

Mathematics

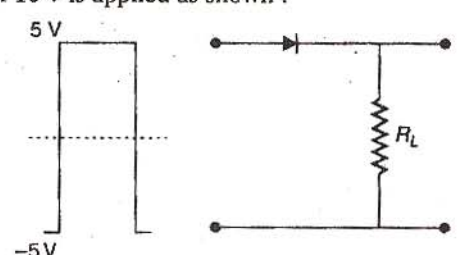
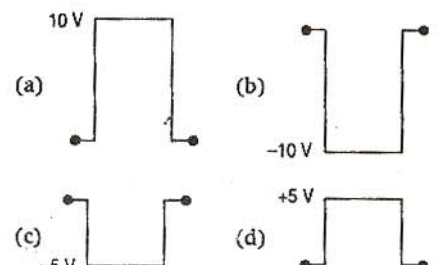
- In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals :
 (a) $\frac{1}{2}(1 - \sqrt{5})$ (b) $\frac{1}{2}\sqrt{5}$
 (c) $\sqrt{5}$ (d) $\frac{1}{2}(\sqrt{5} - 1)$
- If $\sin^{-1}\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, then a value of x is :
 (a) 1 (b) 3
 (c) 4 (d) 5
- In the binomial expansion of $(a - b)^n$, $n \geq 5$, the sum of 5th and 6th terms is zero, then $\frac{a}{b}$ equals :
 (a) $\frac{5}{n-4}$ (b) $\frac{6}{n-5}$
 (c) $\frac{n-5}{6}$ (d) $\frac{n-4}{5}$
- The set $S := \{1, 2, 3, \dots, 12\}$ is to be partitioned into three sets A, B, C of equal size. Thus, $A \cup B \cup C = S$, $A \cap B = B \cap C = A \cap C = \emptyset$. The number of ways to partition S is :
 (a) $12!/3!(4!)^3$ (b) $12!/3!(3!)^4$
 (c) $12!/(4!)^3$ (d) $12!/(3!)^4$
- The largest interval lying in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ for which the function

$$f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$$
 is defined, is :
 (a) $[0, \pi]$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
 (c) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right]$ (d) $\left[0, \frac{\pi}{2}\right]$
- A body weighing 13 kg is suspended by two strings 5 m and 12 m long, their other ends being fastened to the extremities of a rod 13 m long. If the rod be so held that the body hangs immediately below the middle point. The tensions in the strings are :
 (a) 12 kg and 13 kg (b) 5 kg and 5 kg
 (c) 5 kg and 12 kg (d) 5 kg and 13 kg
- A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is :
 (a) $1/729$ (b) $8/9$
 (c) $8/729$ (d) $8/243$
- Consider a family of circles which are passing through the point $(-1, 1)$ and are tangent to x -axis. If (h, k) are the coordinates of the centre of the circles, then the set of values of k is given by the interval :
 (a) $0 < k < 1/2$ (b) $k \geq 1/2$
 (c) $-1/2 \leq k \leq 1/2$ (d) $k \leq 1/2$
- Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angle α with the positive x -axis, then $\cos \alpha$ equals :
 (a) $1/\sqrt{3}$ (b) $1/2$
 (c) 1 (d) $1/\sqrt{2}$
- The differential equation of all circles passing through the origin and having their centres on the x -axis is :
 (a) $x^2 = y^2 + xy \frac{dy}{dx}$
 (b) $x^2 = y^2 + 3xy \frac{dy}{dx}$
 (c) $y^2 = x^2 + 2xy \frac{dy}{dx}$
 (d) $y^2 = x^2 - 2xy \frac{dy}{dx}$

11. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of $(p + q)$ is :
 (a) 2 (b) $\frac{1}{2}$
 (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$
12. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that $AB (= a)$ subtends an angle of 60° at the foot of the tower and the angle of elevation of the top of the tower from A or B is 30° . The height of the tower is :
 (a) $2a/\sqrt{3}$ (b) $2a\sqrt{3}$
 (c) $a/\sqrt{3}$ (d) $a\sqrt{3}$
13. The sum of the series ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \dots - \dots + {}^{20}C_{10}$ is :
 (a) $-{}^{20}C_{10}$ (b) $\frac{1}{2} {}^{20}C_{10}$
 (c) 0 (d) ${}^{20}C_{10}$
14. The normal to a curve at $P(x, y)$ meets the x -axis at G . If the distance of G from the origin is twice the abscissa of P , then the curve is a :
 (a) ellipse (b) parabola
 (c) circle (d) hyperbola
15. If $|z + 4| \leq 3$, then the maximum value of $|z + 1|$ is :
 (a) 4 (b) 10
 (c) 6 (d) 0
16. The resultant of two forces P N and 3 N is a force of 7 N. If the direction of the 3 N force were reversed, the resultant would be $\sqrt{19}$ N. The value of P is :
 (a) 5 N (b) 6 N
 (c) 3 N (d) 4 N
17. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2 , respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is :
 (a) 0.06 (b) 0.14
 (c) 0.2 (d) 0.7
18. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$, then D is :
 (a) divisible by neither x nor y
 (b) divisible by both x and y
 (c) divisible by x but not y
 (d) divisible by y but not x
19. For the Hyperbola $\frac{x^2}{\cos^2 \alpha} - \frac{y^2}{\sin^2 \alpha} = 1$, which of the following remains constant when α varies ?
 (a) Eccentricity
 (b) Directrix
 (c) Abscissae of vertices
 (d) Abscissae of foci
20. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of each of x -axis and y -axis, then the angle that the line makes with the positive direction of the z -axis is :
 (a) $\pi/6$ (b) $\pi/3$
 (c) $\pi/4$ (d) $\pi/2$
21. A value of C for which the conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval $[1, 3]$ is :
 (a) $2 \log_3 e$ (b) $\frac{1}{2} \log_e 3$
 (c) $\log_3 e$ (d) $\log_e 3$
22. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in :
 (a) $(\pi/4, \pi/2)$ (b) $(-\pi/2, \pi/4)$
 (c) $(0, \pi/2)$ (d) $(-\pi/2, \pi/2)$
23. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$
 If $|A^2| = 25$, then $|\alpha|$ equals :
 (a) 5^2 (b) 1
 (c) $\frac{1}{5}$ (d) 5
24. The sum of the series $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots$ upto infinity is :
 (a) e^{-2} (b) e^{-1}
 (c) $e^{-\frac{1}{2}}$ (d) $e^{+\frac{1}{2}}$
25. If \hat{u} and \hat{v} are unit vectors and θ is the acute angle between them, then $2\hat{u} \times 3\hat{v}$ is a unit vector for :
 (a) exactly two values of θ
 (b) more than two values of θ
 (c) no value of θ
 (d) exactly one value of θ
26. A particle just clears a wall of height b at a distance a and strikes the ground at a distance c from the point of projection. The angle of projection is :

- (a) $\tan^{-1} \frac{b}{ac}$ (b) 45°
 (c) $\tan^{-1} \frac{bc}{a(c-a)}$ (d) $\tan^{-1} \frac{bc}{a}$
27. The average marks of boys in a class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is :
 (a) 40 (b) 20
 (c) 80 (d) 60
28. The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is :
 (a) $(-1, 1)$ (b) $(0, 2)$
 (c) $(2, 4)$ (d) $(-2, 0)$
29. If $(2, 3, 5)$ is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the coordinates of the other end of the diameter are :
 (a) $(4, 9, -3)$ (b) $(4, -3, 3)$
 (c) $(4, 3, 5)$ (d) $(4, 3, -3)$
30. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vector \vec{c} lies in the plane of \vec{a} and \vec{b} , then x equals :
 (a) 0 (b) 1
 (c) -4 (d) -2
31. Let $A(h, k)$, $B(1, 1)$ and $C(2, 1)$ be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1, then the set of values which 'k' can take is given by :
 (a) $\{1, 3\}$ (b) $\{0, 2\}$
 (c) $\{-1, 3\}$ (d) $\{-3, -2\}$
32. Let $P = (-1, 0)$, $Q = (0, 0)$ and $R = (3, 3\sqrt{3})$ be three points. The equation of the bisector of the angle PQR is :
 (a) $\sqrt{3}x + y = 0$ (b) $x + \frac{\sqrt{3}}{2}y = 0$
 (c) $\frac{\sqrt{3}}{2}x + y = 0$ (d) $x + \sqrt{3}y = 0$
33. If one of the lines of $my^2 + (1-m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines $xy = 0$, then m is :
 (a) $-\frac{1}{2}$ (b) -2
 (c) ± 1 (d) 2
34. Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where $f(x) = \int_1^x \frac{\log t}{1+t} dt$. Then $F(e)$ equals :
 (a) $\frac{1}{2}$ (b) 0
 (c) 1 (d) 2
35. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \min\{x+1, |x|+1\}$. Then which of the following is true ?
 (a) $f(x) \geq 1$ for all $x \in \mathbb{R}$
 (b) $f(x)$ is not differentiable at $x = 1$
 (c) $f(x)$ is differentiable everywhere
 (d) $f(x)$ is not differentiable at $x = 0$
36. The function $f: \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R}$ given by $f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$ can be made continuous at $x = 0$ by defining $f(0)$ as :
 (a) 2 (b) -1
 (c) 0 (d) 1
37. The solution for x of the equation $\int_{\sqrt{2}}^x \frac{dt}{t\sqrt{t^2-1}} = \pi/2$ is :
 (a) 2 (b) π
 (c) $\sqrt{3}/2$ (d) $2\sqrt{2}$
38. $\int \frac{dx}{\cos x + \sqrt{3} \sin x}$ equals :
 (a) $\frac{1}{2} \log \tan \left(\frac{x}{2} + \frac{\pi}{12} \right) + c$
 (b) $\frac{1}{2} \log \tan \left(\frac{x}{2} - \frac{\pi}{12} \right) + c$
 (c) $\log \tan \left(\frac{x}{2} + \frac{\pi}{12} \right) + c$
 (d) $\log \tan \left(\frac{x}{2} - \frac{\pi}{12} \right) + c$
39. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is :
 (a) $\frac{2}{3}$ (b) 1
 (c) $\frac{1}{6}$ (d) $\frac{1}{3}$
40. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is :
 (a) $(-3, 3)$ (b) $(-3, \infty)$
 (c) $(3, \infty)$ (d) $(-\infty, -3)$

Physics

41. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos \pi t$ metre. The time at which the maximum speed first occurs is :
 (a) 0.5 s (b) 0.75 s
 (c) 0.125 s (d) 0.25 s
42. In an AC circuit the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin \left(\omega t - \frac{\pi}{2} \right)$. The power consumption in the circuit is given by :
 (a) $P = \frac{E_0 I_0}{\sqrt{2}}$ (b) $P = \text{zero}$
 (c) $P = \frac{E_0 I_0}{2}$ (d) $P = \sqrt{2} E_0 I_0$
43. An electric charge $10^{-3} \mu\text{C}$ is placed at the origin (0, 0) of X-Y coordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and (2, 0) respectively. The potential difference between the points A and B will be :
 (a) 9 V (b) zero
 (c) 2 V (d) 4.5 V
44. A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and the work done by the battery will be :
 (a) 1 (b) 2
 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$
45. An ideal coil of 10 H is connected in series with a resistance of 5Ω and a battery of 5 V. 2 s after the connection is made, the current flowing (in ampere) in the circuit is :
 (a) $(1 - e)$ (b) e
 (c) e^{-1} (d) $(1 - e^{-1})$
46. A long straight wire of radius a carries a steady current i . The current is uniformly distributed across its cross-section. The ratio of the magnetic field at $\frac{a}{2}$ and $2a$ is :
 (a) $\frac{1}{4}$ (b) 4
 (c) 1 (d) $\frac{1}{2}$
47. A current I flows along the length of an infinitely long, straight, thin walled pipe. Then :
 (a) the magnetic field is zero only on the axis of the pipe
 (b) the magnetic field is different at different points inside the pipe
 (c) the magnetic field at any point inside the pipe is zero
 (d) the magnetic field at all points inside the pipe is the same, but not zero
48. If M_o is the mass of an oxygen isotope ${}_8\text{O}^{17}$, M_p and M_n are the masses of a proton and a neutron, respectively, the nuclear binding energy of the isotope is :
 (a) $(M_o - 8M_p)c^2$
 (b) $(M_o - 8M_p - 9M_n)c^2$
 (c) $M_o c^2$
 (d) $(M_o - 17M_n)c^2$
49. In γ ray emission from a nucleus :
 (a) both the neutron number and the proton number change
 (b) there is no change in the proton number and the neutron number
 (c) only the neutron number changes
 (d) only the proton number changes
50. If in a p-n junction diode, a square input signal of 10 V is applied as shown :

 Then the output signal across R_L will be :


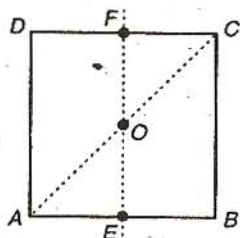
51. Photon of frequency ν has a momentum associated with it. If c is the velocity of light, the momentum is :

(a) ν/c (b) $h\nu c$
(c) $h\nu/c^2$ (d) $h\nu/c$

52. The velocity of a particle is $\nu = \nu_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time ($t = 1$) is :

(a) $\nu_0 + 2g + 3f$ (b) $\nu_0 + g/2 + f/3$
(c) $\nu_0 + g + f$ (d) $\nu_0 + g/2 + f$

53. For the given uniform square lamina ABCD, whose centre is O :



(a) $\sqrt{2}I_{AC} = I_{EF}$ (b) $I_{AD} = 3I_{EF}$
(c) $I_{AC} = I_{EF}$ (d) $I_{AC} = \sqrt{2}I_{EF}$

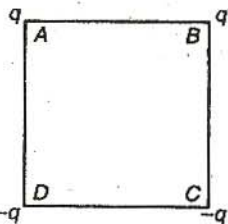
54. A point mass oscillates along the x -axis according to the law $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as :

$a = A \cos(\omega t + \delta)$, then :

(a) $A = x_0, \delta = -\pi/4$
(b) $A = x_0\omega^2, \delta = \pi/4$
(c) $A = x_0\omega^2, \delta = -\pi/4$
(d) $A = x_0\omega^2, \delta = 3\pi/4$

55. Charges are placed on the vertices of a square

as shown. Let \vec{E} be the electric field and V the potential at the centre. If the charges on A and B are interchanged with those on D and C respectively, then :



(a) \vec{E} remains unchanged, V changes
(b) both \vec{E} and V change
(c) \vec{E} and V remain unchanged
(d) \vec{E} changes, V remains unchanged

56. The half-life period of a radioactive element X is same as the mean life time of another radioactive element Y. Initially they have the same number of atoms. Then :

(a) X will decay faster than Y
(b) Y will decay faster than X
(c) Y and X have same decay rate initially
(d) X and Y decay at same rate always

57. A Carnot engine, having an efficiency of $\eta = 1/10$ as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is :

(a) 99 J (b) 90 J
(c) 1 J (d) 100 J

58. Carbon, silicon and germanium have four valence electrons each. At room temperature which one of the following statements is most appropriate ?

(a) The number of free conduction electrons is significant in C but small in Si and Ge
(b) The number of free conduction electrons is negligibly small in all the three
(c) The number of free electrons for conduction is significant in all the three
(d) The number of free electrons for conduction is significant only in Si and Ge but small in C

59. A charged particle with charge q enters a region of constant, uniform and mutually orthogonal fields \vec{E} and \vec{B} with a velocity \vec{v} perpendicular to both \vec{E} and \vec{B} , and comes out without any change in magnitude or direction

of \vec{v} . Then :

(a) $\vec{v} = \vec{E} \times \vec{B} / B^2$ (b) $\vec{v} = \vec{B} \times \vec{E} / B^2$
(c) $\vec{v} = \vec{E} \times \vec{B} / E^2$ (d) $\vec{v} = \vec{B} \times \vec{E} / E^2$

60. The potential at a point x (measured in μm) due to some charges situated on the x -axis is given by : $V(x) = 20 / (x^2 - 4)$ volt

The electric field E at $x = 4 \mu\text{m}$ is given by :

(a) $\frac{5}{3} \text{ V}/\mu\text{m}$ and in the $-ve$ x direction
(b) $\frac{5}{3} \text{ V}/\mu\text{m}$ and in the $+ve$ x direction
(c) $\frac{10}{9} \text{ V}/\mu\text{m}$ and in the $-ve$ x direction
(d) $\frac{10}{9} \text{ V}/\mu\text{m}$ and in the $+ve$ x direction

61. Which of the following transitions in hydrogen atoms emit photons of highest frequency ?

(a) $n = 2$ to $n = 6$ (b) $n = 6$ to $n = 2$
(c) $n = 2$ to $n = 1$ (d) $n = 1$ to $n = 2$

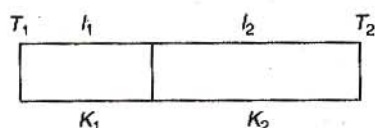
62. A block of mass m is connected to another block of mass M by a spring (massless) of spring constant k . The blocks are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is unstretched. Then a constant force F starts acting on the block of mass M to pull it. Find the force on the block of mass m .

(a) $\frac{mF}{M}$ (b) $\frac{(M+m)F}{m}$
(c) $\frac{mF}{(m+M)}$ (d) $\frac{MF}{(m+M)}$

63. Two lenses of power $-15D$ and $+5D$ are in contact with each other. The focal length of the combination is :

(a) -20 cm (b) -10 cm
(c) $+20$ cm (d) $+10$ cm

64. One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2 . The rod is composed of two sections of lengths l_1 and l_2 and thermal conductivities K_1 and K_2 respectively. The temperature at the interface of the two sections is



(a) $(K_2 l_2 T_1 + K_1 l_1 T_2) / (K_1 l_1 + K_2 l_2)$
(b) $(K_2 l_1 T_1 + K_1 l_2 T_2) / (K_2 l_1 + K_1 l_2)$
(c) $(K_1 l_2 T_1 + K_2 l_1 T_2) / (K_1 l_2 + K_2 l_1)$
(d) $(K_1 l_1 T_1 + K_2 l_2 T_2) / (K_1 l_1 + K_2 l_2)$

65. A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of :

(a) 1000 (b) 10000
(c) 10 (d) 100

66. If C_p and C_v denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then :

(a) $C_p - C_v = R/28$ (b) $C_p - C_v = R/14$
(c) $C_p - C_v = R$ (d) $C_p - C_v = 28R$

67. A charged particle moves through a magnetic field perpendicular to its direction. Then :

- (a) the momentum changes but the kinetic energy is constant
(b) both momentum and kinetic energy of the particle are not constant
(c) both momentum and kinetic energy of the particle are constant
(d) kinetic energy changes but the momentum is constant

68. Two identical conducting wires AOB and COD are placed at right angles to each other. The wire AOB carries an electric current I_1 and COD carries a current I_2 . The magnetic field on a point lying at a distance d from O, in a direction perpendicular to the plane of the wires AOB and COD, will be given by :

(a) $\frac{\mu_0}{2\pi} \left(\frac{I_1 + I_2}{d} \right)^{1/2}$ (b) $\frac{\mu_0}{2\pi d} (I_1^2 + I_2^2)^{1/2}$
(c) $\frac{\mu_0}{2\pi d} (I_1 + I_2)$ (d) $\frac{\mu_0}{2\pi d} (I_1^2 + I_2^2)$

69. The resistance of a wire is 5Ω at 50°C and 6Ω at 100°C . The resistance of the wire at 0°C will be :

(a) 2Ω (b) 1Ω
(c) 4Ω (d) 3Ω

70. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volts. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is :

(a) $\frac{1}{2}(K-1)CV^2$ (b) $CV^2(K-1)/K$
(c) $(K-1)CV^2$ (d) zero

71. If g_E and g_M are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio electronic charge on the moon to be :

(a) 1 (b) zero
(c) g_E/g_M (d) g_M/g_E

72. A circular disc of radius R is removed from a bigger circular disc of radius $2R$, such that the circumference of the discs coincide. The centre of mass of the new disc is $\frac{\alpha}{R}$ from the centre of the bigger disc. The value of α is :

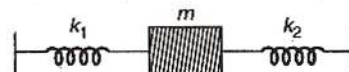
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$
(c) $\frac{1}{6}$ (d) $\frac{1}{4}$

73. A round uniform body of radius R , mass M and moment of inertia I , rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then its acceleration is :

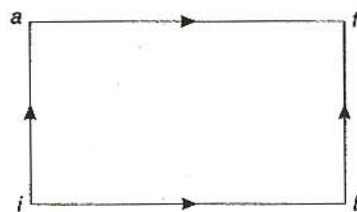
(a) $\frac{g \sin \theta}{1 + I/MR^2}$ (b) $\frac{g \sin \theta}{1 + MR^2/I}$
(c) $\frac{g \sin \theta}{1 - I/MR^2}$ (d) $\frac{g \sin \theta}{1 - MR^2/I}$

74. Angular momentum of the particle rotating with a central force is constant due to :
 (a) constant force
 (b) constant linear momentum
 (c) zero torque
 (d) constant torque
75. A 2 kg block slides on a horizontal floor with a speed of 4 m/s. It strikes a uncompressed spring, and compresses it till the block is motionless. The kinetic friction force is 15 N and spring constant is 10,000 N/m. The spring compresses by :
 (a) 5.5 cm (b) 2.5 cm
 (c) 11.0 cm (d) 8.5 cm
76. A particle is projected at 60° to the horizontal with a kinetic energy K . The kinetic energy at the highest point is :
 (a) K (b) zero
 (c) $K/4$ (d) $K/2$
77. In a Young's double slit experiment the intensity at a point where the path difference is $\frac{\lambda}{6}$ (λ being the wavelength of the light used) is I . If I_0 denotes the maximum intensity, I/I_0 is equal to :
 (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$
 (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

78. Two springs, of force constants k_1 and k_2 , are connected to a mass m as shown. The frequency of oscillation of the mass is f . If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes :
 (a) $f/2$ (b) $f/4$
 (c) $4f$ (d) $2f$



79. When a system is taken from state i to state f along the path iaf , it is found that $Q = 50$ cal and $W = 20$ cal. Along the path ibf $Q = 36$ cal. W along the path ibf is :
 (a) 6 cal (b) 16 cal
 (c) 66 cal (d) 14 cal



80. A particle of mass m executes simple harmonic motion with amplitude a and frequency ν . The average kinetic energy during its motion from the position of equilibrium to the end is :
 (a) $\pi^2 m a^2 \nu^2$ (b) $\frac{1}{4} m a^2 \nu^2$
 (c) $4\pi^2 m a^2 \nu^2$ (d) $2\pi^2 m a^2 \nu^2$

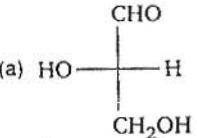
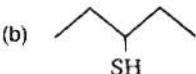
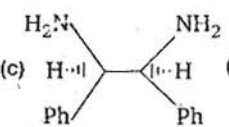
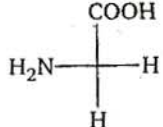
Chemistry

81. The energies of activation for forward and reverse reactions for $A_2 + B_2 \rightleftharpoons 2AB$ are 180 kJ mol^{-1} and 200 kJ mol^{-1} respectively. The presence of a catalyst lowers the activation energy of both (forward and reverse) reactions by 100 kJ mol^{-1} . The enthalpy change of the reaction ($A_2 + B_2 \rightarrow 2AB$) in the presence of catalyst will be (in kJ mol^{-1}) :
 (a) 300 (b) 120
 (c) 280 (d) 20
82. The cell, $\text{Zn} | \text{Zn}^{2+} (1\text{M}) || \text{Cu}^{2+} (1\text{M}) | \text{Cu}$ ($E^\circ_{\text{cell}} = 1.10 \text{ V}$), was allowed to be completely discharged at 298 K. The relative concentration of Zn^{2+} to Cu^{2+} $\left(\frac{[\text{Zn}^{2+}]}{[\text{Cu}^{2+}]} \right)$ is :

- (a) antilog (24.08) (b) 37.3
 (c) $10^{37.3}$ (d) 9.65×10^4

83. The pK_a of a weak acid (HA) is 4.5. The pOH of an aqueous buffered solution of HA in which 50% of the acid ionised is :
 (a) 4.5 (b) 2.5
 (c) 9.5 (d) 7.0
84. Consider the reaction,
 $2A + B \rightarrow \text{products}$

When concentration of B alone was doubled, the half-life did not change. When the concentration of A alone was doubled, the rate increased by two times. The unit of rate constant for this reaction is :

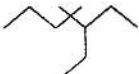
- (a) $\text{L mol}^{-1}\text{s}^{-1}$ (b) no unit
(c) $\text{mol L}^{-1}\text{s}^{-1}$ (d) s^{-1}
85. Identify the incorrect statement among the following :
(a) d-block elements show irregular and erratic chemical properties among themselves
(b) La and Lu have partially filled d orbitals and no other partially filled orbitals
(c) The chemistry of various lanthanoids is very similar
(d) 4f and 5f orbitals are equally shielded
86. Which one of the following has a square planar geometry ?
(a) $[\text{CoCl}_4]^{2-}$ (b) $[\text{FeCl}_4]^{2-}$
(c) $[\text{NiCl}_4]^{2-}$ (d) $[\text{PtCl}_4]^{2-}$
(At. no. Co = 27, Ni = 28, Fe = 26, Pt = 78)
87. Which of the following molecules is expected to rotate the plane of plane-polarised light ?
(a)  (b) 
(c)  (d) 
88. The secondary structure of a protein refers to :
(a) α -helical backbone
(b) hydrophobic interactions
(c) sequence of α -amino acids
(d) fixed configuration of the polypeptide backbone
89. Which of the following reactions will yield, 2, 2-dibromopropane ?
(a) $\text{CH}_3-\text{C}\equiv\text{CH} + 2\text{HBr} \longrightarrow$
(b) $\text{CH}_3\text{CH}=\text{CHBr} + \text{HBr} \longrightarrow$
(c) $\text{CH}\equiv\text{CH} + 2\text{HBr} \longrightarrow$
(d) $\text{CH}_3-\text{CH}=\text{CH}_2 + \text{HBr} \longrightarrow$
90. In the chemical reaction,
 $\text{CH}_3\text{CH}_2\text{NH}_2 + \text{CHCl}_3 + 3\text{KOH} \longrightarrow$
(A) + (B) + $3\text{H}_2\text{O}$, the compounds (A) and (B) are respectively :
(a) $\text{C}_2\text{H}_5\text{CN}$ and 3KCl
(b) $\text{CH}_3\text{CH}_2\text{CONH}_2$ and 3KCl
(c) $\text{C}_2\text{H}_5\text{NC}$ and K_2CO_3
(d) $\text{C}_2\text{H}_5\text{NC}$ and 3KCl
91. The reaction of toluene with Cl_2 in presence of FeCl_3 gives predominantly :
(a) benzoyl chloride
(b) benzyl chloride
(c) o-and p-chlorotoluene
(d) m-chlorotoluene
92. Presence of a nitro group in a benzene ring :
(a) activates the ring towards electrophilic substitution
(b) renders the ring basic
(c) deactivates the ring towards nucleophilic substitution
(d) deactivates the ring towards electrophilic substitution
93. In which of the following ionisation processes, the bond order has increased and the magnetic behaviour has changed ?
(a) $\text{C}_2 \longrightarrow \text{C}_2^+$ (b) $\text{NO} \longrightarrow \text{NO}^+$
(c) $\text{O}_2 \longrightarrow \text{O}_2^+$ (d) $\text{N}_2 \longrightarrow \text{N}_2^+$
94. The actinoids exhibit more number of oxidation states in general than the lanthanoids. This is because :
(a) the 5f orbitals are more buried than the 4f orbitals
(b) there is a similarity between 4f and 5f orbitals in their angular part of the wave function
(c) the actinoids are more reactive than the lanthanoids
(d) the 5f orbitals extend further from the nucleus than the 4f orbitals
95. Equal masses of methane and oxygen are mixed in an empty container at 25°C . The fraction of the total pressure exerted by oxygen is :
(a) $\frac{2}{3}$ (b) $\frac{1}{3} \times \frac{273}{298}$
(c) $\frac{1}{3}$ (d) $\frac{1}{2}$
96. A 5.25% solution of a substance is isotonic with a 1.5% solution of urea (molar mass = 60 g mol^{-1}) in the same solvent. If the densities of both the solutions are assumed to be equal to 1.0 g cm^{-3} , molar mass of the substance will be :
(a) 90.0 g mol^{-1} (b) 115.0 g mol^{-1}
(c) 105.0 g mol^{-1} (d) 210.0 g mol^{-1}
97. Assuming that water vapour is an ideal gas, the internal energy change (ΔU) when 1 mole of water is vapourised at 1 bar pressure and 100°C , (Given : molar enthalpy of vaporisation of water at 1 bar and $373 \text{ K} = 41 \text{ kJ mol}^{-1}$ and $R = 8.3 \text{ J mol}^{-1}\text{K}^{-1}$) will be :

- (a) $4.100 \text{ kJ mol}^{-1}$ (b) $3.7904 \text{ kJ mol}^{-1}$
(c) $37.904 \text{ kJ mol}^{-1}$ (d) $41.00 \text{ kJ mol}^{-1}$
- 98.** In a saturated solution of the sparingly soluble strong electrolyte AgIO_3 (Molecular mass = 283) the equilibrium which sets in is

$$\text{AgIO}_3(s) \rightleftharpoons \text{Ag}^+(aq) + \text{IO}_3^-(aq)$$
 if the solubility product constant K_{sp} of AgIO_3 at a given temperature is 1.0×10^{-8} , what is the mass of AgIO_3 contained in 100 mL of its saturated solution?
 (a) $28.3 \times 10^{-2} \text{ g}$ (b) $2.83 \times 10^{-3} \text{ g}$
 (c) $1.0 \times 10^{-7} \text{ g}$ (d) $1.0 \times 10^{-4} \text{ g}$
- 99.** A radioactive element gets spilled over the floor of a room. Its half-life period is 30 days. If the initial activity is ten times the permissible value, after how many days will it be safe to enter the room?
 (a) 1000 days (b) 300 days
 (c) 10 days (d) 100 days
- 100.** Which one of the following conformations of cyclohexane is chiral?
 (a) Twist boat (b) Rigid
 (c) Chair (d) Boat
- 101.** Which of the following is the correct order of decreasing S_N2 reactivity?
 (a) $\text{RCH}_2\text{X} > \text{R}_3\text{CX} > \text{R}_2\text{CHX}$
 (b) $\text{RCH}_2\text{X} > \text{R}_2\text{CHX} > \text{R}_3\text{CX}$
 (c) $\text{R}_3\text{CX} > \text{R}_2\text{CHX} > \text{RCH}_2\text{X}$
 (d) $\text{R}_2\text{CHX} > \text{R}_3\text{CX} > \text{RCH}_2\text{X}$
 ($\text{X} = \text{a halogen}$)
- 102.** In the following sequence of reactions,

$$\text{CH}_3\text{CH}_2\text{OH} \xrightarrow{\text{P} + \text{I}_2} \text{A} \xrightarrow[\text{ether}]{\text{Mg}} \text{B} \xrightarrow{\text{HCHO}} \text{C} \xrightarrow{\text{H}_2\text{O}} \text{D}$$
 the compound 'D' is:
 (a) butanal (b) *n*-butyl alcohol
 (c) *n*-propyl alcohol (d) propanal
- 103.** Which of the following sets of quantum numbers represents the highest energy of an atom?
 (a) $n = 3, l = 1, m = 1, s = +1/2$
 (b) $n = 3, l = 2, m = 1, s = +1/2$
 (c) $n = 4, l = 0, m = 0, s = +1/2$
 (d) $n = 3, l = 0, m = 0, s = +1/2$
- 104.** Which of the following hydrogen bonds is the strongest?
 (a) $\text{O} \cdots \text{H} \cdots \text{N}$ (b) $\text{F} \cdots \text{H} \cdots \text{F}$
 (c) $\text{O} \cdots \text{H} \cdots \text{O}$ (d) $\text{O} \cdots \text{H} \cdots \text{F}$
- 105.** In the reaction,

$$2\text{Al}(s) + 6\text{HCl}(aq) \rightarrow 2\text{Al}^{3+}(aq) + 6\text{Cl}^-(aq) + 3\text{H}_2(g)$$
 (a) 6 L $\text{HCl}(aq)$ is consumed for every 3 L $\text{H}_2(g)$ produced
 (b) 33.6 L $\text{H}_2(g)$ is produced regardless of temperature and pressure for every mole Al that reacts
 (c) 67.2 L $\text{H}_2(g)$ at STP is produced for every mole Al that reacts
 (d) 11.2 L $\text{H}_2(g)$ at STP is produced for every mole $\text{HCl}(aq)$ consumed
- 106.** Regular use of which of the following fertilisers increases the acidity of soil?
 (a) Potassium nitrate
 (b) Urea
 (c) Superphosphate of lime
 (d) Ammonium sulphate
- 107.** Identify the correct statement regarding a spontaneous process:
 (a) For a spontaneous process in an isolated system, the change in entropy is positive
 (b) Endothermic processes are never spontaneous
 (c) Exothermic processes are always spontaneous
 (d) Lowering of energy in the reaction process is the only criterion for spontaneity
- 108.** Which of the following nuclear reactions will generate an isotope?
 (a) Neutron particle emission
 (b) Positron emission
 (c) α -particle emission
 (d) β -particle emission
- 109.** The equivalent conductances of two strong electrolytes at infinite dilution in H_2O (where ions move freely through a solution) at 25°C are given below:
 $\Lambda^\circ_{\text{CH}_3\text{COONa}} = 91.0 \text{ S cm}^2/\text{equiv}$
 $\Lambda^\circ_{\text{HCl}} = 426.2 \text{ S cm}^2/\text{equiv}$
 What additional information/quantity one needs to calculate Λ° of an aqueous solution of acetic acid?
 (a) Λ° of NaCl
 (b) Λ° of CH_3COOK
 (c) The limiting equivalent conductance of H^+ ($\Lambda^\circ_{\text{H}^+}$)
 (d) Λ° of chloroacetic acid (ClCH_2COOH)
- 110.** Which one of the following is the strongest base in aqueous solution?

- (a) Trimethylamine (b) Aniline
(c) Dimethylamine (d) Methylamine
111. The compound formed as a result of oxidation of ethyl benzene by KMnO_4 is :
(a) benzophenone (b) acetophenone
(c) benzoic acid (d) benzyl alcohol
112. The IUPAC name of  is :
(a) 1, 1-diethyl-2, 2-dimethylpentane
(b) 4, 4-dimethyl-5, 5-diethylpentane
(c) 5, 5-diethyl-4, 4-dimethylpentane
(d) 3-ethyl-4, 4-dimethylpentane
113. Which of the following species exhibits the diamagnetic behaviour ?
(a) O_2^{2-} (b) O_2^+
(c) O_2 (d) NO
114. The stability of dihalides of Si, Ge, Sn and Pb increases steadily in the sequence :
(a) $\text{GeX}_2 < \text{SiX}_2 < \text{SnX}_2 < \text{PbX}_2$
(b) $\text{SiX}_2 < \text{GeX}_2 < \text{PbX}_2 < \text{SnX}_2$
(c) $\text{SiX}_2 < \text{GeX}_2 < \text{SnX}_2 < \text{PbX}_2$
(d) $\text{PbX}_2 < \text{SnX}_2 < \text{GeX}_2 < \text{SiX}_2$
115. Identify the incorrect statement among the following :
(a) Ozone reacts with SO_2 to give SO_3
(b) Silicon reacts with NaOH(aq) in the presence of air to give Na_2SiO_3 and H_2O
(c) Cl_2 reacts with excess of NH_3 to give N_2 and HCl
(d) Br_2 reacts with hot and strong NaOH solution to give NaBr , NaBrO_4 and H_2O
116. The charge/size ratio of a cation determines its polarizing power. Which one of the following sequences represents the increasing order of the polarizing power of the cationic species, K^+ , Ca^{2+} , Mg^{2+} , Be^{2+} ?
(a) $\text{Mg}^{2+} < \text{Be}^{2+} < \text{K}^+ < \text{Ca}^{2+}$
(b) $\text{Be}^{2+} < \text{K}^+ < \text{Ca}^{2+} < \text{Mg}^{2+}$
(c) $\text{K}^+ < \text{Ca}^{2+} < \text{Mg}^{2+} < \text{Be}^{2+}$
(d) $\text{Ca}^{2+} < \text{Mg}^{2+} < \text{Be}^{2+} < \text{K}^+$
117. The density (in g mL^{-1}) of a 3.60 M sulphuric acid solution that is 29% H_2SO_4 (molar mass = 98 g mol^{-1}) by mass will be :
(a) 1.64 (b) 1.88
(c) 1.22 (d) 1.45
118. The first and second dissociation constants of an acid H_2A are 1.0×10^{-5} and 5.0×10^{-10} respectively. The overall dissociation constant of the acid will be :
(a) 5.0×10^{-5} (b) 5.0×10^{15}
(c) 5.0×10^{-15} (d) 0.2×10^5
119. A mixture of ethyl alcohol and propyl alcohol has a vapour pressure of 290 mm at 300 K. The vapour pressure of propyl alcohol is 200 mm. If the mole fraction of ethyl alcohol is 0.6, its vapour pressure (in mm) at the same temperature will be :
(a) 350 (b) 300
(c) 700 (d) 360
120. In conversion of limestone to lime,
 $\text{CaCO}_3(\text{s}) \longrightarrow \text{CaO}(\text{s}) + \text{CO}_2(\text{g})$
the values of ΔH° and ΔS° are $+179.1 \text{ kJ mol}^{-1}$ and 160.2 J/K respectively at 298 K and 1 bar. Assuming that ΔH° and ΔS° do not change with temperature, temperature above which conversion of limestone to lime will be spontaneous is :
(a) 1008 K (b) 1200 K
(c) 845 K (d) 1118 K

ANSWERS

⇒ MATHEMATICS

1. (d)	2. (b)	3. (d)	4. (c)	5. (d)	6. (c)	7. (d)	8. (b)
9. (a)	10. (c)	11. (d)	12. (c)	13. (b)	14. (c)	15. (c)	16. (a)
17. (*)	18. (b)	19. (d)	20. (d)	21. (a)	22. (b)	23. (c)	24. (b)
25. (d)	26. (c)	27. (c)	28. (d)	29. (a)	30. (d)	31. (c)	32. (a)
33. (c)	34. (a)	35. (a)	36. (d)	37. (*)	38. (a)	39. (c)	40. (a)

Note. None of the option is match (*).

PHYSICS

41. (a)	42. (b)	43. (b)	44. (d)	45. (d)	46. (c)	47. (c)	48. (b)
49. (b)	50. (d)	51. (d)	52. (b)	53. (c)	54. (d)	55. (d)	56. (b)
57. (b)	58. (d)	59. (a)	60. (d)	61. (c)	62. (c)	63. (b)	64. (c)
65. (d)	66. (a)	67. (a)	68. (b)	69. (c)	70. (d)	71. (a)	72. (a)
73. (a)	74. (c)	75. (a)	76. (c)	77. (d)	78. (d)	79. (a)	80. (a)

CHEMISTRY

81. (d)	82. (c)	83. (c)	84. (a)	85. (d)	86. (d)	87. (a)	88. (a)
89. (a)	90. (d)	91. (c)	92. (d)	93. (b)	94. (d)	95. (c)	96. (d)
97. (c)	98. (b)	99. (d)	100. (c)	101. (b)	102. (c)	103. (b)	104. (b)
105. (d)	106. (d)	107. (a)	108. (a)	109. (a)	110. (c)	111. (c)	112. (d)
113. (a)	114. (c)	115. (d)	116. (c)	117. (c)	118. (c)	119. (a)	120. (d)

HINTS & SOLUTIONS

Mathematics

1. Since, each term is equal to the sum of two preceeding terms.

$$\begin{aligned} \therefore ar^{n-1} &= ar^n + ar^{n+1} \\ \Rightarrow 1 &= r + r^2 \Rightarrow r^2 + r - 1 = 0 \\ \Rightarrow r &= \frac{\sqrt{5}-1}{2} \quad \left(\because r \neq \frac{-\sqrt{5}-1}{2} \right) \end{aligned}$$

2. $\therefore \sin^{-1}\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$

$$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{4}{5}\right)$$

$$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \cos^{-1}\left(\frac{4}{5}\right)$$

$$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \sin^{-1}\left(\frac{3}{5}\right)$$

$$\Rightarrow x = 3$$

3. Since, in binomial expansion of $(a-b)^n$, $n \geq 5$, the sum of fifth and sixth term is equal to zero.

$$\therefore {}^nC_4 a^{n-4} (-b)^4 + {}^nC_5 a^{n-5} (-b)^5 = 0$$

$$\Rightarrow \frac{n!}{(n-4)!4!} a^{n-4} \cdot b^4 - \frac{n!}{(n-5)!5!} a^{n-5} b^5 = 0$$

$$\Rightarrow \frac{n!}{(n-5)!4!} a^{n-5} \cdot b^4 \left\{ \frac{a}{n-4} - \frac{b}{5} \right\} = 0$$

$$\Rightarrow \frac{a}{b} = \frac{n-4}{5}$$

4. Required number of ways = ${}^{12}C_4 \times {}^8C_4 \times {}^4C_4$

$$\begin{aligned} &= \frac{12!}{8! \times 4!} \times \frac{8!}{4! \times 4!} \times 1 \\ &= \frac{12!}{(4!)^3} \end{aligned}$$

5. 4^{-x^2} is defined for $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$$\cos^{-1}\left(\frac{x}{2} - 1\right) \text{ is defined if } -1 \leq \frac{x}{2} - 1 \leq 1$$

$$\Rightarrow 0 \leq \frac{x}{2} \leq 2 \Rightarrow 0 \leq x \leq 4$$

and $\log(\cos x)$ is defined, if $\cos x > 0$

$$\Rightarrow -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$\text{Hence } f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$$

$$\text{is defined if } x \in \left[0, \frac{\pi}{2}\right)$$

6. $\therefore OC = CA = CB$

$$\Rightarrow \angle AOC = \angle OAC \text{ and } \angle COB = \angle OBC$$

$$\therefore \sin \theta = \sin A = \frac{5}{13}$$

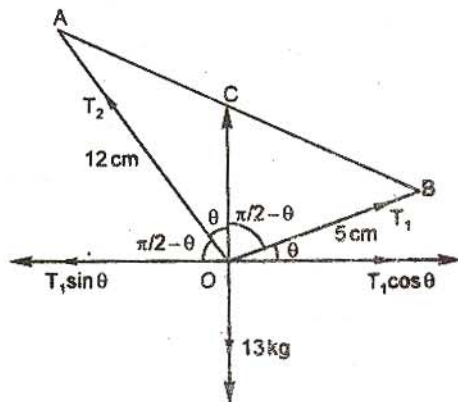
$$\text{and } \cos \theta = \frac{12}{13}$$

Now by Lami's theorem

$$\frac{T_1}{\sin(180^\circ - \theta)} = \frac{T_2}{\sin(90^\circ + \theta)}$$

$$\Rightarrow \frac{T_1}{\sin \theta} = \frac{T_2}{\cos \theta} \Rightarrow T_1 \cos \theta = T_2 \sin \theta$$

$$\Rightarrow T_1 \left(\frac{12}{13} \right) = T_2 \left(\frac{5}{13} \right) \Rightarrow T_1 = \left(\frac{5}{12} \right) T_2$$



Also $T_1 \sin \theta + T_2 \cos \theta = 13$

$$\Rightarrow T_2 \left(\frac{5}{12} \cdot \frac{5}{13} + \frac{12}{13} \right) = 13$$

$$\Rightarrow T_2 \left(\frac{169}{12 \cdot 13} \right) = 13$$

$$\Rightarrow T_2 = 12 \text{ kg and } T_1 = 5 \text{ kg.}$$

7. Probability of getting score 9 in a single throw

$$= \frac{4}{36} = \frac{1}{9}$$

Probability of getting score 9 exactly twice

$$= {}^3C_2 \times \left(\frac{1}{9} \right)^2 \times \frac{8}{9} = \frac{8}{243}$$

8. Equation of circle which touches x-axis and coordinates of centre are (h, k) , is

$$(x-h)^2 + (y-k)^2 = k^2$$

\therefore It is passing through $(-1, 1)$, then

$$(-1-h)^2 + (1-k)^2 = k^2$$

$$\Rightarrow h^2 + 2h - 2k + 2 = 0$$

$$\therefore D \geq 0$$

$$\Rightarrow 2k - 1 \geq 0 \Rightarrow k \geq \frac{1}{2}$$

9. If direction cosines of L be l, m, n , then

$$2l + 3m + n = 0 \quad \dots(i)$$

$$\text{and } l + 3m + 2n = 0 \quad \dots(ii)$$

On solving eqs. (i) and (ii), we get

$$\frac{l}{3} = \frac{m}{-3} = \frac{n}{3}$$

$$\therefore l : m : n = \frac{1}{\sqrt{3}} : \left(-\frac{1}{\sqrt{3}} \right) : \frac{1}{\sqrt{3}}$$

$$\Rightarrow l = \frac{1}{\sqrt{3}} \Rightarrow \cos \alpha = \frac{1}{\sqrt{3}}$$

10. General equation of all such circles which pass through the origin and whose centre lie on x-axis, is

$$x^2 + y^2 + 2gx = 0. \quad \dots(i)$$

On differentiating w.r.t. x , we get

$$2x + 2y \frac{dy}{dx} + 2g = 0$$

$$2g = - \left(2x + 2y \frac{dy}{dx} \right)$$

On putting the value of $2g$ in eq. (i), we get

$$x^2 + y^2 + \left(-2x - 2y \frac{dy}{dx} \right) x = 0$$

$$\Rightarrow x^2 + y^2 - 2x^2 - 2xy \frac{dy}{dx} = 0$$

$$\Rightarrow y^2 = x^2 + 2xy \frac{dy}{dx}$$

Which is required equation.

11. Using $AM \geq GM$

$$\frac{p^2 + q^2}{2} \geq pq$$

$$\Rightarrow pq \leq \frac{1}{2} \quad (\because p^2 + q^2 = 1)$$

Now $(p+q)^2 = p^2 + q^2 + 2pq$

$$\Rightarrow (p+q)^2 = 1 + 2pq$$

$$\Rightarrow (p+q)^2 \leq 1 + 1$$

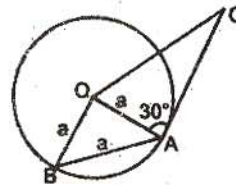
$$\Rightarrow p+q \leq \sqrt{2}$$

12. Let h be the height of a tower.

$$\therefore \angle AOB = 60^\circ$$

$\therefore \triangle OAB$ is equilateral.

$$\therefore OA = OB = AB = a$$



$$\text{Now in } \triangle OAC, \tan 30^\circ = \frac{h}{a} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{a}$$

$$\Rightarrow h = \frac{a}{\sqrt{3}}$$

13. We know that

$$(1+x)^{20} = {}^{20}C_0 + {}^{20}C_1 x + \dots + {}^{20}C_{10} x^{10} + \dots + {}^{20}C_{20} x^{20}$$

On putting $x = -1$ in above expansion, we get

$$0 = {}^{20}C_0 - {}^{20}C_1 + \dots - {}^{20}C_9 + {}^{20}C_{10} - {}^{20}C_{11} + \dots + {}^{20}C_{20}$$

$$\begin{aligned}\Rightarrow 0 &= {}^{20}C_0 - {}^{20}C_1 + \dots - {}^{20}C_9 + {}^{20}C_{10} \\ &\quad - {}^{20}C_9 + \dots + {}^{20}C_0 \\ \Rightarrow 0 &= 2({}^{20}C_0 - {}^{20}C_1 + \dots - {}^{20}C_9) + {}^{20}C_{10} \\ \Rightarrow {}^{20}C_{10} &= 2({}^{20}C_0 - {}^{20}C_1 + \dots + {}^{20}C_{10}) \\ \Rightarrow {}^{20}C_0 - {}^{20}C_1 + \dots + {}^{20}C_{10} &= \frac{1}{2} {}^{20}C_{10}.\end{aligned}$$

14. Equation of normal is $Y - y = -\frac{dx}{dy}(X - x)$

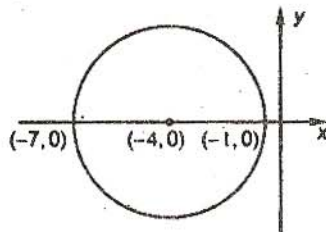
$$\Rightarrow G = \left(x + y \frac{dy}{dx}, 0 \right)$$

According to question

$$\left| x + y \frac{dy}{dx} \right| = |2x|$$

$$\begin{aligned}\Rightarrow y \frac{dy}{dx} &= x \quad \text{or} \quad y \frac{dy}{dx} = -3x \\ \Rightarrow y dy &= x dx \quad \text{or} \quad y dy = -3x dx \\ \Rightarrow \frac{y^2}{2} &= \frac{x^2}{2} + c \quad \text{or} \quad \frac{y^2}{2} = -\frac{3x^2}{2} + c \\ \Rightarrow x^2 - y^2 &= -2c \quad \text{or} \quad 3x^2 + y^2 = 2c.\end{aligned}$$

15. From the Argand diagram maximum value of $|z+1|$ is 6.

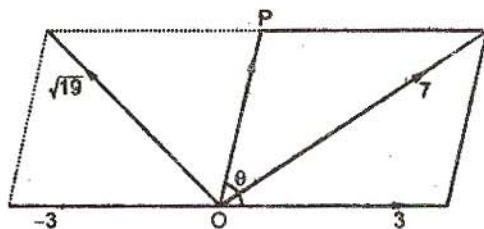


Aliter :

$$|z+1| = |z+4-3| \leq |z+4| + |-3| \leq 6$$

Thus maximum value of $|z+1|$ is 6.

16. $7^2 = p^2 + 3^2 + 2 \times 3 \times p \cos \theta$... (i)



$$\text{and } (\sqrt{19})^2 = p^2 + (-3)^2 + 2 \times (-3) \times p \cos \theta$$

... (ii)

On adding eqs. (i) and (ii), we get

$$68 = 2p^2 + 18 \Rightarrow p = 5.$$

17. The desired probability

$$\begin{aligned}&= (0.7)(0.2) + (0.7)(0.8)(0.7)(0.2) + \\ &\quad (0.7)(0.8)(0.7)(0.8)(0.7)(0.2) + \dots \\ &= 0.14[1 + (0.56) + (0.56)^2 + \dots] \\ &= 0.14 \left[\frac{1}{1-0.56} \right] = \frac{0.14}{0.44} = \frac{7}{22} = 0.32\end{aligned}$$

From above it is clear that no option is correct.

$$18. D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$$

On applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & x & 0 \\ 1 & 0 & y \end{vmatrix} = xy.$$

Hence, D is divisible by both x and y .

19. The given equation of hyperbola is $\frac{x^2}{\cos^2 \alpha} - \frac{y^2}{\sin^2 \alpha} = 1$. Here, $a^2 = \cos^2 \alpha$ and $b^2 = \sin^2 \alpha$

Coordinates of foci are $(\pm ae, 0)$.

$$\therefore e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$\Rightarrow e = \sqrt{1 + \frac{\sin^2 \alpha}{\cos^2 \alpha}} = \sqrt{1 + \tan^2 \alpha}$$

$$\Rightarrow e = \sec \alpha.$$

Hence, abscissae of foci remain constant when α varies.

20. Since, a line makes an angle of $\frac{\pi}{4}$ with positive direction of each of x -axis and y -axis, therefore

$$\alpha = \frac{\pi}{4}, \beta = \frac{\pi}{4}$$

we know that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

$$\Rightarrow \cos^2 \frac{\pi}{4} + \cos^2 \frac{\pi}{4} + \cos^2 \gamma = 1$$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} + \cos^2 \gamma = 1$$

$$\Rightarrow \cos^2 \gamma = 0 \Rightarrow \gamma = 90^\circ$$

21. Using mean value theorem

$$\therefore f'(c) = \frac{f(3) - f(1)}{3 - 1}$$

$$\Rightarrow \frac{1}{c} = \frac{\log 3 - \log 1}{2}$$

$$\Rightarrow c = \frac{2}{\log_3 e} = 2 \log_3 e.$$

22. $\therefore f(x) = \tan^{-1}(\sin x + \cos x)$
 $\therefore f'(x) = \frac{1}{1 + (\sin x + \cos x)^2} (\cos x - \sin x)$
 $= \frac{\sqrt{2} \cos\left(x + \frac{\pi}{4}\right)}{1 + (\sin x + \cos x)^2}$
 $f(x)$ is increasing if $-\frac{\pi}{2} < x + \frac{\pi}{4} < \frac{\pi}{2}$
 $\Rightarrow -\frac{3\pi}{4} < x < \frac{\pi}{4}$
Hence, $f(x)$ is increasing when $x \in \left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$.

23. $\therefore A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$
 $\therefore A^2 = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$
 $= \begin{bmatrix} 25 & 25\alpha + 5\alpha^2 & 10\alpha + 25\alpha^2 \\ 0 & \alpha^2 & 5\alpha^2 + 25\alpha \\ 0 & 0 & 25 \end{bmatrix}$
 $|A^2| = \begin{vmatrix} 25 & 25\alpha + 5\alpha^2 & 10\alpha + 25\alpha^2 \\ 0 & \alpha^2 & 5\alpha^2 + 25\alpha \\ 0 & 0 & 25 \end{vmatrix}$
 $= 25 \begin{vmatrix} 25 & 25\alpha + 5\alpha^2 \\ 0 & \alpha^2 \end{vmatrix} = 625\alpha^2$

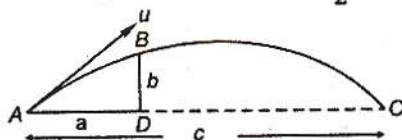
But $|A^2| = 25$
 $625\alpha^2 = 25 \Rightarrow |\alpha| = \frac{1}{5}$

24. $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots$
 $= 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots = e^{-1}$

25. Let $(2\hat{u} \times 3\hat{v})$ is a unit vector.
 $|2\hat{u} \times 3\hat{v}| = 1$
 $\Rightarrow 6|\hat{u}||\hat{v}|\sin\theta = 1$
 $\Rightarrow \sin\theta = \frac{1}{6}$

Hence, there is exactly one value of θ for which $(2\hat{u} \times 3\hat{v})$ is a unit vector.

26. $a = (u \cos \alpha)t$ and $b = (u \sin \alpha)t - \frac{1}{2}gt^2$



$$b = a \tan \alpha - \frac{1}{2}g \frac{a^2}{u^2 \cos^2 \alpha}$$

Also,
 $c = \frac{u^2 \sin 2\alpha}{g}$
 $b = a \tan \alpha - \frac{a^2 g}{2} \left(\frac{\sin 2\alpha}{cg} \right) \sec^2 \alpha$
 $\Rightarrow b = a \tan \alpha - \frac{a^2}{2c} 2 \tan \alpha$
 $\Rightarrow \left(a - \frac{a^2}{c} \right) \tan \alpha = b$
 $\Rightarrow \tan \alpha = \frac{bc}{a(c-a)}$
 $\alpha = \tan^{-1} \frac{bc}{a(c-a)}$

27. Let the number of boys and girls are x and y .

$$\therefore 52x + 42y = 50(x + y)$$

$$\Rightarrow 52x + 42y = 50x + 50y$$

$$\Rightarrow 2x = 8y \Rightarrow x = 4y$$

$$\therefore \text{Total number of students in the class}$$

$$= x + y = 4y + y = 5y$$

$$\therefore \text{Required \% of boys} = \frac{4y}{5y} \times 100\% = 80\%$$

28. \therefore Required point must be on the directrix of the parabola.

Hence the required point is $(-2, 0)$.

29. Equation of sphere is

$$x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$$

whose coordinates of centre are $(3, 6, 1)$

Let the coordinates of the other end of diameter are

$$(\alpha, \beta, \gamma), \text{ then } \frac{\alpha+2}{2} = 3, \frac{\beta+3}{2} = 6, \frac{\gamma+5}{2} = 1$$

Hence $\alpha = 4$, $\beta = 9$ and $\gamma = -3$.

Thus the coordinates of other point are $(4, 9, -3)$.

30. $\therefore \vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$

and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$ are coplanar

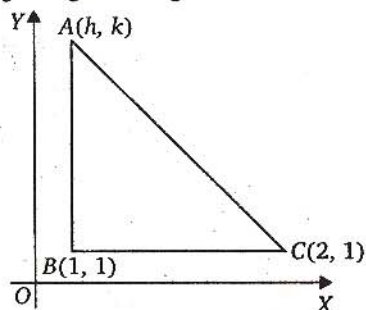
$$\therefore \begin{vmatrix} x & x-2 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 0$$

$$\Rightarrow 3x + 2 - x + 2 = 0$$

$$\Rightarrow 2x = -4$$

$$\Rightarrow x = -2$$

31. $\because A(h, k), B(1, 1)$ and $C(2, 1)$ are the vertices of a right angled triangle ABC .



$$\begin{aligned} \text{Now, } AB &= \sqrt{(1-h)^2 + (1-k)^2} \\ \text{or } BC &= \sqrt{(2-1)^2 + (1-1)^2} = 1 \\ \text{or } CA &= \sqrt{(h-2)^2 + (k-1)^2} \\ \text{Now, Pythagoras theorem} \\ AC^2 &= AB^2 + BC^2 \\ 4 + h^2 - 4h + k^2 + 1 - 2k &= h^2 + 1 - 2h + k^2 + 1 - 2k + 1 \\ 5 - 4h &= 3 - 2h \\ \Rightarrow h &= 1 \quad \dots(i) \end{aligned}$$

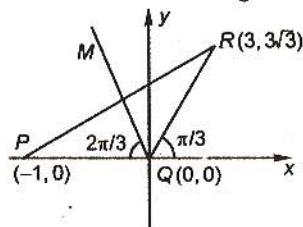
Now, given that area of the triangle is 1.

$$\begin{aligned} \text{Then, area } (\triangle ABC) &= \frac{1}{2} \times AB \times BC \\ 1 &= \frac{1}{2} \times \sqrt{(1-h)^2 + (1-k)^2} \times 1 \\ 2 &= \sqrt{(1-h)^2 + (1-k)^2} \quad \dots(ii) \end{aligned}$$

Putting $h = 1$ from Eq. (i), we get

$$\begin{aligned} 2 &= \sqrt{(k-1)^2} \\ \text{Squaring both the sides, we get} \\ 4 &= k^2 + 1 - 2k \\ \text{or } k^2 - 2k - 3 &= 0 \\ \text{or } (k-3)(k+1) &= 0 \\ \text{So, } k &= -1, 3 \\ \text{Thus the set of values of } k &\text{ is } \{-1, 3\}. \end{aligned}$$

32. Slope of the line $QM = \tan \frac{2\pi}{3} = -\sqrt{3}$



Hence equation is line QM is $y = -\sqrt{3}x$
or $\sqrt{3}x + y = 0$.

33. Equation of bisectors of lines $xy = 0$ are $y = \pm x$
put $y = \pm x$ in $my^2 + (1-m^2)xy - mx^2 = 0$, we get

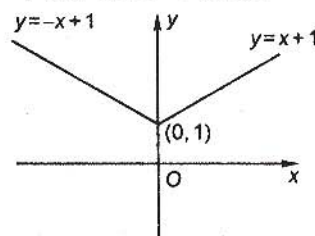
$$\begin{aligned} mx^2 + (1-m^2)x^2 - mx^2 &= 0 \\ \Rightarrow (1-m^2)x^2 &= 0 \Rightarrow m = \pm 1. \end{aligned}$$

34. $\because f(x) = \int_1^x \frac{\log t}{1+t} dt$

and $F(e) = f(e) + f\left(\frac{1}{e}\right)$

$$\begin{aligned} \Rightarrow F(e) &= \int_1^e \frac{\log t}{1+t} dt + \int_1^{1/e} \frac{\log t}{1+t} dt \\ &= \int_1^e \frac{\log t}{1+t} dt + \int_1^e \frac{\log t}{t(1+t)} dt \\ &= \int_1^e \frac{\log t}{t} dt = \left[\frac{(\log t)^2}{2} \right]_1^e \\ &= \frac{1}{2} [(\log e)^2 - (\log 1)^2] = \frac{1}{2} \end{aligned}$$

35. $f(x) = \min \{x+1, |x|+1\}$
 $f(x) = x+1 \quad \forall x \in R.$



From figure it is clear that $f(x) \geq 1$, when $x \in R$.

36. $\lim_{x \rightarrow 0} \left\{ \frac{1}{x} - \frac{2}{e^{2x}-1} \right\}$
 $= \lim_{x \rightarrow 0} \frac{e^{2x}-1-2x}{x(e^{2x}-1)}$
 $= \lim_{x \rightarrow 0} \frac{2e^{2x}-2}{(e^{2x}-1)+2xe^{2x}}$
 $= \lim_{x \rightarrow 0} \frac{4e^{2x}}{4e^{2x}+4xe^{2x}} = 1$

$\therefore f(x)$ is continuous at $x = 0$, then

$$\begin{aligned} \lim_{x \rightarrow 0} f(x) &= f(0) \\ \Rightarrow 1 &= f(0). \end{aligned}$$

37. $\int_{\sqrt{2}}^x \frac{dt}{\sqrt{t^2-1}} = \frac{\pi}{2}$
 $\Rightarrow [\sec^{-1} t]_{\sqrt{2}}^x = \frac{\pi}{2}$

$$\Rightarrow \sec^{-1} x - \frac{\pi}{4} = \frac{\pi}{2}$$

$$\Rightarrow \sec^{-1} x = \frac{3\pi}{4}$$

$$\Rightarrow x = -\sqrt{2}.$$

38. $\int \frac{dx}{\cos x + \sqrt{3} \sin x}$

$$= \frac{1}{2} \int \sec \left(x - \frac{\pi}{3} \right) dx$$

$$= \frac{1}{2} \log \tan \left(\frac{x}{2} - \frac{\pi}{6} + \frac{\pi}{4} \right) + c$$

$$= \frac{1}{2} \log \tan \left(\frac{x}{2} + \frac{\pi}{12} \right) + c.$$

39. Required area = $A = \int_0^1 (\sqrt{x} - x) dx$

$$= \left[\frac{2}{3} x^{3/2} - \frac{x^2}{2} \right]_0^1 = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}.$$

40. Let α and β be the roots of equation $x^2 + ax + 1 = 0$, then

$$\alpha + \beta = -a \text{ and } \alpha\beta = 1$$

Now $|\alpha - \beta| = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

$$\Rightarrow |\alpha - \beta| = \sqrt{a^2 - 4}$$

According to question,

$$\sqrt{a^2 - 4} < \sqrt{5}$$

$$\Rightarrow a^2 - 4 < 5$$

$$\Rightarrow a^2 - 9 < 0$$

$$\Rightarrow a \in (-3, 3).$$

Physics

41. $x = (2 \times 10^{-2}) \cos \pi t$

Here, $a = 2 \times 10^{-2} \text{ m} = 2 \text{ cm}$

At $t = 0$, $x = 2 \text{ cm}$, i.e., the object is at positive extreme, so to acquire maximum speed (i.e., to reach mean position) it takes $\frac{1}{4}$ th of time period.

$$\therefore \text{Required time} = \frac{T}{4}$$

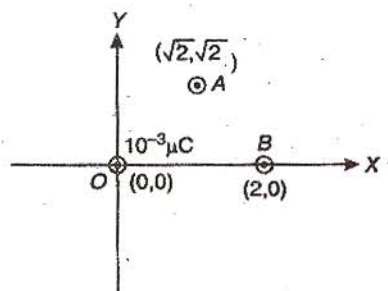
where $\omega = \frac{2\pi}{T} = \pi$

$$\Rightarrow T = 2 \text{ s}$$

So, required time = $\frac{T}{4} = \frac{2}{4} = 0.5 \text{ s}$

42. For given circuit current is lagging the voltage by $\pi/2$, so circuit is purely inductive and there is no power consumption in the circuit. The work done by battery is stored as magnetic energy in the inductor.

43. Potential at A due to charge at O



$$V_A = \frac{1}{4\pi\epsilon_0} \frac{(10^{-3})}{OA}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{(10^{-3})}{\sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}}$$

Potential at B due to charge at O

$$V_B = \frac{1}{4\pi\epsilon_0} \frac{(10^{-3})}{OB}$$

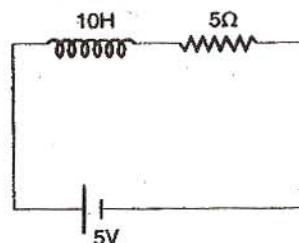
$$= \frac{1}{4\pi\epsilon_0} \frac{(10^{-3})}{2}$$

So, $V_A - V_B = 0$

44. Ratio of energy stored in the capacitor and the work done by the battery

$$= \frac{\frac{1}{2} qV}{qV} = \frac{1}{2}$$

45. Rise of current in L-R circuit is given by



$$I = I_0(1 - e^{-t/\tau})$$

where $I_0 = \frac{E}{R} = \frac{5}{5} = 1 \text{ A}$

Now, $\tau = \frac{L}{R} = \frac{10}{5} = 2s$

After 2s, i.e., at $t = 2s$

Rise of current $I = (1 - e^{-1})A$

46. Current density $J = \frac{i}{\pi a^2}$

From Ampere's circuital law

$$\oint B \cdot dl = \mu_0 \cdot i_{\text{enclosed}}$$

For $r < a$

$$B \times 2\pi r = \mu_0 \times J \times \pi r^2$$

$$\Rightarrow B = \frac{\mu_0 i}{\pi a^2} \times \frac{r}{2}$$

At $r = a/2$

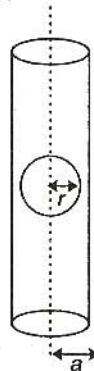
$$B_1 = \frac{\mu_0 i}{4\pi a}$$

For $r > a$

$$B \times 2\pi r = \mu_0 i \Rightarrow B = \frac{\mu_0 i}{2\pi r}$$

At $r = 2a$, $B_2 = \frac{\mu_0 i}{4\pi a}$

So, $\frac{B_1}{B_2} = 1$



47. Using Ampere's circuital law the magnetic field at any point inside the pipe is zero.

48. Binding energy

$$\begin{aligned} BE &= (M_{\text{nucleus}} - M_{\text{nucleons}})c^2 \\ &= (M_o - 8M_p - 9M_n)c^2 \end{aligned}$$

49. In gamma ray emission the energy is released from nucleus, so that nucleus get stabilised.

50. For $V_i < 0$

the diode is reverse biased and hence offer infinite resistance, so circuit would be like as shown in Fig. (2) and $V_o = 0$.

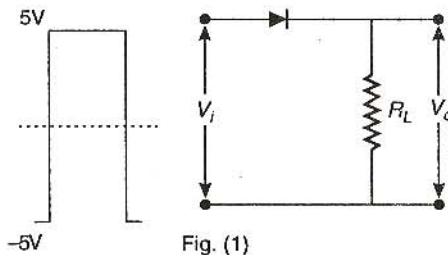


Fig. (1)

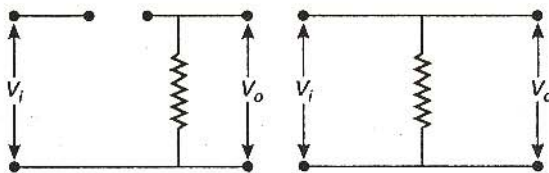


Fig. (2)

Fig. (3)

For $V_i > 0$, the diode is forward biased and circuit would be as shown in Fig. (3) and $V_o = V_i$.

Hence, the opitcal (d) is correct.

51. The momentum of the photon

$$p = \frac{h}{\lambda} = \frac{hv}{c}$$

52. $v = v_0 + gt + ft^2$

or $\frac{dx}{dt} = v_0 + gt + ft^2$

$\Rightarrow dx = (v_0 + gt + ft^2)dt$

So, $\int_0^x dx = \int_0^1 (v_0 + gt + ft^2)dt$

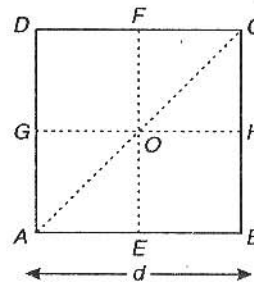
$\Rightarrow x = v_0 + g/2 + f/3$

53. Let the each side of square lamina is d .

So, $I_{EF} = I_{GH}$ (due to symmetry)

and $I_{AC} = I_{BD}$ (due to symmetry)

Now, according to theorem of perpendicular axis,



$$I_{AC} + I_{BD} = I_0$$

$\Rightarrow 2I_{AC} = I_0$... (i)

and $I_{EF} + I_{GH} = I_0$

$\Rightarrow 2I_{EF} = I_0$... (ii)

From Eqs. (i) and (ii), we get $I_{AC} = I_{EF}$

$$\begin{aligned} \therefore I_{AD} &= I_{EF} + \frac{md^2}{4} \\ &= \frac{md^2}{12} + \frac{md^2}{4} \quad \left(\text{as } I_{EF} = \frac{md^2}{12} \right) \end{aligned}$$

So, $I_{AD} = \frac{md^2}{3} = 4I_{EF}$

54. $x = x_0 \cos(\omega t - \pi/4)$

Acceleration, $a = \frac{d^2x}{dt^2}$

$$= -\omega^2 x_0 \cos(\omega t - \pi/4)$$

$$= \omega^2 x_0 \cos(\omega t + 3\pi/4)$$

So, $A = \omega^2 x_0$

and $\delta = 3\pi/4$

55. Direction of \vec{E} reverses while magnitude remains same and V remains unchanged.

$$\begin{aligned} 56. \quad T_{1/2}(X) &= \tau(Y) \\ \Rightarrow \frac{0.693}{\lambda_X} &= \frac{1}{\lambda_Y} \\ \Rightarrow \lambda_Y &= \frac{\lambda_X}{0.693} \\ \Rightarrow \lambda_Y &> \lambda_X \end{aligned}$$

So, Y will decay faster than X .

57. For Carnot engine using as refrigerator

$$W = Q_2 \left(\frac{T_1}{T_2} - 1 \right)$$

$$\text{It is given } \eta = \frac{1}{10}$$

$$\Rightarrow \eta = 1 - \frac{T_2}{T_1}$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{9}{10}$$

$$\text{So, } Q_2 = 90 \text{ J} \quad (\text{as } W = 10 \text{ J})$$

58. The number of free electrons for conduction is significant only in Si and Ge but small in C, as C is an impurity.

59. As \vec{v} of charged particle is remaining constant, it means force acting on charged particle is zero.

$$\text{So, } q(\vec{v} \times \vec{B}) = q\vec{E}$$

$$\Rightarrow \vec{v} \times \vec{B} = \vec{E}$$

$$\Rightarrow \vec{v} = \frac{\vec{E} \times \vec{B}}{B^2}$$

$$60. \quad \vec{E} = -\frac{\partial V}{\partial x} \hat{i} - \frac{\partial V}{\partial y} \hat{j} - \frac{\partial V}{\partial z} \hat{k}$$

$$\Rightarrow E_x = -\frac{\partial V}{\partial x} = -\frac{d}{dx} \left[\frac{20}{x^2 - 4} \right]$$

$$= \frac{40x}{(x^2 - 4)^2}$$

$$\Rightarrow E_x \text{ at } x = 4 \mu\text{m} = \frac{10}{9} \text{ V}/\mu\text{m}$$

and is along positive x -direction.

61. Emission spectrum would be when electron makes a jump from higher energy level to lower energy level.

Frequency of emitted photon is proportional to change in energy of two energy levels, i.e.,

$$v = RcZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$62. \text{ Acceleration of system, } a = \frac{F}{m + M}$$



$$\begin{aligned} \text{So, force acting on mass } m &= ma \\ &= \frac{mF}{m + M} \end{aligned}$$

63. Power of a lens is reciprocal of its focal length.

Power of combined lens is

$$P = P_1 + P_2$$

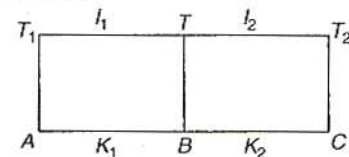
$$= -15 + 5 = -10 \text{ D}$$

$$\therefore f = \frac{1}{P} = \frac{100}{-10} \text{ cm}$$

$$f = -10 \text{ cm}$$

64. Let temperature at the interface is T .

For part AB,



$$\frac{Q_1}{t} \propto \frac{(T_1 - T)K_1}{l_1}$$

For part BC,

$$\frac{Q_2}{t} \propto \frac{(T - T_2)K_2}{l_2}$$

At equilibrium, $\frac{Q_1}{t} = \frac{Q_2}{t}$

$$\therefore \frac{(T_1 - T)K_1}{l_1} = \frac{(T - T_2)K_2}{l_2}$$

$$\Rightarrow T = \frac{T_1 K_1 l_2 + T_2 K_2 l_1}{K_1 l_2 + K_2 l_1}$$

65. Let intensity of sound be I and I' .

Loudness of sound initially

$$\beta_1 = 10 \log \left(\frac{I}{I_0} \right)$$

$$\text{Later, } \beta_2 = 10 \log \left(\frac{I'}{I_0} \right)$$

$$\text{Given, } \beta_2 - \beta_1 = 20$$

$$\therefore 20 = 10 \log \left(\frac{I'}{I} \right)$$

$$\therefore I' = 100 I$$

66. According to Mayer's relation,

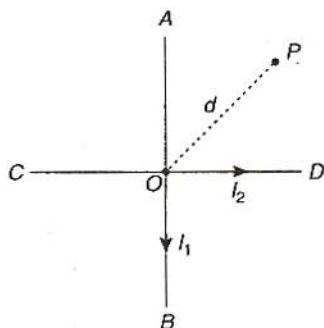
$$C_p - C_v = \frac{R}{m} = \frac{R}{28}$$

67. In case of motion of a charged particle perpendicular to the motion, i.e., displacement, the work done

$$W = \int \vec{F} \cdot d\vec{s} = \int F ds \cos \theta = 0 \quad (\text{as } \theta = 90^\circ)$$

and by work-energy theorem, $W = \Delta KE$, the kinetic energy and hence speed v remains constant. But \vec{v} changes, so, momentum changes.

68. The magnetic field induction at a point P , at a distance d from O in a direction perpendicular to the plane $ABCD$ due to currents through AOB and COD are perpendicular to each other, is



Hence,
$$B = \sqrt{B_1^2 + B_2^2}$$

$$= \left[\left(\frac{\mu_0}{4\pi} \frac{2I_1}{d} \right)^2 + \left(\frac{\mu_0}{4\pi} \frac{2I_2}{d} \right)^2 \right]^{1/2}$$

$$= \frac{\mu_0}{2\pi d} \sqrt{I_1^2 + I_2^2}$$

69. From $R_t = R_0(1 + \alpha t)$
 $5 = R_0(1 + 50\alpha)$... (i)
 and $6 = R_0(1 + 100\alpha)$... (ii)

$$\therefore \frac{5}{6} = \frac{1 + 50\alpha}{1 + 100\alpha} \Rightarrow \alpha = \frac{1}{200}$$

Putting value of α in Eq (i), we get

$$5 = R_0(1 + 50 \times 1/200)$$

$$\therefore R_0 = 4 \Omega$$

70. On introduction and removal and again on introduction, the capacity and potential remain same. So, net work done by the system in this process

$$W = U_f - U_i$$

$$= \frac{1}{2} C V^2 - \frac{1}{2} C V^2$$

$$= 0$$

71. According to Millikan's oil drop experiment, electronic charge is given by,

$$q = \frac{6 \pi \eta r (v_1 + v_2)}{E}$$

which is independent of g .

So,
$$\frac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}} = 1$$

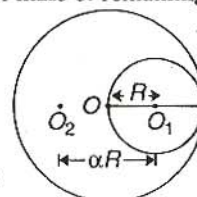
72. In this question distance of centre of mass of new disc from the centre of mass of remaining disc is αR .

Mass of remaining disc

$$= M - M/4 = 3M/4$$

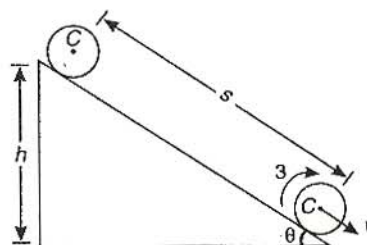
$$\therefore -\frac{3M}{4} \alpha R + \frac{M}{4} R = 0$$

$$\Rightarrow \alpha = 1/3$$



Note : In Q.No. 72, the given distance must be αR for real approach to the solution.

73. Assuming that no energy is used up against friction, the loss in potential energy is equal to the total gain in the kinetic energy.



Thus,
$$Mgh = \frac{1}{2} I(v^2/R^2) + \frac{1}{2} Mv^2$$

or
$$\frac{1}{2} v^2 (M + I/R^2) = Mgh$$

or
$$v^2 = \frac{2Mgh}{M + I/R^2} = \frac{2gh}{1 + I/MR^2}$$

If s be the distance covered along the plane, then

$$h = s \sin \theta$$

$$v^2 = \frac{2gs \sin \theta}{1 + I/MR^2}$$

Now,

$$v^2 = 2as$$

$$\therefore 2as = \frac{2gs \sin \theta}{1 + I/MR^2}$$

or

$$a = \frac{g \sin \theta}{1 + I/MR^2}$$

74. According to the principle of conservation of angular momentum, in the absence of external torque, the total angular momentum of the system is constant.

75.
$$a = \frac{F_k}{m} = \frac{15}{2} = 7.5 \text{ m/s}^2$$

$$\text{Now, } ma = \frac{1}{2} kx^2$$

$$\text{or } 2 \times 7.5 = \frac{1}{2} \times 10000 \times x^2$$

$$\text{or } x^2 = 3 \times 10^{-3}$$

$$\text{or } x = 0.055 \text{ m}$$

$$\Rightarrow x = 5.5 \text{ cm}$$

76. Kinetic energy at highest point

$$(KE)_H = \frac{1}{2} mv^2 \cos^2 \theta$$

$$= K \cos^2 \theta$$

$$= K(\cos 60^\circ)^2$$

$$= \frac{K}{4}$$

77. Phase difference = $\frac{2\pi}{\lambda} \times \text{path difference}$

$$\text{i.e., } \phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{6}$$

$$= \frac{\pi}{3}$$

$$\text{As, } I = I_{\max} \cos^2(\phi/2)$$

$$\text{or } \frac{I}{I_{\max}} = \cos^2(\phi/2)$$

$$\text{or } \frac{I}{I_0} = \cos^2(\pi/6)$$

$$= \frac{3}{4}$$

78.

$$f = \frac{1}{2\pi} \sqrt{\frac{k_1 + k_2}{m}}$$

and

$$f' = \frac{1}{2\pi} \cdot 2 \sqrt{\frac{k_1 + k_2}{m}} = 2f$$

79. From first law of thermodynamics,

$$Q = \Delta U + W$$

For path iaf,

$$50 = \Delta U + 20$$

$$\therefore \Delta U = U_f - U_i = 30 \text{ cal}$$

For path ibf,

$$Q = \Delta U + W$$

$$\text{or } W = Q - \Delta U$$

$$= 36 - 30 = 6 \text{ cal}$$

80. Average kinetic energy of particle

$$= \frac{1}{4} ma^2 \omega^2$$

$$= \frac{1}{4} ma^2 (2\pi v)^2$$

$$= \pi^2 v^2 ma^2$$

Chemistry

81. $A_2 + B_2 \rightleftharpoons 2AB$

$$E_a (\text{forward}) = 180 \text{ kJ mol}^{-1}$$

$$E_a (\text{backward}) = 200 \text{ kJ mol}^{-1}$$

In the presence of catalyst :

$$E_a (\text{forward}) = 180 - 100 = 80 \text{ kJ mol}^{-1}$$

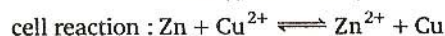
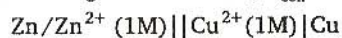
$$E_a (\text{backward}) = 200 - 100 = 100 \text{ kJ mol}^{-1}$$

$$\Delta H = E_a (\text{forward}) - E_a (\text{backward})$$

$$= 80 - 100$$

$$= -20 \text{ kJ mol}^{-1}$$

82. Cell is completely discharged, it means equilibrium gets established, $E_{\text{cell}} = 0$



$$K_{eq} = \frac{[\text{Zn}^{2+}]}{[\text{Cu}^{2+}]}$$

We know,

$$E_{\text{cell}} = E^\circ_{\text{cell}} - \frac{0.0591}{n} \log K_{eq}$$

$$E^\circ_{\text{cell}} = \frac{0.0591}{n} \log K_{eq}$$

$$\text{or } 1.10 = \frac{0.0591}{2} \log K_{eq}$$

$$K_{eq} = \frac{[\text{Zn}^{2+}]}{[\text{Cu}^{2+}]} = \text{antilog } \frac{2.20}{0.0591}$$

$$= \text{antilog } 37.3$$

83. Aqueous buffered solution of H A

50% H A is ionised

$$\Rightarrow [\text{H A}] = [\text{A}^-]$$

Buffer solution of weak acid H A \rightarrow acidic buffer

$$\text{pH} = \text{p}K_a + \log \frac{[\text{A}^-]}{[\text{H A}]}$$

$$\text{or } \text{pH} = \text{p}K_a = 4.5$$

$$\text{pOH} = \text{p}K_w - \text{pH}$$

$$\text{pOH} = 14 - 4.5 = 9.5$$

84. $2A + B \rightarrow \text{products}$

[B] is doubled, half-life didn't change

Half-life is independent of change in conc. of reactant \Rightarrow First order

First order w.r.t. to B

[A] is doubled, rate increased by two times

\Rightarrow First order w.r.t A

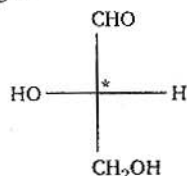
Hence, net order of reaction = $1 + 1 = 2$

$$\begin{aligned}\text{Unit for the rate constant} &= \text{conc.}^{(1-n)} t^{-1} \\ &= (\text{mol} \cdot \text{L}^{-1})^{-1} \cdot \text{s}^{-1} \\ &= \text{L} \cdot \text{mol}^{-1} \text{s}^{-1}\end{aligned}$$

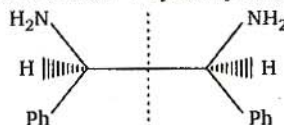
85. 4f and 5f belongs to different shell, experience different amount of shielding.

86. Cl^- is a weak ligand but Cl^- cause the pairing of electron with large Pt^{2+} and consequently give dsp^2 hybridisation and square planar geometry.

87. The molecule, which is optically active, has chiral centre, is expected to rotate the plane of polarised light.



One chiral centre \Rightarrow optically active

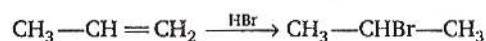
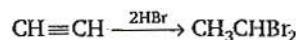
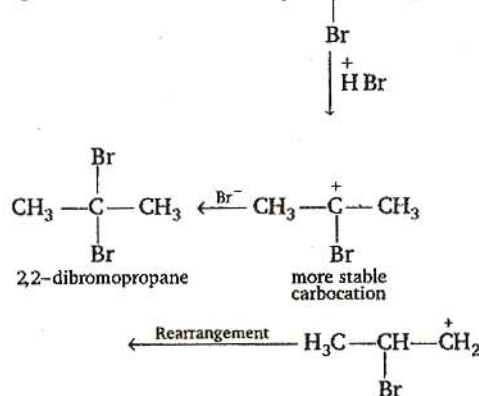


Two chiral centres, but plane of symmetry within molecule \Rightarrow optically inactive

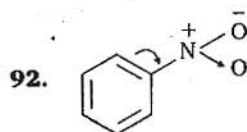
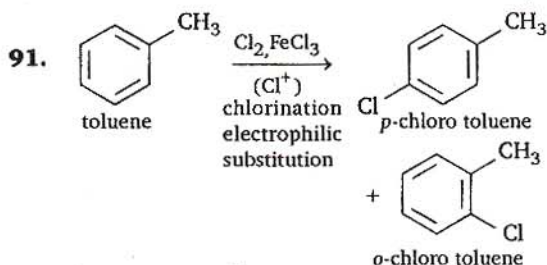
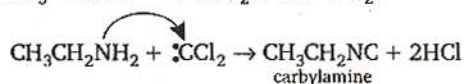
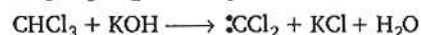
88. Primary structure involves sequence of α -amino acids polypeptide chain.

Secondary structure involves α -helical and β -pleated sheet like structures.

89. $\text{CH}_3-\text{C}\equiv\text{CH} + \text{HBr} \rightarrow \text{CH}_3-\text{C}=\text{CH}_2$



90. $\text{CH}_3\text{CH}_2\text{NH}_2 + \text{CHCl}_3 + \text{KOH} \rightarrow \text{A} + \text{B} + 3\text{H}_2\text{O}$



$-\text{NO}_2$ group withdraw electron from the ring, shows $-M$ effect makes ring electron deficient, thus, deactivates ring for electrophilic substitution.

93. $\text{NO} \rightarrow \text{NO}^+$

$$(\text{NO}) \text{ Total } e^- = 14$$

$$\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \pi 2p_x^{1+1} = 2p_y^{1+1} \sigma 2p_z^2$$

Diamagnetic

$$\text{Bond order} = \frac{10 - 4}{2} = 3$$

$$(\text{NO}) \text{ Total } e^- = 15$$

$$\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2, \sigma 2p_z^2 \pi 2p_x^{1+1} \pi 2p_y^{1+1}$$

$$\pi^* 2p_x^1 = \pi 2p_y$$

Paramagnetic

$$\text{Bond order} = \frac{10 - 5}{2} = 2.5$$

Electron is taken away from non-bonding molecular orbital, that's why bond order increases.

94. The actinoid (5f-elements) exhibits more number of oxidation states in general than the lanthanoid because 5f orbitals extend farther from the nucleus than the 4f orbitals.

95. Suppose the equal mass of methane and oxygen = $w = 1$ g

$$\text{Mole fraction of oxygen} = \frac{w/32}{w/32 + w/16} = \frac{1}{\frac{32}{32} + \frac{16}{32}} = \frac{1}{\frac{48}{32}} = \frac{1}{3}$$

Let the total pressure = P

Pressure exerted by oxygen (partial pressure)

$$= X_{O_2} \times P_{\text{total}} = P \times \frac{1}{3}$$

96. Solution is isotonic.

$$\Rightarrow C_1 RT = C_2 RT \\ C_1 = C_2$$

Density of both the solutions are assumed to be equal to $1.0 \text{ g cm}^{-3} \Rightarrow \text{molality} = \text{molarity}$
5.25%

in 100 g 5.25 g of substance in 1000 g 52.5 g of substance

$$\text{Hence, } \frac{52.5}{M} = \frac{15}{60},$$

M = molecular mass of the substance

$$M = \frac{52.5 \times 60}{15} = 210$$

97. $\text{H}_2\text{O} (l) \longrightarrow \text{H}_2\text{O} (g)$

$$\Delta n = 1 - 0 = 1$$

$$\Delta E = \Delta H - \Delta n RT$$

$$= 41 - 1 \times 8.3 \times 373 \times 10^{-3} \quad (R = 8.3 \times 10^{-3}) \\ = 37.9 \text{ kJ mol}^{-1}$$

98. $\text{AgIO}_3(s) \rightleftharpoons \text{Ag}^+(aq) + \text{IO}_3^-(aq)$

Let solubility of AgIO_3 be S

$$K_{sp} = [\text{Ag}^+][\text{IO}_3^-]$$

$$1.0 \times 10^{-8} = S^2 \text{ or } S = 1 \times 10^{-4} \text{ mol/L}$$

$$\text{In 1000 mL mol of } \text{AgIO}_3 \text{ dissolved} \\ = 1 \times 10^{-4} \text{ mol}$$

$$\text{In 100 mL of mole of } \text{AgIO}_3 \text{ dissolved} \\ = 1 \times 10^{-5} \text{ mol}$$

$$\text{Mass of } \text{AgIO}_3 \text{ in 100 mL} = 1 \times 10^{-5} \times 283 \\ = 2.83 \times 10^{-3}$$

99. Activity $\propto N$

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$$

$$\text{or } \frac{1}{10} = \left(\frac{1}{2}\right)^n$$

$$\text{or } 10 = 2^n$$

Taking log on both sides

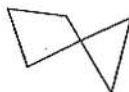
$$\log 10 = n \log 2$$

$$n = \frac{1}{0.301} = 3.32$$

$$\text{time} = n \times \text{half-life}$$

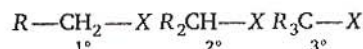
$$= 3.32 \times 30 = 99.6 \text{ days}$$

- 100.



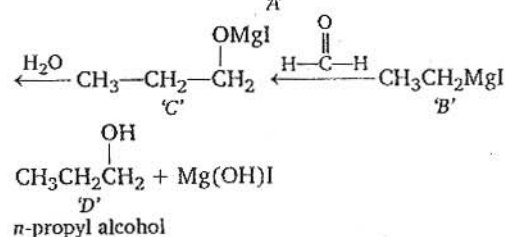
Chair form is unsymmetrical and absence of any element of symmetry.

101. S_N2 reactions are greatly controlled by steric factor.



S_N2 reactivity decreases as bulkiness of alkyl group increases.

102. $\text{CH}_3\text{CH}_2\text{OH} \xrightarrow{\text{P} + \text{I}_2} \text{CH}_3\text{CH}_2\text{I} \xrightarrow{\text{Mg, ether}}$



103. $n + l = 5$ maximum.

104. Hydrogen bond is strongest in HF due to higher electronegativity of F.

105. $2\text{Al}(s) + 6\text{HCl}(aq) \rightarrow 2\text{Al}^{3+}(aq) + 6\text{Cl}^-(aq) + 3\text{H}_2(g)$
1 mol

106. $(\text{NH}_4)_2\text{SO}_4 \longrightarrow 2\text{NH}_4^+ + \text{SO}_4^{2-}$
acidic

107. In an isolated system where either mass and energy are not exchanged with surrounding for the spontaneous process, the change in entropy is positive.

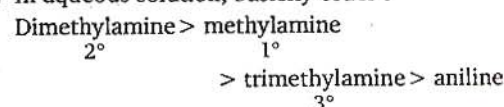
108. ${}_Z^AX \longrightarrow {}_Z^{A-1}X + {}_0^1n$

Isotopes are species having same number of proton, but different number of neutron.

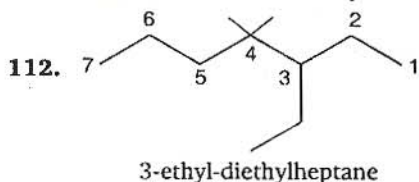
109. We know from Kohlrausch's law

$$\lambda^\circ_{\text{CH}_3\text{COOH}} = \lambda^\circ_{\text{CH}_3\text{COONa}} + \lambda^\circ_{\text{HCl}} - \lambda^\circ_{\text{NaCl}}$$

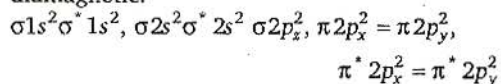
110. In aqueous solution, basicity order :



111. Any aliphatic carbon with hydrogen attached to it, in combination with benzene ring, will be oxidised to benzoic acid by KMnO_4/H^+ .



113. The correct option is O_2^{2-} . This species has 18 electrons, which are filled in such a way that all molecular orbitals are fully filled, so diamagnetic.

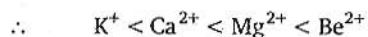


114. Due to inert pair effect, the stability of +2 oxidation state increases as we move down this group.



115. Br_2 reacts with hot and strong NaOH to give NaBr , NaBrO_3 and H_2O .

116. Higher the charge/size ratio, more is the polarising power.



$$117. \text{Molarity} = \frac{10 \times \text{density} \times \text{wt. of solute}}{\text{mol. wt. of the solute}}$$

$$\text{density} = \frac{3.60 \times 98}{10 \times 29} = 1.21$$

$$118. \quad K = k_1 \times k_2 \\ = 1.0 \times 10^{-5} \times 5.0 \times 10^{-10} \\ = 5 \times 10^{-15}$$

119. According to Raoult's law :

$$P = P_A^\circ X_A + P_B^\circ X_B \\ 290 = 200 \times 0.4 + P_B^\circ \times 0.6 \\ P_B^\circ = 350$$

$$120. \Delta S = \frac{\Delta H}{T}$$

$$\Delta S = 160.2 \text{ J/K}$$

$$\Delta H = 179.1 \times 10^3 \text{ J/mol}$$

$$T = \frac{179.1 \times 10^3 \text{ J/mol}}{160.2 \text{ J/K}} \\ = 1117.97 \text{ K} \approx 1118 \text{ K}$$