Exercise – 17A

1. Find the area of triangle whose base measures 24 cm and the corresponding height measure 14.5 cm.

Sol:

Given: base = 24 cm, corresponding height $= 14.5\ cm$

Area of a triangle $= \frac{1}{2} \times base \times$ corresponding height

$= \frac{1}{2} \times 24 \times 14.5$

$= 174\ cm^2$

2. Find the areas of the triangle whose sides are 42 cm, 34 cm and 20 cm. Also, find the height corresponding to the longest side.

Sol:

Let the sides of the triangle be $a = 20\ cm, b = 34\ cm$ and $c = 42\ cm.$

Let s be the semi-perimeter of the triangle.

$s = \frac{1}{2}(a+b+c)$

$s = \frac{1}{2}(20+34+42)$

$s = 48 cm$

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \sqrt{48(48-20)(48-34)(48-42)}$

$\Rightarrow \sqrt{48 \times 28 \times 14 \times 6}$

$\Rightarrow \sqrt{112896}$

$\Rightarrow 336\ cm^2$

Length of the longest side is 42 cm.

Area of a triangle $= \frac{1}{2} \times b \times h$

$\Rightarrow 336 = \frac{1}{2} \times 42 \times h$

$\Rightarrow 672 = 42h$

$\Rightarrow \frac{672}{42} = h$

$\Rightarrow h = 16 cm$

The height corresponding to the longest side is 16 cm.

3. Find the area of the triangle whose sides are 18 cm, 24 cm and 30 cm. Also find the height corresponding to the smallest side.

Sol:

Let the sides of triangle be $a = 18\ cm, b = 24cm$ and $c = 30\ cm$.

Let s be the semi-perimeter of the triangle.

$$s = \frac{1}{2}(a+b+c)$$

$$s = \frac{1}{2}(18+24+30)$$

$$s = 36cm$$

Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{36(36-18)(36-24)(36-30)}$$

$$= \sqrt{36 \times 18 \times 12 \times 6}$$

$$= \sqrt{46656}$$

$$= 216\ cm^2$$

The smallest side is 18 cm long. This is the base.

Now, area of a triangle $= \frac{1}{2} \times b \times h$

$$\Rightarrow 216 = \frac{1}{2} \times 18 \times h$$

$$\Rightarrow 216 = 9h$$

$$\Rightarrow \frac{216}{9} = h$$

$$\Rightarrow h = 24cm$$

The height corresponding to the smallest side is 24 cm.

4. The sides of a triangle are in the ratio 5:12:13 and its perimeter is 150 m. Find the area of the triangle.

Sol:

Let the sides of a triangle be $5x\,m, 12x\,m$ and $13x\,m$.

Since, perimeter is the sum of all the sides,

$$5x + 12x + 13x = 150$$

$$\Rightarrow 30x = 150$$

Or, $x = \frac{150}{30} = 5$

The lengths of the sides are:

$a = 5 \times 5 = 25\ m$

$b = 12 \times 5 = 60\ m$

$c = 13 \times 5 = 65\ m$

Semi-perimeter (s) of the triangle $= \dfrac{Perimeter}{2} = \dfrac{25 = 60 + 65}{2} = \dfrac{150}{2} = 75m$

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{75(75-25)(75-60)(75-65)}$

$= \sqrt{75 \times 50 \times 15 \times 10}$

$= \sqrt{562500}$

$= 750\ m^2$

5. The perimeter of a triangular field is 240m, and its sides are in the ratio 25:17:12. Find the area of the field. Also, find the cost of ploughing the field at ₹ 40 per m^2

Sol:

Let the sides of the triangular field be 25*x*, 17*x* and 12*x*.

As, perimeter = 540 m

$\Rightarrow 25x + 17x + 12x = 540$

$\Rightarrow 54x = 540$

$\Rightarrow x = \dfrac{540}{54}$

$\Rightarrow x = 10$

So, the sides are 250 m, 170 m and 120 m.

Now, semi-perimeter, $s = \dfrac{250 + 170 + 120}{2} = \dfrac{540}{2} = 270m$

So, area of the filed $= \sqrt{270(270-250)(270-170)(270-120)}$

$= \sqrt{270 \times 20 \times 100 \times 150}$

$= \sqrt{3^3 \times 10 \times 2 \times 10 \times 10^2 \times 3 \times 5 \times 10}$

$= 3^2 \times 10^3$

$= 9000\ m^2$

Also, the cost of ploughing the field $= \dfrac{9000 \times 40}{100} = 3,600$

6. The perimeter of a right triangle is 40 cm and its hypotenuse measure 17 cm. Find the area of the triangle.

Sol:

The perimeter of a right-angled triangle = 40 cm

Therefore, $a+b+c=40\ cm$
Hypotenuse = 17 cm
Therefore, c = 17cm
$a+b+c=40cm$
$\Rightarrow a+b+17=40$
$\Rightarrow a+b=23$
$\Rightarrow b=23-a \quad ....(i)$
Now, using Pythagoras theorem, we have:
$a^2+b^2=c^2$
$\Rightarrow a^2+(23-a)^2=17^2$
$\Rightarrow a^2+529-46a+a^2=289$
$\Rightarrow 2a^2-46a+529-289=0$
$\Rightarrow 2a^2-46a+240=0$
$\Rightarrow a^2-23a+120=0$
$\Rightarrow (a-15)(a-8)=0$
$\Rightarrow a=15 \ or \ a=8$
Substituting the value of $a=15,$ in equation (i) we get:
$b=23-a$
$=23-15$
$=8\ cm$
If we had chosen $a=8cm,$ then, $b=23-8=15\ cm$
In any case,
Area of triangle $=\frac{1}{2}\times \text{base}\times \text{height}$
$=\frac{1}{2}\times 8\times 15$
$=60\ cm^2$

7. The difference between the sides at the right angles in a right-angled triangle is 7 cm. the area of the triangle is 60 cm^2 . Find its perimeter.
Sol:
Given:
Area of the triangle $=60\ cm^2$
Let the sides of the triangle be a, b and c, where a is the height, b is the base and c is hypotenuse of the triangle.

$a-b=7\,cm$

$a=7+b \qquad ....(1)$

Area of triangle $=\frac{1}{2}\times b\times h$

$\Rightarrow 60=\frac{1}{2}\times b\times(7+b)$

$\Rightarrow 120=7b+b^2$

$\Rightarrow b^2+7b-120=0$

$\Rightarrow (b+15)(b-8)=0$

$\Rightarrow b=-15\ or\ 8$

Side of a triangle cannot be negative.
Therefore, b = 8 cm.
Substituting the value of b = 8 cm, in equation (1):

$a=7+8=15\ cm$

Now, $a=15\ cm, b=8cm$

Now, in the given right triangle, we have to find third side.

$(Hyp)^2=(First\ \ side)^2+(Second\ \ side)^2$

$\Rightarrow Hyp^2=8^2+15^2$

$\Rightarrow Hyp^2=64+225$

$\Rightarrow Hyp^2=289$

$\Rightarrow Hyp=17\,cm$

So, the third side is 17 cm.
Perimeter of a triangle = a + b + c.
Therefore, required perimeter of the triangle 15 + 8 + 1740 cm

8. The length of the two sides of a right triangle containing the right angle differ by 2 cm. If the area of the triangle is 24 cm^2 , find the perimeter of the triangle.

Sol:

Given:

Area of triangle $=24cm^2$

Let the sides be a and b, where a is the height and b is the base of triangle

$a-b=2\,cm$

$a=2+b \qquad ....(1)$

Area of triangle$=\frac{1}{2}\times b\times h$

$\Rightarrow 24 = \frac{1}{2} \times b \times (2+b)$

$\Rightarrow 48 = b + \frac{1}{2}b^2$

$\Rightarrow 48 = 2b + b^2$

$\Rightarrow b^2 + 2b - 48 = 0$

$\Rightarrow (b+8)(b-6) = 0$

$\Rightarrow b = -8 \ or \ 6$

Side of a triangle cannot e negative.

Therefore, $b = 6cm.$

Substituting the value of b=6 cm in equation (1), we get:

$a = 2+6 = 8 \ cm$

Now, $a = 8cm, b = 6cm$

In the given right triangle we have to find third side. Using the relation

$(Hyp)^2 = (Oneside)^2 + (Otherside)^2$

$\Rightarrow Hyp^2 = 8^2 + 6^2$

$\Rightarrow Hyp^2 = 64 + 36$

$\Rightarrow Hyp^2 = 100$

$\Rightarrow Hyp = 10cm$

So, the third side is 10 cm

So, perimeter of the triangle $= a+b+c$

$= 8+6+10$

$= 24cm$

9. Each side of an equilateral triangle is 10 cm. Find (i) the area of the triangle and (ii) the height of the triangle.

Sol:

(i) The area of the equilateral triangle $= \frac{\sqrt{3}}{4} \times side^2$

$= \frac{\sqrt{3}}{4} \times 10^2$

$= \frac{\sqrt{3}}{4} \times 100$

$= 25\sqrt{3}\, cm^2$

Or $25 \times 1.732 = 43.3cm^2$

So, the area of the triangle is $25\sqrt{3} \ cm^2 \ or \ 43.3 \ cm^2$.

(ii) As, area of the equilateral triangle $=25\sqrt{3}\ cm^2$

$\Rightarrow \frac{1}{2}\times Base\times Height=25\sqrt{3}$

$\Rightarrow \frac{1}{2}\times 10\times Height=25\sqrt{3}$

$\Rightarrow 5\times Height=25\sqrt{3}$

$\Rightarrow Height=\frac{25\sqrt{3}}{5}=5\sqrt{3}$

Or height $=5\times 1.732=8.66\ m$

$\therefore$ The height of the triangle is $5\sqrt{3}\ cm$ or $8.66\ cm.$

10. The height of an equilateral triangle is 6 cm. Find its area.

Sol:

Let the side of the equilateral triangle be *x* cm.

As, the area of an equilateral triangle $=\frac{\sqrt{3}}{4}(side)^2=\frac{x^2\sqrt{3}}{4}$

Also, the area of the triangle $=\frac{1}{2}\times Base\times Height=\frac{1}{2}\times x\times 6=3x$

So, $\frac{x^2\sqrt{3}}{4}=3x$

$\Rightarrow \frac{x\sqrt{3}}{4}=3$

$\Rightarrow x=\frac{12}{\sqrt{3}}$

$\Rightarrow x=\frac{12}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

$\Rightarrow x=\frac{12\sqrt{3}}{3}$

$\Rightarrow x=4\sqrt{3}\ cm$

Now, area of the equilateral triangle $=3x$

$=3\times 4\sqrt{3}$

$=12\sqrt{3}$

$=12\times 1.73$

$=20.76\ cm^2$

11. If the area of an equilateral triangle is $36\sqrt{3}\ cm^2$, find its perimeter.

Sol:

Area of equilateral triangle $= 36\sqrt{3}\ cm^2$

Area of equilateral triangle $= \left(\frac{\sqrt{3}}{4}\times a^2\right)$, where a is the length of the side

$\Rightarrow 36\sqrt{3} = \frac{\sqrt{3}}{4}\times a^2$

$\Rightarrow 144 = a^2$

$\Rightarrow a = 12\,cm$

Perimeter of a triangle $= 3a$

$= 3\times 12$

$= 36\,cm$

12. If the area of an equilateral triangle is $81\sqrt{3}\ cm^2$ find its height.

Sol:

Area of equilateral triangle $= 81\sqrt{3}\ cm^2$

Area of equilateral triangle $= \left(\frac{\sqrt{3}}{4}\times a^2\right)$, where a is the length of the side

$\Rightarrow 81\sqrt{3} = \frac{\sqrt{3}}{4}\times a^2$

$\Rightarrow 324 = a^2$

$\Rightarrow a = 18\ cm$

Height of triangle $= \frac{\sqrt{3}}{2}\times a$

$= \frac{\sqrt{3}}{2}\times 18$

$= 9\sqrt{3}\ cm$

13. The base of a right – angled triangle measures 48 cm and its hypotenuse measures 50 cm. Find the area of the triangle.

Sol:

Base = 48 cm

Hypotenuse 50 cm

First we will find the height of the triangle; let the height be ‘p’.

$\Rightarrow (Hypotenuse)^2 = (base)^2 + p^2$

$\Rightarrow 50^2 = 48^2 + p^2$

$\Rightarrow p^2 = 50^2 - 48^2$

$\Rightarrow p^2 = (50-48)(50+48)$

$\Rightarrow p^2 = 2 \times 98$

$\Rightarrow p^2 = 196$

$\Rightarrow p = 14 cm$

Area of the triangle $= \frac{1}{2} \times base \times height$

$= \frac{1}{2} \times 48 \times 14$

$= 336\ cm^2$

14. The hypotenuse of a right-angled triangle is 65 cm and its base is 60 cm. Find the length of perpendicular and the area of the triangle.

Sol:

Hypotenuse = 65 cm

Base = 60 cm

In a right angled triangle,

$(Hypotenuse)^2 = (Base)^2 + (Perpendicular)^2$

$\Rightarrow (65)^2 = (60)^2 + (perpendicular)^2$

$\Rightarrow (65)^2 - (60)^2 + (perpendicular)^2$

$\Rightarrow (perpendicular)^2 = (65-60)(65+60)$

$\Rightarrow (perpendicular)^2 = 5 \times 125$

$\Rightarrow (perpendicular)^2 = 625$

$\Rightarrow (perpendicular)^2 = 25\ cm$

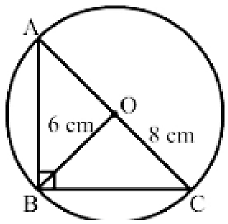
Area of triangle $= \frac{1}{2} \times base \times perpendicular$

$= \frac{1}{2} \times 60 \times 25$

$= 750\ cm^2$

15. Find the area of a right – angled triangle, the radius of whose, circumference measures 8 cm and the altitude drawn to the hypotenuse measures 6 cm.

Sol:



Given: radius = 8 cm
Height = 6 cm
Area = ?
In a right angled triangle, the center of the circumference is the midpoint of the hypotenuse.
Hypotenuse $= 2\times$(radius of circumference) for a right triangle
$= 2\times 8$
$= 16\,cm$
So, hypotenuse = 16 cm
Now, base = 16 cm and height = 6 cm
Area of the triangle $= \frac{1}{2}\times base\times height$
$= \frac{1}{2}\times 16\times 6$
$= 48\ cm^2$

16. Find the length of the hypotenuse of an isosceles right-angled triangle whose area is $200cm^2$. Also, find its perimeter

Sol:
In a right isosceles triangle, base = height = a
Therefore,
Area of a triangle $= \frac{1}{2}\times base\times height = \frac{1}{2}\times a\times a = \frac{1}{2}a^2$
Further, given that area of isosceles right triangle $= 200\,cm^2$
$\Rightarrow \frac{1}{2}a^2 = 200$
$\Rightarrow a^2 = 400$
$or,\ a = \sqrt{400} = 20\ cm$
In an isosceles right triangle, two sides are equal $('a')$ and the third side is the hypotenuse, i.e., ‘c’
Therefore, $c = \sqrt{a^2 + a^2}$
$= \sqrt{2a^2}$
$= a\sqrt{2}$

$= 20 \times 1.41$

$= 28.2\ cm$

Perimeter of the triangle $= a + a + c$

$= 20 + 20 + 28.2$

$= 68.2\,cm$

The length of the hypotenuse is 28.2 cm and the perimeter of the triangle is 68.2 cm.

17. The base of an isosceles triangle measures 80 cm and its area is $360\,cm^2$. Find the perimeter of the triangle.

Sol:

Given:

Base = 80 cm

Area = 360 cm^2

Area of an isosceles triangle $= \left(\frac{1}{4} b\sqrt{4a^2 - b^2}\right)$

$\Rightarrow 360 = \frac{1}{4} \times 80\sqrt{4a^2 - 80^2}$

$\Rightarrow 360 = 20\sqrt{4a^2 - 6400}$

$\Rightarrow 18 = 2\sqrt{a^2 - 1600}$

$\Rightarrow 9 = \sqrt{a^2 - 1600}$

Squaring both the sides, we get:

$\Rightarrow 81 = a^2 - 1600$

$\Rightarrow a^2 = 1681$

$\Rightarrow a = 41\ cm$

Perimeter $= (2a + b)$

$= \left[2(41) + 80\right] = 82 + 80 = 162\ cm$

So, the perimeter of the triangle is 162 cm.

18. Each of the equal sides of an isosceles triangle measure 2 cm more than its height, and the base of the triangle measure 12 cm. Find the area of the triangle.

Sol:

Let the height of the triangle be h cm.

Each of the equal sides measures $a = (h + 2)cm\ and\ b = 12\ cm(base)$

Now,

Area of the triangle = Area of the isosceles triangle

$\Rightarrow \frac{1}{2}\times base\times height=\frac{1}{4}\times b\sqrt{4a^2-b^2}$

$\Rightarrow \frac{1}{2}\times 12\times h=\frac{1}{4}\times 12\times \sqrt{4(h+2)^2-144}$

$\Rightarrow 6h=3\sqrt{4h^2+16h+16-144}$

$\Rightarrow 2h=\sqrt{4h^2+16h+16-144}$

On squaring both the sides, we get:

$\Rightarrow 4h^2=4h^2+16h+16-144$

$\Rightarrow 16h-128=0$

$\Rightarrow h=8$

Area of the triangle $=\frac{1}{2}\times b\times h$

$=\frac{1}{2}\times 12\times 8$

$=48\ cm^2$

19. Find the area and perimeter of an isosceles right angled triangle, each of whose equal sides measure 10cm.

Sol:

Let:

Length of each of the equal sides of the isosceles right-angled triangle = a = 10 cm

And.

Base = Height = a

Area of isosceles right – angled triangle $=\frac{1}{2}\times Base\times Height$

The hypotenuse of an isosceles right – angled triangle can be obtained using Pythagoras' theorem

If *h* denotes the hypotenuse, we have:

$h^2=a^2+a^2$

$\Rightarrow h=2a^2$

$\Rightarrow h=\sqrt{2}a$

$\Rightarrow h=10\sqrt{2}\ cm$

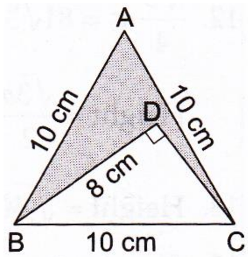
$\therefore$ Perimeter of the isosceles right-angled triangle $=2a+\sqrt{2}a$

$=2\times 10+1.41\times 10$

$=20+14.1$

$=34.1\ cm$

20. In the given figure, ΔABC is an equilateral triangle the length of whose side is equal to 10 cm, and ΔADC is right-angled at D and BD= 8cm. Find the area of the shaded region.



Sol:

Given:

Side if equilateral triangle $ABC = 10\ cm$

$BD = 8\ cm$

Area of equilateral $\Delta ABC = \frac{\sqrt{3}}{4}a^2$ (where $a = 10\,cm$)

Area of equilateral $\Delta ABC = \frac{\sqrt{3}}{4}\times 10^2$

$= 25\sqrt{3}$

$= 25\times 1.732$

$= 43.30\ cm^2$

In the right ΔBDC, we have:

$BC^2 = BD^2 + CD^2$

$\Rightarrow 10^2 = 8^2 + CD^2$

$\Rightarrow CD^2 = 10^2 - 8^2$

$\Rightarrow CD^2 = 36$

$\Rightarrow CD = 6$

Area of triangle $\Delta BCD = \frac{1}{2}\times b\times h$

$= \frac{1}{2}\times 8\times 6$

$= 24\ cm^2$

Area of the shaded region $=$ Area of $\Delta ABC -$ Area of ΔBDC

$= 43.30 - 24$

$= 19.3\ cm^2$

Exercise – 17B

1. The perimeter of a rectangular plot of land is 80 m and its breadth is 16 m. Find the length and area of the plot.

Sol:

As, a perimeter $80\,m$

$\Rightarrow 2(length + breath) = 80$

$\Rightarrow 2(length + 16) = 80$

$\Rightarrow 2 \times length + 32 = 80$

$\Rightarrow 2 \times length = 80 - 32$

$\Rightarrow length = \frac{48}{2}$

$\therefore length = 24\,m$

Now, the area of the plot $= length \times breadth$

$= 24 \times 16$

$= 384 m^2$

So, the length of the plot is 24 m and its area is $384\,m^2$.

2. The length of a rectangular park is twice its breadth and its perimeter is 840 m. Find the area of the park.

Sol:

Let the breadth of the rectangular park be b.

$\therefore$ Length of the rectangular park $= l = 2b$

Perimeter = 840 m

$\Rightarrow 840 = 2(l + b)$

$\Rightarrow 840 = 2(2b + b)$

$\Rightarrow 840 = 2(3b)$

$\Rightarrow 840 = 6b$

$\Rightarrow b = 140\ m$

Thus, we have:

$l = 2b$

$= 2 \times 140$

$= 280\ m$

Area $= l \times b$

$= 280 \times 140$

$= 39200\ m^2$

3. One side of a rectangle is 12 cm long and its diagonal measure 37 cm. Find the other side and the area of the rectangle.

Sol:

One side of the rectangle = 12 cm

Diagonal of the rectangle = 37 cm

The diagonal of a rectangle forms the hypotenuse of a right-angled triangle. The other two sides of the triangle are the length and the breadth of the rectangle.

Now, using Pythagoras' theorem, we have:

$$(one\ \ side)^2+(other\ \ side)^2=(hypotenuse)^2$$

$$\Rightarrow(12)^2+(other\ side)^2=(37)^2$$

$$\Rightarrow 144+(other\ side)^2=1369$$

$$\Rightarrow(other\,side)^2=1329-144$$

$$\Rightarrow(other\ side)^2=1225$$

$$\Rightarrow other\ side=\sqrt{1225}$$

$$\Rightarrow other\ side=35\ cm$$

Thus, we have:

Length = 35 cm

Breadth = 12 cm

Area of the rectangle $=35\times12=420\ cm^2$

4. The area of a rectangular plot is $462m^2$ and is length is 28 m. Find its perimeter.

Sol:

Area of the rectangular plot $=462\ m^2$

Length $(l)=28\ m$

Area of a rectangle = Length$(l)\times$Breath(b)

$$=462=28\times b$$

$$\Rightarrow b=16.5\ m$$

Perimeter of the plot $=2(l+b)$

$$=2(28+16.5)$$

$$=2\times44.5$$

$$=89\,m$$

5. A lawn is in the form of a rectangle whose sides are in the ratio 5 : 3. The area of the lawn is $3375m^2$. Find the cost of fencing the lawn at ₹ 65 per metre.

Sol:

Let the length and breadth of the rectangular lawn be $5x\ m$ and $3x\ m$, respectively.
Given:
Area of the rectangular lawn $= 3375\,m^2$

$\Rightarrow 3375 = 5x \times 3x$

$\Rightarrow 3375 = 15x^2$

$\Rightarrow \frac{3375}{15} = x^2$

$\Rightarrow 225 = x^2$

$\Rightarrow x = 15$

Thus, we have:

$l = 5x = 5 \times 15 = 75\ m$

$b = 3x = 3 \times 15 = 45\ m$

Perimeter of the rectangular lawn $= 2(l + b)$

$= 2(75 + 45)$

$= 2(120)$

$= 240\,m$

Cost of fencing 1 m lawn = Rs 65
$\therefore$ Cost of fencing 240 m lawn $= 240 \times 65 = Rs\ 15,600$

6. A room is 16 m long and 13.5 m broad. Find the cost of covering its floor with 75-m-wide carpet at ₹ 60 per metre.
Sol:
As, the area of the floor = length $\times$ breadth

$= 16 \times 13.5$

$= 216\,m^2$

And, the width of the carpet = 75 m

So, the length of the carpet required $= \frac{Area\ of\ the\ floor}{Width\ of\ the\ carpet}$

$= \frac{216}{75}$

$= 2.88\ m$

Now, the cost of the carpet required $= 2.88 \times 60 = 172.80$
Hence, the cost of covering the floor with carpet is 172.80.
Disclaimer: The answer given in the textbook is incorrect. The same has been rectified above.

7. The floor of a rectangular hall is 24 m long and 18 m wide. How many carpets, each of length 2.5 m and breadth 80 cm, will be required to cover the floor of the hall?

Sol:

Given:

Length = 24 m

Breath = 18 m

Thus, we have:

Area of the rectangular hall $= 24 \times 18$

$= 432\, m^2$

Length of each carpet $= 2.5\, m$

Breath of each carpet $= 80\, cm = 0.80\ m$

Area of one carpet $= 2.5 \times 0.8 = 2\ m^2$

Number of carpets required $= \frac{Area\ of\ the\ hall}{Area\ of\ the\ carpet} = \frac{432}{2} = 216$

Therefore, 216 carpets will be required to cover the floor of the hall.

8. A 36-m-long, 15-m-borad verandah is to be paved with stones, each measuring 6dm by 5 dm. How many stones will be required?

Sol:

Area of the verandah = Length $\times$ Breadth = 36 $\times$ 15 = 540 m^2

Length of the stone = 6 dm = 0. 6 m

Breadth of the stone = 5 dm = 0.5 m

Area of one stone $= 0.6 \times 0.5 = 0.3\ m^2$

Number of stones required $= \frac{Area\ of\ the\ verendah}{Area\ of\ the\ stone}$

$= \frac{540}{0.3}$

$= 1800$

Thus, 1800 stones will be required to pave the verandah.

9. The area of rectangle is $192 cm^2$ and its perimeter is 56 cm. Find the dimensions of the rectangle.

Sol:

Area of the rectangle $= 192\ cm^2$

Perimeter of the rectangle $= 56\ cm$

Perimeter = 2(length + breath)

$\Rightarrow 56 = 2(l + b)$

$\Rightarrow l + b = 28$

$\Rightarrow l = 28 - b$

Area = length $\times$ breath

$\Rightarrow 192 = (28 - b) xb$

$\Rightarrow 192 = 28b - b^2$

$\Rightarrow b^2 - 28b + 192 = 0$

$\Rightarrow (b-16)(b-12) = 0$

$\Rightarrow b = 16 \ or \ 12$

Thus, we have;

$l = 28 - 12$

$\Rightarrow l = 28 - 12$

$\Rightarrow l = 16$

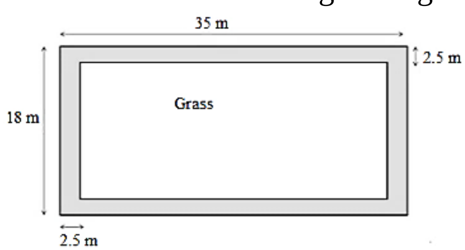
We will take length as 16 cm and breath as 12 cm because length is greater than breath by convention.

10. A rectangular park 358 m long and 18 m wide is to be covered with grass, leaving 2.5 m uncovered all around it. Find the area to be laid with grass.

Sol:

The field is planted with grass, with 2.5 m uncovered on its sides.

The field is shown in the given figure.



Thus, we have;

Length of the area planted with grass $35 - (2.5 + 2.5) = 35 - 5 = 30\,m$

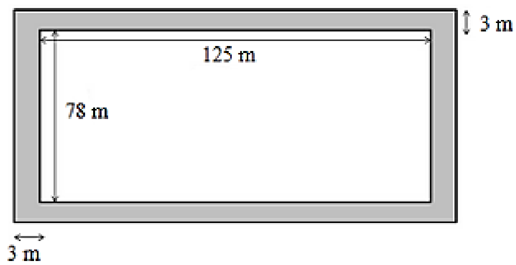
Width of the area planted with grass $= 18 - (2.5 + 2.5) = 18 - 5 = 13\,m$

Area of the rectangular region planted with grass $= 30 \times 13 = 390\,m^2$

11. A rectangular plot measure 125 m by 78 m. It has gravel path 3 m wide all around on the outside. Find the area of the path and the cost of gravelling it at ₹ 75 per m^2

Sol:

The plot with the gravel path is shown in the figure.



Area of the rectangular plot $= l \times b$

Area of the rectangular plot $= 125 \times 78 = 9750\ m^2$

Length of the park including the path $= 125 + 6 = 131m$

Breadth of the park including the path $= 78 + 6 = 84m$

Area of the plot including the path

$= 131 \times 84$

$= 11004\ m^2$

Area of the path $= 11004 - 9750$

$= 1254m^2$

Cost of gravelling 1 m^2 of the path = Rs 75

Cost of gravelling 1254 m^2 of the path $= 1254 \times 75$

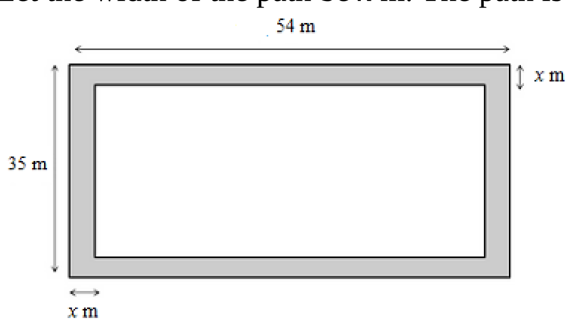
$= Rs\ 94050$

12. A footpath of uniform width runs all around the inside of a rectangular field 54m long and 35 m wide. If the area of the path is $420m^2$, find the width of the path.

Sol:

Area of the rectangular field $= 54 \times 35 = 1890\ m^2$

Let the width of the path be x m. The path is shown in the following diagram:



Length of the park excluding the path $= (54 - 2x)m$

Breadth of the park excluding the path $= (35 - 2x)m$

Thus, we have:

Area of the path $= 420\ m^2$

$\Rightarrow 420 = 54 \times 35 - (54 - 2x)(35 - 2x)$

$\Rightarrow 420 = 1890 - \left(1890 - 70x - 108x + 4x^2\right)$

$\Rightarrow 420 = -4x^2 + 178x$

$\Rightarrow 4x^2 - 178x + 420 = 0$

$\Rightarrow 2x^2 - 89x + 210 = 0$

$\Rightarrow 2x^2 - 84x - 5x + 210 = 0$

$\Rightarrow 2x\left(x - 42\right) - 5\left(x - 42\right) = 0$

$\Rightarrow \left(x - 42\right)\left(2x - 5\right) = 0$

$\Rightarrow x - 42 = 0 \ or \ 2x - 5 = 0$

$\Rightarrow x = 42 \ or \ x = 2.5$

The width of the path cannot be more than the breath of the rectangular field.

$\therefore x = 2.5 \ m$

Thus, the path is $2.5m$ wide.

13. The length and breadth of a rectangular garden are in the ratio 9:5. A path 3.5 m wide, running all around inside it has an area of $1911m^2$. Find the dimensions of the garden.

Sol:

Let the length and breadth of the garden be 9*x* m and 5*x* m, respectively,

Now,

Area of the garden $= \left(9x \times 5x\right) = 45x^2$

Length of the garden excluding the path $= \left(9x - 7\right)$

Breadth of the garden excluding the path $= \left(5x - 7\right)$

Area of the path $= 45x^2 = \left[\left(9x - 7\right)\left(5x - 7\right)\right]$

$\Rightarrow 1911 = 45x^2 - \left[45x^2 - 63x - 35x + 49\right]$

$\Rightarrow 1911 = 45x^2 - 45x^2 + 63x + 35x - 49$

$\Rightarrow 1911 = 98x - 49$

$\Rightarrow 1960 = 98x$

$\Rightarrow x = \frac{1960}{98}$

$\Rightarrow x = 20$

Thus, we have:

Length $= 9x = 20 \times 9 = 180m$

Breadth $= 5x = 5 \times 20 = 100 \ m$

14. A room 4.9 m long and 3.5 m board is covered with carpet, leaving an uncovered margin of 25 cm all around the room. If the breadth of the carpet is 80 cm, find its cost at ₹ 80 per metre.

Sol:

Width of the room left uncovered = 0.25 m

Now,

Length of the room to be carpeted $=4.9-(0.25+0.25)=4.9-0.5=4.4\ m$

Breadth of the room be carpeted $=3.5-(0.25+0.25)=3.5-0.5=3m$

Area to be carpeted $=4.4\times3=13.2\ m^2$

Breadth of the carpet 80 cm = 0.8 m

We know:

Area of the room = Area of the carpet

Length of the carpet = $\dfrac{Area\ of\ the\ room}{Breadth\ of\ the\ carpet}$

$=\dfrac{13.5}{0.8}$

$=16.5m$

Cost of 1 m carpet = Rs 80

Cost of 16.5 m carpet $=80\times16.5=Rs1,320$

15. A carpet is laid on floor of a room 8 m by 5 m. There is border of constant width all around the carpet. If the area of the border is $12m^2$ find its width.

Sol:

Let the width of the border be *x* m.

The length and breadth of the carpet are 8 m and 5 m, respectively.

Area of the carpet $=8\times5=40m^2$

Length of the carpet without border $=(8-2x)$

Breadth of carpet without border $=(5-2x)$

Area of the border $12m^2$

Area of the carpet without border $=(8-2x)(5-2x)$

Thus, we have:

$12=40-[(8-2x)(5-2x)]$

$\Rightarrow 12=40-(40-26x+4x^2)$

$\Rightarrow 12=26x-4x^2$

$\Rightarrow 26x-4x^2=12$

$\Rightarrow 4x^2 - 26x + 12 = 0$

$\Rightarrow 2x^2 - 13x + 6 = 0$

$\Rightarrow (2x-1)(x-6) = 0$

$\Rightarrow 2x - 1 = 0 \ and \ x - 6 = 0$

$\Rightarrow x = \frac{1}{2} \ and \ x = 6$

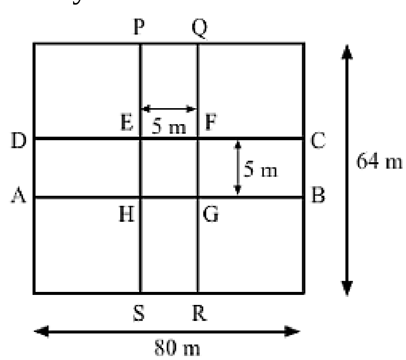
Because the border cannot be wider than the entire carpet, the width of the carpet is $\frac{1}{2}m$, i.e., 50 cm.

16. A 80 m by 64 m rectangular lawn has two roads, each 5 m wide, running through its middle, one parallel to its length and the other parallel to its breadth. Find the cost of gravelling the reads at ₹ $40 \ per \ m^2$.

Sol:

The length and breadth of the lawn are 80 m and 64 m, respectively.

The layout of the roads is shown in the figure below:



Area of the road ABCD = 80 $\times$ 5 = 400 m^2

Area of the road PQRS = 64 $\times$5 = 320 m^2

Clearly, the area EFGH is common in both the roads

Area EFGH = 5 $\times$ 5 = 25 m^2

Area of the roads = 400 + 320 – 25

= 695 m^2

Given:

Cost of gravelling 1 m^2 area = Rs 40

Cost of gravelling 695 m^2 area = 695×40

= Rs 27,800

17. The dimensions of a room are 14 m x 10 m x 6.5 m There are two doors and 4 windows in the room. Each door measures 2.5 m x 1.2 m and each window measures 1.5 m x 1 m. Find the cost of painting the four walls of the room at ₹ $35 \ per \ m^2$.

Sol:

The room has four walls to be painted

Area of these walls $= 2(l\times h)+2(b\times h)$

$=(2\times 14\times 6.5)+(2\times 10\times 6.5)$

$=312\,m^2$

Now,

Area of the two doors $=(2\times 2.5\times 1.2)=6\ m^2$

Area of the four windows $=(4\times 1.5\times 1)=6\ m^2$

The walls have to be painted; the doors and windows are not to be painted.

$\therefore$ Total area to be painted $=312-(6+6)=300\ m^2$

Cost for painting $1m^2 = Rs\ 35$

Cost for painting $300m^2 = 300\times 35 = Rs10,500$

18. The cost of painting the four walls of a room 12 m long at ₹ $30\,per\,m^2$ is ₹ $7560\,per\,m^2$ and the cost of covering the floor with the mat at ₹ $25\,per\,m^2$ is ₹ 2700. Find the dimensions of the room.

Sol:

As, the rate of covering the floor $=$ ₹ $25\ per\ m^2$

And, the cost of covering the floor $=$ ₹ 2700

So, the area of the floor $=\frac{2700}{25}$

$\Rightarrow length\times breadth = 108$

$\Rightarrow 12\times breadth = 108$

$\Rightarrow breadth = \frac{108}{12}$

$\therefore breadth = 9\,m$

Also,

As, the rate of painting the four walls $=$ ₹ $30\ per\ m^2$

And, the cost of painting the four walls $=$ ₹ 7560

So, the area of the four walls $=\frac{7560}{30}$

$\Rightarrow 2(\text{length+breadth})\text{height} = 252$

$\Rightarrow 2(12+9)\text{height} = 252$

$\Rightarrow 2(21)\text{height} = 252$

$\Rightarrow 42\times h\text{eight} = 252$

$\Rightarrow \text{height} = \frac{252}{42}$

$\therefore \text{height} = 6m$

So, the dimensions of the room are $12m \times 9m \times 6m$.

19. Find the area and perimeter of a square plot of land whose diagonal is 24 m long.

Sol:

Area of the square $= \frac{1}{2} \times Diagonal^2$

$= \frac{1}{2} \times 24 \times 24$

$= 288\ m^2$

Now, let the side of the square be $x\ m$.

Thus, we have:

$Area = Side^2$

$\Rightarrow 288 = x^2$

$\Rightarrow x = 12\sqrt{2}$

$\Rightarrow x = 16.92$

Perimeter $= 4 \times \text{Side}$

$= 4 \times 16.92$

$= 67.68m$

Thus, the perimeter of thee square plot is $67.68m$.

20. Find the length of the diagonal of a square whose area is $128cm^2$. Also, find its perimeter.

Sol:

Area of the square $= 128\ cm^2$

Area $= \frac{1}{2}d^2$ (where d is a diagonal of the square)

$\Rightarrow 128 = \frac{1}{2}d^2$

$\Rightarrow d^2 = 256$

$\Rightarrow d = 16cm$

Now,

$Area = Side^2$

$\Rightarrow 128 = Side^2$

$\Rightarrow Side = 11.31cm$

Perimeter $=4(Side)$

$=4(11.31)$

$=45.24\ cm$

21. The area of a square filed is 8 hectares. How long would a man take to cross it diagonally by walking at the rate of 4 km per hour?

Sol: Given, area of square field = 8 hectares

$=8\times 0.01[1\ hectare=0.01km^2]$

$=0.08km^2$

Now, area of square field = $(\text{si}\,de\text{ of square})^2=0.08$

$\Rightarrow$ side of square field = $\sqrt{0.08}=\frac{2\sqrt{2}}{10}=\frac{\sqrt{2}}{5}=km$

Distance covered by man along the diagonal of square field = length of diagonal

$\sqrt{2}\ Side=\sqrt{2}\times\frac{\sqrt{2}}{5}=\frac{2}{5}km$

Speed of walking = 4km/h

$\therefore$ Time taken = $\frac{\text{distane}}{\text{Speed}}=\frac{2}{5\times 4}=\frac{2}{20}=\frac{1}{10}$

= 0.1 hour

= $\frac{1}{10}\times 60\min=6\min utes$

22. The cost of harvesting a square field at ₹ 900 per hectare is ₹ 8100. Find the cost of putting a fence around it at ₹ 18 per meter.

Sol:

As, the rate of the harvesting = ₹900 *per* hectare

And, the cost of harvesting = ₹8100

So, the area of the square field $=\frac{8100}{900}=9$ hectare

$\Rightarrow$ the area $=90000\ m^2$ (As, 1 hectare = $10000\,m^2$)

$\Rightarrow (side)^2=90000$

$\Rightarrow side=\sqrt{90000}$

So, side $=300\,m$

Now, perimeter of the field $=4\times side$

$=4\times 300$

$=1200\ m$

Since, the rate of putting the fence $=₹18\ per\ m$

So, the cost of putting the fence $=1200\times18=₹\,21,600$

23. The cost of fencing a square lawn at ₹ 14 per meter is ₹ 28000. Find the cost of mowing the lawn at ₹ 54 $per\,100\,m^2$

Sol:

Cost of fencing the lawn $Rs\ 28000$

Let l be the length of each side of the lawn. Then, the perimeter is $4l$.

We know:

$Cost = Rate\times Perimeter$

$\Rightarrow 28000=14\times 41$

$\Rightarrow 28000=56l$

$Or,$

$l=\frac{28000}{56}=500m$

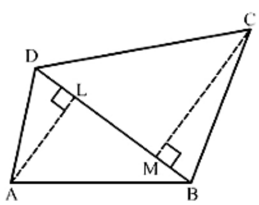
Area of the square lawn $=500\times500=250000\ m^2$

Cost of moving $100\,m^2$ of the lawn $=Rs\ 54$

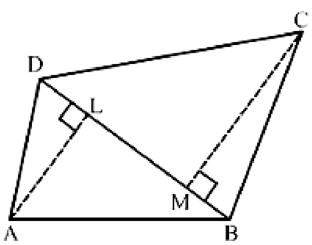
Cost of moving $1\ m^2$ of the lawn $=Rs\,\frac{54}{100}$

$\therefore$ Cost of moving $250000\,m^2$ of the lawn $=\frac{250000\times54}{100}=Rs\ 135000$

24. In the given figure ABCD is quadrilateral in which diagonal BD = 24 cm, $AL\perp BD$ and $CM\perp BD$ such that AL = 9cm and CM = 12 cm. Calculate the area of the quadrilateral.



Sol:



We have,

$BD = 24cm, AL = 9cm, CM = 12cm, AL \perp BD \text{ and } CM \perp BD$

Area of the quadrilateral $= ar(\Delta ABD) + ar(\Delta BCD)$

$= \frac{1}{2} \times BD \times AL + \frac{1}{2} \times BD \times CM$

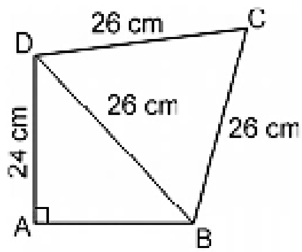
$= \frac{1}{2} \times 24 \times 9 + \frac{1}{2} \times 24 \times 12$

$= 108 + 144$

$= 252\ cm^2$

So, the area of the quadrilateral ABCD is $252\ cm^2$.

25. Find the area of the quadrilateral ABCD in which AD = 24 cm, $\angle BAD = 90°$ and ΔBCD is an equilateral triangle having each side equal to 26 cm. Also, find the perimeter of the quadrilateral.



Sol:

ΔBDC is an equilateral triangle with side $a = 26\ cm.$

Area of $\Delta BDC = \frac{\sqrt{3}}{4} a^2$

$= \frac{\sqrt{3}}{4} \times 26^2$

$= \frac{1.73}{4} \times 676$

$= 292.37\ cm^2$

By using Pythagoras theorem in the right – angled triangle $\Delta DAB,$ we get:

$AD^2 + AB^2 = BD^2$

$\Rightarrow 24^2 + AB^2 = 26^2$

$\Rightarrow AB^2 = 26^2 - 24^2$

$\Rightarrow AB^2 = 676 - 576$

$\Rightarrow AB^2 = 100$

$\Rightarrow AB = 10\, cm$

Area of $\Delta ABD = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 10 \times 24$

$= 120\, cm^2$

Area of the quadrilateral

$= \text{Area of } \Delta BCD + \text{Area of } \Delta ABD$

$= 292.37 + 120$

$= 412.37\, cm^2$

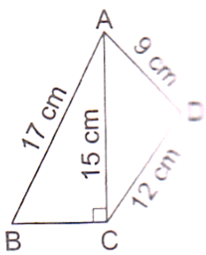
Perimeter of the quadrilateral

$= AB + AC + CD + AD$

$= 24 + 10 + 26 + 26$

$= 86\, cm$

26. Find the perimeter and area of the quadrilateral ABCD in which AB = 17 cm, AD = 9 cm, CD = 12 cm, $\angle ACB = 90°$ and AC = 15 cm.



Sol:

In the right angled ΔACB :

$AB^2 = BC^2 + AC^2$

$\Rightarrow 17^2 = BC^2 + 15^2$

$\Rightarrow 17^2 - 15^2 = BC^2$

$\Rightarrow 64 = BC^2$

$\Rightarrow BC = 8\, cm$

Perimeter $= AB + BC + CD + AD$

$= 17 + 8 + 12 + 9$

$= 46\, cm$

Area of $\Delta ABC = \frac{1}{2}(b \times h)$

$= \frac{1}{2}(8 \times 15)$

$= 60\, cm^2$

In ΔADC :

$AC^2 = AD^2 + CD^2$

So, ΔADC is a right – angled triangle at D.

$$\text{Area of } \Delta ADC = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 9 \times 12$$

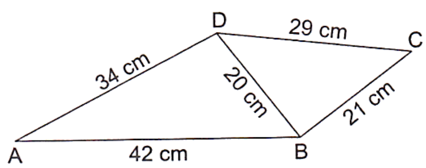
$$= 54\ cm^2$$

$\therefore$ Area of the quadrilateral = Area of ΔABC + Area of ΔADC

$= 60 + 54$

$= 114\ cm^2$

27. Find the area of the quadrilateral ABCD in which in AB = 42 cm, BC = 21 cm, CD = 29 cm, DA = 34 cm and diagonal BD = 20 cm.



Sol:

$$\text{Area of } \Delta ABD = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{1}{2}(a+b+c)$$

$$s = \frac{42+20+34}{2}$$

$$s = 48\ cm$$

$$\text{Area of } \Delta ABD = \sqrt{48(48-42)(48-20)(48-34)}$$

$$= \sqrt{48 \times 6 \times 28 \times 14}$$

$$= \sqrt{112896}$$

$$= 336\ cm^2$$

$$\text{Area of } \Delta BDC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{1}{2}(a+b+c)$$

$$s = \frac{21+20+29}{2}$$

$$s = 35\,cm$$

Area of $\Delta BDC = \sqrt{35(35-29)(35-20)(35-21)}$

$= \sqrt{35 \times 6 \times 15 \times 14}$

$= \sqrt{44100}$

$= 210\ cm^2$

$\therefore$ Area of quadrilateral ABCD = Area of $\Delta ABD +$ Area of ΔBDC

$= 336 + 210$

$= 546\, cm^2$

28. Find the area of a parallelogram with base equal to 25 cm and the corresponding height measuring 16.8 cm.

Sol:

Given:

Base = 25 cm

Height $= 16.8\ cm$

$\therefore$ Area of the parallelogram = Base $\times$ Height $= 25\, cm \times 16.8\ cm = 420\, cm^2$

29. The adjacent sides of a parallelogram are 32 cm and 24 cm. If the distance between the longer sides is 17.4 cm, find the distance between the shorter sides.

Sol:

Longer side = 32 cm

Shorter side = 24 cm

Let the distance between the shorter sides be x cm.

Area of a parallelogram = Longer side $\times$ Distance between the longer sides

= Shorter side $\times$ Distance between the shorter sides

or, $32 \times 17.4 = 24 \times x$

or, $x = \frac{32 \times 17.4}{24} = 23.2\ cm$

$\therefore$ Distance between the shorter sides $= 23.2\, cm$

30. The area of a parallelogram is $392\, m^2$. If its altitude is twice the corresponding base, determined the base and the altitude.

Sol:

Area of the parallelogram $= 392\, m^2$

Let the base of the parallelogram be b m.

Given:

Height of the parallelogram is twice the base

$\therefore$ Height = $2b$ m

Area of a parallelogram = Base $\times$ Height

$\Rightarrow 392 = b \times 2b$

$\Rightarrow 392 = 2b^2$

$\Rightarrow \frac{392}{2} = b^2$

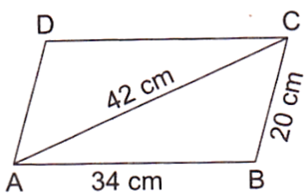
$\Rightarrow 196 = b^2$

$\Rightarrow b = 14$

$\therefore$ Base $= 14m$

Altitude $= 2 \times$ Base $= 2 \times 14 = 28m$

31. The adjacent sides of a parallelogram ABCD measure 34 cm and 20 cm, and the diagonal AC measures 42 cm. Find the area of the parallelogram.



Sol:

Parallelogram ABCD is made up of congruent ΔABC and ΔADC

Area of triangle $ABC = \sqrt{s(s-a)(s-b)(s-c)}$ (Here, s is the semi-perimeter)

Thus, we have:

$s = \frac{a+b+c}{2}$

$s = \frac{34+20+42}{2}$

$s = 48cm$

Area of $\Delta ABC = \sqrt{48(48-34)(48-20)(48-42)}$

$= \sqrt{48 \times 14 \times 28 \times 6}$

$= 336\ cm^2$

Now,

Area of the parallelogram $= 2 \times$ Area of ΔABC

$= 2 \times 336$

$= 672\ cm^2$

32. Find the area of the rhombus, the length of whose diagonals are 30 cm and 16 cm. Also, find the perimeter of the rhombus.

Sol:

Area of the rhombus $= \frac{1}{2} \times d_1 \times d_2$, where d_1 and d_2 are the lengths of the diagonals

$= \frac{1}{2} \times 30 \times 16$

$= 240\ cm^2$

Side of thee rhombus $= \frac{1}{2}\sqrt{d_1^2 + d_2^2}$

$= \frac{1}{2}\sqrt{30^2 + 16^2}$

$= \frac{1}{2}\sqrt{1156}$

$= \frac{1}{2} \times 34$

$= 17\ cm$

Perimeter of the rhombus $= 4a$

$= 4 \times 17$

$= 68\ cm$

33. The perimeter of a rhombus is 60 cm. If one of its diagonal us 18 cm long, find

(i) the length of the other diagonal, and

(ii) the area of the rhombus.

Sol:

Perimeter of a rhombus = 4a (Here, a is the side of the rhombus)

$\Rightarrow 60 = 4a$

$\Rightarrow a = 15\ cm$

(i) Given:

One of the diagonals is 18 cm long

$d_1 = 18 cm$

Thus, we have:

$Side = \frac{1}{2}\sqrt{d_1^2 + d_2^2}$

$\Rightarrow 15 = \frac{1}{2}\sqrt{18^2 + d_2^2}$

$\Rightarrow 30 = \sqrt{18^2 + d_2^2}$

Squaring both sides, we get:

$\Rightarrow 900 = 18^2 + d_2^2$

$\Rightarrow 900 = 324 + d_2^2$

$\Rightarrow d_2^2 = 576$

$\Rightarrow d_2 = 24\ cm$

$\therefore$ Length of the other diagonal = 24 cm

(ii) Area of the rhombus $= \frac{1}{2} d_1 \times d_2$

$= \frac{1}{2} \times 18 \times 24$

$= 216\, cm^2$

34. The area of rhombus is $480\, cm^2$, and one of its diagonal measures 48 cm. Find
(i) the length of the other diagonal,
(ii) the length of each of the sides
(iii) its perimeter

Sol:

(i) Area of a rhombus, $= \frac{1}{2} \times d_1 \times d_2$, where d_1 and d_2 are the lengths of the diagonals.

$\Rightarrow 480 = \frac{1}{2} \times 48 \times d_2$

$\Rightarrow d_2 = \frac{480 \times 2}{48}$

$\Rightarrow d_2 = 20\, cm$

$\therefore$ Length of the other diagonal = 20 cm

(ii) Side $= \frac{1}{2}\sqrt{d_1^2 + d_2^2}$

$= \frac{1}{2}\sqrt{48^2 + 20^2}$

$= \frac{1}{2}\sqrt{2304 + 400}$

$= \frac{1}{2}\sqrt{2704}$

$= \frac{1}{2} \times 52$

$= 26\ cm$

$\therefore$ Length of the side of the rhombus $= 26\ cm$

(iii) Perimeter of the rhombus = $4 \times$ Side

$= 4 \times 26$

$= 104\ cm$

35. The parallel sides of trapezium are 12 cm and 9cm and the distance between them is 8 cm. Find the area of the trapezium.

Sol:

Area of the trapezium $= \frac{1}{2} \times$ (sum of the parallel sides) $\times$ distance between the parallel sides

$= \frac{1}{2} \times (12 + 9) \times 8$

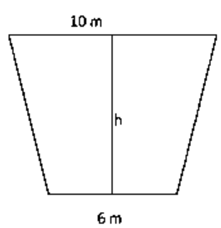
$= 21 \times 4$

$= 84\ cm^2$

So, the area of the trapezium is $84\ cm^2$.

36. The shape of the cross section of a canal is a trapezium. If the canal is 10 m wide at the top, 6 m wide at the bottom and the area of its cross section is $640\, m^2$, find the depth of the canal.

Sol:



Area of the canal $= 640\, m^2$

Area of trapezium $= \frac{1}{2} \times$ (Sum of parallel sides) $\times$ (Distance between them)

$\Rightarrow 640 = \frac{1}{2} \times (10 + 6) \times h$

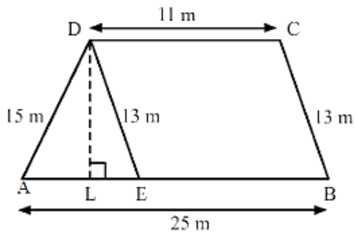
$\Rightarrow \frac{1280}{16} = h$

$\Rightarrow h = 80\, m$

Therefore, the depth of the canal is 80 m.

37. Find the area of trapezium whose parallel sides are 11 m and 25 m long, and the nonparallel sides are 15 m and 13 m long.

Sol:



Draw $DE \parallel BC$ and DL perpendicular to AB.

The opposite sides of quadrilateral DEBC are parallel. Hence, DEBC is a parallelogram

$\therefore DE = BC = 13m$

Also,

$AE = (AB - EB) = (AB - DC) = (25 - 11) = 14m$

For ΔDAE :

Let:

$AE = a = 14m$

$DE = b = 13m$

$DA = c = 15m$

Thus, we have:

$$s = \frac{a+b+c}{2}$$

$$s = \frac{14+13+15}{2} = 21m$$

Area of $\Delta DAE = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21 \times (21 \times 14) \times (21-13) \times (21-15)}$

$= \sqrt{21 \times 7 \times 8 \times 6}$

$= \sqrt{7056}$

$= 84\ m^2$

Area of $\Delta DAE = \frac{1}{2} \times AE \times DL$

$\Rightarrow 84 = \frac{1}{2} \times 14 \times DL$

$\Rightarrow \frac{84 \times 2}{14} = DL$

$\Rightarrow DL = 12\ m$

Area of trapezium $= \frac{1}{2} \times (Sum\ of\ \ parallel\ \ sides) \times (Dis\tan ce\ between\ them)$

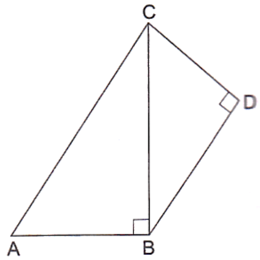
$= \frac{1}{2} \times (11 + 25) \times 12$

$= \frac{1}{2} \times 36 \times 12$

$= 216\ m^2$

Exercise - Formative Assessment

1. In the given figure ABCD is a quadrilateral in which $\angle ABC = 90°, \angle BDC = 90°, AC = 17\,cm, BC = 15\,cm, BD = 12\,cm$ and $CD = 9\,cm$. The area of quadrilateral ABCD is



(a) $102\,cm^2$

(b) $114\ cm^2$

(c) $95\ cm^2$

(d) $57\ cm^2$

Answer: (b) $114\ cm^2$

Sol:

Using Pythagoras theorem in ΔABC, we get:

$AC^2 = AB^2 + BC^2$

$\Rightarrow AB = \sqrt{AC^2 - BC^2}$

$= \sqrt{17^2 - 15^2}$

$= 8\,cm$

Area of $\Delta ABC = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 8 \times 15$

$= 60\ cm^2$

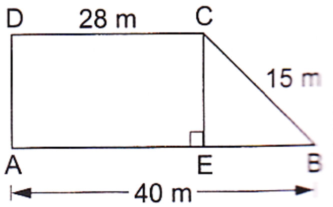
Area of $\Delta BCD = \frac{1}{2} \times BD \times CD$

$= \frac{1}{2} \times 12 \times 9$

$= 54\ cm^2$

$\therefore$ Area of quadrilateral $ABCD = Ar(\Delta ABC) + Ar(\Delta BCD) = 54 + 60 = 114 cm^2$

2. In the given figure $ABCD$ is a trapezium in which $AB = 40m, BC = 15m, CD = 28m,$ $AD = 9m$ and $CE \perp AB.$ Area of trapezium ABCD is



(a) $306\ m^2$

(b) $316\ m^2$

(c) $296\ m^2$

(d) $284\ m^2$

Answer: (a) $306\ m^2$

Sol:

In the given figure, AECD is a rectangle.

Length $AE =$ Length $CD = 28m$

Now,

$BE = AB - AE = 40 - 28 = 12m$

Also,

$AD = CE = 9m$

Area of trapezium $= \frac{1}{2} \times$ sum of parallel sides $\times$ Distance between them

$= \frac{1}{2} \times (DC + AB) \times CE$

$= \frac{1}{2} \times (28 + 40) \times 9$

$= \frac{1}{2} \times 68 \times 9$

$= 306\ m^2$

In the given figure, if DA is perpendicular to AE, then it can be solved, otherwise it cannot be solved.

3. The sides of a triangle are in the ratio 12: 14 : 25 and its perimeter is 25.5 cm. The largest side of the triangle is

(a) 7 cm (b) 14 cm (c) 12.5 cm (d) 18 cm

Answer: (c) 12.5 cm

Sol:

Let the sides of the triangle be $12x\ cm, 14x\ cm\ and\ 25x\ cm$

Thus, we have

$\text{Perimeter} = 12x + 14x + 25x$

$\Rightarrow 25.5 = 51x$

$\Rightarrow x = \frac{25.5}{51} = 0.5$

$\therefore$ Greatest side of the triangle $25x = 25 \times 0.5 = 12.5\ cm$

4. The parallel sides of a trapezium are 9.7cm and 6.3 cm, and the distance between them is 6.5 cm. The area of the trapezium is

(a) $104 cm^2$ (b) $78 cm^2$ (c) $52\ cm^2$ (d) $65 cm^2$

Answer: (c) $52\ cm^2$

Sol:

Area of trapezium $= \frac{1}{2}(\text{Sum of parallel sides}) \times \text{Distance between them}$

$= \frac{1}{2} \times (9.7 + 6.3) \times 6.5$

$= 8 \times 6.5$

$= 52.0\ cm^2$

5. Find the area of an equilateral triangle having each side of length $10 cm.$ (Take $\sqrt{3} = 1.732$)

Sol:

Given:

Side of the equilateral triangle = 10cm

Thus we have:

$\text{Area of the equilateral triangle} = \frac{\sqrt{3}}{4} side^2$

$= \frac{\sqrt{3}}{4} \times 10 \times 10$

$= 25 \times 1.732$

$= 43.3\ cm^2$

6. Find the area of an isosceles triangle each of whose equal sides is 13 cm and whose base is 24 cm.

Sol:

Area of an isosceles triangle:

$= \frac{1}{4} b\sqrt{4a^2 - b^2}$ (Where a is the length of the equal sides and b is the base)

$= \frac{1}{4} \times 24\sqrt{4(13)^2 - 24^2}$

$= 6\sqrt{4 \times 169 - 576}$

$= 6\sqrt{676 - 576}$

$= 6\sqrt{100}$

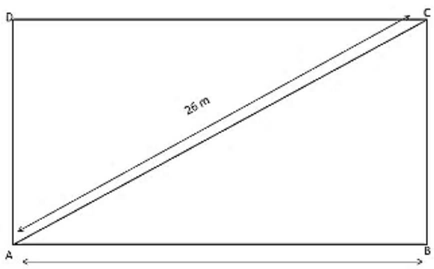
$= 6 \times 10$

$= 60\ cm^2$

7. The longer side of a rectangular hall is 24 m and the length of its diagonal is 26 m. Find the area of the hall.

Sol:

Let the rectangle ABCD represent the hall.



Using the Pythagoras theorem in the right-angled triangle ABC, we have

$Diagonal^2 = Length^2 + Breadth^2$

$\Rightarrow Breadth = \sqrt{Diagonal^2 - Length^2}$

$\Rightarrow \sqrt{26^2 - 24^2}$

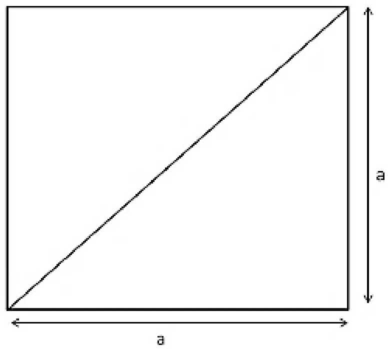
$= \sqrt{676 - 576}$

$= \sqrt{100}$

$= 10\, m$

$\therefore$ Area of the hall = $\text{Length} \times \text{Breadth} = 24 \times 10 = 240\ m^2$

8. The length of the diagonal of a square is 24 cm. Find its area.
Sol:
The diagonal of a square forms the hypotenuse of an isosceles right triangle. The other two sides are the sides of the square of length a cm.



Using Pythagoras theorem, we have:
Diagonal$^2 = a^2 + a^2 = 2a^2$
$\Rightarrow Diagonal = \sqrt{2}a$
Diagonal of the square $= 2\sqrt{a}$
$\Rightarrow 24 = \sqrt{2}a$
$\Rightarrow a = \frac{24}{\sqrt{2}}$
$\Rightarrow a = \frac{24}{\sqrt{2}}$
Area of the square $= Side^2 = \left(\frac{24}{\sqrt{2}}\right)^2 = \frac{24 \times 24}{2} = 288\ cm^2$

9. Find the area of a rhombus whose diagonals are 48 cm and 20cm long.
Sol:
Area of the rhombus $= \frac{1}{2}$(Product of diagonal)
$= \frac{1}{2}(48 \times 20)$
$= 480\ cm^2$

10. Find the area of a triangle whose sides are 42 cm, 34 cm and 20 cm.
Sol:
To find the area of the triangle, we will first find the semiperimeter of the triangle
Thus, we have:

$s=\frac{1}{2}(a+b+c)=\frac{1}{2}(42+34+20)=\frac{1}{2}\times 96=48\ cm$

Now,

Area of the triangle $=\sqrt{a(s-a)(s-b)(s-c)}$

$=\sqrt{48(48-42)(48-34)(48-20)}$

$=\sqrt{48\times 6\times 14\times 28}$

$=\sqrt{112896}$

$=336\ cm^2$

11. A lawn is in the form of a rectangle whose sides are in the ratio 5:3 and its area is $3375\ m^2$. Find the cost of fencing the lawn at ₹ 20 per metre.

Sol:

Let the length and breadth of the lawn be $5x$ m and $3x\,m,$ respectively.

Now,

Area of the lawn $=5x\times 3x=5x^2$

$\Rightarrow 15x^2=3375$

$\Rightarrow x=\sqrt{\frac{3375}{15}}$

$\Rightarrow x=\sqrt{225}=15$

Length $=5x=5\times 15=75m$

Breadth $=3x=3\times 15=45\ m$

$\therefore$ Perimeter of the lawn $=2$ (Length + breadth) $=2(75+45)=2\times 120=240\ m$

Total cost of fencing the lawn at Rs 20 per meter $=240\times 20=Rs\ 4800$

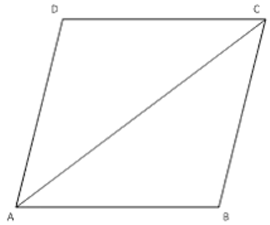
12. Find the area of a rhombus each side of which measures 20 cm and one of whose diagonals is 24 cm.

Sol:

Given:

Sides are 20 cm each and one diagonal is of 24 cm.

The diagonal divides the rhombus into two congruent triangles, as shown in the figure below.



We will now use Hero's formula to find the area of triangle ABC.
First, we will find thee semiperimeter

$s=\frac{1}{2}(a+b+c)=\frac{1}{2}(20+20+24)=\frac{64}{2}=32\,m$

Area of $\Delta ABC=\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{32(32-20)(32-20)(32-24)}$

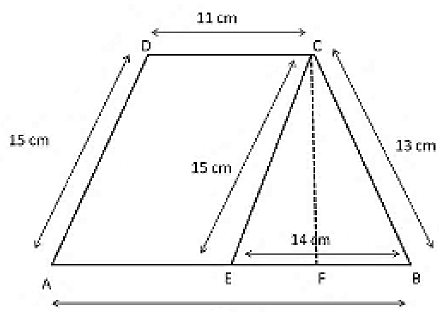
$=\sqrt{32\times12\times12\times8}$

$=\sqrt{36864}$

$=192\ cm^2$

Now,
Area of the rhombus $=2\times$ Area of triangle $ABC=192\times2=384\ cm^2$

13. Find the area of a trapezium whose parallel sides are 11 cm and 25 cm long and non-parallel sides are 15 cm and 13 cm.
Sol:
We will divide the trapezium into a triangle and a parallelogram
Difference in the lengths of parallel sides $=25-11=14\,cm$
We can represent this in the following figure:



Trapezium ABCD is divide into parallelogram AECD and triangle CEB.
Consider triangle CEB.
In triangle CEB, we have,

$EB=25-11=14\,cm$

Using Hero's theorem, we will first evaluate the semi-perimeter of triangle CEB and then evaluate its area.

Semi-perimeter $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(15+13+14) = \frac{42}{2} = 21\ cm$

Area of triangle $CEB = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-15)(21-13)(21-14)}$

$= \sqrt{21\times 6\times 8\times 7}$

$= \sqrt{7056}$

$= 84\,cm^2$

Also,

Area of triangle $CEB = \frac{1}{2}(Base\times height)$

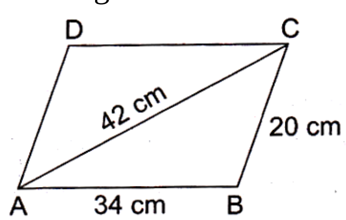
Height of triangle $CEB = \frac{Area\times 2}{Base} = \frac{84\times 2}{14} = 12\ cm$

Consider parallelogram $AECD$.

Area of parallelogram AECD $= Height\times Base = AE\times CF = 12\times 11 = 132\,cm^2$

Area of trapezium $ABCD = Ar(\Delta BEC) + Ar$ (parallelogram AECD) $= 132+84 = 216\ cm^2$

14. The adjacent sides of a ||gm ABCD measure 34 cm and 20 cm and the diagonal AC is 42 cm long. Find the area of the ||gm.



Sol:

The diagonal of a parallelogram divides it into two congruent triangles. Also, the area of the parallelogram is the sum of the areas of the triangles.

We will now use Hero's formula to calculate the area of triangle ABC.

Semiperimeter, $s = \frac{1}{2}(34+20+42) = \frac{1}{2}(96) = 48\,cm$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{48(48-42)(48-34)(48-20)}$

$= \sqrt{48\times 6\times 14\times 28}$

$= \sqrt{112826}$

$= 336\ cm^2$

Area of the parallelogram $= 2\times$ Area $\Delta ABC = 2\times 336 = 672\ cm^2$

15. The cost of fencing a square lawn at 14 per metre is 2800. Find the cost of mowing the lawn at ₹ 54 per 100 m^2.

Sol:

Given:

Cost of fencing = Rs 2800

Rate of fencing = Rs 14

Now,

Perimeter $= \frac{Total\ \cos t}{Rate} = \frac{2800}{14} = 200\,m$

Because the lawn is square, its perimeter is 4 a, where a is the side of the square)

$\Rightarrow 4a = 200 \Rightarrow a = \frac{200}{4} = 50\,m$

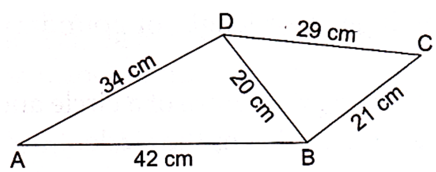
Area of the lawn $= Side^2 = 50^2 = 2500\ m^2$

Cost for mowing the lawn per $100\,m^2 = Rs\,54$

Cost for mowing the lawn per $1m^2 = Rs\,\frac{54}{100}$

Total cost for mowing the lawn per $2500\ m^2 = \frac{54}{100}\times 2500 = Rs\ 1350$

16. Find the area of quadrilateral ABCD in which $AB = 42\,cm, BC = 21\ cm, CD = 29\ cm, DA = 34\ cm$ and diagram $BD + 20\,cm.$



Sol:

Quadrilateral ABCD is divided into triangles $\Delta ABD\ and\ \Delta BCD.$

We will now use Hero's formula

For ΔABD:

Semiperimeter, $s = \frac{1}{2}(42+30+34) = \frac{96}{2} = 48\ cm$

Area of $\Delta ABD = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{48(48-42)(48-34)(48-20)}$

$= \sqrt{48\times 6\times 14\times 28}$

$= \sqrt{112896}$

$= 336\ cm^2$

For ΔBCD:

$s = \frac{1}{2}(20+21+29) = \frac{70}{2} = 35\ cm$

Area of $\Delta BCD = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{35(35-20)(35-21)(35-29)}$

$= \sqrt{35 \times 15 \times 14 \times 6}$

$= \sqrt{44100}$

$= 210\ cm^2$

Thus, we have:

Area of quadrilateral $ABCD = Ar(\Delta ABD) + Ar(BDC) = 336 + 210 = 546\ cm^2$

17. A parallelogram and a rhombus are equal in area. The diagonals of the rhombus measure 120 m and 44 m. If one of the sides of the || gm is 66 m long, find its corresponding altitude.

Sol:

Area of the rhombus $= \frac{1}{2}$ (Product of diagonals) $= \frac{1}{2}(120 \times 44) = 2640\ m^2$

Area of the parallelogram = Base $\times$ Height = 66 $\times$Height

Given:

The area of the rhombus is equal to the area of the parallelogram.

Thus, we have

$66 \times Height = 2640$

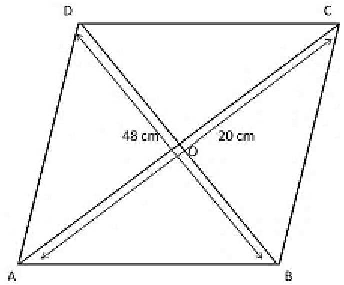
$\Rightarrow Height = \frac{2640}{66} = 40\,m$

$\therefore$ Corresponding height of the parallelogram = 40 m

18. The diagonals of a rhombus are 48 cm and 20 cm long. Find the perimeter of the rhombus.

Sol:

Diagonals of a rhombus perpendicularly bisect each other. The statement can help us find a side of the rhombus. Consider the following figure.



ABCD is the rhombus and AC and BD are the diagonals. The diagonals intersect at point O.

We know

$\angle DOC = 90°$

$DO = OB = \frac{1}{2}DB = \frac{1}{2} \times 48 = 24\ cm$

Similarly,

$AO = OC = \frac{1}{2}AC = \frac{1}{2} \times 20 = 10\,cm$

Using Pythagoras theorem in the right angled triangle ΔDOC, we get

$DC^2 = \sqrt{DO^2 + OC^2}$

$= \sqrt{24^2 + 10^2}$

$= \sqrt{576 + 100}$

$= \sqrt{676}$

$= 26\ cm$

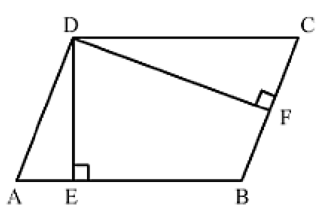
Dc is a side of the rhombus

We know that in a rhombus, all sides are equal.

$\therefore$ Perimeter of $ABCD = 26 \times 4 = 104\ cm$

19. The adjacent sides of a parallelogram are 36 cm and 27 cm in length. If the distance between the shorter sides is 12 cm, find the distance between the longer sides.

Sol:



Area of a parallelogram = Base $\times$Height

$\therefore AB \times DE = BC \times DF$

$\Rightarrow DE = \frac{BC \times DF}{AB}$

$= \frac{27 \times 12}{36}$

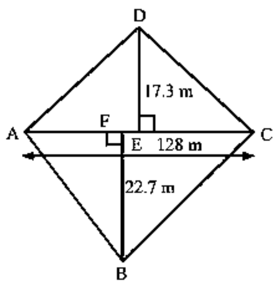
$= 9\ cm$

$\therefore$ Distance between thee longer sides $= 9\ cm$

20. In a four-sided field, the length of the longer diagonal is 128 m. The lengths of perpendiculars from the opposite vertices upon this diagonal are 22.7 m and 17.3 m. Find the area of the field.

Sol:

The field, which is represented as ABCD, is given below



The area of the field is the sum of the areas of triangles ABC and ADC.

Area of the triangle $ABC = \frac{1}{2}(AC \times BF) = \frac{1}{2}(128 \times 22.7) = 1452.8\,m^2$

Area of the triangle $ADC = \frac{1}{2}(AC \times DE) = \frac{1}{2}(128 \times 17.3) = 1107.2\ m^2$

Area of the field = Sum of the areas of both the triangles $= 1452.8 + 1107.2 = 2560\ m^2$