IIT – JEE - 2009

(PAPER – 2, CODE – 1)

Time: 3 Hours **Maximum Marks: 240**

A. Question paper format:

1. The question paper consists of 3 parts (Chemistry, Mathematics and Physics). Each part has 4 sections.

2. **Section I** contains **4** multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **only one is correct.**

3. **Section II** contains **5** multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **one or more is/are correct.**

4. **Section III** contains **2** questions. Each question has four statements (A, B, C and D) given in column I and five statements (p, q, r, s and t) in Column II. Any given statement in Column I can have correct matching with **one or more** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

5. **Section IV** contains **8** questions. The answer to each of the questions is a **single – digit integer,** ranging from 0 to 9. The answer will have to be appropriately bubbled in the ORS as per the instructions given at the beginning of the section.

B. Marking scheme:

6. For each question in **Section I** you will be **awarded 3 marks** if you darken the bubble corresponding to the correct answer and **zero mark** if no bubbles is darkened. In case of bubbling of incorrect answer, **minus one (-1) mark** will be awarded.

7. For each question in **Section II**, you will be **awarded 4 marks** if you darken the bubble(s) corresponding to the correct choice(s) for the answer, and **zero mark** if no bubble is darkened. In all other cases, **Minus one (-1) mark** will be awarded.

8. For each question in **Section III**, you will be **awarded 2 marks** for **each row** in which you have darkened the bubble(s) corresponding to the correct answer. Thus, each question in this section carries a maximum of **8** marks. There is **no negative marking** for incorrect answer(s) for this section.

9. For each question in **Section IV**, you will be **awarded 4 marks** if you darken the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In all other cases, **minus one (-1) mark** will be awarded.

IITJEE 2009 (PAPER-2, CODE-1)

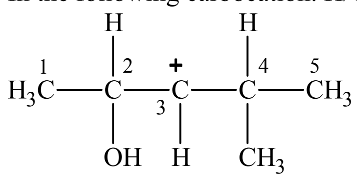
PART I: CHEMISTRY

SECTION-I

Single Correct Choice Type

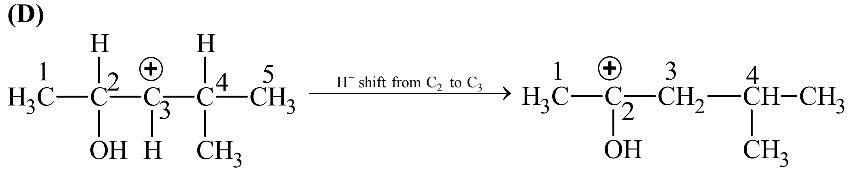
This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out which **ONLY ONE** is correct.

1. In the following carbocation. H/CH_3 that is most likely to migrate to the positively charged carbon is



(A) CH_3 at C-4 (B) H at C-4
(C) CH_3 at C-2 (D) H at C-2

Sol. **(D)**



(driving force is conjugation from oxygen)

2. The correct stability order of the following resonance structures is

$\underset{\text{(I)}}{H_2C=\overset{+}{N}=\overset{-}{N}}$ $\underset{\text{(II)}}{H_2\overset{+}{C}-N=\overset{-}{N}}$ $\underset{\text{(III)}}{H_2\overset{-}{C}-\overset{+}{N}\equiv N}$ $\underset{\text{(IV)}}{H_2\overset{-}{C}-N=\overset{+}{N}}$

(A) (I) > (II) > (IV) > (III) (B) (I) > (III) > (II) > (IV)
(C) (II) > (I) > (III) > (IV) (D) (III) > (I) > (IV) > (II)

Sol. (**B**)
On the basis of stability of resonating structures.

3. The spin only magnetic moment value (in Bohr magneton units) of $Cr(CO)_6$ is
(A) 0 (B) 2.84
(C) 4.90 (D) 5.92

Sol. **(A)**
$[Cr(CO)_6]$
$Cr(24) = [Ar]3d^5\ 4s^1$
Since (CO) is strong ligand, in $Cr(CO)_6$ no unpaired electron is present. So 'spin only' magnetic moment is zero.

4. For a first order reaction $A \rightarrow P$, the temperature (T) dependent rate constant (k) was found to follow the equation $\log k = -(2000)\frac{1}{T} + 6.0$. The pre-exponential factor A and the activation energy E_a, respectively, are
(A) $1.0 \times 10^6\ s^{-1}$ and 9.2 $kJmol^{-1}$ (B) 6.0 s^{-1} and 16.6 $kJmol^{-1}$
(C) $1.0 \times 10^6\ s^{-1}$ and 16.6 $kJmol^{-1}$ (D) $1.0 \times 10^6\ s^{-1}$ and 38.3 $kJmol^{-1}$

Sol. **(D)**

Given, $\log K = 6 - \frac{2000}{T}$

Since, $\log K = \log A - \frac{Ea}{2.303RT}$ So, $A = 10^6\ sec^{-1}$ and $Ea = 38.3$ kJ/mole

SECTION-II

Multiple Correct Choice Type

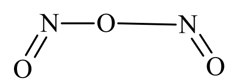
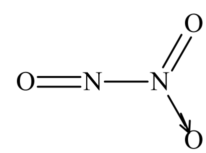
This section contains 5 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out which **ONE OR MORE** is/are correct.

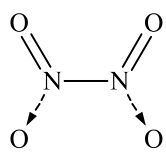
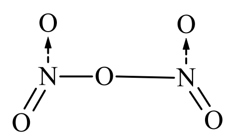
5. The nitrogen oxide(s) that contain(s) N–N bond(s) is(are)

(A) N_2O (B) N_2O_3

(C) N_2O_4 (D) N_2O_5

Sol. **(A, B, C)**

N_2O  , N_2O_3  

N_2O_4  , N_2O_5 

6. In the reaction

$2\mathbf{X} + B_2H_6 \rightarrow [BH_2(\mathbf{X})_2]^+[BH_4]^-$

the amine(s) **X** is(are)

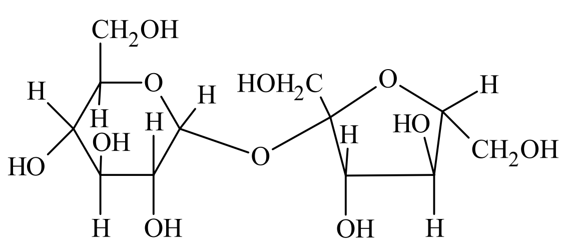
(A) NH_3 (B) CH_3NH_2

(C) $(CH_3)_2NH$ (D) $(CH_3)_3N$

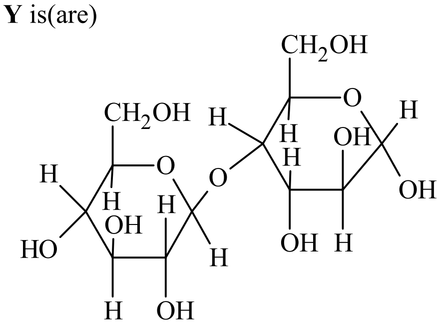
Sol. **(A, B, C)**

Due to bulkiness of trimethylamine, it does not react.

7. The correct statement(s) about the following sugars **X** and **Y** is(are)



X



Y

(A) **X** is a reducing sugar and **Y** is a non-reducing sugar

(B) **X** is a non-reducing sugar and **Y** is a reducing sugar

(C) The glucosidic linkages in **X** and **Y** are α and β, respectively

(D) The glucosidic linkages in **X** and **Y** are β and α, respectively

Sol. **(B, C)**

8. Among the following, the state function(s) is(are)

(A) Internal energy
(B) Irreversible expansion work
(C) Reversible expansion work
(D) Molar enthalpy

Sol. **(A, D)**

9. For the reduction of NO_3^- ion in an aqueous solution, E^0 is +0.96 V. Values of E^0 for some metal ions are given below

$V^{2+}(aq) + 2e^- \rightarrow V \qquad E^0 = -1.19\ V$

$Fe^{3+}(aq) + 3e^- \rightarrow Fe \qquad E^0 = -0.04\ V$

$Au^{3+}(aq) + 3e^- \rightarrow Au \qquad E^0 = +1.40\ V$

$Hg^{2+}(aq) + 2e^- \rightarrow Hg \qquad E^0 = +0.86\ V$

The pair(s) of metals that is(are) oxidized by NO_3^- in aqueous solution is(are)

(A) V and Hg
(B) Hg and Fe
(C) Fe and Au
(D) Fe and V

Sol. **(A, B, D)**

$E^{o}_{\substack{NO_3^-(aq)\\(SRP)}} = 0.96\ V$ All V, Fe, Hg have less SRP w.r.t. NO_3^-.

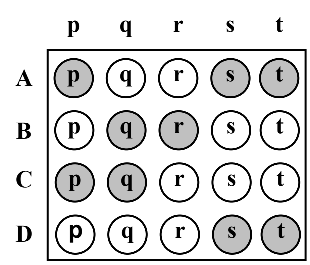
So, V, Fe, Hg can be oxidized by NO_3^- in aqueous solution.

SECTION – III

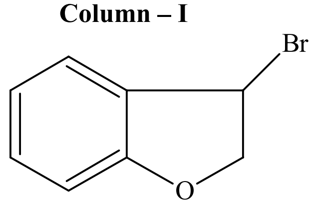
Matrix – Match Type

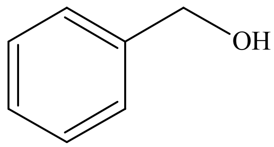
This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II.** The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:

If the correct matches are A – p, s and t; B – q and r; C – p and q; and D – s and t; then the correct darkening of bubbles will look like the following:

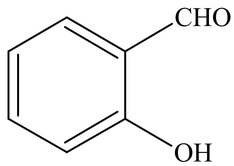


10. Match each of the compounds given in **Column I** with the reaction(s), that they can undergo given in **Column II**.

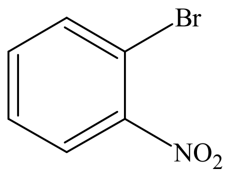
Column – I	Column – II
(A) 	(p) Nucleophilic substitution

(B) 

(q) Elimination

(C) 

(r) Nucleophilic addition

(D) 

(s) Esterification with acetic anhydride

(t) Dehydrogenation

Sol. **((A – p, q, t) (B – p, s, t) (C – r, s) (D – p))**

11. Match each of the reactions given in **Column I** with the corresponding product(s) given in **Column II**.

	Column – I		**Column – II**
(A)	Cu + dil HNO_3	(p)	NO
(B)	Cu + conc HNO_3	(q)	NO_2
(C)	Zn + dil HNO_3	(r)	N_2O
(D)	Zn + conc HNO_3	(s)	$Cu(NO_3)_2$
		(t)	$Zn(NO_3)_2$

Sol. **((A – p, s) (B – q, s) (C – r, t) (D – q, t))**

$$3Cu + dil.8HNO_3 \rightarrow 3Cu\left(NO_3\right)_2 + 4H_2O + 2NO$$

$$Cu + conc.\ 4HNO_3 \rightarrow Cu\left(NO_3\right)_2 + 2H_2O + 2NO_2$$

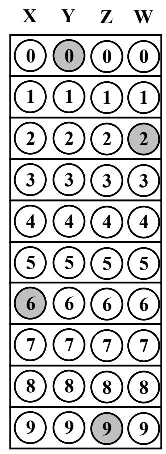
$$4Zn + dil.\ 10HNO_3 \rightarrow 4Zn\left(NO_3\right)_2 + 5H_2O + N_2O$$

$$Zn + conc.\ 4HNO_3 \rightarrow Zn\left(NO_3\right)_2 + 2H_2O + 2NO_2$$

SECTION – IV

Integer Answer Type

This section contains 8 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following:



12. The oxidation number of Mn in the product of alkaline oxidative fusion of MnO_2 is

Sol. **6**

$$2MnO_2 + 4KOH + O_2 \rightarrow \underset{\text{(Potassium manganate)}}{2K_2MnO_4} + 2H_2O$$ O. S. of Mn = +6 in K_2MnO_4

13. The number of water molecule(s) directly bonded to the metal centre in $CuSO_4 \cdot 5H_2O$ is

Sol. **4**

$$CuSO_4 \cdot 5H_2O \rightarrow \left[Cu\left(H_2O\right)_4\right]SO_4 \cdot H_2O$$

So, water molecules directly attached to Cu are 4.

14. The coordination number of Al in the crystalline state of $AlCl_3$ is

Sol. **6**

Coordination number of Al is 6. It exists in ccp lattice with 6 coordinate layer structure.

15. In a constant volume calorimeter, 3.5 g of a gas with molecular weight 28 was burnt in excess oxygen at 298.0 K. The temperature of the calorimeter was found to increase from 298.0 K to 298.45 K due to the combustion process. Given that the heat capacity of the calorimeter is 2.5 kJ K^{-1}, the numerical value for the enthalpy of combustion of the gas in $kJmol^{-1}$ is

Sol. **9**

Energy release at constant volume due to combustion of 3.5 gm of a gas = 2.5×0.45

Hence energy released due to the combustion of 28 gm (i.e., 1 mole) of a gas $= 2.5 \times 0.45 \times \frac{28}{3.5} = 9 \text{ kJmol}^{-1}$

16. The dissociation constant of a substituted benzoic acid at $25^{o}C$ is 1.0×10^{-4}. The pH of a 0.01 M solution of its sodium salt is

Sol. **8**

K_a (C_6H_5COOH) = 1×10^{-4}

pH of 0.01 M C_6H_5COONa

$$\underset{0.01(1-h)}{C_6H_5COO^-} + H_2O \rightleftharpoons \underset{0.01\text{ h}}{C_6H_5COOH} + \underset{0.01\text{ h}}{OH^{-1}}$$

$$K_h = \frac{K_w}{K_a} = \frac{0.01\text{ h}^2}{1-h}$$

$$\frac{10^{-14}}{10^{-4}} = \frac{10^{-2}h^2}{1-h} \quad (1-h \simeq 1)$$

$[OH^-] = 0.01\text{ h} = 0.01 \times 10^{-4} = 10^{-6}$

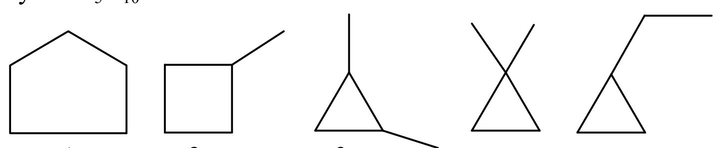
$[H^+] = 10^{-8}$

pH = 8

17. The total number of cyclic structural as well as stereo isomers possible for a compound with the molecular formula C_5H_{10} is

Sol. **7**

Cyclic C_5H_{10}



1 2 3 4 5

For 3rd structure 2 cis-trans and 1 optical isomer are possible.

Total 7 isomers.

18. The total number of α and β particles emitted in the nuclear reaction ${}^{238}_{92}U \rightarrow {}^{214}_{82}Pb$ is

Sol. **8**

${}_{92}U^{238} \xrightarrow{-6\alpha} {}_{80}X^{214} \xrightarrow{-2\beta} {}_{82}Pb^{214}$ (6α, 2β), total 8 paritcles.

19. At 400 K, the root mean square (rms) speed of a gas **X** (molecular weight = 40) is equal to the most probable speed of gas **Y** at 60 K. The molecular weight of the gas **Y** is

Sol. **4**

$$V_{rms_{(X\text{ gas})(400\text{ K})}} = V_{mp_{(Y\text{ gas})(60\text{ K})}}$$

M.W. (X gas) = 40; M.W. (Y gas) = x

$$\sqrt{\frac{3RT_1}{M_1}} = \sqrt{\frac{2RT_2}{M_2}}$$

$$\sqrt{\frac{400 \times 3}{40}} = \sqrt{\frac{2 \times 60}{x}}$$

$$30 = \frac{120}{x}$$

x = 4

PART II: MATHEMATICS

SECTION–I

Single Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

20. The normal at a point P on the ellipse $x^2 + 4y^2 = 16$ meets the x-axis at Q. If M is the mid point of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at the points

(A) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{2}{7}\right)$ (B) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{\sqrt{19}}{4}\right)$

(C) $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$ (D) $\left(\pm 2\sqrt{3}, \pm\frac{4\sqrt{3}}{7}\right)$

Sol. **(C)**

Normal is $4x \sec\phi - 2y \operatorname{cosec}\phi = 12$

$Q \equiv (3\cos\phi, 0)$

$M \equiv (\alpha, \beta)$

$\alpha = \frac{3\cos\phi + 4\cos\phi}{2} = \frac{7}{2}\cos\phi$

$\Rightarrow \cos\phi = \frac{2}{7}\alpha$

$\beta = \sin\phi$

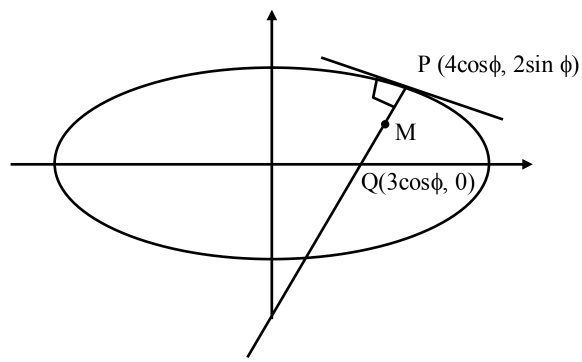
$\cos^2\phi + \sin^2\phi = 1$

$\Rightarrow \frac{4}{49}\alpha^2 + \beta^2 = 1 \Rightarrow \frac{4}{49}x^2 + y^2 = 1$

$\Rightarrow$ latus rectum $x = \pm 2\sqrt{3}$

$\frac{48}{49} + y^2 = 1 \Rightarrow y = \pm\frac{1}{7}$

$(\pm 2\sqrt{3}, \pm 1/7)$.



21. The locus of the orthocentre of the triangle formed by the lines $(1 + p)x - py + p(1 + p) = 0$, $(1 + q)x - qy + q(1 + q) = 0$ and $y = 0$, where $p \neq q$, is

(A) a hyperbola (B) a parabola

(C) an ellipse (D) a straight line

Sol. **(D)**

Intersection point of $y = 0$ with first line is $B(-p, 0)$

Intersection point of $y = 0$ with second line is $A(-q, 0)$

Intersection point of the two lines is $C(pq, (p + 1)(q + 1))$

Altitude from C to AB is $x = pq$

Altitude from B to AC is $y = -\frac{q}{1+q}(x + p)$

Solving these two we get $x = pq$ and $y = -pq$

$\therefore$ locus of orthocentre is $x + y = 0$.

22. A line with positive direction cosines passes through the point P(2, – 1, 2) and makes equal angles with the coordinate axes. The line meets the plane 2x + y + z = 9 at point Q. The length of the line segment PQ equals

(A) 1
(B) $\sqrt{2}$
(C) $\sqrt{3}$
(D) 2

Sol. **(C)**

D.C of the line are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.

Any point on the line at a distance t from P(2, – 1, 2) is $\left(2+\frac{t}{\sqrt{3}},\ -1+\frac{t}{\sqrt{3}},\ 2+\frac{t}{\sqrt{3}}\right)$

which lies on 2x + y + z = 9 $\Rightarrow$ t = $\sqrt{3}$.

23. If the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms is

(A) $\frac{n\left(4n^2-1\right)c^2}{6}$
(B) $\frac{n\left(4n^2+1\right)c^2}{3}$
(C) $\frac{n\left(4n^2-1\right)c^2}{3}$
(D) $\frac{n\left(4n^2+1\right)c^2}{6}$

Sol. **(C)**

$t_n = c\ \{n^2 - (n-1)^2\}$
$= c\ (2n - 1)$
$\Rightarrow\ t_n^2 = c^2\left(4n^2-4n+1\right)$
$\Rightarrow\ \sum_{n=1}^{n} t_n^2 = c^2\left\{\frac{4n(n+1)(2n+1)}{6}-\frac{4n(n+1)}{2}+n\right\}$
$= \frac{c^2n}{6}\{4(n+1)(2n+1)-12(n+1)+6\}$
$= \frac{c^2n}{3}\{4n^2+6n+2-6n-6+3\} = \frac{c^2}{3}n\left(4n^2-1\right)$.

SECTION–II

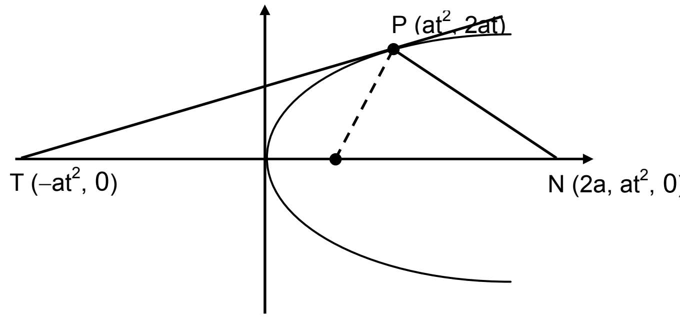
Multiple Correct Choice Type

This section contains 5 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

24. The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose

(A) vertex is $\left(\frac{2a}{3},\ 0\right)$
(B) directrix is x = 0
(C) latus rectum is $\frac{2a}{3}$
(D) focus is (a, 0)

Sol. **(A, D)**



$G \equiv (h, k)$

$\Rightarrow h = \dfrac{2a + at^2}{3}$, $k = \dfrac{2at}{3}$

$\Rightarrow \left(\dfrac{3h - 2a}{a}\right) = \dfrac{9k^2}{4a^2}$

$\Rightarrow$ required parabola is

$\dfrac{9y^2}{4a^2} = \dfrac{(3x - 2a)}{a} = \dfrac{3}{a}\left(x - \dfrac{2a}{3}\right)$

$\Rightarrow y^2 = \dfrac{4a}{3}\left(x - \dfrac{2a}{3}\right)$

Vertex $\equiv \left(\dfrac{2a}{3},\ 0\right)$; Focus $\equiv (a, 0)$

25. For function $f(x) = x\cos\dfrac{1}{x}$, $x \ge 1$,

(A) for atleast one x in interval $[1, \infty)$, $f(x + 2) - f(x) < 2$

(B) $\lim\limits_{x\to\infty} f'(x) = 1$

(C) for all x in the interval $[1, \infty)$, $f(x + 2) - f(x) > 2$

(D) $f'(x)$ is strictly decreasing in the interval $[1, \infty)$

Sol. **(B, C, D)**

For $f(x) = x\cos\left(\dfrac{1}{x}\right)$, $x \ge 1$

$f'(x) = \cos\left(\dfrac{1}{x}\right) + \dfrac{1}{x}\sin\left(\dfrac{1}{x}\right) \to 1$ for $x \to \infty$

also $f''(x) = \dfrac{1}{x^2}\sin\left(\dfrac{1}{x}\right) - \dfrac{1}{x^2}\sin\left(\dfrac{1}{x}\right) - \dfrac{1}{x^3}\cos\left(\dfrac{1}{x}\right)$

$= -\dfrac{1}{x^3}\cos\left(\dfrac{1}{x}\right) < 0 \quad$ for $x \ge 1$

$\Rightarrow f'(x)$ is decreasing for $[1, \infty)$

$\Rightarrow f'(x + 2) < f'(x)$. Also, $\lim\limits_{x\to\infty} f(x+2) - f(x) = \lim\limits_{x\to\infty}\left[(x+2)\cos\dfrac{1}{x+2} - x\cos\dfrac{1}{x}\right] = 2$

$\therefore f(x+2) - f(x) > 2\ \forall\ x \ge 1$

26. For $0 < \theta < \dfrac{\pi}{2}$, the solution(s) of $\sum\limits_{m=1}^{6} \operatorname{cosec}\left(\theta + \dfrac{(m-1)\pi}{4}\right)\operatorname{cosec}\left(\theta + \dfrac{m\pi}{4}\right) = 4\sqrt{2}$ is(are)

(A) $\dfrac{\pi}{4}$ (B) $\dfrac{\pi}{6}$

(C) $\dfrac{\pi}{12}$ (D) $\dfrac{5\pi}{12}$

Sol. **(C, D)**

Given solutions

$$\frac{1}{\sin(\pi/4)}\left[\frac{\sin(\theta + \pi/4 - \theta)}{\sin\theta\cdot\sin(\theta + \pi/4)} + \frac{\sin(\theta + \pi/2 - (\theta + \pi/4))}{\sin(\theta + \pi/4)\cdot\sin(\theta + \pi/2)} + \ldots + \frac{\sin((\theta + 3\pi/2) - (\theta + 5\pi/4))}{\sin(\theta + 3\pi/2)\cdot\sin(\theta + 5\pi/4)}\right] = 4\sqrt{2}$$

$$\Rightarrow \sqrt{2}\left[\cot\theta - \cot(\theta + \pi/4) + \cot(\theta + \pi/4) - \cot(\theta + \pi/2) + \ldots + \cot(\theta + 5\pi/4) - \cot(\theta + 3\pi/2)\right] = 4\sqrt{2}$$

$\Rightarrow \tan\theta + \cot\theta = 4 \Rightarrow \tan\theta = 2 \pm \sqrt{3}$

$\Rightarrow \theta = \frac{\pi}{12}$ or $\frac{5\pi}{12}$.

27. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinates axes, then

(A) equation of ellipse is $x^2 + 2y^2 = 2$
(B) the foci of ellipse are $(\pm 1, 0)$
(C) equation of ellipse is $x^2 + 2y^2 = 4$
(D) the foci of ellipse are $(\pm \sqrt{2}, 0)$

Sol. **(A, B)**

Ellipse and hyperbola will be confocal

$\Rightarrow (\pm ae, 0) \equiv (\pm 1, 0)$

$$\Rightarrow \left(\pm a \times \frac{1}{\sqrt{2}},\ 0\right) \equiv (\pm 1, 0)$$

$$\Rightarrow a = \sqrt{2} \text{ and } e = \frac{1}{\sqrt{2}}$$

$\Rightarrow b^2 = a^2(1 - e^2) \Rightarrow b^2 = 1$

$\therefore$ Equation of ellipse $\frac{x^2}{2} + \frac{y^2}{1} = 1$.

28. If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx$, n = 0, 1, 2, …, then

(A) $I_n = I_{n+2}$
(B) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$
(C) $\sum_{m=1}^{10} I_{2m} = 0$
(D) $I_n = I_{n+1}$

Sol. **(A, B, C)**

$$I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx$$

$$= \int_{0}^{\pi} \left(\frac{\sin nx}{(1+\pi^x)\sin x} + \frac{\pi^x \sin nx}{(1+\pi^x)\sin x} \right) dx = \int_{0}^{\pi} \frac{\sin nx}{\sin x}$$

$$\text{Now, } I_{n+2} - I_n = \int_{0}^{\pi} \frac{\sin(n+2)x - \sin nx}{\sin x} dx$$

$$= \int_{0}^{\pi} \frac{2\cos(n+1)x \cdot \sin x}{\sin x} dx = 0$$

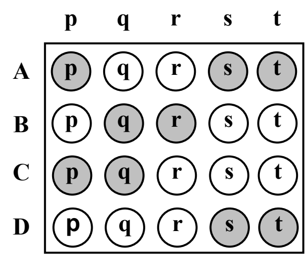
$$\Rightarrow I_1 = \pi,\ I_2 = \int_{0}^{\pi} 2\cos x\, dx = 0$$

SECTION – III

Matrix – Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statement in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement (s) in **Column II.** The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:

If the correct matches are A – p, s and t; B – q and r; C – p and q; and D –s and t; then the correct darkening of bubbles will look like the following.



29. Match the statements/expressions in **Column I** with the values given in **Column II.**

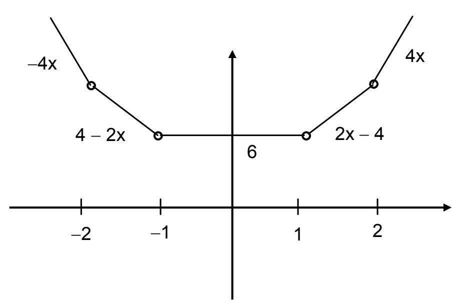
	Column I		**Column II**
(A)	The number of solutions of the equation $xe^{\sin x} - \cos x = 0$ in the interval $\left(0, \frac{\pi}{2}\right)$	(p)	1
(B)	Value(s) of k for which the planes kx + 4y + z = 0, 4x + ky + 2z = 0 and 2x + 2y + z = 0 intersect in a straight line	(q)	2
(C)	Value(s) of k for which $\|x - 1\| + \|x - 2\| + \|x + 1\| + \|x + 2\| = 4k$ has integer solution(s)	(r)	3
(D)	If $y' = y + 1$ and $y(0) = 1$ then value(s) of y(ln2)	(s)	4
		(t)	5

Sol. **(A) → (p)** **(B) → (q, s)** **(C) → (q, r, s, t)** **(D) → (r)**

(A). $f'(x) > 0,\ \forall\ x \in (0, \pi/2)$
$f(0) < 0$ and $f(\pi/2) > 0$
so one solution.

(B). Let (a, b, c) is direction ratio of the intersected line, then
$ak + 4b + c = 0$
$4a + kb + 2c = 0$
$$\frac{a}{8-k} = \frac{b}{4-2k} = \frac{c}{k^2-16}$$
We must have $2(8 - k) + 2(4 - 2k) + (k^2 - 16) = 0$
$\Rightarrow k = 2, 4.$

(C). Let $f(x) = |x + 2| + |x + 1| + |x - 1| + |x - 2|$
$\Rightarrow$ k can take value 2, 3, 4, 5.



(D). $\int \frac{dy}{y+1} = \int dx$

$\Rightarrow f(x) = 2e^x - 1$

$\Rightarrow f(\ln 2) = 3$

30. Match the statements/expressions in **Column I** with the values given in **Column II.**

Column I		Column II	
(A)	Root(s) of the expression $2\sin^2\theta + \sin^2 2\theta = 2$	(p)	$\frac{\pi}{6}$
(B)	Points of discontinuity of the function $f(x) = \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$, where [y] denotes the largest integer less than or equal to y	(q)	$\frac{\pi}{4}$
(C)	Volume of the parallelopiped with its edges represented by the vectors $\hat{i}+\hat{j},\ \hat{i}+2\hat{j}$ and $\hat{i}+\hat{j}+\pi\hat{k}$	(r)	$\frac{\pi}{3}$
(D)	Angle between vectors $\vec{a}$ and $\vec{b}$ where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a}+\vec{b}+\sqrt{3}\vec{c} = \vec{0}$	(s)	$\frac{\pi}{2}$
		(t)	π

Sol. **(A) → (q, s) (B) → (p, r, s, t) (C) → (t) (D) → (r)**

(A). $2\sin^2\theta + 4\sin^2\theta\cos^2\theta = 2$

$\sin^2\theta + 2\sin^2\theta(1 - \sin^2\theta) = 1$

$3\sin^2\theta - 2\sin^4\theta - 1 = 0 \Rightarrow \sin\theta = \pm\frac{1}{\sqrt{2}}, \pm 1$

$\Rightarrow \theta = \frac{\pi}{4}, \frac{\pi}{2}.$

(B). Let $y = \frac{3x}{\pi}$

$\Rightarrow \frac{1}{2} \le y \le 3\ \ \forall x \in \left[\frac{\pi}{6}, \pi\right]$

Now f(y) = [2y] cos[y]

Critical points are $y = \frac{1}{2}, y = 1, y = \frac{3}{2}, y = 3$

$\Rightarrow$ points of discontinuity $\left\{\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \pi\right\}$.

(C). $\begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & \pi \end{vmatrix} = \pi \Rightarrow$ volume of parallelopiped $= \pi$

(D). $|\vec{a}+\vec{b}| = \sqrt{3}$

$\Rightarrow \sqrt{2+2\cos\alpha} = \sqrt{3}$

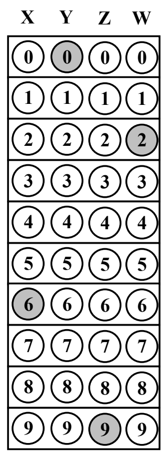
$\Rightarrow 2 + 2\cos\alpha = 3$

$\Rightarrow \alpha = \frac{\pi}{3}.$

SECTION – IV

Integer Answer Type

This section contains 8 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following:



31. Let f: R $\to$ R be a continuous function which satisfies $f(x) = \int_0^x f(t)\,dt$. Then the value of f(ln5) is

Sol. **0**

$f(x) = \int_0^x f(t)\,dt \Rightarrow f(0) = 0$

also, $f'(x) = f(x), x > 0$

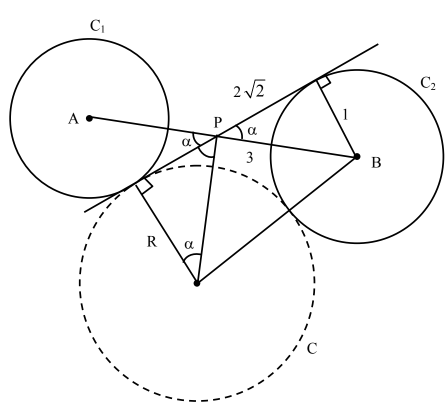
$\Rightarrow f(x) = ke^x, x > 0$

$\because$ f(0) = 0 and f(x) is continuous $\Rightarrow f(x) = 0\ \forall\ x > 0$

$\therefore$ f(ln5) = 0.

32. The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is

Sol. **8**



$\cos\alpha = \frac{2\sqrt{2}}{3}$

$\sin\alpha = \frac{1}{3}$

$\tan\alpha = \frac{2\sqrt{2}}{R}$

$\Rightarrow R = \frac{2\sqrt{2}}{\tan\alpha} = 8$ units.

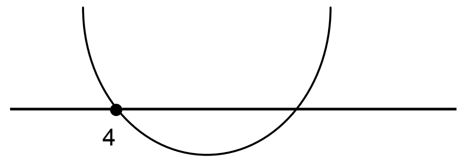
Alternate:

$(R + 1)^2 = (R - 1)^2 + \left(4\sqrt{2}\right)^2$

$\Rightarrow R = 8.$

33. The smallest value of k, for which both the roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4, is

Sol. **2**



$x^2 - 8kx + 16(k^2 - k + 1) = 0$

$D > 0 \Rightarrow k > 1 \qquad \ldots (1)$

$\frac{-b}{2a} > 4 \Rightarrow \frac{8k}{2} > 4$

$\Rightarrow k > 1 \qquad \ldots (2)$

$f(4) \geq 0 \Rightarrow 16 - 32k + 16(k^2 - k + 1) \geq 0$

$k^2 - 3k + 2 \geq 0$

$k \leq 1 \cup k \geq 2 \qquad \ldots (3)$

Using (1), (2) and (3)

$k_{min} = 2.$

34. The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set $A = \{x | x^2 + 20 \leq 9x|\}$ is

Sol. **7**

$f'(x) = 6(x - 2)(x - 3)$

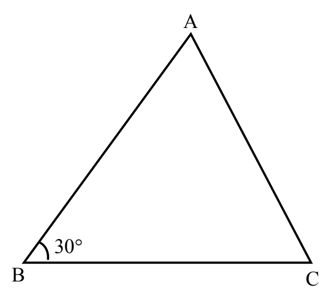
so $f(x)$ is increasing in $(3, \infty)$

Also, $A = \{4 \leq x \leq 5\}$

$\therefore f_{max} = f(5) = 7.$

35. Let ABC and ABC′ be two non–congruent triangles with sides AB = 4, AC = AC′ = $2\sqrt{2}$ and angle B = 30°. The absolute value of the difference between the areas of these triangles is

Sol. **4**



$\cos\beta = \frac{a^2 + 16 - 8}{2 \times a \times 4}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{a^2 + 8}{8a}$

$\Rightarrow a^2 - 4\sqrt{3}\,a + 8 = 0$

$\Rightarrow a_1 + a_2 = 4\sqrt{3}$, $a_1a_2 = 8$

$\Rightarrow |a_1 - a_2| = 4$

$\Rightarrow |\Delta_1 - \Delta_2| = \frac{1}{2} \times 4 \sin 30° \times 4 = 4$.

36. If the function $f(x) = x^3 + e^{x/2}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is

Sol. **2**

$f(0) = 1$, $f'(x) = 3x^2 + \frac{1}{2}e^{x/2}$

$\Rightarrow f'(g(x))\, g'(x) = 1$

Put $x = 0 \Rightarrow g'(1) = \frac{1}{f'(0)} = 2$.

37. Let p(x) be a polynomial of degree 4 having extremum at x = 1, 2 and $\lim_{x\to 0}\left(1+\frac{p(x)}{x^2}\right) = 2$. Then the value of p(2) is

Sol. **0**

Let $P(x) = ax^4 + bx^3 + cx^2 + dx + e$

$P'(1) = P'(2) = 0$

$\lim_{x\to 0}\left(\frac{x^2+P(x)}{x^2}\right) = 2$

$\Rightarrow P(0) = 0 \Rightarrow e = 0$

$\lim_{x\to 0}\left(\frac{2x+P'(x)}{2x}\right) = 2$

$\Rightarrow P'(0) = 0 \ \Rightarrow d = 0$

$\lim_{x\to 0}\left(\frac{2+P''(x)}{2}\right) = 2$

$\Rightarrow c = 1$

On solving, a = 1/4, b = – 1

So $P(x) = \frac{x^4}{4} - x^3 + x^2$

$\Rightarrow P(2) = 0.$

38. Let (x, y, z) be points with integer coordinates satisfying the system of homogeneous equations:

$$3x - y - z = 0$$
$$-3x + z = 0$$
$$-3x + 2y + z = 0.$$

Then the number of such points for which $x^2 + y^2 + z^2 \le 100$ is

Sol. **7**

$3x - y - z = 0$

$-3x + z = 0$

$\Rightarrow y = 0$

and $z = 3x$

$\Rightarrow x^2 + y^2 + z^2 = x^2 + z^2 = x^2 + 9x^2 = 10x^2 \le 100$

$\Rightarrow x^2 \le 10$

$\Rightarrow x = 0, \pm 1, \pm 2, \pm 3$

There are such seven points.

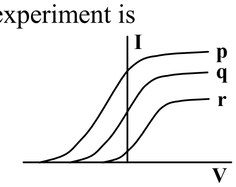
PART III: PHYSICS

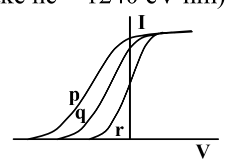
SECTION-I

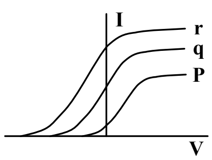
Single Correct Choice Type

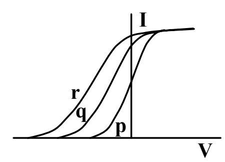
This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out which **ONLY ONE** is correct.

39. Photoelectric effect experiments are performed using three different metal plates p, q and r having work functions $\phi_p = 2.0$ eV, $\phi_q = 2.5$ eV and $\phi_r = 3.0$ eV, respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct I-V graph for the experiment is (Take hc = 1240 eV nm)

(A) 

(B) 

(C) 

(D) 

Sol. **(A)**

$V_B = (1/e)[(hc/\lambda) - \phi]$

$V_P = (1/e)[(1240/550) - 2]eV = 0.2545$ V

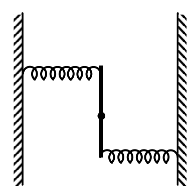
$V_q = (1/e)[(1240/450) - 2.5]eV = 0.255$ V

$V_r = (1/e)[(1240/350) - 3]eV = 0.5428$ V

If n is the number of photons in unit time then $nhc/\lambda = I$

$\Rightarrow \quad i_P : i_q : i_r = n_P : n_q : n_r = \lambda_P : \lambda_q : \lambda_r$

40. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constants k. The springs are fixed to rigid supports as shown in the figure, and rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is



(A) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$

(B) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$

(D) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

Sol. **(C)**

Restoring torque $= -2 \times k\left(\frac{\ell}{2}\theta\right)\frac{\ell}{2} = \frac{Id^2\theta}{dt^2}$

$$\frac{d^2\theta}{dt^2} = \frac{\frac{k\ell^2}{2}(-\theta)}{\frac{M\ell^2}{12}}$$

$$\Rightarrow \omega = \sqrt{\frac{6k}{M}}$$

41. A piece of wire is bent in the shape of a parabola $y = kx^2$ (y-axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x-axis with a constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stays at rest with respect to the wire, from the y-axis is

(A) $\frac{a}{gk}$ (B) $\frac{a}{2gk}$

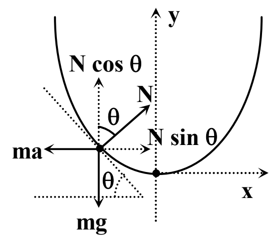
(C) $\frac{2a}{gk}$ (D) $\frac{a}{4gk}$

Sol. (B)

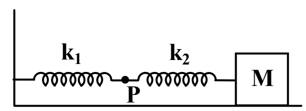
$\tan\theta = \frac{a}{g}$

$\tan\theta = \frac{dy}{dx} = 2kx$

$\Rightarrow \quad x = \frac{a}{2gk}$



42. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is



(A) $\frac{k_1 A}{k_2}$ (B) $\frac{k_2 A}{k_1}$

(C) $\frac{k_1 A}{k_1 + k_2}$ (D) $\frac{k_2 A}{k_1 + k_2}$

Sol. **(D)**

$x_1 + x_2 = A$

$k_1 x_1 = k_2 x_2$

Hence $x_1 = \frac{k_2 A}{k_1 + k_2}$

SECTION -II

Multiple Correct Choice Type

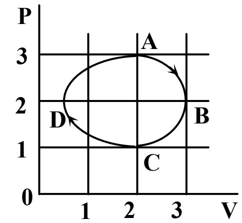
This section contains 5 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

43. Under the influence of the coulomb field of charge +Q, a charge −q is moving around it in an elliptical orbit. Find out the correct statement(s).

(A) The angular momentum of the charge −q is constant
(B) The linear momentum of the charge −q is constant
(C) The angular velocity of the charge −q is constant
(D) The linear speed of the charge −q is constant

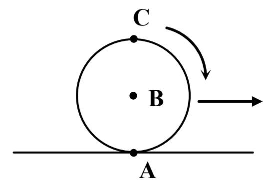
Sol. **(A)**

44. The figure shows the P-V plot of an ideal gas taken through a cycle ABCDA. The part ABC is a semi-circle and CDA is half of an ellipse. Then,
(A) the process during the path $A \rightarrow B$ is isothermal
(B) heat flows out of the gas during the path $B \rightarrow C \rightarrow D$
(C) work done during the path $A \rightarrow B \rightarrow C$ is zero
(D) positive work is done by the gas in the cycle ABCDA



Sol. **(B) & (D)**
$\Delta Q = \Delta U + W$
For process $B \rightarrow C \rightarrow D$
ΔU is negative as well as W is also negative

45. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then,



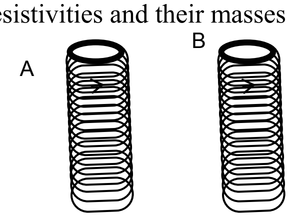
(A) $\vec{V}_C - \vec{V}_A = 2\left(\vec{V}_B - \vec{V}_C\right)$
(B) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$
(C) $\left|\vec{V}_C - \vec{V}_A\right| = 2\left|\vec{V}_B - \vec{V}_C\right|$
(D) $\left|\vec{V}_C - \vec{V}_A\right| = 4\left|\vec{V}_B\right|$

Sol. **(B) & (C)**

46. A student performed the experiment to measure the speed of sound in air using resonance air-column method. Two resonances in the air-column were obtained by lowering the water level. The resonance with the shorter air-column is the first resonance and that with the longer air column is the second resonance. Then,
(A) the intensity of the sound heard at the first resonance was more than that at the second resonance
(B) the prongs of the tuning fork were kept in a horizontal plane above the resonance tube
(C) the amplitude of vibration of the ends of the prongs is typically around 1 cm
(D) the length of the air-column at the first resonance was somewhat shorter than 1/4th of the wavelength of the sound in air

Sol. **(A) & (D)**
Larger the length of air column, feebler is the intensity

47. Two metallic rings A and B, identical in shape and size but having different resistivities ρ_A and ρ_B, are kept on top of two identical solenoids as shown in the figure. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B, respectively, with $h_A > h_B$. The possible relation(s) between their resistivities and their masses m_A and m_B is(are)



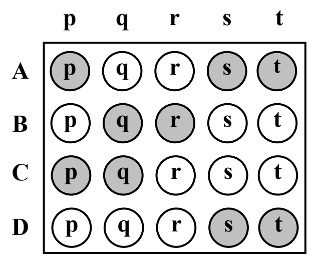
(A) $\rho_A > \rho_B$ and $m_A = m_B$
(B) $\rho_A < \rho_B$ and $m_A = m_B$
(C) $\rho_A > \rho_B$ and $m_A > m_B$
(D) $\rho_A < \rho_B$ and $m_A < m_B$

Sol. **(B) & (D)**
As $-\frac{d\phi}{dt}$ = emf is same, the current induced in the ring will depend upon resistance of the ring. Larger the resistivity smaller the current.

SECTION – III

Matrix-Match Type

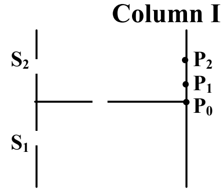
This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II.** The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:
If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s and t; then the correct darkening of bubbles will look like the following.



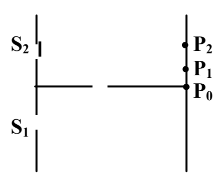
48. **Column I** shows four situations of standard Young's double slit arrangement with the screen placed far away from the slits S_1 and S_2. In each of these cases $S_1P_0 = S_2P_0$, $S_1P_1 - S_2P_1 = \lambda/4$ and $S_1P_2 - S_2P_2 = \lambda/3$, where λ is the wavelength of the light used. In the cases B, C and D, a transparent sheet of refractive index μ and thickness t is pasted on slit S_2. The thicknesses of the sheets are different in different cases. The phase difference between the light waves reaching a point P on the screen from the two slits is denoted by $\delta(P)$ and the intensity by I(P). Match each situation given in **Column I** with the statement(s) in **Column II** valid for that situation.

Column I

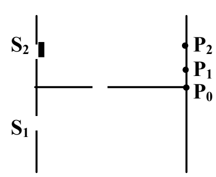
(A)



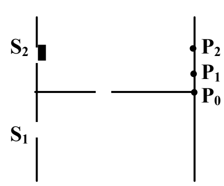
(B) $(\mu - 1)t = \lambda/4$



(C) $(\mu - 1)t = \lambda/2$



(D) $(\mu - 1)t = 3\lambda/4$



Column II

(p) $\delta(P_0) = 0$

(q) $\delta(P_1) = 0$

(r) $I(P_1) = 0$

(s) $I(P_0) > I(P_1)$

(t) $I(P_2) > I(P_1)$

Sol. **A → (p, s); B → (q); C → (t); D → (r, s, t)**

49. **Column II** gives certain systems undergoing a process. **Column I** suggests changes in some of the parameters related to the system. Match the statements in **Column I** to the appropriate process(es) from **Column II.**

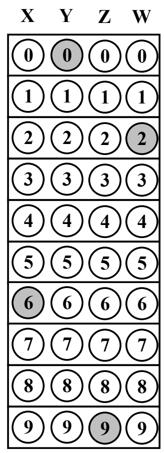
Column I		Column II	
(A) The energy of the system is increased	(p)	System:	A capacitor, initially uncharged
		Process :	It is connected to a battery
(B) Mechanical energy is provided to the system, which is converted into energy of random motion of its parts	(q)	System:	A gas in an adiabatic container fitted with an adiabatic piston
(C) Internal energy of the system is converted in to its mechanical energy		Process:	The gas is compressed by pushing the piston
(D) Mass of the system is decreased	(r)	System:	A gas in a rigid container
		Process:	The gas gets cooled due to colder atmosphere surrounding it
	(s)	System :	A heavy nucleus, initially at rest
		Process:	The nucleus fissions into two fragments of nearly equal masses and some neutrons are emitted
	(t)	System:	A resistive wire loop
		Process:	The loop is placed in a time varying magnetic field perpendicular to its plane

Sol. **A → (p, q, t); B → (q); C → (s); D → (s)**

SECTION –IV

Integer Answer Type

This section contains 8 questions. The answer to each of the questions is a single-digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following



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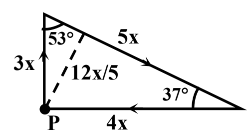
50. A steady current I goes through a wire loop PQR having shape of a right angle triangle with PQ = 3x, PR = 4x and QR = 5x. If the magnitude of the magnetic field at P due to this loop is $k\left(\frac{\mu_0 I}{48\pi x}\right)$, find the value of k.

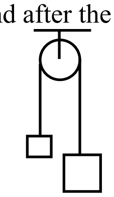
Sol. **7**



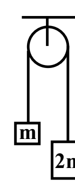
$$B = \frac{\mu_0 I}{4\pi \frac{12x}{5}}[\cos 53° + \cos 37°] = 7\left(\frac{\mu_0 I}{48\pi x}\right)$$

k = 7

51. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking g = 10 m/s^2, find the work done (**in joules**) by the string on the block of mass 0.36 kg during the first second after the system is released from rest.



Sol. **8**



2mg – T = 2ma

T – mg = ma

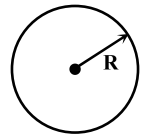
$\Rightarrow$ a = g/3

T = 4mg/3

$W = T.s = T.\frac{1}{2}at^2$ = 8 Joules

52. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = kr^a$, where k and a are constants and r is the distance from its centre. If the electric field at r = R/2 is 1/8 times that at r = R, find the value of a.

Sol. **2**



$\rho = kr^a$

$E\left(r = \frac{R}{2}\right) = \frac{1}{8}E(r = R)$

$\frac{q_{enclosed}}{4\pi\varepsilon_0 (R/2)^2} = \frac{1}{8}\frac{Q}{4\pi\varepsilon_0 R^2}$

$32q_{enclosed} = Q$

$q_{enclosed} = \int_0^{R/2} kr^a 4\pi r^2 dr = \frac{4\pi k}{(a+3)}\left(\frac{R}{2}\right)^{(a+3)}$

$Q = \frac{4\pi k}{(a+3)}R^{(a+3)}$

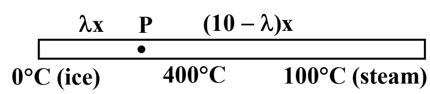
$\frac{Q}{q_{enclosed}} = 2^{a+3}$

$2^{a+3} = 32$

$a = 2$

53. A metal rod AB of length 10x has its one end A in ice at 0°C and the other end B in water at 100°C. If a point P on the rod is maintained at 400°C, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 cal/g and latent heat of melting of ice is 80 cal/g. If the point P is at a distance of λx from the ice end A, find the value of λ. [Neglect any heat loss to the surrounding.]

Sol. **9**



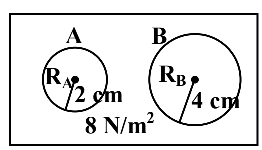
$\frac{dm_{ice}}{dt} = \frac{dm_{vapour}}{dt}$

$\frac{400kS}{\lambda xL_{ice}} = \frac{300kS}{(100-\lambda)xL_{vapour}}$

$\lambda = 9$

54. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 N/m^2. The radii of bubbles A and B are 2 cm and 4 cm, respectively. Surface tension of the soap-water used to make bubbles is 0.04 N/m. Find the ratio n_B/n_A, where n_A and n_B are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity.]

Sol. **6**



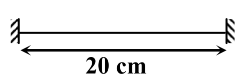
$P_A = P_0 + \frac{4T}{R_A} = 16\ N/m^2$

$P_B = P_0 + \frac{4T}{R_B} = 12\ N/m^2$

$\frac{n_B}{n_A} = \frac{P_B}{P_A}\left(\frac{R_B}{R_A}\right)^3 = 6$

55. A 20 cm long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation (**in cm**) between the successive nodes on the string.

Sol. **5**

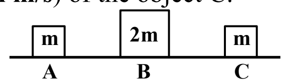


$v = \sqrt{\frac{T}{\mu}} = 10\ m/s$

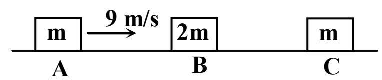
$\lambda = \dfrac{v}{f} = \dfrac{10}{100} = 10$ cm

Distance between the successive nodes = $\lambda/2 = 5$ cm

56. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, 2m and m, respectively. The object A moves towards B with a speed 9 m/s and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (**in m/s**) of the object C.



Sol. 4



After 1st collision

$mv_A = mv'_A + 2mv'_B$

$-1 = \dfrac{v'_B - v'_A}{0 - v_A} \Rightarrow v'_B = 6$ m/s

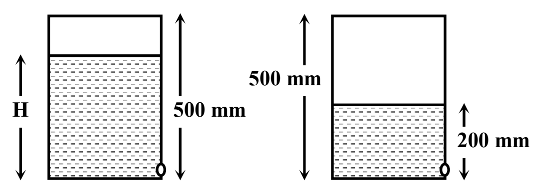
After 2nd collision

$2mv'_B = (2m + m)v_C \Rightarrow \quad v_C = \dfrac{2}{3}v'_B \Rightarrow v_C = 4$ m/s

57. A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it upto height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (**in mm**) of water level due to opening of the orifice.

[Take atmospheric pressure = 1.0×10^5 N/m^2, density of water = 1000 kg/m^3 and g = 10 m/s^2. Neglect any effect of surface tension.]

Sol. **6**



$P = P_0 - \rho gh = 98 \times 10^3$ N/m^2

$P_0V_0 = PV$

$10^5[A(500 - H)] = 98 \times 10^3[A(500 - 200)]$

H = 206 mm

Level fall = 206 − 200 = 6 mm