

(i) Here

$$\begin{array}{l} X: 3 \quad 2 \quad 1 \quad 0 \quad -1 \\ P(X): 0.3 \quad 0.2 \quad 0.4 \quad 0.1 \quad 0.05 \end{array}$$

$$\begin{aligned} & p(X=3) + p(X=2) + p(X=1) + p(X=0) + p(X=-1) \\ &= 0.3 + 0.2 + 0.4 + 0.1 + 0.05 \\ &= 1.05 \neq 1 \end{aligned}$$

So, the given distribution of probabilities is not a probability distribution.

(ii) Here

$$\begin{array}{l} X: 0 \quad 1 \quad 2 \\ P(X): 0.6 \quad 0.4 \quad 0.2 \end{array}$$

$$\begin{aligned} & p(X=0) + p(X=1) + p(X=2) \\ &= 0.6 + 0.4 + 0.2 \\ &= 1.2 \neq 1 \end{aligned}$$

So, the given distribution of probabilities is not a probability distribution.

(iii) Here

$$\begin{array}{l} X: 0 \quad 1 \quad 2 \quad 3 \quad 4 \\ P(X): 0.1 \quad 0.5 \quad 0.2 \quad 0.1 \quad 0.1 \end{array}$$

$$\begin{aligned} & p(X=0) + p(X=1) + p(X=2) + p(X=3) + p(X=4) \\ &= 0.1 + 0.5 + 0.2 + 0.1 + 0.1 \\ &= 1 \end{aligned}$$

So, the given distribution of probabilities is a probability distribution.

(iv) Here

$$\begin{array}{l} X: 0 \quad 1 \quad 2 \quad 3 \\ P(X): 0.3 \quad 0.2 \quad 0.4 \quad 0.1 \end{array}$$

$$\begin{aligned} & p(X=0) + p(X=1) + p(X=2) + p(X=3) \\ &= 0.3 + 0.2 + 0.4 + 0.1 \\ &= 1 \end{aligned}$$

So, the given distribution of probabilities is a probability distribution.

Mean and Variance of a Random Variable Ex 32.1 Q2

Here

$$\begin{array}{l} X: -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \\ P(X): 0.1 \quad k \quad 0.2 \quad 2k \quad 0.3 \quad k \end{array}$$

We know that,

$$\begin{aligned} & p(-2) + p(-1) + p(0) + p(1) + p(2) + p(3) = 1 \\ \Rightarrow & 0.1 + k + 0.2 + 2k + 0.3 + k = 1 \\ \Rightarrow & 4k + 0.6 = 1 \\ \Rightarrow & 4k = 1 - 0.6 \\ \Rightarrow & 4k = 0.4 \\ \Rightarrow & k = \frac{0.4}{4} \\ \Rightarrow & k = \frac{1}{10} \\ \Rightarrow & k = 0.1 \end{aligned}$$

Mean and Variance of a Random Variable Ex 32.1 Q3

Here

$X:$	0	1	2	3	4	5	6	7	8
$P(X):$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

Since $\sum P(X) = 1$

$$\begin{aligned}
 & P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) + P(8) = 1 \\
 \Rightarrow & a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 1 \\
 \Rightarrow & 81a = 1 \\
 \Rightarrow & a = \frac{1}{81}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad P(X < 3) &= P(0) + P(1) + P(2) \\
 &= a + 3a + 5a \\
 &= 9a \\
 &= 9 \left(\frac{1}{81} \right)
 \end{aligned}$$

$$\therefore P(X < 3) = \frac{1}{9}$$

$$P(X \geq 3) = 1 - P(X < 3) = 1 - \frac{1}{9} = \frac{8}{9}$$

$$\begin{aligned}
 P(0 < X < 5) &= P(1) + P(2) + P(3) + P(4) \\
 &= 3a + 5a + 7a + 9a \\
 &= 24a \\
 &= 24 \left(\frac{1}{81} \right)
 \end{aligned}$$

$$\therefore P(0 < X < 5) = \frac{8}{27}$$

Mean and Variance of a Random Variable Ex 32.1 Q4

Here:-

$x:$	0	1	2
$P(X):$	$3c^2$	$4c - 10c^2$	$5c - 1$

Where $c > 0$

$$\begin{aligned}
 \text{(i)} \quad & \text{since } \sum P(X) = 1 \\
 \Rightarrow & P(0) + P(1) + P(2) = 1 \\
 \Rightarrow & 3c^3 + 4c - 10c^2 + 5c - 1 = 1 \\
 \Rightarrow & 3c^3 - 10c^2 + 9c - 2 = 0 \\
 \Rightarrow & 3c^3 - 3c^2 - 7c^2 + 7c + 2c - 2 = 0 \\
 \Rightarrow & 3c^2(c - 1) - 7c(c - 1) + 2(c - 1) = 0 \\
 \Rightarrow & (c - 1)(3c^2 - 7c + 2) = 0 \\
 \Rightarrow & (c - 1)(3c^2 - 6c - c + 2) = 0 \\
 \Rightarrow & (c - 1)(3c(c - 2) - 1(c - 2)) = 0 \\
 & (c - 1)(3c - 1)(c - 2) = 0 \\
 & c = 1, \quad c = 2, \quad c = \frac{1}{3}
 \end{aligned}$$

Only $c = \frac{1}{3}$ is possible. Because if $c = 1$, or $c = 2$ then $P(2)$ will become negative.

$$\begin{aligned}
 \text{(ii)} \quad P(X < 2) &= P(0) + P(1) \\
 &= 3c^3 + 4c - 10c^2 \\
 &= 3\left(\frac{1}{3}\right)^3 + 4\left(\frac{1}{3}\right) - 10\left(\frac{1}{3}\right)^2 \\
 &= \frac{3}{27} + \frac{4}{3} - \frac{10}{9} \\
 &= \frac{1}{9} + \frac{4}{3} - \frac{10}{9} \\
 &= \frac{3}{9}
 \end{aligned}$$

$$\therefore P(X < 2) = \frac{1}{3}$$

$$\begin{aligned}
 \text{(iii)} \quad P(1 < X \leq 2) &= P(2) \\
 &= 5c - 1 \\
 &= 5\left(\frac{1}{3}\right) - 1
 \end{aligned}$$

$$\therefore P(1 < X \leq 2) = \frac{2}{3}$$

Mean and Variance of a Random Variable Ex 32.1 Q5

Here,

$$2P(X_1) = 3P(X_2) = P(X_3) = 5P(X_4)$$

$$\text{Let } P(X_3) = a$$

$$\begin{aligned}
 2P(X_1) &= P(X_3) & \Rightarrow & P(X_1) = \frac{a}{2} \\
 3P(X_2) &= P(X_3) & \Rightarrow & P(X_2) = \frac{a}{3} \\
 5P(X_4) &= P(X_3) & \Rightarrow & P(X_4) = \frac{a}{5}
 \end{aligned}$$

$$\text{Since } P(X_1) + P(X_2) + P(X_3) + P(X_4) = 1$$

$$\begin{aligned}
 \Rightarrow \quad \frac{a}{2} + \frac{a}{3} + \frac{a}{1} + \frac{a}{5} &= 1 \\
 \Rightarrow \quad \frac{15a + 10a + 30a + 6a}{30} &= 1 \\
 \Rightarrow \quad 61a &= 30 \\
 \Rightarrow \quad a &= \frac{30}{61}
 \end{aligned}$$

so,

$$\begin{array}{cccc}
 X & : & X_1 & X_2 & X_3 & X_4 \\
 P(X) & : & \frac{15}{61} & \frac{10}{61} & \frac{30}{61} & \frac{6}{61}
 \end{array}$$

Mean and Variance of a Random Variable Ex 32.1 Q6

Here,

$$P\{X = 0\} = P\{X > 0\} = P\{X < 0\}$$

$$\text{Let } P\{X = 0\} = k$$

$$\Rightarrow P\{X > 0\} = k = P\{X < 0\}$$

$$\text{Since } \sum P(X) = 1$$

$$\Rightarrow P\{X < 0\} + P\{X = 0\} + P\{X > 0\} = 1$$

$$\Rightarrow k + k + k = 1$$

$$\Rightarrow 3k = 1$$

$$\Rightarrow k = \frac{1}{3}$$

$$\text{So, } P\{X < 0\} =$$

$$P\{X = -1\} + P\{X = -2\} + P\{X = -3\} = \frac{1}{3}$$

$$3P\{X = -1\} = \frac{1}{3}, \quad [\because P\{X = -1\} = P\{X = -2\} = P\{X = -3\}]$$

$$P\{X = -1\} = \frac{1}{9}$$

$$\Rightarrow P\{X = -1\} = P\{X = -2\} = P\{X = -3\} = \frac{1}{9} \text{ ---- (i)}$$

$$\Rightarrow P\{X = 0\} = \frac{1}{3} \text{ ---- (ii)}$$

and

$$P\{X > 0\} = k$$

$$P\{X = 1\} + P\{X = 2\} + P\{X = 3\} = \frac{1}{3}$$

$$3P\{X = 1\} = \frac{1}{3}, \quad [\because P\{X = 1\} = P\{X = 2\} = P\{X = 3\}]$$

$$\Rightarrow P\{X = 1\} = \frac{1}{9}$$

$$\Rightarrow P\{X = 1\} = P\{X = 2\} = P\{X = 3\} = \frac{1}{9} \text{ ---- (iii)}$$

From equation (i), (ii), (iii),

X	-3	-2	-1	0	1	2	3
$P(X)$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

Mean and Variance of a Random Variable Ex 32.1 Q7

Let X denote number of aces in a sample of 2 cards drawn.

There are four aces in a pack of 52 cards.

So, X can have values 0, 1, 2

Now,

$$P\{X = 0\} = \frac{{}^{48}C_2}{{}^{52}C_2} = \frac{48 \times 47}{2} \times \frac{2}{52 \times 51} = \frac{188}{221}$$

$$P\{X = 1\} = \frac{{}^{48}C_1 \times {}^4C_1}{{}^{52}C_2} = \frac{48 \times 4 \times 2}{52 \times 51} = \frac{32}{221}$$

$$P\{X = 2\} = \frac{{}^4C_2}{{}^{52}C_2} = \frac{4 \times 3}{2} \times \frac{2}{52 \times 51} = \frac{1}{221}$$

So,

X	0	1	2
$P(X)$	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

Mean and Variance of a Random Variable Ex 32.1 Q8

Probability of getting a Head in one throw of a coin = $\frac{1}{2}$

$$P(H) = \frac{1}{2}$$

$$\Rightarrow P(T) = 1 - \frac{1}{2}$$

$$\Rightarrow P(T) = \frac{1}{2}$$

Let X denote the number of heads obtained in 3 throws of a coin.

Then $X = 0, 1, 2, 3$

Now,

$$P(X = 0) = P(T)P(T)P(T) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$P(X = 1) = P(H)P(T)P(T) + P(T)P(H)P(T) + P(T)P(T)P(H) \\ = \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right)$$

$$P(X = 1) = \frac{3}{8}$$

$$P(X = 2) = P(H)P(H)P(T) + P(H)P(T)P(H) + P(T)P(H)P(H) \\ = \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right)$$

$$P(X = 2) = \frac{3}{8}$$

$$P(X = 3) = P(H)P(H)P(H) \\ = \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) \\ = \frac{1}{8}$$

So,

Required probability distribution is

x	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Mean and Variance of a Random Variable Ex 32.1 Q9

Let x denote number of aces drawn out of 4 cards drawn.

There are four ace aces in a pack of 52.

So, $X = 0, 1, 2, 3, 4$

Now,

$$P(X = 0) = \frac{48C_4}{52C_4}$$

$$P(X = 1) = \frac{48C_3 \times 4C_1}{52C_4}$$

$$P(X = 2) = \frac{48C_2 \times 4C_2}{52C_4}$$

$$P(X = 3) = \frac{48C_1 \times 4C_3}{52C_4}$$

$$P(X = 4) = \frac{4C_4}{52C_4}$$

So,

Required probability distribution is

x	0	1	2	3	4
$P(X)$	$\frac{48C_4}{52C_4}$	$\frac{48C_3 \times 4C_1}{52C_4}$	$\frac{48C_2 \times 4C_2}{52C_4}$	$\frac{48C_1 \times 4C_3}{52C_4}$	$\frac{4C_4}{52C_4}$

Mean and Variance of a Random Variable Ex 32.1 Q10

A bag has 4 red and 6 black balls. Three balls are drawn.

Let X denote number of red balls out of 3 drawn.

Then $X = 0, 1, 2, 3$.

So,

$$P(\text{no red balls}) = P(X = 0) = \frac{{}^6C_3}{{}^{10}C_3} = \frac{6 \times 5 \times 4}{3 \times 2} \times \frac{3 \times 2}{10 \times 9 \times 8} = \frac{1}{6}$$

$$P(\text{one red balls}) = P(X = 1) = \frac{{}^4C_1 \times {}^6C_2}{{}^{10}C_3} = \frac{4 \times 6 \times 5}{2} \times \frac{3 \times 2}{10 \times 9 \times 8} = \frac{1}{2}$$

$$P(\text{two red balls}) = P(X = 2) = \frac{{}^4C_2 \times {}^6C_1}{{}^{10}C_3} = \frac{4 \times 3 \times 6}{2} \times \frac{3 \times 2}{10 \times 9 \times 8} = \frac{3}{10}$$

$$P(\text{all three red}) = P(X = 3) = \frac{{}^4C_3}{{}^{10}C_3} = \frac{4 \times 3 \times 2}{10 \times 9 \times 8} = \frac{1}{30}$$

The required probability distribution is

X	0	1	2	3
$P(X)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

Mean and Variance of a Random Variable Ex 32.1 Q11

Here 5 defective and 15 non-defective mangoes. Let X denote the defective mangoes drawn out of 4 mangoes drawn.

So, $X = 0, 1, 2, 3, 4$.

$$\begin{aligned}P(X = 0) &= \frac{{}^{15}C_4}{{}^{20}C_4} \\&= \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1} \times \frac{4 \times 3 \times 2 \times 1}{20 \times 19 \times 18 \times 17} \\&= \frac{91}{323}\end{aligned}$$

$$\begin{aligned}P(X = 1) &= \frac{{}^5C_1 \times {}^{15}C_3}{{}^{20}C_4} \\&= \frac{5 \times 15 \times 14 \times 13}{3 \times 2 \times 1} \times \frac{4 \times 3 \times 2 \times 1}{20 \times 19 \times 18 \times 17} \\&= \frac{455}{969}\end{aligned}$$

$$\begin{aligned}P(X = 2) &= \frac{{}^5C_2 \times {}^{15}C_2}{{}^{20}C_4} \\&= \frac{5 \times 4}{2} \times \frac{15 \times 14}{2} \times \frac{4 \times 3 \times 2 \times 1}{20 \times 19 \times 18 \times 17} \\&= \frac{70}{323}\end{aligned}$$

$$\begin{aligned}P(X = 3) &= \frac{{}^5C_3 \times {}^{15}C_1}{{}^{20}C_4} \\&= \frac{5 \times 4}{2} \times 15 \times \frac{4 \times 3 \times 2 \times 1}{20 \times 19 \times 18 \times 17} \\&= \frac{10}{323}\end{aligned}$$

$$\begin{aligned}P(X = 4) &= \frac{{}^5C_4}{{}^{20}C_4} \\&= 5 \times \frac{4 \times 3 \times 2 \times 1}{20 \times 19 \times 18 \times 17} \\&= \frac{1}{969}\end{aligned}$$

So, required probability distribution is

x	0	1	2	3	4
$P(X)$	$\frac{91}{323}$	$\frac{455}{969}$	$\frac{70}{323}$	$\frac{10}{323}$	$\frac{1}{969}$

Mean and Variance of a Random Variable Ex 32.1 Q12

Here, X denote the number of sum of two number or two dice thrown together

So, $X = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$.

So,

$$P(X = 2) = \frac{1}{36} \quad [\text{Possible pairs: } (1, 1)]$$

$$P(X = 3) = \frac{2}{36} = \frac{1}{18} \quad [\text{Possible pairs: } (1, 2), (2, 1)]$$

$$P(X = 4) = \frac{3}{36} = \frac{1}{12} \quad [\text{Possible pairs: } (1, 3), (2, 2), (3, 1)]$$

$$P(X = 5) = \frac{4}{36} = \frac{1}{9} \quad [\text{Possible pairs: } (1, 4), (2, 3), (3, 2), (4, 1)]$$

$$P(X = 6) = \frac{5}{36} \quad [\text{Possible pairs: } (1,5), (2,4), (3,3), (4,2), (5,1)]$$

$$P(X = 7) = \frac{6}{36} = \frac{1}{6} \quad [\text{Possible pairs: } (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)]$$

$$P(X = 8) = \frac{5}{36} \quad [\text{Possible pairs: } (2,6), (3,5), (4,4), (5,3), (6,2)]$$

$$P(X = 9) = \frac{4}{36} = \frac{1}{9} \quad [\text{Possible pairs: } (3,6), (4,5), (5,4), (6,3)]$$

$$P(X = 10) = \frac{3}{36} = \frac{1}{12} \quad [\text{Possible pairs: } (4,6), (5,5), (6,4)]$$

$$P(X = 11) = \frac{2}{36} = \frac{1}{18} \quad [\text{Possible pairs: } (5,6), (6,5)]$$

$$P(X = 12) = \frac{1}{36} \quad [\text{Possible pairs: } (6,6)]$$

So, required probability distribution is

x	2	3	4	5	6	7	8	9	10	11	12
$P(X)$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

Mean and Variance of a Random Variable Ex 32.1 Q13

There are 15 students in the class. Each student has the same chance to be chosen.

Therefore, the probability of each student to be selected is $\frac{1}{15}$.

The given information can be compiled in the frequency table as follows.

x	14	15	16	17	18	19	20	21
f	2	1	2	3	1	2	3	1

$$P(X = 14) = \frac{2}{15}, P(X = 15) = \frac{1}{15}, P(X = 16) = \frac{2}{15}, P(X = 17) = \frac{3}{15},$$

$$P(X = 18) = \frac{1}{15}, P(X = 19) = \frac{2}{15}, P(X = 20) = \frac{3}{15}, P(X = 21) = \frac{1}{15}$$

Therefore, the probability distribution of random variable X is as follows.

x	14	15	16	17	18	19	20	21
f	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

Mean and Variance of a Random Variable Ex 32.1 Q14

Here 5 defective and 20 non-defective bolts. Let X denote the number of defective bolts drawn out of 4 bolts drawn. So, X can have values 0,1,2,3,4.

$$\begin{aligned} P(X=0) &= \frac{{}^{20}C_4}{{}^{25}C_4} \\ &= \frac{20 \times 19 \times 18 \times 17}{25 \times 24 \times 23 \times 22} \\ &= \frac{969}{2530} \end{aligned}$$

$$\begin{aligned} P(X=1) &= \frac{{}^5C_1 \times {}^{20}C_3}{{}^{25}C_4} \\ &= \frac{5 \times 20 \times 19 \times 18}{3 \times 2 \times 1} \times \frac{4 \times 3 \times 2}{25 \times 24 \times 23 \times 22} \\ &= \frac{114}{253} \end{aligned}$$

$$\begin{aligned} P(X=2) &= \frac{{}^5C_2 \times {}^{20}C_2}{{}^{25}C_4} \\ &= \frac{5 \times 4}{2} \times \frac{20 \times 19}{2} \times \frac{4 \times 3 \times 2 \times 1}{25 \times 24 \times 23 \times 22} \\ &= \frac{38}{253} \end{aligned}$$

$$\begin{aligned} P(X=3) &= \frac{{}^5C_3 \times {}^{20}C_1}{{}^{25}C_4} \\ &= \frac{5 \times 4}{2} \times \frac{20 \times 4 \times 3 \times 2 \times 1}{25 \times 24 \times 23 \times 22} \\ &= \frac{4}{253} \end{aligned}$$

$$\begin{aligned} P(X=4) &= \frac{{}^5C_4}{{}^{25}C_4} \\ &= 5 \times \frac{4 \times 3 \times 2 \times 1}{25 \times 24 \times 23 \times 22} \\ &= \frac{1}{2530} \end{aligned}$$

So, required probability distribution is

$x :$	0	1	2	3	4
$P(x) :$	$\frac{969}{2530}$	$\frac{114}{253}$	$\frac{38}{253}$	$\frac{4}{253}$	$\frac{1}{2530}$

Mean and Variance of a Random Variable Ex 32.1 Q15

Two cards are drawn successively with replacement from a pack of 52 cards.

Let X be the number of aces obtained. Then $X = 0, 1, 2$.

$$\begin{aligned} P(X=0) &= P(\bar{A}_1) \times P(\bar{A}_2) \\ &= \frac{48}{52} \times \frac{48}{52} \\ &= \frac{144}{169} \end{aligned}$$

$$\begin{aligned} P(X=1) &= P(A_1)P(\bar{A}_2) + P(\bar{A}_1)P(A_2) \\ &= \frac{4}{52} \times \frac{48}{52} + \frac{48}{52} \times \frac{4}{52} \\ &= \frac{24}{169} \end{aligned}$$

$$\begin{aligned} P(X=2) &= P(A_1)P(A_2) \\ &= \frac{4}{52} \times \frac{4}{52} \\ &= \frac{1}{169} \end{aligned}$$

So,

Required probability distribution is

$$\begin{array}{ccc} X : & 0 & 1 & 2 \\ P(X) : & \frac{144}{169} & \frac{24}{169} & \frac{1}{169} \end{array}$$

Mean and Variance of a Random Variable Ex 32.1 Q16

Two cards are drawn successively with replacement from a pack of 52 cards. Let X denote the number of kings drawn out of 2 cards.

So, $X = 0, 1, 2$.

$$\begin{aligned} P(X=0) &= P(\bar{K}_1) \times P(\bar{K}_2) \\ &= \frac{48}{52} \times \frac{48}{52} \\ &= \frac{144}{169} \end{aligned}$$

$$\begin{aligned} P(X=1) &= P(K_1)P(\bar{K}_2) + P(\bar{K}_1)P(K_2) \\ &= \frac{4}{52} \times \frac{48}{52} + \frac{48}{52} \times \frac{4}{52} \\ &= \frac{24}{169} \end{aligned}$$

$$\begin{aligned} P(X=2) &= P(K_1)P(K_2) \\ &= \frac{4}{52} \times \frac{4}{52} \\ &= \frac{1}{169} \end{aligned}$$

So, required probability distribution is

$$\begin{array}{ccc} X : & 0 & 1 & 2 \\ P(X) : & \frac{144}{169} & \frac{24}{169} & \frac{1}{169} \end{array}$$

Mean and Variance of a Random Variable Ex 32.1 Q17

Two cards are drawn without replacement from a pack of 52 cards. Let X denote the number of aces drawn from pack out of 2 cards. So, $X = 0, 1, 2$.

$$\begin{aligned} P(X = 0) &= \frac{{}^{48}C_2}{{}^{52}C_2} \\ &= \frac{48 \times 47}{2} \times \frac{2 \times 1}{52 \times 51} \\ &= \frac{188}{221} \end{aligned}$$

$$\begin{aligned} P(X = 1) &= \frac{{}^4C_1 \times {}^{48}C_1}{{}^{52}C_2} \\ &= \frac{4 \times 48 \times 2}{52 \times 51} \\ &= \frac{32}{221} \end{aligned}$$

$$\begin{aligned} P(X = 2) &= \frac{{}^4C_2}{{}^{52}C_2} \\ &= \frac{4 \times 3}{2} \times \frac{2}{52 \times 51} \\ &= \frac{1}{221} \end{aligned}$$

So, required probability distribution is

$$\begin{array}{ccc} x : & 0 & 1 & 2 \\ P(x) : & \frac{188}{221} & \frac{32}{221} & \frac{1}{221} \end{array}$$

Mean and Variance of a Random Variable Ex 32.1 Q18

Given bag have 4 white and 6 red balls. Let X denote the number of white balls out of 3 balls drawn without replacement, So, $X = 0, 1, 2, 3$.

$$\begin{aligned} P(\text{No white ball}) &= \frac{{}^6C_3}{{}^{10}C_3} \\ &= \frac{6 \times 5 \times 4}{3 \times 2} \times \frac{3 \times 2}{10 \times 9 \times 8} \\ &= \frac{5}{30} \end{aligned}$$

$$\begin{aligned} P(\text{One white ball}) &= \frac{{}^4C_1 \times {}^6C_2}{{}^{10}C_3} \\ &= \frac{4 \times 6 \times 5}{2} \times \frac{3 \times 2}{10 \times 9 \times 8} \\ &= \frac{15}{30} \end{aligned}$$

$$\begin{aligned} P(\text{Two white balls}) &= \frac{{}^4C_2 \times {}^6C_1}{{}^{10}C_3} \\ &= \frac{4 \times 3}{2} \times \frac{6 \times 3 \times 2}{10 \times 9 \times 8} \\ &= \frac{9}{30} \end{aligned}$$

$$\begin{aligned} P(\text{Three white balls}) &= \frac{{}^4C_3}{{}^{10}C_3} \\ &= \frac{4 \times 3 \times 2 \times 1}{10 \times 9 \times 8} \\ &= \frac{1}{30} \end{aligned}$$

So,

Required probability distribution is

$$\begin{array}{cccc} x : & 0 & 1 & 2 & 3 \\ P(x) : & \frac{5}{30} & \frac{15}{30} & \frac{9}{30} & \frac{1}{30} \end{array}$$

Mean and Variance of a Random Variable Ex 32.1 Q19

Since total is 9 when dice has $(3,6)$ $(4,5)$ $(5,4)$ $(6,3)$

$$\therefore P(\text{A total of 9 appears}) = P(A) = \frac{4}{36}$$

Two dice are thrown 2 times.

Here, Y denotes the numbers of times a total of 9 appears.

So, $Y = 0, 1, 2$

$$\begin{aligned} P(Y = 0) &= P(\bar{A}_1) \times P(\bar{A}_2) \\ &= \frac{32}{36} \times \frac{32}{36} \\ &= \frac{64}{81} \end{aligned}$$

$$\begin{aligned} P(Y = 1) &= P(A_1)P(\bar{A}_2) + P(\bar{A}_1)P(A_2) \\ &= \frac{4}{36} \times \frac{32}{36} + \frac{32}{36} \times \frac{4}{36} \\ &= \frac{16}{81} \end{aligned}$$

$$\begin{aligned} P(Y = 2) &= P(A_1)P(A_2) \\ &= \frac{4}{36} \times \frac{4}{36} \\ &= \frac{1}{81} \end{aligned}$$

So,

Required probability distribution is

$$\begin{array}{ccc} X & : & 0 \quad 1 \quad 2 \\ P(X) & : & \frac{64}{81} \quad \frac{16}{81} \quad \frac{1}{81} \end{array}$$

Mean and Variance of a Random Variable Ex 32.1 Q20

Given 25 items in the lot. 5 are defective. Good items are 20.

4 items are chosen at random.

Let X be the random variable that denotes the number of defective items in the selected lot.

$$\begin{aligned} P(X = 0) &= P(4 \text{ non-defective and } 0 \text{ defective}) = {}^5C_0 \cdot {}^{20}C_4 / {}^{25}C_4 \\ &= 4845/12650 \end{aligned}$$

$$\begin{aligned} P(X = 1) &= P(3 \text{ non-defective and } 1 \text{ defective}) = {}^5C_1 \cdot {}^{20}C_3 / {}^{25}C_4 \\ &= 5 \times 1140 / 12650 \end{aligned}$$

$$\begin{aligned} P(X = 2) &= P(2 \text{ non-defective and } 2 \text{ defective}) = {}^5C_2 \cdot {}^{20}C_2 / {}^{25}C_4 \\ &= 10 \times 190 / 12650 \end{aligned}$$

$$\begin{aligned} P(X = 3) &= P(1 \text{ non-defective and } 3 \text{ defective}) = {}^5C_3 \cdot {}^{20}C_1 / {}^{25}C_4 \\ &= 10 \times 20 / 12650 \end{aligned}$$

$$\begin{aligned} P(X = 4) &= P(0 \text{ non-defective and } 4 \text{ defective}) = {}^5C_4 \cdot {}^{20}C_0 / {}^{25}C_4 \\ &= 5 / 12650 \end{aligned}$$

Mean and Variance of a Random Variable Ex 32.1 Q21

Three cards are thrown with replacement. Let X denote the numbers of hearts if three cards are drawn.

So, X has values 0, 1, 2, 3

$$\begin{aligned} P(X=0) &= P(\overline{H_1}) \times P(\overline{H_2}) \times P(\overline{H_3}) \\ &= \frac{39}{52} \times \frac{39}{52} \times \frac{39}{52} \\ &= \frac{27}{64} \end{aligned}$$

$$\begin{aligned} P(X=1) &= P(H_1)P(\overline{H_2})P(\overline{H_3}) + P(\overline{H_1})P(H_2)P(\overline{H_3}) + P(\overline{H_1})P(\overline{H_2})P(H_3) \\ &= \frac{13}{52} \times \frac{39}{52} \times \frac{39}{52} + \frac{39}{52} \times \frac{13}{52} \times \frac{39}{52} + \frac{39}{52} \times \frac{39}{52} \times \frac{13}{52} \\ &= \frac{27}{64} \end{aligned}$$

$$\begin{aligned} P(X=2) &= P(H_1)P(H_2)P(\overline{H_3}) + P(H_1)P(\overline{H_2})P(H_3) + P(\overline{H_1})P(H_2)P(H_3) \\ &= \frac{13}{52} \times \frac{13}{52} \times \frac{39}{52} + \frac{13}{52} \times \frac{39}{52} \times \frac{13}{52} + \frac{39}{52} \times \frac{13}{52} \times \frac{13}{52} \\ &= \frac{9}{64} \end{aligned}$$

$$\begin{aligned} P(X=3) &= P(H_1)P(H_2)P(H_3) \\ &= \frac{13}{52} \times \frac{13}{52} \times \frac{13}{52} \\ &= \frac{1}{64} \end{aligned}$$

So,

Required probability distribution is

x	0	1	2	3
$P(x)$	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

Mean and Variance of a Random Variable Ex 32.1 Q22

Urn has 4 red and 3 blue balls. 3 balls are drawn with replacement.

Let X denote numbers of blue balls drawn out of 3 drawn.

So, X has values 0, 1, 2, 3

$$\begin{aligned} P(X=0) &= P(\overline{B_1}) \times P(\overline{B_2}) \times P(\overline{B_3}) \\ &= \frac{4}{7} \times \frac{4}{7} \times \frac{4}{7} \\ &= \frac{64}{343} \end{aligned}$$

$$\begin{aligned} P(X=1) &= P(B_1)P(\overline{B_2})P(\overline{B_3}) + P(\overline{B_1})P(B_2)P(\overline{B_3}) + P(\overline{B_1})P(\overline{B_2})P(B_3) \\ &= \frac{3}{7} \times \frac{4}{7} \times \frac{4}{7} + \frac{4}{7} \times \frac{3}{7} \times \frac{4}{7} + \frac{4}{7} \times \frac{4}{7} \times \frac{3}{7} \\ &= \frac{144}{343} \end{aligned}$$

$$\begin{aligned} P(X=2) &= P(B_1)P(B_2)P(\overline{B_3}) + P(B_1)P(\overline{B_2})P(B_3) + P(\overline{B_1})P(B_2)P(B_3) \\ &= \frac{3}{7} \times \frac{3}{7} \times \frac{4}{7} + \frac{3}{7} \times \frac{4}{7} \times \frac{3}{7} + \frac{4}{7} \times \frac{3}{7} \times \frac{3}{7} \\ &= \frac{108}{343} \end{aligned}$$

$$\begin{aligned} P(X=3) &= P(B_1)P(B_2)P(B_3) \\ &= \frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} \\ &= \frac{27}{343} \end{aligned}$$

So,

Required probability distribution is

x	0	1	2	3
$P(x)$	$\frac{64}{343}$	$\frac{144}{343}$	$\frac{108}{343}$	$\frac{27}{343}$

Mean and Variance of a Random Variable Ex 32.1 Q23

Two cards are drawn simultaneously .Let X denote the number of spades obtained.

So, X can have values 0,1,2.

$$\begin{aligned} P(X=0) &= \frac{{}^{39}C_2}{{}^{52}C_2} \\ &= \frac{39 \times 38}{52 \times 51} \\ &= \frac{19}{34} \end{aligned}$$

$$\begin{aligned} P(X=1) &= \frac{{}^{39}C_1 \times {}^{13}C_1}{{}^{52}C_2} \\ &= \frac{13 \times 39 \times 2}{52 \times 51} \\ &= \frac{13}{34} \end{aligned}$$

$$\begin{aligned} P(X=2) &= \frac{{}^{13}C_2}{{}^{52}C_2} \\ &= \frac{13 \times 12}{52 \times 51} \\ &= \frac{2}{34} \end{aligned}$$

So,

Required probability distribution is

$x :$	0	1	2
$P(x) :$	$\frac{19}{34}$	$\frac{13}{34}$	$\frac{2}{34}$

Mean and Variance of a Random Variable Ex 32.1 Q24

Let A be the event of occurrence of a number less than 3.

$$P(A) = \frac{2}{6} \quad [\because 1, 2 \text{ are less than } 3.]$$

$$P(\bar{A}) = \frac{1}{3}$$

Let X denote the number of success in 2 throws of die.

So, X has value 0,1,2.

$$\begin{aligned} P(X=0) &= P(\bar{A}_1) \times P(\bar{A}_2) \\ &= \frac{2}{3} \times \frac{2}{3} \\ &= \frac{4}{9} \end{aligned}$$

$$\begin{aligned} P(X=1) &= P(A_1)P(\bar{A}_2) + P(\bar{A}_1)P(A_2) \\ &= \frac{1}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{1}{3} \\ &= \frac{4}{9} \end{aligned}$$

$$\begin{aligned} P(X=2) &= P(A_1)P(A_2) \\ &= \frac{1}{3} \times \frac{1}{3} \\ &= \frac{1}{9} \end{aligned}$$

So,

Required probability distribution is

$x :$	0	1	2
$P(x) :$	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

Mean and Variance of a Random Variable Ex 32.1 Q25

Urn has 5 red and 2 black balls. 2 balls are randomly selected.

Here, X denote the numbers of black balls.

So, possible values of $X = 0, 1, 2$

$$P(X = 0) = P(\bar{B}_1) \times P(\bar{B}_2)$$

$$= \frac{5}{7} \times \frac{5}{7}$$

$$= \frac{25}{49}$$

$$P(X = 1) = P(B_1)P(\bar{B}_2) + P(\bar{B}_1)P(B_2)$$

$$= \frac{2}{7} \times \frac{5}{7} + \frac{5}{7} \times \frac{2}{7}$$

$$= \frac{20}{49}$$

$$P(X = 2) = P(B_1)P(B_2)$$

$$= \frac{2}{7} \times \frac{2}{7}$$

$$= \frac{4}{49}$$

Now,

$$P(X = 0) + P(X = 1) + P(X = 2)$$

$$= \frac{25}{49} + \frac{20}{49} + \frac{4}{49}$$

$$= \frac{49}{49}$$

$$= 1$$

So, $\sum P(X) = 1$

Therefore

X is a random variable

Mean and Variance of a Random Variable Ex 32.1 Q26

Here, coin is tossed 6 times.

So, there can have

1H 5T or 2H 4T or 3H 3T or

4H 2T or 5H 1T or 6H or

6T

Here, X denote the difference between the number of head and number of tails.

So,

$$X = 6, 4, 2, 0, -2, -4, -6$$

Mean and Variance of a Random Variable Ex 32.1 Q27

It is given that out of 10 bulbs, 3 are defective.

Number of non-defective bulbs = $10 - 3 = 7$

2 bulbs are drawn from the lot with replacement.

Let X be the random variable that denotes the number of defective bulbs in the selected bulbs.

$$\therefore P(X = 0) = \frac{{}^7C_2}{{}^{10}C_2}$$

$$= \frac{7}{15}$$

$$\therefore P(X = 1) = \frac{{}^3C_1 \times {}^7C_1}{{}^{10}C_2}$$

$$= \frac{7}{15}$$

$$\therefore P(X = 2) = \frac{{}^3C_2}{{}^{10}C_2}$$

$$= \frac{1}{15}$$

Therefore, the required probability distribution is

X	0	1	2
$P(X)$	$\frac{7}{15}$	$\frac{7}{15}$	$\frac{1}{15}$

Clearly, X can assume values 0, 1, 2, 3, 4 such that

$$P(X = 0) = (\text{Probability of getting no red ball}) = \frac{{}^8C_0 \times {}^4C_4}{{}^{12}C_4} = \frac{1 \times 1}{\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}} = \frac{1}{495}$$

$$P(X = 1) = (\text{Probability of getting one red ball}) = \frac{{}^8C_1 \times {}^4C_3}{{}^{12}C_4} = \frac{8 \times 4}{\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}} = \frac{32}{495}$$

$$P(X = 2) = (\text{Probability of getting two red balls}) = \frac{{}^8C_2 \times {}^4C_2}{{}^{12}C_4} = \frac{\frac{8 \times 7}{2 \times 1} \times \frac{4 \times 3}{2 \times 1}}{\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}} = \frac{168}{495}$$

$$P(X = 3) = (\text{Probability of getting three red balls}) = \frac{{}^8C_3 \times {}^4C_1}{{}^{12}C_4} = \frac{\frac{8 \times 7 \times 6}{3 \times 2 \times 1} \times 4}{\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}} = \frac{224}{495}$$

$$P(X = 4) = (\text{Probability of getting four red balls}) = \frac{{}^8C_4 \times {}^4C_0}{{}^{12}C_4} = \frac{\frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} \times 1}{\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}} = \frac{70}{495}$$

Thus, probability distribution of random variable X is,

X	0	1	2	3	4
P(X)	$\frac{1}{495}$	$\frac{32}{495}$	$\frac{168}{495}$	$\frac{224}{495}$	$\frac{70}{495}$

(i) We know that,

$$P(0) + P(1) + P(2) + P(3) = 1$$

$$k + \frac{k}{2} + \frac{k}{4} + \frac{k}{8} = 1$$

$$15k = 8$$

$$k = \frac{8}{15}$$

$$(ii) P(X \leq 2)$$

$$= P(0) + P(1) + P(2)$$

$$= k + \frac{k}{2} + \frac{k}{4}$$

$$= \frac{8}{15} + \frac{8}{30} + \frac{8}{60}$$

$$= \frac{14}{15}$$

$$P(X > 2) = P(3) = \frac{k}{8} = \frac{1}{15}$$

$$(iii) P(X \leq 2) + P(X > 2)$$

$$= P(0) + P(1) + P(2) + P(3)$$

$$= 1$$

EX - 32.2

Mean and Variance of a Random Variable Ex 32.2 Q1(i)

x_i	p_i	$p_i x_i$	$p_i x_i^2$
2	0.2	0.4	0.8
3	0.5	1.5	4.5
4	0.3	1.2	4.8
		$\Sigma p_i x_i = 3.1$	$\Sigma p_i x_i^2 = 10.1$

$$\text{Mean} = \Sigma p_i x_i = 3.1$$

$$\text{Variance} = \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2 = 10.1 - (3.1)^2 = 0.49$$

$$\text{Standard deviation} = \sqrt{\text{Variance}} = 0.7$$

Mean and Variance of a Random Variable Ex 32.2 Q1(ii)

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
1	0.4	0.4	0.4
3	0.1	0.3	0.9
4	0.2	0.8	3.2
5	0.3	1.5	7.5
		$\Sigma x p = 3$	$\Sigma x^2 p = 12$

$$\text{Mean} = \Sigma x p$$

$$\text{mean} = 3$$

$$\begin{aligned} \text{Standard deviation} &= \sqrt{\Sigma x^2 p - (\text{mean})^2} \\ &= \sqrt{12 - (3)^2} \\ &= \sqrt{3} \end{aligned}$$

$$\text{Standard Deviation} = 1.732$$

Mean and Variance of a Random Variable Ex 32.2 Q1(iii)

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
-5	$\frac{1}{4}$	$-\frac{5}{4}$	$\frac{25}{4}$
-4	$\frac{1}{8}$	$-\frac{1}{2}$	2
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
2	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$
		$\Sigma x p = -1$	$\Sigma x^2 p = \frac{37}{4}$

$$\text{Mean} = \Sigma x p$$

$$\text{mean} = -1$$

$$\text{Standard deviation} = \sqrt{\Sigma x^2 p - (\text{mean})^2}$$

$$= \sqrt{\frac{37}{4} - (-1)^2}$$

$$= \sqrt{\frac{33}{4}}$$

$$= \sqrt{8.25}$$

$$\text{Standard Deviation} = 2.9$$

Mean and Variance of a Random Variable Ex 32.2 Q1(iv)

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
-1	0.3	-0.3	0.3
0	0.1	0	0
1	0.1	0.1	0.1
2	0.3	0.6	1.2
3	0.2	0.6	1.8
		$\Sigma x p = 1$	$\Sigma x^2 p = 3.4$

$$\text{Mean} = \Sigma x p$$

$$\text{mean} = 1$$

$$\text{Standard deviation} = \sqrt{\Sigma x^2 p - (\text{mean})^2}$$

$$= \sqrt{(3.4) - (1)^2}$$

$$= \sqrt{2.4}$$

$$\text{Standard Deviation} = 1.5$$

Mean and Variance of a Random Variable Ex 32.2 Q1(v)

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
1	0.4	0.4	0.4
2	0.3	0.6	1.2
3	0.2	0.6	1.8
4	0.1	0.4	1.6
		$\Sigma x p = 2$	$\Sigma x^2 p = 5$

Mean = $\Sigma x p$

mean = 2

$$\begin{aligned}\text{Standard deviation} &= \sqrt{\Sigma x^2 p - (\text{mean})^2} \\ &= \sqrt{5 - (2)^2}\end{aligned}$$

Standard Deviation = 1

Mean and Variance of a Random Variable Ex 32.2 Q1(vi)

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
0	0.2	0	0
1	0.5	0.5	0.5
3	0.2	0.6	1.8
5	0.1	0.5	2.5
		$\Sigma x p = 1.6$	$\Sigma x^2 p = 4.8$

Mean = $\Sigma x p$

mean = 1.6

$$\begin{aligned}\text{Standard deviation} &= \sqrt{\Sigma x^2 p - (\text{mean})^2} \\ &= \sqrt{4.8 - (1.6)^2} \\ &= \sqrt{4.8 - 2.56} \\ &= \sqrt{2.24}\end{aligned}$$

Standard Deviation = 1.497

Mean and Variance of a Random Variable Ex 32.2 Q1(vii)

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
-2	0.1	-0.2	0.4
-1	0.2	-0.2	0.2
0	0.4	0	0
1	0.2	0.2	0.2
2	0.1	0.2	0.4
		$\Sigma x p = 0$	$\Sigma x^2 p = 1.2$

$$\text{Mean} = \sum xp$$

$$\text{mean} = 0$$

$$\begin{aligned}\text{Standard deviation} &= \sqrt{\sum x^2 p - (\text{mean})^2} \\ &= \sqrt{(1.2)^2 - (0)^2}\end{aligned}$$

$$\text{Standard Deviation} = 1.2$$

Mean and Variance of a Random Variable Ex 32.2 Q1(viii)

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
-3	0.05	-0.15	0.45
-1	0.45	-0.45	0.45
0	0.20	0	0
1	0.25	0.25	0.25
3	0.05	0.15	0.45
		$\sum xp = -0.2$	$\sum x^2 p = 1.6$

$$\text{Mean} = \sum xp$$

$$\text{mean} = -0.2$$

$$\begin{aligned}\text{Standard deviation} &= \sqrt{\sum x^2 p - (\text{mean})^2} \\ &= \sqrt{1.6 - (-0.2)^2} \\ &= \sqrt{1.6 - 0.04} \\ &= \sqrt{1.56}\end{aligned}$$

$$\text{Standard Deviation} = 1.249$$

Mean and Variance of a Random Variable Ex 32.2 Q1(ix)

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	$\frac{1}{6}$	0	0
1	$\frac{5}{18}$	$\frac{5}{18}$	$\frac{5}{18}$
2	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{8}{9}$
3	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{2}$
4	$\frac{1}{9}$	$\frac{4}{9}$	$\frac{16}{9}$
5	$\frac{1}{18}$	$\frac{5}{18}$	$\frac{25}{18}$
		$\sum p_i x_i = \frac{35}{18}$	$\sum p_i x_i^2 = \frac{35}{6}$

$$\text{Mean} = \sum p_i x_i = \frac{35}{18}$$

$$\text{Variance} = \sum p_i x_i^2 - \left(\sum p_i x_i\right)^2 = \frac{35}{6} - \left(\frac{35}{18}\right)^2 = \frac{665}{324}$$

$$\text{Standard deviation} = \sqrt{\text{Variance}} = \frac{\sqrt{665}}{18}$$

Mean and Variance of a Random Variable Ex 32.2 Q2

(i) We know that,

$$P(0.5) + P(1) + P(1.5) + P(2) = 1$$

$$k + k^2 + 2k^2 + k = 1$$

$$3k^2 + 2k - 1 = 0$$

$$3k^2 + 3k - k - 1 = 0$$

$$(3k - 1)(k + 1) = 0$$

$$k = \frac{1}{3} \text{ or } k = -1$$

We know that $0 \leq P(X) \leq 1$

$$\therefore k = \frac{1}{3}$$

(ii)

x_i	p_i	$p_i x_i$
0.5	$\frac{1}{3}$	$\frac{1}{6}$
1	$\frac{1}{9}$	$\frac{1}{9}$
1.5	$\frac{2}{9}$	$\frac{1}{3}$
2	$\frac{1}{3}$	$\frac{2}{3}$
		$\Sigma p_i x_i = \frac{23}{18}$

$$\text{Mean} = \Sigma p_i x_i = \frac{23}{18}$$

Mean and Variance of a Random Variable Ex 32.2 Q3

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
a	p	ap	$a^2 p$
b	q	bq	$b^2 q$

$$\text{Mean} = \Sigma x p$$

$$\text{mean} = ap + bq$$

$$\text{Variance} = \Sigma x^2 p - (\text{mean})^2$$

$$= (a^2 p + b^2 q) - (ap + bq)^2$$

$$= a^2 p + b^2 q - a^2 p^2 - b^2 q^2 - 2abpq$$

$$= a^2 pq + b^2 pq - 2abpq \quad [\because p + q = 1]$$

$$= pq (a^2 + b^2 - 2ab)$$

$$\text{Variance} = pq (a - b)^2$$

$$\text{Standard deviation} = |a - b| \sqrt{pq}$$

Mean and Variance of a Random Variable Ex 32.2 Q4

We know that in a throw of coin.

$$P(H) = \frac{1}{2}, \quad P(T) = \frac{1}{2}$$

Let X denote the number of heads in three tosses of coin.

So, $X = 0, 1, 2, 3$

$$\begin{aligned} P(X=0) &= P(T)P(T)P(T) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

$$\begin{aligned} P(X=1) &= P(H)P(T)P(T) + P(T)P(H)P(T) + P(T)P(T)P(H) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{3}{8} \end{aligned}$$

$$\begin{aligned} P(X=2) &= P(H)P(H)P(T) + P(H)P(T)P(H) + P(T)P(H)P(H) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{3}{8} \end{aligned}$$

$$\begin{aligned} P(X=3) &= P(H)P(H)P(H) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

So,

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
0	$\frac{1}{8}$	0	0
1	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
2	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{12}{8}$
3	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{9}{8}$
		$\Sigma x p = \frac{3}{2}$	$\Sigma x^2 p = 3$

$$\text{Mean} = \Sigma x p$$

$$\text{mean} = \frac{3}{2}$$

$$\text{Variance} = \Sigma x^2 p - (\text{mean})^2$$

$$= 3 - \frac{9}{4}$$

$$\text{Variance} = \frac{3}{4}$$

Mean and Variance of a Random Variable Ex 32.2 Q5

Two cards are drawn simultaneously from a pack of 52 cards.
Let X denotes the number of kings drawn.

So, $X = 0, 1, 2$

$$\begin{aligned} P(X = 0) &= \frac{{}^{48}C_2}{{}^{52}C_2} \\ &= \frac{48 \times 47}{52 \times 51} \\ &= \frac{188}{221} \end{aligned}$$

$$\begin{aligned} P(X = 1) &= \frac{{}^4C_1 \times {}^{48}C_1}{{}^{52}C_2} \\ &= \frac{4 \times 48 \times 2}{52 \times 51} \\ &= \frac{32}{221} \end{aligned}$$

$$\begin{aligned} P(X = 2) &= \frac{{}^4C_2}{{}^{52}C_2} \\ &= \frac{4 \times 3}{52 \times 51} \\ &= \frac{1}{221} \end{aligned}$$

So,

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
0	$\frac{188}{221}$	0	0
1	$\frac{32}{221}$	$\frac{32}{221}$	$\frac{32}{221}$
2	$\frac{1}{221}$	$\frac{2}{221}$	$\frac{4}{221}$
		$\Sigma x p = \frac{34}{221}$	$\Sigma x^2 p = \frac{36}{221}$

$$\text{Mean} = \Sigma x p$$

$$\text{mean} = \frac{34}{221}$$

$$\begin{aligned} \text{Variance} &= \Sigma x^2 p - (\text{mean})^2 \\ &= \frac{36}{221} - \left(\frac{34}{221}\right)^2 \\ &= \frac{7956 - 1156}{48841} \\ &= \frac{6800}{48841} \end{aligned}$$

$$\text{Variance} = \frac{400}{2873}$$

Mean and Variance of a Random Variable Ex 32.2 Q6

We know that ,in a throw of coin.

$$P(T) = \frac{1}{2}, \quad P(H) = \frac{1}{2}$$

Let X denote the number of tails in three throws of coins.

So, X can take values from 0,1,2,3

$$P(X = 0) = P(H)P(H)P(H)$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{8}$$

$$P(X = 1) = P(T)P(H)P(H) + P(H)P(T)P(H) + P(H)P(H)P(T)$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{8}$$

$$P(X = 2) = P(T)P(T)P(H) + P(T)P(H)P(T) + P(H)P(T)P(T)$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{8}$$

$$P(X = 3) = P(T)P(T)P(T)$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{8}$$

So,

$$\text{Mean} = \sum xp$$

$$\text{mean} = \frac{3}{2}$$

$$\text{Variance} = \sum x^2p - (\text{mean})^2$$

$$= 3 - \left(\frac{3}{2}\right)^2$$

$$= 3 - \frac{9}{4}$$

$$\text{Variance} = \frac{3}{4}$$

$$\text{Standard Deviation} = \sqrt{\text{Variance}}$$

$$= \sqrt{\frac{3}{4}}$$

$$\text{Standard Deviation} = 0.87$$

Mean and Variance of a Random Variable Ex 32.2 Q7

Total 12 good and bad eggs. 10 are good and 2 are bad.

3 eggs are drawn from this lot

Let X be the random variable that denotes the number of bad eggs in the lot.

$$P(X = 0) = P(3\text{good and } 0\text{ bad}) = {}^3C_0 \cdot {}^{10}C_3 / {}^{12}C_3$$

$$= 1 \times 120 / 220 = 6/11$$

$$P(X = 1) = P(2\text{good and } 1\text{ bad}) = {}^2C_1 \cdot {}^{10}C_2 / {}^{12}C_3$$

$$= 2 \times 45 / 220 = 9/22$$

$$P(X = 2) = P(1\text{good and } 2\text{ bad}) = {}^2C_2 \cdot {}^{10}C_1 / {}^{12}C_3$$

$$= 1 \times 10 / 220 = 1/22$$

The probability distribution of X is

X	0	1	2
$P(X)$	$\frac{6}{11}$	$\frac{9}{22}$	$\frac{1}{22}$

$$\text{The mean} = 0 \times \frac{6}{11} + 1 \times \frac{9}{22} + 2 \times \frac{1}{22} = \frac{11}{22} = 1/2$$

Mean and Variance of a Random Variable Ex 32.2 Q8

A pair of dice is thrown. And X denote minimum of the two number appeared.

So, X can have values 2,3,4,5,6.

$$P(X = 1) = \frac{11}{36} \quad \left[\begin{array}{l} \text{Possible pairs: } (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (3,1), \\ (4,1), (5,1), (6,1) \end{array} \right]$$

$$P(X = 2) = \frac{9}{36} \quad \left[\text{Possible pairs: } (2,2), (2,3), (2,4), (2,5), (2,6), (3,2), (4,2), (5,2), (6,2) \right]$$

$$P(X = 3) = \frac{7}{36} \quad \left[\text{Possible pairs: } (3,3), (3,4), (3,5), (3,6), (4,3), (5,3), (6,3) \right]$$

$$P(X = 4) = \frac{5}{36} \quad \left[\text{Possible pairs: } (4,4), (4,5), (4,6), (5,4), (6,4) \right]$$

$$P(X = 5) = \frac{3}{36} \quad \left[\text{Possible pairs: } (5,5), (5,6), (6,5) \right]$$

$$P(X = 6) = \frac{1}{36} \quad \left[\text{Possible pairs: } (6,6) \right]$$

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
1	$\frac{11}{36}$	$\frac{11}{36}$	$\frac{11}{36}$
2	$\frac{9}{36}$	$\frac{18}{36}$	$\frac{36}{36}$
3	$\frac{7}{36}$	$\frac{21}{36}$	$\frac{63}{36}$
4	$\frac{5}{36}$	$\frac{20}{36}$	$\frac{80}{36}$
5	$\frac{3}{36}$	$\frac{15}{36}$	$\frac{75}{36}$
6	$\frac{1}{36}$	$\frac{6}{36}$	$\frac{36}{36}$
		$\Sigma x p = \frac{91}{36}$	$\Sigma x^2 p = \frac{301}{36}$

$$\text{Mean} = \Sigma x p$$

$$\text{Mean} = \frac{91}{36}$$

$$\text{Variance} = \Sigma x^2 p - (\text{mean})^2$$

$$\begin{aligned} &= \frac{301}{36} - \left(\frac{91}{36} \right)^2 \\ &= \frac{10836 - 8281}{1296} \\ &= \frac{2555}{1296} \end{aligned}$$

$$\text{Variance} = 1.97$$

Probability distribution is

$$\begin{array}{cccccc} x & : & 1 & 2 & 3 & 4 & 5 & 6 \\ P(X) & : & \frac{11}{36} & \frac{9}{36} & \frac{7}{36} & \frac{5}{36} & \frac{3}{36} & \frac{1}{36} \end{array}$$

Mean and Variance of a Random Variable Ex 32.2 Q9

We know that ,In a toss of coin.

$$P(T) = \frac{1}{2}, \quad P(H) = \frac{1}{2}$$

Let X denote the number of occurring head in 4 throws of coins.

So, X can take values from $X = 0, 1, 2, 3, 4$

$$\begin{aligned} P(X=0) &= P(T)P(T)P(T)P(T) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{16} \end{aligned}$$

$$\begin{aligned} P(X=1) &= P(H)P(T)P(T)P(T) \times {}^4C_1 \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 4 \\ &= \frac{4}{16} \end{aligned}$$

$$\begin{aligned} P(X=2) &= P(H)P(H)P(T)P(T) \times {}^4C_2 \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 6 \\ &= \frac{6}{16} \end{aligned}$$

$$\begin{aligned} P(X=3) &= P(H)P(H)P(H)P(T) \times {}^4C_3 \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 4 \\ &= \frac{4}{16} \end{aligned}$$

$$\begin{aligned} P(X=4) &= P(H)P(H)P(H)P(H) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{16} \end{aligned}$$

So,

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
0	$\frac{1}{16}$	0	0
1	$\frac{4}{16}$	$\frac{4}{16}$	$\frac{4}{16}$
2	$\frac{6}{16}$	$\frac{12}{16}$	$\frac{24}{16}$
3	$\frac{4}{16}$	$\frac{12}{16}$	$\frac{36}{16}$
4	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{16}{16}$
		$\Sigma x p = 2$	$\Sigma x^2 p = 5$

$$\text{Mean} = \Sigma x p$$

$$\text{mean} = 2$$

$$\begin{aligned} \text{Variance} &= \Sigma x^2 p - (\text{mean})^2 \\ &= 5 - (2)^2 \end{aligned}$$

$$\text{Variance} = 1$$

Probability distribution is

$$\begin{array}{c} x : 0 \quad 1 \quad 2 \quad 3 \quad 4 \\ P(X) : \frac{1}{16} \quad \frac{4}{16} \quad \frac{6}{16} \quad \frac{4}{16} \quad \frac{1}{16} \end{array}$$

Mean and Variance of a Random Variable Ex 32.2 Q10

X denotes twice the number appearing on the die.
So, $X = 2, 4, 6, 8, 10, 12$.

Probability distribution is

$$\begin{array}{l} X : \quad 2 \quad 4 \quad 6 \quad 8 \quad 10 \quad 12 \\ P(X) : \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{6} \end{array}$$

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
2	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{4}{6}$
4	$\frac{1}{6}$	$\frac{4}{6}$	$\frac{16}{6}$
6	$\frac{1}{6}$	$\frac{6}{6}$	$\frac{36}{6}$
8	$\frac{1}{6}$	$\frac{8}{6}$	$\frac{64}{6}$
10	$\frac{1}{6}$	$\frac{10}{6}$	$\frac{100}{6}$
12	$\frac{1}{6}$	$\frac{12}{6}$	$\frac{144}{6}$
		$\Sigma x p = 7$	$\Sigma x^2 p = \frac{364}{6}$

$$\begin{aligned} \text{Mean} &= \Sigma x p \\ \text{mean} &= 7 \end{aligned}$$

$$\begin{aligned} \text{Variance} &= \Sigma x^2 p - (\text{mean})^2 \\ &= \left(\frac{364}{6} \right) - (7)^2 \\ &= \frac{364 - 294}{6} \\ &= \frac{70}{6} \end{aligned}$$

$$\text{Variance} = 11.7$$

Mean and Variance of a Random Variable Ex 32.2 Q11

$$\text{Probability of even number} = P(E) = \frac{3}{6} = \frac{1}{2}$$

$$\Rightarrow P(O) = \frac{1}{2}$$

Here, X have values 1 or 3 according as an odd or even number.

So,

$$\begin{array}{l} X : \quad 1 \quad 3 \\ P(X) : \quad \frac{1}{2} \quad \frac{1}{2} \end{array}$$

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
3	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{9}{2}$
		$\Sigma x p = 2$	$\Sigma x^2 p = 5$

$$\text{Mean} = \sum x p$$

$$\text{mean} = 2$$

$$\begin{aligned}\text{Variance} &= \sum x^2 p - (\text{mean})^2 \\ &= 5 - 4\end{aligned}$$

$$\text{Variance} = 1$$

Mean and Variance of a Random Variable Ex 32.2 Q12

Let the event of getting a head = H and getting a tail = T

Let X denote the variable longest consecutive heads occurring in 4 tosses. The possible values are

$$\begin{aligned}X = 0 \text{ (no head)} & \quad \{T, T, T, T\} \\ X = 1 \text{ (1 heads)} & \quad \{H, T, T, T\} \\ X = 2 \text{ (2 heads)} & \quad \{H, H, T, T\} \\ X = 3 \text{ (3 heads)} & \quad \{H, H, H, T\} \\ X = 4 \text{ (4 heads)} & \quad \{H, H, H, H\}\end{aligned}$$

$$n(S) = \{(HHHH), (HHHT), (HHTT), (HTHH), (HTHT), (HTTH), (HTTT), (THHH), (THTH), (THTT), (THTT), (THTT), (TTHH), (TTHH), (TTTH), (TTTT)\}$$

$$P(X=0) = \frac{1}{16}$$

$$P(X=1) = \frac{7}{16}$$

$$P(X=2) = \frac{5}{16}$$

$$P(X=3) = \frac{2}{16}$$

$$P(X=4) = \frac{1}{16}$$

Probability distribution is

X	0	1	2	3	4
$p_i = P(X)$	$\frac{1}{16}$	$\frac{7}{16}$	$\frac{5}{16}$	$\frac{2}{16}$	$\frac{1}{16}$
$p_i x_i^2$	0	$\frac{7}{16}$	$\frac{20}{16}$	$\frac{18}{16}$	1

$$\text{Mean} = \sum_{i=1 \text{ to } n} X_i \times P(X_i)$$

$$\begin{aligned}\text{Mean, } \mu &= 0 \times \frac{1}{16} + 1 \times \frac{7}{16} + 2 \times \frac{5}{16} + 3 \times \frac{2}{16} + 4 \times \frac{1}{16} \\ &= 0 + \frac{7}{16} + \frac{10}{16} + \frac{6}{16} + \frac{4}{16} \\ &= \frac{27}{16} = 1.7\end{aligned}$$

$$\begin{aligned}\text{Variance Var}(X) &= \sum p_i x_i^2 - (\sum p_i x_i)^2 \\ &= \frac{61}{16} - 1.7^2 \\ &= 3.825 - 2.89 \\ &= 0.935\end{aligned}$$

Mean and Variance of a Random Variable Ex 32.2 Q13

Box contains five cards 1,1,2,2,3.

Here,

X denotes the sum of two number on cards drawn.

Y denotes the maximum of the two number.

So, $X = 2, 3, 4, 5$

$Y = 1, 2, 3$

$$P(X = 2) = P(1)P(1)$$

$$= \frac{2}{5} \times \frac{1}{4}$$

$$= 0.1$$

$$P(X = 3) = P(1)P(2) + P(2)P(1)$$

$$= \frac{2}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{2}{4}$$

$$= 0.4$$

$$P(X = 4) = P(2)P(2) + P(1)P(3) + P(3)P(1)$$

$$= \frac{2}{5} \times \frac{1}{4} + \frac{2}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{2}{4}$$

$$= 0.3$$

$$P(X = 5) = P(2)P(3) + P(3)P(2)$$

$$= \frac{2}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{2}{4}$$

$$= 0.2$$

Probability Distribution for X

$$\begin{array}{l} X : \quad 2 \quad 3 \quad 4 \quad 5 \\ P(X) : \quad 0.1 \quad 0.4 \quad 0.3 \quad 0.2 \end{array}$$

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
2	0.1	0.1	0.4
3	0.4	1.2	3.6
4	0.3	1.2	4.8
5	0.2	1.0	5.0
		$\Sigma x p = 3.6$	$\Sigma x^2 p = 13.8$

$$\text{Mean} = \sum x p$$

$$\text{mean} = 3.6$$

$$\begin{aligned}\text{Variance} &= \sum x^2 p - (\text{mean})^2 \\ &= 13.8 - (3.6)^2 \\ &= 13.8 - 12.96\end{aligned}$$

$$\text{Variance} = 0.84$$

$$\begin{aligned}P(Y = 1) &= P(1)P(1) \\ &= \frac{2}{5} \times \frac{1}{4} \\ &= \frac{2}{20} \\ &= 0.1\end{aligned}$$

$$\begin{aligned}P(Y = 2) &= P(1)P(2) + P(2)P(1) + P(2)P(2) \\ &= \frac{2}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{1}{4}\end{aligned}$$

$$P(Y = 2) = 0.5$$

$$\begin{aligned}P(Y = 3) &= P(1)P(3) + P(2)P(3) + P(3)P(1) + P(3)P(2) \\ &= \frac{2}{5} \times \frac{1}{4} + \frac{2}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{2}{4} + \frac{1}{5} \times \frac{2}{4} \\ &= 0.4\end{aligned}$$

Probability distribution for Y is

$$\begin{array}{l} X : \quad 1 \quad \quad 2 \quad \quad 3 \\ P(X) : \quad 0.1 \quad 0.5 \quad 0.4 \end{array}$$

Y_i	p_i	$Y_i p_i$	$Y_i^2 p_i$
1	0.1	0.1	0.1
2	0.5	1.0	2.0
3	0.4	1.2	3.6
		$\sum x p = 2.6$	$\sum x^2 p = 5.7$

$$\text{Mean} = \sum x p$$

$$\text{mean} = 2.3$$

$$\begin{aligned}\text{Variance} &= \sum x^2 p - (\text{mean})^2 \\ &= 5.7 - (2.3)^2\end{aligned}$$

$$\text{Variance} = 0.41$$

Mean and Variance of a Random Variable Ex 32.2 Q14

Probability of getting an odd number = $P(O) = \frac{1}{2}$

$$\Rightarrow P(E) = \frac{1}{2}$$

Die is tossed twice. Let X denote the number of times an odd number occurs.

So, $X = 0, 1, 2$.

$$P(X = 0) = P(E)P(E)$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{4}$$

$$P(X = 1) = P(O)P(E) + P(E)P(O)$$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{2}$$

$$P(X = 2) = P(O)P(O)$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$P(X = 2) = \frac{1}{4}$$

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
0	$\frac{1}{4}$	0	0
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
2	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{4}{4}$
		$\Sigma x p = 1$	$\Sigma x^2 p = \frac{3}{2}$

$$\text{Mean} = \Sigma x p = 1$$

$$\text{Variance} = \Sigma x^2 p - (\text{mean})^2$$

$$= \frac{3}{2} - 1$$

$$\text{Variance} = \frac{1}{2}$$

Mean and Variance of a Random Variable Ex 32.2 Q15

Out of 13 bulbs 5 are defective \Rightarrow 8 bulbs are good.

3 bulbs are drawn without replacement ,

Let X denote number of defective bulbs,

So, X can have values 0,1,2,3

$$P(X = 0) = P(\text{No defective})$$

$$= \frac{{}^8C_3}{{}^{13}C_3}$$

$$= \frac{8 \times 7 \times 6}{13 \times 12 \times 11}$$

$$= \frac{28}{143}$$

$$P(X = 1) = P(\text{Only one defective})$$

$$= \frac{{}^8C_2 \times {}^5C_1}{{}^{13}C_3}$$

$$= \frac{8 \times 7 \times 5}{2} \times \frac{3 \times 2 \times 1}{13 \times 12 \times 11}$$

$$= \frac{70}{143}$$

$$P(X = 2) = P(\text{Only two defective})$$

$$= \frac{{}^8C_1 \times {}^5C_2}{{}^{13}C_3}$$

$$= \frac{8 \times 5 \times 4}{2} \times \frac{3 \times 2 \times 1}{13 \times 12 \times 11}$$

$$= \frac{40}{143}$$

$$P(X = 3) = P(\text{all three are defective})$$

$$= \frac{{}^5C_3}{{}^{13}C_3}$$

$$= \frac{4 \times 5}{2} \times \frac{3 \times 2 \times 1}{13 \times 12 \times 11}$$

$$= \frac{5}{143}$$

So, Probability distribution is

$X :$	0	1	2	3
$P(X) :$	$\frac{28}{143}$	$\frac{70}{143}$	$\frac{40}{143}$	$\frac{5}{143}$

Mean and Variance of a Random Variable Ex 32.2 Q16

$$P(\text{win}) = \frac{1}{13} \Rightarrow P(\text{lose}) = \frac{12}{13}$$

He gains Rs 90 if he wins and loses Rs 10 if his number does not appear.

Let X denote total loss or gain, so,

$X :$	90	- 10
$P(X) :$	$\frac{1}{13}$	$\frac{12}{13}$
$XP :$	$\frac{90}{13}$	$\frac{-120}{13}$

$$E(X) = \sum XP$$

$$= \frac{90}{13} - \frac{120}{13}$$

$$E(X) = -\frac{30}{13}$$

Mean and Variance of a Random Variable Ex 32.2 Q17

Let 'X' be the random variable which can assume values from 0 to 3.

$$P(X=0) = \frac{{}^{26}C_3}{{}^{52}C_3} = \frac{2600}{22100} = \frac{2}{17}$$

$$P(X=1) = \frac{{}^{26}C_1 \times {}^{26}C_2}{{}^{52}C_3} = \frac{8450}{22100} = \frac{13}{34}$$

$$P(X=2) = \frac{{}^{26}C_2 \times {}^{26}C_1}{{}^{52}C_3} = \frac{8450}{22100} = \frac{13}{34}$$

$$P(X=3) = \frac{{}^{26}C_3}{{}^{52}C_3} = \frac{2600}{22100} = \frac{2}{17}$$

Probability distribution of X:

$X = x_i$	0	1	2	3
$p(X = x_i)$	$\frac{2}{17}$	$\frac{13}{34}$	$\frac{13}{34}$	$\frac{2}{17}$

$$\begin{aligned} \text{Mean} &= \sum_{i=0}^3 (x_i \times p_i) \\ &= x_0 p_0 + x_1 p_1 + x_2 p_2 + x_3 p_3 \\ &= 0 \times \frac{2}{17} + 1 \times \frac{13}{34} + 2 \times \frac{13}{34} + 3 \times \frac{2}{17} \\ &= \frac{13 + 26 + 12}{34} \\ &= \frac{51}{34} \\ &= \frac{3}{2} \\ &= 1.5 \end{aligned}$$

Mean and Variance of a Random Variable Ex 32.2 Q18

X can assume values 0, 1, 2.

Yes X is a random variable.

$$P(X=0) = (\text{Probability of getting no black ball}) = \frac{{}^2C_0 \times {}^5C_2}{{}^7C_2} = \frac{1 \times \frac{5 \times 4}{2 \times 1}}{\frac{7 \times 6}{2 \times 1}} = \frac{20}{42}$$

$$P(X=1) = (\text{Probability of getting one black ball}) = \frac{{}^2C_1 \times {}^5C_1}{{}^7C_2} = \frac{2 \times 5}{\frac{7 \times 6}{2 \times 1}} = \frac{20}{42}$$

$$P(X=2) = (\text{Probability of getting two black balls}) = \frac{{}^2C_2 \times {}^5C_0}{{}^7C_2} = \frac{1 \times 1}{\frac{7 \times 6}{2 \times 1}} = \frac{2}{42}$$

Thus, probability distribution of random variable X is,

X	0	1	2
P(X)	$\frac{20}{42}$	$\frac{20}{42}$	$\frac{2}{42}$

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	$\frac{20}{42}$	0	0
1	$\frac{20}{42}$	$\frac{20}{42}$	$\frac{20}{42}$
2	$\frac{2}{42}$	$\frac{4}{42}$	$\frac{8}{42}$
		$\Sigma p_i x_i = \frac{4}{7}$	$\Sigma p_i x_i^2 = \frac{2}{3}$

$$\text{Mean} = \Sigma p_i x_i = \frac{4}{7}$$

$$\text{Variance} = \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2 = \frac{2}{3} - \left(\frac{4}{7}\right)^2 = \frac{50}{147}$$

Mean and Variance of a Random Variable Ex 32.2 Q19

We can select two positive in $6 \times 5 = 30$ different ways.

X denotes the larger number so, X can assume values 3, 4, 5, 6 and 7.

Yes X is a random variable.

$$P(X = 3) = P(\text{larger number is 3}) = \{(2, 3), (3, 2)\} = \frac{2}{30}$$

$$P(X = 4) = P(\text{larger number is 4}) = \{(2, 4), (4, 2), (3, 4), (4, 3)\} = \frac{4}{30}$$

$$P(X = 5) = P(\text{larger number is 5}) = \{(2, 5), (5, 2), (3, 5), (5, 3), (4, 5), (5, 4)\} = \frac{6}{30}$$

$$P(X = 6) = P(\text{larger number is 6}) = \{(2, 6), (6, 2), (3, 6), (6, 3), (4, 6), (6, 4), (5, 6), (6, 5)\} = \frac{8}{30}$$

$$P(X = 7) = P(\text{larger number is 7}) = \{(2, 7), (7, 2), (3, 7), (7, 3), (4, 7), (7, 4), (5, 7), (7, 5), (6, 7), (7, 6)\} = \frac{10}{30}$$

Thus, probability distribution of random variable X is,

x_i	p_i	$p_i x_i$	$p_i x_i^2$
3	$\frac{2}{30}$	$\frac{6}{30}$	$\frac{18}{30}$
4	$\frac{4}{30}$	$\frac{16}{30}$	$\frac{64}{30}$
5	$\frac{6}{30}$	$\frac{30}{30}$	$\frac{150}{30}$
6	$\frac{8}{30}$	$\frac{48}{30}$	$\frac{288}{30}$
7	$\frac{10}{30}$	$\frac{70}{30}$	$\frac{490}{30}$
		$\Sigma p_i x_i = \frac{17}{3}$	$\Sigma p_i x_i^2 = \frac{101}{3}$

$$\text{Mean} = \Sigma p_i x_i = \frac{17}{3}$$

$$\text{Variance} = \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2 = \frac{101}{3} - \left(\frac{17}{3}\right)^2 = \frac{14}{9}$$