Test Booklet Code

E

JEE-MAIN 2016

PAPER - 1 : PHYSICS, CHEMISTRY & MATHEMATICS

Do not open this Test Booklet until you are asked to do so.

Read carefully the Instructions on the Back Cover of this Test Booklet.

Important Instructions :

1. Immediately fill in the particulars on this page of the Test Booklet with ***only Blue/Black Ball Point Pen*** provided by the Board.
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of **3 hours** duration.
4. The Test Booklet consists of **90** questions. The maximum marks are **360.**
5. There are ***three*** parts in the question paper A, B, C consisting of, **Physics, Chemistry and Mathematics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
6. *Candidates will be awarded marks as stated above in instruction No. 5 for correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet*.
7. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.
8. For writing, particulars / marking responses on **Side–1** and **Side–2** of the Answer Sheet use ***only Blue/Black Ball Point Pen only*** provided by the Board.
9. No candidates is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc., except the Admit Card inside the examination hall/room.
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in **three** pages (Pages **21–23**) at the end of the booklet.
11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room / Hall. ***However, the candidates are allowed to take away this Test Booklet with them.***
12. The CODE for this Booklet is **E**. Make sure that the CODE printed on **Side–2** of the Answer Sheet and also tally the serial number of the Test Booklet and Answer Sheet are the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.
13. **Do not fold or make any stray marks on the Answer Sheet.**

Name of the Candidate (in Capital letters) : ____________________

Roll Number : in figures | | | | | | | | |

: in words ____________________

Examination Centre Number : | | | | | | |

Name of Examination Centre (in Capital letters) : ____________________

Candidate's Signature : ____________ Invigilator's Signature : ____________

Read the following instructions carefully :

1. The candidates should fill in the required particulars on the Test Booklet and Answer Sheet (***Side-1***) with ***Blue/Black Ball Point Pen.***

2. For writing/marking particulars on ***Side-2*** of the Answer Sheet, use ***Blue/Black Ball Point Pen only.***

3. The candidates should not write their Roll Numbers anywhere else (except in the specified space) on the Test Booklet/Answer Sheet.

4. Out of the four options given for each question, only one option is the correct answer.

5. For each ***incorrect response, one-fourth (¼)*** of the total marks allotted to the question would be deducted from the total score. ***No deduction*** from the total score, however, will be made ***if no response*** is indicated for an item in the Answer Sheet.

6. Handle the Test Booklet and Answer Sheet with care, *as under no circumstances (except for discrepancy in Test Booklet Code and Answer Sheet Code), another set will be provided.*

7. The candidates are not allowed to do any rough work or writing work on the Answer Sheet. All calculations/writing work are to be done in the space provided for this purpose in the Test Booklet itself, marked 'Space for Rough Work'. This space is given at the bottom of each page and in 3 pages (Pages 21 – 23) at the end of the booklet.

8. On completion of the test, the candidates must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. **However, the candidates are allowed to take away this Test Booklet with them.**

9. Each candidate must show on demand his/her Admit Card to the Invigilator.

10. No candidate, without special permission of the Superintendent or Invigilator, should leave his/her seat.

11. The candidates should not leave the Examination Hall without handing over their Answer Sheet to the Invigilator on duty and sign the Attendance Sheet again. Cases where a candidate has not signed the Attendance Sheet a second time will be deemed not to have handed over the Answer Sheet and dealt with as an unfair means case. **The candidates are also required to put their left hand THUMB impression in the space provided in the Attendance Sheet.**

12. Use of Electronic/Manual Calculator and any Electronic Item like mobile phone, pager etc. is prohibited.

13. The candidates are governed by all Rules and Regulations of the JAB/Board with regard to their conduct in the Examination Hall. All cases of unfair means will be dealt with as per Rules and Regulations of the JAB/Board.

14. No part of the Test Booklet and Answer Sheet shall be detached under any circumstances.

15. **Candidates are not allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, electronic device or any other material except the Admit Card inside the examination room / hall.**

Questions and Solutions

PART- A : PHYSICS

1. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be :

(1) 92 ± 2 s (2) 92 ± 5.0 s (3) 92 ± 1.8 s (4) 92 ± 3 s

1. (1)

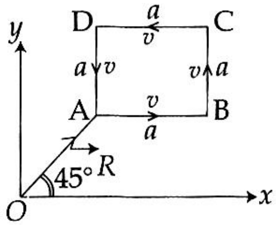
$$\bar{x} = \frac{\Sigma x_i}{N} = \frac{90+91+95+92}{4} = 92$$

Mean deviation = $\dfrac{\Sigma|\bar{x} - x_i|}{N} = \dfrac{2+1+3+0}{4} = 1.5$

L. C. = 1 s.

∴ Required value = 92 ± 2 s

2. A particle of mass m is moving along the side of a square of side 'a', with a uniform speed v in the x–y plane as shown in the figure :



Which of the following statements is false for the angular momentum $\vec{L}$ about the origin?

(1) $\vec{L} = -\dfrac{mv}{\sqrt{2}} R\,\hat{k}$ when the particle is moving from A to B.

(2) $\vec{L} = mv\left[\dfrac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D.

(3) $\vec{L} = m\,v\left[\dfrac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C.

(4) $\vec{L} = \dfrac{mv}{\sqrt{2}} R\,\hat{k}$ when the particle is moving from D to A.

2. (2), (4)

Along CD, ⊥ distance from line of motion = $\left(\dfrac{R}{\sqrt{2}} + a\right)$

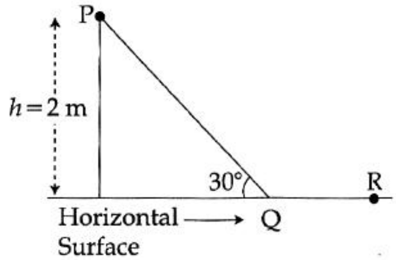
∴ Magnitude of angular momentum = $mv\left[\dfrac{R}{\sqrt{2}} + a\right]$

Hence (2) is incorrect.

In option (4) the direction of $\vec{L}$ is incorrect.

3. A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR.

The values of the coefficient of friction μ and the distance x (= QR), are, respectively close to :



(1) 0.2 and 6.5 m (2) 0.2 and 3.5 m (3) 0.29 and 3.5 m (4) 0.29 and 6.5 m

3. (3)
From work energy theorem and given condition

$$mgh - 2\mu mg\cos\theta\ \frac{h}{\sin\theta} = 0$$

$$\therefore \quad \mu = \frac{1}{2\cot 30} = \frac{1}{2\sqrt{3}} = 0.29$$

again $$\frac{mgh}{2} = \mu\, mg \cdot QR$$

$$\therefore \quad QR = \frac{h}{2\mu} = \frac{2}{2\times 0.29} = 3.5\,\text{m}$$

4. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take g = 9.8 ms^{-2} :

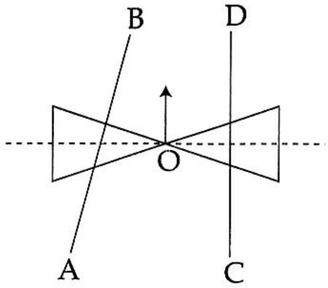
(1) 2.45×10^{-3} kg (2) 6.45×10^{-3} kg
(3) 9.89×10^{-3} kg (4) 12.89×10^{-3} kg

4. (4)

$$0.2\times 3.8\times 10^7 \times m = 10\times g\times 1\times 1000$$

$$m = \frac{10\times 9.8\times 1000}{0.2\times 3.8\times 10^7} = 1.289\times 10^{-2}\ \text{kg} = 12.89\times 10^{-3}\ \text{kg}$$

5. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to :



(1) turn left (2) turn right
(3) go straight (4) turn left and right alternately

5. (1)
If we take 'r' as the distance of IAOR from the axis of rotation, then 'r' decreases on left side as the object moves forward.
So, for left $v = \omega r' < \omega r$ (for right point)
So, the roller will turn to the left as it moves forward.

6. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R; h <<R). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere).

(1) $\sqrt{2gR}$ (2) $\sqrt{gR}$ (3) $\sqrt{gR/2}$ (4) $\sqrt{gR}\,(\sqrt{2}-1)$

6. (4)

Orbital velocity $v = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{GM}{R}}$ as h << R

Velocity required to escape

$\frac{1}{2}mv'^2 = \frac{GMm}{R+h}$; $\quad v' = \sqrt{\frac{2GM}{R+h}} = \sqrt{\frac{2GM}{R}}$ (h << R)

$\therefore$ Increase in velocity

$$v' - v = \sqrt{\frac{2GM}{R}} - \sqrt{\frac{GM}{R}} = \sqrt{2gR} - \sqrt{gR} = \sqrt{gR}\left(\sqrt{2} - 1\right)$$

7. A pendulum clock loses 12 s a day if the temperature is 40°C and gains 4s a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the co–efficient of linear expansion (α) of the metal of the pendulum shaft are respectively :

(1) 25°C; $\alpha = 1.85 \times 10^{-5}$/°C (2) 60°C; $\alpha = 1.85 \times 10^{-4}$/°C

(3) 30°C; $\alpha = 1.85 \times 10^{-3}$/°C (4) 55°C; $\alpha = 1.85 \times 10^{-2}$/°C

7. (1)

Time loss or gain is given by

$$\Delta t = \left(\frac{\Delta T}{T}\right)t = \frac{1}{2}\alpha \cdot \Delta\theta \cdot t$$

$$\therefore \quad 12 = \frac{1}{2}\alpha\left(40 - \theta_0\right) \times 1d \qquad \ldots(1)$$

$$4 = \frac{1}{2}\alpha\left(\theta_0 - 20\right) \times 1d \qquad \ldots(2)$$

$\frac{(1)}{(2)}$ gives

$$3 = \frac{(40 - \theta_0)}{(\theta_0 - 20)}$$

Solving $\theta_0 = 25^\circ C$ and putting in (2)

$$\alpha = \frac{8}{5 \times 29 \times 60 \times 60} = 1.85 \times 10^{-5} / {}^\circ C$$

8. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity Cremains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here C_P and C_V are molar specific heat at constant pressure and constant volume, respectively) :

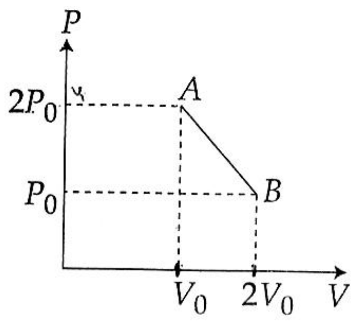
(1) $n = \frac{C_P}{C_V}$ (2) $n = \frac{C - C_P}{C - C_V}$ (3) $n = \frac{C_P - C}{C - C_V}$ (4) $n = \frac{C - C_V}{C - C_P}$

8. (2)

$C = C_v + \frac{R}{1-\eta} \quad \Rightarrow \quad 1 - n = \frac{R}{C - C_V}$

$$n = 1 - \frac{R}{C - C_V} = \frac{C - \left(C_V + R\right)}{C - C_V} = \frac{C - C_P}{C - C_V}$$

9. 'n' moles of an ideal gas undergoes a process A $\rightarrow$ B as shown in the figure. The maximum temperature of the gas during the process will be :



(1) $\frac{9P_0V_0}{4nR}$ (2) $\frac{3P_0V_0}{2nR}$

(3) $\frac{9P_0V_0}{2nR}$ (4) $\frac{9P_0V_0}{nR}$

9. (1)

Equation of line AB

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$P - P_0 = \frac{2P_0 - P_0}{V_0 - 2V_0}(V - 2V_0) \ = -\frac{P_0}{V_0}(V - 2V_0)$$

$$P = -\frac{P_0}{V_0}V + 3P_0$$

$$PV = -\frac{P_0}{V_0}V^2 + 3P_0V$$

$$nRT = -\frac{P_0}{V_0}V^2 + 3P_0V$$

$$T = \frac{1}{nR}\left(-\frac{P_0}{V_0}V^2 + 3P_0V\right)$$

$$\frac{dT}{dV} = 0 \qquad \text{(For maximum temperature)}$$

$$-\frac{P_0}{V_0}2V + 3P_0 = 0$$

$$-\frac{P_0}{V_0}2V = -3P_0$$

$$V = \frac{3}{2}V_0 \qquad \text{(Condition for maximum temperature)}$$

$$T_{max} = \frac{1}{nR}\left(-\frac{P_0}{V_0} \times \frac{9}{4}V_0^2 + 3P_0 \times \frac{3}{2}V_0\right) = \frac{1}{nR}\left(-\frac{9}{4}P_0V_0 + \frac{9}{2}P_0V_0\right) = \frac{9}{4}\frac{P_0V_0}{nR}$$

10. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is :

(1) $\frac{A}{3}\sqrt{41}$ (2) 3A (3) $A\sqrt{3}$ (4) $\frac{7A}{3}$

10. (4)

$m\omega^2 = k$

Total initial energy = $\frac{1}{2}kA^2$

at x = $\frac{2A}{3}$, potential energy = $\frac{1}{2}k\left(\frac{2A}{3}\right)^2 = \left(\frac{1}{2}kA^2\right)\left(\frac{4}{9}\right)$

Kinetic energy at $\left(x = \frac{2A}{3}\right)$ is = $\left(\frac{1}{2}kA^2\right)\cdot\left(\frac{5}{9}\right)$

If speed is tripled, new Kinetic energy = $\frac{1}{2}kA^2 \cdot \frac{5}{9} = \frac{5}{2}kA^2$

$\therefore$ New total energy = $\frac{5}{2}kA^2 + \frac{1}{2}kA^2\left(\frac{4}{9}\right) = \frac{kA^2}{2}\left(\frac{49}{9}\right)$

If next amplitude = A'; then $\frac{1}{2}kA'^2 = \frac{1}{2}kA^2\left(\frac{49}{9}\right) \quad \Rightarrow \quad A' = \frac{7}{3}A$

11. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse in introduced at its lowest end. It starts moving up the string. The time taken to reach the support is :
(take $g = 10\ ms^{-2}$)

(1) $2\pi\sqrt{2}$ s (2) 2 s (3) $2\sqrt{2}$ s (4) $\sqrt{2}$ s

11. (3)

$$\frac{dy}{dt} = \sqrt{\frac{gy\rho A}{\mu}}$$

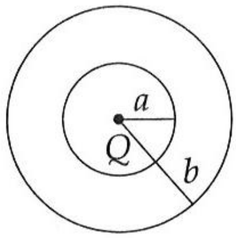
$$\frac{dy}{dt} = \int \sqrt{gy}$$

$$\int \frac{dy}{\sqrt{y}} = \sqrt{g}dt$$

$$\left.\frac{y^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right|_0^{\ell} = \sqrt{g}t\Big|_0^t$$

$$t = 2\sqrt{\frac{20}{10}} = 2\sqrt{2}\ \text{sec}$$

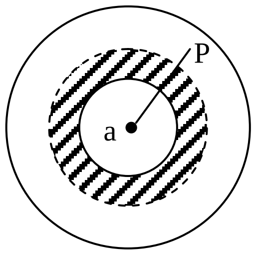
12. The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), has volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is :



(1) $\frac{Q}{2\pi a^2}$ (2) $\frac{Q}{2\pi(b^2 - a^2)}$ (3) $\frac{2Q}{\pi(a^2 - b^2)}$ (4) $\frac{2Q}{\pi a^2}$

12. (1)

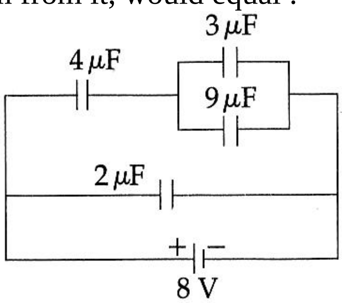
Charge in the shaded region = $\int_a^r 4 \times r^2 \frac{A}{r} \cdot dr = 2\pi A\left(r^2 - a^2\right)$



Total field at P = $\frac{1}{4\pi\varepsilon a} \cdot \frac{Q}{r^2} + \frac{1}{4\pi\varepsilon a} \cdot 2\pi A\left(1 - \frac{a^2}{r^2}\right)$

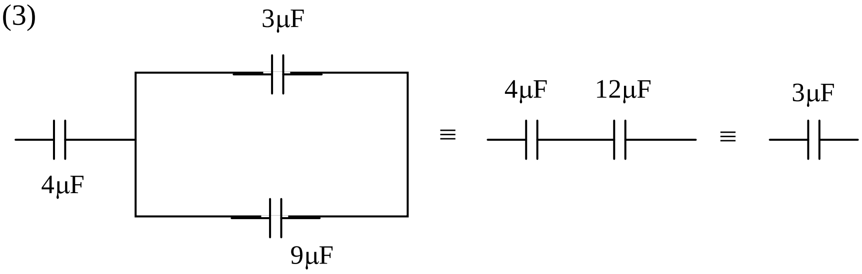
For field to be independent of r : $Q = 2\pi Aa^2$

13. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field due to a point charge Q (having a charge equal to the sum of the charges on the 4 μF and 9μF capacitors), at a point distant 30 m from it, would equal :



(1) 240 N/C (2) 360 N/C (3) 420 N/C (4) 480 N/C

13. (3)



Charges on 3 μF = 3μF × δV = 24 μC
∴ Charge on 4μF = Charge on 12 μF = 24 μC
Charge of 3μF = 3μF × 2V = 6 μC
Charge of 9μF = 9μF × 2V = 18 μC
Charge on 4μF + Charge on 9μF = (24 + 18)μF = 42 μC

∴ Electric field at 30 m = $9 \times 10^3 \times \dfrac{42\times10^{-6}}{30\times30} = 420\dfrac{N}{C}$

14. The temperature dependence of resistances of Cu and undoped Si in the temperature range 300−400 K, is best described by :
(1) Linear increase for Cu, linear increase for Si.
(2) Liner increase for Cu, exponential increase for Si.
(3) Linear increase for Cu, exponential decrease for Si.
(4) Linear decrease for Cu, linear decrease for Si.

14. (3)
Fact based

15. Two identical wires A and B, each of length 'ℓ', carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side 'a'. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\dfrac{B_A}{B_B}$ is :

(1) $\dfrac{\pi^2}{8}$ (2) $\dfrac{\pi^2}{16\sqrt{2}}$ (3) $\dfrac{\pi^2}{16}$ (4) $\dfrac{\pi^2}{8\sqrt{2}}$

15. (4)
<u>For A</u>
$2\pi R = \ell$

$$R = \frac{\ell}{2\pi}$$

$$B_A = \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{2\times\frac{\ell}{2\pi}}$$

$$B_A = \frac{\mu_0 \pi I}{\ell}$$

<u>For B</u>
$4a = \ell$

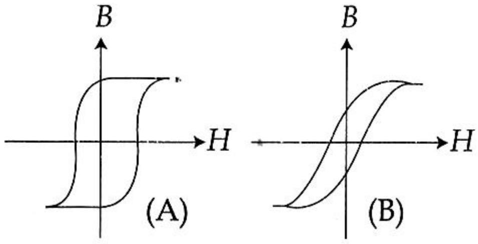
$$a = \frac{\ell}{4}$$

$$B_B = 4\times\frac{\mu_0 I\frac{\ell}{4}}{2\pi\frac{\ell}{8}\sqrt{\left(\frac{\ell}{4}\right)^2+4\left(\frac{\ell}{8}\right)^2}} = \frac{4\mu_0 I}{\pi\cdot\sqrt{2}\frac{\ell}{4}}$$

$$B_B = \frac{16\mu_0 I}{\sqrt{2}\,\pi\,\ell}$$

$$\frac{B_A}{B_B} = \frac{\frac{\mu_0 \pi I}{\ell}}{\frac{16\mu_0 I}{\sqrt{2}\,\pi\ell}} = \frac{\sqrt{2}\,\pi^2}{16} = \frac{\pi^2}{8\sqrt{2}}$$

16. Hysteresis loops for two magnetic materials A and B are given below :

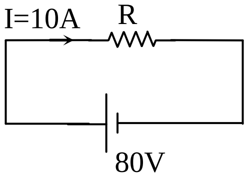


These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then it is proper to use :
(1) A for electric generators and transformers.
(2) A for electromagnets and B for electric generators
(3) A for transformers and B for electric generators
(4) B for electromagnets and transformers

16. (4)
Conceptual (Requires low retentivity and low coercivity)

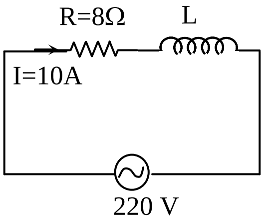
17. An arc lamp requires a direct current of 10 A at 80 V to function. It is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to :
(1) 80 H (2) 0.08 H (3) 0.044 H (4) 0.065 H

17. (4)
For dc



$$R = \frac{80}{10} = 8\,\Omega$$

For ac



$$10 = \frac{220}{\sqrt{R^2 + \omega^2 L^2}}$$

$$R^2 + \omega^2 L^2 = \left(\frac{220}{10}\right)^2$$

$$L^2 = \frac{22^2 - 8^2}{\omega^2} \qquad \therefore \qquad L = \frac{\sqrt{30\times 14}}{2\pi\times 50} = \frac{\sqrt{420}}{100\,\pi} = 0.065\,\text{H}$$

18. Arrange the following electromagnetic radiations per quantum in the order of increasing energy :
A : Blue light B : Yellow light C : X-ray D : Radiowave
(1) D, B, A, C (2) A, B, D, C (3) C, A, B, D (4) B, A, D, C

18. (1)
D, B, A, C

19. An observer looks at a distant tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears :
(1) 10 times taller (2) 10 times nearer (3) 20 times taller (4) 20 times nearer

19. (4)

20. The box of a pin hole camera, of length L, has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say b_{min}) when :

(1) $a = \frac{\lambda^2}{L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$

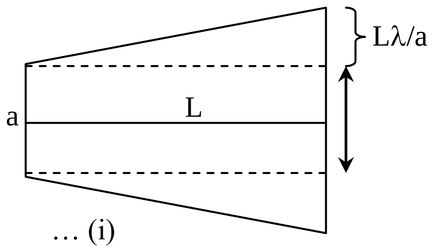
(2) $a = \sqrt{\lambda L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$

(3) $a = \sqrt{\lambda L}$ and $b_{min} = \sqrt{4\lambda L}$

(4) $a = \frac{\lambda^2}{L}$ and $b_{min} = \sqrt{4\lambda L}$

20. (3)

$\sin\theta = \frac{\lambda}{a}$



$B = 2a + \frac{2L\lambda}{a}$ … (i)

$\frac{\partial B}{\partial a} = 0 \Rightarrow 1 - \frac{L\lambda}{a^2} = 0$

$\Rightarrow a = \sqrt{\lambda L}$ … (ii)

$B_{min} = 2\sqrt{\lambda L} + 2\sqrt{\lambda L}$ [By substituting for a from (ii) in (i)]

$= 4\sqrt{\lambda L}$

∴ The radius of the spot $= \frac{1}{2}4\sqrt{\lambda L} = \sqrt{4\lambda L}$

21. Radiation of wavelength λ, is incident on a photocell. The fastest emitted electron has speed v. If the wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be :

(1) $> v\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (2) $< v\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (3) $= v\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (4) $= v\left(\frac{3}{4}\right)^{\frac{1}{2}}$

21. (1)

$\frac{hc}{\lambda} - \phi = \frac{1}{2}mv^2$ ………… (i)

$\frac{4hc}{3\lambda} - \phi = \frac{1}{2}mv'^2$ ………… (ii)

(ii) – (i) gives

$\frac{hc}{3\lambda} = \frac{1}{2}m\left(v'^2 - v^2\right) \Rightarrow v' = \sqrt{v^2 + \frac{2hc}{3\lambda m}}$ ………….. (iii)

also from (i) $\frac{hc}{\lambda} = \phi + \frac{mv^2}{2} \Rightarrow \frac{2hc}{\lambda m} = \frac{2\phi}{m} + v^2$

$\Rightarrow \frac{2hc}{3\lambda m} = \frac{2\phi}{3m} + \frac{r^2}{3} > \frac{v^2}{3}$ …………… (iv)

combining (iii) & (iv)

$v' > \sqrt{\frac{4v^2}{3}}$

22. Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively. Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed numbers of A and B nuclei will be :

(1) 1 : 16 (2) 4 : 1 (3) 1 : 4 (4) 5 : 4

22. (4)

80 minutes = 4 half-lives of A = 2 half-lives of B

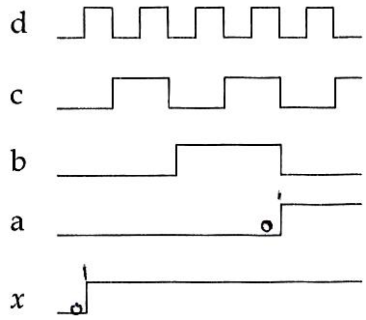
Let the initial number of nuclei in each sample be N

N_A after 80 minutes = $\frac{N}{2^4} \Rightarrow$ Number of A nuclides decayed = $\frac{15}{16}N$

N_B after 80 minutes = $\frac{N}{2^2} \Rightarrow$ Number of B nuclides decayed = $\frac{3}{4}N$

Required ratio = $\frac{15/16}{3/4} = \frac{5}{4}$

23. If a, b, c, d are inputs to a gate and x is its output, then, as per the following time graph, the gate is :



(1) NOT (2) AND (3) OR (4) NAND

23. (3)

x is 1 when at least one of the inputs is 1. Hence x is an OR-Gate.

24. Choose the correct statement :

(1) In amplitude modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(2) In amplitude modulation the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(3) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(4) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the frequency of the audio signal.

24. (1)

25. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of Aluminium. Before starting the measurement, it is found that when the two jaws of the screw gauge and brought in contact, the 45^{th} division coincides with the main scale line and that the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5 mm and the 25^{th} division coincides with the main scale line?

(1) 0.75 mm (2) 0.80 mm (3) 0.70 mm (4) 0.50 mm

25. (2)

L.C. = $\frac{\text{pitch}}{\text{No. of division on circular scale}} = \frac{0.5}{50} = 0.001\text{mm}$

–ve zero error = – 5 × L.C. = –0.005 mm

∴ Measured value

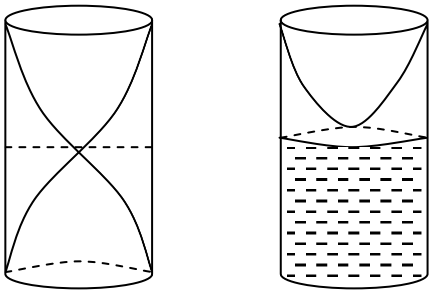
= main scale reading + screw gauge reading – zero error

= 0.5 mm + {25 × 0.001 – (–0.05)} mm = 0.80 mm

26. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now :

(1) $\frac{f}{2}$ (2) $\frac{3f}{4}$ (3) 2f (4) f

26. (4)



27. A galvanometer having a coil resistance of 100 Ω gives a full scale deflection, when a current of 1 mA is passed through it. The value of the resistance, which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A is :

(1) 0.01 Ω (2) 2 Ω (3) 0.1 Ω (4) 3 Ω

27. (1)

Maximum voltage that can be applied across the galvanometer coil = $100\ \Omega \times 10^{-3}$ A = 0.1 V.

If R_S is the shunt resistance :

$R_S \times 10$ A = 0.1 V

$\Rightarrow R_S = 0.01\ \Omega$

28. In an experiment for determination of refractive index of glass of a prism by $i - \delta$, plot, it was found that a ray incident at angle 35°, suffers a deviation of 40° and that it emerges at angle 79°. In that case which of the following is closest to the maximum possible value of the refractive index?

(1) 1.5 (2) 1.6 (3) 1.7 (4) 1.8

28. (1)

$\delta = i + e - A$

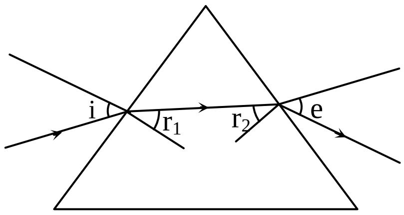
$40 = 35 + 79 - A$

$40 = 114 - A$

$A = 114 - 40 = 74 = r_1 + r_2$

From this we get,

$\mu = 1.5$

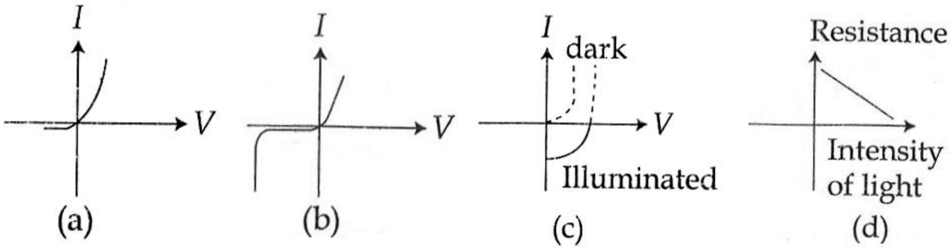


$\therefore \delta_{min} < 40°$

$$\mu < \frac{\sin\left(\frac{70+40}{2}\right)}{\sin 37}$$

$\therefore \mu_{max} = 1.44$

29. Identify the semiconductor devices whose characteristics are given below, in the order (a), (b), (c), (d) :



(1) Simple diode, Zener diode, Solar cell, Light dependent resistance.
(2) Zener diode, Simple diode, Light dependent resistance, Solar cell.
(3) Solar cell, Light dependent resistance, Zener diode, Simple diode.
(4) Zener diode, Solar cell, Simple diode, Light dependent resistance.

29. (1)

30. For a common emitter configuration, if α and β have their usual meanings, the incorrect relationship between α and β is :

(1) $\frac{1}{\alpha} = \frac{1}{\beta} + 1$ (2) $\alpha = \frac{\beta}{1-\beta}$ (3) $\alpha = \frac{\beta}{1+\beta}$ (4) $\alpha = \frac{\beta^2}{1+\beta^2}$

30. (2)

Standard Result

PART- B : CHEMISTRY

31. At 300 K and 1 atm, 15 mL of a gaseous hydrocarbon requires 375 mL air containing 20% O_2 by volume for complete combustion. After combustion the gases occupy 330 mL. Assuming that the water formed is in liquid form and the volumes were measured at the same temperature and pressure, the formula of the hydrocarbon is :

(1) C_3H_6 (2) C_3H_8 (3) C_4H_8 (4) C_4H_{10}

31. (2)

$$C_xH_y + \left(x + \frac{y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$$

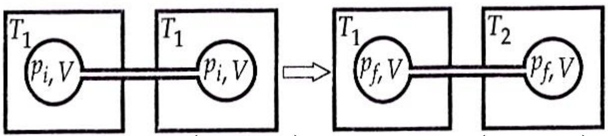
$$15\,\text{mL} \qquad 15\left(x+\frac{y}{4}\right)\text{mL} \qquad 15\,x\,\text{mL}$$

$$V_{O_2} = \frac{20}{100} \times 375 = 75\text{ mL} = 15\left(x + \frac{y}{4}\right)$$

$\Rightarrow$ $x + \frac{y}{4} = 5$

$\Rightarrow$ C_3H_8

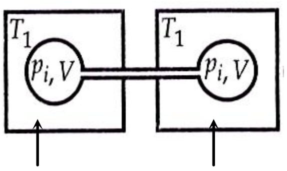
32. Two closed bulbs of equal volume (V) containing an ideal gas initially at pressure p_i and temperature T_1 are connected through a narrow tube of negligible volume as shown in the figure below. The temperature of one of the bulbs is then raised to T_2. The final pressure p_f is :



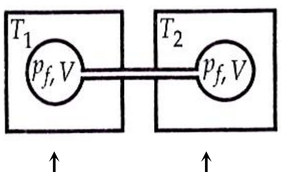
(1) $p_i\left(\frac{T_1T_2}{T_1+T_2}\right)$ (2) $2p_i\left(\frac{T_1}{T_1+T_2}\right)$ (3) $2p_i\left(\frac{T_2}{T_1+T_2}\right)$ (4) $2p_i\left(\frac{T_1T_2}{T_1+T_2}\right)$

32. (3)

Initially



Later on



$n_1 = \frac{p_iV}{RT_1}$ $n_2 = \frac{p_iV}{RT_1}$ $n'_1 = \frac{p_fV}{RT_1}$ $n'_2 = \frac{p_fV}{RT_2}$

$$n_1 + n_2 = n'_1 + n'_2$$

$$\frac{p_i V}{RT_1} + \frac{p_i V}{RT_1} = \frac{p_f V}{RT_1} + \frac{p_f V}{RT_2}$$

$$\frac{2p_i V}{RT_1} = \frac{p_f V}{R}\left(\frac{T_1 + T_2}{T_1 T_2}\right)$$

$$p_f = \frac{2p_i T_2}{T_1 + T_2}$$

33. A stream of electrons from a heated filament was passed between two charged plates kept at a potential difference V esu. If e and m are charge and mass of an electron, respectively, then the value of h/λ (where λ is wavelength associated with electron wave) is given by :

(1) me V (2) 2me V (3) $\sqrt{me\,V}$ (4) $\sqrt{2\,me\,V}$

33. (4)

$$\lambda = \frac{h}{P} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2meV}}$$

$$\Rightarrow \frac{h}{\lambda} = \sqrt{2meV}$$

34. The species in which the N atom is in a state of sp hybridization is :

(1) NO_2^+ (2) NO_2^- (3) NO_3^- (4) NO_2

34. (1)

$$O = \overset{\oplus}{N} = O$$

sp hybridisation

35. The heats of combustion of carbon and carbon monoxide are – 393.5 and –283.5 kJ mol^{-1}, respectively. The heat of formation (in kJ) of carbon monoxide per mole is :

(1) 110.5 (2) 676.5 (3) –676.5 (4) –110.5

35. (4)

$C_{(g)} + O_{2(g)} \longrightarrow CO_{2(g)}$ $\Delta H_1 = -393.5$ kJ/mol

$CO_{(g)} + \frac{1}{2}O_{2\,(g)} \longrightarrow CO_{2\,(g)}$ $\Delta H_2 = -283.5$ kJ/ mol

---

$C_{(g)} + \frac{1}{2}O_{2\,(g)} \longrightarrow CO_{(g)}$ $\Delta H = \Delta H_1 - \Delta H_2 = -110$ kJ/mol

36. 18 g glucose ($C_6H_{12}O_6$) is added to 178.2 g water. The vapor pressure of water (in torr) for this aqueous solution is :

(1) 7.6 (2) 76.0 (3) 752.4 (4) 759.0

36. (3)

$n_{H_2O} = \frac{178.2}{18} = 9.9$; $n_{Glucose} = \frac{18}{180} = 0.1$

$n_{Total} = 10$

$X_{H_2O} = 0.99$

$P = P^0 X_{Solvent} = 760 \times 0.99 = 752.4$ torr.

37. The equilibrium constant at 298 K for a reaction A + B $\rightleftharpoons$ C + D is 100. If the initial concentration of all the four species were 1M each, then equilibrium concentration of D (in mol L^{-1}) will be :

(1) 0.182 (2) 0.818 (3) 1.818 (4) 1.182

37. (3)

$$A + B \rightleftharpoons C + D$$

1	1	1	1
1 – x	1 – x	1 + x	1 + x

$$K_C = \frac{[C][D]}{[A][B]} = \frac{(1+x)^2}{(1-x)^2} = 100 \Rightarrow \frac{1+x}{1-x} = 10$$

$$\Rightarrow 1 + x = 10 - 10x$$

$$\Rightarrow 11x = 9$$

$$x = \frac{9}{11}$$

$$[D] = 1 + x = 1 + \frac{9}{11}$$

$$= 1.818 \text{ M}$$

38. Galvanization is applying a coating of :

(1) Pb (2) Cr (3) Cu (4) Zn

38. (4)

Galvanization is applying a coating of zinc.

$$E^0_{Pb^{+2}/Pb} = -0.13 \text{ V}$$

$$E^0_{Cr^{+3}/Cr} = -0.74 \text{ V}$$

$$E^0_{Cu^{+2}/Cu} = 0.34 \text{ V}$$

$$E^0_{Zn^{+2}/Zn} = -0.76 \text{ V}$$

39. Decomposition of H_2O_2 follows a first order reaction. In fifty minutes the concentration of H_2O_2 decreases from 0.5 to 0.125 M in one such decomposition. When the concentration of H_2O_2 reaches 0.05 M, the rate of formation of O_2 will be :

(1) 6.93×10^{-2} mol min^{-1} (2) 6.93×10^{-4} mol min^{-1}

(3) 2.66 L min^{-1} at STP (4) 1.34×10^{-2} mol min^{-1}

39. (2)

$$t_{3/4} = 2 \times t_{1/2} = 50 \text{ min}$$

i.e. $t_{1/2} = 25$ min

$$k = \frac{0.693}{t_{1/2}} = \frac{0.693}{25} \text{ min}^{-1}$$

Rate of H_2O_2 decomposition $= k[H_2O_2]$

$$= \frac{0.693}{25} \times 0.05 = -\frac{d[H_2O_2]}{dt}$$

$$H_2O_2 \longrightarrow H_2O + \frac{1}{2}O_2$$

$$-\frac{d[H_2O_2]}{dt} = 2\frac{d[O_2]}{dt}$$

$$\Rightarrow \frac{d[O_2]}{dt} = 6.93 \times 10^{-4} \text{ mol min}^{-1}$$

40. For a linear plot of log (x/m) versus log p in a Freundlich adsorption isotherm, which of the following statements is correct ? (k and n are constants)

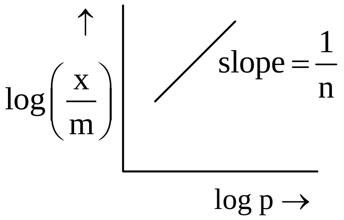
(1) Both k and 1/n appear in the slope term. (2) 1/n appears as the intercept.

(3) Only 1/n appears as the slope. (4) log (1/n) appears as the intercept.

40. (3)

$$\frac{x}{m} = k\,p^{1/n}$$

$$\log\left(\frac{x}{m}\right) = \log k + \frac{1}{n}\log p$$



41. Which of the following atoms has the highest first ionization energy ?
(1) Rb (2) Na (3) K (4) Sc

41. (4)

Element	Ionisation energy (kJ/mol)
Na	496
K	419
Rb	403
Sc	631

Scandium has the highest first Ionisation energy.

42. Which one of the following ores is best concentrated by froth floatation method ?
(1) Magnetite (2) Siderite (3) Galena (4) Malachite

42. (3)
PbS, i.e. Galena is best concentrated by froth floatation method.

43. Which one of the following statements about water is **FALSE** ?
(1) Water is oxidized to oxygen during photosynthesis.
(2) Water can act both as an acid and as a base.
(3) There is extensive intramolecular hydrogen bonding in the condensed phase.
(4) Ice formed by heavy water sinks in normal water.

43. (3)
Water possess intermolecular hydrogen bonding in the condensed phase.

44. The main oxides formed on combustion of Li, Na and K in excess of air are, respectively :
(1) Li_2O, Na_2O and KO_2 (2) LiO_2, Na_2O_2 and K_2O
(3) Li_2O_2, Na_2O_2 and KO_2 (4) Li_2O, Na_2O_2 and KO_2

44. (4)
$Li \rightarrow Li_2O$
$Na \rightarrow Na_2O_2$
$K \rightarrow KO_2$

45. The reaction of zinc with dilute and concentrated nitric acid, respectively produces :
(1) N_2O and NO_2 (2) NO_2 and NO (3) NO and N_2O (4) NO_2 and N_2O

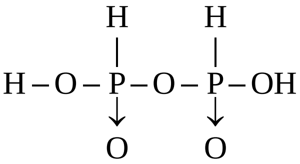
45. (1)
Zn reacts with dil. HNO_3 (20%) to form nitrous oxide (N_2O)
Zn reacts with conc. HNO_3 (70%) to form nitrogen dioxide (NO_2)

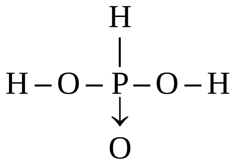
$$4Zn + 10HNO_3(\text{dil.}) \longrightarrow 4Zn(NO_3)_2 + N_2O + 5H_2O$$

$$Zn + 4HNO_3\,(\text{conc.}) \longrightarrow Zn(NO_3)_2 + 2NO_2 + 2H_2O$$

46. The pair in which phosphorous atoms have a formal oxidation state of + 3 is :
(1) Orthophosphorous and pyrophosphorous acids
(2) Pyrophosphorous and hypophosphoric acids
(3) Orthophosphorous and hypophosphoric acids
(4) Pyrophosphorous and pyrophosphoric acids

46. (1)
Phosphorus acid series contain phosphorus in the oxidation state (+ III).


Pyrophosphorus acid

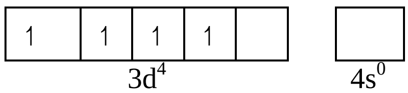

orthophosphorus acid

47. Which of the following compounds is metallic and ferromagnetic ?
(1) TiO_2 (2) CrO_2 (3) VO_2 (4) MnO_2

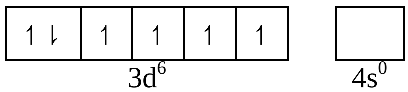
47. (2)
CrO_2 like metal conduct electricity and ferromagnetic.

48. The pair having the same magnetic moment is :
[At. No.: Cr = 24, Mn = 25, Fe = 26, Co = 27]
(1) $[Cr(H_2O)_6]^{2+}$ and $[CoCl_4]^{2-}$ (2) $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$
(3) $[Mn(H_2O)_6]^{2+}$ and $[Cr(H_2O)_6]^{2+}$ (4) $[CoCl_4]^{2-}$ and $[Fe(H_2O)_6]^{2+}$

48. (2)
$Cr^{+2} \rightarrow (Ar)\ 3d^4\ 4s^0$

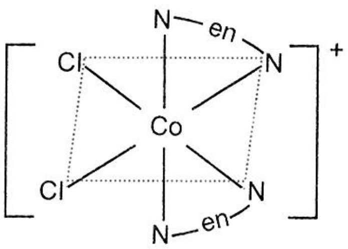


$Fe^{+2} \rightarrow (Ar)\ 3d^6\ 4s^0$



49. Which one of the following complexes shows optical isomerism ?
(1) $[Co(NH_3)_3Cl_3]$ (2) cis$[Co(en)_2Cl_2]Cl$
(3) trans$[Co(en)_2Cl_2]Cl$ (4) $[Co(NH_3)_4Cl_2]$ Cl
(en = ethylenediamine)

49. (2)



Cis-isomer

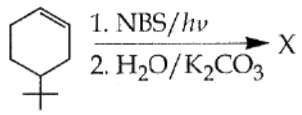
50. The concentration of fluoride, lead, nitrate and iron in a water sample from an underground lake was found to be 1000 ppb, 40 ppb, 100 ppm and 0.2 ppm, respectively. This water is unsuitable for drinking due to high concentration of :
(1) Fluoride (2) Lead (3) Nitrate (4) Iron

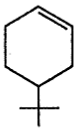
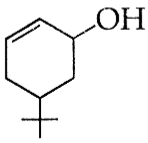
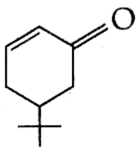
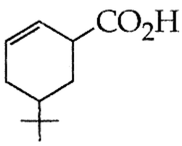
50. (3)
Fluoride – 1000 ppm
Lead – 40 ppm
Nitrate – 100 ppm [> 40 ppm gives blue baby syndrome disease.]
Iron – 0.2 ppm

51. The distillation technique most suited for separating glycerol from spent–lye in the soap industry is :
(1) Simple distillation (2) Fractional distillation
(3) Steam distillation (4) Distillation under reduced pressure

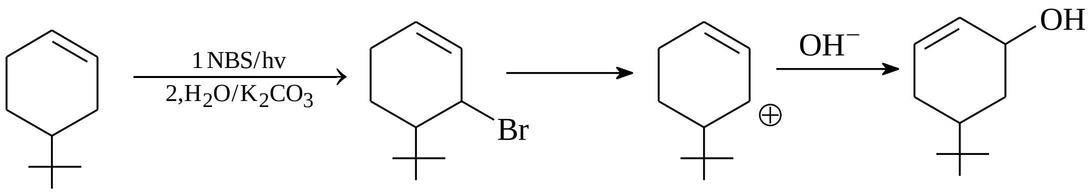
51. (4)
Glycerol and spent-lye can be separated by distillation under reduced pressure.

52. The product of the reaction given below is :

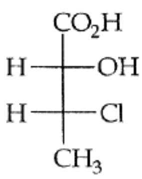


(1)  (2)  (3)  (4) 

52. (2)



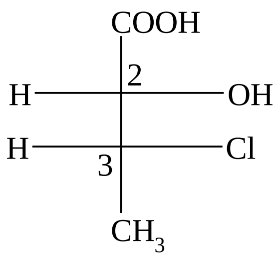
53. The absolute configuration of



is :

(1) (2R, 3S) (2) (2S, 3R) (3) (2S, 3S) (4) (2R, 3R)

53. (2)



(2 S 3 R)

54. 2–chloro–2–methylpentane on reaction with sodium methoxide in methanol yields :

(a) $C_2H_5CH_2\underset{CH_3}{\overset{CH_3}{\overset{|}{\underset{|}{C}}}}-OCH_3$

(b) $C_2H_5CH_2\underset{CH_3}{\underset{|}{C}}=CH_2$

(c) $C_2H_5CH=\underset{CH_3}{\underset{|}{C}}-CH_3$

(1) All of these (2) (a) and (c) (3) (c) only (4) (a) and (b)

54. (1)

$$CH_3-CH_2-CH_2-\underset{Cl}{\overset{CH_3}{\overset{|}{\underset{|}{C}}}}-CH_3 \xrightarrow{CH_3\bar{O}Na^+} CH_3-\underset{OMe}{\overset{CH_3}{\overset{|}{\underset{|}{C}}}}-CH_2-CH_2-CH_3$$

$$C_2H_5-CH_2-\underset{CH_3}{\underset{|}{C}}=CH_2+C_2H_5CH=\underset{CH_3}{\underset{|}{C}}-CH_3$$

55. The reaction of propene with HOCl ($Cl_2 + H_2O$) proceeds through the intermediate :
(1) $CH_3 - CH^+ - CH_2 - OH$ (2) $CH_3 - CH^+ - CH_2 - Cl$
(3) $CH_3 - CH(OH) - CH_2^+$ (4) $CH_3 - CHCl - CH_2^+$

55. (2)
$$CH_3 - CH = CH_2 + HOCl \longrightarrow CH_3 - \underset{\substack{|\\ OH}}{CH} - CH_2 - Cl$$
$$CH_3 - CH = CH_2 + Cl^{\oplus} \longrightarrow CH_3 - \overset{\oplus}{C}H - CH_2 - Cl$$

56. In the Hofmann bromamide degradation reaction, the number of moles of NaOH and Br_2 used per mole of amine produced are :
(1) One mole of NaOH and one mole of Br_2.
(2) Four moles of NaOH and two moles of Br_2.
(3) Two moles of NaOH and two moles of Br_2.
(4) Four moles of NaOH and one mole of Br_2.

56. (4)
$$R - \overset{\overset{O}{||}}{C} - NH_2 + Br_2 + 4NaOH \longrightarrow R - NH_2 + 2NaBr + Na_2CO_3$$

57. Which of the following statements about low density polythene is **FALSE** ?
(1) Its synthesis requires high pressure.
(2) It is a poor conductor of electricity.
(3) Its synthesis requires dioxygen or a peroxide initiator as a catalyst.
(4) It is used in the manufacture of buckets, dust–bins etc.

57. (4)
$$n(H_2C = CH_2) \Rightarrow \frac{200°C, 1500\ atm}{(O_2)} \quad \left[H_2C - CH_2 \right]_n$$

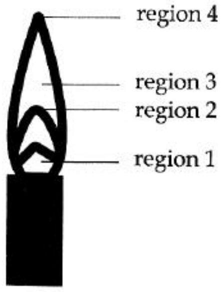
58. Thiol group is present in :
(1) Cytosine (2) Cystine (3) Cysteine (4) Methionine

58. (3)
$$HS - CH_2 - CH \begin{matrix} \diagup NH_2 \\ \diagdown COOH \end{matrix} \quad \text{(Cysteine)}$$

59. Which of the following is an anionic detergent ?
(1) Sodium stearate (2) Sodium lauryl sulphate
(3) Cetyltrimethyl ammonium bromide (4) Glyceryl oleate

59. (2)
$CH_3(CH_2)_{11}OSO_3Na$ = (Sodium Lauryl Sulphate)

60. The hottest region of Bunsen flame shown in the figure below is :



(1) region 1 (2) region 2 (3) region 3 (4) region 4

60. (2)
Region – 2 $\rightarrow$ blue flame

PART- C : MATHEMATICS

61. If $f(x) + 2y\left(\frac{1}{x}\right) = 3x$, $x \neq 0$, and $S = \{x \in R : f(x) = f(-x)\}$; then S:

(1) is an empty set
(2) contains exactly one element
(3) contains exactly two elements.
(4) contains more than two elements.

61. (3)

$$f(x) + 2f\left(\frac{1}{x}\right) = 3x$$

Replace x by $\frac{1}{x}$, $f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x}$

$$\Rightarrow \frac{3x - f(x)}{2} = \frac{\frac{3}{x} - 2f(x)}{1}$$

$$\Rightarrow 3x - f(x) = \frac{6}{x} - 4f(x)$$

$$\Rightarrow f(x) = \frac{2}{x} - x$$

$f(x) = f(-x) \quad \Rightarrow \frac{2}{x} - x = -\frac{2}{x} + x$

$$\Rightarrow \frac{4}{x} = 2x \Rightarrow x = \pm\sqrt{2}$$

62. A value of θ for which $\frac{2 + 3i\sin\theta}{1 - 2i\sin\theta}$ is purely imaginary, is:

(1) $\frac{\pi}{3}$
(2) $\frac{\pi}{6}$
(3) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$
(4) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

62. (4)

$$\frac{2 + 3i\sin\theta}{1 - 2i\sin\theta} \times \frac{1 + 2i\sin\theta}{1 + 2i\sin\theta} = \frac{\left(2 - 6\sin^2\theta\right) + i\left(7\sin\theta\right)}{1 + 4\sin^2\theta}$$

To be purely imaginary if

$$\frac{2 - 6\sin^2\theta}{1 + 4\sin^2\theta} = 0 \Rightarrow 2 = 6\sin^2\theta \Rightarrow \sin^2\theta = \frac{1}{3} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

63. The sum of all real values of x satisfying the equation

$(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$ is:

(1) 3
(2) −4
(3) 6
(4) 5

63. (1)

$$\left(x^2 - 5x + 5\right)^{x^2 + 4x - 60} = 1 = \left(x^2 - 5x + 5\right)^0$$

$\Rightarrow x^2 + 4x - 60 = 0 \qquad [a^x = a^y \Rightarrow x = y \text{ if } a \neq 1, 0, -1]$

$x = -10, 6$

& base $x^2 - 5x + 5 = 0$ or 1 or −1

If $x^2 - 5x + 5 = 0$ But it will not satisfy original equation.	$x^2 - 5x + 5 = 1$ $\therefore x = 4, 1$	$x^2 - 5x + 5 = -1$ $\therefore x = 2, 3$ x = 3 does not satisfy eqn.

Hence solutions are –10, 6, 4, 1, 2

So, sum of solutions = –10 + 6 + 4 + 1 + 2 = 3

64. If A = $\begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A adj A = A A^T, then 5a + b is equal to:

(1) –1 (2) 5 (C) 4 (4) 13

64. (2)

$$A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$$

A.adj A = A . A^T

$$\begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix} = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$$

$$\begin{bmatrix} 10a+3b & 0 \\ 0 & 10a+3b \end{bmatrix} = \begin{bmatrix} 25a^2+b^2 & 15a-2b \\ 15a-2b & 13 \end{bmatrix}$$

Equate, $10a + 3b = 25a^2 + b^2$

& $10a + 3b = 13$

& $15a - 2b = 0$

$\frac{a}{2} = \frac{b}{15}$ = k (let)

Solving a = $\frac{2}{5}$, b = 3

So, 5a + b = 5 × $\frac{2}{5}$ + 3 = 5

65. The system of linear equations

$x + \lambda y - z = 0$

$\lambda x - y - z = 0$

$x + y - \lambda z = 0$

has a non-trivial solution for:

(1) infinitely many values of λ.

(2) exactly one value of λ

(3) exactly two values of λ

(4) exactly three values of λ

65. (4)

$x + \lambda y - z = 0$

$\lambda x - y - z = 0$

$x + y - \lambda z = 0$

For non-trivial solution ⇒ Δ = 0

$$\Rightarrow \begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$\lambda + 1 - \lambda\left\{-\lambda^2 + 1\right\} - (\lambda + 1) = 0$

$\lambda(\lambda^2 - 1) = 0$

$\lambda = 0, \pm 1$

Exactly 3 values of λ.

66. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is:

(1) 46th (2) 59th (3) 52nd (4) 58th

66. (4)

A LL MS

A (LL MS) $\rightarrow \dfrac{4!}{2!} = \dfrac{24}{2} = 12$

L (AL MS) $\rightarrow$ 4! = 24

M (ALLS) $\rightarrow \dfrac{4!}{2!} = \dfrac{24}{2} = 12$

SA (MLL) $\rightarrow \dfrac{3!}{2!} = 3$

SL (ALM) $\rightarrow$ 3! = 6

Total words = 12 + 24 + 12 + 3 + 6 = 57

SMALL 58[th]

$\therefore$ the position of the word SMALL is 58[th].

67. If the number of terms in the expansion of $\left(1-\dfrac{2}{x}+\dfrac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is:

(1) 64 (2) 2187 (3) 243 (4) 729

67. (4)

Number of terms $= \dfrac{(n+1)(n+2)}{2} = 28$

$\Rightarrow$ n = 6

$\therefore \quad a_0 + \dfrac{a_1}{x} + \dfrac{a_2}{x^2} + \ldots + \dfrac{a_{2n}}{x^{2n}} = \left(1-\dfrac{2}{x}+\dfrac{4}{x^2}\right)^n$

Put x = 1, n = 6, $a_0 + a_1 + a_2 + \ldots + a_{2n} = 3^6 = 729$

68. If the 2[nd], 5[th] and 9[th] terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is:

(1) $\dfrac{8}{5}$ (2) $\dfrac{4}{3}$ (3) 1 (4) $\dfrac{7}{4}$

68. (2)

$t_2 = a + d$

$t_5 = a + 4d$

$t_9 = a + 8d$

Given t_2, t_5, t_9 are in G.P.

$(a + 4d)^2 = (a + d)(a + 8d)$

$a^2 + 16d^2 + 8ad = a^2 + 8d^2 + 9ad$

$8d^2 - ad = 0$

$d(8d - a) = 0$

As given non−constant AP. $\Rightarrow d \neq 0$

$\therefore \quad d = \dfrac{a}{8} \Rightarrow a = 8d$

so, A.P. is 8d, 9d, 10d, …..

Common ratio of G.P. = $\dfrac{t_5}{t_2} = \dfrac{a+4d}{a+d} = \dfrac{12d}{9d} = \dfrac{4}{3}$

69. If the sum of the first ten terms of the series

$\left(1\frac{3}{5}\right)^2+\left(2\frac{2}{5}\right)^2+\left(3\frac{1}{5}\right)^2+4^2+\left(4\frac{4}{5}\right)^2+\ldots\ldots$, is $\frac{16}{5}$ m, then m is equal to:

(1) 102 (2) 101 (3) 100 (4) 99

69. (2)

$$\left(1\frac{3}{5}\right)^2+\left(2\frac{2}{5}\right)^2+\left(3\frac{1}{5}\right)^2+4^2+\left(4\frac{4}{5}\right)^2+\ldots\ldots \quad \text{upto 10 terms}$$

$$=\left(\frac{8}{5}\right)^2+\left(\frac{12}{5}\right)^2+\left(\frac{16}{5}\right)^2+\left(\frac{20}{5}\right)^2+\left(\frac{24}{5}\right)^2+\ldots\ldots \text{ upto 10 terms.}$$

$(8)^2 + (12)^2 + (16)^2 + \ldots\ldots\ldots$ up to 10 terms

$T_n = [4(n+1)]^2$ where n varies from 1 to 10.

$= 16(n^2 + 2n + 1)$

$$\sum T_n = \sum_{n=1}^{10} 16\ (n^2 + 2n + 1)$$

$= 16\ [385 + 55(2) + 10]$

$= 16\ (505)$

$$\sum_{n=1}^{10} n^2 = \frac{n(n+1)(2n+1)}{6} = \frac{10\times11\times21}{6} = 385$$

$$\sum_{n=1}^{10} n = \frac{n(n+1)}{2} = \frac{10\times11}{2} = 55$$

$$\sum_{n=1}^{10} 1 = n = 10$$

$$\therefore\ \left(\frac{8}{5}\right)^2+\left(\frac{12}{5}\right)^2+\left(\frac{16}{5}\right)^2+\ldots\ldots \text{ upto 10 terms} = \frac{16\times505}{25}$$

It is given that $\frac{16\times505}{25} = \frac{16}{5}$ m

$\therefore\ m = \frac{505}{5} = 101$

70. Let $p = \lim\limits_{x\to0+}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$ then long p is equal to:

(1) 2 (2) 1 (3) $\frac{1}{2}$ (4) $\frac{1}{4}$

70. (3)

$$p = \lim_{x\to0^+}\left\{1+\tan^2\sqrt{x}\right\}^{\frac{1}{\tan^2\sqrt{x}}\times\frac{\tan^2\sqrt{x}}{2x}}$$

$$= e^{\lim\limits_{x\to0}\frac{\tan^2\sqrt{x}}{(\sqrt{x})^2}\times\frac{1}{2}} = e^{\frac{1}{2}}$$

$$\log_e p = \frac{1}{2}$$

71. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then:

(1) g is not differentiable at x = 0 (2) $g'(0) = \cos(\log 2)$

(3) $g'(0) = -\cos(\log 2)$ (4) g is differentiable at x = 0 and $g'(0) = -\sin(\log 2)$

71. (2)

$g(x) = |\log_e 2 - \sin(|\log_e 2 - \sin x|)|$

At x = 0, $g(x) = \log_e(2) - \sin(\log_e 2 - \sin x)$

$\therefore$ $g'(x) = \cos(\log_e(2) - \sin x) \times \cos(x)$

$\Rightarrow g'(0) = \cos(\log_e(2))$

72. Consider

$f(x) = \tan^{-1}\left(\sqrt{\dfrac{1+\sin x}{1-\sin x}}\right), x \in \left(0, \dfrac{\pi}{2}\right)$.

A normal to y = f(x) at $x = \dfrac{\pi}{6}$ also passes through the point:

(1) (0, 0) (2) $\left(0, \dfrac{2\pi}{3}\right)$ (3) $\left(\dfrac{\pi}{6}, 0\right)$ (4) $\left(\dfrac{\pi}{4}, 0\right)$

72. (2)

$$f(x) = \tan^{-1}(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) = \frac{\pi}{4} + \frac{x}{2}$$

If $x = \dfrac{\pi}{6}$, $f(\dfrac{\pi}{6}) = \dfrac{\pi}{4} + \dfrac{\pi}{12} = \dfrac{4\pi}{12} = \dfrac{\pi}{3}$

$$f'(x) = \frac{1}{2}$$

Normal $y - \dfrac{\pi}{3} = -2\left(x - \dfrac{\pi}{6}\right)$

So, $\left(0, \dfrac{2\pi}{3}\right)$ Satisfy

73. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then:

(1) $2x = (\pi + 4) r$ (2) $(4 - \pi)x = \pi r$ (3) $x = 2r$ (4) $2x = r$

73. (3)

Let length of two parts be ‘a’ and ‘2 – a’

As per condition given, we write

$a = 4x$ and $2 - a = 2\pi r$

$\therefore$ $x = \dfrac{a}{4}$ and $r = \dfrac{2-a}{2\pi}$

$\therefore$ $A(\text{square}) = \left(\dfrac{a}{4}\right)^2 = \dfrac{a^2}{16}$ and

$$A(\text{circle}) = \pi\left[\frac{(2-a)}{2\pi}\right]^2 = \frac{\pi\left(4 + a^2 - 4a\right)}{4\pi^2}$$

$$= \frac{\left(a^2 - 4a + 4\right)}{4\pi}$$

$$f(a)=\frac{a^2}{16}+\frac{a^2-4a+4}{4\pi}$$

$$\therefore \quad f(a)=\frac{a^2\pi+4a^2-16a+16}{16\pi}$$

$$\therefore \quad f'(a)=\frac{1}{16\pi}[2a\pi+8a-16]$$

$f'(a) = 0 \Rightarrow 2a\pi + 8a - 16 = 0 \Rightarrow 2a\,\pi + 8a = 16$

$$\therefore \qquad 2a(\pi+4)=16 \Rightarrow a=\frac{8}{\pi+4}$$

$$x=\frac{a}{4}=\frac{2}{\pi+4} \quad \text{and} \quad r=\frac{2-a}{2\pi}=\frac{2-\frac{8}{\pi+4}}{2\pi}$$

$$=\frac{2\pi+8-8}{2\pi(\pi+4)}=\frac{1}{\pi+4}$$

$$\therefore \quad x=\frac{2}{\pi+4} \quad \text{and} \quad r=\frac{1}{\pi+4} \Rightarrow x=2r$$

74. The integral $\int \frac{2x^{12}+5x^9}{(x^5+x^3+1)^3}\,dx$ is equal to:

(1) $\frac{-x^5}{(x^5+x^3+1)^2}+C$ (2) $\frac{x^{10}}{2(x^5+x^3+1)^2}+C$

(3) $\frac{x^5}{2(x^5+x^3+1)^2}+C$ (4) $\frac{-x^{10}}{2(x^5+x^3+1)^2}+C$

74. (2)

$$\int \frac{2x^{12}+5x^9}{\left[x^5\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)\right]^3}=\int \frac{2x^{12}+5x^9}{x^{15}\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3}\,dx$$

Dividing numerator and denominator by x^{15} we get,

$$= \int \frac{\frac{2}{x^3}+\frac{5}{x^6}}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3}\,dx$$

Put $\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)=t$

$$=\int \frac{-dt}{t^3}$$

$$=\frac{-t^{-3+1}}{-3+1}+C=\frac{1}{2}\times\frac{1}{t^2}+C$$

$$=\frac{1}{2}\frac{1}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^2}+C$$

$$=\frac{1}{2}\frac{x^{10}}{\left(x^5+x^3+1\right)^2}+C$$

75. $\lim\limits_{n\to\infty}\left(\dfrac{(n+1)(n+2)...3n}{n^{2n}}\right)^{1/n}$ is equal to:

(1) $\dfrac{18}{e^4}$ (2) $\dfrac{27}{e^2}$ (3) $\dfrac{9}{e^2}$ (4) $3\log 3 - 2$

75. (2)

Let P $= \lim\limits_{n\to\infty}\left[\dfrac{n+1}{n}\cdot\dfrac{n+2}{n}...\dfrac{n+2n}{n}\right]^{\frac{1}{n}}$

Taking log

Log P $= \lim\limits_{n\to\infty}\dfrac{1}{n}\sum\limits_{r=1}^{2n}\log\left(1+\dfrac{r}{n}\right)$

$= \int_0^2 \log(1+x)\,dx$

$= \log(1+x)\cdot x\ \Big]_0^2 - \int_0^2\dfrac{x}{1+x}\ dx$

$= 2\ln 3 - \int_0^2\left(1-\dfrac{1}{1+x}\right)dx = 2\ln 3 - \left[x\big]_0^2 - 1n(1+x)\big]_0^2\right]$

$= 2\ln 3 - [2 - 1n3] = 3\ln 3 - 2$

$= \ln 3^3 - \ln e^2$

$= \ln\left(\dfrac{27}{e^2}\right)$

P $= \dfrac{27}{e^2}$

76. The area (in sq. units) of the region $\{(x, y) : y^2 \ge 2x \text{ and } x^2 + y^2 \le 4x, x \ge 0, y \ge 0\}$ is:

(1) $\pi - \dfrac{4}{3}$ (2) $\pi - \dfrac{8}{3}$ (3) $\pi - \dfrac{4\sqrt{2}}{3}$ (4) $\dfrac{\pi}{2} - \dfrac{2\sqrt{2}}{3}$

76. (2)

$x^2 + y^2 \le 4x$ & $y^2 \ge 2x$

To find point of intersection,

$x^2 + y^2 = 4x \Rightarrow x^2 + 2x = 4x$

$\Rightarrow x^2 - 2x = 0 \Rightarrow x(x-2) = 0$

$\Rightarrow x = 0$ or $x = 2$

$\therefore$ $y = 0$ or $y = 2$

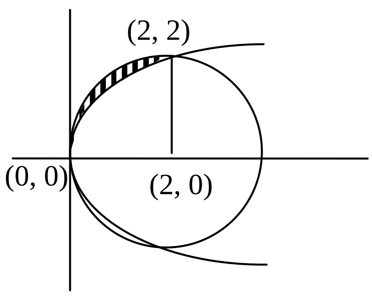
Solve $(x, y) = (0, 0)$ & $(x, y) = (2, 2)$

Area $= \int_0^2 (y_1 - y_2)\,dx = \int_0^2 y_{circle}\ dx - \int_0^2 y_{parabola}\ dx$

$= \dfrac{\pi\times r^2}{4} - \int_0^2\sqrt{2}\cdot x^{1/2}dx$

$= \dfrac{\pi\times 4}{4} - \sqrt{2}\cdot\dfrac{2}{3}\times x^{3/2}\Big|_0^2$

$= \pi - \dfrac{2\sqrt{2}}{3}\cdot\left(2^{3/2} - 0\right) = \pi - \dfrac{8}{3}$



77. If a curve y = f(x) passes through the point (1, –1) and satisfies the differential equation, y(1 + xy) dx = x dy, then $f\left(-\frac{1}{2}\right)$ is equal to:

(1) $-\frac{2}{5}$ (2) $-\frac{4}{5}$ (3) $\frac{2}{5}$ (4) $\frac{4}{5}$

77. (4)

$$\frac{y}{x}(1+xy)=\frac{dy}{dx}$$

$$y = vx \quad \Rightarrow \quad \frac{y}{x}=v$$

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$v(1+vx^2)=v+x\frac{dv}{dx}$$

$$v^2x^2=x\frac{dv}{dx}$$

$$v^2x=\frac{dv}{dx}$$

$$\int x dx=\int\frac{1}{v^2}dv$$

$$\frac{x^2}{2}=-\frac{1}{v}+c$$

$$\frac{x^2}{2}=-\frac{x}{y}+c$$

Put (1, –1)

$$\frac{1}{2}=\frac{1}{1}+c \quad \Rightarrow \quad c=\frac{-1}{2}$$

$$\frac{x^2}{2}=-\frac{x}{y}-\frac{1}{2}$$

We have to find $f\left(-\frac{1}{2}\right)$

Put $x=-\frac{1}{2}$

$$\frac{\left(-\frac{1}{2}\right)^2}{2}=\frac{-\left(-\frac{1}{2}\right)}{y}-\frac{1}{2}$$

$$\frac{1}{8}=\frac{1}{2y}-\frac{1}{2}$$

$$y=\frac{4}{5}$$

78. Two sides of a rhombus are along the lines, x – y + 1 = 0 and 7x – y – 5 = 0. If its diagonals intersect at (–1, –2), then which one of the following is a vertex of this rhombus?

(1) (–3, –9) (2) (–3, –8) (3) $\left(\frac{1}{3},-\frac{8}{3}\right)$ (4) $\left(-\frac{10}{3},-\frac{7}{3}\right)$

78. (3)

Coordinates of A ≡ (1, 2)

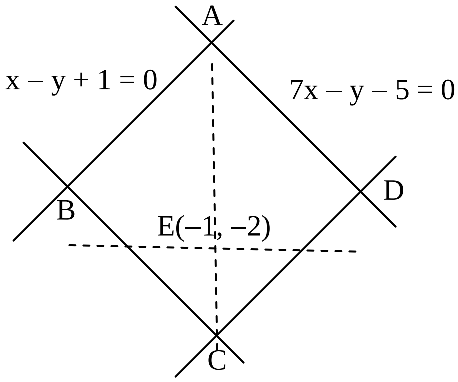
∴ Slope of AE = 2

⇒ Slope of BD = $-\frac{1}{2}$

⇒ Eq. of BD is $\frac{y+2}{x+1} = -\frac{1}{2}$

⇒ x + 2y + 5 = 0

∴ Co-ordinates of D = $\left(\frac{1}{3}, \frac{-8}{3}\right)$



79. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on:

(1) a circle
(2) an ellipse which is not a circle
(3) a hyperbola
(4) a parabola

79. (4)

$x^2 + y^2 - 8x - 8y - 4 = 0$

Centre (4, 4)

Radius $= \sqrt{4^2 + 4^2 + 4} = 6$

Let centre of the circle is (h, k)

$\sqrt{(h-4)^2 + (k-4)^2} = (6 + k)$

$(h-4)^2 + (k-4)^2 = (6+k)^2$

$h^2 - 8h + 16 + k^2 - 8k + 16 = 36 + k^2 + 12k$

$h^2 - 8h - 20k - 4 = 0$

$x^2 - 8x - 20y - 4 = 0$

Which is an equation of parabola

80. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at (–3, 2), then the radius of S is:

(1) $5\sqrt{2}$
(2) $5\sqrt{3}$
(3) 5
(4) 10

80. (2)

$x^2 + y^2 - 4x + 6y - 12 = 0$

Centre (2, –3)

Radius $\sqrt{4+9+12} = 5$

Distance b/w two centres $c_1(2, -3)$ and $c_2(-3, 2)$

$d = \sqrt{(2+3)^2 + (-3-2)^2} = \sqrt{50}$

Radius of (S) $= \sqrt{5^2 + \left(\sqrt{50}\right)^2} = \sqrt{75} = 5\sqrt{3}$

81. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y + 6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is:

(1) $x^2 + y^2 - 4x + 8y + 12 = 0$
(2) $x^2 + y^2 - x + 4y - 12 = 0$
(3) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$
(4) $x^2 + y^2 - 4x + 9y + 18 = 0$

81. (1)

Normal at $P(at^2, 2at)$ is $y + tx = 2at + at^3$

Given it passes $(0, -6)$

$\Rightarrow$ $-6 = 2at + at^3$ $(a = 2)$

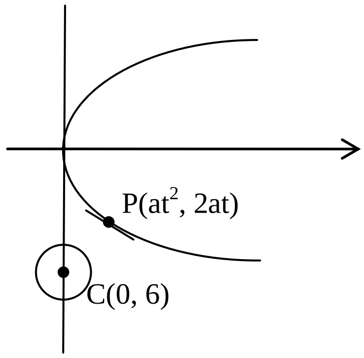
$-6 = 4t + 2t^3$

$t^3 + 2t + 3 = 0$

$t = -1$

so, $P(a, -2a) = (2, -4)$. [a = 1)

radius of circle = CP = $\sqrt{2^2 + (-4+6)^2} = 2\sqrt{2}$



Circle is $(x - 2)^2 + (y + 4)^2 = \left(2\sqrt{2}\right)^2$

$x^2 + y^2 - 4x + 8y + 12 = 0$

82. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half the distance between its foci, is:

(1) $\frac{4}{3}$ (2) $\frac{4}{\sqrt{3}}$ (3) $\frac{2}{\sqrt{3}}$ (4) $\sqrt{3}$

82. (3)

$\ell = \frac{2b^2}{a} = 8 \Rightarrow$ $b^2 = 4a$ … (1)

$2b = \frac{1}{2}(2ae)$

$2b = ae$ … (2)

Squaring eqn. (2), we get

$4b^2 = a^2e^2 \Rightarrow 4\frac{b^2}{a^2} = e^2$ and we know that $e^2 = 1 + \frac{b^2}{a^2} \Rightarrow \frac{b^2}{a^2} = e^2 - 1$

$4(e^2 - 1) = e^2$

$4e^2 - e^2 = 4$

$3e^2 = 4$

$e = \frac{2}{\sqrt{3}}$

83. The distance of the point $(1, -5, 9)$ from the plane $x - y + z = 5$ measured along the line $x = y = z$ is :

(1) $3\sqrt{10}$ (2) $10\sqrt{3}$ (3) $\frac{10}{\sqrt{3}}$ (4) $\frac{20}{3}$

83. (2)

Let Q $(1, -5, 9)$

$$\frac{x-0}{1} = \frac{y-0}{1} = \frac{z-0}{1}$$

Line is $\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = r$ (say)

Any pt on line we can take P $(r + 1, r - 5, r + 9)$

So, Pt satisfy Plane

$\Rightarrow (r + 1) - (r - 5) + (r + 9) = 5$

$r = -10$

So, Point P = $(-9, -15, -1)$

Distance is $PQ = \sqrt{(10)^2 + (10)^2 + (10)^2} = 10\sqrt{3}$

84. If the line, $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in the plane, $\ell x + my - z = 9$, then $\ell^2 + m^2$ is equal to :

(1) 26 (2) 18 (3) 5 (4) 2

84. (4)

Point on line is P = (3, –2, –4)

'P' lies on $\ell x + my - z = 9$

$\Rightarrow 3\ell - 2m + 4 = 9$

$3\ell - 2m = 5$ …(1)

As line lies on plane $\Rightarrow$ $2 \times \ell + m \times (-1) + 3 \times (-1) = 0$

$2\ell - m = 3$ …(3)

Solving $\ell = 1, m = -1$

So, $\ell^2 + m^2 = 2$.

85. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\sqrt{3}}{2}(\vec{b}+\vec{c})$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is :

(1) $\frac{3\pi}{4}$ (2) $\frac{\pi}{2}$ (3) $\frac{2\pi}{3}$ (4) $\frac{5\pi}{6}$

85. (4)

$\vec{a}\times(\vec{b}\times\vec{c})=\frac{\sqrt{3}}{2}(\vec{b}+\vec{c})$

$(\bar{a}\cdot\vec{c})\bar{b}-(\bar{a}\cdot\vec{b})\bar{c}=\frac{\sqrt{3}}{2}\vec{b}+\frac{\sqrt{3}}{2}\vec{c}$

Equate $\vec{a}\cdot\vec{c}=\frac{\sqrt{3}}{2}$ & $-(\bar{a}\cdot\bar{b})=\frac{\sqrt{3}}{2}$

$|\bar{a}|\,|\bar{b}|\cos\theta=-\frac{\sqrt{3}}{2}$

$\cos\theta = -\frac{\sqrt{3}}{2}$ as $\bar{a}$ & $\bar{b}$ unit vectors

$\theta = \frac{5\pi}{6}$

86. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true?

(1) $3a^2 - 26a + 55 = 0$ (2) $3a^2 - 32a + 84 = 0$

(3) $3a^2 - 34a + 91 = 0$ (4) $3a^2 - 23a + 44 = 0$

86. (2)

S.D. = $\sqrt{\frac{\sum d^2}{N}-\left(\frac{\sum d}{N}\right)^2}$

$\Rightarrow \frac{4+9+a^2+121}{4} - \left(\frac{2+3+a+11}{4}\right)^2 = \frac{49}{4}$

$\Rightarrow (4a^2 + 536) - (a^2 + 32a + 256) = 196$

$\Rightarrow 3a^2 - 32a + 84 = 0$

87. Let two fair six–faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT true?

(1) E_1 and E_2 are independent (2) E_2 and E_3 are independent

(3) E_1 and E_3 are independent (4) E_1, E_2 and E_3 are independent

87. (4)

$P(E_1) = \dfrac{1}{6}$

$P(E_2) = \dfrac{1}{6}$

$P(E_3) = \dfrac{1}{2}$

$P(E_1 \cap E_2) = \dfrac{1}{36}$

$P(E_2 \cap E_3) = \dfrac{1}{12}$

$P(E_3 \cap E_1) = \dfrac{1}{12}$

$P(E_1 \cap E_2 \cap E_3) = 0$

$\therefore$ E_1, E_2, E_3 are not independent

88. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x = \cos 3x + \cos 4x = 0$, is :

(1) 3 (2) 5 (3) 7 (4) 9

88. (3)

We have, $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

$$(\cos x + \cos 4x) + (\cos 2x + \cos 3x) = 0$$

$$2\cos\frac{5x}{2}\cdot\cos\frac{3x}{2} + 2\cos\frac{5x}{2}\cdot\cos\frac{x}{2} = 0$$

$$2\cos\frac{5x}{2}\left[\cos\frac{3x}{2} + \cos\frac{x}{2}\right] = 0$$

$$\cos\frac{5x}{2} = 0 \Rightarrow \frac{5x}{2} = (2n+1)\frac{\pi}{2} \Rightarrow x = (2n+1)\frac{\pi}{5} \Rightarrow x = \frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}$$

Or

$$\cos\frac{3x}{2} + \cos\frac{x}{2} = 0$$

$$4\cos^3\frac{x}{2} - 3\cos\frac{x}{2} + \cos\frac{x}{2} = 0 \Rightarrow 4\cos^3\frac{x}{2} - 2\cos\frac{x}{2} = 0 \Rightarrow 2\cos^3\frac{x}{2} - \cos\frac{x}{2} = 0$$

$$2\cos\frac{x}{2}\left[2\cos^2\frac{x}{2} - 1\right] = 0 \Rightarrow 2\cos\frac{x}{2}[\cos x] = 0$$

$$\cos\frac{x}{2} = 0 \Rightarrow \frac{x}{2} = (2n+1)\frac{\pi}{2} \Rightarrow x = (2n+1)\pi \quad \text{or} \quad \cos x = 0 \Rightarrow x = (2n+1)\frac{\pi}{2}$$

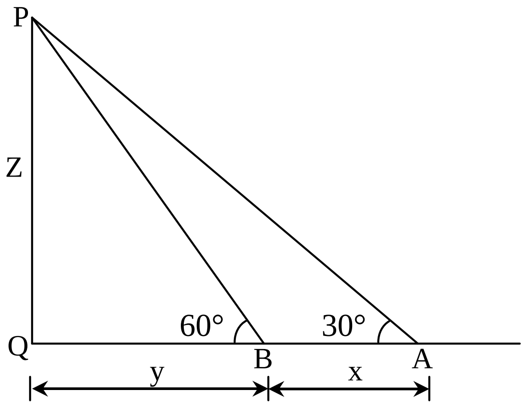
$x = \pi$ or $x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

Solution are $x = \dfrac{\pi}{5}, \dfrac{3\pi}{5}, \pi, \dfrac{7\pi}{5}, \dfrac{9\pi}{5}, \dfrac{\pi}{2}, \dfrac{3\pi}{2}$ … $(0 \le x < 2\pi)$

89. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar, is :

(1) 6 (2) 10 (3) 20 (4) 5

89. (4)



Let AB = x, BQ = y, PQ = z

$$\tan 30° = \frac{z}{x+y} \Rightarrow z = \frac{x+y}{\sqrt{3}}$$

$$\tan 60° = \frac{z}{y} \Rightarrow z = \sqrt{3}y$$

$$\therefore \quad \frac{x+y}{\sqrt{3}} = \sqrt{3}y \Rightarrow x + y = 3y \Rightarrow x = 2y$$

$$\therefore \quad y = \frac{x}{2}$$

To go x, it takes 10 minutes.

$\therefore$ To go y, it takes 5 minutes.

90. The Boolean Expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to :

(1) $\sim p \wedge q$ (2) $p \wedge q$ (3) $p \vee q$ (4) $p \vee \sim q$

90. (3)

$(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \wedge q$	$(p \wedge \sim q) \vee q$	$(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$	$p \vee q$
T	T	F	F	F	F	T	T	T
T	F	F	T	T	F	T	T	T
F	T	T	F	F	T	T	T	T
F	F	T	T	F	F	F	F	F

❑ ❑ ❑ ❑ ❑ ❑