Secondary School Examination

March — 2008

Marking Scheme — Mathematics (Foreign) 30/2/1, 30/2/2, 30/2/3

General Instructions

1. The Marking Scheme provides general guidelines to reduce subjectivity and maintain uniformity among large number of examiners involved in the marking. The answers given in the marking scheme are the best suggested answers.

2. Marking is to be done as per instructions provided in the marking scheme. (It should not be done according to one's own interpretation or any other consideration.) Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. Some of the questions may relate to higher order thinking ability. These questions will be indicated to you separately by a star mark. These questions are to be evaluated carefully and the students' understanding / analytical ability may be judged.

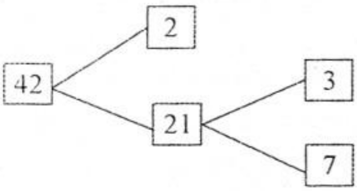
5. The Head-Examiners have to go through the first five answer-scripts evaluated by each evaluator to ensure that the evaluation has been carried out as per the instruction given in the marking scheme. The remaining answer scripts meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.

6. If a question is attempted twice and the candidate has not crossed any answer, only first attempt is to be evaluated. Write EXTRA with second attempt.

7. A full scale of marks 0 to 80 has to be used. Please do not hesitate to award full marks if the answer deserves it.

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

1.  $\frac{1}{2}+\frac{1}{2}$ m

2. 2 — 1 m

3. $(-3)^2 + 6(-3) + 9 = 9 - 18 + 9 = 0$
   $\therefore x = -3$ is a solution of $x^2 + 6x + 9 = 0$ — 1 m

4. $p + 9q$ — 1 m

5. $\frac{17}{12}$ — 1 m

6. 25 cm — 1 m

7. 1:9 — 1 m

8. 7 cm — 1 m

9. $\frac{2}{6}$ or $\frac{1}{3}$ — $\frac{1}{2}+\frac{1}{2}$ m

10. 17.5, 45

SECTION - B

11. $(x + 2)$, $(x - 2)$ are factors of given polynomal

    Getting $\frac{x^4 + x^3 - 34x^2 - 4x + 120}{x^2 - 4} = x^2 + x - 30$ — 1 m

    $x^2 + x - 30 = (x + 6)(x - 5)$ — $\frac{1}{2}$ m

    $\therefore$ The zeroes are $2, -2, -6, 5$ — $\frac{1}{2}$ m

12. Total number of element in the sample space = 36 — $\frac{1}{2}$ m

    Favourable event = 6 — 1 m

    Probability (getting same number on each dice) $= \frac{6}{36} = \frac{1}{6}$ — $\frac{1}{2}$ m

13. $\sec 4A = \operatorname{cosec}(90° - 4A)$ $\frac{1}{2}$ m

$\Rightarrow \quad \operatorname{cosec}(90° - 4A) = \operatorname{cosec}(A - 20°)$ $\frac{1}{2}$ m

$\Rightarrow \quad 90° - 4A = A - 20°$ $\frac{1}{2}$ m

$\Rightarrow \quad A = 22°$ $\frac{1}{2}$ m

or $\tan A = \frac{1}{\sqrt{3}} = \tan 30° \Rightarrow A = 30°$

As $A + B = 90° \Rightarrow B = 60°$ $\frac{1}{2}$ m

$\operatorname{Sin} A \cos B + \cos A \sin B = \frac{1}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}$ 1 m

$= \frac{1}{4} + \frac{3}{4} = 1$ $\frac{1}{2}$ m

14. The points are collnear if the area of triangle formed by the points is zero.

Area of triangle formed by the point $(k, 3)$, $(6, -2)$, $(-3, 4)$ is zero. $\frac{1}{2}$ m

i.e., $k(-2-4) + 6(4-3) - 3(3+2) = 0$ 1 m

or $-6k - 9 = 0 \Rightarrow k = -3/2$ $\frac{1}{2}$ m

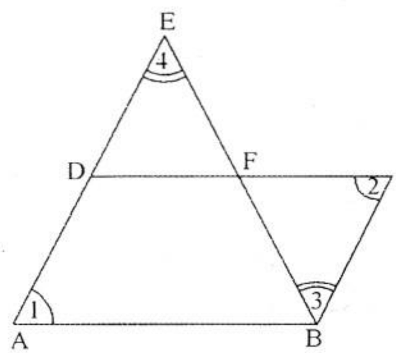
15. 

fig. $\frac{1}{2}$ m

In Δs ABE and CFB

$\angle 1 = \angle 2$ (opposite angles of a $||^{gm}$) 1 m
$\angle 3 = 4$ [Alt. $\angle$ s]

$\therefore \Delta ABE \sim \Delta CFB$ $\frac{1}{2}$ m

SECTION - C

16. Let x be any positive integer, then it is of the form $3q$, $3q + 1$, $3q + 2$ $\frac{1}{2}$ m

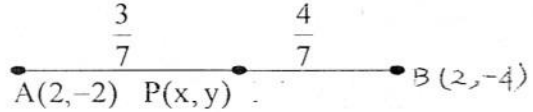
$\therefore \quad x^2 = (3q)^2 = 3 \cdot 3q^2 = 3m$ $\frac{1}{2}$ m

or, $x^2 = (3q+1)^2 = 3(3q^2 + 2q) + 1 = 3m + 1$ 1 m

or, $x^2 = (3q+2)^2 = 3[3q^2 + 4q + 1] + 1 = 3m + 1$ 1 m

17. Drawing correct lines	$1 + 1 = 2$ m
Point of intersection with y-axis	
$(0, 2)$ and $(0, -4)$	$\frac{1}{2} + \frac{1}{2} = 1$ m
18. nth term of A.P. 63, 65, 67, ... $= 63 + 2(n-1)$	1 m
nth term of A.P. 3, 10, 17, ... $= 3 + 7(n-1)$	1 m
$\therefore \quad 63 + 2n - 2 = 3 + 7n - 7$	$\frac{1}{2}$ m
$\Rightarrow \quad n = 13$	$\frac{1}{2}$ m
OR	
Let first term = a and common difference = d	
$\therefore \quad Tm = a + (m-1)d$	$\frac{1}{2}$ m
$Tn = a + (n-1)d$	$\frac{1}{2}$ m
$\therefore \quad m[a + (m-1)d] = n[a + (n-1)d]$	1 m
$\Rightarrow (m-n)[a + (m+n-1)d] = 0$	
As $m \neq n, a + (m+n-1)d = 0$	
or $\quad Tm + n = 0$	1 m
19. Let common difference be d	
$\therefore \quad 8 + (n-1)d = 33 \Rightarrow (n-1)d = 25$	1 m
And, $\quad \frac{n}{2}[16 + (n-1)d] = 123$	
$\Rightarrow \quad \frac{n}{2}(16+25) = 123 \Rightarrow n = 6$	1 m
Also $\quad (n-1)d = 25 \Rightarrow d = 5$	1 m
20. $\text{LHS} = \left(1 + \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}\right)(\sin A - \cos A)$	$\frac{1}{2}$ m
$= \frac{(\sin A \cos A + \cos^2 A + \sin^2 A)(\sin A - \cos A)}{\sin A \cos A}$	$\frac{1}{2}$ m
$= \frac{\sin^3 A - \cos^3 A}{\sin A \cos A} = \frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A}$	$1 + \frac{1}{2}$ m
$= \sin A \tan A - \cos A \cot A$	$\frac{1}{2}$ m
OR $\quad \cos 58° = \cos(90 - 32)° = \sin 32°$, $\text{cosec } 52° = \sec 38°$	
$\tan 75° = \cot 15°$, $\tan 60° = \sqrt{3}$	$1\frac{1}{2}$ m

$\therefore$ Given expression becomes $2 - \sqrt{3} \cdot \frac{1}{\sqrt{3}} = 1$ $1\frac{1}{2}$ m

21. 

$\frac{AP}{PB} = \frac{3}{4}$

or AP : PB = 3:4 1 m

$\therefore$ P divides the join of (–2, – 2) and (2, – 4) in the ratio of 3 : 4

$\therefore$ Coordinates of P are $\left(-\frac{2}{7}, -\frac{20}{7}\right)$ 2 m

22. Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of the given triangle
The mid-point of AB, BC and CA are (3, 4), (4, 6) and (5, 7) respectively

$\therefore$ $x_1 + x_2 = 6, x_2 + x_3 = 8, x_3 + x_1 = 10$

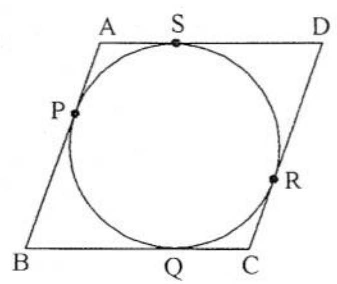
$y_1 + y_2 = 8,\ y_2 + y_3 = 12, y_3 + y_1 = 14$ $1\frac{1}{2}$ m

Solving to get the vertices of Δ ABC as (4, 5), (2, 3), (6, 9) $1\frac{1}{2}$ m

23. Correct construction of right triangle with
sides containing the right angle as 5cm and 4cm 1 m
Constructing correct similar triangle to the given triangle 2 m

24. Correct Figure $\frac{1}{2}$ m

AS = AP, DS = DR, CQ = CR, BQ = BP



Adding we get
(AS + DS) + (BQ + QC) = (AP + BP) + (CR + DR)
$\Rightarrow$ AD + BC = AB + CD 1 m
As ABCD is a $||^{gm} \Rightarrow$ 2AB = 2AD [$\because$ AD = BC, AB = DC]
$\Rightarrow$ AB = AD 1 m

$\therefore$ ABCD is a rhombus $\frac{1}{2}$ m

OR

In right ΔADC, $AC^2 = AD^2 + DC^2 \Rightarrow AD^2 = AC^2 - DC^2$ ....(i) 1 m
Similarly, in right ΔADB, $AB^2 = AD^2 + BD^2 \Rightarrow AD^2 = AB^2 - BD^2$ ....(ii) 1 m
From (i) and (ii), to get

$AC^2 - DC^2 = AB^2 - BD^2$

$\Rightarrow \quad AB^2 + CD^2 = BD^2 + AC^2$ } 1 m

25. Area of quadrant $= \left(\frac{1}{4} \times \frac{22}{7} \times 14 \times 14\right) \text{cm}^2$

$= 154 \text{ cm}^2$ $\frac{1}{2}$ m

Area of Δ ABC $= \left(\frac{1}{2} \times 14 \times 14\right) \text{cm}^2 = 98 \text{ cm}^2$ $\frac{1}{2}$ m

$\therefore$ Area of segment formed with BC $= (154 - 98) \text{ cm}^2 = 56 \text{ cm}^2$ $\frac{1}{2}$ m

Finding BC $= 14\sqrt{2}$ cm $\frac{1}{2}$ m

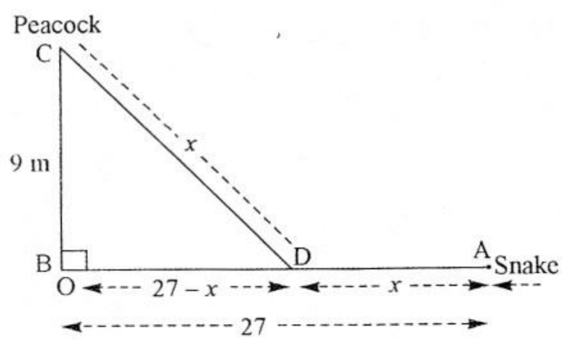
$\therefore$ Area of semi-circle on BC as diameter

$= \left(\frac{1}{2} \times \frac{22}{7} \times 7\sqrt{2} \times 7\sqrt{2}\right) \text{cm}^2 = 154 \text{ cm}^2$ $\frac{1}{2}$ m

$\therefore$ Area of shaded region $= (154 - 56) \text{ cm}^2$ or 98 cm^2 $\frac{1}{2}$ m

SECTION - D

26. Correct figure 1 m



Let $AB = x$ m

As the speeds of Peacock and snake are equal $\Rightarrow$ $CD = AD = x$ (Say) 1 m

and $BD = 27 - x$ 1 m

From right triangle ABC, $9^2 + (27 - x)^2 = x^2$ 1 m

$\Rightarrow$ $54x = 810$ or $x = 15$ 1 m

$\therefore$ AD = 15m and BD = 12m

$\therefore$ Snake is caught at a distance of 12m from its hole 1 m

OR

Let the two numbers be x, $x + 4$ 1 m

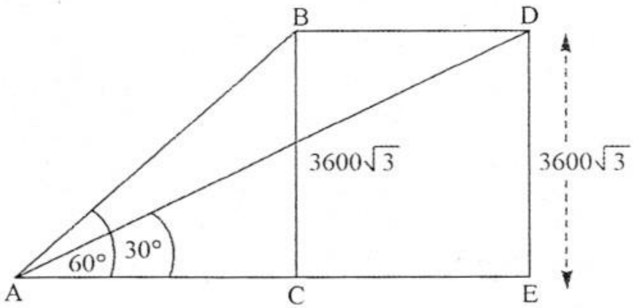
$\therefore$ $\frac{1}{x} - \frac{1}{x+4} = \frac{4}{21}$ 2 m

$\Rightarrow \quad x^2 + 4x - 21 = 0$ 1 m

$\Rightarrow \quad x = -7$ or 3 1 m

$\therefore$ The numbers are (3, 7) or (– 7, – 3) 1 m

27. Figure 1 m



In right Δ ACB, $\dfrac{3600\sqrt{3}}{AC} = \tan 60°$

$\Rightarrow \quad AC = 3600$ $1\frac{1}{2}$ m

In Δ ADE, $\dfrac{3600\sqrt{3}}{AE} = \tan 30°$

$\Rightarrow \quad AE = 10800m$ $1\frac{1}{2}$ m

$\therefore \quad CE = BD = (10800 - 3600)\text{ m} = 7200\text{ m}$ 1 m

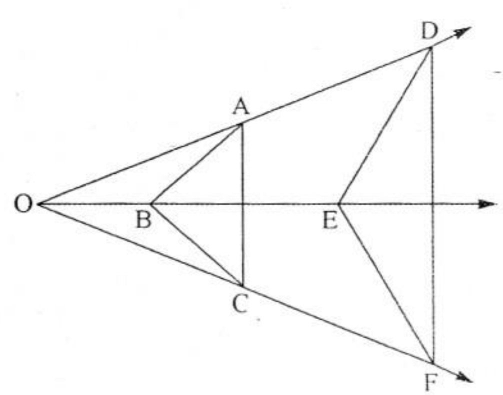
$\therefore \quad$ Speed (in km/hour) $= \dfrac{7200 \times 60 \times 60}{30 \times 1000}$

$= 864$

$\therefore$ The speed of aeroplanes 864 km/hour 1 m

28. Correct figure, given, to prove and construction $\left(\frac{1}{2} \times 4\right)$ 2 m

Correct Proof 2 m



$AB \parallel DE \Rightarrow \dfrac{OA}{AD} = \dfrac{OB}{OE}$ .... (*i*) $\frac{1}{2}$ m

Similarly, $BC \parallel EF \Rightarrow \frac{OC}{CF} = \frac{OB}{OE}$ ....(ii) $\frac{1}{2}$ m

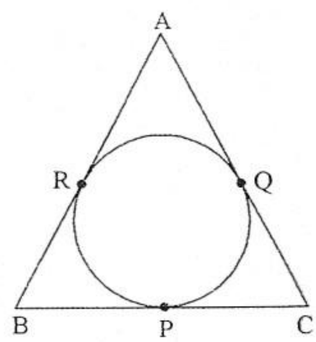
From (i) and (ii), $\frac{OA}{AD} = \frac{OC}{CF}$ $\frac{1}{2}$ m

$\Rightarrow \quad AC \parallel DF$ $\frac{1}{2}$ m

OR

Correct figure, given, to prove and construction $\left(\frac{1}{2} \times 4\right)$ 2 m

Correct proof 2 m



$$AR = AQ$$ $\frac{1}{2}$ m

$$AB = AC \Rightarrow AB - AR = AC - AQ$$

$\Rightarrow \quad BR = CQ \quad ...(i)$ $\frac{1}{2}$ m

Also, from figure, $\quad BR = BP$ and $CQ = CP \quad ...(ii)$ $\frac{1}{2}$ m

From (i) and (ii) $\quad BP = PC$

P bisects base BC $\frac{1}{2}$ m

29. Here $\quad r_1 = 20$ cm, $r_2 = 8$cm, $h = 16$cm

$$l = \sqrt{(16)^2 + (20-8)^2} = 20\text{cm}$$ 1 m

$$\text{Capacity of bucket} = \frac{\pi.16}{3}\,[20^2 + 8^2 + 20 \times 8]\ \text{cm}^3$$ $2\frac{1}{2}$ m

$$= \frac{73216}{7} \text{ or } 10459.43\ \text{cm}^3$$

$$\text{Total surface area} = \left[\frac{22}{7} \times 20\,(20+8) + \frac{22}{7} \times 8 \times 8\right]\ \text{cm}^2$$ $2\frac{1}{2}$ m

$$= 1961.15\text{cm}^2$$

30.

Classes	0-20	20-40	40-60	60-80	80-100	100-120	120-140	Total
mid-value (xi)	10	30	50	70	90	110	130	
cum. freq	6	8	10	12	6	5	3	$50 = \sum fi$
$fixi$	60	240	500	840	540	550	390	$3120 = \sum fixi$

Correct Table as above

2 m

$$\overline{x} = Mean = \frac{\sum fixi}{\sum fi} = \frac{3120}{50} = 62.4$$ 1 m

$$\text{Median} = 60 + \frac{25-24}{12} \times 20 = 60 + 1.67 = 61.67$$ $1\frac{1}{2}$ m

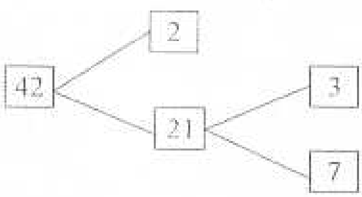
$$\text{Mode} = 60 + \frac{12-10}{24-10-6} \times 20 = 65.0$$ $1\frac{1}{2}$ m

Note: If a candidate finds any two two of the measures of central tendency and finds the third by using empirical formula, give full credit.

30/2/2

SECTION - A

1. $\frac{1}{3}$ 1 m
2. 7 1 m
3. $\frac{1}{9}$ 1 m
4. 17.5, 45 $\frac{1}{2}+\frac{1}{2}$ m
5. $\frac{1}{2}+\frac{1}{2}$ m



6. $a = 2$ 1 m
7. $2(-3)^2 + 6(-3) + 9 = 0 = \text{RHS}$ 1 m
8. $p + 9q$ 1 m
9. $\frac{625}{168}$ 1 m
10. 25cm 1 m

SECTION - B

11. Same as Q No. 15 of 30/2/1
12. Same as Q No. 14 of 30/2/1
13. $\sec 2A = \sec[90° - (A - 42°)]$
$= \sec[132° - A]$ 1 m
$\Rightarrow \quad 2A = 132° - A$
or $\quad A = 44°$ 1 m

OR

$\angle C = 60°, \angle B = 30°$ [$\because \angle A = 90°$] $\frac{1}{2}$ m

$\sin B \cos C + \cos B \sin C = \frac{1}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} = 1$ $1\frac{1}{2}$ m

14. Same as Q.No. 11 of 30/2/1

15. Total number of balls in the bag = 12 $\frac{1}{2}$ m

(*i*) P (yellow ball) $= \frac{3}{12} = \frac{1}{4}$ $\frac{1}{2}$ m

(*ii*) P (not of red colour) $= \frac{8}{12} = \frac{2}{3}$ 1 m

SECTION - C

16. Same as Q.No. 25 fo 30/2/1

17. Same as Q.No. 24 of 30/2/1

18. Same as Q.No. 23 fo 30/2/1

19. Same as Q.No. 22 fo 30/2/1

20. $AB = \sqrt{53}$, $BC = \sqrt{53}$, $CD = \sqrt{53}$, $DA = \sqrt{53}$ 2 m

$\Rightarrow AB = BC = CD = DA$
or ABCD is a rhombus 1 m

21. Same as Q.No. 20 fo 30/2/1

22. Same as Q.No. 16 of 30/2/1

23. Same as Q.No. 17 of 30/2/1

24. Same as Q.No. 18 fo 30/2/1

25. Let common difference is d

Here $a = 25, t_n = -17, S_n = 60$

$\therefore \quad -17 = 25 + (n-1)d \Rightarrow (n-1)d = -42$ ....(*i*) 1 m

$\therefore \quad 60 = \frac{n}{2}[50 + (n-1)d] = \frac{n}{2}[50 - 42] = 4n$ 1 m

$\Rightarrow \quad n = 15$ $\frac{1}{2}$ m

From (*i*), $d = -3$ $\frac{1}{2}$ m

SECTION - D

26.

Classes	0-50	50-100	100-150	150-200	200-250	250-300	300-350
class marks (x_i)	25	75	125	175	225	275	325
f_i	2	3	5	6	5	3	1 : $\sum f = 25$
cum f_i	2	5	10	16	21	24	25
$f_i x_i$	50	225	625	1050	1125	825	325: $\sum f_i x_i = 4225$

(Correct Table) 2 m

$\therefore \quad \bar{x} = \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{4225}{25} = 169$ 1 m

Median $= 150 + \dfrac{\frac{25}{2} - 10}{6} \times 50 = 170.83$ $1\frac{1}{2}$ m

Mode $= 150 + \dfrac{6-5}{12-5-5} \times 50 = 175$ $1\frac{1}{2}$ m

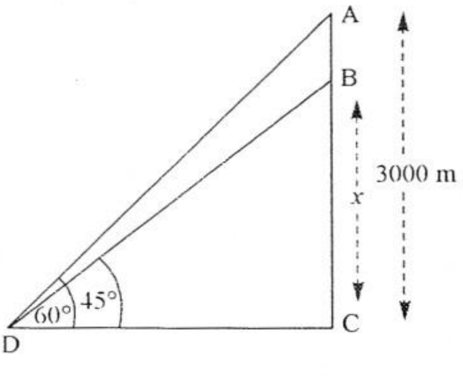
Note: If a candidate finds any two of the measures of central tendency correctly and uses empirical formula to find the third, full credit is to be given.

27. Same as Q. No. 29 of 30/2/1

28. Same as Q. No. 26 of 30/2/1

29. Same as Q. No. 28 of 30/2/1

30. Correct Figure 1 m



Writing trigonometric equations

$\dfrac{3000}{DC} = \tan 60° = \sqrt{3}$ 1 m

$\Rightarrow \quad DC = 1000\sqrt{3} = 1732\text{m}$ 1 m

Also, $\dfrac{x}{DC} = \tan 45° = 1$ 1 m

$x = DC = 1732$ m 1 m

$\therefore$ Distance between aeroplanes $= (3000 - 1732)\text{m} = 1268$ m 1 m

SECTION - A

1. 17.5, 45 $\frac{1}{2}+\frac{1}{2}$ m

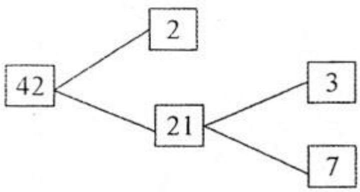
2. $\frac{17}{12}$ 1 m

3. 7cm 1 m

4. 1:9 1 m

5. 25cm 1 m

6. $\frac{1}{2}+\frac{1}{2}$ m



7. $a = 2$ 1 m

8. $3\,(-2)^2 + 13\,(-2) + 14 = 12 - 26 + 14 = 0 = \text{RHS}$ 1 m

9. $p + 4q$ 1 m

10. $\frac{1}{6}$ 1 m

SECTION - B

11. Same as Q. No. 13 of 30/2/1

12. Same as Q. No. 14 of 30/2/1

13. Same as Q. No. 15 of 30/2/1

14. Product of two factors $= x^2 - 2$

Finding $\dfrac{2x^4 + 7x^3 - 19x^2 - 14x + 30}{x^2 - 2} = 2x^2 + 7x - 15$ 1 m

Now $2x^2 + 7x - 15 = (2x - 3)(x + 5)$

$\therefore$ Zeroes of the given polynomial are $\sqrt{2},\ -\sqrt{2},\ -5,\ \frac{3}{2}$ 1 m

15. Number of tickets in the bag = 20 $\frac{1}{2}$ m

(*i*) Multiples of 7 are: 14, 21, 28 (Three in number)

P (multiple of 7) $= \frac{3}{20}$ $\frac{1}{2}$ m

(*ii*) Greater than 15 and multiple 5 : 20, 25, 30 (Three in number)

P (greater than 15 and multiple of 5) $= \frac{3}{20}$ 1 m

SECTION - C

16. Here $a = 22$, $t_n = -11$ and $s_n = 66$, $n = ?$, $d = ?$
$-11 = 22 + (n-1)d \Rightarrow (n-1)d = -33$ ...(i) 1 m

$$66 = \frac{n}{2}[44 + (n-1)d] = \frac{n}{2}(44 - 33)$$ 1 m

$\Rightarrow n = 12$ $\frac{1}{2}$ m

from (i), $d = -3$ $\frac{1}{2}$ m

17. Same as Q. No. 18 of 30/2/1
18. Same as Q. No. 17 of 30/2/1
19. Same as Q. No. 16 of 30/2/1
20. Same as Q. No. 25 of 30/2/1
21. Same as Q. No. 24 of 30/2/1
22. Same as Q. No. 23 of 30/2/1
23. Same as Q. No. 22 of 30/2/1
24. Let the rato be $k : 1$
Let P (x, y) divide the line segment joining (1, 3) and (2, 7) in the ratio of $k : 1$

$$\therefore x = \frac{2k+1}{k+1},\ y = \frac{7k+3}{k+1}$$ 1 m

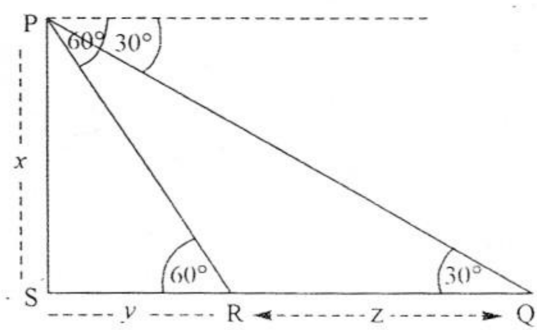
The point P (x, y) lies on the line $3x + y - 9 = 0$
$\Rightarrow (6k+3) + (7k+3) - 9(k+1) = 0$ 1 m

$$4k - 3 = 0 \Rightarrow k = \frac{3}{4}$$ 1 m

$\therefore$ The ratio is 3 : 4
25. Same as Q. No. 20 of 30/2/1

SECTION - D

26. Correct figure 1 m



The distance covered by car in 6 seconds = QR
Getting trigonometric equations

$$\frac{x}{y} = \tan 60° = \sqrt{3}$$ 1 m

$\Rightarrow \quad x = y\sqrt{3}$ $\frac{1}{2}$ m

Again $\frac{x}{y+z} = \tan 30° = \frac{1}{\sqrt{3}}$ 1 m

$\Rightarrow \quad \sqrt{3}\, y \cdot \sqrt{3} = y + z \Rightarrow z = 2y$ 1 m

$\Rightarrow \quad y = \frac{1}{2} z$

For distance QR (z), time taken is 6 seconds

For half the distance (y), it will be 3 seconds $1\frac{1}{2}$ m

27. Same as Q. No. 28 of 30/2/1

28. Same as Q. No. 29 of 30/2/1

29. Same as Q. No. 26 of 3012/1

30.

Classes	0-10	10-20	20-30	30-40	40-50	50-60	60-70
xi	5	15	25	35	45	55	65
fi	6	8	10	15	5	4	$2 : \sum fi = 50$
cum fi	6	14	24	39	44	48	50
$fixi$	30	120	250	525	225	220	$130 : \sum fixi = 1500$

Correct Table 2 m

$$\text{Mean} = \frac{\sum fixi}{\sum fi} = \frac{1500}{50} = 30$$ 1 m

$$\text{Median} = 30 + \frac{25-24}{15} \times 10 = 30.67$$ $1\frac{1}{2}$ m

$$\text{Mode} = 30 + \frac{15-10}{30-15} \times 10 = 33.33$$ $1\frac{1}{2}$ m

Note: If a candidate finds any two of the measures of central tendency correctly and uses empirical formula to find the third, full credit is to be given.