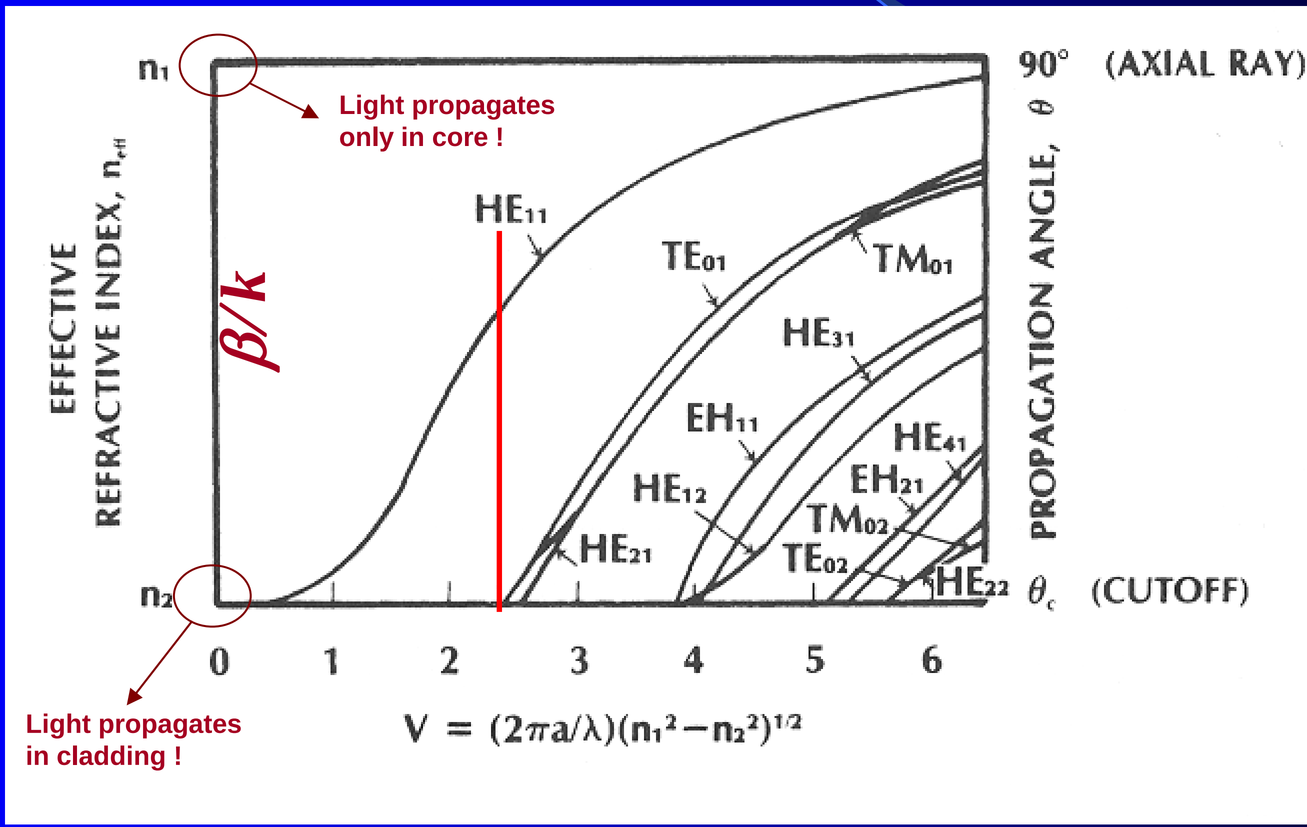
2.4 Mode theory for circular waveguide

- *For planar waveguide (1D), there are TE ($E_z = 0$) and TM ($H_z = 0$) modes exist; single parameter, integer m describe the number of modes*
- *How about Optical fiber (2D), TE, TM, or else modes? How many parameters (integers) to specify modes?*

Exact solution from vector Maxwell' equations :

1. *2 integers ν , m are necessary to specify modes*
2. *Type of modes: $TE_{\nu m}$ $TM_{\nu m}$ $HE_{\nu m}$ $EH_{\nu m}$*
3. *$TE_{\nu m}$ $TM_{\nu m}$: $\nu = 0$, $TE_{\nu m}$ ($E_z = 0$), $TM_{\nu m}$ ($H_z = 0$) correspond to meridional rays traveling within fiber*
4. *$HE_{\nu m}$ $EH_{\nu m}$: hybrid modes, both E_z and H_z are nonzero*
5. *HE modes: E_z dominates*
6. *EH modes: H_z dominates*

2.4 Mode theory for circular waveguide



2.4 Mode theory for circular waveguide

Approximate solution from scalar equations based on weakly guiding fiber approximation:

What is weakly guiding fiber approximation : index difference $\Delta << 1$

1. *Using LP $_{j\,m}$ modes to describe instead of TE $_{\nu m}$ TM $_{\nu m}$ HE $_{\nu m}$ EH $_{\nu m}$*
   LP: linear polarized
2. *Correspondence between LP and traditional exact modes → see table*

Table 2.1 Correspondence between the lower order in linearly polarized modes and the traditional exact modes from which they are formed

Linearly polarized	Exact
LP_{01}	HE_{11}
LP_{11}	HE_{21}, TE_{01}, TM_{01}
LP_{21}	HE_{31}, EH_{11}
LP_{02}	HE_{12}
LP_{31}	HE_{41}, EH_{21}
LP_{12}	HE_{22}, TE_{02}, TM_{02}
LP_{lm}	HE_{2m}, TE_{0m}, TM_{0m}
LP_{lm} ($l \neq 0$ or 1)	$HE_{l+1,m}$, $EH_{l-1,m}$

1. *Each LP $_{0m}$ mode is derived from an HE $_{1m}$ mode*
2. *Each LP $_{1m}$ mode comes from TE $_{0m}$, TM $_{0m}$, and HE $_{2m}$ modes*
3. *Each LP $_{\nu\, m}$ mode (ν >= 2) is from an HE $_{\nu+1,\, m}$ and an EH $_{\nu-1,\, m}$ mode*

We'll talk about this in the following section

2.4 Mode theory for circular waveguide

2.4.1 Over view of Modes

- The stable field distribution in the x direction with only periodic z dependence is known as a mode

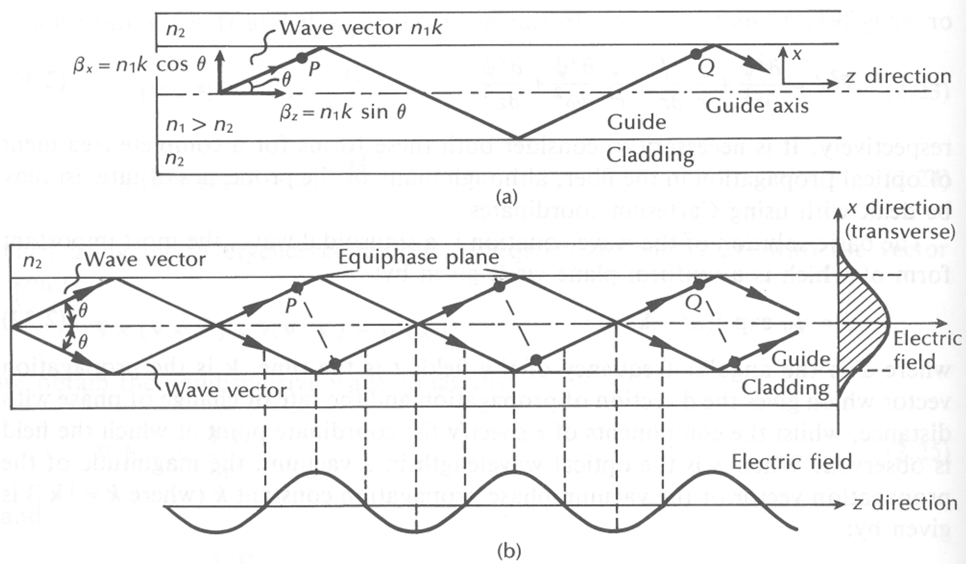


Figure 2.8 The formation of a mode in a planar dielectric guide: (a) a plane wave propagating in the guide shown by its wave vector or equivalent ray – the wave vector is resolved into components in the z and x directions; (b) the interference of plane waves in the guide forming the lowest order mode ($m = 0$).

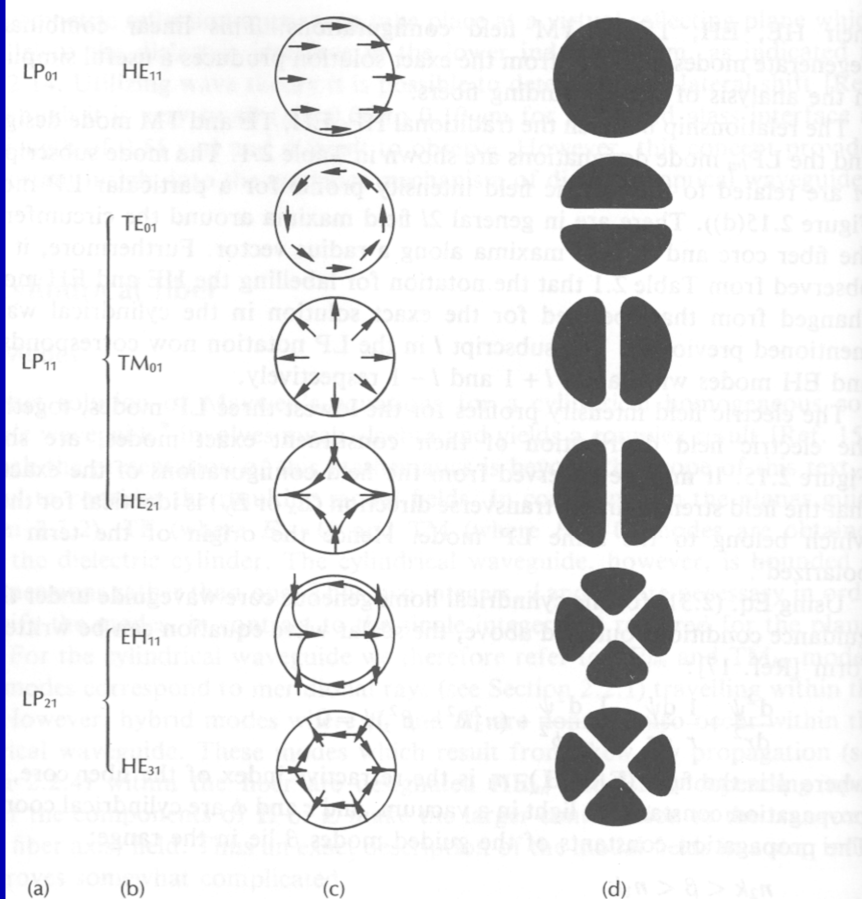


Figure 2.15 The electric field configurations for the three lowest LP modes illustrated in terms of their constituent exact modes: (a) LP mode designations; (b) exact mode designations; (c) electric field distribution of the exact modes; (d) intensity distribution of E_x for the exact modes indicating the electric field intensity profile for the corresponding LP modes.

2.4 Mode theory for circular waveguide

2.4.1 Over view of Modes

- For low-order modes the fields are tightly concentrated near the center of slab (WG, optical fiber), with little penetration into cladding region
- For high-order modes, the fields are distributed more toward the edges of the guide and penetrate further into cladding region

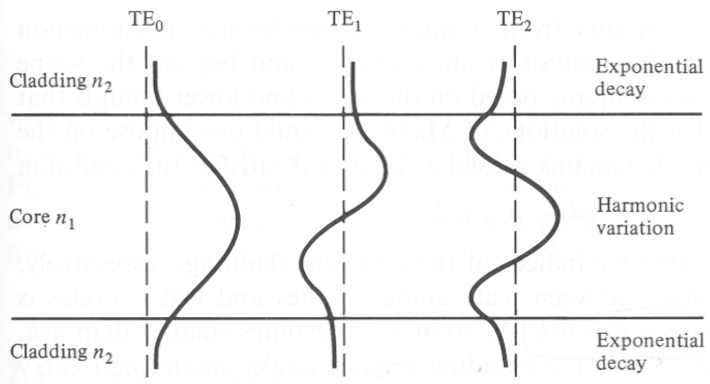
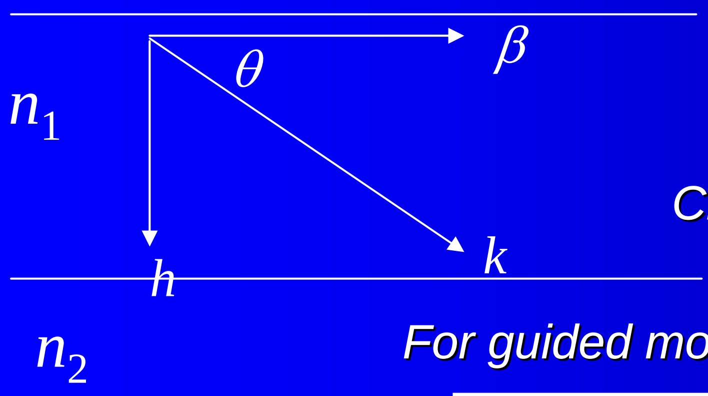


FIGURE 2-14
Electric field distributions for several of the lower-order guided modes in a symmetrical-slab waveguide.

2.4 Mode theory for circular waveguide

2.4.1 Over view of Modes

Cutoff condition : propagation angle for a given mode just equals the critical angle.



$$\beta = k_1 \cos\theta = \frac{2\pi n_1}{\lambda}\cos\theta$$

Critical angle : $\cos\theta = \dfrac{n_2}{n_1}$

For guided modes, propagation constant in the range:

$$\frac{2\pi n_2}{\lambda} \le \beta \le \frac{2\pi n_1}{\lambda} \quad \text{or} \quad k_2 \le \beta \le k_1$$

Cutoff condition: $\beta = k_2 = n_2 k$

Propagation constant

- Guided mode : bound mode guided inside of core $\beta \ge k_2 = n_2 k$
- Radiation modes : refracted mode by cladding $\beta < k_2$
- Leaky mode : partially confined into core $\beta < k_2$

2.4 Mode theory for circular waveguide

2.4.2 Summary of Key Modal Concept

Question : what parameter could determine if the fiber is Single-mode fiber (SMF), or multi-mode fiber (MMF) ?

- *Parameters:* *Fiber : a, n_1, n_2* *Source : λ*
- *V number is an important parameter connected with cutoff condition, determined how many modes a fiber could support.*

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

- *Lowest-order mode : HE_{11}*
- *Single mode condition : V < = 2.405*
- *When V > 10 , the total number of modes :* $M \approx V^2 / 2$
- *Fraction of average optical power residing in cladding :* $\frac{P_{clad}}{P} \approx \frac{4}{3\sqrt{M}}$

2.4 Mode theory for circular waveguide

2.4.3 Maxwell' Equations

Maxwell Equations!

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{D} = 0 \qquad \nabla \times \vec{B} = 0$$

Wave Equations!

$$\nabla^2 \vec{E} - \varepsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\nabla^2 \vec{H} - \varepsilon\mu \frac{\partial^2 \vec{H}}{\partial t^2} = 0$$

- Light is electromagnetic wave

2.4.4 Waveguide Equations

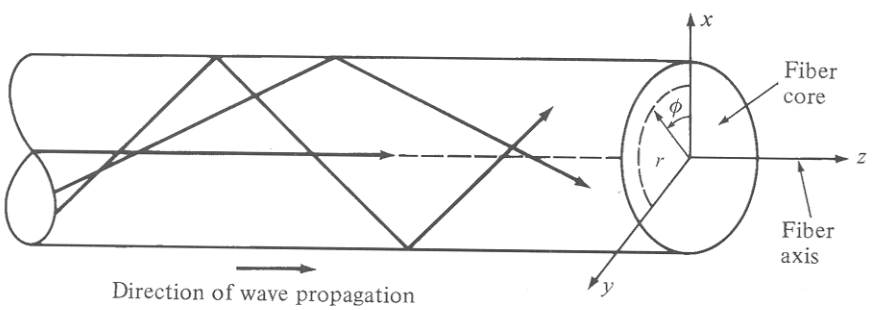


FIGURE 2-15
Cylindrical coordinate system used for analyzing electromagnetic wave propagation in an optical fiber.

Fig 2-15 Cylindrical coordinate
(fiber is a cylinder shape!)

- Goal: Derive wave equation in cylindrical coordinate system:
  - Cylindrical coordinate system: *r*, ϕ, *z*

$$\vec{E} = \vec{E}_0(r,\phi)e^{j(\omega t - \beta z)}$$

$$\vec{H} = \vec{H}_0(r,\phi)e^{j(\omega t - \beta z)}$$

2.4 Mode theory for circular waveguide

2.4.4 Waveguide Equations

- Step 1: Find E components in r, ϕ, z direction

$$\vec{E} = \vec{E}_0(r,\phi)e^{j(\omega t-\beta z)}$$
$$\vec{H} = \vec{H}_0(r,\phi)e^{j(\omega t-\beta z)}$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$
$$\nabla\times\vec{H} = \frac{\partial\vec{D}}{\partial t}$$

B.2.2 Cylindrical Coordinates

Gradient $\nabla f = \frac{\partial f}{\partial r}\mathbf{e}_r + \frac{1}{r}\frac{\partial f}{\partial\phi}\mathbf{e}_\phi + \frac{\partial f}{\partial z}\mathbf{e}_z$

Divergence $\nabla\cdot\mathbf{A} = \frac{1}{r}\frac{\partial(rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$

Curl $\nabla\times\mathbf{A} = \begin{vmatrix} \frac{1}{r}\mathbf{e}_r & \mathbf{e}_\phi & \frac{1}{r}\mathbf{e}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix}$

Laplacian $\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial\phi^2} + \frac{\partial^2 f}{\partial z^2}$

With help of $B=\mu H,\quad D=\varepsilon E$, $\frac{\partial}{\partial t}\to j\omega,\quad \frac{\partial}{\partial z}\to -j\beta$, we can find following Eqs.

E

r direction
$$\frac{1}{r}\left(\frac{\partial H_z}{\partial\phi} + jr\beta H_\phi\right) = j\omega\varepsilon E_r, \qquad (2.33a)$$

ϕ direction
$$j\beta H_r + \frac{\partial H_z}{\partial r} = -j\omega\varepsilon E_\phi \qquad (2.33b)$$

z direction
$$\frac{1}{r}\left(\frac{\partial}{\partial\phi}(rH_\phi) - \frac{\partial H_r}{\partial\phi}\right) = j\omega\varepsilon E_z \qquad (2.33c)$$

H

$$\frac{1}{r}\left(\frac{\partial E_z}{\partial\phi} + jr\beta E_\phi\right) = -j\omega\mu H_r \qquad (2.34a)$$
$$j\beta E_r + \frac{\partial E_z}{\partial r} = j\omega\mu H_\phi \qquad (2.34b)$$
$$\frac{1}{r}\left(\frac{\partial}{\partial\phi}(rE_\phi) - \frac{\partial E_r}{\partial\phi}\right) = -j\omega\mu H_z \qquad (2.34c)$$

2.4.4 Waveguide Equations

- Step 2: Write components E_r, E_ϕ, H_r, H_ϕ in terms of E_z, H_z :

H

$$\frac{1}{r}\left(\frac{\partial E_z}{\partial \phi} + jr\beta E_\phi\right) = -j\omega\mu H_r \quad (2.33a)$$

$$j\beta E_r + \frac{\partial E_z}{\partial r} = j\omega\mu H_\phi \quad (2.33b)$$

$$\frac{1}{r}\left(\frac{\partial}{\partial \phi}(rE_\phi) - \frac{\partial E_r}{\partial \phi}\right) = -j\omega\mu H_z \quad (2.33c)$$

E

$$\frac{1}{r}\left(\frac{\partial H_z}{\partial \phi} + jr\beta H_\phi\right) = j\omega\varepsilon E_r, \quad (2.34a)$$

$$j\beta H_r + \frac{\partial H_z}{\partial r} = -j\omega\varepsilon E_\phi \quad (2.34b)$$

$$\frac{1}{r}\left(\frac{\partial}{\partial \phi}(rH_\phi) - \frac{\partial H_r}{\partial \phi}\right) = j\omega\varepsilon E_z \quad (2.34c)$$

$$E_r = -\frac{j}{q^2}\left(\beta\frac{\partial E_z}{\partial r} + \frac{\mu\omega}{r}\frac{\partial H_z}{\partial \phi}\right) \quad (2.35a)$$

$$E_\phi = -\frac{j}{q^2}\left(\frac{\beta}{r}\frac{\partial E_z}{\partial \phi} - \mu\omega\frac{\partial H_z}{\partial r}\right) \quad (2.35b)$$

$$H_r = -\frac{j}{q^2}\left(\frac{\beta}{r}\frac{\partial H_z}{\partial r} - \frac{\mu\omega}{r}\frac{\partial E_z}{\partial \phi}\right) \quad (2.35c)$$

$$H_\phi = -\frac{j}{q^2}\left(\frac{\beta}{r}\frac{\partial H_z}{\partial \phi} + \omega\varepsilon\frac{\partial E_z}{\partial r}\right) \quad (2.35d)$$

with $$q^2 = \omega^2\varepsilon\mu - \beta^2 = k^2 - \beta^2$$

Using equation 2.33a, 2.34b to find H_r and E_ϕ

Using equation 2.33b, 2.34a to find E_r and H_ϕ

2.4.4 Waveguide Equations

- *Step 3: Find wave equations for E and H:*

$$H_r = -\frac{j}{q^2}\left(\frac{\beta}{r}\frac{\partial H_z}{\partial r} - \frac{\mu\omega}{r}\frac{\partial E_z}{\partial \phi}\right) \quad (2.35c)$$

$$H_\phi = -\frac{j}{q^2}\left(\frac{\beta}{r}\frac{\partial H_z}{\partial \phi} + \omega\varepsilon\frac{\partial E_z}{\partial r}\right) \quad (2.35d)$$

$$\frac{1}{r}\left(\frac{\partial}{\partial \phi}(rH_\phi) - \frac{\partial H_r}{\partial \phi}\right) = j\omega\varepsilon E_z \quad (2.34c)$$

$$\Rightarrow \quad \frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r}\frac{\partial E_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 E_z}{\partial \phi^2} + q^2 E_z = 0 \quad (2.36)$$

$$E_r = -\frac{j}{q^2}\left(\beta\frac{\partial E_z}{\partial r} + \frac{\mu\omega}{r}\frac{\partial H_z}{\partial \phi}\right) \quad (2.35a)$$

$$E_\phi = -\frac{j}{q^2}\left(\frac{\beta}{r}\frac{\partial E_z}{\partial \phi} - \mu\omega\frac{\partial H_z}{\partial r}\right) \quad (2.35b)$$

$$\frac{1}{r}\left(\frac{\partial}{\partial \phi}(rE_\phi) - \frac{\partial E_r}{\partial \phi}\right) = -j\omega\mu H_z \quad (2.33c)$$

$$\Rightarrow \quad \frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r}\frac{\partial H_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 H_z}{\partial \phi^2} + q^2 H_z = 0 \quad (2.37)$$

- **Electric and magnetic field vector solution (mode) could be achieved by solving the wave equation for the field components in z-direction !**

2.4.5 Wave Equations for Step-index Fibers

- Solve Wave equation using separation of variable method :

Wave equation

$$\frac{\partial^2 E_z}{\partial r^2}+\frac{1}{r}\frac{\partial E_z}{\partial r}+\frac{1}{r^2}\frac{\partial^2 E_z}{\partial \phi^2}+q^2 E_z=0$$

$$\frac{\partial^2 H_z}{\partial r^2}+\frac{1}{r}\frac{\partial H_z}{\partial r}+\frac{1}{r^2}\frac{\partial^2 H_z}{\partial \phi^2}+q^2 H_z=0$$

Separation-of-variables method

$$E_z = A F_1(r)F_2(\phi)F_3(z)F_4(t)$$

$$F_3(z)F_4(t)=e^{j(\omega t-\beta z)}$$

$$F_2(\phi)=e^{j\nu\phi}$$

ν is an integer

$$\frac{\partial^2 F_1}{\partial r^2}+\frac{1}{r}\frac{\partial F_1}{\partial r}+\left(q^2-\frac{\nu^2}{r^2}\right)F_1=0 \quad (2.41)$$

This is a well-known differential equation for Bessel functions

- ***Cutoff conditions :***

$$k_2 \le \beta \le k_1$$

- ***In fiber core:*** $q^2=\omega^2\varepsilon\mu-\beta^2=k_1^2-\beta^2>0$
- ***In cladding:*** $q^2=\omega^2\varepsilon\mu-\beta^2=k_2^2-\beta^2<0$

So, solutions for fiber and fiber cladding are different, using Bessel function $J_\nu(ur)$ for core and modified Bessel function of the second kind $K_\nu(wr)$, which is based on conditions: $F_1 \to finite \quad for \quad r\to 0,$ $\quad F_1\to 0 \quad for \quad r\to\infty$

Solution for fiber core

$$E_{z1}(r<a)=AJ_\nu(ur)e^{j\nu\phi}e^{j(\omega t-\beta z)}$$

$$H_{z1}(r<a)=BJ_\nu(ur)e^{j\nu\phi}e^{j(\omega t-\beta z)}$$

$$with \quad u^2=k_1^2-\beta^2 \quad k_1=2\pi n_1/\lambda$$

Solution for fiber cladding

$$E_{z2}(r>a)=CK_\nu(wr)e^{j\nu\phi}e^{j(\omega t-\beta z)}$$

$$H_{z2}(r>a)=DK_\nu(wr)e^{j\nu\phi}e^{j(\omega t-\beta z)}$$

$$with \quad w^2=\beta^2-k_2^2 \quad k_2=2\pi n_2/\lambda$$

2.4 Mode theory for circular waveguide

2.4.6 Modal Equations

$$E_{z1}(r<a) = AJ_\nu(ur)e^{j\nu\phi}e^{j(\omega t-\beta z)} \quad (2.42)$$
$$H_{z1}(r<a) = BJ_\nu(ur)e^{j\nu\phi}e^{j(\omega t-\beta z)} \quad (2.43)$$
$$\text{with} \quad u^2 = k_1^2 - \beta^2 \quad k_1 = 2\pi n_1/\lambda$$

$$E_{z2}(r>a) = CK_\nu(wr)e^{j\nu\phi}e^{j(\omega t-\beta z)} \quad (2.44)$$
$$H_{z2}(r>a) = DK_\nu(wr)e^{j\nu\phi}e^{j(\omega t-\beta z)} \quad (2.45)$$
$$\text{with} \quad w^2 = \beta^2 - k_2^2 \quad k_2 = 2\pi n_2/\lambda$$

Solution β can be determined by Boundary conditions :
Tangential components E_ϕ , E_z , and H_ϕ , H_z at r = a must be continue

At r = a,

$$E_{z1} = E_{z2}$$
$$H_{z1} = H_{z2}$$

$$\Longrightarrow$$

$$AJ_\nu(ua) = CK_\nu(wa) \quad (2.47)$$
$$BJ_\nu(ua) = DK_\nu(wa) \quad (2.51)$$

$$E_{\phi1} = E_{\phi2}$$
$$H_{\phi1} = H_{\phi2}$$

$$E_\phi = -\frac{j}{q^2}\left(\frac{\beta}{r}\frac{\partial E_z}{\partial\phi} - \mu\omega\frac{\partial H_z}{\partial r}\right) \quad (2.35b), \quad H_\phi = -\frac{j}{q^2}\left(\frac{\beta}{r}\frac{\partial H_z}{\partial\phi} + \omega\varepsilon\frac{\partial E_z}{\partial r}\right) \quad (2.35d)$$

$$-\frac{j}{u^2}\left[A\frac{j\nu\beta}{a}J_\nu(ua) - B\omega\mu uJ_\nu^{'}(ua)\right] = \frac{j}{w^2}\left[C\frac{j\nu\beta}{a}K_\nu(wa) - D\omega\mu wK_\nu^{'}(wa)\right] \quad (2.50)$$

$$-\frac{j}{u^2}\left[B\frac{j\nu\beta}{a}J_\nu(ua) + A\omega\varepsilon_1 uJ_\nu^{'}(ua)\right] = \frac{j}{w^2}\left[D\frac{j\nu\beta}{a}K_\nu(wa) + C\omega\varepsilon_2 wK_\nu^{'}(wa)\right] \quad (2.52)$$

2.4 Mode theory for circular waveguide

2.4.6 Modal Equations

- *A set of four Eqs, right side = 0, therefore, only if the determinant of the coefficients is zero, there is a solution exists.*

$$
\begin{array}{l}
(2.47) \rightarrow \\[2ex]
(2.50) \rightarrow \\[2ex]
(2.51) \rightarrow \\[2ex]
(2.52) \rightarrow
\end{array}
\begin{vmatrix}
J_\nu(ua) & 0 & -K_\nu(wa) & 0 \\
\frac{\beta\nu}{au^2}J_\nu(ua) & \frac{j\omega\mu}{u}J_\nu'(ua) & \frac{\beta\nu}{aw^2}K_\nu(wa) & \frac{j\omega\mu}{w}K_\nu'(wa) \\
0 & J_\nu(ua) & 0 & -K_\nu(wa) \\
-\frac{j\omega\epsilon_1}{u}J_\nu'(ua) & \frac{\beta\nu}{au^2}J_\nu(ua) & -\frac{j\omega\epsilon_2}{w}K_\nu'(wa) & \frac{\beta\nu}{aw^2}K_\nu(wa)
\end{vmatrix} = 0 \qquad (2\text{-}53)
$$

- *Eigenvalue equation for β :*

$$(\mathcal{J}_\nu + \mathcal{K}_\nu)(k_1^2\mathcal{J}_\nu + k_2^2\mathcal{K}_\nu) = \left(\frac{\beta\nu}{a}\right)^2\left(\frac{1}{u^2}+\frac{1}{w^2}\right)^2 \qquad (2\text{-}54)$$

where

$$\mathcal{J}_\nu = \frac{J_\nu'(ua)}{uJ_\nu(ua)} \qquad \text{and} \qquad \mathcal{K}_\nu = \frac{K_\nu'(wa)}{wK_\nu(wa)}$$

2.4.7 Modes in Step-Index Fibers Equations

- *2 integers ν , m are necessary to specify modes*
- *We have ν, but Where does m come from ?*

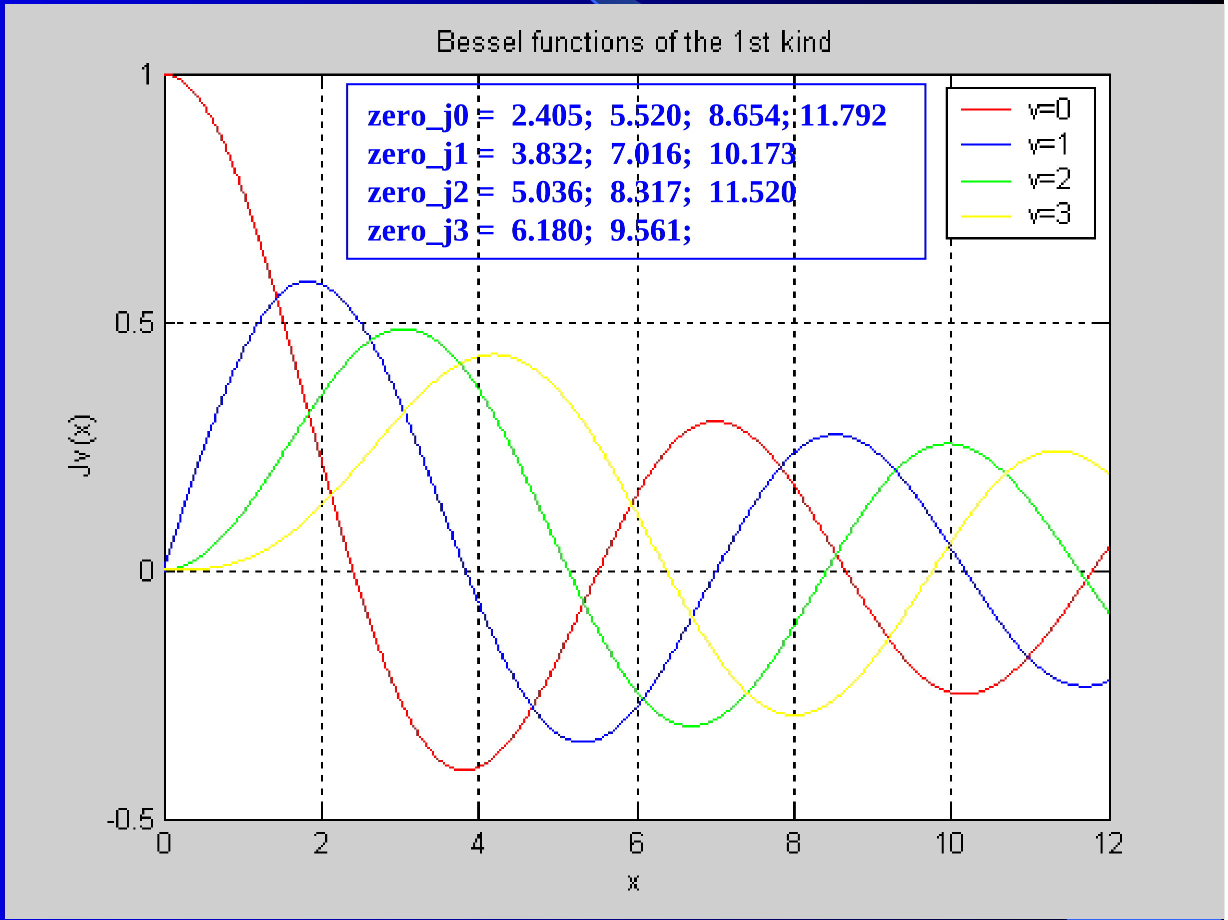
  from oscillatory behavior of J_ν → m roots of Eq.2.54 for a given ν
- *These roots are designed as $\beta_{\nu m}$, and corresponding modes: $TE_{\nu m}$ $TM_{\nu m}$ $HE_{\nu m}$ $EH_{\nu m}$*

```
format long
clear
clc
z = (0:0.001:15)';

j0=besselj(0,z);
j1=besselj(1,z);
j2=besselj(2,z);
j3=besselj(3,z);

figure(1)
plot(z,j0,'r',z,j1,'b',z,j2,'g',z,j3,'y');
ylabel('Jv(x)');xlabel('x');
axis([0 15 -0.5 1]);grid
title('Bessel functions of the 1st
kind');
legend('v=0','v=1','v=2','v=3');

zero_j0=z(find(abs(j0)<1.2e-4))'
z=z(100:end);
j1=j1(100:end);
j2=j2(200:end);
j3=j3(300:end);
zero_j1=z(find(abs(j1)<1.3e-4))'
zero_j2=z(find(abs(j2)<1.5e-4))'
zero_j3=z(find(abs(j3)<1e-4))'
```



2.4.7 Modes in Step-Index Fibers Equations

- Modes: $TE_{\nu m}$ $TM_{\nu m}$ $HE_{\nu m}$ $EH_{\nu m}$
- Special cases: When $\nu = 0$, modes are: TE_{0m} TM_{0m}

$$(\mathcal{J}_\nu + \mathcal{K}_\nu)(k_1^2 \mathcal{J}_\nu + k_2^2 \mathcal{K}_\nu) = \left(\frac{\beta \nu}{a}\right)^2 \left(\frac{1}{u^2} + \frac{1}{w^2}\right)^2 \qquad (2\text{-}54)$$

where

$$\mathcal{J}_\nu = \frac{J_\nu'(ua)}{uJ_\nu(ua)} \qquad \text{and} \qquad \mathcal{K}_\nu = \frac{K_\nu'(wa)}{wK_\nu(wa)}$$

$$\mathcal{J}_0 + \mathcal{K}_0 = 0 \qquad (2\text{-}55a)$$

or, using the relations for J_ν' and K_ν' in App. C,

$$\frac{J_1(ua)}{uJ_0(ua)} + \frac{K_1(wa)}{wK_0(wa)} = 0 \qquad (2\text{-}55b)$$

which corresponds to TE_{0m} modes ($E_z = 0$), and

$$k_1^2 \mathcal{J}_0 + k_2^2 \mathcal{K}_0 = 0 \qquad (2\text{-}56a)$$

or

$$\frac{k_1^2 J_1(ua)}{uJ_0(ua)} + \frac{k_2^2 K_1(wa)}{wK_0(wa)} = 0 \qquad (2\text{-}56b)$$

- Corresponds to TM_{0m} modes ($H_z=0$)

2.4.7 Modes in Step-Index Fibers Equations

- *Cutoff conditions: It means a mode is not longer bound to the fiber core*

$$\longrightarrow \boxed{\beta = k_2 = n_2 k} \longrightarrow \boxed{w = 0}$$

- *Normalized frequency V:* $V^2 = (u^2 + w^2)a^2 = \left(\dfrac{2\pi a}{\lambda}\right)^2 (n_1^2 - n_2^2) = \left(\dfrac{2\pi a}{\lambda}\right)^2 NA^2$

- *HE11 has no cut off, but we have single-mode condition which is:* $\boxed{V <= 2.405}$

- *Question: how do we get this number ?* → *From 1st zero of J_0*

- *When number of mode M is large (>10), we have:*

$$\boxed{M = \frac{V^2}{2}}$$

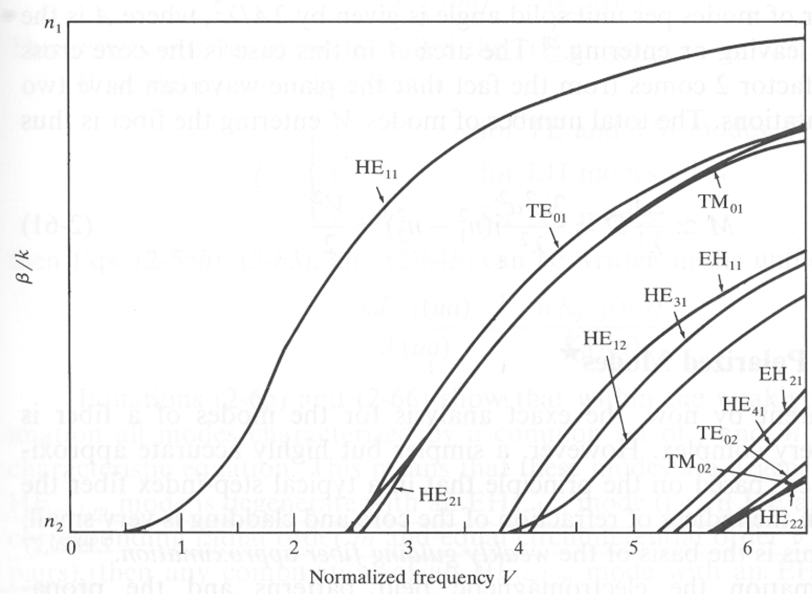


FIGURE 2-18
Plots of the propagation constant (in terms of β/k) as a function of V for a few of the lowest-order modes.

2.4 Mode theory for circular waveguide

2.4.7 Modes in Step-Index Fibers Equations

Example 2-3 (p56)

A step-index fiber has a normalized frequency V=26.6 at a 1300-nm wavelength. If the core radius is 25 μm, let us find the numerical aperture.

Problem 2-19 (p84)

Determine the normalized frequency at 820 nm for a step-index fiber having a 25-μm core radius, n1=1.48, and n2=1.46.

(a) How many modes propagate in this fiber at 820 nm ?

(b) How many modes propagate in this fiber at 1320 nm?

(c) How many modes propagate in this fiber at 1550 nm?

(d) What percent of the optical power flows in the cladding in each case?