

Marking Scheme (Sample Paper)

Section A

Q1. $-\frac{24}{25}$ (1)

Q2. We have $1, 2 \in \mathbb{Z}$ such that $1 * 2 = 8$ and $2 * 1 = 2$. This implies that $1 * 2 \neq 2 * 1$. Hence, $*$ is not commutative. (1)

Q3. $\begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & -2 \\ 3 & 2 & 0 \end{bmatrix}$ (1)

Q4. 4% (1)

Q5. 5 (1)

Q6. -14 (1)

Section B

Q7. Here, $\text{fof} : W \rightarrow W$ is such that, if n is odd, $\text{fof}(n) = f(f(n)) = f(n-1) = n-1+1 = n$ (1+1/2)

and if n is even, $\text{fof}(n) = f(f(n)) = f(n+1) = n+1-1 = n$ (1+1/2)

Hence, $\text{fof} = I$ This implies that f is invertible and $f^{-1} = f$ (1)

OR

Let $(a, b) \in \mathbb{N} \times \mathbb{N}$. Then $\because a^2 + b^2 = b^2 + a^2 \therefore (a, b)R(a, b)$ Hence, R is reflexive. (1)

$$\begin{aligned}
&\text{Let } (a, b), (c, d) \in \mathbb{N} \times \mathbb{N} \text{ be such that } (a, b)R(c, d) \\
&\Rightarrow a^2 + d^2 = b^2 + c^2 \\
&\Rightarrow c^2 + b^2 = d^2 + a^2 \\
&\Rightarrow (c, d)R(a, b)
\end{aligned}$$

Hence, R is symmetric. (1)

$$\begin{aligned}
&\text{Let } (a, b), (c, d), (e, f) \in \mathbb{N} \times \mathbb{N} \text{ be such that } (a, b)R(c, d), (c, d)R(e, f). \\
&\Rightarrow a^2 + d^2 = b^2 + c^2 \text{ and } c^2 + f^2 = d^2 + e^2 \\
&\Rightarrow a^2 + d^2 + c^2 + f^2 = b^2 + c^2 + d^2 + e^2 \\
&\Rightarrow a^2 + f^2 = b^2 + e^2 \\
&\Rightarrow (a, b)R(e, f)
\end{aligned}$$

Hence, R is transitive. (1+1/2)

Since R is reflexive, symmetric and transitive. Therefore R is an equivalence relation. (1/2)

Q8.

$$\tan^{-1} \left(\frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} - \sqrt{1-\cos x}} \right) = \tan^{-1} \left(\frac{\sqrt{2\cos^2 \frac{x}{2}} + \sqrt{2\sin^2 \frac{x}{2}}}{\sqrt{2\cos^2 \frac{x}{2}} - \sqrt{2\sin^2 \frac{x}{2}}} \right) \quad (1)$$

$$= \tan^{-1} \left(\frac{-\sqrt{2}\cos \frac{x}{2} + \sqrt{2}\sin \frac{x}{2}}{-\sqrt{2}\cos \frac{x}{2} - \sqrt{2}\sin \frac{x}{2}} \right) \quad (\pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4} \Rightarrow \cos \frac{x}{2} < 0, \sin \frac{x}{2} > 0) \quad (1+1/2)$$

$$= \tan^{-1} \left(\frac{1 - \tan \frac{x}{2}}{1 + \tan \frac{x}{2}} \right) = \tan^{-1} \left(\tan \left(\frac{\pi}{4} - \frac{x}{2} \right) \right) \quad (1)$$

$$= \frac{\pi}{4} - \frac{x}{2} \quad \left(-\frac{\pi}{4} > \frac{\pi}{4} - \frac{x}{2} > -\frac{\pi}{2} \right) \quad (1/2)$$

$$\text{Q9. } \text{adj}A = \begin{bmatrix} 4 & -3 \\ -1 & -2 \end{bmatrix}' = \begin{bmatrix} 4 & -1 \\ -3 & -2 \end{bmatrix} \quad (2)$$

$$(\text{adj}A)A = \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix} = -11 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (1/2)$$

$$A(\text{adj}A) = \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix} = -11 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (1/2)$$

$$|A| = \begin{vmatrix} -2 & 1 \\ 3 & 4 \end{vmatrix} = -11 \quad (1/2)$$

$$\text{Hence, } A(\text{adj}A) = (\text{adj}A)A = |A|I \text{ verified.} \quad (1/2)$$

$$\begin{aligned} \text{Q(10) LHS} &= \begin{vmatrix} 1 & 1+p & 1+p+q \\ 3 & 4+3p & 2+4p+3q \\ 4 & 7+4p & 2+7p+4q \end{vmatrix} = \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & -1+p \\ 0 & 3 & -2+3p \end{vmatrix} \quad (R_2 \rightarrow R_2 - 3R_1, R_3 \rightarrow R_3 - 4R_1) \\ & \quad (2) \end{aligned}$$

$$= \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & -1+p \\ 0 & 0 & 1 \end{vmatrix} \quad (R_3 \rightarrow R_3 - 3R_2) \quad (1)$$

$$= 1 = \text{RHS. Hence, proved.} \quad (1)$$

OR

$$\text{Let } \Delta = \begin{vmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{vmatrix} = \begin{vmatrix} 0 & -2 & 3 \\ 2 & 0 & -4 \\ -3 & 4 & 0 \end{vmatrix} \quad (\text{interchanging rows and columns}) \quad (1 + 1/2)$$

$$= (-1)(-1)(-1) \begin{vmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{vmatrix} \quad (1 + 1/2)$$

$$= -\Delta \quad (1/2)$$

$$\Rightarrow 2\Delta = 0 \Rightarrow \Delta = 0 \quad (1/2)$$

$$\text{Q11. } AB = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 2I \quad (1/2)$$

$$\Rightarrow A\left(\frac{1}{2}B\right) = I \Rightarrow A^{-1} = \frac{1}{2}B = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \quad (1)$$

$$\text{The given system of equations is equivalent to } A'X = C, \text{ where } X = \begin{bmatrix} x \\ y \end{bmatrix}, C = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \quad (1/2)$$

$$X = (A')^{-1}C = (A^{-1})'C \quad (1)$$

$$= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ -10 \end{bmatrix} \Rightarrow x = 7, y = -10 \quad (1)$$

$$\text{Q12. Since, } f \text{ is differentiable at } x = 2, \text{ therefore, } f \text{ is continuous at } x = 2. \quad (1/2)$$

$$\Rightarrow \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2) \Rightarrow \lim_{x \rightarrow 2^-} x^2 = \lim_{x \rightarrow 2^+} (ax + b) = 4 \Rightarrow 4 = 2a + b \quad (1+1/2)$$

Since, f is differentiable at $x = 2$,

$$\begin{aligned} \therefore Lf'(2) &= Rf'(2) \Rightarrow \lim_{h \rightarrow 0} \frac{f(2-h) - f(2)}{-h} = \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} \quad (h > 0) \\ &\Rightarrow \lim_{h \rightarrow 0} \frac{(2-h)^2 - 4}{-h} = \lim_{h \rightarrow 0} \frac{a(2+h) + b - 4}{h} \\ &\Rightarrow \lim_{h \rightarrow 0} (-h + 4) = \lim_{h \rightarrow 0} \frac{4 + ah - 4}{h} \Rightarrow 4 = a \end{aligned}$$

$$(1+1/2)$$

$$b = -4 \quad (1/2)$$

Q13.

$$\begin{aligned} \text{Let } u &= (\log x)^x. \text{ Then } \log u = x \log(\log x) \Rightarrow \frac{1}{u} \frac{du}{dx} = \frac{1}{\log x} + \log(\log x) \\ &\Rightarrow \frac{du}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x) \right] \end{aligned}$$

(1+1/2)

$$\text{Let } v = x^{x \cos x}. \text{ Then } \log v = x \cos x \log x \Rightarrow \frac{1}{v} \frac{dv}{dx} = \frac{x \cos x}{x} + \cos x (\log x) - x \sin x \log x \\ \Rightarrow \frac{dv}{dx} = x^{x \cos x} [\cos x + \cos x (\log x) - x \sin x \log x]$$

(1+1/2)

$$y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x) \right] + \\ x^{x \cos x} [\cos x + \cos x (\log x) - x \sin x \log x]$$

(1)

OR

$$\frac{dx}{dt} = ap \cos pt, \frac{dy}{dt} = -bp \sin pt, \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -\frac{b}{a} \tan pt \quad (1+1/2)$$

$$\frac{d^2y}{dx^2} = -\frac{b}{a} p \sec^2 pt \times \frac{dt}{dx} \quad (1+1/2)$$

$$\frac{d^2y}{dx^2} = -\frac{b}{a^2 \cos^3 pt} \quad (1/2)$$

$$\Rightarrow \left(\frac{d^2y}{dx^2} \right)_{t=0} = -\frac{b}{a^2} \quad (1/2)$$

Q14. Given integral =

$$\int \frac{1}{\sin x - \sin 2x} dx = \int \frac{1}{\sin x (1 - 2 \cos x)} dx = \int \frac{\sin x}{(1 + \cos x)(1 - \cos x)(1 - 2 \cos x)} dx \\ = -\int \frac{dt}{(1+t)(1-t)(1-2t)} \quad (\cos x = t \Rightarrow -\sin x dx = dt) \quad (1)$$

$$\frac{1}{(1+t)(1-t)(1-2t)} = \frac{A}{1+t} + \frac{B}{1-t} + \frac{C}{1-2t}$$

$$\therefore 1 = A(1-t)(1-2t) + B(1+t)(1-2t) + C(1-t^2) \text{ (An identity)}$$

Putting, $t = 1, \frac{1}{2}, -1$, we get $A = 1/6, B = -1/2, C = 4/3$ (1+1/2)

Therefore, the given integral

$$= -\frac{1}{6} \log|1+t| - \frac{1}{2} \log|1-t| + \frac{4}{6} \log|1-2t| + c$$

$$= -\frac{1}{6} \log|1+\cos x| - \frac{1}{2} \log|1-\cos x| + \frac{2}{3} \log|1-2\cos x| + c$$

(1+1/2)

OR

$$\int \frac{\sin \phi}{\sqrt{\sin^2 \phi + 2 \cos \phi + 3}} d\phi = \int \frac{\sin \phi}{\sqrt{1 - \cos^2 \phi + 2 \cos \phi + 3}} d\phi \quad (1/2)$$

$$= \int \frac{\sin \phi}{\sqrt{-\cos^2 \phi + 2 \cos \phi + 4}} d\phi = \int \frac{-1}{\sqrt{-t^2 + 2t + 4}} dt \quad (\cos \phi = t \Rightarrow -\sin \phi d\phi = dt) \quad (1)$$

$$= -\int \frac{1}{\sqrt{(\sqrt{5})^2 - (t-1)^2}} dt \quad (1+1/2)$$

$$= -\sin^{-1} \frac{t-1}{\sqrt{5}} + c = -\sin^{-1} \frac{\cos \phi - 1}{\sqrt{5}} + c \quad (1)$$

Q15. Let

$$I = \int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx = \int_{-\pi}^{\pi} \frac{2x}{1+\cos^2 x} dx + \int_{-\pi}^{\pi} \frac{2x \sin x}{1+\cos^2 x} dx$$

(as $\frac{2x}{1+\cos^2 x}$ is odd and

$$= 0 + 2 \int_0^{\pi} \frac{2x \sin x}{1+\cos^2 x} dx$$

$$\frac{2x \sin x}{1 + \cos^2 x} \text{ is even) } \quad (1)$$

$$= 4 \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx.$$

Let

$$I_1 = \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx = \int_0^{\pi} \frac{(\pi - x) \sin(\pi - x)}{1 + \cos^2(\pi - x)} dx$$

$$I_1 = \int_0^{\pi} \frac{(\pi - x) \sin x}{1 + \cos^2 x} dx \quad (1)$$

$$\text{Adding, } 2I_1 = \pi \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx$$

$$= -\pi \int_1^{-1} \frac{dt}{1 + t^2} \quad (\cos x = t \Rightarrow -\sin x dx = dt) \quad (1)$$

$$= \pi \left[\tan^{-1} t \right]_{-1}^1 \quad (1/2)$$

$$I_1 = \frac{\pi^2}{4}. \text{ Hence, } I = \pi^2 \quad (1/2)$$

Q16. Given differential equation is

$$\frac{dy}{dx} = \frac{y}{x} + \frac{\sqrt{x^2 + y^2}}{x}, x > 0 \text{ or, } \frac{dy}{dx} = \frac{y}{x} + \sqrt{1 + \left(\frac{y}{x}\right)^2} = f\left(\frac{y}{x}\right), \text{ hence, homogeneous.} \quad (1/2)$$

$$\text{Put } y = v x \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}. \text{ The differential equation becomes } v + x \frac{dv}{dx} = v + \sqrt{1 + v^2} \quad (1)$$

$$\text{or, } \frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x} \quad (1/2).$$

$$\text{Integrating, we get } \log \left| v + \sqrt{1+v^2} \right| = \log |x| + \log k \quad (1)$$

$$\begin{aligned} \Rightarrow \log \left| v + \sqrt{1+v^2} \right| &= \log |x| + \log k \Rightarrow \left| v + \sqrt{1+v^2} \right| = |x|k \\ \Rightarrow v + \sqrt{1+v^2} &= \pm kx \Rightarrow \frac{y}{x} + \sqrt{1 + \left(\frac{y}{x}\right)^2} = cx \\ \Rightarrow y + \sqrt{x^2 + y^2} &= cx^2, \end{aligned}$$

$$\text{which gives the general solution.} \quad (1)$$

$$\text{Q17. We have the following differential equation: } \frac{dx}{dy} = \frac{(\tan^{-1} y - x)}{1+y^2} \text{ or, } \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1} y}{1+y^2},$$

$$\text{which is linear in } x \quad (1/2)$$

$$\text{I.F.} = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y} \quad (1)$$

$$\text{Multiplying both sides by I. F. and integrating we get } xe^{\tan^{-1} y} = \int e^{\tan^{-1} y} \frac{\tan^{-1} y}{1+y^2} dy \quad (1/2)$$

$$\begin{aligned} \Rightarrow xe^{\tan^{-1} y} &= \int e^t dt \quad (\tan^{-1} y = t \Rightarrow \frac{1}{1+y^2} dy = dt) \\ \Rightarrow xe^{\tan^{-1} y} &= e^t - e^t + c \Rightarrow xe^{\tan^{-1} y} = \tan^{-1} y e^{\tan^{-1} y} - e^{\tan^{-1} y} + c \end{aligned}$$

$$\text{which gives the general solution of the differential equation.} \quad (2)$$

Q18. The vector equations of the given lines are

$$\begin{aligned} \vec{r} &= \hat{i} + 2\hat{j} - \hat{k} + \lambda(2\hat{i} - 3\hat{j} + 4\hat{k}), \vec{r} = -2\hat{i} + 3\hat{j} + \mu(-\hat{i} + 2\hat{j} + 3\hat{k}) \\ \vec{a}_1 &= \hat{i} + 2\hat{j} - \hat{k}, \vec{b}_1 = 2\hat{i} - 3\hat{j} + 4\hat{k}, \vec{a}_2 = -2\hat{i} + 3\hat{j}, \vec{b}_2 = -\hat{i} + 2\hat{j} + 3\hat{k} \end{aligned}$$

$$(1)$$

$$\vec{a}_2 - \vec{a}_1 = -3\hat{i} + \hat{j} + \hat{k}, \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 4 \\ -1 & 2 & 3 \end{vmatrix} = -17\hat{i} - 10\hat{j} + \hat{k} \quad (2)$$

$$\text{The required shortest distance} = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} \quad (1/2)$$

$$= \frac{42}{\sqrt{390}} \text{ units} \quad (1/2)$$

Q19. Let us define the following events: E = A solves the problem, F = B solves the problem, G =

C solves the problem, H = D solves the problem (1/2)

(i) The required probability = $P(E \cup F \cup G \cup H)$ (1/2)

$$= 1 - P(\bar{E} \cap \bar{F} \cap \bar{G} \cap \bar{H})$$

$$= 1 - P(\bar{E}) \times P(\bar{F}) \times P(\bar{G}) \times P(\bar{H}) \quad (1)$$

$$= 1 - \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{1}{3} = \frac{13}{15} \quad (1/2)$$

(ii) The required probability =

$$P(\bar{E}) \times P(\bar{F}) \times P(\bar{G}) \times P(\bar{H}) + P(E) \times P(\bar{F}) \times P(\bar{G}) \times P(\bar{H}) + P(\bar{E}) \times P(F) \times P(\bar{G}) \times P(\bar{H}) \\ + P(\bar{E}) \times P(\bar{F}) \times P(G) \times P(\bar{H}) + P(\bar{E}) \times P(\bar{F}) \times P(\bar{G}) \times P(H) \quad (1)$$

$$= \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{1}{3} + \frac{1}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{4} \times \frac{4}{5} \times \frac{1}{3} \\ + \frac{2}{3} \times \frac{3}{4} \times \frac{1}{5} \times \frac{1}{3} + \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{2}{3} = \frac{5}{18} \quad (1/2)$$

Section C

Q20. $f'(x) = (x-1)^2(x+2)(5x+4)$ (1/2)

$$f'(x) = 0 \Rightarrow x = 1, -2, -\frac{4}{5} \quad (1/2)$$

| In the interval | Sign of $f'(x)$ | Nature of the function |
|----------------------|-------------------------|--|
| $(-\infty, -2)$ | $(+ve)(-ve)(-ve) = +ve$ | f is strictly increasing in $(-\infty, -2]$ |
| $(-2, -\frac{4}{5})$ | $(+ve)(+ve)(-ve) = -ve$ | f is strictly decreasing in $[-2, -\frac{4}{5}]$ |
| $(-\frac{4}{5}, 1)$ | $(+ve)(+ve)(+ve) = +ve$ | f is strictly increasing in $[-\frac{4}{5}, 1]$ |
| $(1, \infty)$ | $(+ve)(+ve)(+ve) = +ve$ | f is strictly increasing in $[1, \infty)$ |

(2+1/2)

Hence, f is strictly increasing in $(-\infty, -2]$ and $[-\frac{4}{5}, \infty)$. f is strictly decreasing in $[-2, -\frac{4}{5}]$

(1/2)

In the left nhd of -2 , $f'(x) > 0$, in the right nhd of -2 , $f'(x) < 0$ and $f'(-2) = 0$, therefore, by the first derivative test, -2 is a point of local maximum. (1)

In the left nhd of $-4/5$, $f'(x) < 0$, in the right nhd of $-4/5$, $f'(x) > 0$ and $f'(-4/5) = 0$, therefore, by the first derivative test, $-4/5$ is a point of local minimum. (1)

Q21. We have

$$\vec{a} \cdot \vec{b} = 0, \vec{a} \cdot \vec{c} = 0 \Rightarrow \vec{a} \perp \text{both } \vec{b} \text{ and } \vec{c} \text{ (as } \vec{a}, \vec{b}, \vec{c} \text{ are nonzero vectors)} \\ \Rightarrow \vec{a} \parallel \vec{b} \times \vec{c} \quad (1)$$

$$\text{Let } \vec{a} = \lambda(\vec{b} \times \vec{c}) \quad (1)$$

Then

$$|\vec{a}| = |\lambda| |(\vec{b} \times \vec{c})| \Rightarrow \frac{|\vec{a}|}{|(\vec{b} \times \vec{c})|} = |\lambda| \Rightarrow |\lambda| = \frac{1}{\sin \frac{\pi}{6}} = 2 \Rightarrow \lambda = \pm 2$$

$$\therefore \vec{a} = \pm 2(\vec{b} \times \vec{c})$$

(2)

$$\text{Now } \begin{bmatrix} \vec{a} + \vec{b} & \vec{b} + \vec{c} & \vec{c} + \vec{a} \end{bmatrix} = \left\{ (\vec{a} + \vec{b}) \times (\vec{b} + \vec{c}) \right\} \cdot (\vec{c} + \vec{a}) = (\vec{a} \times \vec{b}) \cdot \vec{c} + (\vec{b} \times \vec{c}) \cdot \vec{a} \quad (\text{As the scalar}$$

triple product = 0 if any two vectors are equal.)

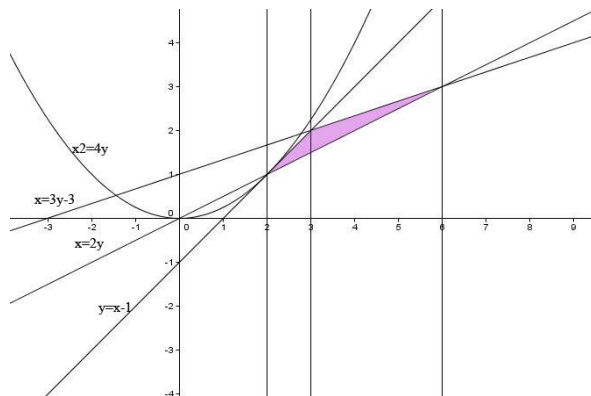
$$\vec{a} \cdot (\vec{b} \times \vec{c}) + (\vec{a} \times \vec{b}) \cdot \vec{c} = \vec{a} \cdot (\vec{b} \times \vec{c}) + \vec{a} \cdot (\vec{b} \times \vec{c}) = 2\vec{a} \cdot (\vec{b} \times \vec{c}) \quad (1+1/2)$$

$$= 2\vec{a} \cdot \left(\pm \frac{1}{2} \vec{a} \right) = \pm 1 \quad (1/2)$$

Q22. We have the curve

$$4y = x^2 \Rightarrow 4 \frac{dy}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{x}{2} \Rightarrow \left(\frac{dy}{dx} \right)_{x=2} = 1 \quad (1)$$

$$\text{The equation of the tangent is } y = x - 1 \quad (1)$$



Graph sketch (1)

The required area = the shaded area =

$$\int_2^3 \left[(x-1) - \frac{x}{2} \right] dx + \int_3^6 \left[\frac{(x+3)}{3} - \frac{x}{2} \right] dx = \int_2^3 (x-1) dx + \frac{1}{3} \int_3^6 (x+3) dx - \frac{1}{2} \int_2^6 x dx \quad (1)$$

$$= \left[\frac{x^2}{2} - x \right]_2^3 + \frac{1}{3} \left[\frac{x^2}{2} + 3x \right]_3^6 - \frac{1}{4} [x^2]_2^6 \quad (1+1/2)$$

$$= 1 \text{ square units} \quad (1/2)$$

Q23. The equation of the line passing through the point(3, -2, 1) and parallel to the given line is

$$\frac{x-3}{2} = \frac{y+2}{-3} = \frac{z-1}{1} \quad (1)$$

$$\text{Any point on this line is } (2\lambda + 3, -3\lambda - 2, \lambda + 1) \quad (1/2)$$

$$\text{If it lies on the plane, we have } 3(2\lambda + 3) - 3\lambda - 2 - \lambda - 1 + 2 = 0 \Rightarrow \lambda = -4 \quad (1)$$

$$\text{Hence, the point common to the plane and the line is } (-5, 10, -3). \quad (1/2)$$

$$\text{Hence, the required distance} = \sqrt{(3+5)^2 + (-2-10)^2 + (1+3)^2} \text{ units} = 4\sqrt{14} \text{ units} \quad (1)$$

The equation of the line passing through (3, -2, 1) and perpendicular to the plane is

$$\frac{x-3}{3} = \frac{y+2}{1} = \frac{z-1}{-1} \quad (1/2)$$

$$\text{Any point on it is } (3\mu + 3, \mu - 2, -\mu + 1) \quad (1/2)$$

$$\text{If it lies on the plane, we get } 3(3\mu + 3) + \mu - 2 + \mu - 1 + 2 = 0 \Rightarrow \mu = \frac{-8}{7} \quad (1/2)$$

$$\text{The required foot of the perpendicular} = \left(\frac{-3}{7}, \frac{-22}{7}, \frac{15}{7} \right) \quad (1/2)$$

OR

Any plane through the line of intersection of the given planes is

$$\begin{aligned}\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 1 + \lambda(\vec{r} \cdot (\hat{i} + \hat{j} - 2\hat{k})) &= 0 \\ \text{or, } \vec{r} \cdot ((2 + \lambda)\hat{i} + (3 + \lambda)\hat{j} + (-1 - 2\lambda)\hat{k}) &= -1\end{aligned}\tag{2}$$

If it contains the point (3, -2, -1), we have

$$(3)(2 + \lambda) + (-2)(3 + \lambda) + (-1)(-1 - 2\lambda) = -1 \Rightarrow \lambda = \frac{-2}{3}\tag{1}$$

The required equation of the plane is

$$\vec{r} \cdot \left(\left(2 - \frac{2}{3}\right)\hat{i} + \left(3 - \frac{2}{3}\right)\hat{j} + \left(-1 + \frac{4}{3}\right)\hat{k} \right) = -1 \text{ or, } \vec{r} \cdot (4\hat{i} + 7\hat{j} + \hat{k}) = -3\tag{1}$$

If θ be the angle between the normals to the two given planes, then θ is the angle between

$$\text{the planes and } \cos \theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| |\vec{n}_2|} = \frac{2 + 3 + 2}{\sqrt{14} \sqrt{6}} = \frac{7}{2\sqrt{21}}\tag{2}$$

Q24. Let us define the following events: E_1 = Two white balls are transferred, E_2 = Two red balls are transferred, E_3 = One red and one white balls are transferred, A = The ball drawn from the Bag II is red

$$P(E_1) = \frac{{}^4C_2}{{}^9C_2} = \frac{4 \times 3}{9 \times 8}\tag{1}$$

$$P(E_2) = \frac{{}^5C_2}{{}^9C_2} = \frac{4 \times 5}{9 \times 8}\tag{1}$$

$$P(E_3) = \frac{{}^5C_1 \times {}^4C_1}{{}^9C_2} = \frac{4 \times 5 \times 2}{9 \times 8}\tag{1}$$

$$P(A / E_1) = \frac{3}{8}, \quad P(A / E_2) = \frac{5}{8}, \quad P(A / E_3) = \frac{4}{8} \quad \left(\frac{1}{2} + \frac{1}{2} + \frac{1}{2}\right)$$

The required probability, $P(E_3 / A)$, by Baye's Theorem,

$$= \frac{P(E_3) \times P(A / E_3)}{P(E_1) \times P(A / E_1) + P(E_2) \times P(A / E_2) + P(E_3) \times P(A / E_3)} \quad (1/2)$$

$$= 20/37 \quad (1/2)$$

OR

Let X represent the random variable. Then $X = 0, 1, 2, 3$ (1/2)

$$P(X = 0) = P(r = 0) = {}^3C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^3 = \frac{125}{216} \quad (1/2)$$

$$P(X = 1) = P(r = 1) = {}^3C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^2 = \frac{75}{216} \quad (1/2)$$

$$P(X = 2) = P(r = 2) = {}^3C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^1 = \frac{15}{216} \quad (1/2)$$

$$P(x = 3) = p(r = 3) = {}^3C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^0 = \frac{1}{216} \quad (1/2)$$

| x_i | p_i | $x_i p_i$ | $(x_i)^2 p_i$ |
|-------|---------|-----------|---------------|
| 0 | 125/216 | 0 | 0 |
| 1 | 75/216 | 75/216 | 75/216 |
| 2 | 15/216 | 30/216 | 60/216 |
| 3 | 1/216 | 3/216 | 9/216 |
| Total | | 1/2 | 2/3 |

(2)

$$\text{Mean} = \sum x_i p_i = \frac{1}{2}, \text{var}(X) = \sum x_i^2 p_i - (\sum x_i p_i)^2 = \frac{2}{3} - \frac{1}{4} = \frac{5}{12} \quad (1)$$

$$\text{Standard deviation} = \sqrt{\text{var}(X)} = \frac{\sqrt{15}}{6} \quad (1/2)$$

Q25. Let the radius of the circular garden be r m and the side of the square garden be x m. Then

$$600 = 2\pi r + 4x \Rightarrow x = \frac{600 - 2\pi r}{4} \quad (1)$$

$$\text{The sum of the areas} = A = \pi r^2 + x^2 \Rightarrow A = \pi r^2 + \left(\frac{600 - 2\pi r}{4} \right)^2 \quad (1)$$

$$\frac{dA}{dr} = 2\pi r + \frac{2}{16} (600 - 2\pi r)(-2\pi) = \frac{\pi}{2} (4r - 300 + \pi r), \frac{dA}{dr} = 0 \Rightarrow r = \frac{300}{\pi + 4} \quad (1)$$

$$\frac{d^2A}{dr^2} = \frac{\pi}{2} (4 + \pi), \left(\frac{d^2A}{dr^2} \right)_{r=\frac{300}{\pi+4}} > 0 \quad (1)$$

$$\text{Therefore, } A \text{ is minimum when } r = \frac{300}{\pi + 4} \text{ For this value of } r, x = 2r \quad (1)$$

To achieve any goal, there is every possibility that energy, time and money are required to be invested. One must plan in such a manner that least energy, time and money are spent.

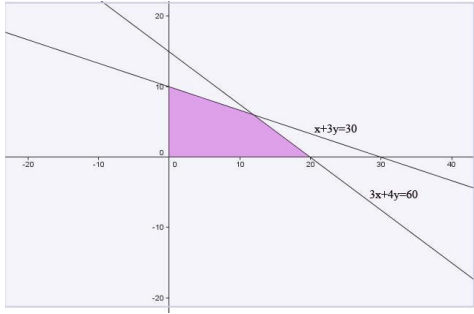
A good planning and execution, therefore, is essentially required. (1)

Q26. Let the number of pieces of model A to be manufactured be $= x$ and the number of pieces

of model B to be manufactured be $= y$. (1/2)

Then to maximize the profit, $P = \text{Rs } (8000x + 12000y)$ (1/2)

subject to the constraints $9x + 12y \leq 180$, or, $3x + 4y \leq 60$, $x + 3y \leq 30$, $x \geq 0, y \geq 0$ (2)



Graph work (on the actual graph paper) (1+1/2)

| At | Profit |
|--------|---------------------|
| (0,0) | Rs 0 |
| (20,0) | Rs 160000 |
| (12,6) | Rs 168000 (maximum) |
| (0,10) | Rs 120000 |

(1)

The number of pieces of model A =12, the number of pieces of model B =6 and the maximum profit = Rs 168000.

(1/2)