

c. A multiple of 6

28. If $P(9a-2, -b)$ divides the line segment joining $A(3a+1, -3)$ and $B(8a, 5)$ in the ratio 3:1. Find the values of a & b .
29. Find the coordinates of the points which divide the line segment joining $A(2, -3)$ and $B(-4, -6)$ into three equal parts.
30. Four equal circles are described at the four corners of a square so that each touches two of the others. The shaded area enclosed between the circles is $\frac{24}{7}\text{cm}^2$. Find the radius of each circle.
31. A right triangle having sides 15 cm and 20 cm is made to revolve about its hypotenuse. Find the Volume and Surface Area of the double cone so formed. (Use $\pi=3.14$).

SAMPLE QUESTION PAPER

Summative Assessment – II

Class – X (2015–16)

Mathematics - Marking Scheme

Section A

Ans 1. $K = 3$ [1]

Ans 2. $\sqrt{200}$ [1]

Ans 3. $\frac{47}{50}$ [1]

Ans 4. $a = -9$ [1]

Section B

Ans 5.

2 is the root of $x^2+kx+12=0$

$$\Rightarrow (2)^2 + 2k+12=0$$

$$\Rightarrow 2k+16=0$$

$$k=-8$$

[1/2]

Put $k=-8$ in $x^2+kx+q=0$

$$\Rightarrow x^2 -8x+q=0$$

[1/2]

For equal roots

$$(-8)^2 - 4(1)q=0$$

[1/2]

$$64 -4q =0$$

$$4q = 64$$

$$q = 16$$

[1/2]

Ans 6.

Two digit numbers which are divisible by 7 are

14,21,28,.....98.

[1/2]

It forms an A.P.

$$a=14, d=7, a_n=98$$

[1/2]

$$a_n=a+(n-1) d$$

$$98=14+(n-1)7$$

[1/2]

$$98-14=7n-7$$

$$84+7=7n$$

$$7n=91$$

$$n=13$$

[1/2]

Ans 7

Let P(x,y) is equidistant from A(-5,3) and B(7,2)

$$AP=BP$$

[1/2]

$$\Rightarrow \sqrt{(x+5)^2 + (y-3)^2} = \sqrt{(x-7)^2 + (y-2)^2}$$

$$\Rightarrow x^2 + 10x + 25 + y^2 - 6y + 9 = x^2 - 14x + 49 + y^2 - 4y + 4$$

[1/2]

$$10x - 6y + 34 = -14x - 4y + 53$$

$$10x + 14x - 6y + 4y = 53 - 34$$

$$24x - 2y = 19$$

$24x - 2y - 19 = 0$ is the required relation

[1]

Ans 8

Perimeter of the shaded region

$$= AD + BC + \text{lengths of semi circles APB \& CPD}$$

[1]

$$= 21 + 21 + 2\left(2\pi \times \frac{22}{7} \times \frac{21}{2}\right)$$

[1/2]

$$= 42 + 2(66)$$

$$= 42 + 132$$

$$= 174 \text{ cm}$$

[1/2]

Ans 9

Let the water level raised in cylindrical vessel be h cm

Volume of Sphere = Volume of water displaced in cylinder [1/2]

$$\frac{4}{3}\pi (3)^3 = \pi (6)^2 h \quad [1]$$

$$\frac{4}{3} \times 27 = 36 h$$

$$36 = 36h$$

$$h = 1 \text{ cm} \quad [1/2]$$

Ans 10

Volume of Coin = $\pi r^2 h$

$$= \frac{22}{7} \times (0.75)^2 \times 0.2 \text{ cm}^3 \quad [1/2]$$

$$\text{Volume of Cylinder} = \frac{22}{7} \times (2.25)^2 \times 10 \text{ cm}^3 \quad [1/2]$$

$$\text{No. of Coins} = \text{Volume of Cylinder} / \text{Volume of Coin} \quad [1/2]$$

$$= \left(\frac{22}{7} \times (2.25)^2 \times 10 \right) / \left(\frac{22}{7} \times (0.75)^2 \times 0.2 \right)$$

$$= 450 \quad [1/2]$$

Section C

Ans 11

$$\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$$

$$\Rightarrow \frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b} \quad [1/2]$$

$$\Rightarrow \frac{x-(a+b+x)}{x(a+b+x)} = \frac{a+b}{ab} \quad [1/2]$$

$$\Rightarrow \frac{x-a-b-x}{x(a+b+x)} = \frac{a+b}{ab}$$

$$\Rightarrow \frac{-(a+b)}{x(a+b+x)} = \frac{a+b}{ab} \quad [1/2]$$

$$\Rightarrow x(a+b+x) = -ab$$

$$\Rightarrow x^2 + (a+b)x - ab = 0$$

$$\Rightarrow (x+a)(x+b) = 0 \quad [1]$$

$$\Rightarrow x = -a \text{ or } x = -b \quad [1/2]$$

Ans 12

$$S_n = \frac{3n^2 + 13n}{2}$$

$$a_n = S_n - S_{n-1} \quad [1]$$

$$\Rightarrow a_{25} = S_{25} - S_{24}$$

$$= \frac{3(25)^2 + 13(25)}{2} - \frac{3(24)^2 + 13(24)}{2} \quad [1]$$

$$= \frac{1}{2} \{3(25^2 - 24^2) + 13(25 - 24)\}$$

$$= \frac{1}{2} (3 \times 49 + 13) = 80 \quad [1]$$

Ans 13

Let the first term of A.P be a and common difference be d .

$$a_9 = 7a_2$$

$$\Rightarrow a + 8d = 7(a + d) \quad \dots\dots\dots (1) \quad [1/2]$$

$$a_{12} = 5a_3 + 2$$

$$\Rightarrow a + 11d = 5(a + 2d) + 2 \quad \dots\dots\dots (2) \quad [1]$$

$$\text{From (1), } a + 8d = 7a + 7d$$

$$-6a + d = 0 \quad \dots\dots\dots (3)$$

$$\text{From (2), } a + 11d = 5a + 10d + 2$$

$$-4a + d = 2 \quad \dots\dots\dots (4)$$

Subtracting (4) from (3)

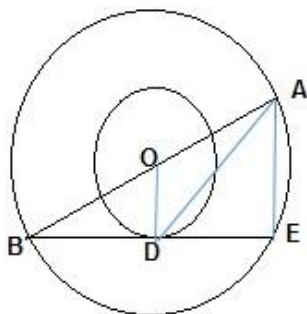
$$-2a = -2$$

$$\Rightarrow a = 1 \quad [1]$$

From (3),

$$-6 + d = 0$$

$$d = 6 \quad [1/2]$$

Ans 14

Join OD and AE [1/2]

$$\angle ODB = 90^\circ \quad \dots\dots\dots (\text{radius is perpendicular to tangent at})$$

point of contact)

$\angle AEB = 90^\circ$ (angle in a semicircle)

$OD \parallel AE$ (Corresponding Angles) [1/2]

$$AE = 2 \times OD$$

$$= 2 \times 8 = 16 \text{ cm} \quad [1/2]$$

$$\text{In right } \triangle ODB, BD^2 = 13^2 - 8^2 \quad [1/2]$$

$$= 169 - 64 = 105$$

$$BD = \sqrt{105} \text{ cm}$$

$$DE = \sqrt{105} \text{ cm} \quad [1/2]$$

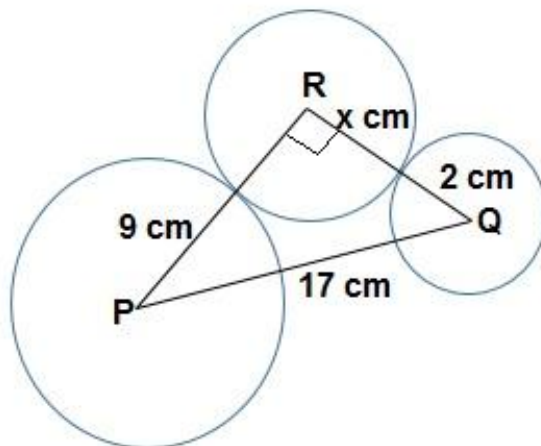
$$\text{In right } \triangle AED, AD^2 = AE^2 + DE^2$$

$$= 16^2 + (\sqrt{105})^2$$

$$= 256 + 105 = 361$$

$$AD = 19 \text{ cm} \quad [1/2]$$

Ans15



[1/2]

In right $\triangle PQR$, by Pythagoras theorem

$$PQ^2 = PR^2 + RQ^2$$

$$\Rightarrow 17^2 = (x+9)^2 + (x+2)^2 \quad [1]$$

$$\Rightarrow x^2 + 11x - 102 = 0 \quad [1/2]$$

$$\Rightarrow x^2 + 17x - 6x - 102 = 0$$

$$\Rightarrow x(x+17) - 6(x+17) = 0$$

$$\Rightarrow (x-6)(x+17) = 0$$

$$\Rightarrow x = 6 \text{ or } x = -17 \quad [1/2]$$

$$\Rightarrow x = 6 \text{ cm (x can't be negative)} \quad [1/2]$$

Ans 16

For correct construction [3]

Ans 17

Total number of cards = 52

Number of non face cards = 52 – 12

= 40

$$P(\text{ non-face cards}) = \frac{40}{52} = \frac{10}{13} \quad [1]$$

Number of black kings = 2

Number of red queens = 2

$$P(\text{ a black King or a red queen}) = \frac{4}{52} \quad [1]$$

Number of spade cards = 13

$$P(\text{ Spade cards}) = \frac{13}{52} \quad [1]$$

Ans 18



$$\angle AOB = 60^\circ$$

[1/2]

Area of shaded region

$$= \text{Area of } \triangle AOB + \text{Area of major sector of circle}$$

[1]

$$= \frac{\sqrt{3}}{4} (12)^2 + \frac{300^\circ}{360^\circ} \times \frac{22}{7} \times (6)^2 \text{ cm}^2$$

[1]

$$= 36\sqrt{3} + \frac{660}{7} \text{ cm}^2$$

[1/2]

Ans 19

Length of water that flows out in 30 minutes

$$= (0.7 \times 100 \times 60 \times 30) \text{ cm}$$

$$= 126000 \text{ cm}$$

[1]

Volume of water that flows out in 30 minutes

$$= \pi (1)^2 \times 126000 \text{ cm}^3$$

$$= 126000 \pi \text{ cm}^3$$

[1/2]

Let the depth of water in the tank be x cm

Volume of water in tank

$$= \pi (40)^2 \times x \text{ cm}^3$$

[1/2]

According to the question

$$\pi (40)^2 \times x = 126000\pi$$

[1/2]

$$\Rightarrow x = 78.75 \text{ cm}$$

[1/2]

Ans 20

Let R and r be the radii of the circular ends of the frustum. ($R > r$)

$$2\pi R = 207.24$$

$$R = 207.24 / (2 \times 3.14)$$

$$R = 33 \text{ cm} \quad [1]$$

$$2\pi r = 169.56 \text{ cm}$$

$$r = 169.56 / (2 \times 3.14)$$

$$r = 27 \text{ cm} \quad [1/2]$$

$$l^2 = h^2 + (R-r)^2$$

$$= 8^2 + (33-27)^2 \quad [1/2]$$

$$l = 10 \text{ cm} \quad [1/2]$$

Whole surface area of the frustum

$$= \pi (R^2 + r^2 + (R+r)l)$$

$$= 3.14 ((33)^2 + (27)^2 + (33+27)10) \quad [1/2]$$

$$= 3.14 (1089 + 729 + 600)$$

$$= 3.14 \times 2418 \text{ cm}^2$$

$$= 7592.52 \text{ cm}^2 [1]$$

Section D

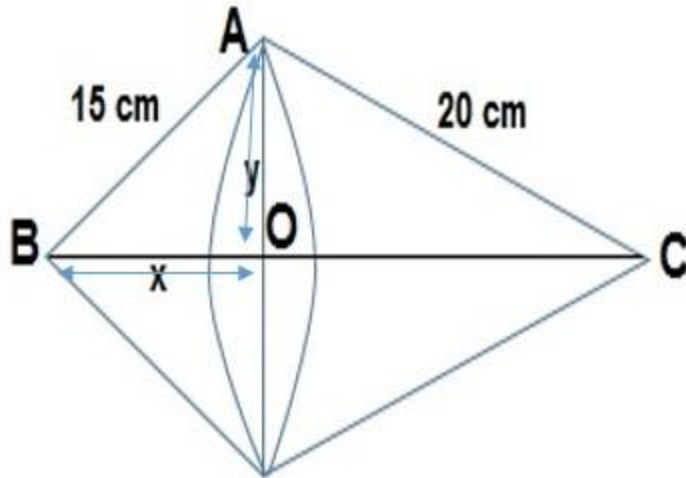
Ans 21

Let the total number of students be x

$$\frac{3}{8}x = 16 + \sqrt{x} \quad [1]$$

$$\Rightarrow \frac{3}{8}x - 16 = \sqrt{x}$$

$$\Rightarrow 3x - 128 = 8\sqrt{x}$$



In right Δ BAC, by Pythagoras theorem

$$\begin{aligned} BC^2 &= AB^2 + AC^2 \\ &= 15^2 + 20^2 \\ &= 225 + 400 \\ &= 625 \end{aligned}$$

$$BC = 25\text{cm}$$

[1/2]

Let OA = y cm and OB = x cm

$$x^2 + y^2 = 15^2$$

[1/2]

$$(25-x)^2 + y^2 = 20^2$$

[1/2]

Solving we get x=9 and y=12

[1/2]

\therefore OA = 12 cm and OB = 9 cm

$$\text{Volume of double cone} = \frac{1}{3}\pi (OA)^2 \times OC + \frac{1}{3}\pi (OA)^2 \times OB$$

$$= \frac{1}{3} \times 3.14 \times (12)^2 \times (OC + OB)$$

[1/2]

$$= \frac{1}{3} \times 3.14 \times 144 \times 25$$

$$= 3768 \text{ cm}^3$$

[1/2]

$$\text{Surface area of double cone} = \pi \times OA \times AC + \pi \times OA \times AB$$

$$= \pi \times 12 \times 20 + \pi \times 12 \times 15$$

[1/2]

$$= 420 \pi \text{ cm}^2$$

$$= 1318.8 \text{ cm}^2$$

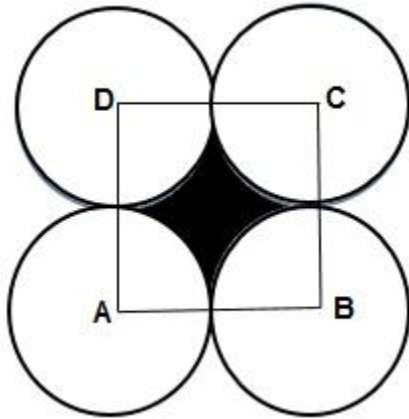
[1/2]

$$Q(x_2, y_2) = \left(\frac{2X(-4) + 1X(2)}{2+1}, \frac{2X(-6) + 1X(-3)}{2+1} \right) \quad [1]$$

$$= \left(\frac{-8+2}{3}, \frac{-12+(-3)}{3} \right)$$

$$= (-2, -5) \quad [1]$$

Ans 30



Let r cm be the radius of each circle.

$$\text{Area of square} - \text{Area of 4 sectors} = \frac{24}{7} \text{ cm}^2 \quad [1/2]$$

$$\Rightarrow (2r)^2 - 4 \left(\frac{90^\circ}{360^\circ} \times \pi r^2 \right) = \frac{24}{7} \quad [1]$$

$$\Rightarrow 4r^2 - \frac{22}{7}r^2 = \frac{24}{7} \quad [1/2]$$

$$\Rightarrow \frac{28r^2 - 22r^2}{7} = \frac{24}{7}$$

$$\Rightarrow 6r^2 = 24$$

$$\Rightarrow r^2 = 4 \quad [1]$$

$$\Rightarrow r = \pm 2$$

$$\Rightarrow \text{radius of each circle is 2 cm (r cannot be negative)} \quad [1]$$

Ans 31

Total number of cards = 48

$$\text{Probability of an event} = \frac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} \quad [1]$$

Number of cards divisible by 7 = 7

$$P(\text{cards divisible by 7}) = \frac{7}{48} \quad [1]$$

Number of cards having a perfect square = 6

$$P(\text{cards having a perfect square}) = \frac{6}{48} = \frac{1}{8} \quad [1]$$

Number of multiples of 6 from 3 to 50 = 8

$$P(\text{multiple of 6 from 3 to 50}) = \frac{8}{48} = \frac{1}{6} \quad [1]$$

Ans 28

By Section formula

$$9a - 2 = \frac{3(8a) + 1(3a + 1)}{3 + 1} \dots\dots\dots(1) \quad [1]$$

$$-b = \frac{3(5) + 1(-3)}{3 + 1} \dots\dots\dots(2) \quad [1]$$

From (2)

$$-b = \frac{15 - 3}{4} = 3$$

$$b = -3 \quad [1]$$

From (1)

$$9a - 2 = \frac{24a + 3a + 1}{4}$$

$$4(9a - 2) = 27a + 1$$

$$36a - 8 = 27a + 1$$

$$9a = 9$$

$$a = 1 \quad [1]$$

Ans 29

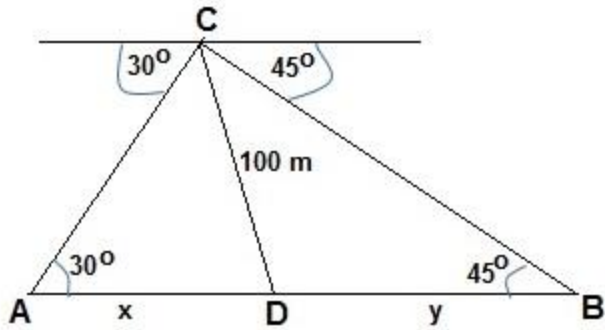
Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points which divide AB in three equal parts.

By Section formula

$$P(x_1, y_1) = \left(\frac{1X(-4) + 2X(2)}{1+2}, \frac{1X(-6) + 2X(-3)}{1+2} \right) \quad [1]$$

$$= \left(\frac{-4 + 4}{3}, \frac{-6 + (-6)}{3} \right)$$

$$= (0, -4) \quad [1]$$



Correct diagram

[1]

In right $\triangle ADC$

$$\tan 30^\circ = \frac{CD}{AD}$$

[1/2]

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{x}$$

$$\Rightarrow x = 100\sqrt{3} \dots\dots\dots (1)$$

[1/2]

In right $\triangle BDC$

$$\tan 45^\circ = \frac{CD}{DB}$$

[1/2]

$$\Rightarrow 1 = \frac{100}{y}$$

[1/2]

$$\Rightarrow y = 100 \text{ m}$$

Distance between two cars

$$= AB = AD + DB$$

[1/2]

$$= (100\sqrt{3} + 100)\text{m}$$

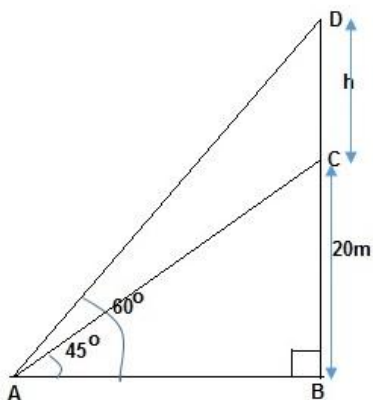
$$= (100 \times 1.73 + 100)\text{m}$$

$$= (173 + 100) \text{ m}$$

$$= 273 \text{ m}$$

[1/2]

Ans 26



Let BC be building of height 20m and CD be the tower of height h m.

Let A be point on the ground at a distance of x m from the foot of the building. [1]

In right $\triangle ABC$,

$$\tan 45^\circ = \frac{BC}{AB}$$

$$\Rightarrow 1 = \frac{20}{x}$$

$$\Rightarrow x = 20 \text{m} \dots\dots\dots(1) \quad [1]$$

In right $\triangle ABD$,

$$\tan 60^\circ = \frac{BD}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{h+20}{x}$$

$$\Rightarrow \sqrt{3} = \frac{h+20}{20} \quad [1]$$

$$\Rightarrow h = 20\sqrt{3} - 20$$

$$= 20(\sqrt{3} - 1)$$

$$= 20 \times 0.732$$

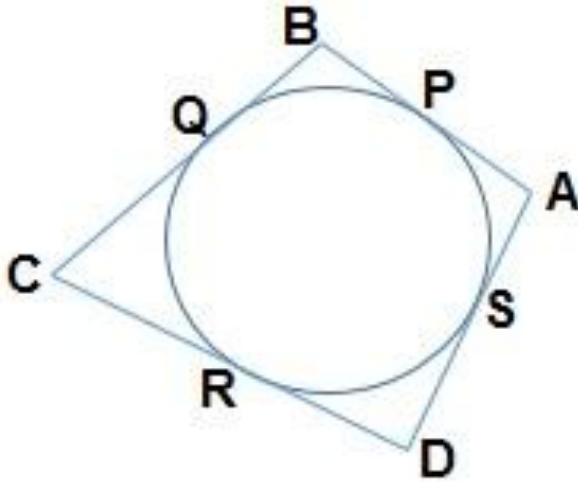
$$= 14.64 \text{ m}$$

$$\text{Height of tower} = 14.64 \text{ m} \quad [1]$$

Ans 27

Correct Proof of the theorem

[2]



In the given figure,

Using the above theorem

$$AP = AS \dots\dots\dots(1)$$

$$BP = BQ \dots\dots\dots(2)$$

$$DR = DS \dots\dots\dots(3)$$

$$CR = CQ \dots\dots\dots(4)$$

[1]

Adding (1), (2),(3) and (4), we get

$$(AB+BP) + (DR+CR) = (AS+DS) + (BQ+CQ)$$

$$\Rightarrow AB + CD = AD + BC$$

[1]

Ans 24

For correct constructions

[4]

Ans 25

$$\Rightarrow 3x - 8\sqrt{x} - 128 = 0 \quad [1/2]$$

Let $\sqrt{x} = y$

$$3y^2 - 8y - 128 = 0 \quad [1/2]$$

$$\Rightarrow 3y^2 - 24y + 16y - 128 = 0$$

$$\Rightarrow 3y(y-8) + 16(y-8) = 0$$

$$\Rightarrow (y-8)(3y+16) = 0$$

$$y = 8 \text{ or } y = -16/3$$

$$y = 8 \Rightarrow x = 64$$

$$y = -16/3 \Rightarrow x = 256/9$$

$$\text{number of students} = 64 \quad [1]$$

$$\text{Values inculcated} \quad [1]$$

Ans 22

$$a = 8, d = 1/3 \text{ years}, S_n = 168 \quad [1/2]$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\Rightarrow 168 = \frac{n}{2} [2(8) + (n-1)\frac{1}{3}] \quad [1/2]$$

$$n^2 + 47n - 1008 = 0 \quad [1]$$

$$\Rightarrow n^2 + 63n - 16n - 1008 = 0$$

$$\Rightarrow (n-16)(n+63) = 0$$

$$\Rightarrow n = 16 \text{ or } n = -63$$

$$n = 16 \quad (n \text{ cannot be negative}) \quad [1]$$

$$\text{Age of the eldest participant} = a + 15d = 13 \text{ years} \quad [1]$$

Ans 23