MATHEMATICS

Time allowed : 3 hours ***Maximum Marks : 100***

GENERAL INSTRUCTIONS :

1. All questions are compulsory.
2. The question paper consists of 34 questions divided into four sections - A, B, C and D.
3. Section A contains 10 questions of 1 mark each, which are multiple choice type questions, Section B contains 8 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 6 questions of 4 marks each.
4. There is no overall choice in the paper. However, internal choice is provided in one question of 2 marks, three questions of 3 marks and two questions of 4 marks.
5. Use of calculators is not permitted.

QUESTION PAPER CODE 30/1/1

SECTION - A

Question Numbers 1 to 10 carry 1 mark each. For each of the questions 1 to 10, four alternative choices have been provided, of which only one is correct. Select the correct choice.

1. If 1 is a root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$, then ab equals:

   (A) 3

   (B) $-\frac{7}{2}$

   (C) 6

   (D) – 3

2. The sum of first 20 odd natural numbers is :

(A) 100

(B) 210

(C) 400

(D) 420

3. In Fig. 1, the sides AB, BC and CA of a triangle ABC, touch a circle at P, Q and R respectively. If PA = 4 cm, BP = 3 cm and AC = 11 cm, then the length of BC (in cm) is :

(A) 11

(B) 10

(C) 14

(D) 15

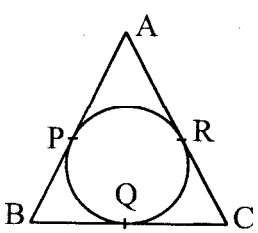

Fig. 1

4. In Fig 2, a circle touches the side DF of Δ EDF at H and touches ED and EF produced at K and M respectively. If EK = 9 cm, then the perimeter of Δ EDF (in cm) is :

(A) 18

(B) 13.5

(C) 12

(D) 9

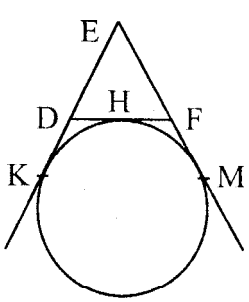

Fig. 2

5. If the radius of the base of a right circular cylinder is halved, keeping the height the same, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is :

(A) 1: 2

(B) 2: 1

(C) 1: 4

(D) 4: 1

6. If the area of a circle is equal to sum of the areas of two circles of diameters 10 cm and 24 cm, then the diameter of the larger circle (in cm) is :

(A) 34

(B) 26

(C) 17

(D) 14

7. The length of shadow of a tower on the plane ground is $\sqrt{3}$ times the height of the tower. The angle of elevation of sun is :

(A) 45°

(B) 30°

(C) 60°

(D) 90°

8. If the coordinates of one end of a diameter of a circle are (2, 3) and the coordinates of its centre are (–2, 5), then the coordinates of the other end of the diameter are:

(A) (–6, 7)

(B) (6, –7)

(C) (6, 7)

(D) (–6, –7)

9. The coordinates of the point P dividing the line segment joining the points A(1, 3) and B(4, 6) in the ratio 2 : 1 are:

(A) (2,4)

(B) (3, 5)

(C) (4,2)

(D) (5, 3)

10. Two dice are thrown together. The probability of getting the same number on both dice is :

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{6}$

(D) $\frac{1}{12}$

SECTION B

Question numbers 11 to 18 carry 2 marks each.

11. Find the value(s) of k so that the quadratic equation $x^2 - 4kx + k = 0$ has equal roots.

12. Find the sum of all three digit natural numbers, which are multiples of 11.

13. Tangents PA and PB are drawn from an external point P to two concentric circles with centre O and radii 8 cm and 5 cm respectively, as shown in Fig. 3. If AP = 15 cm, then find the length of BP.

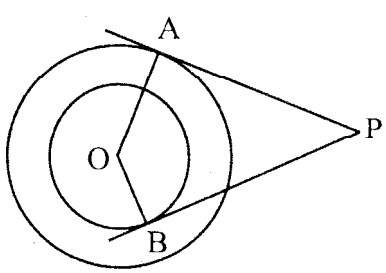

Fig. 3

14. In Fig. 4, an isosceles triangle ABC, with AB = AC, circumscribes a circle. Prove that the point of contact P bisects the base BC.

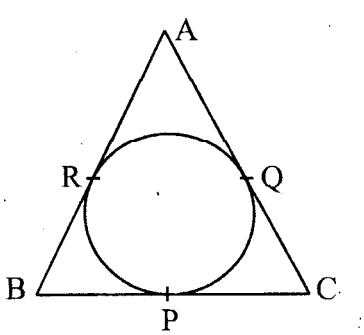

Fig. 4

OR

In Fig. 5, the chord AB of the larger of the two concentric circles, with centre O, touches the smaller circle at C. Prove that AC = CB.

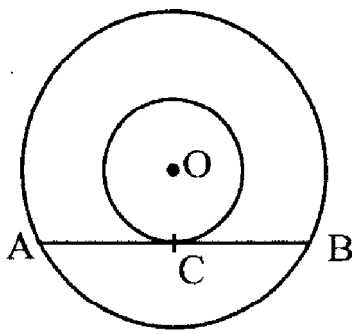

Fig. 5

15. The volume of a hemisphere is $2425\frac{1}{2}$ cm^3. Find its curved surface area.

[Use $\pi = \frac{22}{7}$]

16. In Fig. 6, OABC is a square of side 7 cm. If OAPC is a quadrant of a circle with centre O, then find the area of the shaded region. [Use $\pi = \frac{22}{7}$]

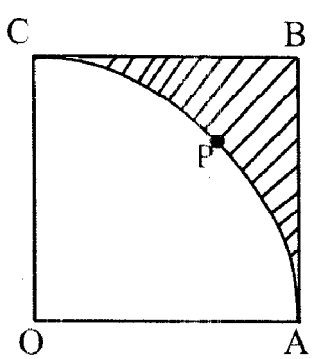

Fig. 6

17. If a point A(0, 2) is equidistant from the points B(3, *p*) and C(*p*, 5), then find the value of *p*.

18. A number is selected at random from first 50 natural numbers. Find the probability that it is a multiple of 3 and 4.

SECTION C

Question numbers 19 to 28 carry 3 marks each.

19. Solve for x : $4x^2 - 4ax + (a^2 - b^2) = 0$

OR

Solve for x : $3x^2 - 2\sqrt{6}\ x + 2 = 0$

20. Prove that the parallelogram circumscribing a circle is a rhombus.

OR

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

21. Construct a right triangle in which the sides, (other than the hypotenuse) are of length 6 cm and 8 cm. Then construct another triangle, whose sides are $\frac{3}{5}$ times the corresponding sides of the given triangle.

22. In Fig. 7, PQ and AB are respectively the arcs of two concentric circles of radii 7 cm and 3.5 cm and centre O. If $\angle$ POQ = 30°, then find the area of the shaded region. [Use $\pi = \frac{22}{7}$]

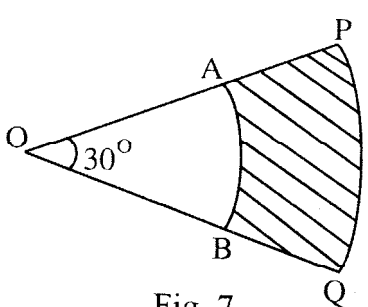

Fig. 7

23. From a solid cylinder of height 7 cm and base diameter 12 cm, a conical cavity of same height and same base diameter is hollowed out. Find the total surface area of the remaining solid. [Use $\pi = \frac{22}{7}$]

OR

A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, then find the radius and slant height of the heap.

24. The angles of depression of two ships from the top of a light house and on the same side of it are found to be 45° and 30°. If the ships are 200 m apart, find the height of the light house.

25. A point P divides the line segment joining the points A(3, –5) and B(–4, 8) such that $\frac{AP}{PB} = \frac{K}{1}$. If P lies on the line $x + y = 0$, then find the value of K.

26. If the vertices of a triangle are (1, –3), (4, *p*) and (–9, 7) and its area is 15 sq. units, find the value(s) of *p*.

27. A box contains 100 red cards, 200 yellow cards and 50 blue cards. If a card is drawn at random from the box, then find the probability that it will be (i) a blue card (ii) not a yellow card (iii) neither yellow nor a blue card.

28. The 17th term of an AP is 5 more than twice its 8th term. If the 11 th term of the AP is 43, then find its *n*th term.

SECTION D

Question numbers 29 to 34 carry 4 marks each.

29. A shopkeeper buys some books for ` 80. If he had bought 4 more books for the same amount, each book would have cost ` 1 less. Find the number of books he bought.

OR

The sum of two numbers is 9 and the sum of their reciprocals is $\frac{1}{2}$.

Find the numbers.

30. Sum of the first 14 terms of an AP is 1505 and its first term is 10. Find its 25th term.

31. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

OR

A quadrilateral ABCD is drawn to circumscribe a circle. Prove that
AB + CD = AD + BC.

32. A solid is in the shape of a cone surmounted on a hemisphere, the radius of each of them being 3.5 cm and the total height of solid is 9.5 cm. Find the volume of the solid. [Use $\pi = \frac{22}{7}$].

33. A bucket is in the form of a frustum of a cone and it can hold 28.49 litres of water. If the radii of its circular ends are 28 cm and 21 cm, find the height of the bucket. [Use $\pi = \frac{22}{7}$].

34. The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of depression from the top of the tower of the foot of the hill. is 30°. If the tower is 50 m high, find the height of the hill.

QUESTION PAPER CODE 30/1

SECTION - A

Question Numbers 1 to 10 carry 1 mark each. For each of the question numbers 1 to 10, four alternative choices have been provided, of which only one is correct. Select the correct choice.

1. The roots of the quadratic equation $2x^2 - x - 6 = 0$ are

   (A) – 2, 3/2

   (B) 2, – 3/2

   (C) – 2, – 3/2

   (D) 2, 3/2

2. If the n^{th} term of an A.P. is (2n + 1), then the sum of its first three terms is

   (A) 6n + 3

   (B) 15

   (C) 12

   (D) 21

3. From a point Q, 13 cm away from the centre of a circle, the length of tangent PQ to the circle is 12 cm. The radius of the circle (in cm) is

(A) 25

(B) $\sqrt{313}$

(C) 5

(D) 1

4. In Figure 1, AP, AQ and BC are tangents to the circle. If AB = 5 cm, AC = 6 cm and BC = 4 cm, then the length of AP (in cm) is

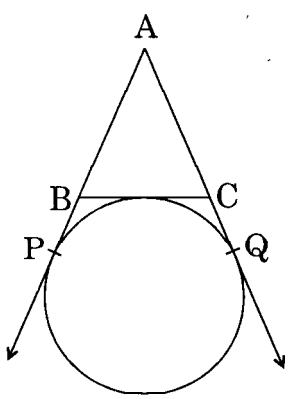

Figure 1

(A) 7.5

(B) 15

(C) 10

(D) 9

5. The circumference of a circle is 22 cm. The area of its quadrant (in cm^2) is

(A) $\frac{77}{2}$

(B) $\frac{77}{4}$

(C) $\frac{77}{8}$

(D) $\frac{77}{16}$

6. A solid right circular cone is cut into two parts at the middle of its height by a plane parallel to its base. The ratio of the volume of the smaller cone to the whole cone is

(A) 1 : 2

(B) 1 : 4

(C) 1 : 6

(D) 1 : 8

7. A kite is flying at a height of 30 m from the ground. The length of string from the kite to the ground is 60 m. Assuming that there is no slack in the string, the angle of elevation of the kite at the ground is

(A) 45°

(B) 30°

(C) 60°

(D) 90°

8. The distance of the point (– 3, 4) from the x-axis is

(A) 3

(B) –3

(C) 4

(D) 5

9. In Figure 2, P(5, - 3) and Q(3, y) are the points of trisection of the line segment joining A(7, – 2) and B(l, – 5). Then y equals

Figure 2

(A) 2

(B) 4

(C) – 4

(D) $-\frac{5}{2}$

10. Cards bearing numbers 2, 3, 4, ..., 11 are kept in a bag. A card is drawn at random from the bag. The probability of getting a card with a prime number is

(A) $\frac{1}{2}$

(B) $\frac{2}{5}$

(C) $\frac{3}{10}$

(D) $\frac{5}{9}$

SECTION B

11. Find the value of p for which the roots of the equation $px(x-2)+6=0$ are equal.

12. How many two-digit numbers are divisible by 3 ?

13. In Figure 3, a right triangle ABC, circumscribes a circle of radius r. If AB and BC are of lengths 8 cm and 6 cm respectively, find the value of r.

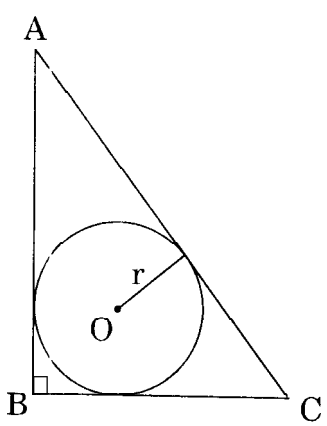

Figure 3

14. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

15. In Figure 4, ABCD is a square of side 4 cm. A quadrant of a circle of radius 1 cm is drawn at each vertex of the square and a circle of diameter 2 cm is also drawn. Find the area of the shaded region. (Use $\pi = 3.14$)

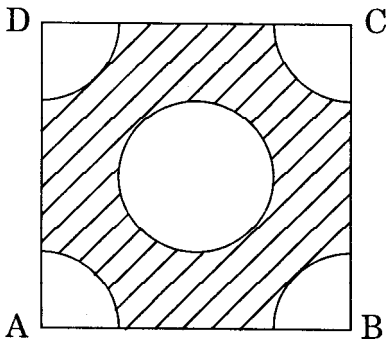

Figure 4

OR

From a rectangular sheet of' paper ABCD with AB = 40 cm and AD = 28 cm, a semi-circular portion with BC as diameter is cut off. Find the area of the remaining paper. (Use $\pi = \frac{22}{7}$)

16. A solid sphere of radius 10.5 cm is melted and recast into smaller solid cones, each of radius 3.5 cm and height 3 cm. Find the number of cones so formed. (Use $\pi = \frac{22}{7}$)

17. Find the value of k, if the point P(2, 4) is equidistant from the points A(5, k) and B(k, 7).

18. A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability of getting

    (i) a red king.

    (ii) a queen or a jack.

SECTION C

Question numbers 19 to 28 carry 3 marks each.

19. Solve the following quadratic equation for x :

    $x^2 - 4ax - b^2 + 4a^2 = 0$

OR

If the sum of two natural numbers is 8 and their product is 15, find the numbers.

20. Find the sum of all multiples of 7 lying between 500 and 900.

21. Draw a triangle ABC with BC = 7 cm, $\angle$ B = 45° and $\angle$ C = 60°.

Then construct another triangle, whose sides are $\frac{3}{5}$ times the corresponding sides of ΔABC.

22. In Figure 5, a circle is inscribed in a triangle PQR with PQ = 10 cm, QR = 8 cm and PR = 12 cm. Find the lengths QM, RN and PL.

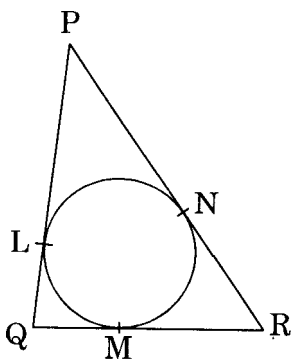

Figure 5

23. In Figure 6, O is the centre of the circle with AC = 24 cm, AB = 7 cm and $\angle$BOD = 90°. Find the area of the shaded region.

[Use π = 3.14]

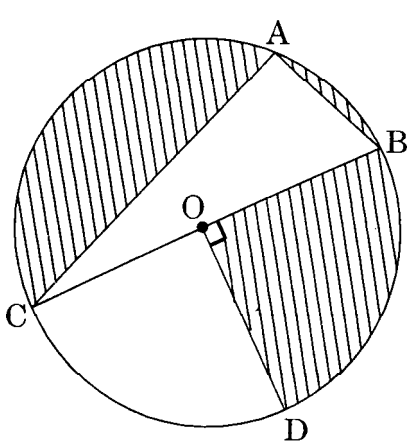

Figure 6

OR

In Figure 7, find the area of the shaded region, if ABCD is a square of side 14 cm and APD and BPC are semicircles.

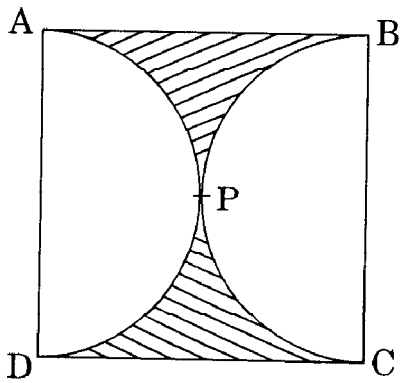

Figure 7

24. A hemispherical bowl of internal radius 9 cm is full of water. Its contents are emptied in a cylindrical vessel of internal radius 6 cm. Find the height of water in the cylindrical vessel.

25. The angles of depression of the top and bottom of a tower as seen from the top of a $60\sqrt{3}$ m high cliff are 45° and 60° respectively. Find the height of the tower.

26. Find the coordinates of a point P, which lies on the line segment joining the points A(– 2, – 2) and B(2, – 4) such that AP = $\frac{3}{7}$ AB.

OR

Find the area of the quadrilateral ABCD whose vertices are

A(– 3, – 1), B(– 2, – 4), C(4, – 1) and D(3, 4).

27. If the points A(x, y), B(3, 6) and C(- 3, 4) are collinear, show that x – 3y + 15 = 0.

28. All kings, queens and aces are removed from a pack of 52 cards. The remaining cards are well shuffled and then a card is drawn from it. Find the probability that the drawn card is

    (i) a black face card.

    (ii) a red card.

SECTION D

Question numbers 29 to 34 carry 4 marks each.

29. The numerator of a fraction is 3 less than its denominator. If 1 is added to the denominator, the fraction is decreased by $\frac{1}{15}$. Find the fraction.

OR

In a flight of 2800 km, an aircraft was slowed down due to bad weather. Its average speed is reduced by 100 km/h and time increased by 30 minutes. Find the original duration of the flight.

30. Find the common difference of an A.P. whose first term is 5 and the sum of its first four terms is half the sum of the next four terms.

31. Prove that the length of tangents drawn from an external point to a circle are equal.

32. A hemispherical tank, full of water, is emptied by a pipe at the rate of $\frac{25}{7}$ litres per sec. How much time will it take to empty half the tank if the diameter of the base of the tank is 3 m ?

OR

A drinking glass is in the shape of the frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm . Find the capacity of the glass. [Use $\pi = \frac{22}{7}$]

33. A military tent of height 8.25 m is in the form of a right circular cylinder of base diameter 30 m and height 5.5 m surmounted by a right circular cone of same base radius. Find the length of the canvas used in making the tent, if the breadth of the canvas is 1.5 m.

34. The angles of elevation and depression of the top and bottom of a light-house from the top of a 60m high building are 30° and 60° respectively. Find

    (i) the difference between the heights of the light-house and the building.

    (ii) the distance between the light-house and the building.