Ex 26.1

Q1

Let $P(x, y)$ be any point on the ellipse whose focus is $S(1, -2)$ and eccentricity $e = \frac{1}{2}$. Let PM be perpendicular from P on the directrix. Then,

$$SP = ePM$$

$$\Rightarrow \quad SP = \frac{1}{2}(PM)$$

$$\Rightarrow \quad SP^2 = \frac{1}{4}(PM)^2$$

$$\Rightarrow \quad 4SP^2 = (PM)^2$$

$$\Rightarrow \quad 4\left[(x-1)^2 + (y+2)^2\right] = \left[\frac{3x - 2y + 5}{\sqrt{(3)^2 + (-2)^2}}\right]^2$$

$$\Rightarrow \quad 4\left[x^2 + 1 - 2x + y^2 + 4 + 4y\right] = \frac{(3x - 2y + 5)^2}{\left(\sqrt{13}\right)^2}$$

$$\Rightarrow \quad 4\left[x^2 + y^2 - 2x + 4y + 5\right] = \frac{(3x - 2y + 5)^2}{13}$$

$$\Rightarrow \quad 52\left[x^2 + y^2 - 2x + 4y + 5\right] = (3x - 2y + 5)^2$$

$$\Rightarrow \quad 52x^2 + 52y^2 - 104x + 208y + 260 = (3x - 2y + 5)^2$$

$$\Rightarrow \quad 52x^2 + 52y^2 - 104x + 208y + 260 = (3x)^2 + (-2y)^2 + (5)^2 + 2 \times 3x \times (-2y) + 2 \times (-2y) \times 5 + 2 \times 5 \times 3x$$

$$\left[\because (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca\right]$$

$$\Rightarrow \quad 52x^2 + 52y^2 - 104x + 208y + 260 = 9x^2 + 4y^2 + 25 - 12xy - 20y + 30x$$

$$\Rightarrow \quad 52x^2 - 9x^2 + 52y^2 - 4y^2 + 12xy - 104x - 30x + 208y + 20y + 260 - 25 = 0$$

$$\Rightarrow \quad 43x^2 + 48y^2 + 12xy - 134x + 228y + 235 = 0$$

This is the required equation of the ellipse.

Q2(i)

Let $P(x, y)$ be a point on the ellips. Then, by definition

$$SP = e\,PM$$

Here $e = \frac{1}{2}$, coordinates of S are $(0,1)$ and the equation of the directrix is

$$x + y = 0.$$

$$\therefore \quad SP = \frac{1}{2}PM$$

$$\Rightarrow \quad SP^2 = \frac{1}{4}(PM)^2$$

$$\Rightarrow \quad 4SP^2 = (PM)^2$$

$$\Rightarrow \quad 4\left[(x-0)^2 + (y-1)^2\right] = \left[\frac{x+y}{\sqrt{1^2+1^2}}\right]$$

$$\Rightarrow \quad 4\left[x^2 + y^2 + 1 - 2y\right] = \frac{(x+y)^2}{2}$$

$$\Rightarrow \quad 4 \times 2\left[x^2 + y^2 - 2y + 1\right] = x^2 + y^2 + 2xy$$

$$\Rightarrow \quad 8x^2 + 8y^2 - 16y + 8 = x^2 + y^2 + 2xy$$

$$\Rightarrow \quad 8x^2 - x^2 + 8y^2 - y^2 - 2xy - 16y + 8 = 0$$

$$\Rightarrow \quad 7x^2 + 7y^2 - 2xy - 16y + 8 = 0$$

This is the required equation of the ellipse.

Q2(ii)

Let $P(x, y)$ be a point on the ellipse. Then, by definition

$$SP = e\,PM$$

Here $e = \frac{1}{2}$, coordinates of S are $(-1, 1)$ and the equation of directrix is

$$x - y + 3 = 0$$

$$\therefore \quad SP = \frac{1}{2} PM$$

$$\Rightarrow \quad SP^2 = \frac{1}{4}(PM)^2$$

$$\Rightarrow \quad 4SP^2 = PM^2$$

$$\Rightarrow \quad 4\left[(x+1)^2 + (y-1)^2\right] = \left[\frac{x - y + 3}{\sqrt{1^2 + (-1)^2}}\right]^2$$

$$\Rightarrow \quad 4\left[x^2 + 1 + 2x + y^2 + 1 - 2y\right] = \frac{(x - y + 3)^2}{2}$$

$$\Rightarrow \quad 8\left[x^2 + y^2 + 2x - 2y + 2\right] = (x - y + 3)^2$$

$$\Rightarrow \quad 8x^2 + 8y^2 + 16x - 16y + 16 = x^2 + (-y)^2 + 3^2 + 2 \times (-y) \times 3 + 2 \times (x) \times (-y) + 2 \times 3 \times x$$

$$\left[\because (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca\right]$$

$$\Rightarrow \quad 8x^2 + 8y^2 + 16x - 16y + 16 = x^2 + y^2 + 9 - 6y - 2xy + 6x$$

$$\Rightarrow \quad 8x^2 - x^2 + 8y^2 - y^2 + 2xy + 16x - 6x - 16y + 6y + 16 - 9 = 0$$

$$\Rightarrow \quad 7x^2 + 7y^2 + 2xy + 10x - 10y + 7 = 0$$

This is the required equation of the ellipse.

Q2(iii)

Let $P(x, y)$ be a point the ellipse. Then, by definition

$$SP = e\,PM$$

Here $e = \frac{4}{5}$, coordinates of S are $(-2, 3)$ and the equation of directrix is

$$2x + 3y + 4 = 0$$

$$\therefore \quad SP = \frac{4}{5}Pm$$

$$\Rightarrow \quad SP^2 = \frac{16}{25}(PM)^2$$

$$\Rightarrow \quad 25SP^2 = 16PM^2$$

$$\Rightarrow \quad 25\left[(x+2)^2 + (y-3)^2\right] = 16\left[\frac{2x+3y+4}{\sqrt{2^2+3^2}}\right]^2$$

$$\Rightarrow \quad 25\left[x^2 + 4 + 4x + y^2 + 9 - 6y\right] = \frac{16(2x+3y+4)^2}{13}$$

$$\Rightarrow \quad 325\left[x^2 + y^2 + 4x - 6y + 13\right] = 16(2x+3y+4)^2$$

This is the required equation of the ellipse.

Q2(iv)

Let $P(x, y)$ be a point on the ellipse. Then, by definition

$SP = e\,PM$

Here $e = \frac{1}{2}$, coordinates of S are $(1, 2)$ and the equation of directrix is

$3x + 4y - 5 = 0$

$\therefore \quad SP = \frac{1}{2} Pm$

$\Rightarrow \quad SP^2 = \frac{1}{4}(PM)^2$

$\Rightarrow \quad 4SP^2 = PM^2$

$\Rightarrow \quad 4\left[(x-1)^2 + (y-2)^2\right] = \left[\frac{3x + 4y - 5}{\sqrt{3^2 + 4^2}}\right]^2$

$\Rightarrow \quad 4\left[x^2 + 1 - 2x + y^2 + 4 - 4y\right] = \frac{(3x + 4y - 5)^2}{25}$

$\Rightarrow \quad 100\left[x^2 + y^2 - 2x - 4y + 5\right] = (3x + 4y - 5)^2$

$\Rightarrow \quad 100x^2 + 100y^2 - 200x - 400y + 500 = (3x + 4y - 5)^2$

$\Rightarrow \quad 100x^2 + 100y^2 - 200x - 400y + 500 = (3x)^2 + (4y)^2 + (-5)^2 + 2 \times 3x \times 4y + 2 \times 4y \times (-5) + 2 \times (-5) \times 3x$

$\Rightarrow \quad 100x^2 + 100y^2 - 200x - 400y + 500 = 9x^2 + 16y^2 + 25 + 24xy - 40y - 30x$

$\Rightarrow \quad 100x^2 - 9x^2 + 100y^2 - 16y^2 - 24xy - 200x + 30x - 400y + 40y + 500 - 25 = 0$

$\Rightarrow \quad 91x^2 + 84y^2 - 24xy - 170x - 360y + 475 = 0$

This is the required equation of the ellipse.

Q3(i)

$4x^2 + 9y^2 = 1$

$\frac{x^2}{\frac{1}{4}} + \frac{y^2}{\frac{1}{9}} = 1$

$eccentricity = \sqrt{\frac{\frac{1}{4} - \frac{1}{9}}{\frac{1}{4}}} = \frac{\sqrt{5}}{3}$

$\text{Length of latus rectum} = \frac{2 \times \frac{1}{9}}{\frac{1}{2}} = \frac{4}{9}$

$Foci\ are\ (\frac{\sqrt{5}}{6}, 0); (-\frac{\sqrt{5}}{6}, 0)$

Q3(ii)

$$5x^2+4y^2=1$$

$$\frac{x^2}{\frac{1}{5}}+\frac{y^2}{\frac{1}{4}}=1$$

$$eccentricity=\sqrt{\frac{\frac{1}{4}-\frac{1}{5}}{\frac{1}{4}}}=\frac{1}{\sqrt{5}}$$

$$\text{Length of latus rectum}=\frac{2\times\frac{1}{5}}{\frac{1}{2}}=\frac{4}{5}$$

$$Foci\ are\ (0,\frac{1}{2\sqrt{5}});(0,-\frac{1}{2\sqrt{5}})$$

Q3(iii)

We have,

$$4x^2 + 3y^2 = 1$$

$$\Rightarrow \quad \frac{x^2}{\frac{1}{4}} + \frac{y^2}{\frac{1}{3}} = 1 \text{ ....(i)}$$

This is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a^2 = \frac{1}{4}$ and $b^2 = \frac{1}{3}$ *i.e.*,

$$a = \frac{1}{2} \text{ and } b = \frac{1}{\sqrt{3}}.$$

Clearly, $b > a$, therefore the major and minor axes of the ellipse (i) are along y and x axes respectively.
Let e be the eccentricity of the ellipse. Then,

$$e = \sqrt{1 - \frac{a^2}{b^2}}$$

$$= \sqrt{1 - \frac{\frac{1}{4}}{\frac{1}{3}}}$$

$$= \sqrt{1 - \frac{3}{4}}$$

$$= \sqrt{\frac{1}{4}}$$

$$\therefore \quad e = \frac{1}{2}$$

The coordinates of the foci are $(0, be)$ and $(0, -be)$ *i.e.*, $\left(0, \frac{1}{2\sqrt{3}}\right)$ and $\left(0, \frac{-1}{2\sqrt{3}}\right)$.

Now,

$$\text{Length of the latus rectum} = \frac{2a^2}{b}$$

$$= 2 \times \frac{\frac{1}{4}}{\frac{1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}}{2}$$

Q3(iv)

We have,

$$25x^2 + 16y^2 = 1600$$

$$\Rightarrow \quad \frac{25x^2}{1600} + \frac{16y^2}{1600} = 1$$

$$\Rightarrow \quad \frac{x^2}{64} + \frac{y^2}{100} = 1 \text{ ....(i)}$$

This is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a^2 = 64$ and $b^2 = 100$ i.e.,

$a = 8$ and $b = 10$.

Clearly, $b > a$, therefore the major and minor axes of the ellipse (i) are along y and x axes respectively.
Let e be the eccentricity of the ellipse. Then,

$$e = \sqrt{1 - \frac{a^2}{b^2}}$$

$$= \sqrt{1 - \frac{64}{100}}$$

$$= \sqrt{\frac{36}{100}}$$

$$= \frac{6}{10}$$

$$\therefore \quad = \frac{3}{5}$$

The coordinates of the foci are $(0, be)$ and $(0, -be)$ i.e., $(0, 6)$ and $(0, -6)$.
Now,

$$\text{Length of the latus rectum} = \frac{2a^2}{b}$$

$$= 2 \times \frac{64}{10}$$

$$= \frac{64}{5}$$

Q4

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \;\ldots\ldots\ldots\ldots (i)$$

$$\therefore \quad e=\sqrt{1-\frac{b^2}{a^2}}$$

$$\Rightarrow \quad \sqrt{\frac{2}{5}}=\sqrt{1-\frac{b^2}{a^2}} \qquad \left[\because \text{ eccentricity}=\sqrt{\frac{2}{5}}\right]$$

$$\Rightarrow \quad \frac{2}{5}=1-\frac{b^2}{a^2}$$

$$\Rightarrow \quad \frac{b^2}{a^2}=1-\frac{2}{5}$$

$$\Rightarrow \quad \frac{b^2}{a^2}=\frac{3}{5}$$

$$\Rightarrow \quad 5b^2=3a^2$$

$$\Rightarrow \quad b^2=\frac{3a^2}{5} \;\ldots\ldots\ldots (ii)$$

Putting the value of $b^2=\frac{3a^2}{5}$ in equation (ii), we get

$$\frac{9}{a^2}+\frac{1}{\frac{3a^2}{5}}=1$$

$$\Rightarrow \quad \frac{9}{a^2}+\frac{5}{3a^2}=1$$

$$\Rightarrow \quad \frac{1}{a^2}\left[9+\frac{5}{3}\right]=1$$

Q5(i)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ .... (i)}$$

The coordinates of the foci are $(\pm 2,0)$. This means that the major and minor axes of the ellipse are along x and y axes respectively and the coordinates of foci are $(\pm ae, o)$

$\therefore \quad ae = 2$

$\Rightarrow \quad a \times \frac{1}{2} = 2 \qquad \left[\because e = \frac{1}{2}\right]$

$\Rightarrow \quad a = 4$

$\Rightarrow \quad a^2 = 16$

Now, $\quad b^2 = a^2\left(1-e^2\right)$

$\Rightarrow \quad b^2 = (4)^2\left[1-\left(\frac{1}{2}\right)^2\right]$

$\Rightarrow \quad b^2 = 16\left[1-\frac{1}{4}\right]$

$\Rightarrow \quad b^2 = 16 \times \frac{3}{4} = 12$

Substituting the value of a^2 and b^2 in (i), we get

$$\frac{x^2}{16}+\frac{y^2}{12}=1$$

$\Rightarrow \quad 3x^2+4y^2 = 48$

required equation of ellipse.

Q5(ii)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \ldots\ldots(i)$$

The length of latus-rectum = 5

$$\therefore \quad \frac{2b^2}{a}=5$$

$$\Rightarrow \quad b^2=\frac{5a}{2} \quad \ldots\ldots(ii)$$

Now, $\quad b^2=a^2\left(1-e^2\right)$

$$\Rightarrow \quad \frac{5a}{2}=a^2\left[1-\left(\frac{2}{3}\right)^2\right] \qquad \left[\because e=\frac{2}{3}\right]$$

$$\Rightarrow \quad \frac{5a}{2}=a^2\left[1-\frac{4}{9}\right]$$

$$\Rightarrow \quad \frac{5}{2}=a\left(\frac{5}{9}\right)$$

$$\Rightarrow \quad \frac{5}{2}\times\frac{9}{5}=a$$

$$\Rightarrow \quad a=\frac{9}{2}$$

$$\Rightarrow \quad a^2=\frac{81}{4}$$

Putting $a=\frac{9}{2}$ in $b^2=\frac{5a}{2}$, we get

$$b^2=\frac{5}{2}\times\frac{9}{2}$$

$$\Rightarrow \quad b^2=\frac{45}{4}$$

Substituting $a^2=\frac{81}{4}$ and $b^2=\frac{45}{4}$ in equation (i), we get

$$\frac{x^2}{\frac{81}{4}}+\frac{y^2}{\frac{45}{4}}=1$$

$$\Rightarrow \quad \frac{4x^2}{81}+\frac{4y^2}{45}=1$$

$$\Rightarrow \quad \frac{4x^2\times 5+4y^2\times 9}{405}=1$$

$$\Rightarrow \quad 20x^2+36y^2=405$$

This is the equation of the required ellipse.

Q5(iii)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ .... (i)}$$

Then, semi-major axis = a

$\therefore \quad a = 4 \qquad [\because \text{ semi-major axis} = 4]$

$\Rightarrow \quad a^2 = 16$

Now,

$$b^2 = a^2\left(1-e^2\right)$$

$$\Rightarrow \quad b^2 = 16\left[1-\left(\frac{1}{2}\right)^2\right] \qquad \left[\because\ e = \frac{1}{2}\right]$$

$$\Rightarrow \quad b^2 = 16\left[1-\frac{1}{4}\right]$$

$$\Rightarrow \quad b^2 = 16\times\frac{3}{4}$$

$$\Rightarrow \quad b^2 = 12$$

Substituting the value of a^2 and b^2 in (i), we get

$$\frac{x^2}{16}+\frac{y^2}{12}=1$$

$$\Rightarrow \quad 3x^2+4y^2 = 48$$

This is the required equation of the ellipse.

Q5(iv)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ where major axis} = 2a \ldots\ldots\ldots\ldots(i)$$

Now,

$$2a = 12 \qquad [\because \text{Major axis} = 12]$$

$$\Rightarrow \quad a = 6$$

$$\Rightarrow \quad a^2 = 36$$

Now,

$$b^2 = a^2\left(1-e^2\right)$$

$$\Rightarrow \quad b^2 = 36\left(1-\frac{1}{4}\right) \qquad \left[\because e = \frac{1}{2}\right]$$

$$\Rightarrow \quad b^2 = 36\times\frac{3}{4}$$

$$\Rightarrow \quad b^2 = 27$$

Substituting the value of a^2 and b^2 in (i), we get

$$\frac{x^2}{36}+\frac{y^2}{27}=1$$

$$\Rightarrow \quad \frac{1}{9}\left[\frac{x^2}{4}+\frac{y^2}{3}\right]=1$$

$$\Rightarrow \quad \frac{3x^2+4y^2}{12}=9$$

$$\Rightarrow \quad 3x^2+4y^2=108$$

This is the equation of the required ellipse.

Q5(v)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ .... (i)}$$

Since the ellipse passes through

$(1,4)$ and $(-6,1)$.

$$\therefore \quad \frac{(1)^2}{a^2}+\frac{(4)^2}{b^2}=1$$

$$\Rightarrow \quad \frac{1}{a^2}+\frac{16}{b^2}=1$$

$$\Rightarrow \quad b^2+16a^2=a^2b^2 \text{ .... (ii)}$$

and $$\frac{(-6)^2}{a^2}+\frac{(1)^2}{b^2}=1$$

$$\Rightarrow \quad \frac{36}{a^2}+\frac{1}{b^2}=1$$

$$\Rightarrow \quad \frac{36}{a^2}+\frac{1}{b^2}=1$$

$$\Rightarrow \quad 36b^2+a^2=a^2b^2 \text{ .... (iii)}$$

Multipliying equation (iii) by 16, we get

$$576b^2+16a^2=16a^2b^2 \text{ .... (iv)}$$

Q5(vi)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The coordinates of its vertices and foci are $(\pm a, 0)$ and $(\pm ae, 0)$ respectively.

$\therefore \quad a = 5$ and $ae = 4 \quad \Rightarrow \quad e = \frac{4}{5}$

Now, $\quad b^2 = a^2\left(1 - e^2\right) \Rightarrow b^2 = 25\left(1 - \frac{16}{25}\right) = 9.$

Substituting the values of $a^2$and b^2 in (i), we get

$\frac{x^2}{25} + \frac{y^2}{9} = 1$, which is the equation of the required ellipse.

Q5(vii)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ ....}(i)$$

The coordinates of its vertices and foci are $(0,\pm b)$ and $(0,\pm be)$ respectively.

$\therefore \quad b=13 \quad [\because \text{vertices: } (0,\pm 13)]$

$\Rightarrow \quad b^2=169$

and $\quad be=5 \quad [\because \text{ foci: } (0,\pm 5)]$

$\Rightarrow \quad 13\times e=5$

$\Rightarrow \quad e=\frac{5}{13}$

Now, $\quad a^2=b^2\left(1-e^2\right)$

$$\Rightarrow \quad a^2=(13)^2\left[1-\left(\frac{5}{13}\right)^2\right]$$

$$\Rightarrow \quad a^2=169\left[1-\frac{25}{169}\right]$$

$$\Rightarrow \quad a^2=169\left[\frac{144}{169}\right]$$

$$\Rightarrow \quad a^2=144$$

Substituting the values of a^2 and b^2 in (i), we get

$$\frac{x^2}{144}+\frac{y^2}{169}=1$$

This is the required equation of ellipse.

Q5(viii)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ ....(i)}$$

The coordinates of its vertices and foci are $(\pm a, 0)$ and $(\pm ae, 0)$ respectively.

$\therefore \quad a = 6 \qquad [\because \text{ vertices: } (\pm 6, 0)]$

$\Rightarrow \quad a^2 = 36$

and $\quad ae = 4 \qquad [\because \text{ foci: } (\pm 4, 0)]$

$\Rightarrow \quad 6 \times e = 4$

$\Rightarrow \quad e = \frac{4}{6} = \frac{2}{3}$

Now, $\quad b^2 = a^2\left(1-e^2\right)$

$\Rightarrow \quad b^2 = 36\left[1-\left(\frac{2}{3}\right)^2\right]$

$= 36 \times \left[1-\frac{4}{9}\right]$

$= 36 \times \frac{5}{9}$

$= 4 \times 5$

$= 20$

Substituting the values of a^2 and b^2 in (i), we get

$$\frac{x^2}{36}+\frac{y^2}{20}=1$$

This is the equation of the required ellipse.

Q5(ix)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ .... } (i)$$

The coordinates of its ends of major axis and minor axis are $(\pm a, 0)$ and $(0, \pm b)$ respectively.

$\therefore \quad a = 3 \qquad \left[\because \text{Ends of major axis} = (\pm 3, 0)\right]$

$\Rightarrow \quad a^2 = 9$

and $\quad b = 2 \qquad \left[\because \text{ Ends of major axis} = (0, \pm 2)\right]$

$\Rightarrow \quad b^2 = 4$

Substituting the values of a^2 and b^2 in (i), we get

$$\frac{x^2}{9}+\frac{y^2}{4}=1$$

This is the equation of the required ellipse.

Q5(x)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ .... } (i)$$

The coordinates of its ends of major axis and minor axis are $(0, \pm b)$ and $(\pm a, 0)$ respectively.

$\therefore \quad b = \sqrt{5} \qquad \left[\because \text{ ends of major axis} = (0, \pm\sqrt{5})\right]$

$\Rightarrow \quad b^2 = 5$

and $\quad a = 1 \qquad \left[\because \text{ ends of major axis} = (\pm 1, 0)\right]$

$\Rightarrow \quad a^2 = 1$

Substituting the values of a^2 and b^2 in (i), we get

$$\frac{x^2}{1}+\frac{y^2}{5}=1$$

This is the equation of the required ellipse.

Q5(xi)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \ldots\ldots\ldots (i)$$

we have,
Length of major axis = 26

$\Rightarrow \quad 2a = 26$

$\Rightarrow \quad a = \frac{26}{2} = 13$

$\Rightarrow \quad a^2 = 169$

The coordinates of foci are $(\pm ae, 0)$.

$\therefore \quad ae = 5$

$\Rightarrow \quad 13 \times e = 5$

$\Rightarrow \quad e = \frac{5}{13}$

Now, $b^2 = a^2(1-e^2)$

$\Rightarrow \quad b^2 = 169\left[1-\left(\frac{5}{13}\right)^2\right]$

$\Rightarrow \quad b^2 = 169\left[1-\frac{25}{169}\right]$

$\Rightarrow \quad b^2 = 169\left[\frac{144}{169}\right]$

$\Rightarrow \quad b^2 = 144$

Q5(xii)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ ....(i)}$$

we have,
Length of major axis = 16

$\Rightarrow \quad 2a = 16$

$\Rightarrow \quad a=\frac{16}{2}=8$

$\Rightarrow \quad a^2 = 64$

The coordinates of foci are $(0, \pm be)$.

$\therefore \quad be = 6 \qquad [\because \text{foci: } (0, \pm 6)]$

$\Rightarrow \quad (be)^2 = 36$

Now, $a^2 = b^2(1-e^2)$

$\Rightarrow \quad a^2 = b^2 - b^2e^2$

$\Rightarrow \quad 64 = b^2 - 36 \qquad [\because (be)^2 = 36 \text{ and } a^2 = 64]$

$\Rightarrow \quad 64 + 36 = b^2$

$\Rightarrow \quad b^2 = 100$

Substituting the values of a^2 and b^2 in (i), we get

$$\frac{x^2}{64}+\frac{y^2}{100}=1$$

This is the equation of the required ellipse.

Q5(xiii)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ .... (i)}$$

we have,

$$a = 4$$

$$\Rightarrow \quad a^2 = 16$$

and, the coordinates of foci are $(\pm 3,0)$

$$\therefore \quad ae = 3$$

$$\Rightarrow \quad 4 \times e = 3$$

$$\Rightarrow \quad e = \frac{3}{4}$$

Now, $b^2 = a^2\left(1-e^2\right)$

$$= 4^2\left[1-\left(\frac{3}{4}\right)^2\right]$$

$$= 16 \times \left(1-\frac{9}{16}\right)$$

$$= 16 \times \frac{7}{16}$$

$$= 7$$

Substituting the values of a^2 and b^2 in (i), we get

$$\frac{x^2}{16}+\frac{y^2}{7}=1$$

This is the equation of the required ellipse.

Q6

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ .... (i)}$$

The coordinates of foci are $(+ae, 0)$ and $(-ae, 0)$.

$\therefore \quad ae = 4 \qquad [\because \text{ focai: } (\pm 4, 0)]$

$\Rightarrow \quad a \times \frac{1}{3} = 4 \qquad \left[\because e = \frac{1}{3}\right]$

$\Rightarrow \quad a = 12$

$\Rightarrow \quad a^2 = 144$

Now, $b^2 = a^2(1-e^2)$

$$\Rightarrow \quad b^2 = 144\left[1-\left(\frac{1}{3}\right)^2\right]$$

$$\Rightarrow \quad b^2 = 144\left[1-\frac{1}{9}\right]$$

$$\Rightarrow \quad b^2 = 144 \times \frac{8}{9}$$

$$\Rightarrow \quad b^2 = 16 \times 8 = 128$$

Substituting $a^2 = 144$ and $b^2 = 128$ in equation (i), we get

$$= \frac{x^2}{144}+\frac{y^2}{128}=1$$

$$\Rightarrow \quad \frac{1}{16}\left[\frac{x^2}{9}+\frac{y^2}{8}\right]=1$$

$$\Rightarrow \quad \frac{x^2}{9}+\frac{y^2}{8}=16$$

This is the equation of the required ellipse.

Q7

The coordinates of foci are $(\pm ae, 0)$.

$\therefore \quad 2ae = 2b \qquad$ [given]

$\Rightarrow \quad ae = b$

$\Rightarrow \quad (ae)^2 = b^2 \ \ldots\ldots\ldots (i)$

The length of latus-rectum is 10.

$\Rightarrow \quad \frac{2b^2}{a} = 10 \qquad \left[\because \text{latus-rectum} = \frac{2b^2}{a}\right]$

$\Rightarrow \quad b^2 = \frac{10a}{2}$

$\Rightarrow \quad b^2 = 5a \ \ldots\ldots\ldots\ldots\ldots (ii)$

Now,

$b^2 = a^2(1 - e^2)$

$\Rightarrow \quad b^2 = a^2 - a^2e^2$

$\Rightarrow \quad b^2 = a^2 - b^2$

$\Rightarrow \quad 2b^2 = a^2$

$\Rightarrow \quad b^2 = \frac{a^2}{2}$

Substituting $b^2 = \frac{a^2}{2}$ in equation (ii), we get

$\frac{a^2}{2} = 5a$

$\Rightarrow \quad a^2 = 10a$

$\Rightarrow \quad a = 10$

$\Rightarrow \quad a^2 = 100$

Q8(i)

Let $2a$ and $2b$ the major and minor axes of the ellipse. Then, its equation is

$$\frac{(x+2)^2}{a^2}+\frac{(y-3)^2}{b^2}=1 \qquad [\because \text{centre}: (-2,3) \ldots\ldots (i)]$$

We have,

semi-major axis $= a = 3$

$\Rightarrow \quad a^2 = 9$

and semi-major axis $= b = 2$

$\Rightarrow \quad b^2 = 4$

Putting $a^2 = 9$ and $b^2 = 4$ in equation (i), we get

$$\frac{(x+2)^2}{9}+\frac{(y-3)^2}{4}=1$$

$$\Rightarrow \quad \frac{4(x+2)^2+9(y-3)^2}{36}=1$$

$$\Rightarrow \quad 4(x+2)^2+9(y-3)^2=36$$

$$\Rightarrow \quad 4\left[x^2+4+4x\right]+9\left[y^2+9-6y\right]=36$$

$$\Rightarrow \quad 4x^2+16+16x+9y^2+81-54y=36$$

$$\Rightarrow \quad 4x^2+9y^2+16x-54y+16+81-36=0$$

$$\Rightarrow \quad 4x^2+9y^2+16x-54y+61=0$$

Q8(ii)

Let $2a$ and $2b$ the minor and major axes of the ellipse. Then, its equation is

$$\frac{(x+2)^2}{a^2}+\frac{(y-3)^2}{b^2}=1 \qquad [\because \text{centre}: (-2,3)\ldots\ldots(i)]$$

We have,

semi-major axis $= a = 2$

$\Rightarrow \quad a^2 = 4$

and semi-major axis $= b = 3$

$\Rightarrow \quad b^2 = 9$

Putting $a^2 = 4$ and $b^2 = 9$ in equation (i), we get

$$\frac{(x+2)^2}{4}+\frac{(y-3)^2}{9}=1$$

$$\Rightarrow \quad \frac{9(x+2)^2+4(y-3)^2}{36}=1$$

$$\Rightarrow \quad 9(x+2)^2+4(y-3)^2=36$$

$$\Rightarrow \quad 9\left[x^2+4+4x\right]+4\left[y^2+9-6y\right]=36$$

$$\Rightarrow \quad 9x^2+36+36x+4y^2+36-24y=36$$

$$\Rightarrow \quad 9x^2+4y^2+36x-24y+36+36-36=0$$

$$\Rightarrow \quad 9x^2+4y^2+36x-24y+36=0$$

Q9(i)

Let $2a$ and $2b$ be the major and minor axes of the ellipse.

(i) when latus-rectum is half of minor axis.

$$\frac{2b^2}{a} = \frac{1}{2} \times 2b$$

$$\Rightarrow \quad 2b^2 = ab$$

$$\Rightarrow \quad \frac{b^2}{b} = \frac{a}{2}$$

$$\Rightarrow \quad b = \frac{a}{2}$$

$$\Rightarrow \quad b^2 = \frac{a^2}{4}$$

Now, $b^2 = a^2\left(1 - e^2\right)$

$$\Rightarrow \quad \frac{a^2}{4} = a^2\left(1 - e^2\right) \qquad \left[\because b^2 = \frac{a^2}{4}\right]$$

$$\Rightarrow \quad \frac{1}{4} = 1 - e^2$$

$$\Rightarrow \quad e^2 = 1 - \frac{1}{4}$$

$$\Rightarrow \quad e^2 = \frac{3}{4}$$

$$\Rightarrow \quad e = \frac{\sqrt{3}}{2}$$

Q9(i)

When latus-rectum is half of major-axis.

$$\frac{2b^2}{a} = \frac{1}{2} \times 2a$$

$\Rightarrow \quad 2b^2 = a^2$

$\Rightarrow \quad a^2 = 2b^2$

Now,

$\Rightarrow \quad b^2 = a^2\left(1 - e^2\right)$

$\Rightarrow \quad b^2 = 2b^2\left(1 - e^2\right) \qquad \left[\because a^2 = 2b^2\right]$

$\Rightarrow \quad 1 = 2\left(1 - e^2\right)$

$\Rightarrow \quad 1 = 2 - 2e^2$

$\Rightarrow \quad 2e^2 = 2 - 1$

$\Rightarrow \quad e^2 = \frac{1}{2}$

$\Rightarrow \quad e = \frac{1}{\sqrt{2}}$

Q10(i)

We have,

$$x^2+2y^2-2x+12y+10=0$$

$$\Rightarrow \quad x^2-2x+2y^2+12y+10=0$$

$$\Rightarrow \quad \left(x^2-2x+1-1\right)+2\left(y^2+6y\right)+10=0$$

$$\Rightarrow \quad \left[(x-1)^2-1\right]+2\left[\left(y^2+2\times y\times 3+9\right)-9\right]+10=0$$

$$\Rightarrow \quad (x-1)^2-1+2\left[(y+3)^2-9\right]+10=0$$

$$\Rightarrow \quad (x-1)^2+2(y+3)^2-18-1+10=0$$

$$\Rightarrow \quad (x-1)^2+2(y+3)^2-19+10=0$$

$$\Rightarrow \quad (x-1)^2+2(y+3)^2-9=0$$

$$\Rightarrow \quad (x-1)^2+2(y+3)^2=9$$

$$\Rightarrow \quad \frac{(x-1)^2}{9}+2\frac{(y+3)^2}{9}=1$$

$$\Rightarrow \quad \frac{(x-1)^2}{9}+\frac{(y+3)^2}{\frac{9}{2}}=1$$

$$\Rightarrow \quad \frac{(x-1)^2}{(3)^2}+\frac{(y+3)^2}{\left(\frac{3}{\sqrt{2}}\right)^2}=1 \text{ ....(i)}$$

$\therefore$ The coordinates of centre of the ellipse are $(1,-3)$.

Shifting the origin at $(1,-3)$ without rotating the coordinate axes and denoting the new coordinates with respect to the new axes by X and Y, we have

$$x=X+1 \qquad \text{and } y=Y-3 \text{ ....(ii)}$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{3^2}+\frac{Y^2}{\left(\frac{3}{\sqrt{2}}\right)^2}=1 \text{ ....(iii)}$$

Q10(ii)

We have,

$$x^2+4y^2-4x+24y+31=0$$

$$\Rightarrow \quad x^2-4x+4\{y^2+6y\}+31=0$$

$$\Rightarrow \quad \left[x^2-2\times x\times 2+2^2-2^2\right]+4\left[y^2+2\times 3\times y+3^2-3^2\right]+31=0$$

$$\Rightarrow \quad \left[(x-2)^2-2^2\right]+4\left[(y+3)^2-9\right]+31=0$$

$$\Rightarrow \quad (x-2)^2-4+4(y+3)^2-36+31=0$$

$$\Rightarrow \quad (x-2)^2+4(y+3)^2-5-4=0$$

$$\Rightarrow \quad (x-2)^2+4(y+3)^2=9$$

$$\Rightarrow \quad \frac{(x-2)^2}{9}+\frac{4(y+3)^2}{9}=1$$

$$\Rightarrow \quad \frac{(x-2)^2}{9}+\frac{(y+3)^2}{\frac{9}{4}}=1$$

$$\Rightarrow \quad \frac{(x-2)^2}{3^2}+\frac{(y+3)^2}{\left(\frac{3}{2}\right)^2}=1 \text{ ....(i)}$$

$\therefore$ The coordinates of centre of the ellipse are $(2,-3)$.

Shifting the origin at $(2,-3)$ without rotating the coordinate axes and denoting the new coordinates with respect to the new axes by X and Y, we have

$$x=X+2 \quad \text{and} \quad y=Y-3 \text{ ....(ii)}$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{3^2}+\frac{Y^2}{\left(\frac{3}{2}\right)^2}=1 \text{ ....(iii)}$$

This is of the form

$$\frac{X^2}{a^2}+\frac{Y^2}{b^2}=1, \text{ where}$$

$$a=3 \quad \text{and} \quad b=\frac{3}{2}$$

Clearly, $a>b$. so, the given equation represents on ellipse whose major and minor axes are along X and Y axes respectively.

Length of the axes:

Q10(iii)

We have,

$$4x^2 + y^2 - 8x + 2y + 1 = 0$$

$$\Rightarrow \quad 4\left(x^2 - 2x\right) + \left(y^2 + 2y\right) + 1 = 0$$

$$\Rightarrow \quad 4\left[\left(x^2 - 2x + 1\right) - 1\right] + \left[\left(y^2 + 2y + 1\right) - 1\right] + 1 = 0$$

$$\Rightarrow \quad 4\left[(x-1)^2 - 1\right] + \left[(y+1)^2 - 1\right] + 1 = 0$$

$$\Rightarrow \quad 4(x-1)^2 - 4 + (y+1)^2 - 1 + 1 = 0$$

$$\Rightarrow \quad 4(x-1)^2 + (y+1)^2 - 4 = 0$$

$$\Rightarrow \quad 4(x-1)^2 + (y+1)^2 = 4$$

$$\Rightarrow \quad \frac{(x-1)^2}{1} + \frac{(y+1)^2}{4} = 1$$

$$\Rightarrow \quad \frac{(x-1)^2}{1^2} + \frac{(y+1)^2}{2^2} = 1 \ldots\ldots\ldots\ldots (i)$$

$\therefore$ The coordinates of centre of the ellipse are $(1, -1)$.

Shifting the origin at $(1, -1)$ without rotating the coordinate axes and denoting the new coordinates with respect to the new axes by X and Y, we have

$$x = X + 1 \quad \text{and} \quad y = Y - 1 \ldots\ldots\ldots\ldots (ii)$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{1^2} + \frac{Y^2}{2^2} = 1$$

This is of the form

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1, \text{ where}$$

$a = 1$ and $b = 2$

Clearly, $b > a$. so, the given equation represents on ellipse whose major and minor axes are along Y and X axes respectively.

Q10(iv)

We have,

$$3x^2 + 4y^2 - 12x - 8y + 4 = 0$$

$$\Rightarrow \quad 3x^2 - 12x + 4y^2 - 8y + 4 = 0$$

$$\Rightarrow \quad 3\left(x^2 - 4x\right) + 4\left(y^2 - 2y\right) + 4 = 0$$

$$\Rightarrow \quad 3\left[x^2 - 2 \times x \times 2 + 2^2 - 2^2\right] + 4\left[y^2 - 2 \times y \times 1^2 - 1^2\right] + 4 = 0$$

$$\Rightarrow \quad 3\left[(x-2)^2 - 4\right] + 4\left[(y+1)^2 - 1\right] + 4 = 0$$

$$\Rightarrow \quad 3(x-2)^2 - 12 + 4(y+1)^2 - 4 + 4 = 0$$

$$\Rightarrow \quad 3(x-2)^2 + 4(y-1)^2 - 12 = 0$$

$$\Rightarrow \quad 3(x-2)^2 + 4(y-1)^2 = 12$$

$$\Rightarrow \quad \frac{3(x-2)^2}{12} + \frac{4(y-1)^2}{12} = 1$$

$$\Rightarrow \quad \frac{(x-2)^2}{4} + \frac{(y-1)^2}{3} = 1$$

$$\Rightarrow \quad \frac{(x-2)^2}{2^2} + \frac{(y-1)^2}{\left(\sqrt{3}\right)^2} = 1 \text{ ....(i)}$$

$\therefore$ The coordinates of centre of the ellipse are $(2, 1)$.

Shifting the origin at $(2, 1)$ without rotating the coordinate axes and denoting the new coordinates w.r.t the new axes by. X and Y, we have

$$x = X + 2 \quad \text{and} \quad y = Y - 1 \text{ ....(ii)}$$

Q10(v)

We have,

$$4x^2 + 16y^2 - 24x - 32y - 12 = 0$$

$$\Rightarrow \quad 4x^2 - 24x + 16y^2 - 32y - 12 = 0$$

$$\Rightarrow \quad 4\left(x^2 - 6x\right) + 16\left(y^2 - 2y\right) - 12 = 0$$

$$\Rightarrow \quad 4\left[x^2 - 2 \times x \times 3 + 3^2 - 3^2\right] + 16\left[y^2 - 2y + 1^2 - 1^2\right] - 12 = 0$$

$$\Rightarrow \quad 4\left[(x-3)^2 - 9\right] + 16\left[(y-1)^2 - 1\right] - 12 = 0$$

$$\Rightarrow \quad 4(x-3)^2 - 36 + 16(y-1)^2 - 16 - 12 = 0$$

$$\Rightarrow \quad 4(x-3)^2 + 16(y-1)^2 - 36 - 28 = 0$$

$$\Rightarrow \quad 4(x-3)^2 + 16(y-1)^2 - 64 = 0$$

$$\Rightarrow \quad 4(x-3)^2 + 16(y-1)^2 = 64$$

$$\Rightarrow \quad \frac{4(x-3)^2}{64} + \frac{16(y-1)^2}{64} = 1$$

$$\Rightarrow \quad \frac{(x-3)^2}{16} + \frac{(y-1)^2}{4} = 1$$

$$\Rightarrow \quad \frac{(x-3)^2}{(4)^2} + \frac{(y-1)^2}{(2)^2} = 1 \text{ ....(i)}$$

$\therefore$ The coordinates of centre of the ellipse are $(3, 1)$.

Shifting the origin at $(3, 1)$ without rotating the coordinate axes and denoting the new coordinates w.r.t the new axes by. X and Y, we have

$$x = X + 3 \quad \text{and } y = Y + 1 \text{ ....(ii)}$$

Using these relations, equation (i) reduces to

Q10(vi)

We have,

$$x^2 + 4y^2 - 2x = 0$$

$$\Rightarrow \quad x^2 - 2x + 4y^2 = 0$$

$$\Rightarrow \quad \left(x^2 - 2x + 1^2 - 1^2\right) + 4y^2 = 0$$

$$\Rightarrow \quad (x-1)^2 - 1 + 4y^2 = 0$$

$$\Rightarrow \quad (x-1)^2 + 4y^2 = 1$$

$$\Rightarrow \quad \frac{(x-1)^2}{1} + \frac{y^2}{\frac{1}{4}} = 1$$

$$\Rightarrow \quad \frac{(x-1)^2}{1^2} + \frac{y^2}{\left(\frac{1}{2}\right)^2} = 1 \text{ .... (i)}$$

$\therefore$ The coordinates of centre of the ellipse are $(1, 0)$.

Shifting the origin at $(1, 0)$ without rotating the coordinate axes and denoting the new coordinates w.r.t the new axes by X and Y, we have

$$x = X + 1 \quad \text{and} \quad y = Y \text{ ....(ii)}$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{1^2} + \frac{Y^2}{\left(\frac{1}{2}\right)^2} = 1, \text{ where}$$

$$a = 1 \quad \text{and} \quad b = \frac{1}{2}$$

Clearly, $a > b$. so, the given equation represents on ellipse whose major and minor axes are along X and Y axes respectively.

Q11

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ ....} (i)$$

The coordinates of its foci are $(\pm ae, 0)$ *i.e.*, $(\pm 3, 0)$.

$\therefore \quad ae = 3$

$\Rightarrow \quad (ae)^2 = 9 \text{ ....} (i)$

The required ellipse passes through $(4, 1)$.

$$\therefore \quad \frac{(4)^2}{a^2}+\frac{(1)^2}{b^2}=1$$

$$\Rightarrow \quad \frac{16}{a^2}+\frac{1}{b^2}=1$$

$\Rightarrow \quad 16b^2 + a^2 = a^2b^2$

$\Rightarrow \quad a^2 + 16b^2 = a^2b^2 \text{ ....} (ii)$

Now,

$b^2 = a^2\left(1-e^2\right)$

$\Rightarrow \quad b^2 = a^2 - a^2e^2$

$\Rightarrow \quad b^2 = a^2 - 9 \qquad$ [Using equation (i)] .... (iii)

Substituting $b^2 = a^2 - 9$ in equation (ii), we get

$a^2 + 16\left(a^2-9\right) = a^2\left(a^2-9\right)$

$\Rightarrow \quad a^2 + 16a^2 - 144 = a^4 - 9a^2$

$\Rightarrow \quad 17a^2 - 144 = a^4 - 9a^2$

$\Rightarrow \quad a^4 - 9a^2 - 17a^2 + 144 = 0$

$\Rightarrow \quad a^4 - 26a^2 + 144 = 0$

$\Rightarrow \quad a^4 - 18a^2 - 8a^2 + 144 = 0$

$\Rightarrow \quad a^2\left(a^2-18\right) - 8\left(a^2-18\right) = 0$

$\Rightarrow \quad \left(a^2-18\right)\left(a^2-8\right) = 0$

$\Rightarrow \quad a^2 = 18 \qquad$ or, $\quad a^2 = 8$

$\Rightarrow \quad a^2 = 18$

Putting $a^2 = 18$ in equation (iii), we get

$b^2 = 18 - 9 = 9$

$\therefore$ The required equation of the ellipse is

$$\frac{x^2}{18}+\frac{y^2}{9}=1$$

Q12

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \ldots\ldots\ldots(i)$$

The length of latus-rectum = 5

$$\therefore \quad \frac{2b^2}{a}=5$$

$$\Rightarrow \quad b^2=\frac{5a}{2} \quad \ldots\ldots\ldots\ldots\ldots(ii)$$

Now,

$$b^2=a^2\left(1-e^2\right)$$

$$\Rightarrow \quad \frac{5a}{2}=a^2\left[1-\left(\frac{2}{3}\right)^2\right] \qquad \left[\because e=\frac{2}{3}\right]$$

$$\Rightarrow \quad \frac{5a}{2}=a^2\left[1-\frac{4}{9}\right]$$

$$\Rightarrow \quad \frac{5}{2}=a\left(\frac{5}{9}\right)$$

$$\Rightarrow \quad \frac{5}{2}\times\frac{9}{5}=a$$

$$\Rightarrow \quad a=\frac{9}{2}$$

$$\Rightarrow \quad a^2=\frac{81}{4}$$

Putting $a=\frac{9}{2}$ in $b^2=\frac{5a}{2}$, we get

$$b^2=\frac{5}{2}\times\frac{9}{2}$$

$$\Rightarrow \quad b^2=\frac{45}{4}$$

Substituting $a^2=\frac{81}{4}$ and $b^2=\frac{45}{4}$ in equation(i), we get

$$\frac{x^2}{\frac{81}{4}}+\frac{y^2}{\frac{45}{4}}=1$$

$$\Rightarrow \quad \frac{4x^2}{81}+\frac{4y^2}{45}=1$$

This is the equation of the required ellipse.

Q13

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ where } a < b \text{ ....(i)} \qquad [\text{foci on y-axis}]$$

Now,

$$a^2 = b^2\left(1 - e^2\right)$$

$$\Rightarrow \quad a^2 = b^2\left[1 - \left(\frac{3}{4}\right)^2\right]$$

$$\Rightarrow \quad a^2 = b^2\left[1 - \frac{9}{16}\right]$$

$$\Rightarrow \quad a^2 = b^2 \times \frac{7}{16}$$

$$\Rightarrow \quad a^2 = \frac{7}{16}b^2 \text{ ....(ii)}$$

The required ellipse through $(6, 4)$.

$$\therefore \quad \frac{(6)^2}{a^2} + \frac{(4)^2}{b^2} = 1$$

$$\Rightarrow \quad \frac{36}{a^2} + \frac{16}{b^2} = 1$$

$$\Rightarrow \quad \frac{36}{\frac{7}{16}b^2} + \frac{16}{b^2} = 1 \qquad \left[\because a^2 = \frac{7}{16}b^2\right]$$

$$\Rightarrow \quad \frac{36 \times 16}{7b^2} + \frac{16}{b^2} = 1$$

$$\Rightarrow \quad \frac{576}{7b^2} + \frac{16}{b^2} = 1$$

Q14

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, \text{ where } a>b \text{ ....(i)}$$

The required ellipse passes through $(4,3)$ and $(-1,4)$.

$$\therefore \quad \frac{(4)^2}{a^2}+\frac{(3)^2}{b^2}=1$$

$$\Rightarrow \quad \frac{16}{a^2}+\frac{9}{b^2}=1$$

$$\Rightarrow \quad 16b^2+9a^2=a^2b^2 \text{ ....(ii)}$$

$$\text{and} \quad \frac{(-1)^2}{a^2}+\frac{(4)^2}{b^2}=1$$

$$\Rightarrow \quad \frac{1}{a^2}\times\frac{16}{b^2}=1$$

$$\Rightarrow \quad b^2+16a^2=a^2b^2 \text{ ....(iii)}$$

Multiplying equation (iii) by 16, we get

$$16b^2+256a^2=16a^2b^2 \text{ ....(iv)}$$

Substracting equation (ii) from equation (iv), we get

$$256a^2-9a^2=16a^2b^2-a^2b^2$$

$$\Rightarrow \quad 247a^2=15a^2b^2$$

$$\Rightarrow \quad \frac{247}{15}=b^2$$

$$\Rightarrow \quad b^2=\frac{247}{15}$$

Putting $b^2=\frac{247}{15}$ in equation (iii) we get

$$\frac{247}{15}+16a^2=a^2\times\frac{247}{15}$$

Q15

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ where } a > b \qquad [\because \text{ axes lie along the coordinates axes}]$$

Now,

$$b^2 = a^2\left(1 - e^2\right)$$

$$\Rightarrow \quad b^2 = a^2\left[1 - \left(\sqrt{\frac{2}{5}}\right)^2\right] \qquad \left[\because e = \sqrt{\frac{2}{5}}\right]$$

$$\Rightarrow \quad b^2 = a^2\left[1 - \frac{2}{5}\right]$$

$$b^2 = a^2 \times \frac{3}{5}$$

$$\Rightarrow \quad b^2 = \frac{3a^2}{5} \quad \ldots\ldots\ldots(ii)$$

The required ellipse passes through $(-3, 1)$

$$\therefore \quad \frac{(-3)^2}{a^2} + \frac{1^2}{b^2} = 1$$

$$\Rightarrow \quad \frac{9}{a^2} + \frac{1}{b^2} = 1 \quad \ldots\ldots\ldots\ldots(ii)$$

Putting $b^2 = \frac{3a^2}{5}$ in equation(ii), we get

$$\frac{9}{a^2} + \frac{1}{\frac{3a^2}{5}} = 1$$

$$\Rightarrow \quad \frac{9}{a^2} + \frac{5}{3a^2} = 1$$

$$\Rightarrow \quad \frac{1}{a^2}\left[\frac{9}{1} + \frac{5}{3}\right] = 1$$

$$\Rightarrow \quad \frac{27 + 5}{3} = a^2$$

Q16

Let the equation of the ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, \ldots\ldots\ldots\ldots (i)$$

We have,

$$2ae = 8 \qquad \text{[given]}$$

$$\Rightarrow \quad e = \frac{8}{2a}$$

$$\Rightarrow \quad e = \frac{4}{a} \ldots\ldots\ldots\ldots (ii)$$

Now,

$$\frac{2a}{e} = 18 \qquad \text{[given]}$$

$$\Rightarrow \quad a = \frac{18e}{2}$$

$$\Rightarrow \quad a = 9e \ldots\ldots\ldots\ldots (iii)$$

Using equation (ii) and equation (iii), we get

$$a = \frac{9\times 4}{a}$$

$$\Rightarrow \quad a^2 = 36$$

Now,

$$b^2 = a^2\left(1-e^2\right)$$

$$\Rightarrow \quad b^2 = 36-(ae)^2$$

$$\Rightarrow \quad b^2 = 36-16 \qquad \text{[Using equation (iii)]}$$

$$\Rightarrow \quad b^2 = 20$$

Putting $a^2 = 36$ and $b^2 = 20$ in equation (i), we get

$$\frac{x^2}{36}+\frac{y^2}{20}=1$$

This is the equation of the required ellipsis.

Q17

Let the equation of the ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, \ldots\ldots\ldots\ldots(i)$$

The coordinates of vertices are $(0, \pm b)$ *i.e.*, $(0, \pm 10)$.

$\therefore \quad b = 10$

$\Rightarrow \quad b^2 = 100$

Now,

$$a^2 = b^2\left(1-e^2\right)$$

$$\Rightarrow \quad a^2 = 100\left[1-\left(\frac{4}{5}\right)^2\right]$$

$$\Rightarrow \quad a^2 = 100\left[1-\frac{16}{25}\right]$$

$$\Rightarrow \quad a^2 = 100\left[\frac{9}{25}\right]$$

$$\Rightarrow \quad a^2 = 4\times 9 = 36$$

Putting $a^2 = 36$ and $b^2 = 100$ in equation(i), we get

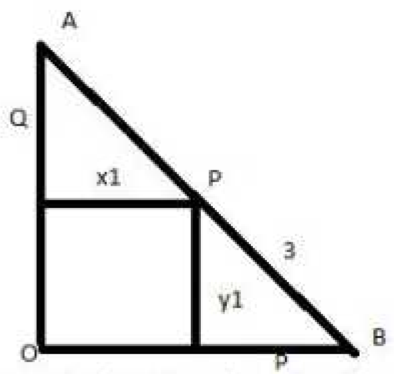
$$\frac{x^2}{36}+\frac{y^2}{100}=1$$

$$\Rightarrow \quad \frac{100x^2+36y^2}{3600}=1$$

$$\Rightarrow \quad 100x^2+36y^2=3600$$

This is the equation of the required ellipsis.

Q18



Using similar triangles principle, we can write

$$\frac{Q}{9}=\frac{y_1}{3}$$

$$Q=3y_1$$

$$Similarly, p=\frac{x}{3}$$

Point P(x,y)

So OB=$x+\frac{x}{3}$

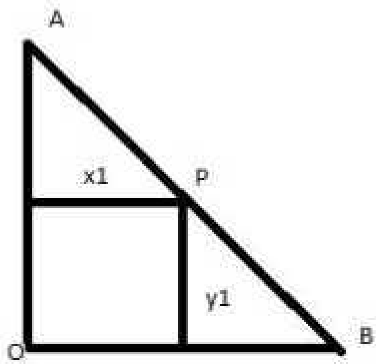
$$OA=y+3y=4y$$

using pythageorus theorem, we get

$$(4y)^2+\left(\frac{4x}{3}\right)^2=12^2$$

$\frac{y^2}{9}+\frac{x^2}{81}=1$ is the equation of ellipse

Q19



From above figure,

Assume length AB=l

$AP = a, PB = b$

$Assume\ \widehat{ABO} = \theta$

so $x_1 = a\cos\theta,\ y_1 = b\sin\theta$

$\Rightarrow \left(\frac{x_1}{a}\right)^2 + \left(\frac{y_1}{b}\right)^2 = 1$

Q20

Let point be (x,y)

Given distances of point from (0, 4) are

2/3 of their distances from the line y = 9

$\sqrt{(x-0)^2 + (y-4)^2} = \frac{2}{3}\left(\sqrt{(y-9)^2}\right)$

Squaring on both sides, we get

$9\left[(x-0)^2 + (y-4)^2\right] = 4\left[(y-9)^2\right]$

$9x^2 + 9y^2 + 144 - 72y = 4y^2 + 324 - 72y$

$9x^2 + 5y^2 = 180$