Ex 9.1

Q1

We have,

$$\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}} = \sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}$$
$$= \frac{\sin\theta}{\cos\theta}$$
$$= \tan\theta = \text{RHS}$$

Q2

LHS,

$$\frac{\sin 2\theta}{1-\cos 2\theta} = \frac{2\sin\theta\cos\theta}{2\sin^2\theta}$$
$$= \frac{\cos\theta}{\sin\theta}$$
$$= \cot\theta = \text{RHS}$$

Q3

LHS,

$$\frac{\sin 2\theta}{1+\cos 2\theta} = \frac{2\sin\theta.\cos\theta}{2\cos^2\theta}$$
$$= \frac{\sin\theta}{\cos\theta}$$
$$= \tan\theta = \text{RHS}$$

Q4

LHS,

$$\sqrt{2+\sqrt{2+2\cos 4\theta}}$$

$$= \sqrt{2+\sqrt{2(1+\cos 4\theta)}}$$

$$= \sqrt{2+\sqrt{2.2\cos^2 2\theta}}$$

$$= \sqrt{2+2\cos 2\theta}$$

$$= \sqrt{2(1+\cos 2\theta)}$$

$$= \sqrt{2.2\cos^2\theta}$$

$$= 2\cos\theta = \text{RHS}$$

Q5

LHS,

$$\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}$$

$$=\frac{2\sin^2\theta+2\sin\theta.\cos\theta}{2\cos^2+2\sin\theta.\cos\theta}$$

$$=\frac{2\sin\theta(\sin\theta+\cos\theta)}{2\cos\theta(\cos\theta+\sin\theta)}$$

$$=\frac{\sin\theta}{\cos\theta}$$

$$=\tan\theta=\text{RHS}$$

Q6

LHS,

$$\frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta}$$

$$=\frac{\sin\theta+2\sin\theta.\cos\theta}{\cos\theta+(1+\cos 2\theta)}$$

$$=\frac{\sin\theta(1+2\cos\theta)}{\cos\theta+2\cos^2\theta}$$

$$=\frac{\sin\theta(1+2\cos\theta)}{\cos\theta(1+2\cos\theta)}$$

$$=\frac{\sin\theta}{\cos\theta}$$

$$=\tan\theta$$

$$=\text{RHS}$$

Q7

LHS,

$$\frac{\cos 2\theta}{1+\sin 2\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\sin^2 + \cos^2\theta + 2\sin\theta\cos\theta} \qquad \left[\begin{array}{l}\because \cos 2\theta = \cos^2\theta - \sin^2\theta \\ \& \ \sin^2\theta + \cos^2\theta = 1\end{array}\right]$$

$$= \frac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{(\cos\theta + \sin\theta)^2} \qquad \left[\because a^2 - b^2 = (a+b)(a-b)\right]$$

$$= \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

Dividing numerator and denomenator by $\cos\theta$

$$= \frac{1-\tan\theta}{1+\tan\theta}$$

$$= \tan\left(\pi/4 - \theta\right) = \text{RHS}$$

Note: $\tan\left(\pi/4 - \theta\right) = \dfrac{\tan \pi/4 - \tan\theta}{1 + \tan \pi/4 \tan\theta}$

$$= \frac{1-\tan\theta}{1+\tan\theta}$$

Q8

$$\frac{\cos\theta}{1-\sin\theta}$$

$$= \frac{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}}{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} \quad \left[\because \cos 2A = \cos^2 A - \sin^2 A \ \& \ \sin^2 A + \cos^2 A = 1\right]$$

$$= \frac{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)^2}$$

$$= \frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}$$

Dividing numerator and denominator by $\cos\frac{\theta}{2}$

$$= \frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}$$

$$= \tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \text{RHS}$$

Q9

LHS,

$$\cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}+\cos^2\frac{5\pi}{8}+\cos^2\frac{7\pi}{8}$$

$$= \cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}+\cos^2\left(\pi-\frac{3\pi}{8}\right)+\cos^2\left(\pi-\frac{\pi}{8}\right)$$

$$= \cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}+\cos^2\frac{3\pi}{8}+\cos^2\frac{\pi}{8}$$

$$= 2\left(\cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}\right)$$

$$= 2\left(\cos^2\frac{\pi}{8}+\cos^2\left(\frac{\pi}{2}-\frac{\pi}{8}\right)\right)$$

$$= 2\left(\cos^2\frac{\pi}{8}+\sin^2\frac{\pi}{8}\right)$$

$= 2$

$=$ RHS

Q10

LHS,

$$\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}+\sin^2\frac{5\pi}{8}+\sin^2\frac{7\pi}{8}$$

$$=\sin^2\frac{\pi}{8}+\sin^2\left(\frac{\pi}{2}-\frac{\pi}{8}\right)+\sin^2\frac{5\pi}{8}+\sin^2\left(\pi-\frac{\pi}{8}\right)$$

$$=\sin^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}+\sin^2\left(\pi-\frac{3\pi}{8}\right)+\sin^2\frac{\pi}{8}$$

$$=1+\sin^2\frac{3\pi}{8}+\sin^2\frac{\pi}{8}$$

$$=1+\sin^2\left(\frac{\pi}{2}-\frac{\pi}{8}\right)+\sin^2\frac{\pi}{8}$$

$$=1+\cos^2\frac{\pi}{8}+\sin^2\frac{\pi}{8}$$

$$=1+1$$

$$=2$$
$$=\text{RHS}$$

Q11

LHS,

$$(\cos\lambda+\cos\beta)^2+(\sin\lambda+\sin\beta)^2$$
$$=\cos^2\lambda+\cos^2\beta+2\cos\lambda\cos\beta+\sin^2\lambda+\sin^2\beta+2\sin\lambda+\sin\beta$$
$$=\left(\cos^2\lambda+\sin^2\lambda\right)+\left(\cos^2\beta+\sin^2\beta\right)+2\left(\cos\lambda\cos\beta+\sin\lambda\sin\beta\right)$$
$$=1+1+2\cos(\lambda-\beta)$$
$$=2+2\cos(\lambda-\beta)$$
$$=2\left(1+\cos(\lambda-\beta)\right)$$
$$=2.2\cos^2\left(\frac{\lambda-\beta}{2}\right)$$
$$=4\cos^2\left(\frac{\lambda-\beta}{2}\right)$$
$$=\text{RHS}$$

Q12

LHS,

$$\sin^2\left(\frac{\pi}{8}+\frac{A}{2}\right)-\sin^2\left(\frac{\pi}{8}-\frac{A}{2}\right)$$

$$=\left[\sin\left(\frac{\pi}{8}+\frac{A}{2}\right)+\sin\left(\frac{\pi}{8}-\frac{A}{2}\right)\right]\left[\sin\left(\frac{\pi}{8}+\frac{A}{2}\right)-\sin\left(\frac{\pi}{8}-\frac{A}{2}\right)\right]$$

$$=\left[\sin\frac{\pi}{8}.\ \cos\frac{A}{2}+\cos\frac{\pi}{8}.\ \sin\frac{A}{2}+\sin\frac{\pi}{8}.\ \cos\frac{A}{2}-\cos\frac{\pi}{8}.\ \sin\frac{A}{2}\right]$$

$$=\left[\sin\frac{\pi}{8}.\ \cos\frac{A}{2}+\cos\frac{\pi}{8}.\ \sin\frac{A}{2}-\sin\frac{\pi}{8}.\ \cos\frac{A}{2}+\cos\frac{\pi}{8}.\ \sin\frac{A}{2}\right]$$

$$=\left(2\sin\frac{\pi}{8}.\ \cos\frac{A}{2}\right)\left(2\cos\frac{\pi}{8}.\ \sin\frac{A}{2}\right)$$

$$=2\sin\frac{\pi}{8}.\ \cos\frac{\pi}{2}\ \ 2\sin\frac{A}{2}\ .\ \cos\frac{A}{2}$$

$$=\sin 2.\frac{\pi}{8}.\ \sin 2.\frac{A}{2}$$

$$=\sin\frac{\pi}{4}.\ \sin A$$

$$=\frac{1}{\sqrt{2}}\sin A$$

$= RHS$

Q13

LHS,

$$1+\cos^2 2\theta$$

$$=1+\left(\cos^2\theta-\sin^2\theta\right)^2 \qquad \left[\because \cos 2\theta=\cos^2\theta-\sin^2\theta\right]$$

$$=1+\cos^4\theta+\sin^4\theta-2\sin^2\theta.\ \cos^2\theta$$

$$=\left(\sin^2\theta+\cos^2\theta\right)^2+\cos^4\theta+\sin^4\theta-2\sin^2\theta\ \cos^2\theta \qquad \left|\because \sin^2\theta+\cos^2\theta=1\right|$$

$$=\sin^4\theta+\cos^4\theta+2\sin^2\theta\cos^2\theta+\cos^4\theta+\sin^4\theta-2\sin^2\theta.\cos^2\theta$$

$$=2\left(\cos^4\theta+\sin^4\theta\right)$$

$= RHS$

Q14

$$\cos^3 2\theta + 3\cos 2\theta = 4\left(\cos^6\theta - \sin^6\theta\right)$$

$$\begin{aligned}
\text{RHS} &= 4\left[\left(\cos^2\theta\right)^3 - \left(\sin^2\theta\right)^3\right] \\
&= 4\left(\cos^2\theta - \sin^2\theta\right)\left[\cos^4\theta + \sin^4\theta + \sin^2\theta\cos^2\theta\right] \\
&= 4\cos^2\theta\left[\left(\cos^2\theta - \sin^2\theta\right)^2 + 2\sin^2\theta\cos^2\theta + \sin^2\theta\cos^2\theta\right] \\
&= 4\cos^2\theta\left[\cos^2 2\theta + 3\sin^2\theta\cos^2\theta\right] \\
&= 4\cos^2\theta\left[\cos^2 2\theta + 3\left(\frac{1-\cos^2\theta}{2}\right)\left(\frac{1+\cos^2\theta}{2}\right)\right] \\
&= 4\cos^2\theta\left[\cos^2 2\theta + \frac{3}{4}\left(1-\cos^2 2\theta\right)\right] \\
&= \cos^2\theta\left[4\cos^2 2\theta + 3 - 3\cos^2 2\theta\right] \\
&= \cos^2\theta\left[\cos^2 2\theta + 3\right] \\
&= \cos^3 2\theta + 3\cos^2\theta \\
&= \text{LHS}
\end{aligned}$$

LHS = RHS

Q15

LHS $= (\sin 3A + \sin A)\sin A + (\cos 3A - \cos A)\cos A$

$\Rightarrow \quad 2\sin 2A.\cos A.\sin A + (-2\sin 2A.\sin A\cos A)$

$$\left[\begin{aligned} &\because \sin C + \sin D = 2\sin\frac{C+D}{2}.\cos\frac{C-D}{2} \\ &\cos C - \cos D = -2\sin\frac{C+D}{2}.\sin\frac{C-D}{2} \end{aligned}\right]$$

$\Rightarrow \quad 2\sin 2A\cos A.\sin A - 2\sin 2A\cos A.\sin A$

$\Rightarrow \quad 0 = \text{RHS}$

Q16

LHS $= \cos^2\left(\frac{\pi}{4} - \theta\right) - \sin^2\left(\frac{\pi}{4} - \theta\right)$

$= \cos 2\left(\frac{\pi}{4} - \theta\right) \qquad \left[\because \cos 2\theta = \cos^2\theta - \sin^2\theta\right]$

$= \cos\left(\frac{\pi}{2} - 2\theta\right) \qquad \left[\because \cos\left(\pi/2 - \theta\right) = \sin\theta\right]$

$= \sin 2\theta$

$=$ RHS

Q17

$$
\begin{aligned}
\text{LHS} &= \cos 4A \\
&= \cos 2\cdot 2A \\
&= 2\cos^2 2A - 1 \quad \left[\because \cos 2\theta = 2\cos^2\theta - 1\right] \\
&= 2\left(2\cos^2 A - 1\right)^2 - 1 \\
&= 2\left(4\cos^4 A - 4\cos^2 A + 1\right) - 1 \\
&= 8\cos^4 A - 8\cos^2 A + 1 \\
&= 1 - 8\cos^2 A + 8\cos^4 A \\
&= \text{RHS}
\end{aligned}
$$

Q18

$$
\begin{aligned}
\text{LHS} &= \sin 4A \\
&= \sin 2.2A \\
&= 2\sin 2A\cos 2A \\
&= 2\left(2\sin A\cos A\right).\left(\cos^2 A - \sin^2 A\right) \\
&= 4\sin A\cos^3 A - 4\sin^3 A\cos A \\
&= \text{RHS}
\end{aligned}
$$

Q19

$$\text{LHS} = 3\left(\sin x - \cos x\right)^4 + 6\left(\sin x + \cos x\right)^2 + 4\left(\sin^6 x + \cos^6 x\right)$$

$$= 3\left[\sin^4 x - 4\sin^3 x\cos x + 6\sin^2 x\cos^2 x - 4\sin x\cos^3 x + \cos^4 x\right]$$
$$+6\left[\sin^2 x + 2\sin x\cos x + \cos^2 x\right] + 4\left(\sin^6 x + \cos^6 x\right)$$
$$\left[\because (a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4 \text{ by binomial expansion}\right]$$

$$= 3\left[\sin^4 x + \cos^4 x - 4\sin x\cos x\left(\sin^2 x + \cos^2 x\right) + 6\sin^2 x\cos^2 x\right]$$

$$+6\left[1 + 2\sin x\cos x\right] + 4\left[\left(\cos^2 x + \sin^2 x\right)\left(\cos^4 x - \cos^2 x\sin^2 x + \sin^4 x\right)\right]$$
$$\left[\because a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)\right]$$

$$= 7\left[\sin^4 x + \cos^4 x\right] + 18\sin^2 x\cos^2 x - 4\sin^2 x\cos^2 x + 6$$

$$= 7\left[\sin^4 x + \cos^4 x + 2\sin^2 x\cos^2 x\right] + 6$$

$$= 7\left[\sin^2 x + \cos^2 x\right]^2 + 6$$

$$
\begin{aligned}
&= 7 + 6 \\
&= 13 \\
&= RHS
\end{aligned}
$$

Q20

$$\text{L.H.S} = 2\left(\sin^6 x + \cos^6 x\right) - 3\left(\sin^4 x + \cos^4 x\right) + 1$$

$$= 2\left(\left(\sin^2 x\right)^3 + \left(\cos^2 x\right)^3\right) - 3\left(\sin^4 x + \cos^4 x\right) + 1 \qquad \left[\because a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)\right]$$

$$= 2\left[\left(\sin^2 x + \cos^2 x\right)\left(\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x\right)\right] - 3\left(\sin^4 x + \cos^4 x\right) + 1$$

$$= -\left[\sin^4 x + \cos^4 x + 2\sin^2 x \cos^2 x\right] + 1$$

$$= -\left[\sin^2 x + \cos^2 x\right] + 1$$

$$= -1 + 1$$

$$= 0$$

$$= \text{RHS}$$

Q21

$$\text{L.H.S} = \cos^6 A - \sin^6 A$$

$$= \left(\cos^2 A\right)^3 - \left(\sin^2 A\right)^3$$

$$= \left(\cos^2 A - \sin^2 A\right)\left(\cos^4 A + \sin^2 A . \cos^2 A + \sin^4 A\right) \quad \left[\because a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)\right]$$

$$= \cos 2A\left(\cos^4 A + 2\sin^2 A \ \cos^2 A + \sin^4 A - \sin^2 A \cos^2 A\right)$$

$$\left[\because \cos^2 A - \sin^2 A = \cos^2 A \ \& \ \text{Adding and subtracling } \sin^2 A \cos^2 A\right]$$

$$= \cos 2A\left[\left(\sin^2 A + \cos^2 A\right)^2 - \frac{4}{4}\sin^2 A \ \cos^2 A\right]$$

$$= \cos 2A\left[1 - \frac{1}{4}\left(2\sin A \cos A\right)^2\right]$$

$$= \cos 2A\left[1 - \frac{1}{4}\sin^2 2A\right]$$

$$= \text{RHS}$$

Q22

$$\text{L.H.S} = \tan\left(\frac{\pi}{4}+\theta\right)+\tan\left(\frac{\pi}{4}-\theta\right)$$

$$= \frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\tan\theta}+\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}$$

$$= \frac{1+\tan\theta}{1-\tan\theta}+\frac{1-\tan\theta}{1+\tan\theta} \qquad \left[\because \tan\frac{\pi}{4}=1\right]$$

$$= \frac{\left(1+\tan^2\theta+2\tan\theta\right)+\left(1+\tan^2\theta-2\tan\theta\right)}{\left(1-\tan\theta\right)\left(1+\tan\theta\right)}$$

$$= \frac{2\left(1+\tan^2\theta\right)}{1-\tan^2\theta}$$

$$= \frac{2\sec^2\theta}{1-\frac{\sin^2\theta}{\cos^2\theta}} \qquad \left[\because \sec^2\theta = 1+\tan^2\theta\right]$$

$$= \frac{2\sec^2\theta.\cos^2\theta}{\cos^2\theta-\sin^2\theta} \qquad \left[\because \sec = \frac{1}{\cos\theta}\right]$$

$$= \frac{2}{\cos 2\theta}$$

$$= 2\sec 2\theta$$

$$= \text{RHS}$$

Q23

$$L.H.S = cot^2 A - ta^2 A$$

$$= \frac{\cos^2 A}{\sin^2 A} - \frac{\sin^2 A}{\cos^2 A}$$

$$= \frac{\left(\cos^2 A\right)^2 - \left(\sin^2 A\right)^2}{\sin^2 A \ \cos^2 A}$$

$$= \frac{\left(\cos^2 A + \sin^2 A\right)\left(\cos^2 A - \sin^2 A\right)}{\left(\sin A \ \cos A\right)^2} \quad \left[\because \ a^2 - b^2 - = (a+b)(a-b)\right]$$

$$= \frac{\cos 2A}{\frac{1}{4}\left(2 \sin A \ \cos A\right)^2} \qquad \left[\because \cos 2A = \cos^2 A - \sin^2 A\right]$$

$$= \frac{4 \cos 2A}{\sin^2 2A}$$

$$= \frac{4 \cos 2A}{\sin 2A} \cdot \frac{1}{\sin 2A} \qquad \left[\because \cos ec\,\theta = \frac{1}{\sin \theta}\right]$$

$$= 4 \cot 2A . \cos ec 2A$$

$$= RHS$$

Q24

$$\cos 4\theta - \cos 4\alpha = 2\cos^2 2\theta - 2\cos^2 2\alpha$$

$$= 2(\cos 2\theta + \cos 2\alpha)(\cos 2\theta - \cos 2\alpha)$$

$$= 2(2\cos^2\theta - 1 + 1 - 2\sin^2\alpha)(2\cos^2\theta - 1 - 2\cos^2\alpha + 1)$$

$$= 8(\cos^2\theta - \sin^2\alpha)(\cos^2\theta - \cos^2\alpha)$$

$$= 8(\cos\theta - \sin\alpha)(\cos\theta + \sin\alpha)(\cos\theta - \cos\alpha)(\cos\theta + \cos\alpha)$$

Q25

$$\sin 3x + \sin 2x - \sin x$$
$$= (\sin 3x - \sin x) + \sin 2x$$
$$= 2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right) + 2\sin x\cos x \ldots\ldots\ldots \left[\begin{array}{l}\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\\ \sin 2x = 2\sin x\cos x\end{array}\right.$$
$$= 2\cos(2x)\sin(x) + 2\sin x\cos x$$
$$= 2\sin x\left[\cos(2x) + \cos x\right]$$
$$= 2\sin x\left[2\cos\left(\frac{2x+x}{2}\right) + \cos\left(\frac{2x-x}{2}\right)\right] \ldots\ldots \left[\because \cos A + \cos B = 2\cos\left(\frac{A-B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$
$$= 4\sin x\cos\frac{3x}{2}\cos\frac{x}{2}$$

Q26

$$\tan 82\frac{1^\circ}{2} = \tan\left(90 - 7\frac{1}{2}\right)^\circ$$
$$= \cot 7\frac{1^\circ}{2}$$
$$= \cot A \qquad \text{If } A = 7\frac{1^\circ}{2}$$

Now

$$\cot A = \frac{\cos A}{\sin A}$$
$$= \frac{2\cos^2 A}{2\sin A\cos A}$$
$$= \frac{1+\cos^2 A}{\sin^2 A}$$
$$\cot A = \frac{1+\cos 15}{\sin 15}$$
$$= \frac{1+\cos(45-30)}{\sin 15}$$
$$= \frac{1+\left(\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\times\frac{1}{2}\right)}{\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\times\frac{1}{2}}$$
$$= \frac{2\sqrt{2}+\left(\sqrt{3}+1\right)}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$$
$$= \frac{2\sqrt{2}\left(\sqrt{3}+1\right)+\left(\sqrt{3}+1\right)^2}{3-1}$$
$$= \frac{2\sqrt{6}+2\sqrt{2}+4+2\sqrt{3}}{2}$$
$$\cot A = \sqrt{6}+\sqrt{2}+2+\sqrt{3} \text{ ------- (1)}$$
$$= \sqrt{2}+2+\sqrt{6}+\sqrt{3}$$
$$= \sqrt{2}\left(1+\sqrt{2}\right)+\sqrt{3}\left(\sqrt{2}+1\right)$$
$$\cot A = \left(\sqrt{2}+1\right)\left(\sqrt{2}+\sqrt{3}\right) \text{ ------- (2)}$$

From equation (1) and (2)

$$\tan 82\frac{1^\circ}{2} = \cot 7\frac{1^\circ}{2} = \sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$$
$$= \left(\sqrt{2}+1\right)\left(\sqrt{2}+\sqrt{3}\right)$$

Q27

We know that,

$$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$$

Put $A = 45°$,

$$\sin 22\frac{1°}{2} = \sqrt{\frac{1-\cos 45°}{2}} \qquad \left\{\text{since } \sin 22\frac{1}{2}, \text{ is positive}\right\}$$

$$= \sqrt{\frac{1-\frac{1}{2}}{2}}$$

$$\sin 22\frac{1°}{2} = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$$

And

$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$$

Put $A = 45°$

$$\cos 22\frac{1°}{2} = \sqrt{\frac{1+\cos 45°}{2}}$$

$$= \sqrt{\frac{1+\frac{1}{2}}{2}}$$

$$\cos 22\frac{1°}{2} = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$$

Now,

$$\cot 22\frac{1°}{2} = \frac{\cos 22\frac{1°}{2}}{\sin 22\frac{1°}{2}}$$

Q28(i)

Since $\cos x = -\frac{3}{5} = \frac{b}{h}$

$\Rightarrow \quad b = 3, h = 5$

$\Rightarrow \quad P = 4$

Now, x lies on third quad.

$\therefore \sin 2x = 2 \sin x . \cos x$

$= 2.\left(\frac{-4}{5}\right).\left(\frac{-3}{5}\right) = \frac{24}{25}$

$\because \pi < x < 3\pi/2 \Rightarrow \quad \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$

Which means $x/2$ lies in second quadrant

so, $\cos x/2 = \sqrt{\frac{1+\cos x}{2}}$ $\qquad [\because 1 + \cos 2\theta = 2\cos^2\theta$

$= \sqrt{\frac{1-3/5}{2}} = \frac{-1}{\sqrt{5}}$ $\qquad$ (−ve sign because of second quad. where $\cos D$ is −ve)

Also,

$\sin x/2 = \frac{\sin x}{2\cos x/2}$ $\qquad [\because \sin 2A = 2 \sin A \cos A]$

$= \left(\frac{-4}{5} \Big/ 2\left(\frac{-1}{\sqrt{5}}\right)\right)$

$= \frac{2}{\sqrt{5}}$

Q28(ii)

$\because x$ lies in II^{nd} quadrant.

$\Rightarrow \quad \frac{\pi}{2} < x < \pi$

$\Rightarrow \quad \pi < 2x < 2\pi \quad \Rightarrow \quad 2x$ lies in I^{st} quad.

Also, $\cos x = \frac{-3}{5} = \frac{b}{h} \qquad \Rightarrow \quad b = 3$

$h = 5$

$\Rightarrow \quad p = 4.$

so, $\sin x = \frac{p}{h} = \frac{4}{5}$

$\therefore \quad \sin 2x = 2 \sin x \cos x$

$= 2.\frac{4}{5}.\left(\frac{-3}{5}\right) = \frac{-24}{25}$

$\sin {x}/{2} = \frac{\sin x}{2 \cos {x}/{2}}$ or $\sqrt{\frac{1 - \cos x}{2}}$

$= \sqrt{\frac{1 - \left(1 - {3}/{5}\right)}{2}}$

$= \frac{2}{\sqrt{5}}$

Q29

$\because x$ lies in II^{nd} quad.

$\Rightarrow \quad \frac{\pi}{2} < x < \pi$

$\Rightarrow \quad \frac{\pi}{4} < x\,\frac{x}{2} < \frac{\pi}{2}$

Which means $\frac{x}{2}$ lies in first quad.

Now, $\sin x = \frac{\sqrt{5}}{3} = \frac{p}{h} \Rightarrow \quad p = \sqrt{5} \quad \Rightarrow b = 2$

$h = 3$

so, $\cos x = \frac{b}{h} = \frac{-2}{3} \quad \left(-\text{ve due to II}^{nd} \text{ quad}\right)$

Thus,

$$\cos\frac{x}{2} = \sqrt{1+\cos x \; 2} = \sqrt{\frac{1-\frac{2}{3}}{2}} = \frac{1}{\sqrt{6}}$$

$$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1+\frac{2}{3}}{2}} = \sqrt{\frac{5}{6}}$$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\sqrt{\frac{5}{6}}}{\frac{1}{\sqrt{6}}} = \sqrt{5}$$

Q30(i)

Since x lies in II^{nd} quadrant

$$\Rightarrow \frac{\pi}{2} < x < \pi$$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}, \text{ which means } \frac{x}{2} \text{ lies in } \text{I}^{st} \text{ quad.}$$

Now,

$$\sin x = \frac{1}{4} = \frac{p}{h} \Rightarrow p = 1 \qquad \Rightarrow b = \sqrt{15}$$
$$h = 4$$

so, $\cos x = \dfrac{b}{h} = \dfrac{-\sqrt{15}}{4}$ $\qquad$ $\left(-\text{ve due to } \text{II}^{nd} \text{ quad}\right)$

Thus,

$$\cos \frac{x}{2} = \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1-\frac{\sqrt{15}}{14}}{2}} = \frac{\sqrt{4-\sqrt{15}}}{8}$$

$$\sin \frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1+\frac{\sqrt{15}}{4}}{2}} = \sqrt{\frac{4+\sqrt{15}}{8}}$$

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\sqrt{\frac{4+\sqrt{15}}{8}}}{\sqrt{\frac{4-\sqrt{15}}{8}}} = \sqrt{\frac{4+\sqrt{15}}{4-\sqrt{15}}}$$

$$= \sqrt{\frac{\left(4+\sqrt{15}\right)\left(4+\sqrt{15}\right)}{\left(4-\sqrt{15}\right)\left(4+\sqrt{15}\right)}}$$

$$= 4+\sqrt{15}$$

Q30(ii)

Since θ in acute, so $0 \le 2\theta < \pi$

Now, $\cos\theta = \frac{4}{5} = \frac{b}{h} \Rightarrow b = 4 \qquad \Rightarrow p = 3$

$h = 5$

$\therefore \sin\theta = \frac{p}{h} = \frac{3}{5}$

$\tan\theta = \frac{p}{b} = \frac{3}{4}$

so, $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2.\,{}^{3}\!/_{4}}{1-\left(\frac{3}{4}\right)^2}$

$= \frac{{}^{6}\!/_{4}}{{}^{7}\!/_{16}} = \frac{24}{7}$

Q30(iii)

$\sin\theta = \frac{4}{5} = \frac{p}{h} \Rightarrow p = 4 \qquad \Rightarrow b = 3$

$h = 5$

$\therefore \cos\theta = \frac{b}{h} = \frac{3}{5}$

Now, $\sin\theta = 2\sin\theta.\cos\theta = 2.\frac{4}{5}.\frac{3}{5} = \frac{24}{25}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = \frac{-7}{25}$

so, $\sin 4\theta = \sin 2.2\theta = 2\sin 2\theta.\cos 2\theta$

$= 2.\frac{24}{25}.\left(\frac{-7}{25}\right)$

$= \frac{-336}{625}$

Q31

$$\sqrt{\frac{a+b}{a-b}}+\sqrt{\frac{a-b}{a+b}}=\frac{(a+b)+(a-b)}{\sqrt{(a-b)(a+b)}}$$

$$=\frac{2a}{\sqrt{a^2-b^2}}$$

$$=\frac{2}{\sqrt{1-\left(\frac{b}{a}\right)^2}}$$

$$=\frac{2}{\sqrt{1-\tan^2 x}}\cdots\cdots\cdots\cdots\left[\because \tan x=\frac{b}{a}\right]$$

$$=\frac{2\cos x}{\sqrt{\cos^2 x-\sin^2 x}}$$

$$=\frac{2\cos x}{\sqrt{\cos 2x}}$$

Q32

We have,

$$\tan A=\frac{1}{7} \quad \& \quad \tan B=\frac{1}{3}$$

$$\therefore \cos 2A=\frac{1-\tan^2 A}{1-\tan^2 A}=\frac{1-\left(\frac{1}{7}\right)^2}{1+\left(\frac{1}{7}\right)^2}=\frac{\frac{48}{49}}{\frac{50}{49}}$$

$$=\frac{48}{50}=\frac{24}{25}\;\cdots\cdots\cdots(A)$$

Also,

$$\sin 4B=\sin 2.2B$$

$$=2\sin 2B.\cos 2B$$

$$=2.\left(\frac{2\tan B}{1+\tan^2 B}\right)\left(\frac{1-\tan^2 B}{1+\tan^2 B}\right)$$

$$=4.\left(\frac{\frac{1}{3}}{1+\frac{1}{9}}\right)\left(\frac{1-\frac{1}{9}}{1+\frac{1}{9}}\right)$$

$$=\frac{4.\frac{1}{3}.\frac{8}{9}}{\frac{10}{9}\times\frac{10}{9}}$$

$$=\frac{32\times 3}{100}$$

$$=\frac{8\times 3}{25}=\frac{24}{25}\;\cdots\cdots(B)$$

form (A) & (B)

$$\cos 2A=\sin 4B$$

Q33

LHS,

$$\cos 7^\circ \ \cos 14^\circ \cos 28^\circ \cos 56^\circ$$

Divide and multiply by $2\sin 7^\circ$, weget

$$\frac{1}{2\sin 7^\circ}.2\sin 7^\circ.\cos 7^\circ.\ \cos 14^\circ.\ \ \cos 28^\circ.\cos 56^\circ$$

$$=\frac{2\sin 14^\circ}{2.2\sin 7^\circ}.\cos 14^\circ.\ \cos 28^\circ.\ \cos 56^\circ \qquad [\because\ 2\sin A\cos A = \sin 2A]$$

$$=\frac{2\sin 28^\circ}{2.4\sin 7^\circ}.\cos 28^\circ.\ \cos 56^\circ$$

$$=\frac{2\sin 56^\circ}{2.8\sin 7^\circ}.\ \cos 56^\circ$$

$$=\frac{\sin 112^\circ}{16\sin 7^\circ}$$

$$=\frac{\sin\left(180^\circ - 68\right)}{16\sin\left(90^\circ - 83^\circ\right)}$$

$$=\frac{\sin 68^\circ}{16\cos 83^\circ}$$

$= \text{RHS}$

Q34

sin(a+b)= sin(a)cos(b)+sin(b)cos(a)   (1)
for a=b, sin(2a)= 2 sin(a) cos(a)   (2)

let a= 16 pi/15   (3)
(so 2a = 32 pi/15)

then using (3) in (2), we have
sin(2a)= 2 sin(a) cos(a)
= 2 (2 sin(a/2) cos(a/2)) cos(a)
= 2 (2 (2 sin(a/4) cos(a/4)) cos(a/2)) cos(a)
= 2 (2 (2 (2 sin(a/8) cos(a/8)) cos(a/4)) cos(a/2)) cos(a)
= 16 sin(a/8) (cos(a/8) cos(a/4) cos(a/2) cos(a))

now note sin(2a) = sin(2 pi/15) and sin(a/8)= sin(2 pi/15)
so,
cos(a/8) cos(a/4) cos(a/2) cos(a)= 1/16
or, replacing a with 16 pi/15,
cos(2pi/15)*cos(4pi/15)*cos(8pi/15)*cos(16pi/15)=1/16

Q35

$$\cos\frac{\pi}{5}\cos\frac{2\pi}{5}\cos\frac{4\pi}{5}\cos\frac{8\pi}{5} = \frac{\sin\frac{2^4\pi}{5}}{2^4\sin\frac{\pi}{5}}$$

$$\left[\because \cos A\cos 2A\cos 2^2A\cos 2^3A\ldots\ldots\cos 2^{n-1}A = \frac{\sin 2^nA}{2^n\sin A}\right]$$

$$= \frac{\sin\frac{16\pi}{5}}{16\sin\frac{\pi}{5}}$$

$$= \frac{\sin\left(3\pi+\frac{\pi}{5}\right)}{16\sin\frac{\pi}{5}}$$

$$= \frac{1\left\{-\sin\left(\frac{\pi}{5}\right)\right\}}{16\sin\frac{\pi}{5}}$$

$$= \frac{-1}{16}$$

Q36

LHS $= \cos\frac{\pi}{65}.\cos\frac{2\pi}{65}.\cos\frac{4\pi}{65}.\cos\frac{8\pi}{15}\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}$

Divide and Multiply by $2\sin\frac{\pi}{65}$, we get

$$= \frac{2.\sin\frac{\pi}{65}}{2\sin\frac{\pi}{65}}.\cos\frac{\pi}{65}.\cos\frac{2\pi}{65}.\cos\frac{4\pi}{65}.\cos\frac{8\pi}{65}.\cos\frac{16\pi}{65}.\cos\frac{32\pi}{65}$$

$$= \frac{2.\sin\frac{2\pi}{65}}{2.2\sin\frac{\pi}{65}}.\cos\frac{2\pi}{65}.\cos\frac{4\pi}{65}.\cos\frac{8\pi}{65}.\cos\frac{16\pi}{65}.\cos\frac{32\pi}{65}$$

$$= \frac{2.\sin\frac{4\pi}{65}}{2.4\sin\frac{\pi}{65}}.\cos\frac{4\pi}{65}.\cos\frac{8\pi}{65}.\cos\frac{16\pi}{65}.\cos\frac{32\pi}{65}$$

$$= \frac{2.\sin\frac{8\pi}{65}}{2.8\sin\frac{\pi}{65}}\cos\frac{8\pi}{65}.\cos\frac{16\pi}{65}.\cos\frac{32\pi}{65}$$

$$= \frac{2.\sin\frac{16\pi}{65}}{2.16\sin\frac{\pi}{65}}\cos\frac{16\pi}{65}.\cos\frac{32\pi}{65}$$

$$= \frac{2.\sin\frac{32\pi}{65}}{2.32\sin\frac{\pi}{65}}\cos\frac{32\pi}{65}$$

$$= \frac{\sin\frac{64\pi}{65}}{64.\sin\frac{\pi}{65}}$$

$$= \frac{1}{64}\cdot\frac{\sin\left(\pi - \frac{\pi}{65}\right)}{\sin\frac{\pi}{65}}$$

$$= \frac{1}{64}\frac{\sin\frac{\pi}{65}}{\sin\pi/65}$$

$$= \frac{1}{64}$$

$= \text{RHS}$

Q37

We have, $2\tan\alpha = 3\tan\beta$

$$\Rightarrow \frac{\tan\alpha}{\tan\beta} = \frac{3}{2}$$

Let $\tan\alpha = 3K$ and $\tan\beta = 2K$

Now, $\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} = \frac{3K - 2K}{1 + 3K.2K} = \frac{K}{1 + 6K^2} \quad \ldots(A)$

Also,

$$\frac{\sin 2\beta}{5 - \cos 2\beta} = \frac{\frac{2\tan\beta}{1 + \tan^2\beta}}{5 - \left(\frac{1 - \tan^2\beta}{1 + \tan^2\beta}\right)}$$

$$= \frac{\frac{2.2K}{1 + 4K^2}}{5 - \left(\frac{1 - 4K^2}{1 + 4K^2}\right)}$$

$$= \frac{4K}{5 + 20K^2 - 1 + 4K^2}$$

$$= \frac{4K}{4 + 24K^2} = \frac{K}{1 + 6K^2} \quad \ldots(B)$$

form (A) & (B)

$$\tan(\alpha - \beta) = \frac{\sin 2\beta}{5 - \cos 2\beta}$$

Q38(i)

We have,

$\sin\alpha + \sin\beta = a$ & $\cos\alpha + \cos\beta = b$ ....(A)

Squaring and adding, we get

$$\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta + \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta = a^2 + b^2$$

$\Rightarrow \quad 1 + 1 + 2(\sin\alpha\sin\beta + \cos\alpha\cos\beta) = a^2 + b^2$

$\Rightarrow \quad 2(\sin\alpha\sin\beta + \cos\alpha\cos\beta) = a^2 + b^2 - 2$

$\therefore \quad 2\cos(\alpha - \beta) = a^2 + b^2 - 2$

Thus, $\cos(\alpha - \beta) = \dfrac{a^2 + b^2 - 2}{2}$ ....(ii)

Again,

$\sin\alpha + \sin\beta = a \quad \Rightarrow \quad 2\sin\dfrac{\alpha+\beta}{2}.\ \cos\dfrac{\alpha-\beta}{2} = a$

$\cos\alpha + \cos\beta = b \quad \Rightarrow \quad 2\cos\dfrac{\alpha+\beta}{2}.\ \cos\dfrac{\alpha-\beta}{2} = b$

$\Rightarrow \quad \tan\dfrac{\alpha+\beta}{2} = \dfrac{a}{b}$ ....(B)

Now,

$$\sin(\alpha+\beta) = \frac{2\tan\frac{\alpha+\beta}{2}}{1 + \tan^2\left(\frac{\alpha+\beta}{2}\right)}$$

$$= \frac{2\,a/b}{1 + \frac{a^2}{b^2}} = \frac{2ab}{a^2 + b^2}$$

Thus,

$$\sin(\alpha+\beta) = \frac{2ab}{a^2 + b^2}$$

Q38(ii)

We have,

$\sin\alpha + \sin\beta = a$ & $\cos\alpha + \cos\beta = b$

Squaring and adding, we get

$$\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta + \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta = a^2 + b^2$$

$$\Rightarrow \quad 1 + 1 + 2(\sin\alpha\sin\beta + \cos\alpha\cos\beta) = a^2 + b^2$$

$$\Rightarrow \quad 2(\sin\alpha\sin\beta + \cos\alpha\cos\beta) = a^2 + b^2 - 2$$

$$\therefore \quad 2\cos(\alpha - \beta) = a^2 + b^2 - 2$$

Thus, $\cos(\alpha - \beta) = \dfrac{a^2 + b^2 - 2}{2}$

Q39

We have,

$$2\tan\alpha/2 = \tan\beta/2$$

$$\Rightarrow \quad \frac{\tan\alpha/2}{\tan\beta/2} = 1/2$$

Let $\tan\alpha/2 = K$ and $\tan\beta/2 = 2K$

Then,

$$\cos\alpha = \frac{1-\tan^2\alpha/2}{1+\tan^2\alpha/2} = \frac{1-K^2}{1+K^2} \quad \ldots\ldots\ldots\ldots\ldots(A)$$

Also,

$$\frac{3+5\cos\beta}{5+3\cos\beta} = \frac{3+5\left(\dfrac{1-\tan^2\beta/2}{1+\tan^2\beta/2}\right)}{5+3\left(\dfrac{1-\tan^2\beta/2}{1+\tan^2\beta/2}\right)}$$

$$= \frac{3+5\left(\dfrac{1-4K^2}{1+4K^2}\right)}{5+3\left(\dfrac{1-4K^2}{1+4K^2}\right)}$$

$$= \frac{8-8K^2}{8+8K^2} = \frac{1-K^2}{1+K^2} \quad \ldots\ldots\ldots\ldots\ldots(B)$$

form (A) & (B)

$$\cos\alpha = \frac{3+5\cos\beta}{5+3\cos\beta}$$

Q40

We have,

$$\cos\theta = \frac{\cos\alpha + \cos\beta}{1+\cos\alpha.\cos\beta}$$

Now,

$$\cos\theta = \frac{1-\tan^2 \theta/2}{1+\tan^2 \theta/2}$$

$$\Rightarrow \quad \frac{1-\tan^2 \theta/2}{1+\tan^2 \theta/2} = \frac{\cos\alpha + \cos\beta}{1+\cos\alpha\cos\beta}$$

by componende and dividendo, we get

$$\frac{\left(1-\tan^2 \theta/2\right)+\left(1+\tan^2 \theta/2\right)}{\left(1-\tan^2 \theta/2\right)-\left(1+\tan^2 \theta/2\right)} = \frac{1+\cos\alpha\cos\beta+\cos\alpha+\cos\beta}{-\left(1+\cos\alpha\cos\beta-\cos\alpha-\cos\beta\right)}$$

$$\Rightarrow \quad \frac{2}{2\tan^2 \theta/2} = \frac{(1+\cos\alpha)(1+\cos\beta)}{(1-\cos\alpha)(1-\cos\beta)}$$

$$\Rightarrow \quad \tan^2 \theta/2 = \frac{(1-\cos\alpha)(1-\cos\beta)}{(1+\cos\alpha)(1+\cos\beta)}$$

$$= \frac{2\sin^2 \alpha/2 . 2\sin^2 \beta/2}{2\cos^2 \alpha/2 . 2\cos^2 \beta/2}$$

$$\Rightarrow \quad \tan \theta/2 = \pm\tan \alpha/2 . \tan \beta/2$$

Q41

We have,

$$\sec(\theta+\alpha)+\sec(\theta-\alpha)=2\sec\theta.$$

$$\Rightarrow \quad \frac{1}{\cos\theta.\cos\alpha-\sin\theta\sin\alpha}+\frac{1}{\cos\theta.\cos\alpha+\sin\theta\sin\alpha}=\frac{2}{\cos\theta}$$

$$\Rightarrow \quad \frac{2\cos\theta\cos\alpha}{\cos^2\theta\cos^2\alpha-\sin^2\theta\sin^2\alpha}=\frac{2}{\cos\theta}$$

$$\Rightarrow \quad \frac{\cos\theta\cos\alpha}{\cos^2\theta\cos^2\alpha-\left(1-\cos^2\theta\right)\sin^2\alpha}=\frac{1}{\cos\theta}$$

$$\Rightarrow \quad \cos^2\theta\cos\alpha=\cos^2\theta\left(\cos^2\alpha+\sin^2\alpha\right)-\sin^2\alpha$$

$$\Rightarrow \quad \cos^2\theta\left(1-\cos\alpha\right)=\sin^2\alpha$$

$$\Rightarrow \quad \cos^2\theta=\frac{\sin^2\alpha}{2\sin^2\alpha/2}$$

$$=\frac{4\sin^2\alpha/2.\cos^2\alpha/2}{2\sin^2\alpha/2}$$

$$\Rightarrow \quad \cos\theta=\pm\sqrt{2}\ \cos\alpha/2$$

Q42

We have,

$$\cos\alpha + \cos\beta = \frac{1}{3} \text{ and } \sin\alpha + \sin\beta = \frac{1}{4}$$

Squaring and adding, we get

$$\left(\cos^2\alpha + \cos^2\beta + 2\cos\alpha\ \cos\beta\right) + \left(\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta\right) = \frac{1}{9} + \frac{1}{16}$$

$$\Rightarrow \quad 1 + 1 + 2\left(\cos\alpha\ \cos\beta + \sin\alpha\ \sin\beta\right) = \frac{25}{144}$$

$$\Rightarrow \quad 2\cos\left(\alpha - \beta\right) = \frac{25}{144} - 2 = \frac{-263}{144}$$

$$\Rightarrow \quad \cos\left(\alpha - \beta\right) = \frac{-263}{288}$$

Now,

$$\cos\left(\frac{\alpha - \beta}{2}\right) = \sqrt{\frac{1 + \cos\left(\alpha - \beta\right)}{2}}$$

$$= \sqrt{\frac{1 - \frac{263}{288}}{2}} = \sqrt{\frac{25}{576}}$$

$$= \pm\frac{5}{24}$$

$$\therefore \quad \cos\left(\frac{\alpha - \beta}{2}\right) = \pm\frac{5}{24}$$

Q43

We have,

$$\sin\alpha = \frac{4}{5} \quad \& \quad \cos\beta = \frac{5}{13} \qquad \Rightarrow \cos\alpha = \frac{3}{5} \ \& \ \sin\beta = \frac{12}{13}$$

$$\therefore \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha.\sin\beta$$

$$= \frac{3}{5}.\frac{5}{13} + \frac{4}{5}.\frac{12}{13}$$

$$= \frac{15}{65} + \frac{48}{65} = \frac{63}{65}$$

Now,

$$\cos\left(\frac{\alpha-\beta}{2}\right) = \sqrt{\frac{1+\cos(\alpha-\beta)}{2}}$$

$$= \sqrt{\frac{1+\frac{63}{65}}{2}}$$

$$= \sqrt{\frac{128}{65\times 2}} = \sqrt{\frac{64}{65}}$$

$$= \pm\frac{8}{\sqrt{65}}$$

$$\therefore \cos\left(\frac{\alpha-\beta}{2}\right) = \frac{8}{\sqrt{65}}$$

Q44

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$Sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

substitute these values in the given equation, it reduces to

$$a(1-\tan^2\theta)+b(2\tan\theta)=c(1+\tan^2\theta)$$

$$(c+a)\tan^2\theta + 2b\tan\theta + c - a = 0$$

As α and β are roots

sum of the roots, $\tan\alpha + \tan\beta = \frac{2b}{c+a}$

Product of roots, $\tan\alpha\ \tan\beta = \frac{c-a}{c+a}$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\ \tan\beta} = \frac{2b}{c+a-c+a} = \frac{b}{a}$$

Q45

$$\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$$

squaring on both sides gives

$$\cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta = \sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta$$

Bring square terms on one side, we get

$$\cos 2\alpha + \cos 2\beta = -2(-\sin\alpha\sin\beta + \cos\alpha\cos\beta) = -2\cos(\alpha+\beta)$$

Ex 9.2

Q1

L.H.S,

$$\sin 5\theta = \sin(3\theta + 2\theta)$$
$$= \sin 3\theta \cos 2\theta + \cos 3\theta . \sin 2\theta$$
$$= (3\sin\theta - 4\sin^3\theta)(1 - 2\sin^2\theta) - (4\cos^3\theta - 3\cos\theta)2\sin\theta\cos\theta.$$
$$= 3\sin\theta - 4\sin^3\theta - 6\sin^3\theta + 8\sin^5\theta + (8\cos^4\theta - 6\cos^2\theta)\sin\theta$$
$$= 3\sin\theta - 10\sin^3\theta + 8\sin^5\theta + 8\sin\theta\left[(1 - \sin^2\theta)^2 - 6\sin\theta(1 - \sin^2\theta)\right]$$
$$= 3\sin\theta - 10\sin^3\theta + 8\sin^5\theta + 8\sin\theta - 16\sin^3\theta + 8\sin^5\theta - 6\sin\theta + 6\sin^3\theta$$
$$= 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta = \text{RHS}$$

Q2

Consider the L.H.S of the given equation

$$4(\cos^3 10° + \sin^3 20°) = 3(\cos 10° + \sin 20°)$$

Since $\sin 30° = \cos 60° = \frac{1}{2}$

and $\sin 60° = \cos 30° = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \sin 3.20° = \cos 3.10°$

$\Rightarrow \quad 3\sin 20° - 4\sin^3 20° = 4\cos^3 10° - 3\cos 10°$

$\Rightarrow \quad 4(\cos^3 10° + \sin^3 20°) = 3(\cos 10° + \sin 20°)$

Q3

$$\cos^3\theta \sin 3\theta + \sin^3\theta \cos 3\theta = \frac{3}{4}\sin 4\theta$$

$$\text{LHS} = \cos^3\theta \sin 3\theta + \sin^3\theta \cos 3\theta$$

$$= \left(\frac{\cos 3\theta + 3\cos\theta}{4}\right)\sin 3\theta + \left(\frac{3\sin\theta - \sin 3\theta}{4}\right)\cos 3\theta \quad \left\{\begin{array}{l}\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta \\ \cos 3\theta = 4\cos^3\theta - 3\cos\theta\end{array}\right\}$$

$$= \frac{1}{4}\left[3(\sin 3\theta\cos\theta + \sin\theta\cos 3\theta) + \cos 3\theta\sin 3\theta - \sin 3\theta\cos 3\theta\right]$$

$$= \frac{1}{4}\left[3\sin(3\theta + \theta) + 0\right]$$

$$= \frac{3}{4}\sin 4\theta$$

So,

$$\cos^3\theta \sin 3\theta + \sin^3\theta \cos 3\theta = \frac{3}{4}\sin 4\theta$$

Q4

We have to prove that

$\sin 5A = 5\cos^4 A \sin A - 10\cos^2 A \sin^3 A + \sin^5 A$

$$
\begin{aligned}
\text{L.H.S} \quad \sin 5A &= \sin(3A + 2A) \\
&= \sin 3A \cos 2A + \cos 3A . \sin 2A \\
&= \left(3\sin A - 4\sin^3 A\right)\left(2\cos^2 A - 1\right) + \left(4\cos^3 A - 3\cos A\right) 2\sin A \cos A \\
&= -3\sin A + 4\sin^3 A - 6\sin A\cos^2 A - 8\sin^3 A\cos^2 A + 8\cos^4 A\sin A - 6\cos^2 A\sin A \\
&= 8\cos^4 A\sin A - 8\sin^3 A\cos^2 A - 3\sin A + 4\sin^3 A \\
&= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A - 3\sin A + 3\cos^4 A\sin A + 4\sin^3 A + 2\sin^3 A\cos^2 A \\
&= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A - 3\sin A(1 - \cos^4 A) + 2\sin^3 A\left(2 + \cos^2 A\right) \\
&= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A - 3\sin A(1 - \cos^2 A)(1 + \cos^2 A) + 2\sin^3 A\left(2 + \cos^2 A\right) \\
&= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A - 3\sin^3 A(1 + \cos^2 A) + 2\sin^3 A\left(2 + \cos^2 A\right) \\
&= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A - \sin^3 A\left[3(1 + \cos^2 A) - 2\left(2 + \cos^2 A\right)\right] \\
&= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A - \sin^3 A\left[3 + 3\cos^2 A - 4 - 2\cos^2 A\right] \\
&= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A - \sin^3 A\left[\cos^2 A - 1\right] \\
&= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A - \sin^5 A \\
&= \text{RHS}
\end{aligned}
$$

Q5

$$\tan A \times \tan(A - 60^\circ) + \tan A \times \tan(A + 60^\circ) + \tan(A + 60^\circ)\tan(A - 60^\circ)$$

$$= \tan(A)\frac{[\tan(A) - \tan(60^\circ)]}{[1 + \tan(A)\tan(60^\circ)]}$$
$$+ \tan(A)\frac{[\tan(A) + \tan(60^\circ)]}{[1 - \tan(A)\tan(60^\circ)]}$$
$$+ \left\{\frac{[\tan(A) - \tan(60^\circ)]}{[1 + \tan(A)\tan(60^\circ)]}\right\}\left\{\frac{[\tan(A) + \tan(60^\circ)]}{[1 - \tan(A)\tan(60^\circ)]}\right\}$$

$$= \tan(A)\frac{[\tan(A) - \tan(60^\circ)][1 - \tan(A)\tan(60^\circ)]}{[1 - \tan^2(A)\tan^2(60^\circ)]}$$
$$+ \tan(A)\frac{[\tan(A) + \tan(60^\circ)][1 + \tan(A)\tan(60^\circ)]}{[1 - \tan^2(A)\tan^2(60^\circ)]}$$
$$+ \frac{[\tan(A) - \tan(60^\circ)][\tan(A) + \tan(60^\circ)]}{[1 - \tan^2(A)\tan^2(60^\circ)]}$$

$$= \tan(A)\frac{[\tan(A) - \sqrt{3}][1 - \sqrt{3}\tan(A)]}{[1 - 3\tan^2(A)]}$$
$$+ \tan(A)\frac{[\tan(A) + \sqrt{3}][1 + \sqrt{3}\tan(A)]}{[1 - 3\tan^2(A)]}$$
$$+ \frac{[\tan(A) - \sqrt{3}][\tan(A) + \sqrt{3}]}{[1 - 3\tan^2(A)]}$$

$$= \tan(A)\frac{[4\tan(A) - \sqrt{3} - \sqrt{3}\tan^2(A)]}{[1 - 3\tan^2(A)]}$$
$$+ \tan(A)\frac{[4\tan(A) + \sqrt{3} + \sqrt{3}\tan^2(A)]}{[1 - 3\tan^2(A)]}$$
$$+ \frac{[\tan^2(A) - 3]}{[1 - 3\tan^2(A)]}$$

$$= \frac{[9\tan^2(A) - 3]}{[1 - 3\tan^2(A)]}$$

Q6

$$\tan A + \tan\left(60^\circ + A\right) - \tan\left(60^\circ - A\right) = 3\tan 3A$$

$$\text{LHS} = \tan A + \tan\left(60^\circ + A\right) - \tan\left(60^\circ - A\right)$$

$$= \tan A + \frac{\tan 60^\circ + \tan A}{1 - \tan 60^\circ \tan A} - \frac{\tan 60^\circ - \tan A}{1 + \tan 60^\circ \tan A}$$

$$= \tan A + \frac{\sqrt{3} + \tan A}{1 - \sqrt{3}\tan A} - \frac{\sqrt{3} - \tan A}{1 + \sqrt{3}\tan A}$$

$$= \tan A + \left[\frac{\sqrt{3} + 3\tan A + \tan A + \sqrt{3}\tan^2 A + \sqrt{3} + 3\tan A + \tan A - \sqrt{3}\tan^2 A}{\left(1 - \sqrt{3}\tan A\right)\left(1 + \sqrt{3}\tan A\right)}\right]$$

$$= \tan A + \frac{8\tan A}{1 - 3\tan^2 A}$$

$$= \frac{\tan A - 3\tan^3 A + 8\tan A}{1 - 3\tan^2 A}$$

$$= \frac{9\tan A - 3\tan^3 A}{1 - 3\tan^2 A}$$

$$= 3\left(\frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}\right)$$

$$= 3\tan 3A$$

so,

$$\tan A + \tan\left(60^\circ + A\right) - \tan\left(60^\circ - A\right) = 3\tan 3A$$

Q7

$$\text{LHS} = \cot A + \cot(60^\circ + A) - \cot(60^\circ - A)$$

$$= \frac{1}{\tan A} + \frac{1}{\tan(60^\circ + A)} - \frac{1}{\tan(60^\circ - A)}$$

$$= \frac{1}{\tan A} + \frac{1 - \sqrt{3}\tan A}{\sqrt{3} + \tan A} - \frac{1 + \sqrt{3}\tan A}{\sqrt{3} - \tan A}$$

$$= \frac{1}{\tan A} - \frac{8\tan A}{3 - \tan^2 A}$$

$$= \frac{3 - \tan^2 A - 8\tan^2 A}{3\tan A - \tan^3 A}$$

$$= \frac{3 - 9\tan^2 A}{3\tan A - \tan^3 A}$$

$$= 3\left(\frac{1 - 3\tan^2 A}{3\tan A - \tan^3 A}\right)$$

$$= \frac{3}{\tan 3A}$$

$$= 3\cot 3A$$

$= \text{RHS}$

$\text{LHS} = \text{RHS}$

Hence proved.

Q8

$$\text{LHS} = \cot A + \cot(60^\circ + A) + \cot(120^\circ + A)$$

$$= \cot A + \cot(60^\circ + A) - \cot[180^\circ - (120^\circ + A)]$$

$$\{\text{since } -\cot\theta = \cot(180^\circ - \theta)\}$$

$$= \cot A + \cot(60^\circ + A) - \cot(60^\circ - A)$$

$$= \frac{1}{\tan A} + \frac{1}{\tan(60^\circ + A)} - \frac{1}{\tan(60^\circ - A)}$$

$$= \frac{1}{\tan A} + \frac{1 - \sqrt{3}\tan A}{\sqrt{3} + \tan A} - \frac{1 + \sqrt{3}\tan A}{\sqrt{3} - \tan A}$$

$$= \frac{1}{\tan A} - \frac{8\tan A}{3 - \tan^2 A}$$

$$= \frac{3 - \tan^2 A - 8\tan^2 A}{3\tan A - \tan^3 A}$$

$$= \frac{3 - 9\tan^2 A}{3\tan A - \tan^3 A}$$

$$= \frac{3(1 - 3\tan^2 A)}{3\tan A - \tan^3 A}$$

$$= \frac{3}{\tan 3A}$$

$$= 3\cot 3A$$

LHS = RHS

Q9

LHS $= \sin^3 A + \sin^3\left(\frac{2\pi}{3} + A\right) + \sin^3\left(\frac{4\pi}{3} + A\right)$

$\left\{\text{we know that } \sin^3 A = \frac{3\sin A - \sin 3A}{4}\right\}$

$$= \left(\frac{3\sin A - \sin 3A}{4}\right) + \left\{\frac{3\sin\left(\frac{2\pi}{3} + A\right) - \sin 3\left(\frac{2\pi}{3} + A\right)}{4}\right\} + \left\{\frac{3\sin\left(\frac{4\pi}{3} + A\right) - \sin 3\left(\frac{4\pi}{3} + A\right)}{4}\right\}$$

$$= \left[\frac{3\sin A - \sin 3A}{4}\right] + \left[\frac{3\sin\left[\pi\left(\frac{2\pi}{3} + A\right)\right] - \sin(2\pi + 3A)}{4}\right] + \left[\frac{3\sin\left(\pi + \left(\frac{\pi}{3} + A\right)\right) - \sin(4\pi + 3A)}{4}\right]$$

$$= \frac{1}{4}\left\{[3\sin A - \sin 3A] + \left[3\sin\left(\frac{\pi}{3} - A\right) - \sin 3A\right] - \left[3\sin\left(\frac{\pi}{3} + A\right) + \sin 3A\right]\right\}$$

$$= \frac{1}{4}\left[3\sin A - \sin 3A + 3\sin\left(\frac{\pi}{3} - A\right) - 3\sin\left(\frac{\pi}{3} + A\right) - \sin 3A - \sin 3A\right]$$

$$= \frac{1}{4}\left[3\sin A - 3\sin 3A + 3\left(\sin\left(\frac{\pi}{3} - A\right) - \sin\left(\frac{\pi}{3} + A\right)\right)\right]$$

$$= \frac{1}{4}\left[3\sin A - 3\sin 3A + 3\left\{2\cos\frac{\frac{\pi}{3} - A + \frac{\pi}{3} + A}{2}\sin\frac{\frac{\pi}{3} - A - \frac{\pi}{3} - A}{2}\right\}\right]$$

$$= \frac{1}{4}\left[3\sin A - 3\sin 3A + 6\cos\frac{\pi}{3}\sin(-A)\right]$$

$$= \frac{1}{4}[3\sin A - 3\sin 3A - 3\sin A]$$

$$= -\frac{3}{4}\sin 3A$$

= RHS

LHS = RHS

Q10

$$\left|\sin\theta \sin(60-\theta)\sin(60+\theta)\right|$$
$$= \left|\sin\theta\left(\sin^2 60 - \sin^2\theta\right)\right|$$
$$\left\{\text{since } \sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B\right\}$$
$$= \left|\sin\theta\left(\frac{3}{4} - \sin^2\theta\right)\right|$$
$$= \left|\frac{1}{4}\sin\theta\left(3 - 4\sin^2\theta\right)\right|$$
$$= \left|\frac{1}{4}\sin 3\theta\right|$$
$$= \frac{1}{4}\left|\sin 3\theta\right|$$
$$\le \frac{1}{4} \qquad \left\{\sin\theta\left|\sin 3\theta\right|\left|\le\right|\right\}$$

So,

$$\left|\sin\theta \sin\left(60^\circ - \theta\right)\sin\left(60^\circ + \theta\right)\right| \le \frac{1}{4}$$

Q11

$$\left|\cos\theta\cos\left(60^\circ-\theta\right)\cos\left(60^\circ+\theta\right)\right|$$

$$=\left|\cos\theta\left(\cos^2 60^\circ-\sin^2\theta\right)\right|$$

$\left\{\text{since } \cos\left(A-B\right)\cos\left(A+B\right)=\cos^2 A-\sin^2 B\right\}$

$$=\left|\cos\theta\left(\frac{1}{4}-\sin^2\theta\right)\right|$$

$$=\left|\cos\theta\,\frac{1}{4}\left(1-4\sin^2\theta\right)\right|$$

$$=\left|\frac{1}{4}\cos\theta\left(1-4\left(1-\cos^2\theta\right)\right)\right|$$

$$=\left|\frac{1}{4}\cos\theta\left(-3+4\cos^2\theta\right)\right|$$

$$=\left|\frac{1}{4}\left(4\cos 3\theta-3\cos\theta\right)\right|$$

$$=\left|\frac{1}{4}\cos 3\theta\right|$$

$$\le\frac{1}{4}\qquad\qquad\left\{\text{since } \left|\cos 3\theta\right|\le 1\right\}$$

So,

$$\left|\cos\theta\cos\left(60^\circ-\theta\right)\cos\left(60^\circ+\theta\right)\right|\le\frac{1}{4}$$

Ex 9.3

Q1

We have,

$$\sin^2 72^\circ - \sin^2 60^\circ.$$

$$= \sin^2(90^\circ - 18^\circ) - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \cos^2 18^\circ - \frac{3}{4}$$

$$= \left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right)^2 - \frac{3}{4} \quad \left[\because \cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}\right]$$

$$= \frac{10+2\sqrt{5}}{16} - \frac{3}{4}$$

$$= \frac{10+2\sqrt{5}-12}{16}$$

$$= \frac{2\sqrt{5}-2}{16}$$

$$= \frac{\sqrt{5}-1}{8}$$

Q2

$$\text{L.H.S} = \sin^2 24^\circ - \sin^2 6^\circ$$

$$= \sin(24+6)\sin(24-6) \quad \left[\because \sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B\right]$$

$$= \sin 30^\circ \; \sin 18^\circ$$

$$= \frac{1}{2}\cdot\frac{\sqrt{5}-1}{4} \quad \left[\because \sin 18^\circ = \frac{\sqrt{5}-1}{4}\right]$$

$$= \frac{\sqrt{5}-1}{8}$$

$$= \text{RHS}$$

Q3

$$\text{L.H.S} = \sin^2 42^\circ - \cos^2 78^\circ$$

$$= \sin^2(90-48) - \cos^2(90-12)$$

$$= \cos^2 48^\circ - \sin^2 12^\circ$$

$$= \cos(48+12).\cos(48-12)$$

$$\left[\because \cos(A+B).\cos(A-B) = \cos^2 A - \sin^2 B\right]$$

$$= \cos 60^\circ.\ \cos 36^\circ$$

$$= \frac{1}{2}.\frac{\sqrt{5}+1}{4} \qquad \left[\because \cos 36^\circ = \frac{\sqrt{5}+1}{4}\right]$$

$$= \frac{\sqrt{5}+1}{8}$$

$= \text{RHS}$

Q4

$$\text{L.H.S} = \cos 78^\circ.\ \cos 42^\circ.\ \cos 36^\circ$$

$$= \frac{\left(2\cos 78^\circ.\cos 42^\circ\right)}{2}.\cos 36^\circ$$

$$= \frac{1}{2}\left(\cos 120^\circ + \cos 36^\circ\right).\cos 36^\circ$$

$$= \frac{1}{2}\left(\frac{-1}{2} + \frac{\sqrt{5}+1}{4}\right)\frac{\sqrt{5}+1}{4}$$

$$= \frac{1}{8}\frac{\left[-2\left(\sqrt{5}+1\right)+5+1+2\sqrt{5}\right]}{4}$$

$$= \frac{1}{8}\left[\frac{4}{4}\right]$$

$$= \frac{1}{8}$$

$= \text{RHS}$

Q5

L.H.S$= \cos\frac{\pi}{15}\cdot\cos\frac{2\pi}{15}\cdot\cos\frac{4\pi}{15}\cdot\cos\frac{7\pi}{15}$

$$= \frac{2\sin\frac{\pi}{15}.\cos\frac{\pi}{15}}{2\sin\frac{\pi}{15}}.\cos\frac{2\pi}{15}.\cos\frac{4\pi}{15}.\ \cos\frac{7\pi}{15}\quad\left[\text{Divide and multiply by } 2\sin\frac{\pi}{15}\right]$$

$$= \frac{2.\sin\frac{2\pi}{15}}{2.2\sin\frac{\pi}{15}}.\ \cos\frac{2\pi}{15}.\ \cos\frac{4\pi}{15}.\cos\frac{7\pi}{15}$$

$$= \frac{2.\sin\frac{4\pi}{15}}{2.4\sin\frac{\pi}{15}}.\cos\frac{4\pi}{15}.\cos\frac{7\pi}{15}$$

$$= \frac{2\sin\frac{8\pi}{15}}{2.8\sin\frac{\pi}{15}}.\cos\left(\frac{7\pi}{15}\right)$$

$$= \frac{\sin\left(\frac{8\pi}{15}+\frac{7\pi}{15}\right)+\sin\left(\frac{8\pi}{15}-\frac{7\pi}{15}\right)}{16\sin\frac{\pi}{15}}$$

$$= \frac{\sin\pi+\sin\pi/15}{16\sin\pi/15}$$

$$= \frac{\sin\pi/15}{16\sin\pi/15}\qquad[\because \sin\pi = 0]$$

$$= \frac{1}{16}$$

$= \text{RHS}$

Q6

$$\cos\frac{5\pi}{15} = \cos\frac{\pi}{3} = \frac{1}{2}$$

$$\cos\frac{7\pi}{15} = \cos\left(\pi - \frac{8\pi}{15}\right)$$

$$\cos\frac{7\pi}{15} = -\cos\frac{8\pi}{15}$$

Now $\text{LHS} = \cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{3\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}$

$$= \left[\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\left(-\cos\frac{8\pi}{15}\right)\right]\left(\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}\right)\frac{1}{2}$$

$$= -\frac{2^3}{2^4\sin\frac{\pi}{15}}\left[2\sin\frac{\pi}{15}\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right]$$

$$\times\frac{2}{8\sin\frac{3\pi}{15}}\left(2\sin\frac{3\pi}{15}\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}\right)$$

$$= -\frac{2^3}{16\sin\frac{\pi}{15}}\left[\sin\frac{2\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right]\times\frac{2}{8\sin\frac{3\pi}{15}}\left(\sin\frac{6\pi}{15}\cos\frac{6\pi}{15}\right)$$

$$= -\frac{2^2}{16\sin\frac{\pi}{15}}\left[2\sin\frac{2\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right]\times\frac{1}{8\sin\frac{3\pi}{15}}\left(2\sin\frac{6\pi}{15}\cos\frac{6\pi}{15}\right)$$

$$= -\frac{2}{16\sin\frac{\pi}{15}}\left[\sin\frac{8\pi}{15}\cos\frac{8\pi}{15}\right]\frac{\sin\frac{12\pi}{15}}{8\sin\frac{3\pi}{15}}$$

$$= -\frac{1}{16\sin\frac{\pi}{15}}\left(\sin\frac{16\pi}{15}\right)\frac{\sin\frac{12\pi}{15}}{8\sin\frac{3\pi}{15}}$$

$$= -\frac{\sin\left(\pi + \frac{\pi}{15}\right)}{128\sin\frac{\pi}{15}}\times\frac{\sin\left(\pi - \frac{3\pi}{15}\right)}{\sin\frac{3\pi}{15}}$$

$$= -\frac{-\sin\frac{\pi}{15}}{128\sin\frac{\pi}{15}}\times\frac{\sin\frac{3\pi}{15}}{\sin\frac{3\pi}{15}}$$

$$= \frac{1}{128}$$

Q7

$$\text{L.H.S} = \cos 6^\circ . \cos 42^\circ . \cos 66^\circ . \cos 78^\circ$$

$$= \frac{1}{4}\left(2\cos 6^\circ . \cos 66^\circ\right)\left(2\cos 42^\circ . \cos 78^\circ\right)$$

$$= \frac{1}{4}\left(\cos 72^\circ + \cos 60^\circ\right)\left(\cos 120^\circ + \cos 36^\circ\right)$$

$$= \frac{1}{4}\left(\sin 18^\circ + \frac{1}{2}\right)\left(-\frac{2}{2} + \frac{\sqrt{5}+1}{4}\right)$$

$$= \frac{1}{4}\left(\frac{\sqrt{5}-1}{4} + \frac{1}{2}\right)\left(\frac{\sqrt{5}+1}{4} - \frac{1}{2}\right)$$

$$= \frac{1}{4}\left(\frac{\sqrt{5}-1+2}{4}\right)\left(\frac{\sqrt{5}+1-2}{4}\right)$$

$$= \frac{1}{64}\left(\sqrt{5}+1\right)\left(\sqrt{5}-1\right)$$

$$= \frac{1}{64}\left(\left(\sqrt{5}\right)^2 - 1^2\right)$$

$$= \frac{1}{64}\left(5-1\right)$$

$$= \frac{1}{16}$$

$$= \text{RHS}$$

Q8

L.H.S $= \sin 6^\circ . \sin 42^\circ . \sin 66^\circ . \sin 78^\circ$

$$= \frac{1}{4}\left(2\sin 6^\circ . \sin 66^\circ\right)\left(2\sin 42^\circ . \sin 78^\circ\right)$$

$$= \frac{1}{4}\left(\cos 60 - \cos 72^\circ\right)\left(\cos 36^\circ - \cos 120^\circ\right)$$

$$= \frac{1}{4}\left(\frac{1}{2} - \sin 18^\circ\right)\left(\frac{\sqrt{5}-1}{4} + \frac{1}{2}\right)$$

$$= \frac{1}{4}\left(\frac{1}{2} - \frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4} + \frac{1}{2}\right)$$

$$= \frac{1}{4}\left(\frac{2-\sqrt{5}+1}{4}\right)\left(\frac{\sqrt{5}+1+2}{4}\right)$$

$$= \frac{1}{64}\left(3^2 - \sqrt{5}^2\right)$$

$$= \frac{1}{64}(9-5)$$

$$= \frac{1}{16}$$

$= \text{RHS}$

Q9

L.H.S $= \cos 36^\circ . \cos 42^\circ . \cos 60^\circ . \cos 78^\circ$

$$= \frac{1}{2}\cos 36^\circ . \cos 60^\circ . \left(2\cos 42^\circ . \cos 78^\circ\right)$$

$$= \frac{1}{2}\left(\frac{\sqrt{5}+1}{4}\right) . \frac{1}{2}\left(\cos 120^\circ + \cos 36^\circ\right)$$

$$= \frac{\left(\sqrt{5}+1\right)}{16}\left(\frac{-1}{2} + \frac{\sqrt{5}+1}{4}\right)$$

$$= \frac{\left(\sqrt{5}+1\right)}{16}\left(\frac{-2+\sqrt{5}+1}{4}\right)$$

$$= \frac{\left(\sqrt{5}+1\right)\left(\sqrt{5}-1\right)}{64}$$

$$= \frac{5-1}{64}$$

$$= \frac{1}{16}$$

$= \text{RHS}$

Q10

L.H.S,

$$\sin 36^\circ . \sin 72^\circ . \sin 108^\circ . \sin 144^\circ$$

$$\left[\begin{array}{l} \because \ \sin 144^\circ = \sin\left(180^\circ - 36^\circ\right) = \sin 36^\circ \\ \quad \text{and } \sin 108^\circ = \sin\left(180^\circ - 78^\circ\right) = \sin 72^\circ \end{array}\right]$$

$$= \sin 36^\circ . \sin 72^\circ . \sin 72^\circ . \sin 36^\circ$$

$$= \frac{1}{4}\left(2 \sin 36^\circ . \sin 72^\circ\right)^2$$

$$= \frac{1}{4}\left(2 \sin 36^\circ \cos 18^\circ\right)^2 \qquad \left[\because \sin 72^\circ = \cos 18^\circ\right]$$

$$= \frac{4}{1}\left(\frac{\sqrt{10 - 2\sqrt{5}}}{4} . \frac{\sqrt{10 + 2\sqrt{5}}}{4}\right)^2$$

$$= \frac{1}{64}\left(10 - 2\sqrt{5}\right)\left(10 + 2\sqrt{5}\right)$$

$$= \frac{100 - 20}{64 \times 4}$$

$$= \frac{80}{256}$$

$$= \frac{5}{16}$$

$$= \text{RHS}$$