x \log \sin x} \quad \left[\text{Using } e^{\log a^b} = a \text{ and } \log a^b = b \log a \right]
 \end{aligned}$$

Differentiating with respect to x using chain rule and product rule,

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} (e^{x \log x}) + \frac{d}{dx} (e^{x \log \sin x}) \\
 &= e^{x \log x} \frac{d}{dx} (x \log x) + e^{x \log \sin x} \frac{d}{dx} (x \log \sin x) \\
 &= e^{\log x^x} \left[x \frac{d}{dx} (\log x) + \log x \frac{d}{dx} (x) \right] + e^{\log(\sin x)^x} \left[x \frac{d}{dx} (\log \sin x) + \log \sin x \frac{d}{dx} (x) \right] \\
 &= x^x \left[x \left(\frac{1}{x} \right) + \log x (1) \right] + (\sin x)^x \left[x \times \left(\frac{1}{\sin x} \right) \frac{d}{dx} (\sin x) + \log \sin x (1) \right] \\
 &= x^x [1 + \log x] + (\sin x)^x \left[x \left(\frac{1}{\sin x} \right) (\cos x) + \log \sin x \right] \\
 \frac{dy}{dx} &= x^x (1 + \log x) + (\sin x)^x [x \cot x + \log \sin x]
 \end{aligned}$$

Differentiation Ex 11.5 Q30

Here,

$$\begin{aligned}
 y &= (\tan x)^{\log x} + \cos^2 \left(\frac{\pi}{4} \right) \\
 y &= e^{\log(\tan x)^{\log x}} + \cos^2 \left(\frac{\pi}{4} \right) \\
 y &= e^{\log x \log \tan x} + \cos^2 \left(\frac{\pi}{4} \right) \quad \left[\text{Since, } e^{\log a^b} = a \text{ and } \log a^b = b \log a \right]
 \end{aligned}$$

Differentiating it using chain rule and product rule,

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} (e^{\log x \log \tan x}) + \frac{d}{dx} \cos^2 \left(\frac{\pi}{4} \right) \\
 &= e^{\log x \log \tan x} \frac{d}{dx} (\log x \log \tan x) + 0 \\
 &= e^{\log(\tan x)^{\log x}} \left[\log x \frac{d}{dx} (\log \tan x) + \log \tan x \frac{d}{dx} (\log x) \right] \\
 &= (\tan x)^{\log x} \left[\log x \left(\frac{1}{\tan x} \right) \frac{d}{dx} (\tan x) + \log \tan x \left(\frac{1}{x} \right) \right] \\
 &= (\tan x)^{\log x} \left[\log x \left(\frac{1}{\tan x} \right) (\sec^2 x) + \frac{\log \tan x}{x} \right] \\
 \frac{dy}{dx} &= (\tan x)^{\log x} \left[\log x \left(\frac{\sec^2 x}{\tan x} \right) + \frac{\log \tan x}{x} \right]
 \end{aligned}$$

Differentiation Ex 11.5 Q31

Here,

$$\begin{aligned}
 y &= x^x + x^{\frac{1}{x}} \\
 &= e^{\log x^x} + e^{\log x^{\frac{1}{x}}} \\
 y &= e^{x \log x} + e^{\left(\frac{1}{x} \log x\right)} \quad \left[\text{Since, } e^{\log a^b} = a, \log a^b = b \log a \right]
 \end{aligned}$$

Differentiating it with respect to x using chain rule and product rule,

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} \left(e^{x \log x} \right) + \frac{d}{dx} \left(e^{\frac{1}{x} \log x} \right) \\
 &= e^{x \log x} + \frac{d}{dx} (x \log x) + e^{\frac{1}{x} \log x} \cdot \frac{d}{dx} \left(\frac{1}{x} \log x \right) \\
 &= e^{\log x^x} \left[x \cdot \frac{d}{dx} (\log x) + \log x \cdot \frac{d}{dx} (x) \right] + e^{\log x^{\frac{1}{x}}} \left[\frac{1}{x} \cdot \frac{d}{dx} (\log x) + \log x \cdot \frac{d}{dx} \left(\frac{1}{x} \right) \right] \\
 &= x^x \left[x \left(\frac{1}{x} \right) + \log x (1) \right] + x^{\frac{1}{x}} \left[\left(\frac{1}{x} \right) \left(\frac{1}{x} \right) + \log x \left(-\frac{1}{x^2} \right) \right] \\
 &= x^x [1 + \log x] + x^{\frac{1}{x}} \left(\frac{1}{x^2} - \frac{1}{x^2} \log x \right) \\
 \frac{dy}{dx} &= x^x [1 + \log x] + x^{\frac{1}{x}} \frac{(1 - \log x)}{x^2}
 \end{aligned}$$

Differentiation Ex 11.5 Q32

$$\text{Let } y = (\log x)^x + x^{\log x}$$

$$\text{Also, let } u = (\log x)^x \text{ and } v = x^{\log x}$$

$$\therefore y = u + v$$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \dots(1)$$

$$u = (\log x)^x$$

$$\Rightarrow \log u = \log [(\log x)^x]$$

$$\Rightarrow \log u = x \log (\log x)$$

Differentiating both sides with respect to x , we obtain

$$\begin{aligned}
 \frac{1}{u} \frac{du}{dx} &= \frac{d}{dx} (x) \times \log (\log x) + x \cdot \frac{d}{dx} [\log (\log x)] \\
 \Rightarrow \frac{du}{dx} &= u \left[1 \times \log (\log x) + x \cdot \frac{1}{\log x} \cdot \frac{d}{dx} (\log x) \right] \\
 \Rightarrow \frac{du}{dx} &= (\log x)^x \left[\log (\log x) + \frac{x}{\log x} \cdot \frac{1}{x} \right] \\
 \Rightarrow \frac{du}{dx} &= (\log x)^x \left[\log (\log x) + \frac{1}{\log x} \right] \\
 \Rightarrow \frac{du}{dx} &= (\log x)^x \left[\frac{\log (\log x) \cdot \log x + 1}{\log x} \right]
 \end{aligned}$$

$$\begin{aligned}
\frac{1}{v} \cdot \frac{dv}{dx} &= \frac{d}{dx} [(\log x)^2] \\
\Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} &= 2(\log x) \cdot \frac{d}{dx} (\log x) \\
\Rightarrow \frac{dv}{dx} &= 2v(\log x) \cdot \frac{1}{x} \\
\Rightarrow \frac{dv}{dx} &= 2x^{\log x} \frac{\log x}{x} \\
\Rightarrow \frac{dv}{dx} &= 2x^{\log x - 1} \cdot \log x \quad \dots(3)
\end{aligned}$$

Therefore, from (1), (2), and (3), we obtain

$$\frac{dy}{dx} = (\log x)^{x-1} [1 + \log x \cdot \log(\log x)] + 2x^{\log x - 1} \cdot \log x$$

Differentiation Ex 11.5 Q33

Here,

$$x^{13}y^7 = (x+y)^{20}$$

Taking log on both the sides,

$$\log(x^{13}y^7) = \log(x+y)^{20}$$

$$13\log x + 7\log y = 20\log(x+y)$$

$$[\text{Since, } \log(AB) = \log A + \log B, \log a^b = b \log a]$$

Differentiating it with respect to x using chain rule,

$$13 \frac{d}{dx} (\log x) + 7 \frac{d}{dx} (\log y) = 20 \frac{d}{dx} \log(x+y)$$

$$\frac{13}{x} + \frac{7}{y} \frac{dy}{dx} = \frac{20}{x+y} \frac{d}{dx} (x+y)$$

$$\frac{13}{x} + \frac{7}{y} \frac{dy}{dx} = \frac{20}{(x+y)} \left[1 + \frac{dy}{dx} \right]$$

$$\frac{7}{y} \frac{dy}{dx} - \frac{20}{(x+y)} = \frac{20}{(x+y)} - \frac{13}{x}$$

$$\frac{dy}{dx} \left[\frac{7}{y} - \frac{20}{(x+y)} \right] = \frac{20}{(x+y)} - \frac{13}{x}$$

$$\frac{dy}{dx} \left[\frac{7(x+y) - 20y}{y(x+y)} \right] = \left[\frac{20x - 13(x+y)}{x(x+y)} \right]$$

$$\frac{dy}{dx} = \left[\frac{20x - 13x - 13y}{x(x+y)} \right] \left[\frac{y(x+y)}{7x + 7y - 20y} \right]$$

$$= \frac{y}{x} \left(\frac{7x - 13y}{7x - 13y} \right)$$

$$\frac{dy}{dx} = \frac{y}{x}$$

Differentiation Ex 11.5 Q34

Here,

$$x^{16}y^9 = (x^2 + y)^{17}$$

Taking log on both the sides,

$$\log(x^{16} \times y^9) = \log(x^2 + y)^{17} \quad \left[\text{Since, } \log(AB) = \log A + \log B, \log a^b = b \log a \right]$$
$$16 \log x + 9 \log y = 17 \log(x^2 + y)$$

Differentiating it with respect to x using chain rule,

$$16 \frac{d}{dx}(\log x) + 9 \frac{d}{dx}(\log y) = 17 \frac{d}{dx} \log(x^2 + y)$$

$$\frac{16}{x} + \frac{9}{y} \frac{dy}{dx} = 17 \frac{1}{(x^2 + y)} \frac{d}{dx}(x^2 + y)$$

$$\frac{16}{x} + \frac{9}{y} \frac{dy}{dx} = \frac{17}{x^2 + y} \left[2x + \frac{dy}{dx} \right]$$

$$\frac{9}{y} \frac{dy}{dx} - \frac{17}{(x^2 + y)} \frac{dy}{dx} = \left(\frac{34x}{x^2 + y} \right) - \frac{16}{x}$$

$$\frac{dy}{dx} \left[\frac{9}{y} - \frac{17}{(x^2 + y)} \right] = \frac{34x^2 - 16x^2 - 16y}{x(x^2 + y)}$$

$$\frac{dy}{dx} \left[\frac{9x^2 + 9y - 17y}{y(x^2 + y)} \right] = \frac{18x^2 - 16y}{x(x^2 + y)}$$

$$\frac{dy}{dx} = \frac{y}{x} \left(\frac{2(9x^2 - 8y)}{9x^2 - 8y} \right)$$

$$\frac{dy}{dx} = \frac{2y}{x}$$

$$x \frac{dy}{dx} = 2y$$

Differentiation Ex 11.5 Q35

Here,

$$y = \sin(x^x) \quad \text{---(i)}$$

$$\text{Let } u = x^x \quad \text{---(ii)}$$

Taking log on both the sides,

$$\log u = \log x^x$$

$$\log u = x \log x$$

Differentiating it with respect to x ,

$$\begin{aligned} \frac{1}{u} \frac{du}{dx} &= \frac{d}{dx}(x \log x) \\ &= x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x) \\ &= x \left(\frac{1}{x}\right) + \log x (1) \end{aligned}$$

$$\frac{1}{u} \frac{du}{dx} = 1 + \log x$$

$$\frac{du}{dx} = u(1 + \log x)$$

$$\frac{du}{dx} = x^x (1 + \log x) \quad \text{---(iii) [Using equation (ii)]}$$

Now, using equation (ii) in equation (i),

$$y = \sin u$$

Differentiating it with respect to x ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}(\sin u) \\ &= \cos u \frac{du}{dx} \end{aligned}$$

Using equation (ii) and (iii),

$$\frac{dy}{dx} = \cos(x^x) \times x^x (1 + \log x)$$

Differentiation Ex 11.5 Q36

Here,

$$x^x + y^y = 1$$

$$e^{\log x^x} + e^{\log y^y} = 1$$

$$e^{x \log x} + e^{y \log y} = 1 \quad \left[\text{Since, } e^{\log a^b} = a, \log a^b = b \log a \right]$$

Differentiating it with respect to x using product rule and chain rule,

$$\begin{aligned} \frac{d}{dx}(e^{x \log x}) + \frac{d}{dx}(e^{y \log y}) &= \frac{d}{dx}(1) \\ e^{x \log x} \frac{d}{dx}(x \log x) + e^{y \log y} \frac{d}{dx}(y \log y) &= 0 \\ e^{\log x^x} \left[x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x) \right] + e^{\log y^y} \left[y \frac{d}{dx}(\log y) + \log y \frac{d}{dx}(y) \right] &= 0 \\ x^x \left[x \left(\frac{1}{x}\right) + \log x (1) \right] + y^y \left[x \left(\frac{1}{y}\right) \frac{dy}{dx} + \log y (1) \right] &= 0 \\ x^x [1 + \log x] + y^y \left(\frac{x}{y} \frac{dy}{dx} + \log y \right) &= 0 \\ y^y \times \frac{x}{y} \frac{dy}{dx} &= -[x^x (1 + \log x) + y^y \log y] \\ (xy^{y-1}) \frac{dy}{dx} &= -[x^x (1 + \log x) + y^y \log y] \\ \frac{dy}{dx} &= - \left[\frac{x^x (1 + \log x) + y^y \log y}{xy^{y-1}} \right] \end{aligned}$$

Differentiation Ex 11.5 Q37

Here,

$$x^y \times y^x = 1$$

Taking on both sides,

$$\log(x^y \times y^x) = \log(1)$$

$$y = \log x + x \log y = \log 1$$

$$\left[\text{Since, } \log(AB) = \log A + \log B, \log a^b = b \log a \right]$$

Differentiating it with respect to x using product rule,

$$\frac{d}{dx}(y \log x) + \frac{d}{dx}(x \log y) = \frac{d}{dx}(\log 1)$$

$$\left[y \frac{d}{dx}(\log x) + \log x \frac{dy}{dx} \right] + \left[x \frac{d}{dx}(\log y) + \log y \frac{d}{dx}(x) \right] = 0$$

$$\left[y \left(\frac{1}{x} \right) + \log x \frac{dy}{dx} \right] + \left[x \left(\frac{1}{y} \frac{dy}{dx} \right) + \log y (1) \right] = 0$$

$$\frac{y}{x} + \log x \frac{dy}{dx} + \frac{x}{y} \frac{dy}{dx} + \log y = 0$$

$$\frac{dy}{dx} \left(\log x + \frac{x}{y} \right) = - \left[\log y + \frac{y}{x} \right]$$

$$\frac{dy}{dx} \left[\frac{y \log x + x}{y} \right] = - \left[\frac{x \log y + y}{x} \right]$$

$$\frac{dy}{dx} = - \frac{y}{x} \left[\frac{x \log y + y}{y \log x + x} \right]$$

Differentiation Ex 11.5 Q38

Here,

$$x^y + y^x = (x+y)^{x+y}$$

$$e^{\log x^y} + e^{\log y^x} = e^{\log(x+y)^{x+y}}$$

$$e^{y \log x} + e^{x \log y} = e^{(x+y) \log(x+y)}$$

$$\left[\text{Since, } e^{\log a^b} = a, \log a^b = b \log a \right]$$

Differentiating it with respect to x using chain rule, product rule,

$$\Rightarrow \frac{d}{dx}(e^{y \log x}) + \frac{d}{dx}(e^{x \log y}) = \frac{d}{dx}(e^{(x+y) \log(x+y)})$$

$$\Rightarrow e^{y \log x} \left[y \frac{d}{dx}(\log x) + \log x \frac{dy}{dx} \right] + e^{x \log y} \left[x \frac{d}{dx} \log y + \log y \frac{d}{dx}(x) \right] = e^{(x+y) \log(x+y)} \frac{d}{dx}[(x+y) \log(x+y)]$$

$$\Rightarrow e^{\log x^y} \left[y \left(\frac{1}{x} \right) + \log x \frac{dy}{dx} \right] + e^{\log y^x} \left[\frac{x}{y} \frac{dy}{dx} + \log y (1) \right] = e^{\log(x+y)^{x+y}} \left[(x+y) \frac{d}{dx} \log(x+y) + \log(x+y) \frac{d}{dx}(x+y) \right]$$

$$\Rightarrow x^y \left[\frac{y}{x} + \log x \frac{dy}{dx} \right] + y^x \left[\frac{x}{y} \frac{dy}{dx} + \log y \right] = (x+y)^{(x+y)} \left[(x+y) \frac{1}{(x+y)} \frac{d}{dx}(x+y) + \log(x+y) \left(1 + \frac{dy}{dx} \right) \right]$$

$$\Rightarrow x^y \times \frac{y}{x} + x^y \log x \frac{dy}{dx} + y^x \times \frac{x}{y} \frac{dy}{dx} + y^x \log y = (x+y)^{(x+y)} \left[1 \times \left(1 + \frac{dy}{dx} \right) + \log(x+y) \left(1 + \frac{dy}{dx} \right) \right]$$

$$\Rightarrow x^{y-1} \times y + x^y \log x \frac{dy}{dx} + y^{x-1} \times x \frac{dy}{dx} + y^x \log y = (x+y)^{(x+y)} + (x+y)^{(x+y)} \frac{dy}{dx} + (x+y)^{(x+y)} \log(x+y) + (x+y)^{(x+y)} \log(x+y) \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} \left[x^y \log x + x y^{x-1} - (x+y)^{(x+y)} (1 + \log(x+y)) \right] = (x+y)^{(x+y)} (1 + \log(x+y)) - x^{y-1} \times y - y^x \log y$$

$$\Rightarrow \frac{dy}{dx} = \frac{(x+y)^{(x+y)} (1 + \log(x+y)) - x^{y-1} \times y - y^x \log y}{x^y \log x + x y^{x-1} - (x+y)^{(x+y)} (1 + \log(x+y))}$$

Differentiation Ex 11.5 Q39

Here,

$$x^m y^n = 1$$

Taking log on both the side,

$$\begin{aligned}\log(x^m y^n) &= \log(1) \\ m \log x + n \log y &= \log(1)\end{aligned}$$

Differentiating it with respect to x ,

$$\begin{aligned}\frac{dy}{dx}(m \log x) + \frac{d}{dx}(n \log y) &= \frac{d}{dx}(\log(1)) \\ \frac{m}{x} + \frac{n}{y} \frac{dy}{dx} &= 0 \\ \frac{dy}{dx} &= -\frac{m}{x} \times \frac{y}{n} \\ \frac{dy}{dx} &= -\frac{my}{nx}\end{aligned}$$

Differentiation Ex 11.5 Q40

Here,

$$y^x = e^{y-x}$$

Taking log on both the sides,

$$\begin{aligned}\log y^x &= \log e^{(y-x)} && \left[\text{Since, } \log a^b = b \log a \text{ and } \log_e e = 1 \right] \\ x \log y &= (y-x) \log e \\ x \log y &= y-x && \text{---(i)}\end{aligned}$$

Differentiating it with respect to x using product rule,

$$\begin{aligned}\frac{d}{dx}(x \log y) &= \frac{d}{dx}(y-x) \\ \left[x \frac{d}{dx}(\log y) + \log y \frac{d}{dx}(x) \right] &= \frac{dy}{dx} - 1 \\ x \left(\frac{1}{y} \right) \frac{dy}{dx} + \log y (1) &= \frac{dy}{dx} - 1 \\ \frac{dy}{dx} \left(\frac{x}{y} - 1 \right) &= -1 - \log y \\ \frac{dy}{dx} \left(\frac{y}{(1+\log y)y} \right) &= -(1+\log y) && \left[\text{Since, from equation (i), } x = \frac{y}{(1+\log y)} \right] \\ \frac{dy}{dx} \left[\frac{1-1-\log y}{(1+\log y)} \right] &= -(1+\log y) \\ \frac{dy}{dx} &= -\frac{(1+\log y)^2}{-\log y} \\ \frac{dy}{dx} &= \frac{(1+\log y)^2}{\log y}\end{aligned}$$

Differentiation Ex 11.5 Q41

Here,

$$(\sin x)^y = (\cos y)^x$$

Taking log on both the sides,

$$\begin{aligned} \log(\sin x)^y &= \log(\cos y)^x & \left[\text{Using } \log a^b &= b \log a \right] \\ y \log(\sin x) &= x \log(\cos y) \end{aligned}$$

Differentiating it with respect to x using product rule and chain rule,

$$\begin{aligned} \frac{d}{dx}[y \log \sin x] &= \frac{d}{dx}[x \log \cos y] \\ y \frac{d}{dx}(\log \sin x) + \log \sin x \frac{dy}{dx} &= x \frac{dy}{dx} \log \cos y + \log \cos y \frac{d}{dx}(x) \\ y \left(\frac{1}{\sin x} \right) \frac{d}{dx}(\sin x) + \log \sin x \frac{dy}{dx} &= \frac{x}{\cos y} \frac{d}{dx}(\cos y) + \log \cos y (1) \\ \frac{y}{\sin x} (\cos x) + \log \sin x \frac{dy}{dx} &= \frac{x}{\cos y} (-\sin y) \frac{dy}{dx} + \log \cos y \\ y \cot x + \log \sin x \frac{dy}{dx} &= -x \tan y \frac{dy}{dx} + \log \cos y \\ \frac{dy}{dx} (\log \sin x + x \tan y) &= \log \cos y - y \cot x \\ \frac{dy}{dx} &= \frac{\log \cos y - y \cot x}{\log \sin x + x \tan y} \end{aligned}$$

Differentiation Ex 11.5 Q42

Here,

$$(\cos x)^y = (\tan y)^x$$

Taking log on both the sides,

$$\begin{aligned} \log(\cos x)^y &= \log(\tan y)^x \\ y \log \cos x &= x \log \tan y & \left[\text{Since, } \log a^b &= b \log a \right] \end{aligned}$$

Differentiating it with respect to x using chain rule and product rule,

$$\begin{aligned} \frac{d}{dx}(y \log \cos x) &= \frac{d}{dx}(x \log \tan y) \\ \left(y \frac{d}{dx} \log \cos x + \log \cos x \frac{dy}{dx} \right) &= \left(x \frac{d}{dx} \log \tan y + \log \tan y \frac{d}{dx}(x) \right) \\ \left(y \left(\frac{1}{\cos x} \right) \frac{d}{dx}(\cos x) + \log \cos x \frac{dy}{dx} \right) &= \left(x \frac{1}{\tan y} \frac{d}{dx}(\tan y) + \log \tan y (1) \right) \\ \left(\frac{y}{\cos x} (-\sin x) + \log \cos x \frac{dy}{dx} \right) &= \left(\frac{x}{\tan y} (\sec^2 y) \right) \frac{dy}{dx} + \log \tan y - y \tan x + \log \cos x \frac{dy}{dx} \\ &= \left(\sec y \operatorname{cosec} y \times x \frac{dy}{dx} + \log \tan y \right) \\ \frac{dy}{dx} [\log \cos x - x \sec y \operatorname{cosec} y] &= \log \tan y + y \tan x \\ \frac{dy}{dx} &= \left[\frac{\log \tan y + y \tan x}{\log \cos x - x \sec y \operatorname{cosec} y} \right] \end{aligned}$$

Differentiation Ex 11.5 Q43

Here,

$$e^x + e^y = e^{x+y} \quad \text{---(i)}$$

Differentiating both the sides using chain rule,

$$\frac{d}{dx}(e^x) + \frac{d}{dx}(e^y) = \frac{d}{dx}(e^{x+y})$$

$$e^x + e^y \frac{dy}{dx} = e^{x+y} \frac{d}{dx}(x+y)$$

$$e^x + e^y \frac{dy}{dx} = e^{x+y} \left[1 + \frac{dy}{dx} \right]$$

$$e^y \frac{dy}{dx} - e^{x+y} \frac{dy}{dx} = e^{x+y} - e^x$$

$$\frac{dy}{dx} = \frac{e^{x+y} - e^x}{e^y - e^{x+y}}$$

$$= \left(\frac{e^x + e^y - e^x}{e^y - e^x - e^y} \right)$$

[Using equation (i)]

$$\frac{dy}{dx} = -e^{y-x}$$

$$\frac{dy}{dx} + e^{y-x} = 0$$

Differentiation Ex 11.5 Q44

Here,

$$e^y = y^x$$

Taking log on both the sides,

$$\log e^y = \log y^x$$

$$y \log e = x \log y$$

[Since, $\log a^b = b \log a, \log_e e = 1$]

$$y = x \log y$$

---(i)

Differentiating it with respect to x using product rule,

$$\frac{dy}{dx} = \frac{d}{dx}(x \log y)$$

$$= x \frac{dy}{dx} (\log y) + \log y \frac{d}{dx}(x)$$

$$\frac{dy}{dx} = \frac{x}{y} \frac{dy}{dx} + \log y (1)$$

$$\frac{dy}{dx} \left(1 - \frac{x}{y} \right) = \log y$$

$$\frac{dy}{dx} \left(\frac{y-x}{y} \right) = \log y$$

$$\frac{dy}{dx} = \frac{y \log y}{y-x}$$

$$\frac{dy}{dx} = \frac{y \log y}{\left(y - \frac{y}{\log y} \right)}$$

[Since, using equation (i)]

$$= \frac{y \log y \times \log y}{y \log y - y}$$

$$= \frac{y (\log y)^2}{y (\log y - 1)}$$

$$\frac{dy}{dx} = \frac{(\log y)^2}{(\log y - 1)}$$

Differentiation Ex 11.5 Q45

Here,

$$e^{x+y} - x = 0$$

$$e^{x+y} = x \quad \text{---(i)}$$

Differentiating it with respect to x using chain rule,

$$\frac{d}{dx}(e^{x+y}) = \frac{d}{dx}(x)$$

$$e^{x+y} \frac{d}{dx}(x+y) = 1$$

$$x \left[1 + \frac{dy}{dx} \right] = 1 \quad \text{[Using equation (i)]}$$

$$1 + \frac{dy}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{x} - 1$$

$$\frac{dy}{dx} = \frac{1-x}{x}$$

Differentiation Ex 11.5 Q46

Here $y = x \sin(a+y)$

Differentiating it with respect to x using the chain rule and product rule,

$$\frac{dy}{dx} = x \frac{d}{dx} \sin(a+y) + \sin(a+y) \frac{dx}{dx}$$

$$\frac{dy}{dx} = x \cos(a+y) \frac{dy}{dx} + \sin(a+y)$$

$$(1 - x \cos(a+y)) \frac{dy}{dx} = \sin(a+y)$$

$$\frac{dy}{dx} = \frac{\sin(a+y)}{(1 - x \cos(a+y))}$$

$$\frac{dy}{dx} = \frac{\sin(a+y)}{\left(1 - \frac{y}{\sin(a+y)} \cos(a+y)\right)} \quad \left[\text{Since } \frac{y}{\sin(a+y)} = x \right]$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin(a+y) - y \cos(a+y)}$$

Differentiation Ex 11.5 Q47

Here $x \sin(a+y) + \sin a \cos(a+y) = 0$

Differentiating it with respect to x using the chain rule and product rule,

$$\frac{d}{dx} [x \sin(a+y) + \sin a \cos(a+y)] = 0$$

$$x \frac{d}{dx} \sin(a+y) + \sin(a+y) \frac{dx}{dx} + \sin a \frac{d}{dx} \cos(a+y) + \cos(a+y) \frac{d}{dx} \sin a = 0$$

$$x \cos(a+y) \left(0 + \frac{dy}{dx} \right) + \sin(a+y) + \sin a \left(-\sin(a+y) \frac{dy}{dx} \right) + 0 = 0$$

$$[x \cos(a+y) - \sin a \sin(a+y)] \frac{dy}{dx} + \sin(a+y) = 0$$

$$\frac{dy}{dx} = -\frac{\sin(a+y)}{x \cos(a+y) - \sin a \sin(a+y)}$$

$$\frac{dy}{dx} = \frac{-\sin(a+y)}{\left(-\frac{\sin a \cos(a+y)}{\sin(a+y)} \right) \cos(a+y) - \sin a \sin(a+y)} \quad \left[\text{Since } x = -\frac{\sin a \cos(a+y)}{\sin(a+y)} \right]$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{(\sin a) \cos^2(a+y) + \sin a \sin^2(a+y)}$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{(\sin a) [\cos^2(a+y) + \sin^2(a+y)]}$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{(\sin a)} \quad \left[\text{Since } \cos^2(a+y) + \sin^2(a+y) = 1 \right]$$

Differentiation Ex 11.5 Q48

Here,

$$(\sin x)^y = x + y$$

Taking log on both the sides,

$$\log(\sin x)^y = \log(x + y)$$

$$y \log(\sin x) = \log(x + y) \quad \left[\text{Since, } \log a^b = b \log a \right]$$

Differentiating it with respect to x using chain rule, product rule,

$$\frac{d}{dx} \{y \log(\sin x)\} = \frac{d}{dx} \log(x + y)$$

$$y \frac{d}{dx} \log \sin x + \log \sin x \frac{dy}{dx} = \frac{1}{x + y} \frac{d}{dx} (x + y)$$

$$\frac{y}{\sin x} \frac{d}{dx} (\sin x) + \log \sin x \frac{dy}{dx} = \frac{1}{(x + y)} \left[1 + \frac{dy}{dx} \right]$$

$$\frac{y (\cos x)}{(\sin x)} + \log \sin x \frac{dy}{dx} = \frac{1}{(x + y)} + \frac{1}{(x + y)} \frac{dy}{dx}$$

$$\frac{dy}{dx} \left(\log \sin x - \frac{1}{x + y} \right) = \frac{1}{(x + y)} - y \cot x$$

$$\frac{dy}{dx} \left(\frac{(x + y) \log \sin x - 1}{(x + y)} \right) = \left(\frac{1 - y(x + y) \cot x}{x + y} \right)$$

$$\frac{dy}{dx} = \left(\frac{1 - y(x + y) \cot x}{(x + y) \log \sin x - 1} \right)$$

Differentiation Ex 11.5 Q49

Here,

$$xy \log(x + y) = 1 \quad \text{---(i)}$$

Differentiating with respect to x using chain rule, product rule,

$$\frac{dy}{dx} \{xy \log(x + y)\} = \frac{d}{dx} (1)$$

$$xy \frac{d}{dx} \log(x + y) + x \log(x + y) \frac{dy}{dx} + y \log(x + y) \frac{d}{dx} (x) = 0$$

$$\frac{xy}{(x + y)} \left(1 + \frac{dy}{dx} \right) + x \log(x + y) \frac{dy}{dx} + y \log(x + y) (1) = 0$$

$$\left(\frac{xy}{x + y} \right) \left(1 + \frac{dy}{dx} \right) + x \log(x + y) \frac{dy}{dx} + y \log(x + y) = 0$$

$$\left(\frac{xy}{x + y} \right) \frac{dy}{dx} + \frac{xy}{x + y} + x \left(\frac{1}{xy} \right) \frac{dy}{dx} + y \left(\frac{1}{xy} \right) = 0 \quad \left[\text{Using equation (i)} \right]$$

$$\frac{dy}{dx} \left[\frac{xy}{x + y} + \frac{1}{y} \right] = - \left[\frac{1}{x} + \frac{xy}{x + y} \right]$$

$$\frac{dy}{dx} \left[\frac{xy^2 + x + y}{(x + y)y} \right] = - \left[\frac{x + y + x^2y}{x(x + y)} \right]$$

$$\frac{dy}{dx} = - \frac{y}{x} \left(\frac{x + y + x^2y}{x + y + xy^2} \right)$$

Differentiation Ex 11.5 Q50

Here,

$$y = x \sin y \quad \text{--- (i)}$$

Differentiating it with respect to x using product rule,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(x \sin y) \\ &= x \frac{d}{dx}(\sin y) + \sin y \frac{d}{dx}(x) \\ &= x \cos y \frac{dy}{dx} + \sin y (1) \\ \frac{dy}{dx} - x \cos y \frac{dy}{dx} &= \sin y \\ \frac{dy}{dx}(1 - x \cos y) &= \sin y \\ \frac{dy}{dx} &= \frac{\sin y}{(1 - x \cos y)}\end{aligned}$$

Put the value of $\sin y = \frac{y}{x}$ from equation (i),

$$\frac{dy}{dx} = \frac{y}{x(1 - x \cos y)}$$

Differentiation Ex 11.5 Q51

Here,

$$f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$$

Differentiating with respect to x using product rule and chain rule,

$$\begin{aligned}\Rightarrow f'(x) &= (1+x)(1+x^2) \frac{d}{dx}(1+x^8) + (1+x)(1+x^2)(1+x^8) \frac{d}{dx}(1+x^4) + (1+x)(1+x^4)(1+x^8) \frac{d}{dx}(1+x^2) \\ &\quad + (1+x^2)(1+x^4)(1+x^8) \frac{d}{dx}(1+x) \\ \Rightarrow f'(x) &= (1+x)(1+x^2)(1+x^4) 8x^7 + (1+x)(1+x^2)(1+x^8) 4x^3 + (1+x)(1+x^4)(1+x^8) 2x \\ &\quad + (1+x^2)(1+x^4)(1+x^8) (1) \\ f'(1) &= (1+1)(1+1)(8) + (1+1)(1+1)(1+1)(4) + (1+1)(1+1)(1+1)(2) + (1+1)(1+1)(1+1) \\ f'(1) &= (2)(2)(8) + (2)(2)(2)(4) + (2)(2)(2)(2) + (2)(2)(2) \\ &= 64 + 32 + 16 + 8 \\ &= 120\end{aligned}$$

So,

$$f'(1) = 120$$

Differentiation Ex 11.5 Q52

Here,

$$y = \log\left(\frac{x^2+x+1}{x^2-x+1}\right) + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{\sqrt{3}x}{1-x^2}\right)$$

Differentiating it with respect to x using chain rule and quotient rule,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \log\left(\frac{x^2+x+1}{x^2-x+1}\right) + \frac{2}{\sqrt{3}} \frac{d}{dx} \tan^{-1}\left(\frac{\sqrt{3}x}{1-x^2}\right) \\ \Rightarrow \frac{dy}{dx} &= \frac{1}{\left(\frac{x^2+x+1}{x^2-x+1}\right)} \frac{d}{dx} \left(\frac{x^2+x+1}{x^2-x+1}\right) + \frac{2}{\sqrt{3}} \left\{ \frac{1}{1+\left(\frac{\sqrt{3}x}{1-x^2}\right)^2} \right\} \frac{d}{dx} \left(\frac{\sqrt{3}x}{1-x^2}\right) \\ \Rightarrow \frac{dy}{dx} &= \left(\frac{x^2-x+1}{x^2+x+1}\right) \left\{ \frac{(x^2-x+1) \frac{d}{dx}(x^2+x+1) - (x^2+x+1) \frac{d}{dx}(x^2-x+1)}{(x^2-x+1)^2} \right\} + \frac{2}{\sqrt{3}} \left\{ \frac{(1-x)^2}{1+x^4-2x^2+3x^2} \right\} \\ &\quad \left\{ \frac{(1-x^2)^2 \frac{d}{dx}(\sqrt{3}x) - \sqrt{3}x \frac{d}{dx}(1-x^2)}{(1-x^2)^2} \right\} \\ \Rightarrow \frac{dy}{dx} &= \left(\frac{1}{x^2+x+1}\right) \left\{ \frac{(x^2-x+1)(2x+1) - (x^2+x+1)(2x-1)}{(x^2-x+1)} \right\} + \frac{2}{\sqrt{3}} \left\{ \frac{(1-x^2)^2}{1+x^2+x^4} \right\} \left\{ \frac{(1-x^2)(\sqrt{3}) - \sqrt{3}x(-2x)}{(1-x^2)^2} \right\} \\ \Rightarrow \frac{dy}{dx} &= \left(\frac{2x^3-2x^2+2x+x^2-x+1-2x^3-2x^2-2x+x^2+x+1}{x^4+2x^2+1-x^2}\right) + \frac{2}{\sqrt{3}} \left(\frac{\sqrt{3}-\sqrt{3}x^2+2\sqrt{3}x^2}{1+x^2+x^4}\right) \\ &= \left(\frac{-2x^2+2}{x^4+x^2+1}\right) + \frac{2\sqrt{3}(x^2+1)}{\sqrt{3}(1+x^2+x^4)} \\ &= \frac{2(1-x^2)}{(x^4+x^2+1)} + \frac{2(x^2+1)}{1+x^2+x^4} \\ &= \frac{2(1-x^2+x^2+1)}{1+x^2+x^4} \\ \frac{dy}{dx} &= \frac{4}{1+x^2+x^4} \end{aligned}$$

Differentiation Ex 11.5 Q53

Here,

$$y = (\sin x - \cos x)^{(\sin x - \cos x)}$$

Taking log on both the sides,

$$\begin{aligned} \Rightarrow \log y &= \log(\sin x - \cos x)^{(\sin x - \cos x)} \\ \Rightarrow \log y &= (\sin x - \cos x) \log(\sin x - \cos x) \end{aligned}$$

Differentiating it with respect to x using product rule, chain rule,

$$\begin{aligned} \Rightarrow \frac{1}{y} \frac{dy}{dx} &= \log(\sin x - \cos x) \frac{d}{dx}(\sin x - \cos x) + (\sin x - \cos x) \frac{d}{dx} \log(\sin x - \cos x) \\ \Rightarrow \frac{1}{y} \frac{dy}{dx} &= \log(\sin x - \cos x) \times (\cos x + \sin x) + \frac{(\sin x - \cos x)}{(\sin x - \cos x)} \frac{d}{dx}(\sin x - \cos x) \\ \Rightarrow \frac{1}{y} \frac{dy}{dx} &= (\cos x + \sin x) \log(\sin x - \cos x) + (\cos x + \sin x) \\ \Rightarrow \frac{1}{y} \frac{dy}{dx} &= (\cos x + \sin x) \{1 + \log(\sin x - \cos x)\} \\ \Rightarrow \frac{dy}{dx} &= y [(\cos x + \sin x) \{1 + \log(\sin x - \cos x)\}] \end{aligned}$$

Using equation (i),

$$\frac{dy}{dx} = (\sin x - \cos x)^{(\sin x - \cos x)} [(\cos x + \sin x) \{1 + \log(\sin x - \cos x)\}]$$

Differentiation Ex 11.5 Q54

The given function is $xy = e^{(x-y)}$

Taking logarithm on both the sides, we obtain

$$\begin{aligned}\log(xy) &= \log(e^{x-y}) \\ \Rightarrow \log x + \log y &= (x-y) \log e \\ \Rightarrow \log x + \log y &= (x-y) \times 1 \\ \Rightarrow \log x + \log y &= x-y\end{aligned}$$

Differentiating both sides with respect to x , we obtain

$$\begin{aligned}\frac{d}{dx}(\log x) + \frac{d}{dx}(\log y) &= \frac{d}{dx}(x) - \frac{dy}{dx} \\ \Rightarrow \frac{1}{x} + \frac{1}{y} \frac{dy}{dx} &= 1 - \frac{dy}{dx} \\ \Rightarrow \left(1 + \frac{1}{y}\right) \frac{dy}{dx} &= 1 - \frac{1}{x} \\ \Rightarrow \left(\frac{y+1}{y}\right) \frac{dy}{dx} &= \frac{x-1}{x} \\ \therefore \frac{dy}{dx} &= \frac{y(x-1)}{x(y+1)}\end{aligned}$$

Differentiation Ex 11.5 Q55

Given that $y^x + x^y + x^x = a^b$.

Putting $u = y^x$, $v = x^y$ and $w = x^x$, we get $u + v + w = a^b$

$$\text{Therefore } \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0 \quad \dots(1)$$

Now, $u = y^x$. Taking logarithm on both sides, we have

$\log u = x \log y$
Differentiating both sides w.r.t. x , we have

$$\begin{aligned}\frac{1}{u} \cdot \frac{du}{dx} &= x \frac{d}{dx}(\log y) + \log y \frac{d}{dx}(x) \\ &= x \cdot \frac{1}{y} \cdot \frac{dy}{dx} + \log y \cdot 1\end{aligned}$$

$$\text{So } \frac{du}{dx} = u \left[\frac{x}{y} \frac{dy}{dx} + \log y \right] = y^x \left[\frac{x}{y} \frac{dy}{dx} + \log y \right] \quad \dots(2)$$

Also $v = x^y$

Taking logarithm on both sides, we have

$\log v = y \log x$
Differentiating both sides w.r.t. x , we have

$$\begin{aligned}\frac{1}{v} \cdot \frac{dv}{dx} &= y \frac{d}{dx}(\log x) + \log x \frac{dy}{dx} \\ &= y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx}\end{aligned}$$

$$\begin{aligned}\text{So } \frac{dv}{dx} &= v \left[\frac{y}{x} + \log x \frac{dy}{dx} \right] \\ &= x^y \left[\frac{y}{x} + \log x \frac{dy}{dx} \right] \quad \dots(3)\end{aligned}$$

Again $w = x^x$

Taking logarithm on both sides, we have

$\log w = x \log x$
Differentiating both sides w.r.t. x , we have

$$\begin{aligned}\frac{1}{w} \cdot \frac{dw}{dx} &= x \frac{d}{dx}(\log x) + \log x \cdot \frac{d}{dx}(x) \\ &= x \cdot \frac{1}{x} + \log x \cdot 1\end{aligned}$$

$$\begin{aligned}\text{i.e. } \frac{dw}{dx} &= w(1 + \log x) \\ &= x^x (1 + \log x) \quad \dots(4)\end{aligned}$$

From (1), (2), (3), (4), we have

$$y^x \left[\frac{x}{y} \frac{dy}{dx} + \log y \right] + x^y \left[\frac{y}{x} + \log x \frac{dy}{dx} \right] + x^x (1 + \log x) = 0$$

$$\text{or } \left(x y^{x-1} + x^y \cdot \log x \right) \frac{dy}{dx} = -x^x (1 + \log x) - y \cdot x^{y-1} - y^x \log y$$

$$\text{Therefore } \frac{dy}{dx} = \frac{-\left[y^x \log y + y \cdot x^{y-1} + x^x (1 + \log x) \right]}{x y^{x-1} + x^y \log x}$$

Differentiation Ex 11.5 Q56

Here $(\cos x)^y = (\cos y)^x$

Taking log on both sides,

$$\log(\cos x)^y = \log(\cos y)^x$$

$$y \log \cos x = x \log \cos y$$

Differentiating it with respect to x using the chain rule and product rule,

$$\frac{d}{dx}(y \log \cos x) = \frac{d}{dx}(x \log \cos y)$$

$$y \frac{d}{dx} \log \cos x + \log \cos x \frac{dy}{dx} = x \frac{d}{dx} \log \cos y + \log \cos y \frac{dx}{dx}$$

$$y \frac{1}{\cos x} (-\sin x) + \log \cos x \frac{dy}{dx} = x \frac{1}{\cos y} (-\sin y) \frac{dy}{dx} + \log \cos y$$

$$\left(\log \cos x + \frac{x \sin y}{\cos y} \right) \frac{dy}{dx} = \log \cos y + y \frac{\sin y}{\cos y}$$

$$(\log \cos x + x \tan y) \frac{dy}{dx} = \log \cos y + y \tan y$$

$$\frac{dy}{dx} = \frac{\log \cos y + y \tan y}{(\log \cos x + x \tan y)}$$

Differentiation Ex 11.5 Q57

Consider the given function,

$$\cos y = x \cos(a + y), \text{ where } \cos a \neq \pm 1$$

Differentiating both sides w.r.t. 'x' we get

$$-\sin y \frac{dy}{dx} = x \left(-\sin(a + y) \frac{dy}{dx} \right) + \cos(a + y)$$

$$\Rightarrow \frac{dy}{dx} [x \sin(a + y) - \sin y] = \cos(a + y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos(a + y)}{x \sin(a + y) - \sin y}$$

Multiplying the numerator and the denominator

by $\cos(a + y)$ on the R.H.S., we have,

$$\begin{aligned} \frac{dy}{dx} &= \frac{\cos^2(a + y)}{x \cos(a + y) \sin(a + y) - \cos(a + y) \sin y} \\ &= \frac{\cos^2(a + y)}{\cos y \sin(a + y) - \cos(a + y) \sin y} \quad [\because \cos y = x \cos(a + y), \text{ given function}] \\ &= \frac{\cos^2(a + y)}{\sin[(a + y) - y]} = \frac{\cos^2(a + y)}{\sin a} \end{aligned}$$

Differentiation Ex 11.5 Q58

Consider the given function, $(x - y)e^{\frac{x}{x-y}} = a$.

We need to prove that $y \frac{dy}{dx} + x = 2y$.

Differentiating the given equation w.r.t. 'x' we get

$$(x - y) \left[e^{\frac{x}{x-y}} \left(\frac{(x - y) - x \left(1 - \frac{dy}{dx} \right)}{(x - y)^2} \right) \right] + e^{\frac{x}{x-y}} \left(1 - \frac{dy}{dx} \right) = 0$$

$$\Rightarrow \frac{(x - y) - x \left(1 - \frac{dy}{dx} \right)}{(x - y)} + \left(1 - \frac{dy}{dx} \right) = 0$$

$$\Rightarrow \left(1 - \frac{dy}{dx} \right) \left(1 - \frac{x}{x - y} \right) + 1 = 0$$

$$\Rightarrow \left(1 - \frac{dy}{dx} \right) \left(\frac{-y}{x - y} \right) + 1 = 0$$

$$\Rightarrow -y + y \frac{dy}{dx} + x - y = 0$$

$$\Rightarrow y \frac{dy}{dx} + x = 2y$$

Differentiation Ex 11.5 Q59

$$x = e^{x/y}$$

$$\log x = \frac{x}{y} \dots \dots \dots (i)$$

$$y = \frac{x}{\log x}$$

$$\frac{dy}{dx} = \frac{\log x \frac{d}{dx}(x) - x \frac{d}{dx}(\log x)}{(\log x)^2}$$

$$\frac{dy}{dx} = \frac{\log x - x \times \frac{1}{x}}{(\log x)^2}$$

$$\frac{dy}{dx} = \frac{\log x - 1}{(\log x)^2}$$

$$\frac{dy}{dx} = \frac{\frac{x}{y} - 1}{(\log x)^2} \dots \dots \dots [\text{from (i)}]$$

$$\frac{dy}{dx} = \frac{x - y}{y(\log x)^2}$$

$$\frac{dy}{dx} = \frac{x - y}{x(\log x)} \dots \dots \dots [\text{from (i)}]$$

Differentiation Ex 11.5 Q60

$$y = x^{\tan x} + \sqrt{\frac{x^2 + 1}{2}}$$

$$y = e^{\tan x \log x} + e^{\frac{1}{2} \log \left(\frac{x^2 + 1}{2} \right)}$$

$$\frac{dy}{dx} = e^{\tan x \log x} \frac{d}{dx}(\tan x \log x) + e^{\frac{1}{2} \log \left(\frac{x^2 + 1}{2} \right)} \frac{d}{dx} \left(\frac{1}{2} \log \left(\frac{x^2 + 1}{2} \right) \right)$$

$$\frac{dy}{dx} = x^{\tan x} \left[\frac{\tan x}{x} + \sec^2 x \log x \right] + \sqrt{\frac{x^2 + 1}{2}} \left(\frac{1}{2} \times \frac{2}{x^2 + 1} \times (x) \right)$$

$$\frac{dy}{dx} = x^{\tan x} \left[\frac{\tan x}{x} + \sec^2 x \log x \right] + \sqrt{\frac{x^2 + 1}{2}} \left(\frac{x}{x^2 + 1} \right)$$

$$\frac{dy}{dx} = x^{\tan x} \left[\frac{\tan x}{x} + \sec^2 x \log x \right] + \frac{x}{\sqrt{2(x^2 + 1)}}$$

Differentiation Ex 11.5 Q61

$$y = 1 + \frac{\alpha}{\left(\frac{1}{x} - \alpha \right)} + \frac{\beta/x}{\left(\frac{1}{x} - \alpha \right) \left(\frac{1}{x} - \beta \right)} + \frac{\gamma/x^2}{\left(\frac{1}{x} - \alpha \right) \left(\frac{1}{x} - \beta \right) \left(\frac{1}{x} - \gamma \right)}$$

$$\left\{ \begin{array}{l} \text{Using the theorem,} \\ \text{If } y = 1 + \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{(x-c)} \text{ then,} \\ \frac{dy}{dx} = \frac{y}{x} \left\{ \frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x} \right\} \end{array} \right\}$$

Here we have $\frac{1}{x}$ instead of x .

So using above theorem we get,

$$\frac{dy}{dx} = \frac{\alpha}{\left(\frac{1}{x} - \alpha \right)} + \frac{\beta}{\left(\frac{1}{x} - \beta \right)} + \frac{\gamma}{\left(\frac{1}{x} - \gamma \right)}$$

Ex 11.6

Differentiation Ex 11.6 Q1

Here,

$$y = \sqrt{x + \sqrt{x + \sqrt{x + \dots \text{ to } \infty}}}$$
$$y = \sqrt{x + y}$$

Squaring both the sides,

$$y^2 = x + y$$

Differentiating it with respect to x ,

$$2y \frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\frac{dy}{dx} (2y - 1) = 1$$

$$\frac{dy}{dx} = \frac{1}{2y - 1}$$

Differentiation Ex 11.6 Q2

Here,

$$y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \dots \text{ to } \infty}}}$$
$$y = \sqrt{\cos x + y}$$

squaring both the sides,

$$y^2 = \cos x + y$$

Differentiating it with respect to x ,

$$2y \frac{dy}{dx} = -\sin x + \frac{dy}{dx}$$

$$\frac{dy}{dx} (2y - 1) = -\sin x$$

$$\frac{dy}{dx} = \frac{-\sin x}{(2y - 1)}$$

$$\frac{dy}{dx} = \frac{\sin x}{1 - 2y}$$

Differentiation Ex 11.6 Q3

Here,

$$y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \dots \text{to } \infty}}}$$
$$y = \sqrt{\log x + y}$$

Squaring both sides,

$$y^2 = \log x + y$$

Differentiating it with respect to x ,

$$2y \frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx}$$
$$\frac{dy}{dx} (2y - 1) = \frac{1}{x}$$

Differentiation Ex 11.6 Q4

Here,

$$y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \dots \text{to } \infty}}}$$
$$y = \sqrt{\tan x + y}$$

Squaring both the sides,

$$y^2 = \tan x + y$$

Differentiating it with respect to x ,

$$2y \frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$$

$$\frac{dy}{dx} (2y - 1) = \sec^2 x$$

$$\frac{dy}{dx} = \frac{\sec^2 x}{2y - 1}$$

Differentiation Ex 11.6 Q5

Here,

$$y = (\sin x)^{(\sin x)^{\sin x}}$$

$$\Rightarrow y = (\sin x)^y$$

Taking log on both the sides,

$$\log y = \log (\sin x)^y$$

$$\log y = y (\log \sin x)$$

Differentiating it with respect to x , using product rule,

$$\frac{1}{y} \frac{dy}{dx} = y \frac{d}{dx} (\log \sin x) + \log \sin x \frac{dy}{dx}$$

$$\frac{1}{y} \frac{dy}{dx} = y \frac{1}{\sin x} \frac{d}{dx} (\sin x) + \log \sin x \frac{dy}{dx}$$

$$\frac{dy}{dx} \left(\frac{1}{y} - \log \sin x \right) = \frac{y}{\sin x} (\cot x)$$

$$\frac{dy}{dx} \left(\frac{1 - y \log \sin x}{y} \right) = y \cot x$$

$$\frac{dy}{dx} = \frac{y^2 \cot x}{(1 - y \log \sin x)}$$

Differentiation Ex 11.6 Q6

Here,

$$y = (\tan x)^{(\tan x)^{\tan x}}$$

$$y = (\tan x)^y$$

Taking log on both the sides,

$$\log y = \log (\tan x)^y$$

$$\log y = y \log \tan x$$

Differentiating with respect to x using product rule and chain rule,

$$\frac{1}{y} \frac{dy}{dx} = y \frac{d}{dx} \log \tan x + \log \tan x \frac{dy}{dx}$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{y}{\tan x} \frac{d}{dx} (\tan x) + \log \tan x \frac{dy}{dx}$$

$$\frac{dy}{dx} \left(\frac{1}{y} - \log \tan x \right) = \frac{y}{\tan x} \sec^2 x$$

$$\left(\frac{dy}{dx} \right)_{x=\frac{\pi}{4}} = \frac{y \sec^2 \left(\frac{\pi}{4} \right)}{\tan \left(\frac{\pi}{4} \right) (1 - y \log \tan \left(\frac{\pi}{4} \right))} \cdot y$$

$$\left(\frac{dy}{dx} \right)_{\frac{\pi}{4}} = \frac{y^2 (\sqrt{2})^2}{1 (1 - y \log \tan 1)}$$

$$= \frac{2(1)^2}{(1-0)}$$

$$\left(\frac{dy}{dx} \right)_{\frac{\pi}{4}} = 2$$

$$\left\{ \begin{array}{l} \text{since,} \\ (y)_{\frac{\pi}{4}} = \left(\tan \frac{\pi}{4} \right)^{\left(\tan \frac{\pi}{4} \right)^{\left(\tan \frac{\pi}{4} \right)}} \\ \Rightarrow y = (1)^1 \\ \Rightarrow y = 1 \end{array} \right\}$$

Differentiation Ex 11.6 Q7

Here,

$$\begin{aligned}
 y &= e^{x^{e^x}} + x^{e^{e^x}} + e^{x^{e^x}} \\
 y &= u + v + w \\
 \frac{dy}{dx} &= \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} \quad \text{---(i)}
 \end{aligned}$$

Where $u = e^{x^{e^x}}, v = x^{e^{e^x}}, w = e^{x^{e^x}}$

Now, $u = e^{x^{e^x}} \quad \text{---(ii)}$

Taking log on both the sides,

$$\begin{aligned}
 \log u &= \log e^{x^{e^x}} \\
 \log u &= x^{e^x} \log e \\
 \log u &= x^{e^x} \quad \text{---(iii)} \quad \left\{ \begin{array}{l} \text{since } \log e = 1, \\ \log a^b = b \log a \end{array} \right.
 \end{aligned}$$

Taking log on both the sides,

$$\begin{aligned}
 \log \log u &= \log x^{e^x} \\
 \log \log u &= e^x \log x
 \end{aligned}$$

Differentiating it with respect to x ,

$$\begin{aligned}
 \frac{1}{\log u} \frac{d}{dx} (\log u) &= e^x \frac{d}{dx} (\log x) + \log x \frac{d}{dx} (e^x) \\
 \frac{1}{\log u} \frac{1}{4} \frac{du}{dx} &= \frac{e^x}{x} + e^x \log x \\
 \frac{du}{dx} &= 4 \log x \left[\frac{e^x}{x} + e^x \log x \right] \\
 \frac{du}{dx} &= e^{x^{e^x}} * x^{e^x} \left[\frac{e^x}{x} + e^x \log x \right] \quad \text{---(A)}
 \end{aligned}$$

Using equation (ii) and (iii)

Now

$$v = x^{e^{e^x}} \quad \text{---(iv)}$$

Taking log on both the sides,

$$\begin{aligned}
 \log v &= \log x^{e^{e^x}} \\
 \log v &= e^{e^x} \log x
 \end{aligned}$$

Differentiating it with respect to x ,

$$\begin{aligned}
 \frac{1}{v} \frac{dv}{dx} &= e^{e^x} \frac{d}{dx} (\log x) + \log x \frac{d}{dx} (e^{e^x}) \\
 \frac{1}{v} \frac{dv}{dx} &= e^{e^x} \left(\frac{1}{x} \right) + \log x e^{e^x} \frac{d}{dx} (e^x) \\
 \frac{dv}{dx} &= v \left[e^{e^x} \left(\frac{1}{x} \right) + \log x e^{e^x} e^x \right] \\
 \frac{dv}{dx} &= x^{e^{e^x}} * e^{e^x} \left[\frac{1}{x} + e^x \log x \right] \quad \text{---(B)}
 \end{aligned}$$

{sinx using equation(4)}

Now, $w = e^{x^{e^x}} \quad \text{---(v)}$

Taking log on both the sides,

$$\begin{aligned}
 \log w &= \log e^{x^{e^x}} \\
 \log w &= x^{e^x} \log e \\
 \log w &= x^{e^x} \quad \text{---(vi)}
 \end{aligned}$$

Taking log on both the sides,

$$\begin{aligned}
 \log \log w &= \log x^{e^x} \\
 \log \log w &= e^x \log x
 \end{aligned}$$

Differentiating it with respect to x ,

$$\frac{1}{\log w} \frac{d}{dx} (\log w) = x^e \frac{d}{dx} (\log x) + \log x \frac{d}{dx} (x^e)$$

$$\frac{1}{\log w} \left(\frac{1}{w} \right) \frac{dw}{dx} = x^e \left(\frac{1}{x} \right) + \log x e x^{e-1}$$

$$\frac{dw}{dx} = w \log w [x^{e-1} + e \log x x^{e-1}]$$

$$\frac{dw}{dx} = e^{x^e} x^{x^e} x^{e-1} (1 + e \log x)$$

---(C) {Using equation (v), (vi)}

Using equation (A), (B) and (C) in equation (i),

$$\begin{aligned} \frac{dy}{dx} &= e^{x^e} x^{x^e} \left[\frac{e^x}{x} + e^x \log x \right] + x^{e^e} e^{e^e} \left[\frac{1}{x} + e^x \log x \right] \\ &+ e^{x^e} x^{x^e} x^{e-1} (1 + e \log x) \end{aligned}$$

Differentiation Ex 11.6 Q8

Here,

$$y = (\cos x)^{(\cos x)^{\cos x}}$$

$$y = (\cos x)^y$$

Taking log on both the sides,

$$\log y = \log (\cos x)^y$$

$$\log y = y \log (\cos x), \{ \text{since } \log a^b = b \log a \}$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{1}{y} \frac{dy}{dx} = y \frac{d}{dx} \log (\cos x) + \log \cos x \frac{dy}{dx}$$

$$\frac{1}{y} \frac{dy}{dx} = y \left(\frac{1}{\cos x} \right) \frac{d}{dx} (\cos x) + \log \cos x \frac{dy}{dx}$$

$$\frac{dy}{dx} \left(\frac{1}{y} - \log \cos x \right) = \frac{y}{\cos x} (-\sin x)$$

$$\frac{dy}{dx} \left(\frac{1 - y \log \cos x}{y} \right) = -y \tan x$$

$$\frac{dy}{dx} = - \frac{y^2 \tan x}{(1 - y \log \cos x)}$$

Ex 11.7

Differentiation Ex 11.7 Q1

Given that $x = at^2$, $y = 2at$

$$\text{So, } \frac{dx}{dt} = \frac{d}{dt}(at^2) = 2at$$

$$\frac{dy}{dt} = \frac{d}{dt}(2at) = 2a$$

$$\text{Therefore, } \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$$

Differentiation Ex 11.7 Q2

Here,

$$x = a(\theta + \sin \theta)$$

Differentiating it with respect to θ ,

$$\frac{dx}{d\theta} = a(1 + \cos \theta) \quad \text{---(i)}$$

And,

$$y = a(1 - \cos \theta)$$

Differentiating it with respect to θ ,

$$\begin{aligned} \frac{dy}{d\theta} &= a(\sin \theta) \\ \frac{dy}{d\theta} &= a \sin \theta \quad \text{---(ii)} \end{aligned}$$

Using equation (i) and (ii),

$$\begin{aligned} \frac{dy}{dx} &= \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} \\ &= \frac{a \sin \theta}{a(1 - \cos \theta)} \\ &= \frac{\frac{2 \sin \theta \cos \theta}{2}}{\frac{2 \sin^2 \theta}{2}}, \quad \left\{ \begin{array}{l} \text{Since, } 1 - \cos \theta = \frac{2 \sin^2 \theta}{2}, \\ \frac{2 \sin \theta \cos \theta}{2} = \sin \theta \end{array} \right\} \\ &= \frac{dy}{dx} = \frac{\tan \theta}{2} \end{aligned}$$

Differentiation Ex 11.7 Q3

Here $x = a \cos \theta$ and $y = b \sin \theta$

Then,

$$\frac{dx}{d\theta} = \frac{d}{d\theta}(a \cos \theta) = -a \sin \theta$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}(b \sin \theta) = b \cos \theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b \cos \theta}{-a \sin \theta} = -\frac{b}{a} \cot \theta$$

Differentiation Ex 11.7 Q4

Here,

$$x = ae^{\theta} (\sin \theta - \cos \theta)$$

Differentiating it with respect to θ ,

$$\begin{aligned}\frac{dx}{d\theta} &= a \left[e^{\theta} \frac{d}{d\theta} (\sin \theta - \cos \theta) + (\sin \theta - \cos \theta) \frac{d}{d\theta} (e^{\theta}) \right] \\ &= a \left[e^{\theta} (\cos \theta + \sin \theta) + (\sin \theta - \cos \theta) e^{\theta} \right] \\ \frac{dx}{d\theta} &= a \left[2e^{\theta} \sin \theta \right] \quad \text{---(i)}\end{aligned}$$

And, $y = ae^{\theta} (\sin \theta + \cos \theta)$

Differentiating it with respect to θ ,

$$\begin{aligned}\frac{dy}{d\theta} &= a \left[e^{\theta} \frac{d}{d\theta} (\sin \theta + \cos \theta) + (\sin \theta + \cos \theta) \frac{d}{d\theta} (e^{\theta}) \right] \\ &= a \left[e^{\theta} (\cos \theta - \sin \theta) + (\sin \theta + \cos \theta) e^{\theta} \right] \\ \frac{dy}{d\theta} &= a \left[2e^{\theta} \cos \theta \right] \quad \text{---(ii)}\end{aligned}$$

Dividing equation (ii) by equation (i),

$$\begin{aligned}\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} &= \frac{a \left[2e^{\theta} \cos \theta \right]}{a \left[2e^{\theta} \sin \theta \right]} \\ \frac{dy}{dx} &= \cot \theta\end{aligned}$$

Differentiation Ex 11.7 Q5

Here $x = b \sin^2 \theta$ and $y = a \cos^2 \theta$

Then,

$$\begin{aligned}\frac{dx}{d\theta} &= \frac{d}{d\theta} (b \sin^2 \theta) = 2b \sin \theta \cos \theta \\ \frac{dy}{d\theta} &= \frac{d}{d\theta} (a \cos^2 \theta) = -2a \cos \theta \sin \theta \\ \therefore \frac{dy}{dx} &= \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-2a \cos \theta \sin \theta}{2b \sin \theta \cos \theta} = -\frac{a}{b}\end{aligned}$$

Differentiation Ex 11.7 Q6

Here $x = a(1 - \cos \theta)$ and $y = a(\theta + \sin \theta)$

Then,

$$\begin{aligned}\frac{dx}{d\theta} &= \frac{d}{d\theta} [a(1 - \cos \theta)] = a(\sin \theta) \\ \frac{dy}{d\theta} &= \frac{d}{d\theta} [a(\theta + \sin \theta)] = a(1 + \cos \theta) \\ \therefore \frac{dy}{dx} &= \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a(1 + \cos \theta)}{a(\sin \theta)} \bigg|_{\theta = \frac{\pi}{2}} = \frac{a(1+0)}{a} = 1\end{aligned}$$

Differentiation Ex 11.7 Q7

Here,

$$x = \frac{e^t + e^{-t}}{2}$$

Differentiating it with respect to t ,

$$\begin{aligned}\frac{dx}{dt} &= \frac{1}{2} \left[\frac{d}{dt}(e^t) + \frac{d}{dt}(e^{-t}) \right] \\ &= \frac{1}{2} \left[e^t + e^{-t} \frac{d}{dt}(-t) \right] \\ \frac{dx}{dt} &= \frac{1}{2} (e^t - e^{-t}) = y \quad \text{---(i)}\end{aligned}$$

And, $y = \frac{e^t - e^{-t}}{2}$

Differentiating it with respect to t ,

$$\begin{aligned}\frac{dy}{dt} &= \frac{1}{2} \left[\frac{d}{dt}(e^t) - \frac{d}{dt}e^{-t} \right] \\ &= \frac{1}{2} \left[e^t - e^{-t} \frac{d}{dt}(e^{-t}) \right] \\ &= \frac{1}{2} (e^t - e^{-t}(-1)) \\ \frac{dy}{dt} &= \frac{1}{2} (e^t + e^{-t}) = x \quad \text{---(ii)}\end{aligned}$$

Dividing equation (ii) by (i),

$$\begin{aligned}\frac{\frac{dy}{dt}}{\frac{dx}{dt}} &= \frac{x}{y} \\ \frac{dy}{dt} &= \frac{x}{y}\end{aligned}$$

Differentiation Ex 11.7 Q8

Here,

$$x = \frac{3at}{1+t^2}$$

Differentiating it with respect to t using quotient rule,

$$\begin{aligned}\frac{dx}{dt} &= \left[\frac{(1+t^2) \frac{d}{dt}(3at) - 3at \frac{d}{dt}(1+t^2)}{(1+t^2)^2} \right] \\ &= \left[\frac{(1+t^2)(3a) - 3at(2t)}{(1+t^2)^2} \right] \\ &= \left[\frac{3a + 3at^2 - 6at^2}{(1+t^2)^2} \right] \\ &= \left[\frac{3a - 3at^2}{(1+t^2)^2} \right] \\ \frac{dx}{dt} &= \frac{3a(1-t^2)}{(1+t^2)^2} \quad \text{---(i)}\end{aligned}$$

And, $y = \frac{3at^2}{1+t^2}$

Differentiating it with respect to t using quotient rule,

$$\begin{aligned}\frac{dy}{dt} &= \left[\frac{(1+t^2) \frac{d}{dt}(3at^2) - 3at^2 \frac{d}{dt}(1+t^2)}{(1+t^2)^2} \right] \\ \frac{dy}{dt} &= \left[\frac{(1+t^2)(6at) - (3at^2)(2t)}{(1+t^2)^2} \right] \\ &= \left[\frac{6at + 6at^3 - 6at^3}{(1+t^2)^2} \right] \\ \frac{dy}{dt} &= \frac{6at}{(1+t^2)^2} \quad \text{---(ii)}\end{aligned}$$

Dividing equation (ii) by (i),

$$\begin{aligned}\frac{\frac{dy}{dt}}{\frac{dx}{dt}} &= \frac{6at}{(1+t^2)^2} \times \frac{(1+t^2)^2}{3a(1-t^2)} \\ \frac{dy}{dx} &= \frac{2t}{1-t^2}\end{aligned}$$

Differentiation Ex 11.7 Q9

The given equations are $x = a(\cos \theta + \theta \sin \theta)$ and $y = a(\sin \theta - \theta \cos \theta)$

$$\begin{aligned}\text{Then, } \frac{dx}{d\theta} &= a \left[\frac{d}{d\theta} \cos \theta + \frac{d}{d\theta} (\theta \sin \theta) \right] = a \left[-\sin \theta + \theta \frac{d}{d\theta} (\sin \theta) + \sin \theta \frac{d}{d\theta} (\theta) \right] \\ &= a [-\sin \theta + \theta \cos \theta + \sin \theta] = a\theta \cos \theta \\ \frac{dy}{d\theta} &= a \left[\frac{d}{d\theta} (\sin \theta) - \frac{d}{d\theta} (\theta \cos \theta) \right] = a \left[\cos \theta - \left\{ \theta \frac{d}{d\theta} (\cos \theta) + \cos \theta \cdot \frac{d}{d\theta} (\theta) \right\} \right] \\ &= a [\cos \theta + \theta \sin \theta - \cos \theta] \\ &= a\theta \sin \theta\end{aligned}$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta} \right)}{\left(\frac{dx}{d\theta} \right)} = \frac{a\theta \sin \theta}{a\theta \cos \theta} = \tan \theta$$

Differentiation Ex 11.7 Q10

Here,

$$x = e^{\theta} \left(\theta + \frac{1}{\theta} \right)$$

Differentiating it with respect to θ using product rule,

$$\begin{aligned} \frac{dx}{d\theta} &= e^{\theta} \frac{d}{d\theta} \left(\theta + \frac{1}{\theta} \right) + \left(\theta + \frac{1}{\theta} \right) \frac{d}{d\theta} (e^{\theta}) \\ &= e^{\theta} \left(1 - \frac{1}{\theta^2} \right) + \left(\frac{\theta^2 + 1}{\theta} \right) e^{\theta} \\ &= e^{\theta} \left(1 - \frac{1}{\theta^2} + \frac{\theta^2 + 1}{\theta} \right) \\ &= e^{\theta} \left(\frac{\theta^2 - 1 + \theta^3 + \theta}{\theta^2} \right) \\ \frac{dx}{d\theta} &= \frac{e^{\theta} (\theta^3 + \theta^2 + \theta - 1)}{\theta^2} \quad \text{---(i)} \end{aligned}$$

And, $y = e^{\theta} \left(\theta - \frac{1}{\theta} \right)$

Differentiating it with respect to θ using product rule and chain rule,

$$\begin{aligned} \frac{dy}{d\theta} &= e^{-\theta} \frac{d}{d\theta} \left(\theta - \frac{1}{\theta} \right) + \left(\theta - \frac{1}{\theta} \right) \frac{d}{d\theta} (e^{-\theta}) \\ &= e^{-\theta} \left(1 + \frac{1}{\theta^2} \right) + \left(\theta - \frac{1}{\theta} \right) e^{-\theta} \frac{d}{d\theta} (-\theta) \\ &= e^{-\theta} \left(1 + \frac{1}{\theta^2} \right) + \left(\theta - \frac{1}{\theta} \right) e^{-\theta} (-1) \\ \frac{dy}{d\theta} &= e^{-\theta} \left[1 + \frac{1}{\theta^2} - \theta + \frac{1}{\theta} \right] \\ &= e^{-\theta} \left[\frac{\theta^2 + 1 - \theta^3 + \theta}{\theta^2} \right] \\ \frac{dy}{d\theta} &= e^{-\theta} \left[\frac{-\theta^3 + \theta^2 + \theta + 1}{\theta^2} \right] \quad \text{---(ii)} \end{aligned}$$

Differentiation Ex 11.7 Q11

Here,

$$x = \frac{2t}{1+t^2}$$

Differentiating it with respect to t using quotient rule,

$$\begin{aligned} \frac{dy}{dx} &= \left[\frac{\left(1+t^2\right) \frac{d}{dt}(2t) - 2t \frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2} \right] \\ &= \left[\frac{\left(1+t^2\right)(2) - 2t(2t)}{\left(1+t^2\right)^2} \right] \\ &= \left[\frac{2+2t^2-4t^2}{\left(1+t^2\right)^2} \right] \\ &= \left[\frac{2-2t^2}{\left(1+t^2\right)^2} \right] \\ \frac{dx}{dt} &= \frac{2\left(1-t^2\right)}{\left(1+t^2\right)^2} \quad \text{---(i)} \end{aligned}$$

And, $y = \frac{1-t^2}{1+t^2}$

Differentiating it with respect to t using quotient rule,

$$\begin{aligned} \frac{dy}{dt} &= \left[\frac{\left(1+t^2\right) \frac{d}{dt}\left(1-t^2\right) - \left(1-t^2\right) \frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2} \right] \\ &= \left[\frac{\left(1+t^2\right)(-2t) - \left(1-t^2\right)(2t)}{\left(1+t^2\right)^2} \right] \\ &= \left[\frac{-2t-2t^3-2t+2t^3}{\left(1+t^2\right)^2} \right] \\ \frac{dy}{dt} &= \left[\frac{-4t}{\left(1+t^2\right)^2} \right] \quad \text{---(ii)} \end{aligned}$$

Dividing equation (ii) by (i),

$$\begin{aligned} \frac{\frac{dy}{dt}}{\frac{dx}{dt}} &= \frac{-4t}{\left(1+t^2\right)^2} \times \frac{\left(1+t^2\right)^2}{2\left(1-t^2\right)} \\ &= \frac{-2t}{1-t^2} \\ \frac{dy}{dx} &= -\frac{x}{y} \quad \left[\text{Since, } \frac{x}{y} = \frac{2t}{1+t^2} \times \frac{1+t^2}{1-t^2} = \frac{2t}{1-t^2} \right] \end{aligned}$$

Differentiation Ex 11.7 Q12

Here,

$$x = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

Differentiating it with respect to t using chain rule,

$$\begin{aligned}\frac{dx}{dt} &= \frac{-1}{\sqrt{1-\left(\frac{1}{1+t^2}\right)^2}} \frac{d}{dt}\left(\frac{1}{\sqrt{1+t^2}}\right) \\ &= \frac{-1}{\sqrt{1-\frac{1}{1+t^2}}} \left\{ \frac{-1}{2(1+t^2)^{\frac{3}{2}}} \right\} \frac{d}{dt}(1+t^2) \\ &= \frac{(1+t^2)^{\frac{1}{2}}}{\sqrt{1+t^2}-1} \times \frac{-1}{2(1+t^2)^{\frac{3}{2}}} (2t) \\ &= \frac{-t}{\sqrt{t^2} \times (1+t^2)} \\ \frac{dx}{dt} &= \frac{-1}{1+t^2} \quad \text{---(i)}\end{aligned}$$

Now, $y = \sin^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$

Differentiating it with respect to t using chain rule,

$$\begin{aligned}\frac{dy}{dt} &= \frac{1}{\sqrt{1-\frac{1}{(1+t^2)^2}}} \times \frac{d}{dt}\left(\frac{1}{\sqrt{1+t^2}}\right) \\ &= \frac{(1+t^2)^{\frac{1}{2}}}{\sqrt{1+t^2}-1} \times \left\{ \frac{-1}{2(1+t^2)^{\frac{3}{2}}} \right\} \frac{d}{dt}(1+t^2) \\ &= \frac{-1}{2\sqrt{t^2}(1+t^2)} \times (2t) \\ \frac{dy}{dt} &= \frac{-1}{(1+t^2)} \quad \text{---(ii)}\end{aligned}$$

Dividing equation (ii) by (i),

$$\begin{aligned}\frac{\frac{dy}{dt}}{\frac{dx}{dt}} &= -\frac{1}{(1+t^2)} \times \frac{(1+t^2)}{-1} \\ \frac{dy}{dx} &= 1\end{aligned}$$

Here,

$$x = \frac{1-t^2}{1+t^2}$$

Differentiating it with respect to t using quotient rule,

$$\begin{aligned}\frac{dx}{dt} &= \left[\frac{(1+t^2) \frac{d}{dt}(1-t^2) - (1-t^2) \frac{d}{dt}(1+t^2)}{(1+t^2)^2} \right] \\ &= \left[\frac{(1+t^2)(-2t) - (1-t^2)(2t)}{(1+t^2)^2} \right] \\ &= \left[\frac{-2t - 2t^3 - 2t + 2t^3}{(1+t^2)^2} \right] \\ \frac{dx}{dt} &= \left(\frac{-4t}{(1+t^2)^2} \right) \quad \text{---(i)}\end{aligned}$$

And, $y = \frac{2t}{1+t^2}$

Differentiating it with respect to t using quotient rule,

$$\begin{aligned}\frac{dy}{dt} &= \left[\frac{(1+t^2) \frac{d}{dt}(2t) - (2t) \frac{d}{dt}(1+t^2)}{(1+t^2)^2} \right] \\ \frac{dy}{dt} &= \left[\frac{(1+t^2)(2) - (2t)(2t)}{(1+t^2)^2} \right] \\ &= \left[\frac{2+2t^2-4t^2}{(1+t^2)^2} \right] \\ \frac{dy}{dt} &= \frac{2(1-t^2)}{(1+t^2)^2} \quad \text{---(ii)}\end{aligned}$$

Differentiation Ex 11.7 Q14

Here, $x = 2\cos\theta - \cos 2\theta$

Differentiating it with respect to θ using chain rule,

$$\begin{aligned}\frac{dx}{d\theta} &= 2(-\sin\theta) - (-\sin 2\theta) \frac{d}{d\theta}(2\theta) \\ &= -2\sin\theta + 2\sin 2\theta \\ \frac{dx}{d\theta} &= 2(\sin 2\theta - \sin\theta) \quad \text{---(i)}\end{aligned}$$

And, $y = 2\sin\theta - \sin 2\theta$

Differentiating it with respect to θ using chain rule,

$$\begin{aligned}\frac{dy}{d\theta} &= 2\cos\theta - \cos 2\theta \frac{d}{d\theta}(2\theta) \\ &= 2\cos\theta - \cos 2\theta (2) \\ &= 2\cos\theta - 2\cos 2\theta \\ \frac{dy}{d\theta} &= 2(\cos\theta - \cos 2\theta) \quad \text{---(ii)}\end{aligned}$$

Dividing equation (ii) by equation (i),

$$\begin{aligned}\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} &= \frac{2(\cos\theta - \cos 2\theta)}{2(\sin 2\theta - \sin\theta)} \\ &= \frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta} \\ \frac{dy}{dx} &= \frac{-2\sin\left(\frac{\theta+2\theta}{2}\right)\sin\left(\frac{\theta-2\theta}{2}\right)}{2\cos\left(\frac{2\theta+\theta}{2}\right)\sin\left(\frac{2\theta-\theta}{2}\right)} \quad \left[\begin{array}{l} \text{Since, } \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \\ \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \end{array} \right] \\ &= \frac{-\sin\left(\frac{3\theta}{2}\right)\left(\sin\left(-\frac{\theta}{2}\right)\right)}{\cos\left(\frac{3\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)} \\ &= \frac{-\sin\left(\frac{3\theta}{2}\right)\left(-\sin\frac{\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right)\left(\sin\frac{\theta}{2}\right)} \\ &= \frac{\sin\left(\frac{3\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right)} \\ \frac{dy}{dx} &= \tan\left(\frac{3\theta}{2}\right)\end{aligned}$$

Differentiation Ex 11.7 Q15

Here,

$$x = e^{\cos 2t}$$

Differentiating it with respect to t using chain rule,

$$\begin{aligned}\frac{dx}{dt} &= \frac{d}{dt} (e^{\cos 2t}) \\ &= e^{\cos 2t} \frac{d}{dt} (\cos 2t) \\ &= e^{\cos 2t} (-\sin 2t) \frac{d}{dt} (2t) \\ &= -\sin 2t e^{\cos 2t} (2) \\ \frac{dx}{dt} &= -2 \sin 2t e^{\cos 2t} \quad \text{---(i)}\end{aligned}$$

And, $y = e^{\sin 2t}$

Differentiating it with respect to t using chain rule,

$$\begin{aligned}\frac{dy}{dt} &= \frac{d}{dt} (e^{\sin 2t}) \\ &= e^{\sin 2t} \frac{d}{dt} (\sin 2t) \\ &= e^{\sin 2t} (\cos 2t) \frac{d}{dt} (2t) \\ &= e^{\sin 2t} (\cos 2t) (2) \\ \frac{dy}{dt} &= 2 \cos 2t e^{\sin 2t} \quad \text{---(ii)}\end{aligned}$$

Dividing equation (ii) by (i),

$$\begin{aligned}\frac{\frac{dy}{dt}}{\frac{dx}{dt}} &= \frac{2 \cos 2t e^{\sin 2t}}{-2 \sin 2t e^{\cos 2t}} \\ \frac{dy}{dx} &= -\frac{y \log x}{x \log y} \quad \left[\begin{array}{l} \text{Since, } x = e^{\cos 2t} \Rightarrow \log x = \cos 2t \\ y = e^{\sin 2t} \Rightarrow \log y = \sin 2t \end{array} \right]\end{aligned}$$

Differentiation Ex 11.7 Q16

Here,

$$x = \cos t$$

Differentiating it with respect to t ,

$$\frac{dx}{dt} = \frac{d}{dt}(\cos t)$$

$$\frac{dx}{dt} = -\sin t \quad \text{---(i)}$$

and, $y = \sin t$

Differentiating it with respect to t ,

$$\frac{dy}{dt} = \frac{d}{dt}(\sin t)$$

$$\frac{dy}{dt} = \cos t \quad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t}{-\sin t}$$

$$\frac{dy}{dx} = -\cot t$$

$$\begin{aligned} \left(\frac{dy}{dx}\right) &= -\cot\left(\frac{2\pi}{3}\right) \\ &= -\cot\left(\pi - \frac{\pi}{3}\right) \\ &= -\left[-\cot\left(\frac{\pi}{3}\right)\right] \\ &= \cot\left(\frac{\pi}{3}\right) \end{aligned}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{3}}$$

Differentiation Ex 11.7 Q17

Here,

$$x = a \left(t + \frac{1}{t} \right)$$

Differentiating it with respect to t ,

$$\begin{aligned} \frac{dx}{dt} &= a \frac{d}{dt} \left(t + \frac{1}{t} \right) \\ &= a \left(1 - \frac{1}{t^2} \right) \\ \frac{dx}{dt} &= a \left(\frac{t^2 - 1}{t^2} \right) \end{aligned} \quad \text{---(i)}$$

And, $y = a \left(t - \frac{1}{t} \right)$

Differentiating it with respect to t ,

$$\begin{aligned} \frac{dy}{dt} &= a \frac{d}{dt} \left(t - \frac{1}{t} \right) \\ &= a \left(1 + \frac{1}{t^2} \right) \\ \frac{dy}{dt} &= a \left(\frac{t^2 + 1}{t^2} \right) \end{aligned} \quad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = a \frac{\left(\frac{t^2 + 1}{t^2} \right)}{a \left(\frac{t^2 - 1}{t^2} \right)} \times \frac{t^2}{t^2}$$

$$\frac{dy}{dx} = \frac{t^2 + 1}{t^2 - 1}$$

$$\frac{dy}{dx} = \frac{x}{y}$$

$$\left[\text{Since, } \frac{x}{y} = \frac{a \left(\frac{t^2 + 1}{t^2} \right)}{a \left(\frac{t^2 - 1}{t^2} \right)} = \left(\frac{t^2 + 1}{t^2 - 1} \right) \right]$$

Differentiation Ex 11.7 Q18

Here,

$$x = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$$

Put $t = \tan \theta$

$$x = \sin^{-1}\left(\frac{2 \tan \theta}{1 + \tan^2 \theta}\right)$$

$$= \sin^{-1}(\sin 2\theta)$$

$$= 2\theta$$

$$\left[\text{Since, } \sin 2x = \frac{2 \tan x}{1 + \tan^2 x} \right]$$

$$x = 2(\tan^{-1} t)$$

$$[\text{Since, } t = \tan \theta]$$

Differentiating it with respect to t ,

$$\frac{dx}{dt} = \frac{2}{1+t^2} \quad \text{---(i)}$$

Now,

$$y = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$$

Put $t = \tan \theta$

$$y = \tan^{-1}\left(\frac{2 \tan \theta}{1 - \tan^2 \theta}\right)$$

$$= \tan^{-1}(\tan 2\theta)$$

$$\left[\text{Since, } \tan 2x = \frac{2 \tan x}{1 - \tan^2 x} \right]$$

$$= 2\theta$$

$$y = 2 \tan^{-1} t$$

$$[\text{Since, } t = \tan \theta]$$

Differentiating it with respect to t ,

$$\frac{dy}{dt} = \frac{2}{1+t^2} \quad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2}{1+t^2} \times \frac{1+t^2}{2}$$

$$\frac{dy}{dx} = 1$$

Differentiation Ex 11.7 Q19

The given equations are $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$ and $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$

$$\text{Then, } \frac{dx}{dt} = \frac{d}{dt} \left[\frac{\sin^3 t}{\sqrt{\cos 2t}} \right]$$

$$= \frac{\sqrt{\cos 2t} \cdot \frac{d}{dt}(\sin^3 t) - \sin^3 t \cdot \frac{d}{dt} \sqrt{\cos 2t}}{\cos 2t}$$

$$= \frac{\sqrt{\cos 2t} \cdot 3 \sin^2 t \cdot \frac{d}{dt}(\sin t) - \sin^3 t \times \frac{1}{2\sqrt{\cos 2t}} \cdot \frac{d}{dt}(\cos 2t)}{\cos 2t}$$

$$= \frac{3\sqrt{\cos 2t} \cdot \sin^2 t \cos t - \frac{\sin^3 t}{2\sqrt{\cos 2t}} \cdot (-2 \sin 2t)}{\cos 2t}$$

$$= \frac{3 \cos 2t \sin^2 t \cos t + \sin^3 t \sin 2t}{\cos 2t \sqrt{\cos 2t}}$$

$$\begin{aligned}
 \frac{dy}{dt} &= \frac{d}{dt} \left[\frac{\cos^3 t}{\sqrt{\cos 2t}} \right] \\
 &= \frac{\sqrt{\cos 2t} \cdot \frac{d}{dt}(\cos^3 t) - \cos^3 t \cdot \frac{d}{dt}(\sqrt{\cos 2t})}{\cos 2t} \\
 &= \frac{\sqrt{\cos 2t} \cdot 3 \cos^2 t \cdot \frac{d}{dt}(\cos t) - \cos^3 t \cdot \frac{1}{2\sqrt{\cos 2t}} \cdot \frac{d}{dt}(\cos 2t)}{\cos 2t} \\
 &= \frac{3\sqrt{\cos 2t} \cdot \cos^2 t (-\sin t) - \cos^3 t \cdot \frac{1}{2\sqrt{\cos 2t}} \cdot (-2 \sin 2t)}{\cos 2t} \\
 &= \frac{-3 \cos 2t \cdot \cos^2 t \cdot \sin t + \cos^3 t \sin 2t}{\cos 2t \cdot \sqrt{\cos 2t}}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \frac{dy}{dx} &= \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{-3 \cos 2t \cdot \cos^2 t \cdot \sin t + \cos^3 t \sin 2t}{3 \cos 2t \sin^2 t \cos t + \sin^3 t \sin 2t} \\
 &= \frac{-3 \cos 2t \cdot \cos^2 t \cdot \sin t + \cos^3 t (2 \sin t \cos t)}{3 \cos 2t \sin^2 t \cos t + \sin^3 t (2 \sin t \cos t)} \\
 &= \frac{\sin t \cos t [-3 \cos 2t \cdot \cos t + 2 \cos^3 t]}{\sin t \cos t [3 \cos 2t \sin t + 2 \sin^3 t]} \\
 &= \frac{[-3(2 \cos^2 t - 1) \cos t + 2 \cos^3 t]}{[3(1 - 2 \sin^2 t) \sin t + 2 \sin^3 t]} \quad \begin{bmatrix} \cos 2t = (2 \cos^2 t - 1), \\ \cos 2t = (1 - 2 \sin^2 t) \end{bmatrix} \\
 &= \frac{-4 \cos^3 t + 3 \cos t}{3 \sin t - 4 \sin^3 t} \\
 &= \frac{-\cos 3t}{\sin 3t} \quad \begin{bmatrix} \cos 3t = 4 \cos^3 t - 3 \cos t, \\ \sin 3t = 3 \sin t - 4 \sin^3 t \end{bmatrix} \\
 &= -\cot 3t
 \end{aligned}$$

Differentiation Ex 11.7 Q20

Here,

$$x = \left(t + \frac{1}{t}\right)^a$$

Differentiating it with respect to t using chain rule,

$$\begin{aligned}\frac{dx}{dt} &= \frac{d}{dt} \left(\left(t + \frac{1}{t}\right)^a \right) \\ &= a \left(t + \frac{1}{t}\right)^{a-1} \frac{d}{dt} \left(t + \frac{1}{t}\right) \\ \frac{dx}{dt} &= a \left(t + \frac{1}{t}\right)^{a-1} \left(1 - \frac{1}{t^2}\right) \quad \text{--- (i)}\end{aligned}$$

And, $y = a^{\left(t + \frac{1}{t}\right)}$

Differentiating it with respect to t using chain rule,

$$\begin{aligned}\frac{dy}{dt} &= \frac{d}{dt} \left(a^{\left(t + \frac{1}{t}\right)} \right) \\ &= a^{\left(t + \frac{1}{t}\right)} \times \log a \frac{d}{dt} \left(t + \frac{1}{t}\right) \\ \frac{dy}{dt} &= a^{\left(t + \frac{1}{t}\right)} \times \log a \left(1 - \frac{1}{t^2}\right) \quad \text{--- (ii)}\end{aligned}$$

Dividing equation (ii) by (i),

$$\begin{aligned}\frac{\frac{dy}{dt}}{\frac{dx}{dt}} &= \frac{a^{\left(t + \frac{1}{t}\right)} \times \log a \left(1 - \frac{1}{t^2}\right)}{a \left(t + \frac{1}{t}\right)^{a-1} \left(1 - \frac{1}{t^2}\right)} \\ \frac{dy}{dx} &= \frac{a^{\left(t + \frac{1}{t}\right)} \times \log a}{a \left(t + \frac{1}{t}\right)^{a-1}}\end{aligned}$$

Differentiation Ex 11.7 Q21

Here,

$$x = a \left(\frac{1+t^2}{1-t^2} \right)$$

Differentiating it with respect to t using chain rule,

$$\begin{aligned} \frac{dx}{dt} &= a \left[\frac{(1+t^2) \frac{d}{dt}(1+t^2) - (1-t^2) \frac{d}{dt}(1-t^2)}{(1-t^2)^2} \right] \\ &= a \left[\frac{(1-t^2)(2t) - (1+t^2)(-2t)}{(1-t^2)^2} \right] \\ &= a \left[\frac{2t - 2t^2 + 2t + 2t^3}{(1-t^2)^2} \right] \end{aligned}$$

$$\frac{dy}{dt} = \frac{4at}{(1-t^2)^2} \quad \text{---(i)}$$

And, $y = \frac{2t}{1-t^2}$

Differentiating it with respect to t using quotient rule,

$$\begin{aligned} \frac{dy}{dt} &= 2 \left[\frac{(1-t^2) \frac{d}{dt}(t) - t \frac{d}{dt}(1-t^2)}{(1-t^2)^2} \right] \\ &= 2 \left[\frac{(1-t^2)(1) - t(-2t)}{(1-t^2)^2} \right] \\ &= 2 \left[\frac{1-t^2+2t^2}{(1-t^2)^2} \right] \\ \frac{dy}{dt} &= \frac{2(1+t^2)}{(1-t^2)^2} \quad \text{---(ii)} \end{aligned}$$

Differentiation Ex 11.7 Q22

It is given that, $y = 12(1 - \cos t)$, $x = 10(t - \sin t)$

$$\therefore \frac{dx}{dt} = \frac{d}{dt} [10(t - \sin t)] = 10 \cdot \frac{d}{dt} (t - \sin t) = 10(1 - \cos t)$$

$$\frac{dy}{dt} = \frac{d}{dt} [12(1 - \cos t)] = 12 \cdot \frac{d}{dt} (1 - \cos t) = 12 \cdot [0 - (-\sin t)] = 12 \sin t$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt} \right)}{\left(\frac{dx}{dt} \right)} = \frac{12 \sin t}{10(1 - \cos t)} = \frac{12 \cdot 2 \sin \frac{t}{2} \cdot \cos \frac{t}{2}}{10 \cdot 2 \sin^2 \frac{t}{2}} = \frac{6}{5} \cot \frac{t}{2}$$

Differentiation Ex 11.7 Q23

Here $x = a(\theta - \sin \theta)$ and $y = a(1 + \cos \theta)$

Then,

$$\frac{dx}{d\theta} = \frac{d}{d\theta} [a(\theta - \sin \theta)] = a(1 - \cos \theta)$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta} [a(1 + \cos \theta)] = a(-\sin \theta)$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-a \sin \theta}{a(1 - \cos \theta)} \bigg|_{\theta = \frac{\pi}{3}} = -\frac{\sin \frac{\pi}{3}}{1 - \cos \frac{\pi}{3}} = \frac{\frac{\sqrt{3}}{2}}{1 - \frac{1}{2}} = -\sqrt{3}$$

Differentiation Ex 11.7 Q24

Consider the given functions,

$$x = a \sin 2t (1 + \cos 2t) \text{ and } y = b \cos 2t (1 - \cos 2t)$$

Rewriting the above function, we have,

$$x = a \sin 2t + \frac{a}{2} \sin 4t$$

Differentiating the above function w.r.t. 't', we have,

$$\frac{dx}{dt} = 2a \cos 2t + 2a \cos 4t \dots (1)$$

$$y = b \cos 2t (1 - \cos 2t)$$

$$y = b \cos 2t - b \cos^2 2t$$

$$\frac{dy}{dt} = -2b \sin 2t + 2b \cos 2t \sin 2t = -2b \sin 2t + b \sin 4t \dots (2)$$

From (1) and (2),

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2b \sin 2t + b \sin 4t}{2a \cos 2t + 2a \cos 4t}$$

$$\therefore \left. \frac{dy}{dx} \right|_{\pi/4} = \left. \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \right|_{t=\pi/4} = \frac{-2b}{-2a} = \frac{b}{a}$$

Differentiation Ex 11.7 Q25

Consider the given functions,

$$x = \cos t (3 - 2 \cos^2 t)$$

$$x = 3 \cos t - 2 \cos^3 t$$

$$\frac{dx}{dt} = -3 \sin t + 6 \cos^2 t \sin t \dots (1)$$

$$y = \sin t (3 - 2 \sin^2 t)$$

$$y = 3 \sin t - 2 \sin^3 t$$

$$\frac{dy}{dt} = 3 \cos t - 6 \sin^2 t \cos t \dots (2)$$

$$\frac{dy}{dx} = \left(\frac{dy}{dt} \right) / \left(\frac{dx}{dt} \right) \dots [\text{From equations (1) and (2)}]$$

$$= \frac{3 \cos t - 6 \sin^2 t \cos t}{-3 \sin t + 6 \cos^2 t \sin t}$$

$$= \frac{3 \cos t (1 - 2 \sin^2 t)}{3 \sin t (2 \cos^2 t - 1)}$$

$$= \cot t \frac{(1 - 2(1 - \cos^2 t))}{(2 \cos^2 t - 1)}$$

$$= \cot t$$

$$\left. \frac{dy}{dx} \right|_{\pi/4} = \cot \frac{\pi}{4} = 1$$

Differentiation Ex 11.7 Q26

$$x = \frac{1 + \log t}{t^2}, y = \frac{3 + 2 \log t}{t}$$

$$\frac{dx}{dt} = \frac{t^2 \left(\frac{1}{t} \right) - (1 + \log t)(2t)}{t^4} = \frac{t - 2t - 2t \log t}{t^4} = \frac{-2 \log t - 1}{t^3}$$

$$\frac{dy}{dt} = \frac{t \left(\frac{2}{t} \right) - (3 + 2 \log t)(1)}{t^2} = \frac{2 - 3 - 2 \log t}{t^2} = \frac{-2 \log t - 1}{t^2}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{-2 \log t - 1}{t^2}}{\frac{-2 \log t - 1}{t^3}} = t$$

Differentiation Ex 11.7 Q27

$$x = 3 \sin t - \sin 3t, y = 3 \cos t - \cos 3t$$

$$\frac{dx}{dt} = 3 \cos t - 3 \cos 3t$$

$$\frac{dy}{dt} = -3 \sin t + 3 \sin 3t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-3 \sin t + 3 \sin 3t}{3 \cos t - 3 \cos 3t}$$

$$\text{When } t = \frac{\pi}{3}$$

$$\frac{dy}{dx} = \frac{-3 \sin\left(\frac{\pi}{3}\right) + 3 \sin(\pi)}{3 \cos\left(\frac{\pi}{3}\right) - 3 \cos(\pi)} = \frac{-3 \times \frac{\sqrt{3}}{2} + 0}{3 \times \frac{1}{2} - 3(-1)} = -\frac{1}{\sqrt{3}}$$

Differentiation Ex 11.7 Q28

$$\sin x = \frac{2t}{1+t^2}, \tan y = \frac{2t}{1-t^2}$$

$$\Rightarrow x = \sin^{-1}\left(\frac{2t}{1+t^2}\right) \text{ and } y = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$$

$$\frac{dx}{dt} = \frac{1}{\sqrt{1 - \left(\frac{2t}{1+t^2}\right)^2}} \times \frac{2(1+t^2) - (2t)(2t)}{(1+t^2)^2}$$

$$\frac{dx}{dt} = \frac{2}{(1+t^2)}$$

$$\frac{dy}{dt} = \frac{1}{\left(\frac{2t}{1-t^2}\right)^2 + 1} \times \frac{2(1-t^2) - (2t)(-2t)}{(1-t^2)^2}$$

$$\frac{dy}{dt} = \frac{2}{(1+t^2)}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{2}{(1+t^2)}}{\frac{2}{(1+t^2)}} = 1$$

Ex 11.8

Differentiation Ex 11.8 Q1

Let $u = x^2$, $v = x^3$

Differentiating u with respect to x ,

$$\frac{du}{dx} = 2x \quad \text{---(i)}$$

Differentiating v with respect to x ,

$$\frac{dv}{dx} = 3x^2 \quad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2x}{3x^2}$$

$$\frac{du}{dv} = \frac{2}{3x}$$

Differentiation Ex 11.8 Q2

Let $u = \log(1+x^2)$

Differentiating it with respect to x using chain rule,

$$\begin{aligned} \frac{du}{dx} &= \frac{1}{(1+x^2)} \frac{d}{dx}(1+x^2) \\ &= \frac{1}{(1+x^2)} (2x) \\ \frac{du}{dx} &= \frac{2x}{(1+x^2)} \quad \text{---(i)} \end{aligned}$$

Let $v = \tan^{-1} x$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{1}{1+x^2} \quad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2x}{(1+x^2)} \times \frac{(1+x^2)}{1}$$

$$\frac{du}{dv} = 2x$$

Differentiation Ex 11.8 Q3

Let $u = (\log x)^x$

Taking log on both the sides,

$$\log u = \log(\log x)^x$$

$$\log u = x \log(\log x) \quad \left[\text{Since, } \log a^b = b \log a \right]$$

Differentiating it with respect to x using chain rule, product rule,

$$\frac{1}{u} \frac{du}{dx} = x \frac{d}{dx} \log(\log x) + \log(\log x) \frac{d}{dx}(x)$$

$$\frac{1}{u} \frac{du}{dx} = x \left(\frac{1}{\log x} \right) \frac{d}{dx}(\log x) + \log \log x (1)$$

$$\frac{du}{dx} = u \left[\frac{x}{\log x} \left(\frac{1}{x} \right) + \log \log x \right]$$

$$\frac{du}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log \log x \right] \quad \text{---(i)}$$

Again, let $v = \log x$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{1}{x} \quad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{(\log x)^x \left[\frac{1}{\log x} + \log \log x \right]}{\frac{1}{x}}$$

$$\frac{du}{dv} = \frac{(\log x)^x \left[\frac{1 + \log x \times \log \log x}{\log x} \right]}{\frac{1}{x}}$$

$$\frac{du}{dv} = (\log x)^{x-1} (1 + \log x \times \log \log x) \times x$$

Differentiation Ex 11.8 Q4(i)

Let $u = \sin^{-1} \sqrt{1-x^2}$

Put $x = \cos \theta$, so,

$$u = \sin^{-1} \sqrt{1 - \cos^2 \theta}$$

$$u = \sin^{-1} (\sin \theta) \quad \text{---(i)}$$

And, $v = \cos^{-1} x \quad \text{---(ii)}$

Now, $x \in (0, 1)$

$$\Rightarrow \cos \theta \in (0, 1)$$

$$\Rightarrow \theta \in \left(0, \frac{\pi}{2}\right)$$

So, from equation (i),

$$u = \theta \quad \left[\text{Since, } \sin^{-1} (\sin \theta) = \theta \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \right]$$

$$u = \cos^{-1} x \quad [\text{Since, } \cos \theta = x]$$

Differentiating it with respect to x ,

$$\frac{du}{dx} = \frac{-1}{\sqrt{1-x^2}} \quad \text{---(ii)}$$

From equation (ii),

$$v = \cos^{-1} x$$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}} \quad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{-1}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{-1}$$

$$\frac{du}{dv} = 1$$

Differentiation Ex 11.8 Q4(ii)

Let $u = \sin^{-1} \sqrt{1-x^2}$

Put $x = \cos \theta$, so,
 $u = \sin^{-1} \sqrt{1 - \cos^2 \theta}$
 $u = \sin^{-1} (\sin \theta)$

---(i)

And, $v = \cos^{-1} x$ ---(ii)

Here,

$x \in (-1, 0)$
 $\Rightarrow \cos \theta \in (-1, 0)$
 $\Rightarrow \theta \in \left(\frac{\pi}{2}, \pi\right)$

So, from equation (i),

$u = \pi - \theta$ $\left[\text{Since, } \sin^{-1}(\sin \theta) = \pi - \theta, \theta \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \right]$
 $u = \pi - \cos^{-1} x$ $[\text{Since, } x = \cos \theta]$

Differentiating it with respect to x ,

$\frac{du}{dx} = 0 - \left(\frac{-1}{\sqrt{1-x^2}} \right)$
 $\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$ ---(v)

And, from equation (ii),

$v = \cos^{-1} x$

Differentiating it with respect to x ,

$\frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}}$ ---(vi)

Dividing equation (v) by (vi)

$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{1}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{-1}$
 $\frac{du}{dv} = -1$

Differentiation Ex 11.8 Q5(i)

Let $u = \sin^{-1} \left(4x\sqrt{1-4x^2} \right)$

Put $2x = \cos \theta$, so

$$u = \sin^{-1} \left(2 \times \cos \theta \sqrt{1 - \cos^2 \theta} \right)$$

$$= \sin^{-1} (2 \cos \theta \sin \theta)$$

$$u = \sin^{-1} (\sin 2\theta) \quad \text{---(i)}$$

Let $v = \sqrt{1-4x^2} \quad \text{---(ii)}$

Here,

$$x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}} \right)$$

$$\Rightarrow 2x \in \left(-1, -\frac{1}{\sqrt{2}} \right)$$

$$\Rightarrow \theta \in \left(\frac{3\pi}{4}, \pi \right)$$

So, from equation (i),

$$u = \pi - 2\theta \quad \left[\text{Since, } \sin^{-1} (\sin \theta) = \pi - \theta \text{ if } \theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2} \right] \right]$$

$$u = \pi - 2 \cos^{-1} (2x) \quad [\text{Since, } 2x = \cos \theta]$$

Differentiating it with respect to x using chain rule,

$$\frac{du}{dx} = 0 - 2 \left(\frac{-1}{\sqrt{1-(2x)^2}} \right) \frac{d}{dx} (2x)$$

$$= \frac{2}{\sqrt{1-4x^2}} (2)$$

$$\frac{du}{dx} = \frac{4}{\sqrt{1-4x^2}} \quad \text{---(vi)}$$

From equation (iv)

$$\frac{dv}{dx} = \frac{-4x}{\sqrt{1-4x^2}}$$

but, $x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}} \right)$

$$\frac{dv}{dx} = \frac{-4(-x)}{\sqrt{1-4(-x)^2}}$$

$$\frac{dv}{dx} = \frac{4x}{\sqrt{1-4x^2}} \quad \text{---(vii)}$$

Differentiating equation (ii) with respect to x using chain rule,

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1-4x^2}} \frac{d}{dx} (1-4x^2)$$

$$= \frac{1}{2\sqrt{1-4x^2}} (-8x)$$

$$\frac{dv}{dx} = \frac{-4x}{\sqrt{1-4x^2}} \quad \text{---(iv)}$$

Divide equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{4}{\sqrt{1-4x^2}} \times \frac{\sqrt{1-4x^2}}{-4x}$$

$$\frac{du}{dv} = -\frac{1}{x}$$

Differentiation Ex 11.8 Q5(ii)

Let $u = \sin^{-1} \left(4x\sqrt{1-4x^2} \right)$

Put $2x = \cos \theta$, so

$$u = \sin^{-1} \left(2 \times \cos \theta \sqrt{1 - \cos^2 \theta} \right)$$

$$= \sin^{-1} (2 \cos \theta \sin \theta)$$

$$u = \sin^{-1} (\sin 2\theta) \quad \text{---(i)}$$

Let $v = \sqrt{1-4x^2} \quad \text{---(ii)}$

Here,

$$x \in \left(\frac{1}{2\sqrt{2}}, \frac{1}{2} \right)$$

$$\Rightarrow 2x \in \left(\frac{1}{\sqrt{2}}, 1 \right)$$

$$\Rightarrow \cos \theta \in \left(\frac{1}{\sqrt{2}}, 1 \right)$$

$$\Rightarrow \theta \in \left(0, \frac{\pi}{4} \right)$$

So, from equation (i)

$$u = 2\theta \quad \left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right]$$

$$u = 2 \cos^{-1} (2x) \quad [\text{Since, } 2x = \cos \theta]$$

Differentiate it with respect to x using chain rule,

$$\frac{du}{dx} = 2 \left(\frac{-1}{\sqrt{1-(2x)^2}} \right) \frac{d}{dx} (2x)$$

$$= \left(\frac{-2}{\sqrt{1-4x^2}} (2) \right)$$

$$\frac{du}{dx} = \frac{-4}{\sqrt{1-4x^2}} \quad \text{---(v)}$$

Dividing equation (v) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{-4}{\sqrt{1-4x^2}} \times \frac{\sqrt{1-4x^2}}{-4x}$$

$$\frac{du}{dv} = \frac{1}{x}$$

Differentiation Ex 11.8 Q5(iii)

Let $u = \sin^{-1} \left(4x\sqrt{1-4x^2} \right)$

Put $2x = \cos \theta$, so

$$u = \sin^{-1} \left(2 \times \cos \theta \sqrt{1 - \cos^2 \theta} \right)$$

$$= \sin^{-1} (2 \cos \theta \sin \theta)$$

$$u = \sin^{-1} (\sin 2\theta) \quad \text{---(i)}$$

Let $v = \sqrt{1-4x^2} \quad \text{---(ii)}$

Here,

$$x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}} \right)$$

$$\Rightarrow 2x \in \left(-1, -\frac{1}{\sqrt{2}} \right)$$

$$\Rightarrow \theta \in \left(\frac{3\pi}{4}, \pi \right)$$

So, from equation (i),

$$u = \pi - 2\theta \quad \left[\text{Since, } \sin^{-1}(\sin \theta) = \pi - \theta \text{ if } \theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2} \right] \right]$$

$$u = \pi - 2 \cos^{-1}(2x) \quad \left[\text{Since, } 2x = \cos \theta \right]$$

Differentiating it with respect to x using chain rule,

$$\frac{du}{dx} = 0 - 2 \left(\frac{-1}{\sqrt{1-(2x)^2}} \right) \frac{d}{dx}(2x)$$

$$= \frac{2}{\sqrt{1-4x^2}} (2)$$

$$\frac{du}{dx} = \frac{4}{\sqrt{1-4x^2}} \quad \text{---(vi)}$$

From equation (iv)

$$\frac{dv}{dx} = \frac{-4x}{\sqrt{1-4x^2}}$$

but, $x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}} \right)$

$$\frac{dv}{dx} = \frac{-4(-x)}{\sqrt{1-4(-x)^2}}$$

$$\frac{dv}{dx} = \frac{4x}{\sqrt{1-4x^2}} \quad \text{---(vii)}$$

Dividing equation (vi) by (vii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{4}{\sqrt{1-4x^2}} \times \frac{\sqrt{1-4x^2}}{4x}$$

$$\frac{du}{dv} = \frac{1}{x}$$

Differentiation Ex 11.8 Q6

Let $u = \tan^{-1} \left(\frac{\sqrt{1+x^2}-1}{x} \right)$

Put $x = \tan \theta$, so

$$u = \tan^{-1} \left(\frac{\sqrt{1+\tan^2 \theta}-1}{\tan \theta} \right)$$

$$= \tan^{-1} \left(\frac{\sec \theta - 1}{\tan \theta} \right)$$

$$= \tan^{-1} \left(\frac{1 - \cos \theta}{\sin \theta} \right)$$

$$= \tan^{-1} \left(\frac{\frac{2 \sin^2 \theta}{2}}{\frac{2 \sin \theta \cos \theta}{2}} \right)$$

$$\begin{aligned}
 &= \tan^{-1} \left(\frac{\frac{\sin \theta}{2}}{\frac{\cos \theta}{2}} \right) \\
 u &= \tan^{-1} \left(\frac{\tan \theta}{2} \right) \quad \text{---(i)}
 \end{aligned}$$

And,

$$\begin{aligned}
 \text{Let } v &= \sin^{-1} \left(\frac{2x}{1+x^2} \right) \\
 &= \sin^{-1} \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right) \\
 v &= \sin^{-1} (\sin 2\theta) \quad \text{---(ii)}
 \end{aligned}$$

Here,

$$\begin{aligned}
 &-1 < x < 1 \\
 \Rightarrow &-1 < \tan \theta < 1 \\
 \Rightarrow &-\frac{\pi}{4} < \theta < \frac{\pi}{4} \quad \text{---(A)}
 \end{aligned}$$

So, from equation (i),

$$\begin{aligned}
 u &= \frac{\theta}{2} && \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right] \\
 u &= \frac{1}{2} \tan^{-1} x && [\text{Since, } x = \tan \theta]
 \end{aligned}$$

Differentiating it with respect to x ,

$$\begin{aligned}
 \frac{du}{dx} &= \frac{1}{2} \left(\frac{1}{1+x^2} \right) \\
 \frac{du}{dx} &= \frac{1}{2(1+x^2)} \quad \text{---(iii)}
 \end{aligned}$$

Now, from equation (ii) and (A)

$$\begin{aligned}
 v &= 2\theta && \left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right] \\
 v &= 2 \tan^{-1} x && [\text{Since, } x = \tan \theta]
 \end{aligned}$$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = 2 \left(\frac{1}{1+x^2} \right) \quad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\begin{aligned}
 \frac{\frac{du}{dx}}{\frac{dv}{dx}} &= \frac{1}{2(1+x^2)} \times \frac{(1+x^2)}{2} \\
 \frac{du}{dv} &= \frac{1}{4}
 \end{aligned}$$

Differentiation Ex 11.8 Q7(i)

Let $u = \sin^{-1} \left(2x\sqrt{1-x^2} \right)$

Put $x = \sin \theta$, so

$$\begin{aligned} u &= \sin^{-1} \left(2 \sin \theta \sqrt{1 - \sin^2 \theta} \right) \\ &= \sin^{-1} (2 \sin \theta \cos \theta) \\ u &= \sin^{-1} (\sin 2\theta) \end{aligned} \quad \text{---(i)}$$

And,

$$\begin{aligned} \text{Let } v &= \sec^{-1} \left(\frac{1}{\sqrt{1-x^2}} \right) \\ &= \sec^{-1} \left(\frac{1}{\sqrt{1-\sin^2 \theta}} \right) \\ &= \sec^{-1} \left(\frac{1}{\cos \theta} \right) \\ &= \sec^{-1} (\sec \theta) \\ &= \cos^{-1} \left(\frac{1}{\cos \theta} \right) \quad \left[\text{Since, } \sec^{-1} x = \cos^{-1} \left(\frac{1}{x} \right) \right] \\ v &= \cos^{-1} (\cos \theta) \end{aligned} \quad \text{---(ii)}$$

Here,

$$\begin{aligned} x &\in \left(0, \frac{1}{\sqrt{2}} \right) \\ \Rightarrow \sin \theta &\in \left(0, \frac{1}{\sqrt{2}} \right) \\ \Rightarrow \theta &\in \left(0, \frac{\pi}{4} \right) \end{aligned}$$

So, from equation (i),

$$u = 2\theta \quad \left[\text{Since, } \sin^{-1} (\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right]$$

$$\text{Let } u = 2 \sin^{-1} x \quad \left[\text{Since, } x = \sin \theta \right]$$

Differentiating it with respect to x ,

$$\begin{aligned} \frac{du}{dx} &= 2 \left(\frac{1}{\sqrt{1-x^2}} \right) \\ \frac{du}{dx} &= \frac{2}{\sqrt{1-x^2}} \end{aligned} \quad \text{---(iii)}$$

And, from equation (ii),

$$\begin{aligned} v &= \theta \quad \left[\text{Since, } \cos^{-1} (\cos \theta) = \theta, \text{ if } \theta \in [0, \pi] \right] \\ v &= \sin^{-1} x \quad \left[\text{Since, } x = \sin \theta \right] \end{aligned}$$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}} \quad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\begin{aligned} \frac{\frac{du}{dx}}{\frac{dv}{dx}} &= \frac{2}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{1} \\ \frac{du}{dv} &= 2 \end{aligned}$$

Differentiation Ex 11.8 Q7(ii)

Let $u = \sin^{-1} \left(2x\sqrt{1-x^2} \right)$

Put $x = \sin \theta$, so

$$\begin{aligned} u &= \sin^{-1} \left(2 \sin \theta \sqrt{1 - \sin^2 \theta} \right) \\ &= \sin^{-1} (2 \sin \theta \cos \theta) \\ u &= \sin^{-1} (\sin 2\theta) \end{aligned} \quad \text{---(i)}$$

And,

Let $v = \sec^{-1} \left(\frac{1}{\sqrt{1-x^2}} \right)$

$$\begin{aligned} &= \sec^{-1} \left(\frac{1}{\sqrt{1 - \sin^2 \theta}} \right) \\ &= \sec^{-1} \left(\frac{1}{\cos \theta} \right) \\ &= \sec^{-1} (\sec \theta) \\ &= \cos^{-1} \left(\frac{1}{\sec \theta} \right) \quad \left[\text{Since, } \sec^{-1} x = \cos^{-1} \left(\frac{1}{x} \right) \right] \\ v &= \cos^{-1} (\cos \theta) \end{aligned} \quad \text{---(ii)}$$

Here,

$$\begin{aligned} x &\in \left(\frac{1}{\sqrt{2}}, 1 \right) \\ \Rightarrow \sin \theta &\in \left(\frac{1}{\sqrt{2}}, 1 \right) \\ \Rightarrow \theta &\in \left(\frac{\pi}{4}, \frac{\pi}{2} \right) \end{aligned}$$

So, from equation (i),

$$\begin{aligned} u &= 2\theta \quad \left[\text{Since, } \sin^{-1} (\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right] \\ u &= 2 \sin^{-1} x \quad \left[\text{Since, } x = \sin \theta \right] \end{aligned}$$

Differentiating it with respect to x ,

$$\frac{du}{dx} = 2 \left(\frac{1}{\sqrt{1-x^2}} \right) \quad \text{---(iv)}$$

From equation (ii)

$$\begin{aligned} v &= \theta \quad \left[\text{Since, } \cos^{-1} (\cos \theta) = \theta, \text{ if } \theta \in [0, \pi] \right] \\ v &= \sin^{-1} x \end{aligned}$$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}} \quad \text{---(v)}$$

Dividing equation (iv) by (v),

$$\begin{aligned} \frac{\frac{du}{dx}}{\frac{dv}{dx}} &= \frac{2}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{1} \\ \frac{du}{dv} &= 2 \end{aligned}$$

Differentiation Ex 11.8 Q8

Let $u = (\cos x)^{\sin x}$

Taking log on both the sides,

$$\log u = \log (\cos x)^{\sin x}$$

$$\log u = \sin x \log (\cos x)$$

Differentiating it with respect to x using product and chain rule,

$$\frac{1}{u} \frac{du}{dx} = \sin x \frac{d}{dx} (\log \cos x) + \log \cos x \frac{d}{dx} (\sin x)$$

$$\frac{1}{u} \frac{du}{dx} = \sin x \left(\frac{1}{\cos x} \right) \frac{d}{dx} (\cos x) + \log \cos x (\cos x)$$

$$\frac{du}{dx} = 4 [(\tan x) \times (-\sin x) + \log \log x \times (\cos x)]$$

$$\frac{du}{dx} = (\cos x)^{\sin x} [\cos x \log \cos x - \sin x \tan x] \quad \text{---(i)}$$

Let $v = (\sin x)^{\cos x}$

Taking log on both the sides,

$$\log v = \log (\sin x)^{\cos x}$$

$$\log v = \cos x \log (\sin x)$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{1}{v} \frac{dv}{dx} = \cos x \frac{d}{dx} (\log \sin x) + \log \sin x \frac{d}{dx} (\cos x)$$

$$\frac{1}{v} \frac{dv}{dx} = \cos x \left(\frac{1}{\sin x} \right) \frac{d}{dx} (\sin x) + \log \sin x (-\sin x)$$

$$\frac{dv}{dx} = v [\cot x (\cos x) - \sin x \log \sin x]$$

$$\frac{dv}{dx} = (\sin x)^{\cos x} [\cot x (\cos x) - \sin x \log \sin x] \quad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{du}{dv} = \frac{(\cos x)^{\sin x} [\cos x \log \cos x - \sin x \tan x]}{(\sin x)^{\cos x} [\cot x (\cos x) - \sin x \log \sin x]}$$

Differentiation Ex 11.8 Q9

$$\text{Let } u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$\text{Put } x = \tan \theta,$$

$$u = \sin^{-1}\left(\frac{2 \tan \theta}{1 + \tan^2 \theta}\right)$$

$$u = \sin^{-1}(\sin 2\theta) \quad \text{---(i)}$$

$$\text{Let } v = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$= \cos^{-1}\left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}\right)$$

$$v = \cos^{-1}(\cos 2\theta) \quad \text{---(ii)}$$

$$\text{Here, } 0 < x < 1$$

$$\Rightarrow 0 < \tan \theta < 1$$

$$\Rightarrow 0 < \theta < \frac{\pi}{4}$$

So, from equation (i),

$$u = 2\theta \quad \left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \right]$$

$$u = 2 \tan^{-1} x \quad \left[\text{Since, } x = \tan \frac{\pi}{2} \right]$$

Differentiating it with respect to x ,

$$\frac{du}{dx} = \frac{2}{1+x^2} \quad \text{---(iii)}$$

From equation (ii),

$$v = 2\theta \quad \left[\text{Since, } \cos^{-1}(\cos \theta) = \theta, \text{ if } \theta \in [0, \pi] \right]$$

$$v = 2 \tan^{-1} x \quad \left[\text{Since, } x = \tan \theta \right]$$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{2}{1+x^2} \quad \text{---(iv)}$$

Differentiation Ex 11.8 Q10

Let $u = \tan^{-1}\left(\frac{1+ax}{1-ax}\right)$

Put $ax = \tan \theta$

$$u = \tan^{-1}\left(\frac{1+\tan \theta}{1-\tan \theta}\right)$$

$$= \tan^{-1}\left(\frac{\frac{\tan \pi}{4} + \tan \theta}{1 - \frac{\tan \pi}{4} \tan \theta}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \theta\right)\right)$$

$$= \frac{\pi}{4} + \theta$$

$$u = \frac{\pi}{4} + \tan^{-1}(ax) \quad [\text{Since, } \tan \theta = ax]$$

Differentiate it with respect to x using chain rule,

$$\frac{du}{dx} = 0 + \frac{1}{1+(ax)^2} \frac{d}{dx}(ax)$$

$$\frac{du}{dx} = \frac{a}{1+a^2x^2} \quad \text{---(i)}$$

Now,

Let $v = \sqrt{1+a^2x^2}$

Differentiating it with respect to x using chain rule,

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1+a^2x^2}} \frac{d}{dx}(1+a^2x^2)$$

$$= \frac{1}{2\sqrt{1+a^2x^2}} (2a^2x)$$

$$\frac{dv}{dx} = \frac{a^2x}{\sqrt{1+a^2x^2}} \quad \text{---(ii)}$$

Differentiation Ex 11.8 Q11

$$\begin{aligned}
 \text{Let } u &= \sin^{-1} \left(2x\sqrt{1-x^2} \right) \\
 \text{Put } x &= \sin \theta, \\
 u &= \sin^{-1} \left(2 \sin \theta \sqrt{1 - \sin^2 \theta} \right) \\
 &= \sin^{-1} (2 \sin \theta \cos \theta) \\
 u &= \sin^{-1} (\sin 2\theta) \quad \text{---(i)}
 \end{aligned}$$

$$\begin{aligned}
 \text{Let } v &= \tan^{-1} \left(\frac{x}{\sqrt{1-x^2}} \right) \\
 &= \tan^{-1} \left(\frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}} \right) \\
 &= \tan^{-1} \left(\frac{\sin \theta}{\cos \theta} \right) \\
 v &= \tan^{-1} (\tan \theta) \quad \text{---(ii)}
 \end{aligned}$$

$$\begin{aligned}
 \text{Here, } -\frac{1}{\sqrt{2}} &< x < \frac{1}{\sqrt{2}} \\
 \Rightarrow -\frac{1}{\sqrt{2}} &< \sin \theta < \frac{1}{\sqrt{2}} \\
 \Rightarrow \left(-\frac{\pi}{4} \right) &< \theta < \left(\frac{\pi}{4} \right)
 \end{aligned}$$

So, from equation (i),

$$\begin{aligned}
 u &= 2\theta && \left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right] \\
 u &= 2 \sin^{-1} x
 \end{aligned}$$

Differentiating it with respect to x ,

$$\frac{du}{dx} = \frac{2}{\sqrt{1-x^2}} \quad \text{---(iii)}$$

From equation (ii),

$$\begin{aligned}
 v &= \theta && \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right] \\
 v &= \sin^{-1} x && [\text{Since, } x = \sin \theta]
 \end{aligned}$$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}} \quad \text{---(iv)}$$

Dividing equation (iii) by (iv)

$$\begin{aligned}
 \frac{\frac{du}{dx}}{\frac{dv}{dx}} &= \frac{2}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{1} \\
 \frac{du}{dv} &= 2
 \end{aligned}$$

Differentiation Ex 11.8 Q12

$$\text{Let } u = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$\text{Put } x = \tan \theta, \text{ so}$$

$$u = \tan^{-1}\left(\frac{2 \tan \theta}{1 - \tan^2 \theta}\right)$$

$$u = \tan^{-1}(\tan 2\theta) \quad \text{---(i)}$$

$$\text{Let } v = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$= \cos^{-1}\left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}\right)$$

$$v = \cos^{-1}(\cos 2\theta) \quad \text{---(ii)}$$

$$\text{Here, } 0 < x < 1$$

$$\Rightarrow 0 < \tan \theta < 1$$

$$\Rightarrow 0 < \theta < \frac{\pi}{4}$$

So, from equation (i),

$$u = 2\theta \quad \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \right]$$

$$u = 2 \tan^{-1} x \quad [\text{Since, } x = \tan \theta]$$

Differentiating it with respect to x ,

$$\frac{du}{dx} = \frac{2}{1+x^2} \quad \text{---(iii)}$$

From equation (ii),

$$v = \theta \quad \left[\text{Since, } \cos^{-1}(\cos \theta) = \theta, \text{ if } \theta \in [0, \pi] \right]$$

$$v = 2 \tan^{-1} x \quad [\text{Since, } x = \tan \theta]$$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{2}{1+x^2} \quad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{1+x^2} \times \frac{1+x^2}{2}$$

$$\frac{du}{dv} = 1$$

Differentiation Ex 11.8 Q13

Let $u = \tan^{-1}\left(\frac{x-1}{x+1}\right)$

Put $x = \tan \theta$, so

$$u = \tan^{-1}\left(\frac{\tan \theta - 1}{\tan \theta + 1}\right)$$

$$= \tan^{-1}\left(\frac{\tan \theta - \frac{\tan \pi}{4}}{1 + \tan \theta \frac{\tan \pi}{4}}\right)$$

$$u = \tan^{-1}\left(\tan\left(\theta - \frac{\pi}{4}\right)\right) \quad \text{---(i)}$$

Here, $-\frac{1}{2} < x < \frac{1}{2}$

$$\Rightarrow -\frac{1}{2} < \tan \theta < \frac{1}{2}$$

$$\Rightarrow -\tan^{-1}\left(\frac{1}{2}\right) < \theta < \tan^{-1}\left(\frac{1}{2}\right)$$

So,

$$u = \theta - \frac{\pi}{4} \quad \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \right]$$

$$u = \tan^{-1} x - \frac{\pi}{4}$$

Differentiating it with respect to x ,

$$\frac{du}{dx} = \frac{1}{1+x^2} - 0$$

$$\frac{du}{dx} = \frac{1}{1+x^2} \quad \text{---(ii)}$$

And,

Let $v = \sin^{-1}(3x - 4x^3)$

Put $x = \sin \theta$, so

$$v = \sin^{-1}(3 \sin \theta - 4 \sin^3 \theta)$$

$$v = \sin^{-1}(\sin 3\theta) \quad \text{---(iii)}$$

Now, $-\frac{1}{2} < x < \frac{1}{2}$

$$\Rightarrow -\frac{1}{2} < \sin \theta < \frac{1}{2}$$

$$\Rightarrow -\frac{\pi}{6} < \theta < \frac{\pi}{6}$$

So, from equation (iii),

$$v = 3\theta \quad \left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \right]$$

$$v = 3 \sin^{-1} x \quad [\text{Since, } x = \sin \theta]$$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{3}{\sqrt{1-x^2}} \quad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{1}{1+x^2} \times \frac{\sqrt{1-x^2}}{3}$$

$$\frac{du}{dv} = \frac{\sqrt{1-x^2}}{3(1+x^2)}$$

Differentiation Ex 11.8 Q14

$$\begin{aligned}
 \text{Let } u &= \tan^{-1} \left(\frac{\cos x}{1 + \sin x} \right) \\
 &= \tan^{-1} \left(\frac{\frac{\cos^2 x}{2} - \frac{\sin^2 x}{2}}{\frac{\cos^2 x}{2} + \frac{\sin^2 x}{2} + \frac{2 \sin x \cos x}{2}} \right) \\
 &= \tan^{-1} \left(\frac{\left(\frac{\cos x}{2} + \frac{\sin x}{2} \right) \left(\frac{\cos x}{2} - \frac{\sin x}{2} \right)}{\left(\frac{\cos x}{2} + \frac{\sin x}{2} \right)^2} \right) \\
 &= \tan^{-1} \left(\frac{\frac{\cos x}{2} - \frac{\sin x}{2}}{\frac{\cos x}{2} + \frac{\sin x}{2}} \right) \\
 &= \tan^{-1} \left[\frac{\frac{\frac{\cos x}{2}}{\cos x} - \frac{\frac{\sin x}{2}}{\cos x}}{\frac{\frac{\cos x}{2}}{\cos x} + \frac{\frac{\sin x}{2}}{\cos x}} \right] \\
 &= \tan^{-1} \left[\frac{1 - \frac{\tan x}{2}}{1 + \frac{\tan x}{2}} \right] \\
 &= \tan^{-1} \left[\frac{\frac{\tan \frac{\pi}{4}}{4} - \frac{\tan x}{2}}{1 + \frac{\tan \frac{\pi}{4}}{4} \times \frac{\tan x}{2}} \right] \\
 &= \tan^{-1} \left[\tan \left(\frac{\pi}{4} - \frac{x}{2} \right) \right] \\
 u &= \frac{\pi}{4} - \frac{x}{2}
 \end{aligned}$$

Differentiating it with respect to x ,

$$\begin{aligned}
 \frac{du}{dx} &= 0 - \left(\frac{1}{2} \right) \\
 \frac{du}{dx} &= -\frac{1}{2} \quad \text{---(i)}
 \end{aligned}$$

$$\text{Let } v = \sec^{-1} x$$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{1}{x\sqrt{x^2 - 1}} \quad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = -\frac{1}{2} \times \frac{x\sqrt{x^2 - 1}}{1}$$

$$\frac{du}{dv} = \frac{-x\sqrt{x^2 - 1}}{2}$$

Differentiation Ex 11.8 Q15

$$\text{Let } u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$\text{Put } x = \tan \theta \Rightarrow \theta = \tan^{-1} x, \text{ so}$$

$$u = \sin^{-1}\left(\frac{2 \tan \theta}{1 + \tan^2 \theta}\right)$$

$$u = \sin^{-1}(\sin 2\theta) \quad \text{---(i)}$$

$$\text{Let } v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$= \tan^{-1}\left(\frac{2 \tan \theta}{1 - \tan^2 \theta}\right)$$

$$v = \tan^{-1}(\tan 2\theta) \quad \text{---(ii)}$$

$$\text{Here, } -1 < x < 1$$

$$\Rightarrow -1 < \tan \theta < 1$$

$$\Rightarrow -\frac{\pi}{4} < \theta < \frac{\pi}{4}$$

So, from equation (i),

$$u = 2\theta \quad \left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \right]$$

$$u = 2 \tan^{-1} x$$

Differentiating it with respect to x ,

$$\frac{du}{dx} = \frac{2}{(1+x^2)} \quad \text{---(iii)}$$

From equation (ii),

$$v = 2\theta \quad \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \right]$$

$$v = 2 \tan^{-1} x$$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{2}{1+x^2} \quad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{1+x^2} \times \frac{1+x^2}{2}$$

$$\frac{du}{dv} = 1$$

Differentiation Ex 11.8 Q16

Let $u = \cos^{-1}(4x^3 - 3x)$

Put $x = \cos \theta \Rightarrow \theta = \cos^{-1} x$, so

$$u = \cos^{-1}(4 \cos^3 \theta - 3 \cos \theta)$$

$$u = \cos^{-1}(\cos 3\theta) \quad \text{---(i)}$$

Let $v = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

$$= \tan^{-1}\left(\frac{\sqrt{1-\cos^2 \theta}}{\cos \theta}\right)$$

$$= \tan^{-1}\left(\frac{\sin \theta}{\cos \theta}\right)$$

$$v = \tan^{-1}(\tan \theta) \quad \text{---(ii)}$$

Here,

$$\frac{1}{2} < x < 1$$

$$\Rightarrow \frac{1}{2} < \cos \theta < 1$$

$$\Rightarrow 0 < \theta < \frac{\pi}{3}$$

So, from equation (i),

$$u = 3\theta \quad \left[\text{Since, } \cos^{-1}(\cos \theta) = \theta, \text{ if } \theta \in [0, \pi] \right]$$

$$u = 3 \cos^{-1} x$$

Differentiating it with respect to x ,

$$\frac{du}{dx} = \frac{-3}{\sqrt{1-x^2}} \quad \text{---(iii)}$$

From equation (ii),

$$v = \theta \quad \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \right]$$

$$v = \cos^{-1} x$$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}} \quad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \left(\frac{-3}{\sqrt{1-x^2}} \right) \left(-\frac{\sqrt{1-x^2}}{1} \right)$$

$$\frac{du}{dv} = 3$$

Differentiation Ex 11.8 Q17

$$\text{Let } u = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

$$\text{Put } x = \cos \theta \Rightarrow \theta = \sin^{-1} x, \text{ so}$$

$$\begin{aligned} u &= \tan^{-1}\left(\frac{\sin \theta}{\sqrt{1-\sin^2 \theta}}\right) \\ &= \tan^{-1}\left(\frac{\sin \theta}{\cos \theta}\right) \\ u &= \tan^{-1}(\tan \theta) \end{aligned} \quad \text{---(i)}$$

And,

$$\begin{aligned} \text{Let } v &= \sin^{-1}(2x\sqrt{1-x^2}) \\ v &= \sin^{-1}(2\sin \theta \sqrt{1-\sin^2 \theta}) \\ &= \sin^{-1}(2\sin \theta \cos \theta) \\ v &= \sin^{-1}(\sin 2\theta) \end{aligned} \quad \text{---(ii)}$$

$$\begin{aligned} \text{Here, } -\frac{1}{\sqrt{2}} &< x < \frac{1}{\sqrt{2}} \\ \Rightarrow -\frac{1}{\sqrt{2}} &< \sin \theta < \frac{1}{\sqrt{2}} \\ \Rightarrow -\frac{\pi}{4} &< \theta < \frac{\pi}{4} \end{aligned}$$

So, from equation (i),

$$\begin{aligned} u &= \theta && \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \right] \\ u &= \sin^{-1} x \end{aligned}$$

Differentiation Ex 11.8 Q18

$$\text{Let } u = \sin^{-1}(\sqrt{1-x^2})$$

$$\text{Put } x = \cos \theta \Rightarrow \theta = \cos^{-1} x, \text{ so}$$

$$u = \sin^{-1}(\sin \theta) \quad \text{---(i)}$$

And,

$$\begin{aligned} \text{Let } v &= \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) \\ &= \cot^{-1}\left(\frac{\cos \theta}{\sqrt{1-\cos^2 \theta}}\right) \\ &= \cot^{-1}\left(\frac{\cos \theta}{\sin \theta}\right) \\ v &= \cot^{-1}(\cot \theta) \end{aligned} \quad \text{---(ii)}$$

$$\begin{aligned} \text{Here, } 0 &< x < 1 \\ \Rightarrow 0 &< \cos \theta < 1 \\ \Rightarrow 0 &< \theta < \frac{\pi}{2} \end{aligned}$$

So, from equation (i),

$$\begin{aligned} u &= \theta && \left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \right] \\ u &= \cos^{-1} x \end{aligned}$$

Differentiation Ex 11.8 Q19

Let $u = \sin^{-1} \{2ax\sqrt{1-a^2x^2}\}$

Put $ax = \sin \theta \Rightarrow \theta = \sin^{-1}(ax)$

$$u = \sin^{-1} \{2 \sin \theta \sqrt{1 - \sin^2 \theta}\}$$

$$= \sin^{-1} \{2 \sin \theta \cos \theta\}$$

$$u = \sin^{-1}(\sin 2\theta) \quad \text{---(i)}$$

And,

Let $v = \sqrt{1-a^2x^2}$

Differentiating it with respect to x using chain rule,

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1-a^2x^2}} \times \frac{d}{dx}(1-a^2x^2)$$

$$= \left(\frac{0-2a^2x}{2\sqrt{1-a^2x^2}} \right)$$

$$\frac{dv}{dx} = \frac{-a^2x}{\sqrt{1-a^2x^2}} \quad \text{---(ii)}$$

Here, $-\frac{1}{\sqrt{2}} < ax < \frac{1}{\sqrt{2}}$

$$\Rightarrow -\frac{1}{\sqrt{2}} < \sin \theta < \frac{1}{\sqrt{2}}$$

$$\Rightarrow -\frac{\pi}{4} < \theta < \frac{\pi}{4}$$

So, from equation (i),

$$u = 2\theta$$

$$\left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right]$$

$$u = 2 \sin^{-1} ax$$

Differentiation Ex 11.8 Q20

Let $u = \tan^{-1}\left(\frac{1-x}{1+x}\right)$

Put $x = \tan \theta \Rightarrow \theta = \tan^{-1} x$, so

$$\begin{aligned} u &= \tan^{-1}\left(\frac{1-\tan \theta}{1+\tan \theta}\right) \\ &= \tan^{-1}\left(\frac{\frac{\tan \pi}{4} - \tan \theta}{1 + \frac{\tan \pi}{4} \tan \theta}\right) \\ u &= \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \theta\right)\right) \end{aligned} \quad \text{---(i)}$$

Here, $-1 < x < 1$

$\Rightarrow -1 < \tan \theta < 1$

$\Rightarrow -\frac{\pi}{4} < \theta < \frac{\pi}{4}$

So, from equation (i),

$$\begin{aligned} u &= \left(\frac{\pi}{4} - \theta\right) && \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right] \\ u &= \frac{\pi}{4} - \tan^{-1} x \end{aligned}$$

Differentiating it with respect to x ,

$$\begin{aligned} \frac{du}{dx} &= 0 - \left(\frac{1}{1+x^2}\right) \\ \frac{du}{dx} &= -\frac{1}{1+x^2} \end{aligned} \quad \text{---(ii)}$$

And,

Let $v = \sqrt{1-x^2}$

Differentiating it with respect to x using chain rule,

$$\begin{aligned} \frac{dv}{dx} &= \frac{1}{2\sqrt{1-x^2}} \times \frac{d}{dx}(1-x^2) \\ &= \frac{1}{2\sqrt{1-x^2}} (-2x) \\ \frac{dv}{dx} &= \frac{-x}{\sqrt{1-x^2}} \end{aligned} \quad \text{---(iii)}$$

Dividing equation (ii) by (iii),

$$\begin{aligned} \frac{\frac{du}{dx}}{\frac{dv}{dx}} &= -\frac{1}{(1+x^2)} \times \frac{\sqrt{1-x^2}}{-x} \\ \frac{du}{dv} &= \frac{\sqrt{1-x^2}}{x(1+x^2)} \end{aligned}$$