Triangle
Exercise 7.1

In each of the following:

1. Which of the following is not a criterion for congruence of triangles?

(A) SAS
(B) ASA
(C) SSA
(D) SSS

Sol. SSA is not a criterion for congruence of triangles.
Hence, (c) is the correct answer.

2. If AB = QR, BC = PR and CA = PQ, then

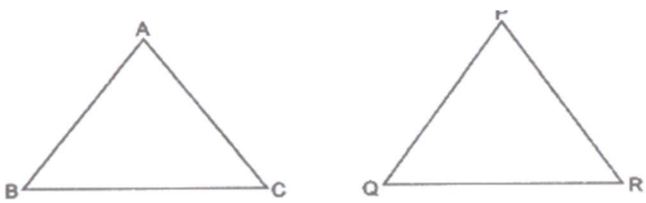
(A) Δ ABC ≅ ΔPQR

(B) ΔCBA ≅ ΔPRQ

(C) ΔBAC ≅ ΔRPQ

(D) ΔPQR ≅ Δ BCA

Sol.



We have AB = QR, BC = PR and
CA = PQ, then
There is one-one corresponding between the vertices. That is, P correspondence to C, Q to A and R to B which is written as

$$P \leftrightarrow C, Q \leftrightarrow A, R \leftrightarrow B$$

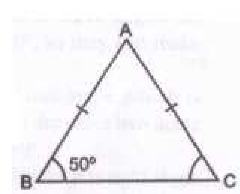
Under this correspondence, we have

$$\Delta CBA \cong \Delta PRQ$$

Hence, (b) is the correct answer.

3. In Δ ABC, AB = AC and ∠B = 50°. Then ∠C is equal to

(A) 40°
(B) 50°
(C) 80°
(D) 130°



Sol. In Δ ABC, we have
AB = AC [Given]
$\therefore$ $\angle C = \angle B$ [$\because$ Angles to opposite to equal sides are equal]
But, $\angle B = 50^0$
$\therefore$ $\angle C = 50^0$

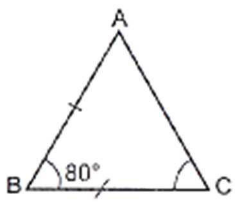
Hence, (b) is the correct answer.

4. In Δ ABC, BC = AB and ∠B = 80°. Then ∠A is equal to

(A) 80°
(B) 40°
(C) 50°
(D) 100°

Sol. In Δ ABC, we have

BC = AB [Given]



$\therefore \quad \angle A = \angle C \quad$ [∵ Angles opposite to equal sides are equal]

But, $\angle B = 80^0$

$\therefore \quad \angle A + \angle B + \angle C = 180^0$

$\Rightarrow \quad \angle A = 80^0 + \angle A = 180^0$

$\Rightarrow \quad 2\angle A = 100^0$

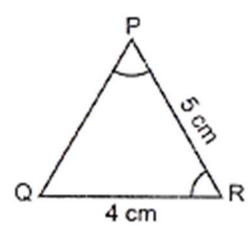
$\Rightarrow \quad \angle A = 100^0 \div 2 = 50^0$

Hence, (c) is the correct answer.

5. In ΔPQR, ∠R = ∠P and QR = 4 cm and PR = 5 cm. Then the length of PQ is

(A) 4 cm
(B) 5 cm
(C) 2 cm
(D) 2.5 cm

Sol. In ΔPQR, we have ∠R = ∠P [Given]



$\therefore \quad PQ + QR$

[∵ Sides opposite to equal angles are equal]

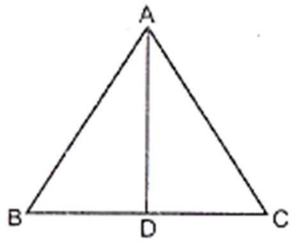
Now, QR = 4cm, therefore, PQ = 4cm.

Hence, (a) is the correct answer.

6. D is a point on the side BC of a Δ ABC such that AD bisects ∠BAC. Then

(A) BD = CD
(B) BA > BD
(C) BD > BA
(D) CD > CA

Sol. In Δ ADC,



Ext. ∠ADB > Int. opp. ∠DAC

$\therefore \quad \angle ADB > \angle BAD \qquad [\because \angle BAD = \angle DAC]$

$\Rightarrow \quad AB > BD \qquad$ [∵ Side opposite to greater angle is longer.]

7. It is given that Δ ABC ≅ Δ FDE and AB = 5 cm, ∠B = 40° and ∠A = 80°. Then which of the following is true?

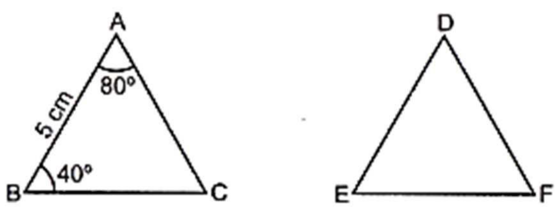
(A) DF = 5 cm, ∠F = 60°

(B) DF = 5 cm, ∠E = 60°

(C) DE = 5 cm, ∠E = 60°

(D) DE = 5 cm, ∠D = 40°

Sol.



It is given that Δ ABC ≅ Δ FDE and AB = 5cm, ∠B = 40° and ∠A = 80°, So ∠C = 60°.

The sides of Δ ABC fall on corresponding equal sides of Δ FDE. A corresponding to F, B corresponds to D, and C corresponds to E.

So, Only DF = 5cm, ∠E = 60° is true.

Hence, (b) is the correct answer.

8. Two sides of a triangle are of lengths 5 cm and 1.5 cm. The length of the third side of the triangle cannot be

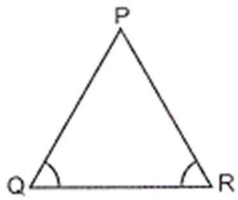
(A) 3.6 cm
(B) 4.1 cm
(C) 3.8 cm
(D) 3.4 cm

Sol. Since sum of any two sides of triangle is always greater than the third side, so their side of the triangle cannot be 3.4 cm because then
1.5 cm + 3.4 cm = 4.9 < third side (5cm).
Hence, (d) is the correct answer.

9. In Δ PQR, if ∠R > ∠Q, then

(A) QR > PR
(B) PQ > PR
(C) PQ < PR
(D) QR < PR

Sol. In Δ PQR, we have ∠R > ∠Q



$\therefore$ $PQ > PR$ [$\because$ Side opposite to greater angle is longer]

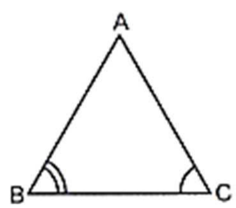
Hence, (b) is the correct answer.

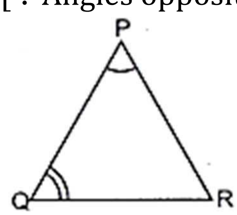
10. In triangles ABC and PQR, AB = AC, ∠C = ∠P and ∠B = ∠Q. The two triangles are

(A) isosceles but not congruent
(B) isosceles and congruent
(C) congruent but not isosceles
(D) neither congruent nor isosceles

Sol. AB = AC [Given]

$\therefore$ $\angle B = \angle C$ [$\because$ Angles opposite to equal sides are equal]





Also, $\angle B = \angle Q$ and $\angle C = \angle P$ [Given]

$\therefore$ $\angle Q = \angle P$

$\Rightarrow$ $PR = RQ$ [$\because$ Sides opp. To equal $\angle s$ equal]

Hence, (a) is the correct answer.

11. In triangles ABC and DEF, AB = FD and ∠A = ∠D. The two triangles will be congruent by SAS axiom. Then,

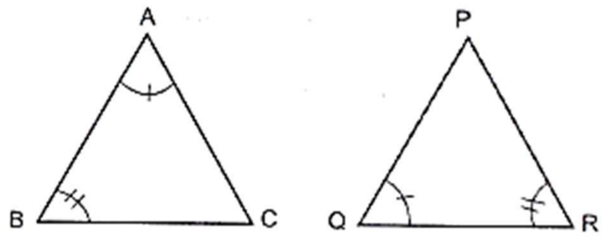
(A) BC = EF
(B) AC = DE
(C) AC = EF
(D) BC = DE

Sol. (b) AC = DE

Triangle
<u>**Exercise 7.2**</u>

1. In triangles ABC and PQR, ∠A= ∠Q and ∠B = ∠R. Which side of ΔPQR should be equal to side AB of Δ ABC so that the two triangles are congruent? Give reason for your answer.

Sol.



In triangle ABC and PQR, we have

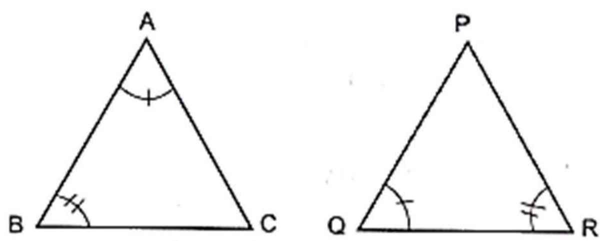
$\angle A = \angle Q$ [Given]

$\angle B = \angle R$ [Given]

For the triangle to be congruent, we must AB = QR. They will be congruent by ASA congruence rule.

2. In triangles ABC and PQR, ∠A= ∠Q and ∠B = ∠R. Which side of ΔPQR should be equal to side BC of Δ ABC so that the two triangles are congruent? Give reason for your answer.

Sol.



In triangle ABC and PQR, we have

$\angle A = \angle Q$ and $\angle B = \angle R$ [Given]

For the triangles to be congruent, we must have

BC = RP

They will be congruent By AAS congruence rule.

3. "If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the two triangles must be congruent." Is the statement true? Why?

Sol. This statement is not true. Angles must be the included angles.

4. **"If two angles and a side of one triangle are equal to two angles and a side of another triangle, then the two triangles must be congruent." Is the statement true? Why?**

Sol. This statement is true. Sides must be corresponding sides.

5. **Is it possible to construct a triangle with lengths of its sides as 4 cm, 3 cm and 7 cm? Give reason for your answer.**

Sol. We know that the sum of any two sides of a triangle is always greater than the third side.
Here, the sum of two sides whose lengths are
4 cm and 3 cm = 4 cm + 3 cm = 7 cm,
Which is equal to the length of third side, i.e., 7 cm.
Hence, it is not possible to construct a triangle with lengths of sides 4 cm, 3 cm and 7 cm.

6. **It is given that Δ ABC ≅ ΔRPQ. Is it true to say that BC = QR? Why?**

Sol. It is False that BC = QR because BC = PQ as ΔABC ≅ ΔRPQ.

7. **It is given that ΔPQR ≅ ΔEDF, then is it true to say that PR = EF? Give reason for your answer.**

Sol. Yes, PR = EF because they are the corresponding sides of ΔPQR and ΔEDF.

8. **In ΔPQR, ∠P = 70° and ∠R = 30°. Which side of this triangle is the longest? Give reason for your answer.**

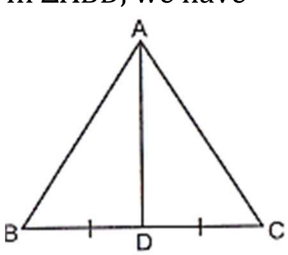
Sol. In ΔPQR, we have

$$\angle Q = 180^0 - (\angle P + \angle R)$$
$$= 180^0 - (70^0 + 30^0) = 180^0 - 100^0 = 80^0$$

Now, in ΔPQR, ∠Q is the larger (greater) and side opposite to greater angle is longer.
Hence, PR is the longest side.

9. **AD is a median of the triangle ABC. Is it true that AB + BC + CA > 2 AD? Give reason for your answer.**

Sol. In ΔABD, we have



AB + BD > AD ...(1)
[∵ Sum of the lengths of any two sides of a triangle must be greater than the third sides]
Now, in ΔADC, we have

AC + CD > AD ...(2)

[∵ Sum of the lengths of any two sides of a triangle must be greater that the third side]

Adding (1) and (2), we get

AB + BD + CD + AC > 2AD

$\Rightarrow AB + BC + CA > 2AD$ [∵ BD = CD as AD is median of ΔABC]

10. M is a point on side BC of a triangle ABC such that AM is the bisector of ∠BAC. Is it true to say that perimeter of the triangle is greater than 2 AM? Give reason for your answer.

Sol. We have to prove that

AB + BC + AC > 2AM.

As sum of any two sides of a triangle is greater than the third side, so in ΔABM, we have

AB + BM > AM ...(1)

And in Δ ACM, AC + CM > AM ...(2)

Adding (1) and (2), we get

AB + BM + AC + CM > 2AM

Or AB + (BM + CM) + AC > 2AM

$\Rightarrow$ $AB + BC + AC > 2AM$

Hence, it is true to say that perimeter of the triangle is greater than 2AM.

11. Is it possible to construct a triangle with lengths of its sides as 9 cm, 7 cm and 17 cm? Give reason for your answer.

Sol. No, it is not possible to construct a triangle whose sides are 9cm, 7cm and 17cm because

9cm + 7cm = 16cm < 17cm

Whereas sum of any two sides of a triangle is always greater than the third side.

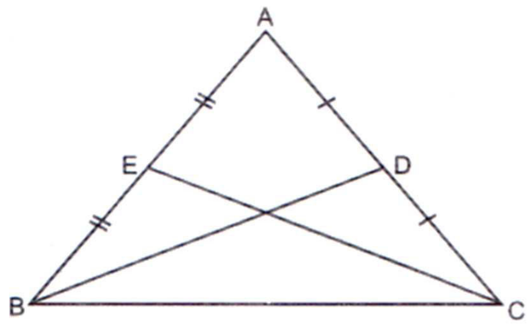
12. Is it possible to construct a triangle with lengths of its sides as 8 cm, 7 cm and 4 cm? Give reason for your answer.

Sol. Yes, it is possible to construct a triangle with lengths of sides as 8 cm, 7 cm and 4 cm as sum of any two sides of a triangle is greater than the third side.

Triangle
<u>Exercise 7.3</u>

1. ABC is an isosceles triangle with AB = AC and BD and CE are its two medians. Show that BD = CE.

Sol. Given: ΔABC with AB = AC



And AD = CD, AE = BE.
To prove: BD = CE
Proof: In ΔABC we have
AB = AC [Given]
$\Rightarrow \frac{1}{2}AB = \frac{1}{2}AC$
$\Rightarrow AE = AD$
[$\because$ D is the mid-point of AC and E is the mid-point of AB]
Now, in ΔABD and ΔACE, we have
$\Delta ABD \cong \Delta ACE$
$\Rightarrow BD = CE$ [CPCT]
Hence, proved.

2. In Fig .7.4, D and E are points on side BC of a Δ ABC such that BD = CE and AD = AE. Show that Δ ABD ≅ Δ ACE.

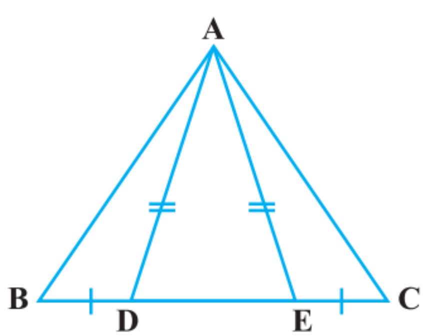


Fig. 7.4

Sol. Given: ΔABC in which BD = CE and AD = AE.
To Prove: Δ ABD ≅ Δ ACE
Proof: In Δ ADE, we have

AD = AE [Given]

$\Rightarrow$ $\angle 2 = 1$

[$\because$ Angle opposite to equal sides of a triangle are equal]

Now, $\angle 1 + \angle 3 = 180^0$ ...(1)

[Linear pair axiom]

$\angle 2 + \angle 4 = 180^0$ ...(2)

[Linear pair axiom]

From equations (1) and (2), we get

$\angle 1 + \angle 3 = \angle 2 + \angle 4$

$\Rightarrow$ $\angle 3 = \angle 4$ $[\because \angle 1 = \angle 2]$

Now, in Δ ABD and Δ ACE, we have

AD = AE [Given]

$\angle 3 = \angle 4$ [Proved above]

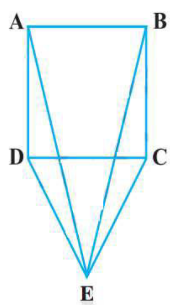
BD = CE [Given]

So, by SAS criterion of congruence, we have

Δ ABD ≅ Δ ACE

Hence, proved

3. CDE is an equilateral triangle formed on a side CD of a square ABCD (Fig.7.5). Show that Δ ADE ≅ Δ BCE.



Sol. Given: An equilateral triangle CDE formed on side CD of square ABCD.

To prove: Δ ADE ≅ Δ BCE

Proof: In square ABCD, we have

$\angle 1 = \angle 2$ ...(1) [$\because$ Each = 90^o]

Now, in Δ DCE, we have

$\angle 3 = \angle 4$ ...(2) [$\because$ Each = 60^o]

Adding (1) and (2), we get

$\angle 1 + \angle 3 = \angle 2 + \angle 4$

$\Rightarrow$ $\angle ADE + \angle BCE$

Now, in Δ ADE and Δ BCE, we have

DE = CE [Sides of an equilateral triangle are equal]

$\angle ADE = \angle BCE$ [Hence proved]

AD = BC [Sides of a square are equal in length]

So, by SAS criterion of congruence, we have

Δ ADE ≅ Δ BCE

Hence, proved

4. In Fig.7.6, BA ⊥ AC, DE ⊥DF such that BA = DE and BF = EC. Show that Δ ABC ≅ Δ DEF.

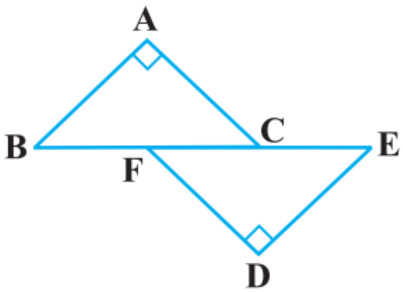


Fig. 7.6

Sol. We have BF = EC

$\therefore \quad BF + FC = CE + FC \Rightarrow BC = EF$

In $\Delta ABC, \angle A = 90^0$ and in $\Delta DEF, \angle D = 90^0$.

$\therefore \Delta ABC$ and ΔDEF are right triangles.

Now, in right triangles ABC and DEF, we have

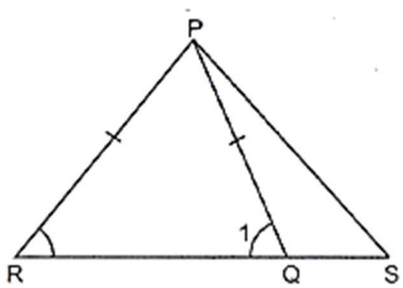
BA = DE [Given]

And BC = EF [Proved above]

$\therefore \quad \Delta ABC \cong \Delta DEF$ [By RHS congruence rule]

5. O is a point on the side SR of a ΔPSR such that PQ = PR. Prove that PS > PQ.

Sol. Given: PQ = PR



To prove: PS > PQ

Proof: In Δ PRQ, we have

PR = PQ [Given]

$\Rightarrow \quad \angle 1 = \angle R$

[$\because$ Angles opposite to equal side of triangle are equal]

But, $\angle 1 > \angle S$

[$\because$ Exterior angle of a triangle is greater than each of the remote interior angles]

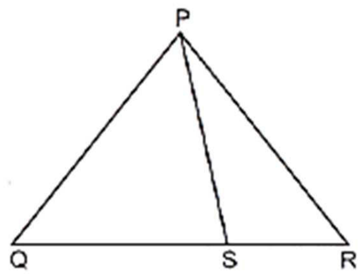
$\Rightarrow \quad \angle R > \angle S$ [$\because \angle 1 = \angle R$]

$\Rightarrow \quad PS < PR$ [$\because$ In a triangle, side opposite to the large is longer]

Hence, proved.

6. S is any point on side QR of a ΔPQR. Show that: PQ + QR + RP > 2 PS.

Sol. Given: A Point S on side QR of ΔPQR.



To prove: PQ + QR + RP > 2PS

Proof: In ΔPQS, we have

PQ + QS > PS ...(1)

[∵ Sum of the length of any two sides of a triangle must be greater than the third side]

Now, in ΔPSR, we have

RS + RP > PS ...(2)

[∵ Sum of the length of any two sides of triangle must be greater than the third side]

Adding (1) and (2), we get

PQ + QS + RS + RP > 2PS

$\Rightarrow \quad PQ + QR + RP > 2PS$

Hence, proved.

7. D is any point on side AC of a Δ ABC with AB = AC. Show that CD < BD.

Sol. In ΔABC, we have

AB = AC [Given]

$\therefore \quad \angle ABC = \angle ACB$

[∵ Angles opp. To equal sides of a triangle are equal]

Now, $\angle DBC < \angle ABC$

$\therefore \quad \angle DBC < \angle ACB$ or $\angle DBC < \angle DCB$

Hence, CD > BD. [∵ Side opposite a greater angle is longer]

8. In Fig. 7.7, l||m and M is the mid-point of a line segment AB. Show that M is also the mid-point of any line segment CD, having its end points on l and m, respectively.

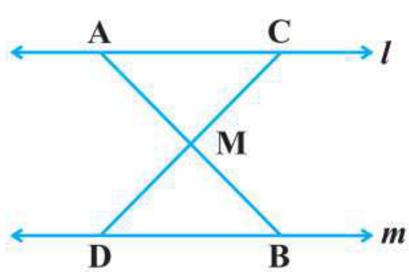
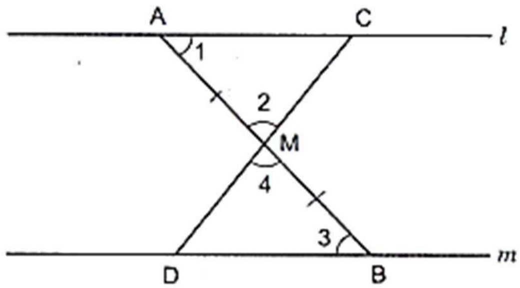


Fig. 7.7

Sol. In ΔAMC and ΔBMD, we have

$\angle 1 = \angle 3$ [Alt. $\angle s$ because l||m]

$\angle 2 = \angle 4$ [Vert. opp. $\angle s$]



AM = BM [Given]

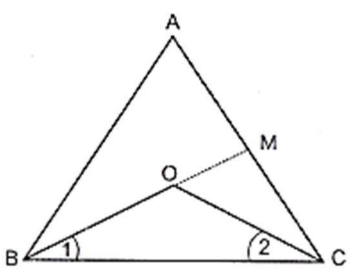
$\therefore$ $\Delta AMC \cong \Delta BMD$ [By ASS congruence rule]

$\therefore$ $CM = DM$ [CPCT]

Hence, M is also the mid-point of CD.

9. Bisectors of the angles B and C of an isosceles triangle with AB = AC intersect each other at O. BO is produced to a point M. Prove that ∠MOC = ∠ABC.

Sol. Bisector of the angles B and C of an isosceles triangle ABC and AB = AC intersect each other at O. BO is produced to a point M.



In ΔABC, we have

AB = AC

$\therefore$ $\angle ABC = \angle ACB$

[$\because$ Angles opposite to equal sides of a triangle are equal]

$\Rightarrow$ $\frac{1}{3}\angle ABC = \frac{1}{2}\angle ACB,$

i.e., $\angle 1 = \angle 2$ [$\because$ BO and CO are bisectors of $\angle B$ and $\angle C$]

In ΔOBC, Ext. $\angle MOC = \angle 1 + \angle 2$

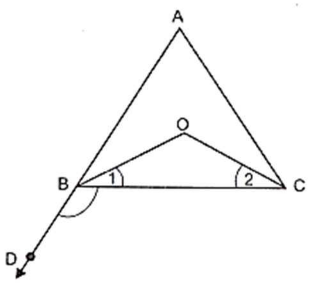
[$\because$ Exterior angle of a triangle is equal to the sum of interior opposite angles]

$\Rightarrow$ $Ext.\angle MOC = 2\angle 1$

Hence, $\angle MOC = \angle ABC.$

10. Bisectors of the angles B and C of an isosceles triangle ABC with AB = AC intersect each other at O. Show that external angle adjacent to ∠ABC is equal to ∠BOC.

Sol. In ΔABC we have



AB = AC

$\therefore \quad \angle B = \angle C$ [$\because$ Angles opposite to equal sides of a triangle are equal]

$\therefore \quad \frac{1}{2}\angle B = \frac{1}{2}\angle C$ ...(1)

In ΔOBC, we have

$$\angle 1 = \frac{1}{2}\angle B$$

And $\angle 2 = \frac{1}{2}\angle C$

$\therefore \quad \angle 1 = \angle 2$ [By (1)]

$\angle DBC + \angle 1 + \angle OBA = 180^0$ [$\because$ ABD is a straight line]

$\Rightarrow \quad \angle DBC + 2\angle 1 = 180^0$ [$\because \angle 1 = \angle OBA$] ...(1)

In ΔOBC,

$$\angle 1 + \angle 2 + \angle BOC = 180^0$$

$\Rightarrow \quad 2\angle 1 + \angle BOC = 180^0$ [$\because \angle 1 = \angle 2$] ...(2)

From (1) and (2), we get

$$\angle DBA + 2\angle 1 = 2\angle 1 + \angle BOC$$

$\Rightarrow \quad \angle DBC = \angle BOC$

11. In Fig. 7.8, AD is the bisector of ∠BAC. Prove that AB > BD.

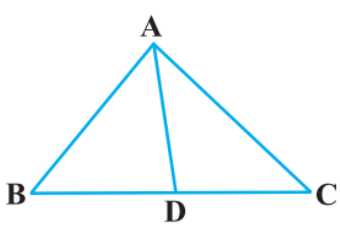


Fig. 7.8

Sol. Since exterior angle of a triangle is greater than either of the interior opposite angles, therefore, in ΔACD,

Ext $\angle 3 > \angle 2 \Rightarrow \angle 3 > \angle 1$

[$\because$ AD is the bisector of $\angle BAC$, so $\angle 1 = \angle 2$]

Now, in ΔABD, we have

$$\angle 3 > \angle 1$$

Hence, AB > BD. [$\because$ In a triangle, side opposite to greater angle is longer]

Triangle
Exercise 7.4

1. Find all the angles of an equilateral triangle.

Sol. In ΔABC, we have

AB = AC

$\Rightarrow \quad \angle C = \angle B \quad$ ...(1)

[$\because$ Angles opposite to equal sides of a triangle are equal]

BC = AC

$\Rightarrow \quad \angle A = \angle B \quad$ ...(2)

[$\because$ Angles opposite to equal sides of a triangle are equal]

Now, $\angle A + \angle B + \angle C = 180^0$ [$\because$ Angle sum property of a triangle]

$\Rightarrow \quad \angle A + \angle A + \angle A = 180^0$ [From (1) and (2)]

$\Rightarrow \quad 3\angle A = 180^0$

$\Rightarrow \quad \angle A = \frac{180^0}{3} = 60^0$

$\therefore \quad \angle A = \angle B = \angle C = 60^0$

2. The image of an object placed at a point A before a plane mirror LM is seen at the point B by an observer at D as shown in Fig. 7.12. Prove that the image is as far behind the mirror as the object is in front of the mirror.

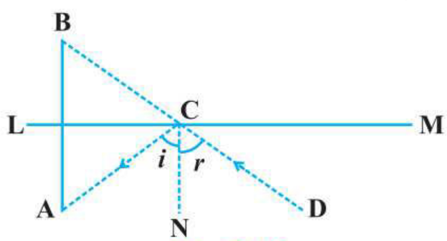


Fig. 7.12

Sol. Let AB intersect LM at O. We have to prove that AO = BO.

Now, $\angle i = \angle r \quad$ ...(1)

[$\because$ Angle of incidence = Angle of reflection]

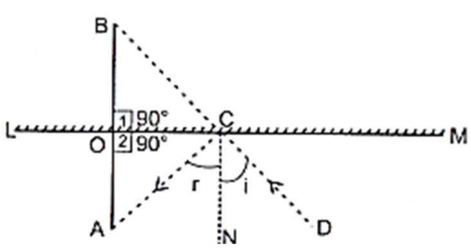
$\angle B = \angle i \quad$ [Corres. $\angle s$] ...(2)

And $\angle A = \angle r \quad$ [Alternate int. $\angle s$] ...(3)

From (1), (2) and (3), we get

$\angle B = \angle A$

$\Rightarrow \quad \angle BCO = \angle ACO$



In ΔBOC and ΔAOC we have

$\angle 1 = \angle 2$ [Each = 90°]

OC = OC [Common side]

And $\angle BCO = \angle ACO$ [Proved above]

$\therefore$ $\Delta BOC \cong \Delta AOC$ [ASA congruence rule]

Hence, AO = BO [CPCT]

3. ABC is an isosceles triangle with AB = AC and D is a point on BC such that AD⊥BC (Fig. 7.13). To prove that ∠BAD = ∠CAD, a student proceeded as follows:

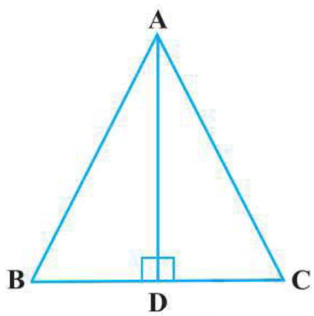


Fig. 7.13

In Δ ABD and Δ ACD,

AB = AC (Given)

∠B = ∠C (because AB = AC)

and ∠ADB = ∠ADC

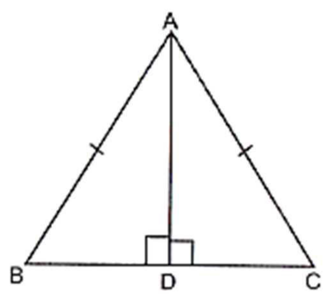
Therefore, Δ ABD = Δ ACD (AAS)

So, ∠BAD = ∠CAD (CPCT)

What is the defect in the above arguments?

[Hint: Recall how ∠B = ∠C is proved when AB = AC].

Sol. In Δ ABD and Δ ADC, we have



∠ADB = ∠ADC [∵ Each equal to 90°]

AB = BC [Given]

AD = AD [Common side]

So, by RHS criterion of congruence, we have

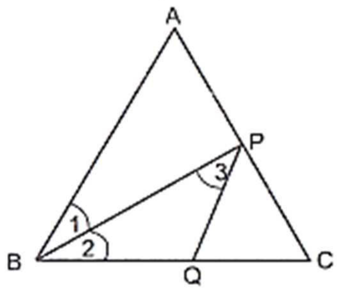
Δ ABD ≅ Δ ACD

$\therefore$ $\angle BAD = \angle CAD$ [CPCT]

Hence, proved.

4. P is a point on the bisector of ∠ABC. If the line through P, parallel to BA meet BC at Q, prove that BPQ is an isosceles triangle.

Sol. We have to prove that BPQ is an isosceles triangle.



$\angle 1 = \angle 2$ ...(1)

[$\because$ BP is the bisector of $\angle ABC$]

Now, PQ is parallel to BA and BP cuts them

$\therefore$ $\angle 1 = \angle 3$ [Alt. $\angle s$] ...(2)

From (1) and (2), we get

$\angle 2 = \angle 3$

In $\Delta BPQ,$ we have

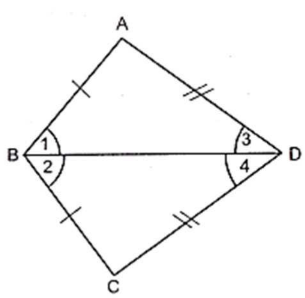
$\angle 2 = \angle 3$ [Proved above]

$\therefore$ $PQ = BQ$ [$\because$ Side of opp. To equal angles are equal]

Hence, BPQ is an isosceles triangle.

5. ABCD is a quadrilateral in which AB = BC and AD = CD. Show that BD bisects both the angles ABC and ADC.

Sol. In ΔABC and ΔCBD, We have



AB = BC [Given]

AD = CD [Given]

BD = BD [Common side]

$\therefore \Delta ABC \cong \Delta CBD$ [By SSS congruence rule]

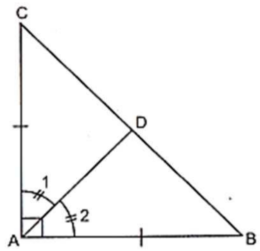
$\Rightarrow$ $\angle 1 = \angle 2$ [CPCT]

And $\angle 3 = \angle 4$

Hence, BD bisects both the angle ABC and ADC.

6. ABC is a right triangle with AB = AC. Bisector of ∠A meets BC at D. Prove that BC = 2 AD.

Sol. Given: A right angles triangle with AB = AC bisector of ∠A meets BC at D.



To prove: BC = 2AD

Proof: In right ΔABC,

AB = AC [Given]

$\Rightarrow$ BC is hypotenuse

[$\because$ Hypotenuse is the longest side.]

$\therefore$ $\angle BAC = 90^0$

Now, in ΔCAD and ΔBAD we have

AC = AB [Given]

$\angle 1 = \angle 2$ [$\because$ AD is the bisector of $\angle A$]

AD = AD [Common side]

So, By SAS criterion of congruence, we have

$\Delta CAD \cong \Delta BAD$

$\therefore$ $CD = BD$ [CPCT]

$\Rightarrow$ $AB = BD = CD$ ...(1)

[$\because$ Mid-point of hypotenuse of a rt. Δ is equidistant from the three vertices of a Δ]

Now, BC = BD + CD

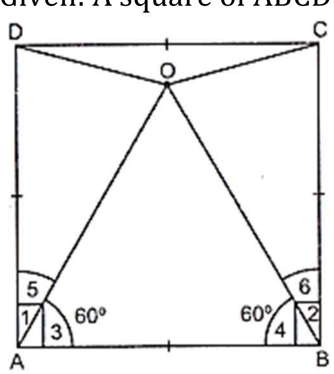
$\Rightarrow$ $BC = AD + AD$ [Using (1)]

$\Rightarrow$ $BC = 2AD$

Hence, proved.

7. O is a point in the interior of a square ABCD such that OAB is an equilateral triangle. Show that Δ OCD is an isosceles triangle.

Sol. Given: A square of ABCD and OA = OB = AB.



TO prove: ΔOCD is an isosceles triangle.

Proof: In square ABCD,

$\angle 1 = \angle 2$ ...(1)

[$\because$ Each equal to 90^o]

Now, in ΔOAB, we have

$\angle 3 = \angle 4$ ...(2)

[$\because$ Each equal to 60^o]

Subtracting (2) from (1), we get

$\angle 1 - \angle 3 = \angle 2 - \angle 4$

$\Rightarrow$ $\angle 5 = \angle 6$

Now, in ΔDAO and ΔCBO,

AD = BC [Given]

$\angle 5 = \angle 6$ [Proved above]

OA = OB [Given]

So, By SAS criterion of congruence, we have

$\Delta DAO \cong \Delta CBO$

$\therefore$ $OD = OC$

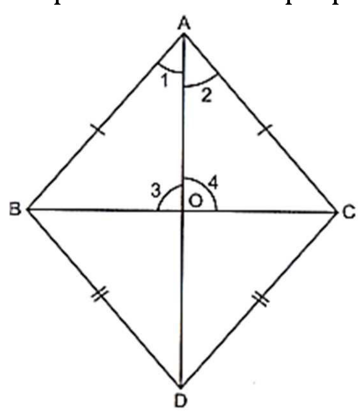
$\Rightarrow \Delta OCD$ is an isosceles triangle.

Hence, proved.

8. ABC and DBC are two triangles on the same base BC such that A and D lie on the opposite sides of BC, AB = AC and DB = DC. Show that AD is the perpendicular bisector of BC.

Sol. Given: ΔABC and ΔDBC on the same base BC. Also, AB = AC and BD = DC.

To prove: AD is the perpendicular bisector of BC i.e., OB = OC



Proof: In ΔBAD and ΔCAD we have

AB = AC [Given]

BD = CD [Given]

AD = AD [Given]

[common side]

So, by SSS criterion of congruence, we have

$\Delta BAD \cong \Delta CAD$

$\therefore$ $\angle 1 = \angle 2$ [CPCT]

Now, in ΔBAO and ΔCAO, we have

AB = AC [Given]

$\angle 1 = \angle 2$ [Proved above]

AO = AO [Common side]

So, by SAS criterion of congruence, we have

$\Delta BAO \cong \Delta CAO$

$\therefore$ $BO = CO$ [CPCT]

And, $\angle 3 = \angle 4$ [CPCT]

But, $\angle 3 + \angle 4 = 180^0$ [Linear pair axiom]

$\Rightarrow$ $\angle 3 + \angle 3 = 180^0$

$\Rightarrow$ $2\angle 3 = 180^0$

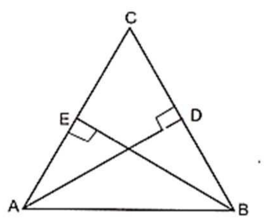
$\Rightarrow$ $\angle 3 = \frac{180^0}{2} = 90^0$

$\therefore$ AD is perpendicular bisector of BC [$\because$ BO = CO and $\angle 3 = 90^0$]

Hence, proved.

9. ABC is an isosceles triangle in which AC = BC. AD and BE are respectively two altitudes to sides BC and AC. Prove that AE = BD.

Sol. In ΔADC and ΔBEC we have



AC = BC [Given] ...(1)

$\angle ADC = \angle BEC$ [Each = 90^o]

$\angle ACD = \angle BCE$ [Common angle]

$\therefore$ $\Delta ADC \cong \Delta BEC$ [By SSS congruence rule]

$\therefore$ $CE = CD$ ...(2) [CPCT]

Subtracting (2) from (1), we get

AC − CE = BC − CD

$\Rightarrow$ $AE = BD$

Hence, proved.

10. Prove that sum of any two sides of a triangle is greater than twice the median with respect to the third side.

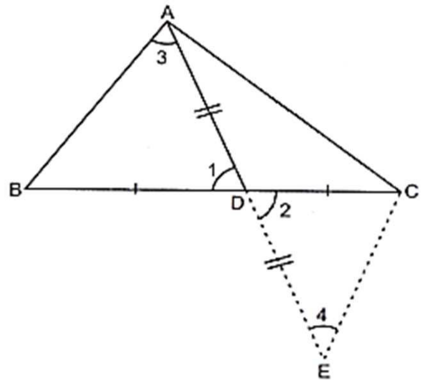
Sol. Given: ΔABC with median AD.

To prove:

AB + AC > 2AD

AB + BC > 2AD

BC + AC > 2AD



Construction: produce AD to E such that DE = AD and join EC.
Proof: In ΔADB and ΔEDC ,

AD = ED [By construction]

$\angle 1 = \angle 2$ [Vertically opposite angles are equal]

DB = DC [Given]

So, by SAS criterion of congruence, we have

$\Delta ADB \cong \Delta EDC$

$\therefore$ $AB = EC$ [CPCT]

And $\angle 3 = \angle 4$ [CPCT]

Now, in ΔAEC, we have

AC + CE > AE [$\because$ Sum of the lengths of any two sides of a triangle must be greater than the third side]

$\Rightarrow$ $AC + CE > AD + DE$

$\Rightarrow$ $AC + CE > AD + AD$ [$\therefore AD = DE$]

$\Rightarrow$ $AC + CE > 2AD$

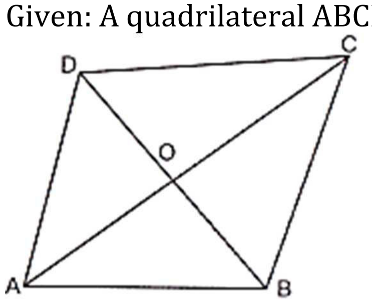
$\Rightarrow$ $AC + AB > 2AD$ [$\because AB = CE$]

Hence, proved.

Similarly, AB + BC > 2AD and BC + AC >2AD.

11. Show that in a quadrilateral ABCD, AB + BC + CD + DA < 2 (BD + AC).

Sol. Given: A quadrilateral ABCD.



To prove: AB + BC + CD + DA <2(BD + AC)

Proof: In ΔAOB we have

$\Rightarrow OA + OB > AB$ ...(1)

[$\because$ Sum of the lengths of any two sides of a triangle must be greater than the third side]

In ΔBOC, we have

OB + OC > BC ...(2) [Same reason]

In ΔCOD, we have

OC + OD > CD ...(3) [Same reason]

In ΔDOA, we have

OD + OA > DA ...(4) [Same reason]

Adding (1), (2), (3) and (4), we get

OA + OB + OB + OC + OC + OD + OD + OA > AB + BC + CD + DA

$\Rightarrow$ 2(OA + OB + OC + OD) > AB + BC + CD + DA

$\Rightarrow$ 2{(OA + OC) + (OC + OD)} > AB + BC + CD + DA

$\Rightarrow$ 2(AC + BD) > AB + BC + CD + DA

$\Rightarrow$ AB + BC + CD + DA < 2(BD + AC)

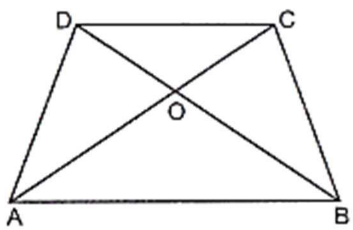
Hence, proved.

12. Show that in a quadrilateral ABCD,
AB + BC + CD + DA > AC + BD

Sol. Given: A quadrilateral ABCD.

To prove: AB + BC + CD + DA > AC + BD

Proof: ΔABC, we have



AB + BC > AC ...(1)

[$\because$ Sum of the lengths of any two sides of a triangle must be greater than the third side]

In ΔBCD, we have

BC + CD > BD ...(2) [Same reason]

In ΔCDA, we have

CD + DA > AC ...(3) [Same reason]

In ΔDAB, we have

AD + AB > BD ...(4) [Same reason]

Adding (1), (2), (3) and (4), we get

AB + BC + BC + CD + CD + DA + AD + AB > AC + BD + AC + BD

$\Rightarrow$ 2AB + 2BC + 2CD + 2DA > 2AC + 2BD

$\Rightarrow$ 2(AB + BC + CD + DA) > 2(AC + BD)

$\Rightarrow$ AB + BC + CD + DA > AC + BD

Hence, proved.

13. In a right triangle, ABC, D is the mid-point of side AC such that BD = $\frac{1}{2}$ AC. Show that $\angle$ABC is a right angle.

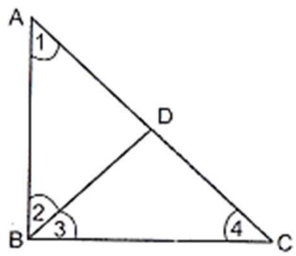
Sol. We have to prove that $\angle$ABC = 90^o.

As D is the mid-point of AC,

So, AD = DC

Also, $BD = \frac{1}{2}AC = AD$ [$\because$ D is the mid-point of AC]

$\therefore$ BD = AD = DC



In ΔABD, we have

BD = AD

$\therefore$ $\angle 1 = \angle 2$ ...(1)

[$\because$ Angles opposite to equal sides are equal]

In ΔBCD, we have

BD = DC,

$\therefore$ $\angle 3 = \angle 4$ ...(2)

In ΔABC, we have

$\angle 1 + \angle ABC + \angle 4 = 180^0$

$\Rightarrow$ $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^0$ [$\because \angle ABC + \angle 3 + \angle 2$]

$\Rightarrow$ $2(\angle 2 + \angle 3) = 180^0$

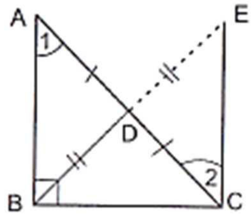
$\Rightarrow$ $\angle 2 + \angle 3 = 90^0$

$\Rightarrow$ $\angle ABC = 90^0$

Hence proved.

14. In a right triangle, prove that the line-segment joining the mid-point of the hypotenuse to the opposite vertex is half the hypotenuse.

Sol. ABC is right triangle, right angles at B and D is the mid-point of AC. We have to prove that $BD = \frac{1}{2}AC.$



Now, produce BD to E such that BD = DE. Join EC.

In ΔADB and b ΔCDE, we have

AD = CD [$\because$ D is the mid-point of AC]

$\angle ADB = \angle CDE$ [Vertically opposite $\angle s$]

BD = DE [By construction]

$\therefore$ $\Delta ADB \cong CDE$ [By SAS criterion of congruence]

$\therefore$ $AB = EC$ [CPCT]

And $\angle 1 = \angle 2$ [CPCT]

But, $\angle 1$ and $\angle 2$ are alternate angles.

$\therefore$ $EC \parallel BA$

Now, EC is parallel to BA and BC is the transversal

$\therefore$ $\angle ABC + \angle BCE = 180^0$

$\Rightarrow$ $90^0 + \angle BCE = 180^0$

$\Rightarrow$ $\angle BCE = 180^0 - 90^0 = 90^0$

In ΔABC and ΔEBC, we have

BC = BC [Common side]

AB = EC [Proved above]

$\angle CBA = \angle BCE$ [$\because$ Each = 90°]

$\therefore$ $\Delta ABC \cong \Delta EBC$ [By SAS criterion of congruence]

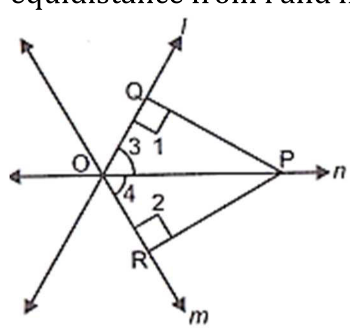
$\therefore$ $AC = EB$ [CPCT]

$\Rightarrow$ $\frac{1}{2}AC = \frac{1}{2}EB \Rightarrow \frac{1}{2}AC = BD$

Hence, $BD = \frac{1}{2}AC.$

15. Two lines l and m intersect at the point O and P is a point on a line n passing through the point O such that P is equidistant from l and m. Prove that n is the bisector of the angle formed by l and m.

Sol. Given: Lines, l and m and n intersect at point O. P is a point on line n and such that P is equidistance from l and n.



To prove: n is the bisector of $\angle QOR$.

Proof: In ΔOQP and ΔORP, we have

$\angle 1 = \angle 2$

[$\because$ Each equal to 90°]

$OP = OP$ [Common side]
$PQ = QR$ [Given]

So, by RHS criterion of congruence, we have

$\Delta OQP \cong \Delta ORP$

$\therefore \angle 3 = \angle 4$ [CPCT]

So, n is bisector of $\angle QOR$

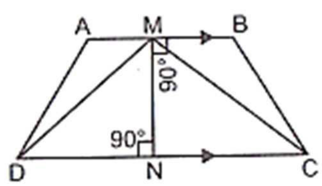
Hence, proved.

16. Line segment joining the mid-points M and N of parallel sides AB and DC, respectively of a trapezium ABCD is perpendicular to both the sides AB and DC. Prove that AD = BC.

Sol. Join MD and CM.

We have, $\angle DNM = \angle NMB$ [Alt. $\angle s$]

$\therefore AB \parallel CD$



Now, in ΔDMN and ΔCNM,

$CN = DN$

[$\because$ N is the mid-point of DC]

$\angle DNM = \angle CNM$ [Each = 90^o]

$NM = NM$ [Common side]

$\therefore \Delta DMN \cong \Delta CNM$ [By SAS congruence rule]

$\therefore DM = CM$ and $\angle NMC = \angle NMD$ ...(1)[CPCT]

Now, $\angle AMN = \angle BMN$ [Each = 90^o]

And $\angle NMD = \angle NMC$ [Proved above]

$\therefore \angle AMN - \angle NMD = \angle BMN - \angle NMC$ [On subtraction]

$\Rightarrow \angle AMD = \angle BMC$ ...(2)

$AM = BM$ [Given]

$DM = CM$ [From (1)]

$\angle AMD = \angle BMC$ [From (2)]

$\Delta AMD \cong \Delta BMC$ [By SAS congruence rule]

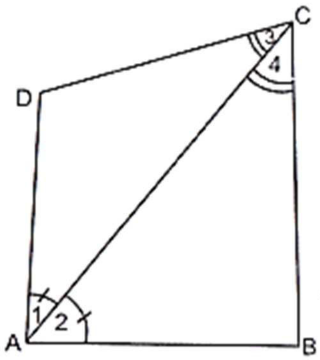
$\therefore AD = BC$ [CPCT]

17. ABCD is a quadrilateral such that diagonal AC bisects the angles A and C. Prove that AB = AD and CB = CD.

Sol. Given: A quadrilateral ABCD such that $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

To prove: AB = AD and CB = CD

Proof: In ΔABC and ΔADC, we have



$\angle 1 = \angle 2$ [Given]

AC = AC [Common side]

$\angle 3 = \angle 4$ [Given]

So, by SAS criterion of congruence, we have

$\Delta ABC \cong \Delta ADC$

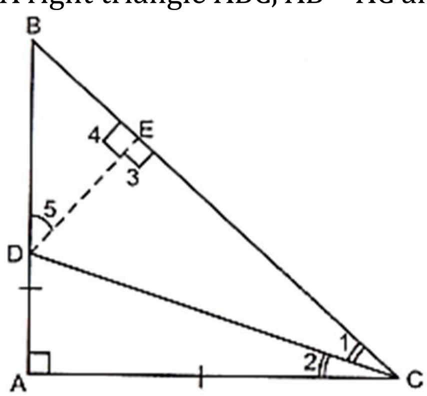
$\therefore$ $AB = AD$ [CPCT]

And CB = CD [CPCT]

Hence, proved.

18. ABC is a right triangle such that AB = AC and bisector of angle C intersects the side AB at D. Prove that AC + AD = BC.

Sol. Given: A right triangle ABC, AB = AC and CD is the bisector of $\angle C$.



To prove: AC + AD = BC

Construction: Draw $DE \perp BC$.

Proof: In right triangle ABC, we have

AB = AC [Given]

$\therefore$ BC is hypotenuse

$\Rightarrow \angle A = 90^0$

In ΔDAC and ΔDEC, we have

$\angle A = \angle 3$ [$\because$ Each Equal to 90^o]

$\angle 1 = \angle 3$ [Given]

DC = DC [Common side]

So, by AAS criterion of congruence, we have

$\Delta DAC \cong \Delta DEC$

$\therefore$ $DA = DE$ ...(1) [CPCT]

And CA = CE ...(2) [CPCT]

In ΔBAC, we have

AB = AC [Given]

$\Rightarrow$ $\angle C = \angle B$ [$\because$ Angles opposite to equal sided of a triangle are equal]

Now, $\angle A + \angle B + \angle C = 180^0$ [Angle sum property of a triangle]

$\Rightarrow 90^0 + \angle B + \angle B = 180^0$ $[\because \angle B = \angle C]$

$\Rightarrow$ $2\angle B = 90^0$

$\Rightarrow$ $\angle B = \dfrac{90^0}{2} = 45^0$

Now, in $\angle BED$, we have

$\Rightarrow \angle 4 + \angle 5 + \angle B = 180^0$ [Angle sum property of a triangle]

$\Rightarrow 90^0 + \angle 5 + \angle 45 = 180^0$

$\Rightarrow$ $\angle 5 = 180^0 - 135^0$

$\Rightarrow$ $\angle 5 = 45^0$

$\therefore$ $\angle B = \angle 5$

$\Rightarrow$ $DE = BE$ ...(3) [$\because$ Side opposite to equal angles of triangle are equal]

From (1) and (3), we get

DA = DE = BE ...(4)

Now, BC = CE + BE

$\Rightarrow$ $BC = CA + DA$ [Using (2), (3) and (4)]

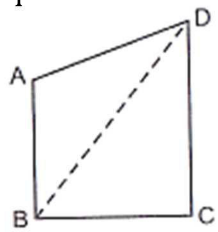
$\Rightarrow$ $BC = AC + AD$

$\Rightarrow$ $AC + AD = BC$

Hence proved.

19. AB and CD are the smallest and lar gest sides of a quadrilateral ABCD. Out of $\angle$B and $\angle$D decide which is greater.

Sol. Given: a quadrilateral ABCD is which AB and CD are the smallest and largest sides of quadrilateral ABCD.



To prove: $\angle B > \angle D$

Construction: Join BD.

Proof: In ΔABD, we have

$\Rightarrow$ $AB > AD$

[$\because$ AB is the smallest side of quadrilateral ABCD]

$\Rightarrow$ $AD > AB$

$\Rightarrow \quad \angle ABD > \angle ADB$ ...(1) [$\because$ Angle opposite to longest side is greater]

Again, in ΔCBD, we have

CD > BC [$\because$ CD is the longest side of quadrilateral ABCD]

$\Rightarrow \quad \angle CBD > \angle BDC$ ...(2) [$\because$ Angle opposite to longest side is greater]

Adding (1) and (2), we get

$\angle ABD + \angle CBD > \angle ADB + \angle BDC$

$\Rightarrow \quad \angle ABC > \angle ADC$

$\Rightarrow \quad \angle B > \angle D$

Hence, proved.

20. Prove that in a triangle, other than an equilateral triangle, angle opposite the longest side is greater than $\frac{2}{3}$ of a right angle.

Sol. Given: A triangle ABC, other than and equilateral triangle.

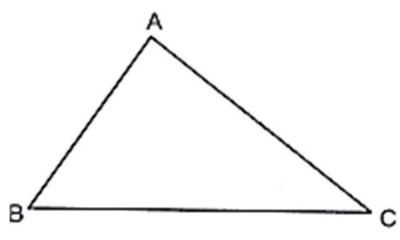
To prove: $\angle A > \frac{2}{3} rt.\angle$

Proof: In ΔABC, we have

BC > AB

$\Rightarrow \quad \angle A > \angle C$ ...(1) [$\because$ In a triangle, angle opposite to the longer side is larger]

BC > AC



$\Rightarrow \quad \angle A > \angle B$ ...(2) [$\because$ In a triangle, angle opposite to the longer side is larger]

Adding (1) and (2), we get

$A + \angle A > \angle B + \angle C$

$\Rightarrow \quad 2\angle A > \angle B + \angle C$

Now, adding $\angle A$ on both sides, we get

$2\angle A + \angle A > \angle A + \angle B + \angle C$

$\Rightarrow \quad 3\angle A > \angle A + \angle B + \angle C$

$\Rightarrow \quad 3\angle A > 180^0$ [Angle sum property of a triangle]

$\Rightarrow \quad \angle A > \frac{180^0}{3}$

$\Rightarrow \quad \angle A > \frac{2}{3} \times 90^0$

$\Rightarrow \quad \angle A > \frac{2}{3} rt.\angle$

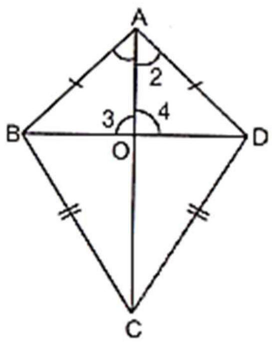
Hence, proved.

21. ABCD is quadrilateral such that AB = AD and CB = CD. Prove that AC is the perpendicular bisector of BD.

Sol. Given: A quadrilateral ABCD in which AB = AD and CB = CD.

To prove: AC is the perpendicular bisector of BD.

Proof: In ΔABC and ΔADC we have



AB = AD [Given]

BC = CD [Given]

AC = AC [Common side]

So, By SSS criterion of congruence, we have

$\Delta ABC \cong \Delta ADC$

$\therefore$ $\angle 1 = \angle 2$ [CPCT]

Now, in ΔAOB and ΔAOD we have

AB = AD [Given]

$\angle 1 = \angle 2$ [Proved above]

AO = AO [Common side]

So, By SAS criterion of congruence, we have

$\Delta AOB \cong \Delta AOD$

$\therefore$ $BO = DO$ [CPCT]

And $\angle 3 = \angle 4$ [CPCT]

But, $\angle 3 + \angle 4 = 180^0$ [Linear pair axiom]

$\Rightarrow$ $\angle 3 + \angle 3 = 180^0$ [$\because \angle 3 = \angle 4$]

$\Rightarrow$ $2\angle 3 = 180^0$

$\Rightarrow$ $\angle 3 = \frac{180^0}{2} = 90^0$

$\therefore$ Ac is perpendicular bisector of BC [$\because \angle 3 = 90^0$ and BO = DO]

Hence, proved.