Time & space Complexity - 2

Q1
```
int c=0;                      → i=i+1 → i=i*2 → i*=2
for(int i=1; i<n; i+=i) {
    c++;
}
```

$i = 1, 2, 4, 8, 16 \text{ ---} n$

no. of iterations = no. of values of i

$i = \underbrace{1, 2^1, 2^2, 2^3, 2^4 \text{ --- } 2^x}_{x+1 \text{ terms}}$

$2^x = n$

$T.C = O(x+1) = O(x) = O(\log n)$

$2^x = n$

$\log_2 n = x$

Q2
```
int c=0
for(int i=1; i<=n; i+=i) {
    for (int j=0; j<i; j++) {
        c++;   ↳ i terms.
    }
}
```

$i=1, \ j=0 \rightarrow 1$

$i=2, \ j=0,1 \rightarrow 2$

$i=4, \ j=0,1,2,3 \rightarrow 4$

$i=8, \ j=0,1,2,3,4,5,6,7 \rightarrow 8$

⋮

$i=n, \ j=0,1,2,3 \text{ --- } n-1 \rightarrow n$

(M-1) $n = 2^k$

no of iterations $= 1+2+4+8+\cdots+n$

$= 1+2^1+2^2+2^3+2^4+\cdots+2^x$ ($x+1$) terms

$$\boxed{S_n = \frac{a(r^n-1)}{r-1}}$$

$$= \frac{1(2^{x+1}-1)}{2-1}$$

$$= 2^{x+1}-1$$

$\Rightarrow$ $T.C = O(2^{x+1}-1) = O(2\cdot 2^x) = O(2^x)$

$$\boxed{T.C = O(n)}$$

(M-2) Total no of iterate :

$\Rightarrow 1+2+4+8+16 \cdots + \frac{n}{2} + n$

$\Rightarrow 2+2+4+8+16 \cdots + \frac{n}{2} + n - 1$

$\Rightarrow 2n-1$

$T.C = O(2n-1)$

$T.C = O(n)$

ex $= n = 16$

$1+1+2+4+8+16-1$

$2+2+4+8+16-1$

$4+4+8+16-1$

$8+8+16-1$

$16+16-1$

$32-1$

$2n-1$

Q.3 int c = 0;

$i *= 2$

```
for (int i = 1; i < n; i += i) {     → T.C = O(log n)
    for (int j = n; j >= 0; j--) {    → T.C = O(n)
        c++;
    }
}
```

$j >= 0$ → 'j' ka koi lena - Dena hi nhi hai

$\boxed{T.C: O(n \cdot \log n)}$

i = 1 → n+1

i = 2 → n+1

⋮

i = n → n+1

→ O(log n)

Q4 int c = 0;

```
for (int i = 1; i < n; i *= 2) {
    for (int j = 0; j < i; j++) {
        c++;
    }
}
```

$j < i$ → 'i' tens

$\boxed{T.C: O(n)}$

i = 1 → j = 0, ~~0~~ → 1

i = 2, j = 0, 1 → 2

i = 3, j = 0, 1, 2, 3 → 4

$i^2 < n \rightarrow i < \sqrt{n}$

Q5 int c = 0;
= for(int i = 1; i * i < n; i * = 2) {
for (int j = 0; j < i; j++) {
c++
}
}

$i = 1, 2, 4, 8, 16, \dots 2^x$
$\downarrow$
$\sqrt{n}$

$\rightarrow \boxed{2^x \cdot 2^x = n}$

$\Rightarrow 2^x = \sqrt{n}$

Total n° of iteration = $1 + 2 + 3 + 4 + \dots 2^x$

$= 2^{x+1} - 1$

$\rightarrow T \cdot C = O(2^x) = O(\sqrt{n})$

$\boxed{T \cdot C : = O(\sqrt{n})}$

Q6 int c = 0;
= for (int i = 1; i*i < n; i += i) {
for (int j = n; j > i; j--) {
c++;
}
}

$\rightarrow i < \sqrt{n}$ $\rightarrow i *= 2$

$i = 1, 2, 4, 8 \dots 2^x \rightarrow \sqrt{n}$
$\Rightarrow 2^x = \sqrt{n}$

$2^x = \sqrt{n}$
$x = \log_2 \sqrt{n}$

$i = 1, j = n, n-1, \dots 3, 2 = n-1$
$i = 2, j = n, n-1, \dots 4, 3 = n-2$
$i = 4, j = n, n-1, \dots 6, 5 = n-4$
$\vdots$
$i = \sqrt{n}, j =$ $\qquad = n - \sqrt{n}$

Total iteration $= (n-1) + (n-2) + (n-4) + \cdots (n-\sqrt{n})$

$$= \underbrace{(n-1) + (n-2) + (n-4) + (n-8) + \cdots + (n-2^x)}_{(x+1)\text{ terms}}$$

$$= \underbrace{(n+n+n+\cdots)}_{x+1} - \underbrace{(1+2+4+8+\cdots 2^x)}_{x+1}$$

$$= n(x+1) - \left[2^{x+1} - 1\right]$$

$$= n\cdot x + n - 2\cdot 2^x + 1$$

$$T\cdot C = O(n\cdot \log_2 \sqrt{n} + n - \sqrt{n})$$

$$\boxed{T\cdot C = O(n\cdot \log \sqrt{n})}$$

$$\left\{\begin{array}{l} * * \\ \log_a b^m = m \log_a b \\ \log_2 \sqrt{n} = \log_2 n^{1/2} = \frac{1}{2} \log_2 n \end{array}\right\}$$

$$T\cdot C = O(n\cdot \log n^{1/2}) = O(n\cdot \tfrac{1}{2} \log n)$$

$$\boxed{T\cdot C = O(n\cdot \log n)}$$

$\rightarrow T.C = \log(\log n))$

$i = i * i$

Q6 int c = 0

for(int i = 2; i < n; i *= i) {

c++;

}

$i = 2, 4, 16, 256, \ldots\ldots 65536$

khatm

$i = 2^1, 2^2, 2^4, 2^8, \ldots\ldots 2^{16}$

$i = 2^1, 2^2, 2^4, 2^8, \ldots\ldots 2^{x/2}, 2^x$

$\Rightarrow 2^1, 2^2, 2^4, 2^8, 2^{16}, 2^{32}, 2^{64} \ldots$

$\underbrace{\underbrace{2^1}_{1}, \underbrace{2^{2^1}, 2^{2^2}, 2^{2^3}, 2^{2^4}, 2^{2^5}, 2^{2^6} \ldots 2^{2^x}}_{x}}_{(x+1)\text{ terms}}$

$2^x = n$

$\log_2 n = x$

$2^{2^x} = n$

$T.C = O(x+1) \simeq O(x)$

$2^{2^x} = n \Rightarrow \log_2 n = 2^x$

$2^x \log_2(2) = \log(n)$ $\quad 2^x = K$

$\log_2 K = x$

$T.C = O(\log(\log n))$

$\boxed{\log_2(\log_2 n) = x}$

Q) T.C —

```
int c = 0;
for (int i = 2 ; i*i <= n ; i *= i) {
    c++;
}
```

$\rightarrow i <= \sqrt{n}$

$i = 2, 4, 16, 256, 65536$

$i = 2, 2^2, 2^4, 2^8, 2^{16}$

$i = \underbrace{2^1, 2^{2^1}, 2^{2^2}, 2^{2^3}, 2^{2^4} \ldots 2^{2^x}}_{x+1}$

$T.C = O(x+1)$

$= O(x)$

$= O\left(\log\left(\log \sqrt{n}\right)\right)$

$= O\left(\log\left(\log n^{1/2}\right)\right)$

$= O\left(\log\left(\frac{1}{2}\log n\right)\right)$

$= O\left(\log \frac{1}{2} + \log(\log n)\right)$

$\boxed{= O\left(\log(\log n)\right)}$ Ans

$2^{2^x} = \sqrt{n}$

$\log_2 \sqrt{n} = 2^x$

$K = 2^x$

$\boxed{\log_2 K = x}$

$x = \log_2\left(\log_2 \sqrt{n}\right)$