Poisson Model and Bradley Terry Model for Predicting Multiplayer Online Battle Games

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Abstract. There has been increasing interest for predicting game results both in lottery industry and in academia, however there have been few literature for predicting Aeon Of Strife (AOS) type game match results. Moreover, few studies have been conducted for comparing different prediction models for game matches. In this paper, we compare Poisson Model and Bradley Terry Model for League of Legends (LOL) matches from 2013 to 2014 in South Korea. For Poisson model, we adopt time dependent bivariate Poisson regression model proposed by Dixon and Coles. For Bradley Terry Model, we add Davidson method to allow tie count. From the constructed models, we estimate maximum likelihood values, and present performance evaluation results on the training data. The performance evaluation results indicate that the adopted models in this paper are effective in prediction of actual LOL match results.

Keywords: Poisson Model, Bradley Terry Model, Multiplayer Online Battle Games

1 Introduction

In PC game industry, there has been increasing interest in Aeon Of Strife (AOS) type games popularized by League of Legends (LOL) of Riot Games, Co. started on October 27, 2009. After the advent of LOL, there have been considerable amount of new AOS type games released. There has been increasing interest for predicting game results both in lottery industry and in academia, however there have been few literature for predicting Aeon Of Strife (AOS) type game match results. Moreover, few studies have been conducted for comparing different prediction models for PC game matches.

AOS can be considered as a mixed genre of Real Time-Strategy (RTS), Action Role Playing Game (Action RPG) and siege warfare. As shown in the figure 1, its basic setting includes three main lanes and jungles. In this setting, players choose their camp, their heroes (champion). The chosen heroes kill monsters to raise their level and skill, and they acquire items. Players' final goal is to conquer their

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opponent's camp. Usually, five players comprise one team, and two teams fight against each other in one map. This characteristic requires communication and teamwork among team mates. Also, players need to understand available maps and champions. In this sense, AOS games can be considered difficult for beginners.

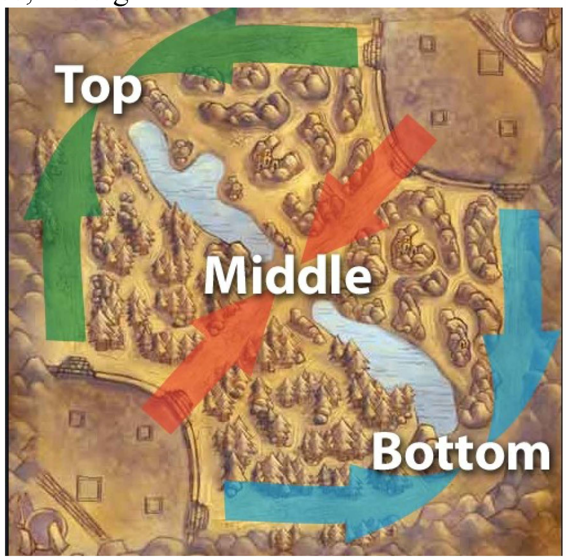

Figure 1 Typical map of AOS type game

In LOL, there are league systems, such as Spring season, Summer season, Winder season, and Masters league. These leagues are open for professional teams in South Korea. As for international leagues, there are All Star League and World Championship, where North American teams, Chinese teams, Korean teams and European teams compete in tournament style. There are sixteen famous teams from eight companies in South Korea, including NaJin W Shield (Najin e-mFire), NaJin B Sword (Najin e-mFire), CJ Entus Frost (CJ Entus), CJ Entus Blaze (CJ Entus), SKT T1 S (SK Telecom T1), SKT T1 K (SK Telecom T1), Jin Air Falcons (Jin Air GreenWings), Jin Air Stealths(Jin Air GreenWings), IM #1 (Incredible Miracle), IM #2 (Incredible Miracle), Samsung Blue (Samsung Galaxy), Samsung Ozone (Samsung Galaxy), KT Arrows (KT Rolster), KT Bullets (KT Rolster), Xenics Storm (Xenics Storm/Blast), Xenics Blast (Xenics Storm/Blast).

It is widely known that LOL has ranked #1 in terms of installation and usage in PC caf es, and in terms of simultaneous connections in game servers. It is constantly rumored that there are private "toto" system for gambling and lottery business of LOL games. Although, there have been many researches for constructing models and predicting results of association footballs, baseballs, basketballs, etc. [1,2], there have been few researches for modelling and analyzing e-sports games such as LOL.

Against these backgrounds, in this paper, we compare Poisson Model and Bradley Terry Model for League of Legends (LOL) matches from 2013 to 2014 in South Korea. The performance evaluation results indicate that the adopted model in this paper is somewhat effective in prediction of actual LOL match results.

2 Methods

We explain Poisson model [1] and Bradley Terry Model [2] for prediction games results.

2.1 Dixon and Coles Poisson Model

Dixon and Coles [1] have let the score of home team *i* and away team *j* as $X_{i,j}$ and $Y_{i,j}$ respectively, and assume the following Poisson distribution.

$$\mathrm{X}_{i,j}\sim\mathrm{Poisson}(\alpha_i,\beta_j,\gamma),\mathrm{Y}_{i,j}\sim\mathrm{Poisson}(\alpha_j,\beta_i)$$

Here, α_i and β_i denote attack and defense of team i, and γ denotes home advantage, thus if $\gamma > 1$, then the team has advantage at home game. Using this assumption and bivariate Poisson distribution, Dixon and Coles [1] have constructed the probability of home team score *x* and away team score *y* with bias τ as follows:

$$Pr(\mathrm{X}_{i,j}=x,\mathrm{Y}_{i,j}=y)=\tau_{\lambda,\mu}\times\frac{\lambda^x e^{-\lambda}}{x!}\times\frac{\mu^y e^{-\mu}}{y!}$$

They let $\lambda=\alpha_i\beta_j\gamma$ as the mean of home team *i*'s score, and $\mu=\alpha_j\beta_i$ as the mean of away team *j*'s score. The bias τ between two teams is calculated as follows:

$$\tau=\begin{cases}1-\lambda\mu\rho, \text{if } x=y=0\\ 1+\lambda\rho, \text{if } x=0,y=1\\ 1+\mu\rho, \text{if } x=1,y=0\\ 1-\rho, \text{if } x=y=1\\ 1, \text{otherwise}\end{cases}$$

The parameter ρ has the following limitation:

$$max\left(\frac{-1}{\lambda},\frac{-1}{\mu}\right)\le\rho\le max\left(\frac{1}{\lambda\mu},1\right)$$

When $\rho=0$, the score distributions of home team and away team are independent.

We denote home team score and away team score of k^{th} game (out of N games) as x_k and y_k respectively. Then the maximum likelihood function to infer the parameters are as follows:

$$L(\alpha_i,\beta_i,\gamma,\rho;i=1\ldots n)=\prod_{k=1}^{N}\tau_{\lambda_k,\mu_k}(x_k,y_k)e^{-\lambda_k}\lambda_k^{x_k}e^{-\mu_k}\mu_k^{y_k}$$

Here, $\lambda_k=\alpha_{i(k)}\beta_{i(k)}\gamma$ and $\mu_k=\alpha_{j(k)}\beta_{j(k)}$. To prevent α_i from being unbounded, we define $n^{-1}\sum_{i=1}^{n}\alpha_i=1$.

We also consider that each team's parameters are changing over time and their past game results. Also, recent game results influence more on current parameters. Therefore, the likelihood function at time t with weights is as follows:

$$L(\alpha_i,\beta_i,\gamma,\rho;i=1\ldots n)=\prod_{k\epsilon A_t}^{N}\left(\tau_{\lambda_k,\mu_k}(x_k,y_k)e^{-\lambda_k}\lambda_k^{x_k}e^{-\mu_k}\mu_k^{y_k}\right)^{\phi(t-t_k)}$$

Here, t_k is the play time of game k. $A_t=\{k|t_k<t\}$ and ϕ is a non-increasing function of time. In our experiment, we adopt $\phi(t)=e^{-\xi t}$, where $\xi>0$.

1.2 Bradley Terry Model

In Bradley Terry's paired comparison model [2], they consider worth parameter π_i. Let us assume that there are *p* teams. Each team *i* is related with parameter π_i, which denotes ability or strength of the team *i*. Then we can denote the probability of team *i* defeats team *j* as follows:

$$p_{ij} = Pr(i \text{ defeats } j) = \frac{\pi_i}{\pi_i + \pi_j}$$

Here, π_i and π_j are the parameters of positive value, which denotes each team's ability. This model does not consider tie cases, so $p_{ij} + p_{ji} = 1$. The model can be represented as follows:

$$p_{ij} = Pr(i \text{ defeats } j) = \frac{e^{\gamma_i}}{e^{\gamma_i} + e^{\gamma_j}} \text{ where } \gamma_i = \log(\pi_i)$$

There are many ways to induce this model. In one way proposed by Davidson[3], he assume that, when team *i* plays, the team has the following cumulative distribution function and obtain unobserved score S_i which is independent to the other team.

$$S_i \sim F_i(s) = e^{-e^{-(s - \log\pi_i)}}$$

Therefore team i obtains random score from limit distribution with location parameter $\log\pi_i$. The distribution of score difference $S_i - S_j$ follows logistic distribution as follows.

$$S_i - S_j \sim F_{ij}(s) = \frac{1}{1 + e^{-\left(s - (\log\pi_i - \log\pi_j)\right)}}$$

This means the following can be derived.

$$Pr(S_i > S_j) = Pr(S_i - S_j > 0) = 1 - \frac{1}{1 + e^{\log\pi_i - \log\pi_j}}$$

With this model, if we assume team *i* and *j* have n_{ij} games and team *i* wins y_{ij} times and loses $n_{ij} - y_{ij} = y_{ji}$ times. Then the distribution of $y = (y_{ji},\ i, j = 1, 2, \dots, p)$ is as follows:

$$f(y|\pi) = \prod_{i<j} \binom{n_{ij}}{y_{ij}} \left(\frac{\pi_i}{\pi_i + \pi_j}\right)^{y_{ij}} \left(\frac{\pi_j}{\pi_i + \pi_j}\right)^{y_{ji}}$$

where, $\pi = (\pi_1, \dots, \pi_p)$ is Bradley-Terry worth parameters.
Likelihood for π can be modeled as follows:

$$L(y|\pi) \propto \frac{\prod_{i=1}^{p} \pi_i^{y_i}}{\prod_{i<j} (\pi_i + \pi_j)^{n_{ij}}}$$

Maximum likelihood estimate for Likelihood for π can be calculated using Newton-Raphson algorithm

3 Experimental results

For model construction and simulation, we have used domestic LOL game results in 2013. Original data collected include 399 observations, but we preserve 333 observations which are statically meaningful. One instance has five attributes which are date, name of team A, score of team A, name of team B, and score of team B.

From the collected data, Poisson model is constructed following Dixon and Coles[1] to estimate ranks and parameters of top twenty teams, shown in table 1.

Table 1. Ranks and parameters of top twenty teams from Dixon and Coles Poisson Model.

Rank	Team	Overall	Attack	Defense	Matches
1	SKT T1 K	3.60	3.18	3.81	26
2	NaJin Sword	3.58	2.54	4.72	13
3	team we	3.45	3.67	2.99	8
4	ig	3.05	2.86	2.90	10
5	CJ Entus Blaze	2.98	2.36	3.33	28
6	tps	2.91	2.56	2.94	4
7	Samsung Ozone	2.75	2.45	2.75	19
8	Samsung Blue	2.55	2.64	2.21	19
9	cloud 9	2.18	1.50	3.02	5
10	NaJin W Shield	2.16	2.34	1.92	19
11	omg	2.15	1.96	2.27	6
12	NaJin B Sword	2.14	2.06	2.14	21
13	KT Arrows	1.98	1.87	2.11	22
14	KT Bullets	1.98	2.08	1.89	20
15	KT Rolster B	1.88	2.39	1.54	18
16	AHQ Korea	1.88	1.54	2.39	1
17	Azubu tps	1.88	2.27	1.62	2
18	SKT T1 S	1.88	1.39	2.64	20
19	Prime Optimus	1.87	1.73	2.12	9
20	NaJin Shield	1.80	2.33	1.49	8

Confusion matrix of training error is shown in table 2.

Table 2. Confusion matrix of training error from Dixon and Coles Poisson Model.

Actual \ Predicted	Win	Lose	Tie
Win	231	54	9
Lose	0	0	0
Tie	19	18	2

In table 3, results from Bradley Terry model are shown. And its corresponding confusion matrix of training error is shown in table 4.

4 Conclusion

In this paper, we compare Poisson Model and Bradley Terry Model for League of Legends (LOL) matches from 2013 to 2014 in South Korea. Overall, Dixon and Coles model shows higher performance over Bradley Terry model, however it is premature to make a conclusion only for these experiments. More detailed experiments on broader data sets will be our future direction.

Table 3. Ranks and worth parameter of top twenty teams from Bradley Terry Model.

Rank	Team	Worth
1	Royal	74.58
2	AHQ Korea	50.3
3	Evil Geniuses	42.88
4	SKT T1 Masters	42.72
5	Mook Secret	41.37
6	JinAir Masters	2.51
7	Club KT Rolster	2.221
8	Samsung Masters	2.192
9	Mook	1.994
10	Anexis	1.888
11	Cloud 9	1.878
12	NaJin Masters	1.8
13	Msh	1.634
14	Millennium	1.605
15	Fnatic RaidCall	1.419
16	Keyd	1.361
17	Insight	1.345
18	Alienware Arena	1.188
19	Mym	1.174
20	Curse NA	1.161

Table 4. Confusion matrix of training error from Bradley Terry Model.

Actual \ Predicted	Win	Lose	Tie
Win	157	203	0
Lose	0	0	0
Tie	12	27	0

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References

1. Dixon, M. J., Coles, S.G.: Modelling Association Football Scores and Inefficiencies in the Football Betting Market. Journal of the Royal Statistical Society. Series C (Applied Statistics). 46, 265--280 (1997)
2. Bradley R.A., Terry M.E.: Rank Analysis of Incomplete Block Designs I: The Method of Paired Comparisons. Biometrika. 39, 324--345 (1952)
3. Davidson R.R.: On Extending the Bradley-Terry Model to Accommodate Ties in Paired Comparison Experiments. Journal of the American Statistical Association. 65, 317—328 (1970)