

# Solutions of JEE(MAIN)-2013

## XYZ

PAPER – 1 : CHEMISTRY, MATHEMATICS & PHYSICS

Test Booklet Code

Q

Read carefully the Instructions on the Back Cover of this Test Booklet.

### ***Important Instructions:***

1. Immediately fill in the particulars on this page of the Test Booklet with *Blue/Black Ball Point Pen*. Use of pencil is strictly prohibited.
2. The test is of **3 hours duration**.
3. The Test Booklet consists of **90** questions. The maximum marks are **360**.
4. There are **three** parts in the question paper A, B, C consisting of **Chemistry, Mathematics** and **Physics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for correct response.
5. *Candidates will be awarded marks as stated above in instruction No. 4 for correct response of each question. (1/4) (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
6. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 5 above.

## PART A –CHEMISTRY

1. An unknown alcohol is treated with the "Lucas reagent" to determine whether the alcohol is primary, secondary or tertiary. Which alcohol reacts fastest and by what mechanism:
- (1) tertiary alcohol by  $S_N1$  (2) secondary alcohol by  $S_N2$   
 (3) tertiary alcohol by  $S_N2$  (4) secondary alcohol by  $S_N1$

**Sol. (1)**

Reaction proceeds through carbocation formation as  $3^\circ$  carbocation is highly stable, hence reaction proceeds through  $S_N1$  with  $3^\circ$  alcohol.

2. The first ionization potential of Na is 5.1 eV. The value of electron gain enthalpy of  $Na^+$  will be:

- (1) -5.1 eV (3) -10.2 eV  
 (2) +2.55 eV (4) -2.55 eV

**Sol. (1)**

$Na \xrightleftharpoons[\Delta H = -5.1 \text{ eV}]{\Delta H = +5.1 \text{ eV}} Na^+ + e^-$ , here the backward reaction releases same amount of energy and known as Electron gain enthalpy.

3. Stability of the species  $Li_2$ ,  $Li_2^-$  and  $Li_2^+$  increases in the order of:

- (1)  $Li_2^- < Li_2^+ < Li_2$  (3)  $Li_2 < Li_2^- < Li_2^+$   
 (2)  $Li_2^- < Li_2 < Li_2^+$  (4)  $Li_2 < Li_2^+ < Li_2^-$

**Sol. (1)**

$$Li_2(6) = \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2$$

$$B.O. = \frac{4-2}{2} = 1$$

$$Li_2^+(5) = \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^1$$

$$B.O. = \frac{3-2}{2} = 0.5$$

$$Li_2^-(7) = \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^1$$

$$B.O. = \frac{4-3}{2} = 0.5$$

$Li_2^+$  is more stable than  $Li_2^-$  because  $Li_2^-$  has more numbers of antibonding electrons.

4. The molarity of a solution obtained by mixing 750 mL of 0.5 (M) HCl with 250 mL of 2(M)HCl will be:

- (1) 1.00 M (2) 1.75 M  
 (3) 0.975 M (4) 0.875 M

**Sol. (4)**

$$M_1V_1 + M_2V_2 = MV$$

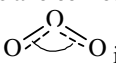
$$M = \frac{M_1V_1 + M_2V_2}{V} = \frac{0.5 \times 750 + 2 \times 250}{1000}$$

$$M = 0.875$$

5. Which of the following is the wrong statement?

- (1)  $O_3$  molecule is bent (2) Ozone is violet-black in solid state  
 (3) Ozone is diamagnetic gas (4)  $ONCl$  and  $ONO^-$  are not isoelectronic

**Sol. (All the options are correct statements)**

(1) Correct, as  is bent.

- (2) Correct, as ozone is violet-black solid.  
 (3) Correct, as ozone is diamagnetic.  
 (4) Correct, as  $\text{ONCl} = 32$  electrons and  $\text{ONO}^- = 24$  electron hence are not isoelectronic.  
 All options are correct statements.

6. Four successive members of the first row transition elements are listed below with atomic numbers. Which one of them is expected to have the highest  $E_{M^{3+}/M^{2+}}^0$  value?

- (1) Mn(Z = 25) (2) Fe(Z = 26)  
 (3) Co(Z = 27) (4) Cr(Z = 24)

**Sol. (3)**

$$E_{\text{Mn}^{3+}/\text{Mn}^{2+}}^0 = 1.57 \text{ V}$$

$$E_{\text{Fe}^{3+}/\text{Fe}^{2+}}^0 = 0.77 \text{ V}$$

$$E_{\text{Co}^{3+}/\text{Co}^{2+}}^0 = 1.97 \text{ V}$$

$$E_{\text{Cr}^{3+}/\text{Cr}^{2+}}^0 = -0.41 \text{ V}$$

7. A solution of (–) –1 – chloro –1 – phenylethane is toluene racemises slowly in the presence of a small amount of  $\text{SbCl}_5$ , due to the formation of :

- (1) carbene (2) carbocation  
 (3) free radical (4) carbanion

**Sol. (2)**



8. The coagulating power of electrolytes having ions  $\text{Na}^+$ ,  $\text{Al}^{3+}$  and  $\text{Ba}^{2+}$  for arsenic sulphide sol increases in the order:

- (1)  $\text{Na}^+ < \text{Ba}^{2+} < \text{Al}^{3+}$  (2)  $\text{Ba}^{2+} < \text{Na}^+ < \text{Al}^{3+}$   
 (3)  $\text{Al}^{3+} < \text{Na}^+ < \text{Ba}^{2+}$  (4)  $\text{Al}^{3+} < \text{Ba}^{2+} < \text{Na}^+$

**Sol. (1)**

$\text{As}_2\text{S}_3$  is an anionic sol (negative sol) hence coagulation will depend upon coagulating power of cation, which is directly proportional to the valency of cation (Hardy-Schulze rule).

9. How many litres of water must be added to 1 litre of an aqueous solution of  $\text{HCl}$  with a pH of 1 to create an aqueous solution with pH of 2?

- (1) 0.9 L (2) 2.0 L  
 (3) 9.0 L (4) 0.1 L

**Sol. (3)**

Initial pH = 1, i.e.  $[\text{H}^+] = 0.1$  mole/litre

New pH = 2, i.e.  $[\text{H}^+] = 0.01$  mole/litre

In case of dilution:  $M_1V_1 = M_2V_2$

$$0.1 \times 1 = 0.01 \times V_2$$

$$V_2 = 10 \text{ litre.}$$

Volume of water added = 9 litre.

10. Which one of the following molecules is expected to exhibit diamagnetic behaviour?

- (1)  $\text{N}_2$  (2)  $\text{O}_2$   
 (3)  $\text{S}_2$  (4)  $\text{C}_2$

**Sol. (1) & (4) both are correct answers.**

$\text{N}_2 \rightarrow$  Diamagnetic

$\text{O}_2 \rightarrow$  Paramagnetic

$\text{S}_2 \rightarrow$  Paramagnetic

$C_2 \rightarrow$  Diamagnetic

11. Which of the following arrangements does **not** represent the correct order of the property stated against it ?
- (1)  $Ni^{2+} < Co^{2+} < Fe^{2+} < Mn^{2+}$  : ionic size
  - (2)  $Co^{3+} < Fe^{3+} < Cr^{3+} < Sc^{3+}$  : stability in aqueous solution
  - (3)  $Sc < Ti < Cr < Mn$  : number of oxidation states
  - (4)  $V^{2+} < Cr^{2+} < Mn^{2+} < Fe^{2+}$  : paramagnetic behaviour

**Sol. (2) & (4) both are correct answers)**

The exothermic hydration enthalpies of the given trivalent cations are:

$Sc^{+3} = 3960$  kJ/mole

$Fe^{+3} = 4429$  kJ/mole

$Co^{+3} = 4653$  kJ/mole

$Cr^{+3} = 4563$  kJ/mole

Hence  $Sc^{+3}$  is least hydrated; so least stable (not most stable)

$Fe^{+2}$  contains 4 unpaired electrons where as  $Mn^{+2}$  contains 5 unpaired electrons hence (4) is incorrect.

12. Experimentally it was found that a metal oxide has formula  $M_{0.98}O$ . Metal M, is present as  $M^{2+}$  and  $M^{3+}$  in its oxide. Fraction of the metal which exists as  $M^{3+}$  would be:
- (1) 4.08%
  - (2) 6.05%
  - (3) 5.08%
  - (4) 7.01%

**Sol. (1)**

Metal oxide =  $M_{0.98}O$

If 'x' ions of M are in +3 state, then

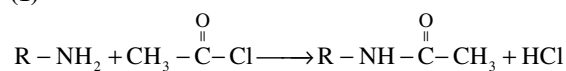
$3x + (0.98 - x) \times 2 = 2$

$x = 0.04$

So the percentage of metal in +3 state would be  $\frac{0.04}{0.98} \times 100 = 4.08\%$

13. A compound with molecular mass 180 is acylated with  $CH_3COCl$  to get a compound with molecular mass 390. The number of amino groups present per molecule of the former compound is:
- (1) 5
  - (2) 4
  - (3) 6
  - (4) 2

**Sol. (1)**



Each  $CH_3-\overset{\overset{O}{\parallel}}{C}$  addition increases the molecular wt. by 42.

Total increase in m.wt. =  $390 - 180 = 210$

Then number of  $NH_2$  groups =  $\frac{210}{42} = 5$

14. Given

$$E_{Cr^{3+}/Cr}^0 = -0.74 \text{ V}; E_{MnO_4^-/Mn^{2+}}^0 = 1.51 \text{ V}$$

$$E_{Cr_2O_7^{2-}/Cr^{3+}}^0 = 1.33 \text{ V}; E_{Cl^-/Cl_2}^0 = 1.36 \text{ V}$$

Based on the data given above, strongest oxidising agent will be:

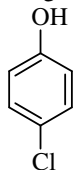
- (1)  $Cr^{3+}$
- (2)  $Mn^{2+}$
- (3)  $MnO_4^-$
- (4)  $Cl^-$

**Sol. (3)**

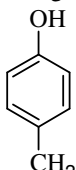
As per data mentioned

$MnO_4^-$  is strongest oxidising agent as it has maximum SRP value.

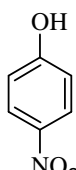
15. Arrange the following compounds in order of decreasing acidity:



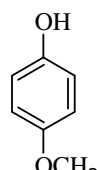
(I)



(II)



(III)



(IV)

(1)  $I > II > III > IV$

(3)  $IV > III > I > II$

(2)  $III > I > II > IV$

(4)  $II > IV > I > III$

**Sol.** (2)

Correct order of acidic strength is  $III > I > II > IV$

16. The rate of a reaction doubles when its temperature changes from 300K to 310K. Activation energy of such a reaction will be:

( $R = 8.314 \text{ JK}^{-1} \text{ mol}^{-1}$  and  $\log 2 = 0.301$ )

(1)  $48.6 \text{ kJ mol}^{-1}$

(3)  $60.5 \text{ kJ mol}^{-1}$

(2)  $58.5 \text{ kJ mol}^{-1}$

(4)  $53.6 \text{ kJ mol}^{-1}$

**Sol.** (4)

As per Arrhenius equation:

$$\ln \frac{k_2}{k_1} = -\frac{E_a}{R} \left( \frac{1}{T_2} - \frac{1}{T_1} \right)$$

$$2.303 \log 2 = -\frac{E_a}{8.314} \left( \frac{1}{310} - \frac{1}{300} \right)$$

$$\Rightarrow E_a = 53.6 \text{ kJ/mole}$$

17. Synthesis of each molecule of glucose in photosynthesis involves:

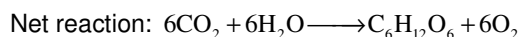
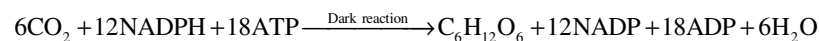
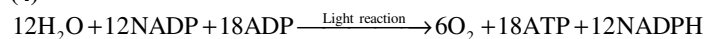
(1) 10 molecules of ATP

(2) 8 molecules of ATP

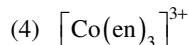
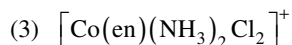
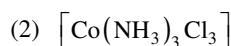
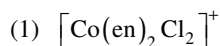
(3) 6 molecules of ATP

(4) 18 molecules of ATP

**Sol.** (4)

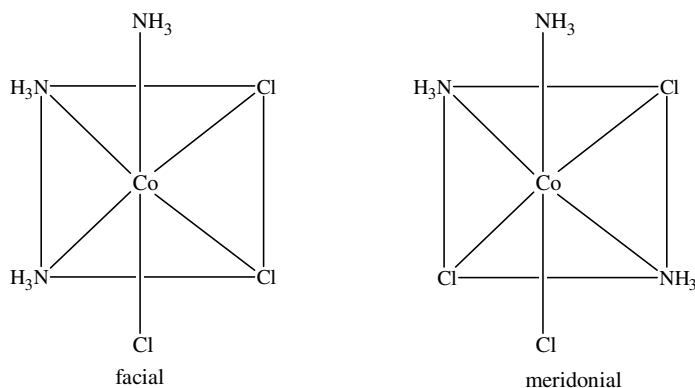


18. Which of the following complex species is not expected to exhibit optical isomerism?



**Sol.** (2)

$[\text{Co}(\text{NH}_3)_3\text{Cl}_3]$  exists in two forms (facial and meridional)



Both of these forms are achiral. Hence,  $[\text{Co}(\text{NH}_3)_3\text{Cl}_3]$  does not show optical isomerism.

19. A piston filled with 0.04 mol of an ideal gas expands reversibly from 50.0 mL to 375 mL at a constant temperature of  $37.0^\circ\text{C}$ . As it does so, it absorbs 208 J of heat. The values of  $q$  and  $w$  for the process will be: ( $R = 8.314 \text{ J/mol K}$ ) ( $\ln 7.5 = 2.01$ )
- (1)  $q = -208 \text{ J}$ ,  $w = -208 \text{ J}$  (2)  $q = -208 \text{ J}$ ,  $w = +208 \text{ J}$   
 (3)  $q = +208 \text{ J}$ ,  $w = +208 \text{ J}$  (4)  $q = +208 \text{ J}$ ,  $w = -208 \text{ J}$

**Sol.** (4)

Process is isothermal reversible expansion, hence  $\Delta U = 0$ .

$\therefore q = -W$

As  $q = +208 \text{ J}$

Hence  $W = -208 \text{ J}$

20. A gaseous hydrocarbon gives upon combustion 0.72 g of water and 3.08 g of  $\text{CO}_2$ . The empirical formula of the hydrocarbon is:
- (1)  $\text{C}_3\text{H}_4$  (2)  $\text{C}_6\text{H}_5$   
 (3)  $\text{C}_7\text{H}_8$  (4)  $\text{C}_2\text{H}_4$

**Sol.** (3)

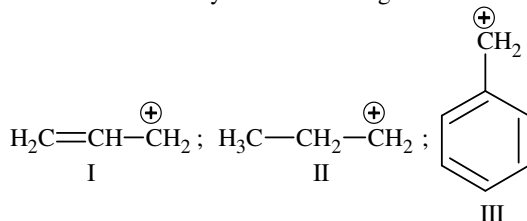
$$\text{C}_x\text{H}_y + \frac{x}{2}\text{O}_2 \rightarrow x\text{CO}_2 + \frac{y}{2}\text{H}_2\text{O}$$

Weight (g)	3.08 g	0.72 g
moles	0.07	0.04

$$\frac{x}{y/2} = \frac{0.07}{0.04}$$

$$\therefore \frac{x}{y} = \frac{7}{8}$$

21. The order of stability of the following carbocations:



is:

- (1)  $\text{II} > \text{III} > \text{I}$  (2)  $\text{I} > \text{II} > \text{III}$   
 (3)  $\text{III} > \text{I} > \text{II}$  (4)  $\text{III} > \text{II} > \text{I}$

**Sol.** (3)

Order of stability is  $\text{III} > \text{I} > \text{II}$ .

(Stability  $\propto$  extent of delocalization)

22. Which of the following represents the correct order of increasing first ionization enthalpy for Ca, Ba, S, Se and Ar?

- (1)  $S < Se < Ca < Ba < Ar$  (2)  $Ba < Ca < Se < S < Ar$   
 (3)  $Ca < Ba < S < Se < Ar$  (4)  $Ca < S < Ba < Se < Ar$

**Sol.** (2)

Increasing order of first ionization enthalpy is  
 $Ba < Ca < Se < S < Ar$

23. For gaseous state, if most probable speed is denoted by  $C^*$ , average speed by  $\bar{C}$  and mean square speed by  $C$ , then for a large number of molecules the ratios of these speeds are:

- (1)  $C^* : \bar{C} : C = 1.128 : 1.225 : 1$  (2)  $C^* : \bar{C} : C = 1 : 1.128 : 1.225$   
 (3)  $C^* : \bar{C} : C = 1 : 1.125 : 1.128$  (4)  $C^* : \bar{C} : C = 1.225 : 1.128 : 1$

**Sol.** (2)

$$C^* = \sqrt{\frac{2RT}{M}}, \quad \bar{C} = \sqrt{\frac{8RT}{\pi M}}, \quad C = \sqrt{\frac{3RT}{M}}$$

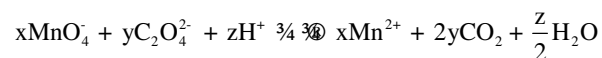
24. The gas leaked from a storage tank of the Union Carbide plant in Bhopal gas tragedy was:

- (1) Methylamine (2) Ammonia  
 (3) Phosgene (4) Methylisocyanate

**Sol.** (4)

It was methyl isocyanate ( $CH_3NCO$ )

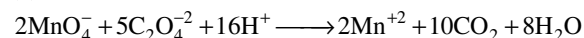
25. Consider the following reaction:



The values of x, y and z in the reaction are, respectively:

- (1) 2, 5 and 8 (2) 2, 5 and 16  
 (3) 5, 2 and 8 (4) 5, 2 and 16

**Sol.** (2)



$$x = 2, y = 5, z = 16$$

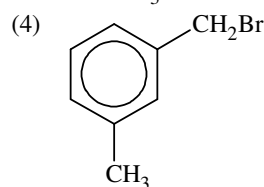
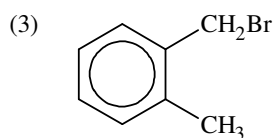
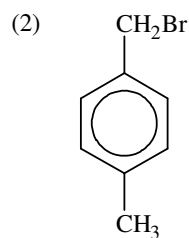
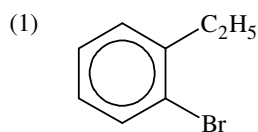
26. Which of the following exists as covalent crystals in the solid state?

- (1) Silicon (2) Sulphur  
 (3) Phosphorous (4) Iodine

**Sol.** (1)

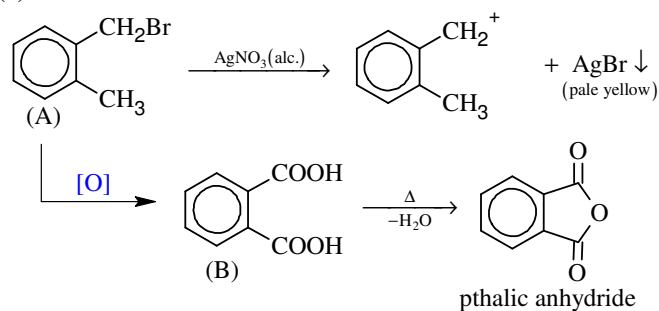
Silicon (Si) – covalent solid  
 Sulphur ( $S_8$ ) – molecular solid  
 Phosphorous ( $P_4$ ) – Molecular solid  
 Iodine ( $I_2$ ) – Molecular solid

27. Compound (A),  $C_8H_9Br$ , gives a white precipitate when warmed with alcoholic  $AgNO_3$ . Oxidation of (A) gives a acid (B),  $C_8H_6O_4$ . (B) easily forms anhydride on heating. Identify the compound (A).



**Sol.**

(3)



28. Energy of an electron is given by  $E = -2.178 \times 10^{-18} \text{ J} \times \frac{Z^2}{n^2}$ . Wavelength of light required to excite an electron in an hydrogen atom from level  $n = 1$  to  $n = 2$  will be ( $h = 6.62 \times 10^{-34} \text{ Js}$  and  $c = 3.0 \times 10^8 \text{ ms}^{-1}$ )
- (1)  $2.816 \times 10^{-7} \text{ m}$  (2)  $6.500 \times 10^{-7} \text{ m}$   
 (3)  $8.500 \times 10^{-7} \text{ m}$  (4)  $1.214 \times 10^{-7} \text{ m}$

**Sol.**

(4)

$$E = \frac{hc}{\lambda} = 2.178 \times 10^{-18} \times Z^2 \left[ \frac{1}{1^2} - \frac{1}{2^2} \right]$$

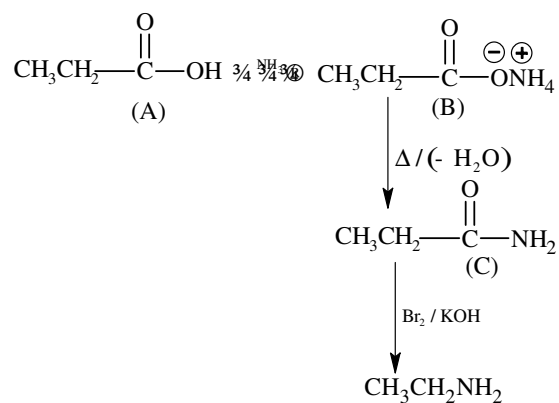
$$\Rightarrow \lambda = 1.214 \times 10^{-7} \text{ m}$$

29. An organic compound A upon reacting with  $\text{NH}_3$  gives B. On heating B gives C. C in presence of KOH reacts with  $\text{Br}_2$  to give  $\text{CH}_3\text{CH}_2\text{NH}_2$ . A is
- (1)  $\text{CH}_3\text{CH}_2\text{CH}_2\text{COOH}$  (2)   
 (3)  $\text{CH}_3\text{CH}_2\text{COOH}$  (4)  $\text{CH}_3\text{COOH}$

**Sol.**

(3)





30. In which of the following pairs of molecules/ions, both the species are not likely to exist?

- |                                       |   |
|---------------------------------------|---|
| (1) $\text{H}_2$ , $\text{He}_2^{2-}$ | (2) $\text{H}_2^{2+}$ , $\text{He}_2$   |
| (3) $\text{H}_2$ , $\text{He}_2^{2+}$ | (4) $\text{H}_2^+$ , $\text{He}_2^{2-}$ |

**Sol.** (2)

Bond order of  $\text{H}_2^{2+}$  and  $\text{He}_2$  is zero, thus their existence is not possible.

## PART B – MATHEMATICS

31. The circle passing through  $(1, -2)$  and touching the axis of  $x$  at  $(3, 0)$  also passes through the point  
 (1)  $(2, -5)$  (2)  $(5, -2)$   
 (3)  $(-2, 5)$  (4)  $(-5, 2)$

**Sol.**

(2)

$$(x - 3)^2 + y^2 + \lambda y = 0$$

The circle passes through  $(1, -2)$ 

$$\Rightarrow 4 + 4 - 2\lambda = 0 \Rightarrow \lambda = 4$$

$$(x - 3)^2 + y^2 + 4y = 0 \Rightarrow \text{Clearly } (5, -2) \text{ satisfies.}$$

32. ABCD is a trapezium such that AB and CD are parallel and  $BC \perp CD$ . If  $\angle ADB = \theta$ ,  $BC = p$  and  $CD = q$ , then AB is equal to

$$(1) \frac{p^2 + q^2 \cos \theta}{p \cos \theta + q \sin \theta}$$

$$(2) \frac{p^2 + q^2}{p^2 \cos \theta + q^2 \sin \theta}$$

$$(3) \frac{(p^2 + q^2) \sin \theta}{(p \cos \theta + q \sin \theta)^2}$$

$$(4) \frac{(p^2 + q^2) \sin \theta}{p \cos \theta + q \sin \theta}$$

**Sol.**

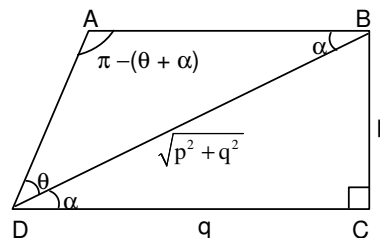
(4)

Using sine rule in triangle ABD

$$\frac{AB}{\sin \theta} = \frac{BD}{\sin(\theta + \alpha)}$$

$$\Rightarrow AB = \frac{\sqrt{p^2 + q^2} \sin \theta}{\sin \theta \cos \alpha + \cos \theta \sin \alpha} = \frac{\sqrt{p^2 + q^2} \sin \theta}{\frac{\sin \theta \cdot q}{\sqrt{p^2 + q^2}} + \frac{\cos \theta \cdot p}{\sqrt{p^2 + q^2}}}$$

$$\Rightarrow AB = \frac{(p^2 + q^2) \sin \theta}{(p \cos \theta + q \sin \theta)}.$$



33. Given : A circle,  $2x^2 + 2y^2 = 5$  and a parabola,  $y^2 = 4\sqrt{5}x$ .

**Statement – I :** An equation of a common tangent to these curves is  $y = x + \sqrt{5}$ .**Statement – II :** If the line,  $y = mx + \frac{\sqrt{5}}{m}$  ( $m \neq 0$ ) is their common tangent, then  $m$  satisfies  $m^4 - 3m^2 + 2 = 0$ .

- (1) Statement - I is True; Statement -II is true; Statement-II is **not** a correct explanation for Statement-I  
 (2) Statement -I is True; Statement -II is False.  
 (3) Statement -I is False; Statement -II is True  
 (4) Statement -I is True; Statement -II is True; Statement-II is a **correct** explanation for Statement-I

**Sol.**

(1)

Let the tangent to the parabola be  $y = mx + \frac{\sqrt{5}}{m}$  ( $m \neq 0$ ).

Now, its distance from the centre of the circle must be equal to the radius of the circle.

$$\text{So, } \left| \frac{\sqrt{5}}{m} \right| = \frac{\sqrt{5}}{\sqrt{2}} \sqrt{1 + m^2} \Rightarrow (1 + m^2) m^2 = 2 \Rightarrow m^4 + m^2 - 2 = 0.$$

$$\Rightarrow (m^2 - 1)(m^2 + 2) = 0 \Rightarrow m = \pm 1$$

So, the common tangents are  $y = x + \sqrt{5}$  and  $y = -x - \sqrt{5}$ .

34. A ray of light along  $x + \sqrt{3}y = \sqrt{3}$  gets reflected upon reaching  $x$ -axis, the equation of the reflected rays is

(1)  $\sqrt{3}y = x - \sqrt{3}$

(2)  $y = \sqrt{3}x - \sqrt{3}$

(3)  $\sqrt{3}y = x - 1$

(4)  $y = x + \sqrt{3}$

**Sol. (1)**Slope of the incident ray is  $-\frac{1}{\sqrt{3}}$ .So, the slope of the reflected ray must be  $\frac{1}{\sqrt{3}}$ .The point of incidence is  $(\sqrt{3}, 0)$ . So, the equation of reflected ray is  $y = \frac{1}{\sqrt{3}}(x - \sqrt{3})$ .

35. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given ?

(1) median

(2) mode

(3) variance

(4) mean

**Sol. (3)**

Variance is not changed by the change of origin.

**Alternate Solution:**

$$\sigma = \sqrt{\frac{\sum |x - \bar{x}|^2}{n}} \text{ for } y = x + 10 \Rightarrow \bar{y} = \bar{x} + 10$$

$$\sigma_1 = \sqrt{\frac{\sum |y + 10 - \bar{y} - 10|^2}{n}} = \sqrt{\frac{\sum |y - \bar{y}|^2}{n}} = \sigma.$$

36. If  $x, y, z$  are in A.P. and  $\tan^{-1}x, \tan^{-1}y$  and  $\tan^{-1}z$  are also in A.P., then

(1)  $2x = 3y = 6z$

(2)  $6x = 3y = 2z$

(3)  $6x = 4y = 3z$

(4)  $x = y = z$

**Sol. (4)**If  $x, y, z$  are in A.P.

$2y = x + z$

and  $\tan^{-1}x, \tan^{-1}y, \tan^{-1}z$  are in A.P.

$2 \tan^{-1}y = \tan^{-1}x + \tan^{-1}z \Rightarrow x = y = z.$

**Note: If  $y = 0$ , then none of the options is appropriate.**

37. If  $\int f(x) dx = \Psi(x)$ , then  $\int x^5 f(x^3) dx$  is equal to

(1)  $\frac{1}{3} x^3 \Psi(x^3) - 3 \int x^3 \Psi(x^3) dx + C$

(2)  $\frac{1}{3} x^3 \Psi(x^3) - \int x^2 \Psi(x^3) dx + C$

(3)  $\frac{1}{3} \left[ x^3 \Psi(x^3) - \int x^3 \Psi(x^3) dx \right] + C$

(4)  $\frac{1}{3} \left[ x^3 \Psi(x^3) - \int x^2 \Psi(x^3) dx \right] + C$

**Sol. (2)**

$\int f(x) dx = \Psi(x)$

Let  $x^3 = t$

$3x^2 dx = dt$

then  $\int x^5 f(x^3) dx = \frac{1}{3} \int t f(t) dt$

$$= \frac{1}{3} \left[ t \int f(t) dt - \int \{1 \cdot \int f(t) dt\} dt \right] = \frac{1}{3} x^3 \Psi(x^3) - \int x^2 \Psi(x^3) dx + C.$$

38. The equation of the circle passing through the foci of the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$ , and having centre at (0, 3) is

- (1)  $x^2 + y^2 - 6y + 7 = 0$  (2)  $x^2 + y^2 - 6y - 5 = 0$   
 (3)  $x^2 + y^2 - 6y + 5 = 0$  (4)  $x^2 + y^2 - 6y - 7 = 0$

**Sol.** (4)

foci  $\equiv (\pm ae, 0)$

We have  $a^2e^2 = a^2 - b^2 = 7$

Equation of circle  $(x - 0)^2 + (y - 3)^2 = (\sqrt{7} - 0)^2 + (0 - 3)^2$

$\Rightarrow x^2 + y^2 - 6y - 7 = 0$ .

39. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1) (1, 1) and (1, 0) is

- (1)  $2 - \sqrt{2}$  (2)  $1 + \sqrt{2}$   
 (3)  $1 - \sqrt{2}$  (4)  $2 + \sqrt{2}$

**Sol.** (1)

$$\text{x-coordinate} = \frac{ax_1 + bx_2 + cx_3}{a + b + c}$$

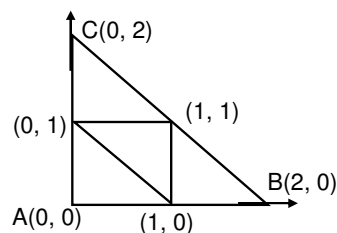
$$= \frac{2 \times 2 + 2\sqrt{2} \times 0 + 2 \times 0}{2 + 2 + 2\sqrt{2}}$$

$$= \frac{4}{4 + 2\sqrt{2}} = \frac{2}{2 + \sqrt{2}} = 2 - \sqrt{2}.$$

**Alternate Solution:**

x-coordinate  $= r = (s - a) \tan A/2$

$$= \left( \frac{4 + 2\sqrt{2}}{2} - 2\sqrt{2} \right) \tan \frac{\pi}{4} = 2 - \sqrt{2}.$$



40. The intercepts on x-axis made by tangents to the curve,  $y = \int_0^x |t| dt$ ,  $x \in \mathbb{R}$ , which are parallel to the line

$y = 2x$ , are equal to

- (1)  $\pm 2$  (2)  $\pm 3$   
 (3)  $\pm 4$  (4)  $\pm 1$

**Sol.** (4)

$$\frac{dy}{dx} = |x| = 2 \Rightarrow x = \pm 2 \Rightarrow y = \int_0^2 |t| dt = 2 \text{ for } x = 2$$

$$\text{and } y = \int_0^{-2} |t| dt = -2 \text{ for } x = -2$$

$\therefore$  tangents are  $y - 2 = 2(x - 2) \Rightarrow y = 2x - 2$

and  $y + 2 = 2(x + 2) \Rightarrow y = 2x + 2$

Putting  $y = 0$ , we get  $x = 1$  and  $-1$ .

41. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ..., is

- (1)  $\frac{7}{9}(99 - 10^{-20})$  (2)  $\frac{7}{81}(179 + 10^{-20})$   
 (3)  $\frac{7}{9}(99 + 10^{-20})$  (4)  $\frac{7}{81}(179 - 10^{-20})$

**Sol.** (2)

$t_r = 0.777 \dots r \text{ times}$

$$= 7(10^{-1} + 10^{-2} + 10^{-3} + \dots + 10^{-r})$$

$$= \frac{7}{9}(1 - 10^{-r})$$

$$S_{20} = \sum_{r=1}^{20} t_r = \frac{7}{9} \left( 20 - \sum_{r=1}^{20} 10^{-r} \right) = \frac{7}{9} \left( 20 - \frac{1}{9}(1 - 10^{-20}) \right) = \frac{7}{81}(179 + 10^{-20})$$

42. Consider :

**Statement – I :**  $(p \wedge \sim q) \wedge (\sim p \wedge q)$  is a fallacy.

**Statement – II :**  $(p \rightarrow q) \Leftrightarrow (\sim q \rightarrow \sim p)$  is a tautology.

- (1) Statement - I is True; Statement -II is true; Statement-II is **not** a correct explanation for Statement-I  
 (2) Statement -I is True; Statement -II is False.  
 (3) Statement -I is False; Statement -II is True  
 (4) Statement -I is True; Statement -II is True; Statement-II is a **correct** explanation for Statement-I

**Sol.**

(1)

S1:

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \wedge q$	$(p \wedge \sim q) \wedge (\sim p \wedge q)$
T	T	F	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	F	F

Fallacy

S2:

p	q	$\sim p$	$\sim q$	$p \Rightarrow q$	$\sim q \Rightarrow \sim p$	$(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Tautology

S2 is not an explanation of S1

43. The area (in square units) bounded by the curves  $y = \sqrt{x}$ ,  $2y - x + 3 = 0$ , x-axis, and lying in the first quadrant is

- (1) 36  
 (2) 18  
 (3)  $\frac{27}{4}$   
 (4) 9

**Sol.**

(4)

$$2\sqrt{x} = x - 3$$

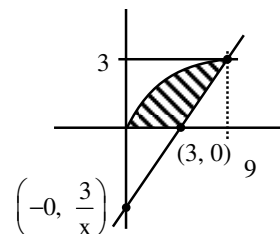
$$4x = x^2 - 6x + 9$$

$$x^2 - 10x + 9$$

$$x = 9, x = 1$$

$$\int_0^3 (2y + 3) - y^2 dy$$

$$\left[ y^2 + 3y - \frac{y^3}{3} \right]_0^3 = 9 + 9 - 9 = 9$$



44. The expression  $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$  can be written as

- (1)  $\sec A \operatorname{cosec} A + 1$   
 (2)  $\tan A + \cot A$   
 (3)  $\sec A + \operatorname{cosec} A$   
 (4)  $\sin A \cos A + 1$

**Sol.**

(1)

$$\frac{1}{\cot A (1 - \cot A)} - \frac{\cot^2 A}{(1 - \cot A)} = \frac{1 - \cot^3 A}{\cot A (1 - \cot A)} = \frac{\cos^2 A + \cot A}{\cot A} = 1 + \sec A \operatorname{cosec} A$$

45. The real number  $k$  for which the equation,  $2x^3 + 3x + k = 0$  has two distinct real roots in  $[0, 1]$
- (1) lies between 2 and 3 (2) lies between  $-1$  and 0  
(3) does not exist (4) lies between 1 and 2

**Sol. (3)**

If  $2x^3 + 3x + k = 0$  has 2 distinct real roots in  $[0, 1]$ , then  $f'(x)$  will change sign  
but  $f'(x) = 6x^2 + 3 > 0$   
So no value of  $k$  exists.

46.  $\lim_{x \rightarrow 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$  is equal to

- (1)  $\frac{1}{2}$  (2) 1  
(3) 2 (4)  $-\frac{1}{4}$

**Sol. (3)**

$$\lim_{x \rightarrow 0} \frac{(1 - \cos 2x)}{x (\tan 4x)} (3 + \cos x)$$

$$\lim_{x \rightarrow 0} 2 \left( \frac{\sin x}{x} \right)^2 \cdot \frac{1}{4} \left( \frac{4x}{\tan 4x} \right) (3 + \cos x) = 2 \times 1 \times \frac{1}{4} \times 1 \times (3 + 1) = 2.$$

47. Let  $T_n$  be the number of all possible triangles formed by joining vertices of an  $n$ -sided regular polygon. If  $T_{n+1} - T_n = 10$ , then the value of  $n$  is
- (1) 5 (2) 10  
(3) 8 (4) 7

**Sol. (1)**

$${}^{n+1}C_3 - {}^nC_3 = 10 \Rightarrow {}^nC_2 = 10 \Rightarrow n = 5.$$

48. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production  $P$  w.r.t. additional number of workers  $x$  is given by  $\frac{dP}{dx} = 100 - 12\sqrt{x}$ . If the firm employs 25 more workers, then the new level of production of items is
- (1) 3000 (2) 3500  
(3) 4500 (4) 2500

**Sol. (2)**

$$\int_{2000}^P dP = \int_0^{25} (100 - 12\sqrt{x}) dx$$

$$(P - 2000) = 25 \times 100 - \frac{12 \times 2}{3} (25)^{3/2}$$

$$P = 3500.$$

49. **Statement – I :** The value of the integral  $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$  is equal to  $\frac{\pi}{6}$ .

$$\text{Statement – II : } \int_a^b f(x) dx = \int_a^b f(a + b - x) dx.$$

- (1) Statement - I is True; Statement -II is true; Statement-II is **not** a correct explanation for Statement-I  
(2) Statement -I is True; Statement -II is False.

- (3) Statement -I is False; Statement -II is True  
 (4) Statement -I is True; Statement -II is True; Statement-II is a **correct** explanation for Statement-I

**Sol.** (3)

$$I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\tan x}}{1 + \sqrt{\tan x}} dx$$

$$2I = \frac{\pi}{6}$$

$$I = \frac{\pi}{12}.$$

50. If  $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$  is the adjoint of a  $3 \times 3$  matrix A and  $|A| = 4$ , then  $\alpha$  is equal to

- (1) 11 (2) 5  
 (3) 0 (4) 4

**Sol.** (1)

$$P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$$

$$|Adj A| = |A|^2$$

$$|Adj A| = 16$$

$$1(12 - 12) - \alpha(4 - 6) + 3(4 - 6) = 16.$$

$$2\alpha - 6 = 16.$$

$$2\alpha = 22.$$

$$\alpha = 11.$$

51. The number of values of k, for which the system of equations

$$(k + 1)x + 8y = 4k$$

$$kx + (k + 3)y = 3k - 1$$

has no solution, is

- (1) 1 (2) 2  
 (3) 3 (4) infinite

**Sol.** (1)

For no solution

$$\frac{k+1}{k} = \frac{8}{k+3} \neq \frac{4k}{3k-1} \quad \dots (1)$$

$$\Rightarrow (k + 1)(k + 3) - 8k = 0$$

$$\text{or } k^2 - 4k + 3 = 0 \Rightarrow k = 1, 3$$

But for  $k = 1$ , equation (1) is not satisfied

Hence  $k = 3$ .

52. If  $y = \sec(\tan^{-1}x)$ , then  $\frac{dy}{dx}$  at  $x = 1$  is equal to

- (1)  $\frac{1}{2}$  (2) 1  
 (3)  $\sqrt{2}$  (4)  $\frac{1}{\sqrt{2}}$





$${}^5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^1 + {}^5C_5 \left(\frac{1}{3}\right)^5$$

$$\frac{5 \times 2}{(3)^5} + \frac{1}{(3)^5} = \frac{11}{3^5}.$$

57. If  $z$  is a complex number of unit modulus and argument  $\theta$ , then  $\arg\left(\frac{1+z}{1+\bar{z}}\right)$  equals

- (1)  $\frac{\pi}{2} - \theta$  (2)  $\theta$   
 (3)  $\pi - \theta$  (4)  $-\theta$

**Sol.** (2)  
 $|z| = 1 \Rightarrow z\bar{z} = 1$   
 $\frac{1+z}{1+\bar{z}} = \frac{1+z}{1+\frac{1}{z}} = z.$

58. If the equations  $x^2 + 2x + 3 = 0$  and  $ax^2 + bx + c = 0$ ,  $a, b, c \in \mathbb{R}$ , have a common root, then  $a : b : c$  is

- (1)  $3 : 2 : 1$  (2)  $1 : 3 : 2$   
 (3)  $3 : 1 : 2$  (4)  $1 : 2 : 3$

**Sol.** (4)  
 For equation  $x^2 + 2x + 3 = 0$   
 both roots are imaginary.  
 Since  $a, b, c \in \mathbb{R}$ .  
 If one root is common then both roots are common  
 Hence,  $\frac{a}{1} = \frac{b}{2} = \frac{c}{3}$   
 $a : b : c = 1 : 2 : 3.$

59. Distance between two parallel planes  $2x + y + 2z = 8$  and  $4x + 2y + 4z + 5 = 0$  is

- (1)  $\frac{5}{2}$  (2)  $\frac{7}{2}$   
 (3)  $\frac{9}{2}$  (4)  $\frac{3}{2}$

**Sol.** (2)  
 $4x + 2y + 4z = 16$   
 $4x + 2y + 4z = -5$   
 $d_{\min} = \frac{21}{\sqrt{36}} = \frac{21}{6} = \frac{7}{2}.$

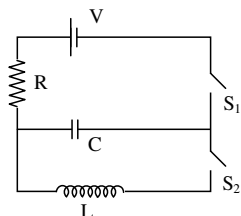
60. The term independent of  $x$  in expansion of  $\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10}$  is

(1) 120 (2) 210  
 (3) 310 (4) 4

**Sol.** (2)  
 $\left(\frac{(x^{1/3} + 1)(x^{2/3} - x^{1/3} + 1)}{x^{2/3} - x^{1/3} + 1} - \frac{1}{\sqrt{x}} \cdot \frac{(\sqrt{x} + 1)(\sqrt{x} - 1)}{(\sqrt{x} - 1)}\right)^{10} = (x^{1/3} - x^{-1/2})^{10}$   
 $T_{r+1} = (-1)^r {}^{10}C_r x^{\frac{20-5r}{6}} \Rightarrow r = 4$   
 ${}^{10}C_4 = 210.$

## PART C – PHYSICS

61. In an LCR circuit as shown below both switches are open initially. Now switch  $S_1$  is closed,  $S_2$  kept open. ( $q$  is charge on the capacitor and  $\tau = RC$  is capacitive time constant). Which of the following statement is correct?



- (1) At  $t = \tau$ ,  $q = CV/2$   
 (2) At  $t = 2\tau$ ,  $q = CV(1 - e^{-2})$   
 (3) At  $t = \frac{\tau}{2}$ ,  $q = CV(1 - e^{-1})$   
 (4) Work done by the battery is half of the energy dissipated in the resistor.

**Sol.****(2)**

Charge on the capacitor at any time 't' is

$$q = CV(1 - e^{-t/\tau})$$

at  $t = 2\tau$ 

$$q = CV(1 - e^{-2})$$

62. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 pico farad in parallel with a load resistance 100 kilo ohm. Find the maximum modulated frequency which could be detected by it.
- (1) 10.62 kHz (2) 5.31 MHz  
 (3) 5.31 kHz (4) 10.62 MHz

**Sol.****(3)**

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2 \times 3.14 \times 100 \times 10^3 \times 250 \times 10^{-12}} = 6.37 \text{ kHz}$$

 $f_c$  = cut off frequencyAs we know that  $f_m \leq f_c$  $\therefore$  (3) is correctNote: The maximum frequency of modulation must be less than  $f_m$ , where

$$f_m = f_c \frac{\sqrt{1-m^2}}{m}$$

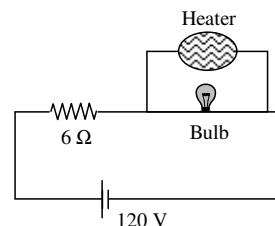
 $m \Rightarrow$  modulation index

63. The supply voltage to a room is 120 V. The resistance of the lead wires is  $6 \Omega$ . A 60 W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240 W heater is switched on in parallel to the bulb?
- (1) 2.9 Volt (2) 13.3 Volt  
 (3) 10.04 Volt (4) zero volt

**Sol.****(3)**

$$\text{Resistance of bulb} = \frac{120 \times 120}{60} = 240 \Omega$$

$$\text{Resistance of Heater} = \frac{120 \times 120}{240} = 60 \Omega$$



Voltage across bulb before heater is switched on,  $V_1 = \frac{120}{246} \times 240$

Voltage across bulb after heater is switched on,  $V_2 = \frac{120}{54} \times 48$

Decrease in the voltage is  $V_1 - V_2 = 10.04$  (approximately)

Note: Here supply voltage is taken as rated voltage.

64. A uniform cylinder of length  $L$  and mass  $M$  having cross-sectional area  $A$  is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density  $\sigma$  at equilibrium position. The extension  $x_0$  of the spring when it is in equilibrium is:

(1)  $\frac{Mg}{k} \left( 1 - \frac{LA\sigma}{M} \right)$  (2)  $\frac{Mg}{k} \left( 1 - \frac{LA\sigma}{2M} \right)$

(3)  $\frac{Mg}{k} \left( 1 + \frac{LA\sigma}{M} \right)$  (4)  $\frac{Mg}{k}$

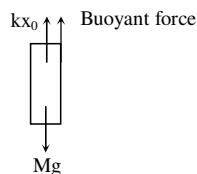
(Here  $k$  is spring constant)

**Sol.** (2)

At equilibrium  $\Sigma F = 0$

$$kx_0 + \left( \frac{AL}{2} \sigma g \right) - Mg = 0$$

$$x_0 = Mg \left[ 1 - \frac{LA\sigma}{2M} \right]$$



65. Two charges, each equal to  $q$ , are kept at  $x = -a$  and  $x = a$  on the  $x$ -axis. A particle of mass  $m$  and charge  $q_0 = \frac{q}{2}$  is placed at the origin. If charge  $q_0$  is given a small displacement ( $y \ll a$ ) along the  $y$ -axis, the net force acting on the particle is proportional to:

(1)  $-y$  (2)  $\frac{1}{y}$

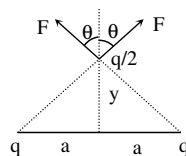
(3)  $-\frac{1}{y}$  (4)  $y$

**Sol.** (4)

$$F_{\text{net}} = 2F \cos \theta$$

$$= 2 \frac{k \cdot q \cdot q / 2}{(\sqrt{a^2 + y^2})^2} \cdot \frac{y}{\sqrt{a^2 + y^2}}$$

$$= \frac{kq^2 y}{a^3} \quad (y \ll a)$$



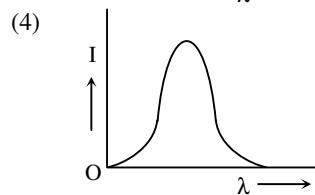
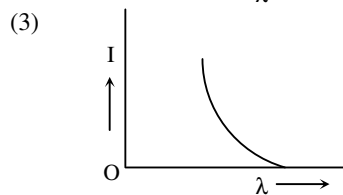
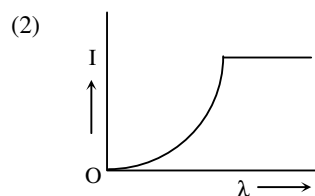
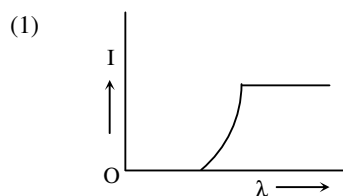
66. A beam of unpolarised light of intensity  $I_0$  is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of  $45^\circ$  relative to that of A. The intensity of the emergent light is:

(1)  $I_0/2$  (2)  $I_0/4$

(3)  $I_0/8$  (4)  $I_0$

**Sol.** (2)

67. The anode voltage of a photocell is kept fixed. The wavelength  $\lambda$  of the light falling on the cathode is gradually changed. The plate current  $I$  of the photocell varies as follows:

**Sol.**

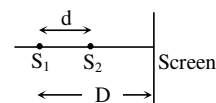
68. (3) Two coherent point sources  $S_1$  and  $S_2$  are separated by a small distance 'd' as shown. The fringes obtained on the screen will be:

(1) straight lines

(2) semi-circles

(3) concentric circles

(4) points

**Sol.**

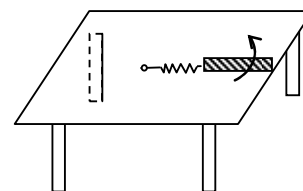
69. (3) A metallic rod of length ' $\ell$ ' is tied to a string of length  $2\ell$  and made to rotate with angular speed  $\omega$  on a horizontal table with one end of the string fixed. If there is a vertical magnetic field 'B' in the region, the e.m.f. induced across the ends of the rod is:

(1)  $\frac{3B\omega\ell^2}{2}$

(2)  $\frac{4B\omega\ell^2}{2}$

(3)  $\frac{5B\omega\ell^2}{2}$

(4)  $\frac{2B\omega\ell^2}{2}$

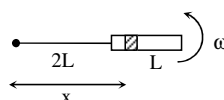
**Sol.**

(3)

$$de = B(\omega x) \cdot dx$$

$$e = B\omega \int_{2L}^{3L} x dx$$

$$= \frac{5B\omega L^2}{2}$$



70.

In a hydrogen like atom electron makes transition from an energy level with quantum number  $n$  to another with quantum number  $(n - 1)$ . If  $n \gg 1$ , the frequency of radiation emitted is proportional to

(1)  $\frac{1}{n^2}$

(2)  $\frac{1}{n^{3/2}}$

(3)  $\frac{1}{n^3}$

(4)  $\frac{1}{n}$

**Sol.**

(3)

$$v \propto \left[ \frac{1}{(n-1)^2} - \frac{1}{n^2} \right]$$

$$\propto \frac{(2n-1)}{n^2(n-1)^2}$$

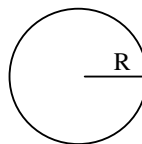
$$\propto \frac{1}{n^3} \quad (\text{since } n \gg 1)$$

71. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is  $T$ , density of liquid is  $\rho$  and  $L$  is its latent heat of vaporization.

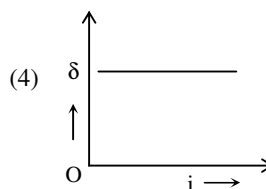
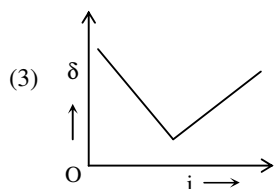
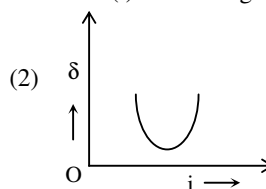
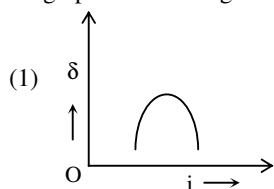
- (1)  $\sqrt{T/\rho L}$  (2)  $T/\rho L$   
(3)  $2T/\rho L$  (4)  $\rho L/T$

**Sol.**

(3)  
 $\rho 4\pi R^2 \Delta R L = T 4\pi [R^2 - (R - \Delta R)^2]$   
 $\rho R^2 \Delta R L = T [R^2 - R^2 + 2R\Delta R - \Delta R^2]$   
 $\rho R^2 \Delta R L = T 2R\Delta R$  ( $\Delta R$  is very small)  
 $R = \frac{2T}{\rho L}$



72. The graph between angle of deviation ( $\delta$ ) and angle of incidence ( $i$ ) for a triangular prism is represented by:



**Sol.** (2)

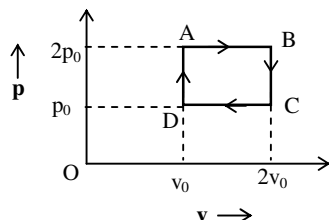
73. Let  $[\epsilon_0]$  denote the dimensional formula of the permittivity of vacuum. If  $M$  = mass,  $L$  = length,  $T$  = time and  $A$  = electric current, then:

- (1)  $[\epsilon_0] = [M^{-1}L^{-3}T^4A^2]$  (2)  $[\epsilon_0] = [M^{-1}L^2T^{-1}A^{-2}]$   
(3)  $[\epsilon_0] = [M^{-1}L^2T^{-1}A]$  (4)  $[\epsilon_0] = [M^{-1}L^{-3}T^2A]$

**Sol.** (1)

73.  $\frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2} = F$   
 $\epsilon_0 = \frac{[A^2T^2]}{[MLT^{-2}L^2]} = [M^{-1}L^{-3}A^2T^4]$

- 74.



The above  $p$ - $v$  diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat extracted from the source in a single cycle is

- (1)  $\left(\frac{13}{2}\right)p_0v_0$  (2)  $\left(\frac{11}{2}\right)p_0v_0$   
(3)  $4p_0v_0$  (4)  $p_0v_0$

**Sol.**

(2)  
Heat is extracted from the source in path DA and AB is

$$\Delta Q = \frac{3}{2}R \left( \frac{P_0 V_0}{R} \right) + \frac{5}{2}R \left( \frac{2P_0 V_0}{R} \right) = \frac{13}{2}P_0 V_0$$

75. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1 %. What is the fundamental frequency of steel if density and elasticity of steel are  $7.7 \times 10^3 \text{ kg/m}^3$  and  $2.2 \times 10^{11} \text{ N/m}^2$  respectively?

- (1) 178.2 Hz (2) 200.5 Hz  
(3) 770 Hz (4) 188.5 Hz

**Sol.** (1)

$$\begin{aligned} \text{Fundamental frequency } f &= \frac{1}{2\ell} \sqrt{\frac{T}{\mu}} \\ &= \frac{1}{2\ell} \sqrt{\frac{T}{A\rho}} \\ &= \frac{1}{2\ell} \sqrt{\frac{\text{stress}}{\rho}} = \frac{1}{2 \times 1.5} \sqrt{\frac{2.2 \times 10^{11} \times 10^{-2}}{7.7 \times 10^3}} \end{aligned}$$

76. This question has statement I and statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement- I: Higher the range, greater is the resistance of ammeter.

Statement- II: To increase the range of ammeter, additional shunt needs to be used across it.

- (1) Statement – I is true, Statement – II is true, Statement – II is not the correct explanation of Statement–I.  
(2) Statement – I is true, statement – II is false.  
(3) Statement – I is false, Statement – II is true  
(4) Statement – I is true, Statement – II is true, Statement – II is the correct explanation of statement- I .

**Sol.** (3)

$$\text{For Ammeter, } S = \frac{I_g G}{I - I_g}$$

So for I to increase, S should decrease, so additional S can be connected across it.

77. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R?

- (1)  $\frac{2GmM}{3R}$  (2)  $\frac{GmM}{2R}$   
(3)  $\frac{GmM}{3R}$  (4)  $\frac{5GmM}{6R}$

**Sol.** (4)

$$\text{T. } E_f = -\frac{GMm}{6R}$$

$$\text{T. } E_i = -\frac{GMm}{R}$$

$$\Delta W = T.E_f - T.E_i = \frac{5GMm}{6R}$$

78. A projectile is given an initial velocity of  $(\hat{i} + 2\hat{j}) \text{ m/s}$ , where  $\hat{i}$  is along the ground and  $\hat{j}$  is along the vertical. If  $g = 10 \text{ m/s}^2$ , the equation of its trajectory is:

- (1)  $y = 2x - 5x^2$  (2)  $4y = 2x - 5x^2$   
(3)  $4y = 2x - 25x^2$  (4)  $y = x - 5x^2$

**Sol.** (1)

$$x = t$$

$$y = 2t - 5t^2$$

Equation of trajectory is  $y = 2x - 5x^2$

79. Two capacitors  $C_1$  and  $C_2$  are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then :

- (1)  $3C_1 = 5C_2$  (2)  $3C_1 + 5C_2 = 0$   
 (3)  $9C_1 = 4C_2$  (4)  $5C_1 = 3C_2$

**Sol.** (1)  
 $120C_1 = 200C_2$   
 $6C_1 = 10C_2$   
 $3C_1 = 5C_2$

80. A hoop of radius  $r$  and mass  $m$  rotating with an angular velocity  $\omega_0$  is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip?

- (1)  $\frac{r\omega_0}{3}$  (2)  $\frac{r\omega_0}{2}$   
 (3)  $r\omega_0$  (4)  $\frac{r\omega_0}{4}$

**Sol.** (2)  
 From conservation of angular momentum about any fix point on the surface

$$mr^2\omega_0 = 2mr^2\omega$$

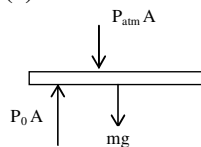
$$\therefore \omega = \frac{\omega_0}{2}$$

$$\therefore V_{CM} = \frac{\omega_0 r}{2}$$

81. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass  $M$ . The piston and cylinder have equal cross sectional area  $A$ . When the piston is in equilibrium, the volume of the gas is  $V_0$  and its pressure is  $P_0$ . The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency:

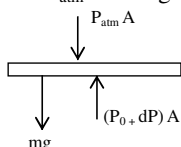
- (1)  $\frac{1}{2\pi} \frac{V_0 M P_0}{A^2 \gamma}$  (2)  $\frac{1}{2\pi} \sqrt{\frac{A^2 \gamma P_0}{M V_0}}$   
 (3)  $\frac{1}{2\pi} \sqrt{\frac{M V_0}{A \gamma P_0}}$  (4)  $\frac{1}{2\pi} \frac{A \gamma P_0}{V_0 M}$

**Sol.** (2)



FBD of piston at equilibrium

$$\Rightarrow P_{\text{atm}} A + mg = P_0 A \quad \dots(1)$$



FBD of piston when piston is pushed down a distance  $x$

$$P_{\text{atm}} + mg - (P_0 + dP) A = m \frac{d^2 x}{dt^2} \quad \dots(2)$$

$$\text{Process is adiabatic} \Rightarrow PV^\gamma = C \Rightarrow -dP = \frac{\gamma P dV}{V}$$

$$\text{Using 1, 2, 3 we get } f = \frac{1}{2\pi} \sqrt{\frac{A^2 \gamma P_0}{MV_0}}$$

82. A charge  $Q$  is uniformly distributed over a long rod AB of length  $L$  as shown in the figure. The electric potential at the point O lying at a distance  $L$  from the end A is:



- (1)  $\frac{3Q}{4\pi\epsilon_0 L}$  (2)  $\frac{Q}{4\pi\epsilon_0 L \ln 2}$   
 (3)  $\frac{Q \ln 2}{4\pi\epsilon_0 L}$  (4)  $\frac{Q}{8\pi\epsilon_0 L}$

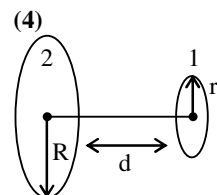
**Sol.**

(3)

$$V = \int_{x=L}^{x=2L} \frac{k}{x} \left( \frac{Q}{L} \right) dx = \frac{Q \ln 2}{4\pi\epsilon_0 L}$$

83. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is
- (1)  $6 \times 10^{-11}$  weber (2)  $3.3 \times 10^{-11}$  weber  
 (3)  $6.6 \times 10^{-9}$  weber (4)  $9.1 \times 10^{-11}$  weber

**Sol.**



Let  $M_{12}$  be the coefficient of mutual induction between loops

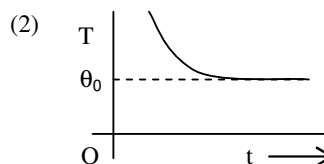
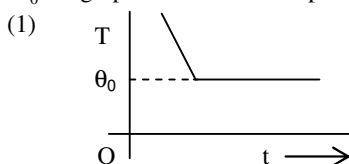
$$\phi_1 = M_{12} i_2$$

$$\Rightarrow \frac{\mu_0 i_2 R^2}{2(d^2 + R^2)^{3/2}} \pi r^2 = M_{12} i_2$$

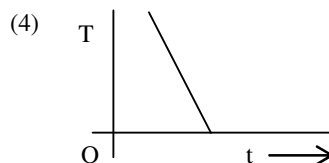
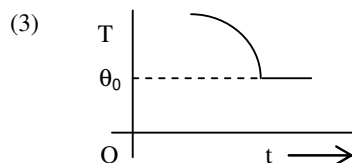
$$\Rightarrow M_{12} = \frac{\mu_0 R^2 \pi r^2}{2(d^2 + R^2)^{3/2}}$$

$$\phi_2 = M_{12} i_1 \Rightarrow \phi_2 = 9.1 \times 10^{-11} \text{ weber}$$

84. If a piece of metal is heated to temperature  $\theta$  and then allowed to cool in a room which is at temperature  $\theta_0$  the graph between the temperature  $T$  of the metal and time  $t$  will be closest to :

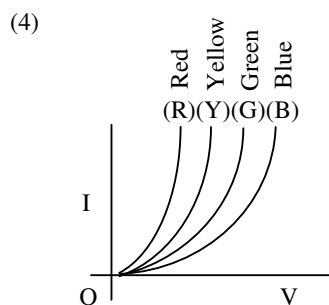
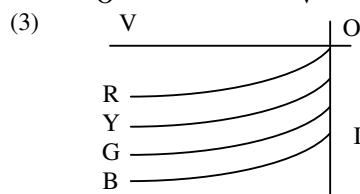
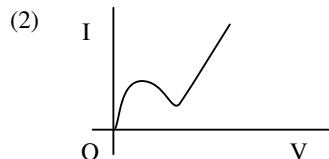
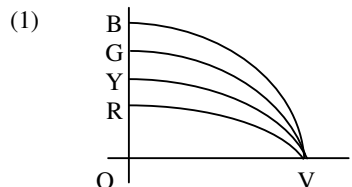




**Sol.****(2)**

The temperature goes on decreasing with time (non-linearly) The rate of decrease will be more initially which is depicted in the second graph.

85. The I – V characteristic of an LED is

**Sol.****(4)**

For LED, in forward bias, intensity increases with voltage.

86. This question has Statement I and Statement II. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement – I : A point particle of mass  $m$  moving with speed  $v$  collides with stationary point particle of mass  $M$ . If the maximum energy loss possible is given as  $f \left( \frac{1}{2}mv^2 \right)$  then  $f = \left( \frac{m}{M+m} \right)$ .

Statement – II : Maximum energy loss occurs when the particles get stuck together as a result of the collision.

- (1) Statement – I is true, Statement – II is true, Statement – II is not a correct explanation of Statement – I.
- (2) Statement – I is true, Statement – II is false.
- (3) Statement – I is false, Statement – II is true
- (4) Statement – I is true, Statement – II is true, Statement – II is a correct explanation of Statement – I.

**Sol.****(3)**

Loss of energy is maximum when collision is inelastic as in an inelastic collision there will be maximum deformation.

$$\text{KE in COM frame is } \frac{1}{2} \left( \frac{Mm}{M+m} \right) V_{rel}^2$$

$$\text{KE}_i = \frac{1}{2} \left( \frac{Mm}{M+m} \right) V^2 \quad \text{KE}_f = 0 \quad (\because V_{rel} = 0)$$

$$\text{Hence loss in energy is } \frac{1}{2} \left( \frac{Mm}{M+m} \right) V^2$$

$$\Rightarrow f = \frac{M}{M+m}$$

87. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s it will decrease to  $\alpha$  times its original magnitude, where  $\alpha$  equals.

(1) 0.81 (2) 0.729  
(3) 0.6 (4) 0.7

**Sol.** (2)

$$A = A_0 e^{-kt}$$

$$\Rightarrow 0.9A_0 = A_0 e^{-5k}$$

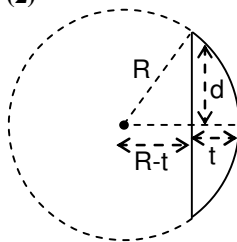
$$\text{and } \alpha A_0 = A_0 e^{-15k}$$

$$\text{solving } \Rightarrow \alpha = 0.729$$

88. Diameter of plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is  $2 \times 10^8$  m/s, the focal length of the lens is :

(1) 20 cm (2) 30 cm  
(3) 10 cm (4) 15 cm

**Sol.** (2)



$$R^2 = d^2 + (R-t)^2$$

$$R^2 - d^2 = R^2 \left\{ 1 - \frac{t}{R} \right\}^2$$

$$1 - \frac{d^2}{R^2} = 1 - \frac{2t}{R}$$

$$R = \frac{(3)^2}{2 \times (0.3)} = \frac{90}{6} = 15 \text{ cm}$$

$$\frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{f} = \left( \frac{3}{2} - 1 \right) \left( \frac{1}{15} \right)$$

$$f = 30 \text{ cm}$$

89. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is :

(1) 6 V/m (2) 9 V/m  
(3) 12 V/m (4) 3 V/m

**Sol.** (1)

$$E_0 = cB_0$$

$$= 3 \times 10^8 \times 20 \times 10^{-9}$$

$$= 6 \text{ V/m}$$

90. Two short bar magnets of length 1 cm each have magnetic moments  $1.20 \text{ Am}^2$  and  $1.00 \text{ Am}^2$  respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centres is close to (Horizontal component of earth's magnetic induction is  $3.6 \times 10^{-5} \text{ Wb/m}^2$ )

(1)  $2.56 \times 10^{-4} \text{ Wb/m}^2$   
(3)  $5.80 \times 10^{-4} \text{ Wb/m}^2$

(2)  $3.50 \times 10^{-4} \text{ Wb/m}^2$   
(4)  $3.6 \times 10^{-5} \text{ Wb/m}^2$

**Sol.**

**(1)**

$$B_{net} = B_{M_1} + B_{M_2} + B_H$$

$$= \frac{\mu_0 M_1}{4\pi x^3} + \frac{\mu_0 M_2}{4\pi x^3} + B_H$$

$$= \frac{\mu_0}{4\pi x^3} (M_1 + M_2) + B_H$$

$$= \frac{10^{-7}}{10^{-3}} \times 2.2 + 3.6 \times 10^{-5}$$

$$= 2.56 \times 10^{-4} \text{ Wb/m}^2$$