

Joint Entrance Exam/IITJEE-2017

Paper Code - C

2nd April 2017 | 9.30 AM – 12.30 PM

CHMISTRY, MATHEMATICS & PHYSICS

Important Instructions:

1. Immediately fill in the particulars on this page of Test Booklet **with only Blue / Black Ball Point Pen** provided in the examination hall.
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of **3 hours** duration.
4. The Test Booklet consists of **90** questions. The maximum marks are **360**.
5. There are **three** parts in the question paper A, B, C, consisting of **Chemistry, Mathematics and Physics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
6. *Candidates will be awarded, marks as stated above in instruction No. 5 for correct response of each question. $\frac{1}{4}$ (one fourth) marks of the total marks allotted to the question (i.e. 1 mark) will be deducted for indicating incorrect response for each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
7. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.
8. For writing particulars/markings responses on **Side -1** and **Side-2** of the Answer Sheet use **only Black Ball Point Pen** provided in the examination hall.
9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination room/hall.
10. Rough work is not to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in **four** pages (Pages **20-23**) at the end of the booklet.
11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. **However, the candidates are allowed to take away this Test Booklet with them.**
12. The CODE for this booklet is **C**. Make sure that the CODE printed on **Side – 2** of the Answer Sheet and also tally the serial number of the Test Booklet and Answer Sheet are the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.
13. **Do not fold or make any stray mark on the Answer Sheet.**

Joint Entrance Exam/IITJEE-2017

CHEMISTRY, MATHEMATICS & Physics

PART-A

CHEMISTRY

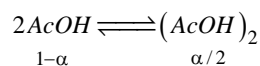
1. The freezing point of benzene decreases by 0.45°C when 0.2 g of acetic acid is added to 20 g of benzene. If acetic acid associates to form a dimer in benzene, percentage association of acetic acid in benzene will be : (K_f for benzene = $5.12 \text{ K kg mol}^{-1}$)

- (1) 94.6 % (2) 64.6 % (3) 80.4% (4) 74.6%

1.(1) $\Delta T_f = i \times K_f \times m$

$$\Rightarrow 0.45 = i \times 5.12 \times \frac{0.2}{60} \times \frac{1000}{20}$$

$$i = 0.527$$



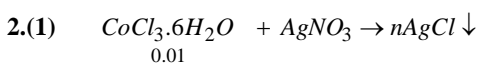
$$i = 1 - \frac{\alpha}{2}$$

$$0.527 = 1 - \frac{\alpha}{2}$$

$$\alpha = 0.946 \text{ or } 94.6 \%$$

2. On treatment of 100 mL of 0.1 M solution of $\text{CoCl}_3 \cdot 6\text{H}_2\text{O}$ with excess AgNO_3 ; 1.2×10^{22} ions are precipitated. The complex is:

- (1) $[\text{Co}(\text{H}_2\text{O})_5\text{Cl}]\text{Cl}_2 \cdot \text{H}_2\text{O}$ (2) $[\text{Co}(\text{H}_2\text{O})_4\text{Cl}_2]\text{Cl} \cdot 2\text{H}_2\text{O}$
 (3) $[\text{Co}(\text{H}_2\text{O})_3\text{Cl}_3] \cdot 3\text{H}_2\text{O}$ (4) $[\text{Co}(\text{H}_2\text{O})_6]\text{Cl}_3$



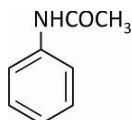
$$\frac{1.2 \times 10^{22}}{6.0 \times 10^{23}} = 2 \times 10^{-2} \text{ moles} = 0.02$$

$$0.01 \times n = 0.02$$

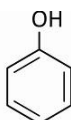
$$n = 2$$

$$\therefore \text{Complex is } [\text{Co}(\text{H}_2\text{O})_5\text{Cl}]\text{Cl}_2 \cdot \text{H}_2\text{O}$$

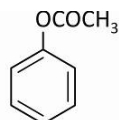
3. Which of the following compounds will form significant amount of meta product during mono-nitration reaction?



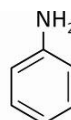
(1)



(2)

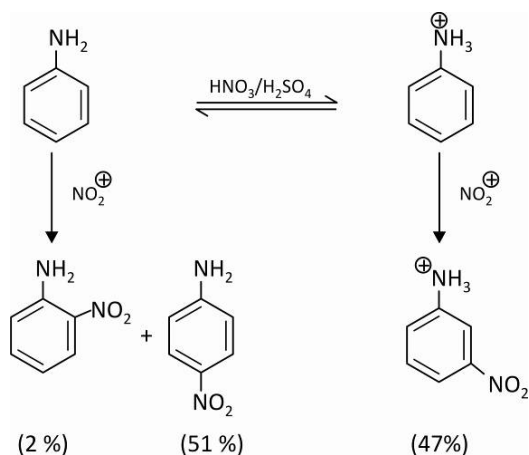


(3)

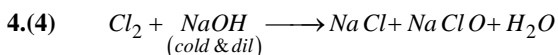
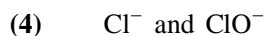
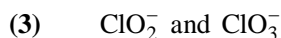
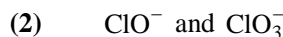


(4)

3.(4)



4. The products obtained when chlorine gas reacts with cold and dilute aqueous NaOH are:



5. Both lithium and magnesium display several similar properties due to the diagonal relationship; however, the one which is incorrect, is:

(1) nitrates of both Li and Mg yield NO_2 and O_2 on heating

(2) both form basic carbonates

(3) both form soluble bicarbonates

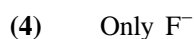
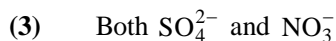
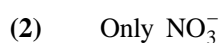
(4) both form nitrides

5.(2) 1, 3 & 4 are correct statements.

6. A water sample has ppm level concentration of following anions

$$\text{F}^- = 10; \text{SO}_4^{2-} = 100; \text{NO}_3^- = 50$$

The anion/anions that make/makes the water sample unsuitable for drinking is/are:

6.(4) Permissible limit of F^- is 1 ppm.

7. The formation of which of the following polymers involves hydrolysis reaction ?

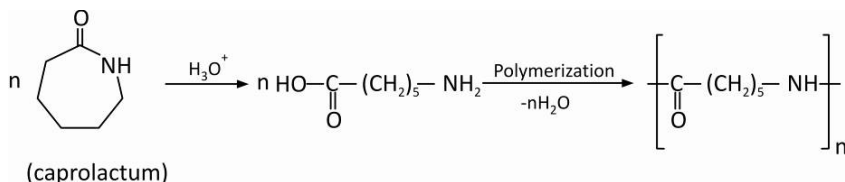
(1) Terylene

(2) Nylon 6

(3) Bakelite

(4) Nylon 6, 6

7.(2)



8. The Tyndall effect is observed only when following conditions are satisfied :
- The diameter of the dispersed particles is much smaller than the wavelength of the light used.
 - The diameter of the dispersed particle is not much smaller than the wavelength of the light used.
 - The refractive indices of the dispersed phase and dispersion medium are almost similar in magnitude.
 - The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude.
- (1) (b) and (c) (2) (a) and (d) (3) (b) and (d) (4) (a) and (c)

8.(3) Refer NCERT Class XII Page no. 139

Tyndall effect is observed only when the following two conditions are satisfied.

- The diameter of the dispersed particle is not much smaller than the wavelength of the light used.
- The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude.

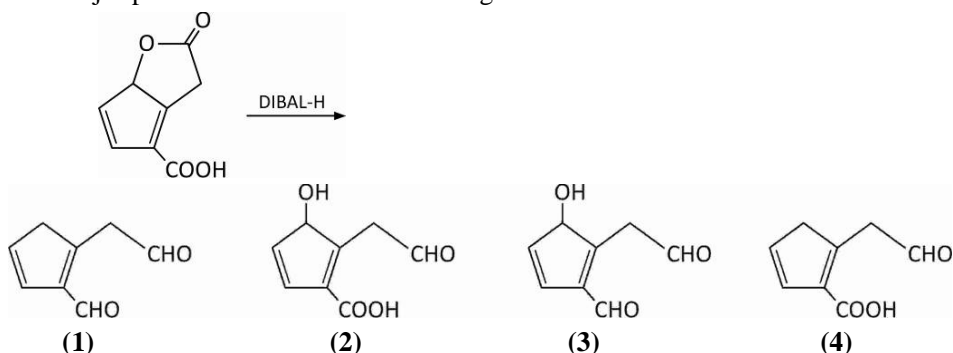
9. pK_a of weak acid (HA) and pK_b of a weak base (BOH) are 3.2 and 3.4, respectively. The pH of their salt (AB) solution is:

- (1) 1.0 (2) 7.2 (3) 6.9 (4) 7.0

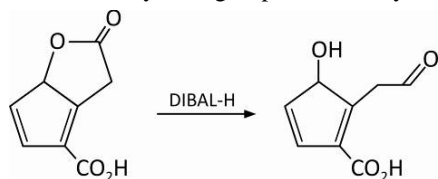
9.(3) pH of a salt made up of weak acid and weak base is calculated by using expression

$$pH = 7 + \frac{1}{2} pK_a - \frac{1}{2} pK_b = 7 + \frac{3.2}{2} - \frac{3.4}{2} = 6.9$$

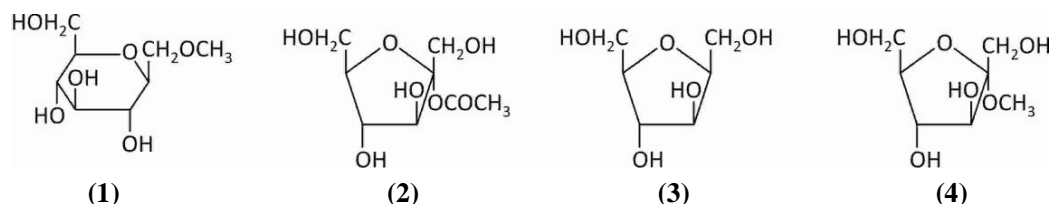
10. The major product obtained in the following reaction is :



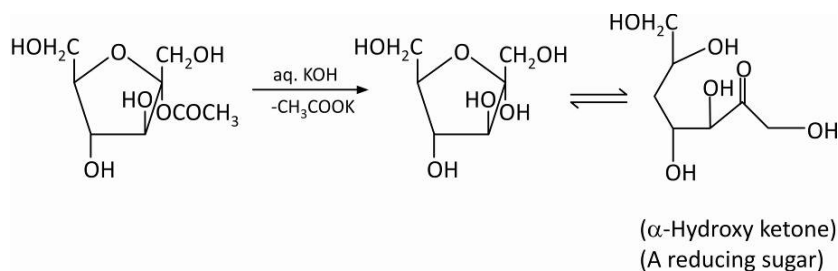
10.(2) DIBAL-H reduces only ester group to an aldehyde and alcohol.



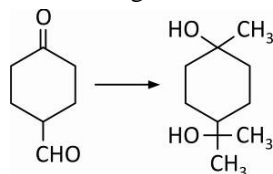
11. Which of the following compounds will behave as a reducing sugar in an aqueous KOH solution.



11.(2)

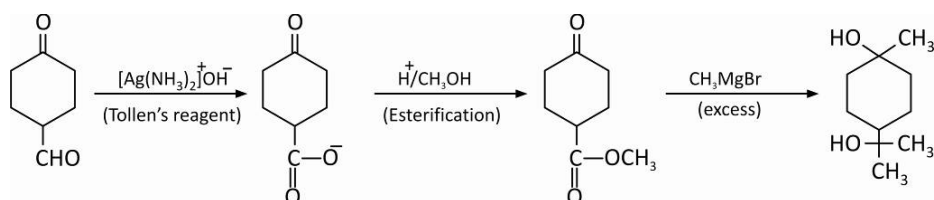


12. The correct sequence of reagents for the following conversion will be



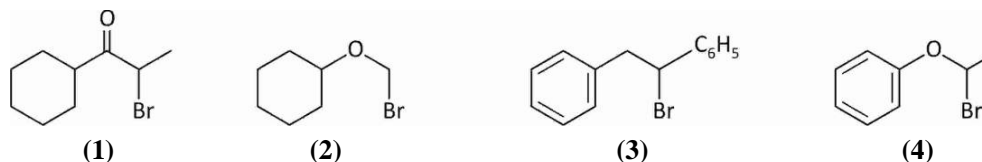
- (1) $[\text{Ag}(\text{NH}_3)_2]^+ \text{OH}^-$, CH_3MgBr , H^+ / CH_3OH
 (2) $[\text{Ag}(\text{NH}_3)_2]^+ \text{OH}^-$, H^+ / CH_3OH , CH_3MgBr
 (3) CH_3MgBr , H^+ / CH_3OH , $[\text{Ag}(\text{NH}_3)_2]^+ \text{OH}^-$
 (4) CH_3MgBr , $[\text{Ag}(\text{NH}_3)_2]^+ \text{OH}^-$, H^+ / CH_3OH

12.(2)

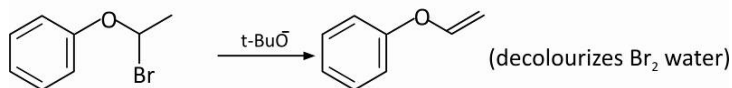
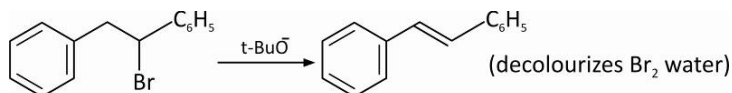
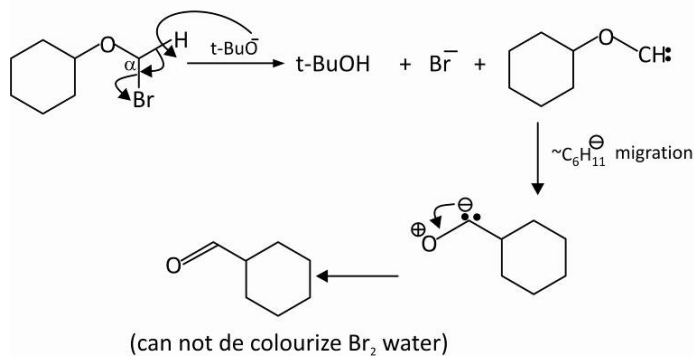
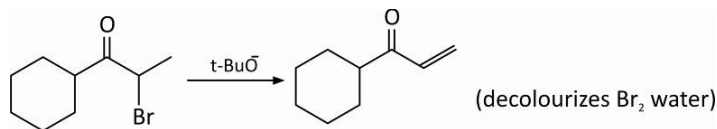


13. Which of the following species is not paramagnetic?

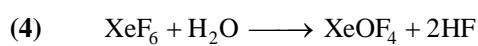
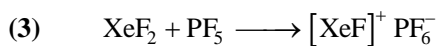
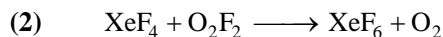
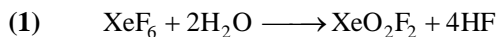
- (1) B_2 (2) NO (3) CO (4) O_2

13.(3) (1) $\text{B}_2 \longrightarrow \sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \pi_{2p_x}^1, \pi_{2p_y}^1$ B_2 contains two unpaired electrons hence paramagnetic.(2) $\text{NO} \longrightarrow$ odd electron species hence paramagnetic(3) $\text{CO} \longrightarrow \sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \pi_{2p_x}^2, \pi_{2p_y}^2, \sigma_{2p_z}^2$ CO contains no unpaired electron hence diamagnetic.(4) $\text{O}_2 \longrightarrow \sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \sigma_{2p_z}^2, \pi_{2p_x}^2, \pi_{2p_y}^2, \pi_{2p_x}^{*1}, \pi_{2p_y}^{*1}$ O_2 contains two unpaired electron hence paramagnetic.14. Which of the following, upon treatment with *tert*-BuONa followed by addition of bromine water, fails to decolourize the colour of bromine?

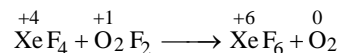
14.(2)



15. Which of the following reactions is an example of a redox reaction?



15.(2) Any reaction having a substance in its elemental form is a redox reaction.

16. ΔU is equal to:

(1) Isothermal work

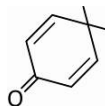
(2) Isochoric work

(3) Isobaric work

(4) Adiabatic work

16.(4) For adiabatic process : $q = 0$ So from 1st law $\Delta U = q + w$ We can write $\Delta U = w$

17. Which of the following molecules is least resonance stabilized?



(1)



(2)

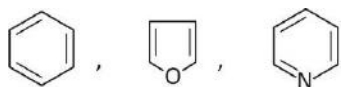


(3)

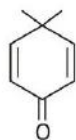


(4)

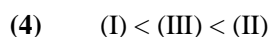
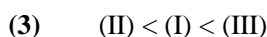
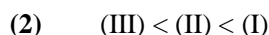
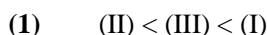
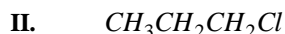
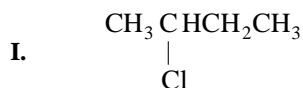
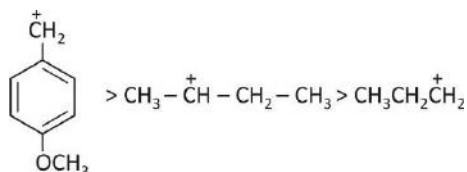
17.(1)



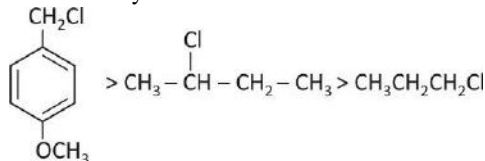
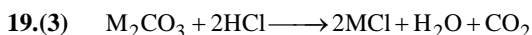
are aromatic and stabilized by according to Huckel's Rule.



is non aromatic hence least stabilized by resonance

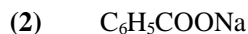
18. The increasing order of the reactivity of the following halides for the S_N1 reaction is:18.(3) Order of reactivity for S_N1 mechanism is in accordance with order of stability of carbocation involved :

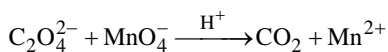
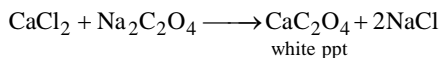
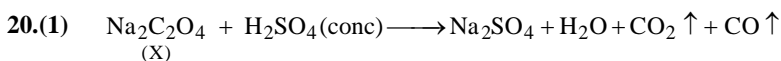
So, order of reactivity

19. 1 gram of a carbonate (M_2CO_3) on treatment with excess HCl produces 0.01186 mole of CO_2 . The molar mass of M_2CO_3 in g mol^{-1} is:No. of moles of M_2CO_3 = No. of moles of CO_2 evolved

$$\frac{1}{M} = 0.01186 \quad (M = \text{molar mass of } \text{M}_2\text{CO}_3)$$

$$\Rightarrow M = \frac{1}{0.01186} = \frac{10^5}{1186} = 84.3$$

20. Sodium salt of an organic acid 'X' produces effervescence with conc. H_2SO_4 . 'X' reacts with the acidified aqueous CaCl_2 solution to give a white precipitate which decolourises acidic solution of KMnO_4 . 'X' is :



21. The most abundant elements by mass in the body of a healthy human adult are: Oxygen (61.4%); Carbon (22.9%), Hydrogen (10.0%); and Nitrogen (2.6%). The weight which a 75 kg person would gain if all ^1H atoms are replaced by ^2H atoms is:

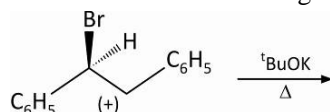
- (1) 10 kg (2) 15 kg (3) 37.5 kg (4) 7.5 kg

21.(4) Mass of ^1H in body $= 75 \times 10^3 \times \frac{10}{100} \text{ g} = 7.5 \times 10^3 \text{ g}$

No. of moles of ^1H replaced by $^2\text{H} = 7.5 \times 10^3$

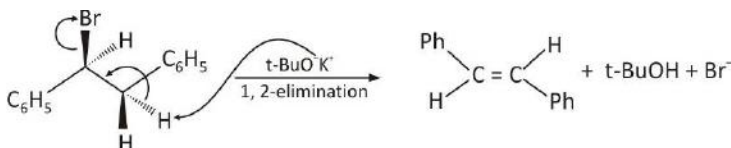
So mass increased $= 7.5 \times 10^3 \text{ g} = 7.5 \text{ kg}$

22. The major product obtained in the following reaction is :

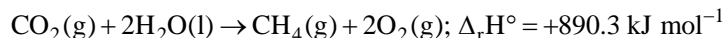
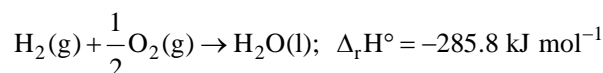


- (1) $(-)\text{C}_6\text{H}_5\text{CH}(\text{O}^i\text{Bu})\text{CH}_2\text{C}_6\text{H}_5$ (2) $(\pm)\text{C}_6\text{H}_5\text{CH}(\text{O}^i\text{Bu})\text{CH}_2\text{C}_6\text{H}_5$
(3) $\text{C}_6\text{H}_5\text{CH}=\text{CHC}_6\text{H}_5$ (4) $(+)\text{C}_6\text{H}_5\text{CH}(\text{O}^i\text{Bu})\text{CH}_2\text{C}_6\text{H}_5$

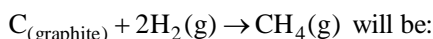
22.(3)



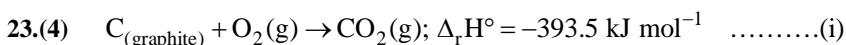
23. Given : $\text{C}_{(\text{graphite})} + \text{O}_2(\text{g}) \rightarrow \text{CO}_2(\text{g}); \Delta_r\text{H}^\circ = -393.5 \text{ kJ mol}^{-1}$

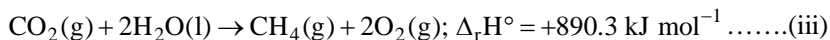
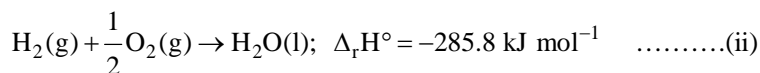


Based on the above thermochemical equations, the value of $\Delta_r\text{H}^\circ$ at 298 K for the reaction



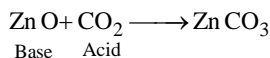
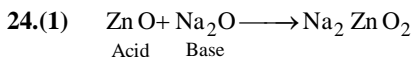
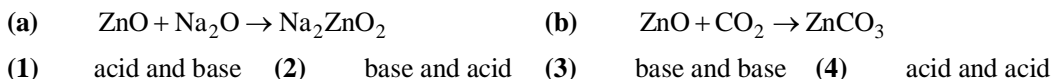
- (1) $-144.0 \text{ kJ mol}^{-1}$ (2) $+74.8 \text{ kJ mol}^{-1}$
(3) $+144.0 \text{ kJ mol}^{-1}$ (4) $-74.8 \text{ kJ mol}^{-1}$





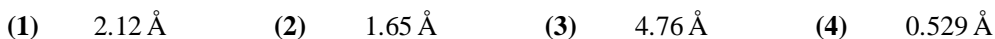
$$\begin{aligned} \Delta_r H_4 &= \Delta_r H_1 + 2\Delta_r H_2 + \Delta_r H_3 \\ &= -393.5 + (-285.8 \times 2) + 890.3 \\ &= -74.8 \text{ KJ / mol} \end{aligned}$$

24. In the following reactions, ZnO is respectively acting as a/an:



Non metal oxides are generally acidic while alkali metal oxides are basic.

25. The radius of the second Bohr orbit for hydrogen atom is: (Planck's Const. $h = 6.6262 \times 10^{-34} \text{ Js}$; mass of electron = $9.1091 \times 10^{-31} \text{ kg}$; charge of electron $e = 1.60210 \times 10^{-19} \text{ C}$; permittivity of vacuum $\epsilon_0 = 8.854185 \times 10^{-12} \text{ kg}^{-1} \text{ m}^{-3} \text{ A}^2$)



25.(1) $r_n = 0.529 \times \frac{n^2}{Z}$

$$r_2 = 0.529 \times 4 = 2.116 \text{ \AA}$$

26. Two reactions R_1 and R_2 have identical pre-exponential factors. Activation energy of R_1 exceeds that of R_2 by 10 kJ mol^{-1} . If k_1 and k_2 are rate constants for reactions R_1 and R_2 respectively at 300 K , then

$\ln(k_2/k_1)$ is equal to : ($R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$)



26.(1) From Arrhenius equation $k = Ae^{-E_a/RT}$

$$\text{Since } \ln\left(\frac{k_2}{k_1}\right) = \frac{1}{RT}[(E_a)_1 - (E_a)_2]$$

$$= \frac{1 \times 10 \times 10^3}{8.314 \times 300} = 4$$

27. A metal crystallizes in a face centred cubic structure. If the edge length of its unit cell is 'a', the closest approach between two atoms in metallic crystal will be:

(1) $\frac{a}{\sqrt{2}}$ (2) $2a$ (3) $2\sqrt{2}a$ (4) $\sqrt{2}a$

- 27.(1) In FCC distance of closest approach between two atoms = $2r$

In FCC, atoms are in close contact along face diagonal of FCC unit cell.

$$\therefore 4r = \sqrt{2}a$$

$$2r = \frac{\sqrt{2}a}{2} = \left(\frac{a}{\sqrt{2}} \right)$$

28. The group having isoelectronic species is:

(1) O^-, F^-, Na^+, Mg^{2+} (2) $O^{2-}, F^-, Na^+, Mg^{2+}$
 (3) O^-, F^-, Na, Mg^+ (4) O^{2-}, F^-, Na, Mg^{2+}

- 28.(2) Isoelectronic species have same no. of electrons. All these species contains 10 electrons.

29. Given $E_{Cl_2/Cl^-}^\circ = 1.36V$, $E_{Cr^{3+}/Cr}^\circ = -0.74V$

$$E_{Cr_2O_7^{2-}/Cr^{3+}}^\circ = 1.33V, E_{MnO_4^-/Mn^{2+}}^\circ = 1.51V$$

Among the following, the strongest reducing agent is:

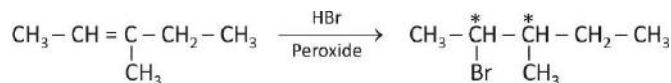
(1) Cl^- (2) Cr (3) Mn^{2+} (4) Cr^{3+}

29.(2) $E_{Red}^\circ \propto \frac{1}{\text{Reducing strength}}$

30. 3-Methyl-pent-2-ene on reaction with HBr in presence of peroxide forms an addition product. The number of possible stereoisomers for the product is:

(1) Four (2) Six (3) Zero (4) Two

- 30.(1)



$$\text{No of stereoisomers} = 2^n = 2^2 = 4 \quad [n = \text{No. of chiral carbon atoms}]$$

PART-B

MATHEMATICS

31. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to :

- (1) 4 (2) -1 (3) -2 (4) 2

31.(4) $I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$

$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1-\cos x}$ using $[a+b-x]$ property

$2I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{2}{1-\cos^2 x} dx$

$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \operatorname{cosec}^2 x \, dx = \cot x \Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}} = 1 - (-1) = 2$

32. Let $I_n = \int \tan^n x \, dx$, ($n > 1$). If $I_4 + I_6 = a \tan^5 x + bx^5 + C$, where C is a constant of integration, then the ordered pair (a, b) is equal to :

- (1) $\left(\frac{1}{5}, -1\right)$ (2) $\left(-\frac{1}{5}, 0\right)$ (3) $\left(-\frac{1}{5}, 1\right)$ (4) $\left(\frac{1}{5}, 0\right)$

32.(4) $\int \tan^4 x \, dx + \int \tan^6 x \, dx = a \tan^5 x + bx^5 + c$

Differentiating both sides

$\tan^4 x + \tan^6 x = 5a \tan^4 x \sec^2 x + 5bx^4 = 5a \tan^4 x (\tan^2 x + 1) + 5bx^4 = 5a \tan^4 x + 5a \tan^6 x + 5bx^4$

$\Rightarrow a = \frac{1}{5} \quad b = 0$

33. The area (in sq. units) of the region $\{(x, y) : x \geq 0, x + y \leq 3, x^2 \leq 4y \text{ and } y \leq 1 + \sqrt{x}\}$ is :

- (1) $\frac{7}{3}$ (2) $\frac{5}{2}$ (3) $\frac{59}{12}$ (4) $\frac{3}{2}$

33.(2) Solving $x^2 = 4y$ and $x + y = 3$

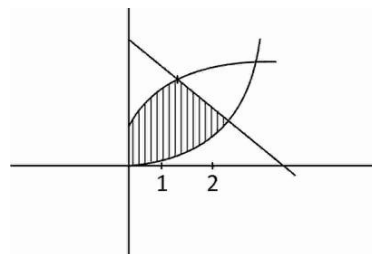
We get, $\frac{x^2}{4} + x = 3$

$\Rightarrow x^2 + 4x - 12 = 0$

$\Rightarrow (x+6)(x-2) = 0$

$\Rightarrow x = 2$
 $y = 1$

Solving $y = 1 + \sqrt{x}$ and $y = 3 - x$



We get, $1 + \sqrt{x} = 3 - x \Rightarrow x = 1 \quad y = 2$

$$\therefore \text{Area} = \int_0^1 (1 + \sqrt{x}) dx + \int_1^2 (3 - x) dx - \int_0^2 \frac{x^4}{4} dx = x + \frac{2}{3} x^{3/2} \Big|_0^1 + 3x - \frac{x^2}{2} \Big|_1^2 - \frac{x^5}{20} \Big|_0^2 = \frac{5}{2}$$

34. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is :

(1) 4 (2) $\frac{6}{25}$ (3) $\frac{12}{5}$ (4) 6

34.(3) $n = 10$

$$p = \frac{15}{25}$$

$$q = \frac{10}{25}$$

$$\text{var}(X) = npq$$

$$= 10 \times \frac{15}{25} \times \frac{10}{25} = 10 \times \frac{3}{5} \times \frac{2}{5} = \frac{12}{5}$$

35. If $(2 + \sin x) \frac{dy}{dx} + (y + 1) \cos x = 0$ and $y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ is equal to :

(1) $-\frac{1}{3}$ (2) $\frac{4}{3}$ (3) $\frac{1}{3}$ (4) $-\frac{2}{3}$

35.(3) $\frac{dy}{dx} = \frac{-(y+1) \cos x}{2 + \sin x}$

$$\int \frac{dy}{y+1} = - \int \frac{\cos x}{2 + \sin x} dx$$

$$\ln(y+1) = -\ln(2 + \sin x) + \ln c$$

$$(y+1)(2 + \sin x) = c$$

$$y(0) = 1$$

$$(2)(2) = c \Rightarrow c = 4$$

$$y(\pi/2) = ?$$

$$(y+1)(2+1) = 4$$

$$y = \frac{1}{3}$$

36. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$.

If $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to :

(1) -1 (2) 1 (3) $-z$ (4) z

36.(3) $2\omega + 1 = z$

$$\omega = \frac{\sqrt{3}i - 1}{2}$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix} = 3k$$

$$R_1 \rightarrow R_1 + R_2 + R_3$$

$$\begin{vmatrix} 3 & 0 & 0 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$$

$$\equiv 3[\omega(-\omega^2 - 1) - \omega^4] \equiv 3[-\omega^3 - \omega - \omega] = 3[-1 - 2\omega]$$

$$= -3[2\omega + 1] = -3z = 3k \Rightarrow k = -z$$

37. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. Let \vec{c} be a vector such that $|\vec{c} - \vec{a}| = 3$, $|\left(\vec{a} \times \vec{b}\right) \times \vec{c}| = 3$ and the angle between \vec{c} and $\vec{a} \times \vec{b}$ be 30° . Then $\vec{a} \cdot \vec{c}$ is equal to :

- (1) 5 (2) $\frac{1}{8}$ (3) $\frac{25}{8}$ (4) 2

37.(4) $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$

$$\vec{b} = \hat{i} + \hat{j}$$

$$|\vec{c} - \vec{a}| = 3$$

$$|\left(\vec{a} \times \vec{b}\right) \times \vec{c}| = 3$$

$$|\vec{a} \times \vec{b}| |\vec{c}| \sin \theta = 3$$

$$|\vec{a} \times \vec{b}| |\vec{c}| = 6 \quad \dots(i)$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} \equiv \hat{i}(2) - \hat{j}(2) + \hat{k}(1) = 2\hat{i} - 2\hat{j} + \hat{k}$$

$$|\vec{a} \times \vec{b}| = 3$$

From (i)

$$3 \cdot |\vec{c}| = 6$$

$$|\vec{c}| = 2$$

Now $|\vec{c} - \vec{a}| = 3$

$$|\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a} \cdot \vec{c} = 9$$

$$4 + 9 - 2\vec{a} \cdot \vec{c} = 9$$

$$\vec{a} \cdot \vec{c} = 2$$

38. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines $y = |x|$ is :

- (1) $4(\sqrt{2} - 1)$ (2) $4(\sqrt{2} + 1)$ (3) $2(\sqrt{2} + 1)$ (4) $2(\sqrt{2} - 1)$

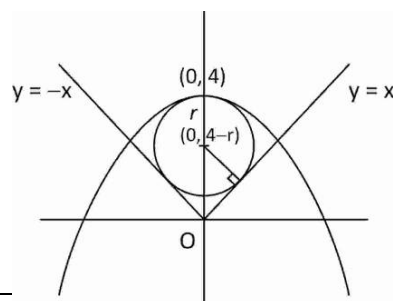
38.(1) $x^2 + x - 4 = 0$

$$x = \frac{-1 \pm \sqrt{1 + 16}}{2} = \frac{-1 \pm \sqrt{17}}{2}$$

$$\frac{(4 - r) - 0}{\sqrt{2}} = r$$

$$4 - r = r\sqrt{2}$$

$$r = \frac{4}{\sqrt{2} + 1} = \frac{4(\sqrt{2} - 1)}{1}$$



39. If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x} \cdot g(x)$, then $g(x)$ equals :

- (1) $\frac{3x}{1-9x^3}$ (2) $\frac{3}{1+9x^3}$ (3) $\frac{9}{1+9x^3}$ (4) $\frac{3x\sqrt{x}}{1-9x^3}$

39.(3) $x \in \left(0, \frac{1}{4}\right)$

$$3x^{3/2} \in \left(0, \frac{3}{8}\right)$$

$$\theta \in \left(0, \tan^{-1}\left(\frac{3}{8}\right)\right)$$

$$y = \tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$$

$$\text{Let } \tan^{-1}(3x^{3/2}) = \theta = \tan^{-1}\left(\frac{2 \tan \theta}{1 - \tan^2 \theta}\right) = \tan^{-1}(\tan 2\theta) = 2\theta = 2 \tan^{-1}(3x^{3/2})$$

$$y' = \frac{2}{1+9x^3} \times 3 \cdot \frac{3}{2} x^{1/2} = \frac{9}{1+9x^3} \sqrt{x}$$

40. If two different numbers are taken from the set $\{0, 1, 2, 3, \dots, 10\}$; then the probability that their sum as well as absolute difference are both multiple of 4, is :

- (1) $\frac{14}{45}$ (2) $\frac{7}{55}$ (3) $\frac{6}{55}$ (4) $\frac{12}{55}$

40.(3) $A \equiv \{0, 4, 8\} \equiv 0$ remainder when divided by 4.

$$B \equiv \{2, 6, 10\} \equiv 2$$

$$C \equiv \{1, 5, 9\} \equiv 1$$

$$D \equiv \{3, 7\} \equiv -1$$

$$\Rightarrow x, y \in A \text{ or } x, y \in B$$

$$\frac{{}^3C_2 + {}^3C_2}{{}^{11}C_2} = \frac{6}{11 \cdot \frac{10}{2}} = \frac{6}{55}$$

41. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals :

- (1) $\frac{1}{8}$ (2) $\frac{1}{4}$ (3) $\frac{1}{24}$ (4) $\frac{1}{16}$

41.(4) $\lim_{x \rightarrow \pi/2} \frac{\cot x - \cos x}{(\pi - 2x)^3}$

$$\lim_{h \rightarrow 0} \frac{\cot(\pi/2 - h) - \cos[\pi/2 - h]}{8h^3}$$

$$\frac{1}{8} \lim_{h \rightarrow 0} \frac{\sin h [1 - \cos h]}{h^3} = \frac{1}{16}$$

42. The value of $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + \dots + ({}^{21}C_{10} - {}^{10}C_{10})$, is :

- (1) $2^{20} - 2^9$ (2) $2^{20} - 2^{10}$ (3) $2^{21} - 2^{11}$ (4) $2^{21} - 2^{10}$

$$42.(2) \quad \binom{21}{1}C_1 - \binom{10}{1}C_1 + \binom{21}{2}C_2 - \binom{10}{2}C_2 + \binom{21}{3}C_3 - \binom{10}{3}C_3 + \binom{21}{4}C_4 - \binom{10}{4}C_4 + \dots + \binom{21}{10}C_{10} - \binom{10}{10}C_{10}$$

$$= \left(\frac{2^{21} - 2}{2} \right) - (2^{10} - 1) = 2^{20} - 1 - 2^{10} + 1 = 2^{20} - 2^{10}$$

43. For three events A , B and C , $P(\text{Exactly one of } A \text{ or } B \text{ occurs}) = P(\text{Exactly one of } B \text{ or } C \text{ occurs}) = P(\text{Exactly one of } C \text{ or } A \text{ occurs}) = \frac{1}{4}$ and $P(\text{All the three events occur simultaneously}) = \frac{1}{16}$.

Then the probability that at least one of the events occurs, is :

- (1) $\frac{7}{64}$ (2) $\frac{3}{16}$ (3) $\frac{7}{32}$ (4) $\frac{7}{16}$

$$43.(4) \quad P(A) + P(B) - 2P(A \cap B) = \frac{1}{4}$$

$$P(B) + P(C) - 2P(B \cap C) = \frac{1}{4}$$

$$P(C) + P(A) - 2P(C \cap A) = \frac{1}{4}$$

$$P(A \cap B \cap C) = \frac{1}{16}$$

$$P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) = \frac{3}{8}$$

$$P(A \cup B \cup C) = \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$$

44. Let a vertical tower AB have its end A on the level ground. Let C be the mid point of AB and P be a point on the ground such that $AP = 2AB$. If $\angle BPC = \beta$, then $\tan \beta$ is equal to :

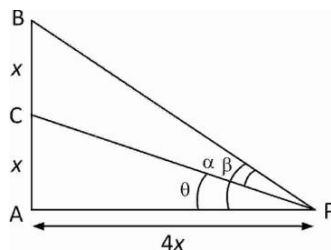
- (1) $\frac{2}{9}$ (2) $\frac{4}{9}$ (3) $\frac{6}{7}$ (4) $\frac{1}{4}$

$$44.(1) \quad \tan \beta$$

$$\tan \alpha = \frac{1}{2}; \quad \tan \theta = \frac{1}{4}$$

$$\tan \beta = \tan(\alpha - \theta)$$

$$\frac{\frac{1}{2} - \frac{1}{4}}{1 + \frac{1}{8}} = \frac{\frac{1}{4}}{\frac{9}{8}} = \frac{2}{9}$$



45. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is $x = -4$, then

the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is :

- (1) $4x + 2y = 7$ (2) $x + 2y = 4$ (3) $2y - x = 2$ (4) $4x - 2y = 1$

$$45.(4) \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{a}{\frac{1}{2}} = 4 \quad \Rightarrow \quad a = 2$$

$$e^2 = 1 - \frac{b^2}{a^2} \Rightarrow b^2 = 3$$

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

Equation of normal

$$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$$

$$\frac{4x}{1} - \frac{3y}{\left(\frac{3}{2}\right)} = 1$$

$$4x - 2y = 1$$

46. If, for a positive integer n , the quadratic equation,

$$x(x+1) + (x-1)(x+2) + \dots + (x+n-1)(x+n) = 10n$$

has two consecutive integral solutions, then n is equal to :

- (1) 10 (2) 11 (3) 12 (4) 9

46.(2) $\sum_{r=1}^n (x+(r-1))(x+r) = 10n$

$$\sum x^2 + (2r-1)x + r(r-1) = 10n$$

$$nx^2 + x.n^2 + \frac{n(n^2-31)}{3} = 0$$

$$3x^2 + 3nx + (n^2 - 31) = 0$$

$$\frac{\sqrt{D}}{|\alpha|} = \frac{\sqrt{9n^2 - 12n^2 + 372}}{3} = 1$$

$$372 - 3n^2 = 9$$

$$3n^2 = 372 - 9 = 363$$

$$n^2 = 121$$

$$n = 11$$

47. The following statement $(p \rightarrow q) \rightarrow [(\sim p \rightarrow q) \rightarrow q]$ is :

- (1) equivalent to $p \rightarrow \sim q$ (2) a fallacy
(3) a tautology (4) equivalent to $\sim p \rightarrow q$

47.(3) $(p \rightarrow q) \rightarrow [(\sim p \rightarrow q) \rightarrow q]$

p	q	$(p \rightarrow q)$	$(\sim p \rightarrow q)$	$(\sim p \rightarrow q) \rightarrow q$	$(p \rightarrow q) \rightarrow [(\sim p \rightarrow q) \rightarrow q]$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	T	T

48. The normal to the curve $y(x-2)(x-3) = x+6$ at the point where the curve intersects the y-axis, passes through the point :

(1) $\left(\frac{1}{2}, -\frac{1}{3}\right)$ (2) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (3) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$ (4) $\left(\frac{1}{2}, \frac{1}{2}\right)$

48.(4) $6y = 6$ $(0, 1)$

$$y = 1$$

$$(x-2)(x-3)y' + (x-3) + (x-2)y = 1$$

$$6y' - 3 - 2 = 1$$

$$y' = 1$$

$$y'_{(x=0)} = 1 \Rightarrow \text{Slope of normal} = -1$$

$$(y-1) = -x$$

$$y+x=1$$

49. For any three positive real numbers a, b and c , $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$. Then :

(1) a, b and c are in A.P.

(2) a, b and c are in G.P.

(3) b, c and a are in G.P.

(4) b, c and a are in A.P.

49.(4) $225a^2 + 9b^2 + 25c^2 - 75ac = 15b(3a + c)$

$$225a^2 + 9b^2 + 25c^2 = 75ac + 45ab + 15bc$$

$$(15a)^2 + (3b)^2 + (5c)^2 = 45ab + 75ac + 15bc$$

$$15a = 3b = 5c = k$$

$$a = \frac{k}{15} = \frac{k}{15}$$

$$b = \frac{k}{3} = \frac{5k}{15}$$

$$c = \frac{k}{5} = \frac{3k}{15}$$

$$b, c, a \text{ are in A.P.}$$

50. If the image of the point $P(1, -2, 3)$ in the plane, $2x + 3y - 4z + 22 = 0$ measured parallel to the line,

$$\frac{x}{1} = \frac{y}{4} = \frac{z}{5} \text{ is } Q, \text{ then } PQ \text{ is equal to :}$$

(1) $\sqrt{42}$

(2) $6\sqrt{5}$

(3) $3\sqrt{5}$

(4) $2\sqrt{42}$

50.(4) $\frac{x-1}{1} = \frac{y+2}{4} = \frac{z-3}{5} = \lambda$

$$(\lambda+1, 4\lambda-2, 5\lambda+3)$$

$$2\lambda + 2 + 12\lambda - 6 - 20\lambda - 12 + 22 = 0$$

$$-6\lambda + 6 = 0$$

$$\lambda = -1$$

$$(2, 2, 8)$$

$$\sqrt{1^2 + 4^2 + 5^2} = \sqrt{42}$$

51. If $5(\tan^2 x - \cos^2 x) = 2 \cos 2x + 9$, then the value of $\cos 4x$ is :

- (1) $\frac{2}{9}$ (2) $-\frac{7}{9}$ (3) $-\frac{3}{5}$ (4) $\frac{1}{3}$

51.(2) $5(\tan^2 x - \cos^2 x) = 2 \cos 2x + 9$

$$5 \left(\frac{\sin^2 x - \cos^4 x}{\cos^2 x} \right) = 2 \cos^2 x + 9$$

$$\cos^2 x = t$$

$$5 \left(\frac{1-t-t^2}{t} \right) = 2(2t-1) + 9$$

$$5 - 5t - 5t^2 = 4t^2 + 7t$$

$$9t^2 + 15t - 3t - 5 = 0$$

$$3t(3t+5) - (3t+5) = 0$$

$$t = \frac{1}{3}$$

$$\cos 2x = 2 \cdot \frac{1}{3} - 1 = \frac{-1}{3}$$

$$\cos 4x = 2 \cdot \left(\frac{-1}{3} \right)^2 - 1 = \frac{2}{9} - 1 = \frac{-7}{9}$$

52. Let $a, b, c \in R$. If $f(x) = ax^2 + bx + c$ is such that $a + b + c = 3$ and $f(x+y) = f(x) + f(y) + xy$,

$\forall x, y \in R$, then $\sum_{n=1}^{10} f(n)$ is equal to :

- (1) 190 (2) 255 (3) 330 (4) 165

52.(3) $f(x) = \frac{1}{2}x^2 + x^2bx$

$$f(1) = \frac{1}{2} + b = 3 \Rightarrow b = 3 \cdot \frac{-1}{3} = \frac{5}{2}$$

$$f(x) = \frac{1}{2}x^2 + \frac{5}{2}x$$

$$\begin{aligned} \sum f(n) &= \frac{1}{2} \sum n^2 + \frac{5}{2} \sum n \\ &= \frac{1}{2} \cdot \frac{10 \cdot 11 \cdot 1}{6} + \frac{5}{2} \cdot \frac{10 \cdot 11}{2} = \frac{385}{2} + \frac{275}{2} = \frac{660}{2} = 330 \end{aligned}$$

53. The distance of the point $(1, 3, -7)$ from the plane passing through the point $(1, -1, -1)$, having normal

perpendicular to both the lines $\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-4}{3}$ and $\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z+7}{-1}$ is :

- (1) $\frac{5}{\sqrt{83}}$ (2) $\frac{10}{\sqrt{74}}$ (3) $\frac{20}{\sqrt{74}}$ (4) $\frac{10}{\sqrt{83}}$

53.(4) $\begin{vmatrix} 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix}$

$$\equiv 2+3:6+1:-1+4$$

$$5:7:3$$

$$5x+7y+3z = 5-7-3 = -5$$

$$5x+2y+3z+5=0$$

$$\left| \frac{5+21-21+5}{\sqrt{25+49+9}} = \frac{10}{\sqrt{83}} \right|$$

54. If S is the set of distinct values of ' b ' for which the following system of linear equations

$$x + y + z = 1$$

$$x + ay + z = 1$$

$$ax + by + z = 0$$

has no solution, then S is :

(1) a finite set containing two or more elements

(2) a singleton

(3) an empty set

(4) an infinite set

$$54.(2) \quad \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = (a-b) - (1-a) + (b-a^2)$$

$$= a - b - 1 + a - b + a^2 = -(a^2 - 2a + 1) = -(a-1)^2 = 0$$

$$a = 1$$

$$x + y + z = 1$$

$$x + by + z = 0$$

$$\Rightarrow \quad \text{Two plane should be parallel} \quad \Rightarrow \quad b = 1$$

55. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then $\text{adj}(3A^2 + 12A)$ is equal to :

(1) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$

(2) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

(3) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

(4) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

$$55.(4) \quad A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} \quad A^2 = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 4+12 & -6-3 \\ -8-4 & 12+1 \end{bmatrix} = \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix}$$

$$3A^2 = \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix} : 12A = \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix}$$

$$3A^2 + 12A = \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$$

$$\text{Adj} (3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$$

56. A hyperbola passes through the point $P(\sqrt{2}, \sqrt{3})$ and has foci at $(\pm 2, 0)$. Then the tangent to this hyperbola at P also passes through the point :

(1) $(\sqrt{3}, \sqrt{2})$ (2) $(-\sqrt{2}, -\sqrt{3})$ (3) $(3\sqrt{2}, 2\sqrt{3})$ (4) $(2\sqrt{2}, 3\sqrt{3})$

56.(4) Clearly $ae = 2$; for $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\frac{2}{a^2} - \frac{3}{a^2(e^2-1)} = 1$$

$$\frac{2}{a^2} - \frac{3}{a^2 e^2 - a^2} = 1$$

$$\frac{2}{a^2} - \frac{3}{4-a^2} = 1$$

Solve to get $a^2 = 1, 8$

$a^2 = 8$ Rejected as e can't be less than 1

$$a^2 = 1, \quad b^2 = 3$$

$$\frac{x^2}{1} - \frac{y^2}{3} = 1$$

$$x(\sqrt{2}) - y \frac{(\sqrt{3})}{3} = 1$$

57. Let k be an integer such that the triangle with vertices $(k, -3k)$, $(5, k)$ and $(-k, 2)$ has area 28 sq. units. Then the orthocentre of this triangle is at the point :

(1) $\left(1, -\frac{3}{4}\right)$ (2) $\left(2, \frac{1}{2}\right)$ (3) $\left(2, -\frac{1}{2}\right)$ (4) $\left(1, \frac{3}{4}\right)$

57.(2) $\frac{1}{2} \begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix} = 28$

$$\begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix} = 56$$

$$1 \left[k(k-2) + 3k(5+k) + 1(10+k^2) \right] = 56$$

$$1 \left[5k^2 + 3k + 10 \right] = 56$$

$$5k^2 + 3k + 10 = \pm 56$$

or $5k^2 + 3k + 66 = 0$ or $5k^2 + 3k - 46 = 0$

$$D < 0$$

Solving we get $k = 2$

Hence, the vertices are $(2, -6)$, $(5, 2)$, $(-2, 2)$

Solving the equation of two altitudes we get orthocentre as $\left(2, \frac{1}{2}\right)$

58. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is :

(1) 25 (2) 30 (3) 12.5 (4) 10

58.(1) Given $2r + r\theta = 20$... (i)

$$\text{Area} = \frac{1}{2} r^2 \theta = A$$

From (i)

$$\theta = \frac{20 - 2r}{r}$$

$$A = \frac{1}{2} r^2 \frac{(20 - 2r)}{r} = \frac{20r - 2r^2}{2} = 10r - r^2$$

$$A = 10r - r^2$$

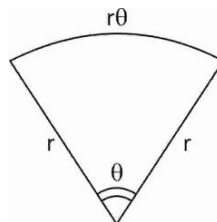
$$\frac{dA}{dr} = 10 - 2r = 0$$

$$r = 5$$

$$\frac{d^2 A}{dr^2} = -2 < 0$$

$r = 5$ will give maximum area

$$\theta = \frac{20 - 2(5)}{5} = 2 \text{ rad}$$



$$A = \frac{1}{2}(5)^2 \cdot 2 = 25$$

59. The function $f: R \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$, is :

- (1) Surjective but not injective (2) Neither injective nor surjective
(3) Invertible (4) Injective but not surjective

59.(1) $f(x) = \frac{x}{1+x^2}$

$$f: R \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$$

$$f'(x) = \frac{(1+x^2) \cdot 1 - x(2x)}{(1+x^2)^2}$$

Non-monotonic \therefore not injective

$$y = \frac{x}{1+x^2}$$

$$x^2(y) - x + y = 0$$

$$D \geq 0$$

$$1 - 4y^2 \geq 0$$

$$y \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

60. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in the party, is :

- (1) 469 (2) 484 (3) 485 (4) 468

60.(3) Let M_m denotes male relative of man $X = 3$
 M_w denotes female relative of man $X = 4$
 W_m denotes male relative of woman $Y = 4$
 W_w denotes female relative of woman $X = 3$

Case I $3M_m + 3W_w$ 1

Case II $3M_w + 3W_m$ ${}^4C_3 \times {}^4C_3 = 16$

Case III $2M_m + 1M_w + 2W_w + 1W_m$ ${}^3C_3 \times {}^4C_1 \times {}^3C_2 \times {}^4C_1 = 144$

Case IV $1M_m + 2M_w + 2W_m + 1W_w$ ${}^3C_1 \times {}^4C_2 \times {}^4C_2 \times {}^3C_1 = 324$

$$\text{Total} = 324 + 144 + 16 + 1 = 485$$

PART-C

PHYSICS

61. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer?

(speed of light = $3 \times 10^8 \text{ ms}^{-1}$)

- (1) 12.1 GHz (2) 17.3 GHz (3) 15.3 GHz (4) 10.1 GHz

- 61.(2) This question involves the use of relativistic Doppler's effect. The usual non-relativistic Doppler formula will NOT be applicable here as the velocity of observer is not small as compared to light.

The relativistic Doppler's formula is $v(\text{observed}) = v(\text{actual}) \sqrt{\frac{1+\beta}{1-\beta}}$

$$\text{Where } \beta = \frac{V}{C}$$

V is relative velocity of observer w.r.t. the source and is taken to be positive if observer and source are moving towards each other. So, here $v(\text{observed}) = (10\text{GHz}) \sqrt{\frac{1+1/2}{1-1/2}} = 17.3 \text{ GHz}$.

62. The following observations were taken for determining surface tension T of water by capillary method:

Diameter of capillary, $D = 1.25 \times 10^{-2} \text{ m}$ rise of water, $h = 1.45 \times 10^{-2} \text{ m}$.

Using $g = 9.80 \text{ m/s}^2$ and the simplified relation $T = \frac{r h g}{2} \times 10^3 \text{ N/m}$, the possible error in surface tension is closest to:

- (1) 1.5% (2) 2.4% (3) 10% (4) 0.15%

62.(1) $T = \frac{r h g}{2} \times 10^3 \text{ N/m} = \frac{d h g}{4} \times 10^3 \text{ N/m}$

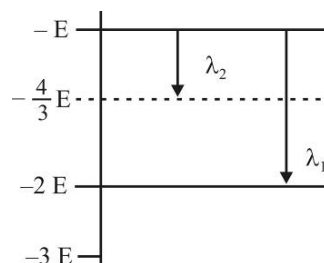
$$\% \text{ error } \frac{\Delta T}{T} = \frac{\Delta d}{d} + \frac{\Delta h}{h}$$

$$\Rightarrow \frac{\Delta T}{T} \times 100 = \left(\frac{0.01 \times 10^{-2}}{1.25 \times 10^{-2}} + \frac{0.01 \times 10^{-2}}{1.45 \times 10^{-2}} \right) \times 100 = 0.8\% + 0.7\% = 1.5\%$$

(There is no error in g because its value is not calculated through experiments)

63. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r_1 = \lambda_1 / \lambda_2$ is given by:

- (1) $r = \frac{2}{3}$ (2) $r = \frac{3}{4}$
(3) $r = \frac{1}{3}$ (4) $r = \frac{4}{3}$



63.(3) $\frac{hc}{\lambda_2} = -E - \left(-\frac{4E}{3} \right) = \frac{E}{3}$

$$\frac{hc}{\lambda_1} = -E - (-2E) = E \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{1}{3}$$

64. A body of mass $m = 10^{-2} \text{ kg}$ is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10 \text{ ms}^{-1}$. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be:

- (1) $10^{-3} \text{ kg s}^{-1}$ (2) $10^{-4} \text{ kg m}^{-1}$ (3) $10^{-1} \text{ kg m}^{-1} \text{ s}^{-1}$ (4) $10^{-3} \text{ kg m}^{-1}$

64.(2) $F = -kv^2$

$$ma = -kv^2$$

$$a = -\frac{k}{m}v^2 \Rightarrow \frac{dv}{dt} = -\frac{k}{m}v^2$$

$$\Rightarrow \int_{10}^v \frac{dv}{v^2} = -\frac{k}{m} \int_0^t dt \Rightarrow \left(-\frac{1}{v}\right)_{10}^v = -\frac{k}{m}t$$

$$\Rightarrow \frac{1}{v} = 0.1 + \frac{kt}{m} \Rightarrow v = \frac{1}{0.1 + \frac{kt}{m}} = \frac{1}{0.1 + 1000k}$$

$$\frac{1}{2} \times m \times v^2 = \frac{1}{8} \times v_0^2$$

$$v = \frac{v_0}{2} = 5$$

$$\Rightarrow \frac{1}{0.1 + 1000k} = 5 \Rightarrow 1 = 0.5 + 5000k$$

$$\Rightarrow k = \frac{0.5}{5000} \Rightarrow \boxed{k = 10^{-4} \text{ kg/m}}$$

65. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that

$$C_p - C_v = a \text{ for hydrogen gas}$$

$$C_p - C_v = b \text{ for nitrogen gas}$$

The correct relation between a and b is:

(1) $a = b$ (2) $a = 14b$ (3) $a = 28b$ (4) $a = \frac{1}{14}b$

65.(2) If C_p & C_v are specific heat capacity per gram

$$a = C_p - C_v = \frac{R}{2} \text{ (for hydrogen) (As R is per mole of H}_2 \text{ i.e. 2 gm of H}_2\text{)}$$

$$b = C_p - C_v = \frac{R}{28} \text{ (for Nitrogen) (As R is per mole of N}_2 \text{ i.e. 28 gm of N}_2\text{)}$$

$$\Rightarrow a = 14b$$

66. The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is I . What is the ratio l/R such that the moment of inertia is minimum?

(1) $\frac{\sqrt{3}}{2}$ (2) 1 (3) $\frac{3}{\sqrt{2}}$ (4) $\sqrt{\frac{3}{2}}$

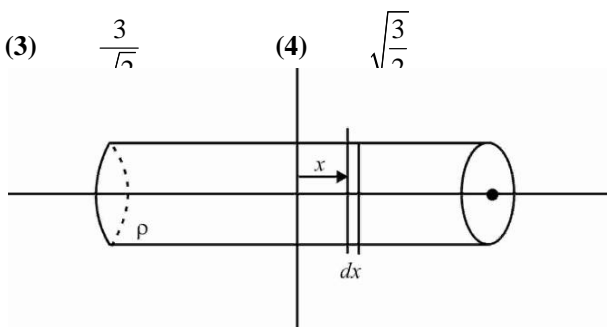
66.(4) $dm = \rho \pi R^2 dx$

$$dI = \frac{dmR^2}{4} + dm x^2 \text{ (Parallel axis theorem)}$$

$$I = \int_{-\frac{l}{2}}^{\frac{l}{2}} dI$$

$$= \int_{-\frac{l}{2}}^{\frac{l}{2}} \rho \frac{\pi R^2 \times R^2}{4} dx + \int_{-\frac{l}{2}}^{\frac{l}{2}} \rho \pi R^2 x^2 dx = \rho \frac{\pi R^2 \times R^2}{4} [x]_{-\frac{l}{2}}^{\frac{l}{2}} + \rho \pi R^2 \left(\frac{x^3}{3} \right)_{-\frac{l}{2}}^{\frac{l}{2}}$$

$$= \rho \frac{\pi R^2 \times R^2}{4} \times l + \rho \frac{\pi R^2}{3} \times \frac{l^3}{4} = \frac{mR^2}{4} + \frac{ml^2}{12}$$



$$I = \frac{mR^2}{4} + \frac{ml^2}{12} = \frac{m^2}{4\pi\rho l} + \frac{ml^2}{12} \quad (\rho\pi r^2 \ell = m)$$

For I to be max.

$$\frac{dI}{dl} = -\frac{m^2}{4\pi\rho} \left(\frac{1}{l^2}\right) + \frac{ml}{6} = 0 \quad \Rightarrow \quad \frac{m^2}{4\pi\rho} = \frac{ml^3}{6}$$

$$\Rightarrow \quad l^3 = \frac{3m}{2\pi\rho} \Rightarrow l = \left(\frac{3}{2}\right)^{1/3} \left(\frac{m}{\pi\rho}\right)^{1/3}$$

$$\rho = \frac{m}{\pi R^2 l} \quad \Rightarrow \quad R^2 = \frac{m}{\pi\rho l}$$

$$R^2 = \frac{m}{\pi\rho} \left(\frac{2}{3}\right)^{1/3} \left(\frac{\pi\rho}{m}\right)^{1/3} = \left(\frac{n}{\pi\rho}\right)^{2/3} \left(\frac{2}{3}\right)^{1/3} \Rightarrow R = \left(\frac{m}{\pi\rho}\right)^{1/3} \left(\frac{2}{3}\right)^{1/6}$$

$$\frac{l}{R} = \frac{\left(\frac{3}{2}\right)^{1/3} \left(\frac{m}{\pi\rho}\right)^{1/3}}{\left(\frac{m}{\pi\rho}\right)^{1/3} \left(\frac{2}{3}\right)^{1/6}} = \left(\frac{3}{2}\right)^{1/3} + \left(\frac{3}{2}\right)^{1/6} \Rightarrow \boxed{\frac{l}{R} = \sqrt{\frac{3}{2}}}$$

67. A radioactive nucleus A with a half life T, decays into a nucleus B. At t = 0, there is no nucleus B. At sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by:

(1) $t = T \frac{\log 1.3}{\log 2}$ (2) $t = T \log(1.3)$ (3) $t = \frac{T}{\log(1.3)}$ (4) $t = \frac{T}{2} \frac{\log 2}{\log 1.3}$

67.(1) $T = \frac{\ln 2}{\lambda}$

$$\frac{N_0 - N}{N} = 0.3 \Rightarrow N = \frac{N_0}{1.3}$$

$$N = N_0 e^{-\lambda t}$$

$$\frac{1}{1.3} = e^{-\lambda t}$$

$$t = \frac{\ln(1.3)}{\lambda} = T \frac{\ln(1.3)}{\ln(2)}$$

68. Which of the following statements is false?

- (1) In a balanced Wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed.
 (2) A rheostat can be used as a potential divider
 (3) Kirchhoff's second law represents energy conservation
 (4) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude

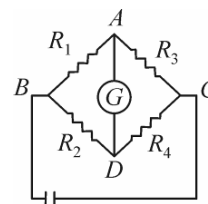
- 68.(1) From the given figure it is clear that if galvanometer is connected between AD and cell between BC, then

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad \dots(i)$$

If cell and galvanometer are interchanged, then for balance

$$\frac{R_1}{R_3} = \frac{R_2}{R_4} \quad \dots(ii)$$

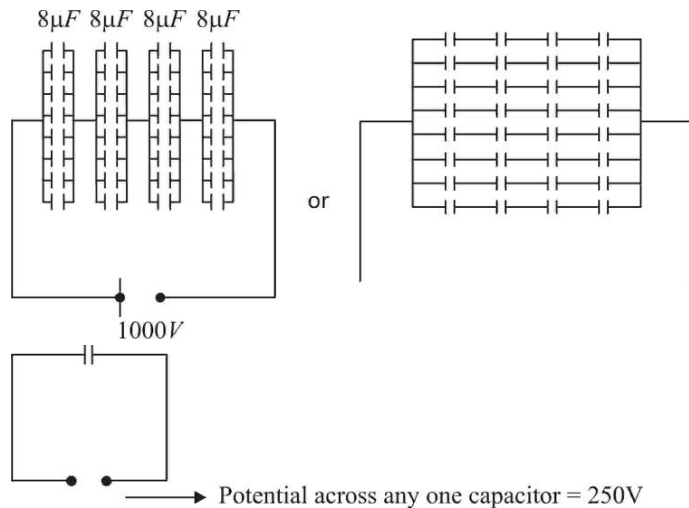
Since equations (i) and (ii) are same, null point is undisturbed if cell and galvanometer are interchanged



69. A capacitance of $2 \mu\text{F}$ is required in an electrical circuit across a potential difference of 1.0 kV . A large number of $1 \mu\text{F}$ capacitors are available which can withstand a potential difference of not more than 300 V . The minimum number of capacitors required to achieve this is:

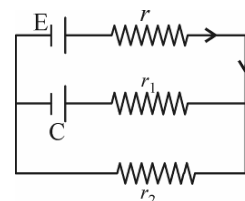
(1) 16 (2) 24 (3) 32 (4) 2

69.(3) $C_{eq} = \frac{8\mu\text{F}}{4} = 2\mu\text{F}$



70. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be :

(1) $CE \frac{r_1}{(r_2 + r)}$ (2) $CE \frac{r_2}{(r + r_2)}$
 (3) $CE \frac{r_1}{(r_1 + r)}$ (4) CE



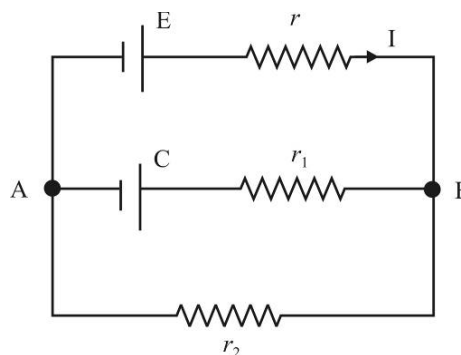
70. (2) At steady state current through the capacitor = 0

$$I = \frac{E}{r + r_2}$$

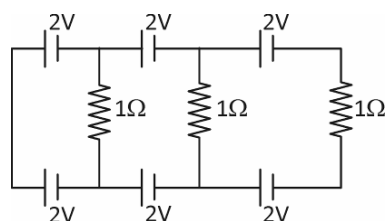
$$\text{P.d. across AB} = \frac{Q}{C}$$

$$E - Ir = \frac{Q}{C}$$

$$\Rightarrow E - \frac{Er}{r + r_2} = \frac{Q}{C} \Rightarrow \boxed{Q = CE \frac{r_2}{r + r_2}}$$



71.



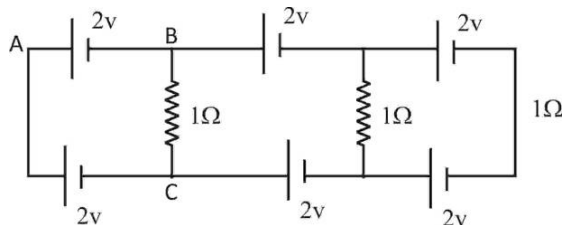
In the above circuit the current in each resistance is:

(1) 0.25 A (2) 0.5 A (3) 0 A (4) 1 A

- 71.(3) If $V_A = V \Rightarrow V_B = V - 2$ and $V_C = V - 2$

Potential difference across each resistor = 0

\Rightarrow current across each resistor = 0



72. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m . The bandwidth ($\Delta\omega_m$) of the signal is such that $\Delta\omega_m \ll \omega_c$. Which of the following frequencies is not contained in the modulated wave?

(1) ω_c (2) $\omega_m + \omega_c$ (3) $\omega_c - \omega_m$ (4) ω_m

- 72.(4) The frequencies in amplitude modulated wave is between $\omega_c - \omega_m$ and $\omega_c + \omega_m$.

73. In a common emitter amplifier circuit using an n-p-n transistor, the phase difference between the input and the output voltages will be:

(1) 90° (2) 135° (3) 180° (4) 45°

- 73.(3) $v_0 = \Delta(V_{cc} - i_c R_c) = -\Delta i_c R_c$

$$v_i = \Delta(v_{BE} + i_B R_B) = \Delta i_B R_B$$

$$A_v = \frac{v_0}{v_i} = -\frac{\Delta i_c}{\Delta i_B} \frac{R_c}{R_B} = -\beta \frac{R_c}{R_B}$$

“-ve” sign indicates output is exactly out of phase i.e. phase difference = 180°

74. A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C . T is given by: (Given : room temperature = 30°C , specific heat of copper = $0.1 \text{ cal/gm}^\circ\text{C}$)

(1) 885°C (2) 1250°C (3) 825°C (4) 800°C

74. (1) Heat gained by (water + calorimeter) = Heat lost by copper ball

$$\Rightarrow 100 \times 0.1 \times (75 - 30) + 170 \times 1 \times (75 - 30) = 100 \times 0.1 \times (T - 75)$$

$$\Rightarrow 450 + 7650 = 10(T - 75) \Rightarrow T - 75 = 810 \Rightarrow \boxed{T = 885^\circ\text{C}}$$

75. In a Young's double slit experiment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is:

(1) 7.8 mm (2) 9.75 mm (3) 15.6 mm (4) 1.56 mm

- 75.(1) $y_1 = \frac{n_1 \lambda_1 D}{d}$, $y_2 = \frac{n_2 \lambda_2 D}{d}$

$$\text{Given } y_1 = y_2$$

$$\Rightarrow n_1 \lambda_1 = n_2 \lambda_2$$

$$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{4}{5}$$

$$\text{Therefore } y_1 = \frac{4 \times 650 \times 10^{-9} \times 1.5}{0.5 \times 10^{-3}}$$

$$y_1 = 7.8 \text{ mm}$$

76. An electric dipole has a fixed dipole moment \vec{P} , which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E}_1 = E\hat{i}$, it experiences a torque $\vec{T}_1 = \tau\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E_1\hat{j}$ it experiences a torque $\vec{T}_2 = -\vec{T}_1$. The angle θ is:

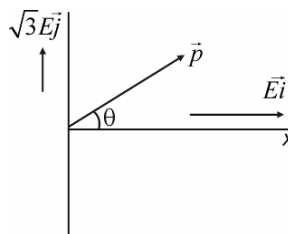
- (1) 45° (2) 60° (3) 90° (4) 30°

76.(2) $\vec{T} = \vec{p} \times \vec{E}$

$$\vec{p} = p \cos \theta \hat{i} + p \sin \theta \hat{j}$$

$$pE \sin \theta = T \text{ and } p\sqrt{3}E \cos \theta = T$$

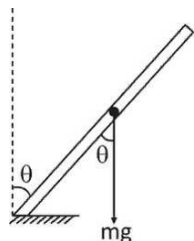
$$\Rightarrow \tan \theta = \sqrt{3} \Rightarrow \theta = 60^\circ$$



77. A slender uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is:

- (1) $\frac{2g}{3l} \sin \theta$ (2) $\frac{3g}{2l} \cos \theta$ (3) $\frac{2g}{3l} \cos \theta$ (4) $\frac{3g}{2l} \sin \theta$

77.(4) $mg \times \frac{l}{2} \sin \theta = \frac{ml^2}{3} \alpha$



$$\alpha = \frac{3g \sin \theta}{2l}$$

78. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by:

- (1) $\frac{P}{\alpha K}$ (2) $\frac{3\alpha}{PK}$ (3) $3PK\alpha$ (4) $\frac{P}{3\alpha K}$

78.(4) $K = \frac{P}{(-\Delta V/V)} \Rightarrow \frac{\Delta V}{V} = \frac{P}{K} \Rightarrow -\Delta V = \frac{PV}{K}$

$$\frac{PV}{K} = V(3\alpha)\Delta T \Rightarrow \Delta T = \frac{P}{3\alpha K}$$

79. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is:

- (1) Virtual and at a distance of 40 cm from convergent lens.
- (2) Real and at a distance of 40 cm from the divergent lens.
- (3) Real and at a distance of 6 cm from the convergent lens.
- (4) Real and at a distance of 40 cm from convergent lens.

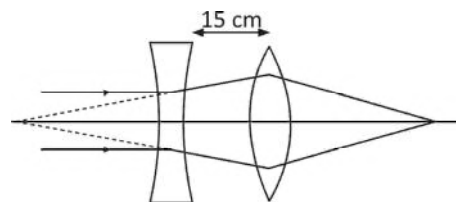
79.(4) $f_1 = -25\text{cm}$, $f_2 = 20\text{cm}$

For diverging lens $V = -25\text{cm}$

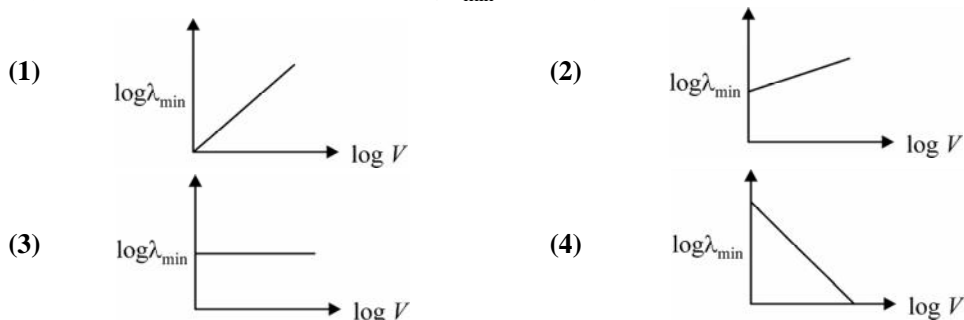
For converging lens, $u = -(15 + 25) = -40\text{cm}$

$$\frac{1}{v} - \frac{1}{-40} = \frac{1}{+20} \Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{40} = \frac{1}{40} \Rightarrow v = 40\text{cm}$$

\Rightarrow Image is real



80. An electron beam is accelerated by a potential difference V to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If λ_{\min} is the smallest possible wavelength of X-ray in the spectrum, the variation of $\log \lambda_{\min}$ with $\log V$ is correctly represented in:



80.(4) $\lambda_{\min} = \frac{hc}{eV}$

$$\ln \lambda_{\min} = \ln \frac{hc}{e} - \ln V \quad (y = k - x)$$

81. The temperature of an open room of volume 30m^3 increases from 17°C to 27°C due to the sunshine. The atmospheric pressure in the room remains $1 \times 10^5\text{Pa}$. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be :

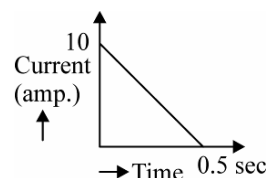
- (1) 1.38×10^{23}
- (2) 2.5×10^{25}
- (3) -2.5×10^{25}
- (4) -1.61×10^{23}

81.(3) $n_f - n_i = \frac{PV}{RT_f} \times N_0 - \frac{PV}{RT_i} \times N_0$, (N_0 = Avogadro's number)

$$N_0 = \frac{10^5 \times 30}{8.314} \times 6.02 \times 10^{23} \left(\frac{1}{300} - \frac{1}{290} \right) = -2.5 \times 10^{25}$$

82. In a coil of resistance 100Ω , a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is:

- (1) 225Wb
- (2) 250Wb
- (3) 275Wb
- (4) 200Wb



$$82.(2) \quad q = \frac{\Delta\phi}{R} \Rightarrow \text{area of } I-t \text{ graph} = \frac{\Delta\phi}{R}$$

$$\Rightarrow \frac{1}{2} \times 10 \times 0.5 = \frac{\Delta\phi}{100} \Rightarrow \Delta\phi = \frac{1}{2} \times 10 \times 0.5 \times 100 = 250 \text{ Wb}$$

83. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15Ω , it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter of range $0-10 \text{ V}$ is :

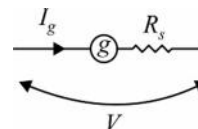
(1) $2.045 \times 10^3 \Omega$ (2) $2.535 \times 10^3 \Omega$ (3) $4.005 \times 10^3 \Omega$ (4) $1.985 \times 10^3 \Omega$

83.(4) $I_g = 5 \text{ mA}$

$$V = I_g (R_g + R_s)$$

$$\Rightarrow 10 = 5 \times 10^{-3} (15 + R_s) \Rightarrow 15 + R_s = 2 \times 10^3 = 2000$$

$$\Rightarrow R_s = 1985 = 1.985 \times 10^3 \Omega$$



84. A time dependent force $F = 6t$ acts on a particle of mass 1 kg . If the particle starts from rest, the work done by the force during the first 1 sec. will be:

(1) 22 J (2) 9 J (3) 18 J (4) 4.5 J

84.(4) $F = 6t$

$$\int F dt = \Delta P \Rightarrow P = \int_0^1 6t dt = \frac{6}{2} = 3$$

$$\Delta K = \frac{P^2}{2m} = \frac{3^2}{2 \times 1} = 4.5$$

$$W = \Delta K = 4.5 \text{ J}$$

85. A magnetic needle of magnetic moment $6.7 \times 10^{-2} \text{ Am}^2$ and moment of inertia $7.5 \times 10^{-6} \text{ kg m}^2$ is performing simple harmonic oscillations in a magnetic field of 0.01 T . Time taken for 10 complete oscillations is:

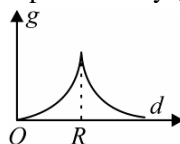
(1) 8.89 s (2) 6.98 s (3) 8.76 s (4) 6.65 s

85.(4) $T = MB \sin \theta = MB\theta$ (if θ is small)

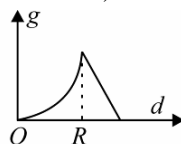
$$T = 2\pi \sqrt{\frac{I}{MB}} = 2\pi \sqrt{\frac{7.5 \times 10^{-6}}{6.7 \times 10^{-2} \times 0.01}} = 0.665$$

$$\text{Time taken for 10 oscillation } 6.65$$

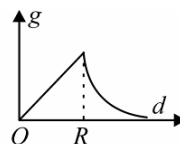
86. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius) :



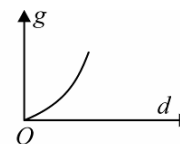
(1)



(2)



(3)

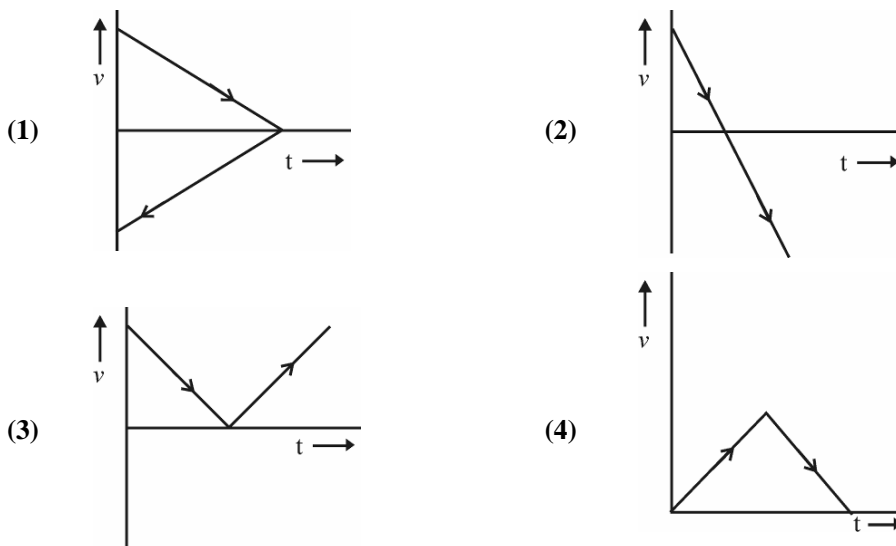


(4)

86.(3) $g = \frac{GM}{R^3} r, 0 \leq r \leq R$

$$g = \frac{GM}{r^2}, \quad r \geq R$$

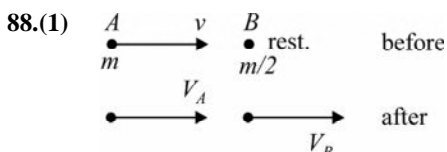
87. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time?



- 87.(2) During the whole journey acceleration remains constant ($a = -g$) $\Rightarrow V = V_0 - gt$

88. A particle A of mass m and initial velocity v collides with a particle B of mass $\frac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is:

(1) $\frac{\lambda_A}{\lambda_B} = 2$ (2) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$ (3) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$ (4) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$



$$mv = mV_A + \frac{m}{2}V_B \quad (\text{Conservation of momentum})$$

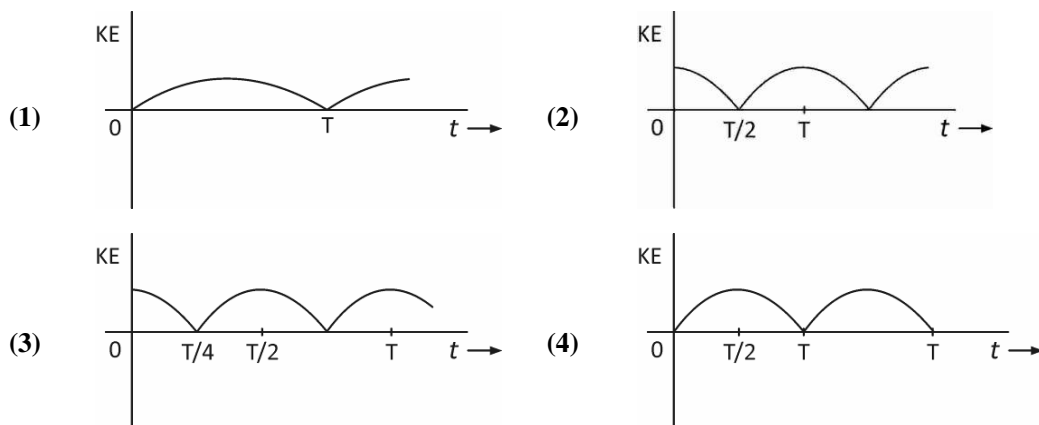
Also $V_B - V_A = v \quad (e = 1)$

$$\frac{\lambda_A}{\lambda_B} = \frac{h/p_A}{h/p_B} = \frac{p_B}{p_A} = \frac{\frac{m}{2}V_B}{mV_A} = \frac{1}{2} \frac{V_B}{V_A}$$

Solving (i) and (ii) $V_A = \frac{v}{3}, V_B = \frac{4v}{3}$

$$\Rightarrow \frac{\lambda_A}{\lambda_B} = \frac{1}{2} \times \frac{4v/3}{v/3} = 2$$

89. A particle is executing simple harmonic motion with a time period T . At time $t = 0$, it is at its position of equilibrium. The kinetic energy – time graph of the particle will look like:



89.(3) K is maximum at mean position and minimum at extreme position and extreme position is reached at $T/4$.

90. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of :

- (1) $\frac{1}{9}$ (2) 81 (3) $\frac{1}{81}$ (4) 9

90.(4)
$$\text{Stress} = \frac{\text{weight}}{\text{area}} = \frac{(9)^3 \times W_0}{(9)^2 \times A_0} = 9 \left(\frac{W_0}{A_0} \right)$$

As volume increases by $(9)^3$ times and area increases by $(9)^2$ times.